Leture week 8

Two-state model (multi-) →

Binomial model →

Poisson model →

u

q

d.

final target is surving probability

Example

$$L = \overbrace{q_x^{\delta}(1-q_x)^{N-w-\delta}}^{\text{contribution from uncensored}} \underbrace{(1-q_x)^{0.5w}}_{\text{contribution from censored}}$$

$$= q_x^{\delta}(1-q_x)^{N-0.5w-\delta}$$

$$l = \delta \cdot \log q_x + (N-0.5w-\delta)\log(1-q_x)$$

$$l' = \frac{\delta}{q_x} + \frac{N-0.5w-\delta}{1-q_x}\cdot(-1)$$

$$l' = 0 \Rightarrow \delta(1-q_x) = (N-0.5w-\delta)q_x$$

$$\delta = (N-0.5w)q_x$$

$$\Rightarrow \hat{q}_x = \frac{\delta}{N-0.5w}$$

More details on Poisson model:

X: total number of deaths

δ: observed number of deaths.

$$P(X=j) = \frac{e^{-\lambda}\lambda^j}{j!}$$

$E(X) = \lambda$ $\qquad \hat{\lambda} = \delta$

observed

e.g. total number of car accidents in an intersection is 10 in last year.

Assuming no. of car accidents this year ~ poisson (λ) $\quad \hat{\lambda} = 10$

But if you know no. of car accidents over last 5 years are, 12, 11, 8, 10, 10.

Then $\hat{\lambda} = \frac{12+11+8+10+10}{5}$

Now further define $\lambda = \mu \cdot E_x^c$

$$\hat{\mu} = \frac{\hat{\lambda}}{E_x^c} = \frac{\delta}{E_x^c}$$

If $X \sim \text{Poisson}(\lambda)$

when λ is large.

X is approximately normal

$$X \sim \text{Normal}(\underbrace{\lambda}_{\text{mean}}, \underbrace{\lambda}_{\text{variance}})$$

By CLT.

Intuition behind Poisson Model

$X = \sum_{i=1}^{n} X_i$, $X_i \sim \text{Bern}$

when $n \to \infty$ $X \sim \text{Poisson}$

Exercise 1

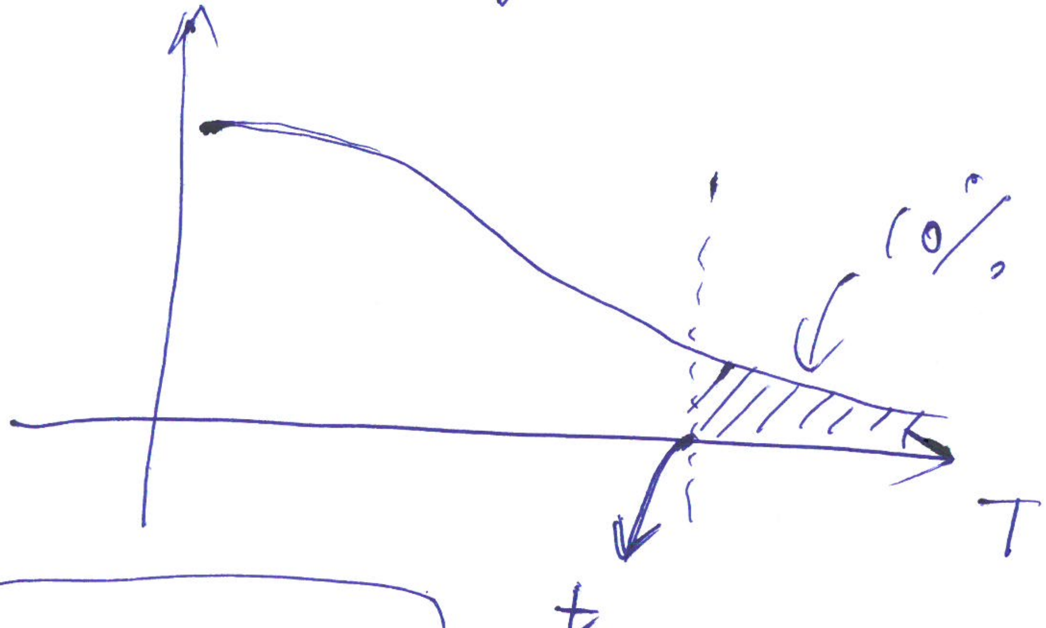
(a) $1000 \times 0.3 = 300$

(b) $${}_3p^{\overline{AA}} = e^{-3(\underline{0.3+0.003+0.001})}$$

$$= 0.40172$$

(c)

(d) T: remaining time that Jennie stays in "intern"



$$\boxed{P(T \geq t)} = 0.1$$

$${}_tp^{\overline{II}} = \underline{e^{-t(0.003+0.001+0.3)}} = \underline{0.1}$$