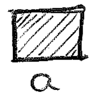
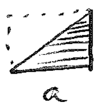
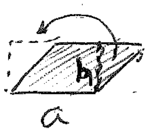
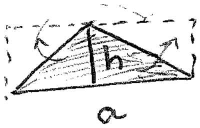
Question of Area

- Area is a mathematical Concept That is defined for a rectangle: 

Area = ab

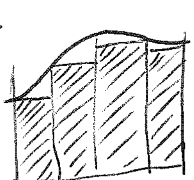
- Area is further extended to Shapes like  $A=\frac{1}{2}ab$

or  $A = ah$

 $A = \frac{ah}{2}$ etc.

But unconventional Shapes and regions like  don't have area. We can define area for These Shapes as a limit of some rectangles

Thus The area of The region bounded by a curve.. is The limit of sums of The areas of rectangles below it, or above it (lower sums or upper sums



Integrability

given $f(x)$, bounded on $[a\,b]$ and an arbitrary partition $\wp = \{x_0, x_1, \dots, x_n\}$ of $[a\,b]$, That is $a = x_0$, $b = x_n$ & $x_{i-1} < x_i$

as f is bounded, The Set $\{f(x);\ x \in [x_{i-1}, x_i]\}$ is bdd, so it has lub & glb, Then we define M_i m_i

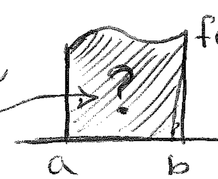
upper sum $S_\wp f = \sum_{i=1}^{n} M_i \, \Delta x_i$ & outer area

lower sum $\mathcal{S}_\wp f = \sum m_i \, \Delta x_i$ inner area

And if $S_\wp f - \mathcal{S}_\wp f = \sum (M_i - m_i)\, \Delta x_i$ Can be controlled, Then we can Say area exists (or f is integrable)

So integrability reduces to The ability to Control Max-min of The function

We can Control $(M_i - m_i)\Delta x_i$ if The Large $M_i - m_i$ takes place on a Small Δx_i.

But, does it even make Sense to Speak of This area 

This is Integrability!

Yes! it does make Sense to Speak of area if The outer area (upper sums) & inner area (lower sum) approach one another