STAT 6046 Tutorial Week 10

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Today's plan

- Brief review of course material
- Go through selective tutorial questions

Arbitrage

- Arbitrage in financial mathematics is generally described as a risk-free trading profit. Arbitrage opportunity exists if either:

- a) an investor can make a deal that would give her or him an immediate profit, with no risk of future loss; or
- b) an investor can make a deal that has zero initial cost, no risk of future loss, and a possibility of a future profit.

No Arbitrage/Law of One Price

- The concept of arbitrage is very important because we generally assume that in modern developed financial markets arbitrage opportunities don't exist.

- WHY? The arbitrage position would soon disappear however because of the increased demand for the cheaper security and the lack of demand for the more expensive security. This would force the security prices back into line.

- The "No Arbitrage" assumption enables us to find the price of complex instruments by "replicating" the payoffs.

Forward

- A **forward contract** is an agreement made at some time $t = 0$ between two parties under which one agrees to buy from the other a specified amount of an asset (denoted by S) at a specified price on a specified future date.

- The investor agreeing to sell the asset is said to hold a ***short*** forward position in the asset, and the buyer is said to hold a ***long*** forward position.
- K is the **forward price**
- S_T is the **spot price** at time T

- Payoff for short: $K - S_T$
- Payoff for Long: $S_T - K$

Pricing forward contract

- Securities with no income
- **Portfolio A**: Enter a forward contract to buy one unit of an asset S , with forward price K , maturing at time T ; simultaneously invest an amount $Ke^{-\delta T}$ in the risk-free investment. (a constant force of interest of δ)
- **Portfolio B**: Buy one unit of the asset S , at the current price 0 at S_0
- Both get S_T at maturity.

$$S_0 = Ke^{-\delta T}$$

$$K = S_0 e^{\delta T}$$

Pricing forward contract

- Securities with income
- **Portfolio A**: Enter a forward contract to buy one unit of an asset S , with forward price K , maturing at time T ; simultaneously invest an amount $Ke^{-\delta T} + \mathrm{PV}_I$ in the risk-free investment. (a constant force of interest of δ)
- **Portfolio B**: Buy one unit of the asset S , at the current price 0 at S_0
- Both get $S_T + \mathrm{FV}_I$ at maturity.

$$K = (S_0 - \mathrm{PV}_I)e^{\delta T}$$

Forward contract value

- Long forward: At time 0, value=0 (No arbitrage)
- **Portfolio A**: Buy the existing long forward contract for price V_L at time r . Invest $K_0 e^{-\delta(T-r)}$at time r in the risk-free investment for $T-r$ years.
- **Portfolio B**: Buy a new long forward contract maturing at the same date, forward price K_r . Invest $K_r e^{-\delta(T-r)}$ in the risk-free investment for $T-r$ years.
- Both get S_T at maturity.

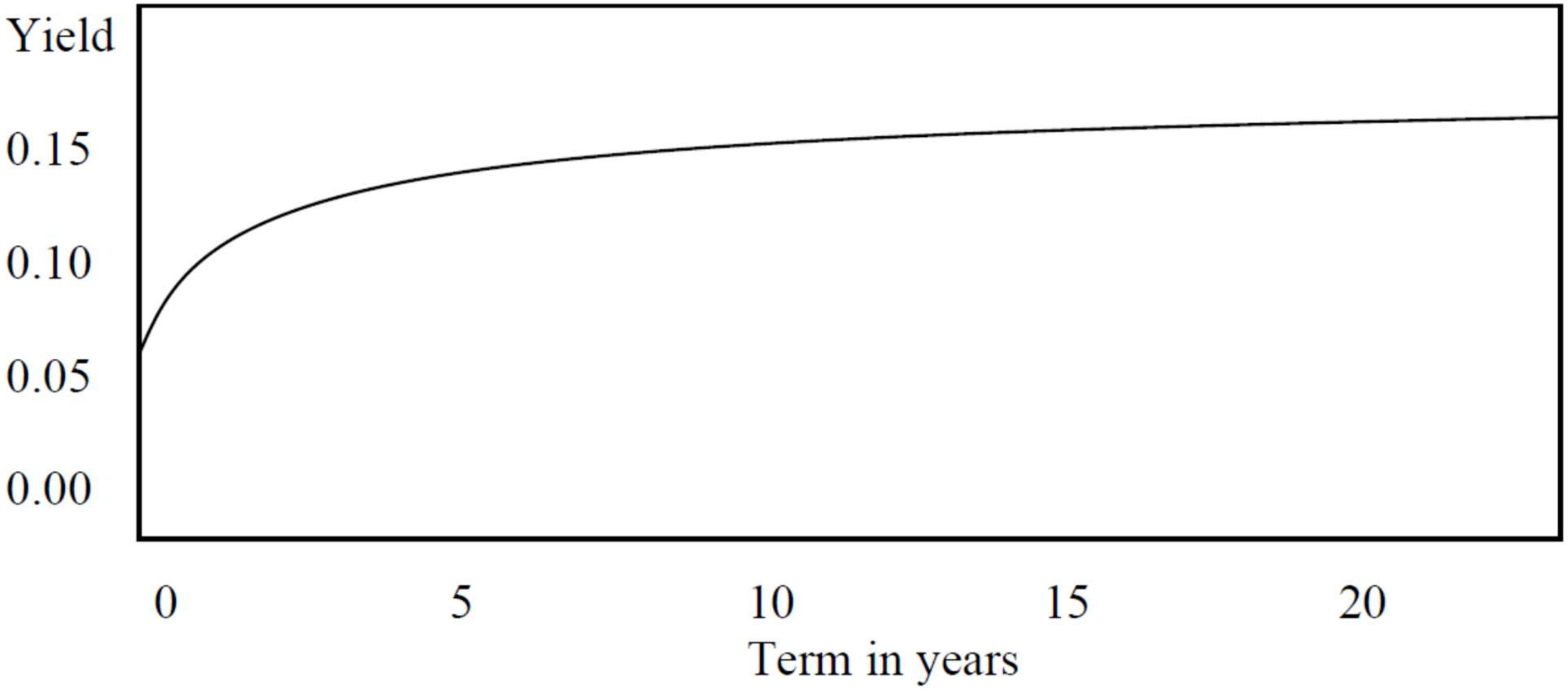
$$\mathrm{V}_L = (K_r - K_0)e^{-\delta(T-r)}$$

$$\mathrm{V}_L = S_r - S_0 e^{\delta r}$$

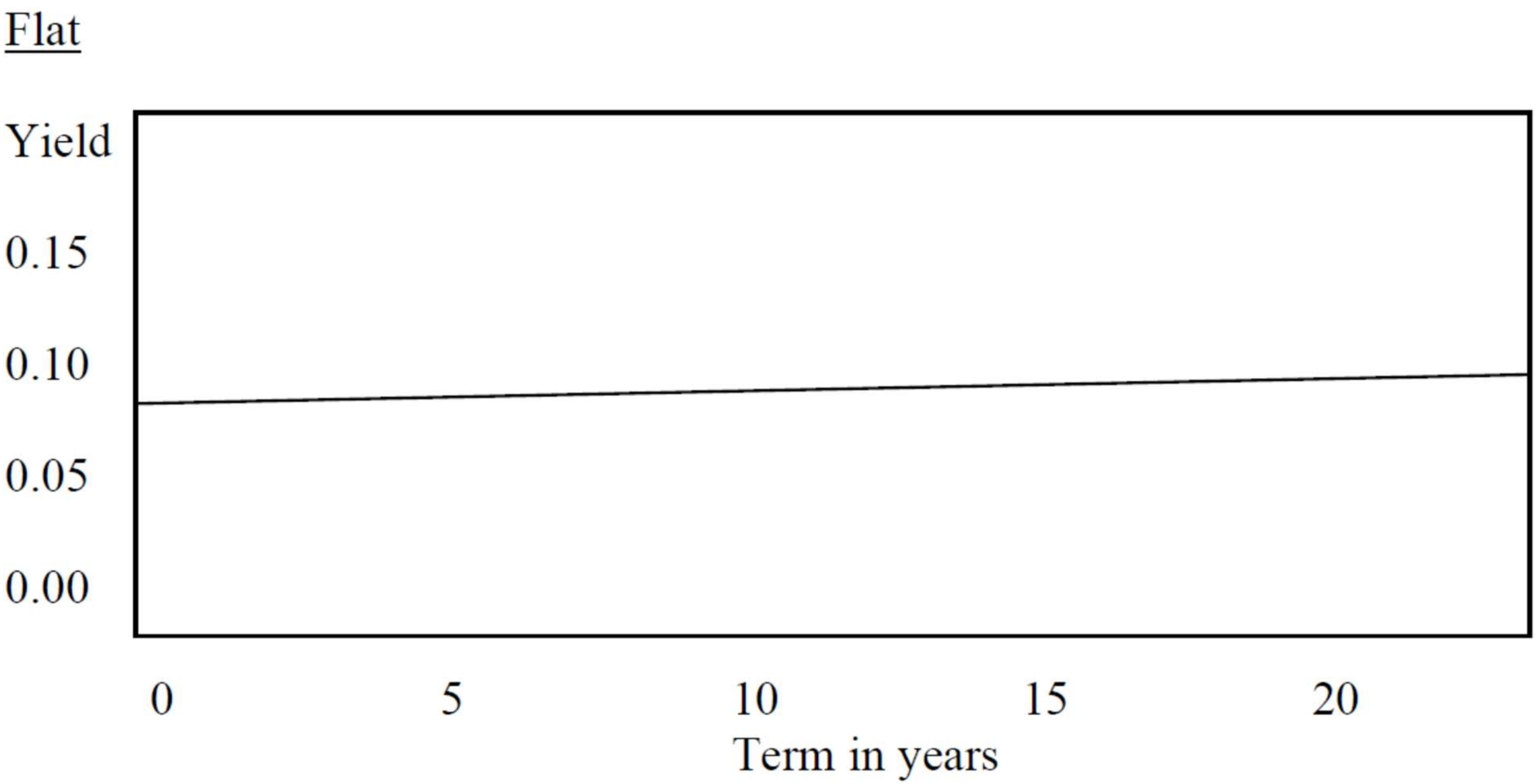
$$\mathrm{V}_S = -\mathrm{V}_L$$

Yield Curve

Upward sloping

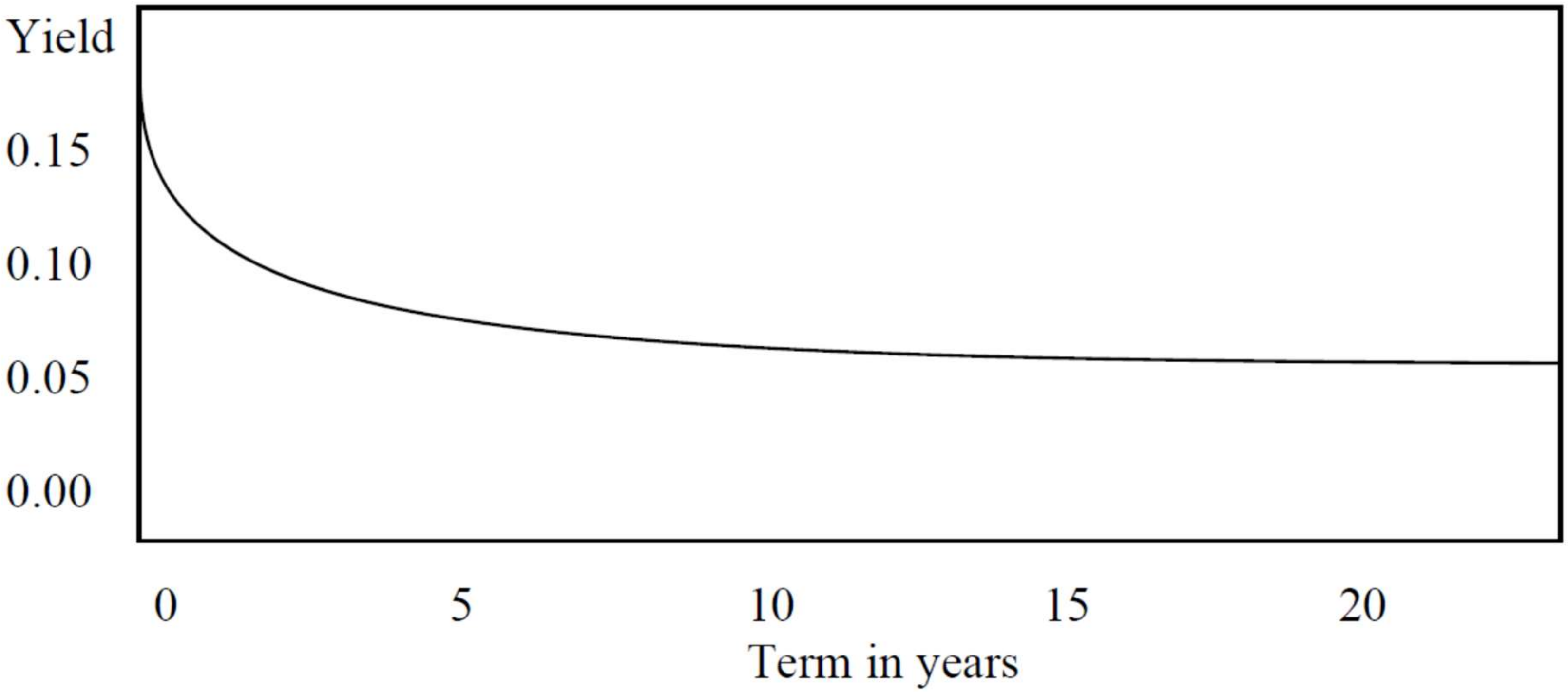


Yield Curve

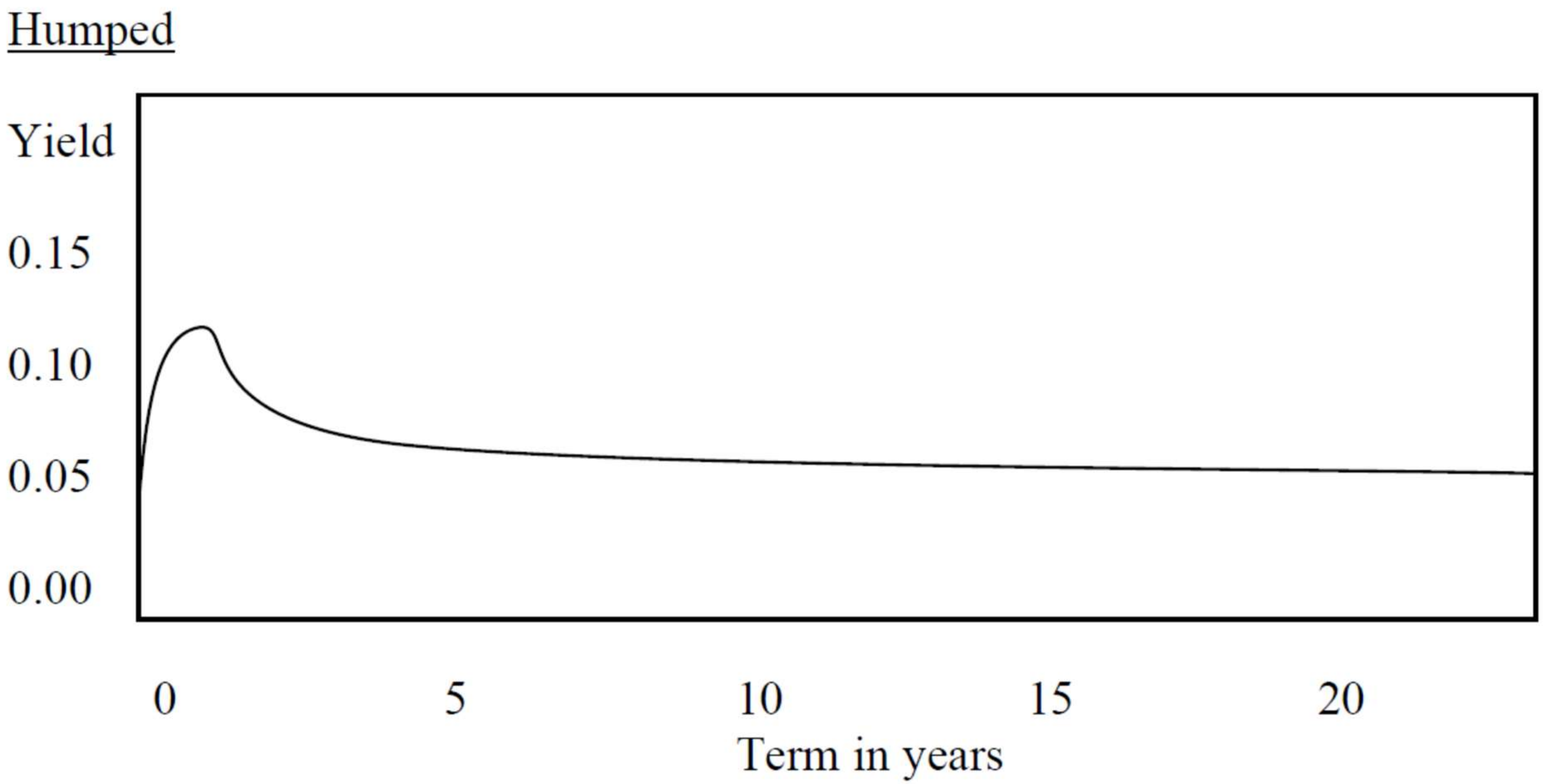


Yield Curve

Downward sloping



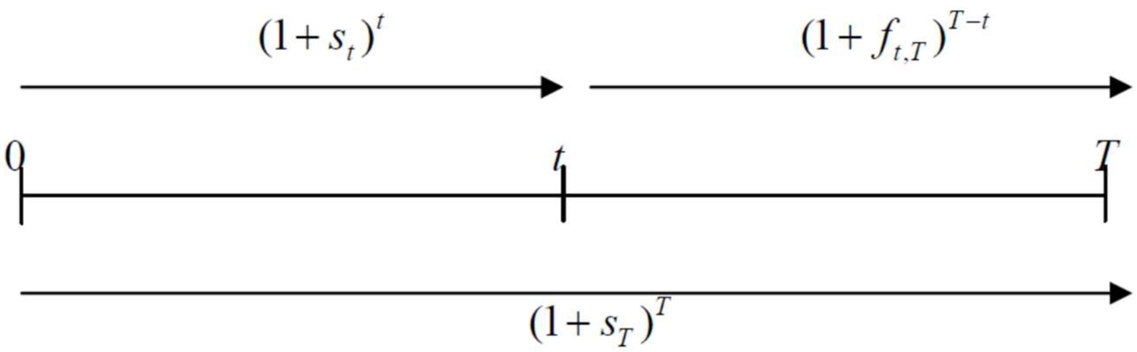
Yield Curve



Spot rate and forward rate

$$(1+f_{t,T})^{T-t} = \frac{(1+s_T)^T}{(1+s_t)^t}$$

The relationship between spot rates and forward rates can be represented on a time line.



$$P = \frac{Fr}{(1+s_1)} + \frac{Fr}{(1+s_2)^2} + \ldots + \frac{Fr+C}{(1+s_n)^n}$$

$$P = \frac{Fr}{(1+f_{0,1})} + \frac{Fr}{(1+f_{0,1})(1+f_{1,2})} + \ldots + \frac{Fr+C}{(1+f_{0,1})(1+f_{1,2})\ldots(1+f_{n-1,n})}$$