University of Toronto
Department of Mathematics

MAT224H1S
Linear Algebra II

Final Examination
August 17, 2012

Z. Amir-Khosravi

Duration: 3 hours

Last Name: ______________________

Given Name: ______________________

Student Number: ______________________

Tutorial Group: ______________________

No calculators or other aids are allowed.

FOR MARKER USE ONLY	
Question	Mark
1	/10
2	/10
3	/10
4	/10
5	/10
6	/10
TOTAL	/60

(10) 1. Define the inner product on $P_2(\mathbb{C})$ by

$$\langle p(x), q(x) \rangle = \int_0^2 p(t)\overline{q(t)}dt.$$

(a) Find an orthonormal basis for $P_2(\mathbb{C})$ by applying the Gram-Schmidt process to the standard basis $\{1, x, x^2\}$.

(b) Find the shortest distance between the vector x^2 and the subspace spanned by $\{1, x\}$.

EXTRA PAGE FOR QUESTION 1 - do not remove.

(10) 2. Consider $M_{2\times 2}(\mathbb{R})$ as a real vector space and define an inner product

$$\langle A, B\rangle = \operatorname{Tr}(B^t A).$$

Let

$$W_1 = \{A \in M_{2\times 2}(\mathbb{R}) : A^t = A\},\ W_2 = \{A \in M_{2\times 2}(\mathbb{C}) : A^t = -A\}.$$

(a) Show that $W_2 = W_1^{\perp}$.

(b) Find an orthonormal basis for W_1, and a formula for the orthogonal projection P_{W_1}.

EXTRA PAGE FOR QUESTION 2 - do not remove.

(10) 3. Let $T : \mathbb{C}^2 \to \mathbb{C}^2$ be given by

$$T(z_1, z_2) = (-2iz_1 + (1+2i)z_2, (-1+2i)z_1 + 2iz_2).$$

(a) Show T is normal.

(b) Find an orthonormal basis of eigenvectors of T for $\mathbb{C}^2$.

EXTRA PAGE FOR QUESTION 3 - do not remove.

(10) 4. Let $T : \mathbb{C}^3 \to \mathbb{C}^3$ be the linear operator whose matrix in standard coordinates is

$$A = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & -i \end{pmatrix}.$$

Find the spectral decomposition of T relative to the standard basis of $\mathbb{C}^3$.

EXTRA PAGE FOR QUESTION 4 - do not remove.

(10) 5. Let V be a finite dimensional inner product space, and $T : V \to V$ a projection operator. Prove that T is normal if and only if T is an orthogonal projection.

(10) 6. Let $T : P_3(\mathbb{C}) \to P_3(\mathbb{C})$ be the linear operator given by:

$$\begin{array}{ll} T(1) = ix + x^2 - ix^3, & T(x) = x^2, \\ T(x^2) = -x, & T(x^3) = i(1 - 2x^3) + (1+i)(x + x^2). \end{array}$$

Find a basis β for $P_3(\mathbb{C})$ such that $[T]_\beta^\beta$ is in Jordan canonical form.

EXTRA PAGE FOR QUESTION 6 - do not remove.