STAT3032 SURVIVAL MODELS

TUTORIAL WEEK FOUR

Question One

We make fifteen observations of a discrete non negative integer random variable T. The data is

4, 6, 9, 4, 2, 0, 8, 7, 14, 6, 1, 3, 4, 12, 10.

$\hat{F}(3.5) = \frac{4}{15}$ $\quad\hat{F}(8.5) = \frac{11}{15}$

(a) Calculate the empirical distribution function at time 3.5 and time 8.5.

(b) Calculate the variance of the estimated empirical distribution function at time 3.5 and time 8.5.

$Var(\hat{F}(3.5)) = \frac{\frac{4}{15}\cdot\frac{11}{15}}{15} = \frac{44}{3375} = \cdots Var(\hat{F}(8.5))$

(c) Form a 95% confidence interval for the empirical distribution function at time 3.5. State any assumptions made.

$\frac{4}{15} \pm 1.96\sqrt{\frac{44}{3375}}$

$\frac{11}{15} \pm 1.96\sqrt{\frac{44}{3375}}$

Question Two

Suppose T is a continuous survival random variable with hazard function

$a + bt,\ a > 0,\ b > 0,\ t > 0.$

Find the *sdf* of T.

$$S(t) = \exp\left(-\int_0^t (a+by)\,dy\right) = \exp\left(-\left[ay + \tfrac{1}{2}by^2\right]_0^t\right) = \exp\left(-at - \frac{bt^2}{2}\right)$$

Question Three

Under a particular survival model, μ_x is constant and equal to 0.01 for all ages. Calculate:

(a) the probability that a 43 year old survives to age 48 $\quad {}_5P_{43} = \exp(-5(0.01)) = \exp(-0.05) = 0.95123$

(b) the complete expectation of life at age 20 $\quad \mathring{e}_{20} = \int_0^\infty {}_tP_{20}\,dt = \int_0^\infty e^{-0.01t}\,dt = -100\left[e^{-0.01t}\right]\Big|_0^\infty = 100$

(c) the curtate expectation of life at age 20 $\quad e_{20} = \sum_{t=1}^\infty {}_tP_{20} = \sum_{t=1}^\infty e^{-0.01t} = \frac{e^{-0.01}}{1-e^{-0.01}} = 99.5008$

(d) the central rate of mortality at age 35 $\quad m_x = \frac{d_x}{\int_0^1 l_{x+t}\,dt} = \frac{q_x}{\int_0^1 {}_tP_x\,dt} = \frac{\int_0^1 {}_tP_x\,\mu_{x+t}\,dt}{\int_0^1 {}_tP_x\,dt} = \frac{\int_0^1 e^{-0.01t}\,0.01\,dt}{\int_0^1 e^{-0.01t}\,dt} = 0.01$

(e) the initial rate (q-rate) of mortality at age 35

Question Four $\quad q_x = \int_0^1 {}_tP_x\,\mu_{x+t}\,dt = \int_0^1 e^{-0.01t}\,0.01\,dt = -e^{-0.01t}\Big|_0^1 = 0.00995$

Derive an approximate formula for q_x in terms of m_x.

$$m_x = \frac{d_x}{\int_0^1 l_{x+t}\,dt} \approx \frac{d_x}{l_x - \frac{1}{2}d_x} \quad \text{under UDD}$$

$$m_x = \frac{q_x}{1-\frac{1}{2}q_x} \qquad q_x \approx \frac{m_x}{1+\frac{1}{2}m_x}$$