$$= \sum_{\beta_1+\gamma_1=\alpha_1} \cdots \sum_{\beta_n+\gamma_n=\alpha_n} \frac{\alpha_1! \cdots \alpha_n!}{\beta_1!\gamma_1! \cdots \beta_n!\gamma_n!} x_{11}^{\beta_1} \cdots x_{1n}^{\beta_n} x_{21}^{\gamma_1} \cdots x_{2n}^{\gamma_n}$$

$$= \sum_{\beta+\gamma=\alpha} \frac{\alpha!}{\beta!\,\gamma!} \vec{x}_1^{\,\beta} \vec{x}_2^{\,\gamma}$$ as $\beta+\gamma=\alpha \Leftrightarrow \beta_i+\gamma_i=\alpha_i \ \forall i$

$\blacksquare$

6) $f(x,y) = (y^2-y)x$

$\partial_x f = y^2 - y = 0 = y(y-1)$ so $y=0$ or $y=1$

$\partial_y f = (2y-1)x = 0 \Rightarrow x=0$ or $y=\frac{1}{2}$

$y=0 \Rightarrow x=0$, $y=1 \Rightarrow x=0$, $x=0$ pairs with $y=0, y=1$

$y=\frac{1}{2}$ isn't zero for 1st eqn.

so $(0,0)$ & $(0,1)$ are the CPs.

$\partial_x^2 f = 0$ $\qquad \partial_x\partial_y f = 2y-1 = \partial_y \partial_x f$

$\partial_y^2 f = 2x$

so $H(x,y) = \begin{pmatrix} 0 & 2y-1 \\ 2y-1 & 2x \end{pmatrix}$ so $\det H = -(2y-1)^2$

$D = \det H = -1$ for $(0,0) \Rightarrow$ saddle

$D = \det H = -1$ for $(0,1) \Rightarrow$ saddle.

$f(\vec{a}+\vec{k}) = f(\vec{a}) + 0 + \frac{1}{2}\vec{k}^T H \vec{k}$ at CPs.

$$H(x,y)\vec{k} = \begin{pmatrix} (2y-1)k_2 \\ (2y-1)k_1 + 2xk_2 \end{pmatrix}$$

so $\vec{k}^T H k = (2y-1)k_1k_2 + (2y-1)k_1k_2 + 2xk_2^2$

$= 2(2y-1)k_1k_2 + 2xk_2^2$

At $(0,0)$: $f(k_1,k_2) = -2k_1k_2$ + high order

At $(0,1)$: $f(k_1, 1+k_2) = 2k_1k_2$ + high order.