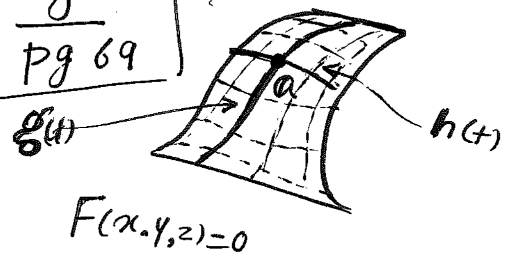
Chain Rule

Our main tool for translating calculus of one variable to mult-variate calculus

When $f : \mathbb{R}^n \rightarrow \mathbb{R}$, we will introduce a new function $g : \mathbb{R} \rightarrow \mathbb{R}^n$ and let f get involved with $g(t)$; This way a new function $\varphi : \mathbb{R} \rightarrow \mathbb{R}$ is born $\varphi(t) = f(g(t))$. Now we impose our Single variable Calculus on $\varphi(t)$ and we get some result about f.

$$\varphi(t_0) = f(g(t_0)) = f(a)$$
$$\varphi'(t_0) = \nabla f(a) \cdot g'(t_0)$$

eg1
pg 69



$\nabla F(a) \cdot g'(t_0) = 0$
$\nabla F(a) \cdot h'(t_0) = 0$

so $\nabla F(a) \perp$ surface $F(x,y,z)=0$

eg2
MVT

eg3
2.5

implicit differentiation

eg4
pg90

proof of multivariate Version of Taylor from Single variable Version

eg5
pg 102-103

Lagrange multiplier

$\nabla G \parallel \nabla f$