

# STAT3032 SURVIVAL MODELS

## TUTORIAL SOLUTIONS WEEK TEN

### Question One

#### Solution:

#### *Chi-square test*

As an example, here we group the final two rows to get an expected value greater than 5.

The test statistic (after grouping) =

$$22.723 - 0.417 - 0.565 + \frac{(10 - 10.384)^2}{10.384} = 21.755$$

Degrees of freedom = 24-3 (parameters) = 21

Critical value (test is one-tailed) is 33.92

Since  $21.755 < 33.92$ , do not reject the null hypothesis at 5% significance.

#### *Individual Standardised Deviations*

Range	Expected	Observed
$(-\infty, -1)$	$0.15866 * 25 = 3.9665$	3
$(-1, 0)$	$0.34134 * 25 = 8.5335$	6
$(0, 1)$	$0.34134 * 25 = 8.5335$	12
$(1, \infty)$	$0.15866 * 25 = 3.9665$	4

$$\chi^2 = 2.396.$$

Degrees of freedom =  $4 - 1 = 3$ ; critical value (test is one tailed) is  $\chi^2_{3,0.05} = 7.815$ .

Since  $2.396 < 7.815$ , do not reject the null hypothesis at 5% significance.

#### *Sign Test*

If  $N$  = number of positive deviations out of  $n$  observed age groups,  $N \sim \text{Bin}(n, 1/2)$ .

$N = 16$ .

$$T = \frac{16 - 12.5 - 0.5}{\sqrt{\frac{25}{4}}} = 1.20$$

The test is two tailed, since we are concerned both with too many or too few positive deviations.

Since  $-1.96 < 1.20 < 1.96$ , do not reject at 5% significance.

### ***Cumulative Deviation***

Test cumulative deviations for whole range and the ranges 65-77, 78-89.

$$\frac{\sum_{65}^{89} (\theta_x - E_x q_x^s)}{\sqrt{\sum_{65}^{89} E_x q_x^s p_x^s}} = 0.32 < 1.96 \rightarrow \text{Do not reject at 5\%}$$

$$\frac{\sum_{65}^{77} (\theta_x - E_x q_x^s)}{\sqrt{\sum_{65}^{77} E_x q_x^s p_x^s}} = 0.22 < 1.96 \rightarrow \text{Do not reject at 5\%}$$

$$\frac{\sum_{77}^{89} (\theta_x - E_x q_x^s)}{\sqrt{\sum_{77}^{89} E_x q_x^s p_x^s}} = 0.92 < 1.96 \rightarrow \text{Do not reject at 5\%}$$

### ***Run's Test***

$n_1$  = number of positive deviations = 16

$n_0$  = number of negative deviations = 9

$g$  = number of positive groups = 5

Expected number of positive groups =

$$\frac{n_1 (n_0 + 1)}{(n_1 + n_0)} = 6.40$$

$$\text{Variance of the number of positive groups} \approx \frac{(n_1 n_0)^2}{(n_1 + n_0)^3} = 1.327$$

The test statistic is  $\frac{5 - 6.40 + 0.5}{\sqrt{1.327}} = -0.781$

The critical region is  $< -1.65$ , since the test is one-tailed.

Since  $-0.781 > -1.65$ , do not reject the null hypothesis at 5% significance.

## Question Two

**Answer:**

$$\log\left(\frac{q}{p}\right) = b_0 + b_1x$$

$$\Rightarrow q = \frac{1}{1 + e^{-b_0 - b_1x}}$$

Age	Exposed	Observed	q	Expected	Chi
60	35000	450	0.013387	469	0.77
61	35000	550	0.01492	522	1.5
62	35000	525	0.016626	582	5.58
63	35000	700	0.018524	648	4.17
64	35000	650	0.020633	722	7.18
					19.2

This test-statistic has  $5-2=3$  degrees of freedom. We lose 2 degrees of freedom due to the parameters estimated for the graduation. The rejection region (at the 5% level of significance) is 7.815 or greater). It does not appear that the graduation fits the data well.