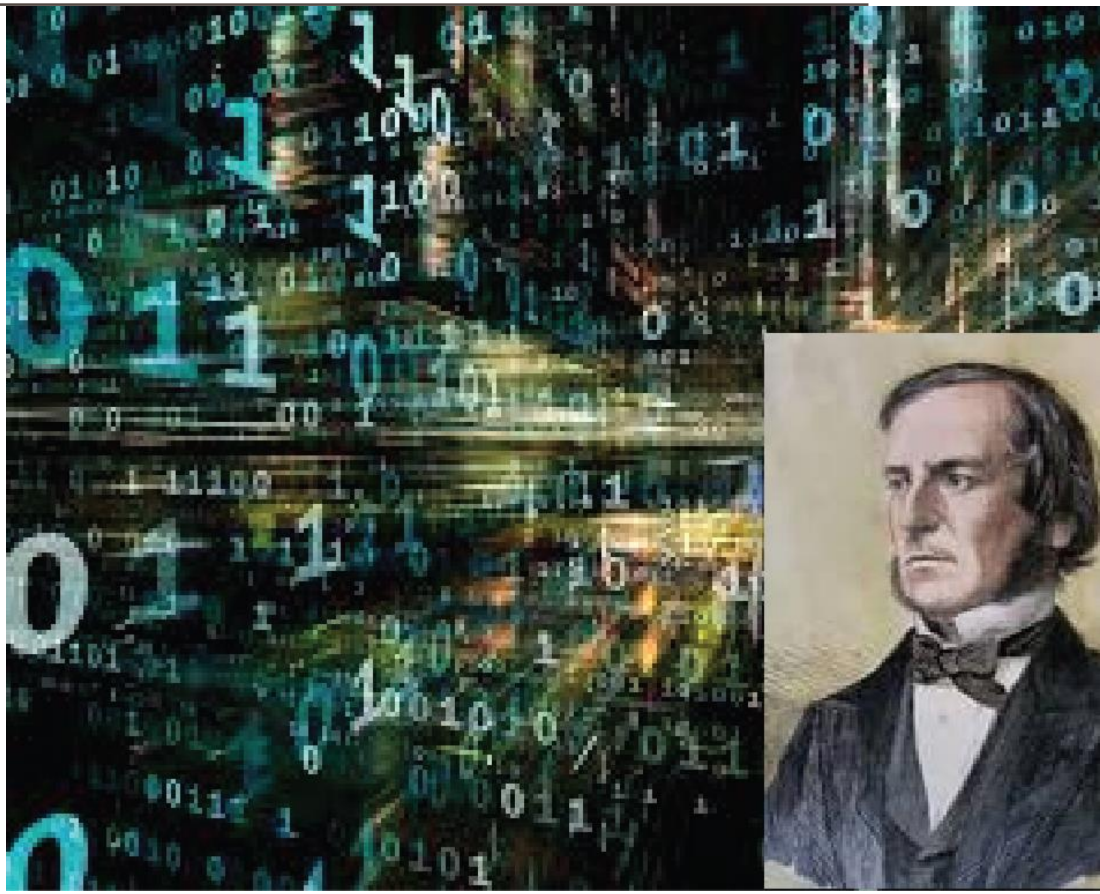
DIGITAL CIRCUITS

Week-6, Lecture-1
Arithmetic and Codes



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Digital Circuits: Announcements/Revision

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Arithmetic of signed numbers

Diminished radix complement: $(r-1)$'s complement

- Let the number be P in base r and is represented with $n-\text{digits}$
- $(r-1)'s$ complement of N is defined as:
  - ➢ $K=(r^n-1)-P$

- Subtract each digit with $(r-1)$:
  - ➢ For binary, subtract with 1
  - ➢ For octal, subtract with 7
  - ➢ For decimal subtract with 9
  - ➢ For hexadecimal subtract with F

Problem: Given $n=4$. Find 9's complement of $(1583)_{10}$

Solution:

- ➢ $K=(10^4-1)_{10}-(1583)_{10}=(9999)_{10}-(1583)_{10}$
- ➢ $=(8416)_{10}$

Problem: Given $n=3$. Find 15's complement of $(1A3)_{16}$

Solution:

- ➢ $K=(FFF)_{16}-(1A3)_{16}$
- ➢ $=(E5C)_{16}$

Radix complement: r's complement

- Let the number be P in base r and is represented with $n - \text{digits}$
- r's complement of N is defined as:
  - ➢ $K = r^n - P$ for $P \neq 0$
  - ➢ $K = 0$ for $P = 0$
- r's complement of N can be found by adding 1 to the $(r-1)'s$ complement of N

Problem: Given $n = 4$. Find 10's complement of $(1583)_{10}$

Solution:

➢ $K = (8416)_{10} + 1 = (8417)_{10}$

Problem: Given $n = 4$. Find 16's complement of $(1A3)_{16}$

Solution:

➢ $K = ((FFFF)_{16} - (01A3)_{16}) + 1$

➢ $= (FE5D)_{16}$

r's or $(r-1)$'s complement

- If the original number P contains a radix point (.)
  - Temporarily remove the radix point from P and then find the r's or $(r-1)$'s complement
  - The radix point is restored to the complemented number in the same relative position

Problem: Given $n = 3$. Find 10's complement of $(15.1)_{10}$

Solution:

- 10's complement of $(151)_{10} = (848 + 1\ )_{10} = (849\ )_{10}$
- 10's complement of $(15.1)_{10} = (84.9\ )_{10}$

- Complement of a complement of given number, restores the original number
  - First Complement: $K = r^n - P$
  - Second complement: $K' = r^n - (r^n - P) = P$

Radix r: Addition of unsigned numbers

Problem:

- Given two n-digit unsigned numbers M and N in radix r
- To find $(M + N)$

Steps:

- Add two numbers digit-wise starting with LSB (right) and proceed to MSB (left)
- Carry is generated at a position i if the sum of the $i-th$ digits of the two numbers and the carry-in from $(i-1)$ position is greater than or equal to r

Problem:

Find: $(23)_8$+$(56)_8$

Answer: $(101)_8$

Problem:

Find: $(AA)_{16}$+$(19)_{16}$

Answer: $(C3)_{16}$

Radix r: Subtraction of unsigned number

Problem:

- Given two n-digit unsigned numbers M and N in radix r
- To find $(M - N)$

Steps:

- Add M to the (r's complement of N)
  - $S = M + r^n - N = r^n + (M - N)$
- Two cases:
  - $M \geq N$: Result non-negative
  - $M < N$: Result negative

Subtraction of unsigned number: result non-negative

- Add M to the (r's complement of N)
  - $S = M + r^n - N = r^n + (M - N)$
- $M \geq N$:
  - S will produce overflow r^n at $(n + 1)$ position, which can be discarded.
  - The other n-digits contain the result

Problem:

Given $n = 5$. Using 10's complement find $(72532)_{10} - (3250)_{10}$.

M	=	72532
10's complement of N	=	+ 96750
Sum	=	169282
Discard end carry 10^5	=	−100000
Answer	=	69282

Subtraction of unsigned number: result negative

- Add M to the (r's complement of N)
  - $S = M + r^n - N = r^n + (M - N)$
- $M < N$:
  - S will contain n digits only.
  - It represents the r's complement of $(N - M)$ because $r^n - (N - M)$.
  - To get the answer in familiar form, take the r's complement of S and place a (–) sign before the number

Problem:

Given $n = 5$. Using 10's complement find $(3250)_{10} - (72532)_{10}$.

$$\begin{aligned} M &= 03250 \\ \text{10's complement of } N &= \underline{+27468} \\ \text{Sum} &= 30718 \end{aligned}$$

- 10's complement of $(30718)_{10}$ is $(69282)_{10}$
- The final answer is $(-69282)_{10}$

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Binary Codes

Binary codes: Introduction

- Binary codes are patterns/group of zeros and ones
- Any discrete information that is distinct within a group can be represented using binary codes
- Binary codes merely change the symbol representing data: the meaning of data is not changed
- Motivation for using binary code is ease of manipulation, storage or transmission of data

Binary codes: Number of bits

- An $n - bit$ binary code is a group of $n - bits$ that assumes up to 2^n combination of zeroes and ones
- Each combination represent one element of the set that is being encoded: code should be unique for each element
- Though minimum number of bits required to represent 2^n elements is n, there is no maximum number of bits that can be used in encoding (some bits may be unused)