Question One - Solutions

a)

Company	Start	End	Bankrupt	Sold	waiting time
1	0	1	1	0	1
2	0	0.7	0	0	0.7
3	0	0.3	1	0	0.3
4	0.5	1.5	0	0	0.5
5	0.8	1	0	1	0.2
6	0.2	1	0	0	0.8
7	0	2.3	0	0	1
			2	1	4.5

$$\hat{\mu} = \frac{2}{4.5}; \hat{\nu} = \frac{1}{4.5}$$

b)

$${}_tP_x^{12} = \frac{\mu}{\mu+\nu}[1-e^{-(\mu+\nu)t}] \text{ (from notes)}$$

$${}_{0.5}P_x^{12} = \frac{2}{3}[1-e^{-(\frac{3}{4.5})0.5}] = 0.19$$

bank ~bin(100,0.19)

bank ~N(19,3.9)

$$p(X>14)=p(Z>\frac{14-19}{3.9}) = p(Z>\frac{14-19}{3.9}) = p(Z>-1.3)$$

Question Two - Solution

a)

$$\exp(-0.82) = 0.44 \text{ reduced by over one-half.}$$

b)

$$TS = \frac{-0.82}{0.17} < -2 \text{ It is an important covariate.}$$

c)

$$\exp(-0.1x + 0.0011x^2 - 0.82) = \exp(-0.1\times 65 + 0.0011\times 65^2)$$
$$\Rightarrow -0.1x + 0.0011x^2 = -1.03$$
$$\Rightarrow -0.1x + 0.0011x^2 + 1.03 = 0$$
$$\Rightarrow 0.0011x^2 - 0.1x + 1.03 = 0$$
$$\Rightarrow x = \frac{0.1 \pm \sqrt{0.1^2 - 4\times 0.0011\times 1.03}}{2\times 0.0011} = 79 \text{ or } 11.84$$

d)

$$\frac{(\alpha_0 + \alpha_1 Z)(\lambda_0 + \lambda_1 Z_2)t^{\alpha_0 + \alpha_1 Z_1 - 1}}{(\alpha_0 + \alpha_1 \hat{Z})(\lambda_0 + \lambda_1 \hat{Z}_2)t^{\alpha_0 + \alpha_1 \hat{Z}_1 - 1}}$$
$$= \frac{(\alpha_0 + \alpha_1 Z)(\lambda_0 + \lambda_1 Z_2)t^{\alpha_1 (Z_1 - \hat{Z}_1)}}{(\alpha_0 + \alpha_1 \hat{Z})(\lambda_0 + \lambda_1 \hat{Z}_2)}$$

need $\alpha_1 = 0$ for proportional hazards

Question Three - Solution

$$
\begin{aligned}
e_x &= \sum_{k=1}^{\infty} {}_k p_x \\
&= {}_1 p_x + \sum_{k=2}^{\infty} {}_k p_x \\
&= {}_1 p_x + \sum_{k=2}^{\infty} {}_1 p_x \cdot {}_{k-1} p_{x+1} \\
&= {}_1 p_x + \sum_{k=1}^{\infty} {}_1 p_x \cdot {}_k p_{x+1} \\
&= {}_1 p_x (1 + \sum_{k=1}^{\infty} {}_k p_{x+1}) \\
&= {}_1 p_x (1 + e_{x+1})
\end{aligned}
$$

4. You are interested in the mortality experience of female elite athletes. At both the start and end of 2011 there were 547 female elite athletes in your county. You also have information on the number of elite athletes at 1 June 2011 (325 athletes) and 1 September 2011 (613 athletes). Approximately, calculate the central exposed to risk over the year.

$$
E_{55}^{c} = \frac{5}{12}\frac{(547+325)}{2} + \frac{3}{12}\frac{(325+613)}{2} + \frac{4}{12}\frac{(613+547)}{2} = 492.25
$$

4. Given that $l_x = \sqrt{1 - x/100}$, find an expression for μ_x.

$$
\begin{aligned}
\mu_x &= -\frac{1}{l_x}\frac{dl_x}{dx} \\
&= \frac{1}{\sqrt{1-x/100}}\frac{1}{2}\frac{1}{\sqrt{1-x/100}}\frac{1}{100} \\
&= \frac{1}{200}\frac{1}{1-\frac{x}{100}}
\end{aligned}
$$