March 1st

Complex Numbers

$ax^2+bx+c=0$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$ax^3+bx^2+cx+d=0$ also exist formulas for roots

$ax^4+bx^3+cx^2+dx+e=0$

- in order to derive these formulas have to work with $\sqrt{\ }$ of negative number

introduce number i such that $i^2=-1$

$(2i)^2=-4$

Complex numbers: $\{a+bi \mid a,b \text{ real numbers}\}$

$1+2i$, $\frac{1}{3}-5i$, $\sqrt{2}-\pi i$ complex numbers

$z_1=a+bi$
$z_2=c+di$
$z_1+z_2=a+c+(b+d)i$
$z_1-z_2=(a-c)+(b-d)i$
$z_1z_2=(a+bi)(c+di)=ac+(ad+bc)i-bd=(ac-bd)+(ad+bc)i$

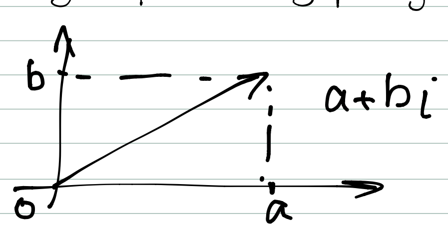
$(1+2i)(3-i)=\cdots$

properties: $z_1+z_2=z_2+z_1$
$z_1z_2=z_2z_1$
$z_1(z_2+z_3)=z_1z_2+z_1z_3$
$z_1(z_2z_3)=(z_1z_2)z_3$

$$\frac{1+2i}{3-i}=\frac{(1+2i)(3+i)}{(3-i)(3+i)}=\frac{3+7i-2}{9+1}=\frac{1+7i}{10}$$

$$\frac{a+bi}{c+di}=\frac{(a+bi)(c-di)}{(c+di)(c-di)}=\frac{ac+bd+(bc-ad)i}{c^2+d^2}$$

presenting complex number graphically



$|z|=\sqrt{a^2+b^2}$

Def: if $z=a+bi \Rightarrow \bar{z}=a-bi$ complex conjugate of z

$\overline{1+2i}=1-2i$

$\overline{3-\sqrt{2}i}=3+\sqrt{2}\,i$

Properties: ① $z\cdot\bar{z}=|z|^2$

$z=a+bi$, $\bar{z}=a-bi$

$$\frac{1}{z}=\frac{1}{a+bi}=\frac{\bar{z}}{a^2+b^2}=\frac{a-bi}{a^2+b^2}$$

② $\bar{\bar{z}}=z$

③ $\overline{z_1+z_2}=\bar{z}_1+\bar{z}_2$

④ $\overline{z_1-z_2}=\bar{z}_1-\bar{z}_2$

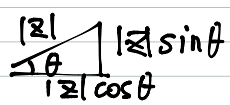
⑤ $\overline{z_1 z_2}=\bar{z}_1\,\bar{z}_2$

Cor: $|z_1\cdot z_2|=|z_1|\,|z_2|$

Pf. $|z_1z_2|^2=z_1z_2\,\overline{z_1z_2}=z_1z_2\bar{z}_1\,\bar{z}_2=|z_1|^2|z_2|^2$

$\sqrt{|z_1z_2|^2}=\sqrt{|z_1|^2}\sqrt{|z_2|^2}$ v.

$$\boxed{\text{Prove } \frac{|z_1|}{|z_2|}=\left|\frac{z_1}{z_2}\right| \quad \text{HW}}$$



$z=a+bi=|z|\cos\theta+|z|\sin\theta\, i=|z|(\cos\theta+i\sin\theta)$

$z_1z_2=|z_1||z_2|(\cos\theta_1+i\sin\theta_1)(\cos\theta_2+i\sin\theta_2)=|z_1||z_2|(\cos(\theta_1+\theta_2)+i\sin(\theta_1+\theta_2))$