NAME (PRINT): _____

Last/Surname   First /Given Name

STUDENT #: _____ SIGNATURE: _____

UNIVERSITY OF TORONTO MISSISSAUGA
DECEMBER 2011 FINAL EXAMINATION
STA257H5F
Probability and Statistics I
Alison Weir
Duration - 3 hours
Aids: Statistical Calculators; Statistical Tables and Formula Sheet

The University of Toronto Mississauga and you, as a student, share a commitment to academic integrity. You are reminded that you may be charged with an academic offence for possessing any unauthorized aids during the writing of an exam, including but not limited to any electronic devices with storage, such as cell phones, pagers, personal digital assistants (PDAs), iPods, and MP3 players. Unauthorized calculators and notes are also not permitted. Do not have any of these items in your possession in the area of your desk. Please turn the electronics off and put all unauthorized aids with your belongings at the front of the room before the examination begins. If any of these items are kept with you during the writing of your exam, you may be charged with an academic offence. A typical penalty may cause you to fail the course.

Please note, you CANNOT petition to re-write an examination once the exam has begun.

1. (8 marks) The random variables X and Y have joint probability mass function given in the table below.

		X		
		-1	0	3
Y	-1	0.2	0.1	0.2
	0	0.0	0.4	0.1

Evaluate each of the following. Put your answers in the chart below. Only answers in the chart will be marked.

a. $E(X)$	
b. $F_{yx}(0,-1)$	
c. $E(|X-Y|)$	
d. $P(X<2|Y=-1)$	

2. (14 marks) A jar contains 5 silver and 7 gold balls. Balls are selected at random from the jar.

Answer each of the following questions. Put your answers in the chart below. Only answers in the chart will be marked.

a. 4 balls are selected with replacement. What is the probability exactly 2 are gold?	

b. 4 balls are selected with replacement. How many do you expect to be silver?

c. Balls are selected sequentially with replacement. What is the probability you need to select 4 balls in order to get 2 gold ones?

d. Balls are selected sequentially with replacement. How many do you expect to sample in order to get 2 silver ones?

e. 4 balls are selected without replacement. What is the probability exactly 2 are silver?

f. Balls are selected sequentially without replacement. What is the probability the 4th ball you select is the 2nd silver one selected?

3. (5 marks) A bin contains three components from supplier A, four from supplier B, and five from supplier C. If four of the components are randomly selected for testing, what is the probability that each supplier would have at least one tested?

4. (5 marks) As items come to the end of a production line, an inspector chooses which items are to go through a complete inspection. Ten percent of all items produced are defective. 60% of all defective items go through a complete inspection, 20% of all good items go through a complete inspection. Given that an item is completely inspected, what is the probability it is not defective?

5. (5 marks) The number of imperfections in the weave of a certain textile has a Poisson distribution with mean 4 per square meter. What is the probability a 3 square meter sample will contain at least one imperfection?

6. (5 marks) In responding to a survey question on a sensitive topic, such as "*Have you ever tried marijuana?*", many people will prefer to answer "*no*". Suppose that 80% of the population have not tried marijuana and that all of these people will truthfully answer the question. The remaining 20% of the population have tried marijuana and 70% of these individuals will lie. Derive the probability distribution of Y, the number of people you would need to question in order to obtain a single "*yes*" answer.

7. (5 marks) A manufacturing plant uses 3000 light bulbs whose lifetimes are independently normally distributed with mean 500 hours and standard deviation 50 hours. To minimize the number of bulbs that burn out during production hours, all the bulbs are replaced after a given period of operation. How often should the bulbs be replaced if we want no more than 1% to burn out between replacements?

8. (5 marks) Three events, A, B, and C, are mutually independent if

$$P(AB) = P(A)\ P(B) \qquad\qquad P(BC) = P(B)\ P(C)$$
$$P(AC) = P(A)\ P(C) \qquad\qquad P(ABC) = P(A)\ P(B)\ P(C)$$

Suppose a balanced coin is tossed (independently) two times. Define these events

  A: Head comes up on first toss
  B: Head comes up on second toss
  C: Both tosses have the same outcome

Show the events A and (B ∪ C) are independent.

9. (6 marks) Find the distribution of the random variables that have each of these moment generating functions.
Note: if the distribution has a parameter, be sure to state it. If the distribution needs a chart, make a chart for your answer.

Put your answers in the chart below. Only answers in the chart will be marked.

$m(t) = \left(\frac{1}{3}e^t + \frac{2}{3}\right)^5$	
$m(t) = \dfrac{e^t}{2 - e^t}$	
$m(t) = e^{3(e^t - 1)}$	

10. (14 marks) Let X and Y have joint probability density function

$$f(x, y) = \begin{cases} 3x, & 0 < y < x < 1 \\ 0, & otherwise \end{cases}$$

a. Sketch the region of positive density.

b. Find $P(X \leq \frac{3}{4} \mid Y \leq \frac{1}{2})$

c. Find $P(X \leq \frac{3}{4} \mid Y = \frac{1}{2})$

d. Find $P(X + Y \leq \frac{1}{2})$

11. (5 marks)   The random variable X has uniform distribution on the interval -1 to 3. Find
the distribution (CDF or pdf) of $Y = X^2$

Note: $f(x) = \begin{cases} \frac{1}{4}, & -1 < x < 3 \\ 0, & otherwise \end{cases}$

12. (6 marks)   Random variables X and Y are independent and identically exponentially
distributed with mean 1. Find the joint distribution of $U = X + Y$ and $V = X - Y$.

Note: $f(x,y) = \begin{cases} e^{-x-y}, & x > 0, y > 0 \\ 0, & otherwise \end{cases}$

13. (7 marks) Let X_1, X_2 and X_3 be independent Poisson random variables with means 2, 4, and 8 respectively. Let $Y = \sum_1^3 X_i$

    a.   Use moment generating functions to determine the distribution of Y.

    b.   Use the moment generating function from part (a) to show $E(Y) = 14$.

Total Marks: 90