

1. We prove that  $\forall n \geq 2, 2^n + 3^n < 4^n$ , using simple induction.

**Base Case:**  $2^2 + 3^2 = 4 + 9 = 13 < 16 = 4^2$ .

**Ind. Hyp.:** Assume  $n \in \mathbb{N}$  and  $n \geq 2$  and  $2^n + 3^n < 4^n$ .

$$\begin{aligned}
 \text{Ind. Step: } 2^{n+1} + 3^{n+1} &= 2 \cdot 2^n + 3 \cdot 3^n \\
 &< 4 \cdot 2^n + 4 \cdot 3^n \\
 &= 4 \cdot (2^n + 3^n) \\
 &< 4 \cdot 4^n && \text{(by the I.H.)} \\
 &= 4^{n+1}
 \end{aligned}$$

Hence, by induction,  $\forall n \geq 2, 2^n + 3^n < 4^n$ .

2. **Predicate:** Let  $P(n)$  be the statement: “any  $n$  squares can be dissected and rearranged to form one square”.

**Base Case:** Any one square is already a square. Hence,  $P(1)$ .

**Ind. Hyp.:** Assume  $n \geq 1$  and  $P(n)$  (any  $n$  squares can be dissected and rearranged to form one square).

**Ind. Step:** Suppose  $S_1, S_2, \dots, S_n, S_{n+1}$  are  $n + 1$  squares.

Since  $n + 1 \geq 2$  (because  $n \geq 1$ ) and  $P(2)$  is true (given),  $S_n$  and  $S_{n+1}$  can be dissected and rearranged to form one new square  $S'$ .

Then,  $S_1, S_2, \dots, S_{n-1}, S'$  is a collection of  $n$  squares. By the inductive hypothesis, they can be dissected and rearranged into one square.

During this dissection,  $S'$  will be cut by a finite number of straight lines, which adds a finite number of further cuts to the original squares  $S_n$  and  $S_{n+1}$ .

Hence,  $S_1, \dots, S_{n+1}$  can be dissected and rearranged to form one square.

Since  $S_1, \dots, S_{n+1}$  were arbitrary,  $P(n + 1)$ .

**Conclusion:** By induction,  $\forall n \geq 1, P(n)$ .

NOTE: There were many other correct ways to prove the induction step, *e.g.*, we could have used the induction hypothesis first, to conclude that  $S_1, S_2, \dots, S_n$  can be dissected and rearranged to form one square  $S'$ , then  $P(2)$  to conclude that  $S', S_{n+1}$  can be dissected and rearranged to form one square.