A series of 3 examples of different type

Eg: 1. A problem having no feasible solution

Maximize $z = 3x - 5y$ s.t.

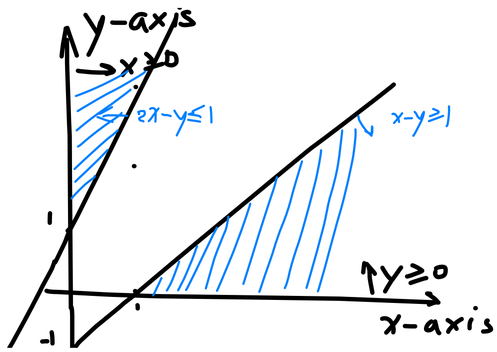
$x - y >= 1$

$2x - y <= -1$

$x >= 0$

$y >= 0$

Graphical solution (see Kolman and Beck Section 3.1)



graphical solution

The shaded regions are disjoint:
no x and y satisfy all 4 constraints
The feasible region is empty.
This problem is infeasible.
(and we have verified this is the case)

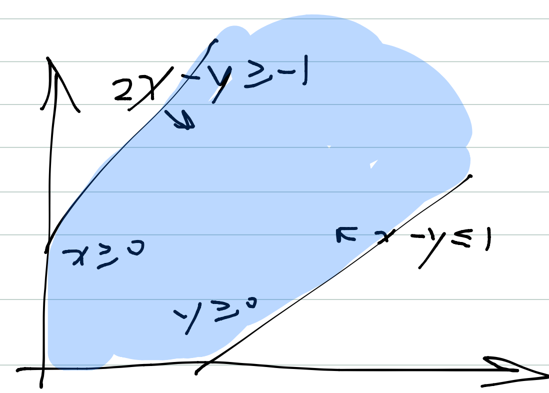
Eg: 2. a problem which is unbounded above. Kolman and Beck would say the problem has "no finite optimal solution". This means: for any M, there is a feasible x and y where z > M.

Maximize $z = x + 3y$ s.t.

$x - y <= 1$

$2x - y >= -1$

$x >= 0, y >= 0.$



This shaded area is the feasible region.
If $M >= 0$, $x = M$, $y = M$ is feasible:

$x - y = M - M <= 0$

$2x - y = 2M - M = M >= -1$

at $x=M, y=M$, $z=M+3M=4M$

x = M >= 0, y = M >= 0

Definition:

A set B in $\mathbb{R}^n$ (consisting of points like $\begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$) is bounded

if there is some real M such that $|x_1| \leq M$, $|x_2| \leq M$ $\dots$ $|x_n| \geq M$ for all $\begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$ in B. B is unbounded.

not bounded

Remark: if a problem is unbounded ("has no finite optimal solution"), then its feasible region is unbounded in the geometrical sense (that is, according to the definition)