Ordinary Least squares model

$$y_{ij} = \beta_{0j} + \beta_{1j} x_{ij} + \varepsilon_{i,j}$$

$i = 1 \cdots n_j$.

$j = 1 \cdots m$.

σ^2 /or

$$\varepsilon_{ij} \sim N(0, \underset{\sim}{\sigma_j^2})$$

model each school separately

Problem: some school has small sample size

$$y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$$

combine all data, $i = 1 \cdots n$. $n = \sum_{j=1}^{m} n_j$

Problem: school behaves differently

Hierarchical Model

$$y_{ij} = \beta_{0j} + \beta_{1j} x_{ij} + \varepsilon_{ij} \qquad \varepsilon_{ij} \sim N(0, \sigma^2)$$

$$\beta_j = (\beta_{0j}, \beta_{1j}) \sim MVN(\theta, \Sigma)$$

Priors and Posteriors shown in slides $\underbrace{\beta_j = \theta + \gamma_j}$

γ_j is random

$$y_{ij} = \theta x_{ij} + \gamma_j x_{ij} + \varepsilon_{ij}$$

Random effect

fixed effect