University of Toronto

Faculty of Arts and Sciences

Sample Final Exam, April-May 2014

MAT 337 H1

Intro Real Analysis

Instructor: Regina Rotman

Duration - 3 hours

No aids allowed

Total marks for this paper is 400

Please write your name in the space provided as well as on the Blue Book

Student Number: ______________________________

Last Name: ______________________________

Given Name: ______________________________

FOR MARKER ONLY	
Question	Mark
1	
2	
3	
4	
5	
6	
TOTAL	

[90] **Problem 1.**

Is there

[10] (a) a function that is uniformly continuous on the interval $[0,1]$, but is not Lipschitz there,

[10] (b) a function that is Lipschitz on the interval $[0,\infty)$, but is not uniformly continuous there,

[10] (c) a differentiable function whose derivative is bounded on the interval $[0,1]$, but the function is not Lipschitz on $[0,1]$,

[10] (d) a function that is continuous on $[0,1]$, but does not attain its minimum value on $[0,1]$,

[10] (e) a function that is continuous on $\mathbb{R}$, but is nowhere differentiable.

[10] (f) a function f that is defined on $[0,1]$, not continuous at any point of $[0,1]$, but f^2 is continuous at every point of $[0,1]$,

[10] (g) a function that is defined on $[0,1]$ and is continuous only at the irrational numbers of $[0,1]$,

[10] (h) a nonconstant continuous function $f:\mathbb{R}\longrightarrow\mathbb{R}$, that has only irrational numbers in its range,

[10] (i) a continuous function $f:\mathbb{R}\longrightarrow\mathbb{R}^n$ such that $\lim_{n\to\infty} f(\frac{1}{n}) \neq f(0)$,

You may explain your answers either by stating the relevant theorem or by giving an example, when it exists, but you do not have to do it to get a full credit for a correct answer.

[70] **Problem 2.**

[35] (a) A normed vector space V is strictly convex if $||u|| = ||v|| = ||\frac{(u+v)}{2}|| = 1$ for vectors u,v implies that $u=v$. Show that an inner product space is always strictly convex.

[35] (b) Let K be a compact subset of $\mathbb{R}^n$. Let $C(K)$ denote the vector space of all continuous functions on K . For $f\in C(K)$, denote $||f||_\infty = \sup_{x\in K}|f(x)|$. Show that this is a norm on $C(K)$.

[70] **Problem 3.** Prove that a compact subset of a normed vector space is closed and bounded.

[70] **Problem 4.** Prove that an inner product space V satisfies the triangle inequality $||x+y|| \leq ||x|| + ||y||$ for all $x,y\in V$.

[50] **Problem 5.**

Prove that the series $f(x) = \Sigma_{n=1}^{\infty} \frac{x}{n(1+nx^2)}$ converges uniformly on $\mathbb{R}$.

[50] **Problem 6.**

Find the Fourier series for $\sin^3\theta$ on $[-\pi,\pi]$.