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Recall from one variable Calculus that the Mean Value Theorem (for $\mathbb{R}$) is used to replace the difference $f(b) - f(a)$ with the expression $f'(c)(b - a)$ where c is some number between a and b. But since in $\mathbb{R}^n$ there is no ordering of the points then MVT will not work the way it is formulated for $\mathbb{R}$ (see the bottom of page 51). However in $\mathbb{R}^n$ the idea of between two points is translated to the on the line segment between the two points. This is the reason why we should work with convex sets (in which the line segment connecting any two points are still in the set.)

Here is a list of places where MVT is used in our textbook. Note that in all cases we are trying to replace $f(b) - f(a)$ with $f'(c)(b - a)$ or with $\nabla f(\boldsymbol{c})$ in higher dimensions.

- in Corollary 2.40, to place a bound on the expression $|f(\boldsymbol{b}) - f(\boldsymbol{a})|$. This bound is very useful in discussion of continuity (and implies uniform continuity)
- in corollary 2.41 this bound is used to show the condition of $\nabla f(\boldsymbol{x}) = \mathbf{0}$ implies the function is a constant function. This is important for the integration applications.
- in the proof of theorem 2.45 (equality of the mixed partial derivatives)
- of course Taylor's theorem is generalization of MVT, and lemma 2.62 is a generalization of Role's theorem
- Theorem 2.88 and its proof
- Proof of IFT (theorem 3.1)
- Proof of FTC (theorem 4.15)
- Proof of theorem 4.47 page 190