UNIT 5 Part 1
SYMBOLIZATION: PREDICATES AND QUANTIFIERS

5.1: THE LOGIC OF SUB-SENTENTIAL RELATIONS

Consider this argument:

All humans are mortal.
Socrates is human.
____
∴ Socrates is mortal.

It is clearly a valid argument, but if we try to symbolize it using an abbreviation scheme for sentential logic we will not be able to derive the conclusion from the premises:

P: All humans are mortal. Q: Socrates is human. R: Socrates is mortal.

P
Q
______
∴ R

Despite the failure of this symbolization scheme to capture the validity of the argument, it is obviously valid. The symbolization techniques that we've learned won't show it because the logical relations in this example concern the properties and terms that occur *within* a sentence ("all", "Socrates", being mortal, being human.) In our sentential logic we were concerned with the logical relations *between* atomic sentences (those symbolized with a single letter) but we never considered the logical relations that stem from structures *within* an atomic sentence. There must be another way to symbolize arguments!

We want to extend our symbolization techniques so we can capture these types of subsentential logical structures. We want to be able to express the logical form:

Every thing that is '*S*' has property '*P*'.
'*a*' is '*S*'.
____
'*a*' has property '*P*'.

The logical form has to do with the internal structure of the sentence — the relations between the ***subjects*** and the ***predicates***. (These are 'subsentential' relationships, relationships between parts of a sentence.)

Any properly constructed assertion has:

- a subject: the object(s), person(s) or place(s) that the sentence is about.
- a predicate: what is being said about (or predicated of) the subject — often a property or an action.

We want to be able to consider the logical relations between subjects and predicates of different sentences.

The Subject – Singular Terms & General Terms:

Singular Terms: words or phrases that refer to specific people, places, things, states of affairs, abstract objects, etc.

Proper names: Aristotle, Plato, Toronto, Daphne, Count Dracula, The CN Tower, Venus.

Definite descriptions: "The Prime Minister of Canada", "Brad's only sister", "The Tallest Freestanding Building," "The Morning Star."

Pronouns: He, she, it, I …

General Terms: words or phrases that refer to people, places or things in general.

$\forall$ **Universally quantified terms:** the subject includes all of a certain group: all people, every dog, nobody (negatively applies to everyone),...

$\exists$ **Existentially quantified terms:** the subject includes at least one of a certain group: some people, at least one dog, …

The Predicate in Relation to Singular Terms

Predicates can operate on singular terms. They are functions telling us what properties an individual has.

One-place or **monadic** predicates operate on **one** singular term:

____________ is human.

____________ is a student of Plato.

One-place predicates have one 'blank' that can be filled with any singular term. Some of those terms will make the sentences true. Other terms will make the sentences false.

Two-place or **dyadic** predicates operate on **two** singular terms:

____________ is a student of ______________.

__________ is sitting between __________ and Daphne.

Two-place predicates have two 'blanks' that can be filled with singular terms.
Some ordered pairs of terms will make the sentences true. Others will make them false.

Three-place or **triadic** predicates operate on **three** singular terms:

__________ is sitting between ___________ and _____________.

__________ borrows ____________ from _____________.

Three-place predicates have three 'blanks' that can be filled with singular terms.
Some 'ordered triples' will make the sentences true. Others will make them false.

Predicates can operate on any number of terms – for instance the predicate could be used to name all the members of a team or class. We will rarely consider predicates more complex than three-places.

5.2 SYMBOLIZATION FOR PREDICATE LOGIC

We are going to extend the symbolic language for sentential logic so that it can also handle predicates.

Every sentence from sentential logic is still a sentence in predicate logic.

Our symbolic language already included atomic sentence letters, logical connectives ($\sim$, $\rightarrow$, $\vee$, $\wedge$, $\leftrightarrow$), parentheses and brackets.

The extended symbolic language will also include: name letters, variables, predicates and quantifiers.

Symbols for Predicate Logic:

Individual constants or name letters:	lower case roman letters, a-h with or without numerical subscripts.	a, b, c, d, e, f, g, h (a_1, a_2, b_1, ...)
Variables:	lower case roman letters, i-z, $\pm$ numerical subscripts.	i-z (x_1, x_2, y_1, ...)
Predicate letters:	upper case roman letters, A-O. may have number of places indicated	A, B, C, D … O F^0, F^1, F^2, F^3
Operation letters:	lower case roman letters, a-h, may have number of places indicated	a, b, c, d, e, f, g, h a^0, a^1, a^2,…
Quantifiers:	Universal quantifier Existential quantifier	$\forall$ $\exists$
Identity sign:		=

And of course we also include:

Atomic Sentence letters:	Uppercase letters P-Z, $\pm$ numerical subscripts.	P, Q, R, …Z.
Logical connectives:		$\sim$, $\rightarrow$, $\wedge$, $\vee$, $\leftrightarrow$
Parentheses and brackets:		( ), [ ]

Symbolizing: Names and Predicates

Sentences that have a singular term as the subject and a simple predicate can be easily symbolized using a name letter and a predicate letter. This will create a new type of atomic sentence or formula.

Consider the sentence: Aristotle is human.

Instead of using an atomic sentence letter (P-Z), we can use a predicate letter (A-O) such as H for the predicate, "is human", and a name letter (a-h) such as a, for the name, "Aristotle", creating a new type of atomic sentence or formula.

Abbreviation scheme:

H^1: *a* is human. a: Aristotle. b: Plato

We can use this abbreviation scheme to symbolize two atomic formulas or sentences:

Atomic formula:	Ha	Aristotle is human.
Atomic formula:	Hb	Plato is human.

We can use the logical connectives with the atomic formulas (including the atomic formulae or sentences) from sentential logic) to build more and more complex sentences. Any complex sentence formed using the symbols and syntactical rules of Unit 2 together with any atomic formulas are molecular formulas:

Molecular formula:	~Ha	Aristotle is not human.
Molecular formula:	$Ha \rightarrow Hb$	If Aristotle is human, so is Plato.
Molecular formula:	$Ha \wedge Hb$	Both Aristotle and Plato are human.
Molecular formula:	$Ha \vee Hb$	Either Aristotle or Plato is human.
Molecular formula:	$Ha \leftrightarrow Hb$	Aristotle is human if and only if Plato is.

Now look at the abbreviation scheme again:

H^1: *a* is human. a: Aristotle. b: Plato

H^1 The superscript '1' shows that this is a one place predicate. In the abbreviation scheme the superscript number specifies how many 'places' the predicate has.

A one-place predicate will have a superscript '1'; a two-place predicate will have a superscript '2', etc. Since it does not lead to ambiguity, the superscript '1' is often left off the monadic predicates in abbreviation schemes: When you symbolize, the superscript numeral is left off!

a "*a* is human." The letter *a* (italicized) in the abbreviation scheme is functioning as the 'blank' or place-holder in the one-place predicate. In two or three place predicates there will also be italicized letters *b* and *c*. They are NOT part of the symbolic language for predicate logic.

a "a: Aristotle" The letter 'a' (not italicized) is being defined (as is 'b', in "b: Plato"). These are name letters or individual constants and are a part of the symbolic language for predicate logic.

EG 5.2 Let's try some!

Abbreviation Scheme:

B^1:	*a* drinks beer.	a:	Adam
C^1:	*a* is Canadian.	b:	Stephen Harper
H^1:	*a* plays Hockey.	c:	Carol
P:	Stereotypes shouldn't be believed.		

a) Adam is Canadian. Ca

b) Stephen Harper is Canadian. Cb

c) Adam and Stephen Harper are both Canadian, but Carol is not. $Ca \wedge Cb \wedge \sim Cc$

d) If Stephen Harper is Canadian then he plays hockey. $Cb \rightarrow Hb$

e) Adam is Canadian if and only if he drinks beer and plays hockey. $Ca \leftrightarrow (Ba \wedge Ha)$

f) Carol doesn't drink beer but she is still Canadian. $\sim Bc \wedge Cc$

g) Although both Adam and Stephen Harper are Canadian, only the latter drinks beer.
$(Ca \wedge Cb) \wedge (\sim Ba \wedge Bb)$

h) Either Stephen Harper drinks beer if he is Canadian or stereotypes shouldn't be believed.
$((Cb \rightarrow Bb) \vee P$

i) At least one of them (Adam, Stephen and Carol) drink beer.

$Ba \vee Bb \vee Bc$

Quantifiers:

Consider terms that pick out individuals in general, rather than particular individuals:

All, every, each, everyone ...

These terms pick out all individuals, relative to a universe of discourse (the set of things being talked about), and are used to say something about all of these individuals.

Some, at least one, something, someone ...

These terms pick out at least one individual but doesn't specify which individual(s) in particular. The terms pick out the individual(s) relative to a universe of discourse (the set of things being talked about), and are used to say something about this individual(s).

None, no one, nothing, ...

These terms pick out all individuals within to a universe of discourse (the set of things being talked about), and are used to deny something of all these individuals.

How do these general terms work with predicates?

Sentence 1 uses a 'blank' to capture the logical form of a group of related sentences (2-6).

1: ________________ is good.

2: ____Aristotle_____ is good.

3: ____Everything___ is good.

4: ____Something___ is good.

5: ____Nothing______ is good.

6: __Not everything__ is good.

We can ask: How are sentences 2, 3, 4, 5, and 6 related to sentence 1?

2: ____Aristotle_____ is good.	2 is true ... if 1 is true for a singular term, "Aristotle".
3: ____Everything___ is good.	3 is true .... if 1 is true for every singular term.
4: ____Something___ is good.	4 is true ... if 1 is true for at least one singular term.
5: ____Nothing______ is good.	5 is true ... if there is no singular term that 1 is true of. (It is false for every singular term.)
6: __Not everything__ is good.	6 is true ... if it is not the case that 1 is true of every singular term. (It is false for some singular terms.)

We can capture these ideas with: **Quantifiers and One-Place Predicates**

$\forall$ is the Universal Quantifier – all things.

It is always used with a variable (i-z) and a predicate (A-O). The variable functions as the 'blank'.

$\exists$ is the Existential Quantifier – some things.

It is always used with a variable (i-z) and a predicate (A-O). The variable functions as the 'blank'.

When giving an abbreviation scheme, letters a-f are used for the predicate's 'places' or arguments. When symbolizing the sentences, the arguments will be singular terms or variables.

Abbreviation scheme: G^1: *a* is good.
a: Aristotle

singular term	Ga	Aristotle is good.
all singular terms	$\forall x\ Gx$	All things are good.
some singular term(s)	$\exists x\ Gx$	Something is good.
no singular term(s)	$\sim\exists x\ Gx$	Nothing is good.
not all singular terms	$\sim\forall x\ Gx$	Not all things are good.

Note:

- "Some" means "at least one", so it is consistent with "all".
- "Nothing" or "none" is the same as "there are not some".
- "Not all" means "at least one is not …"

We can also specify a universe of discourse (U) – the set of things being talked about, or being predicated. If we don't specify the universe, then we assume that everything means everything! Other times we may want to restrict the universe to the set of natural numbers, or whole numbers, to people, etc.

Universe (U) or Universe of Discourse (UD): the set of things that we are talking about (predicating).

F^1: *a* is on Earth.

U: unrestricted

$\forall xFx$ Everything is on Earth. (False!)

$\sim\forall xFx$ Not everything is on Earth. (True!)

F^1: *a* is on Earth.

U: students in PHL 245

$\forall xFx$ All students in PHL 245 are on Earth. (True!)

$\sim\forall xFx$ Not all students in PHL 245 are on Earth. (False!)

5.3 QUANTIFIERS AND COMPLEX PREDICATES

Now that we have a sense of how to make simple quantified sentences (quantified atomic predicates), we can add in the logical connectives from Sentential Logic. This will allow us to quantify more complex sentences.

Working with the Universal Quantifier:

We use the universal quantifier, $\forall$, to say that every member of the universe has certain characteristics.

Abbreviation scheme:

C: *a* is material. D: *a* is phenomenal.

We can symbolize:

Everything is material.	$\forall x\ Cx$
Everything is phenomenal.	$\forall x\ Dx$
Everything is material or phenomenal.	$\forall x\ (Cx \lor Dx)$

But these types of universal sentences are rarely going to be true. Here the universe is unrestricted. There aren't too many characteristics that apply to *everything* in the universe (except perhaps existence itself). But, even if we restrict the universe to people, animals, or some other set of things, there will still be few characteristics that apply to every member of the universe. If we want to say something interesting or informative, we need to be able to restrict the property to certain members the universe.

The universal quantifier is often used with a conditional. The conditional acts to restrict the predication to certain members of the universe. It states that everything has the property of the consequent IF it is the sort of thing that is picked out by the antecedent.

For example:

D: *a* is a dog. M: *a* is a mammal.

All dogs are mammals.

Paraphrase: Every object x is such that if it is a dog, then it is a mammal.

$\forall x\ (Dx \rightarrow Mx)$

The subject of the sentence restricts the sentence to those members of the universe that are dogs (subject term) and the predicate of the sentence predicates of them the property of being a mammal (predicate term). If something is has the subject property then it has the predicate property. The canonical form of the universally quantified sentence is:

$\forall x$ (**Subj** x $\rightarrow$ **Pred** x) All things in the extension of the subject term have the predicate property.

Note: we can use x, y or any other letter i-z as the variable without changing the meaning. The variable is just a 'blank'. $\forall y(Dy \rightarrow My)$ means exactly the same thing as $\forall x(Dx \rightarrow Mx)$!

Stylistic Variants:

You need to think about whether the statement you are translating is a universal statement or not. Sometimes it will use a word like ‘all’ or ‘every’ but often it will not. The following are all symbolized with the universal quantifier:

All dogs are mammals.	Dogs are mammals.
An elephant is a mammal.	The whale is a mammal.
Any bat is a mammal.	If something’s a bat, then it’s a mammal.

CANONICAL FORM OF THE UNIVERSAL QUANTIFIED SENTENCE:

$\forall x\,(Fx \rightarrow Gx)$

Everything, x, is such that if x has the property of the restricting antecedent then it has the property of the predicating consequent.

All things with property F have property, G.

All F’s are G.

Complex Restricting Clauses:

The restricting antecedent clause need not be simple. We can use logical connectives such as conjunction, disjunction and negation to give more details about the members of the universe that we are predicating something of.

C: *a* is a cat. D: *a* is a dog. G: *a* is gray. M: *a* is a mammal.

Gray cats are mammals.

Every x such that x is gray and x is a cat is a mammal.

Restricting antecedent: is gray <u>and</u> is a cat Predicating consequent: is a mammal

$\forall x((Gx \wedge Cx) \rightarrow Mx)$

Dogs that aren’t gray are mammals.

Every x such that x is a dog and x is not gray is a mammal.

Restricting antecedent: is a dog <u>and</u> is <u>not</u> gray Predicating consequent: is a mammal

$\forall x((Dx \wedge {\sim}Gx) \rightarrow Mx)$

All cats and dogs are mammals.

We can symbolize this in either of two ways:

Paraphrase 1: Each x is such that if it is a cat then it is a mammal <u>and</u> each x is such that if it is a dog then it is a mammal.

Restricting antecedent: is a cat   Predicating consequent: is a mammal.

Restricting antecedent: is a dog   Predicating consequent: is a mammal.

$\forall x\ (Cx \rightarrow Mx)\ \wedge \forall y\ (Dy \rightarrow My)$

Paraphrase 2: Each x is such that if it is a cat <u>or</u> a dog, then it is a mammal.

Restricting antecedent: is a cat <u>or</u> is a dog   Predicating consequent: is a mammal.

$\forall x\ ((Cx \vee Dx) \rightarrow Mx)$

In this second form, the restricting antecedent clause is a disjunction. Anything that has either of the properties (being a dog or being a cat) is a mammal.

Be careful! If you use AND, $\wedge$, in the restricting clause like this: $\forall x((Cx \wedge Dx) \rightarrow Mx)$ ERROR! You would be saying that everything that is both a cat AND a dog is a mammal!

Complex Predicating Clauses:

The predicating consequent clause may also be complex. We can use logical connectives such as conjunction, disjunction, negation and biconditional to predicate more complex properties of the subjects.

A: *a* is an animal.   B: *a* is a bird.   C: *a* is a cat.

F: *a* has feathers.   M: *a* is a mammal.

Cats don't have feathers.

Each x is such that <u>if</u> it is a cat, <u>then</u> it does <u>not</u> have feathers.

Restricting antecedent: is a cat   Predicating consequent: does <u>not</u> have feathers.

$\forall x\ (Cx \rightarrow \sim Fx)$

No cats have feathers.

This is also a universal statement: it says something about each and every cat – that it does not have feathers. This is a stylistic variant of the sentence above, "Cat's don't have feathers."

Each x is such that <u>if</u> it is a cat, <u>then</u> it does <u>not</u> have feathers.

Restricting antecedent: is a cat   Predicating consequent: does <u>not</u> have feathers.

$\forall x\ (Cx \rightarrow \sim Fx)$

Birds are animals but they aren't mammals.

Paraphrase 1: Each x is such that <u>if</u> it is a bird <u>then</u> it is an animal <u>and</u> each x is such that <u>if</u> it is a bird <u>then</u> it is <u>not</u> a mammal.

Restricting antecedent: is a bird   Predicating consequent: is an animal.

Restricting antecedent: is a bird   Predicating consequent: is <u>not</u> a mammal.

$\forall x\ (Bx \rightarrow Ax) \wedge \forall x\ (Bx \rightarrow \sim Mx)$

Paraphrase 2: Each x is such that <u>if</u> it is a bird <u>then</u> it is an animal <u>and</u> it is <u>not</u> a mammal.

Restricting antecedent: is a bird   Predicating consequent: is an animal <u>and</u> is <u>not</u> a mammal.

$\forall x\ (Bx \rightarrow (Ax \wedge \sim Mx))$

If something is an animal then it is a bird or it doesn't have feathers.

Each x is such that <u>if</u> it is a animal <u>then</u> it is a bird <u>or</u> it does <u>not</u> have feathers.

Restricting antecedent: is an animal

Predicating consequent: is a bird <u>or</u> does <u>not</u> have feathers.

$\forall x\ (Ax \rightarrow (Bx \vee \sim Fx))$

Note: in this example even though the sentence uses the phrase 'something', it is a universal statement – it is saying of ALL things, if it is an animal, then it is a bird or it doesn't have feathers.

Animals are birds if they have feathers.

Each x is such that <u>if</u> it is an animal <u>then</u> it is a bird <u>if</u> it has feathers.

Restricting antecedent: is an animal

Predicating consequent: <u>if</u> it has feathers <u>then</u> it is a bird.

$\forall x\ (Ax \rightarrow (Fx \rightarrow Bx))$

Animals with feathers are birds, not mammals.

Each x is such that <u>if</u> it's an animal <u>and</u> it has feathers <u>then</u> it's a bird and it's <u>not</u> a mammal.

Restricting antecedent: is an animal <u>and</u> has feathers

Predicating consequent: is a bird <u>and</u> is <u>not</u> a mammal.

$\forall x\ ((Ax \wedge Fx) \rightarrow (Bx \wedge \sim Mx))$

NOTE: the more complicated a sentence is, the more correct symbolizations there are. For instance, one can symbolize this last sentence like this: $\forall x\ (Ax \rightarrow (Fx \rightarrow (Bx \wedge \sim Mx)))$. Any animal is such that if it has feathers then it is a bird and not a mammal.

5.3 EG1 Symbolize the following:

Abbreviation scheme:

F: *a* is a snake. I: *a* is native to Canada.

G: *a* is a rattlesnake. J: *a* is venomous.

a) Rattlesnakes are venomous snakes.

Every object x is such that <u>if</u> x is a rattlesnake <u>then</u> x is venomous <u>and</u> x is a snake.

RA (restricting antecedent): is a rattlesnake

PC (predicating consequent): is venomous <u>and</u> is a snake

$\forall x\ (Gx \rightarrow Jx \wedge Fx)$

b) The only venomous snake native to Canada is the rattlesnake.

$\forall x((Jx \wedge Fx \wedge Ix) \rightarrow Gx)$

c) A snake native to Canada is not venomous unless it is a rattlesnake.

$\forall x((Fx \wedge Ix) \rightarrow (\sim Jx \vee Gx))$

d) Among venomous snakes, only the rattlesnake is native to Canada.

$\forall x((Fx \wedge Jx) \rightarrow (Ix \rightarrow Gx))$

e) Snakes native to Canada are venomous if and only if they are rattlesnakes.

$\forall x((Fx \wedge Iy) \rightarrow (Jx \leftrightarrow Gx))$

f) Any snake native to Canada that is not a rattlesnake is not venomous.

$\forall x((Fx \wedge Ix \wedge \sim Gx) \rightarrow \sim Jx)$

Working with the Existential Quantifier:

We use the existential quantifier, $\exists$, to say that some (at least one) member of the universe exists, or that it has certain characteristics.

An simple existential statement can be used to symbolize such sentences as:

Snakes exist.	$\exists x\ Fx$
Some things are venomous.	$\exists z\ Jz$

A complex existential statement can be used to predicate properties to some members of a group.

Some snakes are venomous.

Paraphrase: there exists at least one thing that is a snake and that is venomous.

$\exists x(Fx \wedge Jx)$

The subject of the sentence states that at least one member of the universe is a snake (subject term) and the predicate of the sentence predicates of them the property of being venomous (predicate term). Something has the subject property and the predicate property. The canonical form of the universally quantified sentence is:

$\exists x(Sx \wedge Px)$ Some things exist that are in the extension of the subject term and have the predicate property.

The existential quantifier is often used with a conjunction. We can think about the conjunction the same way we think about the conditional in the universal statements: one conjunct restricts the statement to certain members of the universe and the other conjunct predicates something of those members. However, in the case of the existential quantifier (and unlike the case with the universal quantifier), it doesn't matter which is the restricting conjunct and which is the predicating conjunct.

CANONICAL FORM OF THE EXISTENTIAL QUANTIFIED SENTENCE:

$\exists x\ (Fx \wedge Gx)$

~~At least one x exists such that x has the property of the restricting conjunct~~ and also has the property of the predicating conjunct.

Something with property F has property G.

Some F's are G.

Some snakes are venomous.

At least one thing exists that is both a snake and is venomous. $\exists x\ (Fx \wedge Jx)$

Some snakes are not venomous.

At least one thing exists that is both a snake and is not venomous. $\exists x\ (Fx \wedge {\sim}Jx)$

Complex Clauses:

The two clauses need not be simple. We can use logical connectives such as conjunction, disjunction and negation to give more details about the members of the universe that we are making an existential claim about. Often, although not always, the properties are all simple properties that are to be conjoined with 'and'. Thus, parentheses are not always needed (and often make it difficult to read). But, do remember, by convention, when the parentheses are left off the main connective is the one on the right.

F: *a* is a snake. I: *a* is native to Canada. J: *a* is venomous.

Some snakes that are native to Canada are venomous.

> There is at least one object x that is a snake <u>and</u> that is native to Canada <u>and</u> that is venomous.
>
> $\exists y(Fy \wedge Iy \wedge Jy)$

Some snakes are venomous but are not native to Canada.

> There is at least one object x that is a snake <u>and</u> that is venomous <u>and</u> that is <u>not</u> native to Canada.
>
> $\exists x(Fx \wedge Jx \wedge {\sim}Ix)$

Some snakes are venomous if they are not native to Canada.

> There is at least one object x that is a snake <u>and</u> <u>if</u> it is <u>not</u> from Canada, <u>then</u> it is venomous.
>
> $\exists y(Fy \wedge ({\sim}Iy \rightarrow Jy))$

Negating Quantified Sentences:

We saw that when we want to say that no members of a group have a property, we can do that with a universal quantifier:

No snakes are venomous. $\forall x(Fx \rightarrow {\sim}Jx)$

We can also use the existential quantifier to symbolize "No snakes are venomous".

"No snakes are venomous" is the negation of: Some snakes are venomous.

> It is not the case that some snakes are venomous.
>
> <u>It is not the case</u> that there is at least one object x that is a snake <u>and</u> it is venomous.
>
> ${\sim}\exists x\ (Fx \wedge Jx)$

Likewise, we symbolized "some snakes are not venomous": $\exists x(Fx \wedge {\sim}Jx)$

But "some snakes are not venomous" is the negation of: It is not the case that all snakes are venomous.

> <u>It is not the case</u> that for all things x, <u>if</u> x is a snake <u>then</u> x is venomous.
>
> ${\sim}\forall x(Fx \rightarrow Jx)$

5.3 EG2 Symbolize the following:

Abbreviation scheme:

A: *a* is an animal. B: *a* is a bird. C: *a* is a cat. D: *a* is a dog.
F: *a* has feathers. G: *a* is gray. H: *a* can fly. M: *a* is a mammal.

a) Some mammals are dogs.

$\exists x(Mx \wedge Dx)$

b) Some animals with feathers are birds, but there aren't any mammals with feathers.

$\exists x(Ax \wedge Bx \wedge Fx) \wedge \sim\exists y(My \wedge Fy)$

c) Some animals, if they don't have feathers, are mammals.

$\exists x(Ax \wedge (\sim Fx \rightarrow Mx))$

d) Although there are gray cats, some cats aren't gray.

$\exists x((Cx \wedge Gx) \wedge \exists y(Cx \wedge \sim G)$

e) Some cats and dogs are gray.

$\exists x((Cx \wedge Gx) \wedge \exists y(Dy \wedge Gy)$

f) There are gray animals that have feathers, but they cannot fly.

$\exists x(((Gx \wedge Ax) \wedge (Fx \wedge \sim Hx))$

g) Mammals that fly exist but they don't have feathers.

$\exists x(Mx \wedge (Hx \wedge \sim Fx))$

5.4 SYLLOGISTIC LOGIC AND THE SQUARE OF OPPOSITION

Categorical syllogistic logic, the earliest formal system of logic, was developed by Aristotle in his Prior Analytics (~350 BC).

Syllogistic or Categorical Logic was the traditional predicate logic until the modern logic of the 20th century (the Greek, *categoria*, means 'predication'). Syllogisms of this type were discussed by Aristotle. His ideas were formalized and further developed by the medieval philosophers. The four types of quantified sentences (universal affirmative, universal negative, existential affirmative and existential negative) were the basis of categorical syllogistic logic. The medieval schools named the four forms of quantified sentences, 'A', 'E', 'I' and 'O'.

A	All A's are B.	All professors are intelligent.
E	No A's are B.	No professors are intelligent.
I	Some A's are B.	Some professors are intelligent.
O	Some A's are not B.	Some professors are not intelligent.

SQUARE OF OPPOSITION

Universal affirmative All A's are B. $\forall x\,(Fx \rightarrow Gx)$	**A**	←contraries→	**E**	Universal negative No A's are B. $\forall x\,(Fx \rightarrow {\sim}Gx)$
		↖ ↗ contradictions ↙ ↘		
Some A's are B. Particular affirmative $\exists x\,(Fx \wedge Gx)$	**I**	←subcontraries→	**O**	Some A's are not B. Particular negative $\exists x\,(Fx \wedge {\sim}Gx)$

Contradictions (A & O; I & E): The diagonals – if one is true, the other is false.

Contraries (A & E): In traditional syllogistic logic, they can both be false but cannot both be true.

Subcontraries (I & O): In traditional syllogistic logic, they can both be true but cannot both be false.

Categorical Syllogisms:

A 'categorical syllogism' is an argument form consisting of two premises and a conclusion, each of which is an A, E, I or O sentence. Each of these sentences has two terms (subject and predicate). The subject term of the conclusion (minor term) occurs in one of the premises; the predicate term of the conclusion (major term) occurs in the other premise; and the two premises share a term with each other (the middle term). There are 256 ways of combining A,E,I and O sentences in this way – but at most 24 of these forms are valid. (How many are valid depends on some logical assumptions. Aristotle identified 19 valid syllogisms, the Medieval philosophers took them to 24. On our system, only 15 are valid forms.)

DIFFERENCES BETWEEN TRADITIONAL AND MODERN LOGIC:

Modern Logic:

Consider a universal affirmative sentence (A sentence) such as: All unicorns are white. According to modern logic, assuming that no unicorns exist, this is a true sentence. Modern logic understands this sentence as: For everything, *x*, if *x* is a unicorn then *x* is white. Since nothing is a unicorn, the sentence is true for all things. This makes sense – this A sentence is the negation of the particular negative (O sentence), "There is at least one thing such that it is a unicorn and is not white". And, if there are no unicorns, then nothing is both a unicorn and non-white.

But if "All unicorns are white" is true, so is "No unicorns are white." This E sentence can be paraphrased: for everything *x*, if *x* is a unicorn then *x* is not white. Again, since there are no unicorns, it is true that everything that is a unicorn is not white.

According to modern logic, if no objects satisfy the subject property, both the A and the E sentence are true. This violates the traditional claim about the categorical statements that A and E sentences are contraries and cannot both be true.

Traditional Syllogistic Logic:

In traditional syllogistic or categorical logic, there is an existential presupposition – the universal A and E sentences presuppose the existence of at least one thing that satisfies the subject property. Indeed, if you do presuppose that the subject term is satisfied by at least one member of the universe, the traditional syllogistic logic and modern predicate logic give you the same results.

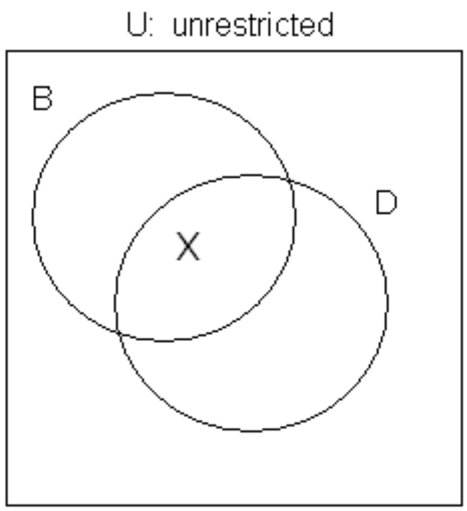
In categorical logic, it isn't clear how to treat a sentence such as "All unicorns are white." Such a sentence cannot be true. But, it cannot be false either – for that would imply that some unicorns are not-white. We might conclude that a sentence like, "All unicorns are white," is neither true nor false. It is not a good assertion. There are no unicorns so nothing can be said about them. This was Aristotle's view. Medieval logicians generally considered syllogisms with A sentences to be valid only if they had the existential import. Thus, an argument such as, "All dragons breath fire. All fire-breathers are dangerous. Therefore, dragons are dangerous." would be only conditionally valid – valid only if dragons exist.

Another difference between modern logic and categorical logic concerns the expression of the universal A and E sentences. In modern logic, we express these as universally quantified conditionals: $\forall x(Fx \rightarrow Gx)$ and $\forall x(Fx \rightarrow {\sim}Gx)$. Traditionally, universal sentences were *never* expressed as conditionals. Indeed, even when the conditional was used in other (non-categorical) syllogistic logic, it was not a material conditional, as it is in modern logic. The ancient and medieval logicians' concept of the conditional was not straight-forwardly truth-functional; hence, its meaning cannot be completely captured in a truth-table nor expressed as a disjunction. Actually, a truth-functional material conditional, or something very similar, was developed in the ancient world by Philo of Megara (early 3rd century BC), but it didn't catch on and remained a minority view.

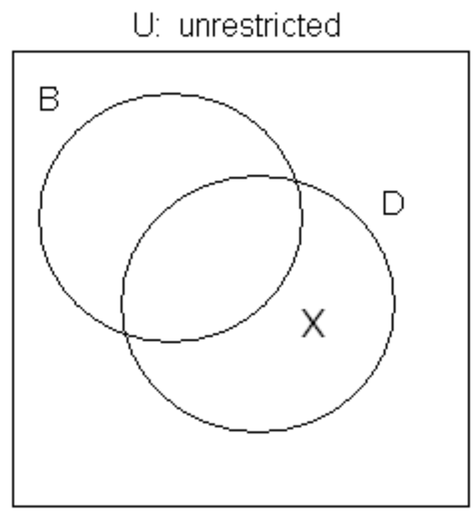
If the conditional is not a truth-functional notion in traditional syllogistic logic, then the symbolic sentences, $\forall x(Fx \rightarrow Gx)$ and $\forall x(Fx \rightarrow {\sim}Gx)$, are not true to the meaning of the A and E sentences in traditional syllogistic logic.

Using Venn Diagrams to illustrate quantified sentences.

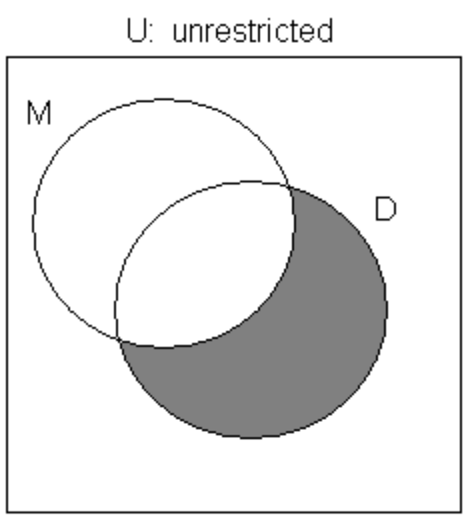
Use a square for the universe of discourse.
Use a circle for each one-place predicate.
Use an X to show some in an area (at least one).
Use shading to show areas where there are none.



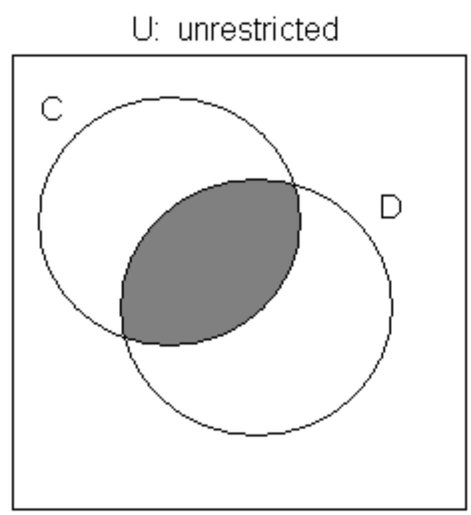
Some dogs are beagles.
(At least one thing is in both B & D).



Some dogs are not beagles.
(At least one thing is in D but not B.)



All dogs are mammals.
(Everything in D is in M.)



No cats are dogs.
(Nothing is in both C and D.)

5.5 SYMBOLIZATION TIPS:

- It is a good idea to paraphrase, underlining logical operators.
- Use the canonical forms of the existential and universal sentences.
- Use expressions like, "For everything x, if x ..." for the universal quantifier, $\forall$.
- Use expressions like, "there is at least one object x such that ..." for the existential quantifier, $\exists$.
- Think about whether it is an A, E, I and O sentence or the negation of one.
- Watch the Universe (U). A restricted universe can make life simpler.

  F: *a* is mortal G: *a* is a person.

  U: people

  All people are mortal: $\forall x\ Fx$

  U: everything/unrestricted.

  All people are mortal: $\forall x\ (Gx \rightarrow Fx)$

- Be careful with words like "any" which can be used for both universally and existentially quantified terms.

  Anyone who races will win a prize. (F: *a* races, G: *a* wins a prize)

  $\forall z\ (Fz \rightarrow Gz)$

  If anyone races, someone will win a prize.

  $\exists x\ Fx \rightarrow \exists y\ Gy$

- Be careful with words like "some" which usually indicate an existential, but can be used to express a universal.

  If some animal has feathers then it is a bird. (A: *a* is an animal, B: *a* is a bird, F: *a* has feathers.)

  $\forall x(Ax \rightarrow (Fx \rightarrow Bx))$

  $\forall x(Ax \wedge Fx \rightarrow Bx)$

- Watch the main connectives – especially in informal notation.
- Be careful with expressions that appear to be predicative. A brown animal is both brown and an animal, but a party animal is neither a party nor an animal. Similarly, a bloodsucking mosquito is both a bloodsucker and a mosquito, but a large mosquito (even a very large one) is not large!

5.5 E1 Let's try some more:

Abbreviation scheme:

B: *a* is basalt. F: *a* glitters. G: *a* is gold. I: *a* is iron pyrite (fool's gold)

J: *a* is quartz. K: *a* is rock. M: *a* is a metal. O: *a* is used for an Olympic first.

a: my wedding ring.

a) Some rocks glitter. $\exists x(Kx \wedge Fx)$

b) Anything gold is metal. $\forall x(Gx \rightarrow Mx)$

c) Basalt and quartz are rocks. $\forall x(Bx \rightarrow Kx) \wedge \forall x(Jx \rightarrow Kx)$

d) Quartz and iron pyrite glitter. $\forall x(Jx \vee Ix \rightarrow Fx)$

e) Some rocks are basalt and some are quartz. $\exists x(Kx \wedge Bx) \wedge \exists x(Kx \wedge Jx)$

f) Some rocks and metals glitter. $\exists x(Kx \wedge Fx) \wedge \exists y(My \wedge Fy)$

g) Gold is a metal but some metals are not gold. $\forall(Gx \rightarrow Mx) \wedge \exists x(Mx \wedge \sim Gx)$

h) Not all rocks are metal. $\sim \forall x(Kx \rightarrow Mx)$

i) No quartz is metal. $\sim \exists x(Jx \wedge Mx)$

j) Some, but not all, rocks are quartz. $\exists x(Kx \wedge Jx) \wedge \sim \forall x(Kx \rightarrow Jx)$

k) Some rocks glitter but they are not gold. $\exists x(Kx \wedge Fx \wedge \sim Gx)$

l) Iron pyrite glitters but is not gold. $\forall x(Ix \rightarrow Fx \wedge \sim Gx)$

m) Gold, and only gold, is used for an Olympic first. $\forall x(Gx \leftrightarrow Ox)$

n) Gold is a metal that glitters. $\forall x(Gx \rightarrow (Mx \wedge Fx))$

o) Gold, which is a metal, glitters. $\forall(Gx \rightarrow (Mx \wedge Fx))$

p) Rocks that are iron pyrite glitter. $\forall x((Kx \wedge Ix) \rightarrow Fx)$

q) Gold and quartz both glitter, however (of the two of them) only gold is a metal. $\forall x(Gx \rightarrow (Fx \wedge Mx))$

r) All that glitters is not gold. $\forall x(Fx \rightarrow \sim Gx)$

s) Only gold is used for an Olympic first but not all gold is used for Olympic firsts. $\forall x(Ox \rightarrow Gx) \wedge \sim \forall y(Gy \rightarrow Oy)$

t) My wedding ring is gold. Ga

u) My wedding ring does not glitter. $\sim Fa$

v) If anything is gold then my wedding ring is. $\forall x(Gx \rightarrow Ga)$

w) Nothing is used for Oympic firsts except gold. $\forall x(Ox \rightarrow Gx)$

5.6 THE SCOPE OF A QUANTIFIER

The scope of a quantifier in a formula is the subformula ϕ of which that quantifier is the main logical operator. ($\forall x\ \phi$ or $\exists x\ \phi$)

$\exists x\ (Fx \wedge Gx)$

Here the main logical operator of the subformula $(Fx \wedge Gx)$ is $\exists x$.

Everything within the brackets following $\exists x$ is within the scope of the existential quantifier, $\exists$.

$\forall y\ (Gy \rightarrow Fy)$

Here the main logical operator of the subformula $(Gy \rightarrow Fy)$ is $\forall y$.

Everything within the brackets following $\forall y$ is within the scope of the universal quantifier, $\forall$.

$\forall x(Gx \rightarrow \exists y(Hy \wedge Fy))$

Here the main logical operator of the subformula $(Gx \rightarrow \exists y(Hy \wedge Fy))$ is $\forall x$.

Everything within the brackets following $\forall x$ is within the scope of the universal quantifier.

The main logical operator of the subformula $(Hy \wedge Fy)$ is $\exists y$. Only $(Hy \wedge Fy)$ is in the scope of the existential quantifier.

Bound variable: A variable, *x*, is bound if and only if it occurs within the scope of an *x*-quantifier.

Free variable: A variable, *x*, is free if and only if it is not bound.

$\forall x\ Fx \rightarrow Gy$

Fx is within the scope of the quantifier $\forall x$, but Gy is not. Thus, 'x' is bound (it is a bound variable) but 'y' is not bound – it is a free variable.

$\exists x\ Fx \wedge Gx$ Free

Fx is within the scope of the quantifier $\exists x$, but Gx is not. Thus, the first occurrence of the variable 'x' is bound, but the second occurrence of 'x' is free.

$\exists x(Fx \wedge Gx)$

Due to the brackets, both occurrences of the variable 'x' are bound – neither is free.

What is within the scope of the quantifier?

Ask … Is there a left bracket right after the quantifier?

NO: then only the atomic formula is within its scope.

YES: everything between L & R bracket is in its scope.

Not all formulas are sentences!

Symbolic Sentence: A symbolic formula that contains no free variables (all variables are bound.)

Open Formula: a formula that does contain free variables. Open formulas are NOT sentences.

Symbolic Name: a formula that does not contain any variables.

5.7 TWO-PLACE (DYADIC) AND THREE-PLACE (TRIADIC) PREDICATES:

A two-place predicate is a sentence that relates two terms – $F^2(xy)$

Adam loves Betty: L(ab)　　　　Carol is taller than Daniel: F(cd)

A three-place predicate is a sentence that relates three terms – $F^3(xyz)$

Adam is standing between Betty and Carl: G(abc)

The terms in a multi-place predicate follow the predicate letter in parentheses.

The terms are ordered pairs or triplets.

We can indicate how many places a predicate has by putting a superscript number after the predicate letter. (The number of places must be indicated in the abbreviation scheme, but when symbolizing a sentence the superscript number is not necessary.) Note that this is done ONLY in the abbreviation scheme – not when symbolizing!

P^0: A zero-place predicate	P^0: John is happy. (atomic sentence).
F^1: A one-place predicate.	F^1: *a* is a person.
F^2: A two-place predicate.	F^2: *a* loves *b*.
F^3: A three-place predicate.	F^3: *a* loans *b* to *c*.

Often the superscript '0' and '1' are left off the zero-place and one-place predicates.

5.7 EG1 Symbolizing with names is straightforward:

G^1: *a* is graceful.　　D^2: *a* dances with *b*.　　B^3: *a* stands between *b* and *c*.
a: Adam　　b: Betty　　c: Carol　　d: Darren

a) Carol dances with Darren.

D(cd)

b) Adam dances with Betty only if she is graceful.

D(ab) → Gb

c) If neither Adam nor Darren is graceful, then Betty dances with Carol.

~(Ga v Gd) → D(bc)

d) Provided Carol is graceful, Darren or Adam will dance with her.

Gc → (D(dc) v D(ac))

e) Adam is standing between Betty and Carol.

B(abc)

f) If Adam is standing between Betty and Carol, then he dances with Betty or with Carol.

B(abc) → (D(ab) v D(ac))

Two-Place Predicates and The Quantifiers

U: People

G^2: *a* loves *b*. f: Frank e: Emma

If one of the variables is a individual constant, it is just a matter of getting the order right... in this case it will determine who loves whom.

G(fe)	Frank loves Emma.
∀x G(fx)	Frank loves everyone.
∃x G(ex)	Emma loves someone.
∀x G(xe)	Everyone loves Emma.
∃x G(xf)	Someone loves Frank.
~ ∀x G(fx)	Frank does not love everyone.
~ ∃x G(ex)	Emma loves no one.
~ ∀x G(xe)	Not everyone loves Emma.
~ ∃x G(xf)	Nobody loves Frank.

NOTE: We could use any individual variables without any change of meaning. ∀x G(ex) is logically equivalent to: ∀y G(ey) and to: ∀z G(ez)

Both variables can be quantified. Now it is a bit trickier – the order of the quantifiers AND the order of the variables must be considered. The quantifiers determine the scope, but the order of the variables in the two place predicate determine who is doing the loving and who is getting loved!

∀x ∀y L(xy)	Everyone loves everyone.
∃x ∃y L(xy)	Somebody loves somebody.
∀x ∃y L(xy)	Everybody has someone that they love.
∃x ∀y L(xy)	Some one person loves everyone.
~∀x ∀y L(xy)	Not everyone loves everyone.
~∃x ∃y L(xy)	Nobody loves anybody.
~∀x ∃y L(xy)	Not everybody has someone that they love.
~∃x ∀y L(xy)	Nobody loves everyone.

Quantifiers with Overlapping Scope

F^1: *a* is a person. L^2: *a* loves *b*. e: Emma

When two or more quantifiers are used, the scopes of the quantifiers often overlap. If the scopes overlap, then two different variables (i-z) must be used.

If somebody loves Emma then Emma loves somebody.

> Here the person who loves Emma may or may not be the same person that Emma loves. The scope of the quantifiers do not overlap. The variable 'x' can be used in both antecedent and consequent; however, one could also use a new variable such as 'y' in the consequent.
>
> $\exists x\ (Fx \wedge L(xe)) \rightarrow \exists x\ (Fx \wedge L(ex))$ or $\exists x(Fx \wedge L(xe)) \rightarrow \exists y(Fy \wedge L(ey))$

If somebody loves Emma then Emma loves that person.

> Here the person who loves Emma is the same person Emma loves. There is only one quantifier (and this time it is a universal since it is true about all people who love Emma.) All occurrences of 'x' are within the scope of the quantifier.
>
> $\forall x\ (Fx \rightarrow (L(xe) \rightarrow L(ex)))$

Somebody loves everybody.

> Here we must use two quantifiers, but the second quantifier is within the scope of the first quantifier. A new variable ***must*** be used.
>
> $\exists x\ (Fx \wedge \forall y\ (Fy \rightarrow L(xy)))$

If we restrict the universe to people, it is easy to see the basic pattern. In all of them L(xy) is in the scope of both quantifiers. In two- and three-place predicates, the order in which the quantifiers occur can matter for it determines which subformulas are within the scope of which quantifiers.

The order of the quantifiers doesn't matter if the quantifiers are the same:

$\forall x\ \forall y\ L(xy)$	Everybody loves everybody.	$\forall y$ is within the scope of $\forall x$
$\forall y\ \forall x\ L(xy)$	Everyone is loved by everyone.	$\forall x$ is within the scope of $\forall y$
$\exists x\ \exists y\ L(xy)$	Someone loves somebody.	$\exists y$ is within the scope of $\exists x$
$\exists y\ \exists x\ L(xy)$	Somebody is loved by somebody.	$\exists x$ is within the scope of $\exists y$

The order of the quantifiers does matter if they are different!

$\forall x\ \exists y\ L(xy)$	Everybody has someone that they love.	$\exists y$ is within the scope of $\forall x$.
$\exists x\ \forall y\ L(xy)$	There is a person who loves everyone.	$\forall y$ is within the scope of $\exists x$.

But we don't have to quantify 'x' before 'y'. When the two quantifiers are different, the meaning changes when the order of the quantifiers changes.

$\exists y\ \forall x\ L(xy)$	There is a person who is loved by everyone.	$\forall x$ is within the scope of $\exists y$
$\forall y\ \exists x\ L(xy)$	Everyone has someone who loves them.	$\exists x$ is within the scope of $\forall y$

In the previous examples, the universe was restricted to people. But the universe won't always be restricted. The restriction must be done with predicates. This is done the same way it was done for simple universal and existential sentences: using $\rightarrow$ for the universal, and $\wedge$ for the existential.

We can symbolize all of these with the following abbreviation scheme:

F^1: *a* is a person L^2: *a* loves *b*

Note that in each of these the canonical forms are used. Every universal quantifier uses a conditional and has a restricting antecedent and a predicating consequent. Every existential quantifier uses a conjunction.

$\forall x\ (Fx \rightarrow \forall y(Fy \rightarrow L(xy)))$	Everyone loves everyone.
$\exists x\ (Fx \wedge \exists y(Fy \wedge L(xy)))$	Somebody loves somebody.
$\forall x\ (Fx \rightarrow \exists y(Fy \wedge L(xy)))$	Everybody has somebody that they love.
$\exists x\ (Fx \wedge \forall y(Fy \rightarrow L(xy)))$	There's a person who loves everyone.
$\exists y\ (Fy \wedge \forall x(Fx \rightarrow L(xy)))$	Some one person is loved by everyone.
$\forall y\ (Fy \rightarrow \exists x(Fx \wedge L(xy)))$	Everyone has someone who loves them.

Some English sentences are ambiguous – when translating to English, avoid using such expressions:

Everybody loves somebody.

This could mean:

Everybody has somebody that they love. OR There's a person who is loved by everyone.

$\forall x\ (Fx \rightarrow \exists y(Fy \wedge L(xy)))$ Everybody has someone that they love

Restricting antecedent of the universal, $\forall x$: Fx
Every x that is a person is such that…

Predicating consequent of the universal, $\forall x$: $\exists y(Fy \wedge L(xy)$
Restricting conjunct: a person, y
Predicating conjunct: x loves y.
There is a person, y, whom x loves.

$\exists y\ (Fy \wedge \forall x(Fx \rightarrow L(xy)))$ There is one person that everyone loves.

Restricting conjunct of the existential, $\exists y$: Fy
There is some y such that y is a person and…

Predicating conjunct of the existential, $\exists y$: $\forall x(Fx \rightarrow L(xy)$
Restricting antecedent: a person, x
Predicating consequent: x loves y.
Every x, such that x is a person, loves y.

'Someone loves everyone.' and 'Everyone is loved by someone.' are similarly ambiguous.

Now let's see what happens when we negate them ...

$\sim \forall x\,(Fx \rightarrow \forall y(Fy \rightarrow L(xy)))$	Not everyone loves everyone.
$\sim \exists x\,(Fx \wedge \exists y(Fy \wedge L(xy)))$	Nobody loves anybody.
$\sim \forall x\,(Fx \rightarrow \exists y(Fy \wedge L(xy)))$	Not everybody has someone that they love.
$\sim \exists x\,(Fx \wedge \forall y(Fy \rightarrow L(xy)))$	No one loves everybody.
$\sim \exists y\,(Fy \wedge \forall x(Fx \rightarrow L(xy)))$	Nobody is loved by everyone.
$\sim \forall y\,(Fy \rightarrow \exists x(Fx \wedge L(xy)))$	Not everyone is loved by somebody.

In the above sentences, the main logical operator is a negation. But, all of them can be symbolized such that the main logical operator is a quantifier.

Notes:

The negation of a universal is an existential: Not all = Some are not.

The negation of an existential is a universal: Not some = All are not.

Thus the restricting elements (the conditional and the conjunction) will also change accordingly.

$\exists x\,(Fx \wedge \sim \forall y(Fy \rightarrow L(xy)))$ $\exists x\,(Fx \wedge \exists y(Fy \wedge \sim L(xy)))$	Someone is not such that he/she loves everyone. Someone doesn't love some person.	Not everyone loves everyone.
$\forall x\,(Fx \rightarrow \sim \exists y(Fy \wedge L(xy)))$ $\forall x\,(Fx \rightarrow \forall y(Fy \rightarrow \sim L(xy)))$	For everyone, there is not someone he/she loves. Everyone fails to love anyone.	Nobody loves anybody.
$\exists x\,(Fx \wedge \sim \exists y(Fy \wedge L(xy)))$ $\exists x\,(Fx \wedge \forall y(Fy \rightarrow \sim L(xy)))$	Somebody has nobody that they love. Someone fails to love anyone.	Not everybody loves somebody
$\forall x\,(Fx \rightarrow \sim \forall y(Fy \rightarrow L(xy)))$ $\forall x\,(Fx \rightarrow \exists y(Fy \wedge \sim L(xy)))$	Everybody is such that they don't love everyone. Everyone has someone they don't love.	No one loves everybody.
$\forall y\,(Fy \rightarrow \sim \forall x(Fx \rightarrow L(xy)))$ $\forall y\,(Fy \rightarrow \exists x(Fx \wedge \sim L(xy)))$	Everyone is such that they aren't loved by everyone. Everyone is such that there's a person who doesn't love them.	Nobody is loved by everyone.
$\exists y\,(Fy \wedge \sim \exists x(Fx \wedge L(xy)))$ $\exists y\,(Fy \wedge \forall x(Fx \rightarrow \sim L(xy)))$	Somebody is such that nobody loves him/her. Someone is such that everybody fails to love him/her.	Not everyone is loved by somebody.

Things get even more complicated when we add in the logical operators and include three-place predicates. However, the principles are the same. Again, you must watch the order of the quantifiers (especially if they are different) and the order of the variables. It helps to stick with the canonical forms: universal quantifier is followed by $\rightarrow$ and the existential quantifier is followed by $\wedge$.

5.7 EG2: Symbolize the following:

F^1: *a* is a person	H^2: *a* teaches *b*
G^1: *a* is a time	J^3: *a* bores *b* at *c*

a) Everyone teaches everyone.

$$\forall x(Fx \rightarrow \forall y(Fy \rightarrow H(xy)))$$

b) Somebody teaches somebody.

$$\exists x(Fx \wedge \exists y(Fy \wedge H(xy)))$$

c) Everyone is a teacher.

$$\forall x(Fx \rightarrow \exists y(Fy \wedge Hxy)))$$

d) Everyone has a teacher.

$$\forall x(Fx \rightarrow \exists y(Fy \wedge H(yx)))$$

e) Someone teaches everybody.

$$\exists x(Fx \wedge \forall y(Fy \rightarrow H(xy)))$$

f) Someone is taught by everyone.

$$\exists(Fx \wedge \forall y(Fy \rightarrow H(yx)))$$

g) Everyone is bored by someone some of the time.

$$\forall x(Fx \rightarrow \exists y(Fy \wedge \exists z(Gz \wedge J(yxz))))$$

h) Some people bore people all of the time.

$$\exists x(Fx \wedge \forall y(Gy \rightarrow \exists z(Fz \wedge J(xzy))))$$

i) Although it's always the case that someone is bored by someone, nobody bores all of the people all of the time.

5.7 Exercises Predicate symbolization

NOTE: you might want to read units 5.9 (Symbolizing: step by step and Equivalencies) for further symbolization tips.

5.7 E1: Symbolize the following quotes about friendship: (Some of these quotes have been altered a little to make symbolization easier.)

A^1: *a* is a person	E^2: *a* is equal to *b*.	F^2: *a* is a friend to *b*
K^2: *a* knows all about *b*.	L^2: *a* likes *b*.	C^3: *a* has *b* in common with *c*.

O^1: *a* is doing what one ought to do (*a* is doing what one should do)

a) A person who is a friend to all is a friend to none.   (Aristotle)

b) To have a have a friend is to be one. (Ralph Waldo Emerson)

c) True friends are those who like and dislike the same things. (Sallust – a Roman Historian, 1st century BC.)

d) Friends have all things in common. (Plato)

e) You should have no friends who are not equal to yourself.   (Confucious)

f) A friend is someone who knows all about you and still likes you. (Elbert Hubbard)

g) Everyone is someone's friend. (Ambiguous – symbolize two distinct ways.)

5.7 E2: Symbolize the following:

A^1: *a* is an act.	F^1: *a* is a person/human.	G^1: *a* is good.
D^2: *a* does b.	H^2: *a* harms *b*.	B^3: *a* borrows *b* from *c*.

a) Good people don't harm people.

b) If you harm anyone then you harm yourself.

c) A person is not good who does things that aren't good.

d) Only people who do not harm others are good.

e) The person whose every act is good is not human.

f) A good person does not perform acts that harm others.

g) No act is good that causes a person harm unless person being harmed is harming people.

h) Everyone borrows things from people, but nobody lends things to everyone.

i) Some people borrow things from people who don't borrow things from anyone.

j) Neither a borrower nor a lender be. (A good person neither borrows nor lends things to others.)

k) You are not doing a good thing if you lend to someone something that you borrowed from someone.

l) Everyone borrows something from somebody. (Ambiguous – symbolize four distinct ways.)

5.7 E3: Symbolize the following:

A^1: *a* is an astronaut	B^1: *a* is a space shuttle	C^1: *a* is a car
E^1: *a* is a time	H^1: *a* is a person	J^1: *a* is a jet
I^1: *a* is a vehicle	D^2: *a* drives *b*	F^2: *a* is faster than *b*
G^2: *a* rides in *b*	K^3: *a* flies in *b* at *c*	L^3: *a* drives *b* at *c*
a: Adam	b: Betty	e: the Endeavor (a space shuttle)

a) Cars and jets are vehicles.

b) A space shuttle is a vehicle if and only if a jet is.

c) People who ride in space shuttles are astronauts.

d) No car is faster than any jet.

e) Adam has never flown in the Endeavor, but he sometimes flies in a space shuttle.

f) Any vehicle that Betty drives is faster than those that Adam rides in.

g) Some people ride in vehicles that are faster than any car.

h) Some people drive vehicles that are faster than any car that they ride in but do not drive.

i) Some people don't ride in cars unless they are driving them.

j) Betty won't ride in any jet that Adam has ever flown in.

k) Astronauts who ride in the Endeavor never fly in space shuttles that are faster than it.

l) People who never fly in space shuttles sometimes fly in jets.

m) Everyone flies in jets some of the time, but never is everyone flying in jets.

n) Somebody is always driving some car. (Ambiguous – symbolize four distinct ways.)

5.7 E4: Symbolize the following arguments:

a)

B^1: *a* is pleasurable	F^1: *a* is a person	G^1: *a* is good
C^2: *a* pursues *b*	D^2: *a* desires *b*	c: Callicles

Everything that people desire is good.

Callicles (who is a person) pursues anything pleasurable.

Some pleasurable things are not good.

Therefore, Callicles pursues things that he does not desire.

(based on Plato, *Gorgias*)

b)

A^1: *a* is within the scope of the physical sciences.		B^1: *a* is about colour vision.
C^1: *a* is a colour	D^1: *a* is a time.	F^1: *a* is a fact.
H^1: *a* is a person.	K^2: *a* knows *b*	D^3: *a* sees *b* at *c.*
a: Mary	P: Physicalism is true	

If Physicalism is true then all facts are within the scope of the physical sciences.

Mary knows all the facts about colour vision within the scope of the physical sciences.

Although Mary is a person, Mary has never seen any colour.[1]

For a person to know certain facts about color vision it is necessary that the person sees something in color at some time or another.[2]

Thus, some facts about colour vision are not within the scope of the physical sciences and physicalism is false.

(based on Jackson, *Epiphenomenal Qualia*.)

1. Mary has never been allowed to see colour, being raised so that everything she sees she sees through a black and white monitor. But she becomes a scientist specializing in colour vision (and learns everything physical there is to know about color vision!)
2. What a colour looks like to you is (according to Jackson in this article) a fact. You know what that colour looks like, you know what it is like to see that colour.

c)

E^1: *a* is evil	F^1: *a* is omnipotent.
H^1: *a* happens	G^1: *a* is perfectly good
A^2: *a* allows *b* to happen.	B^2: *a* is able to prevent *b* from happening.
C^2: *a* is a cause of *b*	K^2: *a* knows *b*

An omniscient being (a being who knows everything) knows the causes of all evils.

An omnipotent being who knows the causes of an evil is able to prevent it from happening.

A perfectly good being who is able to prevent an evil thing from happening does not allow it to happen.

Things that are not allowed to happen (by some being) do not happen.

Evil things happen.

Therefore, there are no omniscient, omnipotent, perfectly good beings.

(based on The Problem of Evil, many sources from Epicurus on.)

To be is to be the value of a bound variable…

Willard Van Orman Quine. 'On What There Is, *Review of Metaphysics* (1948): 21–28.

Willard Van Orman Quine

5.8 SYMBOLIC FORMULAS AND SENTENCES

Atomic sentences: sentence letters or n-place predicates followed by n individual terms (constants or variables).

Logical operators: quantifiers and truth-functional connectives.

What counts as a symbolic formula?

1. Every atomic sentence is a symbolic formula.
2. If ϕ is a symbolic formula then so is $\sim\phi$.
3. If ϕ and ψ are symbolic formulas, then so are: $(\phi \wedge \psi)$, $(\phi \vee \psi)$, $(\phi \rightarrow \psi)$, $(\phi \leftrightarrow \psi)$
4. If ϕ is a symbolic formula that contains an individual variable, *x*, but no quantifier for that variable, then $\forall x\ \phi$ and $\exists x\ \phi$ are both symbolic formulas.
5. Nothing else is a symbolic formula, unless it can be formed by repeated application of the above.

Symbolic Sentences:

Any symbolic formula that does not contain a free variable is a symbolic sentence. (All variables in symbolic sentences must be bound by a quantifier.)

Substitution Instances

Sentences which have bound individual variables can be turned into sentences with individual constants (name letters) in place of those variables. The main quantifier is dropped, and all instances of the variable bound by the quantifier must be replaced by the same individual constant letter.

Fa is a substitution instance of $\forall x$ Fx.

$(Fc \rightarrow Gc)$ is a substitution instance of $\forall y(Fy \rightarrow Gy)$.

$(Gb \wedge \forall y(Ly \rightarrow F(yb)))$ is a substitution instance of $\exists x(Gx \wedge \forall y(Ly \rightarrow F(yx)))$

5.8 E1

a) Which of the following are substitution instances of: $\forall x(Fx \rightarrow Gx)$?

i) $Fa \rightarrow Ga$
ii) $Fx \rightarrow Gx$
iii) $Fx \rightarrow Gb$
iv) $Fb \rightarrow Gb$
iv) $\forall x(Fx \rightarrow Gb)$

b) Which of the following are substitution instances of: $\forall x(Fx \rightarrow \exists y(Gy \wedge L(xy)))$?

i) $Fa \rightarrow \exists y(Gy \wedge L(xy))$
ii) $Fb \rightarrow \exists y(Gy \wedge L(by))$
iii) $\forall x(Fx \rightarrow (Ga \wedge L(xa))$
iv) $Fa \rightarrow \exists y(Ga \wedge L(ay))$
v) $Fd \rightarrow \exists y(Gy \wedge L(dy))$

5.9 SYMBOLIZATION STEP BY STEP

Paraphrasing:

Paraphrase restricted universal sentences: For everything, x, if x is … then x is …

Paraphrase restricted existential sentences: There is at least one x such that x is … and x is ….

Main Logical Operator:

Watch the main logical operator. Is it a truth-functional connective or is it a quantifier?

Scope considerations:

Consider the sentence and symbolization scheme:

If Dan bores somebody then Dan bores himself. F^1: *a* is a person. G^2: *a* bores *b* d: Dan

<u>If</u> <u>there exists some x such that</u> x is a person and Dan bores x, <u>then</u> Dan bores himself.

$\exists x(Fx \wedge G(dx)) \rightarrow G(dd)$

What if we want the whole sentence within the scope of the quantifier?

Wrong: $\exists x\ [(Fx \wedge G(dx)) \rightarrow G(dd)]$

Right: $\forall x\ [(Fx \rightarrow (G(dx) \rightarrow G(dd))]$

The wrong version: there is some x such that if x is a person that Dan bores then Dan bores himself. Thus it is true if anything exists that isn't a person that Dan bores.

The right version says that for each x, if x is a person then if Dan bores that person, Dan bores himself.

What do pronouns refer to?

You need to consider how are nouns and pronouns related to one another. Do they refer back to an earlier occurrence? If so, you need to use the same variable, bound by the same quantifier. If not, you should use a new variable (or bind it to a new quantifier).

If somebody loves Betty, then Betty loves someone.

$\exists x\ F(xB) \rightarrow \exists y\ F(By)$ OR $\exists x\ F(xB) \rightarrow \exists x\ F(Bx)$

If anybody loves Betty, then Betty loves him/her.

$\forall x\ ((F(xB) \rightarrow F(Bx))$

And or Or?

Is the sentence about different kinds of things or things with two or more properties? Think hard about whether you want to use "or" or "and".

Witches and goblins are spooky.

<u>Everything x is such that</u> <u>if</u> x is a witch <u>or</u> a goblin, <u>then</u> x is spooky. $\forall x(Wx \vee Gx \rightarrow Sx)$

One Place Predicates – the basic forms:

Restricted Universal Generalization: symbolized with a conditional $\rightarrow$

$\forall$x (Restricting antecedent $\rightarrow$ Predicating consequent)

Example 1: All humans are mortal. (F: *a* is human. G: *a* is mortal.)

The restricting antecedent is 'is human' since the sentence is about all humans.

The predicating consequent is 'is mortal' since the sentence is predicating that of humans.

Paraphrase: for each x, <u>if</u> x is a human <u>then</u> x is mortal.

$\forall x\ (Fx \rightarrow Gx)$

Example 2: Students that study hard will pass. (F: *a* is a student. G: *a* studies hard. H: *a* will pass.)

The restricting antecedent is a conjunction of 'is a student' and 'studies hard' since the sentence is about students who study hard.

The predicating consequent is 'will pass' since the sentence is predicating that of all students who study hard.

Paraphrase: for each x, <u>if</u> x is a student <u>and</u> x studies hard, <u>then</u> x will pass.

$\forall x\ ((Fx \wedge Gx) \rightarrow Hx)$

This can also be symbolized as follows:

The restricting antecedent is 'is a student' since the sentence says something about all students.

The predicating consequent is a conditional – if the student studies hard, then the student will pass.

Paraphrase: for each x, <u>if</u> x is a student <u>then</u> <u>if</u> x studies hard <u>then</u> x will pass.

$\forall x\ (Fx \rightarrow (Gx \rightarrow Hx))$

Example 3: Students that study hard will pass and will enjoy the course.
(F: *a* is a student. G: *a* studies hard. H: *a* will pass. I: *a* will enjoy the course.)

In this example, the restricting antecedent is the same as in the previous example.

The predicating consequent is now a conjunction of 'will pass' and 'will enjoy the course.

Paraphrase 1: for each x, <u>if</u> x is a student <u>and</u> x studies hard, <u>then</u> x will pass and x will enjoy the course.

$\forall x\ ((Fx \wedge Gx) \rightarrow (Hx \wedge Ix))$

Paraphrase 2: for each x, <u>if</u> x is a student, <u>then</u> <u>if</u> x studies hard, <u>then</u> x will pass <u>and</u> x will enjoy the course.

$\forall x\ (Fx \rightarrow (Gx \rightarrow (Hx \wedge Ix)))$

Restricting Expressions:

There are many English expressions that restrict:

F: *a* is a dog. G: *a* is a mammal. H: *a* is aggressive. I: *a* is dangerous.

In these sentences, the restricting antecedent is ‘is a dog’.

All of these are symbolized: $\forall x\ (Fx \rightarrow Gx)$

All dogs are mammals.	The dog is a mammal.
Every dog is a mammal.	Dogs are mammals.
A dog is a mammal.	Any dog is a mammal.
Only mammals are dogs.	

In these sentences, the restricting antecedent is a conjunction of ‘is a dog’ and ‘is aggressive’.

All of these are symbolized: $\forall x\ ((Fx \wedge Hx) \rightarrow Ix)$ OR $\forall x\ (Fx \rightarrow (Hx \rightarrow Ix))$

Aggressive dogs are dangerous.	Dogs that are aggressive are dangerous.
An aggressive dog is dangerous.	Any dog is dangerous if it is aggressive.
If a dog is aggressive, it’s dangerous.	Dogs, if aggressive, are dangerous.

Example 4: Fruits and vegetables are good for you.
(F: *a* is a fruit. G: *a* is a vegetable. H: *a* is good for you.)

In this example, the restricting antecedent is a disjunction of ‘is a fruit’ and ‘is a vegetable’ since the sentence is about everything that is either a fruit or a vegetable.

The predicating consequent is ‘is good for you’.

Paraphrase: for each x, <u>if</u> x is a fruit <u>or</u> x is a vegetable <u>then</u> x is good for you.

$\forall x\ ((Fx \vee Gx) \rightarrow Hx)$

Alternate symbolization: This can be symbolized as a conjunction of two restricted universals – the first says that if x is a fruit x is good for you, the second says that if y is vegetable then y is good for you.

Paraphrase: for each x, <u>if</u> x is a fruit <u>then</u> x is good for you, <u>and</u> for each y, <u>if</u> y is a vegetable <u>then</u> y is good for you.

$\forall x\ (Fx \rightarrow Hx) \wedge \forall y\ (Gy \rightarrow Hy)$

Example 5: Only fruits and vegetables are good for you.

This says that if something is good for you, then it must be a fruit or a vegetable.

The restricting antecedent is ‘is good for you’. The predicating consequent is a disjunction of ‘is a fruit’ and ‘is a vegetable’.

Paraphrase: for each x, <u>if</u> x is good for you <u>then</u> x is a fruit <u>or</u> x is a vegetable.

$\forall x\ (Hx \rightarrow (Fx \vee Gx))$

Example 6: Either vegetables or chocolates are good for you.
(G: *a* is a vegetable. H: *a* is good for you. I: *a* is a chocolate.)

This says that either vegetables are good for you or chocolates are good for you. It does *not* say that anything that is a vegetable or a chocolate is good for you.

This is a disjunction of two restricted universals. The first has a restricting antecedent of ‘is a vegetable’ and the second has a restricting antecedent of ‘if a chocolate’. Both have the predicating consequent ‘is good for you’.

Paraphrase: for each x, <u>if</u> x is a vegetable <u>then</u> x is good for you <u>or</u> for each x, <u>if</u> x is a chocolate <u>then</u> x is good for you.

$\forall x\ (Gx \rightarrow Hx) \vee\ \forall y\ (Iy \rightarrow Hy)$

Restricted Existential Generalizations: symbolized with a conjunction $\wedge$

$\exists x$ (Restricting conjunct $\wedge$ Predicating conjunct)

Example 7: Some dogs are aggressive. (F: *a* is a dog. G: *a* is aggressive.)

The restricting conjunct is ‘is a dog’ since the sentence is about some dogs.

The predicating conjunct is ‘is aggressive’ since the sentence is predicating that of some dogs.

Paraphrase: there exists at least one x such that x is a dog <u>and</u> x is aggressive.

$\exists x\ (Fx \wedge Gx)$

Example 8: Some aggressive dogs are dangerous.
(F: *a* is a dog. G: *a* is aggressive. H: *a* is dangerous.)

The restricting conjunct is a conjunction of ‘is a dog’ and ‘is aggressive’ since the sentence is about some aggressive dogs.

The predicating conjunct is ‘is dangerous’ since the sentence is predicating that of some aggressive dogs.

Paraphrase: there exists at least one x such that x is a dog <u>and</u> x is aggressive <u>and</u> x is dangerous

$\exists x\ (\ Fx \wedge Gx \wedge Hx)$

Example 9: A man is wearing a red hat. (F: *a* is a man. G: *a* is wearing a red hat.)

Unlike the sentence, 'A dog is a mammal' which says something about all dogs, this sentence says something about a particular man.

The restricting conjunct is 'a man'.

The predicating conjunct is 'is wearing a red hat' since the sentence is predicating that of some man.

Paraphrase: there exists at least one x such that x is a man <u>and</u> x is wearing a red hat.

$\exists x\ (Fx \wedge Gx)$

Example 10: There are black bears and brown bears at the zoo.

(F: *a* is a bear. G: *a* is black. H: *a* is brown. I: *a* is at the zoo.)

This sentence says that there are black bears at the zoo and brown bears at the zoo. It is the conjunction of two existentials.

Paraphrase: there exists at least one x such that x is black, <u>and</u> x is a bear <u>and</u> x is at the zoo <u>and</u> there exists at least one y such that y is brown, <u>and</u> y is a bear <u>and</u> y is at the zoo.

$\exists x\ (Gx \wedge Fx \wedge Ix) \wedge \exists y\ (Hy \wedge Fy \wedge Iy)$

Note: we cannot symbolize it: $\exists x\ (Fx \wedge Ix \wedge (Gx \vee Hx))$ since this would mean that there are bears at the zoo that are brown or black. Although this is true if there are both black and brown bears at the zoo, it is also true if there are just brown bears at the zoo (and the sentence says there are brown bears and black bears there.)

Note: we cannot symbolize it: $\exists x\ (Fx \wedge Ix \wedge Gx \wedge Hx)$ since this would mean that there are bears at the zoo that are brown and black (one bear that is both brown and black would make this true.)

Example 11: If anyone enters the race, somebody will receive a prize.

(F: *a* is a person. G: *a* enters the race. H: *a* will receive a prize.)

This sentence is a conditional, with an existential statement for the antecedent and an existential statement for the consequent. If at least one person enters the race then at least one person will receive a prize.

Paraphrase: <u>if</u> there exists at least one x such that x is a person <u>and</u> x enters the race <u>then</u> there exists at least one y such that y is a person <u>and</u> y receives a prize.

$\exists x\ (Fx \wedge Gx) \rightarrow \exists y\ (Fy \wedge Hy)$

Note: we cannot symbolize it: $\exists x\ (Fx \wedge (Gx \rightarrow Hx))$ This would mean that someone is a person and if that person enters the race then he/she will a prize. But the sentence doesn't say that the person receiving the prize entered the race! Furthermore, the English sentence is true if there are no people at all (since if there are no people, then nobody enters the race and the antecedent is false, thus the conditional is true.) The wrong symbolization is not true if no people exist (although it is true if people exist but nobody enters the race.)

Negative Statements:

With negative statements it is important to identify the restricting predicate and then to determine whether the negative predicate applies to all of the restricted set of individuals or to some of the restricted set of individuals. Once you do that, use a universal quantifier if it applies to all and use an existential quantifier if it applies to some.

F: *a* is a politician. G: *a* is honest. H: *a* is successful.

Sentence	Restrict-ing Predicate	Negative Predicate	Applies to some or all of restricted set.	Symbolic Sentence
No politicians are honest.	F	~G	all	$\forall$x (Fx $\rightarrow$ ~Gx)
Politicians are not honest.	F	~G	all	$\forall$x (Fx $\rightarrow$ ~Gx)
Some politicians aren't honest.	F	~G	some	$\exists$x (Fx $\wedge$ ~Gx)
There's a politician who's dishonest.	F	~G	some	$\exists$x (Fx $\wedge$ ~Gx)
There are no honest politicians.	F	~G	all	$\forall$x (Fx $\rightarrow$ ~Gx)
A politician isn't honest. (ambiguous)	F F	~G ~G	all some	$\forall$x (Fx $\rightarrow$ ~Gx) $\exists$x (Fx $\wedge$ ~Gx)
Successful politicians aren't honest.	F $\wedge$ H	~G	all	$\forall$x ((Fx $\wedge$ Hx) $\rightarrow$ ~Gx)
Some successful politicians aren't honest.	F $\wedge$ H	~G	some	$\exists$x (Fx $\wedge$ Hx $\wedge$ ~Gx)
There are no successful politicians who are honest.	F $\wedge$ H	~G	all	$\forall$x ((Fx $\wedge$ Hx) $\rightarrow$ ~Gx)
Not all honest politicians are successful.	F $\wedge$ G	~H	some	$\exists$x (Fx $\wedge$ Gx $\wedge$ ~Hx)
Of politicians, the successful ones aren't honest.	F $\wedge$ H	~G	all	$\forall$x ((Fx $\wedge$ Hx) $\rightarrow$ ~Gx)
Politicians that are honest aren't successful.	F $\wedge$ G	~H	all	$\forall$x ((Fx $\wedge$ Gx) $\rightarrow$ ~Hx)
Only dishonest politicians are successful.	F $\wedge$ H	~G	all	$\forall$x ((Fx $\wedge$ Hx) $\rightarrow$ ~Gx)
Politicians are neither successful nor honest.	F	~(G $\vee$ H) ~G $\wedge$ ~H	all	$\forall$x (Fx $\rightarrow$ ~(Gx $\vee$ Hx)) $\forall$x (Fx $\rightarrow$ (~Gx $\wedge$~Hx))
No politician is both honest and successful.	F	~(G $\wedge$ H)	all	$\forall$x (Fx $\rightarrow$ ~(Gx $\wedge$ Hx))
Some politicians are neither honest nor successful.	F	~(G $\vee$ H) ~G $\wedge$ ~H	some	$\exists$x (Fx $\wedge$ ~(Gx $\vee$ Hx)) $\exists$x (Fx $\wedge$ ~Gx $\wedge$ ~Hx)
There's a politician who is honest but she is not successful.	F $\wedge$ G	~H	some	$\exists$x (Fx $\wedge$ Gx $\wedge$ ~ Hx)

Negative sentences can also be symbolized as the negation of a universal or existential:

Example 12: Not all politicians are honest.

This sentence is the negation of the universal, ‘All politicians are honest’.

Paraphrase: it is not the case that all politicians are honest.

Symbolization of ‘All politicians are honest’:

$\forall x\ (Fx \rightarrow Gx)$ (Restricting Antecedent $\rightarrow$ Predicating Consequent)

Symbolization of ‘Not all politicians are honest’:

$\sim \forall x\ (Fx \rightarrow Gx)$ (Negation of all politicians are honest.)

Note that this sentence is equivalent to ‘Some politicians are dishonest.’
$\exists x\ (Fx \wedge \sim Gx)$

Example 13: No politicians are honest.

This sentence is the negation of the existential, ‘Some politicians are honest.’

Paraphrase: It is not the case that some politicians are honest.

Symbolization of ‘Some politicians are honest.’

$\exists x\ (Fx \wedge Gx)$ (Restricting conjunct $\wedge$ Predicating conjunct)

Symbolization of ‘No politicians are honest’:

$\sim \exists x\ (Fx \wedge Gx)$

Note: this sentence is equivalent to ‘All politicians are dishonest.’
$\forall x\ (Fx \rightarrow \sim Gx)$

Two- and Three-Place Predicates

When symbolizing two- and three-place predicates, as much as possible stick with the standard forms for restricted quantified sentences ($\forall$ with $\rightarrow$, $\exists$ with $\wedge$.)

- Paraphrase Universals: for each x, if x is … then ….
- Paraphrase Existentials: there is at least one x such that x is… and ….
- Use a new variable for each new quantifier (not always necessary, but keeps it simple).
- Think about what you want to quantify first – this is especially important when some terms will be quantified with existential quantifiers and some will be quantified with universal quantifiers.
- Check! Look at each quantifier. After the quantifier is a variable then a left bracket. In most cases, there will be a restricting predicate immediately after the left bracket that contains that variable.
    - Do the variables match? At least one variable in the restricting predicate should be the same as that in the immediately preceding quantifier.
    - If the quantifier is a universal, then after the restricting predicate should be a $\rightarrow$.
    - If the quantifier is an existential, then after the restricting predicate should be a $\wedge$.

  Note: sometimes the restricting quantifier is complex (uses $\wedge$, $\vee$, $\sim$, $\rightarrow$ or $\leftrightarrow$).
  Note: sometimes there is no restricting predicate.
- Check! Do you have the right number of right brackets? Are there any free (unbound) variables? Did you get all the variables in the right order?

Identify the main logical operator:

Is it a negation? $\sim$

If so, then you should be able to paraphrase the sentence, "It is not the case that ϕ". Symbolize ϕ and put a negation sign in front. Remember negated quantified sentences can also be symbolized as unnegated quantified sentences ($\sim\forall \leftrightarrow \exists\sim$, $\sim\exists \leftrightarrow \forall\sim$)

Is it a sentential connective? $\vee$, $\wedge$, $\rightarrow$ or $\leftrightarrow$

If so, then the sentence will have two clauses and no pronouns or other terms in the second clause will refer back to the first clause.

You should be able to separate two complete sentences, symbolize them independently and then connect them with the sentential connective.

Is it a quantifier? $\forall$ or $\exists$

If so, you should be able to identify the main restricting predicate (which is the subject of the sentence). The entire sentence is about that restricted set of individuals, thus any pronouns or other terms in the second clause should refer back to the first clause.

Example 14: If somebody loves anybody then everybody is loved by somebody.

F^1: *a* is a person. G^2: *a* loves *b*.

Main logical operator is $\rightarrow$

No term in the second clause refers back to the first clause.

Paraphrase: <u>If</u> [there is at least one x such that x is a person <u>and</u> there is at least one y such that y is a person <u>and</u> x loves y] <u>then</u> [for each z, <u>if</u> z is a person <u>then</u> there is at least one w such that w is a person <u>and</u> w loves z.)

$$\exists x\ (Fx \wedge \exists y\ (Fy \wedge G(xy))) \rightarrow \forall z\ (Fz \rightarrow \exists w\ (Fw \wedge G(wz)))$$

Example 15: If someone loves a person then that person loves him/her back.

Main logical operator is $\forall$.

Terms in the second clause 'that person', 'him/her' refer back to the first clause.

The restricting predicate for main quantifier: person (x)

Predication of x: for every person that x loves, that person loves x back.

Applies to: *All* members of the restricted set (x). (The main quantifier is $\forall$.)

So the sentence will begin: $\forall x$ ([restricting predicate for x] $\rightarrow$ [predication of x])

$\forall x\ (Fx \rightarrow \ldots)$

Now look at predication of x: for every person that x loves, that person loves x back.

The restricting predicate: person (y)

Predication of y: if x loves y then y loves x.

Applies to: *All* members of the restricted set (y) (The second quantifier is $\forall$.)

The predication of x is: $\forall y$ ([restricting predicate for y] $\rightarrow$ [predication of y])

$\forall y\ (Fy \rightarrow (G(xy) \rightarrow G(yx)))$

Paraphrase: for each x, <u>if</u> x is a person <u>then</u> for each y, <u>if</u> y is a person <u>then</u> <u>if</u> x loves y <u>then</u> y loves x.

$\forall x\ (Fx \rightarrow \forall y\ (Fy \rightarrow (G(xy) \rightarrow G(yx))))$

OR $\forall x\ (Fx \rightarrow \forall y\ ((Fy \wedge G(xy)) \rightarrow G(yx)))$

(Restricting predicate for second quantifier is person (y) who x loves.)

Example 16: Some students love every subject that they study.

F^1: *a* is a student. G^1: *a* is a subject. H^2: *a* loves *b*. I^2: *a* studies *b*.

The entire sentence is about students (the second last word is 'they' and it refers back to some students.)

Restricting predicate for main quantifier: is a student (x).
Predication of x: for each subject that x studies, x loves that subject.
Predication applies to: some members of the restricted set (x). Main quantifier is ∃.

So the sentence will begin: $\exists x$ ([restricting predicate for x] $\wedge$ [predication of x])
$\exists x (Fx \wedge \ldots)$

For each subject that x studies, x loves that subject.

Restricting predicate for second quantifier: is a subject (y)
Predication of y: <u>if</u> y is studied by x <u>then</u> x loves y.
Predication applies to: all members of restricted set (y). Second quantifier is ∀.

The predication of x will be: $\forall y$ ([restricting predicate for y] $\rightarrow$ [predication of y])
$\forall y (Gy \rightarrow (I((xy) \rightarrow H(xy)))$

Paraphrase: There is at least one x, such that x is a student <u>and</u> for each y, <u>if</u> y is a subject <u>then</u> <u>if</u> x studies y <u>then</u> x loves y.

$\exists x (Fx \wedge \forall y (Gy \rightarrow (I(xy) \rightarrow H(xy))))$

OR Paraphrase: There is at least one x, such that x is a student <u>and</u> for each y, <u>if</u> y is a subject <u>and</u> x studies y, <u>then</u> x loves y.

$\exists x (Fx \wedge \forall y((Gy \wedge I(xy)) \rightarrow H(xy)))$

Example 17: If no student is bored by every professor who teaches them any subject, then professors who bore all their students don't teach every subject to any student.

F^1: *a* is a student. G^1: *a* is a professor. H^1: *a* is a subject. I^2: *a* bores *b*. J^3: *a* teaches *b* to *c*. K^2: *a* is a student of *b*.

No term in the second clause refer back to the first clause, so the main connective is $\rightarrow$.

Symbolize: No student is bored by every professor who teaches them any subject.

Symbolize: Professors who bore all their students don't teach every subject to any student.

Then use $\rightarrow$ to connect them.

Example 17 continued:
F^1: *a* is a student. G^1: *a* is a professor. H^1: *a* is a subject. I^2: *a* bores *b*. J^3: *a* teaches *b* to *c*.
K^2: *a* is a student of *b*.

No student is bored by every professor who teaches them some subject.

Negation of: Some student is bored by every professor who teaches him/her any subject.

Symbolize: Some student is bored by every professor who teaches him/her any subject.

Then use ~ in front of it.

Some student is bored by every professor who teaches him/her some subject.
Restricting predicate for main quantifier: is a student (x)
Predication of x: for each prof who teaches x any subject, x is bored by that prof.
Predication applies to: some members of restricted set x. (Main quantifier is $\exists$)

$\exists x(Fx \wedge [\text{ predication of x}])$

Predication of x: for each prof who teaches x some subject, x is bored by the prof.
Restricting predicate for second quantifier: is a prof (y) and there is some subject (z) such that y teaches z to x.
Predication of y: y bores x.
Predication applies to: all members of restricted set y. (Second quantifier is $\forall$)

Predication of x: $\forall y\,((Gy \wedge \exists z\,(Hz \wedge J(xzy))) \rightarrow I(yx))$

$\exists x(Fx \wedge \forall y\,((Gy \wedge \exists z\,(Hz \wedge J(xzy))) \rightarrow I(yx))\,)$

Symbolization of: No student is bored by every professor who teaches them some subject:

$\sim\exists x(Fx \wedge \forall y\,((Gy \wedge \exists z\,(Hz \wedge J(xzy))) \rightarrow I(yx))\,)$

Alternate symbolization: $\forall x\,(Fx \rightarrow \sim\forall y\,((Gy \wedge \exists z\,(Hz \wedge J(xzy))) \rightarrow I(yx))\,)$

Symbolize: Professors who bore all their students don't teach every subject to any student.
Restricting predicate for main quantifier: is a professor (x) and for each y, <u>if</u> y is a student of x then x bores y.
Predication of x: it is not the case that there is some student (w) such that for every z, <u>if</u> z is a subject then x teaches z to w.
Predication applies to: all members of restricted set x. (Main quantifier is $\forall$)

$\forall x\,(\,[\text{Restricting predicate for main quantifier}] \rightarrow [\text{predication of x}]\,)$
$\forall x\,((Gx \wedge \forall y(K(yx) \rightarrow I(xy))) \rightarrow \sim \exists w\,(Fw \wedge \forall z\,(Hz \rightarrow J(xzw))))$

Alternate symbolization: $\forall x\,((Gx \wedge \forall y(K(yx) \rightarrow I(xy))) \rightarrow \forall w\,(Fw \rightarrow \exists z\,(Hz \wedge \sim J(xzw))))$

Symbolization of the entire sentence:

$\sim\exists x(Fx \wedge \forall y\,((Gy \wedge \exists z\,(Hz \wedge J(xzy))) \rightarrow I(yx))) \rightarrow \forall x\,((Gx \wedge \forall y(K(yx) \rightarrow I(xy))) \rightarrow \sim \exists w\,(Fw \wedge \forall z\,(Hz \rightarrow J(xzw))))$

5.10 EQUIVALENCIES

As in sentential logic, there are often many different ways to symbolize the same sentence.

We've already looked at different ways to symbolize negations:

No F's are G.	Not all F's are G.
$\forall x\ (Fx \rightarrow \sim Gx)$	$\exists x\ (Fx \wedge \sim Gx)$
$\sim\exists x\ (Fx \wedge Gx)$	$\sim\forall x\ (Fx \rightarrow Gx)$

Why are these pairs of sentences equivalent? Quantifier Negation!

All things are F <u>if and only if</u> it is not the case that something is not F

$\forall x\ Fx \leftrightarrow \sim\exists\sim Fx$

Some things are F <u>if and only if</u> it is not the case that all things are non-F

$\exists x\ Fx \leftrightarrow \sim\forall x \sim Fx$

If we put ~Fx for Fx, we get:

$\forall x \sim Fx \leftrightarrow \sim\exists x \sim\sim Fx$ and $\exists x \sim Fx \leftrightarrow \sim\forall x \sim\sim Fx$

We can eliminate the double negative in front of Fx on the right side and get:

$\forall x \sim Fx \leftrightarrow \sim\exists x\ Fx$ and $\exists x \sim Fx \leftrightarrow \sim\forall x\ Fx$

Let's look at the sentences again. We can see why they are equivalent:

No F's are G. $\forall x\ (Fx \rightarrow \sim Gx)$ OR $\sim\exists x\ (Fx \wedge Gx)$

The canonical form of the universal negative is: $\forall x\ (Fx \rightarrow \sim Gx)$

Now use quantifier negation to change it to: $\sim\exists x \sim(Fx \rightarrow \sim Gx)$
Negation of the conditional tells us: $\sim(Fx \rightarrow \sim Gx) \leftrightarrow (Fx \wedge \sim\sim Gx)$
Getting rid of the double negative gives us: $(Fx \wedge \sim\sim Gx) \leftrightarrow (Fx \wedge Gx)$
So we can substitute $(Fx \wedge Gx)$ for $\sim(Fx \rightarrow \sim Gx)$

This gives us: $\sim\exists x\ (Fx \wedge Gx)$

Not all F's are G. $\exists x\ (Fx \wedge \sim Gx)$ OR $\sim\forall x\ (Fx \rightarrow Gx)$

The canonical form of the negative existential is: $\exists x\ (Fx \wedge \sim Gx)$

Now use quantifier negation to change it to: $\sim\forall x \sim(Fx \wedge \sim Gx)$
De Morgan's Law tells us: $\sim(Fx \wedge \sim Gx) \leftrightarrow (\sim Fx \vee \sim\sim Gx)$
Getting rid of the double negative gives us: $(\sim Fx \vee \sim\sim Gx) \leftrightarrow (\sim Fx \vee Gx)$
Conditional as disjunction tells us: $(\sim Fx \vee Gx) \leftrightarrow (Fx \rightarrow Gx)$
So we can substitute $(Fx \rightarrow Gx)$ for $\sim(Fx \wedge \sim Gx)$

This gives us: $\sim\forall x\ (Fx \rightarrow Gx)$

Quantifier Negation:

$\forall x\ \phi \leftrightarrow \sim\exists x\sim \phi$	$\forall x \sim\phi \leftrightarrow \sim\exists x\ \phi$
$\exists x\ \phi \leftrightarrow \sim\forall x\sim \phi$	$\exists x \sim\phi \leftrightarrow \sim\forall x\ \phi$

These give us the following:

$\forall x\ (Fx \rightarrow \sim Gx) \leftrightarrow \sim \exists x\ (Fx \wedge Gx)$

$\exists x\ (Fx \wedge \sim Gx) \leftrightarrow \sim \forall x\ (Fx \rightarrow Gx)$

$\sim\forall x\ (Fx \rightarrow \sim Gx) \leftrightarrow \exists x\ (Fx \wedge Gx)$

$\sim\exists x\ (Fx \wedge \sim Gx) \leftrightarrow \forall x\ (Fx \rightarrow Gx)$

Other Equivalencies:

The biconditional theorems of sentential logic give us a number of equivalent forms of expression.

We will get an equivalent symbolic sentence if we substitute expressions of the form on one side for expressions of the form on the other side (PROVIDED THE THEOREM IS A BICONDITIONAL!).

Here are some common equivalent forms for predicate symbolization:

Exportation:

Rabid chipmunks are dangerous. (F: *a* is a chipmunk, G: *a* is rabid, H: *a* is dangerous)

Paraphrase 1: For each x, <u>if</u> x is a chipmunk <u>then</u> <u>if</u> x is rabid <u>then</u> x is dangerous.
Paraphrase 2: For each x, <u>if</u> x is a chipmunk <u>and</u> x is rabid, <u>then</u> x is dangerous.

1. $\forall x\ (Fx \rightarrow (Gx \rightarrow Hx))$
2. $\forall x\ ((Fx \wedge Gx) \rightarrow Hx)$

Recall theorem 27 from sentential logic (exportation): $((P \wedge Q) \rightarrow R) \leftrightarrow (P \rightarrow (Q \rightarrow R))$
This theorem accounts for the equivalency between sentences 1 and 2.

Complex Subjects:

Chipmunks and raccoons are dangerous if they are rabid. (I: *a* is a raccoon.)

Paraphrase 1: For each x, <u>if</u> x is a chipmunk <u>then</u> <u>if</u> x is rabid <u>then</u> x is dangerous <u>and</u> for each x, <u>if</u> x is raccoon <u>then</u> <u>if</u> x is rabid <u>then</u> x is dangerous.
Paraphrase 2: For each x, <u>if</u> x is a chipmunk <u>or</u> a raccoon <u>then</u> <u>if</u> x is rabid <u>then</u> x is dangerous.

1. $\forall x(Fx \rightarrow (Gx \rightarrow Hx)) \wedge \forall x\ (Ix \rightarrow (Gx \rightarrow Hx)$
2. $\forall x\ ((Fx \vee Ix) \rightarrow (Gx \rightarrow Hx))$

Recall theorem 50 from sentential logic: $((P \rightarrow R) \wedge (Q \rightarrow R)) \leftrightarrow ((P \vee Q) \rightarrow R)$
This theorem accounts for the equivalency between sentences 1 and 2.

NOTE: using exportation on 2 gives us: $\forall x\ (((Fx \vee Ix) \wedge Gx) \rightarrow Hx)$

Confinement:

What happens when you want the scope of the quantifier to extend further or less far?

For instance: $\exists x\ Gx \rightarrow P$

This sentence says that IF there is some x, such that x is green THEN P is true.

If something is green then P is true.

Here the scope of the quantifier does not include P.

What happens if we just extend the scope of the quantifier to include P?

$\exists x\ (Gx \rightarrow P)$

Translation: there is some x, such that if x is green then P is true.

Problem! A pink pig is not green, thus it is an object such that if it is green then P is true. Thus, this sentence is true as long as there are objects that are not green.

We need to change the quantifier if we want to pull P within the scope of the quantifier:

$\forall x\ (Gx \rightarrow P)$

Translation: for each x, if x is green then P is true.

This is correct. "If something is green then P is true" is equivalent to "Everything is such that if it is green then P is true."

Likewise we can show that: $\forall x\ Fx \rightarrow P$ is equivalent to $\exists x\ (Fx \rightarrow P)$

The first can be paraphrased: if everyone fails then the course is impossible to pass.

The second can be paraphrased: there is some person such that if that person fails then the course is impossible to pass. (This is right. After all, if the person who is least likely to fail does fail, then the course is impossible to pass!)

General Rule: Confinement for conditionals when the antecedent is quantified.

If a sentence is a conditional and the **consequent** is not within the scope of the quantifier, then you MUST change the quantifier if you bring the consequent within the scope of the quantifier.

$\exists x\ Fx \rightarrow \phi \leftrightarrow \forall x\ (Fx \rightarrow \phi)$ $\qquad$ $\forall x\ Fx \rightarrow \phi \leftrightarrow \exists x(Fx \rightarrow \phi)$

General Rule: Confinement for conditionals when the consequent is quantified.

If a sentence is a conditional and the **antecedent** is not within the scope of the quantifier, then DO NOT change the quantifier if you bring the quantifier out to the front to include the antecedent within its scope.

$\phi \rightarrow \exists x\ Fx \leftrightarrow \exists x\ (\phi \rightarrow Fx)$ $\qquad$ $\phi \rightarrow \forall x\ Fx \leftrightarrow \forall x\ (\phi \rightarrow Fx)$

Confinement for Biconditionals:

What if the sentence is a biconditional and the right side is not within the scope of the quantifier?

$\forall x\ Fx \leftrightarrow \phi$

This sentence is equivalent to the conjunction of two conditionals:

$(\forall x\ Fx \leftrightarrow \phi) \wedge (\phi \rightarrow \forall x\ Fx)$

The first conjunct is equivalent to: $\exists x\ (Fx \rightarrow \phi)$

The second conjunct is equivalent to: $\forall x\ (\phi \rightarrow Fx)$

So the conjunction is equivalent to: $\exists x\ (Fx \rightarrow \phi) \wedge \forall x\ (\phi \rightarrow Fx)$

Thus, the sentence is *not* equivalent to: $\exists x\ (Fx \leftrightarrow \phi)$ OR $\forall x\ (Fx \leftrightarrow \phi)$

> **General Rule: Confinement for biconditionals**
>
> $(\forall x\ Fx \leftrightarrow \phi) \leftrightarrow (\exists x\ (Fx \rightarrow \phi) \wedge \forall x\ (\phi \rightarrow Fx))$
>
> $(\exists x\ Fx \leftrightarrow \phi) \leftrightarrow (\forall x\ (Fx \rightarrow \phi) \wedge \exists x\ (\phi \rightarrow Fx))$
>
> There is no easy way to bring the right side of a biconditional within the scope of the quantifier on the left side. It is better just to leave them in the simple biconditional form.

Confinement for Conjunctions and Disjunctions

What if the sentence is a conjunction or a disjunction and one conjunct or disjunct is not within the scope of the quantifier.

$\forall xFx \wedge \phi$ $\exists xFx \wedge \phi$ $\forall xFx \vee \phi$ $\exists xFx \vee \phi$

In all these cases there is no problem with just bringing the second conjunct or disjunct within the scope of the quantifier:

> **General Rule: Confinement for conjunctions and disjunctions**
>
> $(\forall xFx \wedge \phi) \leftrightarrow \forall x(Fx \wedge \phi)$ $(\exists xFx \wedge \phi) \leftrightarrow \exists x(Fx \wedge \phi)$
>
> $(\forall xFx \vee \phi) \leftrightarrow \forall x(Fx \vee \phi)$ $(\exists xFx \vee \phi) \leftrightarrow \exists x(Fx \vee \phi)$

It is clear that this is true for the conjunctions: if everything is F and ϕ is true, then everything is such that it is F and ϕ is true. If something is F and ϕ is true, then something is such that it is F and ϕ is true.

An Example Proving Equivalency:

By considering the general rules of confinement and the other equivalencies, we can now see different ways of symbolizing even very complex sentences:

Not everyone is bored by someone some of the time.

F^1: *a* is a person G^1: *a* is a time J^3: *a* bores *b* at *c*

Paraphrase: It is not the case that for each x, if x is a person then there is some y such that y is a person and some z such that z is a time and y bores x at z.

$\sim\forall x\ (Fx \rightarrow \exists y(Fy \wedge \exists z(Gz \wedge J(yxz))))$

Now let's change it into an existential by using quantifier negation:

$\exists x\ (Fx \wedge \sim\exists y(Fy \wedge \exists z(Gz \wedge J(yxz))))$

Now let's change the $\sim\exists y$ into a universal negative:

$\exists x\ (Fx \wedge \forall y(Fy \rightarrow \sim\exists z(Gz \wedge J(yxz))))$

And change the $\sim\exists z$ into a universal negative.

$\exists x\ (Fx \wedge \forall y(Fy \rightarrow \forall z(Gz \rightarrow \sim J(yxz))))$

Now let's bring $\forall y$ out to the front. Since it is a quantified conjunct, we can take it out to the front without changing the quantifier.

$\exists x\ \forall y\ (Fx \wedge (Fy \rightarrow \forall z(Gz \rightarrow \sim J(yxz))))$

We can also bring $\exists z$ to the front without changing the quantifier since it is in the consequent position of a conditional, and a conjunct.

$\exists x\ \forall y\ \forall z\ (Fx \wedge (Fy \rightarrow (Gz \rightarrow \sim J(yxz))))$

Now we can use exportation to bring Fy and Gz together in the antecedent of the conditional:

$\exists x\ \forall y\ \forall z\ (Fx \wedge ((Fy \wedge Gz) \rightarrow \sim J(yxz))))$

Translation: There is some x such that x is a person and for each y and each z if y is a person and G is a time then it is not the case that y bores x at w.

In other words, some person is never bored by anyone. That's the same as "Not everyone is bored by someone some of the time."

NOTE: as we bring the quantifiers out to the front, the relative order of the quantifiers for x, y and z stay the same.

Anyone who has a teacher is bored by somebody some of the time.

F^1: *a* is a person G^1: *a* is a time H^2: a teaches b. J^3: *a* bores *b* at *c*

Paraphrase: For each x, if x is a person and there is some y such that y teaches x, then there is some z such that z is a person and there is some w such that w is a time and z bores x at w.

$$\forall x((Fx \wedge \exists yH(yx)) \rightarrow \exists z(Fz \wedge \exists w(Gw \wedge J(zxw))))$$

We can take $\exists y$ out to the front of the conjunction $(Fx \wedge \exists yH(yx))$ … $\exists y(Fx \wedge H(yx))$

Then, we can take $\exists y$ out to the front of the conditional, but we must change the quantifier since $\exists y$ is in the antecedent position of the conditional.

$$\forall x\forall y((Fx \wedge H(yx)) \rightarrow \exists z(Fz \wedge \exists w(Gw \wedge J(zxw))))$$

Now we can take $\exists w$ out beside $\exists z$:

$$\forall x\forall y((Fx \wedge H(yx)) \rightarrow \exists z\, \exists w(Fz \wedge (Gw \wedge J(zxw))))$$

Paraphrase: For each x and y, if x is a person and y teaches x, then there is some z and some w such that z is a person and w is a time and z bores x at w.

If we wanted to, we could also take $\exists z$ and $\exists w$ out to the front.

$$\forall x\forall y\exists z\, \exists w((Fx \wedge H(yx)) \rightarrow (Fz \wedge (Gw \wedge J(zxw))))$$