Chapter 0 (review)
0.2 Gauss-Jordan
0.3 Matrix Inverse
0.5 Linear Independence
$\left.\begin{array}{l}0.2\\0.3\\0.5\end{array}\right\}$ stressed
(You're still responsible for sections 0.1 and 0.4)

§ 0.2 Gauss-Jordan reduction is a method for solving systems of linear eqtn

e.g. $x_1+2x_2-x_3-3x_4-4x_5=11$
$3x_1-x_2-x_3+x_4+2x_5=-13$

Augmented Matrix ①

$$\begin{array}{c} \begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & x_5 \end{array} \\ \underbrace{\left[\begin{array}{ccccc|c} 1 & 2 & -1 & -3 & -4 & 11 \\ 3 & -1 & -1 & 1 & 2 & -13 \end{array}\right]}_{\text{coefficient matrix}} \end{array} \to \text{right hand side}$$

Augmented Matrix ②

$$\simeq \left[\begin{array}{ccccc|c} -2 & 3 & 0 & -4 & -6 & 24 \\ -3 & 1 & 1 & -1 & -2 & 13 \end{array}\right]$$

Matrix ③

$$\simeq \left[\begin{array}{ccccc|c} \frac{1}{3} & -\frac{1}{2} & 0 & \frac{2}{3} & 1 & -4 \\ -\frac{7}{3} & 0 & 1 & \frac{1}{3} & 0 & 5 \end{array}\right]$$

The strategy just used
① to solve for a variable, pivot (put a circle on) to column
② Never pivot on 0, never pivot on the same row twice (the simplex method does)

Matrix ③ represents the system

$$\frac{1}{3}x_1-\frac{1}{2}x_2+\frac{2}{3}x_4+x_5=-4$$
$$-\frac{7}{3}x_1+x_3+\frac{1}{3}x_4=5$$

By transposing terms we get a solution for x_3 and x_5 in terms of x_1, x_2 & x_4.

$$x_5=-4-\frac{1}{3}x_1+\frac{1}{2}x_2-\frac{2}{3}x_4$$
$$x_3=5+\frac{7}{3}x_1-\frac{1}{3}x_4$$

(a 3-parameter family of solution)

New goal: solve for x_4 and x_5 in terms of x_1, x_2 & x_3. Could start with matrix ① and do like before (more easily) could start with matrix ③

$$\begin{array}{c} \begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & x_5 \end{array} \\ \left[\begin{array}{ccccc|c} \frac{1}{3} & -\frac{1}{2} & 0 & \frac{2}{3} & 1 & -4 \\ -\frac{7}{3} & 0 & 1 & \frac{1}{3} & 0 & 5 \end{array}\right] \end{array} \simeq \begin{array}{c} \begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & x_5 \end{array} \\ \left[\begin{array}{ccccc|c} 5 & -\frac{1}{2} & -2 & 0 & 1 & -14 \\ -7 & 0 & 3 & 1 & 0 & 15 \end{array}\right] \end{array}$$

We have solved ~~for~~ x_4 and x_5 in terms of x_1, x_2 and x_3.

$$x_5=-14-5x_1+\frac{1}{2}x_2+2x_3$$
$$x_4=15+7x_1-3x_3$$