Ch 2 :

#60 : $X \sim N(\mu, \sigma^2)$

$Y = e^X$ $\qquad f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

Find $f_Y(y)$.

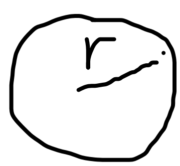
Sol'n: e^X - lognormal r.v. as ,

$\log e^X = X \sim$ Normal

$$f_Y(y) = f_X(\log y) \cdot [\log y]'$$

$$= \frac{1}{y\sqrt{2\pi\sigma^2}} e^{-\frac{(\log y - \mu)}{2\sigma^2}}.$$

#68 .



$r \sim \exp(1)$

f_{Area} - ? $\qquad S = \pi r^2$, $f_r(r) = \begin{cases} e^{-r}, & r \geq 0 \\ 0, & ow \end{cases}$

Sol'n: $f_S(s) = f_r\left[\sqrt{\frac{1}{\pi} s}\right] \left[\sqrt{\frac{s}{\pi}}\right]'$

$$= e^{-\sqrt{\frac{s}{\pi}}} \cdot \frac{1}{2\sqrt{\pi s}}.$$

Ch. 4 :

#32 : $X \sim \text{Gamma}(\alpha, \lambda)$

$Y = \frac{1}{X}$. Find $E(Y)$

Sol'n :

$$E[Y] = \boxed{\int_0^\infty \frac{1}{x} \frac{\lambda^\alpha x^{\alpha-1} e^{-\lambda x}}{\Gamma(\alpha)} dx}$$

$$= \int_0^\infty \frac{\lambda^\alpha x^{\alpha-2} e^{-\lambda x}}{\Gamma(\alpha)} dx$$

$$= \frac{\lambda}{\Gamma(\alpha)} \underbrace{\int_0^\infty \frac{\lambda^{\alpha-1} x^{\alpha-1} e^{-\lambda x}}{\Gamma(\alpha-1)} dx}_{\text{Gamma}(\alpha-1,\lambda)} \cdot \Gamma(\alpha-1)$$

$$\text{''} 1$$

$$= \frac{\lambda \cdot \Gamma(\alpha-1)}{\Gamma(\alpha)} = \frac{\lambda\, \cancel{\Gamma(\alpha-1)}}{(\alpha-1)\cancel{\Gamma(\alpha-1)}} = \boxed{\frac{\lambda}{\alpha-1}}$$

∎