

Course outline for 2011F MAT240 Algebra I

1 Pre-history

1.1 Sets

1.1.1 Maps between sets

1.1.2 Finite sets

2 Vector spaces

2.1 Definition of field

2.1.1 Examples of fields

2.1.2 Subfields

2.1.3 simple properties of fields (usual arithmetic!)

2.1.4 Finite fields

2.2 Definition of vector space

2.2.1 Examples of vector spaces

2.2.2 Diagrams of axioms

2.2.3 Simple properties coming from axioms

2.2.4 Definition of Subspaces

2.2.5 Two ways to get subspaces: span and constraints

2.2.6 Redundancy, i.e. linear (in)dependence

2.3 Dimension and finite-dimensional vector spaces

2.3.1 Span and linear dependence

2.3.2 Definition of basis and dimension

2.3.3 Coordinates, using uniqueness from linear independence

2.3.4 Construction of a basis in a finite dimensional space

2.3.5 Examples of bases

2.3.6 Simple properties of dimension

2.3.7 Algorithm for determining linear independence of vectors in \mathbb{F}^n

3 Linear maps

3.1 Definition of linear map

3.2 Examples of linear maps

3.3 The vector space of linear maps

3.4 Composition of linear maps

3.5 Algebra of linear operators

3.6 Isomorphism

3.6.1 Classification of finite dimensional vector spaces

3.7 Null space and Image of a linear map

3.7.1 Theorem about $\dim U = \dim \mathrm{null} T + \dim \mathrm{ran} T$

3.8 Linear maps vs. matrices

3.8.1 Isomorphism from $L(V,W)$ to $M(n,k,\mathbb{F})$.

3.8.2 Matrix multiplication

3.8.3 Change of basis: Writing the identity operator with respect to two bases: $[\mathrm{id}]_{\beta}^{\sim\beta}$.

3.8.4 Rank-nullity theorem as a consequence of above theorem

3.8.5 Row reduction as matrix multiplication

3.8.6 Matrix inverses and bookkeeping

3.8.7 Einstein summation convention and diagrammatic version

4 Understanding a fixed linear operator

4.1 Eigenvectors and eigenvalues

4.2 Invariant subspaces

4.3 Differential equations

4.4 Polynomials applied to a linear operator

4.4.1 Minimal polynomial

4.5 Upper-triangular matrices

4.6 Diagonal matrices

4.7 Decomposition theorem

4.8 Classification of linear operators

4.8.1 Jordan canonical form (classification theorem for linear operators over

\mathbb{C})

4.9 Characteristic polynomial

5 Miscellaneous topics covered in exercises

5.1 Stochastic matrices and the Markov process

5.2 The determinant of a linear operator

5.2.1 Relation to Euclidean volume

5.3 The dual vector space

5.3.1 The dual of a linear transformation