CSC165H1S Exercise2

Yizhou Sheng, Student# 999362602

Rui Qiu, Student# 999292509

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Question 1:

<u>Solution</u>:

(a) $\forall x \in D(x), T(x) \Rightarrow \neg L(x)$

(b) Some exam question is long.

(c) $\exists x \in D, T(x) \wedge \forall y \in D, E(y) \Rightarrow H(x, y)$

(d) No exam question is harder than every test question.

Question 2:

<u>Solution</u>:
(a) Every prime number except 2 is odd.
(b) Some prime number is larger than or equal to every prime number.

Question 3:

<u>Solution</u>:
(a) This pair of statements are equivalent.
Proof:

When the statement $\forall x \in D, (P(x) \wedge Q(x))$ is true,

for every x in D, x is in P and x is in Q.
So, for every x in D, x is in P and for every x in D, x is in Q.

Thus the statement $(\forall x \in D, P(x)) \wedge (\forall x \in D, Q(x))$ is true.

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for every x in D, x is in P and for every x in D, x is in Q.
So for every x in D, x is P and x is in Q.

Thus the statement $\forall x \in D, (P(x) \wedge Q(x))$ is true.

Therefore, $\forall x \in D, (P(x) \wedge Q(x))$ and $(\forall x \in D, P(x)) \wedge (\forall x \in D, Q(x))$ are equivalent.

(b) This pair of statements are not equivalent.

Counterexample:

Suppose D=R, P(x):x is positive, and Q(x):x is non-positive.

Then $\forall x \in R, x > 0 \vee x \leq 0$ is true $\Rightarrow$ $\forall x \in D, (P(x) \vee Q(x))$ is true.

But both $\forall x \in R, x > 0$ and $\forall x \in R, x \leq 0$ are false

$\Rightarrow$ $(\forall x \in D, P(x)) \vee (\forall x \in D, Q(x))$ is false.

Therefore, the two statements are not equivalent.

(c) This pair of statements are equivalent.

Proof:

When the statement $\exists x \in D, (P(x) \vee Q(x))$ is true,
for some x in D, x is in P or x is in Q.

So for some x in D, x is in P or for some x in D, x is in Q.

Thus the statement $(\exists x \in D, P(x)) \vee (\exists x \in D, Q(x))$ is true.

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for some x in D, x is in P or for some x in D, x is in Q.

So for some x in D, x is in P or x is in Q.

Thus the statement $\exists x \in D, (P(x) \vee Q(x))$ is true,

Therefore, $\exists x \in D, (P(x) \vee Q(x))$ and $(\exists x \in D, P(x)) \vee (\exists x \in D, Q(x))$
are equivalent.

(d) This pair of statements are not equivalent.

Counterexample:

Suppose D=R, P(x):x>1, and Q(x):x>2.

Both $(\forall x \in R, x > 1)$ and $(\forall x \in R, x > 2)$ are false,
then $(\forall x \in R, x > 1) \Rightarrow (\forall x \in R, x > 2)$ is true.
so $(\forall x \in D, P(x)) \Rightarrow (\forall x \in D, Q(x))$ is true.

But $\forall x \in R, x > 1 \Rightarrow x > 2$ is false,
so the statement $\forall x \in D, P(x) \Rightarrow Q(x)$ is false.

Therefore the two statements are not equivalent.

Question 4:

Solution:

$$\neg(\forall \varepsilon \in R^{+}, \exists \delta \in R^{+}, \forall x \in R, 0 < |x-a| < \delta \Rightarrow |f(x)-L| < \varepsilon)$$

$$\Leftrightarrow \exists \varepsilon \in R^{+}, \forall \delta \in R^{+}, \exists x \in R, \quad \neg(0 < |x-a| < \delta \Rightarrow |f(x)-L| < \varepsilon)$$

$$\Leftrightarrow \exists \varepsilon \in R^{+}, \forall \delta \in R^{+}, \exists x \in R, \quad 0 < |x-a| < \delta \wedge |f(x)-L| \geq \varepsilon$$