2017-02-24-lec03

Operations with sets

Product: Given set S, T, their product is defined as $S \times T : \{(x, y) : x \in S, y \in T\}$. e.g. $S^k = S \times S \times \cdots \times S = \{(x_1, \ldots, x_k) : x_i \in S\}$

Example: Cartesian Plane $\mathbb{R}^2 = \{(x, y) : x, y \in \mathbb{R}$, also notated as $(\mathbb{R} \times \mathbb{R})$.

$\mathbb{R} \times \mathbb{Q} \subseteq \mathbb{R}^2$

$\mathbb{Q} \times \mathbb{R} \subseteq \mathbb{R}^2$

$\mathbb{Q}^2 \subseteq \mathbb{R} \times \mathbb{Q}$

$\mathbb{Q}^2 \subseteq \mathbb{Q} \times \mathbb{R}$

Power set: given a set S, the power set of S is defined as $2^S = \{T | T \subseteq S\}$

Remark: There is a unique set with no elements called the empty set $\emptyset$. Note that $\emptyset$ is a subset of every subset.

$S = \{Sydney, Melbourne\}$

$2^S = \{\emptyset, \{Sydney\}, \{Melbourne\}, S\}$

More generally, if S is a finite set with n elements, then 2^S has 2^n elements .

$S = \{x_1, \ldots, x_n\}, T \subseteq S$?

Either $x_1 \in T$ or $x_1 \notin T$,

$x_2 \in T$ or $x_2 \notin T, \ldots$

$x_n \in T$ or $x_n \notin T$. All together, 2^n.

Unions, intersections, difference, complement

Suppose $A, B \subseteq U, A \cup B = \{x \in U | x \in A$ or $x \in B\}$.

$A \cap B = \{x \in U | x \in A$ and $x \in B\}$.

$A - B = \{x \in U | x \in A$ and $x \notin B\}$.

$A^C = \{x \in U | x \notin A\}$.

Example:

$E = \{2k : k \in \mathbb{Z}\}$ even integer.

$O = \{2k + 1 : k \in \mathbb{Z}\}$ odd integer.

$E \cup O = \mathbb{Z}$

$E \cap O = \emptyset$

$E^C = O$

$O^C = E$

$E - O = E$

Functions: Let A, B be sets, a function $f : A \to B$ assigns to each $a \in A$ an element $f(a) \in B$. We call A the domain of f, B the target of f.

$\forall S \subseteq A$, can consider $f(S) := \{f(a) : a \in S\} \subseteq B$

We call $f(A)$ the image of f, or the range of f.

Example: Let $[n] = \{1, 2, \dots, n\}$. How many distinct functions from $[n]$ to $[n]$?

n choices for $f(1)$, n choices for $f(2)$, $\cdots$, n choices for $f(n)$.

n^n possible functions

The **graph of a function** $f : A \to B$ is the subset $\{(a, f(a)) : a \in A\} \subseteq A \times B$. In fact, f is completely determined by its graph, i.e., I could equivalently define a function $f : A \to B$ to be a subset $S \subseteq A \times B$ such that $\forall a \in A, \exists$ a unique $b \in B$ such that $(a, b) \in S$.

Reading: Chapter 1 (skip quadratic formula and arithmetic/geometric inequality) sets, functions...