Last time: quiz:

$X \quad p(x) = \begin{cases} \frac{k}{x^2}, & x = 1, 2, 3 \cdots \\ 0 & o.w. \end{cases}$

$E(X) = \sum_{x=1}^{\infty} x \cdot \frac{k}{x^2} = \sum_{x=1}^{\infty} \cdot \frac{k}{x} = k \sum_{x=1}^{\infty} \frac{1}{x} = k\left(1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} + \cdots\right)$

$1 + \frac{1}{2} + \left(\frac{1}{3} + \frac{1}{4}\right) + \left(\frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8}\right) + \cdots$

$> 1 + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \cdots + \frac{1}{2} + \cdots = 1 + \sum_{m=1}^{\infty} \frac{1}{2} = 1 + \frac{m}{2}$

$m \to \infty$

So $E(x)$ does not exist. $\therefore$

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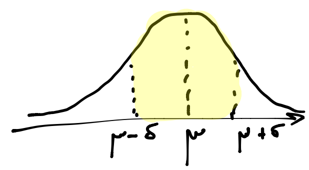
Skip Assignment # 1




#2 $\quad Y \sim N(\mu, \delta^2)$

$P(|Y - \mu| \le 2\delta) = P(\mu - 2\delta \le Y \le \mu + 2\delta) = P(Y \le \mu + 2\delta) - P(Y \le \mu - 2\delta)$

$= \Phi(\mu + 2\delta) - \Phi(\mu - 2\delta)$

$= 1 - 2\Phi(\mu - 2\delta)$ $\quad$ use Appendix B

$P(|Y - \mu| \le 3\delta) = 1 - 2\Phi(\mu - 3\delta)$

#3. $\quad Y \sim G(\alpha, \beta) \quad f_Y(y) = \frac{y^{\alpha-1} e^{-\frac{y}{\beta}}}{\beta^\alpha \Gamma(\alpha)}, \quad 0 \le y < \infty, \quad \alpha, \beta > 0$

$\Gamma(\alpha) = (\alpha - 1)!$

$\Gamma(\alpha + 1) = \Gamma(\alpha) \cdot \alpha$

$E(Y^k) \overset{\text{def'n}}{=} \int_0^\infty y^k \cdot \frac{y^{\alpha-1} e^{-\frac{y}{\beta}}}{\beta^\alpha \Gamma(\alpha)}\, dy$

$= \frac{1}{\beta^\alpha \Gamma(\alpha)} \int_0^\infty y^{\alpha+k-1} \cdot e^{-\frac{y}{\beta}}\, dy =$

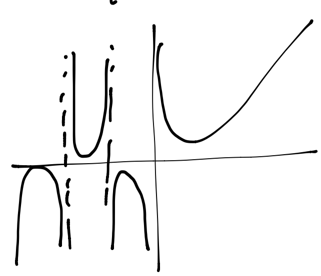
Note:

$\int_0^\infty t^{\theta-1} e^{-t} dt$

$e^{-t} \Rightarrow \frac{y}{\beta} = t$

$= \frac{\beta \cdot \beta^{\alpha+\beta-1}}{\beta^\alpha \Gamma(\alpha)} \int_0^\infty \left(\frac{y}{k}\right)^{\alpha+k-1} e^{-\left(\frac{y}{\beta}\right)} \cdot d\left(\frac{y}{\beta}\right)$

$= \frac{\beta^k}{\Gamma(\alpha)} \cdot \Gamma(\alpha + k)$ ?



$$E(\sqrt{Y}) = \frac{\beta^{\frac{1}{2}}\Gamma(\alpha+\frac{1}{2})}{\Gamma(\alpha)} = \frac{\sqrt{\beta\pi}}{B(\alpha,\frac{1}{2})}$$

note: $\Gamma(\frac{1}{2}) = \sqrt{\pi}$

$$E(\frac{1}{Y}) = \frac{\beta^{-1}\Gamma(\alpha-1)}{\Gamma(\alpha)} \qquad \alpha > 1$$

$\Gamma(\alpha - \frac{1}{2})$ $\alpha > \frac{1}{2}$

$\Gamma(\alpha - 2)$ $\alpha > 2$

#4 (a) (b)

$$E(Y^k) = \int_0^\infty y^k \frac{1}{2^{\frac{v}{2}}\Gamma(\frac{v}{2})} y^{\frac{v}{2}-1} e^{-\frac{y}{2}}\, dy$$

$$= \frac{1}{2^{\frac{v}{2}}\Gamma(\frac{v}{2})} \int_0^\infty y^{\frac{v}{2}+k-1} e^{-\frac{y}{2}}\, dy \qquad \beta = 2,\ \alpha = \frac{v}{2}$$

$$= \frac{2^k\Gamma(\frac{v}{2}+k)}{\Gamma(\frac{v}{2})} \qquad v > -2k$$

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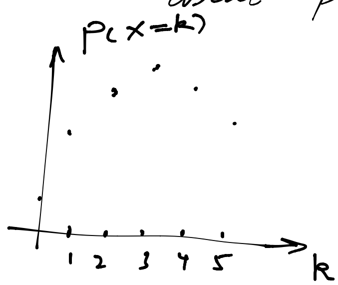
Chapter 2 #30 about poisson distribution

a). $\lambda_1 = 3.96$




b). $\lambda_2 = 0.0825$

$P(X=2, \lambda_2) = 0.003$

#31. $\lambda = 2/\text{hour}$

$\lambda_1 = \frac{1}{3}/10\text{ min}$

(a) $P(X=0;\lambda') = \frac{\lambda_1^0 \cdot e^{-\lambda_1}}{0!} = e^{-\frac{1}{3}} = 0.72$

$P = 1 - P(X=0) = 0.28$

(b). $P(X=0, \lambda_1) = 0.5$

$e^{-n\cdot\frac{1}{3}} \leq 0.5$

$\Rightarrow n \geq 2.079$

so 20.79 minutes