Course outline for 2011F MAT240 Algebra I

1 Pre-history

- **1.1 Sets**
  - **1.1.1 Maps between sets**
  - **1.1.2 Finite sets**

2 Vector spaces

- **2.1 Definition of field**
  - **2.1.1 Examples of fields**
  - **2.1.2 Subfields**
  - **2.1.3 simple properties of fields (usual arithmetic!)**
  - **2.1.4 Finite fields**
- **2.2 Definition of vector space**
  - **2.2.1 Examples of vector spaces**
  - **2.2.2 Diagrams of axioms**
  - **2.2.3 Simple properties coming from axioms**
  - **2.2.4 Definition of Subspaces**
  - **2.2.5 Two ways to get subspaces: span and constraints**
  - **2.2.6 Redundancy, i.e. linear (in)dependence**
- **2.3 Dimension and finite-dimensional vector spaces**
  - **2.3.1 Span and linear dependence**
  - **2.3.2 Definition of basis and dimension**
  - **2.3.3 Coordinates, using uniqueness from linear independence**
  - **2.3.4 Construction of a basis in a finite dimensional space**
  - **2.3.5 Examples of bases**
  - **2.3.6 Simple properties of dimension**
  - **2.3.7 Algorithm for determining linear independence of vectors in \(\mathbb{F}^n\)**

3 Linear maps

- **3.1 Definition of linear map**
- **3.2 Examples of linear maps**
- **3.3 The vector space of linear maps**
- **3.4 Composition of linear maps**
- **3.5 Algebra of linear operators**
- **3.6 Isomorphism**
  - **3.6.1 Classification of finite dimensional vector spaces**
- **3.7 Null space and Image of a linear map**
  - **3.7.1 Theorem about $dim U = dim\mathrm{null} T + dim\ran T$**
- **3.8 Linear maps vs. matrices**
  - **3.8.1 Isomorphism from $L(V,W)$ to $M(n,k,\mathbb{F})$.**
  - **3.8.2 Matrix multiplication**
  - **3.8.3 Change of basis: Writing the identity operator with respect to two bases: $[\id]_{beta}^{~beta}$.**