

The estimated variance model is detailed in the Analysis of Variance table (ANOVA table) ①

ANOVA table

| Source (of variability) | df | SS | MS | F | p |
|-------------------------|-------|--------------------------------------|---------------------------------------|---|---|
| Model/Regression | 1 | $SS_{Reg} = SS_{Total} - SS_{Error}$ | ... | | |
| Error/Residual | $n-2$ | $SS_{Errors} = \sum e_i^2$ | $MS_{Error} = \frac{SS_{Error}}{n-2}$ | | |
| Total | $n-1$ | $S_{yy} = SS_{Total}$ | | | |

degrees of freedom
sum of squares
mean square
(later....)
 $\hat{\sigma}^2$ est. of σ^2

this line is equivalent to the Null model $Y = \beta_0 + \epsilon$

We could calculate

$$MS_{Total} = \frac{SS_{Total}}{n-1} = s_y^2$$

this line is equivalent to the

SLR model $Y = \beta_0 + \beta_1 X + \epsilon$ $\epsilon \stackrel{iid}{\sim} N(0, \sigma^2)$

Our key estimate of the error variance σ^2 is the MS_{Error}

To calculate this:

1. find $\hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$ for all $i=1, 2, \dots, n$ (the sample)

2. find $e_i = Y_i - \hat{Y}_i$ (the residuals)

3. find $\sum e_i^2 = SS_{Errors}$ & "average" over the $df = n-2$ (for SLR)
to get $s^2 = \hat{\sigma}^2 = \frac{\sum e_i^2}{n-2}$

In our current example

| Source | df | SS | MS | F | p |
|-------------------|-----|---------|---------|--------|-----------------------|
| Regression (Year) | 1 | 10.8685 | 10.8685 | 419.53 | 2.2×10^{-16} |
| Residuals (Error) | 136 | 3.5232 | 0.0259 | | |
| Total | 137 | 14.3917 | 0.1050 | | |

Type I & Type II errors

| | | Null Hypothesis | |
|-----------------|---------------------|--------------------------------|---------------------------------|
| | | H_0 valid | H_0 not valid |
| Outcome of test | Do not reject H_0 | ✓ | False negative Type II error |
| | Reject H_0 | False positive Type I error | ✓ |

$$P(\text{Type I error}) = \alpha \text{ (significance level)}$$

$1 - \alpha$ is called the confidence level

$$P(\text{Type II error}) = \beta$$

$1 - \beta$ is called the power

A powerful test is one in which we are more likely to correctly reject a false null hypothesis

NB: about the only way we can reduce both α & β at the same time is to increase the sample size

NB (2): a one-tailed hypothesis test is more powerful than the equivalent two-tailed test (for the same sample size)