STA457 PRACTICE QUESTION BY TOPICS

A. VECTOR AUTOREGRESSIVE PROCESS

1. Check whether the following vector autoregressive processes are stationary:

   a) Model 1

$$\begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix} = \begin{bmatrix} 0.4 \\ 0.2 \end{bmatrix} + \begin{bmatrix} 1.1 & -0.6 \\ 0.3 & 0.2 \end{bmatrix} \begin{bmatrix} y_{1,t-1} \\ y_{2,t-1} \end{bmatrix} + \begin{bmatrix} a_{1t} \\ a_{2t} \end{bmatrix},$$

   b) Model 2

$$\begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix} = \begin{bmatrix} 0.5 & 0.1 \\ 0.4 & 0.5 \end{bmatrix} \begin{bmatrix} y_{1,t-1} \\ y_{2,t-1} \end{bmatrix} + \begin{bmatrix} a_{1t} \\ a_{2t} \end{bmatrix},$$

   c) Model 3

$$\begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix} = \begin{bmatrix} 0.5 \\ 0.2 \end{bmatrix} + \begin{bmatrix} 0.21 & -0.11 \\ 0.11 & 0.51 \end{bmatrix} \begin{bmatrix} y_{1,t-1} \\ y_{2,t-1} \end{bmatrix} + \begin{bmatrix} a_{1t} \\ a_{2t} \end{bmatrix},$$

   where $(a_{1t}, a_{2t})'$ follows a bivariate normal random variable with mean vector $(0,0)'$ and covariance matrix $\begin{bmatrix} 1 & 0.5 \\ 0.5 & 2 \end{bmatrix}$.

2. Describe how to check the stationarity of the following VAR(2) model.

$$\begin{bmatrix} X_{1,t} \\ X_{2,t} \end{bmatrix} = \begin{bmatrix} \phi_{1,11} & \phi_{1,12} \\ \phi_{1,21} & \phi_{1,22} \end{bmatrix} \begin{bmatrix} X_{1,t-1} \\ X_{2,t-1} \end{bmatrix} + \begin{bmatrix} \phi_{2,11} & \phi_{2,12} \\ \phi_{2,21} & \phi_{2,22} \end{bmatrix} \begin{bmatrix} X_{1,t-2} \\ X_{2,t-2} \end{bmatrix} + \begin{bmatrix} a_{1,t} \\ a_{2,t} \end{bmatrix}.$$

   (Hint: Transform the above VAR(2) model into a VAR(1) model.)

B. GRANGER CAUSALITY

1. Define Granger Causality. State the procedure of using the following VAR(p) model to test if y_{1t} Granger causes y_{2t}

$$\begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} + \sum_{j=1}^{p} \begin{bmatrix} \phi_{j,11} & \phi_{j,12} \\ \phi_{j,21} & \phi_{j,22} \end{bmatrix} \begin{bmatrix} y_{1,t-1} \\ y_{2,t-2} \end{bmatrix} + \begin{bmatrix} a_{1t} \\ a_{2t} \end{bmatrix}.$$

2. Can we test Granger Causality without using the VAR approach? If your answer is yes, describe how to implement the test.

3. Define Granger causality and the two tests introduced in class for checking Granger causality.

C. COINTEGRATION

1. Describe the Dickey Fuller unit root test.
2. Define $I(0)$, $I(1)$ and $I(d)$ processes (processes of integrated of order zero, one and d)
3. Define Spurious regression and $I(1)$ processes
4. Define cointegration.
5. Define cointegration between two time series $\{X_t\}$ and $\{Y_t\}$. Describe how to test the existence of the cointegration.
6. Describe the Engle-Granger method to test cointegration.
7. State the Granger representation theorem. Discuss its implication on Vector autoregressive (VAR) modeling.

D. TRANSFER FUNCTION NOISE MODEL

1. Box, Jenkins, and Reinsel (1994) fit a transfer function model to data from a gas furnace. The input variable x_t is the volume of methane entering the chamber in cubic feet per minute and the output is the concentration of carbon dioxide emitted y_t. The transfer function model is

$$y_t = \frac{-(0.53 + 0.37B + 0.51B^2)}{1 - 0.57B} x_t + \frac{1}{1 - 0.53B + 0.63B^2} \varepsilon_t$$

where the input and output variables are measured every nine seconds.

a) What are the value of b, s, and r for this model?

b) What is the form of the ARIMA model for the errors?

c) If the methane input was increased, how long would it take before the carbon dioxide concentration in the output is impacted?

2. An input and output time series consists of 250 observations. The prewhitened input series is modeled by an AR(2) model $y_t = 0.4y_{t-1} + 0.2y_{t-2} + a_t$. Suppose that we have estimated $\hat{\sigma}_\alpha = 0.3$ and $\hat{\sigma}_\beta = 0.35$. The estimated cross-correlation function between the prewhitened input and output time series is shown below.

Lag	0	1	2	3	4	5	6	7	8	9	10
$\rho_{\alpha\beta}$	0.01	0.03	-0.03	0.5	-0.4	-0.09	-0.05	-0.03	-0.02	0.09	-0.01

a) Find the approximate standard error of the cross-correlation function.

b) Which spikes on the cross-correlation function appear to be significant?

c) Estimate the impulse response function. Tentatively identify the form of the transfer function models—i.e. the values of b, r, s in a transfer function model.

E. FORECAST

1. Consider an ARIMA(1,1,0) model,

$$(1-0.5B)(1-B)=a_t,\ \ a_t\sim NID(0,1).$$

   a) Write down the forecast function for origin t.

   b) What is the variance of the 1-step-ahead forecast error?

2. Consider the AR(1) model as follows:

$$(1-0.6B)(X_t-9)=a_t,\quad a_t\sim NID(0,1),\qquad (1)$$

   Suppose that we observe $(X_{97},X_{98},X_{99},X_{100})=(9.6,9,9,8.9)$.

   c) Is the process in eqn. (1) stationary? Why?

   d) Forecast $\{X_t\}$, $t=101,102,103$ and 104 and their associated 95% forecast limits.

   e) Suppose now that the observation at $t=101$ turns out to be $X_{101}=8.8$. Calculate $\hat{X}_{101}(l)$ for lead time, $l=1,2,3$, using "updating forecast".

3. Forecast an ARMA(1,1) model

$$X_t-0.5X_{t-1}=a_t+0.25a_{t-1},\quad a_t\sim NID(0,1),\qquad (*)$$

   Suppose that $(X_{97},X_{98},X_{99},X_{100})=(-0.7,-1,-0.8,-0.4)$. Answer the following questions:

   a) Write down the forecasting function for eqn. (*).

   b) Calculate the best linear forecast of $\mathrm{X}_{101}+\mathrm{X}_{102}+\mathrm{X}_{103}$. For simplicity, assume that $\hat{X}_{99}(1)=0$.

   c) Calculate the 95% forecast (confidence) interval of the forecast in question 3b). For simplicity, use $Z_{0.975}=1$ in your calculation. (10%)

4. Consider the ARIMA(1,1,0) model

$$(1-B)(1+0.9B)X_t=a_t,\quad a_t\sim NID(0,\sigma^2).$$

   The most recent 8 observations for 1989 to 1996 were

   $(X_{89},\dots,X_{96})=(0,-0.1,-1.5,-2.2,-4.3,-4.9,-7.2,-6.3)$.

   a) Write out the recursive formula for forecasting X_{t+l} at original t. Consider $l=1,2,3,\cdots$.

   b) Is this process stationary? Why or why not?

   c) Is this process invertible? Why or why not?

   d) Derive the formulas for predictions for 1997 to 1999 in terms of previously observed values. (These may be expressed in terms of other predictions, as long as you describe how to calculate each term before you use it another formula.)

   e) Let $\bar{X}$ be the average of X_{97}, X_{98} and X_{99}. Use the values above and your formulas to calculate the estimate $\hat{\bar{X}}$ of $\bar{X}$. (You should give this estimate both as a formula and numerically.) Assume that all earlier values of the series are zero if you need them in your predictions.

   f) Calculate the variance of the forecast error $\bar{X}-\hat{\bar{X}}$ in terms of σ^2.

5. Describe the Granger-Newbold test for assessing forecast accuracy and the underlying assumptions.

F. OTHERS

1. Explain the AIC criterion and the BIC criterion for model selection. Which criterion would you choose if the purpose of your analysis is to forecast the time series of interest? **(10%)**

2. Discuss a method taught in class for removing (or modeling) seasonality of time series data.

3. Define generalized autoregressive conditional heteroscedasticity (GARCH) processes

4. An AR(1) model was fitted to a series of 80 observations. The corresponding residual autocorrelation functions are listed below.

k	1	2	3	4	5
$\hat{\rho}_k$	0.75	0.54	0.35	0.13	−0.02

   a) Calculate the Ljung-Box test for diagnostic checking the fitted AR(1) model using all of the residual autocorrelation functions listed above.
   b) What is the theoretical (asymptotic) distribution of the Ljung-Box test asked in question (a). For example, t-distribution with 5 degrees of freedom.
   c) Suppose that you figure out the correct distribution of the test and successfully calculate the p-value of the test, say 0.049. Will you reject the fitted AR(1) model? Why?

5. Show that an autoregressive process of order 1 below is a white noise.

$$X_t = \sigma_t Z_t, \quad Z_t \sim NID(0,1),$$
$$\sigma_t^2 = w_0 + w_1 X_{t-1}^2,$$

   where NID(0,1) denote IID standard normal random variables over time, and we also assume that $w_0 > 0, and\ w_1 \geq 0$.

6. Consider an ARCH(1) process $X_t = \sigma_t Z_t$, where $Z_t \sim NID(0,1)$, $\sigma_t^2 = w_0 + w_1 X_{t-1}^2$, $w_0 > 0$, and $w_1 < 1$. Show that the above ARCH(1) process is "fat-tailed". Hint: Kurtosis greater than 3.