# Introduction to Bayesian Data Analysis

## Tutorial 1

(1) Redo example I from class (estimating the probability of a rare event) but try different prior distributions and assess how your posterior inference changes. For example, you might try a beta(40,400). Also try a beta(2,2) prior. Interpret your findings.

(2) Conditional probability: suppose that if $\theta = 1$, the $y$ has a normal distribution with mean 1 and standard deviation $\sigma$, and if $\theta = 2$, then y has a normal distribution with mean 2 and standard deviation $\sigma$. Also, suppose $\Pr(\theta = 1)$=0.5 and $\Pr(\theta = 2)$=0.5

(a) For $\sigma = 2$, write the formula for the marginal probability density for $y$ and sketch it.

(b) What is $\Pr(\theta = 1|y = 1)$, again supposing $\sigma = 2$?

(c) Describe how the posterior density $\Pr(\theta = 1|y = 1)$ changes in shape as $\sigma$ is increased and as it is decreased

(3) The following problem is loosely based on the television game show *Let's Make a Deal*. At the end of the show, a contestant is asked to choose one of three large boxes, where one box contains a fabulous prize and the other two boxes contain lesser prizes. After the contestant chooses a box, Monty Hall, the host of the show, opens one of the two boxes containing smaller prizes. (In order to keep the conclusion suspenseful, Monty does not open the box selected by the contestant). Monty offers the contestant the opportunity to switch from the chosen box to the remaining unopened box. Should the contestant switch or stay with the original choice? Calculate the probability the contestant wins under each strategy. This is an exercise in being clear about the information that should be conditioned on when constructing a probability judgement. See Selvin (1975) and Morgan et al. (1991) for further discussion of this problem.

(4) An airline company uses past data to estimate the probability that a passenger who is scheduled to take a particular flight, fails to show up. Let $p$ denote this probability. You are given the following prior distribution on $p$:

| $p$ | 0 | 0.025 | 0.05 | 0.075 | 0.10 |
|---|---|---|---|---|---|
| $g(p)$ | 0.80 | 0.10 | 0.05 | 0.035 | 0.015 |

(a) Suppose the for the next flight, 60 tickets have been sold, but only 55 passengers turned up. What is the posterior probability that all passengers show up? What is the posterior probability that 10% of passengers don't show up?

(b) Suppose for the next ten flights, 60 tickets have been sold on each flight, and the number of passengers who actually turned up on each flight is shown in the table below:

| Flight | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| No. passengers on flight | 58 | 55 | 59 | 54 | 56 | 57 | 57 | 50 | 52 | 60 |

What is the posterior mode?

(c) Suppose the airline does not update its assumptions on boarding rates of sold tickets. Discuss the implications on profitability of the company in light of your Bayesian analysis.