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MAT 334H
SUMMER 2014
QUIZ 2

Problem	1	2	3	Total
Points	5	5	5	15
Score	5	3	$4\frac{1}{2}$	12.5

- Solve the following problems, and write up your solutions neatly, in **black or** blue ink, in the space provided.
- Please make sure your name is entered at the top of this page.
- This quiz contains 4 pages. Please ensure they are all there.
- Please do not tear out any pages.
- You have 30 minutes to complete this quiz.
- There are *no* aids allowed.

$20+5\times\cancel{a}\frac{4.5}{15}+5+5\times\cancel{a}\frac{55}{65}+5\times\frac{46}{65}$

GOOD LUCK!

(1) Determine whether the statement is True or False. Circle your answer. (No justification required.)

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(a) The function $f(z) = (1 - \cos z)(z^3 - z^2)$ has a zero of order 5 at $z_0 = 0$. True **False**

2 3

sinz

cosz

(b) If f has a zero of order 3 at z_0, and g has a zero of order 2 at z_0, then $\frac{f^2}{g^3}$ has a removable singularity at z_0. **True** False

(c) If $f(z) = \sum_{k=0}^{\infty} \frac{k}{k^2+1}(z - z_0)^k$, then $\text{Res}(\frac{f(z)}{(z - z_0)^3} : z_0) = \frac{3}{10}$. True **False**

(d) If $\lim_{z \to z_0} \frac{1}{|f(z)|} = \infty$, then f has a removable singularity . **True** False

(e) If $\lim_{z \to z_0} (z - z_0)^5 f(z) = 5$, then z_0 is a pole of f. **True** False

(2) Let $f(z) = \dfrac{e^{\frac{1}{z+1}}(z^3-27)^3}{(z^4-81)^4}$.

(a) Find the zeroes of f, and determine their orders.

Since $e^{\frac{1}{z+1}} \neq 0 \;\forall z \in \mathbb{C}$. 4.

$(z^4-81)^4 \neq 0$ if it is defined. $1\frac{1}{2}$.

so only $(z^3-27)^3 = 0 = z^3-27$

$z^3 = 27$

$(3e^{i\theta})^3 = 27$

$e^{i\theta} = 1$

So $\theta = 0 + 2k\pi$, or $\frac{2}{3}\pi + 2k\pi$ or $\frac{4}{3}\pi + 2k\pi$ $\forall k \in \mathbb{Z}$ (only 3.) $\Rightarrow z = \begin{cases} 3e^{i(2k\pi)} \\ 3e^{i(\frac{2}{3}\pi + 2k\pi)} \\ 3e^{i(\frac{4}{3}\pi + 2k\pi)} \end{cases}$ $\| 3$

orders: let $w = z^3 - 27$.

$w' = 3z^2 \neq 0$.

So, the order is 1. ?

not a zero.

zeros of order 3.

(b) Find and classify each isolated singularity of f. If there are any poles, determine their orders.

The isolated singularity of f ~~is~~ the

the solutions to the following $(z^4-81)^4 = 0$ $1\frac{1}{2}$.

so $z^4 = 81$

$\Rightarrow z_0 = \pm 3, \pm 3i$

essential

finite. ✓

We don't need to care about $e^{\frac{1}{z+1}}$, since for certain $z = z_0$, $e^{\frac{1}{z+1}}$ is

just focus on $\dfrac{(z^3-27)^3}{(z^4-81)^4}$. note that $z_0 = 3$ is one of

zeroes of f. so use L'Hopital rule when $z = 3$.

$\lim_{z \to 3} \left| \dfrac{(z^3-27)^3}{(z^4-81)^4} \right| < \infty$. so it's a removable singularity.

order 1

But for other z_0's, i.e. $z_0 = -3, \pm 3i$.

We get poles with order 1. 4.

why pole? Because $\lim_{z \to z_0} \left| \dfrac{(z^3-27)^3}{(z^4-81)^4} \right| = \infty$ where $z_0 = -3, \pm 3i$

why order is 1? Because by decomposition the denominator contains at most 1 of these terms:

$(z+3), (z-3i), (z+3i)$.

And for $z = -1$, it's an essential singularity. ok.

And we cannot get rid of it by multiplying terms.

(3) Consider the function $f(z) = \dfrac{e^{z^2-4z+4}}{(z-2)^3}$.

(a) Find a power series for e^{z^2-4z+4} centred at $z_0 = 2$.

$$e^{z^2-4z+4} = e^{(z-2)^2},$$

$$= \sum_{n=0}^{\infty} \frac{((z-2)^2)^n}{n!} = \sum_{n=0}^{\infty} \frac{(z-2)^{2n}}{n!}$$ ✓

(b) Compute Res$(f : 2)$.

$f(z) = \sum_{n=0}^{\infty} \dfrac{(z-2)^{2n-3}}{n!}$, pole at 2 with order 3.

$g(z) = e^{z^2-4z+4} = \sum_{n=0}^{\infty} \dfrac{(z-2)^{2n}}{n!}$

So the (3-1)nd coefficient of $g(z)$ is

$C_{3-1} = C_2 = \dfrac{1}{2!} = \dfrac{1}{2}$.

So. $Res(f : 2) = \dfrac{1}{2}$.

X $C_2 = 1$

$4\frac{1}{2}$.