STAT6039 week 12 lecture 1 additional material

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Appendix 1: Continuity Correction

Example: A die is rolled $n = 120$ times. Find the probability that at least 27 sixes come up.

Analysis:

1. $Y \sim Bin(120, \frac{1}{6})$.

2. $Bin(n,p) \dot{\sim} N(np, np(1-p))$.

3. $P(Y \geq 27) \approx P(U \geq 27)$.

4. $P(Y \geq 27) = \sum_{y=27}^{120} \binom{120}{y} \left(\frac{1}{6}\right)^y \left(\frac{5}{6}\right)^{120-y} = 0.0597$.
   This is the exact probability.

5. $P(U \geq 27) = P\left(Z \geq \frac{27-20}{\sqrt{16.667}}\right) = 0.0436$.
   U is normal, Y is binomial.
   This approximation is not very precise/accurate.

6. $P(U \geq 27 - 0.5) = P\left(Z \geq \frac{27-0.5-20}{\sqrt{16.667}}\right) = P(Z \geq 1.59) = 0.0559$.
   Do "continuity correction" by shifting the distribution.
   (actually shifting the normal distribution vertically by 0.5?)

Appendix 2: Buffon's needle problem

Problem: A kitchen floor has a pattern of parallel lines that are 10 cm apart. You have a needle in your hand that is also 10 cm long. If you randomly throw the needle onto the floor, what is the probability p that it will cross a line?

Analysis: Monte Carlo method

1. Throw the needle on the floor $n = 1000$ times and find that the needle crosses a line 651 times.
2. An estimator for p is $\hat{p} = \frac{651}{1000} = 0.651$.
3. A 95% CI for p is

$$\left(0.651 \pm 1.96\sqrt{0.651(1-0.651)/1000}\right) = (0.621, 0.681).$$

Analytical Method of finding p (rather tedious)

Analysis:

1. X: perpendicular distance from centre of needle to nearest line in units of 5 cm. Y: acute angle between lines and needle in radians. A: needle crosses a line.
2. $X \sim U(0,1), Y \sim U(0, \pi/2), X \perp Y$.
3. $f(x) = 1, 0 < x < 1, f(y) = 2/\pi, 0 < y < \pi/2, f(x,y) = f(x)f(y) = 2/\pi, 0 < x < 1, 0 < y < \pi/2$.
4. $p = P(A) = \int ...$