March 25th

Ruler & Compass constructions
① trisecting a angle easy to bisect
② squaring a circle
③ doubling a cube

setup, have interval [0, 1]
what length can we constructing?
- any natural #s
- constant $\frac{1}{2}, \frac{1}{3}, \ldots$ rational #s
- can divide any interval in any number of equal pieces

$\Rightarrow$ can construct any $\frac{m}{n}, m, n \in \mathbb{N}$.

- some irrationals, $\sqrt{2}, \sqrt{3}, \ldots$

Let C = be the set of all constructable numbers

If $x, y \in C$, then $x+y, x-y, x \cdot y, \frac{x}{y}$ $(y \neq 0)$ all constructable.

in other words, C – the set of constructable numbers is a number field

Def: a subset $F \subset \mathbb{R}$ is called a number field if ① $0, 1 \in F$
② $x, y \in F \Rightarrow x+y, x-y, x \cdot y, x/y \in F$ also.

Ex: $\mathbb{Q}$ – set of rational #s
$\mathbb{Q} \subset \mathbb{R}$
$\mathbb{Q}$ is a number field. 1, 0 are rational
if q_1, q_2 are rational $\Rightarrow q_1+q_2, q_1-q_2, \ldots$ are also rational