Lecture 33

Chapter 15 complex functions

- i is a solution of $x^2=-1$ (and $-i$ is the other), we write $i=\sqrt{-1}$.
- A complex number is of the form $a+ib$, for $a,b\in\mathbb{R}$
- The set of all complex numbers is $\mathbb{C}$.

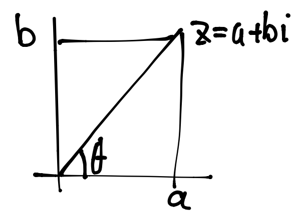
Ex: $1+2i\in\mathbb{C}$, $4=4+0i\in\mathbb{C}$

- if $z=a+bi$, a is the

  b is the
- $|z|=\sqrt{z\cdot\bar{z}}=\sqrt{(a+bi)(a-bi)}=\sqrt{a^2+b^2}$

- $\bar{z}=a-bi$
- we can write a complex # z as

  $z=|z|(\cos\theta+i\sin\theta)=|z|e^{i\theta}$ where $\theta=\arctan\left(\frac{y}{x}\right)$



- Recall Euler's Formula

  $e^{i\theta}=\cos\theta+i\sin\theta$ $\qquad e^{i\pi}=-1$

polar representation

$re^{i\theta}=r\cos\theta+ir\sin\theta$

- When we multiply 2 complex numbers:

  $z=re^{i\theta}$

  $w=se^{i\phi}$

  $\Rightarrow z\cdot w=rse^{i(\theta+\phi)}$

  so z^2 is doubling the angle $\quad z^2=r^2\cdot e^{i2\theta}$

Chapter 16 Julia Set

- Recall the quadratic map: $Q_c(z)=z^2+c$

But now we will consider $z, c\in\mathbb{C}$.

- if $c=0$, then this is just a squaring function

  if we only allow $|z|=1$, then this is the "other doubling function", which is chaotic.

Def: The orbit of $z\in\mathbb{C}$ under Q_c is bounded if $\exists\, k$ s.t. $|Q_c^n(z)|\leq k$, $\forall n\in\mathbb{R}$

otherwise, it is unbounded.

- Def: The filled Julia set of a Q_c is the set of all points, whose orbit is bound-ed.

  $K_c=\{z\in\mathbb{C} : \exists k, \forall n\in\mathbb{N}, |Q_c^n(z)|\leq k\}$

The Julia set is the boundary of the filled Julia set.

$J_c=K_c-K_c$