Survival Models: Week 12

Exposed to Risk

This week we will talk about how to obtain crude mortality rates.

- Crude mortality rates must be computed before a graduation can take place.

The calculation of mortality rates requires a measure of both the number of deaths and the population which was at risk of dying - the exposed-to-risk - over the same period. In a nutshell, the crude rates are obtained by dividing the number of observed deaths by the relevant exposed to risk. For example:

- $\hat{q}_x = \frac{d_x}{E_x}$
- $\hat{\mu}_x = \frac{d_x}{E_x^c}$

where, d_x is the number of deaths aged x and E_x and E_x^c are, respectively, the initial and central exposed to risk.

$$m_x = \frac{d_x}{\int_0^1 l_{x+t}\,dt} = \frac{q_x}{\int_0^1 {}_tp_x\,dt}$$

$$= \frac{1-e^{-\mu}}{\int_0^1 e^{-\mu t}\,dt} = \frac{1-e^{-\mu}}{\frac{1}{\mu}\left[-e^{-\mu t}\right]_0^1}$$

$$= \frac{\cancel{1-e^{-\mu}}}{\frac{1}{\mu}(\cancel{1-e^{-\mu}})}$$

$$= \mu$$

under constant force-of-mortality,

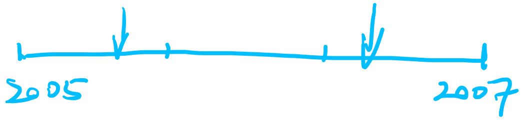
central mortality = force of mortality

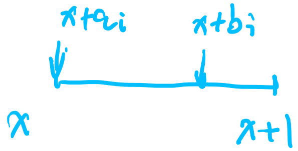
Exposed to Risk

- E_x^c: is the total time that all individuals under study are observed. This is the same as the waiting time.
- E_x: A less natural quantity than E_x^c. E_x requires adjustment for individuals who die while under observation. l_x? Different!

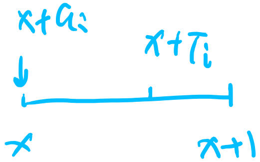
Initial exposed to risk is more complex than central exposed to risk, and is typically more difficult to interpret. Central exposed to risk is more naturally computed from the types of data that are collected in practice (e.g. census data). Details see ALT 2005-2007 page 26-28 and APPENDIX C.

E_x is generally not observed.





individual 1



individual 2

$$\delta = \sum_{i=1}^{n} \delta_i \qquad \mu_x = \frac{\delta}{\nu}$$

$$\nu = \sum_{i=1}^{n} \nu_i$$

$$E_x^c = \nu$$

Exposed to Risk

The following approximation can be used to estimate the initial exposed to risk:

$$E_x \approx E_x^c + 0.5 d_x.$$ no censoring

This approximation assumes that, on average, deaths occur half way through the year.

x $\quad$ $x+0.5$ $\quad$ $x+1$

E_x : total time observed without any deaths

E_x^c : total time observed with deaths

Computation of Exposed to Risk

Exact computation of E_x^c requires knowledge of the following:

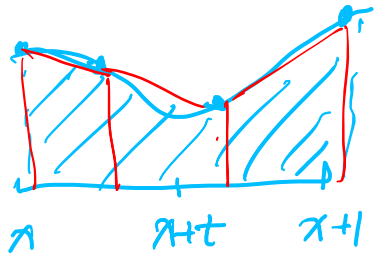
- Exact date of birth.
- Exact date of death.
- Exact date of movement between states (for a multi-state model).

Typically, we do not have the above data (e.g. only have census data) and approximations will be used.

Computation of Exposed to Risk

In many situations we only have information at one time (or a few times) on the number of individuals aged x at time t denoted $P(x,t)$. [Note: If we had $P(x,t)\ \forall\ t$ over some age range then $E_x^c = \int P(x,t)dt$.]

- The Trapezium approximation can be used to estimate the central exposed to risk based on one or more values of $P(x,t)$.
- Area of trapezium is: (sum of sides / 2) * height.



Computation of Exposed to Risk

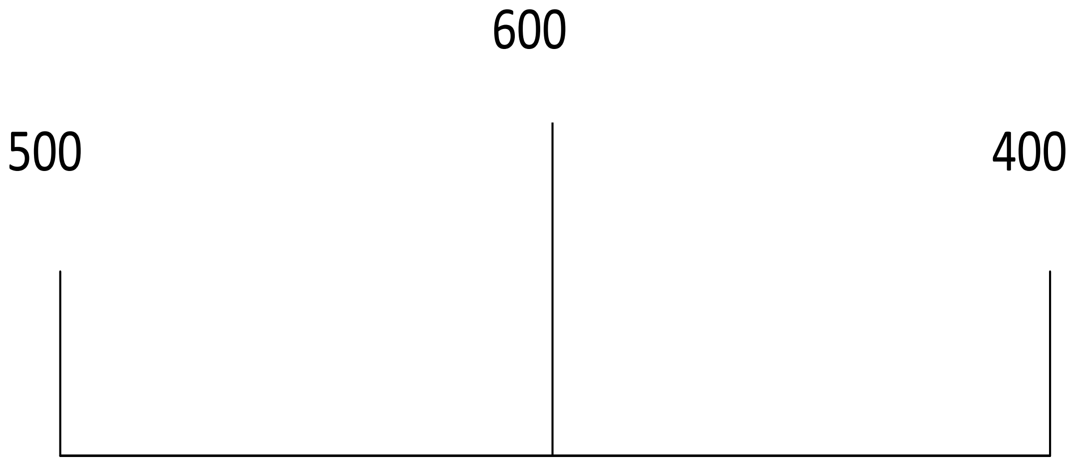
Small study conducted to estimate m_{20} for single men. The following data is collected:

Date	Number aged 20 last b'day
1/1/2008	500
1/1/2009	600
1/1/2010	400

20 deaths were observed over the period of investigation.

Computation of Exposed to Risk

Use of trapezium approximation to estimate E^c_{20}:



$$E^c_{20} = (500 + 600)/2 + (600 + 400)/2 = 1050$$

$$\hat{m}_{20} = 20/1050.$$

Computation of Exposed to Risk

Interested in the mortality experience of elite athletes aged 30. The following data are available from a one year study.

Date	Number aged 30 last b'day
1/1/2008	600
1/6/2008	500
1/9/2008	550
31/12/2008	500

5 deaths were observed over the period of investigation.

5m 3m 4m

1.1 2008 6.1 2008 9.1 2008 12.31 2008

600 500 550 500

$$\tilde{E}^{c}_{30} = \frac{(600+500)\times\frac{5}{12}}{2} + \cdots$$

Computation of Exposed to Risk

Use trapezium approximation to estimate E_{30}^c:

$$E_{30}^c = 5/12(600 + 500)/2 + 3/12(500 + 550)/2 + 4/12(550 + 500)/2$$

$$m_{30} = 5/535$$

Note: the length of each interval is not 1 year!

Rate Intervals

- These are intervals of time over which an individual's age remains the same.
- If an individual dies in the rate interval for age x their death contributes to d_x.

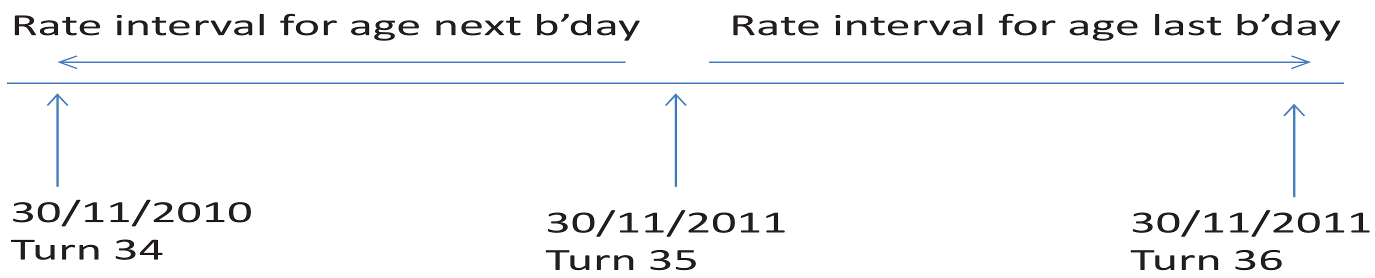
Life Year Rate Intervals

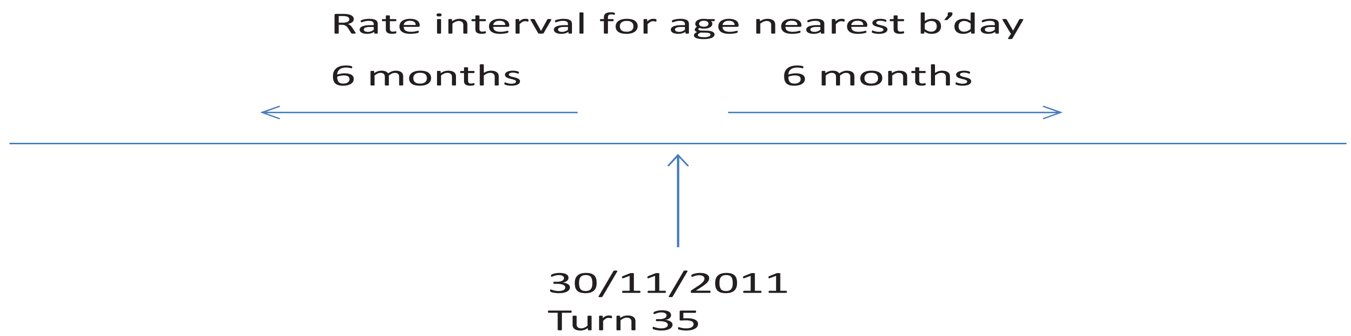
Some possible definitions of "age x" are:

- Age x last birthday.
- Age x next birthday.
- Age x nearest birthday.

Life Year Rate Intervals

Consider the situation of a person born on 30/11/1976. We will focus on being age 35.





Rate Intervals

Rate intervals are important for determining exactly what our crude rates of mortality are estimating.

initial rate of mortality

- The "x" in $\hat{q}_x$ corresponds to the age at the start of the rate interval.
- The "x" in $\hat{\mu}_x$ corresponds to the age at the middle of the rate interval.

central rate of mortality

Example

We have initial exposed to risk data E_x and mortality data d_x relating to age 35. Consider, again, the three life year rate intervals and what they mean for our estimate of the initial rate of mortality: d_x/E_x

- age next birthday: we estimate q_{34}
- age last birthday: we estimate q_{35}
- age nearest birthday: we estimate $q_{34.5}$

Interpret. How age is defined is important!

Principle of Correspondence

- Age definition of exposed to risk must match the age definition used for classifying deaths.
- An individuals whose death would be included in d_x must currently be contributing to E_x^c
- If definitions do not match E_x^c needs to be adjusted accordingly.

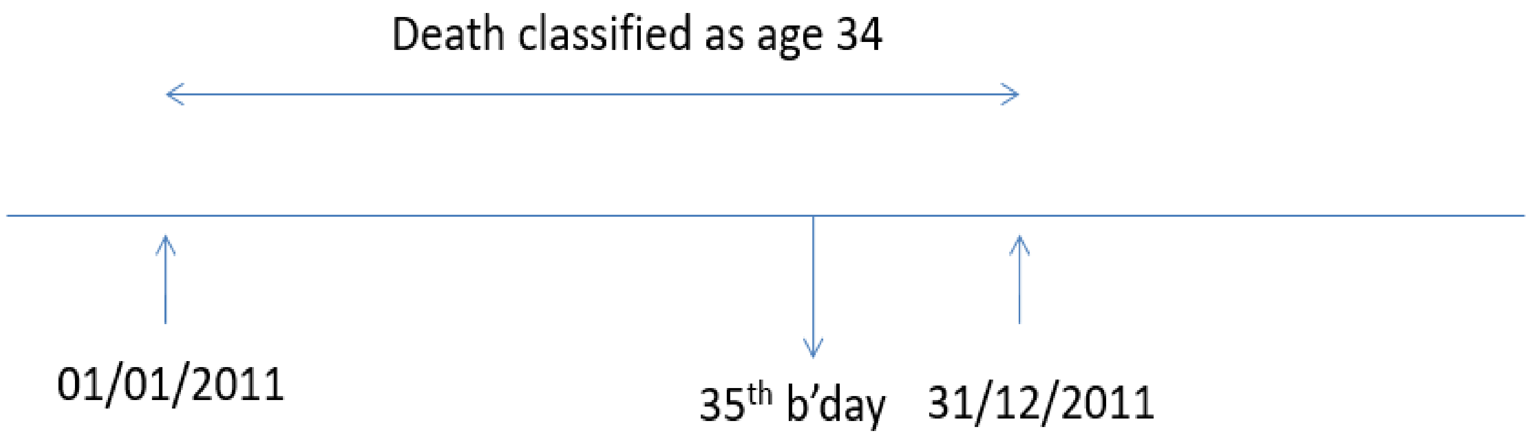
Example

Final exam 2014 Question 7

Calendar year Rate Intervals

For calendar year RIs the rate interval runs for a calendar year (and is the same for all individuals). **Not assessable**.

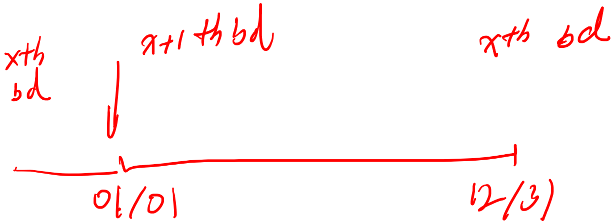
- Example: Age last birthday at 1 January proceeding death. [Consider a person born on 30/11/1976.]



Calendar year Rate Intervals

What does d_x/E_x estimate?

- Rate interval runs from 01/01 to 31/12.
- Ages at start of rate interval range from x (person just had x^{th} bday) to $x+1$ (person will had x^{th} bday one year ago).
- Average age at start of rate interval is $x + 0.5$ - assuming bdays uniformly spread over the year.
- In this situation we estimate $q_{x+0.5}$.



x

$x+1$

$\frac{x+x+1}{2} = x+0.5$

Policy year Rate Intervals

Age is determined by the date at which an insurance policy was taken out. **Not assessable**.

Policy duration is one type of rate interval, and measures the length of time since the individual took out the policy. Integer policy durations are called policy anniversaries. If a policy was purchased on Dec 1, 1999 the second policy anniversary would be on Dec 1, 2001.