VP150-RC6

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Moment of Inertia

Rigid Body

Definition

The distance between any two points of the rigid body remains constant. (No compression, stretch, etc.)

Angular Quantities

- Angular displacement θ
- Angular velocity ω
- Angular acceleration ε

Comments

- Note that θ, ω, and ε are vectors.
- When deriving the relationship between θ, ω, and ε, compare them with x, v, and a.
- When the axis of rotation is not fixed, $\vec{\varepsilon} \not\parallel \vec{\omega}$.

Moment of Inertia

Definition

$$I = \int r_{\perp}^2 \mathrm{d}m$$

Relationship with Kinetic Energy

$$K = \frac{1}{2} I \omega^2$$

Comments

- I is the moment of inertia about the fixed axis of rotation A.
- Moment of inertia depends on the distribution (arrangement) of mass.

> How to find the moment of inertia?
>
> **General calculation:**
>
> An object rotate along z-axis with the density function $\rho(x, y, z)$
>
> $$I = \iiint_V \left(x^2 + y^2\right) \cdot \rho(x, y, z) dxdydz$$
>
> **Practical calculation steps:**
>
> - Determine the axis. Find out how the body is symmetrical. Construct the equation based on the symmetry.
> - Find out the $\mathrm{d}m$. Do integration.
> - Substitute $\mathrm{d}m$ with m and other quantities.

Theories

Parallel Axis Theorem

$$I_{A'} = I_A + mb^2$$

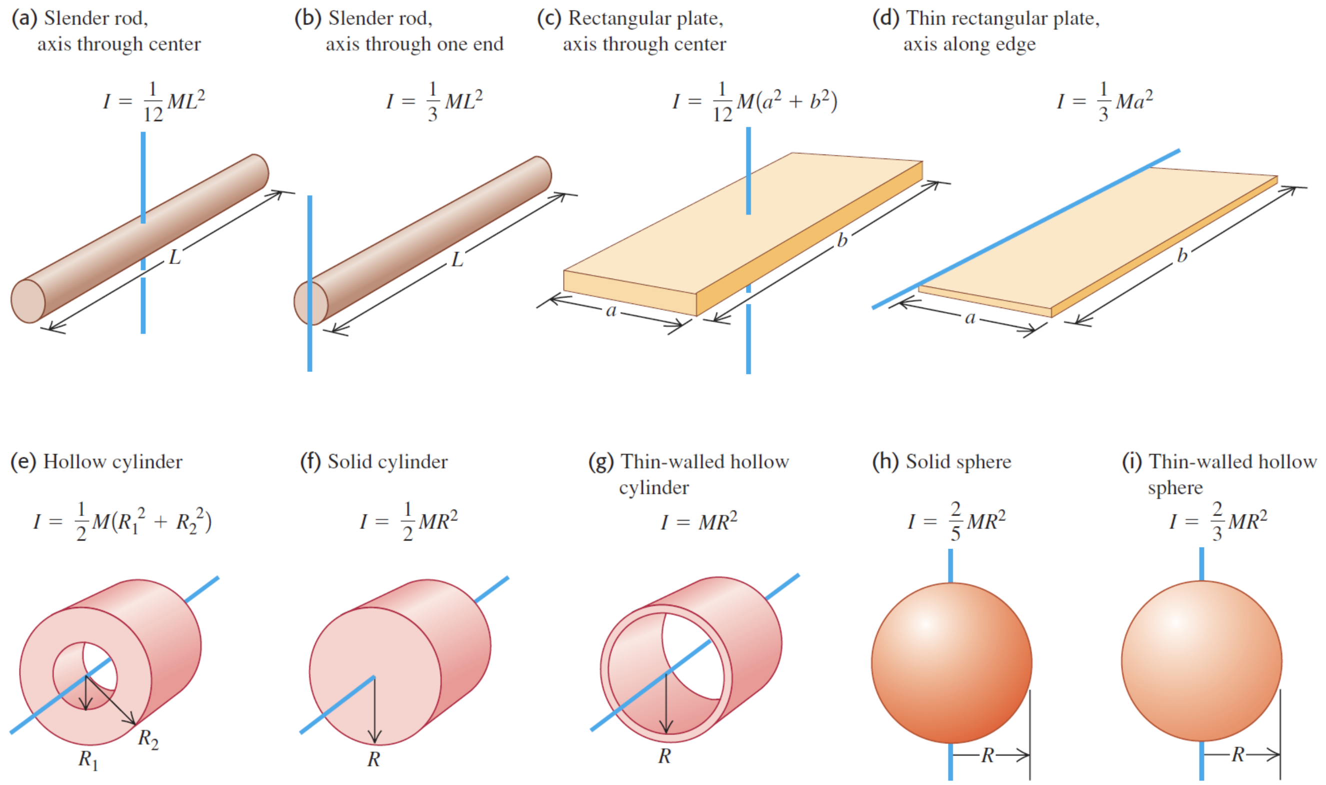
Caution: I_A is the MoI with the axis passing through the center of mass.

Perpendicular Axis Theorem

$$I_Z = I_X + I_Y$$

Caution: The rigid body is only on the plane of XoY.

MoI Table



> Try to calculate the MoI listed above (hint: you may use Parallel/Perpendicular Axis Theorem for some MoI):
>
> (a)
>
> (b)
>
> (c)
>
> (i)

Exercise

Ex.1

Two thin, uniform rods with mass m and length l are symmetrically connected to form a T-square ruler. Each part of the ruler is provided with a rotating shaft perpendicular to the ruler plane and the moment of inertia is I. Find I_{min} and I_{max}.

Ex.2

A uniform square thin plate has a mass of m and each side length of a. Take the rotating axis through center O on the plane of the plate, and find the moment of inertia of the plate with respect to the axis.

Ex.3

The semi-major axis length of an elliptical ring is a, the semi-minor axis length is b, and the mass is m (not necessarily homogeneous). The moment of inertia about the major axis is I_a, find the moment of inertia I_b about the minor axis.

Reference

1. He Yinghui, 2022SU VP150 RC.
2. Qu Zhemin, 2021SU VP150 RC.
3. Mateusz Krzyzosiak, 2023SU VP150 Slides.