VP150-RC7

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Torque

Definition

$$\overline{\tau} = \overline{r} \times \overline{F}$$

Torque and Angular Acceleration

$$\tau_z = I_z \varepsilon_z$$

> **Question 4.** (*5 points*) What is the magnitude of a horizontal force **F** (applied at the axle) able to push a wheel of weight w and radius R over a perfectly rigid step of height h?
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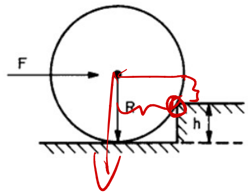
> 
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>
> $\exists \varepsilon \to \exists \tau$ $\quad$ $F \cdot (R-h) - mg \cdot \sqrt{R^2-(R-h)^2} > 0$

Dynamics for Rotation

Rotation with Fixed Axis

The net force can be decomposed into three components: radial, tangential and along the z-axis $\bar{F}_i = \bar{F}_{i,rad} + \bar{F}_{i,tan} + \bar{F}_{i,z}$

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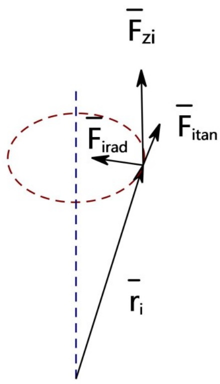
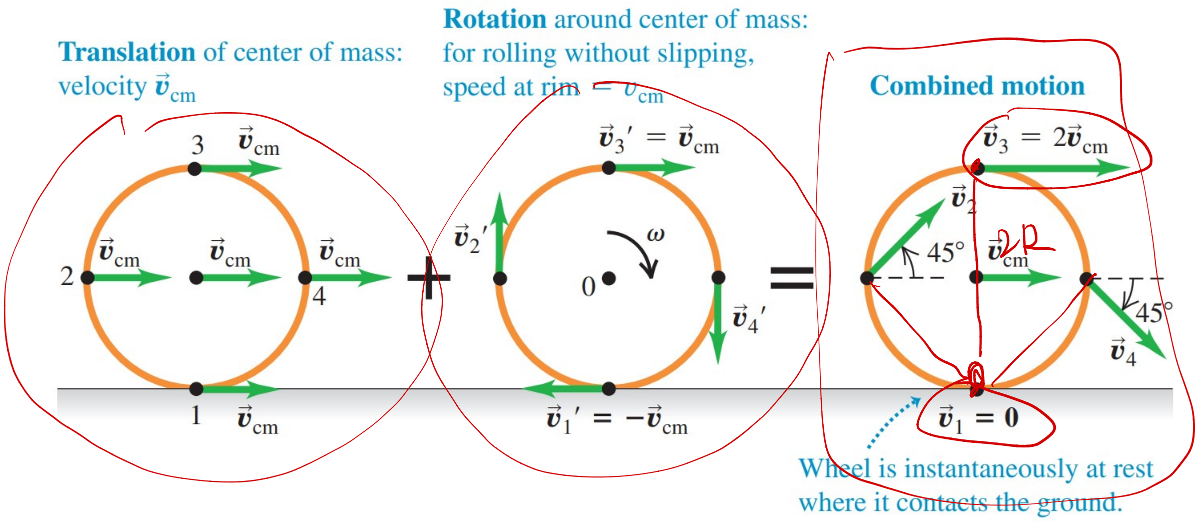


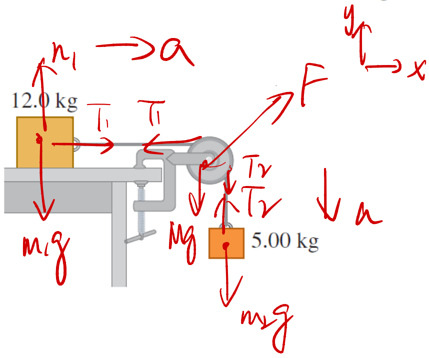
Rolling without slipping

Relationship between velocity of CoM and angular velocity



10.17 •• A 12.0-kg box resting on a horizontal, frictionless surface is attached to a 5.00-kg weight by a thin, light wire that passes over a frictionless pulley (Fig. E10.17). The pulley has the shape of a uniform solid disk of mass 2.00 kg and diameter 0.500 m. After the system is released, find (a) the tension in the wire on both sides of the pulley, (b) the acceleration of the box, and (c) the horizontal and vertical components of the force that the axle exerts on the pulley.

Figure **E10.17**



$$\begin{cases} m_2 g - T_2 = m_2 a \\ T_1 = m_1 a \\ (T_2 - T_1)\cdot R = I\cdot\alpha \\ a = \alpha\cdot R \\ I = \frac{1}{2}MR^2 \end{cases}$$

(a) $T_1 = 32.6\ N$

$T_2 = 35.4N$

(b) $a = 2.72\ m/s^2$

(c) $F_x = 32.6N$

$F_y = 55.0N$

Work and Conservation of Angular Momentum

Work in Rotational Motion

Definition and Formula

$$W_{12} = \int_{\theta_1}^{\theta_2} \tau_z \mathrm{d}\theta$$

$$P = \tau_z \omega_z$$

Work-Kinetic Energy Theorem

$$W = K_2 - K_1$$

where $K = \frac{1}{2} I\omega^2$

Angular Momentum

Definition and Formula

The rate of change of the angular momentum of a particle equals the torque of the net force acting on it.

$$\boxed{\begin{gathered} \overline{\tau} = \frac{\mathrm{d}\overline{L}}{\mathrm{d}t} \\ \overline{L} = \overline{r} \times \overline{p} \end{gathered}}$$

Especially, when for rotation about an **axis of symmetry**, or for a planar object contained in the x–y plane:

$$\vec{L} = I\vec{\omega}$$

Note that $\overline{L} \nparallel \overline{\omega}$ in general.

Conservation of Angular Momentum

Definition and Formula

For a rigid body, valid for any axis of rotation:

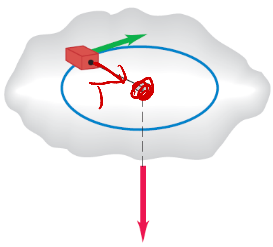
$$\boxed{\overline{\tau}^{ext} = \frac{\mathrm{d}\overline{L}}{\mathrm{d}t}}$$

When the net external torque on a system is zero, then the total angular momentum of the system is conserved.

$$\overline{L}_z = I_z \overline{\omega}_z = const$$

10.42 • **CP** A small block on a frictionless, horizontal surface has a mass of 0.0250 kg. It is attached to a massless cord passing through a hole in the surface (Fig. E10.42). The block is originally revolving at a distance of 0.300 m from the hole with an angular speed of 1.75 rad/s. The cord is then pulled from below, shortening the radius of the circle in which the block revolves to 0.150 m. Model the block as a particle. (a) Is the angular momentum of the block conserved? Why or why not? (b) What is the new angular speed? (c) Find the change in kinetic energy of the block. (d) How much work was done in pulling the cord?

Figure **E10.42**



(a) Conserved

(b) $L_1 = L_2$

$I_1\omega_1 = I_2\omega_2 \quad I = mr^2$

$mr_1^2\omega_1 = mr_2^2\omega_2$

$\Rightarrow \omega_2 = 7\ \text{rad/s}$

(c) $K_1 = \frac{1}{2}I_1\omega_1^2$

$= \frac{1}{2}m_1(\underbrace{\omega_1 r_1}_{v_1})^2$

$= \frac{1}{2}m_1 v_1^2$

$K_2 = \frac{1}{2}m_2 v_2^2$

$\Rightarrow \Delta K = K_2 - K_1 = 0.0103\ \text{J}$

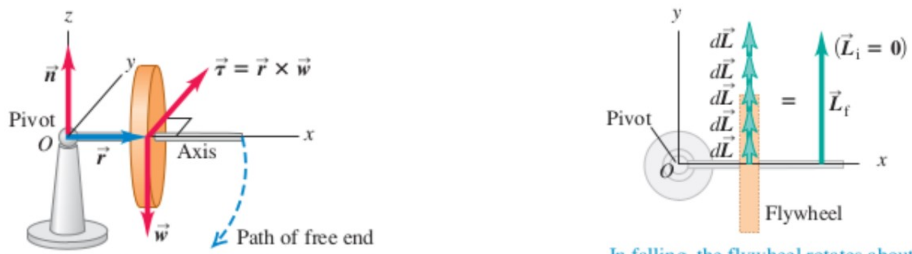
(d) $W_{tot} = \Delta K$

The Gyroscopic Effect

Non-Rotating Flywheel

(a) Nonrotating flywheel falls

(b) View from above as flywheel falls



When the flywheel is not rotating, its weight creates a torque around the pivot, causing it to fall along a circular path until its axis rests on the table surface.

In falling, the flywheel rotates about the pivot and thus acquires an angular momentum $\vec{L}$. The *direction* of $\vec{L}$ stays constant.

$$\boxed{\boldsymbol{\tau}^{\text{ext}} = \frac{d\mathbf{L}}{dt} \Longleftrightarrow d\mathbf{L} = \boldsymbol{\tau}^{\text{ext}}dt}$$

Rotating Flywheel - Precession

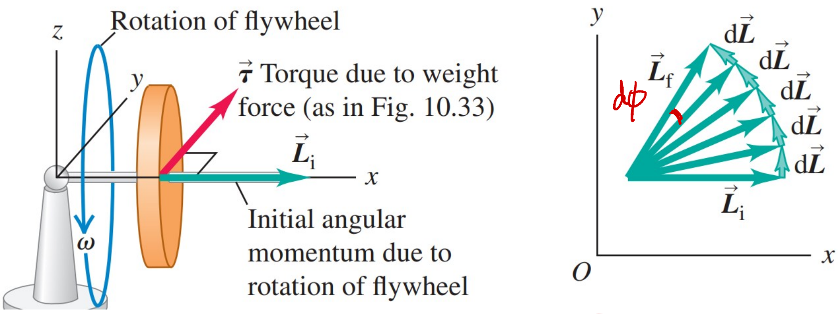
Always remember: $d\mathbf{L} = \boldsymbol{\tau}^{\text{ext}}dt$.

(a) Rotating flywheel

When the flywheel is rotating, the system starts with an angular momentum $\vec{L}_i$ parallel to the flywheel's axis of rotation.

(b) View from above

Now the effect of the torque is to cause the angular momentum to precess around the pivot. The gyroscope circles around its pivot without falling.



$$\Omega = \frac{d\phi}{dt} = \frac{\frac{|d\mathbf{L}|}{|\mathbf{L}|}}{dt} = \boxed{\frac{\tau^{\text{ext}}}{L} = \frac{mgr}{I\omega}}$$

rpm $= r/min$

> The Hubble Space Telescope is stabilized to within an angle of about 2-millionths of a degree by means of a series of gyroscopes that spin at 19,200 rpm. Although the structure of these gyroscopes is actually quite complex, we can model each of the gyroscopes as a thin-walled cylinder of mass $2.0\ \mathrm{kg}$ and diameter $5.0\ \mathrm{cm}$, spinning about its central axis. How large a torque would it take to cause these gyroscopes to precess through an angle of 1.0×10^{-6} degree during a 5.0-hour exposure of a galaxy?

$\omega = \frac{19200}{60} \cdot 2\pi = 640\pi$ rad/s

$\Omega = \frac{1.0\times10^{-6} \cdot \frac{\pi}{180}}{5\times3600} = 9.696\times10^{-13}$ rad/s

$\tau = L\cdot\Omega$
$= I\cdot\omega\cdot\Omega$
$= m\cdot(\frac{1}{2}d)^2\cdot\omega\cdot\Omega$
$= 2.44\times10^{-12}\ kg\cdot m^2/s^2$

Equilibrium

- Equation for torque equilibrium (rotational motion)
- Equation for force equilibrium (translational motion)

Common Procedure

- Sketch the situation; identify the object in equilibrium.
- Draw a free-body diagram with forces attached to the points they act onto.
- Choose an appropriately placed coordinate system (can save calculations by eliminating torques of certain forces).
- Write down equilibrium conditions for forces and torques.
- Solve for unknowns.

Reference

1. Wu Yufan, 2022SU VP150 RC.
2. Qu Zhemin, 2021SU VP150 RC.
3. Mateusz Krzyzosiak, 2023SU VP150 Slides.