CORRECTION SHEET
AN INTRODUCTION TO RANDOM MATRICES
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This list collects the most current list of corrections for the above book. Items marked by $*$ are important, that is, are more than typos. We thank the following people for their comments: Florent Benaych-Georges, Jun Chen, Amir Dembo, Svante Janson, Toby Johnson, Achim Klenke, Mylène Maïda, Edouard Maurel Segala, Alain Rouault.

(1) Page 10, line 2, replace $2z$ in the denominator by 2.
(2) $*$ Page 15, line 22 and 30, replace "the next-to-last letter of w_{i-1}" by "the entry preceding the first occurence of the last letter of w_{i-1}".
(3) $*$ Page 13, line 23, replace "$\ell(w)-1$ edges" by "at most $\ell(w)-1$ edges".
(4) Page 23, line 3, add a missing $|$ after $f(y)$.
(5) Page 32, line 4, replace "fail" by "fails".
(6) Page 36, line -3, replace $T_{\mathbf{i}}^N$ by $\bar{T}_{\mathbf{i}}^N$.
(7) Page 37, (2.2.4), replace "$g_w(1,1)$" by "$g_w(1,1)^{k/2}$".
(8) Page 40, line -2, replace "$G\wedge(-1/\epsilon)\vee(1/\epsilon)$" by "$G\wedge(1/\epsilon)\vee(-1/\epsilon)$".
(9) Page 41, line 3, replace "$= M\ldots$" by "$\leq \sqrt{M}\ldots$". In (2.3.8), replace E in left side by E_P.
(10) Page 49, Equation (2.4.14), replace $S_\mu(\lambda+\epsilon)$ by $S_\mu(\lambda+\mathrm{i}\epsilon)$.
(11) Page 59 line -4, replace $\Delta(x)^{2c}$ by $|\Delta(x)|^{2c}$.
(12) Page 73, in (2.6.10), replace "for all" by "for (Lebesgue) almost all". (In fact, the set where the strict inequality does not hold has vanishing logarithmic capacity.)
(13) Page 76, line -7, Page 77, lines 6 and 17, replace $Z_N^{\beta,V}$ by $Z_{V,\beta}^N$.
(14) Page 78 line -4, -2, -1, page 79 line 3, replace ν by μ.
(15) Page 79 line 7, replace ϵ by δ.
(16) Page 79, line -6, display, replace "$\lim_{N\to\infty}$" by "$\lim_{\delta\to 0}$".
(17) Page 79 line -4, replace $\lambda_i<\lambda_{i-1}$ by $\lambda_i<\lambda_{i+1}$.
(18) Page 80, lines 14, 17, replace $Z_{\beta,V}^N$ by $Z_{V,\beta}^N$.
(19) Page 81, line 7, replace (2.4.6) by (2.4.7).
(20) $*$ Page 81, Assumption 2.6.5: add the assumption that either $J_\beta^V(\cdot)$ achieves its minimum at x^* only, or that under $P_{NV/(N-1),\beta}^{N-1}$, the top eigenvalue converges in probability to x^*. In the former case, one need to use Jensen's inequality in order to show that $J_V^\beta(\cdot)\geq 0$.
(21) $*$ Page 81, display in Theorem 2.6.6 has a sign error, and should read

$$J_\beta^V(x)=\begin{cases} -\beta\int\log|x-y|\sigma_\beta^V(dy)+V(x)-\alpha_{V,\beta} & \text{if } x\geq x^*\,,\\ \infty & \text{otherwise}\,,\end{cases}$$

(22) Page 91, line 6, replace $\sqrt{n}$ by $\sqrt{N}$.
(23) Page 93, line -4, replace Corollary 3.1.5 by Theorem 3.1.5.

(24) Page 97, equation (3.2.10) up to page 98, 2 lines above Lemma 3.2.4: replace $\tilde{C}_{p,N}$ by $\hat{C}_{p,N} = (N-p)!\tilde{C}_{p,N}$ throughout.
(25) Page 103, "Proof of Lemma 2.1.7" should be replaced by "Proof of Lemma 2.1.6".
(26) Page 105, Exercise 3.3.4, line -2: replace x^{2k} by $(x^{2k} - x^k y^k)$.
(27) Page 106, line 2, replace $K(x,y)$ by $K^{(N)}(x,y)$. Line 6, replace Section 3.2.1 by Section 3.2.2.
(28) Page 113, (3.4.25), +tr in the first line of the display should be −tr.
(29) Page 116, line 2 of proof of Theorem 3.5.3, replace $s\varepsilon(s)^2 \to_{s\to\infty} \infty$ by $s\varepsilon(s)^2/\log s \to_{s\to\infty} \infty$.
(30) Page 123, (3.6.5), replace $\Delta_\ell(x,y)$ by Δ_ℓ.
(31) Page 137, first display, replace $O(t^{4/3})$ by $O(t^{2/3})$, replace $O(t^{1/3})$ by $O(t^{-1/3})$.
(32) Page 147, Equation (3.8.22), the limit in t should be as $t \to -\infty$.
(33) Page 151, line 3, add that $n' = n$ if n is even and $n' = n+1$ if n is odd.
(34) $*$ Page 151, line -6: the claim that $\Phi_A(z)$ arises from $\Psi_A(z)$ by column operations is true only when n is odd. For n even, the conclusion that

$$\int_{A_+^r} \det \Psi_A(z) \prod_1^r dz_i = c_1 \int_{A_+^r} \det \Phi_A(z) \prod_1^r dz_i \tag{0.1}$$

is arrived at differently, as we now describe. Set

$$\Theta_A(z) = \left[\begin{array}{cc} f_i(z_j) & \epsilon(\mathbf{1}_A f_i)|_{z_j}^{\infty} \end{array}\right]|_{n,r}.$$

By applying column operations we have $\det \Psi_A(z) = \det \Theta_A(z)$. Define

$$\dot{\Theta}_A(z) = \\ \left[\ \left[\begin{array}{cc} f_i(z_j) & \epsilon(\mathbf{1}_A f_i)|_{z_j}^{\infty} \end{array}\right]|_{n,r-1} \quad \left[\begin{array}{cc} f_i(z_r) & -\epsilon(\mathbf{1}_A f)(\infty) \end{array}\right]|_{n,1}\ \right]$$

and the "deformed matrix"

$$\tilde{\Psi}_A(z) := (\Theta_A + \dot{\Theta}_A)(z) \\ = \left[\ \left[\begin{array}{cc} f_i(z_j) & \epsilon(\mathbf{1}_A f_i)|_{z_j}^{\infty} \end{array}\right]|_{n,r-1} \quad \left[\begin{array}{cc} f_i(z_r) & -\epsilon(\mathbf{1}_A f)(z_r) \end{array}\right]|_{n,1}\ \right].$$

Applying obvious column operations one gets $\det \Phi_A(z) = c_1 \det \tilde{\Psi}_A(z)$, for a nonzero constant c_1 independent of A and z. Finally, one has

$$\begin{aligned} & \int_{A\cap[z_{r-1},\infty)} (\det \widetilde{\Psi}_A(z_1,\ldots,z_{r-1},t)dt - \det \Psi_A(z_1,\ldots,z_{r-1},t))dt \\ = & \int_{A\cap[z_{r-1},\infty)} \det \dot{\Theta}_A(z_1,\ldots,z_{r-1},t)dt \\ = & \det \left[\ \left[\begin{array}{cc} f_i(z_j) & \epsilon(\mathbf{1}_A f_i)|_{z_j}^{\infty} \end{array}\right]|_{n,r-1} \quad \left[\begin{array}{cc} \epsilon(\mathbf{1}_A f_i)|_{z_{r-1}}^{\infty} & -\epsilon(\mathbf{1}_A f)(\infty) \end{array}\right]|_{n,1}\ \right] \\ = & 0. \end{aligned}$$

Integrating over the remaining coordinates yields (0.1).

(35) Page 190, line 18, display: replace $e^{-\beta x_i/4}$ by $e^{-\beta x_i/2}$ (recall that *standard Normal* is defined in pages 188-189).

(36) Page 208, line 4, replace $\mathrm{Mat}_{p\times q}$ by $\mathrm{Mat}_{p\times(q-p)}$.

(37) $*$ Page 208, Proposition 4.1.33: Unfortunately, Assumption (IId) in the definition of the Weyl quadruple is not satisfied in the setup described in Proposition 4.1.33. The following correction shows that the conclusion of the proposition still holds.

The only use of Assumption (IId) is in the proof of

$$\mathbb{T}_{I_n}(f_\lambda)(\mathbb{T}_{I_n}(G)) \subset \mathbb{T}_\lambda(M) \cap \mathbb{T}_\lambda(\Lambda)^\perp \tag{4.1.24}$$

in Lemma 4.1.26, see (4.1.27). Since (4.1.23) already shows that $\mathbb{T}_{I_n}(f_\lambda)(X) = [X, \lambda]$, and since the inclusion in $\mathbb{T}_\lambda(M)$ is automatic, it is enough to verify that

$$[X, \lambda] \cdot \tau = 0 \ \text{ for } X \in T_{I_n}(G),\ \lambda \in \Lambda \text{ and } \tau \in T_\lambda(\Lambda). \tag{0.2}$$

In particular, (0.2) should replace Assumption (IId) in the definition of the Weyl quadruple.

We now check (0.2) under the assumptions of Proposition (4.1.33). Recall that $0 < p \le q$, $n = p + q$, $0 \le r \le q - p$ and $q = p + r + s$. Fix a point

$$\lambda = \mathrm{diag}(\begin{bmatrix} x & y \\ y & I_p - x \end{bmatrix}, I_r, 0_s) \in \Lambda.$$

Recall that $x^2 + y^2 = x$. By implicit differentiation of the last matrix equation we deduce that

$$\begin{aligned} \mathbb{T}_\lambda(\Lambda) \quad = \quad & \{\mathrm{diag}\left(\begin{bmatrix} \xi & \eta \\ \eta & -\xi \end{bmatrix}, 0_{r+s}\right) \mid \ \xi, \eta \in \mathrm{Mat}_p \text{ are diagonal} \\ & \text{and } (2x - I_p)\xi + 2y\eta = 0\}. \end{aligned}$$

Fix a tangent vector

$$\tau = \mathrm{diag}\left(\begin{bmatrix} \xi & \eta \\ \eta & -\xi \end{bmatrix}, 0_{r+s}\right) \in \mathbb{T}_\lambda(\Lambda).$$

We have

$$\mathbb{T}_{I_n}(G) = \{\mathrm{diag}(P, Q) | P \in T_{I_p}(\mathrm{U}_p(\mathbb{F})), Q \in T_{I_q}(\mathrm{U}_q(\mathbb{F}))\}.$$

We arbitrarily fix

$$\begin{array}{cc} & \begin{array}{cccc} p & p & r & s \end{array} \\ \begin{array}{c} p \\ p \\ r \\ s \end{array} & X = \begin{bmatrix} a & 0 & 0 & 0 \\ 0 & b & * & * \\ 0 & * & * & * \\ 0 & * & * & * \end{bmatrix} \in \mathbb{T}_{I_n}(G) \end{array}$$

where we have broken down into blocks of the indicated sizes, and the blocks marked by $*$ will be irrelevant to the computation, and therefore we do not specify their values. We get

$$\begin{bmatrix} a & 0 & 0 & 0 \\ 0 & b & * & * \\ 0 & * & * & * \\ 0 & * & * & * \end{bmatrix} \begin{bmatrix} x & y & 0 & 0 \\ y & I_p - x & 0 & 0 \\ 0 & 0 & I_r & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} ax & ay & 0 & 0 \\ by & b(I_p - x) & * & 0 \\ * & * & * & 0 \\ * & * & * & 0 \end{bmatrix},$$

$$\begin{bmatrix} x & y & 0 & 0 \\ y & I_p - x & 0 & 0 \\ 0 & 0 & I_r & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} a & 0 & 0 & 0 \\ 0 & b & * & * \\ 0 & * & * & * \\ 0 & * & * & * \end{bmatrix} = \begin{bmatrix} xa & yb & * & * \\ ya & (I_p - x)b & * & * \\ 0 & * & * & * \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

and therefore

$$[X, \lambda] \cdot \tau = \Re \mathrm{tr} \left(\begin{bmatrix} [a,x] & ay - yb & * & * \\ by - ya & -[b,x] & * & * \\ * & * & * & * \\ * & * & * & * \end{bmatrix} \begin{bmatrix} \xi & \eta & 0 & 0 \\ \eta & -\xi & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}^* \right).$$

In turn, by exploiting all the zeroes above, we get $[X, \lambda] \cdot \tau = [a, x] \cdot \xi + [b, x] \cdot \xi + (ay - yb) \cdot \eta + (by - ya) \cdot \eta$. Taking into account that x, y, ξ and η are real and diagonal, it implies that indeed $[X, \lambda] \cdot \tau = 0$ as required by (0.2).

(38) Page 214, second display, the expressions in the definitions of $D(\alpha, \beta)$ and $C(\alpha)$ should be squared.

(39) $*$ Page 219, equation (4.2.8), replace $\prod_{i=1}^k D_i \times D^r$ by

$$\prod_{i=1}^k D_i \times (D \setminus \cup_{i=1}^k D_i)^r.$$

(40) $*$ Page 244, display (4.2.51): add the assumption that μ^N is a positive measure of total mass $N!$.

(41) Page 284, 3 lines above Corollary 4.4.4, replace $a^2 m^{-1}$ by $a^{-2} m$.

(42) Page 303, Theorem 4.5.35, replace Edelman–Dumitriu by Dumitriu–Edelman.

(43) Page 429, line 4 of Lemma D.9, replace A by B.