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The most important result is this section is the equality of mixed partial derivatives when the function is C^2 (that is any partial derivatives of order two are continuous.) This is theorem 2.45. Note that the proof of this theorem uses the MVT a couple of times. So this means that we need to assume the convexity of the underlying set, but in this case since we are working in a neighborhood of a point $\boldsymbol{a}$ then we can always assume there is an open ball on which we are working (and the open balls are convex.) Of course this equality is not necessarily true in general. This equality becomes very important when we deal with Taylor polynomials, and in particular working with the second degree Taylor polynomials, as a result of the equality of the mixed partial derivative Hessian of a function (which is like the second derivative) is a symmetric matrix (see page 96). And this is of important implications in deciding the nature of critical points of a function (like the second derivative test.) Indeed any symmetric matrix is orthogonally diagonalizable, and this is of importance in the proof of theirem 2.81.

Also we have a version of this theorem which suggests the equality any k many mixed partial derivative in case the function is C^k. Now this generalization is crucial for the theory of Taylor polynomials. Since we assume our functions are C^k then the equality of the mixed derivatives with the help of the Multi-index notation can simplify the way we shall write the Taylor polynomials. See Theorem 2.68 and the discussion prior to it.