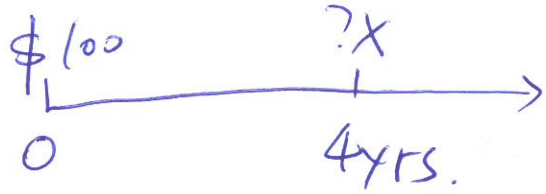
LN2.2.

Example.



1). compound interest 5%

2). simple interest 5%

1). $X = 100(1+i)^{4}$

2). $X = 100\cdot(1+i\cdot 4)$.

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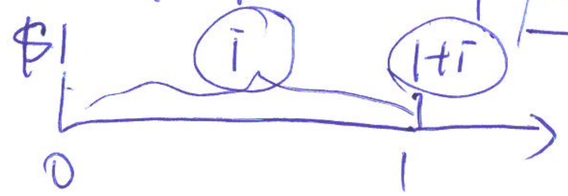
① Nominal Rates of Interest $i^{(m)}$

② Nominal Rates of $\boxed{\text{discount}}$. $d^{(m)}$.



$$d = \frac{i}{1+i}$$

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①. $\boxed{i^{(m)} = 12\%}$ *

m	
1	$i = \left(1+\frac{i^{(m)}}{m}\right)^{m} - 1$
2	:
3	:
4	:
6	:
12	:
52	:
365.. ∞	:

②

$$\lim_{m\to\infty} i = \lim_{m\to\infty} \boxed{\left(1+\frac{i^{(m)}}{m}\right)^{m}} - 1$$

$$= e^{i^{(m)}} - 1$$

$$= e^{12\%} - 1 = 0.127497$$

② effective $i = 12\%$.

m	$i^{(m)} = m \cdot \left((1+i)^{1/m} - 1\right)$
1	.
2	.
3	.
.	.
.	.
.	.
365	.
.	.
∞	.

$$\boxed{\lim_{m\to\infty} i^{(m)}} = \lim_{m\to\infty} m\left((1+i)^{\frac{1}{m}} - 1\right)$$

L'Hopital rule

$$= \lim_{m\to\infty} \frac{(1+i)^{\frac{1}{m}} - 1}{\frac{1}{m}} \quad \begin{cases} 0 \\ -\infty \\ +\infty \end{cases}$$

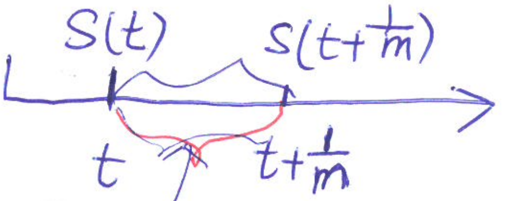
$$= \lim_{m\to\infty} \frac{(1+i)^{\frac{1}{m}} \cdot \ln(1+i) \cdot \cancel{\left(-\frac{1}{m}\right)^{2}}}{\cancel{(-m)^{-2}}}$$

force of interest

$$= \ln(1+i)$$

Force of Interest. δ.

$$\boxed{\lim_{m\to\infty} i^{(m)}} = \delta.$$



$$\boxed{i^{(m)}}$$

$$\boxed{\frac{i^{(m)}}{m} = \frac{S(t+\frac{1}{m}) - S(t)}{S(t)}}.$$

$$\Rightarrow \quad i^{(m)} = \frac{S(t+\frac{1}{m}) - S(t)}{\frac{1}{m}\, S(t)}$$

$$\Rightarrow \quad \delta = i^{(\infty)} = \lim_{m\to\infty} \frac{S(t+\frac{1}{m}) - S(t)}{\frac{1}{m}\cdot S(t)}.$$

$$= \frac{1}{S(t)} \cdot \boxed{\lim_{m\to\infty} \frac{S(t+\frac{1}{m}) - S(t)}{\frac{1}{m}}}.$$

$$\overset{\frac{1}{m} \triangleq h}{=} \frac{1}{S(t)} \cdot \boxed{\lim_{h\to 0} \frac{S(t+h) - S(t)}{h}}.$$

$$\boxed{\delta = \boxed{\frac{S'(t)}{S(t)}} = \frac{d}{dt}\left[\ln[S(t)]\right]} \quad *$$

④

$\delta_t = f(t)$ / $\delta_t = \delta$

$$\lim_{m\to\infty} d^{(m)} = \delta = \lim_{m\to\infty} i^{(m)}$$

Pf: Wattle discussion!

$$\boxed{i > \cancel{d^{(1)}}\ i^{(1)} > i^{(2)} > \cdots > \boxed{\delta} > \cdots > d^{(2)} > d^{(1)} > d}$$

Accumulated V / PV

① $\delta_t = \boxed{\dfrac{S'(t)}{S(t)}} = \boxed{\dfrac{d}{dt}\ln[S(t)]}$

$\boxed{\cancel{S(t)}}$

$\boxed{\text{Simple interest rate } i} \to S(t) = S_0(1+it)$

$$\delta(t) = \boxed{\frac{S'(t)}{S(t)}} = \frac{\cancel{S_0}\cdot i}{\cancel{S_0}(1+it)} = \frac{i}{1+it}.$$

$\boxed{\text{Compound interest rate } i} \to S(t) = \boxed{S_0(1+i)^t}$

$$\delta(t) = \frac{d}{dt}\ln[S(t)] = \frac{d}{dt}\left[\ln S_0 + t\cdot\ln(1+i)\right]$$
$$= \ln(1+i)$$

$$\delta t = \ln(1+\bar{i})$$

$$\Rightarrow \boxed{\bar{i} = e^{\delta t} - 1}$$

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