MAT 337

Sample Midterm Exam

NAME

NO AIDS ALLOWED

Total: 250 points, not including a bonus problem

Problem 1 [30 points]

(a) Give an example of a function $f : \mathbb{R} \longrightarrow \mathbb{R}$ that is continuous at exactly one point of its domain.

(b) Show that $f(x, y) = \min\{x, y\}$ is continuous on $\mathbb{R}^2$.

Explain.

Problem 2 [45 points]

(a) Prove that any Lipschitz function is uniformly continuous.

(b) Is $f(x) = x^2$ uniformly continuous on $[0, 1]$? Is it uniformly continuous on $[0, \infty)$?

(c) Show that $f(x) = \cos x$ is Lipschitz on $\mathbb{R}$.

Problem 3 [30 points] Let $f : (X, \rho) \longrightarrow (Y, \sigma)$ be a continuous map between two metric spaces. Let $C \subset X$ be a connected subset of a metric space X. Show that $f(C)$ is connected.

Problem 4 [40 points] Prove that any closed subset of a compact metric space (X, ρ) is compact.

Problem 5 [45 points] Let $f : [0,1] \longrightarrow [0,1]$ be a continuous map. Show that there exists a point $x \in [0,1]$ such that $f(x) = x$.

Problem 6 [30 points] Show that a connected metric space having more than one point is uncountable.

Problem 7 [30 points] Let $f : (X, \rho) \longrightarrow (Y, \sigma)$ be a continuous map from a compact metric space (X, ρ) to a metric space (Y, σ). Prove that f is uniformly continuous.