THE FACULTY OF ARTS AND SCIENCE
University of Toronto

FINAL EXAMINATIONS, APRIL-MAY 2011

MAT246H1S-LEC5101
Concepts in Abstract Mathematics

Examiners: V. Kapovitch and P. Rosenthal
Duration: 3 hours

NO AIDS ALLOWED.   **Total: 100 marks**

Family Name: _____

(Please Print)

Given Name(s): _____

(Please Print)

Please sign here: _____

Student ID Number: _____

You may not use calculators, cell phones, or PDAs during the exam. Partial credit will be given for partially correct work. Write your answer in the space provided. Use the back sides of the pages for scrap work DO NOT tear any pages from this test.

FOR MARKER'S USE ONLY	
Problem 1:	/10
Problem 2:	/10
Problem 3:	/10
Problem 4:	/10
Problem 5:	/10
Problem 6:	/10
Problem 7:	/10
Problem 8:	/10
Problem 9:	/10
Problem 10:	/10
TOTAL:	/100

1. (10 pts) Let $p = 3, q = 11$ and $e = 7$. Let $N = 3 \cdot 11 = 33$. The receiver broadcasts the numbers $N = 33, e = 7$. The sender sends a secret message M to the receiver using RSA encryption. What is sent is the number $R = 6$.

   Decode to find the original message M.

2. (10 pts) Let $p > 5$ be an integer which is NOT prime. Prove that $(p-1)! \equiv 0(\mod p)$

3. (10 pts)

a) Find $\phi(28^{25})$.

b) Find the remainder when 3^{2463} is divided by 8.

4. (10 pts) Write in the form $a + bi$:

$$\left(-\frac{1}{2} + \frac{i}{2}\right)^{13}$$

5. (10 pts) Prove that the acute angle whose cosine is $\frac{1}{4}$ cannot be trisected with straightedge and compass.

6. (10 pts) Prove that if S is infinite and T is finite then $|S \cup T| = |S|$.

7. (10 pts) Using only the definition of having the same cardinality, prove that the set of all integers and the set of natural numbers have the same cardinality.

8. (10 pts) Prove that the set of finite subsets of $\mathbb{R}$ has the same cardinality as $\mathbb{R}$.

9. (10 pts) Suppose a and b are relatively prime natural numbers such that ab is a perfect cube (i.e., is the cube of a natural number). Prove that both a and b are perfect cubes.

10. (10 pts) For each of the following answer "true" or "false". Justify your answer.

a) If $\frac{x}{y}$ is constructible then both x and y are constructible.

b) If x is constructible then $\frac{1}{x}$ is constructible.

c) There is an angle θ such that $\cos\theta$ is constructible but $\sin\theta$ is not constructible.

d) $\sqrt[3]{\frac{10}{27}}$ is constructible.