May 30

P29 #5 Use Monotone Seq. Thm

Proof: ① Induction

Base case $x_1 = \sqrt{2} < 2$

Inductive step $x_{k+1} = \sqrt{2 + x_k} < \sqrt{2+2} = 2$

So $\underline{x_k < 2}$

② To show $x_k < x_{k+1}$

$x_{k+1} = \sqrt{x_k + 2}$

$x_{k+1}^2 = x_k + 2 > 2x_k > x_k^2$

$x_{k+1} > x_k$

Then use Monotone Seq. Thm.

$\lim_{k\to\infty} x_{k+1} = \lim_{k\to\infty} \sqrt{2 + x_k}$

$\Downarrow \quad\quad \Downarrow$

$a \quad = \quad \sqrt{2+a}$

using the fact that $f(x) = \sqrt{2+x}$ is cont. $f(x) \to f(a)$

$\therefore a = 2$ or -1

since not this and $x_1 = \sqrt{2}$

Hence $a = 2$ $\blacksquare$

P33. #2

(a) give an e.g. of a bounded set $S \subset \mathbb{R} \setminus \{0\}$ and a real-valued function f that is defined and continuous on $\mathbb{R} \setminus \{0\}$ s.t. $f(S)$ is not bounded.

(b). However, if $f\ \mathbb{R}^n \to \mathbb{R}^m$ is cont. everywhere, $S \subset \mathbb{R}^n$ is bounded then $f(S)$ is bounded.

$f(x) = \frac{1}{x}$, $S = (0, 1]$

$f(S) = [1, \infty)$

P38 #3

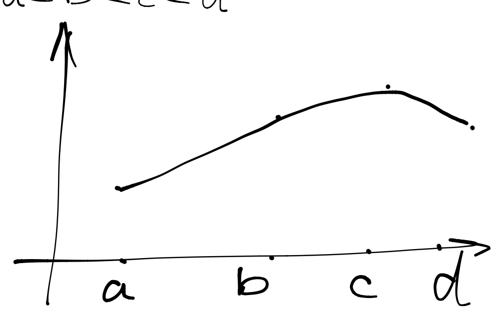
Sps I is an interval . $f : I \to \mathbb{R}$ continuous & 1-1. Show f must be monotone on I.

Use IVT as a contraposition.

Proof: Sps not $a < b$, $c < d$

$f(a) < f(b) \quad f(c) > f(d)$

$a < b < c < d$



Case 1 $f(c) > f(b)$

IVT $\exists\ d > \max(f(b), f(d))$

s.t. d is achieved by a point x_1 in (b, c).

by a point x_2 in (c, d)

$f(x_1) = f(x_2)$

Similarly - Case 2

$x_1 \cdots (a,b)$

$x_2, \cdots (b,c)$

$f(x_1) = f(x_2)$

both $\Rightarrow\!\!\times\!\!\Leftarrow$.

$\blacksquare$