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Here is a list of important proofs which are implicitly or explicitly presented in the textbook. Please read and make sure all of these blocks of proofs are understood. You will be asked to reproduce some of these proofs in test 1.

1.1:
- proof of 1.1, please pay attention to the selection of a function $f : \mathbb{R} \longrightarrow \mathbb{R}$. Think whether this selection was just an accident, or was it somehow planned? What is the geometric interpretation of this function. There is a number of ideas which is incorporated in this design.
- proof of 1.2, and exercise 6
- a Proof of inequality 1.3

1.2: The only proof in this section is 1.4, and it has 4 components: (a) $\Rightarrow$, (a) $\Leftarrow$, (b) $\Rightarrow$, (b) $\Leftarrow$. Also question 9 is important, and we have seen applications of it several times in sections 1.4-1.6

please note that you may use the facts presented in the notes and the maps. For example characterization of $\boldsymbol{x} \in \overline{S}$ is that $\forall r\ B(r, \boldsymbol{x} \cap S \neq \emptyset$. Another series of facts were presented in PS2 (the inverse image).

1.3:
- Proof of 1.7 using 1.6 and 1.3.
- Similarly the proof of the statement in the middle of page 14 using 1.8 and 1.3.
- Proof of one of the components of 1.10.
- proof of 1.13 (the open version).

1.4:
- The last paragraph of page 21 (just before example 4).
- Theorem 1.14 has two blocks of arguments: *if* and *only if*.
- Theorem 1.15 also has two blocks in it: $a \Rightarrow b$ and $b \Rightarrow a$. You may be asked to reproduce these proofs the way they are proceed and not exactly the way their statements are presented. So please learn the idea of the proofs and the techniques of argument that you may find present in them.
- Exercise 4.

1.5:
- Proof of 1.16,
- proof of 1.17 (has two blocks: existence and uniqueness),
- proof of 1.18 has three ideas: selection of the intervals, selection of the sequence, and proving that the sequence converges.)
- proof of 1.19 (for n=2 or 3)
- proof of 1.20 (has three blocks: convergent implies Cauchy, Cauchy is bounded, Cauchy + subsequence converge imply the entire sequence converges.)
- exercise 7

1.6:
- 1.21 has five blocks: $(a) \Rightarrow (b)$, S is not bounded then a sequence in S tends to infinity, any subsequence also tends to infinity, , then no converging subsequence, not closed then exists a subsequence which does not converge in S.

- proof of Remark at the bottom of page 30.
- proof of 1.22, (now you need to memorize the idea or statement or number of each of the theorems used in these arguments so that you can refer to them.)
- Proof of 1.23 has three blocks: inf and sup exist, and they belong to S, and there exist points $\boldsymbol{a}$ and $\boldsymbol{b}$ in S.

1.7:

- proof of 1.25 has five blocks: defining S_1 and S_2, showing (S_1, S_2) is a disconnection of S, selection of $c \in [a, b]$, showing that $c \notin S$, using $[a, b]$ to define T_1 and T_2 and using the proof for the case $[a, b]$.
- 1.26: proof by contraposition.
- 1.27, again one needs to know how to refer to the theorems involved in the proof.
- proof of 1.28.
- Exercises 3, 5, 7, 8