University of Toronto
Faculty of Arts and Science

MAT224H1S
Linear Algebra II

Final Examination
April 2011

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Duration: 3 hours

PLEASE HAND IN

Last Name: ______________________

Given Name: ______________________

Student Number: ______________________

No calculators or other aids are allowed.

FOR MARKER USE ONLY	
Question	Mark
1	/10
2	/10
3	/10
4	/10
5	/10
6	/10
TOTAL	/60

[10] **1.** Find an orthonormal basis of $P_1(\mathbb{C})$, the vector space of linear polynomials with complex coefficients, with respect to the inner product

$$< p(x), q(x) >= \overline{p(0)}q(0) + \overline{p(i)}q(i).$$

[10] **2.** Consider $P_1(\mathbb{R})$, the vector space of real linear polynomials, with inner product

$$< p(x), q(x) >= \int_0^1 p(x)q(x)\,dx.$$

Let T: $P_1(\mathbb{R}) \to P_1(\mathbb{R})$ be defined by $T(p(x)) = p'(x) + p(x)$. Find $T^*(p(x))$ for an arbitrary $p(x) = a + bx \in P_1(\mathbb{R})$.

EXTRA PAGE FOR QUESTION 2 - please do not remove.

[10] **3.** Let $A = \begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix}$. Prove that A is normal and find the spectral decomposition of A.

[10] **4.** Let $P_2(\mathbb{R})$ be the vector space of real polynomials of degree at most 2 with inner product

$$< a_0 + a_1 x + a_2 x^2, b_0 + b_1 x + b_2 x^2 >= a_0 b_0 + a_1 b_1 + a_2 b_2.$$

Find the matrix of the orthogonal projection onto

$$W = \{p(x) \in P_2(\mathbb{R}) \mid p(1) = 0\}$$

relative to the basis $\{1, x, x^2\}$ of $P_2(\mathbb{R})$.

EXTRA PAGE FOR QUESTION 4 - please do not remove.

[10] **5.** Let $T\colon \mathbb{R}^3 \to \mathbb{R}^3$ be the linear operator that has the matrix

$$A = \begin{pmatrix} 0 & 0 & 2 \\ -2 & 2 & 1 \\ -1 & 0 & 3 \end{pmatrix}$$

realtive to the standard basis of $\mathbb{R}^3$. Find a basis of $\mathbb{R}^3$ such that the matrix of T relative to this basis is Jordan canonical form of A, and find the matrix of T relative to this basis.

EXTRA PAGE FOR QUESTION 5 - please do not remove.

[10] **6.** Let T be a Hermitian operator on a finite dimensional complex inner product space V. Suppose T has only two distinct eigenvalues λ_1 and λ_2. Prove $E_{\lambda_1} = E_{\lambda_2}^{\perp}$.