Last name (print), First name (print): Student #:

UNIVERSITY OF TORONTO
Faculty of Arts and Science

DECEMBER 2011 EXAMINATIONS

STA257H1F L0101

PLEASE HAND IN

Duration - 3 hours

Examination Aids:
Non-Programmable Calculator. No aid sheet allowed.

Show your work in the space provided. You may use page backs for preparations.
In every part of a question, clearly circle your final answer.

Read questions carefully; results without work shown are not acceptable. Algebraic expressions should be simplified. Numerical results that are fractions need not be given in decimal form, but should be reduced, whenever possible.

Circle your tutorial section

Section	**A**	**B**	**C**	**D**	**E**	**F**
Room:	**SS2106**	**SS1083**	**SS2110**	**UC328**	**UC144**	**MP188**

Marking

Question	1	2	3	4	5	6	
Max Mark	12	12	12	12	12	16	
Mark earned							
Question	7	8	9				Total
Max Mark	12	10	10				108
Mark earned							

Question 1. [12]
A box contains m black and n white balls. Two balls are selected at random. Then one of these two balls is selected at random and returned to the box.

(a) Find probabilities of all possible contents of the box at the end of the experiment, in terms of numbers of black and white balls.

(b) Find the expected number of black balls in the box at the end.

(c) If it is known that at least one white ball was selected in the first selection of two balls, what is the probability that a black ball was returned to the box?

Question 3. [12]
A player shoots a ball to try to score a point. Probability of a successful shoot is p. The player shoots up to 4 times and stops, and also stops after two consecutive points. Let X be the total number of unsuccessful shots, and Y the total number of successful shots in the game (experiment).

(a) Describe the sample space of the experiment and write all sample points.

(b) What is the range of possible values for (X, Y)?

(c) Find the joint probability distribution function for (X,Y). **(continued)**

(d) Are X and Y independent random variables?

Question 4. [12]
From the box with n marbles, all of different color, j marbles are drawn at random one at the time, with replacement (observed and returned).

(a) What is the probability that all j marbles selected are of different color?

(b) What is the probability that the last drown marble will be one that was previously selected?
(continued)

(c) What is the expected number of different colors which will appear in this selection? If $j = n$, and n is large, show that this expected value is approximately $0.632\ n$. (this part (c) may be tricky; if you cannot do it fast, leave it for later)

Question 5. [12]

Let X be a randomly selected real number form the interval $[0,1]$. Let Y be a randomly selected real number from the interval $[X,1]$.

(a) Find the joint density function for X and Y,

(b) Find the marginal density for Y. **(continued)**

(c) Does $E(Y)$ exist? Explain without calculation. Then find $E(Y)$. (hint: using definition of $E(Y)$ directly could be tricky)

Question 6. [16]
The joint density function for continuous random variables X and Y is $f(x,y) = k(x-y)$, for $0 < y < x < 1$.

(a) Find constant k.

(b) Find the marginal densities for X and Y. **(continued)**

(c) Find $E(Y|X)$.

(d) Find $P(XY < 1/4 | X > 1/2)$.

Question 7. [12]

Random variables X and Y are independent and with $E(1)$ (exponential) distributions ($f(x) = \exp(-x)$, $x \geq 0$).

(a) Find the joint PDF for random variables $Y_1 = X + Y$ and $Y_2 = X - Y$ (be careful with range of (Y_1, Y_2)) **(continued)**

(b) Find the marginal PDFs for Y_1 and Y_2 .

(c) Find $Cov(Y_1,Y_2) = E[(Y_1 - E(Y_1))(Y_2 - E(Y_2))]$ in terms of X and Y, and then apply it to this case. Are Y_1 and Y_2 independent?

Question 8. [10]

A soft drink machine can be regulated so that it discharges an average of μ millilitres (mL) per cup (30 mL = 1 ounce). (hint: $\Phi(1.96) = 0.975$, $\Phi(2.326) = 0.99$)

(a) If the mLs of fill are normally distributed with standard deviation 10 mL, give the setting for μ so that 250 mL - cups will overflow when filled only 1% of the time.

(b) The machine actually has standard deviation σ of fill that can be fixed at certain level by carefully adjusting the machine. What is the largest value of σ that will allow the actual amount dispensed to fall within 30 mL of the mean with probability at least 0.95.

Question 9. [10]
Two events, A and B, are defined on the same sample space. Show

(a) $P(A\bar{B} \cup B\bar{A}) = P(A) + P(B) - 2P(AB)$

(b) If $P(A) \geq 1 - \epsilon, 0 < \epsilon < 1$, then $|P(B) - P(B|A)| \leq \epsilon$.

Total marks = 108