University of Toronto
Department of Mathematics

MAT224H1F
Linear Algebra II

Midterm Examination
October 23, 2012

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Duration: 1 hour 50 minutes

Last Name: ______________________

Given Name: ______________________

Student Number: ______________________

Tutorial Group: ______________________

No calculators or other aids are allowed.

FOR MARKER USE ONLY	
Question	Mark
1	/10
2	/10
3	/10
4	/10
5	/10
6	/10
TOTAL	/60

[10] **1.** Let $T\colon \mathbb{R}^3 \to \mathbb{R}^2$ be the linear transformation that has the matrix

$$\begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 1 \end{bmatrix}$$

relative to the bases $\alpha = \{(1,-1,1),(0,1,0),(1,0,0)\}$ of $\mathbb{R}^3$ and $\beta = \{(3,2),(2,1)\}$ of $\mathbb{R}^2$. Find $T(x,y,z)$ for any $(x,y,z) \in \mathbb{R}^3$.

[10] **2.** Let $T\colon P_2(\mathbb{R}) \to \mathbb{R}^3$ be the linear transformation defined by

$$T(a + bx + cx^2) = (a + b, b + c, a - c).$$

Find bases for the kernel and image of T.

[10] **3.** Let $W = \{(x, y, z) \in \mathbb{R}^3 \mid 3x - 2y + z = 0\}$. Show W is isomorphic to $\mathbb{R}^2$ and find an isomorphism $T\colon W \to \mathbb{R}^2$.

[10] **4.** Let $T\colon \mathbb{C}^2 \to \mathbb{C}^2$ be the linear transformation whose matrix with respect to some basis α for $\mathbb{C}^2$ is

$$\begin{bmatrix} 1+i & 1-i \\ 1-i & 2 \end{bmatrix}.$$

Find the matrix of T^{-1} with respect to α, if possible.

[10]**5.** Let $T\colon \mathbb{Z}_3^3 \to \mathbb{Z}_3^3$ be defined by

$$T(x_1, x_2, x_3) = (2x_1 + x_2, x_1 + x_2 + x_3, x_2 + 2x_3).$$

Show that there is no basis α for $\mathbb{Z}_3^3$ such that $[T]_{\alpha\alpha}$ is diagonal.

6. Let V and W be vector spaces over a field F, and $T\colon V \to W$ a linear transformation. Let $\alpha = \{v_1, v_2, \ldots, v_n\}$ be a basis for V. Prove $\dim(\mathrm{Ker}(T)) = 0$ if and only if $\{T(v_1), T(v_2), \ldots, T(v_n)\}$ is linearly independent.