Jan 9th

Vector spaces over a field $\mathbb{F}$:

Definition of a field:

A field is a set $\mathbb{F}$ with two operations
addition
multiplication which obey the following rules
1) $+$ has an additive identity denoted by $0_{\mathbb{F}}$.
2) Commutativity of addition:
$x+y=y+x$ for any "scalars" x, y in $\mathbb{F}$.
3). each $x \in \mathbb{F}$ admits an additive inverse $-x$.
4). $\mathbb{F}$ admits a multiplicative identity $1_{\mathbb{F}}$
5). For any $x \in F \setminus \{0\}$ x admits a multiplicative inverse x^{-1}
6). $x(y+z) = xy + xz$ for any $x, y, z \in F$
7) $x(yz) = (xy)z$ (Associativity)
8). $xy = yx$ for any $x, y \in F$

Definition: If F is a field then one calls the elements of F as scalars.

Definition of a vector space over a field F:

A vector space is a set V with 2 operations $+$ and "multiplication by scalars" (elements of F)
Elements of V are called vectors.
scalar multiplication need an $\alpha \in F$ and a vector $x \in V$ to produce a new vector αx.
Further more those 2 operations must satisfy
1) $(x+y)+z = x+(y+z)$ for any x, y, z. vectors in V.
2) There exists $0 \in V$ such that $x+0=0+x=x$ for any $x \in V$.
3). $x+y=y+x$
4). $\lambda(x+y) = \lambda x + \lambda y$ for $\lambda \in F$ & $x, y \in V$
5). $\underbrace{1}_{\text{scalar}} \cdot \underbrace{x}_{\text{vector}} = x$ for any vector $x \in V$

6). $(c+d)x = cx + dx$, $\forall c, d \in F$, & $\forall x \in V$.

7). $(cd)x = c(dx)$, $\forall c, d \in F$ & $\forall x \in V$.

8). $x + (-x) = 0$ $\forall x \in V$.

Examples: ① $\mathbb{R}^2$ is a V-S over $\mathbb{R}$.
② $\mathbb{F}^m$ is a V-S over $\mathbb{F}$
For any $m = 1, 2, 3, \ldots$
$F^m = \{(x_1, x_2, \ldots, x_m)$ s.t. $x_i \in F, i = 1, 2, \ldots, m\}$

③ $\mathbb{C}$ is a vector space over $\mathbb{C}$
④ $\mathbb{C}$ is a vector space over $\mathbb{R}$
an element $z \in \mathbb{C}$ can be viewed as:
$z = a + bi = (a, b)$ where $a \in \mathbb{R}$. $b \in \mathbb{R}$.
$\{1, i\}$ is a basis of $\mathbb{C}$ over $\mathbb{R}$.