STAT6046 week 3 lecture 5

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Recall what we learned last week, δ is the **force of interest rate.**

Suppose we have \$1 amount of dollar, invested. And we know after 1 year, we have an amount of $S(1)$. **There could be multiple ways/paths from $S(0)$ to $S(1)$.**

But note that the effective rate of interest is always $i = \frac{S(1)-1}{1}$.

Why not $S'(t)$? The 1st derivative of $S(t)$ is not a good measurement of growth. (example)

So we better use the force of interest:

$$\frac{S'(t)}{S(t)} = \delta.$$

1. Accumulated Value using δ.

$$\delta_t = \frac{S'(t)}{S(t)} = \frac{d}{dt}\ln(S(t))$$

$$S(n) = S(0) \cdot \exp\left(\int_0^n \delta_t dt\right)$$

Proof:

$$\int_{t=0}^{t=n} \delta_t dt = \int_0^n \frac{d}{dt} \cdot \ln\left[S(t)\right] dt$$
$$= \ln[S(n)] - \ln[S(0)]$$
$$S(n) \implies S(0) \cdot \exp\left[\int_0^n \delta_t dt\right]$$

And some other review stuffs.

Annuities   In this class we mainly focus on 3 types of annuities: **immediate annuity, annuity due and deferred annuity.**

Immediate Annuity