SOME RULES BREAK DOWN MOLECULAR SENTENCES

Use them when you have a premise, assumption or derived sentence whose main logical operator is a binary connective:

$\wedge \quad \rightarrow \quad \vee \quad \leftrightarrow$

Look for uses of S, MP, MT, MTP, BC

SOME RULES BUILD MOLECULAR SENTENCES

You can use these rules to build sentences with main connective:

$\wedge \quad \vee \quad \leftrightarrow \quad \sim\sim$

Look for uses of ADJ, ADD, CB, DN
Mostly these rules are used to match a goal sentence,
to match an antecedent (to use with MP)
or a negated consequent or disjunct (to use with MT or MTP)

Look for sentence parts that you can build with these rules!

ALWAYS: On line 1, write: “Show” then the conclusion

NOW: Analyze your show line. What is the main logical operator?

EVERY TIME YOU HAVE A SHOW LINE…

Step 1: Analyze show line…

What is the main operator/connective?

$\rightarrow$ $\leftrightarrow$ $\sim$ $\wedge$ $\vee$

or

Atomic Sentence

Your main logical operator is:

→

You are going to begin a CD.

On next line, write the antecedent (part before the →)

Justification: ASS CD

New Goal: the consequent.

Give your new goal (the consequent) a show line. On the next line, write: “Show” consequent.

If you have a new show line, go back to step 1 and analyze your new show line!

$\rightarrow$

1	~~Show~~ $P \rightarrow (Q \rightarrow R)$		
2	P	ASS CD	First ASS CD
3	~~Show~~ $Q \rightarrow R$		Then Show the consequent
4	Q	ASS CD	Here we use a conditional proof to show the consequent
5			
6	R	CD	
7		CD	

1	~~Show~~ $P \rightarrow Q$		
2	P	ASS CD	First ASS CD
3	~~Show~~ Q		Then Show the consequent
4	$\sim Q$	ASS ID	Here we use an indirect proof to show the consequent
5			
6	CONTRADICTION	ID	
7		CD	

Your main logical operator is:

$\leftrightarrow$

You probably want to derive each of the two conditionals separately.
Then use CB to build the biconditional.

Give a show line for the first conditional:

Show Left Side $\rightarrow$ Right Side

Complete the proof (use CD)

Put in a show line for the second conditional:

Show Right Side $\rightarrow$ Left Side

Complete the proof (use CD)

Now use CB to join the two sentences together!

↔

1	~~Show~~ P ↔ Q		
2	~~Show~~ P → Q		First Show Left Side → Right Side
3	P	ASS CD	Complete the CD. Follow the steps for a conditional show line (previous slides)
4			
5			
6	Q	CD	
7	~~Show~~ Q → P		Now Show Right Side → Left Side
8	Q	ASS CD	Complete the CD Follow the steps for a conditional show line (previous slides)
9			
10			
11	P	CD	
12	P ↔ Q	2 7 CB	Use CB to build the biconditional to match the biconditional show line.
13		DD	

~ or ATOMIC SENTENCE

You are going to begin an ID

On next line, write the opposite of the show line.
add or remove: ~

Goal: any contradiction

Your main logical operator is:

$\wedge$

Can you derive the two conjuncts together?

- Only if the sentence appears in a premise (eg. as the consequent of a conditional sentence.)
- Also look for DM and NC variants if you have derived rules

You probably want to derive each of the two conjuncts separately. Then use ADJ to join them.

Give a show line for the first conjunct:
Show Left Side
Complete the proof (use CD or ID)

Put in a show line for the second conjunct:
Show Right Side
Complete the proof (use CD or ID)

Now use ADJ to join the two sentences together!

$\wedge$

1	~~Show~~ $P \wedge (Q \to S)$	
2	~~Show~~ P	
3	$\sim P$	ASS ID
4		
5		
6	CONTRADICTION	ID
7	~~Show~~ $Q \to S$	
8	Q	ASS CD
9		
10		
11	S	CD
12	$P \wedge (Q \to S)$	2 7 ADJ
13		DD

First Show Left Conjunct

Complete the subderivation.

Here the conjunct is an atomic sentence, so we do an indirect derivation (ID). Follow the steps for a negated or atomic sentence show line (previous slides)

Now Show Right Conjunct

Complete the subderivation.

Here the conjunct is a conditional sentence, so we do a conditional derivation (CD). Follow the steps for a conditional show line (previous slides)

Use ADJ to build the conjunction to match the initial show line.

Your main logical operator is:

$\vee$

Show $P \vee Q$

Can you derive the full disjunctive sentence as a whole?

- Only if the sentence appears in a premise (eg. as the consequent of a conditional sentence.)
- Also look for DM and NC variants if you have derived rules

Can you derive one disjunct and then ADD the other?

- Only if the other disjunct doesn't appear anywhere in the premises!
- Give a show line for the one disjunct that appears in the premises.
- Complete your subderivation and use ADD to build the OR sentence to match the show line.

If you can't do either of the above, do an indirect proof!

(see next slide)

Your main logical operator is:

$\vee$

Show $P \vee Q$

To show with an indirect proof…

First: Make the assumption for ID

$\sim(P \vee Q)$ Ass ID

If you are allowed the derived rules use DM next:

$\sim(P \vee Q)$ Ass ID

$\sim P \wedge \sim Q$ DM

This gives you two things to work with. Your new goal is a contradiction!

If you are NOT allowed the derived rules you can still show $\sim P$ and $\sim Q$:

$\sim(P \vee Q)$ Ass ID

Show $\sim P$ or Show $\sim Q$

P Ass ID Q Ass ID

In both cases it will be an indirect proof. See next page…

1	Show $P \vee Q$	
2	$\sim(P \vee Q)$	ASS ID
3	~~Show~~ $\sim P$	
4	P	ASS ID
5	$P \vee Q$	4 ADD
6	$\sim(P \vee Q)$	2 R
7		5 6 ID
8		

∨

First make assumption for ID

Now show the negation of one of the disjuncts using an indirect derivation. Make an assumption for ID. Use ADD to match the disjunction. Use R to bring down the negated disjunction to meet the box requirements for ID (contradiction).

Now you can show $\sim Q$ the same way.

Neither P or Q means not P and not Q, so if you have $\sim(P \vee Q)$ on an available line you can derive $\sim P$ and $\sim Q$ from it using ID & ADD.

1	Show $P \vee Q$	
2	$\sim(P \vee Q)$	ASS ID
3	$\sim P \wedge \sim Q$	DM

If you have the derived rules, you can get $\sim P$ and $\sim Q$ very quickly if you use DM after making the assumption for ID.

Alternatively, if you have the derived rules, you can show the correlated conditional. If so, put a show line for the conditional.

Then, when you complete your CD, use CDJ to turn it into the disjunction.

~~Show~~ $\sim P \rightarrow Q$

$P \vee Q$ CDJ