UNIVERSITY OF TORONTO
Faculty of Arts and Sciences

AUGUST 2011 EXAMINATIONS

MAT244H1Y

Duration - 3 hours

No Aids Allowed

Last Name: _____

Given Name: _____

Student Number : _____

Question	Points	Score
1	10	
2	15	
3	15	
4	20	
5	20	
6	20	
Total:	100	

1. (10 points) Find the general solution of the differential equation

$$y' = \frac{3x^2}{\sqrt{x^3 + 1}y}$$

and solve the initial value problem $y(2) = 1$.

2. (15 points) Find the general solution of the equation

$$t^2y'' + 2ty' - 6y = 10t^2 - 6, \qquad t > 0.$$

3. (15 points) Find the general solution of the differential equation

$$y^{(4)} - 16y = 3t + \cos t + \sin(2t).$$

4. (20 points) Solve the system of differential equations

$$x' = 3x - 4y + 1z,$$
$$y' = 9y,$$
$$z' = -4y + 3z.$$

5. For the following equation:

$$x'' = -4x^3 + 4x$$

(a) (2 points) Reduce to a first order system with $y = x'$

(b) (5 points) Find solution in the form $H(x, y) = C$

(c) (8 points) Find critical points and classify them (i.e. specify whether they are nodes, saddles, etc. and stability)

(d) (5 points) On (x, y)-plane sketch the phase portrait

6. For the following equation:
$$x'' = -4x^3 + 4x - \alpha x$$

(a) (2 points) Reduce to a first order system with $y = x'$

(b) (12 points) Find the critical points and classify them depending on $\alpha > 0$ (note the case $\alpha = 0$ is above)

(c) (6 points) Sketch a phase portrait in the (x, y) plane for each case depending in $\alpha > 0$