Lecture 8

Example: $F(x) = \frac{-3}{2}x^2 + \frac{5}{2}x + 1$ has a 3-cycle: 0, 1, 2

and $F'(0)F'(1)F'(2) = 35/8 > 1 \Rightarrow$ 3-cycle is repelling

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Exercise: The doubling function has many cycles. Are they repelling or attracting? Repelling (keep the pts between 0 & 1

CHAPTER 6 BIFURCATIONS

The quadratic map. Let $Q_c(x) = x^2 + c$ where c is a constant

§6.1 Dynamics of Q_c

Fixed points. The map Q_c has the fixed points:

$$Q_c(x) = x$$
$$x^2 - x + c = 0$$
$$x = \frac{1 \pm \sqrt{1-4c}}{2}$$

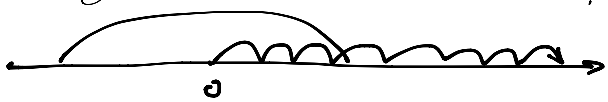
• we have:

if $c > \frac{1}{4}$, no fixed points
$= \frac{1}{4}$, 1 fixed pt.
$< \frac{1}{4}$, 2 pts. $p_1 = \frac{1-\sqrt{1-4c}}{2}$, $p_2 = \frac{1+\sqrt{1-4c}}{2}$

This is what we call a saddle-node bifurcation at $c = 1/4$.
As c decreases, from above to below 1/4, the dynamics change.

$\boxed{\text{CASE } c > \frac{1}{4}}$ The graph of Q_c never intersects the line $y = x$.

For Any x_0, its orbit under Q_c escapes to $+\infty$



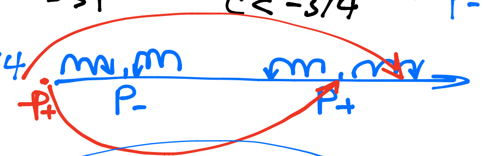
$\boxed{\text{CASE } c = \frac{1}{4}}$ The fixed point $p = \frac{1}{2}$ is neutral $Q_c'(x) = 2x$
$Q_c'(1/2) = 1$

$\boxed{\text{CASE } c < 1/4}$

so $Q_c'(P_+) = 1 + \sqrt{1-4c} > 1$
which means that P_+ is a repelling fixed point
and $Q_c'(P_-) = 1 - \sqrt{1-4c}$
so $|Q_c'(P_-)| = |1 - \sqrt{1-4c}|$ $=1$ if $\sqrt{1-4c} = 2$

$$|Q_c'(P_-)| \begin{cases} <1 & \text{if } -3/4 < c < 1/4 \rightarrow P_- \text{ is attracting} \\ =1 & \text{if } c = -3/4 \rightarrow P_- \text{ is neutral} \\ >1 & \text{if } c < -3/4 \rightarrow P_- \text{ is repelling} \end{cases}$$

$-3/4 < c < 1/4$



$c < -3/4$