Solutions for these problems are only presented during the Problem Solving Sessions W5-6 in SS 2135. You are strongly encouraged to work through the problems ahead of time, and our TA Yiannis will cover the questions you are most interested in. These sessions are very valuable at developing the proper style to present cogent and rigorous mathematical solutions.

This problem solving session contains material from 3.4, 4.1.

Problems:

1. §3.4: For the transformation $u = \frac{x+y}{2\sqrt{2}}, v = \frac{y-x}{2\sqrt{2/3}}$,

    a) If the "before" sketch is a grid in Euclidean space, draw the "after" sketch.

    b) Determine the inverse of this transformation and see the effect of this transformation on the ellipse
    $$x^2 - xy + y^2 = 2.$$

    c) For the transformation $u = x \tan y, v = xy$, draw the effect of this transformation on the lines
    $$x = 1, x = -1, x = 2, x = 0$$
    and
    $$y = 0, y = 1rad, y = -1rad,$$
    as well as determining the Fréchet derivative and discuss possibility of finding the inverse, but no need to locally solve for the inverse.

2. §4.1: Consider the ultimate step function, $f(x)$ defined on $[0, 1]$ as follows: $f(0) = 1$, and $f(x) = \frac{1}{n}$ for $\frac{1}{n+1} < x < \frac{1}{n}$ for all $n = 1, 2, 3, \cdots$. Use Lemma 4.5 to prove that f is Riemann integrable on the interval $[0, 1]$, and then calculate the Riemann integral $I_0^1 f$.

3. §4.1: Theorem 4.9.

4. §3.4: Question 6.