Comp3620/Comp6320 Artificial Intelligence

Quiz 2: Logic and Constraints

April 24–27, 2018

Student Number
u6139152

1. Logic and Deduction

True or False: (1pt per correct answer)

- True or False: $(p \vee \neg q) \wedge (r \wedge (\neg p \vee s))$ is in conjunctive normal form.
- True or False: $(p \vee \neg q) \wedge (r \wedge (\neg p \vee s))$ is in disjunctive normal form.
- True or False: $(p \vee \neg q) \wedge (r \wedge (\neg p \vee s))$ is in negation normal form.

True or False: (2pts per correct answer)

- True or False: $(p \wedge q) \rightarrow r$ logically implies $(p \rightarrow r) \vee (q \rightarrow r)$.
- True or False: $\exists x(F(x) \vee G(x))$ logically implies $\exists x F(x) \vee \exists x G(x)$.
- True or False: $\forall x(F(x) \vee G(x))$ logically implies $\forall x F(x) \vee \forall x G(x)$.

2. Constraint Solving

For this question, suppose we are using backtracking search to look for solutions to a CSP, but in fact it has no solutions (though we don't know that until the search eventually returns "inconsistent").

True or False: (2pts per correct answer)

- True or False: Using pure (naïve) backtracking, the size of the search tree does not depend on variable ordering.
- True or False: Using forward checking with a fixed variable order, the size of the search tree does not depend on value ordering.

arc consistency
$O(d^2)$
d is the size of the largest domain.

n variables
only n^2 constraints
$O(n^2)$
$O(n^2d^2 + n^2d^3)$
$O(n^2d^3)$

3. Arc Consistency

(True) or False: (1pt)

X

- True or False: In the worst case, the runtime of the AC-3 algorithm is exponential in the number of decision variables.

$O(n^2 + n^2d)$
size of the square at the beginning.

Let $\gamma = (V, D, C)$ be a CSP and suppose that X and Y are two variables in V, with domains $D_X = \{1, 2, 3, 4, 5, 6, 7\}$ and $D_Y = \{2, 4, 6\}$ respectively. The constraints relating X and Y are $X < Y$ and $X + Y > 8$. Let $\gamma' = (V, D', C')$ be the network after the pairs (X, Y) and (Y, X) have been made arc consistent. Write out: (1 pt each)

$D'_X = \{ 3, 4, 5 \}$ ✓ ✓ ✓

$D'_Y = \{ 6 \}$ ✓

Let $\gamma^{ac} = (V, D^{ac}, C^{ac})$ be the constraint network after arc consistency has been enforced (i.e. to all of γ, not just to X and Y). Let n be the number of solutions to γ, and let n^{ac} be the number of solutions to γ^{ac}. Circle all of the following statements that could be true: (4 pts)

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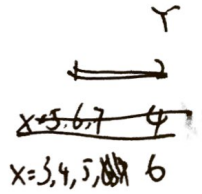
- ✓ (a) $n^{ac} = 0$ but no D_i^{ac} are empty — no solution, but no domains empty
- ✓ (b) $n^{ac} > 0$ but some D_i^{ac} is empty — × one solution, but one domain is empty
- ✗ (c) $n^{ac} > 0$ but a backtracking solver could still backtrack — × won't backtrack 2 times
- ✓ (d) $n^{ac} < n$ — one solution, but backtracking solver could backtrack.
- ✓ (e) $n^{ac} = n$ — solutions after < before
- ✓ (f) $n^{ac} > n$

$x < y$
$x + y > 8$



$X = \{3, 4, 5\}$
$Y = \{6\}$