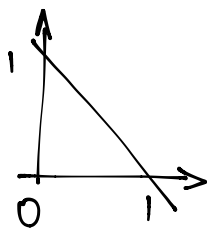


July 18th

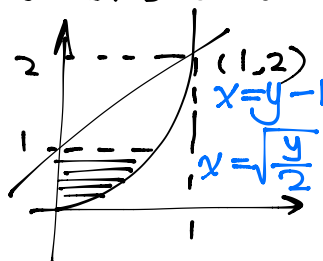
Find the volume of region above the Δ in xy -plane with vertices $(0,0)$, $(1,0)$, $(0,1)$ and below $z=6xy(1-x-y)$



$$\int_0^1 \int_0^{1-x} \int_0^{6xy(1-x-y)} 1 \, dz \, dy \, dx$$

always starting from 1

$$\int_0^1 \int_{x+1}^{x+2} f(y) \, dy \, dx = \int_0^1 [?] \, dy + \int_1^2 [?] \, dy$$



$$\textcircled{1} \int_0^1 \left[\int_0^{\sqrt{\frac{y}{2}}} f(y) \, dx \right] dy$$

$$\textcircled{2} \int_1^2 \left[\int_{y-1}^{\sqrt{\frac{y}{2}}} f(y) \, dx \right] dy$$

$$= f(y) \left(\sqrt{\frac{y}{2}} - y + 1 \right)$$

give $g: \mathbb{R} \rightarrow \mathbb{R}$ is continuous

$$h(x) = \int_0^x \int_0^y g(t) \, dt \, dy$$

$$\text{prove } h'(x) = \int_0^x (x-t)g(t) \, dt$$

integration by parts

$$\begin{aligned} \text{Proof: } h'(x) &= \int_0^x xg(t) \, dt - \int_0^x t g(t) \, dt \\ &= x \int_0^x g(t) \, dt - \left(t \int_0^x g(t) \, dt \right) \Big|_0^x - \int_0^x \int_0^y g(t) \, dt \, dy \end{aligned}$$

$$\int f g' = f g - \int f' g$$

