Practice Problems

MAT 335 – Chaos, Fractals, and Dynamics – Fall 2013

Not to be submitted

Here is a list of problems for practice from the textbook.

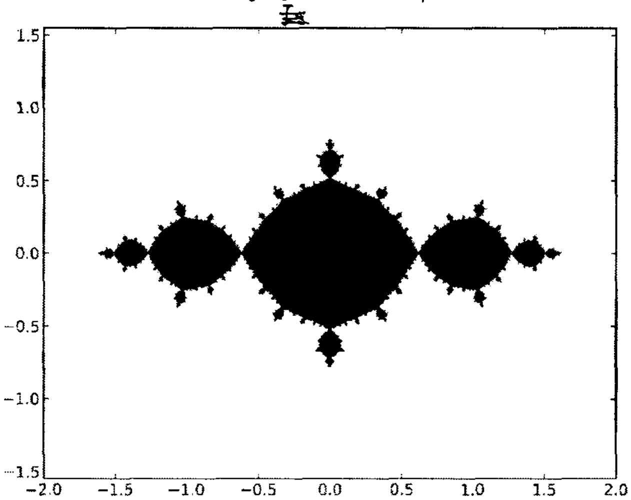
2. find polar representation of $1+\sqrt{3}i$, $2+2i$, $-7i$, $6i$, ...
4. What's the formula for the quotient of two complex #s in polar representation?
5. Let $L_\alpha(z) = \alpha z$, sketch the orbit of 1 in the plane for each of the following values of α: ① $\alpha = \frac{i}{2}$ ② $\alpha = 2i$ ③ $\alpha = 1+\sqrt{3}i$ ④ $\alpha = i$ ⑤ $\alpha = e^{2\pi i/9}$ ⑥ $\alpha = e^{\sqrt{2}\pi i}$

Chapter 15. ~~1~~/~~2~~/~~4~~/~~5~~

Chapter 16. 7 (first part), 8 (first part)

- Find complex neutral fixed points of Q_c: Find $z \in \mathbb{C}$ such that $|Q'_c(z)| = 1$.
- How do K_c and J_c compare?
- Given K_{-1} below

7. Consider complex function $G_\lambda(z) = \lambda(z - z^3)$. Show the points $P_\pm(\lambda) = \pm\sqrt{\frac{\lambda+1}{\lambda}}$ lie on a cycle of period 2 unless $\lambda = 0$ or -1.
8. $Q_i(z) = z^2 + i$, prove that the orbit of 0 is eventually periodic.



find three points $z_1 = a \in \mathbb{R}$, $z_2 = bi$ with $b \in \mathbb{R}$ and $z_3 = x + iy$, with $x, y \neq 0$ such that the orbit of z_i under Q_{-1} is bounded.

- Find points $z_1, z_2, z_3 \in \mathbb{C}$ of the form of the previous ones such that the orbit of z_i under Q_{-1} is unbounded.

Prove that the Mandelbrot set is symmetric about the real axis.
Hint: show that by proving Q_c is conjugate to $Q_{\bar{c}}$. Show that your conjugacy takes 0 to 0. Therefore the orbit of 0 has similar faces for both Q_c & $Q_{\bar{c}}$.

Chapter 17. 3

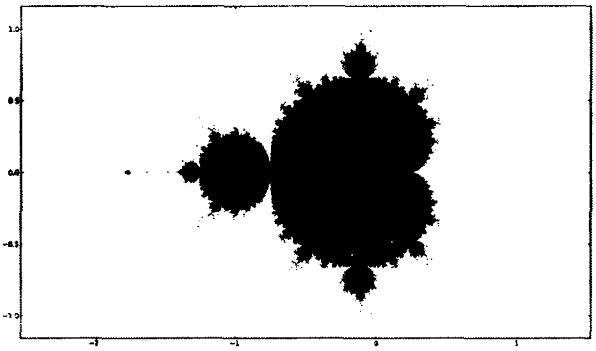
- For which values of c, there exists $z \in \mathbb{C}$, which is a neutral fixed point of Q_c.

- Sketch that set and compare it with the Mandelbrot set $\mathcal{M}$.

- What happens to the orbit of 0 under Q_{-2}? Does $-2 \in \mathcal{M}$?

- What happens to the orbit of 0 under Q_i? Does $i \in \mathcal{M}$?t

- Given the Mandelbrot set $\mathcal{M}$ below



  find complex values of find three points $c_1 = a \in \mathbb{R}$, $c_2 = bi$ with $b \in \mathbb{R}$ and $c_3 = x + iy$, with $x, y \neq 0$ such that the orbit of 0 under Q_{c_i} is bounded.

- Find complex values of $c_1, c_2, c_3 \in \mathbb{C}$ of the form of the previous ones for which the orbit of 0 under Q_{c_i} is unbounded.

- If $c = -1.8 + 1.8i$, is K_c connected or disconnected?

Ch15. $\mathbb{C}$ #s

2. $e^{i\theta}=\cos\theta+i\sin\theta$, $e^{i\pi}=-1$

$x=r\cos\theta=0$

$-7i$ $\quad y=r\sin\theta=-7$

$$\begin{cases}\theta=\frac{3}{2}\pi\\ r=7\end{cases}$$

...

4. $z_1=r_1e^{\theta_1 i}$

$z_2=r_2e^{\theta_2 i}$

$$\frac{z_1}{z_2}=\frac{r_1}{r_2}e^{(\theta_1-\theta_2)i}$$

$r_2\neq 0$

this is a third complex #. with $r_3=\frac{r_1}{r_2}$, $\theta_3=\theta_1-\theta_2$.

5. ① $\alpha=\frac{i}{2}$

$r=\sqrt{0+(\frac{1}{2})^2}=\frac{1}{2}<1$; $\theta=\frac{\pi}{2}$

$L_\alpha^1(1)=L_\alpha(1)=\frac{i}{2}1=\frac{i}{2}$

$L_\alpha^2(1)=-\frac{1}{4}$

$L_\alpha^3(1)=-\frac{i}{8}$

$L_\alpha^4(1)=\frac{1}{16}$

$\vdots$

$L_\alpha^n(1)=(\frac{i}{2})^n$

...

Ch16.

Julia Set

7. $G_\lambda(z)=\lambda(z-z^3)$

$=\lambda z(1-z^2)$

$=\lambda z(1+z)(1-z)$

$$G_\lambda\left(\sqrt{\tfrac{\lambda+1}{\lambda}}\right)=\lambda\cdot\sqrt{\tfrac{\lambda+1}{\lambda}}\left(1-\tfrac{\lambda+1}{\lambda}\right)$$

$$=\lambda\cdot\sqrt{\tfrac{\lambda+1}{\lambda}}\cdot\frac{-1}{\lambda}$$

$$=-\sqrt{\tfrac{\lambda+1}{\lambda}}$$

vice versa.

$\lambda\neq 0$, and $\lambda\neq -1$ (o.w. it's fixed)

8. $z=0$

$Q_i(0)=i$

$Q_i^2(0)=-1+i$

$Q_i^3(0)=(-1+i)^2+i=1-1-2i+i=-i$

$Q_i^4(0)=(-i)^2+i=-1+i=Q_i^2(0)$

...

eventually periodic.

Ch17. Mandelbrot set.

3. Note a complex # is symmetric to its conjugate w.r.t. the real axis

Define. $H:\mathbb{C}\to\mathbb{C}$ s.t. $H(z)=\bar{z}$

$Q_{\bar{c}}\circ H(z)=\bar{z}^2+\bar{c}=\overline{z^2}+\bar{c}=\overline{z^2+c}=H\circ Q(z)$

since $\bar{z}_1\bar{z}_2=\overline{z_1z_2}$

$\bar{z}_1+\bar{z}_2=\overline{z_1+z_2}$

$\therefore Q_c$ is conjugate to $Q_{\bar{c}}$ via H.

i.e. $Q_{\bar{c}}(\bar{z})=\overline{Q_c(z)}$

in fact $Q_{\bar{c}}^n(\bar{z})=\overline{Q_c^n(z)}$

b/c $|Q_{\bar{c}}^n(0)|=|\overline{Q_c^n(0)}|=|Q_c^n(0)|$

$$\begin{tikzcd}
C \arrow[r, "Q_c"] \arrow[d, shift right, "H"'] & C \arrow[d, shift left, "H"] \\
C \arrow[r, "Q_{\bar{c}}"] \arrow[u, shift right, "H"'] & C \arrow[u, shift left, "H"]
\end{tikzcd}$$