Name: SOLUTIONS.

MAT 334H
SUMMER 2014
QUIZ 2

Problem	1	2	3	Total
Points	5	5	5	15
Score				

- Solve the following problems, and write up your solutions neatly, in black or blue ink, in the space provided.
- Please make sure your name is entered at the top of this page.
- This quiz contains 4 pages. Please ensure they are all there.
- Please do not tear out any pages.
- You have 30 minutes to complete this quiz.
- There are *no* aids allowed.

GOOD LUCK!

(1) Determine whether the statement is True or False. Circle your answer. (No justification required.)

(a) The function $f(z) = (1 - e^z)\sin z$ has a zero of order 2 at $z_0 = 0$. $\boxed{\text{True}}$ False

(b) If f has a zero of order 2 at z_0, and g has a zero of order 2 at z_0, then $\frac{f}{g}$ has a pole at z_0. True $\boxed{\text{False}}$

(c) If f has a removable singularity at z_0, then $\mathrm{Res}(f : z_0) = 2\pi i$. True $\boxed{\text{False}}$

(d) If f has an essential singularity at z_0, then $\lim_{z \to z_0} |f(z)| = \infty$. True $\boxed{\text{False}}$

(e) If $f(z_0) \neq 0$, then $\dfrac{f(z)}{(z - z_0)^n}$ has a pole of order n at z_0. $\boxed{\text{True}}$ False

(2) Let $f(z) = \dfrac{e^{\frac{1}{z^2+1}}(z^3-8)^4}{(z^4-16)^3}$.

(a) Find the zeroes of f, and determine their orders.

$e^{\frac{1}{z^2+1}} \neq 0$, so we just need to solve $z^3-8=0$, or $z^3=8$

$z^3 = 8 \Rightarrow z = 2,\ 2e^{2\pi i/3},\ 2e^{4\pi i/3}$

$z=2$ is a zero of order $4-3=1$ (@ $z_0=2$, denom equals 0 with order 3)

$z = 2e^{2\pi i/3}, 2e^{4\pi i/3}$ are zeros of order 4.

(b) Find and classify each isolated singularity of f. If there are any poles, determine their orders.

$z^4-16 = (z^2-4)(z^2+4)$ So we get denom $=0$ at $z = \pm 2, \pm 2i$

$z=2$ is a removable singularity (see above)

$z=-2, \pm 2i$ are poles of order 3.

$z = \pm i$ are esssential singularities

↳ occur when $z^2+1=0$.

(3) Consider the function $f(z) = \dfrac{e^{2z}}{(z-1)^5}$.

(a) Find a power series for e^{2z} centred at $z_0 = 1$.

$$e^{2z} = e^{2(z-1)} \cdot e^2 = e^2 \cdot \sum_{n=0}^{\infty} \frac{[2(z-1)]^n}{n!} \qquad \left(e^w = \sum \frac{w^n}{n!} \right)$$

$$= e^2 \cdot \sum_{n=0}^{\infty} \frac{2^n}{n!} (z-1)^n$$

$$= \sum_{n=0}^{\infty} \frac{e^2 2^n}{n!} (z-1)^n$$

(b) Compute Res$(f : 1)$.

$z_0 = 1$ is a pole of order 5 so we get:

$$\cancel{z} \quad \mathrm{Res}(f : 1) = C_{5-1} = C_4 = \frac{e^2 \cdot 2^4}{4!}$$

$$= \frac{2e^2}{3}$$