STAT3032 SURVIVAL MODELS

TUTORIAL WEEK THREE

$\boxed{l_{40} = l_{35} \cdot p_{35}\, p_{36} \cdot {}_3p_{37}}$

$= l_{35}(1-q_{35})(1-q_{36}) \cdot {}_3p_{37}$

${}_{2|3}q_{35} = p_{35} \cdot p_{36}(1-{}_3p_{37})$

solved

1. Given the following:

$$q_{35} = 0.05 \quad q_{36} = 0.06 \quad {}_{2|3}q_{35} = 0.19 \quad l_{40} = 55{,}444$$

$p_{35} \cdot p_{36}(1-{}_3p_{37})$

${}_2p_{35}(1-{}_3p_{37})$

find l_{35}.

2. Given that ${}_tp_x = \left(\frac{1+x}{1+x+t}\right)^3$ for t>0, calculate the complete life expectancy of a person aged 45.

$\mathring{e}_{45} = \int_0^\infty {}_tp_{45}\,dt = \int_0^\infty \left(\frac{46}{46+t}\right)^3 dt = 46^3 \int_0^\infty \left(\frac{1}{46+t}\right)^3 dt = 46^3 \left[\frac{-1}{-2(46+t)^2}\right]_0^\infty = 23$

3. A life aged 50 is subject to a constant force of mortality of 0.048790. Find the probability that a life aged 50 will die between ages 70 and 80.

${}_tp_x = \exp\left(-\int_0^t \mu_{x+r}\,dr\right) = \exp(-t \times 0.4\cdots)$

4. Using the following find the standard deviation of K_{100}. You may assume that no life survives beyond age 110.

${}_{20}p_{50}(1-{}_{10}p_{70})$

$= e^{-20\times 0.048790}(1-e^{-10\times 0.048790})$

Age x	d_x
100	188
101	133
102	94
103	65
104	45
105	31
106	21
107	14
108	9
109	6

$l_{100} = \sum = 606$

$Var = E(K_{100}^2) - [E(K_{100})]^2$

$E(K_{100}^2) = \sum_{i=1}^{9} \frac{d_{x+i}}{l_x} \cdot k^2 = 8.403$

$E(K_{100}) = \sum_{i=1}^{9} \frac{d_{x+i}}{l_x} \cdot k = 1.9818$

$\Rightarrow SD = \sqrt{8.403 - 1.9818^2}$

$SD(K_{100}) = 2.116$ years

5. The lifetime t (in weeks) of a light bulb can be defined by the force of mortality $0.25t^2+0.4t$.

A hotel has five hundred rooms, and has just opened with four new light bulbs in each room. Light bulbs are checked exactly once a week and all defective bulbs are replaced immediately.

(a) Calculate the number of bulbs that the hotel would expect to replace at the end of each of the first, second, third and fourth weeks of opening.

(b) Calculate the expected curtate future lifetime of a light bulb in weeks.

${}_tp_x = e^{-\int_0^t \mu_{x+s}\,ds}$

$p_{t-1} = \exp\left(-0.25\frac{t^3}{3} - 0.4\frac{t^2}{2} + 0.25\frac{(t-1)^3}{3} + 0.4\frac{(t-1)^2}{2}\right)$

(b). $\sum_t {}_tp_0 = \cdots$

6. Challenge Problem

A mortality table has been constructed for a certain population on the assumption that

$l_x = l_1(1 - 0.005x\log_{10} x) \qquad 1 < x \le 100$

$l_x = 0 \qquad x > 100$

(a) Calculate the complete expectation of life at age 1.

(b) Calculate the average age at death of those dying between ages 1 and 20.

$$\mathring{e}_1 = \int_0^\infty {}_tP_1\,dt$$

$${}_tP_1 = \frac{l_{1+t}}{l_1} = 1 - 0.005(1+t)\log_{10}(1+t)$$

$$\mathring{e}_1 = \int_0^{99} 1 - 0.005(1+t)\log_{10}(1+t)\,dt$$

$$= t\Big|_0^{99} - 0.005\left\{\frac{(1+t)^2}{2}\log_{10}(1+t)\Big|_0^{99} - \int_0^{99}\frac{1}{\ln 10}\frac{(1+t)^2}{2(1+t)}\,dt\right\}$$

$$= 99 - 0.005[5000\log_{10}100] + \frac{0.005}{2\ln 10}\left(t + \frac{t^2}{2}\right)\Big|_0^{99}$$

$$= 54.426$$

(b). $\int_0^{19} 1 - 0.005(1+t)\log_{10}(1+t)\,dt = 17.916$

$$\frac{17.916\,l_1 - 19\,l_{20}}{l_1 - l_{20}} + 1 = 11.67$$

10 $\quad l_1 = 10 \quad l_{20} = 1$

9 dies within 20 years