Laboratorio 3 Álgebra Lineal

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1) Sea $u = 3i - 2j + 4k$

$v = -6i + 4j - 5k$

$w = i + j + 2k$

$x = 2i + aj - \beta k$

• Encuentre magnitud y ángulos dictores

$|u| = \sqrt{(3)^2 + (-2)^2 + (4)^2} = \underline{5.385}$

$a = \cos^{-1} \frac{3}{|u|}$ $\quad \beta = \cos^{-1} \frac{-2}{|u|}$ $\quad \gamma = \cos^{-1} \frac{4}{|u|}$

$\underline{a = 56.1°}$ $\quad \underline{\beta = 111.8°}$ $\quad \underline{\gamma = 42.03°}$

$|v| = \sqrt{(-6)^2 + 4^2 + (-5)^2} = \underline{8.77}$

$a = \cos^{-1} \frac{-6}{|v|}$ $\quad \beta = \cos^{-1} \frac{4}{|v|}$ $\quad \gamma = \cos^{-1} \frac{-5}{|v|}$

$\underline{a = 133.13°}$ $\quad \underline{\beta = 62.88°}$ $\quad \underline{\gamma = 124.73}$

$|w| = \sqrt{1^2 + 1^2 + 2^2} = \underline{2.44}$

$a = \cos^{-1} \frac{1}{|w|} = \underline{65.9°}$ $\quad \beta = \cos^{-1} \frac{1}{|w|} = \underline{65.9°}$ $\quad \gamma = \cos^{-1} \frac{2}{|w|} = \underline{35.26}$

• ¿Para qué valores de a y β los vectores u y x son paralelos?

$u \times x = 0$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -2 & 4 \\ 2 & a & -\beta \end{vmatrix} = \hat{i}(-2(-\beta)-(4a)) - \hat{j}(-3\beta-8) + \hat{k}(3a+4)$$

$$= \hat{i}(2\beta - 4a) + \hat{j}(3\beta+8) + \hat{k}(3a+4) = 0\hat{i} + 0\hat{j} + 0\hat{k}$$

$0 = 2\beta - 4a$

$0 = 2\left(-\frac{8}{3}\right) - 4\left(-\frac{4}{3}\right)$

$0 = -\frac{16}{3} + \frac{16}{3}$

$\underline{0 = 0}$

$0 = 3\beta + 8$

$0 = 3\left(-\frac{8}{3}\right) + 8$

$\underline{0 = 0}$

$0 = 3a + 4$

$0 = 3\left(-\frac{4}{3}\right) + 4$

$0 = 0$

$\underline{\beta = -\frac{8}{3}}$ $\quad$ $\underline{a = -\frac{4}{3}}$

• ¿Que relación deben cumplir a y β para ser ortogonales?

$u \cdot x = (3 \cdot 2 + (-2)a + 4(-\beta)) = 6\hat{i} - 2a\hat{j} - 4\beta\hat{k} = 0$

$3 - a - 2\beta = 0$

$\underline{\frac{3-a}{2} = \beta}$ //

- Calcule $w \cdot (u + v)$

$$u + v = \hat{\imath}(3-6) + \hat{\jmath}(4-2) + \hat{k}(4-5)$$

$$u + v = -3\hat{\imath} + 2\hat{\jmath} - \hat{k}$$

$$w \cdot (u+v) = (-3 \cdot 1) + (2 \cdot 1) + (-1 \cdot 2)$$

$$= -3 + 2 - 2 = \underline{-3}$$

- Calcule el ángulo entre u y w

$$\cos^{-1} \frac{u \cdot w}{|u||w|} = \frac{(3 \cdot 1) + (-2 \cdot 1) + (4 \cdot 2)}{\sqrt{3^2 + (-2)^2 + 4^2} \cdot \sqrt{1^2 + 1^2 + 2^2}}$$

$$\cos^{-1}\left(\frac{3 - 2 + 8}{\sqrt{29} \cdot \sqrt{6}}\right) = \underline{46.98°}$$

- Calcule $proy_u\, v$

$$proy_u\, v = \frac{v \cdot u}{|u|^2} u = \frac{(-6 \cdot 3) + (-2 \cdot 4) + (-5 \cdot 4)}{3^2 + (-2)^2 + 4^2} = \underline{-\frac{46}{29}}$$

$$= \frac{46}{29} 3\hat{\imath} - \frac{46}{29} 2\hat{\jmath} + \frac{46}{29} 4\hat{k}$$

$$\underline{proy_u\, v = \frac{138}{29}\hat{\imath} - \frac{92}{29}\hat{\jmath} + \frac{184}{29}\hat{k}}$$

2) Encuentre dos vectores unitarios ortogonales tanto a $u = -4\hat{i} - 3\hat{j} + 5\hat{k}$ como a $v = -2\hat{i} - \hat{j} + \hat{k}$

$u \cdot x = 0 \qquad x = a\hat{i} + b\hat{j} + c\hat{k}$

$u \cdot x = (-4a\hat{i} - 3b\hat{j} + 5c\hat{k}) = 0$

$a = \frac{3b - 5c}{-4} = \frac{5}{4}c - \frac{3}{4}b$

$a_1 = 1 \qquad a_2 = 1$

$b_1 = -\frac{4}{3} \qquad b_2 = 0$

$c_1 = 0 \qquad c_2 = \frac{4}{5}$

$\frac{\hat{i} - \frac{4}{3}\hat{j}}{\sqrt{1^2 + (\frac{4}{3})^2}} = \underline{\frac{3}{5}\hat{i} - \frac{20}{9}\hat{j}} \qquad \frac{\hat{i} + \frac{4}{5}\hat{k}}{\sqrt{1^2 + (\frac{4}{5})^2}} = \underline{\frac{5\sqrt{41}}{41}\hat{i} + \frac{4\sqrt{41}}{41}\hat{k}}$

$$\boxed{\begin{aligned} x_1 &= \frac{3}{5}\hat{i} - \frac{20}{9}\hat{j} \\ x_2 &= \frac{5\sqrt{41}}{41}\hat{i} + \frac{4\sqrt{41}}{41}\hat{k} \end{aligned}}$$

$v \cdot y = -2a\hat{i} - b\hat{j} + c\hat{k} = 0$

$a = -\frac{b - c}{2} = \frac{1}{2}c - \frac{1}{2}b$

$a_1 = 1 \qquad a_2 = 1$

$b_1 = -2 \qquad b_2 = 0$

$c_1 = 0 \qquad c_2 = 2$

$\frac{\hat{i} - 2\hat{j}}{\sqrt{1^2 + (-2)^2}} = \underline{\frac{\sqrt{5}}{5}\hat{i} - \frac{2\sqrt{5}}{5}\hat{j}} \qquad \frac{\hat{i} + 2\hat{k}}{\sqrt{1^2 + 2^2}} = \underline{\frac{\sqrt{5}}{5}\hat{i} + \frac{2\sqrt{5}}{5}\hat{k}}$

$$\boxed{\begin{aligned} y_1 &= \frac{\sqrt{5}}{5}\hat{i} - \frac{2\sqrt{5}}{5}\hat{j} \\ y_2 &= \frac{\sqrt{5}}{5}\hat{i} + \frac{2\sqrt{5}}{5}\hat{k} \end{aligned}}$$

3) Calcule el volumen del paralepípedo determinado por los vectores en $\mathbb{R}^3$

$u = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}$, $v = 2\hat{\imath} + \hat{k}$, $w = 4\hat{\jmath}$

$$\text{Volumen} = |u \times v| \cdot w = \det \begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

$$\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & -2 & 3 \\ 2 & 0 & 1 \end{vmatrix} = \hat{\imath}(-2\cdot1 - 3\cdot0) - \hat{\jmath}(1\cdot1 - 3\cdot2) + \hat{k}(1\cdot0 + 2\cdot2)$$

$$= \hat{\imath}(-2) - \hat{\jmath}(-5) + \hat{k}(4) = -2\hat{\imath} + 5\hat{\jmath} + 4\hat{k}$$

$$(-2\cdot0 + 5\cdot4 + 4\cdot0) = \underline{20}$$

Doy mi palabra que he realizado esta actividad con integridad académica.

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