March 12th.

About PS 5

Q6:

- $T: \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ linear
- $\alpha = \{(1,1,1),(1,-1,0),(0,1,-1)\}$ basis of $\mathbb{R}^3$ consisting of eigenvectors of T.
- eigenvalues $a, b, c \in \mathbb{R}$

Prove that T is self-adjoint iff $b=c$

Recall: The adjoint of T is the linear map $T^*: \mathbb{R}^3 \to \mathbb{R}^3$ determined by the condition $\langle T(x), y\rangle = \langle x, T^*(y)\rangle \cdot \forall x, y \in \mathbb{R}^3$

Def: T is called self-adjoint if $T^* = T$

Note T is self-adjoint $\Leftrightarrow \langle T(x), y\rangle = \langle x, T(y)\rangle \quad \forall x, y \in \mathbb{R}^3$

($\Rightarrow$) Assume that T is self-adjoint

$T(1,-1,0) = b(1,-1,0)$

$T(0,1,-1) = c(0,1,-1)$

Know: $\langle T(1,-1,0), (0,1,-1)\rangle = \langle (1,-1,0), T(0,1,-1)\rangle$

$\langle b(1,-1,0),(0,1,-1)\rangle = \langle (1,-1,0), c(0,1,-1)\rangle$

$b\langle (1,-1,0),(0,1,-1)\rangle = c\langle (1,-1,0),(0,1,-1)\rangle$

($\Leftarrow$) It suffices to prove that $\langle T(x), y\rangle = \langle x, T(y)\rangle \ \forall x, y \in \alpha$

Compare $[T]_\alpha^\alpha$ and $[T^*]_\alpha^\alpha$

It will suffices to show that $[T]_\alpha^\alpha = [T^*]_\alpha^\alpha$

WRONG!!!

Q5

$P_2(\mathbb{R})$, inner product $\langle p(x), q(x)\rangle = p(-1)q(-1) + p(0)q(0) + p(1)q(1)$

Let $T: P_2(\mathbb{R}) \to P_2(\mathbb{R})$ be defined by $T(p(x)) = p'(x)$. Find $T^*(p(x))$ for arbitrary $p(x) \in P_2(\mathbb{R})$

Consider the standard basis $\alpha = \{1, x, x^2\}$ of $P_2(\mathbb{R})$. We know that $[T^*]_\alpha^\alpha = ([T]_\alpha^\alpha)^T$

Find $[T]_\alpha^\alpha$ $\quad T(1) = 0$, $T(x) = 1$, $T(x^2) = 2x$ $\rightsquigarrow$ $[T]_\alpha^\alpha = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{pmatrix}$

$$[T^*]_\alpha^\alpha = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{pmatrix}^T = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 2 & 0 \end{pmatrix}$$

<u>know</u> $[T^*(p(x))]_\alpha = [T^*]_\alpha^\alpha [p(x)]_\alpha$

$p(x) = a + bx + cx^2 \rightsquigarrow [p(x)]_\alpha = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$

$[T^*(p(x))]_\alpha = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 2 & 0 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} \cdot & - & - \end{pmatrix}$