Code: 0888

FACULTY OF ARTS AND SCIENCE
University of Toronto

FINAL EXAMINATIONS, APRIL/MAY 2009

MAT 135Y1Y
Calculus I

Duration – 3 hours

PLEASE HAND IN

NAME
(Please PRINT full name, and UNDERLINE surname): _____

STUDENT NO: _____

SIGNATURE OF STUDENT
(in INK or BALL-POINT PEN): _____

This Exam has 2 Parts:

PART A: 8 questions (55 marks).
PART B: 18 multiple choice questions (45 marks).
Indicate your answer to each multiple-choice question in PART B by completely filling in the appropriate circle in the ANSWER BOX on this front page. (Use a dark pencil!)

NOTE:

1. Before you start, check that this Exam has **20 pages**.
2. No aids allowed.
   NO CALCULATORS!
3. **DO NOT TEAR OUT THIS PAGE OR ANY OTHER PAGE.**

COMPUTER CARDS AND ANSWER BOOKS WILL NOT BE USED.

NO SCRAP PAPER!

FOR MARKERS ONLY	
A1	/ 5
A2	/ 5
A3	/ 7
A4	/ 7
A5	/ 7
A6	/ 8
A7	/ 8
A8	/ 8
B	/ 45
TOTAL	/ 100

ANSWER BOX FOR PART B	
1.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
2.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
3.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
4.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
5.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
6.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
7.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
8.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
9.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
10.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
11.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
12.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
13.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
14.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
15.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
16.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
17.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
18.	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

PART A *[55 marks]*

Answer all questions in PART A in spaces provided. Show all your work for PART A. Any answer in PART A without proper justification may receive very little or no credit. Use the back of each page for rough work. Marks for each question in PART A are indicated by [ ]. **DO NOT TEAR OUT ANY PAGES.**

1. Find $\int x \cos x \, dx$.

[5]

2. Find $\int \tan^{82} x \sec^4 x \, dx$.

[5]

3. Find $\int \frac{1}{x^2\sqrt{x^2-4}}\,dx$.

[7]

4. Find the arc length of the curve $y = \ln(\sec x)$, $0 \leq x \leq \frac{\pi}{4}$.

[7]

5. Find the solution of the differential equation $\frac{dy}{dx} = 2x(2x^2+1)e^{-3y}$ that satisfies the condition $y(1) = \ln 2$.

[7]

6. Find $\int \frac{\sqrt{x}}{1+x^3}\,dx$.

[8]

7. Find the interval of convergence of the series $\sum_{n=4}^{\infty} \frac{4^n x^n}{\ln(n-2)}$. Remember to fully justify your answer.

[8]

8. **NOTE: This is a hard problem and will be marked extremely strictly. Very little or no credit will be given unless your solution is completely correct.**

Find $\int \sqrt{x - \sqrt{x^2 - 25}}\, dx$.

Hint: Investigate $\sqrt{x+5} - \sqrt{x-5}$.

[8]

PART B *[45 marks]*
18 multiple choice questions

PLEASE READ CAREFULLY: Each of the following multiple-choice questions has exactly one correct answer. **Indicate your answer to each question by completely filling in the appropriate circle in the ANSWER BOX on the front page. Use a dark pencil.**

MARKING SCHEME: $2\frac{1}{2}$ marks for a correct answer,
0 for no answer, a wrong answer or giving more than one answer.

You are not required to justify your answers in PART B.

NOTE: If there is any discrepancy between the circles you darken on these inside pages and those you darken on the front page, the circles you darken on the front page will be regarded as your final answers. Note that only the circles you darken will count. For Part B, your computations and answers (other than the circles you darken) will NOT count.

WARNING: If you darken the circles on these inside pages but do not darken the circles on the front page, you will still get credit for your correct answers, but there will be a PENALTY of minus 4 marks.

YOU MUST NOT TEAR OUT ANY PAGES OF THIS EXAM.

1. Find the value of $\lim_{x\to 0} \dfrac{x-\sin x}{x^3}$.

   (A) $\frac{1}{3}$

   (B) undefined

   (C) 0

   (D) $\frac{1}{6}$

   (E) $-\frac{1}{3}$

2. Let $f(x) = x^4 - 2x^3 - 36x^2 + 5x - 4$, for all x. Then the graph of f has a point of inflection at

   (A) $x = -2$ and $x = 3$ only.

   (B) $x = 3$ only.

   (C) $x = -2$ and $x = -3$ only.

   (D) $x = -2$ only.

   (E) $x = 2$ and $x = 3$ only.

INDICATE YOUR ANSWERS ON THE FRONT PAGE.
Penalty for not doing so is MINUS 4 marks.

3. The length of a rectangle is increasing at 5 ft/sec while its width is decreasing at 4 ft/sec. At what rate will the area of the rectangle be changing when the length of the rectangle is 500 ft and the width is 300 ft?

   (A) increasing at 400 sq ft/sec.

   (B) decreasing at 500 sq ft/sec.

   (C) increasing at 450 sq ft/sec.

   (D) increasing at 500 sq ft/sec.

   (E) decreasing at 400 sq ft/sec.

4. The graph of $y = \dfrac{4x^3 + 3x^2 - 10x + 1}{x^2 + 2x}$ has two vertical aysmptotes and one slant (i.e. oblique) asymptote. The slant asymptote is the line

   (A) $y = 4x + \dfrac{3}{2}$

   (B) $y = 4x - 2$

   (C) $y = 4x$

   (D) $y = 4x + 3$

   (E) $y = 4x - 5$

INDICATE YOUR ANSWERS ON THE FRONT PAGE.
Penalty for not doing so is MINUS 4 marks.

5. The sum of two positive numbers is 16. What is the smallest possible value of the sum of their squares?

   (A) 132

   (B) 126

   (C) 134

   (D) 128

   (E) 130

6. Find the area of the region enclosed between the curves $y = 1 + 2x$ and $y = 1 + x + \sqrt{2x}$.

   (A) $\frac{2}{3}$

   (B) $\frac{4}{5}$

   (C) $\frac{1}{4}$

   (D) $\frac{5}{6}$

   (E) $\frac{3}{5}$

INDICATE YOUR ANSWERS ON THE FRONT PAGE.
Penalty for not doing so is MINUS 4 marks.

7. Let R be the region enclosed between the curves $y = x$ and $y = x^2$. Find the volume of the solid generated by revolving R about the $x-axis$.

   (A) $\frac{5\pi}{12}$

   (B) $\frac{\pi}{3}$

   (C) $\frac{2\pi}{5}$

   (D) $\frac{3\pi}{10}$

   (E) $\frac{2\pi}{15}$

8. Find the average value of the function $f(x) = 3x^2 + 8x^3$ on $[1, 3]$.

   (A) 190

   (B) 80

   (C) 93

   (D) 65

   (E) 127

9. Consider the predator-prey system $\frac{dR}{dt} = 5R - 4RW$, $\frac{dW}{dt} = -3W + 2RW$. When the system is in equilibrium with $W \neq 0$, $R \neq 0$, then $RW =$

   (A) $\frac{15}{8}$

   (B) $\frac{5}{6}$

   (C) $\frac{7}{3}$

   (D) $\frac{7}{4}$

   (E) $\frac{5}{2}$

INDICATE YOUR ANSWERS ON THE FRONT PAGE.
Penalty for not doing so is MINUS 4 marks.

10. Suppose that a population grows according to a logistic model, i.e. the growth is modeled by the differential equation $\frac{dP}{dt} = kP\left(1 - \frac{P}{K}\right)$. Suppose that the carrying capacity K is $10,000$. Suppose further that the initial population is $2,000$ and that it grows to $4,000$ after one year. What will be the population after another year (i.e. 2 years from the beginning)?

(A) $6,800$

(B) $6,400$

(C) $7,000$

(D) $6,600$

(E) $6,500$

INDICATE YOUR ANSWERS ON THE FRONT PAGE.
Penalty for not doing so is MINUS 4 marks.

11. Consider the following three series:

I. $\displaystyle\sum_{n=0}^{\infty} \frac{1}{(n+2)\sqrt{\ln(n+2)}}$

II. $\displaystyle\sum_{n=1}^{\infty} \left(e^{\frac{1}{n}} - 1 - \frac{1}{n^2} \right)$

III. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{n^5} + \left(\frac{3}{4}\right)^{n+1} \right)$

Decide which of the series converge (or converages).

Ⓐ II only

Ⓑ II and III only

Ⓒ I, and II and III

Ⓓ III only

Ⓔ I and III only

INDICATE YOUR ANSWERS ON THE FRONT PAGE.
Penalty for not doing so is MINUS 4 marks.

12. Consider the following series:

I. $\displaystyle\sum_{n=0}^{\infty}(-1)^n \frac{n^3+n+5}{n^4+n^2+25}$

II. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1} \frac{(1+n)^n}{n!}$

Which one of the following statements is correct?

Ⓐ I and II both converge conditionally.

Ⓑ I converges absolutely and II converges conditionally.

Ⓒ I converges conditionally and II diverges.

Ⓓ I converges conditionally and II converges absolutely.

Ⓔ I and II both converge absolutely.

INDICATE YOUR ANSWERS ON THE FRONT PAGE.
Penalty for not doing so is MINUS 4 marks.

13. Find the coefficient of x^4 of the Maclaurin series for $f(x) = \sin^2 x$.

Ⓐ $-\frac{1}{12}$

Ⓑ $-\frac{1}{4}$

Ⓒ $-\frac{3}{4}$

Ⓓ $-\frac{1}{6}$

Ⓔ $-\frac{1}{3}$

14. Let $a_n = \left(1 + \frac{\ln 2}{n}\right)^n$. Then the sequence $\{a_n\}$

Ⓐ converges to 1.

Ⓑ converges to 2.

Ⓒ diverges.

Ⓓ converges to $\ln 2$.

Ⓔ converges to e^2.

INDICATE YOUR ANSWERS ON THE FRONT PAGE.
Penalty for not doing so is MINUS 4 marks.

15. If y is a differentiable function of x such that $(1+x^3)y^2+4\int_{2x}^{xy}\sqrt{5x^2+t^2}\,dt = 112$, find the value of $\frac{dy}{dx}$ at the point where $y=2$ (not $x=2$).

Ⓐ $-\frac{27}{55}$

Ⓑ not determinable due to insufficient information

Ⓒ $-\frac{18}{35}$

Ⓓ $-\frac{23}{44}$

Ⓔ $-\frac{28}{33}$

INDICATE YOUR ANSWERS ON THE FRONT PAGE.
Penalty for not doing so is MINUS 4 marks.

16. Find the value of $\displaystyle\int_0^{\infty} \frac{x \arctan x}{(1+x^2)^2}\, dx$

Ⓐ $\dfrac{\pi}{12}$

Ⓑ $\dfrac{\pi}{8}$

Ⓒ $\dfrac{\pi}{6}$

Ⓓ $\dfrac{\pi}{2}$

Ⓔ $\dfrac{\pi}{4}$

INDICATE YOUR ANSWERS ON THE FRONT PAGE.
Penalty for not doing so is MINUS 4 marks.

17. The improper integral $\displaystyle\int_1^{\infty} \frac{1+8x-2x^2}{8x^4+2x^3+4x^2+x}\,dx$

Ⓐ converges to $\ln\left(\frac{6}{5}\right)$.

Ⓑ converges to $\ln\left(\frac{5}{2}\right)$.

Ⓒ diverges.

Ⓓ converges to $\ln\left(\frac{9}{8}\right)$.

Ⓔ converges to $\ln\left(\frac{5}{4}\right)$.

INDICATE YOUR ANSWERS ON THE FRONT PAGE.
Penalty for not doing so is MINUS 4 marks.

18. Let $f^{(n)}(a)$ denote the value of the n th derivative of f at a. If $f(x) = \dfrac{3+x}{2-8x^2}$, then $f^{(95)}(0) =$

Ⓐ $(95!)2^{94}$

Ⓑ $(95!)2^{95}$

Ⓒ $(95!)2^{97}$

Ⓓ $(95!)2^{93}$

Ⓔ $(95!)2^{96}$