April 19th

$|N| = |Z| = |N^2| = |N^k|$

Schroeder-Bereinstein Thm

$\left.\begin{array}{l}|S| \leq |T| \\ |T| \leq |S|\end{array}\right\} \Rightarrow |S| = |T|$

· union of countably many countable sets is countable

$|P(S)| > |S|$ for any set S (Cantor's Thm)

$\overset{\shortparallel}{\{f: S \longrightarrow \{0,1\}\}}$

If S is infinite, A is countable $\Rightarrow |A \cup S| = |S|$

$|R| = |P(N)|$

① $|R| \leq |P(N)|$

$\shortparallel$

$|(0,1)|$

$0 < x < 1 \quad x = 0.a_1a_2a_3\cdots \rightarrow$ digits

$\{a_1, a_1a_2, a_1a_2a_3, \cdots\}$

$x = 0.10230479\ldots$

$\rightarrow \{1, 10, 102, 1023, 10230, \cdots\}$

this gives a map $(0,1) \longrightarrow P(N)$

$\Rightarrow |R| = |(0,1)| \leq |P(N)|$

② $|P(N)| \leq |R|$ we want a 1-1 map $P(N) \longrightarrow R$

given $A \subseteq N$ subject

$A = \{n_1, n_2, n_3, \cdots\}$

$A = 0.0\cdots 010\cdots 1\cdots\}$ n_1th n_2th

$A = \{3, 6, 9, 12\cdots\}$

$\Rightarrow 0.001001001001\cdots$

$\Rightarrow$ this implies a 1-1 map $\Rightarrow |P(N)| \leq |R|$

By S-B $|P(N)| = |R|$

$2\sqrt{2}x^3 - 5\sqrt{2}x + 1 = 0$

$(\sqrt{2}x)^3 - 5(\sqrt{2}x) + 1 = 0$

$y^3 - 5y + 1 = 0$ Sps x is constructible $\Rightarrow \sqrt{2}x$ is also constructible.

if $\exists$ a constructible root $\Rightarrow$ there is also a rational root

$p/q \quad (p,q) = 1$

$p \mid 1 \quad q \mid 1 \qquad p/q = \pm 1 \qquad$ But neither of them is a root

$\Rightarrow y = \sqrt{2}x$ is not constructible $\Rightarrow x$ is not constructible.