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In section 1.5 a recurring theme is the passage from a sequence to a subsequence. " ... for any sequence $\{x_k : k = 1, 2, \dots\}$ ... there is a subsequence $\{x_{k_j} : j = 1, 2, \dots\}$ which converges ... "

A sequence can be looked at as a garden which is not very well kept. There may be a very strong design which may be hidden under lots of unwanted herbs and weed. If we get rid of the weed then we can see the strong design that the gardener had in mind. This is the process of *passing to a subsequence,* that is, to get rid of the unessential thing.

The notation for subsequences may be somewhat confusing, and it is the purpose of this note to elaborate on the notation of subsequence.

Given a sequence $\{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11} \dots\}$ one can extract a subsequence $\{x_{k_j} : j = 1, 2 \dots\}$ arbitrary as follows:

$$\{x_3, x_4, x_7, x_{11}, x_{25}, x_{76}, x_{78}, x_{1034}, \dots\}$$

. In this case the first term of the subsequence, indexed by k_1 is x_3, that is the index $k_1 = 3$. Similarly the second term (naturally indexed by k_2, is the term x_4, that is $k_2 = 4$, and similarly $k_3 = 7, k_4 = 11, k_5 = 25, k_6 = 76, k_7 = 78, k_8 = 1034$ etc.

In the proof of theorem 1.19 this process of reducing to a subsequence is repeated several time (exactly n times.) So let's try to repeat this process twice, that is let's reduce the above subsequence once more to another subsequence. Say the new subsequence of the old subsequence is

$$\{x_3, x_{11}, x_{78}, \dots\}$$

In this case, the new indexes must be be doubly indexed to refer to the process of selection from a subsequence. That is the new subsequence is $\{x_{k_{j_i}} : i = 1, 2, 3 \dots$. In this case we can see that the first term of the new subsequnce, for $i = 1$ is 3, that is $x_{k_{j_1}} = x_3$ or that the index $k_{j_1} = 3$, and similarly $k_{j_2} = 11$ while $k_{j_3} = 78$.