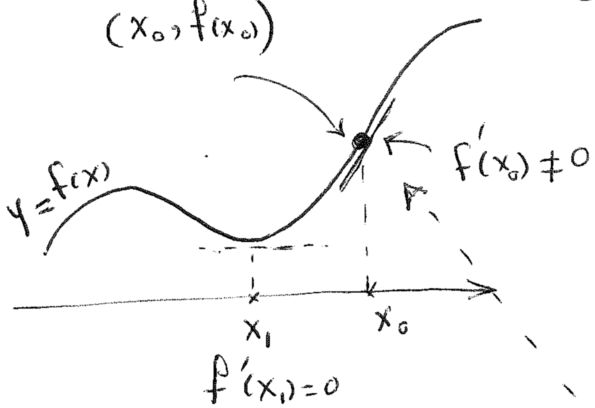
Solving $y = f(x)$ for x in terms of y

a recurring Theme in Chapter 3



Define $F(x,y) = y - f(x)$

any x and y, x and y indep of one another } not only Those x, y That satisfy $y = f(x)$

Choose $y_0 = f(x_0)$

$$F(x_0, y_0) = y_0 - f(x_0) = 0$$

one Condition of IFT

$$\partial_x F(x_0, y_0) = 0 - f'(x_0) \neq 0$$

The other Condition of IFT.

IFT implies $\exists r_1, \exists r_0 < r_1$ st $\forall y$ $|y - y_0| < r_0 \Rightarrow \exists ! x$ s.t. $|x - x_0| < r_1$ & $F(x,y) = 0$

Call it $x = g(y)$

$y = f(x)$ Can be inverted to $x = g(x)$ on a nbd of (x_0, y_0)

$y - f(x) = 0 \Rightarrow y = f(x)$ which means (x,y) is on The Curve $y = f(x)$

$$F(g(y), y) = 0 \Rightarrow y - f(g(y)) = 0 \Rightarrow y = f(g(y))$$

so g is f^{-1}. locally

Note: $\frac{d}{dy} y = \frac{d}{dy} f(g(y)) \Rightarrow 1 = f'(g(y))\, g'(y)$

so $g'(y) = \dfrac{1}{f'(g(y))}$.

See also IMT Thm 3.18