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A Bayesian Approach to the Two-period Style-goods Inventory Problem with Single Replenishment and Heterogeneous Poisson Demands

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This paper presents a model of the two-period style-goods inventory problem for a firm which stocks many hundreds of distinctive items having heterogeneous Poisson demands. The model uses a Bayesian procedure for forecast and probability revisions based on an aggregation-by-items scheme. These revised forecasts are then incorporated into a model which is used to derive the optimal inventory-stocking policies which maximize expected profit during the season. The model is illustrated using an actual case study of inventory planning for unframed poster art.

Key words: forecasting, inventory, statistics

INTRODUCTION

The style-goods inventory problem has received considerable attention in the management science and operational research literatures. Although a large portion of the work on this problem appeared in the late 1960s and early 1970s, recent articles by Chambers and Eglese[1,2] have indicated a renewed interest in the problem. The term 'style-goods' has come to encompass a broad class of inventory problems, including the classical newsboy/Christmas tree problems as well as the mail order, catalogue sales and fashion-goods inventory problems. The problem is generally characterized by the following circumstances (see, for example, Silver and Peterson[3]):

(i) a relatively short, well-defined selling season where the seller/producer must be committed to a large order prior to the start of the season,
(ii) one or more stock-replenishment opportunities during the season, after the initial order,
(iii) attendant underage and overage costs when (and if), respectively, demand exceeds or falls short of the on-hand stock,
(iv) possible substitution of style goods within a line or style, and
(v) moderate to extreme uncertainty in the demand forecasts.

Considering its importance and complexity, it is not surprising that the vast majority of research in this area has focused on the problem of demand forecasting (see, for example, Chang and Fyffe,[4] Goyal,[5] Green and Harrison,[6] Hausman and Peterson,[7] Hausman and Sides,[8] Hertz and Schaffir,[9] Murray and Silver,[10] Ravindran[11] and Silver[12]).

This paper presents a mathematical model of the two-period style-goods inventory problem and its application to a case study in inventory planning and control for a retail firm selling unframed poster art. The selling season is divided into two equal time-periods (periods 1 and 2), and a single replenishment opportunity is allowed at the end of period 1. The model is a direct extension of the single-period newsboy/Christmas tree problem, which allows for inventory carry-over from period 1 into period 2.

The model utilizes a Bayesian forecasting procedure whereby period 1 demands are used to update the prior parameters and revise the period 2 forecasts. These forecasts are then incorporated into the model to derive optimal stocking policies which maximize expected profit over the season. The model assumes that

(i) demand for each item (individual) during the arbitrary time-interval $[0, t]$ is Poisson with unknown but stationary mean rate λt; and
(ii) λ is distributed according to a gamma distribution across the individual items (aggregate).

As the model uses an aggregation by items scheme, (ii) is necessary to capture the heterogeneity of aggregate demand. The consequence of the gamma mixture of Poisson distributions from (i) and (ii) is that aggregate demand follows a negative binomial distribution (NBD). In addition to the 'pure' gamma mixing distribution, a gamma distribution augmented by a spike at the origin to account for those items having true zero demand during the season is also considered.

The mathematics underlying the Bayesian procedure were originally derived over 60 years ago in conjunction with accident-proneness models.[13] While numerous authors[4,5,8,10,14] have previously suggested Bayesian approaches for tackling this problem, Chambers and Eglese[1] have noted that they do not appear to have gained wide acceptance in practice. The formulation in this paper provides more extensive managerial insights into the problem and allows for paper-and-pencil forecasts even when dealing with thousands of unique items.

THE FIRM AND DATA

This case study uses a small retail firm which sells framed and unframed poster art through several outlets in three large metropolitan areas in the eastern United States. The data consisted of the complete demand histories for 667 distinctive unframed poster art titles over 8 months (the season) from a single outlet. The data were split into two consecutive 4-month periods corresponding to periods 1 and 2, respectively. Figure 1 gives the histogram of aggregate demand for all titles over the season. Total seasonal demand was 2,011 units (mean 3.015 units, s.d. 3.236 units). Individual demands ranged from 0 to 24 units and 150 titles (22.5%) recorded no demand during the season. Demands for periods 1 and 2 were 1,012 units (mean 1.517 units, s.d. 1.803 units) and 999 units (mean 1.498 units, s.d. 1.924 units), respectively.

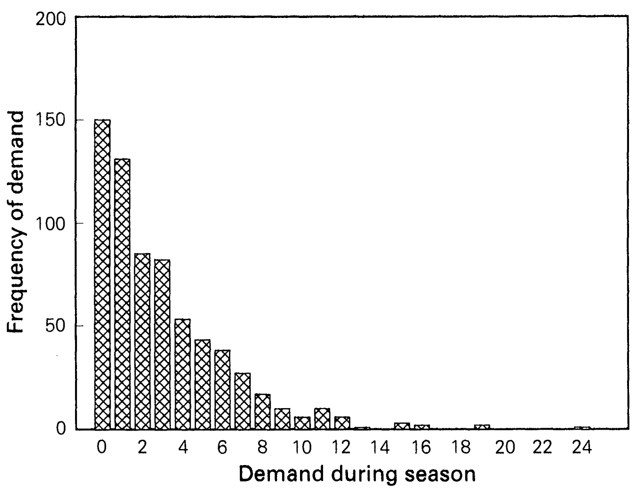

FIG. 1. *Histogram of aggregate demand for the season.*

For several reasons, this product represents a unique and particularly appropriate type of style good. First, it is not a heavily branded product in that rarely does a customer request a specific poster by title. Second, it is not a repeat-purchase item and is usually purchased in a unit batch-size. Lastly, the retail display inventory allocated to each poster title is the same regardless of the amount of on-hand inventory carried for each item.

FORECASTING PERIOD 2 DEMAND FROM PERIOD 1 DEMAND

Period 1 NBD behaviour

Demand for each poster during the arbitrary time-interval $[0, t]$ is assumed to be Poisson with unknown but stationary mean rate λt. The *conditional* probability of demand $X_1 = x$ units during period 1 is

$$p(X_1 = x \mid \lambda) = (\lambda t)^x e^{-\lambda t}/x!, \qquad x = 0, 1, 2, \ldots, \tag{1}$$

where X_1 is the period 1 random distribution of demand and period 1 is of length $[0, t]$. Furthermore, the heterogeneity of aggregate demand is captured by assuming that λ is distributed gamma with density:

$$f_y(\lambda) = (\alpha^r \lambda^{r-1} e^{-\alpha\lambda})/(r-1)!, \qquad \lambda > 0, \tag{2}$$

where r and α are, respectively, the shape and scale parameters of the gamma mixing distribution. This particular parameterization has expectation $E[\lambda] = r/\alpha$ and variance $V[\lambda] = r/\alpha^2$.

Combining (1) and (2) and assuming period 1 is of arbitrary unit length, the unconditional probability of demand $X_1 = x$ is

$$\begin{aligned} p(X_1 = x) &= \int_0^\infty [(\lambda t)^x e^{-\lambda t}/x!] \cdot f_y(\lambda)\, d\lambda \\ &= \binom{r+x-1}{r-1}\left(\frac{\alpha}{\alpha+1}\right)^r\left(\frac{1}{\alpha+1}\right)^x, \qquad x = 0, 1, 2, \ldots, \end{aligned} \tag{3}$$

where X_1 is NB(r, α) with expectation $E[X_1] = r/\alpha$ and variance $V[X_1] = r(\alpha + 1)/\alpha^2$.

Period 2 predictions: the conditional bivariate NBD

The conditional period 2 distribution of λ given $X_1 = x$ is found by the application of Bayes' rule as follows:[14]

$$f_y(\lambda \mid X_1 = x) = \frac{p(X_1 = x \mid \lambda) \cdot f_y(\lambda)}{\int_0^\infty p(X_1 = x \mid \lambda) \cdot f_y(\lambda)\, d\lambda}. \tag{4}$$

Substituting (1) and (2) into (4) yields the *posterior* distribution:

$$f_y(\lambda \mid X_1 = x) = [(\alpha + 1)^{r+x} \lambda^{r+x-1} e^{-(\alpha+1)\lambda}]/(r + x - 1)!, \tag{5}$$

which is also the gamma distribution but with updated parameters $r + x$ and $\alpha + 1$.

The relationship among (1), (2) and (5), when interpreted within a Bayesian context, provides some additional managerial insights into the nature of the two-period prediction problem.[13] In words, Bayes' rule of (4) is

$$\text{posterior} \propto \text{likelihood} \cdot \text{prior}. \tag{6}$$

Suppose that at the beginning of period 1 a poster title is picked at random and nothing is known of its past demand history. The best estimate or guess of its *prior* λ would be the mean of the gamma distribution (2), namely $E[\lambda] = r/\alpha$. During period 1 additional information is acquired on λ in the form of its realized demand, $X_1 = x$. The *likelihood* function, which is the probability of $X_1 = x$, given λ, is the Poisson distribution (1). Substituting (1) and (2) into (6) yields the *posterior* distribution (5), and from the previous results of (3) the period 2 conditional expected demand is

$$E[X_2 \mid X_1 = x] = (r + x)/(\alpha + 1) = r/(\alpha + 1) + x/(\alpha + 1), \qquad x = 0, 1, 2, \ldots, \tag{7}$$

where X_2 is NB($r + x$, $\alpha + 1$), i.e. the period 2 random distribution of demand.

Note that (7) is linear in x with intercept $r/(\alpha + 1)$ and slope $1/(\alpha + 1)$. Since $\alpha > 0$, $1/(\alpha + 1) < 1$, forcing a regression to the mean effect for the period 2 predictions.[15] The regression to the mean effect occurs since each prediction from (7) is a weighted combination of the likelihood of realized demand $X_1 = x$ from (1) and the prior information in (2), the latter of which implies a prior mean demand rate $E[\lambda] = r/\alpha$. Therefore, poster titles which had period 1 demand less (greater) than r/α will tend to have increased (decreased) period 2 demand.

The augmented NBD: the zero category problem

The zero category problem refers to the situation where some proportion of poster titles will have zero expected demand, implying $\lambda = 0$—an obvious violation of the gamma mixing distribution (2). Unfortunately, at the end of period 1, for those posters having demand $X_1 = 0$, it is not possible to determine if this was due by chance to their Poisson behaviour or if they do have truly zero expected demand. Regardless, the implications are that the mixing distribution may

(i) have a spike at the origin, leading to biased predictions from (7), and
(ii) be bi-modal, thus invalidating the gamma assumption.

For the augmented NBD, the analogue to (7) is[16]

$$E[X_2 \mid X_1 = x] = [r/(\alpha + 1)] \cdot [1 - (\phi/\phi_0)], \qquad x = 0,$$
$$E[X_2 \mid X_1 = x] = r/(\alpha + 1) + x/(\alpha + 1), \qquad x = 1, 2, \ldots, \qquad (8)$$

where ϕ is the proportion of posters which have zero expected demand and ϕ_0 is the proportion having $X_1 = 0$. For the $x > 0$ categories, (7) and (8) are, with appropriate r and α, equivalent, and (7) need only be modified for the $x = 0$ category.

FORECASTING AND DATA ANALYSIS

Forecasts of the period 2 conditional expected demands for the pure and augmented NBD models of (7) and (8) were calculated and are shown in Table 1 along with the frequencies of period 1 demand and the period 2 conditional average demands. Also included in Table 1 are the appropriate sets of parameter estimates for r and α (see the Appendix for details on these calculations) as well as the weighted mean squared error and bias statistics for each set of forecasts.

TABLE 1. *Predicting period 2 demand from period 1 demand*

			Forecasted demand	
Actual demand period 1 (x)	Frequency	Average demand period 2	Pure NBD	Augmented NBD
0	260	0.688	0.699	0.554
1	154	1.182	1.225	1.434
2	94	1.840	1.752	1.882
3	66	2.424	2.278	2.330
4	43	2.907	2.805	2.778
5	22	2.773	3.331	2.225
6	17	3.176	3.858	3.673
7^+	11	5.909	—	—
	Weighted mean square error		0.027	0.038
	Weighted bias		0.017	0.022

(i) Forecasts for the pure NBD are based on $\hat{r} = 1.327$ and $\hat{\alpha} = 0.875$ and the sample period 1 mean and variance $\bar{X}_1 = 1.517$ and $S_1^2 = 3.251$.
(ii) Forecasts for the augmented NBD are based on $\hat{r} = 2.204$, $\hat{\alpha} = 1.205$, $\hat{\phi} = 0.171$ and, from the period 1 frequency distribution of demand, $\phi_0 = 0.390$.
(iii) Each forecast was multiplied by the correction factor 0.9872 to reflect the change in aggregate demand from 1,012 units in period 1 to 999 units in period 2.

The period 2 conditional average demands exhibit the regression to the mean effect. Titles which had demand less (greater) than the aggregate period 1 mean ($\bar{X}_1 = 1.517$) tended to have increased (decreased) period 2 demand. Somewhat surprising was the relatively poor performance of the augmented NBD in predicting period 2 demand for the $x = 0$ category. Although the pure NBD produced the highest overall predictive accuracy, both methods gave reasonably accurate forecasts which were better than those obtained using either a naive approach or the aggregate period 1 mean (i.e. $\bar{X}_1$) as the period 2 forecast for each category. Although not shown here, forecasts using the latter two approaches yielded weighted mean square error statistics of 0.684 (naive) and 0.647 (mean) and weighted bias statistics of 0.015 (naive) and 0.093 (mean).

The null hypothesis that the period 1 data are distributed NB(r, α) was tested using the minimum chi-square estimation procedure.[17] Using the parameter estimates $\hat{r} = 1.327$ and $\hat{\alpha} = 0.875$ (see Table 1), the appropriate probabilities from the negative binomial distribution were calculated along with the expected frequencies of demand for each $X_1 = x$ demand category. This yielded the chi-square statistic $\hat{\chi}^2 = 9.21$ (d.f. $= 7$, $p = 0.238$), which is not statistically significant at the 0.05 level of significance, and tends to support the null hypothesis.

Overall, the unconditional (period 1) and conditional (period 2) negative binomial distributions appear to be adequate descriptors of aggregate demand. In the next section, this result is incorporated into the optimal stocking-policy model.

THE TWO-PERIOD STOCKING-POLICY MODEL

The objective of the model is to determine the optimal stocking policies in both periods 1 and 2 which maximize total profit during the season. In developing the model, the following assumptions are made:

A1. An order is placed at the beginning of period 1, and a second opportunity for replenishment occurs at the beginning of period 2. In addition, management uses an 'order up to' policy.

A2. The replenishment lead time is instantaneous, and there is no capacity restriction placed on the order size for any poster title.

A3. For the initial ordering decision, management has knowledge of the prior distribution of aggregate demand.

A4. Each title has the same retail selling price R, acquisition cost v, salvage value g, and lost sale cost B. The cost of overstocking in period 1 is deferred until the end of the season, at which time any remaining on-hand items are assumed to be a total loss.

Period 1 stocking policies

At the beginning of period 1, the management naively stocks H units of each poster title based on knowledge of the prior distribution of aggregate demand. The expected profit per poster title during the period is

$$\Pi_1 = R \cdot \sum_{x=0}^{H} p(X_1 = x) \cdot x + R \cdot H \cdot \sum_{x=H+1}^{\infty} p(X_1 = x) - B \cdot \sum_{x=H+1}^{\infty} (x - H) \cdot p(X_1 = x) - v \cdot H, \tag{9}$$

where $p(X_1 = x)$ is NB(r, α) and R, v and B are as previously defined by A4.

Period 2 stocking policies

At the end of period 1, r and α are updated, and the conditional period 2 probabilities and forecasts are revised accordingly. The period 2 conditional expected profit for each poster title, given $X_1 = x$, is

$$\begin{aligned}\pi_x = R \cdot \sum_{y=0}^{H_x} p(y \mid X_1 = x) \cdot y + R \cdot H_x \cdot \sum_{y=H_x+1}^{\infty} p(y \mid X_1 = x) - B \cdot \sum_{y=H_x+1}^{\infty} (y - H_x) \cdot p(y \mid X_1 = x) \\ + g \cdot \sum_{y=0}^{H_x} (H_x - y) \cdot p(y \mid X_1 = x) - v \cdot (H_x - H + x), \qquad x = 0, 1, 2, \ldots, H,\end{aligned} \tag{10}$$

where y is the period 2 conditional demand, H_x is the conditional stocking level for each title in this category, and $p(y \mid X_1 = x)$ is NBD$(r + x, \alpha + 1)$.

The period 2 expected profit per poster title is a weighted combination of each π_x, as follows:

$$\Pi_2 = \sum_{x=0}^{H-1} p(X_1 = x) \cdot \pi_x + \left[\sum_{x=H}^{\infty} p(X_1 = x) \right] \cdot \pi_H, \tag{11}$$

and the seasonal expected profit per poster title is

$$\Pi_s = \Pi_1 + \Pi_2. \tag{12}$$

The optimal solution to (12) may be found by iterating over various values of H and H_x ($x = 0, 1, 2, \ldots, H$) until Π_s is maximized. Although it is well-known (see, for example, Silver and Peterson[3]) that the optimal stocking policy H^*, which maximizes (9), is

$$\sum_{x=0}^{H^*-1} p(X_1 = x) < C_u/(C_0 + C_u) < \sum_{x=0}^{H^*} p(X_1 = x), \tag{13}$$

where C_u and C_o are the costs of understocking and overstocking, respectively, Π_1 and Π_2 cannot be maximized independently since Π_2 is an implicit function of H through each π_x ($x = 0, 1, \ldots, H$) from (10). As an illustration, at the end of period 1, explicit demand information is revealed only for those posters which had demand of $x \leqslant H$ in period 1, while those posters which would have had period 1 demand $x > H$ are implicitly grouped into the $x = H$ category. The consequence of this is that the calculation of Π_2 is limited to $\pi_0, \pi_1, \ldots, \pi_H$, while the implicit contribution to period 2 profit for those posters for which $x > H$ is captured in the last term of equation (11). Since the period 2 conditional expected demand of equation (7) is monotonically increasing with respect to x, it is evident that $\pi_H < \pi_{H+1} < \pi_{H+2} < \ldots$; therefore, Π_2 is a monotonically increasing function of H. This implies that it may be necessary to overstock in period 1 (i.e. suboptimize Π_1) in order to maximize seasonal profits.

Using the NBD predictions, Π_1, Π_2 and Π_s were calculated for various values of H and are shown in Figure 2. Each policy used the revenue and cost parameters $R = \$25.00$, $v = \$5.00$, $B = \$20.00$ and $g = -\$5.00$. The optimal policy which maximizes Π_s is $H = 6$ units, with corresponding expected profits of $\Pi_1 = \$10.30$, $\Pi_2 = \$34.41$ and $\Pi_s = \$44.71$. Using the empirical relative frequencies in lieu of the NBD predictions yields similar results, with the optimal period 1 policy again being H = 6 units and $\Pi_1 = \$10.96$, $\Pi_2 = \$32.64$ and $\Pi_s = \$43.60$. It is also of interest to note from Figure 2 that the value of H which maximizes Π_1 is $H^* = 4$ units.

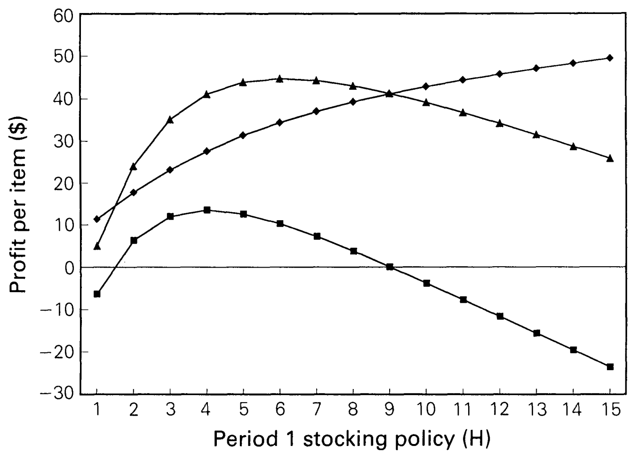

FIG. 2. *Expected profit per item for periods 1 (—■—), 2 (—◆—) and the season (—▲—).*

The optimal period 2 conditional stocking levels (H_x^*) and expected profits (π_x^*) for these two policies are given in Table 2 along with the period 2 conditional service levels (SL_x) obtained for each H_x^*. The latter are the percentages of demand satisfied for all items in a given demand category under the assumption that posters are purchased in unit batch sizes and are defined mathematically as

$$SL_x = 1 - \left[(\hat{\alpha} + 1) \cdot \sum_{x = H_x* + 1}^{\infty} (x - H_x^*) \cdot p(y \mid X_1 = x) \right] \Big/ (\hat{r} + x), \qquad x = 0, 1, 2, \ldots, \mathrm{H}.$$

TABLE 2. *Period 2 stocking policies, conditional expected profit and service levels for $H = 6$*

	NBD predictions			Period 2 empirical data		
x	H_x^*	π_x^*	SL_x	H_x^*	π_x^*	SL_x
0	nc	$22.90	0.997	nc	$22.53	1.000
1	nc	33.84	0.983	nc	33.08	0.995
2	nc	41.02	0.929	nc	39.28	0.890
3	5	44.95	0.946	5	43.85	0.912
4	6	49.01	0.959	6	51.67	0.968
5	7	53.14	0.967	6	43.55	0.967
6	7	57.60	0.952	8	37.65	0.944

(i) nc implies no change from the on-hand units retained from period 1.
(ii) Decreases in π_5^* and π_6^* for the empirical results are due to small sample sizes. They are included here for completeness.

The results from Table 2 indicate that when compared with the empirical data, the model provides very good predictions of expected profits for each period and the season as well as the expected conditional service levels. Expected profits are within 4–5% of the actual profits which would have been obtained had the empirical relative frequencies been used, while the theoretical conditional service levels tend to be within 1–2% of the actual levels.

CONCLUSION

This paper has proposed a pragmatic approach to the two-period style-goods inventory problem for items having heterogeneous Poisson demands. The results of this study are summarized as follows. First, the unconditional (period 1) and conditional (period 2) negative binomial distributions provide very good descriptive models of aggregate demand behaviour. Model predictions of period 2 conditional demands, seasonal profits and service levels were, on average, within 1–5% of the comparable values obtained using the empirical data from the case study. The second finding has to do with the regression to the mean effect for period 2 demands. Theoretically, this effect is due to the uncertain nature of individual demands and the heterogeneity of aggregate demand due to the Bayesian formulation; however, it is not clear why this effect should, necessarily, be observed empirically. Nonetheless, the information that it provides is extremely useful, and a failure to recognize it could lead to a tendency to understock the low- and overstock the high-demand posters. Lastly, expected profit in period 2 was found to be an implicit function of the period 1 stocking policy since the number of units initially stocked for each poster reveals explicit and implicit information on period 2 demands. This implies that it may be necessary to overstock in period 1 in order to maximize profit during the season.

The model developed here could easily be extended to other situations. For example, Chambers and Eglese[2] have suggested a very appealing methodology based on preview exercises whereby empirical demand information is obtained prior to the start of the season. Although the nature of this problem did not allow for previewing, the model would be very compatible with the methodology, particularly when there is no replenishment opportunity during the season. The model might also find practical use in A-B-C analyses for managing the B-C items, thus freeing management to focus on detailed forecasts for the more important A items. As such, it would be especially welcome in a small business environment where computer and managerial resources are limited, as it would alleviate the need for making hundreds or thousands of periodic forecasts.

APPENDIX

Parameter Estimation

Parameter estimates of r and α for the pure NBD of equation (7) were obtained using the method of moments, as follows:

$$\hat{\alpha} = \bar{X}_1/(S_1^2 - \bar{X}_1),$$

$$\hat{r} = \hat{\alpha} \cdot \bar{X}_1,$$

where $\bar{X}_1$ and S_1^2 are the sample mean and variance of the period 1 observed demands, respectively.

Parameter estimates of r, α and ϕ for the augmented NBD of (8) were obtained using the iterative algorithm suggested by Morrison.[16] This involves solving the following three equations:

$$\hat{r} = \bar{X}_1^2/[S_1^2 - \bar{X}_1 + \hat{\phi} \cdot (\bar{X}_1 - S_1^2 - \bar{X}_1^2)], \tag{A1}$$

$$\hat{\alpha} = (1 - \hat{\phi}) \cdot \hat{r}/\bar{X}_1, \tag{A2}$$

$$\phi_0 = \hat{\phi} + (1 - \hat{\phi}) \cdot [\hat{\alpha}/(\hat{\alpha} + 1)]^{\hat{r}}, \tag{A3}$$

as follows. A value of $\hat{\phi}$ is selected from the interval $0 \leqslant \hat{\phi} \leqslant \phi_0$ and is used to obtain $\hat{r}$ and $\hat{\alpha}$ directly from (A1) and (A2). These estimates are then used to evaluate (A3). If (A3) is satisfied, then $\hat{r}$, $\hat{\alpha}$ and $\hat{\phi}$ are the appropriate estimates; otherwise a new value of $\hat{\phi}$ is selected and the procedure is repeated.

REFERENCES

1. M. L. Chambers and R. W. Eglese (1988) Forecasting demand for mail order catalogue lines during the season. *Eur. J. Opl Res.* **34,** 131–138.
2. M. L. Chambers and R. W. Eglese (1986) Use of preview exercises to forecast demand for new lines in mail order. *J. Opl Res. Soc.* **37,** 267–273.
3. E. A. Silver and R. Peterson (1985) *Decision Systems for Inventory Management and Production Planning*, 2nd edn. Wiley, New York.
4. S. H. Chang and D. E. Fyffe (1971) Estimation of forecast errors in seasonal-style-goods sales. *Mgmt Sci.* **18,** 1389–1396.
5. S. Goyal (1973) Optimal decision rules for producing greeting cards. *Opl Res. Q.* **24,** 391–401.
6. M. Green and P. J. Harrison (1973) Fashion forecasting for a mail order company using a Bayesian approach. *Opl Res. Q.* **24,** 193–205.
7. W. H. Hausman and R. Peterson (1972) Multi-product production scheduling for style goods with limited capacity, forecast revisions and terminal delivery. *Mgmt Sci.* **18,** 370–383.
8. W. H. Hausman and R. St. G. Sides (1973) Mail-order demands for style goods: theory and data analysis. *Mgmt Sci.* **20,** 191–202.
9. D. B. Hertz and K. H. Schaffir (1960) A forecasting method for management of seasonal style goods inventories. *Opns Res.* **8,** 45–52.
10. G. R. Murray and E. A. Silver (1966) A Bayesian analysis of the style goods inventory problem. *Mgmt Sci.* **12,** 785–797.
11. A. Ravindran (1972) Management of seasonal style-goods inventory. *Opns Res.* **20,** 265–275.
12. E. A. Silver (1965) Bayesian determination of the re-order point of a slow moving item. *Opns Res.* **13,** 989–997.
13. D. G. Morrison (1968) An analysis of consumer purchase data: a Bayesian approach. *Ind. Mgt R.* **9,** 31–40.
14. J. B. Popovic (1987) Decision making on stock levels in cases of uncertain demand rate. *Eur. J. Opl Res.* **32,** 276–290.
15. D. G. Morrison and D. C. Schmittlein (1981) Predicting future random events based on past performance. *Mgmt Sci.* **27,** 1006–1023.
16. D. G. Morrison (1969) Conditional trend analysis: a model that allows for nonusers. *J. Marketing Res.* **6,** 342–346.
17. U. Wagner and A. Taudes (1987) Stochastic models of consumer behaviour *Eur. J. Opl Res.* **29,** 1–23.