Lecture 1

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ES4146 Tu14, Th13

Txt: A first course in Chaotic Dynamical Systems. by Robert L. Devaney.

6 assignments. best 5 of 6 ⇒ average.

1 Term test. TBA (50 min)

Final (3 h)

Marking Scheme: 25% + 25% + 50%

No Tut.

CHAPTER 3 ORBITS

§3.1 ITERATION

Def: For a function $F(x)$, we define

· its second iterate $F^2(x) = F(F(x)) = F \circ F(x)$

· its third iterate $F^3(x) = F(F(F(x))) = F \circ F \circ F(x) = F(F^2(x))$

...

The nth iterate $F^n(x) = \underbrace{F \circ F \circ \cdots \circ F}_{n \text{ times}}(x) = F \circ F^{n-1}(x)$

Attention: The nth iterate $F^n(x)$ is not $F(x)$ raise to the nth power.

Example: $F(x) = x^2$

$F^2(x) = (x^2)^2 = x^4$

$F^3(x) = (x^4)^2 = x^8$

...

$F^n(x) = x^{2^n}$

Example: $F(x) = x^2 + 1$

$F^2(x) = (x^2+1)^2 + 1 = x^4 + 2x^2 + 2$

$F^3(x) = (x^4+2x^2+2)^2 + 1 = x^8 + 4x^6 + 8x^4 + 8x^2 + 5$

§3.2 OBRBITS

· def: Given $x_0 \in \mathbb{R}$, we define the orbit of x_0 under F to be the sequence of points:

$x_0,\ x_1 = F(x_0),\ x_2 = F^2(x_0) = F(x_1),\ x_3 = F^3(x_0) = F(x_2) \cdots$

And the point x_0 is called the seed of the orbit.

Ex: Let $F(x)=x^2+1$, and $x_0=1$

Then

$x_0=1$
$x_1=2$
$x_2=5$
$x_3=26$
$x_4=677$
$\cdots$

Ex: Let $F(x)=\cos x$ and $x_0=1$

§3.3 Types of orbits

fixed points (most important type of orbit)

def: a fixed pt x_0 is a pt which satisfies $F(x_0)=x_0$

So the orbit of a fixed point x_0 is x_0, $x_1=F(x_0)=x_0$, $x_2=F(x_1)=F(x_0)=x_0$

Ex: ~~For~~ $F(x)=x^2$

The fixed pt satisfy $F(x)=x$

$$x^2=x$$
$$x=0 \text{ or } x=1$$

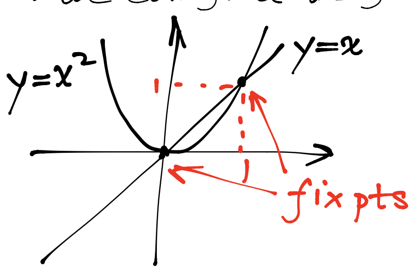
$F(x)=x^2$ has 2 fixed pts 0 & 1.

Ex: $F(x)=x^2-x-8$

The fixed pts satisfy

$$x^2-x-8=x$$
$$x^2-2x-8=0$$
$$x=4 \text{ or } x=-2$$

$\rightarrow$ we can find the fixed pts of a function $F(x)$ graphically.



For $F(x)=\cos x$