Today – finish "A simplex Optimization"

- θ ratio
- the optimality criterion

A routine pivot on the indicated number leads to Tableau ③ (after entering x_1, exiting x_3):

	x_1	x_2	x_3	x_4	x_5	z	
x_1	1	0	$\frac{2}{7}$	0	$-\frac{5}{7}$	0	4
x_4	0	0	$-\frac{1}{7}$	1	$\frac{6}{7}$	0	6
x_2	0	1	$\frac{1}{7}$	0	$\frac{1}{7}$	0	3
	0	0	$\frac{13}{7}$	0	$-\frac{8}{7}$	1	33

θ ratio of x_5 column

$4/(-5/7)$ ✗

$6/(6/7)=7 \longrightarrow x_4$ exits

$3/(1/7)=21$

About the θ-ratio test, given Tableau ⓟ

$$\begin{array}{c|c}
x_j & \\
\hline
a_1 & b_1 \geq 0 \\
\vdots & \vdots \\
a_i & b_i \geq 0 \\
\vdots & \vdots \\
a_m & b_m \geq 0
\end{array}$$

Here, x_j will enter, and a_i is the pivot ($a_i \neq 0$). If $a_i < 0$ next tableau will be feasible if and only if $b_i = 0$. (For x_j will enter with value $\frac{b_i}{a_i} < 0$ (if $b_i > 0$))

If $a_i > 0$, the next tableau will be Tableau (p+1)

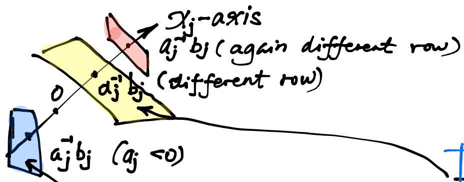
$$\begin{array}{r|c|l}
 & x_j & \\
\hline
 & 0 & b_1 - a_1(a_i^{-1}b_i) \\
 & 0 & \vdots \\
i^{th} \text{ row} & & a_i^{-1}b_i \leftarrow a_i^{-1}b \\
\to x_j & 1 & \vdots \\
 & 0 & b_m - a_m(a_i^{-1}b) \\
 & 0 & \\
\hline
\end{array}$$

Given k in $1, 2, 3, \cdots, m$ with $k \neq i$, b_k has been replaced with $b_k - a_k\underbrace{(a_i^{-1}b_i)}_{\geq 0} \geq b_k \geq 0$ if $a_k \leq 0$

if $a_k > 0$ $b_k - a_k(a_i^{-1}b) = a_k \underbrace{(a_k^{-1}b_k - a_i^{-1}b_i)}_{\text{a difference between } \theta\text{-ratios}}$

and will be positive for all k provided

$a_i^{-1}b_i$ is $<$ any θ-ratio having a positive denominator.



the constraint having this hyperplane is the first to block further increase in x_j.

this ~~never~~ never block any increase in x_j

Eg Tableau ③ represents :
(continued)

① A system of equations

② A basic feasible solution

③ The problem

$$\text{Maximize } z = \frac{-13}{7}x_3 + \frac{8}{7}x_5 + 33 \quad \text{(from the objective row)}$$

$$\text{s.t. } \frac{2}{7}x_3 - \frac{5}{7}x_5 \leq 4$$

$$-\frac{1}{7}x_3 + \frac{6}{7}x_5 \leq 6$$

$$\frac{1}{7}x_3 + \frac{1}{7}x_5 \leq 3$$

$$x_3 \geq 0,\ x_5 \geq 0$$