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Here is a list of important proofs which are implicitly or explicitly presented in the textbook. Please read and make sure all of these blocks of proofs are understood. You will be asked to reproduce some of these proofs in Midterm 1.

Compared to the list for Test 1, there are many new proofs from 1.8 and Chapter 2. However, the proofs from Chapter 1 have been narrowed in the following ways: All exercises that used to be on this list are eliminated (although they are still good to know and good practice and you can find them on the Test 1 proofs list). I will only ask you to reproduce these main theorems, not any exercises. Secondly, I have eliminated theorems that appeared on the first test or makeup test. Of course, you should still know the statements of the theorems, and how to apply them, and I strongly recommend understanding them well, you will just not be asked to reproduce their proofs.

HOW TO STUDY: As I have discussed in class, my goal is not to have you blindly memorize this list. I want you to understand the proofs on this list. Conveying the correct idea, even with wrong details, will be important for part marks. My suggestion is to first carefully read each theorem. See if you can see the big picture ideas in the proof. Then reread the proof making sure you can verify why every detail is true. Can you see where the assumptions come in, what theorems it relies on and why those are necessary, any little facts it is being unclear about? Then write out a short summary of the big ideas of the proof. This is what you should make sure you understand. On the test, you can reproduce this big structure, and hopefully fill in most of the details. Time permitting, you can go back and try writing out the theorems in full based on your shorter summary of the major structure of the proof. This is, of course, just my suggestion on how to study.

1.1:
- proof of 1.1
- proof of 1.2

1.2:
- The only proof in this section is 1.4, and it has 4 components: (a) $\Rightarrow$, (a) $\Leftarrow$, (b) $\Rightarrow$, (b) $\Leftarrow$.

1.3:
- Proof of 1.7 using 1.6 and 1.3.
- Proof 1.10 part i only

1.4:
- Theorem 1.14 has two blocks of arguments: *if* and *only if*.

1.5:
- Proof of 1.16,
- proof of 1.18 has three ideas: selection of the intervals, selection of the sequence, and proving that the sequence converges.)
- proof of 1.20 (has three blocks: convergent implies Cauchy, Cauchy is bounded, Cauchy + subsequence converge imply the entire sequence converges.)

1.6:
- 1.21 has five blocks: $(a) \Rightarrow (b)$, S is not bounded then a sequence in S tends to infinity, any subsequence also tends to infinity, , then no converging subsequence, not closed then exists a subsequence which does not converge in S.

- Proof of 1.23 has three blocks: inf and sup exist, and they belong to S, and there exist points $\boldsymbol{a}$ and $\boldsymbol{b}$ in $S$.

1.7:
- proof of 1.27
- proof of 1.28.

1.8:
- proof of 1.33

2.1:
- proof of 2.5
- proof of 2.6
- proof of 2.7 (useful to draw a sketch of what is going on)
- proof of 2.8 (parts a,b,c)

2.2:
- proof of 2.18
- proof of 2.23

2.3:
- proof of 2.26 (The proof is long, but uses many important ideas. If you can fully understand this proof, you will likely understand most of the key ideas from 2.1 and 2.2)
- proof of 2.37 (The proof is found in the preceding paragraph. You do not need to worry about precisely what a curve or surface is, just the basic analytic argument for why the various things are orthogonal).

2.4:
- proof of 2.39 (and corollaries 2.40, 2.41)
- proof of 2.42 (The proof structure here is very similar to 1.30 which I didn't ask for, but it can be nice to read the two theorems right after each other).

2.5:
- The derivation starting on the bottom of page 75 and continuing onto the top of page 76. The higher dimensional version was asked in the homeworks but you only need to reproduce (or use) the case in the book.

2.6:
- Proof of 2.45 (Think of why D makes sense as the "difference of the differences")(also Corollary 2.46)

2.7:
- Proof of 2.63 (Taylor's Theorem with Lagrange's Remainder)(Can you see how this theorem is a generalization of the proof of MVT1?)
- Proof of 2.68 (Taylor's Theorem in several variables with Lagrange's Remainder)(You only need to know the 2.69 and 2.72 parts that use the Lagrange Remainder, the proof is preceeding)(Can you see how this theorem is a generalization of the proof of MVT3?)
- Proof of 2.75 (But only the Lagrange Remainder parts. See my class notes for a bit of an expansion on relating $||h||$ and $|h|$, although you don't have to reproduce what I did in class, just the part in Folland)
- Proof of 2.77

2.8:
- Proof of 2.78 (see how it extends 2.5)

- Proof of 2.81
- Proof of 2.82 (This is effectively a corollary of 2.81, but will be very useful for applications)

2.9:
- Proof of 2.83 (two parts, a and b)
- The discussion on page 103 deriving the Lagrange Method.

2.10:
- Proof of 2.85 (proof in preceding discussion)

As we didn't have any quiz (and hence no quiz info) for 2.8-2.10, please try exercises 1 (a few) from 2.8, exercises 1,3,5,7,9 (you can do even ones if you want too), and 15,16a from 2.9, and 1 and 2 but only the compute Df parts from 2.10.