University of Toronto
FACULTY OF ARTS AND SCIENCE

FINAL EXAMINATIONS, DECEMBER 2010

APM 236H1F
Applications of Linear Programming

Examiner: P. Kergin
Duration: 3 hours

PLEASE HAND IN

FAMILY NAME _____

GIVEN NAME(S) _____

STUDENT NO _____

SIGNATURE _____

INSTRUCTIONS:

NO calculators or other aids allowed. There are 6 questions, each worth 20 marks. Questions 2, 5, and 6 have part-questions, whose values are stated within the part-questions themselves. Total marks = 120.

This exam consists of 12 pages, printed on both sides of the paper. Write solutions in spaces provided. Pages 3 and 12 are blank and may be used for the solution(s) of any of the problems, or for rough work. Aspects of any question which are indicated in **boldface** will be regarded as crucial during grading. **Show your work.**

GRADER'S REPORT	
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2	
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6	
TOTAL	

1. A confectioner has on hand 6 kg butter, 5 kg flour, and 3 kg sugar, which he may use to make either pastry or soft candy or hard candy. A 4 kg batch of pastry requires 1 kg butter, 2 kg flour, and 1 kg sugar, and sells for \$20, while a 4 kg batch of soft candy requires 1 kg butter, 1 kg flour, and 2 kg sugar, and sells for \$30. Because of waste during cooking, a 4 kg batch of hard candy, which sells for \$10, requires 2 kg butter, 3 kg flour, and 4 kg sugar. **Set up a linear programming model** in **standard form** which will determine how many 4 kg batches of pastry, soft candy, and hard candy the confectioner should make to maximize his revenue (money earned by selling the confections). After setting up the problem, use the **simplex method** to **solve** it.

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2.(a)(10 marks) Consider the following simplex tableau, which represents an optimal solution of a linear programming problem.

	x_1	x_2	x_3	x_4	x_5	x_6	
x_5	5	6	0	−7	1	3	2
x_3	7	−1	1	−2	0	−3	0
	1	4	0	3	0	8	9

Prove that the problem solved by the above tableau has **only one optimal solution**.

2.(b)(10 marks) Consider the following simplex tableau, which represents an optimal solution of a linear programming problem.

	x_1	x_2	x_3	x_4	x_5	x_6	
x_2	-7	1	5	0	3	-4	8
x_4	-4	0	6	1	-2	8	5
	6	0	1	0	0	3	-9

Find **all optimal solutions** of the problem.

3. Suppose in solving a linear programming problem by the simplex method we encounter a tableau, part of which is given below, where $a_1 > 0$ and $a_m > 0$.

	x_j	
x_1	a_1	b_1
$\vdots$	$\vdots$	$\vdots$
x_m	a_m	b_m
	-1	0

In the next iteration of the simplex method, x_j will enter. Now suppose that the θ-ratio for the x_m row is less than the θ-ratio for the x_1 row but, contrary to the rules of the simplex method, we exit x_1. **Prove** that the next tableau will be infeasible.

4. **Solve** the following problem.

Minimize $z = 4x_1 + 2x_2 + 6x_3 + x_4$ subject to the constraints

$$\begin{array}{rcrcrcrcl} x_1 & + & x_2 & + & 2x_3 & - & x_4 & \geq & 5 \\ 3x_1 & + & 2x_2 & + & 2x_3 & - & 4x_4 & \leq & 7 \end{array} , \quad x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0.$$

5. Consider the following primal problem.

Minimize $z = x_1 - x_2$ subject to the constraints

$$\begin{array}{rcrcl} x_1 & + & x_2 & \leq & 4 \\ -x_1 & + & 2x_2 & \geq & 2 \\ -4x_1 & + & 3x_2 & \leq & 12 \end{array} \quad , \; x_1 \text{ unrestricted}, x_2 \geq 0.$$

5.(a)(5 marks) **Solve** the primal problem **graphically**.

5.(b)(5 marks) **State the dual** of the primal problem.

5.(c)(10 marks) **Solve** the dual problem.

6. Consider the following primal problem.

Maximize $z = 7x_1 + x_2 - 23x_3 + 6x_4$ subject to the constraints

$$\begin{array}{rcrcrcl} 2x_1 & & - & 5x_3 & + & 6x_4 & \leq & 3 \\ x_1 & & - & 3x_3 & + & 2x_4 & \leq & 1 \\ -x_1 & + & x_2 & - & x_3 & - & 9x_4 & \leq & 2 \end{array} \quad , \; x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0.$$

6.(a)(5 marks) The third tableau of the simplex solution of the primal problem has basic variables $\{x_5, x_1, x_2\}$ (in that order, where x_5 denotes the slack variable for the first constraint). **Use this information** to find the matrix $\mathbf{B}^{-1}$ which corresponds the third tableau.

6.(b)(15 marks) **Beginning from the third tableau**, use the **revised simplex method** to **solve** the primal problem.

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