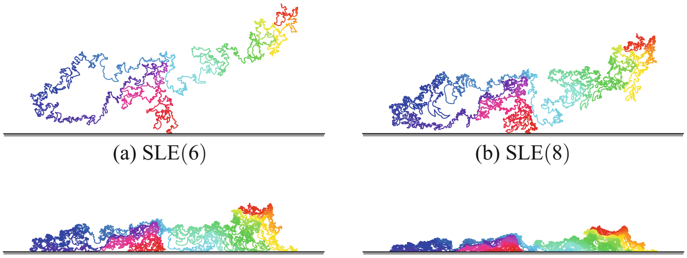

(a) $\mathrm{SLE}(6)$ (b) $\mathrm{SLE}(8)$

(c) $\mathrm{SLE}(16)$ (d) $\mathrm{SLE}(32)$

Fig. 5.2 Instances of $\mathrm{SLE}(\kappa)$, for $\kappa = 6, 8, 16, 32$, driven by the instance of a standard Brownian motion drawn in Fig. 5.1e multiplied by a factor $\sqrt{\kappa}$.

$t \in \mathbb{R}_{\geq 0}$ and for any measurable set B in the space of curves (in what ever way that space is defined…)

$$\mu^{(U,a,b)}(\gamma|_{[t,\infty)} \in B \mid \mathscr{F}_t) = \mu^{(U\setminus\gamma([0,t]),\gamma(t),b)}(\gamma \in B).$$

3. Assume that we can describe the curve γ by the Loewner equation in the sense that there is a $\mu^{(\mathbb{H},0,\infty)}$-almost sure event on which γ satisfies Theorem 4.2.

In Schramm's principle, we'll investigate the consequences of these assumptions.

The first observation is that we need to describe only one of the measures in the family. Then CI fixes the rest of them. Let us choose to work with $\mu^{(\mathbb{H},0,\infty)}$. By Theorem 4.2 for each realization of γ there is a driving term $(W_t(\gamma))_{t\in\mathbb{R}_{\geq 0}}$ such that the corresponding conformal maps g_t satisfy the Loewner equation. Here we also make a reparameterization with the half-plane capacity.[3] Let's call the stochastic driving term $(W_t)_{t\in\mathbb{R}_{\geq 0}}$ as *driving process* of the random curve γ.

Fix some $t \in \mathbb{R}_{\geq 0}$. Define $\hat{\gamma}(s) = g_t(\gamma(t+s)) - W_t$ for all $s \in \mathbb{R}_{\geq 0}$. By CI and the DMP, $\hat{\gamma}$ is distributed as γ and independent of the realization of $\gamma|_{[0,t]}$. The conformal map associated to the hull $\hat{\gamma}([0,s])$ is

$$\hat{g}_s(z) = \tilde{g}_{t,s}(z + W_t) - W_t = g_{t+s} \circ g_t^{-1}(z + W_t) - W_t.$$

Now by differentiating this with respect to s

$$\begin{aligned}\partial_s \hat{g}_s(z) &= (\partial_s g_{t+s})(g_t^{-1}(z + W_t)) \\ &= \frac{2}{g_{t+s}(g_t^{-1}(z + W_t)) - W_{t+s}} = \frac{2}{\hat{g}_s(z) - (W_{t+s} - W_t)}\end{aligned}$$

[3] By an argument which we leave as an exercise, when CI and DMP are satisfied, the half-plane capacity of the hull $\gamma[0,t]$ will tend to infinity as t tends to infinity. Therefore the reparameterized curve will be parametrized by the set $\mathbb{R}_{\geq 0}$.