PS #6.

2. (a). $\dfrac{\binom{p}{n_1}\binom{q}{n_2}\binom{r}{n_3}}{\binom{N}{n}}$, $N=p+q+r$

(b). $\dfrac{\binom{p}{n_1}\binom{q}{n_2}\binom{r}{n-n_1-n_2}}{\binom{N}{n}}$

(c). $\dfrac{\binom{q}{n_2}\binom{p+r}{n-n_2}}{\binom{N}{n}}$

8. a). $\frac{1}{2}$. $\frac{3}{14}$; $\frac{2}{7}$

b). $f_X(x)=\frac{2}{7}+\frac{6}{7}x+\frac{6}{7}x^2$

$f_Y(y)=\frac{2}{7}+\frac{6}{7}y+\frac{6}{7}y^2$

c). $f_{X|Y}(x|y)=\dfrac{\frac{6}{7}(x+y)^2}{\frac{2}{7}+\frac{6}{7}y+\frac{6}{7}y^2}$

$f_{Y|X}(y|x)=\dfrac{\frac{6}{7}(x+y)^2}{\frac{2}{7}+\frac{6}{7}x+\frac{6}{7}x^2}$

12. a). $c=\frac{1}{8}$

b). $f_X(x)=\frac{1}{6}x^3e^{-x}$

$f_Y(y)=\begin{cases}\frac{1}{4}e^{-y}(1+y), & y>0\\ \frac{1}{4}e^{y}(1-y), & y<0\end{cases}$

c). $f_{X|Y}(x|y)=\begin{cases}\dfrac{(x^2-y^2)e^{-x}}{2e^{-y}(y-1)}, & y>0\\ \dfrac{(x^2-y^2)e^{-x}}{2e^{y}(1-y)}, & y<0\end{cases}$. $f_{Y|X}(y|x)=\frac{3}{4}\frac{x^2-y^2}{x^3}$

14. a). $f_X(x)=e^{-x}$

$f_Y(y)=\frac{1}{(1+y)^2}$, NOT INDEPENDENT

b). $f_{Y|X}(y|x)=xe^{-xy}$, $f_{X|Y}(x|y)=(y+1)^2xe^{-x(y+1)}$

20. $f_{X_1,X_2}(x_1,x_2)=\frac{1}{x_1}$, $0\le x_2<x_1\le 1$

$f_{X_2}(x_2)=-\log x_2$, $0\le x_2\le 1$

#2. p black, q white, r red. in a urn. pick out n balls without replacement.

$N = p+q+r$

$n = n_1+n_2+n_3$

a). $p = \dfrac{\binom{p}{n_1}\binom{q}{n_2}\binom{r}{n_3}}{\binom{N}{n}}$ ~ density

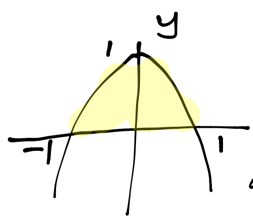
b). $P(B=n_1, W=n_2)$

$= \sum_{R}^{n-n_1-n_2} \dfrac{\binom{p}{n_1}\binom{q}{n_2}\binom{r}{n_3}}{\binom{N}{n}} = \dots$

c). $p(W=n_2) = \sum_B \sum_R \dfrac{\binom{p}{n_1}\binom{q}{n_2}\binom{r}{n_3}}{\binom{N}{n}} = \dfrac{\binom{q}{n_2}\binom{p+r}{n-n_2}}{\binom{N}{n}}$

#9. (X, Y) uniformally distributed

$0 \le y \le 1-x^2, \quad -1 \le x \le 1$



area is $\frac{4}{3}$

a). $f_X(x) = \int_0^{1-x^2} \frac{3}{4}\, dy = \frac{3}{4}(1-x^2)$

$-1 \le x \le 1$

$f_Y(y) = \int_{-\sqrt{1-y}}^{0} \varphi(x)\, dx + \int_0^{\sqrt{1-y}} \varphi(x)\, dx$

why do we write like this? $= 2\int_0^{\sqrt{1-y}} \frac{3}{4}\, dx = \frac{3}{2}\sqrt{1-y}, \quad 0 \le y \le 1$

Since $\begin{cases} x = \sqrt{1-y}, & x>0 \\ x = -\sqrt{1-y}, & x<0 \end{cases}$

b). Conditional density $= \dfrac{\text{joint density}}{\text{marginal density}}$

if marginal density is 0,

#12.

a). $\int_0^{\infty} \int_{-x}^{x} c\,(x^2-y^2)e^{-x}\, dy\, dx = 1$

$\boxed{\begin{array}{l} F(\infty)=1 \\ F(-\infty)=0 \end{array}}$

b). $f_Y(y) = \int_y^{\infty} \frac{1}{8}(x^2-y^2)e^{-x}\, dy, \quad y>0$

$0<x<\infty$! is not the only restriction on x.

$\underline{-x<y<x}$!!!

$f_Y(y) = \int_{-y}^{\infty} \frac{1}{8}(x^2-y^2)e^{-x}\,dx\ ,\ y \leq 0$

#20. X_1 is uniform on $[0, 1]$ and conditional on X_1; X_2 is uniform on $[0, X_1]$. Find joint density & marginal density for Both X_1 & X_2.

① $f_{X_1}(x_1) = 1\ ,\ 0 \leq x \leq 1$

② $f_{X_2|X_1}(x_2|x_1) = \frac{1}{x_1}\ ,\ 0 \leq x_2 \leq x_1 \leq 1$

$f_{X_1,X_2}(x_1, x_2) = f_{X_1}(x_1) \cdot f_{X_2|X_1}(x_2|x_1) = \frac{1}{x}\ ,\ 0 \leq x_2 \leq x_1 \leq 1$

$f_{X_2}(x_2) = \int_{x_2}^{1} f_{X_1,X_2}(x_1, x_2)\,dx_1 = \int_{x_2}^{1} \frac{1}{x_1}\,dx_1$

$= \log x_1 \Big|_{x_2}^{1}$

$= \log 1 - \log x_2 = -\log x_2\ ,\ 0 \leq x \leq \underline{1}$

use upb.
not x_1.