MAT 237, Quiz 7

NAME: **STUDENT ID NUMBER :**

Check your tutorial:	○ TUT5101 TA: Boris	○ TUT5102 TA: James	○ TUT5103 TA: Nan

Part A: (2 marks) Present the definition of $\int_C F_1 dx + F_2 dy + F_3 dz$, where $\mathbf{F} = (F_1, F_2, F_3)$ is a vector field on $\mathbb{R}^3$ and C is a curve in $\mathbb{R}^3$.

$= \int_C (F_1, F_2, F_3) \cdot (dx, dy, dz)$

(1) $= \int_C \vec{F} \cdot d\vec{x}$

(1) $= \int_a^b \vec{F}(\vec{g}(t)) \cdot \vec{g}'(t)\,dt$, where g is a C' parametrization of C.

Part B: (3 marks) Let C be the unit circle centered at the origin. Calculate the line integral $\int_C y dx + x dy$.

(1) C is $\vec{g}(t) = (\cos t, \sin t)$, $0 \le t \le 2\pi$

(1) $x = \cos t$, $y = \sin t$. $dx = -\sin t\,dt$, $dy = \cos t\,dt$

(1) $\int_C y\,dx + x\,dy = \int_0^{2\pi} -\cos t \sin t + \sin t \cos t\,dt = 0$

Part C: (5 marks) Prove that for a vector field in $\mathbb{R}^n$, $|\int_a^b \mathbf{F} dt| \le \int_a^b |\mathbf{F}| dt$.

Let $\vec{u}$ be a unit vector in the direction of $\int_a^b \vec{F}\,dt$ (1)

Then

$\| \int_a^b \vec{F}\,dt \| \overset{(1)}{=} | \int_a^b \vec{F}\,dt \cdot \vec{u} | = | \int_a^b \vec{F} \cdot \vec{u}\,dt | \overset{(1)}{\le} \int_a^b |\vec{F} \cdot \vec{u}|\,dt$

norm

absolute value

$\le \int_a^b |\vec{F}| \cdot |u|\,dt = \int_a^b |\vec{F}|\,dt.$

(1) Cauchy ineqn

(1)