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Solutions for these problems are only presented during the Problem Solving Sessions W5-6 in SS 2135. You are strongly encouraged to work through the problems ahead of time, and our TA Yiannis will cover the questions you are most interested in. These sessions are very valuable at developing the proper style to present cogent and rigorous mathematical solutions.

This problem solving session contains material from 1.4-1.7 and should provide good practice for the midterm on Thursday May 30th.

Problems:

1. Prove that for a bounded subset S of real numbers, if $s = lub(S)$, $s \notin S$, then there exists an increasing sequence of numbers $\{x_n\}_{n=1}^{\infty}$ that converges to s.

2. Prove that if $lub(S)$ or $glb(S)$ exists, it must be unique.

3. Let p>0 and try to write the expression

$$\sqrt{p+\sqrt{p+\sqrt{p+\sqrt{p+\ldots}}}}$$

as the limit of a sequence. Use Monotone sequence theorem to show that the limit must exist and then calculate it.

4. Apply the definition of a disconnection to prove:

a) The set of rationals is disconnected

b) If the axiom of completeness fails (for $\mathbb{R}$) then the set of real numbers $\mathbb{R}$ becomes disconnected.

c) The empty set is connected

5. Read Question 6, section 1.6, and prove that for two sets U and V, d(U,V)>0 then the sets U and V are disconnected. Also prove that two compact sets are either connected or their distance is larger than 0.

6. Read the Nested Interval Theorem and present an example to show that the condition that the intervals I_n must be closed is necessary.