FINANCIAL MATHEMATICS
STAT 2032 / STAT 6046

LECTURE NOTES WEEK 10

INTEREST RATE RISK

We will now cover simple measures used to quantify the level of vulnerability of fixed interest securities to interest rate movements, and we will also introduce the technique of immunisation, which is a method of minimising interest rate risk.

A manager responsible for the investment of a fixed interest portfolio will be concerned about how the portfolio would be affected if there was a change in interest rates.

In this section we consider simple measures of vulnerability to interest rate movements.

There are a number of measures used to track the sensitivity of cash flows to changes in interest rates: effective duration (or *volatility*), duration and convexity.

There are two assumptions underlying the theory of this section:

- The yield curve is flat. ie. $s_t = f_{t,T} = i$ for all t and T.
- Any change in interest rates affects all interest rates by the same amount. ie. the yield curve stays flat when interest rates change.

EFFECTIVE DURATION AND DURATION

The ***effective duration*** (or ***volatility***) of a series of cash flows is a measure of the rate of change of the present value of a series of cash flows as the interest rate changes.

Consider a series of cash flows $\{C_{t_k}\}$ for $k = 1,2,....,n$. Let PV be the present value of the cash flows at interest rate (yield to maturity) i, so that:

$$PV = \sum_{k=1}^{n} C_{t_k} v_i^{t_k} = \sum_{k=1}^{n} C_{t_k} (1+i)^{-t_k}$$

In this case, the effective duration is defined to be:

$$\upsilon = -\frac{1}{PV}\frac{d}{di}PV = \frac{\sum_{k=1}^{n} C_{t_k} t_k (1+i)^{-t_k-1}}{\sum_{k=1}^{n} C_{t_k} (1+i)^{-t_k}} = \frac{\sum_{k=1}^{n} C_{t_k} t_k v_i^{t_k+1}}{\sum_{k=1}^{n} C_{t_k} v_i^{t_k}} = \frac{\sum_{k=1}^{n} C_{t_k} t_k v_i^{t_k+1}}{PV}$$

Another measure of interest rate sensitivity is the ***duration***, also called the ***discounted mean term*** (DMT) or Macaulay's duration.

This is the mean term of the cash flows, weighted by the present values of the cash flows. It can be regarded as a weighted average time to maturity, where the weighting factor is the ratio of the present value of each cash flow to the present value of the total cash flows.

$$\text{DMT} = \tau = \frac{\sum_{k=1}^{n} C_{t_k} t_k v_i^{t_k}}{\sum_{k=1}^{n} C_{t_k} v_i^{t_k}} = \frac{\sum_{k=1}^{n} C_{t_k} t_k v_i^{t_k}}{PV} = (1+i)\upsilon$$

For a bond with a face value of F, redemption amount of C, paying coupons of Fr per half-year, we know that the price is:

$$P = Fr \cdot a_{\overline{n}|j} + C \cdot v_j^n = \sum_{t=1}^{n} Fr \cdot v_j^t + C \cdot v_j^n$$

where n is the number of coupon payments and j is the half-yearly redemption yield.

The duration of this bond is:

$$\tau = \frac{\sum_{t=1}^{n} t \cdot Fr \cdot v_j^t + n \cdot C \cdot v_j^n}{\sum_{t=1}^{n} Fr \cdot v_j^t + C \cdot v_j^n} = \frac{Fr \cdot \left(Ia\right)_{\overline{n}|j} + n \cdot C \cdot v_j^n}{P}$$

Note that this formula calculates duration in half years. The value in years is simply half of the above calculation. This approach applies to any duration calculation where payments are more frequent than annually, and you use an interest rate equivalent to the payment frequency. However, this approach does not work for volatility as the time for discounting and multiplication are not consistent within the formula.

Note that the duration of an n-year zero-coupon bond is just equal to the term to maturity of the bond:

$$\tau = \frac{n \cdot C \cdot v_i^n}{C \cdot v_i^n} = n$$

This last result should be obvious - the average term of a series of cash flows that has only one payment must be the time of that cash flow.

There are other measures of duration, some which allow for a yield curve that is not flat. These are beyond the scope of this course.

Duration can be used to compare the risk of losses or gains for different bonds:

- If interest rates increase and bond values decline, an investor will have a greater risk of loss in value for bonds with greater duration.

- If interest rates decline and bond values increase, an investor will have a greater potential gain for bonds with greater duration.

In other words, the larger the duration (or volatility), the larger the sensitivity of a series of cash flows to an interest rate movement.

EXAMPLE
Find the duration for a $100 zero-coupon bond redeemable at par in 20 years. Assume a half-yearly effective yield of 3%.

Solution

$\tau = \dfrac{n \cdot C \cdot v_i^n}{C \cdot v_i^n} = n = 40$ half-years, or 20 years.

EXAMPLE
Find the duration for a $100 bond paying half-yearly coupons of 4% per annum, redeemable at par in 20 years. Assume a nominal yield compounded half-yearly of 6% p.a.

Solution
The present value of this bond is:

$$P = Fr \cdot a_{\overline{n}|j} + C \cdot v_j^n = (100)(0.02) \cdot a_{\overline{40}|0.03} + 100 \cdot v_{0.03}^{40} = 76.89$$

The duration is given by:

$$\tau = \frac{Fr \cdot (Ia)_{\overline{n}|j} + n \cdot C \cdot v_j^n}{P} = \frac{2 \cdot (Ia)_{\overline{40}|0.03} + 4000 \cdot v_{0.03}^{40}}{76.89} = 25.96 \text{ half-years (or 12.98 years)}$$

In this example the duration is less than the time to maturity. This arises because duration is the weighted average of the time to maturity of each of the cash flows. Apart from the redemption value and the last coupon payment, each cash flow occurs before maturity, which ensures that the duration is less than the time to maturity of the bond.

Note that the duration is dependent on the interest rate used to calculate the present value, as well as the amounts and timings of cash flows.

An implication is that a movement in interest rates will not only affect the price of a bond, but will also affect the duration, and the impact on the duration will vary depending on the initial interest rate.

EXAMPLE
Find the duration for a \$100 bond paying half-yearly coupons of 4% per annum, redeemable at par in 20 years. Assume a nominal yield compounded half-yearly of 20% p.a.

Solution
The present value of this bond is:

$$P = Fr \cdot a_{\overline{n}|j} + C \cdot v_j^n = (100)(0.02) \cdot a_{\overline{40}|0.10} + 100 \cdot v_{0.10}^{40} = 21.77$$

The duration is given by:

$$\tau = \frac{Fr \cdot \left(Ia\right)_{\overline{n}|j} + n \cdot C \cdot v_j^n}{P} = \frac{2 \cdot \left(Ia\right)_{\overline{40}|0.10} + 4000 \cdot v_{0.10}^{40}}{21.77} = 13.13 \text{ half-years (or 6.56 years)}$$

The duration is much lower than the previous example, despite the fact that the timing of the coupons and redemption payments are the same in both examples. This is because the higher interest rate for the second example has the effect of giving more weight to the earlier payments than later payments, thereby reducing the duration.

APPROXIMATIONS
In addition to acting as a measure for comparing the sensitivity of different cash flows, duration and volatility can be used to approximate the present value of a series of cash flows following a small change in interest rates.

To show this we first need to introduce Taylor's approximation which is an expansion of a function into a series of terms. Taylor's theorem states that for small ε, the function $f(x+\varepsilon)$ can be written as:

$$f(x+\varepsilon) = f(x) + \varepsilon f'(x) + \frac{\varepsilon^2}{2!} f''(x) + \ldots$$

Although the function can be expanded into a much larger number of terms, we concentrate on the second order expansion (ie. the expansion that includes the first and second derivatives). With each additional term the accuracy of the approximation increases, however, the magnitude of each additional term is much smaller than the last.

We wish to find the present value of a series of cash flows (calculated originally at interest rate i_0) if there is a shift to a new interest rate $i_0 + \varepsilon$. If we write the present value of a series of cash flows calculated at interest rate i_0 as $PV(i_0)$, from Taylor's theorem, we can expand the present value calculated at interest rate $i_0+\varepsilon$ as:

$$PV(i_0+\varepsilon)=PV(i_0)+\varepsilon PV'(i_0)+\frac{\varepsilon^2}{2}PV''(i_0)+\ldots$$

where $PV'(i_0)$ and $PV''(i_0)$ are the first and second derivatives of $PV(i_0)$ respectively.

Subtracting $PV(i_0)$ from both sides and dividing by $PV(i_0)$ gives:

$$\frac{PV(i_0+\varepsilon)-PV(i_0)}{PV(i_0)}=\varepsilon\frac{PV'(i_0)}{PV(i_0)}+\frac{\varepsilon^2}{2}\frac{PV''(i_0)}{PV(i_0)}+\ldots$$

where the left-hand-side of this equation is the percentage change in present value.

If we ignore the second term in the expansion for the time being, and if we know the duration τ or the volatility υ at interest rate i_0, then the percentage change in present value can be approximated by:

$$\varepsilon\frac{PV'(i_0)}{PV(i_0)}=-\varepsilon\upsilon=-\varepsilon\frac{\tau}{(1+i_0)}\,.$$

EXAMPLE
The price of a particular bond is $21.77 valued using a nominal yield of 20% p.a. convertible half-yearly, and has a duration of 13.13 half-years.

Find the approximate value of the bond if interest rates move to 11% per half-year using the one-term Taylor series expansion introduced above.

Solution
For the purposes of this question, we work with effective half-yearly interest rates and duration. Using the approximation method just introduced, the original interest rate $i_0 = 10\%$, the duration $\tau = 13.13$ and the change in interest rate $\varepsilon = 0.01$. The percentage change in price is approximately:

$$-\varepsilon \frac{\tau}{(1+i_0)} = -0.01 \frac{13.13}{(1.10)} = \text{-}0.1194$$

Therefore, the approximate value of the bond if interest rates move to 11% is:

$$\$21.77(1-0.1194) = \$19.17$$

Note that you must work with interest rates and duration that are consistent in their effective period to obtain the correct answer. In the above example the interest rate and duration were both in half years. Alternatively, you could compute the answer based on years;

where $i_0 = 21\%$ and $\varepsilon = 0.0221$:

$$-\varepsilon \frac{\tau}{(1+i_0)} = -0.0221 \frac{6.56}{(1.21)} = \text{-}0.1198$$

Therefore, the approximate value of the bond if interest rates move to 11% is:

$$\$21.77(1-0.1198) = \$19.16$$

The actual value at 11% based on the bond from the previous example is $19.44.

The inaccuracy of the approximation is partly due to the omission of the second derivative in the Taylor expansion. The second derivative is related to the spread of a set of cash flows about their duration.

This second derivative is used to calculate a measure known as ‘convexity’ which is introduced next.

<u>CONVEXITY</u>

To determine more precisely the impact of a change in the interest rate on the present value of a series of cashflows, we need to introduce a quantity called ***convexity***.

Convexity is a measure of the change in volatility of a bond when the interest rate changes.

The convexity of the cash flow series $\{C_{t_k}\}$ is defined as:

$$c(i) = \frac{1}{PV}\frac{d^2}{di^2}PV = \frac{1}{PV}\frac{d}{di}\left(-\sum_{k=1}^{n} C_{t_k} t_k (1+i)^{-t_k-1}\right) = \frac{\sum_{k=1}^{n} C_{t_k} t_k (t_k+1) v_i^{t_k+2}}{PV}$$

(where $\frac{d}{di}PV = -\sum_{k=1} C_{t_k} t_k (1+i)^{-t_k-1}$)

For cash flow series with the same duration, a cash flow series consisting of payments paid close together will have a low convexity, whereas a series that is more spread out over time will have a higher convexity.

EXAMPLE

Calculate, using 10% interest, the convexity of the following assets, each of which have a duration of 11 years.

- Asset A is an 11-year zero coupon bond.
- Asset B will provide a lump sum payment of \$9,663 in 5 years' time and a lump sum payment of \$26,910 in 20 years' time.
- Asset C is a level perpetuity payable annually in arrears.

Solution

The present value of \$100 nominal of Asset A at interest rate i is:

$$P_A(i) = 100(1+i)^{-11}$$

Using the derivative formula above, the convexity is:

$$c_A(i) = \frac{P_A''(i)}{P_A(i)} = \frac{100(-11)(-12)(1+i)^{-13}}{100(1+i)^{-11}} = \frac{132}{(1+i)^2}$$

When $i = 10\%$, this gives:

$$c_A(0.1) = \frac{132}{(1.1)^2} = 109.1$$

The present value of Asset B at interest rate i is:

$$P_B(i) = 9,663(1+i)^{-5} + 26,910(1+i)^{-20}$$

So its convexity is:

$$c_B(i) = \frac{P_B''(i)}{P_B(i)} = \frac{9663(-5)(-6)(1+i)^{-7} + 26,910(-20)(-21)(1+i)^{-22}}{9,663(1+i)^{-5} + 26,910(1+i)^{-20}}$$

When $i = 10\%$, this gives:

$$c_B(0.1) = \frac{148,759 + 1,388,430}{6,000 + 4,000} = 153.7$$

The present value of Asset C (assumed to consist of payments of 1 unit) at interest rate i is:

$$P_C(i) = \frac{1}{i}$$

So its convexity is:

$$c_C(i) = \frac{P_C''(i)}{P_C(i)} = \frac{(-1)(-2)i^{-3}}{i^{-1}} = \frac{2}{i^2}$$

When $i = 10\%$, this gives:

$$c_C(0.1) = \frac{2}{0.1^2} = 200$$

Since Asset A is a single payment the convexity is lowest for A. The convexity is highest for Asset C because the payments are spread out over an infinite time period.

Note that the above calculation is based on annual time periods and interest rates. Thus, for calculations involving payments more frequently than annually, such as coupon paying bonds, the convexity should be calculated with reference to yearly time periods (i.e. t_k should be in terms of years). This is of particular importance when combining duration and convexity in Taylor expansions and Immunisation (as described below) as it is necessary to ensure that duration and convexity are both calculated with reference to the same time period.

EXAMPLE
Find the convexity of a $10,000 bond maturing in two years, paying half-yearly coupons on 13% p.a. at a nominal yield of 6% p.a. convertible half-yearly.

Solution

$$c(i) = \frac{\sum_{k=1}^{n} C_{t_k} t_k (t_k + 1) v_i^{t_k+2}}{PV}$$

$$= \frac{0.065 \times 10,000\left(0.5 \times 1.5 \times v_{0.03}^{2.5\times 2} + 1 \times 2 \times v_{0.03}^{3\times 2} + 1.5 \times 2.5 \times v_{0.03}^{3.5\times 2} + 2 \times 3 \times v_{0.03}^{4\times 2}\right) + 10,000 \times 2 \times 3 \times v_{0.03}^{4\times 2}}{0.065 \times 10,000 \times a_{\overline{4}|0.03} + 10,000 v_{0.03}^{4}}$$

$$= \frac{53,934.41}{11,300.98} = 4.77254$$

If the convexity of a series of cash flows is known, combining duration and convexity in the Taylor expansion introduced previously gives a more accurate approximation than duration alone:

$$\frac{PV(i_0+\varepsilon)-PV(i_0)}{PV(i_0)} \cong \varepsilon\frac{PV'(i_0)}{PV(i_0)}+\frac{\varepsilon^2}{2}\frac{PV''(i_0)}{PV(i_0)} = -\varepsilon\frac{\tau}{(1+i_0)}+\frac{\varepsilon^2}{2}c$$

EXAMPLE
The price of a particular bond is \$21.77 valued using a yield of 10% per half-year, and the bond has a duration of 6.56 years and a convexity of 66.

Find the approximate value of the bond if interest rates move to 11% per half-year using the two-term Taylor series expansion.

Solution

$$-\varepsilon\frac{\tau}{(1+i_0)} = -0.0221\frac{6.56}{(1.21)} = \text{-}0.1198$$

$$\frac{\varepsilon^2}{2}c = \frac{(0.0221)^2}{2}66 = 0.0161$$

The percentage change is –0.1198 + 0.0161 = -0.1037

Therefore, the approximate value of the bond if interest rates move to 11% is:

$$\$21.77(1-0.1037) = \$19.51.$$

This is very close to the actual value of \$19.44.

Convexity and duration calculations are needed in order to carry out a technique called immunisation.

IMMUNISATION

So far we have introduced some simple measures of a cash flow's vulnerability to interest rate movements.

These measures are used when immunising a portfolio.

Immunisation is the process of selecting a portfolio of assets that will protect a fund's surplus against small changes in interest rates.

We will first define what we mean by 'surplus' and 'deficit'.

Suppose an institution holds assets of value V_A and has liabilities V_L. Since both V_A and V_L are the discounted value of future cash flows, both are sensitive to the rate of interest. If interest rates fall then both V_L and V_A will rise. If interest rates rise then both V_L and V_A will fall.

If $V_A > V_L$ then we say that the fund is in surplus. If $V_A < V_L$ then the fund is in deficit.

Of particular concern is the risk that a change in interest rates causes a surplus to become a deficit. This could occur if interest rates rise and the subsequent decline in V_A is greater than the decline in V_L, or conversely if interest rates fall and the rise in V_L is greater than the rise in V_A.

Let the surplus of a fund be $S(i)$ when the present value of the assets and liabilities are calculated at a valuation rate of interest i:

$$S(i) = V_A(i) - V_L(i)$$

A fund is said to be immunised against small changes in the interest rate if:

- The surplus in the fund at the current interest rate is zero and
- Any small change in the interest rate (in either direction) would lead to a positive surplus.

Thus, at rate of interest i_0 the fund is immunised against small movements in the rate of interest of ε if and only if $V_A(i_0) = V_L(i_0)$ and $V_A(i_0 + \varepsilon) \geq V_L(i_0 + \varepsilon)$

In the 1950s the actuary Frank Redington derived three conditions that are required to achieve immunisation.

We derive these conditions by using Taylor's theorem.
From Taylor's theorem we can expand the surplus calculated at interest rate $i_0 + \varepsilon$ as:

$$S(i_0 + \varepsilon) = S(i_0) + \varepsilon S'(i_0) + \frac{\varepsilon^2}{2} S''(i_0) + \ldots$$

Immunisation is achieved if $S(i_0) = 0$ and $S(i_0 + \varepsilon) \geq 0$. One way this can be achieved is by setting $S(i_0)$ and $\varepsilon S'(i_0)$ equal to zero, and by ensuring that $\frac{\varepsilon^2}{2} S''(i_0) \geq 0$.

The first condition is that the surplus at the current interest rate is zero. That is, $S(i_0) = 0$.

The second term $\varepsilon S'(i_0)$ will be equal to zero if and only if $S'(i_0) = 0$. This is satisfied if $V_A'(i_0) = V_L'(i_0)$.

Since the first condition requires $V_A(i_0) = V_L(i_0)$, $V_A'(i_0) = V_L'(i_0)$ is equivalent to:

$$-\frac{V_A'(i_0)}{V_A(i_0)} = -\frac{V_L'(i_0)}{V_L(i_0)}$$

Since the volatility is $\upsilon = -\frac{1}{PV}\frac{d}{di}PV = -\frac{PV'}{PV}$, the expression above equates the volatilities of the assets and liabilities.

Therefore, the second condition is that the assets and liabilities must have the same volatility. Since the duration $\tau = (1+i)\upsilon$, this is equivalent to requiring that the durations of the two cash flow series are the same:

$$\tau_A(i_0) = \tau_L(i_0)$$

The third condition is that $\frac{\varepsilon^2}{2} S''(i_0) \geq 0$. ε^2 is always positive, so we need to ensure that $S''(i_0) \geq 0$ or equivalently that $V_A''(i_0) \geq V_L''(i_0)$.

As before, since the first condition requires $V_A(i_0) = V_L(i_0)$, $V_A''(i_0) \geq V_L''(i_0)$ is equivalent to:

$$\frac{V_A''(i_0)}{V_A(i_0)} \geq \frac{V_L''(i_0)}{V_L(i_0)}$$

Since the convexity $c = \frac{1}{PV}\frac{d^2}{di^2}PV = \frac{PV''}{PV}$, the expression above is equivalent to requiring that the convexity of the assets is equal to or greater than the convexity of the liabilities:

$$c_A(i_0) \geq c_L(i_0)$$

Summarising,

The conditions for immunisation are as follows:

1. $S(i_0) = V_A(i_0) - V_L(i_0) = 0$. The present value of the assets and liabilities should be equal at the starting rate of interest.

2. The volatilities or durations of the asset and liability cash flow series should be equal: $\tau_A(i_0) = \tau_L(i_0)$ or $\upsilon_A(i_0) = \upsilon_L(i_0)$

3. The convexity of the asset cash flow series should be greater than or equal to the convexity of the liability cash flow series: $c_A(i_0) \geq c_L(i_0)$

EXAMPLE
A fund must make payments of $50,000 at the end of the sixth and eighth years. Show that, if interest rates are currently 7% *(at all durations)*, immunisation to small changes in interest rates can be achieved by holding an appropriately chosen combination of a 5 year zero coupon bond and a 10 year zero coupon bond.

Solution
Let P and Q denote the maturity values of the 5 year and 10 year zero coupon bonds.

Then the present value of the assets is:
$V_A(0.07) = Pv_{0.07}^5 + Qv_{0.07}^{10} = (0.71299)P + (0.50835)Q$

The present value of the liabilities is:
$V_L(0.07) = 50{,}000(v_{0.07}^6 + v_{0.07}^8) = 62{,}418$

The first condition for immunisation is that the PV of the asset and liabilities must be equal:

$$\Rightarrow (0.71299)P + (0.50835)Q = 62{,}418 \qquad (1)$$

The second condition for immunisation is that the volatilities or durations of the assets and liabilities must be equal:

$$\upsilon_A(0.07) = -\frac{V_A'(0.07)}{V_A(0.07)} = -\frac{-5Pv^6 - 10Qv^{11}}{Pv^5 + Qv^{10}} = \frac{(3.3317)P + (4.7509)Q}{62{,}418}$$

$$\upsilon_L(0.07) = -\frac{V_L'(0.07)}{V_L(0.07)} = -\frac{50{,}000(-6v^7 - 8v^9)}{62{,}418} = \frac{404{,}398}{62{,}418}$$

$$\Rightarrow (3.3317)P + (4.7509)Q = 404{,}398 \qquad (2)$$

(Remember that $\dfrac{dv^n}{di} = \dfrac{d(1+i)^{-n}}{di} = -n(1+i)^{-n-1} = -nv^{n+1}$)

Solving equations (1) and (2) we get $P = \$53{,}710$ and $Q = \$47{,}454$.

This determines the portfolio of assets we require. We now need to check the third condition. With these values of P and Q, the convexity of the assets is:

$$c_A(0.07) = \frac{V_A''(i_0)}{V_A(i_0)} = \frac{30Pv^7 + 110Qv^{12}}{62{,}418} = \frac{3{,}321{,}152}{62{,}418} = 53.21$$

The convexity of the liabilities is:

$$c_L(0.07) = \frac{V_L''(0.07)}{V_L(0.07)} = \frac{50,000(42v^8 + 72v^{10})}{62,418} = \frac{3,052,277}{62,418} = 48.90$$

Since $c_A(0.07) \geq c_L(0.07)$, all three conditions for immunisation are satisfied and the fund is immunised against small changes in the interest rate around 7%.

Although we have not been asked to verify the result, we could do a rough check that the fund is immunised by showing that the surplus is positive for interest rates on either side of 7%, say 6.5% and 7.5%:

$$V_A(0.065) = Pv_{0.065}^5 + Qv_{0.065}^{10} = (53,710)v_{0.065}^5 + (47,454)v_{0.065}^{10} = 64,482$$
$$V_L(0.065) = 50,000\left(v_{0.065}^6 + v_{0.065}^8\right) = 64,478$$
$$\Rightarrow S(0.065) = 4$$

$$V_A(0.075) = Pv_{0.075}^5 + Qv_{0.075}^{10} = (53,710)v_{0.075}^5 + (47,454)v_{0.075}^{10} = 60,437$$
$$V_L(0.075) = 50,000\left(v_{0.075}^6 + v_{0.075}^8\right) = 60,433$$
$$\Rightarrow S(0.075) = 4$$

In practice there are difficulties with implementing an immunisation strategy based on these principles. For example the method requires continuous rebalancing of portfolios to keep the assets and liability volatilities equal. Other limitations include:

- Assets may not exist to provide the necessary overall asset volatility to match the liability volatility.

- The theory relies upon small changes in interest rates. The fund may not be protected against large changes.

- The theory assumes a flat yield curve and requires the same change in interest rates at all terms. In practice this is rarely the case.

In addition, it should be recognised that although immunisation removes the risk of fund deficit from interest rate movements, it also reduces the likelihood of making large profits.

Because of its restrictions, actuaries rarely apply Redington's immunisation theory directly. A technique called asset-liability modelling is often used instead. Asset-liability modelling involves investigating the future financial outcomes of a company where the assets, liabilities or both may vary. It often involves the use of stochastic assumptions and modelling techniques. You will meet asset-liability modelling in later year subjects.