Chloy_2447116_P2

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[ ]:
```python
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns

sns.set_theme(style="whitegrid")
```

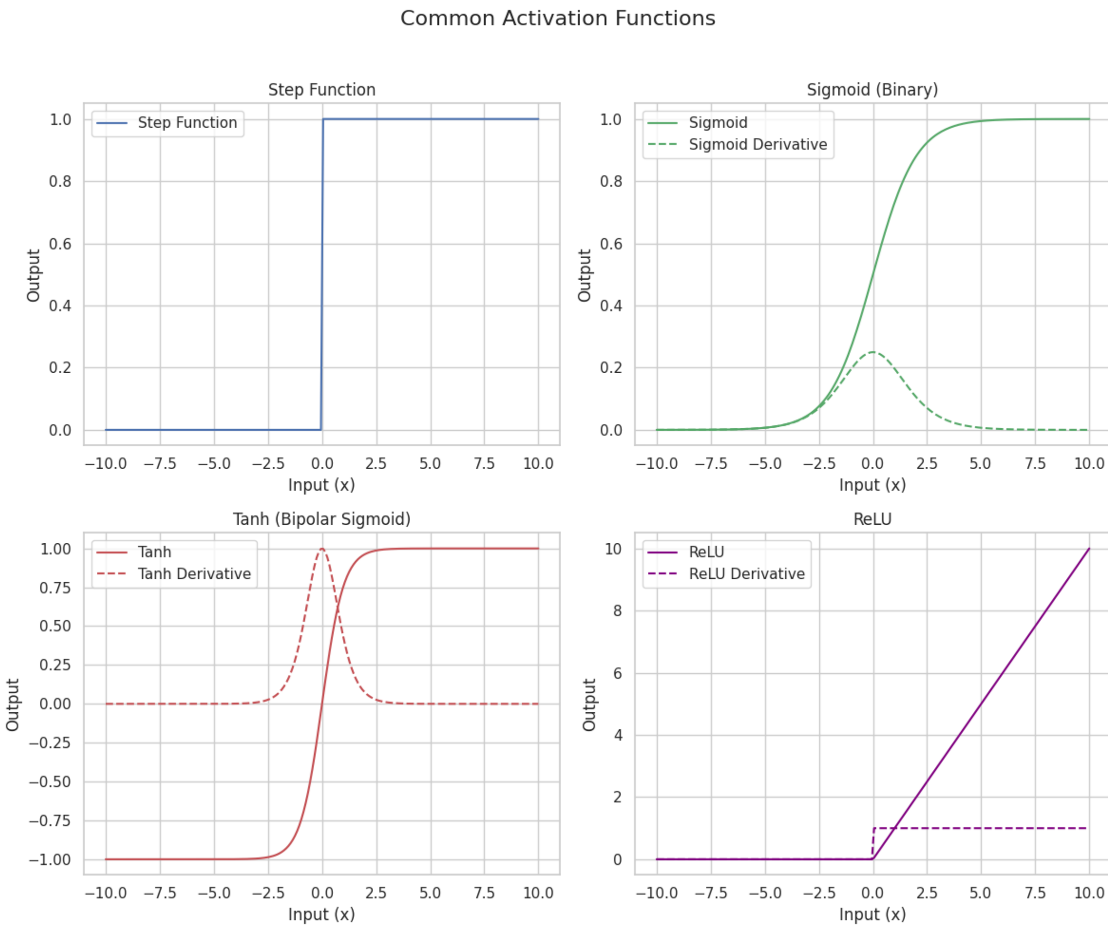
1 Activation Function Definitions

[ ]:
```python
def step_function(x):
    """Returns 1 if x >= 0, else 0."""
    return np.where(x >= 0, 1, 0)

def sigmoid(x):
    """Binary Sigmoid function: maps input to a range between 0 and 1."""
    return 1 / (1 + np.exp(-x))

def sigmoid_derivative(x):
    """Derivative of the sigmoid function."""
    return sigmoid(x) * (1 - sigmoid(x))

def tanh(x):
    """
    Hyperbolic Tangent (Tanh) function: maps input to a range between -1 and 1.
    This is also known as a 'Bipolar Sigmoid' function.
    """
    return np.tanh(x)

def tanh_derivative(x):
    """Derivative of the Tanh function."""
    return 1 - np.tanh(x)**2

def relu(x):
    """Rectified Linear Unit (ReLU) function: returns max(0, x)."""
    return np.maximum(0, x)

def relu_derivative(x):
    """Derivative of the ReLU function."""
```

```python
    return np.where(x > 0, 1, 0)
```

2 Visualization

[ ]:
```python
# Generate a range of input values
x = np.linspace(-10, 10, 200)

# Create a 2x2 grid of subplots
fig, axs = plt.subplots(2, 2, figsize=(12, 10))
fig.suptitle('Common Activation Functions', fontsize=16)

# Step Function
axs[0, 0].plot(x, step_function(x), label='Step Function', color='b')
axs[0, 0].set_title('Step Function')
axs[0, 0].legend()

# Sigmoid Function
axs[0, 1].plot(x, sigmoid(x), label='Sigmoid', color='g')
axs[0, 1].plot(x, sigmoid_derivative(x), label='Sigmoid Derivative', color='g',␣
↪linestyle='--')
axs[0, 1].set_title('Sigmoid (Binary)')
axs[0, 1].legend()

# Tanh Function
axs[1, 0].plot(x, tanh(x), label='Tanh', color='r')
axs[1, 0].plot(x, tanh_derivative(x), label='Tanh Derivative', color='r',␣
↪linestyle='--')
axs[1, 0].set_title('Tanh (Bipolar Sigmoid)')
axs[1, 0].legend()

# ReLU Function
axs[1, 1].plot(x, relu(x), label='ReLU', color='purple')
axs[1, 1].plot(x, relu_derivative(x), label='ReLU Derivative', color='purple',␣
↪linestyle='--')
axs[1, 1].set_title('ReLU')
axs[1, 1].legend()

# Add labels and grid to all subplots
for ax in axs.flat:
    ax.set_xlabel('Input (x)')
    ax.set_ylabel('Output')
    ax.grid(True)

plt.tight_layout(rect=[0, 0, 1, 0.96])
plt.show()
```



3 Neural Network Class

[ ]:
```
class NeuralNetwork:
    def __init__(self, input_nodes, hidden_nodes, output_nodes, learning_rate=0.
↪1):
        # Network architecture
        self.input_nodes = input_nodes
        self.hidden_nodes = hidden_nodes
        self.output_nodes = output_nodes
        self.learning_rate = learning_rate

        # Initialize weights and biases randomly
        self.weights_ih = np.random.uniform(-1, 1, (self.hidden_nodes, self.
↪input_nodes))
        self.weights_ho = np.random.uniform(-1, 1, (self.output_nodes, self.
↪hidden_nodes))
        self.bias_h = np.random.uniform(-1, 1, (self.hidden_nodes, 1))
```

```python
        self.bias_o = np.random.uniform(-1, 1, (self.output_nodes, 1))

        self.activation = 'sigmoid' # Default activation

    def _apply_activation(self, x, derivative=False):
        if self.activation == 'sigmoid':
            if derivative: return sigmoid_derivative(x)
            return sigmoid(x)
        elif self.activation == 'tanh':
            if derivative: return tanh_derivative(x)
            return tanh(x)
        elif self.activation == 'relu':
            if derivative: return relu_derivative(x)
            return relu(x)

    def feedforward(self, inputs):
        # --- FIX 1 ---
        # This method now assumes 'inputs' is already a correctly shaped column vector.
        # The line that reshaped the input has been removed.

        # Calculate signals into hidden layer
        hidden_inputs = np.dot(self.weights_ih, inputs) + self.bias_h
        # Calculate the signals emerging from hidden layer
        hidden_outputs = self._apply_activation(hidden_inputs)

        # Calculate signals into final output layer
        final_inputs = np.dot(self.weights_ho, hidden_outputs) + self.bias_o
        # Calculate the signals emerging from final output layer
        final_outputs = sigmoid(final_inputs) # Sigmoid for binary classification output

        return hidden_outputs, final_outputs

    def train(self, inputs_list, targets_list, activation='sigmoid'):
        self.activation = activation

        # --- FIX 2 ---
        # Convert inputs and targets to correctly shaped column vectors at the start.
        inputs = np.array(inputs_list, ndmin=2).T
        targets = np.array(targets_list, ndmin=2).T

        # --- Feedforward ---
        # The correctly shaped 'inputs' is passed to feedforward.
        hidden_outputs, final_outputs = self.feedforward(inputs)
```

```
        # --- Backpropagation ---
        output_errors = targets - final_outputs
        gradients_o = output_errors * sigmoid_derivative(final_outputs)
        weights_ho_delta = self.learning_rate * np.dot(gradients_o,␣
↪hidden_outputs.T)

        self.weights_ho += weights_ho_delta
        self.bias_o += self.learning_rate * gradients_o

        hidden_errors = np.dot(self.weights_ho.T, output_errors)
        gradients_h = hidden_errors * self._apply_activation(hidden_outputs,␣
↪derivative=True)

        # The 'inputs' variable is now correctly shaped (2,1), so its transpose␣
↪is (1,2).
        # np.dot((4,1), (1,2)) -> (4,2), which is the correct shape for the␣
↪weight update.
        weights_ih_delta = self.learning_rate * np.dot(gradients_h, inputs.T)

        self.weights_ih += weights_ih_delta
        self.bias_h += self.learning_rate * gradients_h

    def predict(self, inputs_list):
        # --- FIX 3 ---
        # The predict method must also shape its input before calling␣
↪feedforward.
        inputs = np.array(inputs_list, ndmin=2).T
        _, final_outputs = self.feedforward(inputs)
        return final_outputs
```

4 XOR Data and Training Loop

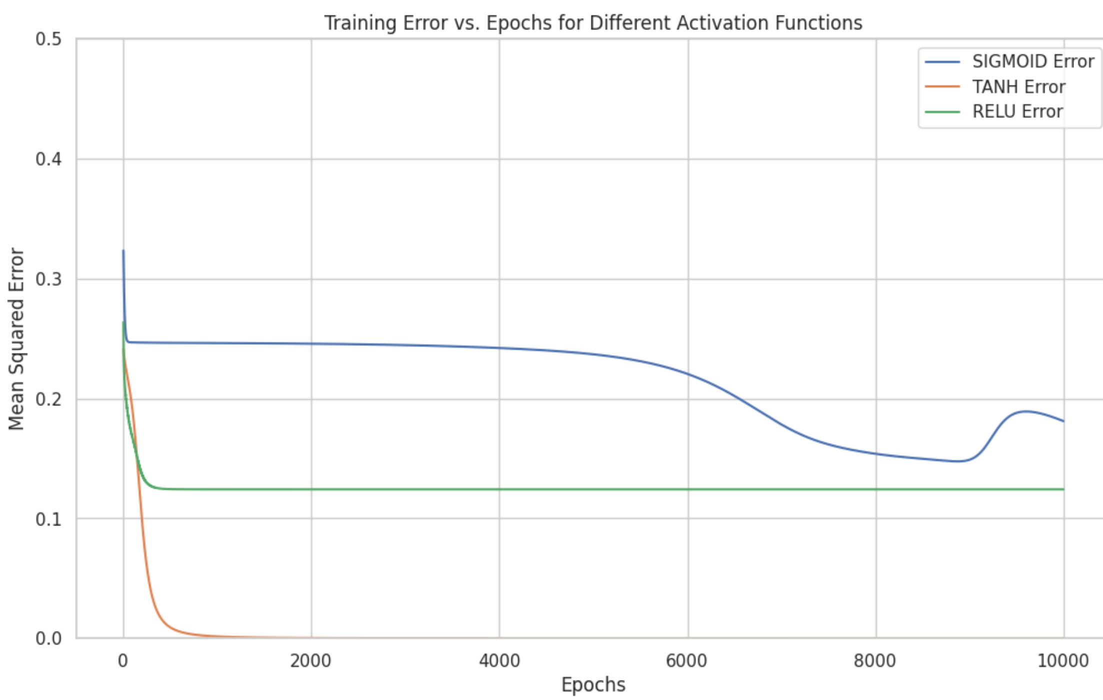
[ ]:
```
# XOR Dataset
X_xor = np.array([[0, 0], [0, 1], [1, 0], [1, 1]])
y_xor = np.array([[0], [1], [1], [0]])

# Training parameters
epochs = 10000
activations_to_test = ['sigmoid', 'tanh', 'relu']
all_errors = {}

for activation_func in activations_to_test:
    print(f"--- Training with {activation_func.upper()} activation ---")

    # Initialize the network
    # 2 input nodes, 4 hidden nodes, 1 output node
```

```python
    nn = NeuralNetwork(input_nodes=2, hidden_nodes=4, output_nodes=1,␣
↪learning_rate=0.1)

    errors = []
    for e in range(epochs):
        epoch_error = 0
        for i in range(len(X_xor)):
            nn.train(X_xor[i], y_xor[i], activation=activation_func)
            # Calculate error for plotting
            prediction = nn.predict(X_xor[i])
            epoch_error += np.mean((y_xor[i] - prediction) ** 2) # Mean Squared␣
↪Error

        errors.append(epoch_error / len(X_xor))

    all_errors[activation_func] = errors

    # Test the network
    print("Test Results:")
    for i in range(len(X_xor)):
        prediction = nn.predict(X_xor[i])
        print(f"Input: {X_xor[i]}, Target: {y_xor[i][0]}, Predicted:␣
↪{prediction[0][0]:.4f} -> Rounded: {round(prediction[0][0])}")
    print("-" * 40)
```

```
--- Training with SIGMOID activation ---
Test Results:
Input: [0 0], Target: 0, Predicted: 0.4597 -> Rounded: 0
Input: [0 1], Target: 1, Predicted: 0.6077 -> Rounded: 1
Input: [1 0], Target: 1, Predicted: 0.4544 -> Rounded: 0
Input: [1 1], Target: 0, Predicted: 0.3459 -> Rounded: 0
----------------------------------------
--- Training with TANH activation ---
Test Results:
Input: [0 0], Target: 0, Predicted: 0.0019 -> Rounded: 0
Input: [0 1], Target: 1, Predicted: 0.9963 -> Rounded: 1
Input: [1 0], Target: 1, Predicted: 0.9963 -> Rounded: 1
Input: [1 1], Target: 0, Predicted: 0.0035 -> Rounded: 0
----------------------------------------
--- Training with RELU activation ---
Test Results:
Input: [0 0], Target: 0, Predicted: 0.5015 -> Rounded: 1
Input: [0 1], Target: 1, Predicted: 0.9998 -> Rounded: 1
Input: [1 0], Target: 1, Predicted: 0.5015 -> Rounded: 1
Input: [1 1], Target: 0, Predicted: 0.0003 -> Rounded: 0
----------------------------------------
```

5 Plotting Performance Comparison

[ ]:
```
plt.figure(figsize=(12, 7))

for activation_func, errors in all_errors.items():
    plt.plot(range(epochs), errors, label=f'{activation_func.upper()} Error')

plt.title('Training Error vs. Epochs for Different Activation Functions')
plt.xlabel('Epochs')
plt.ylabel('Mean Squared Error')
plt.legend()
plt.grid(True)
plt.ylim(0, 0.5) # Limit y-axis for better visibility
plt.show()
```



5.0.1 Analysis and Conclusion

Based on the training process and the outputs generated, we can compare the performance of the three activation functions on the XOR problem.

- – Sigmoid: This activation function failed to correctly solve the XOR problem. While it made three correct predictions, it incorrectly predicted the output for the input [1, 0]. The training error plot shows that the Sigmoid function's error converged very slowly and unstably, remaining at a high Mean Squared Error for most of the epochs before settling at a suboptimal value.

- - Tanh (Bipolar Sigmoid): The Tanh function was the only one to successfully solve the XOR problem, with all four input combinations being predicted correctly. The training error plot corroborates this, showing that the Tanh function's error dropped rapidly to nearly zero within the first 1000 epochs and remained there, indicating fast and stable convergence.
- - ReLU (Rectified Linear Unit): The ReLU function also failed to correctly solve the XOR problem. It incorrectly predicted the output for the input [0, 0]. According to the plot, while ReLU's error dropped quickly, it plateaued at a Mean Squared Error of approximately 0.125 and did not improve further. This indicates that although it learned quickly, it converged to an incorrect solution.
- Conclusion: For this specific binary classification task, the Tanh activation function demonstrated a clear superiority over the others. It was the only function to achieve 100% accuracy on the test data and showed the fastest and most stable convergence to a near-zero error during training. In contrast, both Sigmoid and ReLU failed to correctly model the XOR logic under these specific training conditions.