Lecture 10
Feb 10th, 2015

Binary Search of a sorted list of n Elements

- split into 2 pieces, look at one of them, recurse.
- The left side is $\lfloor \frac{n}{2} \rfloor$, the right side is $\lceil \frac{n}{2} \rceil$
- $\lfloor \frac{n}{2} \rfloor, \lceil \frac{n}{2} \rceil \in \mathbb{N}$ and $\lceil \frac{n}{2} \rceil + \lfloor \frac{n}{2} \rfloor = n$
- For $n \in \mathbb{N}$, let $T(n)$ be worst-case number of steps
- $T(n) = 1 + \max(T(\lfloor \frac{n}{2} \rfloor), T(\lceil \frac{n}{2} \rceil))$, $n \geq 2$ or 1 when $n=1$

e.g. $T(236) = 1 + \max(T(118), T(118)) = 1 + \max(1 + \max(T(59), T(59)), 1 + \max(T(59), T(59))) = \cdots$

$T(256) = 1 + \max(T(128), T(128)) = 1 + T(128) = 1 + 1 + \max(T(64), T(64)) = \cdots$
$= 1+1+1+\max(T(32), T(32)) = 1+1+1+1+\max(T(16), T(16))$
$= 5 + T(8) = 6 + T(4) = 7 + T(2) = 8 + T(1) = 9$

For $k \in \mathbb{N}$, $T(2^k) = 1 + \max(T(\lfloor \frac{2^k}{2} \rfloor), T(\lceil \frac{2^k}{2} \rceil)) = 1 + \max(T(\lfloor 2^{k-1} \rfloor), T(\lceil 2^{k-1} \rceil))$
$k \in \mathbb{N}$ and $k-1 \geq 0$, so $2^{k-1} \in \mathbb{N} = 1 + T(2^{k-1})$

$T(2^8) = 1 + T(2^{8-1}) = 1 + 1 + T(2^{8-1-1}) = 1+1+1+T(2^{8-1-1-1}) = \cdots = 8 + T(2^{8-8}) = 8 + T(1) = 9$

$T(2^k) = 1 + T(2^{k-1}) = 2 + T(2^{k-2}) = \cdots$ // onrolling rewinding
$= k + T(2^{k-k}) = k + T(1) = k+1$

For $n = 2^k$, i.e. $k = \log_2 n$ s.t. $k \geq 1$: $T(n) = k + 1 = 1 + \log_2 n$

$k \leq 1$
$T(k) = 1 + \max(T(\lfloor \frac{k}{2} \rfloor), T(\lceil \frac{k}{2} \rceil))$
$T(1) = 1 + \max(T(\lfloor \frac{1}{2} \rfloor), T(\lceil \frac{1}{2} \rceil))$

$T(236) \leq T(239)$
$T(236) = 1 + \max(T(118), T(118))$
$T(239) = 1 + \max(T(120), T(120))$
For $n \in \mathbb{N}$, $n \geq 1$, let $P(n)$ be T is non-decreasing from 1 up to (but not including?) n.