Derivation Questions for Unit 6 using only UI, EG and EI

6.001 $\forall x(Fx \rightarrow Gx)$. $\forall y(Hy \rightarrow \sim Gy)$. Fc. $\therefore \exists x \sim Hx$

1	~~Show~~ $\exists x \sim Hx$	
2	$Fc \rightarrow Gc$	pr1 ui
3	$Hc \rightarrow \sim Gc$	pr2 ui
4	Gc	pr3 2 mp
5	$\sim\sim Gc$	4 dn
6	$\sim Hc$	3 5 mt
7	$\exists x \sim Hx$	6 eg

6.002 $\forall x(Bx \rightarrow Cx)$. $\forall y(Ay \vee By)$. $\therefore \sim Cb \rightarrow \exists xAx$

1	~~Show~~ $\sim Cb \rightarrow \exists xAx$	
2	$\sim Cb$	ass cd
3	$Bb \rightarrow Cb$	pr1 ui
4	$Ab \vee Bb$	pr2 ui
5	$\sim Bb$	2 3 mt
6	Ab	4 5 mtp
7	$\exists xAx$	6 eg

6.003 $\forall x(Dx \leftrightarrow \sim Cx)$. $\forall x(Cx \vee \sim Dx)$. $\therefore \sim \exists yDy$

1	~~Show~~ $\sim \exists yDy$	
2	$\exists yDy$	ass id
3	Di	2 ei
4	$Di \leftrightarrow \sim Ci$	pr1 ui
5	$Di \rightarrow \sim Ci$	4 bc
6	$\sim Ci$	3 5 mp
7	$Ci \vee \sim Di$	pr2 ui
8	$\sim Di$	6 7 mtp
9		3 8 id

6.004 $\therefore \forall x(Fx \wedge Gx \wedge Hx) \rightarrow \exists xFx \wedge \exists y(Gy \wedge Hy) \wedge \exists z(Hz \vee Bz)$

1	~~Show~~ $\forall x(Fx \wedge Gx \wedge Hx) \rightarrow \exists xFx \wedge \exists y(Gy \wedge Hy) \wedge \exists z(Hz \vee Bz)$		
2	$\forall x(Fx \wedge Gx \wedge Hx)$	ass cd	
3	$Fx \wedge Gx \wedge Hx$	2 UI	You can instantiate to any term here.
4	Fx	3 SL SL	
5	$\exists xFx$	4 EG	
6	Gx	3 SL SR	
7	Hx	3 SR	
8	$Gx \wedge Hx$	6 7 ADJ	
9	$\exists y(Gy \wedge Hy)$	8 EG	
10	$Hx \vee Bx$	7 ADD	
11	$\exists z(Hz \vee Bz)$	10 EG	
12	$\exists xFx \wedge \exists y(Gy \wedge Hy) \wedge \exists z(Hz \vee Bz)$	5 9 ADJ 12 ADJ	
13		12 CD	

6.005 $\forall x(\sim Bx \to Dx)$. $\forall x(Cx \wedge Fx \to \sim Dx)$. Fa. $\forall y(Cy \vee \sim(Fy \wedge By))$. $\therefore \exists x(Cx \leftrightarrow Bx)$

1	~~Show~~ $\exists x(Cx \leftrightarrow Bx)$		You need to show that C...↔B… for some individual. You have Fa (premise 3). So show Ca ↔ Ba!
2	~~Show~~ $Ca \to Ba$		
3	Ca	ass cd	
4	$Ca \wedge Fa \to \sim Da$	pr2 ui	
5	$Ca \wedge Fa$	3 pr3 adj	
6	$\sim Da$	4 5 mp	
7	$\sim Ba \to Da$	pr1 ui	
8	$\sim\sim Ba$	6 7 mt	
9	Ba	8 dn cd	
10	Show $Ba \to Ca$		
11	Ba	ass cd	
12	$Ca \vee \sim(Fa \wedge Ba))$	pr4 ui	
13	$Fa \wedge Ba$	pr3 11 adj	
14	$\sim\sim(Fa \wedge Ba)$	13 dn	
15	Ca	12 14 mtp, cd	
16	$Ba \leftrightarrow Ca$		
17	$\exists x(Cx \leftrightarrow Bx)$		

6.006 $\forall x(Fx \vee Gx) \wedge \forall y(Gy \to Hy)$. $\sim Ha \vee \sim Hb$. $\therefore \exists yFy$

1	~~Show~~ $\exists yFy$		Although it may be very easy to see that this is valid, it may be hard to see how to show it directly. So do an indirect derivation.
2	$\sim\exists yFy$	ass id	
3	$\forall x(Fx \vee Gx)$	pr1 s	
4	$\forall y(Gy \to Hy)$	pr1 s	
5	~~Show~~ $\sim\sim Ha$		If you show this, you can use MPT on PR 2.
6	$\sim Ha$	ass id	
7	$Ga \to Ha$	4 ui	
8	$\sim Ga$	6 7 mt	
9	$Fa \vee Ga$	3 ui	
10	Fa	8 9 mtp	
11	$\exists yFy$	10 eg	
12	$\sim\exists yFy$	2 r, id	
13	$\sim Hb$	5 pr2 mtp	Now repeat 7-11 using b instead of a.
14	$Gb \to Hb$	4 ui	
15	$\sim Gb$	13 14 mt	
16	$Fb \vee Gb$	3 ui	
17	Fb	15 16 mtp	
18	$\exists yFy$	17 eg, 2 id	

Alternate strategy: Show $\sim Ha \to \exists yFy$ and $\sim Hb \to \exists yFy$ and use SC

6.007 $\exists x(\sim Fx \vee Gx) \to \forall y(Ay \to Hy)$. $\therefore \sim(Fb \vee \sim Ab) \to \exists wHw$

1	~~Show~~ $\sim(Fb \vee \sim Ab) \to \exists wHw$		
2	$\sim(Fb \vee \sim Ab)$	ass cd	The show line is a conditional, so assume the antecedent. Now your goal is the consequent of the show line ∃wHw. So you need to show that something is H. To get that you need to show the antecedent of PR1.
3	$\sim Fb \wedge \sim\sim Ab$	2 dm	
4	$\sim Fb$	3 s	
5	$\sim Fb \vee Gb$	4 add	
6	$\exists x(\sim Fx \vee Gx)$	5 eg	

7	$\forall y(Ay \rightarrow Hy)$	6 pr1 mp	
8	$Ab \rightarrow Hb$	7 ui	
9	Ab	3 s, dn	
10	Hb	9 8 mp	
11	$\exists wHw$	10 eg cd	

6.008 $\forall x(Ax \leftrightarrow Bx \vee Cx)$. $\forall x({\sim}Cx \rightarrow (Fx \vee Gx))$. ${\sim}Ga(b)$. $\therefore \exists x({\sim}Fx \rightarrow Ax)$

1	~~Show~~ $\exists x({\sim}Fx \rightarrow Ax)$		
2	~~Show~~ ${\sim}Fa(b) \rightarrow Aa(b)$		This is easier than it looks. a(b) is just a singular term. So instantiate the first to premises to that singular term, and show that if a(b) is not F, then it is A.
3	${\sim}Fa(b)$	ass cd	
4	$Aa(b) \leftrightarrow Ba(b) \vee Ca(b)$	pr1 ui	
5	${\sim}Ca(b) \rightarrow (Fa(b) \vee Ga(b))$	pr2 ui	
6	${\sim}Fa(b) \wedge {\sim}Ga(b)$	pr3 3 adj	
7	${\sim}(Fa(b) \vee Ga(b))$	6 dm	
8	${\sim}{\sim}Ca(b)$	5 7 mt	
9	$Ca(b)$	8 dn	
10	$Ba(b) \vee Ca(b)$	9 add	
11	$Ba(b) \vee Ca(b) \rightarrow Aa(b)$	4 bc	
12	$Aa(b)$	10 11 mp	
13	$\exists x({\sim}Fx \rightarrow Ax)$	13 dd	

6.009 $\forall xGb(cx) \therefore \exists yGb(yy) \wedge \exists zGz$

1	~~Show~~ $\exists yGa(yy) \wedge \exists zGz$		
2	$Gb(cc)$	pr1 ui	UI to c so that the operation b is acting reflexively on something.
3	$\exists yGb(yy)$	3 eg	Replace each instance of c with y
4	$\exists zGz$	3 eg	This time replace b(cc) with z
5	$\exists yGa(yy) \wedge \exists zGz$	3 4 adj, dd	

6.0010 $\forall xG(xx)$. $\forall x(\exists yG(xy) \rightarrow {\sim}Cx \wedge Ax)$. $\forall x(Cx \leftrightarrow Bx)$. $\therefore {\sim}\forall x(Ax \rightarrow Bx)$

1	~~Show~~ ${\sim}\forall x(Ax \rightarrow Bx)$		
2	$\forall x(Ax \rightarrow Bx)$	ass id	You can't show it directly so do an ID
3	$G(ii)$	pr1 ui	It doesn't matter what order you do these, or what term you instantiate to, but they should all match each other!
4	$\exists yG(iy) \rightarrow {\sim}Ci \wedge Ai$	pr2 ui	
5	$Ci \leftrightarrow Bi$	pr3 ui	
6	$Ai \rightarrow Bi$	2 ui	
7	$\exists yG(iy)$	3 eg	Now generalize to get the antecedent of 4.
8	${\sim}Ci \wedge Ai$	4 7 mp	
9	${\sim}Ci$	8 s	
10	$Bi \rightarrow Ci$	5 bc	
11	${\sim}Bi$	9 10 mt	
12	Ai	8 s	
13	Bi	6 12 mp, 11id	

6.0011 $\sim\exists x\exists yB(xy)$. $\forall x\forall y(C(yx) \rightarrow B(yx))$. $\therefore C(aa) \rightarrow \exists x{\sim}C(xx)$

1	~~Show~~ $C(aa) \rightarrow \exists x{\sim}C(xx)$		
2	$C(aa)$	ass cd	After the assumption for cd is made, a
3	~~Show~~ $\exists x{\sim}C(xx)$		contradiction will follow from the premises. Thus,
4	$\sim\exists x{\sim}C(xx)$		it doesn't matter whether or you have a show line
5	$\forall yC(ya) \rightarrow B(ya))$	pr2 ui	for the consequent and/or an assumption for id.
6	$C(aa) \rightarrow B(aa)$	5 ui	
7	$B(aa)$	2 6 mp	
8	$\exists yB(ay)$	7 eg	Make sure you generalize to y (the inner variable)
9	$\exists x\exists yB(xy)$	8 eg	before you generalize to x – since the quantifier
10	$\sim\exists x\exists yB(xy)$	pr1, 9 id	always goes at the beginning when using EG.
14			

6.0012 $\forall x(Fx \rightarrow \forall y(Gy \rightarrow {\sim}L(xy))$. $\exists x(Fx \vee {\sim}Fx) \rightarrow \forall z(Fz \leftrightarrow Gz)$. Ga.
$\therefore \sim\forall x\forall y(Fx \wedge Fy \rightarrow L(xy))$

1	~~Show~~ $\sim\forall x\forall y(Fx \wedge Fy \rightarrow L(xy))$		
2	$\forall x\forall y(Fx \wedge Fy \rightarrow L(xy))$	ass id	
3	~~Show~~ $Fk \vee {\sim}Fk$		You need to show that this is true for some
4	$\sim(Fk \vee {\sim}Fk)$	ass id	individual in order to get the antecedent of
5	$\sim Fk \wedge {\sim\sim}Fk$	4 dm	pr2. There are many ways to do this. The
6	$\sim Fk$	5 s	easiest is to just use RT59.
7	$Fk \vee {\sim}Fk$	6 add, 4 id	
8	$\exists x(Fx \vee {\sim}Fx)$	3 eg	
9	$\forall z(Fz \leftrightarrow Gz)$	pr2 8 mp	
10	$Fa \leftrightarrow Ga$	9 ui	
11	$Fa \rightarrow \forall y(Ga \rightarrow {\sim}L(ay))$	pr1 ui	
12	$Ga \rightarrow Fa$	10 bc	
13	Fa	pr3 12 mp	
14	$\forall y(Fa \wedge Fy \rightarrow L(ay))$	2 ui	instantiate to 'a' to match 13.
15	$Fa \wedge Fa \rightarrow L(aa)$	14 ui	instantiate to 'a' again since nothing else is F
16	$\forall y(Ga \rightarrow {\sim}L(ay))$	11 13 mp	
17	$Ga \rightarrow {\sim}L(aa)$	14 ui	instantiate to 'a' to match
18	$Fa \wedge Fa$	13 13 adj	
19	$L(aa)$		
20	$\sim L(aa)$		

6.0013 ∀x(A(bx) → B(ax)).   ∃x∃yA(xy) → ∀w∀z(B(wz) ∨ B(bb) → C(zw)).   ∴ A(bb) → ∃x∃yC(xy)

1	~~Show~~ A(bb) → ∃x∃yC(xy)		
2	A(bb)	ass cd	
3	A(bb) → B(ab)	pr1 ui	Instantiate to: b to match 2.
4	B(ab)	2 3 mp	
5	∃yA(by)	2 eg	Generalize to get the antecedent of pr2.
6	∃x∃yA(xy)	5 eg	Inner quantifier first!
7	∀w∀z(B(wz) ∨ B(bb) → C(zw))	6 pr2 mp	
8	∀z(B(az) ∨ B(bb) → C(za))	7 ui	Instantiate to w to a and z to b in order
9	B(ab) ∨ B(bb) → C(ba)	8 ui	to match line 4.
10	B(ab) ∨ B(bb)	4 add	
11	C(ba)	9 10 mp	
12	∃yC(by)	11 eg	Generalize to get the consequent of
13	∃x∃yC(xy)	12 eg	show line. Inner quantifier first!
14		13 cd	

For these you will need UI, EG and EI.

6.0014 ∀x(Fx →~Gx). ∀y(Hy ∨ Gy)   ∃xFx.   ∴ ∃xHx

1	~~Show~~ ∃xHx		
2	Fi	pr3 ei	instantiate pr3 first – to any arbitrary term
3	Fi → ~Gi	pr1 ui	now ui to match (after ei)
4	Hi ∨ Gi	pr2 ui	
5	~Gi	2 3 mp	
6	Hi	4 5 mtp	
7	∃xHx	6 eg, dd	

6.0015 ∃x(Ax ∧ ~Bx). ∀z(Cz ∨ Bz). ∀x(Ax ↔ Mx).   ∀x(Cx ∨ Fx → Gx). ∴ ∃y(Gy ∧ My)

1	~~Show~~ ∃x(Gy ∧ My)		
2	Ai ∧ ~Bi	pr1 ei	instantiate pr1 first – to any arbitrary term
3	Ci ∨ Bi	pr2 ui	now ui to match (after ei)
4	Ai ↔ Mi	pr3 ui	
5	Ci ∨ Fi → Gi	pr4 ui	
6	Ai	2 sl	
7	~Bi	2 sr	
8	Ai → Mi	4 bc	
9	Mi	6 8 mp	
10	Ci	3 7 mtp	
11	Ci ∨ Fi	10 add	
12	Gi	11 5 mp	
13	Gi ∧ Mi	9 12 adj	
14	∃y(Gy ∧ My)	13 eg, dd	

6.0016 $\forall x(Ax \to Bx)$. $\exists x(Cx \wedge \sim Dx)$. $\forall x \sim(Bx \leftrightarrow Cx)$. $\therefore \forall y(Dy \vee Fy) \to \exists x(Fx \wedge \sim Ax)$

1	~~Show~~ $\forall y(Dy \vee Fy) \to \exists x(Fx \wedge \sim Ax)$		
2	$\forall y(Dy \vee Fy)$		
3	$Ci \wedge \sim Di$	pr2 ei	instantiate pr2 first – to any arbitrary term.
4	$Di \vee Fi$	2 ui	now ui to match (after ei)
5	$\sim Di$	3 s	
6	Fi	4 5 mtp	
7	$\sim(Bi \leftrightarrow Ci)$	pr3 ui	
8	$Bi \leftrightarrow \sim Ci$	7 nb	
9	$Bi \to \sim Ci$	8 bc	
10	$\sim\sim Ci$	3 s dn	
11	$\sim Bi$	9 10 mt	
12	$Ai \to Bi$	pr1 ui	
13	$\sim Ai$	11 12 mt	
14	$Fi \wedge \sim Ai$	5 13 adj	
15	$\exists x(Fx \wedge \sim Ax)$	14 eg, cd	

6.0017 $\forall x(Fx \leftrightarrow Bx)$. $\forall x \sim(Cx \to Dx)$. $\forall y(By \wedge \sim Dy \to \sim Cy)$. $\sim\exists yGy$. $\therefore \sim\exists x(Fx \vee Gx)$

1	~~Show~~ $\sim\exists x(Fx \vee Gx)$		With just EG and UI there is no way to do this directly, so do an ID!
2	$\exists x(Fx \vee Gx)$	ass id	
3	$Fi \vee Gi$	2 ei	Use ei first, then ui on all the premises to match the individual term.
4	$Fi \leftrightarrow Bi$	pr1 ui	
5	$\sim(Ci \to Di)$	pr2 ui	
6	$Bi \wedge \sim Di \to \sim Ci$	pr3 ui	
7	$Ci \wedge \sim Di$	5 nc	A negated conditional gives you something to work with.
8	Ci	7 s	
9	$\sim\sim Ci$	8 dn	
10	$\sim(Bi \wedge \sim Di)$	6 9 mt	
11	$\sim Bi \vee \sim\sim Di$	10 dm	
12	$\sim Di$	7 s	
13	$\sim\sim\sim Di$	12 dn	
14	$\sim Bi$	11 13 mtp	
15	$Fi \to Bi$	4 bc	
16	$\sim Fi$	14 15 mt	
17	Gi	16 3 mtp	
18	$\exists yGy$	17 eg	
19	$\sim\exists yGy$	pr4 r 18 id	

6.0018 $\therefore\ \exists xL(xa) \wedge \forall x\forall y(L(xy)\leftrightarrow L(yx)) \rightarrow \exists yL(ay)$

1	~~Show~~ $\exists xL(xa) \wedge \forall x\forall y(L(xy)\leftrightarrow L(yx)) \rightarrow \exists yL(ay)$		
2	$\exists xL(xa) \wedge \forall x\forall y(L(xy)\leftrightarrow L(yx))$	ass cd	
3	$\exists xL(xa)$	2 sl	
4	$\forall x\forall y(L(xy)\leftrightarrow L(yx))$	2 sr	
5	$L(ia)$	3 ei	instantiate to a NEW term
6	$\forall y(L(iy)\leftrightarrow L(yi))$	4 ui	use ui to match i or a.
7	$L(ia)\leftrightarrow L(ai)$	6 ui	use ui to match the other!
8	$L(ia)\rightarrow L(ai)$	7 bc	
9	$L(ai)$	5 8 mp	
10	$\exists yL(ay)$	9 eg cd	

6.0019 $\exists x(Fx \wedge Gx)$. $\exists y(Fy \wedge \sim Gy)$. $\therefore\ \forall x(Hx \leftrightarrow Gx) \rightarrow (\exists yHy \wedge \exists y\sim Hy)$

1	~~Show~~ $\forall x(Hx \leftrightarrow Gx) \rightarrow (\exists yHy \wedge \exists y\sim Hy)$		
2	$\forall x(Hx \leftrightarrow Gx)$		
3	$Fi \wedge Gi$		Instantiate pr1 to arbitrary term: i
4	$Fk \wedge \sim Gk$		Instantiate pr2 to arbitrary term: k
5	Gi		
6	$\sim Gk$		
7	$Hi \leftrightarrow Gi$	2 ui	Instantiate 2 to match i on line 5
8	$Hk \leftrightarrow Gk$	2 ui	Instantiate again to match k on line 6
9	Hi	7 bc 5 mp	
10	$\sim Hk$	8 bc 6 mt	
11	$\exists yHy$	9 eg	
12	$\exists y\sim Hy$	10 eg	
13	$\exists yHy \wedge \exists y\sim Hy$	11 12 adj	
14		13 cd	

6.0020 $\forall x(Fx \vee Hx \rightarrow \forall yL(xy))$. $\sim\exists x(Gx \wedge L(xx))$. $\therefore\sim\exists x(Fx \wedge Gx)$

1	~~Show~~ $\sim\exists x(Fx \wedge Gx)$		
2	$\exists x(Fx \wedge Gx)$	ass id	
3	$Fi \wedge Gi$	2 ei	instantiate 2 to any term
4	Fi	3 sl	
5	$Fi \vee Hi \rightarrow \forall yL(iy)$	pr1 ui	
6	$Fi \vee Hi$	4 add	
7	$\forall yL(iy)$	5 6 mp	
8	$L(ii)$	7 ui	instantiate to match pr2 (a reflexive instance of L)
9	Gi	3 sr	
10	$Gi \wedge L(ii)$	8 9 adj	
11	$\exists x(Gx \wedge L(xx))$	10 eg	
12	$\sim\exists x(Gx \wedge L(xx))$	pr2 11 id	

6.0021 ∃y∀xF(b(y)x) ∴∃xF(xx)

1	~~Show~~ ∃xF(xx)		
2	∀xF(b(i)x)	pr1 ei	instantiate to any term (all terms are arbitrary here)
3	F(b(i)b(i))	2 ui	instantiate to match the complex term created in 2: b(i)
4	∃xF(xx)	3 eg	generalize 3. Replace every instance of complex term
5		4 dd	created in 2, b(i), with x.

6.0022 ∴∀x(Ax →∀y(By → ~C(xy))) → ~∃w∃z(Az ∧ Bw ∧ C(zw))

1	~~Show~~ ∀x(Ax →∀y(By → ~C(xy))) → ~∃w∃z(Az ∧ Bw ∧ C(zw))		
2	∀x(Ax →∀y(By → ~C(xy)))	ass cd	
3	~~Show~~ ~∃w∃z(Az ∧ Bw ∧ C(zw))		
4	∃w∃z(Az ∧ Bw ∧ C(zw))	ass id	
5	∃z(Az ∧ Bi ∧ C(zi))	4 ei	instantiate to any term
6	Ak ∧ Bi ∧ C(ki))	5 ei	instantiate to a NEW term
7	Ak ∧ Bi	6 sl	
8	C(ki)	6 sr	
9	Ak	7 sl	
10	Bi	7 sr	
11	Ak →∀y(By → ~C(ky))	2 ui	instantiate to match 9
12	∀y(By → ~C(ky))	9 11 mp	
13	Bi → ~C(ki)	12 ui	instantiate to match 10
14	~C(ki)	13 10 mp 8 id	
15		3 cd	

6.0023 ∀x∀y(B(xxy)→ L(yx)). ∴ ∃x∀y∃z B(xyz) → ∃x∃yL(xy)

1	~~Show~~ ∃x∀y∃z B(xyz) → ∃x∃yL(xy)		
2	∃x∀y∃z B(xyz)	ass cd	
3	∀y∃zB(iyz)	2 ei	instantiate to any term
4	∃zB(iiz)	3 ui	instantiate to match term introduced in line 3
5	B(iik)	4 ei	instantiate to a new term
6	∀y(B(iiy)→L(yi))	pr1 ui	instantiate to match term introduced in line 3
7	B(iik)→L(ki)	6 ui	instantiate to match term introduced in line 5
8	L(ki)	6 7 mp	
9	∃yL(ky)	8 eg	
10	∃x∃yL(xy)	9 eg cd	

6.0024 ∀x∃y~(Ax → ~By). ∴∃x(Ax ∧ Bx)

1	~~Show~~ ∃x(Ax ∧ Bx)	
2	∃y~(Az → ~By)	pr1 ui
3	~(Az → ~Bi)	2 ei
4	∃y~(Ai → ~By)	pr1 ui
5	~(Ai → ~Bk)	4 ei
6	Az ∧ ~~Bi	3 nc
7	Ai ∧ ~~Bk	5 nc
8	Ai	7 sl
9	~~Bi	6 sr
10	Bi	9 dn
11	Ai ∧ Bi	8 10 adj
12	∃x(Ax ∧ Bx)	11 eg dd

It might seem impossible to make the two terms match WHILE obeying the rules: EI must have a new term and you can only use EI and UI on the main logical operator (it has to be the first thing on the line, and the whole sentence in its scope).

Use UI first (since that is all you can do with the premise!)
Now use EI to a NEW term (line 3).
Use UI AGAIN on the premise, but match the term introduced in line 3. Now use EI again to a new term.

Take the conjuncts that match and put them together!