Sta347 Probability I
Selected Practice Problems for Midterm
Oct. 12, 2013

(1) Problem 2 on Page 24 of the Textbook.

(2) Problem 4 on Page 36 of the Textbook.

(3) Show that if A and B are events, then
$$P(\bar{A}\bar{B}) = P(\bar{A}) - P(B) + P(AB).$$

(4) (a) Show that if A and B are events and $P(A) = P(B) = 0$, then $P(A \bigcup B) = 0$.
   (b) Show that if $A_1, A_2, \cdots, A_n$ are events and $P(A_i) = 0$ $i = 1, 2, \cdots, n$, then $P(\bigcup_{i=1}^{n} A_i) = 0$.
   (c) Show that if infinitely many events $A_1, A_2, \cdots$, all have probability 0, then $P(\bigcup_{i=1}^{\infty} A_i) = 0$.

(5) Show that for any random variable X such that $E(|X|^3) < \infty$, we have
$$(E|X^2|)^2 \leq E|X| \times E(|X|^3).$$

(6) Let X be a Binomial(25, 0.6) random variable. Find $P(X \geq 2)$, $P(X = 10)$, $E(X)$ and $V(X)$.

(7) A recruiting firm finds that 20% of the applications are fluent in both English and French. Applicants are selected randomly from a pool and interviewed sequentially. Find the probability that at least five applicants are interviewed before finding the first applicant who is fluent in both English and French.

(8) A swimming pool repair person has three check valves in stock. Then percent of the service calls require a check valve. What is the expected number and standard deviation of the number of service calls she will make before running out of check valves?

(9) Let X be a negative binomial random variable with parameters r and p, and let Y be a binomial random variable with parameters n and p. Show that
$$P(X > n) = P(Y < r).$$

(10) The number of car accidents in a city in a certain week follows a Poisson distribution with a mean of two accidents per square kilometer.

   (a) If four one-square-kilometer regions from the city are selected independently, find the probability that at least one region will contain at least one car accident in a particular week.
   (b) How many one-square-kilometer regions should be selected in order to have probability of approximately 95% of containing at least one car accident in a particular week?