Assignment X: week of March 25th

This is the 10th (and last!) assignment. You are encouraged to work on this by coming to the help sessions (Thursday 12-1, Friday 1-2 at MP202) and **grouping** *up with a few other students. Teaching assistants will be at hand to help. You do not have to hand this one in.*

1. Nuclear fusion and the lifetime of the Sun. The mass of a helium nucleus is 6.6447×10^{-27} kg, while that of a hydrogen nucleus is 1.6737×10^{-27} kg. So when four hydrogen nucleus fuse (in a long chain) to eventually form a helium nucleus, some amount of mass is missing. This missing mass is the source of the Sun's energy. Now assume that the Sun turns all its hydrogen ($\sim 75\%$ of its total mass) into helium, and that it continues to shine at its current luminosity (3.84×10^{26} Watts), how long can it shine?

2. Gas temperature and star formation. Gas that is immersed in the CMB photon bath is heated up to a temperature at least comparable to the CMB temperature (see last assignment). CMB photons are a blackbody with temperature $\sim$ 3000K at redshift 1100. And the blackbody has been cooling down as $T(t) \propto 1/a(t)$, since the expansion of the universe redshifts the wavelengths of CMB photons. If gas has to be cooled below, say, 100 K, in order for stars to form, what is the highest redshift for star formation? For reference, the galactic halo star HD 1523-0901 is determined to have an age of 13.2 Gyrs, corresponding to a redshift of ~ 10 in our current cosmological model.

3. Jupiter is the most massive planet in the Solar System. So in this question we will ignore all other planets in the Solar System but Jupiter. Jupiter and the Sun circle around the center of mass in a 2-body dance. Jupiter's motion is a circle with radius 5.2 astronomical units, while the Sun's motion a circle that is $M_\odot/M_J \sim 1000$ times smaller ($M_\odot$, M_J are the masses of the Sun and Jupiter, respectively). This arises from the fact that the center of mass remains fixed in space. Jupiter moves with a velocity of 13 km/s in its orbit, how large is the velocity of the Sun in its orbit? For an alien observing the Sun (since Jupiter is too faint to be seen) and measuring the Doppler shifts in its spectral lines, how large (express in $\Delta\lambda/\lambda$) is the Doppler shift?