Gibbs is a special case of MH

we use the full conditionals as our proposal distributions in MH.

we have $J_u(u^* / u^{(s)}, v^{(s)}) = p_0(u^* / v^{(s)})$

where $p_0$ is defined as posterior probability density

$$r = \frac{p_0(u^*, v^{(s)})}{p_0(u^{(s)}, v^{(s)})} \times \frac{J_u(u^{(s)} / u^*, v^{(s)})}{J_u(u^* / u^{(s)}, v^{(s)})}$$

$$= \frac{p_0(u^*, v^{(s)}) \cdot p_0(u^{(s)} / v^{(s)})}{p_0(u^{(s)}, v^{(s)}) \cdot p_0(u^* / v^{(s)})}$$

$$= \frac{p_0(u^* / v^{(s)}) \cdot p_0(v^{(s)}) \cdot p_0(u^{(s)} / v^{(s)})}{p_0(u^{(s)} / v^{(s)}) \cdot p_0(v^{(s)}) \cdot p_0(u^* / v^{(s)})}$$

$$= 1$$ (all terms cancel out)

Always accept.

①

$$p(x^{(s)} = x_a, x^{(s+1)} = x_b)$$

$$= p_0(x_a) \times J_s(x_b / x_a) \times \frac{p_0(x_b) J_s(x_a / x_b)}{p_0(x_a) J_s(x_b / x_a)}$$

$$= p_0(x_b) J_s(x_a / x_b)$$

②

$$p(x^{(s)} = x_b, x^{(s+1)} = x_{(a)}) = p_0(x_b) J_s(x_a / x_b)$$

because the acceptance rate is 1

(we assumed $p_0(x_a) J_s(x_b / x_a) \geq p_0(x_b) J_s(x_a / x_b)$)

③ $$p(x^{(s+1)} = x) = \sum_{x_a} Pr(x^{(s+1)} = x, x^{(s)} = x_a)$$

$$= \sum_{x_a} Pr(x^{(s+1)} = x_a, x^{(s)} = x)$$

$$= Pr(x^{(s)} = x)$$

That compelets the proof.