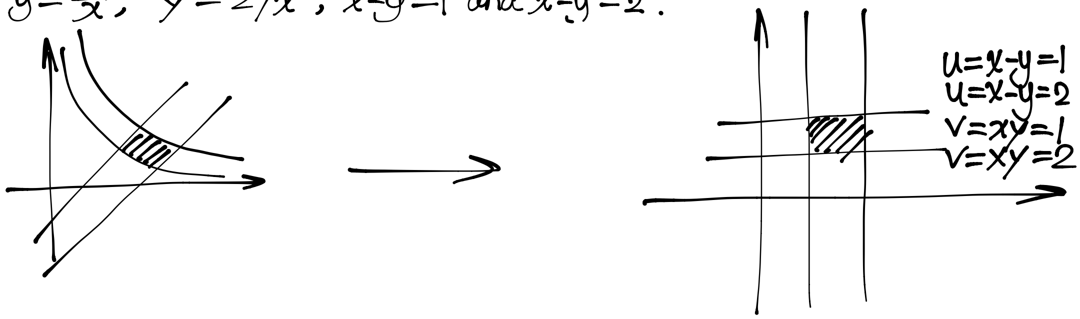
July 23rd

Some problems from past tests

(a). Consider $(u,v)=f(x,y)=(x-y,xy)$
Demonstrate the effect of this transf. of the region of x,y plane bdd by $y=\frac{1}{x}$, $y=2/x$, $x-y=1$ and $x-y=2$.

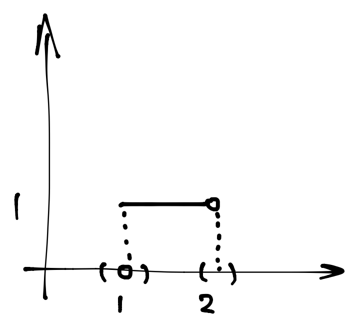


(b). investigate the possibility of finding of an inverse for this transf.

$$Df=\begin{pmatrix}1 & -1\\ y & x\end{pmatrix} \qquad \det Df = x+y$$

$$Df^{-1}=\frac{1}{x+y}\begin{pmatrix}x & 1\\ -y & 1\end{pmatrix}$$

$f(x)=[x]$ on $[0,2]$



General idea: "dig holes at the discontinuities".

Find parametric description of the intersection of the plane $x+z=1$ with $z^2=x^2+y^2$.

$$x+z-1=0$$
$$x^2+y^2-z^2=0$$
$$y=t \Rightarrow (1-x)^2=x^2+y^2 \Rightarrow -2x+1=y^2=t^2 \Rightarrow x=\frac{1-t^2}{2}$$

$$y=t$$

$$z=1-\frac{1-t^2}{2}=\frac{1+t^2}{2}$$

Determine whether $f(u,v)=(u\cos v, u\sin v, u^2)$ $-\pi\le v\le\pi$, $u\in\mathbb{R}$ satisfies the regularity condition at all points.

$$\left.\begin{array}{l} f_u = (\overset{k_1}{\cos v}, \overset{k_2}{\sin v}, \overset{k_2}{2u}) \\ f_v = (-u\sin v, u\cos v, 0) \end{array}\right\} \det \Rightarrow -2u^2\cos v\, k_1 + \cdots$$