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0.1 Hypothesis Test

1. Step I (Specify hypotheses)
   $H_0 : \beta_1 = 0$ which is a specific hypothesis
   $H_A : \beta_1 > 0$ however, is a general hypothesis.

2. Step II (Test statistics)
   Estimated value $\hat{\beta}_1$.
   And under H_0, $E[\beta_1|H_0]$ is 0.
   So the
   $$\frac{\hat{\beta}_1 - E[\beta_1|H_0]}{\hat{se}(\hat{\beta}_1)} \sim t_{n-2}$$
   is the standardized test statistics, where
   $$\hat{se}(\hat{\beta}_1) = \sqrt{\frac{\hat{\sigma}^2}{s_{xx}}} = \frac{\hat{\sigma}}{\sqrt{s_{xx}}}$$

3. Step III (Decision rule)
   Reject H_0 at $\alpha = 0.05$ if observed test statistic $> t_{n-2=136, 0.95} = 1.66$
   Note that $0.95 = 1 - \alpha$

4. Step IV (Calculations)
   Observed test statistic
   $$t = \frac{\hat{\beta}_1 - 0}{\hat{se}(\hat{\beta}_1)} = \frac{0.007045 - 0}{\sqrt{\frac{\hat{\sigma}^2}{s_{xx}}}} = \frac{0.007045}{\sqrt{0.0259/218994.5}} = 20.48$$

5. Step V (Conclusion)
   As $20.48 \gg 1.66$, reject H_0 in favour of H_A and conclude $\beta_1 > 0$.
   So the slope of the linear regression model relating average tempera-
   ture anomalies(?) to year (time) is significantly positive $\implies$ temper-
   atures have been increasing over time!

Finally, ew need to assess the analysis and the results

- Is the data we used the best available to address the research question?
  (issue of the science)

- Is the model a good fit?
  (is the model not only appropriate, but also useful?)

$$R^2 = \frac{SS_{Regression}}{SS_{Total}} = 75.5\%$$

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is the coefficient of determination.

Just a summary measure! better answer is look at the assumptions underlying the model and check if they are satisfied.

Check the residual plots (here that means we still have a caveat!

- (!) Linear models just address association (correlation), they do not necessarily reflect causation (no measure of possible causal factors in these data).

- (!) Also this analysis does not necessarily rule out alternative explanations.
  Like time series (seasonal trend maybe?)

In summary, model and analysis are indicative of a positive (association), an increase in global av. temp over time, but the research question is wider than that and we need to do further research (more data, more analysis).