Lecture 31

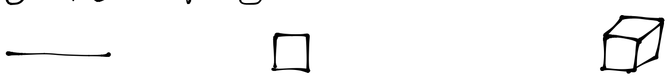
§14.4 Koch Snowflake

What is the perimeter of the Koch snowflake?

Step 0: 3 segments length 1, total perimeter=3
Step 1: 3*4 segments of length 1/3, total perimeter=3*(4/3)
Step 2: 3*4^2 segments of length 1/3^2, total perimeter=3*(4/3)^2
...
Step k: 3*4^k segments of length 1/3^k, total perimeter=3*(4/3)^k

As k goes to infinity! we obtain an infinite perimeter.

§14.5 Topological Dimension



What about the sierpinski triangle?
- at each step of its construction it has 2 dimensions.
- but in the end, we remove all the 2-dimensional parts and are left with line segments, which are 1 dimensional.

→ Fractals don't really fit neatly into an integer dimension.

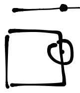
def: a fractal is a subset of $\mathbb{R}^N$ which is self-similar and whose fractal dim exceeds its topological dimension.

def: A set S has topological dimension 0 if every point in S has small neighbourhoods whose boundaries don't intersect S.

A set S has topological dim k if every point in S has small neighbourhoods whose boundaries intersect S in a set of dim $k-1$ and k is the smallest such #.

e.g.

top: topological, not TOP

① : : top dim 0
② top dim 1
③  top dim 2

....

④ Cantor set. it doesn't contain any open sets, so there are "gaps" between each point.

For every point, there are arbitrarily small neighbourhoods whose boundary doesn't intersect the cantor set. Top dim 0

⑤ Sierpinski Triangle

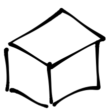
top dim 1

§ 14.6 Fractal dimension

def: A set S is called self-similar if it can be subdivided into k congruent subsets, each of which may be magnified by a constant factor M to yield the whole set S.

A line is self-similar. $k=2, M=2$, dim $=1$
$k'=3, M'=3$, dim $=1$

□ is self-similar (if we divide into 4) $k=4, M=2$, dim $=2$
$k'=9, M'=3$, dim $=2$



$k=8, M=2$, dim $=3$
$k'=27, M'=3$