STAT3032 SURVIVAL MODELS

SOLUTIONS TO TUTORIAL WEEK THREE

1.

$l_{40} = l_{35}\, p_{35}\, p_{36} \cdot {}_3 p_{37}$
$= l_{35} \left(1 - q_{35}\right)\left(1 - q_{36}\right) {}_3 p_{37}$

$$\therefore l_{35} = \frac{l_{40}}{\left(1 - q_{35}\right)\left(1 - q_{36}\right) {}_3 p_{37}}$$

Also ${}_{2|3}q_{35} = p_{35} \cdot p_{36}\left(1 - {}_3 p_{37}\right)$

$$\therefore \left(1 - {}_3 p_{37}\right) = \frac{{}_{2|3}q_{35}}{p_{35} \cdot p_{36}} = \frac{0.19}{\left(1 - 0.05\right)\left(1 - 0.06\right)} = 0.21277$$

$$l_{35} = \frac{55444}{\left(1 - 0.05\right)\left(1 - 0.06\right)\left(1 - 0.21277\right)} = 78868$$

2.

$$\overset{0}{e}_{45} = \int_0^\infty {}_t p_{45}\, dt$$

$$= \int_0^\infty \left(\frac{1 + 45}{1 + 45 + t}\right)^3 dt$$

$$46^3 \int_0^\infty \frac{1}{(46 + t)^3}\, dt$$

$$= 46^3 \left[\frac{-1}{-2(46 + t)^2}\right]_0^\infty$$

$= 23$ years

3.

Using the formula derived in lectures, ${}_t p_x = \exp(-\int_0^t \mu_{x+r} dr),$ and noting that the probability of a 50 year old dying between 70 and 80 is ${}_{20}p_{50}(1-{}_{10}\,p_{70}),$ we get

$${}_{20|10}q_{50} = e^{-20(0.048790)}.(1-e^{-10(0.048790)})$$

$$= 0.14551$$

4.

First calculate $l_{100} = \sum_{x=100}^{109} d_x = 606.$

Standard deviation of K_{100} is $\sqrt{\mathrm{E}(\mathrm{K}_{100}^2) - [E(K_{100})]^2}$

$$E(K_{100}^2) = \sum_{k=1}^{9} k^2 \frac{d_{x+k}}{l_x} = 8.4043$$

$$E(K_{100}) = \sum_{k=1}^{9} k \frac{d_{x+k}}{l_x} = 1.9818$$

Standard deviation of $\mathrm{K}_{100} = 2.116$ years

5.

Since ${}_t p_x = e^{-\int_0^t \mu_{x+s} ds}$

The probability of a light surviving from t-1 to t, given that it reaches t-1 is:

$$\exp(-0.25\frac{\mathrm{t}^3}{3} - 0.4\frac{t^2}{2} + 0.25\frac{(t-1)^3}{3} + 0.4\frac{(t-1)^2}{2})$$

This gives probabilities of:

Week	Prob of surviving the week, given that it starts the week
1	0.7533
2	0.3063
3	0.0755
4	0.0113
5	0.0010
6	0.0001

Therefore, at the end of the first week the number of bulbs replaced is equal to 2000*(1-0.7533) ie 493.4 bulbs replaced.

At the end of the second week, the number of surviving bulbs will be 493.4*0.7533 + 1506.6*0.3063, ie 833.2. Therefore the number replaced will be 1167.

At the end of the third week, the number of surviving bulbs will be 1166.8*0.7533+493.4*0.7533*0.3063+461.5*0.0755, ie 1027.64. Hence 972 will be replaced.

At the end of the fourth week, the number of surviving bulbs will be 972.4*0.7533+1166.8*0.7533*0.3063+493.4*0.7533*0.3063*0.0755+2000*.7533*.3063*.0755*.0113 = 1011. Therefore 989 will be replaced.

(b) The expectation will be approximately, the sum of the probabilities ($\sum_t {}_t p_0$) . Therefore the expected life is 1.00165 weeks.

6.
(a) We can calculate the expectation of life at age 1 using

$$\overset{0}{e}_1 = \int_0^\infty {}_t p_1 dt$$

Now here ${}_t p_1$ is 1-0.005$(1+t)\log_{10}(1+t)$

So $\overset{0}{e}_1 = \int_0^{99} 1 - 0.005(1+t)\log_{10}(1+t)dt$

$$= [t]_0^{99} - 0.005\left\{\left[\frac{(1+t)^2}{2}\log_{10}(1+t)\right]_0^{99} - \int_0^{99} \frac{1}{\ln 10}\frac{(1+t)^2}{2(1+t)}dt\right\}$$

$$= 99 - 0.005\left[5000\log_{10}(100)\right] + \frac{0.005}{2\ln(10)}\left[t + \frac{t^2}{2}\right]_0^{99}$$

$$= 54.428 \text{ years}$$

(b) Time lived between ages 1 and 20 $= \int_0^{19} 1 - 0.005(1+t)\log_{10}(1+t)dt$

$$= [t]_0^{19} - 0.005\left\{\left[\frac{(1+t)^2}{2}\log_{10}(1+t)\right]_0^{19} - \int_0^{19}\frac{1}{\ln 10}\frac{(1+t)^2}{2(1+t)}dt\right\}$$

$$= 19 - 0.005[200\log_{10}(20)] + \frac{0.005}{2\ln(10)}\left[t + \frac{t^2}{2}\right]_0^{19}$$

$= 17.916$ years

Since we are interested only in those who die between 1 and 20 we should remove the time lived by those who survive to get :

$17.916l_1 - 19l_{20}$ years lived by those who die before age 20

The average age at death is therefore given by the number of years lived by those who die before age 20 divided by the total number who died by age 20. This is the number of years lived after age 1. So to convert to an average age at death we need to add 1 to get :

$$1 + \frac{17.916l_1 - 19l_{20}}{l_1 - l_{20}}$$

$= 11.67$