Worth: 3%   **Due:** By 12 noon on Tuesday 7 February.

Remember to write the the *full name* and *student number* of each member of your group prominently on your submission. Your submission must be a PDF file named e3.pdf and it must be handed-in using the MarkUs system. You may create the PDF file using a typesetting system (export to PDF) or by scanning in handwritten work to create a PDF file.

Each exercise may be completed in groups of 1 – 2 students who are in the **same** tutorial section.

*Please read and understand the policy on Collaboration given on the Course Information Sheet. Then, to protect yourself, list on the front of your submission **every** source of information you used to complete this homework (other than your own lecture and tutorial notes, and materials available directly on the course webpage). For example, indicate clearly the **name** of every student with whom you had discussions, the **title** of every additional textbook you consulted, the **source** of every additional web document you used, etc.*

For each question, please write up detailed answers carefully. Make sure that you use notation and terminology correctly, and that you explain and justify what you are doing. Marks **will** be deducted for incorrect or ambiguous use of notation and terminology, and for making incorrect, unjustified, ambiguous, or vague claims in your solutions.

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1. Use truth tables to determine which of the following formulas are equivalent:

   (a) $P \Rightarrow (Q \Rightarrow R)$.
   (b) $Q \Rightarrow (P \Rightarrow R)$.
   (c) $(P \Rightarrow Q) \wedge (P \Rightarrow R)$.
   (d) $(P \wedge Q) \Rightarrow R$.
   (e) $P \Rightarrow (Q \wedge R)$.

2. Give a derivation to show that $(P \Rightarrow Q) \vee (P \Rightarrow R)$ is equivalent to $P \Rightarrow (Q \vee R)$. Justify each step of your derivation. Do not use truth tables or Venn diagrams.

3. Give a derivation to show that $(P \Rightarrow Q) \vee (Q \Rightarrow R)$ is a tautology. Justify each step of your derivation. Do not use truth tables or Venn diagrams.

4. Give a derivation to show that $P \iff P \wedge (P \vee Q)$. Justify each step of your derivation. Do not use truth tables or Venn diagrams. (You may **not** use an "Absorption" law.)

5. Four classmates (William, Xavier, Youssef and Zachary) were suspected of cheating on their homework. At separate meetings with their instructor, they said the following:

   William: If Xavier cheated, so did Zachary.
   Xavier: William cheated, but Zachary did not.
   Youssef: I did not cheat, but at least one of William or Zachary did.
   Zachary: If William did not cheat, then Youssef did.

   Let W represent the statement "William cheated", X represent the statement "Xavier cheated", Y represent the statement "Youssef cheated", Z represent the statement "Zachary cheated", and note that each student either cheated or they did not cheat.

   (a) If each student is telling the truth, which student(s) cheated?
   (b) If the students who cheated did not tell the truth at the meeting, and the students who did not cheat did tell the truth, which student(s) cheated?

   To answer the questions, you may find it helpful to express each students' statement using a logical expression. Do not use truth tables.