PLEASE HAND IN

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University of Toronto
Faculty of Arts and Science

Final Examinations
MAT301H1F – Groups and Symmetry
Tuesday, December 14, 2010

Instructor: Prof. J. W. Lorimer

Duration – 3 hours

NO AIDS ALLOWED

LAST NAME: ______________________________

FISRT NAME: ______________________________

STUDENT NUMBER: ______________________________

INSTRUCTIONS:

1. DO **THREE** questions out of **FOUR** from **PART A** and the **TWO** questions from **PART B.**
2. Write the final solutions in the pages provided.
3. There are 16 pages and 6 questions in this examination paper.

FOR EXAMINER ONLY		
Question	**Value**	**Mark**
PART A		
1.	20	
2.	20	
3.	20	
4.	20	
PART B		
5.	20	
6.	20	
TOTAL	**100**	

PART A

Do *any* **THREE QUESTIONS**.

[20] 1. DEFINE the following terms:

1. (a) The order of an element in a group G.

1. (b) An epimorphism.

1. (c) The alternating group.

1. (d) The centralizer of an element of a group G.

1. (e) The length of an orbit of a permutation

[20] 2. Let $\varphi = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 4 & 5 & 1 & 8 & 7 & 9 & 3 & 6 \end{pmatrix}$ be an element of S_9.

(a) Write φ as a product of disjoint cycles.

2. *(b)* Prove that φ is an element of the alternating group.

2. *(c)* Determine the order of φ.

[20] 3. STATE and PROVE the first Isomorphism theorem for groups.

CONT'D…

[20] 4. Let A_4 be the alternating group on {1,2,3,4} and K_4={1,(12)(34),(13)(24),(14)(23)} the Klein-4-subgroup of A_4.

4. (a) Show that $A_4 = K_4 < (123) >$.

CONT'D…

4. (b) Prove that K_4 is a normal subgroup of A_4.

CONT'D…

4. (c) Prove that A_4 is NOT the internal direct product of K_4 and $< (123) >$.

CONT'D...

4. (d) Prove that A_4/K_4 is isomorphic to Z_3.

CONT'D...

PART B

DO BOTH QUESTIONS.

[20] 5. (a) STATE and PROVE the product Isomorphism theorem for groups.

CONT'D...

CONT'D…

5. (b) Let G be a group. If G is the internal direct product of the normal subgroups H and K prove that G/H is isomorphic to K.

CONT'D...

CONT'D…

[20] 6. Let G be a group of order pq where p and q are distinct primes.

(a) Prove that G is abelian if and only if $Z_G \neq \{e\}$.

CONT'D…

6. (b) Give an example of a group of order pq for distinct primes p and q that is not abelian.

CONT'D...

6. (c) If G is abelian, prove that G is cyclic.