Australian National University

STAT 6046 Tutorial Week 3

By Isaac Pan

The Australian National University

Today's plan

- Brief review of course material

- Go through selective tutorial questions

Nominal Interest

- We define $i^{(m)}$ as the nominal rate of interest per annum convertible m times per year.
- $i^{(m)}$ is payable in equal instalments of $\frac{i^{(m)}}{m}$ at the ***end*** of each subinterval of length $\frac{1}{m}$ years (i.e. at times 1/m,2/m,…,1).

Nominal Interest VS Effective Interest

- A nominal rate of interest convertible m times per year is equivalent to an effective rate of $\frac{i^{(m)}}{m}$ over a time period of $\frac{1}{m}$ years.
- Force of interest: the notation δ(i.e. $i^{(\infty)} = \delta$).Compound continuously.

$$\boxed{1+i=\left(1+\frac{i^{(m)}}{m}\right)^m}$$

$$i=\left(1+\frac{i^{(m)}}{m}\right)^m-1 \longrightarrow \lim_{m\to\infty}\left(1+\frac{0.12}{m}\right)^m-1=e^{0.12}-1$$

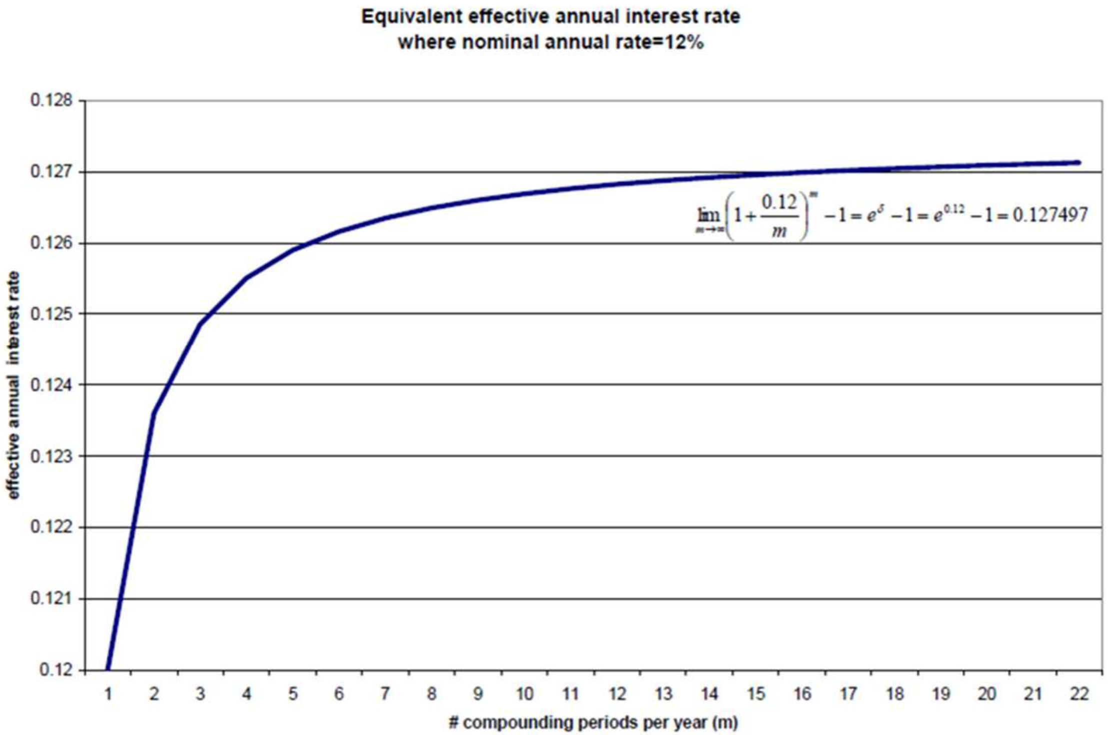
$$i^{(m)}=m\left[(1+i)^{\frac{1}{m}}-1\right]$$

Nominal Interest VS m
(given fixed nominal rate)

$$i^{(m)} = 0.12$$

$$i = \left(1 + \frac{i^{(m)}}{m}\right)^m - 1$$

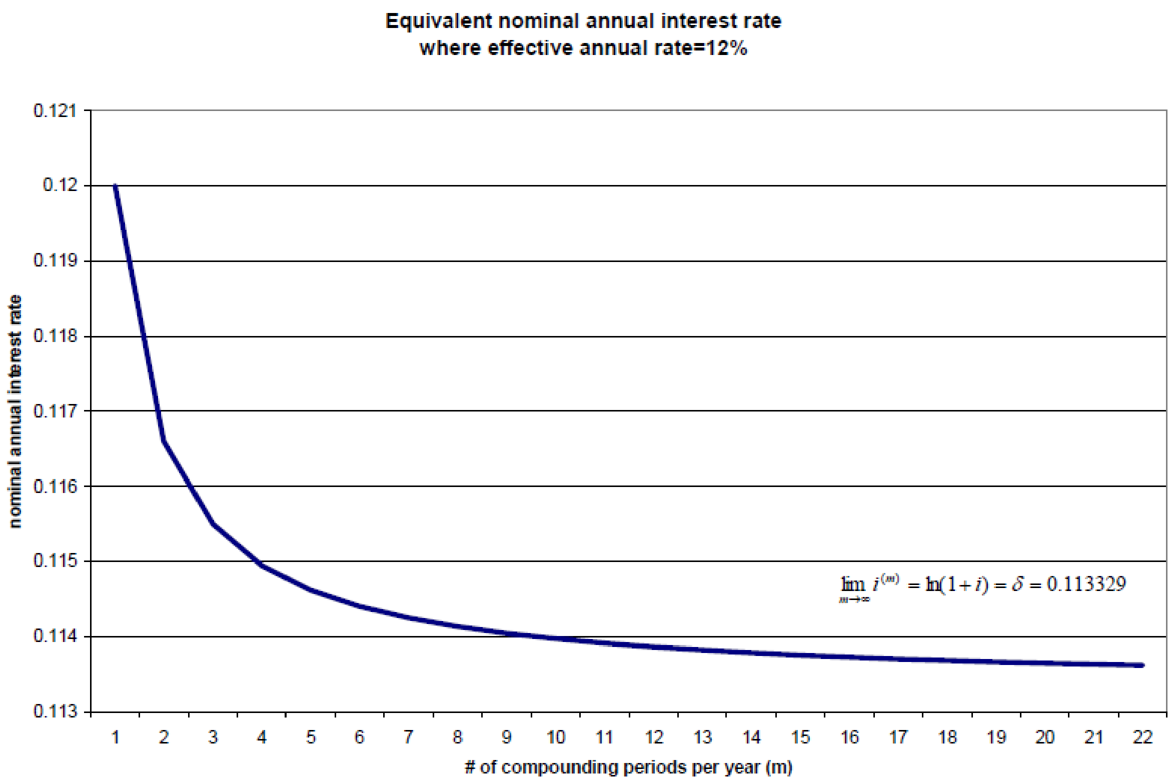
$$i^{(m)} = m\left[(1+i)^{\frac{1}{m}} - 1\right]$$



Nominal Interest VS m (given fixed effective rate)

$i = 0.12$

$$\boxed{i > i^{(2)} > i^{(3)} > \ldots > \delta}$$



Discount Rate

- ***Interest payable in arrears***.
  - Interest paid at the end of an interest compounding Period
- ***Interest payable in advance.***
  - Interest which is payable at the start of the period.
- Discount rate: d

$$d = \frac{\text{amount of interest for the period}}{\text{balance at the end of the period.}}$$

$$i = \frac{\text{amount of interest for the period}}{\text{balance at the start of the period.}}$$

$$\longrightarrow \quad \boxed{d = \frac{i}{1+i}}$$

Discount Rate

- Nominal discount rate

$$\boxed{1-d=\left(1-\frac{d^{(m)}}{m}\right)^{m}}$$

$$\boxed{d<d^{(2)}<d^{(3)}<\ldots<\delta}$$

Using a similar approach to the one above it can also be shown that

$$\lim_{m\to\infty} d^{(m)}=\delta\,.$$

- Question: Why nominal interest rate and nominal discount rate have the same limit?

Force of interest

- Definition

$$\lim_{m \to \infty} i^{(m)} = \delta$$

- Convert between effective interest and force of interest

$$\delta_t = \ln(1 + i)$$
$$i = e^{\delta_t} - 1$$

- Accumulated value:

$$\boxed{S(n) = S(0) \cdot \exp\left(\int_0^n \delta_t \, dt\right)}$$