Australian National University

STUDENT NUMBER.....

Mid-Semester Examination, First Semester 2015

Financial Mathematics

STAT2032

Writing period : 1.5 Hours duration
Study period : 0 minutes duration
Permitted materials : Non-programmable calculators
Dictionaries (must not contain material added by the student)

Total Marks available: 38

Instructions to Candidates:

- ***Please write your student number in the space provided at the top of this page.***
- ***Attempt ALL questions. There are four questions, not equally weighted***
- ***All answers are to be written on the exam paper.***
- ***Please hand in the exam paper before you leave the room.***
- ***A formula sheet and selected actuarial tables are attached to the exam paper. You may detach these for your convenience.***
- ***To ensure full marks show all the steps in working out your solution. Marks may be deducted for failure to show appropriate calculations or formulae.***
- ***If you need additional space, please use the rear of the page and state clearly on the front that you have done so.***

	Q1	Q2	Q3	Q4	Total
Pages	**2-4**	**5**	**6-7**	**8-9**	
Marks	**14**	**8**	**8**	**8**	**38**
Score					

QUESTION 1 (10 marks)

(a) [3 marks] Given that $i^{(m)}$=0.1844144 and $d^{(m)}$=0.1802608, find m.

(b) [2 marks] Indicate whether each statement is TRUE or FALSE. That is, circle the word TRUE if you think the statement is TRUE, or circle the word FALSE if you think the statement is FALSE. One mark will be awarded for each correct response.

(i) $(1 + i \cdot t)^{-1} < (1 + i)^{-t}$ **if t<1**

TRUE / FALSE

(ii) $a_{\overline{m+n}|} = v^n a_{\overline{n}|} + a_{\overline{m}|}$ **if** $m \neq n$

TRUE / FALSE

(iii) $v = \left(1 + \frac{d^{(m)}}{m}\right)^m$

TRUE / FALSE

(iv) $\ddot{s}_{\overline{n+1}|} = 1 + a_{\overline{n}|}(1 + i)^n$

TRUE / FALSE

QUESTION 1 (continued)

c) [3 marks] At the beginning of every 3 years, \$100 is paid into an account which earns interest at a constant rate. Find the accumulated amount of the account immediately before the sixth payment is made, given that the interest rate is 10% per annum effective.

d) [3 marks] Annuities X and Y provide the following payments:

End of Year	Annuity X	Annuity Y
1-10	1	K
11-20	2	0
21-30	1	K

Annuities X and Y have equal present values and an annual effective interest rate of i such that $v^{10} = 0.5$. Determine K.

QUESTION 1 (continued)

e) [3 marks] A bank makes payments continuously at a rate of $400 a year. The payments are made between 5 and 7 years. Find the current value of these payments at time 2 years using an annual rate of discount of 4%.

QUESTION 2 (8 marks)

a) **[6 marks] An annuity is payable for 20 years. The amount of the annuity in the t^{th} year is t^2. On the basis of an effective rate of interest of 5% per annum, find the present value of the annuity assuming that it is payable annually in advance.**

b) **(2 marks) Suppose the annuity in part a) is paid quarterly in advance, the payments for each year being made in four equal instalments. Will the present value of the revised annuity be greater than or less than your answer in part a)? Provide a written explanation for your answer. [Note: you do not need to provide a numerical answer].**

QUESTION 3 (8 marks)

a) [4 marks] Assume that δ_t, the force of interest per annum at time t (years) is given by the formula

$$\delta_t = \begin{cases} 0.08 & for\ 0 \le t < 5 \\ 0.06 & for\ 5 \le t < 10 \\ 0.04 & for\ t \ge 10 \end{cases}$$

Derive expressions for the present value of 1 due at time t.

QUESTION 3 (continued)

b) [4 marks] An investor effects a contract under which he will pay 15 premiums annually in advance into an account which will accumulate according to the force of interest in part a). Each premium will be of amount $600, and the first premium will be paid at time 0. In return, the investor will receive the accumulated amount of the account 1 year after the final premium is paid. Calculate the lump sum payment (that is, the amount the investor will receive 1 year after the final premium is paid).

QUESTION 4 (8 marks)

a) **[5 marks] A borrower is repaying a loan with level payments of $3000 at the end of every year over an unknown period of time. If the amount of interest in the third instalment is $2169.23, find the amount of principal in the sixth instalment. Assume that interest is 10% per annum convertible quarterly.**

QUESTION 4 (continued)

b) [3 marks] A loan of $20,000 is repaid with 15 annual repayments. The first five payments are equal to the interest due only. The remaining ten payments are level amounts.

If interest is charged at 9.6455%pa convertible quarterly, what is the amount of each of the remaining ten payments?

END OF EXAMINATION