Name:________________________ **Student number:**________________________

Term Exam: Tuesday October 19

i) The exam consists of 6 pages. All questions must be answered **on the question sheet itself.** You may use the scratch paper but it is not to be handed in.

ii) This is a closed-book exam with no materials allowed except the exam paper, the scratch paper, and your writing utensil.

iii) Duration: 1 hour and 50 minutes.

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Question 1 (30 points)**.** True or false (no justification required, grade=3(correct) + 0(incorrect))

i) Any sub-list of a linearly dependent list of vectors is also linearly dependent.

ii) Any sub-list of a linearly independent list of vectors is also linearly independent.

iii) If $(v_1, \dots, v_n)$ is a linearly dependent list of vectors, then each vector in the list is a linear combination of other vectors in the list.

iv) A vector space cannot have more than one basis.

v) Any vector space which is the span of a finite number of vectors has a finite basis.

vi) If $(v_1, \dots, v_n)$ is linearly independent, and $a_1 v_1 + \cdots + a_n v_n = 0$ for scalars $a_1, \dots, a_n$ in the field, then all the a_i must be zero.

vii) (v_1, v_2, v_3) is linearly independent if and only if v_3 is not contained in $\mathrm{span}(v_1, v_2)$.

viii) If $V = \mathrm{span}(v_1, \dots, v_n)$ and $\dim V = n$, then $(v_1, \dots, v_n)$ is a basis for V.

ix) $\mathrm{span}(v_1, \dots, v_m, u_1, \dots, u_n) = \mathrm{span}(v_1, \dots, v_m) + \mathrm{span}(u_1, \dots, u_n)$.

x) $\mathrm{span}(v_1, \dots, v_m) + \mathrm{span}(u_1, \dots, u_n)$ is a direct sum if and only if $(v_1, \dots, v_m, u_1, \dots, u_n)$ is linearly independent.

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Question 2 (20 points)**.** Short answers:

i) State the definition of linear independence of a list $(v_1, \ldots, v_n)$ of vectors.

ii) State the definition of a basis for a vector space.

iii) State the definition of the dimension of a vector space.

iv) State the definition of the sum $U_1 + U_2$ of two subspaces U_1, U_2 of V.

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Question 3 (20 points). Short answers:

i) Give the condition on $h \in \mathbb{Q}$ which ensures that the vectors $((1,2,1),(0,1,1),(1,0,h))$ are linearly independent in $\mathbb{Q}^3$.

ii) Show that $(x^2+x+2,\ \ 2x^2+3x+5,\ \ 3x^2+5x+8)$ is linearly dependent in the vector space $\mathcal{P}_2(\mathbb{R})$ of polynomials of degree ≤ 2 with real coefficients, by giving a linear relation among the three vectors.

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Question 4 (30 points)**.** Let U be a vector subspace of a finite-dimensional vector space V. Prove that U is also finite-dimensional.

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Question 5 (Bonus: 10 points)**.** How many 2-dimensional subspaces are there in $(\mathbb{F}_3)^4$?