Symbolization Exercises

Symbolizations with one-place predicates (Unit 5 Part 1: Sections 5.1-5.6)

Symbolize each of the following sentences using the abbreviation scheme provided:

A^1: *a* is an athlete.	B^1: *a* is beautiful.	C^1: *a* works out.
F^1: *a* is famous.	H^1: *a* is a person.	J^1: *a* is a movie star.
K^1: *a* competes in the Olympics.	L^1: *a* gets caught in a sex scandal	a: Wayne Gretzky

S5.01 Everything is beautiful.

$\forall$xBx (note: you can use any variable… $\forall$yBy, $\forall$iBi, $\forall$wBw…)

S5.02 Something is beautiful.

$\exists$yBy (note: you can use any variable … $\exists$xBx, $\exists$iBi, $\exists$wBw….)

S5.03 Nothing is beautiful.

~$\exists$xBx (It is not the case that something is beautiful.) or

$\forall$x~Bx (Everything is such that it is non-beautiful.)

S5.04 Something is not beautiful.

$\exists$x~Bx (Something is such that it is not beautiful.) or

~$\forall$xBx (It is not the case that everything is beautiful.)

NOTE: In more complex sentences, the universal quantifier $\forall$ is generally used with $\to$; the existential quantifier $\exists$ is generally used with $\wedge$:

S5.05 Some people are famous.

$\exists$x(Hx $\wedge$ Fx) (There exists at least one thing that is a person and is famous.)

S5.06 Nobody is famous.

~$\exists$x(Hx $\wedge$ Fx) (It is not the case that some people are famous.)

$\forall$x(Hx $\to$ ~Fx) (Everything is such that if it is a person, then it is not famous.)

S5.07 Everybody is famous.

$\forall$x(Hx $\to$ Fx) (Everything is such that if it is a person, then it is famous.)

S5.08 Some people are not famous.

$\exists$x(Hx $\wedge$ ~Fx) (There exists at least one thing that is a person and is not famous.) or

~$\forall$x(Hx $\to$ Fx) (It is not the case that every person is famous.)

S5.09 Not every athlete is famous.

~$\forall$x(Ax $\to$ Fx) (It is not the case that every athlete is famous.)

$\exists$x(Ax $\wedge$ ~Fx) (There exists at least one athlete who is not famous.)

S5.10 Only movie stars are beautiful.

$\forall x(Bx \rightarrow Jx)$ (All things are such that if it is beautiful then it is a movie star.)

NOTE: only is indicating the consequent here. If only movie stars are beautiful, then if something is beautiful then it must be a movie star.

S5.11 None but the beautiful are famous.

$\forall x(Fx \rightarrow Bx)$ (Everything is such that if it is famous then it is beautiful.)

S5.12 All and only movie stars are famous.

$\forall x(Jx \leftrightarrow Fx)$

S5.13 Not all famous people are beautiful.

$\sim\forall x(Fx \wedge Hx \rightarrow Bx)$ or $\exists x(Fx \wedge Hx \wedge \sim Bx)$

S5.14 Only beautiful movie stars are famous.

$\forall x(Jx \rightarrow (Fx \rightarrow Bx)$ or $\forall x((Jx \wedge Fx) \rightarrow Bx)$ This is the more natural interpretation – among movie stars, only the beautiful ones are famous.

$\forall x(Fx \rightarrow Bx \wedge Jx)$ (The only famous individuals are beautiful movie stars)

S5.15 Not all beautiful people are famous.

$\exists x(Hx \wedge Bx \wedge \sim Fx)$ or $\sim\forall x(Hx \wedge Bx \rightarrow Fx)$

S5.16 Anybody who works out is beautiful.

$\forall x(Hx \wedge Cx \rightarrow Bx)$ or $\forall x(Hx \rightarrow (Cx \rightarrow Bx))$

S5.17 Although not all athletes are famous, Gretzky, who is an athlete, is.

$\sim\forall x(Ax \rightarrow Fx) \wedge (Aa \wedge Fa)$

S5.18 If Wayne Gretzky is not a famous athlete, then nobody is.

$\sim(Aa \wedge Fa) \rightarrow \sim\exists x(Hx \wedge Fx \wedge Ax)$ or $\sim(Aa \wedge Fa) \rightarrow \forall x(Hx \rightarrow \sim(Fx \wedge Ax))$

S5.19 Among athletes, only those who compete in the Olympics are famous.

$\forall x(Ax \rightarrow (Fx \rightarrow Kx))$

S5.20 Movie stars who are beautiful work out.

$\forall x(Jx \wedge Bx \rightarrow Cx)$

S5.21 Movie stars and athletes work out.

$\forall x(Jx \vee Ax \rightarrow Cx)$ or $\forall x(Jx \rightarrow Cx) \wedge \forall x(Ax \rightarrow Cx)$

S5.22 Athletes, who work out, are beautiful.

$\forall x(Ax \rightarrow Cx) \wedge \forall x(Ax \rightarrow Bx)$ OR $\forall x(Ax \rightarrow Cx \wedge Bx)$

S5.23 It is necessary to work out to be an Olympic athlete.

$\forall x(Ax \wedge Kx \rightarrow Cx)$

S5.24 Some people who get caught in sex scandals are not beautiful.

$\exists x(Hx \wedge Lx \wedge \sim Bx)$

S5.25 Olympic athletes aren't famous unless they are caught in sex scandals.

$\forall x(Ax \wedge Kx \rightarrow \sim Fx \vee Lx)$

S5.26 Only famous athletes get caught in sex scandals.

This is the natural interpretation: Of athletes, only the famous ones get caught in sex scandals.

$\forall x(Ax \rightarrow (Lx \rightarrow Fx))$ or $\forall x(Ax \wedge Lx \rightarrow Fx)$

The other interpretation: The only individuals who get caught in sex scandals are famous athletes.

$\forall x(Lx \rightarrow Fx \wedge Ax)$

A^1: *a* is an action.	B^1: *a* is random.	C^1: *a* is carefree.
D^1: *a* is determined by prior states.	E^1: *a* is an event.	F^1: *a* is free.
G^1: *a* is wise	H^1: *a* is happy.	I^1: *a* has ideas
J^1: *a* is a person.	L^1: *a* lives a good life.	a: Anna

S5.27 Anybody who lives a good life is happy.

$\forall x(Jx \wedge Lx \rightarrow Hx)$ OR $\forall x(Jx \rightarrow (Lx \rightarrow Hx))$

S5.28 If only carefree people are happy, then Anna is not a happy person.

$\forall x(Jx \wedge Hx \rightarrow Cx) \rightarrow \sim Ha$ OR $\sim\exists x(Jx \wedge Hx \wedge \sim Cx) \rightarrow \sim Ha$

OR $Ha \rightarrow \exists x(Jx \wedge Hx \wedge \sim Cx)$

S5.29 Not everyone who is free is carefree.

$\sim\forall x(Jx \wedge Fx \rightarrow Cx)$ or $\exists x(Jx \wedge Fx \wedge \sim Cx)$

S5.30 An action is an event, but not all events are actions.

$\forall x(Ax \rightarrow Ex) \wedge \sim\forall x(Ex \rightarrow Ax)$ OR $\forall x(Ax \rightarrow Ex) \wedge \exists x(Ex \wedge \sim Ax)$

S5.31 Some happy people aren't wise, and some wise people aren't happy.

$\exists x(Hx \wedge \sim Gx) \wedge \exists y(Gy \wedge \sim Hy)$ OR $\sim\forall x(Hx \rightarrow Gx) \wedge \sim\forall y(Gy \rightarrow Hy)$

S5.32 A person who has ideas and lives a good life is wise.

$\forall x(Jx \wedge Ix \wedge Lx \rightarrow Gx)$

S5.33 No free action is determined by prior states.

$\sim\exists x(Fx \wedge Dx)$

S5.34 If someone is unhappy then not everyone lives a good life.

$\exists x(Jx \wedge \sim Hx) \rightarrow \sim\forall x(Jx \rightarrow Lx)$

S5.35 If someone is unhappy, he or she will not live a good life.

$\forall x(Jx \wedge \sim Hx \rightarrow \sim Lx)$

S5.36 If anyone is unhappy, Anna is.

$\exists x(Jx \wedge \sim Hx) \rightarrow \sim Ha$ OR $\forall x(Jx \wedge \sim Hx \rightarrow \sim Ha)$

S5.37 People who are happy only if they are free don't live a good life.

$\forall x(Jx \wedge (Hx \rightarrow Fx) \rightarrow \sim Lx)$ or $\forall x(Jx \rightarrow ((Hx \rightarrow Fx) \rightarrow \sim Lx))$

S5.38 If all events are determined by prior states then no actions are free.

$\forall x(Ex \rightarrow Dx) \rightarrow \sim\exists y(Ax \wedge Fx)$ OR $\forall x(Ex \rightarrow Dx) \rightarrow \forall x(Ax \rightarrow \sim Fx)$

S5.39 Having ideas is not sufficient for a person to be wise.

This is trickier than it seems…

It is not the case that having ideas is sufficient for a person to be wise:

$\sim\forall x(Jx \rightarrow (Ix \rightarrow Gx))$ OR $\sim\forall x(Jx \wedge Ix \rightarrow Gx)$ OR $\exists x(Jx \wedge Ix \wedge \sim Gx)$

S5.40 Among wise people, only those who live a good life are happy.

$\forall x(Jx \wedge Gx \rightarrow (Hx \rightarrow Lx))$ OR $\forall x(Jx \wedge Gx \wedge Hx \rightarrow Lx)$

S5.41 If people have ideas if and only if they are happy, then nobody who doesn't have ideas lives a good life.

$\forall x(Jx \rightarrow (Ix \leftrightarrow Hx)) \rightarrow \sim\exists y(Jy \wedge \sim Iy \wedge Ly)$ OR consequent: $\forall y(Jy \wedge \sim Iy \rightarrow \sim Ly)$

Note… you can also do the antecedent like this: $\forall x(Jx \wedge Ix \leftrightarrow Jx \wedge Hx)$

S5.42 There are no random events if all events are determined by prior states.

$\forall x(Ex \rightarrow Dx) \rightarrow \sim\exists x(Ex \wedge Bx)$ OR $\forall x(Ex \rightarrow Dx) \rightarrow \forall y(Ey \rightarrow \sim By)$

S5.43 People who aren't carefree don't live a good life if they aren't happy.

$\forall x(Jx \wedge \sim Cx \rightarrow (\sim Hx \rightarrow \sim Lx))$

S5.44 Although it is necessary that an event not be determined by prior states in order for it to be free, nothing that is random is free.

$\forall x(Ex \rightarrow (Fx \rightarrow \sim Dx)) \wedge \sim\exists x(Bx \wedge Fx)$

S5.45 Since all actions are events, no action is random or free unless events aren't all determined by prior states.

"Since" is a bit tricky here. There is a presupposition that what follows the 'since' is true AND that if it is true, the rest follows from it.

$\forall x(Ax \rightarrow Ex) \wedge (\forall x(Ax \rightarrow Ex) \rightarrow (\sim\exists x(Ax \wedge (Bx \vee Fx)) \vee \sim\forall y(Ey \rightarrow Dy)))$

The antecedent is redundant… so you can just use AND

$\forall x(Ax \rightarrow Ex) \wedge (\exists x(Ax \wedge (Bx \vee Fx)) \rightarrow \sim\forall y(Ey \rightarrow Dy))$

OR $\forall x(Ax \rightarrow Ex) \wedge (\exists x(Ax \wedge (Bx \vee Fx)) \rightarrow \exists y(Ey \wedge \sim Dy))$

OR $\forall x(Ax \rightarrow Ex) \wedge (\forall y(Ey \rightarrow Dy) \rightarrow \forall x(Ax \rightarrow \sim Bx \wedge \sim Fx))$

S5.46 If it's not the case that people are happy if they are free, then some people don't live good lives. (Ambiguous: symbolize two logically distinct ways.)

If it's not the case that *if people are free then they are happy*, then some people don't live good lives.

$\sim\forall x(Jx \wedge Fx \rightarrow Hx) \rightarrow \exists y(Jy \wedge \sim Ly)$ OR antecedent: $\exists x(Jx \wedge Fx \wedge \sim Hx)$

If if people are free then it's not the case that *they are happy*, then some people don't lead good lives.

$\forall x(Jx \wedge Fx \rightarrow \sim Hx) \rightarrow \exists y(Jy \wedge \sim Ly)$ OR antecedent: $\sim\exists x(Jx \wedge Fx \wedge Hx)$

S5.47 Everybody is not happy. (Ambiguous: symbolize two logically distinct ways.)

Everybody is unhappy. $\forall x(Jx \rightarrow \sim Hx)$

Not everybody is happy. $\exists x(Jx \wedge \sim Hx)$ or $\sim\forall x(Jx \rightarrow Hx)$

S5.48 Wise and happy people live a good life. (Ambiguous: symbolize two distinct ways.)

Wise people live a good life and happy people live a good life:

$\forall x(Jx \wedge Gx \rightarrow Lx) \wedge \forall x(Jx \wedge Hx \rightarrow Lx)$ OR $\forall x(Jx \wedge (Gx \vee Hx) \rightarrow Lx)$

People who are BOTH wise and happy live a good life:

$\forall x(Jx \wedge Gx \wedge Hx \rightarrow Lx)$ OR $\forall x(Jx \rightarrow (Gx \wedge Hx \rightarrow Lx))$

S5.49 Only happy people are carefree. (Ambiguous: symbolize two logically distinct ways. Which one do you think is the more natural interpretation?)

Of people only the happy ones are carefree: (This is the natural interpretation!)

$\forall x(Jx \rightarrow (Cx \rightarrow Hx))$ OR $\forall x(Jx \wedge Cx \rightarrow Hx)$

The only carefree things in the universe are happy people.

$\forall x(Cx \rightarrow Jx \wedge Hx)$

A^1: *a* is alive.
B^1: *a* has a brain.
C^1: *a* is conscious.
D^1: *a* deserves a scholarship.
E^1: *a* is successful.
F^1: *a* works hard.
G^1: *a* is a plant.
H^1: *a* is a person.
I^1: *a* is intelligent.
J^1: *a* is a professor.
K^1: *a* is at U of T.
L^1: *a* is a student

S5.50 If every living thing is conscious then plants are intelligent.

$\forall x(Ax \to Cx) \to \forall y(Gy \to Iy)$

S5.51 For a living thing to be conscious it is necessary for it to have a brain.

$\forall x(Ax \wedge Cx \to Bx)$ OR $\forall x(Ax \to (Cx \to Bx))$

S5.52 For students to be successful it is sufficient that they work hard.

$\forall x(Lx \wedge Fx \to Ex)$ OR $\forall x(Lx \to (Fx \to Ex))$

S5.53 Plants are not intelligent unless they have a brain and are conscious.

$\forall x(Gx \to \sim Ix \vee (Bx \wedge Cx))$ OR $\forall x(Gx \to (Ix \to Bx \wedge Cx))$

OR $\forall x(Gx \wedge Ix \to Bx \wedge Cx)$

S5.54 Plants and other living things are not conscious unless they have brains.

$\forall x(Gx \vee (Ax \wedge \sim Gx) \to (\sim Cx \vee Bx))$

$\forall x(Gx \vee (Ax \wedge \sim Gx) \to (Cx \to Bx))$

Logically equivalent: $\forall x(Ax \to (Cx \to Bx))$ OR $\forall x(Ax \to (\sim Cx \vee Bx))$

S5.55 Some professors are not successful even though they work hard and are intelligent.

$\exists x(Jx \wedge \sim Ex \wedge Fx \wedge Ix)$

S5.56 Nobody who is not intelligent is a professor at U of T.

$\sim\exists x(\sim Ix \wedge Jx \wedge Kx)$

S5.57 Only U of T students work hard.

Natural interpretation: among students, only the U of T ones work hard.

$\forall x(Lx \wedge Fx \to Kx)$ or $\forall x(Lx \to (Fx \to Kx))$

Other interpretation: the only individuals that work hard are U of T students.

$\forall x(Lx \wedge Fx \to Kx)$

S5.58 Living things are conscious only if they are not brainless.

$\forall x(Lx \to (Cx \to \sim\sim Bx))$ or $\forall x(Lx \wedge Cx \to Bx)$

S5.59 Living things are conscious if they are not brainless.

$\forall x(Lx \to (\sim\sim B \to Cx))$ or $\forall x(Lx \wedge Bx \to Cx)$

S5.60 Intelligent students who study at U of T deserve scholarships unless they don't work hard.

$\forall x(Ix \wedge Lx \wedge Kx \rightarrow (Dx \vee \sim Fx))$

S5.61 Professors and students at U of T work hard or they are not successful.

$\forall x((Jx \vee Lx) \rightarrow (Fx \vee \sim Ex)$

S5.62 Without a brain, no living thing is conscious.

$\sim\exists x(Ax \wedge \sim Bx \wedge Cx)$ OR $\forall x(Ax \wedge \sim Bx \rightarrow \sim Cx)$

S5.63 To be a successful professor it is not sufficient that you work hard, for you must also be intelligent.

Natural interpretation: about professors, working hard is insufficient for success, but together, hard work and intelligence are sufficient. For this to be true, some professors work hard but are not successful (it is not the case that all professors who work hard are successful), but all professors who work hard and are intelligent are successful.

$\exists x(Jx \wedge Fx \wedge \sim Ex) \wedge \forall x(Jx \wedge Ix \wedge Fx \rightarrow Ex)$

$\sim\forall x(Jx \wedge Fx \rightarrow Ex) \wedge \forall x(Jx \rightarrow (Ix \wedge Fx \rightarrow Ex))$

Alternate interpretation of second conjunct: being intelligent and working hard is both necessary and sufficient for professors to be successful.

$\forall x(Jx \rightarrow (Ix \wedge Fx \leftrightarrow Ex))$

S5.64 Although some intelligent students deserve a scholarship, others don't work hard.

$\exists x(Ix \wedge Lx \wedge Dx) \wedge \exists y(Ix \wedge Lx \wedge \sim Fx)$

S5.65 Of students, those, and only those, who are hardworking and intelligent deserve scholarships.

$\forall x(Lx \rightarrow (Fx \wedge Ix \leftrightarrow Dx))$

S5.66 Some successful people work hard, but not all who work hard are successful.

$\exists x(Hx \wedge Ex \wedge Fx) \wedge \sim\forall x(Fx \rightarrow Ex)$ or $\exists x(Hx \wedge Ex \wedge Fx) \wedge \exists x(Fx \wedge \sim Ex)$ OR

$\exists x(Hx \wedge Ex \wedge Fx) \wedge \sim\forall x(Fx \wedge Hx \rightarrow Ex)$ or $\exists x(Hx \wedge Ex \wedge Fx) \wedge \exists x(Fx \wedge Hx \wedge \sim Ex)$

S5.67 Among living things, all and only those with brains are conscious

$\forall x(Ax \rightarrow (Bx \leftrightarrow Cx))$

S5.68 Professors and students at U of T who fail to work hard are not successful.

$\forall x((Jx \vee Lx) \wedge Kx \wedge \sim Fx \rightarrow \sim Ex)$

S5.69 Even though he/she is intelligent, unless a student works hard, he will not be successful.

$\forall x(Ix \wedge Lx \rightarrow Fx \vee \sim Ex)$ OR $\forall x(Ix \rightarrow (Lx \rightarrow (Ex \rightarrow Fx)))$

S5.70 Students and professors who are intelligent are successful only if they also work hard.

$\forall x((Jx \lor Lx) \land Ix \to (Ex \to Fx))$

S5.71 Students and professors, who are intelligent, are successful only if they also work hard.

$\forall x(Jx \lor Lx \to Ix) \land \forall x(Jx \lor Lx \to (Ex \to Fx))$ OR $\forall x(Jx \lor Lx \to(Ix \land (Ex \to Fx)))$

S5.72 If a living thing is conscious, then, given that all conscious things have brains, it will be intelligent.

$\forall x((Ax \land Cx) \to (\forall y(Cy \to By) \to Ix))$ OR $\forall y(Cy \to By) \to \forall x((Ax \land Cx) \to Ix)$

S5.73 Living things have brains unless they are plants, and in that case, they are neither intelligent nor conscious.

$\forall x(Ax \to ((Bx \lor Gx)) \land (Gx \to \sim(Ix \lor Cx))))$