TUTORIAL 9

(1) If G is a graph with m edges, prove that
$$\sum_{v \in V(G)} d(v) = 2m.$$
In other words, the sum of the vertex degrees of G is exactly twice the number of edges.

(2) Prove that any graph has an even number of vertices of odd degree. Conclude that there is an even number of facebook users with an odd number of facebook friends.

(3) Prove that if G is a graph in which every vertex has even degree, then there is no edge in G such that deleting that edge breaks G into two connected components.

(4) The d-dimensional cube Q_d is the graph defined as follows. The set of vertices of Q_d is just the set of length d binary strings of 0's and 1's, and two vertices are connected if and only if the corresponding strings differ in exactly one place.
  (a) Draw a picture of Q_2 and Q_3.
  (b) Prove that Q_d has $d2^{d-1}$ edges.

(5) If G and H are simple graphs, we say that G and H are *isomorphic* if there is a bijection $f : V(G) \to V(H)$ such that the following condition holds: For all pairs of vertices $u, v \in V(G)$, u and v are adjacent (i.e. connected by an edge) in G if and only if $f(u)$ and $f(v)$ are adjacent in H. Prove that isomorphism defines an equivalence relation on simple graphs.

(6) How many isomorphism classes of simple graphs with 1, 2, 3 vertices are there?

Just for fun.

(1) Can a simple graph have all distinct vertex degrees?

(2) Prove or disprove
  (a) Deleting a vertex of maximum degree (and its adjacent edges) cannot raise the average degree of a graph.
  (b) Deleting a vertex of minimum degree (and its adjacent edges) cannot reduce the average degree of a graph.

(3) How many isomorphism classes of simple graphs with 4 vertices are there?