STAT 6046 Tutorial Week 7

By Isaac Pan
The Australian National University

Today's plan

- Brief review of course material
- Go through selective tutorial questions

Loan repayments

- Repayments can be decomposed into a principal and interest component.
- Most loans operate like a **repayment mortgage**, where the initial principal is repaid during the term of the loan.
- This is done by making repayments that are **greater than** the amount of interest due. The remainder of the payment is used to repay part of the principal (or capital).

Loan repayments

- AMORTISATION METHOD:
  - Present value of all amounts loaned out = Present value of all payments made to repay the loan
- Loan schedule

t	Payment	Interest due	Principal Repaid	Outstanding balance
0				$L = OB_0$
1	K_1	$I_1 = OB_0 \cdot i$	$PR_1 = K_1 - I_1$	$OB_1 = OB_0 - PR_1$
2	K_2	$I_2 = OB_1 \cdot i$	$PR_2 = K_2 - I_2$	$OB_2 = OB_1 - PR_2$
…				
t	K_t	$I_t = OB_{t-1} \cdot i$	$PR_t = K_t - I_t$	$OB_t = OB_{t-1} - PR_t$
…				
n	K_n	$I_n = OB_{n-1} \cdot i$	$PR_n = K_n - I_n$	$OB_n = OB_{n-1} - PR_n$ =0

Calculating outstanding balance

- Retrospective method:
  - Calculate the accumulated value of the original loan and subtract the accumulated value of payments made to date.

$$\boxed{OB_t = L(1+i)^t - \sum_{a=1}^{t}(1+i)^{t-a}K_a}$$

- prospective method:
  - Calculate the present value of future payments that are required to pay off the loan.

$$\boxed{OB_t = vK_{t+1} + v^2K_{t+2} + \ldots + v^{n-t}K_n = \sum_{a=1}^{n-t} v^a K_{a+t}}$$

Loan and repayments

- Loan with level payments:
  - Principal: $$Ka_{\overline{n|}} = L$$
  - Interests:

$$K(1-v^n)+K(1-v^{n-1})+K(1-v^{n-2})+\ldots+K(1-v)=K\left(n-a_{\overline{n|}}\right)$$

- Loan with nominal payments:
  - Principal:

$$L = mKa_{\overline{n|}}^{(m)}$$

  - Interests:

$$nmK - L = nmK - mKa_{\overline{n|}}^{(m)}$$

NPV

- The net present value at the rate of interest *i* of the net cash flows is usually denoted by NPV(*i*).

$$NPV(i) = \sum_t c_t (1+i)^{-t} + \int_0^T \rho(t)(1+i)^{-t}\,dt$$

- The rate of interest *i* used to calculate the net present value is often referred to as the **risk discount rate**.

IRR

- The yield rate or internal rate of return (IRR) is the effective rate of interest that equates the present value of income and outgo, i.e. makes the net present value of the cash flows equal to zero.

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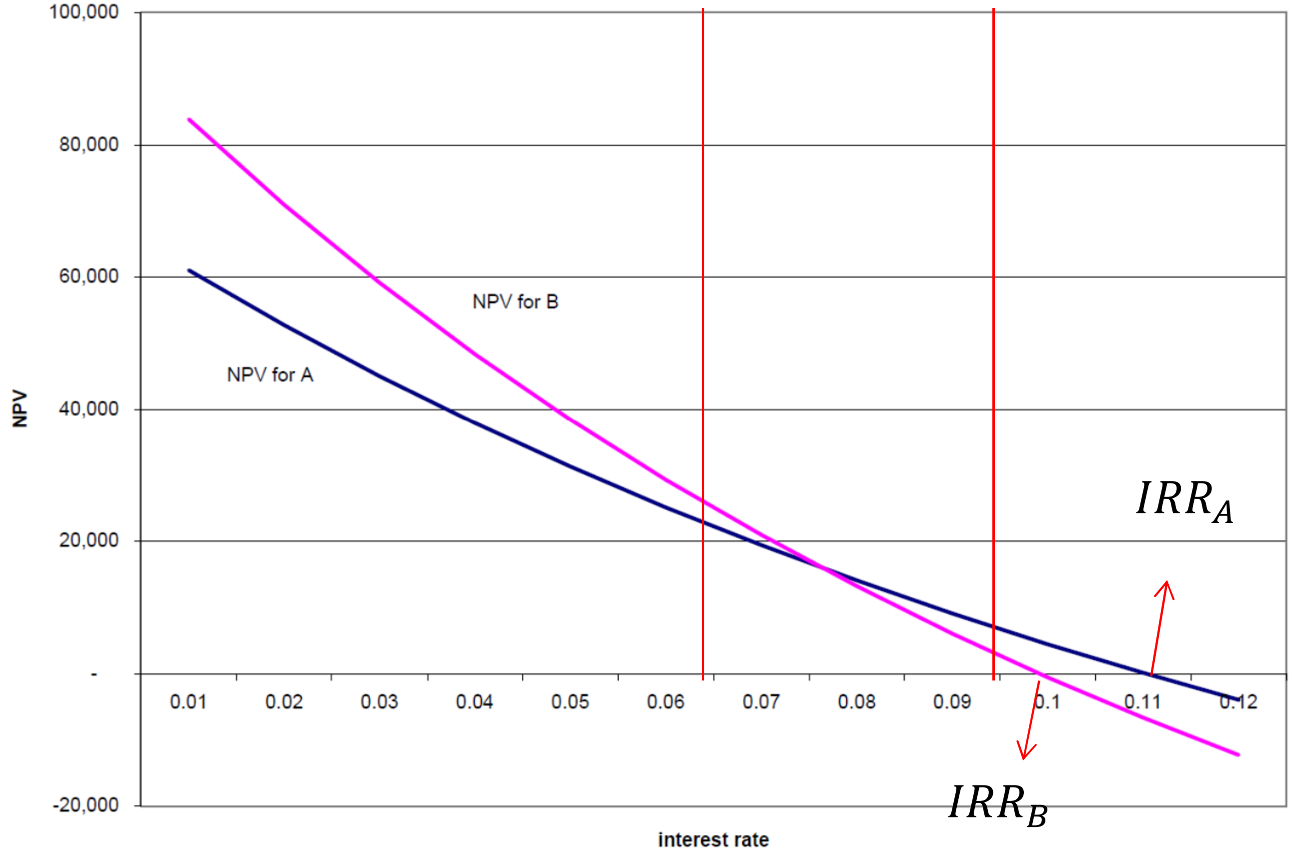
NPV VS IRR

- If the internal rate of return on a project is i_0, then $NPV(i_0) = 0$. Assuming that our inflows are relatively in the future compared to our outflows, the NPV decreases as the risk discount rate increases, the NPV will fall below zero when $i > i_0$.
- Note: If the IRR for project A is greater than that for project B: $i_A > i_B$, this **DOES NOT** imply that $\mathrm{NPV}_A(i_1) > \mathrm{NPV}_B(i_1)$. This is because the NPV for each project depends on i_1 (hurdle rate).

NPV VS IRR



100,000
80,000
60,000
40,000
20,000
-
-20,000
NPV
NPV for B
NPV for A
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.1
0.11
0.12
interest rate
IRR_A
IRR_B

Reinvestment rate & Discounted payback period

- Reinvestment rate $i_S \neq$ yield/borrowing rate/risk discount rate i_D
  - When Account surplus/ incomes, use i_S.
  - When Account deficit/ borrowed funds, use i_D.
- the discounted payback period t_1 is the smallest value of *t* such that the accumulated value *AV*(*t*) of the net cash flows is greater than or equal to zero.
  - $t < t_1$, use $i_D \rightarrow$ Account deficit
  - $t > t_1$, use $i_S \rightarrow$ Account surplus