STAT6039 week 3 tutorial 2 notes

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Q4 Basically you have 10 minutes (2min to transfer, 8min to find the donor). Define:

A : a donor with the right blood is found within 8 minutes.

A_i : the ith person is the first identified with correct blood type. $i = 1, 2, 3, 4$.

A_i's are disjoint $\rightarrow A_i \cap A_J = \emptyset$ i.e. A_i's are mutually exclusive.

$A = A_1 \cup A_2 \cup A_3 \cup A_4$.

If A_1 and A_2 are disjoint, then $P(A_1 \cup A_2) = P(A_1) + P(A_2)$.

So $P(A) = P(A_1 \cup A_2 \cup A_3 \cup A_4) = P(A_1) + P(A_2) + P(A_3) + P(A_4) = 0.4 + 0.6 \times 0.4 + 0.6^2 \times 0.4 + 0.6^3 \times 0.4 = 0.8704$.

The other way to think it is that:

$$\begin{aligned} P(A) &= 1 - P(\overline{A}) \\ &= 1 - (0.6)^4 \\ &= 1 - 0.1296 \\ &= 0.8704 \end{aligned}$$

Consider A^* as the event that if all 4 blood type test fail, just pick a random 5th person without testing and do the blood donation.

$$\begin{aligned} P(A^*) &= 1 - P(\overline{A^*}) \\ &= 1 - (0.6)^5 \\ &= 0.9222 \end{aligned}$$

And there is also a assumption underlying that there is a very large (or infinite) population, so that picking one out does not affect the probability of 40% (success rate).

Q3 pump with 3 components, will stop iff all 3 fail. each has a prob of 0.1 failure rate (independent).

A : event that the pump fails

$C_i = i$th component is functional, $i = 1, 2, 3.$

C_i's are independent of each other.

$A = C_1 \cup C_2 \cup C_3$

$P(A) = 1 - P(\overline{A}) = 1 - P(\overline{C_1})P(\overline{C_2})P(\overline{C_3}) = 0.999$

$P(\overline{C_1}|A)$

Note that

$$P(A|B) = \frac{P(AB)}{P(B)}$$

$$P(AB) = P(A|B)P(B)$$

$$\begin{aligned}
P(\overline{C_1}|A) &= 1 - P(C_1|A) \\
&= 1 - \frac{P(A|C_1)P(C_1)}{P(A)} \\
&= 1 - \frac{1 \cdot 0.9}{0.999} \\
&= 1 - \frac{100}{111} \\
&= \frac{11}{111}
\end{aligned}$$

Q2 (a)

$$P(\text{Two nondefectives}) = \frac{\binom{16}{2}\binom{4}{0}}{\binom{20}{2}} = \frac{12}{19}$$

(b)

$$P(\text{At least one nondefective}) = 1 - P(\text{No nondefectives}) = 1 - \frac{\binom{16}{0}\binom{4}{2}}{\binom{20}{2}} = \frac{92}{95}$$

(c)

$$P(\text{Two nondefectives, given at least one nondefective}) = P(A|B) = \frac{P(AB)}{P(B)} = \frac{\frac{12}{19}}{\frac{92}{95}} = \frac{15}{23}.$$