Chapter 0
Bayes' Theorem & Statistics

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Bayesian Statistics

What is Bayesian Statistics?

- What is Bayes' Theorem?
- What is Statistics?

What is Bayes' Theorem?

Example: We want to calculate the Positive and Negative Predictive Value of a diagnostic test.

Notation:

S: Sick + positive
H: Healthy - negative

Given the accuracy of a test and the prevalence of the disease:

Sensitivity: P(+|S) = 95% => 5% false negatives
Specificity: P(-|H) = 99% => 1% false positives

Prevalence: P(S) = 1% => P(H) = 1-P(S) = 99%

We want to calculate:

Positive Predictive Value: P(S|+)
Negative Predictive Value: P(H|-)

What is Bayes' Theorem?

Example: We want to calculate the Positive and Negative Predictive Value of a diagnostic test.

To calculate the **Positive and Negative Predictive Value** we must use **Bayes' Theorem:**

$$P(S|+) = \frac{P(+|S)P(S)}{P(+|S)P(S) + p(+|H)P(H)}$$

$$P(H|-) = \frac{P(-|H)P(H)}{P(-|H)P(H) + P(-|S)P(S)}$$

What is Bayes' Theorem?

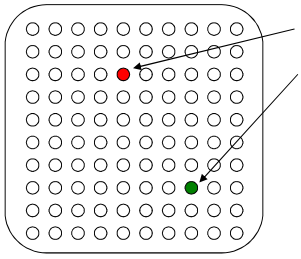
Example: We want to calculate the Positive and Negative Predictive Value of a diagnostic test.

Let us deduce the **Positive Predictive Value P(S|+)** :

Prevalence: P(S) = 1% => Out of every 100 people there is a positive

Sensitivity: P(+|S) = 95% => It is almost certain that we will detect the positive

Specificity: P(-|H) = 99% => Out of every 100 people there is a false positive



Hence, the probability of being really sick if the test is positive will be approximately **50%**. This value is the **Positive Predictive Value, P(S|+)**.

What is Bayes' Theorem?

Example: We want to calculate the Positive and Negative Predictive Value of a diagnostic test.

Given the **accuracy** of a test and the **prevalence** of the disease:

Sensitivity: P(+|S) = 95% => 5% false negatives
Specificity: P(-|H) = 99% => 1% false positives

Prevalence: P(S) = 1% => P(H) = 1-P(S) = 99%

Then:

Positive Predictive value: P(S|+) ≈ 49,0 %
Negative Predictive value: P(H|-) ≈ 99,9 %

Bayes' Theorem

What is Bayes' Theorem?

Example: We want to calculate the Positive and Negative Predictive Value of a diagnostic test.

Sensitivity: P(+|S) = 95%
Specificity: P(-|H) = 99%
Prevalence: P(S) = 1%

The probability of being Sick:

Without Test P(S) = 1% → Bayes' Theorem → With Positive Test P(S|+) = 49%

The probability of being Healthy:

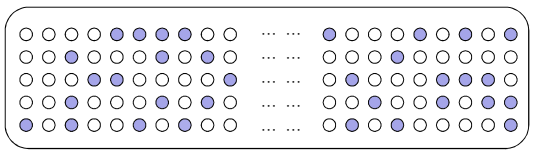
Without Test P(H) = 99% → Bayes' Theorem → With Negative Test P(H|-) = 99,9%

Bayes' theorem allows us to update probabilities after getting new information

What is Statistics?

Example: 20% of the population has a specific feature (e.g. blue eyes)

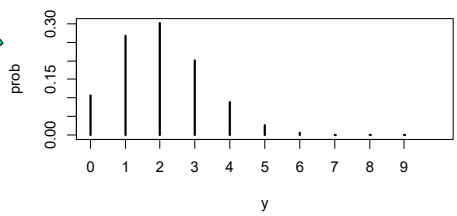
Poputaltion



$p = 20\%$

Probability

If we know the **% population**, we can calculate, for example, the probaility of observing y individuals with the characteristic in a sample of size $n=10$, where $y \in \{0,1,2,\ldots,10\}$.



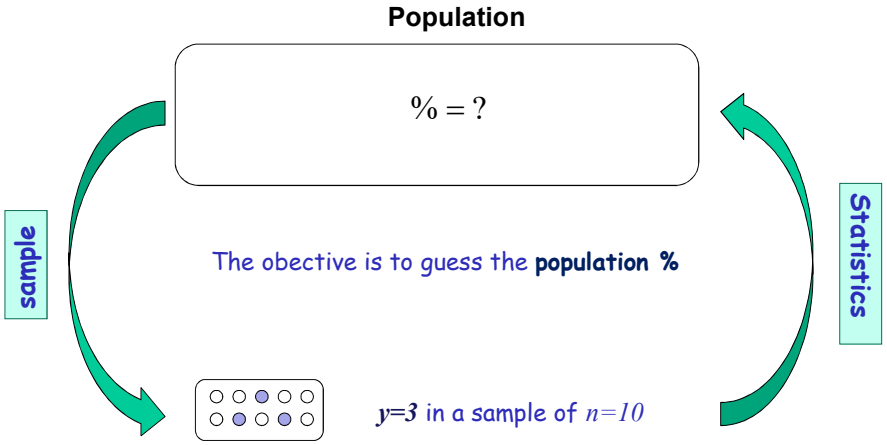
The $y|\theta$ distribution:

$$p(y \mid \theta = 0.2) = \binom{10}{y} 0.2^{y} (1-0.2)^{10-y}$$

Binomial Model

What is Statistics?

Example: We want to know what percentage of the population has a certain feature



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