

**FINANCIAL MATHEMATICS**  
**(STAT 2032 / STAT 6046)**

**TUTORIAL EXERCISES WEEK 3**

**Question 1**

Find the present value of \$1000 due at the end of 10 years if

(a)  $i^{(2)} = 0.09$ , (b)  $i^{(6)} = 0.09$ , and (c)  $i^{(12)} = 0.09$ .

**Question 2**

Mountain Bank pays interest at a nominal rate convertible half-yearly of  $i^{(2)} = 0.15$ .

River Bank pays interest compounded daily. What minimum nominal annual rate convertible daily must River Bank pay in order to be as attractive as Mountain Bank?

**Question 3**

Bank A has an effective annual rate of 18%. Bank B has a nominal annual rate of 17% convertible  $m$  times per year. What is the smallest whole number of times per year ( $m$ ) that Bank B must compound its interest in order that the rate at Bank B be at least as attractive as that at Bank A on an effective annual basis? Repeat the exercise with a nominal rate of 16% per annum at Bank B.

Hint: Use trial and error to check the whole numbers.

**Question 4**

Nominal interest can be defined even if  $m$  is not an integer. The algebraic definition

$1+i = \left(1 + \frac{i^{(m)}}{m}\right)^m$  is still valid. Suppose a bank advertises a nominal rate of 10% per

annum convertible every 45 days on short-term deposits. Find  $m$  and the equivalent effective annual rate of interest.

**Question 5**

The nominal rate of interest  $i^{(m)}$  can be defined for values of  $m < 1$ . Algebraically the

definition follows the relationship in the equation  $1+i = \left(1 + \frac{i^{(m)}}{m}\right)^m$

(a) If  $i = 0.10$ , find the equivalent  $i^{(0.5)}$ ,  $i^{(0.25)}$ ,  $i^{(0.1)}$ , and  $i^{(0.01)}$ . Rank the values in increasing size, and compare with the relationship  $i^{(m)} < i$  for  $m > 1$ .

(b) Find the equivalent effective annual rate  $i$  if (i)  $i^{(0.5)} = 0.10$ , (ii)  $i^{(0.25)} = 0.10$ , (iii)  $i^{(0.1)} = 0.10$ , and (iv)  $i^{(0.01)} = 0.10$ .

### **Question 6**

If the effective rate of interest is 10% per annum, calculate (a)  $d$  and (b)  $d^{(12)}$ .

### **Question 7**

Find the accumulated value of \$100 at the end of two years if:

- (a) the nominal annual rate of interest is 6% convertible quarterly.
- (b) the nominal annual rate of discount is 4% convertible monthly.
- (c) the nominal annual rate of discount is 6% convertible once every four years.

### **Question 8**

An investment of \$1,000 accumulates to \$1,360.86 at the end of 5 years. If the force of interest is  $\delta$  during the first year and  $1.5\delta$  in each subsequent year, find the equivalent effective annual interest rate in the second year.

### **Question 9**

Smith forecasts that interest rates will rise over a 5-year period according to a force of interest function given by  $\delta_t = 0.08 + \frac{0.025t}{t+1}$  for  $0 \leq t \leq 5$ .

(a) According to this scheme, what is the average annual compound effective rate for the 5-year period?

(b) What is the present value at  $t=2$  of \$1,000 due at  $t=4$ ?

Hint:  $\int \left( \frac{t}{t+1} \right) dt = t - \ln(t+1)$

### **Question 10**

The present value of  $K$  payable after 2 years is \$960. If the force of interest is cut in half the present value becomes \$1,200. Find  $K$ .

What is the present value if the effective annual discount rate is cut in half?

**Past Exam Question – 2005 Final Exam Q1**

- (a) Find the accumulated value of \$100 at the end of two years if the nominal annual rate of interest is 6% per annum convertible weekly.
- (c) An investment is made at a compound force of interest of 8% p.a. for a period of 5 years. Find the equivalent rate of *simple interest* per annum.