Code: 0286

DEPARTMENT OF MATHEMATICS
University of Toronto
MAT 135Y
Term-Test #2
Thursday, December 6, 2007
Time allowed: 1 hour, 45 minutes

PLEASE PRINT in INK or BALL-POINT PEN:

NAME OF STUDENT:
(Please PRINT full name
and UNDERLINE surname): _____

STUDENT NO.: _____

SIGNATURE OF STUDENT
(in INK or BALL-POINT PEN): _____

TUTORIAL CODE (e.g. M4A, R5D, etc.): _____

TUTORIAL TIME (e.g. T4, R6, F3, etc.): _____

NAME OF YOUR T.A.: _____

ANSWER BOX	
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NOTE:

1. Before you start, check that this test has 14 pages.
   There are NO blank pages.
2. This test consists of 20 multiple-choice questions.
   Indicate your answer to each question by completely filling in the appropriate circle in the ANSWER BOX on this front page. (Use a sharp dark pencil!)
3. No aids allowed. No calculators!

AT THE END OF THE TEST, THE EXAM SUPERVISOR WILL COLLECT FROM YOU ONLY THIS FRONT PAGE.

THE REST OF THIS EXAM BOOKLET YOU CAN TAKE HOME.

PLEASE READ CAREFULLY:
Each of the following 20 multiple-choice questions has exactly one correct answer. **Indicate your answer to each question by completely filling in the appropriate circle in the ANSWER BOX on the front page. Use a sharp dark pencil!**

MARKING SCHEME: 5 marks for a correct answer, 0 for no answer, a wrong answer or an unclear answer or indicating more than one answer. You are not required to justify your answers.

ADVICE: Once you have done a question, you should indicate your answer on the front page immediately. Don't wait till the end of the test to transfer your answers from the inside pages to the front page!

WARNING: Your computations and answers indicated on these inside pages will NOT count. **Only the final answers indicated in the ANSWER BOX on the front page will count.** If you have done a question correctly but have indicated a wrong answer on the front page due to carelessness (or whatever reason), you will get a zero for that question.

1. Find the value of $\lim_{x\to 0} \dfrac{1-\cos x}{x^2}$.

   (A) 1

   (B) undefined

   (C) 0

   (D) $\sqrt{2}$

   (E) $\dfrac{1}{2}$

2. Find the value of $\lim_{x\to \infty} \dfrac{x^4 - x + \sin x}{-x^3 - 3x^4 - 2\cos x}$.

   (A) 1

   (B) $-\dfrac{1}{3}$

   (C) $-\dfrac{1}{2}$

   (D) -1

   (E) $-\infty$

INDICATE YOUR ANSWERS ON THE FRONT PAGE IMMEDIATELY.

For your own record, you may also want to indicate your answers on these inside pages.

3. Let

$$f(x) = \begin{cases} 4(x-2) - \dfrac{\sin(8x)}{kx} & \text{if} \quad x < 0 \\ 2(x+k) & \text{if} \quad x \geq 0. \end{cases}$$

Find the value of the constant k so that f is continuous everywhere.

Ⓐ -2

Ⓑ $-\frac{1}{4}$

Ⓒ 0

Ⓓ $\frac{1}{2}$

Ⓔ $\frac{1}{4}$

4. The graph of $f(x) = \dfrac{x}{(x+6)^2}$ has a horizontal tangent line at $x =$

Ⓐ 4

Ⓑ 3

Ⓒ 2

Ⓓ 5

Ⓔ 6

INDICATE YOUR ANSWERS ON THE FRONT PAGE IMMEDIATELY.

5. A ball is being thrown vertically upward so that its height (above ground) t seconds after it is thrown is $(25 + 16t - 16t^2)$ feet. What is the maximum height (above ground) attained by the ball?

   (A) 32 feet

   (B) 31 feet

   (C) 30 feet

   (D) 28 feet

   (E) 29 feet

6. If $xy^3 + xy = 6$, find the value of $\frac{dy}{dx}$ at the point where $x = 3$, $y = 1$.

   (A) $\frac{1}{2}$

   (B) 0

   (C) $\frac{1}{3}$

   (D) $-\frac{1}{6}$

   (E) $-\frac{1}{4}$

INDICATE YOUR ANSWERS ON THE FRONT PAGE IMMEDIATELY.

7. Find the value of $\lim_{x\to\infty} \dfrac{2\sinh x + \cosh x}{e^x}$.

   Ⓐ 3

   Ⓑ 0

   Ⓒ $\dfrac{3}{2}$

   Ⓓ -1

   Ⓔ undefined

8. A rectangular box has a square base. If the height of the box is increasing at 3 cm/min and each edge of its base is increasing at 2 cm/min, how fast will the volume of the box be increasing when the height of the box is 4 cm and the area of its base is 9 sq cm?

   Ⓐ 85 cc/min.

   Ⓑ 65 cc/min.

   Ⓒ 80 cc/min.

   Ⓓ 75 cc/min.

   Ⓔ 70 cc/min.

INDICATE YOUR ANSWERS ON THE FRONT PAGE IMMEDIATELY.

9. Find the number c which satisfies the conclusion of the Mean Value Theorem for the function $f(x) = \frac{1}{x}$ on $[1, 3]$.

   Ⓐ $\frac{3}{2}$

   Ⓑ $\sqrt{3}$

   Ⓒ 2

   Ⓓ $\frac{5}{2}$

   Ⓔ $\frac{5}{4}$

10. The function $f(x) = (x^2 - 4)^2$ has a local maximum at $x =$

    Ⓐ 0

    Ⓑ 2

    Ⓒ $-\sqrt{2}$

    Ⓓ -2

    Ⓔ $\sqrt{2}$

INDICATE YOUR ANSWERS ON THE FRONT PAGE IMMEDIATELY.

11. How many points of inflection does the graph of $y = x^6 + x^4 + x^2 - 5x - 4$ have?

   (A) two

   (B) one

   (C) none

   (D) three

   (E) more than three

12. The graph of $y = x^4 - 6x^3 - 3x + 4$ is concave downward on

   (A) $(-\infty, 0) \bigcup (3, \infty)$

   (B) $(0, 3)$

   (C) $(2, 5)$

   (D) $(-2, 1)$

   (E) $(1, 4)$

INDICATE YOUR ANSWERS ON THE FRONT PAGE IMMEDIATELY.

13. The graph of $y = \dfrac{3x^3 + x^2 - 7x - 4}{x^2 + 2x + 1}$ has one vertical asymptote and one other asymptote. This other asymptote is the line

Ⓐ $y = 3x + \dfrac{1}{2}$

Ⓑ $y = 3x + 1$

Ⓒ $y = 3x$

Ⓓ $y = 3x - 5$

Ⓔ $y = 3x - 4$

14. The sum of two positive real numbers is 12. What is the smallest possible value of the sum of their squares?

Ⓐ 72

Ⓑ 74

Ⓒ 68

Ⓓ 76

Ⓔ 70

15. If $f(x) = \ln(\ln x)$, find the value of $f'(\frac{1}{e})$.

Ⓐ $\dfrac{1}{e}$

Ⓑ $\sqrt{e}$

Ⓒ $2e$

Ⓓ $-\dfrac{1}{e}$

Ⓔ undefined

INDICATE YOUR ANSWERS ON THE FRONT PAGE IMMEDIATELY.

16. Given that the tangent line to the graph of f at $(0,0)$ has equation $2y = x$ and that f has a horizontal asymptote at ∞ with equation $y = 2$, find the value of $\lim_{x\to 0+} \left\{ \frac{\sin(2x)}{f(x)} - x^2 f(\frac{1}{x}) \right\}$.

Ⓐ $\frac{1}{2}$

Ⓑ 4

Ⓒ 2

Ⓓ 0

Ⓔ not determinable due to insufficient information

17. Let $f(x) = 2^{x+1} - 2^{-x}$. If $g(x) = f^{-1}(x)$, i.e. if g is the inverse function of f, what is the value of $g'(1)$?

Ⓐ $\frac{3}{2\ln 2}$

Ⓑ $\frac{2}{3\ln 2}$

Ⓒ $\frac{1}{2\ln 2}$

Ⓓ $\frac{1}{3\ln 2}$

Ⓔ $\frac{2}{9\ln 2}$

18. Find the value of $\lim_{x\to\frac{\pi}{4}} \frac{\tan 2x}{\cot(\frac{\pi}{4} - x)}$.

Ⓐ 1

Ⓑ $\frac{1}{4}$

Ⓒ $\frac{1}{2}$

Ⓓ 2

Ⓔ undefined

INDICATE YOUR ANSWERS ON THE FRONT PAGE IMMEDIATELY.

19. If $f = \log_x 2$ (i.e. logarithm of 2 with base x), find the value of $\dfrac{d^2y}{dx^2}$ at $x = 2$.

Ⓐ $\dfrac{2+\ln 2}{4(\ln 2)^2}$

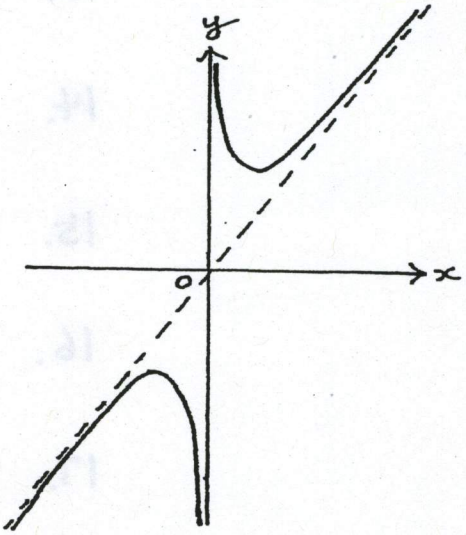
Ⓑ $\dfrac{4+\ln 2}{2(\ln 2)^2}$

Ⓒ $\dfrac{2+\ln 2}{2(\ln 2)^2}$

Ⓓ undefined

Ⓔ $\dfrac{4+\ln 2}{4(\ln 2)^2}$

20.



To which one of the following functions does the above graph correspond?

Ⓐ $f(x) = \dfrac{x^3+1}{x^3}$

Ⓑ $f(x) = \dfrac{x^3+1}{x^2}$

Ⓒ $f(x) = \dfrac{x^4+1}{x^3}$

Ⓓ $f(x) = \dfrac{x^5+1}{x^3}$

Ⓔ $f(x) = \coth x$