Tutorial 7 Solution

The questions for this tutorial have been revised directly from the class text "The Statistical Sleuth"

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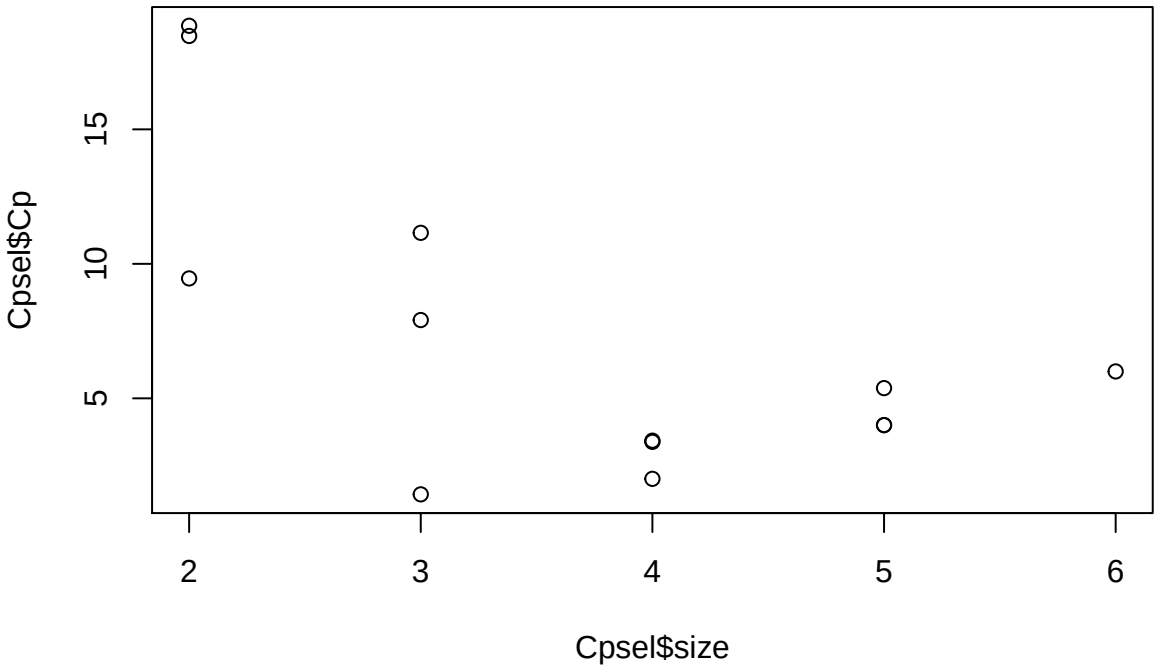
Question 3 in Tutorial 6 (Con'd, revised based on the exercise in Chapter 12 from class text "The Statistical Sleuth")

Blood-Brain Barrier. Please use "install.packages('Sleuth3')" and "library(Sleuth3)" to call the dataset in this problem. Using the data stored in the object "case1102" of the R library "Sleuth3", perform the following variable selection techniques to find a subset of the covariates-days after inoculation (Days), tumor weight (Tumor), weight loss (Loss), initial weight (Weight), and sex (Sex)-for explaining log of the ratio of brain tumor antibody count (Brain) to liver antibody count (Liver).

a) Cp plot among all subsets (showing at most 3 subsets for each size).

Solution:

```
rm(list=ls())
library(Sleuth3)

attach(case1102)
IndSex=ifelse(Sex=='Female',1,0)
Variable=c("Days","Tumor","Loss", "Weight", "IndSex")
p=length(Variable)
X=data.frame(Days,Tumor,Loss, Weight, IndSex)
Y=log(Brain/Liver)
detach(case1102)

#a)
library(leaps)
Cpsel=leaps(X,Y,method="Cp",nbest=3)
plot(Cpsel$size,Cpsel$Cp)
```




```
cbind(Cpsel$which, Cpsel$size, Cpsel$Cp)
```

```
  1 2 3 4 5
1 0 0 0 0 1 2  9.457598
1 1 0 0 0 0 2 18.472485
1 0 0 0 1 0 2 18.850283
2 1 0 0 0 1 3  1.430200
2 1 0 0 1 0 3  7.910826
2 0 0 0 1 1 3 11.153109
3 1 0 0 1 1 4  2.006538
3 1 0 1 0 1 4  3.381547
3 1 1 0 0 1 4  3.426240
4 1 1 0 1 1 5  4.000835
4 1 0 1 1 1 5  4.006424
4 1 1 1 0 1 5  5.379458
5 1 1 1 1 1 6  6.000000
```

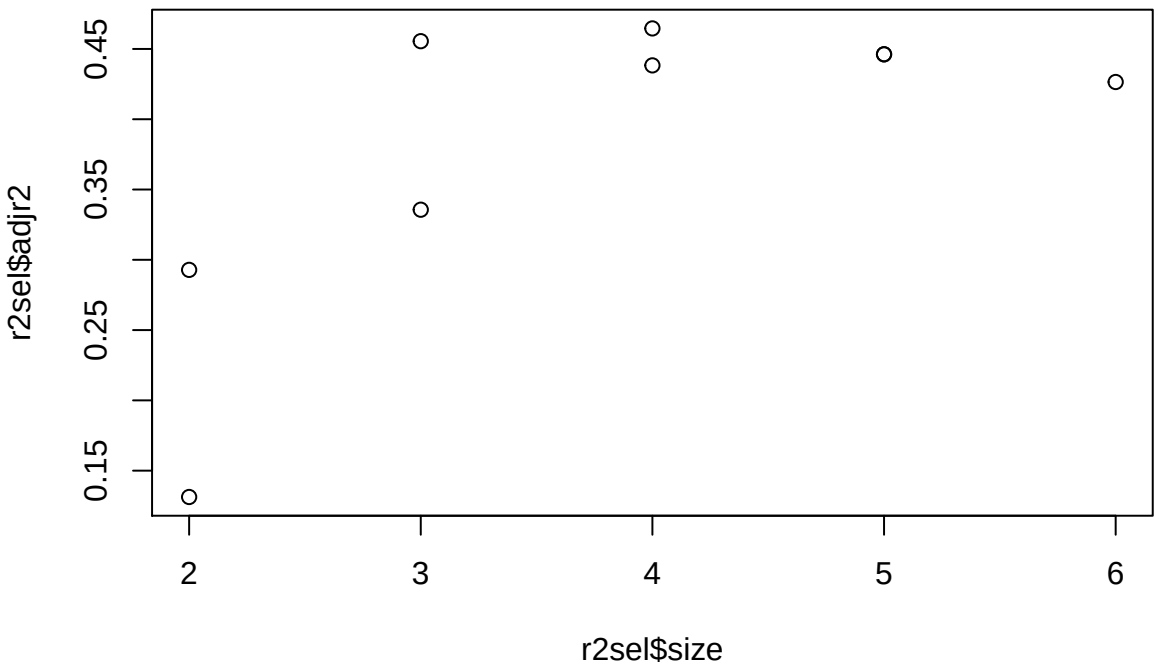
```
result=cbind(Cpsel$which, Cpsel$size, Cpsel$Cp)
result=result[which(result[,7]==min(result[,7])),][1:p]

#Selected Variables
Variable[as.logical(result)]
```

```
[1] "Days"   "IndSex"
```

b) Adjusted R^2 plot among all subsets (showing at most 2 subsets for each size).

Solution:

```
r2sel=leaps(X,Y,method="adjr2",nbest=2)
plot(r2sel$size,r2sel$adjr2)
```




```
cbind(r2sel$which, r2sel$size, r2sel$adjr2)
```

```
  1 2 3 4 5
1 0 0 0 0 1 2 0.2928357
1 1 0 0 0 0 2 0.1312697
2 1 0 0 0 1 3 0.4555335
2 1 0 0 1 0 3 0.3356402
3 1 0 0 1 1 4 0.4646007
3 1 0 1 0 1 4 0.4383147
4 1 1 0 1 1 5 0.4462514
4 1 0 1 1 1 5 0.4461409
5 1 1 1 1 1 6 0.4264918
```

```
result=cbind(r2sel$which, r2sel$size, r2sel$adjr2)
result=result[which(result[,7]==max(result[,7])),][1:p]

#Selected Variables
Variable[as.logical(result)]
```

```
[1] "Days"   "Weight" "IndSex"
```

c) Forward selection by using AIC.

Solution:

```
library(MASS)
fit<-lm(Y~1,data=X)
#stepAIC(fit,direction="forward",scope=list(lower=~1,upper=~ Days+Tumor+Loss+Weight+IndSex))
result=stepAIC(fit,direction="forward",scope=list(lower=~1,upper=~ Days+Tumor+Loss+Weight+IndSex))
```

```
Start:  AIC=56.08
Y ~ 1

         Df Sum of Sq    RSS    AIC
+ IndSex  1    52.421 114.39 45.249
+ Days    1    26.288 140.52 52.245
+ Weight  1    25.193 141.61 52.509
<none>                166.81 56.076
+ Tumor   1     8.070 158.74 56.390
+ Loss    1     0.189 166.62 58.037

Step:  AIC=45.25
Y ~ IndSex

         Df Sum of Sq     RSS    AIC
+ Days    1   29.0686  85.316 37.280
<none>                114.385 45.249
+ Weight  1    0.8827 113.502 46.986
+ Tumor   1    0.3262 114.058 47.152
+ Loss    1    0.0076 114.377 47.247

Step:  AIC=37.28
Y ~ IndSex + Days

         Df Sum of Sq    RSS    AIC
<none>                85.316 37.280
+ Weight  1    4.1271 81.189 37.594
+ Loss    1    0.1410 85.175 39.224
+ Tumor   1    0.0115 85.304 39.275
```

```
names(result)
```

```
 [1] "coefficients"  "residuals"     "effects"       "rank"
 [5] "fitted.values" "assign"        "qr"            "df.residual"
 [9] "xlevels"       "call"          "terms"         "model"
[13] "anova"
```

```r
#Selected Variables
names(result$coef)[-1]
```

```
[1] "IndSex" "Days"
```

d) Backward elimination by using BIC.

Solution:

```r
n=length(Y)
fit<-lm(Y~.,data=X)
#stepAIC(fit,direction="backward",data=X,k=log(n))
result=stepAIC(fit,direction="backward",data=X,k=log(n))
```

```
Start:  AIC=50.74
Y ~ Days + Tumor + Loss + Weight + IndSex

          Df Sum of Sq     RSS    AIC
- Loss     1     0.002  81.172 47.219
- Tumor    1     0.019  81.188 47.226
- Weight   1     3.999  85.169 48.853
<none>                  81.170 50.744
- IndSex   1    19.865 101.035 54.662
- Days     1    31.800 112.970 58.458

Step:  AIC=47.22
Y ~ Days + Tumor + Weight + IndSex

          Df Sum of Sq     RSS    AIC
- Tumor    1     0.017  81.189 43.700
- Weight   1     4.132  85.304 45.381
<none>                  81.172 47.219
- IndSex   1    20.935 102.107 51.494
- Days     1    31.973 113.146 54.984

Step:  AIC=43.7
Y ~ Days + Weight + IndSex

          Df Sum of Sq     RSS    AIC
- Weight   1     4.127  85.316 41.859
<none>                  81.189 43.700
- IndSex   1    22.914 104.103 48.626
- Days     1    32.313 113.502 51.565
```

```
Step:  AIC=41.86
Y ~ Days + IndSex

         Df Sum of Sq     RSS    AIC
<none>                  85.316 41.859
- Days    1    29.069 114.385 48.302
- IndSex  1    55.202 140.518 55.298
```

```r
names(result)
```

```
 [1] "coefficients"  "residuals"     "effects"       "rank"
 [5] "fitted.values" "assign"        "qr"            "df.residual"
 [9] "xlevels"       "call"          "terms"         "model"
[13] "anova"
```

```r
#Selected Variables
names(result$coef)[-1]
```

```
[1] "Days"   "IndSex"
```

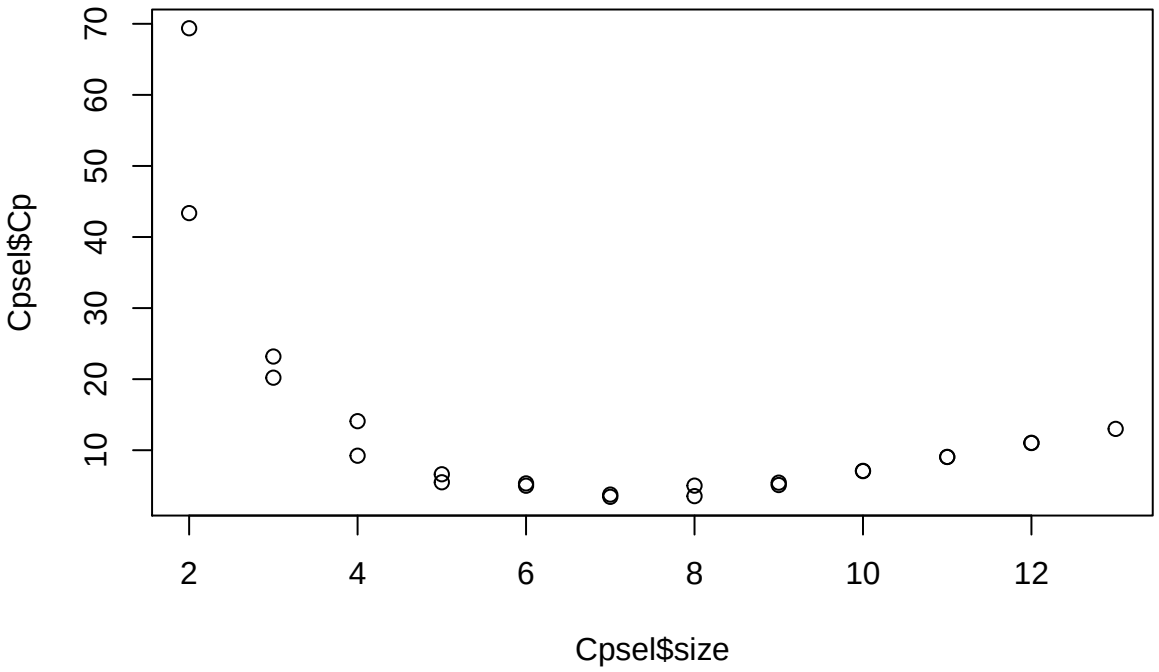
Question 1 (revised based on the exercise in Chapter 12 from class text "The Statistical Sleuth")

Pollution and Mortality. Dataset is stored in the object "ex1217"" of the R library "Sleuth3". The 15 variables for each of 60 cities are (1) Precip: mean annual precipitation (in inches); (2) Humidity: percent relative humidity (annual average at 1 P.M.); (3) JanTemp: mean January temperature (in degrees Fahrenheit); (4) JulyTemp: mean July temperature (in degrees Fahrenheit); (5) Over65: percentage of the population aged 65 years or over; (6) House: population per household; (7) Educ: median number of school years completed by persons of age 25 years or more; (8) Sound: percentage of the housing that is sound with all facilities; (9) Density: population density (in persons per square mile of urbanized area); (10) NonWhite: percentage of 1960 population that is nonwhite; (11) WhiteCol: percentage of employment in white-collar occupations; (12) Poor: percentage of households with annual income under $3,000 in 1960; (13) relative pollution potential of hydrocarbons (HC); (14) relative pollution potential of oxides of nitrogen (NOX); and (15) relative pollution potential of sulphur dioxide (SO2). It is desired to determine whether the pollution variables (13, 14, and 15) are associated with mortality, after the other climate and socioeconomic variables are accounted for. (Note: These data have problems with influential observations and with lack of independence due to spatial correlation; these problems are ignored for purposes of this exercise.)

With mortality as the response, use Cp and adjusted R^2 plots among all subsets (showing at most 2 subsets for each size) to select a good-fitting regression model involving weather and socioeconomic variables as explanatory, respectively. To the model with the lowest Cp (highest adjusted R^2), add the three pollution variables (transformed to their logarithms) and obtain the p-value from the extra-sum-of-squares F-test due to their addition. Based on the R output, are the pollution variables (13, 14, and 15) associated with mortality or not, after the other climate and socioeconomic variables are accounted for? Are the results different for Cp and adjusted R^2?

Solution: We try Cp statistic first.

```
Xf=ex1217[,3:17]
Variablef=colnames(Xf)
pf=length(Variablef)
Y=ex1217$Mortality

#Cp
X=Xf[,1:12]
Variable=colnames(X)
p=length(Variable)


Cpsel=leaps(X,Y,method="Cp",nbest=2)
plot(Cpsel$size,Cpsel$Cp)
```




```
cbind(Cpsel$which, Cpsel$size, Cpsel$Cp)
```

```
   1 2 3 4 5 6 7 8 9 A B C
1  0 0 0 0 0 0 0 0 0 1 0 0  2 43.365661
1  0 0 0 0 0 0 1 0 0 0 0 0  2 69.375569
2  0 0 0 0 0 0 1 0 0 1 0 0  3 20.205143
2  0 0 1 0 0 0 0 0 0 1 0 0  3 23.185052
3  0 0 1 0 0 0 1 0 0 1 0 0  4  9.228953
3  0 0 0 0 0 0 1 0 0 1 0 1  4 14.086062
```

```
4  0 0 1 0 0 1 1 0 0 1 0 0  5  5.488796
4  0 0 1 0 0 0 1 0 1 1 0 0  5  6.615457
5  0 0 1 1 0 1 1 0 0 1 0 0  6  5.001899
5  0 0 1 0 1 0 1 0 1 1 0 0  6  5.329705
6  1 0 1 1 0 0 1 0 1 1 0 0  7  3.443264
6  1 0 1 1 0 1 1 0 0 1 0 0  7  3.756277
7  1 0 1 1 0 1 1 0 1 1 0 0  8  3.538724
7  1 0 1 1 0 1 1 0 0 1 0 1  8  5.016251
8  1 0 1 1 1 1 1 0 1 1 0 0  9  5.100351
8  1 0 1 1 0 1 1 0 1 1 0 1  9  5.435193
9  1 0 1 1 1 1 1 0 1 1 1 0 10  7.060519
9  1 0 1 1 1 1 1 0 1 1 0 1 10  7.091894
10 1 0 1 1 1 1 1 0 1 1 1 1 11  9.048951
10 1 0 1 1 1 1 1 1 1 1 1 0 11  9.053624
11 1 0 1 1 1 1 1 1 1 1 1 1 12 11.010756
11 1 1 1 1 1 1 1 0 1 1 1 1 12 11.043002
12 1 1 1 1 1 1 1 1 1 1 1 1 13 13.000000
```

```
result=cbind(Cpsel$which, Cpsel$size, Cpsel$Cp)
lengthr=length(result[1,])
result=result[which(result[,lengthr]==min(result[,lengthr])),][1:p]

#Selected Variables
Variable[as.logical(result)]
```

```
[1] "Precip"   "JanTemp"  "JulyTemp" "Educ"     "Density"  "NonWhite"
```

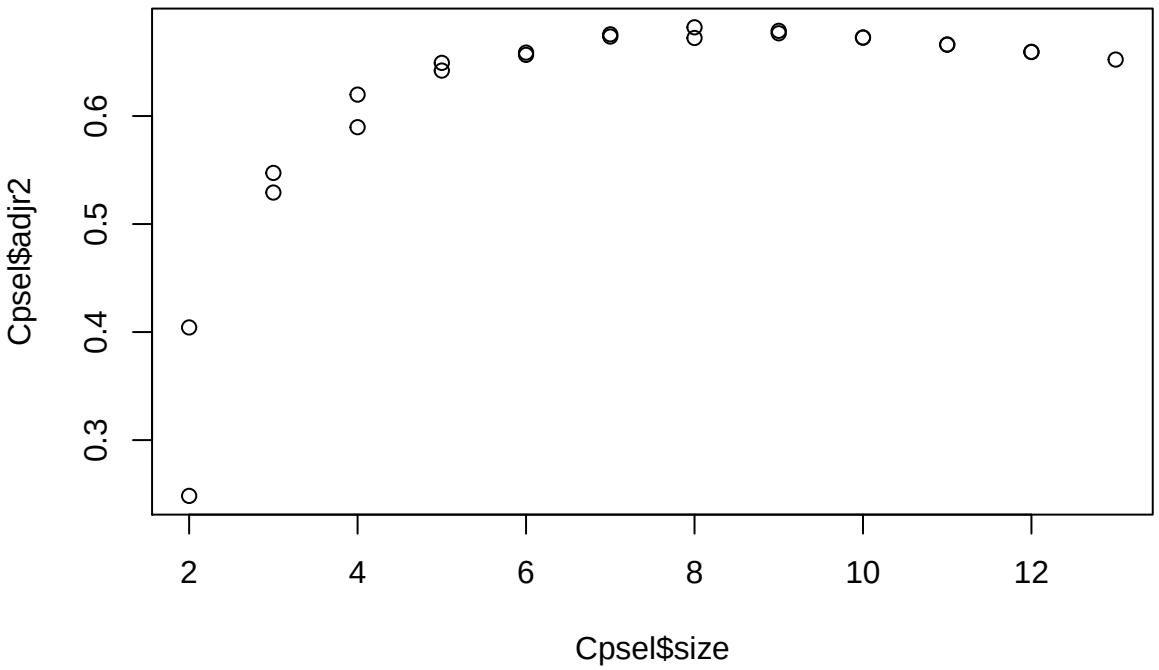
```
Svariable=Variable[as.logical(result)]

Xn=cbind(X[,Svariable],log(Xf[,13:15]))
fit=lm(Y~.,data=Xn)

fitr=lm(Y~.,data=X[,Svariable])
anova(fitr,fit,test='F')
```

```
Analysis of Variance Table

Model 1: Y ~ Precip + JanTemp + JulyTemp + Educ + Density + NonWhite
Model 2: Y ~ Precip + JanTemp + JulyTemp + Educ + Density + NonWhite +
    HC + NOX + SO2
  Res.Df   RSS Df Sum of Sq     F   Pr(>F)
1     53 66518
2     50 52712  3     13806 4.365 0.008313 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

F=4.365 p-value=0.008313, the answer for the first question is Yes when we consider Cp statistic.

We next try adjusted R^2.

```
#Adj R2
X=Xf[,1:12]
Variable=colnames(X)
p=length(Variable)


Cpsel=leaps(X,Y,method="adjr2",nbest=2)
plot(Cpsel$size,Cpsel$adjr2)
```




```
cbind(Cpsel$which, Cpsel$size, Cpsel$adjr2)
```

```
   1 2 3 4 5 6 7 8 9 A B C
1  0 0 0 0 0 0 0 0 0 1 0 0  2 0.4042961
1  0 0 0 0 0 0 1 0 0 0 0 0  2 0.2483649
2  0 0 0 0 0 0 1 0 0 1 0 0  3 0.5473304
2  0 0 1 0 0 0 0 0 0 1 0 0  3 0.5291523
3  0 0 1 0 0 0 1 0 0 1 0 0  4 0.6198185
3  0 0 0 0 0 0 1 0 0 1 0 1  4 0.5896599
4  0 0 1 0 0 1 1 0 0 1 0 0  5 0.6491958
4  0 0 1 0 0 0 1 0 1 1 0 0  5 0.6420730
5  0 0 1 1 0 1 1 0 0 1 0 0  6 0.6587130
5  0 0 1 0 1 0 1 0 1 1 0 0  6 0.6566022
6  1 0 1 1 0 0 1 0 1 1 0 0  7 0.6756205
```

```
6  1 0 1 1 0 1 1 0 0 1 0 0  7 0.6735669
7  1 0 1 1 0 1 1 0 1 1 0 0  8 0.6821177
7  1 0 1 1 0 1 1 0 0 1 0 1  8 0.6722378
8  1 0 1 1 1 1 1 0 1 1 0 0  9 0.6788735
8  1 0 1 1 0 1 1 0 1 1 0 1  9 0.6765906
9  1 0 1 1 1 1 1 0 1 1 1 0 10 0.6727280
9  1 0 1 1 1 1 1 0 1 1 0 1 10 0.6725098
10 1 0 1 1 1 1 1 0 1 1 1 1 11 0.6661311
10 1 0 1 1 1 1 1 1 1 1 1 0 11 0.6660979
11 1 0 1 1 1 1 1 1 1 1 1 1 12 0.6594522
11 1 1 1 1 1 1 1 0 1 1 1 1 12 0.6592186
12 1 1 1 1 1 1 1 1 1 1 1 1 13 0.6522860
```

```
result=cbind(Cpsel$which, Cpsel$size, Cpsel$adjr2)
lengthr=length(result[1,])
result=result[which(result[,lengthr]==max(result[,lengthr])),][1:p]

#Selected Variables
Variable[as.logical(result)]
```

```
[1] "Precip"   "JanTemp"  "JulyTemp" "House"    "Educ"     "Density"
[7] "NonWhite"
```

```
Svariable=Variable[as.logical(result)]

Xn=cbind(X[,Svariable],log(Xf[,13:15]))
fit=lm(Y~.,data=Xn)

fitr=lm(Y~.,data=X[,Svariable])
anova(fitr,fit,test='F')
```

```
Analysis of Variance Table

Model 1: Y ~ Precip + JanTemp + JulyTemp + House + Educ + Density + NonWhite
Model 2: Y ~ Precip + JanTemp + JulyTemp + House + Educ + Density + NonWhite +
    HC + NOX + SO2
  Res.Df   RSS Df Sum of Sq      F   Pr(>F)
1     52 63955
2     49 50403  3     13552 4.3915 0.008162 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

F=4.3915 p-value=0.008162, the answer for the first question is Yes when we consider adjusted R^2.

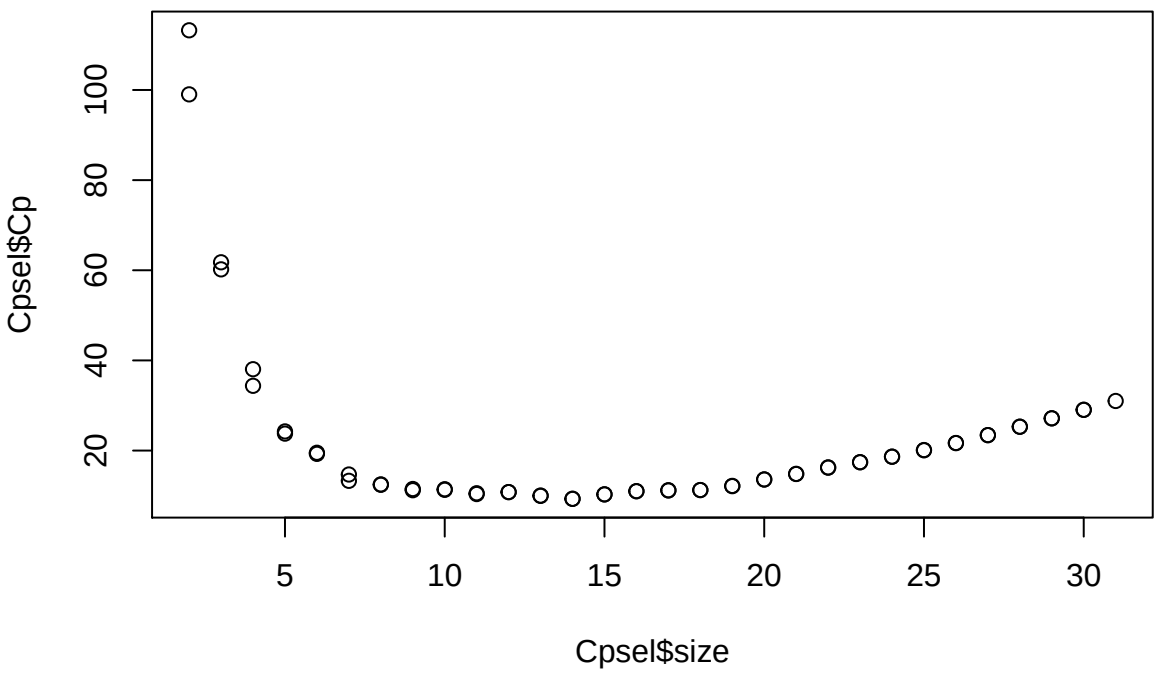
Hence, the answer for the second question is no and the results are not different for Cp and adjusted R^2.

Question 2 (revised based on the exercise in Chapter 12 from class text "The Statistical Sleuth")

Pollution and Mortality. For the dataset in Question 1, please include all the 15 explanatory variables (the three pollution variables transformed to their logarithms) and the squares of them in the multiple linear regression model with mortality as the response. Perform the following variable-selection techniques to find a subset of the covariates. a) Cp plot among all subsets (showing at most 2 subsets for each size); b) forward selection by using BIC; c) stepwise regression by using AIC. Compare the selection results first and then use the R function "system.time()" to compare the computation time for each of the techniques.

Solution:

```
X=cbind(X,log(Xf[,13:15]))
X2=X^2
colnames(X2)=paste(colnames(X),'2',sep='')
X=cbind(X,X2)
Variable=colnames(X)
p=length(Variable)

#a) Cp
Cpsel=leaps(X,Y,method="Cp",nbest=2)
plot(Cpsel$size,Cpsel$Cp)
```



```
cbind(Cpsel$which, Cpsel$size, Cpsel$Cp)
```

```
   1 2 3 4 5 6 7 8 9 A B C D E F G H I J K L M N O P Q R S T U
1  0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0  2
1  0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0  2
2  0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0  3
2  0 0 0 0 0 0 0 1 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0  3
3  0 0 0 0 0 0 0 0 0 0 1 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 1 0  4
3  1 0 0 0 0 0 0 0 0 0 1 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0  4
4  1 0 0 0 0 0 0 0 0 0 1 0 0 0 1 0 0 0 1 0 0 0 0 0 0 0 0 0  5
4  1 0 1 0 0 0 0 0 0 0 1 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0  5
5  1 0 1 0 0 0 0 0 0 0 1 0 0 0 1 0 0 0 0 0 0 0 1 0 0 0 0 0  6
5  1 0 0 0 0 0 0 0 0 0 1 0 0 0 1 0 0 0 1 0 0 0 1 0 0 0 0 0  6
6  1 0 1 0 0 0 0 0 0 0 1 0 1 0 1 0 0 0 0 0 0 0 0 0 0 0 1 0  7
6  0 0 1 0 0 0 0 0 0 0 1 0 1 0 1 0 1 0 0 0 0 0 0 0 0 0 1 0  7
7  1 0 1 0 0 0 0 0 1 0 1 0 0 0 1 0 0 0 0 0 0 0 1 1 0 0 0 0  8
7  0 0 1 0 0 0 0 0 1 0 1 0 0 0 1 0 1 0 0 0 0 0 1 1 0 0 0 0  8
8  0 0 1 0 0 0 0 0 1 0 1 0 0 0 1 0 1 0 0 0 0 0 1 1 0 1 0 0  9
8  0 0 1 0 0 0 0 1 1 0 1 0 0 0 1 0 1 0 0 0 0 0 0 1 0 1 0 0  9
9  0 0 1 0 0 0 0 1 1 0 1 0 0 0 1 0 1 0 0 0 0 0 1 1 0 1 0 0 10
9  0 0 0 0 0 0 0 1 1 0 1 0 0 0 1 0 1 0 1 0 0 0 1 1 0 1 0 0 10
10 0 0 1 0 1 0 1 1 0 1 0 0 0 1 0 1 0 0 0 0 1 1 1 0 0 0 0 0 11
10 0 0 1 0 1 1 1 1 0 1 0 0 0 1 0 1 0 0 0 0 0 1 1 0 0 0 0 0 11
11 0 0 1 1 1 0 0 0 0 1 0 1 1 1 0 1 0 0 0 0 1 1 0 0 0 0 1 0 12
11 0 0 1 0 1 0 0 0 0 1 0 1 1 1 0 1 0 0 1 0 1 1 0 0 0 0 1 0 12
12 0 0 1 0 1 0 0 0 0 1 0 1 1 1 1 1 0 0 1 0 1 1 0 0 0 0 1 0 13
12 0 0 1 1 1 0 0 0 0 1 0 1 1 1 1 1 0 0 0 0 1 1 0 0 0 0 1 0 13
13 1 0 1 0 1 0 1 0 0 1 0 1 0 1 1 0 0 0 1 0 1 1 0 0 0 0 1 1 0 0 14
13 1 0 1 0 0 0 1 0 0 1 0 1 0 1 1 0 0 0 1 1 1 1 0 0 0 0 1 1 0 0 14
14 0 0 1 1 1 0 1 0 0 1 0 1 1 1 1 1 0 0 0 0 1 1 0 0 0 0 1 0 1 0 15
14 0 0 1 1 1 0 1 0 0 1 0 1 0 1 1 1 0 0 0 0 1 1 0 0 0 1 1 1 0 0 15
15 0 0 1 1 1 1 1 0 0 1 0 1 1 1 1 1 0 0 0 0 0 1 0 0 0 1 1 0 1 0 16
15 0 0 1 0 1 1 1 0 0 1 0 1 1 1 1 1 0 0 1 0 0 1 0 0 0 1 1 0 1 0 16
16 0 0 1 1 1 0 1 0 1 1 0 1 1 1 1 1 0 0 0 0 0 1 1 0 1 0 1 1 0 0 0 17
16 0 0 1 0 1 0 1 0 1 1 0 1 1 1 1 1 0 0 1 0 1 1 0 1 0 1 1 0 0 0 17
17 0 0 1 0 1 0 1 0 1 1 0 1 1 1 1 1 0 0 1 0 1 1 1 1 0 1 1 0 0 0 18
17 0 0 1 1 1 0 1 0 1 1 0 1 1 1 1 1 0 0 0 0 0 1 1 1 1 0 1 1 0 0 0 18
18 0 0 1 0 1 0 1 0 1 1 0 1 1 1 1 1 0 0 1 0 1 1 1 1 1 1 1 0 0 0 19
18 0 0 1 1 1 0 1 0 1 1 0 1 1 1 1 1 0 0 0 0 0 1 1 1 1 1 1 0 0 0 19
19 0 0 1 0 1 0 1 0 1 1 0 1 1 1 1 1 0 0 1 0 1 1 1 1 1 1 1 0 1 0 20
19 0 0 1 0 1 0 1 0 1 1 1 1 1 1 1 1 0 0 1 0 1 1 1 1 1 1 1 0 0 0 20
20 0 1 1 0 1 0 1 0 1 1 0 1 1 1 1 1 1 0 1 0 1 1 1 1 1 1 1 0 0 0 21
20 0 0 1 0 1 0 1 0 1 1 0 1 1 1 1 1 0 0 1 0 1 1 1 1 1 1 1 1 0 21
21 0 1 1 0 1 0 1 0 1 1 1 1 1 1 1 1 1 0 1 0 1 1 1 1 1 1 1 0 0 0 22
21 0 1 1 0 1 0 1 1 1 1 1 1 1 1 1 1 1 0 1 0 1 1 0 1 1 1 1 0 0 0 22
22 1 1 1 0 1 0 1 0 1 1 1 1 1 1 1 1 1 0 1 0 1 1 1 1 1 1 1 0 0 0 23
22 1 0 1 0 1 1 1 0 1 1 0 1 1 1 1 1 1 0 0 1 0 0 1 1 1 1 1 1 1 1 1 23
```

```
23 1 1 1 0 1 1 1 0 1 1 1 1 1 1 1 1 1 0 1 0 1 1 1 1 1 1 1 0 0 0 24
23 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1 1 1 0 0 0 1 1 1 1 1 1 1 0 0 0 24
24 0 1 1 0 1 1 1 0 1 1 1 1 1 1 0 1 1 1 1 0 1 1 1 1 1 1 1 0 1 25
24 0 1 1 1 1 1 1 0 1 1 1 1 1 1 0 1 1 1 0 0 1 1 1 1 1 1 1 0 1 25
25 1 1 1 0 1 1 1 1 1 1 1 1 1 1 0 1 1 0 1 0 1 1 1 1 1 1 1 0 1 26
25 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1 1 0 0 0 1 1 1 1 1 1 1 0 1 26
26 1 1 1 0 1 1 1 1 1 1 1 1 1 1 0 1 1 1 1 0 1 1 1 1 1 1 1 0 1 27
26 1 1 1 0 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1 0 1 1 1 1 1 1 1 1 0 27
27 1 1 1 0 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1 1 1 1 0 28
27 1 1 1 0 1 1 1 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1 1 1 1 1 1 0 1 28
28 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1 1 1 1 1 1 0 1 29
28 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1 1 1 1 0 29
29 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 30
29 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1 30
30 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 31

1    99.027619
1   113.241070
2    60.199739
2    61.772858
3    34.366253
3    38.050215
4    23.757811
4    24.246182
5    19.262993
5    19.513280
6    13.285215
6    14.686431
7    12.403663
7    12.488899
8    11.197001
8    11.474956
9    11.288280
9    11.419604
10   10.345959
10   10.532119
11   10.779565
11   10.782153
12    9.941117
12   10.016920
13    9.283503
13    9.296917
14   10.266683
14   10.290041
15   10.979764
15   10.984126
16   11.142537
16   11.197929
```

```
17  11.234115
17  11.242342
18  12.098540
18  12.161725
19  13.583309
19  13.586643
20  14.796291
20  14.825419
21  16.206266
21  16.261237
22  17.403039
22  17.427081
23  18.633121
23  18.644442
24  20.060113
24  20.114628
25  21.655733
25  21.673895
26  23.421683
26  23.421700
27  25.281260
27  25.289137
28  27.146867
28  27.155997
29  29.025347
29  29.050560
30  31.000000
```

```
result=cbind(Cpsel$which, Cpsel$size, Cpsel$Cp)
lengthr=length(result[1,])
result=result[which(result[,lengthr]==min(result[,lengthr])),][1:p]

#Selected Variables
Variable[as.logical(result)]
```

```
 [1] "Precip"    "JanTemp"   "Over65"    "Educ"      "NonWhite"
 [6] "Poor"      "NOX"       "SO2"       "JulyTemp2" "House2"
[11] "Educ2"     "Poor2"     "HC2"
```

```
#Time
system.time(leaps(X,Y,method="Cp",nbest=2))
```

```
   user  system elapsed
  2.548   0.031   2.989
```

```r
#b) BIC
n=length(Y)
fit<-lm(Y~1,data=X)
name=''
for (i in 1:(p-1)){
  name=paste(name,colnames(X)[i],'+',sep='')
}
name=paste(name,colnames(X)[p],sep='')
name
```

```
[1] "Precip+Humidity+JanTemp+JulyTemp+Over65+House+Educ+Sound+Density+NonWhite+WhiteCol+Poor+HC+NOX+
```

```r
result=stepAIC(fit,direction="forward",scope=list(lower=~1,upper=~ Precip+Humidity
                                                    +JanTemp+JulyTemp
                                                    +Over65+House+Educ+Sound
                                                +Density+NonWhite+WhiteCol+Poor
                                                +HC+NOX+SO2+Precip2+Humidity2
                                                +JanTemp2+JulyTemp2+Over652+House2
                                                +Educ2+Sound2+Density2+NonWhite2
                                                +WhiteCol2+Poor2+HC2+NOX2+SO22),k=log(n))
```

```
Start:  AIC=498.73
Y ~ 1

             Df Sum of Sq    RSS    AIC
+ NonWhite    1     94596 133680 470.72
+ NonWhite2   1     82339 145936 475.98
+ Educ2       1     61923 166352 483.84
+ Educ        1     59604 168672 484.67
+ Precip      1     59256 169019 484.79
+ Precip2     1     49427 178848 488.19
+ SO22        1     45615 182660 489.45
+ Sound       1     41584 186691 490.76
+ Sound2      1     41224 187051 490.88
+ Poor        1     38459 189816 491.76
+ Poor2       1     37896 190379 491.93
+ SO2         1     37098 191178 492.19
+ House2      1     29398 198877 494.55
+ House       1     29146 199130 494.63
+ NOX         1     19464 208812 497.48
+ WhiteCol2   1     19296 208979 497.53
+ WhiteCol    1     18518 209757 497.75
+ JulyTemp    1     17515 210760 498.04
+ JulyTemp2   1     16076 212200 498.44
+ Density     1     15898 212377 498.50
<none>                   228275 498.73
+ Density2    1     10485 217790 500.01
```

```
+ Over652    1      7689 220586 500.77
+ Over65     1      6958 221318 500.97
+ HC         1      5188 223087 501.45
+ NOX2       1      5040 223235 501.49
+ Humidity2  1      3465 224810 501.91
+ Humidity   1      1789 226486 502.35
+ JanTemp2   1      1446 226829 502.45
+ JanTemp    1       206 228069 502.77
+ HC2        1         5 228271 502.83

Step:  AIC=470.72
Y ~ NonWhite

            Df Sum of Sq    RSS    AIC
+ Educ2      1     35206  98474 456.48
+ Educ       1     33849  99830 457.30
+ SO2        1     31234 102446 458.85
+ JanTemp2   1     30933 102746 459.02
+ JanTemp    1     29840 103839 459.66
+ SO22       1     29708 103972 459.73
+ Over65     1     21434 112246 464.33
+ Over652    1     19994 113685 465.09
+ WhiteCol2  1     18801 114879 465.72
+ WhiteCol   1     18153 115527 466.06
+ Density    1     16590 117090 466.86
+ Precip     1     16322 117358 467.00
+ Density2   1     12440 121240 468.95
<none>                   133680 470.72
+ Precip2    1      8605 125075 470.82
+ Sound2     1      8113 125567 471.06
+ Sound      1      7263 126417 471.46
+ NOX        1      6836 126844 471.67
+ JulyTemp2  1      3978 129701 473.00
+ JulyTemp   1      2973 130707 473.47
+ House      1      2113 131567 473.86
+ House2     1      1957 131723 473.93
+ Poor2      1      1554 132126 474.11
+ Poor       1       851 132828 474.43
+ HC         1       813 132867 474.45
+ HC2        1       750 132929 474.48
+ NOX2       1       619 133061 474.54
+ Humidity2  1       290 133390 474.68
+ NonWhite2  1       285 133395 474.69
+ Humidity   1        37 133643 474.80

Step:  AIC=456.48
Y ~ NonWhite + Educ2
```

```
            Df Sum of Sq   RSS    AIC
+ SO2        1   17997.6 80476 448.46
+ SO22       1   17332.2 81142 448.95
+ JanTemp    1   16589.0 81885 449.50
+ JanTemp2   1   16574.0 81900 449.51
+ Poor2      1   11941.9 86532 452.81
+ Poor       1   11102.6 87371 453.39
+ NOX        1    9251.0 89223 454.65
+ JulyTemp2  1    7583.9 90890 455.76
+ Density    1    7518.2 90956 455.80
+ Over65     1    6991.6 91482 456.15
+ JulyTemp   1    6883.2 91591 456.22
+ Educ       1    6802.2 91672 456.27
<none>                   98474 456.48
+ Over652    1    6354.0 92120 456.57
+ HC         1    4380.6 94093 457.84
+ Density2   1    3453.5 95021 458.43
+ NOX2       1    3370.1 95104 458.48
+ Precip     1    2143.0 96331 459.25
+ NonWhite2  1    1968.5 96505 459.36
+ HC2        1     821.9 97652 460.07
+ House2     1     580.6 97893 460.21
+ Humidity   1     526.4 97948 460.25
+ House      1     514.9 97959 460.26
+ Humidity2  1     415.0 98059 460.32
+ Precip2    1     278.8 98195 460.40
+ Sound      1     233.9 98240 460.43
+ Sound2     1      95.8 98378 460.51
+ WhiteCol   1      11.8 98462 460.56
+ WhiteCol2  1       7.2 98467 460.57

Step:  AIC=448.46
Y ~ NonWhite + Educ2 + SO2

            Df Sum of Sq   RSS    AIC
+ Precip     1    8775.1 71701 445.63
+ Educ       1    7818.5 72658 446.42
+ JanTemp    1    7352.9 73123 446.81
+ JanTemp2   1    6787.4 73689 447.27
<none>                   80476 448.46
+ HC2        1    5094.7 75382 448.63
+ Precip2    1    4915.2 75561 448.77
+ Poor2      1    3112.5 77364 450.19
+ Over652    1    2712.8 77764 450.50
+ Over65     1    2563.0 77913 450.61
+ Poor       1    2386.6 78090 450.75
+ NOX2       1    2121.4 78355 450.95
+ HC         1    2047.6 78429 451.01
```

```
+ NonWhite2  1    1283.6 79193 451.59
+ Density    1     899.1 79577 451.88
+ Sound2     1     701.7 79775 452.03
+ Humidity   1     694.9 79781 452.03
+ Humidity2  1     470.4 80006 452.20
+ Sound      1     418.1 80058 452.24
+ JulyTemp2  1     229.8 80247 452.38
+ NOX        1     192.6 80284 452.41
+ SO22       1     184.5 80292 452.42
+ JulyTemp   1     153.6 80323 452.44
+ Density2   1     150.0 80326 452.44
+ WhiteCol2  1     141.6 80335 452.45
+ WhiteCol   1     103.1 80373 452.48
+ House2     1      48.7 80428 452.52
+ House      1      27.0 80449 452.53

Step:  AIC=445.63
Y ~ NonWhite + Educ2 + SO2 + Precip

            Df Sum of Sq   RSS    AIC
+ JanTemp    1    5516.4 66185 444.92
+ JanTemp2   1    5249.9 66451 445.16
+ Precip2    1    5077.3 66624 445.31
+ Educ       1    4981.7 66720 445.40
<none>                   71701 445.63
+ Poor2      1    4317.9 67383 445.99
+ Poor       1    2995.9 68705 447.16
+ NonWhite2  1    2501.9 69199 447.59
+ NOX        1    1194.0 70507 448.71
+ JulyTemp2  1    1119.7 70582 448.78
+ JulyTemp   1     993.5 70708 448.88
+ Density    1     820.4 70881 449.03
+ SO22       1     681.8 71019 449.15
+ Humidity   1     518.1 71183 449.29
+ HC2        1     504.0 71197 449.30
+ Humidity2  1     429.9 71271 449.36
+ WhiteCol2  1     288.9 71412 449.48
+ WhiteCol   1     240.3 71461 449.52
+ Sound2     1     217.1 71484 449.54
+ Density2   1     153.9 71547 449.59
+ Sound      1      87.9 71613 449.65
+ House      1      33.1 71668 449.69
+ House2     1      18.6 71683 449.71
+ Over652    1       2.4 71699 449.72
+ Over65     1       2.3 71699 449.72
+ HC         1       1.9 71699 449.72
+ NOX2       1       0.0 71701 449.72
```

```
Step:  AIC=444.92
Y ~ NonWhite + Educ2 + SO2 + Precip + JanTemp

            Df Sum of Sq   RSS    AIC
+ NOX        1    8403.6 57781 440.86
+ NOX2       1    4635.3 61550 444.66
<none>                   66185 444.92
+ SO22       1    2758.0 63427 446.46
+ NonWhite2  1    2660.6 63524 446.55
+ HC         1    2423.1 63762 446.77
+ Educ       1    2008.5 64176 447.16
+ House2     1    1904.2 64281 447.26
+ House      1    1845.3 64340 447.32
+ HC2        1    1578.1 64607 447.56
+ Density    1    1537.6 64647 447.60
+ Poor2      1    1410.2 64775 447.72
+ JulyTemp2  1    1094.3 65091 448.01
+ JulyTemp   1    1091.8 65093 448.01
+ Humidity2  1     960.8 65224 448.13
+ Humidity   1     949.1 65236 448.15
+ Precip2    1     722.7 65462 448.35
+ Poor       1     540.8 65644 448.52
+ Density2   1     517.3 65668 448.54
+ Sound      1      85.4 66099 448.93
+ Over652    1      63.3 66122 448.95
+ Over65     1      52.9 66132 448.96
+ Sound2     1      28.7 66156 448.99
+ WhiteCol2  1      19.6 66165 448.99
+ WhiteCol   1       4.1 66181 449.01
+ JanTemp2   1       0.5 66184 449.01

Step:  AIC=440.86
Y ~ NonWhite + Educ2 + SO2 + Precip + JanTemp + NOX

            Df Sum of Sq   RSS    AIC
+ NonWhite2  1    5615.8 52165 438.82
<none>                   57781 440.86
+ House2     1    3068.7 54712 441.68
+ House      1    2985.1 54796 441.78
+ Educ       1    2890.9 54890 441.88
+ Precip2    1    2623.0 55158 442.17
+ HC2        1    2148.0 55633 442.69
+ HC         1    1957.8 55823 442.89
+ Density    1    1467.1 56314 443.42
+ SO22       1    1356.0 56425 443.53
+ Poor2      1     811.0 56970 444.11
+ Density2   1     570.5 57211 444.36
+ NOX2       1     247.8 57533 444.70
```

```
+ JanTemp2   1     156.6 57625 444.80
+ Sound2     1      48.1 57733 444.91
+ Poor       1      43.4 57738 444.91
+ WhiteCol   1      41.9 57739 444.92
+ JulyTemp   1      40.6 57741 444.92
+ JulyTemp2  1      35.6 57746 444.92
+ WhiteCol2  1      29.2 57752 444.93
+ Humidity2  1      24.5 57757 444.93
+ Humidity   1      14.0 57767 444.94
+ Sound      1       4.1 57777 444.95
+ Over652    1       0.0 57781 444.96
+ Over65     1       0.0 57781 444.96

Step:  AIC=438.82
Y ~ NonWhite + Educ2 + SO2 + Precip + JanTemp + NOX + NonWhite2

            Df Sum of Sq   RSS    AIC
<none>                   52165 438.82
+ HC2        1   3113.68 49052 439.23
+ House2     1   2626.48 49539 439.82
+ House      1   2559.98 49605 439.90
+ HC         1   2522.52 49643 439.94
+ Educ       1   1658.41 50507 440.98
+ Over652    1    932.03 51233 441.84
+ Sound2     1    823.08 51342 441.96
+ Over65     1    736.30 51429 442.07
+ Density    1    686.32 51479 442.12
+ NOX2       1    626.51 51539 442.19
+ Sound      1    621.79 51544 442.20
+ Poor       1    497.41 51668 442.34
+ Precip2    1    471.29 51694 442.37
+ JulyTemp   1    318.50 51847 442.55
+ JulyTemp2  1    302.85 51863 442.57
+ WhiteCol   1    275.96 51889 442.60
+ WhiteCol2  1    271.25 51894 442.61
+ Density2   1    157.62 52008 442.74
+ JanTemp2   1    111.70 52054 442.79
+ SO22       1     71.55 52094 442.84
+ Poor2      1     31.42 52134 442.88
+ Humidity2  1      4.36 52161 442.91
+ Humidity   1      1.68 52164 442.92
```

```
names(result)
```

```
 [1] "coefficients"  "residuals"     "effects"       "rank"
 [5] "fitted.values" "assign"        "qr"            "df.residual"
 [9] "xlevels"       "call"          "terms"         "model"
[13] "anova"
```

```r
#Selected Variables
names(result$coef)[-1]
```

```
[1] "NonWhite"  "Educ2"     "SO2"       "Precip"    "JanTemp"   "NOX"
[7] "NonWhite2"
```

```r
#Time
system.time(stepAIC(fit,direction="forward",scope=list(lower=~1,upper=~ Precip+Humidity
                                                       +JanTemp+JulyTemp
                                                     +Over65+House+Educ+Sound
                                                +Density+NonWhite+WhiteCol+Poor
                                                +HC+NOX+SO2+Precip2+Humidity2
                                                +JanTemp2+JulyTemp2+Over652+House2
                                                +Educ2+Sound2+Density2+NonWhite2
                                                +WhiteCol2+Poor2+HC2+NOX2+SO22),k=log(n)))
```

```
Start:  AIC=498.73
Y ~ 1

            Df Sum of Sq    RSS    AIC
+ NonWhite   1     94596 133680 470.72
+ NonWhite2  1     82339 145936 475.98
+ Educ2      1     61923 166352 483.84
+ Educ       1     59604 168672 484.67
+ Precip     1     59256 169019 484.79
+ Precip2    1     49427 178848 488.19
+ SO22       1     45615 182660 489.45
+ Sound      1     41584 186691 490.76
+ Sound2     1     41224 187051 490.88
+ Poor       1     38459 189816 491.76
+ Poor2      1     37896 190379 491.93
+ SO2        1     37098 191178 492.19
+ House2     1     29398 198877 494.55
+ House      1     29146 199130 494.63
+ NOX        1     19464 208812 497.48
+ WhiteCol2  1     19296 208979 497.53
+ WhiteCol   1     18518 209757 497.75
+ JulyTemp   1     17515 210760 498.04
+ JulyTemp2  1     16076 212200 498.44
+ Density    1     15898 212377 498.50
<none>                   228275 498.73
+ Density2   1     10485 217790 500.01
+ Over652    1      7689 220586 500.77
+ Over65     1      6958 221318 500.97
+ HC         1      5188 223087 501.45
+ NOX2       1      5040 223235 501.49
+ Humidity2  1      3465 224810 501.91
```

```
+ Humidity   1      1789 226486 502.35
+ JanTemp2   1      1446 226829 502.45
+ JanTemp    1       206 228069 502.77
+ HC2        1         5 228271 502.83

Step:  AIC=470.72
Y ~ NonWhite

            Df Sum of Sq    RSS    AIC
+ Educ2      1     35206  98474 456.48
+ Educ       1     33849  99830 457.30
+ SO2        1     31234 102446 458.85
+ JanTemp2   1     30933 102746 459.02
+ JanTemp    1     29840 103839 459.66
+ SO22       1     29708 103972 459.73
+ Over65     1     21434 112246 464.33
+ Over652    1     19994 113685 465.09
+ WhiteCol2  1     18801 114879 465.72
+ WhiteCol   1     18153 115527 466.06
+ Density    1     16590 117090 466.86
+ Precip     1     16322 117358 467.00
+ Density2   1     12440 121240 468.95
<none>                   133680 470.72
+ Precip2    1      8605 125075 470.82
+ Sound2     1      8113 125567 471.06
+ Sound      1      7263 126417 471.46
+ NOX        1      6836 126844 471.67
+ JulyTemp2  1      3978 129701 473.00
+ JulyTemp   1      2973 130707 473.47
+ House      1      2113 131567 473.86
+ House2     1      1957 131723 473.93
+ Poor2      1      1554 132126 474.11
+ Poor       1       851 132828 474.43
+ HC         1       813 132867 474.45
+ HC2        1       750 132929 474.48
+ NOX2       1       619 133061 474.54
+ Humidity2  1       290 133390 474.68
+ NonWhite2  1       285 133395 474.69
+ Humidity   1        37 133643 474.80

Step:  AIC=456.48
Y ~ NonWhite + Educ2

            Df Sum of Sq   RSS    AIC
+ SO2        1   17997.6 80476 448.46
+ SO22       1   17332.2 81142 448.95
+ JanTemp    1   16589.0 81885 449.50
+ JanTemp2   1   16574.0 81900 449.51
```

```
+ Poor2      1   11941.9 86532 452.81
+ Poor       1   11102.6 87371 453.39
+ NOX        1    9251.0 89223 454.65
+ JulyTemp2  1    7583.9 90890 455.76
+ Density    1    7518.2 90956 455.80
+ Over65     1    6991.6 91482 456.15
+ JulyTemp   1    6883.2 91591 456.22
+ Educ       1    6802.2 91672 456.27
<none>                   98474 456.48
+ Over652    1    6354.0 92120 456.57
+ HC         1    4380.6 94093 457.84
+ Density2   1    3453.5 95021 458.43
+ NOX2       1    3370.1 95104 458.48
+ Precip     1    2143.0 96331 459.25
+ NonWhite2  1    1968.5 96505 459.36
+ HC2        1     821.9 97652 460.07
+ House2     1     580.6 97893 460.21
+ Humidity   1     526.4 97948 460.25
+ House      1     514.9 97959 460.26
+ Humidity2  1     415.0 98059 460.32
+ Precip2    1     278.8 98195 460.40
+ Sound      1     233.9 98240 460.43
+ Sound2     1      95.8 98378 460.51
+ WhiteCol   1      11.8 98462 460.56
+ WhiteCol2  1       7.2 98467 460.57

Step:  AIC=448.46
Y ~ NonWhite + Educ2 + SO2

            Df Sum of Sq   RSS    AIC
+ Precip     1    8775.1 71701 445.63
+ Educ       1    7818.5 72658 446.42
+ JanTemp    1    7352.9 73123 446.81
+ JanTemp2   1    6787.4 73689 447.27
<none>                   80476 448.46
+ HC2        1    5094.7 75382 448.63
+ Precip2    1    4915.2 75561 448.77
+ Poor2      1    3112.5 77364 450.19
+ Over652    1    2712.8 77764 450.50
+ Over65     1    2563.0 77913 450.61
+ Poor       1    2386.6 78090 450.75
+ NOX2       1    2121.4 78355 450.95
+ HC         1    2047.6 78429 451.01
+ NonWhite2  1    1283.6 79193 451.59
+ Density    1     899.1 79577 451.88
+ Sound2     1     701.7 79775 452.03
+ Humidity   1     694.9 79781 452.03
+ Humidity2  1     470.4 80006 452.20
```

```
+ Sound      1     418.1 80058 452.24
+ JulyTemp2  1     229.8 80247 452.38
+ NOX        1     192.6 80284 452.41
+ SO22       1     184.5 80292 452.42
+ JulyTemp   1     153.6 80323 452.44
+ Density2   1     150.0 80326 452.44
+ WhiteCol2  1     141.6 80335 452.45
+ WhiteCol   1     103.1 80373 452.48
+ House2     1      48.7 80428 452.52
+ House      1      27.0 80449 452.53

Step:  AIC=445.63
Y ~ NonWhite + Educ2 + SO2 + Precip

            Df Sum of Sq   RSS    AIC
+ JanTemp    1    5516.4 66185 444.92
+ JanTemp2   1    5249.9 66451 445.16
+ Precip2    1    5077.3 66624 445.31
+ Educ       1    4981.7 66720 445.40
<none>                   71701 445.63
+ Poor2      1    4317.9 67383 445.99
+ Poor       1    2995.9 68705 447.16
+ NonWhite2  1    2501.9 69199 447.59
+ NOX        1    1194.0 70507 448.71
+ JulyTemp2  1    1119.7 70582 448.78
+ JulyTemp   1     993.5 70708 448.88
+ Density    1     820.4 70881 449.03
+ SO22       1     681.8 71019 449.15
+ Humidity   1     518.1 71183 449.29
+ HC2        1     504.0 71197 449.30
+ Humidity2  1     429.9 71271 449.36
+ WhiteCol2  1     288.9 71412 449.48
+ WhiteCol   1     240.3 71461 449.52
+ Sound2     1     217.1 71484 449.54
+ Density2   1     153.9 71547 449.59
+ Sound      1      87.9 71613 449.65
+ House      1      33.1 71668 449.69
+ House2     1      18.6 71683 449.71
+ Over652    1       2.4 71699 449.72
+ Over65     1       2.3 71699 449.72
+ HC         1       1.9 71699 449.72
+ NOX2       1       0.0 71701 449.72

Step:  AIC=444.92
Y ~ NonWhite + Educ2 + SO2 + Precip + JanTemp

            Df Sum of Sq   RSS    AIC
+ NOX        1    8403.6 57781 440.86
```

```
+ NOX2       1    4635.3 61550 444.66
<none>                   66185 444.92
+ SO22       1    2758.0 63427 446.46
+ NonWhite2  1    2660.6 63524 446.55
+ HC         1    2423.1 63762 446.77
+ Educ       1    2008.5 64176 447.16
+ House2     1    1904.2 64281 447.26
+ House      1    1845.3 64340 447.32
+ HC2        1    1578.1 64607 447.56
+ Density    1    1537.6 64647 447.60
+ Poor2      1    1410.2 64775 447.72
+ JulyTemp2  1    1094.3 65091 448.01
+ JulyTemp   1    1091.8 65093 448.01
+ Humidity2  1     960.8 65224 448.13
+ Humidity   1     949.1 65236 448.15
+ Precip2    1     722.7 65462 448.35
+ Poor       1     540.8 65644 448.52
+ Density2   1     517.3 65668 448.54
+ Sound      1      85.4 66099 448.93
+ Over652    1      63.3 66122 448.95
+ Over65     1      52.9 66132 448.96
+ Sound2     1      28.7 66156 448.99
+ WhiteCol2  1      19.6 66165 448.99
+ WhiteCol   1       4.1 66181 449.01
+ JanTemp2   1       0.5 66184 449.01

Step:  AIC=440.86
Y ~ NonWhite + Educ2 + SO2 + Precip + JanTemp + NOX

            Df Sum of Sq   RSS    AIC
+ NonWhite2  1    5615.8 52165 438.82
<none>                   57781 440.86
+ House2     1    3068.7 54712 441.68
+ House      1    2985.1 54796 441.78
+ Educ       1    2890.9 54890 441.88
+ Precip2    1    2623.0 55158 442.17
+ HC2        1    2148.0 55633 442.69
+ HC         1    1957.8 55823 442.89
+ Density    1    1467.1 56314 443.42
+ SO22       1    1356.0 56425 443.53
+ Poor2      1     811.0 56970 444.11
+ Density2   1     570.5 57211 444.36
+ NOX2       1     247.8 57533 444.70
+ JanTemp2   1     156.6 57625 444.80
+ Sound2     1      48.1 57733 444.91
+ Poor       1      43.4 57738 444.91
+ WhiteCol   1      41.9 57739 444.92
+ JulyTemp   1      40.6 57741 444.92
```

```
+ JulyTemp2  1      35.6 57746 444.92
+ WhiteCol2  1      29.2 57752 444.93
+ Humidity2  1      24.5 57757 444.93
+ Humidity   1      14.0 57767 444.94
+ Sound      1       4.1 57777 444.95
+ Over652    1       0.0 57781 444.96
+ Over65     1       0.0 57781 444.96

Step:  AIC=438.82
Y ~ NonWhite + Educ2 + SO2 + Precip + JanTemp + NOX + NonWhite2

            Df Sum of Sq   RSS    AIC
<none>                  52165 438.82
+ HC2        1   3113.68 49052 439.23
+ House2     1   2626.48 49539 439.82
+ House      1   2559.98 49605 439.90
+ HC         1   2522.52 49643 439.94
+ Educ       1   1658.41 50507 440.98
+ Over652    1    932.03 51233 441.84
+ Sound2     1    823.08 51342 441.96
+ Over65     1    736.30 51429 442.07
+ Density    1    686.32 51479 442.12
+ NOX2       1    626.51 51539 442.19
+ Sound      1    621.79 51544 442.20
+ Poor       1    497.41 51668 442.34
+ Precip2    1    471.29 51694 442.37
+ JulyTemp   1    318.50 51847 442.55
+ JulyTemp2  1    302.85 51863 442.57
+ WhiteCol   1    275.96 51889 442.60
+ WhiteCol2  1    271.25 51894 442.61
+ Density2   1    157.62 52008 442.74
+ JanTemp2   1    111.70 52054 442.79
+ SO22       1     71.55 52094 442.84
+ Poor2      1     31.42 52134 442.88
+ Humidity2  1      4.36 52161 442.91
+ Humidity   1      1.68 52164 442.92

   user  system elapsed
  0.118   0.001   0.120
```

```r
#c) AIC
fit<-lm(Y~.,data=X)
#stepAIC(fit,direction="both",data=X)
result=stepAIC(fit,direction="both",data=X)
```

```
Start:  AIC=423.95
Y ~ Precip + Humidity + JanTemp + JulyTemp + Over65 + House +
```

```
    Educ + Sound + Density + NonWhite + WhiteCol + Poor + HC +
    NOX + SO2 + Precip2 + Humidity2 + JanTemp2 + JulyTemp2 +
    Over652 + House2 + Educ2 + Sound2 + Density2 + NonWhite2 +
    WhiteCol2 + Poor2 + HC2 + NOX2 + SO22

            Df Sum of Sq   RSS    AIC
- SO22       1      21.9 25028 422.01
- NOX2       1      43.6 25050 422.06
- Over652    1     117.1 25124 422.23
- SO2        1     122.1 25129 422.25
- JanTemp2   1     131.2 25138 422.27
- JulyTemp   1     149.5 25156 422.31
- JulyTemp2  1     194.4 25201 422.42
- Precip     1     261.9 25268 422.58
- HC2        1     363.1 25370 422.82
- Sound      1     395.4 25402 422.89
- Over65     1     471.0 25478 423.07
- Sound2     1     498.6 25505 423.14
- House      1     555.4 25562 423.27
- House2     1     647.7 25654 423.49
- WhiteCol   1     770.3 25777 423.77
<none>                   25007 423.95
- WhiteCol2  1     958.1 25965 424.21
- Precip2    1    1005.1 26012 424.32
- Humidity   1    1087.3 26094 424.51
- Humidity2  1    1144.6 26151 424.64
- Poor2      1    1196.7 26203 424.76
- Poor       1    1212.1 26219 424.79
- Density2   1    1594.9 26602 425.66
- Density    1    2148.9 27156 426.90
- Educ       1    2169.1 27176 426.94
- HC         1    2216.9 27224 427.05
- NonWhite2  1    2236.8 27243 427.09
- JanTemp    1    2277.9 27284 427.18
- Educ2      1    2509.6 27516 427.69
- NOX        1    2732.5 27739 428.18
- NonWhite   1    5425.7 30432 433.73

Step:  AIC=422.01
Y ~ Precip + Humidity + JanTemp + JulyTemp + Over65 + House +
    Educ + Sound + Density + NonWhite + WhiteCol + Poor + HC +
    NOX + SO2 + Precip2 + Humidity2 + JanTemp2 + JulyTemp2 +
    Over652 + House2 + Educ2 + Sound2 + Density2 + NonWhite2 +
    WhiteCol2 + Poor2 + HC2 + NOX2

            Df Sum of Sq   RSS    AIC
- JanTemp2   1     112.7 25141 420.27
- Over652    1     120.3 25149 420.29
```

```
- JulyTemp   1     154.7 25183 420.38
- JulyTemp2  1     206.6 25235 420.50
- NOX2       1     250.6 25279 420.60
- Precip     1     339.8 25368 420.81
- Sound      1     389.2 25418 420.93
- Over65     1     463.1 25492 421.11
- Sound2     1     493.9 25522 421.18
- House      1     614.4 25643 421.46
- House2     1     730.9 25759 421.73
- HC2        1     748.7 25777 421.77
- WhiteCol   1     759.8 25788 421.80
<none>                   25028 422.01
- WhiteCol2  1     956.6 25985 422.26
- Precip2    1    1158.7 26187 422.72
- Poor2      1    1175.7 26204 422.76
- Poor       1    1190.7 26219 422.79
- Humidity   1    1467.7 26496 423.42
- Density2   1    1573.4 26602 423.66
- Humidity2  1    1582.5 26611 423.68
+ SO22       1      21.9 25007 423.95
- Density    1    2127.1 27156 424.90
- Educ       1    2185.0 27213 425.03
- NonWhite2  1    2221.9 27250 425.11
- SO2        1    2225.2 27254 425.12
- JanTemp    1    2339.4 27368 425.37
- Educ2      1    2558.8 27587 425.85
- HC         1    3162.0 28190 427.14
- NOX        1    4727.6 29756 430.39
- NonWhite   1    5639.3 30668 432.20

Step:  AIC=420.27
Y ~ Precip + Humidity + JanTemp + JulyTemp + Over65 + House +
    Educ + Sound + Density + NonWhite + WhiteCol + Poor + HC +
    NOX + SO2 + Precip2 + Humidity2 + JulyTemp2 + Over652 + House2 +
    Educ2 + Sound2 + Density2 + NonWhite2 + WhiteCol2 + Poor2 +
    HC2 + NOX2

            Df Sum of Sq   RSS    AIC
- JulyTemp   1     108.0 25249 418.53
- Over652    1     131.8 25273 418.59
- JulyTemp2  1     151.3 25292 418.63
- NOX2       1     169.0 25310 418.68
- Over65     1     488.5 25630 419.43
- Sound      1     503.5 25645 419.46
- Sound2     1     616.5 25758 419.73
- House      1     628.5 25770 419.76
- HC2        1     638.2 25779 419.78
- House2     1     746.8 25888 420.03
```

```
- Precip     1     839.6 25981 420.25
<none>                   25141 420.27
- WhiteCol   1     863.9 26005 420.30
- WhiteCol2  1    1087.9 26229 420.82
- Poor2      1    1225.0 26366 421.13
- Poor       1    1275.6 26417 421.24
- Humidity   1    1358.5 26500 421.43
- Humidity2  1    1469.9 26611 421.68
- Density2   1    1491.9 26633 421.73
+ JanTemp2   1     112.7 25028 422.01
+ SO22       1       3.3 25138 422.27
- Density    1    2040.8 27182 422.96
- SO2        1    2219.5 27361 423.35
- Precip2    1    2220.3 27361 423.35
- NonWhite2  1    2277.9 27419 423.48
- Educ       1    2371.2 27512 423.68
- Educ2      1    2763.9 27905 424.53
- HC         1    3125.7 28267 425.31
- NOX        1    4803.0 29944 428.76
- NonWhite   1    5595.4 30737 430.33
- JanTemp    1    7207.4 32349 433.40

Step:  AIC=418.53
Y ~ Precip + Humidity + JanTemp + Over65 + House + Educ + Sound +
    Density + NonWhite + WhiteCol + Poor + HC + NOX + SO2 + Precip2 +
    Humidity2 + JulyTemp2 + Over652 + House2 + Educ2 + Sound2 +
    Density2 + NonWhite2 + WhiteCol2 + Poor2 + HC2 + NOX2

            Df Sum of Sq   RSS    AIC
- Over652    1     121.1 25370 416.82
- NOX2       1     252.6 25502 417.13
- Sound      1     438.2 25687 417.56
- Over65     1     461.7 25711 417.62
- Sound2     1     546.9 25796 417.82
- HC2        1     603.8 25853 417.95
- House      1     688.1 25937 418.15
- House2     1     806.5 26056 418.42
- WhiteCol   1     854.7 26104 418.53
<none>                   25249 418.53
- WhiteCol2  1    1072.7 26322 419.03
- Precip     1    1236.9 26486 419.40
- Humidity   1    1265.9 26515 419.47
- Humidity2  1    1396.3 26645 419.76
- Poor       1    1500.9 26750 420.00
- Poor2      1    1527.3 26776 420.06
- Density2   1    1556.7 26806 420.12
+ JulyTemp   1     108.0 25141 420.27
+ JanTemp2   1      66.0 25183 420.38
```

```
+ SO22       1       8.0 25241 420.51
- JulyTemp2  1    2030.2 27279 421.17
- SO2        1    2121.6 27371 421.37
- Density    1    2184.9 27434 421.51
- NonWhite2  1    2292.5 27542 421.75
- Educ       1    2636.6 27886 422.49
- HC         1    3019.6 28269 423.31
- Precip2    1    3037.6 28287 423.35
- Educ2      1    3050.9 28300 423.38
- NOX        1    4980.0 30229 427.33
- NonWhite   1    5547.5 30797 428.45
- JanTemp    1    8054.2 33303 433.14

Step:  AIC=416.82
Y ~ Precip + Humidity + JanTemp + Over65 + House + Educ + Sound +
    Density + NonWhite + WhiteCol + Poor + HC + NOX + SO2 + Precip2 +
    Humidity2 + JulyTemp2 + House2 + Educ2 + Sound2 + Density2 +
    NonWhite2 + WhiteCol2 + Poor2 + HC2 + NOX2

            Df Sum of Sq   RSS    AIC
- NOX2       1     247.2 25617 415.40
- Sound      1     388.4 25759 415.73
- Sound2     1     491.3 25861 415.97
- HC2        1     601.6 25972 416.23
- House      1     668.0 26038 416.38
- House2     1     782.7 26153 416.64
<none>                   25370 416.82
- Precip     1    1117.4 26488 417.41
- WhiteCol   1    1138.0 26508 417.45
- Humidity   1    1302.4 26673 417.82
- WhiteCol2  1    1406.6 26777 418.06
- Humidity2  1    1435.7 26806 418.12
- Density2   1    1532.0 26902 418.34
+ Over652    1     121.1 25249 418.53
+ JulyTemp   1      97.3 25273 418.59
+ JanTemp2   1      76.1 25294 418.64
+ SO22       1       8.8 25361 418.80
- JulyTemp2  1    1917.8 27288 419.19
- Density    1    2148.4 27519 419.70
- Poor2      1    2290.6 27661 420.01
- Poor       1    2318.6 27689 420.07
- NonWhite2  1    2319.7 27690 420.07
- SO2        1    2373.1 27743 420.18
- Over65     1    2395.0 27765 420.23
- HC         1    2911.9 28282 421.34
- Precip2    1    2917.5 28288 421.35
- Educ       1    3008.6 28379 421.54
- Educ2      1    3437.2 28807 422.44
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- NOX        1    4895.3 30266 425.41
- NonWhite   1    6322.2 31692 428.17
- JanTemp    1    7953.0 33323 431.18

Step:  AIC=415.4
Y ~ Precip + Humidity + JanTemp + Over65 + House + Educ + Sound +
    Density + NonWhite + WhiteCol + Poor + HC + NOX + SO2 + Precip2 +
    Humidity2 + JulyTemp2 + House2 + Educ2 + Sound2 + Density2 +
    NonWhite2 + WhiteCol2 + Poor2 + HC2

            Df Sum of Sq   RSS    AIC
- HC2        1     447.2 26065 414.44
- Sound      1     493.3 26111 414.55
- Sound2     1     609.0 26226 414.81
<none>                   25617 415.40
- Precip     1    1087.4 26705 415.90
- House      1    1144.4 26762 416.02
- House2     1    1311.6 26929 416.40
- Density2   1    1366.8 26984 416.52
- WhiteCol   1    1500.1 27117 416.82
+ NOX2       1     247.2 25370 416.82
+ SO22       1     208.8 25409 416.91
+ JulyTemp   1     176.7 25441 416.99
- Humidity   1    1617.8 27235 417.08
+ Over652    1     115.7 25502 417.13
+ JanTemp2   1       4.1 25613 417.39
- JulyTemp2  1    1765.5 27383 417.40
- WhiteCol2  1    1781.7 27399 417.44
- Humidity2  1    1812.9 27430 417.50
- Density    1    1949.9 27567 417.80
- NonWhite2  1    2100.3 27718 418.13
- SO2        1    2277.6 27895 418.51
- Poor2      1    2293.5 27911 418.55
- Poor       1    2304.6 27922 418.57
- Over65     1    2512.0 28129 419.01
- Precip2    1    2835.5 28453 419.70
- Educ       1    2963.0 28580 419.97
- Educ2      1    3392.3 29010 420.86
- HC         1    4342.0 29959 422.80
- NonWhite   1    6088.2 31706 426.19
- JanTemp    1    7795.8 33413 429.34
- NOX        1   15588.0 41205 441.92

Step:  AIC=414.44
Y ~ Precip + Humidity + JanTemp + Over65 + House + Educ + Sound +
    Density + NonWhite + WhiteCol + Poor + HC + NOX + SO2 + Precip2 +
    Humidity2 + JulyTemp2 + House2 + Educ2 + Sound2 + Density2 +
    NonWhite2 + WhiteCol2 + Poor2
```

```
            Df Sum of Sq   RSS    AIC
- Sound      1     350.3 26415 413.24
- Sound2     1     443.1 26508 413.45
- House      1     785.8 26850 414.22
- Precip     1     823.3 26888 414.31
<none>                   26065 414.44
- House2     1     938.4 27003 414.56
- Density2   1    1022.7 27087 414.75
- Humidity   1    1237.9 27302 415.22
+ HC2        1     447.2 25617 415.40
- WhiteCol   1    1360.6 27425 415.49
- Humidity2  1    1395.9 27460 415.57
- Density    1    1544.3 27609 415.89
- WhiteCol2  1    1630.8 27695 416.08
+ Over652    1     123.6 25941 416.15
+ NOX2       1      92.8 25972 416.23
+ SO22       1      46.2 26018 416.33
+ JanTemp2   1       7.0 26058 416.42
+ JulyTemp   1       4.9 26060 416.43
- JulyTemp2  1    1972.4 28037 416.82
- Poor       1    2043.9 28108 416.97
- Poor2      1    2059.0 28124 417.00
- NonWhite2  1    2085.8 28150 417.06
- Over65     1    2153.2 28218 417.20
- Precip2    1    2395.5 28460 417.71
- SO2        1    2759.9 28824 418.48
- Educ       1    3219.6 29284 419.43
- Educ2      1    3658.7 29723 420.32
- HC         1    5194.3 31259 423.34
- NonWhite   1    6312.8 32377 425.45
- JanTemp    1    7879.0 33944 428.29
- NOX        1   16331.2 42396 441.63

Step:  AIC=413.24
Y ~ Precip + Humidity + JanTemp + Over65 + House + Educ + Density +
    NonWhite + WhiteCol + Poor + HC + NOX + SO2 + Precip2 + Humidity2 +
    JulyTemp2 + House2 + Educ2 + Sound2 + Density2 + NonWhite2 +
    WhiteCol2 + Poor2

            Df Sum of Sq   RSS    AIC
- House      1     663.9 27079 412.73
- Precip     1     720.9 27136 412.86
- House2     1     810.4 27225 413.05
<none>                   26415 413.24
- WhiteCol   1    1173.4 27588 413.85
- WhiteCol2  1    1428.1 27843 414.40
+ Sound      1     350.3 26065 414.44
```

```
- Humidity   1    1466.9 27882 414.48
+ HC2        1     304.2 26111 414.55
- Density2   1    1605.5 28020 414.78
- Humidity2  1    1699.0 28114 414.98
+ SO22       1      77.1 26338 415.06
+ Over652    1      72.3 26343 415.08
+ JanTemp2   1      33.0 26382 415.17
+ NOX2       1      26.3 26389 415.18
- JulyTemp2  1    1794.5 28209 415.18
+ JulyTemp   1       2.1 26413 415.24
- NonWhite2  1    1974.6 28389 415.57
- Sound2     1    2058.2 28473 415.74
- Precip2    1    2273.3 28688 416.19
- Density    1    2343.6 28758 416.34
- Over65     1    2598.5 29013 416.87
- SO2        1    2835.5 29250 417.36
- Educ       1    3162.8 29578 418.03
- Poor       1    3580.7 29995 418.87
- Educ2      1    3596.4 30011 418.90
- Poor2      1    3906.9 30322 419.52
- NonWhite   1    6030.9 32446 423.58
- HC         1    6785.2 33200 424.96
- JanTemp    1    9193.4 35608 429.16
- NOX        1   18550.3 44965 443.16

Step:  AIC=412.73
Y ~ Precip + Humidity + JanTemp + Over65 + Educ + Density + NonWhite +
    WhiteCol + Poor + HC + NOX + SO2 + Precip2 + Humidity2 +
    JulyTemp2 + House2 + Educ2 + Sound2 + Density2 + NonWhite2 +
    WhiteCol2 + Poor2

            Df Sum of Sq   RSS    AIC
- Precip     1     692.6 27771 412.24
- WhiteCol   1     753.3 27832 412.38
<none>                   27079 412.73
- WhiteCol2  1    1030.4 28109 412.97
- Humidity   1    1148.2 28227 413.22
+ House      1     663.9 26415 413.24
- Humidity2  1    1329.9 28409 413.61
+ Sound      1     228.4 26850 414.22
- Density2   1    1731.0 28810 414.45
+ JulyTemp   1     108.8 26970 414.49
- JulyTemp2  1    1756.5 28835 414.50
+ SO22       1     102.2 26977 414.50
+ Over652    1      56.7 27022 414.60
+ HC2        1      56.4 27022 414.60
+ NOX2       1      52.3 27026 414.61
- NonWhite2  1    1826.7 28905 414.65
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+ JanTemp2   1       0.7 27078 414.73
- Sound2     1    2004.6 29083 415.01
- Precip2    1    2278.4 29357 415.58
- Density    1    2337.3 29416 415.70
- Educ       1    3022.0 30101 417.08
- SO2        1    3235.4 30314 417.50
- Over65     1    3391.9 30471 417.81
- Educ2      1    3504.0 30583 418.03
- Poor       1    3599.7 30678 418.22
- Poor2      1    3991.0 31070 418.98
- House2     1    4659.3 31738 420.26
- NonWhite   1    5545.6 32624 421.91
- HC         1    7430.2 34509 425.28
- JanTemp    1   11234.1 38313 431.55
- NOX        1   20081.8 47161 444.02

Step:  AIC=412.24
Y ~ Humidity + JanTemp + Over65 + Educ + Density + NonWhite +
    WhiteCol + Poor + HC + NOX + SO2 + Precip2 + Humidity2 +
    JulyTemp2 + House2 + Educ2 + Sound2 + Density2 + NonWhite2 +
    WhiteCol2 + Poor2

            Df Sum of Sq   RSS    AIC
- WhiteCol   1     508.8 28280 411.33
- WhiteCol2  1     731.7 28503 411.81
<none>                  27771 412.24
- Humidity   1    1117.3 28889 412.61
- Humidity2  1    1163.3 28935 412.71
+ Precip     1     692.6 27079 412.73
+ House      1     635.6 27136 412.86
- NonWhite2  1    1393.6 29165 413.18
- Sound2     1    1394.3 29166 413.18
+ JulyTemp   1     413.1 27358 413.35
+ SO22       1     221.3 27550 413.76
- Density2   1    1681.4 29453 413.77
+ JanTemp2   1     182.2 27589 413.85
+ Sound      1     151.8 27620 413.92
+ NOX2       1     116.9 27654 413.99
+ HC2        1       4.0 27767 414.24
+ Over652    1       0.0 27771 414.24
- Density    1    2353.9 30125 415.13
- JulyTemp2  1    2430.3 30202 415.28
- Poor       1    2978.4 30750 416.36
- Educ       1    2979.4 30751 416.36
- Poor2      1    3300.1 31071 416.98
- Over65     1    3340.7 31112 417.06
- Educ2      1    3440.4 31212 417.25
- SO2        1    4507.7 32279 419.27
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- House2     1    4510.7 32282 419.28
- NonWhite   1    4853.0 32624 419.91
- HC         1    7017.8 34789 423.76
- JanTemp    1   11188.4 38960 430.56
- Precip2    1   16479.8 44251 438.20
- NOX        1   19544.1 47315 442.21

Step:  AIC=411.33
Y ~ Humidity + JanTemp + Over65 + Educ + Density + NonWhite +
    Poor + HC + NOX + SO2 + Precip2 + Humidity2 + JulyTemp2 +
    House2 + Educ2 + Sound2 + Density2 + NonWhite2 + WhiteCol2 +
    Poor2

             Df Sum of Sq   RSS    AIC
<none>                    28280 411.33
- Humidity   1    1057.1 29337 411.54
- NonWhite2  1    1090.7 29371 411.60
- Humidity2  1    1099.3 29379 411.62
- Sound2     1    1283.8 29564 412.00
+ WhiteCol   1     508.8 27771 412.24
+ Precip     1     448.1 27832 412.38
+ JulyTemp   1     301.4 27979 412.69
+ House      1     274.9 28005 412.75
+ SO22       1     191.7 28088 412.93
+ JanTemp2   1     173.5 28107 412.96
+ NOX2       1     107.0 28173 413.11
+ Sound      1      94.9 28185 413.13
+ Over652    1      71.5 28209 413.18
+ HC2        1      16.8 28263 413.30
- WhiteCol2  1    1938.9 30219 413.31
- Density2   1    2198.6 30479 413.83
- Educ       1    2484.8 30765 414.39
- JulyTemp2  1    2796.9 31077 414.99
- Density    1    2893.2 31173 415.18
- Educ2      1    2936.8 31217 415.26
- Poor       1    3025.5 31306 415.43
- Poor2      1    3441.4 31721 416.22
- SO2        1    4073.8 32354 417.41
- NonWhite   1    4476.5 32757 418.15
- Over65     1    4562.4 32843 418.31
- House2     1    5032.8 33313 419.16
- HC         1    6785.7 35066 422.24
- JanTemp    1   11525.9 39806 429.85
- Precip2    1   17375.5 45656 438.07
- NOX        1   19154.8 47435 440.37
```

```r
names(result)
```

```
 [1] "coefficients"  "residuals"     "effects"       "rank"
 [5] "fitted.values" "assign"        "qr"            "df.residual"
 [9] "xlevels"       "call"          "terms"         "model"
[13] "anova"
```

```
#Selected Variables
names(result$coef)[-1]
```

```
 [1] "Humidity"  "JanTemp"   "Over65"    "Educ"      "Density"
 [6] "NonWhite"  "Poor"      "HC"        "NOX"       "SO2"
[11] "Precip2"   "Humidity2" "JulyTemp2" "House2"    "Educ2"
[16] "Sound2"    "Density2"  "NonWhite2" "WhiteCol2" "Poor2"
```

```
#Time
system.time(stepAIC(fit,direction="both",data=X))
```

```
Start:  AIC=423.95
Y ~ Precip + Humidity + JanTemp + JulyTemp + Over65 + House +
    Educ + Sound + Density + NonWhite + WhiteCol + Poor + HC +
    NOX + SO2 + Precip2 + Humidity2 + JanTemp2 + JulyTemp2 +
    Over652 + House2 + Educ2 + Sound2 + Density2 + NonWhite2 +
    WhiteCol2 + Poor2 + HC2 + NOX2 + SO22

            Df Sum of Sq   RSS    AIC
- SO22       1      21.9 25028 422.01
- NOX2       1      43.6 25050 422.06
- Over652    1     117.1 25124 422.23
- SO2        1     122.1 25129 422.25
- JanTemp2   1     131.2 25138 422.27
- JulyTemp   1     149.5 25156 422.31
- JulyTemp2  1     194.4 25201 422.42
- Precip     1     261.9 25268 422.58
- HC2        1     363.1 25370 422.82
- Sound      1     395.4 25402 422.89
- Over65     1     471.0 25478 423.07
- Sound2     1     498.6 25505 423.14
- House      1     555.4 25562 423.27
- House2     1     647.7 25654 423.49
- WhiteCol   1     770.3 25777 423.77
<none>                   25007 423.95
- WhiteCol2  1     958.1 25965 424.21
- Precip2    1    1005.1 26012 424.32
- Humidity   1    1087.3 26094 424.51
- Humidity2  1    1144.6 26151 424.64
- Poor2      1    1196.7 26203 424.76
- Poor       1    1212.1 26219 424.79
- Density2   1    1594.9 26602 425.66
```

```
- Density    1    2148.9 27156 426.90
- Educ       1    2169.1 27176 426.94
- HC         1    2216.9 27224 427.05
- NonWhite2  1    2236.8 27243 427.09
- JanTemp    1    2277.9 27284 427.18
- Educ2      1    2509.6 27516 427.69
- NOX        1    2732.5 27739 428.18
- NonWhite   1    5425.7 30432 433.73

Step:  AIC=422.01
Y ~ Precip + Humidity + JanTemp + JulyTemp + Over65 + House +
    Educ + Sound + Density + NonWhite + WhiteCol + Poor + HC +
    NOX + SO2 + Precip2 + Humidity2 + JanTemp2 + JulyTemp2 +
    Over652 + House2 + Educ2 + Sound2 + Density2 + NonWhite2 +
    WhiteCol2 + Poor2 + HC2 + NOX2

            Df Sum of Sq   RSS    AIC
- JanTemp2   1     112.7 25141 420.27
- Over652    1     120.3 25149 420.29
- JulyTemp   1     154.7 25183 420.38
- JulyTemp2  1     206.6 25235 420.50
- NOX2       1     250.6 25279 420.60
- Precip     1     339.8 25368 420.81
- Sound      1     389.2 25418 420.93
- Over65     1     463.1 25492 421.11
- Sound2     1     493.9 25522 421.18
- House      1     614.4 25643 421.46
- House2     1     730.9 25759 421.73
- HC2        1     748.7 25777 421.77
- WhiteCol   1     759.8 25788 421.80
<none>                   25028 422.01
- WhiteCol2  1     956.6 25985 422.26
- Precip2    1    1158.7 26187 422.72
- Poor2      1    1175.7 26204 422.76
- Poor       1    1190.7 26219 422.79
- Humidity   1    1467.7 26496 423.42
- Density2   1    1573.4 26602 423.66
- Humidity2  1    1582.5 26611 423.68
+ SO22       1      21.9 25007 423.95
- Density    1    2127.1 27156 424.90
- Educ       1    2185.0 27213 425.03
- NonWhite2  1    2221.9 27250 425.11
- SO2        1    2225.2 27254 425.12
- JanTemp    1    2339.4 27368 425.37
- Educ2      1    2558.8 27587 425.85
- HC         1    3162.0 28190 427.14
- NOX        1    4727.6 29756 430.39
- NonWhite   1    5639.3 30668 432.20
```

```
Step:  AIC=420.27
Y ~ Precip + Humidity + JanTemp + JulyTemp + Over65 + House +
    Educ + Sound + Density + NonWhite + WhiteCol + Poor + HC +
    NOX + SO2 + Precip2 + Humidity2 + JulyTemp2 + Over652 + House2 +
    Educ2 + Sound2 + Density2 + NonWhite2 + WhiteCol2 + Poor2 +
    HC2 + NOX2

            Df Sum of Sq   RSS    AIC
- JulyTemp   1     108.0 25249 418.53
- Over652    1     131.8 25273 418.59
- JulyTemp2  1     151.3 25292 418.63
- NOX2       1     169.0 25310 418.68
- Over65     1     488.5 25630 419.43
- Sound      1     503.5 25645 419.46
- Sound2     1     616.5 25758 419.73
- House      1     628.5 25770 419.76
- HC2        1     638.2 25779 419.78
- House2     1     746.8 25888 420.03
- Precip     1     839.6 25981 420.25
<none>                   25141 420.27
- WhiteCol   1     863.9 26005 420.30
- WhiteCol2  1    1087.9 26229 420.82
- Poor2      1    1225.0 26366 421.13
- Poor       1    1275.6 26417 421.24
- Humidity   1    1358.5 26500 421.43
- Humidity2  1    1469.9 26611 421.68
- Density2   1    1491.9 26633 421.73
+ JanTemp2   1     112.7 25028 422.01
+ SO22       1       3.3 25138 422.27
- Density    1    2040.8 27182 422.96
- SO2        1    2219.5 27361 423.35
- Precip2    1    2220.3 27361 423.35
- NonWhite2  1    2277.9 27419 423.48
- Educ       1    2371.2 27512 423.68
- Educ2      1    2763.9 27905 424.53
- HC         1    3125.7 28267 425.31
- NOX        1    4803.0 29944 428.76
- NonWhite   1    5595.4 30737 430.33
- JanTemp    1    7207.4 32349 433.40

Step:  AIC=418.53
Y ~ Precip + Humidity + JanTemp + Over65 + House + Educ + Sound +
    Density + NonWhite + WhiteCol + Poor + HC + NOX + SO2 + Precip2 +
    Humidity2 + JulyTemp2 + Over652 + House2 + Educ2 + Sound2 +
    Density2 + NonWhite2 + WhiteCol2 + Poor2 + HC2 + NOX2

            Df Sum of Sq   RSS    AIC
```

```
- Over652    1     121.1 25370 416.82
- NOX2       1     252.6 25502 417.13
- Sound      1     438.2 25687 417.56
- Over65     1     461.7 25711 417.62
- Sound2     1     546.9 25796 417.82
- HC2        1     603.8 25853 417.95
- House      1     688.1 25937 418.15
- House2     1     806.5 26056 418.42
- WhiteCol   1     854.7 26104 418.53
<none>                   25249 418.53
- WhiteCol2  1    1072.7 26322 419.03
- Precip     1    1236.9 26486 419.40
- Humidity   1    1265.9 26515 419.47
- Humidity2  1    1396.3 26645 419.76
- Poor       1    1500.9 26750 420.00
- Poor2      1    1527.3 26776 420.06
- Density2   1    1556.7 26806 420.12
+ JulyTemp   1     108.0 25141 420.27
+ JanTemp2   1      66.0 25183 420.38
+ SO22       1       8.0 25241 420.51
- JulyTemp2  1    2030.2 27279 421.17
- SO2        1    2121.6 27371 421.37
- Density    1    2184.9 27434 421.51
- NonWhite2  1    2292.5 27542 421.75
- Educ       1    2636.6 27886 422.49
- HC         1    3019.6 28269 423.31
- Precip2    1    3037.6 28287 423.35
- Educ2      1    3050.9 28300 423.38
- NOX        1    4980.0 30229 427.33
- NonWhite   1    5547.5 30797 428.45
- JanTemp    1    8054.2 33303 433.14

Step:  AIC=416.82
Y ~ Precip + Humidity + JanTemp + Over65 + House + Educ + Sound +
    Density + NonWhite + WhiteCol + Poor + HC + NOX + SO2 + Precip2 +
    Humidity2 + JulyTemp2 + House2 + Educ2 + Sound2 + Density2 +
    NonWhite2 + WhiteCol2 + Poor2 + HC2 + NOX2

            Df Sum of Sq   RSS    AIC
- NOX2       1     247.2 25617 415.40
- Sound      1     388.4 25759 415.73
- Sound2     1     491.3 25861 415.97
- HC2        1     601.6 25972 416.23
- House      1     668.0 26038 416.38
- House2     1     782.7 26153 416.64
<none>                   25370 416.82
- Precip     1    1117.4 26488 417.41
- WhiteCol   1    1138.0 26508 417.45
```

```
- Humidity   1    1302.4 26673 417.82
- WhiteCol2  1    1406.6 26777 418.06
- Humidity2  1    1435.7 26806 418.12
- Density2   1    1532.0 26902 418.34
+ Over652    1     121.1 25249 418.53
+ JulyTemp   1      97.3 25273 418.59
+ JanTemp2   1      76.1 25294 418.64
+ SO22       1       8.8 25361 418.80
- JulyTemp2  1    1917.8 27288 419.19
- Density    1    2148.4 27519 419.70
- Poor2      1    2290.6 27661 420.01
- Poor       1    2318.6 27689 420.07
- NonWhite2  1    2319.7 27690 420.07
- SO2        1    2373.1 27743 420.18
- Over65     1    2395.0 27765 420.23
- HC         1    2911.9 28282 421.34
- Precip2    1    2917.5 28288 421.35
- Educ       1    3008.6 28379 421.54
- Educ2      1    3437.2 28807 422.44
- NOX        1    4895.3 30266 425.41
- NonWhite   1    6322.2 31692 428.17
- JanTemp    1    7953.0 33323 431.18

Step:  AIC=415.4
Y ~ Precip + Humidity + JanTemp + Over65 + House + Educ + Sound +
    Density + NonWhite + WhiteCol + Poor + HC + NOX + SO2 + Precip2 +
    Humidity2 + JulyTemp2 + House2 + Educ2 + Sound2 + Density2 +
    NonWhite2 + WhiteCol2 + Poor2 + HC2

            Df Sum of Sq   RSS    AIC
- HC2        1     447.2 26065 414.44
- Sound      1     493.3 26111 414.55
- Sound2     1     609.0 26226 414.81
<none>                   25617 415.40
- Precip     1    1087.4 26705 415.90
- House      1    1144.4 26762 416.02
- House2     1    1311.6 26929 416.40
- Density2   1    1366.8 26984 416.52
- WhiteCol   1    1500.1 27117 416.82
+ NOX2       1     247.2 25370 416.82
+ SO22       1     208.8 25409 416.91
+ JulyTemp   1     176.7 25441 416.99
- Humidity   1    1617.8 27235 417.08
+ Over652    1     115.7 25502 417.13
+ JanTemp2   1       4.1 25613 417.39
- JulyTemp2  1    1765.5 27383 417.40
- WhiteCol2  1    1781.7 27399 417.44
- Humidity2  1    1812.9 27430 417.50
```

```
- Density    1    1949.9 27567 417.80
- NonWhite2  1    2100.3 27718 418.13
- SO2        1    2277.6 27895 418.51
- Poor2      1    2293.5 27911 418.55
- Poor       1    2304.6 27922 418.57
- Over65     1    2512.0 28129 419.01
- Precip2    1    2835.5 28453 419.70
- Educ       1    2963.0 28580 419.97
- Educ2      1    3392.3 29010 420.86
- HC         1    4342.0 29959 422.80
- NonWhite   1    6088.2 31706 426.19
- JanTemp    1    7795.8 33413 429.34
- NOX        1   15588.0 41205 441.92

Step:  AIC=414.44
Y ~ Precip + Humidity + JanTemp + Over65 + House + Educ + Sound +
    Density + NonWhite + WhiteCol + Poor + HC + NOX + SO2 + Precip2 +
    Humidity2 + JulyTemp2 + House2 + Educ2 + Sound2 + Density2 +
    NonWhite2 + WhiteCol2 + Poor2

            Df Sum of Sq   RSS    AIC
- Sound      1     350.3 26415 413.24
- Sound2     1     443.1 26508 413.45
- House      1     785.8 26850 414.22
- Precip     1     823.3 26888 414.31
<none>                   26065 414.44
- House2     1     938.4 27003 414.56
- Density2   1    1022.7 27087 414.75
- Humidity   1    1237.9 27302 415.22
+ HC2        1     447.2 25617 415.40
- WhiteCol   1    1360.6 27425 415.49
- Humidity2  1    1395.9 27460 415.57
- Density    1    1544.3 27609 415.89
- WhiteCol2  1    1630.8 27695 416.08
+ Over652    1     123.6 25941 416.15
+ NOX2       1      92.8 25972 416.23
+ SO22       1      46.2 26018 416.33
+ JanTemp2   1       7.0 26058 416.42
+ JulyTemp   1       4.9 26060 416.43
- JulyTemp2  1    1972.4 28037 416.82
- Poor       1    2043.9 28108 416.97
- Poor2      1    2059.0 28124 417.00
- NonWhite2  1    2085.8 28150 417.06
- Over65     1    2153.2 28218 417.20
- Precip2    1    2395.5 28460 417.71
- SO2        1    2759.9 28824 418.48
- Educ       1    3219.6 29284 419.43
- Educ2      1    3658.7 29723 420.32
```

```
- HC         1    5194.3 31259 423.34
- NonWhite   1    6312.8 32377 425.45
- JanTemp    1    7879.0 33944 428.29
- NOX        1   16331.2 42396 441.63

Step:  AIC=413.24
Y ~ Precip + Humidity + JanTemp + Over65 + House + Educ + Density +
    NonWhite + WhiteCol + Poor + HC + NOX + SO2 + Precip2 + Humidity2 +
    JulyTemp2 + House2 + Educ2 + Sound2 + Density2 + NonWhite2 +
    WhiteCol2 + Poor2

            Df Sum of Sq   RSS    AIC
- House      1     663.9 27079 412.73
- Precip     1     720.9 27136 412.86
- House2     1     810.4 27225 413.05
<none>                   26415 413.24
- WhiteCol   1    1173.4 27588 413.85
- WhiteCol2  1    1428.1 27843 414.40
+ Sound      1     350.3 26065 414.44
- Humidity   1    1466.9 27882 414.48
+ HC2        1     304.2 26111 414.55
- Density2   1    1605.5 28020 414.78
- Humidity2  1    1699.0 28114 414.98
+ SO22       1      77.1 26338 415.06
+ Over652    1      72.3 26343 415.08
+ JanTemp2   1      33.0 26382 415.17
+ NOX2       1      26.3 26389 415.18
- JulyTemp2  1    1794.5 28209 415.18
+ JulyTemp   1       2.1 26413 415.24
- NonWhite2  1    1974.6 28389 415.57
- Sound2     1    2058.2 28473 415.74
- Precip2    1    2273.3 28688 416.19
- Density    1    2343.6 28758 416.34
- Over65     1    2598.5 29013 416.87
- SO2        1    2835.5 29250 417.36
- Educ       1    3162.8 29578 418.03
- Poor       1    3580.7 29995 418.87
- Educ2      1    3596.4 30011 418.90
- Poor2      1    3906.9 30322 419.52
- NonWhite   1    6030.9 32446 423.58
- HC         1    6785.2 33200 424.96
- JanTemp    1    9193.4 35608 429.16
- NOX        1   18550.3 44965 443.16

Step:  AIC=412.73
Y ~ Precip + Humidity + JanTemp + Over65 + Educ + Density + NonWhite +
    WhiteCol + Poor + HC + NOX + SO2 + Precip2 + Humidity2 +
    JulyTemp2 + House2 + Educ2 + Sound2 + Density2 + NonWhite2 +
```

```
    WhiteCol2 + Poor2

            Df Sum of Sq   RSS    AIC
- Precip     1     692.6 27771 412.24
- WhiteCol   1     753.3 27832 412.38
<none>                   27079 412.73
- WhiteCol2  1    1030.4 28109 412.97
- Humidity   1    1148.2 28227 413.22
+ House      1     663.9 26415 413.24
- Humidity2  1    1329.9 28409 413.61
+ Sound      1     228.4 26850 414.22
- Density2   1    1731.0 28810 414.45
+ JulyTemp   1     108.8 26970 414.49
- JulyTemp2  1    1756.5 28835 414.50
+ SO22       1     102.2 26977 414.50
+ Over652    1      56.7 27022 414.60
+ HC2        1      56.4 27022 414.60
+ NOX2       1      52.3 27026 414.61
- NonWhite2  1    1826.7 28905 414.65
+ JanTemp2   1       0.7 27078 414.73
- Sound2     1    2004.6 29083 415.01
- Precip2    1    2278.4 29357 415.58
- Density    1    2337.3 29416 415.70
- Educ       1    3022.0 30101 417.08
- SO2        1    3235.4 30314 417.50
- Over65     1    3391.9 30471 417.81
- Educ2      1    3504.0 30583 418.03
- Poor       1    3599.7 30678 418.22
- Poor2      1    3991.0 31070 418.98
- House2     1    4659.3 31738 420.26
- NonWhite   1    5545.6 32624 421.91
- HC         1    7430.2 34509 425.28
- JanTemp    1   11234.1 38313 431.55
- NOX        1   20081.8 47161 444.02

Step:  AIC=412.24
Y ~ Humidity + JanTemp + Over65 + Educ + Density + NonWhite +
    WhiteCol + Poor + HC + NOX + SO2 + Precip2 + Humidity2 +
    JulyTemp2 + House2 + Educ2 + Sound2 + Density2 + NonWhite2 +
    WhiteCol2 + Poor2

            Df Sum of Sq   RSS    AIC
- WhiteCol   1     508.8 28280 411.33
- WhiteCol2  1     731.7 28503 411.81
<none>                   27771 412.24
- Humidity   1    1117.3 28889 412.61
- Humidity2  1    1163.3 28935 412.71
+ Precip     1     692.6 27079 412.73
```

```
+ House      1     635.6 27136 412.86
- NonWhite2  1    1393.6 29165 413.18
- Sound2     1    1394.3 29166 413.18
+ JulyTemp   1     413.1 27358 413.35
+ SO22       1     221.3 27550 413.76
- Density2   1    1681.4 29453 413.77
+ JanTemp2   1     182.2 27589 413.85
+ Sound      1     151.8 27620 413.92
+ NOX2       1     116.9 27654 413.99
+ HC2        1       4.0 27767 414.24
+ Over652    1       0.0 27771 414.24
- Density    1    2353.9 30125 415.13
- JulyTemp2  1    2430.3 30202 415.28
- Poor       1    2978.4 30750 416.36
- Educ       1    2979.4 30751 416.36
- Poor2      1    3300.1 31071 416.98
- Over65     1    3340.7 31112 417.06
- Educ2      1    3440.4 31212 417.25
- SO2        1    4507.7 32279 419.27
- House2     1    4510.7 32282 419.28
- NonWhite   1    4853.0 32624 419.91
- HC         1    7017.8 34789 423.76
- JanTemp    1   11188.4 38960 430.56
- Precip2    1   16479.8 44251 438.20
- NOX        1   19544.1 47315 442.21

Step:  AIC=411.33
Y ~ Humidity + JanTemp + Over65 + Educ + Density + NonWhite +
    Poor + HC + NOX + SO2 + Precip2 + Humidity2 + JulyTemp2 +
    House2 + Educ2 + Sound2 + Density2 + NonWhite2 + WhiteCol2 +
    Poor2

            Df Sum of Sq   RSS    AIC
<none>                    28280 411.33
- Humidity   1    1057.1 29337 411.54
- NonWhite2  1    1090.7 29371 411.60
- Humidity2  1    1099.3 29379 411.62
- Sound2     1    1283.8 29564 412.00
+ WhiteCol   1     508.8 27771 412.24
+ Precip     1     448.1 27832 412.38
+ JulyTemp   1     301.4 27979 412.69
+ House      1     274.9 28005 412.75
+ SO22       1     191.7 28088 412.93
+ JanTemp2   1     173.5 28107 412.96
+ NOX2       1     107.0 28173 413.11
+ Sound      1      94.9 28185 413.13
+ Over652    1      71.5 28209 413.18
+ HC2        1      16.8 28263 413.30
```

```
- WhiteCol2  1    1938.9 30219 413.31
- Density2   1    2198.6 30479 413.83
- Educ       1    2484.8 30765 414.39
- JulyTemp2  1    2796.9 31077 414.99
- Density    1    2893.2 31173 415.18
- Educ2      1    2936.8 31217 415.26
- Poor       1    3025.5 31306 415.43
- Poor2      1    3441.4 31721 416.22
- SO2        1    4073.8 32354 417.41
- NonWhite   1    4476.5 32757 418.15
- Over65     1    4562.4 32843 418.31
- House2     1    5032.8 33313 419.16
- HC         1    6785.7 35066 422.24
- JanTemp    1   11525.9 39806 429.85
- Precip2    1   17375.5 45656 438.07
- NOX        1   19154.8 47435 440.37

   user  system elapsed
  0.281   0.003   0.290
```

Conclusion is that for time: forward $<$ stepwise $<<$ among all subset Cp. (It is noted that the system.time() results can be different for different computers, but this order should be the same.)