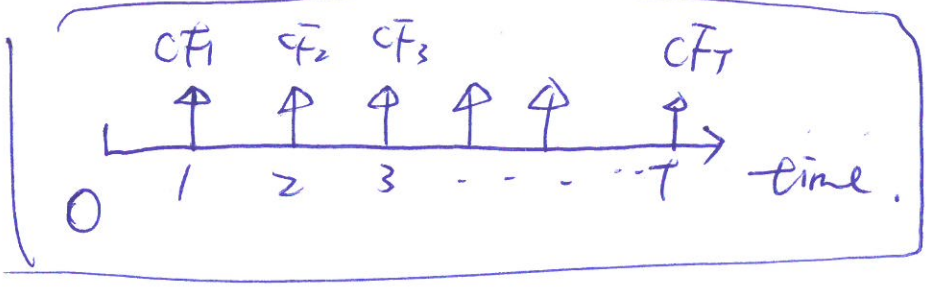
LN1

Finance = Mathematics + $$$

Time

Uncertainty

Sequence of CFs. : Assets.



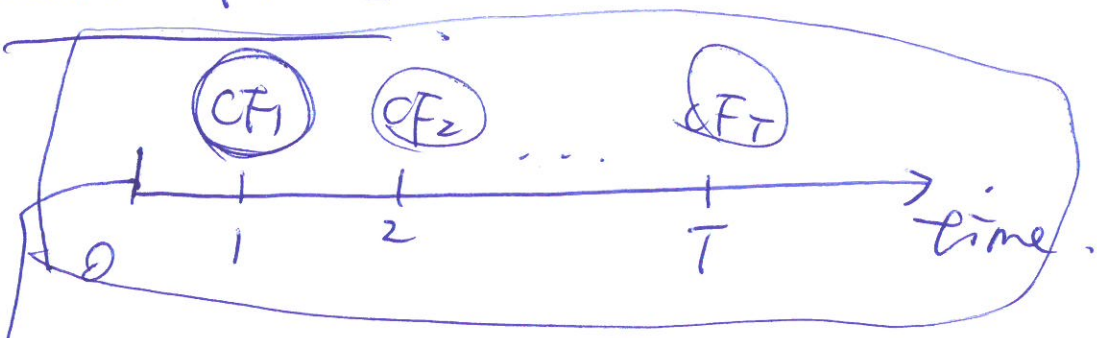
Course Outline.

Week 1-2. CFs. Interest rates.

week 3 → 9. Applications
- Annuities
- loans
- Bonds.
- forwards.
- Capital Budgeting
- Invest performance

week 10 → 11:
- manage interest risk
- stochastic interest.

Cash Flows



CFs. { + / −

Uncertainties { Time. / frequency. / severity

Assumptions: CFs: certain.
interest rate: certain.

→ Value of (CFs) = Asset

Value of Asset (t) = $V_t(CF_t, CF_{t+1}, \cdots CF_T)$

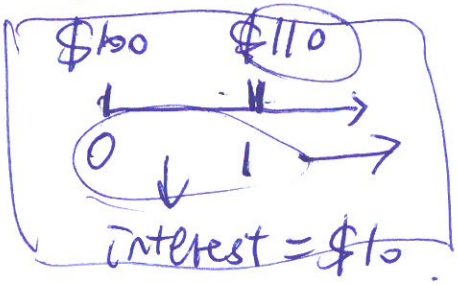
Interest rate

① effective interest rate ③
(nominal . . .)

② Simple interest

③ Compound interest.

① . effective interest rate.



110 > 100.

effective interest rate = $\frac{\$10}{\$100} = 0.1 = 10\%$.

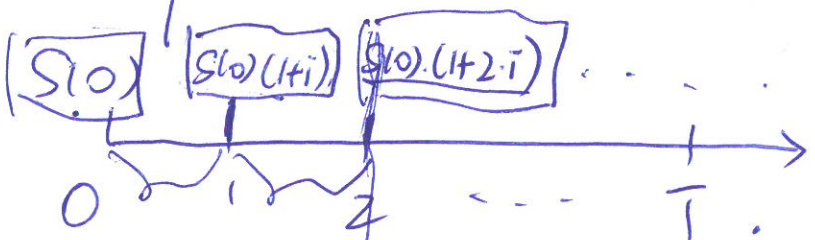
effective interest rate
- annual
- weekly
- .
- .
- .

② . $S(u)$ $S(u+1)$.

u $u+1$.

$$i_{u+1} = \frac{S(u+1) - S(u)}{S(u)}.$$

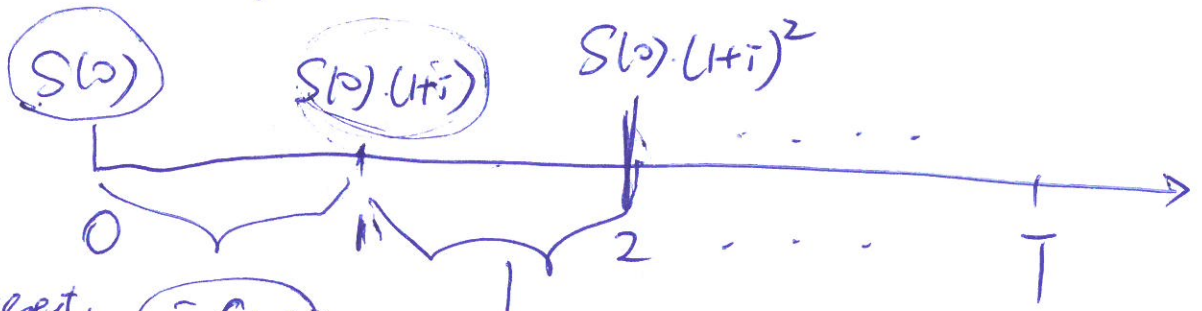
② Simple interest rate. $\bar{i}$.



Interest: $\bar{i} \cdot S(0)$, $\bar{i} \cdot S(0)$

$$S(T) = S(0) \cdot (1 + \cdot T \cdot i)$$

③ Compound interest rate $\bar{i}$.

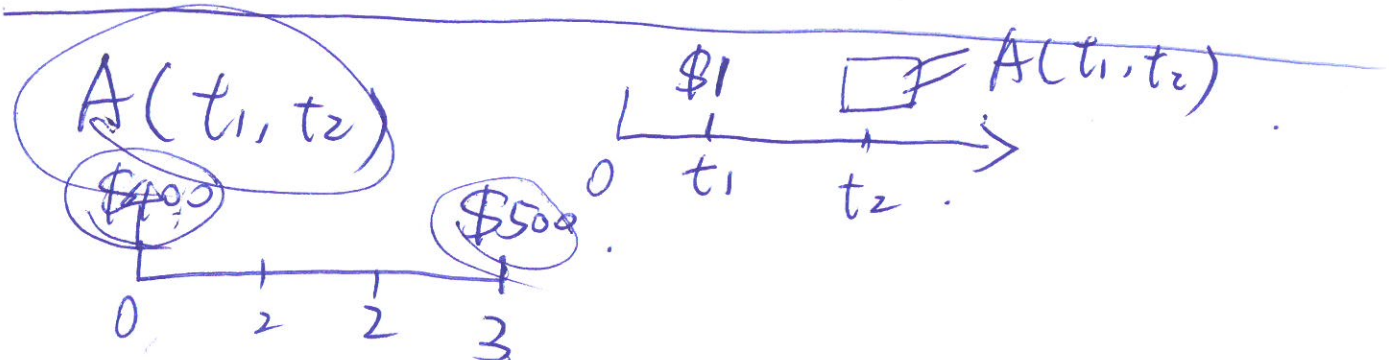


Interest: $\bar{i} \cdot S(0)$

$S(0)(1+\bar{i}) \cdot \bar{i}$

$$S(T) = S(0) \cdot (1+\bar{i})^T$$

$A(t_1, t_2)$



$$A(0,3) = \frac{\$500}{\$400} = 1.25$$

Q1: $50 at 0, $150 at 20.

$150 = 50 × (1 + 20 × i) ⇒ i = 10%.

Q2: $Z at t=0, $X at $t_1=6$, $Y at $t_2=19$.

$$\Rightarrow \boxed{A(6,19) = \frac{\$Y}{\$X}}$$

$$\Rightarrow \boxed{A(0,19) = A(0,6) \times A(6,19)}$$

$$\$Z \times A(0,19) = \$Z\, A(0,6) \times A(6,19)$$

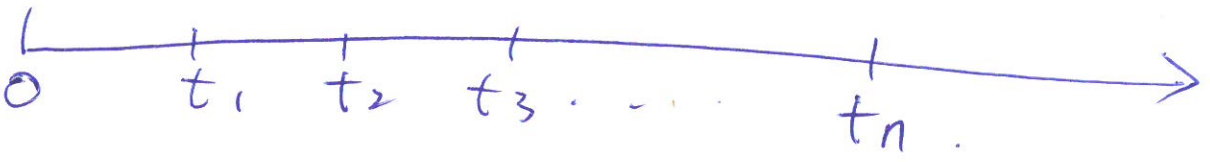
$$\| $$

$S(t_2)$ at $t_2=19$; $S(0)$, $S(t_1)$ at 0, t_1, t_2; $\bar{i}$.

$$\boxed{S(t_2) = S(t_1)\cdot(1+i)^{t_2-t_1}}$$

$$= S(0)\cdot(1+i)^{t_1}(1+i)^{t_2-t_1}$$

$0 < t_1 \leq t_2 \cdots \leq t_n$.



$A(0, t_n) = A(0, t_1) \cdot A(t_1, t_2) \cdots A(t_{n-1}, t_n)$

$A(0, 19) = A(0, 6) \cdot A(6, 19)$

$A(6, 19) = \dfrac{A(0, 19)}{A(0, 6)}$

Q3

$10000 \times (1+i)(1+i-0.05) = 12093.75$

$\Rightarrow i^2 + 1.95i + 0.95 = 1.209375$

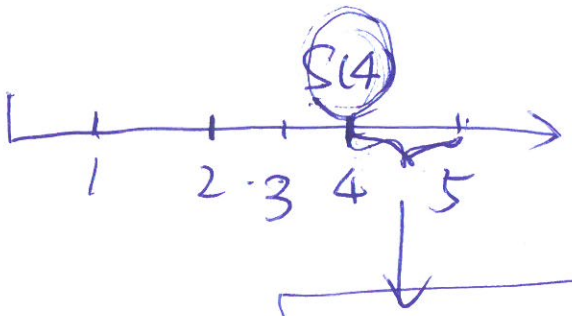
$\Rightarrow \boxed{i = 0.125}$

Q4:

option 1: $800 \times (1.1) - 900 = -20$.

option 2: $1000 \times (1.1) - 1120 = -20$.

Q5:



Interest $= 1000 \times 6\% = \$60$

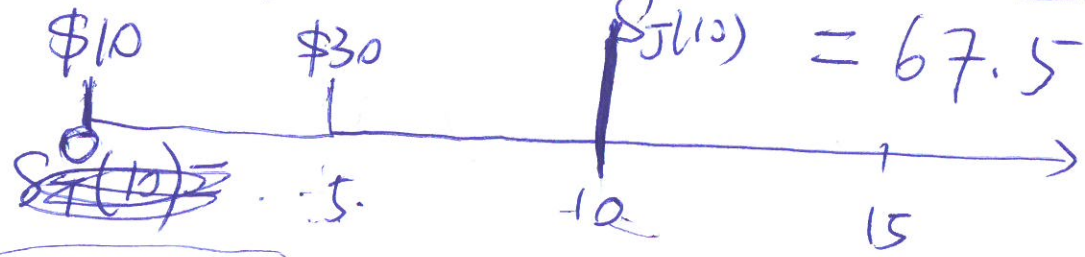
$S(4) = 1000(1 + 0.06 \times 4) = 1240$.

$$\frac{60}{1240} = 4.8\%$$

Q6: $S_J(10) = 10(1 + 0.11 \times 10) + 30(1 + 0.11 \times 5)$

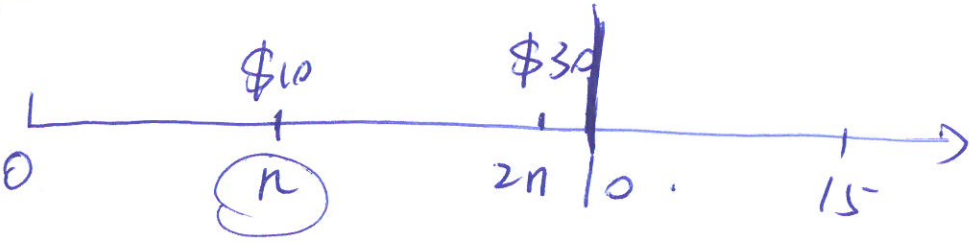
$S_J(10) = 67.5$.

Joe:



Simple 11%

Tina:



$S_J(10)$

$$S_T(10) = 10(1.0915)^{10-n} + 30.(1.0915)^{10-2n}$$

$$= S_J(10) = 67.5.$$

$$\Rightarrow \boxed{1.0915^{-n}} + 3 \times \boxed{1.0915^{-2n}} = 2.81233.$$

$$1.0915^{-n} = a > 0.$$

$$\Rightarrow a + 3 \cdot a^2 = 2.81233$$

$$\Rightarrow \boxed{a = 0.81579}$$

$$\Rightarrow 1.0915^{-n} = 0.81579.$$

$$\Rightarrow -n \cdot \ln(1.0915) = \ln(0.81579).$$

$$\Rightarrow n = -\frac{\ln 0.81579}{\ln 1.0915}$$

$$= 2.325$$