APM462H1S: Nonlinear optimization, Winter 2014.

Summary of March 17 lecture.

The March 17 lecture went quickly over more or less the whole of chapter 2 of the Evans notes on Optimal Control (but totally skipping Section 2.4) mostly omitting proofs or only discussing the ideas of the proofs without many details missing.

For this class, the most important parts of the chapter are the parts that tell you how to do computations to answer rather concrete questions.

In particular:

- matrix exponentials. In the lecture we showed, for several examples of specific matrices M, how to find explicit formulas for the matrix exponential e^{tM}. For example,

$$\text{if } M = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \qquad \text{then } e^{tM} = \begin{pmatrix} 1 & t \\ 0 & 1 \end{pmatrix},$$

$$\text{if } M = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \qquad \text{then } e^{tM} = \begin{pmatrix} \cos t & \sin t \\ -\sin t & \cos t \end{pmatrix},$$

$$\text{if } M = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} \qquad \text{then } e^{tM} = \begin{pmatrix} e^{at} & 0 \\ 0 & e^{bt} \end{pmatrix},$$

  Also, if Q is any matrix such that $Q^TQ = I$, then

$$\text{if } M = Q^T \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} Q \qquad \text{then } e^{tM} = Q^T \begin{pmatrix} e^{at} & 0 \\ 0 & e^{bt} \end{pmatrix} Q.$$

  (Recall that every symmetric matrix can be written in the form Q^TDQ, where $Q^TQ = QQ^T = I$ and D is diagonal.)

- How to use matrix exponentials to solve systems of linear ODEs (see Theorem 2.1 in Section 2.2).

- Section 2.3, and especially the conclusion Theorem 2.6. You should be able to use this to determine, for specific matrices M and N, whether the system of ODEs

$$x'(t) = Mx(t) + N\alpha(t) \text{ for } t > 0$$

  is controllable.

- Do not worry about the proof of Theorem 2.8, which is actually too advanced for this class, but you should know the conclusion.