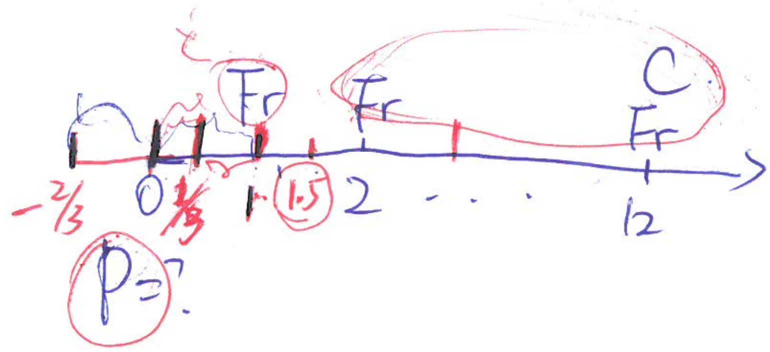
LN 8.2

Ex: $i = 10\%$

$F = 10{,}000 = C$

$n = 6 \times 2 = 12$

$r = \frac{13\%}{2} = 6.5\%$



$j = (1+i)^{\frac{1}{2}} - 1$

$$P = Fr \cdot a_{\overline{n}|j} + C \cdot v_j^n$$

$$= 10{,}000 \cdot 0.065 \cdot a_{\overline{12}|j} + 10{,}000 \cdot v_j^{12}$$

$$= \$\,11{,}445$$

$$P = \boxed{Fr \cdot 2 \cdot a^{(2)}_{\overline{6}|i} + C \cdot v_i^6}$$

Bond Prices Between Coupon Dates.

Ex: ① $P \cdot (1+j)^{\frac{1}{3}} \longrightarrow t = \frac{1}{3}$

② $P \cdot (1+j)^{-\frac{2}{3}} \longrightarrow t = -\frac{2}{3}$

③ $P' \overset{(t=1)}{=} Fr \cdot a_{\overline{n-1}|j} + C \cdot v_j^{n-1} \Rightarrow t = 1.5$

$P = \boxed{P'} \cdot (1+j)^{0.5}$ $(t = 1.5)$

① $P = Fr \cdot a_{\overline{n}|j} + C \cdot v_j^{\,n}$

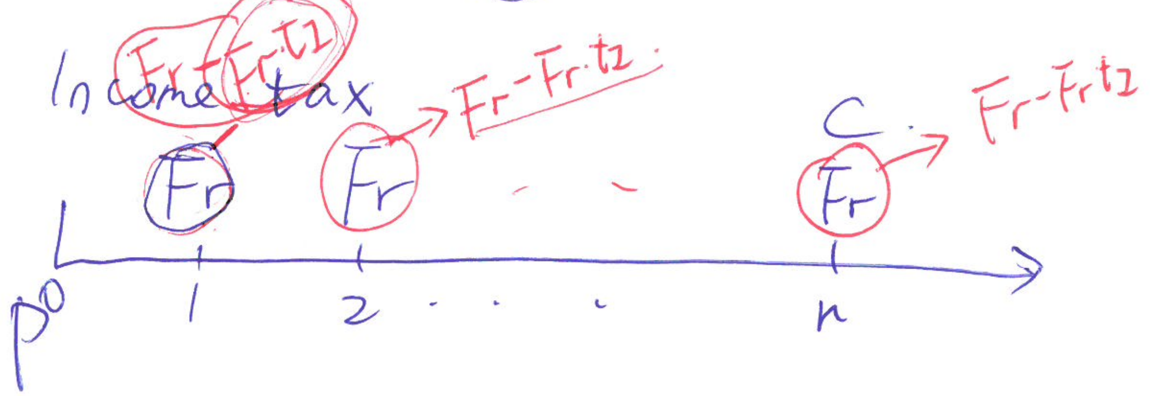
② $P \cdot (1+j)^{\frac{1}{3}(-\frac{2}{3})}$

③ tax $\to$ Income tax, t_1 $\to$ coupon

tax $\to$ capital gain tax, t_c $(P < C)$

P v.s C

1). Income tax



$$P = [Fr - Fr\,t_1] \cdot a_{\overline{n}|j} + C \cdot v_j^{\,n}$$

$$= Fr(1-t_2) \cdot a_{\overline{n}|j} + C \cdot v_j^{\,n}$$

Ex: $P = Fr(1-t_2) \cdot a_{\overline{n}|j} + C\,v_j^{\,n}$

$= 10{,}000 \cdot 0.065 \cdot (1 - 33\%) \cdot a_{\overline{12}|j} + 10{,}000 \cdot v_j^{\,12}$

$= \$9531$

2). Capital gain tax. t_c.

$\boxed{P \text{ vs. } C}$

Step 1: $P = Fr(1-t_2) \cdot a_{\overline{n}|j} + C v_j^n$

Step 2:

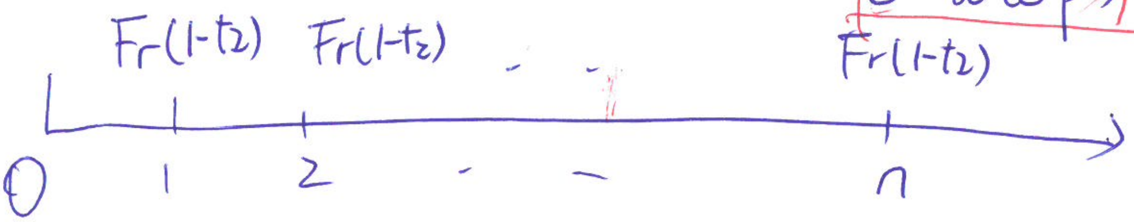
if $P \geq C \Rightarrow$ No capital gain $\Rightarrow P' = P =$

if $P < C \Rightarrow$ capital gain $\Downarrow$

$P' < P < C$

$$P' = \boxed{Fr \cdot (1-t_2) a_{\overline{n}|j} + C \cdot v_j^n} - t_c (C - P') v_j^n$$

$C - t_c (C - P')$



P'

$$\Rightarrow \boxed{P' = P - t_c (C - P') \cdot v_j^n}$$

Ex: $i = 10\%$,

$F = 10{,}000 = C$

$n = 6 \times 2 = 12$

$r = \frac{13\%}{2} = 6.5\%$

$t_I = 33\%$

$t_c = 33\%$

$j = (1.1)^{\frac{1}{2}} - 1$

Step1: $P = Fr(1-t_2)\, a_{\overline{n}|j} \cdot C \cdot v_j^n = \$9531$.

step2: P v.s $C = F = 10{,}000$.

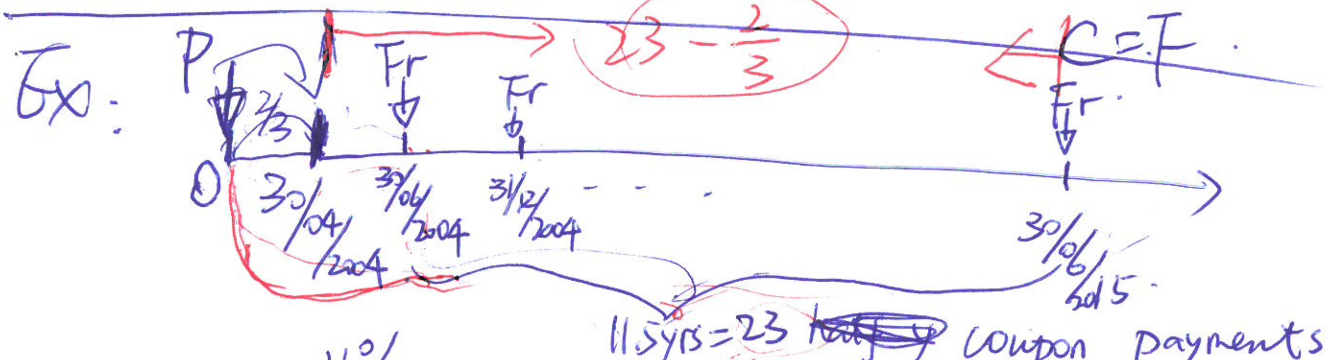
$P < C \Rightarrow$

$P' = P - t_c(C-P')\, v_j^n \leftarrow \boxed{\begin{array}{l} n = 12 \\ j = (1.1)^{\frac{1}{2}} - 1 \end{array}}$

$P' = 9531 - 33\% \cdot (10{,}000 - P') \cdot 0.56447$.

$\Rightarrow P' = 9423 < P$

Ex:



$r = \frac{11\%}{2} = 0.055$

$F = \$100{,}000 = C$.

$n = 23$.

$i^{(2)} = 9\%$ p.a.

$\Downarrow$

$j = \frac{9\%}{2} = 4.5\%$

$t_I = 30\%$

$t_C = 30\%$

$P_1(t = \overset{31/12/2003}{1/1/2004}) = Fr \cdot a_{\overline{n}|j} + C\, v_j^n$.

$P_2(t = 30/04/2004) = P_1 \cdot (1+j)^{2/3}$

$$P_1 = 100{,}000 \cdot 0.055 \cdot (1-0.3) \cdot a_{\overline{23}|j}^{\,0.045} + 100{,}000 \cdot v_{j}^{23} \quad j=0.045$$

$$= \$90{,}803.95.$$

$$P_2 = P_1 (1+0.045)^{2/3} = \boxed{\$93{,}508.03}.$$

$$\boxed{P} = P_2 - t_c (C - \boxed{P}) \cdot \boxed{v_j^{n}} \qquad j = 0.045, \quad n = 23 - \tfrac{2}{3}.$$

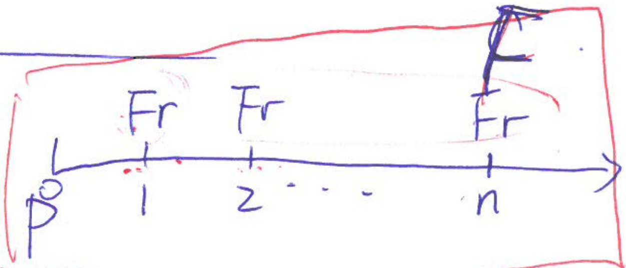
$$\Rightarrow P = 92{,}687.14.$$

Yields

- redemption yield.
- net yield (allow for tax).

(1). $C = F$.

① no income tax



$$\boxed{\begin{aligned} P = F = C &\Longleftrightarrow j = r \\ P > F = C &\Longleftrightarrow j < r \quad \text{capital loss} \\ P < F = C &\Longleftrightarrow j > r \quad \text{capital gain} \end{aligned}}$$

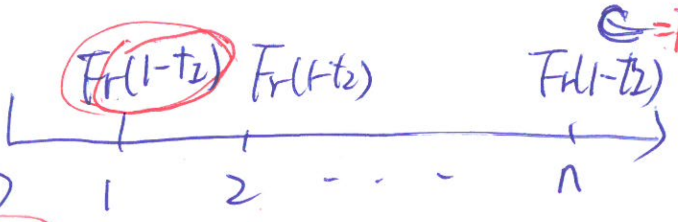
$$P = Fr \cdot a_{\overline{n}|j} + F v_n^j$$

$$\cancel{F} = \cancel{F}r \cdot \frac{1 - v_n^j}{j} + \cancel{F} v_n^j$$

$$\cancel{(1 - v_n^j)}\, j = r \cdot \cancel{(1 - v_n^j)}$$

$$\Rightarrow j = r$$

② income tax.



$P = F = C \iff j = r\cdot(1-t_2)$

$P > F = C \iff j < r(1-t_2)$

$P < F = C \iff j > r(1-t_2)$.

(2). $C \neq F$.

① no income tax

$Fr = Cg$

$\Downarrow$

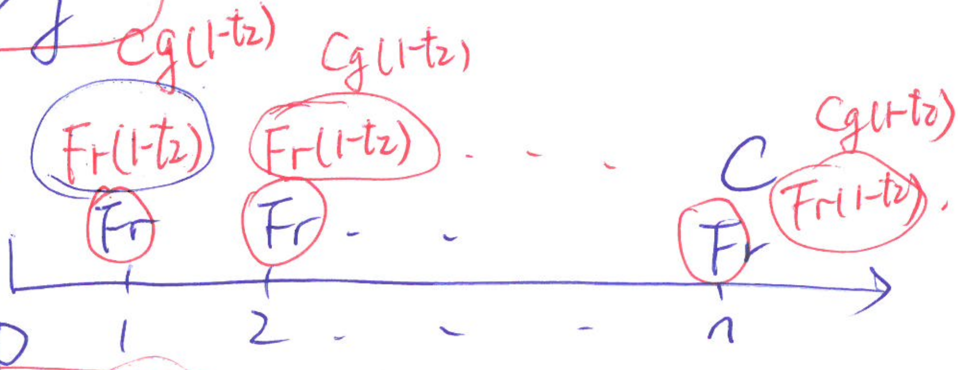
$\boxed{g = \frac{Fr}{C}}$

$P = C \iff j = g$

$P > C \iff j < g$

$P < C \iff j > g$

② income tax



$P = C \iff j = g(1-t_2)$

$P > C \iff j < g(1-t_2)$

$P < C \iff j > g(1-t_2)$.

Ex:

$P = \$12{,}000$

$F = \$10{,}000 = C$

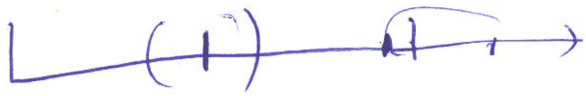
$n = 6 \times 2 = 12$

$r = \frac{13\%}{2} = 6.5\%$

$\Rightarrow i^{(2)} = j \times 2$

Sol: $P = Fr\, a_{\overline{n}|j} + C \cdot v_j^n$

$$\boxed{12{,}000 = 10{,}000 \cdot 6.5\% \cdot a_{\overline{12}|j} + 10{,}000 \cdot v_j^{12}}$$

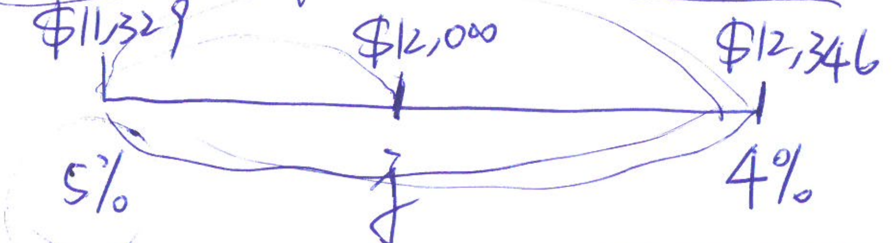


$\Rightarrow \because P > C \Rightarrow j < 6.5\%$

Trial & Error:

$j = 5\% \Rightarrow P = \$11{,}329 < 12{,}000$

$j = 4\% \Rightarrow P = \$12{,}346 > 12{,}000$



$$\Rightarrow j \simeq 5\% + \frac{12{,}000 - 11{,}329}{12{,}346 - 11{,}329} \times (4\% - 5\%)$$

$\Rightarrow j = 4.34\%$

$\Rightarrow i^{(2)} = 2 \times j = 8.68\%$.

The Effect of n on j

$\boxed{P = C} \Rightarrow j = r \Rightarrow$ n ~~don't~~ doesn't affect j.

$\boxed{P \neq C} \Rightarrow$ n has an effect on j.

① $P > C$. capital loss

$$\boxed{\begin{array}{l} n \uparrow \Rightarrow j \uparrow \\ n \downarrow \Rightarrow j \downarrow \end{array}}$$

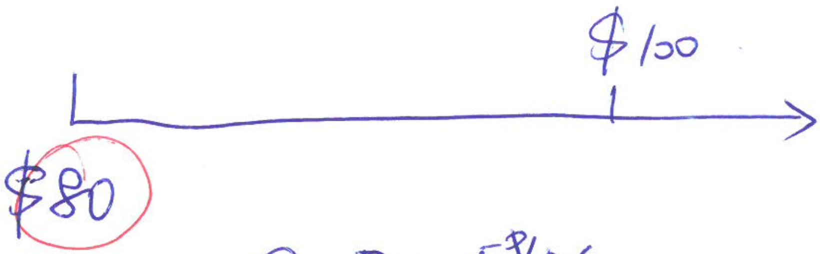
② $P < C$. capital gain.

$$\boxed{\begin{array}{l} n \uparrow \Rightarrow j \downarrow \\ n \downarrow \Rightarrow j \uparrow \end{array}}$$

① . $P > C$., capital loss., choose bond redeemed later.

② $P < C$, capital gain, choose bond redeemed earlier

Ex:



yield $\triangleq$ i $\Leftarrow$ $\begin{cases} ① \ n = 5 \text{ yrs} \\ ② \ n = 10 \text{ yrs.} \end{cases}$

①: $n = 5$, $\quad 80 = 100 \cdot (1+i)^{-5}$.

$\Rightarrow i = 4.6\%$ p.a.

② $n = 10$, $\quad 80 = 100 \cdot (1+i)^{-10}$.

$\Rightarrow i = 2.3\%$ p.a.

P $\quad$ C

$80 < 100$ $\Rightarrow$ capital gain.