Quiz and Solutions

2.2 Suppose that A and B are two events. Write expressions involving unions, intersections, and complements that describe the following:

- **a** Both events occur.
- **b** At least one occurs.
- **c** Neither occurs.
- **d** Exactly one occurs.

2.2 **a.** $A \cap B$ **b.** $A \cup B$ **c.** $\overline{A \cup B}$ **d.** $(A \cap \overline{B}) \cup (\overline{A} \cap B)$

2.42 A personnel director for a corporation has hired ten new engineers. If three (distinctly different) positions are open at a Cleveland plant, in how many ways can she fill the positions?

2.42 There are three *different* positions to fill using ten engineers.
Then, there are P_3^{10} = 10!/7! = 10*9*8 = 720 different ways to fill the positions.

2.76 A survey of consumers in a particular community showed that 10% were dissatisfied with plumbing jobs done in their homes. Half the complaints dealt with plumber A, who does 40% of the plumbing jobs in the town. Find the probability that a consumer will obtain

- **a** an unsatisfactory plumbing job, given that the plumber was A.
- **b** a satisfactory plumbing job, given that the plumber was A.

2.76 Define the events: U: job is unsatisfactory A: plumber A does the job
a. $P(U|A) = P(A \cap U)/P(A) = P(A|U)P(U)/P(A) = .5 * .1/.4 = 0.125$
b. From part a. above, $1 - P(U|A) = 0.875$.

2.124 A population of voters contains 40% Republicans and 60% Democrats. It is reported that 30% of the Republicans and 70% of the Democrats favor an election issue. A person chosen at random from this population is found to favor the issue in question. Find the conditional probability that this person is a Democrat.

2.124 Define the events for the voter: D: democrat R: republican F: favors issue

$$P(D\,|\,F) = \frac{P(F\,|\,D)P(D)}{P(F\,|\,D)P(D) + P(F\,|\,R)P(R)} = \frac{.7(.6)}{.7(.6) + .3(.4)} = 7/9$$

2.148 A bin contains three components from supplier A, four from supplier B, and five from supplier C. If four of the components are randomly selected for testing, what is the probability that each supplier would have at least one component tested?

2.148 Note that $\binom{12}{4} = 495$. P(each supplier has at least one component tested) is given by

$$\frac{\binom{3}{2}\binom{4}{1}\binom{5}{1}+\binom{3}{1}\binom{4}{2}\binom{5}{1}+\binom{3}{1}\binom{4}{1}\binom{5}{2}}{495} = 270/475 = 0.545.$$

2.154 **a** A drawer contains $n = 5$ different and distinguishable pairs of socks (a total of ten socks). If a person (perhaps in the dark) randomly selects four socks, what is the probability that there is no matching pair in the sample?

***b** A drawer contains n different and distinguishable pairs of socks (a total of $2n$ socks). A person randomly selects $2r$ of the socks, where $2r < n$. In terms of n and r, what is the probability that there is no matching pair in the sample?

2.154 Let Y = the number of pairs chosen. Then, the possible values are 0, 1, and 2.

a. There are $\binom{10}{4}$ = 210 ways to choose 4 socks from 10 and there are $\binom{5}{4}2^4$ = 80 ways to pick 4 non-matching socks. So, $P(Y = 0) = 80/210$.

b. Generalizing the above, the probability is $\binom{n}{2r}2^{2r}\Big/\binom{2n}{2r}$.

***2.180** Suppose that the streets of a city are laid out in a grid with streets running north–south and east–west. Consider the following scheme for patrolling an area of 16 blocks by 16 blocks. An officer commences walking at the intersection in the center of the area. At the corner of each block the officer randomly elects to go north, south, east, or west. What is the probability that the officer will

a reach the boundary of the patrol area after walking the first 8 blocks?

b return to the starting point after walking exactly 4 blocks?

2.180 **a.** If the patrolman starts in the center of the 16x16 square grid, there are 4^8 possible paths to take. Only four of these will result in reaching the boundary. Since all possible paths are equally likely, the probability is $4/4^8 = 1/4^7$.

b. Assume the patrolman begins by walking north. There are nine possible paths that will bring him back to the starting point: *NNSS*, *NSNS*, *NSSN*, *NESW*, *NWSE*, *NWES*, *NEWS*, *NSEW*, *NSWE*. By symmetry, there are nine possible paths for each of north, south, east, and west as the starting direction. Thus, there are 36 paths in total that result in returning to the starting point. So, the probability is $36/4^8 = 9/4^7$.