DIGITAL CIRCUITS

Week-7, Lecture-1
K-map

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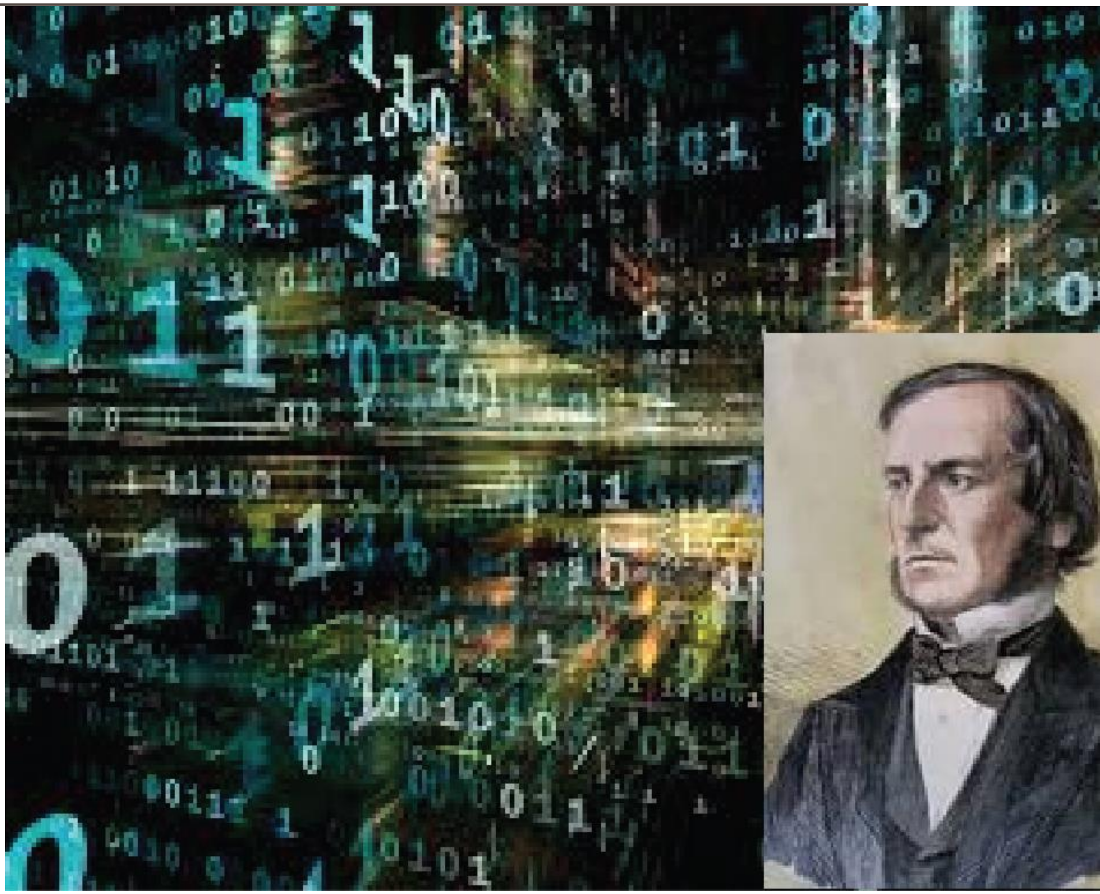
11th September, 2018



Digital Circuits: Announcements/Revision

Digital Circuits: Course Feedback

Total number of students: 223
Number of responses: 88

According to you, what is going well in this course?

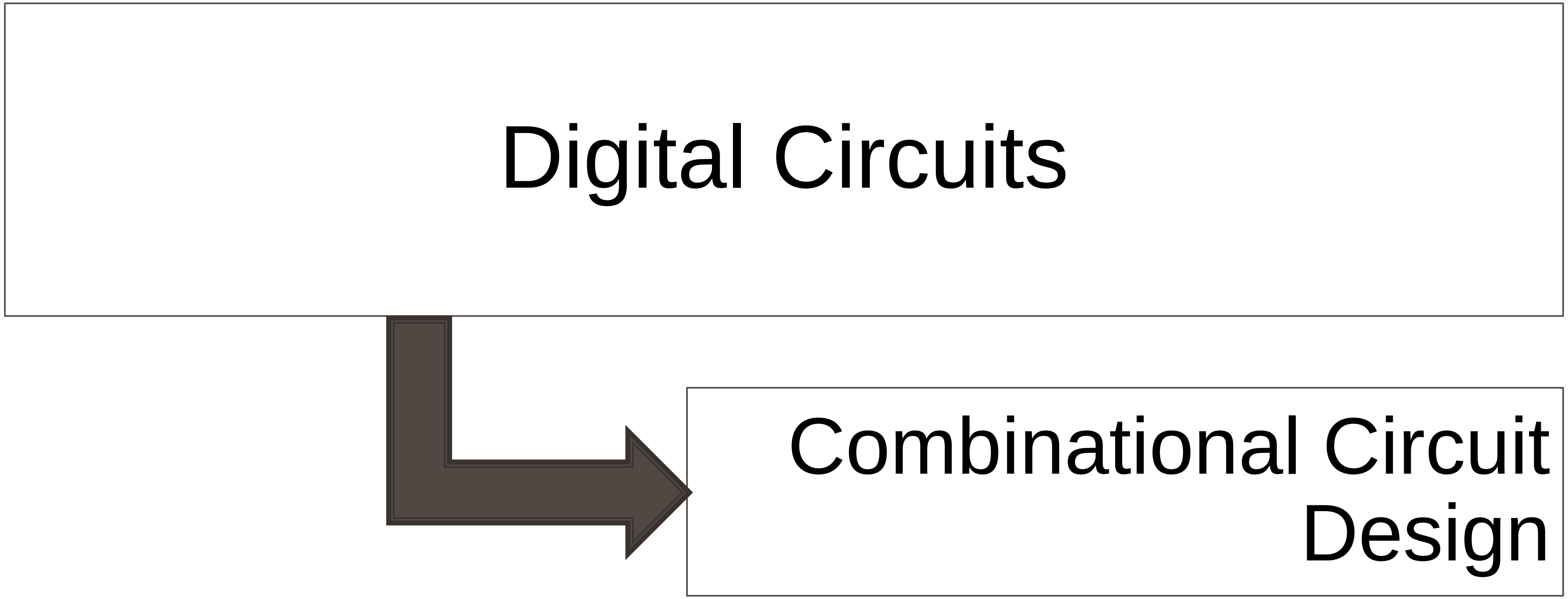
- Many positive feedback
    - Revision, Lectures, Slides, Explanation, Everything etc.

Is there anything you wish to be changed?

- Most students say “Nothing”

Some feedback:

- The level of quiz is more difficult than problems discussed in class
- More problems should be covered in class
- Practice problems

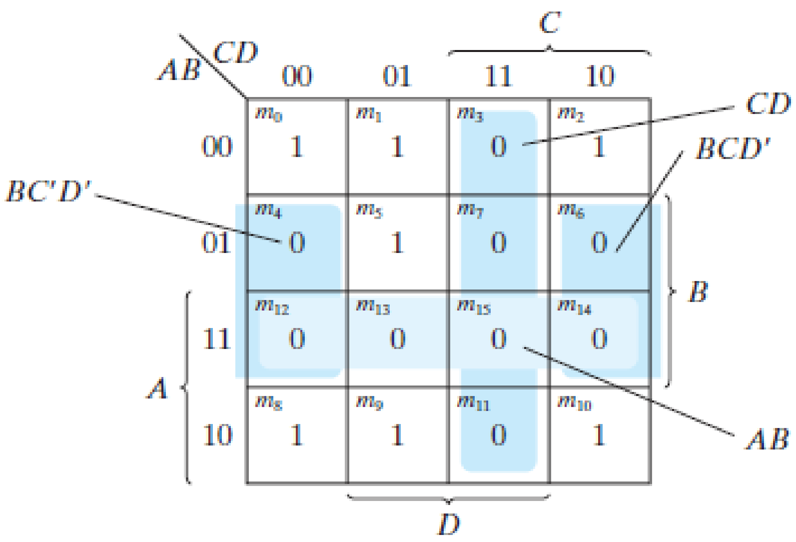

Digital Circuits
Combinational Circuit Design

K-Map: product-of-sums simplification

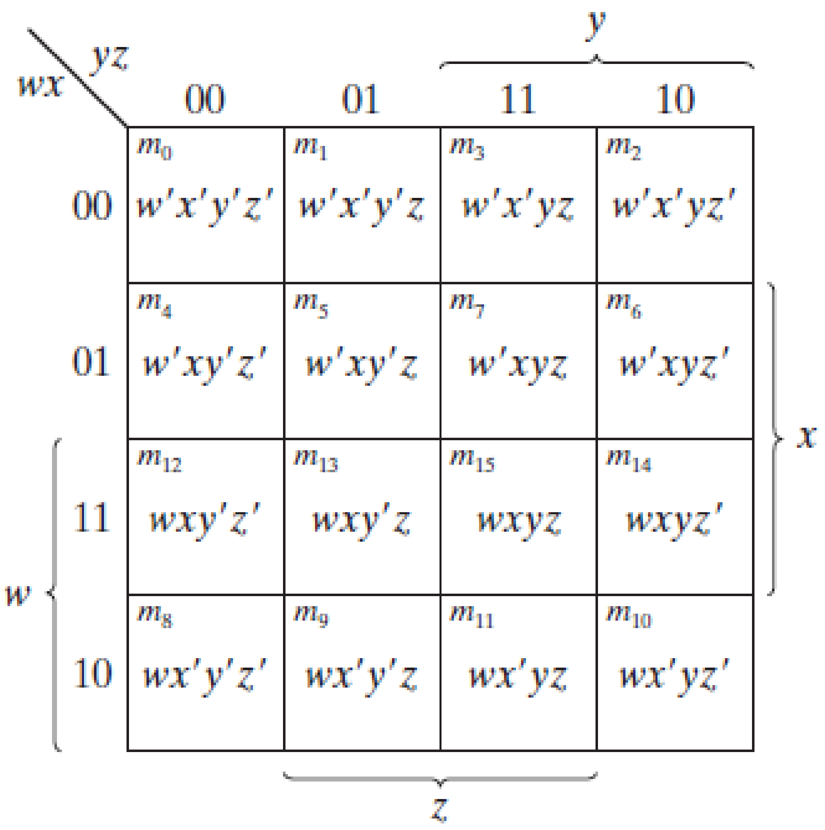
- To find minimized function in POS form instead of SOP

Problem: Find minimized function in POS:

$$F(A,B,C,D) = \Sigma m(0,1,2,5,8,9,10)$$



Note: $BC'D' + BCD' = BD'$

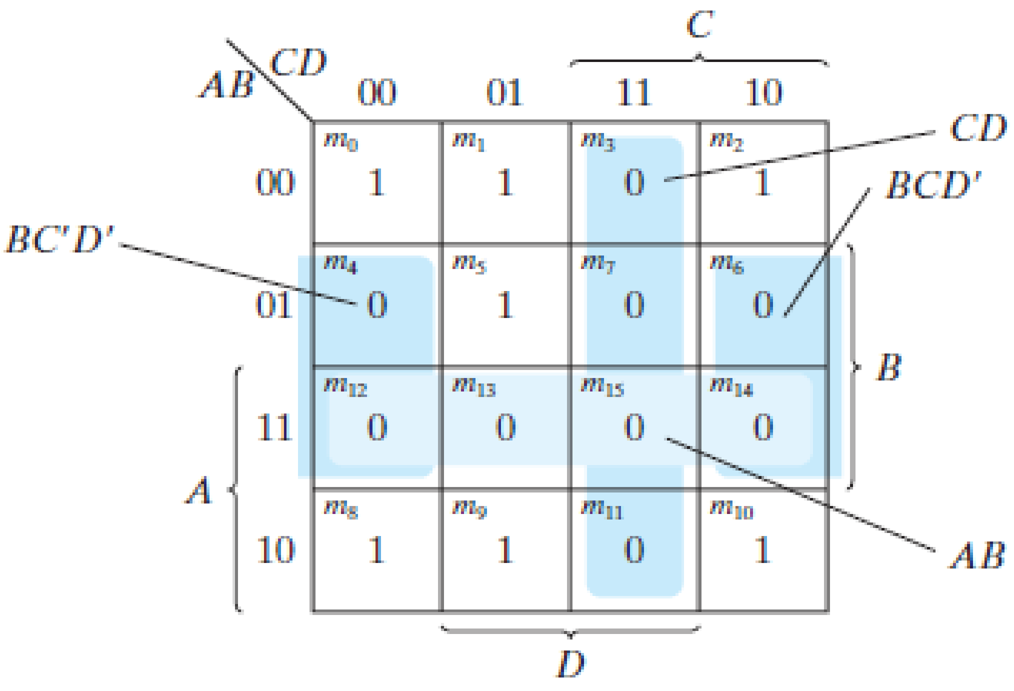


K-Map: product-of-sums simplification

- First find an optimized complement of a function
- Complement of a function is found by determining the missing terms

$F'(A, B, C, D) = \Sigma m(3,4,6,7,,11,12,13,14,15)$

- Find possible adjacent squares in $F'(A, B, C, D)$ i.e. 0 terms in the K-map of $F(A, B, C, D)$
  - m_3, m_7, m_{15} and m_{11}: $= CD$
  - m_{12}, m_{13}, m_{15} and m_{14}: $= AB$
  - m_4, m_{12}, m_6 and m_{14}: $= BD'$
- $F'(A, B, C, D) = CD + AB + BD'$



Note: $BC'D' + BCD' = BD'$

$F(A, B, C, D = (F'(A, B, C, D))' = (CD + AB + BD')'$

$= (C' + D')(A' + B')(B' + D)$

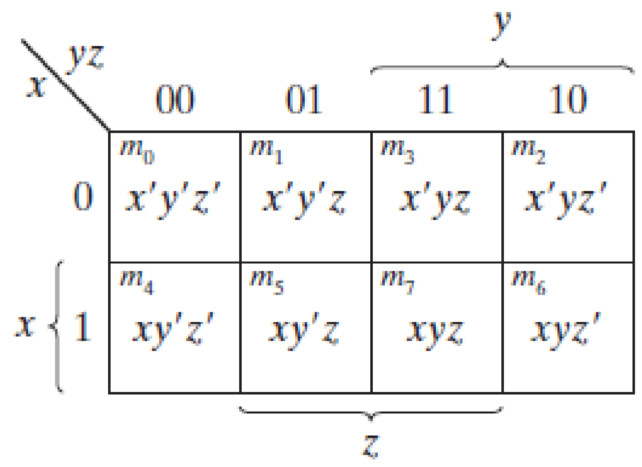
K-Map: using CPOS

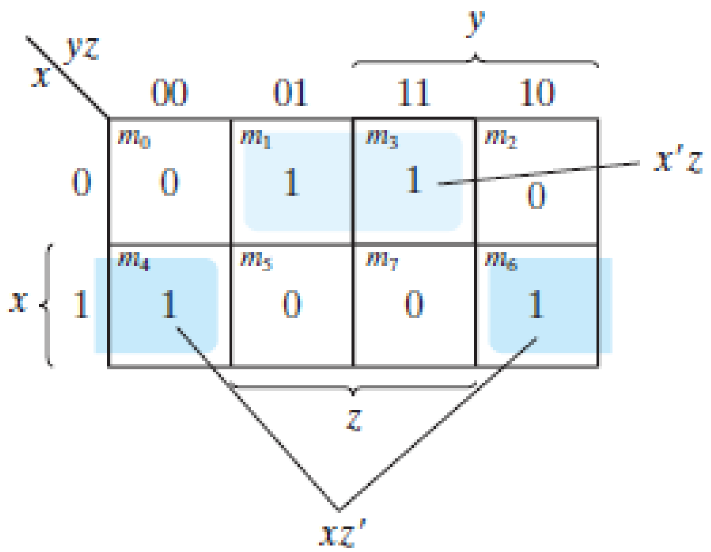
Problem: Find minimized function in SOP:

$F(x, y, z) = \Pi M(0,2,5,7)$

- First represent the given function on the K-map
- Represent a function in CSOP
- $F(x, y, z) = \Pi M(0,2,5,7) = \Sigma m(1,3,4,6)$
- Mark 0's corresponding to terms in CPOS and rest as 1's

$\boldsymbol{f(x, y, z) = x'z + xz'}$





K-map

Principles of Logic Optimization

K-map: Finding minimized SOP

Finding minimized SOP using K-map

1. All the minterms of the function are covered when we combine the squares
2. The number of terms in the expression is minimized
3. Number of literals in each term is minimized

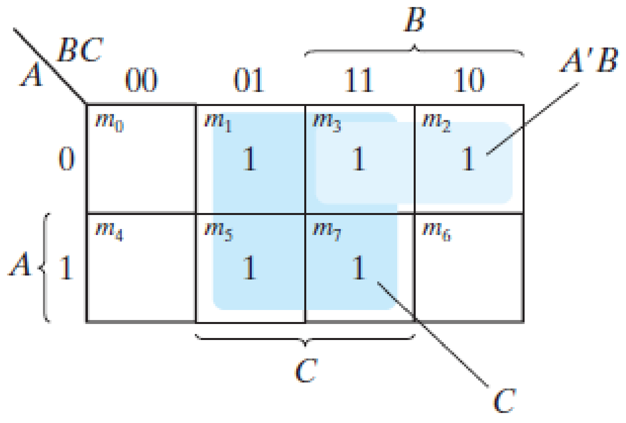
Systematic way of finding minimized expression: ***Principles of Logic Optimization***

Principles of Logic Optimization: *Implicant*

Implicant: Implicant is a ***product term*** that evaluates to 1 only of those combination of inputs for which the given function also evaluates to 1.

Problem:

The K-map of a function $f(A, B, C)$ is shown alongside. List out all the implicants of the function.



Answer:

1. **Squares taken singly (minterms):** $A'B'C$, $A'BC$, $A'BC'$, $AB'C$, ABC

2. **Squares combined in two's:** $A'C$, $A'B$, BC, AC, $B'C$

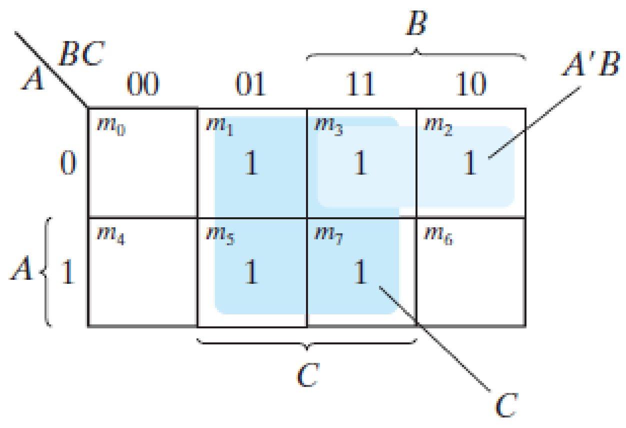
3. **Squares combined in four's:** C

Principles of Logic Optimization: *Cover*

Cover: A cover of a Boolean function is a ***set of implicants*** that covers ***all its minterms***.

Some of the covers are:

1. $\{A'B'C,\ A'BC,\ A'BC',\ AB'C,\ ABC\}$: CSOP
2. $\{A'C, A'B, AC\}$
3. $\{C,\ A'B\}$ etc.



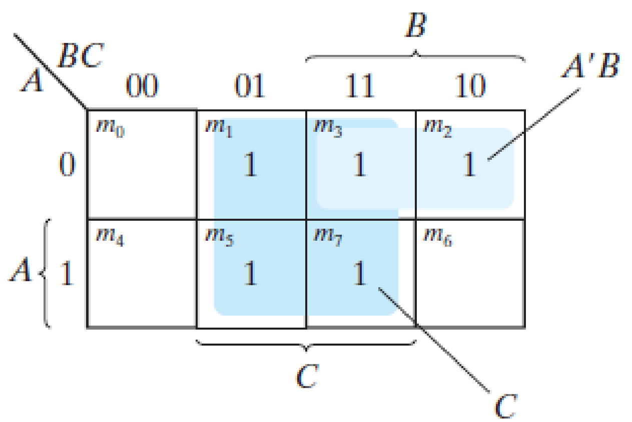
Minimum Cover: A minimum cover is the cover of minimum cardinality (number of elements in the set).

Minimum covers is: $\{C,\ A'B\}$

Principles of Logic Optimization: *Prime Implicant*

Prime Implicant: An implicant is a prime implicant if it is not contained by any other implicant of that function.

- No literal can be dropped from a prime implicant (if some literal is dropped then prime implicant no longer remains an implicant)



Problem:

The K-map of a function $f(A,B,C)$ is shown alongside. List out all the prime implicants of the function.

List of implicants:

$A'B'C$, $A'BC$, $A'BC'$, $AB'C$, ABC, $A'C$, $A'B$, BC, AC, $B'C$ and C

Answer: $A'B$, C

Prime Cover: A cover is prime if all the implicants in its set are prime implicants.

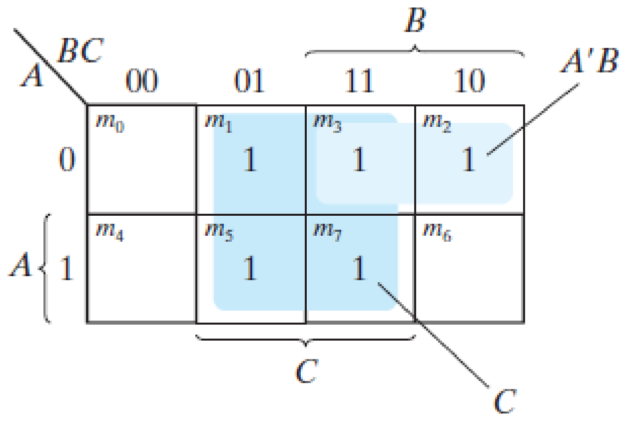
Prime cover is: $\{C, A'B\}$

Principles of Logic Optimization: *Essential Prime Implicant*

Essential Prime Implicant: A prime implicant is essential if there is at least one minterm that is covered by only this prime implicant.

Problem:

The K-map of a function $f(A, B, C)$ is shown alongside. List out all the essential prime implicant of the function.



List of prime implicants: $A'B$, C

Answer (List of essential prime implicants): $A'B$ and C

Principles of Logic Optimization: *Quine's Theorem*

Quine's Theorem: There is a minimum cover that is prime.

Proof:

- Consider a minimum cover that is ***not*** prime.
- Each non-prime implicant can be replaced by a prime implicant that contains it.
- The resulting cover is a prime cover and has the same cardinality as the original cover, hence minimum.

Application:

The theorem allows us to limit the search space for a minimum cover to those covers which consist entirely of ***prime implicants***.

Principles of Logic Optimization: Procedure

Steps:

1. Generate all the ***prime implicants*** of a function.

2. Find the list of ***essential prime implicants***

3. a) If the set of essential prime implicants covers all minterms, then this set is the desired minimum cover

   b) Otherwise, determine the non-essential prime implicants that should be added to form a complete minimum cover.

Digital Circuits: Practice Problems

Problem:

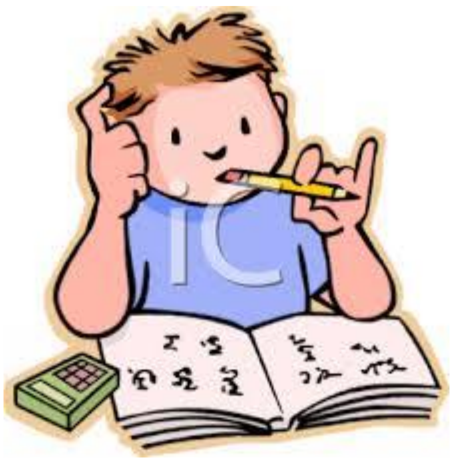
A function $f(x_1, x_2, x_3, x_4) = \Sigma m(2,3,5,6,7,10,11,13,14)$ is given.

a. Draw the K-map of this function.

b. List out all the prime implicants of the function.

c. Identify essential prime implicants.

d. Find the minimum cover.



Do problems 3.1 to 3.13 from “Digital Design”– M. Morris Mano & Michael D. Ciletti