42.

k	exact	chebyshev	
2	0.05	0.25	$\sim \frac{1}{4}$
3	0.02	0.11	$\sim \frac{1}{9}$
4	0.07	0.06	$\sim \frac{1}{16}$

56. $\operatorname{cov}(S,T) = \frac{n(n+1)}{2}\delta^2$

$\operatorname{cov}(S,T) = \sqrt{\frac{3(n+1)}{2(2n+1)}}$

#42. $\Pr(|x-\mu| \geq k\delta) \leq \frac{1}{k^2}$

$p(x) = \lambda \cdot e^{-\lambda x} \qquad x \geq 0$

$1 - F(x) = \int_x^\infty \lambda e^{-\lambda x}\,dx = 1 - (1 - e^{-\lambda x})$

$= e^{-\lambda x}$

$x = \frac{k+1}{\lambda}$

answer : $e^{-(k+1)}$

#47. $\operatorname{cov}(X, Z) = \operatorname{cov}(X, Y-X) = \operatorname{cov}(X,Y) - \operatorname{cov}(X,X)$

$= 0 - \delta x^2 = -\delta x^2$

$\operatorname{cor}(X, Z) = \frac{\operatorname{cov}(X, Y-X)}{\sqrt{\delta x^2}\cdot\sqrt{\delta x^2 + \delta Y^2}} = \frac{-\delta x^2}{\delta x\sqrt{\delta x^2 + \delta y^2}} = \frac{-\delta x}{\sqrt{\delta x^2 + \delta y^2}}$

#55. $E(T) = E\left(\sum_{k=1}^{n} k X_k\right) = \sum_{k=1}^{n} kE(X_k) = \sum_{k=1}^{n} k\mu = \mu\sum_{k=1}^{n} k = \mu\frac{n(n+1)}{2}$

$\operatorname{Var}(T) = \operatorname{Var}\left(\sum_{k=1}^{n} k X_k\right)$

$= \operatorname{Var}(1\cdot X_1 + 2X_2 + \cdots + n\cdot X_n)$

$= \operatorname{Var}(1\cdot X_1) + \operatorname{Var}(2\cdot X_2) + \cdots + \operatorname{Var}(n\cdot X_n)$

$= \operatorname{Var}(X_1) + 4\operatorname{Var}(X_2) + \cdots + n^2\operatorname{Var}(X_n)$

$= \sum_{i=1}^{n} k^2 \cdot \delta^2$

$= \frac{n(n+1)(2n+1)}{6}\delta^2$

#57. $\operatorname{cov}(S,T) = E(ST) - E(S)E(T) = \frac{n(n+1)}{2}(\delta^2 + \mu^2) - \frac{n(n+1)\mu}{2}\mu$

$= \frac{n(n+1)}{2}\delta^2$

$$E\,ST\,E\Big[\sum_{k=1}^{n} X_k \cdot \sum_{k=1}^{n} x_k$$

$$= E\Big[(X_1+X_2+\cdots+X_n)\sum_{k=1}^{n} k\cdot X_k$$

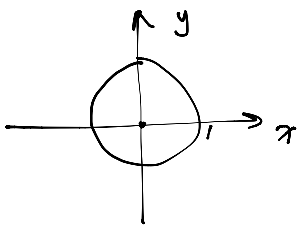
$$= E(X_1+X_2+\cdots+X_n)(1\cdot X_1+2X_2+\cdots+nX_n) \qquad n\times n \text{ terms}$$

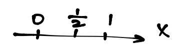
$$= E\; k\cdot X_k^2$$

#59

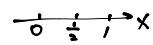
59. Let (X,Y) be a random point uniformly distributed on a unit disk. Show that Cov(X,Y) = 0, but that X and Y are not independent.

$E(X\cdot Y)=0$





$EX = EY = 0$



$\mathrm{cov}(X,Y) = EXY - E X E Y = 0$

#75. $E(U|T) = \frac{t}{2}$

$E(U) = E[E(U|T)] = \frac{1}{2\lambda}$

$Var(U|T) = E(U^2|T) - (E(U|T))^2 = \frac{t^2}{12}$

$Var(U) = Var(E(U|T)) + E(Var(U|T)) = \frac{5}{12\lambda^2}$

#1. $P(|\bar{X}_n - \mu| > \varepsilon) \le \frac{Var(\bar{X}_n)}{\varepsilon^2} = \frac{\frac{1}{n^2}\sum_{i=1}^{n} Var(X_i)}{\varepsilon^2}$ textbk

$$= \frac{\frac{1}{n}\sum_{i=1}^{n} \sigma_i^2}{\varepsilon^2} \longrightarrow 0$$