STAT2001/6039 Final Exam June 2013 Solutions

Solution to Problem 1

(a) Let Y be the number of dollars won on a single game of Luck-Out. Then:

$$EY = 0.01\times 1000 + 0.04\times 100 + 0.95\times 0 = 14 \equiv \mu$$

$$EY^2 = 0.01\times 1000^2 + 0.04\times 100^2 + 0.95\times 0^2 = 10400$$

$$VY = EY^2 - (EY)^2 = 10204 \equiv \sigma^2 .$$

To play n = 250 games will cost 15n = \$3750. Let W be your total winnings in dollars after the n games. Then by the central limit theorem, $W \sim N(n\mu, n\sigma^2)$. Therefore

$$P(Y > 3750) \approx P(W > 3750) = P\left(Z > \frac{3750 - n\mu}{\sqrt{n\sigma^2}} \right) \text{ where } Z \sim N(0,1)$$

$$= P\left(Z > \frac{3750 - 250\times 14}{\sqrt{250\times 10204}} \right) = P(Z > 0.1565) \approx P(Z > 0.16) = \boxed{\mathbf{0.4364}}$$

(using standard normal tables).

Note: A more accurate approximation of 0.4378 could be obtained by using statistical software such as *R*. One may also apply a continuity correction, as follows. After 250 games you will win a multiple of \$100. So you will have more money at the end than when you begin if you win at least \$3800. But the continuity correction in this situation is minus \$50. Therefore

$$P(Y > 3750) = P(Y \geq 3800) \approx P(W > 3800 - 50),$$

which leads to the same answer obtained above without a continuity correction.

(b) Let: X be the number of dollars won by Jim during eight games
A_i be the event that Jim wins some money exactly i times
B be the event that Jim wins some money at least twice.

Then $EX = P(A_0)E(X \mid A_0) + P(A_1)E(X \mid A_1) + P(B)E(X \mid B)$,

where:

$EX = 8\times 14$ (by (a))

$P(A_0) = (0.95)^8$

$E(X \mid A_0) = 0$

$P(A_1) = \binom{8}{1}(0.05)^1(0.95)^7$

$E(X \mid A_1) = 0.2\times 1000 + 0.8\times 100 = 280$ (see the Note below)

$P(B) = 1 - P(A_0) - P(A_1)$ = 0.057245.

It follows that

$$E(X \mid B) = \frac{EX - P(A_0)E(X \mid A_0) - P(A_1)E(X \mid A_1)}{P(B)}$$

$$= \frac{8\times 14 - (0.95)^8 \times 0 - 8\times 0.05\times (0.95)^7 \times 280}{1-(0.95)^8 - 8\times 0.05\times (0.95)^7} = \boxed{\mathbf{590.21}}.$$

Note: If Jim wins something on exactly one of the eight games then (conditional on this) there is a 1/5 chance he wins \$1000 and a 4/5 chance he wins \$100 (on that game, and so overall). It follows that $E(X \mid A_1) = 0.2\times 1000 + 0.8\times 100 = 280$.

Alternative working

Let I be the number of games out of 8 won by Jim, as a random variable.

Then, conditional on B, I has density $f(i \mid B) = \binom{8}{i}\frac{0.05^i 0.95^{8-i}}{P(B)},\ i = 2,...,8$.

Also, using previous logic, observe that $E(X \mid BA_i) = E(X \mid A_i) = 280i$.

It follows by the law of iterated expectation that

$$E(X \mid B) = E\{E(X \mid BA_I) \mid B\} = \sum_{i=2}^{8} 280i \binom{8}{i}\frac{0.05^i 0.95^{8-i}}{P(B)} = 590.21.$$

R Code for Problem 1 (not required, only for interest)

```
PB=1-pbinom(1,8,0.05); PB # 0.05724465
xv=2:8; (280/PB)*sum(xv*dbinom(xv,8,0.05))  # 590.2075
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Solution to Problem 2

(a) We equate $\mu_1' = \mu = EY_i = \frac{c+10}{2}$ with $m_1' = \bar{y} = \frac{1}{3}(2.2+4.8+5.0) = 4.0$ to get the method of moments estimate, $\hat{c} = 2\bar{y} - 10 = 2\times 4 - 10 = \boxed{\mathbf{-2}}$.

Now $E\hat{c} = 2E\bar{Y} - 10 = 2\left(\frac{c+10}{2}\right) - 10 = c$. So $MSE(\hat{c}) = V\hat{c} + (B(\hat{c}))^2$

$$= V\hat{c} + 0^2 = 2^2 V\bar{Y} = 4\frac{VY_i}{n} = 4\left(\frac{(10-c)^2/12}{n}\right) = 4\left(\frac{(10-2)^2/12}{3}\right) = \frac{64}{9} = \boxed{\mathbf{7.111}}.$$

(b) The joint density of the observations is $f(y) = \left(\frac{1}{10-c}\right)^n, y_1,...,y_n \in [c,10]$.

So the likelihood function is $L(c) = \left(\frac{1}{10-c}\right)^n, c \le m$ where $m = \min(y_1,...,y_n)$.

We see that $L(c)$ is a strictly increasing function. So the maximum likelihood estimate of c is $\hat{c} = m = \min(2.2, 4.8, 5.0)$ = $\boxed{\mathbf{2.2}}$. Thus $r = \frac{(1/(10-2.2))^3}{(1/(10-0))^3}$ = $\boxed{\mathbf{2.107}}$.

(c) Observe that $M = \min(Y_1,...,Y_n)$ is an order statistic with cdf

$$F(m) = P(M \le m) = 1 - P(M > m) = 1 - P(Y_1 > m,...,Y_n > m)$$
$$= 1 - P(Y_1 > m)^n = 1 - (1 - F_{Y_1}(m))^n, c \le m \le 10.$$

So M has pdf $f(m) = F'(m) = 0 - n(1 - F_{Y_1}(m))^{n-1} f_{Y_1}(m)$

$$= n\left(1 - \left(\frac{m-c}{10-c}\right)\right)^{n-1} \frac{1}{10-c}, c \le m \le 10.$$

Thus $EM = \int_c^{10} m \left\{ n\left(1 - \left(\frac{m-c}{10-c}\right)\right)^{n-1} \frac{1}{10-c} \right\} dm$.

Let $t = \frac{m-c}{10-c}$. Then $m = (10-c)t + c$ and $\frac{dm}{dt} = 10 - c$.

Hence $EM = \int_0^1 \{(10-c)t + c\} n(1-t)^{n-1} dt$.

But $n(1-t)^{n-1}$ is the pdf of the *Beta*(1,*n*) distribution with mean 1/(1 + *n*).

So $EM = (10-c) \times \frac{1}{1+n} + c \times 1 = \frac{10}{n+1} + \frac{nc}{n+1}$.

So $E\left(M - \frac{10}{n+1}\right) = \frac{nc}{n+1}$. So $E\left(\frac{n+1}{n}\left(M - \frac{10}{n+1}\right)\right) = c$.

So an unbiased estimate of c is $\hat{c} = \left(\frac{n+1}{n}\right)m - \frac{10}{n} = \left(\frac{3+1}{3}\right)2.2 - \frac{10}{3}$ = $\boxed{\mathbf{-0.4}}$.

(d) Observe that $Y-c \sim U(0,10-c)$. So a suitable pivot is $X=\dfrac{Y-c}{10-c} \sim U(0,1)$.

So $1-\alpha = P(\alpha/2 < X < 1-\alpha/2)$

$$\begin{aligned}
&= P\left(\frac{\alpha}{2} < \frac{Y-c}{10-c} < 1-\frac{\alpha}{2}\right) = P\left(\frac{\alpha}{2} < \frac{Y-c}{10-c}, \frac{Y-c}{10-c} < 1-\frac{\alpha}{2}\right) \\
&= P\left(\frac{\alpha}{2}(10-c) < Y-c, Y-c < \left(1-\frac{\alpha}{2}\right)(10-c)\right) \\
&= P\left(c\left(1-\frac{\alpha}{2}\right) < Y-10\frac{\alpha}{2}, Y-10\left(1-\frac{\alpha}{2}\right) < c\frac{\alpha}{2}\right) \\
&= P\left(c < \frac{Y-10\frac{\alpha}{2}}{1-\frac{\alpha}{2}}, \frac{Y-10\left(1-\frac{\alpha}{2}\right)}{\frac{\alpha}{2}} < c\right) = P\left(\frac{Y-10\left(1-\frac{\alpha}{2}\right)}{\frac{\alpha}{2}} < c < \frac{Y-10\frac{\alpha}{2}}{1-\frac{\alpha}{2}}\right),
\end{aligned}$$

and so a central $1-\alpha$ CI for c is $\left(\dfrac{y-10(1-\alpha/2)}{\alpha/2}, \dfrac{y-10\alpha/2}{1-\alpha/2}\right)$.

If y = 4.0 and $\alpha = 0.05$, the CI is $\left(\dfrac{4-10(0.975)}{0.025}, \dfrac{4-10(0.025)}{0.975}\right) = \boxed{\mathbf{(-230, 3.846)}}$.

(e) Since n = 50 is large, the central limit theorem implies that

$$\bar{Y} \,\dot\sim\, N\left(\frac{c+10}{2}, \frac{(10-c)^2/12}{n}\right),$$

and so a suitable pivot is $Z = \dfrac{\bar{Y}-(c+10)/2}{(10-c)/\sqrt{12n}} \,\dot\sim\, N(0,1)$.

Thus, with $z = z_{\alpha/2}$, we write

$$\begin{aligned}
1-\alpha \approx P(-z<Z<z) &= P\left(-z < \frac{\bar{Y}-(c+10)/2}{(10-c)/\sqrt{12n}} < z\right) \\
&= P\left(-z\left(\frac{10-c}{\sqrt{12n}}\right) < \bar{Y}-\frac{c}{2}-5 < z\left(\frac{10-c}{\sqrt{12n}}\right)\right) \\
&= P\left(c\left(\frac{1}{2}+\frac{z}{\sqrt{12n}}\right) < \bar{Y}-5+\frac{10z}{\sqrt{12n}}, \bar{Y}-5-\frac{10z}{\sqrt{12n}} < c\left(\frac{1}{2}-\frac{z}{\sqrt{12n}}\right)\right) \\
&= P\left(\frac{\bar{Y}-5-\frac{10z}{\sqrt{12n}}}{\frac{1}{2}-\frac{z}{\sqrt{12n}}} < c < \frac{\bar{Y}-5+\frac{10z}{\sqrt{12n}}}{\frac{1}{2}+\frac{z}{\sqrt{12n}}}\right).
\end{aligned}$$

So an approximate central $1-\alpha$ CI for c is $\left(\dfrac{\bar{y}-5-\dfrac{10z_{\alpha/2}}{\sqrt{12n}}}{\dfrac{1}{2}-\dfrac{z_{\alpha/2}}{\sqrt{12n}}}, \dfrac{\bar{y}-5+\dfrac{10z_{\alpha/2}}{\sqrt{12n}}}{\dfrac{1}{2}+\dfrac{z_{\alpha/2}}{\sqrt{12n}}} \right)$.

If n = 50, $\alpha = 0.05$ and $\bar{y}$ =4.0, the CI equals $\left(\dfrac{4-5-\dfrac{10\times 1.96}{\sqrt{12\times 50}}}{\dfrac{1}{2}-\dfrac{1.96}{\sqrt{12\times 50}}}, \dfrac{4-5+\dfrac{10\times 1.96}{\sqrt{12\times 50}}}{\dfrac{1}{2}+\dfrac{1.96}{\sqrt{12\times 50}}} \right)$

$$= \left(\frac{-1-\dfrac{1.96}{\sqrt{6}}}{\dfrac{1}{2}-\dfrac{0.196}{\sqrt{6}}}, \frac{-1+\dfrac{1.96}{\sqrt{6}}}{\dfrac{1}{2}+\dfrac{0.196}{\sqrt{6}}} \right) = \boxed{\mathbf{(-4.2863,\ -0.3445)}}.$$

(f) If n = 50 and { $H_0 : c = 0$ } is true then $\bar{Y} \,\dot{\sim}\, N\left(\dfrac{0+10}{2}, \dfrac{(10-0)^2}{12\times 50} \right) \sim N\left(5, \dfrac{1}{6} \right)$.

This distribution is symmetric about 5, and so an appropriate rejection region is $AR = (5-k, 5+k)$, where k is a value which satisfies

$$0.025 = P(\bar{Y} > 5+k) \approx P\left(Z > \frac{5+k-5}{\sqrt{100/(12\times 50)}} \right) = P\left(Z > \frac{k}{\sqrt{6}} \right) \text{ where } Z \sim N(0,1).$$

But $0.025 = P(Z > 1.96)$. Thus we equate $k/\sqrt{6}$ = 1.96 and obtain k = 0.8002.

So the acceptance region is (4.2, 5.8) and the rejection region is $\boxed{(-\infty, 4.2) \cup (5.8, 10)}$.

We observe $\bar{y}$ = 4.0, which is in the rejection region.
Therefore, we $\boxed{\text{reject } H_0}$ and conclude that $c \neq 0$.

The p-value for this test is

$$P(|\bar{Y} - E\bar{Y}| \geq |\bar{y} - E\bar{Y}| \,|\, H_0) = P(|\bar{Y}-5| \geq |4-5|) \quad \text{where } \bar{Y} \,\dot{\sim}\, N(5, 1/6)$$

$$= P\left(\left| \frac{\bar{Y}-5}{1/\sqrt{6}} \right| \geq \left| \frac{4-5}{1/\sqrt{6}} \right| \right)$$

$$\approx 2P(Z > 2.45) = 2\times 0.0071 = \boxed{\mathbf{0.0142}} \text{ (using standard normal tables).}$$

Alternative form of the hypothesis test

Under $H_0 : c = 0$, $Z = \dfrac{\bar{Y}-5}{\sqrt{1/6}} \,\dot{\sim}\, N(0,1)$. So we reject H_0 if $|Z| > z_{\alpha/2} = 1.96$.

The observed value of Z is $z = \dfrac{4-5}{\sqrt{1/6}} = -2.45$. So we reject H_0.

Solution to Problem 3

(a) The cdf of R is $F(r) = P(R \le r) = P(Y / X < r) = P(Y < rX) = \frac{1}{2} \times c \times rc \times \frac{1}{c^2} = \frac{r}{2}$.

This is true if $0 \le r \le 1$. If $r > 1$ then we find that $F(r) = 1 - \frac{1}{2} \times c \times \frac{c}{r} \times \frac{1}{c^2} = 1 - \frac{1}{2r}$.

Note: These results follow easily after noting that $f(x, y) = 1/c^2, 0 < x < c, 0 < y < c$, and sketching the region under the line $y = rx$ in the square define by (0,0), (0,c), (c,c) and (c,0) in the x-y plane. Two cases need to be considered: $0 \le r \le 1$ and $r > 1$.

Thus, $F(r) = \begin{Bmatrix} \frac{1}{2}r, & 0 \le r \le 1 \\ 1 - \frac{1}{2}r^{-1}, & r > 1 \end{Bmatrix}$, and so R has pdf $\boxed{f(r) = F'(r) = \begin{Bmatrix} \frac{1}{2}, & 0 \le r \le 1 \\ \frac{1}{2r^2}, & r > 1 \end{Bmatrix}}$.

Check: $\int f(r)dr = \frac{1}{2} + \int_1^\infty \frac{1}{2r^2} dr = 1$.

We see that R has a distribution which does not depend on c.
So, for all possible values of c (including 8, for example):

The mean of R is $ER = \int_0^1 r\frac{1}{2}dr + \int_1^\infty r\frac{1}{2r^2}dr = \frac{1}{4} + \frac{1}{2}(\log\infty - \log 1) = \boxed{\infty}$.

The mode of R is 0 or 1 or any value between 0 and 1, or the interval $\boxed{[0,1]}$.

The median of R is $\boxed{1}$ (the unique solution in r of the equation $F(r) = 1/2$).

(b) Observe that R in (a) is a suitable pivot for predicting Y.

Setting $F(r) = 0.1$ gives $\frac{1}{2}r = 0.1$ and hence $r = 1/5$.

Likewise, setting $F(r) = 0.9$ gives $1 - \frac{1}{2}r^{-1} = 0.9$ and hence $r = 5$.

So we write $0.8 = P(1/5 < R < 5) = P\left(\frac{1}{5} < \frac{Y}{X} < 5\right) = P\left(\frac{X}{5} < Y < 5X\right)$.

We see that an 80% prediction interval for Y is
$(x/5, 5x) = (4/5, 5 \times 4) = \boxed{\mathbf{(0.8, 20)}}$.

Check: $P(Y < X/5) = P(R < 1/5) = 0.1$, $P(Y > 5X) = P(R > 5) = 0.1$.

This means that the prediction interval is *central* in the sense required.

Solution to Problem 4

(a) Let A_i be the event that 1 and 2 come up on rolls *i* and *i* + 1, respectively (i = 1,...,6). Also let *A* be the event that the sequence 12 comes up at least once.

Then $A = A_1 \cup ... \cup A_6$ and $P(A) = S_1 - S_2 + S_3$ where $S_1 = \sum_{i=1}^{6} P(A_i)$, $S_2 = \sum_{i<j} P(A_i A_j)$ and $S_3 = \sum_{i<j<k}^{6} P(A_i A_j A_k)$. Let $p_i = P(A_i)$, $p_{ij} = P(A_i A_j)$ and $p_{ijk} = P(A_i A_j A_k)$.

Now, $p_i = 1/36$ for all *i* = 1,...,6, and so $S_1 = 6 \times 1/36$.

Also, p_{ij} = 0 if $j = i + 1$, and $p_{ij} = 1/36^2$ if $j > i + 1$.
So $S_2 = \left(p_{13} + p_{14} + p_{15} + p_{16}\right) + \left(p_{24} + p_{25} + p_{26}\right) + \left(p_{35} + p_{36}\right) + p_{46} = 10 \times 1/36^2$.

Finally, p_{ijk} = 0 if $j = i + 1$ or $k = j + 1$, and $p_{ijk} = 1/36^3$ if $j > i + 1$ and $k > j + 1$.
Thus $S_3 = p_{135} + p_{136} + p_{146} + p_{246} = 4 \times 1/36^3$.

It follows that $P(A) = \frac{6}{36} - \frac{10}{36^2} + \frac{4}{36^3} = \frac{1855}{11664}$ = $\boxed{\mathbf{0.1590}}$.

(b) Let $X_i = I(A_i)$, the indicator event for A_i. Thus, X_i = 1 if 1 and 2 come up on rolls *i* and *i* + 1, respectively, and X_i = 0 otherwise. Then $X_1,...,X_6 \sim Bern(1/36)$.
Next define *X* to be the number of occurrences of the sequence 12 (1 followed by 2).
Now, $X = X_1 + ... + X_6$, and so $EX = EX_1 + ... + EX_6 = 6EX_1$, since $EX_1 = ... = EX_6$.
But $EX_1 = 0P(X_1 = 0) + 1P(X_1 = 1) = 0 + P(A_1) = 1/36$.

Therefore $EX = 6 \times 1/36 = 1/6$ = $\boxed{\mathbf{0.1667}}$.

Next, observe that $VX = \sum_{i=1}^{6} \sigma_i^2 + 2\sum_{i<j} \sigma_{ij}$ where $\sigma_i^2 = VX_i$ and $\sigma_{ij} = C(X_i, X_j)$.
Now, $\sigma_i^2 = (1/36)(1 - 1/36) = 35/36^2$. Also, σ_{ij} = 0 whenever $j > i + 1$.
If $j = i + 1$, then $E(X_i X_j)$ = 0 (since the event $A_i A_{i+1}$ is impossible),
and so in that case, $\sigma_{ij} = E(X_i X_j) - (EX_i)(EX_j) = 0 - (1/36)^2 = -1/36^2$.

It follows that
$$VX = \left(\sigma_1^2 + ... + \sigma_6^2\right) + 2\left(\sigma_{12} + \sigma_{23} + \sigma_{34} + \sigma_{45} + \sigma_{56}\right)$$
$$= 6 \times \frac{35}{36^2} + 2 \times 5 \times \left(-\frac{1}{36^2}\right) = \frac{25}{162} = \boxed{\mathbf{0.1543}}.$$

Solution to Problem 5

(a) Let A be the event that 1 comes up at least once, and let B be the event that 3 comes up at least once.Then the required probability is

$$p = P(AB) = 1 - P(\overline{AB}) = 1 - P(\bar{A} \cup \bar{B}) \quad \text{by De Morgan's laws}$$
$$= 1 - \{P(\bar{A}) + P(\bar{B}) - P(\bar{A} \cap \bar{B})\}.$$

Now, $P(\bar{A}) = (5/6)^5$, $P(\bar{B}) = (3/6)^5$ and $P(\bar{A} \cap \bar{B}) = (2/6)^5$.

So $p = 1 - \{(5/6)^5 + (3/6)^5 - (2/6)^5\}$ = $\boxed{0.5710}$.

(b) Let p be the probability that the last number is 3, let p_i be the probability that the last number is 3 given that the first number is i (i = 1,2,3), and let p_{ij} be the probability that the last number is 3 given that the first two numbers are i and j, in that order (j = 1,2,3). Then, applying a first step analysis we have that:

$$p = \frac{1}{6}p_1 + \frac{2}{6}p_2 + \frac{3}{6}p_3$$
$$p_1 = \frac{1}{6}p_{11} + \frac{2}{6}p_{12} + \frac{3}{6}p_{13} = \frac{1}{6}\times 0 + \frac{2}{6}p_2 + \frac{3}{6}p_3$$
$$p_2 = \frac{1}{6}p_{21} + \frac{2}{6}p_{22} + \frac{3}{6}p_{23} = \frac{1}{6}p_1 + \frac{2}{6}\times 0 + \frac{3}{6}p_3$$
$$p_3 = \frac{1}{6}p_{31} + \frac{2}{6}p_{32} + \frac{3}{6}p_{33} = \frac{1}{6}p_1 + \frac{2}{6}p_2 + \frac{3}{6}\times 1.$$

Thus:

$$6p = p_1 + 2p_2 + 3p_3 \qquad (1)$$
$$6p_1 = 0 + 2p_2 + 3p_3 \qquad (2)$$
$$6p_2 = p_1 + 0 + 3p_3 \qquad (3)$$
$$6p_3 = p_1 + 2p_2 + 3. \qquad (4)$$

We now solve these four equations in four unknowns as follows:

(1) - (2) => $p_1 = \frac{6}{7}p$ (5)

(1) - (3) => $p_2 = \frac{3}{4}p$ (6)

(1) - (4) => $p_3 = \frac{2p+1}{3}$. (7)

Next substitute (5), (6) and (7) into (3) to get p = 14/23 = $\boxed{0.6087}$.

Note: We then also get p_1 = 12/23, p_2 = 21/46 and p_3 = 17/23.

Solution to Problem 6

Let X be the number of rolls until a 2 comes up, let Y be the number of times that 3 comes up, and let A be the event that 3 does *not* come up (i.e., the event $Y = 0$).

Then $X \sim Geometric(1/3)$ with pdf $f(x) = \left(\frac{2}{3}\right)^{x-1}\left(\frac{1}{3}\right)$, $x = 1,2,3,....$

Also, $(Y \mid X = x) \sim Bin(x-1, 3/4)$, and hence

$$P(A \mid X = x) = P(Y = 0 \mid X = 3) = \binom{x-1}{0}\left(\frac{3}{4}\right)^0\left(\frac{1}{4}\right)^{x-1-0} = \left(\frac{1}{4}\right)^{x-1}, \; x = 1,2,..$$

Note: If the first 2 comes up on the xth roll, then each of the previous $x-1$ numbers which come up has a 1/4 chance of being 1 and a 3/4 chance of being 3, independently of the others. If 2 comes up on the first roll, then $x = 1$ and the formula $(1/4)^{x-1}$ correctly gives the probability of 3 not coming up (namely, 1).

It follows that the probability of 3 not coming up is

$$P(A) = \sum_{x=1}^{\infty} P(A \mid X = x)P(X = x) = \sum_{x=1}^{\infty}\left(\frac{1}{4}\right)^{x-1}\left\{\left(\frac{2}{3}\right)^{x-1}\left(\frac{1}{3}\right)\right\}$$
$$= \frac{1}{3}\times\frac{6}{5}\sum_{x=1}^{\infty}\left\{\left(\frac{1}{6}\right)^{x-1}\times\frac{5}{6}\right\} = \frac{2}{5}\times 1 = \frac{2}{5}.$$

So the probability that 3 comes up at least once equals $1-2/5$ = 3/5 = $\boxed{\mathbf{0.6}}$.

Alternative working

More simply, let B be the event that 3 comes up at least once (thus $B = \bar{A}$).
Also let $P(1)$ be the event that 1 comes up on the first roll, etc.

Then, applying a first step analysis, we have that

$$P(B) = P(1)P(B \mid 1) + P(2)P(B \mid 2) + P(3)P(B \mid 3)$$
$$= \frac{1}{6}\times P(B) + \frac{2}{6}\times 0 + \frac{3}{6}\times 1.$$

Solving this one equation in one unknown, we get $P(B) = 3/5$.