Assign. 7

1. Every observation in the last few decades is consistent with a flat universe (and largely inconsistent with a sustantially non-flat universe). If FLRW cosmology is sound (and we have no reason to suspect otherwise), then the universe is infinite. A crit. density universe already expands forever, and dark energy only adds to the expansion.

2. $\rho_{crit,0} = \frac{3H_0^2}{8\pi G}$ $\qquad H_0 = 2.27 \cdot 10^{-18}\ s^{-1}$ normally, but $\underline{H_0 = 1.62 \cdot 10^{-17}\ s^{-1}}$ here

$$\rho_{crit,0}(H_0 = 500\ \tfrac{km/s}{Mpc}) = \frac{3 \cdot (1.62 \cdot 10^{-17} \tfrac{1}{s})^2}{8\pi \cdot 6.67 \cdot 10^{-11} \tfrac{N \cdot m^2}{kg^2}}$$

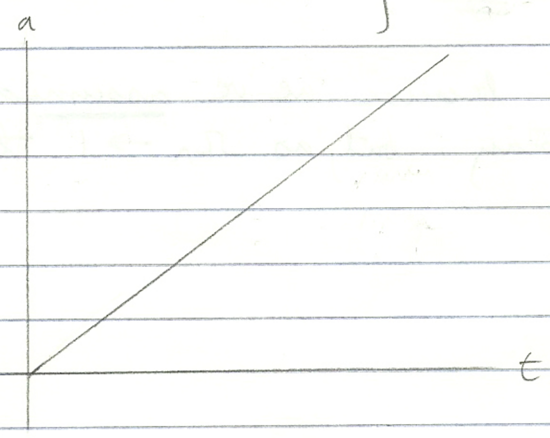
$$= 4.72 \cdot 10^{-25}\ kg/m^3 \ (2.82 \cdot 10^2\ atms/m^3)$$

But our Ω_m would then be (taking $\rho_{m,0} = 0.27 \times \rho_{crit,0}(H_0 = 70 \frac{km/s}{Mpc})$)

$$\Omega_m = \frac{0.27\rho_{crit,0}(H_0 = 70 \frac{km/s}{Mpc})}{\rho_{crit,0}(H_0 = 500 \frac{km/s}{Mpc})} = 0.27 \left(\frac{70}{500}\right)^2 = \underline{0.0055}$$

This is an almost empty universe so (from prev HW) $a(t) \approx ct$ (a nearly linear) from t_0 onward

(note Ω_m will be higher in the past).



The universe expands forever at a constant rate

also from before, $t = \frac{1}{H}$; $t_0 = \frac{1}{H_0}$.
If $H_0 = 500\ \mathrm{km/s/Mpc}$, then $t_0 = \underline{1.40\ \mathrm{Gyr}}$,
less than half the age of solar system.

3 $\Omega_m = \rho/\rho_{crit}$
Now, $\rho = \rho_0\, a_0^3/a^3$ (as from before), but
$\rho_{crit} \neq \rho_{crit,0}\, a_0^3/a^3$, since this is NOT a critical universe.

$$\rho_{crit} = \rho_{crit,0}\, H^2/H_0^2 = \rho_{crit,0} \frac{1}{H_0^2 t^2} \quad \text{(from prev. assign.)}$$

$$\Omega_m = \frac{\rho_0\, a_0^3/a^3}{\rho_{crit,0} \frac{1}{H_0^2 t^2}} = \frac{\rho_0\, t_0^3/t^3}{\rho_{crit,0} \frac{1}{H_0^2 t^2}} = \Omega_{m,0} \frac{t_0^3}{t^3} \cdot t^2/t_0^2$$

$$= \underline{\underline{\Omega_{m,0}\, t_0/t}}$$

So when $t = 0.5 t_0$, $\Omega_m = 2\,\Omega_{m,0} = \underline{\underline{0.6}}$

4. $\Omega_\Lambda = \Lambda c^2/3H^2 = \Omega_{\Lambda,0}\, H_0^2/H^2$
But $H = 1/t$, so $\underline{\Omega_\Lambda = \Omega_{\Lambda,0}(t/t_0)^2}$

When $t = 0.5 t_0$, $\Omega_\Lambda = 0.25\,\Omega_{\Lambda,0} = \underline{\underline{0.175}}$
$t = 2t_0$, $\Omega_\Lambda = \underline{\underline{2.8}}$
$t = 10 t_0$, $\Omega_\Lambda = \underline{\underline{70}}$

$\Omega_\Lambda > 1$ in the future because we've assumed an empty universe expansion history, but as $\Omega_\Lambda \to 1$ this assumption no longer is true.