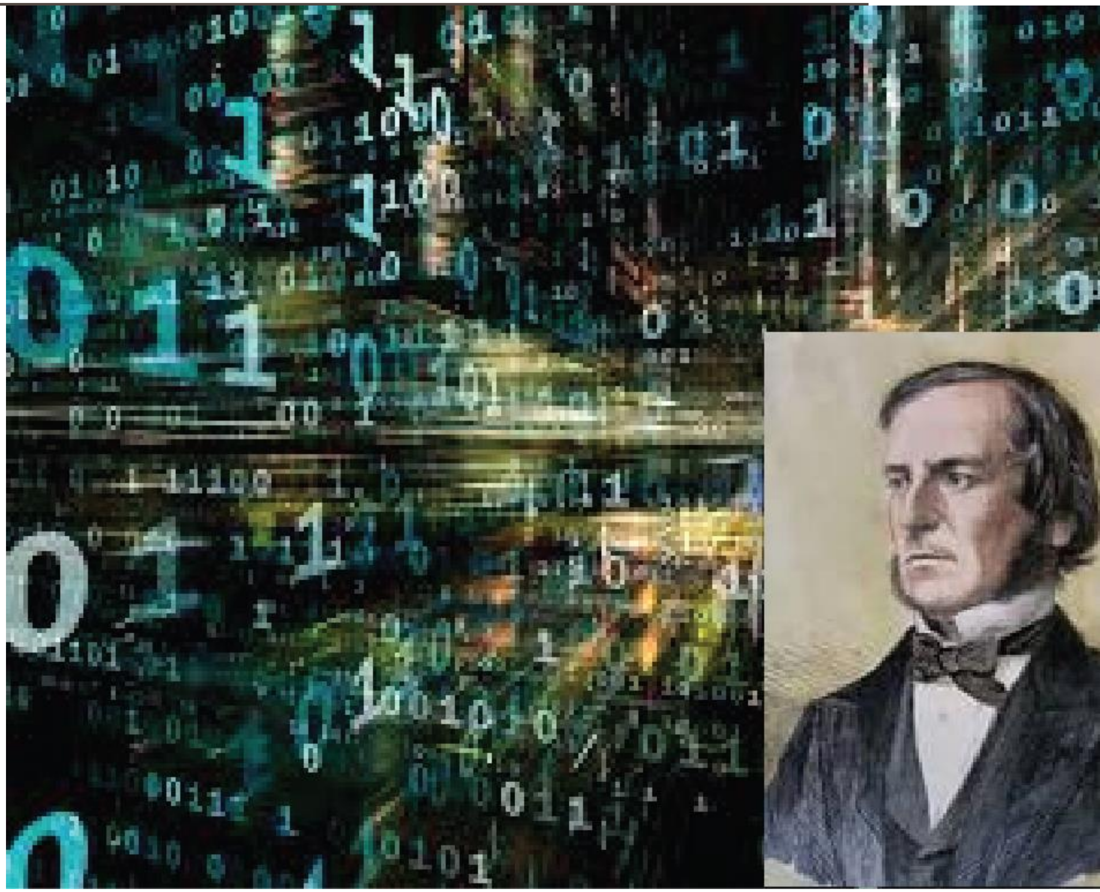
DIGITAL CIRCUITS

Week-3, Lecture-1
Boolean Algebra



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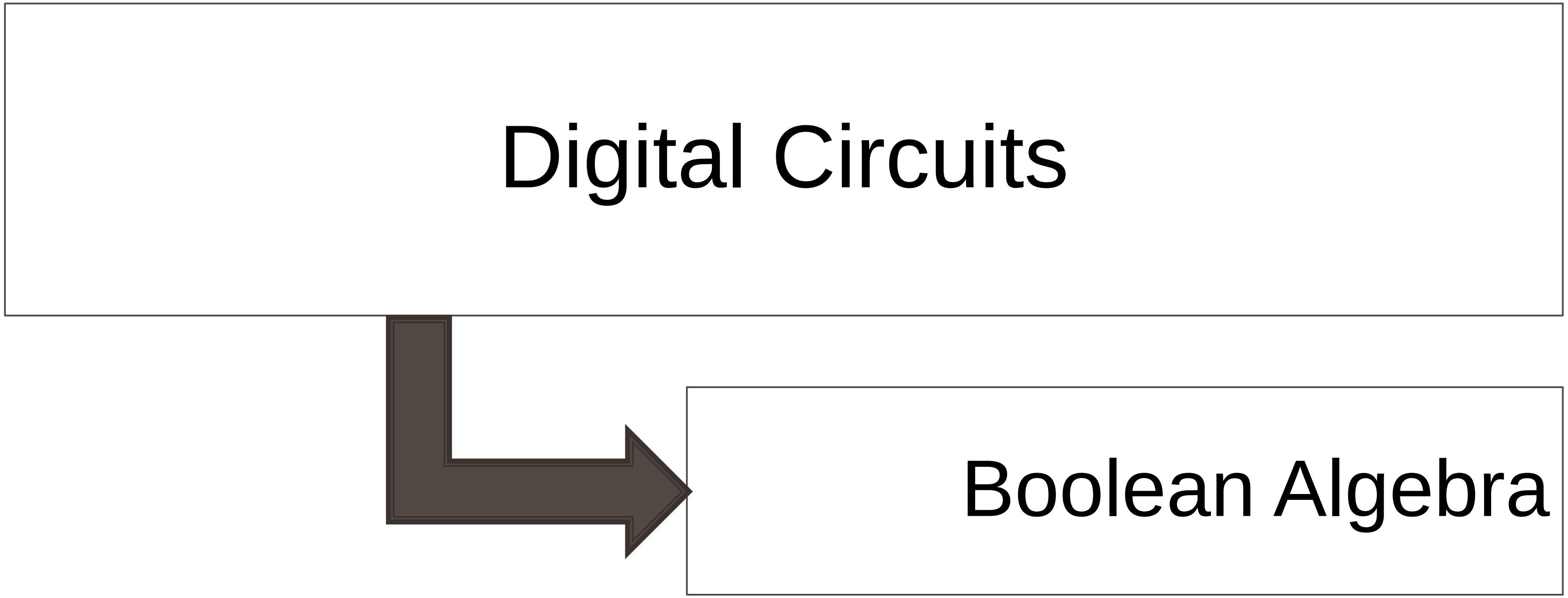
14th August, 2018

Digital Circuits: Announcements/Revision


Digital Circuits
Boolean Algebra

Theorems Proof Using *Huntington's Postulates*

Prove using Huntington's Postulates: $xy + x'z + yz = xy + x'z$

The "." operator can be omitted between variables to make representation look compact

Solution:

$xy + x'z + yz$

$= xy + x'z + yz(\mathrm{x} + \mathrm{x}')$

$= xy + x'z + yz\mathrm{x} + yz\mathrm{x}'$

$= xy.1 + xyz + x'z.1 + x'zy$

$= xy.(1 + z) + x'z.(1 + y)$

$= xy + x'z$

Theorem: $x + 1 = 1$

Proof:

$x + 1 = 1.(x + 1)$

$= (\mathrm{x} + \mathrm{x}').(x + 1)$

$= x + x'.1$

$= x + x' = 1$

2(a) $x + 0 = 0 + x = x$

2(b) $x.1 = 1.x = x$

3(a) $x + y = y + x$

3(b) $x.y = y.x$

4(a) $x.(y + z) = x.y + x.z$

4(b) $x + (y.z) = (x + y).(x + z)$

5(a) $x + x' = 1$

5(b) $x.x' = 0$

Duality Principle: Explanation

The dual of a true ***statement*** is also true.

$x + x' = 1$

Dual Statement

$x . x' = 0$

$x.(y + z) = x.y + x.z$

Dual Statement

$x + (y.z) = (x + y).(x + z)$

$(x_1 + x_2)' = x_1'x_2'$

Dual Statement

$(x_1.x_2)' = x_1' + x_2'$

$x + y$

Dual Expression: $x.y$

They are not equal.

$(x + \bar{z}).y$ Dual Expression: $x.\bar{z} + y$

They are not equal (Verify using Truth Table)

- Statement has LHS and RHS
- Dual has to be taken on both the sides

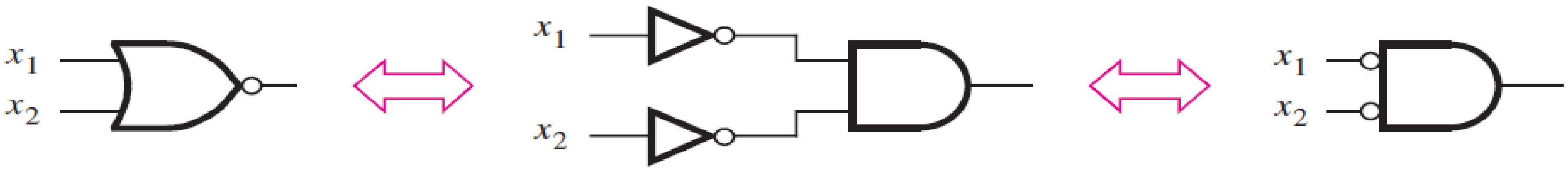
- Principle of duality applies to a ***statement*** and ***NOT to expression***
- In general, ***expression*** and its ***dual expression*** are not EQUAL

IMPORTANT

- Given a function, DO NOT realize the dual of that function.
- In general, they will be different functions

DeMorgan's Theorems (1)

$$(x_1 + x_2)' = x_1'x_2'$$



(b) $\overline{x_1 + x_2} = \bar{x}_1 \bar{x}_2$

x_1	x_2	$(x_1+x_2)'$
0	0	**1**
0	1	**0**
1	0	**0**
1	1	**0**

x_1	x_2	x_1'	x_2'	$x_1'x_2'$
0	0	1	1	**1**
0	1	1	0	**0**
1	0	0	1	**0**
1	1	0	0	**0**

DeMorgan's Theorems (1)

$(x_1 + x_2)' = x_1{}'x_2{}'$

Proof:

Let $A = x_1 + x_2$ and $B = x_1{}'x_2{}'$

$B + A$

$= x_1{}'x_2' + x_1 + x_2 = x_1 + x_1{}'x_2{}' + x_2$

$= (x_1 + x_1')(x_1 + x_2') + x_2$

$= x_1 + x_2' + x_2 = x_1 + 1$

$= 1$(should prove $x + 1 = 1$)

$B.A$

$= x_1{}'x_2'(x_1 + x_2)$

$= x_1{}'x_2'x_1 + x_1{}'x_2'x_2$

$= 0 + 0 = 0$

2(a) $x + 0 = 0 + x = x$

2(b) $x.1 = 1.x = x$

3(a) $x + y = y + x$

3(b) $x.y = y.x$

4(a) $x.(y + z) = x.y + x.z$

4(b) $x + (y.z) = (x + y).(x + z)$

5(a) $x + x' = 1$

5(b) $x.x' = 0$

DeMorgan's Theorems (1)

$(x_1 + x_2)' = x_1' x_2'$

Proof:

Let $A = x_1 + x_2$ and $B = x_1' x_2'$

$B + A = 1$

$B . A = 0$

Postulate 5: Existence of a complement

For every element $x \in B$, there exists $x' \in B$ such that:

a. $x + x' = 1$

b. $x . x' = 0$

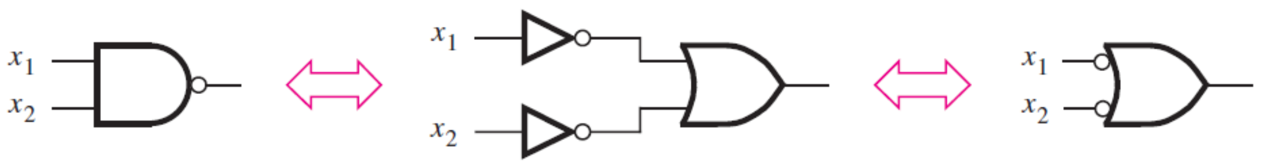
Postulate 5 imply that $A' = B$

$\Rightarrow (x_1 + x_2)' = x_1' x_2'$

DeMorgan's Theorems

$(x_1 + x_2)' = x_1{}'x_2{}'$

$(x_1x_2)' = x_1{}' + x_2{}'$ (Dual)



(a) $\overline{x_1x_2} = \bar{x}_1 + \bar{x}_2$

x_1	x_2	$(x_1x_2)'$
0	0	**1**
0	1	**1**
1	0	**1**
1	1	**0**

x_1	x_2	$x_1{}'$	$x_2{}'$	$x_1{}' + x_2{}'$
0	0	1	1	**1**
0	1	1	0	**1**
1	0	0	1	**1**
1	1	0	0	**0**

DeMorgan's Theorems (More variables)

$$(x_1 + x_2)' = x_1{}'x_2{}'$$

$$(x_1 + x_2 + x_3)' = x_1{}'x_2{}'x_3{}'$$

Proof:

Let $x_2 + x_3 = A$

$(x_1 + x_2 + x_3)'$

$= (x_1 + A)'$

$= x_1'A' = x_1{}'(x_2 + x_3)'$

$= x_1{}'(x_2{}'x_3{}') = x_1{}'x_2{}'x_3{}'$

DeMorgan's Theorems (More variables)

$(x_1 + x_2)' = x_1'x_2'$

$(x_1x_2)' = x_1' + x_2'$ (Dual)

$(x_1 + x_2 + x_3)' = x_1'x_2'x_3'$

$(x_1x_2x_3)' = x_1' + x_2' + x_3'$

$(x_1 + x_2 + x_3 + x_4)' = x_1'x_2'x_3'x_4'$

$(x_1x_2x_3x_4)' = x_1' + x_2' + x_3' + x_4'$

$(x_1 + x_2 + \cdots + x_N)' = x_1'x_2' \ldots x_N'$

$(x_1x_2 \ldots x_N)' = x_1' + x_2' + \cdots + x_N'$

Digital Circuits

Synthesis using Logic Gates

Basic Definitions (1)

Boolean variable: A variable which can take only ‘0’ or ‘1’ values.

Example: $x, y, a, b,$ ...

Literal: A Boolean variable or its complement.

Example: $x, x', y, y', a, a', b, b',$ ...

Product Term: A single literal or a logical AND of two or more literals.

Example: $x, x', xy, yy', abc, a'bc'd,$ ...

Sum Term: A single literal or a logical OR of two or more literals.

Example: $x, x', x + y, y + y', a + b + c, a' + b + c' + d,$ ...

Basic Definitions (2)

Normal Term: A product or sum term in which no variable appears more than once.

Example:

$x, a+b, abc'$, ... are Normal Terms

$a+a'$, $abcb'$, $a+b+b'$ are NOT Normal Terms

In the following expression, identify:

$\boldsymbol{a+a'b+(c+c')+dd'}$

a) Variables and Literals

b) Product terms and Sum Terms

c) Normal Terms and Non-normal terms

a) Variables: a, b, c, d and Literals: a, a', b, c, c', d, d'

b) Product terms: $a, a'b, c, c', dd'$ and Sum Terms: $a, (c+c')$

c) Normal Terms: $a, a'b$ and Non-normal terms: $(c+c'), dd'$

Basic Definitions (3)

Sum of Product (SOP) Expression: A ***logical OR*** of a set of ***product terms.***

Examples:

$ab + a'bc + ac'$

$abc + a'bc + abc' + ab'c'$

$a + b$

Product of Sum (POS) Expression: A ***logical AND*** of a set of ***sum terms***.

Examples:

$(a + b)(a' + b + c)(a + c')$

$(a + b + c)(a' + b + c)(a + b + c')(a + b' + c')$

ab

Synthesis: Using Truth Table (1)

Problem:

Design a network using ***logic gates*** that takes two inputs x_1 and x_2. Assume that x_1 and x_2 represent the states of two switches, either of which may be open (0) or closed (1). The output of the network is 1 when (x_1, x_2) are in the states (0,0), (0,1) or (1,1). In the state (1,0) the output should be 0.

$\boldsymbol{x_1}$	$\boldsymbol{x_2}$	$\boldsymbol{f(x_1, x_2)}$
0	0	1
0	1	1
1	0	0
1	1	1

A sum of product (SOP) expression is formed using truth table.

Synthesis: Truth Table to SOP

x_1	x_2	$f(x_1, x_2)$	Product Term
0	0	1	$x_1'x_2'$
0	1	1	$x_1'x_2$
1	0	0	x_1x_2'
1	1	1	x_1x_2

$$f(x_1, x_2) = x_1'x_2' + x_1'x_2 + x_1x_2$$

How to find the product term:

- For each row find the product term
  - The variable that has value 0 is complemented and the variable that has value 1 is taken without complement

How to find the sum:

- Take the sum of product terms that result in function being 1

Synthesis: SOP to Network

$$f(x_1, x_2) = x_1' x_2' + x_1' x_2 + x_1 x_2$$