map of "$\forall\delta\ \exists x$" (1-4)

The Combination $\forall\delta\ \exists x$ is a relic, for Constructing special Sequences:

f is <u>not</u> Cont at a means
$\exists\epsilon>0\ \underline{\forall\delta>0\ \exists x}\ |x-a|<\delta$ but
$|f(x)-f(a)|>\epsilon$

$a\in\bar{S}$ iff $\forall\delta>0\ B(\delta,a)\cap S\neq\emptyset$
i.e $\underline{\forall\delta>0\ \exists x}\in B(\delta,a)\cap S$

magic of
$\forall\delta>0\ \exists x\ \ldots$

$\forall\delta>0\ \exists x\in B(\delta,a)\ \ldots$

for $\delta=1$ $\quad\exists x_1\in B(1,a)\ldots$
for $\delta=\frac{1}{2}$ $\quad\exists x_2\in B(\frac{1}{2},a)\ldots$
for $\delta=\frac{1}{3}$ $\quad\exists x_3\in B(\frac{1}{3},a)\ldots$
$\vdots$
for $\delta=\frac{1}{n}$ $\quad\exists x_n\in B(\frac{1}{n},a)\ldots$
$\vdots$

f is Cont at a
iff for any Seq $\{x_n\}$
$x_n\to a$ implies
$f(x_n)\to f(a)$:

Pf: ($\Leftarrow$) if f is NOT Cont at a Then
$\exists\epsilon>0\ \underline{\forall\delta>0\ \exists x}$
$x\in B(\delta,a)$ but $|f(x)-f(a)|\geq\epsilon$

Construct a sequence $\{x_n\}$ That converges to a but
$|f(x_n)-f(a)|\geq\epsilon$ for all n.

$a\in\bar{S}$ iff
$\exists\{x_n\}\subset S$ s.t.
$x_n\to a$

Pf: $a\in\bar{S}\Leftrightarrow$
$\forall\delta>0\ \exists x\in B(\delta,a)\cap S$

Construct a Seq $\{x_n\}$ s.t.
$\forall\epsilon>0\ \exists N$ s.t.
$\forall n\ n>N\ \ x_n\in B(\epsilon,a)$
let $N>\frac{1}{\epsilon}$.