

- A surface in three dimensional space (given in parametric form) is an embedding of a plane into a three dimensional space. Similarly a curve in space can be thought of as embedding of a line into a three dimensional space.
- The treatment of the surfaces in three dimensional space is similar to the treatment of the curves in plane. Here we are searching for a local representation of the form $z = f(x, y)$ or other permutations of the variables, as in item (i) of the definition of the smooth surface in page 126. This similarity can be generalized: if we are to embed a k dimensional space into a $k + 1$ dimensional space then we can use the same reasoning and the same condition.
- However the treatment of curves in space consists of two cases: parametric (version (iii)) and non-parametric (version (ii)). The non parametric version requires the system's version of the IFT. While a surface in space can be thought of as the locus of the relation $F(x, y, z) = 0$, a curve is to be considered as the intersection of two surfaces, $F(x, y, z) = 0$ and $G(x, y, z) = 0$. Of course intersection of two surfaces is obtained by solving the system $F(x, y, z) = 0 = G(x, y, z)$. Such a system is solved for two variables and the solution is written in terms of the third variable. Hence representation (i) is obtained. Of course it is the nature of the IFT that this solution is obtained only locally. The condition given in the first line of the second paragraph of page 130 is exactly the condition of theorem 3.9 (that is $\det(B) \neq 0$).
- See how this regularity condition is extended to higher dimensions when we are embedding a k dimensional space into an n dimensional space: in page 131 in the middle of the page the condition is that $n - k$ gradient vectors (each of which n dimensional vectors) to be linearly independent. This means that some $(n - k) \times (n - k)$ submatrix B will be discovered with $\det(B) \neq 0$. Now this is the condition of theorem 3.9.
- However the proof of the case (iii) \implies (i) as presented in the middle of page 130 requires only theorem 3.1. Indeed it is the same situation when we are embedding a curve in any higher dimensional space. But to generalize this to higher dimensions when we need to embed a k dimensional space into an n dimensional space ($n > k + 1$) we need the system's version of the IFT (as discussed at the bottom of page 131) as there are k different variables in the vector \mathbf{u} to be solved in terms of k of the other variables. This requirement is translated to the matrix $D\mathbf{f}(\mathbf{u})$ having rank k .