Assignment VI

1. $E = \frac{1}{2}mv^2 - G\frac{Mm}{r} = 0$ (for crit.)

$v = H_0 r$

$\frac{1}{2}\cancel{m}H_0^2 r^2 - \frac{GM\cancel{m}}{r} = 0$

$M = \frac{4}{3}\pi r^3 \rho_{m0}$

$\frac{1}{2}H_0^2 r^2 - \frac{4}{3}G\pi r^{\cancel{3}2}\rho_{m0}/\cancel{r} = 0$

$\rho_{m0} = \frac{3H_0^2}{8\pi G}$; take $H_0 = 70 \frac{km/s}{Mpc} = 2.27 \cdot 10^{-18}\,s^{-1}$

$\rho_{m0} = 9.2 \cdot 10^{-27}\,kg/m^3 = 5.51$ H atoms$/m^3$

2. $r(t) = r_0 a(t)/a_0$

$\rho(t) = \rho_0\left(\frac{r_0}{r(t)}\right)^3$ (since $\rho(t) = \frac{3M}{4\pi r(t)^3}$)

$= \rho_0 \frac{a_0^3}{a(t)^3}$

But a universe that is critical at some pt. is critical at all pts, which gives:

$\frac{3H(t)^2}{\cancel{8\pi G}} = \frac{3H_0^2}{\cancel{8\pi G}}\frac{a_0^3}{a(t)^3}$

$H^2 = \frac{H_0^2 a_0^3}{a(t)^3}$

$H = v/r = \dot{a}(t)/a(t)$ so:

$\frac{\dot{a}^2}{a^2} = H_0^2 a_0^3 a^{-3}$

$\dot{a}^2 a = H_0^2 a_0^3$

$\dot{a}\sqrt{a} = H_0 a_0^{3/2}$

$\int \sqrt{a}\,da = \int H_0 a_0^{3/2}\,dt$

$\frac{2}{3}a^{3/2} = H_0 a_0^{3/2} t + C$

At $t=0$, $a=0$, so $C=0$

$a = \left(\frac{3}{2}H_0 a_0^{3/2} t\right)^{2/3} = At^{2/3}$

So $H = \dot{a}/a = \frac{2}{3}\cancel{A}t^{-\frac{1}{3}} \Big/ \cancel{A}t^{2/3}$

$= \underline{\underline{\frac{2}{3}t^{-1}}}$

$t_0 = \frac{2}{3H_0} = \frac{2}{3 \cdot 2.27 \cdot 10^{-18}\,s^{-1}} = \underline{\underline{2.94 \cdot 10^{17}\,s}} = \underline{\underline{9.32\ \text{Gyr}}}$

3. $a = At^{2/3}$ $\qquad A = \left(\frac{3}{2}H_0\, a_0^{3/2}\right)$

$1 + z = \frac{a_0}{a(t)} = \frac{a_0}{\left(\frac{3}{2}H_0 \cancel{a_0^{3/2}}\right)^{2/3} t^{2/3}}$

$t = \left(\frac{1}{\left(\frac{3}{2}H_0\right)^{2/3}(1+z)}\right)^{3/2} = \frac{1}{\frac{3}{2}H_0\,(1+z)^{3/2}}$

$= 2.55 \cdot 10^{8}\ \text{yrs} = \underline{\underline{255\ \text{Myr}}}$

OR (easier!)

$1 + z = \left(\frac{t_0}{t}\right)^{2/3}$

$t = t_0 \big/ (1+z)^{3/2}$ $\qquad t_0 = 9.32\ \text{Gyr} \rightarrow \underline{\underline{t = 255\ \text{Myr}}}$

or $\qquad t_0 = 13.7\ \text{Gyr} \rightarrow \underline{\underline{t = 376\ \text{Myr}}}$

4. $E = -\frac{k\cancel{m}c^2 r_0^2}{2a_0^2} = \frac{mv^2}{2} - \frac{GMm}{r}$

$-\frac{kc^2 r_0^2}{a_0^2 r^2} = \frac{v^2}{r^2} - \frac{2GM}{r^3} = H^2 - \frac{8\pi G\rho}{3}$

$H^2 = \frac{8\pi G\rho}{3} - k\frac{c^2 r_0^2}{a_0^2 r^2}$

$r(t) = r_0\frac{a(t)}{a_0}$, so $\frac{r_0^2}{a_0^2 r^2} = \frac{1}{a^2}$

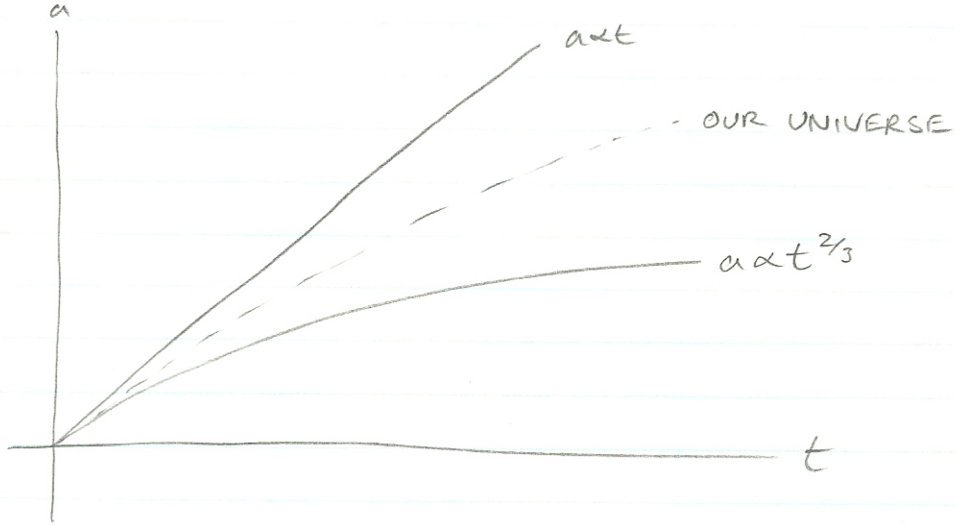
$\underline{\underline{H^2 = \frac{8\pi G\rho}{3} - k\frac{c^2}{a^2}}}$

If $\rho = 0$, $k = -1$, $H^2 = c^2/a^2$ $\qquad H = c/a = \dot{a}/a$

So $\dot{a} = c$, $a = ct$

$\underline{H = \dot{a}/a = \frac{1}{t}}$ $t_0 = \frac{1}{H_0} = \underline{14.0 \text{ Gyr}}$

5.



Our universe will expand forever, though rate will slow