PROBLEM-SOLVING AND PROOFS: ASSIGNMENT 7
DUE FRIDAY, APRIL 28, 4PM.

Problems to be handed in. Solve three of the following four problems.

(1) (a) Reduce 2^{100} modulo 13.
    (b) Reduce 11^{1000} modulo 8.

(2) Let $a, b, c \in \mathbb{Z}$, and suppose that 5 divides $a^2 + b^2 + c^2$. Prove that 5 divides at least one of a, b, or c.

(3) Prove that every year (including leap years) has at least one Friday the 13th. What is the maximum number of Friday the 13ths in a year?

(4) *Divisibility by 11*
    (a) Formulate and prove a criterion for an integer n to be divisible by 11 in terms of the digits of n.
    (b) A number is *palindromic* if it reads the same backwards and forwards, e.g. 183381. Prove that a palindromic number with an even number of digits is always divisible by 11.