STANDARD EQUIVALENCES

- Commutativity

  $P \land Q \iff Q \land P$
  $P \lor Q \iff Q \lor P$
  $P \iff Q \iff Q \iff P$

- Associativity

  $P \land (Q \land R) \iff (P \land Q) \land R$
  $P \lor (Q \lor R) \iff (P \lor Q) \lor R$

- Identity

  $P \land (Q \lor \neg Q) \iff P$

  $P \lor (Q \land \neg Q) \iff P$

- Absorption

  $P \land (Q \land \neg Q) \iff Q \land \neg Q$
  $P \lor (Q \lor \neg Q) \iff Q \lor \neg Q$

- Idempotency

  $P \land P \iff P$
  $P \lor P \iff P$

- Double Negation

  $\neg\neg P \iff P$

- DeMorgan's Laws

  $\neg(P \land Q) \iff \neg P \lor \neg Q$

  $\neg(P \lor Q) \iff \neg P \land \neg Q$

- Distributivity

  $P \land (Q \lor R) \iff (P \land Q) \lor (P \land R)$

  $P \lor (Q \land R) \iff (P \lor Q) \land (P \lor R)$

- Implication

  $P \Rightarrow Q \iff \neg P \lor Q$

- Biconditional

  $P \iff Q \iff (P \Rightarrow Q) \land (Q \Rightarrow P)$

- Renaming (where $P(x)$ does not contain variable y)

  $\forall x, P(x) \iff \forall y, P(y)$

  $\exists x, P(x) \iff \exists y, P(y)$

- Quantifier Negation

  $\neg \forall x, P(x) \iff \exists x, \neg P(x)$

  $\neg \exists x, P(x) \iff \forall x, \neg P(x)$

- Quantifier Commutativity

  $\forall x, \forall y, S(x,y) \iff \forall y, \forall x, S(x,y)$
  $\exists x, \exists y, S(x,y) \iff \exists y, \exists x, S(x,y)$

- Quantifier Distributivity (where S does not contain variable x)

  $S \land \forall x, Q(x) \iff \forall x, S \land Q(x)$
  $S \lor \forall x, Q(x) \iff \forall x, S \lor Q(x)$
  $S \land \exists x, Q(x) \iff \exists x, S \land Q(x)$
  $S \lor \exists x, Q(x) \iff \exists x, S \lor Q(x)$

USE TRUTH TABLE TO VERIFY :

$\cancel{(P \Rightarrow (Q \Rightarrow R)) \iff ((P \Rightarrow Q) \Rightarrow R)}$

$\cancel{P \Rightarrow R \quad Q \Rightarrow R \quad P \Rightarrow (Q \Rightarrow R)}$