Ex (continued)

All 4 tableaux in this example represent 4 different problems:

① having the same feasible region

② the same z-value on the feasible region.

From tableau ③, x_5 will enter

x_5 column θ-ratios

$$\begin{array}{c|l} x_1 & \\ x_4 & \frac{6}{6/7} = 7 \leftarrow x_4 \text{ will exit} \\ x_2 & \frac{3}{1/7} = 21 \end{array}$$

A pivot on $\frac{6}{7}$ produces Tableau ④

	x_1	x_2	x_3	x_4	x_5	z	
x_1	1	0	$\frac{1}{6}$	$\frac{5}{6}$	0	0	9
x_5	0	0	$-\frac{1}{6}$	$\frac{7}{6}$	1	0	7
x_2	0	1	$\frac{1}{6}$	$-\frac{1}{6}$	0	0	2
	0	0	$\frac{5}{3}$	$\frac{4}{3}$	0	1	41

This tableau is optimal because it satisfies the optimality criterion:

① The objective row coefficients of basic variables are all 0.

② The objective row coefficients (not including the objective value) are all ≥ 0.

Tableau ④ represents the problem

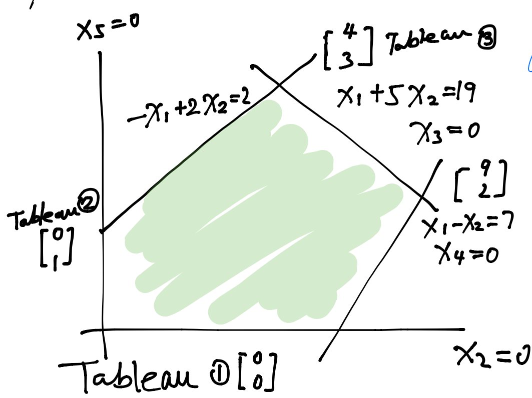
$$\text{Maximize } z = 41 - \frac{5}{3}x_3 - \frac{4}{3}x_4 \text{ s.t.}$$
$$\frac{1}{6}x_3 + \frac{5}{6}x_4 \leq 9$$
$$-\frac{1}{6}x_3 + \frac{7}{6}x_4 \leq 7$$
$$\frac{1}{6}x_3 - \frac{1}{6}x_4 \leq 2, \quad x_3 \geq 0, \ x_4 \geq 0$$

Any feasible change in x_3 or x_4 is an increase (from 0), causing z to decrease. The optimal solution is $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 9 \\ 2 \\ 0 \\ 0 \\ 7 \end{bmatrix}$

This problem has only 1 optimal solution.

because the coefficients of x_3 and x_4 in the objective row are $\neq 0$

If $z = 41 - 0 \cdot x_3 - \frac{4}{3}x_4$ were the case (where x_3 has "0" in the objective row), x_3 could enter without any change in the objective (giving another optimal solution)



Graphical Solution

Note: In the simplex solution,
x_5 exited, then re-entered.
In many problems, a variable will enter and later exit.

From now on, we drop the z column but any objective row represents an equation including the term "$+z$".

Notes on "An unbounded Problem" which is:

Maximize $z = 9x_1+10x_2-8x_3-9x_4$ s.t.

$$2x_1+2x_2-3x_3-2x_4+x_5=6$$
$$-6x_1-x_2+9x_3-x_4+x_6=7$$
$$x_1\geq 0,\ x_2\geq 0,\ x_3\geq 0,\ x_4\geq 0,\ x_5\geq 0,\ x_6\geq 0.$$

From Tableau ①, x_2 will enter

The x_2-column θ-ratios are

$$\begin{array}{c|c} x_5 & \frac{6}{2} \longleftarrow x_5 \text{ exits} \\ x_6 & -\frac{7}{6} \end{array}$$

A routine pivot leads to Tableau ②

Now x_3 enters; x_3-column θ-ratios are

$$\begin{array}{c|c} x_2 & \cancel{\frac{3}{-3/2}} \\ x_6 & \frac{10}{15/2} \longleftarrow x_6 \text{ will exit} \end{array}$$

From Tableau ③ we would enter x_1. θ-ratios are both have denominators ≤ 0. No variable can feasibly exit. The $-10<0$ in the objective row and the coefficient of x_1 otherwise ≤ 0, indicates the problem is unbounded.

Next day, bring

① An Unbounded Problem

② A Degenerate Optimal Solution