2017-02-20-lec01

We basically talk about some fields of the beautiful mathematics in the first lecture as an introduction.

I. Number Theory

primes.

FACT: Any integer n can be decomposed uniquely as a product of primes.

So Euclid asked, **are there infinitely many primes?**

Proof:

Suppose $2, 3, 5, 7$ were all primes we know of.
Can we come up a number that is not divisible by $2, 3, 5, 7$?
Yep, $2 \times 3 \times 5 \times 7 + 1 = 211$ which is a new prime.
Proof by contradiction.
Suppose there are finitely many primes $p_1, \ldots p_k + 1$
Consider the integer $N = p_1 \cdot p_2 \cdots p_k + 1$
Two cases:

- If N is prime, then done.
- If N is not prime, then let q be any prime divisor of N. Then q must be distinct from $p_1, \ldots, p_k$ (based on the fact above). Contradiction done.

Twin Prime Conjecture (which is still unproved yet!): Progess, in 2013 Zhang had proved that $\exists\infty$ many pairs of primes (p_i, p_j) such that $|p_i - p_j| < 7 \times 10^8$.... (346 currently!)

Geometry

Pythagorean Theorem

Analysis

$$l^2 = 2 \ (l = \sqrt{2})$$

For Pythagoreans, the only numbers they had were rationals $\frac{q}{p}$.

Theorem: There does not exist a rational number $\frac{q}{p}$ such that $\left(\frac{q}{p}\right)^2 = 2$

$$\left(\frac{7}{5}\right)^2 = \frac{49}{25} \neq 2$$

$$\left(\frac{99}{70}\right)^2 = \frac{9801}{4900} \neq 2$$

Proof:

Recall that if a is even, then a^2 is even. $(2 = 2k$ for some integer k, $a^2 = 4k^2 = 2(2k^2)$, a^2 is even)

Claim a is odd, then a^2 is also odd. $(a = 2k+1, a^2 = 4k^2 + 4k + 1$, a^2 is odd.)

So suppose there exists integers a, b such that $\left(\frac{a}{b}\right)^2 = 2 \iff a^2 = 2b^2$

WLOG: Assume at least one of a, b is odd.

1. Suppose a, b both odd, a^2 odd, b^2 odd, $2b^2$ even, contradiction.
2. Suppose a odd, b even, a^2 odd, $2b^2$ even, contradiction.
3. Suppose a even, b odd, a^2 even, b^2 odd, $2b^2$ even, $a^2 = (2k)^2 = 4k^2 = 2b^2$. Then $2k^2 = b^2$, $2k^2$ is even but b^2 is odd, contradiction.