PROBLEM-SOLVING AND PROOFS: ASSIGNMENT 5
DUE FRIDAY, MARCH 31, 4PM.

Warm-up problems. These are completely optional.

(1) How many ways are there to pick two cards from a standard 52-card deck such that the first card is a space and the second card is not an Ace.

(2) Determine the coefficient of x^4y^5 in $(x+y)^9$.

Problems to be handed in. Solve four of the following five problems. Do not attempt Problem 5 prior to lecture on Monday, March 27.

(1) The summation identity for binomial coefficients states that:

$$\sum_{i=0}^{n} \binom{i}{k} = \binom{n+1}{k+1}$$ 5.30

Give two proofs of this identity, one using the bug-path model for binomial coefficients, and one using induction.

(2) Give short, insightful proofs of the following formulae:

(a) $\binom{n}{k}\binom{k}{j} = \binom{n}{j}\binom{n-j}{k-j}$,

(b) $\sum_{k=0}^{n} k\binom{n}{k} = n2^{n-1}$ 5.39

(3) Count the sets of six cards from a standard deck of 52 cards that have at least one card in every suit.

5.18

(4) Count the number of ways to group $2n$ people into n distinct pairs. (For example, the answer is 3 when $n=2$).

5.29

(5) (a) Count the solutions in *positive* integers to the equation $x_1 + \ldots + x_k = n$.
(b) Count the solutions in *non-negative* integers to the equation $x_1 + \ldots + x_k \leq n$.

5.28