

General Mathematics Reviewer

Topic: Rational Functions and Rates

Discussion

- A rational function is a function in the form $f(x) = P(x)/Q(x)$, where P and Q are polynomials and $Q(x) \neq 0$.
- Zeros of a rational function: values of x that make the numerator zero.
- Vertical asymptotes: values of x that make the denominator zero.
- Horizontal asymptotes: compare the degree of numerator and denominator.
 - If degree numerator < degree denominator $\rightarrow y=0$.
 - If degrees are equal \rightarrow ratio of leading coefficients.
- Slant (oblique) asymptote: occurs when numerator's degree is exactly one more than denominator's degree.
- Work/Rate problems:
 - Work = Rate \times Time
 - Combined work rate: $1/T = 1/T1 + 1/T2$

Parallel Questions

1. Find the vertical asymptote of $f(x)=\frac{x}{x^2-25}$.

Solution:

2. Find the x-intercept of $f(x)=\frac{x-2}{x+5}$.

Solution:

3. A car travels 300 km at speed v. Express time as a rational expression.

Solution:

4. A worker completes a task in 6 hours, assistant in 12 hours. Together, how long will it take?

Solution:

5. Pipe A fills tank in 5 hours, Pipe B in 10 hours. Together, how long?

Solution:

6. Worker A: 8 days, Worker B: 12 days. Together, how long?

Solution:

7. Two painters finish a wall in 4 hours together. Helper alone: 8 hours. How long for main painter alone?

Solution:

8. Drain A empties pool in 4 hours, Drain B in 12 hours. Together, how long?

Solution:

9. What causes a vertical asymptote in rational functions?

10. . Find the horizontal asymptote of $f(x)=\frac{4x^2+3}{2x^2+5}$.

Solution:

11. Find the horizontal asymptote of $f(x)=9x-7$.

Solution:

12. When does a rational function have a slant asymptote?

13. When does a rational function have a horizontal asymptote?

Answers with Explanations

1. Vertical asymptote at $x=\pm 5$ (denominator zero).
2. x-intercept: set numerator=0 $\rightarrow x=2$.
3. Time = $300/v$ (distance/speed).
4. Combined rate= $1/6+1/12=1/4 \rightarrow 4$ hours.
5. Combined rate= $1/5+1/10=3/10 \rightarrow 10/3=3.33$ hours.
6. Combined rate= $1/8+1/12=5/24 \rightarrow 24/5=4.8$ days.
7. Combined rate= $1/4$, helper= $1/8 \rightarrow$ main= $1/4-1/8=1/8 \rightarrow 8$ hours.
8. Combined rate= $1/4+1/12=1/3 \rightarrow 3$ hours.
9. Vertical asymptotes occur when denominator=0.
10. Degrees equal, ratio= $4/2=2 \rightarrow y=2$.
11. Linear function, no horizontal asymptote.
12. When numerator's degree is one more than denominator's degree.
13. When numerator's degree \leq denominator's degree.