MORE PREDICATE LOGIC DERIVATIONS FOR UNIT 6

TRY A GOOD NUMBER FROM EACH SECTION

Construct derivations to validate each of the following arguments.

One-Place Predicates:

1. $\exists x(Gx \wedge \sim Hx)$. $\forall x(Hx \leftrightarrow \sim Bx)$. $\therefore\ \forall x(Gx \rightarrow \exists y(Gy \wedge By)$

2. $\exists xAx \rightarrow \exists xGx$. $\forall y(Jy \rightarrow Hy)$. $\sim\exists x(\sim Jx \vee Cx) \vee \forall xFx \therefore \forall x(Gx \rightarrow \sim Hx) \rightarrow \forall x(Ax \rightarrow Fx)$

3. $\exists xGx$. $\forall x(Gx \leftrightarrow \sim Dx)$. $\sim\forall yDy \rightarrow \forall x(\sim Cx \rightarrow Ax)$ $\therefore \forall y(\sim Ay \rightarrow Cy)$

4. $\exists x(Bx \vee Cx)$. $\forall x(Fx \vee Hx)$. $\forall xFx \rightarrow \forall x\sim Cx$. $\therefore\ \sim\exists zHz \rightarrow \exists xBx$

5. $\therefore\ (\forall x(Ax \rightarrow \sim Bx) \wedge\ \exists x(Bx \vee \sim Aa)) \rightarrow\ \exists x\sim(Ax \wedge Cx)$

6. $\exists x(Ax \wedge Bx)$. $\exists y(Gy \vee Hy)$. $\exists xAx \rightarrow \forall y(By \rightarrow \sim Hy)$ $\therefore$ $\sim\exists xGx \rightarrow \exists x\exists y(\sim Hx \wedge Hy)$

7. $\forall x(Ax \rightarrow(\forall y(By\rightarrow Cy) \rightarrow Dx))$. $\forall x(Dx \rightarrow (\forall z(Bz\rightarrow Ez) \rightarrow Fx))$.
   $\therefore \forall y(By \rightarrow(Cy \wedge Ey)) \rightarrow \forall x(Ax \rightarrow Fx)$

8. $\forall x(Fx \rightarrow \forall y(Gy \vee Hx)) \rightarrow \sim\forall xAx \therefore$ $\forall x(Fx \rightarrow \forall zGz) \rightarrow \exists x(Cx \rightarrow \sim Ax)$

9. $\forall xBa(b(x)) \rightarrow$ $(\exists xFx \vee Ga(e))$. $\forall x(Ba(x) \wedge Ca(x))$. $\therefore \forall x(\sim Gx \rightarrow \sim\exists yFy) \rightarrow \exists zGa(z)$.

10. $\exists x\sim(Fx \rightarrow \sim Gx) \rightarrow \exists x\sim Hx$ $\therefore$ $\forall x\exists y\sim(Fy \rightarrow \sim Gx) \rightarrow \sim\forall yHy$

Try some of these using ONLY the basic rules (S, ADJ, ADD, MTP, MP, MT, BC, CB, DN, EG, EI, UI).

11. $\exists xBx$. $\forall x(\sim Bx \vee Cx)$. $\forall y((Ay \vee \sim Dy)\rightarrow \sim Cy)$. $\therefore \exists xDx \wedge \exists y\sim Ay$.

12. $\therefore (\sim Ba \vee Ga) \rightarrow (\forall x\sim(Cx \rightarrow Gx) \rightarrow \exists x(Cx \wedge \sim Bx))$

13. $\forall y(By \rightarrow \sim(Dy \rightarrow Ey))$. $\forall x(Dx \rightarrow \sim(Fx \wedge \sim Cx))$. $\forall x(Ex \vee Fx)$. $\therefore \forall x(\sim Bx \vee Cx)$

14. $\forall x(\sim Bx \rightarrow Cx)$. $Ba \leftrightarrow \forall y\sim(By \wedge Cy)$ $\therefore$ $\exists x(\sim Cx \leftrightarrow Bx)$

15. $\forall y(By \wedge Fy \rightarrow Cy)$. $\exists xFx \rightarrow \forall x(Bx \vee Ax)$. Fb. $\therefore$ $\forall x(\sim Ax \vee Bx) \rightarrow Cb$

16. $\forall x(\sim Cx \vee (Aa \leftrightarrow \sim Fx))$. $\forall x(\sim Fx \rightarrow(\sim Cx \rightarrow Ax))$ $\therefore$ $\exists x(Ax \vee Fx)$

17. $A(ab) \vee B(ba)$. $\forall x\forall y(B(xy) \rightarrow C(yx))$. $\forall w\forall z(C(wz) \leftrightarrow A(wz))$. $\forall x\sim(G(xx) \wedge C(xb))$.
    $\therefore\sim\forall x\forall y(A(xy) \rightarrow G(xx))$

18. $\exists x\forall y\sim(B(xy) \vee F(yx))$. $\forall x\forall y(Gx \rightarrow B(xy))$. $\exists x\forall y(F(xy) \vee H(yy))$. $\therefore$ $\exists x(\sim Gx \wedge H(xx))$

Multi-Place Predicates

19. $\forall x\forall y\forall z(F(xy) \wedge F(yz) \rightarrow F(xz))$. $\sim\forall x\forall y\sim F(xy)$. $\forall x\forall y(F(xy) \rightarrow F(yx))$ $\therefore \exists xF(xx)$

20. $\forall x\exists y\sim(G(xy) \wedge H(xy))$ $\therefore \forall x(\sim\forall yG(xy) \vee \sim\forall yH(xy))$

21. $\forall x(Ax \rightarrow \forall yL(xy))$. $\forall y((Cy \wedge L(yy)) \vee \sim By)$. $\therefore \exists x(Ax \vee Bx) \rightarrow \exists xL(xx)$

22. $\forall x\exists y(Gx \rightarrow L(xy))$. $\forall z(\sim Fz \vee Gz)$. $\forall x(Cx \vee \sim\exists yL(yx))$. $\therefore \forall x(Fx \rightarrow \exists y(Cy \wedge L(xy)))$

23. $\forall x\forall y(B(xy) \rightarrow A(yx))$. $\forall x\exists y(Fy \wedge B(yx))$. $\exists x(Fx \vee Hx) \rightarrow \forall x(Fx \rightarrow Hx)$. $\therefore \forall x(Gx \rightarrow \exists y(A(xy) \wedge Hy))$

24. $\therefore$ $\forall x\exists y\forall z(A(xz) \wedge \sim B(zy)) \rightarrow \exists x(\sim A(xx) \leftrightarrow B(xx))$

25. $\exists x\forall y(L(yx) \rightarrow \forall zB(xyz)$. $\forall y(\exists xB(xyy) \rightarrow \forall zH(yz))$. $\forall y\exists xH(xy)\rightarrow \sim\exists x\exists yG(xy)$.
$\therefore \sim\exists x\forall y(L(xy) \wedge G(yx))$

26. $\exists x\forall y(Hx \wedge L(xy))$. $\forall x(Gx \wedge \forall yL(yx))$. $\exists y\forall x\ (Gx \wedge Hy \wedge L(xy) \wedge L(yx)) \rightarrow \sim\exists z(Fz \wedge Gz)$.
$\therefore \forall x(Fz \rightarrow \sim Gz)$

27. $\exists x(Ax \wedge \forall yH(xy))$. $\forall x(Ax \rightarrow Fx)$. $\exists xFx \rightarrow \forall y\forall z(By \wedge H(zz) \rightarrow G(yz))$.
$\therefore \forall x(Bx \rightarrow \exists y(Fy \wedge G(xy)))$

28. $\forall x(Ax \rightarrow \exists y(Gy \wedge F(xy))) \rightarrow \forall xC(xa)$. $\forall x(\sim Ax \vee Bx)$. $\forall x(Bx \rightarrow Gx)$.
$\therefore \forall y\forall z(Gy \rightarrow F(zy)) \rightarrow \exists xC(xx)$

29. $\exists x\exists yL(xy)\rightarrow \forall x\forall y\forall z(L(xy) \wedge L(yz) \rightarrow L(xz))$. $\therefore$ $\forall x\forall y(L(xy)\rightarrow \sim L(yx)) \leftrightarrow \sim\exists xL(xx)$

30. $\forall x\exists y(Ax \wedge By)$. $\exists x(Ax \wedge Bx) \rightarrow\exists x\forall yH(a(x)y)$. $\therefore \exists xH(xx)$

31. $\forall x\forall y(Fx\rightarrow\exists zG(zy)) \rightarrow \forall x\exists y\forall zH(xyz)$. $\exists x\forall y(H(xyy) \rightarrow\forall z\sim B(xz))$. $\therefore$ $\forall x(\ \exists zFz \rightarrow G(bx)) \rightarrow\sim\forall yB(yy)$

32. $\exists x\forall y\exists z(B(xyz) \rightarrow C(yzx))$. $\exists x\exists y\exists zC(xyz) \rightarrow \exists x\forall yL(a(x)y)$ $\therefore$ $\forall x\exists y\forall zB(xyz) \rightarrow \exists xL(\exists xL(xa(x))$

33. $\forall x\exists y\sim(Fx \vee Gy)$. $\exists x(Fx \leftrightarrow Gx) \rightarrow \forall x\exists y\forall zL(xyz)$ $\therefore$ $\exists x\exists yL(xyy)$

34. $\forall x\exists y\forall z(B(xyz) \rightarrow G(xy) \wedge \sim G(yz))$. $\forall x\forall y\forall z(G(xy) \wedge\sim G(zx)\rightarrow H(yz))$
$\therefore$ $\exists x\forall y\forall zB(xyz) \rightarrow$ $\exists x(H(xx) \wedge \sim G(xx))$

35. $\therefore$ $\forall x\exists y\sim(Fy \vee \forall zG(zx)) \rightarrow \exists y\exists x\sim(\sim G(yx) \rightarrow Fx)$

36. $\exists x\forall y\forall z(A(a(x)y) \wedge B(b(y)z))$. $\forall x\forall y(A(xx) \wedge B(yy) \rightarrow C(xy))$. $\therefore$ $\exists x\exists yC(xy)$

37. $\therefore$ $\exists x\forall yL(b(x)yb(y)) \rightarrow\exists xL(xxb(x))$

38. $\forall xI(a(x)x)$. $\forall x\forall y\forall z(I(xy) \wedge I(yz) \rightarrow I(xz))$. $\therefore$ $\forall xI(a(a(a(x))a(x))$

There are lots of theorems for Predicate Logic listed in the text and on Logic 2010. (Remember, there are an infinite number of theorems!)

203-206 are the theorems for Quantifier Negation. (DON'T USE QN TO DERIVE THESE!)

221 and 221 are theorems for containment (259 and 261 also involve containment.)

242 and 243 are theorems for the equivalencies between not all/some not and none/all not.

Make sure you try these and at least some others from 201-248 (one-place predicates) and some others from 249-272 (multi-place predicates).

T203	$\therefore \sim\forall xFx \leftrightarrow \exists x\sim Fx$
T204	$\therefore \sim\exists xFx \leftrightarrow \forall x\sim Fx$
T205	$\therefore \forall xFx \leftrightarrow \sim\exists x\sim Fx$
T206	$\therefore \exists xFx \leftrightarrow \sim\forall x\sim Fx$
T221	$\therefore \forall x(Fx \rightarrow P) \leftrightarrow (\exists xFx \rightarrow P)$
T222	$\therefore \exists x(Fx \rightarrow P) \leftrightarrow (\forall xFx \rightarrow P)$
T242	$\therefore \sim\forall x(Fx \rightarrow Gx) \leftrightarrow \exists x(Fx \wedge \sim Gx)$
T243	$\therefore \sim\exists x(Fx \wedge Gx) \leftrightarrow \forall x(Fx \rightarrow \sim Gx)$