

July 9th

Smooth curve in \mathbb{R}^2

① Connected

② locally the graph of a C^1 function

(1) $x=f(y)$ or $y=f(x)$

$$(2) F(x, y, z) = 0$$

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(3) $f: (a, b) \rightarrow \mathbb{R}^2$
 $t \mapsto (\varphi(t), \psi(t))$

Thm :

(2) $\nabla F \neq 0$, at (a, b)

"connectedness" is not guaranteed

(3) $f'(t_0) \neq 0$, \exists open interval I with $t_0 \in I$ s.t. the set $\{f(t) | t \in I\}$ is the graph of a C^1 function.

A horizontal line segment with endpoints labeled a and b . A point labeled t_0 is marked on the segment, approximately in the middle. The segment is enclosed in large parentheses $($ and $)$.

not 1-1

$f(t_0) = f(t_0) = p$

P125 #2

$$S_p = \{(x, y) \mid x^p + y^p = 1\} \quad p \in \mathbb{Z}^+$$


(a) Show S_p is smooth curve $\forall p$.

(b) Sketch Sp.

(b)

$p=1$

$p=2$



$p=4$
(a "square circle")

$y = (1-x^p)^{\frac{1}{p}}$ for p is odd
 \rightarrow not diff at $x=1$

A graph of a function on a Cartesian coordinate system. The x-axis and y-axis are shown. A vertical dashed line is drawn at $x=1$, representing a vertical asymptote. The function curve is defined for $x < 1$ and $x > 1$. For $x < 1$, the curve starts from the top left, passes through the y-axis at a positive value, and approaches the horizontal asymptote $y=1$ as x approaches 1 from the left. For $x > 1$, the curve starts from the bottom right, passes through the x-axis at a positive value, and approaches the horizontal asymptote $y=1$ as x approaches 1 from the right.