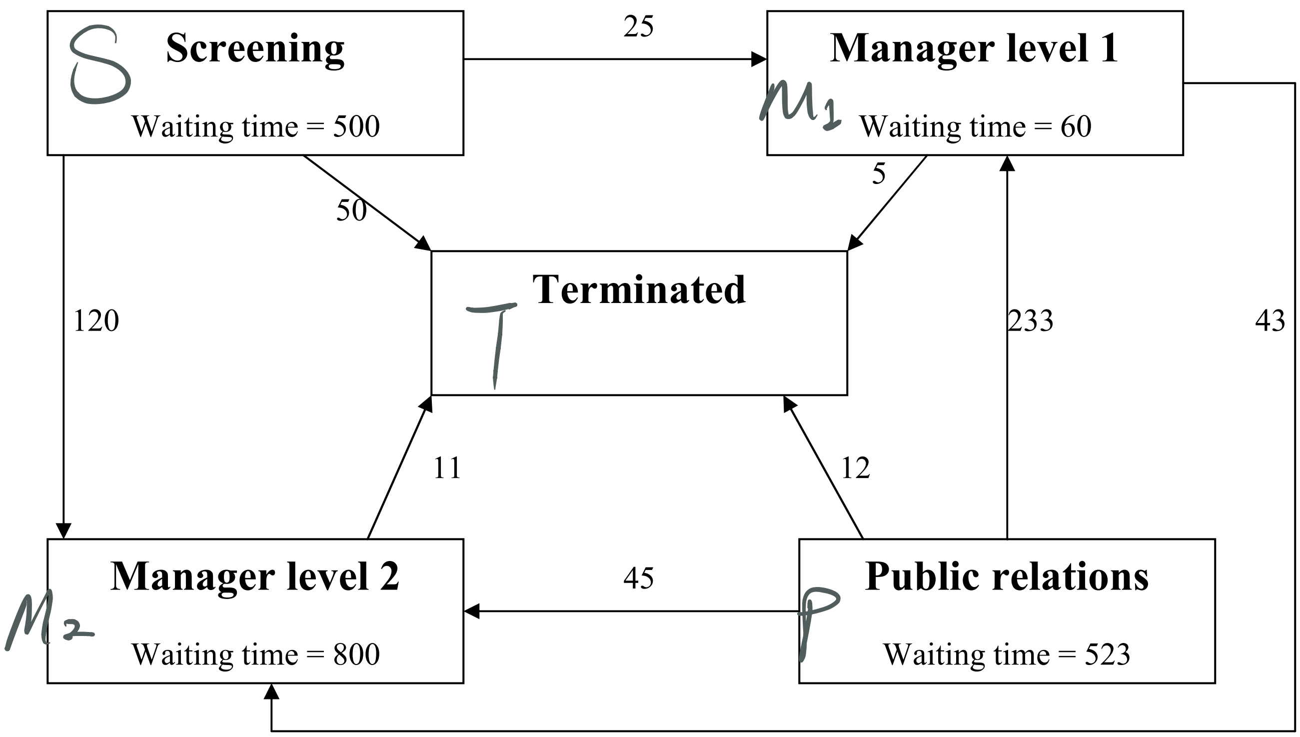
Question One

A multi-state Markov model has been used to describe the transition of complaints sent to a particular company's customer complaints section. Complaints sent to this section of the company are classified into 5 categories depicted below.

unit: years



The values in the diagram above depict the observed waiting times and number of transitions. For example, the waiting time in state "Manager level 1" was ~~100~~ 60 years and the number of transitions from state "Manager level 1" to "Terminated" was 5.

Provide an approximate 95% confidence interval for the probability that a complaint currently in the "screening" state will remain in that state for the next three months.

0.25 years

95% CI for

$$ {}_{0.25}P_x^{\overline{SS}} = e^{-\int_0^{0.25} \sum_{r \neq S} \mu^{Sr}\, dr} $$

multi-state, constant.

$$ {}_{0.25}\hat{p}_x^{\overline{SS}} = e^{-0.25(\hat{\mu}^{SM_1} + \hat{\mu}^{SM_2} + \hat{\mu}^{ST})} $$

let this term be Y.

$$ = e^{-0.25\left(\frac{25+120+50}{500}\right)} $$

$$ = 0.90710234l6\ldots $$

Var?, use δ-method.

sps iid. 1

$$ Var(Y) = Var(\hat{\mu}^{SM_1} + \hat{\mu}^{SM_2} + \hat{\mu}^{ST}) $$

Y is Normal.

$$ = Var(\hat{\mu}^{SM_1}) + Var(\hat{\mu}^{SM_2}) + Var(\hat{\mu}^{ST}) $$

$$ = \frac{25+120+50}{500^2} = 0.00078. $$

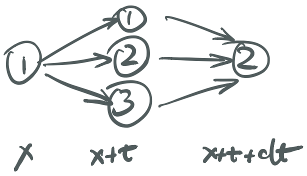
approaches
① 95% CI for Y → exp. (easier)
② Delta → 95% CI.

$$ SD = \sqrt{Var({}_{0.25}\hat{p}_x^{\overline{SS}})} \approx \sqrt{(-0.25e^{-0.25Y})^2 \cdot V(Y)} = \sqrt{0.25^2 (0.9071023416)^2 \cdot 0.00078} $$

delta method

$$ = 0.0063\ldots $$

95% CI : $\hat{p} \pm 1.96\,SD$ (note: (0,1) range!)

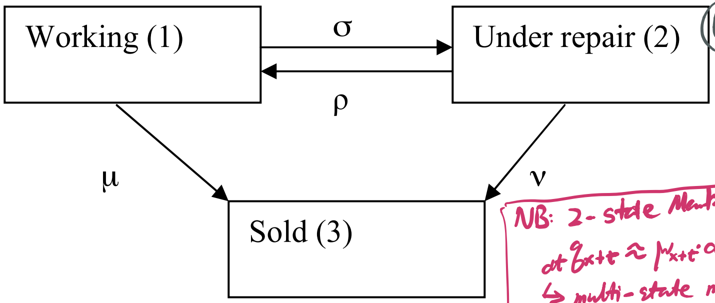


Question Two (prove the forward equation directly)

The following diagram represents the possible states of a newly purchased car. "Working" means that the car is currently operating normally; "Under repair" means that the car is undergoing maintenance to fix a problems; and "Sold' means the car has been sold.

1 3 dt 2

x x+t x+t+dt



$${}_{t+dt}P_x^{12}$$

(law of total prob)

$$= {}_tP_x^{11}\cdot {}_{dt}P_{x+t}^{12} + {}_tP_x^{12}\cdot {}_{dt}P_{x+t}^{22} + \cancel{{}_tP_x^{13}\, {}_{dt}P_{x+t}^{32}}$$

$$= {}_tP_x^{11}\cdot {}_{dt}P_{x+t}^{12} + {}_tP_x^{12}(1-{}_{dt}P_{x+t}^{21}-{}_{dt}P_{x+t}^{23})$$

impossible 3→2 in this case

NB: 2-state Markov

${}_{dt}q_{x+t} \approx \mu_{x+t}\,dt$

↳ multi-state markov

${}_{dt}P_{x+t}^{gr} \approx \mu_{x+t}^{gr}\cdot dt$

$$\approx {}_tP_x^{11}\cdot \sigma\cdot dt + {}_tP_x^{12} - {}_tP_x^{12}(\rho\cdot dt + \nu\cdot dt)$$

$$\lim_{dt\to 0}\frac{{}_{t+dt}P_x^{12} - {}_tP_x^{12}}{dt} = {}_tP_x^{11}\cdot\sigma - {}_tP_x^{12}(\rho+\nu)$$

$$= \frac{d}{dt}{}_tP_x^{12}$$

For this particular model show that: $\frac{d}{dt}\,{}_tp_x^{12} = \sigma\,{}_tp_x^{11} - (\rho+\nu)\,{}_tp_x^{12}$.

Question Three similar method.

g g g

x x+t x+t+dt

Derive the following two results for the multi-state Markov model.

1. $\frac{\partial}{\partial t}\,{}_tp_x^{\overline{gg}} = -\,{}_tp_x^{\overline{gg}}\sum_{r\neq g}\mu_{x+t}^{gr}$
2. $p_x^{\overline{gg}} = \exp\left(-\int_0^t \sum_{r\neq g}\mu_{x+s}^{gr}\,ds\right)$.

$${}_{t+dt}P_x^{\overline{gg}} = {}_tP_x^{\overline{gg}}\cdot {}_{dt}P_x^{\overline{gg}}$$

$$= {}_tP_x^{\overline{gg}}\cdot\Big(1-\sum_{r\neq g}{}_{dt}P_{x+t}^{gr}\Big)$$

$$= {}_tP_x^{\overline{gg}}\Big(1-\sum_{r\neq g}\mu_{x+t}^{gr}\cdot dt\Big)$$

$${}_{t+dt}P_x^{\overline{gg}} - {}_tP_x^{\overline{gg}} = -\,{}_tP_x^{\overline{gg}}\sum \mu_{x+t}^{gr}\cdot dt$$

$$\lim_{dt\to 0}\frac{{}_{t+dt}P_x^{\overline{gg}} - {}_tP_x^{\overline{gg}}}{dt} = -\,{}_tP_x^{\overline{gg}}\sum_{g\neq r}\mu_{x+t}^{gr}$$

$$\frac{d}{dt}\,{}_tP_x^{\overline{gg}} = -\,{}_tP_x^{\overline{gg}}\sum_{g\neq r}\mu_{x+t}^{gr}$$

✓

Use part 1 to prove part 2.

LHS RHS divided ${}_tP_x^{\overline{gg}}$ => ln(·)'s derivative.

integration