STA261H1S: Solutions of first quiz

Question 1. $P(A \cup B) = P(A) + P(B) - P(A)P(B) = 0.4 + 0.6 - 0.4 \times 0.6 = 0.76.$ (**2 marks**)

Question 2. $(n-1)S^2/\sigma^2 \sim \chi_{n-1}$ (**1 mark**), we have $E(S^2) = \sigma^2$ and $Var(S^2) = 2\sigma^4/(n-1)$ (**2 marks**). Since for any $\varepsilon > 0$,

$$P(|S^2 - \sigma^2| > \varepsilon) \leq Var(S^2)/\varepsilon^2 \to 0, \tag{1}$$

as $n \to \infty$ by Chebyschev's Lemma, S^2 is a consistent estimate of σ^2 (**1 mark**).

Question 3. The moment estimate is $\hat{\theta}_m = 2\bar{X}$ (**1 mark**) and is unbiased since $E(\hat{\theta}_m) = \theta$ (**1 mark**). The mle is $\hat{\theta}_{mle} = \max\{X_1, \cdots, X_n\}$ (**1 mark**). The pdf is

$$P(\hat{\theta}_{mle} \leq x) = P(X_1 \leq x, \cdots, X_n \leq x) = \prod_{i=1}^{n} P(X_i \leq x) = \frac{x^n}{\theta^n}$$

$$E(\hat{\theta}_{mle}) = \int_0^\theta xn \frac{x^{n-1}}{\theta^n} = \frac{n\theta}{n+1}.$$

Hence, $\hat{\theta}_{mle}$ is biased (**1 mark**).