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UNIVERSITY OF TORONTO
Faculty of Arts and Science

DECEMBER 2011 EXAMINATIONS

CSC 236 H1F
Instructor: S. Cohen

Duration – 3 hours

Examination Aid: One sheet of paper, handwritten on both sides.

Last (Family) Name: _____

First (Given) Name: _____

Student Number: _____

Do NOT turn the page until instructed to do so.

This exam has 10 pages, including this one and a blank page at the end. When you receive the signal to begin, please make sure that your copy of the exam is complete and fill in your student number at the bottom of every page.

If your answer to a question extends past the space provided on that page, indicate **clearly** where your solution continues.

You will receive 20% of the marks for any question if you leave it blank or write "I do not know" for the answer.

For Marker Use Only

Question	Marks
1	/ 10
2	/ 15
3	/ 10
4	/ 20
5	/ 10
6	/ 10
7	/ 15
TOTAL	**/ 90**

Continued...

Question 1 – 10 marks

The following simple statement is often used, but rarely proved. **Use induction** to show that it is true. This is not a tricky question, so your form will be marked strictly – including properly defining and using a predicate in your proof.

$$a_1 + a_2 + \cdots + a_n = a_n + a_{n-1} + \cdots + a_1$$

Question 2 – 15 marks

Let F_n be the n^{th} Fibonacci number. Prove for all integers m and n, where $n > m > 1$, that

$$F_n = F_{n-m+1}F_m + F_{n-m}F_{m-1}$$

Hint: this is not a double induction problem, but you should proceed carefully.

Question 3 – 10 marks

Let F be the set of all functions that are in $\Theta(p(n))$ for some polynomial p. Show that big-theta is an equivalence relation on F.

Question 4 – 20 marks (5 for part (a), 5 for part (b), 10 for part (c))

Run through the following pseudocode to get a sense of its output.

```
function What (n)
      If n ≤ 1 then
            return 1
      elsif n ≡ 0 (mod 3) then
            return 8 *What(n + 1)
      else
            return (What (n − 2))/2
      endif
end What
```

(a) Find a closed form function that gives the output of the algorithm.

(b) Prove that the algorithm terminates. [There is more room for this on the next page.]

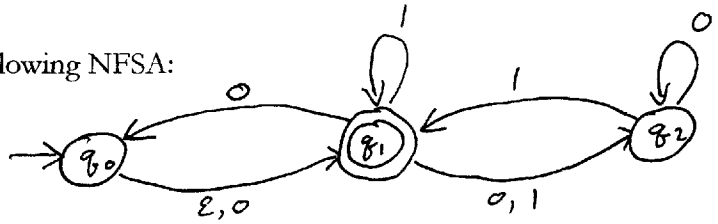
(c) Prove the correctness of your function from part (a).

Question 5 – 10 marks

Let L_1 and L_2 be regular languages. Explain how to create a FSA that accepts the language $L = \{w|$ there exists some $a \epsilon \Sigma$ such that $wa \epsilon (L_1)^2$ or $(wa)^R \epsilon L_2\}$ and (briefly) why your construction is correct. You do not need to re-prove any theorems from class.

Question 6 – 10 marks

Examine the following NFSA:



Create a DFSA with a minimal number of states that accepts the same set of strings as the automaton above and then write a regular expression for the language accepted by both. Justify your answers.

Question 7 – 15 marks total (7 for (a) and 8 for (b))

A *run* of some symbol in a string is an uninterrupted block of length **greater than one** composed of only that symbol. For example, 011000111101001 has runs of 0s with lengths two and three and runs of 1s with lengths two and four. Let L be the language composed of all strings over $\Sigma = \{0,1\}$ that have exactly two runs of zeroes.

(a) Give a regular expression that describes the language L and explain why it is correct.

(b) Create a DFSA that accepts exactly the language L and explain why it is correct.

[This page left blank for extra work]