Department of Mathematics	Practice problems
University of Toronto	
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(1) Let G be a simple graph on n vertices and m edges.
  (a) How many induced subgraphs does G have?
  (b) Show that every shortest cycle in G is an induced subgraph.

(2) The *diameter* of a graph G is the greatest distance between two vertices of G.
  (a) Let G be a simple graph of diameter greater than 3. Show that $\bar{G}$ has diameter less than 3.
  (b) Deduce that any self-complementary graph has diameter at most three.

(3) The *centre* of a graph is a vertex u such that $\max\{d(u,v)|v \in V(G)\}$ is as small as possible ($d(u,v)$ denotes the minimal number of edges separating vertex u from vertex v).
  (a) Let T be a tree on at least three vertices, and let T' be the tree obtained from T by deleting all its leaves. Show that T and T' have the same centres.
  (b) Deduce that every tree has exactly one centre or two, adjacent, centres.

(4) Let $G \stackrel{\text{def}}{=} G[X,Y]$ be a bipartite graph in which each vertex is of odd degree. Suppose any two vertices of X have an even number of common neighbours. Show that G has a matching which covers all vertices of X.

(5) Suppose (S,T) is the *unique* minimal cut in some network with source s and sink t. Prove that it is also a minimal (u,v)–cut for *any* two vertices $u \in S$ and $v \in T$.

(6) Draw a girth 6 cubic planar graph or prove that none exists.

(7) A *block* of a graph G is a subgraph B which is nonseparable (B is connected, and there does not exist a vertex v such that $B - \{v\}$ is disconnected) and is maximal with respect to this property. Prove that

$$\chi(G) = \max\{\chi(B)|B \text{ is a block of } G\}.$$