Announcements

① See the website for a finalized term test 2 announcement (including MAC hours).

②

today { Finish the 2-phase example
Start §3.1 }

Remarks

① The only reason an artificial variable is useful is that it is basic. On existing, they are useless. When an artifical variable exits, you can (and should) omit its columns.

② If an artificial variable (y_i) is basic for the j^{th} constraint, then the legitimate variables (the x_i) satisfy the jth constraint when (and only when) $y_i = 0$.

- Eg. The problem being solved in "A Two-Phase Solution" is (phase 1, tableau 0)

Maximize $z = -2x_1 - 3x_2 - 2x_3$ s.t.

$$
\begin{aligned}
6x_1 - x_2 \quad\quad &= 32 \\
-2x_1 + 4x_2 + 3x_3 &= 12 \\
7x_1 - 5x_2 - 3x_3 &\geq 20 \\
3x_1 + 3x_2 + 3x_3 &= 44 \\
x_1 \geq 0, x_2 \geq 0, x_3 &\geq 0
\end{aligned}
$$

With x_4 as slack in the $\geq$ constraints, and artificial variables, y_1, y_2, y_3, y_4, the phase 1 auxiliary problem is:

Maximize: $z = -y_1 - y_2 - y_3 - y_4$ s.t.

$$
\begin{aligned}
6x_1 - x_2 \quad\quad\quad\quad + y_1 \quad\quad\quad\quad &= 32 \\
-2x_1 + 4x_2 + 3x_3 \quad\quad\quad + y_2 \quad\quad &= 12 \\
7x_1 - 5x_2 - 3x_3 - x_4 \quad\quad\quad + y_3 \quad &= 20 \\
3x_1 + 3x_2 + 3x_3 \quad\quad\quad\quad\quad\quad + y_4 &= 44
\end{aligned}
$$

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0, y_1 \geq 0, y_2 \geq 0, y_3 \geq 0, y_4 \geq 0$$

This leads to Phase 1, Tableau ⓪ (where the optimality criterion does not apply)

To eliminate the coefficients of the y_i, Tableau ① objective row

= Tableau 0 objective row − $\sum_{i=1}^{4}$ all y_i row

All routine until Tableau 2.

From tableau 2, x_2 will enter.

x_2-column θ-ratios:

y_1	$\frac{124}{18}$
y_2	$\frac{124}{18}$
y_3	X
y_4	$\frac{248}{36}$

a 3 way tie

We automatically choose y_1 to exit.

To set up table 0 of phase 2, drop all non-basic variables, but keep the basic artifical variables

Set up the objective row according to the original (objective function:

$$z = -2x_1 - 3x_2 - 2x_3$$

(Maximize)

In tableau 0, the optimality criterion does not apply, but it does in Tableau 1, where objective row = Tableau 0 objectiveness

−3 × the x_2 row

−2 × the x_1 row

Tableau 1 is not optimal and x_4 should enter.

θ-ratio for the x_4-column

x_2	$\frac{124}{5}$ ← smaller, x_2 would normally exit.
y_2	$\frac{0}{-1}$
x_1	140
y_4	$\frac{0}{-1}$

Existing x_2 would cause an artificial variable to become positive, indicating an <u>infeasible</u> solution. To prevent this, exit an artificial variable (using a negative pivot if necessary). Existing y_2 leads to Tableau 2. One more step leads to Tableau 3 (optimal)