Worth: 8% **Due:** *Before* 10pm on **Thursday 5 April** 2012.

Remember to write the *full name*, *student number*, and *CDF/UTOR email address* of *each group member* prominently on your submission.
*Please read and understand the policy on Collaboration given on the Course Information Sheet. Then, to protect yourself, list on the front of your submission **every** source of information you used to complete this homework (other than your own lecture and tutorial notes, and materials available directly on the course webpage). For example, indicate clearly the **name** of every student with whom you had discussions (other than group members), the **title** of every additional textbook you consulted, the **source** of every additional web document you used, etc.*

For each question, please write up detailed answers carefully. Make sure that you use notation and terminology correctly, and that you explain and justify what you are doing. Marks **will** be deducted for incorrect or ambiguous use of notation and terminology, and for making incorrect, unjustified, ambiguous, or vague claims in your solutions.

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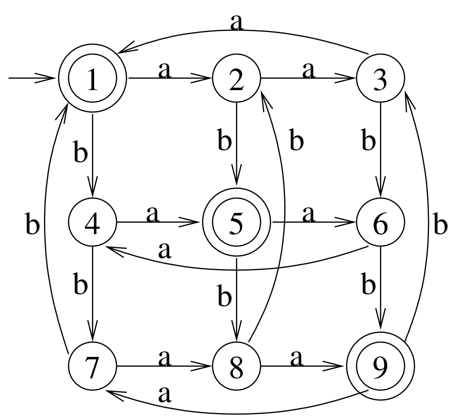
1. Consider the following DFA.



   (a) Give a one-sentence description of the language accepted by this DFA.
   (b) Give another DFA that is equivalent to the one above but using as few states as possible.
   (c) Justify that your DFA accepts the same language as the one above. (You do not have to write a formal proof.)

2. Give an NFA and a RE for each of the following languages. Give a brief justification that your answers are correct, *i.e.*, explain why your NFA accepts every string in the language but no other, and why your RE describes every string in the language but no other — **no formal proof required** — just brief English explanations.

   (a) $L_1 = \{s \in \{a,b,d\}^* : s$ contains *exactly one* occurrence of the substring "*dab*" **and** *no* occurrence of the substring "*bad*"$\}$
   (b) $L_2 = \{s \in \{0,1,2\}^* : s$ contains some pair of 2's separated by exactly five characters other than a $2\}$
   (c) $L_3 = \{s \in \{0,1,2\}^* :$ the integer value of s (in ternary notation) is one less than a multiple of $4\}$
   (For example, $21 \in L_3$ because the integer value of "21" in base three is $2 \cdot 3 + 1 \cdot 1 = 7 = 8 - 1$.)

3. For every language $L \subseteq \Sigma^*$, define $\text{INIT}(L) = \{x \in \Sigma^* : \exists y \in \Sigma^*, xy \in L\}$. (Intuitively, $\text{INIT}(L)$ is the set of all strings that can be "completed" to become a string in L.) For example, $\text{INIT}(\{a, ab, bab\}) = \{\varepsilon, a, ab, b, ba, bab\}$.

   Prove that the class of regular languages is closed under the INIT operation, *i.e.*, that for every regular expression R, there exists a regular expression R_I such that $L(R_I) = \text{INIT}(L(R))$.