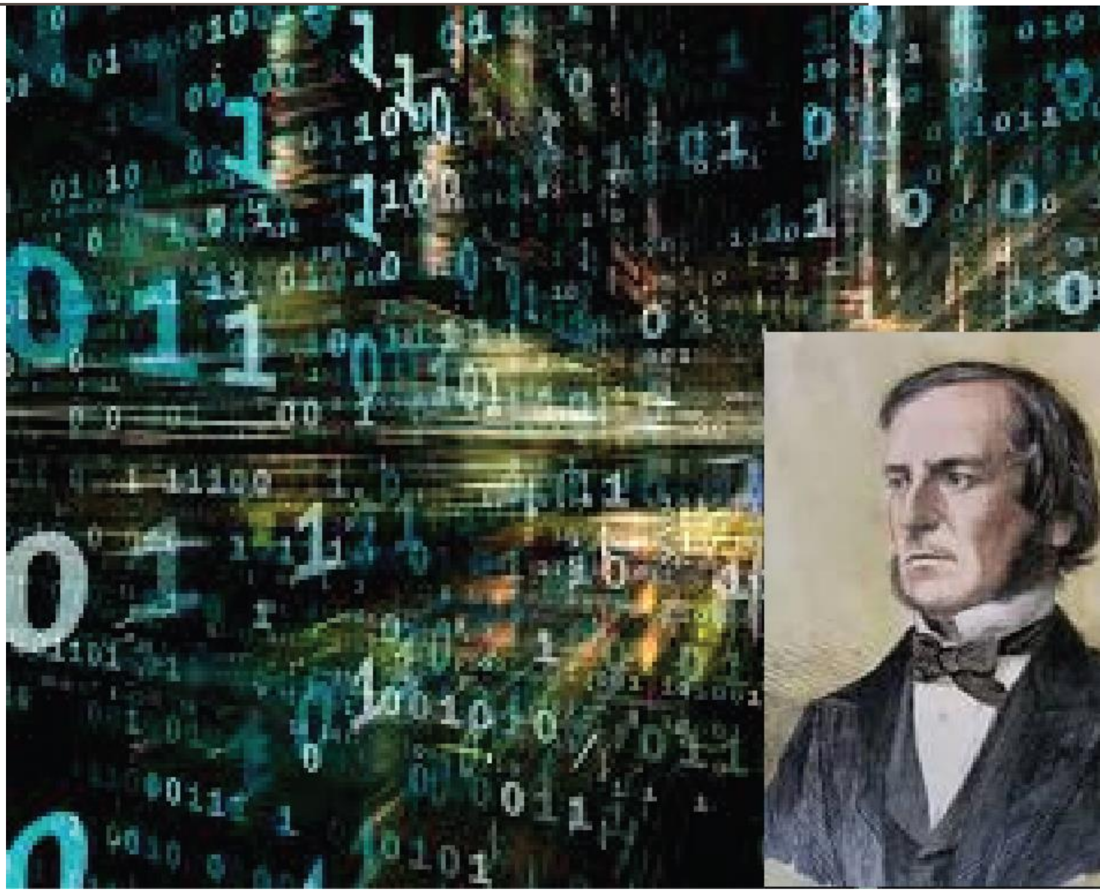
DIGITAL CIRCUITS

Week-2, Lecture-1
Introduction



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7th August, 2018

Digital Circuits: Announcements/Revision

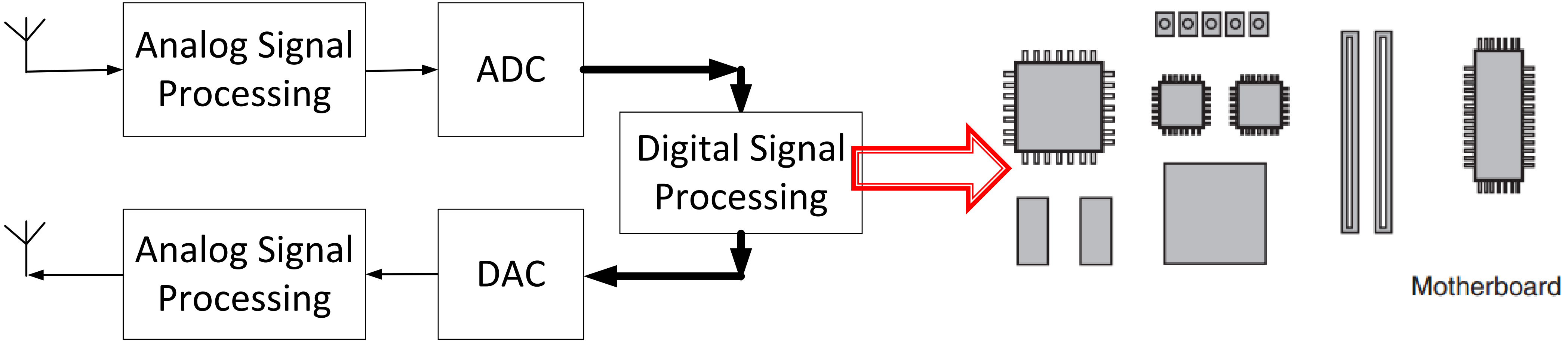
Digital Circuits

Introduction

Analog System vs. Digital System (4)

- Quality of service
- Maintenance, Flexibility
- Delay (?)

Digital Systems (1)



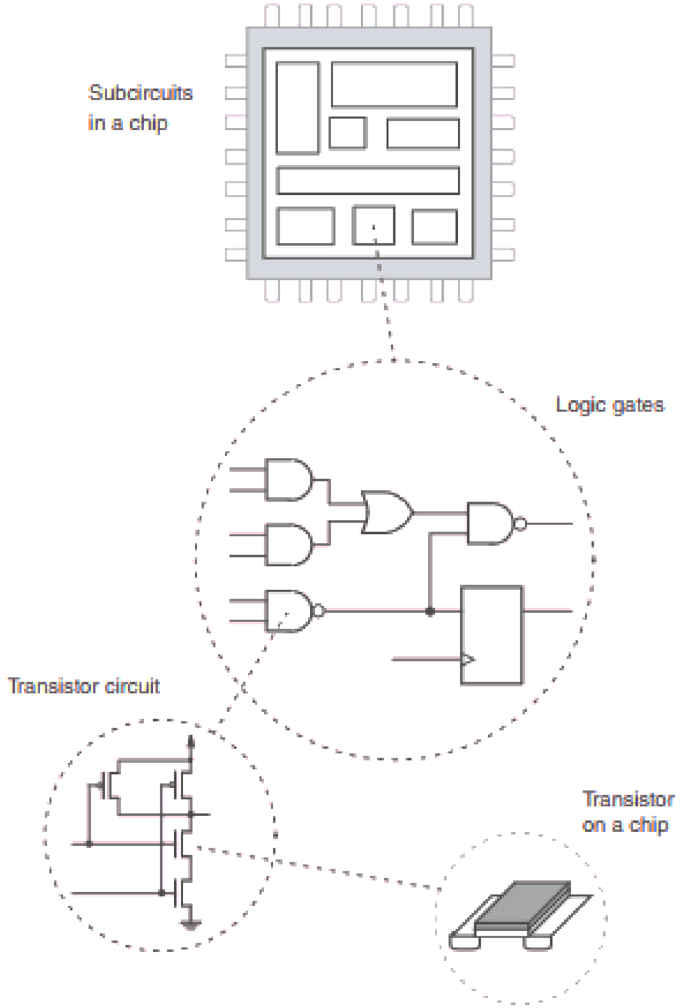
Software and Hardware

- ADC: Analog to Digital Converter

- Digital Processing: Signal is processed in digital domain using software and hardware

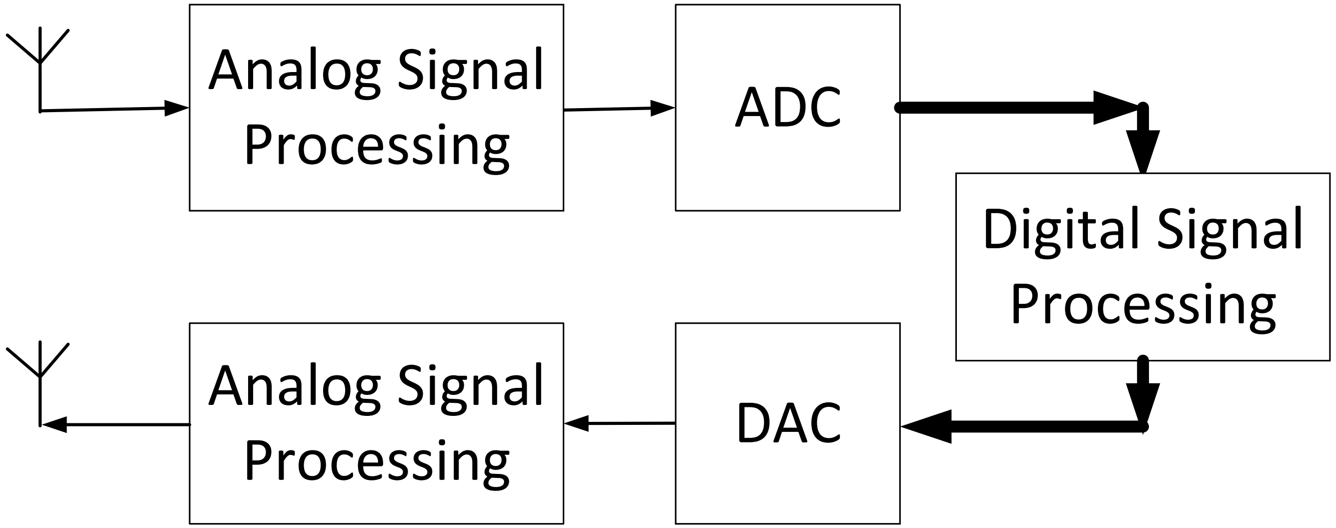
- DAC: Digital to Analog Converter

Digital Systems (2)



- Digital Systems are realized as chips
- **Chips consists of Logic Gates (often millions of logic gates)**
- Logic gates are made up of transistors

Why Analog Circuits are needed?



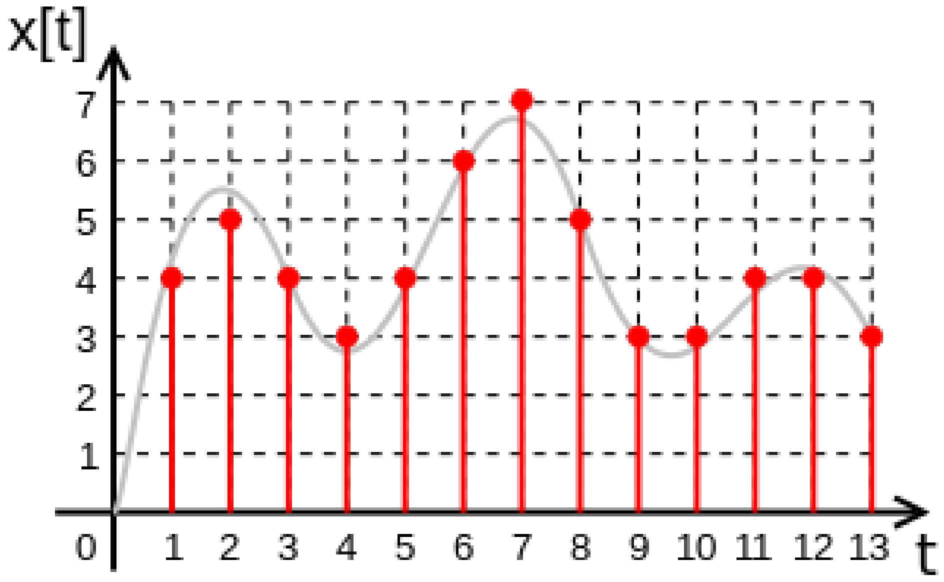
- Analog circuits are important since real signals are analog

- Some initial processing reduces the burden on Digital Signal Processing and overall more-efficient system is realized

Digital Circuits

Zeros and Ones

Digital signals: “Digits”



- y can taken values from a discrete set of values. For example: $\{0, 1, 2, 3, 4, 5, 6, 7\}$

- In this example, signal is represented as: $\{4, 5, 4, 3, 4, 6, 7, 5, 3, 3, 4, 4, 3, .., \}$

Why do we call it “Digital” signal?

- Any digital signal is represented in the form of a number or “digits”

- **A number can be represented in many different ways**
- In normal counting we use only “Decimal Numbers”
- Other number systems are possible

Digital signals: Decimal Number

What is a Decimal Number?

- Decimal Number consists of digits $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$
- Decimal Number is said to be in “base 10”
  - Ten digits are used
  - $5723 = 5 \times 10^3 + 7 \times 10^2 + 2 \times 10^1 + 3 \times 10^0$ [Place value of each digit is in power of tens]

Digital signals: Binary Number

- In Digital systems we use "Binary Number"

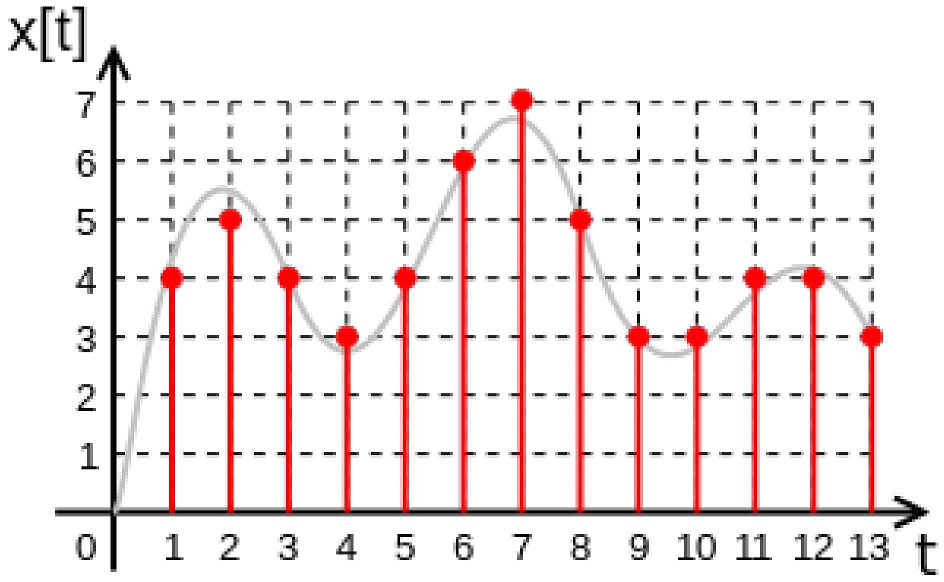
What is a Binary Number?

- Binary Number consists of digits $\{0, 1\}$
- Binary Number is said to be in "base 2"
    - ➢ Two digits are used
    - ➢ $(1001)_2 = 1 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 9$ [Place value of each digit is in power of two]

Why are Binary Numbers used in Digital System?

- $\{0, 1\}$ can conveniently be represented as two voltage levels
- Easy to process binary numbers

Digital signals: In Zeros and Ones (Binary)



- y can taken values from a discrete set of values. For example: $\{0, 1, 2, 3, 4, 5, 6, 7\}$

- In this example, signal is represented as: $\{4, 5, 4, 3, 4, 6, 7, 5, 3, 3, 4, 4, 3, . . , \}$

- In this example, signal is represented in Binary number as: $\{100, 101, 100, 011, 100, 110, 111, 101, 011, 011, 100, 100, 011, . . , \}$

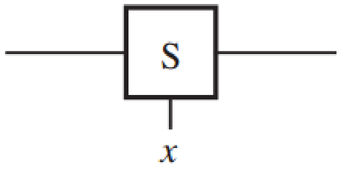
In the following lectures we will deal with signals that can take values “0” or “1”

Digital Circuits

Logic Functions and Logic Gates

Two states of a Binary Number: Switch

Two states of a switch

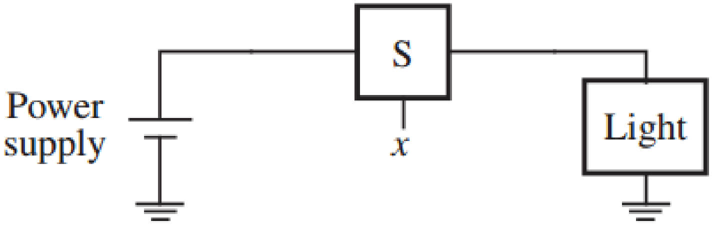


Symbol for a switch

- Two states of a binary number can be represented by a **switch**
  - ON-state and OFF-state

- Assume that the switch is controlled by a variable x
  - If $x = 0$, switch is OFF or “Open”
  - If $x = 1$, switch is ON or “Closed”

Switch/Light bulb: Input/output

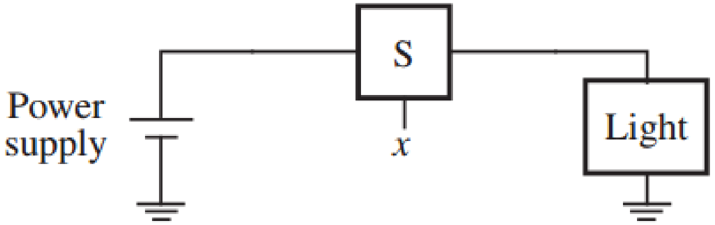


- Switch is used to control Light bulb
- Light bulb glows when current passes through it

- **Light bulb has two states**
  - OFF-state (does not glow) and ON-state (glows)
  - The state of light bulb can be denoted by L
    - OFF-state: $L = 0$
    - ON-state: $L = 1$

- Switch is used to control the state of Light bulb
- In this system:
  - Input: switch variable x
  - Output: state of the light bulb L

Logic function: relationship between Input/output

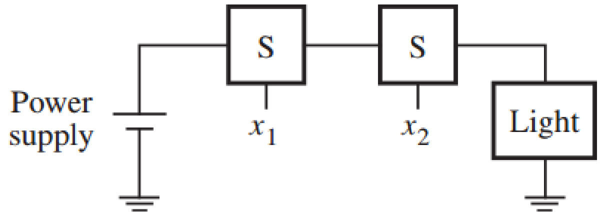


- Derive a relationship between Input and Output

- When $x = 0$, then $L = 0$
- When $x = 1$, then $L = 1$

- $L = x$ is the relationship between the input and the output
- $L(x) = x$
- $\boldsymbol{L(x)}$ is a ***logic function***
- $\boldsymbol{x}$ is the ***input variable***

Logic function: AND function

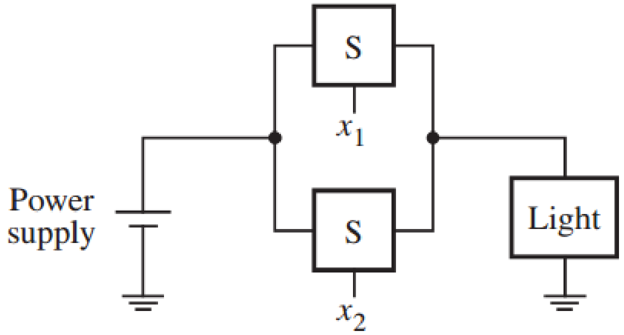


- Inputs are: (x_1, x_2)
- Output is: L
- Derive a relationship between Input and Output

- The light bulb will glow if switch "x_1 ***and*** x_2" are closed
- $L = 1$ if $x_1 = 1$ ***and*** $x_2 = 1$, $L = 0$ otherwise
- $L(x_1, x_2) = x_1 . x_2$

- Symbol "**.**" is known as ***AND operator*** and is said to implement logical ***AND*** function

Logic function: OR function



- Inputs are: (x_1, x_2)
- Output is: L
- Derive a relationship between Input and Output

- The light bulb will glow if switch "x_1 ***or*** x_2" are closed
- $L = 1$ if $x_1 = 1$ ***or*** $x_2 = 1$ ***or*** $x_1 = x_2 = 1$, $L = 0$ if $x_1 = x_2 = 0$
- $L(x_1, x_2) = x_1 + x_2$

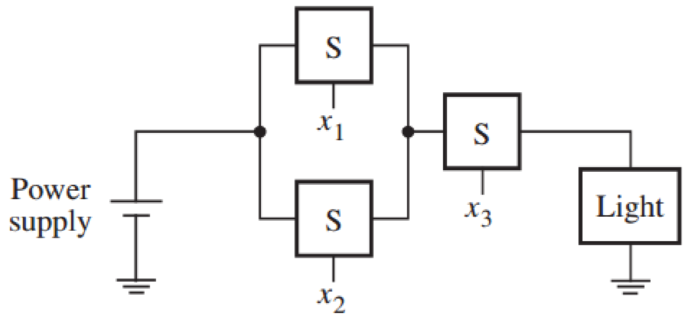
- Symbol "**+**" is known as ***OR operator*** and is said to implement logical ***OR*** function

Logic function: Complex Logic

- AND and OR operations can be used to implement complex logic circuit/function/expression

- The light bulb will glow if switch "$(x_1$ ***or*** $x_2)$ and x_3" are closed

- $L(x_1, x_2, x_3) = (x_1 + x_2).\, x_3$

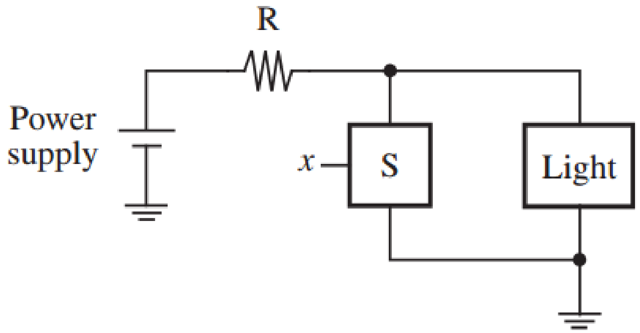


- Inputs are: (x_1, x_2, x_3)

- Output is: L

- Derive a relationship between Inputs and Output

Logic function: Inversion/Complement/NOT



- Switch is connected in parallel with the light

- Current always flow through "***low resistance path***"

- Switch has ***infinite resistance*** when "OPEN"
- Switch has ***zero resistance*** when "CLOSED"

- Light bulb has some ***finite resistance***

- When $x = 0$, then $L = 1$
- When $x = 1$, then $L = 0$

- $L = \bar{x} = x' = !x = \sim x$
- L is the complement of x
- This is also known as ***NOT*** operation
- $\boldsymbol{x}$ is the ***input variable***

Truth Table (1)

- Logic operations can also be defined in form of a table
- These tables are known as ***Truth Tables***

x	$\bar{x}$
0	1
1	0

x_1	x_2	$x_1 . x_2$	$x_1 + x_2$
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	1

Truth Table (2)

For $3\ variable$ inputs, how many entries will be there in the truth table?

x_1	x_2	x_3	$x_1 . x_2 . x_3$	$x_1 + x_2 + x_3$
0	0	0	0	0
0	1	0	0	1
1	0	0	0	1
1	1	0	0	1
0	0	1	0	1
0	1	1	0	1
1	0	1	0	1
1	1	1	1	1

Truth Table (3)

Question:

For N *variable* inputs, how many entries will be there in the truth table?

Answer:

Each input can take 2 values (0 and 1).

Total possibilities:

$$2 \times 2 \times 2 \ldots N \; times = 2^N$$

Inference:

- The number of entries increases exponentially with N
- Truth table representation of a function is feasible for small N
- For large N truth table representation will be too big and not feasible
- Other representations such as logic expression is more compact for large N