Lecture 14

§ 7.1

$Q_{-2}(x)=x^2-2$ $\quad Q_{-2}: I \to I$ For $I=[-2,2]$ $\quad\quad P_+=2$ for $c=-2$

From observing the graphs:

- Q_{-2} has 2 fixed pts
- Q_{-2} has 1 2-cycle
- Q_{-2} has 2 3-cycles
- Q_{-2} has 3 4-cycles

Theorem: Q_{-2} has at least 2^n periodic points of period n (might not be prime) in the interval I

Q: Why can't we find these periodic points on the computer?
The main reason is that all the cycles are repelling

Q: How did we get ∞ fixed pts from a few periodic pts for $c>-5/4$ to infinitely many at $c=-2$? To be covered in next chapter

Example: $F_4: I \to I$ for $I=[0,1]$

just like Q_{-2}, F_4 has infinitely many periodic pts, which are repelling. The orbits are chaotic (most of them)

§ 7.2 $c<-2$

When $c<-2$, the minimum value of $Q_c(x)$ is at $x=0$,

$$Q_c(0)=c<-P_+$$

$$c<-P_+ \iff c<-\frac{1+\sqrt{1-4c}}{2}$$

$$\iff -(2c+1)>\sqrt{1-4c}$$

$$\iff (2c+1)^2>1-4c$$

$$\iff 4c^2+4c+1>1-4c$$

$$\iff 4c(c+2)>0$$

$$\iff c<-2 \text{ or } c>0$$

so $c<-2$ implies $c<-P_+ \to$ left endpoint of $I=[-P_+,P_+]$

min of Q_c

This implies that the image of Q_c is not contained in I for $c<-2$.

Let $c=-2.5$ (we will use the value for the rest of §7.2)

Orbits that escape to infinity.

We want to find all the points $x_0 \in I$ s.t. its orbit under Q_c escapes to infinity.