Duration:

60 minutes (2:15pm - 3:15pm)

Aids Allowed:

Student Number: 999292509

Family Name(s): Qiu

Given Name(s): Ruj

Do not turn this page until you have received the signal to start. In the meantime, please fill out the identification section above.

This term test consists of 5 questions on 10 pages (including this one), printed on both sides of the paper. When you receive the signal to start, please make sure that your copy of the test is complete, fill in the identification section above, and write your name on the back of the last page.

This test is double-sided.

MARKING GUIDE # 2: 10.5/11 # 5: 5 5/6

♥ Good Luck and Have a Happy St. Valentine's Day ♥

Use the space on this "blank" page for scratch work, or for any answer that did not fit elsewhere.

Clearly label each answer with the appropriate question and part number.

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Question 1. [7 MARKS]

Consider the statement:

(S1) 
$$A \Rightarrow (B \lor C)$$
.

Assuming that statement (S1) is true, give the best answer for each of the following questions:

Part (a) [1 MARK]

(BVC) is true.

What can be concluded from (S1), if A is true?

If A is true, (BVC) may be true, if so, then B is true. Cistme; (BVC) may be false, if so, then at least one of B, C is false.

Part (b) [1 MARK]

What can be concluded from (S1), if B is true? Nothing (on the consequent is true) If B is true, C may be true, if SO, A is true;

C may be false, if so, A is false.

Part (c) [1 MARK]

What is the converse of (S1)?

$$(BVC) \Longrightarrow A$$

Part (d) [2 MARKS]

What is the contrapositive of (S1)? (Work the negation(s) all the way in.)

Part (e) [2 MARKS]

What is the negation of (S1)? (Work the negation(s) all the way in.)

 $\neg A = \neg \neg B \land \neg C$ 



7 (A=>(BVC)) C=> 7 (7AV (BVC))
(=> 77A N7(BVC)
(=> ANBNAC)

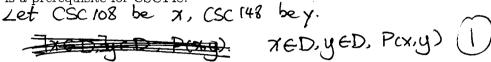
Question 2. [11 MARKS]

Consider the domain  $D = \{\text{all CSC courses and all MAT courses}\}$ , and the predicate symbols C(x): "x is a CSC course", M(x): "x is a MAT course", and P(x, y): "course x is a prerequisite for course y".

Using only these symbols (in addition to appropriate connectives and quantifiers), give a clear symbolic statement that corresponds to each given English sentence. Quantifiers may only be over the domain D.

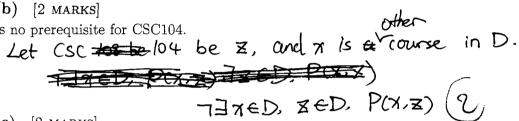
Part (a) [1 MARK]

CSC108 is a prerequisite for CSC148.



Part (b) [2 MARKS]

There is no prerequisite for CSC104.



Part (c) [2 MARKS]

Every course has a prerequisite.

Part (d) [2 MARKS]

No course is a prerequisite for itself.

Part (e) [2 MARKS]

Some CSC course has a prerequisite.

$$\exists x \in D, C(x) \Rightarrow \exists y \in D, \Rightarrow P(y,x)$$

Part (f) [2 MARKS]

Every MAT course has a prerequisite.

$$\forall x \in D, M(x) = \ge \exists y \in D, P(y, \pi)$$

## Question 3. [12 MARKS]

The following terms are used frequently to describe logical statements in CSC165. For each of them:

- (i) Write a definition of the term, in English.
- (ii) Write a statement in English that is true and is an example of a statement that meets the definition of the term.
- (iii) After defining suitable domain(s) and/or predicate(s), write a statement in logic that is true and is an example of a statement that meets the definition of the term.

doesn't need to be true Part (a) [4 MARKS] A universally quantified statement. (i) definition: A statement which is holds for all elements in its domain. A universally quantified statement is a statement that makes a claim about all (ii) statement in English: Every room in Toronto has a door. (iii) domains, predicates and statement in logic: Donain: Let R be the set of all rooms in Terronto. Redicate: Let D be the event that a room has actor. A tautology.

(i) definition: A statement

which has two-sided implication. A cindependent of the olomains on

which has two-sided implication. A cindependent of the olomains on

predicates involved.)

(ii) statement in English:

The season after spring is subsect summer.

just

(iii) domains, predicates and statement in logic: Donain: Let S be the set of seasons in Toronto

PV-Pon VDED, VPEP,

Predicate: Let A be "A season is after spring."

ion

Part (c) [4 MARKS]

PARKS]

I was a season of summer.

Let B ho "A coason is summer.

Part (c) [4 MARKS] Let 7 be a random door.

is the set of all Let B be "A season is after sp possible domains and P is the Let x be a season. possible domains and P is the Let x be a season. cet of all possible for disples on D, A(x) <=> B(x)

Part (c) [4 MARKS] A vacuous truth.

the beaute (i) flefinition: A statement whose inverse is also true if the statement itself is true.

a statement with a false anteredant

always (ii) statement in English:

All unicorns are pink and purple.

Three angles.

\ (iii) domains, predicates and statement in logic: Domain: Le G be the set of supers The "it is a triangle" angles.

A be "it has 3 tracks.

The a random shape.

OVER YXEIR, x2=0=>X=42

Statement: NEG, TOA), A(X).

3.5

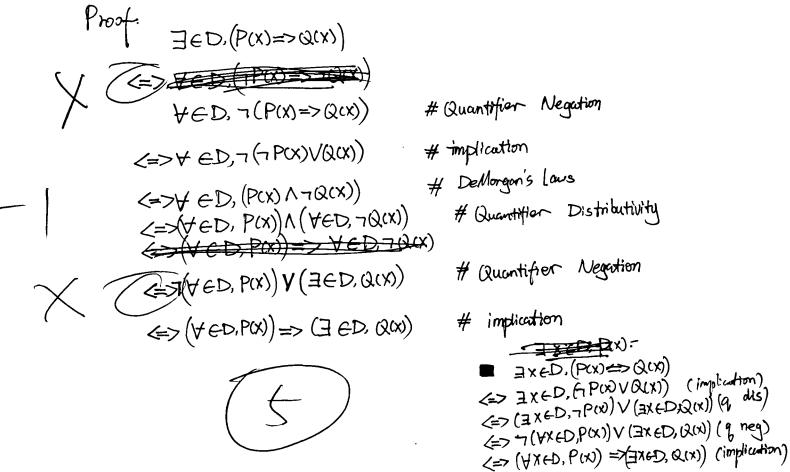
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## Question 4. [6 MARKS]

Show that  $\exists x \in D, (P(x) \Rightarrow Q(x))$  is equivalent to  $(\forall x \in D, P(x)) \Rightarrow (\exists x \in D, Q(x))$ . Justify each step of your derivation. You may use the list of standard equivalences given below.



Standard Equivalences (where P, Q, P(x), Q(x), etc. are arbitrary sentences. All quantifications are over a domain D.)

- Commutativity
  - $P \wedge Q \iff Q \wedge P$  $P \lor Q \iff Q \lor P$

- $P \Leftrightarrow Q \iff Q \Leftrightarrow P$
- Associativity
  - $P \wedge (Q \wedge R) \iff (P \wedge Q) \wedge R$  $P \lor (Q \lor R) \iff (P \lor Q) \lor R$
- Identity
  - $P \wedge (Q \vee \neg Q) \iff P$
  - $P \lor (Q \land \neg Q) \iff P$
- Absorption
- $P \wedge (Q \wedge \neg Q) \iff Q \wedge \neg Q$
- $P \lor (Q \lor \neg Q) \iff Q \lor \neg Q$
- Idempotency
  - $P \wedge P \iff P$
  - $P \lor P \iff P$

- Double Negation
  - $\neg \neg P \iff P$
  - DeMorgan's Laws
  - $\neg (P \land Q) \iff \neg P \lor \neg Q$  $\neg (P \lor Q) \iff \neg P \land \neg Q$
- Distributivity
- $P \wedge (Q \vee R) \iff (P \wedge Q) \vee (P \wedge R)$
- $P \lor (Q \land R) \iff (P \lor Q) \land (P \lor R)$
- Implication
  - $P \Rightarrow Q \iff \neg P \lor Q$
- Biconditional
  - $P \Leftrightarrow Q \iff (P \Rightarrow Q) \land (Q \Rightarrow P)$
- Renaming (where P(x) does not contain variable y)
  - $\forall x, P(x) \iff \forall y, P(y)$
  - $\exists x, P(x) \iff \exists y, P(y)$

- Quantifier Negation
  - $\neg \forall x, P(x) \iff \exists x, \neg P(x)$
  - $\neg \exists x, P(x) \iff \forall x, \neg P(x)$
- Quantifier Commutativity
  - $\forall x, \forall y, S(x,y) \iff \forall y, \forall x, S(x,y)$
- $\exists x, \exists y, S(x,y) \iff \exists y, \exists x, S(x,y)$ • Quantifier Distributivity (where S
- does not contain variable x)
- $S \wedge \forall x, Q(x) \iff \forall x, S \wedge Q(x)$
- $S \lor \forall x, Q(x) \iff \forall x, S \lor Q(x)$
- $S \wedge \exists x, Q(x) \iff \exists x, S \wedge Q(x)$
- $S \vee \exists x, Q(x) \iff \exists x, S \vee Q(x)$
- $(\forall x, P(x)) \land (\forall x, Q(x)) \iff$
- $\forall x, (P(x) \land Q(x))$
- $(\exists x, P(x)) \lor (\exists x, Q(x)) \iff$
- $\exists x, (P(x) \lor Q(x))$

## Question 5. [6 MARKS]

At a murder trial, four witnesses give the following testimony.

Alice: If either Bob or Carol is innocent, then so am I.

**Bob:** Alice is guilty, and either Carol or Dan is guilty.

Carol: If Bob is innocent, then Dan is guilty.

Dan: If Bob is guilty, then Carol is innocent. In addition, Bob is innocent.

Is it possible that everyone is telling the truth? Justify your response.

Proof Star: Write everyone's testimony in symbolic statement: Let Go be "guilty", hence 7G is "innocent". \$\\ A <=> \( \bar{\partial} \left( \gamma \text{G(B)} \right) \left( \gamma \text{G(C)} \right) => \gamma \text{G(A)} \( \bar{\partial} \) B => G(A) A (G(C) VG(D)) / (<=> 76(B)=>6(D) V D <=> (G(B)=>7G(C)) 17G(B) Now suppose everyone is telling a truth:

If D is true, then 7G(B) and  $7G(B) \Rightarrow G(C)$  X.  $(G(B) \Rightarrow 7(G(C)) = (G(C) \Rightarrow 7(G(C))$ 

If C is true, 7G(B) => G(D) # we already have 7G(B)

If B is true, G(A) = # G(C)VG(D) is frue

If A is true, (7 G(B))V(1 G(C))=>7 G(A) # we already have 7 G(B)

Thus the cochision from A contradicts that from B.

Therefore, it is not possible that everyone is telling a truth.