MAT 327 Midterm Test, University of Toronto July 3, 2013

Name (LAST, First): ________________________

Student Number: ________________________

- Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page, and write "Solution continued on the back of this page".
- The test is from 4:10 pm - 6:00 pm. You have 110 minutes.
- The test is out of 100 marks. With bonus questions it is possible to earn a total of 109 marks.

FOR TA USE ONLY	
Question	Score
1	/30
2	/10
3	/10
4	/20
5	/30
BONUS	/9
TOTAL	/100

1. For each of the following questions (a)-(k), answer the question and provide **one or two sentences** of explanation (unless otherwise stated).

   (a) [2 marks] Which T_i property is equivalent to "points are closed"? (No explanation needed.)

   (b) [2 marks] Which T_i property implies that "each sequence converges to at most one point"? (No explanation needed.)

   (c) [2 marks] Let $\mathcal{T}_{\text{usual}}$ be the usual topology on $\mathbb{R}$. Is $\mathcal{T}_{\text{usual}} \times \mathcal{T}_{\text{usual}}$ a topology on $\mathbb{R}^2$?

   (d) [3 marks] Is every continuous function $f : \mathbb{R}_{\text{usual}} \longrightarrow \mathbb{R}_{\text{usual}}$ an open function?

   (e) [3 marks] Is $\mathbb{Z}_{\text{discrete}} \cong \mathbb{Q}_{\text{discrete}}$?

   (f) [3 marks] Which of the following properties does ω_1 (with the order topology) have? First Countable, Second Countable, T_2, Separable. (No explanation needed.)

(g) [3 marks] Does $\mathbb{R}_{\text{usual}}$ refine $\mathbb{R}_{\text{co-countable}}$?

(h) [3 marks] Without proof, write down a countable basis for the usual topology on $\mathbb{R}$.

(i) [3 marks] Is S^{4327} (as a subspace of $\mathbb{R}^{4328}$) a T_3 space?

(j) [3 marks] Which of the following two statements can be false in a topological space $(X, \mathcal{T})$, for $A \subseteq X$: (1) "$\overline{A \cap B} = \overline{A} \cap \overline{B}$" or (2) "$\overline{A \cup B} = \overline{A} \cup \overline{B}$". Provide a counterexample to the false statement.

(k) [3 marks] Give an example of a continuous, surjective, open function

$$f : X \longrightarrow Y$$

that is not a homeomorphism. (Make sure to choose explicit topological spaces X and Y, and an explicit function.)

2. (a) [4 marks] State the definition of "$(X, \mathcal{T})$ is first countable space". Make sure to define all the terms you use.

   (b) [6 marks] Suppose that $(X, \mathcal{T})$ and $(Y, \mathcal{U})$ are both first countable spaces. Prove that $X \times Y$ is a first countable space when given the product topology.

3. This question will be about $\mathbb{R}_{\text{Sorgenfrey}}$, the Sorgenfrey Line. **Prove all assertions you make about the Sorgenfrey line** when answering these two questions.

   (a) [4 marks] Does the sequence $\{ (0, -\frac{1}{n}) : n \in \mathbb{N} \}$ converge to $(0,0)$ in $\mathbb{R}_{\text{Sorgenfrey}} \times \mathbb{R}_{\text{Sorgenfrey}}$ with the product topology?

   (b) [6 marks] Prove or disprove: $\mathcal{B} := \{ [p, q) : p, q \in \mathbb{Q} \}$ is a basis for $\mathbb{R}_{\text{Sorgenfrey}}$.

4. For this question, suppose that $\mathbb{R}$ has its usual topology, and ω_1 is given the order topology. You may reference, without proof, any theorems or propositions from class, or any assignment questions.

   (a) [5 marks] Give two (infinite) countable subspaces of $\mathbb{R}$ that are not homeomorphic.

   (b) [5 marks] Give two uncountable subspaces of $\mathbb{R}$ that are not homeomorphic.

   (c) [5 marks] Give two (infinite) countable subspaces of ω_1 that are not homeomorphic.

   (d) [5 marks] Give two uncountable subspaces of ω_1 that are not homeomorphic.

5. Define a topological space $(X, \mathcal{T})$ to be **super second countable** if *every* basis $\mathcal{B}$ that generates $\mathcal{T}$ is countable.

   (a) [5 marks] Prove that every finite topological space is super second countable.

   (b) [5 marks] Show that a super second countable space has only countably many open sets.

   (c) [5 marks] Show that a topological space with infinitely many mutually disjoint open sets has uncountably many open sets.

(d) [10 marks] Show that any infinite T_2 topological space contains infinitely many mutually disjoint open sets.

(e) [5 marks] From the previous 4 facts, state and prove a new theorem that characterizes when a T_2 space is super second countable, based on its cardinality. (This theorem should contain the words "if and only if".)

6. **These are BONUS questions**

   (a) [1 mark (bonus)] Spell the course instructor's (complete) first name and last name.

   (b) [1 mark (bonus)] Spell the TA Ivan's last name.

   (c) [1 mark (bonus)] Spell the TA Ali's last name.

   (d) [1 mark (bonus)] What was Hausdorff's first name?

   (e) [1 mark (bonus)] What was DeMorgan's first name?

   (f) [1 mark (bonus)] Complete the classic "joke": "A topologist is a person who can't tell a __________________ from a __________________."

   (g) [1 mark (bonus)] Name a famous Australian tennis player.

   (h) [1 mark (bonus)] Name a type of "loose-fitting baggy form of trousers favoured by members of the University of Oxford, especially undergraduates, in England during the early 20th century from the 1920s to around the 1950s."

   (i) [1 mark (bonus)] What word does the "T" stand for in the T_i property?