CLASSICAL GEOMETRIES (MAT402H)

Spring 2014

COURSE INFORMATION

Instructor:	Professor Askold Khovanskii.
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Office hours	R 5–6 at BA6232 (If possible let me know that you are coming beforehand.)
Class:	R 6–9 at SF1101
Textbook:	"Lecture Notes" on our web page.
Teaching Assistants:	Liudmyla Kadets, **e-mail:** ludkad@gmail.com

Marking Scheme

The course mark will be determined by 4 quizzes (24%), one Term Test (26%) and the Final Exam (50%).

Term Test:	February 27.
Final Exam:	Date and time to be announced.

Quizzes Schedule

Quiz #1	January 23.
Quiz #2	February 6.
Quiz #3	March 13.
Quiz #4	March 27.

COURSE SYLLABUS

1. *Affine geometry*

Ceva's theorem and its proof that uses calculation of areas. Three heights, three medians and three bisectors of a triangle. The center of masses, its properties including calculation of its coordinates. Proof of Ceva's theorem by means of the center of masses. Menelaus's theorem. Affine geometries over arbitrary fields.

2. *Convex geometry*

Separation of a convex body from a point. Finite-dimensional Banach space. The Minkowski theorem about an integer point in a convex body. The finite-dimensional Krein–Milman theorem. A simple polyhedron in $\mathbf{R}^n$, its h-vector. Index of a linear function at a vertex of a simple polyhedron. The Dehn–Sommerville duality. Euler's formula for 3-dimensional convex polyhedra.

3. *Reflections, billiards, geometric problems on maxima and minima*

Schwarz's triangle (i.e. triangle of minimal perimeter inscribed in a given triangle). A point which minimizes the sum of the distances from three given points. The above mentioned problems from calculus's point of view. Isoperimetric problem (if a solution exists it must be a circle). Ellipses and hyperbolas as sections of circular cones, their optical properties (reflections of rays coming from their foci). Billiard trajectories on an elliptic billiard.

4. *Inversions*

General properties of inversions on a plane and in space: angle preservation, invariance of circles and lines as well as spheres and planes. Apollonius's problem. Existence of inversions mapping a pair of non-intersecting circles into a pair of concentric circles. Stereographic projection of a sphere onto a plane and its properties (as an application of inversion).

5. *Projective geometry*

Desargues's theorem. Cross ratios of four points on a line, of four lines on a plane passing through a point, and of four planes in space containing a common line. The invariance of cross ratios under projective transformations. Projective transformation of a line mapping a given triple of points into another triple of points. Projective transformation of a plane mapping a given generic quadruple of points into another generic quadruple of points. Coordinate formulas of projective transformations of a line and of a plane. Bundles of lines passing through given points and projective maps of such bundles. (Let F be a projective map from the bundle of lines passing through one point into the bundle of lines passing through another point that maps the line joining these points into itself; describe the locus of points of the form $L \cap F(L)$. Formulate the dual statement.) Cross ratio of four points on a conic section. Its direct and dual descriptions. Conic section as a locus of points of intersection of L and $F(L)$, where F is a projective correspondence between bundles of lines passing through two given points. A dual description of conic sections. Theorems of Pascal and Brianchon, including degenerate cases (Pappus theorem and its dual statement). General duality principle. Homogeneous coordinates. Projective geometries over arbitrary fields.

6. *Spherical and elliptic geometries*

Three heights, three medians and three bisectors of a spherical triangle. Areas of spherical polygons and a proof of Euler characteristic formula for convex polyhedra. Five regular polyhedra (Platonic solids). Duality principle in elliptic geometry.

7. *Elements of hyperbolic geometry*

Linear transformations preserving a circular cone and Klein's model of Lobachevsky's plane. Distances in Klein's model. Inversions preserving a circle and Poincare's model of Lobachevsky's plane. Angles in Poincare's model.