Tutorial 11 Solutions

STAT 3013/4027/8027

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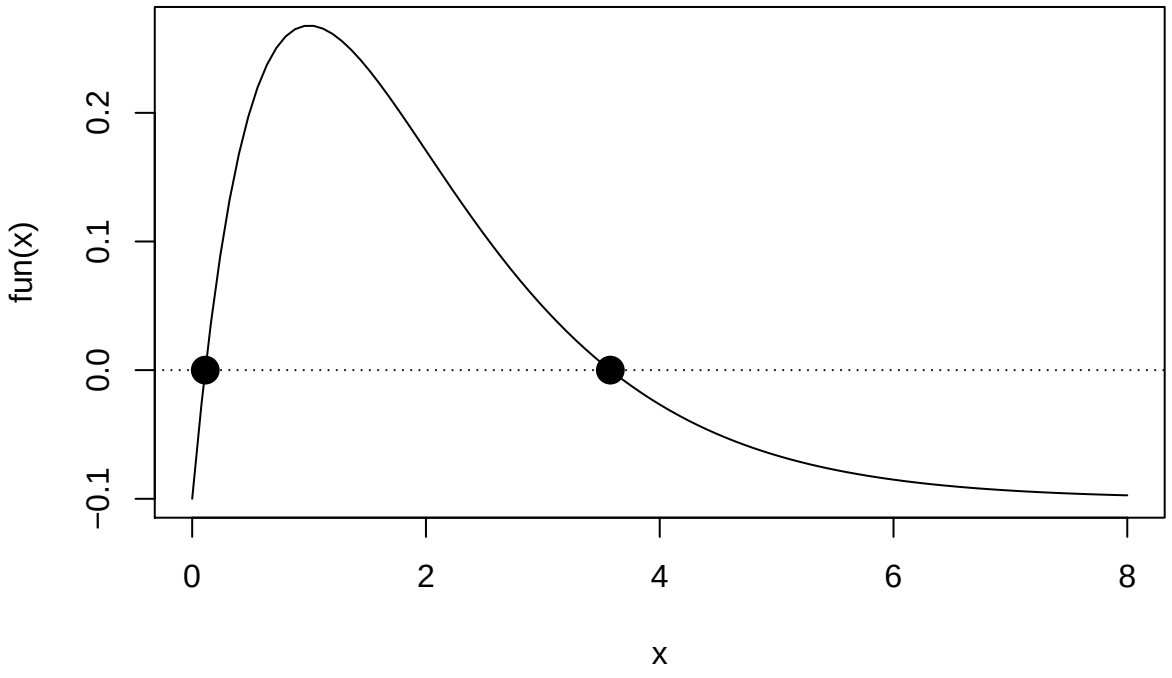
1. **Chapter 9 Question 13:** Computational portion of part Let's check the value of α for a single value of $c = 0.1$. I will use the

```
n <- 10

##
library(rootSolve)
c <- 0.10
fun <- function(x.bar){x.bar* exp(-x.bar) - c}
curve(fun(x), 0, 8)
abline(h = 0, lty = 3)
All <- uniroot.all(fun, c(0, 8))
points(All, y = rep(0, length(All)), pch = 16, cex = 2)
```




```
##
pgamma(All[1], n, rate=n) + 1 - pgamma(All[2], n, rate=n)
```

```
## [1] 4.080553e-07
```

- We can range c from 0 to $\max(x * exp(-x))$. Note the maximum is at $\bar{x} = 1$.

```
1* exp(-1)
```

```
## [1] 0.3678794
```

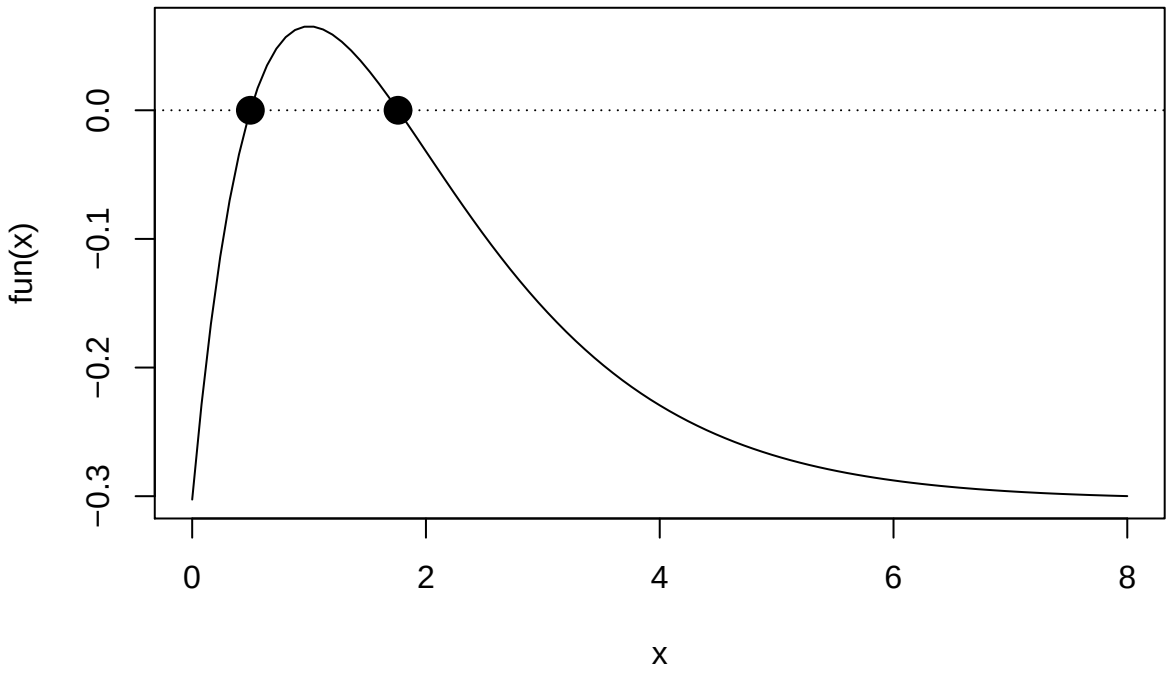
- Now let's use the `while` loop in R:

```
alpha <- 0.9
fun.c <- sort(seq(0.01, 0.366, by=0.00001), decreasing=TRUE)
i <- 1

while(alpha >0.05){
  c <- fun.c[i]
  fun <- function(x.bar){x.bar* exp(-x.bar) - c}
  All <- uniroot.all(fun, c(0, 8))
  ##
  alpha <- pgamma(All[1], n, rate=n) + 1 - pgamma(All[2], n, rate=n)
  i <- i+1
}



curve(fun(x), 0, 8)
abline(h = 0, lty = 3)
points(All, y = rep(0, length(All)), pch = 16, cex = 2)
```



- Let's examine quantities of interest:

```
c
```

```
## [1] 0.30262
```

```
All
```

```
## [1] 0.4978634 1.7613539
```

```
alpha
```

```
## [1] 0.04998751
```

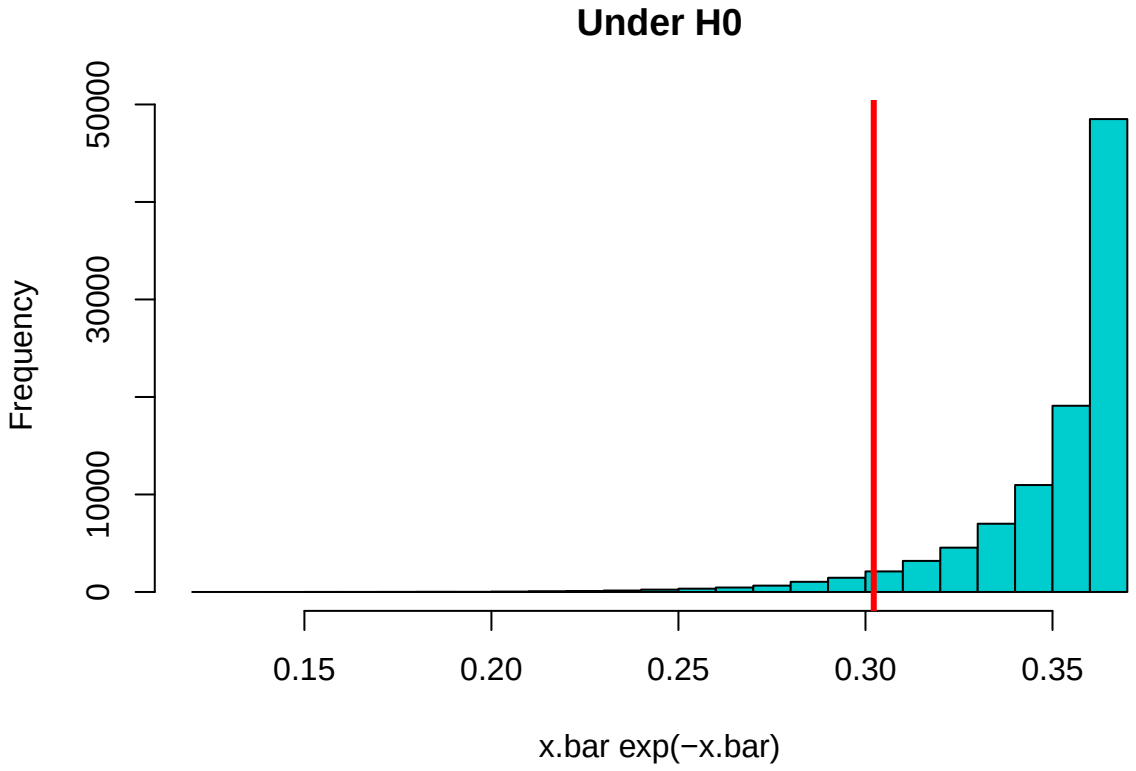
- Part d. Let's examine the statistic $\bar{X}exp(-\bar{X})$ under H_0. Generate repeated samples of size $n = 10$ from and exponential distribution with $\theta_0 = 1$.

```
set.seed(1001)
S <- 100000
out <- rep(0, S)
n <- 10

for(s in 1:S){
    x <- rexp(10, rate=1)
    x.bar <- mean(x)
```

```
    out[s] <- x.bar*exp(-x.bar)
}


hist(out, col="cyan3", main="Under H0", xlab="x.bar exp(-x.bar)")
c <- quantile(out, 0.05)
abline(v=c, col="red", lwd=3)
```



- What was the estimated value of c?

```
c
```

```
##         5%
## 0.3022125
```

2. **Chapter 9 Question 24:**

- Let's examine part (b), consider the following function $h(n/2+y)$:

$$h(n/2+y) = \left(\frac{n}{2}-y\right)^{-(n/2-y)} \left(\frac{n}{2}+y\right)^{-(n/2+y)}$$

```r
n <- 10

fun.y <- function(y){
 out <-  ( (n/2) - y)^(-((n/2)-y)) * ( (n/2) + y)^(-((n/2)+y))
}

y <- -10:10
plot(y, fun.y(y), type="l", lwd=3)
```