藍色代表水流過岩石的

open

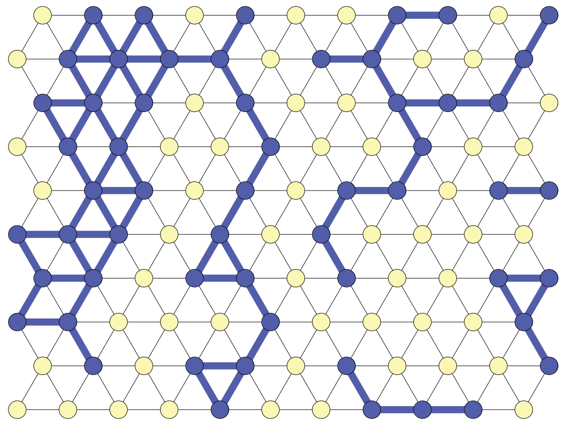

Fig 1: Site percolation on T

open subgraph is induced by the open edges. Denote the resulting probability measure on $\{0,1\}^E$ by $\mathbb{P}_p^{\mathrm{b}}$. In the physical interpretation, in site percolation, water is held mainly in the sites or "pockets" and flows between adjacent pockets — whereas in bond percolation, water is held mainly in the bonds or "channels," and the *sites* express the adjacency of channels. Historically, bond percolation has been studied more extensively than site percolation; however, bond percolation on a graph G is equivalent to site percolation on the covering graph of G.

Percolation is a minimal model which nonetheless captures aspects of the mechanism of interest — that of water seeping through rock — and provides qualitative and quantitative predictions. Since its introduction it has developed into an extremely rich subject, extensively studied by physicists and mathematicians. In this article we focus on one special facet of the theory, and so will certainly miss mentioning many interesting results. The interested reader is referred to the books [23, 47, 48, 65] for accounts of the mathematical theory. For an introduction to the extensive physics literature see [112].

1.1. Critical percolation

In percolation, a natural property to consider is the existence of an infinite open cluster: suppose $G = (V, E)$ is countably infinite and connected, and consider site percolation on G. For each site $x \in G$ let C_x denote the open cluster containing x (with $C_x = \varnothing$ if x does not belong to an open cluster). We say that x **percolates** if $|C_x| = \infty$; the physical interpretation is that x "gets wet." The **(site) percolation probability** is defined to be

$$\theta_x(p) \equiv \theta_x^{\mathrm{s}}(p) \equiv \mathbb{P}_p^{\mathrm{s}}(|C_x| = \infty),$$

a non-decreasing function of p. Because G is connected, for any $x, y \in G$, $\theta_x(p)$ and $\theta_y(p)$ must be both positive or both zero. Thus we can define a **critical**