Tutorial 6

STAT 3013/8027

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1. Consider the following simple linear regression model:

$$\begin{aligned} Y_i &= \beta_0 + \beta_1 x_i + \epsilon_i \\ \epsilon_i &\sim iid \quad n(0, \sigma^2), \quad i = 1, \ldots, n. \end{aligned}$$

Using a least-squares approach show that (See Rice Chapter 14):

$$\begin{aligned} \hat{\beta}_0 &= \bar{y} - \hat{\beta}_1 \bar{x} \\ \hat{\beta}_1 &= S_{xy}/S_{xx} = \sum_{i=1}(x_i - \bar{x})(y_i - \bar{y}) / \sum_{i=1}(x_i - \bar{x})^2 \end{aligned}$$

In R (without using the `lm()` function) provide estimates for $\hat{\beta}_0, \hat{\beta}_1$, based on the GDP and labor data discussed in class. You may use the `lm()` function to check your results.

- For the least-squares estimates of β_0 and β_1 derived last time, find:

$$E[\hat{\beta}_0], \quad E[\hat{\beta}_1], \quad V[\hat{\beta}_0], \quad V[\hat{\beta}_1].$$

2. Let's consider a regression model where we are estimating estimating the means for J different groups (categorical data). Consider the model:

$$\begin{aligned} Y_{ij} &= \mu_j + \epsilon_{ij} \\ \epsilon_{ij} &\sim iid \quad \text{normal}(0, \sigma^2), \quad i = 1, \ldots, n; \; j = 1, \ldots, J. \end{aligned}$$

   a. Find the least-squares estimators: $\hat{\mu}_1, \ldots, \hat{\mu}_J$.

   b. Three groups of six guinea pigs were each randomly injected, respectively, with 0.5 mg, 1.0 mg, and
   1.5 mg of a new tranquilizer. The following data present the number of minutes it took them to fall asleep:

0.5 mg	21	23	19	24	25	23
1.0 mg	19	21	20	18	22	20
1.5 mg	15	10	13	14	11	15

Construct side-by-side box plots to visualize the data and provide the estimates: $\hat{\mu}_1, \hat{\mu}_2, \hat{\mu}_3$.