Question One

Gompertz' Law states that $\mu_x = BC^x$. In answering this question assume that mortality experience conforms to Gompertz' Law.

a) Show that $-\log(-\log p_x) = \log\{\frac{\log C}{B(C-1)}\} - x\log C$.

b) Could the result in part (a) be used to estimate B and C? If you answer "yes" to this question be sure to state exactly how the result could be used.

Question Two

The ordered survival times of N individuals after a particular operation are $t_1, t_2, t_3, ..., t_N$. Assume that each individual was observed until death and that each time of death was unique. In this situation does the empirical survival function $(1 - \frac{d(t)}{N})$ for $t_3 < t < t_4$ yield the same result as the Kaplan-Meier estimate of the survival function for $t_3 < t < t_4$? You must show complete working for your answer and define any notation that you use.

Question Three

Answer each question "TRUE" or "FALSE". In each case, write the whole word. It is **not** acceptable to write only "T" or "F" and answers presented in this form **will be graded incorrect**.

a) ${}_5p_{34}$ must be less than ${}_7p_{33}$.

b) Parameter estimates obtained using method of moments estimation will be the same as those obtained from maximum likelihood estimation.

c) $m_x = \frac{q_x}{\int_0^1 {}_tp_x \, dt}$ can never be less than q_x.

d) For human populations a force of mortality function (μ_x) for which $\lim_{x\to\infty} \int_0^x \mu_s ds \neq \infty$ is plausible.

e) For human populations the force of mortality must be an increasing function of age.

SOLUTIONS

Question One

Gompertz' Law states that $\mu_x = BC^x$. In answering this question assume that mortality experience conforms to Gompertz' Law.

c) Show that $-\log(-\log p_x) = \log\{\frac{\log C}{B(C-1)}\} - x\log C$.

Solution:

$$p_x = \exp(-\int_0^1 \mu_{x+t} dt)$$

$$\Rightarrow -\log p_x = \int_0^1 \mu_{x+t} dt = \int_0^1 BC^{x+t} dt = BC^x \int_0^1 C^t dt = \frac{BC^x(C-1)}{\log C}$$

$$\Rightarrow -\log(-\log p_x) = -\log B - x\log C - \log(C-1) + \log\log C = \log\{\frac{\log C}{B(C-1)}\} - x\log C$$

d) Could the result in part (a) be used to estimate B and C? If you answer "yes" to this question be sure to state exactly how the result could be used.

Solution:

Yes. A regression model could be fit to $-\log(-\log p_x)$ and x. The estimated slope from this regression could be set equal to $-\log C$ and the intercept set equal to $\log\{\frac{\log C}{B(C-1)}\}$.

Question Two

The ordered survival times of N individuals after a particular operation are $t_1, t_2, t_3, ..., t_N$. Assume that each individual was observed until death and that each time of death was unique. In this situation does the empirical survival function $(1-\frac{d(t)}{N})$ for $t_3 < t < t_4$ yield the same result as the Kaplan-Meier estimate of the survival function for $t_3 < t < t_4$? You must show complete working for your answer and define any notation that you use.

Solution:

Assuming no censoring the Kaplan-Meier estimator can be written:

$$\hat{S}(t) = \frac{N-1}{N} \times \frac{(N-1)-1}{N-1} \times \frac{(N-2)-1}{(N-2)}$$
$$= \frac{N-3}{N} = \text{empirical survival function.}$$

Question Three

Answer each question "TRUE" or "FALSE". In each case, write the whole word. It is **not** acceptable to write only "T" or "F" and answers presented in this form **will be graded incorrect**.

f) ${}_5p_{34}$ is less than ${}_7p_{33}$.

FALSE

g) Parameter estimates obtained using method of moments estimation will be the same as those obtained from maximum likelihood estimation.

FALSE

h) $m_x = \frac{q_x}{\int_0^1 {}_tp_x\, dt}$ can never be less than q_x.

TRUE

i) For human populations a force of mortality function (μ_x) for which $\lim_{x\to\infty} \int_0^x \mu_s ds \neq \infty$ is plausible.

FALSE

j) For human populations the force of mortality must be an increasing function of age.

FALSE