University of Toronto
Department of Mathematics

MAT224H1S
Linear Algebra II

Midterm Examination I
Feb. 9, 2011

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Duration: 1 hour 30 minutes

Last Name: _____

Given Name: _____

Student Number: _____

Tutorial Code: _____

No calculators or other aids are allowed.

FOR MARKER USE ONLY	
Question	Mark
1	/10
2	/10
3	/10
4	/10
5	/6
6	/4
TOTAL	/50

1. Let T: $P_2(\mathbb{R}) \to P_2(\mathbb{R})$ be defined by

$$T(p(x)) = p(x-1).$$

(a) Show that T is a linear operator.

(b) Find the matrix of T relative to the basis $\alpha = \{1, 1+x, 1+x+x^2\}$ of $P_2(\mathbb{R})$.

2. Is the set $\{(i, 1, 2i), (1, 1+i, i), (1, 3+5i, -4+3i)\}$ a basis for $\mathbb{C}^3$? Justify your answer.

3. Let T: $\mathbb{R}_{2\times 2} \to \mathbb{R}_{2\times 2}$ be the linear transformation be defined by

$$T(A) = A\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} A.$$

(a) Find a basis for the kernel of T.

(b) Find a basis for the range of T.

4. Let T: $\mathbb{R}^3 \to \mathbb{R}^3$ be defined by

$$T(x_1, x_2, x_3) = (x_1 + x_2 + x_3, cx_2, x_1 + x_3)$$

where $c \in \mathbb{R}$ is a constant. For what values of c does there exist a basis α such that $[T]_{\alpha\alpha}$ diagonal? Justify your answer.

5. Let V and W be vector spaces over a field F, let $\alpha = \{v_1, \ldots, v_n\}$ and $\beta = \{w_1, \ldots, w_n\}$ be bases for V and W respectively, and let $T\colon V \to W$ be a linear transformation. Prove that T is an isomorphism iff $[T]_{\beta\alpha}$ is an invertible matrix.

6. Let $V = M_{22}$, the set of all 2×2 matrices. Let the operation of vector addition in V be the usual matrix addition but let scalar multiplication in V be defined by

$$c \cdot A = cA^T.$$

Is V a vector space? Justify your answer.