1. Let

$$A = \begin{bmatrix} -2 & 3 & 2 \\ 3 & -2 & 0 \\ 0 & -1 & 2 \end{bmatrix}, \quad b = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}.$$

(a) Compute the LU factorization of A using Gauss elimination with partial (row) pivoting. Indicate the L and U factors as they are built at each step of GE. Moreover, indicate the permutation matrices P_k arising at each step of GE, as well as the total permutation matrix P that captures the permutations at all steps. Verify that $P \cdot A = L \cdot U$.

(b) Using L, U, P and b, and applying back and forward substitutions, compute the solution to $Ax = b$. Indicate any intermediate vector arising, and the final solution vector.

Note: In both (a) and (b), use exact (fractional) arithmetic.

2. Show that, for any invertible matrix A, if $\hat{x}$ is an approximate solution to $Ax = b$, $r = b - A\hat{x}$ the respective residual, and $e = x - \hat{x}$ the associated error, then

$$\kappa^{-1} \frac{||r||}{||b||} \leq \frac{||e||}{||x||} \leq \kappa \frac{||r||}{||b||}$$

where κ is the condition number of A in the norm $|| \cdot ||$.
Note: In this question, you don't worry about representation error for A and b. (You can assume they are zero.)