STAT3032 SURVIVAL MODELS

SOLUTIONS TO TUTORIAL WEEK TWO

1. (a) $P(E) = \dfrac{90490}{91217} = 0.99203$

(b) $P(E) = 1 - \dfrac{79293}{91217} = 0.13072$

(c) $P(E) = \dfrac{806 + 892}{91217} = 0.01862$

(d) $P(E) = \dfrac{86714 - 79293}{91217} = 0.08136$

2. (a) P(survives to age 70) $=$ $\mathrm{P(T > 70) = 1\text{-}P(T \le 70)}$ $=$ $\mathrm{1\text{-}F_0(70)} = 1 - [1 - e^{-0.015*70}] = 0.34994$

(b) P(dies by 35) $=$ $\mathrm{P(T \le 35) = F_0(35)} = 1 - e^{-0.015*35} = 0.40845$

(c) $\mathrm{P(T > 50 | T > 25)} = \dfrac{\mathrm{P(T > 50)}}{P(T > 25)} = \dfrac{1 - (1 - e^{-0.015*50})}{1 - (1 - e^{-0.015*25})} = 0.68729$

(d) $\mathrm{P(T < 70 | T > 30)} = \dfrac{\mathrm{P(T < 70 \text{ and } T > 30)}}{P(T > 30)} = \dfrac{(1 - e^{-0.015*70}) - (1 - e^{-0.015*30})}{e^{-0.015*30}} = 0.45119$

3. ${}_3p_x = {}_2p_x p_{x+2}$

$\therefore 0.912285 = \mathrm{(1\text{-}0.0398).p_{x+2}}$

$\therefore p_{x+2} = 0.95010$

4. If we define T_{50} to be a random variable for the future lifetime of a 50 year old, F_{50} to be the associated cumulative distribution function and S_{50} to be the associated survival function, we can write the probability that a 50 year old dies between ages 70 and 80 as:

$$P\left(20 < T_{50} \le 30\right) = F_{50}\left(30\right) - F_{50}(20) = S_{50}(20) - S_{50}(30)$$

If we define T_0 to be a random variable for the future lifetime of a newborn, F_0 to be the associated cumulative distribution function and S_0 to be the associated survival function, we can write the probability that a 50 year old dies between ages 70 and 80 as:

$$P\left(70 < T_0 \le 80 \mid T_0 > 50\right) = \frac{F_0(80) - F_0(70)}{S_0(50)}$$

Using actuarial notation the required probability can be written as

$${}_{20}p_{50} \cdot {}_{10}q_{70} \text{ or as } \frac{l_{70} - l_{80}}{l_{50}} = \frac{\sum_{i=0}^{9} d_{70+i}}{l_{50}}$$

Note for after week two lectures

The required probability can also be written as ${}_{20|10}q_{50}$.

5. It is not a probability. From lectures $\mu_x = -\frac{1}{l_x}\frac{dl_x}{dx}$ and there is no reason why the rate of decline of the l_x curve can not exceed the value of the l_x function.

(a) 6. It measures the expected amount of life lived in the future by a life who has already survived t years.
(b) Since t is a constant it can be taken outside the (conditional) expectation.

(c)

$$f_{T|T>t}(y) = \frac{f_T(y)}{S_T(t)}$$

$$= \frac{\lambda e^{-\lambda y}}{\int_t^\infty \lambda e^{-\lambda y} dy}$$

$$= \frac{\lambda e^{-\lambda y}}{e^{-\lambda t}}$$

(d)

$$r(t) = E\left(T \mid T > t\right) - t$$

$$= e^{\lambda t} \int_t^\infty \lambda y e^{-\lambda y} dy - t$$

$$= e^{\lambda t} \left[\left[-y e^{-\lambda y} \right]_t^\infty + \int_t^\infty e^{-\lambda y} dy \right] - t$$

$$= e^{\lambda t} \left[t e^{-\lambda t} - \frac{1}{\lambda} \left[e^{-\lambda y} \right]_t^\infty \right] - t$$

$$= e^{\lambda t}\left[te^{-\lambda t} + \frac{1}{\lambda}e^{-\lambda t}\right] - t$$

$$= t + \frac{1}{\lambda} - t$$

$$= \frac{1}{\lambda}$$

(e) $E(T) = \frac{1}{\lambda}$.

(f) Memoryless property of the exponential density. If a life has survived a certain number of years the expected future number of years lived is unaffected by the number of years survived up to that time.