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Solutions for these problems are only presented during the Problem Solving Sessions W5-6 in SS 2135. You are strongly encouraged to work through the problems ahead of time, and our TA Yiannis will cover the questions you are most interested in. These sessions are very valuable at developing the proper style to present cogent and rigorous mathematical solutions.

This problem solving session contains material from 2.7-2.8.

Problems:

1. a) Consider $f(x,y) = x^3y^2$. With multi-index notation as in the text, compute $\partial^\alpha f(x,y)$ for all $|\alpha| \leq 3$ and use this to write the Taylor polynomial of degree 3 (k=3) for f at the point (x,y) as per Equation 2.69.

b) Now use the expansion for f(x+h, y+k) and Equation 2.69 to determine the remainder $R_{(x,y),3}(h,k)$. Now verify Equation 2.72 by considering all α such that $|\alpha| = 4$ and computing all $\partial^\alpha f(x,y)$.

c) Determine the Taylor polynomial of degree 2 of $g(x,y) = x^2 + y$ at $(1,2)$.

d) Present the 3rd-order Taylor polynomial for $\frac{1}{2-x^2-y}$ near (0,1). (See example 2 on page 93).

2. a) Determine and classify all the critical points of $f(x,y) = x^3y^2$ according to theorem 2.82.

b) At the point (0,1) determine ∇f and the Hessian. Use your third degree expansion from question 1 to see if you can draw any conclusions about the behaviour of f near the point (0,1). If the degree is zero then you must move to the 4th degree polynomial.

c) Repeat b) for (0,0). This time you may need to go all the way to the 5th degree.

d) Use your expansion from 1a to write the degree 2 Taylor polynomial in the form $\nabla f \cdot \mathbf{h} + 1/2\mathbf{h}^T H\mathbf{h}$ as in 2.80

3. Understanding Proofs: If you have not already, read my description about creating proof summaries to break down complex proofs into their key ideas on the last Problem Solving Session. Now carefully read through the proofs of MVT I, MVT III, Taylor Theorem with Lagrange Remainer for one variable and Taylor Theorem with Lagrange Remainder for multi variables. We want to be able to tell a "story" of how these four theorems and their proofs flow together. In particular, let's create a brief summary of the big ideas of each of the two Taylor theorems, perhaps in bullet point form. You should feel that after reading your summary of the proof that you understand the larger structure of the proof and would be able to fill in the details of your summary if asked. Your proof summaries should make it clear how the different theorems relate to each other, and how the major proof ideas for the two MVT theorems are generalized to become the major proof ideas for the two Taylor theorems.

Incidentally, if you have not yet done so, you should sit down on your own at some point and see if you can reproduce the full proofs for the theorems you created a proof sketch of last week.

4. Try several of the parts from Exercise 1 from 2.8 (although you certainly won't have time to do all of these in tutorial).