SCHOOL OF FINANCE AND APPLIED STATISTICS

FINANCIAL MATHEMATICS
(STAT 2032 / STAT 6046)
TUTORIAL EXERCISES WEEK 9

Question 1
You invest $75,000 on 30 April 2005 on a $100,000 nominal bond redeemable at par on 31 December 2020. Coupons are paid annually at 5% p.a., with the next coupon due on 30 June 2005 (the last coupon payment is on 30 June 2020). Find the net effective redemption yield if tax on income and capital gains is 25%.

Question 2
Calculate the net effective yield on the bond in Question 1, if it is redeemable at par on 31 December 2015 and the last coupon payment is on 30 June 2015. Why has the net yield increased/decreased?

Question 3
A pension fund is about to purchase a fixed-interest stock, which has an optional redemption date. The fund's actuary wishes to calculate an appropriate price for the stock using a specified interest rate, but based on conservative assumptions. The following statements have been made:

I   If the calculated price is less than the total redemption payment for the entire range of possible redemption dates, the price will not depend on the assumed redemption date.
II   If the price is always greater than the total redemption payment, the earliest possible redemption date should be assumed.

Which of these assertions are necessarily true?

Question 4
A 10% p.a. coupon payable half yearly bond with face amount $100 is callable on any coupon date from 15.5 years after issue up to the maturity date which is 20 years from issue.
   (a) Find the price of the bond to yield a minimum nominal annual rate convertible half-yearly of (i) 12%, (ii) 10%, and (iii) 8%.
   (b) Find the minimum annual nominal yield (convertible half-yearly) to maturity if the bond is purchased for (i) 80, (ii) 100, and (iii) 120.

Question 5
A bond of nominal amount $200,000 is redeemable at par in 4 equal annual installments of $50,000, the first being in 11 years' time.

Coupons of 7.5% per annum are payable half yearly in arrears. The coupons are payable on the nominal amount outstanding at the time of coupon payment. For example, the coupons payable during year 12 after the first redemption payment of $50,000 are based on the nominal amount outstanding at that time of $150,000.

All investors require a net effective yield of 9% per annum.

(a)
Calculate the price P_1 for the whole bond for an investor.
(b)
Calculate the revised price P_2 for an investor if the final installment is redeemable at a price of 1.2 per unit nominal.
(c)
Calculate the revised price P_3 for an investor if coupon payments are made at a rate of 9% for the first 3 years (and 7.5% thereafter).

Question 6
On June 15, 1990 a corporation issues an 8% bond with a face value of $1,000,000. The bond can be redeemed, at the option of the corporation, on any coupon date in 2001 or 2002 at par, on any coupon date in 2003 through 2005 for amount $1,200,000, or on any coupon date in 2006 through June 15, 2008 at redemption amount $1,300,000. Coupons are payable half-yearly.

   (a) Find the price to yield a minimum nominal annual rate convertible half-yearly of
       (i) 10% and (ii) 6.5%

   (b) Find the minimum nominal annual yield convertible half-yearly if the bond is
       bought for (i) $800,000, (ii) $1,000,000, or (iii) $1,200,000.

Question 7
An inflation linked bond is purchased on 30 April 1994 for $900. It has a face value of $1,000 and pays half yearly coupons indexed to inflation of 5% p.a. It is redeemable at an indexed value to par at 30 April 2018. Calculate the gross redemption yield assuming a constant rate of inflation of 3% p.a.

Past Exam Question – 2005 Final Exam Q4

You are currently running a small business from a property you own. You decide to mortgage the property in order to finance further expansion of the business. The terms of the mortgage are as follows:

- You receive a lump sum payment of X on 15 July 2004.
- You must make payments of $80,000 every 6 months, the first payment being on 15 January 2005.
- The institution holding the mortgage may demand a closing payment of $1,000,000 on any regular payment date from 15 January 2015 – 15 January 2020 (inclusive). This is paid in addition to the regular $80,000 on that date. All regular payments cease from the date the mortgage is closed.

The maximum interest rate at which you wish to take out the mortgage is 9.0% p.a. convertible half-yearly.

a) Using the techniques of valuing a callable bond, calculate X, the initial lump sum payment which you will receive. (5 marks)

b) The mortgage holder decides to close out the mortgage on 15 January 2017. Assuming you received the lump-sum payment as per your answer to a), and using linear interpolation, calculate the annual interest rate, convertible half-yearly, that is effective on the mortgage (to one decimal place). (5 marks)

Past Exam Question – 2005 Final Exam Q6(a)

You inherit an empty block of land from a distant relative and decide to build a house on it with a view to living in it in the future. The house you wish to build costs $300,000 and is expected to be completed in two years from now. You are required to pay the $300,000 on the completion of the house.

In the meantime you wish to set aside an amount of money to ensure you will have enough to pay for the house in two years time. You are considering investing in inflation-linked bonds with the following characteristics:

- Term = 3 years
- Indexed Coupon rate = 8% p.a.
- Coupon Frequency = Annual in arrears
- Redemption is at the indexed amount of the initial par value

Assuming an interest rate of 6% p.a. and an expected future inflation rate of 3% p.a.:

a) Show that the initial face value of the indexed linked bond that will need to be purchased to ensure that the present value of the inflation linked bond is equal to the present value of the house payment is $233,361.25. (4 marks)