UNIVERSITY OF TORONTO
Faculty of Arts and Sciences
APRIL 2011 EXAMINATIONS
APM236H1S
Applied Linear Programming
Duration- 3 hours
Aids allowed: Non-programmable calculators only

PLEASE HAND IN

Please note: This exam contains 133 marks including 33 bonus marks, however there are more questions than one is expected to fully answer in a 3 hour exam, therefore you need to carefully select and answer those questions you are confident about. No full mark will be given to an unjustified answer!

NAME: (last first) ______________________

STUDENT NUMBER: ______________________

SIGNATURE: ______________________

MARKER'S REPORT:

Question	MARK
Q1	
Q2	
Q3	
Q4	
Q5	
Q6	
Q7	
Q8	
Q10	
TOTAL	

1. (10 marks) Consider the following information about a LPP in the standard format:

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 3 & 4 \\ 6 & 6 & 2 \end{bmatrix} \quad \boldsymbol{c} = \begin{bmatrix} 8 \\ 9 \\ 5 \end{bmatrix} \quad \boldsymbol{b} = \begin{bmatrix} 2 \\ 3 \\ 8 \end{bmatrix}$$

The optimal solution is reached with the basic variable x_4, x_2, and x_1 in this order.

Determine the matrix B^{-1} which was responsible for this solution and determine the range of values for Δb_3 which do not affect the feasibility of the optimal solution. Is there a change in Δb_3 that can transform the same optimal solution to a new, integer valued solution? If so present the new solution, and the new objective value. (This is an alternative way to solving an integer programming problem.)

cont'd...

2. Consider the following matrix T (the content of the final tableau of a LPP) as well as the matrices c and x_B:

$$T = \begin{bmatrix} 0 & -\frac{1}{2} & 1 & 1 & 2 & 0 & -1 \\ 1 & \frac{1}{2} & 0 & 2 & -1 & 0 & -2 \\ 0 & 2 & 0 & -1 & -\frac{1}{2} & 1 & 3 \end{bmatrix} \quad x_B = \begin{bmatrix} 4 \\ 2 \\ 1 \end{bmatrix} \quad c = \begin{bmatrix} 1 \\ 3 \\ 2 \\ 1 \\ 1 \\ 3 \\ 2 \end{bmatrix}$$

a) (4 marks) determine the range of changes Δc_2 that do not affect the optimality of the solution.

b) (2 marks) Repeat part (a) for Δc_4

cont'd...

c) (4 marks) Repeat part (a) for Δc_1

cont'd...

d) (6 marks) We like to modify our solution to make sure that $x_1 \geq 3$. Introduce a new constraint to this tableau to carry out the requirement. Present your final tableau (including your optimal solution and the optimal value of the objective function.)

cont'd...

e) (8 marks) In the original problem change the matrix $\boldsymbol{x}_B$ to $\boldsymbol{x}_B^T = [3\frac{4}{5}, 2\frac{1}{6}, 1]$. And introduce a cutting plane that make (only) x_1 into an integer.

cont'd...

3. Consider the following LPP: maximize $4x_1 + 2x_2 + 3x_3$ subject to

$$\begin{aligned} 2x_1 + 3x_2 + x_3 &\leq 12 \\ x_1 + 4x_2 + 2x_3 &\leq 10 \\ 3x_1 + x_2 + x_3 &\leq 10 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

a) (6 marks) First demonstrate that the point $(2, 0, 4)$ is an extreme point of the feasible region, and then determine a matrix B^{-1} that corresponds to this point.

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b) (7 marks) Use B^{-1} to decide if this solution is optimal. If the solution is not optimal use the η matrix to move forward with the simplex tableau, and if the solution is optimal determine the solution to the dual problem and demonstrate that the complementary slackness holds.

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4. Here is the cost matrix, and the supply and demand vectors of a Transportation problem.

$$C = \begin{bmatrix} 4 & 6 & 7 & 5 \\ 4 & 4 & 7 & 8 \\ 5 & 3 & 6 & 5 \end{bmatrix} \quad s = \begin{bmatrix} 90 \\ 60 \\ 50 \end{bmatrix} \quad d = \begin{bmatrix} 40 \\ 50 \\ 70 \\ 40 \end{bmatrix}$$

a) (5 marks) Apply the vogel algorithm to determine an initial basic solution for the problem, and determine the cost associated to this solution.

b) (3 marks) Apply The Greedy algorithm (row by row please,) to find an initial basic solution and determine the associated cost.

cont'd...

c) (8 marks) use the simplex method, version presented in section 5.1, starting with the greedy solution, to determine an entering variable. Make sure you present your calculations, and indicate at which stage of your calculations and how you use complementary slackness.

cont'd...

d) (6 marks) apply a loop to accommodate the entering variable and determine the departing variable as a result of this loop. Present the new tableau and determine the saving as a result of this operation.

cont'd...

5. (6 points) Prove that if in a LPP the point $(0, 0, 1, 2, 3)$ is an extreme point of the FR then columns A_3, A_4 and A_5 of the coefficient matrix A (in the cannonical format) must be linearly independent. (This is a theorem in the book; please do not quote the theorem but prove it directly for this case.)

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6. Consider the LPP of maximizing $z = 2x_1 + 3x_2$ subject to the constraints: $x_1 + 2x_2 \leq 45$, $x_1 + x_2 \leq 40$, and $x_i \geq 0$. Demonstrate how you would incorporate integer programming to deal with the following additional requirements:

   a) (6 marks) There is a fixed cost of 50 to start the production of x_1, and if the production level reaches the level of 25 units then we will be compensated and will receive the fixed charge of 50 back. Justify your design please.

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b) (6 marks) We must select an additional constraint, and we are given three option from which we shall select the one that works the best. Of course you are asked to design a set of conditions which, together, they communicates this idea to the simplex method:) $x_1 \leq 20,\ x_2 \leq 21$ or $2x_1 + 3x_2 \leq 72$. (Note: please don't be too sensitive to the actual constraints and the coefficients as they are randomly selected to help you find bounds that you need in this procedure. For part marks you may present the choice between two of the constraints only.)

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7. (10 marks) present the dual of the following LPP, draw the feasible region of the dual and use it to solve the dual problem. Then use this solution to solve the LPP (without the simplex method, and only by observing the relationship between the two.) (Hint: You can geometrically look for some of the extreme points.)

maximize $z = 8x_1 + 2x_2 + 5x_3$ subject to

$$\begin{cases} 2x_1 + x_3 \leq 4 \\ x_1 + x_2 + x_3 \leq 3 \\ x_1, x_2 \geq 0, x_3 \text{ unrestricted} \end{cases}$$

cont'd...

8. Consider S to be the feasible region of a LPP defined by the constraints $A\boldsymbol{x} = \boldsymbol{b}$.

a) (5 marks) What does it mean for a set $S \subset \mathbb{R}^3$ to be convex, and what does it mean for the point $\boldsymbol{x}_0$ to be an extreme point of S.

b) (3 marks) prove that the feasible region S as in (a) is a convex set.

cont'd...

c) (6 marks) Assume that the matrix A is 2×6 and that the columns A_5, and A_6 are linearly independent. Assume also that for some positive values a, and b, we have $aA_5 + bA_6 = \boldsymbol{b}$. Prove that the point $(0, 0, 0, 0, a, b)$ is an extreme point of S. (Note: this is a theorem in the book and you are asked to present the complete algebraic proof without quoting the same theorem please.)

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9. (15 marks) State and prove the duality theorem.

cont'd...

10. (7 marks) State and prove the complementary slackness principle.

extra sheet; please do not remove