HAI WEN NI

haiwen.ni@yahoo.com

·— @~~uto~~mail.utoronto.ca

Tel: 647-881-2573

Office hours : Friday 11-12 SS 1091

Assignment 1:

12. $N = C^{25}_{52} C^{5}_{25} C^{5}_{20} C^{5}_{15} C^{5}_{10} C^{5}_{5}$

20. $P = \frac{49!\,4!}{52!}$

26. $P = 1/6^4$

30. (1) $P_1 = 0.05$ (2). $P_2 = 0.3$ (3). $P = 0.06$

36. 35

38. (1). $N_1 = 20$ (2). $N_2 = \binom{9}{3\,3\,3} = \cdots$

42. $N = \binom{11}{4\,3\,3\,1}$

7. Prove $P(A \cap B) \geq P(A) + P(B) - 1$

iff $1 \geq P(A) + P(B) - P(A \cap B) = P(A \cup B)$

8. Prove $P\left(\bigcup_{i=1}^{n} A_i\right) \leq \sum_{i=1}^{n} P(A_i)$

Pf: Math Induction

$n = 2 \quad P(A_1 \cup A_2) \leq P(A_1) + P(A_2)$

if the statement holds for n, then:

$P(A_1 \cup A_2 \cup \cdots \cup A_n) \leq P(A_1) + \cdots + P(A_n)$

For the n+1 case :

$P(A_1 \cup A_2 \cup \cdots \cup A_n \cup A_{n+1}) = P(\bar{A} \cup A_{n+1}) \leq P(\bar{A}) + P(A_{n+1})$

$\leq \sum_{i=1}^{n} P(A_i) + P(A_{n+1}) = \sum_{i=1}^{n+1} P(A_i)$ ∎

12. 5 player 5 cards/each , 52 cards.

(no difference)

$\binom{52}{25}\binom{25}{5}\binom{20}{5}\binom{15}{5}\binom{10}{5}\binom{5}{5}$

20. deck of 52 , 4 aces next to each other , P = ?

"4A" ⟹ 1 card $\quad 1 + 48 = 49 \quad \frac{49!\,4!}{52!}$

Other method: 48 individual cards and 49 "slots"

$\frac{48!\,49 \cdot 4!}{52!}$

#30. 60 students into 2 30-people class. 5 friends. M. S. I. K. C

5 out of 60.

① Pick 2 students out of 6. □△①②③④

$$6 \to 3 \quad 3$$

□△① ②③④

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⑩ all together

$\binom{6}{3}\binom{3}{2}=20$? (exchange 2 classes)

$$\frac{\binom{6}{3}\binom{3}{3}}{\binom{2}{1}}=10$$

$$\frac{\binom{55}{25}\binom{30}{30}}{\binom{30}{30}\binom{30}{30}/\binom{2}{1}}\approx 0.05$$

note : $\binom{n}{k}=\binom{n}{n-k}$

② Take M as an example.

$$60 \to 30 \quad 30$$

M   SIKC

$$\frac{\binom{55}{26}\binom{29}{29}\times 5}{\binom{60}{30}\binom{60}{30}/\binom{2}{1}}$$

③ ~~3~~ $P_2\times\frac{1}{5}$

#38. Ⓡ Ⓖ Ⓡ Ⓖ Ⓡ Ⓖ 6 Blocks

$\binom{6}{3}$ Chose 3 out of 6 spaces to put in red

done!

Method II: _Ⓡ_Ⓡ_Ⓡ_ $\Rightarrow$ 4 slots

3 situations:
1. 3G ~~together~~ $\binom{4}{1}$
2. 2G + 1G $\binom{4}{2}\binom{2}{1}$
3. G+G+G $\binom{4}{3}$

$$\binom{4}{1}+\binom{4}{2}\binom{2}{1}+\binom{4}{3}$$

$\binom{9}{3\ 3\ 3} = \cdots$

or $20 \times \left(\binom{7}{1} + \binom{7}{2}\binom{2}{1} + \binom{7}{3} \right)$