March 27th

Today damped & forced Oscillations

Object on a spring & resistance or damping is not zero

$(F_{net})_x = (F_{sp})_x + D_x$

D_x = damping or resistance force $= -k(x - x_e) - bv_x$

b = damping constant

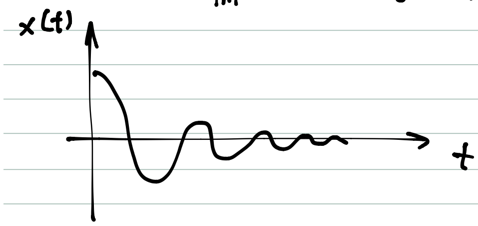
$(F_{net})_x = m a_x = -k(x - x_e) - bv_x$

Let $x_e = 0 \Rightarrow m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$

$$\frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}x = 0$$

General solution to the above equation is $x(t) = [Ae^{-bt/2m}]\cos(\omega t + \phi_0)$

$\omega = \sqrt{\omega_0^2 - \frac{b^2}{4m^2}}$ $\quad$ $\omega_0 = \sqrt{\frac{k}{m}}$



τ = "tau" = time constant $= \frac{m}{b}$

A damped oscillator, τ, when does it half its energy?

$E = \frac{1}{2}k[A(t)]^2$, $A(t) = Ae^{-bt/2m} = Ae^{-t/2\tau}$

$E = \frac{1}{2}k[A(t)]^2 = [\frac{1}{2}kA^2]e^{-t/\tau} = E_0 e^{-t/\tau}$

$E = \frac{1}{2}E_0 = E_0 e^{-t/\tau}$

$e^{-t/\tau} = \frac{1}{2}$

$-t/\tau = \ln(\frac{1}{2})$

$t = -\ln(\frac{1}{2})\tau$

$\approx 0.69\tau$