The length of time for a particular chemical reaction to take place, denoted t, is to be modeled using a parametric approach. The pdf of the parametric distribution that is to be used for modeling reaction time is:

$$\frac{\alpha}{t^{\alpha+1}}\beta^{\alpha}, \ t \geq \beta$$

$L(\alpha) = \prod_{i=1}^{n} \frac{\alpha \cdot 3^{\alpha}}{t_i^{\alpha+1}}$

$l(\alpha) = \sum_{i=1}^{n} \log \alpha + \sum_{i=1}^{n} \alpha \log 3 - \sum_{i=1}^{n} (\alpha+1) \log t_i$

Eleven reactions were observed to take the following times 4, 5, 5, 6, 7.5, 8, 9, 11, 12, 14, 14. Further, it is known that $\beta = 3$.

$= n \log \alpha + n\alpha \log 3 - (\alpha+1)\sum_{i=1}^{n} \log t_i$

$l'(\alpha) = \frac{n}{\alpha} + n \log 3 - \sum_{i=1}^{n} \log t_i = 0$

a) Compute the maximum likelihood estimate of α.

$\frac{n}{\alpha} = \sum_{i=1}^{n} \log t_i - n \log 3$

$\hat{\alpha} = \frac{n}{\sum_{i=1}^{n} \log t_i - n \log 3}$ $(n = 11, t_i = , \ldots)$

b) Provide an estimate of the variance of your maximum likelihood estimate $\hat{\alpha}$.

$l''(\alpha) = -\frac{n}{\alpha^2} < 0$ $\quad E(-l''(\alpha)) = \frac{n}{\alpha^2} = I(\alpha)$ $\quad Var(\hat{\alpha}) = \frac{1}{I(\hat{\alpha})} = \frac{\hat{\alpha}^2}{11} \doteq 0.095$ $\quad = 1.023$

c) Based on the observed data, use a non-parametric approach to estimate the survival function at time 10. Provide a standard error for your estimate.

not a very confident variance as n=11, CLT not working well here.

$\hat{S}(10) = 1 - \hat{F}(10)$ $\quad d_{10}$: # of death before time 10

$\hat{F}(10) = \frac{d_{10}}{N} = \frac{7}{11}$

$\hat{S}(10) = \frac{4}{11}$

$SD(\hat{S}(t)) = \sqrt{Var(\hat{S}(t))} = \sqrt{Var(\hat{F}(t))}$

$= \sqrt{\frac{\hat{F}(t)(1-\hat{F}(t))}{N}}$

$= \sqrt{\frac{\frac{4}{11} \cdot \frac{7}{11}}{11}}$

$= 0.145$