Unit 3 Part 2: Derivation Exercises

Derivations with AND, OR and BICONDITIONAL

NOTE: there are 3 sections. Make sure you do a good number from each section. (In each section, the first ones tend to be easier than the later ones.)

SECTION 1: Construct derivations to validate each of the following arguments. Use only the basic rules: MP, MT, R, DN, ADD, S, ADJ, MTP, BC and CB.

1. $(P \wedge Q) \wedge (R \wedge S)$. $\therefore$ $(P \wedge S) \wedge Q$

2. $P \vee (Q \vee S)$. $\sim S$. $\sim P$ $\therefore$ $(Q \vee T) \vee (W \rightarrow Z)$

3. $P \leftrightarrow Q$. $\sim Q \leftrightarrow \sim R$. $\therefore$ $R \leftrightarrow P$

4. $R \vee \sim S$. $S \wedge \sim T$. $W \leftrightarrow T$. $\therefore$ $R \wedge \sim W$

5. $W \vee P$. $P \vee S \rightarrow Q$. $X \leftrightarrow \sim Q$. $\therefore$ $\sim W \rightarrow \sim X$

6. $(\sim S \wedge T) \wedge (W \vee S)$. $\therefore$ $(\sim T \leftrightarrow \sim X) \rightarrow (X \leftrightarrow W)$

7. $P \leftrightarrow (Q \vee R)$. $R \rightarrow S$. $\sim S \wedge P$. $W \vee Q \rightarrow R$. $\therefore$ T

8. $P \vee (Q \vee R)$. $Q \leftrightarrow (X \wedge Z)$. $(W \rightarrow Z) \rightarrow S$. $\therefore$ $\sim P \rightarrow (\sim R \rightarrow S)$

9. $W \leftrightarrow Q$. $(\sim Q \vee S) \wedge W$. $S \rightarrow T \wedge U$. $\therefore U \vee \sim T$

10. $(\sim P \vee R) \wedge (\sim Q \rightarrow \sim R)$. $Q \leftrightarrow (S \wedge W)$. $S \vee R \rightarrow T$. $T \wedge W \rightarrow P$. $\therefore$ $P \leftrightarrow Q$

11. $\sim(P \wedge Q)$. $\sim P \rightarrow T$. $\sim T \rightarrow Q$. $\therefore$ T

12. $\therefore$ $(T \wedge S) \rightarrow ((T \vee W) \rightarrow \sim S) \rightarrow \sim W)$

13. $P \vee Q \rightarrow R$. $\sim T \wedge (P \rightarrow S)$. $S \leftrightarrow \sim R$. $\sim P \rightarrow T$. $\therefore$ W

14. $\sim(P \rightarrow Q)$. $P \leftrightarrow R$. $Q \vee S$. $\therefore$ $R \wedge S$

15. $\sim(P \wedge R)$. $S \rightarrow R$. $\sim P \rightarrow \sim S$. $\therefore$ $T \vee \sim S$

16. $\sim(T \rightarrow W)$. $(R \rightarrow T) \rightarrow Z$. $\therefore$ $(W \leftrightarrow Z) \rightarrow P$

17. $\sim Z \rightarrow S$. $\sim(S \leftrightarrow P)$. $Q \vee P$. $\therefore$ $\sim Q \rightarrow (Z \vee T)$

18. $\therefore$ $\sim S \vee Q \rightarrow ((Q \rightarrow \sim(P \rightarrow T)) \rightarrow \sim(T \wedge S))$

19. $\therefore$ $\sim(P \rightarrow Q) \rightarrow (\sim(R \vee S) \rightarrow \sim(S \vee \sim P))$

20. $\sim(Q \rightarrow R)$. $(Q \vee S) \rightarrow T$. $\sim P \rightarrow R$. $T \wedge P \rightarrow W$. $\therefore$ W

SECTION 2: THEOREMS

These are some of the theorems from unit 3.2, section 3.15. (If you want more practice, derive all the theorems!)

Construct a derivation to prove the following theorems. Use only the basic rules:

MP, MT, R, DN, ADD, S, ADJ, MTP, BC and CB

Some of these justify the derived rules (Theorems: 33, 40, 46, 49, 63, 64, 65, 66, 90 – they have the rule in parentheses after). It is especially important that you use only the basic rules – otherwise you may be justifying the rule with the rule you are justifying!

T33	$\therefore (P \to Q) \wedge (\sim P \to Q) \to Q$	(Separation of cases, special case)
T40	$\therefore \sim(P \to Q) \leftrightarrow P \wedge \sim Q$	(Negation of conditional)
T46	$\therefore (P \to Q) \leftrightarrow \sim P \vee Q$	(Conditional as disjunction)
T49	$\therefore (P \vee Q) \wedge (P \to R) \wedge (Q \to R) \to R$	(Separation of cases)
T63	$\therefore P \wedge Q \leftrightarrow \sim(\sim P \vee \sim Q)$	(De Morgan's Law)
T64	$\therefore P \vee Q \leftrightarrow \sim(\sim P \wedge \sim Q)$	(De Morgan's Law)
T65	$\therefore \sim(P \wedge Q) \leftrightarrow \sim P \vee \sim Q$	(De Morgan's Law)
T66	$\therefore \sim(P \vee Q) \leftrightarrow \sim P \wedge \sim Q$	(De Morgan's Law)
T90	$\therefore \sim(P \leftrightarrow Q) \leftrightarrow (P \leftrightarrow \sim Q)$	(Negation of biconditional)

T25	$\therefore P \wedge (Q \wedge R) \leftrightarrow (P \wedge Q) \wedge R$	**T26**	$\therefore (P \to Q) \wedge (Q \to R) \to (P \to R)$
T27	$\therefore (P \wedge Q \to R) \leftrightarrow (P \to (Q \to R))$	**T36**	$\therefore \sim(P \wedge \sim P)$
T37	$\therefore (P \to Q) \leftrightarrow \sim(P \wedge \sim Q)$	**T45**	$\therefore P \vee Q \leftrightarrow (\sim P \to Q)$
T54	$\therefore P \vee (Q \vee R) \leftrightarrow (P \vee Q) \vee R$	**T59**	$\therefore P \vee \sim P$
T61	$\therefore P \wedge (Q \vee R) \leftrightarrow (P \wedge Q) \vee (P \wedge R)$	**T62**	$\therefore P \vee (Q \wedge R) \leftrightarrow (P \vee Q) \wedge (P \vee R)$
T67	$\therefore \sim P \wedge \sim Q \to \sim(P \vee Q)$	**T80**	$\therefore (P \leftrightarrow Q) \vee (P \leftrightarrow \sim Q)$
T83	$\therefore (P \leftrightarrow Q) \leftrightarrow (P \wedge Q) \vee (\sim P \wedge \sim Q)$	**T84**	$\therefore P \wedge Q \to (P \leftrightarrow Q)$
T87	$\therefore \sim(P \leftrightarrow Q) \leftrightarrow (P \wedge \sim Q) \vee (\sim P \wedge Q)$	**T88**	$\therefore P \wedge \sim Q \to \sim(P \leftrightarrow Q)$
T93	$\therefore (P \leftrightarrow Q) \wedge (Q \leftrightarrow R) \to (P \leftrightarrow R)$	**T96**	$\therefore (P \leftrightarrow Q) \leftrightarrow (\sim P \leftrightarrow \sim Q)$
T107	$\therefore (P \to (Q \to R)) \leftrightarrow (Q \to (P \to R))$	**T110**	$\therefore P \leftrightarrow \sim\sim P$
T111	$\therefore (P \to Q) \leftrightarrow (\sim Q \to \sim P)$	**T112**	$\therefore (P \to \sim Q) \leftrightarrow (Q \to \sim P)$

T116 $\therefore (P \wedge Q) \vee (R \wedge S) \leftrightarrow (P \vee R) \wedge (P \vee S) \wedge (Q \vee R) \wedge (Q \vee S)$

T118 $\therefore (P \to Q) \wedge (R \to S) \leftrightarrow (\sim P \wedge \sim R) \vee (\sim P \wedge S) \vee (Q \wedge \sim R) \vee (Q \wedge S)$

SECTION 3

Construct derivations to validate each of the following arguments. Use any of the rules, basic or derived. However, you should not be using the derived rules until AFTER you have derived the corresponding theorem (Theorems: 33, 40, 46, 49, 63, 64, 65, 66, 90) with only the basic rules. You can also use any theorem that you have already proved as a rule.

21. $S \rightarrow W.\quad T \vee (\sim S \rightarrow R)\ \therefore\ (W \vee T) \vee R$

22. $\sim(\sim P \rightarrow Q).\quad \sim(R \vee S \leftrightarrow Q).\quad R \rightarrow \sim T.\quad \sim S \vee \sim T.\quad \therefore\ \sim(T \vee P)$

23. $Q \vee S.\quad P \rightarrow (Q \rightarrow R).\quad P \rightarrow \sim R.\quad (T \rightarrow S) \rightarrow P.\quad \therefore\ \sim(P \leftrightarrow Q)$

24. $P \vee Q.\quad \sim(P \wedge Q).\quad P \vee \sim Q.\quad \therefore\ \sim(\sim P \vee Q)$

25. $\sim T \rightarrow \sim(S \rightarrow \sim R)\ \therefore\ (R \wedge S) \vee T$

26. $(R \leftrightarrow S) \vee Q.\quad \sim(Q \rightarrow \sim R) \rightarrow S.\quad \therefore\ R \rightarrow S$

27. $\therefore\ \sim(R \vee S) \wedge \sim(\sim P \rightarrow Q) \leftrightarrow \sim((P \vee Q) \vee (R \vee S))$

28. $\sim(P \rightarrow Q) \wedge (\sim T \vee Q).\quad \therefore\ (S \rightarrow P) \wedge (Q \leftrightarrow T)$

29. $P \vee Q.\quad P \rightarrow \sim(T \vee R).\quad Q \rightarrow (\sim R \leftrightarrow Q).\quad \therefore\ \sim R \vee \sim T.$

30. $\sim R \vee W.\quad X \wedge S \rightarrow T.\quad R \vee W.\quad \sim W \vee X.\quad \therefore\ S \rightarrow T \vee P$

31. $(\sim S \vee \sim T) \rightarrow (Q \vee P).\quad \sim(R \wedge Z) \rightarrow (P \rightarrow \sim Q).\quad \sim(\sim R \rightarrow (S \wedge T)).\quad \therefore\ \sim(P \leftrightarrow Q) \vee W$

32. $\sim(P \vee Q).\quad (R \rightarrow S) \rightarrow \sim(P \rightarrow T).\quad (S \vee Q) \vee W.\quad \therefore\ R \vee W$

33. $\sim(T \rightarrow Q) \rightarrow \sim(P \wedge X).\quad \sim(Q \vee S).\quad \sim P \rightarrow \sim W.\quad \sim(T \leftrightarrow S)\ \therefore\ \sim(W \rightarrow \sim X) \rightarrow S$

34. $(S \rightarrow (\sim P \vee T)) \rightarrow W.\quad R \leftrightarrow W \wedge \sim R\ .\quad \sim(S \rightarrow T) \rightarrow Q.\quad \therefore\ P \wedge Q$

35. $(T \rightarrow R) \rightarrow W.\quad (\sim S \vee P) \wedge \sim(S \leftrightarrow Q).\quad P \vee \sim R \rightarrow \sim Q.\quad P \vee \sim W.\quad \therefore P \leftrightarrow (R \vee S)$