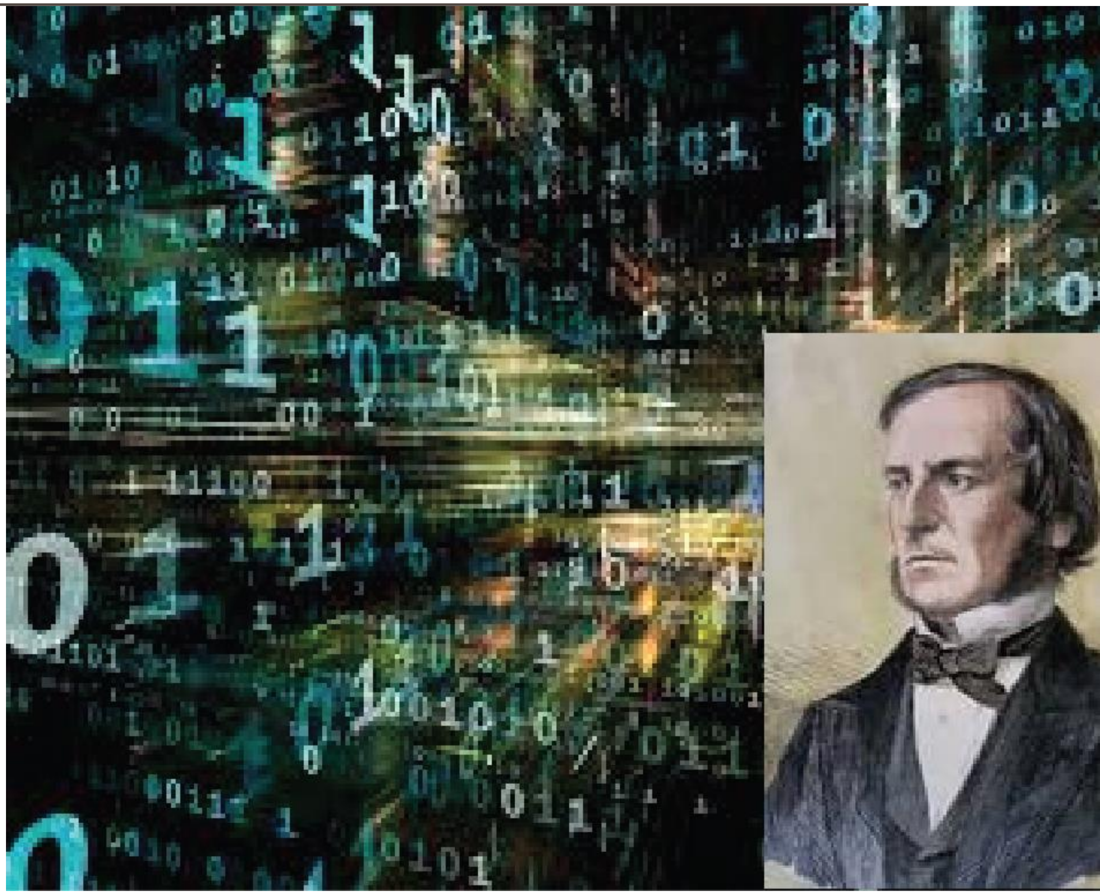
DIGITAL CIRCUITS

Week-4, Lecture-1
Boolean Algebra



Sneh Saurabh
23rd August, 2018

Digital Circuits: Announcements/Revision

Digital Circuits

Boolean Algebra

Duality Principle: More Explanation (1)

$(x_1 + x_2)' = x_1'x_2'$

$B + A = 1$

Can using the duality principle, we can simply state?

$A + B = 1 \Rightarrow B.A = 0$

Proof:

Let $A = x_1 + x_2$ and $B = x_1'x_2'$

$B.A$

$= x_1'x_2'(x_1 + x_2)$

$= x_1'x_2'x_1 + x_1'x_2'x_2$

$= 0 + 0 = 0$

Answer: No

- Duality Principle can **only** be applied on a statement that is TRUE in general
- It implies that the statement must be TRUE for ***ALL combination of value of the variables*** involved in the statement
- If A and B are arbitrary Boolean variables then $A + B = 1$ is not TRUE for $A = B = 0$

Duality Principle: More Explanation (2)

- Here $B + A = 1$ is always true because B and A are related due to assumption:
  - Let $A = x_1 + x_2$ and $B = x_1'x_2'$
- And because of this assumption "$B.A = 0$" turns out to be TRUE.

A counter example:

- Let $A = x_1$ and $B = x_1' + x_2$
- It is easy to see that $B + A = 1$,
  - But $B.A = x_2x_1 \neq 0$
- $\Rightarrow$ Principle of duality cannot be directly applied to: $B + A = 1$

$A + B = 1$

$\Rightarrow x_1 + x_2 + x_1'x_2' = 1$

Principle of Duality can be applied:

$$x_1.x_2.(x_1' + x_2') = 0$$

Complement of a function: Using Dual Expression

To find a complement of a function expressed as AND/OR/NOT:

1. Find the dual of the expression
2. Replace each literal by complement of the literal

Given: $f(a, b, c) = ab + a'bc$

Find $f'(a, b, c)$.

Can easily be checked using De Morgan's Theorem or Truth Table

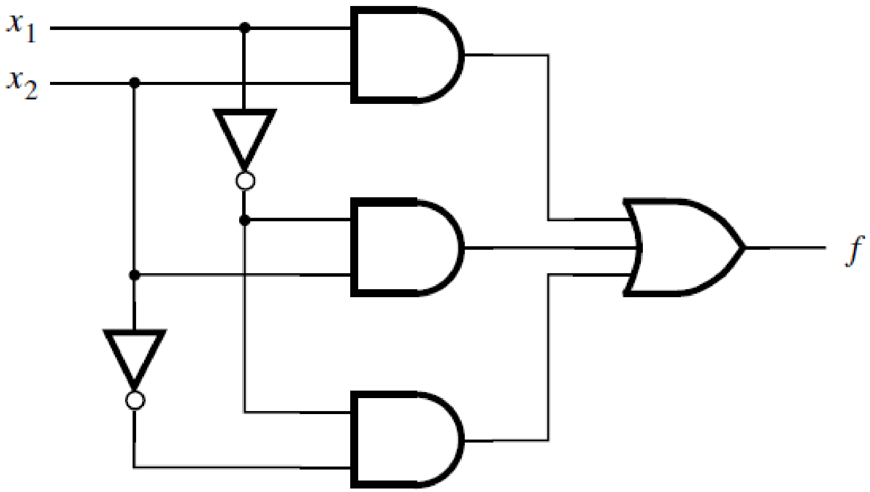
1. Dual of $ab + a'bc$ is $(a + b)(a' + b + c)$
2. $f'(a, b, c) = (a' + b')(a + b' + c')$

Digital Circuits

Synthesis using Logic Gates

Synthesis: SOP to Network

$f(x_1, x_2) = x_1'x_2' + x_1'x_2 + x_1x_2$



Synthesis: SOP Simplification

$f(x_1, x_2) = x_1' x_2{}' + x_1{}' x_2 + x_1 x_2$

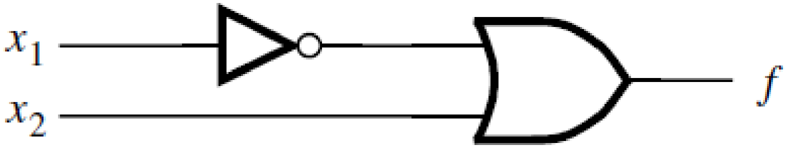
$x_1' x_2{}' + x_1{}' x_2 + x_1 x_2$

$= x_1'(x_2' + x_2) + x_1 x_2$

$= x_1' + x_1 x_2$

$= (x_1' + x_1)(x_1' + x_2)$

$= (x_1' + x_2)$



2(a) $x + 0 = 0 + x = x$

2(b) $x.1 = 1.x = x$

3(a) $x + y = y + x$

3(b) $x.y = y.x$

4(a) $x.(y + z) = x.y + x.z$

4(b) $x + (y.z) = (x + y).(x + z)$

5(a) $x + x' = 1$

5(b) $x.x' = 0$

Some important definition: Minterm

Product Term: A single literal or a logical AND of two or more literals.

Example: $x, x', xy, yy', abc, a'bc'd, \ldots$

Minterm: For a function of N variables, a minterm is a ***normal product*** term with $\boldsymbol{N}$ ***literals***.

Example: Consider a function of 3 variables: a, b, c

Which of the following are minterms $abc, (a + b + c), abc', ab, ab'c', c, a'bb'$

Answer: $abc, abc',\ ab'c'$

For a function of N variables, 2^N minterms exist.

Some important definition: Maxterm

Sum Term: A single literal or a logical OR of two or more literals.

Example: $x, x', x+y, y+y', a+b+c, a'+b+c'+d, \ldots$

Maxterm: For a function of N variables, a maxterm is a ***normal sum*** term with $\boldsymbol{N}$ ***literals***.

Example: Consider a function of 3 variables: a, b, c

Which of the following are maxterms:

$(a+b+c), abc, (a+b+c'), (a+b), (a+b'+c'), c, (a'+b+b')$

Answer: $(a+b+c), (a+b+c'),\ (a+b'+c')$

For a function of N variables, 2^N maxterms exist.

Minterm and Maxterm of 3 variables (1)

			Minterms		Maxterms	
x	y	z	Term	Designation	Term	Designation
0	0	0	$x'y'z'$	m_0	$x + y + z$	M_0
0	0	1	$x'y'z$	m_1	$x + y + z'$	M_1
0	1	0	$x'yz'$	m_2	$x + y' + z$	M_2
0	1	1	$x'yz$	m_3	$x + y' + z'$	M_3
1	0	0	$xy'z'$	m_4	$x' + y + z$	M_4
1	0	1	$xy'z$	m_5	$x' + y + z'$	M_5
1	1	0	xyz'	m_6	$x' + y' + z$	M_6
1	1	1	xyz	m_7	$x' + y' + z'$	M_7

Minterms

- Bit is complemented if it is 0, else without complement
- Designated as m_0,m_1, …
- m_j where j is the decimal equivalent of corresponding binary number

Maxterms

- Bit is complemented if it is 1, else without complement
- Designated as M_0,M_1, …
- M_j where j is the decimal equivalent of corresponding binary number

Minterm and Maxterm of 3 variables (2)

			Minterms		Maxterms	
x	y	z	Term	Designation	Term	Designation
0	0	0	$x'y'z'$	m_0	$x + y + z$	M_0
0	0	1	$x'y'z$	m_1	$x + y + z'$	M_1
0	1	0	$x'yz'$	m_2	$x + y' + z$	M_2
0	1	1	$x'yz$	m_3	$x + y' + z'$	M_3
1	0	0	$xy'z'$	m_4	$x' + y + z$	M_4
1	0	1	$xy'z$	m_5	$x' + y + z'$	M_5
1	1	0	xyz'	m_6	$x' + y' + z$	M_6
1	1	1	xyz	m_7	$x' + y' + z'$	M_7

Maxterms and corresponding Minterms are complement

- $(x + y + z) = (x'y'z')'$
  - $\Rightarrow M_0 = m_0'$
- Similarly, $M_1 = m_1'$, …. $M_j = m_j'$

Canonical Sum of Product (CSOP)

Canonical Sum of Product (CSOP): A Boolean function in sum of product (SOP) form in which each product term is a minterm.

Example: Consider a function of 3 variables: a, b, c

Which of the following are CSOP:

$f_1(a,b,c) = abc + a'bc + a'b'c$

$f_2(a,b,c) = abc + a'b$

$f_3(a,b,c) = a'b'c' + a'b'c + a'bc' + a'bc + ab'c' + ab'c + abc' + abc$

$f_4(a,b,c) = a'b'c' + (a'+c')(b'c + a')$

Answer: $f_1(a,b,c)$ and $f_3(a,b,c)$

CSOP: Representation

A Boolean function in CSOP form can easily be derived from a truth table

x	y	z	**Minterm**	**Maxterm**	$f(x,y,z)$
0	0	0	$m_0 = x'y'z'$	$M_0 = x + y + z$	0
0	0	1	$m_1 = x'y'z$	$M_1 = x + y + z'$	1
0	1	0	$m_2 = x'yz'$	$M_2 = x + y' + z$	0
0	1	1	$m_3 = x'yz$	$M_3 = x + y' + z'$	0
1	0	0	$m_4 = xy'z'$	$M_4 = x' + y + z$	1
1	0	1	$m_5 = xy'z$	$M_5 = x' + y + z'$	0
1	1	0	$m_6 = xyz'$	$M_6 = x' + y' + z$	0
1	1	1	$m_7 = xyz$	$M_7 = x' + y' + z'$	1

Take the sum of minterms that produces 1.

$$\boldsymbol{f(x,y,z) = x'y'z + xy'z' + xyz}$$

$$= m_1 + m_4 + m_7$$

CSOP: Compact Representation

$$\boldsymbol{f(x,y,z)} = \Sigma m(1,4,7)$$

Σm: denotes SUM of minterms

Numbers: are indices of minterms

Canonical Product of Sum (CPOS)

Canonical Product of Sum (CPOS): A Boolean function in product of sum (POS) form in which each sum term is a maxterm.

Example: Consider a function of 3 variables: a, b, c

Which of the following are CSOP:

$f_1(a,b,c) = (a+b+c)(a'+b+c)(a'+b'+c)$

$f_2(a,b,c) = (a+b+c)(a'+b)$

$f_3(a,b,c) = a'b'c' + (a'+c')(b'c+a')$

$f_4(a,b,c) = (a'+b'+c')(a'+b'+c)(a'+b+c')(a'+b+c)(a+b'+c')(a+b'+c)(a+b+c')(a+b+c)$

Answer:$f_1(a,b,c)$ and $f_4(a,b,c)$

CPOS: Representation

x	y	z	Minterm	Maxterm	$f(x,y,z)$
0	0	0	$m_0 = x'y'z'$	$M_0 = x + y + z$	0
0	0	1	$m_1 = x'y'z$	$M_1 = x + y + z'$	1
0	1	0	$m_2 = x'yz'$	$M_2 = x + y' + z$	0
0	1	1	$m_3 = x'yz$	$M_3 = x + y' + z'$	0
1	0	0	$m_4 = xy'z'$	$M_4 = x' + y + z$	1
1	0	1	$m_5 = xy'z$	$M_5 = x' + y + z'$	0
1	1	0	$m_6 = xyz'$	$M_6 = x' + y' + z$	0
1	1	1	$m_7 = xyz$	$M_7 = x' + y' + z'$	1

A Boolean function in CPOS form can easily be derived from a truth table

Take the product of maxterms that produces 0.

CPOS: Compact Representation

$$\boldsymbol{f(x,y,z) = \Pi M(0,2,3,5,6)}$$

ΠM: denotes PRODUCT of maxterms

Numbers: are indices of maxterms

$$\boldsymbol{f' = x'y'z' + x'yz' + x'yz + xy'z + xyz'}$$

$$\boldsymbol{f = (f')' = (x+y+z)(x+y'+z)(x+y'+z')(x'+y+z')(x'+y'+z)}$$

CSOP and CSOP: function and complement

- For complement: take the missing terms

CSOP

$$f(x, y, z) = \Sigma m(1,3,4,7)$$

$$f'(x, y, z) = \Sigma m(0,2,5,6)$$

- CSOP to CPOS or vice-versa: take the missing terms

CPOS

$$f(x, y, z) = \Pi M(0,2,5,6)$$

$$f'(x, y, z) = \Pi M(1,3,4,7)$$

CSOP

$$\mathbf{F}(A, B, C, D) = \Sigma m(1,2,5,7,10,15)$$

$$\mathbf{F}'(A, B, C, D) = \Sigma m(0,3,4,6,8,9,11,12,13,14)$$

CSOP

$$\mathbf{F}(A, B, C, D) = \Pi M(0,3,4,6,8,9,11,12,13,14)$$

$$\mathbf{F}'(A, B, C, D) = \Pi M(1,2,5,7,10,15)$$

- From CSOP to CPOS and vice-versa can be done
- From function in CSOP/CPOS to its complement