UNIVERSITY OF TORONTO
Faculty of Arts and Science

EXAMINATION DECEMBER 2012

PHL 245 H1F
L0101 - Niko Scharer

Duration - 3 hours

Examination Aid: Sheet with rules (provided)

Last Name ______________________________

First Name ______________________________

Student Number___________________________

Answer **all** questions on the exam paper.

Use the supplied examination booklet for rough work OR if you need further space.

The exam consists of fourteen (14) pages. Pages 2-12 have questions on them.

The final two pages (13-14) are an aid sheet and may be detached from the rest of the exam.

1. Suppose there are three sentences: ϕ, ψ and χ. On every interpretation that ϕ is true, ψ is false. What can you conclude (if anything) about the following argument? Explain. (3%)

$$\frac{\phi \vee \chi}{\therefore\ \psi \rightarrow \chi}$$

2. Here is a truth-table for the NEW symbol: ✳

P	Q	P ✳ Q
T	**T**	**F**
T	**F**	**T**
F	**T**	**T**
F	**F**	**T**

a) Given this truth-table, what ordinary English expression can this new truth-functional connective (✳) be used to symbolize? (1 pt.)

b) Using the definition of the new symbol, ✳ , as defined by the truth-table above, provide a shortened truth-table and truth-value assignment that shows that the following sentence is NOT a tautology. (3 pts.)

$$((P \leftrightarrow \sim Q) \wedge (S \ast W)) \rightarrow ((Q \ast Z) \vee \sim(S \vee P))$$

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3. Provide an English language interpretation that shows that the following argument is invalid. Your interpretation should specify the universe of discourse and a symbolization scheme. (4 pts.)

$\forall x(Bx \rightarrow \exists y{\sim}H(xy))$. $\exists x\forall y(Ax \wedge H(xy))$. $\therefore\ \forall z(Bz \rightarrow {\sim}H(zz))$

4. Explain why the following sentence is a contradiction. (4 pts.)

$$\exists x\forall y(Fx \wedge {\sim}L(xy)) \wedge \forall y(Fy \rightarrow \exists xL(yx))$$

5. Use this symbolization scheme to symbolize the following sentences: 36 pts. total

A^1: *a* is ambitious.	C^1: *a* is a citizen.	D^1: *a* is a politician
F^1: *a* is a time.	G^1: *a* gets elected.	H^1: *a* is a person.
J^2: *a* is more popular than *b*.	K^2: *a* votes for *b*.	L^2: *a* likes *b*.
M^3: *a* makes a promise to *b* at *c*.	a^0: Aaron	c^1: the cousin of *a*.

a) Some people who are ambitious are politicians. (2 pts.)

b) Every politician is a citizen, but not all politicians get elected. (3 pts.)

c) Only ambitious people are politicians. (3 pts.)

d) For a person to get elected, it is necessary that he/she is more popular than any politician that people vote for. (4 pts.)

5 continued.
Use this symbolization scheme to symbolize the following sentences:

A^1: *a* is ambitious.	C^1: *a* is a citizen.	D^1: *a* is a politician
F^1: *a* is a time.	G^1: *a* gets elected.	H^1: *a* is a person.
J^2: *a* is more popular than *b*.	K^2: *a* votes for *b*.	L^2: *a* likes *b*.
M^3: *a* makes a promise to *b* at *c*.	a^0: Aaron	c^1: the cousin of *a*.

e) Assuming that no politician is liked by everyone who votes for him/her, every politician makes promises to people some of the time. (4 pts.)

f) Any politician who makes a promise to all citizens at the same time gets elected unless everybody dislikes him/her. (4 pts.)

g) Only Aaron likes exactly those people who vote for him. (4 pts.)

5 continued.

Use this symbolization scheme to symbolize the following sentences:

A^1: *a* is ambitious.	C^1: *a* is a citizen.	D^1: *a* is a politician
F^1: *a* is a time.	G^1: *a* gets elected.	H^1: *a* is a person.
J^2: *a* is more popular than *b*.	K^2: *a* votes for *b*.	L^2: *a* likes *b*.
M^3: *a* makes a promise to *b* at *c*.	a^0: Aaron	c^1: the cousin of *a*.

h) Neither Aaron nor Aaron's cousin votes for the one politician that Aaron likes. (4 pts.)

i) Using the symbolization scheme above, provide an idiomatic English sentence that expresses: (4 pts.)

$$\exists x(Dx \wedge \forall y(Dy \wedge \sim x=y \rightarrow J(xy)) \wedge \sim\forall z(Hz \rightarrow L(zx)))$$

j) Using the symbolization scheme above, symbolize the following ambiguous sentence **two** logically distinct ways. For each, provide an English sentence that explains exactly what the symbolized sentence means:

Somebody doesn't like every politician. (4 pts.)

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6. Provide a derivation that shows the following theorem is valid **using only the 10 basic rules from SL** (R, DN, MP, MT, ADJ, S, ADD, MTP, BC, CB) (9 pts.)

$$\therefore\ \sim(P \vee Q) \wedge \sim(R \rightarrow S)\ \rightarrow\ (\sim Q \leftrightarrow \sim S)$$

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7. Provide a derivation that shows that this is a valid argument **using only the 10 basic rules from SL** (R, DN, MP, MT, ADJ, S, ADD, MTP, BC, CB) **and the 3 basic rules from PL** (UI, EG, EI) (9 pts.)

$\exists z(Fz \wedge \forall y L(zy))$. $\exists z(Bz \vee Cz) \rightarrow \forall x \forall y\ (Fx \leftrightarrow G(yx))$.

$\therefore \forall x(Bx \rightarrow \exists y(G(xy) \wedge L(yx)))$

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8. Provide a derivation to show that this is a valid argument (use any rules). (9 pts.):

$\forall x \forall y(F(yx) \rightarrow \sim B(xy)) \rightarrow \sim\exists y \sim A(yy)$. $\exists x \forall y(L(xyy) \rightarrow \forall z \sim A(xz))$.

$\forall z(\exists w B(wz) \rightarrow H(zz))$. $\forall x(H(xx) \rightarrow \sim\exists z F(xz))$. $\therefore \sim\forall x \exists y \forall z L(xyz)$

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9. Show that the following is a valid argument (use any rules). (9 pts.):

$\exists x(Bx \wedge \sim Cx) \rightarrow \exists x \forall y F(a(x)y)$. $\quad \therefore \forall x \exists y \sim (Bx \rightarrow Cy) \rightarrow \exists x F(xa(x)) \wedge \exists y F(yy)$

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10. Use a finite model to demonstrate that this set of three sentences is consistent (8 pts.):

$\{\exists x(Bx \wedge \forall y L(xy)).$ $\forall x \exists y(L(xy) \rightarrow \sim L(yx)).$ $\sim \forall x(Cx \rightarrow L(xx))\}$

i) provide a truth-functional expansion (to two individuals) for each sentence in this set.

ii) define a finite model with a universe of two individuals that shows that the set is consistent.

$\exists x(Bx \wedge \forall y L(xy)).$ $\forall x \exists y(L(xy) \rightarrow \sim L(yx)).$ $\sim \forall x(Cx \rightarrow L(xx)).$

11. Consider the following derivation rule (which is *not* a rule in our derivation system):

$$\frac{\begin{array}{l}\phi \vee \psi \\ \phi\end{array}}{\sim \psi} \qquad\qquad \frac{\begin{array}{l}\phi \vee \psi \\ \psi\end{array}}{\sim \phi}$$

Explain how this rule works.
What are the advantages (if any) and disadvantages (if any) of adding this rule to our system.
Overall, do you think that it would be good to add this rule to our derivation system?
Explain why or why not. (5 %)

=100 pts. total.

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AID SHEET: DERIVATION RULES (SEE BOTH SIDES)

Derivation Types:

Direct Derivation (DD)

Conditional Derivation (CD)

Indirect Derivation (ID)

Universal Derivation (UD) — Restriction: the instantiating term cannot occur unbound in any previous line.

Basic Rules for Sentential Operators:

Modus Ponens (MP)

$$\frac{\begin{array}{l}(\phi \to \psi)\\ \phi\end{array}}{\psi}$$

Modus Tollens (MT)

$$\frac{\begin{array}{l}(\phi \to \psi)\\ \sim\psi\end{array}}{\sim\phi}$$

Double Negation (DN)

$$\frac{\phi}{\sim\sim\phi} \qquad \frac{\sim\sim\phi}{\phi}$$

Repetition (R)

$$\frac{\phi}{\phi}$$

Simplification (S)

$$\frac{\phi \wedge \psi}{\phi} \qquad \frac{\phi \wedge \psi}{\psi}$$

Adjunction (ADJ)

$$\frac{\begin{array}{l}\phi\\ \psi\end{array}}{\phi \wedge \psi}$$

Addition (ADD)

$$\frac{\phi}{\phi \vee \psi} \qquad \frac{\psi}{\phi \vee \psi}$$

Modus Tollendo Ponens (MTP)

$$\frac{\begin{array}{l}\phi \vee \psi\\ \sim\phi\end{array}}{\psi} \qquad \frac{\begin{array}{l}\phi \vee \psi\\ \sim\psi\end{array}}{\phi}$$

Biconditional-Conditional (BC)

$$\frac{\phi \leftrightarrow \psi}{\phi \to \psi} \qquad \frac{\phi \leftrightarrow \psi}{\psi \to \phi}$$

Conditional-Biconditional (CB)

$$\frac{\begin{array}{l}\phi \to \psi\\ \psi \to \phi\end{array}}{\phi \leftrightarrow \psi}$$

Derived Rules for Sentential Operators:

Negation of Conditional (NC)

$$\frac{\sim(\phi \to \psi)}{\phi \land \sim\psi} \qquad \frac{\phi \land \sim\psi}{\sim(\phi \to \psi)}$$

Conditional as Disjunction (CDJ)

$$\frac{\phi \to \psi}{\sim\phi \lor \psi} \qquad \frac{\sim\phi \lor \psi}{\phi \to \psi} \qquad \frac{\sim\phi \to \psi}{\phi \lor \psi} \qquad \frac{\phi \lor \psi}{\sim\phi \to \psi}$$

Separation of Cases (SC)

$$\begin{array}{l} \phi \lor \psi \\ \phi \to \chi \\ \psi \to \chi \\ \hline \chi \end{array} \qquad \begin{array}{l} \phi \to \chi \\ \sim\phi \to \chi \\ \hline \chi \end{array}$$

Negation of Biconditional (NB)

$$\frac{\sim(\phi \leftrightarrow \psi)}{\phi \leftrightarrow \sim\psi} \qquad \frac{\phi \leftrightarrow \sim\psi}{\sim(\phi \leftrightarrow \psi)}$$

De Morgan's (DM)

$$\frac{\sim(\phi \lor \psi)}{\sim\phi \land \sim\psi} \qquad \frac{\sim\phi \land \sim\psi}{\sim(\phi \lor \psi)} \qquad \frac{\sim(\phi \land \psi)}{\sim\phi \lor \sim\psi} \qquad \frac{\sim\phi \lor \sim\psi}{\sim(\phi \land \psi)} \qquad \frac{\sim(\sim\phi \lor \sim\psi)}{\phi \land \psi} \qquad \frac{\phi \land \psi}{\sim(\sim\phi \lor \sim\psi)} \qquad \frac{\sim(\sim\phi \land \sim\psi)}{\phi \lor \psi} \qquad \frac{\phi \lor \psi}{\sim(\sim\phi \land \sim\psi)}$$

Derivation Rules for Predicate Logic:

Existential Generalization (EG)

$$\frac{\phi_\zeta}{\exists\alpha\phi_\alpha}$$

Universal Instantiation (UI)

$$\frac{\forall\alpha\phi_\alpha}{\phi_\zeta}$$

Restriction: ζ does not occur as a bound variable in ϕ_α

Existential Instantiation (EI)

$$\frac{\exists\alpha\phi_\alpha}{\phi_\zeta}$$

Restriction: ζ does not occur in any previous line or premise.

Quantifier Negation (QN)

$$\frac{\sim\forall\alpha\phi}{\exists\alpha\sim\phi} \qquad \frac{\sim\exists\alpha\,\phi}{\forall\alpha\sim\phi}$$

$$\frac{\exists\alpha\sim\phi}{\sim\forall\alpha\phi} \qquad \frac{\forall\alpha\sim\phi}{\sim\exists\alpha\,\phi}$$