MAT135H1S Calculus I(A)

Solution to even-numbered problems in Section 2.6 and 2.7

(Section 2.6, Q18)

$$\lim_{x\to-\infty}\frac{4x^3+6x^2-2}{2x^3-4x+5}=\lim_{x\to-\infty}\frac{4+\frac{6}{x}-\frac{2}{x^3}}{2-\frac{4}{x^2}+\frac{5}{x^3}}=\frac{4+0-0}{2-0+0}=2$$

(Section 2.6, Q26)

$$\begin{aligned}\lim_{x\to-\infty}\left(x+\sqrt{x^2+2x}\right) &= \lim_{x\to-\infty}\left(x+\sqrt{x^2+2x}\right)\cdot\frac{x-\sqrt{x^2+2x}}{x-\sqrt{x^2+2x}}\\ &= \lim_{x\to-\infty}\frac{x^2-(x^2+2x)}{x-\sqrt{x^2+2x}}\\ &= \lim_{x\to-\infty}\frac{-2x}{x-\sqrt{x^2+2x}}\\ &= \lim_{x\to-\infty}\frac{-2}{1-\frac{\sqrt{x^2+2x}}{x}}\\ &= \lim_{x\to-\infty}\frac{-2}{1+\sqrt{\frac{x^2+2x}{x^2}}} \qquad \left(\text{note that } \sqrt{x^2}=-x,\ \text{since } x<0\right)\\ &= \lim_{x\to-\infty}\frac{-2}{1+\sqrt{1+\frac{2}{x}}}\\ &= \frac{-2}{1+\sqrt{1+0}}=-1\end{aligned}$$

(Section 2.6, Q34)

$$\begin{aligned}\lim_{x\to\infty}\frac{e^{3x}-e^{-3x}}{e^{3x}+e^{-3x}} &= \lim_{x\to\infty}\frac{e^{3x}-e^{-3x}}{e^{3x}+e^{-3x}}\cdot\frac{e^{-3x}}{e^{-3x}}\\ &= \lim_{x\to\infty}\frac{1-e^{-6x}}{1+e^{-6x}}\\ &= \frac{1-0}{1+0} \qquad \left(\text{since} \lim_{x\to\infty}e^{-6x}=0\right)\\ &= 1\end{aligned}$$

(Section 2.7, Q24)
By definition,

$$\begin{aligned} g'(1) &= \lim_{x\to 1} \frac{g(x)-g(1)}{x-1} \\ &= \lim_{x\to 1} \frac{x^4-2-(1^4-2)}{x-1} \\ &= \lim_{x\to 1} \frac{x^4-1}{x-1} \\ &= \lim_{x\to 1} \frac{(x-1)(x^3+x^2+x+1)}{x-1} \\ &= \lim_{x\to 1}(x^3+x^2+x+1) = 4 \end{aligned}$$

Therefore, the equation of the tangent line to the curve $y = x^4 - 2$ at the point $(1, -1)$ is

$$y - (-1) = 4(x-1)$$

or

$$y = 4x - 5$$