Australian National University

Venue _____

Student Number |__|__|__|__|__|__|__|__|

Research School of Finance, Actuarial Studies & Statistics

EXAMINATION

Semester 1 - Mid-Semester, 2017

INTRODUCTORY MATHEMATICAL STATISTICS (STAT2001)

PRINCIPLES OF MATHEMATICAL STATISTICS (STAT6039)

Writing Time: 90 minutes
Reading Time: None

Exam Conditions:
Central Examination
Students must return the examination paper at the end of the examination
This examination paper is not available to the ANU Library archives

Materials Permitted In The Exam Venue:
(No electronic aids are permitted e.g. laptops, phones)
Any material except electronic devices
Calculator (non-programmable)

Materials to Be Supplied To Students:
Script book (10 pages)

Instructions to Students:

- Every student should attempt all parts of all five problems.
- The exam is out of 100 marks, with marks for each part as indicated.
- Show all working and present all final numerical results correct to four significant digits. Draw a box around each final answer.

Problem 1 ***[Total 30 marks]***

A committee of 7 persons was selected randomly from 10 men and 12 women. Then a sub-committee of 3 persons was selected randomly from the 7 committee members.

(a) Find the (single and unconditional) probability that the committee has at least two men and the sub-committee has at least one woman. ***[10 marks]***

(b) Given that the sub-committee has two men and one woman whose name is Jill, find the probability that Jill is the only woman on the committee. ***[10 marks]***

(c) Given that the committee has at least one man, find the expected number of women on the sub-committee. ***[10 marks]***

Problem 2 ***[Total 25 marks]***

A standard six-sided die will be rolled repeatedly until each of the numbers 1 to 6 has come up at least once. (For example, the numbers that come up might be 41226325.)

(a) Find the probability that the die will be rolled exactly six times. ***[5 marks]***

(b) Find the probability that the die will be rolled exactly seven times. ***[10 marks]***

(c) Calculate the expected number of times that the die will be rolled. ***[10 marks]***

Problem 3 ***[15 marks]***

Ann and Jim are about to play a game by taking turns at tossing a fair coin, starting with Jim. The first person to get heads twice in a row will win. (For example, Ann will win if the sequence *HHTTHHTH* occurs.) Find the probability that Jim will win.

Problem 4 ***[Total 10 marks]***

In *Tigeria* (a country), each person has a 20% chance of having *criplea* (a disease), independently of everyone else. The *Waskit* family in Tigeria has three persons.

(a) Suppose that at least one person in the Waskit family has criplea. Find the probability that everyone in the Waskit family has criplea. ***[5 marks]***

(b) Suppose that one person is randomly selected from the Waskit family and found to have criplea. Find the probability that everyone in the Waskit family has criplea. ***[5 marks]***

Problem 5 ***[Total 20 marks]***

Consider a discrete random variable X with probability mass function

$$f(x) = \frac{1}{2^{|x|+1}}, \quad x = \pm 1, \pm 2, \pm 3, \ldots$$

(a) Find the moment generating function of X and evaluate this function at $t = 0.1$.

[10 marks]

(b) Using results in (a), or otherwise, find the mean and variance of X. ***[10 marks]***

END OF EXAMINATION