Solutions for these problems are only presented during the Problem Solving Sessions W5-6 in SS 2135. You are strongly encouraged to work through the problems ahead of time, and our TA Yiannis will cover the questions you are most interested in. These sessions are very valuable at developing the proper style to present cogent and rigorous mathematical solutions.

This problem solving session contains material from 5.1 and 5.2.

Problems:

1. Consider the following two paths that connect point $A(0,0)$ to $B(1,1)$:

$$C_1 : \mathbf{g}(t) = (x(t), y(t)) = (t, t^2), \quad 0 \le t \le 1,$$

and

$$C_2 : \mathbf{h}(t) = (x(t), y(t)) = (1 - 2t, 4t^2 - 4t + 1), \quad 0 \le t \le \frac{1}{2}.$$

   a) Draw these curves (indicate the orientation in each case), and then calculate the line integrals $\int_{C_1} d\mathbf{x}$ and $\int_{C_2} d\mathbf{x}$ as well as the arc length of each curve.
   b) Let $f(x, y) = x$ and integrate $\int_{C_1} f(x, y)ds$ and $\int_{C_2} f(x, y)ds$.
   c) Let $\mathbf{F}(x, y) = (y, x)$ and integrate $\int_{C_1} \mathbf{F} \cdot d\mathbf{x}$ as well as $\int_{C_2} \mathbf{F} \cdot d\mathbf{x}$.
   d) First draw the vector field and then use the $\mathbf{F}_{tang}$ formula as in Section 5.1 to calculate the work of the vector field $\mathbf{F}(x, y) = (x, x)$ along the above curves.

2. Let C be the boundary of the square with corners placed at (0,0), (1,0), (1,1), and (0,1), and let this be the orientation of the curve. Calculate the line integral:

$$\int_C \mathbf{F} \cdot d\mathbf{x} = \int_C F_1 dx + F_2 dy,$$

where $\mathbf{F}(x, y) = (-y, x)$.

3.Repeat the previous question with the curve C replaced by the boundary of the quarter of the disc of radius 1 and centered at the origin in the first quadrant oriented counterclockwise.