Lecture 29

Definition: Suppose $F: X \to X$ and $G: Y \to Y$ are two dynamic systems. A mapping $h: X \to Y$ is called a semi-conjugacy if h is onto, continuous, and at most n-to-one, and satisfies.

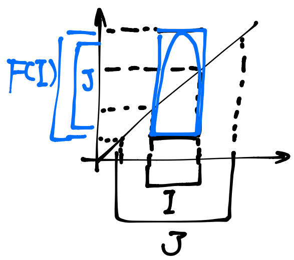
$$h \circ F(x) = G \circ h(x)$$

Remarks: Because a semi-conjugacy h is at most n-to-one, it takes periodic points of F into periodic points of G.
But the prime period may become smaller.

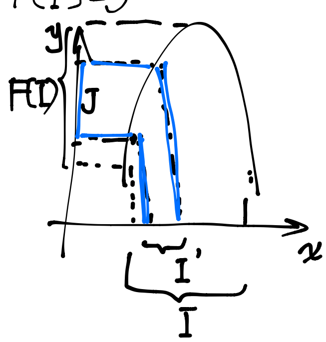
CHAPTER 11 SARKOVSKII'S THM

Thm: sps $F: \mathbb{R} \to \mathbb{R}$ is continuous and has a periodic point of period 3, then F has periodic points of all other periods.
"period 3 $\Rightarrow$ chaos"

Observations
Let $F: \mathbb{R} \to \mathbb{R}$ be a continuous function
① Let $I, J \subseteq \mathbb{R}$ be closed intervals and $I \subseteq J$. if $J \subseteq F(I)$, then F has a fixed point in I.



② Let $I, J \subseteq \mathbb{R}$ be closed intervals and $J \subseteq F(I)$. Then there is a closed interval $I' \subset I$ s.t. $F(I') = J$



The Sarkovskii ordering

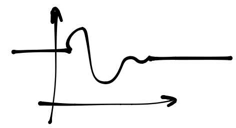
write the natural numbers in a different order

$3, 5, 7, 9, 11, 13, \cdots$ $\to q$ increasing
$6, 10, 14, 18, 22, 26, \cdots$ $\to 2q$ increase q stands for odd numbers
$12, 20, 28, 36, 44, 52, \cdots$ $\to 2^2 q$ increase starting at 3
$24, 40, 56, 72, 88, 104, \cdots$ $\to 2^3 q$ increase
$\vdots$ (write down all odd number $\times 2^n$)
$\cdots 2^n, \cdots, 16, 8, 4, 2, 1.$ $\to 2^n$ decrease (done)

Sarkovskii's Thm : Sps that $F:\mathbb{R}\to\mathbb{R}$ is continuous. sps also that F has a periodic point of prime period n and n precedes k in the Sarkovskii ordering. Then F has a periodic point of prime period k.

Remarks: ① The first number in Sarkovskii ordering is 3, so the period 3 Thm is a consequence of this one.
② The only condition is that F is continuous.
③ The theorem also applies to functions $F:[a,b]\to\mathbb{R}$, by extending F:

$$F(x)=\begin{cases} F(x): \text{if } a\le x\le b \\ F(x)=F(a): \text{if } x<a \\ F(x)=F(b): \text{if } x>b \end{cases}$$



Theorem: for any number n. $\exists$ a function, $F:\mathbb{R}\to\mathbb{R}$ continuous which has a periodic point with prime period n, but no periodic points of prime period k for all k's preceding in the Sarkovskii ordering.

Applications of Sarkovskii Thm
① If F has a 6-cycle, then has have periodic points of every even prime period. but it may not have points of odd prime period.
② If F has a 126-cycle, since $124=2^2\times 31$ (on the 3rd line), so it must have a 40-cycle. b/c $40=2^3\times 5$ and $2^3>2^2$.