Worth: 3% **Due:** By 12 noon on Tuesday 27 March.

Remember to write the the *full name* and *student number* of each member of your group prominently on your submission. Your submission must be a PDF file named e7.pdf and it must be handed-in using the MarkUs system. You may create the PDF file using a typesetting system (export to PDF) or by scanning in handwritten work to create a PDF file.

Each exercise may be completed in groups of 1 – 2 students who are in the **same** tutorial section.

*Please read and understand the policy on Collaboration given on the Course Information Sheet. Then, to protect yourself, list on the front of your submission **every** source of information you used to complete this homework (other than your own lecture and tutorial notes, and materials available directly on the course webpage). For example, indicate clearly the **name** of every student with whom you had discussions, the **title** of every additional textbook you consulted, the **source** of every additional web document you used,* etc.

For each question, please write up detailed answers carefully. Make sure that you use notation and terminology correctly, and that you explain and justify what you are doing. Marks **will** be deducted for incorrect or ambiguous use of notation and terminology, and for making incorrect, unjustified, ambiguous, or vague claims in your solutions.

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1. Use a detailed structured proof to prove or disprove each of the following statements:
   (a) $\forall a \in \mathbb{R}, \forall b \in \mathbb{R}, a \leqslant b \Rightarrow n^a \in \mathcal{O}(n^b)$.
   (b) $\forall a \in \mathbb{R}, \forall b \in \mathbb{R}, 1 < a \leqslant b \Rightarrow a^n \in \mathcal{O}(b^n)$.
   (c) $\forall a \in \mathbb{R}^+, \forall b \in \mathbb{R}^+, a \neq 1 \wedge b \neq 1 \Rightarrow \log_a(n) \in \Theta(\log_b(n))$.
2. If marbles are arranged to form an equilateral triangle shape, with n marbles on each side, a total of $\sum_{i=0}^{n} i$ marbles will be required. In lecture, we proved that $\sum_{i=0}^{n} i = n(n+1)/2$. Numbers t_n, $n \in \mathbb{N}$, of the form $t_n = n(n+1)/2$ are called *triangular numbers*. Use the Principle of Simple Induction to prove that $\sum_{j=0}^{n} t_j = n(n+1)(n+2)/6$.