UNIVERSITY OF TORONTO
Faculty of Arts and Science

PLEASE HAND IN

APRIL 2011 - Final Examination

MAT244 H1S - ODEs

Duration - 3 hours

Instructors: P. Milman, B. Fontaine & V. Ivrii

NO AIDS ALLOWED

Last Name _____ **First Name** _____ **ID** _____

#	Mark	#	Mark	#	*Mark*
1 [10 points]		**4** [20 points]		**5c** [10 points]	
2 [20 points]		**5a** [4 points]		**5d** [10 points]	
3 [20 points]		**5b** [6 points]		**TOTAL**	
TERM		*FINAL*			

MAT244H1S – Final Examination, April 2011

Student Name: _____ **Student Number:** _____

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1 (10 pts) Find the general solution of the differential equation

$$xy' = y - xy'$$

and solve the initial value problem $y(1) = -2$.

Student Name: _____ **Student Number:** _____

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2 (20 pts) Find the general solution of the equation

$$t^2y'' - 6y = 10t^{-2} - 6\ , \qquad t > 0\ .$$

Student Name: _____ **Student Number:** _____

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3 (20 pts) Find the general solution of the differential equation

$$y^{(4)} - y = 3t + \cos t \ .$$

Student Name: _____ **Student Number:** _____

4 (20 pts) Solve the system of differential equations

$$\begin{cases} x' = 2x + y + 2z \ , \\ y' = 2y + 2z \ , \\ z' = 4y \ . \end{cases}$$

Student Name: _____ **Student Number:** _____

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5 (30 pts) Equation

$$x'' = -4x^3 + 4x$$

(a) (4 pts) Reduce to the first order system in variables (t, x, y) with $y = x'$;

(b) (6 pts) Find solution in the form $H(x, y) = C$;

(c) (10 pts) Find critical points and classify them (i.e. specify whether they are nodes, saddles, etc. and stability);

(d) (10 pts) On (x, y)-plane sketch the phase portrait.