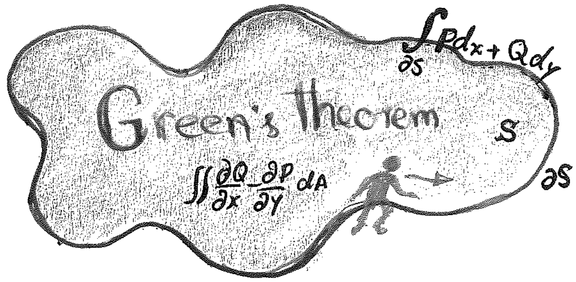


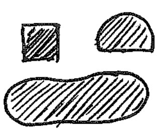
S is a $\boxed{\text{regular}}$ region on the plane

$S = \overline{S^{int}}$ i.e. it has a clear boundary & no false boundary

S is cpt

bad S:

good S:



Let $\mathbb{F}(x,y) = \langle P(x,y), Q(x,y) \rangle$ be C' on an open set containing $\overline{S}$.

$$\iint_S \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA =$$

$$\int_{\partial S} P\,dx + Q\,dy = \int_{\partial S} \mathbb{F} \cdot dx$$

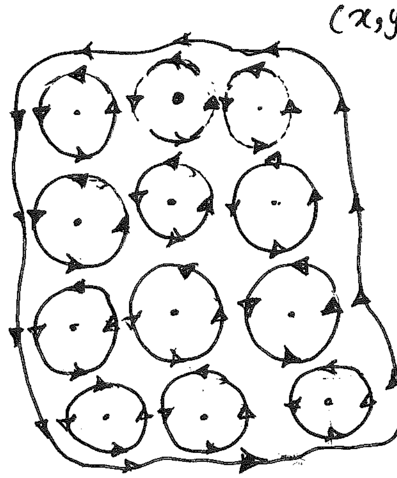
$$= \text{work of } \mathbb{F} \text{ along } \partial S$$

∂S is piecewise smooth curve ← and oriented counter clockwise

so that line integral is possible.

$$\int_{\partial S} \mathbb{F} \cdot dx = \int_{\partial S} P\,dx + Q\,dy$$

$\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}$ measures a twist at the pt (x,y)



Sum of all the twists ~~on~~ S =

= effect of $\mathbb{F}$ along ∂S

eg: if $\mathbb{F} = \nabla f = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right\rangle$ & f is C^2,

Then $\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = 0$ & $\int_{\partial S} \nabla f \cdot dx = 0$

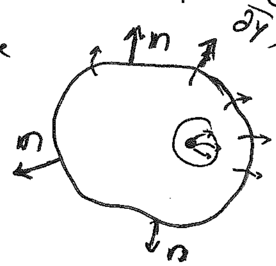
∂S ← closed curve

$= 0$ ← use chain rule

Divergence Thm Version of Green's Thm

$$\int_{\partial S} \mathbb{F} \cdot n\, ds = \iint_S \left(\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} \right) dA = \iint_S \nabla \cdot \mathbb{F}\, dA$$

flux of $\mathbb{F}$ across the boundary



total flux created inside S

Bad Example

$\mathbb{F}$ is not C' on S:

S is the unit disc

$$\mathbb{F}(x,y) = \left\langle \frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2} \right\rangle$$

$\int_{\partial S} P\,dx + Q\,dy = 2\pi$

∂S ← unit circle $x = \cos t$, $y = \sin t$

$\iint_S \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA$ not defined.