DERIVATION PROBLEMS USING BASIC RULES

The first sets of questions use only the following rules: MP, MT, S and ADJ. If they seem easy, that's great! MP, MT and S (or SL and SR) are rules that break down sentences. ADJ is a building rule.

You can do these questions on paper or, if you want, you can use Logic 2010 (and it will check your work). To use Logic 2010, open the derivations module, then click user. Now type the question in.

These are all Basic DD (no sub-derivations):

W → R. W. R → X. ∴ X

P ∧ Q. Q → S. ∴ S ∧ P

~Y. S → Z. Z → Y. ∴ ~S

~W → R. ~R ∧ T. ∴ ~~W ∧ T

(W → T) → S. (W → T). S → Z. ∴ Z

R → ~Q. ~P → Q. R. ∴ ~~P

W. W → (P ∧ Q). (P ∧ W) → T. ∴ Q ∧ T

~(P ∧ Q). S → R. R → (P ∧ Q). ∴ ~S ∧ ~R

These are all Basic CD: Don't forget that you can use your assumption for CD as well as the premises!

Q → S. T → ~S. ∴ T → ~Q

P → (Q → S). Q. ∴ P → S

(S → T) ∧ (W → S). X → W. ∴ ~T → ~X

W → (Q ∧ ~R). Q → S. T → R. ∴ W → (S ∧ ~T)

S → (P → R). Q→S. ∴ (Q ∧ ~R) → ~P

S. T → (W ∧ ~X). R → X. ∴ (S → T) → (W ∧ ~R)

These are DD and CD mixed: Don't forget to analyze your show line. If it is a → sentence, assume antecedent for CD!

Q → ~R. P. ~S →R. ∴ (P → Q) → S

T → X. ~X ∧ R. (~T ∧ R) → Z ∴ Z ∧ ~X

X. Q → W. P →Q. ∴ (W → ~X) → ~P

~(Y→W). (S ∧ T) → Z. Z → (Y →W). ∴ ~(S ∧ T)

T → ~S. ~R → T. T → (Q → S). ~R. ∴ ~Q

R → (~S → T). R ∧ ~S. ~W → S. ∴ T ∧ ~~W

P → (W → Z). ~S → P. ∴ (~Z ∧ ~S) → ~W

~Q → (T → S). P ∧ (R → ~Q). P → (T ∧ R). ∴ S

~~W. (X → P) ∧ (R →X). ∴ (P → ~W) → ~R

P ∧ (P → T). (S ∧ T) → ~Q. ∴ (P → S) → ~Q

The following are all basic ID's. Indirect Derivation, no sub-derivations. (Still just MP, MT, S and ADJ)

$\sim X \rightarrow P.\;\; Q \rightarrow X.\;\; P \rightarrow Q.\;\; \therefore X$

$S \rightarrow T.\;\; T \rightarrow R.\;\; \sim R \wedge S.\;\; \therefore Z$

$Q \rightarrow (S \wedge \sim T).\;\; (R \rightarrow T) \wedge (S \rightarrow R).\;\; \therefore \sim Q$

$T.\;\; \sim W \rightarrow (S \rightarrow R).\;\; R \rightarrow W.\;\; (\sim W \wedge T) \rightarrow S.\;\; \therefore W$

$S \rightarrow (R \wedge \sim W).\;\; P \rightarrow W.\;\; (R \wedge \sim P) \rightarrow \sim T.\;\; \therefore \sim (S \wedge T)$

$S \rightarrow Q.\;\; \sim Q \wedge T.\;\; T \rightarrow X \wedge Y.\;\; Y \wedge \sim S \rightarrow \sim X.\;\; \therefore P$

$\sim T \wedge P \rightarrow Q.\;\; P \rightarrow \sim S.\;\; P \wedge (T \rightarrow Z).\;\; Z \rightarrow Q.\;\; \therefore \sim (Q \rightarrow S)$

Now Mixed DD, CD and ID (Still no sub-derivations)

$(Q \wedge \sim R).\;\; S \rightarrow (T \rightarrow R).\;\; Q \rightarrow S.\;\; \sim T \rightarrow \sim P.\;\; \therefore \sim P$

$X \rightarrow (\sim T \wedge R).\;\; X.\;\; (R \wedge W) \rightarrow \sim P.\;\; S \rightarrow T.\;\; \sim S \rightarrow W.\;\; \therefore \sim P \wedge \sim T$

$R \rightarrow \sim S.\;\; P.\;\; P \rightarrow R.\;\; \therefore (T \rightarrow S) \rightarrow \sim T$

$(Q \rightarrow S) \wedge (\sim S \wedge T).\;\; \sim P \rightarrow W.\;\; W \wedge T \rightarrow Q.\;\; \therefore \sim (P \rightarrow Q)$

$\sim X \rightarrow (W \wedge \sim Y).\;\; W \rightarrow S.\;\; \sim X.\;\; \therefore (T \rightarrow Y) \rightarrow (\sim T \wedge S)$

Now add in MTP and ADD – both rules involve the disjunction $\vee$.

$P \vee Q.\;\; P \rightarrow R.\;\; \sim R.\;\; \therefore Q$

$R.\;\; (R \vee S) \rightarrow T.\;\; \therefore T$

$\sim W.\;\; X \vee W.\;\; X \rightarrow P.\;\; \therefore P \vee Z$

$P \vee T \rightarrow \sim Z.\;\; Z \vee S.\;\; \therefore P \rightarrow R \vee S$

$P \vee (S \rightarrow T).\;\; R \vee Q \rightarrow \sim T.\;\; \sim S \rightarrow P.\;\; \therefore \sim (\sim P \wedge Q)$

Now add in DN: it is mostly needed to build matching sentences to use with MT and MTP. MT requires a line which is the consequent with ~ in front of it. MTP requires a line which is one disjunct with ~ in front of it.

$\sim (P \rightarrow Q) \rightarrow R.\;\; P.\;\; \therefore \sim R \rightarrow Q$

$\sim R \vee \sim S.\;\; \sim S \rightarrow \sim T.\;\; S \rightarrow R.\;\; \therefore \sim T$

$Z \vee T \rightarrow S.\;\; \sim P \rightarrow (Z \wedge \sim Q).\;\; \sim P.\;\; \sim (R \rightarrow Q) \rightarrow P\;\; \therefore \sim (R \vee \sim S)$

$\sim (R \rightarrow S) \rightarrow \sim T.\;\; P \rightarrow T.\;\; \sim R \rightarrow \sim W.\;\; \sim W \vee P.\;\; \therefore W \rightarrow S$

These also use BC and CB. So that's ALL the basic rules and derivation types. (But no sub-derivations!)

$P \wedge Q.\;\; (R \vee P) \rightarrow \sim S.\;\; S \vee T.\;\; (V \rightarrow W) \leftrightarrow Q.\;\; W \rightarrow V.\;\; \therefore T \wedge (V \leftrightarrow W)$

$(W \rightarrow V) \leftrightarrow (X \vee P).\;\; T \wedge \sim Q \rightarrow (V \rightarrow W).\;\; (V \leftrightarrow W) \rightarrow S.\;\; P \vee Q.\;\; Q \rightarrow \sim R.\;\; R \wedge T.\;\; \therefore S$

$(P \wedge \sim R) \rightarrow (Q \rightarrow Z).\;\; (Z \leftrightarrow Q) \rightarrow W.\;\; \sim R \rightarrow (Z \rightarrow Q).\;\; S \leftrightarrow T.\;\; P \rightarrow \sim S.\;\; \sim P \vee \sim R\;\; \therefore P \rightarrow (\sim T \wedge W)$

$S \rightarrow (W \wedge (S \rightarrow P)).\;\; V \wedge S.\;\; X \leftrightarrow (V \wedge W).\;\; (X \vee Q) \rightarrow (P \rightarrow S).\;\; \sim (P \leftrightarrow S) \vee Z.\;\; \therefore Z \vee R$

All of the following problems involve some subderivations.

Subderivations begin when you put a show line after line 1 of a derivation.

The first set of problems all involve subderivations for one of two reasons:

Initial show line is a conditional but the consequent is also a conditional.
After show line 1, assume the antecedent (ASS CD)
On the line after ASS CD, begin a new conditional subderivation: Show CONSEQUENT

Initial show line is a biconditional ↔
There will be two conditional subderivations .
Show LEFT SIDE → RIGHT SIDE (This should be on line 2)
Show RIGHT SIDE → LEFT SIDE (This goes after you finish the previous subderivation.)
Now use CB to derive the initial show sentence from the two canceled show lines.

$P \rightarrow R$. $S \vee T$. $(T \wedge R) \rightarrow W$. $\therefore \sim S \rightarrow (P \rightarrow W)$

$\sim\sim W$. $X \rightarrow P$. $\therefore$ $(P \rightarrow \sim W) \rightarrow ((R \rightarrow X) \rightarrow \sim R)$

$\therefore (\sim R \vee S) \rightarrow (R \rightarrow S)$

$\therefore (Q \rightarrow \sim P) \rightarrow ((\sim R \vee Q) \rightarrow (P \rightarrow \sim R))$

$S \vee W$. $W \rightarrow T$. $(T \rightarrow \sim R) \wedge (S \rightarrow R)$. $\therefore \sim S \leftrightarrow T$

$\sim(P \rightarrow R) \rightarrow Q$. $T \vee S$. $\therefore P \rightarrow (\sim T \rightarrow (\sim Q \rightarrow (R \wedge S)))$

$\sim T \vee S \rightarrow \sim Y$. $T \rightarrow \sim W$. $\sim(Q \rightarrow W) \rightarrow Y$. Q. $\therefore W \leftrightarrow \sim Y$

$\therefore (\sim P \rightarrow Q) \leftrightarrow (\sim Q \rightarrow P)$

$(S \wedge T) \rightarrow \sim Q$. $\therefore$ $(P \rightarrow T) \rightarrow ((P \rightarrow S) \rightarrow (P \rightarrow \sim Q))$

$(Z \vee W) \rightarrow \sim R$. $\sim X \vee S$. $(S \rightarrow R) \wedge (S \leftrightarrow \sim X)$. $\sim Z \rightarrow \sim S$. $\therefore Z \leftrightarrow X$

$\therefore$ $((P \vee Q) \wedge (S \rightarrow \sim Q)) \rightarrow ((\sim S \rightarrow R) \rightarrow (\sim R \rightarrow P))$

$\therefore (P \rightarrow (Q \rightarrow R)) \leftrightarrow ((P \wedge Q) \rightarrow R)$

$(P \wedge \sim R) \rightarrow W$. $(P \rightarrow \sim S)$. $\sim P \vee \sim R$ $\therefore P \rightarrow ((S \leftrightarrow T) \rightarrow (\sim T \wedge W))$

$\sim S \vee W$. $T \rightarrow X$. $X \wedge W \rightarrow R$. $R \rightarrow Y \wedge Z$. $Y \vee Q \rightarrow S$. $\sim T \rightarrow \sim Z$. $\therefore (S \wedge T) \leftrightarrow R$

$(\sim P \vee R) \wedge (\sim Q \rightarrow \sim R)$. $Q \leftrightarrow (S \wedge W)$. $S \vee Y \rightarrow T$. $T \wedge W \rightarrow P$. $\therefore P \leftrightarrow Q$

These problems all involve subderivations for the following reason:

Initial show line is a conjunction: There will be two subderivations.
Show LEFT CONJUNCT (This should be on line 2)
Show RIGHT CONJUNCT (This goes after you finish the previous subderivation.)
Now use ADJ to derive the initial show sentence from the two canceled show lines.

For each subderivation, make an assumption for ID or an assumption for CD immediately after the show line!

$R \leftrightarrow S$. $\sim X \rightarrow P$. $Q \rightarrow X$. $P \rightarrow Q$. $R \vee S$. $\therefore X \wedge R$

$Q \rightarrow (S \wedge \sim T)$. $(R \rightarrow T) \wedge (S \rightarrow R)$. $\sim R \vee W$. $\therefore \sim Q \wedge (S \rightarrow W)$

$R \rightarrow (\sim S \wedge \sim Z)$. $\sim X \rightarrow S$. $X \vee Y \rightarrow Z$. $W \vee (\sim Z \wedge Y)$. $\therefore W \wedge \sim R$

$\sim(P \vee S)$. $\therefore \sim P \wedge \sim S$

$S \wedge R \rightarrow \sim Z$. $Z \vee X$. $W \vee \sim Z$. $X \rightarrow W$. $\therefore (R \rightarrow (S \rightarrow X)) \wedge W$

$T \wedge \sim X$. $\sim W \rightarrow (S \rightarrow R)$. $Z \vee T \rightarrow \sim Q$. $R \rightarrow W$. $(\sim W \wedge T) \rightarrow S$. $\therefore W \wedge \sim(Q \vee X)$

$\sim R \rightarrow (S \wedge Q)$. $\therefore (\sim R \rightarrow S) \wedge (\sim Q \rightarrow R)$

$\sim(P \rightarrow Q)$ $\therefore P \wedge \sim Q$

These problems all involve subderivations that are a bit trickier:

When an available line has a conditional with a complex antecedent (sometimes even a simple one!)	Show ANTECEDENT of the available line
When you are trying to find a contradiction.	
and one available line is $\sim(\rightarrow)$	Show the unnegated conditional: $(\rightarrow)$
or one available line is $\sim(\vee)$	Show the unnegated sentence: $(\vee)$

Always look at your show lines! After every show line, you should make an assumption for ID or for CD unless you are showing a conjunction or biconditional (then you might want two new show lines.)

$(P \rightarrow Q) \rightarrow (R \vee S)$. $\therefore (Q \wedge \sim S) \rightarrow R$

$(Z \rightarrow S) \rightarrow (R \wedge \sim W)$. $P \rightarrow W$. $(R \wedge \sim P) \rightarrow \sim T$. $\therefore \sim(S \wedge T)$

$\sim(S \rightarrow T)$. $W \vee T$. $\therefore \sim W \rightarrow Z$

$\therefore ((R \rightarrow T) \rightarrow S) \rightarrow (T \rightarrow S)$

$\therefore \sim(P \rightarrow Q) \rightarrow P \wedge \sim Q$

$\therefore \sim(P \vee Q) \rightarrow \sim P \wedge \sim Q$

$(T \rightarrow R) \rightarrow \sim W$. $W \vee S$. $(\sim S \rightarrow Z) \rightarrow R$. $\therefore R \leftrightarrow S$

$\sim(\sim P \rightarrow Q)$. $\sim(W \rightarrow \sim X)$ $\therefore \sim P \wedge X$

$(W \leftrightarrow X) \rightarrow R$. $Q \rightarrow S$. $\sim Q \wedge X \rightarrow W$. $\therefore (\sim W \vee X) \rightarrow (\sim S \rightarrow R)$