VP150-RC1

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Introduction

Something about VP150

1. Not so easy
2. Trust in your instructor
3. Exercise makes perfect
4. Refer to syllabus on Canvas for grading policy and course discription
5. Please **box** your answer in your homework. It will help your TAs find your answers a lot!

Something about my RC

1. Concept Review
2. Example: Little time to think, usually following concept
3. Exercise: Sufficient time to work, in the end of RC
4. Challenge: An optional take-home problem, much harder.

RC is not mandatory and contents of the three RC sessions would be highly similar. You may just attend depending on your own schedule.

Scientific Notation and Unit

Scientific Notation

$A = a \cdot 10^k$ where $|a| \in [1, 10)$

The Prefix of Units

Factor	Name	Symbol	Factor	Name	Symbol
10^{24}	yotta	Y	10^{-1}	deci	d
10^{21}	zetta	Z	10^{-2}	centi	c
10^{18}	exa	E	10^{-3}	milli	m
10^{15}	peta	P	10^{-6}	micro	μ
10^{12}	tera	T	10^{-9}	nano	n
10^{9}	giga	G	10^{-12}	pico	p
10^{6}	mega	M	10^{-15}	femto	f
10^{3}	kilo	k	10^{-18}	atto	a
10^{2}	hecto	h	10^{-21}	zepto	z
10^{1}	deka	da	10^{-24}	yocto	y

SI Units

SI units are divided into SI Base units and SI derived units. There are only 7 SI Base units, which are shown following:

SI Base Units		
Mass	kilogram	*kg*
Length	meter	*m*
Time	second	*s*
Amount of Substance	mole	*mol*
Electric Current	ampere	*A*
Thermodynamic Temperature	kelvin	*K*
Luminous Intensity	candela	*cd*

All the SI Derived units are derived from the SI Base units.

> 1. Express the unit of force N in the form of SI Units. $[kg]\cdot[m][s]^{-2}$
> 2. Is km SI Unit? No

Unit Conversions

Conversation of Dimension

We use equations to express relationships among physical quantities, represented by algebraic symbols. Each algebraic symbol always denotes both a number and a unit.

Common Steps

1. Denote the target unit in the form of SI
2. Denote the known unit in the form of SI
3. List equations to make sure that the exponent corresponding to the same SI unit the identical.
4. Solve the system of equations

> A simple pendulum consists of a light inextensible string AB with length L, with the end A fixed, and a point mass M attached to B. The pendulum oscillates with a small amplitude, and the period of oscillation is T. It is suggested that T is proportional to the product of powers of M, L and g, where g is the acceleration due to gravity. Use dimensional analysis to find this relationship.

$[T]=[s]$ $[L]=[m]$ $[g]=[m]\cdot[s]^{-2}$ $[M]=[kg]$ Assume $T=kM^{\alpha}L^{\beta}g^{\gamma}$

$\Rightarrow T=k\sqrt{\frac{L}{g}}$ $(T=2\pi\sqrt{\frac{L}{g}})$

Uncertainty and Significant Figures

Uncertainty

The uncertainty indicates the maximum difference there is likely to be between the measured value and the true value. The uncertainty in commonly expressed in absolute error and relative error:

1. $x = a \pm \epsilon$
2. $x = a \pm \delta$ where δ = $\frac{\epsilon}{a}$ $\times 100\%$

An example from the textbook (TFAE):

1. $mass = 1.6454 \pm 0.0021\ [kg]$
2. $mass = 1.6454(21)\ [kg]$
3. $mass = 1.6454 \pm 0.13\%\ [kg]$

> T or F:
>
> 1. $mass = 1.6454 \pm 0.002\ [kg]$ F
> 2. $mass = 1.6454 \pm 0.0021\ [kg]$ T
> 3. $mass = 1.6454 \pm 0.00210\ [kg]$ F

Uncertainty of non-constant value

In many cases, the uncertainty is indicated by the number of significant figures in the measured value. You will learn more in VP141.

Uncertainty of constant value

The uncertainty for a constant is treated as 0. For example, c, π, G

Significant Figures

Basic Rules

1. Non-zero digits are always significant.
2. Any zeros between two significant digits are significant.
3. A final zero or trailing zeros in the decimal portion ONLY are significant.

Common Steps

1. Transform it in the Scientific Notation ($a\times\ 10^k$).
2. Directly count the "a" part.

> 6 3 5 3
>
> 153000 / 1.53 / 1.5300 / 0.00153

Rules for Multiplication and Division

When numbers are multiplied or divided, the result can have no more significant figures than the factor with the fewest significant figures has.

e.g. $142.6 \times 32/51937 = 0.088$

Rules for Addition and Subtraction

When numbers are added or subtracted, the result can have no smaller uncertainty than the addend with the greatest uncertainty.

e.g. $312.67 + 145.8 + 21.742 = 480.2$

Back-of-the-Envelope Calculation

> How many senior high school Chinese teachers are there in Shanghai?
>
> Example
>
> 1. According to the population survey, there are about 15 million people living in Shanghai with household registration.
> 2. Assuming the population follows the uniform distribution with age, then for teenagers aged from 16 to 18, the population is about $1.5 \times 10^7 \times \frac{3}{80} = 5.6 \times 10^5$.
> 3. However, not every teenager in this age group studies in senior high school. About half of the students drop out of school or go to vocational school after junior high school. Thus, there are about $5.6 \times 10^5 \times \frac{1}{2} = 2.8 \times 10^5$ senior high school students. In fact, the actual number is about 1.5×10^5, while the error is accepetable.
> 4. Suppose every Chinese teacher teaches two classes, each class with 40 students, then there are $\frac{2.8\times10^5}{2\times40} = 3.5 \times 10^3$ senior high school Chinese teachers.

Vectors and Basic Vector Operations

Scalar

Scalar Quantities

Scalar quantities are defined by a single number.

e.g. time, mass, length, volume, density of matter, electric charge, potential energy, pressure, kinetic/potential energy...

Property

$+$ or $-$	only compatible units allowed as arguments
$*$ or $/$	may involve quantities with different units
sin(...), ln(...), exp(...), ...	only dimensionless arguments allowed

Vector

Vector Quantities

Vector quantities have both a magnitude and a direction.

e.g. velocity, force, linear momentum, angular velocity, angular momentum, electric/magnetic field, electric current density...

Basic Operation

1. Vector Addition (parallelogram rule)
2. Multiplication by a scalar
3. Scalar (Dot) Product
4. Vector (Cross) Product (right hand rule)

Property

1. The dot product of two non-zero vectors is zero **iff** the vectors are perpendicular,

$$\overline{u} \circ \overline{w} = 0 \Leftrightarrow \overline{u} \perp \overline{w}$$

2. The cross product is anticommutative,

$$\overline{u} \times \overline{w} = -\overline{w} \times \overline{u}$$

3. The cross product is a zero vector **iff** the two non-zero vectors are parallel (or antiparallel)

$$\overline{u} \times \overline{w} = 0 \Leftrightarrow \overline{u} \parallel \overline{w}$$

> **Q1.25** (a) If $\vec{A} \cdot \vec{B} = 0$, does it necessarily follow that $A = 0$ or $B = 0$? Explain. (b) If $\vec{A} \times \vec{B} = \mathbf{0}$, does it necessarily follow that $A = 0$ or $B = 0$? Explain.
> **Q1.26** If $\vec{A} = \mathbf{0}$ for a vector in the *xy*-plane, does it follow that $A_x = -A_y$? What *can* you say about A_x and A_y?

Q1.25 (a) No. $\vec{A} \perp \vec{B}$

(b) No. $\vec{A} \parallel \vec{B}$ or $\vec{A} \parallel -\vec{B}$

Q1.26 $A_x = A_y = 0$

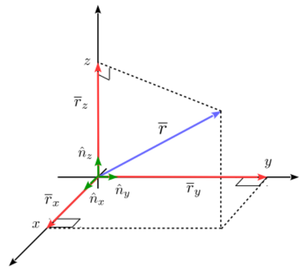
Cartesian Coordinate System

Definition

Defining <u>fixed</u> **unit vectors** (unit = of unit length) $\hat{n}_x$, $\hat{n}_y$, and $\hat{n}_z$ along the axes x, y, z, the position vector (or any other vector) can be represented as

$$\overline{r} = x\hat{n}_x + y\hat{n}_y + z\hat{n}_z.$$

The numbers x, y, z are called the Cartesian coordinates of the position vector.



Property

- mutually perpendicular
  $\hat{n}_x \circ \hat{n}_y = \hat{n}_y \circ \hat{n}_z = \hat{n}_z \circ \hat{n}_x = 0$
- unit length $|\hat{n}_x| = |\hat{n}_y| = |\hat{n}_z| = 1$, or equivalently
  $\hat{n}_x \circ \hat{n}_x = \hat{n}_y \circ \hat{n}_y = \hat{n}_z \circ \hat{n}_z = 1$
- For a right-handed system (note the cyclic permutation)

$$\begin{aligned} \hat{n}_x \times \hat{n}_y &= \hat{n}_z \\ \hat{n}_y \times \hat{n}_z &= \hat{n}_x \\ \hat{n}_z \times \hat{n}_x &= \hat{n}_y \end{aligned}$$

Basic Operation in Cartesian

Scalar Product

For $\overline{u} = (u_x, u_y, u_z)$ and $\overline{w} = (w_x, w_y, w_z)$, the dot product

$$\boxed{\overline{u} \circ \overline{w} = u_x w_x + u_y w_y + u_z w_z}$$

Vector Product

$$\begin{aligned} \overline{u} \times \overline{w} &= (u_y w_z - u_z w_y)\hat{n}_x + (u_z w_x - u_x w_z)\hat{n}_y + (u_x w_y - u_y w_x)\hat{n}_z \\ &= \begin{vmatrix} \hat{n}_x & \hat{n}_y & \hat{n}_z \\ u_x & u_y & u_z \\ w_x & w_y & w_z \end{vmatrix}. \end{aligned}$$

Differentiation

$$\begin{aligned} \boxed{\frac{d\overline{u}}{dt}} &= \frac{d}{dt}\left(u_x(t)\hat{n}_x + u_y(t)\hat{n}_y + u_z(t)\hat{n}_z\right) \\ &= \boxed{\dot{u}_x(t)\hat{n}_x + \dot{u}_y(t)\hat{n}_y + \dot{u}_z(t)\hat{n}_z} \end{aligned}$$

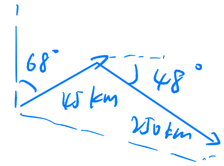
Integration

$$\int_{t_0}^{t_1} \overline{u}dt = \left(\int_{t_0}^{t_1} u_x(t)dt\right)\hat{n}_x + \left(\int_{t_0}^{t_1} u_y(t)dt\right)\hat{n}_y + \left(\int_{t_0}^{t_1} u_z(t)dt\right)\hat{n}_z$$

Exercise

Ex. 1

1.62 ••• Emergency Landing. A plane leaves the airport in Galisteo and flies 145 km at 68.0° east of north; then it changes direction to fly 250 km at 48.0° south of east, after which it makes an immediate emergency landing in a pasture. When the airport sends out a rescue crew, in which direction and how far should this crew fly to go directly to this plane?



Ex. 2

Consider two vectors $\mathbf{u} = 3\hat{n}_x + 4\hat{n}_y$ and $\mathbf{w} = 6\hat{n}_x + 16\hat{n}_y$. Find the components of the vector $\mathbf{w}$ that are, respectively, parallel and perpendicular to the vector $\mathbf{u}$.

For the same vectors, find a vector perpendicular to both

Ex. 3

A river flows from south to north at 5 km/h. On this river, a boat is heading east to west, perpendicular to the current at 7 km/h. As viewed by an eagle hovering at rest over the shore, how fast and in what direction is this boat traveling?

Ex. 4 (Challenge)

Verify the following equation:

$$\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B})$$

Reference

1. Wu Yufan, 2022SU VP150 RC.
2. Qu Zhemin, 2021SU VP150 RC.
3. Mateusz Krzyzosiak, 2023SU VP150 Slides.

Ex4.

Verify: $\vec{A}\times(\vec{B}\times\vec{c}) = \vec{B}\cdot(\vec{A}\cdot\vec{c}) - \vec{c}\cdot(\vec{A}\cdot\vec{B})$

$$\vec{B}\times\vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix} = (B_yC_z - B_zC_y)\hat{i} + (B_zC_x - B_xC_z)\hat{j} + (B_xC_y - B_yC_x)\hat{k}$$

$$LHS = \vec{A}\times(\vec{B}\times\vec{C}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_yC_z - B_zC_y & B_zC_x - B_xC_z & B_xC_y - B_yC_x \end{vmatrix}$$

$$= [A_y(B_xC_y - B_yC_x) - A_z(B_zC_x - B_xC_z)]\hat{i} + \cdots$$

$$\vec{B}\cdot(\vec{A}\cdot\vec{c}) - \vec{c}\cdot(\vec{A}\cdot\vec{B}) = [B_x\cdot(\cancel{A_xC_x} + A_yC_y + A_zC_z) - C_x(\cancel{A_xB_x} + A_yB_y + A_zB_z)]\hat{i}$$

$$+ \cdots$$