Assignment VII: week of Mar. 5th

This is the 7th assignment. You are encouraged to work on this by coming to the help sessions (Thursday 12-1, Friday 1-2 at MP202) and **grouping** *up with a few other students. Teaching assistants will be at hand to help. You do not have to hand this one in.*

1. Is the universe finite or infinite? It turns out that matter is not the only form of energy in our universe. Most recent measurements indicate that our universe is composed of $\sim 30\%$ matter, and $\sim 70\%$ dark energy. The two densities add up, miraculously, to almost exactly the critical density. This makes us a 'flat' (or nearly flat) universe. Based on your readings, what do you think is our current answer to the following questions: is the universe spatially finite or infinite? will expansion of the universe continue forever or will it stall and turn-back? Present brief reasoning to your answers (no more than 200 words).

2. Hubble's original measurement for the Hubble constant is $H_0 \sim 500$ km/s/Mpc. This is ~ 7 times greater than the modern measurement ($H_0 \sim 70$ km/s/Mpc). This is because he used the wrong Cepheid calibration – his Cepheids have higher luminosities than he had assumed. If we have taken Hubble's original value, what would be our conclusion about the fate of the universe? You can ignore dark energy, and assume that the true density in the universe is the same as we measure today. First estimate the critical density if H_0 takes the above value, and then estimate Ω_m, the dimensionless density. From this, sketch out the future fate of the universe. Similarly, estimate the age of the universe if H_0 takes the above value. To put this in context, the age of the Solar system is known to be $\sim 4.568 \pm 0.004$ billion years.

3. Define a dimensionless parameter $\Omega_m = \rho/\rho_{\rm crit}$ that measures the true density relative to the critical density. At the current epoch, $t = t_0 = 13.7$ Gyrs, $\Omega_{m0} = \rho_0/\rho_{\rm crit,0} \approx 0.3$, where ρ_0 is the current density, and t_0 the current age. What was the value of Ω_m when the universe was half its current age? For this exercise, assume that the universe expands roughly as an empty one (even though it does have mass). From your last problem set, we have $a(t) = (a_0/t_0)t$, $H(t) = \dot{a}/a = 1/t$. And density at any time is, $\rho(t) = \rho_0 a_0^3/a^3$, here, a_0 is the current scale factor.

4. Define similarly a dimensionless parameter $\Omega_\Lambda = \Lambda^2 c^2/(8\pi G)/\rho_{\rm crit} = \Lambda c^2/(3H^2)$ that measures the importance of the dark energy relative to the critical density. Let Λ be a constant that does not vary. Today, $\Omega_\Lambda \approx 0.7$. How big was Ω_Λ when the above empty universe was half its current age? What about when the universe is twice its current age? or ten times its current age?