University of Toronto
Faculty of Arts and Science

MAT224H1S
Linear Algebra II

Final Examination
April 2010

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Duration: 3 hours

PLEASE HAND IN

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Last Name: ______________________

Given Name: ______________________

Student Number: ______________________

No calculators or other aids are allowed.

FOR MARKER USE ONLY	
Question	Mark
1	/10
2	/10
3	/10
4	/10
5	/10
6	/10
TOTAL	/60

[10] **1.** Consider the subspace $W = span\{(1, i.1 - i), (i, -1, 0)\}$ of $\mathbb{C}^3$. Find an orthonormal basis for $W^{\perp}$.

EXTRA PAGE FOR QUESTION 1 - please do not remove.

[10] **2.** Let T be the linear operator on $\mathbb{C}^2$ defined by

$$T(z_1, z_2) = (-iz_1 - z_2, z_1 + iz_2).$$

(a) Show that T is normal.

(a) Find an orthonormal basis for $\mathbb{C}^2$ consisting of eigenvectors of T.

EXTRA PAGE FOR QUESTION 2 - please do not remove.

[10] **3.** Show that if T is a normal linear operator on an inner product space V, then

$$ker(T) = ker(T^*).$$

[10] **4.** Let $T\colon \mathbb{R}^3 \to \mathbb{R}^3$ be the linear operator that has the matrix

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$$

realtive to the standard basis of $\mathbb{R}^3$. Find the spectral decomposition of T (relative to the standard basis of $\mathbb{R}^3$).

EXTRA PAGE FOR QUESTION 4 - please do not remove

[10] **5.** Let $P\colon \mathbb{R}^n \to \mathbb{R}^n$ be a linear operator, and W a subpsace of $\mathbb{R}^n$. Show that if $P^2 = P$ and P is symmetric, then P is the orthogonal projection onto W.

[10] **6.** Let $T\colon \mathbb{C}^4 \to \mathbb{C}^4$ be the linear operator that has the matrix

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & -1 \\ 0 & 0 & i & 1 \\ 0 & 0 & 0 & i \end{pmatrix}$$

realtive to the standard basis of $\mathbb{C}^4$. Find a basis of $\mathbb{C}^4$ such that the matrix of T relative to this basis is the Jordan canonical matrix J for T, and find J.

EXTRA PAGE FOR QUESTION 6 - please do not remove.