Assignment IV: week of Jan. 30th

This is the 4th of the 10 assignments. You are encouraged to work on this by coming to the help sessions (Thursday 12-1 MP202, Friday 1-2 MP102) and **grouping** *up with a few other students. Teaching assistants will be at hand to help. You do not have to hand this one in.*

1. Uncertainty Principle.

- The de Broglie wavelength of a particle $\lambda_{\rm dB} = h/p$, where h is the Planck constant and p the particle momentum. Show that for a photon, with energy $E = hc/\lambda$ and zero rest mass, $\lambda_{\rm dB}$ is just its wavelength. For this exercise, you will need to use the equation that relates momentum to energy for relativistic particles, $E^2 = p^2c^2 + m^2c^4$.
- The uncertainty principle can be written in the following form,

$$(\Delta E)(\Delta t) \geq h \tag{1}$$

  where ΔE is the accuracy with which one measures the energy and Δt the precision of time measurement. And h is the Planck constant. The above equation says that, if Δt is very short, the energy of vacuum is not known with enough precision to limit its behavior to a single history. Generation of pair particles is allowed (and even expected). These pairs are called virtual pairs. Alternatively, you can imagine that energy in vaccum rapidly oscillates about zero, and if Δt is sufficiently small, we may catch the vaccum at a high energy state, with $\Delta E \approx h/\Delta t$. Calcualte the "lifetime" of a virtual proton/anti-proton pair as allowed by the uncertainty principle.

2. Light from hydrogen atoms. H-α line emission of a hydrogen atom occurs when its electron jumps from an $n = 3$ level down to an $n = 2$ level.

- Knowing that the ground state has an energy of -13.6 electron volts, calculate the energy of the $n = 2$ and $n = 3$ state. What is the energy for a H-α photon (in unit of ev, $n = 3$ to $n = 2$ transition)? Convert this to wavelength (a convenient unit will be $\mathring{A}$, angstrom).
- When spectra of a single star are taken, it is discovered that the H-α line wobbles around your above answer periodically, with an amplitude of $5\mathring{A}$. The star is likely orbiting around an invisible object. What is its apparent orbital velocity (in unit of m/s)?
- When spectra of a far-away galaxy is taken, the H-α line has been found to be shifted to a wavelength of $20,000\mathring{A}$. If you use the classical Doppler shift formula to calculate the velocity this galaxy is receding (becaues light is 'redshifted') from us, what is the value?
- We have stressed that nothing moves faster than the speed of light. So what is going on? This redshift, called the 'cosmological redshift', is actually due to the expansion of the universe – the fabric of the universe itself being stretched, together with it the wavelength of a photon.

3. Light bulb power and you. Assume that both the lightbulb filament and the human body radiate like perfect blackbodies. (Problem 4.12 of Shu. Modified here)

- How much power (in unit of of watts, 1 watt = 1 Joule/sec) would a light bulb give off if it has a filament of radiating area $1\,\mathrm{cm}^2$ and is heated up to $2,000\,\mathrm{K}$?
- If you want to make a lightbulb that is 100 times brighter (brightness means total power), how much bigger the filament area has to be? (assuming the same temperature).
- At what wavelength is the radiation peaked? Is this in the infrared, visible or ultraviolet? Why does an ordinary light bulb feel hot?
- You, with a body area of $\sim 2\,\mathrm{m}^2$ and a body temperature of $\sim 300\,\mathrm{K}$, also radiate like a blackbody. How many lightbulb power are you comparable to? which wavelength range (in unit of μm, micron) would most of your power be radiated in? To put it in context, human eyes are not sensitive to photons beyond $7000\mathring{A}$ (red), or $0.7\mu m$.

4. Warm-up exercises on the calculus we will be needing in later part of the course.

- Obtain derivatives, df/dx, for the following functions: $f(x) = 1/x$; $f(x) = 2\ln(1-x)$; $f(x) = exp(-x)$; $f(x) = \cos(2x)$; $f(x) = (3x+1)^2$; $f(x) = x^2/(3x-1)$.
- Evaluate the following definite integrals: $\int_0^4 (2x^3+3)\,dx$; $\int_1^{10} \ln x\,dx$; $\int_{-10}^{0} e^x\,dx$;
- Evaluate the following indefinite integrals: $\int e^{kx}\,dx$; $\int 7^x\,dx$; $\int x^n\,dx$; $\int e^{kx}x^2\,dx$.
- Solve the following differential equations to obtain $f(x)$ if: $df/dx = -x+2$; $df/dx = -1/x$; $df/dx = exp(x)$; $d^2f/dx^2 = -1/x^2$.

A list of physical constants in MKS (or SI) units, kg is kilogram, m is meter, K is Kelvin, s is second, N is Newton, J is Joule, C is coulomb.

elementary charge $e = 1.602 \times 10^{-19}C$

proton rest mass $m_p = 1.672 \times 10^{-27}kg$

neutron rest mass $m_n = 1.675 \times 10^{-27}kg$

hydrogen atom mass $m_H = 1.673 \times 10^{-27}kg$

electron rest mass $m_e = 9.109 \times 10^{-31}kg$

Planck constant $h = 6.625 \times 10^{-34}J \cdot s$

Speed of light in vacuum $c = 2.998 \times 10^{8}ms^{-1}$

Gravitational constant $G = 6.673 \times 10^{-11}m^3kg^{-1}s^{-2}$

Fine structure constant $\alpha = 1/137.036$

Boltzmann constant $k = 1.380 \times 10^{-23}JK^{-1}$

Electron volt $eV = 1.602 \times 10^{-19}J$

Stefan-Boltzmann constant $\sigma = 5.67 \times 10^{-8}Js^{-1}m^{-2}K^{-4}$

Coulomb constant $k_e = \frac{1}{4\pi\epsilon_0^2} = \frac{c^2\mu_0}{4\pi} = 8.988 \times 10^{9}Nm^2C^{-2}$.