IIIT Vadodara
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MA202 Numerical Techniques
LAB#6 Solution of linear equations[1]

1. **Method 1: Gauss Elimination** Given the system of linear equations, we solve a system of equations $A_{n\times n}x_{n\times 1} = b_{n\times 1}$

Step 1 Construct augmented matrix using A and b: $A^{aug} = [A \mid b]$
Step 2 For A(i,i) as the pivot element
Step 3 Do row transformation $R_j = R_j - \alpha_{i,j}R_i$ where $\alpha_{i,j} = A(j,i)/A(i,i)$ to create zeros in pivot column.
Step 4 Get solutions using **Back-substitution** Example solutions shown for 3×3 matrix is shown below and same can be extended to $n \times n$ matrix

$$x_3 = \frac{b(3)}{A(3,3)} \tag{1}$$

$$x_2 = \frac{b(2) - A(2,3)x_3}{A(2,2)} \tag{2}$$

$$x_1 = \frac{b(1) - A(1,3)x_3 - (A(1,2)x_2}{A(1,1)} \tag{3}$$

Method 2: LU decomposition L U decomposition of a matrix is the factorization of a given square matrix into two triangular matrices, one upper triangular matrix and one lower triangular matrix, such that the product of these two matrices gives the original matrix. A square matrix A can be decomposed into two square matrices L and U such that $A = LU$ where U is an upper triangular matrix formed as a result of applying Gauss Elimination Method on A; and L is a lower triangular matrix with diagonal elements being equal to 1.

$$\text{L} = \begin{bmatrix} 1 & 0 & ... & 0 \\ \alpha_{2,1} & 1 & ... & 0 \\ \alpha_{3,1} & \alpha_{3,2} & ... & 1 \end{bmatrix}$$

Idea of partial pivoting is to use row exchange to ensure the pivot element $A(i,i)$ to be the largest element in the column. It turns out that a proper permutation in rows (or columns) is sufficient for LU factorization. LU factorization with partial pivoting (LUP) refers often to LU factorization with row permutations only.

Q. 1: Solve the following linear system

$$\text{A} = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 3 \\ 3 & 4 & -2 \end{bmatrix} \quad \text{b} = \begin{bmatrix} 4 \\ 7 \\ 9 \end{bmatrix}$$

a. Using Gauss elimination
b. Using LU decomposition
c. Using Gauss elimination+partial pivoting

Iterative methods for solving linear equations

Jacobi method The first iterative technique is called the Jacobi method, after Carl Gustav Jacob Jacobi(1804–1851). This method makes two assumptions: (1) that the system given by

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$a_{11}x1 + a_{12}x2 + ... + a_{1n}x_n = b_1$
$a_{21}x1 + a_{22}x2 + ... + a_{2n}x_n = b_2$
.
.
.
$a_{n1}x1 + a_{n2}x2 + ... + a_{nn}x_n = b_n$
has a unique solution and (2) that the coefficient matrix A has no zeros on its main diagonal. If any of the diagonal entries are zero, then rows or columns must be interchanged to obtain a coefficient matrix that has nonzero entries on the main diagonal, and diagonally dominant. To begin the Jacobi method,
Step 1: Solve the first equation for x_1, the second equation for x_2 and so on.

$$x^{(i+1)} = \frac{b_k - (\sum_{j \neq k} A_{k,j} x_j^{(i)})}{A_{k,k}} \tag{4}$$

2: Then make an initial approximation of the solution, $(x_1, x_2, ..., x_n)$ and substitute these values into the right-hand side of the rewritten equations to obtain the first approximation. After this procedure has been completed, one iteration has been performed.
Step 3: In the same way, the second approximation is formed by substituting the first approximation's x-values into the right-hand side of the rewritten equations.
Step 4: By repeated iterations, you will form a sequence of approximations that often converges to the actual solution.

Gauss-Siedel method You will now look at a modification of the Jacobi method called the Gauss-Seidel method,named after Carl Friedrich Gauss (1777–1855) and Philipp L. Seidel (1821–1896). This modification is no more difficult to use than the Jacobi method, and it often requires feweriterations to produce the same degree of accuracy. With the Jacobi method, the values of x_i obtained in the n^{th} approximation remain unchanged until the entire $(n+1)^{th}$ approximation has been calculated. With the Gauss-Seidel method, on the other hand, you use the new values x_i of each as soon as they are known. That is, once you have determined $x_1$from the first equation, its value is then used in the second equation to obtain the new x_2 Similarly, the new x_1 and x_2 are used in the third equation to obtain the new and so on. In general

$$x^{(i+1)} = \frac{b_k - (\sum_{j=1}^{k-1} A_{k,j} x_j^{(i+1)} + \sum_{k+1}^{n} A_{k,j} x_j^{(i)}}{A_{k,k}} \tag{5}$$

Q. 2: Solve the following linear system

A = $\begin{bmatrix} 1 & 2 & 2 & 1 \\ 2 & 2 & 4 & 2 \\ 1 & 3 & 2 & 5 \\ 2 & 6 & 5 & 8 \end{bmatrix}$ b = $\begin{bmatrix} 1 \\ 0 \\ 2 \\ 4 \end{bmatrix}$

a. Using Jacobi method of iteration

b. Using Gauss-Siedel method

Advantages of iterative methods 1.Faster iteration process. (than other methods),
2. Simple and easy to implement,

3. Low on memory requirements.

Disadvantages are
1. Slower rate of convergence,

2. Requires a large number of iterations to reach the convergence point.