An Optimal Quadrilateral

Tableau 1:

	x_1	x_2	x_3	x_4	x_5	x_6	
x_5	5	1	2	4	1	0	2
x_6	8	1	5	1	0	1	3
	-41	-6	-23	-13	0	0	0

Tableau 2:

	x_1	x_2	x_3	x_4	x_5	x_6	
x_5	0	$\frac{3}{8}$	$-\frac{9}{8}$	$\frac{27}{8}$	1	$-\frac{5}{8}$	$\frac{1}{8}$
x_1	1	$\frac{1}{8}$	$\frac{5}{8}$	$\frac{1}{8}$	0	$\frac{1}{8}$	$\frac{3}{8}$
	0	$-\frac{7}{8}$	$\frac{21}{8}$	$-\frac{63}{8}$	0	$\frac{41}{8}$	$\frac{123}{8}$

Tableau 3 is optimal:

	x_1	x_2	x_3	x_4	x_5	x_6	
x_4	0	$\frac{1}{9}$	$-\frac{1}{3}$	1	$\frac{8}{27}$	$-\frac{5}{27}$	$\frac{1}{27}$
x_1	1	$\frac{1}{9}$	$\frac{2}{3}$	0	$-\frac{1}{27}$	$\frac{4}{27}$	$\frac{10}{27}$
	0	0	0	0	$\frac{7}{3}$	$\frac{11}{3}$	$\frac{47}{3}$

Tableau 4 is also optimal:

	x_1	x_2	x_3	x_4	x_5	x_6	
x_2	0	1	-3	9	$\frac{8}{3}$	$-\frac{5}{3}$	$\frac{1}{3}$
x_1	1	0	1	-1	$-\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
	0	0	0	0	$\frac{7}{3}$	$\frac{11}{3}$	$\frac{47}{3}$

Tableau 5 is also optimal:

	x_1	x_2	x_3	x_4	x_5	x_6	
x_2	3	1	0	6	$\frac{5}{3}$	$-\frac{2}{3}$	$\frac{4}{3}$
x_3	1	0	1	-1	$-\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
	0	0	0	0	$\frac{7}{3}$	$\frac{11}{3}$	$\frac{47}{3}$

Tableau 6 is also optimal:

	x_1	x_2	x_3	x_4	x_5	x_6	
x_4	$\frac{1}{2}$	$\frac{1}{6}$	0	1	$\frac{5}{18}$	$-\frac{1}{9}$	$\frac{2}{9}$
x_3	$\frac{3}{2}$	$\frac{1}{6}$	1	0	$-\frac{1}{18}$	$\frac{2}{9}$	$\frac{5}{9}$
	0	0	0	0	$\frac{7}{3}$	$\frac{11}{3}$	$\frac{47}{3}$

A Doubly Degenerate Optimal Solution

Tableau 1:

	w_1	w_2	w_3	w_4	w_5	w_6	
w_3	-5	-8	1	0	0	0	-41
w_4	-1	-1	0	1	0	0	-6
w_5	-2	-5	0	0	1	0	-23
w_6	-4	-1	0	0	0	1	-13
	2	3	0	0	0	0	0

Tableau 2:

	w_1	w_2	w_3	w_4	w_5	w_6	
w_2	$\frac{5}{8}$	1	$-\frac{1}{8}$	0	0	0	$\frac{41}{8}$
w_4	$-\frac{3}{8}$	0	$-\frac{1}{8}$	1	0	0	$-\frac{7}{8}$
w_5	$\frac{9}{8}$	0	$-\frac{5}{8}$	0	1	0	$\frac{21}{8}$
w_6	$-\frac{27}{8}$	0	$-\frac{1}{8}$	0	0	1	$-\frac{63}{8}$
	$\frac{1}{8}$	0	$\frac{3}{8}$	0	0	0	$-\frac{123}{8}$

Tableau 3 is feasible and optimal:

	w_1	w_2	w_3	w_4	w_5	w_6	
w_2	0	1	$-\frac{4}{27}$	0	0	$\frac{5}{27}$	$\frac{11}{3}$
w_4	0	0	$-\frac{1}{9}$	1	0	$-\frac{1}{9}$	0
w_5	0	0	$-\frac{2}{3}$	0	1	$\frac{1}{3}$	0
w_1	1	0	$\frac{1}{27}$	0	0	$-\frac{8}{27}$	$\frac{7}{3}$
	0	0	$\frac{10}{27}$	0	0	$\frac{1}{27}$	$-\frac{47}{3}$