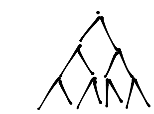
Lecture 7

A binary tree is
- empty, or
- a left binary tree and a right binary tree.

The simplest one is empty.

A binary tree is full iff its left & right subtrees are empty, or both are full binary tree.

Prove that all full binary trees have odd # of nodes.



For $h \in \mathbb{N}$, let $P(h)$ be ALL FULL binary trees of height h have an odd # of nodes

Proof of $\forall n \in \mathbb{N}, P(h)$

By complete induction

Inductive step. Let $h \in \mathbb{N}$, assume all full binary trees of height $< h$ have odd # nodes.
Let t be a full binary tree of height h.

Case: left & right subtrees of t are empty
Then t is just the single node tree, has one node, which is odd.

Case: left & right subtrees of t are full binary trees
Let's call them t_L & t_R, their heights are less than the height of t
By IH, the f.b.t.s t_L & t_R. # of nodes in t_L is odd, # of nodes in t_R is odd.
The # of nodes in t is 1 (the root) + # nodes in t_L + # of nodes in t_R which is 1+odd+odd=odd

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Well ordering Principle

Every non-empty set of natural numbers has a minimal element.

$\emptyset$: NO claim (no min)
$\mathbb{N}$: Min is 0
$\{1,3,4,5,7\}$ min is 1
Primes, min is 2.

Consider sets of real #s
$(0,1)$
$\mathbb{Z}$ No
$[0,1)$ min is 0

$236 / 7 = 33 \cdots\cdots 5$

$a = bq + r$ we like the remainder minimal, say $q > r$

Consider set of all natural numbers
let $R = \{r \in \mathbb{N}, \exists q \in \mathbb{N}, 31415926 = 53589q + r\}$ $31415926 \in R$
not real number set!
$\cap$
$\mathbb{N}$ by def

By WOP, M has a minimum, call it r_0. Let $q_0 \in \mathbb{N}$ be st. $31415926 = 53589 q_0 + r_0$
$r_0 \in \mathbb{N}$ since $r_0 \in M \subset \mathbb{N}$

if $r_0 > 53589$, $r_0 - 53589 \in \mathbb{N}$ and $31415926 = 53589(q+1) + (r_0 - 53589)$
Then r_0 is not the minimum. contradiction
So $r_0 < 53589$