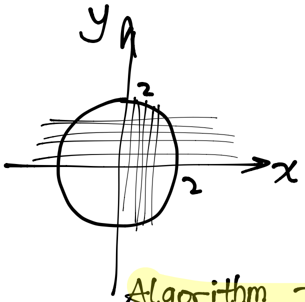
Lecture 34

Chapter 16 Julia Set

$Q_c(z) = z^2 + c$, $z, c \in \mathbb{C}$

filled Julia Set $K_c = \{ z \in \mathbb{C}$, orbit of z under Q_c is bounded $\}$

Julia Set, J_c = boundary of $K_c = \overline{K_c} - K_c$



Theorem: If $|z| > \text{Max}\{|c|, 2\}$, then $|Q^n_c(z)| \to \infty$ as $n \to \infty$

(not Julia set)

Algorithm for filled Juliet Set.

① Choose a max number of iterations N.

② Fix a grid of points

③ For each point z in the grid, compare the first N iterations: z_i for $i = 0, \cdots N$

④ If $|z_i| > \text{Max}\{|c|, 2\}$ for some i, then color z_0 white

⑤ If $|z_i| \leq \text{Max}\{|c|, 2\}$ for all $i = 0, \cdots N$, then color z_0 black.

K_{-1}

$K_{-0.835 - 0.2321i}$

Remarks:

① The filled Julia Sets are self-similar

② K_c sometimes consists of isolated points

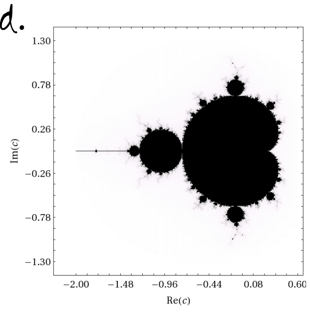
Chapter 17 Mandelbrot Set

$Q_c(z) = z^2 + c$

Remark

① If the orbit of 0 under Q_c escapes to infinity, then K_c consists of infinitely many disjoint components.

② If the orbit of 0 remains bounded, then K_c remains connected.



(points in black are in the Mandelbrot set)

Mandelbrot Set

$\mathcal{M} = \{ c \in \mathbb{C} :$ orbit of 0 under Q_c is bdd $\}$
$= \{ c \in \mathbb{C} : K_c$ is connected $\}$

Algorithm of $\mathcal{M}$:

① Choose N = the max # of iterations

② fix a grid of points

③ for each c in the grind, compute the first N iterations of the orbit of 0 under Q_c . $\underset{0}{\overset{\|}{z_0}}, z_1, \cdots, z_N$.

Remarks:

① From the definition, a plot $\underline{c}$ is black on the Mandelbrot set if the orbit of 0 under Q_c is bounded.

② For real values of C, we know exactly which ones are in $\mathcal{M}$: $-2 \leq C \leq \frac{1}{4}$

③ $\mathcal{M}$ consists of many bulbs

The parameter plane of quadratic polynomials - that is, the plane of possible c-values - gives rise to the famous Mandelbrot set. Indeed, the Mandelbrot set is defined as the set of all c such that J_c is connected.