July 30th

Review of § 5.1

1. Arc length of a curve $C: g:[a,b] \longrightarrow R^n$, g is diff.

$\int_C ds = \int_a^b \sqrt{(x_1'(t))^2+\cdots+(x_n'(t))^2}\, dt$ $\qquad t \longrightarrow (x_1(t),\cdots,x_n(t))$

2. $\int_C d\vec{x} = \int_a^b \vec{g}'(t)dt = \vec{g}(b)-\vec{g}(a)$ $\qquad$ SCALAR

if orientation changes, you will have minus sign.

3. $f: \mathbb{R}^n \longrightarrow \mathbb{R}$, $\vec{g}$ is the same as before.

$\int_C f\, ds = \int_a^b f(\vec{g}(t))\, |g(t)|dt$ $\quad$ invariant under reparametri-zation.

4. $\vec{F}$ is a vector field in $\mathbb{R}^n$. Then

$\int_C \vec{F}d\vec{x} = \int_C F_1dx_1 + F_2dx_2+\cdots+F_ndx_n = \int_a^b \vec{F}(\vec{g}(t))\cdot\vec{g}'(t)dt.$

depends on orientation

VECTOR

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1. $\vec{g}(t) = (a\cos t, a\sin t, bt)$ $\qquad t\in[0,2\pi]$

$\vec{g}'(t) = (-a\sin t, a\cos t, b)$

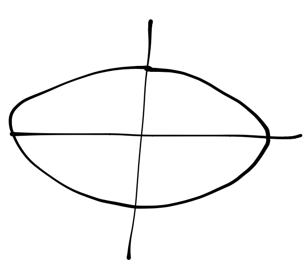
$l = \int_0^{2\pi} |g'(t)|dt = \sqrt{a^2+b^2}\, t\Big|_0^{2\pi} = 2\pi\sqrt{a^2+b^2}$

2. express the arc length of following curves in terms of the integral.

$E(k) = \int_0^{\pi/2} \sqrt{1-k^2\sin^2(t)}\; dt \quad (0<k<1)$

(a) an ellipse

(b). The portion of intersection of the sphere $x^2+y^2+z^2=4$ and the cylinder $x^2+y^2-2y=0$ in first



$x = a\sin t,\ y = b\cos t,\ t\in[0,2\pi]$

$S = 4\int_0^{\pi/2} \sqrt{a^2\cos^2 t + b^2\sin^2 t}\cdot dt$

$= 4\int_0^{\pi/2} \sqrt{a^2\sin^2 t + b^2(1-\sin^2 t)}\; dt$

$= 4\int_0^{\pi/2} \sqrt{(a^2-b^2)\sin^2(t)+b^2}\; dt$

$$=4b\int_0^{\pi/2}\sqrt{1-\left(1-\frac{b^2}{a^2}\right)\sin^2(t)}\cdot dt$$