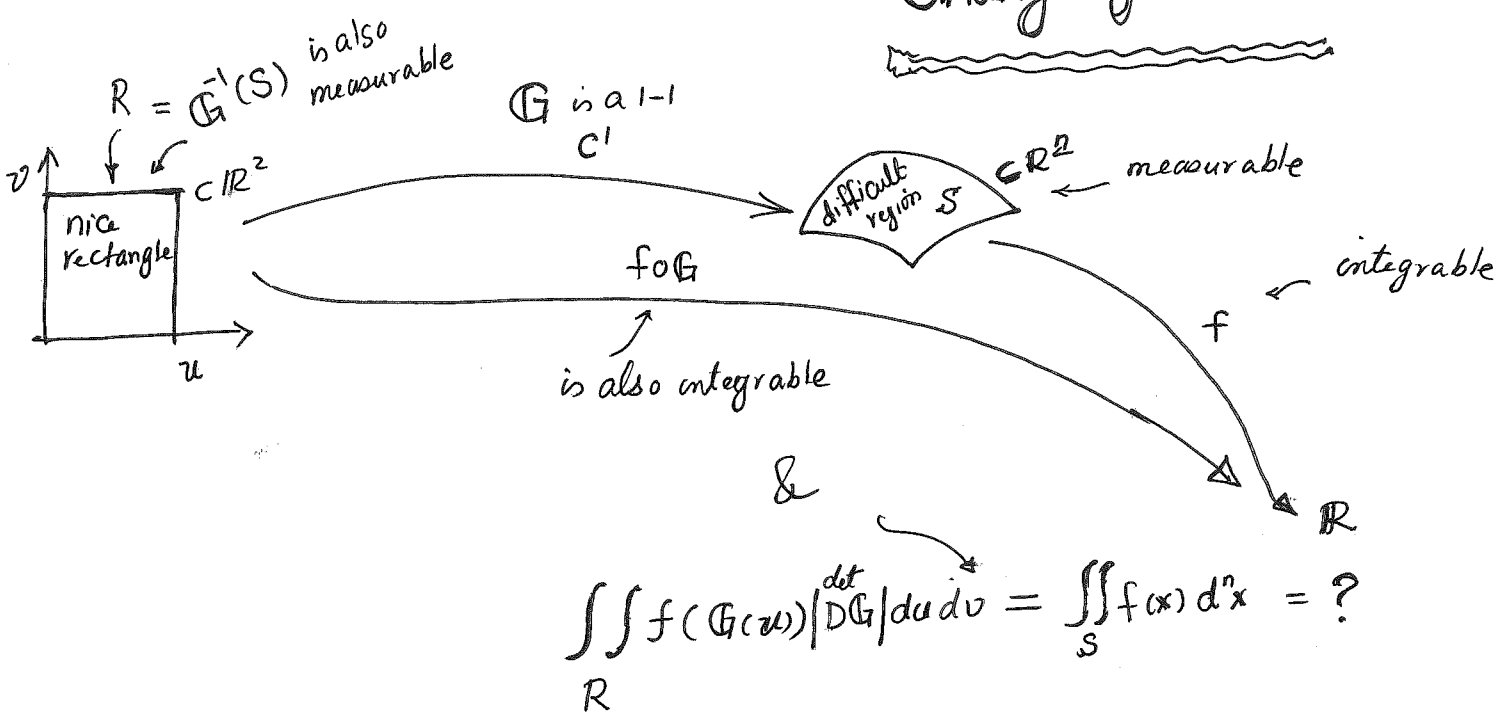
# 4.4

## Change of Variables




&

$$\iint_R f(G(u,v)) \left|\det DG\right| du\,dv = \iint_S f(x)\, d^n x = ?$$

When $G$ is linear: $G(u) = Au$

Then $DG$ is just the matrix $A$ & $\det DG = \det A$

$$\iint_S f(x)\, dA = |\det(A)| \iint_{G^{-1}(S)} f(G(u))\, du\,dv$$

#13 example: When $u = xy$ is given $v = x^2 - y^2$ we don't need to solve for $x, y$ in terms of $u$ and $v$, we can instead work with $G^{-1}(x,y) = (u,v)$

$$\iint_S (x^2+y^2)\, dA = \int_1^4 \int_1^3 (x^2+y^2) \frac{1}{2(x^2+y^2)}\, du\,dv$$

$$= \frac{1}{2} \int_1^4 \int_1^3 du\,dv$$

$$(u,v) = G^{-1}(x,y)$$

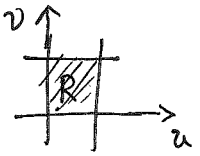
$$DG^{-1}(x,y) = \begin{bmatrix} y & x \\ 2x & -2y \end{bmatrix}$$

$$\det DG^{-1} = -2y^2 - 2x^2$$

$$DG^{-1} = [DG]^{-1}$$

$$\det DG^{-1} = \frac{1}{\det(DG)}$$

or $$\det DG = \frac{1}{\det DG^{-1}}$$




$$|\det DG| = \frac{1}{2(x^2+y^2)}$$