Assignment 8

1. Suppose $\Omega_\Lambda >> \Omega_m$, $\Omega_{tot} = 1$; then $\Omega_\Lambda \sim 1$

$$\rightarrow \quad 1 = \frac{\Lambda c^2}{3H^2} \qquad H = \dot{a}/a$$

$$\frac{\dot{a}}{a} = \sqrt{\frac{\Lambda}{3}}\, c$$

$$\int \frac{1}{a}\, da = \int \sqrt{\frac{\Lambda}{3}}\, c \, dt$$

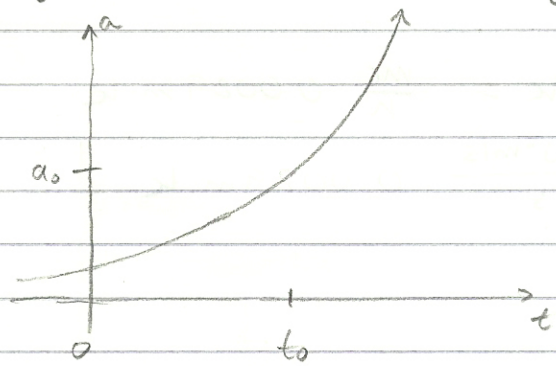
$$\ln(a) = \sqrt{\frac{\Lambda}{3}}\, ct + D$$

$$a = A\exp\left(\sqrt{\Lambda/3}\, ct\right)$$

Where A is an adjustable parameter; ie.

$$a = a_0 \exp\left(\sqrt{\Lambda/3}\, c\,(t - t_0)\right)$$

and you can set a_0 to anything

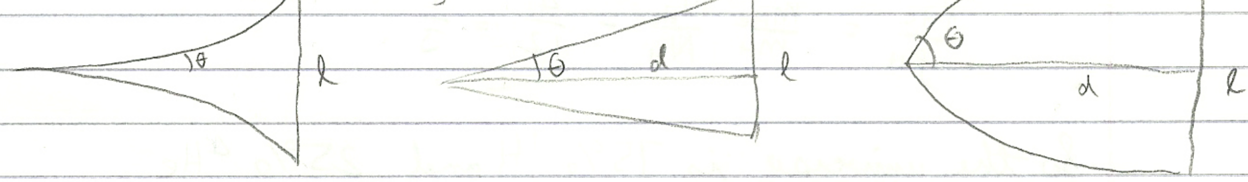


(Also note $a(t=0) \neq 0$! This is because $\Omega \sim 1$ is only true for late times ($t >> 0$))

2. We know the size of the CMB hotspots from careful measurements of the CMB power spectrum (to get Ω_m and $\Omega_{radiation}$) and a basic cosmological model also,

$$\theta = \frac{l}{d}$$

So we can get a theoretical θ_{theory}. We then go measure θ by observing the CMB.



OPEN, $\theta < \theta_{theory}$ FLAT $\theta = \theta_{theory}$ CLOSED $\theta > \theta_{theory}$

3. They would see a different CMB map, but with the same statistical properties.

4. $\cancel{2} E_\gamma = \cancel{2} m_e c^2$ $\quad E_\gamma = 9.1\cdot10^{-31}\,kg\cdot(3\cdot10^8\,m/s)^2$
$= \underline{8.19\cdot10^{-14}\,J}$

$E_\gamma = hc/\lambda$; $\lambda = \frac{0.0029\,m\cdot K}{T}$

$T = \frac{E_\gamma}{hc}\cdot 0.0029\,m\cdot K$

$T = (8.19\cdot10^{-14}\,J / 6.62\cdot10^{-34}\,J\cdot s \cdot 3\cdot10^8\,m/s)\cdot 0.0029\,m\cdot k$
$= \underline{\underline{1.2\cdot10^9\,K}}$

For protons,

$T = \frac{mc^2}{hc}\cdot 0.0029\,m\cdot k = \frac{mc}{h}\cdot 0.0029\,m\cdot k$

$= \frac{1.67\cdot10^{-27}\,kg\cdot 3\cdot10^8\,m/s}{6.62\cdot10^{-34}\,J\cdot s}\cdot 0.0029\,m\cdot k$

$= \underline{\underline{2.2\,10^{12}\,K}}$

5. 1 neutron per 7 protons $\Rightarrow$ 2 neutrons per 14 protons
^4He is $2p^+ + 2n$, so we have one ^4He for every $12\,p^+$

Very roughly, ^4He weighs 4 times ^1H, i.e. $\frac{m_{He}}{m_H} = 4$

$\frac{\rho_{He}}{\rho_H} = \frac{m_{He}}{m_H}\frac{N_{He}}{V}\cdot\frac{V}{N_H}$ (V = volume)

One ^4He for $12\,p^+$ means $N_{He}/N_H = \frac{1}{12}$;

$\frac{\rho_{He}}{\rho_H} = \frac{m_{He}}{m_H}\frac{N_{He}}{N_H} = \frac{4}{12} = \frac{1}{3}$

So the universe is 75% ^1H and 25% ^4He