UNIVERSITY OF TORONTO
Faculty of Arts and Science
AUGUST 2011 EXAMINATIONS
MAT135Y1Y
Duration - 3 hours
No Aids Allowed

LAST NAME: ______________________________

FIRST NAME: ______________________________

STUDENT NUMBER: ______________________________

LECTURE SECTION (morning or evening?): ______________________________

INSTRUCTIONS, PLEASE READ:

Please check that this test has 15 numbered pages. Do not tear out any pages. Scrap paper is not allowed. Use the backs of pages for rough work. If you want the back of a page marked, indicate this clearly on the front of the page.

The test has two parts: short answer and long answer.

For the short answer questions, each question part is worth 2.5 marks. You are not required to show your work, the correct answer gets full credit. If you do choose to show your work, you can get part marks if your reasoning is partially correct.

For the long answer questions L1 to L7, you are REQUIRED TO JUSTIFY YOUR ANSWER. The correct answer without computation or justification is worth no credit.

Question	Points	Score
S1	5	
S2	$7\frac{1}{2}$	
S3	5	
S4	5	
S5	$7\frac{1}{2}$	
S6	5	
S7	5	
L1	8	
L2	9	
L3	8	
L4	8	
L5	8	
L6	10	
L7	9	
Total:	100	

SHORT ANSWER QUESTIONS: Justification optional. Each question part is worth 2.5 marks.

S1. [5]

(a) [2½ marks] $\lim_{x\to 0} x^2 \left(1 + \cot^2(3x)\right)$

(b) [2½ marks] $\lim_{x\to 0} \dfrac{\ln(\sec(x))}{x^2}$

S2. [7½]

(a) [2½ marks] Differentiate $y = (\ln(x))^x, \quad (x > 1)$.

(b) [2½ marks] Differentiate $y = \dfrac{e^x \cos(x)}{\sin(\sqrt{x})}$.

(c) [2½ marks] Find the equation of the line tangent to the curve $\tan(x + 2y) = x^2 y$ at the point $(0, \pi)$.

S3. [5]

(a) [2½ marks] Find the intervals of increase and decrease for the function $f(x) = x^2 + \frac{1}{x}$, where $x \neq 0$.

(b) [2½ marks] A particle is moving on a circular orbit $x^2 + y^2 = 25$. As it passes through the point $(3, 4)$, the y coordinate is decreasing at 2 units per second. At what rate is the x coordinate changing at this moment?

S4. [5]

(a) [2½ marks] Find $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{\pi}{n} \sin(\frac{i\pi}{n})$.

(b) [2½ marks] Find $g'(x)$, where $g(x) = \int_{-x^2}^{0} \sqrt{\sin(2t)}dt$.

S5. [7½]

(a) [2½ marks] Find $\int_0^2 |x^2 + x - 2|dx$.

(b) [2½ marks] Find $\int \frac{1}{x^2+2x+5}dx$.

(c) [2½ marks] Evaluate $\int_{-2}^{2} x\sqrt{|\sin(x)|}dx$. *Hint: this is an easy question!*

S6. [5]

(a) [2½ marks] Evaluate the improper integral $\int_0^\infty x^3 e^{-x^4}\,dx$.

(b) [2½ marks] Find the arc length of the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 1$, between the points $(0,1)$ and $(1,0)$.

Hint: the graph looks almost like a quarter circle. You can express one variable in terms of the other, or use implicit differentiation if you prefer.

S7. [5]

(a) [2½ marks] Find the limit of the sequence $a_n = \cos(\frac{n+1}{n^2})$.

(b) [2½ marks] Find the sum of the series $\sum_{n=1}^{\infty} \frac{2^n}{3^{n+1}}$.

LONG ANSWER QUESTIONS: You are required to justify your answer or show your computation. The right answer by itself is worth no credit.

L1. [8]
If a ball is thrown vertically upward with a velocity of 80 m/s, then its height after t seconds is given by the equation $h = 80t - 16t^2$.

(a) [3 marks] What is the maximum height reached by the ball?

(b) [3 marks] When does the ball hit the ground?

(c) [2 marks] What is the velocity of the ball when it hits the ground?

Remember to show your computation.

L3. [8]
Find $\int \frac{\sin 2x}{8+\cos x} dx$.

Remember to show your reasoning.

L4. [8]
Find the volume of the solid obtained by rotating the region bounded by the curves $y = x^4$ and $y = \sqrt{x}$ about the x axis.

Remember to show your computation.

L5. [8]
Solve the initial value problem $xy' = y^2 + y$, $y(1) = 1$.

Remember to justify your answers.

L6. [10]
Determine whether the following series are absolutely convergent, conditionally convergent, or divergent.

(a) [3 marks] $\sum_{n=1}^{\infty} \frac{\cos^2(n)}{n^2}$

(b) [3 marks] $\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$

(c) [4 marks] $\sum_{n=1}^{\infty} \frac{n^2 2^n}{n!}$

Remember to show your computation.

L7. [9]

(a) [4 marks] Use the definition of MacLaurin series to find the 17th term in the MacLaurin series for the function $f(x) = \sin(5x)$. (I.e. you need to find the coefficient of x^{17} in the MacLaurin series. You need not find the radius of convergence.)

(b) [5 marks] Use any method to find the MacLaurin series and its radius of convergence for the function $f(x) = \ln(1 - x^2)$.