MATH 315; HOMEWORK # 4

Due Feb. 25, 2015

1. (Exercise 14.2) Let $F_n = 2^{2^n} + 1$, so, for example, $F_1 = 5, F_2 = 17, F_3 = 257$ and $F_4 = 65537$. Fermat thought that all of F_n's might be prime, but Euler showed in 1732 that F_5 factors as 641×6700417, and in 1880 Landry showed that F_6 is composite. Primes of the form F_n are called Fermat primes. Show that if $k \neq m$, the numbers F_k and F_m have no common factors; that is, show that $gcd(F_k, F_m) = 1$. (Hint. If $k > m$, show that F_m divides $F_k - 2$.)

2. (Exercise 15.1) If m and n are integers with $gcd(m, n) = 1$, prove that $\sigma(mn) = \sigma(m)\sigma(n)$.

3. (Exercise 15.3 (b, e)) (b) Show that if p is an odd prime, a power p^k can never be a perfect number.

(e) Show that if p and q are distinct odd primes, then a number of the form $q^i p^j$ can never be a perfect number.

4. (Exercise 15.6 (b)) A number is product perfect if the product of all of its divisors (other than itself) is equal to the original number. For example, 6 and 15 are product perfect. Describe all product perfect numbers.

5. (Exercise 16.3 (a)) Compute 7^{7386} (mod 7387) by the method of successive squaring. Is 7387 prime?

6. (Exercise 17.1) Solve the congruence $x^{329} \equiv 452 \pmod{1147}$. [Hint. 1147 is not a prime. Factor it first.]