



Figure 1: On the summer solstice at local noon, in the city Syene, Egypt (which is close to the Tropic of Cancer), the Sun would appear at the zenith, directly overhead (sun elevation of 90°). While in Alexandria, almost due north of Syene, the angle of elevation of the Sun would be 83° (or 7° south of the zenith) at the same time.

Assignment I: week of Jan. 7th

This is one of the 10 problem sets you would be working on in this course. You are encouraged to work on this by coming to the help sessions (Thursday 12-1PM at MP202, Friday 1-2 at MP102) and grouping up with a couple fellow students. Teaching assistants will be at hand to help. You do not hand this one in. However, these problem sets are an essential part of the learning process: I don't have time to cover the quantitative aspects of modern cosmology and they are instead represented in these problem sets. Both the mid-term and the final exam will, assuredly, draw heavily from these problems.

This is meant to be a warm-up exercise leading to more sophisticated concepts and mathematical manipulations later in the course. You can judge your comfort level in the course by this assignment.

1. Ancient greeks measure the size of the Earth. (This is adapted from problem 1.1 in Shu).

From the shape of the shadow cast by the Earth on the Moon during a lunar eclipse, Greeks inferred that Earth is a sphere. Moreover, they understood that the Sun's distance is many times greater than the diameter of the Earth. Eratosthenes (276 BC-194 BC, Greek mathematician, geographer and astronomer) assumed that the Sun was far enough away that the rays from the Sun are virtually parallel when they strike the Earth. He then observed the position of the Sun on the sky at two different locales (see Fig. ??), Syene and Alexandria, separated by 800 kms. This allows him to infer the radius of the Earth, to within 10% of the actual value. Repeat his exercise to get your own value for the Earth's radius.

$$2\pi r \cdot \frac{7}{360} = 800 \text{ km} \quad r = 800 \times \frac{180}{7} \cdot \frac{1}{\pi} = 6548 \text{ km}$$

2. The number of stars in the *observable* universe totals around 10^{22} , with $\sim 10^{11}$ of them sitting in a typical galaxy and a total of $\sim 10^{11}$ galaxies lying within our cosmic horizon.

It is claimed that “the number of grains of sand on all the beaches on Earth is comparable to the number of stars in the observable universe”. We now introduce a skill called ‘order of magnitude estimates’ to check this claim.

To begin with, using your everyday experience to estimate the size of a typical grain and its volume. Now what is the length expanse of a typical sandy beach? what would you guess for the typical depth and width of the sandy beach? and how many sandy beaches do you think are around the globe? your estimates may be inaccurate by about one magnitude, but hopefully not much more. And further, hopefully, all your inaccuracies in the end cancel each other – e.g., you may put in an estimate for the width that is too low, but your estimate for the depth is too high, so it doesn’t matter in the end. Now what do you think about the claim?

Using the same technique, can you estimate the total number of cells in your body? (the answer is $\sim 10^{13}$, see how close you get).

10^{22} STARS

$$10^3 \times 10^2 \times 10^{-1} = 10^4$$

10^5 beaches

$$10^9 \text{ m}^3$$

$$10^9 \times (10^4)^3 = 10^9 \times 10^{12} = 10^{21} \text{ sand grains}$$

$$0.1 \text{ mm} \rightarrow 10^{-4} \text{ m} \rightarrow 10^{-12} \text{ m}^3$$

$$4 \times 10^7 \text{ m} \times 0.5 = 2 \times 10^7 \text{ m} \times 100 \text{ m} \times 1 \text{ m} = 10^{10} \text{ m}^3$$

$$\frac{10^{10}}{10^{-12}} = 10^{22} \text{ grains of sand}$$

3. Basic astronomy. Toronto has a latitude of 43° North (people usually talk about the 49° line as the division between Canada and US, but we are in fact ‘south of 49° ’). What is the highest point on the sky (in degrees away from zenith) the Sun ever gets to?

