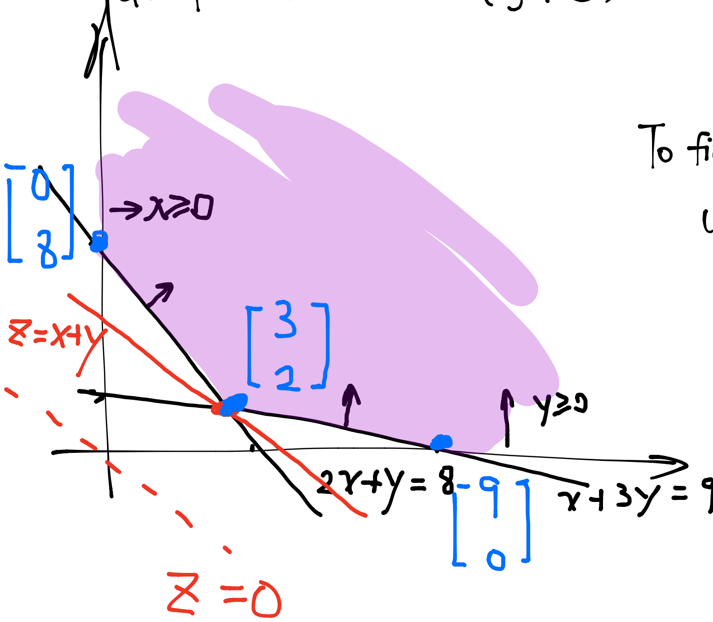
Example: ③ (a problem that has an optimal solution)

Maximize $z = x + y$ s.t.

$$2x + y \geq 8$$
$$x + 3y \geq 9$$
$$x \geq 0,\ y \geq 0$$

Graphical Solution (§ 1.3)



To find the extreme point

which one not on $x=0$ or $y=0$

$x+3y=9$ $\left[\begin{array}{cc|c}1 & 3 & 9\\ 2 & 1 & 8\end{array}\right]$

$2x+y=8$ $\left[\begin{array}{cc|c}1 & 3 & 9\\ 0 & -5 & -10\end{array}\right]$

$\left[\begin{array}{cc|c}1 & 0 & 3\\ 0 & 1 & 2\end{array}\right]$

$\Rightarrow x=3,\ y=2$

This problem has no negative feasible z value $z = x + y \geq 0 + 0 = 0$

This problem has an ~~non~~ non-empty feasible ~~s~~ region ~~is~~ and ~~z~~ is bounded below, so has an optimal solution.

As M moves, the solution of the equation $z = M$ is a line ~~para~~ parallel to $z=0$. As M increases, the line $z=M$ moves to the right.

The solution of the problem (not the optimal z-value) is $\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}3\\2\end{bmatrix}$, with verification in the last diagram.

The optimal z-value is $z = x + y = 3 + 2 = 5$

This problem is now solved ("Solving" means finding and verifying

just one solution

But note: example ③ has just one optimal solution.

Eg ③' (a problem having >1 optimal solution)

Maximize ~~z~~ $z = -2x_1 + x_2 + x_3$ s.t.

$$x_1 - 2x_2 + x_3 = 0$$
$$x_1 + x_2 - 2x_3 = 0$$

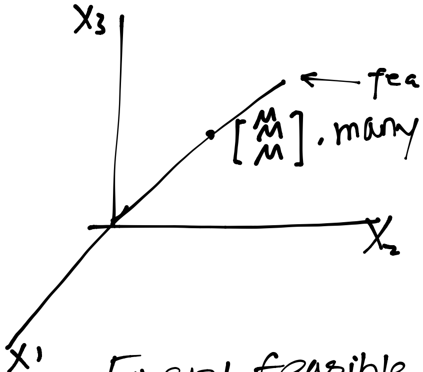
x_1, x_2, x_3 unrestricted.

To find the feasible region

$$\begin{array}{ccc} x_1 & x_2 & x_3 & \\ \end{array}$$
$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 1 & 1 & -2 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 3 & -3 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \end{array}\right] \Rightarrow \begin{array}{l} x_1 - x_3 = 0 \\ x_2 - x_3 = 0 \end{array}$$

$$\Rightarrow \begin{array}{l} x_1 = x_3 \\ x_2 = x_3 \end{array} \longrightarrow \text{a line}$$



(unbounded)

← feasible region is a line $x_1 = x_2 = x_3$

$\begin{bmatrix} M \\ M \\ M \end{bmatrix}$, many real #

At $\begin{bmatrix} M \\ M \\ M \end{bmatrix}$, $z = -2x_1 + x_2 + x_3 = -2M + M + M = 0$

On the feasible region, z never varies

Every feasible pt ($\begin{bmatrix} M \\ M \\ M \end{bmatrix}$) is OPTIMAL