Tutorial 2 Solutions

STAT 3013/8027

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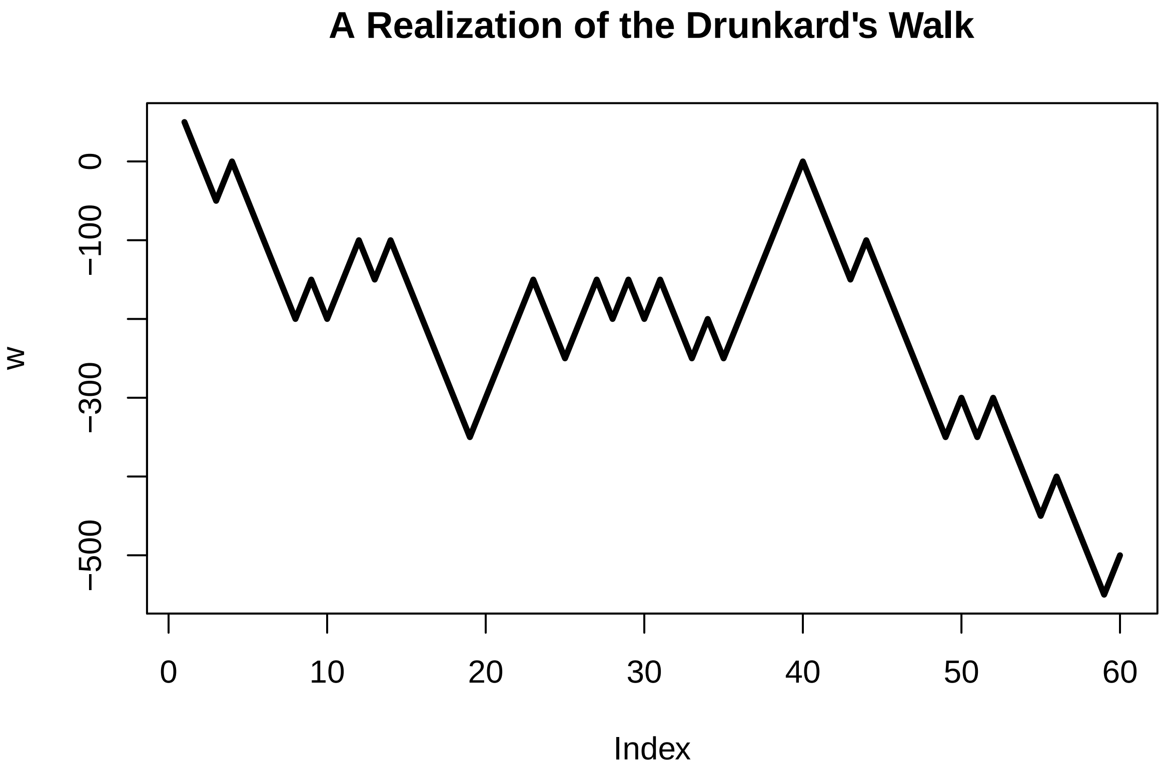
1. Rice: 5.1, 5.13, 5.16 **Ans.** See the handwritten pages.

- **5.13 R code:**

```
##
set.seed(10)
x <- rbinom(60, 1, 0.5)
x[x==0] <- -1
x <- x*50

w <- cumsum(x)

##
plot(w, type="l", lwd=3, main="A Realization of the Drunkard's Walk")
```




```
## Let's examine the sampling distirbution of W
set.seed(10)
S <- 10000
W <- rep(0, S)

for(s in 1:S){
```

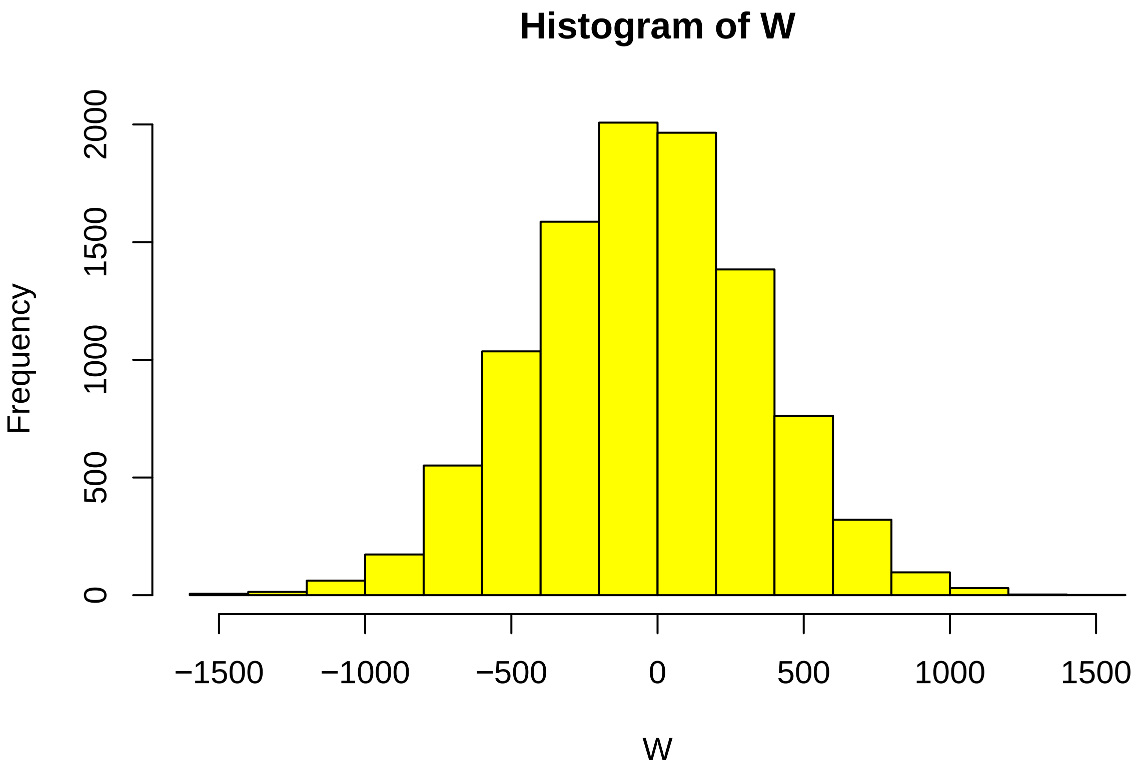
```
  x <- rbinom(60, 1, 0.5)
  x[x==0] <- -1
  x <- x*50
  W[s] <- sum(x)
}

##
mean(W)
```

```
## [1] 1.13
```

```
var(W)
```

```
## [1] 150870.8
```

```
##
hist(W, col="yellow")
```



- **5.19 (a)**

Let's first work out the exact answer.

$$\begin{aligned}\int_0^1 cos(2\pi x)dx &= \frac{1}{2\pi} sin(2\pi x)\Big|_0^1 \\ &= 0\end{aligned}$$

- Note. Suppose we consider $U \sim$ Uniform(0,1). Let's look at the $E[U]$.

$$E[U] = \int_0^1 uf(u)du = \int_0^1 u \; du$$

- Now let's look at the expected value of the function: $cos(2\pi u)$:

$$E[g(U)] = \int_0^1 cos(2\pi u) \; du$$

So our integral of interest is the expected value of the function. We can approximate that via simulation.

$$\hat{I}(g(u)) = \frac{1}{S}\sum_1^S cos(2\pi u)$$

```
set.seed(1001)
S1 <- 100

u <- runif(S1)
g.u <- cos(2*pi*u)
mean(g.u)
```

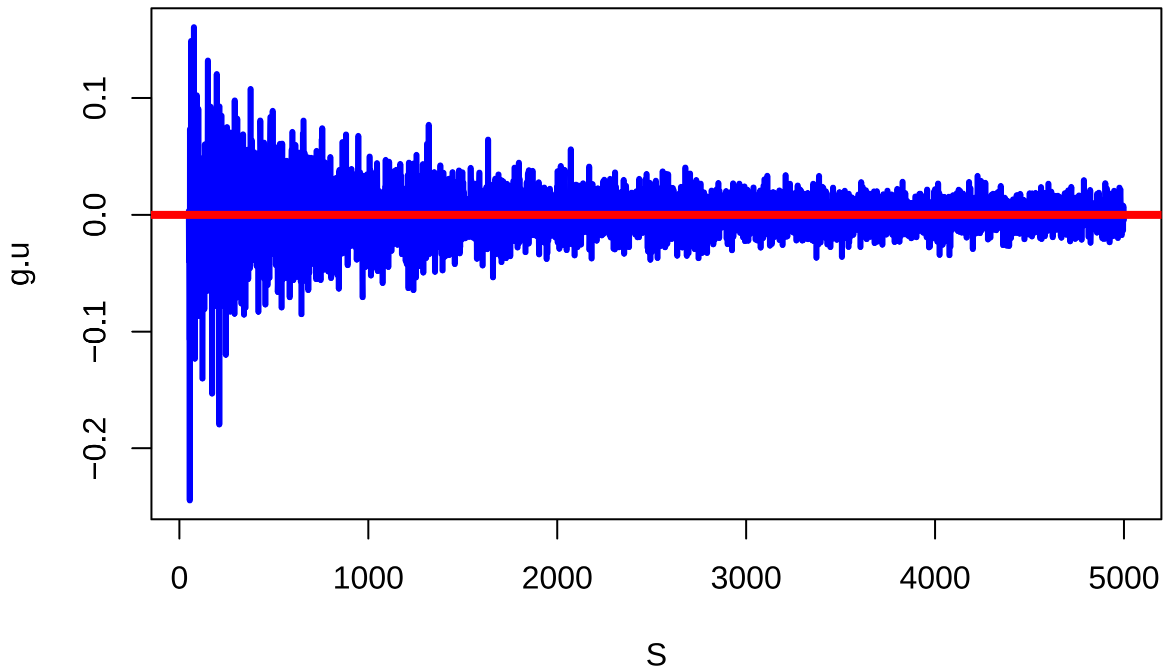
```
## [1] 0.05382692
```

```
set.seed(1001)
S2 <- 1000

u <- runif(S2)
g.u <- cos(2*pi*u)
mean(g.u)
```

```
## [1] 0.006431414
```

- Now let's plot for increasing values of S:

```
S <- 50:5000
g.u <- rep(0, length(S))

c <- 1
for(s in S){
g.u[c] <- mean(cos(2*pi* runif(s)))
c <- c+1
}

plot(S, g.u, type="l", lwd=3, col="blue")
abline(h=0, col="red", lwd=4)
```



- As we can work out the $E[g(u)]$ and the $V[g(u)]$ we could also use the CLT theorem for calculations based on the $\frac{1}{S}\sum_1^S cos(2\pi u)$.
- **5.19 (b) R code**
- For this question we can't work out an analytical solution (some type of approximation must be performed).

$$
\begin{aligned}
E[I(g(u))] &= \int_0^1 cos(2\pi u^2)du \\
&\approx \frac{1}{S}\sum_1^S cos(2\pi u^2)
\end{aligned}
$$

```
set.seed(1001)
S1 <- 100

u <- runif(S1)
g.u <- cos(2*pi*u^2)
mean(g.u)
```

```
## [1] 0.3031969
```

```
set.seed(1001)
S2 <- 1000

u <- runif(S2)
g.u <- cos(2*pi*u^2)
mean(g.u)
```

```
## [1] 0.2592541
```

```r
S <- 50:5000
g.u <- rep(0, length(S))

c <- 1
for(s in S){
g.u[c] <- mean(cos(2*pi* runif(s)^2))
c <- c+1
}

plot(S, g.u, type="l", lwd=3, col="blue")
```