STAT2001 Tutorial 1 Solutions

Problem 1

(a) The elements of the sample space are the possible outcomes of the experiment. These elements may be denoted l = left, r = right, a = ahead.

l, r, a are the sample points, and the sample space is $S = \{l, r, a\}$.

(b) If the three outcomes are equally likely,
it is reasonable to attach a probability of 1/3 to each of them.

To do this formally we let: $L = \{l\}$ = "Vehicle turns left"
$R = \{r\}$ = "Vehicle turns right"
$A = \{a\}$ = "Vehicle continues straight ahead".

The simple events are the singleton sets L, R, A,
and the sample space is the union of these, ie $S = L \cup R \cup A$.

We may now define the probability function by $P(L) = P(R) = P(A) = 1/3$.

(c) Let T = "Vehicle turns". Then $T = \{l, r\}$.

So $P(T) = P(\{l,r\}) = P(\{\mathrm{l}\}) + P(\{r\}) = P(L) + P(R) = 2/3$.

The said assumption might not be reasonable at major-minor intersections.

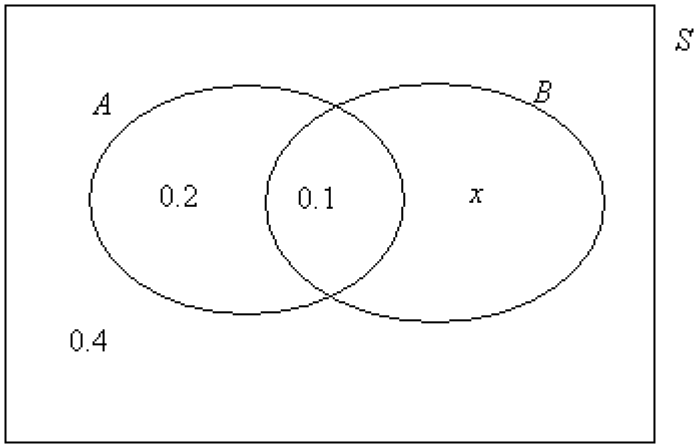
Another solution

In practise the distinction between sample points and simple events is not made.
A briefer solution to the problem is as follows. This second solution is technically flawed but acceptable in this course.

(a) L = left, R = right, A = ahead.
(b) $P(L) = P(R) = P(A) = 1/3$.
(c) Let T = "Vehicle turns". Then $T = \{L, R\}$. So $P(T) = P(L) + P(R) = 2/3$.

Problem 2

We are told that $P(A\bar{B})$ = 0.2, $P(AB) = 0.1$ and $P(\overline{A \cup B})$ = 0.4.
This information can be depicted in a Venn diagram as follows:



We see that 0.4 + 0.2 + 0.1 + x = 1, where x = $P(\bar{A}B)$.
Therefore $P(\bar{A}B)$ = 0.3.

(a) $P(A) = P(A\bar{B}) + P(AB)$ = 0.2 + 0.1 = 0.3.

(b) $P(B) = P(\bar{A}B) + P(AB)$ = 0.3 + 0.1 = 0.4.

(c) $P(A \cup B) = P(A\bar{B}) + P(AB) + P(\bar{A}B)$ = 0.2 + 0.1 + 0.3 = 0.6.

Alternatively, $P(A \cup B) = P(A) + P(B) - P(AB)$ = 0.3 + 0.4 – 0.1 = 0.6.
Or more simply, $P(A \cup B) = 1 - P(\overline{A \cup B})$ = 1 – 0.4 = 0.6.
The theory behind these two alternative solutions may not yet have been covered in lectures.

(d) By De Morgan's laws, $P(\bar{A}\bar{B})$ = $P(\overline{A \cup B})$ = 0.4.

$P(\bar{A}\bar{B})$ can also be obtained by shading $\bar{A}$ and $\bar{B}$ in different ways and then identifying the area in the diagram that is shaded in both ways.

Problem 3

(a) Let 1, 2 and 3 denote the empty cans, and a, b and c the ones with water.

We will let 13*b* (for example) denote the outcome that the expert chooses cans 1, 3 and *b* (in any order), etc.

The sample points are then

123,	12*a*, 12*b*, 12*c*,	13*a*, 13*b*, 13*c*,	23*a*, 23*b*, 23*c*,
abc,	*ab*1, *ab*2, *ab*3,	*ac*1, *ac*2, *ac*3,	*bc*1, *bc*2, *bc*3.

Since the rod is worthless, all 20 outcomes are equally likely, and it is reasonable to assign probability 1/20 to each one. Hence the probability that the expert will correctly identify all 3 of the cans with water is $P(abc) = 1/20$.

This is a small probability. So if the expert *does* find all 3 cans with water, we will be somewhat surprised.

(b) Let A = "The expert will correctly identify at least two of the cans with water".
Then $A = \{abc, \quad ab1, ab2, ab3, \quad ac1, ac2, ac3, \quad bc1, bc2, bc3\}$.
This set has exactly half the elements in S (ie, 10/20). So $P(A) = 1/2$.

This is a very large probability. So if the expert finds 2 of the 3 cans with water, we will not be very impressed by that feat.

Note that $P(A)$ can also be obtained by considering the symmetry in the problem. The probability of finding 2 or 3 cans *with* water is the same as that of finding 2 or 3 cans *without* water (since exactly half of the six cans have water), which in turn is the same as that of finding 1 or 0 cans *with* water. The first and last of these three probabilities must add up to 1 (since 0, 1, 2 or 3 cans with water must be found).
Therefore the probability of finding 2 or 3 cans with water equals 1/2.

Problem 4

Let B = blue, W = white, K = black, G = green.

Then let *BKB*, for example, denote the ordering of a blue auto, a black auto, and another blue auto (in that order), etc.

The sample space is $S = \{BBB, BBW, BBK, ..., GGG\}$.
This set has $n_S = 4^3 = 64$ elements by the *mn* rule.

The 64 sample points are all equally likely.
So it is reasonable to assign a probability of 1/64 to each one.

(a) Let X be the event that one blue, one white and one green are ordered (in any order). Then $X = \{BWG, BGW, GBW, GWB, WGB, WBG\}$ and $n_X = 6$. Therefore $P(X) = n_X / n_S = 6/64 = 3/32$.

(b) Let Y be the event that two blue autos are ordered.
Then $Y = \{BBW,BWB,WBB,BBK,BKB,KBB,BBG,BGB,GBB\}$ and $n_Y = 9$. So $P(Y) = n_Y / n_S = 9/64$.

(c) Let Z be the event that at least one black auto is ordered.
Then $\bar{Z}$, the event that no black autos are ordered, has $n_{\bar{Z}} = 3^3 = 27$ elements. Hence Z has $n_Z = n_S - n_{\bar{Z}} = 64 - 27 = 37$ elements.
Hence $P(Z) = n_Z / n_S = 37/64$.

Alternatively, $P(Z) = 1 - P(\bar{Z}) = 1 - 27/64 = 37/64$.

(d) P(2 have same colour) = P(2 blues) + P(2 whites) + P(2 blacks) + P(2 greens)
= 4P(2 blues)   by symmetry
= 4(9/64)   by (b)
= 9/16.

Alternative working for (d):

P(2 have same colour) = 1 - P(All have same colour) - P(All have different colours)
= 1 - 4(1/64) - 4(3/32)   (*)
= 9/16, as before.

(*) Here, 1/64 is the probability that all 4 autos have the same particular colour, eg blue, and this is multiplied by 4, the number of colours (blue, white, black, green). Also, the 3/32 comes from (a), where black is not ordered, and this is multiplied by 4, the number of colours that might not be ordered (either blue, white, black and green).