STAT 6046 Tutorial Week 9

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Today's plan

- Brief review of course material
- Go through selective tutorial questions

Fixed interests securities

- Bill: short term; less than one year; The yield on government bills is typically quoted as a simple annual rate of discount for the term of the bill.
- Bond: long term; longer than one year;
  - zero-coupon bonds/ coupon paying bonds.
  - Bonds issued by financially and politically stable governments are virtually risk-free and are a safe investment option.
  - Corporate bonds more risky/less liquid. Requires higher yield.

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Bond Yields

- The annual "**redemption yield**“: the internal rate of return or the effective annual rate of interest.
- The "**nominal yield**" is the annual redemption yield expressed as a nominal rate of interest. Normally convertible half-yearly.
- The "**running yield**" (or flat yield) is the ratio of the coupon rate per annum to the original price of the bond per unit nominal.

Coupon paying bonds

- Pricing: $P = Fr \cdot a_{\overline{n}|j} + C \cdot v_j^n$
- j is the effective half-yearly yield.
- If given nominal yield (half yearly)

$$j = \frac{i^{(2)}}{2}$$

- If given annual redemption yield

$$j = (1+i)^{1/2} - 1$$

- Alternative method: $P = 2Fr \cdot a^{(2)}_{\overline{n/2}|i} + C \cdot v_i^{n/2}$
- n here is the number of coupons, not years!

Extensions: bond price between coupon dates

- If P_0 is the value of the bond just after the last coupon, then the value of the bond at time t is:

$$P_t = P_0(1 + j)^t$$

Extensions: Income tax

$$P = Fr(1-t_I)\cdot a_{\overline{n|}j} + C\cdot v_j^n$$

- This formula assumes that tax is payable at the same time as income is incurred.

Extensions: capital gain tax

- Let new price be *P'*

If $P \geq C$ then there is no capital gain, so the price is $P' = P = Fr(1-t_I) \cdot a_{\overline{n|}j} + C \cdot v_j^n$

If $P < C$ then there is a capital gain of $(C - P')$ taxed at t_C and the price is:

$$P' = Fr(1-t_I) \cdot a_{\overline{n|}j} + C \cdot v_j^n - t_C(C - P')v_j^n = P - t_C(C - P')v_j^n$$

Bond price

- If $C = F$

$P = F \Leftrightarrow$ the effective half-yearly yield equals the coupon rate per half-year $j = r$
$P > F \Leftrightarrow j < r$
$P < F \Leftrightarrow j > r$

- With Income Tax

$$P = F \Leftrightarrow j = r(1 - t_I)$$

$$P > F \Leftrightarrow j < r(1 - t_I)$$

$$P < F \Leftrightarrow j > r(1 - t_I)$$

If $P = F$, the bond is said to be bought at par.
If $P > F$, the bond is said to be bought at a premium.
If $P < F$, the bond is said to be bought at a discount.

Bond price

- If $C \neq F$, define a modified coupon rate g:

$$g = \frac{Fr}{C}$$

$$P = C \iff j = g$$
$$P > C \iff j < g$$
$$P < C \iff j > g$$

Yields

- Coupon increases → yield increases
- Term
- When the purchase price is more than the redemption price ($P > C$) :
  - as *n* increases the yield increases. Loss spread over a longer period.
  - as *n* decreases the yield decreases.
- If $P < C$, vice versa.
- Comparing two bonds:
  - $P > C$, longer maturity
  - $P < C$, shorter maturity

Yields

- IRR on a security: redemption yield
- Before tax: gross yield
- After tax: net yield
- To find net yield:

$$P' = Fr(1-t_I)\cdot a_{\overline{n}|j} + C\cdot v_j^n - t_C(C-P')v_j^n$$

$$= Fr(1-t_I)\cdot a_{\overline{n}|j} + \left[C - t_C(C-P')\right]v_j^n$$

Callable bonds

- The discussion that follows assumes that a bond is redeemable at the option of the borrower.
- When a bond is to be redeemed at the option of the issuer:
- For a bond bought at a discount, if an investor requires a particular minimum yield, the price paid should be based on the ***latest*** optional redemption date.
- For a bond bought at a premium, if an investor requires a particular minimum yield, the price paid should be based on the ***earliest*** optional redemption date.
- In both cases, the minimum price is paid by the investor.

Inflation linked bonds

$$P = \sum^{n} Fr(1+y)^{p/2} v_j^p + C(1+y)^{n/2} v_j^n,$$

$$P = \sum_{p=1}^{n} Fr(1+y)^{p/2} v_i^{p/2} + C(1+y)^{n/2} v_i^{n/2} = \sum_{p=1}^{n} Fr \cdot v_{i'}^{p/2} + C \cdot v_{i'}^{n/2}$$

$$v_{i'} = (1+y)v_i \Rightarrow i' = \frac{i-y}{1+y}$$

$$j' = (1+i')^{1/2} - 1$$

$$\boxed{P = Fr \cdot a_{\overline{n}|j'} + C \cdot v_{j'}^n}$$

$$\boxed{P = 2Fr \cdot a_{\overline{n/2}|i'}^{(2)} + C \cdot v_{i'}^{n/2}}$$