Informed search algorithms

Chapter 3, Sections 5–6

Summary

◇ **Goal-based agents**
generate a sequence of actions from an initial to a goal state

◇ **Problem formulation**
initial state, successor function, goal test, path cost

◇ **Tree search algorithm**
build and explore a tree, strategy picks up the order of node expansion

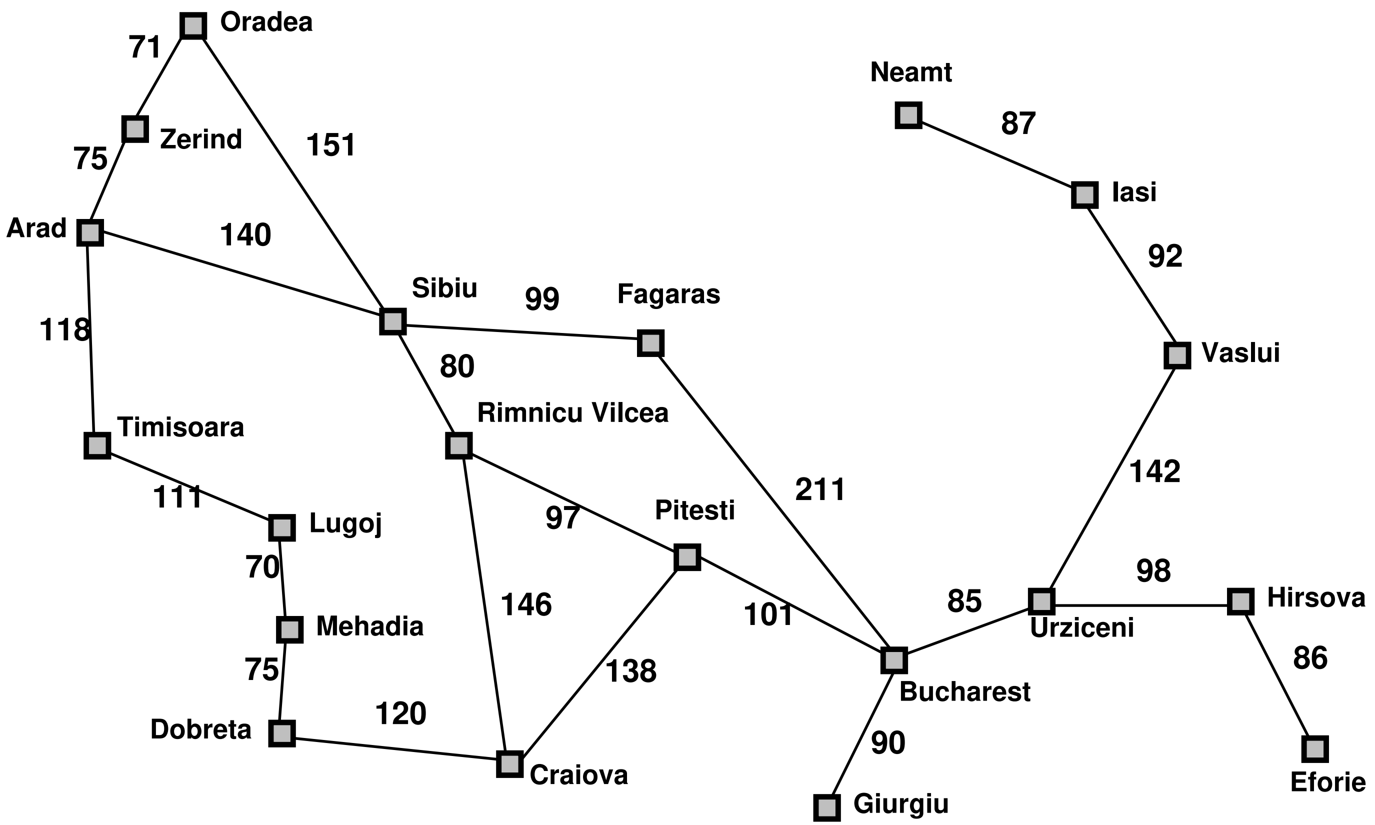
◇ **(Uninformed) strategies** (breadth first, uniform cost, ...)
different ways of ordering nodes on the frontier = priority queue

◇ **Today: Informed strategies**
use problem-specific information given by a heuristic function

Heuristic Function

Estimates the cost from a given state to the goal

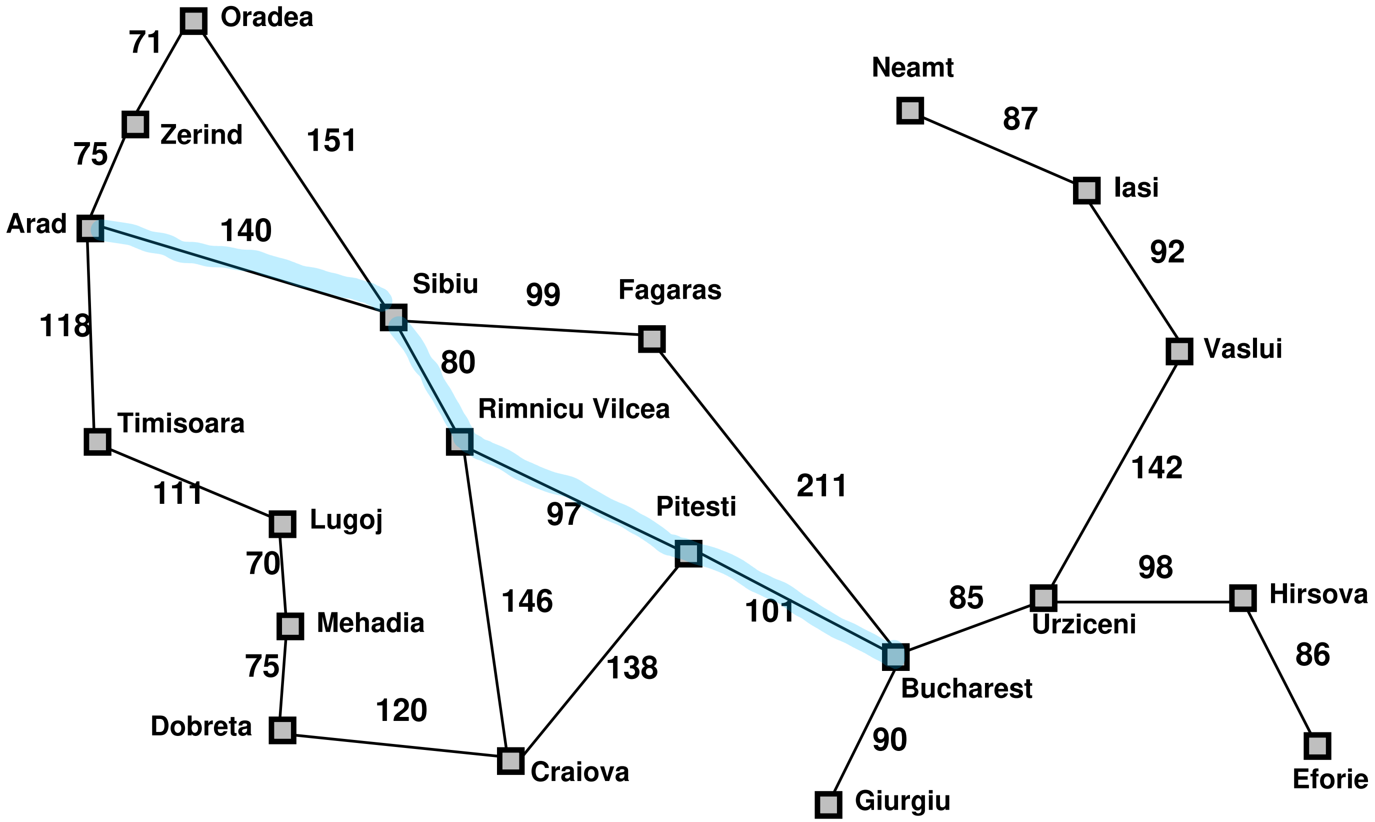
Estimate for the Travel in Romania example??



Heuristic Function

Estimates the cost from a given state to the goal

Estimate for the Travel in Romania example?? Straight Line Distance



Straight−line distance
to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
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Zerind	374

Outline

- ◇ Evaluation functions (use heuristics)
- ◇ Greedy search
- ◇ A* search
- ◇ Designing heuristics
- ◇ Graph search

Review: Tree search

function TREE-SEARCH(*problem, frontier*) **returns** a solution, or failure
    frontier ← INSERT(MAKE-NODE(INITIAL-STATE[*problem*]), *frontier*)
    loop do
        if *frontier* is empty **then return** failure
        node ← REMOVE-FRONT(*frontier*)
        if GOAL-TEST(*problem*, STATE(*node*)) **then return** *node*
        frontier ← INSERTALL(EXPAND(*node*, *problem*), *frontier*)

Note: the goal test is performed when the node is popped from the frontier, **NOT when it is generated during expansion.** This is important when looking for optimal solutions.

A strategy is defined by picking the **order of node expansion**

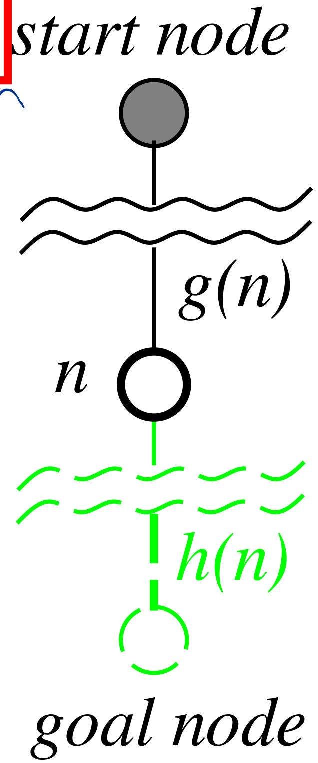
it's about what to expand next?

Evaluation function

Evaluation function $f(n) = g(n) + h(n)$
– estimate of "desirability", usually problem-specific.

any approach to problem solving, learning, or discovery that employs a practical method not guaranteed to be optimal or perfect, but sufficient for the immediate goals.

$g(n)$ = cost so far to reach n
$h(n)$ = estimated cost from n to the closest goal (heuristic)
$f(n)$ = estimated total cost of path through n to goal



cost in the past

cost in the future

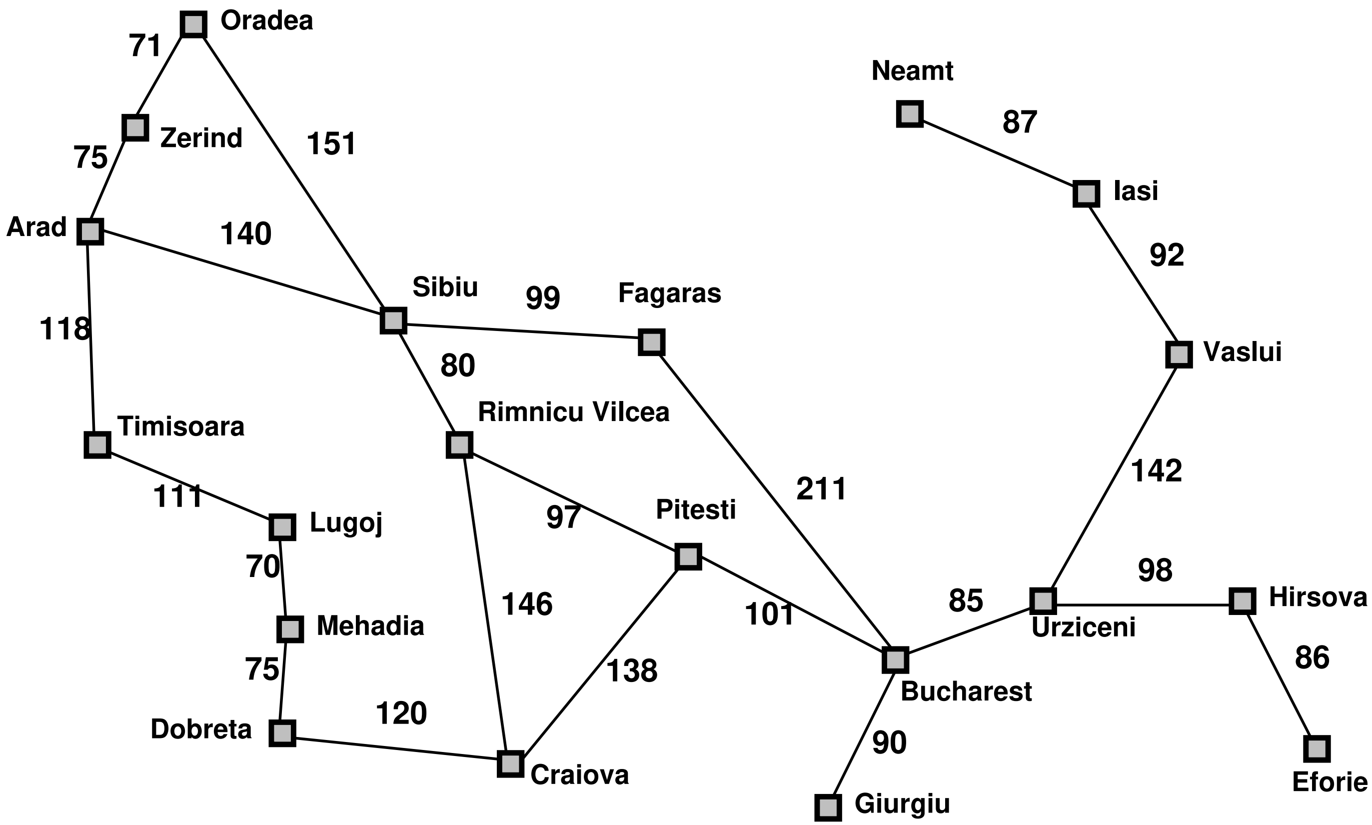
total cost

The lower $f(n)$, the more desirable n is

Implementation:
frontier is a queue sorted in ascending value of $f(n)$

Special cases:
uniform cost search (uninformed): $f(n) = g(n)$
greedy search (informed) $f(n) = h(n)$
A* search (informed) $f(n) = g(n) + h(n)$

Romania with step costs in km



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Greedy search

Evaluation function $f(n) = h(n)$ (entirely heuristic)
$= $ estimate of cost from n to the closest goal

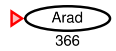
E.g., $h_{\mathrm{SLD}}(n) =$ straight-line distance from n to Bucharest

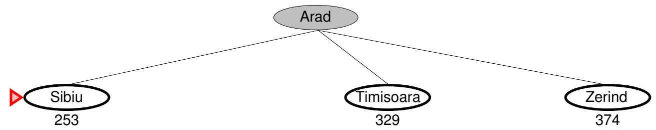
Greedy search expands the node that **appears** to be closest to goal

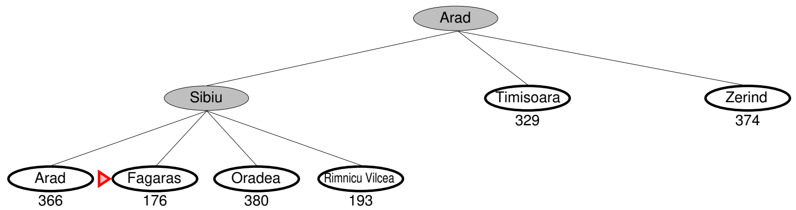
sometimes, not necessarily "really close".

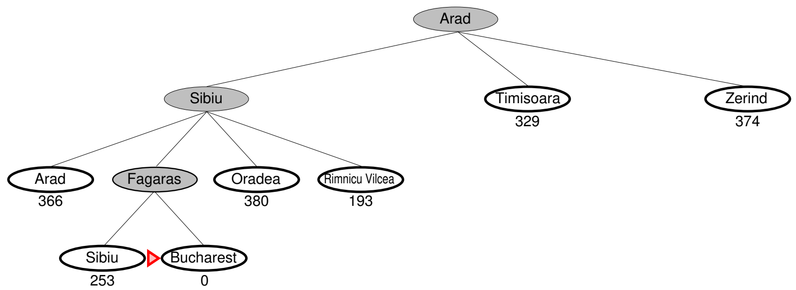
Greedy always pays attention to ~~the~~ ~~things~~ ahead.

Greedy search example









Properties of greedy search

Complete?? No–can get stuck in loops, e.g., with going from Iasi to Fagaras,

Iasi $\rightarrow$ Neamt $\rightarrow$ Iasi $\rightarrow$ Neamt $\rightarrow$

Complete in finite space with repeated-state checking

Time?? $O(b^m)$, but a good heuristic can give dramatic improvement

Space?? $O(b^m)$

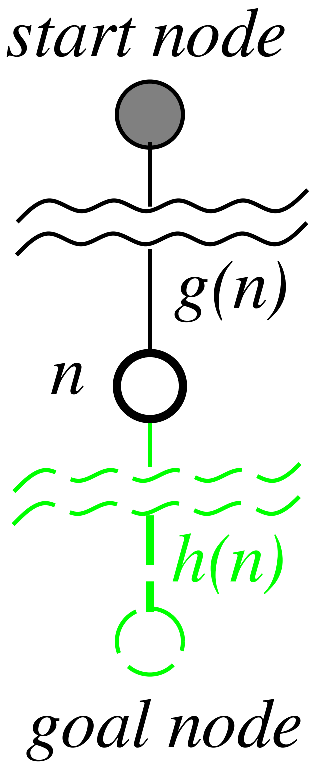
Optimal?? No

might choose a seemingly optimal solution
will go through all the bad decisions.

A* search

APPROVED

Idea: avoid expanding paths that are already expensive



Evaluation function $f(n) = g(n) + h(n)$

$g(n)$ = cost so far to reach n
$h(n)$ = estimated cost from n to the closest goal
$f(n)$ = estimated total cost of path through n to goal

Admissible heuristic:
$\forall n \; h(n) \leq h^*(n)$ where $h^*(n)$ is the **true** cost from n.
(Also require $h(n) \geq 0$, so $h(G) = 0$ for any goal G.)

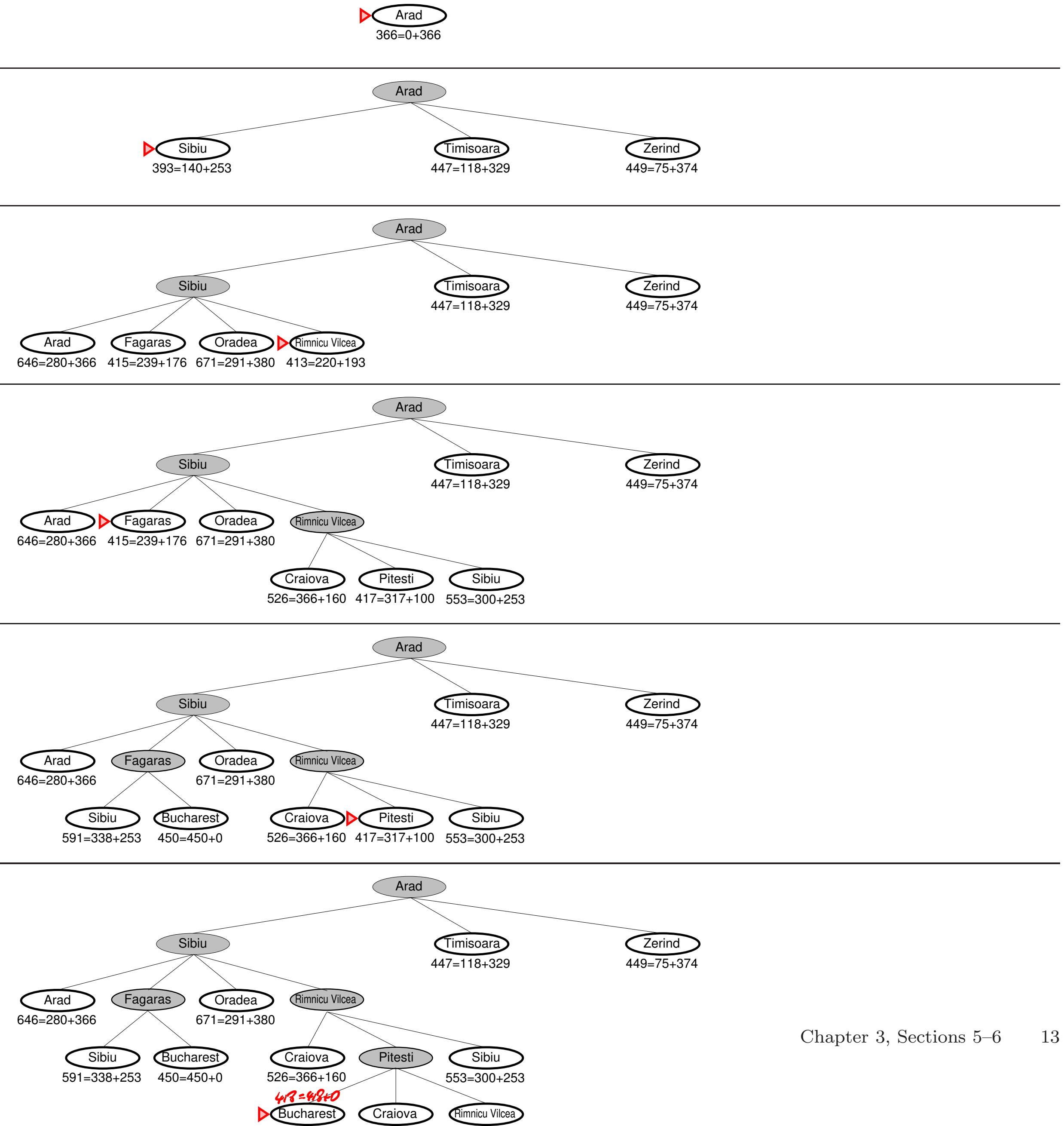
E.g., $h_{\mathrm{SLD}}(n)$ never overestimates the actual road distance

When $h(n)$ is admissible, $f(n)$ never overestimates the total cost of the shortest path through n to the goal

Theorem: if h is admissible, A* search finds the optimal solution

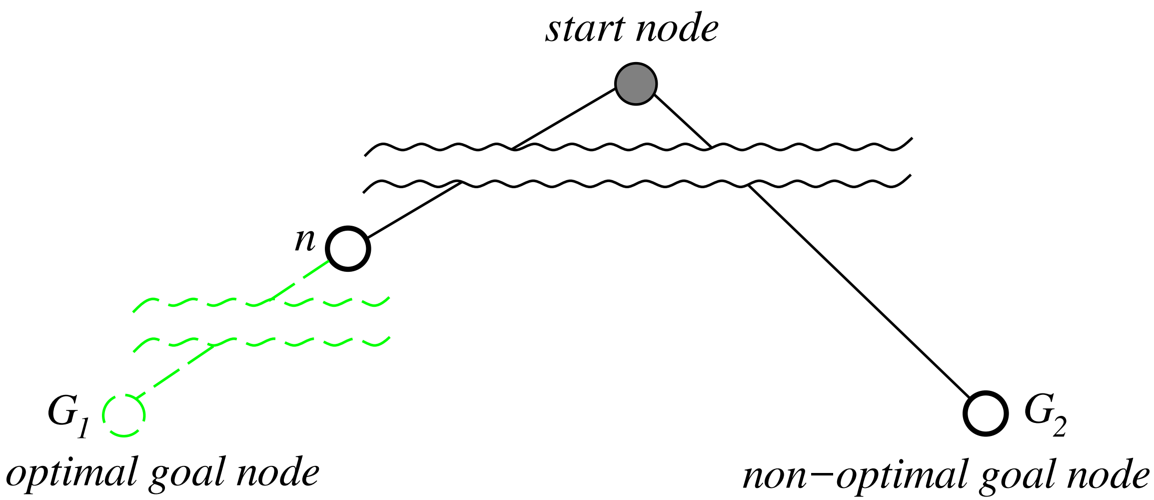
admissible $\Rightarrow$ optimal
(no overestimation)

A* search example



Optimality of A* (based on admissibility)

Suppose some suboptimal goal G_2 has been generated and is in the frontier. Let n be a frontier node on a shortest path to an optimal goal G_1.



$$
\begin{aligned}
f(G_2) &= g(G_2) && \text{since } G_2 \text{ is a goal node, hence, } h(G_2) = 0 \\
&> g(G_1) && \text{since } G_2 \text{ is suboptimal} \\
&\geq f(n) && \text{since } h \text{ is admissible, } f(n) \text{ does not overestimate } g(G_1)
\end{aligned}
$$

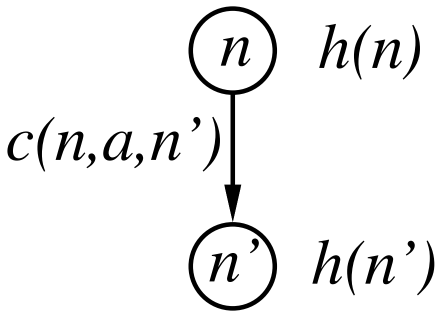
Since $f(G_2) > f(n)$, A* will never select G_2 for expansion

See next page.

Consistency

A heuristic is consistent if

$$h(n) - h(n') \leq c(n, a, n')$$



If h is consistent, then h is admissible, and $f(n)$ is nondecreasing along any path:

$$\begin{aligned} f(n') &= g(n') + h(n') \\ &= g(n) + c(n, a, n') + h(n') \\ &\geq g(n) + h(n) \\ &= f(n) \end{aligned}$$

Consequently, when expanding a node, we cannot get a node with a smaller f, and so the value of the best node on the frontier will never decrease.

otherwise, how could I say it is the "best".

When we dequeue a node labelled with a new state, we found the shortest path to that state: any other node n' labeled with the same state satisfies $f(n) \leq f(n')$ and $h(n) = h(n')$, hence $g(n) \leq g(n')$.

Consistency

(when referring A^*)

A heuristic is consistent if

$$h(n) \leq c(n, a, n') + h(n')$$

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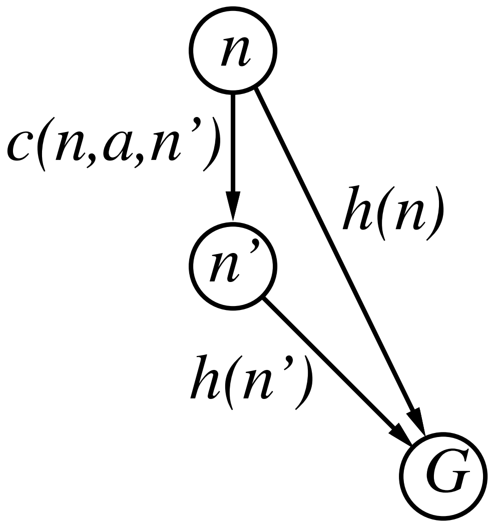

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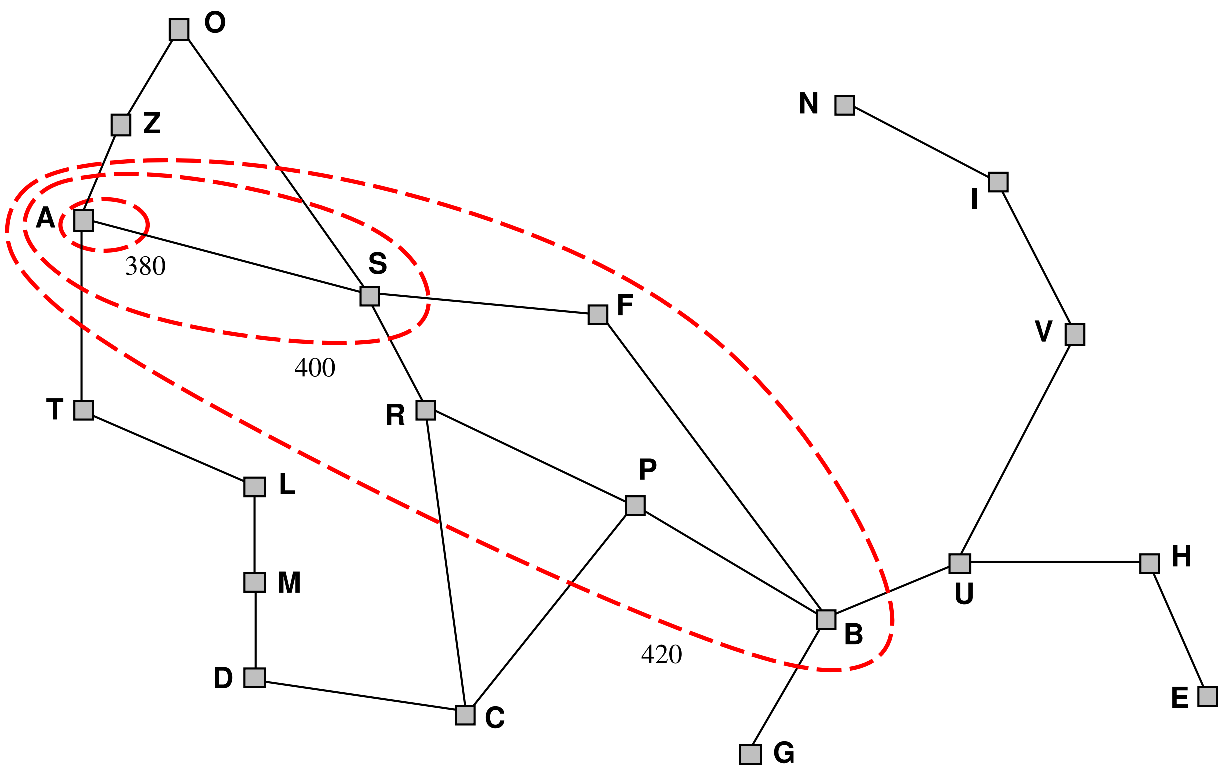
Optimality of A* (based on consistency)

Consistency: A* expands nodes in order of increasing f value

Gradually expands "f-contours" of nodes

Contour i has all nodes with $f = f_i$, where $f_i < f_{i+1}$

(breadth-first expands layers, uniform-cost expands g-contours)



Properties of A*

A* expands all nodes with $f(n) < C^*$
A* expands some nodes with $f(n) = C^*$ (but only one goal node)
A* expands no nodes with $f(n) > C^*$

Complete?? Yes, unless there are infinitely many nodes with $f \leq C^*$

Time?? Exponential in [relative error in h $\times$ length of soln.]

Space?? Exponential

Optimal?? Yes—cannot expand f_{i+1} until f_i is finished

IDA* is an version of iterative deepening with a cutoff on f

Admissible heuristics

E.g., for the 8-puzzle:

$h_1(n)$ = number of misplaced tiles
$h_2(n)$ = total Manhattan distance
(i.e., no. of squares from desired location of each tile)

7	2	4
5		6
8	3	1

Start State

1	2	3
4	5	6
7	8	

Goal State

$h_1(S) =$?? 6
$h_2(S) =$?? 4+0+3+3+1+0+2+1 = 14

Dominance

Given two admissible heuristics h_a, h_b,
if $h_b(n) \geq h_a(n)$ for all n then h_b dominates h_a and is better for search

In the 8-puzzle h_2 dominates h_1. Typical search costs:

$d = 14$	IDS $=$ 3,473,941 nodes	$d = 24$	IDS $\approx$ 54,000,000,000 nodes
	A$^*(h_1) =$ 539 nodes		A$^*(h_1) =$ 39,135 nodes
	A$^*(h_2) =$ 113 nodes		A$^*(h_2) =$ 1,641 nodes

There is a tradeoff between the accuracy of h and the time to compute h

Given two admissible heuristics h_a, h_b,

$$h(n) = \max(h_a(n), h_b(n))$$

is also admissible and dominates h_a, h_b

Is $h_a(n) + h_b(n)$ admissible??

Relaxed problems

Admissible heuristics can be derived from the **optimal** solution cost of a **relaxed** version of the problem

Key point: the optimal solution cost of a relaxed problem is no greater than the optimal solution cost of the real problem

Rules of the 8-puzzle:

> a tile can move from square A to square B if A is adjacent to B and B is blank; get all tiles in their correct positions.

If we relax the rules so that a tile can move **anywhere**, then $h_1(n)$ gives the shortest solution

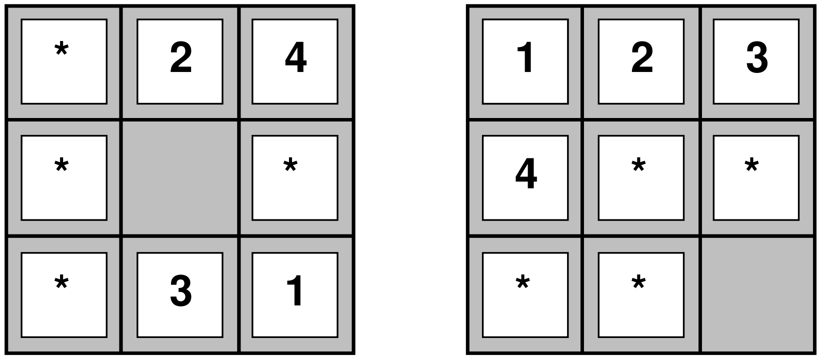
If we relax the rules so that a tile can move to **any adjacent square**, then $h_2(n)$ gives the shortest solution

Relaxed problems

Rules of the 8-puzzle:

> a tile can move from square A to square B if A is adjacent to B and B is blank; get all tiles in their correct positions.

Relaxing the rules so that only **some** tiles need to get in their correct positions and solving the relaxed problem optimally yields another admissible heuristic

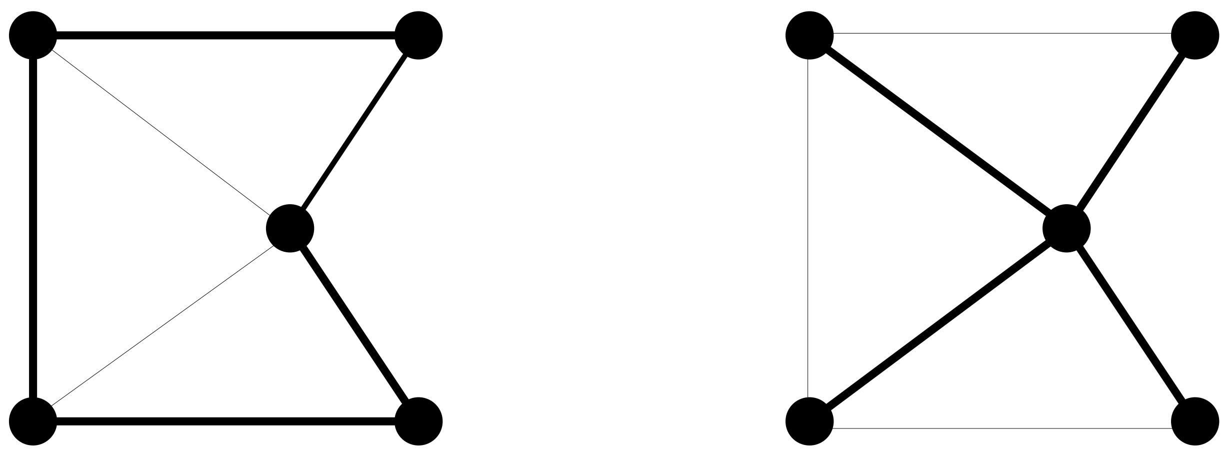


Start State **Goal State**

Heuristics derived from the cost of an optimal solution to a smaller subproblem are used in pattern databases to store solutions for every possible subproblem up to a given size.

Relaxed problems

Well-known example: travelling salesperson problem (TSP)
Find the least-cost tour visiting all cities exactly once



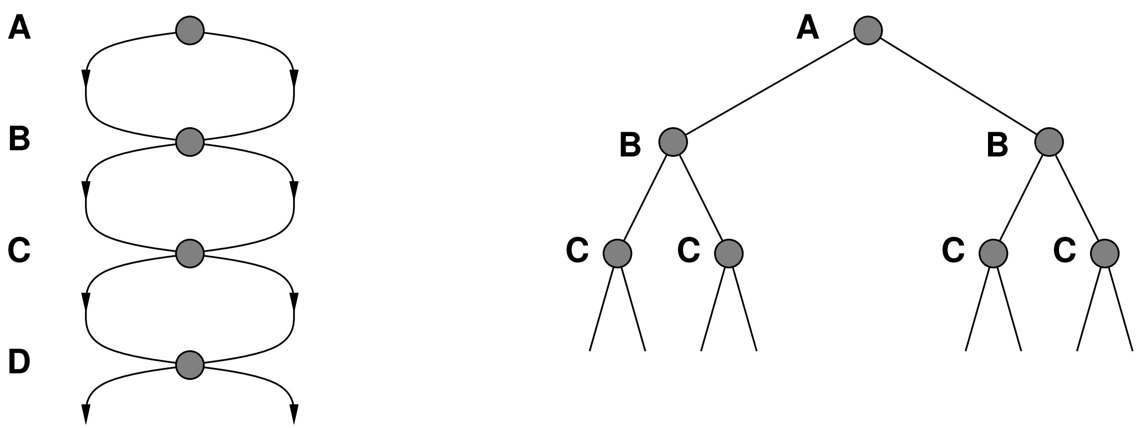
Minimum spanning tree can be computed in $O(n^2)$
and the sum of the edge costs in an MST is a lower bound on the optimal (open) tour cost

Tree Search and Repeated States

♢ For many problems, the state space is a graph rather than a tree

♢ Cycles can prevent termination

♢ Failure to detect repeated states can turn a linear problem into an exponential one!



Graph search

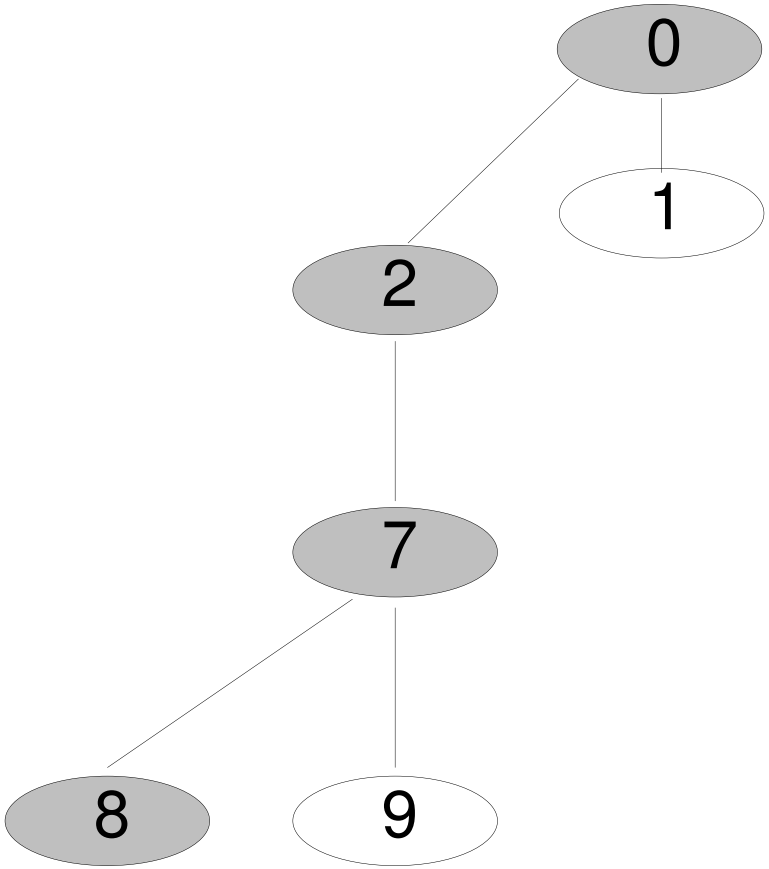
```
function GRAPH-SEARCH( problem, frontier) returns a solution, or failure
    explored ← an empty set of nodes
    frontier ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), frontier)
    loop do
        if frontier is empty then return failure
        node ← REMOVE-FRONT(frontier)
        if GOAL-TEST(problem, STATE[node]) then return node
        add node to explored
        frontier ← INSERTNODES(EXPAND(node, problem), frontier)
```

```
function INSERTNODES( nodes, frontier) returns updated frontier
    for each n in nodes do
      if ∄ m in explored ∪ frontier s.t. STATE[m] = STATE[n] then
        add n to frontier
    return frontier
```

At most one instance of each state in $explored \cup frontier$. All expanded nodes kept in memory!!

Graph search and Optimality

♢ When seeking optimal solutions, mutliple paths to the same state may need to be explored and compared to find the optimal



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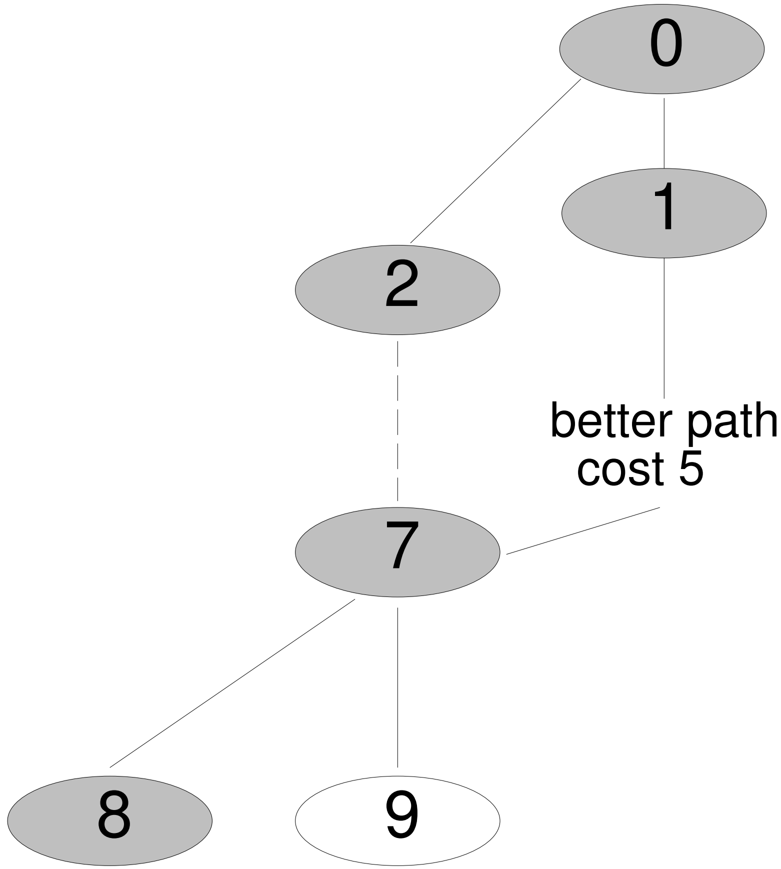
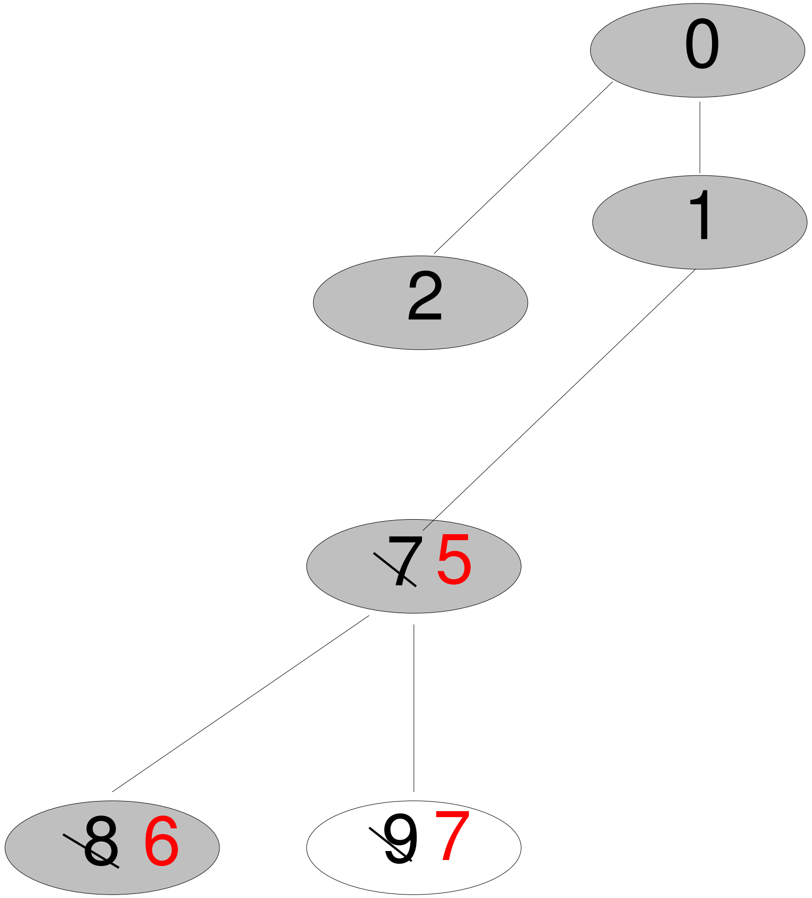


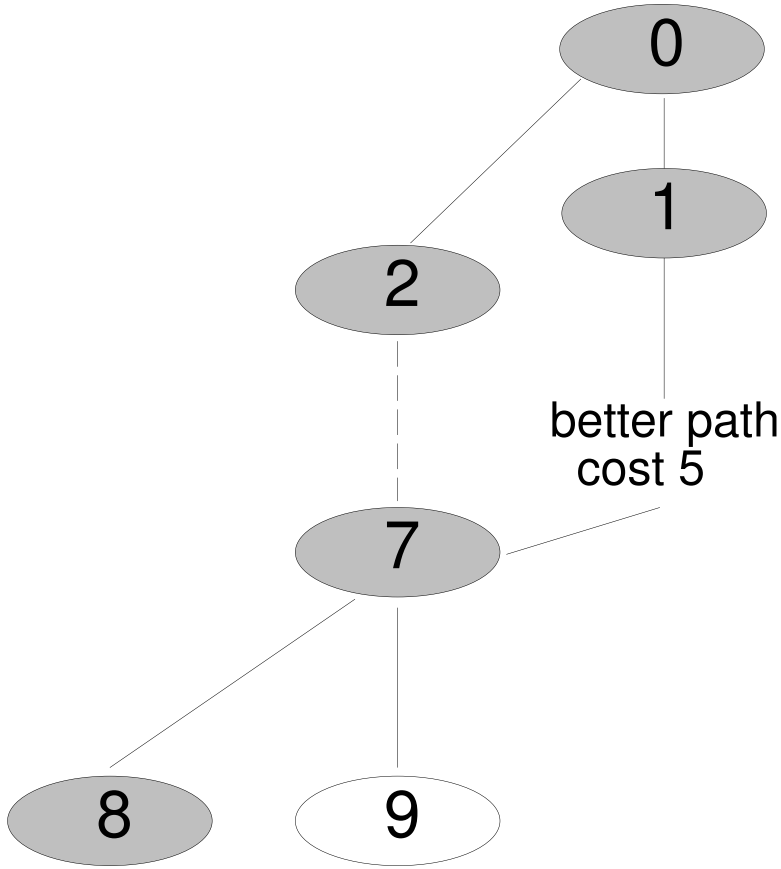
Graph search and Optimality

♢ We may need to keep the better node and update the depths and path-costs of the descendants of the worst one.



Graph search and Optimality

♢ Trick to avoid updating descendants: re-open the explored node; its descendants will be updated when it is re-expanded.



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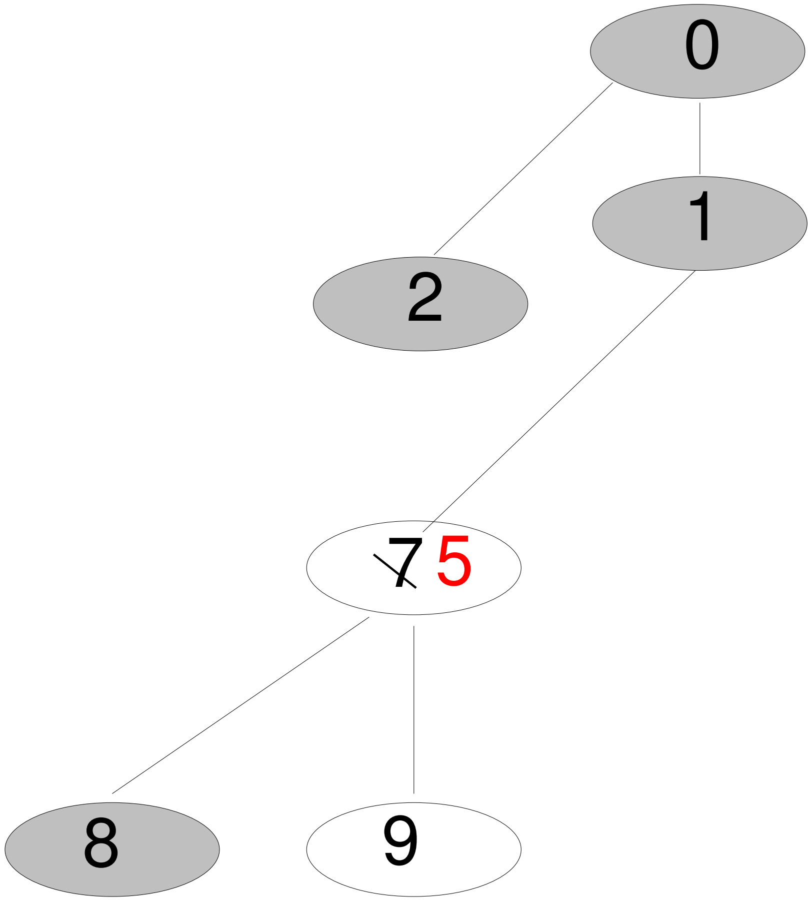

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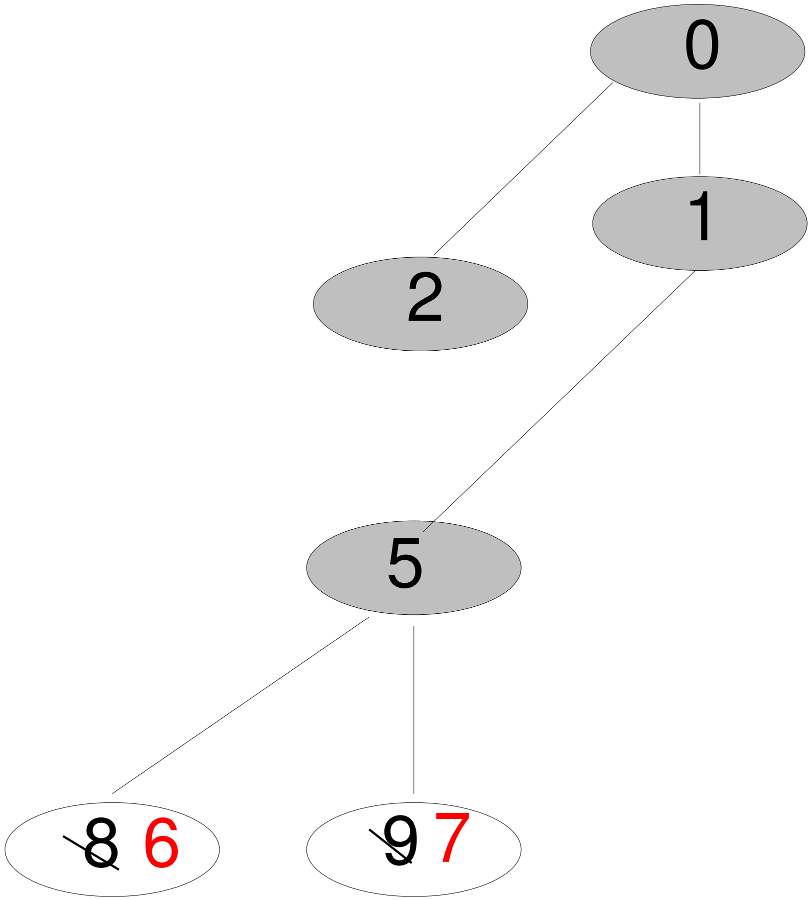

Graph search and Optimality

function INSERTNODES(*nodes, frontier*) **returns** updated frontier

    for each n in *nodes* **do**

      if $\nexists$ m in *explored* $\cup$ *frontier* s.t. STATE[m] $=$ STATE[n] **then**

        add n to *frontier*

      else if PATH-COST[n] $<$ PATH-COST[m]

        PATH-COST[m] $\leftarrow$ PATH-COST[n]

        PARENT[m] $\leftarrow$ PARENT[n]

        ACTION[m] $\leftarrow$ ACTION[n]

        DEPTH[m] $\leftarrow$ DEPTH[n]

        if m in *explored* **then**

          move m back to *frontier* // reopen m

    return *frontier*

The extra square is needed with A*.

It is also needed with uniform cost unless step costs are equal.

If h is consistent (not just admissible), no re-opening (last 2 lines) is needed.

$h = 0$ is consistent so no reopening is needed with uniform cost.

Which algorithm and strategy to use

strategy	**solution**	**useful when**	**frontier**	**algorithm and state space**
DFS	arbitrary	many solutions exist	LIFO	tree-search for finite acyclic graphs recursive algorithm for finite acyclic graphs add cycle detection for finite graphs
BFS	shortest	shallow solutions exist	FIFO	tree search graph search (simple) may improve performance
UC	optimal	good admissible heuristic lacking	priority queue ordered by g	tree search for trees graph-search (simple) for equal step costs graph-search (optimal) for arbitrary step costs (no reopening needed)
A*	optimal	good admissible heuristics exist	priority queue ordered by f	tree search for trees and admissible heuristics graph-search (optimal) for admissible heuristics (no reopening needed for consistent heuristics)
greedy search	arbitrary but maybe good	good (inadmissible) heuristics exist	priority queue ordered by h	tree search for finite acyclic graphs graph search (simple) for finite graphs

Summary

◇ **Heuristic functions** estimate costs of shortest paths
– good heuristics can dramatically reduce search cost

◇ **Greedy best-first** search expands lowest h
– incomplete and not always optimal

◇ **A*** search expands lowest $g + h$
– complete and optimal

◇ **Admissible heuristics** underestimate the optimal cost
– they can be derived from exact solutions to relaxed problems

◇ **Graph search** can be exponentially more efficient than tree search
– often needed to ensure termination and optimality
– stores all expanded nodes and requires extra tests