Department of Mathematics, University of Toronto
MAT224H1S - Linear Algebra II
Winter 2013

Problem Set 7:

- Not to be handed in.

1. Show that for any linear operator T on a vector space V, the subspaces $Ker(T^k)$ and $Im(T^k)$, $k \in \mathbb{Z}^+$, are T-invariant.

2. Let W be a subspace of an inner product space V, and T a linear operator on V. Prove that if W is T-invariant then $W^\perp$ is T^*-invariant.

3. Let $T\colon \mathbb{R}^4 \to \mathbb{R}^4$ be the linear operator given by

$$A = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ -1 & 0 & 2 & 1 \end{pmatrix}.$$

Find a basis α of $\mathbb{R}^4$ such that $[T]_{\alpha\alpha}$ is the Jordan canonical form of A and find $[T]_{\alpha\alpha}$.

4. Let $T\colon M_{2\times 2}\mathbb{R} \to M_{2\times 2}(\mathbb{R})$ be the linear operator defined by

$$T(A) = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} A - A \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}.$$

Find a basis α for $M_{2\times 2}(\mathbb{R})$ such that $[T]_{\alpha\alpha}$ is in Jordan canonical form and determine $[T]_{\alpha\alpha}$.

5. Textbook, Section 6.3, **1, 2, 3, 4, 5, 7, 12, 13**

6. Textbook, Section 6.4, **1, 2, 3, 4, 5**