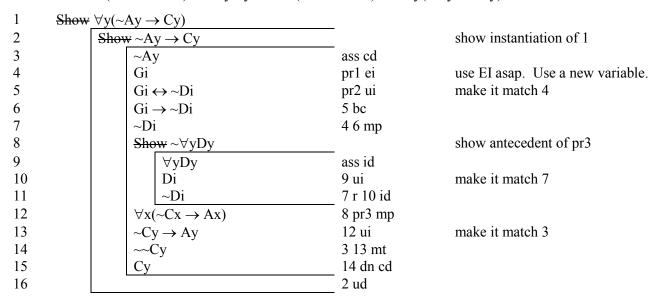
PREDICATE LOGIC DERIVATIONS FOR UNIT 6 ANSWERS for 1-18

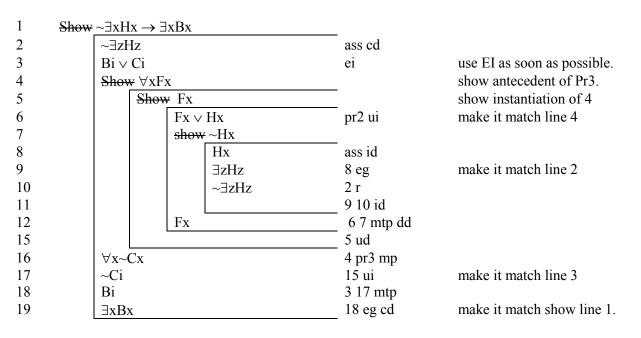
1. $\exists x(Gx \land \sim Hx)$. $\forall x(Hx \leftrightarrow \sim Bx)$. $\therefore \forall x(Gx \rightarrow \exists y(Gy \land By)$ 1 Show $\forall x(Gx \rightarrow \exists y(Gy \land By)$ Show $Gx \to \exists y (Gy \land By)$ 2 3 Gx ass cd Gi ∧ ~Hi (make sure you use a new variable!) 4 pr1 ei (make it match line 4!) 5 pr2 ui Hi ↔ ~Bi 5 bc 6 $\sim Bi \rightarrow Hi$ 7 4 sr ~Hi ~~Bi 8 6 7 mt 9 Bi 8 dn 10 Gi 4 sl 11 $Gi \wedge Bi$ 9 10 adj (make it match show line 1!) 12 11 eg $\exists y (Gy \land By)$ 13 12 cd 14 2 ud 2. $\exists x Ax \rightarrow \exists x Gx$. $\forall y (Jy \rightarrow Hy)$. $\sim \exists x (\sim Jx \lor Cx) \lor \forall x Fx : \forall x (Gx \rightarrow \sim Hx) \rightarrow \forall x (Ax \rightarrow Fx)$

1 Show $\forall x(Gx \rightarrow \sim Hx) \rightarrow \forall x(Ax \rightarrow Fx)$					
2		∀x(0	$Gx \rightarrow \sim Hx$)	ass cd	
3		show	$\forall x(Ax \rightarrow Fx)$		
4			$\overline{\text{show }} Ax \to Fx$		show instantiation of 3
5			Ax	ass cd	
6			∃xAx	5 eg	make it match pr1
7			ЗуGх	6 pr1 mp	
8			Gi	7 ei	use a new variable
9			Gi → ~Hi	2 ui	make it match 8
10			~Hi	8 9 mp	
11			Ji → Hi	pr2 ui	make it match 10
12			∼Ji	10 11 mt	
13			~Ji ∨ Ci	12 add	
14			$\exists x (\sim Jx \vee Cx)$	13 eg	make it match pr3
15			$\sim \exists x (\sim Jx \vee Cx)$	14 dn	
16			∀xFx	15 pr3 mtp	
17			Fx	16 ui cd	
18				4 ud	
19				3 cd	

3. $\exists xGx. \forall x(Gx \leftrightarrow \sim Dx). \sim \forall yDy \rightarrow \forall x(\sim Cx \rightarrow Ax) \therefore \forall y(\sim Ay \rightarrow Cy)$



4. $\exists x(Bx \lor Cx)$. $\forall x(Fx \lor Hx)$. $\forall xFx \to \forall x \sim Cx$. $\therefore \sim \exists zHz \to \exists xBx$

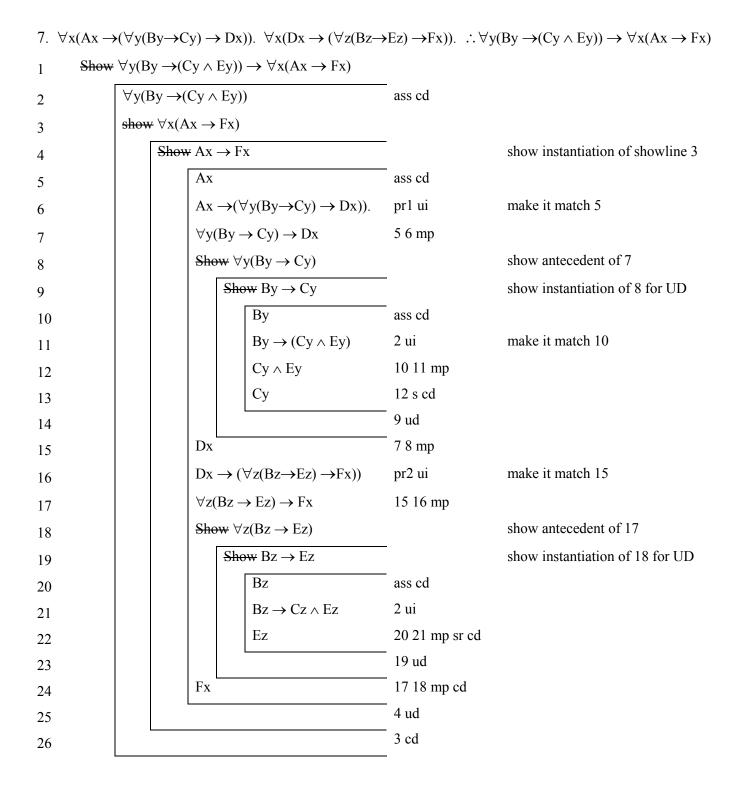


5. $\therefore (\forall x(Ax \rightarrow \sim Bx) \land \exists x(Bx \lor \sim Aa)) \rightarrow \exists x \sim (Ax \land Cx)$

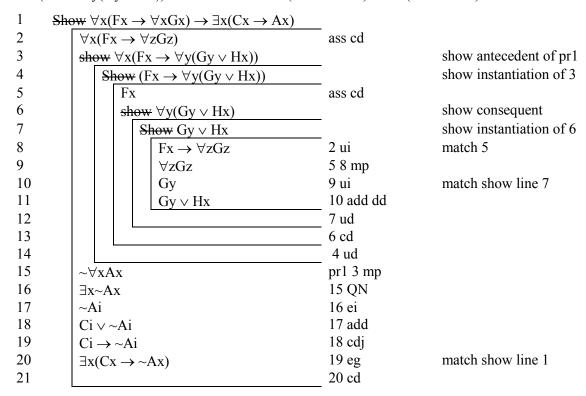
Show $(\forall x(Ax \rightarrow \sim Bx) \land \exists x(Bx \lor \sim Aa)) \rightarrow \exists x \sim (Ax \land Cx)$ 2 $\forall x(Ax \rightarrow \sim Bx) \land \exists x(Bx \lor \sim Aa)$ ass cd 3 2 sl $\forall x(Ax \rightarrow \sim Bx)$ 4 2 sr $\exists x(Bx \vee \sim Aa)$ 5 Show $\sim \exists x \sim (Ax \wedge Cx)$ show consequent 6 $\exists x \sim (Ax \wedge Cx)$ ass id Bi ∨ ~Aa 7 4 ei Do ei asap. New variable. show instantiation to 8 $\frac{\text{show}}{\text{-}(\text{Aa} \land \text{Ca})}$ match 7 (Aa) 9 Aa ∧ Ca ass id 9 sl 10 Aa Bi 10 dn 7 mtp 11 12 Ai → ~Bi 3 ui match 11 13 ~Ai 11 dn 12 mt Since i doesn't have property A (line 13), 14 ~Ai v ~Ci 13 add something (i) is not both 15 \sim (Ai \wedge Ci) 14 dm A and C! You could also use a subderivation to 16 15 eg $\exists x \sim (Ax \wedge Cx)$ derive line 16. 17 $\sim \exists x \sim (Ax \wedge Cx)$ 6 r 16 id 8 eg 6 id make it match 6 18 $\exists x \sim (Ax \wedge Cx)$ 5 cd 19

$6. \ \exists x (Ax \wedge Bx). \ \exists y (Gy \vee Hy). \ \exists x Ax \rightarrow \forall y (By \rightarrow \sim Hy) \ \therefore \ \sim \exists x Gx \rightarrow \exists x \exists y (\sim Hx \wedge Hy)$

1	$\frac{\text{Show}}{\sim} \exists x \in \mathbb{C}$	$Gx \rightarrow \exists x \exists y (\sim Hx \land Hy)$		
2	∼∃x€	Gx	ass cd	
3	Ai ^	. Bi	pr1 ei	Use EI asap! new variable
4	Gk \	/ Hk	pr2 ei	Use EI asap! new variable
5	Ai		3 sl	
6	∃xA	X	5 eg	make it match pr2
7	∀y(]	$By \rightarrow \sim Hy$)	6 7 mp	
8	Bk -	→ ~Hk	7 ui	make it match 4
9	Shor	₩~Gk		
10		Gk	ass id	
11		∃xGx	10 eg	make it match 2
12		∼∃xGx	2 r	
13			11 12 id	
14	Hk		9 4 mtp	
15		→ ~Hi	7 ui	make it match 3
16	Bi		3 s	
17	~Hi		15 16 mp	
18	Hk /	∖ ~Hi	14 17 adj	
19	∃y(~	$-Hx \wedge Hy$)	18 eg	make it match show line
20	∃x∃	$y(\sim Hx \wedge Hy)$	19 eg	make it match show line
21			20 cd	



8. $\forall x(Fx \to \forall y(Gy \lor Hx)) \to \neg \forall xAx$. $\therefore \forall x(Fx \to \forall zGz) \to \exists x(Cx \to \neg Ax)$



9. $\forall x Ba(b(x)) \rightarrow (\exists x Fx \lor Ga(e))$. $\forall x (Ba(x) \land Ca(x))$. $\therefore \forall x (\sim Gx \rightarrow \sim \exists y Fy) \rightarrow \exists z Ga(z)$

```
1
         Show \forall x(Gx \rightarrow \sim \exists yFy) \rightarrow \exists zGa(z)
2
                   \forall x (\sim Gx \rightarrow \sim \exists y Fy)
                                                                          ass cd
3
                                                                                                    Show antecedent of pr1
                   Show \forall x Ba(b(x))
                           \frac{\text{show}}{\text{Ba}(b(x))}
                                                                                                    show instantiation of 3
4
5
                                 Ba(b(x)) \wedge Ca(b(x))
                                                                          pr2 ui
                                                                                                    match show line 4: b(x)/x
                                                                          5 sl dd
6
                                 Ba(b(x))
7
                                                                          4 ud
                   \exists x Fx \vee Ga(e)).
                                                                          3 pr1 cd
8
9
                   Show Ga(e)
10
                                                                          ass id
                           \simGa(e)
                                                                          8 10 mtp
11
                           \exists x Fx
12
                           Fi
                                                                          11 ei
                                                                                                    new variable
13
                           \simGa(e) \rightarrow \sim \exists y F y
                                                                          2 ui
                                                                                                    match 10: a(e)/x
                                                                          10 11 mp
14
                           ~∃yFy
                                                                          14 gn
15
                           \forall x \sim Fx
                                                                          15 ui 12 id
16
                           ~Fi
                                                                          9 eg cd
16
                   \exists zGa(z)
```

10. $\exists x \sim (Fx \rightarrow \sim Gx) \rightarrow \exists x \sim Hx :: \forall x \exists y \sim (Fy \rightarrow \sim Gx) \rightarrow \sim \forall y Hy$

l Sh	$\Theta W \forall x \exists y \sim (Fy \rightarrow \sim Gx) \rightarrow \sim \forall y Hy$		
2	$\forall x \exists y \sim (Fy \rightarrow \sim Gx)$	ass cd	Goal: $\exists x \sim (Fx \rightarrow \sim Gx)$ antecedent of pr1
3	$\exists y \sim (Fy \to \sim Gx)$	2 ui	there is nothing to match! so use any variable.
4	\sim (Fi $\rightarrow \sim$ Gx)	3 ei	use new variable
5	Fi ∧ ~~Gx	4 nc	problem mismatch (i & x)
6	$\exists y \sim (Fy \rightarrow \sim Gi)$	2 ui	solution: UI again to match.
7	\sim (Fk $\rightarrow \sim$ Gi)	6 ei	use a new variable
8	Fk ∧ ~~Gi	7 nc	now Fi & ∼~Gi match!
9	Fi	5 sl	
10	~~Gi	8 sr	
11	Fi ∧ ~~Gi	9 10 adj	
12	\sim (Fi $\rightarrow \sim$ Gi)	11 nc	
13	$\exists x \sim (Fx \rightarrow \sim Gx)$	12 eg	
14	∃x∼Hx	13 pr1 mp	
15	~Hm	14 ei	new variable
16	∃у∼Ну	15 eg	match show line 1
17	∼∀yHy	16 QN cd	

Try some of these using ONLY the basic rules (S, ADJ, ADD, MTP, MP, MT, BC, CB, DN, EG, EI, UI).

11. $\exists x Bx$. $\forall x (\sim Bx \vee Cx)$. $\forall y ((Ay \vee \sim Dy) \rightarrow \sim Cy)$. $\therefore \exists x Dx \wedge \exists y \sim Ay$

1 St	$\exists xDx \land \exists y \sim Ay$					
2	Ba	pr1 ei a/x	Deal with EI first since you need arbitrary term.			
3	~Ba ∨ Ca	pr2 ui a/x	instantiate pr2 using a for x to match 2			
4	$(Aa \lor \sim Da) \rightarrow \sim Ca$	pr3 ui a/y	instantiate pr3 using a for y to match 2 and 3			
5	~~Ba	2 dn				
6	Ca	5 3 mtp				
7	~~Ca	6 dn				
8	~(Aa ∨ ~Da)	7 4 mt	If only you had dm, it would be easy to get ~Aa &			
9	Show ~Aa		~Da. You need them & can get them show them			
10	Aa	ass id				
11	Aa∨~Da	10 add				
12	~(Aa∨~Da)	8 r, 11 id.				
13	Show Da	_				
14	~Da	ass id				
15	Aa∨~Da	14 add				
16	~(Aa∨~Da)	8 r, 15 id				
17	$\exists x Dx$	13 eg	Generalize 13 to match conc.			
18	∃у~Ау	9 eg	Generalize 9 to match conclusion.			
19	$\exists x Dx \land \exists y \sim Ay$	17 18 adj, dd				

12.
$$\therefore$$
 (~Ba \vee Ga) \rightarrow (\forall x~(Cx \rightarrow Gx) \rightarrow \exists x(Cx \wedge ~Bx))

```
1
        Show (\simBa \vee Ga) \rightarrow (\forallx\sim(Cx \rightarrow Gx) \rightarrow \existsx(Cx \wedge \simBx))
2
                                                                                              ass cd
               ~Ba ∨ Ga
3
               Show (\forall x \sim (Cx \rightarrow Gx) \rightarrow \exists x (Cx \land \sim Bx))
4
                         \forall x \sim (Cx \rightarrow Gx)
                                                                                             ass cd
5
                                                                                             4 ui
                                                                                                                    match line 2
                         \sim(Ca \rightarrow Ga)
6
                         Show ~Ga
7
                               Ga
                                                                                             ass id
8
                               \frac{\text{Show}}{\text{Ca}} Ca → Ga
9
                                      Ca
                                                                                             ass cd
                                                                                             7 r cd
10
                                      Ga
                                                                                              5 r 8 id
11
                               \sim(Ca \rightarrow Ga)
                                                                                             6 2 mtp
12
                         ~Ba
                         Show Ca
13
                                                                                              ass id
14
                               \simCa
15
                               \frac{\text{Show}}{\text{Ca}} Ca → Ga
                                      Ca
                                                                                             ass cd
16
                                      ~Ca
                                                                                              14 r 16 id
17
                                                                                             5 r 15 id
18
                               \sim(Ca \rightarrow Ga)
                                                                                             12 13 adj
19
                         Ca ∧ ~Ba
20
                                                                                              19 eg
                                                                                                                    match show line 3
                         \exists x(Cx \land \sim Bx)
21
                                                                                              3 cd
```

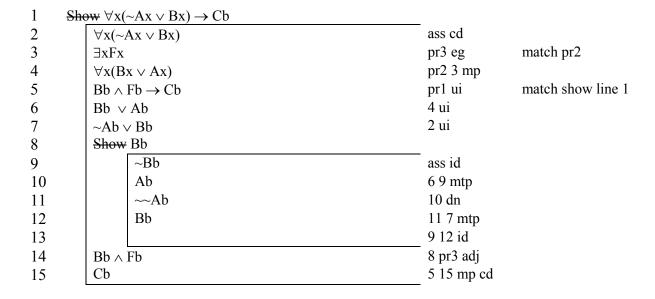
$13. \quad \forall y (By \to {\sim} (Dy \to Ey)). \quad \forall x (Dx \to {\sim} (Fx \land {\sim} Cx)). \quad \forall x (Ex \lor Fx). \quad \therefore \forall x ({\sim} Bx \lor Cx)$

1	Show $\forall x (\sim Bx \vee$	Cx)		
2	Show ∼Bx ∨	Cx		show line 1 is \forall , so show instantiation for UD
3	~(~Bx ∨	Cx)	ass ID	Looks hard to derive this directly, so use ID!
4	Show Bx	(You need Bx and can get it (very easily if you had
5	~Bx		ass id	DM) so show it!
6	~Bx	∨ Cx	5 add	
7	~(~B	$x \vee Cx$	3 r, 6 id	
8	$Bx \rightarrow \sim ($	$\overline{Dx \to Ex}$	pr1 ui x/y	instantiate pr1 to match 4
9	$\sim (Dx \rightarrow$	Ex)	4 8 mp	With NC you could get Dx and ~Ex. You need Dx and
10	Show Dx	ζ.		know you can get it so show it!
11	~Dx		ass id	
12	Show	$+Dx \rightarrow Ex$		Show this to get a contradiction with 9!
13		O _X	ass cd	Show line 12 is \rightarrow so ass antecedent.
14	~	Dx	11 r, id	(If you prefer, show Ex with ID then reiterate 13 & 11)
15	$\sim (\overline{Dx})$	$\rightarrow Ex$)	9 r, 13 id	
16	$Dx \rightarrow \sim (1$	$Fx \wedge \sim Cx$	pr2 ui x/x	instantiate pr2 using x for x to match 10
17	~(Fx ∧ ~	Cx)	10 16 mp	
18	$Ex \vee Fx$]	pr3 ui x/x	instantiate pr3 using x for x to match
19	Show ~E	Ex		You can get this from 9 (easy with NC!)
20	Ex		ass id	
21	Show	$+$ Dx \rightarrow Ex		
22		Dx :	ass cd	
23		E x	20 r, cd	
24	~(Dx	\rightarrow Ex)	9 r, 21 id	
25	Fx		18 19 mtp	
26	Show ~C	Cx		You need this to contradict 17 (with 25) and can get it
27	Cx			with line 3 (DM), so show it!
28	~Bx		27 add	
29	~(~B		3 r,28 id	
30	$Fx \wedge \sim Cx$	<u> </u>	25 26 adj, 17 id	d
31			2 ud	You've shown it for any arbitrary x, so it's true for all!

14. $\forall x (\sim Bx \rightarrow Cx)$. $Ba \leftrightarrow \forall y \sim (By \land Cy) :: \exists x (\sim Cx \leftrightarrow Bx)$

```
1
        \frac{\text{Show}}{\text{Show}} \exists x (\sim Cx \leftrightarrow Bx)
2
                                                                                              show one conditional for the instantiation of
              Show \simCa → Ba
                                                                                              show line (\simCa \leftrightarrow Ba) Match pr2 (Ba \leftrightarrow...)
3
                        ~Ca
                                                                          ass cd
4
                        \simBa \rightarrow Ca
                                                                          pr1 ui
                                                                                              match 3
                                                                          3 4 mt
5
                        ~~Ba
6
                                                                          5 dn cd
                        Ba
7
              Show Ba → ~Ca
8
                        Ba
                                                                          ass cd
9
                                                                          pr2 bc
                        Ba \rightarrow \forall y \sim (By \wedge Cy)
10
                                                                          8 9 mp
                        \forall y \sim (By \wedge Cy)
                                                                          10 ui
                                                                                              match 8
11
                        \sim(Ba \wedge Ca)
12
                        Show ~Ca
13
                              Ca
                                                                          ass id
                                                                          8 13 adj
14
                              Ba \wedge Ca
15
                              \sim(Ba \wedge Ca)
                                                                          11 r 14 id
                                                                          12 cd
16
                                                                          2 7 cb
17
              \simCa \leftrightarrow Ba
                                                                                              match show line 1
18
                                                                          17 eg
              \exists x (\sim Cx \leftrightarrow Bx)
```

15. $\forall y(By \land Fy \rightarrow Cy)$. $\exists xFx \rightarrow \forall x(Bx \lor Ax)$. Fb. $\therefore \forall x(\sim Ax \lor Bx) \rightarrow Cb$



16. $\forall x (\sim Cx \vee (Aa \leftrightarrow \sim Fx))$. $\forall x (\sim Fx \rightarrow (\sim Cx \rightarrow Ax)) :: \exists x (Ax \vee Fx)$

```
1
        Show \exists x (Ax \lor Fx)
                                                                                        pr1 ui
2
                                                                                                             match a in pr1
              \simCa \vee (Aa \leftrightarrow \simFa)
3
              \simFa \rightarrow(\simCa \rightarrow Aa)
                                                                                        pr2 ui
                                                                                                             match 2
4
              \textcolor{red}{\textbf{Show}}~\textbf{Aa} \lor \textbf{Fa}
5
                                                                                        ass id
                       \sim(Aa \vee Fa)
6
                       Show ~Fa
7
                             Fa
                                                                                        ass id
                                                                                        7 add
8
                             Aa \vee Fa
                                                                                        5 r 8 id
9
                             \sim(Aa \vee Fa)
                                                                                        3 6 mp
10
                       \simCa \rightarrow Aa
                       Show ~Aa
11
12
                             Aa
                                                                                        ass id
                                                                                        12 add
13
                             Aa v Fa
14
                             \sim(Aa \vee Fa)
                                                                                        5 r 13 id
                                                                                        11 10 mt
15
                       ~~Ca
                                                                                        2 15 mtp
16
                       Aa ↔ ~Fa
                                                                                        16 bc
17
                       \simFa \rightarrow Aa
                                                                                        6 17 mp
18
                       Aa
                                                                                        11 18 id
19
                                                                                        4 eg dd
20
             \exists x (Ax \vee \overline{Fx})
                                                                                                             match show line
```

17. $A(ab) \lor B(ba)$. $\forall x \forall y (B(xy) \to C(yx))$. $\forall w \forall z (C(wz) \leftrightarrow A(wz))$. $\forall x \sim (G(xx) \land C(xb))$. $\therefore \sim \forall x \forall y (A(xy) \to G(xx))$

1	$\stackrel{Show}{\sim} \forall x \forall y (A(xy) \to G(xx))$				
2	$\forall x \forall y (A(xy) \rightarrow G(xx))$	ass id			
3	$\forall y(A(ay) \rightarrow G(aa))$	2 ui	match pr1		
4	$A(ab) \rightarrow G(aa)$	3 ui	match pr1		
5	$\forall y(B(by) \rightarrow C(yb))$	pr2 ui	match pr1		
6	$B(ba) \rightarrow C(ab)$	5 ui	match pr1		
7	\sim (G(aa) \wedge C(ab))	pr4 ui	match lines 4 and 6		
8	$\forall z(C(az) \leftrightarrow A(az))$	pr3 ui	match lines 4 and 6		
9	$C(ab) \leftrightarrow A(ab)$	8 ui	match lines 4 and 6		
10	Show ~A(ab)		Begin by showing something simple.		
11	A(ab)	ass id	Show negation of one of the disjuncts of pr1.		
12	G(aa)	11 4 mp	Alternatively, show A(ab) or B(ba) – the		
13	$A(ab) \rightarrow C(ab)$	9 bc	antecedents of 4 and 6.		
14	C(ab)	11 13 mp			
15	$G(aa) \wedge C(ab)$	12 14 adj			
16	\sim (G(aa) \wedge C(ab))	7 r 15 id			
17	$C(ab) \rightarrow A(ab)$	9 bc			
18	~C(ab)	10 17 mt			
19	~B(ba)	6 18 mt			
20	A(ab)	19 pr1 mtp			
		10 id			

18. $\exists x \forall y \sim (B(xy) \vee F(yx))$. $\forall x \forall y (Gx \rightarrow B(xy))$. $\exists x \forall y (F(xy) \vee H(yy))$. $\therefore \exists x (\sim Gx \wedge H(xx))$

```
1
       Show ∃x(Ax ∨ Fx)
             \simCa \vee (Aa \leftrightarrow \simFa)
                                                                                     pr1 ui
                                                                                                         match a in pr1
2
3
             \simFa \rightarrow(\simCa \rightarrow Aa)
                                                                                     pr2 ui
                                                                                                         match 2
4
             \textcolor{red}{\textbf{Show}}~\textbf{Aa} \lor \textbf{Fa}
5
                      \sim(Aa \vee Fa)
                                                                                     ass id
6
                      Show ~Fa
7
                            Fa
                                                                                    ass id
                                                                                     7 add
8
                            Aa \vee Fa
                                                                                     5 r 8 id
9
                            \sim(Aa \vee Fa)
                                                                                     3 6 mp
10
                      \simCa \rightarrow Aa
                      Show ~Aa
11
12
                            Aa
                                                                                     ass id
                                                                                     12 add
13
                            Aa \vee Fa
14
                            \sim(Aa \vee Fa)
                                                                                     5 r 13 id
                                                                                     11 10 mt
15
                      ~~Ca
                      Aa ↔ ~Fa
                                                                                     2 15 mtp
16
                                                                                     16 bc
17
                      \simFa \rightarrow Aa
                                                                                    6 17 mp
18
                      Aa
                                                                                     11 18 id
19
             \exists x (Ax \vee Fx)
                                                                                    4 eg dd
                                                                                                         match show line
20
```