MAT 237, Quiz 8

NAME: **STUDENT ID NUMBER :**

Check your tutorial:

○ TUT5101 TA: Boris	○ TUT5102 TA: James	○ TUT5103 TA: Nan

Part A: (2 marks) State the Divergence Theorem.

Suppose R is a regular region in $\mathbb{R}^3$ with piecewise smooth boundary ∂R, oriented so that the positive normal points out of R. Suppose also that $\vec{F}$ is a vector field of class C' on R. Then

$$\iint_{\partial R} \vec{F} \cdot \vec{n} \, dA = \iiint_R \operatorname{div} \vec{F} \, dV$$

Part B: (4 marks) Compute the curl of the vector field $\mathbf{F}(x, y, z) = y^3\mathbf{i} + xy\mathbf{j} - z\mathbf{k}$.

$$\operatorname{curl} \vec{F} = \det \begin{pmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial x & \partial y & \partial z \\ y^3 & xy & -z \end{pmatrix}$$

$$= (y - 3y^2)\, \vec{k}$$

Part C: (4 marks) Suppose R is a regular region in $\mathbb{R}^3$ with piecewise smooth boundary, and f and g are functions of class C^2 on $\bar{R}$. Then show

$$\iint_{\partial R} f\nabla g \cdot \mathbf{n} dA = \iiint_R (\nabla f \cdot \nabla g + f\nabla^2 g) dV.$$

$$\operatorname{div}(f \nabla g) = \nabla f \cdot \nabla g + f(\nabla \cdot \nabla g)$$

$$\Rightarrow \iint_{\partial R} f \nabla g \cdot \vec{n} \, dA = \iiint_R \operatorname{div}(f \nabla g) \, dV$$

$$= \iiint_R (\nabla f \cdot \nabla g + f \nabla^2 g) \, dV$$