June 18th

Show that $|\sin x - x + \frac{1}{6}x^3| < 0.08$ for $|x| \leq \frac{1}{2}\pi$

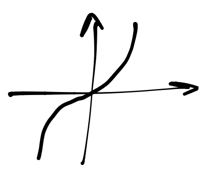
$x - \frac{1}{6}x^3$ is first 5 terms of taylor expansion of $\sin x$ around 0.

$P_{0,4} = x - \frac{1}{6}x^3$

$|\sin x - x + \frac{1}{6}x^3| = |R_{0,4}(x)|$

$\exists c \in (0,h)$, s.t. $|R_{0,4}(x)| = 1 \cdot \frac{x^{4+1}}{(4+1)!}|\sin^{(5)}(c)| \leq \frac{|x|^5}{5!} \leq (\frac{\pi}{2})^5 / 5! \approx 0.08$

$x^3 \Rightarrow 3x^2 \Rightarrow 6x$



neither max nor min

P95 #9

Sps f is C^k on open interval containing point a $f'(a) = f^{(2)}(a) \cdots = f^{(k-1)}(a) = 0$, $f^{(k)}(a) \neq 0$. then (1) f has local maximum $f^{(b)}(a) < 0$ \} k is even

(2) f has - - - minimum $f^{(b)}(a) > 0$

(3) neither max nor min, k is odd.

Proof: $C + A(x-a)^k + R_{a,k}$

$f(x) = C + (A + \frac{R_{a,k}(x)}{(x-a)^k}(x-a)^k)$

$\frac{R(x)}{(x-a)^k} \to 0$ as $x \to a$ i.e. $\forall \varepsilon > 0, \exists \delta > 0$ s.t. $\forall x \in (a-\delta, a+\delta)$

$|\frac{R(x)}{(x-a)^k}| < \varepsilon$ choose $\varepsilon = \frac{1}{2}|A|$

$f(a) = C$ in $(a-\delta, a)$, $(a, a+\delta)$

if k is even, $A > 0$

$(x-a)^k > 0$

$A + \frac{R_{a,k}(x)}{(x-a)^k} > 0 \Rightarrow$ in $(a-\delta, a+\delta)$ $f(x) = C + (\underbrace{A + \cdots}_{>0})\underbrace{(x-a)^k}_{>0}$

$f(x) > C$, min

if k is even, $A < 0$, $A + \frac{R}{(x-a)^k} < 0 \Rightarrow f(x) < C$, max