ENGEN201-22A

Assignment 1 **[20]**

Due Friday 18 March at 1pm

Marks for each question are indicated in square brackets. Marking will be based on the correctness of your answers as well as the correctness of the steps leading to your answers. An incorrect answer with no working will yield minimal marks.

1. Let $u = (3x + 2y)^2 + xy + 1$. Compute partial derivatives u_x, u_y, u_{xx}, u_{xy} and u_{yy}. Do not expand $(3x + 2y)^2$ before differentiating. Cite the chain rule when differentiating $(3x + 2y)^2$. You may expand $(3x + 2y)$ and simplify after differentiating. **[6]**

2. Let $u = f(x) + g(y)$, where f and g are some smooth functions. Show that $u_{xy} = 0$. **[4]**

3. The surface area of a closed cone with radius r and slant height L is $A = \pi r^2 + \pi Lr$. If there are possible errors of $\pm 1\%$ in measuring r and L, estimate the maximum possible error in the calculated value of the surface area A using small increments. **[5]**

4. Let $V = \frac{nRT}{P}$ be the volume of a body of ideal gas with temperature T and pressure P. n and R are constants. Suppose at present the values of T and P are $T = 300$, $P = 1$ (ignoring units), and their present rates of change against time are $\frac{\mathrm{d}T}{\mathrm{d}t} = 1$, $\frac{\mathrm{d}P}{\mathrm{d}t} = 0.001$. What is the value of the rate of change $\frac{\mathrm{d}V}{\mathrm{d}t}$ at present? Express your answer as a multiple of nR. **[5]**

Tutorial problems:

1. **Partial derivatives – higher-order derivatives**

   Find all first- and second-order partial derivatives of $z = 4x^3 - 5xy^2 + 3y^3$.

2. **Partial derivatives – product rule and chain rule**

   If $z = x \cdot f(xy)$, show that $x\dfrac{\partial z}{\partial x} - y\dfrac{\partial z}{\partial y} = z$.

3. **Error estimation from small increments**

   If $\theta = kHLV^{-\frac{1}{2}}$, where k is a constant, and there are possible errors of $\pm 1\%$ in measuring H, L and V, find the maximum possible error in the calculated value of θ.

4. **Rate of change with respect to time – chain rule**

   The voltage V in an electric circuit that satisfies the law $V = IR$ is slowly dropping as the battery wears out. At the same time, the resistant R is increasing as the resistor heats up.

   Express the rate of change of current with respect to time $\dfrac{\mathrm{d}I}{\mathrm{d}t}$, using chain rule. Is the current dropping or rising?