Simplex strategy (§2.1)

– to try to imagine the objective value

– to ensure the next tableau represents a feasible solution.

Tableau ① (From "A simplex Optimization")

	x_1	x_2	x_3	x_4	x_5	z	
x_3	1	5	1	0	0	0	19
x_4	1	-1	0	1	0	0	7
x_5	-1	②	0	0	1	0	2
	-3	-7	0	0	0	1	0

The tableau represents at least 3 things

① Each row represents an equation (The last row now represents the equation
$-3x_1 - 7x_2 + z = 0$ or $z = 3x_1 + 7x_2$)
The entire tableau represents a system of equations

② It represents the basic feasible solution: $x_3 = 19$, $x_4 = 7$, $x_5 = 2$, and $x_1 = 0$, $x_2 = 0$ because x_1 and x_2 are non-basic.

③ As a linear programming problem: Maximize $z = 3x_1 + 7x_2$ s.t.

$$
\begin{aligned}
x_1 + 5x_2 + x_3 \quad &= 19 \\
x_1 - x_2 \quad + x_4 &= 7 \\
-x_1 + 2x_2 \quad + x_5 &= 2 \\
x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, &x_4 \geq 0, x_5 \geq 0.
\end{aligned}
$$

A strategy to maximize z

The objective value is $3x_1 + 7x_2$. If either x_1 or x_2 were to increase, z would increase. This is because the objective row coefficient of x_1 and x_2 are negative. There are 2 possible strategies.

① Search the objective row for any negative coefficient, then increase the first variable encountered, having a negative coefficient.

② Search the entire objective row then make positive any variable having the most negative coefficient. Kolman and Beck do this. This is called the "Dantzig greediest variable criterion".

To increase x_1 or x_2 to a positive value, x_1 or x_2 must become basic. (That is, enters (into the set of basic variables).)

We will enter x_2. In this problem, a basic solution has 3 basic variables (not 4). To enter x_2 one of x_3, x_4, x_5 must ~~exist~~ exits (from the set of basic variables).

The entry of x_2 is accomplished by a Gauss–Jordan row-pivot (as in §0.2): by pivoting on the x_2 column. The existing variable labels the row which is used as the pivotal row.

An attempt to exit x_4 will cause x_2 to enter with value $\frac{7}{-1} = -7 < 0$. This is infeasible.
x_2 column, x_4 row

One exiting rule (for the basic simplex method, §2.1) is: Do not exit a variable that would cause the pivot to be negative. Pivot on positive numbers only. Exception: you may use a negative pivot if the value of the exiting (and entering) variable is 0.

If we exit x_3 (pivot on "5"), x_2 will enter with a value of $19/5$. The new value of x_5 will then be $2 - 2 \times \frac{19}{5} < 0$, again an infeasible choice.

Since we will not exit x_3 or x_4, we will exit x_5 (as indicated).