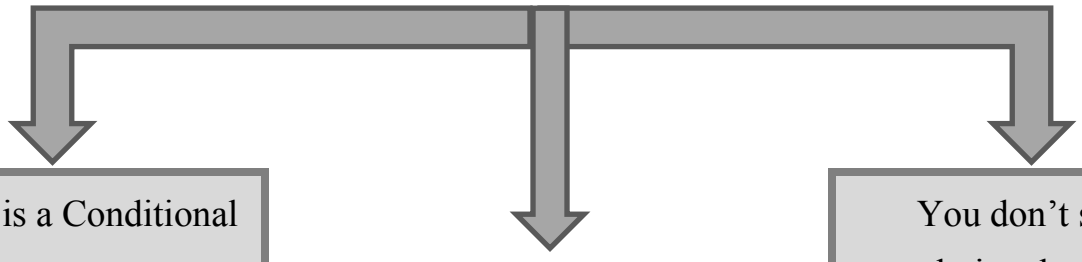
Start with your show line

Analyze your show line!



Show line is a Conditional → CD On the very next line: Antecedent of show line ASS CD Goal for CD: consequent of show line	You can see how to derive the show line just using the premises and the rules. DD Goal for DD: sentence on show line	You don't see how to derive the show line just from the premises. ID On the very next line: Opposite of show line ASS ID Goal for ID: any contradiction
Use the premises, the assumptions and any derived lines Use rules that act on molecular sentences to break things down: S, MT, MP, MTP, BC Use building rules to build sentences to match goals and sentence parts: ADJ, ADD, CB, DN		
CD box requirement: derive consequent of sentence on show line Final Justification: # of line with consequent & CD	DD box requirement: derive sentence on show line (must be exact match) Final Justification: # of line with derived sentence & DD	ID box requirement: derive any contradiction (a sentence and its negation) Final Justification: line #'s of contradictory sentences & ID

Box requirements must be below the show line.

You can use "R" to repeat a line under the show line you want to box and cancel!

SOME RULES BREAK DOWN MOLECULAR SENTENCES

Use them when you have a premise, assumption or derived sentence whose main logical operator is a binary connective: $\rightarrow$ $\wedge$ $\vee$ $\leftrightarrow$

$\rightarrow$ SENTENCES

Modus Ponens

MP

$\phi \rightarrow \psi$
ϕ
———————
ψ

Modus Tollens

MT

$\phi \rightarrow \psi$
$\sim\psi$
———————
$\sim\phi$

$\wedge$ SENTENCES

Simplification to left

SL

$\phi \wedge \psi$
———————
ϕ

Simplification to right

SR

$\phi \wedge \psi$
———————
ψ

Note: the justification 'S' is also acceptable for either SL or SR.

$\vee$ SENTENCES

Modus Tollendo Ponens

MTP

$\phi \vee \psi$
$\sim\phi$
———————
ψ

$\phi \vee \psi$
$\sim\psi$
———————
ϕ

$\leftrightarrow$ SENTENCES

Biconditional to Conditional

BC

$\phi \leftrightarrow \psi$
———————
$\phi \rightarrow \psi$

$\phi \leftrightarrow \psi$
———————
$\psi \rightarrow \phi$

Look at all your available lines (premises, assumptions and newly derived lines.) NOT line 1!

Is $\wedge$ the main connective of any line?

If so, first use SL then use SR (simplification left and right) to derive two new, shorter sentences.

Is $\rightarrow$ the main connective of any line?

If so, is any available line an exact match for the antecedent? Use MP to derive the consequent.

Is any available line the negation of the consequent (the consequent with a ~ in front of it.)? Use MT to derive the negated antecedent (the antecedent with a ~ in front of it.)

Is $\vee$ the main connective of any line?

If so, is any available line the negation of one side or the other side? (Either side with ~ in front of it?) Use MTP to derive the other side.

Is $\leftrightarrow$ the main connective of any line?

Use BC to derive a conditional (left side $\rightarrow$ right side). Then use BC again, to derive the other conditional. (right side $\rightarrow$ left side.) Now you have two new $\rightarrow$ sentences! (Use MP or MT.)

SOME RULES BUILD MOLECULAR SENTENCES

You can use these rules to build sentences that have $\vee$ $\wedge$ $\leftrightarrow$ and $\sim\sim$ as the main connective.

Mostly these rules are used to build a goal sentence
or to build a sentence or negated sentence to use with MP, MT or MTP.

Building a $\wedge$ sentence

from any two sentences…

ADJ

ϕ
ψ
———————
$\phi \wedge \psi$

Building a $\vee$ sentence

from any sentence…

ADD

ϕ ——————— $\phi \vee \psi$

ψ ——————— $\phi \vee \psi$

Building a $\leftrightarrow$ sentence

from two conditionals…

CB

$\phi \rightarrow \psi$
$\psi \rightarrow \phi$
———————
$\phi \leftrightarrow \psi$

Building a $\sim\sim$ sentence

from any sentence…

DN

ϕ
———————
$\sim\sim\phi$

Building a no $\sim\sim$ sentence

from a $\sim\sim$ sentence

DN

$\sim\sim\phi$
———————
ϕ

Look at your goal sentence.

Is it a $\wedge$ sentence? Can you use ADJ to build it from two lines? (You need *both* sides.)

Is is a $\vee$ sentence? Can you use ADD to build it from any line? (You need *one* side.)

Is it a $\leftrightarrow$ sentence? Can you use CB to build it from two lines? (You need both conditionals.)

Look at all your $\rightarrow$ sentences:

If you don't have a line that matches the antecedent, can you use ADJ, ADD, CB or DN to build the antecedent from other lines? After you build the antecedent, you can use MP!

If you don't have a line that is the negation of the consequent, can you use DN to build the negated consequent from another line? After you build the negation of the consequent, you can use MT! (Sometimes you need to build with other rules before using DN.)

Look at all your $\vee$ sentences:

If you don't have a line that is the negation of one side, can you use DN to build the negation of one side from another line? After you build the negation of one side, you can use MTP! (Sometimes you need to build with other rules before using DN.)

Sentential Derivations: Using Subderivations

Subderivations begin when you put a show line after line 1 of a derivation.

The two questions for beginning a subderivation and closely connected:

When do I begin a subderivation?

What do I put on the show line?

The answer to the first question will determine what you put on the show line! Here are some answers:

When do I begin a subderivation?	**What do I put on the show line?**
Immediately after an assumption for CD On the line below ASS CD (the antecedent) put in a show line for your box requirement (the consequent)	Show CONSEQUENT If you have just made an assumption for CD, your goal is to derive the consequent. So give it a show line!
When you are showing a biconditional $\leftrightarrow$ If your show line is a biconditional, then you need to show two conditionals. So, first show one, then show the other. After you have canceled the show lines, use CB (conditional to biconditional) to turn them into the biconditional that you need!	Show LEFT SIDE $\rightarrow$ RIGHT SIDE Show RIGHT SIDE $\rightarrow$ LEFT SIDE You are doing TWO conditional derivations (CD's) and then using CB to make a biconditional.
When you are showing a conjunction $\wedge$ If your show line is a conjunction, then you might want to show each conjunct by itself. So first show one, then show the other. After you have canceled the show lines, use ADJ to join them together into the conjunction that you need.	Show LEFT CONJUNCT Show RIGHT CONJUNCT You are doing TWO derivations – and they will probably be ID's. So after each new show line, make an assumption for ID.
When an available line has a conditional with a complex antecedent (sometimes even a simple one!) If you have an available line that's a conditional, then you can give the antecedent a show line. That allows you to show the antecedent using ID or CD. After you have canceled the show line, you can use MP!	Show ANTECEDENT After you put your show line, analyze it! Do you want to make an assumption for CD (if it is a conditional) or ID (if it isn't)?
When one available line is a negated sentence like: $\sim(\ \rightarrow\ )$ or $\sim(\ \vee\ )$ or $\sim(\ \wedge\ )$ you can show the un-negated sentence. After you have canceled the show line you can use it with the negated sentence to make a contradiction for ID.	Show ($\rightarrow$) or Show ($\vee$) or Show ($\wedge$) Show the sentence without the negation in front of it. Sometimes it will seem too easy (e.g. if you already have the consequent or one disjunct.) Don't forget to use R to move things under your show line. And follow suggestions for show $\rightarrow$ or show $\wedge$ above!
When you have an available line with a conditional or a disjunction, you can show the opposite of the consequent or one disjunct, then use MT or MTP (or use DN and then MT or MTP.)	Show OPPOSITE OF CONSEQUENT. OR Show OPPOSITE OF DISJUNCT

DERIVATIONS STEP BY STEP

Three things to remember:

AVAILABLE LINES: Make sure you know what lines you have available to use. Some lines are not available. Unavailable lines are: uncanceled show lines and boxed lines (lines inside a completed subderivation.) Available lines are: your premises, any unboxed lines that you have derived, any unboxed assumptions you have made and any canceled, but unboxed, show lines.

JUSTIFY: On every line that isn't a show line, you need a justification for that line (ASS CD, ASS ID, MP, MT, DN, MTP, ADD, ADJ, S, BC, CB or R) or a final justification (DD, CD or ID). Remember, rules and the final justifications (DD, CD and ID) need line numbers as part of the justification. The line numbers in your justifications have to be for available lines!

BOX AND CANCEL: When you have met the box requirements, and write a final justification (DD, CD or ID), you have finished a derivation or subderivation. You need to draw a box and cancel the show line. To cancel a show line, you draw a line through the word "show" so it looks like this: ~~show~~. Draw a box that starts just below the show line you just canceled and ends just below the line with the final justification. The sentence is now 'shown' and it is available to use!

Step 1: Show Conclusion! On Line 1 write "show" followed by the symbolic sentence after the ∴.

In Logic 2010, write 'show conc' in the middle box, and press return.
Now, ***go to Step 2.***

Step 2: Look at the sentence on the show line that you just wrote.

Is it a conditional? If it is, ***go to Step 4.***

Is it a biconditional? If it is, ***go to Step 7.***

Is it a conjunction? If it is, ***go to Step 8.***

If you think you can use the rules and available lines to derive it directly, ***go to Step 3***.

Otherwise, ***go to Step 6***.

Step 3: You are doing a direct derivation (DD).

Your goal sentence is the exact sentence after the word "show" on line 1.

To meet the 'box requirements' for a direct derivation, you must derive that exact sentence on some line after line 1. When you have derived it, you can box and cancel.

All the lines after the show line should be indented!

List your premises if you want. Put one on each line. (Justification: Pr1, Pr2,…)

Are there any other lines that you have available to use? (There might be if it's a subderivation.)

Use the rules with premises, assumptions and newly derived lines.
Use S, MP, MT, MTP and BC to break molecular sentences down.
Use DN, ADJ, ADD and CB to build sentences that match goals and sentence parts .

When you've derived the goal sentence you are done the DD.

Justify by writing the line number and DD. Box and cancel. Now, ***go to step 5.***

Step 4: You are doing a CD (conditional derivation). The show line that you just wrote is for a conditional sentence. (The main connective is $\rightarrow$)

a) Assume the antecedent of the show line sentence.

On the line after the show line, write the antecedent of the conditional.
The justification is ASS CD. Don't forget to indent!

In Logic 2010, write "ASS CD" in the middle box, and press return.

b) Now you want to show that the consequent follows. The consequent is your goal sentence.

There are two ways to achieve your goal. Do one of the following two things (the second choice, ii, will always work. But sometimes it is easier to choose the first, i.)

i) Look at your available lines. (Premises, unboxed lines, etc.)
List your premises if you want. Put one on each line.

Are there any other lines that you have available to use? (There might be if it's a subderivation.)

Use the rules with premises, assumptions and newly derived lines.
Use S, MP, MT, MTP and BC to break molecular sentences down.
Use DN, ADJ, ADD and CB to build sentences that match goals and sentence parts .
Use R to bring previous available sentences under your show line.

When you have derived your goal sentence (the consequent of the conditional on the show line), you are done your CD!

Justify with line number of consequent and CD. Box and cancel.
Go to Step 5.

ii) On the next line, write "show" followed by the consequent.

In Logic 2010, write "Show cons" in the middle box, and press return

Go back to Step 2.

Step 5: You have just boxed and canceled. The sentence is now shown.

Is the word "show" that you crossed out on line 1? If so ***you are finished your derivation!***

If the word "show" that you crossed out is not on line 1, the line is now available to use.

Look at the closest show line above the newly canceled one. Is it a conditional? If so, the line you just made available is probably the consequent of that show sentence. You're done that CD.

On the next line justify with line number of the show you just crossed out, and CD.

Box and cancel and ***repeat Step 5*** *(repeat this step).*

If it the closest show line above is NOT a conditional, then you are likely doing an ID and your goal is a contradiction. Does the line that you just made available give you a contradiction? (Remember you might need to use R to get an available line under your assumption for ID.)

If so, on the next line justify with the line numbers of the contradictory sentences, and ID.

Box and cancel ***repeat Step 5*** *(repeat this step).*

If your canceled show line does not immediately reveal a contradiction, keep using the rules and available lines to get a contradiction or ***go to step 9.***

Step 6: You are doing an Indirect Derivation.
You are going to give a "proof by contradiction" for the show sentence that you just wrote.
Your goal (box requirement) is any contradiction (a sentence and the negation of that sentence).

a) On line 2, make the assumption for ID.
Assume the opposite of the sentence on show line 1. Don't forget to indent!

Your assumption is the show sentence with/without a ~ in front of it.
(For example, if you have "Show ~Q", the next line should be: "Q Ass ID")
The justification is ASS ID.
On Logic 2010, write ASS ID in the middle box.

b) Now you want to show that a contradiction follows. Your goal is any contradiction.

If you derive the show sentence, it will contradict line 2 (assumption for ID) That's okay. But, sometimes the two contradicting sentences are totally different from the assumption and show line. That's okay too. Any two contradicting sentences are fine.

List your premises if you want. Put one on each line.
Use the rules with premises, assumptions and newly derived lines.
Use S, MP, MT, MTP and BC to break molecular sentences down.
Use DN, ADJ, ADD and CB to match goals and sentence parts.
Use R to bring previous available lines below your show line.

When you've derived the goal (two sentences that contradict each other), you have met the box requirements for ID.

Justify with line number of the two contradicting sentences and 'ID'.

Box and cancel. ***Go to Step 5***

c) If you can't find a contradiction, ***Go to Step 9.***

Step 7: Your show line is a bi-conditional.

A bi-conditional is just two conditionals. So, do two conditional derivations.

On the next line put a show line: show LEFT SIDE → RIGHT SIDE.
Farther down the page put a show line: show RIGHT SIDE → LEFT SIDE.
For each conditional derivation, ***go to step 4.***

When you are finished both CD's, put them together using CB (Conditional to Biconditional).
Now you have derived the goal sentence (a bi-conditional) and are done the DD.

Justify by writing the line number and DD. Box and cancel. Now, ***go to step 5***

Step 8: Your show line is a conjunction. So you are going to do two subderivations.

On the next line put a show line: show LEFT CONJUNCT
Farther down the page put a show line: show RIGHT CONJUNCT
For each derivation, ***go to step 2 (or go directly to step 6 for an ID or step 4 for a CD).***

When you are finished both subderivations, join them together using ADJ.
Now you have derived the goal sentence (a conjunction) and are done the DD.

Justify by writing the line number and DD. Box and cancel. Now, ***go to step 5***

Step 9: When you are really stuck trying to find a contradiction, sometimes you need to use a new subderivation to get your contradiction. But it can be hard to see what show line to put.

Make sure you know what your goals are! Look at ALL available lines.

Is there a line that you want to contradict? Put in a show line for the contradiction (the opposite).

Is there something that logically follows from an available line? Give it a show line.

Is there a negated conditional? Eg. $\sim(P \rightarrow Q)$

You might want to derive the unnegated conditional to contradict the negated conditional. Definitely do this if you have available, a line with the consequent of the negated conditional.

Write a show line for the unnegated conditional sentence. Show P→Q

P Ass CD

Now can you derive Q or use repetition?

When you have a negated conditional such as $\sim(P \rightarrow Q)$…

You can derive the antecedent of the negated conditional using ID:

Write a show line for the antecedent:	Show P	
Now make an assumption for ID:	~P	Ass ID
Bring down the negated conditional	~(P→Q)	R
Now show P→Q to contradict it!	Show P → Q	
Now make an assumption for CD	P	Ass CD
That contradicts ~P (ass ID above)!		

You can derive the negated consequent of the negated conditional using ID:

Write a show line for negated consequent:	Show ~Q	
Now make an assumption for ID:	Q	Ass ID
Bring down the negated conditional	~(P→Q)	R
Now show P→Q to contradict it!	Show P → Q	
Now make an assumption for CD	P	Ass CD
Now it is easy to finish since you have Q above!	Q	R

Is there a conditional sentence available with a conditional as the antecedent? Eg. $(P \rightarrow \sim S) \rightarrow Q$

Consider putting in a show line for the antecedent. show (P→ ~S)

You have a new show line, ***go back to step 2***. (After you show it, use MP with it & conditional.)

Is there a conditional with a negated conditional as the consequent? Eg. $P \rightarrow \sim(R \rightarrow T)$

Consider putting in a show line for the opposite of the consequent. Show (R→T)

(Then after you show it, you can use DN and MT to get the negation of the antecedent.)

Now you have a new show line, ***go back to step 2.***

Sometimes when you are stuck, the problem is that you made a mistake earlier that is making the problem impossible. Check that you haven't left off a negation or put brackets in the wrong place!