Previous Term Test 2 Questions

This document contains the actual questions found on previous Term Test #2 papers which were offered by Professor P.Rosenthal.

Term Test #2 Questions for 1999, 2000, 2001, 2002, and Final exam questions for 1998,1999,2000,2001 are found below

Term Test #2 January 15, 2002

1. Show that $(32)^{2/3}$ is irrational.

2. Show that cube $\sqrt{4} + \sqrt{7}$ is irrational.

3. a) Write $(1+ i)^{10}$ in the form a + bi.

3 b) Find the square roots of $-i$

4. Find all the complex solutions of $(z^3 +1)/ (z^3 -1) = i$.

5. Find the least upper bound of each of the following sets.

a) $\{ (-1)^n/n^2$: n is a natural number$\}$

b) the set of positive rational numbers whose square roots are less than $\sqrt[3]{4}$.

6. Find the cardinality of the set of all irrational real numbers, and prove your answer is correct.

7. Is there a line in the x-y plane such that both coordinates of every point on the line are rational? Prove your answer.

8. Find the cardinality of the set of all complex numbers, and justify your answer.

9. What is the cardinality of the set of all finite subsets of R? Show that your answer is correct.

10. a)Give the precise definition (as a subset of Q) of a real number

10 b) Describe the subset of Q that, in the construction of R in terms of certain subsets of Q, would correspond to the $\sqrt[3]{17}$.

Term Test #2 January 19,1999

1. Show that $24^{2/3}$ is irrational.

2. Use the Euclidean algorithm to find the greatest common divisor of 1800 and 240

3. Find the Euler function of each of the numbers.

a) 97

b) 36

4. Prove that $\sqrt[3]{4} + \sqrt{6}$ is irrational

5. Show using the precise definitions given in class, that given any natural number k, there is a real number whose square is k.

6. Can a regular polyhedron have an even number of edges, an even number of vertices, and an odd number of faces? Justify your answer.

7. The Cartesian product of the sets A and B is defined to be the set of all ordered pairs (a,b) with a in A and b in B. Prove that the Cartesian product of two countable sets is countable.

8. Find the cardinality of the set of all points in the plane (2 dimensional) which have one rational coordinate and one irrational coordinate and justify your answer.

9. Prove that the cardinality of the set of all finite subsets of the plane is c.

10. Prove that the union of c sets which each have cardinality c has cardinality c.

Old Term Test 2 (2000)

A1 Find the cardinality of $\{ a + b\sqrt{2} : a, b \in Q\}$ and justify your answer.

A2. Find the cardinality of the set of all points in R^3 all of whose coordinates are rational, and justify your answer.

A3. What is the cardinality of

a) the set of all numbers in [0,1] which have decimal expansions with a finite number of non-zero digits?

b) the set of all numbers in [0,1] which have decimal expansions that end with an infinite sequence of 7's?

A4. Prove that the set of all finite subsets of Q is countable.

A5. Show that the set of all polynomials with rational coefficients is countable.

A6. Suppose that there is a function mapping the set S onto the set T. Prove that $|S| \geq |T|$

A7. The Cartesian product of the sets A and B is defined to be the set of all ordered pairs (a,b) with a in A and b in B. Prove that the Cartesian product of two countable sets is countable.

A8. Find the cardinality of the set of all points in the plane which have one rational coordinate and one irrational coordinate, and justify your answer.

A9. Prove that the cardinality of the set of all finite subsets of the plane R^2 is c.

Old Term Test 2 (January 16, 2001)

B1. Show that $(28)^{2/5}$ is irrational.

B3a) Must the sum of an irrational and a rational number be irrational? Prove that your answer is correct.

B3b)Must an irrational number to a rational power be irrational? Prove that your answer is correct.

B4. Prove that $5^{1/3} + 7^{1/2}$ is irrational.

B5. Prove that the set of irrational real numbers has cardinality c.

B6. Find the cardinality of the set of all functions mapping the set of natural numbers into the set {a,b,c} and prove that your answer is correct.

B7. Is there a line in the xy plane such that the coordinates of every point on the line are rational? Prove your answer.

B8. Find the cardinality of the set of all points in the plane which have one rational coordinate and one irrational coordinate, and justify your answer.

B9. Let T be the set of all real numbers of the form a +b□ with a and b algebraic (and □ the ratio of the circumference of a circle to its diameter). Show that T is countable.

B10. Prove that $\sqrt{3} + \sqrt{5} + \sqrt{7}$ is irrational.

Final Exam 1998

3. Prove that $7^{2/3}$ is irrational.

7. Suppose that S and T each have cardinality c (the cardinality of R). Show that S U T has cardinality c.

Final Exam 1999

4. Does the equation

$x^4 + x - 1 = 0$ have a rational root? Justify your answer.

6. Find all cube roots of $8\sqrt{3} + 8i$. (You can leave answers in the form $r(\cos\theta + i \sin\theta)$ where you specify r and θ .)

7. Determine the cardinality of each of the following sets and justify your answer.

a) The set of all functions mapping the set of natural numbers into {3,7}.

b) The set $\{a + bi: a, b \in Q\}$.

c) Omitted (Not applicable)

d) The set $\{(x,y): x \in R, y \in Q\}$.

e) The set of real numbers which have decimal expansion containing only the digits 2 and 4.

10. Prove that a continuous function mapping R into R must be a constant function if its range is countable.

Final Exam 2000.

3b) Let S be the set of all functions mapping R into R. Show that the cardinality of S is greater than c.

4b) Find a polynomial p with integer coefficients such that $p(3 + i\sqrt{7}) = 0$.

5. Let $S = \{x \in R$: x has a decimal representation using only the digits 2 and 6}. Show that the cardinality of S is c.

6. Prove that $3\sqrt{4} + \sqrt{10}$ is irrational

7b) Write in the form a + bi: $(-\sqrt{3}/2 + i/2)^{11}$.

9. Show, using the precise definition of the real numbers that was discussed in class, that there is a real number x such that $x^3 = 4$.

2001 Final Exam (Questions Relevant to Term Test 2)

2 a) Prove that $n^2 - 1$ is divisible by 8 for every odd integer n.

3 a) Let N be the set of natural numbers and a and b be distinct numbers. What is the cardinality of the set of all functions and domain {a,b} and range a subset of N? Justify your answer.

b) Suppose that the set S, T and U satisfy $S \subset T \subset U$ and that S and U both have the same cardinality. Show that T has the same cardinality as S.

4. a) What is the winding number of the curve $f(\phi) = 17 + \cos 17t + i\sin 17t$ for $t \in [0,2]$ about the origin? Explain your answer.

b) Find all the solutions of $z^6 + z^3 + 1 = 0$.

5. Prove the following: if S is uncountable and T is countable subset of S, then the cardinality of S\T is the same as the cardinality of S.

6. Prove the following: if a and b are relatively prime natural numbers and k is a natural number such that

$k^{a/b}$ is rational, then $k^{a/b}$ is a natural number

7. Let S denote the collection of all sequences of real numbers. Show that the cardinality of S is c.