RESEARCH SCHOOL OF FINANCE, ACTUARIAL STUDIES AND APPLIED STATISTICS

INTRODUCTORY MATHEMATICAL STATISTICS (STAT2001)
PRINCIPLES OF MATHEMATICAL STATISTICS (STAT6039)

Mid-Semester Examination – April 2014

Time allowed: *60 minutes*
Reading time: *5 minutes*
Permitted materials: *No restrictions*

Every student should attempt all four problems.
Each problem is worth five marks. The exam is out of 20 marks.
Show all working and present each final numerical answer correct to at least four significant digits. Draw a box around each final answer.

Problem 1 ***(5 marks)***

Consider a discrete random variable X which is equally likely to be any one of the k numbers $c + 1$, $c + 2$, ..., $c + k$, where c is a finite constant and k is a positive integer. Assume that it is not possible for X to take on any value apart from these k numbers. Find a simple closed-form formula for the standardised mean, $\lambda = \mu / \sigma$, where $\mu = E(X)$ and $\sigma^2 = Var(X)$. Then, by applying this formula, or otherwise, calculate λ for the case where $c = 5$ and $k = 7$, and also for the case where $c = 5$ and $k = 10^{50}$.

Problem 2 ***(5 marks)***

A box contains 20 balls, of which exactly 8 are red and 12 are green.

Two balls are randomly selected from the box, without replacement.

If any of these two balls are red, those red balls are put back in the box.

If any of these two balls are green, those green balls are thrown away.

Then, 5 balls are randomly selected from the box, without replacement.

Suppose that, following this procedure, all of these 5 balls are green.

Find the probability that there remain more than 13 balls in the box.

Problem 3 ***(5 marks)***

Consider two events A and B such that:

$$P(B) \geq 2P(A), \quad P(A) \geq 0.2, \quad P(B) \leq 0.8, \quad A \subseteq B.$$

Let X be the number of the two events A and B which occur.

(For example, $X = 0$ if $\bar{A} \cap \bar{B}$.)

Determine, as best you can, the values of $\mu = E(X)$ and $\mu_2' = E(X^2)$.

Problem 4 ***(5 marks)***

Ann and Bob are about to play a game. They will take turns rolling a standard six-sided die. The winner will be the first person to roll a 6 immediately after the other person has rolled a 1. Ann will roll first. Find the probability that Ann will win.