STA347 Assignment 3 Solutions
Total: 30 points

November 26, 2013

(1)(5 points)
Since the four possibilities (bb,bg,gb,gg) are equally probable, then

$$P(bb) = P(bg) = P(gb) = P(gg) = \frac{1}{4}$$

Note that the event bb is a subset of both "elder child a boy" and "at least one boy".

$$P(bb \,|\, elder\, child\, is\, a\, boy) = \frac{P(bb \cap \; elder\, child\, is\, a\, boy)}{P(elder\, child\, is\, a\, boy)}$$

$$= \frac{P(bb)}{P(bb) + P(bg)} = \frac{\frac{1}{4}}{\frac{1}{4} + \frac{1}{4}} = \frac{1}{2}$$

$$P(bb \,|\, at\, least\, one\, boy) = \frac{P(bb \cap \; at\, least\, one\, boy)}{P(at\, least\, one\, boy)} = \frac{P(bb)}{1 - P(gg)} = \frac{\frac{1}{4}}{1 - \frac{1}{4}} = \frac{1}{3}$$

(2)(5 points)
A = The first player holds all four aces
B = He holds the ace of hearts
C = He holds at least one ace
Note that the event A is a subset of both B and C.

$$P(A|B) = \frac{P(AB)}{P(B)} = \frac{P(A)}{P(B)} = \frac{\binom{48}{9}}{\binom{51}{12}} = \frac{132}{12495}$$

$$P(A|C) = \frac{P(AC)}{P(C)} = \frac{P(A)}{P(C)} = \frac{\binom{48}{9}}{\binom{52}{13} - \binom{48}{13}} = \frac{5}{1318}$$

(3)(5 points)

Event	Probability	Outcome
L	$1-p$	Ruined
WW	p^2	3 dollars
WLL	$[p(1-p)](1-p)$	Ruined
WLWW	$[p(1-p)]p^2$	3 dollars
WLWLL	$[p(1-p)]^2(1-p)$	Ruined
WLWLWW	$[p(1-p)]^2p^2$	3 dollars
....	....	....

So the series of events in which the gambler is ruined is:

$$\sum_{n=0}^{\infty}(1-p)[p(1-p)]^n = (1-p)\sum_{n=0}^{\infty}[p(1-p)]^n = \frac{1-p}{1-p(1-p)}$$

(4)(5 points)

$$P(Positive) = P(Positive|Disease)*P(Disease)+P(Positive|NoDisease)*P(NoDisease)$$

$$= 95\% * 1\% + 5\% * 99\% = 0.059$$

$$P(Negative) = P(Negative|Disease)*P(Disease)+P(Negative|NoDisease)*P(NoDisease)$$

$$= 5\% * 1\% + 95\% * 99\% = 0.941$$

a)

$$P(Disease|Positive) = \frac{P(Disease) * P(Positive|Disease)}{P(Positive)} = \frac{1\% * 95\%}{0.059} = 0.161$$

b)

$$P(NoDisease|Negative) = \frac{P(NoDisease) * P(Negative|NoDisease)}{P(Negative)} = \frac{99\% * 95\%}{0.941} = 0.9995$$

(5)(5 points)

$$P(A \cup B|C) = \frac{P[(A \cup B) \cap C]}{P(C)} = \frac{P[AC \cup BC]}{P(C)}$$

$$= \frac{P[AC] + P[BC] - P[ABC]}{P(C)}$$

$$= \frac{P(AC)}{P(C)} + \frac{P(BC)}{P(C)} - \frac{P(ABC)}{P(C)}$$

$$= P(A|C) + P(B|C) - P(AB|C)$$

(6)(5 points)
Sample Space: Ω ={HH,HT,TH,TT}
A={HH,HT}
B={HH,TH}
C={HH,TT}
$P(ABC) = P(HH) = \frac{1}{4}$
$P(A) = P(B) = P(C) = \frac{1}{2}$
$P(ABC) = \frac{1}{4} \neq P(A)P(B)P(C) = \frac{1}{8}$
So, A,B,C are not mutually independent.