CSC165H1 S - Exercise 5
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Question 1: '
Statement S: '' There is no real solution to $x^2+6x+10=0$ '' is True.
Proof:
S: $\neg(\exists x\in R, x^2+6x+10=0)$, S $\Leftrightarrow \forall x\in R, x^2+6x+10\neq 0$
Assume $x\in R$
  Assume $x^2+6x+10=0$ # negation of statement S
    Then $(x+3)^2+1=0$
    Then $(x+3)^2=-1$
    Then $x\in R$
    Then $x-3\in R$
    Then $(x-3)^2\geq 0$
    Then $-1\geq 0$ # contradiction
  Then $x^2-6x+10\neq 0$
Then $\forall x\in R, x^2+6x+10\neq 0$
Then $\neg(\exists x\in R, x^2+6x+10=0)$ # equivalent statement of the above

Then there is no real solution to $x^2+6x+10=0$ is proved to be True.

change '-' to '+'

Question 2: '

(a)

Proof:

Assume $x \in R$, $y \in R$

Case1: $x \geq 0$

Case A: $y \geq 0$

Then $x \geq 0$ and $y \geq 0$

Then $xy \geq 0$

Then $|xy| = xy$

Then $|x| = x$ and $|y| = y$

Then $|x| \cdot |y| = xy$

Then $|x| \cdot |y| = |xy|$

Case B: $y < 0$

Then $x \geq 0$ and $y < 0$

Then $xy \leq 0$

Then $|xy| = -xy$

Then $|x| = x$ and $|y| = -y$

Then $|x| \cdot |y| = x \cdot (-y) = -xy$

Then $|x| \cdot |y| = |xy|$

In either case, $|x| \cdot |y| = |xy|$ is satisfied.

Then $\forall y \in R$, $|x| \cdot |y| = |xy|$

Then $\forall x \geq 0$, $\forall y \in R$, $|x| \cdot |y| = |xy|$

Case2: $x < 0$

Case A: $y \geq 0$

Then $x < 0$ and $y \geq 0$

Then $xy \leq 0$

Then $|xy| = -xy$

Then $|x| = -x$ and $|y| = y$

Then $|x| \cdot |y| = (-x) \cdot y = -xy$

Then $|x| \cdot |y| = |xy|$

Case B: $y < 0$

Then $x < 0$ and $y < 0$

Then $xy \geq 0$

Then $|xy| = xy$

Then $|x| = -x$ and $|y| = -y$

Then $|x| \cdot |y| = (-x) \cdot (-y) = xy$

Then $|x| \cdot |y| = |xy|$

In either case, $|x| \cdot |y| = |xy|$ is satisfied.

Then $\forall y \in R$, $|x| \cdot |y| = |xy|$

Then $\forall x < 0$, $\forall y \in R$, $|x| \cdot |y| = |xy|$

In either case, $\forall y \in R$, $|x| \cdot |y| = |xy|$ is satisfied.

Then $\forall x \in R$, $\forall y \in R$, $|x| \cdot |y| = |xy|$

Question 2: '

(b)

Proof:

Assume $x_1 \in R$, $x_2 \in R$, $y_1 \in R$, $y_2 \in R$

Assume $|x_1| > |x_2| \wedge |y_1| > |y_2|$

Then $|x_1| - |x_2| > 0 \wedge |y_1| - |y_2| > 0$

Then $|x_1| \geq 0$, $|x_2| \geq 0$, $|y_1| \geq 0$, $|y_2| \geq 0$

Then $|y_1| \cdot (|x_1| - |x_2|) > 0 \wedge |x_2| \cdot (|y_1| - |y_2|) > 0$

for positive real number t, $\forall x \in R$, $\forall y \in R$, $x > y \Rightarrow tx > ty$

Then $|y_1| \cdot (|x_1| - |x_2|) + |x_2| \cdot (|y_1| - |y_2|) > 0$

$|y_1| \cdot |x_1| - |y_1| \cdot |x_2| + |x_2| \cdot |y_1| - |x_2| \cdot |y_2| > 0$

$|x_1| \cdot |y_1| - |x_2| \cdot |y_2| > 0$

$|x_1 y_1| > |x_2 y_2|$

Then $|x_1| > |x_2| \wedge |y_1| > |y_2| \Rightarrow |x_1 y_1| > |x_2 y_2|$

Then $\forall x_1 \in R$, $\forall x_2 \in R$, $\forall y_1 \in R$, $\forall y_2 \in R$, $|x_1| > |x_2| \wedge |y_1| > |y_2| \Rightarrow |x_1 y_1| > |x_2 y_2|$

Question 3:

(a)

Solution:

$(1011)_2$ + $(110110)_2$ = $(1000001)_2$

$(1011)_2$ × $(110110)_2$ = $(1001010010)_2$

(b)

Solution:

$(3130)_4$ + $(103)_4$ =$(3233)_4$

$(3130)_4$ × $(103)_4$ = $(1001110)_4$

(c)

Solution:

a = $(342)_8$, b = $(173)_8$

Then a - b = $(342)_8$ - $(173)_8$ = $(147)_8$