LN 6.1. ①

Inflation.

$\bar{i}$: money rate.

r : inflation rate → ① value, ② inflation index.

i' : real rate.

$$\boxed{(1+\bar{i}) = (1+r)\cdot(1+i')}$$

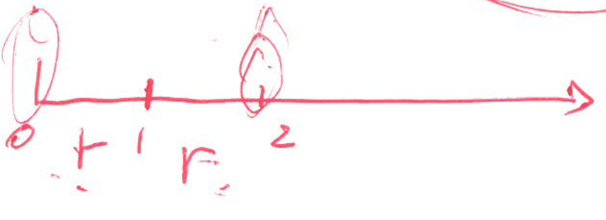
* Lecture Notes Week 5, Page 2

$$\boxed{\sum_{k=1}^{n} I_{t_k}\cdot(1+\bar{i})^{-t_k}} + \int_0^n \rho_I(t)\,(1+\bar{i})^{-t}\,dt = \sum_{k=1}^{n} O_{t_k}\cdot(1+\bar{i})^{-t_k} + \int_0^n \rho_O(t)(1+\bar{i})^{-t}\,dt$$

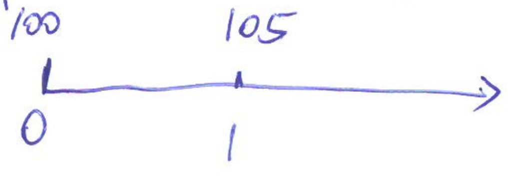
$t_1 \cdots t_k$

$$\sum_{k=1}^{n} \underbrace{(I_{t_k}-O_{t_k})}_{N_{t_k}}\cdot \underbrace{(1+\bar{i})^{-t_k}}_{(1+r)^{-t_k}\cdot(1+i')^{-t_k}} + \int_0^n \underbrace{[\rho_I(t)-\rho_O(t)]}_{\rho_N(t)}\underbrace{(1+\bar{i})^{-t}}_{(1+r)^{-t}\cdot(1-i')^{-t}}\,dt.$$

constant inflation rate.



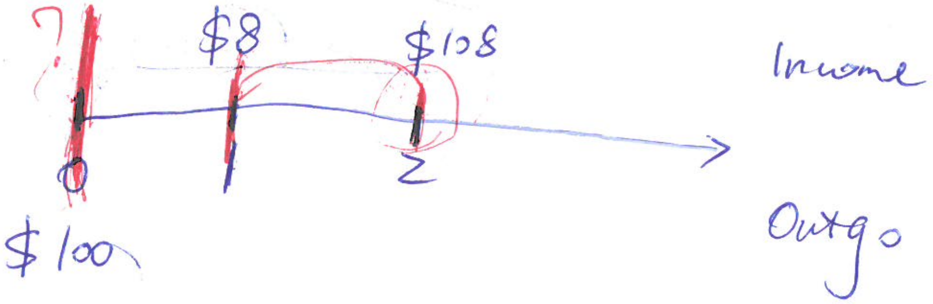
Inflation Index



$$r = \frac{105-100}{100} = 5\%$$

$$1+r = \frac{105}{100}$$

Ex.



$Q_0=150 \quad Q_1=156 \quad Q_2=170$.

1: $(1+r_1)=\frac{156}{150}$ $\qquad (1+i)=(1+r)(1+i')$

2: $(1+r_2)=\cancel{\frac{170}{156}}$? $\frac{170}{150}$ ✓ $=\frac{170}{156}\cdot\frac{156}{150}$

$$100 = 8\cdot\boxed{(1+i)^{-1}} + 108\cdot\boxed{(1+i)^{-2}}$$

different r_K.

$$= 8\left(\frac{156}{150}\right)^{-1}(1+i')^{-1} + 108\cdot\left(\frac{170}{150}\right)^{-1}\cdot(1+i')^{-2}.$$

$$= 8\left(\frac{156}{150}\right)^{-1}\cdot(1+i')^{-1} + 108\left(\frac{170}{150}\right)^{-1}\cdot(1+i')^{-2}$$

$\Rightarrow i' = 0.0154$.

"Goal seek" Excel.

Ex: M1: $1000 \cdot (1.08)^{10} = 21589.25$.

$$\frac{21589.25}{(1+5\%)^{10}} = 13253.93.$$

M2: $$r' = \frac{i-r}{1+r} = \frac{8\%-5\%}{1.05} = 2.857\%$$

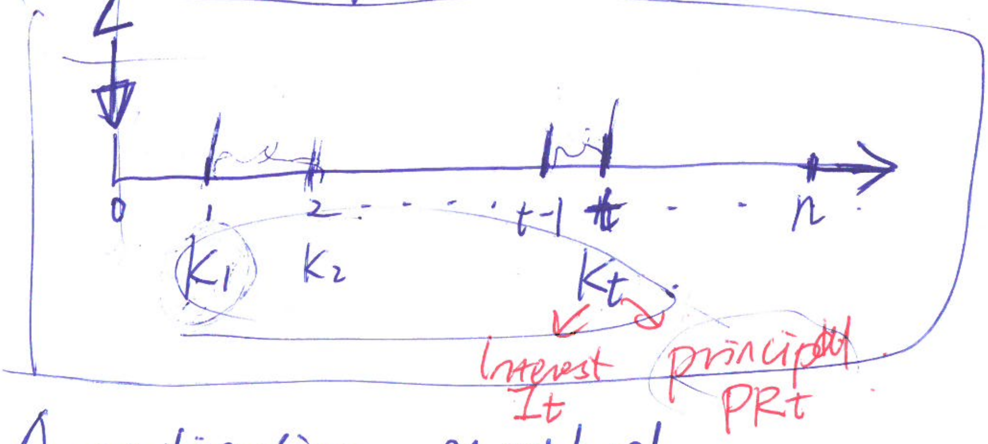
$$\Uparrow$$

$$(1+i) = (1+r)(1+i')$$

$$1000 \cdot (1+i')^{10} = 13253.93.$$

CN. week 6. ⟶ { Loans
Fixed-income Securiti
Forward contracts.

Loan Repayment.



Amortisation method.

* $L = OB_0$, Interest rate $= i$.

* K_t: Amount paid at time $t = K_t$.

* . I_t : Interest at the end of the t^{th} period for period $(t-1, t)$.

* . PR_t: Principal repaid at the end of the t^{th} period, for period $(t-1, t)$.

* . OB_t : Outstanding balance just after payment at time t

$$OB_t = OB_{t-1} + OB_{t-1} \cdot i - K_t$$

$$= OB_{t-1} + I_t - K_t.$$

$$= OB_{t-1} - \boxed{(K_t - I_t)}$$

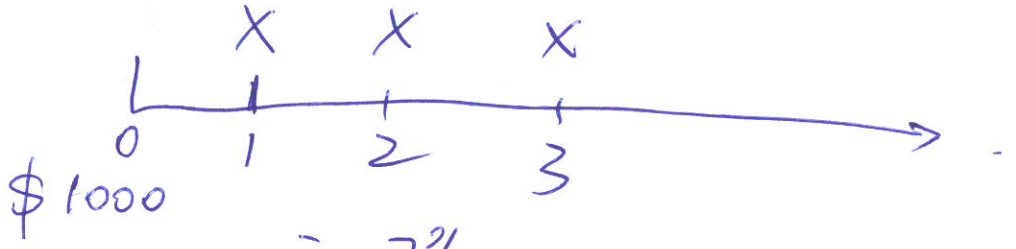
$$= OB_{t-1} - PR_t.$$

Loan Schedule.

t	payment	Interest Due.	Principal Repaid	Outstanding Balance.
0				$L = OB_0$
1	k_1	$I_1 = OB_0 \cdot i$,	$PR_1 = k_1 - I_1$	$OB_1 = OB_0 - PR_1$
2	k_2	$I_2 = OB_1 \cdot i$,	$PR_2 = k_2 - I_2$	$OB_2 = OB_1 - PR_2$
⋯	⋯	⋯	⋯	⋯
t	k_t	$I_t = OB_{t-1} \cdot i$,	$PR_t = k_t - I_t$,	$OB_t = OB_{t-1} - PR_t$.
⋯	⋯	⋯	⋯	⋯
n	k_n	$I_n = OB_{n-1} \cdot i$,	$PR_n = k_n - I_n$	$OB_n = OB_{n-1} - PR_n = 0$

? $PR_t = k_t - I_t < 0. \Rightarrow OB_t > OB_{t-1}$.

Ex



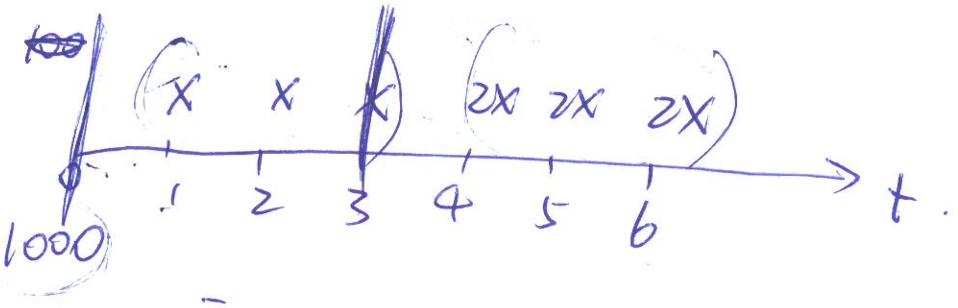
$i = 7\%$.

① . $1000 = X \cdot a_{\overline{3}|7\%} \Rightarrow X = 381.05$.

②

t	k_t	I_t	PR_t	OB_t
0				1000.
1	381.05	$1000 \cdot 7\% = 70$	$381.05 - 70 = 311.05$	$1000 - 311.05 = 688.95$
2	381.05	$688.95 \cdot 7\% = 48.22$	$381.05 - 48.22 = 332.83$	$688.95 - 332.83 = 356.12$
~~3~~				
3	381.05	$356.12 \cdot 7\% = 24.93$	$381.05 - 24.93 = 356.12$	0

Ex.



$i = 1\%$

①

$$1000 = X \cdot a_{\overline{3}|0.01} + 2X \cdot a_{\overline{3}|0.01} \cdot v_{0.01}^{3}$$

$$\Rightarrow \quad X = \boxed{115.61}$$

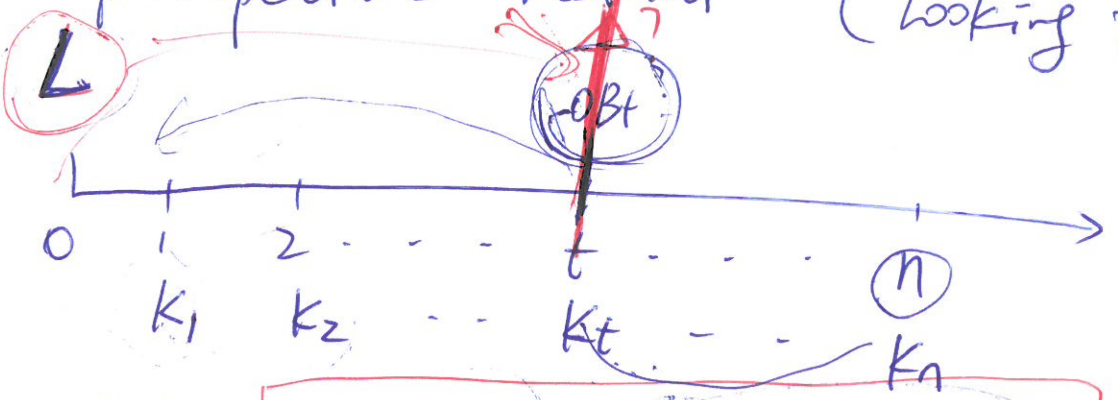
②

t	Kt	It	PRt	OBt
0				1000
1	115.61	1000 · 1% = 10	115.61 − 10 = 105.61	1000 − 105.61 = 894.39
2	115.61	894.39 · 1% = 8.94	115.61 − 8.94 = 106.67	894.39 − 106.67 = 787.72
3				
4				
5				
6				

Calculating the OBt

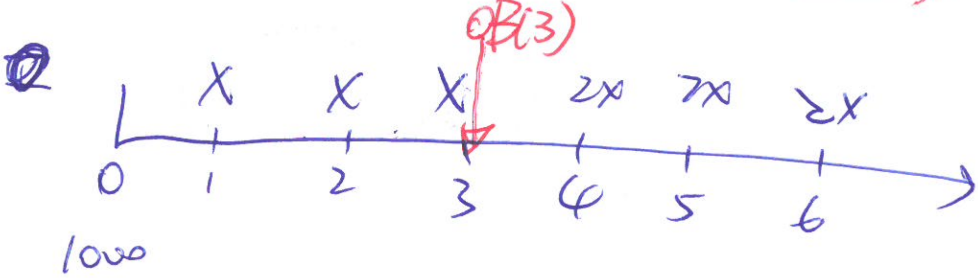
① retrospective method (looking ~~forward~~ back)

② prospective method (looking forward)



① ∴ $$\boxed{OB(t) = L(1+i)^t - \sum_{a=1}^{t} k_a (1+i)^{t-a}}$$

Ex



115.61

$$OB(3) = 1000(1+0.01)^3 - \sum_{a=1}^{3} (1+i)^{3-a} \cdot \boxed{K_a}$$

$$\cong 679.99$$

Pf: $OB_t = OB_{t-1}(1+i) - K_t$

$$\Rightarrow \begin{cases} OB_1 = \boxed{OB_0 (1+i) - K_1} \\ OB_2 = OB_1 (1+i) - K_2 \\ \quad = [OB_0 (1+i) - K_1] \cdot (1+i) - K_2 \\ \quad = \boxed{OB_0 (1+i)^2 - K_1 (1+i) - K_2} \end{cases}$$

$$OB_3 = \overset{\downarrow}{\boxed{OB_2}}(1+i) - K_3 .$$

$$= \left[OB_0(1+i)^2 - K_1(1+i) - K_2\right](1+i) - K_3$$

$$= OB_0(1+i)^3 - K_1(1+i)^2 - K_2(1+i) - K_3$$

$$\cdots$$

$$\vdots$$

$$OB_t = OB_{t-1}(1+i) - K_t$$

$$= OB_0(1+i)^t - \left(K_1(1+i)^{t-1} + K_2(1+i)^{t-2} + \cdots + K_t\right)$$

$$OB_t = OB_0(1+i)^t - \sum_{a=1}^{t}(1+i)^{t-a}K_a .$$