MAT 135 Final Exam Package

We have carefully looked through every test from 2002 to 2010 past Test 2 in the former MAT 135 and have broken the type of problems on the exam to be the following. This package is aimed at to do the following.

i. Focus only on the problems that occur in the past tests so that you won't be wasting your time studying the wrong type of questions.

ii. Every problem in the package is either a question from the past exam or a very similar problem just like the one in the exam.

iii. Problems are categorized in difficulty. If you are just starting, go through the"Easy Ones" first for warm up. If you have been on top of your homework and want to make sure you get your A, jump in to the "Challenging Ones". The great thing is that you don't have to be stuck and give up on any of the problems. Every problem is explained in full detail at www.PrepAnywhere.com !

[I] **Limit Problems Limit Problems:** Limit at a Point, Limit at Infinity

[II] **L'Hospital's Rule & Indeterminate Forms** This topic is very popular for the exam. It will be on it 100%. There will be usually one straight forward one and one challenging one that involves Indeterminate Form.

[III] **Continuity:** Usually asking to find an unknown value which will make the given function(usually piece-wise) to be continuous everywhere.

[IV] **Derivative Problems** Finding Derivative or Finding Derivative and evaluating at a point tangent line which satisfies certain conditions

[V] **Tangent or Perpendicular Line to Function** Finding equation of the tangent line at a point on the line. The difficult ones ask you to find the point(s) for a tangent line which satisfies certain conditions. A lot of tough problems come from here. Make sure you know how to do the hard ones from this category.

[VI] **Exponential Growth and Decay** Usually basic and straight forward. Understanding how to do a few of these should be ok. From the past 8 years of exam we looked at, only two of them had this type of question.

[VII] **Implicit Differentiation** Straight forward problems

[VIII] **Higher Order Derivative Questions** Tough Questions. Requires multiple derivatives and pattern finding.

[IX] **Related Rates** Word Problems of Implicit Differentiation.

[X] **Mean Value Theorem** Always the same type of problem that asks to find the **c** which satisfies MVT.

[XI] **First and Second Derivative Analysis Problems** The MOST popular category for the exam. Usually 4 to 5 of this problems occurs in the exam, ranging from simple to difficult problem.

[XII] **Graphing Related** Usually asks about certain characteristics of the graphs. Slant Asymptotes is a popular question.

[XIII] **Optimization** Optimization word problems. Usually very straight forward. There was only one tough Optimization problem that was quite challenging in the 9 past exams we looked at.

[I] Limit Problems, Limit at a Point, Limit at Infinity

- Simple Ones

Question 1. Find the value of $\lim_{x\to 0^-}\left(\frac{\sin 2x}{3x}+\frac{3x}{\sin 2x}-\frac{\sin 3x}{\sin 4x}\right)$

(A) $\frac{17}{12}$
(B) $+\infty$
(C) $-\infty$
(D) 0
(E) $-\frac{13}{12}$

Question 2. Find the value of $\lim_{x\to\infty}\frac{x^4-x+\sin x}{-x^3-3x^4-2\cos x}$

(A) 1
(B) $-\frac{1}{3}$
(C) $-\frac{1}{2}$
(D) -1
(E) $-\infty$

Question 3. Find the value of $\lim_{x\to 0}\frac{\sin 2x}{3x+4x^2+5x^3}$

(A) $\frac{2}{3}$
(B) undefined
(C) $\frac{2}{5}$
(D) $\frac{1}{2}$
(E) 0

Question 4. Find the value of $\lim_{x\to\infty}\frac{2x^4-x^2+\sqrt{x}}{4+x-3x^4}$

(A) $\frac{1}{4}$
(B) $\frac{1}{2}$
(C) $-\frac{2}{3}$
(D) undefined
(E) $-\frac{1}{3}$

Question 5. Find the value of $\lim_{x\to 0} \dfrac{\sin(3x)}{5x^3 - 4x}$

- **(A)** $+\infty$
- **(B)** $-\frac{3}{4}$
- **(C)** 0
- **(D)** $-\infty$
- **(E)** $\frac{3}{5}$

Question 6. Find the value of $\lim_{x\to \infty} \dfrac{2 + x - x^4}{3x^4 - x + 4}$

- **(A)** $\frac{2}{3}$
- **(B)** $\frac{1}{2}$
- **(C)** $-\frac{1}{3}$
- **(D)** $-\infty$
- **(E)** $-\frac{1}{4}$

- Challenging Ones

Question 7. Given that the tangent line to the graph of f at (0, 0) has equation $2y = x$ and that f has a horizontal asymptote at ∞ with equation $y = 2$, find the value of $\lim_{x\to 0^+}\{\frac{\sin(2x)}{f(x)} - x^2 f(\frac{1}{x})\}$.

(A) $\frac{1}{2}$
(B) 4
(C) 2
(D) 0
(E) not determinable due to insufficient information

Question 8. Find the value of $\lim_{x\to\frac{\pi}{4}} \frac{\tan 2x}{\cot(\frac{\pi}{4} - x)}$.

(A) 1
(B) $\frac{1}{4}$
(C) $\frac{1}{2}$
(D) 2
(E) undefined

Question 9. [*] Find the value of $\lim_{x\to\infty}[x - x^2\ln(\frac{1+x}{x})]$.

(A) $\frac{1}{2}$
(B) undefined
(C) $\frac{1}{\sqrt{2}}$
(D) $\frac{2}{e}$
(E) $\frac{1}{e}$

Question 10. Find the value of $\lim_{x\to 0}(\frac{1}{x^2} - \cot^2 x)$

(A) $\frac{3}{2}$
(B) $\frac{1}{2}$
(C) $\frac{2}{3}$
(D) $-\frac{2}{3}$
(E) $-\frac{3}{2}$

[II] L'Hospital's Rule & Indeterminate Forms

- Simple Ones

Question 11. Find the value of $\lim_{x\to 0} \dfrac{e^x + e^{-x} - 2}{1 - \cos(4x)}$

(A) 0
(B) undefined
(C) $\frac{1}{4}$
(D) $\frac{1}{8}$
(E) $-\frac{1}{4}$

Question 12. Find the value of $\lim_{x\to 0} \dfrac{1 + x - e^x}{x^2}$

(A) $\frac{1}{2}$
(B) 0
(C) $\frac{1}{e}$
(D) $-\frac{1}{2}$
(E) undefined

- Challenging Ones

Question 13. If $f(x) = x^{2x}$, then $f'(e) =$

(A) e^{2e}
(B) $2e^{2e}$
(C) $4e^2$
(D) e^2
(E) $4e^{2e}$

Question 14. If $f(x) = x^x$, then $f'(3) =$

(A) 27
(B) $27(1 + \ln 3)$
(C) $27(3 + \ln 3)$
(D) $27(3 - \ln 3)$
(E) $27 \ln 3$

Question 15. Let $f(x) = x^{x^x}$, i.e., x to the power of x^x. Find $f'(2)$.

(A) $32 + 64\ln 2 + 64(\ln 2)^2$
(B) $32 + 32\ln 2 + 96(\ln 2)^2$
(C) $32 + 32\ln 2 + 128(\ln 2)^2$
(D) $32 + 32\ln 2 + 64(\ln 2)^2$
(E) $32 + 64\ln 2 + 128(\ln 2)^2$

Question 16. Find the value of $\lim_{x\to\infty}(1 + 2^x + 3^x)^{\frac{2}{x}}$

(A) undefined
(B) 6
(C) 9
(D) e^2
(E) $2\ln 3$

Question 17. Let $f(x) = (x^x)^{x^x}$, i.e. x^x to the power of x^x. Find the value of $f'(2)$.

(A) $2^{10}(1 + \ln 2)(1 + 2\ln 2)$
(B) $2^8(1 + \ln 2)(1 + 2\ln 2)$
(C) $2^8(1 + \ln 2)(1 + 4\ln 2)$
(D) $2^8(1 + \ln 2)^2$
(E) $2^{10}(1 + \ln 2)^2$

Question 18. Find the value of $\lim_{x\to\infty}(\frac{\pi}{2} - \arctan x)^{\frac{1}{\ln x}}$

(A) 0
(B) $\frac{1}{e}$
(C) $e - 2$
(D) $\sqrt{e}$
(E) $\frac{e}{2}$

Question 19. Given that $\lim_{x\to 1}\dfrac{ax^4 + bx^3 + 1}{(x-1)\sin(\pi x)}$ (where a and b are constants) exists and is a finite number, find its value

(A) $\frac{3}{\pi}$
(B) $-\frac{6}{\pi}$
(C) $\frac{2}{\pi}$
(D) $-\frac{4}{\pi}$
(E) not determinable due to insufficient information about a and b.

Question 20. If $f(x) = (\tan x)^{\cos x}$, then $f'(\frac{\pi}{4}) =$

(A) $\sqrt{2}$
(B) $\frac{1}{\sqrt{2}}$
(C) $-\sqrt{2}$
(D) $2\sqrt{2}$
(E) undefined

Question 21. Find the value of $\lim_{x\to 0} \dfrac{\sin x - \arctan x}{x^2 \ln(1 + 2x + x^2)}$

(A) $\frac{1}{2}$
(B) $\frac{1}{12}$
(C) $\frac{1}{4}$
(D) $\frac{1}{6}$
(E) $\frac{1}{3}$

[III] **Continuity**: There were only one type of continuity type of question: finding the missing value so that the function is continuous.

Question 22. Let $f(x) = \begin{cases} cx^2 - 4, & \text{if } x < 2 \\ x^3 - c, & \text{if } x \geq 2 \end{cases}$

Find the value of the constant c so that f is continuous everywhere.

(A) $\frac{3}{4}$

(B) $\frac{1}{2}$

(C) $\frac{10}{3}$

(D) $\frac{12}{5}$

(E) $\frac{4}{13}$

Question 23. Let $f(x) = \begin{cases} 4(x-2) - \frac{\sin(8x)}{kx}, & \text{if } x < 0 \\ 2(x+k), & \text{if } x \geq 0 \end{cases}$.

Find the value of the constant k so that f is continuous everywhere.

(A) -2

(B) $-\frac{1}{4}$

(C) 0

(D) $\frac{1}{2}$

(E) $\frac{1}{4}$

Question 24. Let $f(x) = \begin{cases} \frac{\sin(kx)}{3x}, & \text{if } x < 0 \\ (k+x+2)(x-2), & \text{if } x \geq 0 \end{cases}$

Find the value of the constant k so that f is continuous everywhere.

(A) 0

(B) 2

(C) -1

(D) $\frac{5}{2}$

(E) $\frac{1}{2}$

Question 25. Use the Intermediate Value Theorem to show that there is a root of the given equation in the specified interval. $e^x = 3 - 2x$, $(0, 1)$

Question 26. [*] A tibetan monk leaves the monastery at 7:00 AM and takes his usual path to the top of the mountain, arriving at 7:00 PM. The following morning, he starts at 7:00 AM at the top and takes the same path back, arriving at the monastery at 7:00 PM. Use the Intermediate Value Theorem to show that there is a point on the path that the monk will cross at exactly the same time of the day on both days.

[IV] **Derivative Problems**: Finding Derivative or Finding Derivative and evaluating at a point

- Simple Ones

Question 27. If $f(3) = -3$, $f'(3) = 5$ and $g(x) = (2x^2 - 8)f(x)$, then $g'(3) =$

(A) 24
(B) 14
(C) -12
(D) -8
(E) 20

Question 28. If $f(2) = \frac{1}{2}$, $f'(2) = \frac{1}{5}$ and $g(x) = \frac{f(x)}{1+x^2}$, then $g'(2) =$

(A) $\frac{1}{2}$
(B) $-\frac{1}{2}$
(C) $-\frac{1}{25}$
(D) $-\frac{1}{5}$
(E) $\frac{1}{5}$

Question 29. If $f(x) = x \arctan x$, then $f''(2) =$

(A) $\frac{1}{5}$
(B) $\frac{3}{5}$
(C) $\frac{\pi}{4}$
(D) $\frac{2}{25}$
(E) $\frac{8}{25}$

Question 30. If $f(x) = x^2 \arctan x$, then $f'(1) =$

(A) $\frac{\pi}{4}$
(B) $\frac{1}{4}(2+\pi)$
(C) $\frac{1}{4}(1+\pi)$
(D) $\frac{1}{2}(2+\pi)$
(E) $\frac{1}{2}(1+\pi)$

Question 31. If $f(x) = \arctan x^2$, then $f'(2) =$

(A) $\frac{4}{15}$
(B) $\frac{2}{15}$
(C) undefined
(D) $\frac{4}{17}$
(E) $\frac{2}{17}$

Question 32. If $f(x) = \arctan(\dfrac{x}{2})$, then $f''(2) =$

(A) -1
(B) 4
(C) 0
(D) $-\frac{1}{8}$
(E) $\frac{\pi}{4}$

Question 33. If $f(x) = \ln(3 + 3e^{2x})$, then $f'(\ln 3) =$

(A) $\frac{1}{3}\ln 3$
(B) $\frac{9}{5}$
(C) $\frac{1}{2}\ln 3$
(D) undefined
(E) $\frac{3}{4}$

Question 34. If $f(x) = \dfrac{\ln x}{x}$, then $f'(\dfrac{1}{e}) =$

(A) $2e^2$
(B) e^{-2}
(C) e^{-1}
(D) $-e$
(E) $-e^2$

- Challenging Ones

Question 35. If $f(x) = \log_x 2$ (i.e. logarithm of 2 with base x), find the value of $\frac{d^2y}{dx^2}$ at$x = 2$.

(A) $\dfrac{2 + \ln 2}{4(\ln 2)^2}$
(B) $\dfrac{4 + \ln 2}{2(\ln 2)^2}$
(C) $\dfrac{2 + \ln 2}{2(\ln 2)^2}$
(D) undefined
(E) $\dfrac{4 + \ln 2}{4(\ln 2)^2}$

Question 36. Let $f(x) = 2^{x+1} - 2^{-x}$. If $g(x) = f^{-1}(x)$, i.e. if g is the inverse function of f, what is the value of $g'(1)$?

(A) $\frac{3}{2\ln 2}$
(B) $\frac{2}{3\ln 2}$
(C) $\frac{1}{2\ln 2}$
(D) $\frac{1}{3\ln 2}$
(E) $\frac{2}{9\ln 2}$

Question 37. [*] If $f(x) = \dfrac{1 + x + x^2 + x^3 + ... + x^{47}}{1 + x + x^2 + x^3 + ... + x^{98}}$, then $f'(-1) =$

(A) 25
(B) 23
(C) 28
(D) 26
(E) 24

Question 38. [*] Let $f(x) = 3^{-x} + 9^{-x}$, for all x. Let $g(x) = f^{-1}(x)$, i.e., let g be the inverse function of f. Then $g'(2) =$

(A) $\frac{-1}{3\ln 3}$
(B) $\frac{-11\ln 3}{81}$
(C) $\frac{-1}{9\ln 3}$
(D) $\frac{-3}{\ln 3}$
(E) $\frac{-\ln 3}{3}$

Question 39. Let f and g be differentiable functions such that $f(1) = 2, f'(1) = \frac{1}{2}, g(2) = 3$ and $g'(2) = \frac{1}{3}$. Let $h(x) = f(g(f(x)))$. If $h'(1) = 5$, find the value of $f'(3)$.

(A) 24
(B) 20
(C) 30
(D) 12
(E) 18

Question 40. Suppose f is a function that satisfies the equation $f(x+y) = f(x)+f(y)+x^2y+xy^2$ for all real numbers x and y. Suppose also that

$$\lim_{x\to 0} \frac{f(x)}{x} = 1$$

(a) Find $f(0)$.
(b) Find $f'(0)$.
(c) Find $f'(x)$.

[V] **Tangent or Perpendicular Line to Function Problems**: Finding equation of the tangent line at a point on the line. The difficult ones ask you to find the point(s) for a tangent line which satisfies certain conditions

- Simple Ones

Question 41. The tangent line to the curve $y = x^3 - x^2 + 4$ at the point (2,8) will intersect the $y - axis$ at the point.

(A) $(0, -6)$
(B) $(0, -12)$
(C) $(0, -8)$
(D) $(0, -10)$
(E) $(0, -14)$

Question 42. The line **perpendicular** to the curve $y = x^3 - 2x + 1$ at the point (2, 5) will intersect the x-axis at the point

(A) $(-24, 0)$
(B) $(52, 0)$
(C) $(0, 0)$
(D) $(32, 0)$
(E) $(24, 0)$

Question 43. The line tangent to the curve $x^2 + 3y^2 = 1$ at the point $(\frac{1}{2}, \frac{1}{2})$ will intersect the y-axis at the point

(A) $(0, \frac{1}{3})$
(B) $(0, \frac{1}{2})$
(C) $(0, \frac{4}{3})$
(D) $(0, 1)$
(E) $(0, \frac{2}{3})$

- Challenging Ones

Question 44. If the two curves $y = 4\ln x$ and $y = cx^2$ (where c is a positive constant) have exactly one point in common, what must be the value of c?

(A) $\frac{1}{e}$
(B) $\frac{4}{e}$
(C) $\frac{1}{4e}$
(D) $\frac{1}{2e}$
(E) $\frac{2}{e}$

Question 45. There is one line which is tangent to the curve $y = \frac{1}{x}$ at some point A and at the same time tangent to the curve $y = x^2$ at some point B. What is the distance between A and B?

(A) $\sqrt{39}$
(B) $\frac{1}{4}\sqrt{610}$
(C) $\frac{1}{2}\sqrt{155}$
(D) $\frac{1}{2}\sqrt{153}$
(E) $\sqrt{41}$

Question 46. There are exactly two points on the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 9$ where the tangent line has slope $\frac{\sqrt{5}}{2}$. Find the slope of the line joining those two points.

(A) $-\frac{4\sqrt{5}}{5}$
(B) $-\frac{2\sqrt{5}}{3}$
(C) $-\frac{\sqrt{5}}{2}$
(D) $-\frac{5\sqrt{5}}{8}$
(E) $-\frac{3\sqrt{5}}{8}$

Question 47. There is one straight line which is tangent to the curve $4y = x^2$ at a point A and at the same time tangent to the curve $x = 2y^2$ at point B. Find the distance between the points A and B.

(A) $\frac{3\sqrt{5}}{4}$
(B) $\frac{5\sqrt{2}}{3}$
(C) $\frac{5\sqrt{3}}{3}$
(D) $\frac{3\sqrt{6}}{2}$
(E) $\frac{8\sqrt{2}}{3}$

Question 48. [*] (Similar to 45) A straight line passes through the point (1,27) and intersects the positive $x - axis$ at the point A and the positive $y - axis$ at the point B. Find the shortest possible distance between A and B.

(A) $18\sqrt{3}$
(B) $14\sqrt{5}$
(C) $22\sqrt{2}$
(D) $13\sqrt{6}$
(E) $10\sqrt{10}$

Question 49. [*] Let $f(x) = 5^x, g(x) = e^{4x} - 4e^{2x} - 12x + 3 \quad \text{and} \quad h(x) = f(g(x))$.
Then the graph of h must have a horizontal tangent line at $x =$

(A) $\frac{1}{2}\ln 3$.
(B) $\frac{1}{3}\ln 2$.
(C) $\frac{1}{3}\ln 5$.
(D) $\frac{1}{3}\ln 3$.
(E) $\frac{1}{2}\ln 2$.

[VI] **Exponential Growth and Decay** This isn't a very popular question for the exam. It only came up once in the 8 years of exam we have looked at. Try the question below and if you need more practice, try the simple questions in your text book section 3.8.

- Simple Ones

Question 50. At noon, a bacteria culture has 200 bacteria. At 1 p.m., the bacteria population has grown to 800. Assuming exponential growth, when will the bacteria population be 1800?

(A) at 1:30 p.m

(B) at $\frac{\ln 3}{\ln 2}$ hours past noon

(C) at $\frac{\ln 5}{\ln 3}$ hours past noon

(D) at $\ln(\frac{5}{2})$ hours past noon

(E) at $\frac{\ln 5}{\ln 2}$ hours past noon

Question 51. At noon, a bacteria culture has 2000 bacteria. At 1 p.m., the bacteria population has grown to 2200. Assuming exponential growth, what will be the population at 3 p.m.?

(A) 2644

(B) 2640

(C) 2600

(D) 2680

(E) 2662

[VII] **Implicit Differentiation**:

- Simple Ones

Question 52. If $y^3 + xy - 3y^2 - 2x - 2 = 0$, find the value of $\dfrac{dy}{dx}$ at the point where $x = 2, y = 3$.

(A) $-\frac{1}{5}$
(B) $\frac{2}{11}$
(C) $\frac{5}{9}$
(D) $-\frac{1}{9}$
(E) $-\frac{1}{11}$

Question 53. If $xy^3 + xy = 6$, find the value of $\frac{dy}{dx}$ at the point where $x = 3, y = 1$.

(A) $\frac{1}{2}$
(B) 0
(C) $\frac{1}{3}$
(D) $-\frac{1}{6}$
(E) $-\frac{1}{4}$

Question 54. The line tangent to the curve $x^2 + 3y^2 = 1$ at the point $(\frac{1}{2}, \frac{1}{2})$ will intersect the y-axis at the point

(A) $(0, \frac{1}{3})$
(B) $(0, \frac{1}{2})$
(C) $(0, \frac{4}{3})$
(D) $(0, 1)$
(E) $(0, \frac{2}{3})$

- Challenging Ones

Question 55. If $xy = \cot(xy)$, then $\dfrac{d^2y}{dx^2}$

(A) $\frac{2x}{y^2}$
(B) $\frac{y}{x^2}$
(C) $\frac{x}{y^2}$
(D) $\frac{2y}{x^2}$
(E) $\frac{-y}{x^2}$

Question 56. If k is a fixed constant such that the two families of curves $y = ax^5$ and $x^2 + ky^2 = b$ are orthogonal trajectories of each other, what must be the value of k?

(A) $\sqrt{5}$
(B) 5
(C) $\frac{1}{5}$
(D) $\frac{1}{2}$
(E) $5\sqrt{5}$

[VIII] **Higher Order Derivative Questions**: These questions are challenging algebraically. Here are some that appeared in the past Exams.

- Challenging Ones

Question 57. Let $f^{(n)}(a)$ denote the n-th derivative of f at a.
If $f(x) = \ln(x^4 - 2x^2 + 1) - \ln(x^2 - 2x + 1)$, then $f^{49}(3) =$

(A) $(48!)2^{-96}$
(B) $(48!)2^{-100}$
(C) $(48!)2^{-99}$
(D) $(48!)2^{-98}$
(E) $(48!)2^{-97}$

Question 58. Let $f^{(n)}(a)$ denote the n-th derivative of f at a. If $f(x) = \sin^4 x + \sin^2 x \cos^2 x$, then $f^{(75)}(\frac{\pi}{12})$

(A) -2^{76}
(B) -2^{73}
(C) -2^{75}
(D) -2^{77}
(E) -2^{74}

Question 59. Let $f^{(n)}(a)$ denote the n-th derivative of f at a. If $f(x) = \sin x \cos 2x + \cos x \sin 2x$, then $f^{(52)}(\frac{\pi}{18})$

(A) 2^{53}
(B) 3^{52}
(C) 3^{53}
(D) $\frac{1}{2}(3^{53})$
(E) $\frac{1}{2}(3^{52})$

Question 60. [*] Let $f^{(n)}(a)$ denote the n th derivative of f at a. If $f(x) = \dfrac{x^{65}}{x-1}$, then $f^{63}(2) =$

(A) $2(64!)$
(B) $2(63! + 64!)$
(C) $65!$
(D) $2(63!)$
(E) $63! + 64!$

Question 61. Let $f^{(n)}(a)$ denote the n^{th} derivative of f at a. If $f(x) = \frac{1}{1+3x+3x^2+x^3}$, then $f^{(62)}(0)$=

(A) $\frac{1}{2}(65!)$
(B) $\frac{1}{3}(66!)$
(C) $62!$
(D) $\frac{1}{2}(64!)$
(E) $\frac{1}{3}(63!)$

[IX] Related Rates

- Simple Ones

Question 62. The length of a rectangle is increasing at 2ft/sec, while the width is increasing at 1 ft/sec. When the length is 5 feet and the width is 3 feet, how fast is the area of the rectangle increasing?

(A) 12 sq ft/sec^2
(B) 14 sq ft/sec^2
(C) 10 sq ft/sec^2
(D) 11 sq ft/sec^2
(E) 6 sq ft/sec^2

Question 63. A rectangular box has a square base. The width of the box is decreasing at 5 cm/sec, while its height is increasing at 10 cm/sec. At the moment when the height of the box is 30 cm and the area of its base is 400 sq cm, the volume of the box will be

(A) decreasing at 2000 cc/sec.
(B) increasing at 1500 cc/sec.
(C) increasing at 1400 cc/sec.
(D) increasing at 1600 cc/sec.
(E) decreasing at 1800 cc/sec.

Question 64. A light is at the top of a pole whose height is 14 feet. A boy 5 feet tall is walking away from the pole at 3 ft/sec. At what rate is the length of the boy's shadow increasing when he is 25 feet from the pole?

(A) $\frac{5}{3}$ ft/sec
(B) $\frac{4}{5}$ ft/sec
(C) $\frac{3}{4}$ ft/sec
(D) $\frac{5}{4}$ ft/sec
(E) $\frac{4}{3}$ ft/sec

Question 65. The length of a rectangle is *increasing* at 5 cm/min, while its width is *decreasing* at 4 cm/min. At what rate will the area of the rectangle be changing when its lenght is 60 cm and its width is 40 cm?

(A) Increasing at 50 cm^2/min.
(B) Decreasing at 40 cm^2/min.
(C) Increasing at 60 cm^2/min.
(D) Decreasing at 60 cm^2/min.
(E) Increasing at 40 cm^2/min.

Question 66. Each side of a square is increasing at $4cm/sec$. At what rate is the area increasing when the area is 25 sq cm?

(A) 42 sq cm/sec.
(B) 36 sq cm/sec.
(C) 38 sq cm/sec.
(D) 34 sq cm/sec.
(E) 40 sq cm/sec.

- Challenging Ones

Question 67. [*] A ladder leans against a vertical wall and the top of the ladder is sliding down the wall at a constant rate of $\frac{1}{2}$ ft/sec. At the moment when the top of the ladder is 16 feet above the ground, the bottom of the ladder is sliding away from the wall (horizontally) at the rate of $\frac{2}{3}$ ft/sec. At what rate will the bottom of the ladder be sliding away from the wall when the top of the ladder is 12 feet above the ground?

(A) not determinable because of insufficient information
(B) $\frac{3}{8}$ ft/sec.
(C) $\frac{5}{12}$ ft/sec.
(D) $\frac{4}{9}$ ft/sec.
(E) $\frac{3}{10}$ ft/sec.

Question 68. A ladder 30 feet long leans against a vertical wall. The top of the ladder is sliding down the wall at the constant rate of 3 feet per second, resulting in the bottom of the ladder sliding on the ground away from the wall. Find the acceleration of the bottom of the ladder when the top of the ladder hits the ground.

(A) $-\frac{1}{5}ft/sec^2$
(B) $-\frac{3}{5}ft/sec^2$
(C) $-\frac{3}{10}ft/sec^2$
(D) $-\frac{1}{4}ft/sec^2$
(E) $-\frac{2}{5}ft/sec^2$

Question 69. [*] A man 6 feet tall walks at 5 feet per second along one edge of a road that is 30 feet wide. On the other side of the road is a light atop a pole 18 feet high. How fast is the length of the man's shadow (on the horizontal ground) increasing when he is 40 feet beyond the point directly across the road from the pole?

(A) $\frac{5}{3}$ ft/sec
(B) $\frac{7}{4}$ ft/sec
(C) $\frac{3}{2}$ ft/sec
(D) 2 ft/sec
(E) $\frac{5}{4}$ ft/sec

[X] **Mean Value Theorem**: Below are all the same type of problem. If you are confident with one of them, you should be okay with the rest.

- Simple Ones

Question 70. Find the number c that satisfies the conclusion of the Mean Value Theorem for the function $f(x) = \sqrt{x} - \frac{x}{3}$ on the interval [0,9].

(A) $\frac{9}{2}$
(B) $\frac{5}{2}$
(C) $\frac{9}{4}$
(D) $\frac{3}{2}$
(E) $\frac{9}{5}$

Question 71. Let $f(x) = \sqrt{x}$ on [1, 16]. Find the number c in the interval (1, 16) which satisfies the conclusion of the Mean Value Theorem.

(A) $\frac{23}{4}$
(B) $\frac{25}{4}$
(C) $\frac{15}{2}$
(D) $\frac{16}{3}$
(E) $\frac{23}{5}$

Question 72. Find the number c which satisfies the conclusion of the Mean Value Theorem for the function $f(x) = \frac{1}{x}$ on $[1, 3]$.

(A) $\frac{3}{2}$
(B) $\sqrt{3}$
(C) 2
(D) $\frac{5}{2}$
(E) $\frac{5}{4}$

Question 73. Let $f(x) = \dfrac{1}{1+x}$ on $[1, 3]$. Find the number c that satisfies the conclusion of the Mean Value Theorem for $f(x)$ on $[1, 3]$

(A) $3\sqrt{2} - 2$
(B) 2
(C) $2\sqrt{3} - 1$
(D) $2\sqrt{2}$
(E) $2\sqrt{2} - 1$

Question 74. Find the number c which satisfies the conclusion of the Mean Value Theorem for the function $f(x) = x^3 - x + 1$ on the closed interval $[0, 2]$.

(A) 1

(B) $\frac{1}{2}$

(C) $\frac{2}{\sqrt{3}}$

(D) $\frac{2}{3}$

(E) $\frac{1}{3}$

Question 75. [*] Let f be a differentiable function such that $f(2) = 6$ and $1 < f'(x) < 4$ for all x. Consider the following three statements:

I. The function $g(x) = \frac{|f(x)|}{2-\sin x \cos x}$ is continuous everywhere, i.e. continuous for all real x.

II. The function $h(x) = \frac{x^3}{3} - x^2 + f(x)$ is an increasing function on $(-\infty, \infty)$.

III. $9 < f(x) < 18$

(A) I and III are true but II is false.

(B) I and II are true but III is false.

(C) I, II and III are all true.

(D) I is true but II and III are false.

(E) II and III are true but I is false.

[XI] First and Second Derivative Analysis Problems

- Simple Ones

Question 76. The curve $y = 2x^3 + 3x^2 - 12x + 1$ has a horizontal tangent line at
(A) $x = 1$ and $x = -2$
(B) $x = -1$ and $x = -3$
(C) $x = -1$ and $x = 3$
(D) $x = -1$ and $x = -2$
(E) $x = 1$ and $x = 3$

Question 77. The function $f(x) = \frac{x^2+4}{x}$ has
(A) A local maximum at $x = -2$ and no local minimum.
(B) A local minimum at $x = -2$ and a local maximum at $x = 2$.
(C) A local maximum at $x = -2$ and a local minimum at $x = 2$.
(D) A local minimum at $x = -2$ and no local maximum.
(E) A local minimum at $x = 2$ and no local maximum.

Question 78. Let $f(x) = 3x^4 + 20x^3 + 36x^2 + 4$ for all x. Then f will have a local max. at $x =$
(A) 0
(B) -2
(C) -1
(D) -4
(E) -3

Question 79. The function $f(x) = \frac{x^7}{42} - \frac{x^5}{10} + \frac{x^3}{6} + 3x + 4$ has
(A) One point of inflection.
(B) No inflection points.
(C) Four points of inflection.
(D) Three points of inflection.
(E) Two points of inflection.

Question 80. The graph of $y = \frac{x^4}{12} - \frac{x^3}{3} - 4x^2 + 5x - 2$ is concave down on the interval
(A) (-6, 2)
(B) ($-\infty$, -2)
(C) (-2, 4)
(D) (4, ∞)
(E) (-4, 3)

Question 81. On what interval is the graph of $f(x) = (1 - \frac{1}{x})^2$ concave down?

(A) $(\frac{3}{2}, \infty)$
(B) $(1, \infty)$
(C) $(-1, 0)$
(D) $(0, 1)$
(E) $(-\infty, -1)$

Question 82. Let $f(x) = 3x^2 - 2x$ on$[-3, 2]$. Let M be the absolute maximum value of f of on $[-3, 2]$ and m be the absolute minimum value of f on [-3,2]. Then $M - m =$

(A) $\frac{104}{3}$
(B) $\frac{106}{3}$
(C) $\frac{98}{3}$
(D) $\frac{100}{3}$
(E) $\frac{103}{3}$

Question 83. Let $f(x) = x^2 - x + 2$ on [-2, 2]. If M is the absolute maximum value of f on [-2, 2] and m is the absolute minimum value of f on [-2,2], then $M - m =$

(A) $\frac{13}{2}$
(B) 6
(C) 5
(D) $\frac{25}{4}$
(E) 4

Question 84. Find the y-coordinate of the point of inflection of $f(x) = x^3 - x^2$.

(A) $\frac{2}{9}$
(B) $-\frac{2}{27}$
(C) $-\frac{2}{9}$
(D) $-\frac{2}{3}$
(E) $\frac{1}{3}$

Question 85. [*] If f is a function that $f''(x) = (x-1)^3(x+2)(x^2+3x+2)(x^{64}-1)$ for all x, how many points of inflection does the graph of f have?

(A) three
(B) none
(C) two
(D) one
(E) more than three

- Challenging Ones

Question 86. Let $f(x) = x^{\frac{2}{3}}(x+3)^{\frac{1}{3}}$ for all x. How many points of inflection does the graph of f have?

(A) none
(B) two
(C) more than three
(D) one
(E) three

Question 87. Let $f(x) = (x^2 + 2x)^{\frac{2}{3}}$, for $x > 0$. Then f has a point of inflection at$x =$

(A) $\sqrt{3} + 1$
(B) $\sqrt{2} - 1$
(C) $\sqrt{2} + 1$
(D) $2\sqrt{2}$
(E) $\sqrt{3} - 1$

Question 88. The graph of $y = (x^2 - 3x)^{\frac{2}{3}}$, for $x \geq 0$, has a point of inflection at $x =$

(A) $\frac{2}{3}(3 + \sqrt{3})$
(B) $\frac{3}{2}(2\sqrt{3} - 1)$
(C) $\frac{3}{2}(2 + \sqrt{3})$
(D) $\frac{3}{2}(3\sqrt{3} - 1)$
(E) $\frac{3}{2}(1 + \sqrt{3})$

[XII] Graphing Related

- Simple Ones

Question 89. The graph of $y = \frac{3x^3+x^2+2x+1}{x^2+x}$ has two vertical asymptotes and one slant (i.e oblique) asymptote. The slant asymptote is the line

(A) $y = 3x$
(B) $y = 3x - 1$
(C) $y = 3x + 2$
(D) $y = 3x + 1$
(E) $y = 3x - 2$

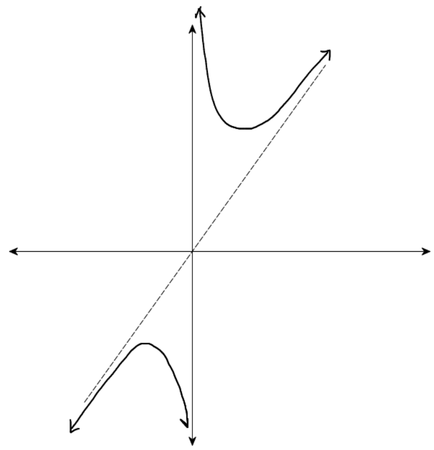
Question 90. The graph of $= \dfrac{3x^3 + x^2 - 7x - 4}{x^2 + 2x + 1}$ has one vertical asymptote and one other asymptote.

(A) $y = 3x + \frac{1}{2}$
(B) $y = 3x + 1$
(C) $y = 3x$
(D) $y = 3x - 5$
(E) $y = 3x - 4$

Question 91. The graph of $y = \dfrac{2x^3 + x^2 - 12x + 16}{x(x+3)}$ has two vertical asymptotes and also has one slant (i.e. oblique) asymptotes, which is the line

(A) y = 2x + 2
(B) y = 2x - 4
(C) y = 2x + 1
(D) y = 2x
(E) y = 2x - 5

- Challenging Ones



Question 92. To which one of the following functions does the above graph correspond?

(A) $f(x) = \dfrac{x^3+1}{x^3}$

(B) $f(x) = \dfrac{x^3+1}{x^2}$

(C) $f(x) = \dfrac{x^4+1}{x^3}$

(D) $f(x) = \dfrac{x^5+1}{x^3}$

(E) $f(x) = \coth x$

Question 93. Let $f(x) = \dfrac{x}{e^x - x}$. Then f has

(A) two horizontal asymptotes and one vertical asymptote

(B) one horizontal asymptote and no vertical asymptotes

(C) one horizontal asymptote and one vertical asymptote

(D) no horizontal asymptote and no vertical asymptotes

(E) two horizontal asymptotes and no vertical asymptotes

Question 94. Let $f(x) = x + \sqrt{x^2 + 6|x| + 4}$, for all x. Which one of the following 5 statements is true?

(A) The graph of f does not have any horizontal asymptotes.

(B) The line $y = 4$ is a horizontal asymptote of the graph of f.

(C) The line $y = 3$ is a horizontal asymptote of the graph of f.

(D) The line $y = 6$ is a horizontal asymptote of the graph of f.

(E) The x- axis is a horizontal asymptote of the graph of f.

[XIII] **Optimization**

- Simple Ones

Question 95. A ball is being thrown vertically upward so that its height (above ground) t seconds after it is thrown is $(25 + 16t - 16t^2)$ feet. What is the maximum height (above ground) attained by the ball?

(A) 32 feet
(B) 31 feet
(C) 30 feet
(D) 28 feet
(E) 29 feet

Question 96. The sum of two positive real number is 12. What is the smallest possible value of the sum of their squares?

(A) 72
(B) 74
(C) 68
(D) 76
(E) 70

Question 97. [This question is exactly like the one above, try it if you need more practice] The product of two positive numbers is 64. The smallest possible value of their sum must be

(A) 17
(B) 16
(C) 14
(D) 15
(E) 18

- Challenging Ones

Question 98. P is a point on the positive $x - axis$, Q is a point on the positive $y - axis$ and O is the origin. What is the smallest possible area of the triangle OPQ if the line passing through P and Q is required to be tangent to the curve $y = 4 - x^2$ at some point?

(A) $\frac{15\sqrt{3}}{4}$
(B) $4\sqrt{3}$
(C) $\frac{33\sqrt{3}}{10}$
(D) $\frac{32\sqrt{3}}{9}$
(E) $\frac{10\sqrt{3}}{3}$

Question 99. A sphere has radius R inches. If the largest cone (i.e. the cone with the largest possible volume) that can be placed inside this sphere has volume $\frac{4\pi}{3}$ cubic inches, find the value of R.
Note : Volume of cone $= \frac{1}{3}\pi r^2 h$, where h=height of cone and r=radius of base (bottom) of cone.

(A) $\frac{7}{6}$
(B) $\frac{3}{2}$
(C) $\frac{4}{3}$
(D) $\frac{5}{4}$
(E) $\frac{5}{3}$

Question 100. A line passes through the point $(1, \frac{1}{8})$ and intersects the positive x-axis at the point A and the positive y-axis at the point B. What is the shortest possible distance between A and B?

(A) $\frac{5\sqrt{5}}{6}$
(B) $\frac{3\sqrt{5}}{4}$
(C) $\frac{5\sqrt{5}}{8}$
(D) $\frac{3\sqrt{3}}{5}$
(E) $\frac{3\sqrt{3}}{8}$

MAT 135 Final Exam Study Pacakge Answer Keys

Q	A	Q	A	Q	A	Q	A	Q	A	Q	A	Q	A	Q	A	Q	A	Q	A
1	A	11	D	21	B	31	D	41	C	51	E	61	C	71	B	81	A	91	E
2	B	12	D	22	D	32	D	42	B	52	E	62	D	72	B	82	D	92	C
3	A	13	E	23	A	33	B	43	E	53	D	63	A	73	E	83	D	93	B
4	C	14	B	24	D	34	A	44	E	54	E	64	A	74	C	84	B	94	C
5	B	15	A	25	see v	35	A	45	D	55	D	65	B	75	C	85	B	95	E
6	C	16	C	26	see v	36	D	46	D	56	B	66	E	76	A	86	D	96	A
7	B	17	A	27	14	37	E	47	A	57	E	67	B	77	C	87	E	97	B
8	C	18	B	28	C	38	A	48	E	58	B	68	C	78	B	88	E	98	D
9	A	19	B	29	D	39	D	49	A	59	E	69	D	79	A	89	E	99	B
10	D	20	A	30	E	40	see v	50	B	60	A	70	C	80	C	90	D	100	C

see v See Video, Not a Multiple Choice Question