STAT3032 SURVIVAL MODELS

TUTORIAL WEEK TWO

1. Using ELT 15 Ultimate mortality, find the probability that a male aged 55 will

   (a) survive to age 56 ✓

   (b) die before reaching age 65 ✓

   (c) die between ages 56 and 58 ✓

   (d) die between ages 60 and 65 ✓

2. The distribution function of the future lifetime of a new-born individual in a certain district is given by:

$$F_0(t) = 1 - e^{-0.015t}$$

   What is the probability that:

   (a) A new-born individual will survive to age 70? $P(T>70) = 1-P(T\leq 70) = 1-F_0(70)$

   (b) A new-born individual dies not later than age 35? $P(T\leq 35) = F_0(35)$

   (c) A person aged 25 survives to age 50? $P(T>50 \mid T>25) = \frac{P(T>50)}{P(T>25)} = \frac{1-F_0(50)}{1-F_0(25)}$

   (d) A person aged 30 dies not later than age 70? $P(T<70 \mid T>30) = \frac{P(T>30 \,\&\, T<70)}{P(T>30)} = \frac{F_0(70)-F_0(30)}{F_0(30)}$

3. If $p_{x+1} = 0.97$, ${}_3p_x = 0.912285$, ${}_2q_x = 0.0398$, find p_{x+2}.

   ${}_3p_x = {}_2p_x \cdot p_{x+2}$ ${}_2p_x + {}_2q_x = 1$

4. Express in as many forms as you can, using both statistical functions (the survival function S and the distribution function F) and actuarial notation (*p*,*q*,*d* and *l* etc) the probability that a person aged 50 will die between 70 and 80.

   $P(20 < T_{50} \leq 30) = F_{50}(30) - F_{50}(20)$

   $= S_{50}(20) - S_{50}(30)$

   $P(70 < T_0 \leq 80 \mid T_0 > 50) = \frac{F_0(80)-F_0(70)}{S_0(50)}$

   ${}_{20}p_{50} \cdot {}_{10}q_{70}$ or $\frac{l_{70}-l_{80}}{l_{50}} = \frac{\sum_{i=0}^{9} d_{70+i}}{l_{50}}$ or ${}_{20|10}q_{50}$

5. Explain why μ_x can be greater than one.

6. **Challenge Problem**

For a continuous lifetime random variable T the mean residual life function is

$\mu_x = -\frac{1}{l_x}\frac{dl_x}{dx}$

$$r(t) = E\left[(T-t) \mid T > t\right]$$

expected lifetime lived in the future who has already survived t years

(a) Explain in non-technical language what the mean residual life function measures.

(b) Explain why $r(t) = E\left[T \mid T > t\right] - t$.

t is constant.

(c) If the random variable T has an exponential distribution, show that the density of T conditional on T being greater than t is

$$\frac{f_T(y)}{S_T(t)} = \frac{\lambda e^{-\lambda y}}{\int_t^\infty \lambda e^{-\lambda y} dy} =$$

$$f_{T|T>t}(y) = \frac{\lambda e^{-\lambda y}}{e^{-\lambda t}}.$$

$r(t) = E(T|T>t) - t = e^{\lambda t}\int_t^\infty \lambda y e^{-\lambda y} dy - t = \frac{1}{\lambda}$

(d) Hence using an appropriate integral for conditional expectation, show that $r(t) = \frac{1}{\lambda}$.

$E(T) = \frac{1}{\lambda}$

(e) Write down the mean of the exponential random variable with parameter λ.

(f) Explain, again using non-technical language, the significance of the fact that parts (d) and (e) yield the same result.

memoryless property