Symbolizations with multi-place predicates (Unit 5: Sections 5.6-5.9)

Symbolize each of the following sentences using the abbreviation scheme provided

- B^1 : a is a mouse. C^1 : a is cat. F^2 : a is afraid of b. G^2 : a chases b. A^1 : a is an animal.
- D^1 : a is a dog.
- a: Astro b: Bandit
- **\$5.74** All dogs chase cats.

$$\forall x(Dx \rightarrow \exists y(Cy \land G(xy)))$$
 (All dogs chase at least one cat.)

\$5.75 All mice are chased by animals.

$$\forall x(Bx \rightarrow \exists y(Ay \land G(yx)))$$
 (All mice are chased by at least one animal.)

S5.76 Some cats are chased by dogs.

$$\exists x(Cx \land \exists y(Dy \land G(yx)))$$
 (At least one cat is chased by at least one dog.)

\$5.77 No dog chases all cats.

$$\sim \exists x(Dx \land \forall y(Cy \rightarrow G(xy)))$$
 (There is no dog that chases all cats.)

$$\forall x(Dx \rightarrow \exists x(Cx \land \sim G(xy)))$$
 (Every dog has at least one cat it doesn't chase.)

S5.78 Some dogs do not chase any cats.

$$\exists x(Dx \land \forall y(Cy \rightarrow \neg G(xy)))$$
 (There is some dog that fails to chase all cats.)

$$\exists x(Dx \land \neg \exists y(Cy \land G(xy)))$$
 (There is some dog such that there's no cat that it chases.)

\$5.79 Some dogs do not chase some cats.

$$\exists x(Dx \land \exists y(Cy \land \sim G(xy)))$$

(There is at least one dog such that there is at least one cat that it doesn't chase.)

\$5.80 Some cats chase mice, but not all mice are chased by cats.

$$\exists x (Cx \land \exists y (By \land G(xy))) \land \neg \forall x (Bx \rightarrow \exists y (Cy \land G(yx)))$$

S5.81 All cats and dogs chase mice.

$$\forall x(Cx \lor Dx \rightarrow \exists y(By \land G(xy)))$$

\$5.82 Some dogs chase cats and mice.

$$\exists x (Dx \land \exists y (Cy \land G(xy)) \land \exists z (Bz \land G(xz)))$$

\$5.83 Mice are afraid of cats and dogs.

$$\forall x(Bx \rightarrow \forall y(Cy \lor Dy \rightarrow F(xy))) OR$$

$$\forall x(Bx \rightarrow \forall y(Cy \rightarrow F(xy))) \land \forall x(Bx \rightarrow \forall y(Dy \rightarrow F(xy)))$$

S5.84 Although Astro is a dog, he doesn't chase cats or mice.

$$Da \wedge \forall x(Cx \vee Bx \rightarrow \neg G(ax))$$
 OR $Da \wedge \neg \exists x((Cx \vee Bx) \wedge \neg G(ax))$

S5.85 No dog that chases Bandit is afraid of cats that chase dogs.

$$\sim \exists x (Dx \land G(xb) \land \forall y (Cy \land \exists z (Dz \land G(yz)) \rightarrow F(xy)))$$

S5.86 Any animal that Bandit chases is afraid of her.

$$\forall x (Ax \land G(bx) \rightarrow F(xb))$$

\$5.87 Any dog that chases mice chases cats.

$$\forall x(Dx \land \exists y(By \land G(xy)) \rightarrow \exists z(Cz \land G(xz)))$$

\$5.88 Not all animals that chase cats are dogs.

$$\exists x(Ax \land \exists y(Cy \land G(xy)) \land \sim Dx) \ OR \sim \forall x(Ax \land \exists y(Cy \land G(xy)) \rightarrow Dx)$$

S5.89 Some dogs chase cats that chase mice.

$$\exists x (Dx \land \exists y (Cy \land \exists z (Bz \land G(yz)) \land G(xy)))$$

\$5.90 Mice are afraid of cats that chase them.

$$\forall x(Bx \rightarrow \forall y(Cy \land G(yx) \rightarrow F(xy)))$$

S5.91 Mice, which are afraid of cats, get chased by them.

$$\forall x(Bx \to \forall y(Cy \to F(xy))) \land \forall x(Bx \to \exists y(Cy \land G(yx)))$$

\$5.92 Some cats are chased by dogs that don't chase mice.

$$\exists x (Cx \land \exists y (Dy \land \sim \exists z (Bz \land G(xz)) \land G(yx)))$$

S5.93 Dogs and cats are animals that chase things.

$$\forall x(Dx \lor Cx \to Ax \land \exists yG(xy))$$

S5.94 Astro chases those and only those cats that don't chase Astro.

$$\forall x(Cx \rightarrow (\sim G(xa) \leftrightarrow G(ax)))$$

\$5.95 Any cat is afraid of a dog that chases it.

$$\forall x(Cx \rightarrow \forall y(Dy \land G(yx) \rightarrow F(xy)))$$

S5.96 Bandit isn't afraid of dogs unless they chase her.

$$\forall x(Dx \rightarrow \sim F(bx) \lor G(xb)) \quad OR \quad \forall x(Dx \rightarrow (F(bx) \rightarrow G(xb)))$$

S5.97 Some cats are not afraid of dogs unless the dogs chase them.

$$\exists z (Cz \land \forall x (Dx \rightarrow \neg F(zx) \lor G(xz))) \ OR \quad \exists z (Cz \land \forall x (Dx \rightarrow (F(zx) \rightarrow G(xz))))$$

\$5.98 No dog is afraid of a cat unless the cat chases them.

$$\sim \exists x (Dx \land \exists y (Cy \land F(xy) \land \sim G(yx)))$$
 or $\forall x (Dx \rightarrow \forall y (Cy \rightarrow (\sim G(yx) \rightarrow \sim F(xy))))$

S5.99 Dogs that are afraid of cats don't chase any animals at all.

$$\forall x(Dx \land \forall y(Cy \rightarrow F(xy)) \rightarrow \neg \exists z(Az \land G(xz)))$$

\$5.100 Bandit chases only those dogs that chase the mice that Bandit chases.

$$\forall x(Dx \land G(bx) \rightarrow \exists y(By \land G(by) \land G(xy)))$$

Symbolize each of the following sentences using the abbreviation scheme provided

 A^1 : a is an author. B^1 : a is a book. C^1 : a is a magazine. F^1 : a is a person. G^1 : a is a time. H^2 : a writes b. I^2 : a is about b. L^2 : a likes b. M^3 : a reads b at c

\$5.101 Nobody likes everyone.

$$\sim \exists x(Fx \land \forall y(Fy \rightarrow L(xy))) \ OR \ \forall x(Fx \rightarrow \exists y(Fy \land \sim L(xy)))$$

\$5.102 Someone likes everyone. (Ambiguous – symbolize two different ways.)

 $\forall x (Fx \rightarrow \exists y (Fy \land L(yx)))$ Everyone has somebody that likes them. $\exists x (Fx \land \forall y (Fy \rightarrow L(xy)))$ There is a person who likes everyone.

\$5.103 Everybody likes somebody. (Ambiguous – symbolize two different ways.)

 $\forall x (Fx \rightarrow \exists y (Fy \land L(xy)))$ Everyone has somebody that they like.

 $\exists x (Fx \land \forall y (Fy \rightarrow L(yx)))$ There is a person whom everyone likes.

\$5.104 Nobody is liked by everyone.

$$\sim \exists x (Fx \land \forall y (Fy \rightarrow L(yx))) \ OR \ \forall x (Fx \rightarrow \exists y (Fy \land \sim L(yx)))$$

\$5.105 Everybody likes those who like them.

$$\forall x(Fx \rightarrow \forall y(Fy \land L(yx) \rightarrow L(xy)))$$

\$5.106 Some people like only those who like them.

$$\exists x(Fx \land \forall y(Fy \land L(xy) \rightarrow L(yx)))$$

\$5.107 Nobody who likes only those who like them likes people who like nobody.

$$\neg \exists x (Fx \land \forall y (Fy \land L(xy) \rightarrow L(yx)) \land \exists z (Fz \land L(xz) \land \forall w (Fw \rightarrow \neg L(zw))))$$

\$5.108 Authors are people who write books.

$$\forall x(Ax \rightarrow Fx \land \exists y(By \land H(xy)))$$

\$5.109 People who write books like at least some books.

$$\forall x(Fx \land \exists y(By \land H(xy)) \rightarrow \exists z(Bz \land L(xz)))$$

\$5.110 People who like exactly the same books as each other like the same magazines too.

$$\forall x \forall y (Fx \land Fy \land \forall z (Bz \rightarrow (L(xz) \leftrightarrow L(yz))) \rightarrow \forall z (Cz \rightarrow (L(xz) \leftrightarrow L(yz)))$$

\$5.111 Some people write books that are about people who write books.

$$\exists x(Fx \land \exists y(By \land H(xy) \land \exists z(Fz \land \exists w(Bw \land H(zw)) \land I(yz))))$$

\$5.112 Any authors who only write books about things that they like, also like things that they don't write books about.

$$\forall x(Ax \land \forall y \forall z(Bz \land H(xz) \land I(zy) \rightarrow L(xy)) \rightarrow \exists y(L(xy) \land \sim \exists z(Bz \land H(xz) \land I(zy))))$$

\$5.113 All books are read by someone or another at some time or another.

$$\forall x(Bx \rightarrow \exists y(Fy \land \exists z(Gz \land M(yxz))))$$

S5.114 Everybody who writes books reads books.

$$\forall x(Fx \land \exists y(By \land H(xy)) \rightarrow \exists z(Bz \land \exists w(Gw \land M(xzw))))$$

\$5.115 Those people who never read books don't write books.

OR
$$\forall x(Fx \land \neg \exists y(Gy \land \exists z(Bz \land M(xzy))) \rightarrow \neg \exists y(By \land H(xy))$$

\$5.116 People who only ever read magazines don't like books.

$$\forall x (Fx \land \forall y (Gy \rightarrow \forall z (M(xzy) \rightarrow Cz)) \rightarrow \forall w (Bw \rightarrow \sim L(xw)))$$

OR
$$\forall x(Fx \land \neg \exists y \exists z(Gy \land \neg Cz \land M(xzy)) \rightarrow \neg \exists w(Bw \land L(xw)))$$

S5.117 Only people who have never read magazines like everything that they have ever read.

$$\forall x(Fx \land \forall y \forall z(Gz \land M(xyz) \rightarrow L(xy)) \rightarrow \neg \exists y(Gy \land \exists z(Cz \land M(xzy))))$$

OR
$$\forall x(Fx \land \forall y(\exists z(Gz \land M(xzy)) \rightarrow L(xz)) \rightarrow \neg \exists y(Gy \land \exists z(Cz \land M(xzy))))$$

\$5.118 Some authors don't ever read magazines, but every magazine gets read by some author.

$$\exists x(Ax \land \neg \exists y(Gy \land \exists z(Cz \land M(xzy)))) \land \forall x(Cx \rightarrow \exists y(Ay \land \exists z(Gz \land M(yxz))))$$

$$OR \ \exists x (Ax \land \sim \exists y (Cy \land \exists z (Gz \land M(xyz)))) \land \ \forall x (Cx \rightarrow \exists y (Gy \land \exists z (Az \land M(zxy))))$$

\$5.119 Anybody who likes every book that he/she ever reads doesn't ever read books about things that he/she doesn't like.

$$\forall x(Fx \land \forall y(By \land \exists z(Gz \land M(xyz)) \rightarrow L(xy)) \rightarrow \neg \exists y(Gy \land \exists z(Bz \land \exists w(I(zw) \land \neg L(xw) \land M(xzy)))))$$

\$5.120 Not all authors write books about things that they have read books about.

$$\sim \forall x(Ax \rightarrow \exists y(By \land H(xy) \land \exists z(I(yz) \land \exists w(Bw \land I(wy) \land \exists v(Gv \land M(xwv)))))$$

\$5.121 Nobody who reads books likes all the books he/she has ever read.

$$\sim \exists x (Fx \land \exists y (By \land \exists z (Gz \land M(xyz))) \land \forall y (By \land \exists z (Gz \land M(xyz)) \rightarrow L(xy)))$$

\$5.122 Only authors like all the books that they have ever read.

$$\forall x(Fx \land \forall y(By \land \exists z(Gz \land M(xyz)) \rightarrow L(xy))) \rightarrow Ax)$$

\$5.123 Although all books are read by people sooner or later, not all books are ever read by the same person.

$$\forall x(Bx \rightarrow \exists y(Fy \land \exists z(Gz \land M(yxz)))) \land \neg \exists x(Fx \land \forall y(By \rightarrow \exists z(Gz \land M(xyz))))$$

\$5.124 Not everyone reads the same book at the same time.

$$\sim \exists x (Bx \land \exists y (Gy \land \forall z (Fz \rightarrow M(zxy))))$$

\$5.125 Someone is always reading a book. (Ambiguous – symbolize 4 diff. ways.)

 $\exists x(Fx \land \exists y(By \land \forall z(Gz \rightarrow M(xyz))))$ The same person is always reading the same book.

 $\exists y(By \land \exists x(Fx \land \forall z(Gz \rightarrow M(xyz))))$ The same person is always reading the same book.

 $\exists x(Fx \land \forall z(Gz \rightarrow \exists y(By \land M(xyz))))$ The same person is always reading some book or another.

 $\exists y(By \land \forall z(Gz \rightarrow \exists x(Fx \land M(xyz))))$ The same book is always being read by someone or another.

 $\forall z(Gz \rightarrow \exists x(Fx \land \exists y(By \land M(xyz))))$ At all times, (different) people are reading (different) books.

 $\forall z(Gz \rightarrow \exists y(By \land \exists x(Fx \land M(xyz))))$ At all times, (different) people are reading (different) books.

Symbolize each of the following sentences using the abbreviation scheme provided

A¹: a is a restaurant. B¹: a is a business. C¹: a is a store. D¹: a is a day. F¹: a is a food. H¹: a is a person. L²: a is more expensive than a. K²: a shops at a. L³: a eats at a on a. M³: a buys a from a. (Or a sells a to a.) a: Honest Ed's b: Holt Renfrew

\$5.126 Some stores are more expensive than any restaurant.

$$\exists x (Cx \land \forall y (Ay \rightarrow J(xy)))$$

S5.127 Some people only buy things from stores that are more expensive than Honest Ed's.

$$\exists x (Hx \land \forall y (Cy \land \exists z M(xzy) \rightarrow J(ya))) \text{ or } \exists x (Hx \land \forall y (Cy \rightarrow (\exists z M(xzy) \rightarrow J(ya))))$$

$$\exists x (Hx \land \forall y \forall z (Cy \land M(xzy) \rightarrow J(ya))) \text{ or } \exists x (Hx \land \forall y \forall z (Cy \rightarrow (M(xzy) \rightarrow J(ya))))$$

\$5.128 Nothing people buy at Honest Ed's is more expensive than anything people buy at Holt Renfrew.

$$\sim \exists x \exists y (Hy \land M(yxa) \land \exists z \exists w (Hz \land M(zwb) \land J(xw)))$$

$$\forall x \forall y (Hy \land M(yxa) \rightarrow {\sim} \exists z \exists w (Hz \land M(zwb) \land J(xw)))$$

$$\forall x \forall y (Hy \land M(yxa) \rightarrow \forall z \forall w (Hz \land M(zwb) \rightarrow {\sim} J(xw)))$$

\$5.129 Stores and restaurants are businesses that sell things to people.

$$\forall x (Cx \vee Ax \rightarrow \exists y \exists z (Hz \wedge M(zyx))) \quad (Description \ of \ stores \ and \ restaurants.)$$

or
$$\forall x \exists y \exists z (Cx \lor Ax \rightarrow (Hz \land M(zyx)))$$

$$\forall x(Cx \lor Ax \leftrightarrow \exists y \exists z(Hz \land M(zyx)))$$
 (Definition of stores and restaurants.)

\$5.130 Some days nobody eats at some restaurants.

$$\exists x (Dx \wedge \exists y (Ay \wedge {\sim} \exists z (Hz \wedge L(zyx)))$$

$$\exists x (Dx \land \exists y (Ay \land \forall z (Hz \rightarrow \sim L(zyx)))$$

\$5.131 For a business to be a store, it is necessary that people shop there.

$$\forall x(Bx \to (Cx \to \exists y(Hy \land K(yx))) \ OR \ \forall x(Bx \land Cx \to \exists y(Hy \land K(yx)))$$

\$5.132 For a business to be a restaurant, it is not sufficient that people by food from it. Either, it is not the case that if people buy food from a business then it's a restaurant

OR Some businesses that people buy food from are not restaurants.

$${\sim} \forall x (Bx \land \exists y (Hy \land \exists z (Fz \land M(yzx))) \rightarrow Ax) \ OR \ \exists x (Bx \land \exists y (Hy \land \exists z (Fz \land M(yzx))) \land {\sim} Ax)$$

\$5.133 Unless a person eats at restaurants every day, that person has to buy food from a store.

$$\forall x(Hx \rightarrow \forall y(Dy \rightarrow \exists z(Az \land L(xzy))) \lor \exists y(Fy \land \exists z(Cz \land M(xyz))))$$

S5.134 If anybody shops at a store, then somebody buys things from somebody.

$$\exists x (Hx \land \exists y (Cy \land K(xy))) \rightarrow \exists x (Hx \land \exists y \exists z (Hz \land M(xyz)))$$

$$\exists x\exists y \ (Hx \land Cy \land K(xy)) \rightarrow \exists x\exists y\exists z (Hx \land Hz \land M(xyz)))$$

\$5.135 If anybody shops at a store, he/she buys something from somebody.

$$\forall x(Hx \land \exists y(Cy \land K(xy)) \rightarrow \exists z \exists w(Hz \land M(xwz)))$$

$$\forall x \forall y (Hx \land Cy \land K(xy)) \rightarrow \exists z \exists w (Hz \land M(xwz)))$$
 or

$$\forall x \forall y \exists z \exists w (Hx \land Cy \land K(xy)) \rightarrow Hz \land M(xwz))$$

S5.136 Everybody buys things from stores, but no store sells things to everybody.

$$\forall x(Hx \rightarrow \exists y \exists z(Cz \land M(xyz))) \land \neg \exists x(Cx \land \forall y(Hy \rightarrow \exists zM(yzx)))$$

\$5.137 Not every business that sells food to people is a restaurant.

$$\exists x (Bx \land \exists y (Fy \land \exists z (Hz \land M(zyx))) \land \sim Ax)$$

\$5.138 Some people don't shop at stores unless those stores are less expensive than Holt Renfrew.

$$\exists x (Hx \land \forall y (Cy \rightarrow \sim K(xy) \lor J(by))) \quad OR \quad \exists x (Hx \land \forall y (Cy \rightarrow (K(xy) \rightarrow J(by))))$$

$$OR \quad \exists x (Hx \land \forall y (Cy \rightarrow (\sim J(by) \rightarrow \sim K(xy))))$$

S5.139 Although Holt Renfrew is more expensive than Honest Ed's, not everything that Holt Renfrew sells to people is more expensive than everything that Honest Ed's sells to people.

$$J(ba) \land \neg \forall x (\exists y (Hy \land M(yxb)) \rightarrow \forall z (\exists w (Hw \land M(wza)) \rightarrow J(xz))))$$

$$J(ba) \land \neg \forall x \forall y (Hy \land M(yxb)) \rightarrow \forall z \forall w (Hw \land M(wza) \rightarrow J(xz)))$$

$$J(ba) \wedge {\sim} \forall x \forall y \forall z \forall w (Hy \wedge M(yxb) \wedge Hw \wedge M(wza) {\rightarrow} J(xz)))$$

$$J(ba) \wedge \exists x \exists y \exists z \exists w (Hy \wedge M(yxb) \wedge Hw \wedge M(wza) \wedge \sim J(xz)))$$

\$5.140 Everybody eats at a restaurant some days, but nobody eats at restaurants every day.

$$\forall x(Hx \rightarrow \exists y(Ay \land \exists z(Dz \land L(xyz)))) \land \neg \exists x(Hx \land \forall y(Dy \rightarrow \exists z(Az \land L(xzy))))$$

$$\forall x \exists y \exists z (Hx \to Ay \land Dz \land L(xyz)) \land \forall x (Hx \to \neg \forall y (Dy \to \exists z (Az \land L(xzy))))$$

$$\forall x (Hx \rightarrow \exists z \exists y (Ay \land Dz \land L(xyz))) \land \forall x (Hx \rightarrow \exists y (Dy \land \sim \exists z (Az \land L(xzy))))$$

$$\forall x\: (Hx \to \exists z (Dz \land \exists y (Ay \land L(xyz)))) \land \forall x (Hx \to \exists y (Dy \land \forall z (Az \to \sim L(xzy))))$$

S5.141 A business that sells things that are more expensive than anything people buy from any store is not a place where people shop.

This was a bit tricky... The subject: all stores. What it says: if the store is such that something it sells is more expensive than anything that any store sells to anybody, then nobody shops at that store.

$$\forall x (Bx \to (\exists y \exists z (Hz \land M(zyx) \land \forall s \forall t \forall w (Cs \land Ht \land M(tws) \to J(yw))) \to \sim \exists i (Hi \land K(ix))))$$
 or
$$\forall x (Bx \land \exists y \exists z (Hz \land M(zyx) \land \forall s \forall t \forall w (Cs \land Ht \land M(tws) \to J(yw))) \to \forall i (Hi \to \sim K(ix)))$$

\$5.142 Some people who shop at exactly the same stores as each other, don't buy anything from the same person.

$$\exists x \exists y (Hx \land Hy \land \forall z (Cz \to (K(xz) \leftrightarrow K(yz))) \land \neg \exists w (Hw \land \exists i M(xiw) \land \exists j M(yjw)))$$
 OR

$$\exists x (Hx \land \exists y (Hy \land \forall z (Cz \land K(xz) \leftrightarrow Cz \land K(yz)) \land \forall w (Hw \rightarrow \sim (\exists i M(xiw) \land \exists j M(yjw))))$$

\$5.143 Nobody buys everything they buy from the same store.

\$5.144 Every store sells things to someone. (Ambiguous – symbolize two different ways.)

 $\forall x (Cx \to \exists y \exists z (Hz \land M(zyx))) \quad \text{All stores sell at least one thing to at least one person.}$

 $\exists x(Hx \land \forall y(Cy \rightarrow \exists zM(xzy)))$ There is somebody who every store sells something to.

\$5.145 There are restaurants where people eat every day. (Ambiguous – symbolize two different ways)

 $\exists x(Ax \land \forall y(Dy \rightarrow \exists z(Hz \land L(zxy)))$ There is at least one restaurant such that everyday people eat there.

 $\exists x(Ax \land \exists z(Hz \land \forall y(Dy \rightarrow L(zxy)))$ There is at least one restaurant such that at least one person eats there every day.

\$5.146 Somebody eats at some restaurant every day. (Ambiguous – symbolize four different ways.)

 $\exists x(Hx \land \exists y(Ay \land \forall z(Dz \rightarrow L(xyz)))$ Some one person eats at some one restaurant every day.

 $\exists x(Hx \land \forall z(Dz \rightarrow \exists y(Ay \land L(xyz))))$ Some one person eats at some restaurant or another every day.

 $\exists y(Ay \land \forall z(Dz \rightarrow \exists x(Hx \land L(xyz)))$ There is a restaurant and every day someone or another eats there.

 $\forall z(Dz \rightarrow \exists x(Hx \land \exists y(Ay \land L(xyz))))$ Every day, someone or another eats at some restaurant or another.