University of Toronto
Department of Mathematics
Faculty of Arts and Science

MAT332H1F, Graph Theory

Final Examination, 12 December 2014

Instructor: Kasra Rafi
Duration: 3 hours

First ____ Last ____ Student Number ____

Instructions: No aids allowed. Write solutions on the space provided. To receive full credit you must show all your work. If you run out of room for an answer, continue on the back of the page. This exam has 8 questions, for a total of 100 points.

Problem #	Grade
1	
2	
3	
4	
5	
6	
7	
Bonus	
Total	

1. (24 points) Define the following terms and expressions:

   (a) Matching

   (b) Network

   (c) Planar graph

   (d) Connected component

(e) Dual of a plane graph

(f) k–vertex colourable

(g) M–augmenting path

(h) f–unsaturated path

Name: Student Number:

2. (20 points) Answer true of false. Justify your answer with an argument or a counter example.

(a) $\square$ Every complete bi-partite graph is vertex transitive.

(b) $\square$ In every simple connected graph, the size of the maximum matching is the same as the size of minimum covering.

(c) $\square$ Every graph is 4–vertex colourable.

(d) $\square$ If G is planar then the complement of G is also planar.

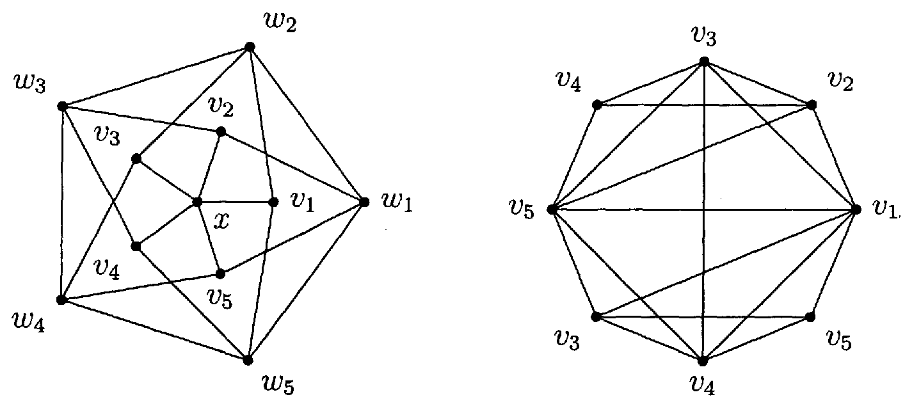
3. (12 points) State and prove the Five Colour Theorem.

Name: Student Number:

4. (12 points) (a) Give an example of a graph with a maximal matching that is not maximum. Then find an M–augmenting in your example.

   (b) Prove that the size of a minimum covering is greater than equal to the size of a maximal matching. Is the converse true (prove or give a counter example).

5. (12 points) Determine if these graphs are planar (For each graph, give an embedding to prove that it is not planar).



Name: Student Number:

6. (10 points) Show that any two longest cycles in a loopless connected graph without cut vertices have at least two vertices in common.

Name: Student Number:

7. (10 points) Let f be a flow on a network N. Show that, if f has integer values, then f can be written as a sum

$$f = f_0 + f_1 + \ldots + f_k,$$

where f_0 is a circulation and the support of each f_i, $1 \leq i \leq k$, is an xy–path.

Name: Student Number:

8. (Bonus Problem) Let S and T be maximal stable sets of a graph G. Show that $G[S \triangle T]$ has a perfect matching. (Recall that S is a stable set, or an independent set, of a graph G if G has no edges with both end points in S.)

End of exam.