See the website for "Solved linear Programming Problem"

§ 2.1

Vague definition: The "Basic Simplex method" or "Single phase simplex method" (in the context of § 2.3 or "primal simplex method" (in the context of Chapter 3) is the method in § 2.1

Remark: This can solve:

Canonical problems:

$$\text{Maximize } Z = c^T x \text{ s.t.}$$
$$Ax = b$$
$$x \geq 0$$

(where A is an $m \times n$ matrix) only when ① A includes the columns of the $m \times n$ identity matrix.

② $b \geq 0 \in R^m$

Standard problems:

$$\text{Maximize } Z = c^T x \text{ s.t.}$$
$$Ax \leq b$$
$$x \geq 0 \in R^n$$

only when $b \geq 0$

Remark: Such a standard problem, after putting in slack variables, lead to a canonical problem satisfying ① & ②

E.g. (A simplex Optimization – see the website)

$$\text{Maximize } Z = 3x_1 + 7x_2 \text{ s.t.}$$
$$x_1 + 5x_2 \leq 19$$
$$x_1 - x_2 \leq 7$$
$$-x_1 + 2x_2 \leq 2$$
$$x_1 \geq 0, x_2 \geq 0$$

In canonical form

$$\text{Maximize } Z = 3x_1 + 7x_2 \text{ s.t.}$$
$$x_1 + 5x_2 + x_3 = 19$$
$$x_1 - x_2 + x_4 = 7$$
$$-x_1 + 2x_2 + x_5 = 2$$
$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0, x_5 \geq 0.$$

Set up tableau ① (treating Σ like any unknown)

The augmented matrix for the equality constraints:

	x_1	x_2	x_3	x_4	x_5	Σ	
x_3	1	5	1	0	0	0	19
x_4	1	−1	0	1	0	0	7
x_5	−1	2	0	0	1	0	2
	−3	−7	0	0	0	1	0

TABLEAU ①

This row represents the equation "$-3x_1-7x_2+\Sigma=0$", which defines Σ as in the problem: "$\Sigma=3x_1+7x_2$"

(incomplete tableau)

Each row represents an equation. So far the tableau represents a system having the basic solution (having basic variables x_3, x_4, x_5):

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 19 \\ 7 \\ 2 \end{bmatrix}$$

Each basic variable is associated with a constraint (a row in the tableau) and vice versa. These are the new label

The tableau (without the last row) represents the basic feasible solution given above. With this solution ($x_1=0, x_2=0$)

$\Sigma=3x_1+7x_2=0$.

Definition: This appears at the lower right of any § 2.1 tableau and is the objective value. The objective row is the rest of the last row (in § 2.1) of any tableau, excluding the coefficient of Σ (=1) and excluding the objective value.