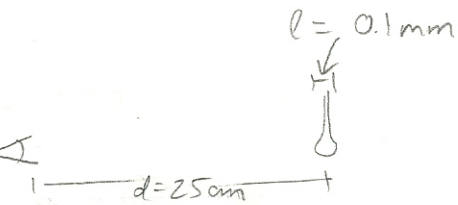
Assignment 5

1. $\theta_\ell = \ell/d$ (use m)

$\theta_\ell = 10^{-4}\,m / 0.25\,m = 4\cdot10^{-4}$ rad

$= 82.5''$



$\theta_\ell(\text{Keck})/\theta_\ell(\text{eye}) = D_{\text{eye}}/D(\text{Keck}) = 5\cdot10^{-3}\,m/10\,m$

$\theta_\ell(\text{Keck}) = 4.1\cdot10^{-2}''$; can see 10^{-4} m from 500 m

2. $V^2/r = GM_{BH}/r^2$

$r = GM_{BH}/V^2$ $V = c$

$r = 6.67\cdot10^{-11}\,N m^2/kg^2 \cdot 19891\cdot10^{30}\,kg / (3\cdot10^8\,m/s)^2$

$= 1.47\cdot10^3$ m $\rightarrow$ 1.47 km

Note that slightly more realistically $v_{escape} = c = \sqrt{2GM/r_{BH}}$

$r_{BH} = 2GM/c^2$

3. $p^2 = 4\pi^2 a^3 / G(M_J + M_{\text{Ganymede}})$ ← tiny

$M_J = \dfrac{4\pi^2\cdot(1.07\cdot10^9\,m)^3}{6.67\cdot10^{-11}\,N m^2/kg^2\cdot(6.134\cdot10^5\,s)^2} = 1.93\cdot10^{27}$ kg

$\sim 0.1\%$ of Sun's mass

4. a) $1'' = 4.85\cdot10^{-6}$ radians $= \theta$

$\theta = \ell/d$

$\ell = \theta d = 4.85\cdot10^{-6}\cdot 20.89 = 1.01\cdot10^{-4}$ Mpc

$= 0.101$ kpc

b) 12 kpc $= 3.72\cdot10^{20}$ m

$\frac{V^2}{r} = \frac{GM_{enc}}{r^2} \rightarrow M_{enc} = \frac{V^2 r}{G}$

$= \dfrac{(3.05\cdot10^5\,m/s)^2\cdot 3.72\cdot10^{20}\,m}{6.67\cdot10^{-11}\,N\cdot m^2/kg^2}$

$= 5.19\cdot10^{41}$ kg $= 2.61\cdot10^{11}\,M_\odot$

c). $2.7 \cdot 10^{10} L_\odot \cdot 2.3 \, M_\odot/L_\odot = 6.21 \cdot 10^{10} M_\odot$

d). Gas is negligible.

$$f_{dm} = \frac{M_{total} - M_{stars}}{M_{total}} = 1 - \frac{M_{stars}}{M_{total}}$$

$$= 1 - 6.21 \cdot 10^{10} M_\odot / 2.61 \cdot 10^{11} M_\odot$$

$$= 0.76$$