Box and Tiao Transformation

Consider a transfer function noise model

$$y_t = \frac{\omega(B)}{\delta(B)} x_t + e_t, \quad (1)$$

where $\omega(B) = w_0 + w_1 B + \cdots + w_s B^s$ and $\delta(B) = 1 - \delta_1 B - \cdots - \delta_r B^r$, and the error term, e_t, follows an $ARMA\ (p, q)$ model

$$\phi(B) e_t = \theta(B) a_t,$$

where B is a backward shift operator and a_t is white noise satisfying

$$\phi(B) = 1 - \phi_1 B - \cdots - \phi_p B^p,$$

and

$$\theta(B) = 1 + \theta_1 B + \cdots + \theta_q B^q.$$

Box and Tiao (1975) suggests transforming our variables as

$$\frac{\phi(B)}{\theta(B)} y_t = \tilde{y}_t, \qquad \frac{\phi(B)}{\theta(B)} x_t = \tilde{x}_t, \qquad (2)$$

where $\tilde{y}_t$ and $\tilde{x}$ are the transformed variables. Applying the $\phi(B)/\theta(B)$ filter on both sides of eqn. (1), we have

$$\tilde{y}_t = \frac{\omega(B)}{\delta(B)} \tilde{x}_t + a_t, \quad (3).$$

Rearranging eqn. (3), we have

$$\tilde{y}_t = \sum_{i=1}^{r} \delta_i \tilde{y}_{t-i} + \sum_{j=1}^{s} w_j \tilde{x}_{t-j} + a_t. \quad (4)$$

Since the error term in eqn. (4) is white noise, we could fit it using the least squares regression.

STEPS OF THE ESTIMATION PROCEDURE

The steps of the estimation procedure may be summarized as follows:

1. Run the OLS regression on

$$y_t = \sum_{i=1}^{r} \delta_i y_{t-i} + \sum_{j=1}^{s} w_j y_{t-j} + e_t. \quad (5)$$

   and collect residuals;
2. Indentify an $ARMA$ model for the residuals collected in step 1;
3. Apply Box and Tiao transformation using the model identified in step 2 to filter $\{y_t\}$ and $\{f_{jt}\}$ for all j, t;
4. Run the OLS regression of eqn. (5) on the transformed variables obtained in Step 3;
5. Check whether the regression residuals on Step 4 are serially uncorrelated.
   i. If not, repeat Step 2 to 4;
   ii. If yes, the model estimation is complete.