5.5 & 5.7

Divergence & Stokes' theorems

Recall **Green's Thm**

$$\iint_S \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA = \int_{\partial S} P\,dx + Q\,dy = \int_{\partial S} \mathbb{F}\cdot d\mathbf{x}$$

S ← Region in $\mathbb{R}^2$

divergence version

generalized to surfaces in $\mathbb{R}^3$

$\nabla\cdot\mathbb{F}$

$$\iint_S \left(\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y}\right) dA = \int_{\partial S} \mathbb{F}\cdot n\,ds$$

generalized to $\mathbb{R}^3$

Divergence Thm

$$\iiint_W \nabla\cdot\mathbb{F}\,dV = \iint_{\partial W} \mathbb{F}\cdot n\,dA$$

W: usual triple integral

∂W: Surface integral oriented outward as a closed surface

Stokes' Thm

$$\iint_S \nabla\times\mathbb{F}\cdot n\,dA = \int_{\partial S} \mathbb{F}\cdot d\mathbf{x}$$

S: Surface integral oriented consistently with

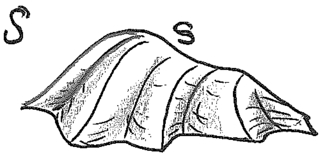
∂S: line integral

Observations: 1. Since $\nabla\cdot\nabla\times\mathbb{G} = 0$ Then $0 = \iiint_W \nabla\cdot\nabla\times\mathbb{G}\,dV = \iint_{\partial W} \nabla\times\mathbb{G}\cdot n\,dA$

$$= \int_{\partial\partial W} \mathbb{G}\cdot d\mathbf{x}$$

$\partial\partial W$ → boundary of a closed surface is a pt.

2. We are given a bad surface S and we need $\iint_S \mathbb{F}\cdot n\,dA$

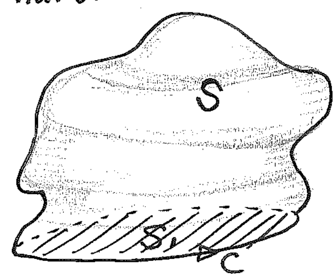


But below S we can find an easier surface S_1 (like xyplane). These two surfaces become surface of a solid W

so $\underbrace{\iiint_W \nabla\cdot\mathbb{F}\,dV}_{A} = \underbrace{\iint_S \mathbb{F}\cdot n\,dA}_{B} + \underbrace{\iint_{S_1} \mathbb{F}\cdot n'\,dA}_{C}$. Now if A & C are easy Then can use $B = A - C$.

3. a trick with using Stokes' Thm:

Surface S is bad but there is an easier surface S_1 that shares same boundary with S:



S_1 is a disc

We are asked to evaluate

$$I_1 = \iint_S \nabla \times \mathbb{F} \cdot n \, dA.$$

by Stokes' $= \int_C \mathbb{F} \cdot dx$

$$= -\iint_{S_1} \underbrace{\nabla \times \mathbb{F} \cdot n_1 \, dA}_{I_2}$$

so I_1 can be avoided & can be replaced by I_2.