The Method of Moments

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On our first tutorial, to show you how to use the R to do the simulation on the Poisson example, I gave you the detail of it. And for parameter estimate, one method, namely, the method of moment is briefly covered with two examples. As requested from two of you, I typed them up and posted it on the portal.

Enjoy it and have fun on learning. :)

1 The method of moments

We recall the definition of the k^{th} moment of the distribution of the random variable X is

$$\mu_k = E(X^k) = h(\theta)$$

if it exists where θ is/are the unknown parameter(s). Suppose that $X_1, ..., X_n$ iid from that a distribution then the method of moments estimate for μ_k will be

$$\hat{\mu}_k = \frac{1}{n}\sum_i X_i^k$$

The method of moments is based on the assumption that the parameters we want to estimate can be written as functions of the moments, may be the first two moments but it could be more. so

$$\hat{\theta} = h^{-1}(\hat{\mu}_k)$$

Example 1: A random sample from Poisson(λ), find a estimate of λ by method of moments.

Solution: Note that E(X)=λ, and $\hat{E}(X) = \frac{1}{n}\sum_i X_i$, hence we have $\hat{\lambda} = \hat{E}(X) = \frac{1}{n}\sum_i X_i$.
Is it an unbiased estimate? Yes, since $E(\hat{\lambda}) = E(\frac{1}{n}\sum_i X_i) = n\lambda/n = \lambda$

Example 2: A random sample from N(μ, σ^2)), μ, σ^2 both unknown. Find a estimate of σ^2 by method of moments.

Solution: We know that

$$E(X) = \mu, \quad E(X^2) = var(X) + (E(X))^2 = \sigma^2 + \mu^2$$

and we could use method of moments to estimate of the first two moments. Hence we have

$$\hat{\mu} = \frac{1}{n}\sum_i X_i = \bar{X}$$

Therefore, we could find the estimate of the variance by applying the method of moment

$$\hat{\sigma}^2 = \frac{1}{n}\sum_i X_i^2 - (\bar{X})^2$$

Question: Is $\hat{\sigma}^2$ is an unbiased estimate? Try it. (hint, $\bar{X} \sim N(\mu, \sigma^2/n)$)