University of Toronto
Department of Mathematics

MAT224H1F
Linear Algebra II

Midterm Examination
October 25, 2011

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Duration: 1 hour 50 minutes

Last Name: ____________________

Given Name: ____________________

Student Number: ____________________

Tutorial Group: ____________________

No calculators or other aids are allowed.

FOR MARKER USE ONLY	
Question	Mark
1	/10
2	/10
3	/10
4	/10
5	/10
6	/10
TOTAL	/60

[10] **1.** Let $T\colon M_{2\times 2}(\mathbb{R}) \to M_{2\times 2}(\mathbb{R})$ be the linear transformation defined by

$$T(A) = \frac{A + A^T}{2}.$$

Find the matrix of T relative to the basis $\alpha = \left\{ \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \right\}$ for $M_{2\times 2}(\mathbb{R})$.

EXTRA PAGE FOR QUESTION 1 - do not remove.

[10] **2.** Let $T\colon P_2(\mathbb{R}) \to P_2(\mathbb{R})$ be the linear transformation defined by

$$T(a + bx + cx^2) = (-2b + 11c) + (-2a + c)x + (3a - b + 4c)x^2.$$

Find bases for the kernel and image of T.

EXTRA PAGE FOR QUESTION 2 - do not remove.

[10] **3.** Let $V = P_4(\mathbb{R})$ and $W = \{p(x) \in P_5(\mathbb{R}) \mid p(1) = 0\}$. Show that V and W are isomorphic and find an isomorphism $T\colon V \to W$.

[10] **4.** Let $T\colon \mathbb{C}^2 \to \mathbb{C}^2$ be the linear transformation whose matrix with respect to the standard basis of $\mathbb{C}^2$ is

$$\begin{bmatrix} 1 & i \\ -i & 1 \end{bmatrix}.$$

Find a basis α for $\mathbb{C}^2$ consisting of eigenvectors of T and find $[T]_{\alpha\alpha}$.

EXTRA PAGE FOR QUESTION 4 - do not remove.

[10]**5.** Let $T\colon P_2(\mathbb{R}) \to P_1(\mathbb{R})$ be the linear transformation defined by

$$T(a + bx + cx^2) = (a - 3b + c) + (2a - 6b + 3c)x.$$

Find bases α' for $P_2(\mathbb{R})$, and β' for $P_1(\mathbb{R})$ such that $[T]_{\beta'\alpha'}$ is the reduced row echelon form of $[T]_{\beta\alpha}$ where α and β are the standard bases for $P_2(\mathbb{R})$ and $P_1(\mathbb{R})$ respectively.

EXTRA PAGE FOR QUESTION 5 - do not remove.

6. Let V and W be vector spaces over a field F. Let $\alpha = \{v_1, v_2, \ldots, v_n\}$ be a basis for V, and $\beta = \{w_1, w_2, \ldots, w_m\}$ a basis for W. Let $T\colon V \to W$ be a linear transformation.

[5] **(a)** Prove that T is surjective if and only if the columns of $[T]_{\beta\alpha}$ span F^m.

[5] **(b)** Prove that T is injective if and only if the columns of $[T]_{\beta\alpha}$ are linearly independent in F^m.