PREDICATE LOGIC DERIVATIONS FOR UNIT 6
ANSWERS for 19-38
Multi-Place Predicates

19. $\forall x \forall y \forall z(F(xy) \wedge F(yz) \rightarrow F(xz))$. $\sim\forall x \forall y \sim F(xy)$. $\forall x \forall y(F(xy) \rightarrow F(yx))$ $\therefore \exists x F(xx)$

1	~~Show~~ $\exists x F(xx)$		
2	$\exists x \sim\forall y \sim F(xy)$	pr2 QN	TO DO WITHOUT QN SEE BELOW
3	$\sim\forall y \sim F(iy)$	2 ei	Use EI asap. New variable.
4	$\exists y \sim\sim F(iy)$	3 QN	
5	$\sim\sim F(ik)$	4 ei	Use EI asap. New variable.
6	$F(ik)$	5 dn	
7	$\forall y(F(iy) \rightarrow F(yi))$	pr3 ui	match 6
8	$F(ik) \rightarrow F(ki)$	7 ui	match 6
9	$F(ki)$	6 8 mp	
10	$\forall y \forall z(F(iy) \wedge F(yz) \rightarrow F(iz))$	pr1 ui	match 6 & 9
11	$\forall z(F(ik) \wedge F(kz) \rightarrow F(iz))$	10 ui	match 6 & 9
12	$F(ik) \wedge F(ki) \rightarrow F(ii)$	11 ui	match 6 & 9
13	$F(ik) \wedge F(ki)$	6 9 adj	
14	$F(ii)$	12 13 mp	
15	$\exists x F(xx)$	14 eg	match show line
16		15 dd	

If you aren't using QN, you could start as follows:

1	~~Show~~ $\exists x F(xx)$		
2	$\sim\forall x \forall y \sim F(xy)$.	pr2	
3	~~Show~~ $\exists x \exists y F(xy)$		
4	$\sim\exists x \exists y F(xy)$	ass id	
5	~~Show~~ $\forall x \forall y \sim F(xy)$		show contradiction of 2
6	~~Show~~ $\forall y \sim F(xy)$		show instantiation of 5 for ud
7	~~Show~~ $\sim F(xy)$		show instantiation of 6 for ud
8	$F(xy)$	ass id	
9	$\exists y F(xy)$	8 eg	
10	$\exists x \exists y F(xy)$	9 eg	
11	$\sim\exists x \exists y F(xy)$	4 r 10 id	
12		7 ud	
13		6 ud	
14		5 ud	
15	$\exists y F(iy)$	3 ei	
16	$F(ik)$	15 ei	now continue above from line 6

20. $\forall x\exists y{\sim}(G(xy) \land H(xy))\ \ \therefore \forall x({\sim}\forall yG(xy) \lor {\sim}\forall yH(xy))$

1	~~Show~~ $\forall x({\sim}\forall yG(xy) \lor {\sim}\forall yH(xy))$		
2	~~Show~~ ${\sim}\forall yG(xy) \lor {\sim}\forall yH(xy)$		show instantiation of 1
3	${\sim}({\sim}\forall yG(xy) \lor {\sim}\forall yH(xy))$	ass id	
4	$\forall yG(xy) \land \forall yH(xy))$	3 dm	
5	$\exists y{\sim}(G(xy) \land H(xy))$	pr1 ui	match 4
6	${\sim}(G(xi) \land H(xi))$	5 ei	New variable.
7	${\sim}G(xi) \lor {\sim}H(xi)$	6 dm	
8	$\forall xG(xy)$	4 sl	
9	$G(xi)$	8 ui	match 7
10	${\sim}{\sim}G(xi)$	9 dn	
11	${\sim}H(xi)$	7 10 mtp	
12	$\forall yH(xy)$	5 sr	
13	$H(xi)$	12 ui	match 11
14		11 13 id	
15		2 ud	

21. $\forall x(Ax \rightarrow \forall yL(xy))$. $\forall y((Cy \land L(yy)) \lor {\sim}By)$. $\therefore \exists x(Ax \lor Bx) \rightarrow \exists xL(xx)$

1	Show $\exists x(Ax \lor Bx) \rightarrow \exists xL(xx)$		
2	$\exists x(Ax \lor Bx)$	ass cd	
3	$Ai \lor Bi$	2 ei	Use EI asap. New variable.
4	$Ai \rightarrow \forall yL(iy))$.	pr1 ui	match 3
5	$(Ci \land L(ii)) \lor {\sim}Bi$	pr2 ui	match 3
6	~~Show~~ $L(ii)$		
7	${\sim}L(ii)$	ass id	
8	$\exists y{\sim}L(iy)$	7 eg	
9	${\sim}\forall yL(iy)$	8 qn	
10	${\sim}Ai$	9 4 mt	
11	Bi	10 3 mtp	
12	$Ci \land L(ii)$	11 dn 5 mtp	
13	$L(ii)$	12 sr	
14		7 13 id	
15	$\exists xL(xx)$	6 eg cd	

22. $\forall x \exists y(Gx \rightarrow L(xy))$. $\forall z(\sim Fz \vee Gz)$. $\forall x(Cx \vee \sim\exists y L(yx))$. $\therefore \forall x(Fx \rightarrow \exists y(Cy \wedge L(xy)))$

1	~~Show~~ $\forall x(Fx \rightarrow \exists y(Cy \wedge L(xy)))$		
2	~~Show~~ $Fx \rightarrow \exists y(Cy \wedge L(xy))$		Show line 1 is $\forall$, so show instantiation for UD
3	Fx	ass cd	Show line 2 is $\rightarrow$ so assume ant. for CD
4	$\exists y(Gx \rightarrow L(xy))$	pr1 ui x/x	Instantiate pr1 to match 2 (x in 1st place of L(xy))
5	$Gx \rightarrow L(xi)$	4 ei i/y	Instantiate 4 to arbitrary term, i.
6	$\sim Fx \vee Gx$	pr2 ui x/z	Instantiate pr2 using x for x to match 3
7	$\sim\sim Fx$	3 dn	
8	Gx	7 3 mtp	
9	$L(xi)$	5 8 mp	
10	$Ci \vee \sim\exists y L(yi)$	pr3 ui i/x	instantiate pr3 to match 9 (i in 2nd place of L(yi))
11	$\exists y L(yi)$	9 eg y/x	generalize 9 to match 10
12	$\sim\sim\exists y L(yi)$	11 dn	
13	Ci	10 12 mtp	
14	$Ci \wedge L(xi)$	13 9 adj	
15	$\exists y(Cy \wedge L(xy))$	14 eg cd	generalize 14 to match cons. of 2
16		2 ud	

23. $\forall x \forall y(B(xy) \rightarrow A(yx))$. $\forall x \exists y(Fy \wedge B(yx))$. $\exists x(Fx \vee Hx) \rightarrow \forall x(Fx \rightarrow Hx)$. $\therefore \forall x(Gx \rightarrow \exists y(A(xy) \wedge Hy)$

1	~~Show~~ $\forall x(Gx \rightarrow \exists y(A(xy) \wedge Hy)$		
2	~~Show~~ $Gx \rightarrow \exists y(A(xy) \wedge Hy)$		
3	Gx	ass cd	
4	$\exists y(Fy \wedge B(yx))$	pr2 ui	Pr1 relates B and A. You will need an 'x' in the A term, so you will need one in the B term.
5	$Fi \wedge B(ix)$	4 ei	Use EI asap. New variable.
6	Fi	5 sl	
7	$B(ix)$	5 sr	
8	$\forall y(B(iy) \rightarrow A(yi))$	pr1 ui	match 7
9	$B(ix) \rightarrow A(xi)$	8 ui	match 7
10	$A(xi)$	7 9 mp	
11	$Fi \vee Hi$	6 add	
12	$\exists x(Fx \vee Hx)$	11 eg	match pr3 antecedent
13	$\forall x(Fx \rightarrow Hx)$	12 pr3 mp	
14	$Fi \rightarrow Hi$	13 ui	match 6
15	Hi	6 14 mp	
16	$A(xi) \wedge Hi$	10 15 adj	
17	$\exists y(A(xy) \wedge Hy)$	16 eg	match show line
18		17 cd	
19		2 ud	

24. $\therefore\ \forall x\exists y\forall z(A(xz) \wedge \sim B(zy)) \rightarrow \exists x(\sim A(xx) \leftrightarrow B(xx))$

1	~~Show~~ $\forall x\exists y\forall z(A(xz) \wedge \sim B(zy)) \rightarrow \exists x(\sim A(xx) \leftrightarrow B(xx))$		
2	$\forall x\exists y\forall z(A(xz) \wedge \sim B(zy))$	ass cd	
3	$\exists y\forall z(A(xz) \wedge \sim B(zy))$	2 ui	nothing to match
4	$\forall z(A(xz) \wedge \sim B(zi))$	3 ei	new variable
5	$A(xi) \wedge \sim B(ii)$	4 ui	match second term in B
6	$\exists y\forall z(A(iz) \wedge \sim B(zy))$	2 ui	repeat 3-5 to get terms in A to match terms in B
7	$\forall z(A(iz) \wedge \sim B(zk))$	6 ei	New variable.
8	$A(ii) \wedge \sim B(ik)$	7 ui	match first term in A
9	$\sim B(ii)$	5 sr	
10	$A(ii)$	8 sl	
11	$A(ii) \vee B(ii)$	10 add	(or show: $\sim A(ii) \rightarrow B(ii)$)
12	$\sim A(ii) \rightarrow B(ii)$	11 cdj	
13	$\sim B(ii) \vee \sim A(ii)$	9 add	(or show: $B(ii) \rightarrow \sim A(ii)$)
14	$B(ii) \rightarrow \sim A(ii)$	13 cdj	
15	$\sim A(ii) \leftrightarrow B(ii)$	12 14 cb	
16	$\exists x(\sim A(xx) \leftrightarrow B(xx))$	15 eg	match show line
17		16 cd	

25. $\exists x\forall y(L(yx) \rightarrow \forall zB(xyz)$. $\forall y(\exists xB(xyy) \rightarrow \forall zH(yz))$. $\forall y\exists xH(xy) \rightarrow \sim\exists x\exists yG(xy)$.

$\therefore \sim\exists x\forall y(L(xy) \wedge G(yx))$

1	~~Show~~ $\sim\exists x\forall y(L(xy) \wedge G(yx))$		
2	$\exists x\forall y(L(xy) \wedge G(yx))$	ass id	
3	$\forall y(L(iy) \wedge G(yi))$	2 ei	Use EI asap. New variable.
4	$\forall y(L(yk) \rightarrow \forall zB(kyz)$.	pr1 ei	Use EI asap. New variable.
5	$L(ik) \wedge G(ki)$	3 ui	match 4
6	$L(ki) \rightarrow \forall zB(kiz)$	4 ui	match 3
7	$L(ik)$	5 sl	
8	$\forall zB(kiz)$	6 7 mp	
9	$\exists xB(xii) \rightarrow \forall zH(iz)$	pr2 ui	match 8
10	$B(kii)$	8 ui	match 9
11	$\exists xB(xii)$	10 eg	match 9
12	$\forall zH(iz)$	9 11 mp	*alternate strategy:
13	$G(ki)$	5 sr	~~Show~~ $\forall y\exists xH(xy)$
14	$\exists yG(ky)$	13 eg	~~Show~~ $\exists xH(xy)$
15	$\exists x\exists yG(xy)$	14 eg	$H(iy)$ 12 ui
16	$\sim\sim\exists x\exists yG(xy)$	15 dn	$\exists xH(xy)$ eg dd
17	$\sim\forall y\exists xH(xy)$	16 pr3 mt	ud
18	$\exists y\sim\exists xH(xy)$	17 qn	
19	$\sim\exists xH(xj)$	18 ei	new variable
20	$\forall x\sim H(xj)$	19 qn	
21	$\sim H(ij)$	20 ui	match 12
22	$H(ij)$	12 ui 21 id	match 21

26. $\exists x \forall y(Hx \wedge L(xy))$. $\forall x(Gx \wedge \forall y L(yx))$. $\exists y \forall x\,(Gx \wedge Hy \wedge L(xy) \wedge L(yx)) \rightarrow \sim\exists z(Fz \wedge Gz)$.
$\therefore \forall x(Fz \rightarrow \sim Gz)$

1	~~Show~~ $\forall z(Fz \rightarrow \sim Gz)$		
2	~~Show~~ $Fz \rightarrow \sim Gz$		show instantiation of 1 for UD
3	Fz	ass cd	
4	$\forall y(Hi \wedge L(iy))$	pr1 ei	Use EI asap. New variable.
5	$Gi \wedge \forall y\, L(yi)$	pr2 ui	match 4
6	~~Show~~ $\forall x(Gi \wedge Hi \wedge L(xi) \wedge L(ix))$		set up UD asap. Match 4 & 5.
7	$Hi \wedge L(ix)$	4 ui	match show line 6
8	$\forall y L(yi)$	5 sr	
9	$L(xi)$	8 ui	match 7
10	$Gi \wedge Hi$	5 sl 7 sr adj	
11	$Gi \wedge Hi \wedge L(xi)$	10 9 adj	
12	$Gi \wedge Hi \wedge L(xi) \wedge L(ix)$	11 7 sr adj	this is an instantiation of 6
13		12 ud	
14	$\exists y \forall x(Gy \wedge Hy \wedge L(xy) \wedge L(yx))$	6 eg	match pr3
15	$\sim\exists z(Fz \wedge Gz)$	14 pr3 mp	
16	$\forall z\sim(Fz \wedge Gz)$	15 QN	
17	$\sim(Fz \wedge Gz)$	16 ui	match 3
18	$\sim Fz \vee \sim Gz$	17 dm	
19	$\sim\sim Fx$	3 dn	
20	$\sim Gz$	18 19 mtp cd	
21		2 ud	

27. $\exists x(Ax \wedge \forall y H(xy))$. $\forall x(Ax \rightarrow Fx)$. $\exists x Fx \rightarrow \forall y \forall z(By \wedge H(zz) \rightarrow G(yz))$. $\therefore \forall x(Bx \rightarrow \exists y(Fy \wedge G(xy)))$

1	~~Show~~ $\forall x(Bx \rightarrow \exists y(Fy \wedge G(xy)))$		
2	~~Show~~ $Bx \rightarrow \exists y(Fy \wedge G(xy))$		
3	Bx	ass cd	
4	$Ai \wedge \forall y H(iy)$	pr1 ei	Use EI asap. New variable.
5	Ai	4 sl	
6	$Ai \rightarrow Fi$	pr2 ui	match 5
7	Fi	5 6 mp	
8	$\exists x Fx$	7 eg	match pr3
9	$\forall y \forall z(By \wedge H(zz) \rightarrow G(yz))$	8 pr3 mp	
10	$\forall z(Bx \wedge H(zz) \rightarrow G(xz))$	9 ui	match 3: Bx
11	$Bx \wedge H(ii) \rightarrow G(xi))$	10 ui	match 4: $\forall y H(iy)$
12	$\forall y H(iy)$	4 sr	
13	$H(ii)$	12 ui	match 11
14	$Bx \wedge H(ii)$	3 13 adj	
15	$G(xi)$	11 14 mp	
16	$Fi \wedge G(xi)$	7 15 adj	
17	$\exists y(Fy \wedge G(xy)$	16 eg	match show line
18		17 cd	
19		2 ud	

28. $\forall x(Ax \rightarrow \exists y(Gy \wedge F(xy))) \rightarrow \forall xC(xa)$. $\forall x(\sim Ax \vee Bx)$. $\forall x(Bx \rightarrow Gx)$.
$\therefore \forall y\forall z(Gy \rightarrow F(zy)) \rightarrow \exists xC(xx)$

1	~~Show~~ $\forall y\forall z(Gy \rightarrow F(zy)) \rightarrow \exists xC(xx)$		
2	$\forall y\forall z(Gy \rightarrow F(zy))$	ass cd	
3	~~Show~~ $\forall x(Ax \rightarrow \exists y(Gy \wedge F(xy)))$		show antecedent of pr1
4	~~Show~~ $Ax \rightarrow \exists y(Gy \wedge F(xy))$		show instantiation of 3 for ud
5	Ax	ass cd	
6	$\sim Ax \vee Bx$	pr2 ui	match 5
7	$\sim\sim Ax$	5 dn	
8	Bx	6 7 mtp	
9	$Bx \rightarrow Gx$	pr3 ui	match 8
10	Gx	8 9 mp	
11	$\forall z(Gx \rightarrow F(zx))$	2 ui	match 10
12	$Gx \rightarrow F(xx)$	11 ui	match show line 4
13	$F(xx)$	10 12 mp	
14	$Gx \wedge F(xx)$	10 14 adj	
15	$\exists y(Gy \wedge F(xy))$	14 eg	match show line 4
16		15 cd	
17		3 ud	
18	$\forall xC(xa)$	3 pr1 mp	
19	$C(aa)$	18 ui	match show line 1 (both terms in C
20	$\exists xC(xx)$	19 eg cd	are the same)

29. $\exists x \exists y L(xy) \rightarrow \forall x \forall y \forall z(L(xy) \wedge L(yz) \rightarrow L(xz))$. $\therefore$ $\forall x \forall y(L(xy) \rightarrow \sim L(yx)) \leftrightarrow \sim \exists x L(xx)$

1	~~Show~~ $\forall x \forall y(L(xy) \rightarrow \sim L(yx)) \leftrightarrow \sim \exists x L(xx)$		
2	~~Show~~ $\sim \exists x L(xx) \rightarrow \forall x \forall y(L(xy) \rightarrow \sim L(yx))$		
3	$\sim \exists x L(xx)$	ass cd	
4	~~Show~~ $\forall x \forall y(L(xy) \rightarrow \sim L(yx))$		
5	~~Show~~ $\forall y(L(xy) \rightarrow \sim L(yx))$		
6	~~Show~~ $L(xy) \rightarrow \sim L(yx)$		
7	$L(xy)$	ass cd	
8	~~Show~~ $\sim L(yx)$		
9	$L(yx)$	ass id	
10	$\exists y L(xy)$	7 eg	
11	$\exists x \exists y L(xy)$	10 eg	
12	$\forall x \forall y \forall z(L(xy) \wedge L(yz) \rightarrow L(xz))$	11 pr1 mp	
13	$\forall y \forall z(L(xy) \wedge L(yz) \rightarrow L(xz))$	12 ui	match 7
14	$\forall z(L(xy) \wedge L(yz) \rightarrow L(xz))$	13 ui	match 7
15	$L(xy) \wedge L(yx) \rightarrow L(xx)$	14 ui	match 9
16	$L(xy) \wedge L(yx)$	7 9 adj	
17	$L(xx)$	15 16 mp	
18	$\exists x L(xx)$	17 eg	
19	$\sim \exists x L(xx)$	3 r 18 id	
20		8 cd	
21		6 ud	
22		5 ud	
23		4 cd	
34	~~Show~~ $\forall x \forall y(L(xy) \rightarrow \sim L(yx)) \rightarrow \sim \exists x L(xx)$		
35	$\forall x \forall y(L(xy) \rightarrow \sim L(yx))$	ass cd	
36	~~Show~~ $\sim \exists x L(xx)$		
37	$\exists x L(xx)$	ass id	
38	$L(ii)$	37 ei	new variable
39	$\forall y(L(iy) \rightarrow \sim L(yi))$	35 ui	match 38
40	$L(ii) \rightarrow \sim L(ii)$	39 ui	match 38
41	$\sim L(ii)$	38 40 mp 38 id	
42		36 cd	
43	$\forall x \forall y(L(xy) \rightarrow \sim L(yx)) \leftrightarrow \sim \exists x L(xx)$	2 34 cb dd	

30. $\forall x\exists y(Ax \wedge By)$. $\exists x(Ax \wedge Bx) \rightarrow \exists x\forall yH(a(x)y)$. $\therefore \exists xH(xx)$

1	~~Show~~ $\exists xH(xx)$		
2	$\exists y(Ax \wedge By)$	pr1 ui	nothing to match. Use any variable.
3	$Ax \wedge Bi$	2 ei	new variable
4	$\exists y(Ai \wedge By)$	2 ui	match 3
5	$Ai \wedge Bk$	4 ei	new variable
6	Bi	3 sr	
7	Ai	5 sl	
8	$Ai \wedge Bi$	6 7 adj	
9	$\exists x(Ax \wedge Bx)$	8 eg	match pr2
10	$\exists x\forall yH(a(x)y)$	9 pr2 mp	
11	$\forall yH(a(m)y)$	10 ei	new variable
12	$H(a(m)a(m))$	11 ui	match first term in 11
13	$\exists xH(xx)$	12 eg	match show line
14		13 dd	

31. $\forall x\forall y(Fx\rightarrow\exists zG(zy)) \rightarrow \forall x\exists y\forall zH(xyz)$. $\exists x\forall y(H(xyy) \rightarrow\forall z{\sim}B(xz))$. $\therefore$ $\forall x(\exists zFz \rightarrow G(bx)) \rightarrow{\sim}\forall yB(yy)$

1	~~Show~~ $\forall x(\exists zFz \rightarrow G(bx)) \rightarrow{\sim}\forall yB(yy)$		
2	$\forall x(\exists zFz \rightarrow G(bx))$	ass cd	
3	~~Show~~ $\forall x\forall y(Fx\rightarrow\exists zG(zy))$		show antecedent of pr1
4	~~Show~~ $\forall y(Fx\rightarrow\exists zG(zy))$		
5	~~Show~~ $Fx\rightarrow\exists zG(zy))$		
6	Fx		
7	$\exists zFz \rightarrow G(by)$	2 ui	match show line 5
8	$\exists zFz$		
9	$G(by)$		
10	$\exists zG(zy)$	9 eg dd	match show line 5
11		5 ud	
12		4 ud	
13	$\forall x\exists y\forall zH(xyz)$	3 pr1 mp	
14	$\forall y(H(iyy) \rightarrow\forall z{\sim}B(iz))$	pr2 ei	new variable
15	$\exists y\forall zH(iyz)$	13 ui	match 14
16	$\forall zH(ikz)$	15 ei	new variable
17	$H(ikk)\rightarrow\forall z{\sim}B(iz)$	14 ui	match 16
18	$H(ikk)$	16 ui	match 17
19	$\forall z{\sim}B(iz)$	17 18 mp	
20	${\sim}B(ii)$	19 ui	match other term in B to match show line
21	$\exists y{\sim}B(yy)$	20 eg	match show line
22	${\sim}\forall yB(yy)$	21 qn cd	

32. $\exists x \forall y \exists z(B(xyz) \rightarrow C(yzx))$. $\exists x \exists y \exists z C(xyz) \rightarrow \exists x \forall y L(a(x)y) \therefore \forall x \exists y \forall z B(xyz) \rightarrow \exists x L(xa(x))$

1	~~Show~~ $\forall x \exists y \forall z B(xyz) \rightarrow \exists x L(xa(x))$		
2	$\forall x \exists y \forall z B(xyz)$	ass cd	
3	$\forall y \exists z(B(iyz) \rightarrow C(yzi))$	pr1 ei	new variable
4	$\exists y \forall z B(iyz)$	2 ui	match 3
5	$\forall z B(ikz)$	4 ei	new variable
6	$\exists z(B(ikz) \rightarrow C(kzi))$	3 ui	match 5
7	$B(ikm) \rightarrow C(kmi)$	6 ei	new variable
8	$B(ikm)$	5 ui	match 7
9	$C(kmi)$	7 8 mp	
10	$\exists z C(kmz)$	9 eg	match pr2
11	$\exists y \exists z C(kyz)$	10 eg	match pr2
12	$\exists x \exists y \exists z C(xyz)$	11 eg	match pr2
13	$\exists x \forall y L(a(x)y)$	12 pr2 mp	
14	$\forall y L(a(o)y)$	13 ei	new variable
15	$L(a(o)a(a(o)))$	14 ui	match first term in consequent of show line 1.
16	$\exists x L(xa(x))$	15 eg cd	match consequent of show line 1

33. $\forall x \exists y \sim(Fx \vee Gy)$. $\exists x(Fx \leftrightarrow Gx) \rightarrow \forall x \exists y \forall z L(xyz) \therefore \exists x \exists y L(xyy)$

1	~~Show~~ $\exists x \exists y L(xyy)$		
2	~~Show~~ $\exists x(Fx \leftrightarrow Gx)$		We need to show $\exists x(Fx \leftrightarrow Gx)$ to use with pr2
3	$\exists y \sim(Fx \vee Gy)$	pr1 ui	
4	$\sim(Fx \vee Gk)$	3 ei	instantiate to NEW term
5	$\sim Fx \wedge \sim Gk$	4 dm	If we had $\sim Fk$ or $\sim Gx$ it would be easy to show 2
6	$\exists y \sim(Fk \vee Gy)$	pr1 ui	instantiate pr1 again using k for x … to get $\sim Fk$
7	$\sim(Fk \vee Gm)$	6 ei	instantiate to NEW term
8	$\sim Fk \wedge \sim Gm$	7 dm	Now we have $\sim Fk$ to use with $\sim Gk$ to show $Fk \leftrightarrow Gk$
9	$\sim Gk$	5 s	
10	$\sim Fk$	8 s	
11	$\sim Gk \vee Fk$	9 add	
12	$Gk \rightarrow Fk$	11 cdj	
13	$\sim Fk \vee Gk$	10 add	
14	$Fk \rightarrow Gk$	13 cdj	
15	$Fk \leftrightarrow Gk$	12 14 cb	
16	$\exists x(Fx \leftrightarrow Gx)$	15 eg x/k dd	generalize 15 to match 2
17	$\forall x \exists y \forall z L(xyz)$	2 pr2 mp	
18	$\exists y \forall z L(xyz)$	17 ui x/x	instantiate 17 using any term
19	$\forall z L(xiz)$	18 ei i/y	instantiate 18 to NEW term
20	$L(xii)$	19 ui i/z	instantiate 19 to same variable as 19 to match conc.
21	$\exists y L(xyy)$	20 eg y/i	generalize 20 to match conc.
22	$\exists x \exists y L(xyy)$	21 eg x/x dd	generalize 21 to match conc.

34. $\forall x\exists y\forall z(B(xyz) \to G(xy) \wedge {\sim}G(yz))$. $\forall x\forall y\forall z(G(xy) \wedge {\sim} G(zx)\to H(yz))$

$\therefore\ \exists x\forall y\forall zB(xyz) \to\ \exists x(H(xx) \wedge {\sim}G(xx))$

1	~~Show~~ $\exists x\forall y\forall zB(xyz) \to\ \exists x(H(xx) \wedge {\sim}G(xx))$		
2	$\exists x\forall y\forall zB(xyz)$	ass cd	now we need ~G(kk)
3	$\forall y\forall zB(iyz)$	2 ei	new variable
4	$\exists y\forall z(B(iyz) \to(G(iy) \wedge {\sim}G(yz))$	pr1 ui	
5	$\forall z(B(ikz) \to G(ik) \wedge {\sim}G(kz))$	4 ei	new variable
6	$\forall zB(ikz)$	3 ui	match 5
7	$\forall y\forall z(G(iy) \wedge{\sim} G(zi)\to H(yz))$	pr2 ui	match 5
8	$\forall z(G(ik) \wedge{\sim} G(zi)\to H(kz))$	7 ui	match 5
9	$G(ik) \wedge {\sim}G(ki) \to H(kk)$	8 ui	match goal $\exists x(H(xx) \wedge\ldots$
10	$B(iki) \to(G(ik) \wedge {\sim}G(ki))$	5 ui	match 9
11	$B(iki)$	6 ui	match 10
12	$G(ik) \wedge {\sim}G(ki)$	10 11 cd	
13	$H(kk)$	9 12 mp	one half of the goal: H(kk)
14	$B(ikk) \to G(ik) \wedge {\sim}G(kk)$	5 ui	
15	$B(ikk)$	6 ui	
16	$G(ik) \wedge {\sim}G(kk)$	14 15 mp	
17	${\sim}G(kk)$	16 sr	
18	$H(kk) \wedge {\sim}G(kk)$	13 17 adj	
19	$\exists x(H(xx) \wedge {\sim}G(xx))$	18 eg cd	

35. $\therefore\ \forall x\exists y{\sim}(Fy \vee \forall zG(zx)) \to \exists y\exists x{\sim}({\sim}G(yx) \to Fx)$

1	~~Show~~ $\forall x\exists y{\sim}(Fy \vee \forall zG(zx)) \to \exists y\exists x{\sim}({\sim}G(yx) \to Fx)$		
2	$\forall x\exists y{\sim}(Fy \vee \forall zG(zx))$	ass cd	
3	$\exists y{\sim}(Fy \vee \forall zG(zx))$	2 ui	nothing to match.
4	${\sim}(Fi \vee \forall zG(zx))$	3 ei	new variable
5	$\exists y{\sim}(Fy \vee \forall zG(zi))$	2 ui	match 4
6	${\sim}(Fk \vee \forall zG(zi))$	5 ei	new variable
7	${\sim}Fi \wedge {\sim}\forall zG(zx)$	4 dm	
8	${\sim}Fk \wedge {\sim}\forall zG(zi)$	6 dm	
9	${\sim}Fi$	7 sl	
10	${\sim}\forall zG(zi)$	8 sr	
11	$\exists z{\sim}G(zi)$	10 qn	
12	${\sim}G(mi)$	11 ei	new variable
13	${\sim}G(mi) \wedge {\sim}Fi$	12 9 adj	
14	${\sim}({\sim}G(mi) \to Fi)$	13 nc	
15	$\exists x{\sim}({\sim}G(mx) \to Fx)$	14 eg	match show line
16	$\exists y\exists x{\sim}({\sim}G(yx) \to Fx)$	15 eg	match show line
17		16 cd	

36. $\exists x\forall y\forall z(A(a(x)y) \wedge B(b(y)z))$. $\forall x\forall y(A(xx) \wedge B(yy) \rightarrow C(xy))$. $\therefore \exists x\exists yC(xy)$

1	~~Show~~ $\exists x\exists yC(xy)$		
2	$\forall y\forall z(A(a(i)y) \wedge B(b(y)z))$	pr1 ei	new variable
3	$\forall z(A(a(i)a(i)) \wedge B(b(a(i))z))$	2 ui	match first term to match pr2: A(xx) the second term must be the same as the first.
4	$\forall y(A(a(i)a(i)) \wedge B(yy) \rightarrow C(a(i)y))$	pr2 ui	match 3
5	$A(a(i)a(i)) \wedge B(b(a(i))b(a(i)))$	3 ui	match first term to match pr2: B(yy) the second term must be the same as the first.
6	$A(a(i)a(i)) \wedge B(b(a(i))b(a(i)))\rightarrow C(a(i)b(a(i)))$	4 ui	match 5
7	$C(a(i)b(a(i)))$	5 6 mp	
8	$\exists yC(a(i)y)$	7 eg	
9	$\exists x\exists yC(xy)$	8 eg dd	

37. $\therefore \exists x\forall yL(b(x)yb(y)) \rightarrow\exists xL(xxb(x))$

1	~~Show~~ $\exists x\forall yL(b(x)yb(y)) \rightarrow\exists xL(xxb(x))$		
2	$\exists x\forall yL(b(x)yb(y))$	ass cd	Show 1 is $\rightarrow$ so assume ant
3	$\forall yL(b(k)yb(y))$	2 ei k/x	New term
4	$L(b(k)b(k)b(b(k)))$	3 ui b(k)/y	Instantiate y to b(k) to match first term so that it matches the consequent.
5	$\exists xL(xxb(x)$	4 eg x/b(k)	
6		5 cd	

38. $\forall xI(a(x)x)$. $\forall x\forall y\forall z(I(xy) \wedge I(yz) \rightarrow I(xz))$. $\therefore$ $\forall xI(a(a(a(x)))a(x))$

1	~~Show~~ $\forall xI(a(a(a(x)))a(x))$		
2	~~Show~~ $I(a(a(a(x)))\ a(x))$		Show 1 is $\forall$, show instance
3	$I(a(a(x))\ a(x))$	pr1 ui a(x)/x	Use ui, making 2^{nd} term match 2^{nd} term of line 2
4	$I(a(a(a(x)))\ a(a(x)))$	pr1 ui a(a(x))/x	use ui again, making the 2^{nd} term of 4 match the 1^{st} term of 3
5	$\forall y\forall z(I(a(a(a(x)))y) \wedge I(yz) \rightarrow I(a(a(a(x)))z))$	pr1 ui	USE UI three times on pr2 so that 3 and 4 form the antecedent and 2 is the consequent.
6	$\forall z(I(a(a(a(x)))a(a(x))) \wedge I(a(a(x))z) \rightarrow I(a(a(a(x)))z))$	4 ui	
7	$I(a(a(a(x)))a(a(x))) \wedge I(a(a(x))a(x)) \rightarrow I(a(a(a(x)))a(x))$	5 ui	
8	$I(a(a(a(x)))a(a(x))) \wedge I(a(a(x))a(x))$	2 3 adj	
9	$I(a(a(a(x)))a(x))$	6 7 mp, dd	
10		2 ud	