STAT8027: Tutorial #0

23 Feb, 2018

Rui Qiu u6139152

Problem 3

(a) Solution:

Since $x > 0, y = x^2 > 0$

$$
\begin{aligned}
F_Y(y) &= P(Y \leq y) = P(x^2 \leq y) = P(0 \leq x \leq \sqrt{y}) = F_x(\sqrt{y}) \\
f_Y(y) &= F'_Y(y) \\
&= F'_X(\sqrt{y}) \cdot \frac{1}{2} y^{-\frac{1}{2}} \\
&= f_X(\sqrt{y}) \cdot \frac{1}{2} y^{-\frac{1}{2}} \\
&= \sqrt{\frac{2}{\pi}} \exp\left(-\frac{1}{2}y\right) \frac{1}{2} \frac{1}{\sqrt{y}}, y > 0 \\
&= \frac{1}{\sqrt{2y} \cdot \sqrt{\pi}} \exp\left(-\frac{1}{2}y\right)
\end{aligned}
$$

(b) Solution:

As $\Gamma(\frac{1}{2}) = \sqrt{\pi}$, the PDF of Y can be written as:

$$
\frac{1}{\sqrt{2y}} \frac{1}{\Gamma(\frac{1}{2})} \exp\left(-\frac{1}{2}y\right) = \frac{y^{-\frac{1}{2}} \frac{1}{2}^{\frac{1}{2}} e^{-\frac{1}{2}y}}{\Gamma(\pi)}
$$

So $Y \sim \text{Gamma}(\alpha = \frac{1}{2}, \lambda = \frac{1}{2})$.

Problem 4

Proof:

Suppose $A = \{(u_1, u_2) : g_1(u_1, u_2) \leq y_1, g_2(u_1, u_2) \leq y_2\}, A_h = \{(v_1, v_2) : v_1 \leq y_1, v_2 \leq y_2\}$. And by definition, we have

$$
\begin{aligned}
v_1 = g_1(u_1, u_2), v_2 = g_2(u_1, u_2) \\
u_1 = h_1(v_1, v_2), u_2 = h_2(v_1, v_2)
\end{aligned}
$$

The CDF of joint distribution of Y_1, Y_2 can be written as:

Problem 4 continued on next page. . .

$$
\begin{aligned}
F_{Y_1Y_2}(y_1, y_2) = P(A_h) &= P(u_1 \leq h_1(y_1, y_2), u_2 \leq h_2(y_1, y_2)) \\
&= F_{X_1X_2}(h_1(y_1, y_2), h_2(y_1, y_2)) \\
&= \int\int_A f_{X_1X_2}(u_1, u_2) du_1 du_2 \\
&= \int\int_{A_h} f_{X_1X_2}(h_1(v_1, v_2), h_2(v_1, v_2)) \mid J(v_1, v_2) \mid dv_1 dv_2
\end{aligned}
$$

Therefore, the PDF of joint distribution of Y_1, Y_2

$$
f_{Y_1Y_2}(y_1, y_2) = f_{X_1X_2}(h_1(y_1, y_2), h_2(y_1, y_2)) \mid J(y_1, y_2) \mid
$$

where $J(y_1, y_2)$is the Jacobian matrix.

■