Lecture 22
April 2nd

REGULAR EXPRESSION

Let Σ be an alphabet

The set of regular expression $\mathbb{R}$ over Σ is defined by $\varepsilon \in \mathbb{R}$

Each symbol from Σ is in R

If $R, S \in \mathbb{R}$, then so are R^*, $\underset{R \circ S}{\underset{|||}{RS}}$, and $R+S$

E.g. with $\Sigma = \{a, b\}$

$\mathbb{R} = \{\varepsilon, a, b, \varepsilon^*, a\varepsilon, b+a, ((a\varepsilon+(b+a))^*)^*, \ldots\}$

For each $R \in \mathbb{R}$ there is a language called $L(R)$

$L: \mathbb{R} \longrightarrow$ Language over Σ

It's defined by: $L(\varepsilon) = \{\varepsilon\}$

For each $\alpha \in \Sigma$, $L(\alpha) = \{\alpha\}$

If $R, S \in \mathbb{R}$, $L(R+S) = L(R) \cup L(S)$

$L(RS) = \{S_1 S_2,\ S_1 \in L(R), S_2 \in L(S)\}$

If $L(RS) = L(R) \circ L(S)$

$L(R^*) = L(R)^*$

$L(a) = \{a\}$
$L(b) = \{b\}$

$L(a+\varepsilon) = L(a) \cup L(\varepsilon) = \{a, \varepsilon\}$

$$
\begin{aligned}
L(((a\varepsilon+(b+a))^*)^*) &= L((a\varepsilon+(b+a))^*)^* \\
&= (L(a\varepsilon+(b+a))^*)^* \\
&= (L(a\varepsilon) \cup L(b+a))^*)^* \\
&= [[L(a) \circ L(\varepsilon) \cup (L(a) \cup L(b))]^*]^* \\
&= (\{a\} \circ \{\varepsilon\} \cup \{a\} \cup \{b\})^*)^* \\
&= ((\{a\} \cup \{b\})^*)^* \\
&= (\{a, b\}^*)^* = \Sigma^*
\end{aligned}
$$

Even # of a's. over $\overset{\Sigma}{\{a, b\}}$

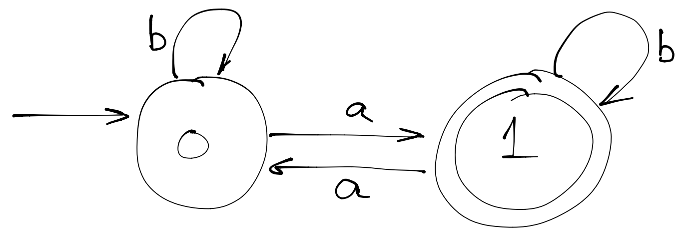
$L((aa)^*) = \{\varepsilon, aa, aaaa, \ldots\}$

$L((aa)^* b^*) = \{\varepsilon, \underset{?}{\underline{aa\,\varepsilon a}}, aabbb, \ldots\}$

$b^*(ab^*ab^*)^*$?
ababbaa

Start and end with different symbols over $\Sigma = \{a, b\}$

$[a(a+b)^* b]$
$+ [b(a+b)^* a]$



$R^k_{i,j}$: Regular expression for strings that if we start in state i we need up in state j , without going through intermediate states above k.

$R^{-1}_{0,0} = \varepsilon + b$
$R^{-1}_{0,1} = a$
$R^{-1}_{1,0}$
$R^{-1}_{1,1}$

$R^0_{0,0}$
$R^0_{0,1}$
$R^0_{1,0}$
$R^0_{1,1}$

$R^1_{0,0}$
$R^1_{0,1}$
$R^1_{1,0}$
$R^1_{1,1}$

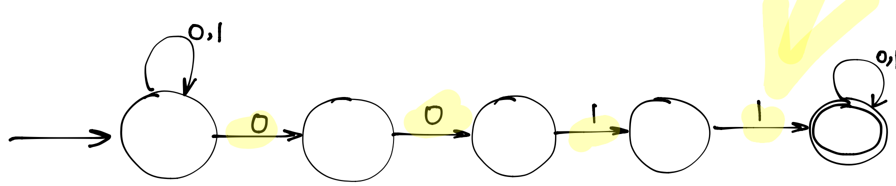
$R^{-1}_{0,0} + [R^{-1}_{0,0}(R^{-1}_{0,0})^* R^{-1}_{0,0}]$

$R^1_{0,1} = R^0_{0,1} + R^0_{0,1}(R^0_{1,1})^k R^0_{1,1}$

$R^{-1}_{1,0} + R^{-1}_{1,0}(R^{-1}_{0,0})^* R^{-1}_{0,1}$

Tutorial : L = { strings w/ 0s & 1s with 0011 substrings }
NFA with only 4 transmissions

L = { strings of 0s & 1s with more 0s than 1s }
prove not regular



Assume S = 0 ∧ n
Assume T = 0 ∧ m , m < n
S' = S + 1 ∧ m
T' = T + 1 ∧ m
then T' has less 0 than 1's
then s' has more 0 than 1s
So s' is accepted

Are there non-regular languages L_1, L_2 s.t. $L_1 \cap L_2$ is regular?

YES

Let $L_1 = \{$ more 0s than 1s $\}$

$L_2 = \{$ more 1s than 0s $\}$

$L_1 \cap L_2 = \{ \}$ empty set which is regular.