Australian National University
Research School of Finance, Actuarial Studies and Applied Statistics

STAT2032/6046: Financial Mathematics

Review Questions (Week 10 – Week 12)

WEEK 10

Question 1

On 1 January 1999 an investor agrees to pay \$3,000 in four years' time for a security. The security pays no interest and the price of the security at the time of the agreement was \$2,680.

On 1 July 2000 the price of the security is \$2,800. Calculate the value of the forward contract on 1 July 2000.

Question 2

The Government issues a fixed-interest stock with a 20 year term, which pays half-yearly coupons in arrears and is purchased and redeemable at par. The risk-free effective interest rate is assumed to be 7% over each of the next 5 years, and 8% thereafter.

i. What is the coupon rate convertible half-yearly for \$200 nominal of the stock?
ii. If an investor enters into a forward contract to buy the security in 4 years, immediately after the coupon payment then due, what is the forward price that should be paid (assuming no arbitrage)?

Question 3

Length of Investment	Interest Rate
1 Year	7.00%
2 years	8.00%
3 Years	8.75%
4 Years	9.25%
5 Years	9.50%

i. Find the price of a \$2000 two-year bond with annual 10% coupons using the spot rates given in the table above
ii. Compute the redemption yield for the bond

Question 4

Based on the yield curve given in the table for the previous question, find the following expected forward rates:

i. The 3-year forward rate that is deferred for 1-year. $f_{1,4}$

ii. The 2-year forward rate that is deferred for 3-years. $f_{3,5}$

WEEK 11

Question 5

Suppose that the yield rate and coupon rate on an n-coupon bond are the same. Show that the duration is $\ddot{a}_{\overline{n}|}$ valued at the yield rate. Find the duration of a 6-coupon bond with coupon rate 10% per coupon period and yield rate 10%.

Question 6

Based on an interest rate of 8% a fund has liabilities with present value \$400,000 and assets with present value \$420,000. The discounted mean terms of the liability and asset cashflows are 4 years and 3 years respectively. Estimate the surplus in the fund if interest rates move to 9% by using the Taylor series approximation for the surplus S:

$$S(i+\varepsilon) \cong S(i) + \varepsilon S'(i)$$

Question 7

Calculate, using an interest rate of 10%, the discounted mean term of a 20-year annuity certain payable annually in arrears under which the first payment is $1000 if

a. the payments remain level throughout the term
b. the payments increase by $50 each year
c. the payments increase by 10% (compound) each year

You are given that $\sum_{t=1}^{20} t^2 1.1^{-t} = 718.027$

Question 8

A financial institution has an obligation to pay $5,000 at the end of each year for 5 years. The institution receives $5000a_{\overline{5}|0.1} = \$18,954$ in exchange for assuming this obligation. The only investments available to the institution are 1, 3, and 5-year zero coupon bonds, all yielding 10%. The institution invests an amount in the 1 year zero coupon bond that is redeemed for X_1 = $6,950, an amount in the 3 year zero coupon bond that is redeemed for X_2 = $8,409 and an amount in the 5 year zero coupon bond that is redeemed for X_3 = $10,175

Verify that this investment strategy is not optimal under immunisation theory.

Question 9

The liabilities of a fund consist of two lump sum payments due at known times in the future. The second lump sum is due for payment 5 years after the first and is twice the amount of the first.

i. If the total present value and duration of the liabilities (both calculated at 6%) are $60,000 and 6 years, determine the timing and amounts of the payments.
ii. If the assets of the fund consist of a single zero coupon bond that will mature 6 years from now with a redemption payment of $85,111.15, is the portfolio immunised?

WEEK 12

Question 10

For the random interest rate $\tilde{i}$, denote $E\left[\tilde{i}\right]=j$ and $Var\left[\tilde{i}\right]=s^2$. Assuming independence of rates, show that:

i. $E\left[\tilde{S}(n)\right]=(1+j)^n$

ii. $Var\left[\tilde{S}(n)\right]=\left(1+2j+j^2+s^2\right)^n-(1+j)^{2n}$

Question 11

A stochastic interest rate model assumes that the interest rates in different years are independent and identically distributed with a normal distribution with mean 10% and standard deviation 3%. Find the standard deviation of the accumulated value of an initial investment of $12,000 at the end of the second year.

Question 12

A stochastic interest rate model assumes that interest rates in different years are independent and conform to the probability distribution:

$$\tilde{i}=\begin{cases}0.035 & p=0.2\\ 0.045 & p=0.4\\ 0.075 & p=0.4\end{cases}$$

Calculate the standard deviation of the annual interest rate.

Question 13

The yield obtained on a company's funds each year is expected to be 2%, 3% or 4% with probabilities 0.2, 0.5 and 0.3 respectively. Find the mean and standard deviation of the accumulated value of an initial sum of $1000 invested for 12 years if the yields in different years are independent.

Question 14

i. An amount of 1 is invested for 10 years. The interest rate earned by the investment for the 10-year period will be either 5% for all 10 years, or 10% for all 10 years, or

15% for all 10 years. Each of the three possible cases is equally likely. Find the expected value and variance of the accumulated value at the end of the 10 years.

ii. Now, suppose that interest rates may be different from year to year, but again for a particular year the rate is either 5%, 10% or 15%. Assuming independent interest rates from year to year, find the expected value and variance of the accumulated value.

Question 15

Annual effective interest rates will be random in years 1 and 2, following a uniform distribution between 6% and 12%. In years 3 and 4 the rates will be uniformly distributed between 5% and 15%. Find $Var\left[\tilde{S}(4)\right]$.

Question 16

Interest rates for the next 25 years are independently and identically distributed according to the following distribution:

$$\tilde{i} = \begin{cases} 0.08 & p = 0.3 \\ 0.13 & p = 0.7 \end{cases}$$

Using the lognormal approximation for $\tilde{S}(25)$, find the approximate probability that the accumulation of \$500 for 25 years will be greater than \$8,000 and also the approximate probability that the accumulation will be greater than $E\left[500\tilde{S}(25)\right]$.