Question One

The following model for the force of mortality of patients following a particular operation has been suggested:

$$\lambda(t, x_1, x_2, x_3) = (0.0002t)\exp\{\beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_3^2\}$$

if no t involved in this part, generally it's a proportional hazard.

$\lambda(t, x_1, x_2, x_3)$ is the force of mortality at time t for an individual with covariate values x_1, x_2 and x_3. x_1, x_2 and x_3 are covariates representing smoking status (1 if smoke; 0 if do not smoke), blood pressure, and weight, respectively.

a). individual 1 : x_1^*, x_2^*, x_3^*

individual 2: x_1', x_2', x_3'

$\frac{\lambda(t, x_1^*, x_2^*, x_3^*)}{\lambda(t, x_1', x_2', x_3')} =$ (t cancelled out)

a) Is the proposed model a proportional hazards model? Provide a reason for your answer.

b) What sign would you expect the parameters β_1 and β_2 to be? Do you see a potential problem with using weight as a covariate? Provide reasons for your answers.

c) You observe the following survival data for 5 patients after the particular operation in question:

Patient	x_1	x_2	x_3	Time	Died/Censored
1	1	150	80	5	Died
2	0	120	65	10	Censored
3	1	110	100	3	Censored
4	1	180	73	7	Died
5	0	100	94	11	Censored

IN EXAM! no need to max here.

Based on the above data, obtain an expression for the partial likelihood. Also, state how you would use the partial likelihood to obtain estimates of the unknown parameter values.

~~the~~ take log-like ⇒ first order ⇒ ·–

d) What is the force of mortality at time 5 for a 75kg smoker with a blood pressure of 135?

Question One - Solutions

a) Yes. The ratio of the hazards does not depend on time.

b) I would expect β_1 and β_2 to both be positive. Both smoking and increases in blood pressure are typically associated with poor health. A problem with weight is that weight means different things depending on height. A better measure might be body mass index.

c) The partial likelihood is the product of the following terms:

$$\frac{e^{\beta_1+150\beta_2+80\beta_3+80^2\beta_4}}{e^{\beta_1+150\beta_2+80\beta_3+80^2\beta_4}+e^{\beta_1+180\beta_2+73\beta_3+73^2\beta_4}+e^{120\beta_2+65\beta_3+65^2\beta_4}+e^{100\beta_2+94\beta_3+94^2\beta_4}}$$

$$\frac{e^{\beta_1+180\beta_2+73\beta_3+73^2\beta_4}}{e^{\beta_1+180\beta_2+73\beta_3+73^2\beta_4}+e^{120\beta_2+65\beta_3+65^2\beta_4}+e^{100\beta_2+94\beta_3+94^2\beta_4}}$$

d) $\lambda(5,1,135,75)=(0.0002\times 5)\exp\{\beta_1+135\beta_2+75\beta_3+75^2\beta_4\}$

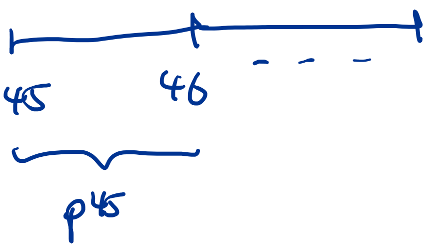
Question Two

(a) Mortality of a group of lives is assumed to follow Gompertz' law, $\mu_x = Bc^x$. Calculate μ_x for a 30-year old and a 70-year old, given that $\mu_{50} = 0.003$ and $\mu_{60} = 0.01$.

(b) For a particular population, $e_{45} = 40.20$ and $e_{46} = 39.27$. Calculate q_{45}.

$e_{45} =$ expected future lifetime aged 45

$e_{46} =$ expected future lifetime aged 46



$$e_{45} = \begin{cases} 1 + e_{46} & \text{with } p = p_{45} \\ & \end{cases}$$

$$e_{45} = 0 \cdot q_{45} + p_{45}(1 + e_{46})$$

$$e_{45} = p_{45}(1 + e_{46})$$

$$e_{45} = \sum_{k=1}^{\infty} {}_kp_{45}$$

$$e_{46} = \sum_{k=1}^{\infty} {}_kp_{46}$$

$$p_{45}\, e_{46} = \sum_{k=1}^{\infty} {}_kp_{46} \cdot p_{45} = {}_1p_{46}\, p_{45} + {}_2p_{46}\, p_{45} + \cdots$$

$$= {}_2p_{45} + {}_3p_{45} + \cdots$$

$$= e_{45} - {}_1p_{45}$$

Hence

$$p_{45} e_{46} + {}_1p_{45} = e_{45}$$

Question Two - Solutions

(a)

$$\begin{aligned}
&\mu_x = Bc^x \\
&0.003 = Bc^{50}; 0.01 = Bc^{60} \\
&c = 1.128; B = 7.29\times10^{-6} \\
&\Rightarrow \mu_x = 7.29\times10^{-6}1.128^x \\
&\Rightarrow \mu_{30} = 0.00027; \mu_{70} = 0.033.
\end{aligned}$$

(b)

$$\begin{aligned}
&e_{45} = q_{45}\times 0 + p_{45}(1+e_{46}) = p_{45}(1+e_{46}) \\
&\Rightarrow 40.20 = p_{45}(1+e_{46}) \\
&\Rightarrow p_{45} = 0.99826; q_{45} = 0.00174
\end{aligned}$$