Bring the degeneracy example on Monday

Eg. "A simplex optimization", solved by the revised simplex method.

$$A=\begin{array}{c} \begin{matrix} x_1 & x_2 & x_3 & x_4 & x_5 \end{matrix} \\ \begin{bmatrix} 1 & 5 & 1 & 0 & 0 \\ 1 & -1 & 0 & 1 & 0 \\ -1 & 2 & 0 & 0 & 1 \end{bmatrix} \end{array},\ b=\begin{bmatrix} 19 \\ 7 \\ 2 \end{bmatrix},\ cT= [\ 3\ 7\ 0\ 0\ 0\ ].$$

Tableau-1 has basic variables {x3, x4, x5}, c_B^T = [0 0 0], $B^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$,

$w_B^T = c_B^T B^{-1}$ = [0 0 0]

$B^{-1}b = \begin{pmatrix} 19 \\ 7 \\ 2 \end{pmatrix} \begin{matrix} x_3 \\ x_4 \\ x_5 \end{matrix}$, $B^{-1}A_2 = \begin{pmatrix} 5 \\ -1 \\ 2 \end{pmatrix}$, θ ratios are $\begin{matrix} 5/19 \\ -1/7 \times \\ 2/2 \checkmark \end{matrix}$ $\therefore$ x_5 exits.

Tableau-2 has basic variables {x3, x4, x2}, c_B^T = [0 0 7].

So $w_B^T = c_B^T B^{-1}$ = [0 0 7/2]

Tableau-2 has objective row:

$w_B^T A - c^T$ = [-7/2 7 0 0 7/-2]- [3 7 0 0 0] = [-13/2 0 0 0 7/2], x1 will exit.

To get new B^{-1}, $\left(\begin{matrix} 5 \\ -1 \\ 2 \end{matrix} \middle| I_3\right) \simeq \left(\begin{matrix} 0 \\ 0 \\ 1 \end{matrix} \middle| \cdots \right)$

$\Rightarrow B^{-1}b = \begin{pmatrix} 14 \\ 8 \\ 1 \end{pmatrix}$, $B^{-1}A_1 = \begin{pmatrix} \frac{7}{2} \\ \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}$ θ-ratios: $\begin{matrix} 4 \checkmark \\ 16 \\ \cancel{-2} \end{matrix}$ $\rightarrow$ x_3 exits

Tableau-3 has basic variables {x1, x4, x2},

To get new B^{-1}, $\left(\begin{matrix} \frac{7}{2} \\ \frac{1}{2} \\ -\frac{1}{2} \end{matrix} \middle| \begin{matrix} 1 & 0 & -\frac{5}{2} \\ 0 & 1 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} \end{matrix}\right) \simeq \left(\begin{matrix} 1 \\ 0 \\ 0 \end{matrix} \middle| \begin{matrix} \frac{2}{7} & 0 & -\frac{5}{7} \\ -\frac{1}{7} & 1 & \frac{6}{7} \\ \frac{1}{7} & \frac{1}{7} & \frac{1}{7} \end{matrix}\right)$

$w_B^T = c_B^T B^{-1} = (3\ 0\ 7) \begin{pmatrix} \frac{2}{7} & 0 & -\frac{5}{7} \\ -\frac{1}{7} & 1 & \frac{6}{7} \\ \frac{1}{7} & \frac{1}{7} & \frac{1}{7} \end{pmatrix} = \left(\frac{13}{7}\ \ 0\ \ -\frac{8}{7}\right)$

Tableau-3 has objective row:

$w_B^T A - c^T$ = [3 7 13/7 0 -8/7] - [3 7 0 0 0] = [0 0 13/7 0 -8/7], x5 will exit.

$B^{-1}b = \begin{pmatrix} 4 \\ 6 \\ 3 \end{pmatrix}$, $B^{-1}A_5 = \begin{pmatrix} -5/7 \\ 6/7 \\ 1/7 \end{pmatrix} \begin{matrix} \leftarrow x_1 \\ \leftarrow x_4 \\ \leftarrow x_2 \end{matrix}$ θ-ratio: $\begin{matrix} -\frac{28}{5} \times \\ 7 \checkmark \\ 21 \end{matrix}$ $\rightarrow$ x_4 exits.

Tableau-4 has basic variables {x1, x5, x2},

To get new B^{-1}, $\left(\begin{matrix} -5/7 \\ 6/7 \\ 1/7 \end{matrix} \middle| \begin{matrix} 2/7 & 0 & -5/7 \\ -1/7 & 1 & 6/7 \\ 1/7 & 0 & 1/7 \end{matrix}\right) \simeq \left(\begin{matrix} 0 \\ 1 \\ 0 \end{matrix} \middle| \underbrace{\begin{matrix} 1/6 & 5/6 & 0 \\ -1/6 & 7/6 & 1 \\ 1/6 & -1/6 & 0 \end{matrix}}_{\text{new } B^{-1}}\right)$

$w_B^T = c_B^T B^{-1} = (3\ 0\ 7)\ \cdots\ ?$

Tableau-4 objective row:

$w_B^T A - c^T$ = [3 7 5/3 4/3 0] - [3 7 0 0 0] = [0 0 5/3 4/3 0]

Optimal values of $\begin{pmatrix} x_1 \\ x_5 \\ x_2 \end{pmatrix}$ are:

$$\begin{pmatrix} 1/6 & 5/6 & 0 \\ -1/6 & 7/6 & 1 \\ 1/6 & -1/6 & 0 \end{pmatrix}\begin{pmatrix} 19 \\ 7 \\ 2 \end{pmatrix} = \begin{pmatrix} 9 \\ 7 \\ 2 \end{pmatrix}$$

, so optimal sltn: $= \begin{pmatrix} 9 \\ 2 \\ 0 \\ 0 \\ 7 \end{pmatrix}$