



2	1	0	4	1	4	9	5
9	0	6	9	0	1	5	9
7	3	4	9	6	6	5	4
0	7	4	0	1	3	1	3
4	7	2	7	1	2	1	1
7	4	2	3	5	1	2	4
4	6	3	5	5	6	0	4
1	9	5	7	8	9	3	7

Machine Learning Labs

Carayon Chloé – Taillieu Victor

Regression Class

```
def normalizeFeatures(self, X, fit=False):  
    if fit:  
        self.scaling = list(zip(X.min(), X.max()))  
  
    minmax = list(zip(*self.scaling))  
    min, max = np.array(minmax[0]), np.array(minmax[1])  
    X = np.matrix((X - min) / (max - min))  
  
    return np.insert(X, 0, 1, axis=1)
```

```
def gradientDescent(self, alpha=0.03, threshold=1e-3, iter=1000, autoAlpha=True):  
    i = 0  
    self.J = []  
    self.w = np.matrix([[1] for i in range(self.n + 1)])  
  
    while True:  
        i += 1  
        self.J.append(self.costFunction())  
  
        self.w = self.w - alpha * self.gradient()  
  
        if len(self.J) > 1:  
            if autoAlpha and self.J[-1] > self.J[-2]:  
                alpha /= 1.1  
  
            if abs(self.J[-1] - self.J[-2]) < threshold or i == iter:  
                break  
  
    self.updateScores()
```

$$X_{norm} = \frac{X - \min(X)}{\max(X) - \min(X)}$$

$$w_k = w_k - \frac{\alpha}{m} \sum_{i=1}^m (h_w(x^{(i)}) - y^{(i)}) x_k^{(i)}$$

$$w = w - \alpha \Delta$$

Linear Regression Class

$$J(w) = \frac{1}{2m} \sum_{i=1}^m (h_w(x^{(i)}) - y^{(i)})^2$$

$$J = \frac{1}{2m} (Xw - y)^T (Xw - y)$$

$$\Delta = \frac{1}{m} \sum_{i=1}^m (h_w(x^{(i)}) - y^{(i)}) x_k^{(i)}$$

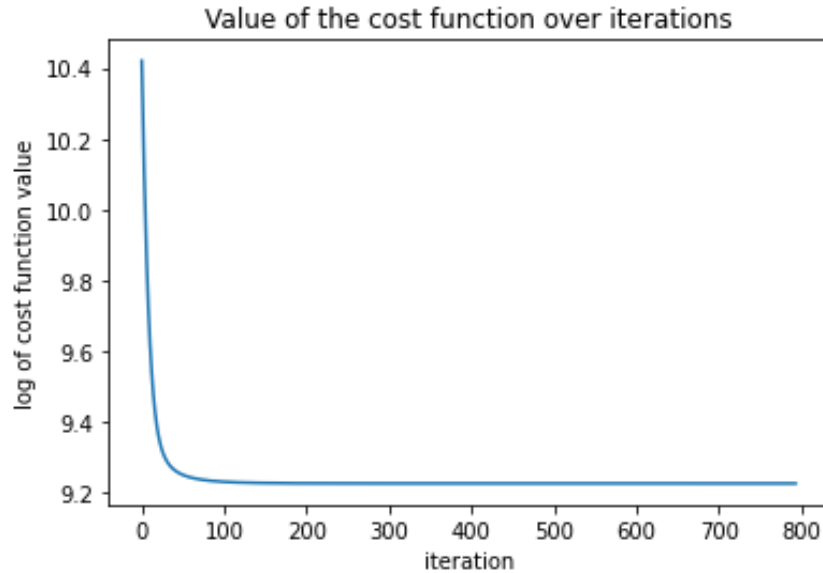
$$\Delta = \frac{1}{m} (X^T (Xw - y))$$

```
class LinearRegression(Regression):  
    def costFunction(self):  
        X, y, w, m = self.X_train, self.y_train, self.w, self.m  
        return float(1 / (2 * m) * (X * w - y).T * (X * w - y))  
  
    def gradient(self):  
        X, y, w, m = self.X_train, self.y_train, self.w, self.m  
        return 1 / m * X.T * (X * w - y)  
  
    def normalEquation(self):  
        X, y = self.X_train, self.y_train  
        self.w = (X.T * X)**-1 * X.T * y  
  
        self.updateScores()
```

$$w = (X^T X)^{-1} \cdot (X^T y)$$

Linear regression Results

Gradient descent



Gradient descent coefficients:

```
[[ 15.25614432]
 [ 62.16363578]
 [ 70.1350214 ]
 [-10.19451843]
 [176.85420204]
 [-27.4104932 ]
 [ 4.30315651]
 [-11.33107788]
 [273.63532282]
 [-198.095751 ]
 [ 20.9882641 ]
 [ -3.36688933]
 [ 24.98548201]]
```

MAE: 106.66332152370752

R2:

Train: 0.38761758431685356

Test: 0.3855197492585182

Normal Equation

Normal equation coefficients:

```
[[ 16.82540449]
 [ 62.01144869]
 [116.58965098]
 [ 32.48776091]
 [176.96206809]
 [-27.50480882]
 [ 4.30596978]
 [-11.2490898 ]
 [273.51027645]
 [-198.28934502]
 [ 20.67036957]
 [ -3.39974329]
 [-67.35622144]]
```

MAE: 106.66142249364468

R2:

Train: 0.3876553087897192

Test: 0.38567137931379636

Scikit Learn

Scikit-learn

```
[[ 16.82540449]
 [ 62.01144869]
 [116.58965098]
 [ 32.48776091]
 [176.96206809]
 [-27.50480882]
 [ 4.30596978]
 [-11.2490898 ]
 [273.51027645]
 [-198.28934502]
 [ 20.67036957]
 [ -3.39974329]
 [-67.35622144]]
```

MAE: 106.6614224936571

Train: 0.38765530878971943

Test: 0.38567137931378104

Logistic Regression Class

$$S(z) = h_w(z) = \frac{1}{1+e^{-z}}, z = Xw$$

$$h_w(Xw) = \frac{1}{1+e^{-Xw}}$$

```
class LogisticRegression(Regression):  
    def sigmoid(self, X):  
        return 1 / (1 + np.exp(-X * self.w))  
  
    def costFunction(self):  
        y, m, sigmoid = self.y_train, self.m, self.sigmoid(self.X_train)  
        return float(-1 / m * (y.T * np.log(sigmoid) + (1 - y.T) * np.log(1 - sigmoid)))  
  
    def gradient(self):  
        X, y = self.X_train, self.y_train  
        return X.T * (self.sigmoid(X) - y)
```

$$J = \frac{-1}{m} (y^T \log(h_w(x^{(i)})) + (1 - y^T) \log(1 - h_w(x^{(i)})))$$

$$\Delta = \sum_{i=1}^m (h_w(x^{(i)}) - y^{(i)}) x_k^{(i)}$$

$$\Delta = (X^T \cdot (\text{sigmoid}(X) - y))$$

Logistic Regression Results

Gradient descent

Logistic model

```
[[ -2.82598959]  
 [ -0.07721819]  
 [  0.45676453]  
 [  0.22917433]  
 [  0.63103143]  
 [  0.06463871]  
 [  0.7269864 ]  
 [  0.75345777]  
 [ -0.30125548]  
 [ -0.59108348]  
 [  3.85793586]  
 [  0.16918771]]
```

Confusion matrix:

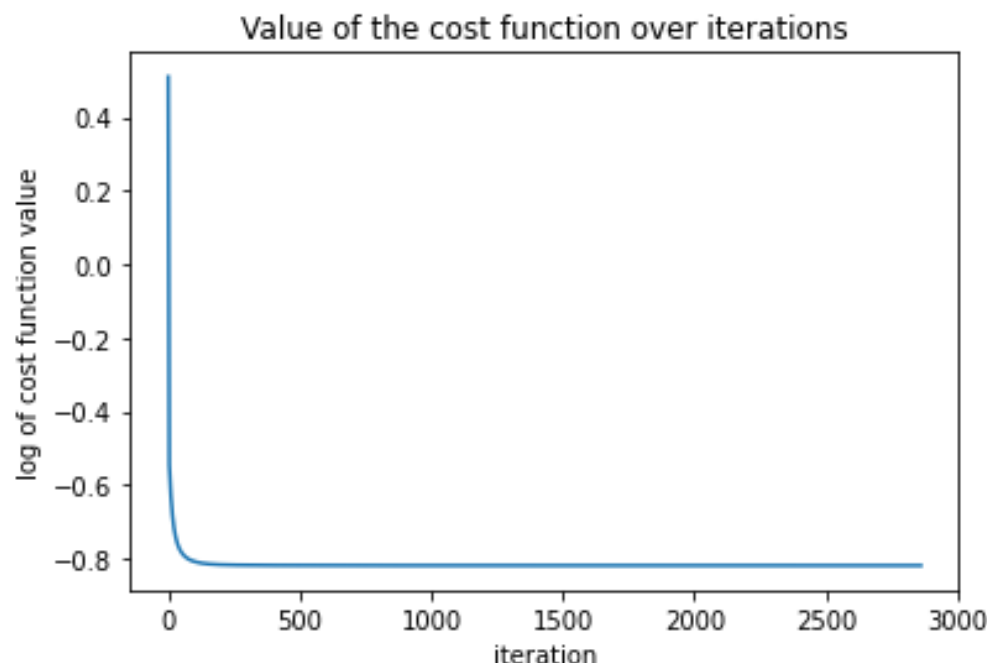
```
[[ 71  76]  
 [  7 334]]
```

Accuracy: 0.8299180327868853

Precision: 0.8146341463414634

Recall: 0.9794721407624634

F1-score: 0.8894806924101198



Scikit Learn

Scikit-Learn

```
[[ -2.44816876]  
 [ -0.06222258]  
 [  0.4205428 ]  
 [  0.17793724]  
 [  0.57452944]  
 [  0.08444219]  
 [  0.11094913]  
 [  0.09099877]  
 [ -0.06057049]  
 [ -0.33916367]  
 [  3.32797682]  
 [  0.15472534]]
```

Confusion matrix:

```
[[ 71  76]  
 [  7 334]]
```

Accuracy: 0.8299180327868853

Precision: 0.8146341463414634

Recall: 0.9794721407624634

F1-score: 0.8894806924101198

Neural Network Layers

$$a^i = \text{sigmoid}(z^i)$$
$$z^i = W^{i-1} a^{i-1}$$

```
class OutputLayer(Layer):
    def forward(self, inputs):
        z = self.w * inputs
        a = self.sigmoid(z)
        self.output = a

    def backward(self, y):
        self.error = self.output - y
```

$$\delta^n = a^n - y$$

```
class Layer:
    def __init__(self, n_inputs, n_neurons, random_state=42):
        np.random.seed(random_state)
        self.w = np.matrix(np.random.randn(n_neurons, n_inputs + 1))

    def sigmoid(self, x):
        return 1 / (1 + np.exp(-x))

    def sigmoid_deriv(self, x):
        return np.multiply(x, (1 - x))

    def forward(self, inputs):
        z = self.w * inputs
        a = self.sigmoid(z)
        self.output = np.insert(a, 0, 1, axis=0)

    def backward(self, next_w, next_error):
        deriv = self.sigmoid_deriv(self.output)
        error = np.multiply(next_w.T * next_error, deriv)
        self.error = np.delete(error, 0, axis=0)
```

```
class InputLayer(Layer):
    def __init__(self):
        pass

    def forward(self, inputs):
        self.output = np.insert(inputs, 0, 1, axis=0)
```

$$i < n: \delta^i = (W^i)^T \cdot \delta^{i+1} * \text{sigmoid}'(W^i a^i)$$

Deep Neural Network Class

$$J = \frac{-1}{m} [\sum_{i=1}^m \sum_{k=1}^k y_k^i \log(h_w(x^i))_k + (1 - y_k^i) \log(1 - h_w(x^i))_k]$$

```
def gradientDescent(self, alpha=1e-3, threshold=1e-5, epochs=1000):
    i = 0
    self.J = []
    self.train_accuracy = []
    self.test_accuracy = []

    while True:
        i += 1
        self.forward(self.X_train)
        self.backward()

        self.J.append(self.costFunction(self.layers[-1].output))

        grads = self.gradient()

        for l in range(len(self.layers) - 1):
            self.layers[l + 1].w -= alpha * grads[l]

        self.train_accuracy.append(self.accuracy(self.X_train, self.labels_train))
        self.test_accuracy.append(self.accuracy(self.X_test, self.labels_test))

        if len(self.J) > 1:
            if abs(self.J[-1] - self.J[-2]) < threshold or i == epochs:
                break
```

$$w^l = w^l - \alpha \Delta^l$$

```
def costFunction(self, y_pred):
    y, m = self.y_train, self.m

    return -1 / m * np.sum(np.multiply(y, np.log(y_pred))
                            + np.multiply((1 - y), np.log(1 - y_pred)))

def forward(self, x):
    inputs = x
    for l in range(len(self.layers)):
        self.layers[l].forward(inputs)
        inputs = self.layers[l].output

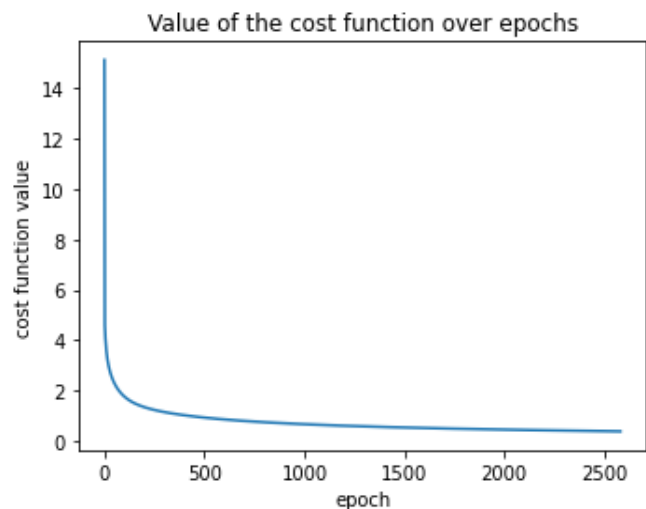
def backward(self):
    self.layers[-1].backward(self.y_train)
    next_w, next_error = self.layers[-1].w, self.layers[-1].error
    for l in range(len(self.layers) - 2, 0, -1):
        self.layers[l].backward(next_w, next_error)
        next_w, next_error = self.layers[l].w, self.layers[l].error

def gradient(self):
    grads = []
    for l in range(len(self.layers) - 1):
        grads.append(self.layers[l + 1].error * self.layers[l].output.T)

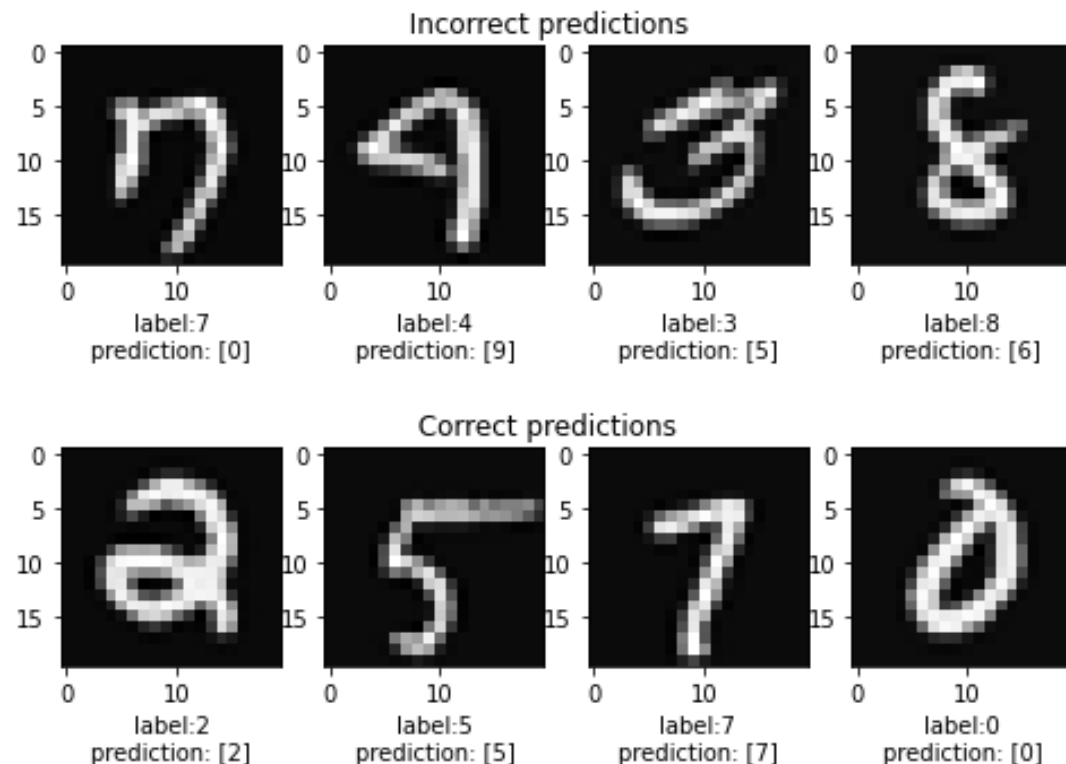
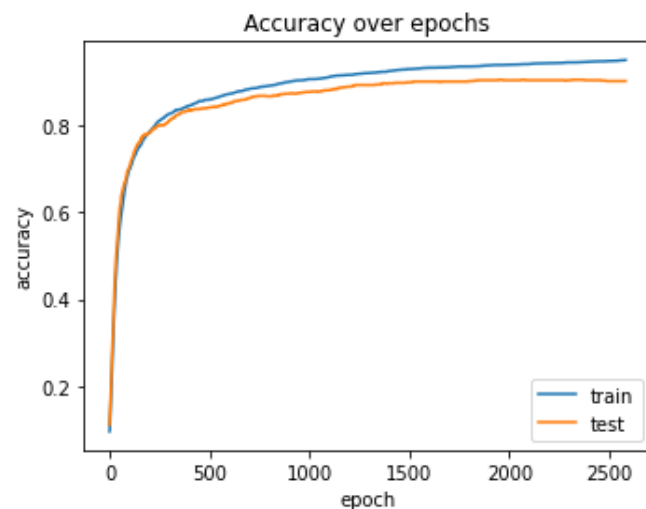
    return grads
```

$$\Delta_{ij}^l = \delta_i^{l+1} (a_j^l)^T$$

Neural Network Results



Train accuracy: 0.95175
Test accuracy: 0.903



DNN:

Test accuracy: 0.903

```
[[106  0  0  0  0  1  2  0  1  1]
 [  0 100  1  0  0  4  0  0  0  0]
 [  1  0 94  4  3  0  1  0  1  1]
 [  0  0  3 74  1  2  1  1  2  1]
 [  0  0  2  0 83  0  3  0  1  5]
 [  2  0  0  1  1 90  1  0  3  0]
 [  2  0  4  0  2  1 88  0  1  0]
 [  1  2  3  1  2  0  0 100  1  2]
 [  1  1  0  3  0  1  2  0 73  0]
 [  0  0  1  1  5  1  0  5  3 95]]
```

Keras:

Test accuracy: 0.9229999780654907

```
[[109  0  0  0  0  1  0  0  0  1]
 [  0 101  1  0  0  2  0  0  0  1]
 [  1  1 91  3  5  1  1  1  1  0]
 [  0  0  3 76  0  1  1  1  2  1]
 [  0  0  2  0 84  0  2  0  1  5]
 [  0  2  0  0  1 93  0  0  1  1]
 [  2  0  0  0  0  2 93  0  1  0]
 [  1  3  2  0  2  0  0 100  0  4]
 [  1  2  1  0  0  4  1  0 72  0]
 [  0  0  0  2  0  1  0  3  1 104]]
```



2	1	0	4	1	4	9	5
9	0	6	9	0	1	5	9
7	3	4	9	6	6	5	4
0	7	4	0	1	3	1	3
4	7	2	7	1	2	1	1
7	4	2	3	5	1	2	4
4	6	3	5	5	6	0	4
1	9	5	7	8	9	3	7

Conclusion