Lecture 4

n	3^n	n^3
0	1	0
1	3	1
2	9	8
$\boxed{3}$	$\boxed{27}$	$\boxed{27}$ ***
4	81	64
5	243	125
...		

$3^{n+1} = 3\cdot 3^n \qquad (n+1)^3 = (1+\frac{1}{n})^3 \cdot n^3$

(*) For $n \geq 3$, $\frac{1}{n} \leq \frac{1}{3}$

so $1+\frac{1}{n} \leq \frac{4}{3}$

so $(1+\frac{1}{n})^3 \leq \frac{64}{27} \leq \frac{81}{27} = 3$

If $n \geq 3$ and $\underbrace{n^3 \leq 3^n}_{(**)}$

then $(n+1)^3 = (1+\frac{1}{n})^3 \cdot n^3 \leq 3n^3$ by (*) and $n \geq 3$

$\leq 3\cdot 3^n$ by (**)

$= 3^{n+1}$

For $n \in \mathbb{N}$, let $P(n)$ be $n^3 \leq 3^n$

n	$P(n)$
0	
1	
2	by ***
3	T
4	T
5	T
6	T
...	

chain of application

$\Rightarrow \forall n \in \mathbb{N}, [(n \geq 3 \wedge P(n)) \rightarrow P(n+1)]$

$(3 \geq 3 \wedge P(3)) \rightarrow P(4)$

$(4 \geq 3 \wedge P(4)) \rightarrow P(5)$

$(0 \geq 3 \wedge P(0)) \rightarrow P(1)$ ..... This is True (False → True) is True

$[P(3) \wedge \forall n \in \mathbb{N}, (n \geq 3) \wedge P(n)) \rightarrow P(n+1)] \rightarrow \forall n \in \mathbb{N}, (n \geq 3 \rightarrow P(n))$

An inductive principle we believe / accept

(A) Proof of $\forall n \in \mathbb{N}, n \geq 3 \rightarrow P(n)$

By Induction

Base case: $P(3)$

$3^3 \leq 3^3$

Inductive step: $\forall n \in \mathbb{N}, (n \geq 3 \wedge P(n)) \rightarrow P(n+1)$

Assume $n \geq 3$

$P(n)$, i.e. $n^3 \leq 3^n$ (IH)

Then $(n+1)^3 = (1+\frac{1}{n})^3 \cdot n^3$

$\leq 3 \cdot n^3$ since $n \geq 3$

(so $\frac{1}{n} \leq \frac{1}{3}$, so $1+\frac{1}{n} \leq \frac{4}{3}$, so $(1+\frac{1}{n})^3 \leq \frac{64}{27} \leq 3$)

$\leq 3 \cdot 3^n$ by (IH)

$= 3^{n+1}$

BTW, $\forall n \in \mathbb{N}, P(n)$

$P(0)$: $0^3 = 0 \leq 1 = 3^0$

$P(1)$:

$P(2)$:

Rest by (A)

What amounts of money can you make from \$3 & \$5 bills?

0, 3, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15, 16, ...

if you know 233 is possible, then so is 236.

For $n \in \mathbb{N}$, let $P(n)$ be: n can be made from 3s AND 5s

$\forall n \in \mathbb{N}, (P(n) \rightarrow P(n+3))$

$$\left[P(8) \wedge P(9) \wedge P(10) \wedge \forall n \in N, \big([n \geq 8; P(n)] \rightarrow P(n+3)\big)\right] \rightarrow \underbrace{\forall n \in N, (n \geq 8 \rightarrow P(n))}_{\circledast}$$