UNIT 2
SENTENTIAL LOGIC: SYMBOLIZATION

Answers to Exercises

2.4 E1 UNDERSTANDING THE MATERIAL CONDITIONAL… A LITTLE LOGIC PUZZLE: Discussion & Answers

Every card has a number on one side and a letter on the other. Suppose there is a rule:
If one side of a card has an odd number on it, then the other side has a vowel on it.
Which of the following cards must you turn over in order to test whether the rule was broken?

11	20	B	E

You need to turn over the 11 (to make sure the other side has a vowel on it) AND you must turn over the B (to make sure that the other side doesn't have an odd number on it.) We test the affirmation of the antecedent (11) and the denial of the consequent (B)

Now consider the following rule: If one side has a consonant on it, the other side has an even number on it. We turn over the same two cards to test it! (11 and B) The two rules are equivalent.

The antecedent of the second rule is the negation of the consequent of the first rule, and the consequent is the negation of the antecedent.

$P \rightarrow Q$ is logically equivalent to $\sim Q \rightarrow \sim P$.

If you have a true antecedent, you need to check if the consequent is true as well – if not, the conditional is false and the rule is broken! If you have a false consequent, check if the antecedent is false as well – if not, the conditional is false and the rule is broken.

Now consider this rule:

A person can drink alcohol only if he/she is 19 years or older.

Each of the following people is drinking a beverage. Which of the following people must you learn more about in order to determine whether the rule was broken?

Adam: age 23	Betty: age 17	Carol: drinking soda	Darren: drinking beer

In this case, we have to learn more about Betty (what she is drinking) and more about Darren (how old he is.) Again, we test the affirmation of the antecedent (those drinking alcohol) and the negation of the consequent (under 19 years old.)

This rule is logically equivalent to:

If a person is drinking alcohol then he/she must be 19 years or older.

If a person is not 19 years or older, then he/she must not drink alcohol.

2.5 E1

P: I am alert
Q: I have had coffee.
R: I bump into things.
S: I have had a good night's sleep.
T: I am in a hurry.

Using the abbreviation scheme above, symbolize the following sentences:

(a) I don't bump into things.

$\sim R$

(b) It's not the case that I don't bump into things.

$\sim\sim R$

(c) If I'm alert then I've had coffee.

$P \rightarrow Q$

(d) If I haven't had coffee, I'm not alert.

$\sim Q \rightarrow \sim P$ or $P \rightarrow Q$

(e) I bump into things if I haven't had a good night's sleep.

$\sim S \rightarrow R$

(f) It's not the case that whenever I'm in a hurry I bump into things.

$\sim(T \rightarrow R)$

(g) Assuming I'm in a hurry, I bump into things if I am not alert.

$T \rightarrow (\sim P \rightarrow R)$ or $\sim P \rightarrow (T \rightarrow R)$

NOTE: these are logically equivalent. They are both false ONLY when T is true, P is false and R is false. (As is also the case in questions (i) and (j).)

(h) Coffee is sufficient for my being alert only if I have had a good night's sleep.

$(Q \rightarrow P) \rightarrow S$

(i) If I haven't had a good night's sleep then coffee is necessary for my being alert.

$\sim S \rightarrow (P \rightarrow Q)$ or $P \rightarrow (\sim S \rightarrow Q)$

(j) Assuming I'm not in a hurry, only if I haven't had coffee do I bump into things.

$\sim T \rightarrow (R \rightarrow \sim Q)$ or $R \rightarrow (\sim T \rightarrow \sim Q)$

(k) It is not the case that whenever I bump into things I haven't had coffee.

$\sim(R \rightarrow \sim Q)$

(l) It's not the case that if I'm alert I don't bump into things if I have had coffee.

<u>It's not the case that</u> (<u>if</u> I am alert <u>then</u> (<u>if</u> I have had coffee <u>then</u> <u>it is not the case that</u> I bump into things.))

$\sim ((P \rightarrow (Q \rightarrow \sim R))$

OR

<u>It's not the case that</u> ((<u>if</u> I have had coffee <u>then</u> (<u>if</u> I am alert <u>then</u> <u>it is not the case that</u> I bump into things.)

$\sim((Q \rightarrow (P \rightarrow \sim R)$

These first two are logically equivalent

OR you might symbolize it this way, although without a comma after 'things' it is not the natural reading of the sentence.

If I have had coffee then <u>it's not the case that</u> (<u>if</u> I am alert <u>then</u> <u>it is not the case that</u> I bump into things.))

$Q \rightarrow \sim(P \rightarrow \sim R)$

This is not logically equivalent to the first two. The English sentence is ambiguous!

(m) Provided that I have had a good night's sleep, it's not the case that only if I have had coffee am I alert.

$S \rightarrow \sim (P \rightarrow Q)$

(n) If it is necessary that I have coffee in order to be alert then having a good night's sleep is sufficient for my not bumping into things.

$(P \rightarrow Q) \rightarrow (S \rightarrow \sim R)$

2.9 E1 Symbolize each of the following sentences using the abbreviation scheme provided:

P: I exist. S: I think U: Determinism is true.

Q: God exists. T: The bible is the word of God. V: I am free.

R: Angels exist.

(a) Determinism is true but I am free. $U \wedge V$

(b) Either determinism is true or I am free. $U \vee V$

(c) Determinism is false, however I am not free. $\sim U \wedge \sim V$

(d) Angels exist if, but only if, God does. $R \leftrightarrow Q$

(e) I exist if I think. $S \rightarrow P$

(f) I am free just in the case that determinism is false. $V \rightarrow \sim U$

(g) God exists if and only if the bible is the word of God, but it is not. $(Q \leftrightarrow T) \wedge \sim T$

(h) Either the bible is the word of God or God doesn't exist. $T \vee \sim Q$

(i) Although God exists, determinism is true. $Q \wedge U$

(j) God's existence is a necessary and sufficient condition for the existence of angels.
$Q \leftrightarrow R$

(k) Determinism is true or I am free, but not both.

$U \vee V \wedge \sim(U \wedge V)$ OR $(U \wedge \sim V) \vee (\sim U \wedge V)$ OR $U \leftrightarrow \sim V$ OR $\sim(U \leftrightarrow V)$

(l) Both angels and God exist; however, the bible is not the word of God.

$R \wedge Q \wedge \sim T$

(m) The bible is the word of God, who exists. $T \wedge Q$

(n) If determinism is true, then neither am I free nor does God exist.

$U \rightarrow \sim(V \vee Q)$ OR $U \rightarrow (\sim V \wedge \sim Q)$ OR $U \rightarrow \sim V \wedge \sim Q$

(o) Neither angels nor God exists. $\sim(R \vee Q)$ OR $\sim R \wedge \sim Q$

(p) I am not free unless determinism is false.

$\sim V \vee \sim U$ OR $V \rightarrow \sim U$ OR $U \rightarrow \sim V$

(q) Provided both that I think and if I think then I exist, I exist.

$(S \wedge (S \rightarrow P)) \rightarrow P$

(r) I am not free unless God exists, and in that case, only if determinism is false am I free.

$(\sim V \vee Q) \wedge (Q \rightarrow (V \rightarrow \sim U))$ or… $(V \rightarrow Q) \wedge (Q \wedge V \rightarrow \sim U))$

Incorrect answers include things like: $\boldsymbol{\sim V \vee (Q \rightarrow (V \rightarrow \sim U)).}$ $\boldsymbol{V \rightarrow (Q \wedge (V \rightarrow \sim U)).}$

2.10 E1

Disambiguate the following sentences by providing two logically distinct symbolizations:

(a) It is not the case that God exists if angels do.

$R \rightarrow \sim Q$ If angels exist then it is not the case that God exists.

$\sim(R \rightarrow Q)$ It is not the case that (if angels exist then God exists.)

(b) I am free if and only if determinism is false or God exists.

$V \leftrightarrow (\sim U \vee Q)$ I am free if and only if (determinism is false or God exists.)

$(V \leftrightarrow \sim U) \vee Q$ (I am free if and only if determinism is false) or God exists.

(c) If it's not the case that determinism is true only if I am free then I do not exist.

$(\sim U \rightarrow V) \rightarrow \sim P$ If (if it's not the case that determinism is true then I am free) then it's not the case that I exist.

$\sim(U \rightarrow V) \rightarrow \sim P$ If (it's not the case that if determinism is true then I am free) then it's not the case that I exist.

2.10 E2

P: Professor Plum teaches philosophy.

Q: Doctor Quimby teaches philosophy.

R: Professor Rosenblum teaches philosophy.

S: Professor Plum is boring.

T: Doctor Quimby is an easy grader.

Translate the following symbolic sentences into idiomatic English sentences. (Writing idiomatic English sentences means writing sentences that sound natural in English – sentences people might actually use.)

(a) $\sim (P \vee Q)$

Neither Prof. Plum nor Dr. Quimby teach philosophy.

(b) $P \wedge Q \wedge \sim R$

Prof. Plum and Dr. Quimby teach philosophy, but Prof. Rosenblum doesn't.

(c) $(P \rightarrow \sim S) \wedge (Q \wedge T)$

If Prof. Plum teaches philosophy, he isn't boring; however, Dr. Quimby teaches philosophy and he's an easy grader.

(d) $P \leftrightarrow \sim Q \wedge T$

Prof. Plum teaches philosophy if and only if Dr. Quimby is an easy grader but doesn't teach philosophy.

(e) $\sim P \vee \sim Q \vee \sim R$

Plum, Quimby and Rosenblum don't all teach philosophy.

At least one of the three doesn't teach philosophy.

(f) $(P \wedge Q) \vee (Q \wedge R) \vee (R \wedge P)$

At least two of them (Plum, Quimby, Rosenblum) teach philosophy.

(g) $\sim((P \wedge Q) \vee (Q \wedge R) \vee (R \wedge P))$

At most one of them teaches philosophy.

It's not true that at least two of them teach philosophy.

2.10 E3

Symbolize the following sentences using this abbreviation scheme:

P: Paul is present.
R: Robin is present.
S: Sonia is present.
T: The meeting will take place.

U: The meeting will start on time.
W: Somebody is going to be late.
X: The vote will take place.
Y: The motion will pass.

(a) Unless both Robin and Sonia are present, the meeting will not take place.

$(R \wedge S) \vee \sim T$ or $T \rightarrow (R \wedge S)$ or $\sim(R \wedge S) \rightarrow \sim T$

(b) The meeting will take place but it won't start on time although nobody is going to be late.

$T \wedge \sim U \wedge \sim W$ (okay to include parentheses)

(c) Either Paul is present as well as Sonia, or both Robin is not present and someone is going to be late.

$(P \wedge S) \vee (\sim R \wedge W)$

(d) The vote will take place exactly on condition that the motion will pass.

$X \leftrightarrow Y$

(e) The meeting will take place only if both Robin and Sonia are present.

$T \rightarrow (R \wedge S)$

(f) The meeting will start on time unless someone is late, but the meeting won't take place if Paul is not present.

$(U \vee W) \wedge (\sim P \rightarrow \sim T)$ or $(\sim W \rightarrow U) \wedge (\sim P \rightarrow \sim T)$

(g) The meeting will start on time if and only if no one is late and Sonia is there.

$U \leftrightarrow (\sim W \wedge S)$

(h) If and only if Paul is present, will the meeting take place and the motion will pass.

$(T \wedge Y) \leftrightarrow P$

(i) If the meeting doesn't start on time, then the vote will take place only if nobody is going to be late.

$\sim U \rightarrow (X \rightarrow \sim W)$

(j) The motion will not pass unless both Robin is present and the vote takes place.

$\sim Y \vee (R \wedge X)$ or $\sim(R \wedge X) \rightarrow \sim Y$ or $(\sim R \vee \sim X) \rightarrow \sim Y$

(k) At least one of them (Paul, Robin and Sonia) is present.

$P \vee R \vee S$

(l) At least two of them (Paul, Robin and Sonia) are present.

$(P \wedge R) \vee (R \wedge S) \vee (S \wedge P)$

(m) No more than two of them (Paul, Robin and Sonia) are present.

$\sim(P \wedge R \wedge S)$ or $\sim P \vee \sim R \vee \sim S$

(n) No more than one of them (Paul, Robin and Sonia) are present.

$\sim((P \wedge R) \vee (R \wedge S) \vee (S \wedge P))$ or $\sim(P \wedge R) \wedge \sim(R \wedge S) \wedge \sim(S \wedge P)$

(o) None of them (Paul, Robin and Sonia) are present.

$\sim P \wedge \sim R \wedge \sim S$ or $\sim(P \vee R \vee S)$

(p) Not all of them (Paul, Robin and Sonia) are present.

$\sim(P \wedge R \wedge S)$ or $\sim P \vee \sim R \vee \sim S$

(q) Exactly one of them (Paul, Robin and Sonia) is present.

$(P \vee R \vee S) \wedge \sim((P \wedge R) \vee (R \wedge S) \vee (S \wedge P))$ or

$(P \wedge \sim R \wedge \sim S) \vee (\sim P \wedge R \wedge \sim S) \vee (\sim P \wedge \sim R \wedge S)$

(r) Exactly two of them (Paul, Robin and Sonia) are present.

$((P \wedge R) \vee (R \wedge S) \vee (S \wedge P)) \wedge \sim(P \wedge R \wedge S)$ or

$(P \wedge R \wedge \sim S) \vee (\sim P \wedge R \wedge S) \vee (P \wedge \sim R \wedge S)$

P: Paul is present.
R: Robin is present.
S: Sonia is present.
T: The meeting will take place.
U: The meeting will start on time.
W: Somebody is going to be late.
X: The vote will take place.
Y: The motion will pass.

Disambiguate the following sentences by providing two logically distinct symbolizations:

(s) Although the motion will pass, the vote will not take place unless Robin is present.

$(Y \wedge \sim X) \vee R$ (The motion will pass and it is not the case that the vote will take place) or Robin is present. or … $\sim R \rightarrow (Y \wedge \sim X)$

$Y \wedge (\sim X \vee R)$ The motion will pass and (it is not the case that the vote will take place) or Robin is present). or $Y \wedge (X \rightarrow R)$

(t) Paul is present if and only if Sonia is but Robin is not.

$P \leftrightarrow (S \wedge \sim R)$ …. in more informal notation: $P \leftrightarrow S \wedge \sim R$ Paul is present if and only if (Sonia is present and it is not the case that Robin is present).

$(P \leftrightarrow S) \wedge \sim R$ (Paul is present if and only if Sonia is present) and it is not the case that Robin is present.

(u) Only if nobody is late will the meeting start on time and the vote will take place.

If (the meeting will start on time and the vote will take place) then it is not the case that somebody is late.

$(U \wedge X) \rightarrow \sim W$

(If the meeting will start on time then it is not the case that somebody is late) and the vote will take place.

$(U \rightarrow \sim W) \wedge X$

2.11 E1 SYMBOLIZING ARGUMENTS

Symbolize the following arguments using the abbreviation scheme given:

(a)

I realized, as I lay in bed thinking, that we are not responsible for what we do. This is because either determinism or indeterminism must be true. Provided that determinism is true, we cannot do other than we do. If so, we are but puppets on strings – our actions are not free. If indeterminism is true, then human actions are random, and hence not free. If our actions are not free, it must be conceded that we are not responsible for what we do.

P: Determinism is true.

Q: Indeterminism is true.

R: We can do other than we do.

S: Our actions are free.

T: Our actions are random.

U: We are responsible for what we do.

Either determinism is true or indeterminism is true.

If determinism is true then we cannot do other than we do.

If we cannot do other than we do then our actions are not free.

If indeterminism is true, then human actions are random.

If human actions are random, then they are not free.

If our actions are not free then we are not responsible for what we do.

∴We are not responsible for what we do.

$P \vee Q$
$P \rightarrow {\sim}R$
${\sim}R \rightarrow {\sim}S$
$Q \rightarrow T$
$T \rightarrow {\sim}S$
${\sim}S \rightarrow {\sim}U$

$\therefore {\sim}U$

(b)

In our world, there are conscious experiences. Yet, there is a logically possible world physically identical to ours, and in that world there are no conscious experiences. If there are conscious experiences in our world, but not in a physically identical world, then facts about consciousness are further facts about our world, over and above the physical facts. If this is so, not all facts are physical facts. It follows, then, that materialism is false. For, in virtue of the meaning of materialism, materialism is true only if all facts are physical facts.

(Based on David Chalmers, *The Conscious Mind: In Search of a Fundamental Theory*. Oxford: Oxford University Press, 1996. 123-129.)

P: There are conscious experiences in our world.

Q: There is a logically possible world that is physically identical to ours in which there are no conscious experiences.

R: Facts about consciousness are not physical facts.

S: All facts are physical facts.

T: Materialism is true.

There are conscious experiences in our world.

There is a logically possible world that is physically identical to ours in which there are no conscious experiences.

If there are conscious experiences in our world and there is a logically possible world that is physically identical to ours in which there are no conscious experiences, then facts about consciousness are not physical facts.

If facts about consciousness are not physical facts then not all facts are physical facts.

If materialism is true then all facts are physical facts.

$\therefore$ Materialism is false. (or it is not the case that materialism is true.)

P
Q
$P \wedge Q \rightarrow R$
$R \rightarrow \sim S$
$T \rightarrow S$

$\therefore \sim T$

(c)

Next we must consider what virtue is. Since things that are found in the soul are of three kinds —passions, faculties, states of character — virtue must be one of these. We are not called good or bad on the ground of our passions, but are so called on the ground of our virtues. And if we are called good or bad on the grounds of the one, but not the other, then virtues cannot be passions. Likewise, virtues are faculties only if we are called good or bad on the grounds of our faculties as we are so called on the grounds of our virtues. If we have the faculties by nature (which we do) but we are not made good or bad by nature (which we are not) then we cannot be called good or bad on the grounds of our faculties. And since this shows that the virtues are neither passions nor faculties, all that remains is that they should be states of character.

P: Virtues are passions   Q: Virtues are faculties

R: Virtues are states of character

S: We are called good or bad on the ground of our passions.

T: We are called good or bad on the ground of our faculties.

U: We are called good or bad on the ground of our virtues.

V: We have the faculties by nature.

W: We are made good or bad by nature.

Virtues are passions or virtues are faculties or virtues are states of character.
We are not called good or bad on the ground of our passions.
We are called good or bad on the grounds of our virtues.
If we are called good or bad on the ground of our virtues, but we are not called good or bad on the ground of our passions then virtues are not passions.
If virtues are faculties then we are called good or bad on the grounds of our faculties.
If we have the faculties by nature but are not made good or bad by nature then we cannot be called good or bad on the grounds of our faculties.
We have the faculties by nature.
We are not made good or bad by nature.

∴Virtues are states of character.

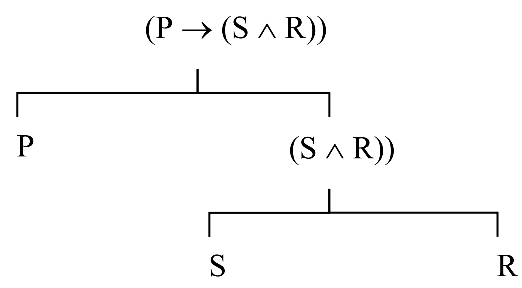
$P \vee Q \vee R$.
$\sim S$
U
$U \wedge \sim S \rightarrow \sim P$
$Q \rightarrow T$
$V \wedge \sim W \rightarrow \sim T$
V
$\sim W$

$\therefore R$

2.12 E1

a) $(P \rightarrow (S \wedge R))$

Official notation.

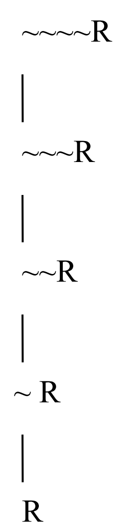


b) T & Q

Not well-formed. & is not a symbol we are using.

c) ~ ~ ~ ~ R

Official notation.

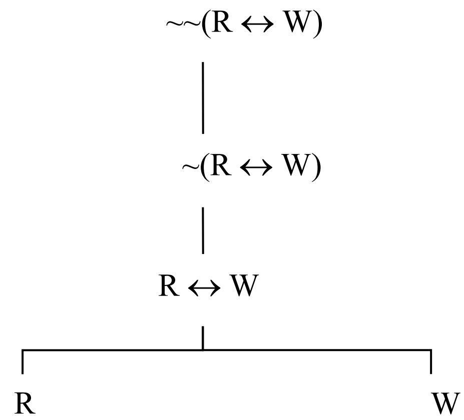


d) (~P)

Not well-formed. Don’t add parentheses when using the negation sign.

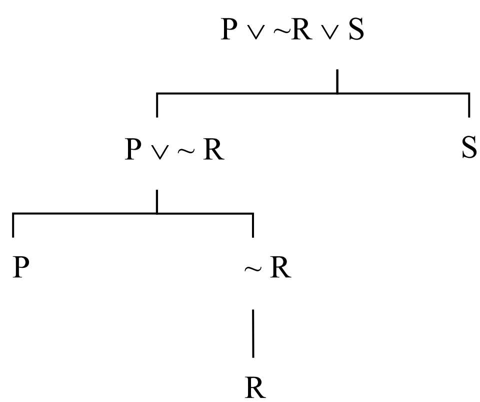
e) ~ ~ (R $\leftrightarrow$W)

Official notation.



f) P $\vee$ ~R $\vee$ S

Informal notation. Official notation: ((P $\vee$ ~R) $\vee$ S)



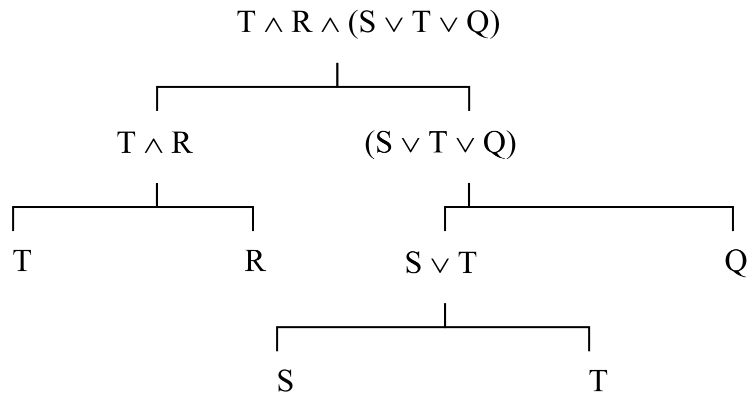
g) S $\wedge$ T $\vee$ D

Not well-formed.

A mix of conjunctions and disjunctions needs parentheses. D is not a sentence letter.

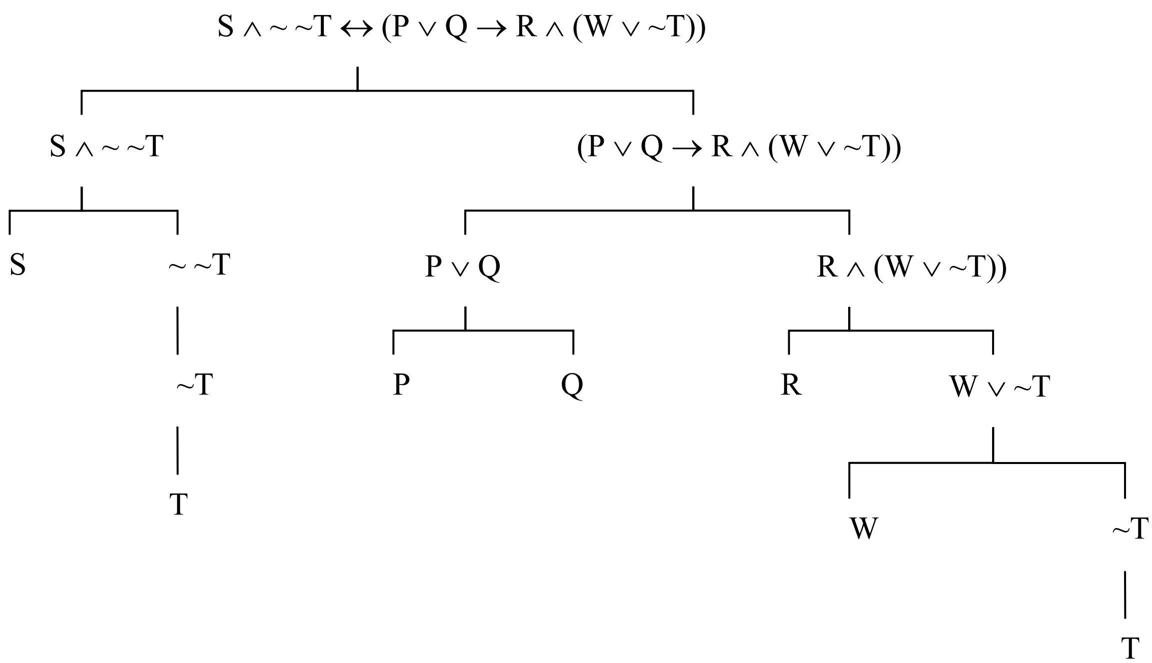
h) $T \wedge R \wedge (S \vee T \vee Q)$

Informal notation. Official notation: $((T \wedge R) \wedge ((S \vee T) \vee Q))$



i) $S \wedge {\sim}{\sim}T \leftrightarrow (P \vee Q \rightarrow R \wedge (W \vee {\sim}T))$

Informal notation. Official notation: $((S \wedge {\sim}{\sim}T) \leftrightarrow ((P \vee Q) \rightarrow (R \wedge (W \vee {\sim}T))))$



j) ${\sim}({\sim}\ ((R \wedge P) \rightarrow {\sim}\ S \wedge T \wedge (P \vee Q)))$

not well-formed. The parentheses between the two ~~ shouldn't be there.

k) ${\sim}({\sim}\ ((R \wedge P) \vee T) \leftrightarrow ((S \wedge P) \rightarrow {\sim}\ T))$

official notation. ~ main connective. $\leftrightarrow$ main connective of negated sentence.

l) $(S \wedge T \rightarrow P) \wedge ({\sim}T \leftrightarrow R \vee P) \rightarrow P \vee Q \vee R$

Informal notation. Official notation: $((((S \wedge T) \rightarrow P) \wedge ({\sim}T \leftrightarrow (R \vee P))) \rightarrow ((P \vee Q) \vee R))$