In-class Exercises: Projection and Minimal Basis

1. Suppose we have these FDs: : $S = \{ABE \to CF,\ \ DF \to BD,\ \ C \to DF,\ \ E \to A,\ \ AF \to B\}$

   Project the FDs onto: $L = CDEF$

Attributes to take all subsets X of:				Closure of the subset X^+	Functional dependencies inferred
C	D	E	F		
✓				$C^+ = CDFBD$	$C \to DF$
	✓			$D^+ = D$	$\emptyset$
		✓		$E^+ = EA$	$\emptyset$
			✓	$F^+ = F$	$\emptyset$
✓	✓			$CD^+ = CDFB$	$\cancel{CD \to F}$ weaker
✓		✓		$CE^+ = CEDFAB$	$\cancel{CE \to DF}$ weaker
✓			✓	$CF^+ =$	"
	✓	✓		$DE^+ = DEA$	$\emptyset$
	✓		✓	$DF^+ = DFB$	$\emptyset$
		✓	✓	$EF^+ = EFAB$	$\emptyset$
✓	✓	✓		$CDE^+ =$	weaker
✓	✓		✓		
✓		✓	✓		
	✓	✓	✓	$DEF^+ = DEFBAC$	$DEF \to C$

Final answer: The projection of S onto L is

$C \to DF,\ \ DEF \to C$

2. Find a minimal basis for this set of FDs: $S = \{ABF \to G,\ \ BC \to H,\ \ BCH \to EG,\ \ BE \to GH\}$.

1. $ABF \to G$	$ABF^+_{S-\{\#1\}} = ABF$	
2. $BC \to H$	$BC^+_{S-\{\#2\}} = BC$	
3. $BCH \to E$	$BCH^+_{S-\{\#3\}} = BCHG$	
~~4. $BCH \to G$~~	$BCH^+_{S-\{\#4\}} = BCHEG$	discard the FD
5. $BE \to G$	$BE^+_{S-\{\#4, \#5\}} = BEH$	
6. $BE \to H$	$BE^+_{S-\{\#4, \#6\}} = BEG$	

S_2:

1. $ABF \to G$	$AB^+_{S_2} = AB$ $\quad AF^+_{S_2} = AF$ $\quad BF^+_{S_2} = BF$	
2. $BC \to H$	$B^+_{S_2} = B$ $\quad C^+_{S_2} = C$	
3. $BC\cancel{H} \to E$	$BC^+_{S_2} = BCHEG$	$\therefore$ Don't need H on LHS
5. $BE \to G$	$B^+_{S_2} = B$ $\quad E^+_{S_2} = E$	
6. $BE \to H$	" $\quad$ "	Not done!!

S_3:

1. $ABF \to G$	$ABF^+_{S_3 - \{\#1\}} =$	
~~2. $BC \to H$~~	$BC^+_{S_3 - \{\#2\}} = BCHEG$	discard the FD
3'. $BC \to E$	$BC^+_{S_3 - \{\#2, \#3'\}} = BC$	
5. $BE \to G$	$BE^+_{S_3 - \{\#2, \#5\}} = BEH$	
6. $BE \to H$	$BE^+_{S_3 - \{\#2, \#6\}} = BEG$	

Final answer: A minimal basis for S is

$ABF \to G,\ \ BC \to E,\ \ BE \to G,\ \ BE \to H$