

STAT6039 week 5 lecture 12

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About mixed random variables (see textbook section 4.11)

Definition: Suppose a random variable has cdf

$$F(y) = cF_1(y) + (1 - c)F_2(y)$$

where $0 < c < 1$, F_1 is the cdf of discrete random variable X_1 , F_2 is the cdf of discrete random variable X_2 .

Then we say that Y is a **mixed random variable** and Y has a ...

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The cdf of Y is like the cdf of a continuous random variable but it also has "jumps".

Example 1: 2 coins are tossed. If 2 tails then $Y = 0$, if 2 heads then $Y = 1$. Otherwise, Y is a number chosen randomly uniformly between 0 and 1.

For $y < 0$, $P(Y \leq y) \dots$

For

For $0 < y < 1$

$P(Y \leq y) = ?$ (use LTP)

$$\begin{aligned} P(Y \leq y) &= P(TT)P(Y \leq y|TT) + P(HH)P(Y \leq y|HH) + P(HT \cup TH)P(Y \leq y|HT \cup TH) \\ &= \frac{1}{4}P(Y = 0|TT) + \frac{1}{4} \cdot 0 + \frac{1}{2}P(Z \leq y) \\ &= \frac{1}{4} \times 1 + 0 + \frac{1}{2}y \end{aligned}$$

So

$$F(y) = \begin{cases} 0, & y < 0 \\ \frac{1}{4} + \frac{1}{2}y, & 0 \leq y < 1 \\ 1, & y \geq 1 \end{cases}$$

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