König's Thm: The max cardinality of a matching of G = the min cardinality of a vertex cover of its edges.

Hall's marriage Thm: G contains a matching of A iff $|N(S)| \geq |S|$ for all $S \subseteq A$.

Stable marriage Thm: For every set of preferences, G has a stable matching

Tutte's necessary / sufficient condition for an arbitrary graph to have 1-factor: A graph G has 1-factor iff $q(G-S) \leq |S|$ for all $S \subseteq V(G)$.

Petersen's Thm on 1-factor in bridgeless cubic graph:
Every bridgeless cubic graph has a 1-factor.

Tutte's characterization of 3-connected graph:
A graph G is 3-connected iff there exists a sequence $G_0, \ldots G_n$ of graphs s.t.

(i). $G_0 = K^4$, $G_n = G$

(ii). G_{i+1} has an edge e s.t. $G_i = G_{i+1} \div e$ for every $i < n$.

Moreover, the graphs in any such seq are all 3-connected.

...

(ii). ... edge xy s.t. $d(x), d(y) \geq 3$ & $G_i = G_{i+1}/xy$ for every $i < n$.

Moreover, ...

Menger's thm: $G = (V, E)$, $A, B \subseteq V$. The minimum number of vertices separating A from B in G = the max number of disjoint A-B paths.

Brook's thm: G be a connected graph. If G neither complete nor odd cycle, $\chi(G) \leq \Delta(G)$

Hajós' thm on k-constructible graphs:
G, $k \in \mathbb{N}$, $\chi(G) \geq k$ iff G has a k-constructible subgraph.

König's thm on $\chi'(G)$ of bipartite graphs.
Every bipartite graph G satisfies $\chi'(G) = \Delta(G)$

Vizing's thm: Every graph G, $\Delta(G) \leq \chi'(G) \leq \Delta(G)+1$

Turan's thm: For all integers r, n, $r>1$, every graph $G \not\supseteq K^r$ with n vertices and $ex(n, K^r)$ edges is a $T^{r-1}(n)$.

unique complete $(r-1)$-partite graphs on $n \geq r-1$ vertices whose partition sets differ in size by at most 1.

Erdos & Stone thm: For all integer $r \geq 2$, $s \geq 1$, $\varepsilon > 0$
$\exists$ n_0 s.t. every graph with $n \geq n_0$ vertices and at least $t_{r-1}(n) + \varepsilon n^2$ edges contains K^r_s as a subgraph.

Prop: Every graph of average degree at least 2^{r-2} has a K^r minor.

Hadwiger's conjecture: $\forall r > 0$, $\chi(G) \geq r \Rightarrow G \succcurlyeq K^r$

König's infinity lemma: $V_0, V_1, \cdots$ inf seq of disjoint non-empty finite sets, G be a graph on their union. Assume every vertex v in a set V_n with $n \geq 1$ has a neighbour $f(v)$ in V_{n-1}.
Then G contains a ray $v_0 v_1 \cdots$ with $v_n \in V_n$ $\forall n$.

de Bruijin & Erdös Thm: $G = (V, E)$, $k \in \mathbb{N}$. If every finite subgraph of G has chromatic # at most k, so does G.

2^{2r-3}

Ramsey's
Thm: $\forall r \in N, \exists n \in N$ s.t. every graph of order at least n contains either K^r or $\overline{K^r}$ as an induced subgraph.

$s, t > 0$. T be a tree of order t.
$R(T, K^s) = (s-1)(t-1) + 1$

CRST: $\forall \Delta > 0 \ \exists c$ s.t. $R(H) \leq c|H| \ \forall H$ with $\Delta(H) \leq \Delta$

Dirac's thm: Every graph with $n \geq 3$ vertices and minimum degree at least $\frac{n}{2}$ has a Hamiltonian cycle.

Erdös lower bound for Ramsey numbers: $\forall k \geq 3$, $R(k) > 2^{k/2}$

Erdös thm on graphs with large $\chi(G)$ & $g(G)$:
$\forall$ int k $\exists H$ with $g(H) > k$ and $\chi(H) > k$.

Seymour & Robertson's thm: The finite graphs are W-Q-O by $\preccurlyeq$.