NAME: **STUDENT ID NUMBER :**

Check your tutorial:	○ TUT5101 TA: Boris	○ TUT5102 TA: James	○ TUT5103 TA: Nan

Instructions: No Calculators. Please clearly circle the correct answer. Rough work can be done on the sides or on the additional pieces of paper provided (which does not need to be handed in). Enjoy!

1. What is the standard distance between the points (1,2,5) and (3,0,4)? $d(\vec{a},\vec{b}) = |\vec{a}-\vec{b}|$

   A 1 **B** $\sqrt{101}$ **C** 3 **D** 101 **E** 9

   so $\sqrt{(3-1)^2+(0-2)^2+(4-5)^2} = 3$

2. How many of the following are true for $\mathbf{x}, \mathbf{y}, \mathbf{z} \in \mathbb{R}^3$?
   i) $\mathbf{x} \times \mathbf{y} = -\mathbf{y} \times \mathbf{x}$ True (anti-commutivity)
   ii) $(\mathbf{x} \times \mathbf{y}) \times \mathbf{z} = \mathbf{x} \times (\mathbf{y} \times \mathbf{z})$ False (See Jacobi)
   iii) $\mathbf{x} \times \mathbf{x} = \mathbf{0}$ True
   iv) $\mathbf{x} \cdot (\mathbf{y} \times \mathbf{z}) = \mathbf{0}$ False (triple product)
   v) $(\mathbf{x} \times \mathbf{y}) \cdot \mathbf{x} = \mathbf{0}$ True (orthogonality of $x \times y$)

   A 0 or 1 **B** 2 **C** 3 **D** 4 **E** 5

3. Which of the following vectors is perpendicular to (1,1,2)?
   A (1,0,0)
   B (1,-1,1)
   C (2,0,1)
   D (3,1,-2) $(3,1,-2)\cdot(1,1,2) = 3+1-4=0$
   E (1,1,2)

4. Consider the vector space $\mathbb{R}^n$ and let $\mathbf{x}, \mathbf{y}, \mathbf{z} \in \mathbb{R}^n$, $a, b \in \mathbb{R}$. How many of the following are properties of this vector space? $\mathbf{1}$ is ambiguous notation. Hence I accept either T or F as correct. i) is true if $\mathbf{1}$ means $(1,1,\dots,1)$ but false if it means some form of "multiplicative identity" which we don't have.
   i) $\forall \mathbf{x} \exists \mathbf{y}$ such that $\mathbf{x} + \mathbf{y} = \mathbf{1}$
   ii) $|\mathbf{x}| = \sqrt{\mathbf{x} \cdot \mathbf{x}}$ False
   iii) if $\mathbf{x} \cdot \mathbf{y} = \mathbf{x} \cdot \mathbf{z}$, $\mathbf{x} \neq \mathbf{0}$ then $\mathbf{y} = \mathbf{z}$ False, Ex $(1,0,0)$, $(0,1,0)$, $(0,0,1)$
   iv) $((a+b)\mathbf{x}) \cdot \mathbf{y} = (a\mathbf{x}) \cdot \mathbf{y} + b(\mathbf{x} \cdot \mathbf{y})$ True (distributivity)
   v) Pythagoras Theorem True

   A 0 or 1 **B** 2 **C** 3 **D** 4 **E** 5

   Both are fine.

5. Describe the set of all vectors in $\mathbb{R}^3$ perpendicular to (1,1,2) and (2,0,3)

A $t(3,-1,2), t \in \mathbb{R}$

B $t(2,0,6), t \in \mathbb{R}$

C $t(3,1,5), t \in \mathbb{R}$

D $t(3,-1,5), t \in \mathbb{R}$

E $t(3,1,-2), t \in \mathbb{R}$

$$(1,1,2)\times(2,0,3) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 2 \\ 2 & 0 & 3 \end{vmatrix} = \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}$$

which is $\perp$ to (1,1,2) & (2,0,3)

6. What is the angle between (0,1,1) and (0,1,0)?

A 0 B $\pi/4$ C $\pi/3$ D $\pi/2$ E π

$$\theta = \arccos\left(\frac{(0,1,1)\cdot(0,1,0)}{|(0,1,1)||(0,1,0)|}\right) = \arccos\left(\frac{1}{\sqrt{2}\cdot 1}\right) = \frac{\pi}{4}$$

7. Find an equation of the line passing through the points (1,2) and (3,5).

A $(x,y) = t(2,3) + (1,2), t \in \mathbb{R}$

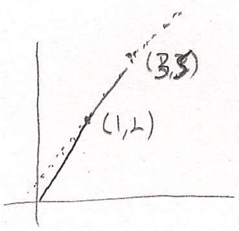
B $(x,y) = t(1,2) - (3,5), t \in \mathbb{R}$

C $(x,y) = t(-2,-3), t \in \mathbb{R}$

D $(x,y) = t(3,10) + (3,5), t \in \mathbb{R}$

E $(x,y) = t(3,5) + (1,2), t \in \mathbb{R}$

$(3,5) - (1,2) = (2,3)$



8. Consider the vector space $\mathbb{R}^n$ and let $\mathbf{x}, \mathbf{y}, \mathbf{z} \in \mathbb{R}^n$, $c \in \mathbb{R}$. How many of the following inequalities are true?

i) $|\mathbf{x} \cdot \mathbf{y}| \leq |\mathbf{x}| + |\mathbf{y}|$ False

ii) $|\mathbf{x}|^2 \leq max\{|x_i|^2\}_{1 \leq i \leq n} \cdot n$ False

iii) $|\mathbf{z} - \mathbf{x}| \leq |\mathbf{z} - \mathbf{y}| + |\mathbf{y} - \mathbf{x}|$ True Triangle

iv) $|c\mathbf{x}| < |c||\mathbf{x}|$ False, should be =

v) $|\mathbf{x} \times \mathbf{y}| \leq$ Area of Parallelogram determined by $\mathbf{x}$ and $\mathbf{y}$ True, but $\leq$ should be =

A 0 or 1 B 2 C 3 D 4 E 5