

## DERIVATION STRATEGY FOR PREDICATE LOGIC

FIRST: On line 1, write: “Show” then the conclusion

EVERY TIME YOU HAVE A SHOW LINE...

Step 1: Analyze show line... What is the main operator/connective?

- set up CD (assume the antecedent, show the consequent)
- ↔ set up two CDs (then use CB for direct derivation of biconditional)
- ∧ show both conjuncts then use ADJ (you might want to give them each a show line and show them with ID)
- ~ consider doing ID (assume the opposite – no negation – and then find a contradiction!)
- ∨ sometimes you can show one disjunct and add the other (usually only if one of the disjuncts doesn't appear anywhere in the premises), consider doing ID then DM, consider showing correlated conditional then ADJ.

For more help with these connectives (→, ↔, ∧, ~) see Sentential Derivations Step by Step.

- ∀ Set up Universal Derivation (UD) (Set this up as soon as possible – it needs an arbitrary variable!)  
on next line, write “Show” and then an instantiation of the sentence  
(you show that it is true for ALL things by showing it is true for any thing/ a random x)  
(for an instantiation, you usually just drop the first ∀)

NOTE to self: show instance of Line # (show line) for UD

Make sure you are using an arbitrary term. The variable cannot occur unbound on any previous line!

Now you have a new show line, start step 1 again on this show line!

Usually the new show line will be a conditional or a universal. So set up another UD or a CD!

- ∃ You need to show a particular instance... but be careful, you don't know what letter it will be instantiated to. There are a few options:

1. Often when your show line begins with ∃, you want to derive it directly using the rules.  
NOTE to self: goal : instance of ∃ (write it with a blank!)  
Use the rules (starting with EI, then UI, etc...) until you derive the instantiation.  
Then use EG – make sure that you match the variable on the show line!
2. If you are allowed any rules (including QN) ... you might want to begin an ID.  
~∃x ... ass ID (on the next line, make assumption for ID)  
∀x~... QN (on the line after, use QN) Now derive a contradiction!
3. Sometimes you need to put in a show line for the instantiated sentence.  
BUT, be very careful. Often there are individual terms your show line must match.  
So look for what unbound variables or other singular terms that you should be matching.  
If you can use EI on other sentences first, make sure you do that.  
If you can set up a UD first, do that too. Both EI and UD require arbitrary individuals.  
You might need to match those individual terms in your show line.

AND... if you don't know what else to do, set up an ID. Make your assumption for ID and then show a contradiction.

You may have to run through steps 2 to 6 a number of times! Always go back to step 1 when you have a new show line!

Step 2: Look for any possible use of EI (existential instantiation) ALWAYS USE A NEW TERM (i, k, m, ...)

You can use EI if the MAIN operator is  $\exists$  (it has to be at the front with the whole sentence in its scope)

How to use EI: take away  $\exists$ ; replace each instance of the variable (x, y, ...) with NEW individual term (i, k, m, ...)

Step 3: Look for any possible use of UI (universal instantiation)



STOP before UI and MATCH! Match an unbound term...

You can use UI if the MAIN operator is  $\forall$  (it has to be at the front with the whole sentence in its scope)

You can replace the variable with any term (x, y, i, k, a, b, a(i), b(k)...) except any variable that is still bound in the sentence.

Often when you use UI it matters what term you are instantiating to...

STOP. Figure out what term you want to MATCH!

Match an unbound term. (What you used for EI, a letter in premise, or any unbound variable.)

How to use UI... take away  $\forall$ ; replace each instance of the variable (x or y ...) with the term.

Step 4: Look for uses of S, MP, MT, MTP and BC... rules that break things down.

Look for uses of EG, ADJ, ADD, DN and CB... rules that build matching sentences or sentence parts.

Look at premises and all available lines. Do you want to show an antecedent? (For MP)

Do you want to show the negation of a consequent (for MT) or of a disjunction (for MTP)?

Can you build what you need? If not, do you want to make a show line for what you need (then use ID, CD or UD).

Step 5: Look for uses of EG (existential generalization)

When you use EG you want to MATCH a bound variable (maybe in a goal sentence or a complex sentence for using with MT, MP or MTP.)

Replace the individual term with the variable you want and put  $\exists$  in front of the sentence.

Step 6: Look for uses of QN to use with steps 2, 3, 4, and/or 5.

$\sim\exists$  or  $\exists\sim$  or  $\sim\forall$  or  $\forall\sim$  HAVE to be at the front of the sentence with whole sentence in scope.

$\sim\exists$  becomes  $\forall\sim$   $\sim\forall$  becomes  $\exists\sim$   $\exists\sim$  becomes  $\sim\forall$   $\forall\sim$  becomes  $\sim\exists$ .

### When stuck:

Put check beside EVERY available line (go line by line - don't forget the assumptions for CD/ID)

Ask what lines you have NOT used already.

What is the main connective?

IF  $\wedge$  or  $\leftrightarrow$ , use S twice and BC twice. Does that help?

IF  $\rightarrow$  or  $\vee$  Ask... what rules work with it? (MP, MT or MTP) What do you need to use that rule?

Don't forget to use DN, EG, ADD, ADJ and CB to build sentences you need.

You may also want to put in a show line for something you need (antecedent, negation of consequent...)

If you have a complex antecedent: show antecedent. Then use MP when you have boxed & canceled.

If you have a complex consequent: show  $\sim$  consequent. Use MT when you have boxed & canceled.

If you have a complex disjunct: show  $\sim$  disjunct. Use MT when you have boxed & canceled.