Lecture 22

$\Sigma = \{(s_0 s_1 s_2 \cdots),\ s_i \in \{0,1\}\}$
$d[s,t] = \sum_{i=0}^{\infty} \frac{|s_i - k_i|}{2^i}$

Proximity Thm: $\begin{cases} s_i = t_i \text{ for } i=0,\dots,n \Rightarrow d[s,t] \le \frac{1}{2^n} \\ d[s,t] < \frac{1}{2^n} \Rightarrow s_i = t_i \text{ for } i=0,\dots,n \end{cases}$

Calculus Review
$\lim_{x \to a} f(x) = L \Leftrightarrow \forall \varepsilon > 0, \exists \delta > 0$ s.t. $0 < |x-a| < \delta \Rightarrow |f(x) - L| < \varepsilon$

f cts. at $a \Leftrightarrow \lim_{x \to a} f(x) = f(a)$

Def'n:
① A set $A \subseteq \mathbb{R}$ is open if it is the union of open intervals
② A set is closed if it is the complement of an open set.
③ The closure of a set $A \subseteq \mathbb{R}$ is the intersection of all the closed ~~that~~ sets contain A denoted by $\bar{A}$.
④ If $Y \subseteq \mathbb{R}$ & $f: \mathbb{R} \to \mathbb{R}$ then $f^{-1}(Y) = \{x \in \mathbb{R} \mid f(x) \in Y\}$

Theorem: Sps that $f: \mathbb{R} \to \mathbb{R}$ is a function then following 2 statements are equivalent.
① f is cts in $\mathbb{R}$
② $f^{-1}(A)$ is an open set if $A \subseteq \mathbb{R}$ is an open set.

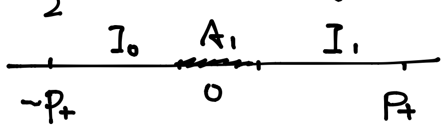
Notes:
① $\mathbb{R}$ and $\emptyset$ are open and closed.
② The union of open sets is open, but only the intersection of a finite number of open sets is open
③ The intersection of closed sets is closed, but only the union of finitely many closed sets is closed
④ $\overline{(a,b)} = [a,b]$
$A = \{\frac{1}{n} : n \in N\} = \{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \cdots\}$
$\Rightarrow$ the closed set $\Rightarrow \bar{A} = A \cup \{0\}$

We have a distance in Σ, so we can define an open ball of radius ε centered at $a \in \Sigma$ as the set $\{s \in \Sigma : d[a,s] < \varepsilon\}$

- An open set in Σ is the union of any number of open balls in Σ
- Let $\varepsilon = \frac{1}{2^n}$, then s is in the open ball of radius ε & centered at a

if $d[s,t] < \varepsilon = \frac{1}{2^n} \Rightarrow s_i = a_i$ for all $i = 0, \dots, n$

$\underline{c < -2}$



SHIFT MAP

$\sigma : \Sigma \to \Sigma$ is defined by $\sigma(s_0 s_1 s_2 \cdots) = (s_1 s_2 s_3 \cdots)$

Example:

$\sigma(01010101\cdots) = (1010 10\cdots)$
$\sigma(1111\cdots) = (1111\cdots)$
$\sigma(0111\cdots) = (1111\cdots)$
$\sigma(101010\cdots) = (010101\cdots)$

2nd iterate

$\sigma^2(s_0 s_1 s_2 \cdots) = (s_2 s_3 s_4 \cdots)$
$\sigma^3(s_0 s_1 s_2 \cdots) = (s_3 s_4 s_5 \cdots)$

So the periodic pts of σ are sequences that satisfy:

period 2: $s = (s_0 s_1 s_0 s_1 s_0 s_1 \cdots)$
period k: $s = (s_0 s_1 \cdots s_{k-1} s_0 s_1 \cdots s_{k-1} \cdots) = (\overline{s_0 \cdots s_{k-1}})$
which satisfies $\sigma^k(s) = s$

Examples:

Fixed pts: $(\overline{0})$ and $(\overline{1})$
2-cycles: $(\overline{01}), (\overline{10}) \Rightarrow$ One 2-cycle
3-cycles: $(\overline{001}), (\overline{010}), (\overline{100}) \Rightarrow$ Two 3-cycles
$(\overline{011}), (\overline{110}), (\overline{101})$
4-cycles: $(\overline{0101}), (\overline{1010})$ (2 cycle not 4-cycle)

so remove all 2 cycles

$(\overline{0001}), (\overline{0010}), (\overline{0100}), (\overline{1000})$
$(\overline{0011}), (\overline{0110}), (\overline{1100}), (\overline{1001})$
$\vdots$

5-cycles : 6 of them
6-cycles : 18 of them