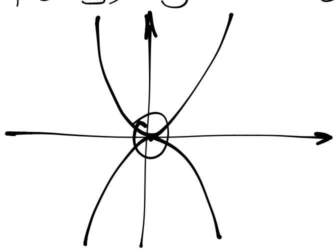
July 4th

1. Given $F(\vec{x}, y)$ in $\mathbb{R}^{n+1}$ it is C' sps $F(\vec{a}, b) = 0$ sps $\partial_y F(\vec{a}, b) \neq 0$
$\forall \vec{x}$ in $|\vec{x} - \vec{a}| < r_0$ $\exists !\, y$ s.t. $|y - b| < r_1$ and $F(\vec{x}, y) = 0$. denote y by $f(\vec{x})$

If $\partial_y f(\vec{a}, b) = 0$

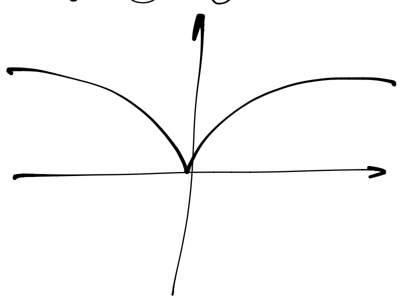
① $f(x,y) = x^2 + y^2 - 1 = 0$ not exists

② $f(x,y) = y^2 - x^4 = 0$ around $(0,0)$. $y = \pm x^2$. exists not unique.



③ exists unique but not diff

$f(x,y) = y^3 - x^2 = 0$



P120 #3
Can the equation on $(x^2+y^2+2z^2)^{1/2} - \cos z = 0$
be solved uniquely for y in terms of x & z near $(0,1,0)$?
For z interms of x & y.

$F_y(0,1,0) = \frac{1}{2}(x^2+y^2+2z^2)^{-\frac{1}{2}} \cdot 2y = 1 \neq 0 \Big|_{(0,1,0)}$

$F_z(0,1,0) = \frac{1}{2}(x^2+y^2+2z^2)^{-\frac{1}{2}} + \sin z = 0$

#5
Sps $F(x,y)$ is a C' func s.t. $F(0,0)=0$
What conditions on F will guarantee that the equation $F(F(x,y),y)$ can be solved for y as a C' func of x near $(0,0)$

$G(x,y) = F(F(x,y),y) = 0$
note $G(0,0) = F(F(0,0),0) = F(0,0) = 0$

$G_y(x,y) = \partial_x F(F(x,y), y) \cdot \partial_y F(x,y) + \partial_y F(F(x,y), y)$
$G_y(0,0) = \partial_x F(0,0) \cdot \partial_y F(0,0) + \partial_y F(0,0) \neq 0$

$\Rightarrow (\partial_x F(0,0)+1)\,\partial_y F(0,0) \neq 0$

$\Rightarrow \partial_x F(0,0) \neq -1$ AND $\partial_y F(0,0) \neq 0$.

#2
Show $x^2+2xy+3y^2=c$ can be solved either for y as a C^1 func of x or x as a C^1 func of y (but perhaps not both) near any (a,b) s.t. $a^2+2ab+3b^2=c$, provided $c>0$. What happens if $c=0$ or $c<0$?

Solution:
$F(x,y) = x^2+2xy+3y^2-c=0$
$\partial_y F = 2x+6y \neq 0 \Rightarrow x \neq -3y$
$\partial_x F = 2x+2y \neq 0 \Rightarrow x \neq -y$

if $c \leq 0$. either $x=-3y$ or $-y$ then unsolvable.