Australian National University

STAT 6046 Tutorial Week 6

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Today's plan

- Brief review of course material
- Go through selective tutorial questions

Equation of value

- Basic relationship

$$\text{PV income} = \text{PV outgo}$$

- General formula

$$\sum_{k=1}^{n} I_{t_k} \cdot (1+i)^{-t_k} + \int_0^n \rho_I(t) \cdot (1+i)^{-t}\, dt = \sum_{k=1}^{n} O_{t_k} \cdot (1+i)^{-t_k} + \int_0^n \rho_O(t) \cdot (1+i)^{-t}\, dt$$

Solving for unknown time

- Annuity payments' value equals to a fixed value

$$A = Bs_{\overline{n}|i} \text{ or } A = Ba_{\overline{n}|i}$$

$$n = \frac{-\ln\left(1-\frac{Ai}{B}\right)}{\ln(1+i)} = \frac{-\ln\left(1-\frac{Ai}{B}\right)}{\delta}$$

$$n = \frac{\ln\left(\frac{Ai}{B}+1\right)}{\ln(1+i)} = \frac{\ln\left(\frac{Ai}{B}+1\right)}{\delta}$$

Solving for unknown time

- Annuity payments' value equals to a time-varying value

$$90 s_{\overline{n}|0.01} > 100n \qquad \Rightarrow 90 \frac{(1.01)^n - 1}{0.01} > 100n$$

- Linear Interpolation:

$$\frac{f(n_0) - f(n_1)}{f(n_2) - f(n_1)} \cong \frac{n_0 - n_1}{n_2 - n_1}$$

$$\Rightarrow n_0 \cong n_1 + \frac{f(n_0) - f(n_1)}{f(n_2) - f(n_1)} \cdot (n_2 - n_1)$$

- Note: This approximation works best if the trial values are close to the true value.

Solving for unknown interest rate

- Quadratic formula:

$$ax^2 + bx + c = 0$$

- Solution:

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Inflation

- Real interest rate:

$$i_{real} = \frac{i - r}{1 + r}$$

- Time varying inflation: CPI
- At time 0, purchasing power of $1 at time t:

$$\$1 * \frac{CPI_0}{CPI_t}$$