

# Model Refinement : Added Variable Plots

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The aim of multiple regression (MR) modelling is to use the  $X$  or explanatory variables (predictors) to explain what is happening with the  $Y$  or response variable.

In analysis of variance terms we want to maximise the variance explained by the model (involving the  $X$  variables) & minimise the unexplained variance (in  $Y$ ). In fact, we would like (and we assume) that this unexplained variance will be just random stochastic variability

$$e \sim \text{iid } N(0, \sigma^2)$$

"the errors are independent & identically (Normally) distributed with constant variance,  $\sigma^2$ "

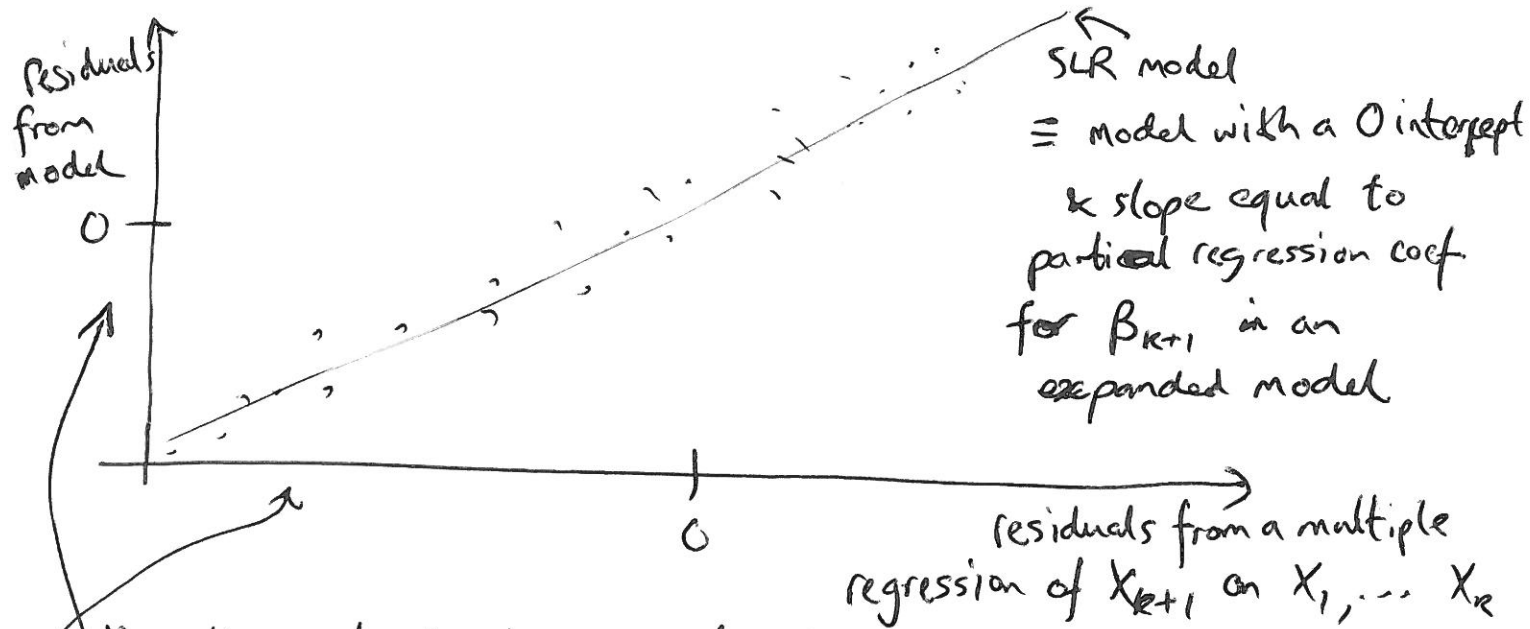
However, the residuals (observed errors) from a multiple regression model often show non-random structure (ie they are not independent) or they might be random, but simply too variable (the unexplained variance is large & the explained variance is relatively small & the overall  $F$ -test is not significant).

If we have other  $X$  variables (in the data, but not in the model), we can try adding them to the model & see if they help explain the unwanted structure and/or some of the unexplained variation.

Added variable plots (AVP) are a way of assessing if, but also how, we might include another  $X$  variable in the model.

Note, if we don't have any other candidate  $X$  variables in the data (there are no other observed  $X$  variables) then the structure and/or excess variability is "unobserved heterogeneity"

AVP for a model  $Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \dots + \beta_k X_k + \varepsilon$   
and an additional variable  $X_{k+1}$



Here the vertical axis represents what is left in  $Y$  that is not explained by  $X_1, \dots, X_k$

& the horizontal axis is what is in  $X_{k+1}$ , that is not explained by  $X_1, \dots, X_k$

We can check if the relationship on the AVP is linear by adding a SLR line

OR we could propose a different way to include  $X_{k+1}$  in the model, i.e. some transformation or higher order term if there is an apparent relationship which is non-linear!