Australian National University

STUDENT NUMBER ....

Mid-Semester Examination, First Semester 2016

Financial Mathematics

STAT2032/STAT6046

Writing period: 90 minutes duration

Study period: 0 minutes duration

Permitted materials: Non-programmable calculators

Dictionaries (must be clear of all annotations)

Total Marks Available: 40

Instructions to Candidates:

- ***Please write your student number in the space provided at the top of this page.***
- ***Attempt ALL questions.***
- ***All answers are to be written on the exam paper.***
- ***Please hand in the exam paper before you leave the room.***
- ***A formula sheet and the compound interest tables are attached at the end of the exam paper. You may detach these for your convenience.***
- ***For Questions 2 to 4, you need to show all the working steps in obtaining the solution. Marks may be deducted for failure to show appropriate calculations or formulae.***
- ***If you need additional space, please use the rear of the page and state clearly on the front that you have done so.***

	Q1	Q2	Q3	Q4	Total
Marks	**16**	**9**	**7**	**8**	**40**
Score					

QUESTION 1 (16 marks)
Please write down your answer (either A, B, C, D or E) clearly in the space provided.

(a) **(2 marks) A continuous payment stream is received for a period of T years. The rate of payment at time t is $e^{-0.03t}$ and the force of interest $\delta(t)$ is a constant value of 0.09. Denote $v(t)$ as the present value at time 0 of a $1 to be payable at time t, which of the following does not represent the present value of this payment stream?**

A. $\int_0^T e^{-0.03t} v(t)\, dt$

B. $\int_0^T e^{-0.03t} e^{-0.09T}\, dt$

C. $e^{-0.09T} \int_0^T \frac{e^{-0.03t}}{v(T-t)}\, dt$

D. $\int_0^T e^{-0.12t}\, dt$

E. $\frac{1}{0.12}\left(1 - e^{-0.12T}\right)$

Answer: ________

(b) **(2 marks) Which of the following relationships is wrong?**

A. $\ddot{a}_{\overline{n}|} = 1 + a_{\overline{n}|} - v^n$

B. $s_{\overline{n}|} = (1+i)^{n-1} \ddot{a}_{\overline{n}|}$

C. $d^{(p)} = \frac{i^p}{1+\frac{i^{(p)}}{p}}$

D. $(Ia)_{\overline{n}|} + (Da)_{\overline{n-1}|} = n a_{\overline{n}|}$

E. $(Ia)_{\overline{\infty}|} = \frac{1}{i^2 v}$

Answer: ________

(c) (2 marks) A loan of \$50,000 is repayable by annual repayments in arrears for the next 12 years at an effective annual rate of interest i. For the first 4-year period, the payments are K per year; for the second 4-year period, the payments are $2K$ per year; and for the last 4-year period, the payments are $3K$ per year. the expression for K is

A. $\frac{50{,}000}{3a_{\overline{12}|}-a_{\overline{8}|}-a_{\overline{4}|}}$

B. $\frac{50{,}000}{3a_{\overline{12}|}-2a_{\overline{8}|}-a_{\overline{4}|}}$

C. $\frac{50{,}000}{4a_{\overline{12}|}-a_{\overline{8}|}-2a_{\overline{4}|}}$

D. $\frac{50{,}000}{4a_{\overline{12}|}-2a_{\overline{8}|}-a_{\overline{4}|}}$

E. None of the above

Answer: _______

(d) (3 marks) A loan of \$20,000 is repayable by level monthly repayments of \$450 made in arrears for as long as necessary. If the nominal rate of interest is 9% per annum compounded monthly, the amount of capital repayment in the 25th repayment is

A. 353.87

B. 356.43

C. 358.92

D. 361.62

E. None of the above

Answer: _______

QUESTION 1 (continued)

(e) **(3 marks) Which of the following statements is wrong?**

A. **Under a positive inflation environment, the real interest rate is always less than the money rate.**

B. **The implied constant force of interest for any given period is always less than the effective periodic rate of interest.**

C. **A perpetuity due is a special case of an annuity due with its term n tends to infinity.**

D. **A continuous annuity is a special case of an annuity due payable p times a year when p tends to infinity.**

E. **Under an identical first annual payment of \$1, a geometrically increasing annuity immediate with a constant growth rate of g is always more valuable than an arithmetically increasing annuity immediate with a fixed payment increment of r when $g > r$.**

Answer: ______

(f) **(4 marks) Consider an increasing annuity immediate that pays \$1 at the end of years 4 to 6, \$2 at the end of years 8 to 10, \$3 at the end of years 12 to 14, ..., \$$k$ at the end of years $4k, 4k+1$ and $4k+2$. Under a constant effective annual rate of interest i, when $k \to \infty$, the present value of this annuity at time 0.**

A. $\frac{a_{\overline{3}|}}{\left((1+i)^4-1\right)}$

B. $\frac{\ddot{a}_{\overline{3}|}}{i^4 d^4}$

C. $\frac{\ddot{a}_{\overline{3}|}}{\left((1+i)^4-1\right)d^4}$

D. $\frac{\ddot{a}_{\overline{3}|}}{(1+i)^4-1+(1+i)^{-4}}$

E. **None of the above**

Answer: ______

QUESTION 2 (9 marks)

(a) **(2 marks) Given a force of interest $\delta(t) = 0.04t$ for $0 \leq t \leq 3$, calculate the value of $s_{\overline{3}|}$.**

QUESTION 2 (continued)

(b) **(2 marks) A 9-year deferred annuity-immediate with \$1,700 payable annually will start after a deferred period of 4 years. If the effective quarterly rate of interest in the first 6 years is 2% and the nominal rate of interest afterwards is 10% compounded semi-annually, calculate the present value of this annuity at time 0.**

QUESTION 2 (continued)

(c) (3 marks) Consider a 10-year continuous annuity that has a payment rate of \$3,500 during the first year, \$4,000 during the second year, \$4,500 during the third year and so on, that is, the payment rate increases by \$500 per annum and will apply throughout every annual period. Given an effective annual rate of interest of 8%, calculate the present value of this annuity at time 0.

QUESTION 2 (continued)

(d) (2 marks) Consider a level (i.e., constant) loan repayment schedule for a fixed rate amortized loan repayable p times a year in arrears for a period of n years, the ratio of the last interest payment to the level repayment can be expressed using only the payment frequency p and the effective annual rate of interest i. In other words, the original loan amount borrowed at time 0 L_0 is irrelevant.

True/False. Write down the expression if it is true, otherwise explain why the statement above is false.

QUESTION 3 (7 marks)

(a) **(1 mark) Consider a loan of \$180,000 to be repayable by level monthly installments of \$2032.46 for a period of n years. Calculate the value of n if the flat rate for this transaction is 4.46%.**

QUESTION 3 (continued)

(b) **(3 marks) Hence, calculate the APR (annual percentage rate of charge) for this loan transaction.**

QUESTION 3 (continued)

(c) **(3 marks) Using the APR obtained above, calculate the total interest payments in the first 2 years.**

QUESTION 4 (8 marks)

(a) **(4 marks) Consider a 30-year annuity that has a monthly payment of \$1 payable in advance in the first year, a monthly payment of \$1.05 payable in advance in the second year, a monthly payment of \$$1.05^2$ payable in advance in the third year and so on, that is, the amount of monthly payment grows geometrically at a rate of 5% for each subsequent year. Given a 7% constant effective annual rate of interest, calculate the present value of this annuity at time 0.**

QUESTION 4 (continued)

(b) **(4 marks) Consider an n-year annuity that pays $t(t+1)$ at the end of year t at an effective annual rate of interest i. From first principles, show that its present value at time 0 can be written as**

$$\sum_{t=1}^{n} 2tv^{t-1} a_{\overline{n+1-t}|}$$

and subsequently be simplified to

$$\frac{2(I\ddot{a})_{\overline{n}|} - n(n+1)v^n}{i}$$

END OF EXAMINATION

Formula Sheet for Mid-Semester Exam

1.
$$A(t_1, t_2) = \exp\Big(\int_{t_1}^{t_2} \delta(t)\, dt\Big)$$

2.
$$1 + i = \Big(1 + \frac{i^{(p)}}{p}\Big)^p = \Big(1 - \frac{d^{(p)}}{p}\Big)^{-p} = (1 - d)^{-1}$$

3.
$$PV_t = \sum_{j: t_j \geq t} c_{t_j} v(t, t_j)$$

4.
$$PV(t, T_2) = \int_t^{T_2} \rho(s) \exp\Big(-\int_t^s \delta(u)\, du\Big)\, ds$$

5.
$$a_{\overline{n}|} = \frac{1 - v^n}{i}$$

6.
$$a^{(p)}_{\overline{n}|} = \frac{1 - v^n}{i^{(p)}}$$

7.
$$\overline{a}_{\overline{n}|} = \frac{1 - v^n}{\delta}$$

8.
$$(Ia)_{\overline{n}|} = \frac{\ddot{a}_{\overline{n}|} - nv^n}{i}$$

9.
$$(Da)_{\overline{n}|} = \frac{n - a_{\overline{n}|}}{i}$$

10.
$$(\overline{I}\overline{a})_{\overline{n}|} = \frac{\overline{a}_{\overline{n}|} - nv^n}{\delta}$$

11.
$$(I\overline{a})_{\overline{n}|} = \frac{\ddot{a}_{\overline{n}|} - nv^n}{\delta}$$

12.
$$i \approx \frac{i_2 f(i_1) - i_1 f(i_2)}{f(i_1) - f(i_2)}$$