Lecture 3

§3.4 Other Types of Orbits

Example: consider $F(x) = x^2 - 2$

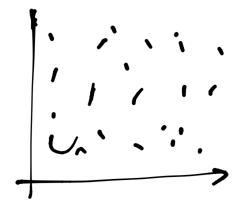
The point $x_0 = 0$ is eventually fixed.

$0, -2, 2, 2, \cdots$

But if we compute the orbit slightly off 0, say $x_0 = 0.01$.

(Then we get a chaos)

We obtained what we call chaotic behaviour.



If take $X_0 = 0.001$, get another totally different plot. (You cannot really "predict" the plot)

Tiny deviations make great changes.

§3.5 Doubling Function

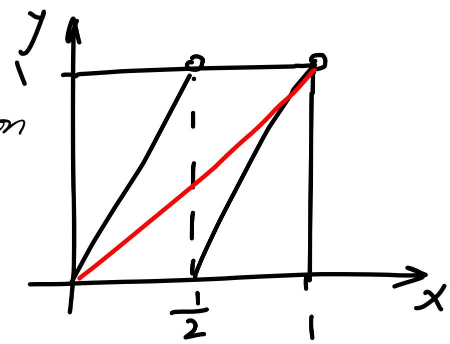
We define the doubling function as $D: [0,1) \longrightarrow [0,1)$

$$D(x) = \begin{cases} 2x & \text{if } 0 \le x < \frac{1}{2} \\ 2x - 1 & \text{if } \frac{1}{2} \le x < 1 \end{cases}$$

First we can express the doubling function in different ways:

$D(x) = 2x \bmod 1 = 2x - [[2x]]$

where $[[x]]$ = integer part of x.



Graph of doubling function

Properties

① There is only one fixed point $x_0 = 0$.

② There are lots of cycles:

$0.2, 0.4, 0.8, 0.6, 0.2 \cdots$ 4 cycles

$\frac{1}{9}, \frac{2}{9}, \frac{4}{9}, \frac{8}{9}, \frac{7}{9}, \frac{5}{9}, \frac{1}{9} \cdots$ 6 cycles $F^6(\frac{1}{9}) = \frac{1}{9}$

*But due to the error of computer, $\frac{1}{9}$ is "fixed". Why? b/c the computer approximates $\frac{1}{9}$ as 0.1111111, which is not correct.

CHAPTER 4 GRAPHICAL ALALYSIS

To sketch a cobweb graph of an orbit of x_0 under F, follow the procedure

① Graph $y = f(x)$ and $y = x$

② Start with the point (x_0, x_0) on the line $y = x$.

③ Go vertically (up or down) to the graph $y = f(x)$ to the point $(x_0, F(x_0)) = (x_0, x_1)$

④ Go horizontally (left or right) to the graph $y = x$ to the point (x_1, x_1)

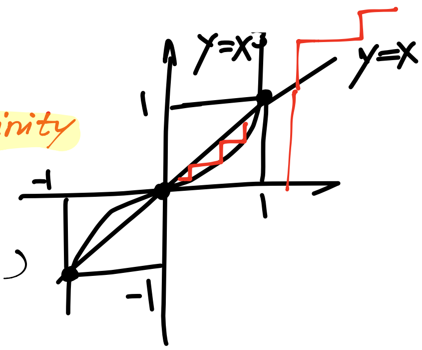
⑤ repeat the process for the next iteration.

§4.1 Orbits Analysis

Orbit analysis is a complete description of each orbit under F for every possible seed x_0.

example: Let $F(x)=x^3$

· There are 3 fixed point, -1, 0, 1.



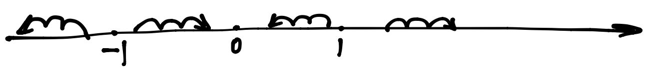
· If $x_0>1$, then the orbit will escape to infinity

· If $x_0<-1$, then - - - - negative infinity

· If $0<x_0<1$ ~~or $-1<x_0<0$~~, then the orbit will converge to 0^+. (but never touches 0)

· If $-1<x_0<0$, · · · · 0^-.

• We can summerize this information



Example: $F(x)=x^2$

The fixed pts are 0 & 1.

The point -1 is eventually fixed.