Week 6 Tutorial - unrolling

Use repeated substitution, AKA **unrolling or unwinding**, to find a closed form for $T(n)$ when $n = 2^k$ and $k \in \mathbb{N}$.

$T(n) = 1$ if $n = 1$

$T(n) = 1 + T(\lceil \frac{n}{2} \rceil) + T(\lfloor \frac{n}{2} \rfloor)$ if $n > 1$

Prove your closed form is correct (for the subset of natural numbers indicated) by Induction.

Solution:

unrolling:
$T(2^k) = 1 + T(2^{k-1}) + T(2^{k-1}) = 1 + 2T(2^{k-1}) = 1 + 2(1 + 2T(2^{k-2})) = 1 + 2 + 2^2 T(2^{k-2}) = 1 + 2 + 2^2 + 2^3 T(2^{k-3})$
$= \ldots 1 + 2 + 2^2 + \ldots + 2^k T(1) = 1 + 2 + 2^2 + \ldots + 2^k = 2^{k+1} - 1$

Base case: $k = 0, T(1) = 1, P(0)$ holds.

Inductive steps:
(IH) assume for $k = m, P(m)$ holds, and we want to show it holds for $P(m+1)$.

$T(2^m) = 2^{m+1} - 1$
then $T(2^{m+1}) = 1 + T(2^m) + T(2^m) = 1 + 2^{m+1} - 1 + 2^{m+1} - 1 = 2 \cdot 2^{m+1} - 1 = 2^{m+2} - 1$

hence $P(m+1)$ holds.

Therefore, conclusion…

■