

INTRODUCTORY MATHEMATICAL STATISTICS
(STAT2001/6039)

Tutorial 12

Problem 1

A random sample of size n is selected from a large number of items produced in a certain factory. The number of defectives X is observed.

Find the MLE (maximum likelihood estimate) of the ratio r of the number of defective to nondefective items produced in the factory.

Problem 2

Consider a random sample Y_1, \dots, Y_n from the Poisson distribution with parameter λ .

- (a) Find the method of moments and maximum likelihood estimators of λ .
- (b) Find the bias and mean square error of λ 's MLE.
- (c) Determine whether or not λ 's MLE is consistent for λ .
- (d) Suppose that the sample observations are 1, 0, 3, 1, 0, 1.

Find the MLE of the probability that the next independent observation from the same Poisson distribution will be zero.

Problem 3

Suppose that 0.31, 0.76, 0.29 and 0.97 are a random sample from the distribution with pdf

$$f(y) = (\theta + 1)y^\theta, \quad 0 < y < 1, \theta > -1.$$

Find the method of moments and maximum likelihood estimates of θ .

Problem 4

A die was rolled 600 times and 123 sixes came up.

We are interested in whether or not the die is fair.

- (a) Conduct an appropriate hypothesis test at the 5% significance level.
- (b) Compute the p -value.
- (c) Find the probability of accepting the die as fair if in fact sixes come up 105 times per 600 rolls on average.