Homework 1
Due by
Tuesday 1 August 2017 17:00

Assume we have $k \times k$ matrices $\mathbf{A} = (a_{ij})$ and $\mathbf{B} = (b_{ij})$ with real entries.

Question 1 [5 Points]
Show that
$$\mathrm{tr}(\mathbf{AB}) = \mathrm{tr}(\mathbf{BA}).$$

Question 2 [5 Points]
Show that
$$\mathrm{tr}(\mathbf{AA}') = \sum_{i=1}^{k} \sum_{j=1}^{k} a_{ij}^2.$$

Question 3 [5 Points]
For $\mathbf{U}$ an orthogonal $k \times k$ matrix, show that
$$\mathrm{tr}(\mathbf{A}) = \mathrm{tr}(\mathbf{U'UA}) = \mathrm{tr}(\mathbf{U'AU}).$$

Question 4 [5 Points]
Set $k = 5$ and demonstrate numerically in R that for a matrix $\mathbf{A}$, that the left-hand side of the equation is equal to the right hand side for the cases of Question 1 and Question 2. The matrix $\mathbf{A}$ can be generated with random entries or non-zero entries of your choice.

This homework is to be submitted through Wattle in digital form only as per ANU policy. If you use any references (note: this will never count against you), please clearly indicate which ones. This homework has 10% weight.