SoftMax + CE 手推 2021.05.09

R_{se} R_{ve} R_{vi}

Soft Max

$$\text{Predicted}_{P_{se}} = \text{SoftMax}_{se}(\text{Raw}_{se}, \text{Raw}_{ve}, \text{Raw}_{vi}) = \frac{e^{se}}{e^{se}+e^{ve}+e^{vi}}$$

Arg Max

0, 1

Cross Entropy 交叉熵

$$-\sum_{c=1}^{M} \underbrace{\text{Observed}_c}_{\text{观察概率}} \times \log(\underbrace{\text{Predicted}_c}_{\text{预测概率}})$$

$M=3 \Rightarrow$

$$= -\sum_{c=1}^{3} \text{Observed}_c \times \log(\text{Predicted}_c)$$

$$= -\text{Observed}_{se} \times \log(\text{Predicted}_{se}) - \text{Observed}_{ve} \times \log(\text{Pre}_{ve}) - \text{Ob}_{vi} \times \log(\text{Pre}_{vi})$$

at Se:

Se. Ve. Vi : $\begin{cases} \text{Ob}_{se}=1 \\ \text{Ob}_{ve}=0 \\ \text{Ob}_{vi}=0 \end{cases} \Rightarrow CE = -\log(\text{Pre}_{se})$

at Ve: ...

at Vi: ...

$$\Rightarrow CE_c = -\log(\text{Predicted}_c)$$

Total $CE = CE_{se} + CE_{ve} + CE_{vi}$

SoftMax + CE + backPropagation 2025.05.09

$$\frac{d(CE_{se})}{db_3} = \frac{d(-\log(P_{se}))}{db_3} = \frac{d\left(-\log\left(\frac{e^{se}}{e^{se}+e^{ve}+e^{vi}}\right)\right)}{db_3}$$

$CE_{se} = -\log(\text{Predicted}_{P_{se}})$

$\text{Predicted}_{P_{se}} = \text{SoftMax}(Raw_{se}, Raw_{ve}, Raw_{vi}) = \frac{e^{se}}{e^{se}+e^{ve}+e^{vi}}$ The Chain Rule!

$Raw_{se} = \text{green} = \text{blue} + \text{orange} + b_3$

$$\frac{dCE_{se}}{db} = \underbrace{\frac{dCE_{se}}{d\,\text{Predicted}_{se}}}_{①} \cdot \underbrace{\frac{d\,\text{Predicted}_{se}}{d\,Raw_{se}}}_{②} \cdot \underbrace{\frac{d\,Raw_{se}}{db_3}}_{③}$$

① : $\frac{dCE_{se}}{dP_{se}} = \frac{d(-\log(P_{se}))}{dP_{se}} = -\frac{1}{P_{se}}$

② : $\frac{dP_{se}}{dRaw_{se}} = \frac{d\left(\frac{e^{se}}{e^{se}+e^{ve}+e^{vi}}\right)}{d(Raw_{se})} = \frac{e^{se}\cdot(e^{se}+e^{ve}+e^{vi}) - e^{se}\cdot e^{se}}{(e^{se}+e^{ve}+e^{vi})^2}$

$$= \frac{e^{se}}{e^{se}+e^{ve}+e^{vi}}\left(\frac{e^{se}+e^{ve}+e^{vi}}{e^{se}+e^{ve}+e^{vi}} - \frac{e^{se}}{e^{se}+e^{ve}+e^{vi}}\right)$$

$$= P_{se}(1-P_{se})$$

③ : $\frac{dRaw_{se}}{db_3} = \frac{d(\text{green})}{db_3} = \frac{d(\text{blue}+\text{orange}+b_3)}{db_3} = 0+0+1 = 1$

$$\Rightarrow \frac{dCE_{se}}{db_3} = \left(-\frac{1}{P_{se}}\right)\cdot P_{se}(1-P_{se})\cdot 1 = P_{se}-1$$

Only Raw_{se} is related to b_3

$$\frac{dCE_{vi}}{db_3} = \underbrace{\frac{dCE_{vi}}{dP_{vi}}}_{①} \cdot \underbrace{\frac{dP_{vi}}{dRaw_{se}}}_{②} \cdot \underbrace{\frac{dRaw_{se}}{db_3}}_{③}$$

① : $\frac{dCE_{vi}}{dP_{vi}} = \frac{d(-\log P_{vi})}{dP_{vi}} = -\frac{1}{P_{vi}}$

② : $\frac{dP_{vi}}{dRaw_{se}} = \frac{d\left(\frac{e^{vi}}{e^{se}+e^{vi}+e^{ve}}\right)}{dRaw_{se}} = e^{vi}\cdot\left(-\frac{1}{(e^{se}+e^{vi}+e^{ve})^2}\cdot e^{se}\right) = -\frac{e^{vi}\cdot e^{se}}{(e^{se}+e^{vi}+e^{ve})^2} = -P_{se}\cdot P_{vi}$

③ : $\frac{dRaw_{se}}{db_3} = \frac{d(\text{green})}{db_3} = \frac{d(\text{blue}+\text{orange}+b_3)}{db_3} = 0+0+1 = 1$

$$\Rightarrow \frac{dCE_{vi}}{db_3} = \left(-\frac{1}{P_{vi}}\right)\cdot(-P_{se}\cdot P_{vi})\cdot 1 = -P_{se}$$