

10 October.

§ 1.1 Two topics:

Mathematical modeling—problem set 2. (with solutions) includes the even-numbered exercises from § 1.1
 - General linear programming problem (pg 51) and standard and canonical form.

Eg: (#11, pg 60)

A local health food store packages 3 types of snack foods — Chewy, Crunchy and Nutty — by mixing seeds, raisins, and peanuts. The specifications for each mixture are given in the table.

Mixture	Seeds	Raisins	Peanuts	Selling price (\$)
Chewy		$\geq 60\%$	$\leq 20\%$	2
Crunchy	$\geq 60\%$			1.6
Nutty	$\leq 20\%$		$\geq 60\%$	1.2

The suppliers of the ingredients can deliver at most 100 kg of seeds at \$1 per kg, 80 kg of raisins at 1.5 \$ per kg, and 60 kg of peanuts at \$ 0.8 per kg.

Set up a linear programming problem which will maximize the store's profit.

Choosing the independent variable Let x_{ij} = # kg of ingredient j used in mixture i .

	Seeds (j=1)	Raisins (j=2)	Peanuts (j=3)
Chewy (i=1)	x_{11}	x_{12}	x_{13}
Crunchy (i=2)	x_{21}	x_{22}	x_{23}
Nutty (i=3)	x_{31}	x_{32}	x_{33}

Constraints

$$.6(x_{11} + x_{12} + x_{13}) \leq x_{12}$$

$$.2(x_{11} + x_{12} + x_{13}) \geq x_{13}$$

$$.6(x_{21} + x_{22} + x_{23}) \leq x_{21}$$

$$.2(x_{31} + x_{32} + x_{33}) \geq x_{31}$$

$$.6(x_{31} + x_{32} + x_{33}) \leq x_{33}$$

Also:

$$x_{11} + x_{21} + x_{31} \leq 100$$

$$x_{12} + x_{22} + x_{32} \leq 80$$

$$x_{13} + x_{23} + x_{33} \leq 60$$

also:

$$x_{ij} \geq 0, \text{ all } i, j = 1, 2, 3$$

Profit = revenue - cost

$$= 2(x_{11} + x_{12} + x_{13}) + 1.6(x_{21} + x_{22} + x_{23}) + 1.2(x_{31} + x_{32} + x_{33}) - (x_{11} + x_{21} + x_{31}) - 1.5(x_{12} + x_{22} + x_{32}) - 0.8(x_{13} + x_{23} + x_{33})$$

A linear programming problem in general form:

$$\text{Maximize } Z = x_{11} + 0.5x_{12} + 1.2x_{13} + 0.6x_{21} + 0.1x_{22} + 0.8x_{23} + 0.2x_{31} - 0.3x_{32} + 0.4x_{33}$$

$$\text{s.t. } 0.6x_{11} - 0.4x_{12} + 0.6x_{13} \leq 0$$

$$2x_{11} + 0.2x_{12} - 0.8x_{13} \geq 0$$

$$-0.4x_{21} + 0.6x_{22} + 0.6x_{23} \leq 0$$

$$-2X_{s1} + 2X_{s2} + 0.2X_{s3} \geq 0$$

$$0.6X_{s1} + 0.6X_{s2} - 1.4X_{s3} \leq 0$$

$$X_{r1} + X_{s1} + X_{y1} \leq 100$$

$$X_{r2} + X_{s2} + X_{y2} \leq 20$$

$$X_{r3} + X_{s3} + X_{y3} \leq 60$$

$$X_{ij} \geq 0 \text{ for } i=1,2,3; j=1,2,3.$$