Causal Inference

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Prerequisite

- Probability, Chain-Rule, Conditional Independence
- Graphs, Graphical Model, and d-separation

Credit

Many slides are inspired from lecture notes by Prof. Elias Bareinboim at Columbia University

Overview

What is Causality?

Definition

“Causality ... is influence by which **one** event, process, state, or object (a cause) **contributes to the production of another** event, process, state, or object (an effect) where the cause is **partly** responsible for the effect, and the effect is **partly** dependent on the cause.” (emphasis mine)*

- (News) Papers: ‘increases’, ’decreases’ vs. linked to, associated with
- Daily Life: because, hence, thus, due to, ...

* Wikipedia `https://en.wikipedia.org/wiki/Causality`

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Why Do We Study Causality?

- Definition of **Science**
  - "Knowledge or a system of knowledge covering general truths or **the operation of general laws** especially as obtained and tested through scientific method."*

- Causality in various academic disciplines
  - Physics, Chemistry, Biology, Climate Science,
  - Psychology, Social Science, Economics,
  - Epidemiology, Public Health
    (COVID-19, mask policy, social distancing, # of vaccination, side effects)

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How is Causality related to & {AI, ML, & DS}?

Artificial Intelligence

a rational agent performing actions to achieve a goal
e.g., reinforcement learning $\pi_\theta(\text{action} \mid \text{state})$

Machine Learning

Currently focused on learning correlations, e.g., $\hat{P}_\theta(y|\mathbf{x}) \approx P(y|\mathbf{x})$

Data Science

Capture, Process, Analyze (e.g., Stat, ML), Communicate with Data

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Pearl's Causal Hierarchy

- Level 1: Associational or Observational
- Level 2: Interventional or Experimental
- Level 3: Counterfactual

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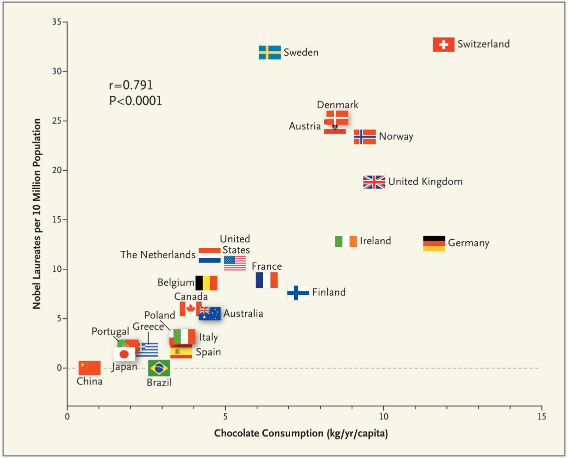
Correlation (Level 1) vs. Causation (Level 2)

Chocolate Consumption vs. Nobel Laureates



Franz H. Messerli, Chocolate Consumption, Cognitive Function, and Nobel Laureates, N Engl J Med 2012; 367:1562-1564, `https://www.alexhahnillustrator.com`

Simpson's Paradox

Consider the following scenario:

1. A Patient with Kidney Stone visits a Hospital.
2. A Doctor examines the Patient and provides a Treatment.
3. The Patient's Health Outcome is later reported.

Healthcare Database!

Simpson's Paradox

		Treatment A	Treatment B
Stone	Small	Group 1 93% (81/ 87)	Group 2 87% (234/270)
	Large	Group 3 73% (192/263)	Group 4 69% (55/ 80)

Each cell represents $P(\text{success} \mid \text{treatment}, \text{stone})$

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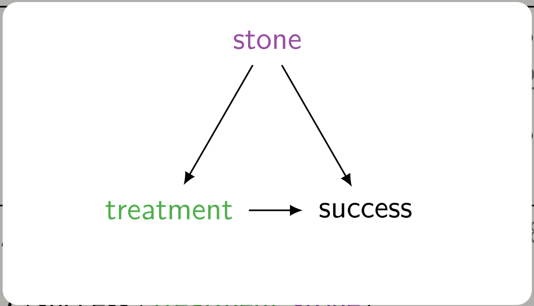
Simpson's Paradox

		Treatment	
		A	B
Stone			up 2
			234/270)
			up 4
			55/ 80)
A			89/350)



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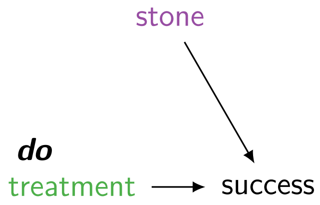

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What if we administer each treatment *randomly*?

Simpson's Paradox

		Treatment A	Treatment B
Stone	Small	Group 1 93% (81/ 87)	Group 2 87% (234/270)
	Large	Group 3 73% (192/263)	Group 4 69% (55/ 80)
	Efficacy	83.2%	78.2%

What if we administer each treatment *randomly*? The causal effect of A

$$P(\text{succ} \mid A) \neq P(\text{succ} \mid do(A)) = \sum_{\text{stone}} P(\text{succ} \mid A, \text{stone}) P(\text{stone})$$

Lesson's Learned from Simpson's Paradox

- Causal analyses need to be guided by subject-matter knowledge 📚.
- Identical data arising from different causal structures need to be analysed differently.
- No purely statistical rules exist to guide causal analyses.

Hernán et al. (2011) The Simpson's paradox unraveled. International Journal of Epidemiology 2011;40:780–-785

Data & Questions

Data scientists should take care of the types of **data** and **question**:

		Question	
		Non-Causal	Causal
Data	Non-Causal	Observational Study, Machine Learning*	**Causal Inference**
	Causal	**Causal Inference**	Experimental Study, Reinforcement Learning

*ML, in general, does not care about the type of data but the question should match the data type.

Formalizing Causality

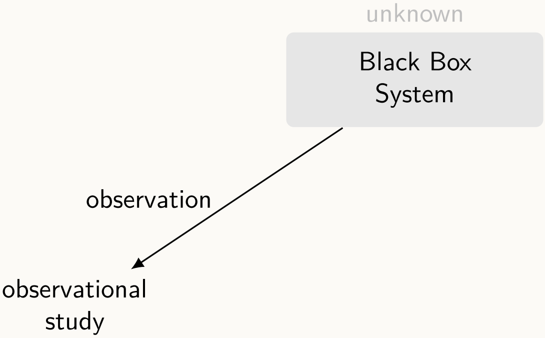
Observation & Intervention (Experiments)

unknown

Black Box
System

Formalizing Causality

Observation & Intervention (Experiments)



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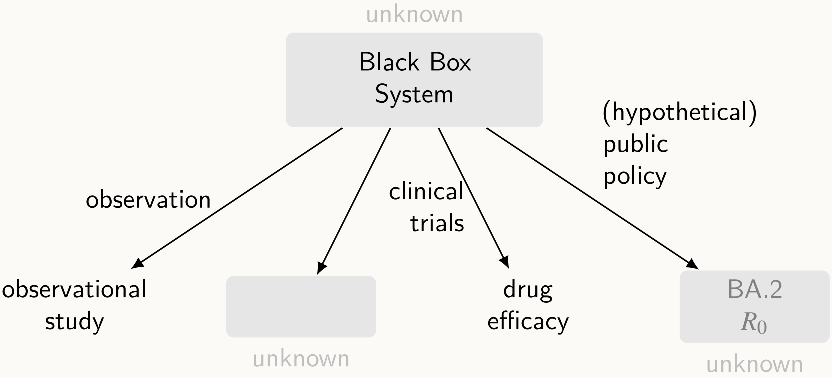

Causal Framework: Structural Causal Model

Structural Causal Model (SCM, Pearl [2000]) is a formal framework to study causality.

unknown

$$
\begin{tikzcd}
 & \text{SCM } \mathscr{M} \;\; \langle \mathbf{U},\mathbf{V},\mathbf{F},P(\mathbf{U})\rangle \arrow[dl, "do(\emptyset)"'] \arrow[d, "do(x_1)"] \arrow[dr, "do(x_2)"] \arrow[drr, "do(x_2,x_4)"] & & \\
P(\mathbf{V}) & P_{x_1}(\mathbf{V}) & P_{x_2}(\mathbf{V}) & P_{x_2,x_4}(\mathbf{V})
\end{tikzcd}
$$

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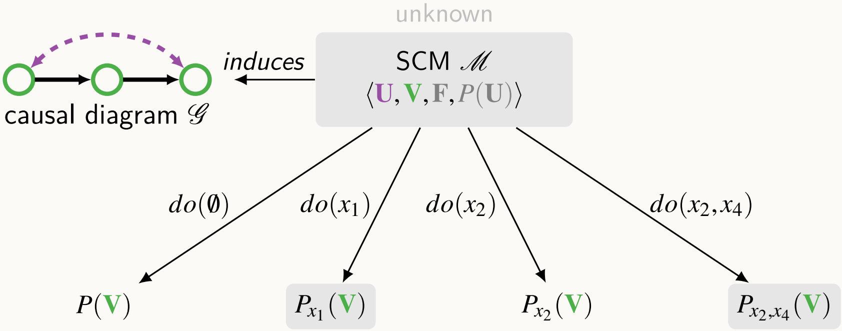

Causal Framework: Structural Causal Models

Definition (Structural Causal Model)

A structural causal model (SCM) $\mathscr{M}$ is a 4-tuple $\langle \mathbf{V}, \mathbf{U}, \mathbf{F}, P(\mathbf{U}) \rangle$, where

- $\mathbf{U}$ is a set of **exogenous** variables;
- $P(\mathbf{U})$ is a distribution over $\mathbf{U}$;
- $\mathbf{V} = \{V_1, \ldots, V_n\}$ are **endogenous** variables;
- $\mathbf{F} = \{f_1, \ldots f_n\}$ are functions determining $\mathbf{V}$,

$$v_i \leftarrow f_i(\mathbf{pa}_i, \mathbf{u}_i)$$

where $\mathbf{Pa}_i \subseteq \mathbf{V} \setminus \{V_i\}$, $\mathbf{U}_i \subseteq \mathbf{U}$.

Causal Diagram $\mathscr{G}$ is a View for Causal Model $\mathscr{M}$

$$\langle \underbrace{\mathbf{V}}_{\text{observed}}, \underbrace{\mathbf{U}}_{\text{unobserved}}, \underbrace{\mathbf{F}}_{\text{mechanisms for } \mathbf{V}}, P(\mathbf{U})\rangle$$

- $\mathbf{V} = \{\text{Smoking}, \text{Cancer}\}$
- $\mathbf{U} = \{U_S, U_C, U_{Gene}\}$
- $\mathbf{F}$:
$$\begin{cases} \text{Smoking} & \leftarrow f_{\text{Smoking}}(U_S, U_{Gene}) \\ \text{Cancer} & \leftarrow f_{\text{Cancer}}(\text{Smoking}, U_C, U_{Gene}) \end{cases}$$

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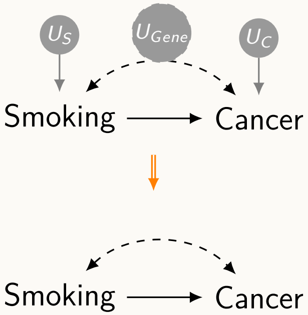

Intervention — $do(\cdot)$ operator

- Given a model $\mathscr{M}$ the action of fixing any observable variable $X \in \mathbf{V}$ to a constant value x is denoted using the $do(\cdot)$ operator as $do(X = x)$.

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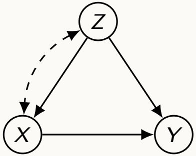
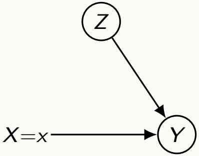
- This operation gives birth to a **submodel** $\mathscr{M}_x$ that looks exactly like $\mathscr{M}$ but with functions where f_x has been replaced with a constant x.
- These two graphs represent the world *before* and *after* an intervention $do(X = x)$.



Causal Graph $\mathscr{G}$



Causal Graph under Intervention $\mathscr{G}_{\overline{X}}$

Intervention — Causal Effects

Definition (Causal Effect)

Given two disjoint sets of variables, $\mathbf{X}$ and $\mathbf{Y}$, the causal effect of $\mathbf{X}$ on $\mathbf{Y}$, denoted as $P(\mathbf{y}|do(\mathbf{x}))$ or $P_{\mathbf{x}}(\mathbf{y})$, is a function from $\mathbf{X}$ to the space of probability distributions of $\mathbf{Y}$.

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Researchers may be interested in

- Expectation: $\mathbb{E}[Y|do(x)]$
- Difference: $\mathbb{E}[Y|do(X=1)] - \mathbb{E}[Y|do(X=0)]$ (Average Treatment Effect, ATE)
- Conditional: $\mathbb{E}[Y|do(X=1),\mathbf{Z}] - \mathbb{E}[Y|do(X=0),\mathbf{Z}]$ (Conditional ATE)

Reading Conditional Independence from Causal Diagram

'Separation' in graph $\mathscr{G}$ implies 'Conditional Independence' in distribution P:

$$(X \perp\!\!\!\perp_{\mathscr{G}} Y \mid Z) \implies (X \perp\!\!\!\perp_{P} Y \mid Z).$$

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$$\begin{tikzcd}
 & \text{season} \arrow[dl] \arrow[dr] & \\
\text{sprinkler} \arrow[dr] & & \text{rain} \arrow[dl] \\
 & \text{wet} \arrow[d] & \\
 & \text{slippery} &
\end{tikzcd}$$

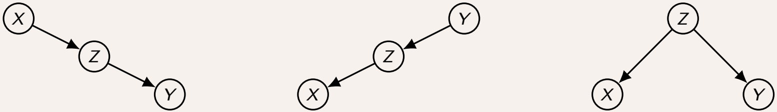
1. ✗ CI: (Wet $\perp\!\!\!\perp$ Sprinkler)
2. ✗ CI: (Wet $\perp\!\!\!\perp$ Season | Sprinkler)
3. ✓ CI: (Rain $\perp\!\!\!\perp$ Slippery | Wet)
4. ✓ CI: (Season $\perp\!\!\!\perp$ Wet | Sprinkler, Rain)
5. ✗ CI: (Sprinkler $\perp\!\!\!\perp$ Rain | Season, Wet)

Reading Conditional Independence from Causal Diagram

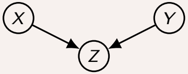
Definition (d-separation)

Two vertices X, Y are said to be d-separated by a set $\mathbf{Z}$ in a directed acyclic graph $\mathscr{G}$, denoted by $(X \perp\!\!\!\perp_{\mathscr{G}} Y \mid \mathbf{Z})$, if every path $\mathbf{p}$ from X to Y are blocked where blockage occurs when one of the following holds:

1. $\mathbf{p}$ contains at least one arrow-emitting node that is in $\mathbf{Z}$, or



2. $\mathbf{p}$ contains at least one collider that is outside $\mathbf{Z}$ and has no descendant in $\mathbf{Z}$.



Generalized to two sets $\mathbf{X}$ and $\mathbf{Y}$ and with bidirected edges.

Summary for Part 1

- **Structural Causal Model** $\mathscr{M} = \langle \mathbf{U}, \mathbf{V}, \mathbf{F}, P(\mathbf{U}) \rangle$ provides a formal framework.
- SCM induces observational, interventional, and counterfactual distributions.
- SCM *induces* a **causal graph** $\mathscr{G}$, which implies **conditional independencies** testable via **d-separation** (blockage).
- The underlying model $\mathscr{M}$ is unknown but the causal graph $\mathscr{G}$ can be given from *common sense* or *domain expertise*.
- **Intervention** $do(\mathbf{X} = \mathbf{x})$ as a **submodel** $\mathscr{M}_{\mathbf{x}}$, which induces a manipulated causal graph $\mathscr{G}_{\overline{\mathbf{X}}}$.
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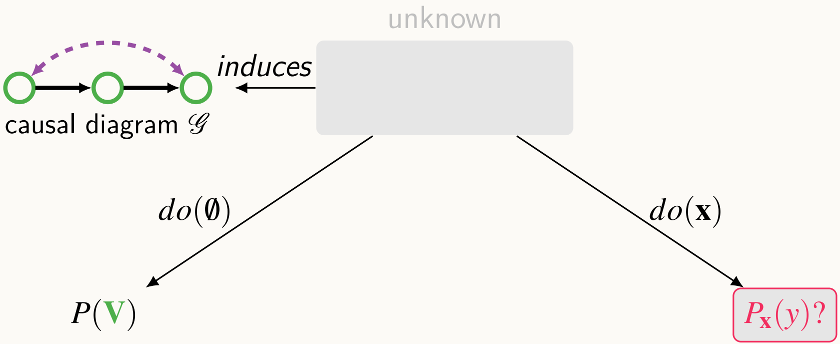
Next Part ...



* illustrative purposes

Overview

Part 1: Causality

Part 2: Causal Effect Identification

- Back-door Criterion
- Do-Calculus

Part 3: Modern Identification

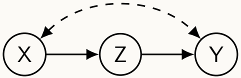
- Generalized Identification
- Transportability
- Recovering from Selection Bias
- Recovering from Missing Data

Causal Effect Identifiability

1. Query

$P_{\mathbf{x}}(\mathbf{y}) = P(\mathbf{y} | do(\mathbf{x}))$

2. Causal Diagram

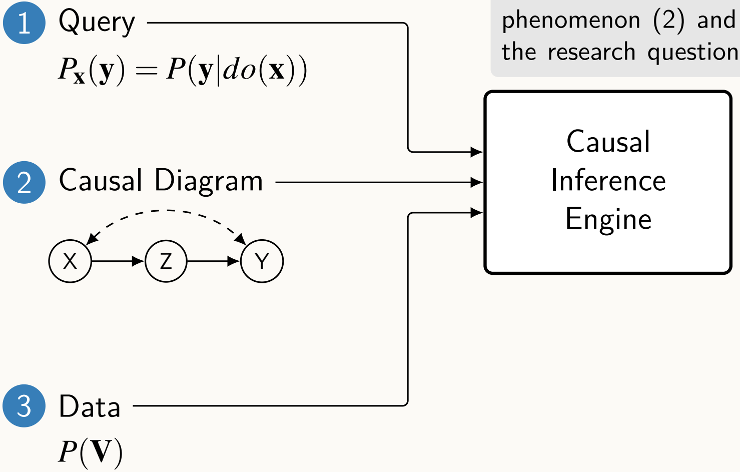


3. Data

$P(\mathbf{V})$

Causal Effect Identifiability

Based on the current knowledge about the phenomenon (2) and the available data (3), is the research question (1) identifiable?



Causal Effect Identifiability

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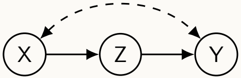


3. Data

   $P(\mathbf{V})$

Causal Inference Engine

Solution

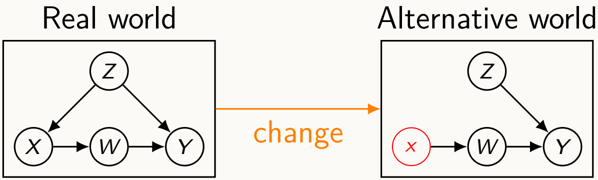
Yes / No

formula

$P_{\mathbf{x}}(\mathbf{y}) = f_{\mathscr{G}}(P(\mathbf{V}))$

evidence

Computing Causal Effects from Observational Data



Z: age, sex
X: action
W: mediator
Y: outcome

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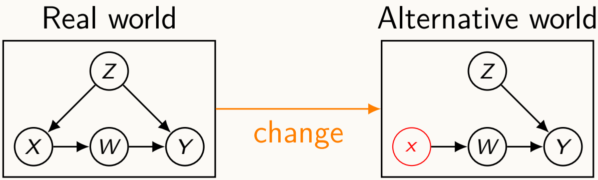

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$$P(\mathbf{v}) = P(z) \times P(x|z) \times P(w|x) \times P(y|w,z)$$

$$\xrightarrow{do(x)}$$

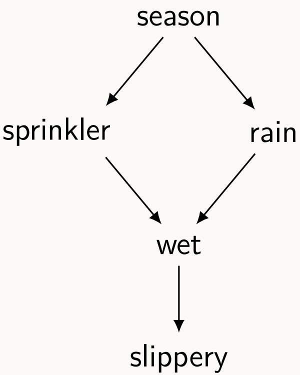
$$P_x(\mathbf{v} \backslash \{X\}) = P(z) \times \cancel{P(x|z)} \times P(w|x) \times P(y|w,z)$$

$\longleftarrow$ equal to 1 in $\mathscr{M}_x$

Computing Causal Effects from Observational Data

This distribution decomposes as

$$P(\mathbf{V}) = P(\mathsf{Sl}|W)P(W|\mathsf{Sp},R)P(\mathsf{Sp}|\mathsf{Sn})P(R|\mathsf{Sn})P(\mathsf{Sn})$$



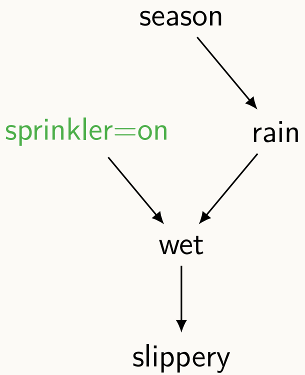
sn:season, sp:sprinkler, sl:slippery

Computing Causal Effects from Observational Data

This distribution decomposes as

$$P(\mathbf{V}) = P(\mathsf{Sl}|W)P(W|\mathsf{Sp},R)P(\mathsf{Sp}|\mathsf{Sn})P(R|\mathsf{Sn})P(\mathsf{Sn})$$

$$P(W \mid do(\mathsf{Sp} = \mathsf{on}))$$

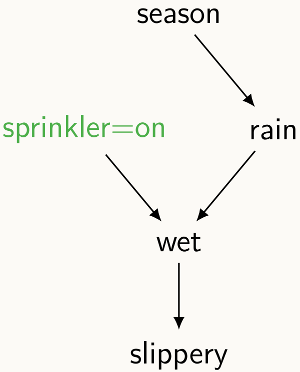


sn:season, sp:sprinkler, sl:slippery

Computing Causal Effects from Observational Data

This distribution decomposes as

$$P(\mathbf{V}) = P(\mathsf{Sl}|W)P(W|\mathsf{Sp},R)P(\mathsf{Sp}|\mathsf{Sn})P(R|\mathsf{Sn})P(\mathsf{Sn})$$



$$P(W \mid do(\mathsf{Sp} = \mathsf{on}))$$
$$= \sum_{\mathsf{sn},r,\mathsf{sl}} P(W, \mathsf{sn}, r, \mathsf{sl} \mid do(\mathsf{Sp} = \mathsf{on}))$$
$$= \sum_{\mathsf{sn},r,\mathsf{sl}} P(\mathsf{sl}|W)P(W|\mathsf{Sp} = on, r)P(r|\mathsf{sn})P(\mathsf{sn})$$

sn:season, sp:sprinkler, sl:slippery

Adjustment by Direct Parents for Singleton Intervention

Theorem

The causal effect $Q = P(\mathbf{y}|do(x))$ *is identifiable whenever* $X, \mathbf{Y}, \mathbf{Pa}_X \subseteq \mathbf{V}$ *(all parents of* X*) are measured.*[1]

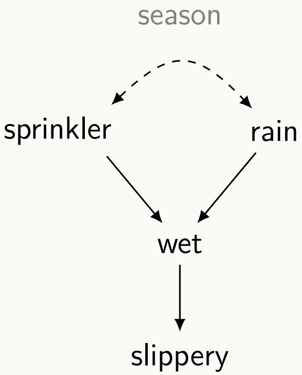
The expression of Q *is then obtained by adjustment for* $\mathbf{Pa}_X$*, or*

$$P(\mathbf{y}|do(x)) = \sum_{\mathbf{pa}_X} P(\mathbf{y}|x, \mathbf{pa}_X)P(\mathbf{pa}_X).$$

e.g.,

$$\sum_{\mathsf{sn}} P(W|\mathsf{Sp} = on, \mathsf{sn})P(\mathsf{sn})$$

If Season is latent, is the effect still computable?



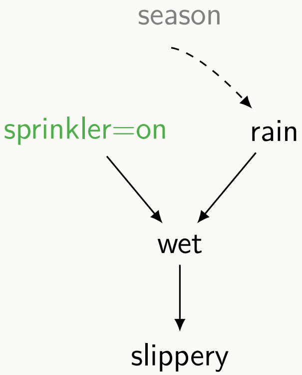
If Season is latent, is the effect still computable?

$$P(W \mid do(\text{Sp} = \text{on}))$$

$$= \sum_{\text{sn},r} P(W \mid \text{Sp} = \text{on}, r)P(\text{sn})P(r|\text{sn})$$

$$= \sum_{r} P(W \mid \text{Sp} = \text{on}, r)\sum_{\text{sn}} P(r, \text{sn})$$

$$= \sum_{r} P(W \mid \text{Sp} = \text{on}, r)P(r)$$



Overview

Back-door Criterion

$$\begin{tikzcd} & \text{kidney stone } Z \arrow[dl] \arrow[dr] & \\ \text{treatment } X \arrow[rr] & & \text{health } Y \end{tikzcd}$$

Definition (Back-door)

Find a set $\mathbf{Z}$ such that it can sufficiently explain 'confounding' between $\mathbf{X}$ and $\mathbf{Y}$. Then,

$$P(\mathbf{y}|do(\mathbf{x})) = \sum_{\mathbf{z}} P(\mathbf{y}|\mathbf{x},\mathbf{z})P(\mathbf{z})$$

Back-door Criterion

Definition (Back-door Criterion)

A set $\mathbf{Z}$ satisfies the back-door criterion with respect to a pair of variables $\mathbf{X}$, $\mathbf{Y}$ in a causal diagram $\mathscr{G}$ if;

(i) no node in $\mathbf{Z}$ is a descendant of $\mathbf{X}$; and

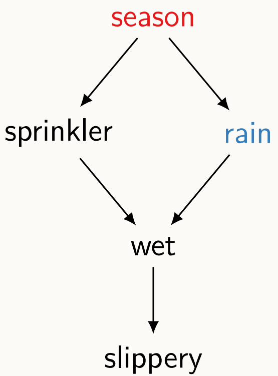
(ii) $\mathbf{Z}$ blocks every path between $X \in \mathbf{X}$ and $Y \in \mathbf{Y}$ that contains an arrow into X.

Back-door sets as substitutes of the direct parents of X

Rain satisfies the back-door criterion relative to Sprinkler and Wet:

(i) Rain is not a descendant of Sprinkler, and

(ii) Rain blocks the only back-door path from Sprinkler to Wet.

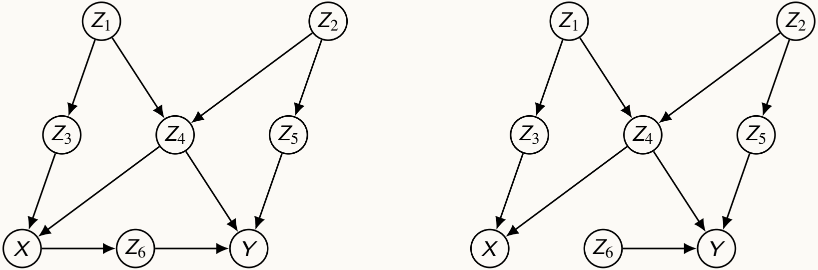


Adjusting for the direct parents of Sprinkler, we have:

$$\begin{aligned} P(\mathsf{wt}|do(\mathsf{sp})) &= \sum_{\mathsf{sn}} P(\mathsf{wt}|\mathsf{sp},\mathsf{sn})P(\mathsf{sn}) \\ &\vdots \\ &= \sum_{\mathsf{rn}} P(\mathsf{wt}|\mathsf{sp},\mathsf{rn})P(\mathsf{rn}) \end{aligned}$$

A Graphical Condition for Back-door Admissible Sets

$P(\mathbf{y}|do(\mathbf{x}))$ is identifiable if (i) & (ii) there is a set that d-sep. $\mathbf{X}$ from $\mathbf{Y}$ in $\mathcal{G}_{\underline{\mathbf{X}}}$.



$$P(y|do(x)) = \sum_{z_1,z_4} P(y|x,z_1,z_4)P(z_1,z_4)$$

Overview

Part 1: Causality

Part 2: Causal Effect Identification

- Back-door Criterion
- Do-Calculus

Part 3: Modern Identification

- Generalized Identification
- Transportability
- Recovering from Selection Bias
- Recovering from Missing Data

Rules of Do-calculus

- Backdoor criterion results in a very specific form of identification formula.
- **Do-Calculus** [Pearl 1995] provides general machinery to manipulate observational and interventional distributions.

$$\text{Level 1 Associational} \Leftrightarrow \text{Level 2 Experimental}$$

Rules of Do-calculus

Theorem (Rules of Do-calculus (simplified))

Rule 1: *Adding/removing Observations*

$$P(\mathbf{y}|do(\mathbf{x}),\mathbf{z}) = P(\mathbf{y}|do(\mathbf{x})) \qquad \textit{if } (\mathbf{Z} \perp\!\!\!\perp \mathbf{Y} \mid \mathbf{X}) \textit{ in } \mathscr{G}_{\overline{\mathbf{X}}}.$$

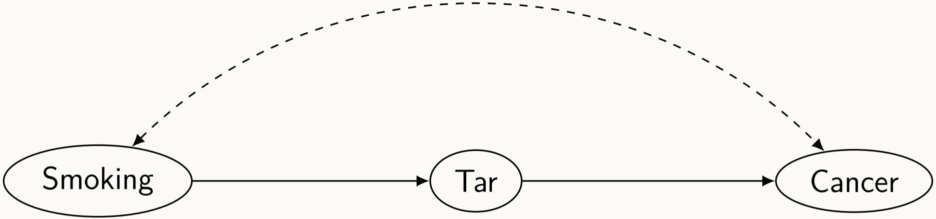
Rule 2: *Action/observation Exchange*

$$P(\mathbf{y}|do(\mathbf{x}),do(\mathbf{z})) = P(\mathbf{y}|do(\mathbf{x}),\mathbf{z}) \qquad \textit{if } (\mathbf{Z} \perp\!\!\!\perp \mathbf{Y} \mid \mathbf{X}) \textit{ in } \mathscr{G}_{\overline{\mathbf{X}}\underline{\mathbf{Z}}}.$$

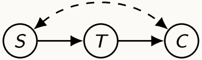
Rule 3: *Adding/removing Actions*

$$P(\mathbf{y}|do(\mathbf{x}),do(\mathbf{z})) = P(\mathbf{y}|do(\mathbf{x})) \qquad \textit{if } (\mathbf{Z} \perp\!\!\!\perp \mathbf{Y} \mid \mathbf{X}) \textit{ in } \mathscr{G}_{\overline{\mathbf{X}\mathbf{Z}}}$$

Do-calculus in Action



Do-calculus in Action

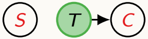


$$P(c|do(s))$$

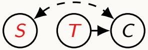
$$= \sum_t P(c|do(s),t)P(t|do(s))$$ Probability Axioms

$$= \sum_t P(c|do(s),do(t))P(t|do(s))$$ Rule 2 $(T \perp\!\!\!\perp C \mid S)_{\mathscr{G}_{\overline{S}\underline{T}}}$

$$= \sum_t P(c|do(t))P(t|do(s))$$ Rule 3 $(S \perp\!\!\!\perp C \mid T)_{\mathscr{G}_{\overline{T,S}}}$



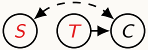
$$= \sum_t P(c|do(t))P(t|s)$$ Rule 2 $(T \perp\!\!\!\perp S)_{\mathscr{G}_{\underline{S}}}$



$$= \sum_t P(t|s) \sum_{s'} P(c|s',do(t))P(s'|do(t))$$ Probability Axioms

$$= \sum_t P(t|s) \sum_{s'} P(c|s',t)P(s'|do(t))$$ Rule 2 $(T \perp\!\!\!\perp C \mid S)_{\mathscr{G}_{\underline{T}}}$

$$= \sum_t P(t|s) \sum_{s'} P(c|s',t)P(s')$$ Rule 3 $(T \perp\!\!\!\perp S)_{\mathscr{G}_{\overline{T}}}$



Smoking, **T**ar deposit in lung, and **C**ancer

Algorithmic Identification

- **Do-calculus** is sound and complete but it has no algorithmic insight.
- A graphical condition and an efficient **algorithmic** procedure have developed for identifiability.

Finite sample and estimation is another important issue.

Summary for Part 2

- Identifiability: Causal Effect may be computable from existing observational data for some causal graphs.
- In a Markovian case and singleton X, a causal effect can be easily derivable by canceling out $P(x|\mathbf{pa}_x)$.
- A back-door adjustment formula is simple and widely used but limited.
- Do-calculus is a set of rules to manipulate observational or interventional probabilities. (Do-calculus is complete)
- There exists a polynomial time algorithm to yield a causal effect formula (whenever identifiable) given an arbitrary causal diagram.

Overview

Various Data Sources

Target ◎

$Q = P^*(y|do(x))$

Various Data Sources

Target $Q = P^*(y\|do(x))$	Dataset 1	Dataset 2	Dataset n

Various Data Sources

Target ◎ $Q = P^*(y \mid do(x))$	Dataset 1	Dataset 2	Dataset n

d_1	Population	**Los Angeles**	**New York**	**Seoul**
d_2	Obs./Exp.	Experimental	Observational	Experimental
	Treatment Assignment	Randomized Z_1	-	Randomized Z_2
d_3	Sampling	Selection on Age	Selection on SES	-
d_4	Measured	$\{X_1, Z_1, W, M, Y_1\}$	$\{X_1, X_2, Z_1, N, Y_2\}$	$\{X_2, Z_1, W, L, M, Y_1\}$

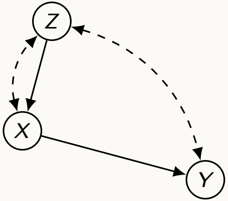
SES = Socio-Economic Status

Modern Identification Tasks

1. Experimental conditions
   Generalized Identification
2. Environmental conditions
   Transportability
3. Sampling conditions
   Recovering from Selection Bias
4. Responding conditions
   Recovering from Missingness

Overview

Generalized Identifiability

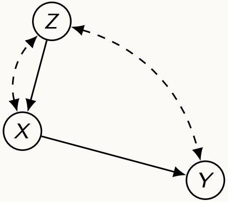


Z: Diet
X: Cholesterol Level
Y: Heart Attack

Measured:
Observational study: $P(X, Y, Z)$

Needed: $Q = P(y|do(x))$?

Generalized Identifiability

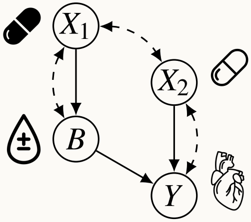


Z: Diet
X: Cholesterol Level
Y: Heart Attack

Measured:
Observational study: $P(X,Y,Z)$
Experimental Study: $P(X,Y|do(Z))$

Needed: $Q = P(y|do(x)) = \dfrac{P(x,y|do(z))}{P(x|do(z))}$

Generalized Identifiability: Drug-Drug Interactions

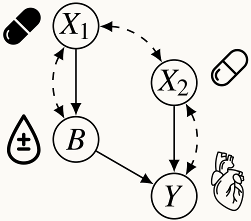


$$P_{x_1,x_2}(y) \Leftarrow \{P_{x_1}(\mathbf{V}), P_{x_2}(\mathbf{V})\}?$$

Y *cardiovascular disease*; B *blood pressure*; X_1 taking an *antihypertensive drug*; and X_2 the use of an *anti-diabetic drug*.

Goal: assess the effect of prescribing **both** treatments ( ) on the risk of cardiovascular diseases from **individual** drug experiments, either or .

Generalized Identifiability: Drug-Drug Interactions



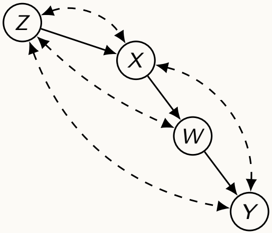
$$P_{x_1,x_2}(y) = \sum_b P_{x_2}(y|b)P_{x_1}(b)$$

Y *cardiovascular disease*; B *blood pressure*; X_1 taking an *antihypertensive drug*; and X_2 the use of an *anti-diabetic drug*.

Goal: assess the effect of prescribing **both** treatments ( ) on the risk of cardiovascular diseases from **individual** drug experiments, either or .

General Identifiability reduced to Calculus

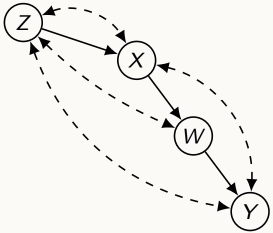
$$
\begin{aligned}
P(y|do(x)) &= \sum_w P(y|do(x), w)P(w|do(x)) \\
&= \sum_w P(y|do(x, w))P(w|do(x)) \\
&= \sum_w \underbrace{P(y|do(w))}_{Q[Y]} \underbrace{P(w|do(x))}_{Q[W]}
\end{aligned}
$$



Both effects are not identifiable from $P(\mathbf{V})$.

General Identifiability reduced to Calculus

$$
\begin{aligned}
Q[Y] &= P(y|do(w)) \\
&= P(y|do(w,z)) \\
&= \sum_x P(y|do(w,z),x)P(x|do(w,z)) \\
&= \sum_x P(y|do(w,z),x)P(x|do(z)) \\
&= \sum_x P(y|do(z),w,x)P(x|do(z)).
\end{aligned}
$$



$$
\begin{aligned}
Q[W] &= P(w|do(x)) \\
&= P(w|do(x,z)) \\
&= P(w|do(z),x).
\end{aligned}
$$

Available from $P(\mathbf{V}|do(z))$!

Summary for General Identifiability

The identifiability of any expression of the form

$$P(\mathbf{y} \mid do(\mathbf{x}), \mathbf{z})$$

can be determined given any causal graph $\mathscr{G}$ and an arbitrary **combination of observational and experimental studies**.

If the query is identifiable, then its estimand can be derived in *polynomial time*.

Lee, Correa, and Bareinboim (2019) *General Identifiability with Arbitrary Surrogate Experiments*, UAI.

Overview

Transportability

Is it possible to compute the effect of $\mathbf{X}$ on $\mathbf{Y}$ in a target environment ◎, using **observational and experimental findings** from different populations?

e.g., applying education policies of U.S. to South Korea.

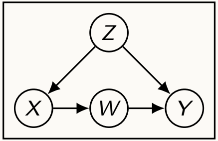
Environment can be understood as population, domain, etc.

How is this Problem seen in other Sciences?

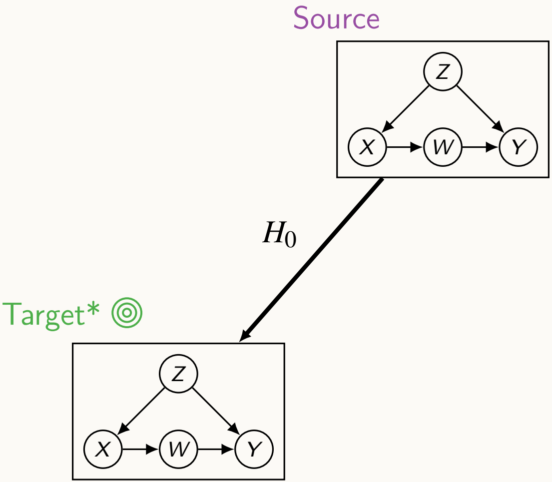
- "**External Validity** asks the question of generalizability: To what populations, settings, treatment variables, and measurement variables can this effect be generalized?" (Shadish, Cook, and Campbell, 2002)
- "**Extrapolation** across studies requires 'some understanding of the reasons for the differences.' " (Cox, 1958)
- "An experiment is said to have "**external validity**" if the distribution of outcomes realized by a treatment group is the same as the distribution of outcome that would be realized in an actual program." (Manski, 2007)

Transportability: the Spectrum

Source

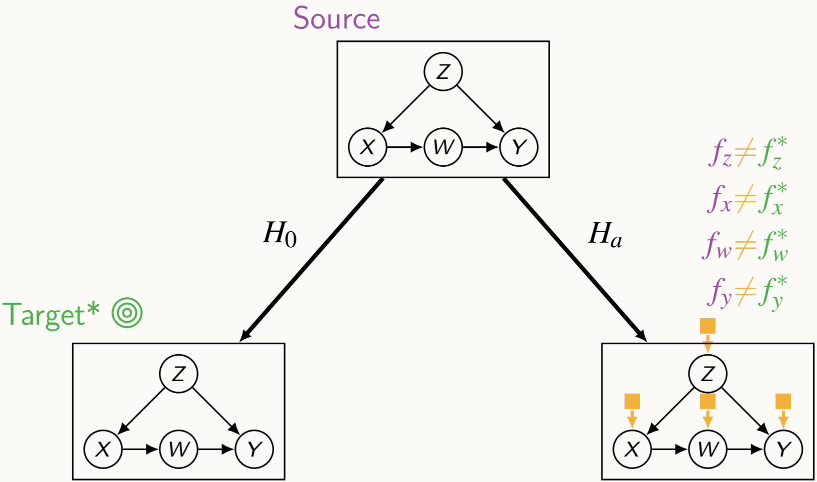


Transportability: the Spectrum



Everything is assumed to be
the same, trivially transportable!

Transportability: the Spectrum



Everything is assumed to be
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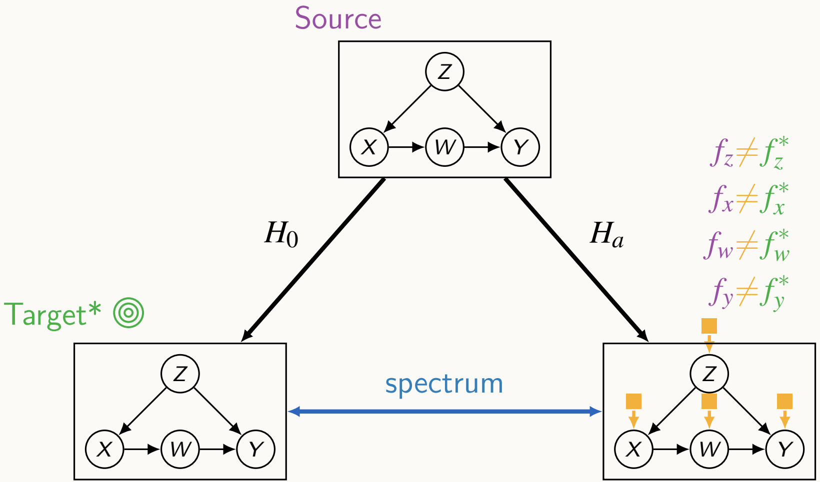
Everything is assumed to be
different, *not* transportable!

Transportability: the Spectrum



Everything is assumed to be
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Everything is assumed to be
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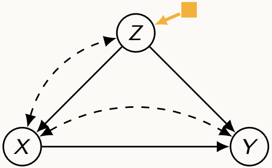
Transportability: Formulas Depend on the Causal Story



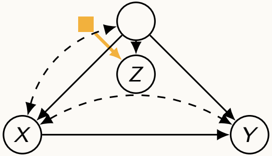
(a)

(a) Z represents **age**

$$P^*(y|do(x)) = \sum_z P(y|do(x),z)P^*(z)$$

P^* target domain & P source domain

Transportability: Formulas Depend on the Causal Story



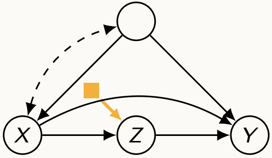
(b)

(b) Z represents **language skill**

$$P^*(y|do(x)) = P(y|do(x))$$

P^* target domain & P source domain

Transportability: Formulas Depend on the Causal Story



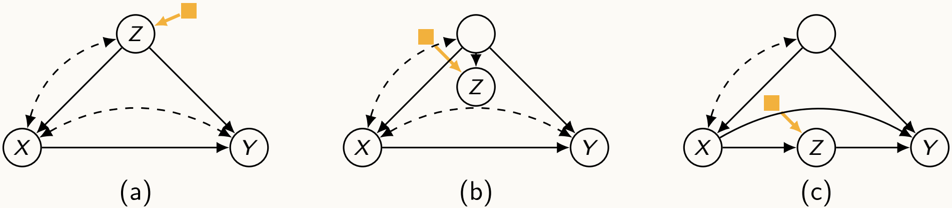
(c)

(c) Z represents **bio-marker**

$$P^*(y|do(x)) = \sum_z P(y|do(x),z)P^*(z|x)$$

P^* target domain & P source domain

Transportability: Formulas Depend on the Causal Story



(a) Z represents **age**

$$P^*(y|do(x)) = \sum_z P(y|do(x),z)P^*(z)$$

(b) Z represents **language skill**

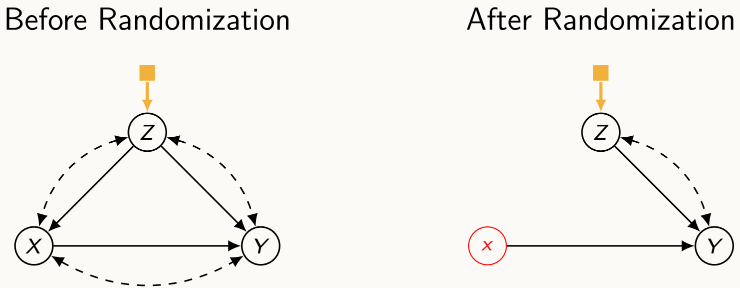
$$P^*(y|do(x)) = P(y|do(x))$$

(c) Z represents **bio-marker**

$$P^*(y|do(x)) = \sum_z P(y|do(x),z)P^*(z|x)$$

P^* target domain & P source domain

Is the Gold Standard Golden? (Generalizability from Trials)



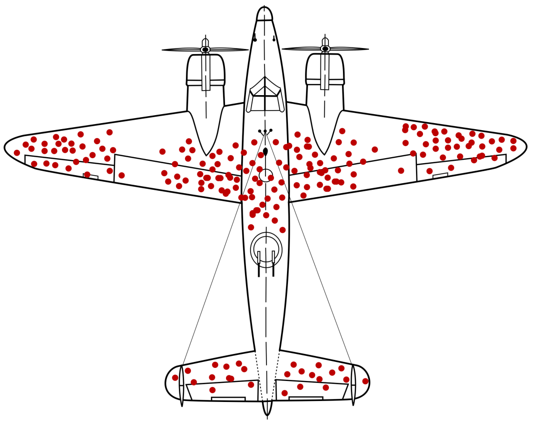
Lesson. Even if we have a perfect RCT, one still needs to exercise transportability.

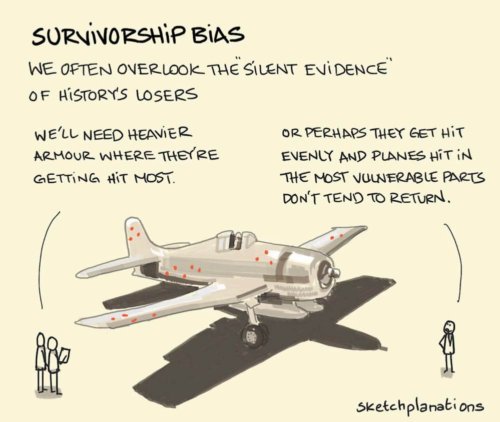
Summary for Transportability

- Non-parametric transportability can be determined provided that the problem instance is encoded in selection diagrams ($= \mathscr{G} + \blacksquare$).
- When transportability is feasible, the transport formula can be derived in polynomial time.
- The causal calculus and the corresponding transportation algorithm are complete.

Overview

Identification under Selection



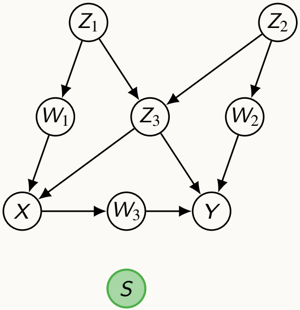


Wikipedia, https://en.wikipedia.org/wiki/Survivorship_bias, sketchplanations, https://sketchplanations.com/survivorship-bias-silent-evidence

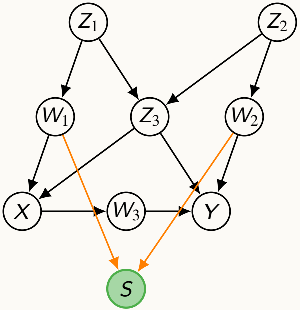
Identification under Selection

- Selection bias, caused by preferential inclusion S of samples from the data, is a major obstacle to valid **causal** and **statistical** inferences;

Without Selection Bias



With Selection Bias

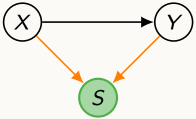


Selection Bias without External Information

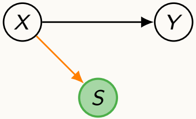
Theorem

$Q = P(y|x)$ is recoverable from selection biased data if and only if

$$(S \perp\!\!\!\perp Y \mid X).$$



$P(y|x)$ is not recoverable



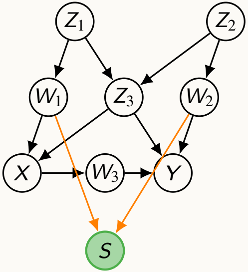
$P(y|x)$ is recoverable

Identification under Selection (with External Data)

Theorem

$P(y|x)$ is recoverable if there is a set $\mathbf{C}$ such that $(Y \perp\!\!\!\perp S \mid \mathbf{C}, X)$ holds in $\mathcal{G}$ and $P(\mathbf{C}, X)$ is estimable. Moreover,

$$P(y|x) = \sum_{\mathbf{c}} P(y|x, \mathbf{c}, S=1) P(\mathbf{c}|x)$$



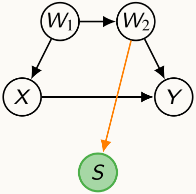
$\mathbf{C} = \{W_1, W_2\}$? Yes

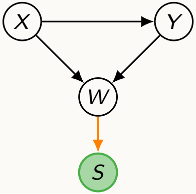
$\mathbf{C} = \{W_1, Z_1, Z_2\}$? No

$\mathbf{C} = \{W_2, Z_3\}$? Yes

Identification under Selection (with External Data)

Goal: recover a causal effect $P(y|do(x))$.





$$P(y|do(x)) = \sum_{w_2} P(y|x, w_2) P(w_2) = \sum_{w_2} P(y|x, w_2, S=1) P(w_2).$$

$$P(y|do(x)) = P(y|x) = \sum_{w} P(y|w, x) P(w|x) = \sum_{w} P(y|w, x, S=1) P(w|x).$$

Summary for Selection Bias

- Nonparametric recoverability of selection bias from causal and statistical settings can be determined provided that an augmented causal graph (w/ the selection mechanism S) is available.
- When recoverability is feasible, the estimand can be derived in **polynomial time**.
- The result is complete for pure recoverability, and sufficient for recoverability with external information.

Correa, Tian, and Bareinboim (2019) *Identification of Causal Effects in the Presence of Selection Bias*. AAAI.

Overview

Identification under Missing Data

Missing data present a challenge in many academic disciplines.

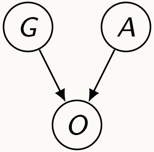
- *Sensors* do not always work reliably.
- *Respondents* do not fill out every question in the questionnaire.
- Medical *patients* are often unable to recall treatments or outcomes.

#	Age	Gender	Obesity*
1	16	F	Obese
2	15	F	N/A
3	15	M	N/A
4	14	F	Not Obese
5	13	M	Not Obese
6	15	M	Obese
7	14	F	Obese

Excerpted mainly from Mohan, K. and Pearl, J. (2021) *Graphical models for processing missing data*, JASA.

Identification under Missing Data: Example

Consider a study conducted in a school with
Age (A), **G**ender (G) and **O**besity (O).



- **Age** and **Gender** are fully observed (obtained from school records).
- **Obesity** however is corrupted by missing values due to some students not reporting their weight.

Identification under Missing Data: Proxy Variable

Modelling the **missingness process** using

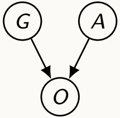
- Obesity O (true, partly-observed),
- a missingness mechanism R_O, and
- a proxy variable O^* (what's observed)

#	Age	Gender	Obesity*	R_O
1	16	F	Obese	0
2	15	F	m	1
3	15	M	m	1
4	14	F	Not Obese	0
5	13	M	Not Obese	0
6	15	M	Obese	0
7	14	F	Obese	0

$$O^* = \begin{cases} O & \text{if } R_O = 0 \\ m & \text{if } R_O = 1 \end{cases}$$

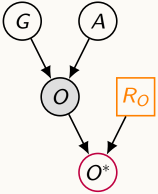
Identification under Missing Data: Reasons for Missingness

Missingness can be *caused* by random processes or can depend on other variables.



Identification under Missing Data: Reasons for Missingness

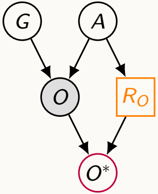
Missingness can be *caused* by random processes or can depend on other variables.



- Students *accidentally* *losing* their questionnaires.

Identification under Missing Data: Reasons for Missingness

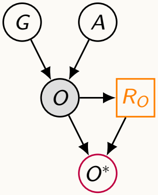
Missingness can be *caused* by random processes or can depend on other variables.



- Teenagers rebelling and not reporting their weight.

Identification under Missing Data: Reasons for Missingness

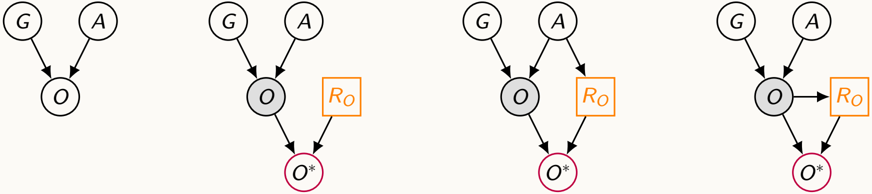
Missingness can be *caused* by random processes or can depend on other variables.



- Obese students who are embarrassed of their obesity and hence reluctant to reveal their weight.

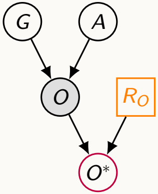
Identification under Missing Data: Reasons for Missingness

Missingness can be *caused* by random processes or can depend on other variables.

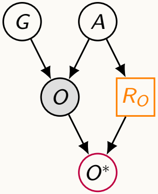


- Students *accidentally losing* their questionnaires.
- Teenagers rebelling and not reporting their weight.
- Obese students who are embarrassed of their obesity and hence reluctant to reveal their weight.

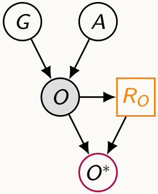
Three Categories of Missingness



MCAR



MAR



MNAR

$$O, A, G \perp\!\!\!\perp R_O$$

$$O \perp\!\!\!\perp R_O \mid A, G$$

$$O \not\!\perp\!\!\!\perp R_O \mid A, G$$

Identification under Missing Data: Example

Factorization:

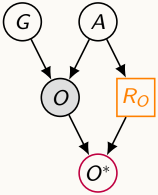
$$P(G,O,A) = P(G,O|A)P(A)$$

Transformation into observables:

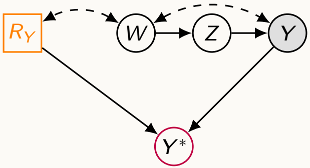
$$= P(G,O|A,R_O = 0)P(A)$$

Conversion

$$= P(G,O^*|A,R_O = 0)P(A).$$



Identification under Missing Data: Example



$$
\begin{aligned}
P(y|do(z)) &= P(y|do(z), r_y) \\
&= P(y^*|do(z), r_y) \\
&= \sum_w P(y^*|w, do(z), r_y) P(w|do(z), r_y) \\
&= \sum_w P(y^*|w, z, r_y) P(w|r_y).
\end{aligned}
$$

Summary for Part 3

Modern Identification

1. **General** Identification: combining data sets of different experimental conditions
2. **Transportability**: combining data sets from different sources ■
3. Identification under **Selection** S
4. Identification under **Missingness** $\boxed{R_O}$

Summary for Causal Inference Lecture

This lecture focused mainly on a basic causal effect identification task (SCM, do-operator, Causal Graph, Conditional Independence ...)

There are many interesting future research directions

- Causal Data Science
- Causal Discovery
- Causal Decision Making
- Causality + Machine Learning