MAT 237, Quiz 2

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Part A: (2 marks) Precisely state the Mean Value Theorem

Suppose f is continuous on $[a,b]$, differentiable on (a,b)

Then $\exists c \in (a,b)$ s.t $f'(c) = \frac{f(b)-f(a)}{b-a}$

Part B: (3 marks) Use the Mean Value Theorem to prove that if f is differentiable on an open interval I, and f'(x)<0 for all $x \in I$ then f is strictly decreasing on I.

For $a,b \in I$, $f(b)-f(a) = f'(c)(b-a)$ as f satisfies M.V.T.

As $f'(x) < 0$, ~~as~~ for $b > a$, $f(b)-f(a) < 0$.

Hence $f(b) < f(a)$ when $b > a$ $\Rightarrow$ strictly decreasing

Part C: (5 marks) Prove Rolle's Theorem (do not use the Mean Value Theorem in your proof)

Suppose f is cont. on $[a,b]$ and diff on (a,b)

As $[a,b]$ is compact, by extremum value theorem, f takes a max & min on $[a,b]$. If the max and min both occur at an endpoint, as $f(a) = f(b)$ by assumption then f is constant on $[a,b]$ (and has $f'(c) = 0$ $\forall c \in (a,b)$)

Else, either max or min occurs at some $c \in (a,b)$

And since max and mins have derivative zero (prop 2.5)

$f'(c) = 0$. Hence, for such f, $f(a) = f(b) \Rightarrow \exists c \in (a,b)$ s.t $f'(c) = 0$