Australian
National
University

# Research School Of Finance, Actuarial Studies and Applied Statistics

## Second Semester Final Examination 2014

## FINANCIAL MATHEMATICS

## (STAT 2032 / STAT 6046)

Study Period: 15 minutes
Writing Period: 3 hours

<u>Permitted Materials:</u>
Non-Programmable Calculators
Dictionaries (must not contain material added by the student)
Compound Interest Rate Tables (provided with this examination paper)
Formula Sheet (provided with this examination paper)

Students have also been provided with a formula sheet in addition to this exam paper.

<u>Total Marks: 100</u>

*<u>Instructions to all candidates:</u>*

- *Attempt all seven questions*
- *Start your solution to each question on a new page*
- *Unless otherwise stated, show all working*

**Question 1** **(10 marks)**

**An investor effects a contract under which he will pay 15 level premiums of $600 annually in advance into an account which accumulates at the force of interest $\delta(t)$, where t is measured in years, given by the formula:**

$$\delta(t)=\begin{cases}0.08 & 0\le t<5\\ 0.06 & 5\le t<10\\ 0.04 & 10\le t\end{cases}$$

a. **Derive an expression for the present value of $1 due at time *t*. (Note: You do not need to provide a numerical answer.)** **(3 marks)**

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The present value for $1 at time *t*.

$$v^t=\begin{cases}e^{-0.08t} & 0\le t<5\\ e^{-0.40}e^{-0.06(t-5)} & 5\le t<10\\ e^{-0.40}e^{-0.30}e^{-0.04(t-10)} & 10\le t\end{cases}$$

b. **At the end of 15 years, what is the balance of the investor's account?** **(4 marks)**

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The annual interest rate for the 15 year period is:

$$i(t)=\begin{cases}e^{0.08}=8.3287\% & 0\le t<5\\ e^{0.06}=6.1837\% & 5\le t<10\\ e^{0.04}=4.0811\% & 10\le t\end{cases}$$

Thus the accumulated value of the 15 level premiums at the end of 15 years:

$$\begin{aligned} A(15) &= 600\left[\ddot{s}_{\overline{5}|8.3\%} e^{0.3} e^{0.2} + \ddot{s}_{\overline{5}|6.2\%} e^{0.2} + \ddot{s}_{\overline{5}|4.1\%}\right] \\ &= 600\left[6.3970 \times 1.349859 \times 1.221403 + 6.0077 \times 1.221403 + 5.6465\right] \\ &= 14{,}118.70 \end{aligned}$$

**c. Suppose at the end of 15 years, the balance, calculated in part b., is withdrawn semi-annually using 8 fixed level amounts. The first withdrawal is made exactly 1 year after the last premium is deposited. Calculate the amount of the fixed semi-annual withdrawal. (3 marks)**

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If W is the amount of each withdrawal then:

$$\begin{aligned} A(15) &= 2W\ddot{a}^{(2)}_{\overline{4}|4.08\%} \\ 14118.70 &= 7.4670W \\ W &= 1{,}890.81 \end{aligned}$$

**Question 2** **(10 marks)**

**An actuarial student has taken out two loans.**

- **Loan A: A 5 year car loan for $8,991 repayable by equal monthly instalments in arrears with an interest rate of 12% pa convertible monthly.**
- **Loan B: A 10 year personal loan of $19,966 repayable by equal monthly instalments in arrears with an interest rate of 7.5% pa convertible monthly.**

**The student continues to pay off both these loans for a period of 2 years.**

**a. What is the total balance outstanding at the end of 2 years for both the loans to the nearest dollar?** **(6 marks)**

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First evaluate the level repayment for the two Bonds. Let K be the monthly instalment for Loan A and M be the monthly instalment for Loan B.

$$\begin{aligned} Ka_{\overline{60}|1\%} &= 8991 \\ 44.9550K &= 8991 \\ K &= 200 \end{aligned} \qquad \begin{aligned} Ma_{\overline{120}|0.63\%} &= 19966 \\ 84.2447M &= 19966 \\ M &= 237 \end{aligned}$$

Thus the balance outstanding for the two loans using prospective approach:

$$\begin{aligned} OB_A &= Ka_{\overline{36}|1\%} \\ &= 200 \times 30.1075 \\ &= 6021.50 \end{aligned} \qquad \begin{aligned} OB_B &= Ma_{\overline{96}|0.63\%} \\ &= 237 \times 72.02602 \\ &= 17070.17 \end{aligned}$$

Thus the total balance outstanding is $23,092.

**b.** **Now if the two loans are rolled into a single loan which is being offered at an effective rate of interest 9% pa effective. If the new monthly repayment is the sum of the two original loan repayments find out the time it takes to payback the new loan. (4 marks)**

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The monthly effective rate is $=(1.09)^{1/12}-1=0.7207\%$

The new repayment will be the sum of the original 2 repayments which is $437.

Let $n$ be the time it takes for the new loan to be repaid:

$$\begin{aligned}437a_{\overline{n}|0.72\%} &= 23092\\ a_{\overline{n}|0.72\%} &= 52.842105\\ \frac{1-1.007207^{-n}}{0.007207} &= 52.842105\\ n &= 66.76\end{aligned}$$

Thus it will take another 67 months (5 years 7 months) for the loan to be repaid.

**Question 3** **(6 marks)**

**A bond with a face value of $250,000 is issued bearing coupons payable quarterly in arrears at a rate of 8% p.a. The bond can be redeemed at 110% if redeemed on a coupon paying date between 10 and 15 years. If the bond is redeemed between 15 and 20 years then the bond can be redeemed at par. The date of redemption is at the option of the buyer of the bond.**

**An investor who is liable to income tax at 25% and capital gains tax at 30% wishes to purchase this bond. Calculate the price to ensure a net effective yield of at least 5% p.a. effective.**

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The various parameters of the bonds are:

$$\begin{aligned} F &= 250,000 \\ C_{10-15} &= 275,000 \\ C_{15-20} &= 250,000 \\ Coupon &= 5000 \end{aligned}$$

$$\begin{aligned} r &= 2\% \\ g_{10-15}\left(1-t_I\right) &= \frac{0.02}{1.1}\times 0.75 = 1.3636\% \\ g_{15-20}\left(1-t_I\right) &= r\left(1-t_I\right) = 1.50\% \end{aligned}$$

$$\begin{aligned} i &= 5\% \text{ pa} \\ j &= 1.05^{0.25}-1 = 1.2272\% \end{aligned}$$

For both redemption periods $j < g$ hence $P > C$. Hence no capital gains tax apply in this situation. Since the bond can be redeemed at the option of the buyer the date for optimal redemption date for both periods is as late as possible since there is a capital loss on this bond.

If the bond is redeemed at time $t$ = 15 years.

$$\begin{aligned} \mathrm{P} &= 20000\left(1-0.25\right)a_{\overline{15}|5\%}^{(4)} + 275000v_{5\%}^{15} \\ &= 15000\times 10.5723 + 27500\times 0.481017 \\ &= 290,864.18 \end{aligned}$$

If the bond is redeemed at time $t$ = 20 years.

$$\begin{aligned} \mathrm{P} &= 20000\left(1-0.25\right)a_{\overline{20}|5\%}^{(4)} + 250000v_{5\%}^{20} \\ &= 15000\times 12.6935 + 250000\times 0.376889 \\ &= 284,624.90 \end{aligned}$$

Thus to achieve at least 5% net yield the price to be offered for the bond must be \$284,624.90

**Question 4** **(18 marks)**

**Dream developers have bought a piece of land on the Sydney harbour side for \$5,000,000 and plan to build a luxury home on this land. The construction will be completed in 2 years' time and will incur a cost of \$7,000,000 paid continuously over the 2 years. Dream developers can borrow or invest funds at a rate of 15% p.a. effective.**

**On completion, Dream developers have 2 possible strategies:**

1. **Sell the house in 3 years' time for \$16,500,000**
2. **Sell the house in 4 years' time for \$18,000,000**

**They are also able to receive a rental income from the house between times of completion to time of sale of \$500,000 p.a. payable quarterly in advance. The rental income will increase by \$50,000 p.a. at the beginning of each year that the rental is paid.**

a. **Derive an expression, using annuity functions, for the present value of \$1 p.a. paid m times a year in advance at *i* effective rate p.a. Alternatively, show that:**

$$(I\ddot{a})_{\overline{n}|}^{(m)} = \frac{i}{d^{(m)}}(Ia)_{\overline{n}|} = \frac{d}{d^{(m)}}(I\ddot{a})_{\overline{n}|}$$ **(5 marks)**

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The annuity will pay $\frac{1}{m}$ every m^th^ period in the 1^st^ year, and $\frac{2}{m}$ every m^th^ period in the 2^nd^ year and so on. Hence the present value is:

$$(I\ddot{a})_{\overline{n}|}^{(m)} = \frac{1}{m}\left[v^0 + v^{\frac{1}{m}} + \cdots + v^{\frac{m-1}{m}}\right] + \frac{2}{m}\left[v^1 + \cdots + v^{1\frac{m-1}{m}}\right] + \cdots + \frac{n}{m}\left[v^{n-1} + \cdots + v^{n-1\frac{m-1}{m}}\right]$$

$$m(I\ddot{a})_{\overline{n}|}^{(m)} = \left[v^0 + v^{\frac{1}{m}} + \cdots + v^{\frac{m-1}{m}}\right] + 2\left[v^1 + \cdots + v^{1\frac{m-1}{m}}\right] + \cdots + n\left[v^{n-1} + \cdots + v^{n-1\frac{m-1}{m}}\right] \quad \ldots(1)$$

Now multiplying both sides by $v^{\frac{1}{m}}$ we get:

$$v^{\frac{1}{m}} m\left(I\ddot{a}\right)_{\overline{n}|}^{(m)} = \left[v^{\frac{1}{m}} + \cdots + v^{\frac{m}{m}}\right] + 2\left[v^{1\frac{1}{m}} + \cdots + v^{2}\right] + \cdots + n\left[v^{n-1\frac{1}{m}} + \cdots + v^{n}\right] \quad \ldots(2)$$

Subtracting (2) from (1) we get:

$$\begin{aligned}
m\left(I\ddot{a}\right)_{\overline{n}|}^{(m)} - v^{\frac{1}{m}} m\left(I\ddot{a}\right)_{\overline{n}|}^{(m)} &= v^{0} + v^{1} + v^{2} + \cdots + v^{n-1} - nv^{n} \\
m\left(I\ddot{a}\right)_{\overline{n}|}^{(m)}\left[1 - v^{\frac{1}{m}}\right] &= \ddot{a}_{\overline{n}|} - nv^{n} \\
\left(I\ddot{a}\right)_{\overline{n}|}^{(m)} &= \frac{\ddot{a}_{\overline{n}|} - nv^{n}}{m\left[1 - v^{\frac{1}{m}}\right]} \\
&= \frac{i \times \left(Ia\right)_{\overline{n}|}}{d^{(m)}} = \frac{d \times \left(I\ddot{a}\right)_{\overline{n}|}}{d^{(m)}}
\end{aligned}$$

**b. Determine the optimum strategy if this is based upon using NPV as the decision criteria. (8 marks)**

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Let's first evaluate the present value of the costs first:

$$PV(Costs) = 5000 + 3500\bar{a}_{\overline{2}|} = 5000 + 3500 \times 1.744789 = 11{,}106.792$$

Present Value of the benefits depend on which strategy is chosen:

$$\begin{aligned}
PV\left(Strategy1\right) &= 500\ddot{a}_{\overline{1}|}^{(4)} v^{2} + 16500v^{3} \\
&= 500 \times 0.949663 \times 0.756144 + 16500 \times 0.657516 \\
&= 359.041 + 10849.018 \\
&= 11208.059
\end{aligned}$$

$$PV\left(Strategy2\right)=\left[450\ddot{a}_{\overline{2}|}^{(4)}+50\left(I\ddot{a}\right)_{\overline{2}|}^{(4)}\right]v^2+18000v^4$$

$$=\left[450\times1.775457+50\times2.601251\right]0.756144+18000\times0.571753$$

$$=500\ddot{a}_{\overline{1}|}^{(4)}v^2+550\ddot{a}_{\overline{1}|}^{(4)}v^3+18000v^4$$
$$=500\times0.949663\times0.756144+550\times0.949663\times0.657516$$
$$+18000\times0.571753$$
$$=10994.030$$

Thus the NPV for strategy 1 is:

$$\begin{aligned}NPV_1&=PV\left(Strategy1\right)-PV\left(Costs\right)\\&=11208.059-11106.792\\&=101.266\\&=\$101,266\end{aligned}$$

Thus the NPV for strategy 2 is:

$$\begin{aligned}NPV_2&=PV\left(Strategy2\right)-PV\left(Costs\right)\\&=10994.030-11106.792\\&=-112.763\\&=-\$112,763\end{aligned}$$

Thus based on NPV we must choose strategy 1.

**c. If instead the house is sold in 6 years' time, the developer believes that they can obtain an IRR of the project of 17.5% p.a. Calculate the sale price that the developer can receive. (5 marks)**

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The *IRR* = 17.5% and let the sale price be *S*.

$$PV\left(Costs\right)=5000+3500\bar{a}_{\overline{2}|17.5\%}=5000+3500\times1.744789=10,983.303$$

$$\begin{aligned}PV\left(Benefits\right)&=\left[450\ddot{a}_{\overline{4}|17.5\%}^{(4)}+50\left(I\ddot{a}\right)_{\overline{4}|17.5\%}^{(4)}\right]v_{17.5\%}^2+Sv_{17.5\%}^6\\&=\left[450\times3.007555+50\times6.617032\right]\times0.724310+0.379991S\\&=1230.784+0.379991S\end{aligned}$$

S will be the price where the $NPV = 0$.

$$\begin{aligned}
PV\left(Costs\right) &= PV\left(Benefits\right) \\
10,983.303 &= 1230.784 + 0.379991S \\
0.379991S &= 9752.519 \\
S &= 25,665.161 \\
&= \$25,665,161
\end{aligned}$$

**Question 5** **(17 marks)**

**A self-managed superannuation fund has two pay-outs due in 2 and 5 years' time of $2.5 million each.**

a. **Calculate the present value of these liabilities at an interest rate of 4% p.a. effective.** **(3 marks)**

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The present value of the liabilities are:

$$\begin{aligned} PV_L &= 2.5\left[v^2 + v^5\right] \\ &= 2.5\left[0.924556 + 0.821927\right] \\ &= 4{,}366{,}208 \end{aligned}$$

b. **Calculate the duration of the liabilities at the same interest rate of 4% p.a. effective.** **(3 marks)**

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The duration of the liabilities are:

$$\begin{aligned} \tau_L &= \frac{2.5\left[2v^2 + 5v^5\right]}{2.5\left[v^2 + v^5\right]} \\ &= \frac{\left[0.924556 \times 2 + 0.821927 \times 5\right]}{\left[0.924556 + 0.821927\right]} \\ &= 3.4119 \end{aligned}$$

**The fund has invested in a 6 year zero coupon bond a 3 year 8% p.a. annual coupon paying bond which can be redeemed at par. The nominal value of the two bonds purchases are set such that the fund is immunised for small changes in interest rates.**

**c. Find the nominal values of the two bonds. (8 marks)**

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Let A be the face value of the zero coupon bond and B be the face value of the coupon paying bond.

The present value of the assets is given as:

$$\begin{aligned} PV_A &= Av^6 + 0.08Ba_{\overline{3}|} + Bv^3 \\ &= 0.790315A + 2.7751 \times 0.08B + 0.888996B \\ &= 0.790315A + 1.111004B \end{aligned}$$

The duration of the assets is given as:

$$\begin{aligned} \tau_A &= \frac{0.08B(Ia)_{\overline{3}|} + 3v^3B + 6v^6A}{PV_A} \\ &= \frac{3.105200B + 4.741887A}{0.790315A + 1.111004B} \end{aligned}$$

Using the first two conditions of immunisation we can find values of A and B

$$\begin{aligned} PV_A = 0.790315A + 1.111004B &= PV_L \\ 0.790315A + 1.111004B &= 4366208 \end{aligned}$$

$$\begin{aligned} \tau_A &= \frac{3.105200B + 4.741887A}{4366208} = \tau_L \\ 3.4119 &= \frac{3.105200B + 4.741887A}{4366208} \\ 3.105200B + 4.741887A &= 14897065 \end{aligned}$$

Thus the value of the two bonds are $A = \$1,063,458$ and $B = \$3,173,473$

**d. Approximate the change in present values of assets and liabilities when interest rate changes to 3.5% using the duration calculated in part (b).**
**(3 marks)**

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The interest rate falls by 0.5%.

$$\Delta P = -\varepsilon \frac{\tau}{1.04} = 0.005 \frac{3.4119}{1.04} = 0.016403 = 1.6403\%$$

Hence the change in present values of both assets and liabilities are:

$$P = 4366208 \times (1 + 0.016403) = \$4,437,827$$

**Question 6** **(5 marks)**

**An investor entered into a short forward contract for a security 8 years ago, and the contract is due to mature in 4 years' time. Eight years ago the price of the security was $94.50 and the current price of the security is $143 for every $100 nominal of the security. The risk free rate of interest can be assumed to be 5% p.a. effective throughout the contract.**

**Calculate the value of the contract if it were known from the outset that the security will pay coupons of $9 in 2 years and $10 in 3 years from now.**

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The different parameters of the forward contract and the underlying security are given below:

$$S_0 = 94.50 \qquad i = 5\%$$
$$S_r = 143 \qquad c_{10} = 9$$
$$r = 8 \qquad c_{11} = 10$$
$$T = 12$$

Value of a short forward contract is given as:

$$V = (K_0 - K_r)(1+i)^{-(T-r)} = K_0(1+i)^{-(T-r)} - (S_r - PV_r)$$

$$\begin{aligned} K_0 &= (S_0 - PV_0)(1+i)^T \\ &= \left(94.50 - 9v^{10} - 10v^{11}\right)1.05^{12} \\ &= (94.50 - 5.5252 - 5.8468)\times 1.7959 \\ &= 149.29 \end{aligned}$$

$$\begin{aligned} K_r &= (S_r - PV_r)(1+i)^{T-r} \\ &= \left(143 - 9v^2 - 10v^3\right)1.05^4 \\ &= (143 - 8.1633 - 8.6384)\times 1.2155 = 126.20 \times 1.2155 \\ &= 153.39 \end{aligned}$$

Value of the short forward contract:

$$V = (149.29 - 153.39)(1.05)^{-4} = 149.29 \times (1.05)^{-4} - 126.20 = -\$3.38$$

**Question 7** **(10 marks)**

**The annual returns, *i*, on a fund is independent and identically distributed. Each year, the distribution of $1+i$ is log-normally distributed with parameters μ = 0.05 and σ² = 0.004.**

**a. Calculate the expected accumulation in 25 years' time if \$300 is invested at the end of each year.** **(4 marks)**

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From the information in the question: $(1+i) \sim LN\left(\mu = 0.05, \sigma^2 = 0.004\right)$

Thus $E(1+i) = e^{\mu+0.5\sigma^2} = e^{0.05+0.5(0.004)} = 1.0533757 \Rightarrow E(i) = 0.0533757$

Since *i* is an iid random variable the expected accumulated value of \$300 invested at the end of each year for 25 years is:

$$\begin{aligned} E\left[300\tilde{s}_{\overline{25|}}\right] &= 300E\left[\tilde{s}_{\overline{25|}}\right] \\ &= 300s_{\overline{25|}E(i)} \\ &= 300\frac{\left(1+E(i)\right)^{25}-1}{E(i)} \\ &= 15,002.855 \end{aligned}$$

**b. Calculate the probability that the accumulation of \$1 invested at time 0, in 25 years' time is greater than its expected value.** **(6 marks)**

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From part a, $E(i) = 0.0533757 \Rightarrow E\left[S(25)\right] = \left(1+E(i)\right)^{25} = 3.669297$

We know that $(1+i) \sim LN\left(\mu, \sigma^2\right) \Rightarrow S(25) \sim LN\left(25\mu, 25\sigma^2\right) = (1.25, 0.10)$

Hence $\ln\left[S(25)\right] \sim N\left(\mu = 1.25, \sigma^2 = 0.1\right)$

Thus the probability that $S(25) > E\left[S(25) = 3.669297\right]$ is:

$$\begin{aligned}
\Pr\left[S(25) > E\left[S(25) = 3.669293\right]\right] &= \Pr\left[\ln\left[S(25)\right] > \ln\left[3.669297\right]\right] \\
&= \Pr\left[\frac{\ln\left[S(25)\right] - 1.25}{\sqrt{0.1}} > \frac{1.30 - 1.25}{\sqrt{0.1}}\right] \\
&= \Pr\left[Z > 0.1581\right] \\
&= 0.4364 \\
&= 43.64\%
\end{aligned}$$

**Question 8** **(14 marks)**

**Consider the following unit prices in two separate accumulation funds over the period 1 Jan 2013 to 1 Jan 2014.**

| Fund | Unit Prices | | | | |
|---|---|---|---|---|---|
| | 2013 | | | | 2014 |
| | 1-Jan | 1-Apr | 1-Jul | 1-Oct | 1-Jan |
| **Property** | 1.24 | 1.31 | 1.48 | 1.58 | 1.64 |
| **Equity** | 1.21 | 0.92 | 1.03 | 1.31 | 1.55 |

a. **Find the time-weighted rate of return for each fund for the year 2013.** **(4 marks)**

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The time-weighted rate of return for the Property fund is $i_p = \frac{1.64}{1.24} - 1 = 32.2581\%$

The time-weighted rate of return for the Equity fund is $i_e = \frac{1.55}{1.21} - 1 = 28.0992\%$

b. **Adam has bought units in the property fund on 1 January 2013, 1 April 2013, 1 July 2013 and 1 October 2013 and sold the units on 1 January 2014. Find the money-weighted return on the completed transaction if**
   i. **He bought 1000 units on each date.** **(5 marks)**

   ii. **He invested \$1000 on each date.** **(5 marks)**

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If Adam has bought 1000 units at the start of each quarter of 2013 then the NPV equation for the Property fund will be:

$$NPV(i) = -1240 - 1310(1+i)^{-0.25} - 1480(1+i)^{-0.5} - 1580(1+i)^{-0.75} + 6560(1+i)^{-1}$$

Using linear interpolation and initial interest rates of $i_1 = 29\%$ and $i_2 = 30\%$ we can solve for *i*, the money-weighted rate of return.

$$NPV(29\%) = -7.6868$$
$$NPV(30\%) = 16.5006$$

$$\begin{aligned} i &= 0.30 - \frac{16.5006 - 0}{16.5006 + 7.6868}(0.30 - 0.29) \\ &= 0.2932 \\ &= 29.32\% \end{aligned}$$

If Adam has purchases units worth $1000 at the start of each quarter of 2013 then the NPV equation for the Property fund will be:

$$NPV(i) = -1000\ddot{a}_{\overline{1}|}^{(4)} + 4720.5718(1+i)^{-1}$$

Where,

$$\begin{aligned} 4720.5718 &= \left(\frac{1000}{1.24} + \frac{1000}{1.31} + \frac{1000}{1.48} + \frac{1000}{1.58}\right) \times 1.64 \\ &= (806.45 + 763.36 + 675.68 + 632.91) \times 1.64 \\ &= 2878.40 \times 1.64 \end{aligned}$$

Using linear interpolation and initial interest rates of $i_1 = 29\%$ and $i_2 = 30\%$ we can solve for *i*, the money-weighted rate of return.

$$NPV(29\%) = -14.4359$$
$$NPV(30\%) = 3.7396$$

$$\begin{aligned} i &= 0.30 - \frac{3.7396 - 0}{3.7396 + 14.4359}(0.30 - 0.29) \\ &= 0.2932 \\ &= 29.79\% \end{aligned}$$

**Question 9** **(10 marks)**

**Assume the following treasury spot rates:**

| Years to Maturity | Spot Rate (p.a.) |
|---|---|
| 0.5 | 5.0% |
| 1.0 | 5.4% |
| 1.5 | 5.8% |
| 2.0 | 6.4% |
| 2.5 | 7.0% |
| 3.0 | 7.2% |
| 3.5 | 7.4% |
| 4.0 | 7.8% |

**Compute:**

a. **The 1 year forward rate 2.5 years from now.** **(2 marks)**

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The 1 year forward rate 2.5 years from now:

$$f_{2.5,3.5} = \frac{\left(1+s_{3.5}\right)^{3.5}}{\left(1+s_{2.5}\right)^{2.5}} - 1 = \frac{1.074^{3.5}}{1.07^{2.5}} - 1 = 8.4066\%$$

b. **The price at the current time of a 2 year \$100 bond with semi-annual coupons of 8% p.a. issued in 1.5 years' time.** **(4 marks)**

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The coupons at each half year will be \$4. Thus the current price, P, will be:

$$P = \left[\frac{4}{1.064^2} + \frac{4}{1.07^{2.5}} + \frac{4}{1.072^3} + \frac{104}{1.074^{3.5}}\right] = 91.1640 = \$91.16$$

**c. The accumulated value at the end of 4 years for half-yearly payments of $50 paid at the end of each half-year for the first 2 years. (4 marks)**

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First we find the PV at t = 0 using the spot rates:

$$\begin{aligned}PV &= 50\left[\left(1+s_{0.5}\right)^{-0.5}+\left(1+s_{1}\right)^{-1}+\left(1+s_{1.5}\right)^{-1.5}+\left(1+s_{2}\right)^{-2}\right]\\ &= 50\left[1.05^{-0.5}+1.054^{-1}+1.058^{-1.5}+1.064^{-2}\right]\\ &= 186.344543\end{aligned}$$

Thus the accumulated value at time t = 4 is:

$$AV\left(4\right)=PV\left(1+s_{4}\right)^{4}=186.344543\times 1.078^{4}=251.647$$

$$AV\left(4\right)=\$251.65$$

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## End of Examination