Comparing two groups

intuitive understanding

$\nu_0 = 0$

$$\sigma_n^2 = \frac{\sum(y_{i,1}-(\mu+\delta))^2 + \sum(y_{i,2}-(\mu-\delta)^2)}{n_1+n_2}$$

a pooled sample estimate of the variance

$\nu_0 = \tau_0^2 = \infty$

$$\mu_n = \frac{\sum(y_{i,1}-\delta)+\sum(y_{i,2}+\delta)}{n_1+n_2}$$

$$\delta_n = \frac{\sum(y_{i,1}-\mu)-\sum(y_{i,2}-\mu)}{n_1+n_2}$$

sample estimator for $\mu$ and $\delta$.

plug in $\mu_n$ for $\mu$, and $\delta_n$ for $\delta$.

you get $\bar{y}_1 = \mu_n + \delta_n$

$\bar{y}_2 = \mu_n - \delta_n$

$\theta_1 \cdots \theta_m, \quad \mu, \tau^2, \sigma^2$

joint conditional

$$p(\theta_1 \cdots \theta_m, \mu, \tau^2, \sigma^2 / y_1 \cdots y_m)$$

$$\propto p(\mu, \tau^2, \sigma^2) \times p(\theta_1 \cdots \theta_m / \mu, \tau^2, \sigma^2)$$

$$\times p(y_1 \cdots y_m / \theta_1 \cdots \theta_m, \mu, \tau^2, \sigma^2)$$

$$= p(\mu)\, p(\tau^2)\, p(\sigma^2) \times \left\{ \prod_{j=1}^{m} p(\theta_j / \mu, \tau^2) \right\}$$

$$\times \left\{ \prod_{j=1}^{m} \prod_{i=1}^{n_j} p(y_{i,j} / \theta_j, \sigma^2) \right\}$$

Full conditional of $\mu$, and $\tau^2$

$$p(\mu / \phi, Y) \propto p(\mu, \phi / Y)\, p(\phi / Y)$$

$$\propto p(\mu) \prod p(\theta_j / \mu, \tau^2)$$

$$p(\tau^2 / \phi, Y) \propto \cancel{p(\sigma^2 /} \; p(\tau^2, \phi / Y) \cdot p(\phi / Y)$$

$$\propto p(\tau^2) \prod p(\theta_j / \mu, \tau^2)$$

$$\{\mu / \theta_1 \cdots \theta_m, \tau^2\} \sim \text{Normal}\left( \frac{m\bar{\theta}/\tau^2 + \mu_0/\gamma_0^2}{m/\tau^2 + 1/\gamma_0^2}, \left[m/\tau^2 + 1/\gamma_0^2\right]^{-1} \right)$$

$$\{1/\tau^2 / \theta_1 \cdots \theta_m, \mu\} \sim \text{Gamma}\left( \frac{\nu_0 + m}{2}, \frac{\nu_0 \tau_0^2 + \sum(\theta_j - \mu)^2}{2} \right)$$

Full conditional of $\theta_j$

$$p(\theta_j | \mu, \tau^2, \sigma^2, y_1 \cdots y_m) \propto p(\theta_j | \mu, \sigma^2) \cdot \prod_{i=1}^{n_j} p(y_{i,j} | \theta_j, \sigma^2)$$

conditionally independent of other $\theta$'s

$$\{\theta_j | y_{1,j} \cdots y_{n_j,j}, \sigma^2, \tau^2, \mu\} \sim \text{Normal}\left(\frac{n_j \bar{y}_j / \sigma^2 + \mu/\tau^2}{n_j/\sigma^2 + 1/\tau^2}, [n_j/\sigma^2 + 1/\tau^2]^{-1}\right)$$

Full conditional of $\sigma^2$

$$p(\sigma^2 | \theta_1 \cdots \theta_m, y_1 \cdots y_m) \propto p(\sigma^2) \prod_{j=1}^{m} \prod_{i=1}^{n_j} p(y_{i,j} | \theta_j, \sigma^2)$$

$$\{1/\sigma^2 | \theta, y_1 \cdots y_m\} \sim \text{gamma}\left(\frac{1}{2}[\nu_0 + \sum_{j=1}^{m} n_j], \frac{1}{2}[\nu_0 \sigma_0^2 + \underbrace{\sum_{j=1}^{m} \sum_{i=1}^{n_j} (y_{i,j} - \theta_j)^2}_{\text{SSR residual across all groups}}]\right)$$

no pf