IFT 3.1 Version $n=1$

Solving $F(x,y)=0$ for y in terms of x : $y=f(x)$ See also next map

assumptions

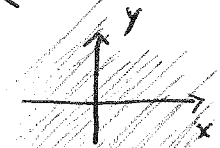
$F: \mathbb{R}^2 \to \mathbb{R}$

C^1

C^1 means partial derivatives are Cont

$F(x,y) \in \mathbb{R}$

indep variables everywhere



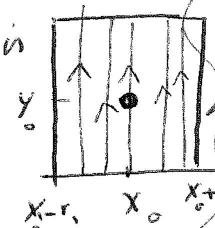
$F(x_0,y_0)=0$

$\partial_y F(x_0,y_0) \neq 0$ say >0

so $\partial_y F$ is Cont near (x_0,y_0) and $\partial_y F(x_0,y_0)>0$

$\exists r_1>0$ st. $\partial_y F(x,y)>0$ as long as $|x-x_0|<r_1$, $|y-y_0|<r_1$

i.e. The values of F are inc w.r.to y, That is



Some result from 137: if f is Cont near x_0, and $f(x_0)>0$ Then $\exists r_1>0$ st $\forall x$ $|x-x_0|<r_1 \Rightarrow f(x)>0$

& $F(x_0,y_0)=0$ implies

$F(x_0,y_0-r_1)<0<F(x_0,y_0+r_1)$

as F is cont $\exists r_0^1$, $\exists r_0^2$ st.

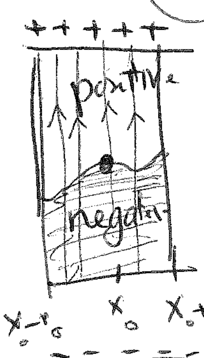
$\forall x$ $|x-x_0|<r_0^1 \Rightarrow$

$F(x,y_0-r_1)<0$

& $\forall x$ $|x-x_0|<r_0^2 \Rightarrow F(x,y_0+r_1)>0$

let $r_0=\min(r_0^1,r_0^2)$ so

$\forall x$ $|x-x_0|<r_0 \Rightarrow F(x,y_0-r_1)<0<F(x,y_0+r_1)$



One may Think That Since F is Cont and $F(x_0,y_0)=0$ Then Close to (x_0,y_0) F will be Close to 0 but it doesn't have to actually be $=0$.

But The third condition $\partial_y F(x_0,y_0)\neq 0$ will imply There is actually a curve passing Through (x_0,y_0) along which F is always $\underline{0}$!!

eg : $F(x,y)=x^2+y^2$, $F(0,0)=0$, but $F(x,y)=0$ implies $x=0$ & $y=0$ so nowhere else is $F=0$

Note : Condition 3 fails $\partial_y F(0,0)=0$ $\partial_y F(0,0)=0$

Now for any x st. $|x-x_0|<r_0$

$\exists!\, y$ st. $F(x,y)=0$ by IVT

unique b/c F is inc

call This $y=f(x)$

$F(x,f(x))=0$