

# Friedmann's equation for matter-dominated universe

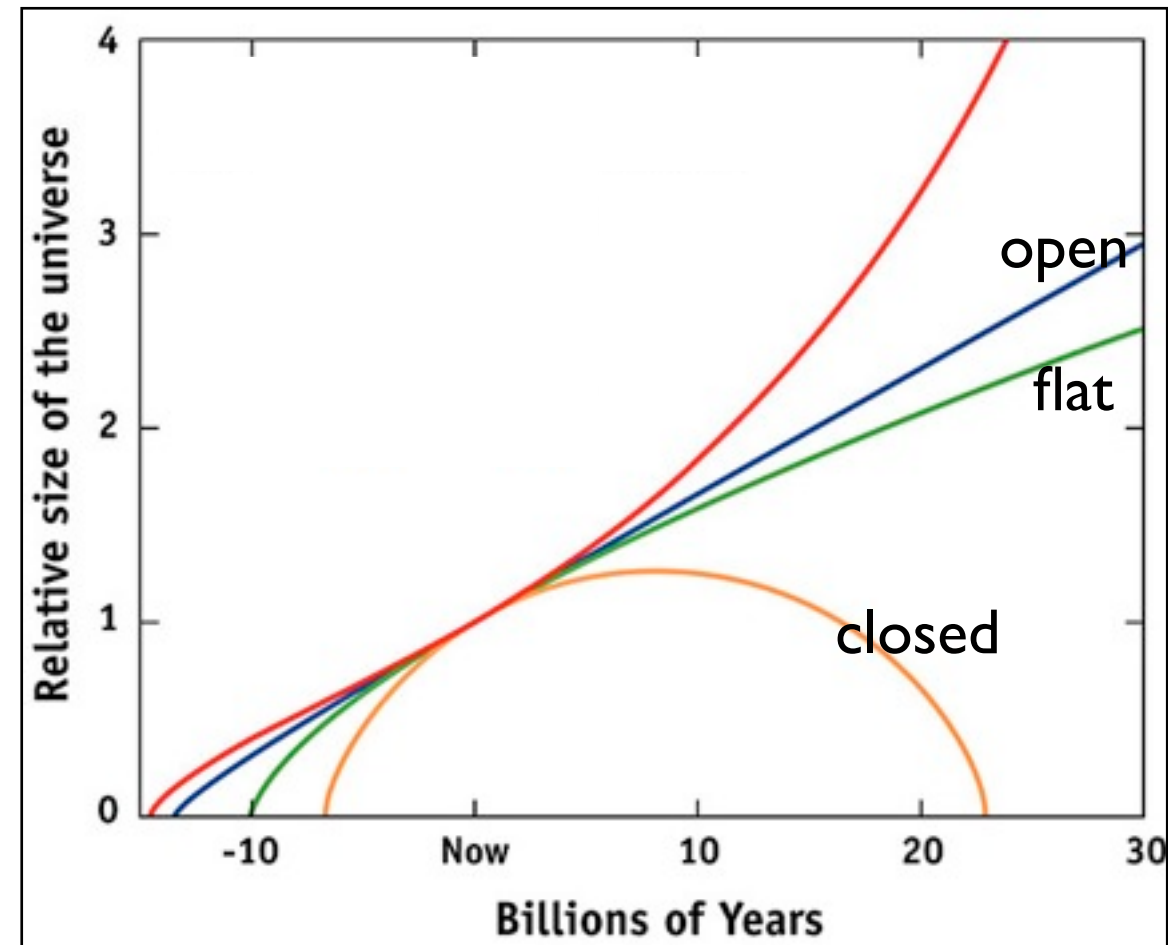
$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2}$$

$$E = \frac{1}{2} m v^2 - GMm/r$$

$$E = 0, \text{ flat, } k = 0$$

$$E > 0, \text{ open, } k = -1$$

$$E < 0, \text{ closed, } k = +1$$



- Expansion of the universe analogous to the motion of a rocket shot upward.
- $H_0$  fixes the current expansion rate. But need to know matter density to tell the future/past.
- In all cases, cosmic expansion slowed down with time as expected --- like the rocket.

# Friedmann's equation for **flat** universe

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} \rightarrow 0$$

$$E = \frac{1}{2} m v^2 - GMm/r = 0$$

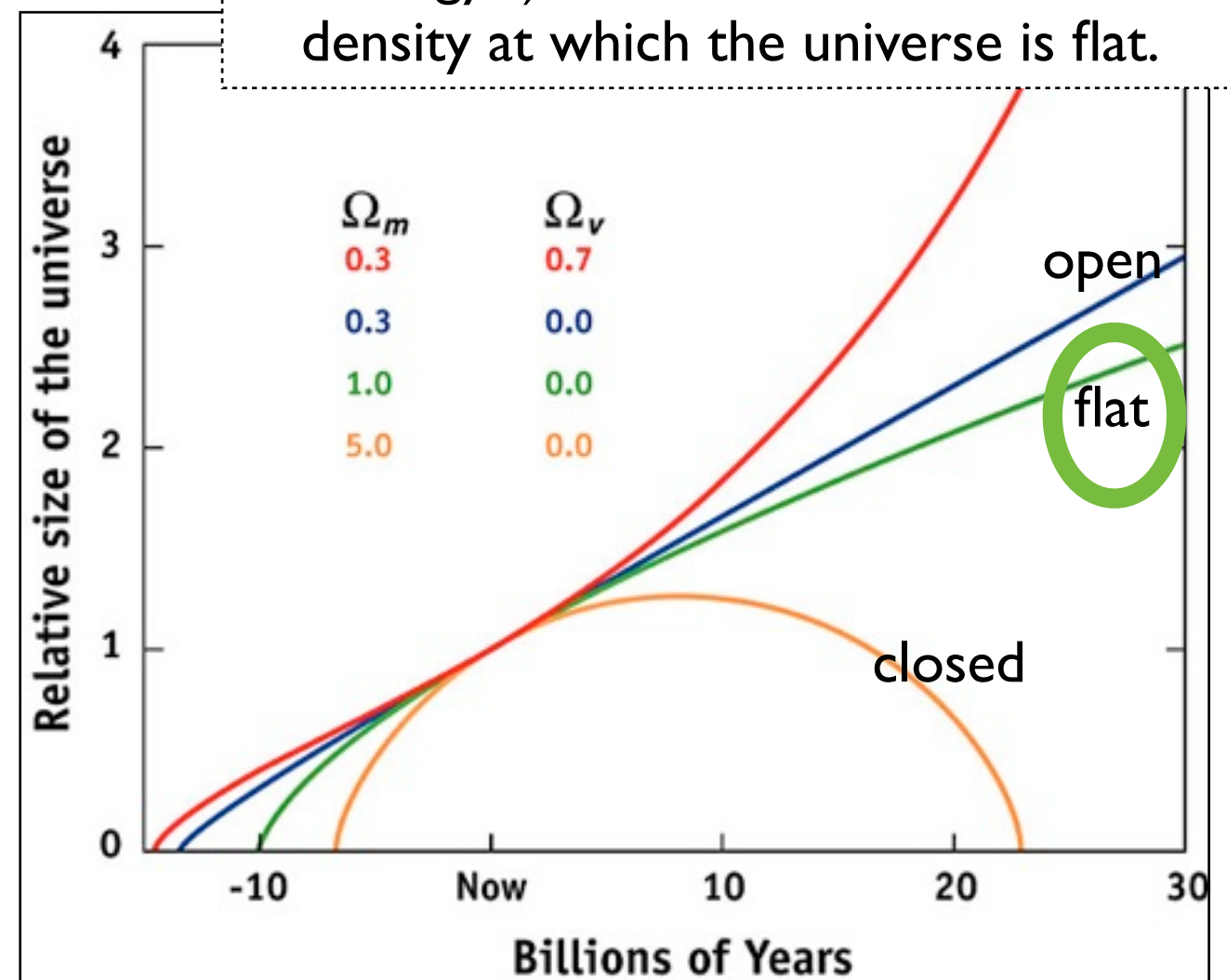
we define  $\rho_{\text{crit}} \equiv \frac{3H^2}{8\pi G}$

as universe ages, expansion  
just so that

$$\rho = \rho_{\text{crit}} \equiv \frac{3H^2}{8\pi G}$$

at all times

**Critical Density** -- in a universe dominated by matter (no radiation, dark energy...), there exists a threshold density at which the universe is flat.



# Defining $\Omega_m$ : gravity against expansion

Friedmann's equation 
$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2}$$

turns into

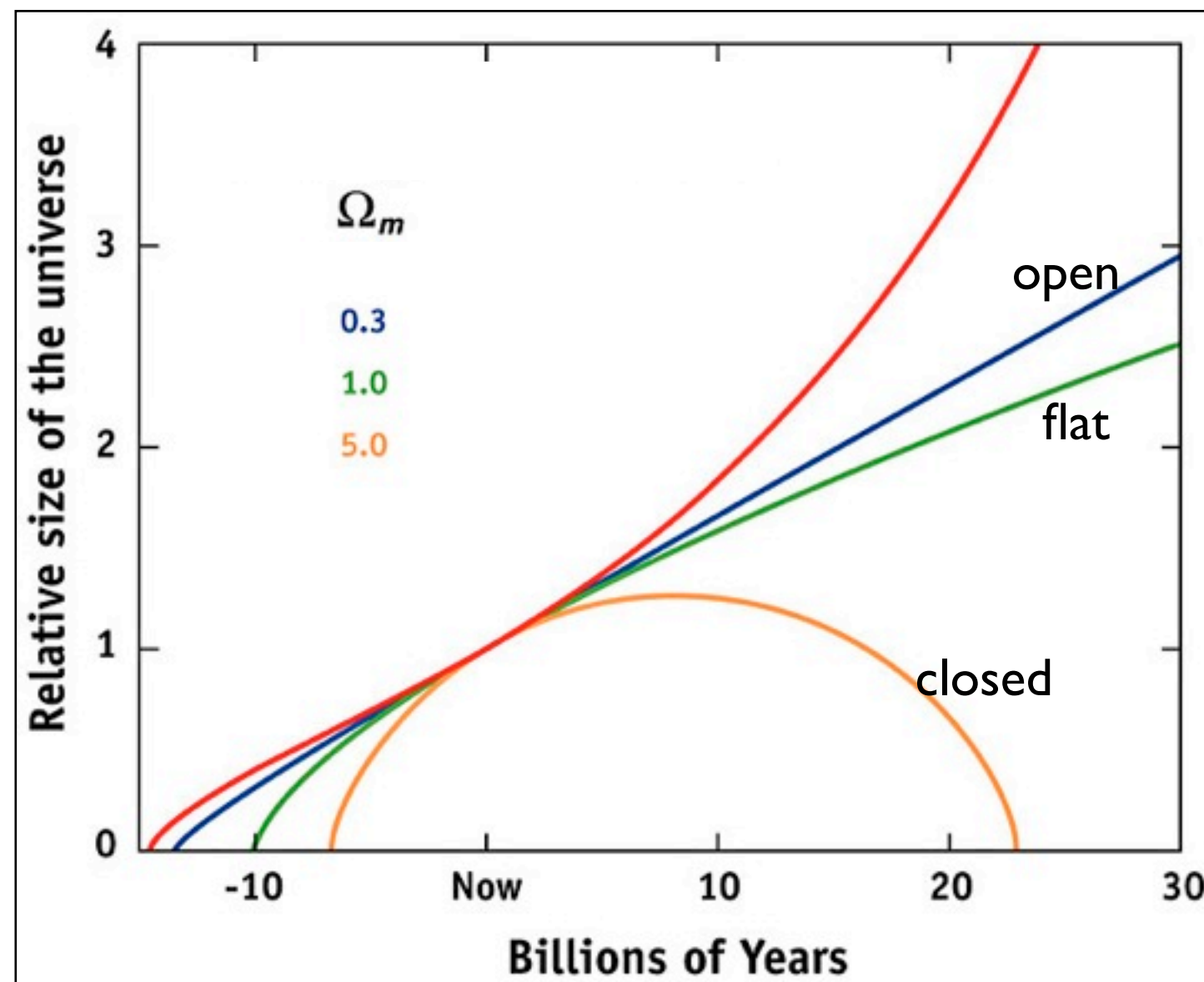
$$\Omega_m \equiv \frac{\rho}{\rho_{\text{crit}}} = 1 + \frac{kc^2}{a^2 H^2}$$

$\Omega_m < 1$  gravity loses, open  
“big freeze”

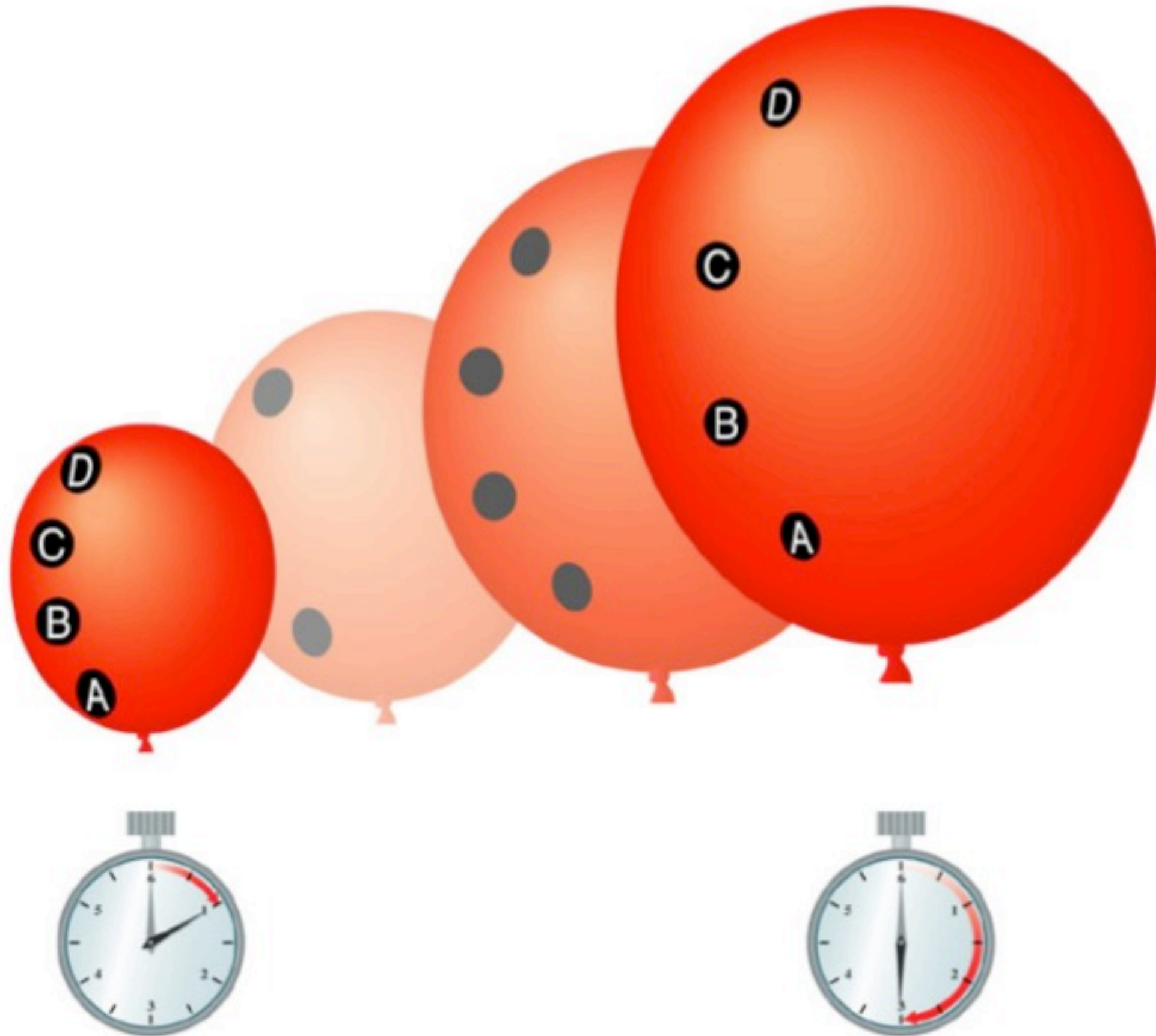
$\Omega_m = 1$  flat

$\Omega_m > 1$  gravity wins, closed  
“big crunch”

(ignoring dark energy for the moment)



# A 2D metaphor for the 3D expansion





# *How does it feel to be living in a ... closed universe*

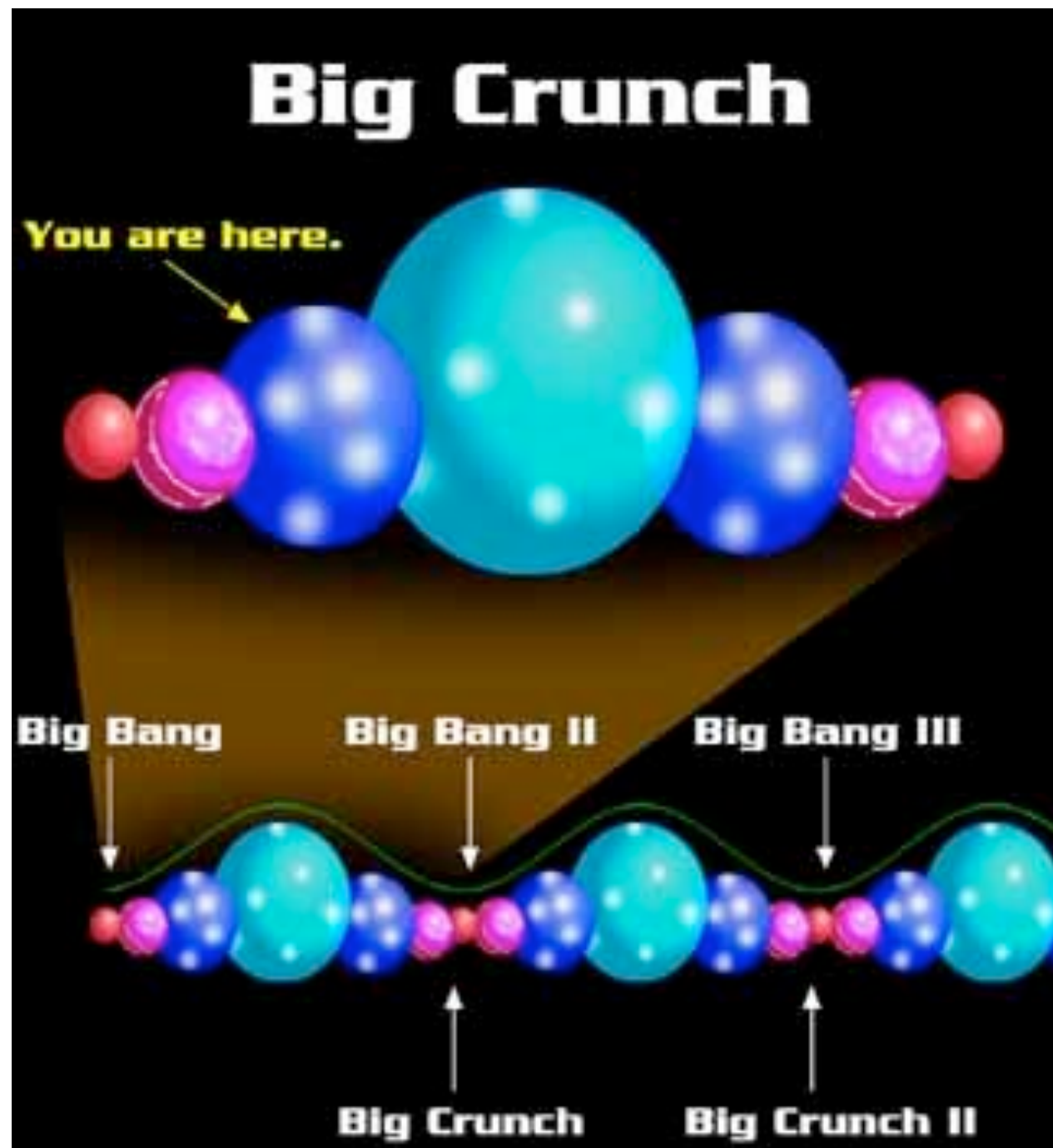
$(\Omega_m > 1)$

to visualize, we imagine the universe sits on a 2-D balloon

- space and number of galaxies finite
- but no edge. nor centre.
- at some point, your horizon encompasses the whole universe
- light travels along geodesics (great circles), you can see your rear, given enough time.
- but universe will contract,  $H < 0$ , big crunch into a single blackhole.



# Life after death? Big Bounce?



some suggest that a big crunch may be the seed for the next big bang, so on, and so on.... cyclical universe

# *How does it feel to be living in an ... open universe* ( $\Omega_m < \text{or} = 1$ )



- space and number of galaxies infinite.
- no edge. nor centre.
- light never returns to same point
- enlargement of your horizon  $<$  expansion of space. finally, only one mega-galaxy left visible.
- with time, the universe cools, all stars run out of nuclear fuel and become dark objects (neutron stars, white dwarfs and blackholes), no more light/heat, cold death (“big freeze”)

# Universe in the far future (Krauss & Scherrer, Sci.American, 2008)

## COSMIC MILESTONES

**10<sup>-30</sup> second**

Cosmic inflation occurs

**100 seconds**

Deuterium and helium are created

**400,000 years**

Microwave background is released

**8 billion years**

Expansion begins to accelerate

**13.7 billion years**

Today

**20 billion years**

Milky Way and Andromeda collide

**100 billion years**

All other galaxies are invisible

**1 trillion years**

Primordial isotopes are lost or diluted

**100 trillion years**

Last star burns out

- future civilization won't discover expansion of the universe
- other galaxies are beyond or approaching horizon, with enormous redshifts
- cosmic microwave background too cold to be observed
- light element ratios lost finger print of big bang
- we live truly in an island universe where all vestiges of the big bang is erased... may come to the wrong idea about the origin of the universe



Show us the number.

# Adding up the mass...

$\Omega_m \sim 0.3$ , gravity loses, open

Fukugita & Peebles, Astrophysical Journal, 2004

Parameter	Components <sup>a</sup>	Totals <sup>a</sup>
Dark sector:		$0.954 \pm 0.003$
Dark energy	$0.72 \pm 0.03$	
Dark matter	$0.23 \pm 0.03$	
Primeval gravitational waves	$\lesssim 10^{-10}$	
Primeval thermal remnants:		$0.0010 \pm 0.0005$
Electromagnetic radiation	$10^{-4.3 \pm 0.0}$	
Neutrinos	$10^{-2.9 \pm 0.1}$	
Prestellar nuclear binding energy	$-10^{-4.1 \pm 0.0}$	
Baryon rest mass:		$0.045 \pm 0.003$
Warm intergalactic plasma	$0.040 \pm 0.003$	
Virialized regions of galaxies	$0.024 \pm 0.005$	
Intergalactic	$0.016 \pm 0.005$	
Intracluster plasma	$0.0018 \pm 0.0007$	
Main-sequence stars: spheroids and bulges	$0.0015 \pm 0.0004$	
Main-sequence stars: disks and irregulars	$0.00055 \pm 0.00014$	
White dwarfs	$0.00036 \pm 0.00008$	
Neutron stars	$0.00005 \pm 0.00002$	
Black holes	$0.00007 \pm 0.00002$	
Substellar objects	$0.00014 \pm 0.00007$	
H I + He I	$0.00062 \pm 0.00010$	
Molecular gas	$0.00016 \pm 0.00006$	
Planets	$10^{-6}$	
Condensed matter	$10^{-5.6 \pm 0.3}$	
Sequestered in massive black holes	$10^{-5.4}(1 + \epsilon_n)$	

# The real density doesn't have to be close to Critical Density

$$\Omega_m = \rho/\rho_{\text{crit}}$$

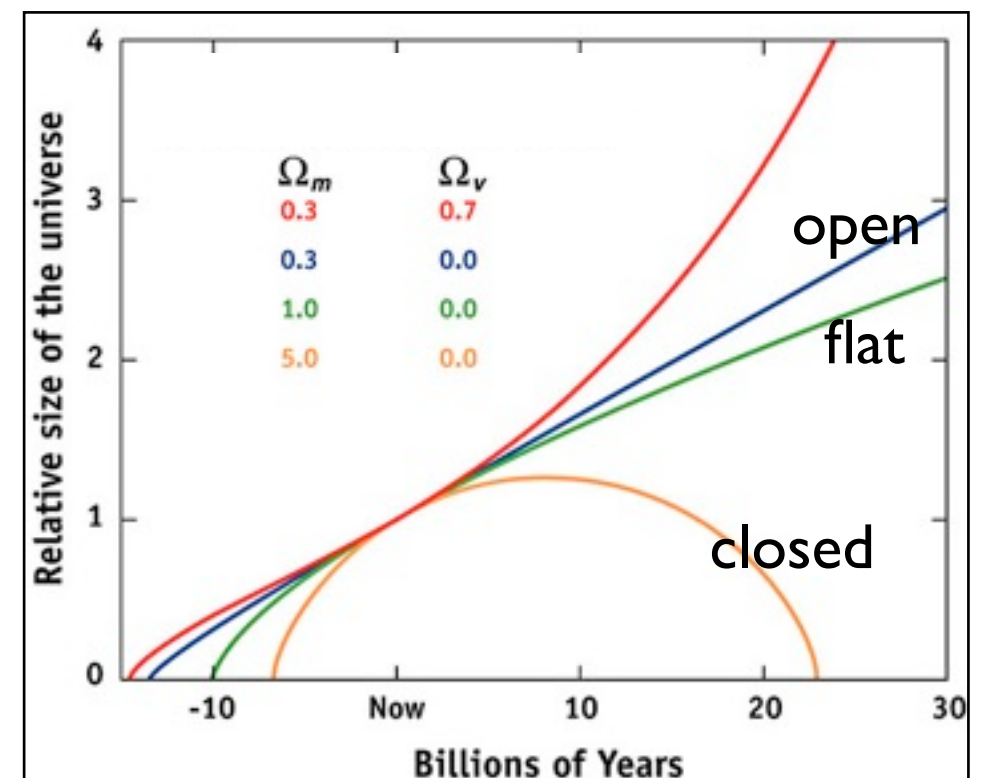
$\Omega_m \ll 1$  expansion too fast, prevent galaxies & stars to form

$\Omega_m \sim 1$ , permitted range for life

$\Omega_m \gg 1$  universe re-collapse too fast before life develops

To have  $\Omega_m \sim 0.3$  today,  $\Omega_m$  will have to be within  $1/10^{15}$  of unity at one second after big-bang

(Rees 'Just six numbers')



Some other ideas...

**Multiverse:** our universe is but one of many universes that occur simultaneously, some matter, some anti... they may collide

avored by some inflation theorists

fundamental constants in our universe conducive to intelligence, other universes?





# density also determines Shape of the universe

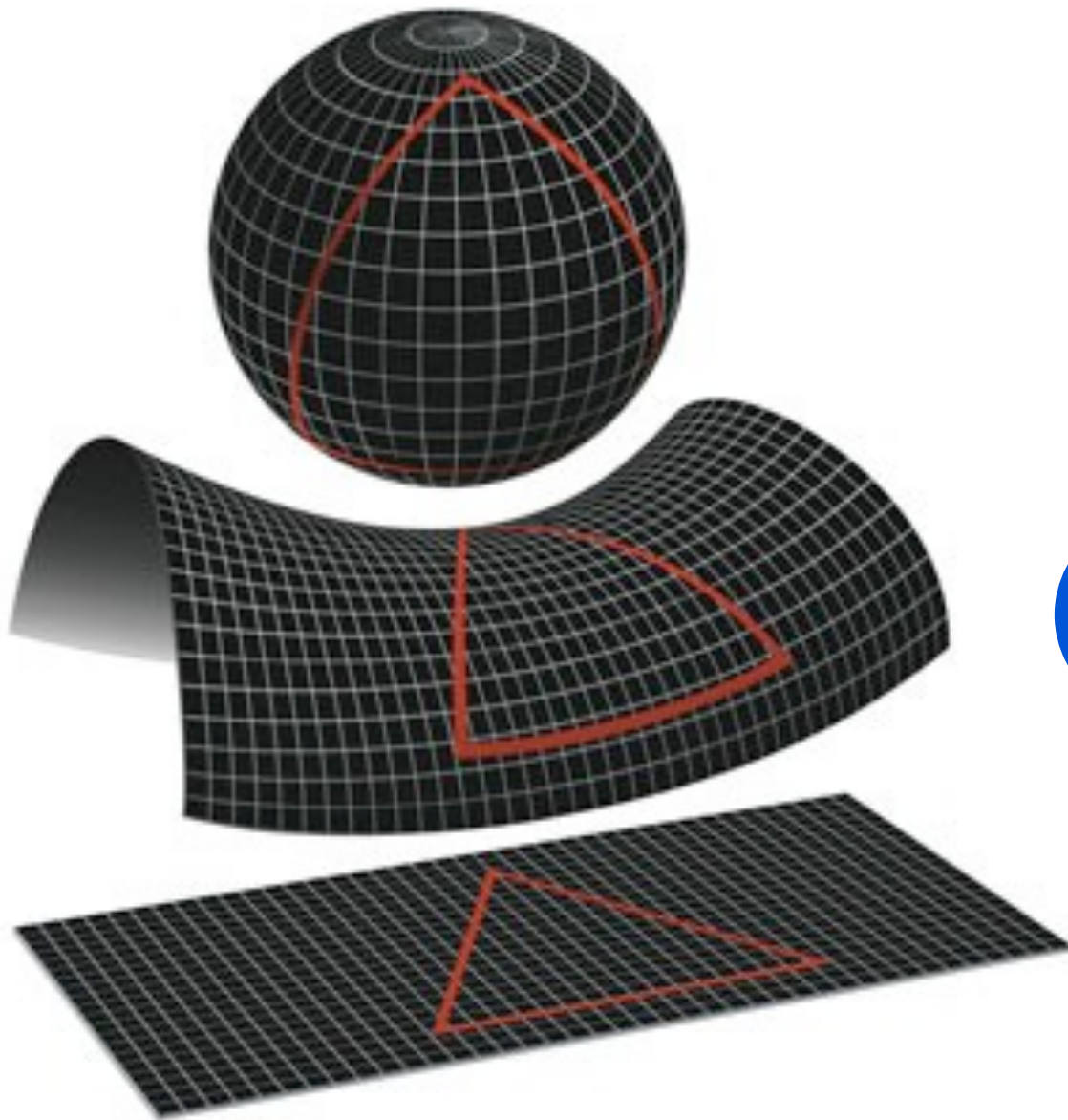
$$\Omega_m \equiv \frac{\rho}{\rho_{\text{crit}}} = 1 + \frac{kc^2}{a^2 H^2}$$

$$\Omega_m = \rho/\rho_{\text{crit}}$$

$\Omega_m > 1$ , matter curves space closed (positive curvature)

$\Omega_m < 1$ , expansion shapes the universe open (negative curvature)

$\Omega_m = 1$ , just the right amount of matter, flat geometry (zero curvature)



with  $\sim 0.3$ , the universe should be open and negative curvature

# density also determines Shape of the universe

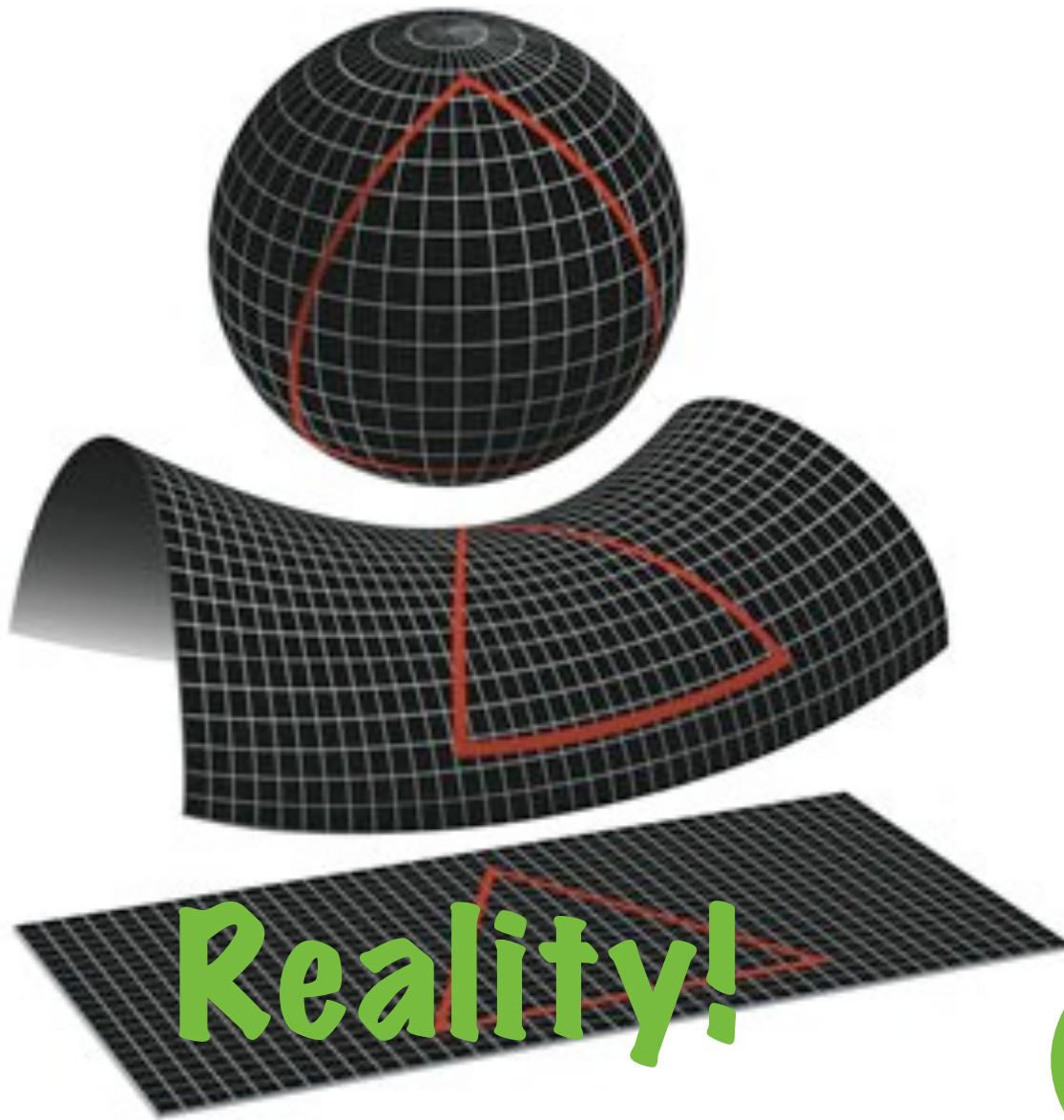
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# density + $H_0$ : also determine age of the universe

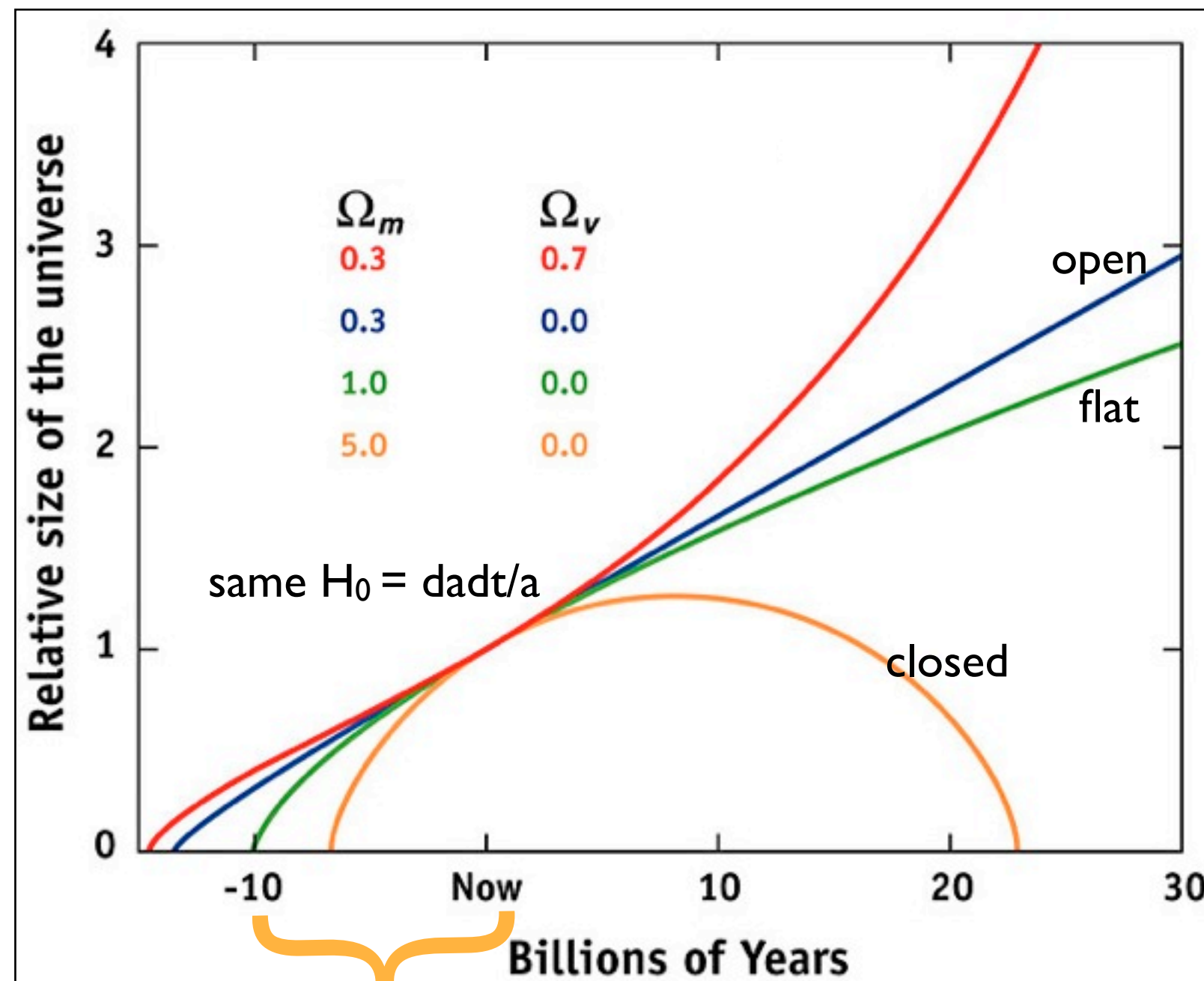
Let  $t_{\text{Hubble}} = H_0^{-1} \sim 13.7$  Gyrs

- $\Omega_m > 1$ ,  $0 < \text{age} < 2/3 t_{\text{Hubble}}$
- $\Omega_m = 1$ ,  $\text{age} = 2/3 t_{\text{Hubble}} = 9.1$  Gyrs
- $\Omega_m < 1$ ,  $2/3 t_{\text{Hubble}} < \text{age} < t_{\text{Hubble}}$

a denser universe is younger

- $\Omega_m = 0.30$ : age = 11.2 Gyrs  
(1 Gyrs =  $10^9$  yrs)

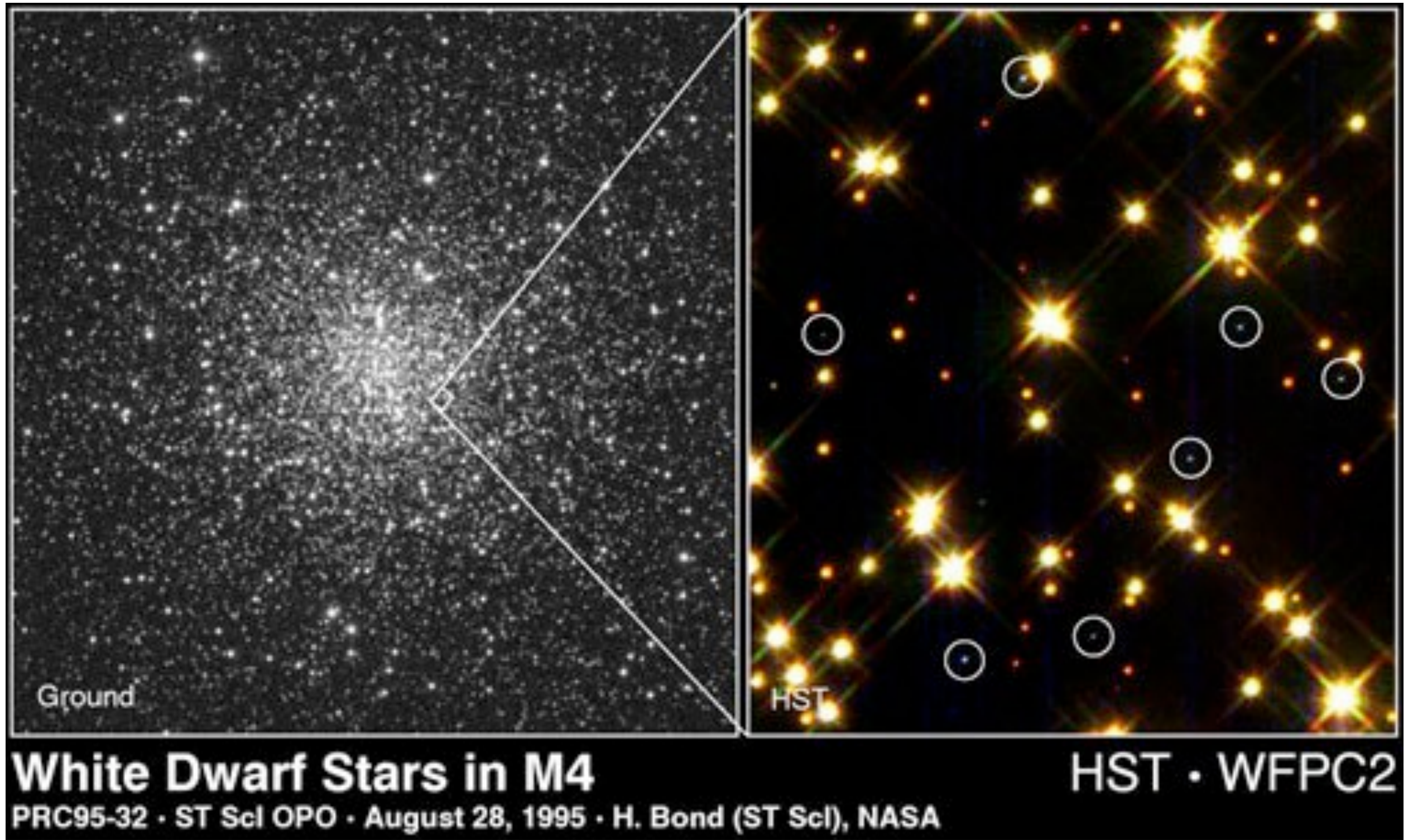
this age, however,  
conflicts with the  
measured age of  
the universe





observations:

Globular star cluster M4:  
age =  $12.7 \pm 0.7$  Gyrs





# The state of our knowledge: 1998

Observations measure:  $\Omega_m = \rho/\rho_{\text{crit}} \sim 0.3$

Theory (using Friedmann eq.) then predicts

- 1) space is open
- 2) universe is 11.7 Gys old

Both predictions **conflict** with further observations:

- 1) space is flat
- 2) universe is at least 12.7 Gyrs old.

Where is the mistake?

*lots of finger pointing  
and incriminations....*

# Enters the Supernova...

some stars go supernova  
at the end of life

sudden brightening ( $\sim$  the  
entire galaxy); can be seen  
to the edge of the visible  
universe

gradual fading over tens  
of days; an intrinsically  
brighter supernova fades  
slower

a great tool for  
determining distances!

*Cepheids are not bright enough  
for galaxies outside  $\sim 100$  million  
light years. For those,  
we use Supernova Type Ia as  
standard candle.*





supernova are rare (~ 1/a few centuries/per galaxy)  
one of the most recent ones in our galaxy: AD1054



Anasazi petrograph of SN1054?



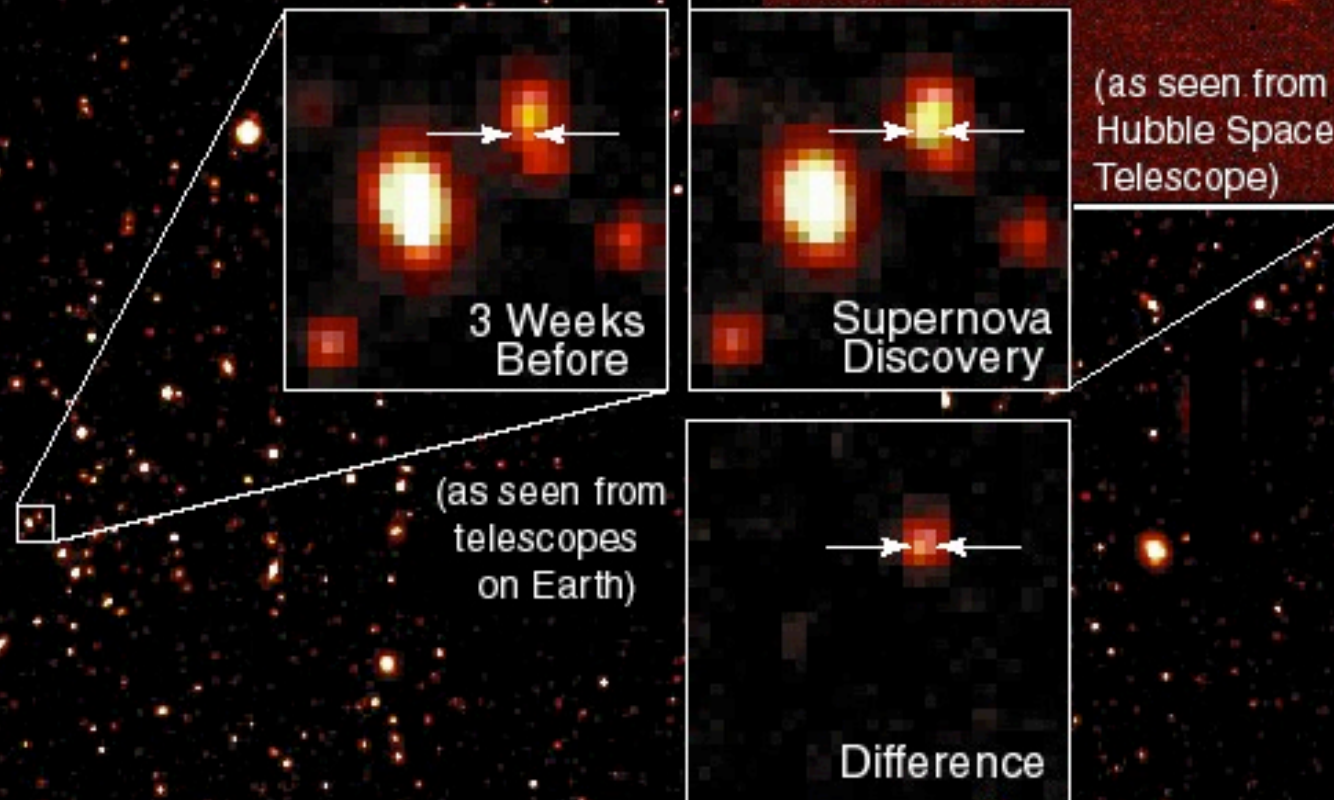
## ‘Guest star’ supernova 1054

".. In the 1st year of the period Chih-ho, the 5th moon, the day chich'ou, a guest star appeared approximately several inches south-east of Tien-Kuan [Zeta Tauri]. After more than a year, it gradually became invisible .." It was visible for a month during daylight.

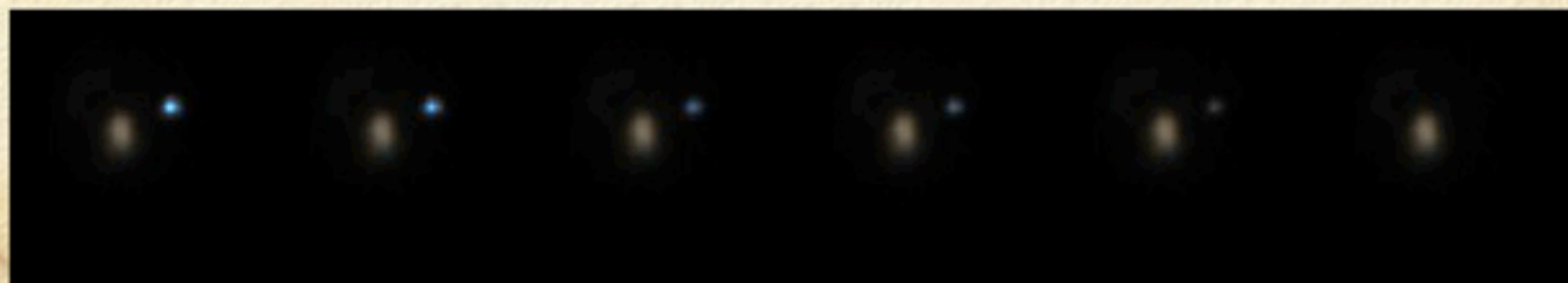


Supernova 1998ba  
Supernova Cosmology Project  
(Perlmutter, *et al.*, 1998)

$z=0.43$



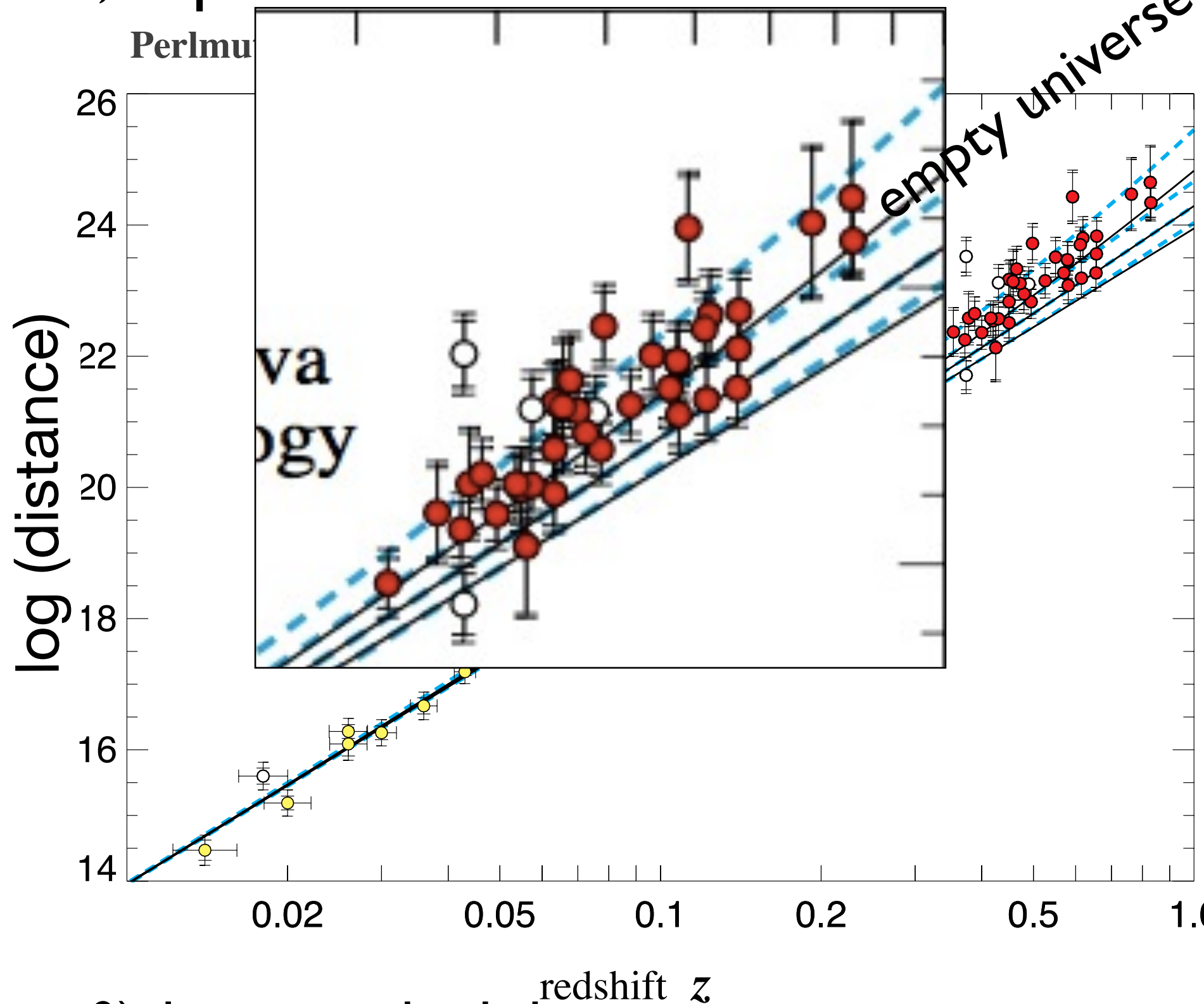
from nearby  
supernova, we know  
that longer lasting SN  
are intrinsically  
brighter; so by  
measuring their time  
sequence....



Time sequence -- measuring light curve.



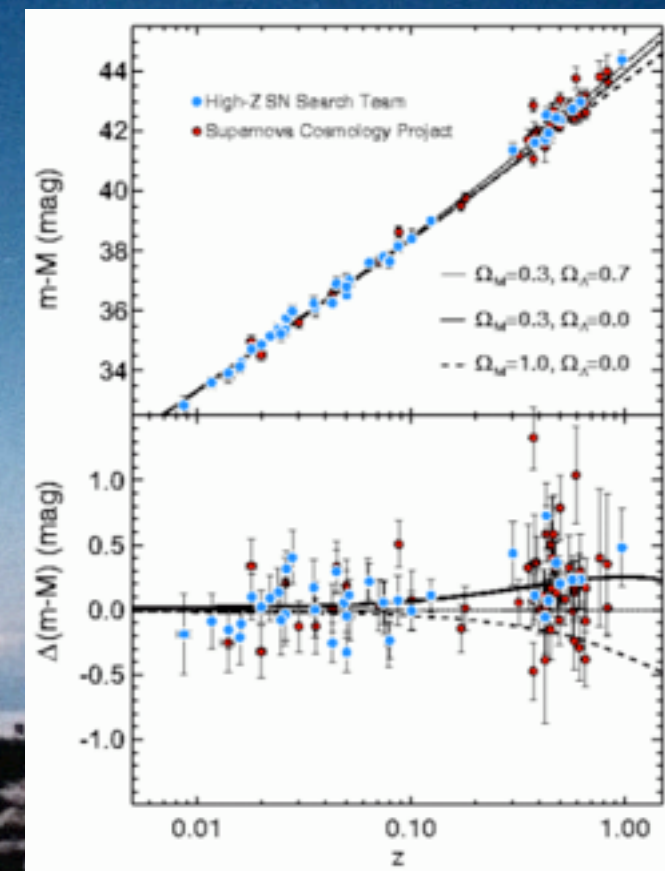
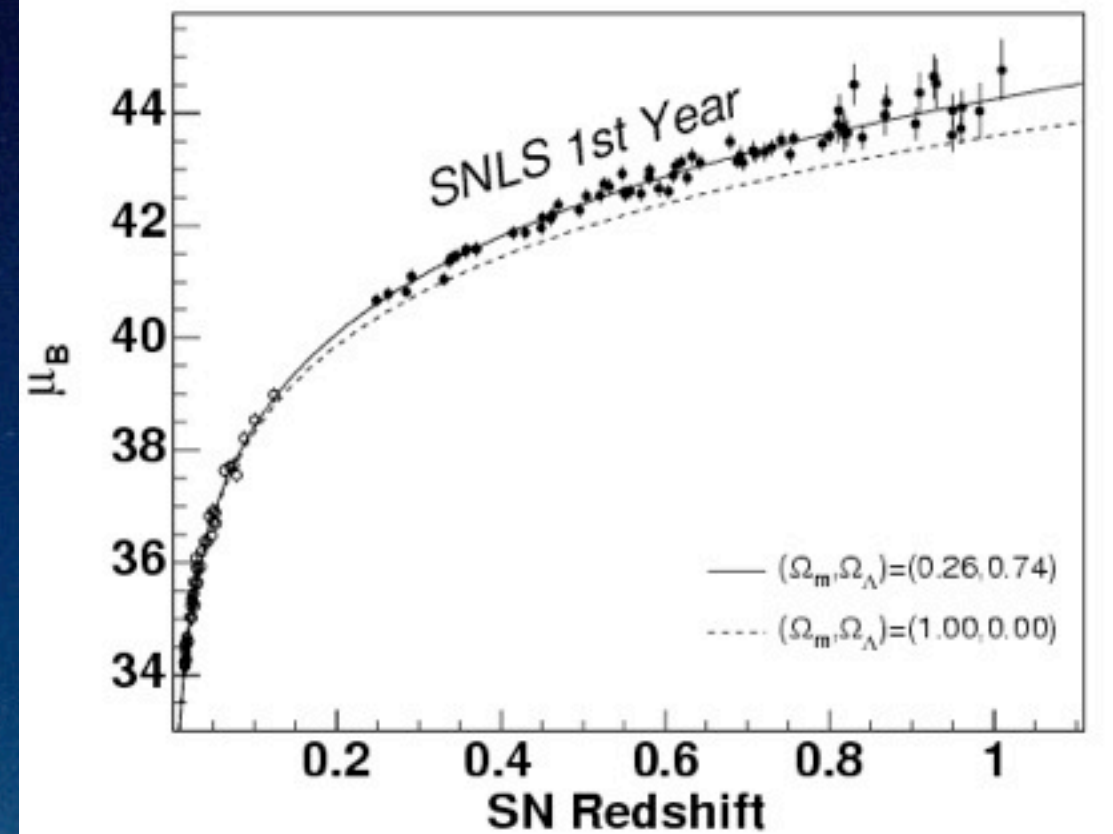
# Like Cepheid Stars, Supernova are 'standard Candles'



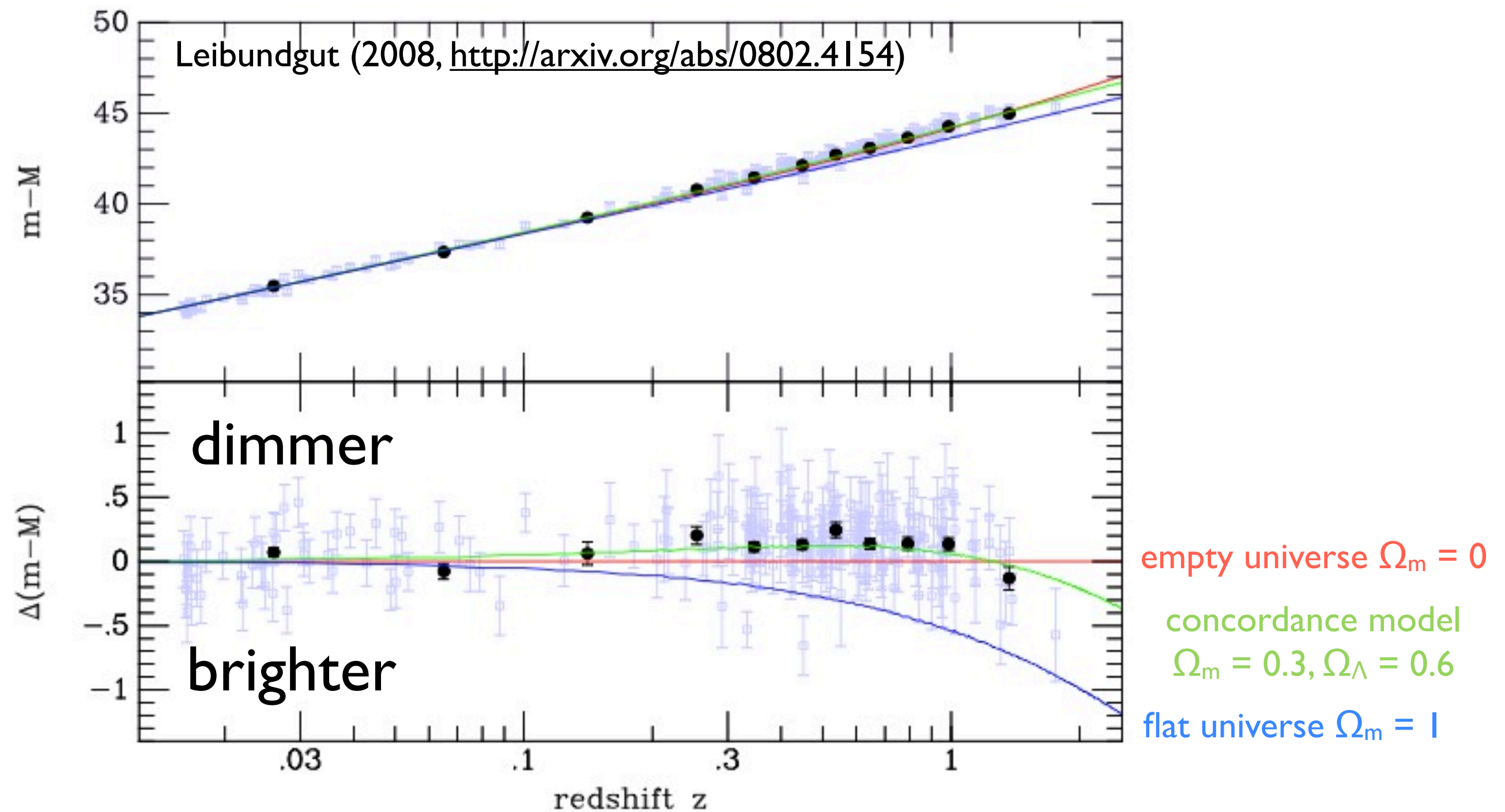
empty universe ( $\Omega_m = 0$ ): has stretched the most; supernova at a given  $z$  should look the dimmest. But real supernova appear even dimmer (1998) --> The universe has stretched even more.



# Canada-France- Hawaii Telescope







**Compared to matter-only model ( $\rho/\rho_{\text{crit}}=0.3$ )**

observed supernova looks dimmer at high redshift

--> further away

-->  $H = c \cdot z / r$  smaller in the past,

$$H^2 = \left( \frac{\dot{a}}{a} \right)^2$$

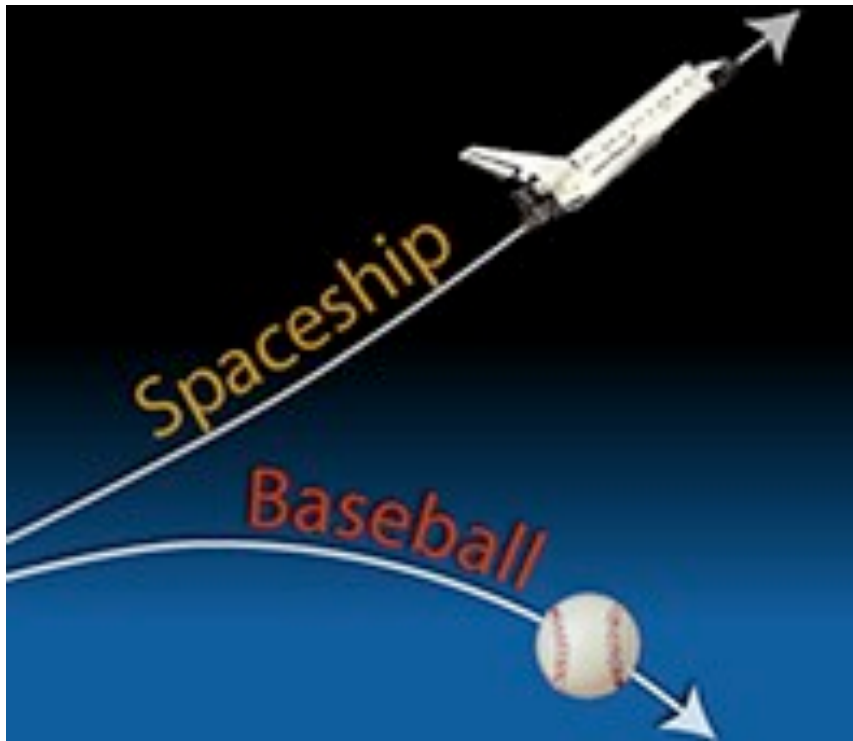
--> expansion of the universe is now **accelerating!**

**How can the universe  
expand faster than  
an empty one?**

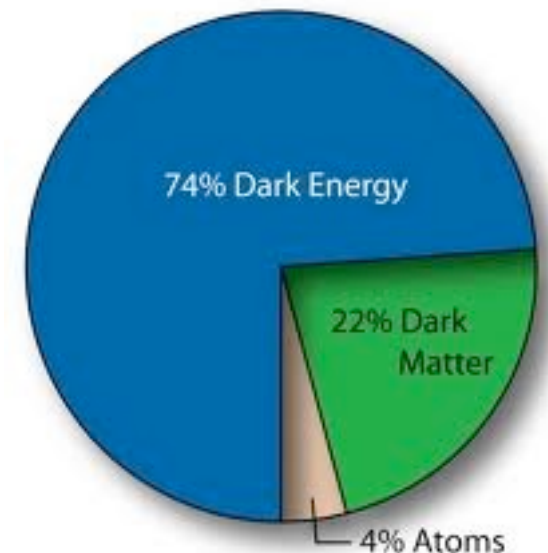


Distance bigger than even in the empty model.

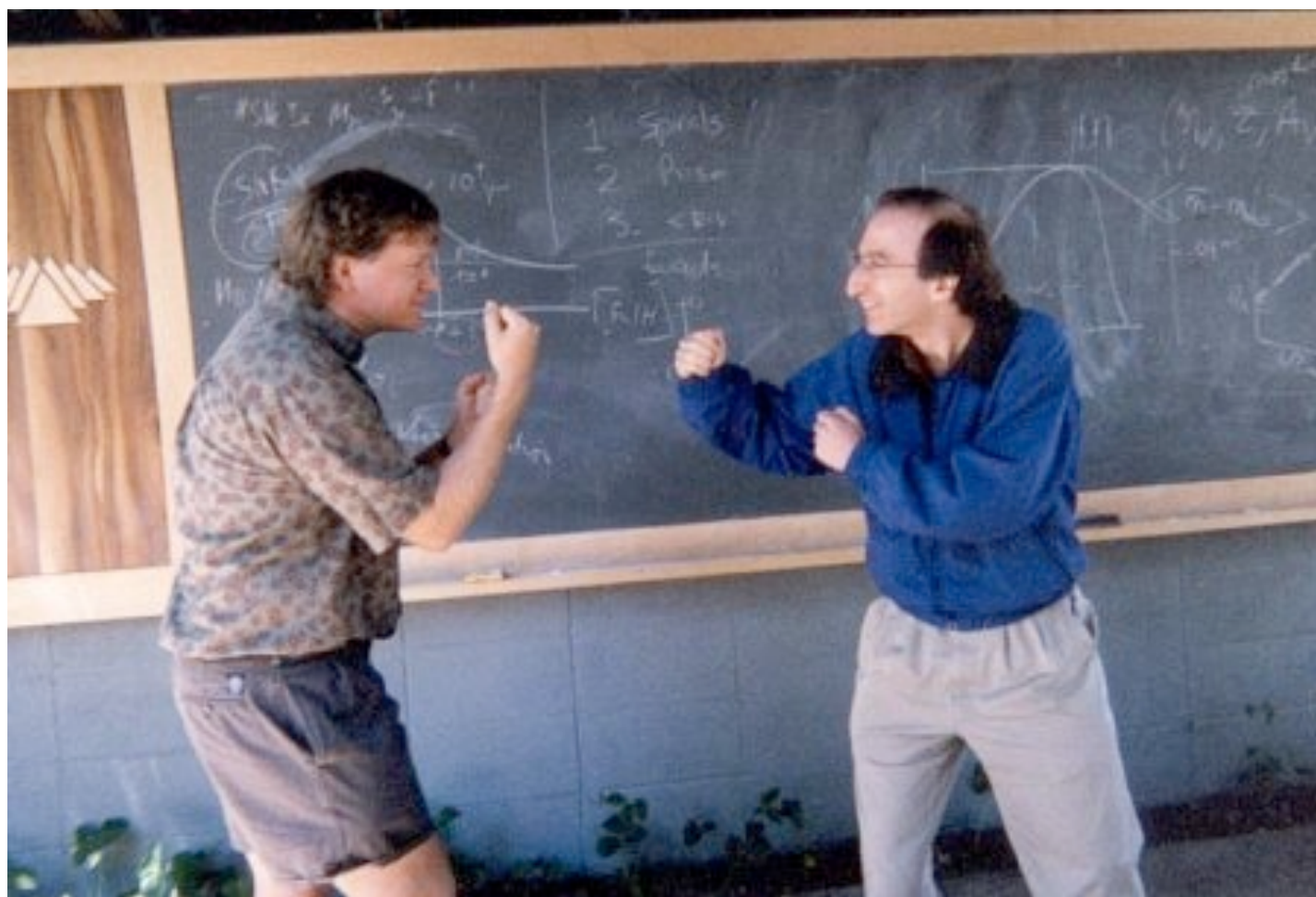
the expansion is picking up (accelerating)



while a ball thrown upward can only decelerate with time, expansion of the universe actually accelerates with time!



So enters the stage: **DARK ENERGY**



first announcements (1998)  
came from two  
independent groups led by  
B. Schmidt & S. Perlmutter,  
respectively...

## Nobel Prize



## Physics 2011

However, supernova data can  
mean a number of things...

**The Exciting:** the Universe is accelerating.

**The Heretical:** General Relativity is as sacred as anything in Physics,  
but it may be wrong.

**The Mundane:** We are simply wrong, perhaps high redshift supernova  
are intrinsically dimmer. They are not standard candles. Perhaps  
there is cosmic 'fog' making them look dimmer.

Observations measure:

$$\Omega_m = \rho/\rho_{\text{crit}} \sim 0.3$$

*Mistake here!  
Dark energy solved  
these two conflicts*

Theory (using Friedmann eq.) then predicts

- 1) space is open
- 2) universe is 11.7 Gys old

Both predictions **conflict** with further observations:

- 1) space is flat
- 2) universe is at least 12.7 Gyrs old.

Where is the mistake?