March 4th

Test:

① Show if $n>m$ and we have n objects in m holes then one hole has more than one object

Hint: Induct on m.

WRONG:

Base: Consider $n=1$

Since $n>m$ we have $n\geq 2$

Thus 2-objects in one hole.

Sps the claim holds for $m\geq 1$

we have $n>m$

$\Rightarrow n-m\geq 1$

So done!

The # of objects in hole $(m+1)$ is $<2 \Rightarrow =0,1$

Good: Place one object into the $(m+1)$th hole

Now, note: we can't put another item in the $(m+1)$th hole.

We now have $n>m$ objects and m holes.

So by induction $\square$

Compute

$2^{(p!)^2} \bmod p, \ p>1$

$p>1 \Rightarrow \gcd(2,p)=1$, thus Fermat $2^{(p!)^2} \equiv (2^{p-1})^k \equiv \cdots$

$p=2 \Rightarrow \gcd(2,2)=2, \quad 2^{(2!)^2} \equiv 2^4 \equiv 16 \bmod 2 \equiv 0 \bmod 2$

$\boxed{\text{HW 6}}$

#8 $Z = \dfrac{(1+3i)^{150}}{(2+2i)^{500}(3+4i)^{75}}$

$a+bi = Re^{i\theta}$

e.g. $2+2i = \sqrt{8}\, e^{i\frac{\pi}{4}}$

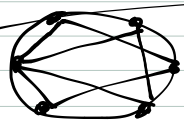
...

Let $n^2 = a^2+b^2$ $\quad a^2-b^2=(a-b)(a+b)$ $\quad a,b,c,d \in \mathbb{Z}$

$m^2 = c^2+d^2$ $\quad a^2+b^2=(a+bi)(a-bi)$

$$nm = (a+bi)(a-bi)(c+di)(c-di)$$
$$= [ac+adi+bci-bd](a-bi)(c-di)$$
$$= [(ac-bd)+(ad+bc)i][(ac-bd)-(ad+bc)i]$$

$$=(ac-bd)^2 - [(bc+ad)i]^2$$
$$=(ac-bd)^2 + (bc+ad)^2$$



n-lights

Goal: Turn on all lights
Moreover, for $d|n$ change state of each d^n light

Consider the set $\mathcal{L}$ as vertices of regular n-gon

$$\mathcal{L} = \{z : z^n - 1 = 0\}$$

Consider the sum of the active lights

If we change the state of the d^{th} lights
we add or substraet
ω . . - - ???

Prove $\sqrt{3}+\sqrt{2}$ is irrational

Proof: $x = \sqrt{3}+\sqrt{2}$

$x^2 = 3+2\sqrt{6}+2$

$(x^2-5)^2 = 24$

$x^4 - 10x^2 + 25 - 24 = 0$

$x^4 - 10x^2 + 1 = 0$

Argue has no rational roots