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This section presents an alternative definition of differentiability which replaces the more common limit of ratios definition. Instead of suggesting the the ratio $\dfrac{f(a+h)-f(a)}{x-a}$ tends to some constant value as the variable h tends to zero, we say that there is some m and some function $E(h)$ (read it as error function) such that $f(a+h)=f(a)+mh+E(h)$ and that $\dfrac{E(h)}{h}$ tends to zero as h tends to zero. That is, $f(a+h)-f(a)$ is just like a line, plus some error which is seems to be of order of magnitude h^{α} for $a>1$. This means the error tends to zero much faster than h would tend to zero. This means that we are safe to assume $f(a+h)$ is more like a line: $f(a)+mh$. This is also known as the linear approximation of f near the point a. There is two very good reason for this:

- as you may readily agree in the case of several variables, and vector valued functions, the limit of ratios definition may be too difficult to work with.
- the new definition reminds us of Taylor polynomials. So we can easily generalize differentiability to higher order Taylor polynomials (in section 2.7). This proves to be very useful in optimization (see section 2.8).

The main idea in this section is to get used to an alternative notation/definition for differentiability and to integrate it into our previous understanding of differentiability from MAT137. Therefore it is important to be able to prove the old facts using the new definition.