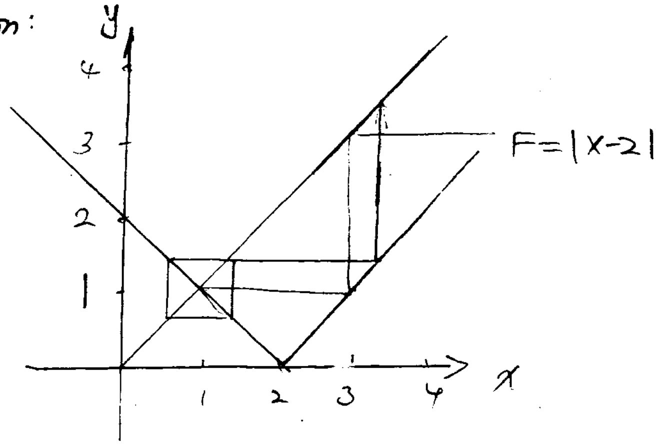
MAT335 ~~Assignment~~ Homework 2 Rui Qiu #999292509

Chapter 4.

Q5. Solution:



red: ~~only 1 is a fixed point~~ 0, 2 are a 2-cycle.

~~green:~~

blue: every odd integer is an eventually ~~period~~ fixed point.

green: every point left is an eventually periodic point.

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Q6. $F(x)=x^2-1.1$

Solution:

$$x^2-1.1=x$$

$$x^2-x-1.1=0$$

$$x=\frac{1\pm\sqrt{1+44}}{2}=\frac{1\pm\sqrt{5.4}}{2}$$

$$F^2(x)=(x^2-1.1)^2-1.1=x^4-2.2x^2+1.21-1.1$$

$$=x^4-2.2x^2+0.11$$

Then $x^4-2.2x^2-x+0.11=0$

$$\Rightarrow (x^2-x-1.1)(x^2+x-0.1)=0$$

$$\Rightarrow x^2-x-1.1=0 \text{ or } x^2+x-0.1=0$$

$$\Rightarrow x=\frac{1\pm\sqrt{5.4}}{2} \text{ or } x=\frac{-1\pm\sqrt{1.4}}{2}$$

Thus $\frac{-1\pm\sqrt{14}}{2}$ is the 2-cycle that we need.

Q7. Solution: $F(x)=ax+b$.

a. $F(x)=ax+b=x$

$$(a-1)x=-b$$

$$x=\frac{-b}{a-1}=\frac{b}{1-a} \quad (a\neq 1)$$

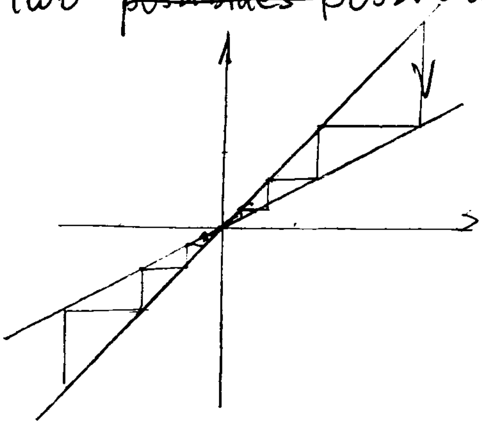
$\frac{b}{1-a}$ is the fixed point.

b. When $F(x)$ is parallel to $y=x$, but not on it!
i.e. $a=1$ but $b\neq 0$.

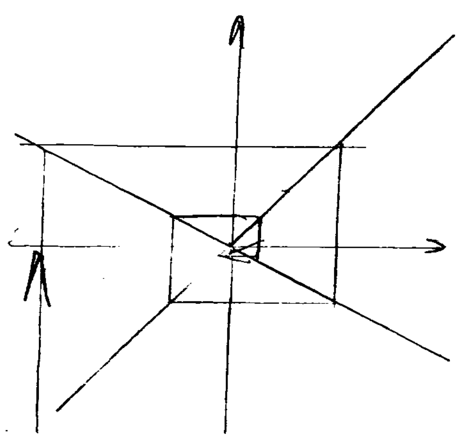
c. When $F(x)$ is the line $y=x$ itself, it has ∞ many intersections.
i.e. $a=1$ and $b=0$.

d. No matter what value of b we have, $F(x)$ has exactly one fixed point iff $a \neq 1$.

e. Two ~~possibilies~~ possibilities:



$0<a<1$



$-1<a<1$

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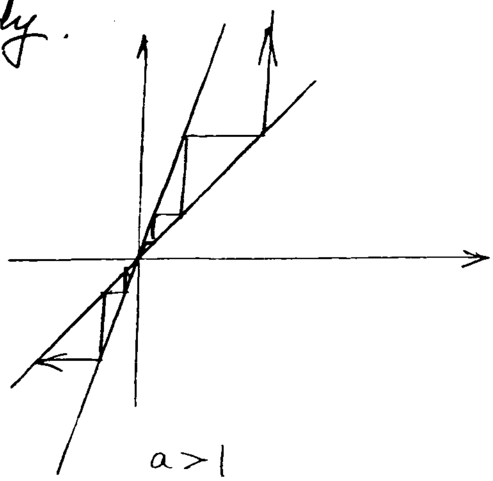
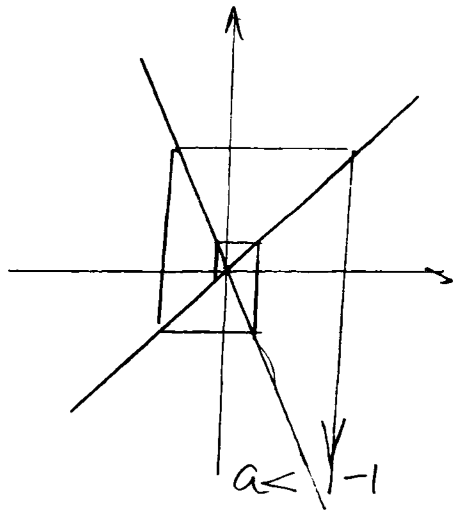
We call ~~it~~ them attracting point b/c all the points near it tends to converge to it.

f. When $a=0$, every point is eventually fixed after at most one iteration.

g. Similarly.



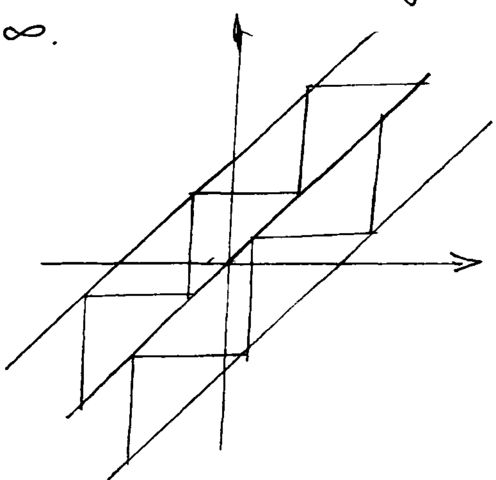
$a>1$



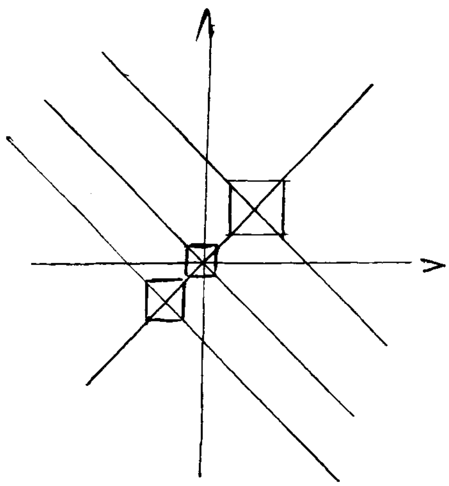
$a<-1$

We say ~~it~~ them repelling points b/c all the points near it tend to escape from it.

h. i. $b=0$, $F(x)=x$, all real numbers are fixed pts.
ii. $b>0$, $F(x)=x+b$, no cycles, and $F^n(x)\to\infty$ as $n\to\infty$.
iii. $b<0$, $F(x)=x+b$, no cycles as well, $F^n(x)\to-\infty$ as $n\to\infty$.



i,



We have $\frac{b}{2}$ as the only fixed point.

And every real number is an eventually periodic point with ~~2~~ 2 cycle.

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Chapter 5.

Q1.

b). $F(x)=x(1-x)$

$x-x^2=x$

$-x^2=0$

$x=0$

So $x=0$ is a fixed point.

$F'(x)=1-2x$

$F'(0)=1 \Rightarrow 0$ is neutral.

d). $F(x)=3x(1-x)$

$3x-3x^2=x$

$3x^2-2x=0$

$x(3x-2)=0$

$x=0$ or $x=\frac{2}{3}$

0 and $\frac{2}{3}$ are two fixed points.

$F'(x)=3-6x$

$F'(0)=3>1 \Rightarrow 0$ is repelling

$F'(\frac{2}{3})=3-4=-1 \Rightarrow \frac{2}{3}$ is neutral.

j). $T(x)=\begin{cases} 2x & \text{if } x\leq\frac{1}{2} \\ 2-2x & \text{if } x>\frac{1}{2} \end{cases}$

$2x=x \Rightarrow \underline{x=0} \Rightarrow T'(x)=2 \Rightarrow 0$ is repelling

fixed point.

$2-2x=x \Rightarrow x=\frac{2}{3} \Rightarrow T'(x)=-2 \Rightarrow \frac{2}{3}$ is also repelling k). $F(x)= 1/x^2$

$\frac{1}{x^2} = x \Rightarrow x = 1$ is a fixed point.

$F'(x) = -2x^{-3} \Rightarrow F'(1) = -2$

$\Rightarrow 1$ is a repelling fixed point. 2

Q3. Solution:

$$D(x) = \begin{cases} 2x & \text{if } 0 \le x < \frac{1}{2} \\ 2x-1 & \text{if } \frac{1}{2} \le x < 1 \end{cases}$$

$\forall x \neq \frac{1}{2}$, $D'(x) = 2$.

Let $x_0 \in \text{per}_n D$, i.e. x_0 is in this n-cycle.
that is to say $D^n(x_0) = x_0$,
Let $x_k = D^k(x_0)$ for $k = 0, 1, 2, 3, \cdots, n-1$.

Then
$$\begin{aligned}(D^n)'(x_0) &= \prod_{k=0}^{n-1} D'(x_k) \\ &= D'(x_0)\,D'(x_1)\cdots D'(x_{n-1}) \\ &= \underbrace{2 \cdot 2 \cdots 2}_{n \text{ times}} \\ &= 2^n > 1 \quad \forall n > 0.\end{aligned}$$

Therefore, all periodic points are repelling in this case

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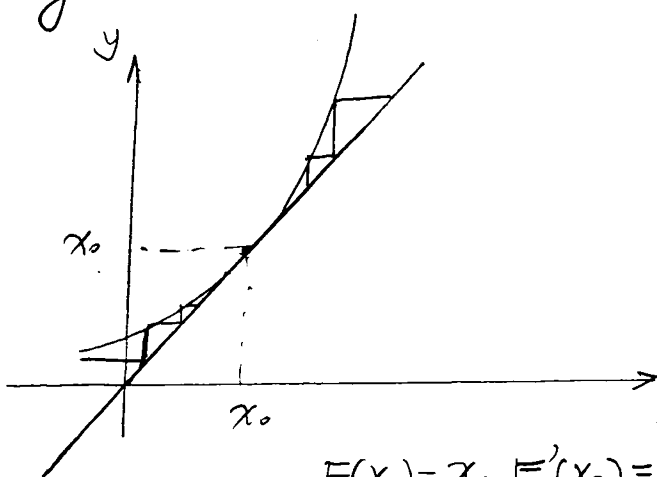
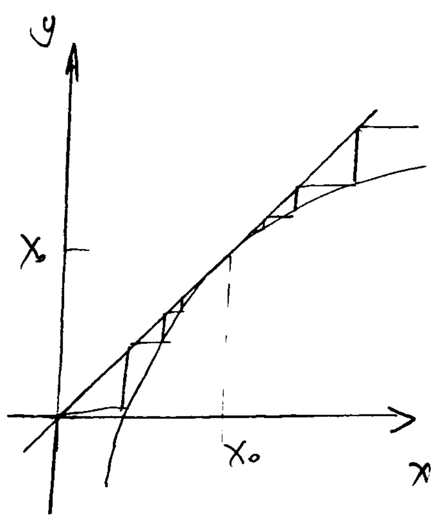
0.5

Q5. Solution:

Since $F''(x_0)>0$, the graph of F concaves up at the netural fixed point x_0.
The graphical analysis also contains the other possibility ~~that~~ which is $F''(x_0)<0$ (concaves down).



$F(x_0)=x_0$, $F'(x_0)=1$, $F''(x_0)>0$



$F(X_0)=X_0$,
$F'(X_0)=1$,
$F''(X_0)<0$

$\frac{8.5}{10}$