PROBLEM-SOLVING AND PROOFS: ASSIGNMENT 2
DUE FRIDAY, MARCH 10, 4PM.

Warm-up problems. These are completely optional.

(1) From outside mathematics, give an example of statements A, B, C such that A and B together imply C, but such that neither A nor B alone implies C. 2.8

(2) The negation of the statement "No kangaroos attend this school" is:
  (a) All kangaroos attend this school.
  (b) All kangaroos do not attend this school. 2.9
  (c) Some kangaroos attend this school.
  (d) Some kangaroos do not attend this school.

(3) Let P and Q be well-defined mathematical statements. Prove that the following are always true, no matter the truth values of P and Q.
  (a) $P \wedge Q \implies P$.
  (b) $P \implies P \vee Q$. 2.44
  (c) $Q \wedge \neg Q \implies P$.

Problems to be handed in. Solve three of the following four problems. 2.28

(1) Consider the equation $x^4y + ay + x - 0$.
  (a) Show that the following statement is false. "For all $a, x \in \mathbb{R}$, there is unique y such that $x^4y + ay + x = 0$."
  (b) Find the set of real numbers a such that the following statement is true. "For all $x \in \mathbb{R}$, there is a unique y such that $x^4y + ay + x = 0$."

(2) Let $P(x)$ be the assertion "x is odd" and let $Q(x)$ be the assertion "$x^2 - 1$ is divisible by 8." Determine whether the following statements are true.
  (a) $(\forall x \in \mathbb{Z})\ (P(x) \implies Q(x))$, 2.47
  (b) $(\forall x \in \mathbb{Z})\ (Q(x) \implies P(x))$.

(3) Using statements about set membership, prove the statements below, where A, B, C are any sets. Use a picture to illustrate the results and guide the proofs.
  (a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
  (b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ 2.51

(4) Consider tokens that have some letter written on one side and some integer written on the other, in unknown combinations. The tokens are laid out, some with the letter side up, some with number side up. Explain which tokens must be turned over to determine whether these statements are true. 2.30
  (a) Whenever the letter side is a vowel, the number side is odd.
  (b) The letter side is a vowel if and only if the number side is odd.