Australian National University

STUDENT NUMBER.....

Mid-Semester Examination, First Semester 2014

Financial Mathematics

STAT2032/STAT6046

Writing period : 1.5 Hours duration
Study period : 15 minutes duration
Permitted materials : Non-programmable calculators
Dictionaries (must not contain material added by the student)

Total Marks available: 60

Instructions to Candidates:

- ***Please write your student number in the space provided at the top of this page.***
- ***Attempt ALL questions. There are five questions, not equally weighted.***
- ***All answers are to be written on the exam paper.***
- ***Please hand in the exam paper before you leave the room.***
- ***A formula sheet and selected actuarial tables are attached to the exam paper. You may detach these for your convenience.***
- ***To ensure full marks show all the steps in working out your solution. Marks may be deducted for failure to show appropriate calculations or formulae.***
- ***If you need additional space, please use the rear of the page and state clearly on the front that you have done so.***

	Q1	Q2	Q3	Q4	Q5	Total
Pages	2-4	5-7	8-10	11-12	13	
Marks	13	14	14	11	8	60
Score						

QUESTION 1 (13 marks)

(a) (i) [1 mark] Paul is saving to buy a house. Bank A offers a savings account which earns interest at a nominal rate of 10% pa payable half-yearly (that is, $i^{(2)} = 10\%$). What amount X must Paul invest at t=0 to accumulate $100,000 after 5 years (that is, at t=5)?

(ii) [2 marks] Bank B also offers a savings account. Funds accumulate at a nominal rate of $d^{(2)}$ per annum payable half-yearly. What rate $d^{(2)}$ should Bank B offer, so that Paul prefers Bank B over Bank A?

QUESTION 1 (continued)

(b) [5 marks] Simon borrows money from Bank C for 1 year at a nominal discount rate of 20% per annum payable half-yearly (that is, $d^{(2)} = 20\%$). In 1 year (that is, at t=1), Simon must repay $100,000 to Bank C. After 6 months (that is, at t=0.5), and just before the payment of any discount, Simon notices that Bank C is also offering loans at the nominal discount rate of $d^{(4)} = 20\%$ per annum payable quarterly.

What action do you think Simon would like to take at t=0.5 (before the payment of any discount at this time)? Provide a written explanation for your answer and quantify any fee Bank C may charge at t=0.5.

QUESTION 1 (continued)

(c) [3 marks] Prove that $\frac{i^{(m)}}{m} - \frac{d^{(m)}}{m} = \frac{i^{(m)}}{m} \times \frac{d^{(m)}}{m}$

(d) [2 marks] It is known that $1 + \frac{i^{(n)}}{n} = \frac{1+\frac{i^{(4)}}{4}}{1+\frac{i^{(5)}}{5}}$**. Find n.**

QUESTION 2 (14 marks)

(a) [5 marks] At an annual effective interest rate of $i, i > 0$, both of the following annuities have a present value of X:

- **a 20-year annuity in arrears with annual payments of 55;**
- **a 30-year annuity in arrears with annual payments that pays 30 per year for the first 10 years, 60 per year for the second 10 years, and 90 per year for the final 10 years.**

Calculate X.

QUESTION 2 (continued)

(b) (i) [5 marks] A 10-year annuity in arrears has the following sequence of annual payments:

1, 1, 2, 2, 3, 3, 4, 4, 5, 5

Derive an algebraic expression for the present value of this annuity at an effective annual interest rate i. Simplify your answer, using actuarial notation where possible.

QUESTION 2 (continued)

(b) (ii) [2 marks] For the annuity in (b)(i), now suppose that payments are made continuously at the rate of

- **1 per annum in years 1 and 2;**
- **2 per annum in years 3 and 4;**
- **3 per annum in years 5 and 6;**
- **4 per annum in years 7 and 8; and**
- **5 per annum in years 9 and 10.**

Will the present value of this continuous annuity be greater than, less than or equal to the present value of the annuity in (b)(i)? Provide a written explanation for your answer.

(iii) [2 marks] Find an algebraic expression for the present value of the annuity in part (b)(ii). Simplify your answer, using actuarial notation where possible.

QUESTION 3 (14 marks)

(a) [5 marks] A family wishes to provide an annuity of $100 at the end of each month to their daughter now entering university. The annuity will be paid for only nine consecutive months each calendar year for four years. Find the present value of the annuity one month before the first payment at an annual effective interest rate of 12%.

QUESTION 3 (continued)

(b) (i) [3 marks] Show that the present value of a continuous perpetuity $\overline{a}_{\overline{\infty}|}$, (that is, an annuity that pays continuously at a rate of $1 per annum forever), is given by $\overline{a}_{\overline{\infty}|} = \frac{1}{\delta}$

(ii) [1 mark] A perpetuity pays continuously at a rate of 100 per year and has a present value of 800. Calculate the annual effective interest rate used to calculate the present value.

QUESTION 3 (continued)

(c) [5 marks] An investment fund is started with an initial deposit of 1 at time 0. New deposits are made continuously at the annual rate $(1 + t)$ at time t over the next n years. The force of interest at time t is given by $\delta_t = (1 + t)^{-1}$. Find the accumulated value in the fund at the end of n years.

QUESTION 4 (11 marks)

(a) [5 marks] An annuity has 15 annual payments made at the end of each year. The first three payments are $45,000 each, the next three payments are 6% smaller, and so on. That is, payments decrease by 6% every three years. Find the accumulated value of the annuity on the date of the last payment, if the effective annual interest rate is 6% per annum.

QUESTION 4 (continued)

(b) [3 marks] Now suppose for the term of the annuity in part (a), inflation occurs at the rate of 3% per annum. What is the annuity worth in 'real' dollars on the date of the final payment? (That is, express the accumulated value of the annuity in terms of purchasing power at time t=0).

(c) [3 marks] For the annuity in part (a), if payments decrease by 7% every three years (instead of 6% every three years), the accumulated value is $933,544.35. If payments decrease by 8% each year, the accumulated value is $918,449.03. Find the percentage decrease required every three years to obtain an accumulated value of $920,000 (on the date of the last payment).

QUESTION 5 (8 marks)

(a) [2 marks] Amy borrows $50,000 from Bank X and agrees to repay it back over six years, in quarterly instalments of principal and interest at the nominal interest rate of 8% per annum payable quarterly. Determine the size of Amy's quarterly payment (assuming she makes payments at the end of every quarter).

(b) [4 marks] Suppose that immediately after the sixth payment, Amy loses her job. Fortunately, it doesn't take long for Amy to find another job, and she misses only two quarterly payments. That is, she recommences payments 9 months after losing her job. If the term of the loan and interest rate remain unchanged, what is Amy's revised quarterly repayment?

(c) [2 marks] What is the total amount of interest Amy pays on the loan?

END OF EXAMINATION