TUTORIAL 10

(1) Prove that if G does not have two disjoint odd cycles, then $\chi(G) \leq 5$.

(2) Let G be a simple graph in which every vertex has degree at least k, where k is an integer at least 2. Prove that G has a path of length at least k and a cycle of length at least $k+1$.

(3) A *component* of G is a connected subgraph that is not contained in any larger connected subgraph. Prove that every graph with n vertices and $n-k$ edges has at least k components.

(4) Let G be a graph. Define a relation R on $V(G)$ by setting $(u,v) \in R$ if and only if G contains a uv-path. Prove that this relation is an equivalence relation. The equivalence classes of this equivalence relation are the vertex sets of the components of G.

(5) Prove that G is connected if and only if the following condition holds: For every partition of $V(G)$ into nonempty subsets S and T, there exists an edge with one endpoint in S and one endpoint in T.

(6) Prove or disprove
  (a) Deleting a vertex of maximum degree (and its adjacent edges) cannot raise the average degree of a graph.
  (b) Deleting a vertex of minimum degree (and its adjacent edges) cannot reduce the average degree of a graph.

Just for fun.

(1) Given a collection of lines in the plane with no three intersecting at a point, form a graph G whose vertices are the intersection points of the lines and the whose edges are the segments along the lines that join intersection points. Prove that $\chi(G) \leq 3$.

(2) Can a simple graph have all distinct vertex degrees?

(3) How many isomorphism classes of simple graphs with 4 vertices are there?