PARTE 2.

$\boxed{5}$ Cajas de 20 juguetes

$C_1 = \{$ sacar una caja sin juguetes defectuosos $\}$ $P(C_1) = 0,80$

$C_2 = \{$ sacar una caja con un juguetes defectuoso $\}$ $P(C_2) = 0,15$

$C_3 = \{$ sacar una caja con dos juguetes defectuosos $\}$ $P(C_3) = 0,05$

$D = \{$ se sacan dos juguetes no defectuosos $\}$

a) $$P(C_1|D) = \frac{P(C_1)P(D|C_1)}{P(D)} = \frac{P(C_1)P(D|C_1)}{P(C_1)P(D|C_1) + P(C_2)P(D|C_2) + P(C_3)P(D|C_3)}$$

$$= \frac{0,80 \cdot 1}{0,80 \cdot 1 + 0,15\,(19/20)(18/19) + 0,05\,(18/20)(17/19)} = \frac{0,80}{0,975} = \boxed{0,82}$$

b) $$P(C_2|D) = \frac{P(C_2)P(D|C_2)}{P(C_2)P(D|C_2) + P(C_1)P(D|C_1) + P(C_3)P(D|C_3)}$$

$$= \frac{0,15 \cdot (19/20)(18/19)}{0,975} = \boxed{0,14}$$

c) $$P(C_3|D) = \frac{P(C_3)P(D|C_3)}{P(D)} = \frac{0,05 \cdot (18/20)(17/19)}{0,975} = \boxed{0,04}$$

$\boxed{6}$

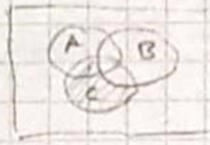
a) $(1-0,0025)^{30}$

b) $(1-0,0025)^{19}(0,0025)$

c) $\sum_{i=0}^{9} (1-0,0025)^{i}(0,0025)$

$\boxed{7}$

a) $P(A\cap B\cap C) = P(A)P(B|A)P(C|A\cap B)$
$= 0{,}3\cdot 0{,}75\cdot 0{,}2 = \boxed{0{,}045}$



b) $P(B^c\cap C) = P(A\cap B^c\cap C) + P(B^c\cap A^c\cap C)$
$= P(A\cap B^c)P(C|A\cap B^c) + P(A^c\cap B^c)P(C|A^c\cap B^c)$
$= P(A)P(B^c|A)\cdot 0{,}8 + P(A^c)P(B^c|A^c)\, 0{,}9$
$= 0{,}3(1-P(B|A)\cdot 0{,}8 + 0{,}7\cdot(1-P(B|A^c))\cdot 0{,}9$
$= 0{,}3(1-0{,}75)\cdot 0{,}8 + 0{,}7(1-0{,}2)\cdot 0{,}9$
$= 0{,}3(0{,}25)\cdot 0{,}8 + 0{,}7(0{,}8)\cdot 0{,}9$
$\boxed{= 0{,}56}$

c) $P(C) = P(A\cap B^c\cap C) + P(A\cap B\cap C) + P(B\cap A^c\cap C) + P(B^c\cap A^c\cap C)$
$= P(A\cap B^c)P(C|A\cap B^c) + 0{,}045 + P(B\cap A^c)P(C|B\cap A^c) + P(B^c\cap A^c)P(C|B^c\cap A^c)$
$= P(A)P(B^c|A)\, 0{,}8 + 0{,}045 + P(A^c)P(B|A^c)\cdot 0{,}15 + P(A^c)P(B^c|A^c)\, 0{,}9$
$= 0{,}3(0{,}25)0{,}8 + 0{,}045 + 0{,}7\cdot 0{,}2\cdot 0{,}15 + 0{,}7\cdot 0{,}8\cdot 0{,}9$
$\boxed{= 0{,}63}$

d) $P(A|C\cap B^c) = \dfrac{P(A)P(C\cap B^c|A)}{P(C\cap B^c)} = \dfrac{P(A)P(C\cap B^c\cap A)}{\cancel{P(A)}P(C\cap B^c)}$

$= \dfrac{P(A\cap B^c)P(C|A\cap B^c)}{P(C\cap B^c)} = \dfrac{P(A)P(B^c|A)\cdot 0{,}8}{0{,}56}$

$= \dfrac{0{,}3\cdot 0{,}25\cdot 0{,}8}{0{,}56} = \boxed{0{,}1}$

$\boxed{8}$ $|X| = 9\cdot 10^9 - 9\cdot 9! = 9(10^9 - 9!)$

$\boxed{9}$ $\underset{k_1}{\overset{n}{\sqcup}}\quad \underset{k_2}{\overset{n-k_1}{\sqcup}}\quad \underset{n-k_1-k_2}{\overset{n-k_1-k_2}{\sqcup}}\qquad \binom{n}{k_1}\binom{n-k_1}{k_2}\binom{n-k_1-k_2}{n-k_1-k_2}$

$$\sum_{k_1=1}^{n}\sum_{k_2=1}^{n}\binom{n}{k_1}\binom{n-k_1}{k_2}\binom{n-k_1-k_2}{n-k_1-k_2}$$

$\boxed{10.}$ $\sum_{k=0}^{n} \binom{n}{k}(-1)^k = 0$

$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k}$

$(-1+1)^n = 0^n = 0 = \sum_{k=0}^{n} \binom{n}{k}(-1)^k \cancel{(1)^{n-k}}^{\,1}$

$0 = \sum_{k=0}^{n} \binom{n}{k}(-1)^k$

$\boxed{11}$. Sea X una v.a. binomial con parametros n y p

Note que $p_X(0) = \binom{n}{0} p^0 (1-p)^{n-0} = (1-p)^n$

Caso 1. $k=0$

$P_X(0+1) = \frac{p}{1-p} \cdot \frac{n-0}{0+1} P_X(0) = \frac{p}{1-p} \cdot \frac{n}{1}(1-p)^n = p\,n(1-p)^{n-1}$

$= \frac{n!}{(n-1)!\,1!} p(1-p)^{n-1} = \binom{n}{1} p(1-p)^{n-1} = P_X(1)$

Caso 2. Supongamos que valido para $k-1$, es decir

$P_X((k-1)+1) = \frac{p}{1-p} \frac{n-(k-1)}{k-1+1} P_X(k-1) = \binom{n}{k} p^k (1-p)^{n-k} = P_X(k)$

demostraremos que se cumple para k.

$P_X(k+1) = \frac{p}{1-p} \frac{n-k}{k+1} p_X(k) = \frac{p}{1-p} \frac{n-k}{k+1} \binom{n}{k} p^k (1-p)^{n-k}$

$= p^{k+1}(1-p)^{n-k-1} \frac{n-k}{k+1} \cdot \frac{n!}{(n-k)!\,k!} = \frac{\cancel{n-k}}{k+1} \cdot \frac{n!}{\cancel{(n-k)}(n-k-1)!\,k!} p^{k+1}(1-p)^{n-k+1}$

$= \frac{n!}{(n-(k+1))!\,(k+1)!} p^{k+1}(1-p)^{n-k-1} = \binom{n}{k+1} p^{k+1}(1-p)^{n-(k+1)}$

$= P_X(k+1)$

$\blacksquare$