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Office hours: Friday 11-12 SS/091

Assignments 1:

12. $N = C_{52}^{25} C_{25}^5 C_{20}^5 C_{15}^5 C_{10}^5 C_5^5$

20. $P = \frac{49! \cdot 4!}{52!}$

26. $P = 1/6^4$

30. (1) $P_1 = 0.05$ (2) $P_2 = 0.3$ (3) $P = 0.06$

36. 35

38. (1) $N_1 = 20$ (2) $N_2 = \binom{7}{3,3,1} = \dots$

42. $N = \binom{11}{4,3,3,1}$

7. Prove $P(A \cap B) \geq P(A) + P(B) - 1$

iff $1 \geq P(A) + P(B) - P(A \cap B) = P(A \cup B)$

8. Prove $P(\bigcup_{i=1}^n A_i) \leq \sum_{i=1}^n P(A_i)$

Pf: Math Induction

$n=2$ $P(A_1 \cup A_2) \leq P(A_1) + P(A_2)$

if the statement holds for n , then:

$P(A_1 \cup A_2 \cup \dots \cup A_n) \leq P(A_1) + \dots + P(A_n)$

For the $n+1$ case:

$P(A_1 \cup A_2 \cup \dots \cup A_n \cup A_{n+1}) \leq P(\bar{A} \cup A_{n+1}) \leq P(\bar{A}) + P(A_{n+1})$

$\leq \sum_{i=1}^n P(A_i) + P(A_{n+1}) = \sum_{i=1}^{n+1} P(A_i)$

12. 5 player 5 cards/each, 52 cards.
(no difference)

$\frac{52!}{25! 5! 5! 5! 5! 5!}$

20. deck of 52, 4 aces next to each other, $P_{\text{ace}} = ?$

"4A" \Rightarrow 1 card $1+48=49$ $\frac{49! \cdot 4!}{52!}$

Other method: 48 individual cards and 4 "slots"

$\frac{48! \cdot 4!}{52!}$

#30. 60 students into 2 30-people class. 5 friends. M.S.I.K.C
5 out of 60.

① Pick 2 students out of 6. $\begin{matrix} 6 \\ \swarrow \searrow \\ 3 \quad 3 \end{matrix}$ $\begin{matrix} \Delta \\ \Delta \end{matrix}$ $\begin{matrix} 1 \\ 2 \\ 2 \\ 4 \end{matrix}$

$\begin{matrix} \Delta \\ \Delta \end{matrix}$ $\begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix}$ $\begin{matrix} 2 \\ 3 \\ 4 \end{matrix}$
 $\begin{matrix} \Delta \\ \Delta \end{matrix}$ $\begin{matrix} 2 \\ 3 \\ 4 \end{matrix}$ $\begin{matrix} 1 \\ 3 \\ 4 \end{matrix}$
 $\begin{matrix} \Delta \\ \Delta \end{matrix}$ $\begin{matrix} 3 \\ 4 \end{matrix}$ $\begin{matrix} 1 \\ 2 \end{matrix}$
 $\begin{matrix} \Delta \\ \Delta \end{matrix}$ $\begin{matrix} 4 \end{matrix}$ $\begin{matrix} 1 \\ 2 \\ 3 \end{matrix}$
 $\begin{matrix} 1 \\ 2 \end{matrix}$ $\begin{matrix} \Delta \\ \Delta \end{matrix}$ $\begin{matrix} 3 \\ 4 \end{matrix}$
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 $\begin{matrix} 3 \\ 4 \end{matrix}$ $\begin{matrix} \Delta \\ \Delta \end{matrix}$ $\begin{matrix} 1 \\ 2 \end{matrix}$
 $\begin{matrix} 1 \\ 2 \end{matrix}$ $\begin{matrix} \Delta \\ \Delta \end{matrix}$ $\begin{matrix} 3 \\ 4 \end{matrix}$

⑩ all together

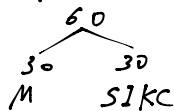
$$\binom{6}{3} \binom{3}{2} = 20 \quad ? \text{ (exchange 2 classes)}$$

$$\frac{\binom{6}{3} \binom{3}{3}}{\binom{1}{1}} = 10$$

$$\frac{\binom{55}{25} \binom{30}{30}}{\binom{30}{30} \binom{30}{30} / \binom{1}{1}} \approx 0.05$$

$$\text{note: } \binom{n}{k} = \binom{n}{n-k}$$

② Take M as an example.



$$\frac{\binom{55}{26} \binom{29}{29} \times 5}{\binom{60}{30} \binom{30}{30} / \binom{1}{1}}$$

$$\textcircled{3} P_2 \times \frac{1}{5}$$

#38. $\begin{pmatrix} R \\ R \\ R \end{pmatrix} \begin{pmatrix} G \\ G \\ G \end{pmatrix}$ 6 Blocks $\binom{6}{3}$ choose 3 out of 6 spaces to put in red done!

Method II: $\underline{R} \underline{R} \underline{R} \underline{\quad} \Rightarrow 4 \text{ slots}$

3 situations: 1. 3 G together $\binom{4}{1}$
 2. 2 G + 1 G $\binom{4}{2} \binom{2}{1}$
 3. G + G + G $\binom{4}{3}$

$$\binom{4}{1} + \binom{4}{2} \binom{2}{1} + \binom{4}{3}$$

$$\binom{9}{3 \ 3 \ 3} = \dots$$

$$\text{or } 20 \times \binom{7}{1} + \binom{7}{2} \binom{7}{1} + \binom{7}{3}$$