

## Summary

The contest rules specify that you should include a one-page summary of your report. This page appears before the rest of the report, and will have a special header attached to it that takes up the top 2.5" of the page.

By typing your summary inside a `summary` environment,  $\text{\TeX}$  will handle the formatting of that page correctly, including leaving space at the top of the page and not numbering the page.

It will also reset the page numbers so that the first page of your report is labeled correctly.

What should you put here? Basically, you want a brief restatement of the problem followed by a largely *non-technical* description of what you've done. Try to avoid using mathematical notation.

You probably want to write a few paragraphs, around half to two-thirds of a page.

In 2016, the COMAP folks said the following:

The summary is an essential part of your MCM/ICM paper. The judges place considerable weight on the summary, and winning papers are often distinguished from other papers based on the quality of the summary.

To write a good summary, imagine that a reader will choose whether to read the body of the paper based on your summary: Your concise presentation in the summary should inspire a reader to learn about the details of your work. Thus, a summary should clearly describe your approach to the problem and, most prominently, your most important conclusions. Summaries that are mere restatements of the contest problem, or are a cut-and-paste boilerplate from the Introduction are generally considered to be weak.

Besides the summary sheet as described each paper should contain the following sections:

**Restatement and Clarification of the Problem** State in your own words what you are going to do.

**Explain Assumptions and Rationale/Justification** Emphasize the assumptions that bear on the problem. Clearly list all variables used in your model.

**Include Your Model Design and Justification** for type model used or developed.

**Describe Model Testing and Sensitivity Analysis** , including error analysis, etc.

**Discuss the Strengths and Weaknesses** of your model or approach.

[1]

Your Report Title  
MCM Contest Question C

Team # 63713

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## 1 Introduction

The authors aimed to model the influence on traffic flow of self-driving car percentage, average traffic volume, and the number of lanes. In particular, interaction between vehicles are implemented by a function of motion states, namely the relative distances and relative velocities of adjacent vehicles, as well as their accelerations. Then the model was applied to the data on the roads of interest, before the authors attempted to determine an optimal strategy towards the following objectives:

1. Guarantee traffic security.
2. Maximize the total traffic flow.
3. Avoid sharp change of vehicle velocities.

In this article, the authors decomposed the problem into the following steps when modeling and simulating:

1. Construct the model of traffic flow on a transportation network consisting of xxx xx and xxx.
2. Use the model to simulate the traffic of given roads of interest, analyze the existence of equilibria and tipping point, then consider possible policy changes of lanes according to the simulation result.
3. Take sensitive cases such as accidents, vibrations of acceleration of vehicles into consideration.

In Section 1 we give our definitions and notation. Section 2 describes our numerical experiments. . . .

We prove our main result, Theorem 6, in Section 5. . . .

## 2 Basic Assumptions

1. Self-driving cars can receive the information of xxxxxx, including velocities, positions, and accelerations; drivers can see the motion state of its adjacent cars, the largest visual range is xxx meters
2. The reaction time of self-driving cars can be ignored, the average reaction time of drivers are set to be 1 second
3. Overtaking is not considered in the basic model

4. Cellular Automata is adopted to simulate the motion of vehicles
5. Basic notations in this article are listed as follow in the label:

| Symbol       | Units   | Meaning  |
|--------------|---------|--|
| $Vol$        |         | Average traffic volume   |
| $P$          | /       | Percentage of self-driving cars  |
| $P_0$        | /       | Equilibria of percentage that xxx  |
| $a_s$        | $m/s^2$ | Acceleration of self-driving car   |
| $a_o$        | $m/s^2$ | Acceleration of ordinary car   |
| $\Delta v_1$ | $m/s$   | Velocity of the front car minus the velocity of the objective car  |
| $\Delta v_2$ | $m/s$   | Velocity of the objective car minus the velocity of the back car   |
| $\Delta v_3$ | $m/s$   | Speed limit minus the velocity of objective car  |
| $d_1$        | $m$     | Distance between the front car and the objective car   |
| $d_2$        | $m$     | Distance between the objective car and the back car  |
| $x$          | /       | A coefficient ranges from -1 to 1 representing the influence on acceleration of the motion status of adjacent cars |
| $Num$        | /       | Number of lanes  |
| $\Delta t$   | $s$     | Reaction time of driver  |

Table 1: Notations

## 2.1 Other Assumptions

1. In the initial model,
2. Under the model of cellular automation, each cellular is set to be xxxxxx. More specifically, the

## 3 Analysis of the Problem

### 3.1 Basic model for motion

Firstly, in order to enlarge the traffic capacity, the behavior of each individual vehicle should be analyzed. To do that, the authors gave initial velocities to vehicles and let the accelerations be functions of some factors. For each ordinary vehicle(i.e., not self-driving), its instantaneous acceleration is positively correlated to  $\Delta v_1$ ,  $\Delta v_3$ , and  $d_1$ , negatively correlated to  $\Delta v_2$  and  $d_2$  because the drivers tend to accelerate if the vehicles in the front is running fast or far away, and vice versa for the vehicles behind. Besides,

if the velocity is lower than the speed limit, the vehicle tends to accelerate. For simplicity, when modeling acceleration, we introduced a variable  $x$  ranging from -1 to 1 which depends linearly on  $\Delta v_1$ ,  $d_1$ ,  $\Delta v_2$  and  $d_2$ . By assigning reasonable values to these four independent values and fitting coefficients, we got the following formula:

$$x = 0.0159(\text{s/m})\Delta v_1 + 0.0010(\text{m}^{-1})d_1 - 0.0054(\text{s/m})\Delta v_2 - 0.0004(\text{m}^{-1})d_2 + 0.0224(\text{s/m})\Delta v_3 - 0.1000$$

Obviously,  $x$  is positively correlated to acceleration. However, a linear equation is not accurate enough to describe the relation. To make the motion as even as possible, we adopted an "S curve" and by fitting data, we got the following formula:

$$a_o = 3\left(\frac{2}{1+e^{-4x}} - 1\right)$$

The figure of this function is as follow:

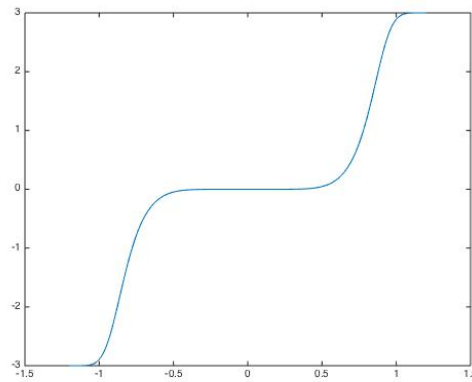


Figure 1: Acceleration

As for self-driving vehicles, since they can detect others' acceleration, if the acceleration of the vehicle ahead changes suddenly, self-driving cars will simultaneously change its acceleration so as to adapt.

### **3.2 Merging and Divergence**

In reality, the number of lanes in a road is not constant, so sometimes vehicles need to merge into fewer lanes and sometimes diverge to more lanes. We assumed that

## **4 Calculating and Simplifying the Model**

## **5 Application of the Model**

## **6 Revise and Improve the Model**

## **7 The Model Results**

### **7.1**

## **8 Conclusions**

## **9 Evaluation of the Model**

### **9.1 Strengths**

1. In our model, the

### **9.2 Weakness**

1. We set the reaction time to be constant, but actually the reaction time of each driver at different time may vary, which may lead to error.
2. We used a formula to determine the accelerations of ordinary cars, but in reality, drivers cannot react so accurately, so an uncertain error may occur.
3. In our model, the accelerations of self-driving vehicles change instantaneously. This may harm smoothness of the system.
4. Overtaking is not considered in our model. If overtaking is allowed, the accelerations of vehicles will be influenced also by motion of vehicles in adjacent lanes.

5. In the formula of acceleration, we assumed that accelerations range from  $-3(\text{m/s}^2)$  to  $3(\text{m/s}^2)$ , where sharp changes of acceleration are not considered, so this model can't be applied to extreme cases such as collision.

## References

- [1] COMAP. Contest registration and instructions. Website, 2016. URL <http://www.comap.com/undergraduate/contests/mcm/instructions.php>. Viewed on 2016 January 22.