UNIVERSITY OF TORONTO Faculty of Arts and Science

EXAMINATION DECEMBER 2011

PHL 245 H1F L0101 - Niko Scharer

Duration - 3 hours

Examination Aid: Sheet with rules (provided)



| Last Name | |
|----------------|------|
| First Name | |
| Student Number | |

Answer all questions on the exam paper.

Use the supplied examination booklet for rough work OR if you need further space.

The exam consists of fourteen (14) pages. Pages 2-12 have questions on them.

The final two pages (13-14) are an aid sheet and may be detached from the rest of the exam.

1. A set of sentences {P, Q, R} is logically inconsistent. Which of the following arguments must be valid? Circle the correct answer.

2 %

b)
$$P \wedge S$$
 Q Valid
 $R \rightarrow T$ Not necessarily valid.

2. Suppose there are three sentences: ϕ , ψ and χ . On every interpretation that ϕ is true, ψ is false. What can you conclude (if anything) about the following argument? Explain. (3%)

$$\frac{\phi \vee \chi}{\therefore \psi \to \chi}$$

3. Consider the following truth-table for the NEW symbol: * (2%)

| P | Q | P * Q |
|--------------|---|--------------|
| T | T | F |
| T | F | \mathbf{T} |
| \mathbf{F} | T | T |
| \mathbf{F} | F | F |

- a) Using the new symbol, and other logical connectives if necessary, symbolize: P iff Q.
- b) What ordinary English expression can this new truth-functional connective (**) be used to symbolize (given its truth-table)?

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|----|-----------------------------------|---|---|--------------------------|-----------------------------|
| 4. | Use | this symbolization scheme to | symbolize the followi | ing sentences: (36 % | $6 = 9 \times 4\%$ |
| | A ¹ : F ¹ : | a has an appointment.a is a person | B^1 : a is a place. G^1 : a is a tour guide. K^2 : a visits b . | D^1 : a is a doctor. | |
| | | • | M^2 : a is more popula | | |
| | _ | Adam | | | d^1 : the daughter of a |
| | a) | Some doctors don't have any | appointments only pro | ovided that everybody | is healthy. |
| | | | | | · |
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| | b) | For a person to visit a doctor, | , it is necessary that he | /she has an appointme | ent. |
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| | | | | | |
| | c) | Only people who are not heal | lthy visit the hospital, | unless they are visiting | g a friend. |
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| | | | | | |
| | d) | If there are days when nobod anyone likes. | y calls a tour guide, the | en some tour guides d | on't visit places that |
| | | | | | |

4 continued. Use this symbolization scheme to symbolize the following sentences: $(36\% = 9 \times 4\%)$ A^1 : a has an appointment. B^1 : a is a place. D^1 : a is a doctor. E^1 : a is a day. F^1 : a is a person G^1 : a is a tour guide. H^1 : a is healthy. K^2 : a visits b. J^2 : a is a friend of b. L^2 : a likes b. C^3 : a calls b on c. M^2 : a is more popular than b. h⁰: the hospital a⁰: Adam b⁰: Dr. Bailey d^1 : the daughter of a.

e) Not all people who visit exactly those friends who visit them ever call people with whom they are not friends.

f) Exactly one doctor visits Adam, and she is neither Dr. Bailey nor Dr. Bailey's daughter.

g) Only Adam's daughter visits only those places that Adam likes.

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|---------------------------------------|--------------------------|------------------------|-------------------------|--|
| 4 continued. $(36 \% = 9 \times 4\%)$ | | | | |
| A^1 : a has an appointment. | B^1 : a is a place. | D^1 : a is a doctor. | E^1 : a is a day. | |
| F^1 : a is a person | G^1 : a is a tour guid | le. | H^1 : a is healthy. | |
| J^2 : a is a friend of b. | K^2 : a visits b. | L^2 : a likes b. | | |
| C^3 : a calls b on c. | M^2 : a is more popu | ılar than <i>b</i> . | | |

 h^0 : the hospital d^1 : the daughter of a.

h) Using the symbolization scheme above, provide an idiomatic English sentence that expresses:

b⁰: Dr. Bailey

$$\exists x (Bx \land \forall y (By \land x \neq y \rightarrow M(xy)) \land \forall z (Fz \land K(zx) \rightarrow L(zx)))$$

i) Using the symbolization scheme above, symbolize the following ambiguous sentence **three** logically distinct ways. For each, provide an English sentence that explains exactly what the symbolized sentence means:

Somebody calls a doctor every day.

a⁰: Adam

| 5. | Provide a derivation that shows the following theorem is valid using only the 10 basic rules fro | m SL |
|----|--|------|
| | (R, DN, MP, MT, ADJ, S, ADD, MTP, BC, CB) and the 3 basic rules from PL (UI, EG, EI) | (9%) |

$$\therefore \ (\forall y \sim (Dy \to Ay) \land \exists x \sim (Bx \lor Fx)) \ \to \ \exists x (Ax \leftrightarrow Bx)$$

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| 6. Pr (R | ovide a derivation that show, DN, MP, MT, ADJ, S, Al | ws that this is a valid argund DD, MTP, BC, CB) and t | ment using only the 10 basic rules from SL he 3 basic rules from PL (UI, EG, EI) (9%) |
| ∃x(~ | $Dx \wedge \forall y F(xy)$). | $\forall x (\sim Hx \to Dx).$ | $\exists x Hx \to \forall y \forall z (By \land F(zz) \to G(yz)).$ |
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| | $\exists x H(xa(x)) \rightarrow \forall x \exists y \sim (Fx \rightarrow Gy).$ | $\therefore \exists x \forall y H(a(x)a(y)) \rightarrow \exists z (Fz \land \sim Gz)$ |
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| 8. Show that t | he following | is a valid argument (use an | y rules). (9%): | |
| $\forall x \exists y \forall z A(x)$ | yz). | $\exists x \forall y (A(xyy) \rightarrow \forall z \sim$ | B(xz)). | $\forall z (\exists w K(wz) \to M(zz))$ |
| $\forall x \forall y (K(xy))$ | $\rightarrow \sim L(yx)$ | $\rightarrow \sim \exists y \sim B(yy).$ | ∴~` | $\forall x (M(xx) \rightarrow \sim \exists z L(xz))$ |
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9. Provide an English language interpretation (including the universe of discourse and a symbolization scheme) that shows that the following set of sentences is consistent. (4%)

$$\exists x (Fx \land \forall y G(xy)). \qquad \exists x \exists y (Bx \land \sim Fy \land G(xy)). \qquad \sim \exists x (Bx \land \forall y (Fy \rightarrow G(yx))).$$

10. Explain why the following sentence is a contradiction. (4 %)

$$(\exists y Fy \land \exists x Gx) \leftrightarrow \forall y (Fy \rightarrow \forall x \sim Gx)$$

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- 11. Use a finite model to demonstrate the invalidity of this argument (8 %):
 - i) provide a truth-functional expansion (to two individuals) for each sentence in this argument with respect to the model
 - ii) define a model with a universe of two individuals that shows that this argument is invalid.

$$\exists x (Ax \land \forall y G(xy)). \qquad \forall x \exists y (Bx \rightarrow \sim G(xy)). \qquad \therefore \sim \exists x (G(xx) \land Bx)$$

12. Consider the following derivation rule (which is *not* a rule in our derivation system):



Explain how this rule works.

What are the advantages (if any) and disadvantages (if any) of adding this rule to our system. Overall, do you think that it would be good to add this rule to our derivation system? Explain why or why not. (5%)

AID SHEET: DERIVATION RULES (SEE BOTH SIDES)

Derivation Types:

Direct Derivation (DD)

Conditional Derivation (CD)

Indirect Derivation (ID)

Universal Derivation (UD)

Restriction: the instantiating term cannot occur unbound in any previous line.

Basic Rules for Sentential Operators:

Modus Ponens (MP)

$$\frac{(\varphi \to \psi)}{\varphi}$$

Modus Tollens (MT)

$$\begin{array}{c} (\phi \to \psi) \\ \sim \psi \\ \hline \sim \phi \end{array}$$

Double Negation (DN)

Repetition (R)

Simplification (S)

Adjunction (ADJ)

Addition (ADD)

$$\frac{\phi}{\phi \vee \psi} \qquad \frac{\psi}{\phi \vee \psi}$$

Modus Tollendo Ponens (MTP)

$$\begin{array}{ccc} \varphi \vee \psi & & & \varphi \vee \psi \\ \sim \varphi & & & \sim \psi \\ \hline \psi & & \varphi \end{array}$$

Biconditional-Conditional (BC)

$$\begin{array}{ccc} \phi \leftrightarrow \psi & & \phi \leftrightarrow \psi \\ \hline \phi \rightarrow \psi & & \psi \rightarrow \phi \end{array}$$

Conditional-Biconditional (CB)

Derived Rules for Sentential Operators:

Negation of Conditional (NC)

Conditional as Disjunction (CDJ)

$$\frac{\sim (\phi \to \psi)}{\cdot}$$

$$\frac{\psi \wedge \sim \psi}{\sim (\phi \rightarrow \psi)}$$

$$\phi \rightarrow \psi$$

$$\sim \varphi \rightarrow \psi$$

Separation of Cases (SC)

$$\begin{array}{ccc} \phi \lor \psi \\ \phi \to \chi & \phi \to \chi \\ \psi \to \chi & \sim \phi \to \chi \\ \hline \chi & \chi \end{array}$$

Negation of Biconditional (NB)

$$\frac{\sim (\phi \leftrightarrow \psi)}{\phi \leftrightarrow \sim \psi} \qquad \frac{\phi \leftrightarrow \sim \psi}{\sim (\phi \leftrightarrow \psi)}$$

De Morgan's (DM)

Derivation Rules for Predicate Logic:

| Existential Generalization (EG) | Universal Instantiation (UI) | Existential Instantiation (EI) | Quantifier Negation | n (QN) |
|---|--|--|---------------------------------------|--|
| φς | $\forall \alpha \varphi_\alpha$ | $\exists \alpha \phi_{\alpha}$ | $\sim \forall \alpha \phi$ | ~∃α φ |
| $\overline{\exists \alpha \phi_{\alpha}}$ | φ _ζ | $\overline{\phi_{\zeta}}$ | ∃α ~φ | $\overline{\forall \alpha \sim \!\!\! \phi}$ |
| | Restriction: ζ does not | Restriction: ζ does not | ∃α ~φ | ∀α ~ φ |
| | occur as a bound variable in ϕ_{α} | occur in any previous line or premise. | $\overline{\sim} \forall \alpha \phi$ | $\overline{\sim \exists \alpha \phi}$ |