Duration: **75 minutes**

Aids Allowed: **none**

Student Number: ⌊_�⌋_⌋_⌋_⌋_⌋_⌋_⌋_⌋

Last (Family) Name(s): _____

First (Given) Name(s): _____

---

*Do **not** turn this page until you have received the signal to start.*
In the meantime, please read the instructions below carefully.

This term test consists of 4 questions on 10 pages (including this one), printed on both sides of the paper. *When you receive the signal to start, please make sure that your copy of the test is complete, fill in the identification section above, write your student number where indicated at the bottom of every odd-numbered page (except page 1), and write your name on the back of the last page.*

Answer each question directly on the test paper, in the space provided, and use the reverse side of the pages for rough work. If you need more space for one of your solutions, use the reverse side of a page and *indicate clearly the part of your work that should be marked.*

In your answers, you may use without proof any result or theorem covered in lectures, tutorials, homework, tests, or the textbook, as long as you give a clear statement of the result(s)/theorem(s) you are using. You must justify all other facts required for your solutions.

Write up your solutions carefully! In particular, use notation and terminology correctly and explain what you are trying to do — part marks *will* be given for showing that you know the general structure of an answer, even if your solution is incomplete.

If you are unable to answer a question (or part), you will get 20% of the marks for that question (or part) if you write "I don't know" and nothing else — you will *not* get those marks if your answer is completely blank, or if it contains contradictory statements (such as "I don't know" followed or preceded by parts of a solution that have not been crossed off).

MARKING GUIDE

1: _____/12

2: _____/12

3: _____/12

4: _____/12

TOTAL: _____/48

Use this page for rough work — clearly indicate any section(s) to be marked.

Question 1. [12 MARKS]

Let P represent the set of all people and M represent the set of all meetings. Let $A \in P$ represent "Alice" and $B \in P$ represent "Bruno", and let $m_1 \in M$ represent "the first meeting" and $m_2 \in M$ represent "the second meeting". Let $S(x)$ represent the open sentence "x is a student" (for $x \in P$), $K(x, y)$ represent the open sentence "x knows y" (for $x, y \in P$), and $M(x, y)$ represent the open sentence "x attended y" (for $x \in P$ and $y \in M$).

Rewrite each of the following sentences symbolically, using only the domains and predicates defined above (in other words, you are *not* allowed to define new sets or predicates in your answers). Of course, you can (and should) use quantifiers and connectives appropriately.

Part (a) [2 MARKS]

"Alice did not attend the first meeting."

Part (b) [2 MARKS]

"Bruno knows everyone who attended the second meeting."

Part (c) [2 MARKS]

"No student attended every meeting."

Part (d) [2 MARKS]

"Bruno knows everyone that Alice knows."

Part (e) [2 MARKS]

"The first and second meetings were attended by exactly the same people."

Part (f) [2 MARKS]

"Every meeting was attended by some student that Alice does not know."

Use this page for rough work — clearly indicate any section(s) to be marked.

Question 2. [12 MARKS]

Consider the following sentences about sequences of natural numbers $a_0, a_1, a_2, \ldots$:

$$(S_1) \quad \forall i \in \mathbb{N}, \exists j \in \mathbb{N}, j < i \wedge a_j < a_i$$
$$(S_2) \quad \forall i \in \mathbb{N}, i > 0 \Rightarrow \exists j \in \mathbb{N}, j < i \wedge a_j < a_i$$
$$(S_3) \quad \exists i \in \mathbb{N}, \forall j \in \mathbb{N}, j < i \Rightarrow a_j < a_i$$

Part (a) [6 MARKS]

Rewrite each of the sentences above in clear, natural English.

(S_1):

(S_2):

(S_3):

Part (b) [3 MARKS]

Determine whether each sentence above is true or false for the following sequence and circle the appropriate answers (no justification required):

$$1, \; 2, 2, \; 3, 3, 3, \; 4, 4, 4, 4, \; 5, 5, 5, 5, 5, \; 6, 6, 6, 6, 6, 6, \; \ldots$$

(S_1) is: TRUE   FALSE   IMPOSSIBLE TO DETERMINE*   I DON'T KNOW (FOR 20%)

(S_2) is: TRUE   FALSE   IMPOSSIBLE TO DETERMINE*   I DON'T KNOW (FOR 20%)

(S_3) is: TRUE   FALSE   IMPOSSIBLE TO DETERMINE*   I DON'T KNOW (FOR 20%)

Part (c) [3 MARKS]

Determine whether each sentence above is true or false for the following sequence and circle the appropriate answers (no justification required):

$$0, \; 1, 0, \; 2, 1, \; 3, 2, \; 4, 3, \; 5, 4, \; 6, 5, \; 7, 6, \; 8, 7, \; 9, 8, \; \ldots$$

(S_1) is: TRUE   FALSE   IMPOSSIBLE TO DETERMINE*   I DON'T KNOW (FOR 20%)

(S_2) is: TRUE   FALSE   IMPOSSIBLE TO DETERMINE*   I DON'T KNOW (FOR 20%)

(S_3) is: TRUE   FALSE   IMPOSSIBLE TO DETERMINE*   I DON'T KNOW (FOR 20%)

---

*$i.e.$, there is not enough information to determine whether the sentence is true or false

Use this page for rough work — clearly indicate any section(s) to be marked.

Question 3. [12 MARKS]

Part (a) [3 MARKS]

Write the *contrapositive* of the sentence from Part (d) of Question 1, both symbolically and in natural English.

Part (b) [4 MARKS]

Rewrite the **negation** of statement (S_1) from Question 2 so that the negation symbol is "worked in" as much as possible, and give a natural English translation of the resulting statement.

Part (c) [5 MARKS]

Give a detailed proof structure that you could use to prove your statement from Part (b).

Question 4. [12 MARKS]
Part (a) [4 MARKS]

Let A, B, C be three subsets of universe U, and $A(x) : x \in A$, $B(x) : x \in B$, $C(x) : x \in C$ be the corresponding predicates. Consider the following statement:

$$(S) \quad \forall x \in U, A(x) \vee B(x) \Leftrightarrow \neg C(x).$$

In the Venn diagram on the right, put a check mark "$\checkmark$" in every region corresponding to elements for which statement (S) is true, and put an "X" in every region corresponding to elements for which statement (S) is false.
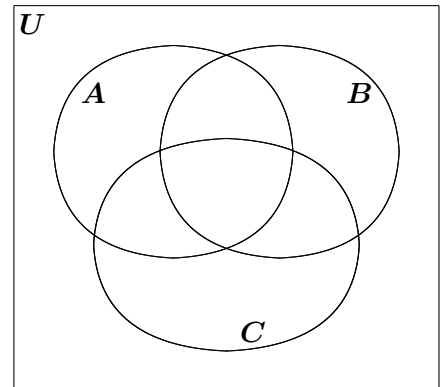
Part (b) [3 MARKS]
At a trial, four witnesses give the following testimony.

    Alice: Either I am innocent, or Danny is guilty.   **Carol:** If I am innocent, then Betty is guilty.
    Bruno: Carol and Danny are not both guilty.   **Danny:** If Alice is innocent, then so is Bruno.

Translate each statement into symbolic notation—clearly state the meaning of all your propositional or predicate symbols. (Note that each person is either innocent or guilty, but not both.)

Part (c) [5 MARKS]
Given the testimonies in Part (b), is it possible that everyone is telling the truth? If so, can you determine who is guilty and who is innocent? Explain your reasoning.

Last (Family) Name(s): _____

First (Given) Name(s): _____