Exam: 3 hours 6 problems

REVIEW

Direction Fields (slope fields)

E.g. $\frac{dy}{dt} = t^2 + 4y^2 - 1 = f(t,y)$

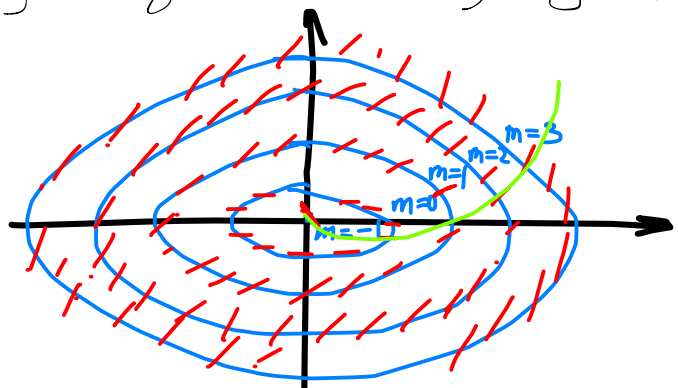
To draw direction fields, first find isoclines $f(t,y) = m$

$t^2 + 4y^2 - 1 = m$

$t^2 + 4y = m + 1$

ellipses

E.g. $m = 0$ $t^2 + 4y = 1$



E.g. $\frac{dt}{dy} = \frac{t-y}{2t+y} = f(t,y)$

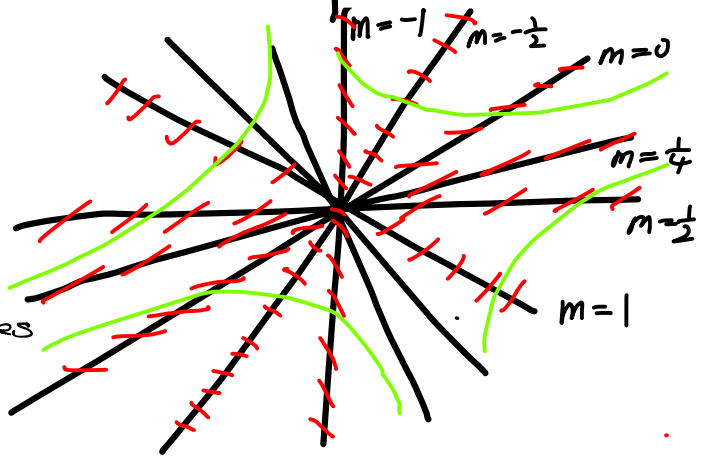
Isoclines $\frac{t-y}{2t+y} = m$

$(t-y) = m(2t+y)$

$0 = (2m-1)t + (m+1)y$

$y = \frac{1-2m}{m+1}t$ + b

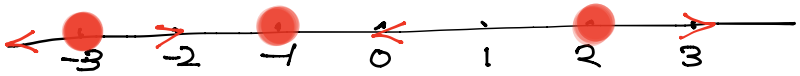
isoclines are straight lines



E.g. Find critical points of $y' = (y+1)(y-2)(y+3)$ and determine their stability

critical points $y = -1, y = 2, y = -3$

Phase line



To find direction, just look where $f>0$ or $f<0$

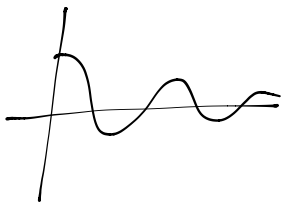
$\Rightarrow$ -1 stable, $2, 3$ unstable

Alternatively, look at $f'(t)$ at the critical point

$mx''+\gamma x'+kx=F(t)$

$x(t)=e^{rt}$

Char. equation: $mr^2+\gamma r+k=0$



E.g.

$(x^2+1)\frac{dy}{dx}+4xy=x$, $y(2)=1$

$(x^2+1)\frac{dy}{dx}=x(1-4y)$

$\frac{dy}{dx}=\frac{x}{x^2+1}(1-4y)$

$\frac{dy}{dx}+\frac{4x}{1+x^2}y=x$

$\frac{dy}{dx}+p(x)y=g(x)$

linear

E.g. $\frac{dy}{dx}=\frac{y-x}{y+x}$ homogeneous

$\frac{dy}{dx}=\frac{\frac{y}{x}-1}{\frac{y}{x}+1}$

for homogeneous eqn

$\frac{dy}{dt}=F(\frac{y}{t})$, substitute $v=\frac{y}{t}$

$y=vt$ $\frac{dy}{dt}=t\frac{dv}{dt}+v$

$\Rightarrow t\frac{dv}{dt}=-v+F(v)$ separable