

UNIT 2: SYMBOLIZATION 2 (WITH CONJUNCTION, DISJUNCTION AND BICONDITIONAL)

Answers to Exercises

P: Polly will pass the course.	T: Quincy will take the exam.
Q: Quincy will pass the course.	U: Quincy will study.
R: Ryan will pass the course.	W: The professor is insane.
S: Polly will study.	V: The course is too difficult.

- S2.01** Polly and Quincy will pass the course. $P \wedge Q$
- S2.02** Although the course is too difficult, Polly will pass it. $V \wedge P$
- S2.03** Polly will pass the course but Quincy will not. $P \wedge \sim Q$
- S2.04** Either the professor is insane or the course is too difficult. $W \vee V$
- S2.05** Even though Polly will study, she will not pass the course. $S \wedge \sim P$
- S2.06** Ryan and Quincy will not both pass the course. $\sim (R \wedge Q)$ OR $\sim R \vee \sim Q$
- S2.07** Neither Polly nor Quincy will study. $\sim (S \vee U)$ OR $\sim S \wedge \sim U$
- S2.08** Quincy will study, take the exam and pass the course.
 $U \wedge T \wedge Q$ OR $(U \wedge T) \wedge Q$ OR $U \wedge (T \wedge Q)$
- S2.09** Only if the professor is not insane, will Quincy take the exam and pass the course.
 $(T \wedge Q) \rightarrow \sim W$
- S2.10** If the course is too difficult, Polly will fail the course even though she studies.
 $V \rightarrow (\sim P \wedge S)$
- S2.11** If Polly, Quincy and Ryan all pass the course then the course is not too difficult.
 $(P \wedge Q \wedge R) \rightarrow \sim V$ OR $((P \wedge Q) \wedge R) \rightarrow \sim V$ OR $P \wedge Q \wedge R \rightarrow \sim V$
- S2.12** If Polly, Quincy or Ryan passes the course then the course is not too difficult.
 $(P \vee Q \vee R) \rightarrow \sim V$ OR $((P \vee Q) \vee R) \rightarrow \sim V$ OR $P \vee Q \vee R \rightarrow \sim V$
- S2.13** Both Quincy and Polly will study, however neither of them will pass the course.
 $(U \wedge S) \wedge \sim (Q \vee P)$ OR $(U \wedge S) \wedge (\sim Q \wedge \sim P)$

- S2.14** If the course is too difficult, then it is not the case that both Polly and Quincy will pass the course.

$$V \rightarrow \sim(P \wedge Q) \qquad \text{OR} \quad V \rightarrow (\sim P \vee \sim Q)$$
- S2.15** Polly will pass the course if and only if Quincy does. $P \leftrightarrow Q$
- S2.16** Unless the professor is insane, Quincy and Polly will pass on the condition that they both study.

$$W \vee ((U \wedge S) \rightarrow (Q \wedge P)) \qquad \text{OR} \quad W \vee (U \wedge S \rightarrow Q \wedge P)$$
- S2.17** Quincy will pass the course if and only if he studies and takes the exam.

$$Q \leftrightarrow (U \wedge T) \qquad \text{OR} \quad Q \leftrightarrow U \wedge T$$
- S2.18** If the professor is not insane and the course is not too difficult, then Polly will pass the course if she studies.

$$(\sim W \wedge \sim V) \rightarrow (S \rightarrow P)$$
- S2.19** If the course is not too difficult, Quincy will take the exam and pass the course just in case he studies.
If it is not the case that the course is too difficult then ((Quincy will take the exam and Quincy will pass the course) if and only if Quincy will study).

$$\sim V \rightarrow ((T \wedge Q) \leftrightarrow U) \qquad \text{OR PERHAPS}$$

If it is not the case that the course is too difficult then (Quincy will take the exam and (Quincy will pass the course) if and only if Quincy will study.))

$$\sim V \rightarrow (T \wedge (Q \leftrightarrow U))$$
- S2.20** Polly, Quincy or Ryan will pass the course, but the other two won't.

$$(P \wedge \sim Q \wedge \sim R) \vee (Q \wedge \sim P \wedge \sim R) \vee (R \wedge \sim P \wedge \sim Q)$$

or $(P \vee Q \vee R) \wedge ((\sim P \wedge \sim Q) \vee (\sim Q \wedge \sim R) \vee (\sim R \wedge \sim P))$
- S2.21** At least two of them (Polly, Quincy and Ryan) will pass the course.

$$(P \wedge Q) \vee (Q \wedge R) \vee (R \wedge P)$$
- S2.22** No more than one of Polly, Quincy or Ryan will pass the course.

$$(\sim P \wedge \sim Q) \vee (\sim Q \wedge \sim R) \vee (\sim R \wedge \sim P)$$

or $(P \rightarrow \sim Q \wedge \sim R) \wedge (Q \rightarrow \sim P \wedge \sim R) \wedge (R \rightarrow \sim P \wedge \sim Q)$
- S2.23** No more than two of them (Polly, Quincy and Ryan) will pass the course.

$$\sim(P \wedge Q \wedge R)$$

- P: The Conservatives win the election.
 Q: The Liberals win the election.
 R: The NDP wins the election.
 S: There will be a majority government.
 T: There will be a minority government.
 U: The Liberals dominate in Ontario.
 V: The PC dominate in Quebec.
 W: The Liberals choose a new leader.
 X: The Conservatives choose a new leader.
 Y: There is an election.

S2.24 If the Liberals choose a new leader, they will win the election only if they dominate in Ontario.

$$W \rightarrow (Q \rightarrow U)$$

S2.25 Assuming that the Liberals don't dominate in Ontario but the PC do in Quebec, then there will be a minority government if there is an election.

$$(\sim U \wedge V) \rightarrow (Y \rightarrow T)$$

S2.26 Neither the NDP nor the Liberals will win the election.

$$\sim(R \vee Q) \text{ OR } \sim R \wedge \sim Q$$

S2.27 The Conservatives won't win the election with a majority government unless the Liberals fail to dominate in Ontario.

$$\sim(P \wedge S) \vee \sim U \text{ OR } (P \wedge S) \rightarrow \sim U$$

S2.28 If the Conservatives choose a new leader then so do the Liberals, and if the Liberals don't win the election then neither does the NDP.

$$(X \rightarrow W) \wedge (\sim Q \rightarrow \sim R)$$

S2.29 Supposing both the Liberals and the Conservatives choose a new leader, there will be a majority government

$$(W \wedge X) \rightarrow S$$

S2.30 Either the Liberals will fail to dominate in Ontario and the Conservatives will win the election, or the PC will fail to dominate in Quebec and the Liberals will win the election.

$$(\sim U \wedge P) \vee (\sim V \wedge Q)$$

S2.31 If there's an election, neither the Conservatives nor the Liberals will win with a majority.

$$Y \rightarrow \sim((P \wedge S) \vee (Q \wedge S)) \text{ OR } Y \rightarrow (\sim(P \wedge S) \wedge \sim(Q \wedge S)) \text{ OR}$$

$$Y \rightarrow ((\sim P \wedge \sim Q) \vee \sim S) \text{ OR } Y \rightarrow (\sim(P \vee Q) \vee \sim S) \text{ OR } Y \rightarrow ((\sim P \wedge \sim Q) \vee \sim S)$$

- S2.32** The Conservatives won't choose a new leader unless they fail to win an election.
 $\sim X \vee \sim P \quad \text{OR} \quad X \rightarrow \sim P$
- S2.33** Exactly one of the Conservatives, the Liberals and the NDP will win the election.
 $(P \wedge \sim Q \wedge \sim R) \vee (\sim P \wedge Q \wedge \sim R) \vee (\sim P \wedge \sim Q \wedge R) \quad \text{OR}$
 $(P \vee Q \vee R) \wedge \sim((P \wedge Q) \vee (Q \wedge R) \vee (R \wedge P)) \quad \text{OR}$
 $(P \vee Q \vee R) \wedge (\sim(P \wedge Q) \wedge \sim(Q \wedge R) \wedge \sim(R \wedge P))$
- S2.34** At least two of the Conservatives, the Liberals and the NDP will fail to win the election.
 $(\sim P \wedge \sim Q) \vee (\sim Q \wedge \sim R) \vee (\sim R \wedge \sim P)$
- S2.35** No more than one of the three main parties (Conservatives, Liberals and NDP) will win provided there is an election.
 $Y \rightarrow ((\sim P \wedge \sim Q) \vee (\sim Q \wedge \sim R) \vee (\sim R \wedge \sim P)) \quad \text{OR}$
 $Y \rightarrow ((P \rightarrow \sim Q \wedge \sim R) \wedge (Q \rightarrow \sim R \wedge \sim P) \wedge (R \rightarrow \sim Q \wedge \sim P))$
- S2.36** The Liberals win the election precisely when they dominate in Ontario.
 $Q \leftrightarrow U$
- S2.37** The Conservatives will win the election if, but only if, both they don't choose a new leader and the Liberals do not dominate in Ontario.
 $P \leftrightarrow (\sim X \wedge \sim U)$
- S2.38** If an election is held, there will be a majority government or a minority government but not both.
 $Y \rightarrow ((S \vee T) \wedge \sim(S \wedge T))$

P: Patricia is majoring in Philosophy.
 Q: Quinn is majoring in philosophy.
 R: Rylan is majoring in philosophy.
 S: Patricia takes a logic course.
 T: Quinn takes a logic course.
 U: Rylan takes a logic course.
 V: Patricia studies hard.
 W: Quinn will get into law school.
 X: Quinn studies hard.
 Y: Quinn will pass.
 Z: Rylan will pass.

S2.39 Both Rylan and Patricia, but not Quinn, are majoring in philosophy.

$$P \wedge R \wedge \sim Q$$

S2.40 If Quinn majors in philosophy, then he will get into law school only provided that he takes a logic course and studies hard.

$$Q \rightarrow (W \rightarrow (T \wedge X))$$

S2.41 Although Rylan takes a logic course and is majoring in philosophy, he will not pass.

$$U \wedge R \wedge \sim Z$$

S2.42 Rylan will pass if, but only if, Quinn, who studies hard, passes.

$$(Z \leftrightarrow Y) \wedge X$$

S2.43 Although Patricia, Quinn and Rylan are all majoring in philosophy, only Quinn is taking a logic course.

$$(P \wedge Q \wedge R) \wedge (T \wedge \sim S \wedge \sim R)$$

S2.44 Neither Rylan nor Quinn take logic, but exactly one of them is majoring in philosophy.

$$\sim(U \vee T) \wedge ((R \wedge \sim Q) \vee (\sim R \wedge Q)) \quad \text{OR}$$

$$(\sim U \wedge \sim T) \wedge (R \vee Q) \wedge \sim(R \wedge Q)$$

S2.45 Patricia, who studies hard, is taking logic even though she isn't majoring in philosophy.

$$V \wedge S \wedge \sim P$$

S2.46 If both Quinn and Patricia take a logic course, but neither studies hard, they won't be majoring in philosophy.

$$((T \wedge S) \wedge (\sim X \wedge \sim V)) \rightarrow (\sim Q \wedge \sim P) \quad \text{OR} \quad ((T \wedge S) \wedge \sim(X \vee V)) \rightarrow \sim(Q \vee P)$$

S2.47 At least one of (Patricia, Quinn and Rylan) are taking a logic course, but none of them are majoring in philosophy.

$$(S \vee T \vee U) \wedge \sim(P \vee Q \vee R) \quad \text{OR} \quad (S \vee T \vee U) \wedge (\sim P \wedge \sim Q \wedge \sim R)$$

S2.48 No more than two of them (Patricia, Quinn and Rylan) are majoring in philosophy, nevertheless at least two of them are taking a logic course.

$$(\sim P \vee \sim Q \vee \sim R) \wedge ((S \wedge T) \vee (T \wedge U) \vee (U \wedge S))$$

$$\sim(P \wedge Q \wedge R) \wedge ((S \wedge T \wedge \sim U) \vee (S \wedge \sim T \wedge U) \vee (\sim S \wedge T \wedge U) \vee (S \wedge T \wedge U))$$

S2.49 If Patricia, Quinn and Rylan don't all major in philosophy, no more than one of them takes a logic course.

$$\sim(P \wedge Q \wedge R) \rightarrow ((\sim S \wedge \sim T) \vee (\sim T \wedge \sim U) \vee (\sim U \wedge \sim S)) \quad \text{OR}$$

$$(\sim P \vee \sim Q \vee \sim R) \rightarrow ((S \rightarrow \sim T \wedge \sim U) \wedge (T \rightarrow \sim U \wedge \sim S) \wedge (U \rightarrow \sim T \wedge \sim S))$$

S2.50 Only if Patricia, Quinn and Rylan all major in philosophy do exactly two of them take a logic course.

$$((S \wedge T \wedge \sim U) \vee (S \wedge \sim T \wedge U) \vee (\sim S \wedge T \wedge U)) \rightarrow (P \wedge Q \wedge R)$$

S2.51 Quinn takes logic if he's majoring in philosophy, but Patricia doesn't whether or not she is.

$$(Q \rightarrow T) \wedge ((P \vee \sim P) \rightarrow \sim S)$$

S2.52 Either Rylan takes a logic course and will pass or he doesn't (and won't).

$$(U \wedge Z) \vee (\sim U \wedge \sim Z)$$

S2.53 If Quinn passes only if Rylan does, then only if both of them take logic will Quinn get into law school.

$$(Y \rightarrow Z) \rightarrow (W \rightarrow (T \wedge U))$$

P: Paul plays guitar.
 Q: Sarah plays guitar.
 R: Robin plays the drums.
 S: Sarah plays bass.
 T: Tammy sings.
 U: Paul is sick.
 V: John plays bass.
 W: The band performs.
 X: The band rehearses.
 Y: The band sounds great.
 Z: Sarah is away.

S2.54 Both Sarah and Paul play guitar only if neither Sarah nor John plays bass.

$$(P \wedge Q) \rightarrow \sim(S \vee V) \quad \text{OR} \quad (P \wedge Q) \rightarrow (\sim S \wedge \sim V)$$

S2.55 Paul plays guitar if Sarah doesn't and vice versa, but if Sarah plays guitar then, only if John plays bass, does the band perform.

$$(P \leftrightarrow Q) \wedge (Q \rightarrow (W \rightarrow V))$$

S2.56 Assuming Paul is sick and doesn't play guitar, Sarah plays guitar unless she plays bass in which case John doesn't.

$$(U \wedge \sim P) \rightarrow ((Q \vee S) \wedge (S \rightarrow \sim V))$$

S2.57 If the band doesn't rehearse, then either the band won't perform or they will but won't sound great.

$$\sim X \rightarrow (\sim W \vee (W \wedge \sim Y))$$

S2.58 The band sounds great only on condition that John or Sarah, but not both, play bass and Robin plays the drums.

$$Y \rightarrow ((S \vee V) \wedge \sim(S \wedge V) \wedge R)$$

S2.59 If both Sarah and Paul play guitar, Tammy sings and John plays bass, then the band sounds great provided Robin isn't playing the drums.

$$(Q \wedge P \wedge T \wedge V) \rightarrow (\sim R \rightarrow Y)$$

S2.60 Neither John nor Sarah is playing bass, nevertheless the band performs and they sound great.

$$\sim(S \vee V) \wedge (W \wedge Y)$$

S2.61 Although Sarah isn't away, she neither plays guitar nor plays bass; however, Paul, being sick, does not play the guitar and the band doesn't rehearse.

$$\sim Z \wedge \sim(Q \vee S) \wedge (U \wedge \sim P \wedge \sim X) \quad \text{OR} \quad (\sim Z \wedge \sim Q \wedge \sim S) \wedge (U \wedge \sim P \wedge \sim X)$$

S2.62 If Tammy sings if and only if Robin plays the drums, either John doesn't play bass or the band doesn't perform.

$$(T \leftrightarrow R) \rightarrow (\sim V \vee \sim W)$$

S2.63 Whenever Sarah is away, John plays bass; and otherwise he plays bass when Sarah plays guitar but not when she plays bass.

$$(Z \rightarrow V) \wedge (\sim Z \rightarrow ((Q \rightarrow V) \wedge (S \rightarrow \sim V)))$$

S2.64 Unless Sarah, who either plays the guitar or plays bass, does play bass, Paul, who is sick, will play the guitar if the band performs.

$$(Q \vee S) \wedge (S \vee (W \rightarrow P)) \wedge U$$

S2.65 On the assumption that neither is Paul is sick nor Sarah away, the band both rehearses and performs.

$$\sim(U \vee Z) \rightarrow (X \wedge W) \quad \text{OR} \quad (\sim U \wedge \sim Z) \rightarrow (X \wedge W)$$

S2.66 John plays bass exactly when Sarah does not; but Paul and Sarah both play guitar or just one of them does.

$$(V \leftrightarrow \sim S) \wedge ((P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge Q)) \quad \text{OR} \quad (V \leftrightarrow \sim S) \wedge (P \vee Q)$$

- P: I have a body.
 Q: I have a soul.
 R: I have a mind.
 S: Dualism is true.
 T: Materialism is true.
 U: Body is material.
 V: The soul is material.
 W: The mind is nothing more than the brain.
 X: A completed physics would explain the mind.
 Y: A completed physics would account for human creativity.
 Z: 'Soul' and 'mind' are synonymous.

S2.67 Although I have both body and mind, unless dualism is true the mind is nothing more than the brain

$$(R \wedge P) \wedge (S \vee W) \quad \text{OR} \quad (R \wedge P) \wedge (\sim S \rightarrow W)$$

S2.68 Although the mind is nothing more than the brain, materialism is true if and only if I do not have a soul or the soul is material.

$$W \wedge (T \leftrightarrow (\sim Q \vee V))$$

S2.69 It cannot be the case that dualism is false if I have both body and soul, unless neither body nor soul are material, in which case neither dualism nor materialism is true.

$$\begin{aligned} &(((P \wedge Q) \rightarrow \sim \sim S) \vee \sim(U \vee V)) \wedge (\sim(U \vee V) \rightarrow \sim(S \vee T)) \quad \text{OR} \\ &(((P \wedge Q) \rightarrow S) \vee (\sim U \wedge \sim V)) \wedge ((\sim U \wedge \sim V) \rightarrow (\sim S \wedge \sim T)) \quad \text{OR} \\ &((U \vee V) \rightarrow ((P \wedge Q) \rightarrow \sim \sim S)) \wedge (\sim(U \vee V) \rightarrow \sim(S \vee T)) \end{aligned}$$

S2.70 A completed physics would neither explain the mind nor account for human creativity, yet materialism is true and dualism, false.

$$\sim(X \vee Y) \wedge (T \wedge \sim S) \quad \text{OR} \quad (\sim X \vee \sim Y) \wedge (T \wedge \sim S)$$

S2.71 It cannot be that materialism is true, assuming that a completed physics would not account for human creativity even though it would explain the mind.

$$(\sim Y \wedge X) \rightarrow \sim T$$

S2.72 Since I have body, mind and soul, it can't be that materialism is true; and dualism is true only if either mind is nothing more than the brain or 'soul' and 'mind' are synonymous.

$$((P \wedge Q \wedge R) \rightarrow \sim T) \wedge (S \rightarrow (W \vee Z))$$

S2.73 That I have a body, which is material, and a soul, which is not, is necessary and sufficient for dualism to be true.

$$(U \vee \sim V) \wedge ((P \wedge Q) \leftrightarrow S)$$

S2.74 Only if a completed physics would account for human creativity would it explain the mind; and if it would not do the former, which it won't, then it would not do the latter and materialism is not true.

$$(X \rightarrow Y) \wedge (\sim X \rightarrow (\sim Y \wedge \sim T)) \wedge \sim X$$

S2.75 Assuming the mind is nothing more than the brain and that materialism is true, then, unless a completed physics would fail to explain the mind, a completed physics would account for human creativity if 'soul' and 'mind' are synonymous.

$$(W \wedge T) \rightarrow (\sim X \vee (Z \rightarrow Y))$$

S2.76 On the assumptions that soul is not material and 'soul' is synonymous with 'mind', if Dualism is false then I have neither mind nor soul have I but, merely, a body.

$$(\sim V \wedge Z) \rightarrow (\sim S \rightarrow (\sim Q \wedge \sim R \wedge P))$$

S2.77 Materialism is true if and only if dualism is not, but even if materialism is true, it may or may not be the case that a completed physics would explain the mind.

$$(T \leftrightarrow \sim S) \wedge (T \rightarrow (X \vee \sim X))$$

S2.78 Materialism is true and the mind is nothing more than the brain, or dualism is true and mind is something more than the brain; yet if either is true, I have both body and mind.

$$(T \wedge W) \vee (S \wedge \sim W) \wedge ((T \vee S) \rightarrow (P \wedge R))$$

The last few sentences to symbolize are ambiguous – they mean different things depending on how you parse them. Symbolize them in different ways to disambiguate them. There are two different ways for 71, 72, 73 and five different ways for 74.

P: The students listen to the lectures.
 Q: The students will learn.
 R: The professor is boring.
 S: The students will skip class

S2.79 The professor is boring or the students will skip class and they will not learn.
 (Symbolize in two different ways.)

$R \vee (S \wedge \sim Q)$ $(R \vee S) \wedge \sim Q$

S2.80 The students listen to the lectures if and only if the professor is not boring assuming that they don't skip class. (Symbolize in two different ways.)

$P \leftrightarrow (\sim S \rightarrow \sim R)$ $\sim S \rightarrow (P \leftrightarrow \sim R)$

S2.81 It is not the case that the students will skip class unless the professor is boring.
 (Symbolize in two different ways.)

$\sim S \vee R$ OR $S \rightarrow R$ $\sim(S \vee R)$ OR $\sim(\sim S \rightarrow R)$

S2.82 The students will listen to the lectures if the professor isn't boring and yet the students will learn unless they skip class. (Symbolize in five different ways.)

(The students will listen to the lectures if the professor isn't boring) **and** (yet the students will learn unless they skip class.)

$(\sim R \rightarrow P) \wedge (Q \vee S)$ OR $(\sim R \rightarrow P) \wedge (\sim Q \rightarrow S)$

The students will listen to the lectures **if** [(the professor isn't boring and yet the students will learn) unless they skip class.]

$((\sim R \wedge Q) \vee S) \rightarrow P$

The students will listen to the lectures **if** [the professor isn't boring and (yet the students will learn unless they skip class.)]

$(\sim R \wedge (Q \vee S)) \rightarrow P$

[(The students will listen to the lectures if the professor isn't boring) and yet the students will learn] **unless** they skip class.

$((\sim R \vee P) \wedge Q) \vee S$

[The students will listen to the lectures if (the professor isn't boring and yet the students will learn)] **unless** they skip class.

$((\sim R \wedge Q) \rightarrow P) \vee S$

Use the abbreviation scheme to give an idiomatic (natural sounding) English sentence:

- P: Paul is a lawyer.
Q: Quade is a lawyer.
R: Rita is a lawyer.
S: Paul studied logic.
T: Quade studied logic.
U: Rita studied logic.

Of course, there are lots of ways to do this – and perhaps they don't all sound natural to you....

S2.83 $P \leftrightarrow \sim Q$

Either Paul or Quade is a lawyer, but not both.

S2.84 $(R \leftrightarrow U) \wedge (P \wedge \sim S)$

Rita is a lawyer if, but only if, she studied logic; however, Paul is a lawyer even though he didn't.

If Rita studied logic then she's a lawyer, and vice versa, but Paul is a lawyer even though he didn't study logic.

S2.85 $P \rightarrow (U \rightarrow S)$

If Paul is a lawyer, then Rita studied logic only if Paul did.

S2.86 $\sim(S \vee T) \rightarrow \sim(P \wedge Q)$

If neither Paul nor Quade studied logic, then they aren't both lawyers.

S2.87 $(\sim(P \wedge Q) \wedge (P \vee Q)) \rightarrow R$

If either Paul or Quade, but not both, is a lawyer then Rita is one too.

If exactly one of Paul and Quade are lawyers, then so is Rita.

S2.88 $P \vee Q \vee R$

At least one of Paul, Quade and Rita is a lawyer.

S2.89 $\sim(P \wedge Q) \wedge \sim(Q \wedge R) \wedge \sim(R \wedge P)$

No more than one of Paul, Quade and Rita is a lawyer.

S2.90 $\sim(S \wedge T \wedge U) \rightarrow \sim[(P \wedge Q) \vee (Q \wedge R) \vee (R \wedge P)]$

If Paul, Quade and Rita didn't all study logic, then no more than one of them is a lawyer.