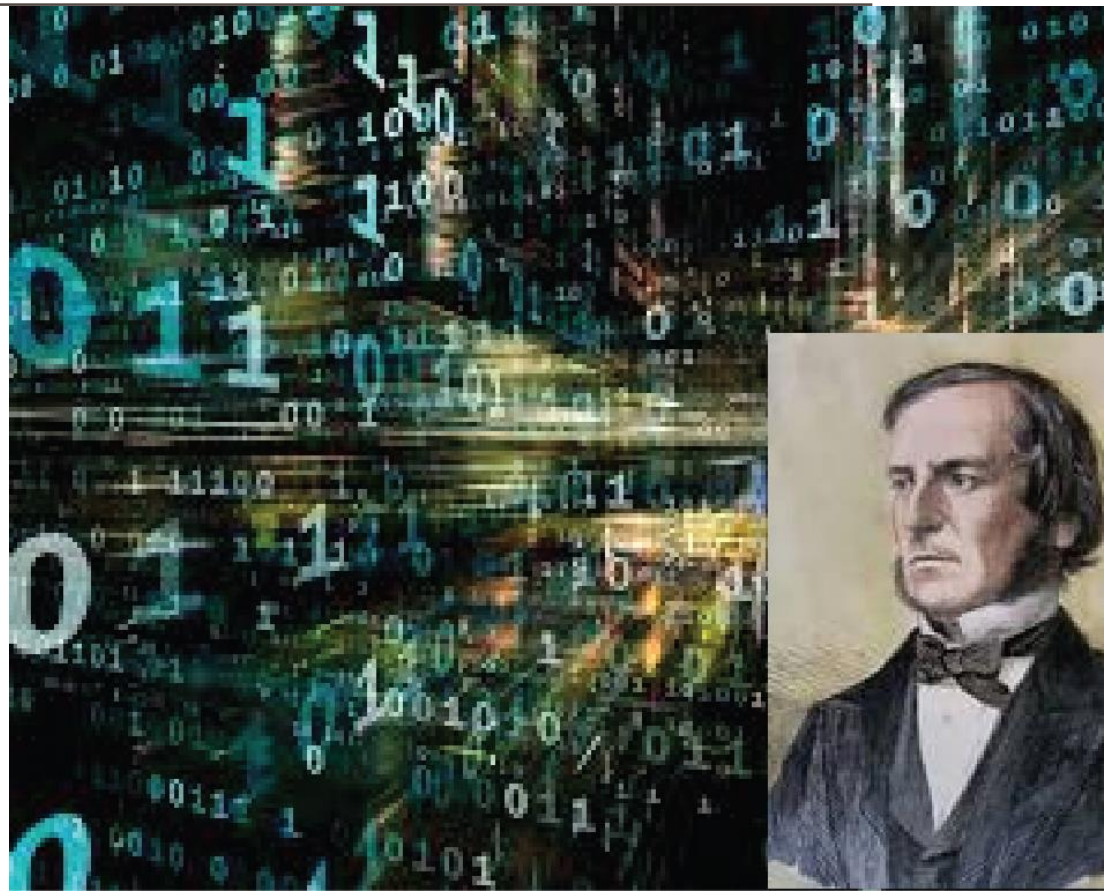
DIGITAL CIRCUITS

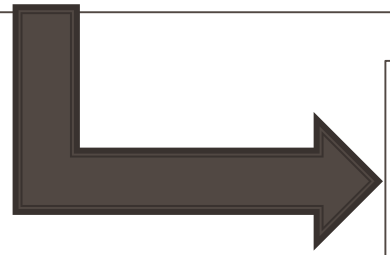
Week-12, Lecture-1
Sequential Circuits



Sneh Saurabh
30th October, 2018

Digital Circuits: Announcements/Revision

Sequential Circuits



Analysis

Sequential Circuit: Behavior

Sequential Circuit Behavior:

- Determined by the inputs, outputs and the state of the circuit.

State of a Sequential Circuit: The value at the output of all the flip-flops in a circuit at any given time defines the state of the circuit

Output of a Sequential Circuit: In general, output depends on the inputs and the current state of the circuit

- **Analysis of Sequential Circuits**: Given a circuit, determine its behavior

- **Behavior expressed as**: State Equation, State Table and State Diagram

Sequential Circuit: State Equation

State Equation (Transition Equation): specifies the next state as a function of the present state and inputs.

Problem:

Write the State Equation for the circuit shown alongside.

$$A(t+1) = f(A(t), B(t), x(t))$$
$$B(t+1) = g(A(t), B(t), x(t))$$

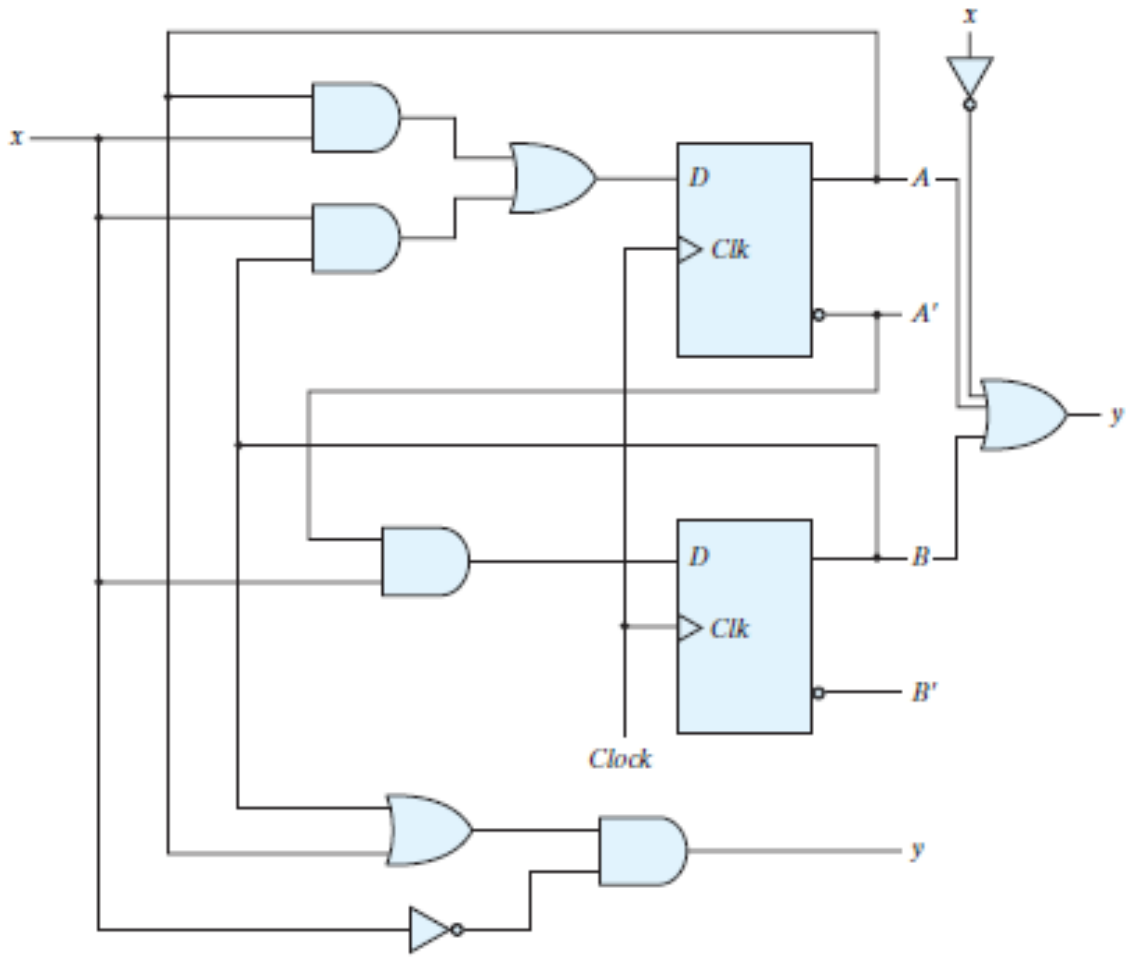
Answer:

$$A(t+1) = A(t)x(t) + B(t)x(t)$$

$$B(t+1) = A'(t)x(t)$$

Output Equation:

$$y(t) = (A(t) + B(t))x'(t)$$



Sequential Circuit: State Table

State Table (Transition Table)

- Enumerates present state, input, next state, and output in a table
- Can be derived from the circuit or state equation

Problem:

Given state equations and output equations as follows, draw the state table

$A(t+1) = Ax + Bx$

$B(t+1) = A'x$

$y(t) = (A + B)x'$

Present State		Input	Next State		Output
A	*B*	*x*	*A*	*B*	*y*
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	0	1
0	1	1	1	1	0
1	0	0	0	0	1
1	0	1	1	0	0
1	1	0	0	0	1
1	1	1	1	0	0

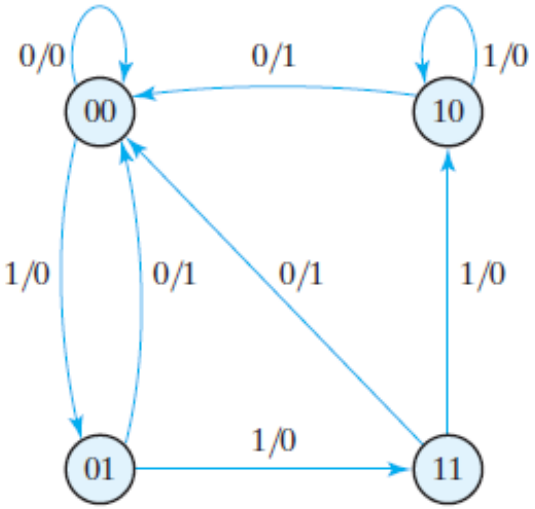
Sequential Circuit: State Diagram (1)

State Diagram (Transition Table)

- Graphical representation of the information available in a state table
- States represented as circle
- Transitions between states are indicated by directed lines connecting circles

Sequential Circuit: State Diagram (2)

Present State		Input	Next State		Output
A	B	x	A	B	y
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	0	1
0	1	1	1	1	0
1	0	0	0	0	1
1	0	1	1	0	0
1	1	0	0	0	1
1	1	1	1	0	0



- States identified by binary numbers inside the circle
- Directed lines are marked as "Input/Output"
- State diagram more suitable for human interpretation

Sequential Circuit: Input Equations

Input Equations of Flip-flops (Excitation Equation)

- Set of Boolean functions describing the functionality at the inputs to the flip-flops in a sequential circuit

For D flip-flops State Equations and Input Equations are the same

- State Equation:

$A(t+1) = Ax + Bx$

$B(t+1) = A'x$

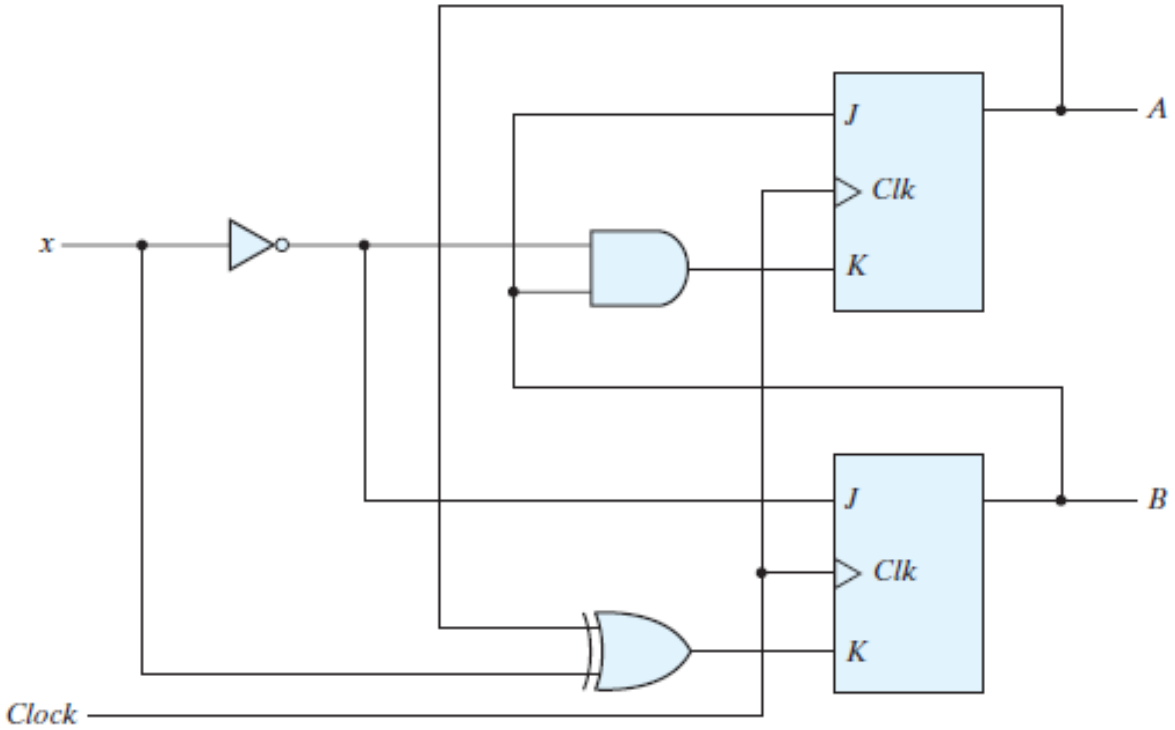
- Inputs Equation:

$D_A = Ax + Bx$

$D_B = A'x$

- For other types of flip-flops State Equations and Input Equations can be different
- Using Input Equations and Characteristics Equation of the flip-flops, State Equations for a circuit can be derived

Sequential Circuit: Input Equations for JK Flip-flops (1)



Problem:

Write the input equations for the flip-flops shown in the circuit alongside.

Answer:

$J_A = B, K_A = Bx'$

$J_B = x', K_B = A \oplus x$

Sequential Circuit: Input Equations to State Table

Problem:

Draw the state table for the circuit whose input equations are given as follows:

$J_A = B, K_A = Bx'$

$J_B = x', K_B = A \oplus x$

Present State		Input	Next State		Flip-Flop Inputs			
A	B	x	A	B	J_A	K_A	J_B	K_B
0	0	0	0	1	0	0	1	0
0	0	1	0	0	0	0	0	1
0	1	0	1	1	1	1	1	0
0	1	1	1	0	1	0	0	1
1	0	0	1	1	0	0	1	1
1	0	1	1	0	0	0	0	0
1	1	0	0	0	1	1	1	1
1	1	1	1	1	1	0	0	0

JK Flip-Flop

J	K	Q(t + 1)	
0	0	$Q(t)$	No change
0	1	0	Reset
1	0	1	Set
1	1	$Q'(t)$	Complement

Sequential Circuit: Input Equations to State Equations

Problem:

Derive the state equation for the circuit whose input equations are given as follows:

$J_A = B, K_A = Bx'$

$J_B = x', K_B = A \oplus x$

Characteristics equation for JK flip-flop (Week 11, Lecture-1):

$Q(t+1) = JQ' + K'Q$

Answer:

$A(t+1) = J_A A' + K_A{}' A$

$= BA' + (Bx')'A$

$= A'B + AB' + Ax$

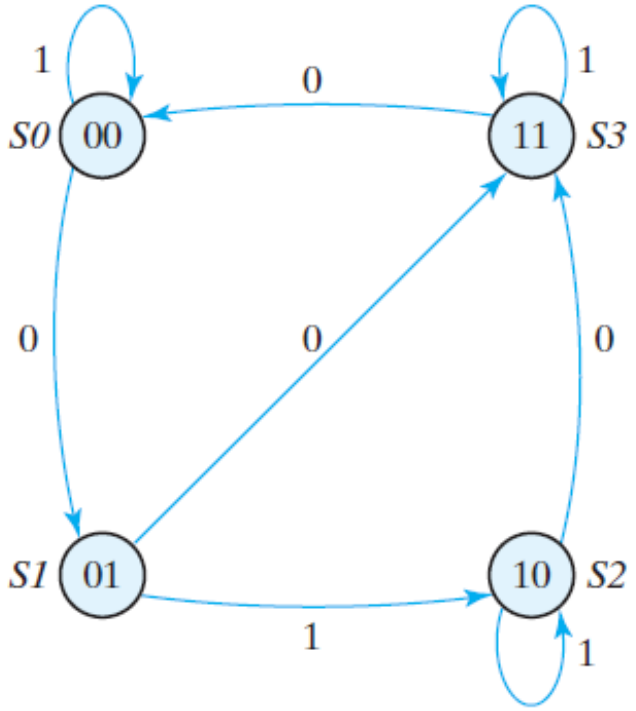
$B(t+1) = x'B' + (A \oplus x)'B$

$= B'x' + ABx + A'Bx'$

Sequential Circuit: State Diagram

Problem:

Draw the state diagram for the circuit whose state table is shown alongside.



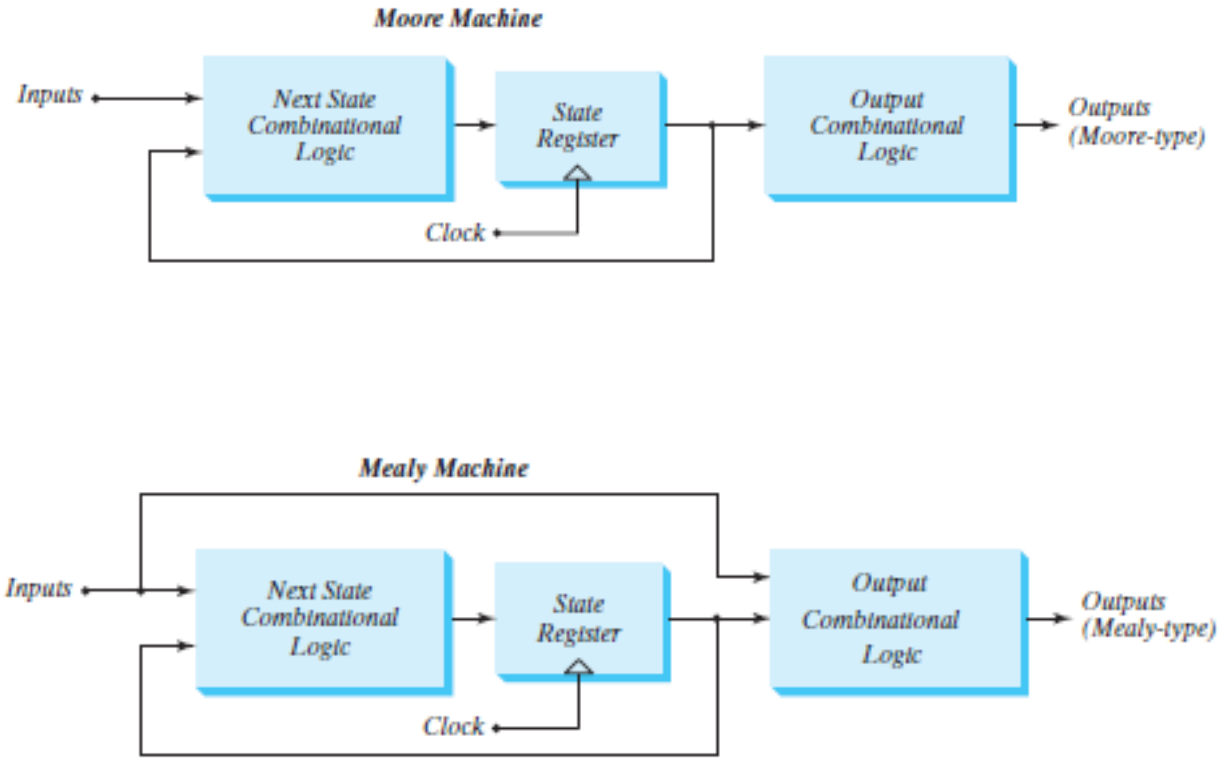
Present State		Input	Next State		Flip-Flop Inputs			
A	B	x	A	B	J_A	K_A	J_B	K_B
0	0	0	0	1	0	0	1	0
0	0	1	0	0	0	0	0	1
0	1	0	1	1	1	1	1	0
0	1	1	1	0	1	0	0	1
1	0	0	1	1	0	0	1	1
1	0	1	1	0	0	0	0	0
1	1	0	0	0	1	1	1	1
1	1	1	1	1	1	0	0	0

No output given in this case

Finite State Machine: Moore and Mealy

Finite State Machine:

- Finite non-empty set of states
- Transition from one state to another
- Inputs and Outputs
- **Two Types:** Moore and Mealy



1. **Moore machine**: output depends on current state only
2. **Mealy machine**: output depends on current state and inputs