Code: 2782

UNIVERSITY OF TORONTO
Faculty of Arts and Science
APRIL 2010 EXAMINATIONS
MAT135Y1Y
Calculus I
Duration — 3 hours

PLEASE HAND IN

NAME
(*Please PRINT full name, and UNDERLINE surname*): _____

STUDENT NO: _____

SIGNATURE OF STUDENT
(in INK or BALL-POINT PEN): _____

This exam has two Parts:
PART A: 7 questions (50 marks).
PART B: 20 multiple choice questions (50 marks).

Indicate your answer to each multiple-choice question in PART B by completely filling in the appropriate circle in the ANSWER BOX on this front page. (Use a dark pencil!)

NOTE:

1. Before you start, check that this test has 21 pages.
2. No aids allowed.
   NO CALCULATORS!
3. **DO NOT TEAR OUT ANY PAGES!**
4. **COMPUTER CARDS AND ANSWER BOOKS WILL NOT BE USED.**
   NO SCRAP PAPER.

FOR MARKERS ONLY	
A1	/6
A2	/7
A3	/7
A4	/7
A5	/7
A6	/8
A7	/8
PART B	/50
TOTAL	/100

ANSWER BOX FOR PART B					
1.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
2.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
3.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
4.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
5.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
6.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
7.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
8.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
9.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
10.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
11.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
12.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
13.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
14.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
15.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
16.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
17.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
18.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
19.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
20.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ

PART A *[50 marks]*

Answer all questions in PART A in spaces provided. Show all your work for PART A. Any answer in PART A without proper justification may receive little or no credit. Use the back of each page for rough work. Marks for each question in PART A are indicated by *[ ]*.

DO NOT TEAR OUT ANY PAGES.

1. Find $\int xe^{5x}\,dx$.

[6]

2. Find $\int \cot^{38} x \csc^4 x\,dx$.

[7]

3. Find $\int \frac{1}{(4-x^2)^{3/2}}\,dx$.

[7]

4. Let R be the region bounded by the curves $y = 2x + 1$ and $y = x^2 + 1$. Find the volume of the solid generated by revolving R about the y-axis.

[7]

5. Find the solution of the differential equation $\frac{dy}{dx} = (9x^2 - 4x)(1 + y^2)$ that satisfies the condition $y(1) = 0$. Express the solution explicitly as a function of x.

[7]

6. Find the interval of convergence of the series $\sum_{n=1}^{\infty} \frac{(2x-1)^n}{\sqrt{n}\, 3^n}$. Remember to fully justify your answer.

[8]

7. **NOTE: This is a hard question and will be marked very strictly. Very little or no credit will be given unless your solution is completely correct.**

   Find $\displaystyle\int \frac{\cos x + x \sin x}{x(x + \cos x)}\, dx$.

[8]

PART B *[50 marks]*

PLEASE READ CAREFULLY: Each of the following multiple-choice questions has exactly one correct answer. **Indicate your answer to each question by completely filling in the appropriate circle in the ANSWER BOX on the front page. Use a dark pencil.**

MARKING SCHEME: $2\frac{1}{2}$ marks for a correct answer,
0 for no answer, a wrong answer or giving more than one answer.

You are not required to justify your answers in PART B.

NOTE: If there is any discrepancy between the circles your darken on these inside pages and those you darken on the front page, the circles you darken on the front page will be regarded as your final answers. Note that only the circles you darken will count. For PART B, your computations and answers (other than the circles your darken) will NOT count.

WARNING: If you darken the circles on these inside pages but do not darken the circles on the front page, you will still get credit for your correct answers, but there will be a PENALTY of minus 4 marks.

YOU MUST NOT TEAR OUT ANY PAGES OF THIS EXAM.

1. $\lim_{x\to 1} \dfrac{x^3+4x^2+x-6}{x^3-7x+6} =$

   Ⓐ 5

   Ⓑ $\dfrac{1}{3}$

   Ⓒ -3

   Ⓓ 0

   Ⓔ undefined

2. The graph of $y = \dfrac{x^3+x^2-20x+12}{x^2+6x}$ has a slant (i.e., oblique) asymptote, which is the line

   Ⓐ $y = x$

   Ⓑ $y = x-5$

   Ⓒ $y = x+1$

   Ⓓ $y = x+2$

   Ⓔ $y = x+\dfrac{1}{6}$

3. The product of two positive numbers is 144. What is the smallest possible value of their sum?

   (A) 26

   (B) 18

   (C) 24

   (D) 20

   (E) 22

4. The length of a rectangle is decreasing at 2 cm / sec, while the width is increasing at 3 cm / sec. At the moment when the length is 10 cm and the width is 20 cm, the ares of the rectangle will be

   (A) increasing at 10 sq cm / sec

   (B) increasing at 8 sq cm / sec

   (C) decreasing at 10 sq cm / sec

   (D) decreasing at 8 sq cm / sec

   (E) neither increasing nor decreasing

5. The function $f(x) = x^3 + 3x^2 - 45x + 8$ has a local maximum at $x =$

   (A) -5
   (B) 2
   (C) -4
   (D) 0
   (E) 4

6. If $h(x) = \int_{3/2}^{x} (t^2 + t + 1)\sqrt{5t - 6}\, dt$, then $h'(2) =$

   (A) 14
   (B) 0
   (C) undefined
   (D) $\frac{23}{2}$
   (E) 10

7. $\int_1^2 \frac{3x+2}{x(x+1)}\,dx =$

   (A) $\frac{1}{6}$

   (B) $\ln 6$

   (C) $3\ln 2$

   (D) $2\ln 2$

   (E) $\frac{1}{3}$

8. Find the area of the region enclosed between the curves $y = x^2$ and $y = 2x + 3$.

   (A) $\frac{19}{2}$

   (B) $\frac{25}{3}$

   (C) $\frac{32}{3}$

   (D) $\frac{23}{3}$

   (E) $\frac{21}{2}$

9. Let $f(x) = \begin{cases} 2x^3 - 1 & \text{if } x < 1, \\ 3x^2 - 2x & \text{if } x \geq 1. \end{cases}$

Then $\int_0^2 f(x)\,dx =$

Ⓐ 4

Ⓑ 5

Ⓒ $\frac{11}{2}$

Ⓓ $\frac{13}{3}$

Ⓔ $\frac{7}{2}$

10. $\int_{-\infty}^{\infty} \frac{x^2}{9 + x^6}\,dx =$

Ⓐ $\frac{\pi}{6}$

Ⓑ $2 \ln 3$

Ⓒ $\frac{\pi}{9}$

Ⓓ 0

Ⓔ $\frac{1}{3}$

11. Consider the predator-prey system $\frac{dR}{dt} = 6R - 2RW, \quad \frac{dW}{dt} = -4W + 5RW.$

When the system is in equilibrium with $W \neq 0$, $R \neq 0$, then $RW =$

Ⓐ $\frac{10}{3}$

Ⓑ $\frac{7}{5}$

Ⓒ $\frac{12}{5}$

Ⓓ $\frac{9}{4}$

Ⓔ $\frac{5}{2}$

12. For any integer $n \geq 1$, $\int (\ln x)^n \, dx =$

Ⓐ $x(\ln x)^n - n \int (\ln x)^{n-1} \, dx$

Ⓑ $(\ln x)^n - n \int (\ln x)^{n-1} \, dx$

Ⓒ $(\ln x)^{n-1} - n \int (\ln x)^{n-1} \, dx$

Ⓓ $x^2(\ln x)^n - n \int (\ln x)^{n-1} \, dx$

Ⓔ $x(\ln x)^{n-1} - n \int (\ln x)^{n-1} \, dx$

13. Consider the following three series:

I. $\sum_{n=1}^{\infty} \left(\frac{1}{n^2} - \left(\frac{\pi}{e} \right)^n \right)$

II. $\sum_{n=1}^{\infty} \frac{\arctan(\sqrt{n}+1)}{\arctan(\sqrt{n}+5)}$

III. $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}\ln(n^3)}$

Decide which of the series converge

Ⓐ II only

Ⓑ II and III only

Ⓒ III only

Ⓓ none

Ⓔ I and II only

14. Consider the following two series:

I. $\sum_{n=1}^{\infty}(-1)^n \frac{1}{\sqrt{\ln(n+1)}}$

II. $\sum_{n=1}^{\infty}(-1)^n \frac{(3n+1)!}{(n!)^3}$

Which of the following statements is correct?

(A) I and II both converge conditionally.

(B) I converges conditionally and II diverges.

(C) I converges conditionally and II converges absolutely.

(D) I and II both diverge.

(E) I converges absolutely and II diverges.

15. Find the coefficient of x^4 of the Maclaurin series for $f(x) = e^{2x} \sin 3x$.

Ⓐ 8

Ⓑ $-\frac{21}{2}$

Ⓒ $\frac{5}{2}$

Ⓓ $\frac{13}{3}$

Ⓔ -5

16. $\lim_{x\to 0} \dfrac{\sin x - \arctan x}{x^2 \ln(1+x)} =$

Ⓐ $\dfrac{1}{3}$

Ⓑ $\dfrac{1}{4}$

Ⓒ $\dfrac{1}{2}$

Ⓓ $\dfrac{1}{6}$

Ⓔ 0

17. Find the arc length of the curve $12xy - 4y^4 = 3$ from the point $(\frac{7}{12}, 1)$ to the point $(\frac{67}{24}, 2)$.

Ⓐ $\frac{59}{24}$

Ⓑ $\frac{29}{12}$

Ⓒ $\frac{37}{14}$

Ⓓ $\frac{37}{16}$

Ⓔ $\frac{43}{18}$

18. $\int_0^{\pi/2} e^{2x} \sin x \cos x \, dx =$

Ⓐ $\frac{1}{4}(e^{\pi} + 1)$

Ⓑ $\frac{1}{8}(2e^{\pi} - 1)$

Ⓒ $\frac{1}{8}(e^{\pi} - 1)$

Ⓓ $\frac{1}{4}(e^{\pi} - 1)$

Ⓔ $\frac{1}{8}(e^{\pi} + 1)$

19. $\int_e^{e^2} \frac{1-\ln x}{(\ln x)^2}\,dx =$

Ⓐ $\frac{e}{4}(1-e)$

Ⓑ $\frac{e}{4}(2-e)$

Ⓒ $\frac{e}{2}(2-e)$

Ⓓ $\frac{e}{4}(1+e)$

Ⓔ $\frac{e}{2}(1-e)$

20. $\lim_{n\to\infty}\left(\frac{1+n}{1^2+n^2}+\frac{2+n}{2^2+n^2}+\frac{3+n}{3^2+n^2}+\frac{4+n}{4^2+n^2}+\cdots+\frac{n+n}{n^2+n^2}\right)=$

(A) $\frac{\pi}{4}+\frac{1}{2}\ln 2$

(B) $\frac{\pi}{4}+2\ln 2$

(C) $\frac{\pi}{2}+\frac{1}{2}\ln 2$

(D) $\frac{\pi}{2}+\ln 2$

(E) $\frac{\pi}{4}+\ln 2$