Multiclass Logistic Regression for Bearing Fault Classification

Biswajit Sahoo

Before discussing multiclass logistic regression, we will briefly mention logistic regression. Logistic regression is a binary classification technique that uses logistic function $(\frac{1}{1+e^{-x}})$ to fit data. Contrary to linear regression where the output is a numerical value, in logistic regression the output is a probability scores of an input belonging to a particular class.

If there are p predictors (also known as independent variables) X_1, X_2, \ldots, X_p , in linear regression, the target (dependent variable) y takes the form

$$y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \dots \beta_p X_p$$

In logistic regression, the target is either 0 (for one class) or 1 (for other class) and p(x) denotes the probability that a point belongs to a particular class. If the probability score exceeds a threshold, it is assigned to one class or other. In logistic regression log odds is modeled as a linear combination of predictors

$$log(\frac{p(x)}{1-p(x)}) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \dots + \beta_p X_p$$

After simplification, the above equation will take the form

$$p(x) = \frac{1}{1 + e^{-(\beta_0 + \beta_1 X_1 + \beta_2 X_2 + \dots + \beta_p X_p)}}$$

Now the task is to learn the parameters $\beta_0, \beta_1, \dots, \beta_p$. This is done by minimizing the negative log-likelihood with respect to training data. Minimizing negative log-likelihood is equivalent to maximizing likelihood.

One point to note here is that, unlike linear regression where a closed form solution exists, logistic regression solution may not converge sometimes. This happens if the classes are well separated from each other. We will not discuss this point further here, rather we will refer the readers to this excellent short article.

As mentioned previously, logistic regression can handle only two classes. For multiclass classification problems there are extensions of logistic regression. One simple strategy is to do one-vs-all classification. This converts multiclass classification problem involving k classes into k binary classification problem. In each of those k binary classification problems, one class is compared against rest all of the classes. For prediction, a point is assigned to a class for which its probability score is maximum.

We will not implement these on our own. Rather we will use R. The beauty of R is that most of the statistical techniques are already implemented in it so that we can just use those for our analysis. In our case we will use 'nnet' package to implement multiclass logistic regression algorithm involving 10 classes.

Description of data

Detailed discussion of how to prepare the data and its source can be found in this post. Here we will only mention about different classes of the data. There are 12 classes and data for each class are taken at a load of 1hp. The classes are:

• C1: Ball defect (0.007 inch)

```
C2: Ball defect (0.014 inch)
C3: Ball defect (0.021 inch)
C4: Ball defect (0.028 inch)
C5: Inner race fault (0.007 inch)
C6: Inner race fault (0.014 inch)
C7: Inner race fault (0.021 inch)
C8: Inner race fault (0.028 inch)
C9: Normal
C10: Outer race fault (0.007 inch, data collected from 6 O'clock position)
C11: Outer race fault (0.014 inch, 6 O'clock)
C12: Outer race fault (0.021 inch, 6 O'clock)
```

Important Note: In the CWRU website, sampling frequency for the normal data is not mentioned. Most research paper take it as 48k. Some authors also consider it as being taken at a sampling frequency of 12k. Some other authors just use it without ever mentioning its sampling frequency. In our application we only need segment of normal data of length 1024. So we will use the normal data segments available at the website without going into the discussion of sampling frequency. Still, to be on the safer side, we will show results including the normal data as a class as well as excluding it.

When we exclude normal data, we won't consider "C9" class and study the rest 11 fault classes. At that time "C09", "C10", and "C11" will correspond to outer race faults of fault depth 0.007, 0.014, and 0.021 inch respectively.

Codes

```
library(reticulate)
use_condaenv("r-reticulate")
```

How to get data?

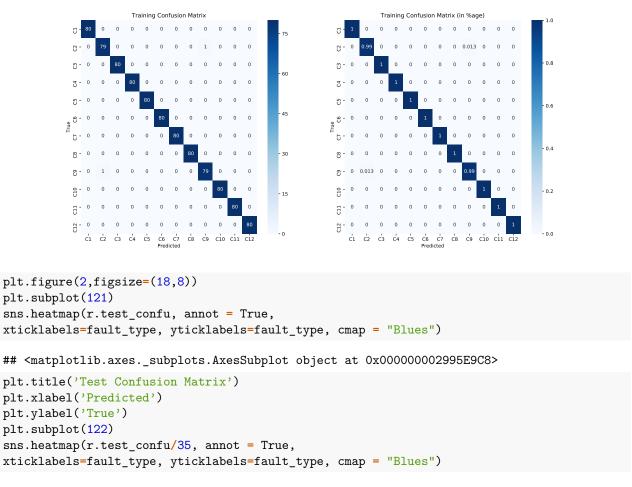
Readers can download the .csv file used in this notebook from here. Another convenient way is to download the whole repository and run the downloaded notebooks.

It should be noted that for some of the deterministic techniques, shuffling of data is not required. But some other techniques like deep learning require the data to be shuffled for better training. So as a recipe we

always shuffle data whether the method is deterministic or not. This doesn't hurt either for a deterministic technique.

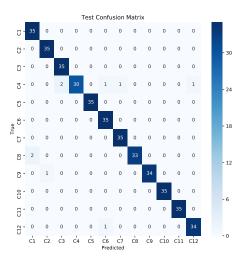
Now we will apply 'multinom' function that does multiclass logistic regression. We will also use the generic predict function that is used for prediction once the model is fit. Finally we will plot the results.

```
multi_logit = multinom(fault~., data = train_data)
## # weights: 120 (99 variable)
## initial value 2385.510384
## iter 10 value 1865.111639
## iter 20 value 1693.472112
## iter 30 value 1400.221938
## iter 40 value 1076.910319
## iter 50 value 925.623202
## iter 60 value 800.794580
## iter 70 value 647.208334
## iter 80 value 395.514696
## iter 90 value 10.948886
## iter 100 value 5.790825
## final value 5.790825
## stopped after 100 iterations
pred_train = predict(multi_logit, newdata = train_data)
pred_test = predict(multi_logit, newdata = test_data)
# Confusion matrix
train_confu = table(train_data$fault, pred_train)
test_confu = table(test_data$fault, pred_test)
import seaborn as sns
import matplotlib.pyplot as plt
fault_type = ['C1', 'C2', 'C3', 'C4', 'C5', 'C6', 'C7', 'C8', 'C9', 'C10', 'C11', 'C12']
plt.figure(1,figsize=(18,8))
plt.subplot(121)
sns.heatmap(r.train confu, annot= True,fmt = "d",
xticklabels=fault_type, yticklabels=fault_type, cmap = "Blues")
## <matplotlib.axes._subplots.AxesSubplot object at 0x000000001B973148>
plt.title('Training Confusion Matrix')
plt.xlabel('Predicted')
plt.ylabel('True')
plt.subplot(122)
sns.heatmap(r.train_confu/80, annot= True,
xticklabels=fault_type, yticklabels=fault_type, cmap = "Blues")
## <matplotlib.axes._subplots.AxesSubplot object at 0x00000000291715C8>
plt.title('Training Confusion Matrix (in %age)')
plt.xlabel('Predicted')
plt.ylabel('True')
plt.show()
```



<matplotlib.axes._subplots.AxesSubplot object at 0x0000000029994DC8>

```
plt.title('Test Confusion Matrix (in %age)')
plt.xlabel('Predicted')
plt.ylabel('True')
plt.show()
```



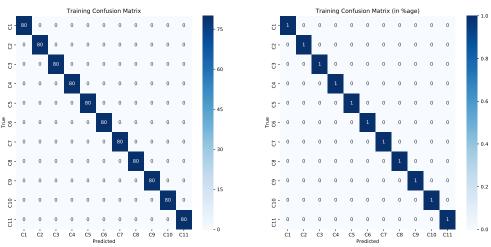
```
overall_test_accuracy = sum(diag(test_confu))/420
sprintf("Overall Test Accuracy: %.4f", overall_test_accuracy*100)
```

[1] "Overall Test Accuracy: 97.8571"

We will also show the results excluding the normal data. The results are as below.

```
## # weights: 110 (90 variable)
## initial value 2110.147840
## iter 10 value 1637.183730
## iter 20 value 1406.059795
## iter 30 value 1128.437364
## iter 40 value 897.278424
## iter 50 value 772.028818
## iter 60 value 594.709477
## iter 70 value 382.593512
## iter 80 value 40.222649
## iter 90 value 0.190876
## iter 100 value 0.010686
## final value 0.010686
```

```
## stopped after 100 iterations
pred_train_new = predict(multi_logit_new, newdata = train_data_new)
pred_test_new = predict(multi_logit_new, newdata = test_data_new)
# Confusion matrix
train_confu_new = table(train_data_new$fault, pred_train_new)
test_confu_new = table(test_data_new$fault, pred_test_new)
import seaborn as sns
import matplotlib.pyplot as plt
fault_type = ['C1','C2','C3','C4','C5','C6','C7','C8','C9','C10','C11']
plt.figure(1,figsize=(18,8))
plt.subplot(121)
sns.heatmap(r.train_confu_new, annot= True,fmt = "d",
xticklabels=fault_type, yticklabels=fault_type, cmap = "Blues")
## <matplotlib.axes._subplots.AxesSubplot object at 0x000000002AC536C8>
plt.title('Training Confusion Matrix')
plt.xlabel('Predicted')
plt.ylabel('True')
plt.subplot(122)
sns.heatmap(r.train_confu_new/80, annot= True,
xticklabels=fault_type, yticklabels=fault_type, cmap = "Blues")
## <matplotlib.axes._subplots.AxesSubplot object at 0x00000000291BED88>
plt.title('Training Confusion Matrix (in %age)')
plt.xlabel('Predicted')
plt.ylabel('True')
plt.show()
```

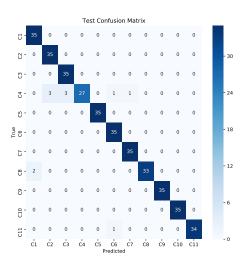


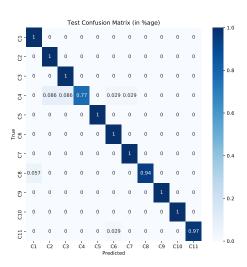
```
plt.figure(2,figsize=(18,8))
plt.subplot(121)
sns.heatmap(r.test_confu_new, annot = True,
xticklabels=fault_type, yticklabels=fault_type, cmap = "Blues")
```

<matplotlib.axes._subplots.AxesSubplot object at 0x0000000029A26488>

```
plt.title('Test Confusion Matrix')
plt.xlabel('Predicted')
plt.ylabel('True')
plt.subplot(122)
sns.heatmap(r.test_confu_new/35, annot = True,
xticklabels=fault_type, yticklabels=fault_type, cmap = "Blues")

## <matplotlib.axes._subplots.AxesSubplot object at 0x0000000029193E88>
plt.title('Test Confusion Matrix (in %age)')
plt.xlabel('Predicted')
plt.ylabel('True')
plt.show()
```





```
overall_test_accuracy_new = sum(diag(test_confu_new))/385
sprintf("New overall Test Accuracy: %.4f", overall_test_accuracy_new*100)
```

[1] "New overall Test Accuracy: 97.1429"

For better results, see this python notebook.

To see results of other techniques applied to public condition monitoring datasets, visit this page.

sessionInfo()

```
## R version 3.6.2 (2019-12-12)
## Platform: x86_64-w64-mingw32/x64 (64-bit)
## Running under: Windows 7 x64 (build 7601) Service Pack 1
##
## Matrix products: default
##
## locale:
## [1] LC_COLLATE=English_United States.1252
## [2] LC_CTYPE=English_United States.1252
## [3] LC_MONETARY=English_United States.1252
## [4] LC_NUMERIC=C
## [5] LC_TIME=English_United States.1252
##
```

```
## attached base packages:
## [1] stats
              graphics grDevices utils datasets methods base
## other attached packages:
## [1] nnet_7.3-12
                    reticulate_1.14
##
## loaded via a namespace (and not attached):
                      digest_0.6.23
## [1] Rcpp_1.0.3
                                     rappdirs_0.3.1 jsonlite_1.6.1
## [5] magrittr_1.5
                      evaluate_0.14 rlang_0.4.4
                                                    stringi_1.4.5
## [9] rmarkdown_2.1 tools_3.6.2
                                     stringr_1.4.0 xfun_0.12
## [13] yaml_2.2.0
                      compiler_3.6.2 htmltools_0.4.0 knitr_1.27
```

Last updated: 14^{th} February, 2020