Practice questions from past final exams

GRANGER CAUSALITY

a) Define Granger causality and the two tests introduced in class for checking Granger causality.

b) Define Granger Causality. State the procedure of using the following VAR(p) model to test if y_{1t} Granger causes y_{2t}

$$\begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} + \sum_{j=1}^{p} \begin{bmatrix} \phi_{j,11} & \phi_{j,12} \\ \phi_{j,21} & \phi_{j,22} \end{bmatrix} \begin{bmatrix} y_{1,t-1} \\ y_{2,t-2} \end{bmatrix} + \begin{bmatrix} a_{1t} \\ a_{2t} \end{bmatrix}.$$

c) Can we test Granger Causality without using the VAR approach? If your answer is yes, describe how to implement the test.

d) Define Granger causality and state how to test Granger causality without using vector autoregressive models

Answer:

1) Granger causality is defined through the enhancement on the prediction of conditional mean. The Granger causality between $\{X_t\}$ and $\{Y_t\}$ may be defined as

   – If Y_t can be better predicted by using the past values of X_t, i.e. $\{X_\tau, \tau < t\}$ than by not doing so, all other relevant information (including the past values of $\{X_t\}$) is the universe being used in either case.

2) Pierce and Haugh (1977) expanded up the work of Granger (1969) and gave a comprehensive survey regarding research on causality in temporal systems. For simplicity, consider two causal and invertible time series $\{X_t\}$ and $\{Y_t\}$. Let both processes be given by

$$\phi_x(B)x_t = \theta_x(B)u_t, \quad u_t \sim NID(0, \sigma_u^2)$$

$$\phi_y(B)y_t = \theta_y(B)v_t, \quad v_t \sim NID(0, \sigma_u^2),$$

where$x_t = X_t - \mu_x, y_t = Y_t - \mu_y,\ \phi_x(B), \phi_y(B), \theta_x(B)$ and $\theta_y(B)$ are polynomials of the backward shift operator B and satisfy all causal and invertible conditions. Pierce and Haugh (1977) explained that there are many possible types of causal interpretation between $\{X_t\}$ and $\{Y_t\}$ which can be characterized by the properties of residual cross-correlation functions between $\hat{u}_t$ and $\hat{v}_t$. An overall (portmanteau) test for testing Granger causality (H_0: X_t does not Granger cause Y_t) is given by

$$Q_L = n^2 \sum_{k=0}^{L} (n-k)^{-1} r_{uv}^2(k).$$

The Q_L statistic follows a chi-squared distribution with $L+1$ degrees of freedom, where L is a user defined integer. We will reject the null hypothesis if the p-value of Q_L is smaller than a predetermined significance level.

COINTEGRATION

e) Define $\mathrm{I}(0)$, $\mathrm{I}(1)$ and $\mathrm{I}(\mathrm{d})$ processes (processes of integrated of order zero, one and d)

f) Define cointegration.

g) Define $\mathrm{I}(\mathrm{d})$ processes and explain how to test the cointegration of two $\mathrm{I}(1)$ processes using a regression approach

Answer:

1) A stochastic process $\{X_t\}$ is said to be an $I(d)$ process if $Y_t = (1-B)^d X_t$ is a weakly stationary process, where B denotes the backward shift operation, $BX_t = X_{t-1}$, and $d > 0$ and $d \in Z$.

2) Steps to test cointegration of two $I(1)$ processes may be given as follows:

   i. Use the unit root tests, such as Augmented Dickey-Fuller test, to test if the given two time series,$\{X_t\}$ and $\{Y_t\}$, follow $I(1)$ processes.

   ii. If both $\{X_t\}$ and $\{Y_t\}$ are stationary, then regress $\{X_t\}$ against $\{Y_t\}$ (or $\{Y_t\}$ against $\{X_t\}$). Let $\{\varepsilon_t\}$ denote the corresponding regression residuals.

   iii. Apply the unit root test again on the regression residuals $\{\varepsilon_t\}$. If the test result shows that $\{\varepsilon_t\}$ is stationary or follows a $I(d)$ process. Then we said $\{X_t\}$ and $\{Y_t\}$ are cointegrated.

h) Define cointegration between two time series $\{X_t\}$ and $\{Y_t\}$. Describe how to test the existence of the cointegration.

i) Describe the Engle-Granger method to test cointegration.

Answer:

1) Test if data of interests are $I(1)$ or nonstationary using unit root tests
2) If the data of interest follows $I(1)$ processes, then run regression using the least square method.
3) Collect the residuals of the aforesaid regression and test if the residuals are stationary using unit root tests. If the residuals do not contain stochastic trend (unit root), we say that the data of interest are cointegrated.

   Remarks: They may exist multiple cointegrated relationships among a set of variables.

j) Describe Granger representation theorem

Answer:

If time series $\{X_t\}$ and $\{Y_t\}$ are cointegrated, an ECM (Error correction mechanism) must be included for their modeling process. Specifically, consider the following bivariate VAR (vector autoregressive) model

$$\Delta X_t = \alpha_1 + \gamma_1 z_{t-1} + \sum_i^{m1} \beta_{1i} \Delta X_{t-i} + \sum_i^{m2} \beta_{2i} \Delta Y_{t-i} + e_{1t}$$

$$\Delta Y_t = \alpha_2 + \gamma_2 z_{t-1} + \sum_i^{m3} \beta_{3i} \Delta X_{t-i} + \sum_i^{m4} \beta_{4i} \Delta Y_{t-i} + e_{2t},$$

where $(e_{1t}, e_{2t})'$ follows a bivariate white noises, and $z_t = X_t - \alpha Y_t$ is an $I(0)$ proces, where $(1, -\alpha)$ is a cointegrated vector between $\{X_t\}$ and $\{Y_t\}$. Otherwise, there will be a model misspecification in the aforesaid bivariate VAR model.

Remark 1: we also require that $\gamma_1 < 0$ and $\gamma_2 > 0$ in our model.

Remark 2: Cointegration between $\{X_t\}$ and $\{Y_t\}$ is a necessary condition for ECM and vice versa.

k) State the Granger representation theorem. Discuss its implication on Vector autoregressive (VAR) modeling.

OTHERS (VECTOR AUTOREGRESSIVE PROCESS, FORECAST ACCURACY AND ELSE)

l) Discuss a method taught in class for removing (or modeling) seasonality of time series data.

m) Describe the Granger-Newbold test for compare forecast accuracy and its assumptions.

Answer:

Granger and Newbold (1976) considers the following transformation

$$x_i = e_{1i} + e_{2i} \text{ and } z_i = e_{1i} + e_{2i}, i = 1, \ldots, H,$$

where e_{ki} stands for the one-step ahead forecast error of model $k, k = 1,2$ at time $t + i$.

Granger and Newbold (1976) assumes that

1) The forecast errors have zero mean and normally distributed ;
2) The forecast errors are serially uncorrelated;

Under the above two assumptions and under the assumption of equal forecast accuracy (H_0), x_i and z_i should be uncorrelated ($\because \rho_{xz} = E(xz) = E(e_1^2 - e_2^2) = 0$).

We can therefore use the sample correlation coefficient between $\{x_i\}$ and $\{z_i\}$, denoted as r_{xz}, to evaluate the accuracy between model 1 and 2. In particular, Granger and Newbold (1976) showed that

$$\frac{r_{xz}}{\sqrt{\frac{(1 - r_{xz}^2)}{H - 1}}} \sim t_{H-1}$$

if assumption 1 and 2 hold.

Thus, if r_{xz} is statistically different from zero, we reject the null hypothesis. Specifically, if $r_{xz} > 0$, model 1 has a larger MPSE (so less accuracy); and if $r_{xz} < 0$, model 2 has a larger MPSE.

n) Consider a two dimensional vector autoregressive process of order one

$$\begin{bmatrix} r_{2t} \\ r_{1t} \end{bmatrix} = \begin{bmatrix} 0.4 \\ 0.2 \end{bmatrix} + \begin{bmatrix} 1.1 & -0.6 \\ 0.3 & 0.2 \end{bmatrix} \begin{bmatrix} r_{2,t-1} \\ r_{1,t-1} \end{bmatrix} + \begin{bmatrix} a_{2t} \\ a_{1t} \end{bmatrix},$$

where $\Sigma = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$ is the covariance matrix between a_{2t} and a_{1t}. Answer whether the above VAR(1) model is (weakly) stationary.

Answer:

Suppose that $\Phi = \begin{bmatrix} 1.1 & -0.6 \\ 0.3 & 0.2 \end{bmatrix}$. For the process to be staionary, the zeros of the determinantal equation $|I - \Phi B|$ must be outside the unit circle. Letting $\lambda = B^{-1}$, we have $|I - \Phi\, B| = 0 \leftrightarrow |\lambda I - \Phi| = 0$. Thus, the zeros of $|I - \Phi B|$ are related to the eigenvalues of Φ. Let λ_1 and λ_2 be the eigenvalues (assume that the eigenvectors are linear independent). Thus, we have $|I - \Phi B| = \prod_{i=1}^{2}(1 - \lambda_i B)$. Hence, the zeros of $|I - \Phi B|$ are outside the unit circle if and only if all the eigenvalues are inside the unit circle. Using R, we have

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> m<-matrix(c(1.1,0.3,-0.6,0.2),2,2,F)
> eigen(m)$values
[1] 0.8 0.5
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Therefore, the process is stationary.

o) **<u>**** Do practice questions in the TFN course note *****</u>**