

UNIVERSITY OF TORONTO  
Faculty of Arts and Sciences

AUGUST 2012 EXAMINATIONS

MAT244H1Y

Instructor: Fang Gu

Duration - 3 hours

No Aids Allowed

Last Name: \_\_\_\_\_

Given Name: \_\_\_\_\_

Student Number : \_\_\_\_\_

Question	Points	Score
1	16	
2	18	
3	18	
4	18	
5	12	
6	18	
Total:	100	

1. (16 points) Find the general solution of the differential equation

$$y' = \frac{y^2 + xy}{x^2}$$

and solve the initial value problem  $y(1) = 1$ .

(Hint: Make a substitution  $z = \frac{y}{x}$ )

2. (18 points) Find the general solution of the equation

$$t^2 y'' - 2ty' + 2y = 3t^2 - t, \quad t > 0.$$

3. (18 points) Find the general solution of the differential equation

$$y^{(4)} - 3y''' + 3y'' - y' = 1 + 2t + 6e^t.$$

4. (18 points) Solve the system of differential equations

$$\mathbf{x}' = \begin{pmatrix} 3 & -2 \\ 2 & -2 \end{pmatrix} \mathbf{x} + \begin{pmatrix} t \\ 3e^t \end{pmatrix}$$

5. For the system of differential equations

$$\begin{cases} x' = \frac{1}{2} \sin(x) - \sin(y), \\ y' = \frac{1}{2} \sin(y) - \sin(x). \end{cases}$$

- (a) (4 points) Linearize the system at a critical point  $(x_0, y_0)$  of your choice;
- (b) (4 points) Describe the type of the critical point  $(x_0, y_0)$  of the linearized and of the original system;
- (c) (4 points) Sketch the phase portraits of the linearized and the original system near this critical point  $(x_0, y_0)$ .

6. For the following equation:

$$x'' = -x^3 + 4x$$

- (a) (2 points) Reduce to a first order system with  $y = x'$
- (b) (4 points) Find solution in the form  $H(x, y) = C$
- (c) (6 points) Find critical points and classify them (i.e. specify whether they are nodes, saddles, etc. and stability)
- (d) (6 points) On  $(x, y)$ -plane sketch the phase portrait

