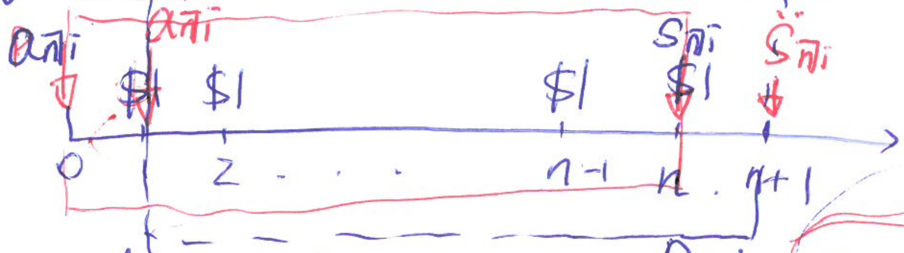
LN 3.2 Mid-Sem Exam: 18th Apr. 9:30am



Immediate Annuity (0 ~ n)

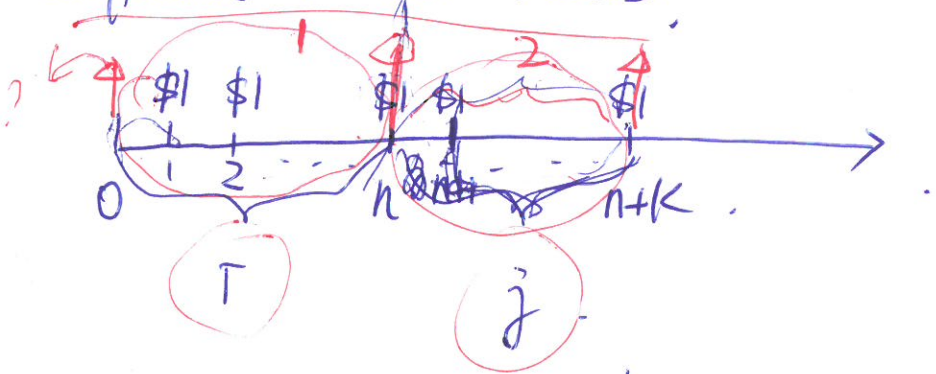
- P.V: $a_{\overline{n}|i} = \dfrac{1-v^n}{i}$
- A.V: $s_{\overline{n}|i} = \dfrac{(1+i)^n-1}{i}$

Annuity Due (1 ~ n+1)

- P.V: $\ddot{a}_{\overline{n}|i} = \dfrac{1-v^n}{d}$
- A.V: $\ddot{s}_{\overline{n}|i} = \dfrac{(1+i)^n-1}{d}$

---

1. different interest rates



$$P.V_2 = a_{\overline{k}|j} \cdot v_i^{\,n}$$

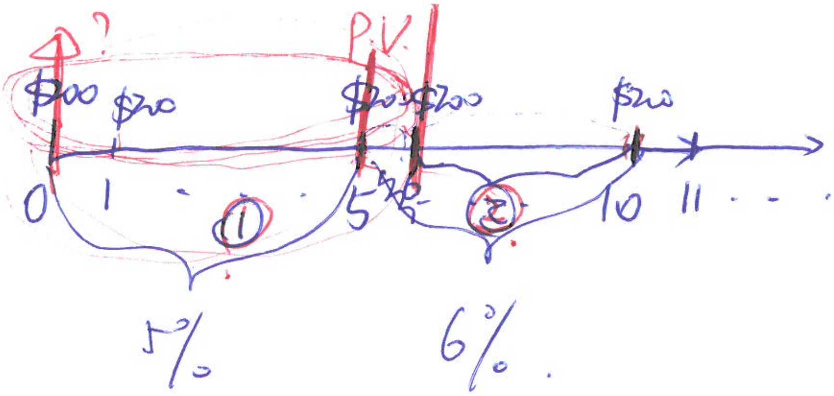
$$P.V_1 = a_{\overline{n}|i}$$

$$P.V = P.V_1 + P.V_2$$

$$A.V_2 = s_{\overline{k}|j}$$

$$A.V_1 = s_{\overline{n}|i} \cdot (1+j)^k$$

Ex:



Sol: $P.V._1 = 200 \cdot \ddot{a}_{\overline{6}|5\%} = \$1065.9$

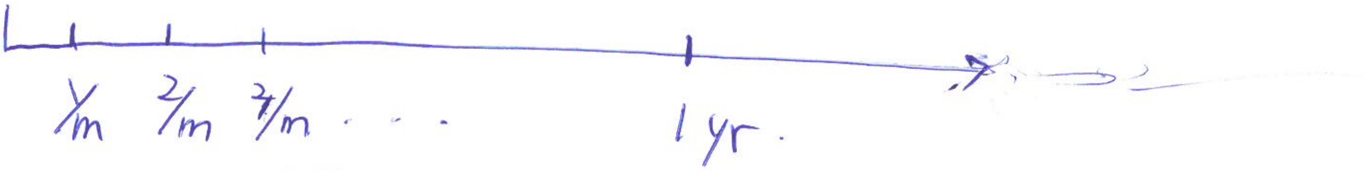
$P.V._2 = 200 \cdot a_{\overline{5}|6\%} \cdot v_{0.05}^{5}$

$= 200 \cdot \ddot{a}_{\overline{5}|6\%} \cdot v_{0.06} \cdot v_{0.05}^{5}$

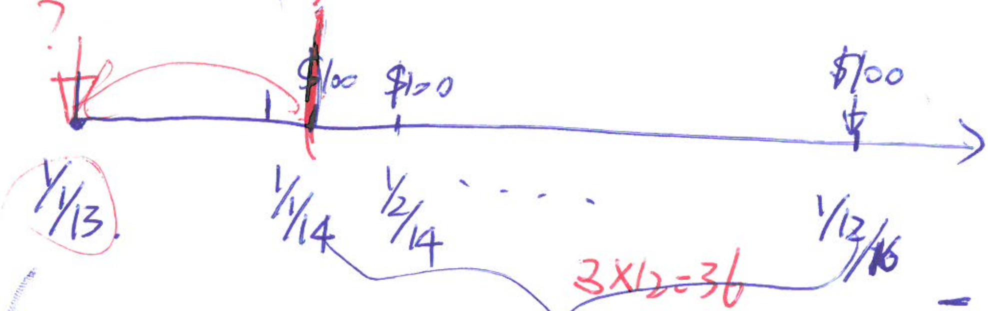
$= \$660.10$

$\boxed{PV_1 + PV_2 = \$1726}$

② Annuities Payable more frequently than Annually.



Ex:

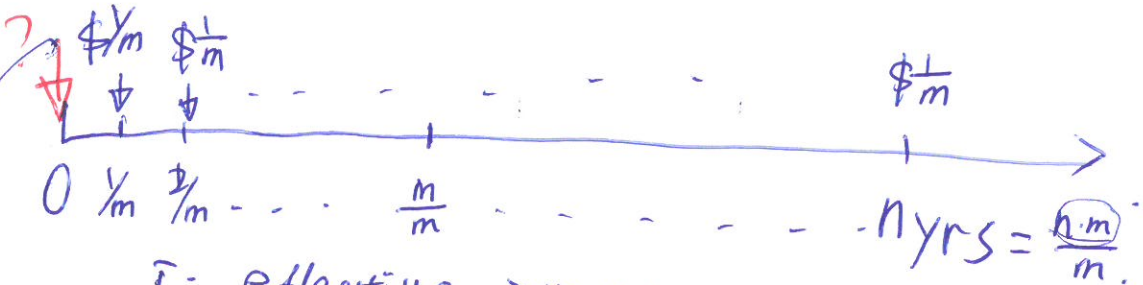


effective interest rate per annum = 8%.

① $j = (1+8\%)^{\frac{1}{12}} - 1 = 0.006434$.

② $P.V.(t=1/1/14) = 100 \cdot \ddot{a}_{\overline{36}|j}$

③ $P.V.(t=1/1/13) = 100 \cdot \ddot{a}_{\overline{36}|j} \cdot v_i$



i: effective interest rate per annum.

$$a_{\overline{n}|}^{(m)} = \frac{1-v^n}{m\left[(1+i)^{\frac{1}{m}}-1\right]} = \frac{1-v^n}{i^{(m)}}$$

Pf: $a_{\overline{n}|}^{(m)} = \frac{1}{m} v^{\frac{1}{m}} + \frac{1}{m} v^{2/m} + \cdots + \frac{1}{m} v^{\frac{nm}{m}=n}$

$$m\, a_{\overline{n}|}^{(m)} = v^{\frac{1}{m}} + v^{\frac{2}{m}} + \cdots + v^{n} \quad (1)$$

$$v^{-\frac{1}{m}} m\, a_{\overline{n}|}^{(m)} = 1 + v^{\frac{1}{m}} + \cdots + v^{\frac{n(m-1)}{m}} \quad (2)$$

Correction: The power of the last term in Equation (2) should be m(n-1)/m.

(2) − (1)

$$\Longrightarrow m\, a_{\overline{n}|}^{(m)}\left(v^{-\frac{1}{m}}-1\right) = 1-v^n.$$

$$\Rightarrow a_{\overline{n}|}^{(m)} = \frac{1-v^n}{m\left[(1+i)^{\frac{1}{m}}-1\right]}$$

$$= \frac{1-v^n}{i^{(m)}}$$

#

$$\begin{cases} s_{\overline{n}|}^{(m)} = \dfrac{(1+i)^n - 1}{i^{(m)}} \\ \ddot{a}_{\overline{n}|}^{(m)} = \dfrac{1-v^n}{d^{(m)}} \\ \ddot{s}_{\overline{n}|}^{(m)} = \dfrac{(1+i)^n - 1}{d^{(m)}} \end{cases}$$

Ex: $P.V(t = 1/1/14) = (100 \times 12) \cdot \ddot{a}_{\overline{3}|}^{(12)}$ $i = 8\%$

$$= 1200 \cdot \frac{1-v^n}{d^{(m)}}$$

$$= 1200 \cdot \frac{1-v^3}{m\left[1-(1+i)^{\frac{-1}{m}}\right]}$$

$$= 1200 \cdot \frac{1-1.08^{-3}}{12 \cdot \left[1-(1.08)^{\frac{-1}{12}}\right]}$$

$$P.V.(t = 1/1/13) = \cancel{\times} \; v = \frac{1}{1.08}$$

$$= \$2986.$$