

University of Toronto, Faculty of Arts and Science
Final Examinations, April 17, 2012, 19:00–22:00, EX200

MAT244 — Differential Equations

Instructors: Prof. Pierre Milman, Prof. Victor Ivrii, Prof. Amir Moradifam

Duration — 3 hours

The problems are independent. Total marks for this paper: 100.

No aids allowed

Last name
First name
ID
Section

Term Mark	
Exam Mark	
Final Mark	

#	points	Mark	#	points	Mark
1	[16]		6a	[2]	
2	[18]		6b	[4]	
3	[18]		6c	[6]	
4	[18]		6d	[4]	
5a	[4]		6e	[2]	
5b	[4]				
5c	[4]		Total	[100]	

1 (16 pts) Find the general solution of the differential equation

$$y' - \frac{y}{x^2} = \frac{1}{x^2}$$

and solve the initial value problem $y(1) = 1$.

Continued on the other side

2 (18 pts) Find the general solution of the equation

$$t^2 y'' - 4ty' + 6y = t^4 e^t, \quad t > 0.$$

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3 (18 pts) Find the general solution of the differential equation

$$y^{(4)} + 3y'' - 4y = 16 + 8 \cos(2t).$$

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4 (18 pts) Solve the system of differential equations

$$\begin{cases} x' = x + 2y + 2z, \\ y' = 2x + 3z, \\ z' = 2x + 3y. \end{cases}$$

Continued on the other side

5 (12 pts) For the system of differential equations

$$\begin{cases} x' = \sin(y) - \frac{1}{2} \sin(x) , \\ y' = \sin(x) - \frac{1}{2} \sin(y) . \end{cases}$$

- (a) (4 pts) Linearize the system at a critical point (x_0, y_0) of your choice;
- (b) (4 pts) Describe the type of the critical point (x_0, y_0) of the linearized and of the original system;
- (c) (4 pts) Sketch the phase portraits of the linearized and the original system near this critical point (x_0, y_0) .

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6 (18 pts) Equation

$$x'' = 2x^3 - 2x$$

- (a) (2 pts) Reduce to the first order system in variables (t, x, y) with $y = x'$;
- (b) (4 pts) Find solution in the form $H(x, y) = C$;
- (c) (6 pts) Find critical points and classify them (i.e. specify whether they are nodes, saddles, etc. and stability);
- (e) (4 pts) Sketch the phase portraits near each of the critical points;
- (d) (2 pts) Sketch solution on (x, y) plane.

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