STA305/1004 Winter 2016 - Homework 1

Due: Electronic submission on UofT Learning Portal course page by Tuesday, January 26, 2016 at 22:00. NB: e-mail submissions will NOT be accepted.

STA305 students are not required to complete the bonus question. If the bonus question is completed then it will be worth extra marks.

If you work with other students on this assignment then:

- indicate the names of the students on your solutions;
- your solutions must be written up independently (i.e., your solutions should not be the same as another students solutions).

1. A medical scientist has four light objects to weigh. She decides to use an old fashioned pan balance scale in the lab since she heard about a design from a colleague that is supposed to increase the accuracy of her measurements and take less time.

   The scientist decides to obtain weight measurements using the following design:

   - weigh all four objects together;
   - weigh objects one and two in one pan, and objects three and four in the other pan;
   - weigh objects one and three in one pan, and objects two and four in the other pan;
   - weigh objects one and four in one pan, and objects two and three in the other pan;

   In a pan balance scale when one object is in one pan and another object is in another pan the measurement obtained is the difference in weight between the two objects.

   Let y_1, y_2, y_3, y_4 be the readings from the scale for determining the unknown weights, $\beta_1, \beta_2, \beta_3, \beta_4$. The standard deviation of each weighing is denoted by σ.

   Answer the following questions.

(a) Write four equations relating the observed weights y_1, y_2, y_3, y_4 to the unknown weights $\beta_1, \beta_2, \beta_3, \beta_4$.

(b) Find the least-squares estimates of $\beta_1, \beta_2, \beta_3, \beta_4$.

(c) Find the standard error of the least-squares estimates of $\beta_1, \beta_2, \beta_3, \beta_4$.

(d) If the scientist measured each object twice would she achieve the same precision as this design? Explain.

(e) (**STA1004 Additional Question/ STA305 Bonus**) Suppose that a highly accurate digital scale is used instead of the pan balance scale to determine the objects' weights. The standard deviation of each weighing is $\sigma/2$, where σ is the standard deviation of a weighing from the pan balance scale. The digital scale cannot provide a measurement of the difference between two objects in a single measurement so the design is modified to:

- weigh all four objects together;
- weigh objects one and two together;
- weigh objects one and three together;
- weigh objects one and four together.

Question: Find the least-squares estimates of the weights and standard error of the weights using this design. Does this design using a digital scale yield estimates of the weights with higher precision compared to the design above using the pan balance?

2. (*Adapted from Box, Hunter, and Hunter*) Fifteen judges were randomly allocated to judge one of two brands of beer, A or B, for taste. Eight judges will be assigned to Brand A and seven judges to brand B. The judges ranked the beer they tasted using a 10-point (Likert) scale with 1 representing 'poor taste' and 10 representing 'outstanding taste'.

The table below shows the rating from each judge. The number in brackets beside the rating indicates which judge gave the rating. For example, judge 1 gave a rating of 2 to brand A, and judge 9 gave a rating of 3 to brand B.

Brand A	2 (1)	4 (2)	2 (3)	1 (4)	9 (5)	9 (6)	2 (7)	2 (10)
Brand B	8 (8)	3 (9)	5 (11)	3 (12)	7 (13)	7 (14)	4 (15)	

Answer the following questions using R.

(a) How many values does the randomization distribution of the average difference in rating between the two brands contain?

(b) Create a histogram of this randomization distribution. What is the P-value of your test? **(Hand in your R code and output for this part)**

(c) Is there evidence of a significant difference in taste between brands A and B? Explain your answer including how you define 'significant'. **(Hand in your R code and output for this part)**

3. Suppose that two drugs A and B are to be tested on five healthy subjects' eyes. The drugs will be randomly assigned to the left eye or right eye based on the flip of a fair coin. If the coin toss is heads then a subject will receive drug A in their right eye. The table below shows measurements of the logarithm of intraoccular pressure (mm Hg) after the drug was administered.

Subject	Drug A	Drug B	Toss
1	7	9	T
2	3	5	H
3	8	12	H
4	11	4	H
5	4	6	H

   (a) Create a table that shows which eye received drugs A and B in each subject.

   (b) What type of design was used in this study? Explain.

   (c) What is the probability that a subject's right eye will receive drug A? Explain.

   (d) The investigator in this study is worried that the treatment allocation is biased since he obtained four heads and only one tail. Is this treatment allocation less likely than a treatment allocation with three tails and two heads or an allocation with three heads and two tails? Explain by evaluating the probability of obtaining a treatment allocation in this type of design.

   (e) Describe the randomization distribution for this comparison. How many values does this distribution contain?

   (f) Use the randomization test to determine if there is evidence of a difference in intraoccular pressure between the two drugs. **(Hand in your R code and output for this part)**

4. Use the data in the previous question to conduct an appropriate t-test. **(Hand in your R code and output for this part)**

   (a) Are the assumptions behind the t-test satisfied?

   (b) Do the results of the t-test agree with the results of the randomization test? Explain.