MATH 315; HOMEWORK # 2

Due Jan 26, 2015

1. (Exercise 6.2 (a)) Describe all integer solutions to $105x + 121y = 1$.

2. (Exercise 6.4 (c)) Find an integer solution of $155x + 341y + 385z = 1$. [Hint: $gcd(341, 385) = 11$. Write the equation as $155x + 11(31y + 35z) = 1$. First solve the equation $155x + 11u = 1$ and then solve $31y' + 35z' = 1$.]

3. (Exercise 7.6) Welcome to $\mathbb{M}$-world, where the only numbers that exist are positive integers that leave a remainder of 1 when divided by 4. In other words, the only $\mathbb{M}$-numbers that exist are $\{1, 5, 9, 13, 17, 21, ...\}$. (Another description is that these are the numbers of the form $4t + 1$ for $t = 0, 1, 2, ...$) In the $\mathbb{M}$-world, we cannot add numbers, but we can multiply them, since if a and b both leave a remainder of 1 when divided by 4, then so does thier product. We say m $\mathbb{M}$-divides n if $n = mk$ for some $\mathbb{M}$-number k. And we say that n is an $\mathbb{M}$-prime if its only $\mathbb{M}$-divisors are 1 and itself. (Of course, we don't consider 1 itself to be an $\mathbb{M}$-prime.)

(1) Find the first six $\mathbb{M}$-primes.
(2) Find an $\mathbb{M}$-number n that has two different factorizations as a product of $\mathbb{M}$-primes.

4. (Exercise 8.5 (c)) Find all incongruent solutions to the following congruence. $21x \equiv 14 \pmod{91}$

5. (Exercise 9.1 (c)) Use Fermat's Little Theorem to solve $x^{39} \equiv 3 \pmod{13}$.

6. (Exercise 9.2) The quantity $(p-1)! \pmod{p}$ appeared in our proof of Fermat's Little Theorem, although we didn't need to know its value.

(1) Compute $(p-1)! \pmod{p}$ for some small values of p, find a pattern, and make a conjecture.
(2) Prove that your conjecture is correct. [Try to discover why $(p-1)!$ (mod p) has the value it does for small values of p, and then generalize your observation to prove the formula for all values of p.]