UNIT 2
SENTENTIAL LOGIC: SYMBOLIZATION

2.1 WHAT IS SENTENTIAL LOGIC?

We only look into the logical relation between sentences and clauses, ignore the subsentences logics.

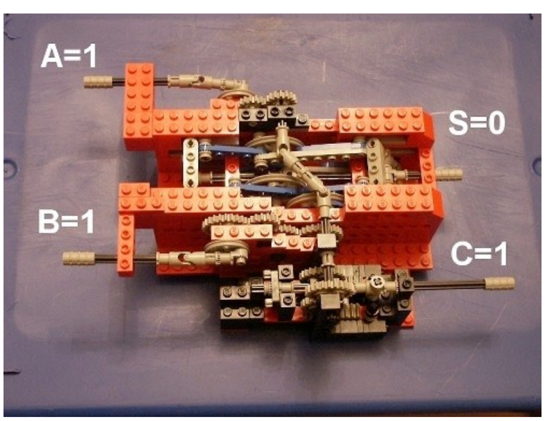
Sentential Logic (SL): A branch of logic in which sentences or propositions are used as the basic units. It is also called Propositional Logic or Propositional Calculus.

We will use a symbolic language that will let us move from English sentences to symbolic sentences and back again. Each truth-valuable English sentence (statements that can be true or false, rather than questions, orders, exclamations, etc.) will be assigned a symbol and then we can use symbols for logical operators to combine those sentences together.

Sentential logic allows us to focus on the logical relations *between* sentences. By symbolizing truth-functional sentences of natural languages (English, French, Mandarin…), we can focus on the logical structures without being distracted by what the sentences mean. Of course, the disadvantage of this is that it ignores the logical structures within a sentence. Some of those logical relations within sentences will be addressed in the second part of the course (Predicate Logic).

The logical connectives that join the simple sentences are 'truth-functional' – they operate on the truth-values of sentences rather than their meanings. For that to work, the atomic sentences need to be simple and (for classical logic) bivalent. The logical operators work on the simple sentences in a systematic way allowing us to calculate the truth-value of complex sentences from the truth-values of simple sentences. Then, we can use the techniques of sentential logic to determine whether sets of complex sentences are consistent and whether arguments are valid or invalid, etc.

The truth-functional nature of the logical operators of sentential logic makes it relatively easy to interpret the logical operations electronically or mechanically. 'Logic gates' (AND, OR, NOT…) control the flow of information (truth-values) and are used in logical circuits, calculators and computers. They work by taking the truth-values of one or two sentences as input and outputting truth-values according to the logical function of the 'gate'. By assembling and arranging such logic gates, so that the output of one gate is the input for another, one can build more and more complex computing devices. Indeed, you can build simple computing machines out of wooden levers and balls, dominoes, Popsicle sticks, or out of Lego® or Meccano®.



Logic gates made out of Lego®.

Babbage difference engine made of Meccano®
Constructed by Tim Robinson

2.2 THE SYMBOLS FOR SL

In sentential logic we need three types of symbols

~R "not R"

1. Symbols for sentences or propositions: capital letters.
2. Symbols for the logical relationships between those sentences: ~ ∧ ∨ → ↔
3. Symbols to keep things clear and organized: brackets and parentheses.

It is just a matter of convention which symbols get used for sentences and logical operators – and there are many different conventions. But, the different conventions share many features, and once you get used to one set of conventions, it is easy to understand symbolic sentences or arguments symbolized according to different conventions.

Sentence Letters: Capital letters P through Z (with or without numerical subscripts)

P, Q, R … Z

or we can use them with numerical subscripts so that we have an infinite supply:
P_1, Q_1, R_1…, P_2, Q_2, R_2, …

Each letter symbolizes a complete sentence or proposition. These are 'atomic sentences', the basic building blocks of the symbolic language we are using.

'P' could symbolize 'Plato is a Greek philosopher.', 'Frank is the fastest runner in town.' or 'Toronto is in Ontario.' or any other sentence.

Likewise, 'Q' could symbolize 'Anteaters eat termites.' or 'Suzy loves Adam.' or 'Plato is a Greek philosopher.' or any other sentence.

Most symbolic languages use letters – some use small letters, others use capitals. We will use the capital letters in the latter part of the alphabet for atomic sentences.

Sentential Connectives or Logical Operators:

→	**Conditional sign (if … then)**
~	**Negation sign (not)**
∧	**Conjunction sign (and)**
∨	**Disjunction sign (or)**
↔	**Biconditional (if and only if)**

These operate syntactically on sentences creating compound symbolic sentences (well-formed formulas). Most of them are binary operators, used to combine two sentences together. the negation sign is unary operator, operating on a single sentence.

In other textbooks or in philosophical articles you might see other symbols. These are other symbols that might be used for the logical operators we are using.		
Negation	$\sim$	$\sim$ $\neg$
Conditional	$\rightarrow$	$\rightarrow$ $\Rightarrow$ $\supset$
Disjunction	$\vee$	$\vee$
Conjunction	$\wedge$	$\wedge$ & $\bullet$
Biconditional	$\leftrightarrow$	$\leftrightarrow$ $\equiv$ $\Leftrightarrow$

Parentheses or Brackets

(round brackets/parentheses) [square brackets]

These keep things clear in complex symbolic sentences by indicating the order of operation for the logical operators. Whatever is inside a set of brackets or parentheses must be done first.

This is just like in math: $3 \times (2 + 4)$ is equal to 3×6.

Connecting Sentences in Ordinary Language

We connect sentences or clauses together informally with words such as ‘and’, ‘or’ and ‘not’. For instance, consider the following two sentences:

1.1 It’s raining.

1.2 It’s windy.

We use logical operators to connect them, to form compound sentences:

2.1 It’s raining *and* it’s windy.

2.2 It’s raining *or* it’s windy

2.3 It’s *not* windy.

The meaning of these new sentences depends on the meaning of the two simple sentences 1.1 and 1.2 *and* on the logical operation performed by the word connecting them.

The difference between 2.1 and 2.2 is due just to the difference in the logical operators. The logical connective (*and*, *or*) determines the inferential relations between sentences. For instance, from 2.1 we can infer either 1.1 or 1.2. (If we are told that it is raining *and* it’s windy, we can infer that it is raining.) From 2.2 we cannot! (If we are told that it is raining or it is windy, we cannot conclude that it is raining and we cannot conclude that it is windy.) However, from 2.2 and 2.3 together, we *can* infer 1.1

The Building Blocks … Atomic Sentences

These are sentences that have truth-value (they can be true or false). Such sentences must be statements or propositions, as opposed to questions, commands, exclamations, promises, etc.

We can symbolize them with capital letters (P-Z). We use an ‘abbreviation scheme’ or ‘symbolization scheme’ to show what letter stands for what English sentence.

P: Plato was a Classical Greek philosopher..

S: Socrates was a Classical Greek philosopher.

T: Thales was a Classical Greek philosopher.

Any letter can be used to stand for any atomic sentence. When making an abbreviation scheme, you can use any letter that hasn’t been already assigned, but it’s more convenient to start at P and put them in alphabetical order, or to use a letter related to the sentence if you can. Although we will rarely do it in this course, you can also use numeric subscripts: P_2 or T_3. Since subscripts are available, you can have an abbreviation scheme for an unlimited number of sentences!

Remember, each letter symbolizes the whole sentence. Each sentence letter is an atomic sentence.

The Mortar … Sentential Connectives or Logical Operators

We can use logical operators to build new, well-formed sentences out of simpler ones:

$(P \wedge S)$	Plato was a Classical Greek philosopher ***and*** Socrates was a Classical Greek philosopher.
$\sim T$	***It is not the case*** that Thales was a Classical Greek philosopher.
$(S \rightarrow \sim T)$	***If*** Socrates was a Classical Greek Philosopher ***then*** Thales was not a Classical Greek philosopher.
$((S \wedge P) \rightarrow \sim T)$	***If*** Socrates and Plato were Classical Greek Philosophers ***then*** Thales was not a Classical Greek philosopher.

“and” “not” and “if ... then” are operating on atomic sentences, connecting them together in increasingly complex ways.

Official Notation

In official notation, a set of parentheses is used every time a binary connective is used to connect two sentences. In informal notation, the outmost set of these parentheses are often left off, and sometimes inner parentheses, provided it does not lead to ambiguity or lack of clarity.

Sentential Connectives are logical operators that can be used to join or operate on atomic sentences to form well-formed formulas.

Compound sentences are the sentences formed when you use sentential connectives to operate on one or more atomic sentences (single sentence letters without any logical operators).

Official notation is the formal, grammatically correct way of symbolizing a sentence.

2.3 NEGATION ~

P	~P
T	F
F	T

This is the characteristic truth table for negation.
It defines the use of the negation sign: ~

The truth table shows the truth-value of the complex sentence given the truth value of the atomic sentences. Here it shows that when the truth-value of P is T (true) then the truth-value of ~P is F (false); and when the truth-value of P is F (false) then the truth-value of ~P is T (true).

Note: the 'squiggle' ~ is called a 'tilde'.

Negation is a unary connective — it acts on a single sentence.

Putting '~' in front of a sentence negates that sentence.

Thus, the negation of ' P ' is ' ~P ' – it is not the case that P.

The negation of any sentence is the sentence with a negation sign in front of it.

> The negation of ' ~P ' is ' ~ ~P '.
>
> ~ ~ P is logically equivalent to P.

Consider the following ***scheme of abbreviation:***

> P: Mental states are identical to brain states.

This abbreviation scheme shows that the atomic sentence, "Mental states are identical to brain states." is symbolized with the sentence letter 'P'.

Now we can symbolize "Mental states are not identical to brain states." using the negation sign.

> First, paraphrase the sentence, "Mental states are not identical to brain states." using "it is not the case that" for the negation.
>
> > <u>It's not the case that</u> mental states are identical to brain states.
>
> Second, replace the sentence with the sentence letter:
>
> > <u>It's not the case that</u> P
>
> Third, replace the logical operator with the negation sign:
>
> > ~P

Stylistic Variances:

Many different English sentences express the same idea.

W: Mary will win the election.

All of the following sentences can be symbolized ~W

Mary won't win the election.
The election won't be won by Mary.
Mary will fail to win the election.
Mary won't be elected.
Mary will lose the election.*

*Note that some words can sometimes be negated with a different word altogether:

win/lose pass/fail succeed/fail stop/continue

You need to be careful using such words when negating sentences since in many contexts they aren't true negations. For example, if you don't win a game you don't necessarily lose (you may tie or draw). But, if you don't win an election, you lose it.

Any properly formed symbolic sentence can be negated by putting a negation sign at the front. Thus, we can negate the same sentence twice:

Mary won't lose the election.

We can paraphrase that:

<u>It's not the case that</u> Mary will lose the election.

Above we symbolized 'Mary will lose the election' with ~ W.

<u>It's not the case that</u> ~ W.

Now we can use the negation sign to replace 'it's not the case that'.

~ ~ W

~~W is logically equivalent to W (they have exactly the same truth-table).

2.4 CONDITIONAL $\rightarrow$

Truth fuctional (meaning doesn't matter)
Can use it join any two sentences
Won't always seem intuitively correct (sentential logic focus on what is between sentences)
Suppose:logical structure within sentences is incorrect, the written English for whole sentence will sound incorrect.

P	Q	P → Q
T	T	T
T	F	F
F	T	T
F	F	T

P Q

It's the only false condition: T→F

This is the characteristic truth table for the conditional (if ... then). (Also called the material conditional)
It defines the use of the conditional sign: $\rightarrow$

The truth table shows the truth-value of the complex sentence given the truth value of the atomic sentences. For instance, when both P and Q are assigned the truth-value T, then the sentence $P \rightarrow Q$ also has the truth-value T (true). You can see that there is only one truth-value assignment on which the sentence $P \rightarrow Q$ is false: line 2, when P has the truth-value T (true) and Q has the truth-value F (false).

P is the antecedent of the conditional (it comes before $\rightarrow$) and Q is the consequent of the conditional (it comes after $\rightarrow$).

antecedent $\rightarrow$ consequent

Unlike negation, which is a unary connective, the conditional is a binary connective, joining two sentences together.

Consider the following scheme of abbreviation:

E.g taxonomic trees
If lower then upper

Lower->upper

Lower is sufficient for upper

T: Tom is taking logic at U of T.
S: Tom is a student.

We can symbolize the complex sentence, "If Tom is taking logic at U of T then he's a student." using the conditional symbol.

<u>If</u> Tom is taking logic at U of T <u>then</u> Tom is a student.

We replace the atomic sentences according to the scheme of abbreviation:

<u>If</u> T <u>then</u> S

Then, we replace the logical operator with the conditional sign and put it in parentheses:

(T $\rightarrow$ S)

The material conditional is similar to the conditional we use in everyday speech; however, in everyday speech the antecedent and the consequent are usually semantically related (their meanings are related) so that the conditional *makes sense* (as it does in this example.) But, in logic, any two sentences can be connected with the material conditional – they don't have to have anything to do with one another. The resulting material conditional statement may sound odd, but it will be a well-formed sentence. For example: If Tom is a student then Harper will win the next election. Like any material conditional sentence, it is a true statement unless the antecedent is true and the consequent is false.

Stylistic variances for "if … then …"

There are many stylistic variances for "if ... then" in English. When using the conditional, think about which clause is the antecedent and which is the consequent.

> Think about when the sentence is false. A material conditional is false only when the antecedent is true and the consequent is false. So if you know what will make it false, you can figure out which clause is the antecedent.
>
> Sarah will quit her job if she wins the lottery.
>
> This is false only if Sarah wins the lottery but she doesn't quit her job. Thus, "Sarah wins the lottery" is the antecedent and "Sarah quits her job" is the consequent. $W \to Q$.

Only

P : pass
S : complete

The word 'only' often causes some confusion! Consider the difference between:

> You will pass the course if you complete the problem sets. $(S \to P)$
>
> You will pass the course ONLY if you complete the problem sets. $(P \to S)$

In the first case: You will pass the course (P) if you complete the problem sets (S).

> If your professor reassures you of this, you know that if you just complete the problem sets, you will get that credit.
>
> $S \to P$

In the second case: You will pass the course (P) ONLY if you complete the problem sets (S).

> If your professor warns you of this, you know that if you end up passing the course, then you will have completed the problem sets. You won't pass if you don't complete them, but completing them won't guarantee a pass.
>
> $P \to S$
>
> The expression 'only if' introduces the consequent not the antecedent!

The word 'only' seems to reverse the antecedent and the consequent in conditional sentences.

You will pass only provided you study.	$P \to S$
Only on the assumption that you study will you pass.	$P \to S$
You pass only when you study.	$P \to S$

Consider the following stylistic variances for the conditional. This list is by no means complete! See if you think of more expressions or create new variants of these by using commas or changing the order of the clauses.

P: I am alert
Q: I have had coffee.

If P then Q	$P \rightarrow Q$	If I am alert then I have had coffee.
P only if Q	$P \rightarrow Q$	I'm alert only if I have had coffee.
Whenever P, Q	$P \rightarrow Q$	Whenever I'm alert, I've had coffee.
Q if P	$P \rightarrow Q$	I have had coffee if I am alert.
P provided that Q	$Q \rightarrow P$	I am alert provided that I have had coffee.
P on the condition that Q.	$Q \rightarrow P$	I am alert on the condition that I have had coffee.
P only on the condition that Q.	$P \rightarrow Q$	I am alert only on the condition that I've had coffee.
Q is necessary for P	$P \rightarrow Q$	Having coffee is necessary for my being alert.
Q is sufficient for P	$Q \rightarrow P$	Having coffee is sufficient for my being alert.

Note, in the table above, there are no parentheses around the conditional sentences. In Official Notation, a pair of parentheses is used for every binary connective. In Informal Notation the parentheses can be left off provided there is no ambiguity.

"How did people ever do philosophy before coffee?"

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[1] This illustration is the property of Gerald Grow, Professor of Journalism, Florida A&M University. http://www.longleaf.net/ggrow/CartoonPhil.html

2.4 E1: UNDERSTANDING THE MATERIAL CONDITIONAL… A LITTLE LOGIC PUZZLE

Every card has a number on one side and a letter on the other. Suppose there is a rule:

If one side of a card has an odd number on it, then the other side has a vowel on it.

P V

Which of the following cards must you turn over in order to test whether the rule was broken?

V P

11	20	B	E

If odd then it must be a vowel

If no vowel then it cannot be ODD

Now consider this rule:

A person can drink alcohol only if he/she is 19 years or older.

Each of the following people is drinking a beverage. Which of the following people must you learn more about in order to determine whether the rule was broken?

Adam: age 23	Betty: age 17	Carol: drinking soda	Darren: drinking beer

Both the rules above are conditionals.

What is the antecedent? What is the consequent?

If the rule is broken, then that conditional is false.

Remember, a material conditional is false only if the antecedent is true and the consequent is false!

2.5 SYMBOLIZATION WITH NEGATION AND CONDITIONAL

Parts of the Sentence:

An ***atomic sentence*** is a sentence letter: P-Z.

A ***molecular sentence*** (or compound sentence) is one that is composed of an atomic sentence or sentences that have been made into more complex sentences through operations of the logical connectives: ~ and →

A Syntax for Sentential Logic with Negation and Conditional:

Now we can provide a syntax or grammar for our symbolic language.

The Greek letters ϕ (phi) and ψ (psi) can represent any sentence, whether atomic or molecular.

Only symbolic sentences formed through the following steps are well-formed formulas in SL:

1. Sentence letters (P-Z with or without numerical subscripts) are symbolic sentences.
2. If ϕ is a sentence then $\sim \phi$ is a symbolic sentence.
3. If ϕ and ψ are sentences then $(\phi \rightarrow \psi)$ is a symbolic sentence.
4. Anything that can be constructed through recursive application of steps 1-3 is a symbolic sentence.

Every sentence we form through steps 1-3 is a well-formed symbolic sentence. Since steps 2 and 3 work on any symbolic sentence, we can use steps 1-3 recursively to make even more complex sentences. Thus, these steps provide a technique for generating all possible truth-valuable sentences in SL.

Official notation: brackets or parentheses must be used whenever two atomic sentences are joined with a binary connective, such as →.

Informal notation: brackets or parentheses must be used to avoid ambiguity (it is customary to leave the outermost brackets off).

When a negation sign, ~, is directly in front of a sentence letter, then it operates on that sentence letter. $\sim P$

When a negation sign, ~, is directly in front of a leftmost parenthesis then it is operating on the molecular sentence inside the parentheses: $\sim (P \rightarrow Q)$

In informal notation, the outermost parentheses or brackets are left off.
Thus $((P \rightarrow Q) \rightarrow \sim R)$ can be written $(P \rightarrow Q) \rightarrow \sim R$

We can symbolize complex sentences using negation and conditional together. When symbolizing English sentences, there are three basic steps:

Step 1: Paraphrase the sentence using standard expressions: "it is not the case that" for negation; "if … then …" for the conditional; and the atomic sentences. Underline the standard logical expressions. Use parentheses to keep the order of operations clear.

Step 2: Replace the atomic sentences with their sentence letters.

Step 3: Replace the logical terms with their symbols.

P: I am alert
Q: I have had coffee.

Example: If I am not alert then I have not had coffee.

Step 1: Paraphrase using standard expressions

<u>If</u> <u>it is not the case that</u> I am alert <u>then</u> <u>it is not the case that</u> I have had coffee.

Step 2: Replace the atomic sentences with their sentence letters.

<u>If</u> <u>it is not the case that</u> P <u>then</u> <u>it is not the case that</u> Q.

Step 3: Replace the logical terms with their symbols.

<u>If</u> ~ P <u>then</u> ~ Q

$\sim P \rightarrow \sim Q$

Example: Having coffee is not sufficient for my being alert.

Step 1: Paraphrase using standard expressions. Use parentheses to keep the order of operations clear.

<u>It is not the case that</u> (having coffee is sufficient for my being alert).

<u>It is not the case that</u> (<u>if</u> I have had coffee <u>then</u> I am alert).

Step 2: Replace the atomic sentences with their sentence letters.

<u>It is not the case that</u> (if Q <u>then</u> P).

Step 3: Replace the logical terms with their symbols.

$\sim (Q \rightarrow P)$

* NOTE: The parentheses here show that the negation acts on the sentence: $Q \rightarrow P$.

If there were no parentheses, $\sim Q \rightarrow P$, then it would mean: If I haven't had coffee then I am alert.

Now, let's expand the abbreviation scheme and try some more:

P: I am alert
Q: I have had coffee.
R: I bump into things.
S: I have had a good night's sleep.
T: I am in a hurry.

Example: Assuming I haven't had a good night's sleep, I bump into things if I haven't had coffee.

Step 1: Paraphrase using standard expressions and parentheses.

<u>If</u> <u>it is not the case that</u> I have had a good night's sleep <u>then</u> (<u>if</u> <u>it is not the case that</u> I have had coffee <u>then</u> I bump into things.)

Step 2: Replace the atomic sentences with their sentence letters.

<u>If</u> <u>it is not the case that</u> S <u>then</u> (<u>if it is not the case that</u> Q <u>then</u> R.)

Step 3: Replace the logical terms with their symbols.

$\sim S \rightarrow (\sim Q \rightarrow R.)$

P: I am alert
Q: I have had coffee.
R: I bump into things.
S: I have had a good night's sleep.
T: I am in a hurry.

2.5 E1

Using the abbreviation scheme above, symbolize the following sentences:

(a) I don't bump into things. $\sim R$

(b) It's not the case that I don't bump into things. $\sim\sim R$

(c) If I'm alert then I've had coffee. $(P \rightarrow Q)$

(d) If I haven't had coffee, I'm not alert. $(\sim Q \rightarrow \sim P)$

(e) I bump into things if I haven't had a good night's sleep. $(\sim S \rightarrow R)$

(f) It's not the case that whenever I'm in a hurry I bump into things. $\sim(T \rightarrow R)$

(g) Assuming I'm in a hurry, I bump into things if I am not alert. $(T \rightarrow (\sim P \rightarrow R))$

(h) Coffee is sufficient for my being alert only if I have had a good night's sleep. $((Q \rightarrow P) \rightarrow S)$

(i) If I haven't had a good night's sleep then coffee is necessary for my being alert. $(\sim S \rightarrow (P \rightarrow Q))$

(j) Assuming I'm not in a hurry, only if I haven't had coffee do I bump into things. $(\sim T \rightarrow (R \rightarrow \sim Q))$

(k) It is not the case that whenever I bump into things I haven't had coffee. $\sim(R \rightarrow \sim Q)$

(l) It's not the case that if I'm alert I don't bump into things if I have had coffee. $\sim(P \rightarrow (Q \rightarrow \sim R))$

(m) Provided that I have had a good night's sleep, it's not the case that only if I have had coffee am I alert. $(S \rightarrow \sim(P \rightarrow Q))$

(n) If it is necessary that I have coffee in order to be alert then having a good night's sleep is sufficient for my not bumping into things. $((P \rightarrow Q) \rightarrow (S \rightarrow \sim R))$

NOW TRY LOGIC 2010 SYMBOLIZATION EXERCISES FOR CHAPTER 1.

2.6: DO WE NEED MORE LOGICAL OPERATORS?

We *could* symbolize everything we want to symbolize in SL just using negation and conditional. Indeed, any possible finite truth-value assignment can be expressed using only these sentential connectives.

However, we will see that things get a little complex (and less like natural language) if we don't introduce some new symbols.

Disjunction - Or

Suppose I want to symbolize: Simon will study **or** he will fail the course.

S: Simon will study.
T: Simon will fail the course.

The sentence is logically equivalent to (means the same as):

If Simon won't study then Simon will fail the course.

If it is not the case that Simon will study then Simon will fail the course.

If it is not the case that S then T.

$\sim S \rightarrow T$

It's easier to use a symbol for **'or'**: $\vee$

Simon will study or he will fail the course.

$S \vee T$

$(\sim S \rightarrow T) \Longleftrightarrow (S \vee T)$

Conjunction – And

Suppose I want to symbolize: Simon will study **and** he will fail the course.

This sentence is logically equivalent to:

It is not the case that if Simon studies then he won't fail the course.

It is not the case that (if Simon will study then it is not the case that Simon will fail the course.)

It is not the case that (if S then it is not the case that T).

$\sim (S \rightarrow \sim T)$

It's easier to use a symbol for **'and'** : $\wedge$

Simon will study and Simon will fail the course.

$S \wedge T$

$[\sim (S \rightarrow \sim T)] \Longleftrightarrow (S \wedge T)$

Biconditional – If and only if

How would we symbolize this? Simon will fail if and only if he doesn't study.

This is the same as a conjunction of two conditionals:

Simon will fail if he doesn't study ***and*** Simon will fail only if he doesn't study.

> <u>If</u> <u>it is not the case that</u> Simon will study <u>then</u> Simon will fail <u>**and**</u> <u>if</u> Simon will fail <u>then</u> <u>it is not the case that</u> Simon will study.

We know how to express conditionals and conjunction with just ~ and →.

Conditional (if ϕ then ψ):	$\phi \rightarrow \psi$
Conjunction (ϕ and ψ):	$\sim(\phi \rightarrow \sim\psi)$
The two conditionals would be:	$(\sim S \rightarrow T)$ and $(T \rightarrow \sim S)$

So the entire sentence, expressed with just negation and conditional would be:

$$\sim[(\sim S \rightarrow T) \rightarrow \sim(T \rightarrow \sim S)]$$

It's much easier to use a symbol for **'if and only if'**: ↔

Simon will fail <u>if and only if</u> he doesn't study.

$T \leftrightarrow \sim S$

All the sentences that we will be able to symbolize in our expanded language for SL *can* be symbolized just with negation and conditional (and you might want to try this just for fun). However, by adding in a few more logical connectives, we can make the symbolization a little easier!

In the meantime, it would be useful to learn more about the three new logical connectives: conjunction, disjunction and biconditional.

2.7 CONJUNCTION $\wedge$

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

This is the characteristic truth table for conjunction – 'and'.
It defines the use of the conjunction sign: $\wedge$

The truth table shows the truth-value of the complex sentence given the truth value of the atomic sentences. For instance, when both *P* and *Q* are assigned the truth-value T, then the sentence $P \wedge Q$ also has the truth-value T (true). You can see that on every other truth-value assignment the sentence is false. A conjunction is true if and only if both conjuncts are true.

If you join two atomic sentences with a conjunction, then the resulting molecular sentence is true if and only if both conjuncts are true.

Plato was a Greek philosopher ***and*** Socrates was a Greek philosopher.

$(P \wedge S)$

"P" and "S" are the conjuncts, and are both atomic sentences.
"$(P \wedge S)$" is the resulting molecular sentence.

Plato and Socrates were Greek philosophers.

can be paraphrased ...

Plato was a Greek philosopher **and** Socrates was a Greek philosopher.

and can be symbolized ... $P \wedge S$

Stylistic variances

The following can all be symbolized using the conjunction sign:

Socrates was a Greek philosopher and so was Plato. $(S \wedge P)$

Plato was a Greek philosopher but Turing was a mathematician. $(P \wedge T)$

Plato was a Greek philosopher as well as Socrates. $(P \wedge S)$

Although Turing was a mathematician, Socrates was a Greek philosopher. $(T \wedge S)$

Plato and Socrates were Greek philosophers; however, Turing was a mathematician. $(P \wedge S \wedge T)$

Other conjunctions include although, yet, also, in addition to, moreover, even though …

2.8 DISJUNCTION $\vee$

P	Q	P $\vee$ Q
T	T	T
T	F	T
F	T	T
F	F	F

This is the characteristic truth table for disjunction – 'or'.
It defines the use of the conjunction sign: $\vee$

The truth table shows that the truth-value of the complex sentence given the truth-value of the atomic sentences. For instance, when both P and Q are assigned the truth-value T, then the sentence $P \vee Q$ also has the truth-value T (true). You can see that the only truth-value assignment on which the sentence is false is when both atomic sentences are false. A disjunction is true if and only if at least one disjunct is true.

If you join two atomic sentences with a disjunction, then the resulting molecular sentence is : true if and only if at least one of the disjuncts is.

Russell was a mathematician ***or*** Turing was a mathematician.

"R" and "T" are the disjuncts and both are atomic sentences.
"(R $\vee$ T)" is the resulting molecular sentence.

Disjunction is equivalent to 'or' used in the ***inclusive sense.***
The inclusive sense of 'or' includes either disjunct or both – the molecular disjunctive sentence is true if and only if at least one of the disjuncts is.

People use the inclusive sense of 'or' when they ask if you would like cream or sugar in your coffee.

You can have cream or sugar

You can have cream or you can have sugar.

R $\vee$ S

You can have cream, sugar ***or both.***

In contrast, when we invite a person to come for dinner on Monday or Tuesday, we are using 'or' in the exclusive sense — we only mean to invite them for one meal, and mean to exclude the possibility they will come both nights!

Stylistic variances: use $\vee$ to symbolize: or, unless, either/or, else, otherwise …

Tom will fail unless he studies. T $\vee$ S (T: Tom will fail. S: Tom studies)
Tom will study else he will fail: S $\vee$ T
Sara or Tom or Uri or Vanna will pass the course.: S $\vee$ T $\vee$ U $\vee$ V
(S: Sara will pass the course. T: Tom will pass. U: Uri will pass. V: Vanna will pass.)

The order in which the conjuncts are listed doesn't matter. (S $\vee$ T) is logically equivalent to (T $\vee$ S)

2.9 BICONDITIONAL ↔

P	Q	P ↔ Q
T	T	T
T	F	F
F	T	F
F	F	T

This is the characteristic truth table for the biconditional (also called the material biconditional or equivalence) – if and only if.

It defines the use of the biconditional sign: ↔

The truth table shows that the truth-value of the complex sentence given the truth-value of the atomic sentences. Here, when both *P* and *Q* are assigned the same truth-value (as on the first and last line), then the sentence *P* ↔ *Q* has the truth-value T (true). Whenever *P* and *Q* are assigned different truth-values (as on the second and third line) the sentence is false. A biconditional is true if and only if the two sides have the same truth-value.

It is called a biconditional because it is equivalent to the conjunction of two conditionals:

$(P \rightarrow Q) \wedge (Q \rightarrow P)$

$P \rightarrow Q$: P only if Q

$Q \rightarrow P$: P if Q

$P \leftrightarrow Q$: P if and only if Q

It also means: either both P and Q *or* neither P nor Q.

Which can be symbolized: $(P \wedge Q) \vee (\sim P \wedge \sim Q)$

Tom passes if and only if he studies.

$T \leftrightarrow S$

"If and only if" is often abbreviated: IFF. Thus, "P iff Q" is symbolized: $P \leftrightarrow Q$.

One might also symbolize the following sentences with the biconditional:

For Tom to pass the course it is necessary and sufficient that he study.

Either Tom studies and passes or he does neither.

Tom passes when and only when he studies.

Tom passes exactly on condition that he studies.

Tom passes just in case he studies. (Although logicians talk this way, 'just in case' is rarely used this way in ordinary language.)

Stylistic variances: sentences symbolized with the biconditional sign may include expressions such as: is equivalent to, is necessary and sufficient for, exactly on the condition that, exactly when...

The biconditional is often used for definitions.

A closed figure is a triangle <u>if and only if</u> it has three sides.

2.9 E1

Symbolize each of the following sentences using the abbreviation scheme provided:

P: I exist.

S: I think

U: Determinism is true.

Q: God exists.

T: The bible is the word of God.

V: I am free.

R: Angels exist.

(a) Determinism is true but I am free. $U \wedge V$

(b) Either determinism is true or I am free. $U \vee V$

(c) Determinism is false, however I am not free. $\sim U \wedge \sim V$

(d) Angels exist if, but only if, God does. $R \leftrightarrow Q$

(e) I exist if I think. $(S \rightarrow P)$

(f) I am free just in the case that determinism is false. $V \leftrightarrow \sim U$

(g) God exists if and only if the bible is the word of God, but it is not. $\sim(Q \leftrightarrow T)$

(h) Either the bible is the word of God or God doesn't exist. $T \vee \sim Q$

(i) Although God exists, determinism is true. $Q \wedge U$

(j) God's existence is a necessary and sufficient condition for the existence of angels. $Q \leftrightarrow R$

(k) Determinism is true or I am free, but not both. $(U \vee V) \wedge \sim(U \wedge V)$

(l) Both angels and God exist; however, the bible is not the word of God.

(m) The bible is the word of God, who exists. $T \wedge Q$

(n) If determinism is true, then neither am I free nor does God exist. $(U \rightarrow (\sim V \wedge \sim Q))$

(o) Neither angels nor God exists. $\sim Q \wedge \sim R$

(p) I am not free unless determinism is false. $\sim V \vee \sim U$

(q) Provided both that I think and if I think then I exist, I exist. $((S \wedge (S \rightarrow P)) \rightarrow P)$

(r) I am not free unless God exists, and in that case, only if determinism is false am I free.

$(V \rightarrow \sim U) \rightarrow Q$

NOW YOU CAN TRY LOGIC 2010 SYMBOLIZATION EXERCISES FOR CHAPTER 2 …

but more tips about complex symbolization in the next section.

2.10 SYMBOLIZING COMPLEX SENTENCES

We use connectives to join atomic sentences (sentences expressed with a single letter). But we can also use them to join molecular sentences (sentences that already contain a connective.) When symbolizing complex sentences, it can be difficult to recognize which sentences are being connected with a given logical connective.

When symbolizing complex sentences, it is useful to first identify the main connective. Once you have done that, proceed by identifying the main connective of each immediate sentential component. Continue until you are down to the atomic sentential components.

> Assuming that the course is not easy, Tom will pass only if he studies or he gets help from his tutor.
>
> P: Tom will pass. .
> Q: Tom will study
> R: Tom will get help from his tutor.
> S: The course is easy.
>
> The main connective is a conditional:
> <u>If</u> the course is not easy <u>then</u> (Tom will pass only if he studies or he gets help from his tutor).
>
> The main connective of the antecedent is a negation:
> <u>If</u> <u>it is not the case that</u> the course is easy <u>then</u> (Tom will pass only if he studies or he gets help from his tutor).
>
> The main connective of the consequent is a conditional:
> <u>If</u> <u>it is not the case that</u> the course is easy <u>then</u> (<u>if</u> Tom will pass <u>then</u> (he studies or he gets help from his tutor)).
>
> The main connective of the consequent of the second conditional is a disjunction:
> <u>If</u> <u>it is not the case that</u> the course is easy <u>then</u> (<u>if</u> Tom will pass <u>then</u> (Tom will study <u>or</u> Tom will get help from his tutor)).
>
> Now we can put in the sentence letters:
> <u>If</u> <u>it is not the case that</u> S <u>then</u> (<u>if</u> P <u>then</u> (Q <u>or</u> R)).
>
> And finally, we can put in the logical connectives:
> $\sim S \rightarrow (P \rightarrow (Q \vee R))$

Symbolizing complex sentences can look difficult, but once you identify the main connective and begin to break it down into simpler chunks it starts to look easier.

The key is being able to identify the main connective, and then, the immediate sentential components. Once you've done that, you can identify the main connective in the components, and then, the immediate sentential components of those, … It's a recursive process that will eventually lead you to the simple atomic components.

Many English expressions use more than one logical symbol to express. Symbolizing complex sentences is easier if you know how to symbolize the common expressions.

Neither …nor

Neither Karl Popper nor W. V. O. Quine is alive.

We can symbolize this in two different ways:

P	**Q**	**$\sim P \wedge \sim Q$**
T	T	F
T	F	F
F	T	F
F	F	T

Popper is not alive and Quine is not alive.

The sentence $(\sim P \wedge \sim Q)$ is true only when both P and Q are false.

Logically equivalent to: $\sim(P \vee Q)$

P	**Q**	**$\sim(P \vee Q)$**
T	T	F
T	F	F
F	T	F
F	F	T

It's not the case that either Popper or Quine is alive.

The sentence $\sim(P \vee Q)$ is true only when both P and Q are false.

Logically equivalent to: $\sim P \wedge \sim Q$

Not both

Popper and Quine were not both British.

We can symbolize this in two different ways:

P	**Q**	**$\sim P \vee \sim Q$**
T	T	F
T	F	T
F	T	T
F	F	T

Popper was not British or Quine was not British.

The sentence $(\sim P \vee \sim Q)$ is false only when both P and Q are true.

Equivalent to $\sim(P \wedge Q)$

P	**Q**	**$\sim(P \wedge Q)$**
T	T	F
T	F	T
F	T	T
F	F	T

It's not the case that both Popper and Quine were British.

The sentence $\sim(P \wedge Q)$ is false only when both P and Q are true.

Equivalent to $\sim P \vee \sim Q$

Exclusive or – exactly one of two.

Exactly one of Popper and Quine was American.
Popper was American or Quine was, but not both

P	**Q**	**$\sim(P \leftrightarrow Q)$**
T	T	F
T	F	T
F	T	T
F	F	F

This is true when either P or Q is true but they aren't both true.

It's the exclusive sense of 'or'.

(On a menu: the sandwich comes with soup <u>or</u> salad.)

We can also symbolize it like this: $(P \leftrightarrow \sim Q)$ or $(\sim P \leftrightarrow Q)$ or $((P \vee Q) \wedge \sim(P \wedge Q))$

Tips for paraphrasing:

When symbolizing long and complicated sentences or passages, it is useful to paraphrase them first.

1. Identify the simple sentences – sentences that have the same meaning should be written with the same words, preferably those used in the abbreviation scheme.

2. Paraphrase the sentences using canonical logical terms – standard logical expressions. For instance, replace 'but' with 'and'; replace 'unless' with 'or', and 'only if' with 'if … then'. You might want to underline the connectives.

   Jessica plays guitar unless Frank does.

   can be rewritten … Jessica plays guitar or Frank plays guitar.

3. Move the negations out to the front of the sentence.

   Rolling stones gather no moss.

   can be rewritten …It is not the case that rolling stones gather moss.

4. Consider whether negatives are acting on simple or complex sentences or clauses.

   Sarah and Tom don't study.

   can be rewritten… It's not the case that Sarah studies and it's not the case that Tom studies.

   Sarah and Tom don't both study.

   can be rewritten… It's not the case that (Sarah studies and Tom studies).

5. Eliminate ambiguities by using brackets to group sentences.

   For example, rewrite 'Jessica will play guitar and Sam will play drums unless he is on bass' as 'Jessica will play guitar and (Sam will play drums or Sam is on bass).'

6. Put arguments in standard form.

7. THINK! You need to think about what the sentences mean. Rules will not be sufficient.

   'Frankie and Johnny are lawyers' means the same as 'Frankie is a lawyer and Johnny is a lawyer'

   BUT

   'Frankie and Johnny are lovers' does ***not*** mean the same as 'Frankie is a lover and Johnny is a lover'

   There is no formal rule for this ... you just have to think about what the sentences mean.

Disambiguating Ambiguous Sentences

Some sentences that are ambiguous – either because the speaker was imprecise in choosing words, or because it was poorly punctuated. Such sentences can be disambiguated by symbolizing it in different ways. By producing all the correct symbolizations of the sentence, we can then try to determine which meaning the speaker intended.

For example disambiguate this sentence by providing two correct but logically distinct symbolizations:

> It's not the case that Tom passes if he doesn't study.
>
> P: Tom passes. . S: Tom studies

This sentence has two logical operators: if then & not

Either "It's not the case that" or "If then" can be considered the main connective.

<u>If then</u> as the main connective:

<u>If</u> <u>it is not the case that</u> Tom studies **<u>then</u>** <u>it is not the case that</u> Tom passes.)

$\sim S \rightarrow \sim P$

This is true if Tom studies or if he fails to pass. It is false if Tom fails to study but passes anyhow.

<u>Not the case that</u> as the main connective:

<u>It is not the case that</u> (<u>if</u> <u>it is not the case that</u> Tom studies <u>then</u> Tom passes).

$\sim(\sim S \rightarrow P)$

This true if Tom neither studies nor passes. It's false if Tom studies or if he fails to pass.

By providing two different symbolizations of the sentence, we disambiguate it. Now we can ask the speaker which way it was intended.

Here is a really difficult one!

> It's not the case that Tom will get help from his tutor if he studies only if the course is easy.
>
> P: Tom will pass. . R: Tom will get help from his tutor.
> Q: Tom will study S: The course is easy.

This can be understood in many ways. There are three logical operators, each of which could be the main connective:

<u>It's not the case that[1]</u> Tom will get help from his tutor <u>if[2]</u> he studies <u>only if[3]</u> the course is easy.

We will look at each possibility.

It's not the case that^1 as the main connective:

It's not the case that (*Tom will get help from his tutor if^2 he studies only if^3 the course is easy.*)

Now, we have to find the main connective of the sentence being negated (in italics):

Take if^2 as the main logical operator, and everything after if^2 is the antecedent of the negated clause.

It's not the case that^1 (if^2 (if^3 Tom studies then^3 the course is easy) then^2 Tom will get help from his tutor).

$\sim((Q \rightarrow S) \rightarrow R)$

Take only if^3 as the main logical operator, and everything after only if^3 is the consequent of the negated clause.

It's not the case that^1 (if^3 (if^2 Tom studies then^2 Tom will get help from his tutor) then^3 the course is easy).

$\sim((Q \rightarrow R) \rightarrow S)$

Now, let's try if^2 as the main connective:

In this case, everything after if^2 is the antecedent of the main clause.

It's not the case that^1 Tom will get help from his tutor if^2 he studies only if^3 *the course is easy.*

If^2 (Tom studies only if^3 the course is easy) then^2 it's not the case that^1 Tom will get help from his tutor.

If^2 (if^3 Tom studies then^3 the course is easy) then^2 it's not the case that^1 Tom will get help from his tutor.

$(Q \rightarrow S) \rightarrow \sim R$

Now, let's try only if^3 as the main connective:

In this case, everything after only if^3 is the consequent of the main clause.

It's not the case that^1 Tom will get help from his tutor if^2 he studies only if^3 the course is easy.

If^3 *(it's not the case that^1 Tom will get help from his tutor if^2 he studies)* then^3 the course is easy.

Now, there are two options for the antecedent (in italics): it's not the case that^1 or if^2 could be the main connective of the antecedent.

If^3 it's not the case that^1 (if^2 Tom studies then^2 Tom will get help from his tutor) then^3 the course is easy.

$\sim(Q \rightarrow R) \rightarrow S$

If^3 (if^2 Tom studies then^2 it's not the case that^1 Tom will get help from his tutor)) then^3 the course is easy.

$(Q \rightarrow \sim R) \rightarrow S$

There were FIVE different symbolic forms of this sentence – each of which is a correct interpretation of the ambiguous sentence! A good reason to try to pick our words carefully and to punctuate properly!

Symbolizing Common Expressions (that sometimes cause difficulty)

Only if P only if Q $P \rightarrow Q$	You can drive legally only if you are 16 or older. Drive legally $\rightarrow$ over 16	'Only if' introduces the consequent. Drive legally $\rightarrow$ over 16 Every legal driver is over 16, but it doesn't go the other direction. It's not the case that everybody over 16 is a legal driver.
Necessary P is necessary for Q $Q \rightarrow P$	Oxygen is necessary for fire. Fire $\rightarrow$ oxygen.	The 'necessary' condition is the consequent. If there is fire then there is oxygen, but it doesn't go the other direction. It's not the case that if there is oxygen then there is fire.
Sufficient P is sufficient for Q $P \rightarrow Q$	50% is sufficient for a pass. 50% $\rightarrow$ pass Cereal with milk is sufficient for a nutritious breakfast. cereal w/ milk $\rightarrow$ nutritious breakfast	The 'sufficient' condition is the antecedent. If you get 50% you pass, but it doesn't go the other direction. It's not the case that if you pass then you get 50% (you could get 90%). If you eat cereal with milk then you eat a nutritious breakfast. But it doesn't go the other way. If you eat a nutritious breakfast then you don't necessarily eat cereal.
Only on condition that P only on condition that Q $P \rightarrow Q$	You'll get a philosophy degree only on condition that you pass PHL 245. Phil. degree $\rightarrow$ PHL 245	'Only on condition Q' is a way to say that Q is a necessary condition for P. If you get a philosophy degree, you must have passed PHL 245. But it doesn't work the other way. If you pass PHL 245, it doesn't follow that you get the degree.
On condition that* P on condition that Q $Q \rightarrow P$	You receive a discount on condition that you use a coupon. Use coupon $\rightarrow$ get discount	A mere condition is the antecedent (it is a sufficient condition.) If you use a coupon, you get a discount. But, it doesn't work the other way round. If you get a discount, it doesn't follow that you used a coupon (sometimes you get a discount without a coupon!)

*NOTE: We ALMOST NEVER use "on condition that" this way in ordinary speech. Usually the context makes it clear that 'exactly on condition that' is implied – that's a biconditional, or necessary and sufficient conditions. This is also somewhat true of "only on condition that": although it can be used to state a necessary but insufficient condition, it often is used in contexts which make it clear that it is *already* a sufficient condition, and thus, it too can function as a biconditional.

Unless	Unless can be symbolized with $\vee$ or $\rightarrow$ Use the method that is most intuitive for you. But, be aware of the other forms. Unless is often used with a negative clause, making it a little trickier. But, remember $\vee$ is the inclusive or. So it is true even if both disjuncts are true. And $\rightarrow$ is a material conditional, so it is true if the antecedent is false or the consequent is true. No matter which way you symbolize it ($\vee$ or $\rightarrow$) the truth table will have one line on which the unless sentence is false and three lines on which the unless sentence is true. In natural English there is often a presupposition that rules out one possibility, so that you might think it is an 'exclusive or' (P or Q but not both) or a biconditional (if not P then Q and if not Q then P). In other words, you might think that the truth table for the 'unless' sentence has two lines on which it is true and two lines on which it is false. But, we will NOT symbolize it that way unless the English sentence is *explicitly* ruling out that possibility. (For instance: you will fail unless you study, in which case, you'll pass.)	
Unless (positive) P unless Q $P \vee Q$ $\sim P \rightarrow Q$ $\sim Q \rightarrow P$	The plant will die unless you water it. plant die $\vee$ water plant doesn't die $\rightarrow$ water no water $\rightarrow$ plant dies	Either the plant dies or you water it. You might water it and it still dies. If the plant doesn't die then you watered it. It doesn't work the other way. If you water it, it may still die. If you don't water it then the plant dies. It doesn't work the other way. If it dies, it doesn't follow that you didn't water it.
Unless (negative) Not P unless Q $\sim P \vee Q$ $P \rightarrow Q$ $\sim Q \rightarrow \sim P$	You won't win unless you buy a ticket. not win $\vee$ buy ticket win $\rightarrow$ buy ticket not buy ticket $\rightarrow$ not win	Either you don't win or you buy a ticket. You might buy a ticket and still not win. If you win then you bought a ticket. It doesn't work the other way. If you buy a ticket you might not win. If you don't buy a ticket you won't win. It doesn't work the other way. If you don't win, it doesn't follow that you didn't buy a ticket.

2.10 E1

P: I exist

Q: God exists.

R: Angels exist.

U: Determinism is true.

V: I am free.

Disambiguate the following sentences by providing two logically distinct symbolizations:

(a) It is not the case that God exists if angels do. $(R \rightarrow \sim Q)$ $\sim(R \rightarrow Q)$

(b) I am free if and only if determinism is false or God exists. $V \leftrightarrow (\sim U \vee Q)$

(c) If it's not the case that determinism is true only if I am free then I do not exist. $(V \leftrightarrow \sim U) \vee Q$

$\sim(U \rightarrow V) \rightarrow \sim P$ $(\sim U \rightarrow V) \rightarrow \sim P$

2.10 E2

P: Professor Plum teaches philosophy.

Q: Doctor Quimby teaches philosophy.

R: Professor Rosenblum teaches philosophy.

S: Professor Plum is boring.

T: Doctor Quimby is an easy grader.

Translate the following symbolic sentences into idiomatic English sentences. (Writing idiomatic English sentences means writing sentences that sound natural in English – sentences people might actually use.)

(a) $\sim (P \vee Q)$ Neither Prof Plum nor Dr. Quimby teach philosophy.

(b) $P \wedge Q \wedge \sim R$ Prof. Plum and Dr. Quimby teach philosophy, but Prof. Rosenblum doesn't.

(c) $(P \rightarrow \sim S) \wedge (Q \wedge T)$ If Prof. Plum teaches philosophy, he isn't boring; however, Dr. Quimby teaches philosophy and he is an easy grader.

(d) $P \leftrightarrow \sim Q \wedge T$ Prof. Plum teaches philosophy if and only if Dr. Quimby is an easy grader but doesn't teach philosophy.

(e) $\sim P \vee \sim Q \vee \sim R$ Plum, Quimby and Rosenblum don't all teach philosophy.

(f) $(P \wedge Q) \vee (Q \wedge R) \vee (R \wedge P)$ At least two of them (Plum, Quimby, Rosenblum) teach philosophy.

(g) $\sim((P \wedge Q) \vee (Q \wedge R) \vee (R \wedge P))$ At most one of them teaches philosophy.

2.10 E3

Symbolize the following sentences using this abbreviation scheme:

P: Paul is present.
R: Robin is present.
S: Sonia is present.
T: The meeting will take place.
U: The meeting will start on time.
W: Somebody is going to be late.
X: The vote will take place.
Y: The motion will pass.

(a) Unless both Robin and Sonia are present, the meeting will not take place. $(R \wedge S) \vee \sim T$

(b) The meeting will take place but it won't start on time although nobody is going to be late. $(T \wedge \sim U) \wedge \sim W$

(c) Either Paul is present as well as Sonia, or both Robin is not present and someone is going to be late. $(P \wedge S) \vee (\sim R \wedge W)$

(d) The vote will take place exactly on condition that the motion will pass. $X \leftrightarrow Y$

(e) The meeting will take place only if both Robin and Sonia are present. $T \rightarrow (R \wedge S)$

(f) The meeting will start on time unless someone is late, but the meeting won't take place if Paul is not present. $(U \vee W) \wedge (\sim P \rightarrow \sim T)$

(g) The meeting will start on time if and only if no one is late and Sonia is there. $U \leftrightarrow (\sim W \wedge S)$

(h) If and only if Paul is present, will the meeting take place and the motion will pass. $P \leftrightarrow (T \wedge Y)$

(i) If the meeting doesn't start on time, then the vote will take place only if nobody is going to be late. $\sim U \rightarrow (X \rightarrow \sim W)$

(j) The motion will not pass unless both Robin is present and the vote takes place. $\sim Y \vee (R \wedge X)$

(k) At least one of them (Paul, Robin and Sonia) is present. $P \vee R \vee S$

(l) At least two of them (Paul, Robin and Sonia) are present. $(P \wedge R) \vee (R \wedge S) \vee (P \wedge S)$

(m) No more than two of them (Paul, Robin and Sonia) are present. $\sim (P \wedge R \wedge S)$

(n) No more than one of them (Paul, Robin and Sonia) are present. $\sim ((P \wedge R) \vee (R \wedge S) \vee (P \wedge S))$

(o) None of them (Paul, Robin and Sonia) are present. $\sim (P \vee R \vee S)$

(p) Not all of them (Paul, Robin and Sonia) are present. $\sim (P \wedge R \wedge S)$

(q) Exactly one of them (Paul, Robin and Sonia) is present. $(P \vee R \vee S) \wedge \sim ((P \wedge R) \vee (R \wedge S) \vee (P \wedge S))$

(r) Exactly two of them (Paul, Robin and Sonia) are present. $((P \wedge R) \vee (R \wedge S) \vee (P \wedge S)) \wedge \sim (P \wedge R \wedge S)$

Disambiguate the following sentences by providing two logically distinct symbolizations:

(s) Although the motion will pass, the vote will not take place unless Robin is present. $Y \wedge (\sim X \vee R)$ $(Y \wedge \sim X) \vee R$

(t) Paul is present if and only if Sonia is but Robin is not. $P \leftrightarrow (S \wedge \sim R)$ $(P \leftrightarrow S) \wedge \sim R$

(u) Only if nobody is late will the meeting start on time and the vote will take place. $(U \rightarrow \sim W) \wedge X$ $(U \wedge X) \rightarrow \sim W$

NOW TRY LOGIC 2010 SYMBOLIZATION EXERCISES FOR CHAPTER 2.

2.11 SYMBOLIZING ARGUMENTS

Since arguments are composed of sentences, we can use our symbolization techniques to symbolize them. Every sentence (premises and conclusion) must be symbolized independently, using the same abbreviation scheme.

Here is a simple one: (slightly altered from Descartes, Meditations on First Philosophy, Meditation II.)

I have convinced myself that there is absolutely nothing in the world, no sky, no earth, no minds, no bodies. Does it now follow that I too do not exist? No. If I can convince myself that there is absolutely nothing (indeed if I can convince myself of anything at all) then I certainly exist.

Abbreviation scheme:

P: I have convinced myself that there is absolutely nothing in the world.
Q: I exist.

We can put this little argument in standard form, using the abbreviation scheme to aid the paraphrase:

I have convinced myself that there is absolutely nothing in the world.
If I have convinced myself that there is absolutely nothing in the world then I exist.

Therefore, I exist.

The first sentence has been simplified, leaving out the details without changing the meaning. The second sentence, "does it now follow that I too do not exist?" is not part of the argument. Rather, taken together with the "No", it tells us what the conclusion is – I exist.

Now it is easy to symbolize:

P
$P \rightarrow Q$

$\therefore Q$

When symbolizing arguments from philosophical texts, letters to the editors, or other natural locations for arguments, you must consider carefully which parts are essential to the argument. When you are paraphrasing, consider your abbreviation scheme. If you are making your own abbreviation scheme, try to develop the paraphrase and abbreviation scheme together, so that you can best capture the argument.

2.11 E1

Symbolize the following arguments using the abbreviation scheme given:

(a)

I realized, as I lay in bed thinking, that we are not responsible for what we do. This is because either determinism or indeterminism must be true. Provided that determinism is true, we cannot do other than we do. If so, we are but puppets on strings – our actions are not free. If indeterminism is true, then human actions are random, and hence not free. If our actions are not free, it must be conceded that we are not responsible for what we do.

P: Determinism is true.

Q: Indeterminism is true.

R: We can do other than we do.

S: Our actions are free.

T: Our actions are random.

U: We are responsible for what we do.

$P \vee Q$
$P \to \sim R$
$\sim R \to \sim S$
$Q \to T$
$T \to \sim S$
$\sim S \to \sim U$
$\therefore \sim U$

(b)

In our world, there are conscious experiences. Yet, there is a logically possible world physically identical to ours, and in that world there are no conscious experiences. If there are conscious experiences in our world, but not in a physically identical world, then facts about consciousness are further facts about our world, over and above the physical facts. If this is so, not all facts are physical facts. It follows, then, that materialism is false. For, in virtue of the meaning of materialism, materialism is true only if all facts are physical facts.

(Based on David Chalmers, *The Conscious Mind: In Search of a Fundamental Theory*. Oxford: Oxford University Press, 1996. 123-129.)

P: There are conscious experiences in our world.

Q: There is a logically possible world that is physically identical to ours in which there are no conscious experiences.

R: Facts about consciousness are not physical facts.

S: All facts are physical facts.

T: Materialism is true.

P
Q
$P \wedge Q \to R$
$R \to \sim S$
$T \to S$
$\therefore \sim T$

(c)

Next we must consider what virtue is. Since things that are found in the soul are of three kinds —passions, faculties, states of character — virtue must be one of these. We are not called good or bad on the ground of our passions, but are so called on the ground of our virtues. And if we are called good or bad on the grounds of the one, but not the other, then virtues cannot be passions. Likewise, virtues are faculties only if we are called good or bad on the grounds of our faculties as we are so called on the grounds of our virtues. If we have the faculties by nature (which we do) but we are not made good or bad by nature (which we are not) then we cannot be called good or bad on the grounds of our faculties. And since this shows that the virtues are neither passions nor faculties, all that remains is that they should be states of character.

(Based on Aristotle, *Nicomachean Ethics,* Book II, Chapter 5, translated by W. D. Ross.)

P: Virtues are passions

Q: Virtues are faculties

R: Virtues are states of character

S: We are called good or bad on the ground of our passions.

T: We are called good or bad on the ground of our faculties.

U: We are called good or bad on the ground of our virtues.

V: We have the faculties by nature.

W: We are made good or bad by nature.

$P \vee Q \vee R$
$\sim S$
U
$U \wedge \sim S \rightarrow \sim P$
$Q \rightarrow T$
$V \wedge \sim W \rightarrow \sim T$
V
$\sim W$

$\therefore R$

Some of these arguments are pretty tricky.

[2]

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[2] This illustration is the property of Gerald Grow, Professor of Journalism, Florida A&M University. http://www.longleaf.net/ggrow/CartoonPhil.html

2.12 SYNTAX: GRAMMAR FOR SL

Parts of the Sentence:

An ***atomic sentence*** is a sentence letter: P-Z.

A ***molecular sentence*** is one that is composed of an atomic sentence or sentences that have been made into more complex sentences through operations of the logical connectives: $\sim$, $\rightarrow$, $\vee$, $\wedge$ and $\leftrightarrow$.

Every molecular sentence can be expressed in one of the following five logical forms:

$\sim \phi$
$(\phi \rightarrow \psi)$
$(\phi \vee \psi)$
$(\phi \wedge \phi)$
$(\phi \leftrightarrow \psi)$

ϕ and ψ can represent any sentence, whether atomic or molecular.

The ***main connective*** is the logical connective in the five logical forms above.

The ***immediate sentential components*** are the sentences that ϕ and ψ stand for in the logical forms above.

Syntax for our Sentential Logic:

Now we can complete the syntax for our symbolic language.

The Greek letters ϕ (phi) and ψ (psi) can represent any sentence, whether atomic or molecular.

In SL, all well-formed formulas are also symbolic sentences. We can generate all possible symbolic sentences by following these steps:

STEP 1: Start with sentence letters (P-Z with or without numerical subscripts)

Each one is a symbolic sentence.

STEP 2: Add the negation sign, if desired.

If ϕ is a sentence then $\sim \phi$ is a symbolic sentence.

STEP 3: Connect two sentences with a binary connective, if desired.

If ϕ and ψ are sentences so are $(\phi \rightarrow \psi)$, $(\phi \vee \psi)$, $(\phi \wedge \psi)$ and $(\phi \leftrightarrow \psi)$.

STEP 4: Repeat steps 1-3 in any order, indefinitely.

Any formula you can form through recursive application of these steps is a sentence in SL, in official notation (using the parantheses).

Every sentence we form through these steps is a well-formed symbolic sentence in official notation. We can use them recursively to make even more complex sentences. Thus, these steps provide a technique for generating all possible truth-valuable sentences in SL.

Official and Informal Notation:

A set of brackets for every binary connective

Never add brackets for negation

Official or formal notation requires that parentheses are used every time a binary connective is used. However, in informal notation the parentheses can be left off if it does not affect clarity.

The outermost parentheses are often left off:

$(S \wedge T)$ can be informally written: $S \wedge T$

Informal notation also allows parentheses to be left off a series of sentence letters joined with conjunctions, or a series of sentence letters joined with disjunctions.

Consider the sentence: Pepper is a cat, Quincy is a cat and Roxy is a cat.

We might symbolize it in official notation: $((P \wedge Q) \wedge R)$

However, in informal notation it can be symbolized: $P \wedge Q \wedge R$

There is no ambiguity caused by omitting the parentheses in a series of conjuncts.

By convention, when using informal notation, the main connective is considered to be the one to the right. Thus, $P \wedge Q \wedge R$ is informal notation for $((P \wedge Q) \wedge R)$.

But, placement of the parentheses does not affect the meaning of the sentence provided all the atomic sentences are joined by conjunction signs. $((P \wedge Q) \wedge R)$ is logically equivalent to $(P \wedge (Q \wedge R))$.

Using formal notation, there is a pair of parentheses or brackets for every binary connective.

Using informal notation, the outermost parentheses are usually left out, and some inner parentheses may be left out provided it does not cause ambiguity.

Be careful – you can only leave out the parentheses when it doesn't result in logical ambiguity.

A series of conjuncts doesn't need parentheses:	$P \wedge Q \wedge R$
	$P \wedge Q \wedge R \wedge S$
These are taken as equivalent to:	$((P \wedge Q) \wedge R)$.
	$(((P \wedge Q) \wedge R) \wedge S)$.
Likewise, a series of disjuncts doesn't need parentheses.	$P \vee Q \vee R$
	$P \vee Q \vee R \vee S$
These are understood to be equivalent to	$((P \vee Q) \vee R)$.
	$(((P \vee Q) \vee R) \vee S)$

A series of sentences joined by conditionals needs parentheses.

$(P \to Q) \to R$ is **not** the same as $P \to (Q \to R)$

A mix of conjunctions and disjunctions needs parentheses.

$P \vee (Q \wedge R)$ is **not** the same as $(P \vee Q) \wedge R$

A mix of conditionals and biconditionals needs parentheses.

$P \to (Q \leftrightarrow R)$ is **not** the same as $(P \to Q) \leftrightarrow R$

A mix of conditionals/biconditionals with disjunctions/conjunctions may not need parentheses!

The conditional or biconditional is the primary connective.

$P \to Q \wedge R$ is an informal notation for $P \to (Q \wedge R)$

$P \vee Q \to R$ is an informal notation for $(P \vee Q) \to R$

$P \leftrightarrow Q \vee R$ is an informal notation for $P \leftrightarrow (Q \vee R)$

$P \wedge Q \leftrightarrow R$ is an informal notation for $(P \wedge Q) \leftrightarrow R$

Negation signs can act on an atomic sentence (no parentheses) or a molecular sentence. If it is acting on a molecular sentence, then the molecular sentence must be in parentheses.

Correct Official Notation:

$\sim P$
$\sim (P \to Q)$
$\sim (P \vee Q)$
$\sim (P \wedge Q)$
$\sim (P \leftrightarrow Q)$

Identifying the main connective in informal notation:

Make sure that you can still identify the main connective when you use informal notation.

$P \wedge Q \wedge R$ Which is the main connective?
$P \vee Q \vee R \vee S$

In a series of conjuncts or disjuncts without any parentheses, the main connective is always the one farthest to the right. It would be the last one that one naturally adds to a list.

$P \to Q \vee R$
$P \wedge Q \leftrightarrow R$ Which is the main connective?
$P \vee Q \to R \wedge S$

In a well-formed mix of a conditional or biconditional with disjunctions and/or conjunctions (without parentheses), the main connective is always the conditional or biconditional.

Parsing Sentences:

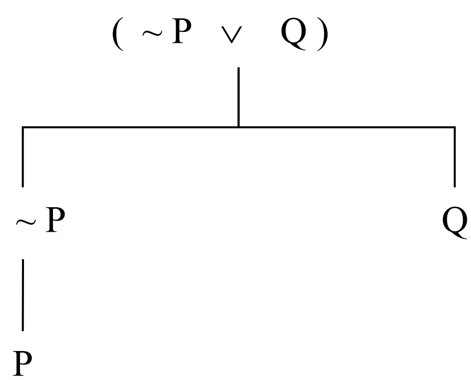
To parse a sentence is to break it down into its component parts. To parse the symbolic sentence is to breaking it down into its atomic sentences and logical connectives. This is essentially a matter of applying steps 1-7 above in reverse. It provides us with an analysis of the logical structure of a symbolic sentence.

1. Identify the main logical connective.
2. Identify the immediate sentential components of that connective. To illustrate the relations, draw a vertical line down from the main logical connective, then split it into the two sentential components (if it is a binary connective).
3. Determine if any of the immediate sentential components are molecular sentences. If they are repeat steps 1 and 2 on those components.
4. Continue applying steps 1-3 until the sentence is broken down into its component parts.

Let's try some:

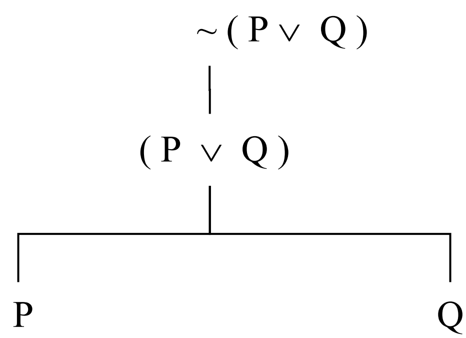
(~P ∨ Q)

This sentence is a well-formed symbolic sentence, in official notation. The main connective is ∨. Thus, the vertical line goes down from ∨ and splits into the two sentential components, ~P and Q. The sentence on the left, ~P, is a molecular sentence, thus a vertical line goes down from the main connective, ~, and ends in the atomic sentence, P.



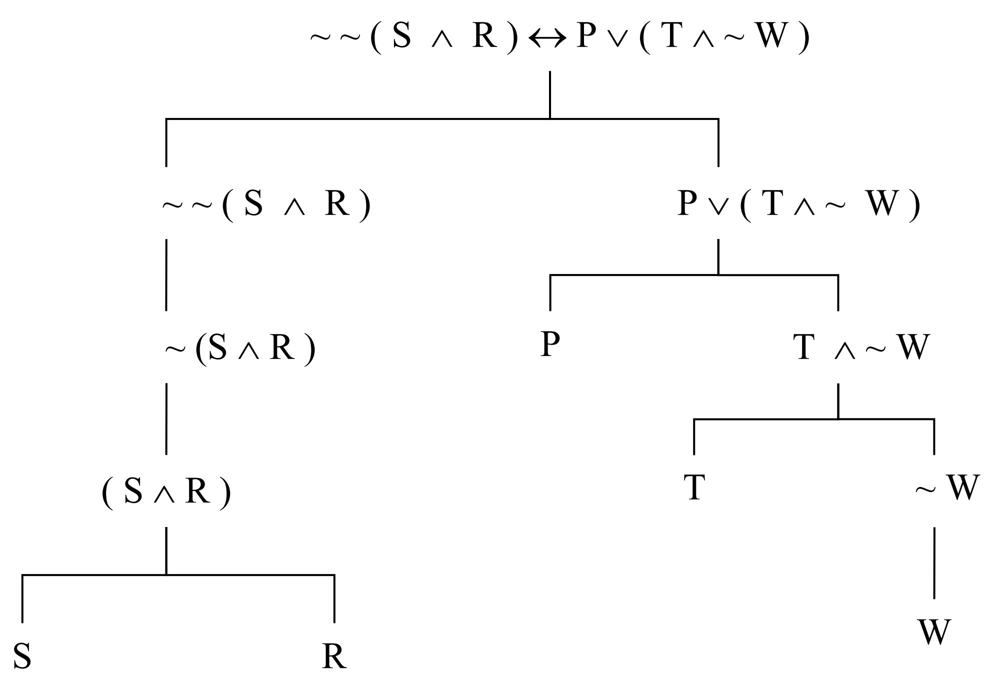
Now try this one: ~ (P ∨ Q)

This sentence is also a sentence of SL, in official notation. It differs from the previous sentence in the location of the parentheses. This makes quite a difference in logical structure! Now the main logical connective is ~. Compare it to the previous parsing diagram.



Here is a more complex sentence: ~ ~ (S ∧ R) ↔ P ∨ (T ∧ ~W)

This sentence is a sentence of SL, but in informal notation. In formal notation, there would be two more sets of parentheses: (~ ~ (S ∧ R) ↔ (P ∨ (T → ~W)))



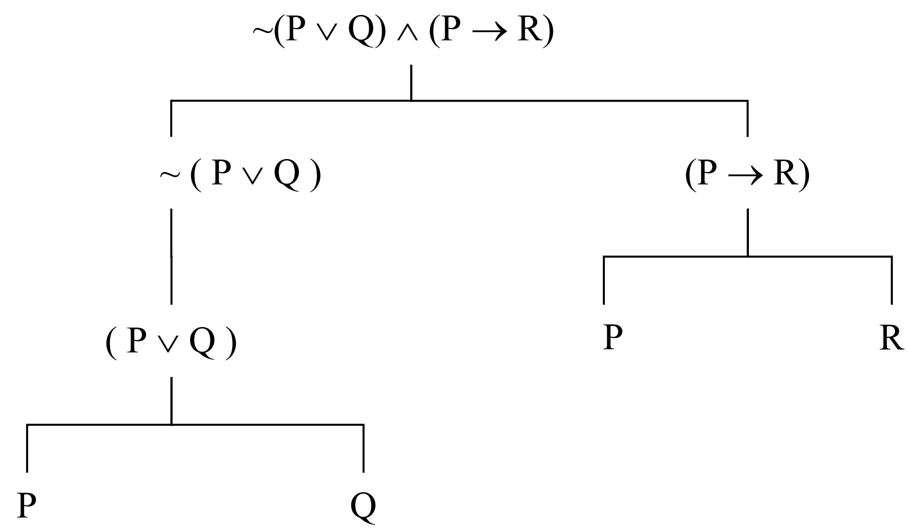
2.12

For each of the following sentences, determine whether it is a well-formed symbolic sentence (a sentence of SL). If not, briefly explain why. If it is, determine whether the sentence is in official notation or informal notation. If it is in informal notation, rewrite it in official notation. Parse the sentence by breaking it down into its atomic sentences.

Examples:

$\sim P \vee Q \wedge R$ — Not a sentence of SL due to lack of parentheses.

$(\sim(P \vee Q) \wedge (P \rightarrow R)$ — Informal notation for... $(\sim(P \vee Q) \wedge (P \rightarrow R))$



(c) $\sim(\sim (S \leftrightarrow B))$ — Not a sentence of SL due to extra parentheses, and use of sentence letter 'B'.

2.12 E1 (instructions above)

a) $(P\rightarrow (S \wedge R))$ Official

b) T & Q Not well formed

c) $\sim \sim \sim \sim R$ Official

d) $(\sim P)$ Not well formed

e) $\sim \sim (R \leftrightarrow W)$ Official

f) $P \vee \sim R \vee S$ Informal

g) $S \wedge T \vee D$ Not well formed

h) $T \wedge R \wedge (S \vee T \vee Q)$ Informal

i) $S \wedge \sim \sim T \leftrightarrow (P \vee Q \rightarrow R \wedge (W \vee \sim T))$ Informal

j) $\sim(\sim ((R \wedge P) \rightarrow \sim S \wedge T \wedge (P \vee Q)))$ Not well formed

k) $\sim(\sim ((R \wedge P) \vee T) \leftrightarrow ((S \wedge P) \rightarrow \sim T))$ Offfcial

l) $(S \wedge T \rightarrow P) \wedge (\sim T \leftrightarrow R \vee P) \rightarrow P \vee Q \vee R$ Informal

NOW TRY LOGIC 2010 PARSING EXERCISES FOR CHAPTERS 1 AND 2.