2015-09-17
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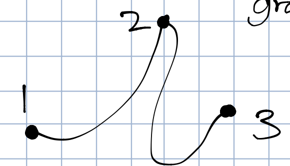
midterm + final + quizzes (based on assigned HW)

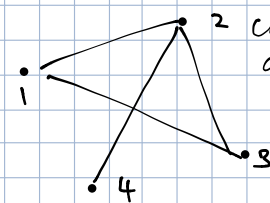
Formal Definition:
V set ("set of vertices)
$E \subset V^2$ = the set of all pairs of vertices
↑ set of edges
$G=(V,E)$ $\quad V=\{1,2,3\}$
$\quad E=\{\{1,2\},\{2,3\}\}$
and an exception $G=(\emptyset,\emptyset)$

Informally:

graph diagram



It shall be continuous curve, but doesn't have to be.



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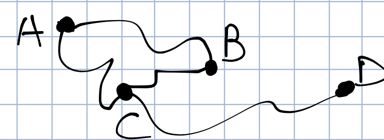
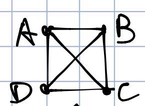


Isomorphism between graphs
$G_1=(V_1,E_1)$, $G_2=(V_2,E_2)$
Isomorphism $f: G_1 \to G_2$
f is bijective map from V_1 to V_2
s.t. ① if $v_1, v_2 \in V_1$, there is an edge $e=\{v_1, v_2\} \in E_1$
then ...

— .



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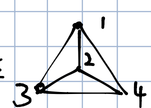
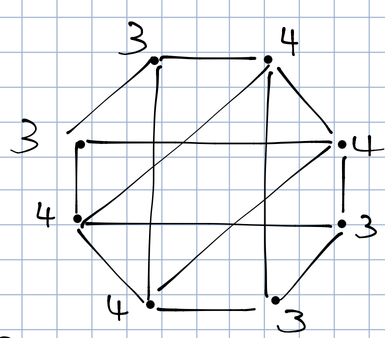
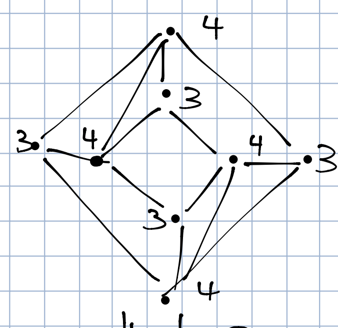


$A \to 1$
$B \to 2$
$C \to 3$
$D \to 4$

K_4 $\qquad K_n$: n vertices, each pair is connected by an edge

degree of a vertex v,
$d(v)=\#$ of edges incident to v
an edge e is incident to v if $v \in e$
If $f: G_1 \to G_2$ is an isomorphism then $\forall v \in G_1, \cdots d(v)=d(f(v))$



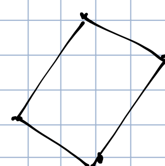


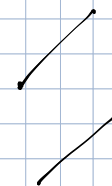
G_1 $\quad |V_1|=8$
$|E_1|=14$

G_2 $\quad |V_2|=8$
$|E_2|=14$

Def: $G=(V,E)$, Let $W\subset V$, $G(W)$ is a subgraph with the set of vertices W and edge $e\in E$ s.t. both __________ INDUCED SUBGRAPH

assume
W_1 the set of vertices of G_1 with degree 4
W_2 the set of vertices of G_2 with degree 4

$G_1(W_1) =$ 

$G_2(W_2) =$ 

Assume that G_1 & G_2 are isomorphism. Any isomorphism preserves degrees

$f: W_1 \to W_2$ It will be 1-1 & onto

↖ all vectors of G_1 of degree 4

f must be an isomorphism between the induce
But these graphs are not isomorphism, graphs, so $G(W_1)$ & $G(W_2)$ are not induced subgraph

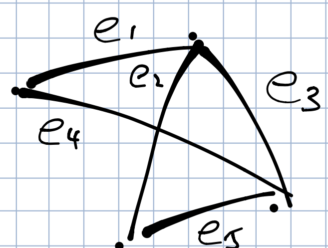
- The subgraphs of K_n is induced subgraph?

$d(v)$ degrees of vertices

$\delta(G)\ \max_{v\in V} d(v)$

$\Delta(G) = \min_{v\in V} d(w)$

$d(V) =$ average degree $= \frac{1}{|V|}\sum_{v\in V} d(v)$ → order of a "graph"?



$e_1, e_2, e_3 \to 3$
$e_3, e_4, e_5 \to 3$
$e_2, e_5 \to 2$
$e_1, e_4 \to 2$

$$\sum_{v\in V} d(v) = d(v_1) + \cdots + d(v_m) = 2|E|$$

$|A|$ = the cardinality of A = # of elements in A

Observation: even
Corrollary: # of vector with odd degrees is even.

$$\varepsilon(G) = \frac{|E|}{|V|} = \frac{1}{2} d(V)$$ lemma next time

Given a graph (finite) G, when is it possible to draw G on the plane so that edges intersect only at their end points.

A graph is planar if one can draw its diagram so that its edges only intersect at its endpoints

Utility graph (UG) is nonplanar.