Duration: **60 minutes (2:15pm – 3:15pm)**
Aids Allowed: **none**

Student Number: 9 9 9 2 9 2 5 0 9

Family Name(s): Qiu

Given Name(s): Rui

*Do **not** turn this page until you have received the signal to start.*
In the meantime, please fill out the identification section above.

This term test consists of 5 questions on 10 pages (including this one), printed on both sides of the paper. *When you receive the signal to start, please make sure that your copy of the test is complete, fill in the identification section above, and write your name on the back of the last page.*

This test is double-sided.

MARKING GUIDE

# 1:	3 / 7
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# 3:	3.5 /12
# 4:	5 / 6
# 5:	5.5 / 6
TOTAL:	27.5 /42

♡ *Good Luck and Have a Happy St. Valentine's Day* ♡

Use the space on this "blank" page for scratch work, or for any answer that did not fit elsewhere.
Clearly label each answer with the appropriate question and part number.

Question 1. [7 MARKS]

Consider the statement:

(S1) $A \Rightarrow (B \vee C)$.

Assuming that statement (S1) is true, give the best answer for each of the following questions:

Part (a) [1 MARK] (BVC) is true.

What can be concluded from (S1), if A is true?

If A is true, (BVC) may be true, if so, then B is true, C is true;
(BVC) may be false, if so, then at least one of B, C is false.

Part (b) [1 MARK]

What can be concluded from (S1), if B is true? Nothing (or the consequent is true)

If B is true, C may be true, if so, A is true;
C may be false, if so, A is false.

Part (c) [1 MARK]

What is the converse of (S1)?

$(B \vee C) \Rightarrow A$

1

Part (d) [2 MARKS]

What is the contrapositive of (S1)? (Work the negation(s) all the way in.)

$\neg B \wedge \neg C \Rightarrow \neg A$

2

Part (e) [2 MARKS]

What is the negation of (S1)? (Work the negation(s) all the way in.)

$\neg A \Rightarrow \neg B \wedge \neg C$

$\neg(A \Rightarrow (B \vee C)) \Leftrightarrow \neg(\neg A \vee (B \vee C))$
$\Leftrightarrow \neg\neg A \wedge \neg(B \vee C)$
$\Leftrightarrow A \wedge \neg B \wedge \neg C$

Question 2. [11 MARKS]

Consider the domain $D = \{$all CSC courses and all MAT courses$\}$, and the predicate symbols $C(x)$: "x is a CSC course", $M(x)$: "x is a MAT course", and $P(x, y)$: "course x is a prerequisite for course y".

Using only these symbols (in addition to appropriate connectives and quantifiers), give a clear symbolic statement that corresponds to each given English sentence. Quantifiers may **only** be over the domain D.

Part (a) [1 MARK]

CSC108 is a prerequisite for CSC148.

Let CSC 108 be x, CSC 148 be y.

~~$\exists x \in D, \exists y \in D, P(x,y)$~~ $x \in D, y \in D, P(x,y)$ 1

Part (b) [2 MARKS]

There is no prerequisite for CSC104.

Let CSC ~~108 be~~ 104 be z, and x is ~~a~~ other course in D.

~~$\neg\exists x \in D, P(x,z)$~~ ~~$\exists x \in D, P(x,z)$~~

$\neg\exists x \in D, z \in D. P(x,z)$ 2

Part (c) [2 MARKS]

Every course has a prerequisite.

$\forall x \in D, \exists y \in D, P(y,x)$ 2

Part (d) [2 MARKS]

No course is a prerequisite for itself.

$\neg\exists x \in D, P(x,x)$ 2

Part (e) [2 MARKS]

Some CSC course has a prerequisite.

$\exists x \in D, C(x) \Rightarrow \exists y \in D,$ ~~$P(x,y)$~~ $P(y,x)$ 1.5

Part (f) [2 MARKS]

Every MAT course has a prerequisite.

$\forall x \in D, M(x) \Rightarrow \exists y \in D, P(y,x)$ 2

Question 3. [12 MARKS]

The following terms are used frequently to describe logical statements in CSC165. For each of them:

(i) Write a **definition** of the term, in English.

(ii) Write a **statement in English** that is true and is an example of a statement that meets the definition of the term.

(iii) After defining suitable domain(s) and/or predicate(s), write a **statement in logic** that is true and is an example of a statement that meets the definition of the term.

Part (a) [4 MARKS]

A universally quantified statement.

doesn't need to be true

(i) definition: A statement which ~~is~~ holds for all elements in its domain.

1 A universally quantified statement is a ~~statement~~ ~~that~~ makes a claim about all objects in a domain

(ii) statement in English: Every room in Toronto has a door.

1

(iii) domains, predicates and statement in logic:

Domain: Let R be the set of all rooms in Toronto.
Predicate: Let D be ~~is~~ the event that a room has a door.

1

Let x be a random door.
Statement: $\forall x \in R$, $D(x)$.

Part (b) [4 MARKS]

A tautology.

(i) definition: A statement which has two-sided implication. ~~A~~

a statement which is always true (independent of the domains or predicates involved)

0

(ii) statement in English: The season just after spring in Toronto is ~~summer~~ Summer.

Either this statement just

1/2

(iii) domains, predicates and statement in logic:

Domain: Let S be the set of seasons in Toronto.
Predicate: Let A be "A season is just after spring."
Let B be "A season is summer".
Let x be a ~~...~~ season.
Statement: $x \in S$, $A(x) \Leftrightarrow B(x)$

0

$P \vee \neg P$ or $\forall D \in D, \forall P \in P$, that is $\forall x \in D, P(x) \vee \neg P(x)$, where D is the set of all possible domains and P is the set of all possible predicates on D

Part (c) [4 MARKS]

A vacuous truth.

An implication always true because its ~~antecedent~~ assumption is always false

(i) definition:

A statement whose ~~inverse~~ converse is also true. if the statement itself is true.

a statement with a false antecedant

(ii) statement in English: ~~All ... ~~ ~~...~~ A triangle ~~is~~ is a ~~...~~ shape with three angles.

All unicorns are pink and purple.

(iii) domains, predicates and statement in logic: Domain: ~~Let~~ G be the set of ~~...~~ shapes.

T be "it is a triangle"
A be "it has 3 ~~...~~ angles".
x be a random shape.

Statement: $x \in G$, $T(x), A(x)$. ~~...~~

$\forall x \in \mathbb{R}, x^2 < 0 \Rightarrow x = 42$

3.5

Question 4. [6 MARKS]

Show that $\exists x \in D, (P(x) \Rightarrow Q(x))$ is equivalent to $(\forall x \in D, P(x)) \Rightarrow (\exists x \in D, Q(x)))$. Justify each step of your derivation. You may use the list of standard equivalences given below.

Proof: $\exists \in D, (P(x) \Rightarrow Q(x))$

$\Leftrightarrow$ ~~$\forall \in D, (\neg P(x) \Rightarrow \neg Q(x))$~~

$\forall \in D, \neg(P(x) \Rightarrow Q(x))$ # Quantifier Negation

$\Leftrightarrow \forall \in D, \neg(\neg P(x) \vee Q(x))$ # implication

$\Leftrightarrow \forall \in D, (P(x) \wedge \neg Q(x))$ # DeMorgan's Laws

$\Leftrightarrow (\forall \in D, P(x)) \wedge (\forall \in D, \neg Q(x))$ # Quantifier Distributivity

~~$\Leftrightarrow (\forall \in D, P(x)) \Rightarrow \forall \in D, \neg Q(x)$~~

$\Leftrightarrow \neg(\forall \in D, P(x)) \vee (\exists \in D, Q(x))$ # Quantifier Negation

$\Leftrightarrow (\forall \in D, P(x)) \Rightarrow (\exists \in D, Q(x))$ # implication

5

~~$\exists x \in D, P(x)$~~ :

■ $\exists x \in D, (P(x) \Leftrightarrow Q(x))$

$\Leftrightarrow \exists x \in D, (\neg P(x) \vee Q(x))$ (implication)

$\Leftrightarrow (\exists x \in D, \neg P(x)) \vee (\exists x \in D, Q(x))$ (q dis)

$\Leftrightarrow \neg(\forall x \in D, P(x)) \vee (\exists x \in D, Q(x))$ (q neg)

$\Leftrightarrow (\forall x \in D, P(x)) \Rightarrow (\exists x \in D, Q(x))$ (implication)

Standard Equivalences (where P, Q, $P(x)$, $Q(x)$, *etc.* are arbitrary sentences. All quantifications are over a domain D.)

- *Commutativity*
  $P \wedge Q \iff Q \wedge P$
  $P \vee Q \iff Q \vee P$
  $P \Leftrightarrow Q \iff Q \Leftrightarrow P$
- *Associativity*
  $P \wedge (Q \wedge R) \iff (P \wedge Q) \wedge R$
  $P \vee (Q \vee R) \iff (P \vee Q) \vee R$
- *Identity*
  $P \wedge (Q \vee \neg Q) \iff P$
  $P \vee (Q \wedge \neg Q) \iff P$
- *Absorption*
  $P \wedge (Q \wedge \neg Q) \iff Q \wedge \neg Q$
  $P \vee (Q \vee \neg Q) \iff Q \vee \neg Q$
- *Idempotency*
  $P \wedge P \iff P$
  $P \vee P \iff P$
- *Double Negation*
  $\neg\neg P \iff P$
- *DeMorgan's Laws*
  $\neg(P \wedge Q) \iff \neg P \vee \neg Q$
  $\neg(P \vee Q) \iff \neg P \wedge \neg Q$
- *Distributivity*
  $P \wedge (Q \vee R) \iff (P \wedge Q) \vee (P \wedge R)$
  $P \vee (Q \wedge R) \iff (P \vee Q) \wedge (P \vee R)$
- *Implication*
  $P \Rightarrow Q \iff \neg P \vee Q$
- *Biconditional*
  $P \Leftrightarrow Q \iff (P \Rightarrow Q) \wedge (Q \Rightarrow P)$
- *Renaming* (where $P(x)$ does not contain variable y)
  $\forall x, P(x) \iff \forall y, P(y)$
  $\exists x, P(x) \iff \exists y, P(y)$
- *Quantifier Negation*
  $\neg\forall x, P(x) \iff \exists x, \neg P(x)$
  $\neg\exists x, P(x) \iff \forall x, \neg P(x)$
- *Quantifier Commutativity*
  $\forall x, \forall y, S(x,y) \iff \forall y, \forall x, S(x,y)$
  $\exists x, \exists y, S(x,y) \iff \exists y, \exists x, S(x,y)$
- *Quantifier Distributivity* (where S does not contain variable x)
  $S \wedge \forall x, Q(x) \iff \forall x, S \wedge Q(x)$
  $S \vee \forall x, Q(x) \iff \forall x, S \vee Q(x)$
  $S \wedge \exists x, Q(x) \iff \exists x, S \wedge Q(x)$
  $S \vee \exists x, Q(x) \iff \exists x, S \vee Q(x)$
  $(\forall x, P(x)) \wedge (\forall x, Q(x)) \iff \forall x, (P(x) \wedge Q(x))$
  $(\exists x, P(x)) \vee (\exists x, Q(x)) \iff \exists x, (P(x) \vee Q(x))$

5.5

Question 5. [6 MARKS]

At a murder trial, four witnesses give the following testimony.

Alice: If either Bob or Carol is innocent, then so am I.

Bob: Alice is guilty, and either Carol or Dan is guilty.

Carol: If Bob is innocent, then Dan is guilty.

Dan: If Bob is guilty, then Carol is innocent. In addition, Bob is innocent.

Is it possible that everyone is telling the truth? Justify your response.

Proof ~~Solution~~: Write everyone's testimony in symbolic statement:
Let "G" be "guilty", hence "¬G" is "innocent".

$A \iff (\neg G(B)) \vee (\neg G(C)) \Rightarrow \neg G(A)$ ✓

$B \iff G(A) \wedge (G(C) \vee G(D))$ ✓

$C \iff \neg G(B) \Rightarrow G(D)$ ✓

$D \iff (G(B) \Rightarrow \neg G(C)) \wedge \neg G(B)$ ✓

Now suppose everyone is telling a truth.

If D is true, then $\neg G(B)$ and $\neg G(B) \Rightarrow \underline{G(C)}$ X.

~~then~~ $(G(B) \Rightarrow \neg G(C)) = (G(C) \Rightarrow \neg G(B))$

If C is true, $\neg G(B) \Rightarrow \underline{G(D)}$ # we already have $\neg G(B)$

If B is true, $\underline{G(A)}$. # $G(C) \vee G(D)$ is true

If A is true, $(\neg G(B)) \vee (\neg G(C)) \Rightarrow \underline{\neg G(A)}$ # we already have $\neg G(B)$

Thus the coclusion from A contradicts that ~~from~~ from B.

Therefore, it is <u>not possible</u> that everyone is telling a truth.

■