Assignment II: week of Jan. 14th

This is the 2nd of the 10 assignments. You are encouraged to work on this by coming to the help sessions (Thursday 12-1 MP202, Friday 1-2 MP102) and **grouping** *up with a few other students. It is helpful to first finish your reading assignment before attempting these. TAs will be at hand to help. You do not have to hand this one in.*

1. Motion of an apple. You are standing on the surface of the Earth and throw an apple upward. The gravitational acceleration goes as GM/r^2, where M is the mass of the Earth, G the universal Newton's constant, and r the distance between the center of the Earth and the object of interest (in this case, the apple). The radius of the Earth is R_0. We will be using simple calculus to derive how the apple moves. This turns out similar to the expansion of the universe.

Your hand gives the apple an upward velocity of v_0 (positive value of v means upward motion;negative value, downward). However, as the apple is climbing up the gravitational potential well, its upward velocity necessarily decreases. And at some point it will turn around.

- Write down the differential equation that describes the variation of velocity due to the gravitational acceleration, $-GM/r^2$.
- Both radius r and velocity $v = dr/dt$ are functions of time, $r = r(t)$, $v = v(t)$. Show that you can express the gravitational acceleration as

$$-\frac{GM}{r^2} = \frac{\frac{d}{dt}\left(\frac{GM}{r}\right)}{v}. \tag{1}$$

  Show that this leads to

$$\frac{d}{dt}\left(\frac{v^2}{2} - \frac{GM}{r}\right) = 0. \tag{2}$$

  This should appear familiar to you. The combination $(v^2/2 - GM/r)$ is the total energy (kinetic and gravitational potential energies) for the apple, and it is conserved throughout the motion.
- Express the maximum height the apple can go before it turns around, as a function of v_0, M, and R_0.
- Let this height be infinity, what is the value of v_0? This is called the escape velocity. On the surface of the Earth, it is ~ 11 km/s, or $40,000$km/hour.

2. Kepler's law on planetary motion. The Earth moves around the Sun once a year, and its mean distance from the Sun is called the 'astronomical unit' (AU). The dwarf planet Eris, which is discovered at 2005 and is actually more massive than our conventional nineth planet, Pluto, orbits the Sun at a mean distance of 67 AUs.

- How long is its orbital period?

- With a diameter of 2400 kilometers, how big (in unit of arcseconds, which is 1/60 of an arcminute, which is 1/60 of a degree) does it appear to Earth-based observer? It used to be said that 'stars twinkle, planets don't'. In the case of Eris, it's less clear.

3. Understanding the strong force (be careful with your physics units). Here, we make an order-of-magnitude estimate for the strength of the strong force inside atomic nuclei. A single proton, with a mass of 1.67×10^{-27} kg, is composed of three quarks. Such a mass, using Einstein's famous equation that relates mass and energy, $E = mc^2$, corresponds to an energy of ~ 1 Gev (giga-electron-volts). Or, the proton has a mass of 1 Gev$/c^2$. This is much greater than the total mass of the three quarks (adding up to ~ 0.01 Gev$/c^2$, see lecture note).

This mass difference arises because, the strong force field which binds the quarks together to a seperation of $\sim 10^{-15}$ meter (or 1 fermi, roughly size of the proton), contains energy, very much like in the first problem where the gravitational field contains energy. And energy is mass. Here, we adopt a very classical view of the physics to gain some flavour for the forces.

- First calculate the electrostatic force (Coulomb force) within a proton,

$$F = k_e \frac{q_1 q_2}{r^2} \tag{3}$$

  where q_1 and q_2 are electric charges and k_e the Coulomb constant. Assume, for simplicity, that the two up (with a fractional electric charge of $+2/3$) and one down (with a fractional electric charge of $-1/3$) quarks are placed at two different ends of the proton. Express your result for the forcing in unit of either 'dyne' or 'Newton'.

- The presence of this electrostatic force over the proton radius indicates that there is an amount of energy stored in the electrostatic field. Force times distance is energy. Calculate this energy and express it in unit of ev.

- Calculate the gravitational attraction between the three quarks (again place two ups and one down at opposite ends of the proton). What is the amount of energy stored in the gravitational field? again express your result in ev.

- How weak is the gravitational force compared to the electrostatic force?

- Neither of these fields can get close to the Gev level of energy. So the bill has to be picked up by the strong force. Assume the rest energy of proton (1 Gev) is entirely due to the strong force, following the same philosophy as above, calculate the strength of the strong force (in unit of 'dyne' or 'Newton'). *Hint: again, the force is energy divided by distance.*

A list of physical constants in MKS (or SI) units, kg is kilogram, m is meter, K is Kelvin, s is second, N is Newton, J is Joule, C is coulomb.

elementary charge $e = 1.602 \times 10^{-19}C$

proton rest mass $m_p = 1.672 \times 10^{-27}kg$

neutron rest mass $m_n = 1.675 \times 10^{-27}kg$

hydrogen atom mass $m_H = 1.673 \times 10^{-27}kg$

electron rest mass $m_e = 9.109 \times 10^{-31}kg$

Planck constant $h = 6.625 \times 10^{-34}J \cdot s$

Speed of light in vacuum $c = 2.998 \times 10^{8}ms^{-1}$

Gravitational constant $G = 6.673 \times 10^{-11}m^3kg^{-1}s^{-2}$

Fine structure constant $\alpha = 1/137.036$

Boltzmann constant $k = 1.380 \times 10^{-23}JK^{-1}$

Electron volt $eV = 1.602 \times 10^{-19}J$

Stefan-Boltzmann constant $\sigma = 5.67 \times 10^{-8}Js^{-1}m^{-2}K^{-4}$

Coulomb constant $k_e = \frac{1}{4\pi\epsilon_0^2} = \frac{c^2\mu_0}{4\pi} = 8.988 \times 10^{9}Nm^2C^{-2}$.