Worth: 3%   **Due:** By 12 noon on Tuesday 6 March.

Remember to write the the *full name* and *student number* of each member of your group prominently on your submission. Your submission must be a PDF file named e5.pdf and it must be handed-in using the MarkUs system. You may create the PDF file using a typesetting system (export to PDF) or by scanning in handwritten work to create a PDF file.

Each exercise may be completed in groups of $1 - 2$ students who are in the **same** tutorial section.

*Please read and understand the policy on Collaboration given on the Course Information Sheet. Then, to protect yourself, list on the front of your submission **every** source of information you used to complete this homework (other than your own lecture and tutorial notes, and materials available directly on the course webpage). For example, indicate clearly the **name** of every student with whom you had discussions, the **title** of every additional textbook you consulted, the **source** of every additional web document you used, etc.*

For each question, please write up detailed answers carefully. Make sure that you use notation and terminology correctly, and that you explain and justify what you are doing. Marks **will** be deducted for incorrect or ambiguous use of notation and terminology, and for making incorrect, unjustified, ambiguous, or vague claims in your solutions.

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1. Without using derivatives, or formulas that you might know about the roots of quadratic equations, write a *detailed structured proof* to prove or disprove the following statement:

$$\text{There is no real solution to } x^2 - 6x + 10 = 0.$$

2. For $x \in \mathbb{R}$, define $|x|$ by

$$|x| = \begin{cases} -x, & x < 0 \\ x, & x \geq 0 \end{cases}$$

   Write *detailed structured proofs* to prove or disprove the following statements:

   (a) $\forall x \in \mathbb{R}, \forall y \in \mathbb{R}, |x||y| = |xy|$.

   (b) $\forall x_1 \in \mathbb{R}, \forall x_2 \in \mathbb{R}, \forall y_1 \in \mathbb{R}, \forall y_2 \in \mathbb{R}, |x_1| > |x_2| \land |y_1| > |y_2| \Rightarrow |x_1 y_1| > |x_2 y_2|$.

   You may assume that for real $t > 0$, $\forall x \in \mathbb{R}, \forall y \in \mathbb{R}, x > y \Rightarrow tx > ty$.

3. (a) Add and multiply $(1011)_2$ and $(110110)_2$ together in base 2.

   (b) Add and multiply $(3130)_4$ and $(103)_4$ together in base 4.

   (c) If $a = (342)_8$ and $b = (173)_8$, find $a - b$ without coverting to base 10.