STAT2032/6046

Solution to Q10, Review Questions Part 4

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Question 10

For the random interest rate $\tilde{i}$, denote $E[\tilde{i}] = j$ and $Var[\tilde{i}] = s^2$. Assuming independence of rates, show that:

i. $E[\tilde{S}(n)] = (1+j)^n$

ii. $Var[\tilde{S}(n)] = (1+2j+j^2+s^2)^n - (1+j)^{2n}$

Solution:

i. $E[\tilde{S}(n)] = (E(1+\tilde{i}))^n = (1+E[\tilde{i}])^n = (1+j)^n$

ii. $E[\tilde{i}^2] = Var[\tilde{i}] + (E[\tilde{i}])^2 = s^2 + j^2$

$E[\tilde{S}(n)^2] = (E[(1+\tilde{i})^2])^n = (E[1+2\tilde{i}+\tilde{i}^2])^n = (1+2E[\tilde{i}]+E[\tilde{i}]^2)^n = 1+2j+j^2+s^2$

$Var[\tilde{S}(n)] = E[\tilde{S}(n)^2] - (E[\tilde{S}(n)])^2 = (1+2j+j^2+s^2)^n - (1+j)^{2n}$