



Question One

A multiple state model has been suggested as a representation of the lifecycle of new boutique fund managers during their first year of operation. The model has three states: *operating* (state 1), *bankrupt* (state 2), and *sold* (state 3). In this model transitions are only possible from the operating state into the bankrupt state and from the operating state into the sold state. The transition intensities for both of these movements are assumed constant. Denote these two intensities by μ and ν , respectively.

a) The following data were observed for 7 new boutique fund managers:

Company	Start	End	Bankrupt	Sold
1	0	1	1	0
2	0	0.7	0	0
3	0	0.3	1	0
4	0.5	1.5	0	0
5	0.8	1	0	1
6	0.2	1	0	0
7	0	2.3	0	0

Key:

Start = The age of the company when it entered observation.

End = The age of the company when it ceased being observed.

Bankrupt = Whether the company ceased being observed due to bankruptcy.

Sold = Whether the company ceased being observed due to it being sold.

Based on this data provide estimates of the two transition intensities.

b) Estimate the probability that 14 or more of a group of 100 new boutique fund managers will go bankrupt during their first 6 months of operation.

Question One - Solutions

a)

Company	Start	End	Bankrupt	Sold	waiting time
1	0	1	1	0	1
2	0	0.7	0	0	0.7
3	0	0.3	1	0	0.3
4	0.5	1.5	0	0	0.5
5	0.8	1	0	1	0.2
6	0.2	1	0	0	0.8
7	0	2.3	0	0	1
			2	1	4.5

$$\hat{\mu} = \frac{2}{4.5}; \hat{\nu} = \frac{1}{4.5}$$

b)

$${}_tP_x^{12} = \frac{\mu}{\mu + \nu} [1 - e^{-(\mu + \nu)t}] \text{ (from notes)}$$

$${}_{0.5}P_x^{12} = \frac{2}{3} [1 - e^{-(\frac{3}{4.5})0.5}] = 0.19$$

$$\# \text{ bank} \sim \text{bin}(100, 0.19)$$

$$\# \text{ bank} \sim N(19, 3.9)$$

$$p(X > 14) = p(Z > \frac{14-19}{3.9}) = p(Z > \frac{14-19}{3.9}) = p(Z > -1.3)$$