June 20th

P100
2. What are the conditions on a, b, c for $f(x,y)=ax^2+bxy+cy^2$ to have a min max or a saddle point at the origin.

$f_x=2ax+by$
$f_y=bx+2cy$

Note $f_x(0,0)=f_y(0,0)=0$

$f_{xx}=2a$
$f_{xy}=f_{yx}=b$
$f_{yy}=2c$

$$H=\begin{pmatrix} 2a & b \\ b & 2c \end{pmatrix}$$

Saddle, $4ac-b^2<0$
local max, $4ac-b^2>0$, $2a+2c<0 \Rightarrow 2a<0$
local min, $4ac-b^2>0$, $2a>0$

1.a.b
① $f(x,y)=x^2+3y^4+4y^3-12y^2$
② $f(x,y)=x^4-2x^2+y^3-6y$

① $f_x=2x=0$
$f_y=12y^3+12y^2-24y=12y(y^2+y-2)=12y(y+2)(y-1)=0$

$\Rightarrow (0,0), (0,-2), (0,1)$

$f_{xx}=2$, $f_{xy}=0=f_{yx}$, $f_{yy}=36y^2+24y-24=12(3y^2+2y-2)$

$$\begin{pmatrix} 2 & 0 \\ 0 & 12(3y^2+2y-2) \end{pmatrix}$$

$(0,0) \Rightarrow \begin{pmatrix} 2 & 0 \\ 0 & -24 \end{pmatrix}$ Saddle point

$(0,-2) \Rightarrow \begin{pmatrix} 2 & 0 \\ 0 & 72 \end{pmatrix}$ local min

$(0,1) \Rightarrow \begin{pmatrix} 2 & 0 \\ 0 & 36 \end{pmatrix}$ local min

② $f_x=4x^3-4x=0 \Rightarrow x=1$ or -1 or 0
$f_y=3y^2-6=0 \Rightarrow y=\pm\sqrt{2}$

$(0,\pm\sqrt{2}), (-1,\pm\sqrt{2}), (1,\pm\sqrt{2})$
$f_{xx}=12x^2-4$
$f_{xy}=f_{yx}=0$
$f_{yy}=6y$

$$\begin{pmatrix} 12x^2-4 & 0 \\ 0 & 6y \end{pmatrix}$$

THM 2.82 !!

1. i

$f(x,y,z) = xyz(4-x-y-z) = 4xyz - x^2yz - xy^2z - xyz^2$

$f_x = 4yz - 2xyz - y^2z - yz^2 = yz(4-2x-y-z) = 0$

$f_y = 4xz - x^2z - 2xyz - xz^2 = xz(4-x-2y-z) = 0$

$f_z = 4xy - x^2y - xy^2 - 2xyz = xy(4-x-y-2z) = 0$

if $y=0$ then $f_y \Rightarrow xz(4-x-z)=0$

i. $x=0$ and $z \in \mathbb{R}$

ii. $z=0$ and $x \in \mathbb{R}$

iii. $4-x-z=0$, $x = 4-z$

See solutions.pdf