CSC236 2015 Winter, Assignment 1

Due Monday February 2nd, 6 p.m.

Notice: The due date for this assignment has been postponed to Monday February 2nd at 6PM.

You may work in groups of up to three people currently enrolled in CSC236.

Submit your solutions as a PDF file a1.pdf via MarkUs (which will accept submissions starting in the evening of Monday January 26th).

Your file must be produced using a word processor or editor that exports files in PDF format (no scanned handwriting accepted).

Late assignments have a deduction of 5% per hour, for up to 20 hours.

You will receive 20% of the marks for any question (or part of a question) that you either leave blank or for which you write "I cannot answer this."

1. Prove by Induction that $1 + mn \leq (1+m)^n$ for all natural numbers m and n.

2. Let the sequence r be defined by:

$$\begin{aligned} r_1 &= 1, \\ r_n &= 1 + r_{\lfloor \sqrt{n} \rfloor}, n \geq 2. \end{aligned}$$

   Prove by Induction that r_n is $O\left(\log_2\left(\log_2 n\right)\right)$.

3. Consider the number of binary trees of height h, where we measure height by number of levels. For example, the empty tree is the only tree of height 0, a single-node tree is the only tree of height 1, and there are 3 trees of height 2.

   (a) Give a recursive algebraic formula for a sequence b, and prove for all natural numbers h that b_h is the number of binary trees of height h.

   (b) Let the sequence a be defined by:

$$\begin{aligned} a_0 &= 0, \\ a_{n+1} &= a_n^2 + 1, n \in \mathbb{N}. \end{aligned}$$

   Prove that $b_{h+1} = a_{h+1}^2 - a_h^2$ for all natural numbers h.