

FINANCIAL MATHEMATICS
(STAT 2032 / STAT 6046)

TUTORIAL EXERCISES WEEK 2

Question 1

\$2,500 is invested. Find the accumulated value of the investment 10 years after it is made for each of the following rates.

- (a) 4% annual simple interest
- (b) 4% effective annual compound interest
- (c) 2% effective 6-month compound interest
- (d) 1% effective 3-month compound interest

Question 2

If the effective 3-month compound interest rate is -3%, what is the accumulated value after 1 year of an initial investment of \$1?

Question 3

It is known that the present value of \$864 due in two years is \$600. Find the accumulated value of \$2000 invested at the same rate of compound interest for three years.

Question 4

What is the present value of \$1000 due in 10 years if the effective annual interest rate is 6% for each of the first 3 years, 7% for the next 4 years, and 9% for the final 3 years?

Question 5

Smith needs to borrow \$5,000 for one year.

Under one scenario (A) he is offered a loan at an effective annual rate of 5%. In other words, he has to pay interest of $5,000 \times 0.05 = 250$ at the end of the year.

Under another scenario (B), he is offered a loan of \$10,000 at a lower effective annual rate of interest denoted by i . If he borrows the \$10,000, he can invest the excess \$5,000 for one year at 3%. In other words, at the end of the year he will have to pay interest of $(10,000 \times i)$, but he receives interest of $5000 \times 0.03 = 150$.

How low must the rate on the \$10,000 loan (scenario B) be in order for Smith to prefer it to the \$5,000 loan (scenario A)?

Question 6

Jones invests \$100,000 in a 180-day short term investment at a bank, based on simple interest at an annual rate of 7.5%. After 120 days, interest rates have risen to 9% and Jones would like to redeem the certificate early and reinvest in a 60-day certificate at the higher rate.

In order for there to be no advantage in redeeming early and reinvesting at the higher rate, what early redemption penalty P should the bank charge at the time of early redemption?

Hint: You can approach this question by equating the accumulated value of two scenarios and solving for P .

- A) Jones invests \$100,000 for 180 days at 7.5% simple interest per annum.
- B) Jones invests \$100,000 for 120 days at 7.5% simple interest per annum and then redeems the accumulated amount. Upon redemption he pays a penalty of P . He then reinvests the balance at 9% simple interest per annum for 60 days.

Question 7

- (a) Show that for an effective annual compound interest rate of i , the amount of interest earned in successive years on an investment of \$1 grows by a factor of $(1 + i)$ and these amounts are i , $(1 + i)i$, $(1 + i)^2 i$, ..., $(1 + i)^{n-1} i$ for the first, second, third, ..., n th year, respectively.
- (b) Using the fact that the amount of interest earned from time 0 to time n is $(1 + i)^n - 1$, derive a formula for the sum $1 + (1 + i) + (1 + i)^2 + \dots + (1 + i)^{n-1}$.

Past Exam Question – 2005 Final Exam Q1(d)

At a certain rate of compound interest, 1 will increase to 2 in a years, and 4 will increase to 20 in b years. If 6 will increase to 15 in n years, show that $n = b - a$. (3 marks)