6.2 EG1 Let’s try a derivation using UI

$\forall x(Fx \to Gx)$. $\forall y(Hy \to {\sim}Gy)$. Fb. $\therefore {\sim}Hb$.

1	~~Show~~ ${\sim}Hb$		
2	$Fb \to Gb$	pr1 ui	We instantiate to premise 1 to ‘b’ – removing the quantifier and substituting b for x. We choose to instantiate to ‘b’ so that it matches the individual in the third premise and the conclusion.
3	Gb	pr3 2 mp	Since the individual matches, we can MP to work with the substitution instance (2) and the third premise.
4	$Hb \to {\sim}Gb$	pr2 ui	Again, we instantiate to ‘b’. this time the second premise.
5	${\sim}{\sim}Gb$	3 dn	
6	${\sim}Hb$	5 4 mt dd	

6.2 EG2 Let’s try one that uses more of our skills from sentential logic.

$\forall x((Fx \wedge {\sim}Gx) \to (Ax \vee Bx))$. $\forall y{\sim}(Gy \wedge Dy)$. $Fa \wedge Da$. $\therefore \forall x(Gx \leftrightarrow Bx) \to Aa$

1	~~show~~ $\forall x(Gx \leftrightarrow Bx) \to Aa$		
2	$\forall x(Gx \leftrightarrow Bx)$	ass CD	Since the show line is a conditional, we do a conditional derivation. We assume the antecedent and our goal is the consequent … Aa
3	$(Fa \wedge {\sim}Ga) \to (Aa \vee Ba)$	pr1 ui	We instantiate to premise 1 to ‘a’ – removing the quantifier and substituting a for x. We choose to instantiate to ‘a’ so that it matches the individual in the third premise and the conclusion.
4	${\sim}(Ga \wedge Da)$	pr2 ui	We need more to work with, we instantiate the second premise – again to ‘a’ so that it matches the other sentences so that we can work with it.
5	$Ga \leftrightarrow Ba$	2 ui	Now line 2. Here every universal has been instantiated – now it is just a sentential logic derivation.

6.2 EG2 cont. Here's the complete derivation.

$\forall x((Fx \wedge \sim Gx) \rightarrow (Ax \vee Bx))$. $\forall y \sim (Gy \wedge Dy)$. $Fa \wedge Da$. $\therefore \forall x(Gx \leftrightarrow Bx) \rightarrow Aa$

1	~~show~~ $\forall x(Gx \leftrightarrow Bx) \rightarrow Aa$	
2	$\forall x(Gx \leftrightarrow Bx)$	ass CD
3	$(Fa \wedge \sim Ga) \rightarrow (Aa \vee Ba)$	pr1 ui
4	$\sim(Ga \wedge Da)$	pr2 ui
5	$Ga \leftrightarrow Ba$	2 ui
6	Fa	pr 3 s
7	Da	pr 3 s
8	$\sim Ga \vee \sim Da$	dm
9	$\sim\sim Da$	7 dn
10	$\sim Ga$	8 9 mtp
11	$Ba \rightarrow Ga$	5 bc
12	$\sim Ba$	10 11 mt
13	$Fa \wedge \sim Ga$	6 10 adj
14	$Aa \vee Ba$	13 3 mp
15	Aa	14 12 mtp cd

6.2 EG3 We can use UI twice on the same sentence!

$\forall x \forall y(Gx \wedge Hy \rightarrow L(xy))$. $\forall x(Bx \vee Hx)$. $Ga \wedge \sim Bb$. $\therefore L(ab)$

1	~~show~~ L(ab)		
2	$Bb \vee Hb$	pr2 ui	We instantiate the second premise. We choose to instantiate to 'b' so that it matches the second conjunct of the third premise, ~Bb.
3	$\forall y(Ga \wedge Hy \rightarrow L(ay))$	pr1 ui (a/x)	Now we instantiate premise 1, working with the main logical operator - $\forall$x. When we instantiate, we replace all instances of x with a individual term – here we choose 'a' so that it matches the first conjunct of the third premise, Ga and the conclusion L(**a**b). The (a/x) is an optional annotation, that says that you substituted 'a' for 'x'.
4	$Ga \wedge Hb \rightarrow L(ab)$	3 ui (b/y)	Now we instantiate the universal on line 3, substituting 'b' for 'y' to match line 2 and the conclusion. All universals have been instantiated – from this point on it is sentential logic!

6.2 EG3 cont. Here's the complete derivation.

$\forall x \forall y(Gx \wedge Hy \rightarrow L(xy))$. $\forall x(Bx \vee Hx)$. $Ga \wedge \sim Bb$. $\therefore L(ab)$

1	~~show~~ $L(ab)$	
2	$Bb \vee Hb$	pr2 ui
3	$\forall y(Ga \wedge Hy \rightarrow L(ay))$	pr1 ui (a/x)
4	$Ga \wedge Hb \rightarrow L(ab)$	3 ui (b/y)
5	$\sim Bb$	pr3 s
6	Hb	2 5 mtp
7	Ga	pr3 s
8	$Ga \wedge Hb$	6 7 adj
9	$L(ab)$	8 4 mp dd

6.2 EG4 When we use UI, we can instantiate to any constant… even one that we have already instantiated to in the same sentence. (This one is easiest as an indirect derivation!)

$\forall x \forall y(Fx \rightarrow (Cy \rightarrow B(xy)))$. $\forall x \sim B(xx)$. $\therefore \sim Fa \vee \sim Ca$

1	~~show~~ $\sim Fa \vee \sim Ca$		
2	$\sim(\sim Fa \vee \sim Ca)$	ass id	This one is easiest as an indirect derivation.
3	$\forall y(Fa \rightarrow (Cy \rightarrow B(ay)))$	pr1 ui	We instantiate the first premise, removing the main logical operator $\forall x$, and substituting 'a' for x. We instantiate to 'a' to match the first disjunct of the conclusion, ~Fa.
4	$Fa \rightarrow (Ca \rightarrow B(aa))$	3 ui	We instantiate line 3, removing the main logical operator $\forall y$, and substituting 'a' for y. We instantiate to 'a' to match the second disjunct of the conclusion, ~Ca … and with UI we can instantiate to *any* individual, even one we have already used!
5	$\sim B(aa)$	pr2 ui	We instantiate premise 2, substituting 'a' for every instance of 'x'. By instantiating to 'a', we get it to match the consequent of 4.
6	$\sim\sim Fa \wedge \sim\sim Ca$	2 dm	
7	$\sim\sim Fa$	6 s	
8	Fa	dn	
9	Ca	6 sr dn	We can combine two steps into one line.
10	$Ca \rightarrow B(aa)$	4 8 mp	
11	$B(aa)$	7 8 mp 2 id	There's the contradiction – lines 5 & 11.

6.3 EG1 Let's try another – although it is more complex, it uses the same general strategy.

$\sim$Fc. $\forall$x(Gx $\rightarrow$ Fx). $\forall$x $\sim$(Hx $\wedge$ $\sim$Gx). $\forall$y(Ly $\vee$ Hy). $\therefore$ $\exists$z (Lz $\vee$ Jz)

1	~~Show~~ $\exists$z(Lz $\vee$ Jz)	The goal is to show an instantiated form of this: Lα $\vee$ Jα	
2	Gc $\rightarrow$ Fc	pr2 ui	Instantiate to 'c', so that it will match pr1.
3	$\sim$(Hc $\wedge$ $\sim$Gc)	pr3 ui	Instantiate to 'c', again to match pr1 and 2.
4	Lc $\vee$ Hc	pr4 ui	Instantiate to 'c' again.
5	$\sim$Gc	pr1 2 mt	
6	$\sim$Hc $\vee$ $\sim\sim$Gc	3 dm	
7	$\sim\sim\sim$Gc	5 dn	
8	$\sim$Hc	6 7 mtp	
9	Lc	4 8 mtp	
10	Lc $\vee$ Jc	9 add	That's the instantiated sentence we want.
11	$\exists$z(Lz $\vee$ Jz)	10 EG dd	Now generalize to match the show line.
12			

6.3 EG2 Sometimes you need to use the rules for sentential logic before you remove the quantifiers and/or after you introduce them.

$\forall$x(Gx $\rightarrow$ (Hx $\wedge$ Jx)) $\wedge$ $\forall$y(Jy $\rightarrow$ Ly). $\forall$xGx $\therefore$ $\exists$xHx $\wedge$ $\exists$yLy

1	~~Show~~ $\exists$xHx $\wedge$ $\exists$yLy	Goal: each conjunct in its instantiated form: Hα, Lα	
2	$\forall$x(Gx $\rightarrow$ (Hx $\wedge$ Jx))	PR1 SL	We need $\forall$ as the main operator to use UI.
3	$\forall$y(Jy $\rightarrow$ Ly)	PR1 SR	We need this conjunct as well.
4	Ga	PR2 UI	We can instantiate to any term – here I've used 'a'.
5	Ga $\rightarrow$ (Ha $\wedge$ Ja)	2 UI	We instantiate to 'a' so it matches 4.
6	Ha $\wedge$ Ja	4 5 MP	
7	Ha	6 S	That's one instantiated conjunct.
8	$\exists$xHx	7 EG	Generalize and we have one goal!
9	Ja $\rightarrow$ La	3 UI	Again instantiate to 'a' to match 6.
10	Ja	6 S	
11	La	9 10 MP	That's the other instantiated conjunct.
12	$\exists$yLy	11 EG	Generalize to get the other goal.
13	$\exists$xHx $\wedge$ $\exists$yLy	8 12 adj DD	That's it … now just box and cancel!

6.3 EG3 Sometimes you need to use UI or EG several times for the same sentence.

Every use of UI is a separate step, likewise every use of EG is a separate step. We can, of course, use the short-cut of combining two steps on one line, but each step will still need its own justification.

Notice that this argument includes a two-place predicate.

$\forall x \forall y((Fx \wedge Gy) \rightarrow L(xy))$. $\sim L(ba)$. $\therefore \exists x \exists y(\sim Fy \vee \sim Gx)$

1	~~show~~ $\exists x \exists y(\sim Fx \vee \sim Gy)$		
2	$\forall y((Fb \wedge Gy) \rightarrow L(by))$	Pr1 UI	We instantiate the universal generalization of premise 1 by removing '$\forall$x' and substituting 'b' for 'x'. We instantiate to 'b' because we want the consequent to match the order of 'b' and 'a' in the second premise (so that later we can use MT).
3	$(Fb \wedge Ga) \rightarrow L(ba)$	2 UI	We instantiate the universal generalization of line 2 by removing '$\forall$y' and substituting 'a' for 'y'. (Instantiate to 'a' so that the consequent matches premise 2.)
4	$\sim(Fb \wedge Ga)$	3 pr2 MT	Now we can use the rules of sentential logic – the second premise is the negation of the consequent of 3, so we can use MT to derive the negation of the antecedent.
5	$\sim Fb \vee \sim Ga$	4 DM	De Morgan's law allows us to derive the disjunction of negations from the negation of a conjunction.
6	$\exists y(\sim Fy \vee \sim Ga)$	5 EG	We can generalize the sentence on line 5 by replacing 'b' with 'y' and quantifying the entire sentence with $\exists$y. Because the quantifier must always go at the front of the whole sentence with EG, always generalize the 'inner' variable first. We replace 'b' with 'y' because that is the form that the desired sentence is in.
7	$\exists x \exists y(\sim Fy \vee \sim Gx)$	6 EG DD	We can generalize the sentence on line 6 by replacing 'a' with 'x' and quantifying the entire sentence with $\exists$x.

In some cases, when using EG, not all instances of the variable should be replaced.

$\forall x L(xx)$. $\therefore \exists x \exists y\ L(xy)$

1	~~Show~~ $\exists x \exists y\ L(xy)$		
2	$L(aa)$	PR1 UI	First we need an instantiated form of PR1
3	$\exists y L(ay)$	2 EG	If a is in relation L to itself, then a is L to something.
4	$\exists x \exists y L(xy)$	3 EG	So something is in relation L to something. (Since, everything is in relation L to itself!)

6.3 EG4 Let's try another:

$\forall x \forall y(F(xy) \rightarrow G(xx))$. $F(ca)$ $\therefore \exists xG(xc) \wedge \exists yG(yy)$

1	~~show~~ $\exists xG(xc) \wedge \exists yG(yy)$	Here the goal is two existentals, connected with $\wedge$.	
2	$\forall y(F(cy) \rightarrow G(cc))$	pr1 UI	Instantiate premise 1, substituting 'c' for every occurrence of 'x' to make it match premise 2.
3	$F(ca) \rightarrow G(cc)$	pr2 UI	Instantiate 2, substituting 'a' for 'y'. Now it matches premise 2.
4	$G(cc)$	pr2 3 MP	
5	$\exists xG(xc)$	4 EG	We generalize, replacing the first occurrence of 'c' with 'x', matching the first conjunct of the show line.
6	$\exists yG(yy)$	4 EG	We generalize, replacing both occurrences of 'c' with 'y', matching the second conjunct of the show line.
7	$\exists xG(xc) \wedge \exists yG(yy)$	5 6 ADJ DD	

Don't let operation letters confuse you! You can use EG in lots of different ways on one or more place operations.

6.3 EG5 Try this one:

$La(b(c))$. $\therefore \exists xLx \wedge \exists yLa(y) \wedge \exists zLa(b(z))$

1	show $\boldsymbol{\exists xLx \wedge \exists yLa(y) \wedge \exists zLa(b(z))}$		
2	$\exists xLx$	pr EG	a(b(c))/x
3	$\exists yLa(y)$	pr EG	b(c)/y
4	$\exists zLa(b(z))$	pr EG	c/y
5	$\boldsymbol{\exists xLx \wedge \exists yLa(y) \wedge \exists zLa(b(z))}$	2 3 adj 4 adj dd	

CAREFUL: When using EG you must replace a singular term (an individual) with the variable.

Consider La(b(c)): we can imagine an abbreviation scheme that would give us this symbolization.

c^0: Carol b^1: the brother of *a* c^1: the neice of *a*

c is a singular term – Carol.

b(c) is a singular term – Carol's brother.

a(b(c)) is a singular term – Carol's brother's neice.

Any of those can be replaced with a variable when using EG. However, the following are NOT legitimate existential generalizations: $\exists xLa(x(c))$ or $\exists xLx(b(c))$. In La(b(c)), the expressions 'a' and 'b' are not singular term letters – so they cannot be replaced with a variable!

6.3 EG6 In this one, we need to show the instantiated form of the conclusion, so that we can use EG. But, it is difficult to do that directly – but we can easily do it with an indirect derivation. First show the instantiated sentence (which is a negated conjunction), then assume the unnegated form (the conjunction) for ID.

$\forall x(Fx \to \forall y(Gy \to \sim H(xy)))$. $\forall x(\sim Gx \vee H(xx))$.

$\therefore\ \exists x \sim(Fx \wedge Gx)$

1	~~Show~~ $\exists x \sim(Fx \wedge Gx)$	
2	~~Show~~ $\sim(Fa \wedge Ga)$	
3	$Fa \wedge Ga$	Ass ID
4	Fa	3 s
5	Ga	3 s
6	$Fa \to \forall y(Gy \to \sim H(ay))$	Pr1 ui
7	$\forall y(Gy \to \sim H(ay))$	4 6 MP
8	$Ga \to \sim H(aa)$	6 ui
9	$\sim H(aa)$	5 8 MP
10	$\sim Ga \vee H(aa)$	pr2 ui
11	$\sim Ga$	9 10 mtp 5 ID
12	$\exists x \sim(Fx \wedge Gx)$	2 eg dd

6.3 EG7 Sometimes we need to use CD inside a derivation.

$(Fa \to \exists y\sim Gy) \to \forall y(Hy \to \exists xL(xy))$. Ha. $\forall y(Fy \to By)$. $\forall y(Gy \to \sim By)$.

$\therefore\ \exists xL(xa)$

1	~~Show~~ $\exists xL(xa)$		
2	~~Show~~ $Fa \to \exists y\sim Gy$		show ant. pr1
3	Fa	ass cd	goal: $\exists y\sim Gy$
4	$Fa \to Ba$	pr3 UI	a/x
5	Ba	3 4 mp	
6	$Ga \to \sim Ba$	pr4 UI	a/y
7	$\sim\sim Ba$	5 dn	
8	$\sim Ga$	6 7 mt	
9	$\exists y\sim Gy$	8 EG cd	
10	$\forall y(Hy \to \exists xL(xy))$	2 pr1 mp	
11	$Ha \to \exists xL(xa)$	10 UI	a/y
12	$\exists xL(xa)$	pr2 11 mp dd	

6.3 E1 Show that the following syllogisms are valid by providing a derivation (Note they are all conditionally valid, moving from universals to existential statements. Thus, the conclusions are conditional and follow from the antecedent that at least one such thing exists with the property of the universal.):

a) 1st figure: Celeront $\forall x(Cx \rightarrow \sim Jx)$. $\forall y(Ky \rightarrow Cy)$. $\therefore$ $Ka \rightarrow \exists z(Kz \wedge \sim Jz)$

1	~~show~~ $Ka \rightarrow \exists z(Kz \wedge \sim Jz)$	
2	Ka	ass cd
3	$Ka \rightarrow Ca$	pr2 UI
4	Ca	2 3 mp
5	$Ca \rightarrow \sim Ja$	pr1 UI
6	$\sim Ja$	4 5 mp
7	$Ka \wedge \sim Ja$	2 6 adj
8	$\exists z(Kz \wedge \sim Jz)$	7 eg cd

b) 2nd figure: Camestrop $\forall x(Bx \rightarrow Fx)$. $\forall y(Dy \rightarrow \sim Fy)$. $\therefore Db \rightarrow \exists x(Dx \wedge \sim Bx)$

1	~~show~~ $Db \rightarrow \exists x(Dx \wedge \sim Bx)$	
2	Db	ass cd
3	$Db \rightarrow \sim Fb$	pr2 UI
4	$\sim Fb$	2 3 mp
5	$Bb \rightarrow Fb$	pr1 UI
6	$\sim Bb$	4 5 mt
7	$Da \wedge \sim Ba$	2 6 adj
8	$\exists x(Dx \wedge \sim Bx)$	7 eg cd

c) 3rd figure: Darapti $\forall x(Ax \rightarrow Fx)$. $\forall x(Ax \rightarrow Ex)$. $\therefore Aa \rightarrow \exists x(Ex \wedge Fx)$

1	~~show~~ $Aa \rightarrow \exists x(Ex \wedge Fx)$	
2	Aa	ass cd
3	$Aa \rightarrow Ea$	pr2 UI
4	Ea	2 3 mp
5	$Aa \rightarrow Fa$	pr1 UI
6	Fa	4 5 mp
7	$Ea \wedge Fa$	4 6 adj
8	$\exists x(Ex \wedge Fx)$	7 eg cd

d) 4^{th} figure: Fesapo $\forall y(Fy \rightarrow \sim By)$. $\forall x(Bx \rightarrow Dx)$ $\therefore Bc \rightarrow \exists y(Dy \wedge \sim Fy)$

1	~~show~~ $Bc \rightarrow \exists y(Dy \wedge \sim Fy)$	
2	Bc	ass cd
3	$Bc \rightarrow Dc$	pr2 UI
4	Dc	2 3 mp
5	$Fc \rightarrow Bc$	pr1 UI
6	$\sim Fc$	2 dn 5 mt
7	$Dc \wedge \sim Fc$	4 6 adj
8	$\exists x(Dx \wedge \sim Bx)$	7 eg cd

6.3 E2 Show that the following arguments are valid by providing a derivation:

a) $\forall x(Fx \vee Gx)$. $\forall y(Hy \rightarrow \sim Fy)$. $\forall z(\sim Bz \rightarrow \sim Gz)$ $\therefore Ha \rightarrow Ba$

1	~~show~~ $Ha \rightarrow Ba$	
2	Ha	ass cd
3	$Ha \rightarrow \sim Fa$	pr2 ui
4	$\sim Fa$	2 3 mp
5	$Fa \vee Ga$	pr1 ui
6	Ga	4 5 mtp
7	$\sim Ba \rightarrow \sim Ga$	pr3 ui
8	$\sim\sim Ga$	6 dn
9	$\sim\sim Ba$	7 8 mt
10	Ba	9 dn cd

b) $\therefore \forall x(Ex \rightarrow (Fx \rightarrow Ax)) \rightarrow ((Ea \wedge Fa) \rightarrow \exists yAy)$

1	~~Show~~ $\forall x(Ex \rightarrow (Fx \rightarrow Ax)) \rightarrow ((Ea \wedge Fa) \rightarrow \exists yAy)$		
2	$\forall x(Ex \rightarrow (Fx \rightarrow Ax))$		ass cd
3	~~show~~ $Ea \wedge Fa \rightarrow \exists yAy$		
4		$Ea \wedge Fa$	ass cd
5		Ea	4 s
6		Fa	4 s
7		$Ea \rightarrow (Fa \rightarrow Aa)$	2 ui
8		$Fa \rightarrow Aa$	5 7 mp
9		Aa	6 8 mp
10		$\exists yAy$	9 eg cd
11			3 cd

c) $\forall x(Ax \wedge (Bx \vee Cx))$. $\forall y(Ay \rightarrow (Cy \rightarrow Dy))$. $\sim(Ab \wedge Bb)$ $\therefore \exists x(Dx \wedge Cx)$

1	~~Show~~ $\exists x(Dx \wedge Cx)$	
2	$\sim Ab \vee \sim Bb$	pr3 dm
3	$Ab \wedge (Bb \vee Cb)$	pr1 ui
4	Ab	3 s
5	$Bb \vee Cb$	3 s
6	$\sim\sim Ab$	4 dn
7	$\sim Bb$	6 2 mtp
8	Cb	5 7 mtp
9	$Ab \rightarrow (Cb \rightarrow Db)$	pr2 ui
10	$Cb \rightarrow Db$	4 9 mp
11	Db	8 10 mp
12	$Db \wedge Cb$	8 11 adj
13	$\exists x(Dx \wedge Cx)$	12 eg dd

d) $\forall x(Ax \rightarrow Bx)$. $\sim Ba(c)$. $\forall z(\sim Az \rightarrow Cz)$. $\therefore \exists yCy$

1	~~Show~~ $\exists yCy$	
2	$Aa(c) \rightarrow Ba(c)$	pr1 ui
3	$\sim Aa(c)$	pr2 2 mt
4	$\sim Aa(c) \rightarrow Ca(c)$	pr3 ui
5	$Ca(c)$	3 4 mp
6	$\exists yCy$	5 eg dd

e) $\forall x(Fx \leftrightarrow \sim Gx \wedge Hx)$. $\forall y(Fy \wedge (\sim Cy \rightarrow \sim Hy))$. $\therefore \exists x(\sim Gx \wedge Cx)$

1	~~Show~~ $\exists x(\sim Gx \wedge Cx)$	
2	$Fa \leftrightarrow \sim Ga \wedge Ha$	pr1 ui
3	$Fa \wedge (\sim Ca \rightarrow \sim Ha)$	pr2 ui
4	Fa	3s
5	$\sim Ca \rightarrow \sim Ha$	3s
6	$Fa \rightarrow \sim Ga \wedge Ha$	2 bc
7	$\sim Ga \wedge Ha$	4 6 mp
8	$\sim Ga$	7s
9	$\sim\sim Ha$	7s dn
10	Ca	5 9 mt dn
11	$\sim Ga \wedge Ca$	8 10 adj
12	$\exists x(\sim Gx \wedge Cx)$	11 eg
13		12 dd

f) $\forall x\forall y(Ax \wedge {\sim}By \rightarrow D(yx))$. ${\sim}(Ae \rightarrow Bd)$. $\forall x\forall y(D(yx) \rightarrow Gx \wedge Hy)$ $\therefore \exists x\exists y(Gy \wedge Hx)$

1	~~show~~ $\exists x\exists y(Gy \wedge Hx)$	
2	$\forall y(Ae \wedge {\sim}By \rightarrow D(ye))$	pr1 ui
3	$Ae \wedge {\sim}Bd \rightarrow D(de)$	2 ui
4	$\forall y(D(ye) \rightarrow Ge \wedge Hy)$	pr3 ui
5	$D(de) \rightarrow Ge \wedge Hd$	4 ui
6	$Ae \wedge {\sim}Bd$	pr2 nc
7	$D(de)$	6 3 mp
8	$Ge \wedge Hd$	5 7 mp
9	$\exists y(Gy \wedge Hd)$	8 eg
10	$\exists x\exists y(Gy \wedge Hx)$	9 eg dd

g) $\forall y((Fy \vee Gy) \rightarrow Hy)$. ${\sim}Hb$. $\forall x(Fx \vee {\sim}Bx)$. $\forall x{\sim}(Ax \leftrightarrow Bx)$ $\therefore \exists z(Az \wedge {\sim}Gz)$

1	~~Show~~ $\exists z(Az \wedge {\sim}Gz)$	
2	$Fb \vee Gb \rightarrow Hb$	pr1 ui
3	${\sim}(Fb \vee Gb)$	2 pr2 mt
4	${\sim}Fb \wedge {\sim}Gb$	3 dm
5	${\sim}Fb$	4s
6	${\sim}Gb$	4s
7	$Fb \vee {\sim}Bb$	pr3 ui
8	${\sim}Bb$	5 7 mtp
9	${\sim}(Ab \leftrightarrow Bb)$	pr4 ui
10	$Ab \leftrightarrow {\sim}Bb$	9 nb
11	${\sim}Bb \rightarrow Ab$	10 bc
12	Ab	8 11 mp
13	$Ab \wedge {\sim}Gb$	6 12 adj
14	$\exists z(Az \wedge {\sim}Gz)$	13 eg dd

h) $\therefore \forall x\forall yL(xy) \rightarrow \exists x\exists y(L(xy) \wedge L(yx))$

1	show $\forall x\forall yL(xy) \rightarrow \exists x\exists y(L(xy) \wedge L(yx))$	
2	$\forall x\forall yL(xy)$	ass cd
3	$\forall yL(ay)$	2 ui
4	$L(ab)$	3 ui
5	$\forall yL(by)$	2 ui
6	$L(ba)$	5 ui
7	$L(ab) \wedge L(ba)$	4 6 adj
8	$\exists y(L(ay) \wedge L(ya))$	7 eg
9	$\exists x\exists y(L(xy) \wedge L(yx))$	8 eg cd

i) $\therefore \forall x \forall y \forall z A(xyz) \rightarrow \exists x \exists y(A(xxy) \wedge A(yxy) \wedge A(yyy))$

1	~~show~~ $\forall x \forall y \forall z A(xyz) \rightarrow \exists x \exists y(A(xxy) \wedge A(yxy) \wedge A(yyy))$	
2	$\forall x \forall y \forall z A(xyz)$	ass cd
3	$\forall y \forall z A(ayz)$	2 ui
4	$\forall z A(aaz)$	3 ui
5	$A(aab)$	4 ui
6	$\forall y \forall z A(byz)$	2 ui
7	$\forall z A(baz)$	6 ui
8	$A(bab)$	7 ui
9	$\forall z A(bbz)$	6 ui
10	$A(bbb)$	9 ui
11	$A(aab) \wedge A(bab)$	5 8 adj
12	$A(aab) \wedge A(bab) \wedge A(bbb)$	11 10 adj
13	$\exists y(A(aay) \wedge A(yay) \wedge A(yyy))$	12 eg
14	$\exists x \exists y(A(xxy) \wedge A(yxy) \wedge A(yyy))$	13 eg cd

j) $\forall x \forall y(Ax \wedge By \rightarrow C(xy))$. Ab. $\forall x \forall y(C(xy) \leftrightarrow D(yx))$. $\therefore Be \rightarrow \exists z D(ez)$

1	~~Show~~ $Be \rightarrow \exists z D(ez)$	
2	Be	Ass cd
3	$\forall y(Ab \wedge Be \rightarrow C(by))$	pr1 ui
4	$Ab \wedge Be \rightarrow C(be)$	3 ui
5	$Ab \wedge Be$	2 pr2 adj
6	$C(be)$	5 4 mp
7	$\forall y(C(by) \leftrightarrow D(yb))$	pr3 ui
8	$C(be) \leftrightarrow D(eb)$	7 ui
9	$C(be) \rightarrow D(eb)$	8 bc
10	$D(eb)$	6 9 mp
11	$\exists z D(ez)$	10 eg cd

k) $\forall x((Dx \wedge Cx) \to Ex)$. $\forall y(Fy \leftrightarrow Ey)$. $\forall y{\sim}(Dy \wedge Fy)$. $\forall z({\sim}Dz \vee {\sim}Cz)$ $\therefore \exists x(Dx \to Ax)$

1	~~Show~~ $\exists x(Dx \to Ax)$	
2	~~Show~~ $Dx \to Ax$	
3	Dx	
4	~~Show~~ Ax	
5	${\sim}Ax$	ass id
6	$(Dx \wedge Cx) \to Ex$	pr1 ui
7	$Fx \leftrightarrow Ex$	pr2 ui
8	${\sim}(Dx \wedge Fx)$	pr3 ui
9	${\sim}Dx \vee Cx$	pr4 ui
10	${\sim}{\sim}Dx$	3 dn
11	Cx	8 9 mtp
12	$Dx \wedge Cx$	3 10 adj
13	Ex	11 5 mp
14	Fx	6 bc 12 mp
15	$Dx \wedge Fx$	3 13 adj 7 id
16		4 cd
17	$\exists x(Dx \to Ax)$	2 eg dd

l) $\forall x\forall y(Fx \to L(xy))$. $\forall z(Gz \to C(zz))$. $\forall y(\exists zC(zy) \to {\sim}\exists xL(xy))$ $\therefore{\sim}(Fa \wedge Gb)$

1	~~Show~~ ${\sim}(Fa \wedge Gb)$	
2	$Fa \wedge Gb$	ass id
3	$\forall y(Fa \to L(ay))$	pr1 ui
4	$Fa \to L(ab)$	2 ui
5	$Gb \to C(bb)$	pr2 ui
6	Fa	2 s
7	$L(ab)$	4 6 mp
8	Gb	2 s
9	$C(bb)$	5 8 mp
10	$\exists zC(zb) \to {\sim}\exists xL(xb)$	pr3 ui
11	$\exists zC(zb)$	9 eg
12	${\sim}\exists xL(xb)$	10 11 mp
13	$\exists xL(xb)$	7 eg 12 id

m) $Fa \wedge Gb$. $\forall x\forall y(Fx \wedge Gy \to L(xy))$. $\forall x\forall y(L(xy) \to (Hy \vee {\sim}L(xx)))$. $\therefore$ ${\sim}Hb \to \exists z{\sim}(L(zz) \vee Gz)$

1	Show ${\sim}Hb \to \exists z{\sim}(L(zz) \vee Gz)$	
2	${\sim}Hb$	ass cd
3	$\forall y(Fa \wedge Gy \to L(ay))$	pr2 ui
4	$Fa \wedge Gb \to L(ab)$	3 ui
5	$L(ab)$	pr1 4 mp
6	$\forall y(L(ay) \to Hy \vee {\sim}L(aa))$	pr3 ui
7	$L(ab) \to Hb \vee {\sim}L(aa))$	6 ui
8	$Hb \vee {\sim}L(aa)$	7 5 mp
9	${\sim}L(aa)$	8 2 mtp
10	$Fa \wedge Ga \to L(aa)$	3 ui
11	${\sim}(Fa \wedge Ga)$	9 10 mt
12	${\sim}Fa \vee {\sim}Ga$	11 dm
13	${\sim}{\sim}Fa$	pr1 s dn
14	${\sim}Ga$	12 13 mtp
15	${\sim}L(aa) \wedge {\sim}Ga$	9 14 adj
16	${\sim}(L(aa) \vee Ga)$	15 dm
17	$\exists z{\sim}(L(zz) \vee Gz)$	16 eg cd

6.4 EG1 $\exists xFx$. $\forall x\exists y(Fx \to (Gy \wedge L(xy)))$. $\forall x\forall y(L(xy) \to {\sim}H(yx))$. $\therefore$ $\exists x\exists y(Gx \wedge {\sim}H(xy))$

1	Show $\exists x\exists y(Gx \wedge {\sim}H(xy))$	
2	Fi	pr1 ei i/x
3	$\exists y(Fi \to (Gy \wedge L(iy)))$	pr2 ui
4	$Fi \to (Gk \wedge L(ik))$	3 ei k/y
5	$Gk \wedge L(ik)$	2 4 mp
6	$\forall y(L(iy) \to {\sim}H(yi))$	pr3 ui
7	$L(ik) \to {\sim}H(ki)$	6 ui
8	$L(ik)$	5 s
9	${\sim}H(ki)$	7 8 mp
10	Gk	5 s
11	$Gk \wedge {\sim}H(ki)$	9 10 adj
12	$\exists y(Gk \wedge {\sim}H(ky))$	11 eg
13	$\exists x\exists y(Gx \wedge {\sim}H(xy))$	11 eg (y/i), eg (x/k, dd

6.4 EG2 Sometimes, when the conclusion is an existential, it is best to first show the substitution instance with a conditional or indirect derivation.

Bc(a). ∃xBx → ∀y(Hy → Gy). ∀x(~Fx ∨ Hx). ∴∃x(Fx → Gx)

1	~~Show~~ ∃x(Fx → Gx)		
2	~~Show~~ Fa → Ga	Show instance of 1: any variable will work here.	
3	Fa	ass cd	Now your show line is a conditional. Assume antecedent and show consequent!
4	∃xBx	pr1 EG x/c(a)	Generalize pr. 1: replace c(a) with x to match pr. 2.
5	∀y(Hy → Gy)	4 pr2 mp	
6	Ha → Ga	5 ui a/y	instantiate 5 to a, matching 3
7	~Fa ∨ Ha	pr3 ui a/x	instantiate pr3 to a, matching 3, 6
8	~~Fa	3 dn	
9	Ha	7 8 mpt	
10	Ga	6 9 mp, cd	
11	∃x(Fx → Gx)	2 eg, dd	

6.4 E1 Show that the following syllogisms are valid by providing a derivation.

a) 1st figure: Ferio ∀x(Mx→~Ax). ∃x(Bx ∧ Mx). ∴∃x(Bx ∧ ~Ax)

1	~~show~~ ∃x(Bx ∧ ~Ax)	
2	Bi ∧ Mi	pr2 ei
3	Mi → ~Ai	pr1 ui
4	Bi	2 s
5	Mi	2 s
6	~Ai	3 5 mp
7	Bi ∧ ~Ai	4 6 adj
8	∃x(Bx ∧ ~Ax)	7 eg dd

These have the same general structure …

b) 2nd figure: Baroco ∀x(Bx → Ex). ∃y(Ay → ~Ey). ∴ ∃z(Az ∧ ~Bz):

c) 3rd figure: Disamis ∃x(Bx →Fx). ∀y(By → Gy). ∴ ∃x(Gx ∧ Fx)

d) 4th figure: Fresison ∀y(Fy → ~Dy). ∃x(Dx ∧ Bx) ∴∃y(By ∧ ~Fy)

6.4 E2 Construct derivations that show that the following are valid arguments:

a) $\therefore \exists x(Fx \wedge Gx) \rightarrow \exists xFx \wedge \exists xGx$

1	~~show~~ $\exists x(Fx \wedge Gx) \rightarrow \exists xFx \wedge \exists xGx$	
2	$\exists x(Fx \wedge Gx)$	ass cd
3	$Fi \wedge Gi$	2 ei
4	Fi	3s
5	Gi	3s
6	$\exists xFx$	4 eg
7	$\exists xGx$	4 eg
8	$\exists xFx \wedge \exists xGx$	6 7 adj cd

b) $\exists x\exists yL(xy) \rightarrow \exists z\exists xL(zx)$

1	~~show~~ $\exists x\exists yL(xy) \rightarrow \exists z\exists xL(zx)$	
2	$\exists x\exists yL(xy)$	ass cd
3	$\exists yL(iy)$	2 ei
4	$L(ik)$	3 ei
5	$\exists xL(ix)$	4 eg
6	$\exists z\exists xL(zx)$	5 eg
7		6 cd

c) $\exists x(Fx \vee {\sim}Gx)$. $\forall y(Fy \rightarrow Ay)$. $\forall z({\sim}Az \rightarrow Gz)$. $\therefore \exists xAx$

1	~~show~~ $\exists xAx$	
2	$Fi \vee {\sim}Gi$	pr1 ei
3	~~Show~~ Ai	
4	${\sim}Ai$	ass id
5	$Fi \rightarrow Ai$	pr2 ui
6	${\sim}Fi$	4 5 mt
7	${\sim}Ai \rightarrow Gi$	pr3 ui
8	Gi	4 7 mp
9	${\sim}Gi$	1 6 mtp 8 id
10	$\exists xAx$	3 eg dd

d) $\forall x({\sim}Cx \vee Jx)$. $\forall y(By \rightarrow Cy)$. $\exists xBx$ $\therefore \exists z(Bz \wedge Jz)$

1	~~Show~~ $\exists z(Bz \wedge Jz)$	
2	Bi	pr3 ei
3	$Bi \rightarrow Ci$	pr2 ui
4	Ci	2 3 mp
5	${\sim}Ci \vee Ji$	pr1 ui
6	${\sim}{\sim}Ci$	4 dn
7	Ji	5 6 mtp
8	$Bi \wedge Ji$	2 7 adj
9	$\exists z(Bz \wedge Jz)$	8 eg dd

e) $\forall x(Bx \rightarrow Fx)$. $\forall y\sim(Dy \wedge Fy)$. $\therefore \exists xDx \rightarrow \exists x\sim(\sim Dx \vee Bx)$

1	~~Show~~ $\exists xDx \rightarrow \exists x\sim(\sim Dx \vee Bx)$	
2	$\exists xDx$	ass cd
3	Di	2 ei
4	$\sim(Di \wedge Fi)$	pr2 ui
5	$\sim Di \vee \sim Fi$	4 dm
6	$\sim\sim Di$	3 dn
7	$\sim Fi$	5 6 mtp
8	$Bi \rightarrow Fi$	pr1 ui
9	$\sim Bi$	7 8 mt
10	$\sim\sim Di \wedge \sim Bi$	6 9 adj
11	$\sim(\sim Di \wedge Bi)$	10 dm
12	$\exists x\sim(\sim Dx \wedge Bx)$	11 eg cd

f) $\exists x(Gx \wedge \forall y(By \rightarrow L(xy)))$. $\exists x(Bx \wedge \forall y(Gy \rightarrow L(xy)))$. $\therefore \exists x\exists y(L(xy) \wedge L(yx))$

1	~~Show~~ $\exists x\exists y(L(xy) \wedge L(yx))$	
2	$Gi \wedge \forall y(By \rightarrow L(iy))$	pr1 ei
3	Gi	2 s
4	$\forall y(By \rightarrow L(iy))$	2 s
5	$Bk \wedge \forall y(Gy \rightarrow L(ky))$	pr2 ei
6	Bk	5 s
7	$\forall y(Gy \rightarrow L(ky))$	5 s
8	$Bk \rightarrow L(ik)$	4 ui
9	$Gi \rightarrow L(ki)$	7 ui
10	$L(ik)$	6 8 mp
11	$L(ki)$	3 9 mp
12	$L(ik) \wedge L(ki)$	10 11 adj
13	$\exists y(L(iy) \wedge L(yi))$	12 eg
14	$\exists x\exists y(L(xy) \wedge L(yx))$	13 eg dd

6.5 EG1 Let's try some:

$\forall x(Fx \rightarrow Gx)$. $\forall y(Gy \rightarrow \sim Hy)$. $\therefore \forall x(Fx \rightarrow \sim Hx)$

1	~~Show~~ $\forall x(Fx \rightarrow \sim Hx)$	
2	~~Show~~ $Fx \rightarrow \sim Hx$	show instance of 1
3	Fx	ass cd
4	$Fx \rightarrow Gx$	pr1 ui x/x
5	$Gx \rightarrow \sim Hx$	pr2 ui x/y
6	Gx	3 4 mp
7	$\sim Hx$	5 6 mp
8		7 cd
9		2 UD

6.5 EG2 Let's try another – a little more complex.

$\exists xFx \rightarrow \forall y(Jy \vee Hy)$. $\sim\exists xHx$. $\forall x(Jx \rightarrow Gx)$ $\therefore$ $\forall x(Fx \rightarrow Gx)$

1	~~Show~~ $\forall x(Fx \rightarrow Gx)$	
2	~~Show~~ $Fx \rightarrow Gx$	
3	Fx	Ass CD
4	$\exists xFx$	3 eg
5	$\forall y(Jy \vee Hy)$	4 pr1 mp
6	$Jx \vee Hx$	5 ui
7	~~Show~~ $\sim Hx$	
8	Hx	Ass ID
9	$\exists xHx$	8 eg
10	$\sim\exists xHx$	pr2 9 ID
11	Jx	6 7 MTP
12	$Jx \rightarrow Gx$	pr3 ui
13	Gx	11 12 mp CD
14		2 UD
15		

6.5 EG3 We can also use UD when we have two place predicates. Here's an easy one:

$\forall x\forall y(F(xy) \rightarrow F(yx))$. $\therefore$ $\forall x(F(ax) \rightarrow F(xa))$.

1	~~Show~~ $\forall x(F(ax) \rightarrow F(xa))$		
2	~~Show~~ $F(ax) \rightarrow F(xa)$		show inst. of 1
3	$F(ax)$	ass cd	
4	$\forall y(F(ay) \rightarrow F(ya))$	pr1 ui a/x	
5	$F(ax) \rightarrow F(xa)$	4 ui x/y	
6	$F(xa)$	3 5 mp cd	
7		2 ud	
8			

6.5 EG4 Here's another:

$\forall x \forall y((Fx \wedge Fy) \rightarrow L(xy)) \therefore \forall x(Fx \rightarrow L(xx))$

1	~~Show~~ $\forall x(Fx \rightarrow L(xx))$		
2	~~Show~~ $Fx \rightarrow L(xx)$		show inst. of 1
3	Fx	ass cd	goal: $L(xx)$
4	$\forall y(Fx \wedge Fy \rightarrow L(xy))$	pr1 ui	x/x
5	$Fx \wedge Fx \rightarrow L(xx)$	4 ui	x/y
6	Fx	3 r	
7	$Fx \wedge Fx$	3 6 adj	
8	$L(xx)$	7 5 mp, cd	
9		2 UD	

6.5 EG5 $\forall x \exists y(Fx \rightarrow (Gy \wedge L(xy)))$. $\forall x \forall y(Gx \wedge {\sim}L(xy) \rightarrow {\sim}L(yx)) \therefore \forall x(Fx \rightarrow \exists y L(yx))$

1	~~Show~~ $\forall x(Fx \rightarrow \exists y L(yx))$		
2	~~Show~~ $Fx \rightarrow \exists y L(yx)$		
3	Fx	ass cd	goal: $L(x\ldots)$
4	$\exists y(Fx \rightarrow (Gy \wedge L(xy))$	pr1 ui	x/x
5	$Fx \rightarrow (Gi \wedge L(xi))$	4 ei	i/y
6	$Gi \wedge L(xi)$	5 3 mp	
7	$\forall y(Gi \wedge {\sim}L(iy) \rightarrow {\sim}L(yi))$	pr2 ui	i/x
8	$Gi \wedge {\sim}L(ix) \rightarrow {\sim}L(xi)$	7 ui	x/y
9	$L(xi)$	6 s	
10	Gi	6 s	
11	${\sim}(Gi \wedge {\sim}L(ix))$	9 dn 8 mt	
12	${\sim}Gi \vee {\sim\sim}L(ix)$	11 dm	
13	${\sim\sim}L(ix)$	10 dn 12 mtp	
14	$L(ix)$	13 dn	
15	$\exists y L(yx)$	14 eg cd	
9		2 ud	

6.5 EG6 $\forall x \forall y(L(xy) \rightarrow G(xy))$. $\forall z({\sim}Fz \vee \forall y L(zy))$. $\therefore \forall x \forall y(Fx \rightarrow G(xy))$

1	~~Show~~ $\forall x \forall y(Fx \rightarrow G(xy))$		
2	~~Show~~ $\forall y(Fx \rightarrow G(xy))$		show inst. for 1
3	~~Show~~ $Fx \rightarrow G(xy)$		show inst. for 2
4	Fx	ass cd	
5	${\sim}Fx \vee \forall y L(xy)$	pr2 ui	
6	${\sim}{\sim}Fx$	4 dn	
7	$\forall y L(xy)$	4 6 mtp	
8	$\forall y(L(xy) \rightarrow G(xy))$	pr1 ui x/x	
9	$L(xy) \rightarrow G(xy)$	8 ui y/y	
10	$L(xy)$	7 ui y/y	
11	$G(xy)$	9 10 cd	
12		3 ud	
13		2 ud	

6.5 EG7 This one is a bit trickier. Be careful in parsing the second premise.
$\exists x(Hx \wedge \forall y(Gy \rightarrow K(xy)))$. $\forall x(Gx \rightarrow (\exists y(Hy \wedge K(yx)) \rightarrow \forall z(Hz \rightarrow L(xz))))$.
$\therefore \forall x \forall y((Gx \wedge Hy) \rightarrow L(xy))$

1	~~Show~~ $\forall x \forall y((Gx \wedge Hy) \rightarrow L(xy))$		
2	~~Show~~ $\forall y((Gx \wedge Hy) \rightarrow L(xy))$		
3	~~Show~~ $(Gx \wedge Hy) \rightarrow L(xy)$		
4	$Gx \wedge Hy$	ass cd	goal:
5	$Hi \wedge \forall y(Gy \rightarrow K(iy))$	pr1 ei	
6	Hi	5s	
7	$\forall y(Gy \rightarrow K(iy))$	5s	
8	$Gx \rightarrow (\exists y(Hy \wedge K(yx)) \rightarrow \forall z(Hz \rightarrow L(xz))))$.	pr2 ui	
9	$\exists y(Hy \wedge K(yx)) \rightarrow \forall z(Hz \rightarrow L(xz)))$	4 sl 8 mp	
10	$Gx \rightarrow K(ix)$	7 ui	x/y
11	$K(ix)$	4 sl 10 mp	
12	$Hi \wedge K(ix)$	6 11 adj	
13	$\exists y(Hy \wedge K(yx))$	12 eg	y/x
14	$\forall z(Hz \rightarrow L(xz))$	9 13 mp	
15	$Hy \rightarrow L(xy)$	14 ui	y/z
16	$L(xy)$	15 4 sr mp	
17		16 cd	
18		3 ud	
19		2 ud	

6.5 EG8

This one looks complex, but most of the work is just removing and then putting back the quantifiers. The logic isn't very difficult at all! In this one, one must use UD to free the consequent of the second premise.

$\forall z\, \forall w(Gz \rightarrow \sim F(wz))$. $\forall x \forall y(F(xy) \rightarrow L(yx)) \rightarrow F(ab)$. $\therefore$ $\forall x \forall z(Gx \vee L(xz)) \rightarrow \exists x \exists y F(xy)$.

1	~~Show~~ $\forall x \forall z(Gx \vee L(xz)) \rightarrow \exists x \exists y F(xy)$.	
2	$\forall x \forall z(Gx \vee L(xz))$	ass cd
3	~~Show~~ $\forall x \forall y(F(xy) \rightarrow L(yx))$	
4	~~Show~~ $\forall y(F(xy) \rightarrow L(yx))$	
5	~~Show~~ $F(xy) \rightarrow L(yx)$	
6	$F(xy)$	Ass CD
7	$\forall w(Gy \rightarrow \sim F(wy))$	pr1 ui y/z
8	$Gy \rightarrow \sim F(xy)$	7 ui x/w
9	$\sim Gy$	6 dn mt
10	$\forall z(Gy \vee L(yz))$	2 ui y/x
11	$Gy \vee L(yx)$	10 ui x/z
12	$L(yx)$	9 11 mtp
13		12 cd
14		5 UD
15		4 UD
16	$F(ab)$	3 pr2 mp
17	$\exists y F(ay)$	16 eg
18	$\exists x \exists y F(xy)$	17 eg
19		

DERIVATION TIP: When you are going to use UD, try to set up the show line as soon as possible. Then immediately put a show line for the instantiated form of the universal that you are trying to prove.

6.5 EG9

We can also prove theorems using our new rules. This theorem makes sense: it says that if something is not F then not all things are F. That's Quantifier Negation! (Theorem 203 to be precise!)

∴ ∃x~Fx ↔ ~∀xFx

1	~~Show~~ ∃x~Fx ↔ ~∀xFx	
2	~~Show~~ ∃x~Fx → ~∀xFx	
3	∃x~Fx	Ass CD
4	~~Show~~ ~∀xFx	
5	∀xFx	Ass ID
6	~Fi	3 ei
7	Fi	5 ui
8		6 7 id
9		4 CD
10	~~Show~~ ~∀xFx → ∃x~Fx	
11	~∀xFx	ASS CD
12	~~Show~~ ∃x~Fx	
13	~∃x~Fx	ASS ID
14	~~Show~~ ∀xFx	
15	~~Show~~ Fx	
16	~Fx	Ass ID
17	∃x~Fx	16 eg
18	~∃x~Fx	13 R
19		17 18 ID
20		15 UD
21	~∀xFx	11 R 14 ID
22		12 CD
23	∃x~Fx ↔ ~∀xFx	2 10 CB DD

6.5 E1 Show that the following syllogisms are valid by providing a derivation.

a) 1st figure: Celarent $\forall x(Mx \to \sim Ax)$. $\forall x(Bx \to Mx)$. $\therefore \forall x(Bx \to \sim Ax)$

1	~~show~~ $\forall x(Bx \to \sim Ax)$	
2	~~Show~~ $Bx \to \sim Ax$	
3	Bx	ass cd
4	$Bx \to Mx$	pr2 ui
5	Mx	3 4 mp
6	$Mx \to \sim Ax$	pr1 ui
7	$\sim Ax$	5 6 mp cd
8		2 ud

These have the same general pattern.

b) 2nd figure: Camestres $\forall x(Bx \to Fx)$. $\forall y(Dy \to \sim Fy)$. $\therefore \forall z(Dz \to \sim Bz)$:

c) 4th figure: Camenes $\forall y(Fy \to By)$. $\forall x(Bx \to \sim Mx)$ $\therefore \forall y(My \to \sim Fy)$

6.5 E2 Construct derivations that show that the following are valid arguments:

d) $\therefore \forall x(Fx \to \forall xGx) \to \forall x(Fx \to Gx)$

1	~~Show~~ $(\forall xFx \to \forall xGx) \to \forall x(Fx \to Gx)$	
2	$\forall x(Fx \to \forall xGx)$	
3	~~Show~~ $\forall x(Fx \to Gx)$	
4	~~Show~~ $Fx \to Gx$	
5	Fx	ass cd
6	$Fx \to \forall xGx$	2 ui
7	$\forall xGx$	5 6 mp
8	Gx	7 ui
9		8 cd
10		4 ud
11		3 cd

e) $\forall x(Fx \to Gx)$. $\forall x((Gx \vee Hx) \to (Ax \vee Bx))$. $\therefore \forall x((Fx \wedge \sim Bx) \to Ax)$

1	~~Show~~ $\forall x((Fx \wedge \sim Bx) \to Ax)$	
2	~~Show~~ $(Fx \wedge \sim Bx) \to Ax$	
3	$Fx \wedge \sim Bx$	ass cd
4	Fx	3 s
5	$Fx \to Gx$	pr1 ui
6	Gx	4 5 mp
7	$(Gx \vee Hx) \to (Ax \vee Bx)$	pr2 ui
8	$Gx \vee Hx$	6 add
9	$Ax \vee Bx$	7 8 mp
10	$\sim Bx$	3 s
11	Ax	9 10 mtp
12		11 cd
13		2 ud

f) $\forall y(Fy \to \sim Ay)$. $\forall z(Bz \vee Cz \to Az)$. $\therefore \forall x(Fx \to \sim Cx)$

1	~~Show~~ $\forall x(Fx \to \sim Cx)$	
2	~~Show~~ $Fx \to \sim Cx$	
3	Fx	ass cd
4	$Fx \to \sim Ax$	3 s
5	$\sim Ax$	pr1 ui
6	$Bx \vee Cx \to Ax$	4 5 mp
7	$\sim(Bx \vee Cx)$	5 6 mt
8	$\sim Bx \wedge \sim Cx$	7 dm
9	$\sim Cx$	8 s
10		9 cd
11		2 ud

g) $\forall x(Fx \rightarrow Gx) \rightarrow \forall y(\sim Ay \rightarrow By)$. $\exists xHx \rightarrow \forall y(\sim Gy \rightarrow \sim Fy)$. Ha. $\therefore$ $\forall y(By \vee Ay)$

1	~~Show~~ $\forall y(By \vee Ay)$	
2	~~Show~~ $By \vee Ay$	
3	$\exists xHx$	pr3 eg
4	$\forall y(\sim Gy \rightarrow \sim Fy)$	pr2 3 mp
5	~~Show~~ $\forall x(Fx \rightarrow Gx)$	
6	~~Show~~ $Fx \rightarrow Gx$	
7	Fx	ass cd
8	$\sim Gx \rightarrow \sim Fx$	4 ui
9	$\sim\sim Gx$	7 dn 8 mt
10	Gx	9 dn cd
11		6 ud
12	$\forall y(\sim Ay \rightarrow By)$	5 pr1 mp
13	$\sim Ay \rightarrow By$	12 ui
14	$Ay \vee By$	13 cdj
15	$By \vee Ay$	14 RT53
16		15 dd
17		2 ud

h) $\therefore$ $\forall x \forall y L(xy) \rightarrow \forall x \forall y L(yx)$ (NOTE: I am showing two different derivations of this.)

1	~~Show~~ $\forall x \forall y L(xy) \rightarrow \forall x \forall y L(yx)$	
2	$\forall x \forall y L(xy)$	ASS CD
3	~~Show~~ $\forall x \forall y L(yx)$	
4	~~show~~ $\forall y L(yx)$	
5	~~show~~ $L(yx)$	
6	~~show~~ $\forall z \forall w L(zw)$	
7	~~show~~ $\forall w L(zw)$	
8	~~Show~~ $L(zw)$	
9	$\forall y L(zy)$	2 UI
10	$L(zw)$	9 UI DD
11		10 UD
12		8 UD
13	$\forall w L(yw)$	6 UI
14	$L(yx)$	13 UI DD
11		5 UD
12		4 UD
13		3 CD

In this version, the universal derivation is done with an arbitrary variable that does NOT match the one

1	~~Show~~ $\forall x\forall yL(xy) \to \forall x\forall yL(yx)$	
2	$\forall x\forall yL(xy)$	ASS CD
3	~~Show~~ $\forall x\forall yL(yx)$	
4	~~show~~ $\forall yL(yz)$	show arbitrary instance of 3 z/x
5	~~show~~ $L(wz)$	show arbitrary instance of 4 2/y
6	$\forall yL(wy)$	2 UI
7	$L(wz)$	6 UI
8		DD
9		5 UD
10		4 UD
11		3 CD

It is also very easy to do with QN. (Ass. antecedent, Show consequent, assume negated consequent for ID, use QN-EI-QN-EI, then generate a contradiction.

i) $\forall x\forall y(B(xy) \to C(yx))$. $\exists xFx \to \forall yB(yy)$. $\therefore Fa \to \forall x\exists yC(xy)$

1	~~Show~~ $Fa \to \forall x\exists yC(xy)$	
2	Fa	ass cd
3	~~Show~~ $\forall x\exists yC(xy)$	
4	~~Show~~ $\exists yC(xy)$	
5	$\exists xFx$	2 eg
6	$\forall yB(yy)$	pr2 5 mp
7	$B(xx)$	6 ui
8	$\forall y(B(xy) \to C(yx))$	pr1 ui
9	$B(xx)\to C(xx)$	8 ui
10	$C(xx)$	7 9 mp
11	$\exists yC(xy)$	10 eg dd
12		4 ud
13		3 cd

j) $\therefore \forall x\forall y(\sim(Fx \vee \sim Gy) \to B(xy)) \leftrightarrow \forall z(\sim Fz \to \forall w(Gw \to B(zw))$

E7.5 j: $\therefore \forall x\forall y(\sim(Fx \vee \sim Gy) \to B(xy)) \leftrightarrow \forall z(\sim Fz \to \forall w(Gw \to B(zw)))$

1	~~Show~~ $\forall x\forall y(\sim(Fx\vee\sim Gy)\to B(xy))\leftrightarrow\forall z(\sim Fz\to\forall w(Gw\to B(zw)))$	"show conc"
2	~~Show~~ $\forall x\forall y(\sim(Fx\vee\sim Gy)\to B(xy))\to\forall z(\sim Fz\to\forall w(Gw\to B(zw)))$	"show cond"
3	$\forall x\forall y(\sim(Fx\vee\sim Gy)\to B(xy))$	ass cd
4	~~Show~~ $\forall z(\sim Fz\to\forall w(Gw\to B(zw)))$	"show cons"
5	~~Show~~ $\sim Fz\to\forall w(Gw\to B(zw))$	"show inst"
6	$\sim Fz$	ass cd
7	~~Show~~ $\forall w(Gw\to B(zw))$	"show cons"
8	~~Show~~ $Gw\to B(zw)$	"show inst"
9	Gw	ass cd
10	$\forall y(\sim(Fz\vee\sim Gy)\to B(zy))$	3 ui
11	$\sim(Fz\vee\sim Gw)\to B(zw)$	10 ui
12	$\sim Fz\wedge\sim\sim Gw$	6 9 dn adj
13	$\sim(Fz\vee\sim Gw)$	12 dm
14	$B(zw)$	11 13 mp
15		14 cd
16		8 ud
17		7 cd
18		5 ud
19		4 cd
20	~~Show~~ $\forall z(\sim Fz\to\forall w(Gw\to B(zw)))\to\forall x\forall y(\sim(Fx\vee\sim Gy)\to B(xy))$	"show cond"
21	$\forall z(\sim Fz\to\forall w(Gw\to B(zw)))$	ass cd
22	~~Show~~ $\forall x\forall y(\sim(Fx\vee\sim Gy)\to B(xy))$	"show cons"
23	~~Show~~ $\forall y(\sim(Fx\vee\sim Gy)\to B(xy))$	"show inst"
24	~~Show~~ $\sim(Fx\vee\sim Gy)\to B(xy)$	"show inst"
25	$\sim(Fx\vee\sim Gy)$	ass cd
26	$\sim Fx\wedge\sim\sim Gy$	25 dm
27	$\sim Fx\to\forall w(Gw\to B(xw))$	21 ui
28	$\forall w(Gw\to B(xw))$	26 sl 27 mp
29	$Gy\to B(xy)$	28 ui
30	Gy	26 sr dn
31	$B(xy)$	29 30 mp
32		31 cd
33		24 ud
34		23 ud
35		22 cd
36	$\forall x\forall y(\sim(Fx\vee\sim Gy)\to B(xy))\leftrightarrow\forall z(\sim Fz\to\forall w(Gw\to B(zw)))$	2 20 cb
37		36 dd

k) $\forall x(\sim Fx \vee Gx) \wedge \exists y\forall x(Gx \to B(yx))$. $\forall x\forall y(B(xy)\leftrightarrow L(yx))$. $\therefore \forall x(Fx \to \exists yL(xy))$

E 7.5 k: $\forall x(\sim Fx \vee Gx) \wedge \exists y\forall x(Gx \rightarrow B(yx))$. $\forall x\forall y(B(xy)\leftrightarrow L(yx))$. $\therefore$ $\forall x(Fx \rightarrow \exists yL(xy))$

1	~~Show~~ $\forall x(Fx\rightarrow\exists yL(xy))$	"show conc"
2	~~Show~~ $Fx\rightarrow\exists yL(xy)$	"show inst"
3	Fx	ass cd
4	$\forall x(\sim Fx\vee Gx)$	pr1 s
5	$\exists y\forall x(Gx\rightarrow B(yx))$	pr1 s
6	$\forall x(Gx\rightarrow B(ix))$	5 ei
7	$\sim Fx\vee Gx$	4 ui
8	$\sim\sim Fx$	3 dn
9	Gx	7 8 mtp
10	$Gx\rightarrow B(ix)$	6 ui
11	$B(ix)$	9 10 mp
12	$\forall y(B(iy)\leftrightarrow L(yi))$	pr2 ui
13	$B(ix)\leftrightarrow L(xi)$	12 ui
14	$B(ix)\rightarrow L(xi)$	13 bc
15	$L(xi)$	11 14 mp
16	$\exists yL(xy)$	15 eg
17		16 cd
18		2 ud

6.6 E1 Construct derivations (using QN) that show that the following are valid arguments:

a) $\therefore \sim\exists x(Fx \wedge \sim Gx) \to \forall x(Fx \to Gx)$

1	~~Show~~ $\sim\exists x(Fx \wedge \sim Gx) \to \forall x(Fx \to Gx)$	
2	$\sim\exists x(Fx \wedge \sim Gx)$	ASS CD
3	~~show~~ $\forall x(Fx \to Gx)$	
4	~~Show~~ $Fx \to Gx$	
5	Fx	
6	$\forall x\sim(Fx \wedge \sim Gx)$	2 QN
7	$\sim(Fx \wedge \sim Gx)$	6 UI
8	$\sim Fx \vee \sim\sim Gx$	7 DM
9	$\sim\sim Fx$	5 DN
10	$\sim\sim Gx$	8 9 MTP
11	Gx	10 DN
12		4 UD
13		3 CD

b) $\sim\forall x(Ax \to Mx).\ \forall x(Rx \to Mx).\ \therefore\ \exists x(Ax \wedge \sim Rx)$

1	~~Show~~ $\exists x(Ax \wedge \sim Rx)$	
2	$\sim\exists x(Ax \wedge \sim Rx)$	ass id
3	$\forall x\sim(Ax \wedge \sim Rx)$	2 qn
4	$\exists x\sim(Ax \to Mx)$	pr1 qn
5	$\sim(Ai \to Mi)$	4 ei
6	$\sim(Ai \wedge \sim Ri)$	2 ui
7	$Ri \to Mi$	pr2 ui
8	$Ai \wedge \sim Mi$	5 nc
9	$\sim Mi$	8 s
10	$\sim Ri$	7 9 mt
11	Ai	8 s
12	$Ai \wedge \sim Ri$	11 9 adj
13		12 6 id

c) $\forall x(Px \rightarrow Sx)$. $(Pa \vee Pb)$. $\therefore\ \exists xSx$ (Try this one as an indirect derivation.)

1	~~Show~~ $\exists xSx$	
2	$\sim\exists xSx$	ass id
3	~~Show~~ Pa	
4	$\sim Pa$	ass id
5	Pb	4 pr2 mtp
6	$Pb \rightarrow Sb$	pr1 ui
7	Sb	5 6 mp
8	$\exists xSx$	7 eg
9	$\sim\exists xSx$	2 r 8 id
10	$Pa \rightarrow Sa$	pr1 ui
11	Sa	3 10 mp
12	$\exists xSx$	11 eg
13		2 12 id

d) $\exists x(\sim Fx \vee \sim Gx)$. $\sim\forall x(Fx \wedge Gx) \rightarrow \exists y\sim Ay$. $\forall x(\sim Gx \rightarrow Ax)$. $\therefore\sim\forall x\sim Gx$

1	~~Show~~ $\sim\forall x\sim Gx$	
2	$\forall x\sim Gx$	ass id
3	$\sim Fi \vee \sim Gi$	pr1 ei
4	$\sim(Fi \wedge Gi)$	3 dm
5	$\exists x\sim(Fi \wedge Gi)$	4 eg
6	$\sim\forall x(Fx \wedge Gx)$	5 qn
7	$\exists y\sim Ay$	6 pr2 mp
8	$\sim Ak$	7 ei
9	$\sim Gk \rightarrow Ak$	pr3 ui
10	Gk	8 9 mt dn
11	$\sim Gk$	2 ui 10 id

e) $\sim\exists x\exists y(B(xy) \wedge C(yx))$. $\exists x\sim Fx \rightarrow \forall yB(yy)$. $\sim\forall xFx$. $\therefore\sim\exists x\forall yC(xy)$

1	~~Show~~ $\sim\exists x\forall yC(xy)$	
2	$\exists x\forall yC(xy)$	ass id
3	$\forall yC(iy)$	2 ei
4	$\exists x\sim Fx$	pr3 qn
5	$\forall yB(yy)$	4 pr2 mp
6	$\forall x\sim\exists y(B(xy) \wedge C(yx))$	pr1 qn
7	$\sim\exists y(B(iy) \wedge C(yi))$	6 ui
8	$\forall y\sim(B(iy) \wedge C(yi))$	7 qn
9	$\sim(B(ii) \wedge C(ii))$	8 ui
10	$C(ii)$	3 ui
11	$B(ii)$	5 ui
12	$B(ii) \wedge C(ii)$	11 10 adj
14		12 9 id

f) $\therefore \forall x \exists y(B(xy) \to \exists w \forall z B(wz))$ (This one is a bit tricky.)

1	~~Show~~ $\forall x \exists y(B(xy) \to \exists w \forall z B(wz))$	
2	~~show~~ $\exists y(B(xy) \to \exists w \forall z B(wz))$	
3	$\sim\exists y(B(xy) \to \exists w \forall z B(wz))$	ass id
4	$\forall y \sim(B(xy) \to \exists w \forall z B(wz))$	3 qn
5	$\sim(B(xy) \to \exists w \forall z B(wz))$	4 ui
6	$B(xy) \wedge \sim\exists w \forall z B(wz)$	5 nc
7	$\sim\exists w \forall z B(wz)$	7 s
8	$\forall w \sim\forall z B(wz)$	7 qn
9	$\sim\forall z B(xz)$	8 ui
10	$\exists z \sim B(xz)$	9 qn
11	$\sim B(xi)$	10 ei
12	$\sim(B(xi) \to \exists w \forall z B(wz))$	4 ui
13	$B(xi) \wedge \sim\exists w \forall z B(wz)$	12 nc
14	$B(xi)$	13 s
15		11 14 id
16		2 ud

6.6 EG1 Let's try it out

$\sim\exists x \sim(Fx \to Gx)$. $\sim\exists y(Gy \wedge Hy)$. $\therefore \forall x \sim(Fx \wedge Hx)$

1	~~Show~~ $\forall x \sim(Fx \wedge Hx)$	
2	~~show~~ $\sim(Fx \wedge Hx)$	
3	$Fx \wedge Hx$	ass id
4	$\forall x \sim\sim(Fx \to Gx)$.	pr1 qn
5	$\sim\sim(Fx \to Gx)$	4 ui
6	$Fx \to Gx$	5 dn
7	Gx	3 sl 6 mp
8	$\forall y \sim(Gy \wedge Hy)$	pr2 qn
9	$\sim(Gx \wedge Hx)$	8 ui
10	Hx	3 sr
11	$Gx \wedge Hx$	7 10 adj
12		9 11 id
13		2 ud
14		

6.6 EG2 Let's try another. Here we have to use QN twice – but in between we need to instantiate in order to make the negation sign the main logical operator.

$\sim\exists x\exists y(F(xy) \wedge F(yx))$. $\sim\forall xF(xd) \rightarrow \sim\forall yLy$ $\therefore F(de) \rightarrow \exists y\sim Ly$

1	~~Show~~ $F(de) \rightarrow \exists y\sim Ly$	
2	$F(de)$	ass cd
3	$\forall x\sim\exists y(F(xy) \wedge F(yx))$	pr1 qn
4	$\sim\exists y(F(dy) \wedge F(yd))$	3 ui
5	$\forall y\sim(F(dy) \wedge F(yd))$	4 qn
6	$\sim(F(de) \wedge F(ed))$	5 ui
7	$\sim F(de) \vee \sim F(ed)$	6 dm
8	$\sim F(ed)$	2 dn 7 mtp
9	$\exists x\sim F(xd)$	8 eg
10	$\sim\forall xF(xd)$	9 qn
11	$\sim\forall yLy$	10 pr2 mp
12	$\exists y\sim Ly$	11 qn cd

Let's try another. Here we have to use QN twice – but in between we need to instantiate in order to make the negation sign the main logical operator.

$\sim\exists x\exists y(F(xy) \wedge F(yx))$. $\sim\forall xF(xd) \rightarrow \sim\forall yLy$ $\therefore F(de) \rightarrow \exists y\sim Ly$

1	~~Show~~ $F(de) \rightarrow \exists y\sim Ly$	
2	$F(de)$	Ass cd
3	$\forall x\sim\exists y(F(xy) \wedge F(yx))$	pr 1 QN
4	$\sim\exists y(F(dy) \wedge F(yd))$	3 ui
5	$\forall y\sim(F(dy) \wedge F(yd))$	4 QN
6	$\sim(F(de) \wedge F(ed))$	5 ui
7	$\sim F(de) \vee \sim F(ed)$	6 dm
8	$\sim F(ed)$	2 dn mtp
9	$\exists x \sim F(xd)$	8 eg
10	$\sim\forall xF(xd)$	9 QN
11	$\sim\forall yLy$	10 pr2 mp
12	$\exists y\sim Ly$	11 QN cd

6.6 E1 Construct derivations (using QN) that show that the following are valid arguments:

a) $\therefore \sim\exists x(Fx \wedge \sim Gx) \rightarrow \forall x(Fx \rightarrow Gx)$

1	~~Show~~ $\sim\exists x(Fx \wedge \sim Gx) \rightarrow \forall x(Fx \rightarrow Gx)$	
2	$\sim\exists x(Fx \wedge \sim Gx)$	ASS CD
3	~~show~~ $\forall x(Fx \rightarrow Gx)$	
4	~~Show~~ $Fx \rightarrow Gx$	
5	Fx	
6	$\forall x\sim(Fx \wedge \sim Gx)$	2 QN
7	$\sim(Fx \wedge \sim Gx)$	6 UI
8	$\sim Fx \vee \sim\sim Gx$	7 DM
9	$\sim\sim Fx$	5 DN
10	$\sim\sim Gx$	8 9 MTP
11	Gx	10 DN
12		4 UD
13		3 CD

b) $\sim\forall x(Ax \rightarrow Mx).\ \forall x(Rx \rightarrow Mx).\ \therefore\ \exists x(Ax \wedge \sim Rx)$

1	~~Show~~ $\exists x(Ax \wedge \sim Rx)$	
2	$\sim\exists x(Ax \wedge \sim Rx)$	ass id
3	$\forall x\sim(Ax \wedge \sim Rx)$	2 qn
4	$\exists x\sim(Ax \rightarrow Mx)$	pr1 qn
5	$\sim(Ai \rightarrow Mi)$	4 ei
6	$\sim(Ai \wedge \sim Ri)$	3 ui
7	$Ri \rightarrow Mi$	pr2 ui
8	$Ai \wedge \sim Mi$	5 nc
9	$\sim Mi$	8 s
10	$\sim Ri$	7 9 mt
11	Ai	8 s
12	$Ai \wedge \sim Ri$	11 9 adj
13		12 6 id

c) $\forall x(Px \rightarrow Sx)$. $(Pa \vee Pb)$. $\therefore \exists xSx$ (Try this one as an indirect derivation.)

1	~~Show~~ $\exists xSx$	
2	$\sim\exists xSx$	ass id
3	~~Show~~ Pa	
4	$\sim Pa$	ass id
5	Pb	4 pr2 mtp
6	$Pb \rightarrow Sb$	pr1 ui
7	Sb	5 6 mp
8	$\exists xSx$	7 eg
9	$\sim\exists xSx$	2 r 8 id
10	$Pa \rightarrow Sa$	pr1 ui
11	Sa	3 10 mp
12	$\exists xSx$	11 eg
13		2 12 id

d) $\exists x(\sim Fx \vee \sim Gx)$. $\sim\forall x(Fx \wedge Gx) \rightarrow \exists y\sim Ay$. $\forall x(\sim Gx \rightarrow Ax)$. $\therefore \sim\forall x\sim Gx$

1	~~Show~~ $\sim\forall x\sim Gx$	
2	$\forall x\sim Gx$	ass id
3	$\sim Fi \vee \sim Gi$	pr1 ei
4	$\sim(Fi \wedge Gi)$	3 dm
5	$\exists x\sim(Fi \wedge Gi)$	4 eg
6	$\sim\forall x(Fx \wedge Gx)$	5 qn
7	$\exists y\sim Ay$	6 pr2 mp
8	$\sim Ak$	7 ei
9	$\sim Gk \rightarrow Ak$	pr3 ui
10	Gk	8 9 mt dn
11	$\sim Gk$	2 ui 10 id

e) $\sim\exists x\exists y(B(xy) \wedge C(yx))$. $\exists x{\sim}Fx \rightarrow \forall yB(yy)$. $\sim\forall xFx$. $\therefore \sim\exists x\forall yC(xy)$

1	~~Show~~ $\sim\exists x\forall yC(xy)$	
2	$\exists x\forall yC(xy)$	ass id
3	$\forall yC(iy)$	2 ei
4	$\exists x{\sim}Fx$	pr3 qn
5	$\forall yB(yy)$	4 pr2 mp
6	$\forall x{\sim}\exists y(B(xy) \wedge C(yx))$	pr1 qn
7	$\sim\exists y(B(iy) \wedge C(yi))$	6 ui
8	$\forall y{\sim}(B(iy) \wedge C(yi))$	7 qn
9	$\sim(B(ii) \wedge C(ii))$	8 ui
10	$C(ii)$	3 ui
11	$B(ii)$	5 ui
12	$B(ii) \wedge C(ii)$	11 10 adj
14		12 9 id

f) $\therefore \forall x\exists y(B(xy) \rightarrow \exists w\forall zB(wz))$ (This one is a bit tricky.)

1	~~Show~~ $\forall x\exists y(B(xy) \rightarrow \exists w\forall zB(wz))$	
2	~~show~~ $\exists y(B(xy) \rightarrow \exists w\forall zB(wz))$	
3	$\sim\exists y(B(xy) \rightarrow \exists w\forall zB(wz))$	ass id
4	$\forall y{\sim}(B(xy) \rightarrow \exists w\forall zB(wz))$	3 qn
5	$\sim(B(xy) \rightarrow \exists w\forall zB(wz))$	4 ui
6	$B(xy) \wedge \sim\exists w\forall zB(wz)$	5 nc
7	$\sim\exists w\forall zB(wz)$	7 s
8	$\forall w{\sim}\forall zB(wz)$	7 qn
9	$\sim\forall zB(xz)$	8 ui
10	$\exists z{\sim}B(xz)$	9 qn
11	$\sim B(xi)$	10 ei
12	$\sim(B(xi) \rightarrow \exists w\forall zB(wz))$	4 ui
13	$B(xi) \wedge \sim\exists w\forall zB(wz)$	12 nc
14	$B(xi)$	13 s
15		11 14 id
16		2 ud