UNIVERSITY OF TORONTO
Faculty of Arts and Science

APRIL 2013 EXAMINATIONS

MAT402H1S

Duration – 3 hours

No Aids Allowed

Total Marks: 50

1. *[10 marks]* Consider a convex 4-gon $ABCD$ with parallel sides AB and DC of different lengths. Let P be the point of intersection of the lines containing AD and BC. Let Q be the point of intersection of the diagonals AC and BD. Prove that the line PQ intersects the sides AD and BC at their midpoints.

2. *[10 marks]* For given points $A = (1,0)$, $B = (2,3)$, $C = (5,0)$ and $D = (6,5)$ find $X = (x,y)$ such the sum $|AX| + |BX| + |CX| + |DX|$ is the smallest.

3. *[10 marks]* Consider points O_1, O_2, A and a segment PQ of length R on plane. Using this data and compass and a straightedge construct two tangent lines to the hyperbola $||O_1X| - |XO_2|| = R$ passing through the point A.

4. *[10 marks]* Let S be the circle $\{(x,y) \in S \Leftrightarrow x^2 + y^2 = 1\}$ and let $M, N \in S$ be points symmetric in the line l, $\{(x,y) \in l \Leftrightarrow y = 0\}$. Let $f_1 : l \to S$ and $f_2 : S \to l$ be projections from M and N correspondingly. Prove that $f_2 \circ f_1 : l \to l$ is the inversion in S restricted to l. *Hint: Prove that $f_2 \circ f_1$ is a projective map. Compute $f_2 \circ f_1$ at $(-1,0)$, $(0,0)$ and $(0,1)$.*

5. *[10 marks]* For triangles on the unit sphere state and prove cosine law for sides and the spherical analogue of the Pythagorean theorem.