

# Table of Common Distributions

taken from *Statistical Inference* by Casella and Berger

## Discrete Distributions

distribution	pmf	mean	variance	mgf/moment
Bernoulli( $p$ )	$p^x(1-p)^{1-x}; x = 0, 1; p \in (0, 1)$	$p$	$p(1-p)$	$(1-p) + pe^t$
Beta-binomial( $n, \alpha, \beta$ )	$\binom{n}{x} \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \frac{\Gamma(x+\alpha)\Gamma(n-x+\beta)}{\Gamma(\alpha+\beta+n)}$	$\frac{n\alpha}{\alpha+\beta}$	$\frac{n\alpha\beta}{(\alpha+\beta)^2}$	
Notes: If $X P$ is binomial ( $n, P$ ) and $P$ is beta( $\alpha, \beta$ ), then $X$ is beta-binomial( $n, \alpha, \beta$ ).				
Binomial( $n, p$ )	$\binom{n}{x} p^x(1-p)^{n-x}; x = 1, \dots, n$	$np$	$np(1-p)$	$[(1-p) + pe^t]^n$
Discrete Uniform( $N$ )	$\frac{1}{N}; x = 1, \dots, N$	$\frac{N+1}{2}$	$\frac{(N+1)(N-1)}{12}$	$\frac{1}{N} \sum_{i=1}^N e^{it}$
Geometric( $p$ )	$p(1-p)^{x-1}; p \in (0, 1)$	$\frac{1}{p}$	$\frac{1-p}{p^2}$	$\frac{pe^t}{1-(1-p)e^t}$
Note: $Y = X - 1$ is negative binomial( $1, p$ ). The distribution is <i>memoryless</i> : $P(X > s X > t) = P(X > s - t)$ .				
Hypergeometric( $N, M, K$ )	$\frac{\binom{M}{x}\binom{N-M}{K-x}}{\binom{N}{K}}; x = 1, \dots, K$ $M - (N - K) \leq x \leq M; N, M, K > 0$	$\frac{KM}{N}$	$\frac{KM}{N} \frac{(N-M)(N-K)}{N(N-1)}$	?
Negative Binomial( $r, p$ )	$\binom{r+x-1}{x} p^r(1-p)^x; p \in (0, 1)$ $\binom{y-1}{r-1} p^r(1-p)^{y-r}; Y = X + r$	$\frac{r(1-p)}{p}$	$\frac{r(1-p)}{p^2}$	$\left(\frac{p}{1-(1-p)e^t}\right)^r$
Poisson( $\lambda$ )	$\frac{e^{-\lambda}\lambda^x}{x!}; \lambda \geq 0$	$\lambda$	$\lambda$	$e^{\lambda(e^t-1)}$
Notes: If $\lambda$ is gamma( $\alpha, \beta$ ), $\lambda$ is Poisson( $\frac{x}{\beta}$ ), and $\alpha$ is an integer, then $P(X \geq \alpha) = P(Y \leq y)$ .				

## Continuous Distributions

distribution	pdf	mean	variance	mgf/moment
Beta( $\alpha, \beta$ )	$\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1} (1-x)^{\beta-1}; x \in (0, 1), \alpha, \beta > 0$	$\frac{\alpha}{\alpha+\beta}$	$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$	$1 + \sum_{k=1}^{\infty} \left( \prod_{r=0}^{k-1} \frac{\alpha+r}{\alpha+\beta+r} \right) \frac{t^k}{k!}$
Cauchy( $\theta, \sigma$ )	$\frac{1}{\pi\sigma} \frac{1}{1+(\frac{x-\theta}{\sigma})^2}; \sigma > 0$	does not exist	does not exist	does not exist
Notes: Special case of Student's $t$ with 1 degree of freedom. Also, if $X, Y$ are iid $N(0, 1)$ , $\frac{X}{Y}$ is Cauchy				
$\chi_p^2$	$\frac{1}{\Gamma(\frac{p}{2})2^{\frac{p}{2}}} x^{\frac{p}{2}-1} e^{-\frac{x}{2}}; x > 0, p \in N$	$p$	$2p$	$\left( \frac{1}{1-2t} \right)^{\frac{p}{2}}, t < \frac{1}{2}$
Notes: Gamma( $\frac{p}{2}, 2$ ).				
Double Exponential( $\mu, \sigma$ )	$\frac{1}{2\sigma} e^{-\frac{ x-\mu }{\sigma}}; \sigma > 0$	$\mu$	$2\sigma^2$	$\frac{e^{\mu t}}{1-(\sigma t)^2}$
Exponential( $\theta$ )	$\frac{1}{\theta} e^{-\frac{x}{\theta}}; x \geq 0, \theta > 0$	$\theta$	$\theta^2$	$\frac{1}{1-\theta t}, t < \frac{1}{\theta}$
Notes: Gamma( $1, \theta$ ). Memoryless. $Y = X^{\frac{1}{\gamma}}$ is Weibull. $Y = \sqrt{\frac{2X}{\beta}}$ is Rayleigh. $Y = \alpha - \gamma \log \frac{X}{\beta}$ is Gumbel.				
$F_{\nu_1, \nu_2}$	$\frac{\Gamma(\frac{\nu_1+\nu_2}{2})}{\Gamma(\frac{\nu_1}{2})\Gamma(\frac{\nu_2}{2})} \left( \frac{\nu_1}{\nu_2} \right)^{\frac{\nu_1}{2}} \frac{x^{\frac{\nu_1-2}{2}}}{(1+(\frac{\nu_1}{\nu_2})x)^{\frac{\nu_1+\nu_2}{2}}}; x > 0$	$\frac{\nu_2}{\nu_2-2}, \nu_2 > 2$	$2(\frac{\nu_2}{\nu_2-2})^2 \frac{\nu_1+\nu_2-2}{\nu_1(\nu_2-4)}, \nu_2 > 4$	$EX^n = \frac{\Gamma(\frac{\nu_1+2n}{2})\Gamma(\frac{\nu_2-2n}{2})}{\Gamma(\frac{\nu_1}{2})\Gamma(\frac{\nu_2}{2})} \left( \frac{\nu_2}{\nu_1} \right)^n, n < \frac{\nu_2}{2}$
Notes: $F_{\nu_1, \nu_2} = \frac{\chi_{\nu_1}^2/\nu_1}{\chi_{\nu_2}^2/\nu_2}$ , where the $\chi^2$ s are independent. $F_{1, \nu} = t_{\nu}^2$ .				
Gamma( $\alpha, \beta$ )	$\frac{1}{\Gamma(\alpha)\beta^{\alpha}} x^{\alpha-1} e^{-\frac{x}{\beta}}; x > 0, \alpha, \beta > 0$	$\alpha\beta$	$\alpha\beta^2$	$\left( \frac{1}{1-\beta t} \right)^{\alpha}, t < \frac{1}{\beta}$
Notes: Some special cases are exponential ( $\alpha = 1$ ) and $\chi^2$ ( $\alpha = \frac{p}{2}, \beta = 2$ ). If $\alpha = \frac{2}{3}$ , $Y = \sqrt{\frac{X}{\beta}}$ is Maxwell. $Y = \frac{1}{X}$ is inverted gamma.				
Logistic( $\mu, \beta$ )	$\frac{1}{\beta} \frac{e^{-\frac{x-\mu}{\beta}}}{\left[ 1+e^{-\frac{x-\mu}{\beta}} \right]^2}; \beta > 0$	$\mu$	$\frac{\pi^2\beta^2}{3}$	$e^{\mu t} \Gamma(1+\beta t),  t  < \frac{1}{\beta}$
Notes: The cdf is $F(x \mu, \beta) = \frac{1}{1+e^{-\frac{x-\mu}{\beta}}}$ .				
Lognormal( $\mu, \sigma^2$ )	$\frac{1}{\sqrt{2\pi\sigma}} \frac{1}{x} e^{-\frac{(\log \frac{x-\mu}{\sigma})^2}{2\sigma^2}}; x > 0, \sigma > 0$	$e^{\mu+\frac{\sigma^2}{2}}$	$e^{2(\mu+\sigma^2)} - e^{2\mu+\sigma^2}$	$EX^n = e^{n\mu+\frac{n^2\sigma^2}{2}}$
Normal( $\mu, \sigma^2$ )	$\frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}; \sigma > 0$	$\mu$	$\sigma^2$	$e^{\mu t+\frac{\sigma^2 t^2}{2}}$
Pareto( $\alpha, \beta$ )	$\frac{\beta\alpha^{\beta}}{x^{\beta+1}}; x > \alpha, \alpha, \beta > 0$	$\frac{\beta\alpha}{\beta-1}, \beta > 1$	$\frac{\beta\alpha^2}{(\beta-1)^2(\beta-2)}, \beta > 2$	does not exist
$t_{\nu}$	$\frac{\Gamma(\frac{\nu+1}{2})}{\Gamma(\frac{\nu}{2})} \frac{1}{\sqrt{\nu\pi}} \frac{1}{(1+\frac{x^2}{\nu})^{\frac{\nu+1}{2}}}$	$0, \nu > 1$	$\frac{\nu}{\nu-2}, \nu > 2$	$EX^n = \frac{\Gamma(\frac{\nu+1}{2})\Gamma(\nu-\frac{n}{2})}{\sqrt{\pi}\Gamma(\frac{\nu}{2})} \nu^{\frac{n}{2}}, n \text{ even}$
Notes: $t_{\nu}^2 = F_{1, \nu}$ .				
Uniform( $a, b$ )	$\frac{1}{b-a}, a \leq x \leq b$	$\frac{b+a}{2}$	$\frac{(b-a)^2}{12}$	$\frac{e^{bt}-e^{at}}{(b-a)t}$
Notes: If $a = 0, b = 1$ , this is special case of beta ( $\alpha = \beta = 1$ ).				
Weibull( $\gamma, \beta$ )	$\frac{\gamma}{\beta} x^{\gamma-1} e^{-\frac{x^{\gamma}}{\beta}}; x > 0, \gamma, \beta > 0$	$\beta^{\frac{1}{\gamma}} \Gamma(1+\frac{1}{\gamma})$	$\beta^{\frac{2}{\gamma}} \left[ \Gamma(1+\frac{2}{\gamma}) - \Gamma^2(1+\frac{1}{\gamma}) \right]$	$EX^n = \beta^{\frac{n}{\gamma}} \Gamma(1+\frac{n}{\gamma})$
Notes: The mgf only exists for $\gamma \geq 1$ .				