# RESEARCH SCHOOL OF FINANCE, ACTUARIAL STUDIES AND STATISTICS

## *INTRODUCTORY MATHEMATICAL STATISTICS*

## *PRINCIPLES OF MATHEMATICAL STATISTICS*

## *(STAT2001/6039)*

## Assignment 2 (2017)

- *Your solutions to the assignment should be placed in the appropriate box in the FAS School foyer by the due time and date (as provided on Wattle).*
- *Attach a cover sheet (as provided on Wattle) which has your ANU ID number.*
- *The assignment is out of 100 and is worth 10% of your overall course mark.*
- *The assignment is to be done alone. Marks may be deducted for any copying.*

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**Problem 1** ***(Total 30 marks)***

A continuous random variable $Y$ has pdf (probability density function)

$$f(y) = \begin{cases} 2e^{ky}, & y > 0 \\ 1/6, & -6 < y < -3 \end{cases}$$

**(a)** Determine the value of the constant $k$. Then sketch the pdf.
Finally, derive and sketch $Y$'s cdf (cumulative distribution function).

**(b)** Find $Y$'s mean, variance and standard deviation.

**(c)** Find $Y$'s mode and median.

**Problem 2** ***(Total 20 marks)***

A continuous random variable $Y$ has cdf $F(y) = \begin{cases} a + ay/2, & -2 < y < 0 \\ 1 - be^{-cy^3}, & y > 0 \end{cases}$

**(a)** Sketch the cdf when $a = 1/2$, $b = 1/2$ and $c = 1$.
Then derive $Y$'s pdf generally, and sketch it when $a = 1/2$, $b = 1/2$ and $c = 1$.
Finally, write down the range of possible values for $a$, $b$ and $c$.

**(b)** Find $P(Y > -1 \mid Y < 1)$ when $a = 1/2$, $b = 1/2$ and $c = 1$.

**Problem 3** ***(Total 20 marks)***

A discrete random variable has pdf $p(y) = bk^y, \quad y = 0,1,2,\ldots$

**(a)** Sketch this pdf when $b = k = 1/2$.
Then derive $Y$'s cdf when $b = k = 1/2$ and sketch it.
Finally, write down the range of possible values for $b$ and $k$.

**(b)** Find $Y$'s mgf (moment generating function, $m(t)$) generally.
Then sketch this mgf when $b = k = 1/2$, showing the points at $t = 0$ and $t = 0.2$.
Using the mgf method or otherwise, find the variance of $Y$ when $b = k = 1/2$.

**Problem 4** ***(Total 20 marks)***

In a certain country, the weights of female adults (ages 18 and up) are approximately normally distributed. About one fifth of the female adult population is heavier than 91 kg, and one quarter is lighter than 46 kg.

**(a)** Find the proportion of female adults in the country who are heavier than 60 kg.

**(b)** A female adult, Lisa, has been randomly chosen from the population of the country. We are told that her weight is below average. Find the probability that Lisa weighs more than 60 kg.

**Problem 5** ***(Total 10 marks)***

A straight wooden plank of length 2 m has been purchased for $34. This plank is about to be sawed into two at a point chosen randomly along it. Each of the two resulting short planks will then be sawed ***lengthwise*** into twelve identical sticks. Next, two perfectly cubic frames will be constructed from the twenty four sticks. Then both frames will be made into huge dice by covering them, each on all six sides, with a type of red plastic sheet which costs $42 per square metre. Next, each die will be filled with a bouncy rubber material which costs $66 per cubic metre. Finally, the usual number of dots will be put onto the dice. For each die whose total surface area is at least 5400 square centimetres, the dots will be made of gold at a cost of $15 each; for each die smaller than this, the dots will be painted on with black paint at negligible cost.

For how many dollars should the two dice be sold (in advance) so that the expected profit is $700?