$y = Mx + b$

$M = -1$

$b = 1$

Learning Rate $= 0.1$

Given points $= (1,3)$ & $(3,6)$

$\hat{y}_1 = (-1)(1) + 1 = 0$

$\hat{y}_2 = (-1)(3) + 1 = -2$

Error 1 $= 3 - 0 = 3$

Error 2 $= 6 - (-2) = 8$

Gradient descent with respect to

$$\frac{\partial J}{\partial M} = \frac{2[(\hat{y}_1 - y_1)x_1 + (\hat{y}_2 - y_2)x_2]}{\cancel{2}\, n}$$

$$= \frac{\cancel{2}\,(0-3)(1) + (-2-6)(3)}{\cancel{2}}$$

$$= -3 + -24 = -27$$

with respect to $\frac{\partial J}{\partial b}$

$$= \frac{2}{n}[(\hat{y}_1 - y_1) + (\hat{y}_2 - y_2)]$$

$$= \frac{\cancel{2}}{\cancel{2}}[(0-3) + (-2-6)]$$

$$= -3 - 8 = -11$$

updating the value of M to get M new & b to get b new

$$M_{new} = M - \alpha \frac{\partial J}{\partial M}$$

$M = -1$

$\alpha = 0.1$

$\frac{\partial J}{\partial M} = -27.$

$$M_{new} = -1 - 0.1(-27)$$
$$= -1 + 2.7$$
$$= 1.7.$$

$$b_{new} = b - \alpha \frac{\partial J}{\partial b}$$

$b = 1$

$\alpha = 0.1$

$\frac{\partial J}{\partial b} = -11$

$$b_{new} = 1 - 0.1(-11)$$
$$= 1 + 1.1$$
$$= 2.1.$$

Iteration 2.

Using the new values of m and b from Iteration 1

$m = 1.7$ and $b = 2.1$

Calculating the new predictions.

from $y = mx + b$.

For points $(1, 3)$.

$\hat{y}_1 = 1.7 \times 1 + 2.1 = 3.8$

For points $(3, 6)$

$\hat{y}_2 = 1.7 \times 3 + 2.1 = 7.2$

$\therefore$ New predictions =

$\hat{y}_1 = 3.8$ and $\hat{y}_2 = 7.2$

Calculating the new cost

$$= \frac{(3-3.8)^2 + (6-7.2^2)}{2}$$

$$= \underline{1.04}$$

Now calculate the errors

Errors:

$\hat{y}_1 = 3 - 3.8 = -0.8$

$\hat{y}_2 = 6 - 7.2 = -1.2$

Computing gradients

with respect to $\frac{\partial J}{\partial m}$

$$\frac{\partial J}{\partial m} = -2\frac{\left[(y_1 - \hat{y}_1)x_1 + (y_2 - \hat{y}_2)x_2\right]}{n}$$

$$= \frac{-\cancel{2}\left[(3-3.8)\times 1 + (6-7.2)3\right]}{\cancel{2}}$$

$$= -(-0.8 - 3.6) = 4.4$$

$$= \underline{4.4}$$

With respect to $\frac{\partial J}{\partial b}$

$$\frac{\partial J}{\partial b} = \frac{-\cancel{2}\left[(3-3.8) + (6-7.2)\right]}{\cancel{2}}$$

$$= -(-0.8 - 1.2) = 2.0$$

$$= \underline{2}$$

Updating values of m and b to get m_{new} and b_{new}

$m_{new} = m - \alpha \frac{\partial J}{\partial m}$

where:

$m = 1.7$, $\alpha = 0.1$, $\frac{\partial J}{\partial m} = 4.4$

$= 1.7 - 0.1 \times 4.4 = 1.26$

$\therefore m_{new} = 1.26$

$b_{new} = b - \alpha \frac{\partial J}{\partial b}$

where

$b = 2.1$, $\alpha = 0.1$ $\frac{\partial J}{\partial b} = 2.0$

$= 2.1 - 0.1 \times 2.0 = 1.9$

$\therefore b_{new} = 1.9$

Answer:

$m_{new} = 1.26$ and $b_{new} = 1.9$

ITERATION 3

Using the new values of m & b

$m_1 = 1.26$

$b_1 = 1.9$

$\bar{y}_1 = (1.26)(1) + 1.9 = 3.16$

$\bar{y}_2 = (1.26)(3) + 1.9 = 5.68$

Error 1 $= 3 - 3.16 = -0.16$

Error 2 $= 6 - 5.68 = 0.32$

Gradient descent

$$\frac{d}{dm} = (\bar{y}_1 - y_1)x_1 + (\bar{y}_2 - y_2)x_2$$

$$= (3.16 - 3)(1) + (5.68 - 6)3$$

$$= 0.16 + (-0.96)$$

$$= -0.8$$

$$\frac{d}{db} = (\bar{y}_1 - y) + (\bar{y}_2 - y_2)$$

$$= (3.16 - 3) + (5.68) - 6)$$

$$= 0.16 - 0.32$$

$$= -0.16$$

New parameters

$$M_2\text{new} = m_1\text{new} - \alpha\frac{d}{dm}$$

$$= 1.26 - (0.1 \times -0.8)$$

$$= 1.26 - -0.08$$

$M_2\text{new} = 1.34$

New parameter for b

$$b_2\text{new} = b_1 - \alpha\frac{d}{db}$$

$$= 1.9 - (0.1).(-0.16)$$

$$= 1.9 + 0.016$$

$$= 1.916$$

New prediction

$$\bar{y}_1\text{new} = 1.34 \times 1 + 1.916$$

$$= 3.256$$

$$\bar{y}_2\text{new} = 1.34 \times 3 + 1.916$$

$$= 5.936$$

Loss calculation

Error 1 $= 3 - 3.256$

$= -0.256$

Error 2 $= 6 - 5.936$

$= 0.064$