INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

12 April 2016 (am)

Subject CT1 – Financial Mathematics
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 12 questions, beginning your answer to each question on a new page.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 List the characteristics of convertible bonds. [3]

2 An insurance company has liabilities of £6 million due in exactly 8 years' time and a further £11 million due in exactly 15 years' time.

The assets held by the insurance company consist of:

- a 5-year zero-coupon bond of nominal amount £5.5088 million; and
- a 20-year zero-coupon bond of nominal amount £13.7969 million.

The current rate of interest is 8% per annum effective at all durations.

(i) Show that the first two conditions of Redington's theory for immunisation against small changes in the rate of interest are satisfied. [5]

(ii) Explain, without doing any further calculations, whether the insurance company will be immunised against small changes in the rate of interest. [2]
[Total 7]

3 At time $t = 0$, the one-year zero-coupon yield is 4% per annum effective and the one-year forward rate per annum effective at time t ($t = 1, 2, \ldots$) is given by:

$$f_{t,1} = (4 + t)\%.$$

(i) Determine the issue price per £100 nominal of a three-year 4% coupon bond issued at time $t = 0$, paying coupons annually in arrear and redeemable at 105%. [4]

(ii) Determine the three-year par yield at time $t = 0$. [3]
[Total 7]

4 A loan of nominal amount £100,000 is to be issued bearing coupons payable quarterly in arrear at a rate of 7% per annum. Capital is to be redeemed at £108 per £100 nominal on a coupon date between 15 and 20 years inclusive after the date of issue. The date of redemption is at the option of the borrower.

An investor who is liable to income tax at 25% and capital gains tax at 40% wishes to purchase the entire loan at the date of issue.

(i) Determine the price which the investor should pay to ensure a net effective yield of at least 5% per annum. [5]

(ii) Explain the significance of the redemption date being at the option of the borrower in relation to your calculation in part (i). [2]
[Total 7]

5 A loan is to be repaid by a series of instalments payable annually in arrear for 15 years. The first instalment is £1,200 and payments increase thereafter by £250 per annum.

Repayments are calculated using a rate of interest of 6% per annum effective.

Determine:

(i) the amount of the loan. [3]

(ii) the capital outstanding immediately after the 9th instalment has been made. [2]

(iii) the capital and interest components of the final instalment. [2]
[Total 7]

6 The force of interest, $\delta(t)$, is a function of time and at any time t, measured in years, is given by the formula:

$$\delta(t) = \begin{cases} 0.06 & 0 \le t \le 4 \\ 0.10 - 0.01t & 4 < t \le 7 \\ 0.01t - 0.04 & 7 < t \end{cases}.$$

(i) Calculate, showing all working, the value at time $t = 5$ of £10,000 due for payment at time $t = 10$. [5]

(ii) Calculate the constant rate of discount per annum convertible monthly which leads to the same result as in part (i). [2]
[Total 7]

7 A one-year forward contract on a share was agreed on 1 September 2015 when the share price was £8.70 and the risk-free force of interest was 7% per annum. The stock was expected to pay a dividend of £1.10 eight months after the date of issue.

The price of the share was £9.90 on 1 February 2016 and the risk-free force of interest was 6.5% per annum. The dividend expectation was unchanged.

Calculate, showing all working, the value of the contract to the holder of the long forward position on 1 February 2016. [7]

8 An individual is planning to purchase £100,000 nominal of a bond on 1 June 2016 which will be redeemable at 110% on 1 June 2020. The bond will pay coupons of 3% per annum at the end of each year.

The individual wishes to invest the coupon payments on deposit until the bond is redeemed. It is assumed that, in any year, there is a 55% probability that the rate of interest will be 6% per annum effective and a 45% probability that it will be 5.5% per annum effective. It is also assumed that the rate of interest in any one year is independent of that in any other year.

(i) Derive the necessary formula to determine the mean value of the total accumulated investment on 1 June 2020. [4]

(ii) Calculate the mean value of the total accumulated investment on 1 June 2020. [2]

[Total 6]

9 In January 2014, the government of a country issued an index-linked bond with a term of two years. Coupons were payable half-yearly in arrear, and the annual nominal coupon rate was 6%. The redemption value, before indexing, was £100 per £100 nominal. Interest and capital payments were indexed by reference to the value of an inflation index with a time lag of six months.

A tax-exempt investor purchased £100,000 nominal at issue and held it to redemption. The issue price was £97 per £100 nominal.

The inflation index was as follows:

Date	*Inflation Index*
July 2013	120.0
January 2014	122.3
July 2014	124.9
January 2015	127.2
July 2015	129.1
January 2016	131.8

(i) Set out a schedule of the investor's cashflows, showing the amount and month of each cashflow. [3]

(ii) Determine the annual effective real yield obtained by the investor to the nearest 0.1% per annum. [5]

[Total 8]

10 The following table gives information concerning a fund held by an investment manager:

Year	*2012*	*2013*	*2014*	*2015*
Value of fund at 30 June	–	12,700,000	13,000,000	14,100,000
Net cash flow received on 1 July	–	2,600,000	–3,700,000	1,800,000
Value of fund at 31 December	12,000,000	13,500,000	12,900,000	17,200,000

(i) Calculate, to the nearest 0.1% and showing all working, the annual effective time-weighted rate of return (TWRR) achieved by the fund during the period from 31 December 2012 to 31 December 2015. [3]

(ii) Show that the annual effective money-weighted rate of return (MWRR) achieved by the fund over the same period is less than the answer obtained in part (i) above. [2]

(iii) Explain why you would expect the outcome described in part (ii) for this fund. [2]

(iv) Explain which of the two measures referred to in parts (i) and (ii) is a better indicator of the investment manager's performance over the period. [2]

[Total 9]

11 An investor is considering the purchase of 10,000 ordinary shares in Enterprise plc.

Dividends from the shares are payable half-yearly in arrear. The next dividend is due in exactly six months and is expected to be 6.5 pence per share.

The required rate of return is 6% per half-year effective and an estimated rate of future dividend growth is 2% per half-year.

(i) Calculate, showing all working, the maximum price that the investor should pay for the shares. [4]

As a result of a recently announced expansion plan, the investor increases the estimated rate of future dividend growth to 2.5% per half-year.

(ii) (a) Calculate, showing all working, the maximum price the investor should now pay for the shares.

(b) Explain the difference between your answers to part (i) and part (ii)(a). [2]

It is rumoured that new legislation may affect the operation of Enterprise plc.

As a result, the investor decides to increase her required rate of return to 7% per half-year effective. The estimated dividend growth rate remains at 2% per half-year

(iii) (a) Explain why it might be appropriate for the investor to increase her required rate of return.

(b) Calculate the maximum price that the investor should now pay for the shares.

(c) Explain the difference between your answers to part (i) and part (iii)(b). [3]

In the prevailing economic circumstances, investors are expecting lower inflation in the wider economy.

As a result, the investor decides to reduce both the assumed rate of dividend growth and her required rate of return to 1% and 5% per half-year effective respectively.

(iv) (a) Explain why it is appropriate for the investor to reduce both the future dividend growth rate and the required rate of return in this case.

(b) Calculate the maximum price that the investor should now pay for the shares.

(c) Explain the difference between your answers to part (i) and part (iv)(b). [5]

[Total 14]

12 (i) Show that:

$$\left(\bar{I}\bar{a}\right)_{\overline{n}|} = \frac{\bar{a}_{\overline{n}|} - nv^n}{\delta}.$$ [4]

A company is considering the purchase of a gold mine which has recently ceased production.

The company forecasts that:

- the cost of re-opening the mine will be \$900,000, which will be incurred continuously over the first twelve months.
- additional costs are expected to be constant throughout the term of the project at \$200,000 per annum, excluding the first year. These are also incurred continuously.
- after the first twelve months, the rate of revenue will grow continuously and linearly from zero per annum to \$3,600,000 per annum at a constant rate of \$300,000 per annum.
- when the rate of revenue reaches \$3,600,000 per annum it will then decline continuously and linearly at a constant rate of \$150,000 per annum until it reaches \$600,000 per annum.
- when the rate of revenue declines to \$600,000 per annum production will stop and the mine will have zero value.

(ii) Determine the overall term of the project. [2]

(iii) Calculate, showing all working, the price that the company should pay in order to earn an internal rate of return (IRR) of 25% per annum effective. [12]

[Total 18]

END OF PAPER

INSTITUTE AND FACULTY OF ACTUARIES

EXAMINERS' REPORT

April 2016

Subject CT1 – Financial Mathematics
Core Technical

Introduction

The Examiners' Report is written by the Principal Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and using past papers as a revision aid and also those who have previously failed the subject.

The Examiners are charged by Council with examining the published syllabus. The Examiners have access to the Core Reading, which is designed to interpret the syllabus, and will generally base questions around it but are not required to examine the content of Core Reading specifically or exclusively.

For numerical questions the Examiners' preferred approach to the solution is reproduced in this report; other valid approaches are given appropriate credit. For essay-style questions, particularly the open-ended questions in the later subjects, the report may contain more points than the Examiners will expect from a solution that scores full marks.

The report is written based on the legislative and regulatory context pertaining to the date that the examination was set. Candidates should take into account the possibility that circumstances may have changed if using these reports for revision.

F Layton
Chair of the Board of Examiners
June 2016

A. General comments on the *aims of this subject and how it is marked*

1. CT1 provides a grounding in financial mathematics and its simple applications. It introduces compound interest, the time value of money and discounted cashflow techniques which are fundamental building blocks for most actuarial work.

2. Please note that different answers may be obtained to those shown in these solutions depending on whether figures obtained from tables or from calculators are used in the calculations but candidates are not penalised for this. However, candidates may be penalised where excessive rounding has been used or where insufficient working is shown.

B. General comments on *student performance in this diet of the examination*

1. The comments that follow the questions concentrate on areas where candidates could have improved their performance. Where no comment is made the question was generally answered well by most candidates.

2. Performance was of a similar standard to most recent diets with the weaker performance in September 2015 being an exception to the general standard. As in previous diets, the non-numerical questions were often answered poorly by marginal candidates.

C. Comparative Pass Rates for the past 3 years for this diet of examination

Year	*%*
April 2016	55
September 2015	44
April 2015	55
September 2014	57
April 2014	60
September 2013	57

Reasons for any significant change in Pass Rates in current diet to those in the past:

The Pass Rate is consistent most recent diets, again noting that September 2015 was an exception to the usual standard.

D. Pass Mark

The Pass Mark for this exam was 60%.

Solutions

Q1 Convertible Securities

- Generally unsecured loan stocks.
- Can be converted into ordinary shares of the issuing company.
- Pay interest/coupons until conversion.
- The date of conversion might be a single date or, at the option of the holder, one of a series of specified dates.
- Risk characteristics of convertible vary as the final date for convertibility approaches (behaviour will tend towards the security into which it is likely to convert)
- Generally less volatility than in the underlying share price before conversion.
- Combine lower risk of debt securities with the potential for gains from equity investment.
- Security and marketability depend upon issuer.
- Generally provide higher income than ordinary shares and lower income than conventional loan stock or preference shares.
- Option to convert will have time value which is reflected in price of the security.

Despite being a bookwork question, this was answered poorly. This performance is consistent with questions in previous years on the same area of the syllabus.

Q2 (i) PV of asset proceeds is:

$$V_A(0.08) = 5.5088v_{8\%}^5 + 13.7969v_{8\%}^{20} = 6.7093$$

PV of liability outgo is:

$$V_L(0.08) = 6v_{8\%}^8 + 11v_{8\%}^{15} = 6.7093 = V_A(0.08)$$

Hence, condition (1) for immunisation is satisfied.

Also, DMT of asset proceeds is:

$$\tau_A(0.08) = \frac{5\times 5.5088v_{8\%}^5 + 20\times 13.7969v_{8\%}^{20}}{6.7093} = 11.618$$

And, DMT of liability outgo is:

$$\tau_L(0.08) = \frac{8\times 6v_{8\%}^8 + 15\times 11v_{8\%}^{15}}{6.7093} = 11.618 = \tau_A(0.08)$$

Hence, condition (2) for immunisation is also satisfied.

(ii) Yes, the insurance company is immunised.

As the asset proceeds are received at times 5 and 20, whereas the liability outgo is paid at times 8 and 15, the spread of the asset proceeds around the DMT is greater than the spread of the liability outgo around the same DMT.

> Part (i) was generally answered well; however, candidates must include sufficient factors and workings to demonstrate that the respective asset and liability values are the same for each of the two conditions. Part (ii) was often answered poorly. To get full marks for this part, candidates were required to make reference to the actual data in the question rather than just repeating the theory (e.g. stating the actual figures for the spread of the assets and the liabilities around the DMT).

Q3 (i) Issue price (per £100 nominal) = $\frac{4}{1+i_i}+\frac{4}{(1+i_2)^2}+\frac{4}{(1+i_3)^3}+\frac{105}{(1+i_3)^3}$

where i_t is the t-year zero coupon rate at time t = 0 and we have that:

$$\left(1+i_{t-1}\right)^{t-1}*(1+f_{t-1,1})=(1+i_t)^t$$

where $f_{t-1,1}$ is the one-year forward rate at time $t-1$

we have $1+i_1 = 1.04$ (i_1 is given)

$$(1+i_2)^2 = (1+i_1)(1+f_{1,1})$$

$$= 1.04*1.05$$

$$(1+i_3)^3 = (1+i_2)^2(1+f_{2,1})$$

$$= 1.04*1.05*1.06$$

$$\Rightarrow \text{Issue Price} = \frac{4}{1.04}+\frac{4}{1.04*1.05}+\frac{4+105}{1.04*1.05*1.06}$$

$$= £101.68$$

(ii) Let y_{c_3} be the 3 year par yield (%). Then y_{c_3} is given by

$$100 = y_{c_3}\left(\frac{1}{1+i_1}+\frac{1}{(1+i_2)^2}+\frac{1}{(1+i_3)^3}\right)+\frac{100}{(1+i_3)^3}$$

$$= y_{c_3}\left(\frac{1}{1.04}+\frac{1}{1.04*1.05}+\frac{1}{1.04*1.05*1.06}\right)+\frac{100}{1.04*1.05*1.06}$$

$$= y_{c_3} * 2.741205336 + 86.39159583$$

$$\Rightarrow y_{c_3} = 4.9644\%$$

Generally well answered.

Q4 (i) $\left(1+\frac{i^{(4)}}{4}\right)^4 = 1.05 \Rightarrow i^{(4)} = 0.049089$

$$\frac{D}{R}(1-t_1) = \frac{0.07}{1.08} \times 0.75 = 0.04861$$

$$\Rightarrow i^{(4)} > (1-t_1)g$$

$\Rightarrow$ Capital gain on contract and we assume loan is redeemed as late as possible (i.e. after 20 years) to obtain minimum yield.

Let price of stock = P

$$P = 0.07 \times 100000 \times 0.75 \times a_{\overline{20}|}^{(4)} + (108000 - 0.40(108000 - P))v^{20} \text{ at } 5\%$$

$$\Rightarrow P = \frac{5250\, a_{\overline{20}|}^{(4)} + 64800\, v^{20}}{1 - 0.40\, v^{20}}$$

$$= \frac{5250 \times 1.018559 \times 12.4622 + 64800 \times 0.37689}{1 - 0.40 \times 0.37689}$$

$$= £107,228.63$$

(above uses factors from Formulae and Tables Book – exact answer is £107,228.67)

(ii) As the redemption date is at the option of the borrower, it is outside the investor's control when the stock will be redeemed. Hence the investor must assume a worst case scenario in pricing the loan.

> Part (i) was answered well. The reasoning of marginal candidates in part (ii) was often unclear. The key point is that the date of redemption is out of the control of the investor.

Q5 (i) Loan = $950\, a_{\overline{15|}} + 250(Ia)_{\overline{15|}}$ at 6%

= 950 × 9.7122 + 250 × 67.2668

= £26,043.29

(ii) Capital outstanding after 9 payments:

$$3200\, a_{\overline{6|}} + 250(Ia)_{\overline{6|}} = 3200 \times 4.9173 + 250 \times 16.3767 = £19,829.54$$

(iii) Capital outstanding after 14 payments = 4700v at 6%
= £4,433.96
= Capital in final payment

⇒ Interest in final payment = 4700 – 4433.96
= £266.04

(above uses factors from Formulae and Tables Book – exact answers are £26,043.34 for (i) and £19,829.61 for (ii))

> The best answered question on the paper.

Q6 (i)

$$pv = 10,000 \times \exp\left[-\int_7^{10}\left(0.01t - 0.04\right)dt\right] \times \exp\left[-\int_5^7\left(0.10 - 0.01t\right)dt\right]$$

$$= 10,000\exp\times\left(-\left[\frac{0.01t^2}{2} - 0.04t\right]_7^{10}\right) \times \exp\left(-\left[0.10t - \frac{0.01t^2}{2}\right]_5^7\right)$$

$$= 10,000 \times \exp\left(-\left[\frac{0.01*51}{2} - 0.04 \times 3\right]\right) \times \exp\left(-\left[0.10*2 - \frac{0.01*24}{2}\right]\right)$$

= 10,000exp(–0.255 + 0.12 – 0.20 + 0.12)

= 10,000exp(–0.215)

= £8,065.41

(ii) Required discount rate p.a. convertible monthly is given by

$$10,000\left(1-\frac{d^{(12)}}{12}\right)^{12\times 5} = 8,065.41$$

$d^{(12)}$ = 4.2923% p.a. convertible monthly.

Generally well answered.

Q7 Forward price of the contract is:

$$K_0 = (S_0 - I)e^{\delta T} = (8.70 - I)e^{0.07}$$

where I is the present value of the income expected during the contract

$$\Rightarrow I = 1.10 \times e^{-0.07\times 8/12} = 1.049846$$

$$\Rightarrow K_0 = (8.70 - 1.049846)\times e^{0.07} = 8.204853$$

Forward price of contract set up at time r (where r = 5 months) is

$$K_r = (S_r - I_r)e^{\delta(T-r)}$$

where I_r is the value at time r of the income expected during the contract

$$= 1.10 \times e^{-0.065\times 3/12} = 1.082269$$

$$\Rightarrow K_r = (9.90 - 1.082269)e^{0.065\times 7/12} = 9.158489$$

Value of original forward contract

$$= (K_r - K_0)e^{-\delta(T-r)}$$

$$= (9.158489 - 8.204853)e^{-0.065\times\frac{7}{12}}$$

= 0.918154

= £0.92

> Although this question was answered better than similar questions in past diets, the workings shown by marginal candidates were often unclear.

Q8 (i) Work in £000's

Let total accumulation at 1/6/20 be X, and i_y = investment return for the year starting from 1 June 2016 + y

$$E(X) = E\left[3(1+i_1)(1+i_2)(1+i_3) + 3(1+i_2)(1+i_3) + 3(1+i_3) + 3 + 110\right]$$

Due to independence:

$$E(X) = 3\left[E(1+i_1)E(1+i_2)E(1+i_3) + E(1+i_2)E(1+i_3) + E(1+i_3)\right] + 113$$

$$= 3\left[(1+E[i_1])(1+E[i_2])(1+E[i_3]) + (1+E[i_2])(1+E[i_3]) + (1+E[i_3])\right] + 113$$

where $E(i_y)$ = 0.55 × 6% + 0.45 × 5.5%

= 5.775%

(ii) $E(X) = 3\,\ddot{s}_{\overline{3}|}^{5.775\%} + 113$

$$= 3\left(\frac{1.05775^3 - 1}{0.05775/1.05775}\right) + 113$$

= 123.080 (= £123,080 for £100,000 nominal)

Whilst the calculations were often correct, relatively few candidates followed the instructions to derive the required formula for these calculations. For full marks, such derivation was required including identifying where the independence assumption is used.

Q9 (i) Cash Flows:

Issue Price:	Jan 14	$-0.97\times 100{,}000 = -£97{,}000$
Interest Payments:	July 14	$0.03\times 100{,}000 \times \frac{122.3}{120.0} = £3{,}057.50$
	Jan 15	$0.03\times 100{,}000 \times \frac{124.9}{120.0} = £3{,}122.50$
	July 15	$0.03\times 100{,}000 \times \frac{127.2}{120.0} = £3{,}180.00$
	Jan 16	$0.03\times 100{,}000 \times \frac{129.1}{120.0} = £3{,}227.50$
Capital redeemed:	Jan 16	$100{,}000 \times \frac{129.1}{120.0} = £107{,}583.33$

(ii) Express all amounts in "January 2014 money", and we get:

$$97000 = 3057.50 \times \frac{122.3}{124.9} v^{\frac{1}{2}} + 3122.50 \times \frac{122.3}{127.2}\, v$$

$$+3180.00 \times \frac{122.3}{129.1} v^{1\frac{1}{2}} + \frac{122.3}{131.8} \times (107583.33 + 3227.50) v^2$$

$$\Rightarrow 97000 = 2993.85 v^{\frac{1}{2}} + 3002.22 v + 3012.50 v^{1\frac{1}{2}} + 102823.71 v^2$$

Try 7%, RHS = 98232.04
8%, RHS = 96499.48

$$i = 0.07 + \left(\frac{98232.04 - 97000}{98232.04 - 96499.48} \right) \times 0.01$$

= 7.7% p.a. effective real yield (exact answer is 7.708%).

> This question seemed to strongly differentiate between stronger and weaker candidates. Common errors from the latter included not correctly allowing for the time lag in part (i) or not uplifting the nominal cashflows for inflation at all.

Q10 (i) TWRR is i such that:

$$(1+i)^3 = \frac{12,700}{12,000} \times \frac{13,000}{12,700+2,600} \times \frac{14,100}{13,000-3,700} \times \frac{17,200}{14,100+1,800}$$

$= 1.474830 \Rightarrow i = 13.8\%$

(ii) If the MWRR achieved by the fund were 13.8% p.a., then fund value at 31 December 2015 would be (in £000's):

$$12000 \times (1.138)^3 + 2600 \times (1.138)^{2\frac{1}{2}} - 3700 \times (1.138)^{1\frac{1}{2}} + 1800 \times (1.138)^{\frac{1}{2}}$$

=18,706 which is greater than 17,200. This means that the MWRR must be less than 13.8% p.a.

(iii) The MWRR is lower because the fund performed badly immediately after receiving the large positive cash flow in July 2013 and also performed well immediately after the large negative cash flow in July 2014.

(iv) The TWRR is not influenced by the amount and timing of the cash flows (which are generally considered to be outside of the control of the fund manager) and, thus, better reflects the manager's performance over the period.

> Parts (i) and (ii) were answered well. In part (ii), it is not necessary to calculate the MWRR.
>
> As in previous diets, candidates had difficulty explaining the relative values for the MWRR and TWRR. For full marks in part (iii), candidates needed to make reference to the actual data in the question. In part (iv), the key point is that the amount and timing of the cash flows are generally considered to be outside of the control of the fund manager.

Q11 (i) 10,000 shares give a total dividend on the next payment date of £650.

Then, working in half-year periods, we have:

$$V = 650 \times \left(v_{6\%} + 1.02 v_{6\%}^2 + 1.02^2 v_{6\%}^3 + \ldots \right)$$

$$= 650 v_{6\%} \times \left(1 + 1.02 v_{6\%} + \left(1.02 v_{6\%}\right)^2 + \ldots \right)$$

$$= 650 v_{6\%} \times \left(\frac{1}{1 - 1.02 v_{6\%}} \right)$$

$$= £16{,}250$$

(ii) (a) We now have

$$v = 650 v_{6\%} \times \left(\frac{1}{1 - 1.025 v_{6\%}} \right) = £18{,}571.43$$

(b) The higher rate of dividend growth means that expected future dividend income is increased and, thus, the investor is prepared to pay a higher price to purchase the shares.

(iii) (a) The rumoured change in legislation might be thought of as increasing the uncertainty of the future growth prospects for the company (without necessarily either increasing or decreasing them).

Thus it is appropriate that the investor requires a higher return to compensate for this greater uncertainty.

(b) We now have:

$$v = 650 v_{7\%} \times \left(\frac{1}{1 - 1.02 v_{7\%}} \right) = £13{,}000$$

(c) The higher risk (as reflected by the higher effective rate of return required) means that the investor is now prepared to pay a lower maximum price to purchase the shares.

(iv) (a) Lower inflation is likely to lead to lower (nominal) profits and, thus, lower (nominal) dividend payments.

Also, as many investors are more concerned with real returns (i.e. in excess of inflation), it is appropriate to reduce the effective rate of return to reflect the lower expected inflation.

(b) We now have:

$$v = 650v_{5\%} \times \left(\frac{1}{1 - 1.01v_{5\%}} \right) = £16,250$$

(c) In this case, the maximum price that the investor is prepared to pay is unchanged. Lower expected inflation leads to lower nominal dividend payments, which are then discounted at a lower nominal interest rate. Thus, the price is unaffected (i.e. equities are a real asset).

> The calculations in this question were relatively simple and generally done well. The explanatory parts of the questions were answered better than expected.

Q12 (i)

$$(\bar{I}\bar{a})_{\overline{n}|} = \int_0^n te^{-\delta t} dt = \left[t \times \frac{e^{-\delta t}}{-\delta} \right]_0^n - \int_0^n \frac{e^{-\delta t}}{-\delta} \, dt$$

$$= -\frac{n.e^{-\delta n}}{\delta} + \frac{1}{\delta} \int_0^n e^{-\delta t} \, dt$$

$$= -\frac{n.e^{-\delta n}}{\delta} + \frac{1}{\delta} \left[-\frac{e^{-\delta t}}{\delta} \right]_o^n$$

$$= -\frac{n.e^{-\delta n}}{\delta} + \frac{1}{\delta} \left[\frac{1 - e^{-\delta n}}{\delta} \right] = \frac{\bar{a}_{\overline{n}|} - nv^n}{\delta}$$

(ii) Project lasts for 33 years as follows:

Time take to reopen mine = 1 year

Time taken for net revenue to go from zero to \$3,600,000 is 12 years

$$\left(\text{from } \frac{3,600,000}{300,000} = 12 \right)$$

Time taken for net revenue to decline to \$600,000 is 20 years

$$\left(\text{from } \frac{3,600,000 - 600,000}{150,000} = 20 \right)$$

(iii) PV of reopening costs and additional costs $= 700,000\ \bar{a}_{\overline{1|}} + 200,000\ \bar{a}_{\overline{33|}}$ at 25%

where

$$\bar{a}_{\overline{1|}}^{25\%} = \frac{i}{\delta} a_{\overline{1|}} = 1.120355 \times 0.8 = 0.896284$$

$$\bar{a}_{\overline{33|}}^{25\%} = \frac{i}{\delta}\ .\ a_{\overline{33|}} = 1.120355 \times 3.9975 = 4.478619$$

$\Rightarrow$ PV = 627,399 + 895,724 = 1,523,123

PV of net revenue

$$v.\ 300,000\left(I\bar{a}\right)_{\overline{12|}} + v^{13}\left\{3,600,000\ \bar{a}_{\overline{20|}} - 150,000\left(I\bar{a}\right)_{\overline{20|}}\right\}$$

where $\bar{a}_{\overline{12|}} = \frac{i}{\delta}.\ a_{\overline{12|}} = 1.120355 \times 3.7251 = 4.173434$

$$\left(I\bar{a}\right)_{\overline{12|}} = \frac{\bar{a}_{\overline{12|}} - 12v^{12}}{\delta} = \frac{4.173434 - 12 \times 0.06872}{0.223144} = 15.0073$$

$$\bar{a}_{\overline{20|}} = \frac{i}{\delta}.\ a_{\overline{20|}} = 1.120355 \times 3.9539 = 4.429772$$

$$\left(I\bar{a}\right)_{\overline{20|}} = \frac{4.429772 - 20v^{20}}{\delta} = 18.818324$$

$\Rightarrow$ PV of net revenue

$$= \frac{300,000}{1.25} \times 15.0073 + 0.05498\{3,600,000 \times 4.429772 - 150,000 \times 18.818324\}$$

= 3,601,752 + 721,581 = 4,323,333

$\Rightarrow$ Price to obtain IRR of 25% p.a. is:

4,323,333 – 1,523,123 = \$2,800,210.

(above uses factors from Formulae and Tables Book – exact answer is 4,323,319 – 1,523,115 = \$2,800,204)

The proof in part (i) was answered very poorly. Also, since the result is given, candidates must provide enough steps in deriving the result to convince the examiners that they haven't just jumped to the result. In part (iii), the workings of many marginal candidates were very unclear. The examiners recommend that candidates set out their working clearly e.g. by calculating each component of the costs and benefits separately. This enables examiners to give full credit for correct working even if errors are made in the calculations.

END OF EXAMINERS' REPORT