Feb 13th

Quiz 2 solutions see the notes.

True or false (pg. 241)

16. a) center of a 95% CI for the mean is a r.v. TRUE

b). 95% CI for μ contains $\bar{X}$ (sample mean) with prob. 0.95. FALSE

c). 95% CI contains 95% of population FALSE only has sth. to do with mean

d). Out of 100 95% CI, 95 will contain μ. FALSE

b. $\bar{X} \pm 1.96\,\sigma_{\bar{X}}$

d. $P(\mu \in CI) = 0.95$ True but cannot say "out of 100, 95 ..."

(pg. 245)

38. $X_1, \dots, X_n$ is$_n$ SRS

Show that $\sum_{i=1}^{n} X^3$ is an unbiased estimator for $\frac{1}{N}\sum_{i=1}^{N} X_i^3$

X_1^3 is unbiased est.

X_1^3, X_2^3

$$E[X_1^3, X_2^3] = E[X_1^3 + E[X_2^3 \mid X_1]]$$

$$= \frac{1}{N}\sum X_i^3 + \underbrace{\frac{1}{N}\sum_{i=1}^{N}\sum_{j\neq i}\frac{1}{N-1}X_j^3}_{= \frac{1}{N}\sum X_i^3}$$

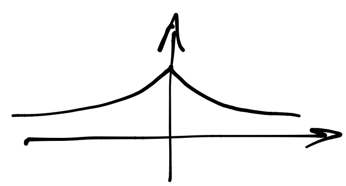
$$E[X_2^3 \mid X_1] = \frac{1}{N-1}\sum_{X_j \neq X_i} X_i^3$$

$$\Rightarrow \frac{1}{2}E[X_1^3, X_2^3] = \frac{1}{N}\sum X_i^3$$

(pg 316)

16. i.i.d. r.v.s. with

$$f(x \mid \sigma) = \frac{1}{2\sigma}e^{-\frac{|x|}{\sigma}}$$



a). Method of moments estimate.

$X_1, \dots, X_n$, $\bar{X} = \sum_{i=1}^{n} X_i / n$

$$\int_{-\infty}^{\infty}\frac{1}{2\sigma}e^{-\frac{|x|}{\sigma}}dx = \frac{1}{2\sigma}\int_{-\infty}^{0} y e^{\frac{y}{\sigma}}dy + \int_0^{\infty} x e^{-\frac{x}{\sigma}}dx$$

$$E[X^2] = \int_{-\infty}^{\infty}\frac{x^2}{2\sigma}e^{-\frac{|x|}{\sigma}}dx$$

$$= \frac{1}{\sigma}\int_0^{\infty} x^2 e^{-x/\sigma}dx$$

$$= \frac{2}{\sigma^2}$$

$$2\sigma^2 = \bar{X} \Rightarrow \hat{\sigma} = \sqrt{\bar{X}/2}$$

b). $L(\sigma) = \prod_{i=1}^{n} \frac{1}{2\sigma} e^{-\frac{|x_i|}{\sigma}}$

$$l(\sigma) = \sum \log(2\sigma) + \frac{|x_i|}{\sigma} = 0$$

$$\Rightarrow \sigma = \frac{\sum |x_i|}{n}$$

c). $\frac{\partial^2}{\partial \sigma^2} \log f(x|\sigma)$

$$I = -E\left[\frac{1}{\sigma^2} - \frac{2|x|}{\sigma^3}\right] = \frac{1}{\sigma^2}$$

$$\text{Variance} = \frac{1}{nI} = \frac{\sigma^2}{n}$$