AST121 Fundamentals of Astrophysics

AST121 TAs

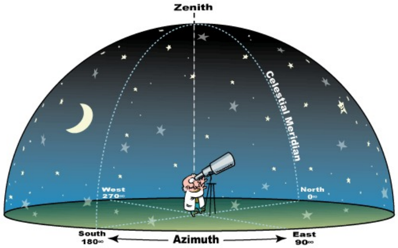
February 10, 2013

Outline

- The celestial sphere
- Parallax and the small angle formula
- The inverse square law of radiation
- The Doppler Effect

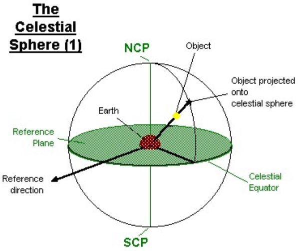
The Celestial Sphere

- Idea that stars are all fixed on a gigantic spherical shell with the Earth at the centre - core feature of many ancient models of the universe
- Today it's used for determining the angular positions of objects on the night sky.
- (Radius of the sphere is much greater than that of the Earth, so can be assumed to be infinite.)

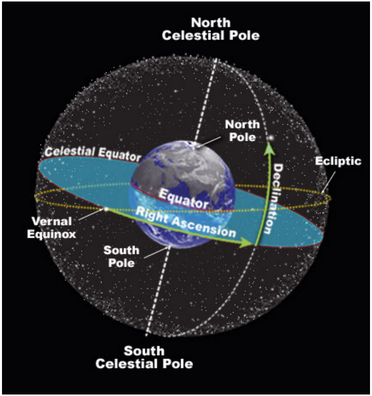


From this page about astrology...

THE CELESTIAL SPHERE



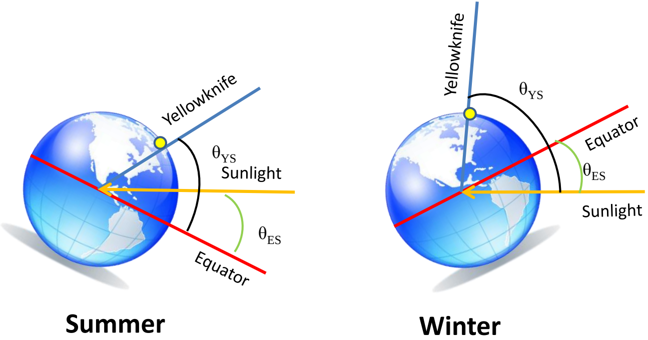
From astunit.com.



From DavidDarling.com.

The Sun in Yellowknife

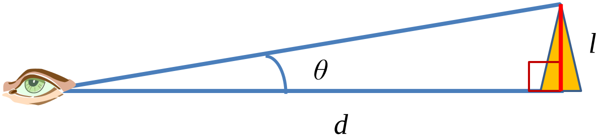
How high and how low in the sky does the Sun get at Yellowknife, Northwest Territories (latitude L = 62^o North)?



On the summer solstice, $\theta_{YS} = L - \theta_{ES} = 62^o - 23^o = 39^o$ (or 51^o above the horizon). On the winter solstice, $\theta_{YS} = L - \theta_{ES} = 62^o + 23^o = 85^o$ (or 5^o above the horizon).

The Small Angle Formula

Assume a right triangle with $l << d$:

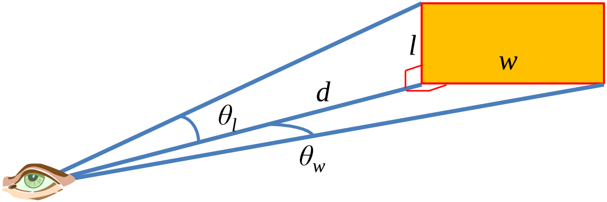


$$\theta \approx \tan\theta = l/d \tag{1}$$

θ is the **angular size of the object**, while l is its physical size, and d the separation between the observer and object. This allows us to determine the angular size of an object if l and d are known, the physical size of the object if d and θ are known, or the distance if l and θ are known. In astrophysics, θ is measured either in radians, degrees, or arcseconds (1/3600 of a degree).

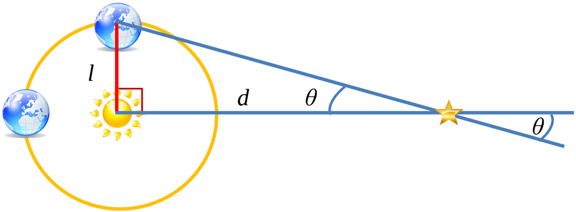
Angular Area

Extending the picture in the previous slide to two dimensions, and assuming $l << d$, $w << d$:



We can then determine $\theta_l = l/d$ and $\theta_w = w/d$. $\theta_l \theta_w = \Omega$, the angular area, measured in units of steradians (square radians), square degrees, or square arcseconds. This picture gets more complicated if l and w are not small compared to d.

PARALLAX



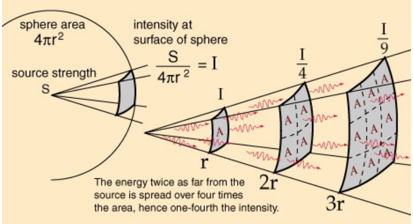
Assuming $l << d$, the above diagram is geometrically identical to the triangle for small angle formula, so $\theta = l/d$. In this case, we know l (1 AU) and we can measure θ by looking at the movement of the star against the background of more distant stars, so we can determine d. This is known as (trigonometric) **parallax**. If both l and d are the same units, θ is in radians, but if we set l in AU and d in parsecs, then θ is in arcseconds:

$$\theta(\text{arcsec}) = \frac{l(\text{AU})}{d(\text{pc})} \tag{2}$$

Suppose we take two measurements of Proxima Centauri, the nearest star to our own Sun. These measurements are taken six months apart, giving us a baseline of 2 AU, and we see Proxima Centauri drift by 1.53″. How far is Proxima Centauri from the Sun?

Using Eqn. 2, $2\,\mathrm{AU}/1.53'' = 1.31\,\mathrm{pc}$, or 4.27 ly. This is the *closest* star to our Sun, so one can imagine the precision necessary to do parallax on stars hundreds of light years away.

Inverse Square Law



From Hyperphysics.

- An emitting body has a luminosity $L = dE/dt$. (For blackbodies $L = A\sigma_{SB}T^4$, where A is the surface area of the emitting body, σ_{SB} is the Stefan-Boltzmann constant and T is temperature.)
- Assume the radiation is isotropic (spherically symmetric); then flux (luminosity/area) a distance d from the emitting body is:

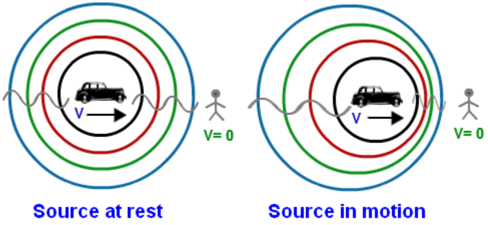
$$F = \frac{L}{4\pi d^2} \tag{3}$$

  Observers further away will see a fainter object.
- In general $L = \int_S F dA$.
- Objects of known luminosity can be used as distance indicators ("standard candles").

Consider a Cepheid variable, a kind of standard candle star, in a nearby galaxy. This particular kind of Cepheid is known to be 20 times solar luminosity ($L_\odot = 3.8 \times 10^{26}$ W), but our telescopes record a flux of 6.35×10^{-19} W/m^2. How far away is the Cepheid (and, in effect, the galaxy)?

The star's luminosity is $L = 7.6 \times 10^{27}$ W. Rearranging Eqn. 3, we get $d = \sqrt{L/4\pi F}$. Plugging in numbers, we obtain 3.09×10^{22} m, or 1 megaparsec (Mpc). Note this is a very hard measurement to make!

Classic Doppler Shift

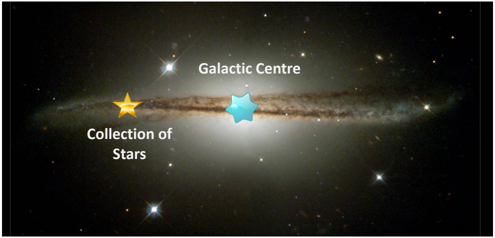


From TutorVista.

Doppler shift is a change in the frequency of emitted radiation due to relative motion between an object and an observer, and a constant radiation propagation speed in the medium (for light, speed is always constant). It obeys the equation

$$\frac{\lambda - \lambda_0}{\lambda_0} = v/c_s \tag{4}$$

where λ_0 is the rest wavelength of the radiation, and c_s is the radiation travel speed, c for light. For light, Eqn. 4 works when $v << c$.



Modified from StarrySkies.com.

The Cepheid from the previous question was part of a collection of stars moving around a galaxy. The collection of stars is situated 34.3′ away from the galactic centre. The Hα absorption line (6562 Å) in the Cepheid spectrum is shifted by 203 Å (1 Å = 10^{-10} m). How far from the galactic centre is the collection of stars orbiting, and how much mass is enclosed within the radius of the orbit? Assume mass is spherically distributed inside the orbit of the collection of stars, and the collection's orbit is circular.

Let's first determine the distance between the collection of stars and the galactic centre. From the last question, we found the Cepheid (and therefore the galaxy) was 1 Mpc away. 34.3′ = 0.01 radians, and use of the small angle formula, Eqn. 1, gives us 10 kpc (3.09×10^{20} m) as the stars - galactic centre distance.

To determine the enclosed mass, we first determine how fast the collection of stars is orbiting. From Eqn. 4, we get $v = c(\lambda - \lambda_0)/(\lambda_0)$, giving us $v = 9.28 \times 10^6$ m/s. We then note that

$$\begin{aligned} \frac{v^2}{r} &= \frac{GM_{\text{enc}}}{r^2} \\ M_{\text{enc}} &= \frac{v^2 r}{G} \end{aligned}$$

We have r and v, and plugging in gives $M_{\text{enc}} = 3.99 \times 10^{44}$ kg, or about $2 \times 10^{11}\ \mathrm{M}_\odot$. If we integrated the light from inside the orbit and apply a mass-to-light ratio, like what is done in one of your assignments, we could determine the fraction of M_{enc} due to dark matter.

Further Reading

- The Celestial Sphere on Wikipedia
- The Celestial Sphere on Astrowiki
- Parallax on Wikipedia
- Inverse Square Law on Hyperphysics
- Doppler Effect on Wikipedia