Lecture 5

Ex: Let $F(x)=\cos x$

We have $F:[0,\pi/2]\longrightarrow[0,\pi/2]$ so $F(x)$ has a fixed pt p.

Then

$F'(p)=-\sin p \Rightarrow |F'(p)|=\sin p<1$

So p is an attracting fixed pt.

Ex: Let $F(x)=2\sin^2 x$, $F:[0,2]\longrightarrow[0,2]$

By a previous thm, $F(x)$ has at least one fixed pt.

In fact $F(x)$ has 3 fixed pts, $p_0=0$, p_1, p_2

So $F'(x)=4\sin x\cos x=2\sin(2x)$

Then $F'(0)=0 \Rightarrow p_0=0$ is an attracting fixed pt.

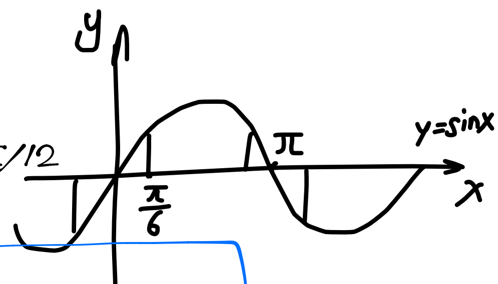
p is an attracting fixed pt iff

$|2\sin(2p)|<1 \Leftrightarrow |\sin(2p)|<\frac{1}{2}$

$\Rightarrow |2p|<\pi/6$

or $|2p-\pi|<\pi/6$

$\Rightarrow -\pi/12<p<\pi/12$ or $5\pi/12<p<7\pi/12$



We want to check

(1) $p_1<\pi/12$?

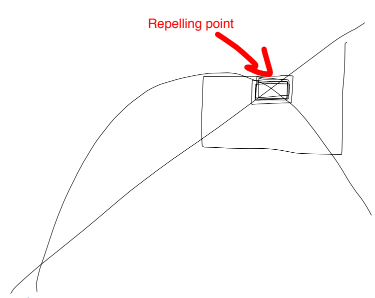
Graphically $p_1>\pi/12 \Rightarrow p_1$ is a repelling fixed pt.

(2) $5\pi/12<p_2<7\pi/12$?

$p_2>7\pi/12 \Rightarrow p_2$ is a repelling fixed pt.

If we have two repelling points, it doesn't mean that the orbit goes to infinity, like we start from me point it escapes from itself and goes to the other repelling point but it will approach the the second point but never get there.

Example: Let $F(x)=x-x^2$



So $F(x)=x \Leftrightarrow x^2=0 \Leftrightarrow x=0$ only 1 fixed pt.

and $F'(x)=1-2x \Rightarrow F'(0)=1$

$\Rightarrow 0$ is a neutral fixed pt.

$F'(x)$ >1 if $x<0$ → orbits with seeds $x_0<0$ escapes to $-\infty$.

$F'(x)$ <1 if $x>0$ → orbits with seeds $x_0>0$ (near 0) converge to 0

neutral ones: you cannot tell what's really happening. unless you study the properties of both sides. (if L attracting + R attracting, then ...)