- The test is **7.00 → 9.00 in EX 310** on Tuesday, June 24, 2014. Please arrive about 5 minutes early, so that we can start on time.
- The test will have 7 questions, covering the material from lectures and assignments up to, and including, June 17.
- More precisely, it covers: 1.1 → 1.6, 2.1 → 2.4. Please make sure you are comfortable with this material. If you're not, ask the instructor or TA during office hours, or ask a friend as soon as possible.
- Extra office hours will be as follows: (TBA)
- There are no calculators, notes, books or other aids allowed.
- Remember to bring your student ID to the test.
- Below is a list of some things that you should know. I make no claim that there is nothing missing from this list. I will promise that nothing on the test is independent from the course. (I'm not trying to trick you with this list, of course. I just do not want 100 people yelling at me after the test. So use the list below as a guide, but make sure you are good with the assignment and practice problems.)

Things to know for the test

- **Basics of Complex Numbers** - You should be familiar with operations on complex numbers, and how to represent complex numbers in regular and polar form. You should be able to know when to use $x + iy$ or $re^{i\theta}$ form, and how to convert between them. You should be familiar with modulus, Re, Im, and basic inequalities. You should be able to solve various equations over complex numbers and sketch their solutions.
- **Geometry and Topology of the Plane** - You should know the basics of topology of $\mathbb{C}$. In particular, you should be able to determine if a given set is open, closed, connected, convex, simply connected. You should be able to determine the boundary of a given set. You should know how to write the equation of circles and lines in complex form, and be able to describe simple sets, such as disks, sectors, annuli and half planes.
- **Complex Functions and Limits** - You should know the basics of complex functions. You should know how to take limits, and determine continuity. You should be familiar with exponential, trig and logarithmic functions, and their properties. You should understand the subtlety of complex logarithms, and how to deal with some of the issues (e.g. non-continuity, multi-valuedness) of complex logarithm. You should be familiar with the principal branch of logarithm, obtained by deleting the negative reals.
- **Line Integrals and Green's Theorem** - You should be able to parametrize line segments, circle arcs, graphs of functions, and piecewise curves made up of line segments, circle arcs and graphs of functions. You should be able to compute line integrals. You should know the complex form of Green's Theorem, and how/when to use it.
- **Analytic and Harmonic Functions, Cauchy-Riemann Equations** - You should know how the derivative of a complex function is defined. You should understand that this is a very strong condition, and there are many functions which are not analytic. You should know the Cauchy-Riemann (C-R) equations, and be able to use them to test/prove analyticity of a given function. You should know what a harmonic function is, and that the real and imaginary parts of an analytic function are harmonic. You should know how to find the "conjugate" harmonic function, to a harmonic function. You should know when a given function is, or is not, the real or imaginary part of an analytic function.
- **Sequences, Series and Power Series** - You should be able to determine convergence of sequences. You should know how to determine convergence of complex series, and be able to use tests such as ratio, comparison, root, etc., to determine absolute convergence. You should know how to determine the radius of convergence of a power series, and that a convergent power series is an analytic function on its domain of convergence. You should know the power series expansions for $e^z, \sin(z), \cos(z), \dfrac{1}{1-z}$, and you should be able to find power series for

easy compositions or products of functions given by power series (e.g. find the power series for e^{z^2} using the power series for e^z) or power series obtained by differentiating.

- **Cauchy Theorem and Cauchy Formula** - You should know the statement of Cauchy's Theorem, as well as the important Theorems in Ch. 2.3 that follow from Cauchy's Theorem. You should be able to use these Theorems to answer problems, and compute line integrals. You should know Cauchy's Formula. You should be able to use it to compute line integrals, or values of a given function. Hint: If you're asked to compute a line integral over a curve you can't parametrize (e.g. an arbitrary curve joining two points) you should know to try to apply Cauchy's Theorem. You should know that the line integral of an analytic function depends only on the endpoints of the curve, and not that actual path taken. This is *very* useful.
- **Application of Cauchy Theorem and Formula** - You should be able to use these as tools to solve other problems, including computing real integrals by realizing them as line integrals (e.g. trig integrals), then using Cauchy's Formula to evaluate them. You should be able to recognize when/how to apply Cauchy's Formula to compute line integrals. You should know the formula for the power series of an analytic function centred at z_0 as well as the coefficients of the series. You should know how to write the kth derivative of a function in terms of a line integral. You should what the order of a zero is, and various ways to determine the order of a zero.

Extra Review Problems

Here are some extra review problems. Do as many as you need to feel comfortable and confident about the material. Don't forget to look back at the problems already assigned, either as HW, or as practice. (There is likely to be some repetition between these problems and the previous assignment and practice problems.)

(1) 1.1 # 3, 4, 5, 6
(2) 1.2 # 1, 3, 11, 13, 23, 25
(3) 1.3 # 2, 5, 7, 8, 15
(4) 1.4 # 3, 11, 13, 31, 39, 41
(5) 1.5 # 3, 5, 11, 13, 23
(6) 1.6 # 1, 3, 4, 6, 12
(7) 2.1 # 9, 10, 18, 20(a), 21
(8) 2.2 # 2, 3, 8, 9, 11
(9) 2.3 # 2, 3, 5, 10, 11
(10) 2.4 # 1, 2, 9, 10, 12, 13

Tips for Studying

Please (please, please, please, ...) don't wait until the night before the test to start studying. If you find out then that you don't know something, it is doubtful that you'll have anybody to help you.
Math is like everything else: you need to practice to succeed. Do as many problems as you can. The more you do, the easier they become.
Aim to get to the point where you can answer problems without looking things up in the book. Practice until you can't make a mistake, rather than until you get it right once.
For many people it can be helpful to write things out. Take your notes, or the book, and make a full list of all the definitions and theorems that we've talked about. The simple act of writing something explicitly yourself can help you internalize it.
Many of the things in this course (definitions, theorems, proofs) has a picture that go with it. For more visual thinkers, make a list of each definition and theorem together with the appropriate picture, if applicable.