Australian National University
Research School of Finance, Actuarial Studies and Applied Statistics

STAT2032/6046: Financial Mathematics

Review Questions (Week 7 – Week 9)

WEEK 7

Question 1

Deposits of \$1100 are made into an investment fund at year 0 and year 1. The fund balance is \$2500 at year 2.

a. Compute annual money-weighted rate of return.
b. Compute the annual time-weighted rate of return given that the fund balance is \$1200 at year 1, just before the \$1100 investment made at year 1.

Solution

a. $1100(1+i)^2 + 1100(1+i) = 2500$

   Solve the quadratic in: $(1 + i)$:

   $$(1+i) = \frac{-1100 \pm \sqrt{1100^2 + 4(1100)(2500)}}{2200} = 1.0883,\ -2.0883$$

   Therefore, the money-weighted rate of return is 8.83%.

b. The progress of the fund was as follows:

   Initial fund balance = \$1100

   Fund balance before cashflow at time 1 = \$1200

   Cashflow at time 1 = \$1100

   Fund balance after cashflow at time 1 = \$2300

   Fund balance at time 2 = \$2500

The annual time-weighted rate of return is: $\left[\left(\frac{1200}{1100}\right)\left(\frac{2500}{2300}\right)\right]^{1/2} - 1 = 8.89\%$

WEEKS 8 & 9

Question 2

An investor owns a \$2000 par value bond with coupons of 10% pa payable half-yearly. The bond will mature at par at the end of 10 years. The investor decides that a 6-year bond would be preferable. Current nominal yield rates are 5% convertible half-yearly for all durations. The investor uses the proceeds from the sale of the 10% bond to purchase a bond paying *annual* coupons of 6% pa, maturing at par at the end of 6 years. Find the par value of the 6-year bond.

Solution

$i^{(2)} = 5\%$

$j = \frac{i^{(2)}}{2} = 2.5\%$

$n = 20$ (number of coupon periods)

The bond with 10% coupons is worth an amount P, where:

$$P = 2000(0.05)a_{\overline{20}|j} + 2000v_j^{20} = 2{,}779.46$$

The investor sells this bond for the amount P and uses the proceeds to purchase a bond with par value X where:

$$P = X(0.06)a_{\overline{6}|i} + Xv_i^6 = 2{,}779.46 \Rightarrow X = \frac{2{,}779.46}{(0.06)a_{\overline{6}|i} + v_i^6} = \$2{,}653.45$$

where $i = 1.025^2 - 1$.

Question 3

An investor buys two 10-year bonds, each having half-yearly coupons and each redeemable at par. For each bond the purchase price produces the same yield rate. One bond has a par value of \$600 and a coupon of \$50. The other bond has a par value of \$1000 and a coupon

of \$30.

The first bond is purchased at a premium (ie. $P > 600$). The second bond is bought at discount (ie. $P < 1000$). The dollar amount of premium on the first bond is twice as great as the dollar amount of discount on the second bond. Find the nominal annual yield rate convertible semi-annually.

Solution

$n = 20$ (number of coupon periods)

The bond with \$50 coupons has a price P_1, where: $P_1 = 50a_{\overline{20}|j} + 600v_j^{20}$

The bond with \$30 coupons has a price P_2, where: $P_2 = 30a_{\overline{20}|j} + 1000v_j^{20}$

The 1st bond is bought at a premium of $P_1 - 600$

The 2nd bond is bought at a discount of $1000 - P_2$

We are told that $P_1 - 600 = 2\left(1000 - P_2\right) \Rightarrow P_1 = 2600 - 2P_2$

So we have two equations with two unknowns.

$$2600 - 2P_2 = 50a_{\overline{20}|j} + 600v_j^{20} \Rightarrow P_2 = 1300 - 25a_{\overline{20}|j} - 300v_j^{20} \quad (1)$$

$$P_2 = 30a_{\overline{20}|j} + 1000v_j^{20} \quad (2)$$

Subtracting (2) from (1)

$$\Rightarrow 0 = 1300 - 55a_{\overline{20}|j} - 1300v_j^{20}$$

Interpolate to find j. $j = 4.23\% \Rightarrow i^{(2)} = 8.46\%$

Question 4

A \$200 par value 10-year bond has coupons at the annual rate of 6% payable continuously. If the bond is bought to yield rate i, find the price of the bond expressed as a function of δ.

Solution

$$P = 200(0.06)\bar{a}_{\overline{10}|} + 200v^{10} = 12\left(\frac{1-v^{10}}{\delta}\right) + 200v^{10}$$

$$= 12\left(\frac{1-e^{-10\delta}}{\delta}\right) + 200e^{-10\delta}$$

since $e^{\delta} = 1+i \Leftrightarrow e^{-\delta} = v$

Question 5

A corporation issues bonds with *annual* 7% coupons maturing in five years, which are quoted at a price that yields 5% effective. It is proposed to replace this issue of bonds with an issue of bonds with annual coupons of 6% at the same price. How long must the new issue run so that the bondholders will still realise 5% effective?

Solution

$$P = 100(0.07)a_{\overline{5}|0.05} + 100v^{5}_{0.05} = 108.66$$

$$108.66 = 100(0.06)a_{\overline{n}|0.05} + 100v^{n}_{0.05} \Rightarrow 108.66 = 120 - 20v^{n} \Rightarrow v^{n} = 0.567$$

$$\Rightarrow n = \frac{\ln(0.567)}{\ln v} = 11.63$$

Note that it is impossible to get an issue in complete years that give an exact yield of 5% effective; the closest period would be 12 years.

Question 6

A $1000 par value 4% bond with half-yearly coupons matures at the end of 10 years. The bond is callable at $1050 at the ends of years 4 through 6, at $1025 at the ends of years 7 through 9, and at $1000 at the end of year 10. Find the maximum price that an investor can pay and still be certain of a yield rate of 5% convertible semi-annually.

Solution

$$P = 1000(0.02)a_{\overline{n}|j} + Cv^{n}_{j}$$

Years 4 – 6 C = 1050

Years 7 – 9 C = 1025

Years 10 C = 1000

We want a minimum effective half-yearly yield of j = 2.5%

Since the half-yearly yield is greater than the coupon rate $\left(j > 0.02\right) \Leftrightarrow P < C$. ie. the price paid needs to be less than the redemption amount in order to generate a minimum yield of 2.5%.

For a bond bought at discount, the minimum yield occurs at the latest redemption date.

If redemption occurs in years 4-6, use n = 12: $P_1 = 1000(0.02)a_{\overline{12}|j} + 1050v_j^{12} = 985.89$

If redemption occurs in years 7-9, use n = 18: $P_1 = 1000(0.02)a_{\overline{18}|j} + 1025v_j^{18} = 944.26$

If redemption occurs in year 10, use n = 20: $P_1 = 1000(0.02)a_{\overline{20}|j} + 1000v_j^{20} = 922.06$

For any redemption date, the lowest of these amounts should be selected. ie. \$922.06 should be paid to guarantee a minimum yield of 5% per annum convertible half-yearly. If an amount greater than \$922.06 was paid and redemption occurred at the end of year 10, then the yield would be less than 5%. eg. if \$944.26 was paid and redemption occurred in year 10 then the yield would be 4.7%.