June 13th

P72 1.

State & prove two analogues of Rolle's theorem for functions of several variables, whose hypotheses are :

a. f is diff. on a set containing the line segment from $\vec{a}$ to $\vec{b}$, and $f(\vec{a})=f(\vec{b})$

b. f diff. on a bounded open set S, continuous on the closure of S, constant on the boundary.

$\Downarrow$ Does this mean $\|\nabla f\| \leq M$ on S?

a. By MVT

$$f(\vec{b})-f(\vec{a})=\nabla f(\vec{c})\cdot(\vec{b}-\vec{a})=\nabla f(\vec{c})\frac{\overrightarrow{b-a}}{\overrightarrow{b-a}}\cdot(\vec{b}-\vec{a})=0$$

— No. Counterexample:

$f=x^{\frac{1}{2}}$

$\vec{c}\in\overline{ba}$

b. continuous on $\bar{S}\Rightarrow f$ has max & min

① max or min is achieved on ∂S

$\Rightarrow f$ is constant on $\bar{S}\Rightarrow\nabla f=0$ everywhere

② ····· ···· $\operatorname{int}S$ say at point $x=(x_1,\cdots,x_n)$

then consider $\partial_j f(x_1,\cdots,x_n)$

claim it vanishes, $f(\text{———}x_j\text{———})$ has max or min at $(x_1,\cdots,x_j,\cdots,x_n)$

$\Rightarrow\partial_j f(\cdot\text{——})=0\Rightarrow\nabla f=0$

P77

3. Compute dy/dt & dz/dt when y,z are determined as functions of t by the equations $y^5+e^{yz}+zt^2=1$ and $y^2+z^4=t^2$

$$\begin{cases} 5y^4\dfrac{dy}{dt}+e^{yz}\cdot\left(z\cdot\dfrac{dy}{dt}+y\dfrac{dz}{dt}\right)+\dfrac{dz}{dt}t^2+z\cdot 2t=0 \\ 2y\dfrac{dy}{dt}+4z^3\dfrac{dz}{dt}=2t \end{cases}$$

Solve it.

P84

3. If $u=F(x+g(y))$, then $u_x u_{xy}=u_y u_{xx}$.

$u_x=F'(x+g(y))$

$u_{xy}=F''(x+g(y))\cdot g'(y)$

$u_y=F'(x+g(y))\cdot g'(y)$

$u_{xx}=F''(x+g(y))$

$$P_{a,k}(h)=\sum_{j=0}^{k}\frac{f^{(j)}(a)}{j!}h^j$$

$R_{a,k}(h) = f(a+h) - P_{a,k}(h)$

$|\sin x - x + \frac{1}{6}x^3| < 0.08$ (P 94.3)

for $|x| \leqslant \frac{1}{2}\pi$

$\underline{(x - \frac{1}{6}x^3)}$
dominant

Thm 2.55 : f is of C^{k+1} on $I \subset \mathbb{R}$, $a \in I$.

$$R_{a,k}(h) = \left|\frac{h^{k+1}}{k!}\right| \left|\int_0^1 (1-t)^k f^{(k+1)}(a+th)\,dt\right|$$

Then $$|R_{0,4}(h)| < \frac{|h|^5}{4!} \left|\int_0^1 (1-t)^4 \sin(th)\,dt\right|$$
$$\leqslant \frac{|h|^5}{4!} \left|\int_0^1 (1-t)^4\,dt\right|$$