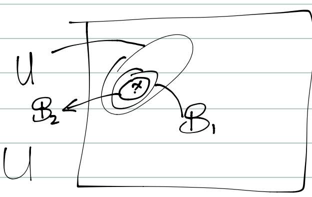
Tutorial 1 May 26th

§2.6 Prove proposititon 9

Let $\mathcal{B}_1, \mathcal{B}_2$ be bases on X, the following are equivalent

1. $\tau_{\mathcal{B}_1} \subset \tau_{\mathcal{B}_2}$
2. $\forall B_1 \in \mathcal{B}_1, x \in B_1, \exists\ B_2 \in \mathcal{B}_2$ s.t. $x \in B_2 \subset B_1$.



$1 \Rightarrow 2$ $\tau_{\mathcal{B}_1} \subset \tau_{\mathcal{B}_2}$

$\forall$ open sets $U \in \tau_{\mathcal{B}_1} \Rightarrow U \in \tau_{\mathcal{B}_2}$

$B_1 \in \mathcal{B}_1 \subset \tau_{\mathcal{B}_1} \Rightarrow B_1 \in \tau_{\mathcal{B}_2}$

$2 \Rightarrow 1$ Let $U \in \tau_{\mathcal{B}_1}$, $\forall x \in U$, $\exists B_1 \in \mathcal{B}_1$ s.t. $x \in B_1 \subset U$

$\exists B_2 \in \mathcal{B}_2$ s.t. $x \in B_2 \subset B_1 \subset U$

so $U \in \tau_{\mathcal{B}_2}$

$\tau_{\mathcal{B}_1} \subset \tau_{\mathcal{B}_2}$

§3.3

Let $\tau = \{U \in P(\mathbb{R}) \mid 0 \notin U \text{ or } U = \mathbb{R}\}$

(a) τ is top on $\mathbb{R}$

(b) closed subsets of $\mathbb{R}$

(c) find $\overline{\{1\}}$

① $\emptyset \in \tau$, $\mathbb{R} \in \tau$

② $A, B \in \tau$

③ $A = \cancel{(a_1, a_2) \setminus \{0\}}$

$B = \cancel{(b_1, b_2) \setminus \{0\}}$ $\quad$ $\cancel{A \cap B = (\max(a_1, b_1), \min(a_2, b_2)) \setminus \{0\}}$

may not be intervals

Tutorial 2 June 2nd

§2.3

A is any subset of $[0,1]$, X is set of all fuction $[0,1] \to [0,1]$.

$B_A = \{f \in X : f(x) = 0, \forall x \in A\}$

WTS $\mathcal{B}$ is a basis for top. on X

Proof: ① Show $\mathcal{B}$ is a basis :

~~(i). $\mathcal{B}$ covers X~~

~~(ii). $\forall B_1, B_2 \in \mathcal{B}$ if $x \in B_1 \cap B_2$, $\exists B \in \mathcal{B}$ containing x st. $B \subseteq B_1 \cap B_2$~~

idea: $B_{[0,\frac{1}{2}]} \cap B_{[\frac{1}{2},1]} = B_{[0,1]}$

So $B_{A_1} \cap B_{A_2} = B_{A_1 \cup A_2}$

§3.9

$\overline{\mathbb{Q}} = \mathbb{R}$

$\forall x \in \mathbb{Q}\ \ x+\sqrt{2} \Rightarrow$ irrational

$A = \mathbb{Q}+\sqrt{2} = \{x+\sqrt{2} : x \in \mathbb{Q}\}$

WTS

$\overline{A} = \mathbb{R}$ und $\mathbb{R}_{usual}$

$\overline{\mathbb{Q}} = \mathbb{R}, \forall x - \sqrt{2} \in \mathbb{R}, B_\varepsilon(x-\sqrt{2}) \cap \mathbb{Q} \neq \emptyset$

$y \in B_\varepsilon(x-\sqrt{2}) \cap \mathbb{Q} \Rightarrow y \in \mathbb{Q}$

$y+\sqrt{2} \in B_\varepsilon(x) \cap \mathbb{Q}+\sqrt{2}$

$\Rightarrow \forall x \in \mathbb{R}$ can find an basic open set that intersects $(\mathbb{Q}+\sqrt{2})$ non-empty

$\Rightarrow x \in \overline{\mathbb{Q}+\sqrt{2}}$

$\Rightarrow \overline{\mathbb{Q}} = \mathbb{R}$

§2.5

In our proof that a basis generates a topology we showed that the topology was closed under finite intersections. Explain this proof in words and pictures. Also explain why this property is not just immediate from the fact that a basis is directed.

Both direction is proved by induction similarly.

$P(n)$ = Given $A_1, \cdots, A_n$ to be arbitrary element in $\mathcal{T}$, then $\bigcap_{i=1}^{n} A_i \in \mathcal{T}$

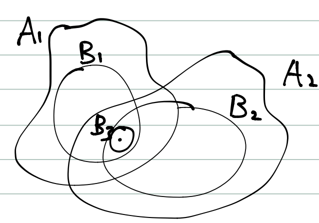
$P(2)$ = Given A_1, A_2, $A_1 \cap A_2 \in \mathcal{T}$

Assume $P(n-1)$ to be true, prove $P(n)$ to be true.

$\therefore$ Just need to show $A_1 \cap A_2 \in \mathcal{T}$

① $\forall x \in X, \exists B \in \mathcal{B}$, s.t. $x \in B$

② Given B_1, B_2, $x \in B_1 \cap B_2$, $\exists B_3 \subseteq B_1 \cap B_2$ s.t. $x \in B_3$ & $B_3 \in \mathcal{B}$.



Wts $\exists$ a bijection from $\mathbb{N}$ to the collection of all binary strings.

The size of the collection is 2^n, $n \in \mathbb{N}$

§4.7

4.8.

$f, g : \mathbb{N} \to \mathbb{N}$

$f \lesssim g : \exists N \in \mathbb{N}, \forall n \geq N, f(n) \leq g(n)$

4.7. $f_1, \cdots, f_{1000}$, $\exists g$, $\forall i, i \in [1, 1000]$. $f_i \lesssim g$

$g(x) = \max(f_1(x), \cdots, f_{1000}(x))$

$f_i(x) \leq g(x)$

4.8. $f_1, f_2, \cdots$

$\forall i \in \mathbb{N}, f_i \lesssim g$

$g(x) = \max(f_1(x), f_2(x), \cdots, f_x(x))$

$f_k \lesssim g$

$f_k(x) \leq g(x)$ $\forall x \geq k$