# STAT3032 SURVIVAL MODELS

# SOLUTIONS TO TUTORIAL WEEK FOUR

1.

(a) $\hat{F}(3.5) = \dfrac{4}{15}$ and $\hat{F}\left(8.5\right) = \dfrac{11}{15}$

(b)

$$Var\left(\hat{F}\left(3.5\right)\right) = \frac{\frac{4}{15}\frac{11}{15}}{15} = \frac{44}{3375}$$

$$Var\left(\hat{F}\left(8.5\right)\right) = \frac{\frac{11}{15}\frac{4}{15}}{15} = \frac{44}{3375}$$

(c)

Confidence Interval for $F(3.5)$ is $\dfrac{4}{15} \pm 1.96\sqrt{\dfrac{44}{3375}} = \left(0.04287, 0.49050\right)$

Confidence Interval for $F(8.5)$ is $\dfrac{11}{15} \pm 1.96\sqrt{\dfrac{44}{3375}} = \left(0.50954, 0.95713\right)$

under the assumption that the distribution of $\hat{F}\left(t\right)$ is normal.

2.

$$S\left(t\right) = \exp\left(-\int_0^t \left(a + by\right) dy\right)$$

$$= \exp\left(-\left[ay + \frac{by^2}{2}\right]_0^t\right)$$

$$= \exp\left(-at - \frac{bt^2}{2}\right)$$

3. (a) ${}_5 p_{43} = \exp(-5(0.01)) = \exp(-0.05) = 0.95123$

(b) $\overset{o}{e}_{20} = \int_0^\infty {}_t p_{20} dt = \int_0^\infty e^{-0.01t} dt = -100\left[e^{-0.01t}\right]_0^\infty = 100$

Note that under this model the expectation of life is the same for all ages.

(c) $e_{20} = \sum_{t=1}^{\infty} {}_t p_{20} = \sum_{t=1}^{\infty} e^{-0.01t} \dfrac{e^{-0.01}}{1-e^{-0.01}} = 99.5008$

It is of course reasonable that this is not a whole number. The curtate future lifetime of an individual is a whole number however the average of a set of whole numbers, ie here the curate expectation of life, does not need to be a whole number.

(d) $m_x = \dfrac{d_x}{\int_0^1 l_{x+t}dt} = \dfrac{q_x}{\int_0^1 {}_t p_x dt} = \dfrac{\int_0^1 {}_t p_x \mu_{x+t} dt}{\int_0^1 {}_t p_x dt} = \dfrac{\int_0^1 e^{-0.01t} 0.01dt}{\int_0^1 e^{-0.01t} dt} = 0.01$

(e) $q_x = \int_0^1 {}_t p_x \mu_{x+t} dt = \int_0^1 e^{-0.01t} 0.01dt = -\left[ e^{-0.01t} \right]_0^1 = 0.00995$

5.

$m_x = \dfrac{d_x}{\int_0^1 l_{x+t}dt} \approx \dfrac{d_x}{l_x - \frac{1}{2}d_x}$ under UDD

$\therefore m_x \approx \dfrac{q_x}{1-\frac{1}{2}q_x}$

Rearranging, we get $q_x \approx \dfrac{m_x}{1+\frac{1}{2}m_x}$