$$\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-x^2/2}\, dx \stackrel{?}{=} 1$$

$$\int_{-\infty}^{\infty} \underbrace{e^{-x^2/2}}_{\text{even}}\, dx = \sqrt{2\pi}$$

$$2\int_{0}^{\infty} e^{-x^2/2}\, dx = \sqrt{2\pi} \qquad \frac{x}{\sqrt{2}} = z, \quad dx = \sqrt{2}\, dz$$

$$I = 2\int_{0}^{\infty} e^{-z^2}\, dz \stackrel{?}{=} \sqrt{\pi}$$

$$I^2 \stackrel{?}{=} \pi$$

$$I^2 = 4\int_{0}^{\infty} e^{-z^2}\, dz \cdot \int_{0}^{\infty} e^{-w^2}\, dw = 4\int_{0}^{\infty}\int_{0}^{\infty} e^{-(z^2+w^2)}\, dz\, dw$$

$$= \begin{bmatrix} w = zs \\ dw = z\, ds \end{bmatrix} = 4\int_{0}^{\infty}\left[\int_{0}^{\infty} e^{-z^2 - z^2 s^2}\, z\, dz\right] ds$$

$$= 4\int_{0}^{\infty}\left[\int_{0}^{\infty} z\, e^{-z^2(1+s^2)}\, dz\right] ds$$

$$= \cancel{4}\int_{0}^{\infty} -\frac{1}{\cancel{2}(1+s^2)} e^{-z^2(1+s^2)}\Big|_{0}^{\infty} ds$$

$$= 2\int_{0}^{\infty} \frac{1}{1+s^2}\, ds = 2\tan^{-1} s\Big|_{0}^{\infty}$$

$$= 2\cdot\frac{\pi}{2} = \pi \qquad \blacksquare$$