Lecture 3

For $n \in \mathbb{N}$, let $a_n = 12^n - 1$

n	a_n
0	$12^0 - 1 = 0$
1	$12^1 - 1 = 11$
2	$12^2 - 1 = 143 = 11 \times 13$
3	$12^3 - 1 = 1727 = 11 \times 157$
4	$12^4 - 1 = (12^3 - 1)12 + 11$
$\vdots$	

$12^{236} - 1$

$12^{237} = 12(12^{236} - 1) + 11$

$$\boxed{12(12^n - 1) + 11 = 12^{n+1} - 1}$$

Claim: a_n is a multiple of 11 for each $n \in \mathbb{N}$

For $n \in \mathbb{N}$, let $P(n)$ be $12^n - 1$ be a multiple of 11.

Claim: $\forall n \in \mathbb{N}, P(n)$

Proof: By Induction.

Base case: $P(0)$

[$P(0)$: $12^0 - 1$ is a multiple of 11]

$12^0 - 1 = 1 - 1 = 0 = 11 \times 0$

Inductive step: $\forall n \in \mathbb{N}, (P(n) \rightarrow P(n+1))$

Let $n \in \mathbb{N}$, Suppose $P(n)$, i.e.

$12^n - 1$ is a multiple of 11 (IH)

$12^{n+1} - 1 = 12(12^n - 1) + 11 = 12(11 \cdot k) + 11$ for some $k \in \mathbb{Z}$ by (IH)

$= 11(12k + 1)$ where $12k + 1 \in \mathbb{Z}$ since $k \in \mathbb{Z}$

is a multiple of 11.

For $n \in \mathbb{N}$, let $P(n)$ be $3^n \geq n^3$

$3^0 = 1$ $\qquad 3^{n+1} = 3^n + 3^n + 3^n$

$3^1 = 3$

$3^2 = 9$

$3^3 = 27$

$3^4 = 81$

$0^3 = 0$

$1^3 = 1$

$2^3 = 8$

$3^3 = 27$

$4^3 = 64$

$5^3 = 64 + 3 \times 16 + 3 \times 4 + 1 = 125$

n^3

$(n+1)^3 = n^3 + 3n^2 + 3n + 1$

$(n+1)^3 = \frac{(n+1)^3}{n^3} \times n^3 = \left(\frac{n+1}{n}\right)^3 \times n^3$

$= \left(1 + \frac{1}{n}\right)^3 \times n^3$

When does $3^n \geq n^3 \rightarrow 3^{n+1} \geq (n+1)^3$?

$3 \geq \left(1 + \frac{1}{n}\right)^3$

For $n \geq 3$: $\left(1 + \frac{1}{n}\right)^3 \leq \left(1 + \frac{1}{3}\right)^3 = \frac{64}{27} \leq 3$