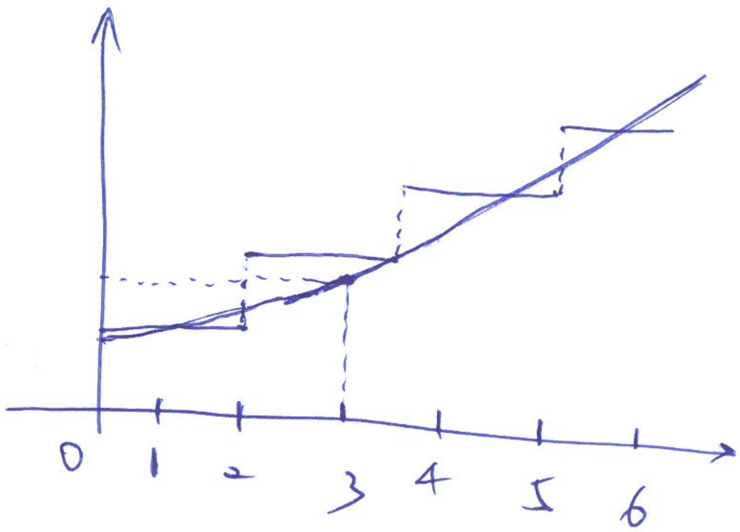
Lecture week 9



week 9 and week 10

crude mortality $\xrightarrow{\text{smoothing method.}}$ smoothed curve (rate)

we will talk more about how to calculate crude rates in week 12

standard tables

the rates you have seen in week 1~3

comparing these rates (week 9)

statistical tests

(e.g. if the standard table is describing the experience)

Chi-Square test

$$\theta_x \sim Bin(E_x \cdot q_x)$$

$$E(\theta_x) = E_x q_x$$

$$Var(\theta_x) = E_x q_x \underbrace{(1-q_x)}_{\simeq 1 \text{ when } q_x \text{ is small}}$$

$$\approx E_x q_x$$

Normal Approximation (CLT)

Observed Death : θ_x

Expected Death : $E_x \cdot q_x$

$$Z_x = \frac{\theta_x - E_x q_x}{\sqrt{E_x q_x}} \sim N(0,1)$$

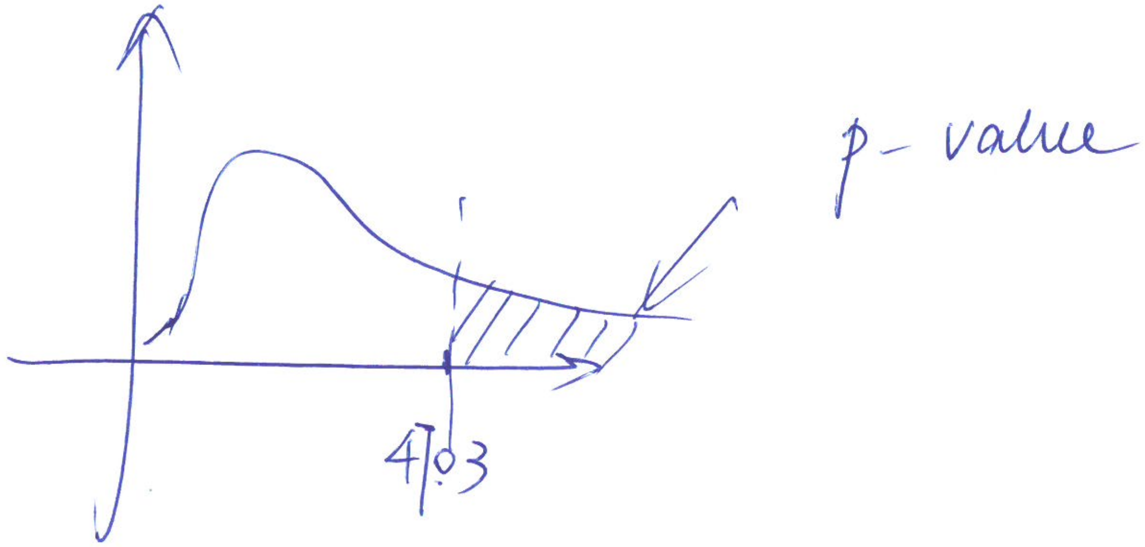
$$Z_i \sim N(0,1)$$

$$\chi_n^2 = \sum_{i=1}^{n} Z_i^2 \sim \chi_n^2$$

$$\underbrace{Z_{20}^2 + Z_{21}^2 + \cdots Z_{58}^2}_{\chi^2 \; df = n} \leftarrow n \text{ age groups}$$

* Graduation

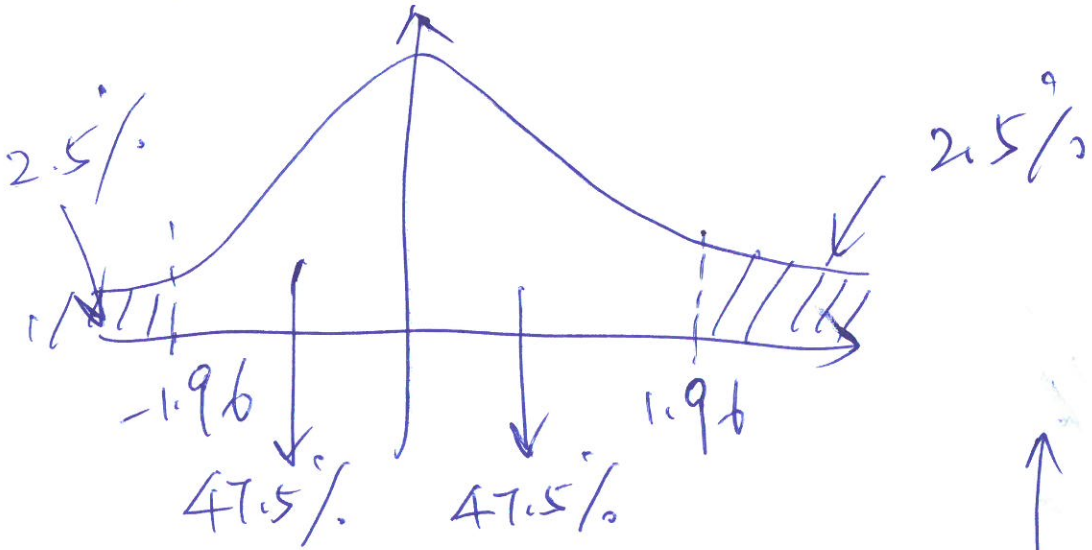
$df = n - p$ p: no. of parameter



standard deviation test

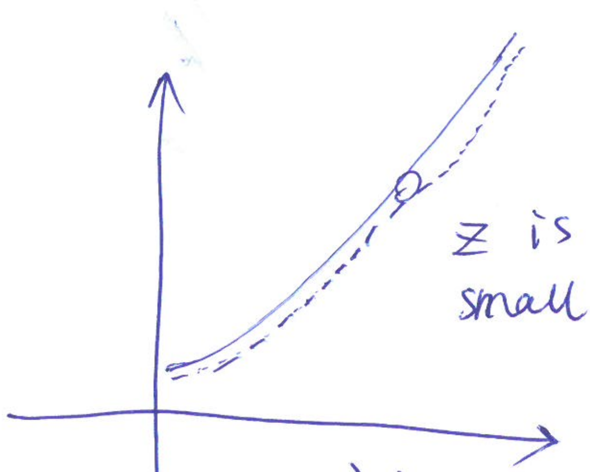
$Z_x \sim N(0,1)$

$Z_{20}, Z_{21} \cdots Z_{58} \sim N(0,1)$



$39 \times 0.025 = 0.975$

$39 \times 0.475 = 18.525$

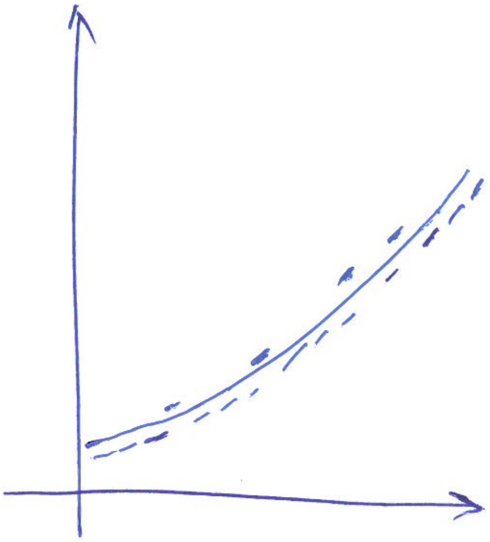


χ^2 is small

but Z_x do not follow a normal

(doesn't work) × Chi-square test

✓ standard deviation test

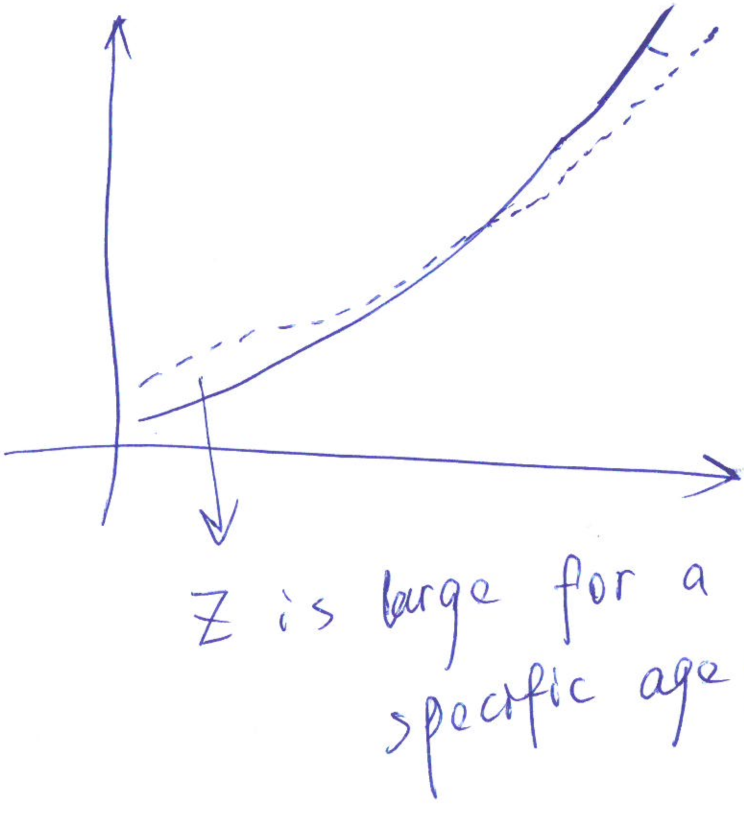


sign test

No. of positive signs

$\sim Bin(N, 0.5)$

$\Rightarrow \sim N(0.5N, 0.25N)$



cumulative devation test.

$$\frac{\sum_a^b (\theta_x - E_x q_x)}{\sqrt{\sum_a^b (E_x q_x p_x)}}$$

Z is large for a specific age group

is this is large, reject null

Runs Test

Definition of "run"

No. of grouping of positive deviation

1, 2, -1, -2, 3, 1

2 runs (positive)

1 negative run

0.1, 0.1, 0.2, -0.1, 0.1, 0.2, -0.1

2 positive runs

Run test → No. of positive runs is large enough?

If No. of positive runs is too small, we reject the null.

e.g. $\underbrace{0.1, 0.1, 0.2, 0.3}_{1}$, $\underbrace{-0.2, -0.5, -0.3}$

n_1 : positive deviations
number of

n_0 : number of negative deviations

m : no. of age groups

N : no. of positive runs

$$P(N=j)=?$$

① $\binom{m}{n_1}$: no. of ways to pick n_1 no. of positive deviations

② : $n_1 \rightarrow j$ groups $= \binom{n_1-1}{j-1}$

e.g. 1 2 | 3 4 $\quad n_1=4$, $j=2$

↑ ↑ ↑

③ : Insert j groups into n_0 numbers

↓ -1 ↓ -2 ↓ -3 ↓

e.g. $j=2$, $n_1=4$, $n_0=3$, $\binom{n_0+1}{j}$

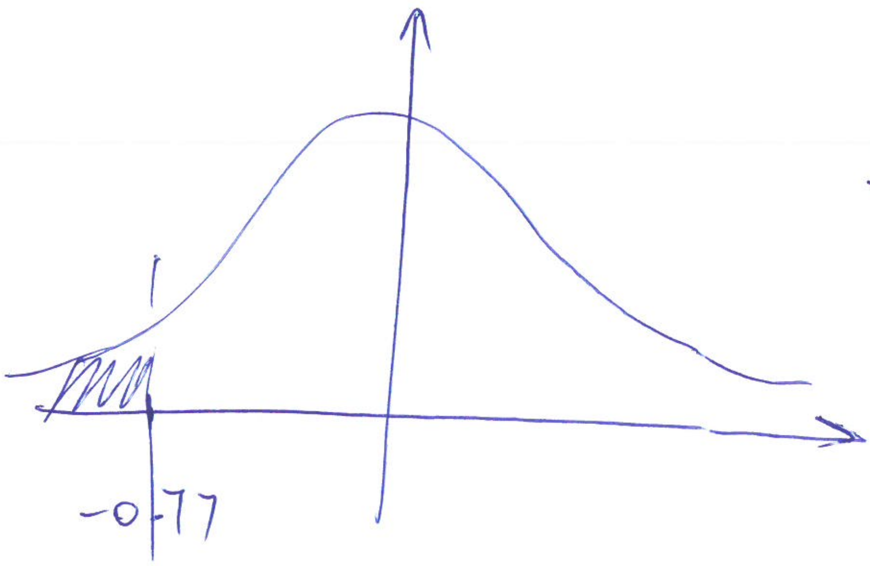
$$\Rightarrow \quad P(N=j) = \frac{\binom{n_1-1}{j-1}\binom{n_0+1}{j}}{\binom{m}{n_1}}$$

$N \sim$ hypergeometric distribution

$n_1 + n_0 = m$

$$E(N) = \frac{n_1(n_0+1)}{n_0+n_1}$$

$$var(N) = \frac{(n_0 n_1)^2}{(n_0+n_1)^3}$$



$$\frac{N - E(N)}{\sqrt{Var(N)}} \sim \text{Normal}$$

P-value $= P(N < t.s)$

χ^2 - test χ^2 → one-side

cum-dev Z → separate groups (two-side)

std. Dev χ^2 → one-side

sign test Z → two-side

runs test Z → one-side