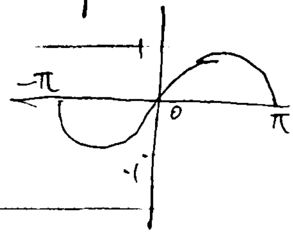
Some tutorial notes

Jan 15.

§7.2. example 2.



$f(x)=\sin x \quad [-\pi,0],[0,\pi]$

$f'(x)=\cos x$

$f'(x)=0$ when $x=-\frac{\pi}{2}$ in $[-\pi,0]$

$f'(x)=0$ when $x=\frac{\pi}{2}$ in $[0,\pi]$

Feasible ~~set~~ direction

$f'(x)\cdot b\geq 0$

$(b=)\ -1\cdots b\geq 0$ possible that π is a minimum

Similarly, at 0

~~$f(x)=b$~~ $f'(x)\cdot b=1\cdot b\geq 0$ possible that 0 is a minimum

二阶
no boundary $\quad f''(x)=-\sin x,\ f''(-\frac{\pi}{2})=1,\ f''(\frac{\pi}{2})=1$

1 dimension
∇^2 = positive

Example: $f(x)=-x^4$, $f'(x)=-4x^3$

$f''(x)=-12x^2$

2 dimension
∇^2 = positive definite

* second order condition becomes sufficient condition if $f''(x)>0$ (strictly positive)

So far, we have only dealt with "local minimum"

Practice Problem P188 skipped (Q3)

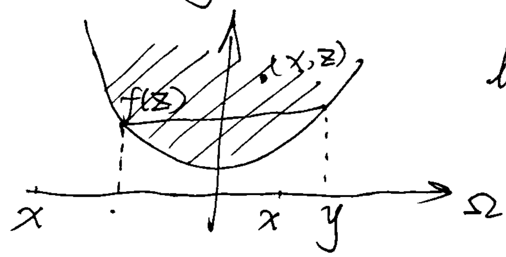
$\left[\begin{pmatrix} a & b \\ c & d \end{pmatrix} (z\ w)^T\right]$?

If f is convex, certain set is convex.

Claim: A function is convex on Ω.
$A: \Omega \to \mathbb{R}$ iff $A = \{(x, z) \in \Omega \times \mathbb{R}, z \geq f(x)\}$

$-f$ is convex: $f(\theta x + (1-\theta)y) \leq \theta f(x) + (1-\theta) f(y)$

value of function lower than the line



linear interpolation

$\nabla^2 > 0$

$X, Y \in A$

$\theta x + (1-\theta) y \in A \Rightarrow$ Suppose f is convex
$f(\theta x + (1-\theta)y) \leq \theta f(x) + (1-\theta) f(y)$

$(x, z), (y, w) \in A$

$z \geq f(x)$, $w \geq f(y)$

$\theta(x, z) + (1-\theta)(y, w)$

$(\theta x + (1-\theta)y, \theta z + (1-\theta) w)$

$\theta z + (1-\theta) w \geq \theta f(x) + (1-\theta) f(y) \geq f(\theta x + (1-\theta) y)$

$\Leftarrow$ Suppose A is convex

$z \geq f(x)$ $\quad (x, z), (y, w) \in A$ $\quad w \geq f(y)$

$(\theta x + (1-\theta) y, \theta z + (1-\theta) w)$

$\theta z + (1-\theta) w \geq f(\theta x + (1-\theta) y)$

E.g. $f(x, y)$