Question (taken from previous final exam)

A smooth of a set of points in two-dimensional space is required. You decide to use kernel smoothing with a kernel of the form $K(t) = e^{-2|t|}$ for all t.

(a) Using the kernel you specified in (a) and the following data, determine the smooth at $x = 6$. Use a bandwidth of 1.

x	y
3	1
5	2
7	4
8	6
9	12

(b) As an alternative method of smoothing you decide to use local averaging with a window width of 4. That is, when determining the smooth at $x = 6$ you average all y-values falling between $x = 4$ and $x = 8$. Specify a kernel function and bandwidth that would lead to the same smoothed values as the local averaging method described here.

Solution:

x	y	$\frac{x_i - x_j}{b}$	$K\left(\frac{x_i - x_j}{b}\right)$
3	1	3	$2.4788*10^{-3}$
5	2	1	0.13534
7	4	-1	0.13534
8	6	-2	0.01832
9	12	-3	$2.4788*10^{-3}$

Sum of kernel values in final column of the above table is 0.29394.

Hence smooth at $x = 6$ is

$$\frac{1}{0.29394}\left[2.4788*10^{-3} + 2(0.1354) + 4(0.1354) + 6(0.1832) + 12(2.4788*10^{-3})\right]$$
$$= 3.2460$$

(a) Any uniform kernel that covers the range $\left[-\frac{2}{b}, \frac{2}{b}\right]$ where any bandwidth parameter, b, is possible.