

Lecture 25

Let $x \in \Lambda$ and $\varepsilon > 0$, choose n s.t. $\frac{1}{2^n} < \varepsilon$

• We write $S(x) = (s_0 s_1 \dots)$

• Recall $I_{s_0 \dots s_n} = \{z \in \Lambda, z \in I_{s_0}, Q_c(z) \in I_{s_1}, Q_c^2(z) \in I_{s_2}, \dots, Q_c^n(z) \in I_{s_n}\}$

which are closed

Also $I_{s_0 s_1 \dots s_n}$ and $I_{t_0 \dots t_n}$ are disjoint
iff $(s_0, s_1, \dots, s_n) \neq (t_0, t_1, \dots, t_n)$

And then $|x - z| > \delta$ for some $\delta > 0$

$$\begin{aligned} x &\in I_{s_0 s_1 \dots s_n} \\ z &\in I_{t_0 t_1 \dots t_n} \end{aligned}$$

so if $y \in \Lambda$ and $|x - y| < \delta$, then $y \in I_{s_0 s_1 \dots s_n}$

Then $S(x)$ and $S(y)$ are the same for $(n+1)$ entries $(0, \dots, n)$

• By the proximity thm, $d[S(x), S(y)] < \frac{1}{2^n} < \varepsilon \Rightarrow S$ is continuous ■

④ Exercise

• Remarks: The sets Λ and Σ are homeomorphic

This means that if p and q are close in Λ , then $S(p)$ and $S(q)$ are close in Σ .

Def: Let $F: X \rightarrow X$ and $G: Y \rightarrow Y$ be 2 functions

F and G are called **conjugate** if there is a homeomorphism $h: X \rightarrow Y$
s.t. $h \circ F = G \circ h$

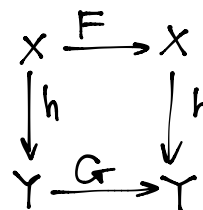
• The map h is called a **conjugacy**

• We have just proved that

* Thm (The Conjugacy Thm)

The shift map $\sigma: \Sigma \rightarrow \Sigma$ is conjugate of $Q_c: \Lambda \rightarrow \Lambda$ when $c < -2$

• This implies that $Q_c^n = S^{-1} \circ \sigma^n \circ S$
 $\sigma^n = S \circ Q_c^n \circ S^{-1}$



Remark:

① The dynamics of σ are identical to the dynamics of Q_c

② S converts orbits of Q_c into orbits under σ .

③ S is a periodic pt of σ iff $S^{-1}(S)$ is a periodic pt of Q_c with the same period.

CHAPTER 10 CHAOS

§10.1 Three properties of a chaotic system

① Density ② Transitivity ③ Sensitivity

Density: Let X be a set and $Y \subseteq X$

Assume that d is a distance in X

We say that Y is dense in X if

① For any open set $A \subseteq X$, $A \cap Y \neq \emptyset$

or ② equivalently, for any $x \in X$, $\exists y_n \in Y$ s.t. $\lim_{n \rightarrow \infty} d(x, y_n) = 0$

or ③ equivalently, $\bar{Y} = X$

Ex: 1). \mathbb{Q} is dense in \mathbb{R} .

$\sqrt{2} = 1.41421356237 \dots$

2) (a, b) is dense in $[a, b]$

3). \mathbb{Z} is not dense in \mathbb{R} or \mathbb{Q} .

$\frac{1}{2} \in \mathbb{R}$ and $(\frac{1}{4}, \frac{3}{4})$ is an open set

$\frac{1}{2} \in (\frac{1}{4}, \frac{3}{4}) \subseteq \mathbb{R}$

and $\mathbb{Z} \cap (\frac{1}{4}, \frac{3}{4}) = \emptyset$