UNIVERSITY OF TORONTO
Faculty of Arts and Science

EXAMINATION APRIL 2012

PHL 245 H1S
L0101 - Niko Scharer

Duration - 3 hours

Examination Aid: Sheet with rules (provided)

Last Name ______________________________

First Name ______________________________

Student Number__________________________

Answer **all** questions on the exam paper.

Use the supplied examination booklet for rough work OR if you need further space.

The exam consists of fourteen (14) pages. Pages 2-12 have questions on them.

The final two pages (13-14) are an aid sheet and may be detached from the rest of the exam.

1. Suppose there are two sentences: ϕ and ψ. On every interpretation that ϕ is true, ψ is false.

What can you conclude (if anything) about the following argument? Briefly explain. (3 pts.)

$$\dfrac{\sim(\phi \rightarrow \chi)}{\therefore \sim\psi \vee \chi}$$

2. Here is a truth-table for the NEW symbol: $\ast$

P	Q	P $\ast$ Q
T	T	F
T	F	T
F	T	T
F	F	T

a) Given this truth-table, what ordinary English expression can this new truth-functional connective ($\ast$) be used to symbolize? (1 pt.)

b) Using the definition of the new symbol, $\ast$, as defined by the truth-table above, provide a shortened truth-table and truth-value assignment that shows that the following sentence is NOT a tautology. (3 pts.)

$$((W \rightarrow R) \ast S) \rightarrow (R \vee (S \ast Y))$$

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3. Provide an English language interpretation that shows that the following argument is invalid. Your interpretation should specify the universe of discourse and a symbolization scheme. (4 pts.)

$\exists x(Ax \wedge \forall y{\sim}F(xy))$. $\exists x \exists y(Bx \wedge {\sim}Ay \wedge F(xy))$. $\therefore {\sim}\forall x(Bx \rightarrow \exists y(Ay \wedge F(yx)))$.

4. Explain why the following sentence is a tautology. (4 pts.)

$$\forall x(Hx \rightarrow \exists y{\sim}L(xy)) \rightarrow {\sim}\exists y(Hy \wedge \forall xL(yx))$$

5. Use this symbolization scheme to symbolize the following sentences: 36 pts. total

A^1: *a* is an amusement park	B^1: *a* is a ride	C^1: *a* is a roller coaster
D^1: *a* is a day	E^1: *a* is exciting	F^1: *a* is a Ferris wheel
H^1: *a* is a person	G^2: *a* goes on *b*	K^2: *a* is a friend of *b*
L^2: *a* likes *b*	M^2: *a* is more popular than *b*	O^3: *a* visits *b* on *c*
a^0: Aaron	b^0: Betsy	f^1: the father of *a*

a) Roller coasters and Ferris wheels are rides. (2 pts.)

b) Only assuming that not all rides are exciting, Aaron doesn't go on any Ferris wheels. (3 pts.)

c) Some people only go on rides that are exciting. (3 pts.)

d) People who don't like amusement parks don't go on exciting rides unless their friends do too. (4 pts.)

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5 continued. Use this symbolization scheme to symbolize the following sentences:

A^1: *a* is an amusement park	B^1: *a* is a ride	C^1: *a* is a roller coaster
D^1: *a* is a day	E^1: *a* is exciting	F^1: *a* is a Ferris wheel
H^1: *a* is a person	G^2: *a* goes on *b*	K^2: *a* is a friend of *b*
L^2: *a* likes *b*	M^2: *a* is more popular than *b*	O^3: *a* visits *b* on *c*
a^0: Aaron	b^0: Betsy	f^1: the father of *a*

e) Every day people visit amusement parks, but on no day is everybody visiting the same amusement park. (4 pts.)

f) The most popular ride is not liked by everyone who goes on it. (4 pts.)

g) Aaron likes exactly one ride, but neither Betsy nor Betsy's father goes on it. (4 pts.)

h) Only Betsy goes on exactly those rides that Aaron dislikes. (4 pts.)

5 continued.

A^1: *a* is an amusement park	B^1: *a* is a ride	C^1: *a* is a roller coaster
D^1: *a* is a day	E^1: *a* is exciting	F^1: *a* is a Ferris wheel
H^1: *a* is a person	G^2: *a* goes on *b*	K^2: *a* is a friend of *b*
L^2: *a* likes *b*	M^2: *a* is more popular than *b*	O^3: *a* visits *b* on *c*
a^0: Aaron	b^0: Betsy	f^1: the father of *a*

i) Using the symbolization scheme above, provide an idiomatic English sentence that expresses: (4 pts.)

$$\forall x(Hx \rightarrow \forall y(Ay \rightarrow \exists z(Dz \wedge O(xyz) \wedge \exists w(Dw \wedge w \neq z \wedge O(xyw)) \rightarrow L(xy))))$$

j) Using the symbolization scheme above, symbolize the following ambiguous sentence **two** logically distinct ways. For each, provide an English sentence that explains exactly what the symbolized sentence means:

Somebody doesn't go on every ride. (4 pts.)

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6. Provide a derivation that shows the following theorem is valid **using only the 10 basic rules from SL** (R, DN, MP, MT, ADJ, S, ADD, MTP, BC, CB) **and the 3 basic rules from PL** (UI, EG, EI) (9 pts.)

$$\therefore\ \forall x((Bx \lor Cx) \rightarrow \sim(Fx \rightarrow Ax)) \rightarrow (\exists x(\sim Bx \rightarrow Cx) \rightarrow \exists x\sim Ax)$$

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7. Provide a derivation that shows that this is a valid argument **using only the 10 basic rules from SL** (R, DN, MP, MT, ADJ, S, ADD, MTP, BC, CB) **and the 3 basic rules from PL** (UI, EG, EI) (9 pts.)

$\exists zFz \rightarrow \exists x({\sim}Gx \wedge \forall yH(xy))$. $\quad$ $\forall x(Gx \vee Dx)$. $\quad$ $\forall w\forall z({\sim}H(wz) \leftrightarrow L(zw))$.

$\therefore \forall x(Fx \rightarrow \exists y(Dy \wedge {\sim}L(xy)))$

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8. Provide a derivation to show that this is a valid argument (use any rules). (9 pts.):

$\exists z(\sim Bz \wedge \sim Cz) \rightarrow \exists x \forall y F(a(x)a(y))$. $\therefore \forall x \exists y \sim(Bx \vee Cy) \rightarrow \exists x F(xa(x))$

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9. Show that the following is a valid argument (use any rules). (9 pts.):

$\forall z(\exists w A(wz) \to B(zz))$. $\forall x \exists y \forall z H(xyz)$. $\exists x \forall y(H(xyy) \to \forall z \sim G(xz))$.

$\forall x \forall y(A(xy) \to \sim M(yx)) \to \sim\exists y \sim G(yy)$. $\therefore \sim\forall x(B(xx) \to \sim\exists z M(xz))$

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10. Use a finite model to demonstrate that this set of three sentences is consistent (8 pts.):

$\{\exists x \forall y(Fx \wedge L(xy)).\ \ \forall x(Gx \rightarrow \exists y{\sim}L(xy)).\ \ {\sim}\forall x(L(xx) \rightarrow {\sim}Gx)\}$

i) provide a truth-functional expansion (to two individuals) for each sentence in this set.

ii) define a finite model with a universe of two individuals that shows that the set is consistent.

$\exists x \forall y(Fx \wedge L(xy)).$ $\forall x(Gx \rightarrow \exists y{\sim}L(xy)).$ ${\sim}\forall x(L(xx) \rightarrow {\sim}Gx).$

11. Is the material conditional a necessary logical connective in our system? Consider whether we would be able to symbolize the same English sentences without the material conditional. Consider whether we would be able to derive the same theorems without the material condition.

Justify your answer with an explanation that considers the role of the material conditional in both symbolization and derivations. (5 pts.)

=100 pts. total.

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AID SHEET: DERIVATION RULES (SEE BOTH SIDES)

Derivation Types:

Direct Derivation (DD)

Conditional Derivation (CD)

Indirect Derivation (ID)

Universal Derivation (UD) — Restriction: the instantiating term cannot occur unbound in any previous line.

Basic Rules for Sentential Operators:

Modus Ponens (MP)

$$\frac{(\phi \rightarrow \psi) \qquad \phi}{\psi}$$

Modus Tollens (MT)

$$\frac{(\phi \rightarrow \psi) \qquad \sim\psi}{\sim\phi}$$

Double Negation (DN)

$$\frac{\phi}{\sim\sim\phi} \qquad \frac{\sim\sim\phi}{\phi}$$

Repetition (R)

$$\frac{\phi}{\phi}$$

Simplification (S)

$$\frac{\phi \wedge \psi}{\phi} \qquad \frac{\phi \wedge \psi}{\psi}$$

Adjunction (ADJ)

$$\frac{\phi \qquad \psi}{\phi \wedge \psi}$$

Addition (ADD)

$$\frac{\phi}{\phi \vee \psi} \qquad \frac{\psi}{\phi \vee \psi}$$

Modus Tollendo Ponens (MTP)

$$\frac{\phi \vee \psi \qquad \sim\phi}{\psi} \qquad \frac{\phi \vee \psi \qquad \sim\psi}{\phi}$$

Biconditional-Conditional (BC)

$$\frac{\phi \leftrightarrow \psi}{\phi \rightarrow \psi} \qquad \frac{\phi \leftrightarrow \psi}{\psi \rightarrow \phi}$$

Conditional-Biconditional (CB)

$$\frac{\phi \rightarrow \psi \qquad \psi \rightarrow \phi}{\phi \leftrightarrow \psi}$$

Derived Rules for Sentential Operators:

Negation of Conditional (NC)

$$\frac{\sim(\phi \rightarrow \psi)}{\phi \wedge \sim\psi} \qquad \frac{\phi \wedge \sim\psi}{\sim(\phi \rightarrow \psi)}$$

Conditional as Disjunction (CDJ)

$$\frac{\phi \rightarrow \psi}{\sim\phi \vee \psi} \qquad \frac{\sim\phi \vee \psi}{\phi \rightarrow \psi} \qquad \frac{\sim\phi \rightarrow \psi}{\phi \vee \psi} \qquad \frac{\phi \vee \psi}{\sim\phi \rightarrow \psi}$$

Separation of Cases (SC)

$$\frac{\begin{array}{l}\phi \vee \psi \\ \phi \rightarrow \chi \\ \psi \rightarrow \chi\end{array}}{\chi} \qquad \frac{\begin{array}{l}\phi \rightarrow \chi \\ \sim\phi \rightarrow \chi\end{array}}{\chi}$$

Negation of Biconditional (NB)

$$\frac{\sim(\phi \leftrightarrow \psi)}{\phi \leftrightarrow \sim\psi} \qquad \frac{\phi \leftrightarrow \sim\psi}{\sim(\phi \leftrightarrow \psi)}$$

De Morgan's (DM)

$$\frac{\sim(\phi \vee \psi)}{\sim\phi \wedge \sim\psi} \qquad \frac{\sim\phi \wedge \sim\psi}{\sim(\phi \vee \psi)} \qquad \frac{\sim(\phi \wedge \psi)}{\sim\phi \vee \sim\psi} \qquad \frac{\sim\phi \vee \sim\psi}{\sim(\phi \wedge \psi)}$$

$$\frac{\sim(\sim\phi \vee \sim\psi)}{\phi \wedge \psi} \qquad \frac{\phi \wedge \psi}{\sim(\sim\phi \vee \sim\psi)} \qquad \frac{\sim(\sim\phi \wedge \sim\psi)}{\phi \vee \psi} \qquad \frac{\phi \vee \psi}{\sim(\sim\phi \wedge \sim\psi)}$$

Derivation Rules for Predicate Logic:

Existential Generalization (EG)

$$\frac{\phi_\zeta}{\exists\alpha\phi_\alpha}$$

Universal Instantiation (UI)

$$\frac{\forall\alpha\phi_\alpha}{\phi_\zeta}$$

Restriction: ζ does not occur as a bound variable in ϕ_α

Existential Instantiation (EI)

$$\frac{\exists\alpha\phi_\alpha}{\phi_\zeta}$$

Restriction: ζ does not occur in any previous line or premise.

Quantifier Negation (QN)

$$\frac{\sim\forall\alpha\phi}{\exists\alpha\sim\phi} \qquad \frac{\sim\exists\alpha\,\phi}{\forall\alpha\sim\phi}$$

$$\frac{\exists\alpha\sim\phi}{\sim\forall\alpha\phi} \qquad \frac{\forall\alpha\sim\phi}{\sim\exists\alpha\,\phi}$$