TUTORIAL 8

(1) Prove that any collection of seven distinct integers contains a pair whose sum or difference is a multiple of 10.

(2) The numbers 1 through 10 appear in some order around a circle. Prove that some set of three consecutive numbers sums to at least 17.

(3) Suppose you decide to flip a coin until heads shows up for the first time, and then stop. Assume the coin shows heads with probability p and tails with probability $(1-p)$.
   (a) What is the underlying probability space for this experiment, i.e. what is the set of possible outcomes and the probability of each outcome.
   (b) What is the expected number of flips until you stop?

   Hint: You will need to use the following infinite summation formulas, which are valid for any real number x satisfying $|x| < 1$.
   - $\sum_{n=0}^{\infty} x^n = \frac{1}{(1-x)}$
   - $\sum_{n=1}^{\infty} nx^{n-1} = \frac{1}{(1-x)^2}$

(4) A drunk has n keys and only one will open his door. He tries keys at random. Under each model below, what is the expected number of selections until he opens the door.
   (a) He selects keys in a random order (without replacement) until one works.
   (b) After each mistake, he replaces the key and selects randomly again.

Just for fun.

(1) Consider two three-year universities X and Y, each having 100 students. Construct an example where, in each of the categories "first-year students," "second-year students," and "third-year students," the proportion of students who are math majors is greater at X than at Y, yet Y has more math majors than X. This phenomenon is sometimes called Simpson's paradox.