Ch 2.

58. $g(y)=2y,\ 0\le y\le 1$

62. ~~ ~~

68. $g(y)=\lambda e^{-\lambda\sqrt{\frac{y}{a}}}\cdot\frac{1}{2\sqrt{\pi a}}$

Ch 4.

2. $\frac{n+1}{2},\ \frac{n^2-1}{12}$

14. a. $\frac{2}{3}$

b. $\frac{1}{2};\ \frac{1}{18}$

16. ~~ ~~

30. $\frac{1}{\lambda}(1-e^{-\lambda})$ $\longrightarrow$

$$\begin{cases} E(X+C)=E(X)+C \quad \cdots\cdots (1) \\ E(X+Y)=E(X)+E(Y) \quad \cdots\cdots (2) \\ E(aX)=aE(X)\cdots (3) \\ Var(X+a)=Var(X) \\ Var(aX)=a^2 Var(X) \end{cases}$$

34. a. $\frac{2}{3}$ b. $\frac{2}{3}$

Ch.2 #57.

$$p\{Y<t\}=p\{aX+b<t\}=p\{x<\frac{t-b}{a}\}$$

$$=\frac{1}{\sqrt{2\pi}\sigma}\int_{-\infty}^{\frac{t-b}{a}} e^{-\frac{(y-\mu)^2}{2\sigma^2}}dy$$

$y=\frac{\theta-b}{a}$

$$=\frac{1}{\sqrt{2\pi}\sigma}\int_{-\infty}^{t}\exp\left\{-\frac{(\frac{\theta-b}{a}-\mu)^2}{2\sigma^2}\right\}d\left(\frac{\theta-b}{a}\right)$$

$$d\left(\frac{\theta-b}{a}\right)=\frac{1}{a}d\theta$$

$$=\frac{1}{\sqrt{2\pi}\sigma}\int_{-\infty}^{t}\exp\left\{-\frac{(\theta-(a\mu+b))^2}{2a^2\sigma^2}\right\}d\theta$$

☆☆☆ if $\underline{\eta=f(X)}$ $\underline{f \text{ is inversible}}$ $E(a)=\{w \mid f(w)<a\}$

pdf of η $\Downarrow$ $g(y)$

$$P(\eta<a)=P\{f(x)<a\}=P\{x\in E(a)\}=\int_{E(a)}p(x)\,dx$$

$$=\int_{-\infty}^{a}p[f^{-1}(y)]\,|[f^{-1}(y)]'|\,dy$$

$$\boxed{\Rightarrow g(y)=p[f^{-1}(y)]\cdot|[f^{-1}(y)]'|}$$

e.g. $P\{Y<t\}\Rightarrow$ $Y=aX+b$, $X=\frac{Y-b}{a}$

$$=\int_{-\infty}^{t}p\left(\frac{y-b}{a}\right)\left|\frac{1}{a}\right|dy$$

$$=\left|\frac{1}{a}\right|\int_{-\infty}^{t}\frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{(\frac{y-b}{a}-\mu)^2}{2\sigma^2}\right)dy$$

#67. a). $p(x) = F'(x) = \cdots$

$$= \frac{\beta}{\alpha^\beta} x^{\beta-1} e^{-(\frac{x}{\alpha})^\beta}$$

b). $F(x) \mid 1 - e^{-(\frac{x}{\alpha})^\beta}$ $\qquad w = (\frac{x}{\alpha})^\beta$

$F(w) = \underline{1 - e^{-w}}$

$w = f(x)$

$f^{-1}(w) = w^{\frac{1}{\beta}} \cdot \alpha$

$$g(w) = \frac{\beta}{\alpha^\beta} (\alpha w^{\frac{1}{\beta}})^{\beta-1} \cdot e^{-w} \left| \frac{\alpha}{\beta} y^{\frac{1}{\beta}-1} \right|$$

$$= y^{\frac{\beta-1}{\beta} + \frac{1-\beta}{\beta}} e^{-y}$$

$$= e^{-y}$$

Ch 4. #2. $\quad E(x) = \sum_{k=1}^{n} k\, p(x=k) = \frac{1}{n} \sum_{k=1}^{n} k = \frac{1}{n} \cdot \frac{n(n+1)}{2}$

$$E(x^2) = \sum_{k=1}^{n} k^2 p(x=k) = \frac{1}{n} \sum_{k=1}^{n} k^2 = \frac{1}{n} \cdot \frac{n(n+1)(2n+1)}{6}$$

$$\boxed{Var(X) = E(X^2) - [E(X)]^2}$$

#30. $X : E(x) \quad y = g(x)$

$E(g(x)) \neq g(E(x))$

$\parallel$

$$\int_\Omega g(x)\, dP = \int_\Omega g(x) p(x)\, dx$$

$$E(g(x)) = \sum g(x) p(x)$$

$$E(\frac{1}{1+k}) = \sum_{k=0}^{n} \frac{1}{1+k} \cdot \frac{\lambda e^{k-\lambda}}{k!}$$

$$= \sum_{k=0}^{n} \frac{\lambda^k \cdot e^{-\lambda}}{(k+1)!}$$

$X = \frac{1}{1+k}$

$m = k+1 = \frac{1}{X} \sum_{m=1}^{\infty} \frac{\lambda^m \cdot e^{-\lambda}}{m!}$ $\qquad \sum_{m=0}^{\infty} \frac{\lambda^m e^{-\lambda}}{m!} = \sum_{m=0}^{\infty} P(x=m) = 1$

$$\sum_{m=0}^{\infty} p(x=m) = p(x=0) + p(x=1) + \cdots + p(x=\infty)$$