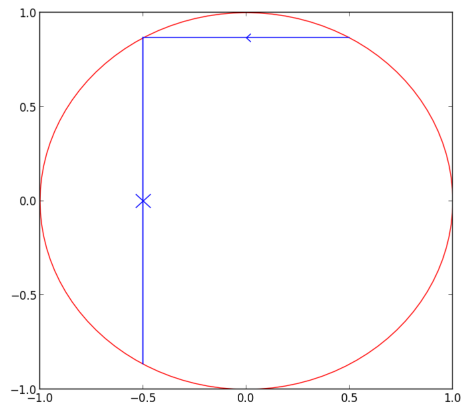
Another Doubling Function. In the textbook he defines a new doubling function called D. We will call it $D_2(z)$ because it is the square function applied to a complex variable z.

1. Let $\mathbb{S}^1 = \{z \in \mathbb{C} : |z| = 1\}$
2. Define $D_2 : \mathbb{S}^1 \to \mathbb{S}^1$ by $D_2(z) = z^2$
3. The orbit of $z = i$ under D_2 is eventually fixed: $i, -1, 1, 1, \ldots$
4. The orbit of $z = \frac{1+\sqrt{3}i}{2}$ under D_2 is eventually a 2-cycle:



`i_orbit_D2(0.5*(1+sqrt(3)*1j),5)`

Q. Why does Devaney call it a doubling function?
If $z = \cos\theta + i\sin\theta$, then $z^2 = \cos(2\theta) + i\sin(2\theta)$. So the argument (angle) of z is doubling.

Try running: `i_orbit_D2(cos(pi/180)+1j*sin(pi/180),500)`

Now consider $B : \mathbb{S}^1 \to [-2,2]$ defined by $B(z) = 2\mathrm{Re}(z)$. Check that $B \circ D_2 = Q_{-2} \circ B$:

$$Q_{-2}\big(B(z)\big) = Q_{-2}(2\cos\theta) = 4\cos^2\theta - 2 = 2\cos(2\theta) = B(z^2)$$

$$\begin{tikzcd}
\mathbb{S}^1 \arrow[r, "D_2"] \arrow[d, "B"'] & \mathbb{S}^1 \arrow[d, "B"] \\
{[-2,2]} \arrow[r, "Q_{-2}"'] & {[-2,2]}
\end{tikzcd}$$

Since B is two-to-one, B is a semiconjugacy and we can use it to show that D_2 is chaotic on $\mathbb{S}^1$, because Q_{-2} is chaotic on $[-2,2]$.

⚠ We need to use B^{-1} (but it is not a function) to get information about D_2 from Q_{-2}. We need to define $B^{-1}(z) = \{z, \overline{z}\}$ to obtain that extra information.

Note. z is a periodic point for D_2 iff $\overline{z}$ is (and $B(z) = B(\overline{z})$).