STAT2001 Tutorial 4 Solutions

Problem 1

(a) A square total means 4 or 9. The probabilities of these events are, respectively:

$$P(4) = P(13) + P(31) + P(22) = 3 \times 1/36$$

$$P(9) = P(36) + P(63) + P(45) + P(54) = 4 \times 1/36$$

So the probability of a square total is 7/36.

Y has the binomial distribution with parameters n = 5 and p =7/36:

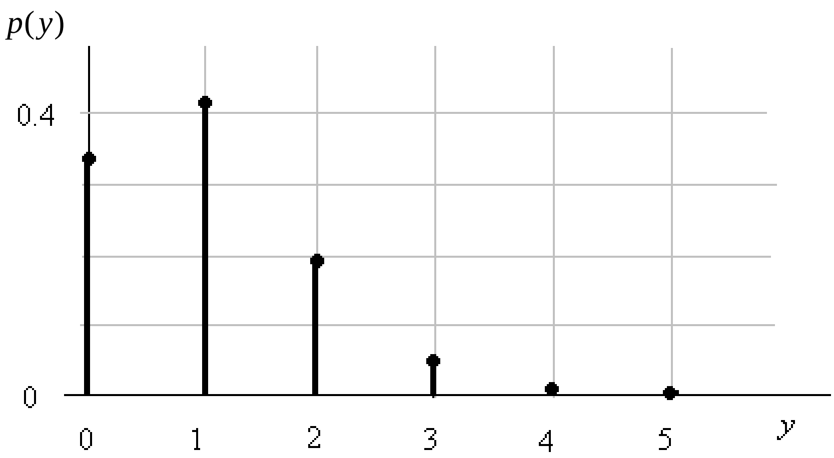
$$Y \sim \text{Bin}(5,\ 7/36), \quad p(y) = \binom{5}{y}\left(\frac{7}{36}\right)^{y}\left(\frac{29}{36}\right)^{5-y}, \quad y = 0, 1, 2, 3, 4, 5.$$

(b) $p(0) = (29/36)^5 = 0.339$, $\quad p(1) = 5\left(\frac{7}{36}\right)\left(\frac{29}{36}\right)^4 = 0.409$

$p(2) = \frac{5\times4}{2}\left(\frac{7}{36}\right)^2\left(\frac{29}{36}\right)^3 = 0.198$, $\quad p(3) = \frac{5\times4}{2}\left(\frac{7}{36}\right)^3\left(\frac{29}{36}\right)^2 = 0.048$,

$p(4) = 5\left(\frac{7}{36}\right)^4\frac{29}{36} = 0.006$, $\quad p(5) = \left(\frac{7}{36}\right)^5 = 0.000$.

y	0	1	2	3	4	5
$p(y)$	0.339	0.409	0.198	0.048	0.006	0.000



(c) $P(Y \geq 1) = 1 - P(Y = 0)$ = 1 – 0.339 = 0.661.

(d) The Bernoulli distribution with parameter p = 7/36. $Y \sim$ Bern(7/36).

Problem 2

(a) The geometric distribution with parameter 7/36. $Y \sim \text{Geo}(7/36)$.
$p(y) = (29/36)^{y-1} 7/36$, $y = 1, 2, 3, \ldots$.

(b) $p(3) = \left(\frac{29}{36}\right)^2 \frac{7}{36} = 0.126$.

(c) P(even number of rolls) = $p(2) + p(4) + p(6) + \ldots$

$$= qp + q^3 p + q^5 p + \ldots \quad \text{where } p = 7/36 \text{ and } q = 29/36$$

$$= qp(1 + q^2 + q^4 + \ldots)$$

$$= qp\{1 + (q^2) + (q^2)^2 + \ldots\}$$

$$= \frac{qp}{1-q^2} = \frac{q(1-q)}{(1-q)(1+q)} = \frac{q}{1+q} = \frac{29/36}{1+29/36} = \frac{29}{65} = 0.4462.$$

Another solution to (c):

P(odd number of rolls) $= p(1) + p(3) + p(5) + \ldots$

$$= p + q^2 p + q^4 p + \ldots$$

$$= \frac{1}{q}(qp + q^3 p + q^5 p + \ldots)$$

$$= \frac{1}{q} P(\text{even number of rolls}). \qquad (1)$$

But P(odd number of rolls) + P(even number of rolls) = 1. (2)

Solving (1) and (2), we get P(even number of rolls) = 0.4462, as before.

Yet another solution to (c), via *first step analysis*:

Let E = "Even number of rolls" and F = "Square total on first roll".

Then $P(E) = P(F)P(E \mid F) + P(\bar{F})P(E \mid \bar{F})$ (by LTP), which implies that

$$P(E) = \frac{7}{36}(0) + \frac{29}{36}(1 - P(E)).$$

Solving, we get $P(E) = 0.4462$, as before.

NB: $P(E \mid \bar{F})$ = P(even no. of rolls, starting from 1st roll, given 1st total not square)

= P(odd no. of rolls, starting from 2nd roll, given 1st total not square)

= P(odd no. of rolls, starting from 2nd roll)

(result on 1st roll is then irrelevant)

= P(odd no. of rolls, starting from 1st roll)

(probabilities regarding the future are the same, wherever you start)

= 1 – P(even no. of rolls, starting from 1st roll)

= 1 – $P(E)$.

Problem 3

(a) The Poisson distribution with parameter λ = 1/9. Y ~ Poi(1/9).

$$p(y) = \frac{e^{-1/9}(1/9)^y}{y!}, \quad y = 0, 1, 2, 3, ...$$

(b) **(i)** $p(1) = \dfrac{e^{-1/9}(1/9)^1}{1!} = 0.0994$.

(ii) $P(Y=0) = \dfrac{e^{-1/9}(1/9)^0}{0!}$ = 0.8948.

So $P(Y \geq 1) = 1 - P(Y=0) = 1 - 0.8948 = 0.1052$.

(iii) Let X be the number of accidents over the next two months.

Then X ~ Poi(2/9).

So $P(X \geq 1) = 1 - P(X=0) = 1 - e^{-2/9} = 0.199$.

Another solution to (iii):

Let A_i = "At least one accident in Month i" (i = 1, 2).

Then $P(X \geq 1) = P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 A_2)$

$= P(A_1) + P(A_2) - P(A_1)P(A_2)$ by independence

= 0.1052 + 0.1052 – $(0.1052)^2$ by (ii)

= 0.199, as before.

Yet another solution to (iii):

$P(X \geq 1) = P(A_1 \cup A_2) = 1 - P(\overline{A_1 \cup A_2})$

$= 1 - P(\bar{A}_1 \bar{A}_2)$ by De Morgan's laws

$= 1 - P(\bar{A}_1)P(\bar{A}_2)$ by independence

= $1 - (0.8948)^2$ by (ii)

= 0.199, as before.

Problem 4

(a) The hypergeometric distribution with parameters N = 12, r = 5 and n = 4.

Y ~ Hyp(12,5,4). $p(y) = \binom{5}{y}\binom{7}{4-y} \Big/ \binom{12}{4}$, y = 0, 1, 2, 3, 4.

(b) $p(0)$ = $\binom{5}{0}\binom{7}{4} \Big/ \binom{12}{4} = 0.0707$, $p(1)$ = $\binom{5}{1}\binom{7}{3} \Big/ \binom{12}{4} = 0.3535$.

So $P(Y \leq 1)$ = 0.0707 + 0.3535 = 0.4242.

So $P(Y \geq 2) = 1 - P(Y \leq 1) = 1 - 0.4242$ = 0.5758.