

Topology of \mathbb{R}^n

geometry deals with distance & relative position of points.

topology studies relationship between points and sets

most of the ideas of calculus are presented and discussed in a topological setting, using terminologies from topology

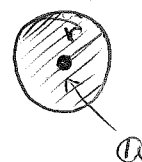
Chapter 1 of Folland is about Topology

Basic tool for studying Topology is The open Ball

$$B(r, a) = \{x \in \mathbb{R}^n : \|x - a\| < r\}$$

radius Center

norm in \mathbb{R}^n euclidean, but it could be any other norm too



interior of a Set

$$a \in S^{\text{int}} \text{ if } \exists r > 0 \ B(r, a) \subset S$$

boundary or edge of S

$$a \in \partial S \text{ if } \forall r > 0 \ B(r, a) \cap S \neq \emptyset \text{ \& } B(r, a) \cap S^c \neq \emptyset$$

$$\partial S = \partial S^c$$

a useful decomposition idea:

$$\text{for any Set } S \quad \mathbb{R}^n = S^{\text{int}} \cup \partial S \cup S^{c \text{int}}$$

and S^{int} , ∂S and $S^{c \text{int}}$ are disjoint

- 1.14:
- a) S is open iff $S = S^{\text{int}}$
 - b) S is closed iff S^c is open

points ∂S could :

1. all inside $S \Rightarrow S$ is said to be closed
2. all outside $S \Rightarrow S$ is said to be open
3. some inside, some outside $\Rightarrow S$ is neither closed nor open