STA437/2005 Methods for Multivariate Data
Practice Problem set #1

Problem 1. Consider the following seven observations of three variables:

V1	V2	V3
5.1	3.3	1.7
6.8	2.8	4.8
5.8	2.7	3.9
6.9	3.1	4.9
5.7	2.5	5.0
5.8	2.8	5.1
6.4	3.2	4.5

(a) Compute the sample mean
(b) Compute the sample variance
(c) Make z-transformation on V3 and investigate whether or not there are any anormalous observations.
(d) Compute the χ^2-statistic of the first observation and determine whether or not it is anormalously big.
(e) Assess a hypothesis $H_0 : \mu = 0$.
(d) Assess whether or not V3 is normally distribute.

Problem 2. $X = (X_1, X_2, X_3)^\top \sim N_3(\mu, \Sigma)$ where $\mu = (1, 2, 3)^\top$ and $\Sigma = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{pmatrix}$
(a) Find the distribution of $2X_1 - 3X_2 + X_3$.
(b) Solve a and b so that X_1 and $aX_2 + bX_3$ are independent.
(c) Solve a and b so that X_1 and $aX_2 + bX_3$ are independent as well as X_2 and $aX_1 + bX_3$ are independent.
(d) Find the conditional distribution of X_3 given (X_1, X_2)

Problem 3. A set of two dimensional data is observed which is assumed to be $\mathbf{x}_j \sim N_2(\mu, \Sigma)$. Sample statistics are given by $n = 40$, $\bar{\mathbf{x}} = (0.7, 0.4)^\top$ and $S = \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$.
(a) Write the equation of 95% confidence region for μ and draw it roughly.
(b) Assess a hypothesis $H_0 : \mu = (1, 0)^\top$.
(c) Assess a hypothesis $H_0 : \mu_1 = 2\mu_2$.
(d) Find a 95% confidence interval for μ_2.
(e) Assess whether or not X_1 and X_2 are independent.

Problem 4. A $n \times p$ random matrix $\mathbf{X}$ and a random vector $Y \in \mathbb{R}^p$ has a relationship $Y = \beta^\top \mathbf{X} + \boldsymbol{\epsilon}$ where $\beta \in \mathbb{R}^p$ is an unknown parameter, $\boldsymbol{\epsilon} \sim N_n(O, I_n)$ and $\mathbf{X}$ and $\boldsymbol{\epsilon}$ are independent.
(a) Find the least square estimator $\hat{\beta}$ of β.
(b) Show the consistency of $\hat{\beta}$.
(c) Show the asymptotic normality of $\hat{\beta}$.
(d) Assess a hypothesis $H_0 : \beta = O$.
(e) Write the required assumptions for (a)-(d).

Problem 5. (a) Describe a procedure of detecting outliers.
(b) Describe a procedure of assessing normality.
(c) Describe Box-Cox transformation.