Lecture 19

- Remarks on the Orbit Diagram

① The lines on the diagram are only the attracting pts
② For $-3/4 < c < 1/4$, There is one attracting fixed pt. so the orbit converges to that fixed pt.
③ For $-5/4 < c < -3/4$, there is one attracting 2-cycle and the fixed pt becomes repelling, so we don't see it on the diagram

The Period-doubling always happens at the same c

④ For $? < c < -5/4$, there is an attracting 4-cycle
≈ -1.36
⑤ We see successive period-doubling bifurcation
2-cycle → 4-cycle → 8-cycle → ... 2^n-cycle
⑥ The period-3 window is the wide white area on the left that is crossed by 3 lines. A 3-cycle.
⑦ Then it's followed by period doublings
3-cycle → 6-cycle → 12-cycle → ... $3 \cdot 2^n$-cycle.
There are other period windows
⑧ In each period-n window, we see the apperance of an n-cycle followed by period doublings.
⑨ There are no coexisting attracting cycles
⑩ The diagram is self-similar

Orbit diagram for the logistic family

$F_\lambda(x) = \lambda x(1-x)$

① $F_\lambda : [0, 1] \longrightarrow [0, 1]$ for $1 \le \lambda \le 4$
② $P = 1 - \frac{1}{\lambda}$ is attracting for $1 < \lambda < 3$
③ F_λ has a 2-cycle $\dfrac{\lambda + 1 \pm \sqrt{\lambda^2 - 2\lambda - 3}}{2\lambda}$

attracting for $3 < \lambda < 1 + \sqrt{6}$

④ $x_0 = 1/2$ is a non-degenerate critical pt for $F_\lambda(x)$.