LN 10.2.

$$v = \frac{\tau}{(1+i)} \iff \underline{\tau} = v \cdot (1+i).$$

$$j = \frac{6\%}{2} = 3\%.$$

<u>Ex</u>: $PV = Fr \cdot a_{\overline{n|}j} + C * v_j^n = 100 \cdot \frac{4\%}{2} \cdot a_{\overline{40|}j} + 100 \cdot v_j^{40}$

$$\boxed{= \$76.89}$$

$$\tau = \frac{\sum_{t=1}^{n} t \cdot Fr \cdot v_j^t + n \cdot C \cdot v_j^n}{\sum_{t=1}^{n} Fr \cdot v_j^t + C \cdot v_j^n}.$$

$$\tau = \frac{Fr \boxed{\left(\sum_{t=1}^{n} t \cdot v_j^t\right)} + n \cdot C \cdot v_j^n}{P}$$

$$\boxed{\tau = \frac{Fr \cdot (Ia)_{\overline{n|}j} + n \cdot C \cdot v_j^n}{P}.}$$

$$\tau = \frac{100 \times \frac{4\%}{2} \cdot (Ia)_{\overline{40|}j} + 100 \times 40 \times v_j^{40}}{76.89}$$

$\tau = 25.96$. half years

$= \frac{25.96}{2}$ years

$= 12.98$ yrs.

Ex: $j = \frac{20\%}{2} = 10\%$ per half

$\tau = 13.13$ half yrs

$= 6.56$ yrs.

Approximate Prices.

Taylor's Series, $f(x), f'(x), f''(x), \dots$

$$f(x+\varepsilon) = f(x) + \varepsilon \cdot f'(x) + \frac{\varepsilon^2}{2!} f''(x) + \cdots$$

$$\Downarrow$$

$$f(x+\varepsilon) \cong f(x) + \varepsilon \cdot \boxed{f'(x)} + $$

$$PV(\bar{i}_0+\varepsilon) \cong PV(\bar{i}_0) + \varepsilon \cdot PV'(\bar{i}_0).$$

$$v = -\frac{PV'(\bar{i}_0)}{PV} \quad ; \quad \tau = v\cdot(1+\bar{i}_0) = -\frac{PV'(\bar{i}_0)}{PV}\cdot(1+\bar{i}_0)$$

$$PV(\bar{i}_0+\varepsilon) \cong PV(\bar{i}_0) + \varepsilon\cdot(-PV(\bar{i}_0)\cdot v) = PV(\bar{i}_0)\cdot(1-\varepsilon v)$$

$$= PV(\bar{i}_0) + \varepsilon\cdot\left(-\frac{PV(\bar{i}_0)\cdot\tau}{(1+\bar{i}_0)}\right)$$

$$= PV(\bar{i}_0)\left(1-\frac{\varepsilon\tau}{(1+\bar{i}_0)}\right)$$

$$\frac{PV(\bar{i}_0+\varepsilon)-PV(\bar{i}_0)}{PV(\bar{i}_0)} \cong \varepsilon \cdot \frac{PV'(\bar{i}_0)}{PV(\bar{i}_0)}$$

$$= -\varepsilon \nu$$

$$= -\varepsilon \cdot \frac{\tau}{(1+\bar{i}_0)}.$$

Ex: $\begin{cases} P = \$21.77. \\ j = \frac{20\%}{2} = 10\%. \\ \tau = 13.13 \text{ half yrs.} \end{cases} \Rightarrow$ P' ? $j' \Rightarrow 11\%$

Sol: M1: $PV(11\%) = PV(\overset{\bar{i}_0}{\underline{10\%}} + \overset{\varepsilon}{\underline{1\%}})$

$$= PV(10\%) \cdot \left(1 - \frac{1\% \cdot 13.13}{1+10\%}\right).$$

$$= 21.77 \cdot (1 - 0.1194)$$

$$= \$19.17.$$

M2: $\bar{i}_0 = (1+10\%)^2 - 1 = 0.21$ p.a.

$$\tau = \frac{13.13}{2} = 6.565 \text{ yrs.}$$

$$\varepsilon = \left[(1+11\%)^2 - 1\right] - \bar{i}_0 = 0.0221.$$

$$PV(i_0+\varepsilon) \simeq PV(i_0)\left(1-\frac{\varepsilon\tau}{(1+i_0)}\right)$$

$$= 21.77\cdot\left(1-\frac{0.0221\times 6.565}{1+0.21}\right)$$

$$= 21.77\,(1-0.1198)$$

$$= \$19.17$$

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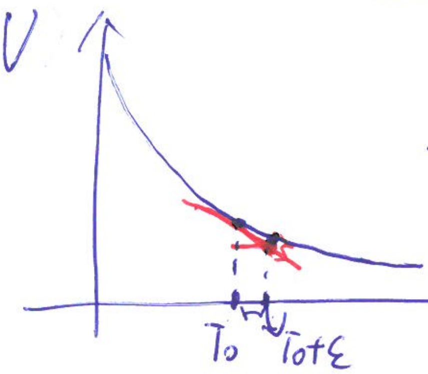
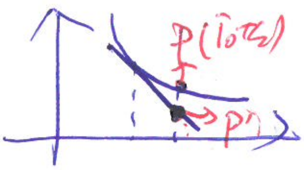
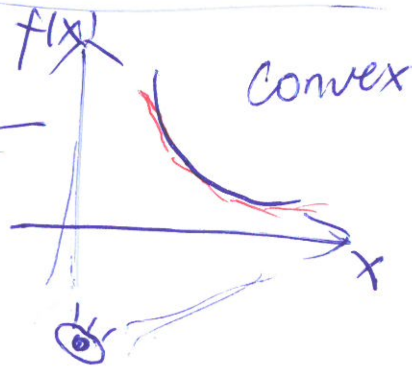
③ Convexity





$$f(x+\varepsilon) \simeq f(x) + \varepsilon\cdot f'(x) + \frac{\varepsilon^2}{2!} f''(x)$$

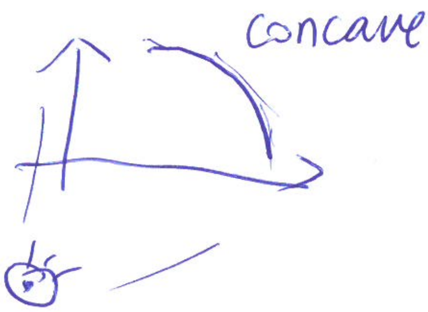
$f''(x) \geq 0$



Convexity:

$$C(i) = \frac{d^2PV}{di^2}\cdot\frac{1}{PV}$$

$$\boxed{C(i) = \frac{PV''(i)}{PV} > 0}$$



$$C(i) = \frac{1}{PV} \cdot \frac{d}{di}\left(\underbrace{\boxed{-\sum_{k=1}^{n} C_{t_k}\, t_k (1+i)^{-t_k-1}}}_{= \frac{dPV}{di}} \right) \quad < 0$$

$$\boxed{PV = \sum_{k=1}^{n} C_{t_k} \cdot (1+i)^{-t_k}}$$

$$C(i) = \frac{\sum_{k=1}^{n} C_{t_k} \cdot t_k\,(t_k+1) \cdot (1+i)^{-t_k-2}}{PV}$$

$$C(i) = \frac{\sum_{k=1}^{n} C_{t_k} \cdot t_k \cdot (t_k+1) \cdot v_i^{t_k+2}}{PV}$$