Lecture 29

Today - § 3.4 (dual simplex method)

Next week - § 3.5

3 December - review (Chapter 3)

§3.4 - The Dual Simplex Method

In one phase, the dual simplex method can only solve problems beginning with "Minimize $z = c_1 x_1 + \dots + c_n x_n \dots$" where $c_1 \geq 0$, $c_2 \geq 0$, ..., $c_n \geq 0$ (as in a cost-minimization problem) or "Maximize $z = c_1 x_1 + \dots + c_n x_n \dots$" where $c_1 \leq 0, c_2 \leq 0, \dots, c_n \leq 0$

Ex We will solve the problem.

Minimize $z = 19w_1 + 7w_2 + 2w_3$ s.t

$w_1 + w_2 - w_3 \geq 3$

$5w_1 - w_2 + 2w_3 \geq 7$

$w_1 \geq 0,\ w_2 \geq 0,\ w_3 \geq 0$

if these were < 0 the dual simplex method would still work

One style of executing the dual simplex method is to solve the dual problem, using the primal simplex method (§2.1), then read the optimal dual variables from the objective row if the optimal primal tableau

The §3.4 style:

Put the problem in primal standard form, then write a §2.1-style primal simplex tableau, Tableau ①. This will satisfy the optimality criterion except it will not represent a feasible solution. (Until the problem is solved)

The strategy: exit one of the most negative variable. Then choose the entering variable to preserve the optimality criterion.

From Tableau ①, we will exit w_5

Then form the Θ-ratios:

$$\frac{\text{objective row coefficients (not the objective value)}}{\text{coefficients of the exiting variable}}$$

w_1	w_2	w_3	w_4	w_5
$-\frac{19}{5}$	$\cancel{\frac{7}{1}}$	$\frac{2}{-2}$	$\cancel{\frac{0}{0}}$	$\cancel{\frac{0}{1}}$

(Delete all ratios whose denominator is not negative).

The entering variable is any variable which has the greatest (least negative) Θ-ratio. Here w_3 enters

A routine pivot (as in §0.2) produces Tableau ②

From tableau ②, w_4 will exit, w_4-row Θ-ratios are

w_1	w_2	w_3	w_4	w_5
$\frac{14}{-7/2}$	$\frac{8}{-1/2}$	$\cancel{\frac{0}{0}}$	$\cancel{\frac{0}{1}}$	$\cancel{\frac{*}{-1/2}}$
-4	-16			

(w_1 enters)

Another row-pivot gets to Tableau ③, from which w_3 exits, w_3-row Θ-ratios:

w_1	w_2	w_3	w_4	w_5
$\cancel{\frac{0}{0}}$	$\frac{6}{-6/7}$	$\cancel{\frac{0}{1}}$	$\cancel{\frac{4}{5/7}}$	$\frac{3}{-1/7}$
	-7 (w_2 enters)			-21

$\frac{4}{5/7}$: not taken because positive

Another routine row pivot leads to Tableau ④, which is optimal and feasible

About "An infeasible Problem"
Routine until Tableau ③, from which w_3 would exit.
The w_3-row Θ-ratios all have ≥ 0 denominators and the w_3-row represents the equation $w_3 + \frac{2}{3} w_5 = -\frac{11}{3}$ which has no solution with $w_3 \geq 0$, $w_6 \geq 0$