SOME PRACTICE PROBLEMS

1. (a) The R output below gives the results of a factor analysis of 5 variables using a single factor. Note that R standardizes the variables to have mean 0 and variance 1.

```
Uniquenesses:
   x1    x2    x3    x4    x5
0.605 0.383 0.389 0.084 0.089
Loadings:
   Factor1
x1 0.628
x2 0.786
x3 0.781
x4 0.957
x5 0.954
               Factor1
SS loadings      3.449
Proportion Var   0.690

Test of the hypothesis that 1 factor is sufficient.
The chi square statistic is 2.6 on 5 degrees of freedom.
The p-value is 0.761
```

Give an estimate of the correlation matrix assuming that the one factor model holds. Does this model seem appropriate?
(b) The output below is for a 2 factor model.

```
Uniquenesses:
   x1    x2    x3    x4    x5
0.005 0.383 0.384 0.074 0.097
Loadings:
   Factor1 Factor2
x1 0.335   0.939
x2 0.734   0.280
x3 0.757   0.206
x4 0.915   0.299
x5 0.886   0.343
               Factor1 Factor2
SS loadings      2.847   1.211
```

```
Proportion Var   0.569   0.242
Cumulative Var   0.569   0.811

Test of the hypothesis that 2 factors are sufficient.
The chi square statistic is 0.46 on 1 degree of freedom.
The p-value is 0.499
```

Suppose we rotate the loadings by the matrix

$$Q = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$$

Is Q an orthogonal matrix? What are the new laodings?

2. Suppose that $f_1(x_1, x_2)$ and $f_2(x_1, x_2)$ are joint density functions defined for $0 < x_1^2 + x_2^2 < 1$ by

$$\begin{aligned} f_1(x_1, x_2) &= 1/\pi \qquad \text{for } 0 < x_1^2 + x_2^2 < 1 \\ f_2(x_1, x_2) &= 2(x_1^2 + x_2^2)/\pi \quad \text{for } 0 < x_1^2 + x_2^2 < 1 \end{aligned}$$

and suppose that $\boldsymbol{X} = (X_1, X_2)$ is generated by $f_1(x_1, x_2)$ with probability 1/3 and by $f_2(x_1, x_2)$ with probability 2/3.

(a) Consider the classification rule that classifies $\boldsymbol{X} = \boldsymbol{x} = (x_1, x_2)$ as coming from f_2 if $x_1 > 0$. Find the probablity of misclassification for this rule.

(b) Find the optimal classification rule (i.e. the Bayes rule).

(c) Suppose only we know only the signs of X_1 and X_2 are known. What can be said about the optimal classification rule in this case.

3. Describe the differences between ICA and PCA.

4. Describe how principal components can be used to construct estimates for factor analysis.

5. Consider the following distance matrix:

```
     1  2  3  4
===============
2    5
3    9  3
4   10  7  4
5   16 11  8  6
```

We want to use this matrix to construct hierarchical clustering trees using single and complete linkage clustering.

(a) Which two observations are in the first cluster?

(b) Construct the single linkage and complete linkage clustering trees.