Example 1

$X \sim beta(a,b)$ $X \in [0,1]$. $p(X) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} x^{a-1}(1-x)^{b-1}$.

$E(X) = \frac{\alpha}{\alpha+\beta}$ $Var(X) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$

$\theta \sim Beta(a=2, b=20)$. $mode(X) = \frac{\alpha-1}{\alpha+\beta-2}$

$p(\theta|y) = \frac{p(\theta,y)}{p(y)} \propto p(\theta,y)$ (only solve up to a proportionality constant).

$= \theta^{a-1}(1-\theta)^{b-1} \theta^y (1-\theta)^{n-y}$

$= \theta^{a+y-1}(1-\theta)^{n-y+b-1}$

$\propto beta(a+y, b+(n-y))$

• more uncertainty in my prior, mode(θ) shifted to left, increased belief that $\theta < 0.10$; data has provided evidence that value of θ is smaller than previously thought.

Exercise 1

$\theta \sim Unif(0.05, 0.20)$. $p(\theta) = \frac{1}{0.15}$. (flat. Beta(1,1) trunc. on interval (0.05, 0.20)).

$p(\theta|y) \propto p(\theta,y) = \theta^y(1-\theta)^{n-y}$ $\propto beta(1+y, 1+n-y)$

$= \theta^{1+y-1}(1-\theta)^{1+n-y}$

proportional to a Beta distribution truncated on this interval $\theta \in [0.05, 0.20]$

undefined when?

wha shrinkage factor: $\frac{a+b}{a+b+n}$ (quantifies shrinkage towards prior mean)

→ can express posterior mean as a lin combin. of prior and sample mean, $\bar{y}/n$ → reliab. when n is small?

→ likelihood

prior assumptions can have an influence on posterior inference

• contour graphs - as w increases, posterior summary becomes more reflective of prior assumption

low prior belief, generally 90% or more certain infect. rate < 10%. High degree of certainty a priori. believe θ_0 ... b/w

→ increasing w means more weight on prior; lines do not go to y=0