3. $S_3 = S_1 \cup S_2$

$x = y^2$

3 expressions according to our pictures def'n of MR

4. a) ① $\begin{cases} x = 1 + 3t \\ y = 2 + 4t \\ z = 3 + 2t \end{cases}$

② ~~$\frac{x-1}{3} = \frac{y-2}{4} = \frac{z-3}{2}$~~

③ $y = \frac{4}{3}x + \frac{2}{3}$

$z = \frac{2}{3}x + \frac{7}{3}$

④ $F(x,y,z) = y - \frac{4}{3}x - \frac{2}{3}$

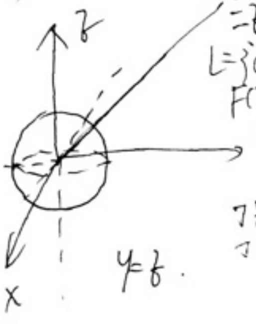
$G(x,y,z) = z - \frac{2}{3}x - \frac{7}{3}$

$L = \{(x,y,z) : F(x,y,z) = 0 = G(x,y,z)\}$

$\nabla F = (-\frac{4}{3}, 1, 0)$

$\nabla G = (-\frac{2}{3}, 0, 1)$

b) $x^2 + y^2 + z^2 = 1$,



$y = z$.

i) $x^2 + 2y^2 = 1$.

ii) $F(x,y,z) = x^2 + y^2 + z^2 - 1 = 0$

$G(x,y,z) = y - z = 0$.

iii) $\vec{f}(t) = \left(t, \pm\sqrt{\frac{1-t^2}{2}}, \pm\sqrt{\frac{1-t^2}{2}}\right)$

IFT: $F(\vec{x}, y)$: $(\vec{a}, b) \in \mathbb{R}^{n+1}$,

suppose $\partial_y F(\vec{a}, b) \neq 0$

$\exists\, r_0, r_1 > 0$ s.t.

A) if $|\vec{x} - \vec{a}| < r_0 \Rightarrow \exists$ unique y s.t. $|y - b| < r_1$ and $F(\vec{x}, y) = 0$

in other words, for $|\vec{x} - \vec{a}| < r_0$, $|y - b| < r_1$. can solve $F(\vec{x}, y) = 0$ for y in terms of $\vec{x}$, $y = f(\vec{x})$

B) $\partial_j f(\vec{x}) = -\frac{\partial_j F}{\partial_y F}(\vec{x}, f(\vec{x}))$

a) $F(x,y) = x - 3y - 5 = 0$ $(a,b) = (8,1)$

$\Rightarrow y = \frac{1}{3}(x - 5)$

Try $r_0 = 10$, $r_1 = 20$. check that with these values A & B hold

check: A) suppose $|x - 8| < 10$

$\Rightarrow -2 < x < 18$

Using $y = \frac{1}{3}(x-5)$ we see $\exists$ a unique sol'n to y for $\forall\, x \in (-2, 18)$.

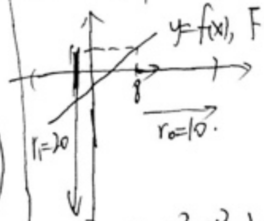
$f(x) = \frac{1}{3}(x - 5)$

$|y - 1| = |\frac{1}{3}(x-5) - 1| = |\frac{1}{3}x - \frac{8}{3}|$

$= \frac{1}{3}|x - 8| < \frac{10}{3} < r_1 = 20$.

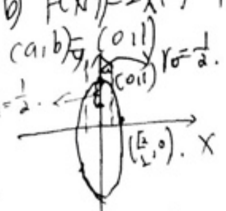
B) $\partial_x f = \frac{1}{3}$, while

LHS= RHS $= -\frac{\partial_x F}{\partial_y F} = \frac{-1}{-3} = \frac{1}{3}$



b) $F(x,y) = 2x^2 + y^2 - 1 = 0$

$(a,b) = (0,1)$



choose $r_0 = \frac{1}{2}$, $r_1 = \frac{1}{2}$.

Notice $\frac{1}{2} < \frac{1}{\sqrt{2}}$.

check (A) for $r_0 = \frac{1}{2} = r_1$

Suppose $|x - 0| < \frac{1}{2}$

Then since $2x^2 + y^2 - 1 = 0$

$\Rightarrow y = \pm\sqrt{1 - 2x^2}$ ... (*)

$f(x) = \sqrt{1-2x^2}$. For each value x s.t. $|x - 0| < \frac{1}{2}$ (*) shows that $\exists$ exactly 2 possible values for y. But only one of these satisfies

$|y - 1| < r_1 = \frac{1}{2}$

$|\sqrt{1-2x^2} - 1| < \frac{1}{2}$ (can see from the graph) also $|-\sqrt{1-2x^2} - 1| > \frac{1}{2}$.

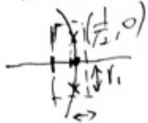
B) $f(x) = \sqrt{1 - 2x^2}$.

$\partial_x f = \frac{1}{2}(1-2x^2)^{-\frac{1}{2}} \cdot (-4x)$

$= -2x(1-2x^2)^{-\frac{1}{2}}$

$-\frac{\partial_x F}{\partial_y F} = \frac{-4x}{2y} = -\frac{2x}{\sqrt{1-2x^2}}$

what if $(a,b) = (\frac{1}{\sqrt{2}}, 0)$



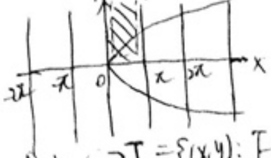
Notice $\partial_y F(\frac{1}{\sqrt{2}}, 0) = 2y = 0$

So IFT does not guarantee that we can write $y = f(x)$ near $(\frac{1}{\sqrt{2}}, 0)$

Notice #2. $\partial_x F(\frac{1}{\sqrt{2}}, 0) \neq 0$

$\Rightarrow$ IFT says can solve x in terms of y near $(\frac{1}{\sqrt{2}}, 0)$ $(x = f(y))$

③ $S = \{(x,y) : (x - y^2)\tan x = 0\}$



But $S \supset T = \{(x,y) : F(x,y) = 0,\ 0 < x < \pi,\ 0 < y\}$

exercise: S is smooth (on shaded curve, $x = y^2$)

Problem Session 6.

1. $\forall x$ $|x-3|<r_0$ $\exists$ unique y s.t. $|y-1|<r_1$. We denote this y by $f(x)$ & $f(x)=$

$F(x,y)=x-3y$

$\forall x$ $|x-3|<r_0$ $\exists$ y: $|y-1|<r_1$

$F(x,))=0$. $y=\frac{x-3}{3}$

$3-r_0<x<3+r_0$

$\frac{3-r_0}{3}<\frac{x-3}{3}<\frac{3+r_0}{3}$

$1+r_1 \geq \frac{3+r_0}{3}$

$1-r_1 \leq \frac{3-r_0}{3}$

$1-r_1<y<1+r_1$

$\frac{r_0}{3}\leq r_1$

this explains why f is not 3.

C at the point where the curve crosses the axis

$\partial_1 F(F(x,y),y)\cdot F_2(x,y)$
$+ F_2(F(x,y),y)\neq 0$

$x=y=0$

$\Rightarrow \partial_1(F(0,0),0)\cdot F_2(0,0)$
$+\partial_2 F(F(0,0),0)\neq 0$

$\Rightarrow \partial_1(0,0)\cdot F_2(0,0)+\partial_2 F(0,0)\neq 0$

$F_2(0,0)[1+F_1(0,0)]\neq 0.$

$\S 3.2.$ 3. a) $F_3=F_1F_2$ b) $\nabla F(\vec{x})$

$\partial_y F=-3y^2$ at $y=0$, $\partial_y F=0\Rightarrow$ IFT doesn't apply at where $y=0$ on the curve (it does apply to other points)

$F(x,y)=x-e^{1-x}y^3$

$\vec{x}\in S_1\cup S_2$

$\Rightarrow \vec{x}\in S_1$ or $\vec{x}\in S_2$

$\Rightarrow F_1(\vec{x})=0$ or $F_2(\vec{x})=0$

$F_3(\vec{x})=F_1F_2(\vec{x})=0.$

$\vec{x}\in S_3$

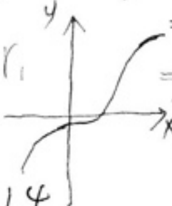
$\Rightarrow S_1\cup S_2\subseteq S_3$

$\vec{x}\in S_3$

i.e. $F_3(\vec{x})=0$

$\Rightarrow F_1F_2(\vec{x})=0$

$\vec{x}\in S_1\cup S_2$



b)

2. 3.1.4.

$f(x,y)=x-e^{1-x}y^3$

$\partial_x f=1+e^{1-x}>0$

$\partial_y f=-3y^2\leq 0.$

But $\partial_x f$ is not defined every where (tangent becomes vertical where curve crosses the x-axis)

3.1.5. $\partial_y F=\partial_F(F(x,y),y)\cdot F_y(x,y)$
$+\partial_y F(F(x,y),y)\neq 0.$

plug $(0,0)$ in

$\partial_1 F(0,0),0)\cdot F_2(0,0)$
$+\partial_2 F(F(0,0),0)\neq 0$

Note that the graph passes the vertical test

$\Rightarrow y=f(x)$

$\Rightarrow \partial_1 F(0,0)F_2(0,0)$
$+\partial_2 F(0,0)$