SSY281 Model Predictive Control

Assignment 4 - MPT and persistent feasibility

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Question 1 MPT and polyhedral sets

(a) we use the Polyhedron() function and plot the graph as below:

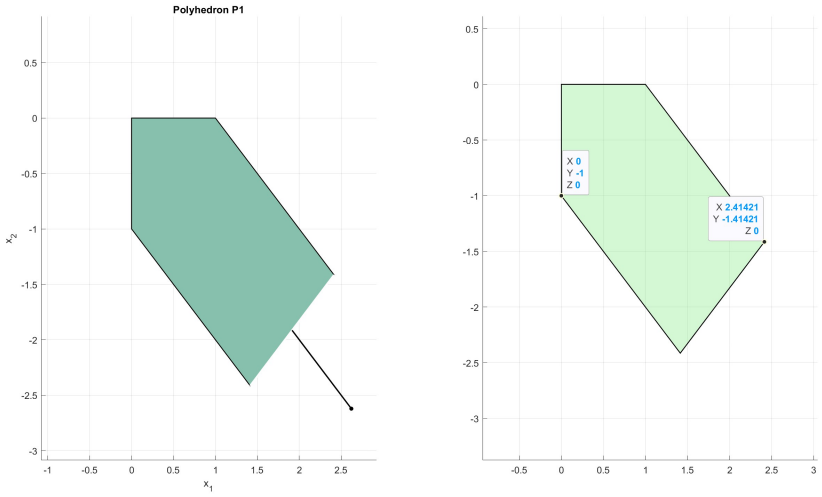

Figure 1: Enter Caption

(b) P, Q, P + Q, P Q, (P Q) + Q, (P + Q) Q, (Q P) + P, (Q + P) P are plotted as below:

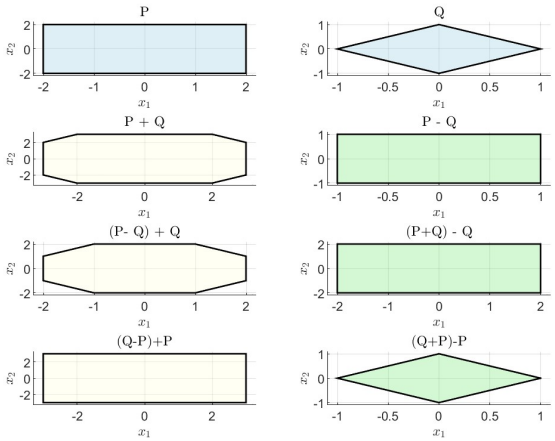

Figure 2: Minkowski sum and the Pontryagin difference operations

According to the Hint, when S1= and S2 not empty, the S1+S2 will incorrectly return S2, so we can see the left down corner graph(Q-P)+P , MPT incorrectly return P.

Question 2 Forward and backward reachability

(a) The set S is positively invariant.

By definition, the control invariant set C for system x(k+1) = f(x(k),u(k)) if:

$$x(k) \in C \rightarrow \exists u(k) \in U \ such \ that \ x(k+1) = f(x(k), u(k))$$

we can see that the 'AS' is contained in 'S' set, which means the x is control invariant.

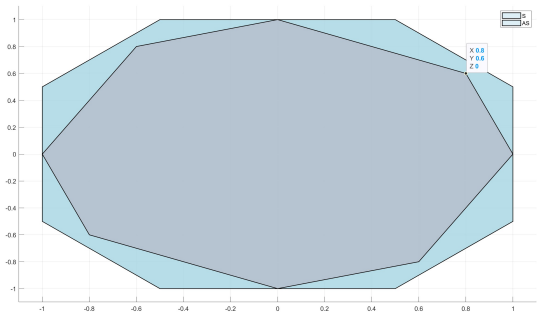

Figure 3: Control invariant

(b) By definition, the reachable set can be computed by affine mapping operation:

$$reach(X) = (A \circ X) \oplus (B \circ U) \tag{1}$$

the S and one-step reachable set are plotted as:

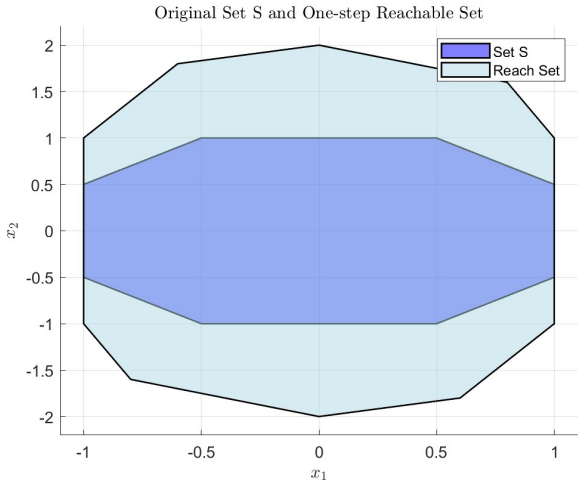

Figure 4: S and Reach_S

(c) The Pre(S) set contains all states from which the system can reach the set S in one time step,

$$Pre(S) = \{x \in R^2 : \forall u \in U, \exists x^+ \in S, suchthatx^+ = Ax + Bu\}$$

To calculate the Pre set, we need:

(1)express the condition A_{in}(Ax+Bu) $\leq b_{in}$;

(2)express -1$\leq$ u $\leq$ 1;

then we use the "projection" command to show the PreS set projection.

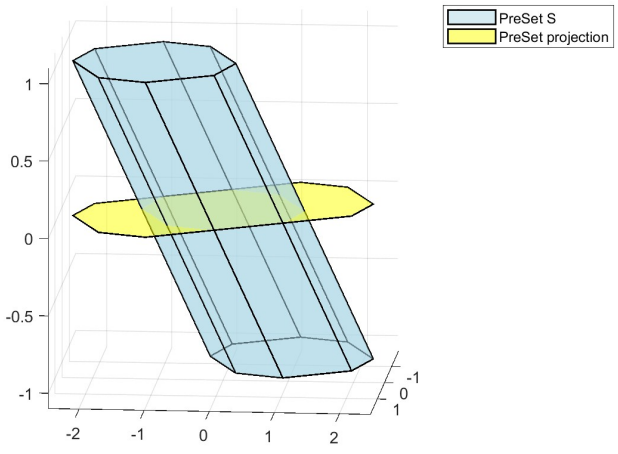

Figure 5: PreS set projection

Question 3 Persistent feasibility

(a) The shortest N to find a feasible controller is 26.

(b) By setting the horizon N=2 , and using 'InvariantSet' command to define $X_f = C_{inf}$, the RH controller is still feasible to the origin.

(c) By function of 'reachableSet', we can find the feasible set for these 2 controllers as below:

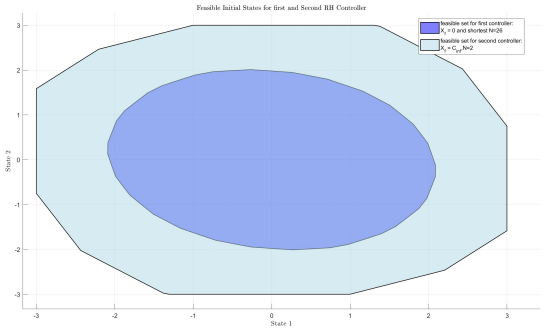

Figure 6: feasible sets of 2 controllers

The first controller has smaller set range due to the more strict terminal set X_f =0, the first controller has N=26 longer prediction horizon thus leads to a larger feasible set.

Both controller have 2 states and 1 control input.

The first controller has 52 Inequalities, 0 equality;

the second controller has 14 inequalities, 0 equality.