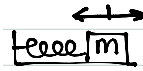
March 25th

Last time ...



Simple harmonic motion

$$F_{sp} = -k(x - x_e)$$

$$F_{NET} = F_{sp} = ma_x$$

$$ma_x = -k(x - x_e) \qquad x_e : \text{equilibrium position}$$

$$\frac{m\,d^2x}{dt^2} = -k(x - x_e)$$

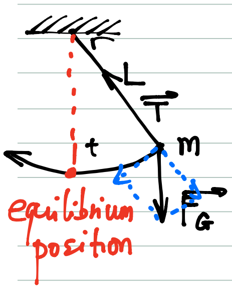
$$\frac{d^2x}{dt^2} = -\frac{k}{m}(x - x_e)$$

call this $\omega = \sqrt{\frac{k}{m}}$

$$x(t) = A\sin(\omega t + \phi_0)$$

amplitude, angular frequency, phase constant

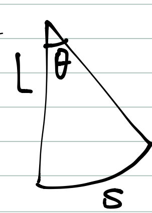
Today Pendulum



$$(F_{net})_r = T - (F_G)_r = 0$$

$$(F_{net})_t = -(F_G)_t = -mg\sin\theta$$

$(F_{net})_t = ma$ ← acceleration along the circle



$$s = \theta L$$

$$\theta = \frac{s}{L}$$

$$a = \frac{d^2s}{dt^2} = \frac{d^2}{dt}(\theta L) = L\frac{d^2\theta}{dt^2}$$

$$(F_{net})_t = mL\frac{d^2\theta}{dt^2} = -mg\sin\theta$$

$$\frac{d^2\theta}{dt} = \frac{-mg}{mL}\sin\theta = -\frac{g}{L}\sin\theta$$

$$\approx -\omega^2\theta$$

compare with $\frac{d^2x}{dt^2} = -\frac{k}{m}(x - x_e)$ for the object spring system.

For small angles, $\sin\theta \approx \theta$

$\theta(t) = A\cos(\omega t + \phi_0)$ satisfies

$$\frac{d^2\theta}{dt^2} = -\omega^2\theta, \text{ where } \omega(\text{"omega"}) = \sqrt{\frac{g}{L}} = \text{angular frequency}$$

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energy $= \frac{1}{2}mv_x^2 + \frac{1}{2}kx^2$ ... object on spring

$x = A\cos(t)$ or $x = A\sin(t)$

$V_x = \frac{dx}{dt} = -\omega A\sin(\omega t)$ or $V_x = \frac{dx}{dt} = \omega A\cos(\omega t)$

} E_{TOTAL} is constant