Poisson Matting

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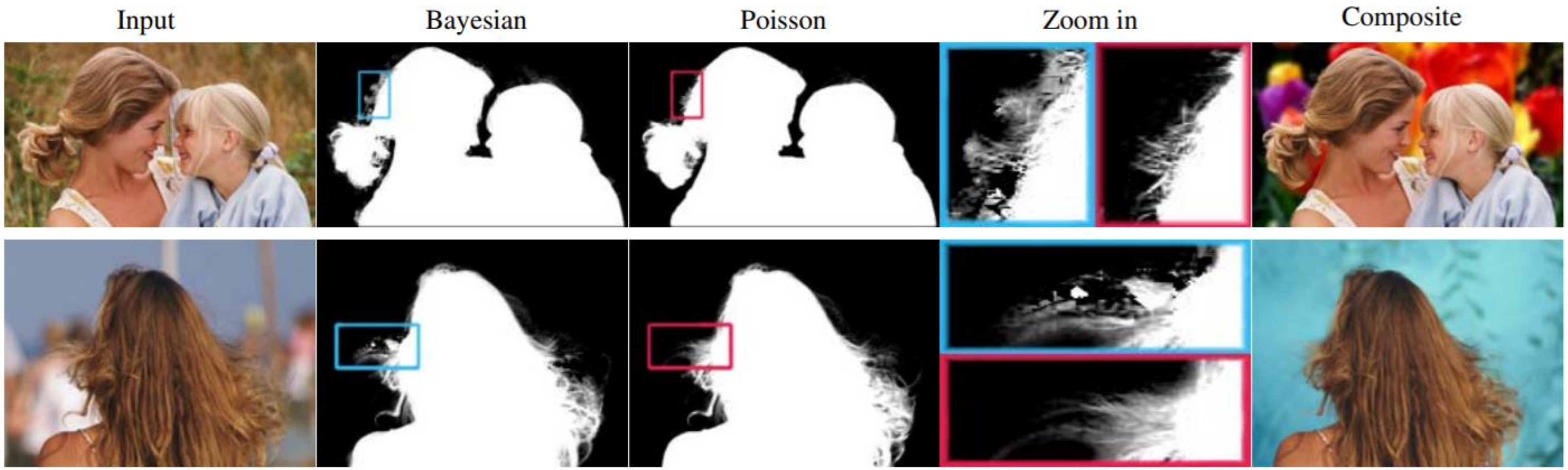
ACM Transactions on Graphics (TOG) Volume 23 Issue 3, August 2004

Pages 315-321

What is Poisson matting?

Figure 1: Pulling of matte from a complex scene. From left to right: a complex natural image for existing matting techniques where the color background is complex, a high quality matte generated by Poisson matting, a composite image with the extracted koala and a constant-color background, and a composite image with the extracted koala and a different background.

What is Poisson matting?



Advantages

- The matte is directly reconstructed from a continuous matte gradient field by solving Poisson equations <u>using boundary information from a user-supplied trimap.</u>

- By interactively manipulating the matte gradient field using a number of <u>filtering tools</u>, the user can further <u>improve</u> Poisson matting results locally <u>until he or she is satisfied</u>

Applications

Multi-background

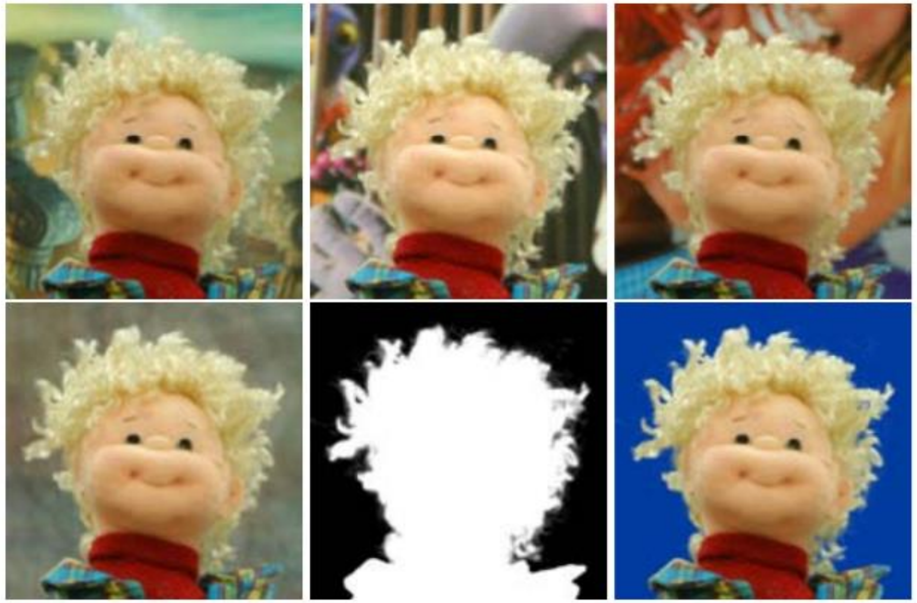


Figure 7: Multi-Background. Top row: three input images with different complex backgrounds. Bottom row: the mean of all eight input images, computed alpha matte and composite image using Poisson matting.

De-fogging

Figure 8: De-fogging. The de-fogged image is obtained using the boosting brush with several local operations.

Introduction

$$I = \alpha F + (1 - \alpha) B$$

- I(x, y): a new image
- F(x, y): a foreground image
- B(x, y): a background image
- α(x, y): a alpha matte

Poisson

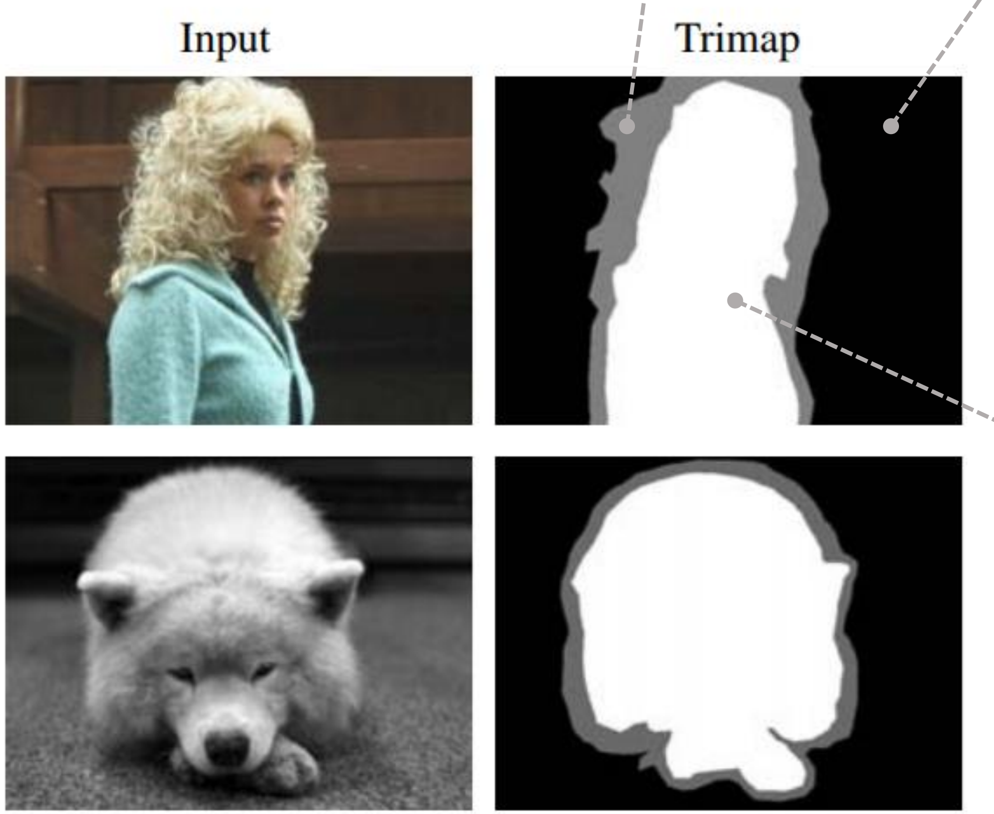
Alpha compositing

In computer graphics, alpha compositing is the process of combining an image with a background to create the appearance of partial or full transparency.

Wikipedia, "Alpha compositing", 24 June 2018, <https://en.wikipedia.org/wiki/Alpha_compositing>, 06 July 2018

Introduction

Trimap



"unknown region"

"definitely background"

Matting is inherently under-constrained because the matting equation has too many unknowns.

The user is required to supply a trimap that partitions the image into three regions:

"definitely foreground"

In the unknown region, the matte can be estimated using the color statistics in the known foreground and background regions.

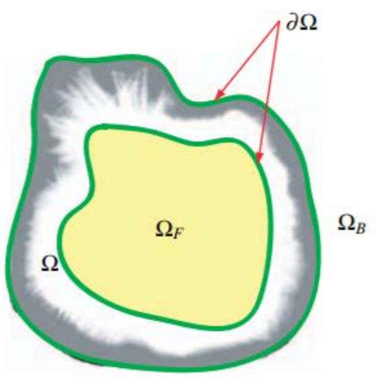
Poisson Matting

Global Poission matting

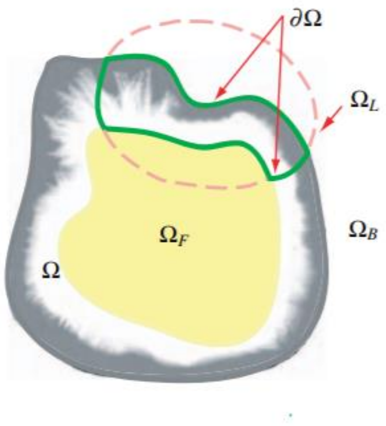
If it fails to produce high quality mattes

due to a complex background

Local Poission matting



(a)



(b)

Global Poisson Matting

$$I = \alpha F + (1-\alpha)B$$

$$\nabla I = \nabla(\alpha F + (1-\alpha)B)$$

$$\nabla I = (F-B)\nabla\alpha + \alpha\nabla F + (1-\alpha)\nabla \mathrm{B}$$

Foreground F and background B are smooth.
$\alpha\nabla F + (1-\alpha)\nabla \mathrm{B}$ is relatively small with respect to $(F-B)\nabla\alpha$

$$\nabla\alpha \approx \frac{1}{F-B}\nabla I$$

Global Poisson Matting

$$\nabla\alpha \approx \frac{1}{F-B}\nabla I$$

For each pixel $p = (x, y)$,

$$\alpha^* = \arg\min_{\alpha} \int\int_{p\in\Omega} \left\| \nabla\alpha_p - \frac{1}{F_p - B_p}\nabla I_p \right\|^2 dp$$

with $\alpha|_{\partial\Omega} = \hat{\alpha}|_{\partial\Omega}$

which is defined $\widehat{\alpha_p}|_{\partial\Omega} = \begin{cases} 1 & p \in \Omega_F \\ 0 & p \in \Omega_B \end{cases}$

$$\Delta\alpha = div\left(\frac{\nabla I}{F-B}\right)$$

Iterative optimization

Global Poisson matting is an iterative optimization process:

1. (F - B) initialization : For each pixel p in Ω, F_p and B_p are approximated by corresponding the nearest foreground pixel in Ω_F and background pixel in Ω_B.
2. α reconstruction
3. F, B refinement

$$\Omega_F^+ = \{p \in \Omega | \alpha_p > 0.95, I_p \approx F_p\}$$

$$\Omega_B^+ = \{p \in \Omega | \alpha_p < 0.05, I_p \approx B_p\}$$

Local Poisson Matting

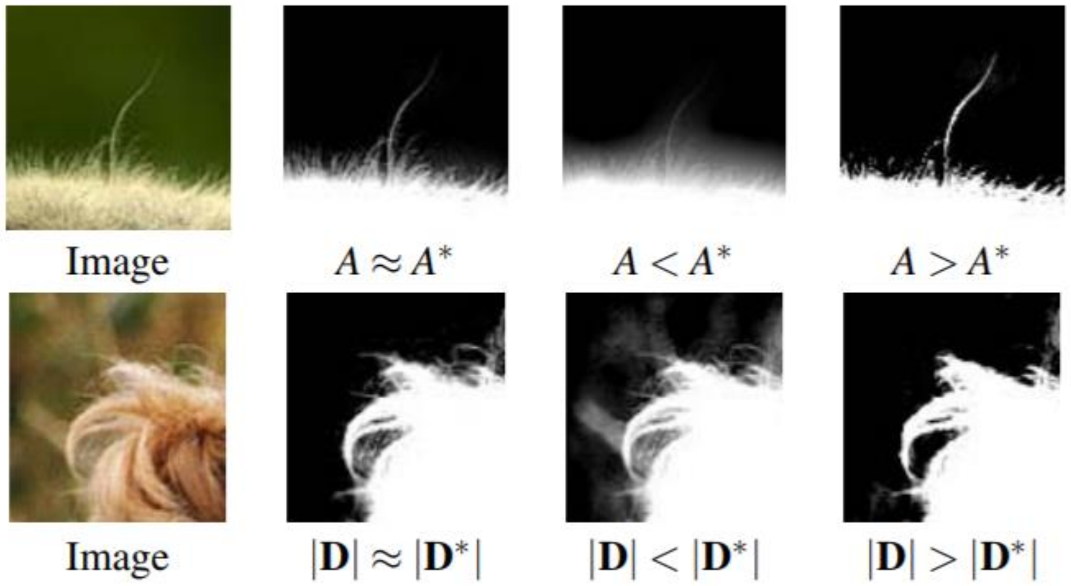
$$\nabla I = (F - B)\nabla\alpha + \alpha\nabla F + (1 - \alpha)\nabla B$$

↓

$$\nabla\alpha = \boldsymbol{A}(\nabla I - \boldsymbol{D})$$

$$\boldsymbol{A} = \frac{1}{F - B}$$

$$\boldsymbol{D} = [\alpha\nabla F + (1 - \alpha)\nabla B]$$



A → Sharpen boundaries

D → A gradient field

Local Poisson Matting

For each pixel $p = (x, y)$,

$$\alpha^* = \arg\min_{\alpha} \int \int_{p\in\Omega_L\cap\Omega} \left\|\nabla\alpha_p - A_p(\nabla I_p - D_p)\right\|^2 dp$$

with $\alpha|_{\partial\Omega} = \hat{\alpha}|_{\partial\Omega}$

which is defined $\widehat{\alpha_p}|_{\partial\Omega} = \begin{cases} 1 & p \in \Omega_F \\ 0 & p \in \Omega_B \\ \alpha_g & p \in \Omega \end{cases}$

Usually, the local region size we use is small (fewer than 200×200 pixels), where a Poisson solver very quickly generates the matting result.

Channel selection

To reduce the error of D

Local filtering

To manipulate A and D

1. Boosting brush
2. Highpass filtering
3. Diffusion filtering
4. Clone brush

Erase brush / Inverse Brush

Local Poisson Matting

Channel Selection

$$\min_{a,b,c} \sum_i [(a\ b\ c) \cdot (R_i\ G_i\ B_i)^T\ - (a\ b\ c) \cdot (\bar{R}\ \bar{G}\ \bar{B})\ ]^2\ \ s.t.\ a + b + c = 1$$

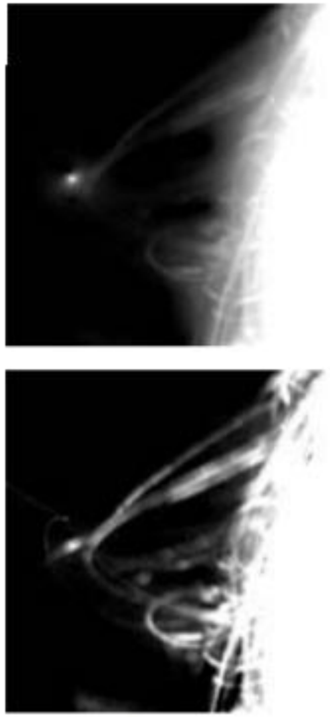
(a) (b) (c)

A new channel $\gamma = aR + bG + cB$

The error of D is reduced and the hair shape is better recovered.

Local Poisson Matting

Local Filtering


(a) Boosting

Boosting brush.

$$A'_p = \left[1 + \lambda \exp\left(-\frac{||p - p_0||^2}{2\sigma^2}\right)\right] \cdot A_p$$

- p_0: the coordinate of the brush center
- σ : the size of the boosting effect, which is defined by user
- λ : the strength of the boosting effect, which is defined by user

When the matting result is smoother or sharper than what users expect, a boosting brush can be used to increase or decrease **A** directly.

Local Poisson Matting

Local Filtering

(b) Highpass

Highpass filtering.

D can be estimated using the low-frequency part of the image gradient.

$$D = K * \nabla I$$

- $K = N(p; p_0, \sigma^2)$: a Gaussian filter

$$\nabla\alpha = A(\nabla I - \boldsymbol{D})$$

$$\nabla\alpha = A(\nabla I - K * \nabla I)$$

Local Poisson Matting

Local Filtering

(c) Diffusion

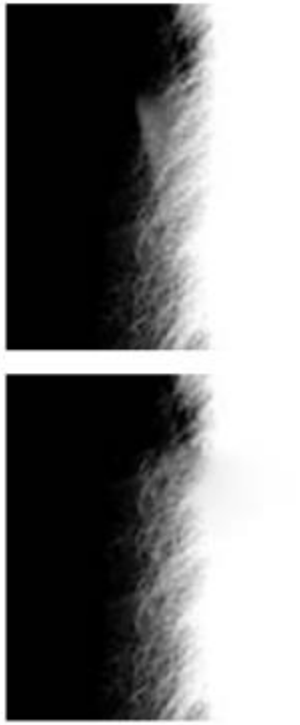
Diffusion filtering.

Adopt anisotropic diffusion [Perona and Malik. 1990] to diffuse the image. It is an edge-preserving blurring process to remove small scale noise.

Pepper & Sault

Clone brush.

The clone brush can be used to directly copy the matte gradient $A(\nabla I - D)$ from a user selected source to a target region.



(d) Clone

Local Poisson Matting

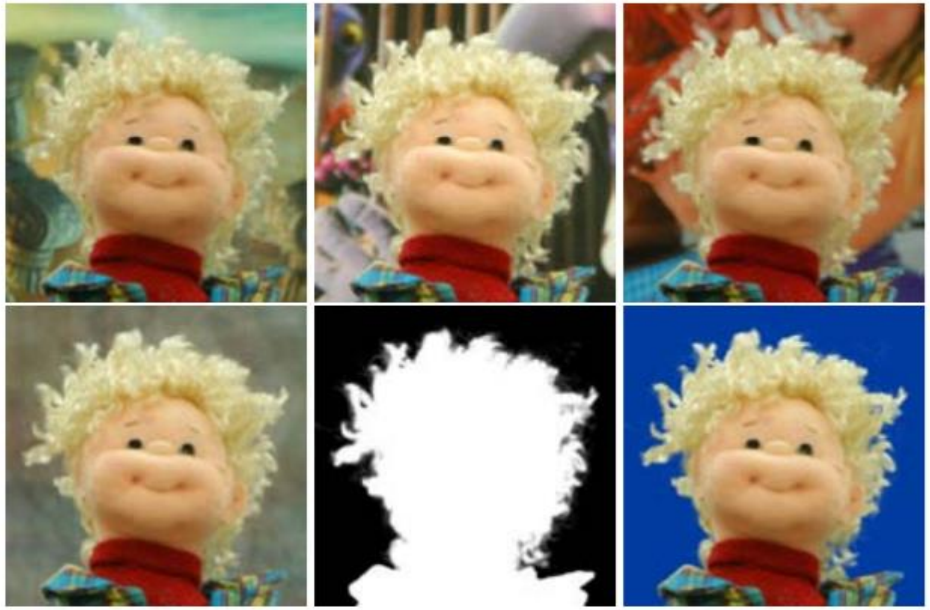
Refinement Process

1. Apply channel selection to reduce the errors in D. For solid object boundaries, apply the diffusion filter to remove possible noise.
2. Apply the highpass filtering to obtain an approximation of D.
3. Apply the boosting brush to manipulate A.
4. Possibly apply the clone brush if gradients are indistinguishable.

Optional) Erase brush / Inverse brush

Applications

Multi-background



Without any information about backgrounds, we calculate the mean image.

$$\bar{I} = \frac{1}{T}\sum_{t}^{T}(\alpha F + (1-\alpha)B_t)$$

$$\frac{1}{T}\cdot T\alpha F + (1-\alpha)\frac{1}{T}\sum_{t}^{T} B_t$$

$$\alpha F + (1-\alpha)\bar{B}$$

Poisson matting works better in the mean image than in any individual image in $\{I_t\}_{t=1}^{T}$.

Applications

De-fogging

$$I = I_c \cdot e^{-\beta d} + Fog \cdot \left(1 - e^{-\beta d}\right)$$

$$\nabla I_c = \nabla I \cdot e^{\beta d}$$

- Fog: the color of fog
- Ic: the clear image without fog
- β: the scattering coefficient of the atmosphere
- d: the depth value.

Use the boosting brush to locally modify $e^{\beta d}$ in the selected region to fine tune the defogged image gradient.

Conclusions

In this paper, we have presented a new digital matting method, Poisson matting. By solving Poisson equations, Poisson matting reconstructs a faithful matte from its approximated gradient field estimated from an input image semi-automatically. Given a few hints using local operations, Poisson matting is capable of producing impressive results for many complex images problematic to previous natural image matting methods.

Limitations

1. When the foreground and background colors are very similar, the matting equation becomes ill-conditioned.
2. When the matte gradient estimated in global Poisson matting largely biases the true values, so that small regions need to be processed for local refinements in local Poisson matting, which increases user interaction.
3. When the matte gradients are highly interweaved with the gradients of the foreground and background within a very small region.