

CSC 236 2015 Winter

Program Correctness Exercise

1. Consider the following function:

```
# PREcondition: n is a positive natural number
# POSTcondition: return the floor of the log_2 of n, i.e.
# a natural number g with  $2^g \leq n < 2^{(g+1)}$ .
def f(n):
    k = n
    d = 0
    while k > 1:
        k = k // 2
        d = d + 1
    return d
```

- (a) Write down the corresponding mathematical recurrence for the values of the changing variables.
- (b) Develop a Loop Invariant.
- (c) Assuming the PREcondition, prove that the Invariant is always true.
- (d) Assuming the PREcondition and that the loop terminates, prove that the POSTcondition is true.
- (e) Develop a Variant.
- (f) Assuming the PREcondition, prove that the Variant is a decreasing sequence of natural numbers.

2. Prove that the following function is correct:

```
# PREcondition: n is a positive natural number
# POSTcondition: return the floor of the log_2 of n, i.e.
# a natural number g with  $2^g \leq n < 2^{(g+1)}$ .
def f(n):
    if n > 1:
        return f(n // 2) + 1
    else:
        return 0
```