JAN 7, 2013
Vitali Kapovitch 416-978-6786 HU1023
email: vtk@math.toronto.edu

TA: Parker Glynn-Adey 416-978-3646 HU 1026
email: parker.glynn.adey@utoronto.ca

course homepage: www.math.toronto.edu/vtk/246Winter2013
Text: "Beautiful Mathematics"

Course Grade:
Homework 20% + Term Test 30% + Final Exam 50%

Assignment:
Posted every Friday, due next Friday at the lecture.

Exam dates:
Term test: Wednesday, Feb 27 3-5 pm
Final: TBA

Induction

$1 = 1 = 1^2$
$1+3 = 4 = 2^2$
$1+3+5 = 9 = 3^2$
$1+3+5+7 = 16 = 4^2$
$1+3+5+7+9 = 25 = 5^2$
$\dots$

$1+3+5+\dots+(2n-1) = n^2$ (General guess) Claim: This is True

Natural numbers: 1, 2, 3, 4, 5, ...
Suppose we want to prove a certain statement for all natural n.

① Check for $n=1$ (Case of induction)
② prove that if the statement is true for n then it is true for $n+1$ where $n \geq 1$ any natural number.

→ the statement holds for all $n \geq 1$.
check for $n=1$.
put $n=1 \Rightarrow$ by part ② the statement holds for $1+1=2$
put $n=2 \Rightarrow$ by ...... $2+1=3$
put $n=3 \Rightarrow$ by ...... $3+1=4$, etc.

① 2 3 4 5 ...
② ② ② ② ② part②
Back to →

$\forall n \geq 1 \quad 1+3+5+\cdots+(2n-1)=n^2$ Claim

① check for $n=1$

$1=1^2$ ✓

② suppose holds for some $n\geq 1 \Rightarrow$ we want to show it holds for $n+1$

$1+3+5+\cdots+(2n-1)+(2(n+1)-1)=n^2+2n+2-1=n^2+2n+1=(n+1)^2$ ✓

Ex: $1+2+2^2+2^3+\cdots+2^n=2^{n+1}-1$

$n=1,2,3,\cdots$

$1+2=3=4-1=2^2-1$

$1+2+2^2=7=8-1=2^3-1$

$1+2+2^2+2^3=15=16-1=2^4-1$

Proof: ① Check for $n=1$ $\quad 1+2^1=3=2^2-1$ ✓

② if the formula holds for $n \Rightarrow$ it holds for $n+1$

Let $n\geq 1$ and suppose

$1+2^1+2^2+\cdots+2^n=2^{n+1}-1$

look at the formula for $n+1$

$1+2+\cdots+2^n+2^{n+1}=2^{n+1}+2^{n+1}-1=2\cdot 2^{n+1}-1=2^{n+2}-1=2^{(n+1)+1}-1$ ✓

$1+a+a^2+\cdots+a^n=\frac{a^{n+1}-1}{a-1}$ Claim this holds for all $n\geq 1$ and any $a\neq 1$.

$$\boxed{1+a+a^2+\cdots+a^n=\begin{cases}\frac{a^{n+1}-1}{a-1} & \text{if } a\neq 1\\ n+1 & \text{if } a=1\end{cases}}$$

Let $a\neq 1$

proof by induction

① For $n=1$, $1+a=\frac{a^{1+1}-1}{a-1}=\frac{a^2-1}{a-1}=a+1$

② induction step Suppose $1+a+\cdots+a^n=\frac{a^{n+1}-1}{a-1}$

want to prove $1+\cdots+a^n+a^{n+1}\overset{?}{=}\frac{a^{(n+1)+1}-1}{a-1}$

$1+\cdots+a^n+a^{n+1}$

$$\frac{a^{n+1}-1}{a-1}+a^{n+1}=\frac{a^{n+1}-1+a^{n+1}(a-1)}{a-1}=\frac{a^{n+1}-1+a^{n+1}\cdot a-a^{n+1}}{a-1}=\frac{a^{n+2}-1}{a-1}=\frac{a^{(n+1)+1}-1}{a-1}$$