1. Write a 2^3 design in blocks of size 2 such that no main effect is confounded with block differences.

   Use block generator $B_1=12$, $B_2=13$.

Run	1	2	3	12	13
1	-1	-1	-1	+1	+1
2	+1	-1	-1	-1	-1
3	-1	+1	-1	-1	+1
4	+1	+1	-1	+1	-1
5	-1	-1	+1	+1	-1
6	+1	-1	+1	-1	+1
7	-1	+1	+1	-1	-1
8	+1	+1	+1	+1	+1

The blocks will be formed by using the signs of $B_1=12$, $B_2=13$.

Block I: $B_1=+$, $B_2=+$ (runs 1, 8)
Block II: $B_1=+$, $B_2=-$ (runs 4, 5)
Block III: $B_1=-$, $B_2=+$ (runs 3, 6)
Block IV: $B_1=-$, $B_2=-$ (runs 2, 7)

The block effects are obtained by multiplying the B_i: $B_1B_2=1213=23$. So the block effects are confounded with two-factor interactions and no main effects.

2. Write a 2^4 design in four blocks such that the main effects are not confounded with block differences.

   Use block generator $B_1=124$, $B_2=134$.

Run	1	2	3	4	124	134
1	-1	-1	-1	-1	-1	-1
2	+1	-1	-1	-1	+1	+1
3	-1	+1	-1	-1	+1	-1
4	+1	+1	-1	-1	-1	+1
5	-1	-1	+1	-1	-1	+1
6	+1	-1	+1	-1	+1	-1
7	-1	+1	+1	-1	+1	+1
8	+1	+1	+1	-1	-1	-1
9	-1	-1	-1	+1	+1	+1
10	+1	-1	-1	+1	-1	-1
11	-1	+1	-1	+1	-1	+1
12	+1	+1	-1	+1	+1	-1
13	-1	-1	+1	+1	+1	-1
14	+1	-1	+1	+1	-1	+1
15	-1	+1	+1	+1	-1	-1
16	+1	+1	+1	+1	+1	+1

   The four blocks are defined using the signs of $B_1=124$, $B_2=134$.

   Block I $B_1=+$, $B_2=+$ (runs 2, 7, 9, 16)
   Block II $B_1=+$, $B_2=-$ (runs 3, 6, 12, 13)
   Block III $B_1=-$, $B_2=+$ (runs 4, 5, 11, 14)
   Block IV $B_1=-$, $B_2=-$ (runs 1, 8, 10, 15)

   The block effects are $B_1B_2=124134=23$. The blocks are not confounded with the main effects.

3. In a research study two different treatments were applied to the two eyes of patients by flipping a fair coin. But the treatment applied to one eye had an influence on the response in the other eye. Is this an appropriate experimental design to study compare the effectiveness of the treatments?

   A matched pairs design assumes that treatment applied to one eye has a negligible effect on the response in the other eye. If this is not the case then a matched pairs design should not be used. Instead a design that randomized subjects to one of the two treatments is appropriate since this would lead to responses that were independent.

4. Describe the design in the following scenarios:

    a) Suppose that four medical treatments (A, B, C, D) were to be compared at four clinical research sites. But each clinical research site could only collect data on three treatments. The consulting statistician suggested using hospital as a blocking variable. The design is shown in the following table:

Hospital	Treatments		
1	A	B	C
2	A	B	D
3	A	C	D
4	B	C	D

    Randomized incomplete block design.

    b) Fifteen judges rated two randomly allocated brands of beer, A and B, according to taste (scale: 1 to 10).

    Completely randomized design.

    c) Twenty judges each rated two brands of beer, A and B, according to taste (scale: 1 to 10).

    Randomized paired design.

    d) You have 16 benches in a greenhouse, that differ in light and temperature such that each bench has different conditions associated with 4 distinct light and 4 temperature levels: 25%, 50%, 75% and 100% of full sun; 15, 20, 25, 30 C. 4 fertilizer treatments are applied to the benches (75, 150, 225 and 300 mM nitrogen), randomized in such a way that every fertilizer treatment is tested under all 4 light levels, and all 4 temperature levels.

    Latin square. Sun and fertilizer will be the 4 rows and columns of the Latin Square. The only way to make the randomization true is for every treatment to appear once in every "row" and "column.

5. The following experimental results were obtained from a welding study.

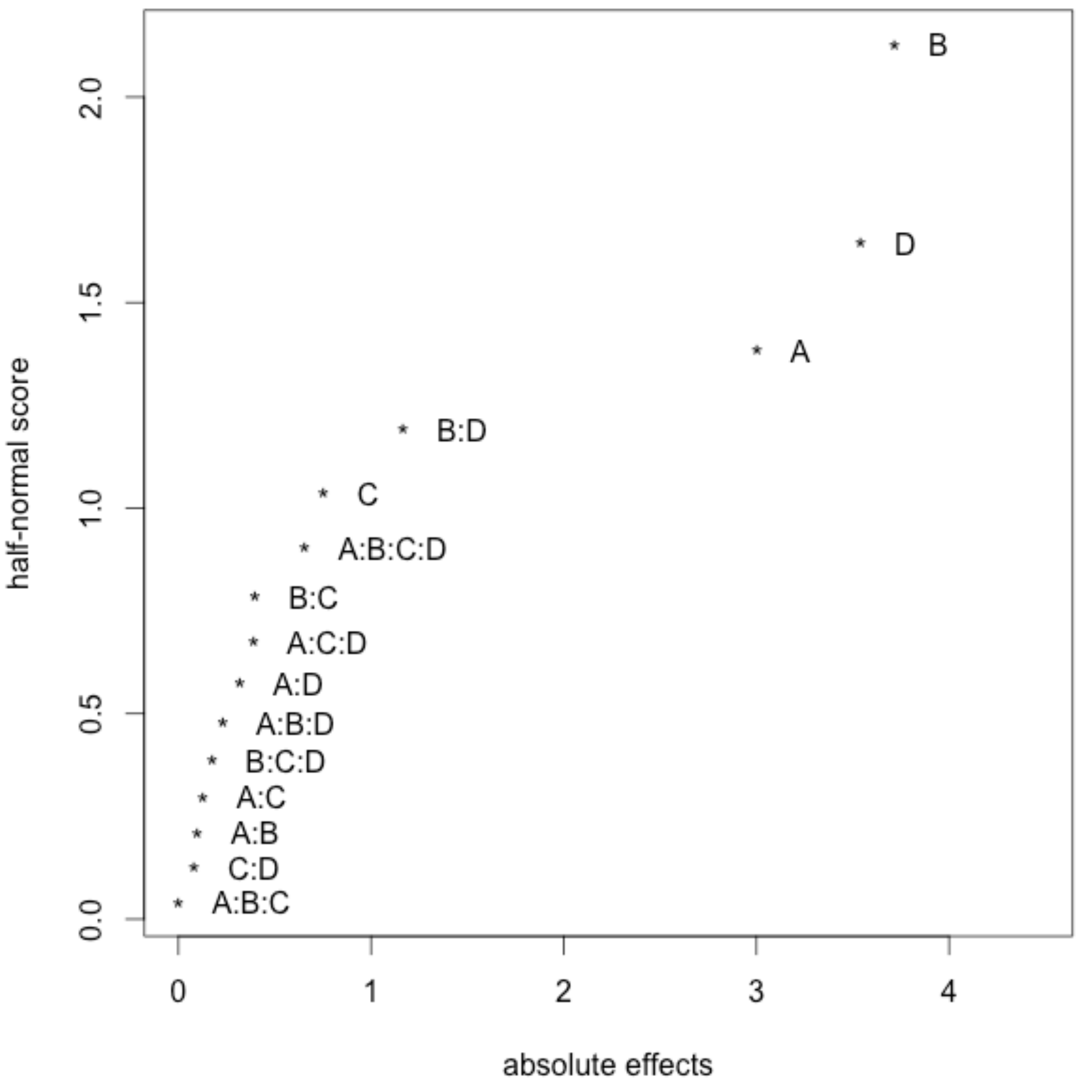
A: open circuit voltage
B: slope
C: electrode melt-off rate (ipm)
D: electrode diameter (in)
E: electrode extension (in)

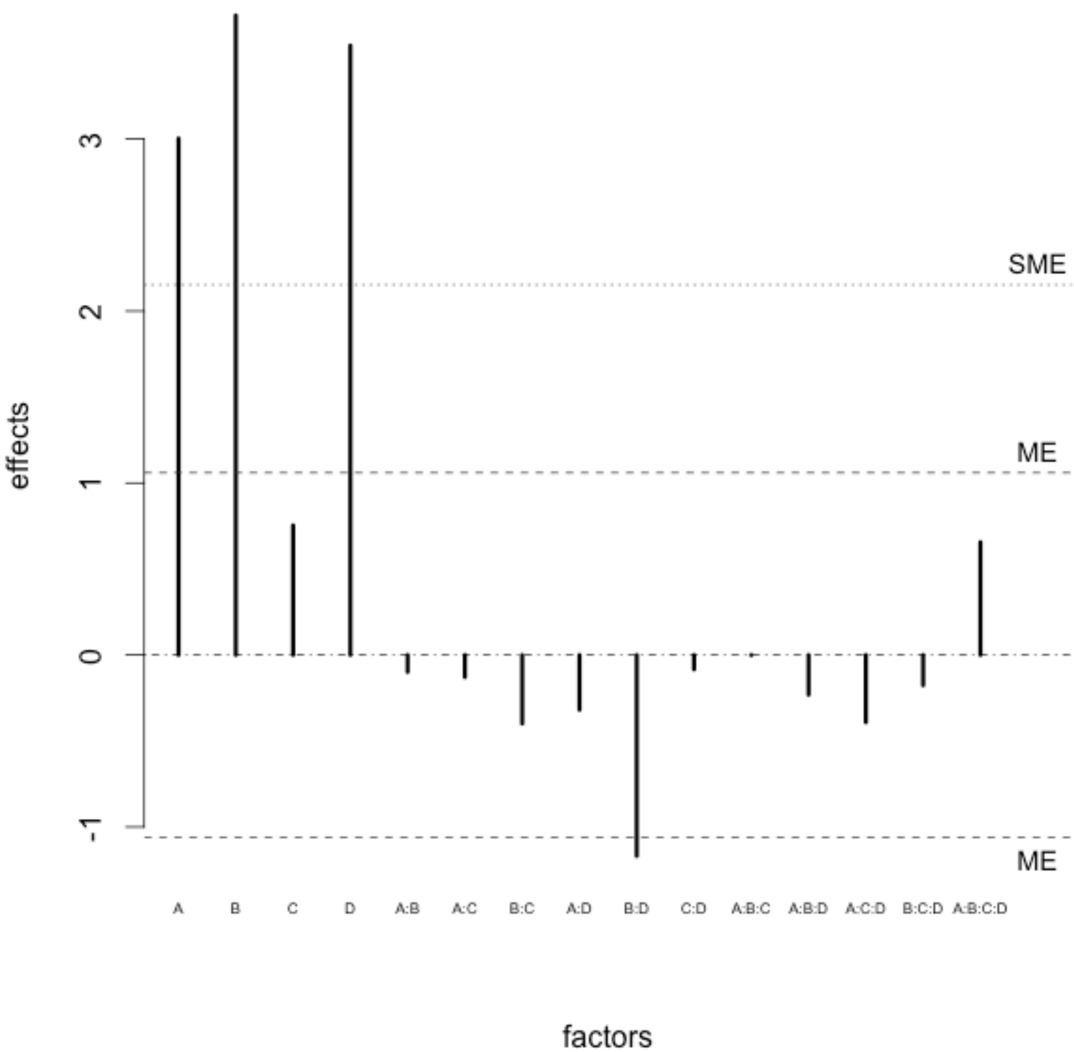
	runs	A	B	C	D	E	y1
1	12	-1	-1	-1	-1	1	23.43
2	1	1	-1	-1	-1	-1	25.70
3	2	-1	1	-1	-1	-1	27.75
4	6	1	1	-1	-1	1	31.60
5	15	-1	-1	1	-1	-1	23.57
6	8	1	-1	1	-1	1	27.68
7	7	-1	1	1	-1	1	28.76
8	4	1	1	1	-1	-1	31.82
9	11	-1	-1	-1	1	-1	27.09
10	14	1	-1	-1	1	1	31.28
11	3	-1	1	-1	1	1	31.20
12	16	1	1	-1	1	-1	33.42
13	13	-1	-1	1	1	1	29.51
14	10	1	-1	1	1	-1	31.35
15	5	-1	1	1	1	-1	31.16
16	9	1	1	1	1	1	33.65

The data were analysed in R.

```
> fact1 <- lm(y1~A*B*C*D,data=prb0609)
> summary(fact1)

Call:
lm.default(formula = y1 ~ A * B * C * D, data = prb0609)

Residuals:
ALL 16 residuals are 0: no residual degrees of freedom!

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept) 29.310625         NA      NA       NA
A            1.501875         NA      NA       NA
B            1.859375         NA      NA       NA
C            0.376875         NA      NA       NA
D            1.771875         NA      NA       NA
A:B         -0.049375         NA      NA       NA
A:C         -0.064375         NA      NA       NA
B:C         -0.199375         NA      NA       NA
```

```
A:D         -0.159375         NA      NA       NA
B:D         -0.584375         NA      NA       NA
C:D         -0.041875         NA      NA       NA
A:B:C       -0.000625         NA      NA       NA
A:B:D       -0.115625         NA      NA       NA
A:C:D       -0.195625         NA      NA       NA
B:C:D       -0.088125         NA      NA       NA
A:B:C:D      0.328125         NA      NA       NA

Residual standard error: NaN on 0 degrees of freedom
Multiple R-squared:      1,   Adjusted R-squared:    NaN
F-statistic:   NaN on 15 and 0 DF,  p-value: NA

> fact2 <- aov(y1~A*B*C*D,data=prb0609)
> DanielPlot(fact2,half=T)
> LenthPlot(fact2,cex.fac=0.5)
   alpha      PSE       ME      SME
0.050000 0.412500 1.060365 2.152694
```


B
D
A
B:D
C
A:B:C:D
B:C
A:C:D
A:D
A:B:D
B:C:D
A:C
A:B
C:D
A:B:C
half-normal score
absolute effects
0.0
0.5
1.0
1.5
2.0
0
1
2
3
4



effects
3
2
1
0
-1
SME
ME
ME
A
B
C
D
A:B
A:C
B:C
A:D
B:D
C:D
A:B:C
A:B:D
A:C:D
B:C:D
A:B:C:D
factors

a) What type of experimental design was implemented? Explain

Fractional factorial design 2^{5-1}. A 16 (=2^4) run design that studied 5 factors.

b) Is there any aliasing present in the design? If aliasing is present then specify the defining relation and all the aliasing relations.

Aliasing is present. The factor E is aliased with ABCD. The defining relation is I=ABCDE.

There are 15 aliasing relations:

E=ABCD, B=ACDE, C=ABDE, D=ABCE, A=BCDE, AB=CDE, AC=BDE, AD=BCE, AE=BCD, BC=ADE, BD=ACE, BE=ACD, CD=ABE, CE=ABD, DE=ABC.

c) Is this design replicated? If it is replicated then estimate the standard error of the effects.

The design is not replicated.

d) Which factors are significant according to Lenth's method? Are the same factors statistically significant at the 5% level if you use the half-normal plot instead of Lenth's method? Explain.

A, B, D, and B:D are all significant at the 5% level according to Lenth's method using ME.

The half-normal plot gives similar results to Lenth's method except the interaction B:D only shows a slight deviation from the other factors and might not be judged to be significant if only the half-normal plot was used.

e) What is estimated effect of the interaction between B and D?

Estimated effect of B:D is 2-0.584375 = -1.16875*

6. The following experiment investigated two factors, pulp preparation method and temperature, on the tensile strength of paper. Temperature was to be set at four levels, and there were three preparation methods. It was desired to run three replicates, but only 12 runs could be made per day. One replicate was run on each of the three days.

   On each day, three batches of pulp were prepared by the three different methods. Each of the three batches was subdivided into four equal parts, and processed at a different temperature. The data are given in the table below.

		Day 1			Day 2			Day 3		
Prep. method		1	2	3	1	2	3	1	2	3
	1	30	34	29	28	31	31	31	35	32
Temp	2	35	41	26	32	36	30	37	40	34
	3	37	38	33	40	42	32	41	39	39
	4	36	42	36	41	40	40	40	44	45

The data were analysed in R:

```
> temp <- factor(c(rep(1,9),rep(2,9),rep(3,9),rep(4,9)))
> prep <- factor(c(rep(c(1,2,3),12)))
> day <- factor(rep(c(rep(1,3),rep(2,3),rep(3,3)),4))
> strength <-
c(30,34,29,28,31,31,31,35,32,35,41,26,32,36,30,37,40,34,37,38,33,40,42,
32,41,39,39,36,42,36,41,40,40,40,44,45)
>
> tensdata <- data.frame(day,prep,temp,strength)
>
> model1 <- aov(strength~temp*prep,data=tensdata)
> summary(model1)
            Df Sum Sq Mean Sq F value   Pr(>F)    
temp         3  434.1  144.69  18.737 1.76e-06 ***
prep         2  128.4   64.19   8.313  0.00181 ** 
temp:prep    6   75.2   12.53   1.622  0.18426    
Residuals   24  185.3    7.72                     
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
>
> model2 <- aov(strength~ temp*prep*day,data=tensdata)
> summary(model2)
              Df Sum Sq Mean Sq
temp           3  434.1  144.69
prep           2  128.4   64.19
day            2   77.6   38.78
temp:prep      6   75.2   12.53
temp:day       6   20.7    3.44
prep:day       4   36.3    9.07
temp:prep:day 12   50.8    4.24
>
> model3 <- aov(strength~
prep+day+prep:day+temp+prep:temp+Error(day/prep),data=tensdata)
```

```
> summary(model3)

Error: day
    Df Sum Sq Mean Sq
day  2  77.56   38.78

Error: day:prep
         Df Sum Sq Mean Sq
prep      2 128.39   64.19
prep:day  4  36.28    9.07

Error: Within
          Df Sum Sq Mean Sq F value   Pr(>F)
temp       3  434.1  144.69  36.427 7.45e-08 ***
prep:temp  6   75.2   12.53   3.154   0.0271 *
Residuals 18   71.5    3.97
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
> model.tables(model3,type="mean")
Tables of means
Grand mean

36.02778

 prep
prep
    1     2     3
35.67 38.50 33.92

 day
day
    1     2     3
34.75 35.25 38.08

 temp
temp
    1     2     3     4
31.22 34.56 37.89 40.44


prep:day
    day
prep 1     2     3
   1 34.50 35.25 37.25
   2 38.75 37.25 39.50
   3 31.00 33.25 37.50
```

```
 prep:temp
    temp
prep 1     2     3     4
   1 29.67 34.67 39.33 39.00
   2 33.33 39.00 39.67 42.00
   3 30.67 30.00 34.67 40.33
```

a) What is the name of the design used in this experiment?

The design is a split plot. The whole plots are preparation method since this is the 'hard to change' factor, and temperature is the subplot factor since each preparation method is run at four different temperatures.

b) Is there a significant effect due to: (1) temperature; (2) preparation method at the 5% significance level?

Use model 3 for all analyses.

The main effect of preparation method can be evaluated with an F test. The observed F-value is 64.19/9.07 = 7.077, referred to an $F_{2,4}$ distribution. The 5% critical value is 6.944. Therefore, there is evidence of a difference in preparation methods at the 5% level.

The main effect of temperature can be evaluated using the subplot error. This is recorded in the `Error: Within` *section of the ANOVA table. The observed F-value is 36.427, referred to a $F_{3,18}$. The p-value is available as part of the output 0. Therefore, there is evidence of a difference in temperature.*