Ex. $X \sim \exp(\lambda)$ $f_X(x) = \lambda e^{-\lambda x}, \, x > 0$

$Y = \frac{1}{X+1} \longrightarrow$ decreasing)

$F_Y(y) = P(Y \leq y) = P\left(\frac{1}{X+1} \leq y\right) = P\left(X \geq \frac{1}{y} - 1\right)$

$= 1 - P\left(X \leq \frac{1}{y} - 1\right) = 1 - \int_0^{1/y - 1} \lambda e^{-\lambda x} \, dx$

$= 1 + e^{-\lambda x} \Big|_0^{1/y - 1} = e^{-\lambda\left(\frac{1}{y} - 1\right)}$

$f_Y(y) = \frac{\lambda}{y^2} e^{-\lambda\left(\frac{1}{y} - 1\right)}$

Formula $= -f_X\left(\frac{1}{y} - 1\right) \cdot \frac{d}{dy}\left(\frac{1}{y} - 1\right)$

$= -\lambda e^{-\lambda\left(\frac{1}{y} - 1\right)} \cdot \left(-\frac{1}{y^2}\right)$

$= \frac{\lambda}{y^2} e^{-\lambda\left(\frac{1}{y} - 1\right)}$