Duration: **75 minutes**
Aids Allowed: **none**

Student Number: |_|_|_|_|_|_|_|_|_|

Family Name(s): ____________________

Given Name(s): ____________________

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*Do **not** turn this page until you have received the signal to start.*
In the meantime, please read the instructions below *carefully*.

This term test consists of 3 questions on 8 pages (including this one), printed on both sides of the paper. *When you receive the signal to start, please make sure that your copy of the test is complete, fill in the identification section above, write your student number where indicated at the bottom of every odd-numbered page (except page 1), and write your name on the back of the last page.*

Answer each question directly on the test paper, in the space provided, and use the reverse side of the pages for rough work. If you need more space for one of your solutions, use the reverse side of a page and *indicate clearly the part of your work that should be marked.*

In your answers, you may use without proof any result or theorem covered in lectures, tutorials, homework, tests, or the textbook, as long as you give a clear statement of the result(s)/theorem(s) you are using. You must justify all other facts required for your solutions.

Write up your solutions carefully! In particular, use notation and terminology correctly and explain what you are trying to do — part marks *will* be given for showing that you know the general structure of an answer, even if your solution is incomplete.

If you are unable to answer a question (or part), you will get 20% of the marks for that question (or part) if you write "I don't know" and nothing else — you will *not* get those marks if your answer is completely blank, or if it contains contradictory statements (such as "I don't know" followed or preceded by parts of a solution that have not been crossed off).

Marking Guide

1: ______/18

2: ______/18

3: ______/12

TOTAL: ______/48

 Good Luck!

Use this page for rough work — clearly indicate any section(s) to be marked.

Question 1. [18 MARKS]

Part (a) [10 MARKS]

Prove that $\forall x \in \mathbb{Z}, \forall y \in \mathbb{Z}, x^2 + y = 13 \wedge y \neq 4 \Rightarrow x \neq 3$.

Part (b) [8 MARKS]

For this part, you may refer to any equivalence from the following list by its number.

(1) $p \Rightarrow q \iff \neg p \vee q$ (2) $\neg \forall x \in D, p(x) \iff \exists x \in D, \neg p(x)$

(3) $(\exists x \in D, p(x) \vee q(x)) \iff (\exists x \in D, p(x)) \vee (\exists x \in D, q(x))$

Simplify the formula $(\forall y \in D, P(y)) \Rightarrow (\exists y \in D, Q(y))$ to use as few quantifiers and connectives as possible.

Use this page for rough work — clearly indicate any section(s) to be marked.

Question 2. [18 MARKS]

Write a detailed structured proof that $\forall i \in \mathbb{N}, \exists j \in \mathbb{N}, (j < i \wedge a_j > a_i) \vee (j > i \wedge a_j < a_i)$ for the sequence defined by $a_n = 4\lfloor n/2 \rfloor + 1 - n$ (*i.e.*, $1, 0, 3, 2, 5, 4, 7, 6, 9, 8, 11, 10, 13, 12, \ldots$).
(HINT: You may need the following fact: $(*)\ \forall n \in \mathbb{N}, \exists k \in \mathbb{N}, n = 2k \vee n = 2k + 1$.)

Use this page for rough work — clearly indicate any section(s) to be marked.

Question 3. [12 MARKS]

Describe two different strategies/approaches you could use to attempt to solve the following problem, and explain what you hope to obtain from each strategy.

In the game of "chomp", chocolate chips are laid out in an $m \times n$ grid, and the players take turns selecting one chocolate chip (any one they want), then eating it and every chocolate chip to its right and below. The player who eats the last chocolate chip (in the upper-left corner) loses—in some descriptions, that last chocolate chip in said to be poisoned, but we won't be so morbid! For example, if player A selects the chocolate chip marked "A" in the grid on the left, the result is the grid in the middle; if player B then selects the chocolate chip marked "B", the result is the grid on the right; and so on...

$$\begin{array}{ccccc} X & O & O & O & O \\ O & O & O & O & O \\ O & O & O & A & O \\ O & O & O & O & O \end{array} \qquad \begin{array}{ccccc} X & O & O & O & O \\ O & B & O & O & O \\ O & O & O & & \\ O & O & O & & \end{array} \qquad \begin{array}{ccccc} X & O & O & O & O \\ O & & & & \\ O & & & & \\ O & & & & \end{array} \qquad \cdots$$

Your problem is to determine a winning strategy for the first player (player A), given the dimensions $m \times n$ of the playing grid.

On this page, please write nothing except your name.

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