15.09.11

Lecture 2 handout

Handshake Lemma

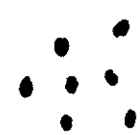
$$\sum_{v \in V} \deg(v) = 2|E|$$

Corollary

A graph has an even number of odd valence vertices.

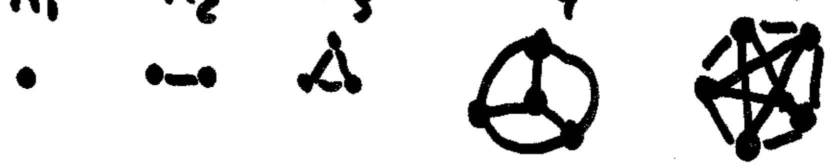
Special families of graphs

Null graph: $(\emptyset, \emptyset, \psi)$

Empty graphs: $(V, \emptyset, \psi)$ 

Complete graphs: Any two vertices are neighbours.

K_1 K_2 K_3 K_4 K_5



...

Q: How many edges does K_n have?

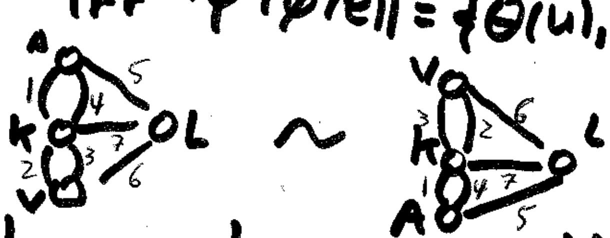
Paths: A_n  $V=\{v_1,\dots,v_n\}$, $E=\{e_1,\dots,e_{n-1}\}$, $\psi(e_i)=\{v_i,v_{i+1}\}$

Cycles: C_n  $V=\{v_1,\dots,v_n\}$, $E=\{e_1,\dots,e_n\}$, $\psi(e_i)=\{v_i,v_{(i+1)\bmod n}\}$

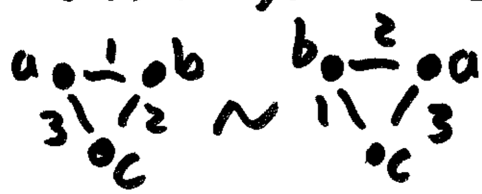
(1.2)

Graph isomorphism

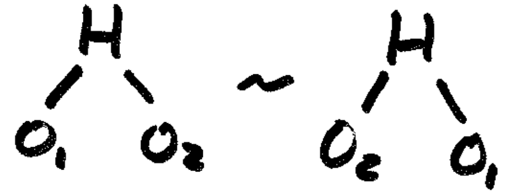
Defn Two graphs $G=(V,E,\psi)$ and $G'=(V',E',\psi')$ are <u>isomorphic</u> if there are bijections $\Theta:V\to V'$ and $\phi:E\to E'$ such that $\psi(e)=\{u,v\}$ iff $\psi'(\phi(e))=\{\Theta(u),\Theta(v)\}$.



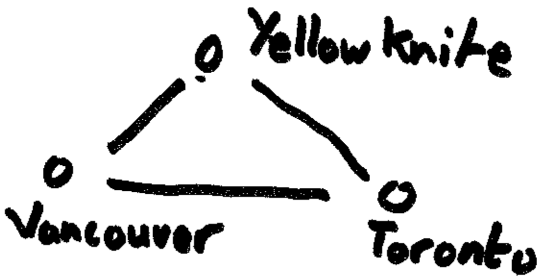
<u>Simple graph version</u>: Notation as above. $G\sim G'$ if there is a bijection $\Theta:V\to V'$ preserving adjacency.



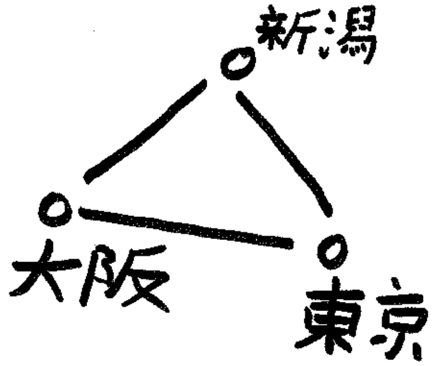
We consider isomorphic graphs the same unless they are <u>labeled</u> <u>graphs</u>.



might as well be considered the same

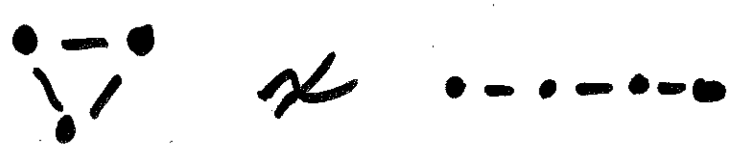


and

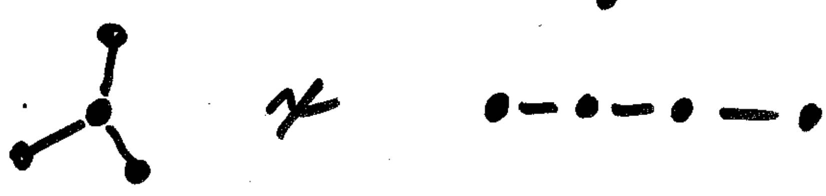


should be distinguished.

Distinguishing between non-isomorphic graphs:

|V|: 

|E|: 

Degree Sequence: 

Connected?: 

Next time: paths, walks, and cycles; Deletion, contraction, complement (1.4), Bipartite graphs.