CH 8 Transition to chaos

consider the quadratic map $Q_c(x) = x^2 + c$

$\rightarrow$ saddle bifurcation
(FP neutral)

$c > \frac{1}{4} \Rightarrow$ no FP

analyze $\copyright \rightarrow$ response x.
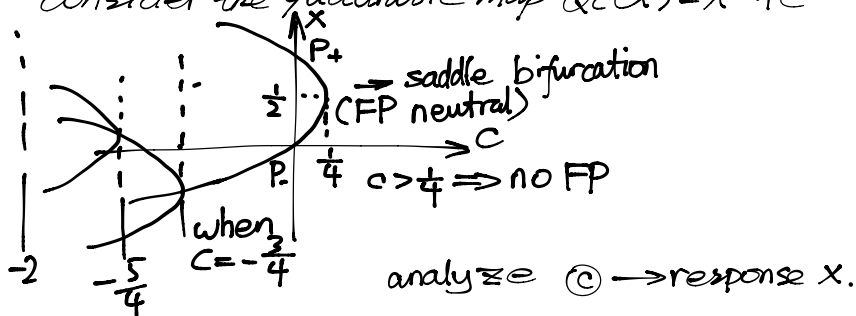
we want sketch a diagram automatically:

① values of c on the horizontal axis.

② values of x on the vertical axis.

$\rightarrow$ Don't plot all iterations

$\rightarrow$ ignore the first 100 iterations to let orbit stablize.

$\rightarrow$ plot the value of x_n for $100 < n < 300$

③ the choice of x_0 is important. it needs to be a non-degenerate critical point of Q_c

Definition: let $F = \mathbb{R} \to \mathbb{R}$. then x_0 is a critical point of F if $F'(x_0) = 0$.

The Critical pt x_0 is non-degenerate if $F''(x_0) \neq 0$, o.w. it's called degenerate.

Orbit diagram for the Quadratic Family.

• We need to find x_0 which is a non-de critical pt of Q_c

• $Q_c'(x_0) = 2x_0 = 0 \Rightarrow x_0 = 0$

• $Q_c''(x_0) = 2 \neq 0$

• We need to choose which value of c to sketch.

$\quad \hookrightarrow$ For $c > 1/4$, the orbit goes to ∞.

$\hookrightarrow$ For $c < -2$, the orbit goes to ∞

$\hookrightarrow$ For $-2 \leq c \leq 1/4$, the orbit of 0 will stay on $[-P_+, P_+] \subseteq [-2, 2]$