Lecture 14

```
def: pow(b,n):
    m=0
    r=1
    while m<n:
        m=m+1
        r=r*b
    return r
```

$m_{i-1} < n$

m_i

For $i \in \mathbb{N}$, let $I(i)$ be: if there are at least i iterations then $m_i = i$, $r_i = b^i$.

I: "Loop Invariant"

Proved $\forall i \in \mathbb{N}, I(i)$ by simple Induction.

Now we need TERMINATION.

Make a "variant": a decreasing sequence of natural numbers. Must be finite.

$\langle n - m_i \rangle$

need: (A) If at least i iterations then $n - m_i \in \mathbb{N}$

(B) If at least $i+1$ iterations then $n - m_{i+1} < n - m_i$

(B): Let $i \in \mathbb{N}$, at least i iterations

$n - m_{i+1} = n - (m_i + 1) = (n - m_i) - 1 < n - m_i$

(A): Let $i \in \mathbb{N}$, at least i iterations

by $I(i)$: $m_i = i \in \mathbb{N}$, so $n - m_i \in \mathbb{Z}$ by PRE for n.

$n \geq m_i$

$m_i \geq n$

$r_i = b^{m_i}$

$n\, n \geq m_i$

Case $i=0$ by PRE: $n \in \mathbb{N}$, so $n - m_i = n - 0 = n \in \mathbb{N}$

$i=1$ $i-1 \in \mathbb{N}$, having at least i iterations, meant

$m_{i-1} < n$ so $i-1 < n$ by $I(i-1)$

so $n > i-1$, so $n - i > -1$, so $n - i \geq 0$ $\because n - i \in \mathbb{Z}$ so $n - m_i \in \mathbb{N}$

so it terminates. Let t be the iteration after which it terminates

returns $r_t = b^t$ terminated when $m_t \geq n$, i.e. $m_t - n \geq 0$

but $n - m_t \in \mathbb{N}$, so $m_t - n \leq 0$. so $m_t - n = 0$ so $m_t = n$.

by $I(t)$, $m_t = t$, so $r_t = b^t = b^{m_t} = b^n$.

$n \in \mathbb{N}$

```
def c(n):
    m=n
    while m>1:
        if m%2==0:
            m=m/2
        else:
            m=3*m+1
    return m
```

unable to prove termination