Lecture 4

CHAPTER 5
FIXED & PERIODIC POINTS

Recall

$\boxed{\text{Intermediate value theorem}}$

Let $f:[a,b]\to\mathbb{R}$ be continuous, then $\forall k$ between $f(a)$ and $f(b)$, $\exists c\in[a,b]$ s.t. $f(c)=k$

fixed point theorem. Let $F:[a,b]\to[a,b]$ be a continuous function. Then F has at least one fixed point $x_0\in[a,b]$.

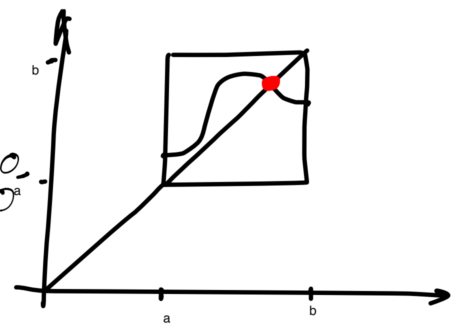
Proof: define $h(x)=F(x)-x$

so $h(a)=F(a)-a\geq a-a=0$

$h(b)=F(b)-b\leq b-b=0$

we can use IVT with $h(x)$ & $k=0$. To deduce that $\exists x\in[a,b]$ s.t. $h(x_0)=k=0$

so x_0 is a fixed pt of $F(x)$. ∎



Remarks:

① The thm states that there is at least one fixed pt.

② It is important that range $\subseteq$ domain.

③ The domain of F is closed interval of $[a,b]$. The thm doesn't hold for open intervals.

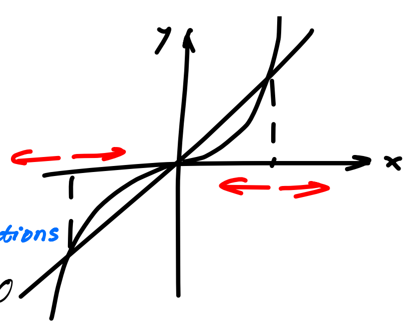
④ From this proof, we don't know what the fixed point is.

Counter example for ③: $F(x)=x^2$ in $(0,1)$

Ex: $F(x)=x^3$

- So the fixed point $x=1$ repels orbits.

- the fixed point $x=0$ attracts orbits



Example: Let $L_m(x)=mx$ for $m\neq 1$ ← it's a family of functions

These functions have 1 fixed point $x_0=0$

Let $x_0\neq 0$, then $x_1=F(x_0)=mx_0$

$x_2=F(x_1)=m^2x_0$

$x_3=F(x_2)=m^3x_0$

$\vdots$

$x_n=m^nx_0$

Then

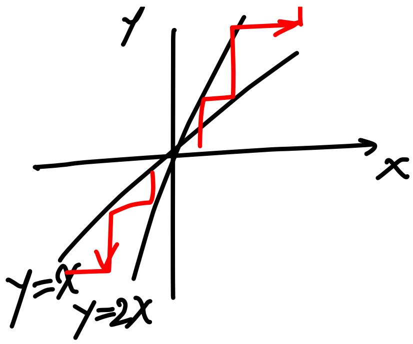
if $|m|>1$, then x_n escapes to infinity

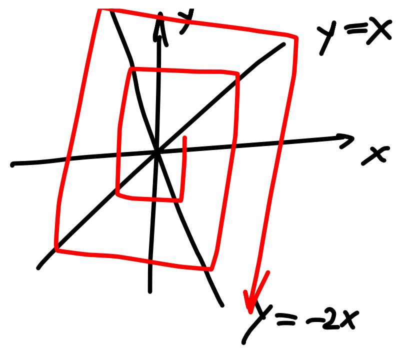
if $|m|<1$, then x_n converges to 0.

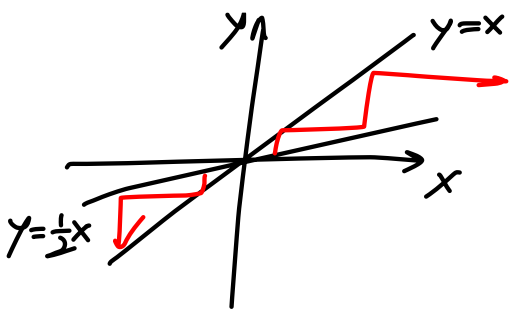
if $m=-1$, then the orbit is a 2-cycle.
if $m=1$, then the orbit is fixed.

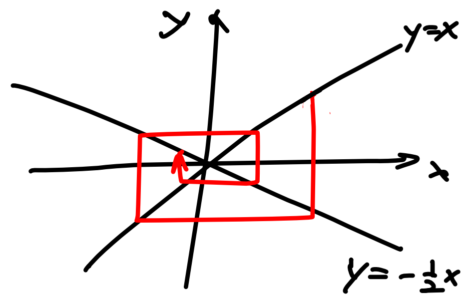
So the fixed point $x=0$ is:

- attracting if $|m|<1$
- repelling if $|m|>1$
- netural if $|m|=1$









§5.3 Calculus of fixed points

def: Let p be a fixed point of $F(x)$ then
(i) if $|F'(p)|<1$, then p is called an attractive fixed point.
(ii) if $|F'(p)|>1$, then p is called a repelling fixed pt
(iii). $\cdots\cdot = 1$, $\cdots$ neutural / indifferent fixed pt.