

1 Temperature

1.1 micro state

Definition:

$$\frac{1}{k_B T} = \frac{d \ln \Omega}{dE} \quad (1.1)$$

where Ω is the number of microstate, and k_B is Boltzmann constant:

$$k_B = 1.3807 \times 10^{-23} \text{ J K}^{-1} \quad (1.2)$$

$$\frac{1}{T} = \frac{dS}{dE} \quad (1.3)$$

hence:

$$dS = k_B \ln \Omega \quad (1.4)$$

1.2 Boltzmann factor

The probability $P(\epsilon)$ of a system at energy ϵ is:

$$P(\epsilon) \propto e^{-\epsilon/k_B T} \quad (1.5)$$

We define the quantity to simplify:

$$\beta \equiv \frac{1}{k_B T} \quad (1.6)$$

then the Boltzmann factor is $e^{-\beta E}$, and

$$\beta = \frac{d \ln \Omega}{dE} \quad (1.7)$$

2 Maxwell distribution

2.1 velocity distribution

$$g(v_x) = \sqrt{\frac{m}{2\pi k_B T}} e^{-mv_x^2/2k_B T} \quad (2.1)$$

2.2 speed distribution

For 3 dimensional speed distribution:

$$f(v)dv = \frac{4}{\sqrt{\pi}} \left(\frac{m}{2k_B T} \right)^{3/2} v^2 dv e^{-mv^2/2k_B T} \quad (2.2)$$

which implies the average and rms speed:

$$\bar{v} = \langle v \rangle = \sqrt{\frac{8k_B T}{\pi m}} \quad (2.3)$$

$$v_{\text{rms}} = \sqrt{\langle v^2 \rangle} = \sqrt{\frac{3k_B T}{m}} \quad (2.4)$$

3 Entropy

$$dS = \frac{dQ_{\text{rev}}}{T} \quad (3.1)$$

4 Potentials

4.1 Internal energy

$$dU = TdS - pdV \quad (4.1)$$

which implies:

$$T = \left(\frac{\partial U}{\partial S} \right)_V, p = - \left(\frac{\partial U}{\partial V} \right)_S \quad (4.2)$$

4.2 Enthalpy

Definition:

$$H = U + pV \quad (4.3)$$

then

$$dH = TdS + Vdp \quad (4.4)$$

which implies:

$$T = \left(\frac{\partial H}{\partial S} \right)_p, V = \left(\frac{\partial H}{\partial p} \right)_S \quad (4.5)$$

4.3 Helmholtz function

Definition:

$$F = U - TS \quad (4.6)$$

then

$$dF = -SdT - pdV \quad (4.7)$$

which implies:

$$S = - \left(\frac{\partial F}{\partial T} \right)_S, p = - \left(\frac{\partial F}{\partial V} \right)_T \quad (4.8)$$

4.4 Gibbs function

Definition:

$$G = H - TS \quad (4.9)$$

$$dH = -SdT + Vdp \quad (4.10)$$

which implies:

$$S = - \left(\frac{\partial G}{\partial T} \right)_p, V = \left(\frac{\partial G}{\partial p} \right)_T \quad (4.11)$$

5 Maxwell's relations

$$\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial p}{\partial S}\right)_V \quad (5.1)$$

$$\left(\frac{\partial T}{\partial p}\right)_S = \left(\frac{\partial V}{\partial S}\right)_p \quad (5.2)$$

$$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial p}{\partial T}\right)_V \quad (5.3)$$

$$\left(\frac{\partial S}{\partial p}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_p \quad (5.4)$$

6 Susceptibility

6.1 Expansivity

$$\alpha = \beta_p = \frac{1}{V} \left(\frac{\partial V}{\partial T}\right)_p \quad (6.1)$$

6.2 Compressibility

$$\kappa = \kappa_T = -\frac{1}{V} \left(\frac{\partial V}{\partial p}\right)_T \quad (6.2)$$

7 Heat capacity

$$C_V \equiv \left(\frac{\partial Q}{\partial T}\right)_V = T \left(\frac{\partial S}{\partial T}\right)_V \quad (7.1)$$

$$C_p \equiv \left(\frac{\partial Q}{\partial T}\right)_p = T \left(\frac{\partial S}{\partial T}\right)_p \quad (7.2)$$

$$C_p - C_V = \left[\left(\frac{\partial U}{\partial V}\right)_T + p \right] \left(\frac{\partial V}{\partial T}\right)_p \quad (7.3)$$

$$= \frac{VT\alpha^2}{\kappa} \quad (7.4)$$

adiabatic index:

$$\gamma \equiv \frac{C_p}{C_V} \quad (7.5)$$

8 Partition function

There are some states α in a system. Define the partition function Z :

$$Z = \sum_{\alpha} e^{-\beta E_{\alpha}} \quad (8.1)$$

which implies:

$$U = -\frac{d \ln Z}{d\beta} \quad (8.2)$$

$$S = \frac{U}{T} + k_B \ln Z \quad (8.3)$$

$$F = -k_B T \ln Z \quad (8.4)$$

$$p = -\left(\frac{\partial F}{\partial V}\right)_T = k_B T \left(\frac{\partial \ln Z}{\partial V}\right)_T \quad (8.5)$$

8.1 Density of states

$$g(k)dk = \frac{V k^2 dk}{2\pi^2} \quad (8.6)$$

8.2 Quantum concentration

partition function of one particle:

$$Z_1 \equiv \int_0^{\infty} e^{-\beta E(k)} g(k) dk = V n_Q \quad (8.7)$$

where

$$n_Q = \frac{1}{h^3} \left(\frac{mk_B T}{2\pi}\right)^{3/2} \quad (8.8)$$

and thermal wavelength:

$$\lambda_{th} = n_Q^{-1/3} = \frac{h}{\sqrt{2\pi m k_B T}} \quad (8.9)$$

8.3 N particles

For N indistinguishable particles:

$$Z_N = \frac{(Z_1)^N}{N!} \quad (8.10)$$

which is satisfied when

$$[\text{number density}] \ll n_Q \quad (8.11)$$

9 Black body

9.1 Plank distribution

$$u_{\nu} = \frac{8\pi h}{c^3} \frac{\nu^3}{e^{\beta h \nu} - 1} \quad (9.1)$$

9.2 Stefan-Boltzmann law

power incident per unit area:

$$J = \sigma T^4 \quad (9.2)$$

where

$$\sigma = \frac{\pi^2 k_B^4}{60 c^2 \hbar^3} \approx 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4} \quad (9.3)$$

10 van der Waals gas

molar volume $V_m = V/n$. Definition:

$$\left(p + \frac{a}{V_m^2}\right)(V_m - b) = RT \quad (10.1)$$

critical point satisfies:

$$\left(\frac{\partial p}{\partial V}\right)_T = 0, \left(\frac{\partial^2 p}{\partial T^2}\right)_T = 0 \quad (10.2)$$

hence:

$$V_c = 3b \quad (10.3)$$

$$T_c = \frac{8a}{27Rb} \quad (10.4)$$

$$p_c = \frac{a}{27b^2} \quad (10.5)$$

11 Phase transition

11.1 Latent heat

When changing from phase 1 to phase 2 at constant temperature T_c , it needs "latent heat":

$$L = \Delta Q_{\text{rev}} = T_c(S_2 - S_1) \quad (11.1)$$

11.2 Clausius-Clapeyron

$$\frac{dp}{dT} = \frac{L}{T(V_2 - V_1)} \quad (11.2)$$

12 Bose-Einstein and Fermi-Dirac distribution

12.1 bosons and fermions

When exchange particles, if the wave function is symmetric, the particles are called bosons. On the other hand, if it's antisymmetric, they are fermions.

12.2 Partion function

$$Z = \sum_n e^{-n\beta E} \quad (12.1)$$

For fermions, because of Pauli exclusion principle, $n = 0, 1$. And for bosons, $n = [0, \infty)$. Therefore:

$$\ln Z = \pm \ln(1 \pm e^{-\beta E}) \quad (12.2)$$

where $+$ for fermions and $-$ for bosons.

The mean number of particles, same as the distribution function $f(E)$:

$$\langle n \rangle = f(E) = \frac{1}{e^{\beta E} \pm 1} \quad (12.3)$$

where $+$ for Fermi-Dirac and $-$ for Bose-Einstein distribution function.