理论题 1

- 在回归分析中，对数据进行变换

$$\tilde{y}_i = \frac{y_i - c_1}{d_1}, \quad \tilde{x}_i = \frac{x_i - c_2}{d_2}, \quad i = 1, 2, \cdots, n,$$

其中，选取 c_1, c_2, d_1, d_2 为适当的常数。请回答：

  - 试建立由原始数据和变换后数据得到的最小二乘估计、总偏差平方和、回归平方和以及残差平方和之间的关系；
  - 证明：由原始数据和变换后数据得到的 F 统计量的值保持不变。

解：(1) $\bar{\tilde{x}} = \frac{1}{n}\sum_{i=1}^{n}\tilde{x}_i = \frac{1}{d_2}(\bar{x}-c_2)$， $\bar{\tilde{y}} = \frac{1}{n}\sum_{i=1}^{n}\tilde{y}_i = \frac{1}{d_1}(\bar{y}-c_1)$，

$$l_{\tilde{x}\tilde{y}} = \sum_{i=1}^{n}(\tilde{x}_i-\bar{\tilde{x}})(\tilde{y}_i-\bar{\tilde{y}}) = \frac{1}{d_1 d_2}\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y}) = \frac{1}{d_1 d_2} l_{xy},$$

$$l_{\tilde{x}\tilde{x}} = \sum_{i=1}^{n}(\tilde{x}_i-\bar{\tilde{x}})^2 = \frac{1}{d_2^2}\sum_{i=1}^{n}(x_i-\bar{x})^2 = \frac{1}{d_2^2} l_{xx},$$

$$l_{\tilde{y}\tilde{y}} = \sum_{i=1}^{n}(\tilde{y}_i-\bar{\tilde{y}})^2 = \frac{1}{d_1^2}\sum_{i=1}^{n}(y_i-\bar{y})^2 = \frac{1}{d_1^2} l_{yy},$$

$$\therefore \begin{cases} \tilde{\beta}_1 = \dfrac{l_{\tilde{x}\tilde{y}}}{l_{\tilde{x}\tilde{x}}} = \dfrac{d_2^2 l_{xy}}{d_1 d_2 l_{xx}} = \dfrac{d_2}{d_1}\hat{\beta}_1, \\ \tilde{\beta}_0 = \bar{\tilde{y}} - \tilde{\beta}_1\bar{\tilde{x}} = \dfrac{1}{d_1}(\bar{y}-c_1) - \dfrac{1}{d_1}\hat{\beta}_1(\bar{x}-c_2) = \dfrac{1}{d_1}\hat{\beta}_0 - \dfrac{1}{d_1}(c_1 - \hat{\beta}_1 c_2). \end{cases}$$

$$\hat{\beta}_1 = \frac{d_1}{d_2}\tilde{\beta}_1, \quad \hat{\beta}_0 = d_1\tilde{\beta}_0 + c_1\left(1 - \frac{d_1/c_1}{d_2/c_2}\tilde{\beta}_1\right).$$

$$S_T = l_{yy} = d_1^2 l_{\tilde{y}\tilde{y}} = d_1^2 \tilde{S}_T,$$

$$S_R = \hat{\beta}_1^2 l_{xx} = \frac{d_1^2}{d_2^2}\tilde{\beta}_1^2 \cdot d_2^2 l_{\tilde{x}\tilde{x}} = d_1^2 \tilde{S}_R,$$

$$S_e = d_1^2 \tilde{S}_e.$$

(2) $$F = \frac{S_R}{S_e/(n-2)} = \frac{\tilde{S}_R}{\tilde{S}_e/(n-2)} = \tilde{F}$$

即由原始数据和变换后数据得到的 F 检验统计量的值保持不变.

理论题 2

- 对给定的 n 组数据 $(x_i, y_i), i = 1, 2, \cdots, n$, 若我们关心的是 y 如何依赖 x 的取值而变动，则可以建立回归方程
$$\hat{y} = a + bx.$$
反之，若我们关心的是 x 如何依赖 y 的取值而变动, 则可以建立另一个回归方程
$$\hat{x} = c + dy.$$
试问这两条直线在直角坐标系中是否重合? 为什么? 若不重合, 它们有无交点? 若有, 试给出交点的坐标。

解: 不重合.

$$\hat{y} = a + bx \Longrightarrow \hat{y} - \bar{y} = \frac{l_{xy}}{l_{xx}}(x - \bar{x})$$

$$\hat{x} = c + dy \Longrightarrow \hat{x} - \bar{x} = \frac{l_{xy}}{l_{xx}}(y - \bar{y})$$

当且仅当 $l_{xy}^2 = l_{xx} l_{yy}$ 时两条直线重合

几乎不可能.

交点: $(\bar{x}, \bar{y})$