UNIT 5 PART 2
PREDICATE SYMBOLIZATION:
COMPLEX PARTICULAR TERMS AND IDENTITY

5.11 Complex Particular Terms

We have been using name letters to pick out particular objects.

a: Aristotle b: Bob F^2: *a* studies the work of *b*.

F(ba): Bob studies the work of Aristotle.

However, there are times when we want to refer to a particular person or object but we don't have a name for that individual.

Bob studies the work of Plato's most famous student.

Bob is studying philosophy with the man in the corner wearing a suit and tie.

Bob is studying philosophy with the tallest woman in the department.

In addition to the name letters, we can refer to individuals with…

Operation letters: used to refer to a unique individual through relations to other individuals

Identity: = A special two-place predicate that we can use to pick out a unique individual (among other things).

Iota: ι a symbol for the definite descriptor (used like a quantifier, to pick out 'the …"
(Note: you will not be required to use the definite descriptor, ι, in this course.)

Operation letters:

Operation letters function in much the same way as name letters – to identify particular objects. The objects that they identify are ones that we would use a noun phrase to identify rather than a name:

the square of 4

the oldest child of Tom and Mary

the sum of 5 and 8 and 7

Like predicates, operations have 'blanks' or 'places' where terms can be put:

the square of ___________

the oldest child of __________ and ____________

the sum of __________ and __________ and ___________

Operation letters are the lower case letters a-h with the number of places indicated with a superscript number. They can also have a subscript number so that there are an infinite number of possible operation letters. (We won't ever need the subscripts.)

Zero-place operation letters are name letters as in the earlier part of this unit (we will also be using them to name numbers!). Here the superscript number, 0, shows that they have no places. When using the name letters in a sentence, the superscript can be dropped.

a^0: Tom b^0: Toronto c^0: Ontario d^0: 3 e^0: 4

F^1: *a* is Canadian. L^2: *a* is in *b* G^2: *a* is greater than *b*.

Tom is Canadian: Fa
Toronto is in Ontario: L(bc)
4 is greater than 3: G(ed)

One-place operation letters can be used for operations with one place (blank). Two-place operation letters can be used for operations with two places (blanks). Use brackets or parentheses after all operation letters (even one-place operation letters). When using the operation letters in a sentence, the superscripts can be left off.

a^0: Tom b^0: Toronto c^0: Ontario d^0: 3 e^0: 4

a^1: the capital of *a*. b^1: the father of *a*. c^2: the sum of *a* plus *b*.

Tom's father is Canadian:

To symbolize Tom's father (the father of Tom), we use operation letter 'b' to refer to the father of an individual, in this case, Tom, who we use name letter, 'a', to refer to.
Tom's father: b(a) Tom's father is Canadian: Fb(a)

The capital of Ontario is in Ontario:

The capital of Ontario: a(c) The capital of Ontario is in Ontario: L(a(c)c)

The sum of 3 plus 4 is greater than 3.

The sum of 3 plus 4: c(de) The sum of 3 plus 4 is greater than 3: G(c(de)d)

We can put any term that designates an individual in the places of an operation letter. Since all operation letters designate terms, we can put other operation letters in those places!

Tom's father's father is Canadian.
Tom's father: b(a) Tom's father's father: b(b(a))
Tom's father's father is Canadian: Fb(b(a))

NOTE: Operation letters always pick out a unique individual. Thus, if Tom has two sisters, we cannot use an operation letter such as h^1: the sister of *a*, to refer to Tom's sister: h(a). There is no unique individual that we can call, "the sister of Tom".

NOTE: expressions such as b(a), a(c), c(de), b(b(a), etc. are just noun-phrases. They are not complete sentences! (They lack a predicate – every sentence needs a subject and a predicate!)

5.11 E1 Let's try a few:

a^0: Tom b^0: Ontario c^0: Mary d^0: 3 e^0: 4 f^0: Sarah

a^1: the square of *a*. b^1: the capital city of *a*. c^1: the father of *a*.

a^2: the product of *a* and *b* b^2: the oldest child of *a* and *b* c^2: the sum of *a* and *b*

a^3: the sum of *a* and *b* and *c*

Symbolize the following noun phrases using operation letters:

a) the capital city of Ontario b(a)

b) the father of Tom c(a)

c) Mary's father. c(c)

d) 4 squared a(e)

e) the product of 3 and 4. a(de)

f) the oldest child of Tom and Mary. b(ac)

g) the oldest child of Tom's father and Mary. b(c(a)c)

h) the sum of 3 and 4. c(de)

i) the sum of 4 and 3 and 3 a(edd)

j) the sum of the square of 3 and the square of four. c(a(d)a(e))

k) the oldest child of Mary's father and Sarah and Tom's oldest child. b(c(c)b(fa))

l) the sum of the square of three, the sum of three and four, and four. a(a(d)c(de)e)

m) the sum of three squared, four squared and the square of the sum of three and four

a(a(d)a(e)a(c(de)))

5.11 E2 Symbolize the following sentences using operation letters:

F^1: *a* is a person. G^2: *a* is the spouse of *b*. L^2: *a* loves *b*. M^2: *a* is less than *b*

a^0: Tom b^0: Sarah c^0: Mary d^0: 3 e^0: 4

a^1: the square of *a* b^1: the father of *a* c^2: the product of *a* and *b*

d^2: the oldest child of *a* and *b* e^2: the sum of *a* and *b* f^3: the sum of *a* and *b* and *c*

a) Mary's father is Sarah's spouse. $G(b(c)b)$

b) Sarah loves Tom's father. $L(b\,b(a))$

c) Mary's spouse is the oldest child of Tom and Sarah. $G(d(ab)c)$

d) The sum of three squared and four squared is less than the square of the sum of three $M(e(a(d)a(e))a(e(de)))$

e) The product of three and four is not less than the sum of three plus four. $\sim M(c(de)e(de))$

f) Tom and Mary's oldest child has no spouse. $\sim\exists x G(x\,d(ac))$

g) Not everybody loves Mary's father. $\sim\forall x(Fx \rightarrow L(x\,b(c)))$

h) No spouse of Sarah is the father of Mary. $\sim G(b\,b(c))$

i) Anything less than the sum of 4 plus 4 is less than the square of 3. $\forall x(M(x\,e(ee)) \rightarrow M(x\,a(d)))$

j) Mary's father is not the spouse of the spouse of Sarah's father. $\sim\exists x(G(b(c)x) \wedge G(x\,b(b)))$

5.12 Properties of Binary Predicates or Relations

We will talk about binary predicates as relations between two individuals. Binary predicates often have special properties.

Symmetry: A binary relation is symmetrical if and only if it is the case that if one thing stands in that relation to another, then the latter stands in that relation to the first.

Is a cousin of:	If A is B's cousin, then B is A's cousin.
Is beside:	If A is beside B, then B is beside A
Is equal to:	If A is equal to B, then B is equal to A.

If F is a symmetrical relation: $\forall x \forall y\, (F(xy) \rightarrow F(yx))$

This can be expressed as a biconditional.

If F is a symmetrical relation: $\forall x \forall y\, (F(xy) \leftrightarrow F(yx))$

Asymmetry: A binary relation is asymmetrical if and only if it is the case that if one thing stands in that relation to another, the latter cannot stand in that relation to the first.

Is taller than:	If A is taller than B, then B cannot be taller than A.
Is the mother of:	If A is the mother of B, then B cannot be the mother of A.
Is greater than:	If A is greater than B, then B cannot be greater than A.

If F is an asymmetrical relation: $\forall x \forall y\, (F(xy) \rightarrow \sim F(yx))$

Asymmetry cannot be expressed in terms of a biconditional. If a relation is asymmetrical, then if one thing stands in that relation to another, then it follows that the second does not stand in that relation to the first. However, if one thing does not stand in the asymmetrical relation to another, it does not follow that the second thing stands in that relation to the first! For example, it doesn't follow from the fact that A is not the mother of B that B is the mother of A.

Of course, some relations are neither symmetrical nor asymmetrical.

Loves: if A love B, then B might love A or B might not love A.

Is afraid of: if A is afraid of B, then B might be afraid of A or B might not be afraid of A.

Is bored by: if A is bored by B, B might be bored by A or B might not be bored by A.

Transitivity: A binary relation is transitive if and only if it is the case that if one thing stands in the relation to a second, and the second stands in the same relation to a third, then the first stands in that relation to the third.

Is taller than: If A is taller than B, and B is taller than C, then A is taller than C.

Is west of: If A is west of B, and B is west of C, then A west of C.

If F is transitive: $\forall x \forall y \forall z\, [(F(xy) \wedge F(yz)) \rightarrow F(xz)]$

Intransitivity: A binary relation is intransitive if and only if it's the case that if one thing stands in the relation to a second, and the second stands in the same relation to a third, then the first cannot stand in that relation to the third.

Is the father of: If A is the father of B, and B is the father of C, then A cannot be the father of C.

Is $5 more than: If A is $5 more than B, and B is $5 more than C, then A is not $5 more than C.

If F is intransitive: $\forall x \forall y \forall z\, [(F(xy) \wedge F(yz)) \rightarrow \sim F(xz)]$

Of course, some relations are neither transitive nor intransitive:

Loves: If A loves B and B loves C, then A may or may not love C.

Is bored by: If A is bored by B and B is bored by C, then A may or may not be bored by C.

Reflexivity: A binary relation is unrestrictedly reflexive if it is the case that everything stands in that relation to itself.

Is identical to: A is identical to A

If R is an unrestricted reflexive relation: $\forall x\, F(xx)$

Being identical to is the only unrestrictively reflexive relation.

Similar relations, such as 'is the same size as' or 'is the same color as' are not unrestrictively reflexive since not all things have a size or a color. The universe matters. Many relations are reflexive within a restricted universe.

U: people H^2: *a* is the same height as *b*

H is reflexive in this universe because all people have a height and everyone is the same height as his/herself.

U: things with color H^2: *a* is the same color as *b*

You can also express restricted reflexivity in an unrestricted universe by putting the restriction in the antecedent of a universally predicated conditional.

Is the same height as: For all things, x, if x is the same height as anything (thus x has a height) then x is the same height as x.

Is the same color as: For all things x, if x is the same color as anything (thus x has a color), then x is the same color as itself.

If H is a restricted reflexive relation: $\forall x[\exists y H(xy) \rightarrow H(xx)]$

This says …

For each x, if there is some y such that x stands in that relation to y, x stands in that relation to itself.

A relation is a restricted reflexive relation if each thing that can stand in that relation to something also stands in that relation to itself.

Irreflexivity: A binary relation is irreflexive if and only if it's the case that nothing can stand in that relation to itself.

Is the parent of: Nothing is its own parent.

Is faster than: Nothing is faster than itself.

If F is irreflexive: $\forall x \sim F(xx)$

Some binary relations are neither reflexive nor irreflexive.

Loves: A person may or may not love him/herself.
Praises: A person may or may not praise him/herself.

Implicit Premises

Some arguments are valid only because we take such properties of relations for granted.

Mike's cousin Tony is taller than Mary.

All of Tony's cousins are taller than Tony.

Thus, Mike is taller than Mary.

The conclusion cannot be drawn from the premises without the facts that cousinhood is a symmetrical relation (thus if Tony is Mike's cousin, then Mike is Tony's cousin,) and 'is taller than' is a transitive relation (thus, since Mike is taller than Tony and Tony is taller than Mary, Mike is taller than Mary.)

If we were symbolizing the argument, we might want to make these properties of the relations explicit. If we don't, we cannot derive the conclusion from the premises.

5.13 Identity

One thing is identical to another thing if they have exactly the same properties, i.e. if and only if everything that is a property of one thing is a property of the other.

We could use a predicate… I: *a* is identical to *b*

We will express identity with "=":

$a = b$: *a* is identical to *b*

This allows us to recognize that identity is a special relation, in that it is symmetrical, transitive and reflexive.

$\forall x \forall y\ (x = y \rightarrow y = x)$

$\forall x \forall y \forall z\ (x = y \wedge y = z \rightarrow x = z)$

$\forall x\ \ x = x$

The negation of identity, not identical to, is symbolized: $\sim x = y$ OR $x \neq y$

Identity is a two place predicate, an atomic sentence. Thus, there are not brackets around it.

Leibniz's Law aka The principle of the Identity of Indiscernibles

x is identical to y, $x = y$, if and only if every property of x is a property of y, and no property that does not belong to x belongs to y.

This makes sense: if Superman and Clark Kent have ***all*** the same properties (from Krypton, faster than a speeding bullet, in the exact same places at exactly the same times, then they are really one and the same person. If Superman loves Lois Lane, then Clark Kent loves Lois Lane. And if Superman isn't from Earth, then Clark Kent isn't from Earth.

Identity allows the symbolization of many new ideas:

The (definite description – the one and only)

Only

Exactly one, exactly two, exactly three …

At most (no more than) one, at most two, at most three, …

At least two, at least three, …

Superlatives (the fastest, the smallest, …)

We will focus on the use of identity to express uniqueness (including 'the', 'only', superlatives...)

UNIQUENESS:

Two names… one unique person.

Sometimes we want to say that two different names or descriptions belong to one person. For instance, "Superman is Clark Kent". Or, "Anne's father is Bob".

We can use identity for that.

a: Anne. b: Bob. c: Clark Kent. d: Superman. f^1: the father of *a.*

d=c	Superman is Clark Kent.	Superman is identical to Clark Kent.
f(a)=b.	Anne's father is Bob.	The father of Anne is identical to Bob.

Only

Only Betty helps Adam.

Betty is the unique helper of Adam.

Betty helps Adam AND

everyone who helps Adam is identical to Betty (if a person helps Adam, then that person is Betty.)

$H(ba) \wedge \forall x(H(xa) \rightarrow x=b)$

For everything, x, if x helps Adam, x is identical to Betty and if x is identical to Betty, then x helps Adam.

We can also use a bi-conditional for this.

$\forall x(x=b \rightarrow H(xa))$ means, literally, Anything in the universe that is identical to Betty helps Adam.

In other words, Betty helps Adam. So this is logically equivalent to H(ba) (Betty helps Adam.)

So $H(ba) \wedge \forall x(H(xa) \rightarrow x=b)$ is logically equivalent to: $\forall x(x=b \rightarrow H(xa)) \wedge \forall x(H(xa) \rightarrow x=b)$

And that's the same as: $\forall x(x=b \leftrightarrow H(xa))$

Betty helps only Adam.

Adam is the unique individual whom Betty helps.

Betty helps Adam and anyone who Betty helps *is* Adam.

$H(ba) \wedge \forall x(H(bx) \rightarrow x=a)$ OR $\forall x(H(bx) \leftrightarrow x=a)$

Exactly one ...

What is true if exactly one thing with a certain property (or set of properties exists)?

Something exists.

That thing has those properties.

Everything with those properties is identical to that thing.

There is exactly one god. (G^1: *a* is a god.)

Something exists: $\exists x$

That thing is a god: Gx

Everything that is a god is identical to x: $\forall y(Gy \rightarrow y=x)$

Putting it together: There exists a god, and all gods are the same being as that god.

$\exists x(Gx \wedge \forall y(Gy \rightarrow y=x))$

If all gods are identical to x, then anything identical to x is a god, and anything that is a god is identical to x. So, we can symbolize it this way:

$\exists x\forall y(Gy \leftrightarrow y=x))$

There is something that exists and everything in the universe with certain properties is identical to it. Thus, there can only be one thing with those properties.

CANONICAL FORM OF THE UNIQUE INDIVIDUAL SENTENCE:

$\exists x\forall y\ (Fy \leftrightarrow x=y)$

There is exactly one individual with property F.

$\exists x(\forall y(Fy \leftrightarrow x=y) \wedge Gx)$

The one individual that has property F also has property G.

Exactly one person wins a gold medal. (H^1: *a* is a person. G^1: *a* wins a gold medal.)

$\exists x\forall y(Hy \wedge Gy \leftrightarrow x=y)$

Somebody, x, wins a gold medal and any person, y, who wins a gold medal is identical to x.

Just one person wins a gold medal, and that person is French. (F^1: *a* is French.)

$\exists x(\forall y(Hy \wedge Gy \leftrightarrow x=y) \wedge Fx)$

Note: the further property applies to x. It is not one of the properties that make the winner unique.

Definite Description ... THE exactly 1

Consider the expression: The professor for PHL 245.

The use of "the" in this expression is a way to say that there is exactly one professor for PHL 245.

The professor for PHL 245 is Niko Scharer.

F^2: *a* is a professor for *b*.   *a*: PHL 245   *b*: Niko Scharer

We can say: there is exactly one professor for PHL 245 and that person is identical to Niko Scharer.

$\exists x(\forall y(F(ya) \leftrightarrow y{=}x) \wedge y{=}b)$

or: $\forall y(F(ya) \leftrightarrow y{=}b)$

The person who won the lottery is now a millionaire.

H^1: *a* is a person. G^1: *a* is a now a millionaire. L^1: *a* won the lottery.

$\exists x(\forall y(Hy \wedge Ly \leftrightarrow y{=}x) \wedge Gx)$

The person drinking a martini is Sarah's sister's husband.

F^1: *a* is a person. G^1: *a* is drinking a martini.
a^0: the sister of *a*. b^0: the husband of *a*. c^0: Sarah.

$\exists x(\forall y((Fy \wedge \exists z(Mz \wedge D(yz))) \leftrightarrow x{=}y) \wedge x{=}b(a(c)))$

Definite Descriptions of Non-Existent Things

This way of symbolizing "The" makes every sentence about a unique individual an existential sentence. Thus, if that unique individual doesn't exist, the sentence is false.

Consider the sentence: The king of France is bald.

It seems that 'The king of France' refers to some individual x, who satisfies the description 'x is the king of France'. But now we have a problem – there is no king of France!

According to Bertrand Russell, 'the king of France' presupposes that there is an object x such that x is the king of France. Thus, it says two things: There is an x such that x is the king of France (the existential claim) AND there is exactly one x such that x is the king of France (the uniqueness claim). This is a false conjunction since the existential claim fails – there is no king of France.

This method of symbolizing the definite description with identity is really Russell's method. It makes it explicit that there exists such a being with the property of being the king of France and that that being is the only one. There is exactly one king of France:

$\exists x \forall y(K(ya) \leftrightarrow x{=}y)$

OR: $\exists x(K(xa) \wedge \forall y(K(ya) \rightarrow y{=}x))$

Existential claim   Uniqueness claim

Gottlob Frege's way: Consider the sentence, "The king of France is bald." According to Frege, 'the king of France' is the object x such that x is the king of France. Since there is no king of France, this phrase does not refer to anything! It is an improper definite description – an empty name. Thus, we can't say anything about the king of France, and thus, sentences containing this phrase are neither true nor false.

There is a way to symbolize definite descriptions this way. We can use the descriptive operator, ι, iota. The descriptive operator picks out the one and only thing, but there is nothing about the operator that says that such a thing exists. It just refers. Thus, there is a presupposition that such an object exists. (Using the descriptive operator with a variable, ιx, is a way of saying "the 'x' such that…" All the properties that we use to pick out 'the x' follow in parentheses, much like the syntax of a quantifier.)

> K^2: *a* is king of *b*. a: France. B^1: *a* is bald.
>
> $\iota x(K(xa))$ … the x such that x is the king of France.

A phrase with iota is not a sentence… it is just a noun phrase. It gets used like a name letter or operation letter.

> $B\iota x(K(xa))$ The king of France is bald.
>
> $K(\iota x(K(xa))a)$ The king of France is the king of France.

Negating identity $\sim x = y$ OR $x \neq y$

We often want to talk about things that are NOT the same as a given individual.

> a different person, everybody else, anyone but….

Barney loves Amy, but Amy loves somebody else. (H^1*: a* is a person. L^2: *a* loves b.)

> Amy loves somebody who is not identical to Barney.
>
> $L(ba) \wedge \exists x(Hx \wedge L(ax) \wedge \sim x = b)$

Cameron will work for anyone but Adam. (H^1*: a* is a person. B^2: *a* will work for b.)

> Anybody not identical to Adam is such that Cameron will work for that person.
>
> $\forall x(Hx \wedge x \neq a \rightarrow B(cx))$

Darren is the oldest person in the company. (C^1: *a* is in the company. O^2: *a* is older than *b*)

> Darren is in the company and Darren is older than everyone else in the company.
>
> $Cd \wedge \forall x(Hx \wedge Cx \wedge x \neq d \rightarrow O(dx))$

We can also use the negation of identity to symbolize numbers greater than one.

> At least two different people are older than Darren. $\exists x\exists y (Hx \wedge Hy \wedge x \neq y \wedge O(xd) \wedge O(yd))$
>
> Exactly two people are older than Darren. $\exists x\exists y(x \neq y \wedge \forall z(Hz \wedge O(zd) \leftrightarrow z=x \vee z=y))$

Superlatives:

Things are not faster, higher or stronger than themselves! When symbolizing that somebody or something is the best or the worst, you must specify that it is better or worse than everything ***else*** – better than everything that is not identical to it.

> Anna is the fastest rower.
>
> Anna is a rower, and for every x, if x is a rower and x is not identical to Anna then Anna is faster than x.
>
> $Ga \wedge \forall x\ ((Gx \wedge {\sim}x = a) \rightarrow F(ax))$

G^1: *a* is a rower F^2: *a* is faster than b a: Anna

The tallest teacher is not married to a teacher.

G^1: a is a teacher F^2: *a* is taller than *b*. H^2: *a* is married to *b*.

There is some x such that x is a teacher and for every y, if y is a teacher and y is not identical to x then x is taller than y and there is not some z such that z is a teacher and x is married to z.

> $\exists x(Gx \wedge \forall y\ ((Gy \wedge y \neq x) \rightarrow F(xy)) \wedge {\sim}\exists z(Gz \wedge H(xz)))$

Complex Uniqueness:

Recall that the canonical form for uniqueness is:

$\exists x \forall y(Fy \leftrightarrow y=x)$	Exactly one F
$\exists x(\forall y(Fy \leftrightarrow y=x) \wedge Gx)$	Exactly one F has property G.

But both property F and property G can be complex properties! They may themselves involve uniqueness claims or other complex expressions.

For instance consider the following statement: Exactly one student solved the hardest problem.

> This sentence says that there is exactly thing that:
> is a student & solved the hardest problem

So instead of "y has property F", we need "y is a student and y solved the hardest problem"!

C^1: *a* is a student D^1: *a* is a problem H^2: *a* is harder than b K^2: *a* solved b

y is a student:	Cy
y solved the hardest problem:	There is some 'z' that y solved & z is the hardest problem $\exists z(K(yz) \wedge Dz \wedge \forall w(Dw \wedge z \neq w) \rightarrow H(zw)))$

Now we put them together: $\exists x \forall y((\ Cy\ \wedge\ \ \exists z(K(yz) \wedge Dz \wedge \forall w(Dw \wedge z \neq w) \rightarrow H(zw)))\ \ \leftrightarrow\ \ y=x)$

"There is exactly one student and he/she solved the hardest problem":

> In this the uniqueness property (student) is simple, but the further property is complex:
>
> $\exists x(\forall y(Cy \leftrightarrow\ \ y=x)\ \wedge\ \ \exists z(K(yz) \wedge Dz \wedge \forall w(Dw \wedge z \neq w) \rightarrow H(zw)))$

5.14 Step by step: Complex Symbolizations

Symbolization is, to a large extent, recursive. Thus, complex symbolizations can be broken down into simpler steps.

Step 1: Analyze the sentence: Identify the main connective and sentence type. If you have a quantified sentence or clause, identify the subject (possibly complex), the predicate (possibly complex).

Step 2: Sketch out the symbolized sentence, using the canonical form that matches the sentence type. Leave blanks for the subject and predicate.

Step 3: Symbolize the subject and predicate (if they are complex, then repeat steps one, two and three with respect to the subject and/or predicate.)

Step 4: Put the subject and predicate into the sketch of step 2.

Step 1: Analyzing the sentence

The first step in analyzing the sentence is identifying the main connective or logical operator!

Compound sentences:
The main logical operator is one of the binary connectives from sentential logic. The binary connective ($\wedge, \vee, \rightarrow, \leftrightarrow$) links two clauses.

Quantified sentences:
The main logical operator is a quantifier or the negation of the quantifier. These are the four types of sentences from the square of opposition: universal affirmative (ALL: All S's are P), existential affirmative (SOME: Some S's are P.) universal negative (NONE: All S's are non-P. *or* It is not the case that some S's are P.), existential negative (NOT ALL: Some S's are non-P. *or* It is not the case that all S's are P.) 'S' is the subject, and 'P' is the predicate. The two negative types have two canonical forms.

Uniqueness sentences:
These state that there exists exactly one thing with certain properties or, in more complex sentences, it states that there is exactly one thing with certain properties and that they also have other properties.

Consider all the properties that apply to the subject: those are the restricting subject properties.

The/Exactly one (simple), Canonical form: $\exists x \forall y$(Subject properties applied to y $\leftrightarrow x=y$)

There is exactly one person in a blue coat. (F: *a* is a person, B: *a* wears a blue coat.)

Subject properties applied to y: $Fy \wedge By$

So put that into the canonical form: $\exists x \forall y((Fy \wedge By) \leftrightarrow x=y)$

The/Exactly one (complex), Canonical form:

$\exists x(\forall y$(Subject properties applied to y $\leftrightarrow x=y) \wedge$ Predicate properties applied to x)

One person is wearing a blue coat and that person is late.

Subject properties applied to y: $Fy \wedge By$ Predicate property applied to x: Lx

In the canonical form: $\exists x(\forall y((Fy \wedge Ry) \leftrightarrow x=y) \wedge Lx)$

Negative sentences:

You can put a negation sign in front of any type of sentence. But be careful when analyzing the sentence – make sure you know whether it is the whole sentence being negated, or just one clause or predicate!

Sentence Type	Canonical Form	What to watch out for.	Strategy
Compound Sentences			
Conjunction Disjunction Conditional Biconditional	$\varphi \wedge \psi$ $\varphi \vee \psi$ $\varphi \rightarrow \psi$ $\varphi \leftrightarrow \psi$	If there are terms or pronouns in ψ that refer back to φ, then it is probably a quantified sentence not a compound sentence.	Symbolize the two clauses, φ and ψ, independently and then connect them according to the canonical form.
Quantified Sentences			
All Some None Not all	$\forall\alpha(S\alpha \rightarrow P\alpha)$ $\exists\alpha(S\alpha \wedge P\alpha)$ $\forall\alpha(S\alpha \rightarrow \sim P\alpha)$ $\sim\exists\alpha(S\alpha \wedge P\alpha)$ $\exists\alpha(S\alpha \wedge \sim P\alpha)$ $\sim\forall\alpha(S\alpha \rightarrow P\alpha)$	Sα is the subject – and it might be complex. Think about what properties pick out the things that the sentence is about. Pα is the predicate – and it might be complex. Think about what things are being said about the subject(s). To identify the canonical form or sentence type, ask: How many of the subjects is the sentence true about? (All, some, none or not all.)	Sketch in the canonical form. Symbolize the subject and the predicate independently (making sure the variables are working together!) then put them into the canonical form.
Uniqueness Sentences			
There is one…	$\exists\alpha\forall\sigma\ (S\sigma \leftrightarrow \alpha = \sigma)$	Sσ is the subject property (or properties) that makes that thing unique.	Sketch in the canonical form. What properties pick out the unique individual?
The one and only…	$\exists\alpha(\forall\sigma\ (S\sigma \leftrightarrow \alpha = \sigma) \wedge P\alpha)$	Sσ is the subject property (or properties) that makes that thing unique. Pα may be complex… what is being said about the unique individual?	What else (if anything) is being said about that individual?

Make sure you think about what terms pronouns refer to.

Step 2: Paraphrasing the quantified sentences.

First Identify:

1. Subject (Restricting): What/who is the sentence about?
2. Predicate (Descriptive): What is the sentence saying about the subject?
3. How many of the subjects is this true of: All/At least one/Not all/None.

Paraphrase with Canonical form: Start with the answer to 3 – how many is it true of?

All: $\forall$ α (Restricting Subject $\rightarrow$ Descriptive Predicate)

Some: $\exists$ α (Restricting Subject $\wedge$ Descriptive Predicate)

Not all: ~$\forall$ α (Restricting Subject $\rightarrow$ Descriptive Predicate)
OR $\exists$ α (Restricting Subject $\wedge$ ~ Descriptive Predicate)

None: ~$\exists$ α (Restricting Subject $\wedge$ Descriptive Predicate)
OR $\forall$ α (Restricting Subject $\rightarrow$ ~Descriptive Predicate)

Step 3: Focusing on the properties

Identify the subject properties: the things that pick out the subject

Identify the predicate properties: the things that are being said about the subject.

Which properties are simple? (One place predicates – if there is more than one, are they joined by 'and' or by 'or'?) Which are complex? (Two or more place predicates involved, other quantifiers needed.)

Step 4: Sketch your answer:

Put it in canonical form, making sure all the simple properties are in the right place. Then, leave blanks with descriptions attached for the complex predicates.

Check your paraphrase:

Say it to yourself: All/At least one/No/Not all things with these characteristics (Subject Properties) also have these characteristics (Predicate Properties.) Does it capture the meaning? Fix any mistakes.

Step 5: Completing the symbolization

Now fill in the complicated bits – repeat the analysis (steps 2-4) for those parts.

Dealing with Singular Terms:

Singular terms all pick out individuals (names and operations).

Jones, Adam, Betty, etc. (Symbolized with a, b, c, d, e)

Adam's sister, Betty's husband, etc. Often symbolized with operations a(b), c(ab), etc.

Step 1:	Find all the primary singular terms, abbreviate and symbolize them. Abbreviate them: ST1, ST2 Symbolize them.
Step 2:	Paraphrase the sentence using the abbreviations (ST1, ST2)
Step 3:	Sketch your symbolization of step 2, using blanks for the singular terms.
Step 4:	Put the results of step 1 together with the results of step 3.

5.14 EG 1 Let's try a few examples:

a) Adam's sister's husband doesn't own a car.

b) Any friend of Adam is a friend of Adam's sister.

c) Adam's sister's husband is Daniella's husband's brother.

d) Adam introduces all of his friends to his sister, but his sister doesn't introduce some of her friends to him.

C^1: *a* is a car	F^2: *a* is a friend of *b*.	O^2: *a* owns *b*	I^3: *a* introduces *b* to *c*.
a^1: Adam	d^1: Daniella	b^1: *a*'s brother	
c^1: *a*'s sister	h^1: *a*'s husband.	e^2: the person sitting between *a* and *b*.	

a) Adam's sister's husband doesn't own a car.

Step 1: Find all the primary singular terms in the sentence, abbreviate and symbolize them: .

ST1: Adam's sister's husband. h(c(a))

Step 2: Paraphrase the sentence using the abbreviations

ST1 doesn't own a car. (There is no car that ST1 owns.)

Step 3: Sketch your symbolization of step 2.

$\sim\exists x(Cx \wedge O(\underset{ST1}{\underline{\quad}}\ x))$

Step 4: Put the results of step 3 together with the results of step 1.

$\sim\exists x(Cx \wedge O(h(c(a))x))$

b) Any friend of Adam is a friend of Adam's sister.

Step 1: Find all the primary singular terms in the sentence, abbreviate and symbolize them: .

ST1=Adam a ST2=Adam's sister. c(a)

Step 2: Paraphrase the sentence using the abbreviations

For all x, if x is a friend of ST1 then x is a friend of ST2.

Step 3: Sketch your symbolization of step 2.

$\forall x(F(x \underset{ST1}{\underline{\quad}}) \rightarrow F(x \underset{ST2}{\underline{\quad}}))$

Step 4: Put the results of step 3 together with the results of step 4.

$\forall x(F(xa) \rightarrow F(xc(a))$

c) Adam's sister's husband is Daniella's husband's brother.

Step 1: Find all the primary singular terms in the sentence, abbreviate and symbolize them: .

ST1 = Adam's sister's husband h(c(a)) ST2= Daniella's husband's brother. b(h(d))

Step 2: Paraphrase the sentence using the abbreviations

ST1 is ST2.

Step 3: Sketch your symbolization of step 2.

$\underset{ST1}{\underline{\quad}} = \underset{ST2}{\underline{\quad}}$

Step 4: Put the results of step 3 together with the results of step 1.

h(c(a)) = b(h(d))

d) Adam introduces all of his friends to his sister, but his sister doesn't introduce some of her friends to him.

Step 1: Find all the primary singular terms in the sentence, abbreviate and symbolize them: .

ST1=Adam a ST2=Adam's sister. c(a)

Step 2: Paraphrase the sentence using the abbreviations

For all x, if x is a friend of ST1 then ST1 introduces x to ST2, <u>and</u> there is some y such that y is a friend of ST2 and it is not the case that ST2 introduces y to ST1.

Step 3: Sketch your symbolization of step 2.

$\forall x(F(x \underset{ST1}{\underline{\quad}}) \rightarrow I(\underset{ST1}{\underline{\quad}} x \underset{ST2}{\underline{\quad}})) \wedge \exists y(F(y \underset{ST2}{\underline{\quad}}) \wedge \sim I(\underset{ST2}{\underline{\quad}} y \underset{ST1}{\underline{\quad}})$

Step 4: Put the results of step 3 together with the results of step 4.

$\forall x(F(xa) \rightarrow I(axc(a))) \wedge \exists y(F(yc(a)) \wedge \sim I(c(a)ya))$

5.14 E1 Symbolize:

A^2:	a is taller than b.	B^1:	a is a baseball player.	D^2:	a is a daughter of b.
E^2:	a is a son of b.	F^1:	a is a person	G^1:	a is a physician
H^1:	a is a marine biologist.	L^2:	a is a brother of b.	L^1:	a lives in town.
M^2:	a is married to b.	O^2:	a is older than b.		
a^1:	the boss of a.	b^1:	the ex-husband of a.	a^0:	Adam.
b^0:	Bryan.	c^0:	Carrie	d^0:	Doreen.

a) Bryan is Carrie's son. $E(bc)$

b) None of the Adam's sons are baseball players, but some of his daughters are. $\sim\exists x(E(xa)\land Bx)\land\exists y\exists z(y\neq z \land D(ya)\land D(za) \land By\land Bz)$

c) Bryan is Carrie's only son. $\forall x(E(xc)\leftrightarrow x=b)$

d) Adam is the only person in town taller than Bryan. $\forall x(Fx\land Lx\land A(xb)\leftrightarrow x=a)$

e) Adam's boss's ex-husband is married to Doreen's ex-husband's boss. $M(b(a(a))a(b(d)))$

f) Adam is Carrie's ex-husband's boss's ex-husband. $a=b(a(b(c)))$

g) Carrie is not Adam's boss's ex-husband's boss. $\neg c=a(b(a(a)))$

h) Doreen's only daughter is a physician. $\exists x(\forall y(D(yd)\leftrightarrow y=x)\land Gx)$

i) Although Carrie and Bryan have at least one son together, Carrie is married to somebody else. $\exists x(E(xc)\land E(xb))\land\exists y(y\neq b\land M(cy))$

j) All of Carrie's sons and daughters live in town. $\forall x(D(xc)\lor E(xc)\rightarrow Lx)$

k) Doreen's ex-husband is married to Carrie only if Doreen is married to one of Carrie's brothers. $M(b(d)c)\rightarrow\exists x(L(xc)\land M(dx))$

l) Bryan's sons and Adam's daughters are baseball players. $\forall x(E(xb)\lor D(xa)\rightarrow Bx$

m) Carrie is married to Adam's only son. $\exists x(\forall y(E(ya)\leftrightarrow =xy)\land M(cx))$

n) There is only one physician in town other than Bryan. $\exists x(\forall y(Gy\land Ly\land y\neq b\leftrightarrow x=y)\land Gb\land Lb$

o) Carrie and Adam have one son.

p) Carrie and Adam each have exactly one son.

q) Carrie's ex-husband is married to Doreen.

r) Carrie's boss's ex-husband is the person who is married to Bryan's boss.

s) Just one of Doreen's daughters is a physician.

t) The physician who lives in town is also a marine biologist.

u) Bryan's daughter is a baseball player.

v) Adam's son has one son.

w) One of Adam's sons is a baseball player, but none of his other children are.

x) The oldest person in town is not married to anybody. NOTE: the oldest person in town is the person in town who is older than everyone else in town.

y) The only marine biologist in town is married to one of Doreen's sons.