Lecture 17
March 17th, 2015

TUT + Quizzes
  L0101 : RW 117
  L0301: GB 120 Mar. 19
        GB 119 Mar 26
        GB 119 Apr 02

Binary Search of $n=2^m$ elements:

2^5 elements	$\Theta(1)$
2^4	$\Theta(1)$ Divide the problem in half
2^3	$\Theta(1)$ $\Theta(1)$ works at each level
2^2	$(\log_2 2^5 + 1)$ levels $\Theta(1)$ Total Time: $\Theta(\# \text{ levels}) = \Theta(\log_2 n) = \Theta(\log n)$
2^1	$\Theta(1)$
2^0	$\Theta(1)$

Unary Tree, half the problem in each child.

Mergesort of $n = 2^m$ elements
    Sort two halves
    merge sorted results
      Linear in the
      total length of both lists

11 23 45 69
31 41 5926
31 41 5926
31 41 5926

Total work for level is $\Theta(\# \text{ elements})$,
Total work is $\Theta(\# \text{ levels} \cdot \# \text{elements}) = \Theta((\log n) \cdot n)$

Binary Tree, half the problem in each child.

Multiplication of natural numbers

        31415926 $n = 2^m$
    $\times$ 53589793

} add up 8 things

generate 8 digits #s 8^2
addition: 16 columns of 8 digits
    Time for generating n n-digit #s
        $\Theta(n^2)$
    Add $\Theta(n)$ colums of $\Theta(n)$ digits $= \Theta(n^2)$
    Total $\Theta(n^2)$

x_0 3141 5926 x_1
  $\times$ y_0 5358 9793 y_1

$\left(\begin{array}{c}3141 \\ \times 9793\end{array}\right) 10^4 + \begin{array}{c}5926 \\ \times 9793\end{array}$

$+ \left(\begin{array}{c}5926 \\ \times 5358\end{array}\right) \times 10^4 + \left(\begin{array}{c}3141 \\ \times 5358\end{array}\right) \times 10^8$

$10^8 (x_0 y_0) + 10^4 (x_1 y_0 + x_0 y_1) + x_1 y_1$

$10^8 (x_0 y_0) + 10^4 [(x_0 + x_1)(y_0 + y_1) - x_0 y_0 - x_1 y_1] + x_1 y_1$

Now only need to do 3 multiplications

$\log_2 n$ ... $3^{\log_2 n} = n^{\log_2 3}$?