VP150-RC4

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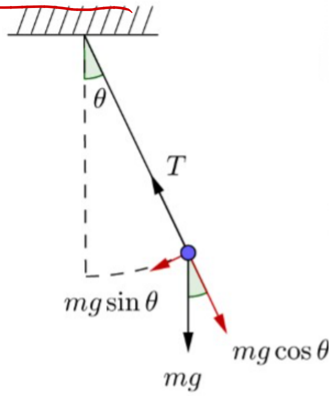
Simple Harmonic Oscillation

Motion Equation

$$\ddot{x}+\omega_0^2 x=0$$

- **Spring:** $\ddot{x}+\frac{k}{m}x=0$
- **Simple pendulum:** $\ddot{\theta}+\frac{g}{l}\theta=0$



General Solution

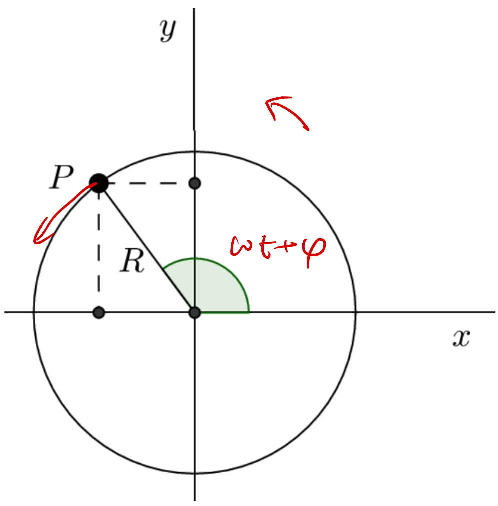
$$x(t)=A\cos(\omega_0 t+\varphi)$$

- natural Frequency: ω_0
- Frequency: $f=\frac{\omega_0}{2\pi}$
- Period: $T=\frac{1}{f}=\frac{2\pi}{\omega_0}$

e.g.

- **Spring:** $\omega_0=\sqrt{k/m}$
- **Simple Pendulum:** $\omega_0=\sqrt{g/l}$

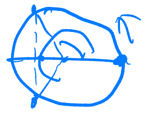
SHM and Uniform Circular Motion



$$x=R\cos(\omega t+\varphi), y=R\sin(\omega t+\varphi)$$

$$\begin{cases} a_x=-R\omega^2\cos\omega_0 t=-\omega_0^2 x \\ a_y=-\omega_0^2 y \end{cases}$$

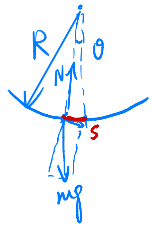
$\varphi = 0$



1. $A = 6cm$ **14.66** ••• An object is undergoing SHM with period 0.300 s and amplitude 6.00 cm. At $t = 0$ the object is instantaneously at rest at $x = 6.00$ cm. Calculate the time it takes the object to go from $x = 6.00$ cm to $x = -1.50$ cm.

$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.3s} = 20.9 rad/s$ $\cos^{-1}$ $+nT$

2. Discuss the motion of a particle that is placed on the inner surface of a spherical pot, close to its bottom, and released from hold (no friction).



$mg \sin\theta \approx mg\theta$

$g \cdot \theta = -\ddot{s}$ $R \cdot \ddot{\theta} + g\theta = 0$

$s = R \cdot \theta$

Damped Oscillation

Motion Equation

$$\ddot{x} + \frac{b}{m}\dot{x} + \omega_0^2 x = 0$$

Classification

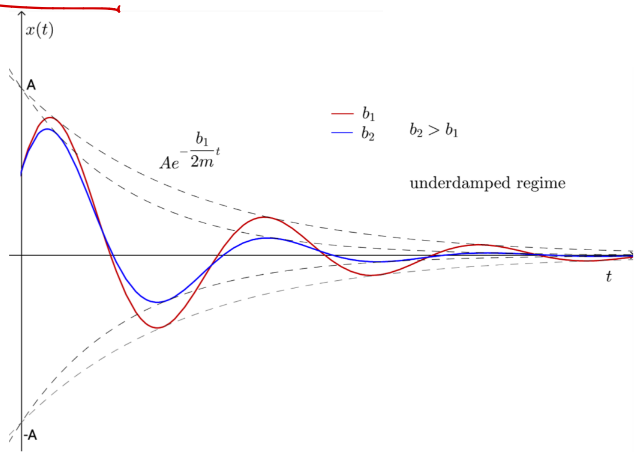
- Underdamped Regime: $b < 2m\omega_0$
- Overdamped Regime: $b > 2m\omega_0$
- Critical Damping: $b = 2m\omega_0$

Underdamped Regime: $\left(\frac{b}{m}\right)^2 < 4\omega_0^2$

General Solution

$$x(t) = \underbrace{Ae^{-\frac{b}{2m}t}}_{A'} \cos\left(\underbrace{\sqrt{\omega_0^2 - \left(\frac{b}{2m}\right)^2}\, t}_{\omega t} + \varphi\right)$$

- Amplitude: $Ae^{-\frac{b}{2m}t}$
- Angular frequency: $\omega = \sqrt{\omega_0^2 - \left(\frac{b}{2m}\right)^2}$



Comments

- Motion is still periodic.
- The amplitude of oscillations decreases exponentially with time ($Ae^{-\frac{b}{2m}t}$).
- Angular frequency $\omega = \sqrt{\omega_0^2 - \left(\frac{b}{2m}\right)^2} < \omega_0$.
- As b increases, the angular frequency decreases, and the amplitude decreases faster.

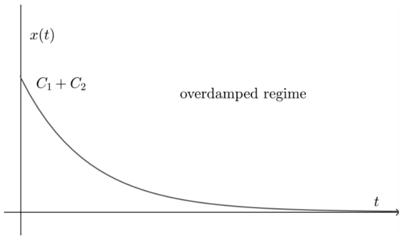
Overdamped Regime: $\left(\frac{b}{m}\right)^2 > 4\omega_0^2$

General Solution

$$x(t) = C_1 e^{-\left(\frac{b}{2m}-\sqrt{\left(\frac{b}{2m}\right)^2-\omega_0^2}\right)t} + C_2 e^{-\left(\frac{b}{2m}+\sqrt{\left(\frac{b}{2m}\right)^2-\omega_0^2}\right)t}$$

Effects of strong damping (overdamped regime)

* No periodic behavior.
* Strong damping results in aperiodic motion: the particle returns aperiodically to the equilibrium position.



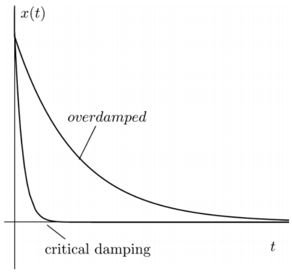
Critical Damping: $\left(\frac{b}{m}\right)^2 = 4\omega_0^2$

General Solution

$$x(t) = D_1 e^{-\frac{b}{2m}t} + D_2 t e^{-\frac{b}{2m}t}$$

Effects of critical damping

* No periodic behavior.
* The system may pass through the equilibrium position at most once (see Problem Set).



Driven Oscillation

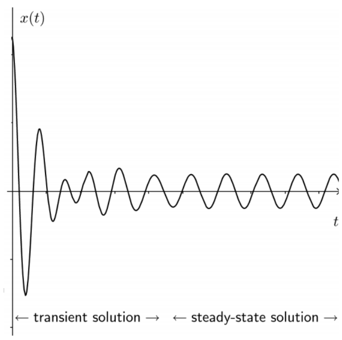
Motion Equation

m/s²　　　　m/s²

$$\ddot{x} + \frac{b}{m}\dot{x} + \omega_0^2 x = \boxed{\frac{F_0}{m}\cos\omega_{dr}t}$$

Solution

$$x(t) = \underbrace{\boxed{\text{solution to the equation of motion for damped oscillator}}}_{\text{(vanishes as } t\to\infty)} + \underbrace{\boxed{\text{periodic steady-state oscillations with angular frequency } \omega_{dr}}}_{x_s(t)}$$



Comments

- The final frequency will be dominated by the frequency of the imposed force. (like a frequency injection)
- Phase response: ϕ corresponds to a delay between input and output.

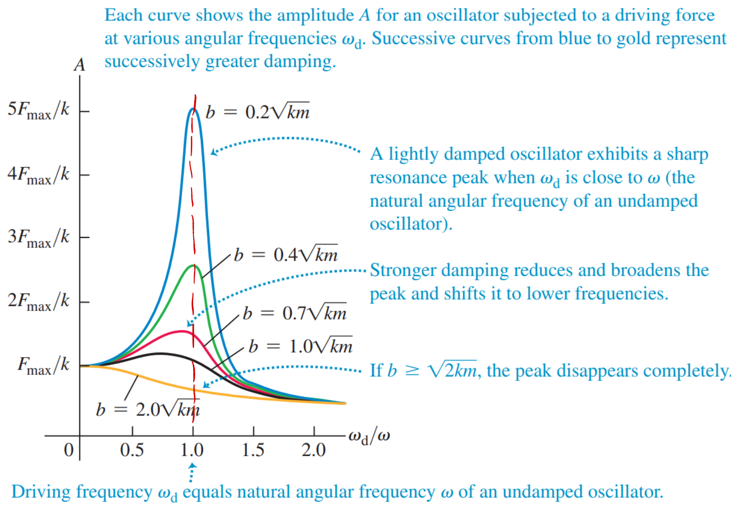
Steady State Solution

$$x_s(t) = A\cos(\omega_{dr}t + \phi)$$

Features

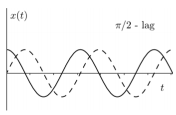
- Amplitude: $A(\omega_{dr}) = \frac{F_0}{m\sqrt{(\omega_0^2-\omega_{dr}^2)^2+\left(\frac{b\omega_{dr}}{m}\right)^2}}$
- Phase Shift: $\tan\varphi = \frac{b\omega_{dr}}{m(\omega_{dr}^2-\omega_0^2)}$
- Resonance Frequency: $\omega_{res} = \sqrt{\omega_0^2 - b^2/2m^2}$

Mechanical Resonance



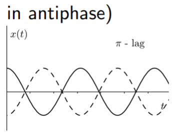
Phase Shift

- If $\omega_{dr} \to \omega_0$ (close to resonance[3]), then $\phi \to -\pi/2$.



The response ($x_s(t)$) lags the drive ($F_{dr}(t)$) by 1/4 of the cycle.

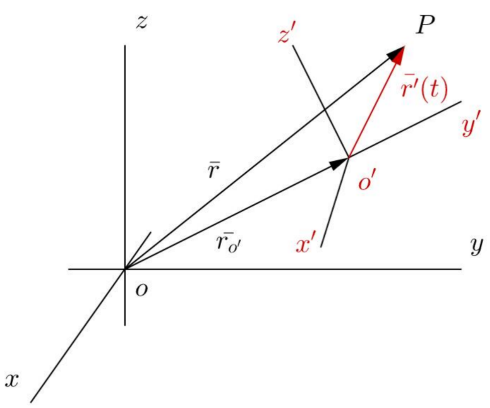
- If $\omega_{dr} \to \infty$ (high frequencies), then $\phi \to -\pi$ the response lags the drive by 1/2 of the cycle (displacement and drive are in antiphase)



The response ($x_s(t)$) lags the drive ($F_{dr}(t)$) by 1/2 of the cycle.

Non-inertial Frame of Reference

Concepts



$$
\begin{cases}
\vec{r} = \vec{r}_{O'} + \vec{r}' \\
\vec{v} = \vec{v}_{O'} + \vec{v}' + \vec{\omega} \times \vec{r}' \\
\vec{a} = \vec{a}_{O'} + \vec{a}' + 2\vec{\omega} \times \vec{v}' + \frac{\mathrm{d}\vec{\omega}}{\mathrm{d}t} \times \vec{r}' + \vec{\omega} \times (\vec{\omega} \times \vec{r}')
\end{cases}
$$

Pseudo-Forces

$$m\vec{a}' = \vec{F} - m\vec{a}_{O'} - 2m\vec{\omega} \times \vec{v}' - m\frac{\mathrm{d}\vec{\omega}}{\mathrm{d}t} \times \vec{r}' - m\vec{\omega} \times (\vec{\omega} \times \vec{r}')$$

- **d' Alembert "Force":** $-m\vec{a}_{O'}$
- **Coriolis "Force":** $-2m\vec{\omega} \times \vec{v}'$
- **Euler "Force":** $-m\frac{\mathrm{d}\vec{\omega}}{\mathrm{d}t} \times \vec{r}'$
- **Centrifugal "Force":** $-m\vec{\omega} \times (\vec{\omega} \times \vec{r}')$

Pay attention to the signs and coefficients!

Application of Coriolis Force

> Suppose that you are completely lost on the Earth. The only things you have are a small ball and a length of a string. How, using your knowledge of dynamics, can you determine whether you are closer to the North Pole or closer to the South Pole?

e.g.

- Foucault pendulum
- Tornado

Earth as Frame of Reference

The Motion of Earth

- Rotation
- Revolution

To the right

To the left

Pseudo-Forces

- **d' Alembert "Force":** $-m\vec{a}_{O'}$
- **Coriolis "Force":** $-2m\vec{\omega} \times \vec{v}'$
- **Centrifugal "Force":** $-m\vec{\omega} \times (\vec{\omega} \times \vec{r}')$

> Consider the direction of Coriolis "Force" on earth.
>
> *Hint: Different for the Northern Hemisphere and the Southern.*

> (*5 points*) A platform rotates clockwise about the axis perpendicular to the page and through the platform's center O, as shown in the top-view figure. The angular speed of rotation is constant. Suppose that you are standing at point A, throwing a ball towards the center of the platform O, along the radial direction AO.
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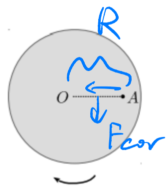
> 
>
>
> $\otimes \vec{\omega}$
>
> (a) An observer, standing on the platform at O, will see the object being deflected to the left or to the right with respect to the radial direction AO (as he looks from O)? right
>
> (b) How will an immobile observer outside of the platform explain this deflection?
>
> v
>
> ωR
>
> $\frac{\omega R}{r}$

Reference

1. He Yinghui, 2022SU VP150 RC.
2. Qu Zhemin, 2021SU VP150 RC.
3. Mateusz Krzyzosiak, 2023SU VP150 Slides.