1 March, 2018.

Exercise

If Gompertz' law of mortality holds prove that ${}_t p_x = g^{c^x(c^t-1)}$ where $g = \exp(\frac{-B}{\log c})$.

$\Downarrow$

$$\log(\mu(t)) = \log(B) + t\log(C)$$

$$\mu(t) = \cancel{\frac{f(t)}{S(t)}} = B \cdot C^{x+t}$$

$$ {}_tP_x = \cancel{\frac{l_{x+t}}{l_x}} = S_{T_x}(t) = \exp\left(-\int_0^t \mu_x(s)\, ds\right)$$

$$= \exp\left(-\int_0^t B \cdot C^{x+s}\, ds\right)$$

$$= \exp\left(-BC^x \int_0^t c^s\, ds\right)$$

$$= \exp\left(-BC^x \int_0^t e^{s\ln c}\, ds\right)$$

$$= \exp\left(-BC^x \frac{1}{\ln C}\left[e^{s\ln c}\right]_0^t\right)$$

$$= \exp\left(-BC^x \frac{1}{\ln C}(C^t - 1)\right)$$

$$= g^{c^x(c^t-1)} \quad \text{where } g = \exp\left(\frac{-B}{\ln C}\right)$$