PHL 245 H1S

Test 2: Thursday, March 20, 2014

Aid sheet given. No other aids allowed.

100 minutes.

Part marks will be given for ALL questions. When solving symbolizations and derivations show the overall structure and as much work as possible even if you can't solve them completely.

There are seven pages with questions (pages 2-8). Pages 9 and 10 are for rough notes or in case you need extra space.

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1. COMPLETE a FULL truth-table for the following argument. State whether or not the argument is valid and briefly explain how you know. (13 %)

$$P \wedge Q \leftrightarrow R. \qquad \sim(R \vee Q). \qquad \therefore \sim R \rightarrow P$$

	P	Q	R	P ∧ Q ↔ R	~(R ∨ Q)	∴ ~R → P
1	T	T	T	T T T T T	F T T T	F T T T
2	T	T	F	T T T F F	F F T T	T F T T
3	T	F	T	T F F F T	F T T F	F T T T
4	T	F	F	T F F T F	T F F F	T F T T
5	F	T	T	F F T F T	F T T T	F T T F
6	F	T	F	F F T T F	F F T T	T F F F
7	F	F	T	F F F F T	F T T F	F T F F
8	F	F	F	F F F T F	T F F F	T F T F

Valid: when premises are true, conclusion is true.
So we consider the TVA of ↔ in PR1, ~ in PR2 & → in Conclusion

10

Note that in case 4 & 8, which satisfy our standard,
and in either case conclusion is true.
Hence the argument is VALID.

2. Provide a shortened truth-table of one line, including a truth-value assignment, that shows that the following set of sentences is consistent. (10 %)

$$\{\ (\sim Q \vee P \rightarrow R), \quad \sim(Q \wedge S \rightarrow R), \quad (\sim S \vee W \leftrightarrow \sim P)\ \}$$

Consistent: A TVA such that, those ~~threes~~ sentences are all true or all false.

P	Q	R	S	W
F	T	F	T	T

10

(~Q ∨ P → R),	~(Q ∧ S → R),	(~S ∨ W ↔ ~P)
F T F F T F	T T T T F F	F T T T T T F

In this case, those 3 sentences are true. So consistency proved.

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Name: ______________________

A^1: *a* has a motor.	B^1: *a* is a boat.	C^1: *a* is a canoe.
D^1: *a* is a day.	H^1: *a* is a person.	K^1: *a* is a kayak.
G^2: *a* goes in *b*.	L^2: *a* likes *b*.	M^3: *a* paddles *b* on *c*.
e^0: Elliott	a^1: the aunt of *a*.	something time

3. Using the above abbreviation scheme, symbolize the following: (5 × 5% = 25%)

a) Canoes and kayaks are boats, but no canoe has a motor.

$$\forall x(Cx \to Bx) \land \forall x(Kx \to Bx) \land (\sim\exists x(Cx \land Ax))$$

5

b) Some people only go in boats that have motors.

$$\exists x(Hx \land \forall y(By \to (Gxy \to Ay)))$$

5

c) Provided that Elliott doesn't go in boats unless they have motors, he never paddles a canoe.

$$\forall x(Bx \to \sim G(ex) \lor Ax) \to \forall y(Cy \to \forall z(Dz \to \sim M(eyz)))$$

5

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A^1: *a* has a motor. B^1: *a* is a boat. C^1: *a* is a canoe.

D^1: *a* is a day. H^1: *a* is a person. K^1: *a* is a kayak.

G^2: *a* goes in *b*. L^2: *a* likes *b*. M^3: *a* paddles *b* on *c*.

e^0: Elliott a^1: the aunt of *a*.

3. (continued) Using the above abbreviation scheme, symbolize the following: (5 × 5% = 25%)

d) Not everyone who paddles a canoe and a kayak on the same day likes all motorless boats.

$$\sim\forall x(Hx\rightarrow(\exists z(Dz\wedge\exists m\exists n(Cm\wedge Kn\wedge M(xmz)\wedge M(xnz)))\rightarrow\forall y(By\wedge\sim Ay\rightarrow L(xy)))$$

$$\sim\forall x(Hx\rightarrow(\exists z(Dz\wedge\exists m\exists n(Cm\wedge Kn\wedge M(xmz)\wedge M(xnz)))\rightarrow\forall y(By\wedge\sim Ay\rightarrow L(xy)))$$

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e) The one boat that Elliot goes in is a kayak that both he and his aunt like.

$$\underline{\forall x((Bx\rightarrow\exists y(}G(ey)\wedge Ky\leftrightarrow L(ey)\wedge L(a(e)y))\leftrightarrow x=y)$$

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Name: ______________________________

A^1: *a* has a motor. B^1: *a* is a boat. C^1: *a* is a canoe.
D^1: *a* is a day. H^1: *a* is a person. K^1: *a* is a kayak.
G^2: *a* goes in *b*. L^2: *a* likes *b*. M^3: *a* paddles *b* on *c*.
e^0: Elliott a^1: the aunt of *a*.

4. a) Using the symbolization scheme above, provide an idiomatic English sentence that expresses: 5 %

$$\forall x(Hx \land \forall y(Hy \land L(yx) \to L(xy)) \leftrightarrow x=e)$$

5

Elliott is the person ~~thats~~ that likes everyone who likes him.

4. b) Using the symbolization scheme above, symbolize the following ambiguous sentence **two** logically distinct ways and, for each, write an English sentence that clarifies the meaning: 5 %

Someone paddles a canoe every day.

① ~~Someone~~, The same person paddles a (different, maybe) canoe every day.

$\exists x(Hx \land \forall y(Cy \to \forall z(Dz \to M(xyz))))$

$3\frac{1}{2}$

② There are always people (different people) paddles the canoe (doesn't matter which or whose) every day.

$\forall x(Hx \to \exists y(Cy \land \forall z(Dz \to M(xyz))))$

$8\frac{1}{2}$

5. Provide a derivation that shows that the following argument is valid, using only the basic rules: DN, R, MP, MT, MTP, ADD, S, ADJ, BC, CB, EI, EG, UI. 14 %

$\exists x(Hx \wedge \forall y{\sim}M(xy))$. $\exists yFy \rightarrow \forall w\forall z(B(zw) \rightarrow M(wz))$.

$\therefore \forall x(Fx \rightarrow \exists y(Hy \wedge {\sim}B(xy)))$

1	~~Show~~ $\forall x(Fx \rightarrow \exists y(Hy \wedge {\sim}B(xy)))$	show conc
2	~~Show~~ $Fx \rightarrow \exists y(Hy \wedge {\sim}B(xy))$	show inst
3	Fx	ass cd
4	~~Show~~ $\exists y(Hy \wedge {\sim}B(xy))$	show cons
5	$Hi \wedge \forall y{\sim}M(iy)$	pr1 ei
6	Hi	5 sl
7	$\forall y{\sim}M(iy)$	5 sr
8	$\exists yFy$	3 eg
9	$\forall w\forall z(B(zw) \rightarrow M(wz))$	pr2 8 mp
10	${\sim}M(ix)$	7 ui
11	$\forall z(B(zi) \rightarrow M(iz))$	9 ui
12	$B(xi) \rightarrow M(ix)$	11 ui
13	${\sim}B(xi)$	10 12 mt
14	$Hi \wedge {\sim}B(xi)$	6 13 adj
15	$\exists y(Hy \wedge {\sim}B(xy))$	14 eg
16		15 dd
17		4 cd
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Name: ______________________________

6. Provide a derivation that shows that the following argument is valid. (Use any rules.) 14 %

$\exists zM(a(z)z) \rightarrow \exists x\forall y(B(xy) \wedge L(xy)). \quad \therefore \exists x\forall yM(ya(x)) \rightarrow \sim\forall x\exists y(B(xy) \rightarrow \sim L(xx))$

1	~~Show~~ $\exists x\forall yM(ya(x)) \rightarrow \sim\forall x\exists y(B(xy)\rightarrow\sim L(xx))$	show conc
2	$\exists x\forall yM(ya(x))$	ass cd
3	~~Show~~ $\sim\forall x\exists y(B(xy)\rightarrow\sim L(xx))$	show cons
4	$\forall x\exists y(B(xy)\rightarrow\sim L(xx))$	ass id
5	$\forall yM(ya(i))$	2 ei
6	$M(a(a(i))a(i))$	5 ui
7	$\exists zM(a(z)z)$	6 eg
8	$\exists x\forall y(B(xy)\wedge L(xy))$	7 pr lmp
9	$\forall y(B(jy)\wedge L(jy))$	8 ei
10	$\exists y(B(jy)\rightarrow\sim L(jj))$	4 ui
11	$B(jj)\wedge L(jj)$	9 ui
12	$L(jj)$	11 s
13	$B(jk)\rightarrow\sim L(jj)$	10 ei
14	$\sim\sim L(jj)$	12 dn
15	$\sim B(jk)$	13 14 mt
16	$B(jk)$	9 ui sl
17		15 bid
18		3 cd
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T4

7. Show that the following is a valid argument. (Use any rules.) 14 %

$\forall x\exists y(F(yxy) \rightarrow H(yx))$. pr1 $\quad$ $\forall x\forall y(L(xx)\rightarrow G(xy)) \rightarrow \exists x\forall y\forall z F(yxz)$. pr2

$\forall x(\exists y L(xy) \rightarrow \forall z(G(xz) \vee G(zz)))$. pr3 $\quad$ $\therefore \sim\exists x G(xx) \rightarrow \exists x\exists y H(xy)$

pc

1	Show $\sim\exists x G(xx) \rightarrow \exists x\exists y H(xy)$	show conc
2	$\sim\exists x G(xx)$	ass cd
3	Show $\exists x\exists y H(xy)$	show cons
4	$\sim\exists x\exists y H(xy)$	ass id
5	$\forall x \sim\exists y H(xy)$	4 qn
6	$\sim\exists y H(xy)$	show inst
7	$\exists y H(xy)$	ass id
8	$\exists y(L(yy) \rightarrow \forall z(G(yz) \vee G(zz)))$	pr3 ui
9	$L(ii) \rightarrow \forall z(G(iz) \vee G(zz))$	8 ei
10	$\sim L(ii) \vee \forall z(G(iz) \vee G(zz))$	9 cdj
11	rough idea $\sim(L(ii) \wedge \forall z(G(iz) \vee G(zz)))$	10 dm
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23	Basically, idea is to use pr3 first, then use L & G to get	
24	F in pr2, back to pr1 finally get H,	
25	And should use show cont pr2 somewhere	
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$2\frac{1}{2}$