Worth: 3% **Due:** By 12 noon on Tuesday 31 January.

Remember to write the the *full name* and *student number* of each member of your group prominently on your submission. Your submission must be a PDF file named e2.pdf and it must be handed-in using the MarkUs system. You may create the PDF file using a typesetting system (export to PDF) or by scanning in handwritten work to create a PDF file.

Each exercise may be completed in groups of 1 – 2 students who are in the **same** tutorial section.

*Please read and understand the policy on Collaboration given on the Course Information Sheet. Then, to protect yourself, list on the front of your submission **every** source of information you used to complete this homework (other than your own lecture and tutorial notes, and materials available directly on the course webpage). For example, indicate clearly the **name** of every student with whom you had discussions, the **title** of every additional textbook you consulted, the **source** of every additional web document you used,* etc.

For each question, please write up detailed answers carefully. Make sure that you use notation and terminology correctly, and that you explain and justify what you are doing. Marks **will** be deducted for incorrect or ambiguous use of notation and terminology, and for making incorrect, unjustified, ambiguous, or vague claims in your solutions.

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1. Consider the domain $D = \{\text{all test and exam questions}\}$, and the predicate symbols $E(x)$: "x is an exam question", $T(x)$: "x is a test question", $H(x, y)$: "x is harder than y", and $L(x)$: "x is long". (Assume every question is either "long" or "short".)

   Using only these symbols (in addition to appropriate connectives and quantifiers), translate each sentence below. That is, give a natural English sentence that corresponds to each given symbolic sentence, and give a clear symbolic sentence that corresponds to each given English sentence.

   (a) Every test question is short.
   (b) $\exists x \in D, E(x) \wedge L(x)$
   (c) Some test question is harder than every exam question.
   (d) $\neg\exists x \in D, E(x) \wedge \forall y \in D, T(y) \Rightarrow H(x, y)$

2. Translate the following logical statements into English statements. The English statements will be about mathematics and might not be true.

   (a) $\forall x \in \mathbb{N}, P(x) \wedge \neg(x = 2) \Rightarrow O(x)$,
   where $P(x)$ means "x is a prime number" and $O(x)$ means "x is odd."
   (b) $\exists x \in \mathbb{N}, P(x) \wedge \forall y \in \mathbb{N}, P(y) \Rightarrow y \leq x$, where $P(x)$ means "x is a prime number."

3. Determine whether or not each of the following pairs of statements are equivalent. Give a convincing reason for each conclusion. To show that a pair is equivalent, show that the left statement being true means that the right statement is true, and vice versa. To show that a pair is not equivalent, construct a counter-example.

   (a) $\forall x \in D, (P(x) \wedge Q(x))$ $\quad$ $(\forall x \in D, P(x)) \wedge (\forall x \in D, Q(x))$.
   (b) $\forall x \in D, (P(x) \vee Q(x))$ $\quad$ $(\forall x \in D, P(x)) \vee (\forall x \in D, Q(x))$.
   (c) $\exists x \in D, (P(x) \vee Q(x))$ $\quad$ $(\exists x \in D, P(x)) \vee (\exists x \in D, Q(x))$.
   (d) $\forall x \in D, (P(x) \Rightarrow Q(x))$ $\quad$ $(\forall x \in D, P(x)) \Rightarrow (\forall x \in D, Q(x))$.

4. The definition of the limit of a function, $\lim_{x \to a} f(x) = L$, can be expressed using quantifiers as

$$\forall \epsilon \in \mathbb{R}^+, \exists \delta \in \mathbb{R}^+, \forall x \in \mathbb{R}, 0 < |x - a| < \delta \Rightarrow |f(x) - L| < \epsilon.$$

   Express the negation of this statement (*i.e.*, the definition of $\lim_{x \to a} f(x) \neq L$) by working the negation "in" — your final answer should *not* contain the negation symbol.