Analytical calculations:

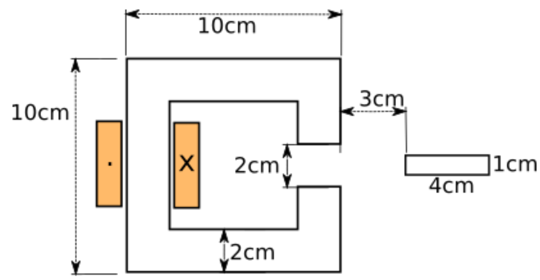


Figure 1: Magnetic Circuit

The purpose of this analysis is to calculate total magnetic energy stored in the system, horizontal force acting on the plunge, and to calculate inductance with respect to position of the plunge.
Assumptions:
-Iron plunge and core are infinitely permeable.
-Leakage flux and fringing factors are neglected.
-The calculations are carried out in per $1m$ Depth. Calculate N and i such that the average flux density in the air gap is equal to $0.5(T)$:

Calculating the airgap flux density (assuming infinitely permeable core):

$$\oint \mathbf{H}.\overline{dl} = Ni \Rightarrow H_g.l_g = Ni \Rightarrow H_g = \frac{Ni}{l_g} \tag{1}$$

$$B_g = \mu_0 H_g \Rightarrow B_g = \mu_0 \frac{Ni}{l_g} \tag{2}$$

$$\begin{cases} B_g = 0.5 \\ \\ l_g = 2cm \end{cases} \Rightarrow i = 120(A) \quad \& \quad N = 66 \tag{3}$$

Case A (0.02<x & x<-0.04):

$$R = \frac{l}{\mu_0 A_{gap}} \tag{4}$$

$$L = \frac{N^2}{R} \tag{5}$$

$$W_{fld} = \frac{L(x)i^2}{2} \tag{6}$$

$$f_{fld} = \frac{i^2}{2}\frac{dL(x)}{dx} \tag{7}$$

Reluctance:

$$R = \frac{l}{\mu_0 A_{gap}} = \frac{2 \times 10^{-2}}{\mu_0 \times 2 \times 10^{-2}} = 795774$$

Inductance:

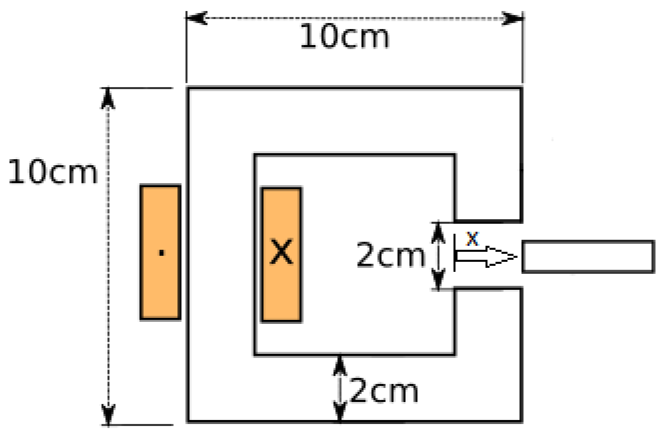
$$L = \frac{N^2}{R} = \frac{66^2}{795774} = 5.47 \quad (mH)$$

Energy:

$$W_{fld} = \frac{L(x)i^2}{2} = \frac{1}{2} \times 5.47 \times 10^{-3} \times 120^2 = 38.38 \quad (J)$$

Force:
Since I have neglected the fringing effect to magnetic field is present out side of the air gap. And as the energy is a constant number the force on the bar outside of the gap is Zero.

Case B(0<x<0.02):
x is shown on the graph.



Reluctance:

$$R(x) = \frac{l}{\mu_0 A_{gap}} = \frac{0.01}{\mu_0 \times (0.02 - x)} = \frac{7957}{0.02 - x} \quad 0 < x < 0.02$$

Inductance:

$$L(x) = \frac{N^2}{R(x)} = 0.547 \times (0.02 - x) \quad 0 < x < 0.02$$

Energy:

$$W_{fld} = \frac{L(x)i^2}{2} = \frac{1}{2} \times 0.547 \times (0.02 - x) \times 120^2 = 3938(0.02 - x) \quad 0 < x < 0.02$$

Force:

$$F = 3938 \quad (N) \quad 0 < x < 0.02$$

Case C(-0.02<x<0):
Reluctance:

$$R(x) = 397887$$

Inductance:

$$L(x) = 10 \quad (mH)$$

Energy:

$$W_{fld} = 28.8 \quad (J)$$

Force:

$$F = 0$$