Question 1

The force of mortality for a particular population from birth (that is, $\lambda(t)$ is the hazard *t* years after birth) is assumed to take the following form:

$$\lambda(t) = (\theta - t)^{-1},$$

where, $0 \le t \le \theta$. Based on this form of the force of mortality, find an expression for ${}_t q_x$.

Solution:

$$S(t) = \exp(-\int_0^t \frac{1}{\theta - s} ds) = \exp\{\log(\theta - t) - \log(\theta)\} = \frac{\theta - t}{\theta}.$$

$${}_t p_x = \frac{S(x+t)}{S(x)} = \frac{\theta - (x+t)}{\theta} \frac{\theta}{\theta - x} = \frac{\theta - (x+t)}{\theta - x}.$$

$$\Rightarrow {}_t q_x = 1 - \frac{\theta - (x+t)}{\theta - x} = \frac{t}{\theta - x}.$$

Question 2

The observed survival times (in years) of 7 patients after a heart transplant operation are provided below. Values denoted with "*" correspond to times of censoring, rather than times of death. The censoring is uninformative, right censoring.

1, 2, 3*, 4, 5*, 6, 7

a) Based on this data, what is the Kaplan-Meier estimate of the survival function at time 6.5? Your answer must also include an estimate of the standard error of the survival function at time 6.5.

Solution:

"cheating" and using R I get the following table:

time	n.risk	n.event	survival	std.err	lower 95% CI	upper 95% CI
1	7	1	0.857	0.132	0.598	1.000
2	6	1	0.714	0.171	0.380	1.000
4	4	1	0.536	0.201	0.142	0.929
6	2	1	0.268	0.214	0.000	0.688
7	1	1	0.000	NaN	NaN	NaN

This gives an estimate of the survival function of 0.268 and a standard error of 0.214.

b) Provide an estimate of the mean survival time after a heart transplant operation.

Solution:

One way to answer this question is to approximate the integral $\int_0^\infty {}_t p_x dt$. This can be done by computing the area under the estimated KM survival function. This area is equal to $(1-0)\times 1+(2-1)\times(0.857)+(4-2)\times(0.714)+(6-4)\times(0.536)+(7-6)\times 0.268.$ = 4.625.

c) How does the Nelson-Aalen estimate of the survival function at time 6.5 compare to the Kaplan-Meier estimate at this time?

Solution:

The NA estimator is computed as $\exp(-(\frac{1}{7}+\frac{1}{6}+\frac{1}{4}+\frac{1}{2})).$ =0.3466

Question 3

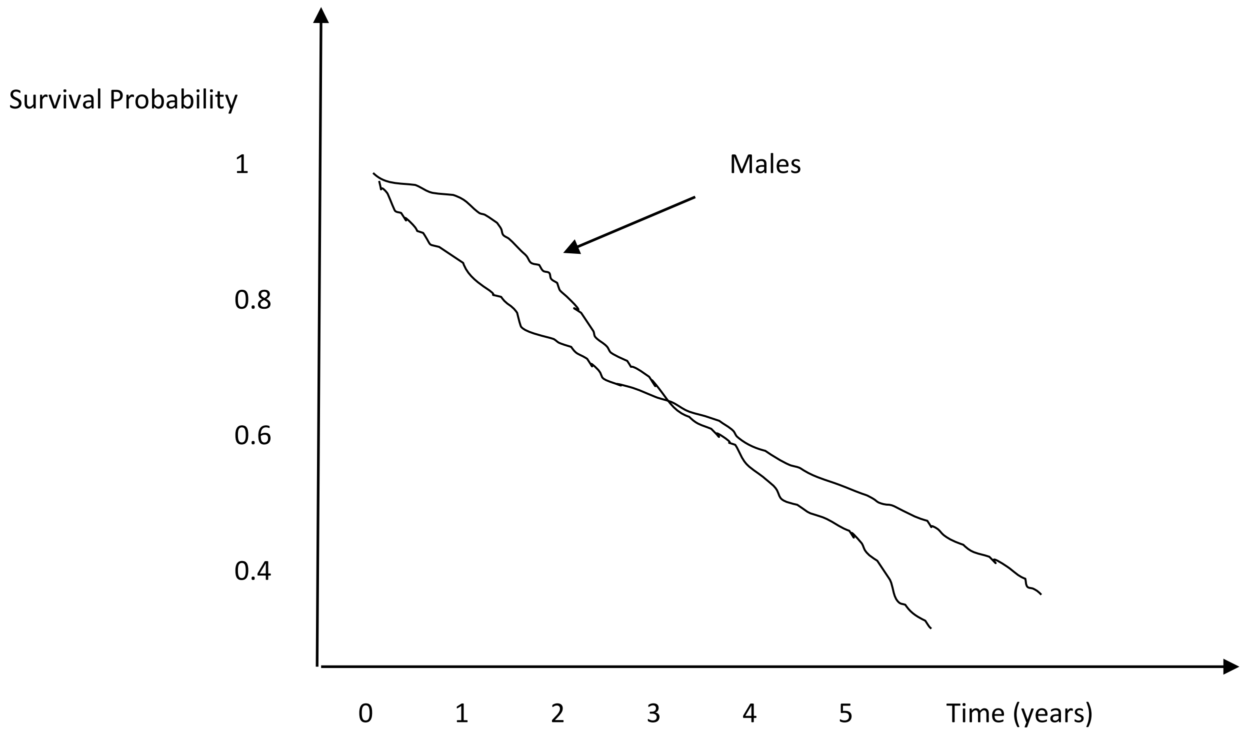
Based on a given set of data you have computed the maximum likelihood estimate of the parameter θ, denoted $\hat{\theta},$ to be 1.5. You have also computed an estimate of the standard error to be 2. Using this information compute an estimate of the variance of the quantity $\log(\hat{\theta})$. Also, provide an approximate 95% confidence interval for θ.

Solution:

$$\text{var}\{\log(\hat{\theta})\} \approx \frac{1}{\hat{\theta}^2}4 = \frac{1}{1.5^2}4$$
$$\Rightarrow SE\{\log(\hat{\theta})\} \approx 1.33$$
$$95\% \text{ CI for } \theta = 1.5 \pm 2\times 2$$

Question 4

The figures below contain (rough) plots of estimated survival curves for males and females after a particular operation.



Based on this figure answer the following questions:

a) Provide a rough estimate of the median survival time for males.
b) Comment on any concerns you might have modeling this data using the Cox regression formulation of the hazard given in class.

Solution:

a) About 4.5 years.
b) The survival functions cross – this is cannot happen with the Cox regression formulation of the hazard function given in lectures.