May 23rd

$\boxed{\text{Textbook P12 . 8}}$. Give an example of a set S such that the interior of S is unequal to the interior of closure of S.

① $B(1,\vec{0}) \setminus \{\vec{0}\}$

$\text{int}(B(1,\vec{0}) \setminus \{\vec{0}\}) = B(1,\vec{0}) \setminus \{\vec{0}\}$

$\overline{B(1,\vec{0}) \setminus \{\vec{0}\}}$ = closed ball at $\vec{0}$ with radius 1

$\Rightarrow \text{int}(\overline{B(1,\vec{0}) \setminus \{0\}}) = B(1,\vec{0})$

② $\mathbb{Q}$

$\text{int}(\mathbb{Q}) = \emptyset$

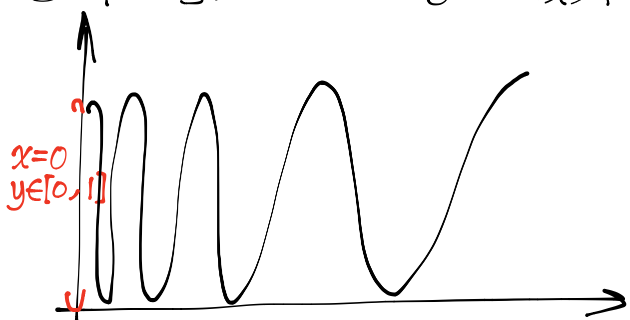
$\partial \mathbb{Q} = \mathbb{R}$

$\overline{\mathbb{Q}} = \mathbb{R}$

$\text{int}(\overline{\mathbb{Q}}) = \text{int}(\mathbb{R}) = \mathbb{R}$

1.(e) (P12)

$S = \{(x,y) : x > 0 \text{ and } y = \sin(\frac{1}{x})\}$



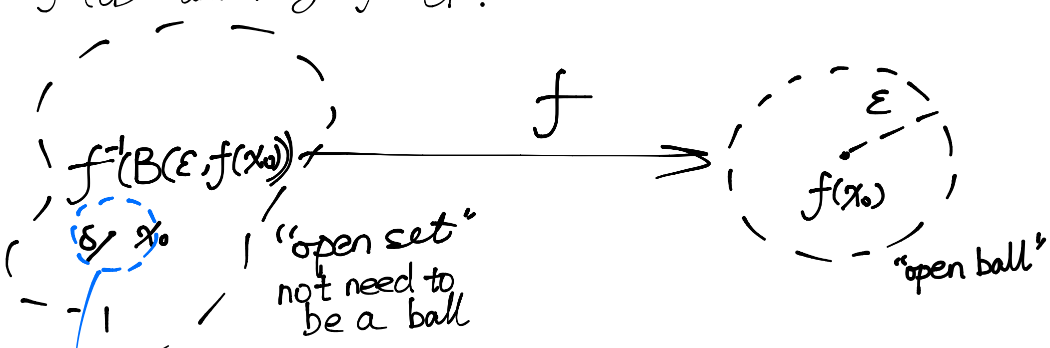
$\text{int}(S), \partial S, \overline{S} = ?$

$\overline{S} = S \cup \{(x,y) \mid x=0, y \in [0,1]\}$

P19. #8

Sps $\vec{f}: \mathbb{R}^n \to \mathbb{R}^k$ has the following property: For any open set $U \subset \mathbb{R}^k$, $\{\vec{x} : \vec{f}(\vec{x}) \in U\}$ is an open set in $\mathbb{R}^n$. Show that $\vec{f}$ is continuous on $\mathbb{R}^n$. Show also that the same result holds for "closed".

$f^{-1}(U)$ "anti-image of U".



we can always get an open ball. Why? B/c the ~~open~~ set is open.

NOTE :

$(f^{-1}(A))^c = f^{-1}(A^c)$

P19 #6, #7 ($\varepsilon - \delta$ argument)

P23 #2

$x_k = \frac{3k+4}{k-5}$ then $\lim_{k\to\infty} x_k = 3$. Given $\varepsilon > 0$. find K s.t. $|x_k - 3| < \varepsilon$ when $k > K$.

$$\left|\frac{3k+4}{k-5} - 3\right| < 0 \iff \left|\frac{19}{k-5}\right| < \varepsilon \qquad \iff |k-5| > 19/\varepsilon$$

$$\iff K \geq 5 + 19/\varepsilon$$