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Why $\mathbb{R}^n$:

This course attempts at generalizing functions one variable like $y = f(x)$ to functions of several variables; this means functions that take more than one variables as their argument (or their input:) denoted by $f(x, y), f(x, y, z)$ or $f(x_1, x_2, \ldots x_n)$. So the expression $f(x, y, z)$ stands for the value that f assigns to the point of inputs (x, y, z). However such inputs are often interpreted as points in the palne, in the space or just $n-$tuples. The main task of a multi-variate Calculus would be to extend the main ideas and techniques of one variable Calculus to the multi-variate case. It is a common practice in one variable Calculus to work with expressions such as $f(-x), f(x + h)$, or $f(kx)$, and these make perfect sense since the range of the variable x is real numbers, and in the real numbers we have defined algebraic operations such as addition and multiplications, etc. In one variable Calculus one does not realize how conveniently a *point* or an input to the function is a *real number* and as such they are conveniently open to *algebra* and possess convenient Algebraic properties. Notions such as addition, subtraction and magnification, which are naturally defined on the real numbers, are all absent when we deal with the points in the plane or with the n-tuples which are to be inputs to the multi-variate functions. Therefore, $f((x, y) + (a, b))$ or $f(k(x, y)$ are not meaningful. To deal this problem we (the textbook) decided to work with $\mathbb{R}^2$ instead of the plane, and working with $\mathbb{R}^3$ instead of space. In general $\mathbb{R}^n$ is considered to be the collection of n-tuples with additional Algebraic properties (please read page 4 very carefully and see how the textbook defines addition and scalar multiplication for n-tuples.) The advantage of working with $\mathbb{R}^2$, $\mathbb{R}^3$ or $\mathbb{R}^n$ is that they are structures which *consistently* embody algebraic properties; the elements of these spaces are known as *vectors*. (Note: here, the term *consistently* means all the theory of Linear Algebra!)