July 9th

Smooth curve in $\mathbb{R}^2$

① connected

② locally the graph of a $C^1$ function

(1) $x=f(y)$ or $y=f(x)$

(2) $F(x,y,z)=0$

(3). $f:(a,b)\to\mathbb{R}^2$

$t\to(\varphi(t),\psi(t))$

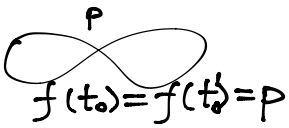
Thm:

(2) $\nabla F\neq 0$, at $(a,b)$

"connectedness" is not guaranteed

(3) $f'(t_0)\neq\vec{0}$, $\exists$ open interval $I$ with $t_0\in I$ s.t. the set $\{f(t)\mid t\in I\}$ is the graph of a $C^1$ function.


not 1-1



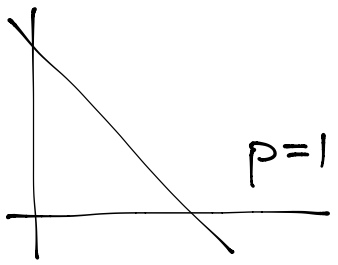

P125 #2

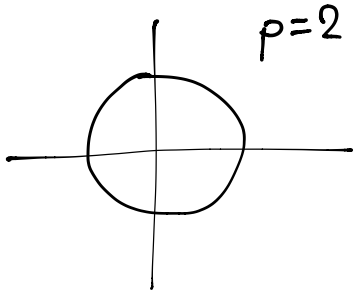
$S_p=\{(x,y)\mid x^p+y^p=1\}$ $p\in\mathbb{Z}^+$

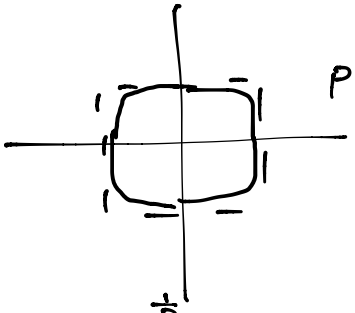
(a) Show $S_p$ is smooth curve $\forall p$.

(b) Sketch $S_p$.

(b)




(a "square circle")

$y=(1-x^p)^{\frac{1}{p}}$

for p is odd

↳ not diff at $x=1$