$$\frac{mv^2}{r} = \frac{m\left(\frac{2\pi r}{T}\right)^2}{r} = \frac{m\, 4\pi^2 r^2}{T^2 r}$$

AST121 assignment 3

1. a). $r = R_e + R_{iss} =$ ($R_{iss} = 380$ km)

VIA UNIFORM CIRCULAR MOT:

$$m\frac{4\pi^2 r}{P^2} = m\frac{GM_e}{r^2}$$

$$P = \left(\frac{4\pi^2 r^3}{GM_e}\right)^{0.5}$$

VIA KEPLER:

$$P = \sqrt{\frac{4\pi^2 r^3}{G(M_e + M_{iss})}} \approx \sqrt{\frac{4\pi^2 r^3}{GM_e}}$$

$$P = \left(\frac{4\pi^2 (3.8\cdot 10^5 m + 6.38\cdot 10^6 m)^3}{6.67\cdot 10^{-11}\frac{m^3}{kg\, s^2}\cdot 5.97\cdot 10^{24} kg}\right)^{0.5}$$

$= 5.55\cdot 10^3 s = \underline{1.54 \text{ hours}}$ (92.4 min, 5544 s)

b) $$\frac{dr}{dt} = -K\left(\frac{1}{r-R_e}\right)^6 \qquad K = 0.4\text{ km/day}\cdot(380\text{ km})^6$$

$$(r-R_e)^6 dr = -K dt$$

$$\to \frac{1}{7}(r-R_e)^7 = -Kt + C$$

$$r - R_e = (-7Kt + C)^{1/7}$$

at $t=0$, $r = R_e + R_{iss}$, so $C = R_{iss}^7 = 1.144\cdot 10^{18}\text{ km}^7$

$$\underline{r = R_e + (-7Kt + C)^{1/7}}$$

$-7Kt + C = 0$ is when $r = R_e$

$$t = R_{iss}^7 / 7K = \frac{(380\text{ km})^7}{7\cdot 0.4\text{ km/day}\cdot(380\text{ km})^6}$$

$= \underline{135 \text{ days}}$

c). $$\left(\frac{P_{gps}}{P_{iss}}\right)^2 = \left(\frac{a_{gps}}{a_{iss}}\right)^3$$

$$a_{gps} = (R_e + R_{iss})\left(P_{gps}/P_{iss}\right)^{2/3} = (3.8\cdot 10^5 m + 6.38\cdot 10^6 m)\cdot\left(\frac{12\text{ hrs}}{1.54\text{ hrs}}\right)^{2/3}$$

$= 2.66\cdot 10^7 m$

$R_{gps} = a_{gps} - R_e = 2.02\cdot 10^7 m \to \underline{2.02\cdot 10^4 km}$

$$t = R_{gps}^7 / 7K = \frac{(20200\text{ km})^7}{7\cdot 0.4\text{ km/day}\cdot(380\text{ km})^6}$$

$= 1.62\cdot 10^{14}$ days $= 444$ Gyr.

2 a). $G = \frac{F_g r^2}{M_1 M_2} \rightarrow \frac{kg\, m/s^2 \cdot m^2}{kg \cdot kg} \rightarrow [L]^3[M]^{-1}[T]^{-2}$

$h \rightarrow J \cdot s \rightarrow kg \frac{m^2}{s^2} \cdot s \rightarrow kg \frac{m^2}{s} \rightarrow [M][L]^2[T]^{-1}$

$c \rightarrow m/s \rightarrow [L][T]^{-1}$

b). $[T] = [G]^\alpha [h]^\beta [c]^\gamma$

$= [L]^{3\alpha}[M]^{-\alpha}[T]^{-2\alpha}[M]^\beta[L]^{2\beta}[T]^{-\beta}$
$[L]^\gamma[T]^{-\gamma}$

$\rightarrow \quad 3\alpha + 2\beta + \gamma = 0 \quad (L)$

$-\alpha + \beta = 0 \quad (M)$

$-2\alpha - \beta - \gamma = 1 \quad (T)$

$\alpha = \beta \qquad \gamma = -5\alpha$

$-2\alpha - \alpha + 5\alpha = 1 \qquad \alpha = \frac{1}{2} \quad \beta = \frac{1}{2} \quad \gamma = -\frac{5}{2}$

c). $t_{planck} = (6.67 \cdot 10^{-11} \frac{N m^2}{kg^2})^{\frac{1}{2}} (6.62 \cdot 10^{-34} J \cdot s)^{\frac{1}{2}} (3 \cdot 10^8 m/s)^{-5/2}$

$= \underline{1.35 \cdot 10^{-43}}\ s$

d). $3\alpha + 2\beta + \gamma = 1$

$-\alpha + \beta = 0 \qquad \alpha = \beta$

$-2\alpha - \beta - \gamma = 0 \qquad \gamma = -3\alpha$

$3\alpha + 2\alpha - 3\alpha = 1 \qquad \alpha = \frac{1}{2} \qquad \beta = \frac{1}{2} \qquad \gamma = -\frac{3}{2}$

$l_{planck} = (6.67 \cdot 10^{-11} \frac{N m^2}{kg^2})^{\frac{1}{2}} (6.62 \cdot 10^{-34} J \cdot s)^{\frac{1}{2}} (3 \cdot 10^8 m/s)^{-\frac{3}{2}}$

$= \underline{4.04 \cdot 10^{-35}\ m}$

e). $3\alpha + 2\beta + \gamma = 0$

$-\alpha + \beta = 1 \qquad \alpha + \beta = 0$

$-2\alpha - \beta - \gamma = 0 \qquad \beta = \frac{1}{2} \quad \alpha = -\frac{1}{2} \quad \gamma = \frac{1}{2}$

$m_{planck} = (6.67 \cdot 10^{-11} \frac{N m^2}{kg^2})^{-\frac{1}{2}} (6.62 \cdot 10^{-34} J \cdot s)^{\frac{1}{2}} (3 \cdot 10^8 m/s)$

$= 5.46 \cdot 10^{-8}\ kg \ (341\ GeV)$