$3.3 - examples
$3.2 - complementary slackness

Eg. Consider the primal problem:

Maximize z=26x1-19x2-76x3+7x4 s.t.

2x1-4x2+78x3+2x4<=12, 5x1-3x2-13x3+x4<=4, x1-2x2+34x3+x4<=5,

x1, x2, x3, x4 >=0

[Sltn]

Let x5, x6, x7 be the slacks for the 1st, 2nd, 3rd constraints.

Suppose a later tableau has basic variables x5, x4, x2 (in that order),

we'll reconstruct the later tableau, and if it is optimal, solve the dual problem.

Let A= $\begin{array}{c} \begin{matrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 \end{matrix} \\ \begin{pmatrix} 2 & -4 & 78 & 2 & 1 & 0 & 0 \\ 5 & -3 & -13 & 1 & 0 & 1 & 0 \\ 1 & -2 & 34 & 1 & 0 & 0 & 1 \end{pmatrix} \end{array}$, b= $\begin{pmatrix} 12 \\ 4 \\ 5 \end{pmatrix}$

$$C^T = (26\ \ -19\ \ 76\ \ -7\ \ 0\ \ 0\ \ 0)$$

The m the later tableau, the x5, x4, x2 columns of A, $\begin{array}{c} \begin{matrix} x_5 & x_4 & x_2 \end{matrix} \\ \begin{pmatrix} 1 & 2 & -4 \\ 0 & 1 & -3 \\ 0 & 1 & -2 \end{pmatrix} \end{array}$

Are replaced by $\begin{array}{c} \begin{matrix} x_5 & x_4 & x_2 \end{matrix} \\ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \end{array}$

Let B = $\begin{pmatrix} 1 & 2 & -4 \\ 0 & 1 & -3 \\ 0 & 1 & -2 \end{pmatrix}$. then $B^{-1}A$ will equal the LHS of the constraint of the later tableau.

(side calculation (invert B)

$$\Big(\underbrace{\begin{matrix} 1 & 2 & -4 \\ 0 & 1 & -3 \\ 0 & 1 & -2 \end{matrix}}_{B} \Big| \begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}\Big) \simeq \left(\begin{matrix} 1 & 0 & 2 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{matrix} \middle| \begin{matrix} 1 & -2 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{matrix}\right) \simeq \Big(\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix} \Big| \underbrace{\begin{matrix} 1 & 0 & -2 \\ 0 & -2 & 3 \\ 0 & -1 & 1 \end{matrix}}_{B^{-1}}\Big)$$

The constraint part of the later tableau is $\Big(\overset{x_1 x_2 x_3 x_4 x_5 x_6 x_7}{B^{-1}A} \,\Big|\, B^{-1}b\Big)$

$$= \begin{array}{c|c} & \begin{matrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 \end{matrix} \\ \begin{matrix} x_5 \\ x_4 \\ x_2 \end{matrix} & \left(\begin{matrix} 0 & 0 & 10 & 0 \\ -7 & 0 & 128 & 1 \\ -4 & 1 & 47 & 0 \end{matrix}\ \underbrace{\begin{matrix} 1 & 0 & -2 \\ 0 & -2 & 3 \\ 0 & -1 & 1 \end{matrix}}_{B^{-1}} \middle| \underbrace{\begin{matrix} 2 \\ 7 \\ 1 \end{matrix}}_{B^{-1}b}\right) \end{array}.$$

Let $C_B^T = \underbrace{(0\ \ 7\ \ -19)}_{x_5, x_4, \text{ and } x_2 \text{ components of } c^T}$

Now let $W_B^T = C_B^T B^{-1} = (0\ \ 7\ \ -19)\begin{pmatrix} 1 & 0 & -2 \\ 0 & -2 & 3 \\ 0 & -1 & 1 \end{pmatrix} = (0\ \ 5\ \ 2)$

The objective row of the later tableau is:

$$ x_2 \quad \cancel{x_4}\, x_5$$

$$W_B^T A - c^T = (27\ -19\ 3\ 7\ \underbrace{0\ 5\ 2}_{W_B^T}) - (26\ -19\ -76\ 7\ 0\ 0\ 0) = (1\ 0\ 79\ 0\ 0\ 5\ 2)$$

and the later tableau is optimal.

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For the dual problem, let w1, w2, w3 be the dual variables corresponding to the 1st, 2nd, 3rd primal constraints.
An optimal dual solution is (w1 w2 w3) = (0 5 2)
The optimal objective value is $c_B^T (B^{-1} b) = (0\ 7\ -19) \begin{pmatrix} 2 \\ 7 \\ 1 \end{pmatrix} = (30)$
It is also equal to $(c_B^T B^{-1}) b = \underbrace{W_B^T b = b^T w}_{1 \times 1 \text{ matrix}} = (12\ 4\ 5) \begin{pmatrix} 0 \\ 5 \\ 2 \end{pmatrix} = (30)$