LN 9.2.

1. Forward price K.
2. Value of a forward contract V_L

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1. $K = ?$

(1). no income

	Portfolio A	Portfolio B.
$t=0$	* long forward contract to buy one unit of S * Invest $\$K\cdot e^{-\delta T} + ce^{-\delta t_1}$	buy one unit of S. ($\$S_0$)
$t=T$	* pay K to get one unit of S * $\$K\cdot e^{-\delta T}\cdot e^{\delta T} = \$K$	* ~~get~~ have one unit of S (S_T) $S_T + c\cdot e^{\delta(T-t_1)}$

$\Updownarrow$

one unit of S

$S_T + c\cdot e^{\delta(T-t_1)}$

$$\Rightarrow \quad K e^{-\delta T} = S_0 \quad \Rightarrow \quad \boxed{K = S_0\cdot e^{\delta T}}$$

	A	B
$t=0$	long forward contract to buy one S	borrow $\$S_0$ to buy one S
$t=T$	pay $\$K$ to get one S	repay $\$S_0 e^{\delta T}$ have one S

$$\Rightarrow \quad K = S_0\cdot e^{\delta T}.$$

Ex:

$$\left.\begin{aligned} S_0 &= \$50 \\ T &= \tfrac{1}{2}\text{ yrs} \\ i &= 9\%\text{ p.a.} \end{aligned}\right\} \Rightarrow K$$

Sol:

$\delta = \ln(1+i)$

$$K = S_0 e^{\delta T} \doteq S_0 \cdot (1+i)^T = 50 \cdot (1+9\%)^{\frac{1}{2}}$$

$$= \$52.2$$

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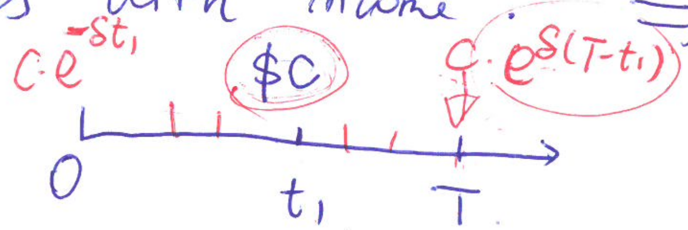
(2). Securities with Income $\Rightarrow K$?

Assume.



$$\Rightarrow \quad K \cdot e^{-\delta T} + c \cdot e^{-\delta t_1} = S_0$$

$$\Rightarrow \quad \boxed{K = S_0 e^{\delta T} - c \cdot e^{\delta(T-t_1)}}$$

Ex: $r = \frac{10\%}{2} = 5\%$

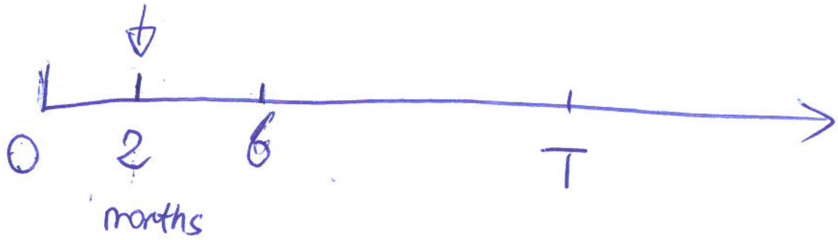
$F = 100 = C$

$P = \$50.2 = S_0$ per \$100 nominal

\$20,000 nominal

$$S_0 = \$50.2 \times \frac{20{,}000}{100} = \underline{\$10{,}040}$$

$$T = \frac{1}{2}\text{ yrs}$$



$C = 20{,}000 \times 5\% = \$1000$.

$\delta = 0.07$ p.a.

Sol:

$\Rightarrow$ $K = S_0 \cdot e^{\delta T} - C \cdot e^{\delta (T - t_1)}$

$= 10{,}040 \cdot e^{0.07 \cdot \frac{1}{2}} - 1000 \cdot e^{0.07 \cdot (\frac{1}{2} - \frac{2}{12})}$

$= 9{,}374$.

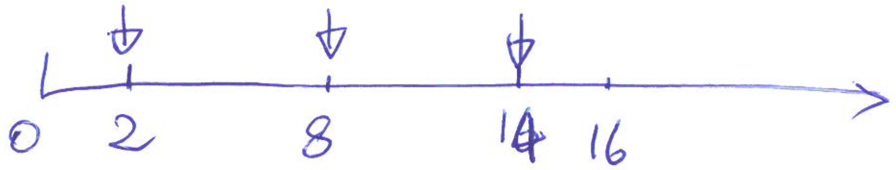
$\Rightarrow$ $K = S_0 \cdot e^{\delta T} - (PV_2) \cdot e^{\delta T}$

$\boxed{K = (S_0 - PV_2) \cdot e^{\delta T}}$

Ex: $PV_2 = 1000 \cdot e^{-\delta \times \frac{2}{12}}$

$\Rightarrow$ $K = (S_0 - PV_2) \cdot e^{\delta \times \frac{1}{2}}$

Ex:



$S_0 = 50{,}000 \times \frac{50.2}{100} = \$25{,}100$.

$T = \frac{16}{12}$ yrs.

$C = 50{,}000 \times \frac{0.1}{2} = \$2500$.

$$K = S_0 \cdot e^{\delta T} - C_1 \cdot e^{\delta(T-t_1)} - C_2 \cdot e^{\delta(T-t_2)} - C_3 \cdot e^{\delta(T-t_3)}$$

$$K = (S_0 - PV_2) \cdot e^{\delta T}.$$

PV_2
$\parallel$

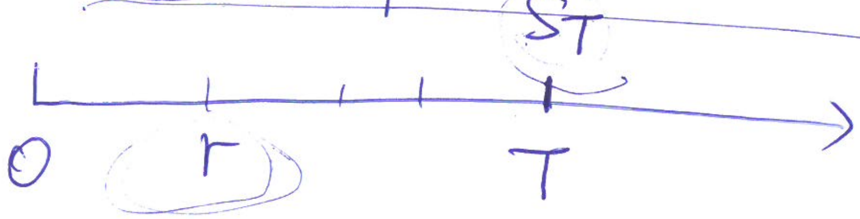
$$= \left[\$25{,}100 - \left(2500 \cdot e^{-\delta \cdot \frac{2}{12}} + 2500 \cdot e^{-\delta \cdot \frac{8}{12}} + 2500 \cdot e^{-\delta \cdot \frac{14}{12}}\right)\right]$$

$$* \; e^{\delta \times \frac{16}{12}}.$$

$\delta = 0.07$

$$= \$19{,}693.97$$

② The Value of a forward contract.



$t=0$: $V_S = V_L = 0$

$t=T$: $V_L = S_T - K$: long.

$V_S = K - S_T$: short