Lecture 30

Sarkovskii ordering:

3, 5, 7, 9, ... odd numbers

6, 10, 14, 18, ... $2\cdot$(odd #s)

12, 20, 28, 36, ... 2^2(odd #s)

} starting at 3

⋮

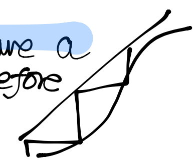
... 128, 64, 32, 16, 8, 4, 2, 1 powers of 2 (decreasing)

Applications:

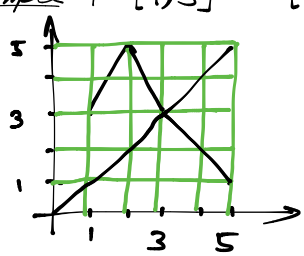
- IF F has any finitely many periodic points, then they are consecutive powers of 2.

- If F is increasing, then it can only have fixed pts

because an increasing function cannot have a 2-cycle (then it cannot have any cycle before 2.



Example: $F: [1,5] \longrightarrow [1,5]$



The orbit of $x_0 = 1$:
$\boxed{1, 3, 4, 2, 5}, 1, 3, \ldots$
5 cycle!

By the Sarkovskii Thm, F has all cycles except a 3-cycles (in front of 5)

$[1,2] \xrightarrow{F} [3,5] \xrightarrow{F} [1,4] \xrightarrow{F} [2,5]$

So $F^3([1,2]) = [2,5]$

So 2 is the only possible point of period 3 in [1,2]

but 2 has period 5.

Similarly (check!), there are no periodic points of period 3 in [2,3] and [4,5]

In [3,4], F is decreasing, F^2 is increasing and F^3 is decreasing, so F^3 can only have 1 fixed point but that fixed point of F^3 is not a periodic point of period 3, it is a fixed pt of F. Thus, F has no 3-cycles.

(Skip ch 12 & 13)

CHAPTER 14 FRACTALS

§14.1 Chaos Game

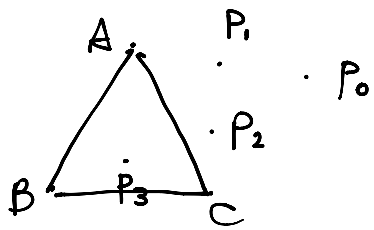
① Begin with 3 vertices of an equalateral △

② Choose a point P_0

③ Randomly 1 of A, B or C,

④ move the point P_0 half way towards that vertex

⑤ Repeat



§5.3 Sierpinski Triangle

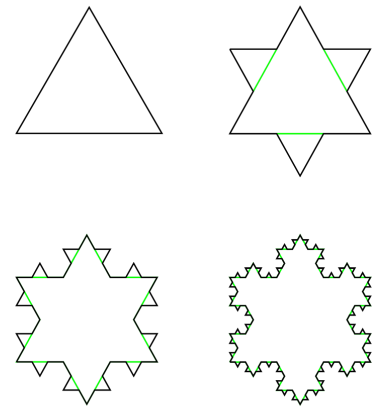
① Start with any triangle in a plane (any closed, bounded region in the plane will actually work). The canonical Sierpinski triangle uses an equilateral triangle with a base parallel to the horizontal axis (first image).

② Shrink the triangle to ½ height and ½ width, make three copies, and position the three shrunken triangles so that each triangle touches the two other triangles at a corner (image 2). Note the emergence of the central hole - because the three shrunken triangles can between them cover only 3/4 of the area of the original. (Holes are an important feature of Sierpinski's triangle.)

③ Repeat step 2 with each of the smaller triangles (image 3 and so on).

§14.4 The Koch Snowflake

The Koch snowflake can be constructed by starting with an equilateral triangle, then recursively altering each line segment as follows:



1. divide the line segment into three segments of equal length.
2. draw an equilateral triangle that has the middle segment from step 1 as its base and points outward.
3. remove the line segment that is the base of the triangle from step 2.

After one iteration of this process, the resulting shape is the outline of a hexagram.