PLEASE HAND IN

UNIVERSITY OF TORONTO
Faculty of Arts and Science

EXAMINATION APRIL 2011

PHL 245 H1S
L0101 - Niko Scharer

Duration - 3 hours

Examination Aid: Sheet with rules (provided)

Last Name ______________________________

First Name ______________________________

Student Number___________________________

Answer **all** questions on the exam paper.

Use the supplied examination booklet for rough work OR if you need further space.

The exam consists of fourteen (14) pages. Pages 2-12 have questions on them.

The final two pages (13-14) are an aid sheet and may be detached from the rest of the exam.

1. An argument consists of two premises and a conclusion. On every interpretation that the conclusion is false, so are the premises. Does it follow that the argument is valid? Briefly explain. (3%)

2. Consider the following truth-table for the NEW symbol: ✳ (4%)

P	Q	P ✳ Q
T	T	F
T	F	T
F	T	T
F	F	F

a) What ordinary English expression can this new truth-functional connective (✳) be used to symbolize (given its truth-table)?

b) Using the new symbol, ✳, as defined by the truth-table above, symbolize: $P \vee Q$. In addition to ✳, you may also use $\sim$ and $\wedge$ (negation and conjunction), if you need them. In other words, symbolize $P \vee Q$ using only: P, Q, ✳, $\sim$, $\wedge$.

3. Provide an English language interpretation that shows that the following set of sentences is consistent. Your interpretation should specify the universe of discourse and a symbolization scheme. (4%)

$\forall x \exists y(Ax \to L(xy))$. $\exists x(Ax \wedge \forall y(\sim Ay \to L(xy)))$. $\sim\forall x(Bx \to \exists y L(yx))$

4. Explain why the following sentence is a tautology. (4 %)

$$(\forall y Fy \to \exists x \sim Gx) \to \sim\forall y \forall x(Fy \wedge Gx)$$

5. Use this symbolization scheme to symbolize the following sentences: (36 % = 9 × 4%)

A^1: *a* is a store.	B^1: *a* is a barber.	C^1: *a* gets a haircut.	D^1: *a* is a day.
E^1: *a* is bald.	F^1: *a* is a person	G^1: a is prosperous.	
H^2: *a* cuts *b*'s hair.	K^2: *a* shaves *b*.	L^2: *a* likes *b*.	
M^2: *a* is more successful than *b*.		O^3: *a* shops at *b* on *c*.	
a^0: Anna	b^0: Ben	g^0: Gus	c^1: the cousin of *a*.

a) Everybody gets a haircut only provided that nobody is bald.

b) No barber shaves exactly those people who do not shave themselves.

c) On the assumption that some people shave themselves if anybody shaves them, for a barber to be prosperous it is necessary that people like him.

d) Any barber who just shaves people he likes is not prosperous unless he cuts people's hair.

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5 continued. Use this symbolization scheme to symbolize the following sentences: (36 % = 9 × 4%)

A^1: *a* is a store.	B^1: *a* is a barber.	C^1: *a* gets a haircut.	D^1: *a* is a day.
E^1: *a* is bald.	F^1: *a* is a person	G^1: a is prosperous.	
H^2: *a* cuts *b*'s hair.	K^2: *a* shaves *b*.	L^2: *a* likes *b*.	
M^2: *a* is more successful than *b*.		O^3: *a* shops at *b* on *c*.	
a^0: Anna	b^0: Ben	g^0: Gus	c^1: the cousin of *a*.

e) If not all stores have people shopping there every day, then it's not the case that everyone likes the same store.

f) Exactly one barber cuts Anna's hair, and he is neither Gus nor Gus's cousin, Ben.

g) Only Anna's cousin likes only stores that Anna likes.

5 continued. (36 % = 9 × 4%)

A^1: *a* is a store. B^1: *a* is a barber. C^1: *a* gets a haircut. D^1: *a* is a day.
E^1: *a* is bald. F^1: *a* is a person G^1: a is prosperous.
H^2: *a* cuts *b*'s hair. K^2: *a* shaves *b*. L^2: *a* likes *b*.
M^2: *a* is more successful than *b*. O^3: *a* shops at *b* on *c*.
a^0: Anna b^0: Ben g^0: Gus c^1: the cousin of *a*.

g) Using the symbolization scheme above, provide an idiomatic English sentence that expresses:

$$\exists x(Bx \wedge \forall y(By \wedge x \neq y \rightarrow M(xy)) \ \wedge \forall z(Fz \wedge H(xz) \rightarrow L(zx)))$$

h) Using the symbolization scheme above, symbolize the following ambiguous sentence **three** logically distinct ways. For each, provide an English sentence that explains exactly what the symbolized sentence means:

A person shops at a store every day.

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6. Provide a derivation that shows the following theorem is valid **using only the 10 basic rules from SL** (R, DN, MP, MT, ADJ, S, ADD, MTP, BC, CB) **and the 3 basic rules from PL** (UI, EG, EI) (9%)

$\therefore$ $(\exists x{\sim}(Ax \vee Gx) \wedge \forall y{\sim}(By \rightarrow Hy)) \rightarrow \exists x(Hx \leftrightarrow Ax)$

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7. Provide a derivation that shows that this is a valid argument **using only the 10 basic rules from SL** (R, DN, MP, MT, ADJ, S, ADD, MTP, BC, CB) **and the 3 basic rules from PL** (UI, EG, EI) (9%)

$\forall x(Fx \rightarrow Hx)$. $\quad \exists x(Fx \wedge \forall yB(xy))$. $\quad \exists xHx \rightarrow \forall y\forall z(Ay \wedge B(zz) \rightarrow G(yz))$.

$\therefore \forall x(Ax \rightarrow \exists y(Hy \wedge G(xy)))$

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8. Provide a derivation to show that this is a valid argument (use any rules). (9 %):

$\forall z(\exists w F(wz) \rightarrow B(zz))$. $\forall i \forall k(F(ik) \rightarrow \sim L(ki)) \rightarrow \forall x \exists y \sim(Ax \rightarrow Gy)$.

$\therefore \forall x(B(xx) \rightarrow \sim\exists y L(xy)) \rightarrow \sim\forall z(\sim Az \vee Gz)$

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9. Show that the following is a valid argument (use any rules). (9%):

$\exists x \forall y(H(xyy) \rightarrow \forall z{\sim}F(xz))$. $\qquad {\sim}\forall x Kx \rightarrow \forall x \exists y \forall z H(xyz)$.

$\exists y{\sim}F(yy) \rightarrow \exists x \forall y G(b(x)b(y))$. $\qquad \therefore\ {\sim}\exists x G(xb(x)) \rightarrow Ka$

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10. Use a finite model to demonstrate the invalidity of this argument (8 %):

    i) provide a truth-functional expansion (to two individuals) for each sentence in this argument with respect to the model

    ii) define a model with a universe of two individuals that shows that this argument is invalid.

$\exists x \forall y(Fx \wedge L(xy))$. $\quad\quad \forall x(Gx \rightarrow \exists y{\sim}L(xy))$. $\quad\quad \therefore\ {\sim}\exists x(L(xx) \wedge Gx)$

11. Is the material conditional a necessary logical connective in our system? Would we be able to symbolize the same English sentences without the material conditional? Would we be able to derive the same theorems without the material condition?

Justify your answer with an explanation that considers the role of the material conditional in both symbolization and derivations. (5 %)

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AID SHEET: DERIVATION RULES (SEE BOTH SIDES)

Derivation Types:

Direct Derivation (DD)

Conditional Derivation (CD)

Indirect Derivation (ID)

Universal Derivation (UD) Restriction: the instantiating term cannot occur unbound in any previous line.

Basic Rules for Sentential Operators:

Modus Ponens (MP)

$$\frac{\begin{array}{l}(\phi \to \psi)\\ \phi\end{array}}{\psi}$$

Modus Tollens (MT)

$$\frac{\begin{array}{l}(\phi \to \psi)\\ \sim\psi\end{array}}{\sim\phi}$$

Double Negation (DN)

$$\frac{\phi}{\sim\sim\phi} \qquad \frac{\sim\sim\phi}{\phi}$$

Repetition (R)

$$\frac{\phi}{\phi}$$

Simplification (S)

$$\frac{\phi \wedge \psi}{\phi} \qquad \frac{\phi \wedge \psi}{\psi}$$

Adjunction (ADJ)

$$\frac{\begin{array}{l}\phi\\ \psi\end{array}}{\phi \wedge \psi}$$

Addition (ADD)

$$\frac{\phi}{\phi \vee \psi} \qquad \frac{\psi}{\phi \vee \psi}$$

Modus Tollendo Ponens (MTP)

$$\frac{\begin{array}{l}\phi \vee \psi\\ \sim\phi\end{array}}{\psi} \qquad \frac{\begin{array}{l}\phi \vee \psi\\ \sim\psi\end{array}}{\phi}$$

Biconditional-Conditional (BC)

$$\frac{\phi \leftrightarrow \psi}{\phi \to \psi} \qquad \frac{\phi \leftrightarrow \psi}{\psi \to \phi}$$

Conditional-Biconditional (CB)

$$\frac{\begin{array}{l}\phi \to \psi\\ \psi \to \phi\end{array}}{\phi \leftrightarrow \psi}$$

Derived Rules for Sentential Operators:

Negation of Conditional (NC)

$$\frac{\sim(\phi \to \psi)}{\phi \wedge \sim\psi} \qquad \frac{\phi \wedge \sim\psi}{\sim(\phi \to \psi)}$$

Conditional as Disjunction (CDJ)

$$\frac{\phi \to \psi}{\sim\phi \vee \psi} \qquad \frac{\sim\phi \vee \psi}{\phi \to \psi} \qquad \frac{\sim\phi \to \psi}{\phi \vee \psi} \qquad \frac{\phi \vee \psi}{\sim\phi \to \psi}$$

Separation of Cases (SC)

$$\frac{\begin{array}{l}\phi \vee \psi \\ \phi \to \chi \\ \psi \to \chi\end{array}}{\chi} \qquad \frac{\begin{array}{l}\phi \to \chi \\ \sim\phi \to \chi\end{array}}{\chi}$$

Negation of Biconditional (NB)

$$\frac{\sim(\phi \leftrightarrow \psi)}{\phi \leftrightarrow \sim\psi} \qquad \frac{\phi \leftrightarrow \sim\psi}{\sim(\phi \leftrightarrow \psi)}$$

De Morgan's (DM)

$$\frac{\sim(\phi \vee \psi)}{\sim\phi \wedge \sim\psi} \qquad \frac{\sim\phi \wedge \sim\psi}{\sim(\phi \vee \psi)} \qquad \frac{\sim(\phi \wedge \psi)}{\sim\phi \vee \sim\psi} \qquad \frac{\sim\phi \vee \sim\psi}{\sim(\phi \wedge \psi)}$$

$$\frac{\sim(\sim\phi \vee \sim\psi)}{\phi \wedge \psi} \qquad \frac{\phi \wedge \psi}{\sim(\sim\phi \vee \sim\psi)} \qquad \frac{\sim(\sim\phi \wedge \sim\psi)}{\phi \vee \psi} \qquad \frac{\phi \vee \psi}{\sim(\sim\phi \wedge \sim\psi)}$$

Derivation Rules for Predicate Logic:

Existential Generalization (EG)

$$\frac{\phi_\zeta}{\exists\alpha\phi_\alpha}$$

Universal Instantiation (UI)

$$\frac{\forall\alpha\phi_\alpha}{\phi_\zeta}$$

Restriction: ζ does not occur as a bound variable in ϕ_α

Existential Instantiation (EI)

$$\frac{\exists\alpha\phi_\alpha}{\phi_\zeta}$$

Restriction: ζ does not occur in any previous line or premise.

Quantifier Negation (QN)

$$\frac{\sim\forall\alpha\phi}{\exists\alpha\sim\phi} \qquad \frac{\sim\exists\alpha\,\phi}{\forall\alpha\sim\phi}$$

$$\frac{\exists\alpha\sim\phi}{\sim\forall\alpha\phi} \qquad \frac{\forall\alpha\sim\phi}{\sim\exists\alpha\,\phi}$$