MAT 237, Problem Session 3

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Solutions for these problems are only presented during the Problem Solving Sessions W5-6 in SS 2135. You are strongly encouraged to work through the problems ahead of time, and our TA Yiannis will cover the questions you are most interested in. These sessions are very valuable at developing the proper style to present cogent and rigorous mathematical solutions.

This problem solving session contains material from 1.8-2.2.

Problems:

1. For $f : S \subset \mathbb{R}^n \to \mathbb{R}^m$ uniformly continuous and $g : f(S) \subset \mathbb{R}^m \to \mathbb{R}^k$ uniformly continuous, prove (with a rigorous $\epsilon - \delta$ argument) that the composition $g(f(x))$ is uniformly continuous on S.

2. Using the definition of differentiability described in Folland, show that $f(x) = \sqrt{x}$ is differentiable at x = 4.

3. Using the definition of differentiability described in Folland, show that f(x) = sin x is differentiable at the $\pi/3$.

Hint: For the above two questions, first use some algebraic tricks like taking conjugates or the additional formula for sin, then add and substract whatever you need to make the definition work.

4. Use the definition of differentiability (as in 2.16) to prove that the function $f(x, y, z) = 3x^2yz^3 - xy^2$ is differentiable at the point $\mathbf{a} = (1, 2, 3)$. Please carefully specify the derivative, $\mathbf{c}$, and the function E($\mathbf{h}$). Also find $grad(f)(\mathbf{a})$. Then, use the comment presented in the last paragraph on page 58 to prove differentiability of f at the point $\mathbf{a}$.

5. Use the idea of differentials to estimate the value of f(1.023, 1.992) where $f(x, y) = ln(x(y-1))\frac{x^2y^3}{e^{2y-4}}$

6. Exercise 8 from 2.2. Hint: Modify the main idea of Theorem 2.19.