Eg (revised simplex method, continued)

$$A=\begin{array}{c} \begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ -1 & 2 & -2 & 0 & 1 & 0 \\ 2 & 1 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array} \qquad b=\begin{array}{cc} \begin{bmatrix} 4 \\ 6 \\ 5 \end{bmatrix} & \begin{array}{c} \leftarrow x_4 \\ \leftarrow x_5 \\ \leftarrow x_6 \end{array} \end{array}$$

$$C^T=[1 \;\; 2 \;\; -1 \;\; 0 \;\; 0 \;\; 0]$$

In tableau ③ basic variables are $\{x_1, x_2, x_6\}$

B^{-1} is best! (The following procedure is used to fight round-off)

$$B=\begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_6 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 \\ -1 & 2 & 0 \\ 2 & 1 & 1 \end{bmatrix} \end{array}$$

Inversion of B:

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ -1 & 2 & 0 & 0 & 1 & 0 \\ 2 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\simeq \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 3 & 0 & 1 & 1 & 0 \\ 0 & -1 & 1 & -2 & 0 & 1 \end{array}\right]$$

$$\simeq \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{2}{3} & -\frac{1}{3} & 0 \\ 0 & 1 & 0 & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & 0 & 1 & -\frac{5}{3} & \frac{1}{3} & 1 \end{array}\right]$$

B^{-1} for tableau ③

$$C_B^T=\begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_6 \end{array} \\ [1 \;\; 2 \;\; 0] \end{array}, \quad W_B^T=[1, 2, 0]\begin{bmatrix} \frac{2}{3} & -\frac{1}{3} & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 \\ -\frac{5}{3} & \frac{1}{3} & 1 \end{bmatrix}=\left[\frac{4}{3} \;\; \frac{1}{3} \;\; 0\right]$$

$$W^TA-C^T=\begin{array}{c} \begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \end{array} \\ \left[1 \;\; 2 \;\; 0 \;\; \frac{4}{3} \;\; \frac{1}{3} \;\; 0\right] \end{array}-[1 \;\; 2 \;\; -1 \;\; 0 \;\; 0 \;\; 0]$$

$$=\left[0 \;\; 0 \;\; 1 \;\; \frac{4}{3} \;\; \frac{1}{3} \;\; 0\right]$$

Tableau ③ objective row

Check that the basic variables actually do have 0 for their coefficients, just after finding any tableau.

Tableau ③ is optimal, with optimal solution $B^{-1}b=\begin{bmatrix} \frac{2}{3} & -\frac{1}{3} & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 \\ -\frac{5}{3} & \frac{1}{3} & 1 \end{bmatrix}\begin{bmatrix} 4 \\ 6 \\ 5 \end{bmatrix}=\begin{bmatrix} \frac{2}{3} \\ \frac{10}{3} \\ \frac{1}{3} \end{bmatrix}=\begin{bmatrix} x_1 \\ x_2 \\ x_6 \end{bmatrix}$

That is, $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix}=\begin{bmatrix} \frac{2}{3} \\ \frac{10}{3} \\ 0 \\ 0 \\ 0 \\ \frac{1}{3} \end{bmatrix}$ is the optimal solution.

The optimal objective value is $x_1 + 2x_2 - x_3 = \frac{2}{3} + 2 \cdot \frac{10}{3} - 0 = \frac{22}{3}$

(This is $c_B (B^{-1} b)$.)

Alternatively, the optimal dual solution is w_B^T for tableau ③ :
$[w_1 \ \ w_2 \ \ w_3] = [\frac{4}{3} \ \ \frac{1}{3} \ \ 0]$

The dual objective function is "Maximize $Z = 4w_1 + 6w_2 + 5w_3$". where the value
$[\frac{4}{3} \ \ \frac{1}{3} \ \ 0]$ is $4 \cdot \frac{4}{3} + 6 \cdot \frac{1}{3} + 5 \cdot 0 = \frac{22}{3}$

This is $b^T w_B = w_B^T b = (c_B^T B^{-1}) \cdot b$