STA305/1004 - Exam Jam Review Problems- Solutions

April 6, 2015

Problem 1

An engineer is intrested in the effects of cutting speed (A), tool geometry (B), and cutting angle (C) on the life of a machine tool. Two levels of each factor are chosen and all factor-level combinations were run three times.

(a) What type of design did the engineer use?

A 2^3 factorial design.

R Output for problem 1

```
                                          Estimate Std. Error t value
(Intercept)                                  40.83       1.12   36.42
Cutting.Speed                                 0.17       1.12    0.15
Tool.Geometry                                 5.67       1.12    5.05
Cutting.Angle                                 3.42       1.12    3.05
Cutting.Speed:Tool.Geometry                  -0.83       1.12   -0.74
Cutting.Speed:Cutting.Angle                  -4.42       1.12   -3.94
Tool.Geometry:Cutting.Angle                  -1.42       1.12   -1.26
Cutting.Speed:Tool.Geometry:Cutting.Angle    -1.08       1.12   -0.97
                                          Pr(>|t|)
(Intercept)                                   0.00
Cutting.Speed                                 0.88
Tool.Geometry                                 0.00
Cutting.Angle                                 0.01
Cutting.Speed:Tool.Geometry                   0.47
Cutting.Speed:Cutting.Angle                   0.00
Tool.Geometry:Cutting.Angle                   0.22
Cutting.Speed:Tool.Geometry:Cutting.Angle     0.35
```

Problem 1 (cont'd)

(b) Which effects are statistically significant at the 5% level?

Tool geometry, cutting angle, Cutting.Speed:Cutting.Angle

(c) Estimate the factorial effect of cutting angle? What is it's standard error?

$2 \times 3.42 = 6.84$ and $2 \times 1.12 = 2.24$ are the estimates of the main effect and standard error.

(d) Interpret the main effect of cutting angle.

The average change in life hours when cutting angle is high versus low is 6.84. But, since there is a significant interaction between cutting angle and cutting speed, cutting angle should be interpreted at different levels of cutting speed.

(e) Would using a half-normal plot be useful in determining which effects are significant?

No. The design is replicated so the p-value and confidence interval can be estimated.

(f) Suppose that the engineer wanted to study an additional factor using the same 2^3 factorial design. Suggest a design where the engineer would have the ability to estimate all the main effects and two-factor interactions. What is the design matrix of this design?

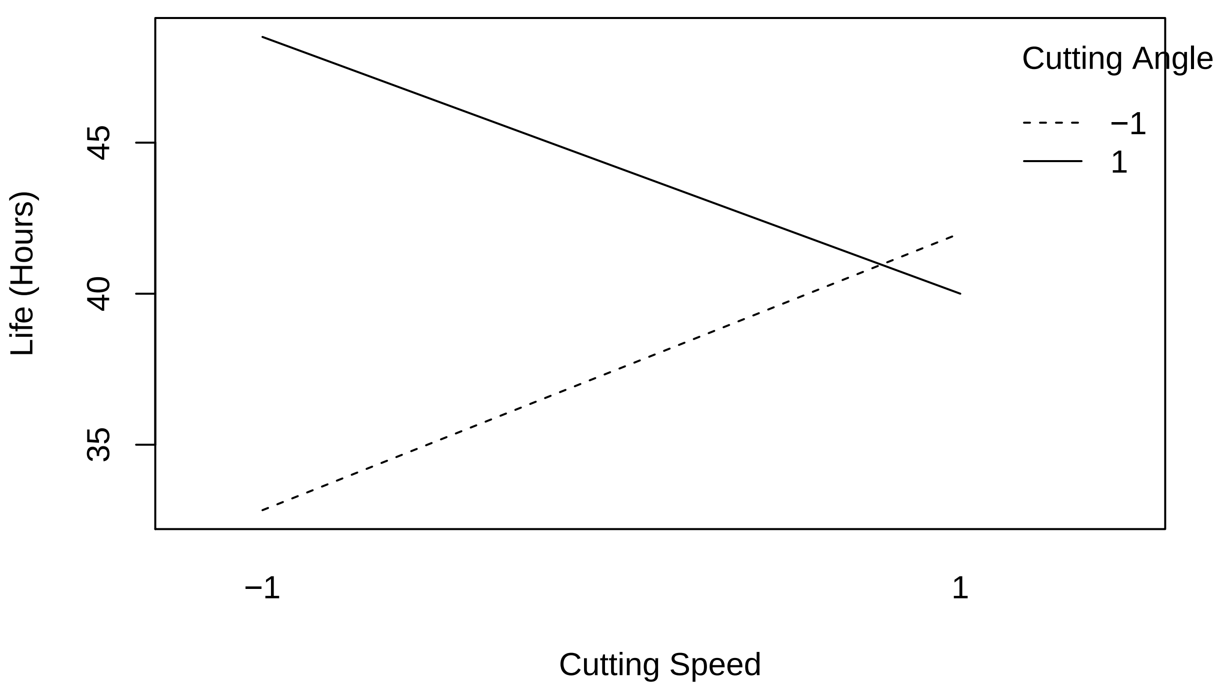
Use a 2^{4-1} fractional factorial design with $4 = 123$. This will give a design with defining relation $I = 1234$ with aliasing relations:

$$1 = 234, 2 = 134, 3 = 124, 4 = 123, 12 = 34, 13 = 24, 14 = 23.$$

The design matrix (derived in the review session) is:

```
   A  B  C  D
1  1  1 -1 -1
2 -1  1 -1  1
3  1  1  1  1
4  1 -1  1 -1
5  1 -1 -1  1
6 -1  1  1 -1
7 -1 -1  1  1
8 -1 -1 -1 -1
class=design, type= FrF2.generators
```

Problem 1 Interaction plot



Problem 2

To compare the effects of five different assembly methods (A, B, C, D, E) on the throughput (the number of completed pieces per day), an experiment was conducted involving three blocking variables: day, operator, and machine type. Five operators were tested over five days on five machine types. Each pair of assembly method and machine type is tested on one day by one operator (i.e. once in the experiment). What is the name of this experimental design? Explain your reasoning.

> This is a Graeco-Latin square. There are three blocking variables with five levels and each pair of assembly method and machine type appears once in the experiment.

Problem 3

The following are weights (in grams) of six rock samples (randomly collected from different lakes) measured on two different scales (I, II). The purpose of this data collection is to test whether the two scales are different.

Scale	1	2	3	4	5	6
I	8	14	16	19	18	12
II	11	16	20	18	20	15

What type of statistical design was used in this experiment? What statistical test could be used to test whether the two scales are different?

> This is a paired design (blocks of size two) since each sample is weighed on two scales. An appropriate analysis would be a paired t-test.