UNIVERSITY OF TORONTO
Faculty of Arts and Science

EXAMINATION DECEMBER 2011

PHL 245 H1F
L0101 - Niko Scharer

PLEASE HAND IN

Duration - 3 hours

Examination Aid: Sheet with rules (provided)

Last Name ______________________________

First Name ______________________________

Student Number __________________________

Answer **all** questions on the exam paper.

Use the supplied examination booklet for rough work OR if you need further space.

The exam consists of fourteen (14) pages. Pages 2-12 have questions on them.

The final two pages (13-14) are an aid sheet and may be detached from the rest of the exam.

1. A set of sentences {P, Q, R} is logically inconsistent.
Which of the following arguments must be valid? Circle the correct answer. 2 %

a)
$$\begin{array}{l} \sim P \\ \sim R \\ \hline Q \end{array}$$
Valid
Not necessarily valid

b)
$$\begin{array}{l} P \wedge S \\ Q \\ \hline R \rightarrow T \end{array}$$
Valid
Not necessarily valid.

c)
$$\begin{array}{l} P \rightarrow Q \\ \sim R \vee P \\ \hline S \vee \sim R \end{array}$$
Valid
Not necessarily valid

d)
$$\begin{array}{l} \sim(P \rightarrow \sim Q) \\ R \\ \hline S \end{array}$$
Valid
Not necessarily valid.

2. Suppose there are three sentences: ϕ, ψ and χ. On every interpretation that ϕ is true, ψ is false.
What can you conclude (if anything) about the following argument? Explain. (3%)

$$\begin{array}{l} \phi \vee \chi \\ \hline \therefore \psi \rightarrow \chi \end{array}$$

3. Consider the following truth-table for the NEW symbol: $*$ (2%)

P	**Q**	**P $*$ Q**
T	**T**	**F**
T	**F**	**T**
F	**T**	**T**
F	**F**	**F**

a) Using the new symbol, and other logical connectives if necessary, symbolize: P iff Q.

b) What ordinary English expression can this new truth-functional connective ($*$) be used to symbolize (given its truth-table)?

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4. Use this symbolization scheme to symbolize the following sentences: (36 % = 9 × 4%)

A^1: *a* has an appointment.	B^1: *a* is a place.	D^1: *a* is a doctor.	E^1: *a* is a day.
F^1: *a* is a person	G^1: *a* is a tour guide.		H^1: *a* is healthy.
J^2: *a* is a friend of *b*.	K^2: *a* visits *b*.	L^2: *a* likes *b*.	
C^3: *a* calls *b* on *c*.	M^2: *a* is more popular than *b*.		
a^0: Adam	b^0: Dr. Bailey	h^0: the hospital	d^1: the daughter of *a*.

a) Some doctors don't have any appointments only provided that everybody is healthy.

b) For a person to visit a doctor, it is necessary that he/she has an appointment.

c) Only people who are not healthy visit the hospital, unless they are visiting a friend.

d) If there are days when nobody calls a tour guide, then some tour guides don't visit places that anyone likes.

4 continued. Use this symbolization scheme to symbolize the following sentences: (36 % = 9 × 4%)

A^1: *a* has an appointment.	B^1: *a* is a place.	D^1: *a* is a doctor.	E^1: *a* is a day.
F^1: *a* is a person	G^1: *a* is a tour guide.		H^1: *a* is healthy.
J^2: *a* is a friend of *b*.	K^2: *a* visits *b*.	L^2: *a* likes *b*.	
C^3: *a* calls *b* on *c*.	M^2: *a* is more popular than *b*.		
a^0: Adam	b^0: Dr. Bailey	h^0: the hospital	d^1: the daughter of *a*.

e) Not all people who visit exactly those friends who visit them ever call people with whom they are not friends.

f) Exactly one doctor visits Adam, and she is neither Dr. Bailey nor Dr. Bailey's daughter.

g) Only Adam's daughter visits only those places that Adam likes.

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4 continued. (36 % = 9 × 4%)

A^1: *a* has an appointment.	B^1: *a* is a place.	D^1: *a* is a doctor.	E^1: *a* is a day.
F^1: *a* is a person	G^1: *a* is a tour guide.		H^1: *a* is healthy.
J^2: *a* is a friend of *b*.	K^2: *a* visits *b*.	L^2: *a* likes *b*.	
C^3: *a* calls *b* on *c*.	M^2: *a* is more popular than *b*.		
a^0: Adam	b^0: Dr. Bailey	h^0: the hospital	d^1: the daughter of *a*.

h) Using the symbolization scheme above, provide an idiomatic English sentence that expresses:

$$\exists x(Bx \wedge \forall y(By \wedge x \neq y \rightarrow M(xy)) \wedge \forall z(Fz \wedge K(zx) \rightarrow L(zx)))$$

i) Using the symbolization scheme above, symbolize the following ambiguous sentence **three** logically distinct ways. For each, provide an English sentence that explains exactly what the symbolized sentence means:

Somebody calls a doctor every day.

5. Provide a derivation that shows the following theorem is valid **using only the 10 basic rules from SL** (R, DN, MP, MT, ADJ, S, ADD, MTP, BC, CB) **and the 3 basic rules from PL** (UI, EG, EI) (9%)

$$\therefore\ (\forall y{\sim}(Dy \rightarrow Ay) \wedge \exists x{\sim}(Bx \vee Fx)) \rightarrow \exists x(Ax \leftrightarrow Bx)$$

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6. Provide a derivation that shows that this is a valid argument **using only the 10 basic rules from SL** (R, DN, MP, MT, ADJ, S, ADD, MTP, BC, CB) **and the 3 basic rules from PL** (UI, EG, EI) (9%)

$\exists x(\sim Dx \wedge \forall y F(xy))$. $\forall x(\sim Hx \rightarrow Dx)$. $\exists x Hx \rightarrow \forall y \forall z(By \wedge F(zz) \rightarrow G(yz))$.

$\therefore \forall x(Bx \rightarrow \exists y(Hy \wedge G(xy)))$

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7. Provide a derivation to show that this is a valid argument (use any rules). (9 %):

$$\exists x H(xa(x)) \rightarrow \forall x \exists y \sim(Fx \rightarrow Gy). \qquad \therefore \exists x \forall y H(a(x)a(y)) \rightarrow \exists z(Fz \wedge \sim Gz)$$

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8. Show that the following is a valid argument (use any rules). (9%):

$\forall x \exists y \forall z A(xyz)$. $\exists x \forall y(A(xyy) \rightarrow \forall z{\sim}B(xz))$. $\forall z(\exists w K(wz) \rightarrow M(zz))$.

$\forall x \forall y(K(xy) \rightarrow {\sim}L(yx)) \rightarrow {\sim}\exists y{\sim}B(yy)$. $\therefore {\sim}\forall x(M(xx) \rightarrow {\sim}\exists z L(xz))$

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9. Provide an English language interpretation (including the universe of discourse and a symbolization scheme) that shows that the following set of sentences is consistent. (4%)

$\exists x(Fx \wedge \forall y G(xy))$. $\exists x \exists y(Bx \wedge {\sim}Fy \wedge G(xy))$. ${\sim}\exists x(Bx \wedge \forall y(Fy \rightarrow G(yx)))$.

10. Explain why the following sentence is a contradiction. (4 %)

$$(\exists y Fy \wedge \exists x Gx) \leftrightarrow \forall y(Fy \rightarrow \forall x{\sim}Gx)$$

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11. Use a finite model to demonstrate the invalidity of this argument (8 %):

    i) provide a truth-functional expansion (to two individuals) for each sentence in this argument with respect to the model

    ii) define a model with a universe of two individuals that shows that this argument is invalid.

$\exists x(Ax \wedge \forall y\, G(xy))$. $\forall x \exists y(Bx \rightarrow \sim G(xy))$. $\therefore\ \sim\exists x(G(xx) \wedge Bx)$

12. Consider the following derivation rule (which is *not* a rule in our derivation system):

$$\begin{array}{l} \phi \vee \psi \\ \phi \\ \hline \sim \psi \end{array} \qquad\qquad \begin{array}{l} \phi \vee \psi \\ \psi \\ \hline \sim \phi \end{array}$$

Explain how this rule works.
What are the advantages (if any) and disadvantages (if any) of adding this rule to our system.
Overall, do you think that it would be good to add this rule to our derivation system?
Explain why or why not. (5 %)

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AID SHEET: DERIVATION RULES (SEE BOTH SIDES)

Derivation Types:

Direct Derivation (DD)

Conditional Derivation (CD)

Indirect Derivation (ID)

Universal Derivation (UD) Restriction: the instantiating term cannot occur unbound in any previous line.

Basic Rules for Sentential Operators:

Modus Ponens (MP)

$$\frac{\begin{array}{l}(\phi \to \psi)\\ \phi\end{array}}{\psi}$$

Modus Tollens (MT)

$$\frac{\begin{array}{l}(\phi \to \psi)\\ \sim\psi\end{array}}{\sim\phi}$$

Double Negation (DN)

$$\frac{\phi}{\sim\sim\phi} \qquad \frac{\sim\sim\phi}{\phi}$$

Repetition (R)

$$\frac{\phi}{\phi}$$

Simplification (S)

$$\frac{\phi \wedge \psi}{\phi} \qquad \frac{\phi \wedge \psi}{\psi}$$

Adjunction (ADJ)

$$\frac{\begin{array}{l}\phi\\ \psi\end{array}}{\phi \wedge \psi}$$

Addition (ADD)

$$\frac{\phi}{\phi \vee \psi} \qquad \frac{\psi}{\phi \vee \psi}$$

Modus Tollendo Ponens (MTP)

$$\frac{\begin{array}{l}\phi \vee \psi\\ \sim\phi\end{array}}{\psi} \qquad \frac{\begin{array}{l}\phi \vee \psi\\ \sim\psi\end{array}}{\phi}$$

Biconditional-Conditional (BC)

$$\frac{\phi \leftrightarrow \psi}{\phi \to \psi} \qquad \frac{\phi \leftrightarrow \psi}{\psi \to \phi}$$

Conditional-Biconditional (CB)

$$\frac{\begin{array}{l}\phi \to \psi\\ \psi \to \phi\end{array}}{\phi \leftrightarrow \psi}$$

Derived Rules for Sentential Operators:

Negation of Conditional (NC)

$$\frac{\sim(\phi \rightarrow \psi)}{\phi \wedge \sim\psi} \qquad \frac{\phi \wedge \sim\psi}{\sim(\phi \rightarrow \psi)}$$

Conditional as Disjunction (CDJ)

$$\frac{\phi \rightarrow \psi}{\sim\phi \vee \psi} \qquad \frac{\sim\phi \vee \psi}{\phi \rightarrow \psi} \qquad \frac{\sim\phi \rightarrow \psi}{\phi \vee \psi} \qquad \frac{\phi \vee \psi}{\sim\phi \rightarrow \psi}$$

Separation of Cases (SC)

$$\frac{\begin{array}{l}\phi \vee \psi \\ \phi \rightarrow \chi \\ \psi \rightarrow \chi\end{array}}{\chi} \qquad \frac{\begin{array}{l}\phi \rightarrow \chi \\ \sim\phi \rightarrow \chi\end{array}}{\chi}$$

Negation of Biconditional (NB)

$$\frac{\sim(\phi \leftrightarrow \psi)}{\phi \leftrightarrow \sim\psi} \qquad \frac{\phi \leftrightarrow \sim\psi}{\sim(\phi \leftrightarrow \psi)}$$

De Morgan's (DM)

$$\frac{\sim(\phi \vee \psi)}{\sim\phi \wedge \sim\psi} \qquad \frac{\sim\phi \wedge \sim\psi}{\sim(\phi \vee \psi)} \qquad \frac{\sim(\phi \wedge \psi)}{\sim\phi \vee \sim\psi} \qquad \frac{\sim\phi \vee \sim\psi}{\sim(\phi \wedge \psi)}$$

$$\frac{\sim(\sim\phi \vee \sim\psi)}{\phi \wedge \psi} \qquad \frac{\phi \wedge \psi}{\sim(\sim\phi \vee \sim\psi)} \qquad \frac{\sim(\sim\phi \wedge \sim\psi)}{\phi \vee \psi} \qquad \frac{\phi \vee \psi}{\sim(\sim\phi \wedge \sim\psi)}$$

Derivation Rules for Predicate Logic:

Existential Generalization (EG)

$$\frac{\phi_\zeta}{\exists\alpha\phi_\alpha}$$

Universal Instantiation (UI)

$$\frac{\forall\alpha\phi_\alpha}{\phi_\zeta}$$

Restriction: ζ does not occur as a bound variable in ϕ_α

Existential Instantiation (EI)

$$\frac{\exists\alpha\phi_\alpha}{\phi_\zeta}$$

Restriction: ζ does not occur in any previous line or premise.

Quantifier Negation (QN)

$$\frac{\sim\forall\alpha\phi}{\exists\alpha\sim\phi} \qquad \frac{\sim\exists\alpha\,\phi}{\forall\alpha\sim\phi}$$

$$\frac{\exists\alpha\sim\phi}{\sim\forall\alpha\phi} \qquad \frac{\forall\alpha\sim\phi}{\sim\exists\alpha\,\phi}$$