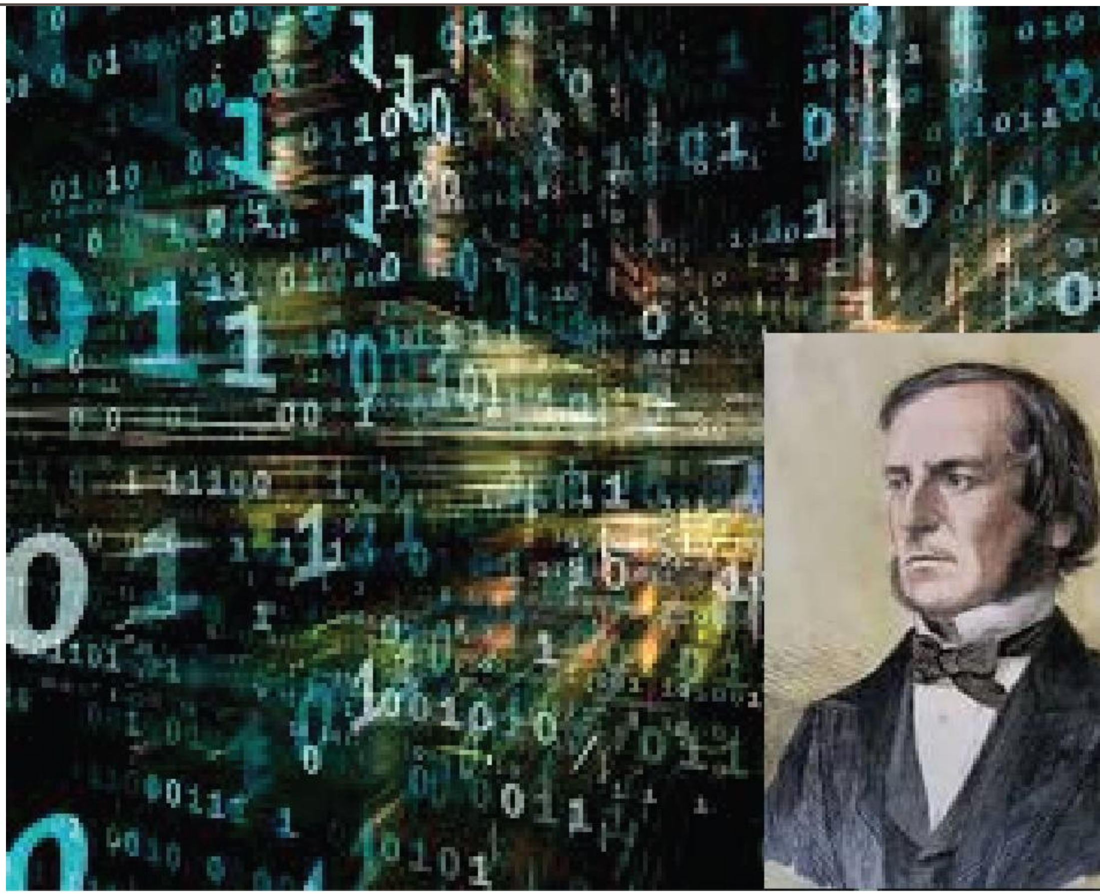
DIGITAL CIRCUITS

Week-2, Lecture-3
Boolean Algebra



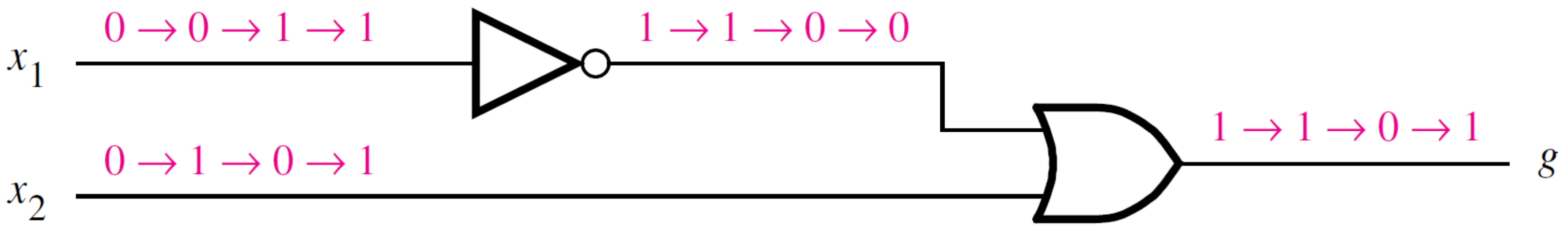
Sneh Saurabh
10th August, 2018

Digital Circuits: Announcements/Revision

Digital Circuits

Logic Functions and Logic Gates

Logic Network Analysis: Function $g(x_1, x_2)$



Analysis: Find the function that is represented by the above Logic Network

Approach:

a) Write the Logic Function

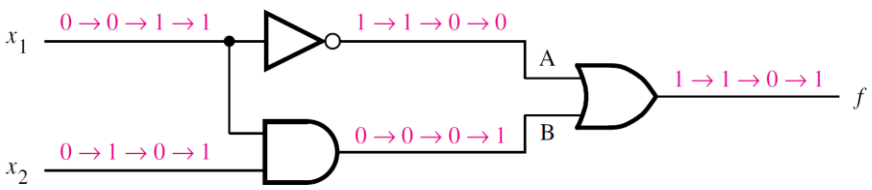
b) Write the Truth Table

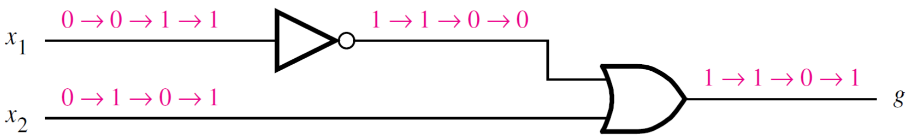
Logic Function:

$$g(x_1, x_2) = \overline{x_1} + x_2$$

x_1	x_2	$g(x_1, x_2)$
0	0	1
0	1	1
1	0	0
1	1	1

Functionally Equivalent Networks (1)



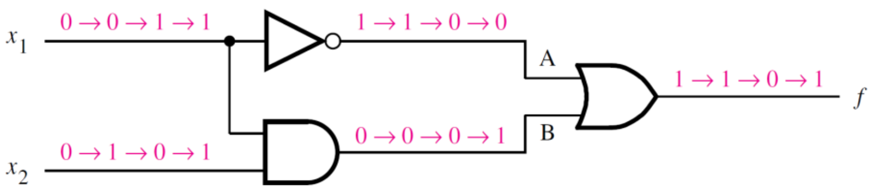


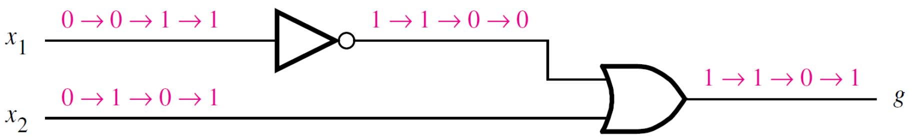
x_1	x_2	A	B	$f(x_1, x_2)$
0	0	1	0	1
0	1	1	0	1
1	0	0	0	0
1	1	0	1	1

x_1	x_2	$g(x_1, x_2)$
0	0	1
0	1	1
1	0	0
1	1	1

- **Truth table of two logic networks are exactly same**
- Therefore, the two networks implement the same function
- Such networks are called ***functionally equivalent network***

Functionally Equivalent Networks (2)





Which of the two logic networks is preferable?

- The function $g(x_1, x_2)$ is preferred because it has less number of logic gates

How to find whether two logic networks are functionally equivalent?

- We have established equivalence by comparing truth table
- $\overline{x_1} + x_1.x_2 \equiv \overline{x_1} + x_2$

Can we establish equivalence by algebraic techniques?

- Yes, using ***Boolean Algebra***

Digital Circuits

Boolean Algebra

Boolean Algebra: Introduction

- *George Boole: scheme for algebraic description of processes involved in* ***logical thought and reasoning (1849)***

- This scheme later got refined and became known as ***Boolean Algebra***

- In 1930s Claude Shannon showed that Boolean algebra can be used to describe circuits built with switches

- Effective tool for ***analysis and synthesis*** of digital circuits

- Boolean algebra is based on a set of rules that are derived from a small number of basic assumptions (***axioms***)

Boolean Algebra: *Axioms*

- Elements can take one of the two values: $\{0, 1\}$
- Operators "**.**" and "**+**"

S.No.	(a)	(b)
1	$0.0 = 0$	$1 + 1 = 1$
2	$1.1 = 1$	$0 + 0 = 0$
3	$0.1 = 1.0 = 0$	$1 + 0 = 0 + 1 = 1$
4	If $x = 0$, then $\bar{x} = 1$	If $x = 1$, then $\bar{x} = 0$

Boolean Algebra: *Axioms*

S.No.	(a)	(b)
1	$0.0 = 0$	$1 + 1 = 1$
2	$1.1 = 1$	$0 + 0 = 0$
3	$0.1 = 1.0 = 0$	$1 + 0 = 0 + 1 = 1$
4	If $x = 0$, then $\bar{x} = 1$	If $x = 1$, then $\bar{x} = 0$

Using above axioms, theorems can be proven by perfect induction.

Prove by induction: $x.0 = 0$

Solution:

If $x = 0$, then $x.0 = 0.0 = 0$ [By 1(a)]

If $x = 1$, then $x.0 = 1.0 = 0$ [By 3(a)]

Prove by induction (For your practice):

$x + 1 = 1$

$x.1 = x$ $\qquad$ $x + 0 = x$

$x.x = x$ $\qquad$ $x + x = x$

$x.x' = 0$ $\qquad$ $x + x' = 1$

$(x')' = x$

Boolean Algebra: *Dual* of a logical expression

- Given a logical expression, its dual expression is obtained by:
  - Replace all “+” operator by “.” operator and vice versa
  - Replace all zeros with ones, and vice versa

Find the dual for the following expression: $x.\bar{z} + y$

Ans: $(x + \bar{z}).y$ [care should be taken to account for precedence]

Find the dual for the following expression: $(x.y + 0).z$

Ans: $\left((x + y).1\right) + z$

Boolean Algebra: *Principle of Duality*

The dual of a true statement is also true.

S.No.	(a)	(b)
1	$0.0 = 0$	$1 + 1 = 1$
2	$1.1 = 1$	$0 + 0 = 0$
3	$0.1 = 1.0 = 0$	$1 + 0 = 0 + 1 = 1$
4	If $x = 0$, then $\bar{x} = 1$	If $x = 1$, then $\bar{x} = 0$

Statements in column (a) and (b) are duals of each other

Boolean Algebra: *Huntington's Postulates*

- In 1904, E. V. Huntington defined Boolean algebra by providing 6 postulates that must be satisfied.

- Set of elements of a Boolean Algebra is represented by B
- Two operators are defined "**.**" and "**+**"

Postulate 1: Closure

a) The structure is closed with respect to the operator +

b) The structure is close with respect to the operator .

Boolean Algebra: *Huntington's Postulates*

Postulate 2: Identity Element

a) The element 0 is the identity element with respect to +

$$x + 0 = 0 + x = x$$

b) The element 1 is the identity element with respect to .

$$x.1 = 1.x = x$$

Postulate 3: Commutative Property

a) The structure is commutative with respect to +

$$x + y = y + x$$

b) The structure is commutative with respect to .

$$x.y = y.x$$

Boolean Algebra: *Huntington's Postulates*

Postulate 4: Distributive Property

a) The operator . is distributive over +

$$x.(y+z) = x.y + x.z$$

b) The operator + is distributive over .

$$x + (y.z) = (x+y).(x+z)$$

Postulate 5: Existence of a complement

For every element $x \in B$, there exists $x' \in B$ such that:

a. $x + x' = 1$

b. $x.x' = 0$

Boolean Algebra: *Huntington's Postulates*

Postulate 6: Distinct elements

- There exist at least two elements $x, y \in B$ such that $x \neq y$
- ***Two valued Boolean algebra***: $B = \{0, 1\}$, $0 \neq 1$
- In this course we will consider only ***Two valued Boolean algebra***

We will use Huntington's Postulate to:

- Prove Theorems in Boolean Algebra
- Prove equivalence of two circuits
- Minimize circuits [represent with less number of logic gates]

Theorems Proof Using *Huntington's Postulates*

Prove using Huntington's Postulates:
$x + 1 = 1$

Solution:

$x + 1$

$= 1.(x + 1)$

$= (\mathrm{x} + \mathrm{x}').(x + 1)$

$= x + x'.1$

$= x + x'$

$= 1$

2(a) $x + 0 = 0 + x = x$

2(b) $x.1 = 1.x = x$

3(a) $x + y = y + x$

3(b) $x.y = y.x$

4(a) $x.(y + z) = x.y + x.z$

4(b) $x + (y.z) = (x + y).(x + z)$

5(a) $x + x' = 1$

5(b) $x.x' = 0$

By Duality: $x.0 = 0$

Theorems Proof Using *Huntington's Postulates*

Prove using Huntington's Postulates:
$x + x = x$

Solution:

$x + x$

$= (x + x).1$

$= (\mathrm{x} + \mathrm{x}).(x + x')$

$= x + xx'$

$= x + 0$

$= x$

2(a) $x + 0 = 0 + x = x$

2(b) $x.1 = 1.x = x$

3(a) $x + y = y + x$

3(b) $x.y = y.x$

4(a) $x.(y + z) = x.y + x.z$

4(b) $x + (y.z) = (x + y).(x + z)$

5(a) $x + x' = 1$

5(b) $x.x' = 0$

By Duality: $x.x = x$

Theorems Proof Using *Huntington's Postulates*

Prove using Huntington's Postulates:
$x.(x' + y) = xy$

Solution:

$x.(x' + y)$

$= x.x' + xy$

$= 0 + xy$

$= xy$

2(a) $x + 0 = 0 + x = x$

2(b) $x.1 = 1.x = x$

3(a) $x + y = y + x$

3(b) $x.y = y.x$

4(a) $x.(y + z) = x.y + x.z$

4(b) $x + (y.z) = (x + y).(x + z)$

5(a) $x + x' = 1$

5(b) $x.x' = 0$

Theorems Proof Using *Huntington's Postulates*

Prove using Huntington's Postulates:
$x + x'.y = x + y$

Solution:

$x + x'.y$

$=(x + x').(x + y)$

$=1.(\mathrm{x} + \mathrm{y})$

$=x + y$

2(a) $x + 0 = 0 + x = x$

2(b) $x.1 = 1.x = x$

3(a) $x + y = y + x$

3(b) $x.y = y.x$

4(a) $x.(y + z) = x.y + x.z$

4(b) $x + (y.z) = (x + y).(x + z)$

5(a) $x + x' = 1$

5(b) $x.x' = 0$