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STAPLES!
PEN (Remarks) neatly

PS 2.

#(7)

Claim: If $m > 1$ is not prime then there are a, b, c s.t.
$$\left.\begin{array}{l} c \not\equiv 0 \mod m \\ ac \equiv bc \mod m \\ a \not\equiv b \mod m \end{array}\right\}$$

<u>Recall:</u>

- m not prime $\Rightarrow m$ composite
  $\Rightarrow m = xy$ (x, y integers) $x, y \neq 1$

$\boxed{6 = 2 \cdot 3}$

- $A \equiv B \mod C \Leftrightarrow C$ divides $A - B$

$m = xy$

we have, $c \not\equiv 0 \mod xy$
$ac \equiv bc \mod xy$
$a \not\equiv b \mod xy$

$xy \equiv 0 \mod xy$

Take $c = x$, note $c \not\equiv 0 \mod xy$ since $0 < c < xy$

Now

$\underbrace{x}_{a}\underbrace{y}_{c} \equiv \underbrace{0}_{b}\underbrace{y}_{c} \mod xy$

since $0 < x, y < xy$
so we have $0 \not\equiv x, y \mod xy$

#12

21 divides $3n^7 + 7n^3 + 11n$. for all n.

$21 = 3 \times 7$

By problem (2). 21 divides .. iff 3 & 7 divides.

Check 3 divides $3n^7 + 7n^3 + 11n$
.... 7 .... ✓

#19. 133 divides $11^{n+1} + 12^{2n-1}$ for every natural number n.

$133 = 7 \cdot 19$

Check 7 divides $11^{n+1}+12^{2n-1}$

$$\equiv 4^{n+1}+5^{2n-1}$$
$$\equiv 4^{n+1}+(5^2)^n\cdot 5^{-1}$$
$$\equiv 4^{n+1}+(4)^n\cdot 5^{-1}$$
$$\equiv 4^{n+1}+(4)^n\cdot 3$$
$$\equiv (4+3)4^n$$
$$\equiv 7\cdot 4^n$$
$$\equiv 0 \mod 7$$

✓

Check 19 divides $11^{n+1}+12^{2n-1}$

$$11^{n+1}+(12^2)^n\ 12^{-1} \mod 19$$
$$\equiv 11^{n+1}+11^n\cdot 8 \mod 19$$
$$\equiv 11^n(11+8) \mod 19$$
$$\equiv 0 \mod 19$$

✓

12^{-1}: $12x\equiv 1 \mod 19$

$x=8$

$144\equiv ? \mod 19$

$?=11$