Lecture 20

- In the orbit diagram for $-1.54 < c < 0.75$, we expect to see a doubling of the diagram for $-2 < c < 0.25$
- The same behavior happens for all period-n windows.

CHAPTER 9 SYMBOLIC DYNAMICS

Recall $Q_c(x) = x^2 + c$ where we know about Q_c for $c < -2$.

① Fixed points for Q_c are $P_- = \frac{1-\sqrt{1-4c}}{2}$ and $P_+ = \frac{1+\sqrt{1-4c}}{2}$

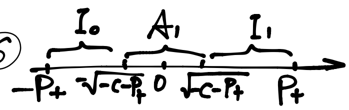
② $I = [-P_+, P_+]$

③ if $x_0 \notin I$, then $x_n \to \infty$

④ $\Lambda = \{x \in I,\ Q_c^n(x) \in I \text{ for all } n \in \mathbb{N}\}$

⑤ $A_1 = (-\sqrt{-c-P_+}, \sqrt{-c-P_+})$

x_0 whose orbit Q_c exits I in 1 iteration

⑥ 

A_1 divides I in two disjoint closed intervals I_0 & I_1

Definition: Let $x_0 \in \Lambda \subseteq I_0 \cup I_1$. The itinerary of x_0 is the sequence $S(x_0)$ of 0's and 1's, given of

$$S(x_0) = (s_0 s_1 s_2 \cdots)$$

$$s_i = \begin{cases} 0 & \text{if } Q_c(x) \in I_0 \\ 1 & \text{if } Q_c(x) \in I_1 \end{cases}$$

example: $(11111\cdots)$ b/c $x_n = P_+ \in I_1$

$S(-P_+) = (0\,1111\cdots)$ b/c

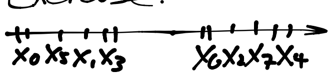
$x_0 = -P_+ \in I_0$

$x_1 = Q_c(-P_+) = P_+ \in I_1$

$x_n = P_+,\ n = 1, 2, \cdots$

$S(2) = (0000\cdots)$ b/c $x_n = P_- \in I_n$

Exercise:



$S(x_0) = (0010011)$

§9.2 Sequence Space

Definition: the sequence space on two symbols is the set

$$\Sigma = \{(s_0 s_1 s_2 \cdots) \ s_i \in \{0, 1\}\}$$

Definition: The distance between $s = (s_0 s_1 s_2 \cdots)$ and $t = (t_0 t_1 t_2 \cdots)$ is given by $d[s,t] = \sum_{i=0}^{\infty} \frac{|s_i - t_i|}{2^i}$

Remark : the theory always converges b/c the distance between s_i and t_i is always 0 or 1. i.e. $|s_i - t_i| \in \{0, 1\}$. So $d[s, t] \leq \sum_{i=0}^{\infty} \frac{1}{2^i} = 2$