The linear regression model. $SSR(\beta) = \sum_{i=1}^{n} (y_i - \beta^T x_i)^2$

$$Y_i = \beta^T x_i + \varepsilon_i \qquad \varepsilon_i \cdots \varepsilon_n \overset{iid}{\sim} \text{Normal}(0, \sigma^2)$$

$$Y_i \sim \text{Normal}(\beta^T x_i, \sigma^2)$$

$$\hat{\beta}_{OLS} = (X^T X)^{-1} X^T y. \quad Var(\hat{\beta}_{OLS}) = (X^T X)^{-1} \sigma^2$$

Bayesian Linear regression (semi-conjugate)

$p(\beta / y, X, \sigma^2)$ Multi-Normal

$$E(\beta / y, X, \sigma^2) = (\Sigma_0^{-1} + X^T X / \sigma^2)^{-1} (\underbrace{\Sigma_0^{-1}}\beta_0 + X^T y / \sigma^2)$$

$$Var(\beta / y, X, \sigma^2) = (\underset{\text{prior}}{\underset{\downarrow}{\Sigma_0^{-1}}} + \underset{\text{from data}}{\underset{\downarrow}{X^T X / \sigma^2}})^{-1}$$

$p(\sigma^2 / y, X, \beta)$ inverse-gamma

$$\left(\frac{\nu_0 + n}{2}, \frac{[\nu_0 \sigma_0^2 + SSR(\beta)]}{2} \right)$$

$$SSR(\beta) = \sum_{i=1}^{n} (y_i - \beta^T x_i)^2 = (y - X\beta)^T (y - X\beta)$$