STAT2032 - Solutions to Mid-Semester Examination - First Semester 2015

1 Question 1 (11 marks)

(a) [3 marks]

$$\left(1+\frac{0.184414}{m}\right)=\left(1-\frac{0.1802608}{m}\right)^{-1}$$

$$\frac{m+0.18144144}{m}=\frac{m}{m-0.1802608}$$

Solving for m, we have $m = 8.003$

(b) [2 marks] (i) TRUE (Marks are given to everyone.) (ii) FALSE (iii) FALSE (iv) FALSE

(c) [2 marks]

Solve for K the following equation of value:

$$a_{\overline{10}|}+2v^{10}a_{\overline{10}|}+v^{20}a_{\overline{10}|}=Ka_{\overline{10}|}+v^{20}Ka_{\overline{10}|}$$

$$1+2\times 0.5+0.5^2=K+0.5^2\times K$$

$$K=1.8$$

(d) [3 marks]

Present value $=(1-d)^3 400\bar{a}_{\overline{2}|}$

d=0.04; i=0.0417; $\delta = 0.0408$

Present value= \$679.67

2 Question 2 (8 marks)

(a) [6 marks]

Let the present value of this annuity be A. We have

$$A = 1 + 4v + 9v^2 + ... + 361v^{18} + 400v^{19}$$

so

$$vA = v + 4v^2 + 9v^3 + ... + 361v^{19} + 400v^{20}$$

and

$$\begin{aligned} v - vA &= 1 + 3v + 5v^2 + ... + 39v^{19} - 400v^{20} \\ &= 1 + v + v^2 + ... + v^{19} + 2(v + 2v^2 + 3v^3 + ... + 19v^{19}) - 400v^{20} \\ &= 1 + a_{\overline{19}|} + 2(Ia)_{\overline{19}|} - 400v^{20} \end{aligned}$$

Hence $A = \frac{1+a_{\overline{19}|}+2(Ia)_{\overline{19}|}-400v^{20}}{1-v}$ at 5% = \$1452.26

(b) [2 marks] The present value will be less than \$1452.26 because of the delay in the receipt of the payments.

3 Question 3 (8 marks)

(a) [4 marks]

$v(t) = \exp\left[-\int_0^t \delta(s)ds\right]$

Now

$$\int_0^t \delta(s)ds = \begin{cases} 0.08t & for \quad 0 \leq t \leq 5 \\ 0.1 + 0.06t & for \quad 5 \leq t \leq 10 \\ 0.3 + 0.04t & for \quad t \geq 10 \end{cases}$$

Hence

$$v(t) = \begin{cases} \exp(-0.08t) & for \quad 0 \leq t \leq 5 \\ \exp(-0.1 - 0.06t) & for \quad 5 \leq t \leq 10 \\ \exp(-0.3 - 0.04t) & for \quad t \geq 10 \end{cases}$$

(b) [4 marks]

Let the single payment be denoted by S. The equation of value, at the present time is,

$$600[v(0) + v(1) + ... + v(14)] = Sv(15)$$

Hence we obtain

$$S = \frac{600(1 + e^{-0.08} + ... + e^{-0.86})}{e^{-0.9}}$$

$$S = 14,119$$

4 Question 4 (8 marks)

(a) [**5 marks**] $i = 1.025^4 - 1 = 0.103813$. Let n be the time of the last payment
$I_3 = i \cdot OB_2 = 0.103813 \cdot (3000 a_{\overline{n-2}|}) = 2169.23$
$a_{\overline{n-2}|} = 6.965195 = \frac{1-1.03813^{-(n-2)}}{0.103813}$
$1.103813^{-(n-2)} = 0.276923$

$$n = -\frac{\ln 0.276923}{\ln 1.103813} + 2$$

$$n = 15$$

$I_6 = OB_5 \cdot i = 3000 a_{\overline{10}|} \times 0.103813 = 18135.58 \times 0.103813 = \$1882.71$
Hence the principal repaid in the sixth instalment is:

$$PR_6 = K_6 - I_6 = 3000 - 1882.71 = \$1117.29$$

(b) [**3 marks**]
Since the amount of interest in the first five year is just equal to the interest,
$I_t = K_t$ for t=1,2,3,4,5 and so $OB_5 = OB_0$
The annual effective interest rate is $(1.096455)^4 - 1 = 10\%$
$OB_5 = 20,000 = K_{\overline{10}|0.10} = 3254.91$