α , use Monte Carlo simulation or analytic soln.

$(2, 20)$. $p(y) = \int_0^1 \frac{\Gamma(2+20)}{\Gamma(2)\Gamma(20)} \binom{20}{y} \theta^{2-1}(1-\theta)^{20-1}$

$\theta^y (1-\theta)^{n-y} \, dy$

$$= \frac{\Gamma(2+20)}{\Gamma(2)\Gamma(20)} \binom{20}{y} \int_0^1 \theta^{2+y-1} (1-\theta)^{20+(n-y)-1} \, dy.$$

$(n=20)$.

$$= \frac{\Gamma(2+20)}{\Gamma(2)\Gamma(20)} \binom{20}{y} \frac{\Gamma(2+y)\Gamma(20+20-y)}{\Gamma(2+20+20)}$$

Unif $(0.05, 0.20)$

$$= \frac{1}{0.15} \binom{n}{y} \int_{0.05}^{0.20} \theta^y (1-\theta)^{n-y} \, d\theta.$$

$$\frac{1}{0.15} \binom{n}{y} \frac{\Gamma(y)\Gamma(n-y)}{\Gamma(n)} \left(F_{Beta(y,n-y)}(0.20) - F_{Beta(y,n-y)}(0.05)\right)$$

Exercise 3

$\theta \sim$ Beta$(2, 20)$

$$p(\tilde{y} \mid y=0) = \int_0^1 p(\tilde{y} \mid \theta)\, p(\theta \mid y=0)\, d\theta$$

$$\frac{\Gamma(a+b+n)}{\Gamma(a)\Gamma(b+n)} \quad = \int_0^1 \theta^{\tilde{y}} (1-\theta)^{1-\tilde{y}}\, \theta^{a-1} (1-\theta)^{b+n-1}\, d\theta.$$

$$= \int_0^1 \theta^{a+\tilde{y}-1} (1-\theta)^{(b+n+1-\tilde{y})-1}\, d\theta.$$

$$= \frac{\Gamma(a+b+n)}{\Gamma(a)\Gamma(b+n)} \times \frac{\Gamma(\tilde{y}+a)\Gamma(b+n+1-\tilde{y})}{\Gamma(b+n+1+a)}$$

$\theta \sim$ Unif$(0.05, 0.20)$

$$p(\tilde{y} \mid y=0) = \int_{0.05}^{0.20} \theta^{\tilde{y}} (1-\theta)^{1-\tilde{y}}\, \theta^{\overset{1+-1}{y}} (1-\theta)^{\overset{H}{n-y-1}}\, d\theta$$

$$= \int_{0.05}^{0.20} \theta^{(\tilde{y}+1)-1} (1-\theta)^{(n+1-\tilde{y}+1)-1}\, d\theta$$

$$= \frac{\Gamma(\tilde{y}+1)\Gamma(n+2-\tilde{y})}{\Gamma(n+3)} \left\{ F_{Beta(\tilde{y}+1,\, n+2-\tilde{y})}(0.20) - F_{Beta(\tilde{y}+1,\, n+2-\tilde{y})}(0.05) \right\}.$$