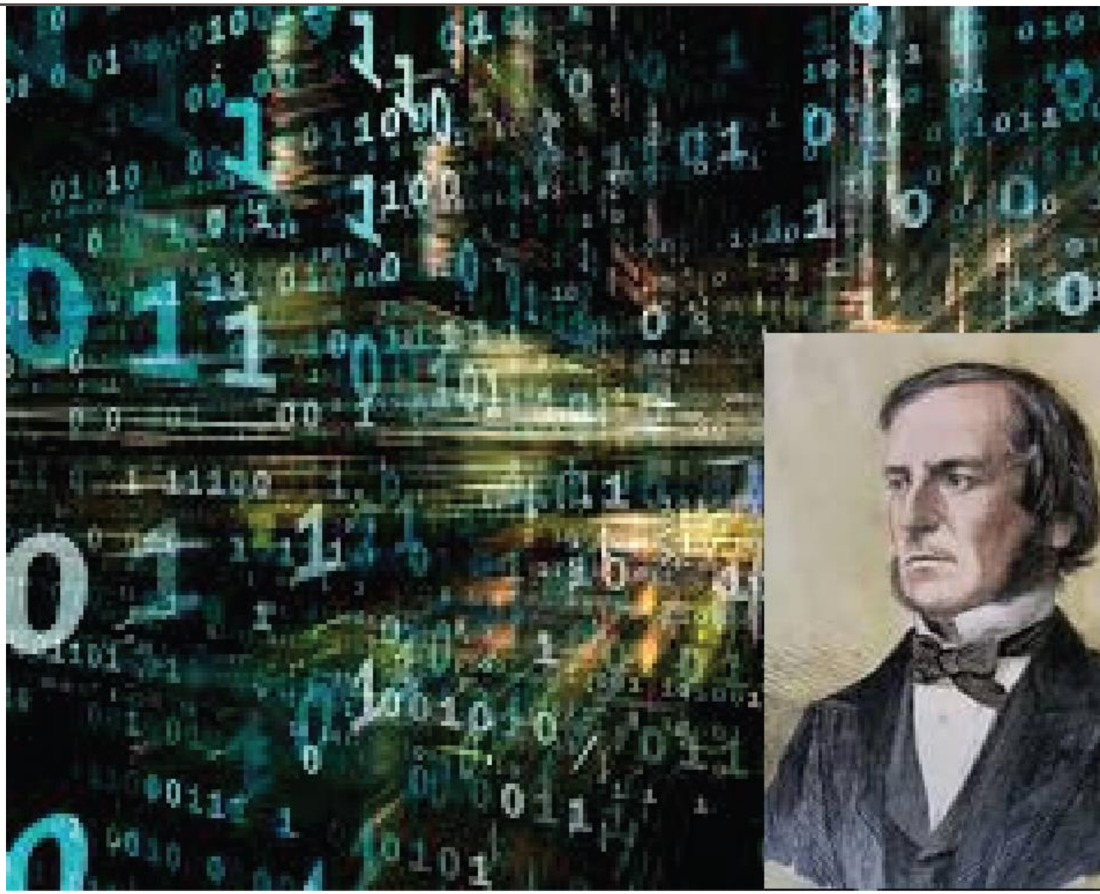
DIGITAL CIRCUITS

Week-6, Lecture-3
K-map



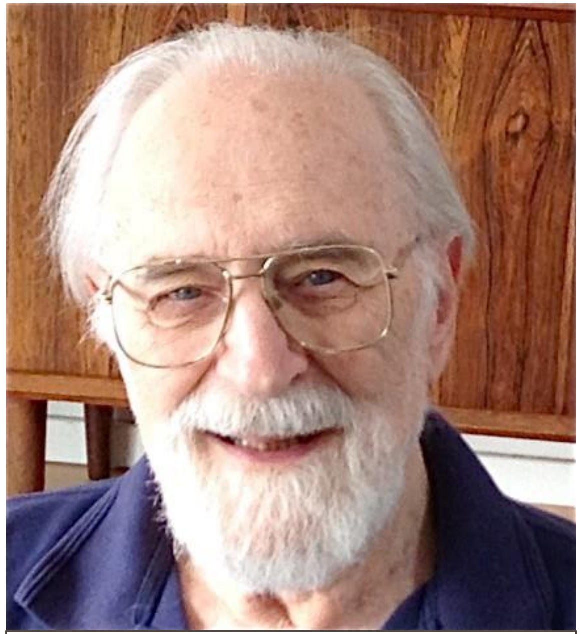
Sneh Saurabh
6th September, 2018

Digital Circuits: Announcements/Revision

Digital Circuits

Combinational Circuit Design

Karnaugh Map Technique



Maurice Karnaugh

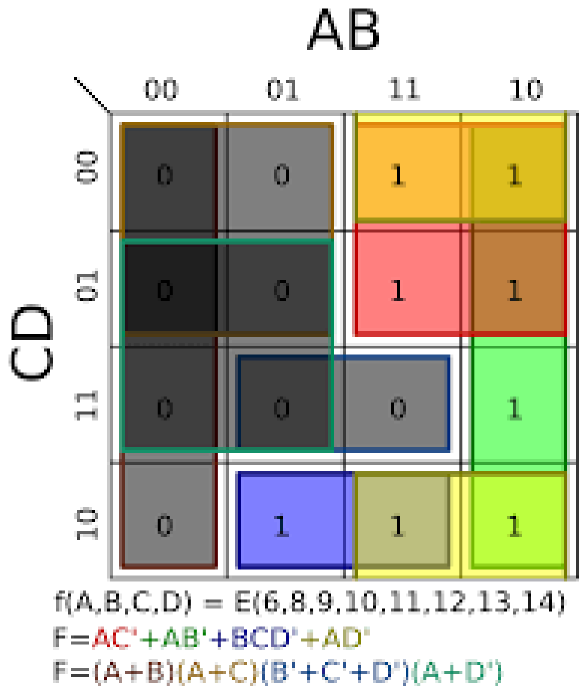
- Goal is to minimize a given logic function
- Algebraic techniques lack rules to predict each successive step
- Karnaugh Map or K-map technique is a simple and straightforward procedure for minimizing Boolean function

- K-map is a pictorial representation of the truth-table of a given function
  - Humans can easily find patterns in the K-map that provide opportunities of minimization

K-Map: Representation

- K-map is a diagram made up of squares
  - Each square represents one minterm of the function

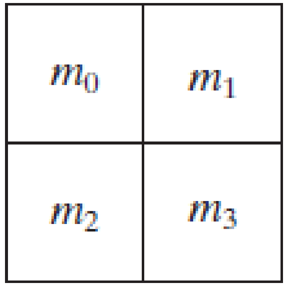
- Any Boolean function can be represented as a sum of minterms
  - A function is represented in the K-map by those squares whose minterms are included in the function



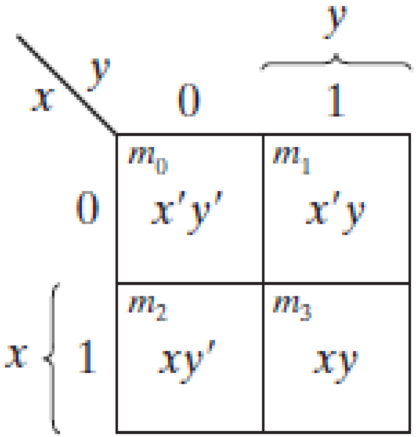
K-Map: Simplification

- Simplified expression obtained using K-map will always be in SOP or POS form
- Simplest expression is one with ***minimum number of terms*** and with ***smallest number of literals in each term***
  - This expression produces a circuit diagram with a minimum number of gates and the minimum number of inputs to each gate
- Simplest expression may not be unique for a given function

K-Map: Two variables



- Two variables:
  - Four minterms: four squares
    - $m_0 = x'y'$
    - $m_1 = x'y$
    - $m_2 = xy'$
    - $m_3 = xy$



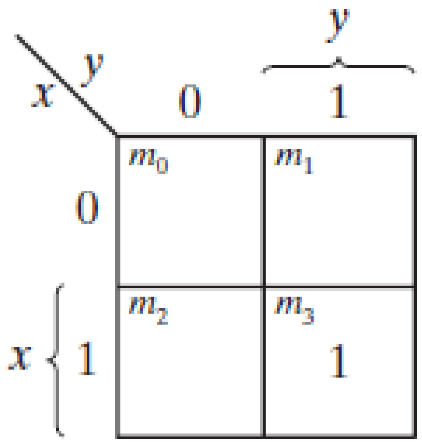
- $0/1$ marked in each row/column represents the value of the variable
- Variable x appears complemented in row 0 and non-complemented in row 1
- Variable y appears complemented in column 0 and non-complemented in column 1

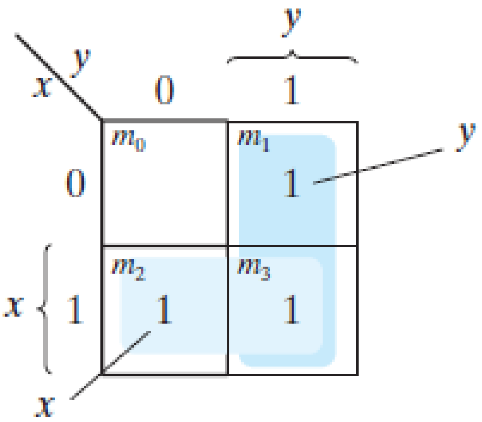
K-Map: Two variables functions

Function is represented on a K-map by marking the square as ‘1’ if the corresponding minterms belong to that function

Problem: Represent the function $f(x,y) = xy$ on a K-map

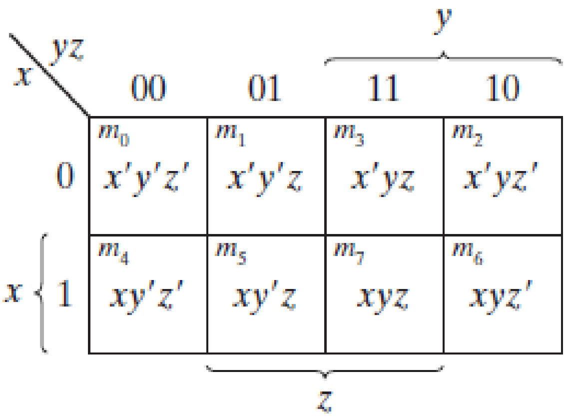
Problem: Represent the function $f(x,y) = x + y$ on a K-map





K-Map: Three variables

m_0	m_1	m_3	m_2
m_4	m_5	m_7	m_6

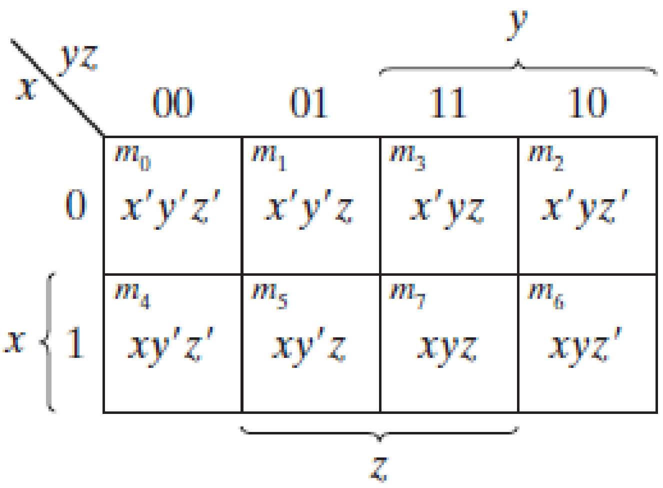


- Three variables:
  - Eight minterms: eight squares
  - Minterms are: $m_0 = x'y'z'$, $m_1 = x'y'z$, $m_2 = x'yz'$, $m_3 = x'yz$, $m_4 = xy'z'$, $m_5 = xy'z$, $m_6 = xyz'$ and $m_7 = xyz$

- Minterms are not arranged in a binary sequence but similar to Gray code
- Only one bit changes in value from one adjacent column to the next

- Any two adjacent squares in the K-map differ by only one variable
  - By adjacent it is meant **horizontally** or **vertically**, but NOT diagonally
- This property is exploited in ***simplification***

K-Map: Basis of simplification



- Consider any two adjacent squares
  - m_5 and m_7 ($xy'z$ and xyz): variable y is complemented and also not-complemented
  - m_1 and m_5: $x'y'z$ and $xy'z$: variable x is complemented and also not-complemented

- Sum of two minterms in adjacent squares can be simplified to a single product term consisting of only two literals.
  - $m_5 + m_7 = xy'z + xyz = xz(y + y') = xz$: variable y is removed
  - $m_1 + m_5 = x'y'z + xy'z = y'z(x + x') = y'z$: variable x is removed

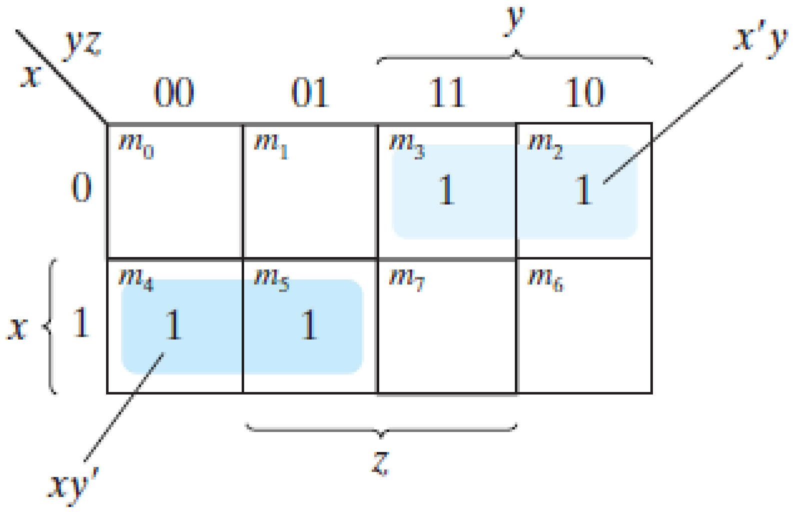
- Any two minterms in adjacent squares that are OR'ed together will cause a removal of the dissimilar variable.

K-Map: Three variables simplification

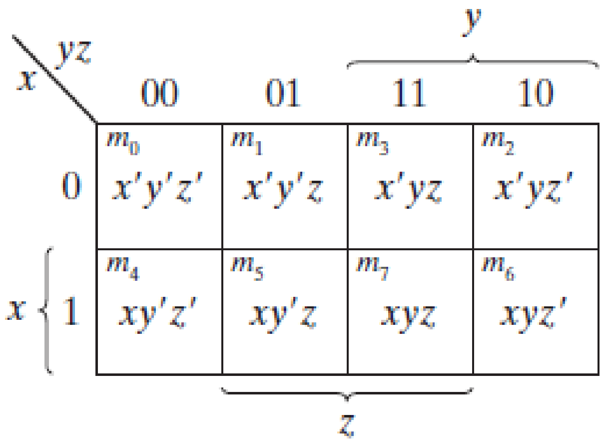
Simplify $\boldsymbol{f(x,y,z)} = \Sigma m(2,3,4,5)$

- First represent the given function on K-map
- Find possible adjacent squares:
  - m_3 and m_2: $= x'yz + x'yz' = x'\ y$
  - m_4 and m_5: $= xy'z' + xy'z = xy'$

$\boldsymbol{f(x,y,z)} = x'y + xy'$



K-Map: adjacent squares

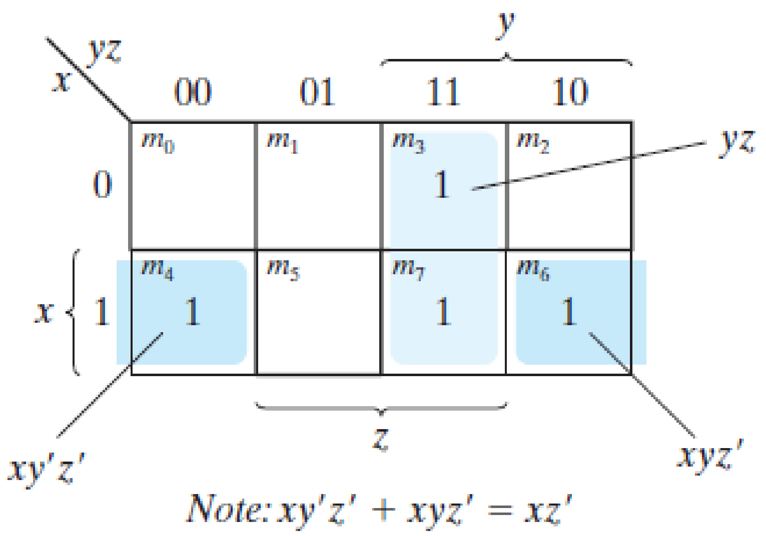


- Consider the squares at the boundaries
  - m_0 and m_2: $x'y'z'$ and $x'yz'$ [Differ in only y' and y]
  - m_4 and m_6: $xy'z'$ and xyz' [Differ in only y' and y]
- Minterms differ by one variable only

- The corresponding squares at the boundaries behave as "adjacent squares"
- Considering the K-map as being drawn on a surface in which the right and left edges touch each other to form adjacent squares

K-Map: Three variables simplification (two squares)

Simplify $F(x, y, z) = \Sigma m(3, 4, 6,7)$



- First represent the given function on K-map
- Find possible adjacent squares:
  - m_3 and m_7: $= x'yz + xyz = yz$
  - m_4 and m_6: $= xy'z' + xyz' = xz'$

For three variables function:

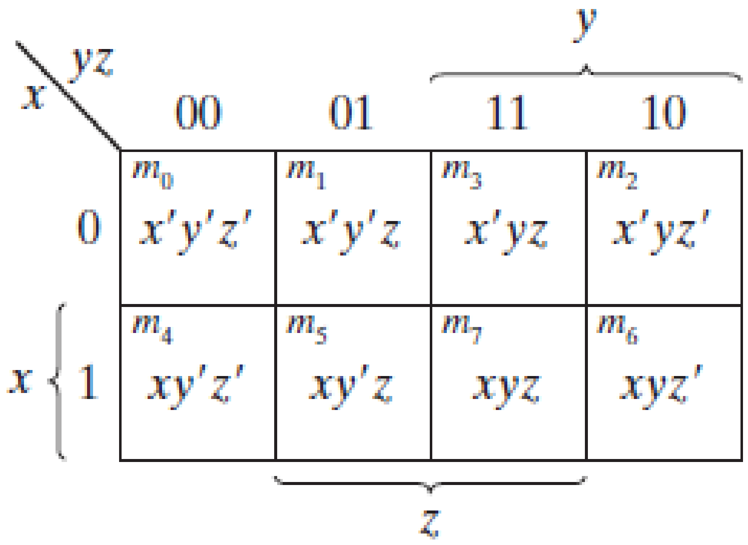
When two adjacent squares are combined, product terms with two literals remain

$\boldsymbol{f(x, y, z) = yz + xz'}$

K-Map: Three variables simplification (four squares)

Simplify $F(x, y, z) = \Sigma m(0, 2, 4,6)$

- First represent the given function on K-map



- Find possible adjacent squares:
  - m_0, m_2, m_4 and m_6: $= x'y'z' + x'yz' + xy'z' + xyz' = x'z' + xz' = z'$

$\boldsymbol{f(x, y, z) = z'}$

For three variables function:

When four adjacent squares are combined, product terms with only one literals remain

K-Map: Bigger combination, fewer literals

- The number of adjacent squares that may be combined must always represent a number that is a power of two, such as 1, 2, 4, and 8 [otherwise number of literals will not reduce]
- As more adjacent squares are combined, we obtain a product term with fewer literals

For three variables function:

- One square represents one minterm, giving a term with three literals
- Two adjacent squares represent a term with two literals
- Four adjacent squares represent a term with one literal
- Eight adjacent squares encompass the entire map and produce a function that is always equal to 1