Nature Index - mathematical Framework

Definition, spatial units, indicator observations, scaling functions and weights

Bård Pedersen

11 4 2021

Definition

The Nature Index (NI) is a weighted average of scaled indicator states

$$NI_{jKt} = \sum_{i=1}^{n} w_{ijK} S_{ijKt}$$

where $S_{ijKt}, i = 1, ..., n$ are the n scaled indicator states included in the calculation. The weights (w_{ijK}) obey the condition $\sum_{i=1}^{n} w_{ijK} = 1$, while the indicator states are scaled to range between zero and 1. The Nature Index is calculated for a specified major habitat (j) in a defined spatial unit (K, here referred to as an *NIunit*) and for a particular year (t) from a set of measurements of indicator states.

Indicators

Indicators are typically the state of individual species. Most often abundance is used to represent state. In addition, indicators may be various types of community indices that represent groups of species with similar ecological function. For a few indicators, substitutes for species and species groups are used to represent state. Such substitutes, or indirect indicators, may be important resources or dominating environmental or biological factors with a negative impact on abundance or community composition. The open web portal naturindeks.no presents each indicator in the Norwegian implementation of the NI-framework.

Spatial units

Measurements of indicator states are collected from indicator-specific spatial units (here referred to as *ICunits*) that may have a different spatial extent than the NIunit. The spatial delineation of indicator areas may also vary among indicators.

That is, for each indicator there is a set of non-overlapping spatial units from which indicator measurements are collected. However, each ICunit and NIunit consist of one or more basic spatial units (*BSunit*). The set of BSunits varies among implementations of the Nature Index framework. The Norwegian implementation of the Nature Index uses municipalities as of 01.01.2010 as BSunits, while the pilot implementation in Costa Rica used a hexagonal grid of spatial units.

Thus, in order to calculate the index, the delineation of each IC- and NIunit must be provided. That is, each ICunit and NIunit must be specified in terms of the BSunits they consist of. Further, all indicator measurements must be linked to the correct ICunit.

Indicator observations

Measurements of indicator states, indicator observations, may be generated in different ways. They may be estimated from monitoring programs, or by expert judgment, or as predictions from models.

All indicators are supposed to be non-negative variables with zero as the minimum value.

Indicator measurements may be associated with errors and are therefore uncertain. Thus, in the context of the Nature Index, measurements are specified as probability distributions where the dispersion of the distribution represents this uncertainty, and the central tendency of the distribution represents the magnitude of the indicator value. Thus, each measurement is treated as a stochastic variable.

Probability distributions representing indicator measurements may be provided in different formats. Measurements generated from model predictions are typically specified as standard probability distributions (e.g. lognormal distribution) together with estimates of the distributions' parameters; or as large sets/samples of observations from the distributions, generated from the models through e.g. simulations or MCMC runs.

Probability distributions representing expert judgments are typically provided through an elicitation process. Elicitation is the term for the process that generates probability distributions for uncertain quantities based on experts' knowledge and beliefs about them. In the context of the Nature Index, this is a two-step process. In the first step, experts estimate the distribution's expected value and its lower and upper quartiles for all indicator values, where the interquartile distance measures the uncertainty in the measurements. In the second step, a probability distribution is fitted from among a number of model distributions for each indicator value. The fitting is based on a least squares criterion. Model distributions are non-negative, univariate distributions. They are the truncated normal-, lognormal-, Weibull-, "zeroinflated" exponential -, and gamma distributions. The truncated normal distribution is left-truncated at zero. All model distributions thus have two parameters.

Scaling

The indicators are measured or observed using measurement scales that are specific to the individual indicator. Scaling indicator values to a common scale is however necessary to calculate a meaningful average. This is done using nonlinear (piecewise linear) scaling functions. The scaling functions contain only one parameter called the reference value (Ref). Reference values are specified for each indicator area in which an indicator is observed. The scaling functions' range is the interval [0,1].

There are two types of scaling functions, LOW and MAX (Figure 1).

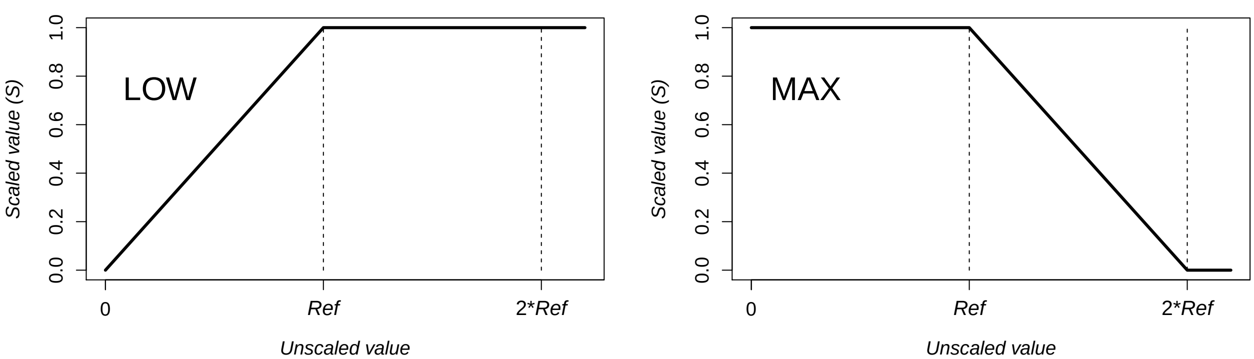

Figure 1: *The scaling models LOW and MAX*

LOW:

$S_i = \frac{U_i}{Ref_i}$, when $0 \leqslant U_i \leqslant Ref_i$, and

$S_i = 1$, when $U_i > Ref_i$.

MAX:

$S_i = 1$, when $0 \leqslant U_i < Ref_i$,

$S_i = 2 - \frac{U_i}{Ref_i}$, when $Ref_i \leqslant U_i \leqslant 2 * Ref_i$, and

$S_i = 0$, when $U_i > 2 * Ref_i$.

U_i is the state of indicator i measured on the original, indicator specific scale.

The choice of scaling function is determined by whether the indicator relates positively or negatively to the aspect of biodiversity that the indicator represents. The LOW model is used when there is a positive relation between the indicator and biodiversity. This holds true for most indicators. The MAX model is used when there is a negative relation between the indicator and biodiversity. This model applies only to certain indirect indicators that represent a negative effect that the measured indicator has on other components of biodiversity.

Weights

Weights are calculated according to an *a priori* defined system. Weights assigned to indicator states depend on the indicators' specificity / fidelity (φ) to the respective major ecosystem, the indicators' ecological function, and the area the major ecosystem in question covers in BSunits included in the NIunit. This means that weights depend on characteristics of indicators and BSunits rather than properties of the indicator observations themselves. Weights can be written as a sum of products of three factors, a fidelity weight ($w^{fidelity}$), a trophic weight ($w^{trophic}$), and an area weight (w^{area}). A grouping of the indicators into trophic groups and key indicators according to their ecological function, is the basis for calculating $w^{fidelity}$ and $w^{trophic}$. Key indicators are either species with key ecological functions in the ecosystem, or community indices representing many species. The precise definitions of area, fidelity, ecological groups and key indicators may vary among implementations of the Nature Index framework.

The weight assigned to a measurement of indicator i from ICunit C, in the calculation of an index for habitat j in NIunit K, where both C and K may include several BSunits, is

$$w_{ijK} = \sum_{k \in K \cap C} w_{jk}^{area} w_{ijk}^{fidelity} w_{ijk}^{trophic}$$

where the sum is over all BSunits included in both C and K.Products between corresponding fidelity- and trophic weights, $w_{ijk}^{BSunit} = w_{ijk}^{fidelity} w_{ijk}^{trophic}$, are referred to as BSunit weights.

The fidelity weight for BSunit k assigned to a measurement of indicator i belonging to trophic group g, is given by

$$w_{ijk}^{fidelity} = \frac{\varphi_{ij}}{\sum_{m \in g} \varphi_{mj}}$$

where the sum in the denominator is over all indicators within the trophic group observed in unit k. The same formula is applied for key indicators that are treated as a separate group in this context.

Trophic weights depend on the presence of key indicators and the number of other functional (trophic) groups (r_{jk})represented in the BSunit. $w_{ijk}^{trophic} = \frac{1}{2}$ if i is a key indicator and nonkey indicators are also present, $w_{ijk}^{trophic} = 1$ if i is a key indicator and nonkey indicators are not present, $w_{ijk}^{trophic} = \frac{1}{2r_{jk}}$ if i is a nonkey indicator and key indicators are also present, $w_{ijk}^{trophic} = \frac{1}{r_{jk}}$ if i is a nonkey indicator and no key indicators are present in BSunit k.

As a consequence of these assignments $\sum_{i=1}^{n} w_{ijk}^{BSunit} = 1$, where the sum is over all indicators recorded in BSunit k with $\varphi_{ij} > 0$. Further, in most cases, both key- and non-key indicators are present in k. In such cases key indicators weigh 50% in the Nature Index for a BSunit. Also, the different trophic groups represented in the BSunit are weighted equally, regardless of the number of indicators represented within each group.

Area weights are calculated as

$$w_{jk}^{area} = \frac{a_{jk}}{\sum_{q \in K} a_{jq}}$$

where a_{jk} is the area that major ecosystem j covers in BSunit k, and $\sum_{k \in K} w_{jk}^{area} = 1$

Calculation

Since the Nature Index is calculated from indicator measurement that are considered stochastic variables, the index itself is also a stochastic variable with an associated probability distribution. Parametric bootstrapping is used to simulate this distribution from the distributions of indicator measurements. An observation from the index's distribution is generated by randomly drawing one observation from each of the distributions of indicator measurements included in the calculation. The Nature Index value is then calculated as a weighted average of the sample of draws. The index's distribution is simulated by repeating this procedure *nsim*=1000 times, for example. Normally, the median of the simulated distribution is given as a point estimate for the Nature Index, while the 95% confidence interval, given by the 2.5% and 97.5% quantiles of the distribution, is used as a measure of uncertainty in the index estimate.

Thematic indices and indicator indices

Nature Index datasets may also be used as a basis for calculation of so-called thematic indices. A thematic index is similar in construction as the Nature Index, but is most often composed of a smaller selection of indicators that form part of the nature index. The system for weighting indicator measurements, described above, has often little or no relevance for the calculation of thematic indices. The system for weighting indicator measurements varies among the various indices, depending on the selection of indicators used to construct them. Most often, fidelities and functional groupings are ignored when calculating the weights for thematic indices.

Indicator indices give an area-weighted summary of the state for individual indicators.

The open web portal naturindeks.no presents examples of thematic- and indicator indices.

Functions in NIcalc for calculating indices

NIcalc::calculateIndex() is a utility function tailored for the Nature Index framework. It calculates the Nature Index, thematic indices, as well as indicator indices from a set of indicator measurements and for a specified set of NIunits.

NIcalc::calculateIndex() does the whole procedure of sampling and scaling indicator observations, calculating weights and the weighted average. It calls a series of other functions from the *NIcalc* package; *NIcalc::sampleObsMat()* draws the bootstrap samples, *NIcalc::scaleObsMat()* does the scaling of indicator measurements, while *NIcalc::calculateWeights()* and *NIcalc::indexCalculation()* calculate weights and the weighted average respectively. Each of these calls yet another sets of functions within the package.

As input, *NIcalc::calculateIndex()* requires lists of class *niInput*. Such lists should contain all the necessary information for the calculations, including indicator observations, reference values, indicator- and BSunit characteristics for calculating weights, as well as the delineation of ICunits and NIunits in terms of BSunits. *NIcalc::calculateIndex()* returns an object of class *niOutput* which contain an extensive output from the calculations for further processing of the results. The vignette *Objects in NIcalc* presents the detailed structure of S3 class objects defined within *NIcalc*.

NIcalc::elicitation() fits two-parameter distributions to indicator observations entered as expert judgments, while *NIcalc::makeDistribution()* generates distribution objects from model predictions. The vignette *Distributions* gives a more detailed account of this function.