University of Toronto
FACULTY OF ARTS AND SCIENCE

FINAL EXAMINATIONS, DECEMBER 2009

APM 236H1F
Applications of Linear Programming

PLEASE HAND IN

Examiner: P. Kergin
Duration: 3 hours

FAMILY NAME ______________________________

GIVEN NAME(S) ______________________________

STUDENT NO ______________________________

SIGNATURE ______________________________

INSTRUCTIONS:

NO calculators or other aids allowed. There are 6 questions, each worth 20 marks. Questions 1, 2, 5, and 6 have part-questions, whose values are stated within the part-questions themselves. Total marks = 120.

This exam consists of 12 pages, printed on both sides of the paper. Write solutions in spaces provided. Page 12 is blank and may be used for the solution(s) of any of the problems, or for rough work. Aspects of any question which are indicated in **boldface** will be regarded as crucial during grading. **Show your work.**

GRADER'S REPORT	
1	
2	
3	
4	
5	
6	
TOTAL	

1. Let **P** denote the following linear programming problem.

Minimize $z = 5x_1 + 2x_2 - x_3$ subject to the constraints

$$\begin{array}{rcrcrcl} 4x_1 & + & 2x_2 & - & 3x_3 & \geq & 72 \\ & & x_2 & & & \leq & 30 \end{array}$$

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$$

1.(a) (3 marks) Put **P** into **canonical form**.

1.(b) (11 marks) Find **all basic solutions** (both feasible and infeasible) of the constraints of the canonical form of **P**.

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1.(c) (3 marks) List the extreme points of the feasible region of **P**. Note that **P** has three decision variables.

1.(d) (3 marks) **Solve** the problem **P**. You may assume that **P** <u>has</u> a solution.

2.(a) (10 marks) Find an **optimal solution** for the following linear programming problem.

Maximize $z = 6x_1 + x_2 + 5x_3$ subject to the constraints

$$\begin{array}{rcrcrcl} 3x_1 & + & x_2 & + & 2x_3 & \leq & 4 \\ x_1 & & & + & x_3 & \leq & 1 \\ & & x_2 & + & x_3 & \leq & 5 \end{array}$$

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$$

2.(b) (5 marks) Find a **second optimal solution** for the problem of question 2.(a).

2.(c) (5 marks) There are then infinitely many optimal solutions for the problem of question 2.(a). What are they?

3. Write **one** linear programming problem (in general form) which **simultaneously** satisfies **all** of the following:

(1) It has two decision variables.

(2) Its feasible region has $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 2 \end{bmatrix}$, and $\begin{bmatrix} -3 \\ 0 \end{bmatrix}$ as extreme points, and these are its only extreme points.

(3) It is unbounded (that is, "has no finite optimal solution").

4. **Solve** the following problem.

Minimize $z = 3x_1 + x_2 + x_3$ subject to the constraints

$$\begin{array}{rcrcrcr} 4x_1 & + & x_2 & + & x_3 & \geq & 5 \\ 5x_1 & + & 5x_2 & + & 2x_3 & \geq & 10 \end{array}$$

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$$

5. Consider the primal problem:

Maximize $z = x_1 - x_2$ subject to the constraints

$$\begin{array}{rcrcr} x_1 & + & 2x_2 & \geq & 1 \\ -x_1 & + & 2x_2 & = & 3 \\ 2x_1 & + & x_2 & \leq & 4 \\ -3x_1 & + & 2x_2 & \leq & 6 \end{array}$$

x_1 unrestricted, $x_2 \geq 0$

5.(a) (5 marks) **Solve** the primal problem **graphically**.

5.(b) (5 marks) State the dual of the primal problem.

5.(c) (10 marks) **Solve** the dual problem.

6. Consider the problem:

Maximize $z = 10x_1 + 15x_2 - 6x_3 + 5x_4$ subject to the constraints

$$\begin{array}{rcrcrcrcr} 2x_1 & + & 3x_2 & - & 3x_3 & + & 2x_4 & \leq & 3 \\ 2x_1 & + & 4x_2 & - & x_3 & + & x_4 & \leq & 5 \\ x_1 & + & 3x_2 & + & 2x_3 & - & x_4 & \leq & 4 \end{array}$$

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0$$

6.(a) (5 marks) The fourth tableau of the simplex solution of this problem has basic variables $\{x_1, x_6, x_3\}$ (in that order, where x_6 denotes the slack variable for the second constraint). **Use this information** to find the matrix B^{-1} which corresponds to the fourth tableau.

6.(b) (15 marks) **Beginning from the fourth tableau**, use the **revised simplex method** to **solve** the problem of question 6. Note: there is no need to write the entire fourth tableau.

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