UNIT 5 Part 1
SYMBOLIZATION: PREDICATES AND QUANTIFIERS

EG 5.2

Abbreviation Scheme:

B^1:	drinks beer.	a:	Adam
C^1:	is Canadian.	b:	Stephen Harper
H^1:	plays Hockey.	c:	Carol
P:	Stereotypes shouldn't be believed.		

Adam is Canadian.	Ca
Stephen Harper is Canadian.	Cb
Adam and Stephen Harper are both Canadian, but Carol is not.	$Ca \wedge Cb \wedge {\sim}Cc$
If Stephen Harper is Canadian then he plays hockey.	$Cb \rightarrow Hb$
Adam is Canadian if and only if he drinks beer and plays hockey.	$Ca \leftrightarrow (Ha \wedge Ba)$
Carol doesn't drink beer but she is still Canadian.	${\sim}Bc \wedge Cc$
Although both Adam and Stephen Harper are Canadian, only the latter drinks beer.	$(Ca \wedge Cb) \wedge ({\sim}Ba \wedge Bb)$
Either Stephen Harper drinks beer if he is Canadian or stereotypes shouldn't be believed.	$(Cb \rightarrow Bb) \vee P$
At least one of them (Adam, Stephen and Carol) drink beer.	$Ba \vee Bb \vee Bc$

5.3 QUANTIFIERS AND COMPLEX PREDICATES

5.3 EG1 Symbolize the following:

Abbreviation scheme:

F:	is a snake.	I:	is native to Canada.
G:	is a rattlesnake.	J:	is venomous.

a) Rattlesnakes are venomous snakes.

Every object x is such that <u>if</u> x is a rattlesnake <u>then</u> x is venomous <u>and</u> x is a snake.

RA (restricting antecedent): is a rattlesnake

PC (predicating consequent): is venomous <u>and</u> is a snake

$\forall x\ (Gx \rightarrow Jx \wedge Fx)$

b) The only venomous snake native to Canada is the rattlesnake.

This sentence tells us that a snake is venomous and native to Canada only if it is a rattlesnake. Thus, we can paraphrase it: for every x, if x is venomous and x is a snake and x is native to Canada then x is a rattlesnake.

RA: is venomous and is a snake and is native to Canada

PC: is a rattlesnake

$\forall x\ ((Jx \wedge Fx \wedge Ix) \rightarrow Gx)$

Or we can paraphrase it: every object, x, is such that if it is a snake, then if it is venomous, then if it is native to Canada, then it is a rattlesnake.

RA: is a snake

PC: if it is venomous, then if it is native to Canada, then it is a rattlesnake.

$\forall x\ (Fx \rightarrow (Jx \rightarrow (Ix \rightarrow Gx)))$

c) A snake native to Canada is not venomous unless it is a rattlesnake.

RA: $Fx \wedge Ix$ Sentence is about Canadian snakes.
PC: $\sim Jx \vee Gx$ It says of the subject: it is not venomous or it's a rattlesnake.

$\forall x((Fx \wedge Ix) \rightarrow (\sim Jx \vee Gx))$

Also: $\forall x((Fx \wedge Ix) \rightarrow (Jx \rightarrow Gx))$

OR:

RA: Fx Sentence is about snakes.

PC: $Ix \rightarrow (\sim Jx \vee Gx)$ It says of subj.: if it's Canadian, then it's not venomous or it's a rattlesnake.

$\forall x(Fx \rightarrow (Ix \rightarrow (\sim Jx \vee Gx)))$

Also: $\forall x(Fx \rightarrow (Ix \rightarrow (Jx \rightarrow Gx)))$ OR $\forall x(Fx \wedge Ix \wedge Jx \rightarrow Gx)$

d) Among venomous snakes, only the rattlesnake is native to Canada.

RA: $(Fy \wedge Jy)$ Sentence is about venomous snakes
PC: $(Iy \rightarrow Gy)$ It says of the subject: if it is native to Canada then it is a rattlesnake.

$\forall y((Fy \wedge Jy) \rightarrow (Iy \rightarrow Gy))$ (NOTE: this sentence is logically equivalent to c!)

e) Snakes native to Canada are venomous if and only if they are rattlesnakes.

RA: $(Fy \wedge Iy)$ Sentence is about Canadian snakes.

PC: $(Jy \leftrightarrow Gy)$ It says of the subject: it is venomous if and only if it's a rattler.

$\forall x((Fy \wedge Iy) \rightarrow (Jy \leftrightarrow Gy))$

f) Any snake native to Canada that is not a rattlesnake is not venomous.

RA: $(Fx \wedge Ix \wedge {\sim}Gx)$ Sentence is about Canadian snakes that aren't rattlers

PC: ${\sim}Jx$ It says of the subject: it is not venomous.

$\forall x((Fx \wedge Ix \wedge {\sim}Gx) \rightarrow {\sim}Jx)$ OR $\forall x((Fx \wedge Ix) \rightarrow ({\sim}Gx \rightarrow {\sim}Jx))$

5.3 EG2 Symbolize the following:

Abbreviation scheme:

A:	is an animal.	B:	is a bird.	C:	is a cat.	D:	is a dog.
F:	has feathers.	G:	is gray.	H:	can fly.	M:	is a mammal.

a) Some mammals are dogs.

RC (restricting conjunct): Mx Sentence is about mammals

PC (predicating conjunct): Dx Says of the subject: some are dogs.

$\exists x(Mx \wedge Dx)$

b) Some animals with feathers are birds, but there aren't any mammals with feathers.

RC1: $Ax \wedge Bx$ Sentence is about animals with feathers

PC1: Fx Says of the subject: some are birds.

RC2: My Sentence is about mammals

PC2: Fy Says of the subject: none have feathers (${\sim}\exists$)

$\exists x(Ax \wedge Bx \wedge Fx) \wedge {\sim}\exists y(My \wedge Fy)$

c) Some animals, if they don't have feathers, are mammals.

RC: Ax Sentence is about animals

PC: ${\sim}Fx \rightarrow Mx$ Says of the subject: some are mammals if featherless.

$\exists x(Ax \wedge ({\sim}Fx \rightarrow Mx))$

d) Although there are gray cats, some cats aren't gray.

RC1: Cx Sentence is about cats

PC1: Gx Says of the subject: some are gray.

RC2: Cy Sentence is about cats

PC2: ${\sim}Gy$ Says of the subject: some are not gray

$\exists x(Cx \wedge Gx) \wedge \exists y(Cy \wedge {\sim}Gy)$

e) Some cats and dogs are gray.

RC1: Cx — Sentence is about cats

PC1: Gx — Says of the subject: some are gray.

RC2: Cy — Sentence is about dogs

PC2: ~Gy — Says of the subject: some are gray

$\exists x(Cx \wedge Gx) \wedge \exists x(Dx \wedge Gx)$

NOTE: can't combine under one quantifier, since we want to say that there are gray cats AND gray dogs.

f) There are gray animals that have feathers, but they cannot fly.

RC: $Gx \wedge Ax$ — Sentence is about gray animals

PC: $Fx \wedge \sim Hx$ — Says of the subject: some have feathers but cannot fly.

$\exists x((Gx \wedge Ax) \wedge (Fx \wedge \sim Hx))$

g) Mammals that fly exist but they don't have feathers.

RC: Mx — Sentence is about mammals

PC: $Hx \wedge \sim Fx$ — Says of the subject: some fly but don't have feathers.

$\exists x(Mx \wedge (Hx \wedge \sim Fx))$

5.5 E1 Let's try some more:

Abbreviation scheme:

B: is basalt.	F: glitters.	G: is gold.	I: is iron pyrite (fool's gold)
J: is quartz.	K: is rock.	M: is a metal.	O: is used for an Olympic first.

a: my wedding ring.

a) Some rocks glitter.

RC: Kx — Sentence is about rocks. Existential (some rocks)
PC: Fx — Says of the subject: it glitters.
$\exists x(Kx \wedge Fx)$

b) Anything gold is metal.

RA: Gx — Sentence is about gold things. Universal (all gold things)
PC: Mx — Says of the subject: it is metal
$\forall x(Gx \rightarrow Mx)$

c) Basalt and quartz are rocks.

RA: $Bx \lor Jx$	Sentence is about anything that is basalt or quartz. Universal.
PC: Kx	Says of the subject: it is a rock.

$\forall x(Bx \lor Jx \to Kx)$

OR $\forall x(Bx \to Kx) \land \forall x(Jx \to Kx)$

d) Quartz and iron pyrite glitter.

$\forall x(Jx \lor Ix \to Fx)$ OR $\forall x(Jx \to Fx) \land \forall x(Ix \to Fx)$ (similar to c)

e) Some rocks are basalt and some are quartz.

$\exists x(Kx \land Bx) \land \exists x(Kx \land Jx)$

NOTE: you cannot combine these like this: $\exists x(Kx \land (Bx \lor Jx))$ since it would not guarantee the existence of both basalt and quartz.

f) Some rocks and metals glitter.

$\exists x(Kx \land Fx) \land \exists y(My \land Fy)$

Similar to e).

g) Gold is a metal but some metals are not gold.

Main connective: $\land$

RA1: Gx	Sentence is about gold.
PC1: Mx	Says of the subject: it is metal.
RC2: Mx	Sentence is about metals
PC2: $\sim Gx$	Says of the subject: some are not gold.

$\forall x(Gx \to Mx) \land \exists x(Mx \land \sim Gx)$

h) Not all rocks are metal.

$\sim\forall x(Kx \to Mx)$ OR $\exists x(Kx \land \sim Mx)$

i) No quartz is metal.

$\forall x(Jx \to \sim Mx)$ OR $\sim\exists x(Jx \land Mx)$

j) Some, but not all, rocks are quartz.

$\exists x(Kx \land Jx) \land \sim\forall y(Kx \to Jx)$ OR $\exists x(Kx \land Jx) \land \exists y(Kx \land \sim Jx)$

k) Some rocks glitter but they are not gold.

$\exists y(Ky \land Fy \land \sim Gy)$

l) Iron pyrite glitters but is not gold.

$\forall x(Ix \to Fx \land \sim Gx)$

m) Gold, and only gold, is used for an Olympic first.

$\forall x(Gx \rightarrow Ox) \wedge \forall y(Oy \rightarrow Gy)$ OR $\forall x(Gx \leftrightarrow Ox)$

NOTE: this states that all gold is used for an Olympic first – and is true in the sense that all gold is the type of metal/thing used for an Olympic first.

Alternate interpretation: $\exists x(Ox \wedge Gx) \wedge \forall y(Oy \rightarrow Gy$

This states that some instances of Gold are used for Olympic firsts and everything used for an Olympic first is gold.

n) Gold is a metal that glitters.

$\forall x(Gx \rightarrow (Mx \wedge Fx))$ OR $\forall x (Gx \rightarrow Mx) \wedge \forall y (Gy \rightarrow Fy)$

o) Gold, which is a metal, glitters.

$\forall x(Gx \rightarrow (Mx \wedge Fx))$ OR $\forall x (Gx \rightarrow Mx) \wedge \forall y (Gy \rightarrow Fy)$

p) Rocks that are iron pyrite glitter.

$\forall x((Kx \wedge Ix) \rightarrow Fx)$

q) Gold and quartz both glitter, however (of the two of them) only gold is a metal.

$\forall x(Gx \rightarrow (Fx \wedge Mx)) \wedge \forall y(Jy \rightarrow (Fy \wedge {\sim}My))$

r) All that glitters is not gold.

$\exists x(Fx \wedge {\sim}Gx)$ OR ${\sim}\forall x(Fx \rightarrow Gx)$

s) Only gold is used for an Olympic first but not all gold is used for Olympic firsts.

$\forall x(Ox \rightarrow Gx) \wedge {\sim}\forall y(Gy \rightarrow Oy)$ OR $\forall x(Ox \rightarrow Gx) \wedge \exists y(Gy \wedge {\sim}Oy)$

t) My wedding ring is gold.

Ga

u) My wedding ring does not glitter.

${\sim}Fa$

v) If anything is gold then my wedding ring is.

$\exists xGx \rightarrow Ga$ OR $\forall x(Gx \rightarrow Ga)$

NOTE: there are no brackets in the first one- so Ga is not within the scope of the quantifier: it means that if something is gold then my wedding ring is gold. In the second one, Ga is in the scope of the quantifier. It means the same thing: for everything, x, if x is gold then my wedding ring is gold. Since it is true for all things, if anything is gold then my wedding ring is.

w) Nothing is used for Oympic firsts except gold.

$\forall x(Ox \rightarrow Gx)$ OR ${\sim}\exists x(Ox \wedge {\sim}Gx)$

5.7 EG1 Symbolizing with names is straightforward:

G^1: is graceful. D^2: dances with . B^3: stands between and .
a: Adam b: Betty c: Carol d: Darren

a) Carol dances with Darren.

$D(cd)$

b) Adam dances with Betty only if she is graceful.

$D(ab) \rightarrow Gb$

c) If neither Adam nor Darren is graceful, then Betty dances with Carol.

$\sim(Ga \vee Gd) \rightarrow D(bc)$ OR $(\sim Ga \wedge \sim Gd) \rightarrow D(bc)$

d) Provided Carol is graceful, Darren or Adam will dance with her.

$Gc \rightarrow (D(dc) \vee D(ac))$

e) Adam is standing between Betty and Carol.

$B(abc)$

f) If Adam is standing between Betty and Carol, then he dances with Betty or with Carol.

$B(abc) \rightarrow (D(ab) \vee D(ac))$

5.6 EG2: Symbolize the following:

F^1: is a person G^1: is a time H^2: teaches
J^3: bores at

a) Everyone teaches everyone. $\forall x(Fx \rightarrow \forall y(Fy \rightarrow H(xy)))$

b) Somebody teaches somebody. $\exists x(Fx \wedge \exists y(Fy \wedge H(xy)))$

c) Everyone is a teacher. $\forall x(Fx \rightarrow \exists y(Fy \wedge H(xy)))$

Everybody teaches somebody.

d) Everyone has a teacher. $\forall x(Fx \rightarrow \exists y(Fy \wedge H(yx)))$

Everybody is taught by somebody.

e) Someone teaches everybody. $\exists x(Fx \wedge \forall y(Fy \rightarrow H(xy)))$

f) Someone is taught by everyone. $\exists x(Fx \wedge \forall y(Fy \rightarrow H(yx)))$

g) Everyone is bored by someone some of the time.

$\forall x(Fx \rightarrow \exists y(Fy \wedge \exists z(Gz \wedge J(yxz))))$

h) Some people bore people all of the time.

This is ambiguous.
Some people always bore someone or another (not the same people). – this is probably the more natural interpretation

$\exists x(Fx \wedge \forall y(Gy \rightarrow \exists z(Fz \wedge J(xzy))))$

Subject: some person x

What sentence says about subject: that at every time, y, there is a person, z, such that x is boring z at y.

Some people are such that there is one person that they are always boring.

$\exists x(Fx \wedge \exists y(Fy \wedge \forall z(Gz \rightarrow J(xyz)))$

Subject: some person x
What sentence says about subject: that there is some person, y, such that at every time, z, x is boring y.

i) Although it's always the case that someone is bored by someone, nobody bores all of the people all of the time.

Note: this first conjunct says: for all things, x, if x is a time then somebody, y, is being bored by somebody, z, at that time. We have to put the universal/time first in order to make it that at all times somebody or another is boring somebody or another (the people boring each other can change!)

Note: the variations on the second conjunct are all just the different ways of doing the negative sentences.

$\forall x(Gx \rightarrow \exists y(Fy \wedge \exists z(Fz \wedge J(zyx)))) \wedge \sim\exists x(Fx \wedge \forall y(Fy \rightarrow \forall z(Gz \rightarrow J(xyz))))$
$\forall x(Gx \rightarrow \exists y(Fy \wedge \exists z(Fz \wedge J(zyx)))) \wedge \forall x(Fx \rightarrow \sim\forall y(Fy \rightarrow \forall z(Gz \rightarrow J(xyz))))$
$\forall x(Gx \rightarrow \exists y(Fy \wedge \exists z(Fz \wedge J(zyx)))) \wedge \forall x(Fx \rightarrow \exists y(Fy \wedge \sim\forall z(Gz \rightarrow J(xyz))))$
$\forall x(Gx \rightarrow \exists y(Fy \wedge \exists z(Fz \wedge J(zyx)))) \wedge \forall x(Fx \rightarrow \exists y(Fy \wedge \exists z(Gz \wedge \sim J(xyz))))$

Some people are always boring their students.

$\exists x(Fx \wedge \forall y(Gy \rightarrow \exists z((Fz \wedge H(xz)) \rightarrow J(xzy)))$

Some people, at all times, are such that some people that they are boring some people that they teach.

5.7 Exercises Predicate symbolization

5.7 E1: Symbolize the following quotes about friendship:

A^1: is a person E^2: is equal to . F^2: is a (true) friend to
K^2: knows all about . L^2: likes . C^3: has in common with .
O^1: is doing what one ought to do (is doing what one should do)

a) A person who is a friend to all is a friend to none. (Aristotle)

$\forall x(Ax \rightarrow (\forall y(Ay \rightarrow F(xy)) \rightarrow \forall z(Az \rightarrow {\sim}F(xz)))$

b) To have a have a friend is to be one. (Ralph Waldo Emerson)

Two interpretations: first, this is true only of people.

$\forall x(Ax \rightarrow (\exists y(Ay \wedge F(yx)) \rightarrow \exists z(Az \wedge F(xz)))$

Second interpretation: it is true of anything – if x has a friend, then x is a friend.

$\forall x(\exists yF(yx) \rightarrow \exists zF(xz))$

c) True friends are those who like and dislike the same things. (Sallust – a Roman Historian, 1st century BC.)

$\forall x(Ax \rightarrow \forall y(F(xy) \rightarrow \forall z(L(xz) \leftrightarrow L(yz))))$

d) Friends have all things in common. (Plato)

$\forall x\forall y(F(xy) \rightarrow \forall zC(xzy))$

e) You should have no friends not equal to yourself. (Confucious)

Anyone who is doing what he/she ought to do has no friend not equal to him/herself.

$\forall x((Ax \wedge Ox \rightarrow {\sim}\exists y(F(xy) \wedge {\sim}E(xy))))$

f) A friend is someone who knows all about you and still likes you. (Elbert Hubbard)

$\forall x\forall y(F(xy) \leftrightarrow K(xy) \wedge L(xy))$ OR $\forall x\forall y(Ax \wedge Ay \rightarrow (F(xy) \leftrightarrow K(xy) \wedge L(xy)))$

On this interpretation, "friend" is being defined. Thus, it is a biconditional. In the second form, it is being restricted to people. You might think it is a conditional rather than a biconditional… if you are a my friend, then you know all about me and still like me!

g) Everyone is someone's friend. (Ambiguous – symbolize two distinct ways.)

$\forall x(Ax \rightarrow \exists y(Ay \wedge F(xy)))$ Everyone is a friend to someone or another.

$\exists x(Ax \wedge \forall y(Ay \rightarrow F(xy)))$ Some one person is a friend to everyone.

5.7 E2: Symbolize the following:

A^1:	is an act.	F^1:	is a person/human.	G^1:	is good.
D^2:	does b.	H^2:	harms .	B^3:	borrows from .

a) Good people don't harm people.

$\forall x((Fx \wedge Gx) \rightarrow \sim\exists y(Fy \wedge H(xy)))$ OR $\forall x((Fx \wedge Gx) \rightarrow \forall y(Fy \rightarrow \sim H(xy)))$

b) If you harm anyone then you harm yourself.

$\forall x(Fx \rightarrow (\exists y(Fy \wedge H(xy)) \rightarrow H(xx)))$

c) A person is not good who does things that aren't good.

$\forall x(Fx \wedge \exists y(\sim Gy \wedge D(xy)) \rightarrow \sim Gx)$ OR $\forall x(Fx \rightarrow (\exists y(\sim Gy \wedge D(xy)) \rightarrow \sim Gx))$

d) Only people who do not harm others are good. (This is equivalent to a!)

$\forall x((Fx \wedge Gx) \rightarrow \sim\exists y(Fy \wedge H(xy)))$ OR $\forall x((Fx \wedge Gx) \rightarrow \forall y(Fy \rightarrow \sim H(xy)))$

e) The person whose every act is good is not human.

$\forall x((Fx \wedge \forall y(Ay \wedge D(xy) \rightarrow Gy) \rightarrow \sim Fx)$

Alternate symbolization: $\forall x(Fx \vee \sim\forall y(Ay \wedge D(xy) \rightarrow Gy))$

f) A good person does not perform acts that harm others.

$\forall x((Fx \wedge Gx) \rightarrow \sim\exists y(Ay \wedge D(xy) \wedge \exists z(Az \wedge H(yz))))$

g) No act is good that causes a person harm unless person being harmed is harming people.

$\forall x(Ax \wedge \exists y(Fy \wedge H(xy) \wedge \sim\exists z(Fz \wedge H(yz))) \rightarrow \sim Gx)$

$\sim\exists x(Ax \wedge Gx \wedge \exists y(Fy \wedge H(xy) \wedge \sim\exists z(Fz \wedge H(yz))))$

h) Everyone borrows things from people, but nobody lends things to everyone.

$\forall x(Fx \rightarrow \exists y\exists z(Fz \wedge B(xyz))) \wedge \sim\exists x(Fx \wedge \forall y(Fy \rightarrow \exists zB(yzx)))$

i) Some people borrow things from people who don't borrow things from anyone.

$\exists x(Fx \wedge \exists y\exists z(Fz \wedge \sim\exists w\exists v(Fv \wedge B(zwv)) \wedge B(xyz))$

Subject: person, x, (some people)

What sentence says about x: there is some thing y and some person z and x borrows y from z and there is no thing, w, and no person, v, such that z borrows w from v.

j) Neither a borrower nor a lender be. (A good person neither borrows nor lends things to others.)

$\forall x((Fx \wedge Gx) \rightarrow \sim\exists y\exists z(Fz \wedge (B(xyz) \vee B(zyx))))$ OR

$\forall x((Fx \wedge Gx) \rightarrow (\sim\exists y\exists z(Fz \wedge (B(xyz)) \wedge \sim\exists y\exists z(Fz \wedge B(zyx))))$

k) You are not doing a good thing if you lend to someone something that you borrowed from someone.

$\forall x(Fx \wedge \exists y(\exists z(Fz \wedge B(xyz) \wedge \exists w(Fw \wedge B(wyx) \rightarrow \sim\exists i(Ai \wedge Gi \wedge D(xi)))$

l) Everyone borrows something from somebody. (Ambiguous – symbolize four distinct ways.)

NOTE: THIS QUESTION IS ABOUT CHANGING THE SCOPE OF THE QUANTIFIER

THE ORDER OF THE QUANTIFIER MATTERS WHEN THE QUANTIFIERS ARE DIFFERENT (SOME $\forall$ AND SOME $\exists$).

Everyone borrows something or another from someone or another:

(They borrow different things from different people.)

$\forall x(Fx \rightarrow \exists y \exists z(Fz \wedge B(xyz)))$

Everyone borrows something or another from some one person.

(They borrow different things from the same person.)

$\exists z(Fz \wedge \forall x(Fx \rightarrow \exists y\, B(xyz)))$

Everyone borrows some one thing from someone or another.

(They borrow the same thing from different people.)

$\exists y \forall x(Fx \rightarrow \exists z(Fz \wedge B(xyz)))$

Everyone borrows some one thing from some one person.

(They borrow the same thing from the same person.)

$\exists y \exists z(Fz \wedge \forall x(Fx \rightarrow B(xyz)))$ OR

$\exists z \exists y(Fz \wedge \forall x(Fx \rightarrow B(xyz)))$

5.7 E3: Symbolize the following:

A^1: is an astronaut	B^1: is a space shuttle	C^1: is a car
E^1: is a time	H^1: is a person	J^1: is a jet
I^1: is a vehicle	D^2: drives	F^2: is faster than
G^2: rides in	K^3: flies in at	L^3: drives at
a: Adam	b: Betty	e: the Endeavor (a space shuttle)

a) Cars and jets are vehicles.

$\forall x(Cx \vee Jx \rightarrow Ix)$ OR $\forall x(Cx \rightarrow Ix) \wedge \forall y(Jy \rightarrow Iy)$

b) A space shuttle is a vehicle if and only if a jet is.

All space shuttles are vehicles if and only if all jets are vehicles.

$\forall x(Bx \rightarrow Ix) \leftrightarrow \forall y(Jy \rightarrow Iy)$

c) People who ride in space shuttles are astronauts.

This is about people who ride in at least one space shuttle. Such people are astronauts.

$\forall x(Hx \wedge \exists y(By \wedge G(xy)) \rightarrow Ax)$

d) No car is faster than any jet.

There is not something that is a car and is such that there is at least one jet it is faster than.

$\sim\exists x(Cx \wedge \exists y(Jy \wedge F(xy)))$

All cars are such that there is nothing that is a jet that the car is faster than.

$\forall x(Cx \rightarrow \sim\exists y(Jy \wedge F(xy)))$

All cars are such that all things that are jets are such that the car is not faster than they are.

$\forall x(Cx \rightarrow \forall y(Jy \rightarrow \sim F(xy)))$

e) Adam has never flown in the Endeavor, but he sometimes flies in a space shuttle.

$\sim\exists x(Ex \wedge K(aex)) \wedge \exists y(Ey \wedge \exists z(Bz \wedge K(azy)))$

f) Any vehicle that Betty drives is faster than those that Adam rides in.

This is about all vehicles that Betty drives. It says of such a vehicle that it is faster than all vehicles that Adam rides in.

$\forall x(Ix \wedge D(bx) \rightarrow \forall y(Iy \wedge G(by) \rightarrow F(xy)))$

g) Some people ride in vehicles that are faster than any car.

There is at least one person and there is at least one vehicle such that that person rides in it and it is faster than all cars.

$\exists x(Hx \wedge \exists y(Iy \wedge G(xy) \wedge \forall z(Cz \rightarrow F(xz))))$

h) Some people drive vehicles that are faster than any car that they ride in but do not drive.

There is at least one person and there is at least one vehicle such that he/she drives it and it is faster than every car that the person rides in but does not drive.

$\exists x(Hx \wedge \exists y(Iy \wedge D(xy) \wedge \forall z((Cz \wedge G(xz) \wedge {\sim}D(xz)) \rightarrow F(yz))))$

i) Some people don't ride in cars unless they are driving them.

There is at least one person, and all cars are such that that person doesn't ride in it or is driving it.

$\exists x(Hx \wedge \forall y(Cy \rightarrow {\sim}G(xy) \vee D(xy)))$

$\exists x(Hx \wedge \forall y(Cy \rightarrow (G(xy) \rightarrow D(xy))))$

There is at least one person such that there is no car that the person rides in but doesn't drive.

$\exists x(Hx \wedge {\sim}\exists y(Cy \wedge G(xy) \wedge {\sim}D(xy)))$

j) Betty won't ride in any jet that Adam has ever flown in.

This is about all jets such that there is some time at which Adam has flown in it. Betty doesn't ride in such jets.

$\forall x(Jx \wedge \exists y(Ey \wedge K(axy)) \rightarrow {\sim}G(bx))$

k) Astronauts who ride in the Endeavor never fly in space shuttles that are faster than it.

This is about all astronauts who ride in the Endeavor. There is no time at which such an astronaut flies in at least one space shuttle that is faster than the Endeavor.

$\forall x(Ax \wedge G(xe) \rightarrow {\sim}\exists y(Ey \wedge \exists z(Bz \wedge K(xzy) \wedge F(ze))))$

l) People who never fly in space shuttles sometimes fly in jets.

There exists at least one person who never flies in space shuttles but sometimes flies in jets.

$\exists x(Hx \wedge {\sim}\exists y(Ey \wedge \exists z(Bz \wedge K(xzy))) \wedge \exists i(Ei \wedge \exists k(Jk \wedge K(xki))))$

Alternate interpretation: This is about people who never fly in space shuttles. They sometime fly in jets.

$\forall x(Hx \wedge \sim\exists y(Ey \wedge \exists z(Bz \wedge K(xzy))) \rightarrow \exists i(Ei \wedge \exists k(Jk \wedge K(xki))))$

m) Everyone flies in jets some of the time, but never is everyone flying in jets.

All people are such that there is some jet and some time such that he/she flies in the jet at that time. But, there is no time such that all people are such that there is a jet that they are flying in at that time.

$\forall x(Hx \rightarrow \exists y(Jy \wedge \exists z(Ez \wedge K(xyz)))) \wedge \sim\exists z(Ez \wedge \forall x(Hx \rightarrow \exists y(Jy \wedge K(xyz)))$

n) Somebody is always driving some car. (Ambiguous – symbolize four distinct ways.)

There is a person and a car and he/she drives the car all the time.

$\exists x(Hx \wedge \exists y(Cy \wedge \forall z(Iz \rightarrow D(xyz))))$ OR $\exists x(Cx \wedge \exists y(Hy \wedge \forall z(Iz \rightarrow D(yxz))))$

There is a person and at all times he/she is driving some car or another.

$\exists x(Hx \wedge \forall z(Iz \rightarrow \exists y(Cy \wedge D(xyz))))$

There is a car and at all times it is being driven by some person or another.

$\exists x(Cx \wedge \forall z(Iz \rightarrow \exists y(Hy \wedge D(yxz))))$

At all times there is some person and some car such that the person is driving the car at that time.

$\forall z(Iz \rightarrow \exists x(Hx \wedge \exists y(Cy \wedge D(xyz))))$ OR $\forall z(Iz \rightarrow \exists x(Cx \wedge \exists y(Hy \wedge D(yxz))))$

a) B^1: is pleasurable F^1: is a person G^1: is good

C^2: persues D^2: desires c: Callicles

Everything that people desire is good.

Callicles (who is a person) pursues anything pleasurable.

Some pleasurable things are not good.

Therefore, Callicles pursues things that he does not desire.

$\forall x(Fx \rightarrow \forall y(D(xy) \rightarrow Gy))$ or $\forall y(\exists x(Fx \wedge D(xy)) \rightarrow Gy)$

$\forall x(Bx \rightarrow C(cx)) \wedge Fc$

$\exists x(Bx \wedge \sim Gx)$

$\therefore \exists x(C(cx) \wedge \sim D(cx))$

b) A^1: is withing the scope of the physical sciences. B^1: is about colour vision.

C^1: is a colour D^1: is a time. F^1: is a fact.

H^1: is a person. K^2: knows D^3: sees at

a: Mary P: Physicalism is true

If Physicalism is true then all facts are within the scope of the physical sciences.

Mary knows all the facts about colour vision within the scope of the physical sciences.

Although Mary is a person, Mary has never seen any colour.

For a person to know certain facts about color vision it is necessary that the person sees something in color at some time or another.

Thus, some facts about colour vision are not within the scope of the physical sciences and physicalism is false.

(based on Jackson, .)

$P \rightarrow \forall x(Fx \rightarrow Ax)$

$\forall x(Fx \wedge Bx \wedge Ax \rightarrow K(ax))$

$Ha \wedge \sim\exists x\exists y(Dx \wedge Cy \wedge O(ayx))$

$\forall x(Hx \rightarrow \exists y(Fy \wedge By \wedge (K(xy) \rightarrow \exists z\exists w(Dz \wedge Cw \wedge O(xwz))))$

$\therefore \exists x(Fx \wedge Bx \wedge \sim Ax) \wedge \sim P$

c) E^1: is evil | F^1: is omnipotent.

E^1: is evil	F^1: is omnipotent.
H^1: happens	G^1: is perfectly good
A^2: allows to happen.	B^2: is able to prevent from happening.
C^2: is a cause of	K^2: knows

An omniscient being (a being who knows everything) knows the causes of all evils.

An omnipotent being who knows the causes of an evil is able to prevent it from happening.

A perfectly good being who is able to prevent an evil thing from happening does not allow it to happen.

Things that are not allowed to happen (by some being) do not happen.

Evil things happen.

Therefore, there are no omniscient, omnipotent, perfectly good beings.

(based on The Problem of Evil, many sources from Epicurus on.)

$\forall x(\forall y K(xy) \rightarrow \forall z(Ez \rightarrow \forall w(C(wz) \rightarrow K(xw))))$

$\forall x(Fx \rightarrow \forall y(Ey \rightarrow \forall z((C(zy) \rightarrow K(xz)) \rightarrow B(xy)))))$

$\forall x(Gx \rightarrow \forall y(Ey \rightarrow (B(xy) \rightarrow \sim A(xy))))$

$\forall x(\exists y \sim A(yx) \rightarrow \sim Hx)$

$\exists x(Ex \wedge Hx)$

$\therefore \sim\exists x(Fx \wedge Gx \wedge \forall y K(xy))$

E: a is evil.

F: a is omnipotent.

G: a is perfectly good.

H: a happens.

A: a allows b to happen.

B: a is able to prevent b from happening.

C: a is a cause of b.

K: a knows b:

5.8 E1

a) Which of the following are substitution instances of: $\forall x(Fx \rightarrow Gx)$?

i)	$Fa \rightarrow Ga$	yes
ii)	$Fx \rightarrow Gx$	no (an individual constant was not substituted)
iii)	$Fx \rightarrow Gb$	no (not all occurrences of x were replaced)
iv)	$Fb \rightarrow Gb$	yes
iv)	$\forall x(Fx \rightarrow Gb)$	no (the quantifier wasn't dropped)

b) Which of the following are substitution instances of: $\forall x(Fx \rightarrow \exists y(Gy \wedge L(xy)))$?

i)	$Fa \rightarrow \exists y(Gy \wedge L(xy))$	no (second occurrence of x wasn't replaced)
ii)	$Fb \rightarrow \exists y(Gy \wedge L(by))$	yes
iii)	$\forall x(Fx \rightarrow (Ga \wedge L(xa))$	no (the initial quantifier wasn't dropped)
iv)	$Fa \rightarrow \exists y(Ga \wedge L(ay))$	no(the first occurrence of y was also replaced.)
v)	$Fd \rightarrow \exists y(Gy \wedge L(dy))$	yes