April 1st

Show $|P(N)| \leq |R|$
Take $S \in P(N)$
write $S = \{s_1, s_2, \cdots\}$

Define $f(S) = 0.s_1 s_2 s_3 \cdots$
Show f injective
$0.999\cdots = 1.00\cdots$
$\parallel$
x $\qquad 10x = 9.999\cdots$
$= 9 + x$

$9x = 9$
$x = 1$

$f(\{9, 99, 999\}) = f(\{1\})$

$f(\{1,2,3\}) \neq f(\{3,2,1\})$
$\parallel \qquad\qquad \parallel$
$0.123 \qquad 0.321$

Define $f: P(N) \to R$ by $f(s) = \sum_{k \in S} 10^{-k}$

$f(\{1,2,3\}) = 10^{-1} + 10^{-2} + 10^{-3} = 0.111000\cdots$
$\parallel$
$f(\{3,2,1\}) = 10^{-3} + 10^{-2} + 10^{-1}$

If $f(S) = f(L)$
then $\sum_{k \in S} 10^{-k} = \sum_{l \in L} 10^{-l}$

$f: N \to Z$ bijective

$$f(n) = \begin{cases} \frac{n}{2} & n \text{ even} \\ -\left(\frac{n+1}{2}\right) & n \text{ odd} \end{cases}$$

Check injective
If $f(n_1) = f(n_2) \geq 0$
then $\frac{n_1}{2} = \frac{n_2}{2} \Rightarrow n_1 = n_2$

If $f(n_1) = f(n_2) < 0$
then $-\frac{(n_1+1)}{2} = -\frac{(n_2+1)}{2}$

HW 10

1 (c)
Is $\{a+b\sqrt{2}+c\sqrt[4]{2}+d\sqrt[4]{8} : a,b,c,d \in \mathbb{R}\}$ a field?

$\mathbb{Q} = F_0 \subset F_1 = \mathbb{Q}(\sqrt{2}) \subset F_2$

$= \{(a+b\sqrt{2}) + \sqrt[4]{2}(c+d\sqrt{2})\}$
$= \{\alpha + \beta\sqrt[4]{2},\ \alpha,\beta \in \mathbb{Q}(\sqrt{2})\}$

$(\alpha - 2\beta^4)(\gamma + \delta\sqrt[4]{2}) = (\alpha - \beta^4\sqrt{2})(\alpha^2 + \beta^2\sqrt{2})$

$\gamma + \delta\sqrt[4]{2} = \dfrac{(\alpha - \beta^4\sqrt{2})(\alpha + \beta^2\sqrt{2})}{\alpha^4 - 2\beta^4}$

$= \left(\dfrac{\alpha^2}{\alpha^4 + 2\beta^4}\right) + (\quad)\sqrt[4]{2}$