INTRODUCTORY MATHEMATICAL STATISTICS (STAT2001/6039)

Tutorial 9

Problem 1

Consider two random variables X and Y with means 10 and –5, respectively, and variances 16 and 4, respectively.

Find the mean and variance of $U = X - 3Y$ if:

(a) X and Y are independent

(b) the correlation between X and Y is 0.65.

Problem 2

Consider two continuous random variables X and Y whose joint pdf is

$$f(x,y) = 2,\ 0 < x < 1,\ 0 < y < 1,\ 0 < x + y < 1.$$

Find:

(a) the covariance and correlation between X and Y

(b) the expected value and variance of $U = X - 2Y$.

Problem 3

Suppose that X and Y have joint pdf

$$f(x,y) = e^{-x}, \ 0 < y < x .$$

Find: **(a)** $P(X + Y > 4)$

(b) $P(X + Y > 4 \mid Y = 2)$

(c) $E(Ye^{-X})$

(d) $E(Ye^{-X} \mid Y = 2)$.

Problem 4

A \$1 coin is going to be tossed twice.

A \$2 coin will then be tossed the same number of times
as the number of heads that come up on the \$1 coin.

Let X be the number of heads that come up on the \$1 coin,
and let Y be the number of heads that come up on the \$2 coin.

(a) Create a table which shows the joint probability distribution of X and Y.

Then explicitly write down the joint pdf of X and Y as a formula.

(b) Find the mean and variance of Y.

(c) Find the correlation between X and Y.

(d) Suppose that the \$1 coin is going to be tossed 20 times, rather than twice.

Find the probability that no heads will come up on the \$2 coin.