

Started on Wednesday, 21 March 2018, 4:00 PM**State** Finished**Completed on** Wednesday, 21 March 2018, 4:25 PM**Time taken** 24 mins 50 secs**Marks** 7.00/7.00**Grade** 10.00 out of 10.00 (100%)**Question 1**

Correct

Mark 1.00 out of 1.00

Let X and Y be independent random variables where $X \sim \text{gamma}(a = r, b = 1)$ and $Y \sim \text{gamma}(a = s, b = 1)$, where the $E[X] = r$.Let $Z_1 = X + Y$ and $Z_2 = X/(X + Y)$. Which of the following statement is true?

Select one:

- ☒ a. Marginally, $Z_1 \sim \text{gamma}(r + s, 1)$ and $Z_2 \sim \text{beta}(r, s)$. ✓
- ☐ b. Marginally, $Z_1 \sim \text{gamma}(r + s, 1)$ and $Z_2 \sim \text{beta}(r/s, 1)$.
- ☐ c. Z_1 and Z_2 are dependent.
- ☐ d. Marginally, $Z_1 \sim \text{gamma}(r + s, 1)$ and $Z_2 \sim \text{gamma}(r/s, 1)$.

The correct answer is: Marginally, $Z_1 \sim \text{gamma}(r + s, 1)$ and $Z_2 \sim \text{beta}(r, s)$.

Question 2

Correct

Mark 1.00 out of 1.00

An original method for generating random standard normal variables based on random uniform variables was through the following transformation:

$$X = \sum_{i=1}^{12} U_i - 6$$
$$U_i \sim \text{iid uniform}(0, 1).$$

The mean of X is?

Select one:

- ☒ a. 0 ✓
- ☐ b. 6
- ☐ c. 12
- ☐ d. -2

The correct answer is: 0

Question 3

Correct

Mark 1.00 out of 1.00

An original method for generating random standard normal variables based on random uniform variables was through the following transformation:

$$X = \sum_{i=1}^{12} U_i - 6$$
$$U_i \sim \text{iid uniform}(0, 1).$$

The variance of X is?

Select one:

- ☒ a. 1 ✓
- ☐ b. 0
- ☐ c. 12^2
- ☐ d. 12

The correct answer is: 1

Question 4

Correct

Mark 1.00 out of 1.00

Let $X_1, \dots, X_N \sim \text{iid } f(x; \theta) = \theta x^{\theta-1}$, where $0 \leq x \leq 1$ and $0 < \theta < \infty$. Maximise the joint density or the log of the joint density with respect to θ . The value of θ which maximises the function (i.e. the MLE) is

Select one:

- ☐ a. Doesn't have an analytical solution.
- ☐ b. $-\frac{n}{\sum_{i=1}^n x_i}$
- ☒ c. $-\frac{n}{\sum_{i=1}^n \log(x_i)}$ ✓
- ☐ d. \bar{x}

The correct answer is: $-\frac{n}{\sum_{i=1}^n \log(x_i)}$

Question 5

Correct

Mark 1.00 out of 1.00

Find the constant c such that the following function is a proper probability density:

$$f(x) = c(1-x)^3; 0 \leq x \leq 3.$$

Select one:

- ☐ a. 1/3
- ☐ b. 10
- ☐ c. -2/3
- ☒ d. -4/15 ✓

The correct answer is: -4/15

Question 6

Correct

Mark 1.00 out of 1.00

Determine the following integral using Monte Carlo integration:

$$\int_0^{\infty} \sin(x) \exp(-x/2) dx.$$

Base the integration on an exponential distribution with $E[X] = 2$. Use 10,000 draws and use the following to set the seed: `set.seed(2000)`. Round to 4 decimal places.

Answer: 0.7938



```
S<-10000
```

```
set.seed(2000)
```

```
x <- rexp(S, 1/2)
```

```
l.hat <- mean(2*sin(x))
```

```
round(l.hat,4)
```

The correct answer is: 0.7938

Question 7

Correct

Mark 1.00 out of 1.00

A researcher from the College of Medicine states: "I just fit a least-squares model to determine the effects of age and gender on blood pressure." This statement is clear and correct.

Select one:

- ☐ True
- ☒ False

Least-squares is a method of estimation and is not a model.

The correct answer is 'False'.