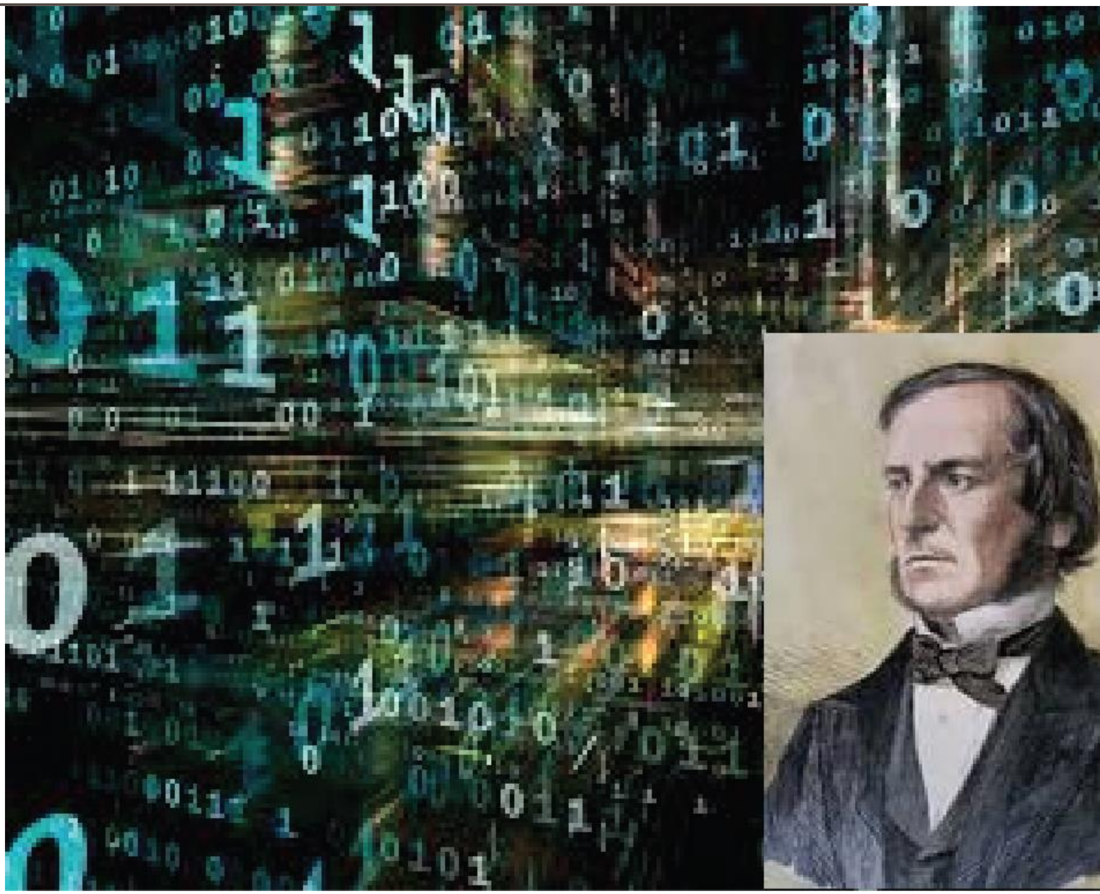
DIGITAL CIRCUITS

Week-5, Lecture-1
Boolean Algebra



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29th August, 2018

Digital Circuits: Announcements/Revision

Digital Circuits

Number System

Decimal Number System: Base/Radix 10

- In decimal system $\{0, 1, 2, \dots, 9\}$ digits are used

- A number is represented in general by a series of coefficients
- The position of the coefficients determines the power of 10 that the coefficient must be multiplied
  - $a_4 a_3 a_2 a_1 a_0 . a_{-1} a_{-2}$
  $= a_4 \times 10^4 + a_3 \times 10^3 + a_2 \times 10^2 + a_1 \times 10^1 + a_0 \times 10^0 + a_{-1} \times 10^{-1} + a_{-2} \times 10^{-2}$
  - Example: 72451.29
  $= 7 \times 10^4 + 2 \times 10^3 + 4 \times 10^2 + 5 \times 10^1 + 1 \times 10^0 + 2 \times 10^{-1} + 9 \times 10^{-2}$
- The decimal number system is said to be of ***base*** or ***radix 10***
  - Because it uses 10 digits $\{0, 1, 2, \dots, 9\}$ and coefficients are multiplied by power of 10

Binary system or Base-2 system or Radix-2 system

- In binary system $\{0, 1\}$ digits are used

- The position of the coefficients determines the power of 2 that the coefficient must be multiplied
  - $a_3 a_2 a_1 a_0 . a_{-1} a_{-2}$

    $= a_3 \times 2^3 + a_2 \times 2^2 + a_1 \times 2^1 + a_0 \times 2^0 + a_{-1} \times 2^{-1} + a_{-2} \times 2^{-2}$
  - Example: $(1101.11)_2$

    $= (1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 + 1 \times 2^{-1} + 1 \times 2^{-2})_{10}$

    $= (8 + 4 + 1 + 0.5 + 0.25)_{10} = (13.75)_{10}$

Base-r or Radix-r system

- In base-r system $\{0, 1, 2, \ldots, (r-1)\}$ digits are used. Beyond $9, \{A, B, C, \ldots\}$ symbols are used

- Example, digits for various base system:
  - $Base - 4$: $\{0, 1, 2, 3\}$
  - $Base - 5$: $\{0, 1, 2, 3, 4\}$
  - $Base - 8\ (Octal)$: $\{0, 1, 2, 3, 4, 5, 6, 7\}$
  - $Base - 16\ (Hexadecimal)$: $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F\}$

- The position of the coefficients determines the power of r that the coefficient must be multiplied
  - $(a_4 a_3 a_2 a_1 a_0 . a_{-1} a_{-2})_r$
  $= a_4 \times r^4 + a_3 \times r^3 + a_2 \times r^2 + a_1 \times r^1 + a_0 \times r^0 + a_{-1} \times r^{-1} + a_{-2} \times r^{-2}$
  - Example: $(213.2)_8$
  $= (2 \times 8^2 + 1 \times 8^1 + 3 \times 8^0 + 2 \times 8^{-1})_{10} = (128 + 8 + 3 + 2 \times 0.125)_{10} = (139.25)_{10}$

Hexadecimal to decimal

Problem:

Convert the number $(FA)_{16}$ to decimal.

Solution:

$$(FA)_{16} = (15 \times 16^{1} + 10 \times 16^{0})_{10} = (250)_{10}$$

Conversion of a decimal number to base-r

- If a decimal number contains both the integer portion and the fraction part, then both these parts are treated differently
- Conversion of integer is done and conversion of fraction is done, and both results are combined

Integer part

Divide the number and all successive quotients by r and accumulate the remainders.

Conversion of a decimal number to binary

Problem:

Convert the decimal number 153 into binary ($r = 2$)

Answer:

$$(153)_{10} = (10011001)_2$$

Integer	Remainder
153	
76	1
38	0
19	0
9	1
4	1
2	0
1	0
0	1

Conversion of a decimal number to Octal/Hexadecimal

Problem:

Convert the decimal number 153 into octal ($r = 8$)

Answer:

$$(153)_{10} = (231)_8$$

Integer	Remainder
153	
19	1
2	3
0	2

Problem:

Convert the decimal number 153 into hexadecimal ($r = 16$)

Answer:

$$(153)_{10} = (99)_{16}$$

Integer	Remainder
153	
9	9
0	9

Unsigned number representation

Decimal (base 10)	Binary (base 2)	Octal (base 8)	Hexadecimal (base 16)
00	0000	00	0
01	0001	01	1
02	0010	02	2
03	0011	03	3
04	0100	04	4
05	0101	05	5
06	0110	06	6
07	0111	07	7
08	1000	10	8
09	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

Conversion of binary to Octal/Hexadecimal

Problem:

Convert the binary number $(10011001)_2$ into octal ($r = 8$)

- Form a group of **three** bits starting from right.
- Replace each group with corresponding octal digit
- Add extra zeroes on the left of MSB to make the number of digits as multiple of three

$(010 \quad 011 \quad 001)_2$

$= (2 \quad 3 \quad 1)_8$

Problem:

Convert the binary number $(10011001)_2$ into hexadecimal ($r = 16$)

- Form a group of **four** bits starting from right.
- Replace each group with corresponding hexadecimal digit
- Add extra zeroes on the left of MSB to make the number of digits as multiple of four

$(1001 \quad 1001)_2$

$= (9 \quad 9)_{16}$

Conversion of fraction in decimal to base-r

Fraction part.

Multiply the fractional part by r and accumulate the integer portion

Problem:

Convert the decimal number $(0.6875)_{10}$ into binary ($r = 2$)

	Integer	Fraction
0.6875×2	1	0.375
0.375×2	0	0.75
0.75×2	1	0.5
0.5×2	1	0

Answer:

$$(0.6875)_{10} = (0.1011)_2$$

Problem:

Convert the decimal number $(153.6875)_{10}$ into binary ($r = 2$)

Answer:

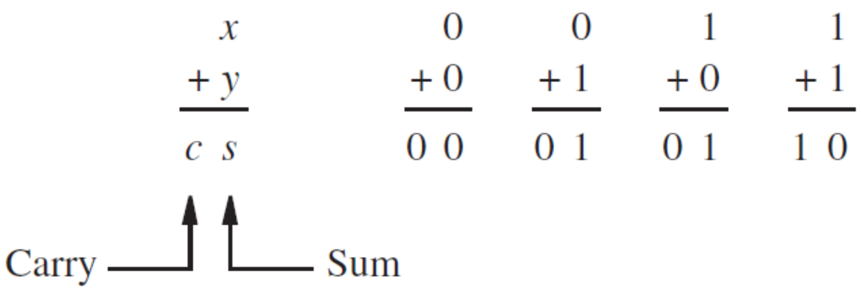
$$(153.6875)_{10} = (10011001.1011)_2$$

Digital Circuits

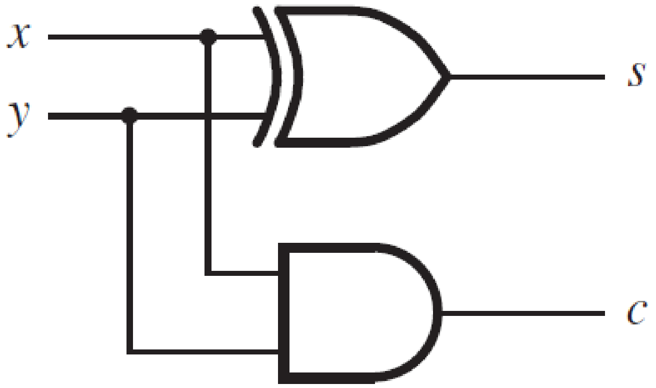
Arithmetic Operations

Addition of unsigned numbers: 2 bits

- Addition of binary numbers is similar to decimal numbers, except that individual digits can take only 0 and 1



x	y	Carry c	Sum s
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0



A circuit which implements the addition of only two bits is known as ***Half Adder (HA)***

Addition of unsigned numbers: Multiple bits

- Each pair is added, similar to two-bits
- At bit position i, carry-in from bit position $(i - 1)$ can come

Generated carries $\longrightarrow$ 1 1 1 0

$$
\begin{array}{rcl}
X = x_4x_3x_2x_1x_0 & 0\,1\,1\,1\,1 & (15)_{10} \\
+\,Y = y_4y_3y_2y_1y_0 & +\,0\,1\,0\,1\,0 & +\,(10)_{10} \\
\hline
S = s_4s_3s_2s_1s_0 & 1\,1\,0\,0\,1 & (25)_{10}
\end{array}
$$

- For each bit position i, the addition involves bits x_i and y_i and carry-in c_i {except for the first bit}
- For each bit position sum s_i and carry for the next stage c_{i+1} are computed

Addition of unsigned numbers: Full Adder

c_i	x_i	y_i	c_{i+1}	s_i
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

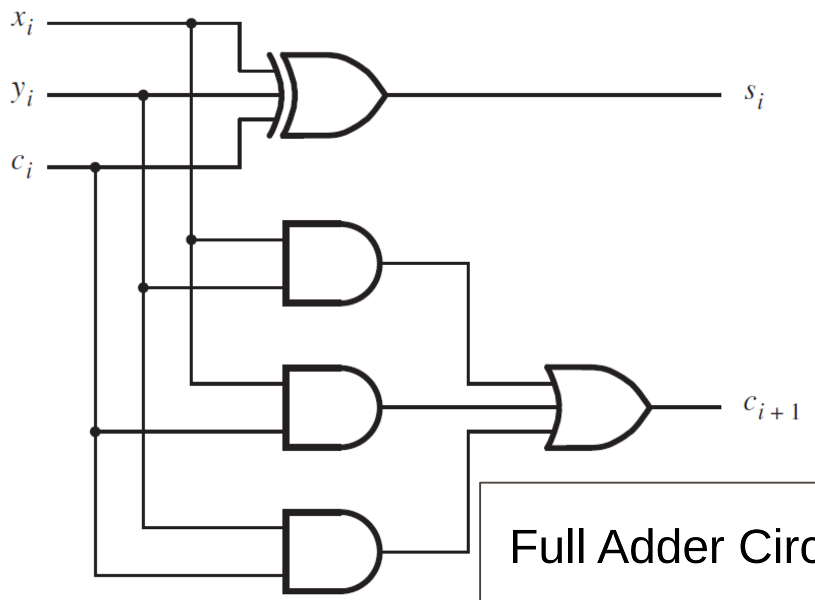
$$s_i = c_i'x_i'y_i + c_i'x_iy_i' + c_ix_i'y_i' + c_ix_iy_i$$

$$= c_i'(x_i'y_i + x_iy_i') + c_i(x_i'y_i' + x_iy_i)$$

$$= c_i'(x_i \oplus y_i) + c_i(x_i \oplus y_i)'$$

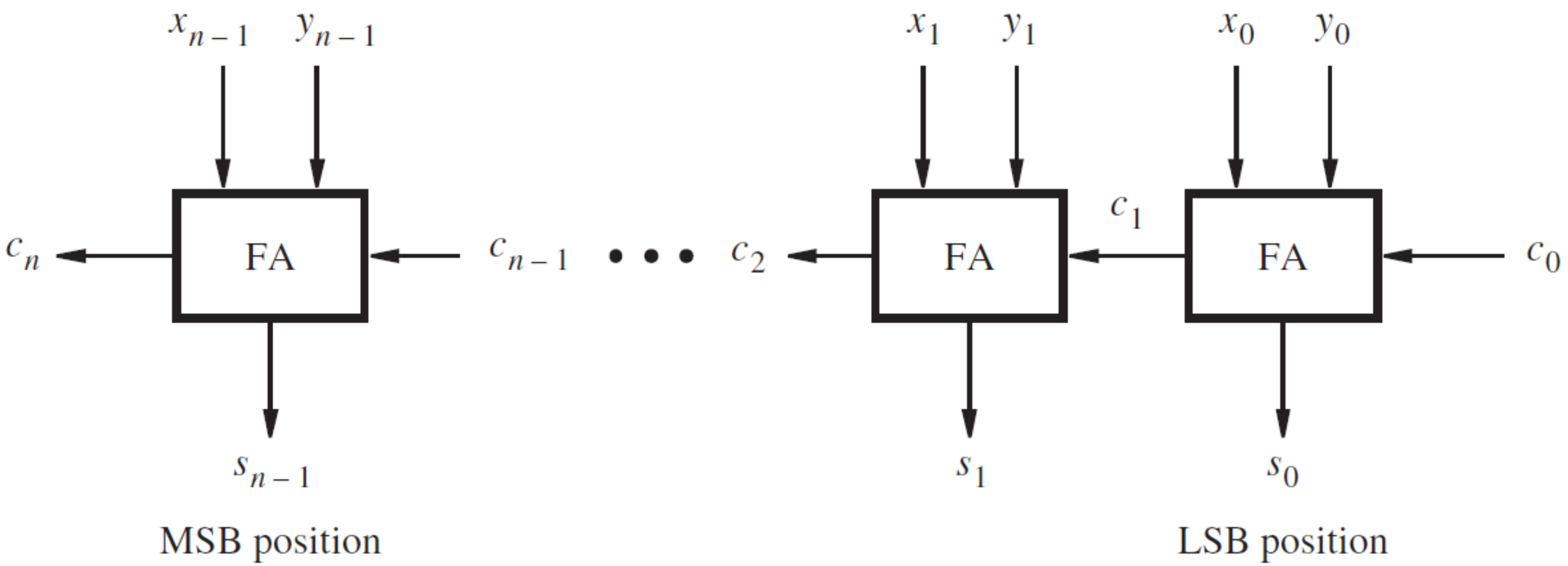
$$= x_i \oplus y_i \oplus c_i$$

$$c_{i+1} = x_iy_i + x_ic_i + y_ic_i$$



Full Adder Circuit

N-bit adder



- To add two unsigned numbers, circuit is designed similar to what is done in hand-calculation
- Least Significant Bit (LSB) is on the right and Most Significant Bit (MSB) is on the left
- Bits are added starting from right using Full Adders
- Carry bits propagate from right to left

Digital Circuits

Signed Numbers

Signed Number Representation in Binary

- Positive numbers are represented by positional number system (as explained)
- Negative numbers can be represented in three ways:
    1. Sign-and-magnitude
    2. 1's complement
    3. 2's complement
- Sign is represented by the leftmost bit