March 6th V Hermitian ips. T:V->V the adjoint of T is the "operator" T*: V->V satisfying < T(v), w>=< v, T*(v)> main property: If d= [V1; ... , Vn] is an orthonormal basis of V-then $[T^*]_{\alpha} = [T]_{\alpha}^*$ means conjugate transpose $EX: V=P_{i}(\mathcal{L})$ $< p(x), q(x)>=p(0) \overline{q(0)} + p(i) \overline{q(i)}$ This defines a Hermitian innor product on V (of HW) T:V->V; T(p(n)=p'(n) Question: what's T *? want T* (ax+b)? $B=\{1.X\}$, standard basis of V $\langle 1, x \rangle = -i$ so not orthonormal

$$\beta = \{1, \chi\}, \text{ standard basis of } \}$$

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orthonormal basi's d=(方, 万文一方) $[T]_{\alpha} = \begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix} \qquad T(\sqrt{2}) = 0, T(\sqrt{2}) = \sqrt{2} = 2(\sqrt{2})$ $[T]_{\alpha} = \begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix} \Rightarrow T^*(\frac{1}{12}) = 2\sqrt{2} \chi - \sqrt{2}i$ $T*(\sqrt{2}x - \frac{1}{2}) = 0$

T*(ax+b)=全丁*(12x-主)+(12b+12)丁*(主)=(12b+12)(212x-主) ax+b= 号(シスノーラ)+c(世) b=-10 + 5 => 12b+19 =c

V Hermitian ips

 $T:V \longrightarrow V$

Tnormal if T.T = T *T

Spectral theorem for normal operators:

T normal. Then V has an orthonormal basis of eigenvalues of T. in particular. T diagonalizable

JORDAN CANONICAL FORM

Z let's say that x, y & Z are equivalent if x=y mod n i.e. n divides x-y

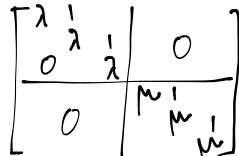
Ex: when n=2, Z=Zeven UZodd
(They have no integers in common.

when n=3... the set of integers equivalent to 0 is

. TORDAN is about to break up these.

Mn ([) recall that A similar to B if there exists on invertible matrix X s.t. A=XBX-1
Big picture: Mn([) get split up into similarity classes and choose the "best" representative for each class.

Will see is that any $A \in M_n(\mathbb{C})$ is similar to a matrix of the form



Will prove the theorem in 3 major steps
1. triangularizability
2. Prove the theorem for "nilpotent" matrices

upper triangular matrix is a matrix of the form

$$\begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_n \end{bmatrix}$$

recall: evalues of an upper \triangle matrix and its diagonal entries $T:V\longrightarrow V$; what does it mean for $[T]_{\alpha}$ to be upper \triangle ?

$$d = \{V_1, \dots, V_n\}$$

$$[T]_{\alpha} = \begin{bmatrix} \alpha_1 & \alpha_{12} & \times \\ 0 & \alpha_{22} & \times \\ \vdots & 0 & \ddots \\ 0 & \vdots & \ddots & \alpha_{nn} \end{bmatrix}$$

$$T(v_i)=\alpha_{11}V_i \\ T(v_2)=\alpha_{22}V_2+*V_i$$

$$T(v_3)=\alpha_{33}V_3+*V_2+*V_i$$

$$T(span\{v_1,v_2,V_3\}) \subset span\{v_1,v_2,V_3\}$$