Lecture 5

We accept the following inductive principle:

$\circledast$ $\{P(8) \wedge P(9) \wedge P(10) \wedge \forall n \in \mathbb{N}[(n \geq 8 \wedge P(n)) \rightarrow P(n+3)]\} \Rightarrow \forall n \in \mathbb{N}, (n \geq 8 \rightarrow P(n))$

For $n \in \mathbb{N}$, let $P(n)$ be: $\exists k, l \in \mathbb{N}, n = 3k + 5l$.

Proof of $\forall n \in \mathbb{N} (n \geq 8 \rightarrow P(n))$ by $\circledast$

Base Cases: $P(8), P(9), P(10)$

$8 = 3 \cdot 1 + 5 \cdot 1$, $1 \in \mathbb{N}$, so $P(8)$

$9 = 3 \cdot 3 + 5 \cdot 0$, $3, 0 \in \mathbb{N}$, so $P(9)$

$10 = 3 \cdot 0 + 5 \cdot 2$, $0, 2 \in \mathbb{N}$, so $P(10)$

Inductive step: $\forall n \in \mathbb{N}, (n \geq 8 \wedge P(n) \rightarrow P(n+3))$

Let $n \in \mathbb{N}$

Assume $n \geq 8$

Assume $P(n)$: $\exists k, l \in \mathbb{N}$, $n = 3k + 5l$ (IH)

Let $a, b \in \mathbb{N}$ s.t. $n = 3a + 5b$ by (IH)

Then $n+3 = 3a + 5b + 3 = 3(a+1) + 5b$

$a+1 \in \mathbb{N}$, $a \in \mathbb{N}$, and $b \in \mathbb{N}$, so $P(n+3)$

The ~~units~~ last digit of 3^n for $n \in \mathbb{N}$

$3^0 = 1$

$3^1 = 3$ Always one of $1, 3, 9, 7$

$3^2 = 9$

$3^3 = 27$ $3^{n+4} = 3^n \cdot 3^4 = 3^n \cdot 81$

$3^4 = 81$ $P(0) \wedge \forall n \in \mathbb{N}, (P(n) \rightarrow P(n+1))$ with cases in IS

$3^5 = 243$ $P(0) \wedge P(1) \wedge P(2) \wedge P(3) \wedge \forall n \in \mathbb{N}, (\cancel{n \geq 0} \wedge P(n)) \rightarrow P(n+4))$

$3^6 = 729$

Proof of $\forall n \in \mathbb{N}, P(n)$ by IP1

Base Case: $3^0 = 1 \in \{1, 3, 7, 9\}$

IS: $\forall n \in \mathbb{N}$, $P(n) \rightarrow P(n+4)$

Let $n \in \mathbb{N}$

Assume $P(n)$, i.e. unit digit of $3^n \in \{1, 3, 7, 9\}$ (IH)

Case: unit digit of 3^n is 1

Then $3^{n+1} = 3 \cdot 3^n$ ends in $3 \in \{1, 3, 7, 9\}$

Case: ...

Case: ...

Case: ...

Proof of IP2

BC: $P(0), P(1), P(2), P(3)$

$3^0 = 1$ ends in $\{1, 3, 7, 9\}$

$3^1 = 3$ ...

$3^2 = 9$ ...

$3^3 = 27$ ...

IS: $\forall n \in \mathbb{N}$, $P(n) \rightarrow P(n+4)$

Let $n \in \mathbb{N}$

Assume units digit of $3^n \in \{1,3,7,9\}$ (IH)
Then $3^{n+4} = 3^n \cdot 3^4 = 81 \cdot 3^n$ has same units digit as 3^n since 81 ends in 1, so ~~that~~ digit also $\in \{1,3,7,9\}$

Every natural # ≥ 2 is a product of primes (FTA)