MAT135H1S Calculus I(A)

Solution to even-numbered problems in Chap. 1

(Section 1.3, Q32)

The domain for both f and g are $\mathbb{R}$, and hence for their composite.

(a) $(f \circ g)(x) = f\big(g(x)\big) = f(x^2 + 3x + 4) = (x^2 + 3x + 4) - 2 = x^2 + 3x + 2$

(b) $(g \circ f)(x) = g\big(f(x)\big) = g(x - 2) = (x - 2)^2 + 3(x - 2) + 4 = x^2 - x + 2$

(c) $(f \circ f)(x) = f\big(f(x)\big) = f(x - 2) = (x - 2) - 2 = x - 4$

(d) $$\begin{aligned}(g \circ g)(x) = g\big(g(x)\big) = g(x^2 + 3x + 4) &= (x^2 + 3x + 4)^2 + 3(x^2 + 3x + 4) + 4 \\ &= x^4 + 6x^3 + 20x^2 + 33x + 32\end{aligned}$$

(Section 1.3, Q38)

$\big(f \circ g \circ h(x)\big) = f\big(g(\sqrt{x})\big) = f(2^{\sqrt{x}}) = |2^{\sqrt{x}} - 4|$

(Section 1.6, Q16)

Since f is increasing, it is necessarily one-to-one. Therefore, it has an inverse f^{-1}. By the property of inverse function, we have $f\big(f^{-1}(2)\big) = 2$. To find $f^{-1}(3)$, one observes (by inspection) that $f(1) = 3$. Hence, it follows by the definition of inverse function that $f^{-1}(3) = 1$.

(Section 1.6, Q22)

Let $y = \dfrac{4x - 1}{2x + 3}$. Then we have

$$\begin{aligned}2xy + 3y &= 4x - 1 \\ 2xy - 4x &= -3y - 1 \\ x(2y - 4) &= -3y - 1 \\ x &= \frac{-3y - 1}{2y - 4}\end{aligned}$$

Interchange x and y, we obtain $y = \dfrac{-3x - 1}{2x - 4}$. Therefore, $f^{-1}(x) = \dfrac{-3x - 1}{2x - 4}$ $\left(\text{or } \dfrac{3x + 1}{4 - 2x}\right)$.

(Section 1.6, Q38)

(a) $e^{-2\ln 5} = e^{\ln 5^{-2}} = 5^{-2} = \dfrac{1}{5^2} = \dfrac{1}{25}$

(b) $\ln\left(\ln e^{e^{10}}\right) = \ln(e^{10}) = 10$