PHL 245: Practice Test: Second Test
Focus on Units 4-6

UPCOMING TEST WILL BE (roughly):
40%: derivations   35%: symbolization   25%: concepts and truth-tables

A^1: *a* advertises.
B^1: *a* is a business
C^1: *a* is a store.
D^1: *a* has diamonds on it.
E^1: *a* is an engagement ring.
F^1: *a* is a person.
G^1: *a* is a restaurant
H^1: *a* stays in business.
L^2: *a* is displayed at *b*.
M^2: *a* is getting married to *b*.
N^2: *a* is more expensive than *b*.
O^3: *a* buys *b* from *c* (*c* sells *b* to *a*).

a^0: Tiffany's
b^0: Brian
c^0: Carol
d^1: the fiancé of *a*
e^1: the sister of *a*
b^2: the best man at the wedding of *a* and *b*.

1. Use the above symbolization scheme to symbolize the following sentences:
(There are more here for extra practice than will be on the second test.)

(a) Although stores and restaurants are businesses, not all businesses advertise.

$\forall x(Cx \vee Gx \to Bx) \wedge \sim\forall x(Bx \to Ax)$ OR $\forall x(Cx \to Bx) \wedge \forall x(Gx \to Bx) \wedge \exists x(Bx \wedge \sim Ax)$

(b) Assuming that people don't buy things from businesses that don't advertise, restaurants stay in business only if they do.

$\forall x(Bx \wedge \sim Ax \to \sim\exists y\exists z(Fy \wedge O(yzx))) \to \forall x(Gx \to (Hx \to Ax))$

or $\sim\exists x\, (Bx \wedge \sim Ax \wedge \exists y\exists z(Fy \wedge O(yzx))) \to \forall x(Gx \wedge Hx \to Ax)$

(c) In order for Carol's fiancé to get married to Carol, it is necessary that he buys her an engagement ring from Tiffany's.

$M(d(c)c) \to \exists x(Ex \wedge O(d(c)xa))$

or $M(cd(c))$ for the first part.

(d) Stores that display engagement rings sell things with diamonds on them to people.

$\forall x(Cx \wedge \exists y(Ey \wedge L(yx)) \to \exists z(Dz \wedge \exists w(Fw \wedge O(wzx))))$

or $\forall x(Cx \to (\exists y(Ey \wedge L(yx)) \to \exists z(Dz \wedge \exists w(Fw \wedge O(wzx)))))$

or $\forall x(Cx \to \forall y(Ey \wedge L(yx) \to \exists z(Dz \wedge \exists w(Fw \wedge O(wzx)))))$

Watch the effect of brackets on the quantifier for y (See confinement in unit 5 part 1: 5.10)

(e) Everyone buys things from stores, but no store sells things to everyone.

$\forall x(Fx \to \exists y\exists z(Cz \wedge O(xyz))) \wedge \sim\exists x(Cx \wedge \forall y(Fy \to \exists z O(yzx))$

OR second conjunct: $\forall x(Cx \to \exists y\, (Fy \wedge \sim\exists z\, O(yzx)))$

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(f) The only store that Brian buys an engagement ring from is Tiffany's.

$\forall x(Cx \wedge \exists y(Ey \wedge O(byx)) \leftrightarrow x{=}a)$

OR $\exists x(\forall y(Cy \wedge \exists z(Ez \wedge O(bzy)) \leftrightarrow x{=}y) \wedge x{=}a)$

(g) If anybody buys an engagement ring from a store then he/she is getting married to somebody.

$\forall x(Fx \wedge \exists y(Ey \wedge \exists z(Cz \wedge O(xyz))) \rightarrow \exists w(Fw \wedge M(xw))$

(h) The best man at Carol and Brian's wedding is Carol's sister's fiancé.

$a(bc){=}d(e(c))$

(i) Give an idiomatic English translation of:

$\exists x(Cx \wedge \forall y(Cy \wedge x \neq y \rightarrow N(xy)) \wedge \forall z(L(zx) \rightarrow Dz))$.

Everything displayed at the most expensive store has diamonds on it.

(j) Disambiguate this ambiguous sentence by providing two symbolizations. For each, provide an English sentence that makes the meaning clear.

Everybody buys something from a store.

The two natural interpretations are:

$\forall x(Fx \rightarrow \exists y\exists z(Cz \wedge O(xyz)))$ Everybody buys something from some store or another. (diff. things, diff. stores)

$\exists z(Cz \wedge \forall x(Fx \rightarrow \exists y O(xyz)))$ There is one store that everybody buys something from. (diff. things, same store)

Two other (less plausible) interpretations:

$\exists y\forall x(Fx \rightarrow \exists z(Cz \wedge O(xyz)))$ Everybody buys the same one thing from some store or another. (same thing, diff. stores)

$\exists y\exists z(Cz \wedge \forall x(Fx \rightarrow O(xyz)))$ Everybody buys the same thing from the same store. (same thing, same store)

3. Use a full truth table to determine whether the following is a tautology, a contradiction or a contingent sentence. State which it is and briefly explain how you know.

$$P \rightarrow (Q \vee R) \wedge \sim(P \leftrightarrow Q)$$

P	**Q**	**R**		**P**	↓ **→**	**(Q**	**∨**	**R)**	**∧**	**~**	**(P**	**↔**	**Q)**	
T	T	T		T	F	T	T	T	F	F	T	T	T	
T	T	F		T	F	T	T	F	F	F	T	T	T	← false on this TVA
T	F	T		T	T	F	T	T	T	T	T	F	F	← true on this TVA
T	F	F		T	F	F	F	F	F	T	T	F	F	
F	T	T		F	T	T	T	T	T	T	F	F	T	
F	T	F		F	T	T	T	F	T	T	F	F	T	
F	F	T		F	T	F	T	T	F	F	F	T	F	
F	F	F		F	T	F	F	F	F	F	F	T	F	

This is a contingent sentence since some lines below the main connective are true and some false.

4. Use a shortened truth-table of one line to show that the following argument is INVALID.

$P \rightarrow Q \vee R$. $\quad \sim(Q \leftrightarrow \sim S \wedge P)$. $\quad \therefore P \rightarrow R$.

P	**Q**	**R**	**S**
T	T	F	T

P	↓ **→**	**Q**	**∨**	**R**	**.**	↓ **~**	**(Q**	**↔**	**~**	**S**	**∧**	**P)**	**∴**	**P**	↓ **→**	**R**
T	T	T	T	F		T	T	F	F	T	F	T		T	F	F

The premises are both true, but the conclusion is false. Therefore, the argument is INVALID.

5. Show that the following arguments are valid:

a) $\forall y(Fy \rightarrow \exists z(Jz \wedge Gz))$. $\exists x(Jx \vee Bx) \rightarrow \forall x\forall yH(xy)$. $\therefore \forall x(Fx \rightarrow \exists y(Gy \wedge H(xy)))$

Tests basic skills with UD, EG, EI and UI.

practest2 deriv1: ∀y(Fy → ∃z(Jz ∧ Gz)). ∃x(Jx ∨ Bx) → ∀x∀yH(xy) ∴∀x(Fx → ∃y(Gy ∧ H(xy)))

1	~~Show~~ ∀x(Fx→∃y(Gy∧H(xy)))	"show conc"
2	~~Show~~ Fx→∃y(Gy∧H(xy))	"show inst"
3	Fx	ass cd
4	Fx→∃z(Jz∧Gz)	pr1 ui
5	∃z(Jz∧Gz)	3 4 mp
6	Ji∧Gi	5 ei
7	Ji	6 sl
8	Gi	6 sr
9	Ji∨Bi	7 add
10	∃x(Jx∨Bx)	9 eg
11	∀x∀yH(xy)	10 pr2 mp
12	∀yH(xy)	11 ui
13	H(xi)	12 ui
14	Gi∧H(xi)	8 13 adj
15	∃y(Gy ∧ H(xy))	14 eg
16		15 cd
17		2 ud

b) ∃x∀y(H(xyy) →∀z~L(xz)). ∀x∀y(Gx→∃zK(zy)) → ∀x∃y∀zH(xyz).
∀x(∃z(Gz ∧ ~Mz) → K(xx)). ∴~∃zMz → ~∀yL(yy)

A bit tougher… You will need to see that you have to derive the antecedent of premise 2 (or the negation of the consequent.) Then it is all about matching.

Always use a new term for EI (ASAP… use a brand new term.)

Use UI to match (always match a free term… often the arbitrary term from EI or from setting up a UD.)

Use EG to match a bound variable (often in show lines, consequents of show lines and antecedents of lines that you haven't used.)

Problem: ∃x∀y(H(xyy) →∀z~L(xz)). ∀x∀y(Gx→∃zK(zy)) → ∀x∃y∀zH(xyz).
∀x(∃z(Gz ∧ ~Mz) → K(xx)) ∴~∃zMz → ~∀yL(yy) C

1	~~Show~~ ~∃zMz→~∀yL(yy)	"show conc"	
2	~∃zMz	ass cd	
3	~~Show~~ ~∀yL(yy)	"show cons"	
4	∀yL(yy)	ass id	
5	∀y(H(iyy)→∀z~L(iz))	pr1 ei/i	This could be done later.
6	~~Show~~ ∀x∀y(Gx → ∃zK(zy))		Need to show ANT of pr2 so you can use MP (or the negation of consequent… so you can use MT.)
7	~~Show~~ ∀y(Gx→∃zK(zy))	"show inst"	
8	~~Show~~ Gx→∃zK(zy)	"show inst"	
9	Gx	ass cd	
10	∃z(Gz∧~Mz)→K(yy)	pr3 ui/y	Match the free term y in the K relation of show line 8.
11	∀z~Mz	2 qn	
12	~Mx	11 ui/x	Match the free term x in line 9 so that you can build the ant. of 10.
13	Gx∧~Mx	9 12 adj	
14	∃z(Gz∧~Mz)	13 eg	
15	K(yy)	10 14 mp	
16	∃zK(zy)	15 eg	Match the bound variable in the consequent of show line 8.
17		16 cd	
18		8 ud	
19		7 ud	
20	∀x∃y∀zH(xyz)	pr2 6 mp	
21	∃y∀zH(iyz)	20 ui/i	Match the free i of line 5.
22	∀zH(ikz)	21 ei/k	EI to a new term.
23	H(ikk)→∀z~L(iz)	5 ui/k	Match the free k of line 22
24	H(ikk)	22 ui/k	Match the free k of line 23
25	∀z~L(iz)	23 24 mp	
26	L(ii)	4 ui/i	Match the free i of line 25
27	~L(ii)	25 ui/i	Match the free i of line 26.
28		26 27 id	
29		3 cd	

c) ∃x∀yF(d(x)yd(y)).   ∃xF(xxd(x)) → ∀w∀z~(A(wz) ↔ B(wz)).

∴ ~ ∀x∃yA(xy) → ~∀x(A(xa) ∨ ~B(xx))

Practest2 Q3: ∃x∀yF(d(x)yd(y)). ∃xF(xxd(x)) → ∀w∀z~(A(wz) ↔ B(wz)) ∴ ~ ∀x∃yA(xy) → ~∀x(A(xa) ∨ ~B(xx))

1	~~Show~~ ~∀x∃yA(xy)→~∀x(A(xa)∨~B(xx))	"show conc"	
2	~∀x∃yA(xy)	ass cd	
3	~~Show~~ ~∀x(A(xa)∨~B(xx))	"show cons"	
4	∀x(A(xa)∨~B(xx))	ass id	
5	∃x~∃yA(xy)	2 qn	
6	~∃yA(iy)	5 ei/i	
7	∀y~A(iy)	6 qn	
8	A(ia)∨~B(ii)	4 ui/i	
9	~A(ia)	7 ui/a	
10	~B(ii)	8 9 mtp	
11	∀yF(d(k)yd(y))	pr1 ei/k	EI ASAP to a new term
12	F(d(k)d(k)d(d(k)))	11 ui/d(k)	Now match the form of the
13	∃xF(xxd(x))	12 eg	antecedent of premise 2:
14	∀w∀z~(A(wz)↔B(wz))	13 pr2 mp	F(_ _ d(_))
15	∀z~(A(iz)↔B(iz))	14 ui/i	The second term must match the
16	~(A(ii)↔B(ii))	15 ui/i	first term. Since the first term is
17	A(ii)↔~B(ii)	16 nb	given in line 11: d(k), that is what you are putting in for all the y's.
18	~B(ii)→A(ii)	17 bc	
19	A(ii)	10 18 mp	
20	~A(ii)	7 ui/i	Use UI on line 7 a second time!
21		19 20 id	
22		3 cd	