Lecture 5~8

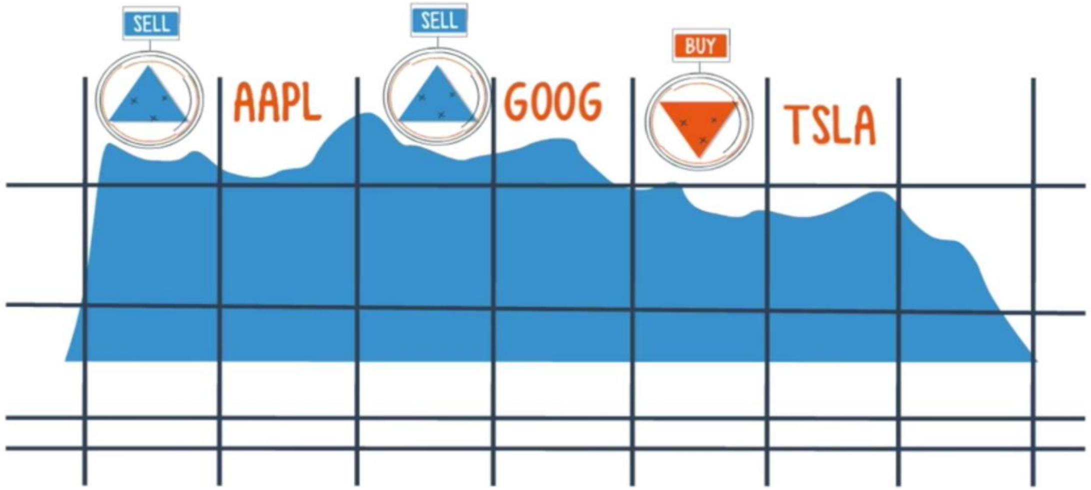
Lecture 5 – Logistic Regression

Classification

- Spam Detection: Spam(1) or Ham(0)
- Facebook feed: show(1) or hide(0)
- Credit Card Fraudulent Transaction detection: legitimate/fraud

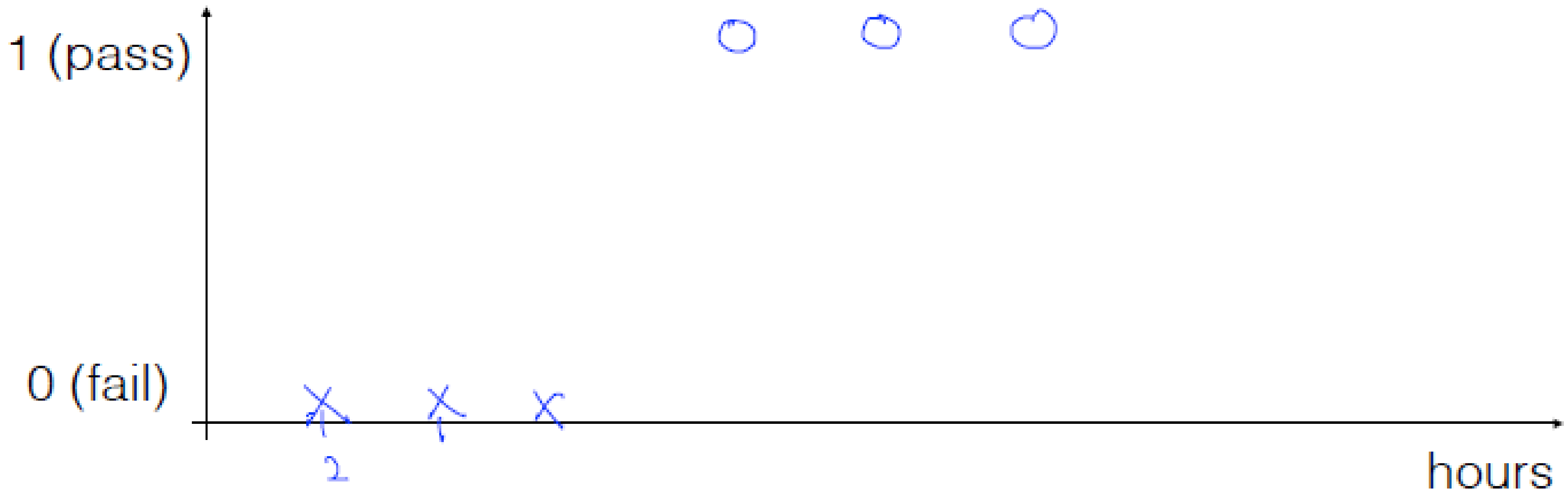
Finance

DWJI	17,499.10
SP500	2,025.51
NASDAQ	4,976.9
AAPL	107.71
GOOG	750.06
TSLA	234.24



TODSI
Become A Better Data Scientist

Pass(1)/Fail(0) based on study hours



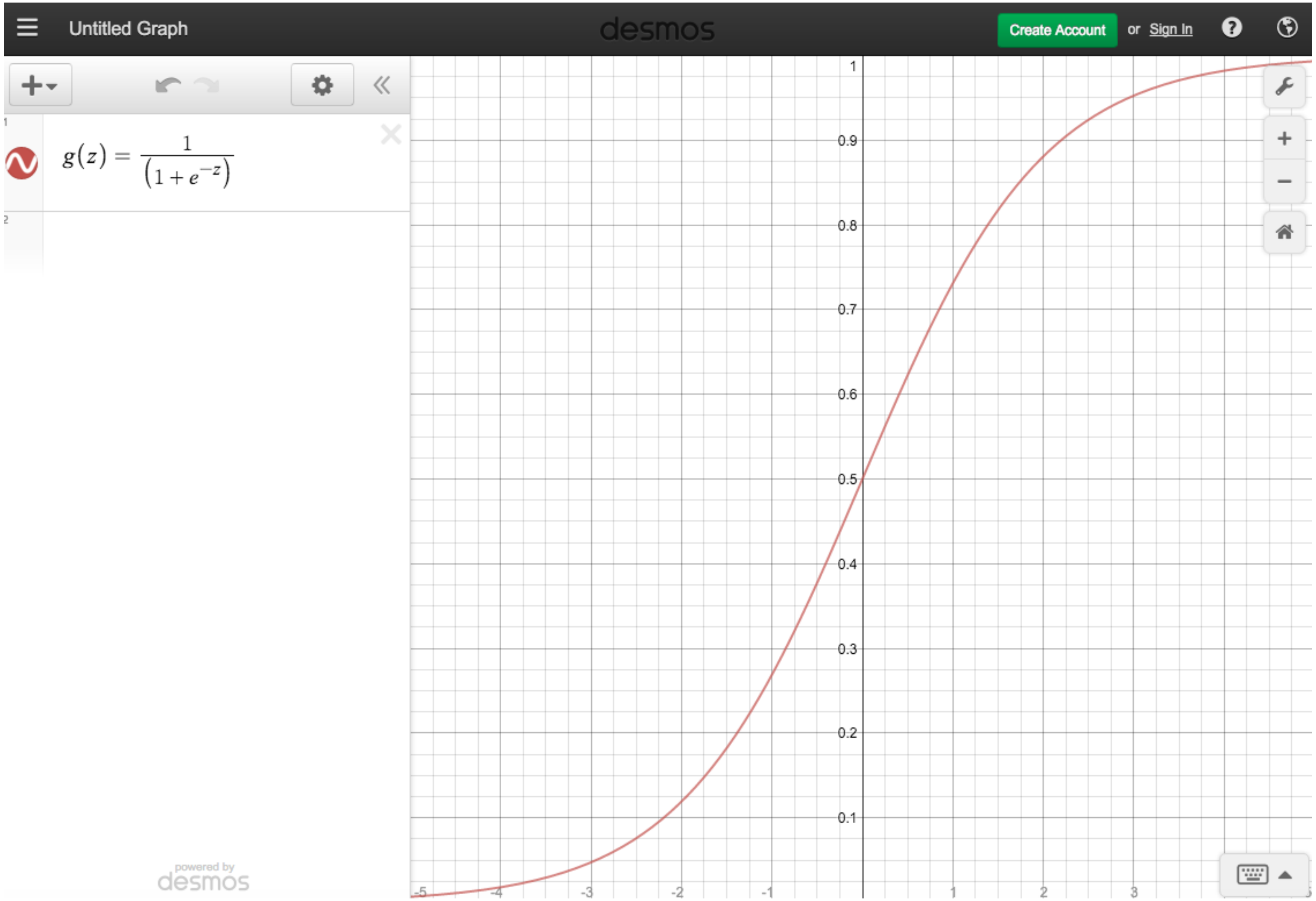
In linear regression...

- We know Y is 0 or 1

  $H(x) = Wx + b$

$W = 0.5$, $b = 0$

- Hypothesis can give values large than 1 or less than 0


Untitled Graph
desmos
Create Account
or Sign In
$g(z) = \frac{1}{(1+e^{-z})}$
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
-5
-4
-3
-2
-1
1
2
3
powered by desmos

Logistic Hypothesis

$$H(X) = \frac{1}{1 + e^{-(W^T X)}}$$

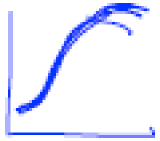
Cost function

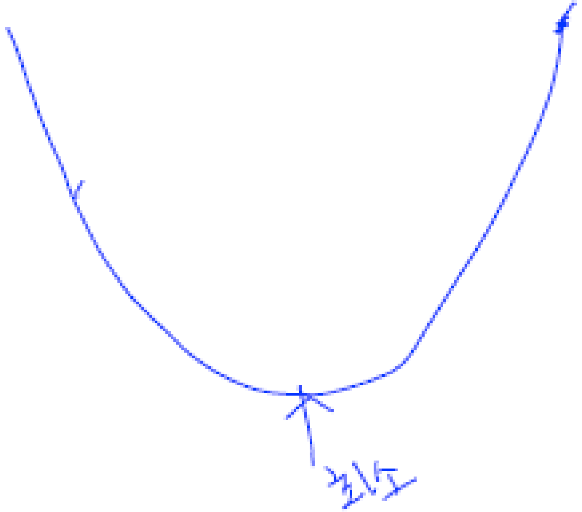
$$cost(W,b) = \frac{1}{m}\sum_{i=1}^{m}(H(x^{(i)}) - y^{(i)})^2$$

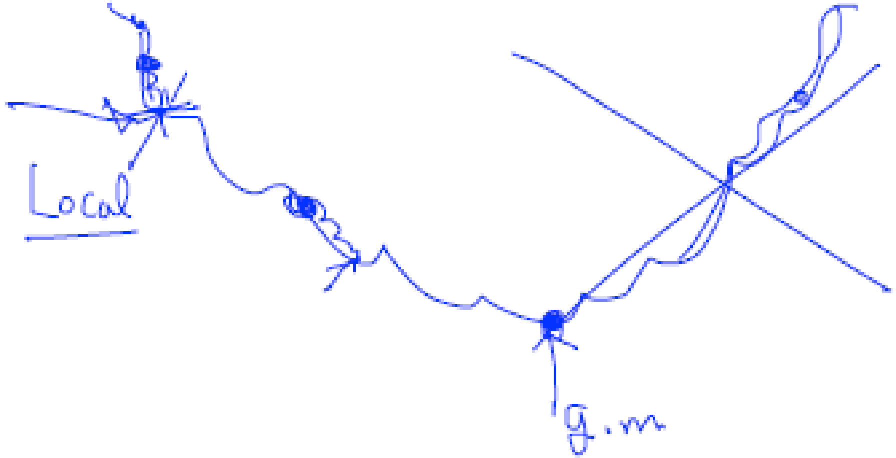
0< ~ <1

$H(x) = Wx + b$

$$H(X) = \frac{1}{1+e^{-W^TX}}$$







New cost function for logistic

$$cost(W) = \frac{1}{m} \sum c(H(x), y)$$

$$c(H(x), y) = \begin{cases} -log(H(x)) & : y = 1 \\ -log(1 - H(x)) & : y = 0 \end{cases}$$

Cost function

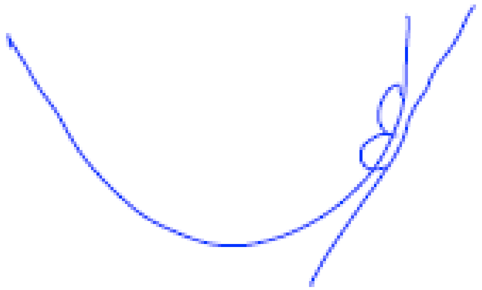
$$cost(W) = \frac{1}{m} \sum c(H(x), y)$$

$$c(H(x), y) = \begin{cases} -log(H(x)) & : y = 1 \\ -log(1 - H(x)) & : y = 0 \end{cases}$$

$$\boxed{c(H(x), y) = -ylog(H(x)) - (1 - y)log(1 - H(x))}$$

Minimize cost - Gradient decent algorithm

$$cost(W) = -\frac{1}{m}\sum y log(H(x)) + (1-y) log(1-H(x))$$



$$W := W - \alpha \frac{\partial}{\partial W} cost(W)$$

Binary Classification

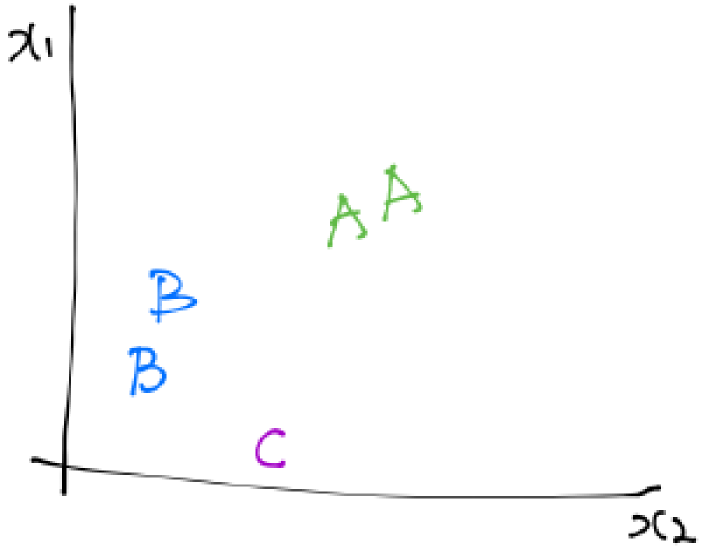
Label	Yes	No
Model 의 예측 결과 ($\bar{y}$)	0.7	0.3
실제 값 (y)	1	0
	$-1 \cdot \log(0.7)$	$-0 \cdot \log(0.3)$

두 확률 분포의 차이 = cross entropy

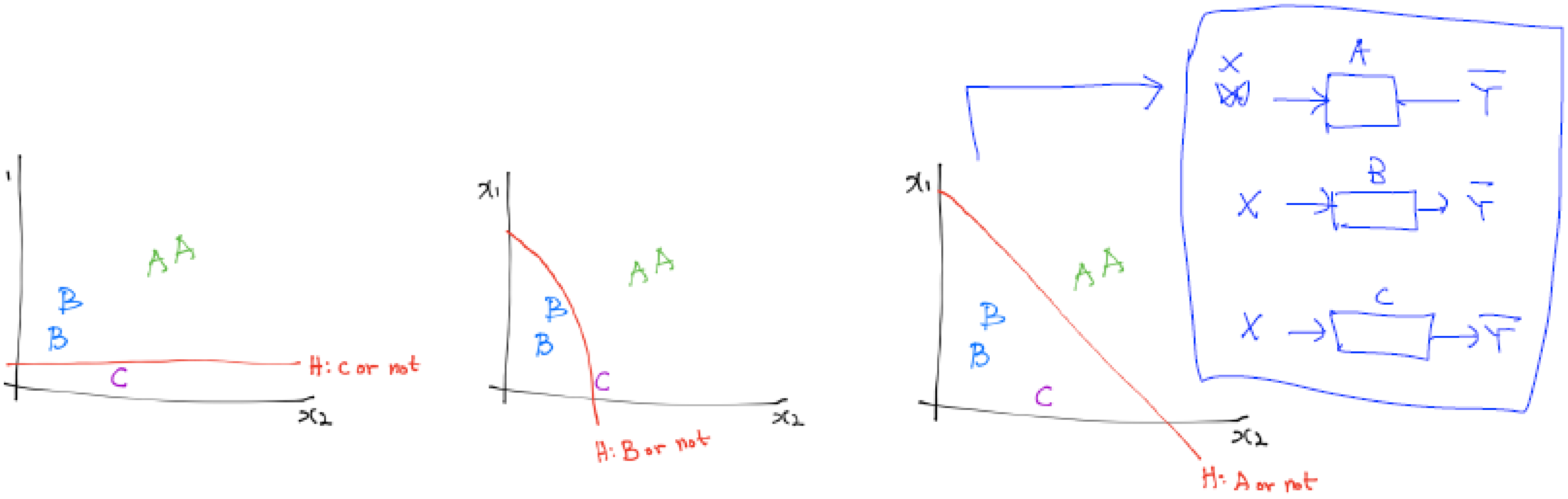
Lecture 6 – Softmax classification

Multinomial classification

x1 (hours)	x2 (attendance)	y (grade)
10	5	A
9	5	A
3	2	B
2	4	B
11	1	C



Multinomial classification

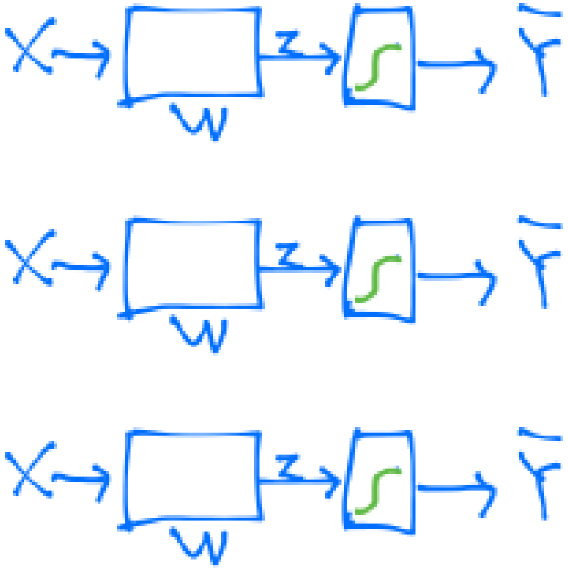


Multinomial classification

$$\begin{bmatrix} w_1 & w_2 & w_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} w_1 x_1 + w_2 x_2 + w_3 x_3 \end{bmatrix}$$

$$\begin{bmatrix} w_1 & w_2 & w_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} w_1 x_1 + w_2 x_2 + w_3 x_3 \end{bmatrix}$$

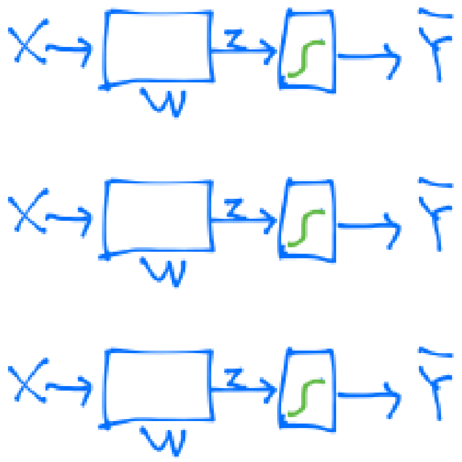
$$\begin{bmatrix} w_1 & w_2 & w_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} w_1 x_1 + w_2 x_2 + w_3 x_3 \end{bmatrix}$$

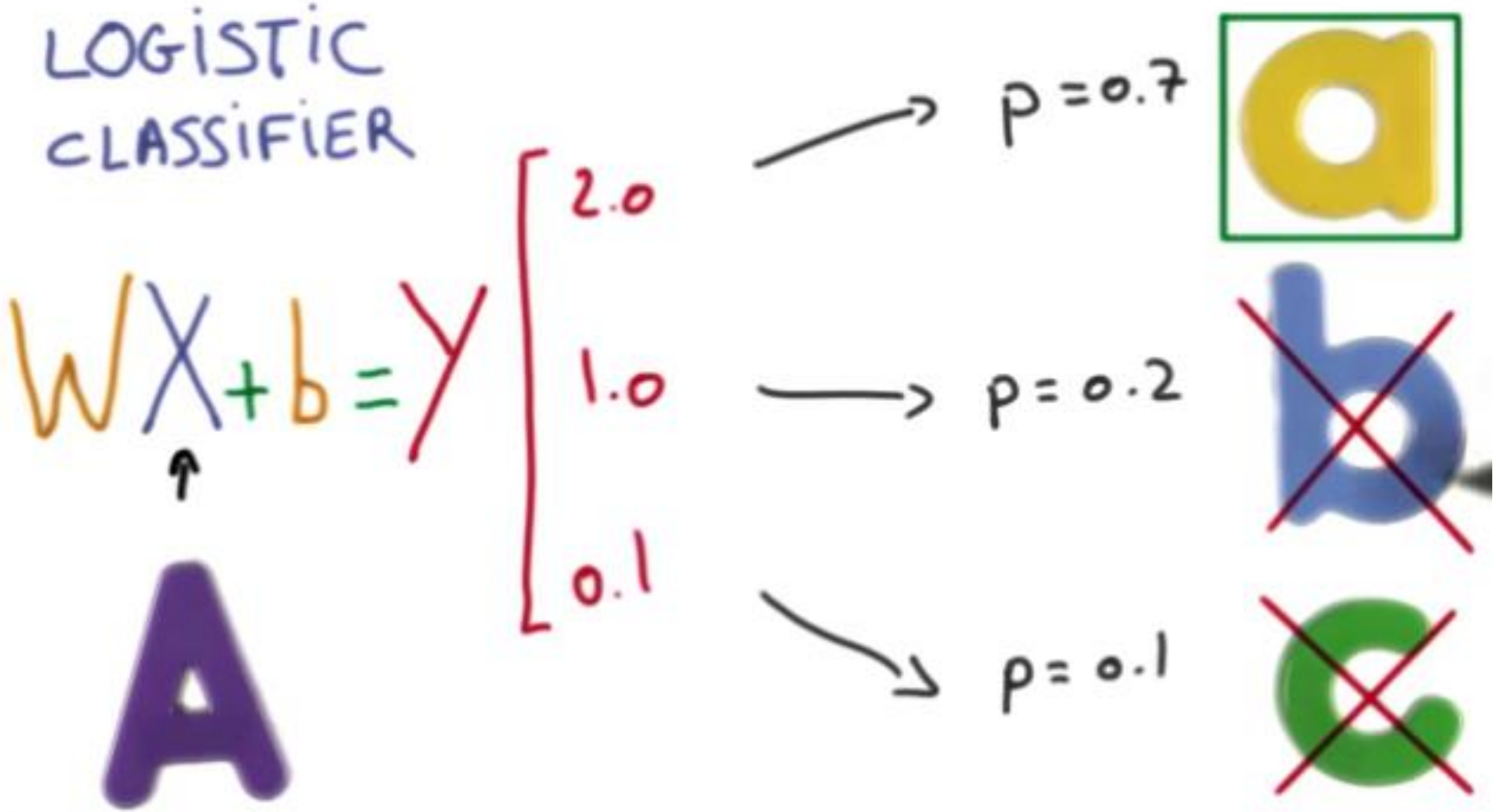


Multinomial classification

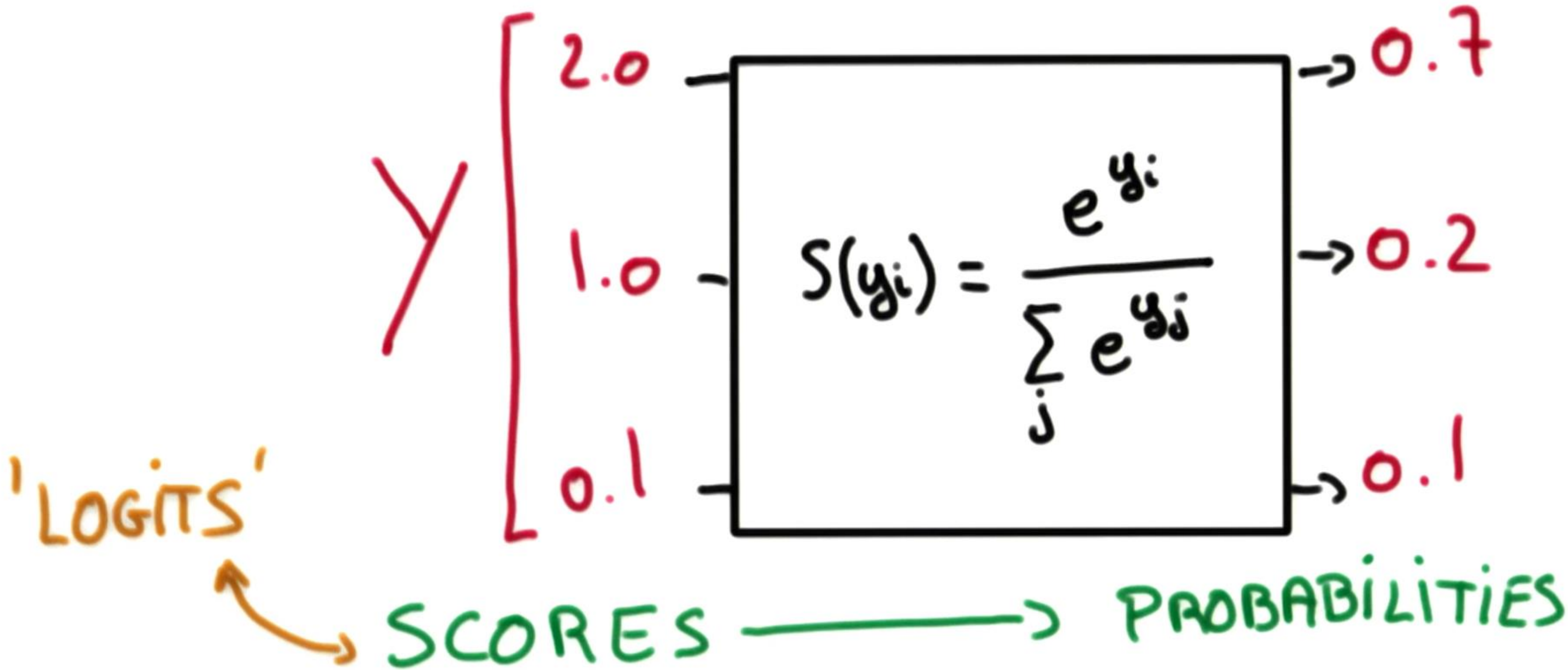
$$\begin{bmatrix} w_1 & w_2 & w_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} w_1x_1 + w_2x_2 + w_3x_3 \end{bmatrix}$$

$$\begin{bmatrix} w_{A1} & w_{A2} & w_{A3} \\ w_{B1} & w_{B2} & w_{B3} \\ w_{C1} & w_{C2} & w_{C3} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} =$$




LOGISTIC
CLASSIFIER
WX+b=Y
2.0
1.0
0.1
p=0.7
p=0.2
p=0.1

SOFTMAX



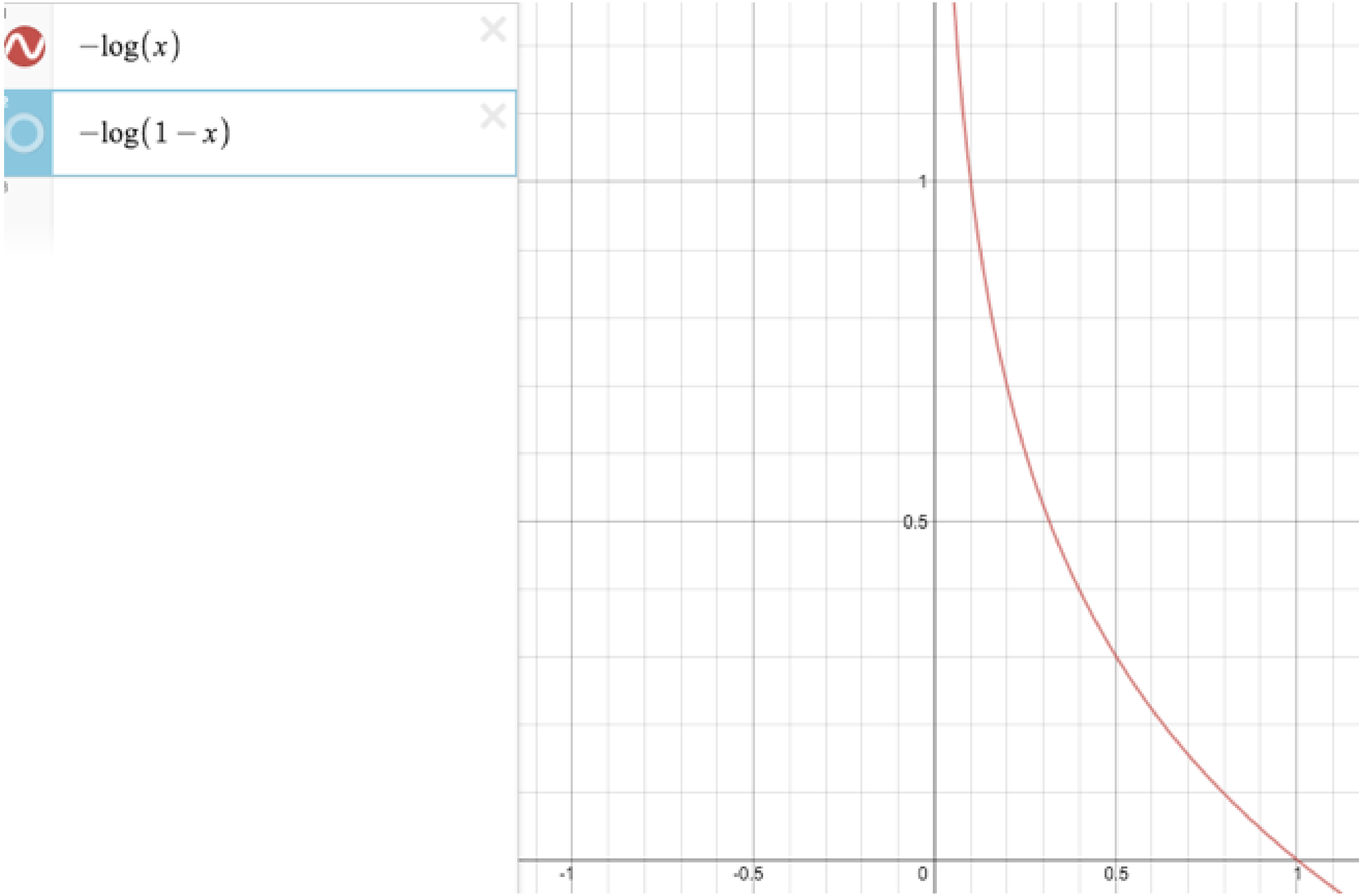
Y
2.0
1.0
0.1
$S(y_i) = \frac{e^{y_i}}{\sum_j e^{y_j}}$
0.7
0.2
0.1
'LOGITS'
SCORES
PROBABILITIES

CROSS-ENTROPY

$$S(Y) = \begin{bmatrix} 0.7 \\ 0.2 \\ 0.1 \end{bmatrix} \qquad L = \begin{bmatrix} 1.0 \\ 0.0 \\ 0.0 \end{bmatrix}$$

$$D(S, L) = -\sum_i L_i \log(S_i)$$

그래서 cross entropy가 MSE보다 좋은게 뭔데?



$$C(H(x), y) = y\log(H(x)) - (1-y)\log(1-H(x))$$

// ?

$$D(S, L) = -\sum_i L_i \log(S_i)$$

LOSS

$$\mathcal{L} = \frac{1}{N} \sum_i D(S(wx_i + b), L_i)$$

TRAINING SET

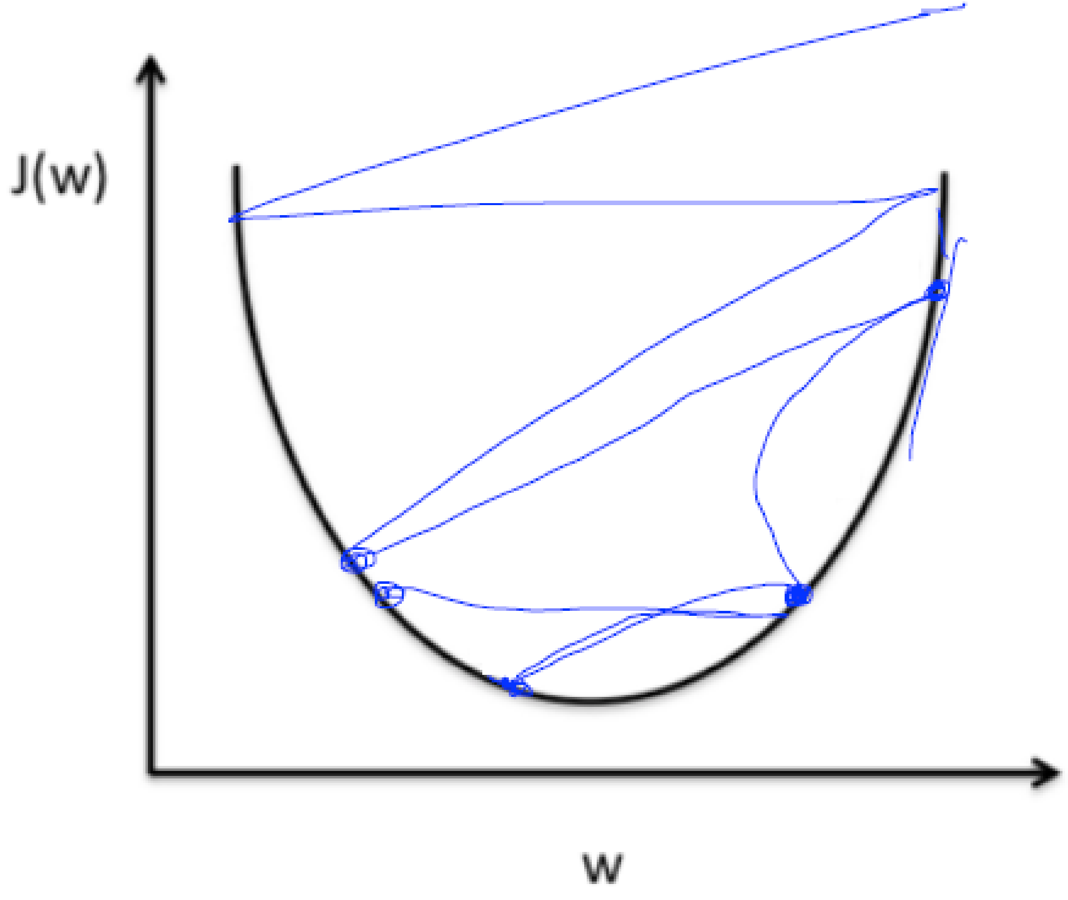
Classification 정리

- MLP의 결과를 Probability distribution 으로 만들기 위해
- Softmax 사용

- 정답 Label의 probability distribution와 softmax 결과의 probability distribution가 얼마나 다른지?
- Cross Entropy Loss 사용

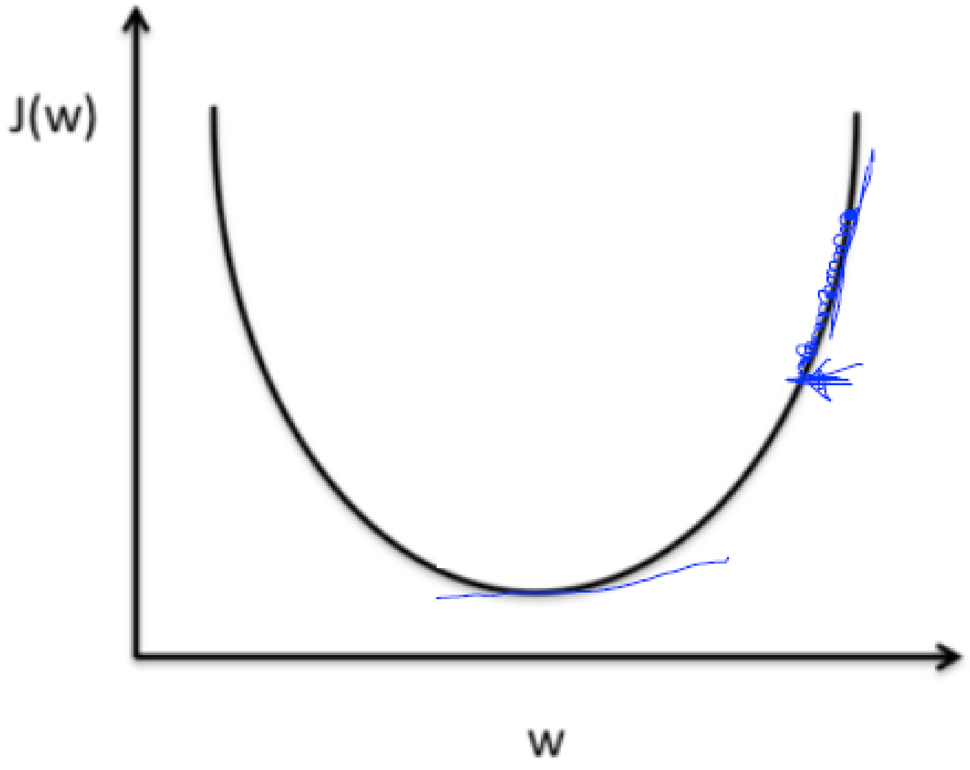
- Binary classification이든 multinomial classification이든 똑같음.

Large learning rate: overshooting



Small learning rate:

takes too long, stops at local minimum



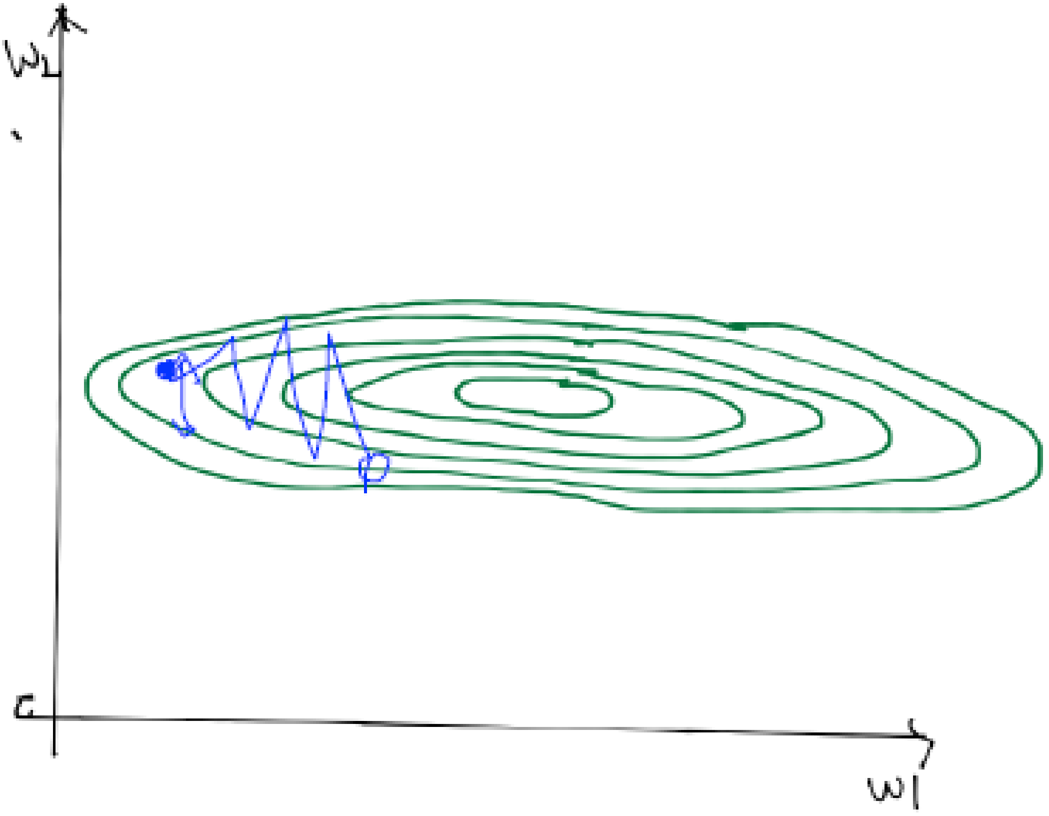
Try several learning rates

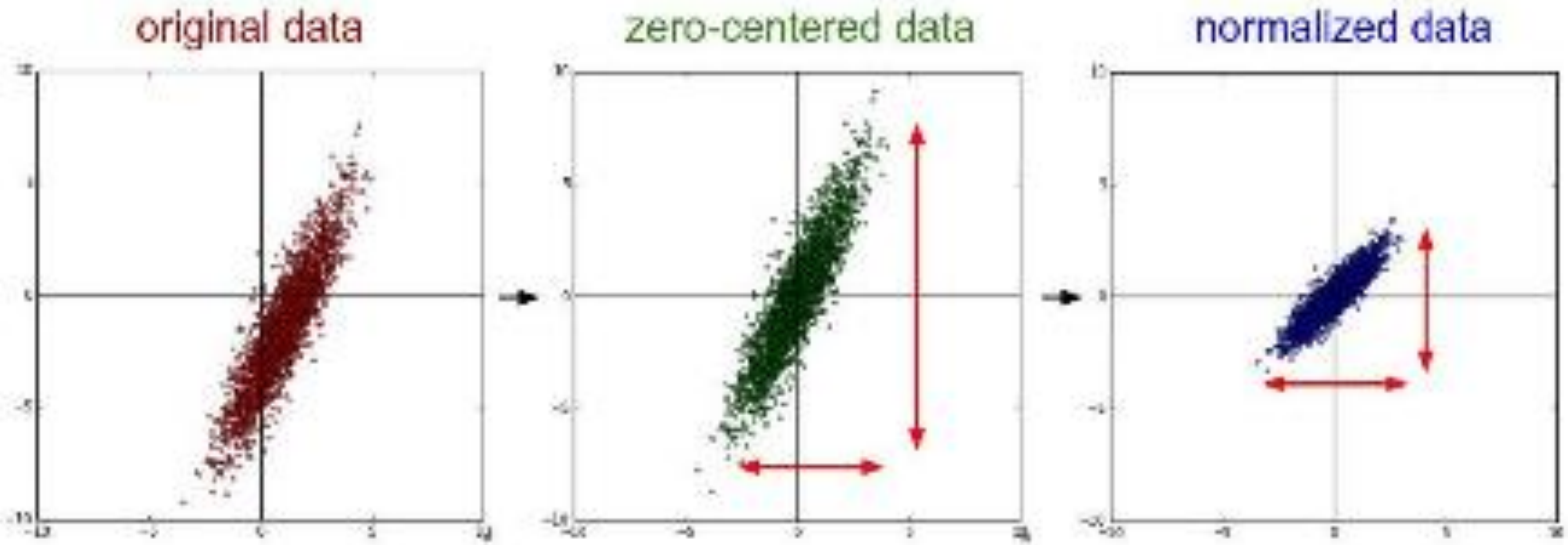
$\boxed{0.01}$

- Observe the cost function
- Check it goes down in a reasonable rate

Data (X) preprocessing for gradient descent

x1	x2	y
1	9000	A
2	-5000	A
4	-2000	B
6	8000	B
9	9000	C




original data
zero-centered data
normalized data

Standardization

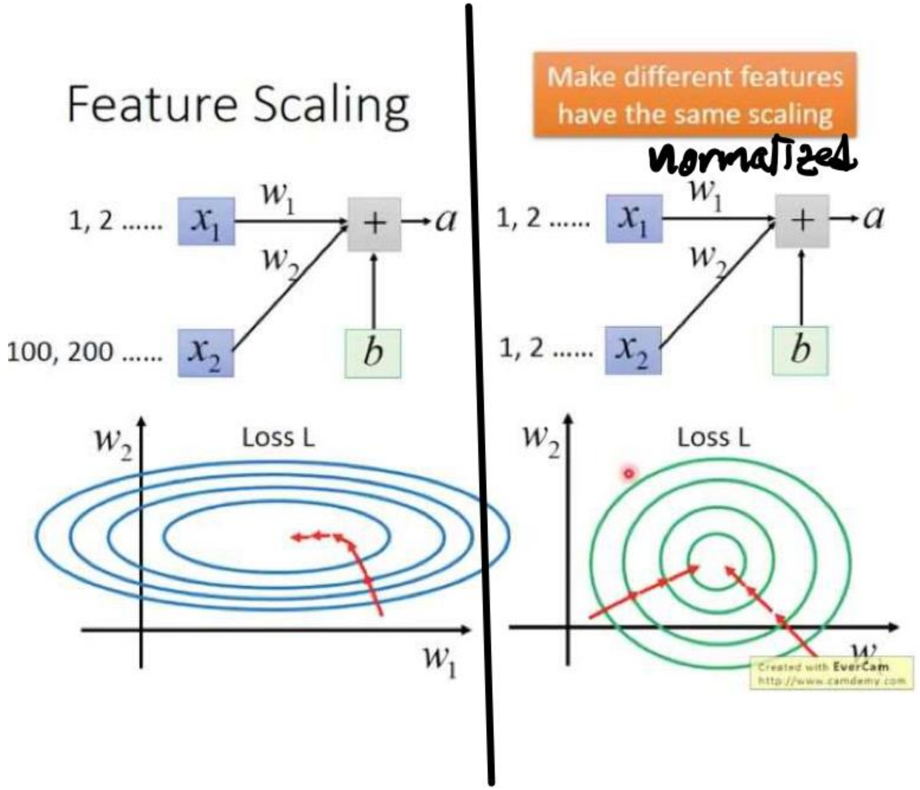
$$x'_j = \frac{x_j - \mu_j}{\sigma_j}$$

```
X_std[:,0] = (X[:,0] - X[:,0].mean()) / X[:,0].std()
```

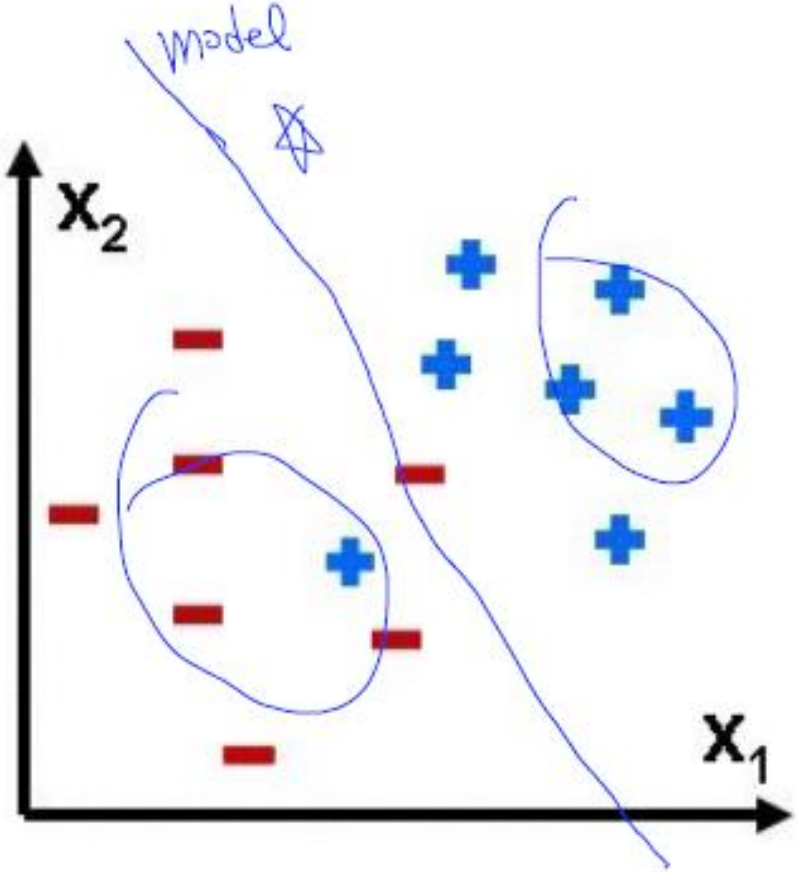
http://sebastianraschka.com/Articles/2015_singlelayer_neurons.html

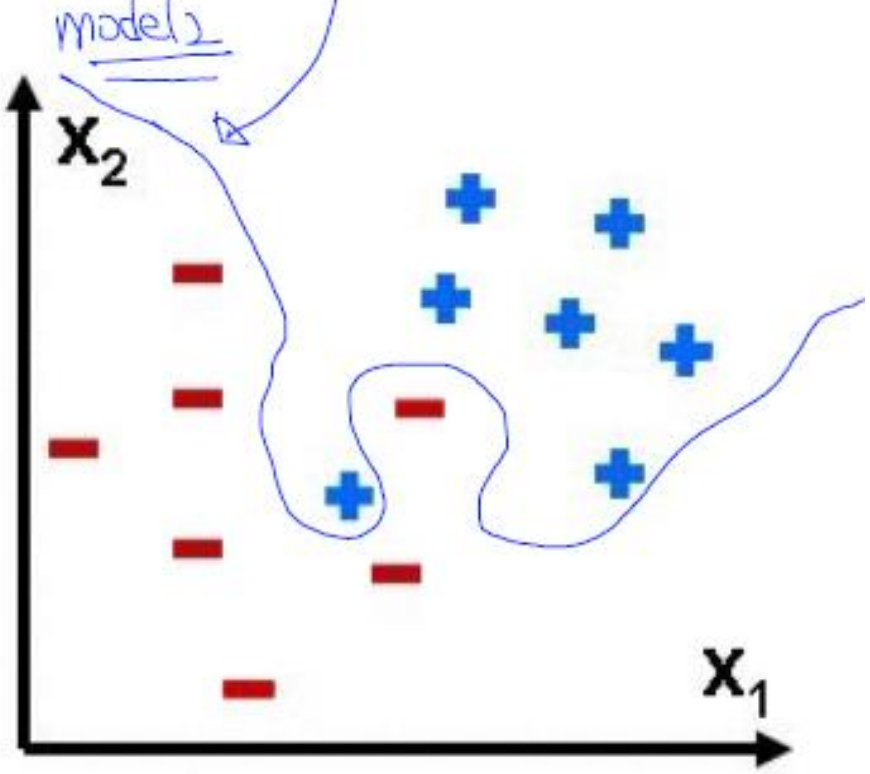
Data Preprocessing: 왜함?

- 모델이 더 빠르게 학습할 수 있다.
- 모델이 학습하기 더 쉬워진다.



Overfitting



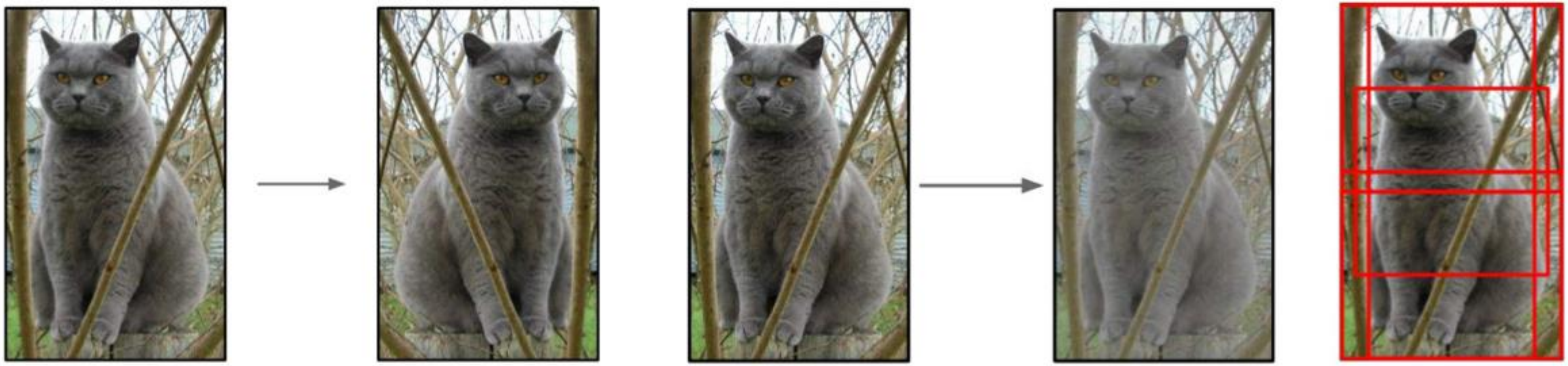


Solutions for overfitting

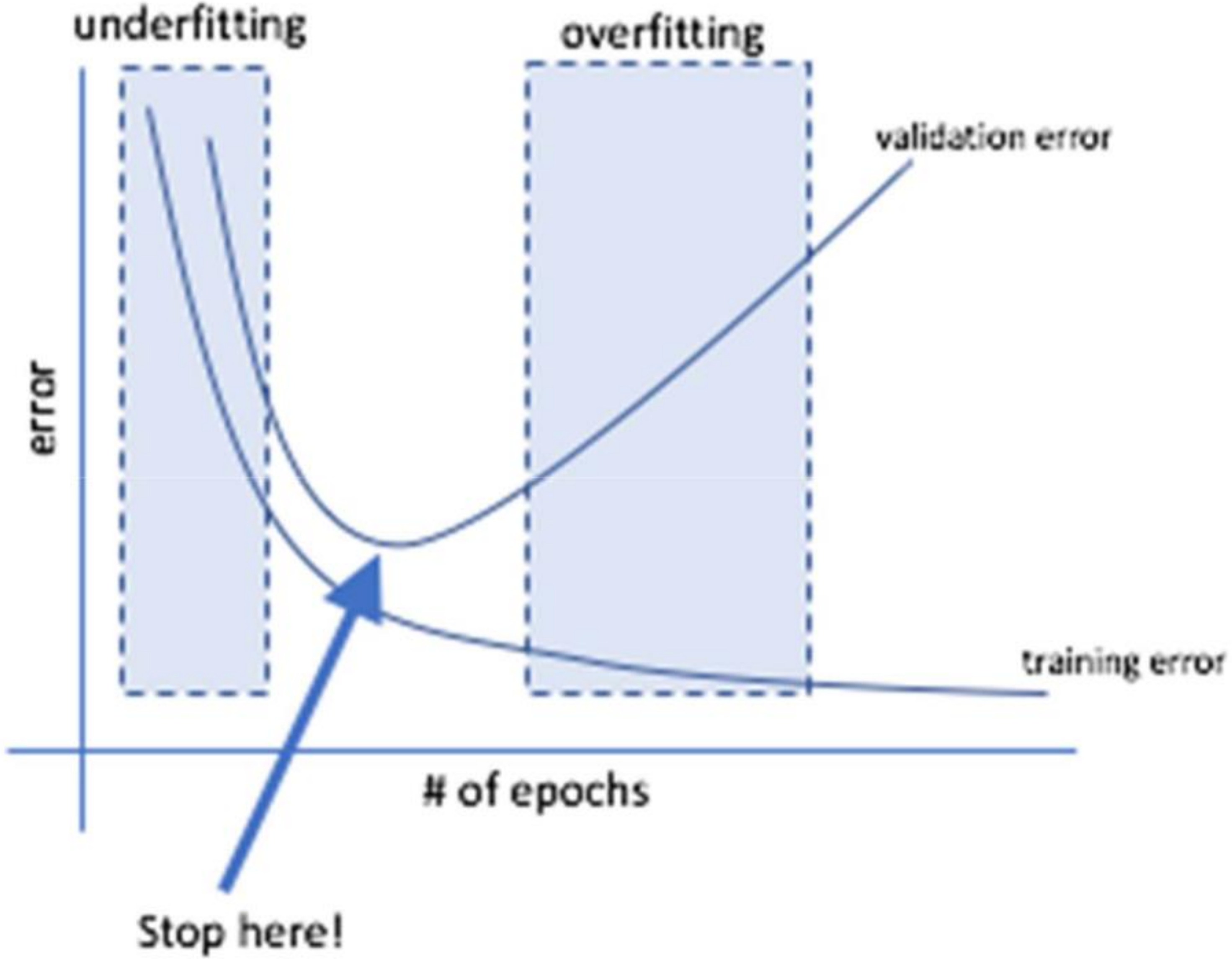
- More training data!
- Reduce the number of features
- **Regularization** - L1, L2, Batch norm, Dropout

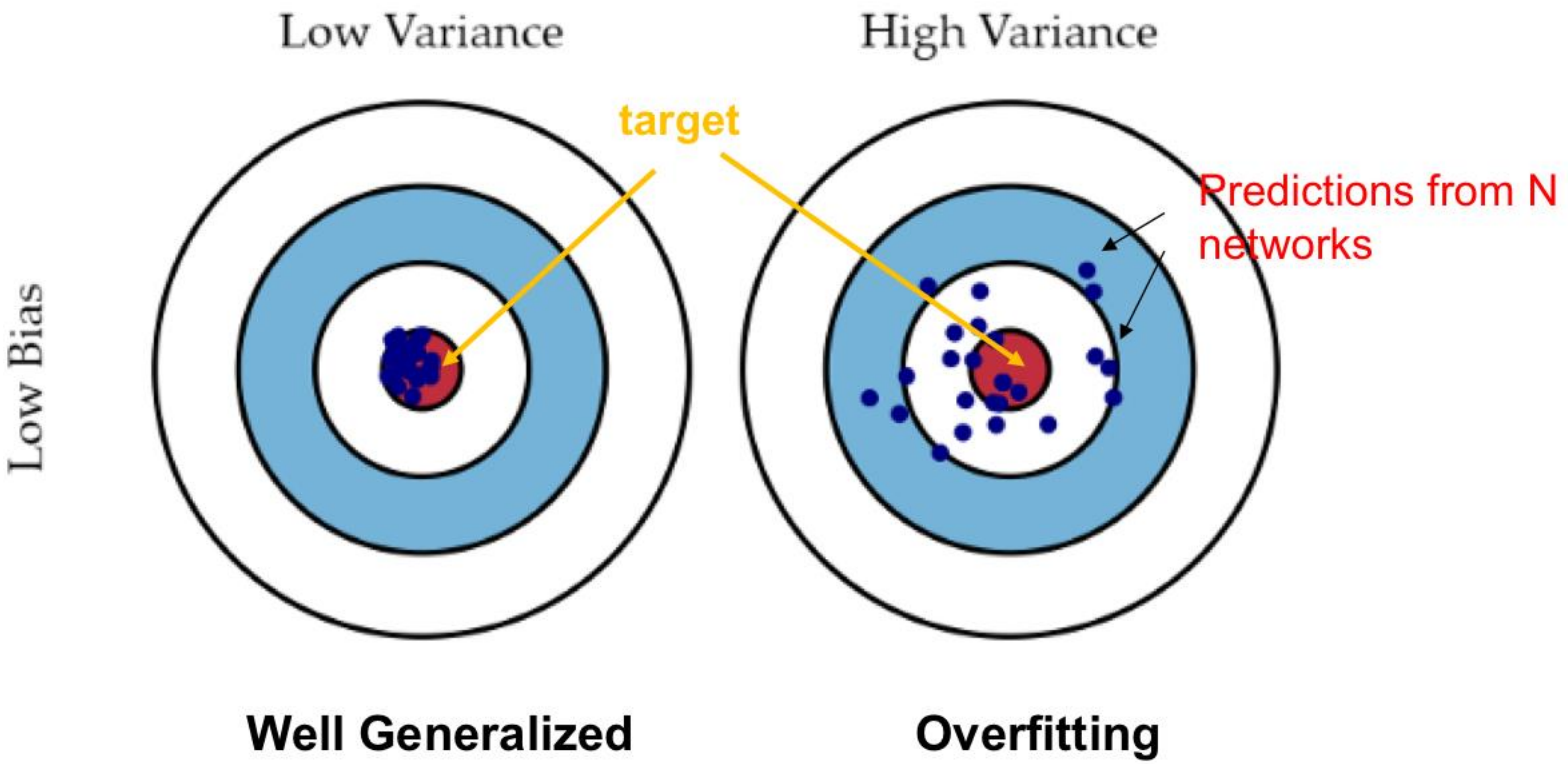
+ Data Augmentation
+ Early Stopping
+ Ensembles

Data Augmentation?



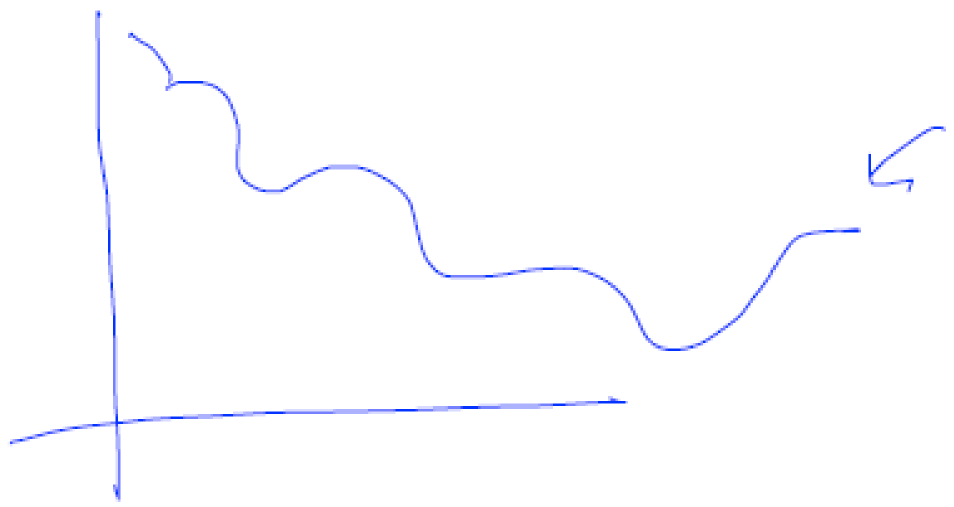
Early Stopping?




Low Variance
High Variance
target
Predictions from N networks
Low Bias
Well Generalized
Overfitting

Regularization

- Let's not have too big numbers in the weight



Weight가 크면 왜 안좋은데?

Regularization

- Let's not have too big numbers in the weight

larization strength

```
l2reg = 0.001 * tf.reduce_sum(tf.square(W))
```

LOSS

$$\mathcal{L} = \frac{1}{N} \sum_i D(S(wx_i + b), L_i) + \lambda \sum W^2$$

TRAINING SET

Evaluation using training set?

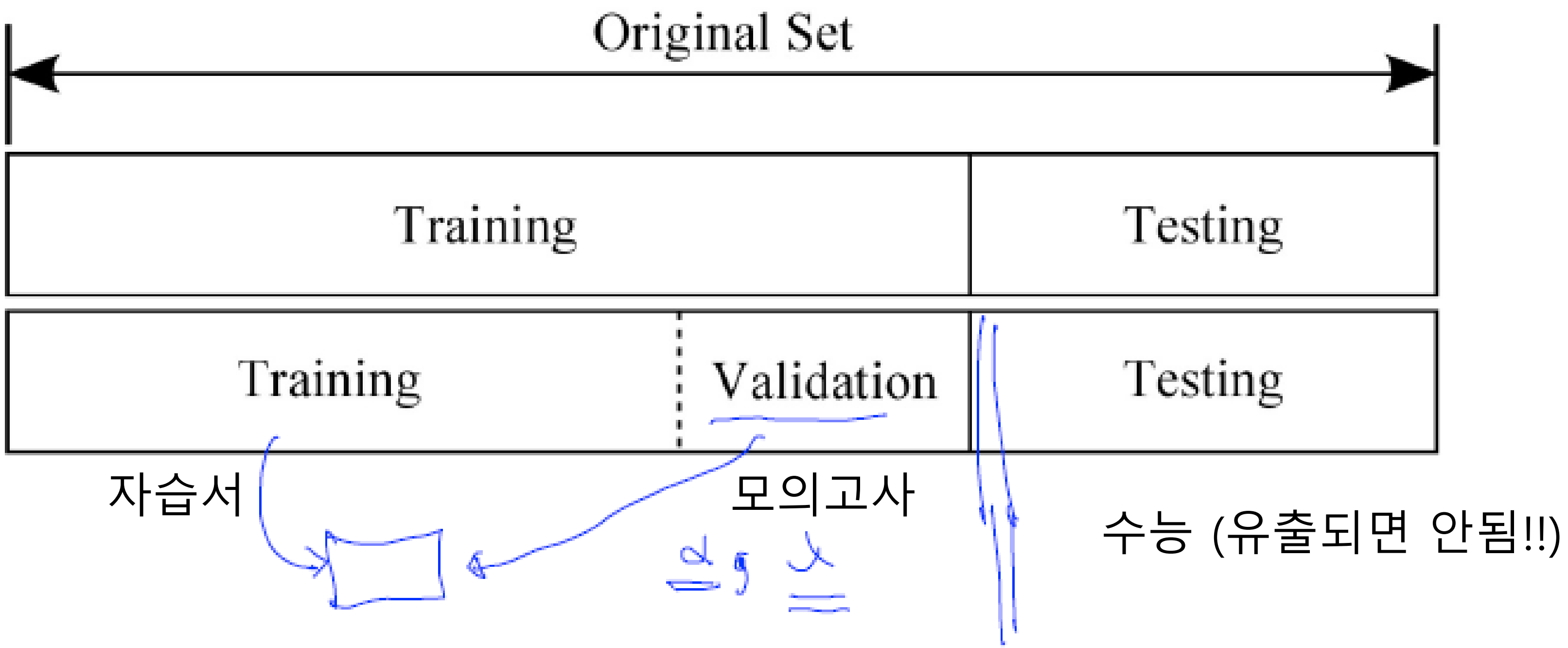
Size	Price
2104	400
1600	330
2400	369
1416	232
3000	540
1985	300
1534	315
1427	199
1380	212
1494	243

training set → model

- 100% correct (accuracy)
- Can memorize

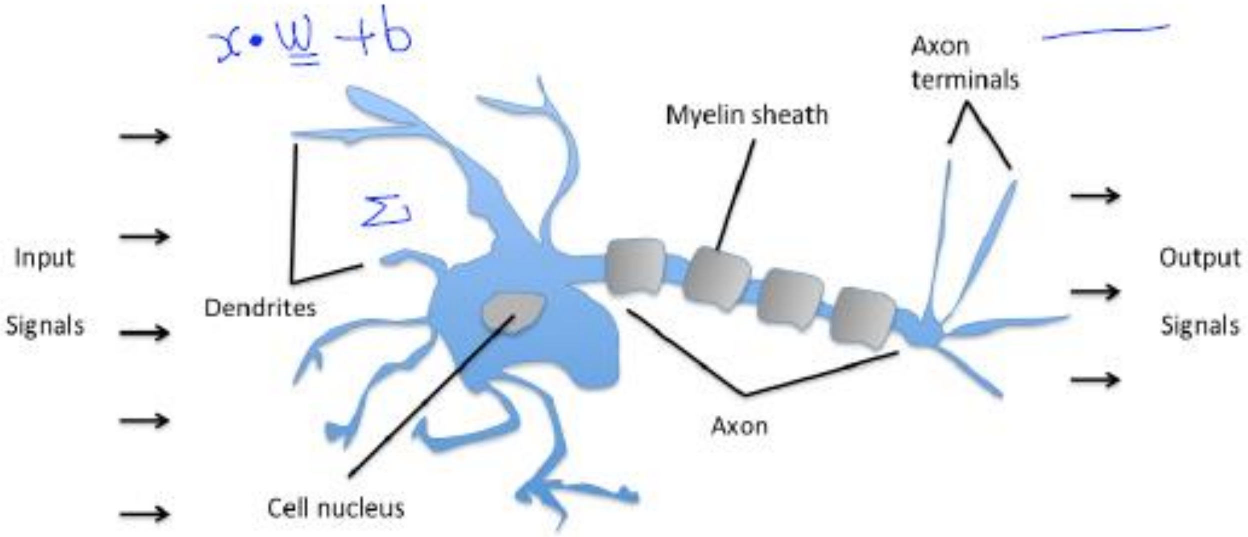
족보 그대로 시험문제가 나옴!! 제대로 평가할 수 없음.

Training, validation and test sets



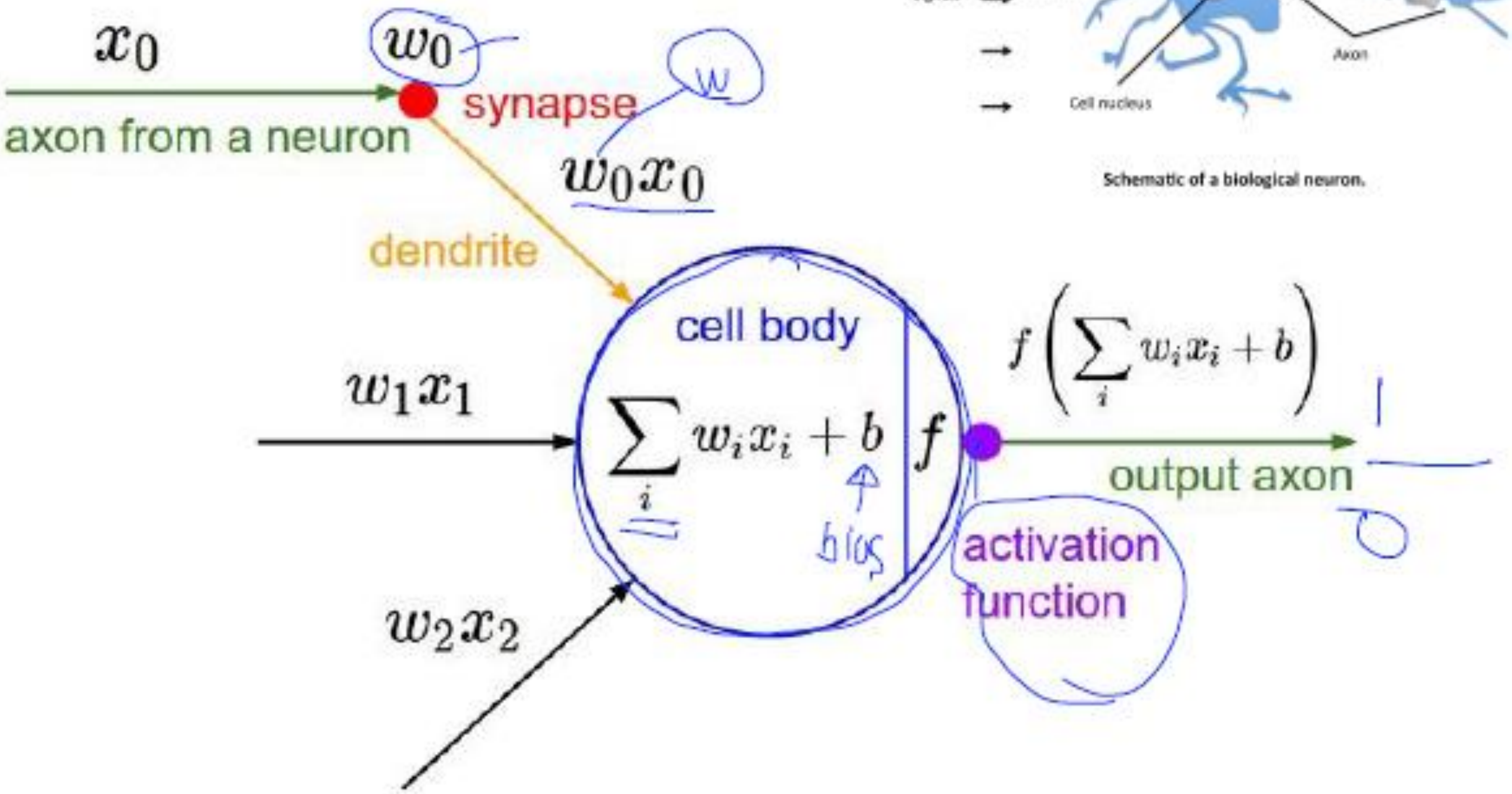
Lecture 8

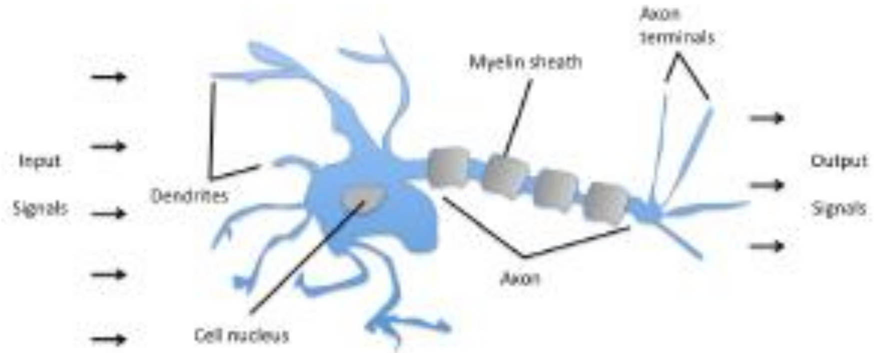
Ultimate dream: thinking machine



Schematic of a biological neuron.

Activation Functions



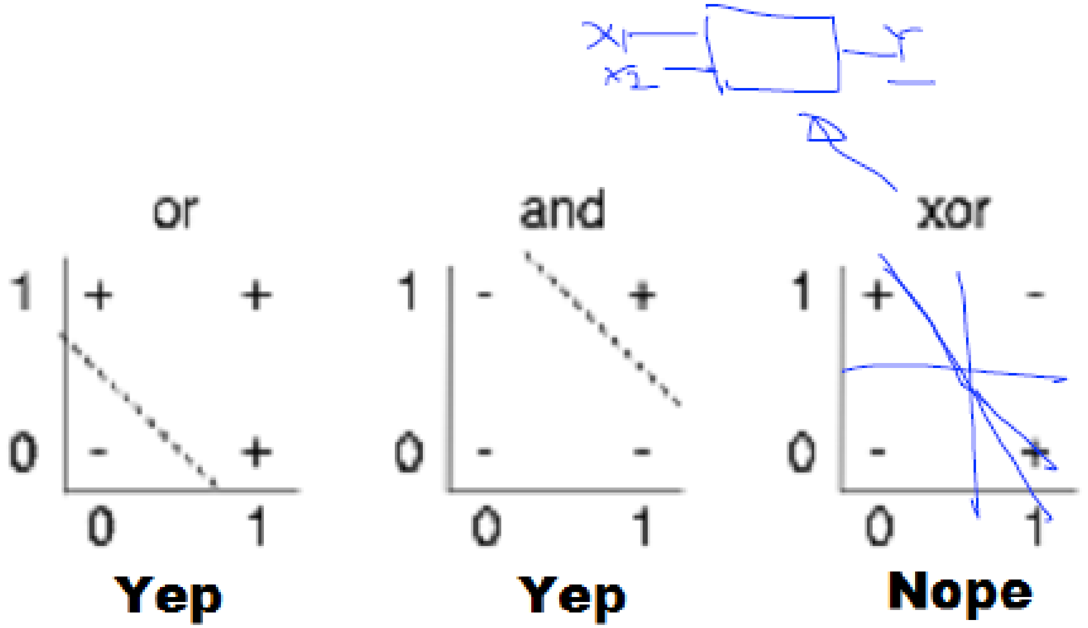


Schematic of a biological neuron.

Perceptron도 문제가 있다?

XOR Problem

(Simple) XOR problem: linearly separable?



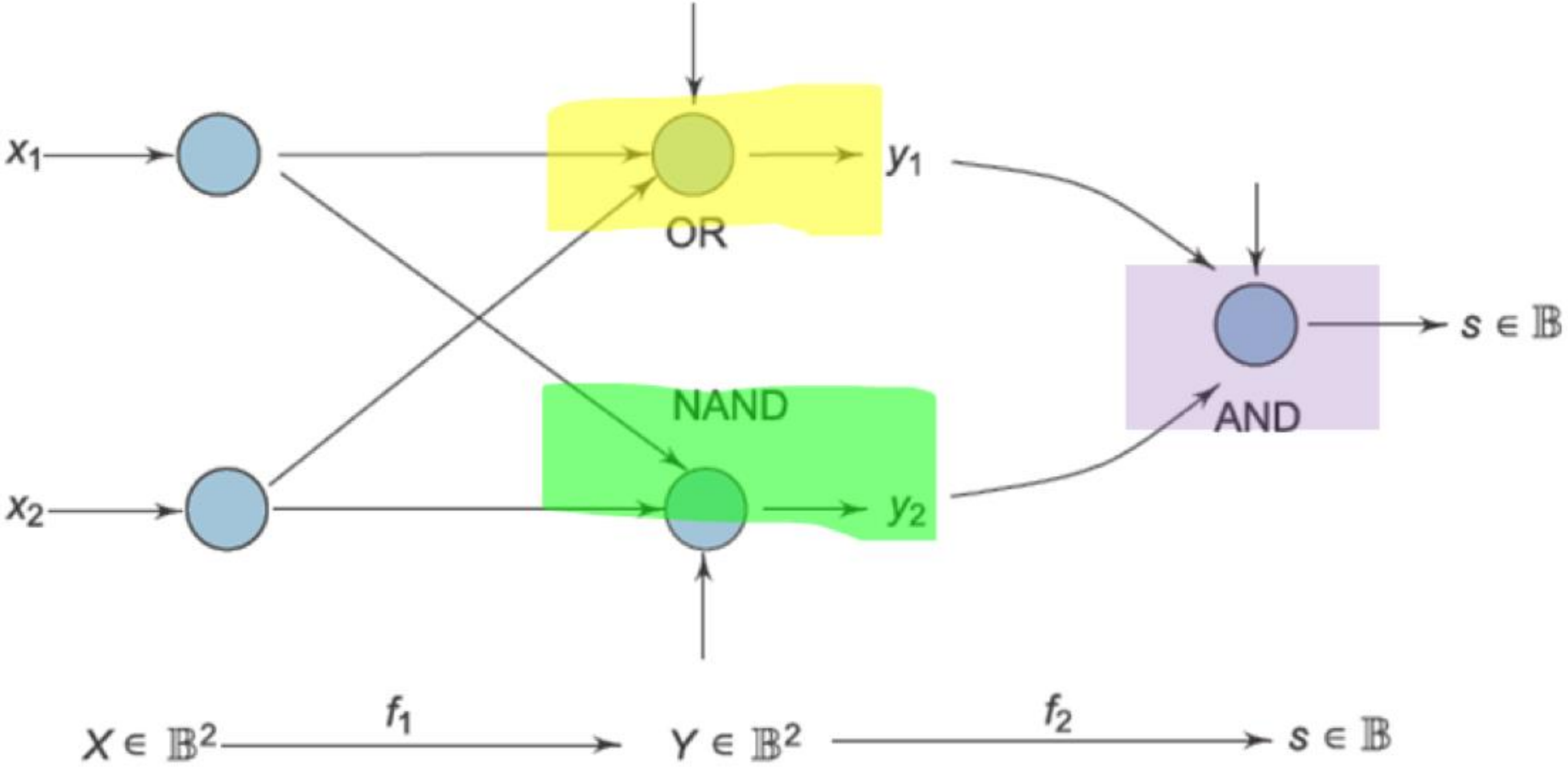
x_1	x_2	
0	0	0
0	1	1
1	0	1
1	1	0

어떻게 해결하지?

- Perceptron은 Linearly separable 한 문제만 풀 수 있다.
- 그럼 문제를 linearly separable하게 만들면 되지 않을까?

- 1. instance 의 숫자를 줄인다.
- 2. feature의 차원을 높인다.

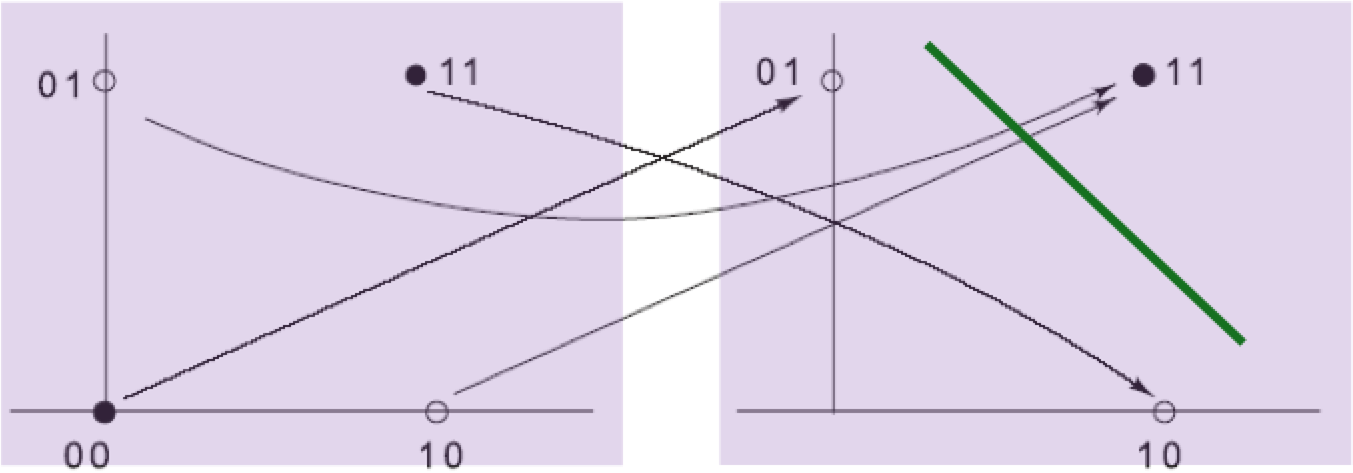
Instance의 개수를 줄인다?



$$y_1 = x_1 + x_2 ; y_2 = \overline{x_1 x_2}$$

x_1	x_2	y_1	y_2	s
0	0	0	1	0
0	1	1	1	1
1	0	1	1	1
1	1	1	0	0

Original input **New input** **Output**

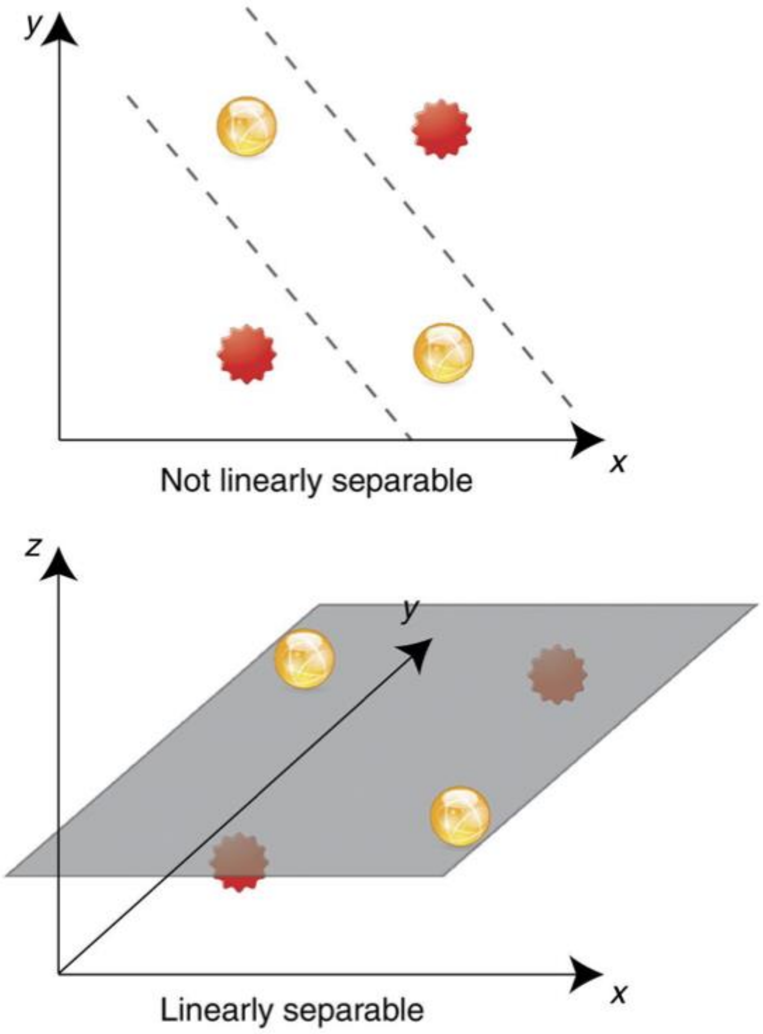


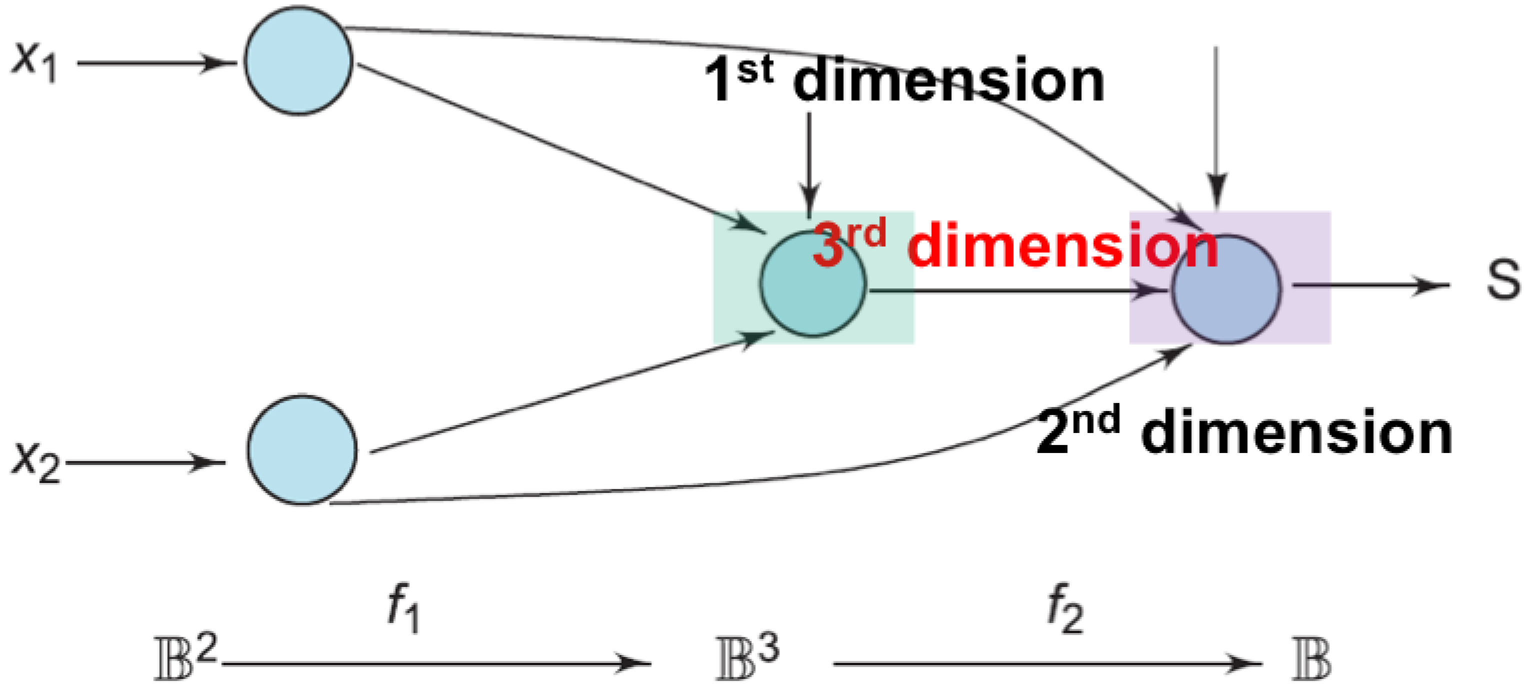
Linearly non-separable

Linearly separable

Feature 의 차원을 높인다?

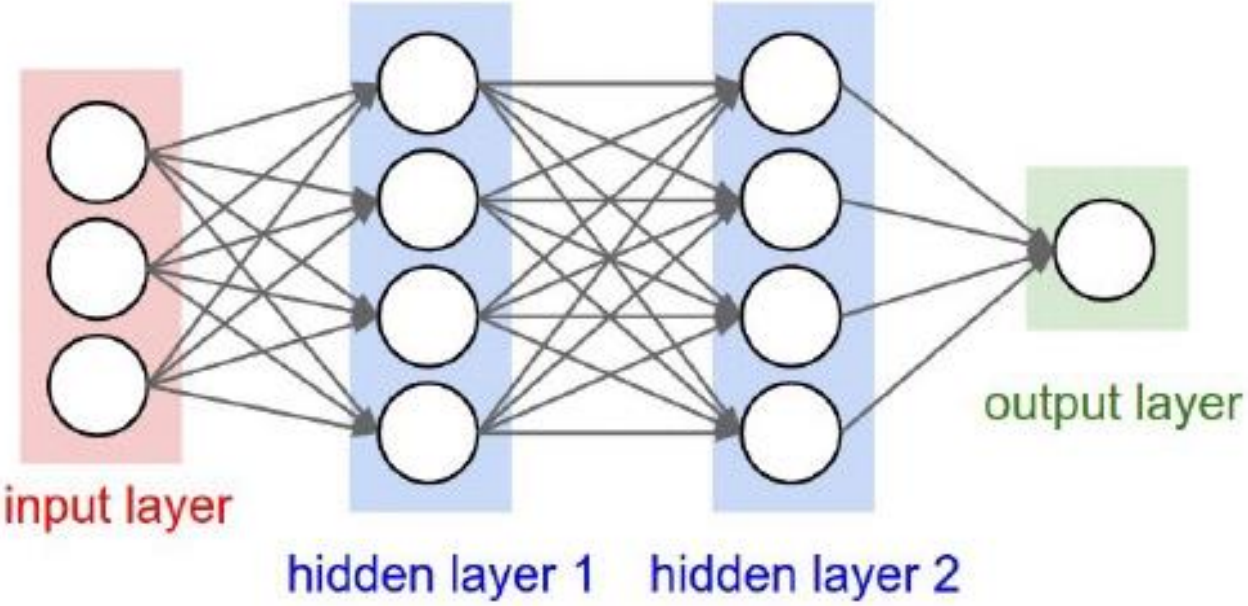
depth





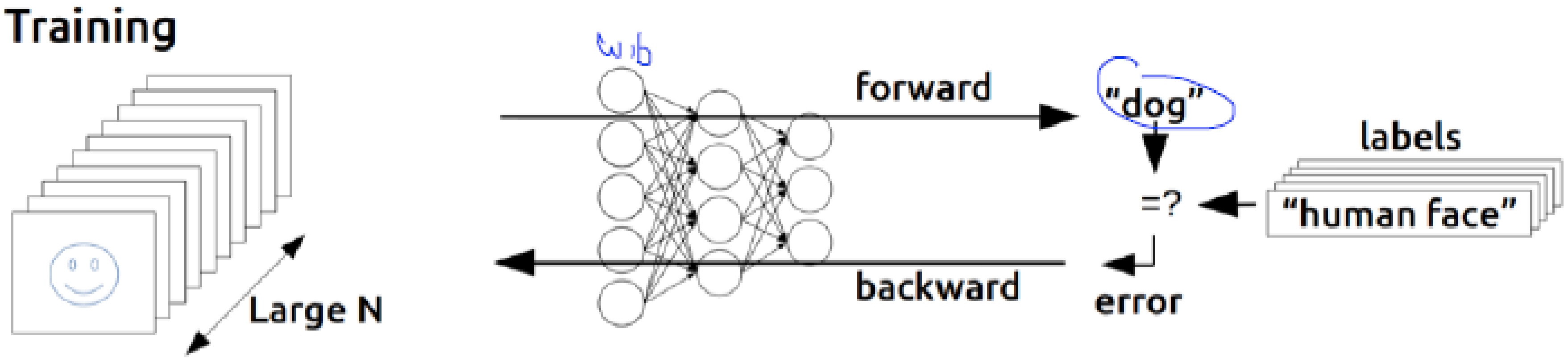
x_1
x_2
1st dimension
3rd dimension
2nd dimension
S
$\mathbb{B}^2 \xrightarrow{f_1} \mathbb{B}^3 \xrightarrow{f_2} \mathbb{B}$

“No one on earth had found a viable way to train*”

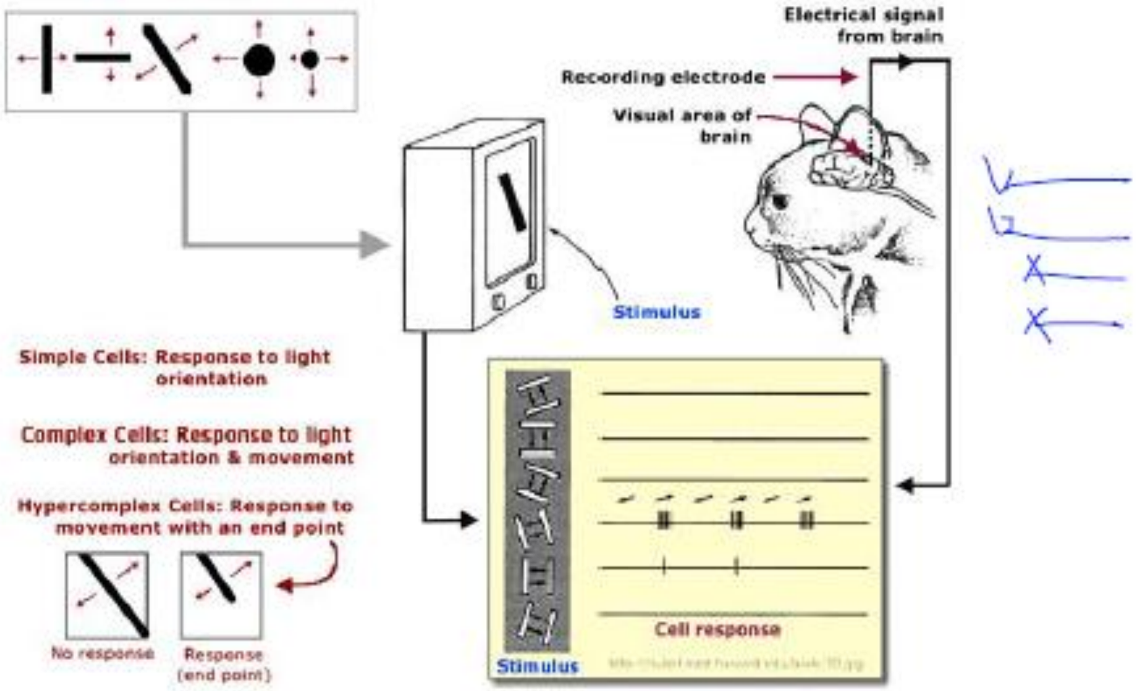


Backpropagation

(1974, 1982 by Paul Werbos, 1986 by Hinton)



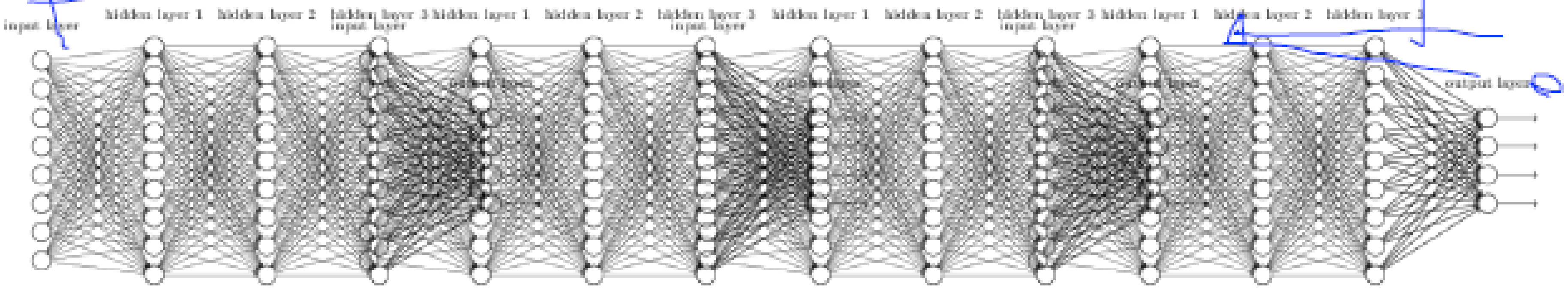
Convolutional Neural Networks



Hubel & Wiesel, 1959

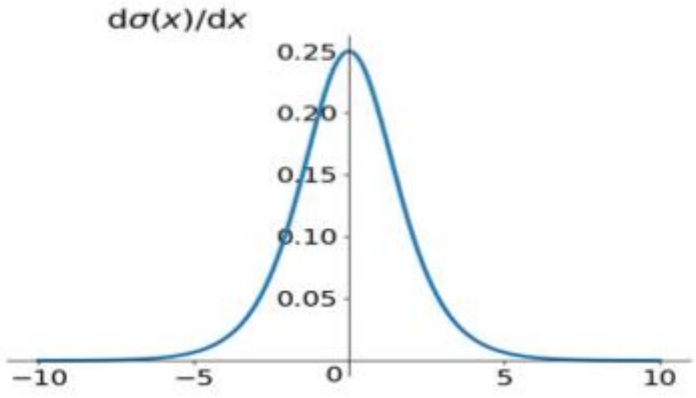
A BIG problem

- Backpropagation just did not work well for normal neural nets with many layers
- Other rising machine learning algorithms: SVM, RandomForest, etc.
- 1995 "Comparison of Learning Algorithms For Handwritten Digit Recognition" by LeCun et al. found that this new approach worked better



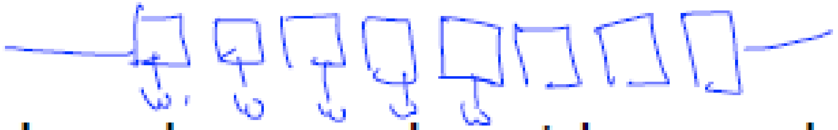
Gradient Vanishing이 일어나는 이유?

- 비 효율적인 Activation function을 썼기 때문.
- 당시 널리 쓰이던 sigmoid로 인한 문제.



Breakthrough

in 2006 and 2007 by Hinton and Bengio



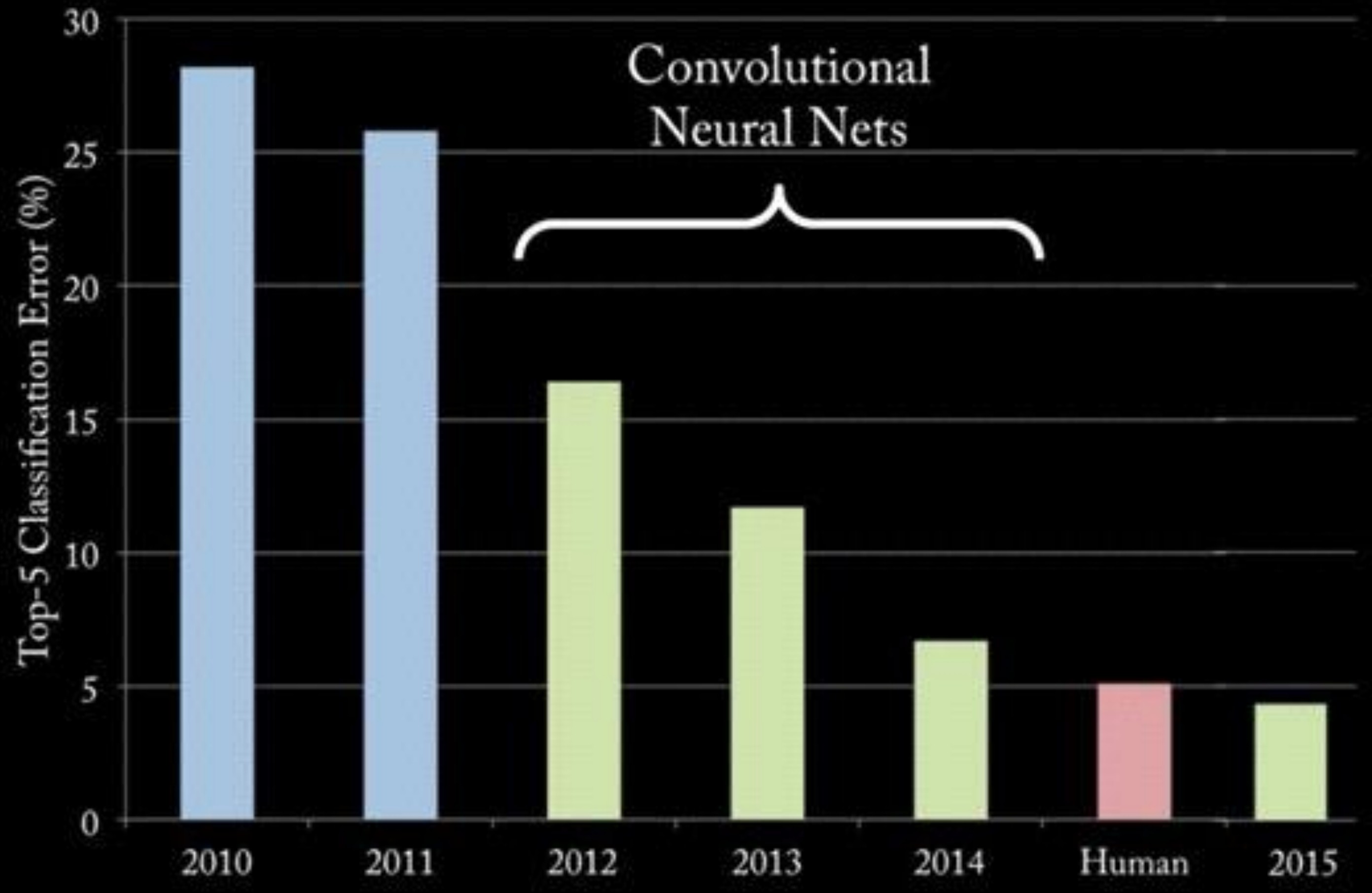
- Neural networks with many layers really could be trained well, *if the weights are initialized in a clever way* rather than randomly.
- Deep machine learning methods are more efficient for difficult problems than shallow methods.
- Rebranding to *Deep Nets*, *Deep Learning*

Initialization이 영향을 많이 미치나?

- 작게 초기화 했을 경우 => 학습이 너무 느림
- 크게 초기화 했을 경우 => bad local minima에 갇힘

- Xavier, He initializer으로 개선.

ImageNet Classification (2010 – 2015)



끝