

**THE AUSTRALIAN NATIONAL UNIVERSITY
RESEARCH SCHOOL OF FINANCE, ACTUARIAL
STUDIES AND APPLIED STATISTICS**

Second Semester Examination 2014

**FINANCIAL MATHEMATICS
(STAT 2032 / STAT 6046)**

Study Period: 15 minutes

Writing Period: 3 hours

Permitted Materials:

Non-Programmable Calculators

Dictionaries (must not contain material added by the student)

Total Marks: 100

Instructions to all candidates:

- *Attempt all nine questions*
- *Start your solution to each question on a new page*
- *To ensure full marks show all the steps in working out your solution. Marks may be deducted for failure to show appropriate calculations or formulae.*
- *Formula sheet, compound interest rate tables and statistical tables are attached at the end of this exam paper.*

Question 1**(10 marks)**

An investor enters into a contract under which he will pay 15 level premiums of \$600 annually in advance into an account which accumulates at the force of interest $\delta(t)$, where t is measured in years, given by the formula:

$$\delta(t) = \begin{cases} 0.08 & 0 \leq t < 5 \\ 0.06 & 5 \leq t < 10 \\ 0.04 & 10 \leq t \end{cases}$$

- a. Derive an expression for the present value of \$1 due at time t . (Note: You do not need to provide a numerical answer.) **(3 marks)**

- b. At the end of 15 years, what is the balance of the investor's account? **(4 marks)**

- c. Suppose at the end of 15 years, the balance, calculated in part (b), is withdrawn semi-annually using 8 fixed level amounts. The first withdrawal is made exactly 1 year after the last premium is deposited. Calculate the amount of the fixed semi-annual withdrawal. **(3 marks)**

Question 2

(10 marks)

An actuarial student has taken out two loans:

- **Loan A:** A 5 year car loan for \$8,991 repayable by equal monthly instalments in arrears with an interest rate of 12% pa convertible monthly.
- **Loan B:** A 10 year personal loan of \$19,966 repayable by equal monthly instalments in arrears with an interest rate of 7.5% pa convertible monthly.

The student continues to pay off both of these loans for a period of 2 years.

- a. What is the total balance outstanding at the end of 2 years for both the loans to the nearest dollar? (6 marks)
- b. Now suppose at the end of 2 years, the two loans are rolled into a single loan which is being offered at an effective rate of interest of 9% pa effective. If the new monthly repayment is the sum of the two original loan repayments, calculate the time it takes to payback the new single loan. (4 marks)

Question 3

(6 marks)

A bond with a face value of \$250,000 is issued bearing coupons payable quarterly in arrears at a rate of 8% p.a. The bond can be redeemed at 110% if redeemed on a coupon paying date between 10 and 15 years. If the bond is redeemed between 15 and 20 years then the bond can be redeemed at par. The date of redemption is at the option of the buyer of the bond.

An investor who is liable to income tax at 25% and capital gains tax at 30% wishes to purchase this bond. Calculate the purchase price to ensure a net effective yield of at least 5% p.a. effective to the investor.

Question 4**(18 marks)**

Dream developers have bought a piece of land on the Sydney harbour side for \$5,000,000 and plan to build a luxury home on this land. The construction will be completed in 2 years' time and will incur a cost of \$7,000,000 paid continuously over the 2 years. Dream developers can borrow or invest funds at a rate of 15% p.a. effective.

On completion, Dream developers have 2 possible strategies:

1. Sell the house in 3 years' time from now for \$16,500,000
2. Sell the house in 4 years' time from now for \$18,000,000

They are also able to receive rental income from the house between the times of completion to the time of sale of \$500,000 p.a. payable quarterly in advance. The rental income will increase by \$50,000 p.a. at the beginning of each year that the rental income is paid.

- a.** Derive an expression, using annuity functions, for the present value of \$1 p.a. paid m times a year in advance at an effective interest rate of $i\%$ p.a. Alternatively, show that:

$$(I\ddot{a})_{\overline{n}|}^{(m)} = \frac{\ddot{a}_{\overline{n}|} - nv^n}{d^{(m)}} = \frac{i}{d^{(m)}} (Ia)_{\overline{n}|} = \frac{d}{d^{(m)}} (I\ddot{a})_{\overline{n}|} \quad \textbf{(5 marks)}$$

- b.** Determine the optimum strategy for the sale of the house using NPV as the decision criteria. **(8 marks)**
- c.** If instead the house is sold in 6 years' time, the developer believes that they can obtain an IRR of the project of 17.5% p.a. Calculate the sale price that the developer will receive to achieve an IRR of 17.5% pa. **(5 marks)**

Question 5**(17 marks)**

A self-managed superannuation fund has two pay-outs due in 2 and 5 years' time of \$2.5 million each.

- a. Calculate the present value of these liabilities at an interest rate of 4% p.a. effective. **(3 marks)**

- b. Calculate the duration of the liabilities at the same interest rate of 4% p.a. effective. **(3 marks)**

The fund's assets are invested in a 6 year zero coupon bond and a 3 year 8% p.a. annual coupon paying bond which can be redeemed at par. The nominal value of the two bond purchases are set such that the fund is immunised for small changes in interest rates.

- c. Find the nominal values of the two bonds. **(8 marks)**

- d. Approximate the change in present values of assets and liabilities when the interest rate changes to 3.5% using the duration calculated in part (b). **(3 marks)**

Question 6**(5 marks)**

An investor entered into a short forward contract for \$100 nominal of a security 8 years ago, and the contract is due to mature in 4 years' time. Eight years ago the price of the security was \$94.50 and the current price of the security is \$143 for every \$100 nominal of the security. The risk free rate of interest can be assumed to be 5% p.a. effective throughout the contract.

Two year from now, the security will pay a coupon of \$9 per \$100 nominal and three years from now, the security will pay a coupon of \$10 per \$100 nominal. Assume that the coupon information was known 8 years ago when the contract was agreed upon. Calculate the value of the short forward contract.

Question 7**(10 marks)**

The annual returns, i , on a fund are independent and identically distributed. Each year, the distribution of $1 + i$ is log-normal with parameters $\mu = 0.05$ and $\sigma^2 = 0.004$.

- a. Calculate the expected accumulated value in 25 years' time, of \$300 invested at the end of each year. **(4 marks)**
- b. Calculate the probability that the accumulation of \$1 invested at time 0 is greater than its expected value in 25 years' time. **(6 marks)**

Question 8**(14 marks)**

Consider the following unit prices in two separate accumulation funds over the period 1 Jan 2013 to 1 Jan 2014.

Fund	Unit Prices				
	2013				2014
	1-Jan	1-Apr	1-Jul	1-Oct	1-Jan
Property	1.24	1.31	1.48	1.58	1.64
Equity	1.21	0.92	1.03	1.31	1.55

- a. Find the time-weighted rate of return for each fund for the year 2013. **(4 marks)**
- b. Adam has bought units in the property fund on 1 January 2013, 1 April 2013, 1 July 2013 and 1 October 2013 and sold the units on 1 January 2014. Find the money-weighted return on the completed transaction if
- i. He bought 1000 units on each date. **(5 marks)**
- ii. He invested \$1000 on each date. **(5 marks)**

Question 9**(10 marks)**

Assume the following treasury spot rates:

Years to Maturity	Spot Rate (p.a.)
0.5	5.0%
1.0	5.4%
1.5	5.8%
2.0	6.4%
2.5	7.0%
3.0	7.2%
3.5	7.4%
4.0	7.8%

Compute:

- a. The 1 year forward rate 2.5 years from now. **(2 marks)**

- b. The price at time 0, of a 2 year \$100 bond with semi-annual coupons of 8% p.a. issued in 1.5 years' time and redeemed at par. **(4 marks)**

- c. The accumulated value at the end of 4 years for half-yearly payments of \$50 paid at the end of each half-year for the first 2 years. **(4 marks)**

End of Examination