Homework 1

Due by
Monday 6 August 2018 10:00

Question 1 [5 Points]

Show that the determinant of the $p \times p$ diagonal matrix $\mathbf{A} = (a_{ij})$ with $a_{ij} = 0$, $i \neq j$, is given by the product of the diagonal elements; that is, $|\mathbf{A}| = a_{11} a_{22} \cdots a_{pp}$.

Question 2 [5 Points]

Show that the determinant of a square symmetric $p \times p$ matrix $\mathbf{A}$ can be expressed as the product of its eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_p$; that is, $|\mathbf{A}| = \prod_{i=1}^{p} \lambda_i$.

Question 3 [10 Points]

Illustrate numerically the universality conjecture that the spectral density of large symmetric matrices formed from independent identically distributed random variables with zero mean and finite variance converges to the density of the Wigner Semicircle distribution. That is:

1. Take $p = 100$ and generate $n = 500$ symmetric matrices with entries sampled from the standard Normal distribution. Hint: generate a matrix `A` with Normal entries and then symmetrise using `A[lower.tri(A)] <- t(A)[lower.tri(A)]`.
2. Calculate the eigenvalues of each of these matrices and plot the histogram of all these eigenvalues together (i.e., obtained from all the matrices).
3. Fit the Wigner semicircle distribution to this histogram (i.e., find the parameters of the distribution).
4. Repeat the experiment taking p and n larger and larger with the ratio p/n fixed.
5. Repeat the experiment with the entries sampled from a Student-T distribution.

Question 4 [5 Points]

Repeat your numerical experiment (from the last question) but with the entries of your random matrices having a distribution whose variance is not finite. Do you obtain a semicircle distribution?

This homework is to be submitted through Wattle in digital form only as per ANU policy. If you use any references (note: this will never count against you), please clearly indicate which ones. This homework has 15% weight.