Selected practice questions from past exams

DEFINITIONS

1. Discuss a method taught in class for removing (or modeling) seasonality of time series data.
2. Describe the Dickey Fuller unit root test.
3. Define strictly and weakly stationary time series.
4. State Wold decomposition and how this theorem supports the use of ARMA model.
5. Define an invertible ARMA(p, q) process and state the reason why we discuss invertible processes in class.
6. Define partial autocorrelation functions (PACFs) and describe how to use PACFs for model identification.

BOX-JENKINS APPROACH (ARMA MODELS)

1. Autoregressive model of order 2

Answer the questions using the following AR(2) process

$$(1 - 0.5B)(1 - 0.1B)X_t = a_t, \quad a_t \sim NID(0,1), \qquad (2)$$

causal

1) Is the AR(2) process in eqn. (2) stationary? Explain why?
2) Find the autocorrelation function at lag 1 and 2 for the above AR(2) process.
3) Find the partial autocorrelation functions at lag 1, 2 and 3.

2. Method of moment estimation

Consider a time series $\{x_t\}, t = 1, \ldots, 100$ with sample autocovariances $\hat{\gamma}(0) = 1800$, $\hat{\gamma}(1) = 1200, \hat{\gamma}(2) = 600$. Suppose that we decide to fit $\{x_t\}$ using an AR(2) process as follows:

$$X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + a_t, \quad a_t \sim NID(0, \sigma^2). \quad (3)$$

1) Find the Yule-Walker estimates of ϕ_1, ϕ_2 and σ^2.
2) Find the 95% confidence intervals for ϕ_1 and ϕ_2.

FORECAST

1. Consider an ARIMA(1,1,0) model,

$$(1 - 0.5B)(1 - B) = a_t, \;\; a_t \sim NID(0,1).$$

   a) Write down the forecast function for origin t.

   b) What is the variance of the 1-step-ahead forecast error.

2. Consider the AR(1) model as follows:

$$(1 - 0.6B)(X_t - 9) = a_t, \quad a_t \sim NID(0,1), \qquad (1)$$

   Suppose that we observe $(X_{97}, X_{98}, X_{99}, X_{100}) = (9.6, 9, 9, 8.9)$.

   c) Is the process in eqn. (1) stationary? Why?

   d) Forecast $\{X_t\}$, t=101, 102, 103 and 104 and their associated 95% forecast limits.

   e) Suppose now that the observation at $t = 101$ turns out to be $X_{101} = 8.8$. Calculate $\hat{X}_{101}(l)$ for lead time, $l = 1,2,3$, using "updating forecast".

3. Forecast an ARMA(1,1) model

$$X_t - 0.5X_{t-1} = a_t + 0.25a_{t-1}, \quad a_t \sim NID(0,1), \qquad (*)$$

   Suppose that $(X_{97}, X_{98}, X_{99}, X_{100}) = (-0.7, -1, -0.8, -0.4)$. Answer the following questions:

   a) Write down the forecasting function for eqn. (*).

   b) Calculate the best linear forecast of $\mathrm{X}_{101} + \mathrm{X}_{102} + \mathrm{X}_{103}$.

   c) Calculate the 95% forecast (confidence) interval of the forecast in question 5b). For simplicity, use $Z_{0.975} = 1$ in your calculation.

4. Consider the ARIMA(1,1,0) model

$$(1 - B)(1 + 0.9B)X_t = a_t, \quad a_t \sim NID(0, \sigma^2).$$

   The most recent 8 observations for 1989 to 1996 were

   $(X_{89}, \dots, X_{96}) = (0, -0.1, -1.5, -2.2, -4.3, -4.9, -7.2, -6.3)$.

   a) Write out the recursive formula for forecasting X_{t+l} at original t. Consider $l = 1, 2, 3$ and m.

   b) Is this process stationary? Why or why not?

   c) Is this process invertible? Why or why not?

d) Derive the formulas for predictions for 1997 to 1999 in terms of previously observed values. (These may be expressed in terms of other predictions, as long as you describe how to calculate each term before you use it another formula.)

e) Let $\bar{X}$ be the average of X_{97}, X_{98} and X_{99}. Use the values above and your formulas to calculate the estimate $\hat{\bar{X}}$ of $\bar{X}$. (You should give this estimate both as a formula and numerically.) Assume that all earlier values of the series are zero if you need them in your predictions.

f) Calculate the variance of the forecast error $\bar{X} - \hat{\bar{X}}$ in terms of σ^2.

5. Consider and ARMA(1,1) model*

$$(1 - 0.5B)(X_t - 4) = (1 + 0.5B)a_t, \quad a_t \sim NID(0,1).$$

Its one-step forecast at origin $t = 99$ is $\hat{X}_{99}(1) = 2.09$, and

$$\{X_{99}, X_{100}, X_{101}, X_{102}, X_{103}, X_{104}, X_{105}\} = \{2.11, 1.39, 2.57, 4.11, 6.28, 4.89, 5.94\}.$$

We shall refer the above $ARMA(1,1)$ model as Model A. Use the above information to answer the following question.

a) Calculate the l step ahead forecast $\hat{X}_{100}(l)$ for $l = 1,2,3$ under Model A.

b) Calculate the forecast error variance with origin $t = 100$ and lead time $l = 2,3$ under Model A.

~~c)~~ State the Granger-Newbold forecast accuracy test taught in class and its assumptions.

~~d)~~ Consider a non-nested competitive model B with the following one step ahead forecast errors: $\{e_{100}(1), e_{101}(1), e_{102}(1), e_{103}(1), e_{104}(1)\} = \{0.3, 0.9, 2.0, -1.5, 1.8\}$. Use the Granger-Newbold test to compare the forecast accuracy between model A and B. (Hint: Calculate for $e_{100}(1), e_{101}(1), e_{102}(1), e_{103}(1)$ and $e_{104}(1)$ for Model A and use 2.13 as the 95% quantile of a Student t distribution with 4 degrees of freedom)

- ***Remark: Forecast evaluation is not yet taught in class and won't be tested in the midterm test. That said, questions c) and d) are skipped.***