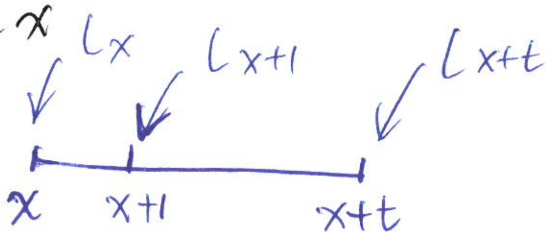
Lecture week 1

① $p_x = \dfrac{l_{x+1}}{l_x}$, $_tp_x = \dfrac{l_{x+t}}{l_x}$



② $d_x = l_x - l_{x+1}$

③ l_x in terms of d_x

$d_x = l_x - l_{x+1}$

$d_{x+1} = l_{x+1} - l_{x+2}$

$\vdots$

$d_{x+T} = l_{x+T} - \boxed{l_{x+T+1}} = 0$

$$l_x = d_x + d_{x+1} \cdots d_{x+T} = \sum_{t=0}^{T} d_{x+t}$$

sum of deaths in all future years

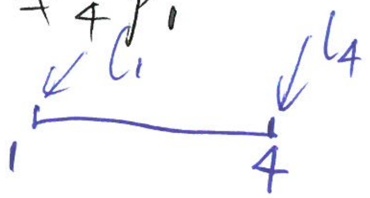
④ $q_x = 1 - p_x = \dfrac{l_x - l_{x+1}}{l_x} = \dfrac{d_x}{l_x}$

$_tq_x = 1 - {_tp_x} = \dfrac{l_x - l_{x+t}}{l_x}$ $\quad \sum_{s=0}^{t} d_{x+s}$

no close expression.

1. $P(\text{aged } 1 \text{ survives to } 4) = \frac{l_4}{l_1} = {}_3p_1$

$\neq {}_4p_1$



2. $E(\text{No. of } 0 \text{ to } 2)$

$Y_i = \begin{cases} 1 & {}_2p_0 \\ 0 & 1 - {}_2p_0 \ ({}_2q_0) \end{cases}$

↓

individual i

number of newborn survive to 2 $= \sum_{i=1}^{l_0} Y_i$

$E\left(\sum_{i=1}^{l_0} Y_i\right) = \sum_{i=1}^{l_0} E(Y_i) = \sum_{i=1}^{l_0} \left({}_2p_0 \cdot 1 + {}_2q_0 \cdot 0\right)$

$= l_0 \times {}_2p_0$

$= l_0 \times \frac{l_2}{l_0} = l_2$ Not suprising!

l_0 ↓ no. of new born

$\frac{l_2}{l_0}$ → prob. new born survives to 2.

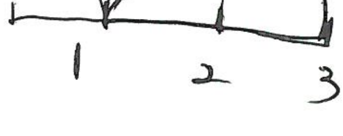
Assumption: every one has the same surviving probability.

3. $P(\text{aged } 1 \text{ dies aged } 2)$

1 dies before age 3

$\neq {}_2q_1 \neq q_1$

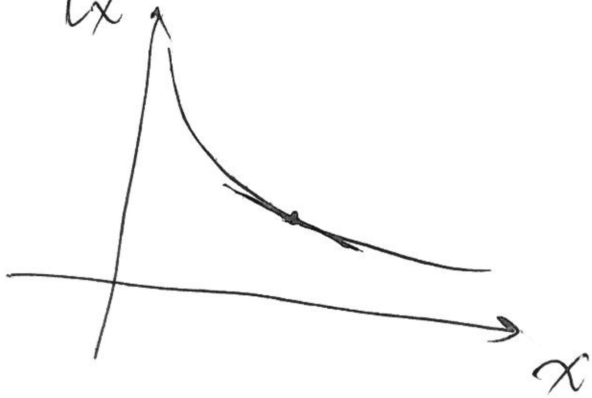
$= {}_{1|}q_1 = \frac{l_2 - l_3}{l_1} = \frac{d_2}{l_1}$



aged 2

survives at age 2 but dies before age 3.

show that $\mu_x = \frac{{}_hq_x}{h}$



①

$$\mu_x = -\frac{1}{l_x}\frac{d\, l_x}{dx}$$

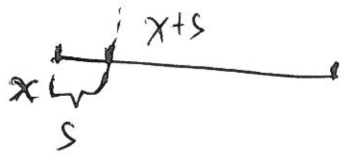
$$= -\frac{1}{l_x} \times \lim_{h\to 0}\frac{l_{x+h}-l_x}{h}$$ (by definition of derivative)

$$\approx -\frac{1}{l_x}\cdot\frac{l_{x+h}-l_x}{h}$$ (for small h)

$$= \frac{1}{h}\cdot\overbrace{\frac{l_x-l_{x+h}}{l_x}}^{{}_hq_x}$$

$$= \frac{{}_hq_x}{h}$$ → annulised mortality at the precise moment of attaining age x

② $\mu_x \rightarrow {}_np_x$



$$\mu_x = -\frac{d\ln l_x}{dx}$$

x is the starting age.

$$\Rightarrow \mu_{x+s} = -\frac{d\ln l_{x+s}}{ds}$$

s changes

$$\Rightarrow \int_0^n \mu_{x+s}\, ds = \int_0^n -\frac{d}{ds} \ln l_{x+s}\, ds$$

$$= -\left[\ln l_{x+s}\right]_0^n$$

$$= -\ln \frac{l_{x+n}}{l_x} \rightarrow {}_n p_x$$

$$= -\ln {}_n p_x$$

$\Rightarrow$

$${}_n p_x = \exp\left(-\int_0^n \mu_{x+s}\, ds\right)$$

③ ${}_n q_x$? $\quad \mu_x \rightarrow {}_n q_x$

$$\mu_{x+s} = -\frac{d\, l_{x+s}}{ds} \cdot \frac{1}{l_{x+s}}$$

$$\frac{d\, l_{x+s}}{ds} = -l_{x+s} \cdot \mu_{x+s}$$

$$\int_0^n \frac{d\, l_{x+s}}{ds} \cdot ds = -\int_0^n l_{x+s}\, \mu_{x+s}\, ds$$

$$\underline{l_{x+n} - l_x} = -\int_0^n l_{x+s}\, \mu_{x+s}\, ds$$

dividing l_x for both sides, and take negative sign for both sides

$$-\frac{l_{x+n}-l_x}{l_x} = +\int_0^n \frac{l_{x+s}}{l_x}\,\mu_{x+s}\,ds$$

$$\Rightarrow \quad {}_nq_x = \int_0^n {}_sp_x\,\mu_{x+s}\,ds$$

Letting $n=1$

$$\underline{l_{x+1}-l_x} = -\int_0^1 l_{x+s}\,\mu_{x+s}\,ds \quad \rightarrow \text{take } \ominus$$

$$\Rightarrow \quad d_x = \int_0^1 l_{x+s}\,\mu_{x+s}\,ds$$

$f(t) = F'(t)$ $\quad \mu(t) = \dfrac{f(t)}{S(t)}$

$$= \lim_{\Delta t\to 0} \frac{\left(\dfrac{P(\text{life dies in }(t, t+\Delta t))}{P(\text{life survives longer than } t)}\right)}{\Delta t}$$

A $= AB$; B ; dies in $(t, t+\infty)$

$$= \lim_{\Delta t\to 0} \frac{\left(P(\text{life dies in }(t, t+\Delta t) / \text{alive at } t)\right)}{\Delta t}$$

$A|B$; 1

Actuarial notation and statistical notation.

T_x : future lifetime for a person aged x

$\frac{l_x - l_{x+t}}{l_x}$

${}_tq_x = Pr(T_x \le t) = F_{T_x}(t)$

$\frac{l_{x+t}}{l_x}$

${}_tp_x = 1 - Pr(T_x \le t) = S_{T_x}(t)$

stat notation

$\mu_{x+t} = -\frac{1}{l_{x+t}} \frac{d l_{x+t}}{dt}$ $\quad \mu(t)$

Actuarial notation

$1 - \frac{l_{x+t}}{l_x}$ $\quad$ //

$$= - \frac{\frac{d l_{x+t}}{dt} / l_x}{l_{x+t} / l_x}$$

$$\left(\frac{d\, {}_tq_x}{dt} = - \frac{d l_{x+t}}{dt} / l_x \right)$$

$F'_{T_x}(t) \rightarrow$ simplify notation

$$= \frac{f_{T_x}(t)}{S_{T_x}(t)} = - \frac{S'(t)}{S(t)}$$

stat notation

$$f(t) = \frac{dF(t)}{dt} = \lim_{\Delta t \to 0} \frac{F(t+\Delta t) - F(t)}{\Delta t} = \lim_{\Delta t \to 0} \frac{P(\text{life dies in } (t, t+\Delta t))}{\Delta t}$$

According to ① and ②

$f(t)$ and $\mu(t)$ are different

$P($life dies in $(t, t+\Delta t))$

$l_x \qquad l_{x+t} \qquad l_{x+t+\Delta t}$

$$= \frac{l_{x+t} - l_{x+t+\Delta t}}{l_x}$$

$P($life dies in $(t, t+\Delta t)$ / alive at $t)$

$$= \frac{l_{x+t} - l_{x+t+\Delta t}}{l_{x+t}}$$

Explanation based on actuarial notation