Tutorial 2

FORECAST

Question 2 $\quad Y_t$

$(1-0.6B)(X_t-9)=a_t \sim NID(0,1)$ mean can be calculated

$X_t-9-0.6X_{t-1}+5.4=a_t$

$\Rightarrow X_t-0.6X_{t-1}-3.6=a_t$

$B=3$ outside unit circle $\Rightarrow$ stationary

(b). $\hat{X}_t(l)=\psi_l a_t+\psi_{l+1}a_{t-1}+\psi_{l+2}a_{t-2}+\cdots$

$\hat{X}_{100}(1)=\psi_1 a_{100}+\psi_2 a_{99}+\psi_3 a_{98}+\psi_4 a_{97}$

$\hat{X}_t(1)=E(X_{t+1} \mid X_t, X_{t-1}, X_{t-2}, X_{t-3})=E(0.6X_t+3.6+a_{t+1} \mid X_t,\cdots,X_{t-3})=3.6+0.6X_t$

$\hat{X}_{101}=3.6+0.6\times 8.9$

$\hat{X}_{102}=E(X_{t+2} \mid X_t,\cdots,X_{t-3})=E(0.6X_{t+1}+3.6+a_{t+2} \mid X_t,\cdots,X_{t-3})=3.6+0.6\hat{X}_t(1)=\cdots$

$\hat{X}_{103}$

$\hat{X}_{104}$

$\gamma(0)=E[(X_t-9)^2]=E(X_t^2-18X_t+81)$

$\sqrt{(1+\sum_j^{l-1}\psi_j^2)\sigma^2}$

$Var(\hat{X}_t(1))=0.6^2 Var(X_t)=0.6^2\gamma(0)$

Q3

$\hat{X}_t(1)=E(X_{t+1} \mid X_t, X_{t-1}, X_{t-3},\cdots)$

$=E(0.5X_t+a_{t+1}+0.25a_t \mid X_t, X_{t-1},\cdots)$

$=0.5X_t+0.25a_t$

$\hat{X}_t(2)=E(X_{t+2} \mid X_t,\cdots)$

$=E(0.5X_{t+1}+a_{t+2}+0.25a_{t+1} \mid X_t,\cdots)$

$=0.5E(X_{t+1} \mid X_t,\cdots)=0.5\hat{X}_t(1)$

$\hat{X}_t(1)=\psi_l a_t+\psi_{l+1}a_{t-1}+\psi_{l+2}a_{t-2}+\cdots$

$a_t=\sum_{j=0}^{\infty}\pi_j X_{t-j}$

$\Phi(B)X_t=\Theta(B)a_t \Rightarrow X_t=\frac{\Theta(B)}{\Phi(B)}a_t, \quad \psi(B)=\frac{\Theta(B)}{\Phi(B)}$