August 1st

$$\frac{\partial(y,z)}{\partial(s,t)} = \frac{\partial(y,z)}{\partial(u,v)} \cdot \frac{\partial(u,v)}{\partial(s,t)}$$

It's like Chain Rule with multi variables

$y(u(s,t), v(s,t))$

$f: \mathbb{R}^m \to \mathbb{R}^n \quad g: \mathbb{R}^k \to \mathbb{R}^m$

$D(f\circ g) = D(f\circ g) D g$

$f\circ g\,(s,t) \longrightarrow (y,z)$

$f(u,v) \to (y,z)$

$g(y,z) \longrightarrow (u,v)$

$\mathrm{Det}(D(f\circ g)) = \mathrm{Det}((f\circ g)\, D(g)) = \mathrm{Det}(D(f\circ g)) \cdot \mathrm{Det}(D(g))$

$\mathrm{Det}(AB) = \mathrm{Det}A\, \mathrm{Det}B$

§5.4 Vector Derivatives.

P239 #9

Show that for any C^2 functions f & g, $\mathrm{div}(\mathrm{grad} f \times \mathrm{grad}\, g) = 0$

Remark: $\mathrm{grad} f = \nabla f = (\partial_1 f, \partial_2 f, \dots, \partial_n f)$

$\mathrm{div} \vec{F} = \nabla \cdot \vec{F} = \partial_1 F_1 + \cdots + \partial_n F_n$

$$\mathrm{curl}\, \vec{F} = \nabla \times \vec{F} = \det\begin{pmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial_1 & \partial_2 & \partial_3 \\ F_1 & F_2 & F_3 \end{pmatrix}$$

$\mathrm{grad} f = (\partial_1 f, \cdots \partial_n f)$

$\mathrm{grad}\, g = (\partial_1 g, \cdots, \partial_n g)$

$$\mathrm{grad} f \times \mathrm{grad}\, g = \det\begin{pmatrix} i & j & k \\ \partial_1 f & \partial_2 f & \partial_3 f \\ \partial_1 g & \partial_2 g & \partial_3 g \end{pmatrix} = (\partial_2 f \partial_3 g - \partial_3 f \partial_2 g) i - (\partial_1 f \partial_3 g - \partial_3 f \cdot \partial_1 g) j + (\partial_1 f \partial_3 g - \partial_3 f \partial_1 g) k$$

$$\mathrm{div}(\mathrm{grad} f \times \mathrm{grad}\, g) = \partial_1(\partial_2 f \partial_3 g - \partial_3 f \partial_2 g) - \partial_2 f(\partial_1 f \partial_3 g - \partial_3 f \partial_1 g) + \partial_3(\partial_1 f \partial_3 g - \partial_3 f \partial_1 g)$$

As $C^2 \to$ can do inter-change s.t. it's done.

$\nabla \cdot (\nabla \times \vec{F}) = 0$

$\nabla \times (\nabla \vec{F}) = 0$

Pf: $\partial_1(\partial_2 F_3 - \partial_3 F_2) + \partial_2(\partial_3 F_1 - \partial_1 F_3) + \partial_3(\partial_1 F_2 - \partial_2 F_1) = 0$ ✓

....

$\Delta f = \partial_1^2 f + \partial_2^2 f + \cdots + \partial_n^2 f = \nabla \cdot \nabla f$

Laplacian

$$\nabla(f\cdot g) = \nabla f \cdot g + f \cdot \nabla g$$

scalar multiplication

... works for divergence as well.

Δ ~~also~~ doesn't work~~s~~ for $\Delta\cancel{(fg) = \Delta f\, g + f\Delta g}$