University of Toronto
Faculty of Arts and Science

MAT224H1F
Linear Algebra II

Final Examination
December 2010

S. Uppal

Duration: 3 hours

PLEASE HAND IN

Last Name: ______________________________

Given Name: ______________________________

Student Number: ______________________________

No calculators or other aids are allowed.

FOR MARKER USE ONLY	
Question	Mark
1	/10
2	/10
3	/10
4	/10
5	/10
6	/10
7	/5
TOTAL	/65

[10] **1.** Let $W_1 = \{A \in \mathbb{R}_{2\times 2} \mid A = A^T\}$ and let $W_2 = \{(x, y, z, w) \in \mathbb{R}^4 \mid x + y + z = 0\}$. Show that W_1 and W_2 are isomorphic and find an isomorphism $T \colon W_1 \to W_2$

EXTRA PAGE FOR QUESTION 1 - please do not remove.

[10] **2.** Let $W = span\{(i, 0, 1)\}$ in $\mathbb{C}^3$. Find an orthonormal basis for $W^\perp$.

EXTRA PAGE FOR QUESTION 2 - please do not remove.

[10] **3.** Let $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ be the linear operator defined by $T(x, y) = (x + y, -x + y)$. Show that T is normal and find the spectral decomposition of T

EXTRA PAGE FOR QUESTION 3 - please do not remove.

[10] **4.** Let V be an inner product space and let $y, z \in V$. Define $T\colon V \to V$ by

$$T(x) = < x, y > z$$

for all $x \in V$.

(a) Show that T is a linear operator.

(b) Find $T^*(x)$.

[10] **5.** Verify the Cayley-Hamilton theorem for

$$A = \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}.$$

[10] **6.** Let $T\colon \mathbb{R}^3 \to \mathbb{R}^3$ be the linear operator that has the matrix

$$A = \begin{pmatrix} -5 & 3 & 1 \\ -4 & 2 & 1 \\ -4 & 3 & 0 \end{pmatrix}$$

realtive to the standard basis of $\mathbb{R}^3$. Find a basis of $\mathbb{R}^3$ such that the matrix of T relative to this basis is block triangular, and find the matrix of T relative to this basis.

EXTRA PAGE FOR QUESTION 6 - please do not remove.

[5] **7.** Let W be a subspace of an inner product space V and let $T \colon V \to V$ be a linear operator. Prove that if W is T-invariant, then $W^{\perp}$ is T^{*}-invariant.