Exerzitien I

Submit your *concise* solutions in the correct order and no later than 2:10 pm on Sept. 22, in your tutorial.

Reading suggestion: **Complex numbers**, **Definition of vector space**, **Properties of vector spaces**, and **Subspaces** - first four sections in Axler, chapter 1.

Exercise 1.

1. *Let $a, b \in \mathbb{R}$, not both zero. Find $c, d \in \mathbb{R}$ such that*
$$(a + bi)^{-1} = c + di.$$
2. *Prove that $\frac{-1+i\sqrt{3}}{2}$ is a cube root of 1.*
3. *Let $S = \{z \in \mathbb{C} \mid z^3 = 1\}$ and $T = \{z \in \mathbb{C} \mid z^4 = 1\}$. List the elements in S and T, and plot them.*

Exercise 2.

1. *Let ℓ_1 and ℓ_2 be the lines in $\mathbb{R}^2$ defined by the linear equations*
$$\ell_1 = \{(x, y) \in \mathbb{R}^2 \mid x + y = 2\},$$
$$\ell_2 = \{(x, y) \in \mathbb{R}^2 \mid 2x - y = 2\}.$$
Draw a graph showing ℓ_1 and ℓ_2, and then find the intersection $\ell_1 \cap \ell_2$ of the two lines.
2. *Let ℓ_3 be the line in $\mathbb{R}^2$ defined by the linear equation*
$$\ell_3 = \{(x, y) \in \mathbb{R}^2 \mid x = y\},$$
and add it to your diagram. Determine the intersections $\ell_1 \cap \ell_3$ and $\ell_2 \cap \ell_3$, as well as the intersection of all three lines $\ell_1 \cap \ell_2 \cap \ell_3$.
3. *Which of ℓ_1, ℓ_2, ℓ_3 are linear subspaces? Prove your claim.*

Exercise 3. *Let V be the set of pairs (x, y) of real numbers and define a modified addition operation*
$$(x, y) + (u, v) = (x + u, 0)$$
as well as a modified scalar multiplication by $c \in \mathbb{R}$ via
$$c(x, y) = (cx, 0).$$
Using these two modified operations, is V a vector space? Justify your answer.