Duration: **60 minutes**
Aids Allowed: **none**

Student Number: ⌞_⌟_⌟_⌟_⌟_⌟_⌟_⌟_⌟_⌟_⌟

Family Name(s): _____

Given Name(s): _____

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*Do **not** turn this page until you have received the signal to start.*
In the meantime, please read the instructions below carefully.

This term test consists of 4 questions on 12 pages (including this one), printed on both sides of the paper. *When you receive the signal to start, please make sure that your copy of the test is complete, fill in the identification section above, and write your name on the back of the last page.*

Answer each question directly on the test paper, in the space provided, and use one of the "blank" pages for rough work. If you need more space for one of your solutions, use a "blank" page and *indicate clearly the part of your work that should be marked.*

In your answers, you may use without proof any theorem or result covered in lectures, tutorials, homework, tests, or the textbook, as long as you give a clear statement of the result(s)/theorem(s) you are using. You must justify all other facts required for your solutions.

Write up your solutions carefully! In particular, use notation and terminology correctly and explain what you are trying to do — part marks *will* be given for showing that you know the general structure of an answer, even if your solution is incomplete.

MARKING GUIDE

1: _____/ 8

2: _____/ 8

3: _____/10

4: _____/10

BONUS
MARKS: _____/ 3

TOTAL: _____/36

Good Luck!

Use the space on this "blank" page for scratch work, or for any solution that did not fit elsewhere.
Clearly label each such solution with the appropriate question and part number.

Question 1. [8 MARKS]

Part (a) [4 MARKS]

Write a detailed proof *structure* for the following statement.

$$\forall x \in D, \big(P(x) \vee Q(x) \Rightarrow \exists y \in D, R(x,y)\big)$$

NOTE: You need only design the *structure* of a proof for a general statement of this form.

Part (b) [4 MARKS]

Fill in every missing conclusion in the following proof structure.

Assume $x \in X$.
    Assume $A(x)$.
        Assume $\neg B(x)$.
            ... derive a contradiction...

        Then _____

        Let $y_0 = \ldots$
        Then $y_0 \in X$.
            $\vdots$
        Then $C(x, y_0)$.

        Then _____

    Then _____

Then _____

Use the space on this "blank" page for scratch work, or for any solution that did not fit elsewhere.
Clearly label each such solution with the appropriate question and part number.

Question 2. [8 MARKS]

Consider the following statement:

> If the product of two real numbers is negative, then one number is positive and the other number is negative.

Part (a) [2 MARKS]

Translate the statement into symbolic notation.

Part (b) [6 MARKS]

Write a detailed proof of the statement. (HINT: Use proof structures and, for part of your proof, consider the sign of one of the numbers.)

Use the space on this "blank" page for scratch work, or for any solution that did not fit elsewhere.
Clearly label each such solution with the appropriate question and part number.

Question 3. [10 MARKS]

Consider the following statement:

For all real numbers x, $x^2 + x < 0$ if and only if $-1 < x < 0$.

NOTE: The statement "$-1 < x < 0$" means "$-1 < x$ and $x < 0$."

Part (a) [2 MARKS]

Translate the statement into symbolic notation.

Part (b) [8 MARKS]

Write a detailed proof of the statement. (HINT: Use proof structures and the statement from Question 2.)

Use the space on this "blank" page for scratch work, or for any solution that did not fit elsewhere.
Clearly label each such solution with the appropriate question and part number.

Question 4. [10 MARKS]

Recall that for all integers m and n, we write "$m \mid n$" when m divides n **exactly**. Two facts that follow from the definition of "$m \mid n$" are that $3 \mid 0$ and $0 \mid 0$.

Consider the following statement:

> For all integers a and b, if $a \mid b$ and $b \mid a$ then $a = \pm b$.

NOTE: The expression "$a = \pm b$" means "$a = b$ or $a = -b$."

Part (a) [2 MARKS]

Translate the statement into symbolic notation.

Part (b) [8 MARKS]

Write a detailed proof of the statement. (HINT: Use proof structures.)

Use the space on this "blank" page for scratch work, or for any solution that did not fit elsewhere.
Clearly label each such solution with the appropriate question and part number.

Bonus. [3 MARKS]

WARNING! This question will be marked harshly: credit will be given only for making *significant* progress toward a correct answer. Please attempt this only *after* you have completed the rest of the test.

Describe *two different approaches* you might use to try and answer the question below. Describe clearly the expected outcome of each of your strategies. (Note that you are **not** being asked to actually answer the question; doing so will earn you **no credit**.)

QUESTION: Determine every integer value of n for which the following loop eventually halts.

```
# Assume n is an integer.
while n > 1:
    if n % 2 == 0:  # i.e., if n is even
        n = n / 2
    else:
        n = 3 * n + 1
```

On this page, please write nothing except your name.

Family Name(s): _____

Given Name(s): _____

Total Marks = 36