Lecture 2

Solving Certain equations

For Linear & quadratic, do as before. I.e. quad formula still works

careful: $az^2+bz+c=0$

$$z=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$\leftarrow$ $a, b, c \in \mathbb{C}$

$\leftarrow$ $\sqrt{b^2-4ac} \in \mathbb{C}$

How to take nth roots of complex numbers?

Suppose we want to find $\sqrt[n]{z}$.

I.e. we want to find all complex numbers w s.t. $w^n=z$

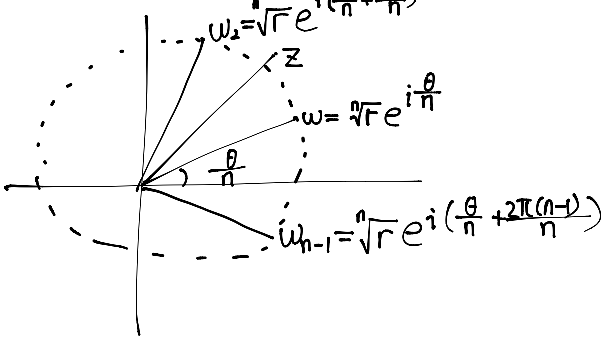
Let's write z, w in polar coordinates $z=re^{i\theta}$, $w=Re^{i\phi}$

$w^n=R^ne^{in\phi}=re^{i\theta}$

$\Rightarrow R=+\sqrt[n]{r}$

$\Rightarrow n\phi=\theta+2\pi k$

$\phi=\frac{\theta}{n}+\frac{2\pi k}{n}$, $k=0,1,2,\cdots,n-1$



see textbook for better pic.

Ex Solve $z^5=i$.

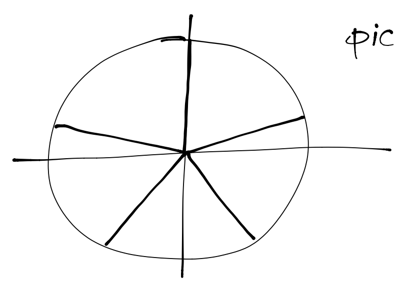
$z^5=e^{i\frac{\pi}{2}}$

$(re^{i\theta})^n=z^5=e^{i\frac{\pi}{2}}$

$R^5=1 \Rightarrow R=1$

$5\phi=\frac{\pi}{2}+2\pi k$

$\phi=\frac{\pi}{10}+\frac{2}{5}2\pi k$, $k=0,1,2,3,4$



pic like this

Lines & circles

A line is given by $Ax+By+C=0$

In complex form, this line is given by:

$$Re(az+b)=0$$

where $a=A-Bi$

b is any complex number so that $Re(b)=C$

in textbk, $y=mx+b$ $0=Re((m+i)z+b)$

Why?

$$z = x+iy$$
$$\begin{aligned} \mathrm{Re}(az+b) &= \mathrm{Re}((A-Bi)(x+iy)+b) \\ &= \mathrm{Re}((A-Bi)(x+iy)) + \mathrm{Re}(b) \\ &= \mathrm{Re}(Ax+(Ay-Bx)i+By)+\mathrm{Re}(b) \\ &= Ax+By+C \\ &= 0 \end{aligned}$$

Circles: One way to describe a circle of radius r centered at z_0 is by the eq'n:

$$|z-z_0| = r$$

The equation $|z-p| = \rho\,|z-q|$ also describes a circle

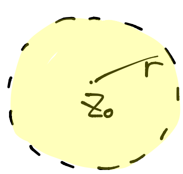
Ex: What circle is described by $|z-i| = \frac{1}{2}|z-1|$?

$$|z-i|^2 = \tfrac{1}{4}|z-1|^2$$
$$(z-i)\overline{(z-i)} = \tfrac{1}{4}(z-1)\overline{(z-1)}$$
$$(z-i)(\bar{z}+i) = \tfrac{1}{4}(z-1)(\bar{z}-1)$$
$$z\cdot\bar{z}+iz-i\bar{z}-i^2 = \tfrac{1}{4}(z\cdot\bar{z}-z-\bar{z}+1)$$
$$4|z|^2+4iz-4i\bar{z}+4 = |z|^2-z-\bar{z}+1$$
$$(z = x+iy \text{ \& simplify})$$

$$3(x^2+y^2)-8y+2x = -3$$
$$3x+2x \qquad +3y^2-8y \qquad = -3$$
$$(x+\tfrac{1}{3})^2+(y-\tfrac{4}{3})^2 = \tfrac{8}{9}$$

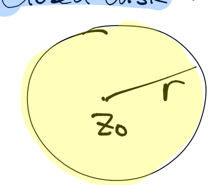
Topology of the plane

Def'n: The open disk centered at z_0 of radius r is the set $\{z\in\mathbb{C},\ |z-z_0|<r\}$



Convention: Exclude dashed pts ......
Include shaded pts ——

Closed disk: $\{z\in\mathbb{C} \mid |z-z_0|\leq r\}$



Def'n: A set S is open if for every $w\in S$, there is a small open disk centered at w which is completely contained in S.

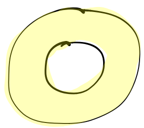
Def'n: A pt w is called a boundary pt of S if every disk centered at w contains pts inside of S & pts not in S. We denote the bdy of S by ∂S.

Def'n: A set is **closed** if it contains its boundary.

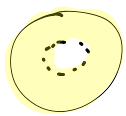
Ex:

C O O

Annulus



$\{z \mid r \leq |z - z_0| \leq R\}$



$\{z \mid r < |z - z_0| \leq R\}$

Neither

not closed

not open