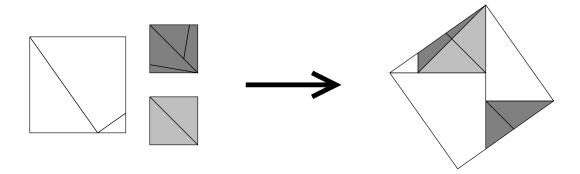
Worth: 2% Due: Before 10pm on Tuesday 17 January 2012.

## Remember to write your full name and student number prominently on your submission.

Please read and understand the policy on Collaboration given on the Course Information Sheet. Then, to protect yourself, list on the front of your submission **every** source of information you used to complete this homework (other than your own lecture and tutorial notes, and materials available directly on the course webpage). For example, indicate clearly the **name** of every student with whom you had discussions, the **title** of every additional textbook you consulted, the **source** of every additional web document you used, etc.

For each question, please write up detailed answers carefully. Make sure that you use notation and terminology correctly, and that you explain and justify what you are doing. Marks **will** be deducted for incorrect or ambiguous use of notation and terminology, and for making incorrect, unjustified, ambiguous, or vague claims in your solutions.

- 1. Write a detailed proof by induction that  $\forall n \geq 2, 2^n + 3^n < 4^n$ .
  - NOTE: There is no specific requirement that your proof be formatted in a particular way. Just make sure that it contains all of the appropriate parts, clearly indicated, and that it is correct—like the proofs presented in lecture and in the textbook.
- 2. Let P(n) be the statement: "any n squares can be dissected (cut in a finite number of straight lines) and rearranged to form one larger square". For example, in the picture below, each of the three squares on the left can be cut as indicated and the pieces can be moved and rotated to form the square on the right.



Obviously P(1) holds—no cut is required! You may use without proof the fact that P(2) holds. Write a detailed proof by induction that P(n) holds for all natural numbers  $n \ge 1$ .