

from \mathbb{R} to \mathbb{R}^n

Cartesian product

\mathbb{R}^2 is $\mathbb{R} \times \mathbb{R}$ together with algebraic structure of Vector space

We are designing properties for \mathbb{R}^n so that we can extend our knowledge and intuition of one variable calculus to multivariate i.e. to $\mathbb{R} \times \dots \times \mathbb{R}$
 n times

Properties of \mathbb{R}^n

order

\mathbb{R}^n is NOT ordered

so we cannot have lub or glb

so Completeness of \mathbb{R}^n is defined differently

Through Cauchy Sequences

See bottom of page 27 &

Thm 1.20

$\cos \theta = \frac{a \cdot b}{|a||b|}$
 $-1 \leq \frac{a \cdot b}{|a||b|} \leq 1$ or

or $-|a||b| \leq a \cdot b \leq |a||b|$

Algebraic structure

\mathbb{R}^n has vectorspace properties

$x+y, kx$ but NOT x/y !!

only extends x in magnitude
 $x \parallel y$ if $y = kx$

Scalar
 kx

dot product

result is a number
 $x \cdot y$

Euclidean norm

$|x| = \sqrt{x \cdot x}$

orthogonality
 $x \perp y$ if $x \cdot y = 0$

distance $d(x,y) = |x-y|$

$|a \cdot b| \leq |a||b|$

triangle \leq

$|a+b| \leq |a|+|b|$
 or $||a|-|b|| \leq |a-b|$

Cauchy inequality

$\begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

in \mathbb{R}^3 only
 Cross product

$a \times b$

$a \perp a \times b$
 $b \perp a \times b$

$|a \times b| = |a||b| \sin \theta$
 = area of parallelogram of a, b

$a \cdot b = |a||b| \cos \theta$
 projection

Properties of \mathbb{R}

is ordered:
 $\forall a, b \in \mathbb{R}$
 $a < b$ or $a > b$
 or $a = b$

has algebraic structure known as field
 $a+b$ ka a/b

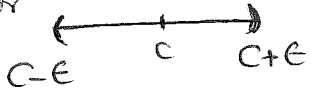
$|a-b|$ is interpreted as distance between a and b

$|x-c| < \epsilon$

means

$c-\epsilon < x < c+\epsilon$

or



Parallel to properties of \mathbb{R}

multiplications on \mathbb{R}^n