June 4th

Sphere in $\mathbb{R}^3$ $\quad \{(x,y,z) \mid x^2+y^2+z^2=1\}$

$a=(0,0,1)$ $\quad b= \cdots$

$\cos\theta\sin\varphi \,.\, \cos\theta\cos\varphi \,,\, \sin\theta$

$\varphi=0 \quad , \theta=\frac{\pi}{2}$

path-connected.

Antipodal point

$(x,y,z) \Rightarrow (-x,-y,-z)$

distribution of temperature on the earth at same time is continuous.

P38. #9

sphere: $S\subset\mathbb{R}^3$, $f:S\to\mathbb{R}$ cont. whether $\exists x$. st. $f(x)=f(-x)$

$g(x)=f(x)-f(-x)$

Want to prove $g(x)$ has a zero on S.

$g(-x)=f(-x)-f(x)=-g(x)$

if $g(x)>0$, then $g(-x)<0$

$g(x)<0$, then $g(-x)>0$

By IVT, $\exists$ a point a s.t. $g(a)=0$.

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§2.1

$g(x)=x^2$, $g'(x)=2x$

$g(x)=\cos x$, $g'(x)=-\sin x$

if $g:\mathbb{R}\to\mathbb{R}$ is diff, is g' cont. or not?

May not be.

P52. #2

$$f(x)=\begin{cases} x^2\sin\left(\frac{1}{x}\right) & , x\neq 0 \\ 0 & . x=0 \end{cases}$$

diff everywhere except 0

Then check 0.

since $|\sin\frac{1}{h}|\leq 1$

so $|h\sin\frac{1}{h}|\leq|h|$

$$\lim_{h\to 0}\frac{f(h)-f(0)}{h}=\lim_{h\to 0}\frac{h^2\sin\frac{1}{h}}{h}=\lim_{h\to 0}h\sin\frac{1}{h}=0$$

so diff. everywhere

$f'(x) = 2x\sin\frac{1}{x} - \cos\frac{1}{x}$, $\forall\ x \neq 0$ which doesn't have a lim.

So $\lim_{x\to 0} \cos\frac{1}{x}$ not exists.

Hence the function is diff but not cont.

P52 #4

$$h(x) = \begin{cases} x^2, & x \in \mathbb{Q} \\ 0, & x \notin \mathbb{Q} \end{cases}$$

$|f(x)| \leq |x^2| = |x|^2 \to 0$ as $|x| \to 0$

$\forall \varepsilon > 0, \exists \delta > 0$ s.t. $\forall x$, $|x-a| < \delta \Rightarrow |f(x) - f(a)| < \varepsilon$

negation: $\exists \varepsilon > 0, \forall \delta > 0$ s.t. $\exists x$ $|x-a| < \delta \Rightarrow |f(x) - f(a)| \geq \varepsilon$

say $\varepsilon = x^2/2$ (works whatever x, $\mathbb{Q}$ or $\neg\mathbb{Q}$)