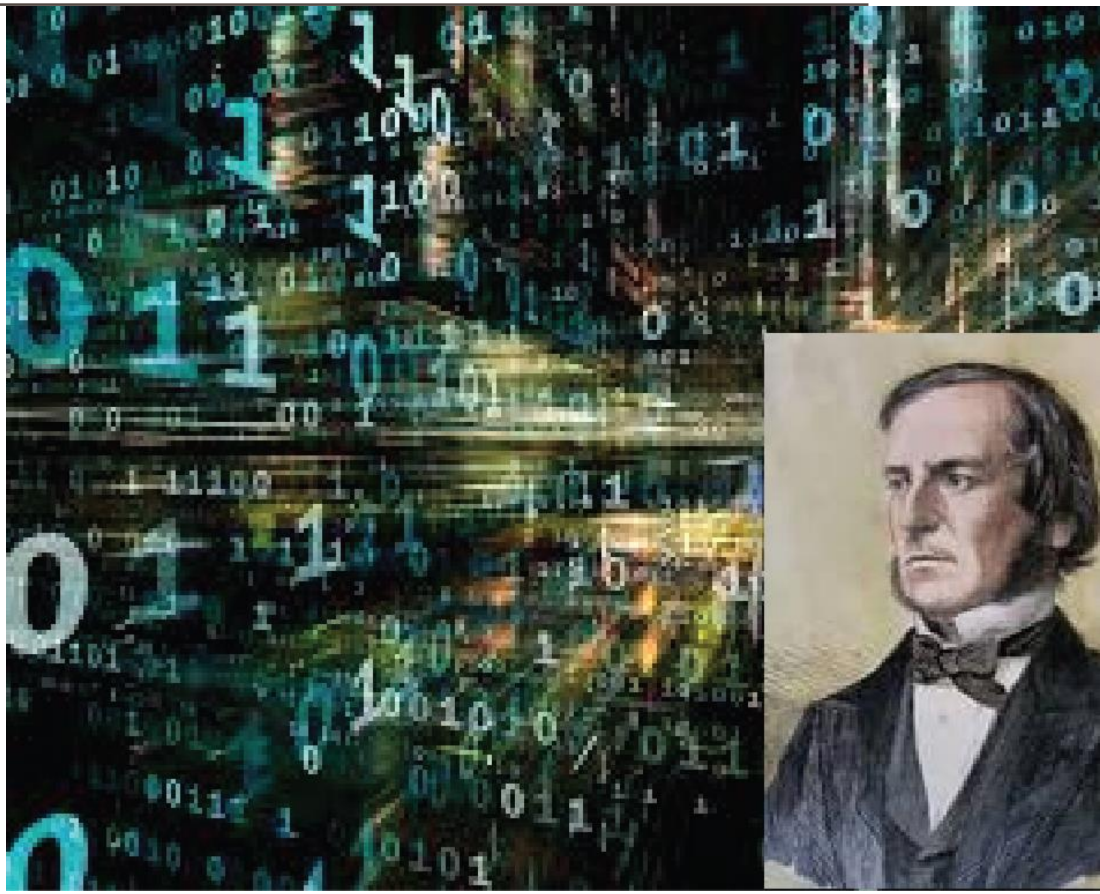
DIGITAL CIRCUITS

Week-6, Lecture-4
K-map



Sneh Saurabh
7th September, 2018

Digital Circuits: Announcements/Revision

Digital Circuits

Combinational Circuit Design

K-Map: Bigger combination, fewer literals

- The number of adjacent squares that may be combined must always represent a number that is a power of two, such as 1, 2, 4, and 8 [otherwise number of literals will not reduce]
- As more adjacent squares are combined, we obtain a product term with fewer literals

For three variables function:

- One square represents one minterm, giving a term with three literals
- Two adjacent squares represent a term with two literals
- Four adjacent squares represent a term with one literal
- Eight adjacent squares encompass the entire map and produce a function that is always equal to 1

K-Map: Three variables simplification (reuse squares)

Simplify $F(x, y, z) = \Sigma m(0, 2, 4, 5, 6)$

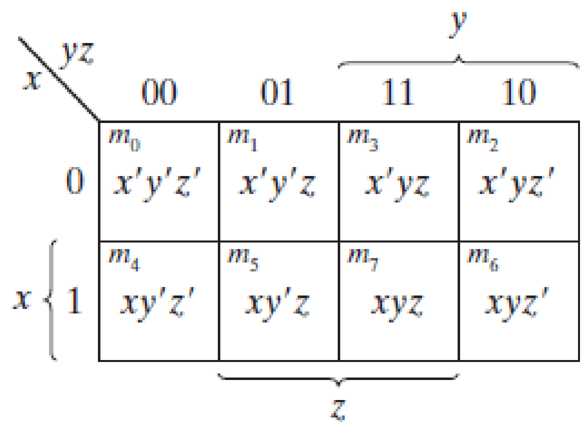
- First represent the given function on K-map

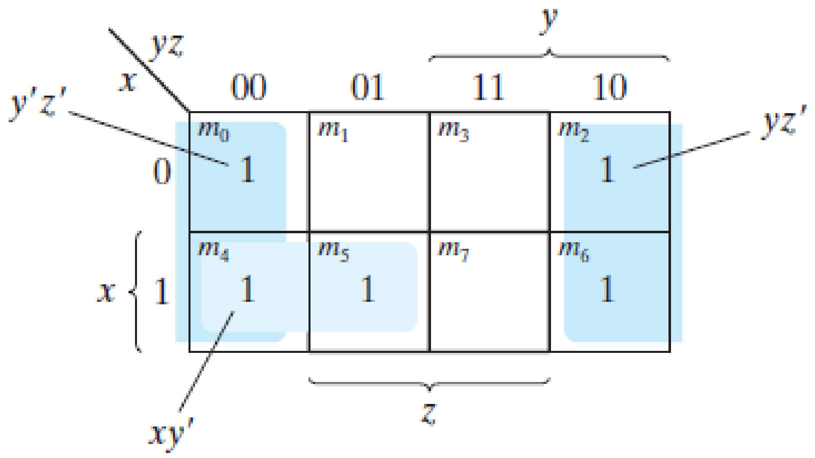
- Find possible adjacent squares:
  - ➢ m_0, m_2, m_4 and m_6: $= z'$
  - ➢ m_4 and m_5: $= xy'z' + xy'z = xy'$

$\boldsymbol{f(x, y, z) = z' + xy'}$

- The same square is allowed to be combined multiple times (in different combinations)

- It is better to combine squares than leave it isolated (combination reduces number of literals)



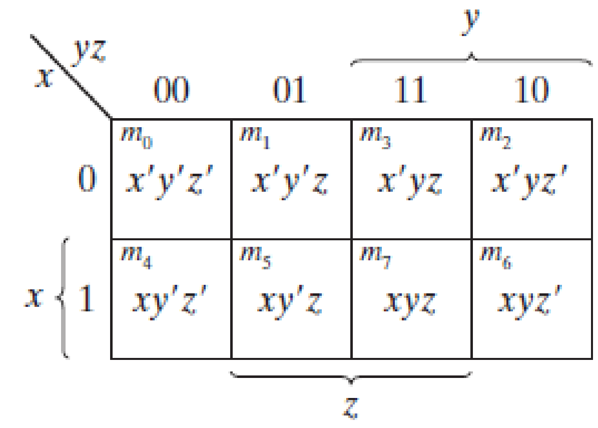


Note: $y'z' + yz' = z'$

K-Map: Three variables simplification (from SOP)

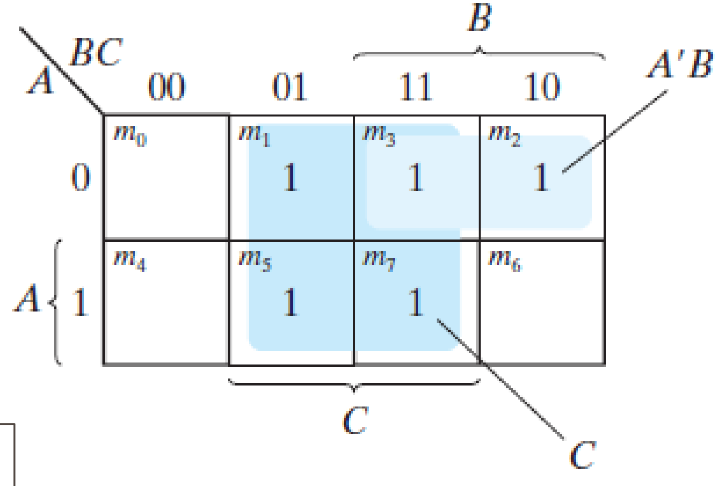
Problem: $\boldsymbol{F(A,B,C) = A'C + A'B + AB'C + BC}$

- Express the above function as a sum of minterms
- Find the minimal sum-of-products expression



- First represent the given function on the K-map
  - ➢ Mark 1 in K-map corresponding to each product term
  - ➢ Product term is identified on K-map by intersection of literals

- Given K-map, expressing a function in minterm is straightforward
  - ➢ $\boldsymbol{F(A,B,C) = \Sigma m(1,2,3,5,7)}$

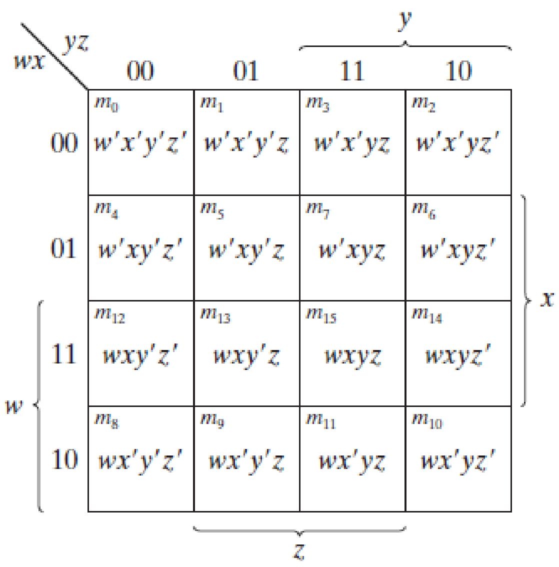


- Combine squares:
  - ➢ m_1, m_3, m_5 and m_7: C
  - ➢ m_3 and m_2: $A'B$

- $\boldsymbol{F(A,B,C) = A'B + C}$

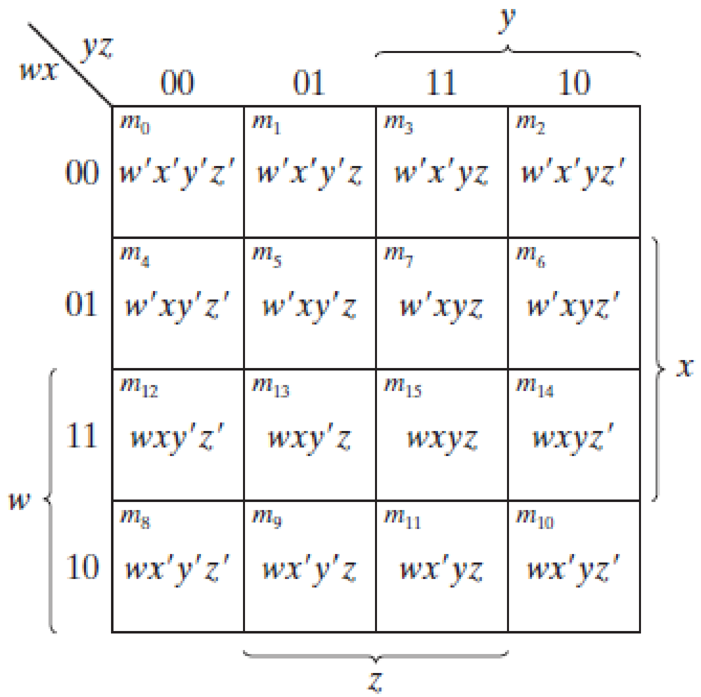
K-Map: Four variables

m_0	m_1	m_3	m_2
m_4	m_5	m_7	m_6
m_{12}	m_{13}	m_{15}	m_{14}
m_8	m_9	m_{11}	m_{10}



- Four variables:
  - Sixteen minterms: 16 squares
- Rows/columns are arranged as Gray code
  - Only one bit changes in value from one adjacent column/row to the next
- Minterm corresponding to each square is obtained by concatenation of row-number and column-number
  - Example: Consider square in second row, third column m_7: row is 01 and column is 11

K-Map: Four variables minimization



- Procedure similar as for three variables
  - Combine adjacent squares to get rid of literals

- K-map is considered to lie on a surface with:
  - top and bottom edges touch each other to form adjacent squares
  - right and left edges touch each other to form adjacent squares
  - m_3 and m_{11} are adjacent
  - m_{12} and m_{14} are adjacent

K-Map: Bigger combination, fewer literals

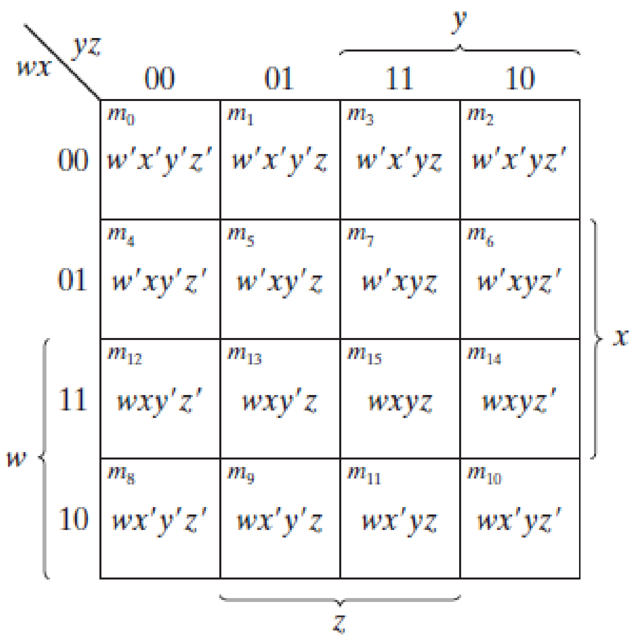
For four variables function:

- One square represents one minterm, giving a term with four literals
- Two adjacent squares represent a term with three literals
- Four adjacent squares represent a term with two literals
- Eight adjacent squares represent a term with one literal
- Sixteen adjacent squares encompass the entire map and produce a function that is always equal to 1

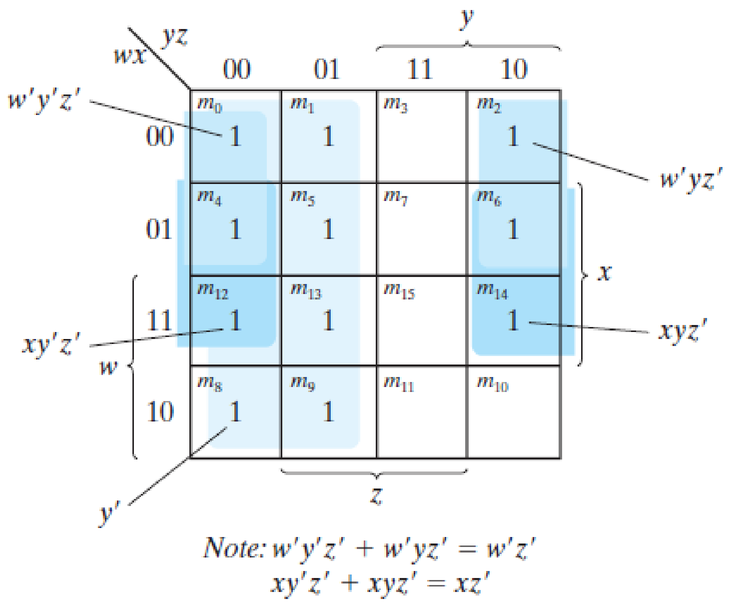
K-Map: Four variables simplification

Simplify: $F(w,x,y,z) = \Sigma m(0,1,2,4,5,6,8,9,12,13,14)$

- First represent the given function on K-map



	00	01	11	10
00	m_0 $w'x'y'z'$	m_1 $w'x'y'z$	m_3 $w'x'yz$	m_2 $w'x'yz'$
01	m_4 $w'xy'z'$	m_5 $w'xy'z$	m_7 $w'xyz$	m_6 $w'xyz'$
11	m_{12} $wxy'z'$	m_{13} $wxy'z$	m_{15} $wxyz$	m_{14} $wxyz'$
10	m_8 $wx'y'z'$	m_9 $wx'y'z$	m_{11} $wx'yz$	m_{10} $wx'yz'$



- Find possible adjacent squares:
  - ➢ $m_0, m_1, m_4, m_5, m_{12}, m_{13}, m_8$ and $m_9 := y'$
  - ➢ m_2, m_6, m_0 and $m_4 := w'z'$
  - ➢ m_6, m_{14}, m_4 and $m_{12} := xz'$

$\boldsymbol{f(x,y,z) = y' + w'z' + xz'}$

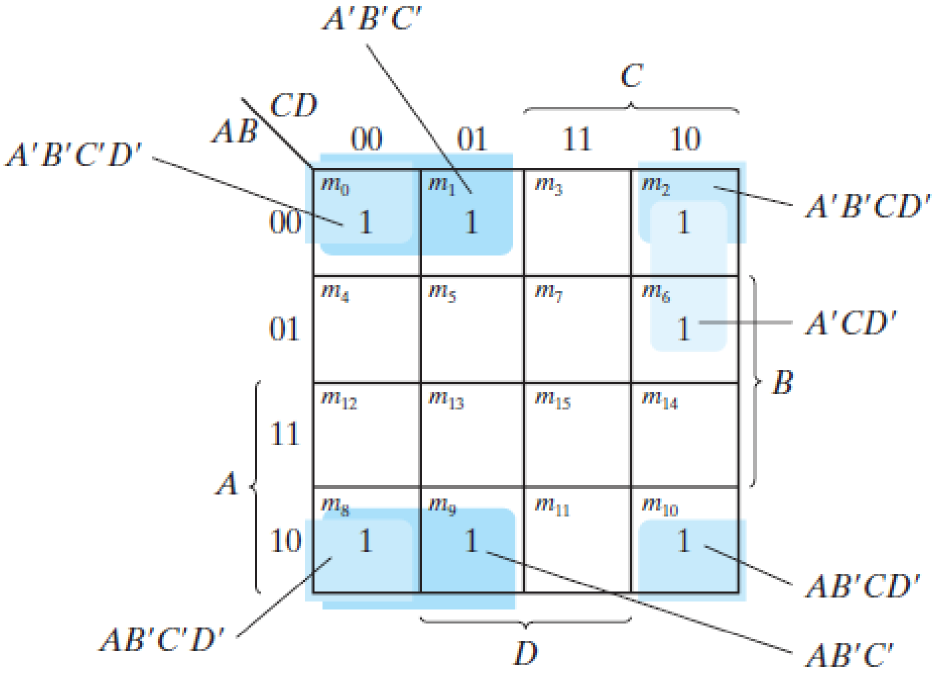
K-Map: Four variables simplification (from SOP)

Problem:

Simplify the Boolean function
$F(A,B,C,D) = A'B'C' + B'CD' + A'BCD' + AB'C'$

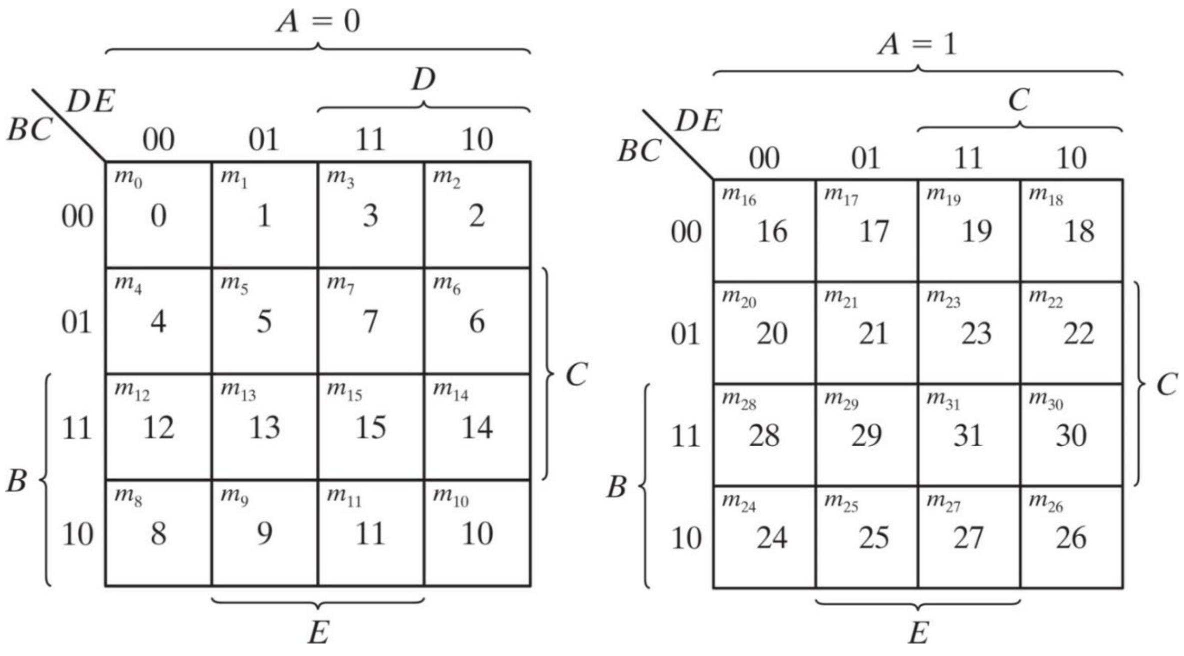
- First represent the given function on the K-map
  - $A'B'C'$: m_0 and m_1, $B'CD'$: m_2 and m_{10}
  - $A'BCD'$: m_6 $AB'C'$: m_8 and m_9

- Combine squares:
  - m_0, m_1, m_8 and m_9: $B'C'$
  - m_0, m_2, m_8 and m_{10}: $B'D'$
  - m_2 and m_6: $A'CD'$
- $\boldsymbol{F(A,B,C) = B'C' + B'D' + A'CD'}$



Note: $A'B'C'D' + A'B'CD' = A'B'D'$
$AB'C'D' + AB'CD' = AB'D'$
$A'B'D' + AB'D' = B'D'$
$A'B'C' + AB'C' = B'C'$

K-Map: Five variables



- Five variables: 32 minterms: 32 squares

- Two four-variable K-maps can be used to represent the function
- Two K-maps differ by one variable: $A = 0$ and $A = 1$
  - Minterms m_0 to m_{15}: belong to $A = 0$
  - Minterms m_{16} to m_{31}: belong to $A = 1$

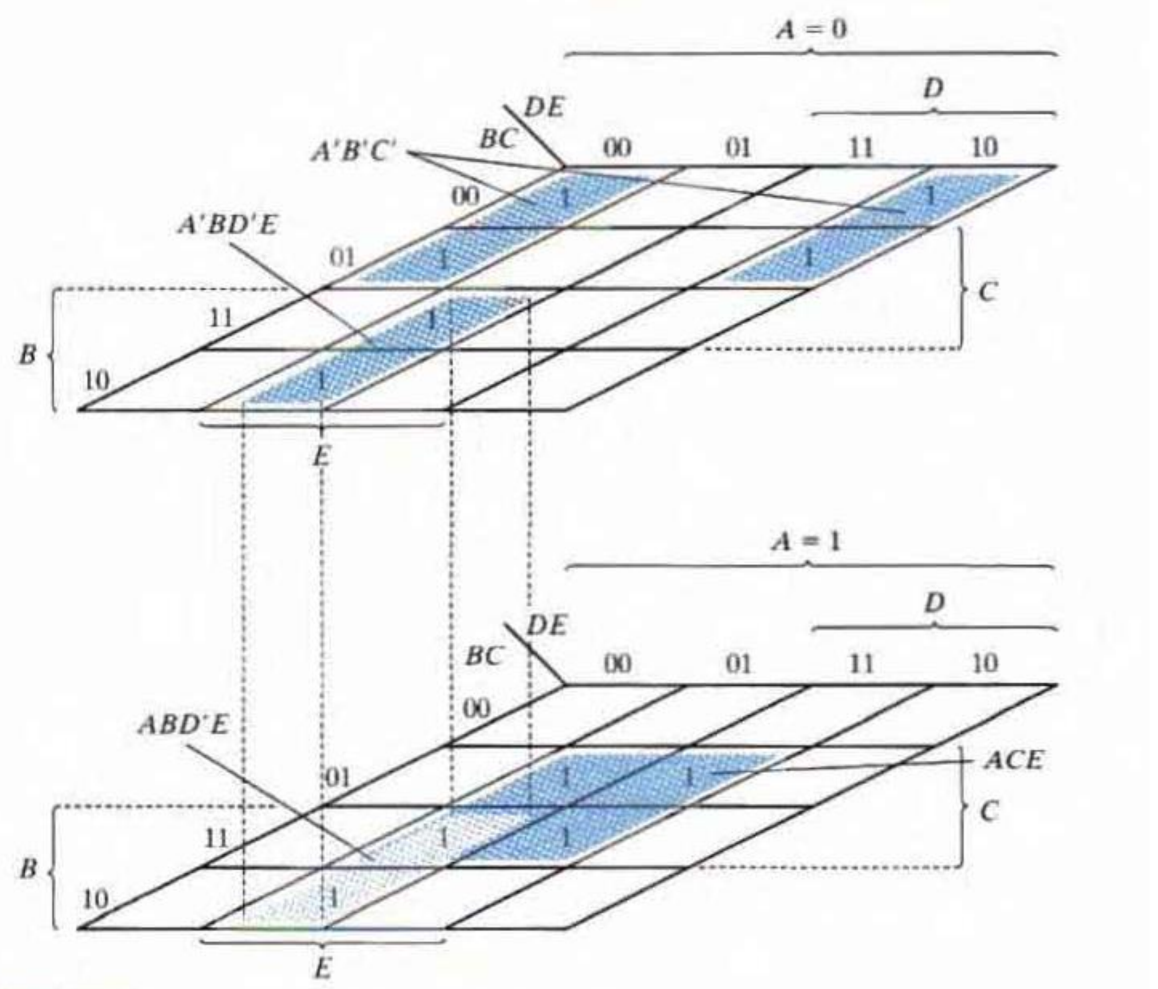
- Each four-variable K-map retains the previously defined adjacency when taken separately
- Each square in $A = 0$ K-map is adjacent to corresponding square in $A = 1$ K-map
  - (m_4 and m_{20}) , (m_{15} and m_{31}), ….

K-Map: Five variables representation

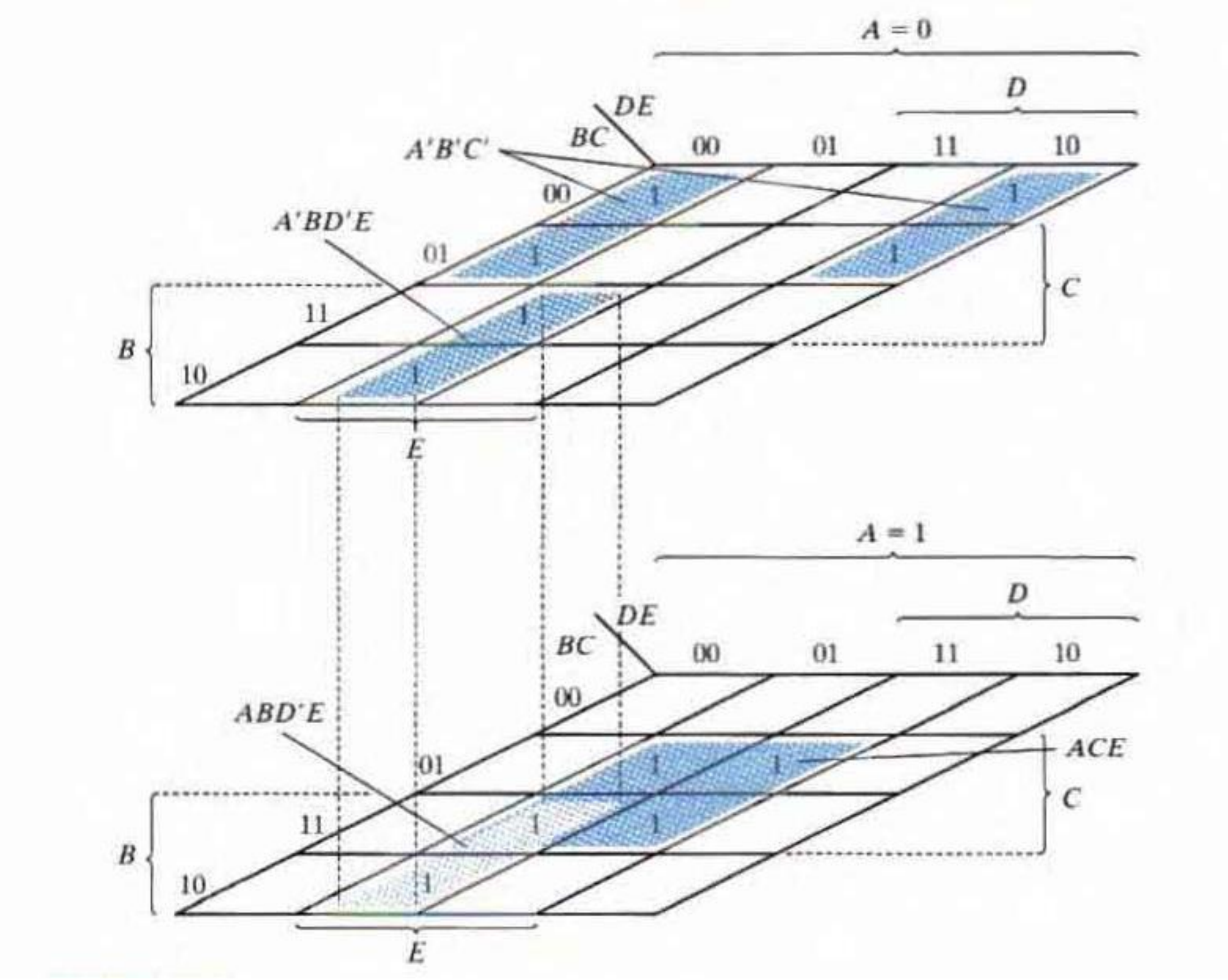
Simplify:

$F(A, B, C, D, E) = \Sigma m(0, 2,4,6,9,13,21,23,25,29,31)$

- First represent the given function on the K-map
- Minterms m_0 to m_{15}: belongs to $A = 0$: Six terms
- Minterms m_{16} to m_{31}: belongs to $A = 1$: Five Terms



K-Map: Five variables simplification



- Find possible adjacent squares
- Four squares in $A = 0$ are combined: $A'B'E'$
- Four squares in the last two rows in both $A = 0$ and $A = 1$ can be combined: $BD'E$
- Four squares in the center for $A = 1$: ACE

$$\boldsymbol{f(A, B.C.D.E) = A'B'E' + BD'E + ACE}$$