TUTORIAL 2

(1) For each of the following functions $f : \mathbb{R} \to \mathbb{R}$, determine whether f is injective and/or surjective. If f is bijective, find a formula for f^{-1}. If f is not bijective, determine nontrivial subsets $S, T \subset \mathbb{R}$ such that the restriction of f to S determines a bijection $S \to T$.
   (a) $f(x) = 3x + 2$.
   (b) $f(x) = \sin x$.
   (c) $f(x) = e^x$.
   (d) $f(x) = x^3 - 1$.
   (e) $f(x) = x^3 - x + 1$.

(2) Determine which of the following statements are true. Give proofs of the true statements and counterexamples for the false statements.
   (a) Every increasing function from $\mathbb{R}$ to $\mathbb{R}$ is surjective.
   (b) Every increasing function from $\mathbb{R}$ to $\mathbb{R}$ is injective.
   (c) Every injective function from $\mathbb{R}$ to $\mathbb{R}$ is either increasing or decreasing.
   (d) Every surjective function from $\mathbb{R}$ to $\mathbb{R}$ is unbounded.
   (e) Every unbounded function from $\mathbb{R}$ to $\mathbb{R}$ is surjective.

(3) Let A be a finite set with n elements, and let $f : A \to A$ be a function.
   (a) Prove that f is injective if and only if f is surjective. Is this equivalence true if A is infinite?
   (b) How many different functions $f : A \to A$ are there?
   (c) How many different bijections $f : A \to A$ are there?

Just for fun.

(1) For which n and m, is it possible to L-tile an $n \times m$ rectangle?