LN 2.1

① * Nominal Rates of Interest.

② * Effective and Nominal Rates of Discount

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① : <u>effective interest rates</u>.

⇒ Nominal interest rates per annum convertible m times a year.

* effective interest rate $= \boxed{\dfrac{i^{(m)}}{m}}$

* nominal interest rate $= \dfrac{i^{(m)}}{m} \times \boxed{m}$

$$= \boxed{i^{(m)}}$$

<u>Ex</u>

effective monthly interest rate = 1%.

<u>nominal interest rate per annum</u>

{ convertible / compounded / payable } monthly $= 1\% \times 12 = 12\%$

$\boxed{i^{(m)}}$.

$\boxed{\dfrac{i^{(m)}}{m}}$

$\Rightarrow$ i : effective annual interest rate.

$$\boxed{1+i = \left(1+\frac{i^{(m)}}{m}\right)^{m}} \quad *$$

Pf.

① $X = \$1 \cdot (1+i)$

② $X = \$1 \cdot \left(1+\dfrac{i^{(m)}}{m}\right)^{m}$

→ principle of consistency

$$\Rightarrow \quad i = \left(1+\frac{i^{(m)}}{m}\right)^{m} - 1$$

Ex: $\dfrac{i^{(12)}}{12} = 1\%$, $\quad i^{(12)} = 12\%$

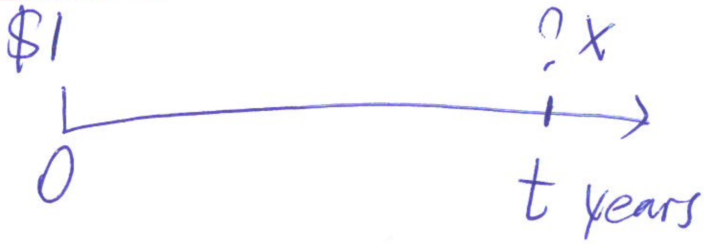
$$\Rightarrow \quad i = \left(1+\frac{i^{(12)}}{12}\right)^{12} - 1$$

$$= (1+1\%)^{12} - 1$$

$$= 12.6825\%$$

$$\boxed{i^{(m)} = m\left[(1+i)^{\frac{1}{m}} - 1\right]}$$ *

$\Rightarrow$ $\boxed{t \text{ years}}$.



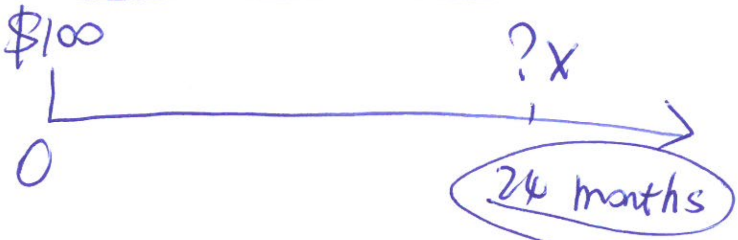
$$X = \$1 \cdot (1+i)^t$$

$$X = \$1 \cdot \left(1+\frac{i^{(m)}}{m}\right)^{mt}.$$

$$\Rightarrow \boxed{(1+i)^t = \left(1+\frac{i^{(m)}}{m}\right)^{mt}} = A(0,t)$$

Ex: Accumulated Values.

(a)



18%: nominal rate per annum convertible $\boxed{\text{quarterly}}$.

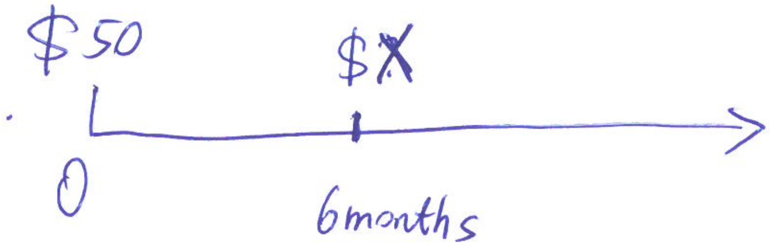
$$S(t) = S(0) \cdot \boxed{\left(1+\frac{i^{(m)}}{m}\right)^{mt}}$$

$$= \$100 \cdot \left(1+\frac{18\%}{4}\right)^{4\cdot 2}$$

$m = 4$
$i^{(m)} = 18\%$
$t = 2$ years
$S(0) = \$100$.

$$\boxed{S(0)} = \boxed{S(t)} \cdot \left(1+\frac{i^{(m)}}{m}\right)^{-mt}.$$

(b)



12%: nominal rate per annum covertible weekly.

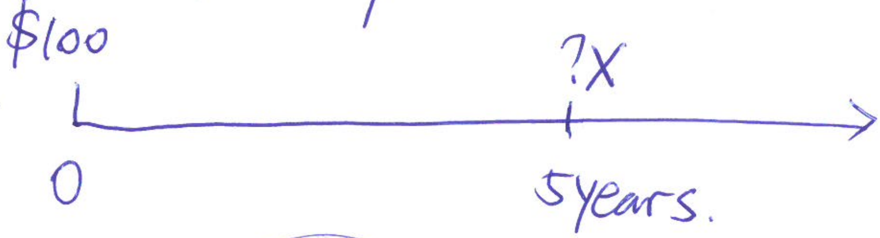
$$\begin{cases} m = 52 \\ i^{(m)} = 12\% \\ t = \frac{1}{2} \text{ year} \\ S(0) = \$50. \end{cases}$$

$$S(t) = S(0) \cdot \left(1 + \frac{i^{(m)}}{m}\right)^{mt}$$

$$= 50 \cdot \left(1 + \frac{12\%}{52}\right)^{52 \cdot \frac{1}{2}}.$$

$$= \$53.09.$$

(C)



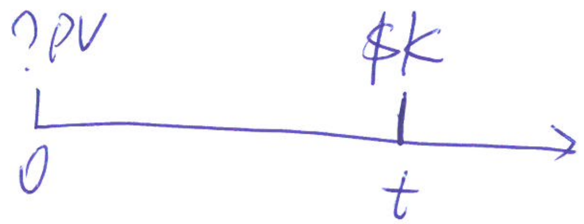
8%: effective half-yearly rate

$$\begin{cases} m = 2 \\ i^{(m)} = 8\% \times 2 = 16\% \\ t = 5 \text{ years} \\ S(0) = \$100 \end{cases}$$

$$S(t) = 100\cdot\left(1+\frac{16\%}{2}\right)^{2\cdot 5} = \$215.89.$$ (5)

8%

PV.



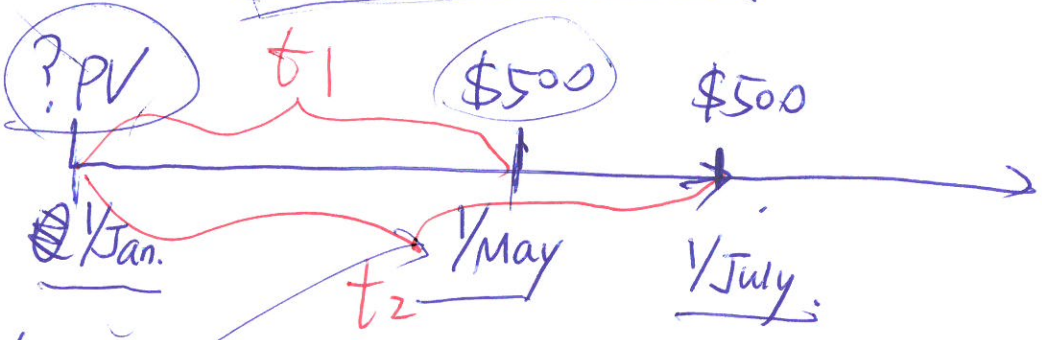
$$v = \frac{1}{1+i}$$

$$v^t = (1+i)^{-t}, \quad PV = K\cdot v^t$$

$i^{(m)}$: nominal rate of interest ...

$$\Rightarrow \boxed{PV = K\cdot\left(1+\frac{i^{(m)}}{m}\right)^{-mt}}$$

Ex:



12%: nominal interest rate per annum convertible monthly.

① $PV_1 = \$500\left(1+\frac{12\%}{12}\right)^{-12\left(\frac{4}{12}\right)}$

$+$

② $PV_2 = \$500\left(1+\frac{12\%}{12}\right)^{-12\cdot\frac{6}{12}}$.

$\| \; PV = \$951.52$

$m = 12$

$i^{(m)} = 12\%$

$t_1 = \frac{4}{12}$ yrs

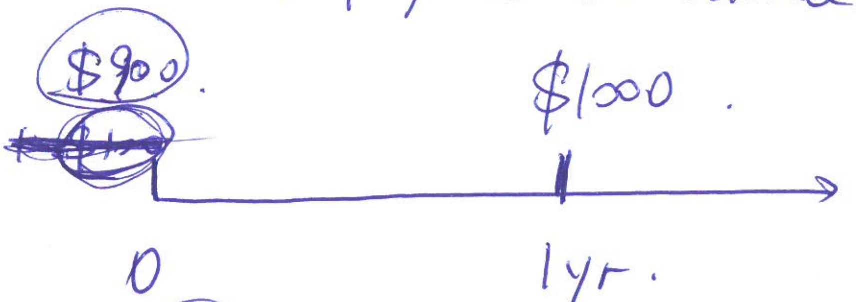
$t_2 = \frac{6}{12}$ yr

$S(t_1) = \$500$

$S(t_2) = \$500$

* Effective and Nominal Rates of Discount

{ interest payable in arrears
{ interest payable in advance.



10% payable in advance

borrow $1000.

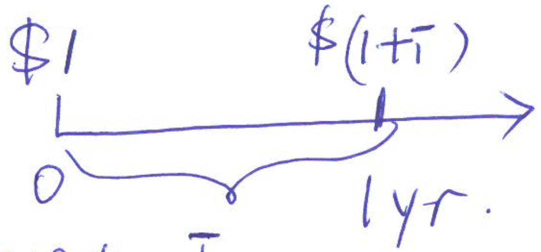
interest = $1000 × 10% = $100.

$$d = \frac{100}{1000} = 10\%$$

$$= \frac{\text{interest}}{\text{balance at the end of the period}}$$

$$i = \frac{100}{900} \cong 11.11\%$$

$$= \frac{\text{interest}}{\text{balance at the beginning of the period}}$$



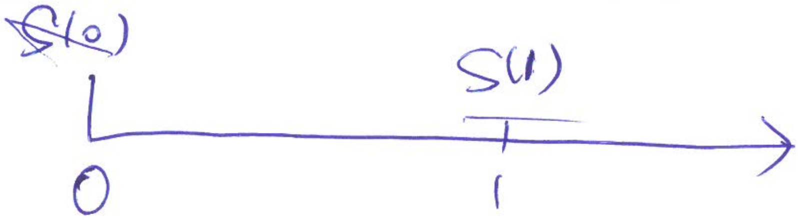
i: annual effective rate of interest

Interest $= i$

$$\boxed{d = \frac{i}{1+i}} \Rightarrow \boxed{i = \frac{d}{1-d}}$$

$$\boxed{v = \frac{1}{1+i}.}$$

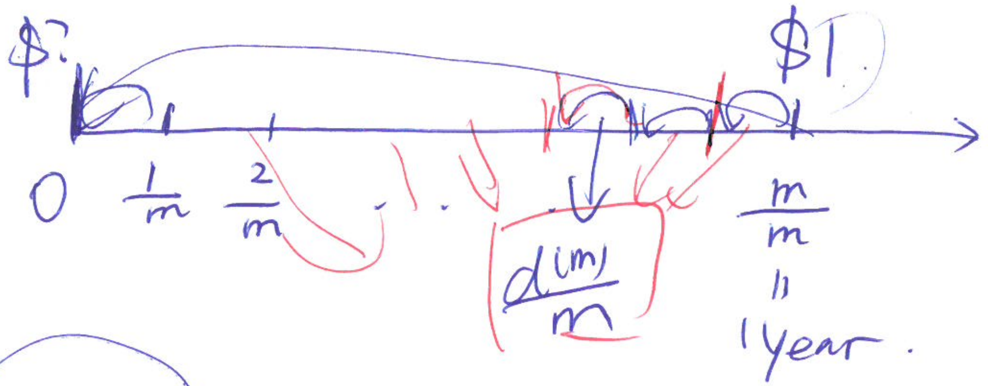
$$\Rightarrow \boxed{\begin{aligned} & d = i \cdot v \\ \Rightarrow\ & d = \frac{i}{1+i} = 1 - \frac{1}{1+i} = 1 - v \end{aligned}}$$



$$d = \frac{S(1) - S(0)}{S(1)}$$

$$i = \frac{S(1) - S(0)}{S(0)}.$$

Nominal Discount Rate



$d^{(m)}$: nominal annual rate of discount convertible m times a year.

$\frac{d^{(m)}}{m}$: effective rate of Discount over $\frac{1}{m}$ yr

d : effective annual rate of discount.

① $PV = \$1 \cdot \left(\frac{1}{1+i}\right) = \$1 \cdot v = 1(1-d)$

$= \boxed{1-d}$

$d = \frac{\text{interest}}{1} \Rightarrow \text{interest} = d$

$\Rightarrow PV = AV - \text{interest} = 1-d.$

② $PV = \$1\left(1-\frac{d^{(m)}}{m}\right)\left(1-\frac{d^{(m)}}{m}\right)\cdots\left(1-\frac{d^{(m)}}{m}\right)$

$= \boxed{\left(1-\frac{d^{(m)}}{m}\right)^m}.$

$$\boxed{d = 1-\left(1-\frac{d^{(m)}}{m}\right)^{m}} \cdots *$$

$$\Updownarrow$$

$$\boxed{d^{(m)} = m\left[1-(1-d)^{\frac{1}{m}}\right]}$$

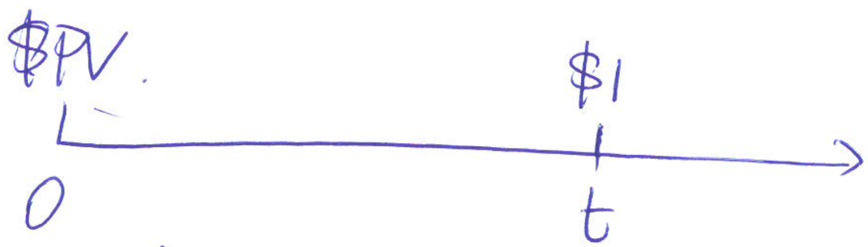
Accumulated Values.

d , $d^{(m)}$

$$X = \boxed{\$1\cdot(1+i)^{t}} \stackrel{?}{=} (1-d)^{-t}$$

$$\stackrel{?}{=} \left(1-\frac{d^{(m)}}{m}\right)^{-mt}$$

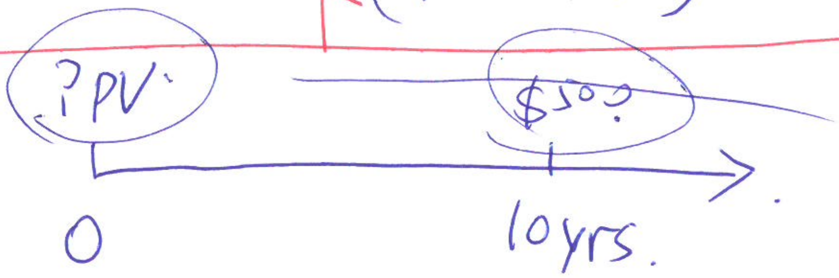
Present Values:



d , $d^{(m)}$.

$$PV = \$1\cdot(1+i)^{-t} = v^t = (1-d)^t$$
$$= \overset{S(t)}{\cancel{K}}\left(1-\frac{d^{(m)}}{m}\right)^{mt}.$$

Ex:



12%: nominal rate of discount - convertible once every two yrs.

Sol:
$$\begin{cases} m = 0.5 \\ d^{(m)} = 12\% \\ t = 10 \text{ yrs} \\ S(t) = \$500 \end{cases}$$

$$S(0) = S(t)\cdot\left(1-\frac{d^{(m)}}{m}\right)^{mt}$$
$$= 500\cdot\left(1-\frac{12\%}{0.5}\right)^{0.5\cdot 10}$$
$$= \$126.78.$$

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i: effective annual interest rate:

$$S(0)\cdot(1+i)^t = 500. \Rightarrow i = 14.71\%$$