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- The main issue in section 3.2 is to determine the condition under which a relation such as $F(x, y) = 0$ can be locally written as $y = f(x)$ or $x = g(y)$. Even though in page 121 there are three common representations of the smooth curves given, the representations (ii) and (iii) must be locally translated to representation (i) in order to establish that we locally have a smooth curve. See top of page 23 for the definition of a **smooth curve**; here the goal is to establish the graph representation for a smooth curve. This is how the IFT becomes useful. Note that the IFT is a local property, and as such the idea of smooth curve is a local idea. That is, we investigate the smooth curves in a neighborhood of a given point. The condition of the IFT (that is corollary 3.3) becomes the condition of smooth curve.
- The notation (i) for a smooth curve is the true representation of the a smooth curve, while the other two representations are very useful in expressing a formula for complex curves. The representation (ii), (see examples 5 and 6 and top of page 124), while representation (i) is very limited in producing shapes.
- Proof of 3.11 (b) is a nice proof to learn. Please pay attention to the notations: the function $F(x, t)$ is defined as $x - \phi(t)$, and it should not be confused with $x = \phi(t)$. F is a function of two variables defined on the x-t plane. However at some point (t_0, x_0) $F(x_0, t_0) = 0$ because we have chosen x_0 to be the value of $\phi(t_0)$. One may think this choice is arbitrary and could be done in any other point t_1 also. That is correct, however we are sure that $\phi'(t_0)$ is not zero, hence at the point (x_0, t_0) we have the conditions of the IFT satisfied for the function F. Then the IFT leads to a collection of points (x, t) where $F(x, t) = 0$, which means $x - \phi(t) = 0$, which means $x = \phi(t)$.