2.31.) X = # of phone calls

Let $X \sim \text{Poisson}(\lambda = 2)$; $\lambda = 2$ per hour

a.) A nice property of the distribution is that we can easily change the time length.

- For 10 minutes: $\gamma = \frac{\lambda}{60} \times 10 = \frac{\lambda}{6} = \frac{2}{6} = 1/3.$

$$P(\text{the phone rings at least once}) = 1 - P(\text{phone does not ring})$$

$$= 1 - P(X = 0)$$

$$= 1 - \frac{\gamma^0 e^{-\gamma}}{0!}$$

$$= 1 - e^{-\gamma} = 1 - e^{-1/3} =$$

$$\approx 0.2835.$$

b.) The prebability of receiving no phone calls to be at most 0.5.

$$P(X=0) \leq 0.5$$

$$e^{-\gamma} \leq 0.5$$

$$-\gamma \leq \log(0.5) = -0.6931$$

$$\gamma > 0.6931$$

Recall $\lambda = 2$ per hour

$\therefore \quad \delta = \frac{2}{60}$ per minute

$\Rightarrow$ How many minutes so $\delta > 0.6931$?

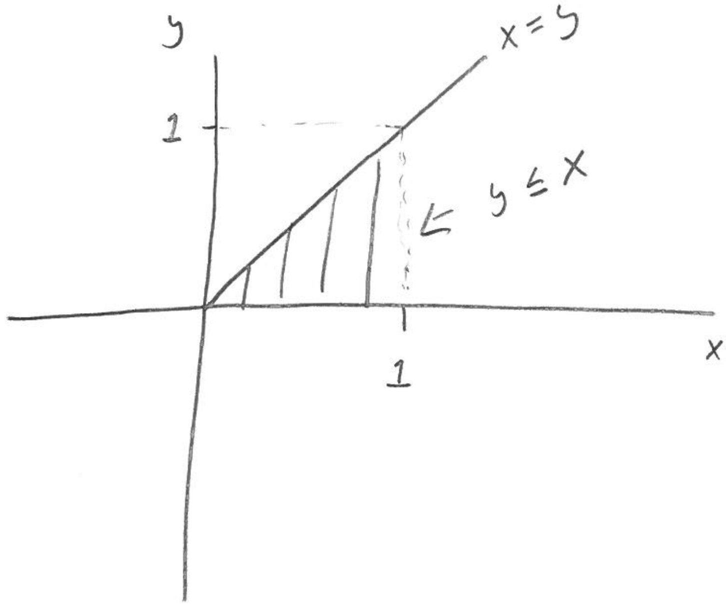
Let $z = \#$ of minutes.

$$\frac{2}{60} z = 0.6931$$

$$z = \frac{0.6931(60)}{2} \approx 20.79 \text{ minutes}$$

3.18) X and Y have a joint density:

$$f(x,y) = \begin{cases} K(x-y) & 0 \le y \le x \le 1 \\ 0 & \text{elsewhere} \end{cases}$$

a.) 

b.) $\int_0^1 \int_0^x f(x,y)\, dy\, dx = 1$

$\Rightarrow \int_0^1 \int_0^x K(x-y)\, dy\, dx$

$= \int_0^1 K \left[\int_0^x x - y\, dy \right] dx = \int_0^1 K \left[xy - \frac{y^2}{2} \right]_0^x dx$

$= \int_0^1 K \left[x^2 - \frac{x^2}{2} \right] dx = \int_0^1 K \frac{x^2}{2}\, dx = \frac{K}{2} \left[\frac{x^3}{3} \right]_0^1$

$= \frac{K}{6} = 1 \Rightarrow K = 6$

$\therefore f(x,y) = 6(x-y) \quad 0 \le y \le x \le 1$

c.) $f_X(x) = \int_0^x 6(x-y)\,dy$

$$= 6\int_0^x (x-y)\,dy = 6\left[xy - \frac{y^2}{2}\right]_0^x$$

$$= 6\left[x^2 - \frac{x^2}{2}\right] = 6\,\frac{x^2}{2}$$

$$= 3x^2 \qquad 0 \le x \le 1$$

$f_Y(y) = \int_y^1 6(x-y)\,dx = 6\int_y^1 (x-y)\,dx$

$$= 6\left[\frac{x^2}{2} - xy\right]_y^1 = 6\left[\left[\frac{1}{2} - y\right] - \left[\frac{y^2}{2} - y^2\right]\right]$$

$$= 6\left[\frac{1}{2} - y + \frac{y^2}{2}\right] = 3\left[y^2 - 2y + 1\right]$$

$$= 3(y-1)^2 \qquad 0 \le y \le 1$$

d.) $f_{Y|X}(y|x) = \dfrac{f(x,y)}{f_X(x)} = \dfrac{6(x-y)}{3x^2} = \dfrac{2(x-y)}{x^2}$

$= 2\left(\dfrac{1}{x} - \dfrac{y}{x^2}\right)$; $0 \leq y \leq x$

$f_{X|Y}(x|y) = \dfrac{f(x,y)}{f_Y(y)} = \dfrac{6(x-y)}{3(y-1)^2} = \dfrac{2(x-y)}{(y-1)^2}$; $y \leq x \leq 1$