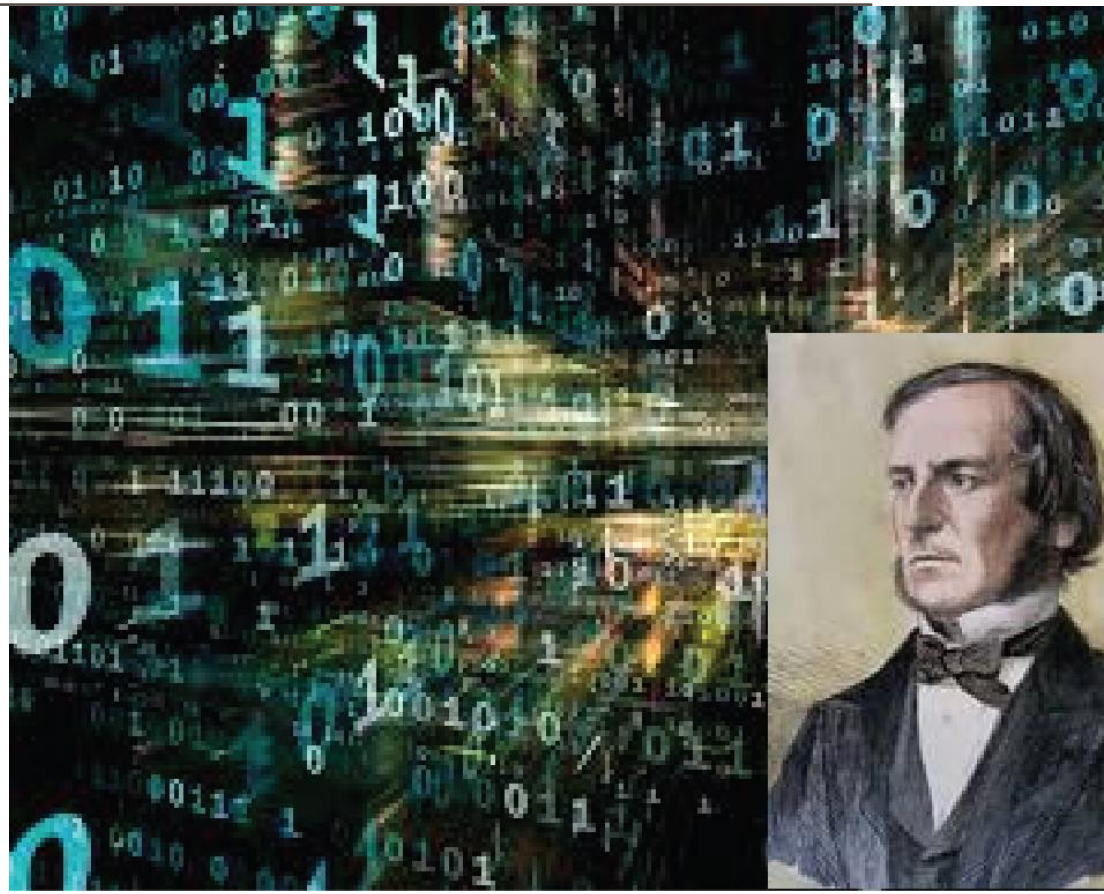
DIGITAL CIRCUITS

Week-5, Lecture-1
Boolean Algebra



Sneh Saurabh
28th August, 2018

Digital Circuits: Announcements/Revision

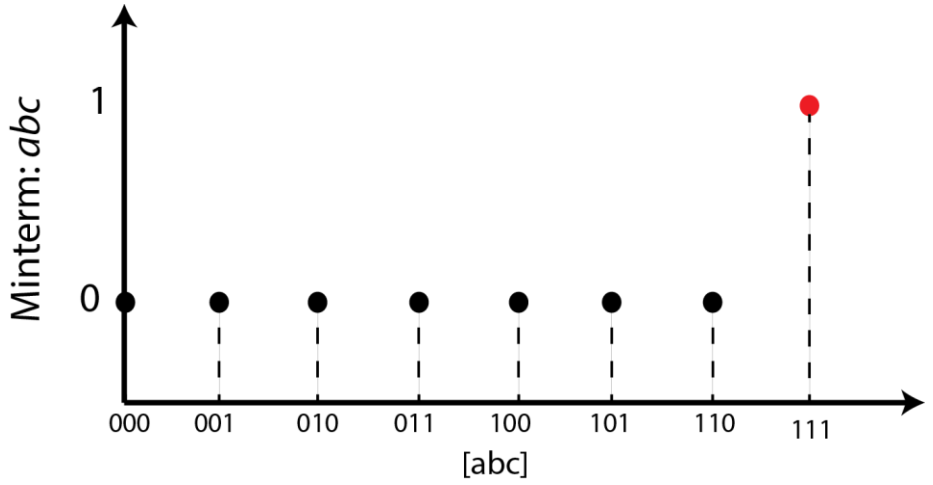
- Compensatory Lecture (lost due to inauguration of Phase-II of building) will be held on Thursday, 6th September, 4:00-5:00 pm in the same room (102)

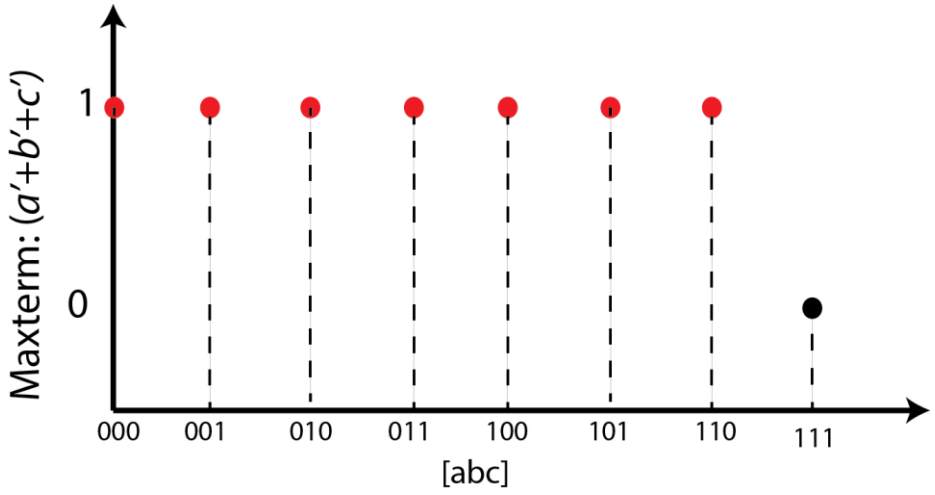
- First Quiz: Tomorrow, Wednesday, 5:30-6:00 pm in the same room 102

Explanation on Minterm and Maxterm

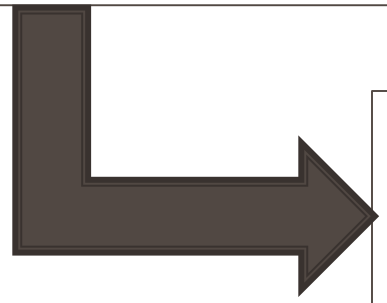
- Let us consider a function of three variables $f(a, b, c)$

- Consider any one minterm and its corresponding maxterm. Example: abc and $(a' + b' + c')$
- What will be the value of the minterm abc and the maxterm $(a' + b' + c')$ at all possible values of a, b and c?





Digital Circuits



Word Problem to Logic Gate

Problem 2

Consider one day cricket match statistics obtained from 2005-2011*.

It is observed that India wins if either Virender Sehwag or Sachin Tendulkar score century provided that the match is played in India.

In case match is played outside India, both should make century for India to win.

Write an expression to represent the outcome of the match.

Solution:

- Let Sehwag making century is represented by V:
  - $V = 0$: Sehwag does not make a century
  - $V = 1$: Sehwag makes a century
- Let Tendulkar making century is represented by T:
  - $T = 0$: Tendulkar does not make a century
  - $T = 1$: Tendulkar makes a century
- Let match played in India is represented by I:
  - $I = 0$: match played outside India
  - $I = 1$: match played in India

Let outcome of a match be represented by $W(V, T, I)$.

$W(V, T, I)$ is 1 if India wins else 0.

$$W(V, T, I) = (V + T)I + VTI'$$

Problem 2 …

$$W(V,T,I) = (V+T)I + VTI'$$

Minimize:

$$= VI + TI + VTI'$$

$$= VI + VTI' + TI$$

$$= V(I + TI') + TI$$

$$= V(I + I')(I + T) + TI$$

$$= VI + VT + TI$$

$$= (V+T)I + VT$$

Problem 3

A student has to decide whether he will buy a textbook prescribed for a course, based upon the following considerations:

(a) Cost of the book – LOW/MEDIUM/HIGH,

(b) Quality of the teacher – AVERAGE/GOOD/EXCELLENT

(c) Quality of lecture notes – GOOD/AVERAGE

(d) The book being useful in other courses also – YES/NO

- Let the quality of the teacher be represented by Q_1Q_0 (2 bits) and assume that
  - $Q_1Q_0 = 00$ or $Q_1Q_0 = 01$: represent AVERAGE
  - $Q_1Q_0 = 10$: represents GOOD
  - $Q_1Q_0 = 11$: represents EXCELLENT

- Let the cost of book be represented by C_1C_0 (2 bits) and assume that
  - $C_1C_0 = 00$ or $C_1C_0 = 01$: represent LOW cost
  - $C_1C_0 = 10$: represents MEDIUM cost
  - $C_1C_0 = 11$: represents HIGH cost

- Let quality of lecture notes be represented by L (1 bit) and assume that $L = 1$ represents GOOD and $L = 0$ represents AVERAGE

- Book being useful in other course be represented by U (1 bit) and assume that $U = 1$ represents YES and $U = 0$ represents NO

Problem 3 …

1. He would like to buy the book if its cost is LOW. However, either if the teacher is EXCELLENT or if the teacher is reasonably GOOD and gives GOOD lecture notes, he feels that he may not have to buy the book.

2. But if the teacher is AVERAGE, and the book is useful in other courses also, he would prefer to buy the book whatever its cost.

3. If the book is useful in other courses, he would buy it in any case provided its cost is MEDIUM.

Obtain a Boolean expression for the output Y representing the decision to buy the book.

Let us assume $Y(C_1, C_0, Q_1, Q_0, L, U) = 1$ if the decision is to buy the book and $Y(C_1, C_0, Q_1, Q_0, L, U) = 0$ if the decision is not to buy the book.

Condition 1:

Cost is low: $C_1'C_0' + C_1'C_0$

Teacher is excellent: Q_1Q_0

Teacher is reasonably GOOD and gives GOOD lecture notes: $Q_1Q_0'L$

Condition for not buying the book is: $(Q_1Q_0 + Q_1Q_0'L)$

Condition to buy the book: $(C_1'C_0' + C_1'C_0).(Q_1Q_0 + Q_1Q_0'L)'$

Problem 3 …

1. He would like to buy the book if its cost is LOW. However, either if the teacher is EXCELLENT or if the teacher is reasonably GOOD and gives GOOD lecture notes, he feels that he may not have to buy the book.
2. But if the teacher is AVERAGE, and the book is useful in other courses also, he would prefer to buy the book whatever its cost.
3. If the book is useful in other courses, he would buy it in any case provided its cost is MEDIUM.

Obtain a Boolean expression for the output Y representing the decision to buy the book.

Condition 2:

Teacher is AVERAGE: $Q_1'Q_0' + Q_1'Q_0$

Book is useful in other courses also: U

Condition to buy the book is: $(Q_1'Q_0' + Q_1'Q_0)U$

Condition 3:

Book is useful in other courses also: U

Cost is MEDIUM: C_1C_0'

Condition to buy the book is: UC_1C_0'

Problem 3 …

Full Condition to buy the book is:

$$(C_1'C_0' + C_1'C_0).(Q_1Q_0 + Q_1Q_0'L)' + (Q_1'Q_0' + Q_1'Q_0)U + UC_1C_0'$$

Minimize:

$$= C_1'(Q_1Q_0 + Q_1Q_0'L)' + Q_1'U + UC_1C_0'$$

Problem 4

A student staying in a hostel has to make up his mind about his dinner.

If he has enough money (M) and at least three of his friends (F) also agree to go out for dinner, and it is not raining (R), he will have dinner with his friends in a restaurant outside the campus.

If he is not able to go out, but at least three of his friends agree to join him (J), if the kind of food he wanted is available on home delivery (K) he will order home delivery of food from outside.

But, or if the general feeling is that the food in the hostel mess is good on that day (G), he will have his dinner in the hostel mess.

Let his decision be denoted by a 2-bit output $D_1 D_0$:

$D_1 D_0 = 00 \implies$ He eats in the hostel mess,

$D_1 D_0 = 01 \implies$ He goes out to have dinner in a restaurant, and

$D_1 D_0 = 10 \implies$ He orders food through home delivery.

Assign binary variables to represent the various conditions using the letters indicated in parentheses above, and obtain both expressions for D_0 and D_1 in terms of these variables, simplifying the expressions as far as possible.

Problem 4 …

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But, or if the general feeling is that the food in the hostel mess is good on that day (G), he will have his dinner in the hostel mess.

Assignment of binary variables

M: 1 has enough money, 0 otherwise

F: 1 when at least three of his friends also agree to go out for dinner, 0 otherwise

R: 1 when raining, 0 otherwise

J: 1 when at least three of his friends agree to join him, 0 otherwise

K: 1 if the kind of food he wanted is available on home delivery, 0 otherwise

G: 1 if the general feeling is that the food in the hostel mess is good on that day

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Decision be denoted by a 2-bit output D_1D_0

$D_1D_0 = 00 \implies$ He eats in the hostel mess,

$D_1D_0 = 01 \implies$ He goes out to have dinner in a restaurant, and

$D_1D_0 = 10 \implies$ He orders food through home delivery.

Assuming that the third condition is valid only when he is not able to go out.

The bit D_0 is 1 when he goes out to have dinner in a restaurant.

$$D_0 = MFR'$$

The bit D_1 is 1 when he orders food through home delivery

$$D_1 = (MFR')'JKG' = (M' + F' + R)JKG'$$