University of Toronto
Faculty of Arts and Sciences

APRIL 2012 Examinations

AST121H1S, Origin and Evolution of the Universe

Duration: 3 Hours

Examination Aids: non-programmable calculators

Name: ____________________________ **Student Number:** ________________________

The test is composed of 20 multiple choice questions (one point each) and 6 short-answer questions (5 points each). For the multiple choices, only one answer is correct and please mark your answers on the scantron sheet. For the short-answer questions, please use the space provided below each question. Hand in **both** the exam paper and the scantron sheet, but don't forget to fill in your name/student number on **both** forms. The exam papers have a total of 11 pages.

A list of physical constants in MKS (or SI) units, kg is kilogram, m is meter, K is Kelvin, s is second, N is Newton, J is Joule, C is coulomb.

parsec $pc = 3.0857 \times 10^{16}$ m

arcsecond 1" $= 4.848 \times 10^{-6}$ radian

elementary charge $e = 1.602 \times 10^{-19}$ C

proton rest mass $m_p = 1.672 \times 10^{-27}$ kg

neutron rest mass $m_n = 1.675 \times 10^{-27}$ kg

hydrogen atom mass $m_H = 1.673 \times 10^{-27}$ kg

electron rest mass $m_e = 9.109 \times 10^{-31}$ kg

mass of the Sun $M_\odot = 1.99 \times 10^{30}$ kg

Planck constant $h = 6.625 \times 10^{-34} J \cdot s$

Speed of light in vacuum $c = 2.998 \times 10^{8} m s^{-1}$

Gravitational constant $G = 6.673 \times 10^{-11} m^3 kg^{-1} s^{-2}$

Fine structure constant $\alpha = 1/137.036$

Boltzmann constant $k = 1.380 \times 10^{-23} J K^{-1}$

Electron volt $eV = 1.602 \times 10^{-19} J$

Stefan-Boltzmann constant $\sigma = 5.67 \times 10^{-8} J s^{-1} m^{-2} K^{-4}$

Coulomb constant $k_e = \frac{1}{4\pi\epsilon_0^2} = \frac{c^2\mu_0}{4\pi} = 8.988 \times 10^{9} N m^2 C^{-2}$.

Part A, 20 multiple-choice questions. Input your answers on the scantron sheet. One point each

1. The Hubble flow cannot be due to the actual movement of the galaxies through space. Which of the following gives the correct argument for this statement?

   A) Galaxies cannot move faster than speed of light so their redshifts cannot correpond to actual movements.

   B) Galaxies are too massive and cannot be accelerated to such high velocites as are implied by their redshifts.

   C) If galaxies are moving apart, this will mean that at some point in the past they are together and at a special location in the universe. The latter violates the cosmological principle.

   D) In the case that galaxies are moving through space, the Hubble flow will look different to different galaxies. This violates the cosmological principle.

2. Hubble's original measurement for the Hubble constant yielded $H_0 \sim 500$ km/s/Mpc. This is much greater than the modern measurement ($H_0 \sim 70$ km/s/Mpc). Which of the following statement explains the discrepancy?

   A) The Cepheids he was observing are much higher in luminosity than he had assumed.

   B) The Cepheids he was observing are much lower in luminosity than he had assumed.

   C) The expansion of the universe had a different speed when he was measuring it.

   D) Hubble was measuring galaxies that are not experience the expansion of the universe, but are orbiting around each other in a galaxy cluster.

3. Which of the following statement correctly captures the chronological sequence among different stages in the big bang?

   A) Eras of Planck, Inflation, Atoms, Nucleosynthesis, Galaxies.

   B) Eras of Atoms, Nucleosynthesis, Galaxies, Nuclei.

   C) Eras of Inflation, Atoms, Nucleosynthesis, CMB, Galaxies.

   D) Eras of Inflation, Hadrons, Nucleosynthesis, Atoms.

4. Dark matter and dark energy are not predicted by the standard model of particle physics. Which of the following statement is FALSE?

   A) Dark matter slows down the expansion of the universe while dark energy accelerates it.

   B) Dark matter particles need to have non-zero masses and zero-charges. Dark energy particles cannot have charges.

   C) The total amount of dark matter is conserved when the universe is expanding, but not so for dark energy.

   D) If it is not for dark energy, the universe will be going toward a "big crunch".

5. The cosmic microwave background is now redshifted to $z \approx 1100$. The size of one of its hot spots corresponds to the size of the horizon at age 300,000 yrs. Assume one such hot spot region has been expanding together with the universe, how big in physical size has it become today?

A) $\sim 10^3$ light-years. B) $\sim 10^6$ light-years. C) $\sim 10^8$ light-years.

D) $\sim 10^{10}$ light-years. E) Not enough information given.

6. During the life of the Sun, what are the longest, the biggest and the most luminous phases called, respectively,

A) White dwarf, planetary nebulae, red giant. B) Main-sequence, red giant, red giant.

C) Red Giant, proto-stellar, planetary nebulae. D) Proto-stellar, red giant, white dwarf.

7. One of the predictions of the Big Bang theory is the primordial ratio for hydrogen and helium in the universe. Neutrons are less abundant than protons by a factor of 7 at the end of the big bang. What do you conclude is the prediction for hydrogen (made up of one proton) and helium (two protons and two neutrons) abundances?

A) by mass, 50% hydrogen and 50% helium.

B) by number, 50% hydrogen and 50% helium.

C) by mass, 75% hydrogen and 25% helium.

D) by number, 75% hydrogen and 25% helium.

8. Today we have measured the Hubble constant to be 70 km/s/Mpc. Which of the following is correct?

A) When the CMB photons were emitted, the Hubble constant was smaller.

B) Even the highest redshift galaxies we observe are receding from us today with a speed less than the speed of light. Else we would not be able to observe them.

C) For an alien civilization currently measuring the Hubble constant, the value they get depends on how far they are away from us.

D) The acceleration of the universal expansion means the Hubble constant is now increasing with time.

9. The early universe was so hot that photons continuously create pairs and pairs quickly annihilate back to photons. What is the typical photon energies for generating proton/anti-proton pairs and electron/positron pairs, respectively?

A) 1 Mev, 1 Gev. B) 1 Gev, 1 Mev. C) 1 ev, 10 Gev. D) 10 Gev, 1 ev.

10. What is the observational motivation for introducing inflation?

A) the universe looks nearly the same in all directions.

B) the universe looks diferent in different directions.

C) distant supernova of a given redshift are brighter than expected.

D) distant supernova of a given redshift are dimmer than expected.

11. Which of the following statement incorrectly relates the fate of the universe with the cosmological parameters?

    A) Universe with too low a density will experience "Big chill" where almost all galaxies recede out of our horizon.

    B) Universe with too high a density will end up in "Big crunch" where the entire universe collects into a blackhole.

    C) If the cosmological constant is increasing with time, we will encounter "Big rip" where even atoms are ripped apart by the expansion.

    D) No matter what happens, gravity within the Solar system will keep it together.

12. We live in a matter universe, not an anti-matter universe. Which of the following statement is correct?

    A) Anti-matter has never been observed in our universe and is a hypothetical particle. Their existence will cause catastrophic consequences.

    B) In the very early universe, frequent photon annihilations produce matter and anti-matter in roughly equal proportion.

    C) Matter attracts matter and anti-matter repels matter. So anti-matter may be responsible for dark energy.

    D) If our universe has equal amounts of matter and anti-matter, their mutual attractions will be sufficiently great to resist the expansion of the universe.

13. Why do dark matter reside preferentially in the outskirts of galaxies?

    A) Dark matter can not lose energy by radiating photons so they maintain their high temperature and are circling at large radii.

    B) Stars collide frequently so they settle to the galactic disk and bulge. Dark matter only interact weakly when they collide so they cannot settle.

    C) Dark matter do not experience collisions so could not get rid of the primordial angular momentum. They are therefore circling at large radii.

    D) Dark matter do not experience gravity as strongly as baryons do. So they are circling at larger radii.

14. Rank the following photons in the correct order of their characteristics.

    A) In increasing wavelength, radio, x-ray, optical, infrared photons.

    B) In increasing wavelength, x-ray, optical, radio, infrared.

    C) In increasing energy, radio, x-ray,optical, infrared photons.

    D) In increasing photon frequency, radio, infared, optical, x-ray photons.

15. The Sun is orbiting the center of the Milky way at a distance of 8 kpc (kilo-parsec), and with a circular velocity of 200 km/s. Assume for simplicity that all mass inside of the Sun's orbit is distributed spherically. What is the amount of mass inside?

    A) $\sim 10^9 M_\odot$. B) $\sim 10^{11} M_\odot$. C) $\sim 10^{13} M_\odot$. D) $\sim 10^{15} M_\odot$.

16. Photons have no mass yet their paths are bent when near a blackhole. Which of the following statement is correct?

    A) Neutrinos have no charges so their path would not be bent like that of photons.

    B) The path of the photon is modified by the local curvature in space.

    C) $E = mc^2$, so photons actually do have masses associated with them since they also have non-zero energies.

    D) The path of the photon is modified by the local vacuum flutuations around the blackhole.

17. Bohr's hydrogen model. Compared to the wavelength of the photon emitted when the electron jumps from the $n = \infty$ (free state) to the $n = 2$ state, how long is the wavelength of the photon emitted when the electron undergoes the $n = 2$ to $n = 1$ transition?

    A) One-third. B) Two-thirds. C) Three. D) Six.

18. Approximately how much brighter (or colder) are the fluctuations on the CMB compared to the average CMB brightness:

    A) 50%; B) 10%; C) 0.1%; D) 0.001%

19. Which of the following observations, if confirmed, will disprove the Big Bang theory?

    A) A star that has an age > 5 Gyr.

    B) The universe was much denser in the past.

    C) The expansion of the universe was decelerating before it started to accelerate.

    D) The universe was contracting before it started expanding.

20. Which of the following statement correctly captures the sequence of domination in the cosmic history?

    A) Energy density in the universe is sequentially dominated by that in baryons, photons, dark matter, and lastly, dark energy.

    B) Sequentially dominated by that in photons, dark matter and dark energy.

    C) In turn dominated by that in dark energy, baryons and photons.

    D) In turn dominated by that in photons, dark energy and baryons.

Part B, 6 short-answer questions. Write your answers in the space provided below. Five points each

1. The matter density (normal matter and dark matter) in the universe satisfies $\rho \approx 0.3\rho_{\rm crit}$, where the critical density $\rho_{\rm crit} = 3H^2/(8\pi G)$.

   a) Estimate the total amount of mass in the observable universe. (express in unit of solar mass, $M_\odot$). Here, simply take the size of the horizon to be the light travel distance over the age of the universe, ignoring the expansion effect.

   b) A typical galaxy in the universe has a total mass of $\sim 10^{12} M_\odot$. Estimate the total number of galaxies in the observable universe.

   c) Assume galaxies are evenly spaced in the universe and each galaxy collects its mass from an original sphere with diameter R. Calculate the diameter of the sphere. (express it in unit of light-years) This is the average distance between two neighbouring galaxies, if our assumption stands.

   d) In reality, our next-door neighbour, the Andromeda galaxy, is at a distance of 2.5×10^6 light-years. Explain in words why this number is smaller than your answer from the above question.

2. The magic of Solar Power. The mass of a hydrogen nucleus is 1.673×10^{-27} kg, and the mass of a helium nucleus is 6.644×10^{-27} kg.

   a) How much energy is released when four hydrogen nucleus is fused into one helium nucleus? (express in unit of J)

   b) Assuming the entire Sun was made of hydrogen initially, how much energy in total can be released as solar energy? (express in unit of J)

   c) Adopt the current solar luminosity of 4×10^{26} J/s. How long can the Sun shine by its nuclear energy if all of its hydrogen can be converted into helium? (express in unit of year)

3. Standard rulers and standard candles. Cepheids and Supernova are called 'standard candles' and the CMB hot/cold spots are called 'standard rulers'.

   a) What is the meaning of a standard candle? (two sentences or fewer)

   b) Explain what discoveries the 'standard candles' have led to in cosmology.

   c) What is the meaning of a standard ruler? (two sentences or fewer)

   d) Explain how the apparent angular sizes of the CMB spots can be used to infer the curvature of the universe. Draw a diagram if you need to.

4. The CMB photons follow a nearly perfect blackbody spectrum. At the present time, the temperature of this blackbody is at 2.7 Kelvin.

   a) Everything in the universe is immersed in the CMB photon bath. For an object that has no internal energy source (also no residual heat), the CMB photons will heat it up to a certain temperature. Can you find out what this temperature is by equating the CMB flux received with the blackbody flux radiated?

   b) As earlier times, the CMB photons had shorter wavelengths so the temperature of the CMB blackbody was hotter. How does this temperature scale with redshift?

   c) If gas in the universe has to be cooled to below 100 Kelvin for stars to start forming, what is the highest redshift we expect to observe star formation?

5. Start from the Friedmann's equation for a flat universe,

$$H^2 = \frac{1}{a^2}\left(\frac{da}{dt}\right)^2 = \frac{8\pi G}{3}\rho_m + \frac{\Lambda c^2}{3}, \tag{1}$$

where ρ_m is the average matter density, and Λ the cosmological constant.

a) Solve for expansion of the universe $a = a(t)$ assuming that matter density dominates (i.e., ignore dark energy in the above equation).

b) What is the present age of such a universe if the the currently measured Hubble constant is $H_0 = 74$ km/s/Mpc?

c) The highest redshift galaxies currently known is at a redshift of $z = 8.4$. What is the age of the universe when the photons we see left it? Again, only consider the matter dominated universe.

d) Define $\Omega_m = \rho/\rho_{\text{crit}}$, and $\Omega_\Lambda = \Lambda c^2/(3H^2)$, and the critical density $\rho_{\text{crit}} = 3H^2/(8\pi G)$. Assuming that Λ is a constant, show that the ratio of Ω_Λ/Ω_m will increase with time.

6. An extra-solar planet is orbiting its star in a circular Keplerian orbit with an orbital period of 3 days. Its host star has the mass of the Sun and its spectral lines (at $6000\mathring{A}$) are observed to wobble with the same period by as much as $0.02\mathring{A}$ to the blue or to the red of the rest-frame wavelength. Assume that the planet's orbit is exactly perpendicular to the plane of the sky,

   a) What is the orbital separation between the planet and the star, assuming for the moment that the planet mass is much smaller than the stellar mass? (express in unit of m) What is the orbital velocity of the planet? (express in unit of m/s)

   b) What is the Doppler velocity of the star (express in unit of m/s)?

   c) The center of mass of the planet-star pair is not moving. Combine this fact with your result for the orbital speed of the planet in the first part to obtain the mass of the planet. (express in unit of solar mass)

Total Marks = 50