STA 304/1003 F   Homework Solutions   Oct 28/09

Exercise 3.15

* Note: (3.6) gives $E(\hat{B}-B)^2 \simeq \cdots$; we're ignoring bias & assuming $E(\hat{B}-B)^2 = V(\hat{B})$.

Since $\hat{\bar{y}}_r = \hat{B}\bar{x}_U$, $V(\hat{\bar{y}}_r) \cong \bar{x}_U^2 \times (3.6)$, i.e.

(1) $V(\hat{\bar{y}}_r) \simeq \left(1-\frac{n}{N}\right)\frac{1}{n}\left(S_y^2 - 2RS_xS_yB + B^2S_x^2\right)$.

from (3.13)

(2) $V(\hat{\bar{y}}_{reg}) \simeq \left(1-\frac{n}{N}\right)\frac{S_y^2}{n}\left(1-R^2\right)$

$$
\begin{aligned}
(1)-(2) &= \left(1-\frac{n}{N}\right)\frac{S_y^2}{n}\left\{1 - \frac{2RS_xB}{S_y} + \frac{B^2S_x^2}{S_y^2} - (1-R^2)\right\} \\
&= \left(1-\frac{n}{N}\right)\frac{S_y^2}{n}\left(R^2 - \frac{2RS_xB}{S_y} + \frac{B^2S_x^2}{S_y^2}\right) \\
&= \left(1-\frac{n}{N}\right)\frac{S_y^2}{n}\left(R - \frac{BS_x}{S_y}\right)^2 \\
&\geq 0 \qquad \blacksquare
\end{aligned}
$$

Example 4.3 - see text & R code from Oct 23

Exercise 4.2 - see R code from Oct 23

$\hat{t}_{str} = 918{,}927{,}973 \qquad \hat{V}(\hat{t}_{str}) = (50{,}149{,}772)^2$

$\hat{t}_{SRS} = 930{,}319{,}422 \qquad \hat{V}(\hat{t}_{SRS}) = (58{,}276{,}069)^2$