Why do Sums of Squares in ANOVA depend on the order you fit the model but the fitted model is the same?

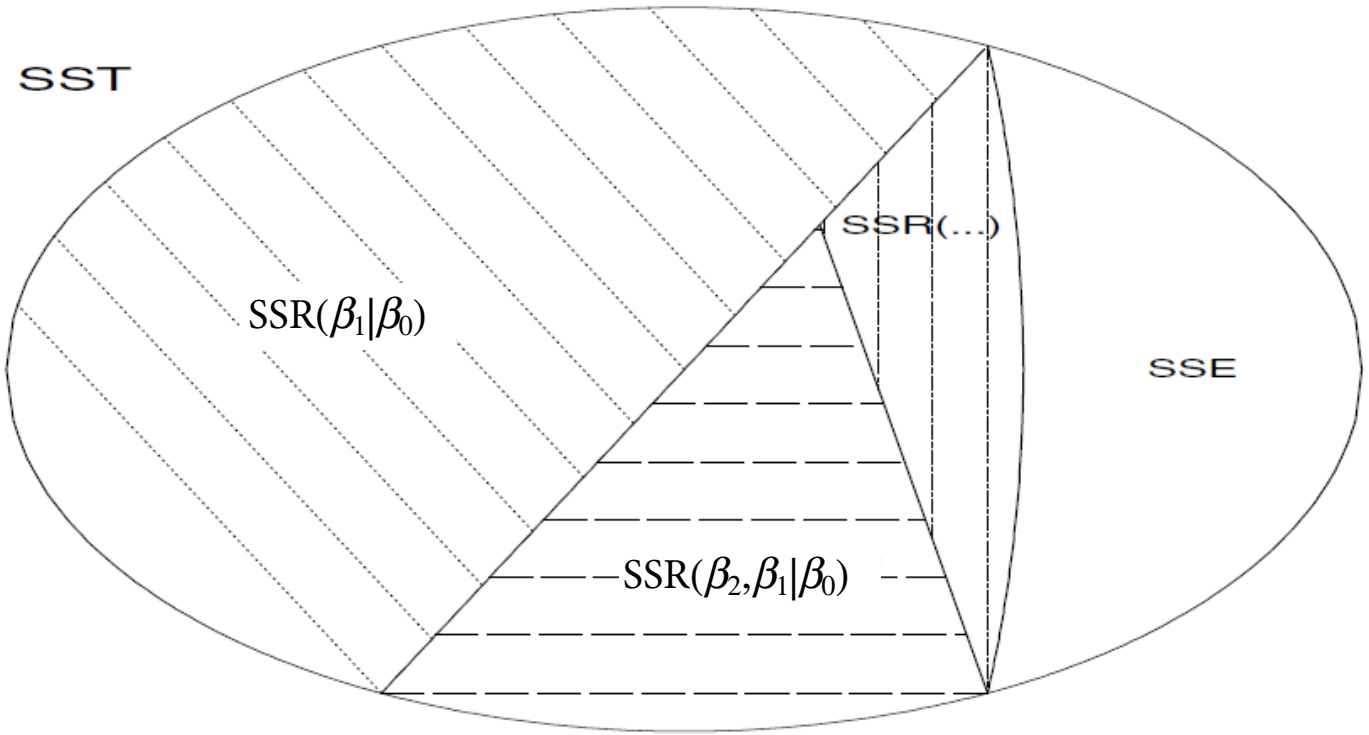
Because they are Sequential Sums of Squares:

The model or regression sum of squares can be partioned as follows:

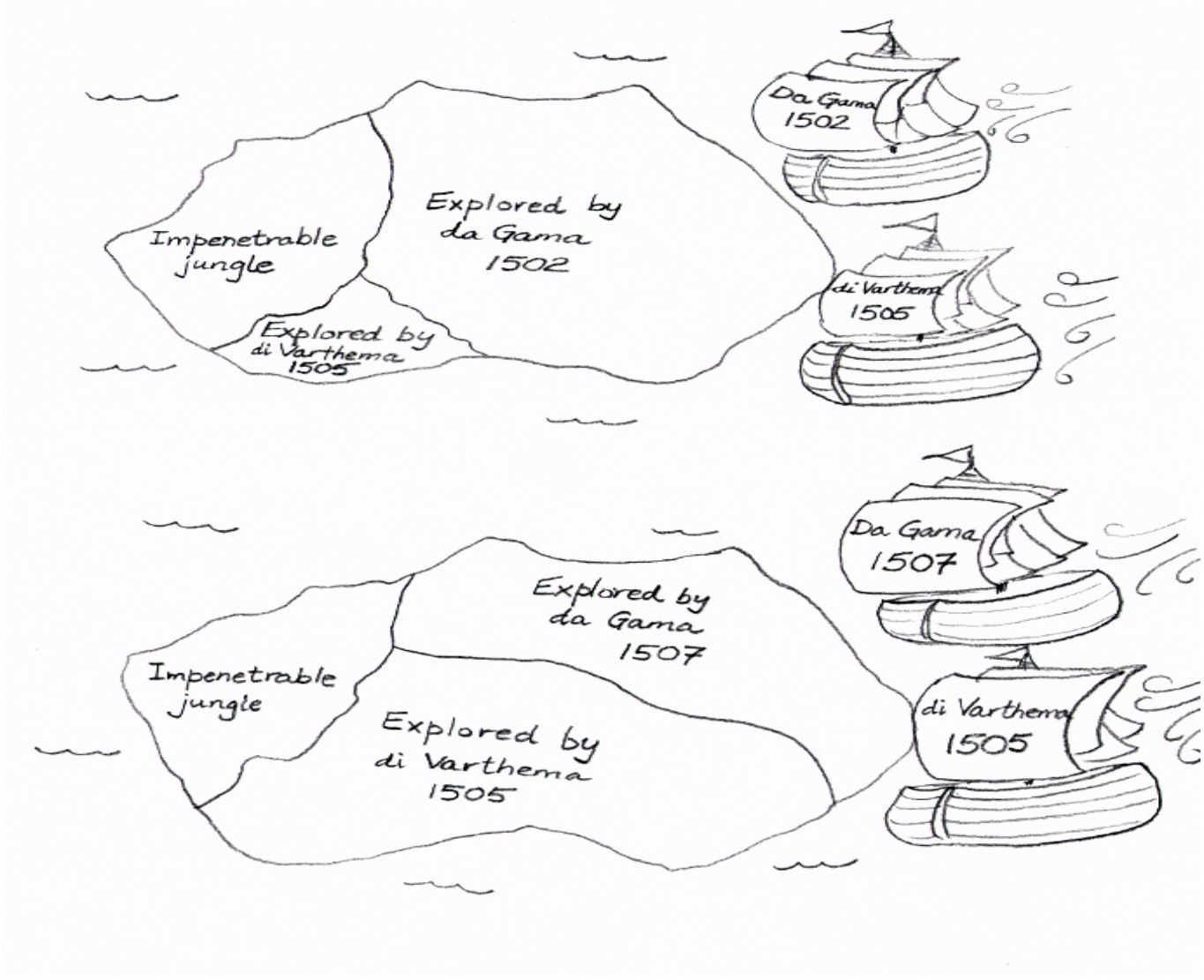
$$\begin{aligned} SSR &= SSR(\beta_1, \beta_2, \ldots, \beta_k|\beta_0) \\ &= SSR(\beta_1|\beta_0) + SSR(\beta_2|\beta_1,\beta_0) + SSR(\beta_3|\beta_2,\beta_1,\beta_0) + \ldots \\ &\quad + SSR(\beta_k|\beta_0,\beta_1,\beta_2, \ldots, \beta_{k-2},\beta_{k-1}) \end{aligned}$$

For example, the sequential $SSR(\beta_2|\beta_1,\beta_0)$ is the amount of unexplained variability from a simple linear regession on x_1 which is subsequently explained by x_2, so it represents the increase in the regression sum of squares obtained by adding the predictor x_2 to a model that already contains x_1.

Diagram from the top of page 8 of the chapter 2 of the lecture notes



Sums of squares – the undiscovered island (whoever gets there first, gets first claim!)



Sums of squares – explanatory variables are like explorers – who gets there first matters!