STAT2032/6046 - Solutions to Selected Questions - Lecture Week 1

1. An investment of \$50 grows to \$150 after 20 years at a simple effective interest rate i. Find i.

   a. 20%

   b. 15%

   c. 10%

   d. 7.5%

Answer=**c.** $150 = 50 \times (1 + 20 \times i) \rightarrow i = 10\%$

Explanation: The total amount of absolute growth over the 20 years is \$100. Under simple interest, the absolute amount of growth per period is constant. That is, the absolute amount of growth is 100/20=\$5 per year. 5/50 implies a simple interest rate of 10%.

The equivalent compound effective annual growth rate is $\left(\frac{150}{50}\right)^{1/20} - 1 = 5.65\%$ effective per annum. Note the difference between the compound effective growth rate and the equivalent effective simple interest rate.

2. Investing \$X at time $t_1 = 6$ grows to \$Y at time $t_2 = 19$. The growth in this investment is best described as:

a. $A(0,19) - A(0,6)$

b. $A(6,19) \times A(0,6)$

c. $\frac{A(19,0)}{A(0,6)}$

d. $A(0,6) + A(6,19)$

e. $\frac{A(0,19)}{A(0,6)}$

Answer=**e.** $A(0,19) = A(0,6) \times A(6,19)$. Therefore $A(6,19) = \frac{A(0,19)}{A(0,6)}$.
Explanation: We are after an expression for A(6,19), to describe the growth factor over the 13 years between $t_1 = 6$ and $t_2 = 19$. (note: because we are assuming interest rates are constant, $A(6,19)$ must equal $A(0,13)$). Answers b. and d. are not correct as they involve additional terms. Answer c. involves A(19,0) which is not defined. Answer a. represents the absolute difference in growth for an investment for 19 years versus an investment for 6 years - this does not describe the growth for an investment for thirteen years, that is $A(0,13) \neq A(0,19) - A(0,6)$.

The expression $A(0,19) = A(0,6) \times A(6,19)$ means the growth for an investment over 19 years is equivalent to the growth from an investment for 6 years, the proceeds of which are then reinvested for another 13 years. This is the principle of consistency.

3. An investor deposits \$10,000 in a bank. During the first year, the bank credits an annual effective interest rate of i. During the second year, the bank credits an annual effective rate of interest $i - 0.05$. At the end of two years, the account balance is \$12,093.75. What would the account balance have been at the end of three years, if the annual effective rate of interest were $i + 0.09$ for each of the three years?

Under compound interest we know that

$$10000 \times (1+i) \times (1+i-0.05) = 12093.75$$

.

We can now form a quadratic expression for i and solve.

$$i^2 + 1.95i + 0.95 = 1.209375$$
$$i^2 + 1.95i - 0.259375 = 0$$
$$i = \frac{-1.95 \pm \sqrt{1.95^2 - 4 \times 1 \times (-0.259375)}}{2}$$
$$i = 0.125 \text{ (taking the positive square root)}$$

Therefore, the account balance after the end of 3 years if the effective interest rate is $i + 0.09$ is

$$10000(1 + 0.125 + 0.09)^3 = 10000 \times 1.215^3 = \$17,936.13$$

.

(note: the account balance almost doubles over three years, not bad!! But the effective compound interest rate is very high at 21.5% so we do expect a high growth factor).

4. Scott wants to have \$800 now. He may obtain it by promising to pay \$900 at the end of one year; or he may borrow \$1,000 and repay \$1,120 at the end of the year. If he invests any cash inflows at 10% effective per annum for the year, which should he choose?

Under the first scenario, the net cashflow at the end of the year is $800 \times (1.1) - 900 = -\$20$. Under the second scenario, the net cashflow at the end of the year is $1000 \times (1.1) - 1120 = -\$20$. Scott is in deficit by \$20 under both scenarios. Therefore, the two options are equivalent.

Also, consider the additional \$200 from the second option. Invested at 10%, this grows to \$220 at the end of the year, equivalent to the extra amount payable at the end of the year under the second option (\$1120-\$900).

5. You invest \$1000 now, at a simple interest rate of 6% per annum for ten years. What is the effective rate of interest in the fifth year of your investment?

a. 6.0%

b. 5.4%

c. 4.8%

d. 5.0%

Answer = **c.** The interest earned in any year is 6% of \$1000, under simple interest. So the interest earned in the fifth year is \$60. To get the effective rate in the fifth year, we also need the investment value at the beginning of the fifth year (or end of the fourth year), that is, after 4 years of simple interest. The investment value at the beginning of the fifth year is $1000 \times (1 + 0.06 \times 4) = 1240$. Therefore, the effective rate of interest for the fifth year is $\frac{60}{1240} = 4.8\%$.
Checks: The rate of growth is not constant under simple interest, therefore answer a. is not correct. Answer b. is the effective compound annual rate for 5 years – check this for yourself.

6. Joe deposits 10 today and another 30 in five years into a fund paying **simple** interest of 11% per year. Tina will make the same two deposits, but the 10 will be deposited n years from today and the 30 will be deposited $2n$ years from today. Tina's deposits earn an annual effective **compound** interest rate of 9.15%. **In** 10 years, the accumulated amount of Tina's deposits equals the accumulated amount of Joe's deposits. Calculate n.

For Joe, we know (i) the amount of each deposit; (ii) the timing of each deposit; and (iii) the interest rate – so we can evaluate Joe's investment value in ten years.
Let $S_J(10)$ be Joe's investment value in ten years.

$$S_J(10) = 10(1 + 0.11 \times 10) + 30(1 + 0.11 \times 5) = 67.5$$

.
Let $S_T(10)$ be Tina's investment value in ten years.

$$S_T(10) = 10(1.0915)^{(10-n)} + 30(1.0915)^{(10-2n)} = 67.5$$

.

Solving for n:

$$10(1.0915)^{10}(1.0915^{-n} + 3 \times 1.0915^{-2n}) = 67.5$$
$$1.0915^{-n} + 3 \times 1.0915^{-2n} = 2.81233$$

Let $a = 1.0915^{-n}$.

$$3a^2 + a - 2.81233 = 0$$
$$a = \frac{-1 \pm \sqrt{1 - 4 \times 3 \times (-2.81233)}}{2 \times 3}$$
$$a = 0.81579 \text{ (taking the positive square root)}$$

Therefore
$$n = -\frac{\ln 0.81579}{\ln 1.0915} = 2.325$$
.

That is, Tina makes her first investment in 2.325 years.

Check: $S_T(10) = 10(1.0915)^{(10-2.325)} + 30(1.0915)^{(10-2\times 2.325)} = 67.5$ as required.

7. Using an effective annual (compound) interest rate of 5%, the present value at $t_1 = 7$, of \$125 due at $t_2 = 10$ is:

a. \$76.74

b. \$107.98

c. \$88.84

d. \$92.59

e. \$108.70

Answer = **b.** Present value = $125 \times v_{0.05}^{(10-7)} = 125(1.05)^{-3} = \$107.98$. (Our discount factor is $v_{0.05}^{(10-7)} = \frac{1}{1.05^3} = 1.05^{-3}$. Multiply the future investment value by the discount factor to obtain the present value).

Checks: Answers a.,c. and d. are too small. Respectively they imply accumulation factors of approximately 1.62, 1.40 and 1.35. These growth factors are too big to achieve at 5% interest for only three years.

Answer e. is the present value assuming simple interest = 5% (check this for yourself). Note \$108.70 > \$107.98. This makes sense because the growth factor is less under simple interest (simple interest does not accrue simple interest, unlike compound interest which accrues compound interest), and therefore, the investment value needs to be greater at $t_1 = 7$ to achieve the same value at $t_2 = 10$.

8. An investment pays a monthly effective (compound) rate of interest $j = 1.5\%$. What is the equivalent quarterly effective (compound) rate of interest?

a. 6.00%

b. 6.14%

c. 4.50%

d. 4.57%

Answer **d.** Let i bet the effective quarterly rate. Consider an investment for 3 months (= 1 quarter). Under the definition of equivalence, the investment value at the end of 3 months must be the same if interest is earned at an effective monthly rate of 1.5% for 3 months, or at an effective quarterly rate i for one quarter. So we solve for i such that $(1 + i) = (1.015)^3$. The answer would be c. if we were dealing with simple interest.