Lecture 25

Let $x \in \Lambda$ and $\varepsilon > 0$, choose n s.t. $\frac{1}{2^n} < \varepsilon$

- we write $S(x) = (s_0 s_1 \cdots)$
- Recall $I_{s_0 \cdots s_n} = \{z \in \Lambda, z \in I_{s_0}, Q_c(z) \in I_{s_1}, Q_c^2(z) \in I_{s_2}, \cdots, Q_c^n(z) \in I_{s_n}\}$

which are closed

Also $I_{s_0 s_1 \cdots s_n}$ and $I_{t_0 \cdots t_n}$ are disjoint
iff $(s_0, s_1, \cdots, s_n) \neq (t_0, t_1, \cdots, t_n)$

And then $|x - z| > \delta$ for some $\delta > 0$
$x \in I_{s_0 s_1 \cdots s_n}$
$z \in I_{t_0 t_1 \cdots t_n}$

So if $y \in \Lambda$ and $|x - y| < \delta$, then $y \in I_{s_0 s_1 \cdots s_n}$
Then $S(x)$ and $S(y)$ are the same for $(n+1)$ entries $(0, \cdots, n)$
- By the proximity thm, $d[S(x), S(y)] < \frac{1}{2^n} < \varepsilon \Rightarrow S$ is continuous $\blacksquare$

④ Exercise

- Remarks: The sets Λ and Σ are homeomorphic
This means that if p and q are close in Λ, then $S(p)$ and $S(q)$ are close in Σ.

Def: Let $F: X \to X$ and $G: Y \to Y$ be 2 functions
F and G are called conjugate if there is a homeomorphism $h: X \to Y$ s.t. $h \circ F = G \circ h$

$$\begin{tikzcd} X \arrow[r, "F"] \arrow[d, "h"] & X \arrow[d, "h"] \\ Y \arrow[r, "G"] & Y \end{tikzcd}$$

- The map h is called a conjugacy

- We have just proved that
* Thm (The Conjugacy Thm)
The shift map $\sigma : \Sigma \to \Sigma$ is conjugate of $Q_c : \Lambda \to \Lambda$ when $c < -2$
- This implies that $Q_c^n = S^{-1} \circ \sigma^n \circ S$
$\sigma^n = S \circ Q_c^n \circ S^{-1}$

Remark:
① The dynamics of σ are identical to the dynamics of Q_c
② S converts orbits of Q_c into orbits under σ.
③ s is a periodic pt of σ iff $S^{-1}(s)$ is a periodic pt of Q_c with the same period.

CHAPTER 10 CHAOS

§10.1 Three properties of a chaotic system

① Density ② Transitivity ③ Sensitivity

Density : Let X be a set and $Y \subseteq X$
Assume that d is a distance in X
We say that Y is dense in X if
① For any open set $A \subseteq X$, $A \cap Y \neq \emptyset$
or ② equivalently, for any $x \in X$, $\exists\, y_n \in Y$ s.t. $\lim\limits_{n \to \infty} d(x, y_n) = 0$
or ③ equivalently, $\bar{Y} = X$

Ex: 1). $\mathbb{Q}$ is dense in $\mathbb{R}$.
$\sqrt{2} = 1.41421356237 \ldots$
2) (a, b) is dense in $[a, b]$
3). $\mathbb{Z}$ is not dense in $\mathbb{R}$ or $\mathbb{Q}$.
$\frac{1}{2} \in \mathbb{R}$ and $(\frac{1}{4}, \frac{3}{4})$ is an open set
$\frac{1}{2} \in (\frac{1}{4}, \frac{3}{4}) \subseteq \mathbb{R}$
and $\mathbb{Z} \cap (\frac{1}{4}, \frac{3}{4}) = \emptyset$