Duration: 60 minutes (2:15pm - 3:15pm)

Aids Allowed: none

Student Number: 9999292509

Family Name(s): Qiu

Given Name(s): Rui

Do **not** turn this page until you have received the signal to start. In the meantime, please fill out the identification section above.

This term test consists of 4 questions on 10 pages (including this one), printed on both sides of the paper. When you receive the signal to start, please make sure that your copy of the test is complete, fill in the identification section above, and write your name on the back of the last page.

This test is double-sided.

Marking Guide

TOTAL: $\frac{1}{2}$ /31

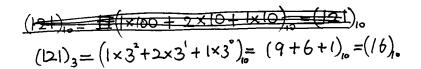
Good Luck!

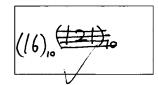
Question 1. [5 MARKS]

The Setun computer was developed in Moscow in the 1950s. It used a ternary (base 3) number system.

Part (a) [1 MARK]

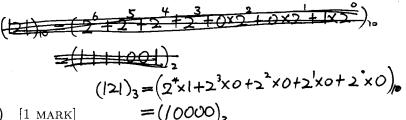
What is the decimal (base 10) representation of the ternary number 121? Show your work and place your final answer in the box.

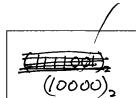




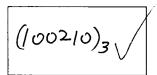
Part (b) [1 MARK]

What is the binary (base 2) representation of the ternary number 121? Show your work and place your final answer in the box.





Using only ternary numbers, determine the sum of the ternary numbers 10101 and 20102. Show your work and place your final answer in the box.



Part (d) [2 MARKS]

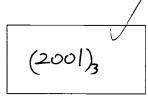
Using only ternary numbers, determine the product of the ternary numbers 12 and 102. Show your work and place your final answer in the box.

$$(12)_{3} \times (102)_{3}$$

$$101$$

$$12$$

$$(2001)_{3}$$



5/5

Question 2. [9 MARKS]

Recall that an integer n is even if and only if $\exists q \in \mathbb{Z}, n = 2q$. Also, an integer n is odd if and only if $\exists q \in \mathbb{Z}, n = 2q + 1$. Integers are either even or odd.

Let us define the predicates E(n): "n is an even number" and O(n): "n is an odd number".

Consider the following statement:

For every integer n, n^3 is even if and only if n is even.

Part (a) [1 MARK]

Translate the statement into symbolic notation. Quantify over the integers (\mathbb{Z}). Use the predicate E(n).

 $(=\forall n \in \mathbb{N})$

\ne Z. 1= E(n) <=>E(n).

Part (b) [8 MARKS]

Write a detailed structured proof of the statement. Part marks will be given for having correct elements of the proof structure.

Assuma F(n3)

Proof: Assume $E \cap E \mathbb{Z}$. # n is a typical integer

Assume E(n). # one direction?

Then $\exists k \in \mathbb{Z}$, n = 2k. # definition of even numbers

Then $n^3 = (2k)^3 = 8k^3 = 2(4k^3)$, # algobra

Let $j = 4k^3$.

Then $n^3 = 2(4k^3) = 2j$,

Then $E(n) = 2(4k^3) = 2j$,

Then $E(n) = E(n^3)$. # implication

Assume $ext{T}(n)$. # the other direction

Assume $ext{T}(n)$. # the other direction

Then $ext{T}(n)$ # definition of odd numbers.

Then $ext{T}(n)$ # definition of odd numbers.

Then $ext{T}(n)$ # definition of odd numbers.

Let $ext{J}$ * $ext{J}$ *

Then n3=21+1

(see the next page)

CONT'D...

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Use the space on this "blank" page for scratch work, or for any answer that did not fit elsewhere.

Clearly label each answer with the appropriate question and part number.

Then
$$\neg E(n) = \neg \neg E(n^3)$$
. # implication

Then $E(n^3) = \neg E(n)$. # contrapositive

Therefore $E(n) < = \neg E(n^3)$. # since $E(n) = \neg E(n^3)$ and $E(n^3) = \neg E(n)$

Then $\forall n \in \mathbb{Z}$, $E(n^3) < = \neg E(n)$

7E(n) (2k+1) (4k²+4k+1)(2k+1) 8k³+8k²+2k+4k²+4k+1

Question 3. [9 MARKS]

Recall that an integer p > 1 is prime if and only if its only positive integer divisors are 1 and p.

Also, an integer n is odd if and only if $\exists q \in \mathbb{Z}, n = 2q + 1$. An integer n is even if and only if $\exists q \in \mathbb{Z}, n = 2q$. Integers are either odd or even.

Let us define the predicates P(n): "n is a prime number", O(n): "n is an odd number" and E(n): "n is an even number".

Consider the following statement:

All prime numbers greater than 2 are odd.

Part (a) [2 MARKS]

Translate the statement into symbolic notation. Quantify over the natural numbers (\mathbb{N}). Use the predicates P(n), O(n) and/or E(n).



 $\forall n \in \mathbb{Z}, n > 2, \bigoplus P(n) \Rightarrow O(n).$

Part (b) [7 MARKS]

Write a detailed structured proof of the statement. Part marks will be given for having correct elements of the proof structure.

Proof: Assume $n \in \mathbb{N}$, #n is a typical integer.

Assume n > 2,

Assume n > 0,

Then E(n), #negation of consequent

Then E(n), # definition and since integers are either even

Then $\exists k \in \mathbb{Z}[N]$, n = 2k # definition of even

Then $\exists P(n)$ # by definition of prime numbers

Then $\exists P(n) \Rightarrow P(n)$ # since $E(n) \Leftrightarrow P(n)$ Then $P(n) \Rightarrow O(n)$ # contrapositive

Then n>2, P(n)=>0(n).

Then \(n \in N, n > 2, P(n) \(> 0 Cn).

You have to show the only sinteger admissing the only sinteger admissing the number is I and of the number is I and I a

Question 4. [8 MARKS]

Recall that for $x \in \mathbb{R}$, we can define |x| by $|x| = \begin{cases} -x, & x < 0, \\ x, & x \ge 0. \end{cases}$

(This is the only definition of |x| that you are allowed to use in your solution to this question.)

Consider the following statement:

For every real number x, if |x-3| < 3 then 0 < x < 6.

This statement is equivalent to the symbolic statement:

$$\forall x \in \mathbb{R}, |x - 3| < 3 \Rightarrow 0 < x < 6.$$

Now consider the following argument:

Assume $x \in \mathbb{R}$.

Assume |x-3| < 3.

Then either $x-3 \ge 0$ or x-3 < 0.

Case 1: Assume $x - 3 \ge 0$.

Then |x-3| = x-3. # by the above definition

Then x - 3 < 3. # since |x - 3| < 3

Then x < 6. # add 3 to both sides

Case 2: Assume x - 3 < 0.

Then |x-3| = -(x-3). # by the above definition

Then -(x-3) < 3. # since |x-3| < 3

Then -x + 3 < 3.

Then -x < 0. # subtract 3 from both sides

Then 0 < x # add x to both sides.

Then we have proven both 0 < x and x < 6.

Then 0 < x < 6.

Then $|x-3| < 3 \Rightarrow 0 < x < 6$.

Then $\forall x \in \mathbb{R}, |x-3| < 3 \Rightarrow 0 < x < 6$.

Part (a) [2 MARKS]

This argument is not a correct proof of the statement. Explain the flaw in the argument.

The argument ignores the and of x in two assumptions \neq , which are $\chi-3\geq0=>\chi>3$ and

7-3<0=>7<3.

Hence the conclusions of two cases should

be $0<\pi<3$ and $3\leq\pi<6$.

Why I this a flow though?

The proof shows that junder the assumption that |x-3|<3, it

follows that for x-3>0.7<6. It also shows that under the

fallows assumption, when x-3<0, x>0. Since either x-3>0 on x-3<0,

we have shown that either as home x-1 and x-1 and x-2 and x-1 and

we have shown that either 926 or 770. But we are CONT'D... required to show OCXCO. That is --- and --. In other words, the disjunction

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Part (b) [6 MARKS]

Give a correct proof of the statement $\forall x \in \mathbb{R}, |x-3| < 3 \Rightarrow 0 < x < 6$.

Proof: Assume $7 \in \mathbb{R}$. Assume |x-3| < 3.

Then either $\chi-3 \ge 0$ on $\pi-3 < 0$.

Case 1 : Assume 7-3≥0.

Then $x \ge 3$. # algebra.

Then |x-3|=x-3. # by the above definition.

Then $\chi-3<3$. # since |x-3|<3Then $\chi<6$. # algebra.

Then 3≤x<6.

Case 2. Assume x-3 <0.

Then 1<3. #algebra

Then -(x-3) < 3. # since |x-3|<3

Then -x+3<3.

Then -x<0. #algebra

Then 0= <7. #add x to both sides.

Then Then $0<\pi<3$. Lither or -1Then we have proven both 0<x<3 and 3<x<6. Then 0<x<6 lwhy is this the case?

Then 1x-31<3=>0<x<6.

Then $\forall x \in \mathbb{R}$, $|x-3| < 3 \Rightarrow 0 < x < 6$.