UNIT 4
SEMANTICS: FUN WITH TRUTH-TABLES

4.3 EG1 ~R ∨ S

R	S	~	R	↓ ∨	S	
T	T	F	T	T	T	
T	F	F	T	F	F	←
F	T	T	F	T	T	
F	F	T	F	T	F	

A contingent sentence.

~(R ∨ S)

R	S	↓ ~	(R	∨	S)	
T	T	F	T	T	T	←
T	F	F	T	T	F	←
F	T	F	F	T	T	←
F	F	T	F	F	F	

A contingent sentence.

4.3 EG2

Let's do a truth-table for the sentence: (Q ∨ ~R) ∧ ~ (P →Q)

Since there are 3 atomic sentences, there will be 2^3 possible TVA's. That's 8 rows.

Truth-table for: (Q ∨ ~ R) ∧ ~ (P → Q)

P	**Q**	**R**	**(Q**	* ∨	**~**	**R)**	↓ ∧	* **~**	**(P**	→	**Q)**	
T	T	T	T	T	F	T	F	F	T	T	T	
T	T	F	T	T	T	F	F	F	T	T	T	
T	F	T	F	F	F	T	F	T	T	F	F	
T	F	F	F	T	T	F	T	T	T	F	F	←
F	T	T	T	T	F	T	F	F	F	T	T	
F	T	F	T	T	T	F	F	F	F	T	T	
F	F	T	F	F	F	T	F	F	F	T	F	
F	F	F	F	T	T	F	F	F	F	T	F	

Not a contradiction. It's contingent, as we can see from the fourth row.

4.4 EG1: Let's try it out.

Are the following sentences tautologous, contradictory or contingent?

a) $\sim P \vee Q \leftrightarrow (P \rightarrow Q)$: removed parentheses for informal notation

TAUTOLOGY

P	Q	(~	P	* ∨	Q)	↓ ↔	(P	* →	Q)
T	T	F	T	T	T	T	T	T	T
T	F	F	T	F	F	T	T	F	F
F	T	T	F	T	T	T	F	T	T
F	F	T	F	T	F	T	F	T	F

b) $\sim(\sim P \vee \sim Q) \wedge (Q \rightarrow \sim P)$

CONTRADICTION

P	Q	* ~	(~	P	∨	~	Q)	↓ ∧	(Q	* →	~	P)
T	T	T	F	T	F	F	T	F	T	F	F	T
T	F	F	F	T	T	T	F	F	F	T	F	T
F	T	F	T	F	T	F	T	F	T	T	T	F
F	F	F	T	F	T	T	F	F	F	T	T	F

c) $\sim(P \leftrightarrow (P \rightarrow Q))$ Contingent.

P	Q	↓ ~	(P	↔	(P	→	Q))
T	T	F	T	T	T	T	T
T	F	T	T	F	T	F	F
F	T	T	F	F	F	T	T
F	F	T	F	F	F	T	F

4.4 EG2

a) Are these sentences equivalent? YES.

$(P \vee \sim Q)$ $(Q \rightarrow P)$

P	Q	(P	↓ ∨	~	Q)	(Q	↓ →	P)
T	T	T	T	F	T	T	T	T
T	F	T	T	T	F	F	T	T
F	T	F	F	F	T	T	F	F
F	F	F	T	T	F	F	T	F

b) Are these sentences consistent? yes

~ (P ∨ Q) (P ↔ Q)

P	Q	~	(P	∨	Q)	(P	↔	Q)	
T	T	F	T	T	T	T	T	T	
T	F	F	T	T	F	T	F	F	
F	T	F	F	T	T	F	F	T	
F	F	T	F	F	F	F	T	F	←

c) Are these sentences consistent, equivalent or neither? neither.

(~ P ∧ Q) (Q → P)

NEITHER, INCONSISTENT

				↓			↓	
P	Q	(~	P	∧	Q)	(Q	→	P)
T	T	F	T	F	T	T	T	T
T	F	F	T	F	F	F	T	T
F	T	T	F	T	T	T	F	F
F	F	T	F	F	F	F	T	F

4.4 EG3: Is this argument valid? (P ∧ ~Q) ∨ R . ~ R ∨ Q. ∴ ~P → Q

Does { ((P ∧ ~Q) ∨ R), (~ R ∨ Q) } tautologically imply (~P →Q)?

VALID , TAUTOLOGICAL IMPLICATION

							↓					↓					↓		
P	Q	R	**((P**	∧	~	**Q)**	∨	**R)**	∧	**(~**	**R**	∨	**Q)**	→	**(~**	**P**	→	**Q)**	
T	T	T	T	F	F	T	T	T	T	F	T	T	T	T	F	T	T	T	Y
T	T	F	T	F	F	T	F	F	F	T	F	T	T	T	F	T	T	T	
T	F	T	T	T	T	F	T	T	F	F	T	F	F	T	F	T	T	F	
T	F	F	T	T	T	F	T	F	T	T	F	T	F	T	F	T	T	F	Y
F	T	T	F	F	F	T	T	T	T	F	T	T	T	T	T	F	T	T	Y
F	T	F	F	F	F	T	F	F	F	T	F	T	T	T	T	F	T	T	
F	F	T	F	F	T	F	T	T	F	F	T	F	F	T	T	F	F	F	
F	F	F	F	F	T	F	F	F	F	T	F	T	F	T	T	F	F	F	

4.4 E1: Construct a full truth-table for each of the following sentences. Determine whether each sentence is a tautology, a contradiction or a contingent sentence.

a) $Q \rightarrow (S \rightarrow Q)$

b) $(T \leftrightarrow \sim T) \rightarrow \sim (T \leftrightarrow \sim T)$

c) $(P \leftrightarrow Q) \rightarrow (\sim P \rightarrow \sim Q)$

d) $[\,(P \rightarrow Q) \wedge (Q \rightarrow R)] \wedge (P \wedge \sim R)$

e) $\sim P \rightarrow ((P \vee Q) \rightarrow Q)$

f) $(W \wedge X) \rightarrow ((Y \wedge \sim Y) \wedge W)$

g) $\sim S \rightarrow ((T \wedge S) \rightarrow U)$

h) $((P \wedge Q) \vee R) \leftrightarrow ((P \vee Q) \wedge (\sim P \rightarrow R))$

i) $(S \rightarrow (Q \rightarrow V)) \leftrightarrow (\sim (V \vee \sim Q) \wedge S)$

a) $Q \rightarrow (S \rightarrow Q)$ TAUTOLOGY

Q	S	Q	↓ →	(S	→	Q)
T	T	T	T	T	T	T
T	F	T	T	F	T	T
F	T	F	T	T	F	F
F	F	F	T	F	T	F

b) $(T \leftrightarrow \sim T) \rightarrow \sim (T \leftrightarrow \sim T)$ TAUTOLOGY

T	(T	↔	~	T)	↓ →	~	(T	↔	~	T)
T	T	F	F	T	T	T	T	F	F	T
F	F	F	T	F	T	T	F	F	T	F

c) $(P \leftrightarrow Q) \rightarrow (\sim P \rightarrow \sim Q))$ TAUTOLOGY

P	Q	(P	↔	Q)	↓ →	(~	P	→	~	Q)
T	T	T	T	T	T	F	T	T	F	T
T	F	T	F	F	T	F	T	T	T	F
F	T	F	F	T	T	T	F	F	F	T
F	F	F	T	F	T	T	F	T	T	F

d) $[\,(P \rightarrow Q) \wedge (Q \rightarrow R)] \wedge (P \wedge \sim R)$ CONTRADICTION

P	Q	R	[(P	→	Q)	∧	(Q	→	R)]	↓ ∧	(P	∧	~	R)
T	T	T	T	T	T	T	T	T	T	F	T	F	F	T
T	T	F	T	T	T	F	T	F	F	F	T	T	T	F
T	F	T	T	F	F	F	F	T	T	F	T	F	F	T
T	F	F	T	F	F	F	F	T	F	F	T	T	T	F
F	T	T	F	T	T	T	T	T	T	F	F	F	F	T
F	T	F	F	T	T	F	T	F	F	F	F	F	T	F
F	F	T	F	T	F	T	F	T	T	F	F	F	F	T
F	F	F	F	T	F	T	F	T	F	F	F	F	T	F

e) $\sim P \rightarrow ((P \vee Q) \rightarrow Q)$ TAUTOLOGY

P	Q	~	P	↓ →	((P	∨	Q)	→	Q)
T	T	F	T	T	T	T	T	T	T
T	F	F	T	T	T	T	F	F	F
F	T	T	F	T	F	T	T	T	T
F	F	T	F	T	F	F	F	T	F

f) $(W \wedge X) \rightarrow ((Y \wedge \sim Y) \wedge W)$ CONTINGENT

W	X	Y	(W	∧	X)	↓ →	((Y	∧	~	Y)	∧	W)
T	T	T	T	T	T	F	T	F	F	T	F	T
T	T	F	T	T	T	F	F	F	T	F	F	T
T	F	T	T	F	F	T	T	F	F	T	F	T
T	F	F	T	F	F	T	F	F	T	F	F	T
F	T	T	F	F	T	T	T	F	F	T	F	F
F	T	F	F	F	T	T	F	F	T	F	F	F
F	F	T	F	F	F	T	T	F	F	T	F	F
F	F	F	F	F	F	T	F	F	T	F	F	F

g) ~S → ((T ∧ S) → U) TAUTOLOGY

S	T	U	~	S	→	((T	∧	S)	→	U)
					↓					
T	T	T	F	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T	T	F	F
T	F	T	F	T	T	F	F	T	T	T
T	F	F	F	T	T	F	F	T	T	F
F	T	T	T	F	T	T	F	F	T	T
F	T	F	T	F	T	T	F	F	T	F
F	F	T	T	F	T	F	F	F	T	T
F	F	F	T	F	T	F	F	F	T	F

h) ((P ∧ Q) ∨ R) ↔ ((P ∨ Q) ∧ (~P → R)) CONTINGENT

P	Q	R	((P	∧	Q)	∨	R)	↔	((P	∨	Q)	∧	(~	P	→	R))
								↓								
T	T	T	T	T	T	T	T	T	T	T	T	T	F	T	T	T
T	T	F	T	T	T	T	F	T	T	T	T	T	F	T	T	F
T	F	T	T	F	F	T	T	T	T	T	F	T	F	T	T	T
T	F	F	T	F	F	F	F	F	T	T	F	T	F	T	T	F
F	T	T	F	F	T	T	T	T	F	T	T	T	T	F	T	T
F	T	F	F	F	T	F	F	T	F	T	T	F	T	F	F	F
F	F	T	F	F	F	T	T	F	F	F	F	F	T	F	T	T
F	F	F	F	F	F	T	T	F	F	F	F	F	T	F	F	T

i) (S → (Q → V)) ↔ (~ (V ∨ ~Q) ∧ S) CONTRADICTION

Q	S	V	(S	→	(Q	→	V))	↔	(~	(V	∨	~	Q)	∧	S)
								↓							
T	T	T	T	T	T	T	T	F	F	T	T	F	T	F	T
T	T	F	T	F	T	F	F	F	T	F	F	F	T	T	T
T	F	T	F	T	T	T	T	F	F	T	T	F	T	F	F
T	F	F	F	T	T	F	F	F	T	F	F	F	T	F	F
F	T	T	T	T	F	T	T	F	F	T	T	T	F	F	T
F	T	F	T	T	F	T	F	F	F	F	T	T	F	F	T
F	F	T	F	T	F	T	T	F	F	T	T	T	F	F	F
F	F	F	F	T	F	T	F	F	F	F	T	T	F	F	F

4.4 E2: Construct a full truth-table for each of the following pairs of sentences. Determine whether each pair is equivalent.

a) not equivalent $\sim(P \wedge Q)$ $\sim P \wedge \sim Q$

		↓						↓		
P	Q	$\sim$	(P	$\wedge$	Q)	$\sim$	P	$\wedge$	$\sim$	Q
T	T	F	T	T	T	F	T	F	F	T
T	F	T	T	F	F	F	T	F	T	F
F	T	T	F	F	T	T	F	F	F	T
F	F	T	F	F	F	T	F	T	T	F

b) equivalent $P \rightarrow (Q \rightarrow P)$ $(R \wedge \sim R) \vee (Q \rightarrow Q)$

				↓								↓			
P	Q	R	P	$\rightarrow$	(Q	$\rightarrow$	P)	(R	$\wedge$	$\sim$	R)	$\vee$	(Q	$\rightarrow$	Q)
T	T	T	T	T	T	T	T	T	F	F	T	T	T	T	T
T	T	F	T	T	T	T	T	F	F	T	F	T	T	T	T
T	F	T	T	T	F	T	T	T	F	F	T	T	F	T	F
T	F	F	T	T	F	T	T	F	F	T	F	T	F	T	F
F	T	T	F	T	T	F	F	T	F	F	T	T	T	T	T
F	T	F	F	T	T	F	F	F	F	T	F	T	T	T	T
F	F	T	F	T	F	T	F	T	F	F	T	T	F	T	F
F	F	F	F	T	F	T	F	F	F	T	F	T	F	T	F

c) equivalent $T \leftrightarrow (S \vee R)$ $\sim T \leftrightarrow (\sim S \wedge \sim R)$

				↓						↓					
R	S	T	T	$\leftrightarrow$	(S	$\vee$	R)	$\sim$	T	$\leftrightarrow$	($\sim$	S	$\wedge$	$\sim$	R)
T	T	T	T	T	T	T	T	F	T	T	F	T	F	F	T
T	T	F	F	F	T	T	T	T	F	F	F	T	F	F	T
T	F	T	T	T	F	T	T	F	T	T	T	F	F	F	T
T	F	F	F	F	F	T	T	T	F	F	T	F	F	F	T
F	T	T	T	T	T	T	F	F	T	T	F	T	F	T	F
F	T	F	F	F	T	T	F	T	F	F	F	T	F	T	F
F	F	T	T	F	F	F	F	F	T	F	T	F	T	T	F
F	F	F	F	T	F	F	F	T	F	T	T	F	T	T	F

d) not equivalent P ∧ (Q ∨ R) (P ∧ Q) ∨ R

P	Q	R	P	↓ ∧	(Q	∨	R)	(P	∧	Q)	↓ ∨	R
T	T	T	T	T	T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	F	T	T	T	T	F
T	F	T	T	T	F	T	T	T	F	F	T	T
T	F	F	T	F	F	F	F	T	F	F	F	F
F	T	T	F	F	T	T	T	F	F	T	T	T
F	T	F	F	F	T	T	F	F	F	T	F	F
F	F	T	F	F	F	T	T	F	F	F	T	T
F	F	F	F	F	F	F	F	F	F	F	F	F

e) not equivalent (P ∨ ~(S ∧T)) → ~S (S ∨ ~(P ∧ T)) → ~ P

P	S	T	(P	∨	~	(S	∧	T))	↓ →	~	S	(S	∨	~	(P	∧	T))	↓ →	~	P
T	T	T	T	T	F	T	T	T	F	F	T	T	T	F	T	T	T	F	F	T
T	T	F	T	T	T	T	F	F	F	F	T	T	T	T	T	F	F	F	F	T
T	F	T	T	T	T	F	F	T	T	T	F	F	F	F	T	T	T	T	F	T
T	F	F	T	T	T	F	F	F	T	T	F	F	T	T	T	F	F	F	F	T
F	T	T	F	F	F	T	T	T	T	F	T	T	T	T	F	F	T	T	T	F
F	T	F	F	T	T	T	F	F	F	F	T	T	T	T	F	F	F	T	T	F
F	F	T	F	T	T	F	F	T	T	T	F	F	T	T	F	F	T	T	T	F
F	F	F	F	T	T	F	F	F	T	T	F	F	T	T	F	F	F	T	T	F

f) equivalent (W ∧ X) ∨ ~(W ∨ X) W ↔ X

W	X	(W	∧	X)	↓ ∨	~	(W	∨	X)	W	↔	X
T	T	T	T	T	T	F	T	T	T	T	T	T
T	F	T	F	F	F	F	T	T	F	T	F	F
F	T	F	F	T	F	F	F	T	T	F	F	T
F	F	F	F	F	T	T	F	F	F	F	T	F

g) not equivalent P ∨ ~(W ∨ ~Y) (Y ↔ ~P) ∨ W

P	W	Y	P	∨	~	(W	∨	~	Y)	(Y	↔	~	P)	↓ ∨	W
T	T	T	T	T	F	T	T	F	T	T	F	F	T	T	T
T	T	F	T	T	F	T	T	T	F	F	T	F	T	T	T
T	F	T	T	T	T	F	F	F	T	T	F	F	T	F	F
T	F	F	T	T	F	F	T	T	F	F	T	F	T	T	F
F	T	T	F	F	F	T	T	F	T	T	T	T	F	T	T
F	T	F	F	F	F	T	T	T	F	F	F	T	F	T	T
F	F	T	F	T	T	F	F	F	T	T	T	T	F	T	F
F	F	F	F	F	F	F	T	T	F	F	F	T	F	F	F

4.4 E3: Construct a full truth-table for each of the following sets of sentences. Determine whether each set is consistent or inconsistent.

a) inconsistent P ∧ (R ∨ ~S). ~(P ∨ ~(S →R)

P	R	S	P	↓ ∧	(R	∨	~	S)	↓ ~	(P	∨	~	(S	→	R))
T	T	T	T	T	T	T	F	T	F	T	T	F	T	T	T
T	T	F	T	T	T	T	T	F	F	T	T	F	F	T	T
T	F	T	T	F	F	F	F	T	F	T	T	T	T	F	F
T	F	F	T	T	F	T	T	F	F	T	T	F	F	T	F
F	T	T	F	F	T	T	F	T	T	F	F	F	T	T	T
F	T	F	F	F	T	T	T	F	T	F	F	F	F	T	T
F	F	T	F	F	F	F	F	T	F	F	T	T	T	F	F
F	F	F	F	F	F	T	T	F	T	F	F	F	F	T	F

b) Inconsistent P → Q. R → P. R ∧ ~Q.

P	Q	R	P	↓ →	Q	R	↓ →	P	R	↓ ∧	~	Q
T	T	T	T	T	T	T	T	T	T	F	F	T
T	T	F	T	T	T	F	T	T	F	F	F	T
T	F	T	T	F	F	T	T	T	T	T	T	F
T	F	F	T	F	F	F	T	T	F	F	T	F
F	T	T	F	T	T	T	F	F	T	F	F	T
F	T	F	F	T	T	F	T	F	F	F	F	T
F	F	T	F	T	F	T	F	F	T	T	T	F
F	F	F	F	T	F	F	T	F	F	F	T	F

c) consistent W ↔ ~Y. (W ∨ Z) ∧ (~Y ∨ Z). Z ↔ ~W.

				↓						↓						↓		
W	Y	Z	W	↔	~	Y	(W	∨	Z)	∧	(~	Y	∨	Z)	Z	↔	~	W
T	T	T	T	F	F	T	T	T	T	T	F	T	T	T	T	F	F	T
T	T	F	T	F	F	T	T	T	F	F	F	T	F	F	F	T	F	T
T	F	T	T	T	T	F	T	T	T	T	T	F	T	T	T	F	F	T
T	F	F	T	T	T	F	T	T	F	T	T	F	T	F	F	T	F	T
F	T	T	F	T	F	T	F	T	T	T	F	T	T	T	T	T	T	F
F	T	F	F	T	F	T	F	F	F	F	F	T	F	F	F	F	T	F
F	F	T	F	F	T	F	F	T	T	T	T	F	T	T	T	T	T	F
F	F	F	F	F	T	F	F	F	F	F	T	F	T	F	F	F	T	F

d) consistent. P ↔ (Q ∨ R). R →(~Q ∨ P). Q ↔ ~R.

				↓					↓						↓			
P	Q	R	**P**	↔	**Q**	∨	**R**	**R**	→	**(~**	**Q**	∨	**P)**	**Q**	↔	**~**	**R**	
T	T	T	T	T	T	T	T	T	T	F	T	T	T	T	F	F	T	
T	T	F	T	T	T	T	F	F	T	F	T	T	T	T	T	T	F	←
T	F	T	T	T	F	T	T	T	T	T	F	T	T	F	T	F	T	←
T	F	F	T	F	F	F	F	F	T	T	F	T	T	F	F	T	F	
F	T	T	F	F	T	T	T	T	F	F	T	F	F	T	F	F	T	
F	T	F	F	F	T	T	F	F	T	F	T	F	F	T	T	T	F	
F	F	T	F	F	F	T	T	T	T	T	F	T	F	F	T	F	T	
F	F	F	F	T	F	F	F	F	T	T	F	T	F	F	F	T	F	

e) see next page

f) consistent (P ∧ (Q → ~S)) → (S → ~P). ~(Q ↔ ~P).

P	Q	S	(P	∧	(Q	→	~	S))	→	(S	→	~	P)	~	(Q	↔	~	P)
T	T	T	T	F	T	F	F	T	T	T	F	F	T	T	T	F	F	T
T	T	F	T	T	T	T	T	F	T	F	T	F	T	T	T	F	F	T
T	F	T	T	T	F	T	F	T	F	T	F	F	T	F	F	T	F	T
T	F	F	T	T	F	T	T	F	T	F	T	F	T	F	F	T	F	T
F	T	T	F	F	T	F	F	T	T	T	T	T	F	F	T	T	T	F
F	T	F	F	F	T	T	T	F	T	F	T	T	F	F	T	T	T	F
F	F	T	F	F	F	T	F	T	T	T	T	T	F	T	F	F	T	F
F	F	F	F	F	F	T	T	F	T	F	T	T	F	T	F	F	T	F

e) CONSISTENT $S \rightarrow (R \vee Q)$. $Q \leftrightarrow \sim T$. $T \rightarrow (\sim R \wedge S)$.

Q	R	S	T	S	↓ →	(R	∨	Q)	Q	↓ ↔	~	T	T	↓ →	(~	R	∧	S)
T	T	T	T	T	T	T	T	T	T	F	F	T	T	F	F	T	F	T
T	T	T	F	T	T	T	T	T	T	T	T	F	F	T	F	T	F	T
T	T	F	T	F	T	T	T	T	T	F	F	T	T	F	F	T	F	F
T	T	F	F	F	T	T	T	T	T	T	T	F	F	T	F	T	F	F
T	F	T	T	T	T	F	T	T	T	F	F	T	T	T	T	F	T	T
T	F	T	F	T	T	F	T	T	T	T	T	F	F	T	T	F	T	T
T	F	F	T	F	T	F	T	T	T	F	F	T	T	F	T	F	F	F
T	F	F	F	F	T	F	T	T	T	T	T	F	F	T	T	F	F	F
F	T	T	T	T	T	T	T	F	F	T	F	T	T	F	F	T	F	T
F	T	T	F	T	T	T	T	F	F	F	T	F	F	T	F	T	F	T
F	T	F	T	F	T	T	T	F	F	T	F	T	T	F	F	T	F	F
F	T	F	F	F	T	T	T	F	F	F	T	F	F	T	F	T	F	F
F	F	T	T	T	F	F	F	F	F	T	F	T	T	T	T	F	T	T
F	F	T	F	T	F	F	F	F	F	F	T	F	F	T	T	F	T	T
F	F	F	T	F	T	F	F	F	F	T	F	T	T	F	T	F	F	F
F	F	F	F	F	T	F	F	F	F	F	T	F	F	T	T	F	F	F

4.4 E4: Construct a full truth-table for each of the following arguments. Determine whether it is valid.

a) valid $P \wedge Q$. $Q \rightarrow R$. $\therefore \sim P \vee R$.

P	Q	R	P	∧	Q	Q	↓ →	R	∴	~	P	↓ ∨	R
T	T	T	T	T	T	T	T	T		F	T	T	T
T	T	F	T	T	T	T	F	F		F	T	F	F
T	F	T	T	F	F	F	T	T		F	T	T	T
T	F	F	T	F	F	F	T	F		F	T	F	F
F	T	T	F	F	T	T	T	T		T	F	T	T
F	T	F	F	F	T	T	F	F		T	F	T	F
F	F	T	F	F	F	F	T	T		T	F	T	T
F	F	F	F	F	F	F	T	F		T	F	T	F

b) invalid $S \rightarrow (T \vee W)$. $\sim T$. $\therefore \sim (S \vee T)$.

				↓				↓			↓			
S	T	W	S	→	(T	∨	W)	~	T	∴	~	(S	∨	T)
T	T	T	T	T	T	T	T	F	T		F	T	T	T
T	T	F	T	T	T	T	F	F	T		F	T	T	T
T	F	T	T	T	F	T	T	T	F		F	T	T	F
T	F	F	T	F	F	F	F	T	F		F	T	T	F
F	T	T	F	T	T	T	T	F	T		F	F	T	T
F	T	F	F	T	T	T	F	F	T		F	F	T	T
F	F	T	F	T	F	T	T	T	F		T	F	F	F
F	F	F	F	T	F	F	F	T	F		T	F	F	F

c) invalid $\sim (P \vee (\sim S \wedge Q))$. $S \rightarrow (P \rightarrow Q)$. $\therefore \sim P \leftrightarrow \sim S$.

			↓								↓							↓		
P	Q	S	~	(P	∨	(~	S	∧	Q))	S	→	(P	→	Q)	∴	~	P	↔	~	S
T	T	T	F	T	T	F	T	F	T	T	T	T	T	T		F	T	T	F	T
T	T	F	F	T	T	T	F	T	T	F	T	T	T	T		F	T	F	T	F
T	F	T	F	T	T	F	T	F	F	T	F	T	F	F		F	T	T	F	T
T	F	F	F	T	T	T	F	F	F	F	T	T	F	F		F	T	F	T	F
F	T	T	T	F	F	F	T	F	T	T	T	F	T	T		T	F	F	F	T
F	T	F	F	F	T	T	F	T	T	F	T	F	T	T		T	F	T	T	F
F	F	T	T	F	F	F	T	F	F	T	T	F	T	F		T	F	F	F	T
F	F	F	T	F	F	T	F	F	F	F	T	F	T	F		T	F	T	T	F

d) valid $\sim R \vee (S \leftrightarrow \sim T)$. $S \rightarrow R$. $\sim R \vee T$. $\therefore \sim S$.

					↓						↓				↓			↓	
R	S	T	~	R	∨	(S	↔	~	T)	S	→	R	~	R	∨	T	∴	~	S
T	T	T	F	T	F	T	F	F	T	T	T	T	F	T	T	T		F	T
T	T	F	F	T	T	T	T	T	F	T	T	T	F	T	F	F		F	T
T	F	T	F	T	T	F	T	F	T	F	T	T	F	T	T	T		T	F
T	F	F	F	T	F	F	F	T	F	F	T	T	F	T	F	F		T	F
F	T	T	T	F	T	T	F	F	T	T	F	F	T	F	T	T		F	T
F	T	F	T	F	T	T	T	T	F	T	F	F	T	F	T	F		F	T
F	F	T	T	F	T	F	T	F	T	F	T	F	T	F	T	T		T	F
F	F	F	T	F	T	F	F	T	F	F	T	F	T	F	T	F		T	F

e) Invalid R → Q. ~(S ∧ T) ↔ R. ∴ ~T ∨ ~Q

					↓						↓					↓		
Q	R	S	T	R	→	Q	~	(S	∧	T)	↔	R	∴	~	T	∨	~	Q
T	T	T	T	T	T	T	F	T	T	T	F	T		F	T	F	F	T
T	T	T	F	T	T	T	T	T	F	F	T	T		T	F	T	F	T
T	T	F	T	T	T	T	T	F	F	T	T	T		F	T	F	F	T
T	T	F	F	T	T	T	T	F	F	F	T	T		T	F	T	F	T
T	F	T	T	F	T	T	F	T	T	T	T	F		F	T	F	F	T
T	F	T	F	F	T	T	T	T	F	F	F	F		T	F	T	F	T
T	F	F	T	F	T	T	T	F	F	T	F	F		F	T	F	F	T
T	F	F	F	F	T	T	T	F	F	F	F	F		T	F	T	F	T
F	T	T	T	T	F	F	F	T	T	T	F	T		F	T	T	T	F
F	T	T	F	T	F	F	T	T	F	F	T	T		T	F	T	T	F
F	T	F	T	T	F	F	T	F	F	T	T	T		F	T	T	T	F
F	T	F	F	T	F	F	T	F	F	F	T	T		T	F	T	T	F
F	F	T	T	F	T	F	F	T	T	T	T	F		F	T	T	T	F
F	F	T	F	F	T	F	T	T	F	F	F	F		T	F	T	T	F
F	F	F	T	F	T	F	T	F	F	T	F	F		F	T	T	T	F
F	F	F	F	F	T	F	T	F	F	F	F	F		T	F	T	T	F

4.5 SHORTENED TRUTH-TABLES

4.5 EG1

Show that this sentence is not a contradiction: $((P \wedge Q) \rightarrow R) \wedge (Q \wedge \sim R)$

			F	F	T		F						
P	**Q**	**R**	**((P**	**$\wedge$**	**Q)**	**$\rightarrow$**	**R)**	**$\wedge$**	**(Q**	**$\wedge$**	**~**	**R)**	
F	T	F	F	F	T	T	F	T	T	T	T	F	

4.5 EG2: Let's try a few:

a) Show that the following sentence is not a tautology:

$$\sim (S \vee P) \vee (Q \leftrightarrow R) \rightarrow \sim(R \leftrightarrow P) \vee (S \rightarrow Q)$$

P	**Q**	**R**	**S**
F	F	F	T

F	T	T	F		F	T	F			F		F				
[~	**(S**	**$\vee$**	**P)**	**$\vee$**	**(Q**	**$\leftrightarrow$**	**R)]**	**$\rightarrow$**	**[~**	**(R**	**$\leftrightarrow$**	**P)**	**$\vee$**	**(S**	**$\rightarrow$**	**Q)]**
			F	T				F	F		T		F	T	F	F

b) Show that these sentences are consistent:

$$R \rightarrow (P \vee Q) \qquad R \leftrightarrow (P \rightarrow S) \qquad \sim (Q \vee S)$$

P	**Q**	**R**	**S**
T	F	F	F

					F					F				
	R	**$\rightarrow$**	**(P**	**$\vee$**	**Q)**	**R**	**$\leftrightarrow$**	**(P**	**$\rightarrow$**	**S)**	**~**	**(Q**	**$\vee$**	**S)**
		T					**T**				T	F	F	F
X	T		T	T		T	F	T	F	F				
	F	T	T	T	F	F	T	T	F		T	F	F	F

c) Show that this argument is invalid:

$$S \rightarrow P \quad Q \rightarrow (R \vee S) \quad \therefore \sim P \rightarrow (Q \wedge R)$$

P	**Q**	**R**	**S**
F	F	F	F

F		F						F							
(S	**→**	**P)**	**∧**	**(Q**	**→**	**(R**	**∨**	**S))**	**→**	**(~**	**P**	**→**	**(Q**	**∧**	**R))**
	T		T	F	T	T/F	T		F	T	F	F	F	F	T/F

4.5 E1: Construct a shortened truth table for each of the following that shows what is asked.

Show that each of the following is not a tautology:

a) $((P \wedge Q) \vee \sim S) \rightarrow (P \vee (Q \wedge \sim S))$

P	**Q**	**S**
F	F	F

						F						
((P	**∧**	**Q)**	**∨**	**~**	**S)**	**→**	**(P**	**∨**	**(Q**	**∧**	**~**	**S))**
F	F	F	T	T	F	F	F	F	F	F	T	F

b) $[\sim P \wedge (Q \rightarrow (S \vee P))] \rightarrow ((S \wedge \sim P) \rightarrow Q)$

P	**Q**	**S**
F	F	T

								F						
[~	**P**	**∧**	**(Q**	**→**	**(S**	**∨**	**P))]**	**→**	**((S**	**∧**	**~**	**P)**	**→**	**Q)**
T	F	T	F	T	T	T	F	F	T	T	T	F	F	F

c) $[(W \leftrightarrow \sim X) \wedge (\sim(W \vee Y) \rightarrow Z)] \rightarrow (\sim Z \rightarrow \sim X)$

W	**X**	**Y**	**Z**
F	T	T	F

											F					
[(W	↔	~	**X)**	∧	**(~**	**(W**	∨	**Y)**	→	**Z)]**	→	**(~**	**Z**	→	**~**	**X)**
F	T	F	T	T	F	F	T	T	T	F	F	T	F	F	F	T

Show that each of the following is not a contradiction:

Show that each of the following is not a contradiction:

d) $[(P \rightarrow Q) \wedge (P \rightarrow R)] \leftrightarrow (Q \leftrightarrow \sim R)$

P	**Q**	**R**
T	F	F

							T				
[(P	→	**Q)**	∧	**(P**	→	**R)]**	↔	**(Q**	↔	**~**	**R)**
T	F	F	F	T	F	F	T	F	F	T	F

e) $((P \wedge Q) \rightarrow (R \wedge \sim R)) \wedge (P \vee Q)$

Any TVA in which P and Q have different truth-values will make this sentence true.

P	**Q**	**R**
T	F	T

								T			
((P	∧	**Q)**	→	**(R**	∧	**~**	**R))**	∧	**(P**	∨	**Q)**
T	F	F	T	T	F	F	T	T	T	T	F

f) [~ (S ∧ T) → (U ∨ S)] ∧ ~ [~U → ~ (T ∨ S)]

S	**T**	**U**
T	F	F

T

[~	**(S**	**∧**	**T)**	**→**	**(U**	**∨**	**S)]**	**∧**	**~**	**[~**	**U**	**→**	**~**	**(T**	**∨**	**S)**
T	T	F	F	T	F	T	T	T	T	T	F	F	F	F	T	T

Show that each of the following is a contingent sentence:

g) (S → T) ∧ (T →R) ↔ (R ↔ S) There are many solutions to this one.

↓

R	**S**	**T**		**(S**	**→**	**T)**	**∧**	**(T**	**→**	**R)**	**↔**	**(R**	**↔**	**S)**
T	T	T		T	T	T	T	T	T	T	T	T	T	T
T	T	F		T	F	F	F	F	T	T	F	T	T	T

h) (P ∨ (~Q ↔ R)) ∨ (R → ~(P ∨ Q))

In this one, only one TVA makes it false. The rest make it true.

↓

P	**Q**	**R**		**(P**	**∨**	**(~**	**Q**	**↔**	**R))**	**∨**	**(R**	**→**	**~**	**(P**	**∨**	**Q))**
T	T	T		T	T	F	T	F	T	T	T	F	F	T	T	T
F	T	T		F	F	F	T	F	T	F	T	F	F	F	T	T

i) ~ [(P ∨ ~S) ∨ ((R∧T) → (T ↔P))]   There are many solutions to this one.

P	**R**	**S**	**T**		**~**	**[(P**	**∨**	**~**	**S)**	**∨**	**((R**	**∧**	**T)**	**→**	**(T**	**↔**	**P))]**
F	F	T	F		T	F	F	F	T	F	F	F	F	T	F	T	F
T	T	T	T		F	T	T	F	T	T	T	T	T	T	T	T	T

Show that the following pairs are not equivalent:

Show that the following pairs are not equivalent:

j) ~(~W ∨ ~(X ∧ Y)).   (X ∨ Y) ∧ ~W.

			↓												↓		
W	**X**	**Y**	**~**	**(~**	**W**	**∨**	**~**	**(X**	**∧**	**Y)**		**(X**	**∨**	**Y)**	**∧**	**~**	**W**
T	T	T	T	F	T	F	F	T	T	T		T	T	T	F	F	T

k) ~(P ∨ Q) → (R → Q).   R → (P ∧ Q).

							↓						↓			
P	**Q**	**R**	**~**	**(P**	**∨**	**Q)**	**→**	**(R**	**→**	**Q)**		**R**	**→**	**(P**	**∧**	**Q)**
F	T	T	F	F	T	T	T	T	T	T		T	F	F	F	T

l) (~P ∧ Q) ∧ ~(R ∨ S).   ~ (R ∨ P) ∧ ~ (Q ∧ S).

P	**Q**	**R**	**S**
F	F	F	T

(~	**P**	**∧**	**Q)**	**∧**	**~**	**(R**	**∨**	**S)**		**~**	**(R**	**∨**	**P)**	**∧**	**~**	**(Q**	**∧**	**S)**
T	F	F	F	F	F	F	T	T		T	F	F	F	T	T	F	F	T

Show that the following sets are consistent:

m) $P \to \sim Q$. $P \leftrightarrow R$. $R \vee Q$.

				↓				↓			↓	
P	**Q**	**R**	**P**	**$\to$**	**~**	**Q**	**P**	**$\leftrightarrow$**	**R**	**R**	**$\vee$**	**Q**
T	F	T	T	T	T	F	T	T	T	T	T	F

n) $\sim (S \vee T) \leftrightarrow (U \wedge W)$. $U \leftrightarrow T$. $\sim S \leftrightarrow \sim W$.

These sentences are also all true if S and W are both true, but U and T are both false.

S	**T**	**U**	**W**
F	T	T	F

				↓					↓				↓		
~	**(S**	**$\vee$**	**T)**	**$\leftrightarrow$**	**(U**	**$\wedge$**	**W)**	**U**	**$\leftrightarrow$**	**T**	**~**	**S**	**$\leftrightarrow$**	**~**	**W**
F	F	T	T	T	T	F	F	T	T	T	T	F	T	T	F

o) $(P \vee Q) \vee \sim(S \vee Q)$. $S \to \sim Q$. $\sim P \to T$. $\sim T \wedge S$.

P	**Q**	**S**	**T**
T	F	T	F

(P	$\vee$	Q)	$\vee$	~	(S	$\vee$	Q)	S	$\to$	~	Q	~	P	$\to$	T	~	T	$\wedge$	S
T	T	F	T	F	T	T	F	T	T	T	F	F	T	T	F	T	F	T	T

p) $(Q \wedge R) \to (S \wedge \sim T)$. $(T \vee Q) \leftrightarrow (S \to \sim R)$. $\sim(P \vee \sim(T \wedge R))$.

P	**Q**	**R**	**S**	**T**
F		T	F	T

			↓								↓					↓						
(Q	$\wedge$	R)	$\to$	(S	$\wedge$	~	T)	(T	$\vee$	Q)	$\leftrightarrow$	(S	$\to$	~	R)	~	(P	$\vee$	~	(T	$\wedge$	R))
F	F	T	T	F	F	F	T	T	T	F	T	F	T	F	T	T	F	F	F	T	T	T

Show that the following are not valid arguments:

q) $P \vee (Q \vee S)$. $\sim S$. $\therefore \sim P$ (This is also shown by TVA: P=T, Q=F, S=F)

P	**Q**	**S**
T	T	F

P	$\vee$	**(Q**	$\vee$	**S)**	**~**	**S**	$\therefore$	**~**	**P**
T	T	T	T	F	T	F		F	T

r) $\sim(T \vee \sim(R \vee \sim S))$. $R \rightarrow (T \rightarrow S)$. $\therefore \sim T \leftrightarrow \sim R$.

This is also shown by TVA: R=T, S=T, T=F

R	**S**	**T**
T	F	F

~	**(T**	$\vee$	**~**	**(R**	$\vee$	**~**	**S))**	**R**	$\rightarrow$	**(T**	$\rightarrow$	**S)**	$\therefore$	**~**	**T**	$\leftrightarrow$	**~**	**R**
T	F	F	F	T	T	T	F	T	T	F	T	F		T	F	F	F	T

s) $(P \rightarrow Q) \wedge (R \rightarrow S)$. $(Q \leftrightarrow \sim S) \wedge (\sim S \rightarrow P)$ $\therefore P$

P	**Q**	**R**	**S**
F	F	T/ F	T

(P	$\rightarrow$	**Q)**	$\wedge$	**(R**	$\rightarrow$	**S)**	**(Q**	$\leftrightarrow$	**~S)**	$\wedge$	**(~**	**S**	$\rightarrow$	**P)**	$\therefore$	**P**
F	T	F	T		T	T	F	T	FT	T	F	T	T	F		F

t) $\sim((P \leftrightarrow T) \rightarrow (S \rightarrow W))$. $\sim(T \leftrightarrow \sim Q)$. $\therefore (P \vee Q) \rightarrow (R \wedge S)$

P	**Q**	**R**	**S**	**T**	**W**
T	T	F	T	T	F

~	**((P**	$\leftrightarrow$	**T)**	$\rightarrow$	**(S**	$\rightarrow$	**W))**	**~**	(T	$\rightarrow$	**~**	**Q)**	$\therefore$	**(P**	$\vee$	**Q)**	$\rightarrow$	**(R**	$\wedge$	**S)**
T	T	T	T	F	T	F	F	T	T	F	F	T		T	T	T	F	F	F	T

4.7 EG3: Now for some more complicated ones.

a) Rhonda says: Will is knight and Sam is a knave.
Will says: Sam is not a knave.
Patty says: Will and Sam are either both knaves or they are both knights.
Sam says: Rhonda is a knave.

R: Rhonda is a knight. W: Will is a knight. P: Patty is a knight. S: Sam is a knight.

P	R	S	W
T	F	T	T

Rhonda is a knave, the rest are knights.

R	**↔**	**W**	**∧**	**~**	**S**	**W**	**↔**	**S**	**P**	**↔**	**(W**	**↔**	**S)**	**S**	**↔**	**~**	**R**
	T						T			T					T		
T		T	T	T	F	T	F	F									
F	T	T	F	F	T	T	T	T	T		T	T	T	T	T	T	F

b) Peter says: Sarah would say that I am a knight.*
Randy says: Of Peter and myself, exactly one is a knight.
Sarah says: Randy is not a knave.

* Peter is a knight if and only if [Sarah would say Peter is a knight] .
We symbolize “Sarah would say Peter is a knight”: S ↔ P
(Sarah is a knight if and only if what she says is true.)
Thus * line is symbolized: P ↔ (S ↔ P) (Peter is a knight if and only if [S ↔ P])

P	R	S
F	T	T

Peter is a knave, the others are knights.

P	**↔**	**(S**	**↔**	**P)**	**R**	**↔**	**(P**	**∧**	**~**	**R)**	**∨**	**(R**	**∧**	**~P)**	**S**	**↔**	**R**
	T					T										T	
T		T	T	T	T	F	T	F	F	T	T	T	F	F	T		T
F		T	F	F	T	T	F	T	F	T	T	T	T	T	T		T

c) Peggy says: Zoe would say that Quinton is a knight.

Quinton says: Shawna and I are not the same.

Ryan says: Shawna is a knave.

Shawna says: Either I am a knight or Zoe is a knave.

Zoe says: Peggy is a knave unless Ryan is.

Again, you need to symbolize Peggy's statement as a biconditional with the right side a further biconditional.

Quinton's statement is that Quinton and Shawna are not the same. Quinton is a knight if and only if Shawna is a knave. This can be symbolized: $Q \leftrightarrow \sim S$ OR $\sim (Q \leftrightarrow S)$. Thus, Quinton is a knight if and only if [Quinton and Shawna are not the same].

$Q \leftrightarrow (Q \leftrightarrow \sim S)$

P	Q	R	S	Z
F	F	T	F	T

Peggy, Quinton and Shawna are knaves. Ryan and Zoe are knights.

P	$\leftrightarrow$	(Z	$\leftrightarrow$	Q)	Q	$\leftrightarrow$	(S	$\leftrightarrow$	~	Q)	R	$\leftrightarrow$	~	S
	T					T						T		
F		T	F	F	F	T	F	F	T	F	T		T	F

S	$\leftrightarrow$	(S	$\vee$	~	Z)	Z	$\leftrightarrow$	(~	P	$\vee$	~	R)
	T						T					
F		F	F	F	T	T		T	F	T	F	T

d) Poppy: Qasim is a knave and Ralph is a knight.

Qasim: Poppy would say I am a knave.

Ralph: Qasim and I aren't the same.

$P \leftrightarrow \sim Q \wedge R$

$Q \leftrightarrow (P \leftrightarrow \sim Q)$

$R \leftrightarrow \sim(Q \leftrightarrow R)$

P	Q	R
T	F	T

All of them are knaves.

P	**↔**	**~**	**Q**	**∧**	**R**	**Q**	**↔**	**(P**	**↔**	**~**	**Q)**	**R**	**↔**	**~**	**(Q**	**↔**	**R)**
	T						T						T				
F		T	F	F	F	F		F	F	T	F	F		F	F	T	F

e) Rianna: Ursula would say that Waldo is a knave.

Stuart: Vinnie would tell you that Trixie is a knave.

Trixie: Rianna and Waldo are both knights or both knaves.

Ursula: Trixie is a knight or I am a knight.

Vinnie: Stuart is a knight or Rianna is a knight, but not both.

Waldo: I am a knight and Vinnie is a knave

R	S	T	U	V	W
F	F	F	T	F	T

Ursula and Waldo are knights, the rest are knaves.

R	↔	(U	↔	~	W)	S	↔	(V	↔	~	T)	T	↔	(R	↔	W)
	T						T						T			
F		T	F	F	T	F		F	F	T	F	F		F	F	T

U	↔	(T	∨	U)	V	↔	~	(S	↔	R)	W	↔	(W	∧	~	V)
	T					T						T				
T		F	T	T	F		F	F	T	F	T		T	T	T	F

f) Paul says: If Rory is a knight then so is Walter.

Queenie says: Uri is a knave or Walter is a knight.

Rory says: Queenie and Uri are both knights.

Suzy says: Val's a knave

Uri says: Walter's a knave but Rory is a knight.

Val says: Paul and Rory are the same.

Walter says: Either I'm a knight or Val's a knave.

Remember: you may have to try two possibilities.

P	Q	R	S	U	V	W
T	T	F	T	F	F	T

Paul, Queenie, Suzy and Walter are knights. The other three are knaves.

P	$\leftrightarrow$	(S	$\rightarrow$	W)
	T			
T		T	T	T
T		F	T	F

Q	$\leftrightarrow$	(~	U	$\vee$	W)
	T				
T		T	F	T	T
T	**F**	F	T	F	F

R	$\leftrightarrow$	(Q	$\wedge$	U)
	T			
F		T	F	F
T		T	T	T

S	$\leftrightarrow$	~	**V**
	T		
T		**T**	**F**
F		**F**	**T**

U	$\leftrightarrow$	(~	W	$\wedge$	R)
	T				
F		F	T	F	F
T		T	F	T	T

V	$\leftrightarrow$	(P	$\leftrightarrow$	R)
	T			
F		T	F	F
T		T	T	T

W	$\leftrightarrow$	(W	$\vee$	~	V)
	T				
T		T	T	T	F
F		F	F	F	T