**CSC165H1 S - Exercise 8**
**Yizhou Sheng, Student# 999362602**
**Rui Qiu, Student# 999292509**
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**Question 1:**

Want to prove: $T(n) \in O(n^2)$ and $T(n) \in \Omega(n)$

Proof:

First show $T(n) \in O(n^2)$ (where n = len(L)):

  Let c′ = 2. Then $c' \in R$
  Let B′ = 1. Then $B' \in R$
  Assume $n \in N$, and L is a list of n numbers, n>= B′
    Case 1: Assume L[0] is even:
      Then the line 1 takes $1 <= n^2$ steps.
      Also, the loop over i iterates exactly $n^2$ times, and each iteration takes 1 step, for a total of $n^2$ steps.
      So the loop over i takes $n^2$ steps.
      So the entire algorithm takes at most $n^2 + n^2 = 2n^2$ steps
    Case 2: Assume L[0] is not even:
      Then line 4 takes $1 <= n^2$ steps.
      Then the loop over i iterates exactly n times, and each iteration takes 1 step, for a total of n steps.
      So the loop body for i takes $n <= n^2$ steps.
      So the entire algorithm takes at most $n^2 + n^2 = 2n^2$ steps
    Then the entire algorithm therefore takes at most $2n^2$ steps.
  Then $\forall n \in N$, $n >= 1 \Rightarrow \forall L \in$ {all lists of numbers}, $len(L)=n \Rightarrow t(L) <= 2n^2$.
  Hence $T(n) \in O(n^2)$.

Then show $T(n) \in \Omega(n)$ (where n = len(L)):

  Let c′ = 1. Then $c' \in R$
  Let B′ = 1. Then $B' \in R$
  Assume $n \in N$, and L is a list of n numbers, n>= B′
    Case 1: Assume L[0] is even:
      Then the line 1 takes 1 steps.
      Also, the loop over i iterates exactly $n^2$ times, and each iteration takes 1 step, for a total of $n^2$ steps.
      So the loop over i takes $n^2 >= n$ steps.
    So the entire algorithm takes at least $n + 1 >= n$ steps
    Case 2: Assume L[0] is not even:
      Then line 4 takes 1 step.
      Then the loop over i iterates exactly n times, and each iteration takes 1 step, for a total of n steps.
      So the loop over i takes at least $1 + n >= n$ steps.
    Then the entire algorithm therefore takes at least n steps.
  Then $\forall n \in N$, $n >= 1 \Rightarrow \forall L \in$ {all lists of numbers}, $len(L)=n \Rightarrow t(L) >= n$.
  Hence $T(n) \in \Omega(n)$.

Therefore $T(n) \in O(n^2)$ and $T(n) \in \Omega(n)$.

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**Question 2:**

Want to prove: $T(n) \in O(n^{1/2})$ and $T(n) \in \Omega(n^{1/2})$

Proof:

First show $T(n) \in O(n^{1/2})$ (where n = len(L)):

Let $c' = 15$. Then $c' \in R$

Let $B' = 1$. Then $B' \in R$

Assume $n \in N$, n = len(L) >0 and L is a list of n numbers.

Then line 1 and line 2 altogether take $2 <= 2(n^{1/2})$ steps.

Then the while loop iterates k times and each iteration takes 3 steps and 1step of condition check

Also it takes 1 step to exit the loop.

Then the loop body takes 4k+1 steps

Then (k-1)k/2<= len(L) < k(k+1)/2

Then (k-1)k/2<= n < k(k+1)/2   # since len(L)=n

Then $(k-1)^2/2 <= n < (k+1)^2/2$

Then $k-1 <= (2n)^{1/2} < k+1$

Then $k <= 1+(2n)^{1/2} <= (n^{1/2})+(4n)^{1/2}=3\ (n^{1/2})$

Then $4k+1 <= 4\ (3(n^{1/2}))+1 <= 12\ (n^{1/2})+ n^{1/2}= 13\ (n^{1/2})$

Then the entire algorithm takes at most $2(n)^{1/2}+13(n)^{1/2} <= 15\ (n^{1/2})$ steps

Then $\forall n \in N, n>=1 \Rightarrow \forall L \in$ {all lists of numbers},len(L)=n $\Rightarrow t(L)<=15(n^{1/2})$

Hence $T(n) \in O(n^{1/2})$.

Then show $T(n) \in \Omega(n^{1/2})$ (where n = len(L)):

Let $c' = 4$. Then $c' \in R$

Let $B' = 1$. Then $B' \in R$

Assume $n \in N$, n = len(L) >0 and L is a list of n numbers.

Then line 1 and line 2 altogether take 2 steps.

Then the while loop iterates k times and each iteration takes 3 steps and 1step of condition check

Also it takes 1 step to exit the loop.

Then the loop body takes 4k+1 steps

Then (k-1)k/2<= len(L) < k(k+1)/2

Then (k-1)k/2<= n < k(k+1)/2   # since len(L)=n

Then $(k-1)^2/2 <= n < (k+1)^2/2$

Then $k-1 <= (2n)^{1/2} < k+1$

Then $k > (2n)^{1/2}-1$

Then $k >= (2n)^{1/2}=(2^{1/2}) * (n^{1/2}) > 1 * (n^{1/2})$

Then $4k+1 >= 4(n^{1/2}) +1 >= 4(n^{1/2})$

Then the entire algorithm takes at least $2+4(n^{1/2}) >= 4(n^{1/2})$ steps.

Then $\forall n \in N, n>=1 \Rightarrow \forall L \in$ {all lists of numbers}, len(L)=n $\Rightarrow t(L)>=4(n^{1/2})$

Hence $T(n) \in \Omega(n^{1/2})$

Therefore $T(n) \in O(n^{1/2})$ and $T(n) \in \Omega(n^{1/2})$

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