NAME (PRINT): ____
Last/Surname First /Given Name

STUDENT #: ______________________ SIGNATURE: ____________________

UNIVERSITY OF TORONTO MISSISSAUGA
DECEMBER 2010 FINAL EXAMINATION
STA257H5F
Probability and Statistics I
Alison Weir
Duration - 3 hours
Aids: Calculators; Statistical Formulas

The University of Toronto Mississauga and you, as a student, share a commitment to academic integrity. You are reminded that you may be charged with an academic offence for possessing any unauthorized aids during the writing of an exam, including but not limited to any electronic devices with storage, such as cell phones, pagers, personal digital assistants (PDAs), iPods, and MP3 players. Unauthorized calculators and notes are also not permitted. Do not have any of these items in your possession in the area of your desk. Please turn the electronics off and put all unauthorized aids with your belongings at the front of the room before the examination begins. If any of these items are kept with you during the writing of your exam, you may be charged with an academic offence. A typical penalty may cause you to fail the course.

*Please note, you **CANNOT** petition to **RE-WRITE** an examination once you have begun writing.*

1. Random variables X and Y have joint probability mass function shown in the table below.

		X		
		-1	0	1
Y	0	0.0	0.2	0.3
	1	0.2	0.3	0.0

Answer each of these questions. Put *only* your final answers in the chart below. Do your calculations in the space below, if your final answer is wrong you *might* earn some part marks for your calculations.

a. *(1 mark)* What is $F_{xy}(1, 0)$?	
b. *(1 mark)* What is $F_{yx}(1, 0)$?	
c. *(2 marks)* What is $P(X \leq 0)$?	
d. *(1 mark)* What is $P(Y = -1)$?	
e. *(1 mark)* What is $E(X)$?	
f. *(1 mark)* What is $E(Y)$?	
g. *(2 marks)* What is $E(XY)$?	
h. *(2 marks)* What is $Cov(X, Y)$?	

Calculations for Q1

2. Twenty percent of STATSU College students own cars. Assume car ownership among students is independent.
Twenty percent of the cars owned by STATSU College students have defective brakes.

Answer each of these questions. Put *only* your final answers in the chart below. Do your calculations in the space below, if your final answer is wrong you *might* earn some part marks for your calculations.

a. *(3 marks)* In a group of 10 students, what is the probability exactly 3 own cars?	
b. *(3 marks)* In a group of 10 students, how many do you expect will own a car?	
c. *(3 marks)* Campus police inspect cars sequentially, looking for defective brakes. What is the probability they need to inspect 10 cars in order to find 3 with defective brakes?	
d. *(3 marks)* How many cars do the campus police expect to inspect in order to find 3 with defective brakes?	
e. *(3 marks)* What is the probability a randomly selected STATSU College student owns a car that does not have defective brakes?	

3. A pair of dice is tossed. Let A be the event that the sum of spots on the two dice is 6. Let B be the event that one of the dice lands with 3 spots up.

   Answer each of these questions. Put *only* your final answers in the chart below. Do your calculations in the space below, if your final answer is wrong you *might* earn some part marks for your calculations.

a. *(2 marks)* Find P(A)	
b. *(2 marks)* Find $P(\bar{A}\|B)$	
c. *(2 marks)* Find $P(B\|\bar{A})$	
d. *(2 marks)* Find P(A U B)	

Calculations for Q3

4. An industrial spool of 1mm diameter copper wire contains 10^6m of wire. The wire has imperfections. The number of imperfections per metre of wire follows a Poisson distribution with mean 0.2.

Answer each of these questions. Put *only* your final answers in the chart below. Do your calculations in the space below, if your final answer is wrong you *might* earn some part marks for your calculations.

a. *(3 marks)* You have a 7m length of this wire. What is the probability there is exactly four imperfections in this piece of wire?	
b. *(3 marks)* You have two lengths of this wire, one piece is 3m and the other is 4m. What is the probability there is a total of four imperfections in these two pieces of wire?	
c. *(3 marks)* What is the probability there is more than 1.5m between two successive imperfections?	

Calculations for Q4

5. The continuous random variable Y has probability density function

$$f(y) = \begin{cases} \frac{3y^2}{8} & 0 < y \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

Answer each of these questions. Put *only* your final answers in the chart below. Do your calculations in the space below, if your final answer is wrong you *might* earn some part marks for your calculations.

a. *(3 marks)* P(Y > 1)	
b. *(2 marks)* F(1)	
c. *(1 mark)* P(Y = 1)	
d. *(3 marks)* E(Y)	

Calculations for Q5

6. Continuous random variables X and Y have joint probability density function

$$f_{xy}(x,y) = \begin{cases} 2x + 2y & 0 < x < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

Answer each of these questions. Put *only* your final answers in the chart below. Do your calculations in the space below, if your final answer is wrong you *might* earn some part marks for your calculations.

a. *(3 marks)* Sketch the support. Do not add other regions to your sketch.	
b. *(5 marks)* Find the marginal density function of X. Don't forget the support.	
c. *(5 marks)* Find the density function of Y, conditional on X = ½ . Don't forget the support.	

7. *(4 marks)* The developing time of photographic prints has a normal distribution with mean 15 seconds and standard deviation 0.5 seconds. What is the probability it will take between 14 and 16 seconds to develop one print? Model solution required, circle your final answer.

8. *(4 marks)* The developing time of photographic prints has an unknown distribution with mean 15 seconds and standard deviation 0.5 seconds. What is the smallest possible probability it takes between 14 and 16 seconds to develop one print? Model solution required, circle your final answer.
Hint – this is not the same question as #7

9. A manufacturer buys sheet metal from four different suppliers, A, B, C, and D. 10% of supplier A's sheet metal contains flaws, 5% of supplier B's sheet metal contains flaws, 3% of supplier C's sheet metal contains flaws, and 7% of supplier D's sheet metal contains flaws. The manufacturer buys 10% of their sheet metal from supplier A, 50% from supplier B, 25% from supplier C, and the rest from supplier D.

   a. *(2 marks)* This question is about two events, name the two events.

   b. *(4 marks)* Sketch, and fully label, a tree diagram for this question.

   c. *(3 marks)* The manufacturer's quality control officer randomly selects a piece of sheet metal. What is the probability it contains a flaw? Circle your final answer

   d. *(3 marks)* The manufacturer's quality control officer randomly selects a piece of sheet metal and observes that it does not contain a flaw. What is the probability this piece of sheet metal came from supplier B? Circle your final answer.

10. Random variable Y_1 has Binomial distribution, $Y_1 \sim B(12, 3/4)$. Random variable Y_2 has Binomial distribution, $Y_2 \sim B(8, 3/4)$.

   a. *(2 marks)* Write down the moment generating function of Y_1.

   b. *(4 marks)* Define $X = Y_1 + Y_2$ What is the moment generating function of X?

11. *(3 marks)* State the three Axioms of Probability.

12. Answer any two of the following. If you answer all three, your best two solutions will be counted.

    a. *(3 marks)* Y has a Poisson distribution. P(Y=0)=P(Y=1). What is E(Y)?

    b. *(3 marks)* X has a Binomial distribution. E(X) = 5 and Var(X) = 4. What is n?

    c. *(3 marks)* W is a random variable with moment generating function $\exp(e^t-1)$. What is E(W)?

Total Marks: 100

Distribution	pmf	Support	Mean	Variance	mgf	Θ
Bernoulli	$f(x) = \theta^x(1-\theta)^{1-x}$	$x = 0, 1$	θ	$\theta(1-\theta)$	$1-\theta+\theta e^t$	$0 \leq \theta \leq 1$
Binomial	$f(x) = \binom{n}{x}\theta^x(1-\theta)^{n-x}$	$x = 0, 1, 2, \ldots, n$	$n\theta$	$n\theta(1-\theta)$	$(1-\theta+\theta e^t)^n$	$0 \leq \theta \leq 1$
Hypergeometric	$f(x) = \frac{\binom{K}{x}\binom{M-K}{n-x}}{\binom{M}{n}}$	$x = 0, 1, 2, \ldots, n$	$\frac{nK}{M}$	$n\frac{K}{M}\frac{M-K}{M}\frac{M-n}{M-1}$	Not Useful	$M = 1, 2, \ldots$ $K = 0, 1, 2, \ldots, M$ $n = 1, 2, \ldots, M$
Poisson	$f(x) = \frac{e^{-\lambda}\lambda^x}{x!}$	$x = 0, 1, 2, \ldots$	λ	λ	$\exp(\lambda(e^t - 1))$	$0 < \lambda < \infty$
Geometric	$f(x) = \theta(1-\theta)^{x-1}$	$x = 1, 2, 3 \ldots$	$\frac{1}{\theta}$	$\frac{1-\theta}{\theta^2}$	$\frac{\theta e^t}{1-(1-\theta)e^t}$	$0 < \theta \leq 1$
Negative Binomial	$f(x) = \binom{x-1}{k-1}\theta^k(1-\theta)^{x-k}$	$x = k, k+1, k+2, \ldots$	$\frac{k}{\theta}$	$\frac{k(1-\theta)}{\theta^2}$	$\left(\frac{\theta e^t}{1-(1-\theta)e^t}\right)^k$	$0 < \theta \leq 1$
Discrete Uniform	$f(x) = \frac{1}{n}$	$x = x_1, x_2, \ldots, x_n$	$\frac{1}{n}\sum x_i$	$\frac{1}{n}\sum(x_i - \mu)^2$	$\frac{1}{n}\sum e^{tx_i}$	Not Applicable

Distribution	pdf or CDF	Support	Mean	Variance	mgf	Θ
Uniform	$f(x) = \frac{1}{\beta-\alpha}$	$\alpha < x \leq \beta$	$\frac{\alpha+\beta}{2}$	$\frac{(\beta-\alpha)^2}{12}$	$\frac{e^{\beta t}-e^{\alpha t}}{(\beta-\alpha)t}$	$-\infty < \alpha < \beta < \infty$
Normal	$f(x) = \frac{1}{\sqrt{2\pi}\sigma} exp(\frac{-1}{2}\frac{(x-\mu)^2}{\sigma^2})$	$-\infty < x < \infty$	μ	σ^2	$\exp(\mu t + \sigma^2 t^2/2)$	$-\infty < \mu < \infty, \quad \sigma > 0$
Exponential	$f(x) = \frac{1}{\beta} e^{-x/\beta}$	$0 < x < \infty$	β	β^2	$(1-\beta t)^{-1}$	$0 < \beta < \infty$
Gamma	$f(x) = \frac{1}{\Gamma(\alpha)\beta^\alpha} x^{\alpha-1} e^{-x/\beta}$	$0 < x < \infty$	$\alpha\beta$	$\alpha\beta^2$	$(1-\beta t)^{-\alpha}$	$0 < \alpha < \infty, 0 < \beta < \infty$
Beta	$f(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}$	$0 < x < 1$	$\frac{\alpha}{\alpha+\beta}$	$\frac{\alpha\beta}{(\alpha+\beta+1)(\alpha+\beta)^2}$	Not Useful	$0 < \alpha < \infty, \quad 0 < \beta < \infty$
Cauchy	$f(x) = \frac{1}{\pi\beta(1+[(x-\alpha)/\beta]^2)}$	$-\infty < x < \infty$	Does Not Exist	Does Not Exist	Does Not Exist	$-\infty < \alpha < \infty, \quad 0 < \beta < \infty$
Wiebull	$f(x) = \alpha\beta x^{\beta-1} e^{-\alpha x^b}$	$0 < x < \infty$	$\alpha^{1/\beta}\Gamma(1+1/\beta)$	$\alpha^{-2/\beta}[\Gamma(1+2/\beta) - \Gamma^2(1+1/\beta)]$	$\alpha^{-t/\beta}\Gamma(1+t/\beta)$	$0 < \alpha < \infty, \quad 0 < \beta < \infty$
Logistic	$F(x) = \left(1+e^{-(x-\alpha)/\beta}\right)^{-1}$	$-\infty < x < \infty$	α	$\frac{\beta^2\pi^2}{3}$	$e^{\alpha t}\pi\beta t \csc(\pi\beta t)$	$-\infty < \alpha < \infty, \quad 0 < \beta < \infty$
Pareto	$f(x) = \frac{\theta a^\theta}{x^{\theta+1}}$	$a < x < \infty$	$\frac{\theta a}{\theta-1}$	$\frac{\theta a^2}{(\theta-1)^2(\theta-2)}$	Does Not Exist	$0 < a < \infty, \quad 0 < \theta < \infty$