

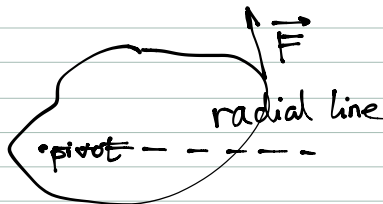
March 18th

Last lecture

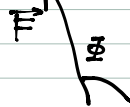
Rotations & Equilibrium

This time

Vector discription of torque and angular momentum



$$\tau = r F \sin \phi$$

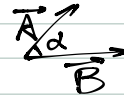


$$\vec{\tau} = \vec{r} \times \vec{F}$$

vector / cross product

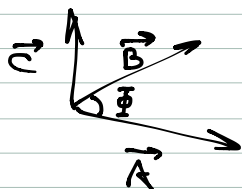
Recall : scalar / dot product of  $\vec{A} \cdot \vec{B}$

$$\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos \alpha$$



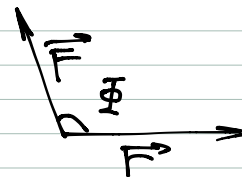
$$\vec{A} \times \vec{B} = \vec{C} = \text{a new vector}$$

$$|\vec{A} \times \vec{B}| = |\vec{A}| |\vec{B}| \sin \phi$$



"right-hand rule"

so for  $\vec{\tau} = \vec{r} \times \vec{F}$  points out the page



with symbol  $\odot \vec{\tau}$

$$\omega = \frac{d\theta}{dt} = \text{angular velocity}$$

( $\otimes$  means points into)

$v = \omega r$  speed of a part of wheel

The small part of wheel at S has velocity  $\vec{v}$  and momentum  $\vec{p} = m\vec{v}$  and an angular momentum  $\vec{L} = \vec{r} \times \vec{p}$

$$\vec{F}_{\text{NET}} = \frac{d\vec{p}}{dt} \leftrightarrow \vec{\tau}_{\text{NET}} = \frac{d\vec{L}}{dt}$$

$I = \text{momentum of inertia}$

$\vec{\omega}$  = direction given by curling fingers  
along rotation direction & using your  
thumb

sitting in a chair with a wheel.  $\vec{L}$  of the wheel is  $(L, \text{up})$ . inverts the wheel  
The  $\vec{L}$  of system consisting of the person + the chair is ?

$$\vec{L}_{\text{TOTAL, INITIAL}} = \vec{L}_{\text{system, initial}} + \vec{L}_{\text{wheel, initial}} = \vec{0} + (L, \text{up}) = (L, \text{up})$$

$$\vec{L}_{\text{TOTAL, FINAL}} = \vec{L}_{\text{system, final}} + \vec{L}_{\text{wheel, final}}$$

$$\vec{L}_{\text{final}} = \vec{L}_{\text{system, final}} + (L, \text{down}) = (L, \text{up}) = \vec{L}_{\text{initial}}$$

$$\Rightarrow \vec{L}_{\text{system, final}} = (2L, \text{up})$$