A Novel Inverse Optimization Approach to the Fourth Down Decision in Football

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A selected project submitted to the faculty of
Brigham Young University
in partial fulfillment of the requirements for the degree of

Master of Science

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April 2023

ABSTRACT

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This paper aims to infer the beliefs and biases that coaches have about the likelihood of various outcomes relative to the fourth down decision in football based on their observed actions. Using eight recent seasons of data, we apply a novel inverse optimization approach by resampling observations to create a reweighted data set that, when used to estimate optimal decisions, is consistent with a risk neutral objective. This reweighted data set can then represent the skewed perspective that coaches may have on the outcome of certain plays. With this reweighted data we can better understand the discrepancy between what statistical models prescribe and the decisions coaches are actually making. The results allow us to conceptualize the historical data as having unequal weights across observations in the coaches' minds; some situations weigh heavier in the minds of coaches, while others leave less of an impression.

Keywords: expected points, resampling methods, nflfastR, transition probability estimation, win probability

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CHAPTER 1

INTRODUCTION

One of the most commonly debated decisions in football is the decision of whether and when teams should "go for it" on fourth down. When the game is potentially on the line and a coach makes a fourth down decision that doesn't result in a win, the post-game talk will often focus on that decision as the reason for the loss. If the decision results in a win, attention will be drawn to the coach's foresight and genius. Due to the popularity of football in America and the financial significance of coaching decisions in the National Football League (NFL), many models, research papers, and blog posts have been written to enhance understanding about the fourth down decision and to prescribe optimal behavior.

Despite the attention and analysis that has been applied to this decision, many of the decisions made (about one third) do not agree with the optimal decisions estimated by statistical models. Figure 1.1 shows the most common decision made by coaches in the NFL on fourth down between 2014 and 2021. Each square is a grouping of fourth down plays that occurred within the specified yards-to-go (yards until a first down is achieved) and yards to opponent end zone bin. In general, beyond 40 yards to the opponent end zone, coaches rarely do anything but punt, whereas within 40 yards of the end zone coaches typically punt unless they are very close to the first down line.

Figure 1.2 shows the most frequently prescribed decision using win probability estimates as given by the `nflfastR` package's fourth down model (Carl et al. 2021). Obviously, there is a vast difference between the model-based prescriptions and the actual coaching decisions. There are many more bins in which coaches should be going for it more frequently than kicking a field goal or punting, according to the model.

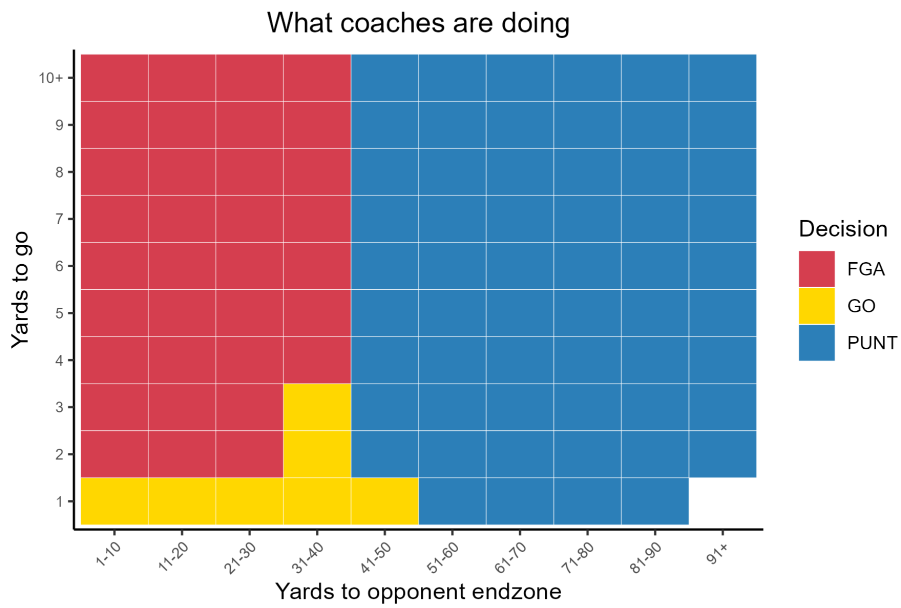

Figure 1.1: Most frequent fourth down decision for each bin in the NFL (2014 - 2021).

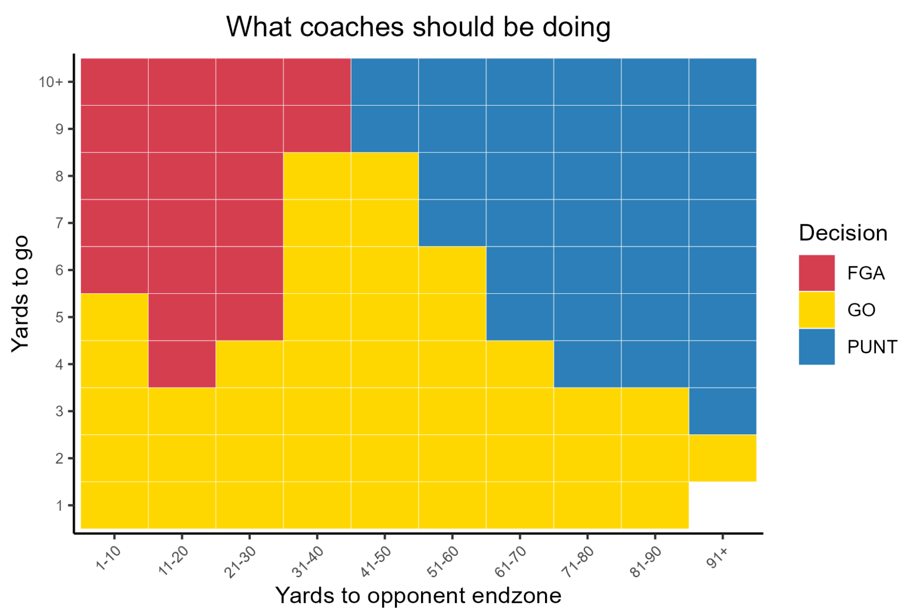

Figure 1.2: Most frequent optimal decision prescribed by `nflfastR` model for each bin (2014 - 2021).

Based on these two figures, we can see that the difference between the actions prescribed by statistical models and the actual actions taken by teams is large. Romer (2006) posits two potential reasons for this gap. The first reason is that the objective function for coaches, owners, players, and others involved in the football teams may not be based on winning alone. Winning may be important, but other factors may make the objective function for these stakeholders more complicated. A common refrain is that job security is more easily retained among coaches who choose to make more "traditional" football decisions, rather than ones that may be controversial if deemed unsuccessful (Owens and Roach 2018). Another more complicated objective function could be when profit maximization and win maximization are not in line with the same team choices.

The second reason he posits is that since coaching staffs often rely on intuition and experience when making this decision, their *beliefs* about the outcome probabilities may vary from the true probabilities. In other words, Romer posits that while staffs may genuinely intend to maximize their teams' chance of winning, "...because they are risk averse in other contexts, experience and intuition may lead them to behave more conservatively than is appropriate for maximizing their chances of winning."

We build on this second reason as motivation for our modeling approach to explain why coaches act they way they do on fourth down decisions. Under this reasoning, the key assumption is that coaches have differing beliefs about the transition dynamics of certain football plays/situations than what would be suggested by the data. In this paper, we essentially treat these beliefs as the estimand of interest.

The goal of this project is to create methodology to infer these skewed beliefs. We will assume that coaches do intend to optimize with respect to win probability, but that they have a biased view of the probabilities of outcomes given certain actions. We will use resampling methods to find a "weighting" of the observations in the data that is more consistent with the coaches' decisions. Then we will use the reweighting to infer the coaches beliefs. This, in turn, will provide a possible explanation for the enigma illustrated in figures 1.1 and 1.2.

CHAPTER 2

LITERATURE REVIEW

Romer (2006), Burke et al. (2014), Owens and Roach (2018), and Yam and Lopez (2018) have all published prior research on the fourth down decision and how the current behavior of coaches is inconsistent with optimal choices. Yam and Lopez estimate the number of additional wins a team could have gained had they followed the strategy from Burke et al. (2014).

As opposed to traditional optimization, inverse optimization takes decisions as the input and infers parameters of the objective function that make these inputs optimal, or at least approximately optimal. Inverse optimization at a high level provides a way to explain the discrepancy illustrated in Section 1. It provides a way to explain the gaps between optimization model prescriptions and observed decisions. Inverse optimization has been widely studied, primarily in the context of estimating objective function parameters of a convex optimization problem, given observed decisions that are assumed to be optimal (Ahuja and Orlin 2001).

Relatedly, reinforcement learning is a sub-field of machine learning methods in which an agent learns an optimal policy through trial and error: incorrect decisions are punished with poor rewards and correct decisions are "reinforced" with positive rewards. Inverse reinforcement learning tries to learn the reward function of a Markov decision process given behavior that is assumed to be optimal (Ng et al. 2000).

In both the case of inverse optimization and inverse reinforcement learning, the estimand typically considered is the reward function (or "cost vector" as commonly referred to in IO). However, in our case, the reward function is fixed and known (e.g., a touchdown has a fixed and known value of 6 points). Although some papers have used inverse optimization and inverse reinforcement learning to learn risk preferences, we take a different approach to the inverse problem. We attempt to learn a coach's beliefs about the transition probabilities of the decision process such that it would render their behavior as optimal.

In another field, a separate but related motivating feature in this paper is the ambiguity effect. Within psychology, the ambiguity effect is a cognitive bias in which people choose a solution or action because of what is "known", even if the actual probability of another solution or action is more desirable. "The effect implies that people tend to select options for which the probability of a favorable outcome is known, over an option for which the probability of a favorable outcome is unknown" (Ellsberg 1961). This is related because we are assuming that the "truth" in going for it on fourth down is not born out in the known probabilities. In this sense, analysts are recommending fourth down strategies that are informed by "known" but misrepresented probabilities, while the current decisions made by coaches can be viewed as optimal based on true but unknown probabilities.

CHAPTER 3

METHODS

We begin this section by discussing how we will attach value to any given game state in football. This is prerequisite to solving the forward optimization problem, which briefly describe in order to set the stage for the inverse optimization problem.

3.1 Models and Metrics

Various metrics that quantify the current state of the game and assess value of specific play outcomes are used as the unit to maximize in models. Among others, win probability (or win probability added) and expected points (or expected points added) are both used as $\hat{v}$, often for different reasons. Expected points and expected points added are typically very consistent throughout the game. These metrics stay considerably stable despite changes in time left in the half/game, game score, time outs remaining by each team, etc. Win probability moves very little at the beginning of the game or when the outcome is very certain. However, when there is not much time left on the clock and changes in game state drastically change the potential outcomes, win probability can vary widely and quickly from one play to the next.

We will use expected points added ($\Delta\hat{v}_i(S_i|a_i)$) as the metric to determine the optimal decision because it is less subject to low to high variability from the beginning to the end of games. In addition, it is essentially situationally agnostic outside of field position and yards to go, which are the factors used in our analysis to determine the current state.

3.2 Forward Optimization

Reviewing forward optimization helps set the stage for how our method will invert the forward process. In addition, we use forward optimization to ensure our inverse optimization method is moving in the right direction (as can be seen in Algorithm 1). While different methods that optimize coaching decisions on fourth down use different models, they generally employ the same objective function—they choose an action that maximizes estimated win probability given the current state and past data.

Objective Function

The generalized objective function used in these forward optimization models is to maximize win probability given the current game state, fourth down decision, and data as follows:

$$\max_{a \in A} \; (\hat{v}|s, a, X) \tag{3.1}$$

where a is the decision variable within the action states A (one of go for it, field goal attempt, or punt). $\hat{v}$ is the estimated team win probability, s is the current state of the team (i.e. field position, yards to go, score, time remaining, etc.), and X is any other historical data used to estimate $\hat{v}$.

Because the true win probability, or value, of each decision on every play is unknown, different models estimate $\hat{v}$ in different ways. The `nfl4th` package in R contains estimates for win probability, win probability added, expected points, and expected points added for each play (Baldwin 2023). In the package, and in general, $\hat{v}$ is estimated conditional on historical data. We are going to bypass the estimation of $\hat{v}$ by leveraging past work. Because all of these metrics are commonly used and accepted in the football analytics community, we rely on these estimates for play value in this project.

Under this context we determine the forward optimal decision as follows:

$$a^{\star} = \arg\max_{a \in A} \frac{\sum_{\{S_i : a_i = a\}} \Delta\hat{v}_i(S_i|a_i)}{N_a} \tag{3.2}$$

where

$$N_a = |\{S_i : a_i = a\}|. \tag{3.3}$$

$\hat{v}_i(S_i|a_i)N_a$ is the change in estimated value of a specific play in a specific game state given the action taken. Action $a^{\star}$ maximizes the change in the estimated value, $\Delta\hat{v}_i(S_i|a_i)$, which depends on state S and action a. N_a is the total amount of plays for a specific action in a specific state. The change in value is calculated as the difference between each play's estimated value on the next play after the fourth down and the estimated value for that fourth down play based on the current state. These differences are then summed and divided by the total plays for that specific action state combination to create an average value $\Delta\hat{v}(S|a)$.

Simply put, for each state (yards to go and yard line bin) and action combination, we calculate the average that play had over the expected value of that play state and choose the decision that maximizes that value. The decision that maximized value in each state can again be seen in figure 1.2.

3.3 Inverse Optimization

In order to infer the beliefs of coaches, we construct an inverse optimization method based on reweighting the data. Essentially, we estimate a weight vector over the observations such that, when equations (3.2) and (3.3) are solved, they harmonize with the coaches' observed decisions. Formally, given a vector of N decisions $\mathbf{a} := (a_1, ..., a_N)$ that were observed in N corresponding fourth down states, we solve

$$\min_{w \in \mathbb{N}^N} \; loss\left(\mathbf{a}, \mathbf{a}^{\star}\left(\mathbf{w}\right)\right) \tag{3.4}$$

where $\mathbf{a}^{\star}(\mathbf{w}) := (a_1^{\star}, ..., a_N^{\star})$, $\mathbf{w} := (w_1, ..., w_{N_a})$, and,

$$a_i^{\star}(w_i) = \arg\max_{a \in A} \; \frac{\sum_{\{S_i : a_i = a\}} w_i \times \Delta\hat{v}_i(S_i|a_i)}{\tilde{N}_a}. \tag{3.5}$$

and

$$\tilde{N}_a = \sum_{i=1}^{N_a} w_i \tag{3.6}$$

We note that that the weights, w, are the estimand of interest. These are adjusted such that the loss function is minimized where the loss is calculated as the difference between the observed actions, $\mathbf{a}$, and the optimal actions $\mathbf{a}^{\star}(\mathbf{w})$ given weight vector $\mathbf{w}$. $\mathbf{a}^{\star}(\mathbf{w})$ in the inverse optimization setting is different than in the forward optimization specification because it maximizes the expected points added conditional on the reweighted data set implied by weight vector $\mathbf{w}$. The $w_i \times \Delta\hat{v}_i(S_i|a_i)$ term is a biased estimate of $\hat{v}$ in equation (3.1) because it has been adjusted by the weights.

Algorithm Implementation

We observed in Figures 1.1 and 1.2 the discrepancy between coaching decisions made on fourth down and the model prescriptions from `nfl4th`. Because our method assumes coaches make the optimal decisions based on lived experience, or the plays that have been observed in our data set, we use a different plot to reconcile what coaches *should* be doing with what coaches *are* doing. Those discrepancies can be seen in figures 3.1 and 3.2. Specifically, figure 3.1 looks different than figure 1.2 because figure 1.2 is smoothed by the model used to construct the decision making. Figure 3.1 looks more choppy because the optimal decision is determined empirically in each bin by choosing the decision that has, on average the highest EPA. Even though these two plots describing what coaches should be doing are different, their trends are the same. A vast majority of the time coaches *should* be going for it on fourth down, while closer to their attacking end zone and with more yards to go kicking a field goal is more frequently deemed optimal. Generally, far from their attacking end zone punting is also more optimal. Even though these two plots don't match exactly, their trends tell the same story and depict in essence what coaches should be doing.

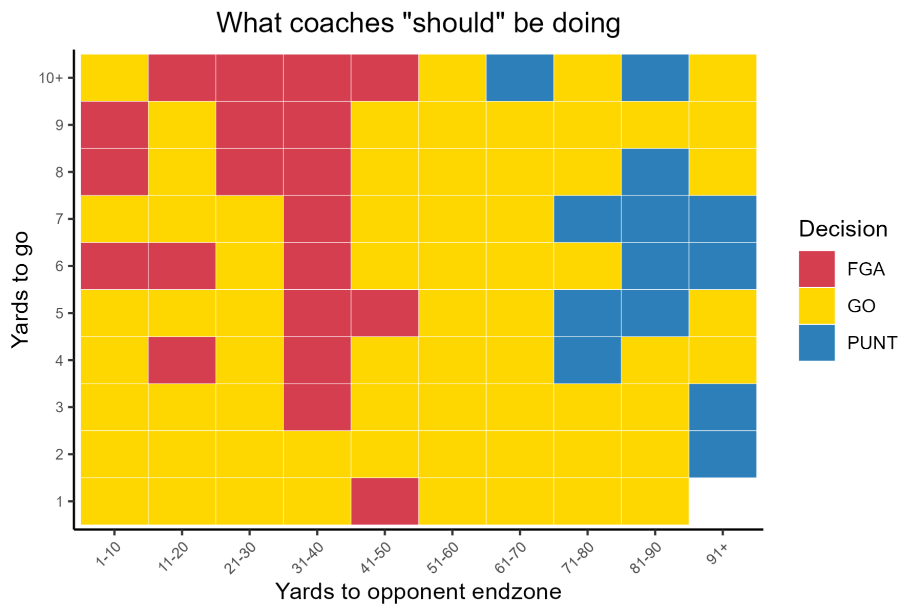

Figure 3.1: What coaches should be doing based not on model output, but on the actual results from the observed data over the eight seasons of data. If model output isn't taken into account, then these are the decisions coaches should be making based on what gives, on average, the highest EPA for each state.

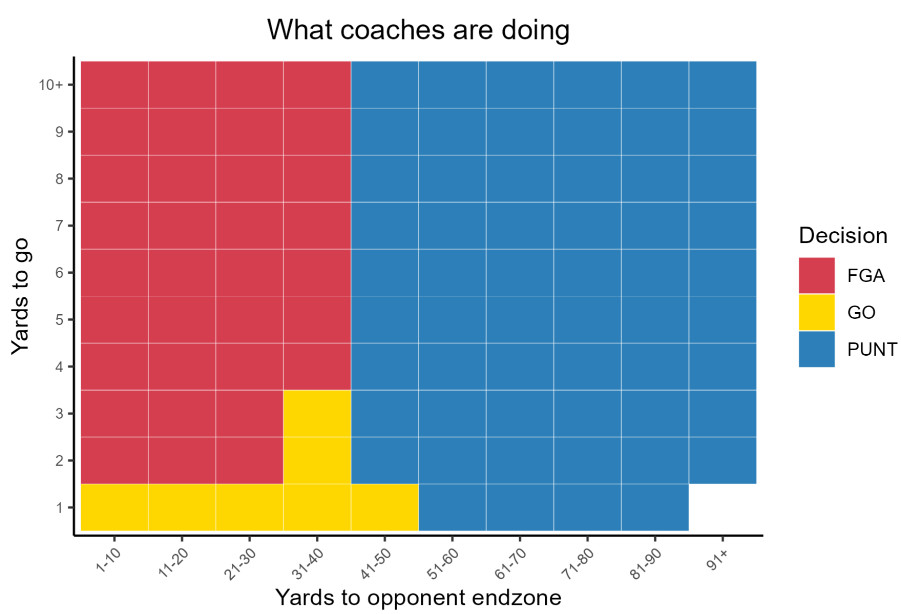

Figure 3.2: Actual most frequent fourth down decisions in the NFL (2014 - 2021).

We specifically target only the play states that have a discrepancy between figures 3.1 and 3.2. The other states are already in line, so we assume that the coaches' decisions are optimal in light of the original historical data. The method we use to find the weights is a random walk in which we sample one play uniformly from a particular yards to go and yard line bin and increment the weight up by one. We then reevaluate the optimal decision for that bin. If the new optimal decision is moving closer to the decision that coaches are making, we keep that weighting. If it does not move closer in this way, we revert to the previous iterations weighting and resample a new play to upweight. This process continues for each play state until the reweighted optimal decisions are in line with the coaches current decisions.

We implemented the general algorithm in Algorithm 1 in order to find a full data set with weights for each play that result in the coaches' decisions on fourth down reflecting the optimal decision for that state.

Algorithm 1 General inverse optimization algorithm

```
for each state (combination of yards to go and position on the field) do
    create a weight vector with all plays having weight = 1
    while optimal decision ≠ coaches' decision do
        randomly sample one play and increase weight by 1
        recalculate mean EPA for each decision
        if mean EPA for each decision is getting closer to coaches' decisions then
            keep the current weighting
        else
            go back to previous step's weighting
        end if
    end while
end for
```

CHAPTER 4

RESULTS

We implemented the algorithm described in the Methods chapter 100 times. Because of the stochastic nature of the algorithm, this results in 100 randomly generated weighted data sets.

The final reweighted data set we used to compare against the original data set was the average of the randomly generated data sets from the inverse algorithm. The final weightings for the punting plays had the highest variability in weightings across the data sets. The largest standard deviation for any specific play was six. Plays in which the decision was to go for it or kick a field goal all had standard deviations around two or below.

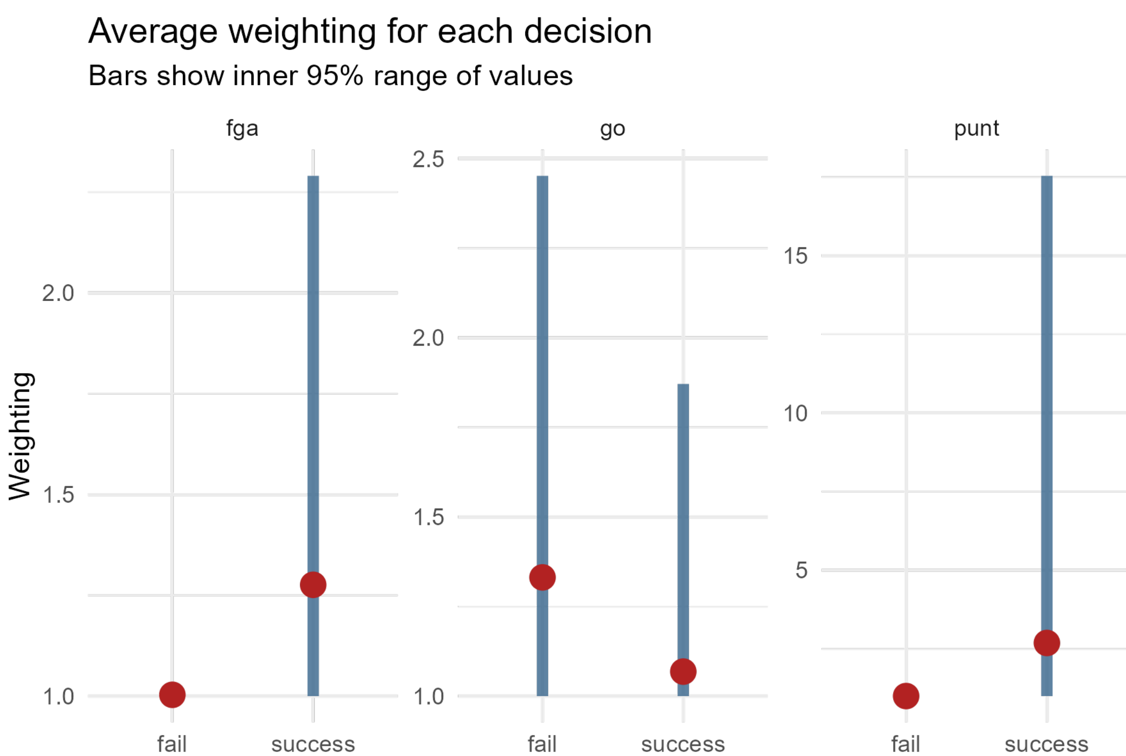

Figure 4.1: Weighting for each decision averaged over the entire reweighted data set with a linerange to represent the lower 2.5% quantile and upper 97.5% quantile of play weightings.

Figure 4.1 shows the average weighting over all plays in the reweighted data set for each decision. The red point represents the overall average weighting while the blue line range represents the lower 2.5% quantile up to the 97.5% quantile of weightings for each action type. Punting plays had a higher average weighting than field goals or going for it, but it also had a much wider range of weightings than the other two actions. We suspect this is because the magnitude of EPA for punting plays is typically very small relative to that of field goal attempts and going for it.

Table 4.2 shows the difference between the original data set and the reweighted data set based on success and failure rates for each action. Successes are determined by a positive EPA play, while failures are determined by a negative EPA play. In the weighted data set, weightings also are taken into account when determining successes. For example, if a positive EPA play had a weighting of three, then the play would count for three successful observations. Thus, the reweighted data set has a greater number of plays corresponding to each action than in the original data set, as can be seen in the final column of Table 4.2. Through this table, the general trend and magnitude of the discrepancy between coaching beliefs and the original observed data is quantified. The greatest difference seems to be the success rate and failure rate of punting. Coaches seem to view punting plays as successful more than 20% more often than in the original data. In other words, coaches might expect better outcomes on punts than they usually get. The reweighted data set reflects that coaches believe going for it is successful less than half the time, 5% less than in the original data. Finally, it seems that coaches' beliefs and observed data line up for field goals relatively similarly, with coaches only slightly overvaluing field goals compared to what the original data would tell us.

Fourth down choices made Based on EPA value on that play			
	Success	Fail	n
Original			
fga	81.5%	18.5%	7,362
go	54.7%	45.3%	4,342
punt	42.9%	57.1%	17,683
Reweighted			
fga	84.7%	15.3%	8,917
go	49.8%	50.2%	5,147
punt	66.8%	33.2%	30,458

Figure 4.2: Table of comparisons between the original data set and the final reweighted data set.

Furthermore, differences between the original data set and the reweighted data set are shown more explicitly for each decision and in each state in Figure 4.3. Overall, field goals have slightly higher value in the reweighted data set than in the original data set. Similarly, punts are valued higher in the reweighted data set than in the original data. In contrast, and with the highest magnitude of disparity, going for it on fourth down is valued much lower in the reweighted data compared to the original data.

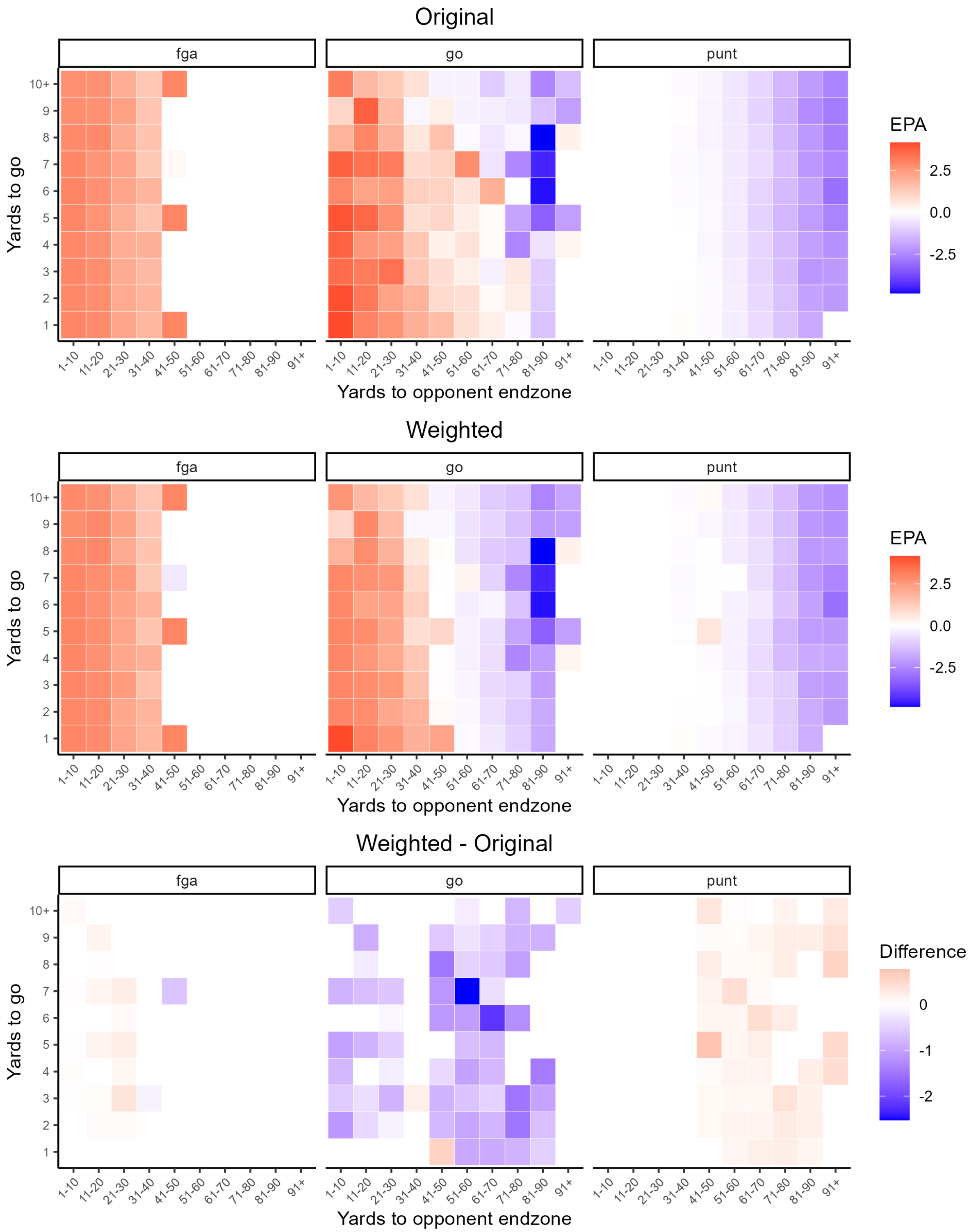

Figure 4.3: Expected Points Added comparison plots along with the difference between the two. Top: Original data set's average EPA for each state given each decision. Middle: Average of the reweighted data sets' EPA for each state given each decision. Bottom: Difference between the reweighted data set and the original data set.

CHAPTER 5

DISCUSSION

Overall, the results from the inverse optimization algorithm indicate the types of biases we would expect in coaches. Based on the common sentiment and from watching football, it is expected that coaches under value going for it on fourth down and over value punting the ball away. Traditional football thinking says to take less risks by ensuring you give the ball to the other team in bad field position by punting the ball away instead of possibly turning the ball over on a failed field goal or by going for it on fourth down and failing.

This analysis, however, sheds even more light on the magnitude of coaches' beliefs regarding the value of certain fourth down decisions. It is insightful that the most controversial action on fourth down, going for it, is successful less than 50% of the time in the reweighted set as opposed to being successful more than 55% of the time in the original data set. In addition, we were able to see that in almost every single yards to go and yard line state, coaches undervalue going for it.

The inverse algorithm also estimated different weights with different variability, depending on the coaches' actions. Specifically, the algorithm was more likely to upweight similar go for it and field goal plays in order to match the coaches' beliefs. On the other hand, the varying weights it used for punting plays was drastically different. One reason for this might be that because most punt plays are all so similar, the algorithm upweighted many different punt plays many times in different iterations of the algorithm in order to match the coaches' beliefs. Further, the marginal gains in EPA that may be made by having a very successful punting play must be upweighted many more times than a successful or unsuccessful attempt to go for it in order to match its total affect in EPA outcome. Thus, a positive punting play may "weigh" 50+ times heavier in the mind of a coach than one successful instance of going for it. Do coaches feel like the likelihood of a significantly positive punting play is that much higher than the likelihood of going for it successfully?

Going deeper than just the overall trends, we can look inside of the data to see examples of what types of plays are heavily weighted in comparison with plays that leave less of an impression. Because reading through every single play description, weighting, and outcome for each state is infeasible, we choose one anecdote from the reweighted data set as a specific example.

Figure 3.1 and 3.2 show a discrepancy in the state of 2 yards to go and within 10 yards of the end zone. Among others, the reweighted data set upweighted two similar missed fourth down plays by a factor of three. In one, out of the shotgun, Aaron Rodgers completed a pass short right to Devonte Adams for no gain, resulting in a turnover and a highly negative EPA. In another, Justin Herbert, again out of the shotgun, completed a pass short right to Keenan Allen for a one yard gain resulting in a turnover on downs and a similarly poor EPA. These overweighting of very adverse plays may be the exact types of plays that coaches overweight in games that pushes them to kick a field goal instead of going for it. In a human sense, it makes sense for a coach to more vividly remember the most extreme circumstances and more improbable for them to remember the less extreme but more common outcomes.

One application of this work is to use this analysis to improve understanding and communication between coaches and analytics personnel. It is easy to explain to someone that they are wrong in their decision making, but it is much harder and more beneficial for both parties to understand where the other is coming from and create solutions. Assuming both parties have the same objective of winning, then the realization of the weightings of certain beliefs can help bridge that gap.

Another application is to use the coaches' beliefs as priors in an analysis. Prior specification can be difficult, particularly for areas with small sample sizes. Using the coaches' beliefs as found by utilizing inverse optimization provides an evidence-based and expert-assumed starting point.

In conclusion, we can see that the beliefs that coaches have about outcomes are much quite dissimilar in some areas of the game and less different in other decision-making areas of the game in a fourth down scenario. Indeed, as Romer said, they do seem to behave more conservatively than is appropriate, but their beliefs about actual outcomes in each decision do inform their consistent decision-making.

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