TP1 : Quelques manipulations élémentaires autour de l'inertie

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Charger dans le logiciel les données relatives aux vins de Loire (wine.csv). Elles contiennent deux variables qualitatives (Appellation: "Label" = {Bourgueil, Chinon, Saumur} et Sol: "Soil" = {Env1, Env2, Env3=référence, Env4}) et 29 variables quantitatives décrivant diverses intensités sensorielles (odeur, arôme, goût, couleur etc). Les vins seront traduits en nuage dans l'espace des 29 variables quantitatives, $\mathbb{R}^{29}$.

```
library(readr)
wine = read.csv("wine.csv",header=TRUE)#Les données relatives aux vins de Loire
```

```
library(knitr)
library(rmarkdown)
library(markdown)
library(tinytex)
```

- **Question 1**

Il faut centrer-réduire les variables quantitatives.

Dans un premier temps, nous allons nommer nos variables quantitatives V.quantitative et V.qualitative :

```
V.quantitative <- wine[4:32]
V.qualitative <- wine[2:3]
```

Maintenant on introduit une fonction qui permet de centrer-réduire.

```
Centre.reduire <- function(x){
  Centre_reduire <- function(x){
    N = 29
    moyenne <- mean(x)
    ecart.type <- sqrt((N-1)/N*var(x))
    y <- (x-moyenne)/(ecart.type)
    return(y)
  }
  y <- as.data.frame((lapply(x,Centre_reduire)))
  return(y)
}
```

Ce qui donnera au final, pour centrer-réduire les variables quantitatives

```
Variables.C_R <- Centre.reduire(V.quantitative)
print(Variables.C_R)
```

```
##    Odor.Intensity.before.shaking Aroma.quality.before.shaking
## 1                    -0.13059236                   -0.2264733
## 2                    -0.51734667                   -1.1123353
## 3                    -0.89355313                   -0.5778487
## 4                    -1.06583460                   -2.2406957
## 5                     1.74340805                    1.8966259
```

```
## 6                -0.89355313                 0.3228600
## 7                 0.36164039                 0.8721934
## 8                 0.03114126                -0.9589178
## 9                -0.89355313                -1.1420290
## 10               -0.76697900                -0.2264733
## 11                0.48821453                 1.1889261
## 12                0.99099513                 0.6593886
## 13                0.23858220                 1.1889261
## 14               -0.14114021                 0.3030642
## 15               -0.01456607                 0.4812264
## 16               -0.64040486                 0.6593886
## 17               -0.26419840                 0.6593886
## 18               -0.14114021                -0.5926956
## 19               -1.64596606                -1.2855485
## 20                2.05632745                 0.7237249
## 21                2.09851883                -0.5926956
##    Fruity.before.shaking Flower.before.shaking Spice.before.shaking
## 1          -0.0002410747             1.5804683           -0.1336605
## 2          -1.7164521550             1.5804683           -1.3226523
## 3          -0.7798767867            -0.6874683            0.3631683
## 4          -1.5038242335            -1.0205714            0.7156194
## 5           2.2272895309             0.6874683            0.2060515
## 6          -0.6938131042            -0.1204841            0.3631683
## 7           1.2552761757             0.4110635            0.2060515
## 8          -1.0836309602             1.0134841            0.8217794
## 9           0.4756404637            -0.9496984            0.3504291
## 10         -0.7241885216            -1.4954207           -1.3226523
## 11         -0.0002410747            -0.9284365           -0.1251677
## 12          0.2782002510            -0.1346587           -0.3035165
## 13          0.3237633770             1.2331905           -0.9829404
## 14          0.0858226077             0.4465000           -0.8130844
## 15          0.6680181069             0.9071746           -0.1251677
## 16          0.6983935243            -0.4039762            0.1933123
## 17          1.6349688925             1.2331905           -1.3778555
## 18          0.1364483033            -0.4039762           -0.4733724
## 19         -0.9013784561            -1.1906667           -0.7791132
## 20          0.6022047027            -1.6371667            1.6710593
## 21         -0.9823795690            -0.1204841            2.8685440
##    Visual.intensity     Nuance Surface.feeling Odor.Intensity Quality.of.odour
## 1         0.6732197  0.5327899      0.32264556     0.05970439       0.32235246
## 2        -1.3623151 -1.4215485     -1.35396831    -0.12811064      -1.08261770
## 3        -0.7807337 -0.6534935     -0.65568228    -0.73724046      -1.40649069
## 4        -1.9716790 -1.8397769     -2.33593305    -1.19408782      -1.63000867
## 5         0.8065759  0.6031461      0.74452671     0.71451895       0.55955522
## 6         0.9380800  1.0389635      0.82453865    -1.09764227       0.67359500
## 7         0.3435334  0.3940318      0.25354434     0.17137819       1.19817802
## 8         0.4750375  0.2533195     -0.37564048     1.31349661      -0.73137516
## 9         0.1472034  0.3236757      0.36265153    -0.19409970       0.49569294
## 10       -0.4695692 -0.6261328     -0.25198566    -0.87937075       0.09883448
## 11       -0.1861872 -0.1649089      0.28627649     1.07492076       0.67359500
## 12        1.4011225  1.5099591      0.51176469     0.43533445       0.67359500
## 13        0.4898548  0.6715479      1.02456850     0.43533445       0.84693548
## 14        1.4011225  1.5803153      0.90455059    -0.19409970       0.94272890
## 15        0.2101772  0.3236757      1.02456850    -0.19409970       0.60517113
```

```
## 16       -0.1269177 -0.5831374     -0.13560465    -0.55450151       0.32235246
## 17       -0.3176912 -0.2352651      0.64269332     0.24751942       1.19817802
## 18       -0.5158734 -0.6534935      0.04260376    -0.12811064       0.55955522
## 19       -2.5013996 -2.3283615     -2.67780226    -2.56970600      -1.99493599
## 20        0.6732197  0.5327899      0.55540757     1.73481140      -0.71769039
## 21        0.6732197  0.7419041      0.28627649     1.68405058      -1.60720072
##          Fruity      Flower       Spice      Plante    Phenolic Aroma.intensity
## 1   0.173862242  1.06570445 -1.66870325  0.22142229  0.71775489      0.24780069
## 2  -1.318728703  1.87583272 -2.16684816  0.22142229 -1.07631474     -0.93833354
## 3  -0.358877911  0.20156763  0.35346916 -1.33475849 -1.95000664     -0.47905598
## 4  -2.094875994 -0.53429888 -0.05589745  2.11373813 -0.44272138     -2.61569509
## 5   1.436823810  0.46485932 -0.14960788 -1.26006181  0.38428470      1.66956429
## 6  -0.216507697  0.52561894 -0.14960788 -1.33475849 -0.44272138      0.06808338
## 7   0.876528133  0.25557618  1.01930642 -0.86167953 -0.44272138      0.21185723
## 8  -1.502432203  1.06570445  1.31030196  0.71940015  2.41845286      0.33166877
## 9   0.702009807 -0.82459485 -0.26304682 -0.55666810 -0.12258999     -0.19550200
## 10  0.362158330 -0.82459485 -0.67241343  0.22142229  1.53809156     -0.47905598
## 11  0.192232592 -0.31826468 -0.09042234 -0.05869025  0.19087199      0.35563108
## 12  0.527491481 -1.09463760  0.10686277  0.48286067  0.01746749      0.49541120
## 13  0.532084069  0.72139994 -0.12001511 -0.04624080  0.19087199      1.13440608
## 14  0.173862242 -0.28450933  0.82695343 -0.86167953  0.05081451      0.28773787
## 15 -0.005248671  0.52561894  0.50143300 -1.33475849 -0.12258999      0.54333582
## 16  1.230157372  0.72139994 -0.87956280 -0.29522973 -0.94959607     -0.62682354
## 17  1.551638498 -0.01446658  0.30414788 -0.32012862  1.38469527      1.29016108
## 18  0.532084069  0.98469162 -1.28892941  0.22142229 -0.75618336      0.21185723
## 19 -1.594283954 -1.35117822 -0.33209661  0.71940015 -1.49648718     -2.06456201
## 20 -0.064952309 -2.62713025  1.29057345  2.03904145  0.19087199      0.95868249
## 21 -1.135025202 -0.53429888  2.12410305  1.30452412  0.71775489     -0.40716906
##    Aroma.persistency Aroma.quality Attack.intensity     Acidity Astringency
## 1        -0.08192571    0.43923128      -0.64273777 -1.18056954 -0.05836254
## 2        -0.72449963   -0.44321871      -0.39998906 -1.18056954 -1.35120359
## 3        -0.75766474    0.04309497       0.21852134 -0.87512444 -0.98403673
## 4        -1.65726824   -1.88605665      -1.50399688  3.36716857 -1.32017541
## 5         1.29857182    1.28303382       1.02324993  0.78785442  0.49497343
## 6         0.68501736    0.82892634       0.43134212  0.03272626  1.04830940
## 7         0.99179459    1.03504605       0.78715187  0.94057696  0.86214029
## 8         0.73476502   -0.32727638       0.31163043 -0.87512444  0.98108167
## 9        -0.37212038    0.78705828       0.43134212 -0.42119909 -0.17213255
## 10       -1.15564601   -0.91987053      -0.87551050 -0.11999629 -0.98403673
## 11        0.22070588    0.51008493       0.54772849  0.18544881  0.67597118
## 12        0.36580322    0.25565591       0.66744018  0.18544881  0.86214029
## 13        1.14518321    0.66145408       0.78715187 -0.42119909  0.30880432
## 14        0.40311397    0.41346631       1.20614279 -1.16360037  0.49497343
## 15        0.22485152    0.98673674       0.09548432 -0.42119909  1.04830940
## 16       -0.09436262    0.43923128       0.78715187  0.03272626 -0.43070076
## 17        1.29857182    1.28303382      -0.28360270  0.78785442 -0.61686987
## 18       -0.26018815   -0.59136725      -0.28360270  0.03272626 -0.61686987
## 19       -2.79731875   -1.63484826      -3.24979236 -0.57392164 -2.46304690
## 20       -0.10265390   -1.49314096      -0.64273777  0.09211836  1.04830940
## 21        0.63526971   -1.67027508       0.58763239  0.78785442  1.17242214
##         Alcohol     Balance      Smooth  Bitterness  Intensity    Harmony
## 1  -1.463213846  0.37003325  0.13953738 -0.77476654 -0.8439357 -0.0114018
## 2  -0.526919114 -0.62118967 -0.42832824 -0.77476654 -0.7455980 -0.4235111
## 3  -0.593797309  0.58724567  0.01170616 -0.37074399 -0.2511775 -0.0114018
```

```
## 4  -1.463213846 -2.43537235 -2.44412825 -0.57275526 -1.9229195 -2.5554287
## 5   0.275619229  1.02472986  0.88931665  0.01689927  1.3031054  1.1397416
## 6   0.707287579  0.48016912  0.44928226  0.60655380  0.8141482  0.8105146
## 7   0.226980541  1.02472986  0.44928226 -0.75838724  1.3031054  0.9394427
## 8   0.226980541  0.15282082  0.27474347 -0.37074399  0.4235288  0.3408481
## 9   0.002026612  0.04268494  0.36078372 -0.56729550 -0.0490388  0.3178252
## 10 -0.222927317  0.25989737 -0.42832824  0.63931239 -0.8439357 -0.4258134
## 11  0.926161673  0.19259211  0.44928226  0.79764555  0.5218665 -0.1771665
## 12  0.488413486 -0.06745094  0.52794762 -0.16873271  0.2295849  0.5641699
## 13  0.488413486  1.13486574  1.50388982 -0.37074399  0.6584468  1.1397416
## 14  0.226980541  0.96354326  1.36868372  0.01689927  1.3686639  1.4689686
## 15  1.145035766  0.48016912  0.36078372  0.21345078  0.4235288  0.3408481
## 16 -0.222927317  0.58724567  0.80081811 -0.37074399  0.1312471  0.1520606
## 17  1.145035766  0.62395763  0.36078372  0.41000229  0.4235288  0.2349429
## 18  0.488413486  0.04268494 -0.16529092  0.41000229 -0.3522469 -0.1702596
## 19 -2.983172828 -2.03460012 -2.09750898 -2.12332829 -2.6959636 -2.3965710
## 20  1.351750188 -1.70725181 -1.48293580  0.84132366 -0.3522469 -0.9369211
## 21 -0.222927317 -1.10150448 -0.90032043  3.27091872  0.4563080 -0.3406288
##    Overall.quality    Typical
## 1        0.1457179  0.1272641
## 2       -0.2759554 -0.3842965
## 3        0.4825854 -0.0424593
## 4       -2.0427429 -2.2632058
## 5        0.9655073  0.5910152
## 6        0.7346471  0.4691012
## 7        0.9019030  0.3830443
## 8        0.1457179 -0.3006300
## 9       -0.3089354  0.7248815
## 10      -0.3584055 -0.5564103
## 11       0.5650355  0.7248815
## 12      -0.4314327  0.8587478
## 13       0.5650355  1.7503931
## 14       1.4083820  0.6794626
## 15       0.7346471  0.2372257
## 16       0.9867088  0.8946049
## 17       0.4825854  0.1726830
## 18       0.3129738  0.5910152
## 19      -2.2641803 -2.0934824
## 20      -1.6210696 -1.4958649
## 21      -1.1287248 -1.0679708
```

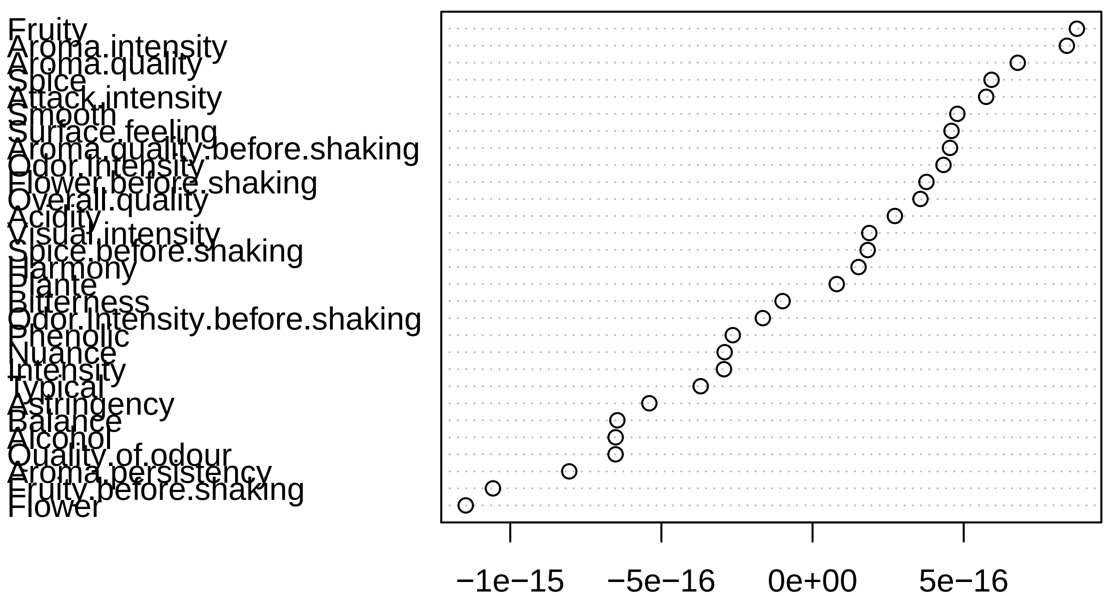
Maintenant, il faut montrer que le barycentre du nuage se trouve à l'origine. On sait que le barycentre est le point d'application de la résultante des actions de la pesanteur sur toute les parties d'un corps ; c'est **le centre de gravité**. On aura donc :

```r
BarycentreVariables.C_R <- colMeans(Variables.C_R)
print(BarycentreVariables.C_R)
```

```
## Odor.Intensity.before.shaking  Aroma.quality.before.shaking
##                 -1.644683e-16                  4.546628e-16
##        Fruity.before.shaking         Flower.before.shaking
##                 -1.057180e-15                  3.766828e-16
##         Spice.before.shaking              Visual.intensity
##                  1.823938e-16                  1.876806e-16
##                       Nuance               Surface.feeling
```

```
##                  -2.907727e-16                  4.596191e-16
##                 Odor.Intensity               Quality.of.odour
##                   4.331852e-16                 -6.515952e-16
##                         Fruity                         Flower
##                   8.744658e-16                 -1.147313e-15
##                          Spice                         Plante
##                   5.921189e-16                  7.996249e-17
##                       Phenolic                Aroma.intensity
##                  -2.638432e-16                  8.412583e-16
##              Aroma.persistency                  Aroma.quality
##                  -8.049117e-16                  6.786899e-16
##               Attack.intensity                        Acidity
##                   5.742761e-16                  2.722690e-16
##                    Astringency                        Alcohol
##                  -5.399120e-16                 -6.513887e-16
##                        Balance                         Smooth
##                  -6.456476e-16                  4.790315e-16
##                     Bitterness                      Intensity
##                  -9.912706e-17                 -2.930857e-16
##                        Harmony                Overall.quality
##                   1.521600e-16                  3.568574e-16
##                        Typical
##                  -3.700743e-16
```

```r
dotchart(sort(BarycentreVariables.C_R))
```



```r
distance_euclidienne <- sqrt(sum((BarycentreVariables.C_R^2)))
distance_euclidienne
```

```
## [1] 2.990468e-15
```

On remarque que le barycentre est composé de valeurs qui sont toutes très proches de 0. On peut donc en conclure que le barycentre du nuages se trouve à l'origine.

Maintenant, il faut montrer que l'inertie totale du nuage est égale au nombre de variables; c'est-à-dire 29.

```r
Inertie = 0
for (i in 1:29){
  Inertie <- Inertie + var(Variables.C_R[,i])*(28/29)
}
print(Inertie)
```

```
## [1] 29
```

Nous trouvons donc bien que l'inertie totale du nuage vaut 29.

De façon formelle, pour K variables centrées réduites v_j, l'inertie totale I_{total} vaut :

$$I_{total} = \sum_{j=1}^{K} \underbrace{Var(v_j)}_{=1} = \sum_{j=1}^{K} 1 = K$$

- **Question 2**

Calculer les poids et les barycentres des trois appellations (Bourgueil, Chinon, Saumur). Puis, calculer les normes euclidiennes carrées de ces trois barycentres.

Pour commencer, calculons les poids des variables qualitatives. Dans un premier temps, nous allons initialiser à 0 les variables "bourgueuil", "Chinon", "Saumur"

```r
Effectif <- 21
Bo <- 0
Ch <- 0
Sa <- 0
```

Puis on définit une boucle for qui determine une somme.

```r
for (i in 1:21){
  if(wine[i,2] == "Bourgueuil")
    Bo <- Bo + 1
  if(wine[i,2] == "Chinon")
    Ch <- Ch + 1
  if(wine[i,2] == "Saumur")
    Sa <- Sa + 1
}
```

Et on remarque que :

```r
Bo
```

```
## [1] 6
```

```r
Ch
```

```
## [1] 4
```

```r
Sa
```

```
## [1] 11
```

Or le poids de chaque variable vaut l'effectif de la variable/l'effectif totale et cet effectif total vaut 21

```
Poids_Bo <- Bo/Effectif
Poids_Ch <- Ch/Effectif
Poids_Sa <- Sa/Effectif
```

Au final , on a :

```
print(Poids_Bo)
```

```
## [1] 0.2857143
```

```
print(Poids_Ch)
```

```
## [1] 0.1904762
```

```
print(Poids_Sa)
```

```
## [1] 0.5238095
```

On aura donc le poids de Bourgueuil = $0.2857143 = \frac{6}{21}$; le poids de Chinon = $0.1904762 = \frac{4}{21}$ et le poids de Saumur = $0.5238095 = \frac{11}{21}$

Maintenant, calculons les barycentres des trois appellations :“Bourgueuil”, “Chinon”, "Saumur.

On intoduit des matrices des variables des appellations :

```
Bourgueuil = Variables.C_R[wine$Label == "Bourgueuil",]
Chinon = Variables.C_R[wine$Label == "Chinon",]
Saumur = Variables.C_R[wine$Label == "Saumur",]
```

Ce qui donnera les barycentres :

```
BarycentreBourgueuil = colMeans(Bourgueuil)
print(BarycentreBourgueuil)
```

```
## Odor.Intensity.before.shaking  Aroma.quality.before.shaking
##                  -0.621067144                   0.078711859
##         Fruity.before.shaking         Flower.before.shaking
##                   0.041103243                  -0.180726192
##          Spice.before.shaking              Visual.intensity
##                   0.036903234                  -0.319543413
##                        Nuance               Surface.feeling
##                  -0.301386900                  -0.227739618
##                Odor.Intensity               Quality.of.odour
##                  -0.830301959                  -0.100355012
##                        Fruity                        Flower
##                  -0.011372121                   0.146433900
##                         Spice                        Plante
##                   0.085490216                  -0.740297430
##                      Phenolic               Aroma.intensity
##                  -0.900687105                  -0.391194184
##             Aroma.persistency                 Aroma.quality
##                  -0.291280439                   0.283031187
##              Attack.intensity                       Acidity
##                  -0.155023472                  -0.144035948
##                    Astringency                       Alcohol
##                  -0.153170883                  -0.286765595
##                       Balance                        Smooth
```

```
##                  0.187493223                  -0.004272747
##                   Bitterness                     Intensity
##                 -0.467199820                  -0.045851933
##                      Harmony               Overall.quality
##                 -0.027517809                   0.262718497
##                      Typical
##                 -0.025327597
```

```r
BarycentreChinon = colMeans(Chinon)
print(BarycentreChinon)
```

```
## Odor.Intensity.before.shaking  Aroma.quality.before.shaking
##                   -0.59118158                   -0.82900791
##         Fruity.before.shaking         Flower.before.shaking
##                    0.18580836                   -0.28526389
##          Spice.before.shaking              Visual.intensity
##                   -0.19629487                   -0.66451006
##                        Nuance               Surface.feeling
##                   -0.60121498                   -0.32199611
##                Odor.Intensity               Quality.of.odour
##                   -0.31719469                    0.15585437
##                        Fruity                        Flower
##                    0.17271410                   -0.09716717
##                         Spice                        Plante
##                   -0.32593145                    0.36459093
##                      Phenolic               Aroma.intensity
##                    0.01580014                   -0.32729470
##             Aroma.persistency                 Aroma.quality
##                   -0.24775124                   -0.10183295
##              Attack.intensity                       Acidity
##                   -0.40996504                    0.94163754
##                   Astringency                       Alcohol
##                   -0.68151193                    0.04306550
##                       Balance                        Smooth
##                   -0.43151121                   -0.47196293
##                    Bitterness                     Intensity
##                   -0.08001154                   -0.47516911
##                       Harmony               Overall.quality
##                   -0.54323006                   -0.38902978
##                       Typical
##                   -0.19365651
```

```r
BarycentreSaumur = colMeans(Saumur)
print(BarycentreSaumur)
```

```
## Odor.Intensity.before.shaking  Aroma.quality.before.shaking
##                   0.553739018                   0.258523682
##         Fruity.before.shaking         Flower.before.shaking
##                  -0.089986626                   0.202310247
##          Spice.before.shaking              Visual.intensity
##                   0.051250917                   0.415936430
##                        Nuance               Surface.feeling
##                   0.383016485                   0.241311104
##                Odor.Intensity               Quality.of.odour
##                   0.568235500                  -0.001935221
```

```
##                  Fruity                  Flower
##            -0.056602150            -0.044539520
##                   Spice                  Plante
##             0.071889500             0.271220080
##                Phenolic         Aroma.intensity
##             0.485538372             0.332394899
##       Aroma.persistency           Aroma.quality
##             0.248971598            -0.117350484
##        Attack.intensity                 Acidity
##             0.233636453            -0.263848587
##             Astringency                 Alcohol
##             0.331370273             0.140757414
##                 Balance                  Smooth
##             0.054644136             0.173953474
##              Bitterness               Intensity
##             0.283931373             0.197798913
##                 Harmony         Overall.quality
##             0.212547917            -0.001835623
##                 Typical
##             0.084235603
```

Maintenant, calculons les normes euclidiennes carrées de ces trois barycentres.

```r
NormeBourgueuil <- crossprod(BarycentreBourgueuil)
print(NormeBourgueuil)
```

```
##          [,1]
## [1,] 3.554499
```

```r
NormeChinon <- crossprod(BarycentreChinon)
print(NormeChinon)
```

```
##          [,1]
## [1,] 5.326501
```

```r
NormeSaumur <- crossprod(BarycentreSaumur)
print(NormeSaumur)
```

```
##          [,1]
## [1,] 2.090938
```

On en déduit l'inertie inter-appellations, puis le R^2 de la partition des vins en appellations.

```r
Inertie_Inter_Appellations <- Poids_Bo*((NormeBourgueuil)) + Poids_Ch*((NormeChinon)) + Poids_Sa*((Norme
print(Inertie_Inter_Appellations)
```

```
##          [,1]
## [1,] 3.125396
```

Le R^2 de la partition des vins en appellations On sait que R2 de la partition = ecart-type au carré inter-appellations / ecart-type au carré total. Or on sait que Les variables centrées réduites ont une **variance = 1**, ce qui donnera:

```r
R2 <- (Inertie_Inter_Appellations)/(Inertie)
print(R2)
```

```
##           [,1]
## [1,] 0.1077723
```

Par ce résultat, on peut donc en déduire que l'appellation n'explique qu'environ 11% des disparités sensorielles entre les vins de Loire. R^2 vaut la dispersion inter-classe/dispersion totale qui est un coefficient qui appartient à [0,1].

- **Question 3**

Démontrons ce résultat **mathématiquement**. Par définition :

$$R^2_{part} = \frac{\sigma_{inter}}{\sigma^2} = \frac{\sum_{j=1}^{J} w^j (\overline{x^j} - \overline{x})^2}{\sum_{j=1}^{J} \sum_{i \in A_j} w_i (x_i - \overline{x})^2}$$

Comme les variables sont centrées réduites i.e $\overline{x} = 0$:

$$\begin{aligned} R^2_{part} = \frac{\sum_{j=1}^{J} w^j (\overline{x^j})^2}{\sum_{j=1}^{J} \sum_{i \in A_j} w_i x_i^2} &= \frac{\sum_{j=1}^{J} w^j (\overline{x^j})^2}{\sum_{i=1}^{n} w_i (x_i^j)^2} \\ &= \frac{\sum_{j=1}^{J} w^j (\overline{x^j})^2}{Var(x^j)} \\ &= \sum_{j=1}^{J} w^j (\overline{x^j})^2 \end{aligned}$$

En faisant **la moyenne arithmétique** avec la somme des poids w_i :

$$\sum_{i=1}^{K} w_i = \sum_{i=1}^{K} \frac{1}{K}$$

On obtient :

$$\begin{aligned} \sum_{i=1}^{K} w_i \sum_{j=1}^{J} w^j \sum_{i=1}^{K} (\overline{x^j})^2 &= \frac{1}{K} \sum_{j=1}^{J} w^j \sum_{i=1}^{K} (\overline{x^j})^2 \\ &= \frac{1}{K} \sum_{j=1}^{J} w^j \|\overline{x^j}\|^2 \\ &= \frac{I_{externe}}{I_{total}} \\ &= R^2_{senso} \end{aligned}$$

avec R^2_{senso}, le R^2 des **variables sensorielles**

On voudrait savoir **informatiquement** quelles sont les variables qui sont les plus liées à l'appellation

Calculons le R^2:

On initialise le vecteur avec des 0 grâce à l'élément : numeric (29)

```
R2.sensorielle <- numeric(29)
```

On sait que le R2.sensorielle = Variance interclasse / Variance totale. Or **la variance totale = 1** car les variables sont **centrées-réduites**. Au final, on aura :

```
for (i in 1:29){
  R2.sensorielle[i]= R2.sensorielle[i]+Poids_Bo*((BarycentreBourgueuil[i])^2)+Poids_Ch*((BarycentreChin
}
print(R2.sensorielle)
```

```
##  [1] 0.337391664 0.167684251 0.011300446 0.046271309 0.009104334 0.203903653
##  [7] 0.171645720 0.065069501 0.385269779 0.007506205 0.007397068 0.008964029
## [13] 0.025029800 0.220433883 0.355316417 0.122001651 0.068402090 0.032076296
## [19] 0.067472614 0.211284761 0.152689085 0.034226892 0.047075036 0.058283976
## [25] 0.105811842 0.064101221 0.080089607 0.048549514 0.011043443
```