Exercise

The future lifetime of an individual aged x, can be described by the random variable T, with cumulative distribution function

$$F_T(t) = 1 - e^{-0.02t} \qquad t > 0$$

(a) Determine the probability density function $f_T(t)$.

$f_T(t) = 0.02e^{-0.02t}$

(b) Calculate the force of mortality.

$\mu(t) = \frac{f(t)}{S(t)} = \frac{f(t)}{1-F(t)} = \frac{0.02e^{-0.02t}}{e^{-0.02t}} = 0.02$

(c) Calculate the probability that a life aged x lives for more than 10 years.

$P(T>10) = S(t=10) = e^{-0.02\times10} = e^{-0.2}$

(d) If a life currently aged x survives a further ten years, calculate the probability that the life will die in the subsequent 15 years.

$P(10<T<25 \mid T>10) = \frac{P(10<T<25)}{P(T>10)} = \frac{S(10)-S(25)}{S(10)} = \frac{e^{-0.2}-e^{-0.5}}{e^{-0.2}}$

$= 0.2592$

(e) Calculate the expected future lifetime for a life aged x.

$E(T) = \int_0^\infty t \cdot f(t)\, dt = \int_0^\infty 0.02\, t\, e^{-0.02t}\, dt$

$u = t$ $\quad du = 1$

$dv = e^{-0.02t}$ $\quad v = -e^{-0.02t}$

$= [-te^{-0.02t}]_0^\infty + \int_0^\infty e^{-0.02t}\, dt$ $\quad \int u\,dv = uv - \int v\,du$

$\lim_{t\to\infty} \frac{t}{e^{0.02t}} \to 0$

$= 0 + [-\frac{1}{0.02} e^{-0.02t}]_0^\infty = 0 + 50 = 50$

(f) Find the number of years lived by a life aged x who outlives exactly 10% of the population of identical lives aged x.

$P(T<t) = 0.1$

t = # of years lived by this person.

e.g. 1, 2, 3, ..., 10, 11, ..., 100

$t=11$, b/c 10 persons lived less than this individual.

$1-e^{-0.02t} = 0.1$

$0.9 = e^{-0.02t}$ $\qquad S(t) = 0.9$

$t = -50 \cdot \ln 0.9$

$= 5.27$