t_j	d_j	r_j	$\frac{r_j-d_j}{r_j}$	$\hat{S}(t_j)=\prod_{i=1}^{j}\frac{r_i-d_i}{r_i}$
4	1	20		
5	1	19		
10	2	15		
11	1	13		
13	1	12		
15	1	10		
17	2	8		
18	2	6		
21	1	2		
22	1	1		

STAT3032 SURVIVAL MODELS

TUTORIAL WEEK FIVE

Question One $N=20$

The survival times after a particular operation for a group of human subjects are provided below (* represent censored observations).

14* 17 18

4, 5, 6*, 7*, 8*, 10, 10, 11, 15, 15*, 17, 18, 18*, 19*, 21, 22

~~17~~, ~~13~~, ~~15*~~, ~~7*~~, 21, ~~18*~~, ~~5~~, ~~18~~, ~~6*~~, 22, 19*, ~~15~~, ~~4~~, ~~11~~, ~~14*~~, ~~18~~, ~~10~~, ~~10~~, ~~8*~~, ~~17~~

Using the notation from lectures, what are N, m and k? Construct a table (similar to the one in lectures) that was used to produce the KM estimator.

Question Two

The results of a study to see whether a particular treatment prolonged survival are provided below. Again, censored observations are denoted with a "*".

Treatment	Control	
23	5	68
47	8	71
69	10	76*
70*	13	105*
71*	18	107*
100*	24	109
101*	26	113
148	26	116*
181	31	118
198*	35	143
208*	40	154*
212*	41	162*
224	48	188*
	50	212*
	59	217*
	61	225*

Use R to calculate the KM estimator both ignoring treatment group (that is, combine both sets of data) and allowing for treatment group. Please provide a 95% confidence intervals for your curve computed ignoring treatment group.

$g(Y)=\log(1+y^2)$ $\quad g'(y)=\frac{2y}{1+y^2}$ $\quad E[\log(1+Y^2)]=\log 37$

Question Three $\quad g'(\mu)=g'(6)=\frac{2(6)}{1+36}=\frac{12}{37}$ $\quad Var[\log(1+Y^2)]=2\left(\frac{12}{37}\right)^2$

(a) Suppose Y is a random variable with mean 6 and variance 2. Use the δ–method to approximate the mean and variance of $\log(1+Y^2)$

(b) Consider a time interval A of length 1. Let the number of individuals known to be alive at the start of A be r and the number of deaths in A be d. Then the usual estimate of the hazard for the interval is

$Var(\hat{q}) = \frac{\frac{d}{r}(1-\frac{d}{r})}{r} = \boxed{\frac{d(r-d)}{r^3}}$

$\hat{q} = 1-\exp(-\hat{\lambda})$

$\boxed{\hat{\lambda} = -\log(1-\hat{q})}$

$\frac{p(1-p)}{n}$

$$\hat{q} = \frac{d}{r}.$$

δ-method $g(y) = -\log(1-y)$ $\boxed{g'(y) = \frac{1}{1-y}}$

$E(\hat{\lambda}) = E[-\log(1-\hat{q})] \approx -\log(1-q)$

$Var(\hat{\lambda}) \approx \boxed{\frac{d(r-d)}{r^3(1-q)^2}}$

If exponential failure with rate λ is assumed for the interval, then the true q satisfies

$$q = 1 - \exp(-\lambda).$$

Use this relationship to suggest an estimator of λ, $\hat{\lambda}$ say and the δ–method to approximate the mean and variance of $\hat{\lambda}$. Hint: Make λ the subject of the above equation and use the delta method on the resulting formula. You will also need the result for the variance of $\hat{q}$ from lectures.

Question Four

The times until rejection (of the transplanted organ) or censoring are provided below for thirty-six patients who received an organ transplant. The patients in the treatment group received a new drug (thought to prolong survival) and those in the control group received a placebo.

The times (in weeks) until rejection or censoring for each group are shown below.

Treatment Group

Times to rejection: 6,6,7,10,13,16,23
Times to censoring: 6,9,11,12,19,20,25,30,32,32,35

Control Group

Times to rejection: 1,1,2,3,5,5,8,9,9,10,10,11,12,18,25
Times to censoring: 3,4,9

Compute the KM estimate of the distribution function, $\hat{F}(t)$. Also, provide an estimate of the standard error of your KM estimate.

TRTMT

$\hat{F}(t)$ is just $P(T<t)$

SE of $\hat{F}(t)$ is ( )