MAT 237, Problem Session 6

Solutions for these problems are only presented during the Problem Solving Sessions W5-6 in SS 2135. You are strongly encouraged to work through the problems ahead of time, and our TA Yiannis will cover the questions you are most interested in. These sessions are very valuable at developing the proper style to present cogent and rigorous mathematical solutions.

This problem solving session contains material from 3.1-3.3.

Problems:

1. §3.1: In each of the following cases consider the relation $F(x, y) = 0$, and a point (a, b). Follow the statement of the Implicit Function, Theorem 3.1, step by step and try to find numbers r_0, r_1, show that that a and b of the theorem hold and demonstrate what the function $y = f(x)$ would be, and finally determine $f'(x)$. In each case draw the diagram of figure 3.1.

   a) The linear equation $F(x, y) = x - 3y - 5 = 0$ and $(a, b) = (8, 1)$.

   b) $F(x, y) = 2x^2 + y^2 - 1 = 0$ and $(a, b) = (0, 1)$. What if we change the point to $(a, b) = (1/\sqrt{2}, 0)$?

2. §3.1: Questions 4 and 5.

3. §3.2: Read question 6 of §3.2 and try to determine a subset of $S = \{(x, y) : (x - y^2)\tan x = 0\}$, which can be a smooth curve. Repeat with $S = \{(x, y) : (x^2 - y^2 - 1)(-x2 + 4y^2 - 1) = 0\}$.

4. §3.3: Curves in R^3:

   a) determine the equation of a line passing through the points $A(1, 2, 3)$ and $B(4, 6, 5)$ in all three representations.

   b) The plane $z = y$ intersects the sphere of radius 1 centered at the origin. Present the equation of the curve of the intersection in three representations. Check the regularity condition for representations (ii) and (iii). (see page 130).