§1.1 - modelling (problem set 2)
- "standard" and "canonical" form } today
§1.2 - matrix notation
§1.3 - geometry (convexity)

For the general linear programming problem (defn), see page 51.

Def'n: A general linear programming problem is in standard form provided it is a Maximization problem with $\leq$ constraints only, except all decision variables are constrained to be ≥ 0 (That is, the constraints $x_i \geq 0$ are present.

A general problem is in canonical form provided it is a Maximization problem with equality constraints only, except $x_i \geq 0$ for any decision variable x_i.

Remark: In linear programming, the word "standard" and "canonical" are not universally defined. Their definitions depend entirely on the book or paper you r reading. We are using the Kolman & Beck definitions. "Standard" will ~~at~~ be also refered to as "Primal Standard".

There are 5 techniques for putting a general problem in either form.

① To change a minimization problem to a maximization problem. (or vice versa) multiply z by -1

"Maximize $z = c_1x_1 + \cdots + c_nx_n + \cdots$" $\longrightarrow$ "Minimize $z' = -c_1x_1 - \cdots - c_nx_n - \cdots$"

This leaves the solution $(x_1, \cdots, x_n)$ unchanged.

It does change the sign of z.

② To change a $\geq$ constraint to a $\leq$ constraint (or vice versa), multiply by -1.

This will not change the solution set of the constraint.

Instead of "$a_1x_1 + \cdots + a_nx_n \geq b$", write "$-a_1x_1 - \cdots - a_nx_n \leq -b$"

③ (To get standard form) we may replace the equality

"$a_1x_1 + \cdots + a_nx_n = b$" $\Longrightarrow$ "$a_1x_1 + \cdots + a_nx_n \leq b$"

"$a_1x_1 + \cdots + a_nx_n \geq b$" (intermediate step)

then with "$a_1x_1 + \cdots + a_nx_n \leq b$

$-a_1x_1 - \cdots - a_nx_n \leq -b$" (technique 2)

④ If a variable (x, say) is unrestricted (that is, "$x \geq 0$" is absent), one introduces 2 new variables, x^+ and x^-, then substitute

$x = x^+ - x^-$, while including the constraints "$x^+ \geq 0$" and "$x^- \geq 0$".

This leads to a problem with more decision variables than the original problem. This is an equivalent problem because a solution (with x^+ and x^-) of the equivalent problem will lead to a solution of the original problem (recalling $x = x^+ - x^-$).

⑤ (See § 1.2 . page 65).

(To get canonical form), one may change the inequality "$a_1x_1 + \cdots a_nx_n \leq b$" to an equality, introduce another variable (a slack variable), x_{n+1}, which is $b - (a_1x_1 + \cdots + a_nx_n)$, and including "$x_{n+1} \geq 0$"

The constraint becomes

"$a_1x_1 + \cdots + a_nx_n + x_{n+1} = b$" (an equality)

"$x_{n+1} \geq 0$"