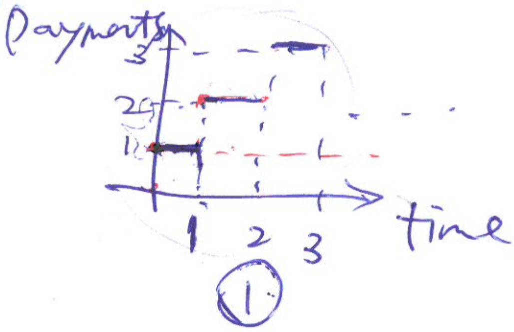
LN 4.2 ①

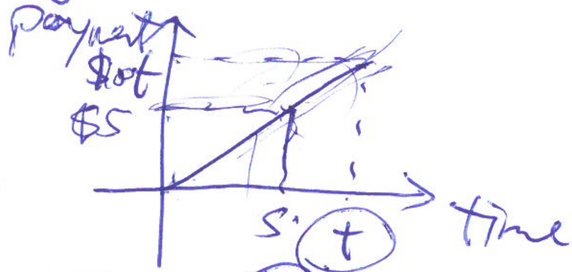
Increasing / Decreasing Annuities.

① payments in Arithmetic Progression.

$A, \; A+B, \; A+2B \; \dots \; A+(n-1)B.$

② Continuos Increasing Annuities





③ payments in Geometric Progression.

(Annuities with Indexation)

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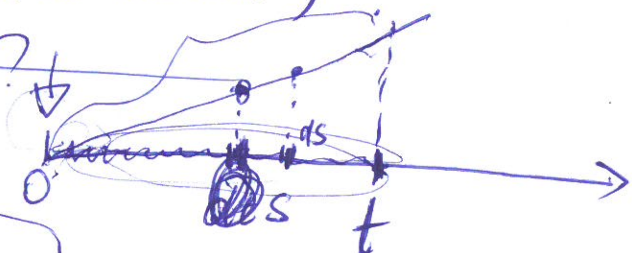
② type 2



$$\left(\bar{I}\bar{a}\right)_{\overline{n}|i} = \boxed{\int_0^{t} v^s\, s \, ds}$$

$$= \int_0^t s \cdot e^{-\delta s}\, ds$$

integration by parts

$$= \frac{-s \cdot e^{-\delta s}}{\delta}\Big|_0^t + \int_0^t \frac{e^{-\delta s}}{\delta}\, ds.$$

$$= -\frac{t v^t}{\delta} + \frac{e^{-\delta s}}{\delta^2}\Big|_0^t$$

Correction: The Positive sign before the second item (of the last line) should be negative.

$$= \frac{\boxed{\frac{1-v^t}{\delta}} - t v^t}{\delta} = \boxed{\frac{\bar{a}_{\overline{t}|i} - t \cdot v^t}{\delta}}$$

① type 1

$$(I\bar{a})_{\overline{n}|i} = \underbrace{\int_0^1 \overbrace{v^t dt}^{=\bar{a}_{\overline{1}|}}}_{yr1} + \underbrace{\int_1^2 2 \cdot v^t dt}_{yr2} + \cdots + \underbrace{\int_{n-1}^n n \cdot v^t dt}_{yn.}$$

$$= \boxed{\sum_{t=1}^n \left(\int_{t-1}^{t} t \cdot v^s ds \right)}$$

$$\boxed{s = q + (t-1)}$$

$$\Rightarrow \begin{cases} s = t-1, \Rightarrow q = 0 \\ s = t, \Rightarrow q = 1 \end{cases}$$

$$= \sum_{t=1}^n t \cdot \int_0^1 \underbrace{v^{(q+t-1)}}_{= v^q \cdot v^{t-1}} \underbrace{d(q+t-1)}_{= dq}$$

\$1 \$1 \$2 ... \$n

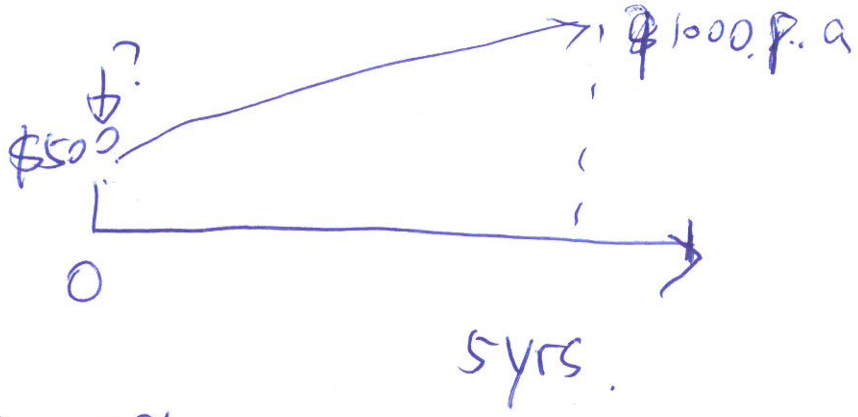
0 1 2 n

$$= \boxed{\sum_{t=1}^n t \cdot v^{t-1}} \boxed{\int_0^1 v^q dq} = \bar{a}_{\overline{1}|}$$

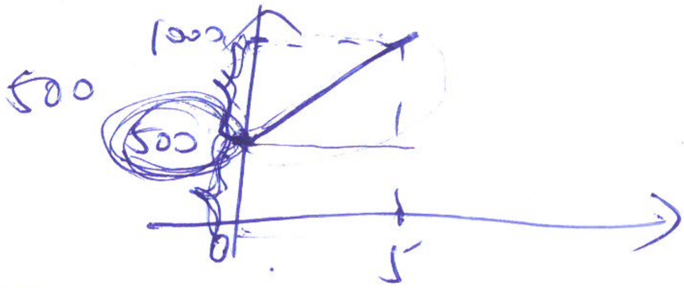
$\ddot{a} \times v$

$$= \boxed{(I\ddot{a})_{\overline{n}|} \cdot \bar{a}_{\overline{1}|}}$$

Ex:



$i = 4\%$.



Sol:

$$500 \cdot \bar{a}_{\overline{5}|0.04} + 100 \cdot (\bar{I}\bar{a})_{\overline{5}|0.04}.$$