June 6th

$$h(x) = \begin{cases} x^2 & \text{if } x \in \mathbb{Q} \\ 0 & \text{o.w.} \end{cases}$$

we proved last time x is discont. except at $x=0$.

$$\lim_{k\to 0} \frac{h(k)-h(0)}{k} = \lim_{k\to 0} \left|\frac{h(k)}{k}\right| \leq \lim_{k\to 0} \frac{|k^2|}{|k|} \longrightarrow 0$$

$f: \mathbb{R}^n \longrightarrow \mathbb{R}$, $\exists \vec{c} \in \mathbb{R}^n$ s.t.

$$\lim_{\vec{h}\to \vec{0}} \frac{f(\vec{a}+\vec{h}) - f(\vec{a}) - \vec{c}\cdot\vec{h}}{|\vec{h}|} = 0$$

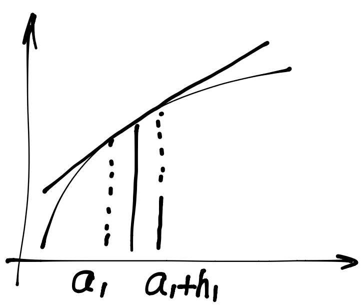
Directional derivative (P 60)

$$\partial_u f(\vec{a}) = \lim_{t\to 0} \frac{f(\vec{a}+\vec{u}t) - f(\vec{a})}{t} \quad \text{exists.}$$

Sps f is defined on an open set S, $\partial_i f$ exists and bounded on S, then f is continuous on S. Prove it. (P62, #8)

Proof:

Sps $\vec{a} \in S$, cont. at $\vec{a} \Leftrightarrow |f(\vec{a}+\vec{h}) - f(\vec{a})| \longrightarrow 0$ when $|\vec{h}| \longrightarrow 0$



USE MVT

in one variable case

$$\frac{|f(a_1+h_1) - f(a_1)|}{|h_1|} = |f'(c)| \quad \text{for a } c$$

Use this in multivariables

$$|f(\vec{a}+\vec{h}) - f(\vec{a})| \longrightarrow 0$$

$= |f(a_1+h_1, a_2+h_2, \ldots, a_n+h_n) - f(a_1, \cdots, a_n)|$

$= |f(a_1+h_1, a_2+h_2, \cdots, a_n+h_n) - f(a_1, a_2+h_2, \cdots, a_n+h_n)$ $\leq C_1$, let C_1 be $\sup \partial_1 f$

$+ f(a_1, a_2+h_2, \cdots, a_n+h_n) - f(a_1, a_2, a_3+h_3, \cdots, a_n+h_n)$ $\leq C_2 = \sup \partial_2 f$

$+ \cdots$

$+ f(a_1, \cdots, a_n+h_n) - f(a_1, a_2, \cdots, a_n)|$ $\leq C_n = \sup \partial_n f$

$\leq C_1|h_1| + C_2|h_2| + \cdots + C_n|h_n| \longrightarrow 0$

$f(x,y) = x^2y + \sin \pi xy$

(i) ∇f

(ii) find $\nabla_u f(1,-2)$ $\vec{u} = (3,4)$

(i) $f_x = 2xy + \cos \pi xy \cdot \pi y$

$f_y = x^2 + \cos \pi xy \cdot \pi x$

$\nabla f = (f_x, f_y)$

(ii) Normalize $\vec{u} = (\frac{3}{5}, \frac{4}{5})$

$\nabla_u f(1,-2) = (f_x, f_y) \cdot \vec{u} = \frac{3}{5} \times (-4-2\pi, 1+\pi) + \frac{4}{5}(-4-2\pi, 1+\pi)$

$f_x(1,-2) = -4 - 2\pi$

$f_y(1,-2) = 1+\pi$

$(x, y, f(x,y))$ at $(0,0)$

$(1, 0, f_x(0,0))$

$(0, 1, f_y(0,0))$

$(-f_x(0,0), -f_y(0,0), 1)$ The gradiant is always $\perp$ to the func.