Assignment 4.

1. a). $E^2 = p^2c^2 + m^2c^4$ $m=0$ for photons

$E = pc = hc/\lambda$

$\lambda = h/p \equiv \lambda_{dB}$

b). $\Delta t \sim h/\Delta E = h/2m_p c^2$

$= 6.62 \cdot 10^{-34} J\cdot s / 2 \cdot 1.67 \cdot 10^{-27} kg \cdot (3 \cdot 10^8 m/s)^2$

$= \underline{2.20 \cdot 10^{-24} s}$

2. a). $\Delta E = h\frac{c}{\lambda} = -2.176 \cdot 10^{-8} J \left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right) = -E_I \left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right)$

$\frac{1}{\lambda} = -\frac{E_I}{hc}\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right)$ $n_i = 3$ $n_f = 2$

$\lambda = 6.57 \cdot 10^{-7} m = \underline{6570 \text{ Å}}$

b). $\Delta\lambda/\lambda = v/c$

$v = c\Delta\lambda/\lambda$

$v = 2.28 \cdot 10^5 m/s = \underline{228 km/s}$

c). $v = c\Delta\lambda/\lambda \to 6.13 \cdot 10^8 m/s = 2.04c$

d). Not due to motion through space. also,

$\lambda_e = \lambda_0 \sqrt{\frac{1+v/c}{1-v/c}}$

3. a). $L = A\sigma T^4$

$L = 10^{-4} m^2 \cdot 5.67 \cdot 10^{-8} \frac{W}{m^2 K^4} (2000K)^4$

$= \underline{90.7 W}$

b). $\underline{100}$

c). $\lambda T = 0.29$ cm K.

$\lambda = \underline{1450 nm}$ (IR)

d). $L = A\sigma T^4$

$$L = 2m^2 \cdot 5.67 \cdot 10^{-8} \frac{W}{m^2 K^4} (300K)^4$$
$$= 918.5\ W = \underline{10.1\ \text{lights}}$$

$$\lambda T = 0.29\ cm \cdot K$$
$$\lambda = 9670\ nm = \underline{9.67\ \mu m}$$

4. a). $f(x) = \frac{1}{x}$ $\quad f'(x) = -\frac{1}{x^2}$

$f(x) = 2\ln(1-x)$ $\quad f'(x) = \frac{2}{1-x}\frac{d}{dx}(1-x)$

$$= -\frac{2}{1-x}$$

$f(x) = e^{-x}$ $\quad f'(x) = -e^{-x}$

$f(x) = \cos(2x)$ $\quad f'(x) = -2\sin(2x)$

$f(x) = (3x+1)^2$ $\quad f'(x) = 2(3x+1) = 6x+2$

$f(x) = x^2/3x-1$ $\quad f'(x) = 2x/3x-1 - \frac{x^2}{(3x-1)^2} \cdot 3$

$$= \frac{2x(3x-1) - 3x^2}{(3x-1)^2}$$

$$= \frac{3x^2 - 2x}{(3x-1)^2}$$

b). $\int_0^4 (2x^3+3)dx = \frac{1}{2}x^4 + 3x \Big]_0^4 = \underline{140}$

$\int_1^{10} \ln x\, dx = x\ln x \Big]_1^{10} - \int_1^{10} dx$

$$= x\ln x - x \Big]_1^{10} = \underline{14.03}$$

$\int_{-10}^{0} e^x dx = e^x \Big]_{-10}^{0} = \underline{0.99995}$

c). $\int e^{kx} dx = \underline{\frac{1}{k}e^{kx}} + C$

$\int 7^x dx = \int e^{x\ln 7} dx = \frac{1}{\ln 7} e^{x\ln 7} + C = \underline{\frac{1}{\ln 7} 7^x + C}$

$\int x^n dx = \underline{\frac{1}{n} x^{n+1}} + C$

$\int e^{kx} x^2 dx = \frac{1}{k} e^{kx} x^2 - \int \frac{2}{k} e^{kx} x\, dx$

$$= \frac{x^2}{k} e^{kx} - \frac{2}{k}\left(\frac{1}{k}e^{kx} x - \int \frac{1}{k} e^{kx} dx\right)$$

$$= \frac{x^2}{k} e^{kx} - \frac{2}{k^2} e^{kx} x + \frac{2}{k^3} e^{kx} + C$$

d). $\frac{df}{dx} = -x + 2$

$f = \int -x + 2 \, dx$

$= \underline{\underline{-\frac{1}{2}x^2 + 2x + C}}$

$\frac{df}{dx} = -\frac{1}{x}$

$f = -\int \frac{1}{x} dx$

$= \underline{\underline{-\ln x + C}}$

$\frac{df}{dx} = \exp(x)$

$f = \underline{\underline{\exp(x) + C}}$

$\frac{d^2f}{dx^2} = -\frac{1}{x^2}$

$\frac{df}{dx} = -\int \frac{1}{x^2} dx$

$= \frac{1}{x} + C$

$f = \int \frac{1}{x} + C \, dx$

$= \underline{\underline{\ln x + Cx + D}}$