STAT 6046 Tutorial Week 5

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Today’s plan

- Brief review of course material
- Go through selective tutorial questions

Perpetuity

- An annuity where payments continue forever is called a ***perpetuity***.

$$\boxed{a_{\overline{n}|} = s_{\overline{n}|} \cdot v^n = \frac{1-v^n}{i}} \longrightarrow \boxed{a_{\overline{\infty}|} = \lim_{n\to\infty} a_{\overline{n}|} = \frac{1}{i}}$$

$$\boxed{\ddot{a}_{\overline{n}|} = \frac{1-v^n}{d} = \frac{i}{d} a_{\overline{n}|}} \longrightarrow \boxed{\ddot{a}_{\overline{\infty}|} = \frac{1}{d}}$$

$$\boxed{a^{(m)}_{\overline{n}|} = \frac{1-v^n}{i^{(m)}} = \frac{i}{i^{(m)}} a_{\overline{n}|}} \longrightarrow \boxed{a^{(m)}_{\overline{\infty}|} = \frac{1}{i^{(m)}}}$$

$$\boxed{\ddot{a}^{(m)}_{\overline{n}|} = \frac{1-v^n}{d^{(m)}} = \frac{i}{d^{(m)}} a_{\overline{n}|}} \longrightarrow \boxed{\ddot{a}^{(m)}_{\overline{\infty}|} = \frac{1}{d^{(m)}}}$$

Continuous Annuities

- Constant force of interest:

$$\boxed{\bar{s}_{\overline{n|}} = \int_0^n (1+i)^{n-t}\,dt = \frac{(1+i)^n - 1}{\delta} = \frac{i}{\delta}\cdot s_{\overline{n|}}}$$

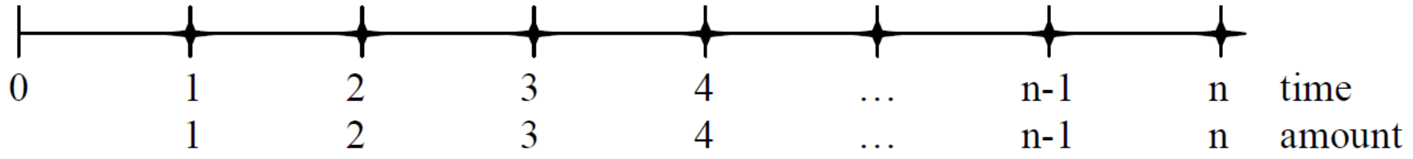
$$\boxed{\bar{a}_{\overline{n|}} = \int_0^n v^t\,dt = \frac{1-v^n}{\delta} = \frac{i}{\delta}\cdot a_{\overline{n|}}}$$

- Changing force of interest:

$$\bar{s}_{\overline{n|}\delta_r} = \int_0^n \exp\left(\int_t^n \delta_r\,dr\right)dt$$

$$\bar{a}_{\overline{n|}\delta_r} = \int_0^n \exp\left(-\int_o^t \delta_r\,dr\right)dt$$

Increasing Annuity



$$(Is)_{\overline{n}|i} = \frac{\ddot{s}_{\overline{n}|i} - n}{i}$$

$$(Ia)_{\overline{n}|i} = \sum_{t=1}^{n} t v^{t} = \frac{\ddot{a}_{\overline{n}|i} - n v^{n}}{i}$$

$$(I\ddot{a})_{\overline{n}|i} = \sum_{t=0}^{n-1} (t+1) v^{t} = \frac{\ddot{a}_{\overline{n}|i} - n v^{n}}{d}$$

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Decreasing Annuity

0	1	2	3	4	…	n-1	n	time
	n	n-1	n-2	n-3	…	2	1	amount

$$(Ds)_{\overline{n}|i} = \frac{n \cdot (1+i)^n - s_{\overline{n}|i}}{i}$$

$$(Da)_{\overline{n}|i} = \sum_{t=1}^{n} (n-t+1)v^t = \frac{n - a_{\overline{n}|i}}{i}$$

$$(D\ddot{a})_{\overline{n}|i} = \sum_{t=0}^{n-1} (n-t)v^t = \frac{n - a_{\overline{n}|i}}{d}$$

General formula for increasing annuity

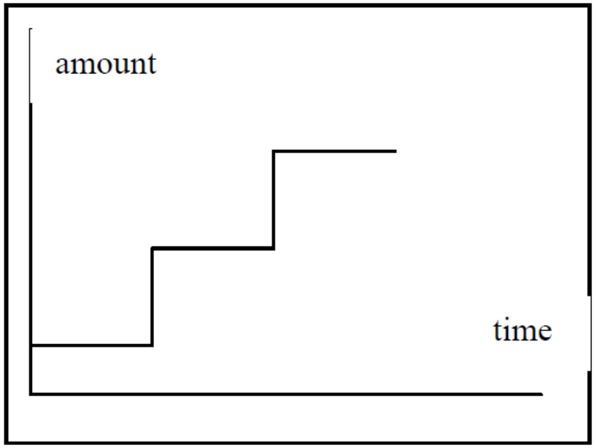
0	1	2	3	4	…	n-1	n	time
Increasing	A	A+B	A+2B	A+3B	…	A+(n-2)B	A+(n-1)B	amount

- Decompose into two parts

$$\boxed{S(n) = (A-B)s_{\overline{n}|i} + B\left(Is\right)_{\overline{n}|i}}$$

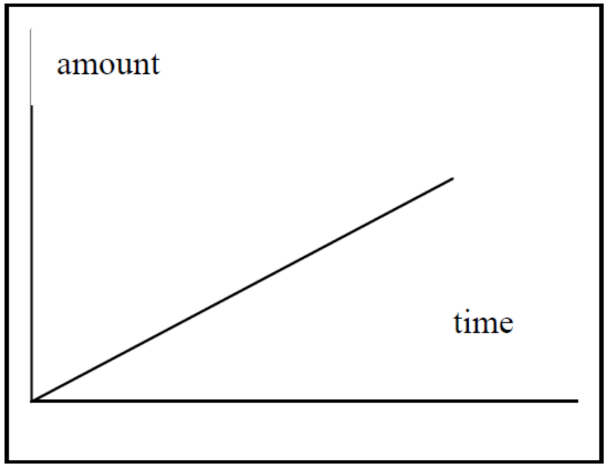
Continuous payments

- Increase at the end of the year



$$(I\bar{a})_{\overline{n}|i} = \int_0^n \lceil t \rceil v^t\,dt = \frac{\ddot{a}_{\overline{n}|i} - nv^n}{\delta} = \frac{i}{\delta}(Ia)_{\overline{n}|i}$$

- Increase continuously



$$(\bar{I}\bar{a})_{\overline{n}|i} = \int_0^n t v^t\,dt = \frac{\bar{a}_{\overline{n}|i} - nv^n}{\delta}$$

Annuities with indexation

- **Annuities in arrears**

0	1	2	3	4	…	n-1	n	time
	1	$(1+r)$	$(1+r)^2$	$(1+r)^3$	…	$(1+r)^{n-2}$	$(1+r)^{n-1}$	amount

$$S(n) = \left(\frac{(1+r)^n - (1+i)^n}{r-i} \right)$$

$$S(0) = S(n) \cdot v_i^n = (1+i)^{-n} \cdot \frac{(1+r)^n - (1+i)^n}{r-i} = \frac{\left(\frac{1+r}{1+i}\right)^n - 1}{r-i}$$

Annuities with indexation

- **Annuities-due**

time	0	1	2	3	4	…	n-1	n
amount	1	$(1+r)$	$(1+r)^2$	$(1+r)^3$	$(1+r)^4$	…	$(1+r)^{n-1}$	

$$S(n)=(1+i)\cdot\left[\frac{(1+i)^n-(1+r)^n}{i-r}\right]$$

$$S(0)=\left[\frac{1-\left(\frac{1+r}{1+i}\right)^n}{1-\left(\frac{1+r}{1+i}\right)}\right]$$

$$v_j^n=\left(\frac{1+r}{1+i}\right)^n=(1+j)^{-n}$$

$$S(0)=\left[\frac{1-\left(\frac{1+r}{1+i}\right)^n}{1-\left(\frac{1+r}{1+i}\right)}\right]=\frac{1-v_j^n}{1-v_j}=\frac{1-v_j^n}{d_j}=\ddot{a}_{\overline{n}|j}$$

Annuities with indexation

- In the situation when payment period and index period don't coincide, the payment period should be modified to coincide with the index period.