PROBLEM-SOLVING AND PROOFS: ASSIGNMENT 10
DUE FRIDAY, MAY 19, 4PM.

Warm-up problems. These are completely optional.

(1) Each year the grievance committee consists of three professors. How many professors must there be in the department to avoid having the same committee in a period of 11 years.

(2) How many integers less than 252 are relatively prime to 252.

Problems to be handed in. Solve four of the following five problems.

(1) Prove that every set of five points in the square of area 1 has two points separated by distance at most $\sqrt{2}/2$. Prove that this is the best possible by exhibiting five points with no pair less than $\sqrt{2}/2$ apart.

(2) A private club has 90 rooms and 100 members. Keys must be given to members such that each set of 90 members can be assigned 90 distinct rooms whose doors they can open. Each key opens one door. The management wants to minimize the total number of keys. Prove that the minimum number of keys is 990. (Hint: Consider the scheme where 90 of the members have one key, and the remaining 10 members have keys to all 90 rooms. Prove that this works, and that no scheme with fewer keys works.)

(3) Given $n, k \in \mathbb{N}$, determine the maximum size of a subset of $[n]$ that has no two numbers differing by exactly k.

(4) Given five types of coins (5 cent, 10 cent, 20 cent, 50 cent, 1 dollar), give a formula for the number of possible collections of n coins which contain no more than four coins of any one type.

(5) Consider a set of $2n$ insects, n male and n female. In each situation below, derive formulas for the number of of ways to partition them into pairs so that the i^{th} largest male is not paired with the i^{th} largest female. (Leave answers as summations)
   (a) Same sex pairs are allowed.
   (b) Each pair has one insect of each sex.