

Marks for problem set 2 have been posted.  
Some very quick notes:

(1) If  $G$  is an infinite group, then it is not guaranteed that there exists an element of infinite order. As an example, take the infinite direct sum of  $\mathbb{Z}_2$ .

(2) If you're wanting to prove if and only if, be careful about contrapositives. For example, if you wanted to prove that  $G$  abelian implies the inversion map is an isomorphism, this is NOT equivalent to " $G$  not abelian implies inversion not an isomorphism".

Remember: " $A$  implies  $B$ " is equivalent to " $\text{Not } B$  implies  $\text{Not } A$ ".

(3) There are many ways to express the identity of  $S_n$ . Examples include (for sufficiently high  $n$ )

$$e = (1) = (12)(12) = (3) = (123)(321) = (1)(2)(3) \text{ \&c}$$

The element  $(12345)$  in  $S_5$ , however, is not the identity. It's easy to see this, since the identity sends 1 to 1, and this permutation sends 1 to 2.

(4) If  $z$  is an element of a finite subgroup  $G$  in  $C^*$ , then Lagrange tells us that it satisfies  $z^{|G|} = 1$ . Bizarrely, a lot of people wrote it satisfied

$$z|G| = 1$$

Clearly for each  $|G|$ , there is only one solution to this,  $z = 1/|G|$ . (ranging over all  $|G|$ , an infinite number of solutions)