VP150-RC8

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Elasticity

Elastic Modulus

$$elastic\ modulus = \frac{stress}{strain}$$

where stress is **force per unit area** and strain is the **fractional deformation due to the stress**.

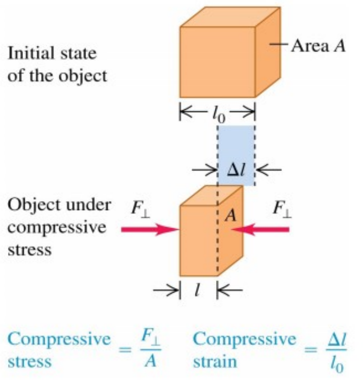
- Young's Modulus (tensile stress) Y
- Bulk Modulus (compressive stress) B
- Shear Modulus (shear stress) S

Shear Modulus is much smaller.

Young's Modulus

Young's modulus is tensile stress divided by tensile strain, and is given by

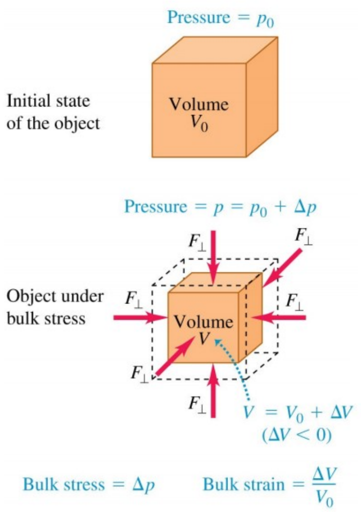
$$Y = \frac{\frac{F_\perp}{A}}{\frac{\Delta L}{L}}$$



Bulk Modulus

Bulk modulus is bulk stress divided by bulk strain and is given by

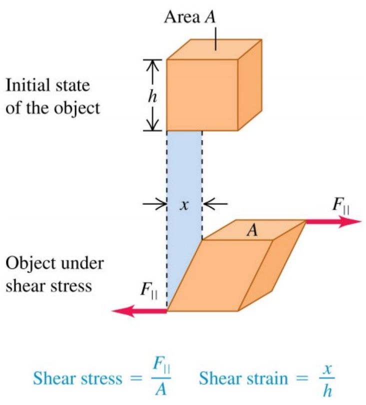
$$B = -\frac{\Delta p}{\frac{\Delta V}{V_0}}$$



Shear Modulus

Shear modulus is shear stress divided by shear strain, and is given by

$$S = \frac{\frac{F_{\parallel}}{A}}{\frac{x}{h}}$$



Fluid

Basic Concepts

Density of Mass

$$\rho = \lim_{\Delta V \to 0} \frac{\Delta m}{\Delta V}$$

Relative Density

$$r = \frac{\rho}{1000[kg/m^3]}$$

Pressure

$$p = \frac{\Delta F_{\perp}}{\Delta A}$$

Pressure in Liquid

$$\mathrm{d}p = -\rho g y$$
$$\Delta p = \int_{p_1}^{p_2} \mathrm{d}p$$

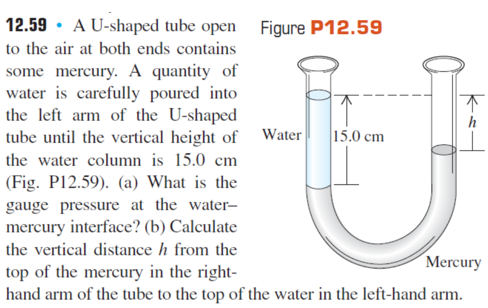
The density of mass ρ and the gravity acceleration g might be a function of depth y.

Pressure with Constant ρ and g

$$p = p_0 + \rho g h$$

atmospheric pressure: $1.01 \times 10^5 \approx 1 \times 10^5 [Pa]$

pressure in 1-meter-depth water: $9.8 \times 10^3 \approx 1 \times 10^4 [Pa]$

> 
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> **12.59** • A U-shaped tube open to the air at both ends contains some mercury. A quantity of water is carefully poured into the left arm of the U-shaped tube until the vertical height of the water column is 15.0 cm (Fig. P12.59). (a) What is the gauge pressure at the water–mercury interface? (b) Calculate the vertical distance h from the top of the mercury in the right-hand arm of the tube to the top of the water in the left-hand arm.
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Pascal's Law

$$\frac{F_2}{F_1} = \frac{A_2}{A_1}$$

*Verify it with:

- the definition of pressure,
- work-kinetic energy theorem.

Archimedes' Principle

Buoyant Force

$$F_b = \rho_{liquid} V_{immersed} g$$

The buoyant force equals the weight of the fluid displaced by the body in magnitude.

Practically, the density and the gravity acceleration can still be a function of depth y. In this case, solve the buoyant force with integration.

For objects with vertical side surfaces, the buoyant force can be derived with $F_b = (p_2 - p_1)A$.

Surface Tension

Origin

The attractive interaction between molecules of the liquid.

Meniscus and capillarity

- adhesion forces dominate $\leftrightarrow$ concave liquid surface
- cohesion forces dominate $\leftrightarrow$ convex liquid surface

Bernouli's Equation

Requirements and Preparation Equations

- The density of mass ρ is constant, which means the liquid cannot be compressed.
- $\delta W = p_1 \mathrm{d}V - p_2 \mathrm{d}V$
- $\mathrm{d}K = \frac{1}{2}\rho(v_2^2 - v_1^2)\mathrm{d}V$
- $\mathrm{d}U_{grav} = \rho g(y_2 - y_1)\mathrm{d}V$

Equations

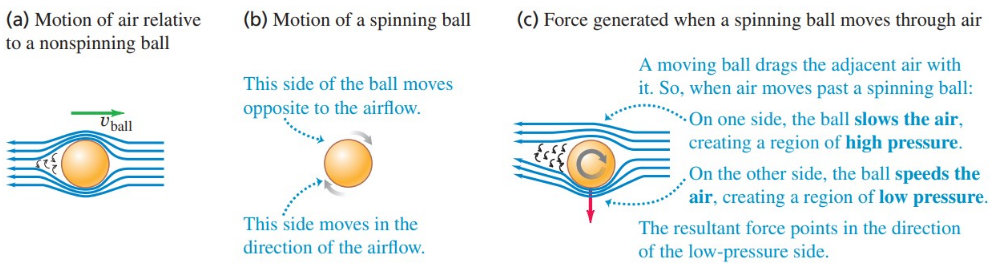
$$p + \frac{1}{2}\rho v^2 + \rho g y = const$$

Viscosity and Turbulence

Definition

- Viscosity is internal friction in a fluid.
- Turbulence is irregular chaotic flow that is no longer laminar.

Application



Reference

1. Wu Yufan, 2022SU VP150 RC.
2. Qu Zhemin, 2021SU VP150 RC.
3. Mateusz Krzyzosiak, 2023SU VP150 Slides.