Lecture 32

$K = M^D$

def: Let S be a self similar set (which can be subdivided into k connect-ed subsets, and each may be magnified by a factor M to yield S.

The fractal dimension of S is

$$\text{Dim} = \frac{\log k}{\log M} = \frac{\log(\#\text{ of } \Delta s)}{\log(\text{magnification})}$$

Ex: Sierpinski Δ

$$D = \frac{\log 3}{\log 2} \approx 1.585$$

Ex: Cantor set

$$D = \frac{\log 2}{\log 3} \approx 0.63$$

Ex: Koch snowflake/line

$$D = \frac{\log 4}{\log 3} \approx 1.26 \qquad D = \frac{\log 16}{\log 9} \doteq 1.26 \text{ same}$$

§14.7 Iterated function thm

Define $A\begin{pmatrix} x \\ y \end{pmatrix} = \beta\begin{pmatrix} x-x_0 \\ y-y_0 \end{pmatrix} + \begin{pmatrix} x_0 \\ y_0 \end{pmatrix}$ where $\beta \in (0,1)$

This function has a fixed point $P_0 = \begin{pmatrix} x_0 \\ y_0 \end{pmatrix}$

and because $0<\beta<1$, it moves each point closer to P_0

The chaos game was played using 3 such functions.

In the end, the sierpinski triangle emerged.

Definition: suppose that $0<\beta<1$, $P_1, \cdots, P_n$ are points in the plane.

And let $A_i(P) = \beta_i(P-P_i)+P_i$

This collection of functions is called an iterated function system.

Definition: sps that $\{A_1, \cdots, A_n\}$ is an iterated function system. The set of points to which an arbitrary orbit in the plane converges is the attractor of the system.

Example: we can obtain the cantor set in this way, using.

$A_1\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{3}\begin{pmatrix} x-0 \\ y-0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$$A_2\begin{pmatrix}x\\y\end{pmatrix}=\frac{1}{3}\begin{pmatrix}x-1\\y-0\end{pmatrix}+\begin{pmatrix}1\\0\end{pmatrix}$$

$\cdot(0,1)$ $\cdot(1,1)$

$\cdot(0,0)$ $\cdot(1,0)$

Variations on the chaos game

We can also define functions A_i which not only move a point towards its fixed point, but also rotates the path.

$$A\begin{pmatrix}x\\y\end{pmatrix}=\beta\begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}x-x_0\\y-y_0\end{pmatrix}+\begin{pmatrix}x_0\\y_0\end{pmatrix}$$