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(**Note:** the more one knows about linear Algebra the more one is able to use these tools in the rest of the course. Please see the optional readings on linear Algebra if you have any doubts about your linear algebra knowledge.)

The Following are few extra practice questions to deal with algebraic manipulations. These exercises must be studied in conjunction with the 8 exercises of section 1.1 Please refer to optional readings or the past Linear Algebra courses for more details.

1. Read Propositions 1.1 and 1.2 and try to see that Cauchy's inequality and triangle inequalities are really establishing relationships between the algebra and norm of $\mathbb{R}^n$ and that of $\mathbb{R}$ This is often the story of Multi-variate Calculus (sections 1.1 - 2.9)
2. Algebraically describe equation of the line passing through two points (a, b) and (c, d).
3. Algebraically describe equation of the line passing through a point (a, b, c) and parallel to the vector (u, v, w).
4. Describe the set of all the vectors in $\mathbb{R}^2$ that is perpendicular to the vector $(1, 2)$.
5. Describe the set of all the vectors in $\mathbb{R}^3$ that is perpendicular to the vector $(1, 2, 3)$.
6. Describe the set of all the vectors in $\mathbb{R}^3$ which are perpendicular to the vectors $(1, 2, 3)$ and $(2, 1, 4)$.
7. Prove if $\boldsymbol{u}$ is perpendicular to both $\boldsymbol{v}$ and $\boldsymbol{w}$ then it is also perpendicular to any linear combination of the two vectors, that is vectors of the form $\alpha\boldsymbol{v} + \beta\boldsymbol{w}$.
8. If $\boldsymbol{a} \cdot \boldsymbol{b} = \boldsymbol{a} \cdot \boldsymbol{c}$ and $\boldsymbol{a} \neq 0$ then is it true that $\boldsymbol{b} = \boldsymbol{c}$?
9. Simplify the expression $(\boldsymbol{u} + \boldsymbol{v}) \times (\boldsymbol{u} - \boldsymbol{v})$, and then geometrically interpret your result.
10. For any two positive numbers a and b prove the geometric mean is no larger than the arithmetic mean, that is prove that
$$\sqrt{ab} \leq \frac{a + b}{2}$$
(Hint: define two vectors $\boldsymbol{x} = (\sqrt{a}, \sqrt{b})$ and $\mathbf{y} = (\sqrt{b}, \sqrt{a})$ then apply an appropriate inequality to these two vectors.
11. Generalize the previous inequality about the means to n numbers $a_1, a_2, \ldots a_n$.
12. Prove
$$\boldsymbol{u} \cdot \boldsymbol{v} = \frac{1}{4}|(\boldsymbol{u} + \boldsymbol{v})|^2 - \frac{1}{4}|(\boldsymbol{u} - \boldsymbol{v})|^2$$
Then determine the value of $\boldsymbol{u} \cdot \boldsymbol{v}$ if $|(\boldsymbol{u} + \boldsymbol{v})| = 5$ and $|(\boldsymbol{u} - \boldsymbol{v})| = 3$

13. page 6 defines the concept of the distance between two points in $\mathbb{R}^n$. The distance between $(0,0)$ and the point $(1,1)$ is just $\sqrt{2}$. Calculate the distance in the 100 dimensional space between the origin and the point $(1,1,\ldots 1)$. Find an intuitive interpretation of your result compared with that of 2 dimensional case. (see also exercise 3 which presents a generalization of the Pythagorean identity.)

14. Similarly, in page 6, the "angle between two vectors" is defined. Use your calculator and calculate the angle between $(1,0)$ and $(1,1)$. Now repeat the same question in the 100 dimensional space: the angle between $(1,0,\ldots .0)$ and $(1,1,\ldots,1)$. Again present an intuitive interpretation for your result.

15. Use the determinant definition of the cross product to prove the following properties of the cross product:

    a) $\boldsymbol{v} \times \boldsymbol{u} = -\boldsymbol{u} \times \boldsymbol{v}$
    b) $\boldsymbol{u} \times (\boldsymbol{v} + \boldsymbol{w}) = (\boldsymbol{u} \times \boldsymbol{v}) + (\boldsymbol{u} \times \boldsymbol{w})$
    c) $\boldsymbol{u} \boldsymbol{.} (\boldsymbol{u} \times \boldsymbol{v}) = 0$
    d) $\boldsymbol{v} \boldsymbol{.} (\boldsymbol{u} \times \boldsymbol{v}) = 0$

16. Evaluate $(x, y, 0) \times (-y, x, 0)$ Interpret your result by drawing a diagram.