3.1(b)

$X_1, \ldots, X_n \overset{iid}{\sim} Unif(-\frac{\theta}{2}, \frac{\theta}{2})$

$$f(x) = \frac{1}{b-a} = \frac{1}{\frac{\theta}{2} - (-\frac{\theta}{2})} = \frac{1}{\frac{2\theta}{2}} = \frac{1}{\theta}$$

$$L(\theta) = \prod_{i=1}^{n} \frac{1}{\theta} = \frac{1}{\theta^n}$$; we want the MLE

· If $\theta \downarrow \Rightarrow L(\theta) \uparrow$

$$-\frac{\theta}{2} < x_{(1)} < x_{(n)} < \frac{\theta}{2} \Rightarrow \frac{\theta}{2} > \max(|x_{(1)}|, |x_{(n)}|)$$

$$\hat{\theta} = 2\max(|x_{(1)}|, |x_{(n)}|)$$

$$\boxed{\begin{array}{c} -\theta < x_{(1)} < x_{(n)} < \theta \\ \quad\ \ \| \qquad\quad \| \\ \ -0.75 \quad\ \ 0.5 \end{array}}$$

$-1.0 \downarrow -0.5$ $\quad$ $1.0 \downarrow 0.5$

MLE $\frac{\partial}{\partial\theta} L(\hat{\theta}) = 0$

$\downarrow$

for uniform
we don't have
this property

3.1(a). $X_1, \ldots, x_n \overset{iid}{\sim} f(x)$

$$f(x) = \theta(1-\theta)^{x-1} \qquad x = 1, 2, \ldots$$

What is the MLE for θ?

$$L(\theta) = \prod_{i=1}^{n} \theta(1-\theta)^{x_i - 1}$$

$$= \theta^n (1-\theta)^{(x_1 - 1) + \cdots + (x_n - 1)}$$

$$= \theta^n (1-\theta)^{\sum x_i - n}$$

$$l(\theta) \Rightarrow n \log\theta + (\sum x_i - n)\log(1-\theta)$$

$$\frac{\partial l}{\partial\theta} = \frac{n}{\theta} - \frac{\sum x_i - n}{1-\theta} = \frac{n(1-\theta) - \theta\sum x_i + n\theta}{\theta(1-\theta)} = \frac{-\theta\sum x_i + n}{\theta(1-\theta)} = 0$$

$$\hat{\theta} = \frac{n}{\sum x_i}$$

(check that this is a max)

3.1(c).

$X_1, \cdots, X_n \overset{iid}{\sim} f(x)$

$$f(x) = \frac{1}{\theta^2} x \exp\left(\frac{-x}{\theta}\right)$$

MLE?

$$L(\theta) = \prod_{i=1}^{n} \frac{1}{\theta^2} x_i \exp\left(-\frac{x_i}{\theta}\right)$$

$$= \frac{1}{\theta^{2n}} \left[\prod x_i\right] \cdot \exp \sum \left(-\frac{x_i}{\theta}\right)$$

$$l(\theta) = \log\left(\frac{\sum x_i}{\theta^{2n}}\right) + \sum\left(-\frac{x_i}{\theta}\right)$$

$$= \log \sum x_i - 2n \log\theta - \frac{\sum x_i}{\theta}$$

$$\frac{-2n\theta}{\theta^2} + \frac{\sum x_i}{\theta^2} = 0 \quad \theta^{-1}$$

$$-\theta^{-2}$$

$$\sum x_i = 2n\theta$$

$$\hat{\theta} = \frac{\sum x_i}{2n}$$

3.11 $X_1, \cdots, X_n \overset{iid}{\sim} \text{Poisson}(\theta)$

$$f(x) = \frac{e^{-\theta}\theta^x}{x!}$$

MLE for θ $\quad \hat{\theta} = \bar{X}$

· What's the MLE for $\phi = \theta^2$

$$L(\theta) = \prod_{i=1}^{n} \frac{e^{-\theta}\theta^x}{x!} = \frac{e^{-\theta n}\theta^{\sum x_i}}{\prod_{i=1}^{n} x_i!} \Rightarrow A = \frac{e^{-\theta}\theta^{\sum x_i}}{A}$$

$$\theta = \phi^{\frac{1}{2}}$$

$$L(\phi) = \frac{e^{-\phi^{\frac{1}{2}} n} \phi^{\frac{1}{2}\sum x_i}}{A}$$

$$l(\phi) = -\phi^{\frac{1}{2}} n + \frac{1}{2} \sum x_i \log(\phi) - \log A$$

$$\frac{\partial l}{\partial \phi} = -\frac{1}{2}\phi^{-\frac{1}{2}} n + \frac{1}{2}\sum x_i \frac{1}{\phi} = 0$$

$$-\frac{1}{\sqrt{\phi}} + \frac{\sum x_i}{\phi} = 0$$

$$\sum x_i = \sqrt{\phi}\, n$$

$$\phi = \left(\frac{\sum x_i}{n}\right)^2 = \bar{X}^2$$