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[1] Que X_i v.a, tal que $X_i = \begin{cases} 1 & \text{si } (x_i, y_i) \in S \\ 0 & \text{dlc} \end{cases}$

y tiene distribucion de bernoulli

definida por $p_X(x) = \begin{cases} \text{area}(S) & \text{si } x_i = 1 \\ 1-\text{area}(S) & \text{si } x_i = 0. \end{cases}$

Note que $E[S_n] = E\left[\frac{x_1 + x_2 + \cdots + x_n}{n}\right] = \frac{1}{n} E\left[\sum_{i=1}^{n} x_i\right]$

$$= \frac{1}{n} \sum_{i=1}^{n} E[X_i] = \frac{1}{\cancel{n}} \cancel{n} \cdot \text{area}(S) = \underline{\underline{\text{area}(S)}}.$$

$$\text{Var}(S_n) = \text{var}\left(\frac{1}{n} \sum_{i=1}^{n} X_i\right) = \frac{1}{n^2} \sum_{i=1}^{n} \text{var}(X_i) = \frac{1}{n^2} \sum_{i=1}^{n} \text{area}(S)(1-\text{area}(S))$$

$$= \frac{1}{n^{\cancel{2}}} \cancel{n}\, \text{area}(S)(1-\text{area}(S)).$$

$$\text{Var}(S_n) = \underline{\frac{\text{area}(S)(1-\text{area}(S))}{n}}.$$

Observe que si en $\text{Var}(S_n)$ $n \to \infty$ el valor de $\text{Var}(S_n) \to 0$.

[2]. $S_n = \dfrac{X_1 + X_2 + \cdots + X_{n-1} + X_n}{n} = \underbrace{\dfrac{X_1}{n} + \dfrac{X_2}{n} + \cdots + \dfrac{X_{n-1}}{n}}_{n-1} + \dfrac{X_n}{n}.$

$S_n = S_{n-1} + \dfrac{X_n}{n}$

luego para calcular S_n solo es necesario conocer S_{n-1} y $\dfrac{X_n}{n}$.

16. X v.a.n con media $\mu = 1$ y varianza $\sigma^2 = 4$

- $P(0 \leq X < 1) = P(X < 1) - P(X < 0)$ $\qquad Y = \dfrac{X - \mu}{\sigma}$

$P(X<1) = P\left(Y < \frac{1-1}{2}\right) = P(Y<0) = \Phi(0) = 0{,}5$

$P(X<0) = P\left(Y < \frac{0-1}{2}\right) = P(Y < -1/2) = \Phi(-1/2) = 1 - \Phi(1/2)$

$= 1 - 0{,}6915$

$= 0{,}3085$

$P(0 \leq X < 1) = P(X<1) - P(X<0)$

$= 0{,}5 - 0{,}3085$

$\underline{\underline{P(0 \leq X < 1) = 0{,}1915}}$

- $P(X^2 > 4) = P(|X| > 2) = P(X>2, X<-2) = P(X<-2) + P(X>2) =$

$= P(X<-2) + 1 - P(X<2)$

$= P\left(Y < \frac{-2-1}{2}\right) + 1 - P\left(Y < \frac{2-1}{2}\right)$

$= P(Y < -3/2) + 1 - P(Y < 1/2)$

$= \Phi(-3/2) + 1 - \Phi(1/2)$

$= 1 - \Phi(3/2) + 1 - \Phi(1/2)$

$= 2 - 0{,}9332 - 0{,}6915$

$\underline{\underline{P(X^2 > 4) = 0{,}3753}}$

17. $P(X > c) = 0{,}1$ $\qquad Y = \dfrac{X - \mu}{\sigma}$ $\qquad$ X v.a. normal con media $\mu = 12$ y varianza $\sigma^2 = 4$

$P\left(Y > \frac{c-12}{2}\right) = 0{,}1$

$1 - P\left(Y < \frac{c-12}{2}\right) = 0{,}1$

$P\left(Y < \frac{c-12}{2}\right) = 0{,}9$

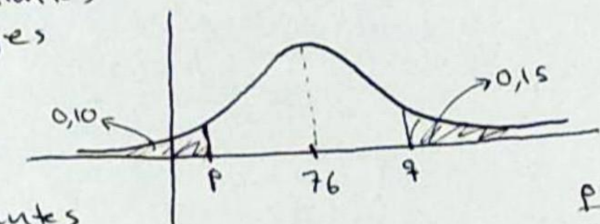
Note que $P(Y < 1{,}29) = \Phi(1{,}29) = 0{,}9015$ luego $\Phi\left(\frac{c-12}{2}\right) \approx \Phi(1{,}29)$

entonces $\frac{c-12}{2} \approx 1{,}29 \rightarrow c \approx 2{,}58 + 12 \rightarrow \underline{\underline{c \approx 14{,}58}}$

$\boxed{8}$ X v.a. normal con media $\mu = 76$ y desviación estandar $\sigma = 15$

X representa los puntajes de un examen.

- 15% de los estudiantes con mejores puntajes obtienen A



p: mínimo para pasar

q: mínimo para sacar A.

- 10% de los estudiantes con peores puntajes perdieron

$$\boxed{Y = \frac{X-\mu}{\sigma}}$$

$P(X<p) = 0{,}10 \qquad P(X>q) = 0{,}15.$

$P(X<p) = P\left(Y < \frac{p-76}{15}\right) = 0{,}10.$ Note que $\Phi(-1{,}28) \approx 0{,}1003$

luego $\Phi\left(\frac{p-76}{15}\right) \approx \Phi(-1{,}28)$ entonces $\frac{p-76}{15} \approx -1{,}28$

$p \approx -19{,}2 + 76$, $\underline{p \approx 56{,}8}$ Puntaje mínimo para pasar

$P(X>q) = 1 - P(X<q) = 1 - P\left(Y < \frac{q-76}{15}\right) = 1 - \Phi\left(\frac{q-76}{15}\right) = 0{,}15$

$\Phi\left(\frac{q-76}{15}\right) = 0{,}85$

Note que $\Phi(1{,}04) = 0{,}8508$ luego $\Phi(1{,}04) \approx \Phi\left(\frac{q-76}{15}\right)$

entonces $\frac{q-76}{15} \approx 1{,}04 \rightarrow q \approx 15{,}6 + 76 = 91{,}6$

$\underline{q \approx 91{,}6}$ Puntaje mínimo para sacar A.

$\boxed{9.}$

$$f_{X,Y}(x,y) = \begin{cases} x+y & 0<x<1;\ 0<y<1 \\ 0 & \text{d/c.} \end{cases}$$

$$F_{X,Y}(x,y) = \int_0^x\int_0^y s+t\ dt\,ds = \int_0^x st + \frac{t^2}{2}\Big|_0^y ds = \int_0^x sy + \frac{y^2}{2}\ ds$$

$$= y\frac{s^2}{2} + \frac{y^2}{2}s\Big|_0^x = y\frac{x^2}{2} + x\frac{y^2}{2}$$

- $$F_{X,Y}(x,y) = \begin{cases} 0 & x<0,\ y<0 \\ \frac{x^2}{2}y + \frac{y^2}{2}x & 0<x,y<1 \\ 1 & x>1,\ y>1. \end{cases}$$

$$f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dy = \int_0^1 x+y\ dy = xy + \frac{y^2}{2}\Big|_0^1 = x + \frac{1}{2}$$

- $$f_X(x) = \begin{cases} x + 1/2 & 0<x<1 \\ 0 & \text{d/c.} \end{cases}$$

de manera análoga

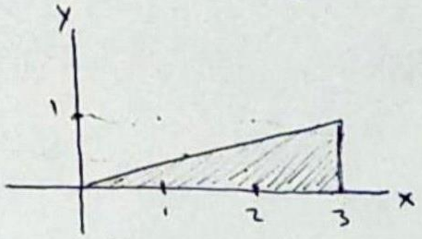
- $$f_Y(y) = \begin{cases} y + 1/2 & 0<y<1 \\ 0 & \text{d/c.} \end{cases}$$

$\boxed{10}$

$$f_{X,Y}(x,y) = \begin{cases} k & 0<x, y<1 ;\ 3y \leq x \\ 0 & \text{dlc.} \end{cases}$$

$f_{X,Y}$ es una función conjunta con distribución uniforme.

$k = \frac{1}{A(S)}$ $\qquad k = 1/\frac{3 \cdot 1}{2} = 2/3$



$$f_{X,Y}(x,y) = \begin{cases} 2/3 & 0<x, y<1 ;\ 3y \leq x \\ 0 & \text{dlc.} \end{cases}$$

$$F_{X,Y}(x,y) = \int_0^y\int_0^x 2/3\, ds\, dt = \int_0^y 2/3\, s\Big|_0^x dt = \int_0^y 2/3\, x\, dt = \frac{2}{3}xt\Big|_0^y = \frac{2}{3}xy$$

- $$F_{X,Y}(x,y) = \begin{cases} 0 & x<0,\ y<0. \\ 2/3\, xy & 0<x<3,\ 0<y<1. \\ 1 & \text{dlc}. \end{cases}$$

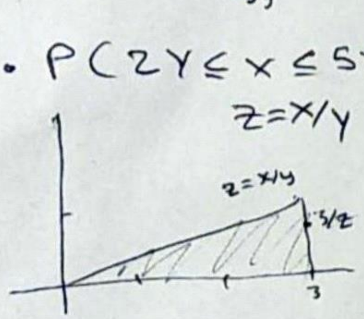
$$f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\, dy = \int_0^{x/3} 2/3\, dy = 2/3\, y\Big|_0^{x/3} = \frac{2}{3}\frac{x}{3} = \frac{2}{9}x$$

- $$f_X(x) = \begin{cases} 2/9\, x & 0<x<3 \\ 0 & \text{dlc.} \end{cases}$$

$$f_Y(y) = \int_{3y}^{3} 2/3\, dx = \frac{2}{3}x\Big|_{3y}^{3} = \frac{2}{\cancel{3}}\cancel{3} - \frac{2}{\cancel{3}}\cancel{3}y = 2 - 2y$$

- $P(2Y \leq X \leq 5Y) = P(X \leq 5Y) - P(X \leq 2Y) = P(X/Y \leq 5) - P(X/Y \leq 2)$

$Z = X/Y$ $\qquad = \frac{9}{2 \cdot 5} - 0 = \underline{\underline{\frac{9}{10}}}$



$$F_Z(z) = P(Z \leq z) = P(X/Y < z) = \begin{cases} \frac{9}{2z} & \text{si } z \geq 3 \\ 0 & \text{dlc} \end{cases}$$