Hypothesis test on β_1

Step I (Specify Hypotheses)

$H_0: \beta_1 = 0$ ← specific

$H_A: \beta_1 > 0$ ← general

Step II (Test statistic)

Standardised test statistic

$$t = \frac{\hat{\beta}_1 - E[\beta_1 \mid H_0]}{\hat{se}(\hat{\beta}_1)} \sim t_{n-2}$$

← under H_0 this is 0

← estimate

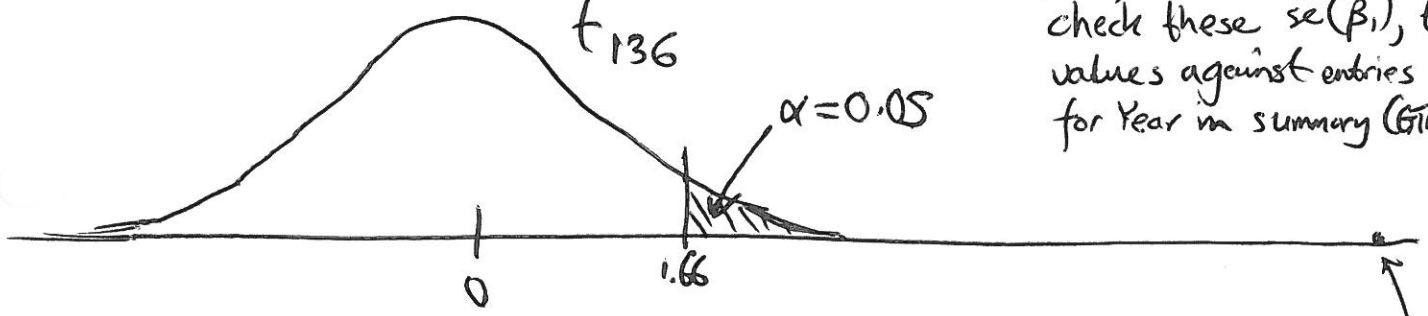
where $\hat{se}(\hat{\beta}_1) = \sqrt{\frac{\hat{\sigma}^2}{S_{xx}}} = \frac{\hat{\sigma}}{\sqrt{S_{xx}}}$

Step III (Decision Rule)

Reject H_0 at $\alpha = 0.05$

if observed test statistic $> t_{n-2=136,\ 0.95}$ ($1-\alpha$) $= 1.66$

in R, check these $\hat{se}(\hat{\beta}_1)$, t values against entries for Year in summary(GTA.lm)



Step IV (Calculations)

Observed test statistic

$$t = \frac{\hat{\beta}_1 - 0}{\hat{se}(\hat{\beta}_1)} = \frac{0.007045 - 0}{\sqrt{\hat{\sigma}^2 / S_{xx}}} = \frac{0.007045}{\sqrt{\frac{0.0259}{218{,}994.5}}} \doteq 20.48$$

in R, use 137*var(Year)

Step V (Conclusion)

As $20.48 >> 1.66$, reject H_0 in favour of H_A & conclude $\beta_1 > 0$

$\Rightarrow$ the slope of the linear regression model relating average temperature anomalies to year (time) is significantly positive $\Rightarrow$ temperatures have been increasing over time!

Finally, we need to assess the analysis & the results

? – is the data we used the best available to address the research question?
(issue of the science)

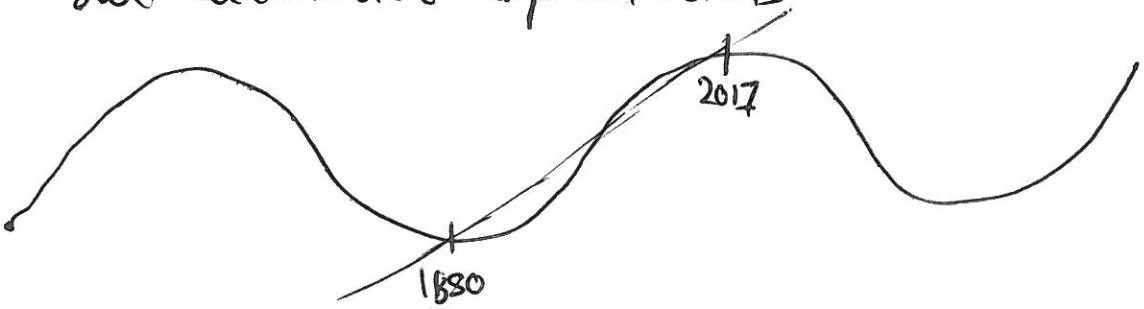
? – is the model a good fit?
(is the model not only appropriate, but also useful?)

coefficient of determination → $R^2 = \frac{SS_{Regression}}{SS_{Total}} = 75.5\%$
(just a summary measure)

↘ better answer is look at the assumptions underlying the model & check if they are satisfied
→ residual plots (here that means we still have a caveat or!)

! – linear models just address association (correlation), they do not necessarily reflect causation
(no measure of possible causal factors in these data)

! – also this analysis does not necessarily rule out alternative explanations



In summary, model & analysis are indicative of a positive (association), an increase in global av. temp. over time, but the research question is wider than that and we need to further research (more data, more analysis) do