

**RESEARCH SCHOOL OF FINANCE, ACTUARIAL STUDIES AND
APPLIED STATISTICS**

Second Semester Mid-Semester Exam 2012

Survival Models / Biostatistics

| **(STAT3032/7042/8003)**

Writing period: 1 Hour duration

Permitted materials: Calculators, lecture notes, dictionary

You must attempt to answer all questions.

All questions are to be completed in the script book provided.

Question 1 (5 marks)

$$S(t) = \exp\left(-\int_0^t \frac{1}{\theta-s} ds\right) = \exp\{\log(\theta-t) - \log(\theta)\} = \frac{\theta-t}{\theta}$$

The force of mortality for a particular population from birth (that is, $\lambda(t)$ is the hazard t years after birth) is assumed to take the following form:

$$\lambda(t) = (\theta - t)^{-1}, \quad {}_t p_x = \frac{S(x+t)}{S(x)} = \frac{\theta-(t+x)}{\theta} \cdot \frac{\theta}{\theta-x} = \frac{\theta-(x+t)}{\theta-x}$$

where, $0 \leq t \leq \theta$. Based on this form of the force of mortality, find an expression for ${}_t q_x$.

Question 2 (10 marks = 3+5+2)

The observed survival times (in years) of 7 patients after a heart transplant operation are provided below. Values denoted with "*" correspond to times of censoring, rather than times of death. The censoring is uninformative, right censoring.

1, 2, 3*, 4, 5*, 6, 7

- a) Based on this data, what is the Kaplan-Meier estimate of the survival function at time 6.5? Your answer must include an estimate of the standard error.

$$\int_0^\infty {}_t p_x dt = (1-0) \times 1 + (2-1) \times 0.857 + \dots + (7-6) \times 0.268 = 4.625$$

- b) Provide an estimate of the mean survival time after a heart transplant operation.

- c) How does the Nelson-Aalen estimate of the survival function at time 6.5 compare to the Kaplan-Meier estimate at this time?

$$NA: \exp\left(-\left(\frac{1}{1} + \frac{1}{2} + \frac{1}{4} + \frac{1}{2}\right)\right) = 0.3466$$

$$g(\hat{\theta}) = \log(\hat{\theta}) \quad \left[g'(\hat{\theta})\right]^2 = \frac{1}{\hat{\theta}^2}$$

Question 3 (5 marks)

$$\hat{\theta} = 1.5, \quad SD(\hat{\theta}) = 2$$

$$\text{Var}(\log(\hat{\theta})) \approx \text{Var}(\hat{\theta}) \cdot \frac{1}{\hat{\theta}^2}$$

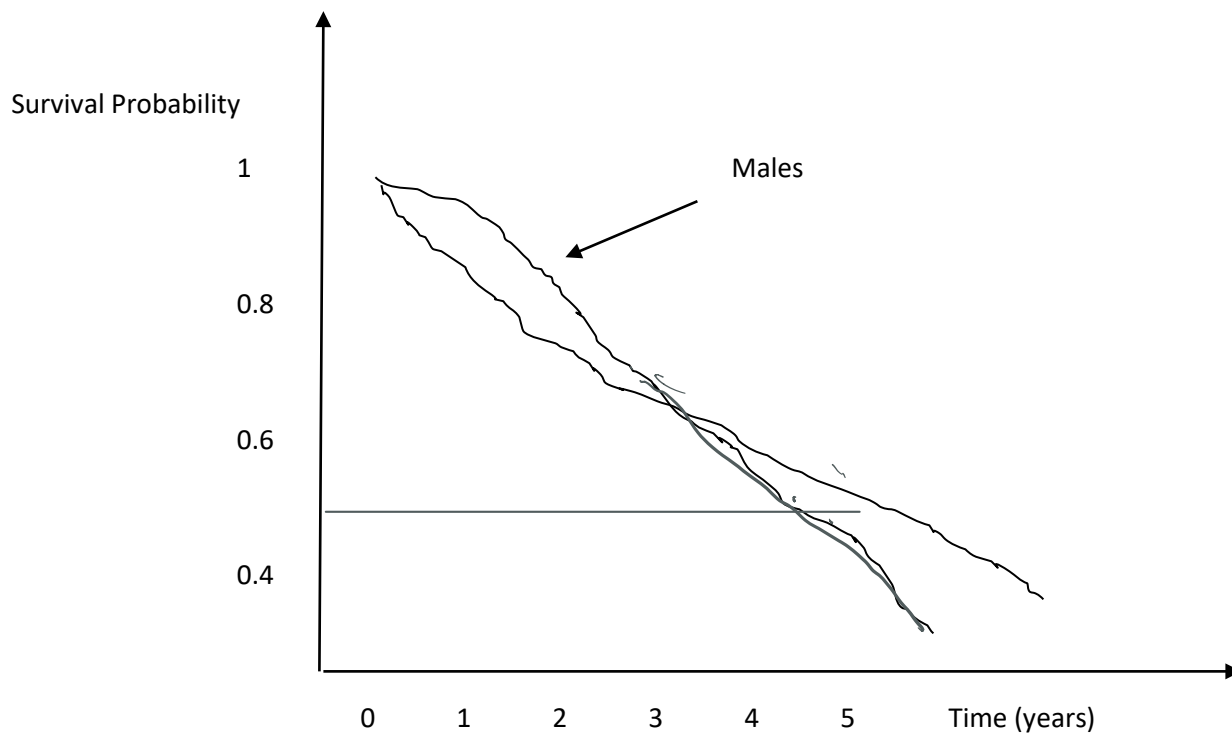
Based on a given set of data you have computed the maximum likelihood estimate of the parameter θ , denoted $\hat{\theta}$, to be 1.5. You have also computed an estimate of the standard error to be 2. Using this information compute an estimate of the variance of the quantity $\log(\hat{\theta})$. Also, provide an approximate 95% confidence interval for θ .

$$SD = \sqrt{\text{Var}(\hat{\theta})}$$

$$\hat{\theta} \pm 1.96 SD$$

Question 4 (5 marks = 2 + 3)

The figure below contains (rough) plots of the estimated survival curves for males and females after a particular operation.



Based on this figure answer the following questions:

- a) Provide a rough estimate of the median survival time for males. 4.5
- b) Comment on any concerns you might have modeling this data using the Cox regression formulation of the hazard function given in class. (Cox regression)