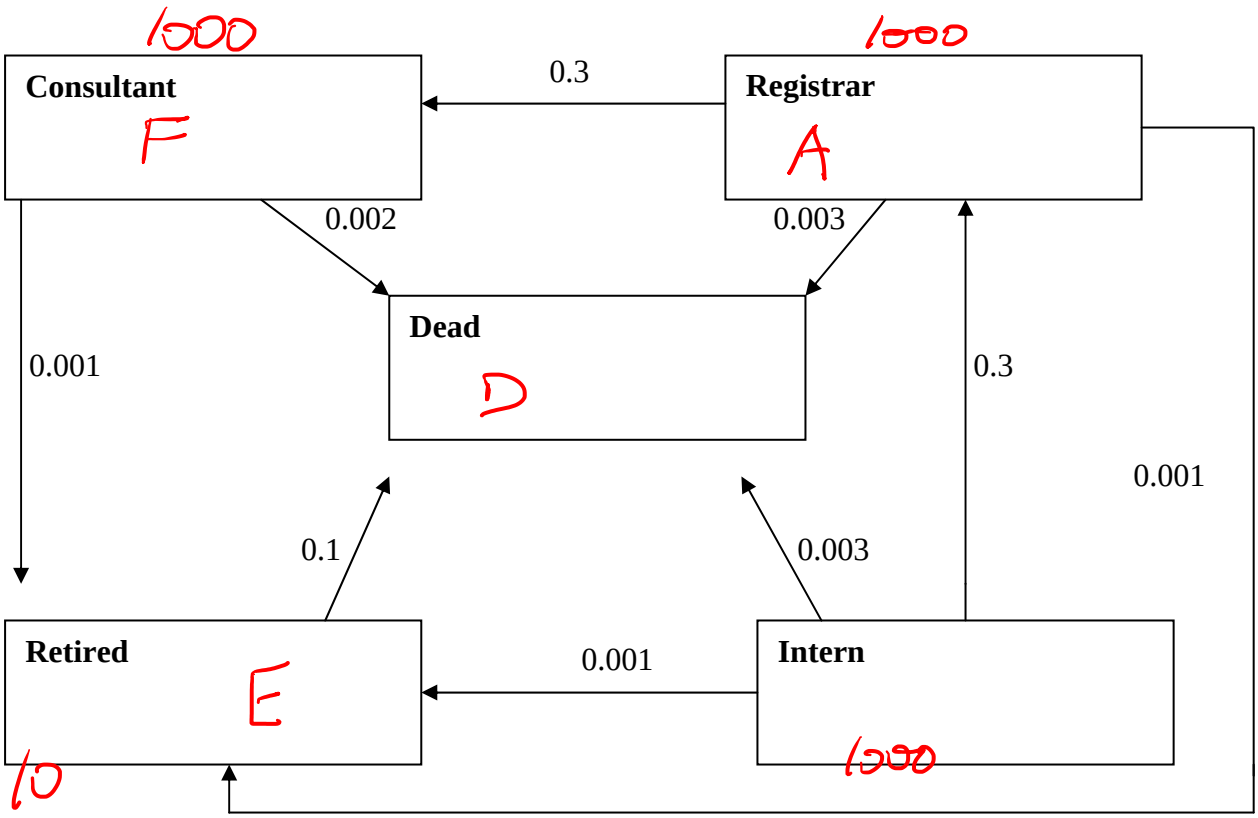
A multi-state Markov model has been used to describe the transition of medical doctors between the various levels of their career. The levels of career are *consultant*, *registrar* and *intern*. In total the Markov model has five states.

The transition intensities given in the figure below were estimated by observing 1000 years of waiting time in each of the three states consultant, registrar and intern and 10 years for the retired state. For example, the estimated transition intensity between registrar and retired is 0.001.



Based on the above information and figure answer the following questions:

(a) How many of registrars were observed to move to become consultants during the period of observation?

$1000 \times 0.3 = 300$

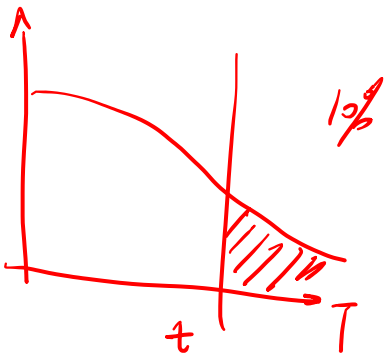
can't come back

(b) Calculate the probability that a new registrar will still be a registrar in 3 years' time.

$${}_3p^{\overline{AA}} = e^{-3(0.3+0.003+0.001)} = 0.40172$$

(c) Calculate a 90% confidence interval for the probability calculated in (b).

(d) Jennifer has been an intern for exactly two years. Calculate the 90th percentile (ie the upper 10th percentile) of the probability distribution of the amount of remaining time that Jennifer will spend as an intern.



$P(T \geq t) = 0.1$

(a) How many of registrars were observed to move to become consultants during the period of observation?

300

(b) Calculate the probability that a new registrar will still be a registrar in 3 years' time.

$$\exp\left(-3(0.304)\right) = 0.40172.$$

(c) Calculate a 90% confidence interval for the probability calculated in (b).

$$\hat{\lambda}_{AF} = 0.3 \quad Var\left(\hat{\lambda}_{AF}\right) = \frac{300}{1000^2} \quad \hat{\lambda}_{AD} = 0.003 \quad Var\left(\hat{\lambda}_{AD}\right) = \frac{3}{1000^2} \quad \hat{\lambda}_{AE} = 0.001 \quad Var\left(\hat{\lambda}_{AE}\right) = \frac{1}{1000^2}$$

$$Var\left(\hat{\lambda}_{AF} + \hat{\lambda}_{AD} + \hat{\lambda}_{AE}\right) = \frac{304}{1000^2}$$

Let $Y = \hat{\lambda}_{AF} + \hat{\lambda}_{AD} + \hat{\lambda}_{AE}$ and using the delta method,

$$Var\left(e^{-3Y}\right) \approx \left(-3e^{-3Y}\right)^2 .Var\left(Y\right) = 9\left(e^{-0.912}\right)^2 \frac{304}{1000^2} = 0.00044$$

Hence an approximate 95% CI is

$$0.40172 \pm 2\sqrt{0.00044}.$$

(d) Jennifer has been an intern for exactly two years. Calculate the 90th percentile (ie the upper 10th percentile) of the probability distribution of the amount of remaining time that Jennifer will spend as an intern.

Force of exit = 0.304.

$$S_T\left(t\right) = e^{-0.304t} = 0.1 \rightarrow t = -\frac{1}{0.304}\ln(0.1) = 7.58 \text{ years.}$$