Assignment VIII: week of Mar. 11th

This is the 8th assignment. You are encouraged to work on this by coming to the help sessions (Thursday 12-1 MP202, Friday 1-2 at MP102) and **grouping** *up with a few other students. Teaching assistants will be at hand to help. You do not have to hand this one in.*

1. Flat universe remains flat. Our current universe seems quite flat and therefore satisfies the zero-curvature Friedman equation. Part of the energy density to keep it flat is matter, and the other part is dark energy. The two parts add up to keep universe flat forever, namely, $\Omega = \Omega_m + \Omega_\Lambda = 1$ at all times. Here, we define $\Omega_m = \rho/\rho_{\rm crit}$, and $\Omega_\Lambda = \Lambda c^2/(3H^2)$, and the critical density $\rho_{\rm crit} = 3H^2/(8\pi G)$. Now consider a flat universe dominated by dark energy ($\Omega_\Lambda \gg \Omega_m$, as will likely occur in the future) and derive the expansion law $a = a(t)$ for a constant Λ (the 'cosmological constant').

2. Convince yourself, using simple diagrams, that the size of CMB spots can be used to measure the curvature of the universe.

3. CMB photons we see today have already started their journey when the universe was 300,000 yrs old. For other observers in the universe, currently looking at the sky, how would their measurement of the CMB map differ or agree with ours?

4. Pair annihilation. The early universe was so hot it is said that particles were continuously created and annihilated. We illustrate this point here using a simple analysis. To generate a pair of electron and positron, we need two photons with a total energy exceeding the rest mass energy of the pair. What is the energy of these photons (assuming identical photons)? If these photons have wavelengths that lie at the blackbody peak of their respective environment, what is the temperature of the environment? Can you obtain the temperature if we want to generate a proton/anti-proton pair? So at these very hot environments, energy is converted into matter and matter into energy continuously.

5. Nucleosynthesis. At the end of the particle era, we are left with unequal numbers of protons and neutrons, with neutrons (as they are slightly heavier and therefore cost slightly more energy to make) being under-abundant by about a factor of 7. As these protons and neutrons combine to form hydrogen and helium nuclei, show that the post-big-bang chemical composition of our universe is roughly 75% of hydrogen-1 (made up of 1 proton) and 25% helium-4 (made up of 2 protons and 2 neutrons) **by mass**.