Previous Term Test 3 Questions

This document contains the actual questions found on previous Term Test #3 papers (and one Term Test # 2 paper), and Exams which were offered by Professor P.Rosenthal.

Term Test #3 March, 2001

1. For each of the following numbers, state whether or not it is constructible and justify your answer.

a) $29^{1/3}$

b) $\sqrt{\frac{\pi}{4}}$

c) $\frac{0.489}{\sqrt{27}}$

d) tan 22.5 degrees

e) arctan 1

2. Explain how to construct a regular polygon with 24 sides using straightedge and compass

3 a) Find a polynomial p with rational coefficients such that

$p(5 - \sqrt{9 + 7\sqrt{5}}) = 0$

b) Suppose x satisfies $x^5 + 17\sqrt{3}\,x^4 - 9x^3 + 27 - \sqrt{27} = 0$. Must x be algebraic? Justify your answer.

4.Define "surd"

5. Can a polynomial of degree 4 with rational coefficients have a constructible root without having a rational root? Justify your answer.

6. Show that the acute angle whose cosine is 1/5 can't be trisected with straightedge and compass.

7. State which of the following angles are constructible and briefly explain why:

a) 120 degrees

b) 75 degrees

c) 7.5 degrees

d) 35 degrees

e) 80 degrees.

8. State the cardinality ("countable" or "c") or each of the following sets:

a) the set of roots of polynomials with constructible coefficients.

b) the set of roots of polynomials with real coefficients

c) the set of constructible angles

d) the set of all points (x,y) in the plane such that x is constructible and y is irrational.

e) the set of constructible angles.

9. Which of the following subsets of R are number fields? Justify your answer in each case.

a) $\{ a\sqrt{2}\ a \in Q\}$
b)$\{ a + b\pi i: a,b \in Q\}$ (you can use the fact that pi is transcendental)

10. Give an outline of the proof that every surd is constructible.

Term Test #3 March 14, 1999

1. For each of the following numbers, state whether it is constructible and justify your answer:

a) 1/ Root of ($3+ 4\sqrt{6}$)

b) $1/ \pi$

c) 0.67893

d) Root of ($\pi^2 +4$)

e) cos(15 degrees)

2. Is there a line in the plane such that every point on it is constructible? Justify your answer

3.a) Find a polynomial p with rational coefficients such

that p($3 - \sqrt{2+7\sqrt{5}}$) = 0.

3 b) Suppose that x satisfies $x^4 + 2x + \sqrt{3} = 0$. Must x be algebraic? Justify your answer.

4. Define "surd".

5. Does $x^3 - 3x^2 - 2x + 6 = 0$ have a constructible root? Explain your answer (Hint: Yes it does)

6. Does $x^3 + 4x + 1 = 0$ have a constructible root? Justify your answer (Hint: No it doesn't)

7. State which of the following angles are constructible and briefly explain why:

a) 120 degrees

b) 75 degrees

c) 7.5 degrees

d) 35 degrees

e) 80 degrees

8. Let $Q\sqrt{2}) = \{a + b\sqrt{2} : a,b \in Q\}$, and let $F = \{c + d\sqrt{3} : c,d \in Q\sqrt{2}$. Prove that every element of F is the root of polynomial of degree 4 with rational coefficients.

9. Prove that the cube can't be "tripled", in the sense that, starting with an edge of a cube of volume 1, an edge of a cube of volume 3 can't be constructed with straightedge and compass.

Term Test #3 March 16, 1999 (Make Up Test)

1. Does the polynomial $p(x) = 5x^9 – 2x^8 + x^2 – x + 1$ have a real root? Justify your answer.

2. Find all the complex roots of the equation $z^7 – z = 0$.

3. State whether each of the following angles can be trisected with straightedge and compass and justify your answers:

a) an angle of 15 degrees

b) an angle of 30 degrees.

4. State whether the following angles can be constructed and justify your answers.

a) an angle of 7.5 degrees

b) an angle of 160 degrees.

5. For each of the following number, state whether or not it is constructible and justify your answer

a) Eighth root of 2

b) Sixth root of 2

6. Find $(1/\sqrt{2} + i/\sqrt{2})^{106}$

7. Prove (you can quote without proof theorems we've proven in class) that the cube root of a natural number is not constructible unless it is an integer.

8. Say that the complex number at a +bi is constructible if the point (a,b) is constructible (equivalently, if a and b are both constructible real numbers). Are any of the cube roots of $½ + i (\sqrt{3}/2)$ constructible? Justify your answer. (Hint: It is not)

9. Say that the complex number is algebraic if it is the root of a polynomial with integer coefficients. Is the set of all algebraic complex numbers countable? Prove your answer. (Hint: Yes they are)

10. Does the equation $x^3 - 2x - 1 = 0$ have a constructible root? Prove that your answer is correct.

(Hint: Yes)

Term Test # 2 (2000) - This is only put because it contains relevant material for Term Test 3 for this offering.

A1 Find the cardinality of $\{ a + b\sqrt{2} : a, b \in Q\}$ and justify your answer.

A2. Find the cardinality of the set of all points in R^3 all of whose coordinates are rational, and justify your answer.

A3. What is the cardinality of

a) the set of all numbers in [0,1] which have decimal expansions with a finite number of non-zero digits?

b) the set of all numbers in [0,1] which have decimal expansions that end with an infinite sequence of 7's?

A4. Prove that the set of all finite subsets of Q is countable.

A5. Show that the set of all polynomials with rational coefficients is countable.

A6. Suppose that there is a function mapping the set S onto the set T. Prove that $|S| \geq |T|$

A7. The Cartesian product of the sets A and B is defined to be the set of all ordered pairs (a,b) with a in A and b in B. Prove that the Cartesian product of two countable sets is countable.

A8. Find the cardinality of the set of all points in the plane which have one rational coordinate and one irrational coordinate, and justify your answer.

A9. Prove that the cardinality of the set of all finite subsets of the plane R^2 is c.

Test 3 Questions from 1998 Exam

5. State which of the following angles are constructible, and justify your answer.

a) 10 degrees

b) 30 degrees

c) 15 degrees

d) 75 degrees

e) 5 degrees

6. Determine whether the equation

$z^3 - z^2 + z + 1 = 0$ has a constructible root, and justify your answer.

7. Suppose that S and T each have cardinality c (the cardinality of R). Show that

S U T has cardinality c.

10. a) State whether each of the following numbers is constructible and justify.

i) 1/ root of $(3 + \sqrt{2})$

ii) $7^{2/3}$

iii) π^{10}

iv) sin20 degrees

v) 0.37219

b) Prove that the acute angle whose cosine is ¼ can't be trisected with straightedge and compass.

Test 3 Questions from 1999 Exam

4. Does the equation $x^4 + x - 1 = 0$ have a rational root? Justify your answer.

5. State which of the following numbers are constructible and justify your answers:

a) $\sqrt{\frac{3792}{1419}}$

b) $7^{2/3}$

c) cos20 degrees

d) cos15 degrees

e) 3.146891

10. Prove that a continuous function mapping R into R must be a constant function if its range is countable.

Test 3 Questions from 2000 Exam

3a) What is the cardinality of the set of all sets of constructible numbers? Justify your answer.

3b) Let S be the set of all functions mapping R into R. Show that the cardinality of S is greater than c.

4 b) Find a polynomial p with integer coefficients such that $p(3 + i\sqrt{7}) = 0$.

8 a) Can a polyhedron have an odd number of edges, an odd number of vertices, and an odd number of faces? Justify your answer.

b) Can a regular polygon with 20 sides be constructed with straightedge and compass? Justify your answer.

Test 3 Questions from 2001 Exam

3 a) Let N be the set of natural numbers and a and b be distinct numbers. What is the cardinality of the set of all functions with domain {a,b} and range a subset of N? Justify your answer.

5. Prove the following: If S is uncountable and T is a countable subset of S, then the cardinality of S\T is the same as the cardinality of S.

7. Let S denote the collection of all sequences of real numbers. Show that the cardinality of S is c.

10. a) State whether each of the following numbers is constructible and justify.

i) root of $(\pi^2 + 4)$

ii) cos10 degrees

iii) root of $(3 + 4\sqrt{2} + \sqrt{5})$

iv)sin75 degrees

v) $\sqrt{7\cos 15^\circ}$

10 b) Prove that the acute angle whose cosine is 2/5 can't be trisected with straightedge and compass.