Australian National University
Research School of Finance, Actuarial Studies and Applied Statistics

STAT2032/6046: Financial Mathematics

Review Questions (Week 10 – Week 12)

WEEK 10

Question 1

On 1 January 1999 an investor agrees to pay \$3,000 in four years' time for a security. The security pays no interest and the price of the security at the time of the agreement was \$2,680.

On 1 July 2000 the price of the security is \$2,800. Calculate the value of the forward contract on 1 July 2000.

Solution

We have been given

$K = \$3,000$

$S_0 = \$2,680$

$T = 4$

The risk-free force of interest can be found from the equation:

$K = S_0 e^{\delta T}$

where K is the forward price and S_0 is the price of the security at issue.

$\delta = \frac{1}{4}\ln\left(\frac{3,000}{2,680}\right) = 2.82\%$

The value of the forward contract at date in time between the date of the initial contract and the maturity date, to the investor holding a long position, is given by the equation:

$V_L = S_r - S_0 e^{\delta r}$.

We want to find the value of the contract on 1 July 2000, which is 1.5 years after the contract was established. $\Rightarrow r = 1.5$.

The price of the security on 1 July 2000 is $S_r = \$2,800$

$\Rightarrow V_L = S_r - S_0 e^{\delta r} = 2,800 - 2,680 e^{1.5(0.0282)} = \$4.20$

Question 2

The Government issues a fixed-interest stock with a 20 year term, which pays half-yearly coupons in arrears and is purchased and redeemable at par. The risk-free effective interest rate is assumed to be 7% over each of the next 5 years, and 8% thereafter.

i. What is the coupon rate convertible half-yearly for $200 nominal of the stock?
ii. If an investor enters into a forward contract to buy the security in 4 years, immediately after the coupon payment then due, what is the forward price that should be paid (assuming no arbitrage)?

Solution

i. Work in periods of half-years:

$$200 = 200r\left[a_{\overline{10}|j} + v_{0.07}^{5} a_{\overline{30}|k}\right] + 200 v_{0.08}^{15} v_{0.07}^{5} = 4157.29r + 44.95$$

$\Rightarrow r = 3.73\%$

where $j = (1.07)^{0.5} - 1$ and $k = (1.08)^{0.5} - 1$

So the coupon rate convertible half-yearly is 2 x 3.73% = 7.46%.

ii. We are only concerned with the risk free force of interest over the next four years which is $\delta = \ln(1.07)$

$K = (S_0 - PV_I)e^{\delta T}$

$PV_I = 200(0.0373)a_{\overline{8}|j} \cong 51.41$ where $j = (1.07)^{0.5} - 1$

$S_0 = 200$

$K = (S_0 - PV_I)e^{\delta T} = (200 - 51.41)e^{4\ln(1.07)} = (200 - 51.41)(1.07)^4 = 194.77$ =

$(200 - 51.41)e^{4\ln(1.07)} = (200 - 51.41)(1.07)^4 = 194.77$

Question 3

Length of Investment	Interest Rate
1 Year	7.00%
2 years	8.00%
3 Years	8.75%
4 Years	9.25%
5 Years	9.50%

i. Find the price of a $2000 two-year bond with annual 10% coupons using the spot rates given in the table above

ii. Compute the redemption yield for the bond

Solution

i. $P_1 = 2000(0.1)v_{0.07} + 2000(0.1)v_{0.08}^2 + 2000v_{0.08}^2 = \$2073.06$

ii. $2073.06 = 200v_i + 2200v_i^2 \Rightarrow 2200v_i^2 + 200v_i - 2073.06 = 0$

Solve the quadratic function to find v

$$v = \frac{-200 \pm \sqrt{40000 + 4(2200)(2073.06)}}{4400} = 0.92633, -1.01724 \text{ ignore negative root}$$

$\Rightarrow i = 1/v - 1 = 7.95\%$

Question 4

Based on the yield curve given in the table for the previous question, find the following expected forward rates:

i. The 3-year forward rate that is deferred for 1-year. $f_{1,4}$

ii. The 2-year forward rate that is deferred for 3-years. $f_{3,5}$

Solution

$$(1 + f_{t,T})^T = \frac{(1 + s_T)^T}{(1 + s_t)^t}$$

a. $\left(1 + f_{1,4}\right)^3 = \dfrac{\left(1 + s_4\right)^4}{\left(1 + s_1\right)^1} = \dfrac{1.0975^4}{1.07} \Rightarrow f_{1,4} = 0.1001$

b. $\left(1+f_{3,5}\right)^2=\dfrac{\left(1+s_5\right)^5}{\left(1+s_3\right)^3}=\dfrac{1.095^5}{1.0875^3}\Rightarrow f_{3,5}=0.1063$

WEEK 11

Question 5

Suppose that the yield rate and coupon rate on an n -coupon bond are the same. Show that the duration is $\ddot{a}_{\overline{n}|}$ valued at the yield rate. Find the duration of a 6-coupon bond with coupon rate 10% per coupon period and yield rate 10%.

Solution

$$D=\frac{\sum_{t=1}^{n} t\cdot Fr\cdot v_j^t+n\cdot F\cdot v_j^n}{\sum_{t=1}^{n} Fr\cdot v_j^t+F\cdot v_j^n}=\frac{Fr(Ia)_{\overline{n}|j}+n\cdot F\cdot v_j^n}{Fr\cdot a_{\overline{n}|j}+F\cdot v_j^n}$$

If $r=j$ then the denominator is

$$Fj\cdot a_{\overline{n}|j}+F\cdot v_j^n=F(1-v_j^n)+F\cdot v_j^n=F$$

$$D=\frac{Fj(Ia)_{\overline{n}|j}+n\cdot F\cdot v_j^n}{F}=j(Ia)_{\overline{n}|j}+n\cdot v_j^n$$

$$D=\frac{Fj(Ia)_{\overline{n}|j}+n\cdot F\cdot v_j^n}{F}=j(Ia)_{\overline{n}|j}+n\cdot v_j^n$$

Now recall from earlier lectures,

$$(Ia)_{\overline{n}|j}=\frac{\ddot{a}_{\overline{n}|j}-n\cdot v_j^n}{j}\Rightarrow D=\ddot{a}_{\overline{n}|j}$$

If $r=j=0.10$ and $n=6$ then $D=\ddot{a}_{\overline{6}|0.10}=4.7908$

Question 6

Based on an interest rate of 8% a fund has liabilities with present value \$400,000 and assets with present value \$420,000. The discounted mean terms of the liability and asset cashflows are 4 years and 3 years respectively. Estimate the surplus in the fund if interest rates move to 9% by using the Taylor series approximation for the surplus *S*:

$$S(i+\varepsilon) \cong S(i) + \varepsilon S'(i)$$

Solution

The surplus at interest rate i is:

$$S(i) = PV_A(i) - PV_L(i)$$

If interest rates move by a small amount ε, the new surplus will be:

$$S(i+\varepsilon) = PV_A(i+\varepsilon) - PV_L(i+\varepsilon)$$

Using Taylor's formula, we get the approximation:

$$S(i+\varepsilon) \cong S(i) + \varepsilon S'(i) = PV_A(i) - PV_L(i) + \varepsilon\left[PV_A{}'(i) - PV_L{}'(i)\right]$$

(We have only included the first two terms of the series).

We can find the derivatives of the present values from the relationship:

$$DMT = (1+i)\left(-\frac{PV'(i)}{PV(i)}\right)$$

$$\Rightarrow PV'(i) = -PV(i)\frac{DMT}{(1+i)}$$

So:

$$S(i+\varepsilon) \cong S(i) + \varepsilon S'(i) = PV_A(i) - PV_L(i) + \varepsilon\left[-PV_A(i)\frac{DMT_A}{(1+i)} + PV_L(i)\frac{DMT_L}{(1+i)}\right]$$

Using the values given in the question:

$$i = 0.08,\ \varepsilon = 0.01,\ PV_A(0.08) = 420{,}000,\ PV_L(0.08) = 400{,}000$$

$DMT_A(0.08) = 3, DMT_L(0.08) = 4$

$$S(0.09) \cong 420{,}000 - 400{,}000 + \frac{0.01}{1.08}\left[-420{,}000(3) + 400{,}000(4)\right] = 23{,}148$$

Question 7

Calculate, using an interest rate of 10%, the discounted mean term of a 20-year annuity certain payable annually in arrears under which the first payment is $1000 if

a. the payments remain level throughout the term
b. the payments increase by $50 each year
c. the payments increase by 10% (compound) each year

You are given that $\sum_{t=1}^{20} t^2 1.1^{-t} = 718.027$

Solution

a. Here we need to find (after cancelling a factor of 1,000):

$$DMT = \frac{\sum_{t=1}^{20} tv^t}{\sum_{t=1}^{20} v^t}$$

The numerator is $(Ia)_{\overline{20|}} = 63.9205$ and the denominator is $a_{\overline{20|}} = 8.5136$.

So the discounted mean term is 7.51 years.

b. Here we need to find

$$DMT = \frac{\sum_{t=1}^{20} t(950+50t)v^t}{\sum_{t=1}^{20} (950+50t)v^t}$$

The numerator is (using the figure given):

$$950(Ia)_{\overline{20|}} + 50\sum_{t=1}^{20} t^2 v^t = 950(63.9205) + 50(718.027) = 96{,}626$$

and the denominator is:

$$950a_{\overline{20|}} + 50(Ia)_{\overline{20|}} = 950(8.5136) + 50(63.9205) = 11{,}284$$

So the discounted mean term is 8.56 years.

c. Here we need to find:

$$DMT = \frac{\sum_{t=1}^{20}(1{,}000)tv^t(1.1)^{t-1}}{\sum_{t=1}^{20}(1{,}000)v^t(1.1)^{t-1}} = \frac{\sum_{t=1}^{20}tv^t(1.1)^t}{\sum_{t=1}^{20}v^t(1.1)^t}$$

But since $i = 0.1$, this is just:

$$DMT = \frac{\sum_{t=1}^{20}t}{\sum_{t=1}^{20}1} = \frac{1+2+3+...+20}{20} = \frac{0.5(20)(21)}{20} = 10.5 \text{ years.}$$

Question 8

A financial institution has an obligation to pay \$5,000 at the end of each year for 5 years. The institution receives $5000a_{\overline{5|}0.1} = \$18{,}954$ in exchange for assuming this obligation. The only investments available to the institution are 1, 3, and 5-year zero coupon bonds, all yielding 10%. The institution invests an amount in the 1 year zero coupon bond that is redeemed for X_1 = \$6,950, an amount in the 3 year zero coupon bond that is redeemed for X_2 = \$8,409 and an amount in the 5 year zero coupon bond that is redeemed for X_3 = \$10,175

Verify that this investment strategy is not optimal under immunisation theory.

Solution

$$PV_L = 5{,}000a_{\overline{5|}0.1} = 5{,}000\sum_{t=1}^{5}v^t = \$18{,}954$$

$$PV_A = X_1v + X_2v^3 + X_3v^5 = (6950)v + (8409)v^3 + (10175)v^5 = \$18{,}954$$

The first condition for immunisation $\left(PV_A = PV_L\right)$ is satisfied.

$$PV_A' = -X_1v^2 - 3X_2v^4 - 5X_3v^6 = -51{,}692$$

$$PV_L' = -5{,}000\sum_{t=1}^{5}tv^{t+1} = -5000v(Ia)_{\overline{5|}} = -48{,}421.9 \neq PV_A'$$

The second condition for immunisation $\left(PV_A' = PV_L'\right)$ is not satisfied.

Alternatively, we can find the surplus at interest rates just above and below 10%; ie. $S(0.11)$ and $S(0.09)$

$$S(0.11) = PV_A(0.11) - PV_L(0.11) = 18{,}448 - 18{,}479 \ = -31$$

$$S(0.09) = PV_A(0.09) - PV_L(0.09) = 19{,}482 - 19{,}448 \ = 34$$

Since the surplus becomes negative with a small shift in interest rates (10% to 11%), the portfolio is not immunised.

Question 9

The liabilities of a fund consist of two lump sum payments due at known times in the future. The second lump sum is due for payment 5 years after the first and is twice the amount of the first.

i. If the total present value and duration of the liabilities (both calculated at 6%) are \$60,000 and 6 years, determine the timing and amounts of the payments.
ii. If the assets of the fund consist of a single zero coupon bond that will mature 6 years from now with a redemption payment of \$85,111.15, is the portfolio immunised?

Solution

i. Let the liabilities consist of a lump sum of amount X payable at time t and a lump sum of amount 2X payable at time *t* + 5. The present value is:
$$PV_L = Xv^t + 2Xv^{t+5} = \$60{,}000 \Rightarrow Xv^t(1+2v^5) = 60{,}000 \Rightarrow Xv^t = 24{,}052.76$$
The duration is:
$$\frac{Xtv^t + 2X(t+5)v^{t+5}}{60{,}000} = 6 \Rightarrow Xv^t\left[t + 2(t+5)v^5\right] = 360{,}000$$
$$\Rightarrow \left[t + 2(t+5)v^5\right] = t(1+2v^5) + 10v^5 = \frac{360{,}000}{24{,}052.76} = 14.967 \Rightarrow t = 3$$
So, X = 28,647. Therefore a payment of \$28,647 is made at time 3 and a payment of \$57,294 at time 8.

ii. The present value of the assets is: PV_A=$(85{,}111.15)v^6 = 60{,}000 = PV_L$; so condition 1 for immunisation is satisfied.
The durations of the assets and the liabilities are the same (6 years) so condition 2 is satisfied.
The third condition is that the convexity of the assets is greater than the convexity of the liabilities, or equivalently that $PV_A'' \geq PV_L''$.

$\mathrm{PV}_A'' = 6(7)(85{,}111.15)v^8 = 2{,}242{,}791$
$PV_L' = -tXv^{t+1} - (t+5)2Xv^{t+6}$
$PV_L'' = t(t+1)Xv^{t+2} + (t+5)(t+6)2Xv^{t+7} = 3(4)Xv^5 + (8)(9)2Xv^{10} = 2{,}560{,}353$

$PV_A'' < PV_L''$, so the portfolio is not immunised. We can see that, since the asset cashflow falls between the liabilities cashflows, the convexity of the assets is less than the convexity of the liabilities (ie. the liability cashflows are more spread out than the asset cashflows).

WEEK 12

Question 10

For the random interest rate $\tilde{i}$, denote $E\left[\tilde{i}\right] = j$ and $Var\left[\tilde{i}\right] = s^2$. Assuming independence of rates, show that:

i. $E\left[\tilde{S}(n)\right] = (1+j)^n$

ii. $Var\left[\tilde{S}(n)\right] = \left(1+2j+j^2+s^2\right)^n - (1+j)^{2n}$

Solution

For the random interest rate $\tilde{i}$, denote $E\left[\tilde{i}\right]$ by j and $Var\left[\tilde{i}\right]$ by s^2. Assuming independence of rates, show that $E\left[\tilde{S}(n)\right] = (1+j)^n$ and $Var\left[\tilde{S}(n)\right] = \left(1+2j+j^2+s^2\right)^n - (1+j)^{2n}$

Question 11

A stochastic interest rate model assumes that the interest rates in different years are independent and identically distributed with a normal distribution with mean 10% and standard deviation 3%. Find the standard deviation of the accumulated value of an initial investment of $12,000 at the end of the second year.

Solution

The variance of the accumulated amount at the end of 2 years is:

$$Var\left(12000\tilde{S}(2)\right) = 12{,}000^2\left(E\left[\tilde{S}(2)^2\right] - \left(E\left[\tilde{S}(2)\right]\right)^2\right)$$
$$E\left[\tilde{S}(2)\right] = E\left[1+\tilde{i}\right]^2 = 1.1^2$$

Now,

$$E\left[\left(1+\tilde{i}\right)^2\right] = Var\left[1+\tilde{i}\right] + \left(E\left[1+\tilde{i}\right]\right)^2 = 0.03^2 + 1.1^2 = 1.2109$$
$$\Rightarrow E\left[\tilde{S}(2)^2\right] = E\left[\left(1+\tilde{i}\right)^2\right]^2 = 1.2109^2$$
$$\Rightarrow Var\left(12000\tilde{S}(2)\right) = 12{,}000^2\left(1.2109^2 - 1.1^4\right) = 313{,}748.6$$
$$\Rightarrow S.D.\left(12000\tilde{S}(2)\right) = \sqrt{313{,}748.6} = 560.1$$

Question 12

A stochastic interest rate model assumes that interest rates in different years are independent and conform to the probability distribution:

$$\tilde{i} = \begin{cases} 0.035 & p = 0.2 \\ 0.045 & p = 0.4 \\ 0.075 & p = 0.4 \end{cases}$$

Calculate the standard deviation of the annual interest rate.

Solution

$$\tilde{i} = \begin{cases} 0.035 & p = 0.2 \\ 0.045 & p = 0.4 \\ 0.075 & p = 0.4 \end{cases}$$

$$E\left[\tilde{i}\right] = 0.035(0.2) + 0.045(0.4) + 0.075(0.4) = 0.055$$
$$E\left[\tilde{i}^2\right] = 0.035^2(0.2) + 0.045^2(0.4) + 0.075^2(0.4) = 0.003305$$

The variance is:

$$Var\left[\tilde{i}\right] = E\left[\tilde{i}^2\right] - E\left[\tilde{i}\right]^2 = 0.003305 - 0.055^2 = 0.00028$$

Hence standard deviation is 0.016733

Question 13

The yield obtained on a company's funds each year is expected to be 2%, 3% or 4% with probabilities 0.2, 0.5 and 0.3 respectively. Find the mean and standard deviation of the accumulated value of an initial sum of $1000 invested for 12 years if the yields in different years are independent.

Solution

$$E\left[1+\tilde{i}\right]=1.02(0.2)+1.03(0.5)+1.04(0.3)=1.031$$

$$E\left[\left(1+\tilde{i}\right)^2\right]=1.02^2(0.2)+1.03^2(0.5)+1.04^2(0.3)=1.06301$$

$$E\left[1000\tilde{S}(12)\right]=1000\left(E\left[1+\tilde{i}\right]\right)^{12}=1000(1.031^{12})=\$1442.46$$

1.031

$$Var\left(1000\tilde{S}(12)\right)=1000^2\left(E\left[\tilde{S}(12)^2\right]-\left(E\left[\tilde{S}(12)\right]\right)^2\right)$$

$$=1000^2\left(\left(E\left[\left(1+\tilde{i}\right)^2\right]\right)^{12}-\left(E\left[1+\tilde{i}\right]\right)^{24}\right)$$

$$=1000^2\left(1.06301^{12}-1.031^{24}\right)$$

$$=1151.272$$

The standard deviation is 33.93.

Question 14

i. An amount of 1 is invested for 10 years. The interest rate earned by the investment for the 10-year period will be either 5% for all 10 years, or 10% for all 10 years, or 15% for all 10 years. Each of the three possible cases is equally likely. Find the expected value and variance of the accumulated value at the end of the 10 years.

ii. Now, suppose that interest rates may be different from year to year, but again for a particular year the rate is either 5%, 10% or 15%. Assuming independent interest rates from year to year, find the expected value and variance of the accumulated value.

Solution

The accumulated value $\tilde{S}$ is either $(1.05)^{10}$, $(1.10)^{10}$ or $(1.15)^{10}$, each with probability $\frac{1}{3}$. The expected accumulated value is:

$$E\left[\tilde{S}\right]=\frac{1}{3}\left[(1.05)^{10}+(1.10)^{10}+(1.15)^{10}\right]=2.7561$$

The variance of $\tilde{S}$ is $E\left[\tilde{S}^2\right]-\left(E\left[\tilde{S}\right]\right)^2=\frac{1}{3}\left[(1.05)^{20}+(1.10)^{20}+(1.15)^{20}\right]-(2.7561)^2=0.9866$

If rates can change from year to year then

$$E\left[1+\tilde{i}\right]=\frac{1.05+1.10+1.15}{3}=1.10$$

$$E\left[\left(1+\tilde{i}\right)^2\right]=\frac{1.05^2+1.10^2+1.15^2}{3}=1.21167$$

$$E\left[\tilde{S}(10)\right]=(1.10)^{10}=2.5937$$

$$E\left[\tilde{S}(10)^2\right]=\left(E\left[(1+\tilde{i})^2\right]\right)^{10}=1.21167^{10}$$

$$Var\left[\tilde{S}(10)\right]=E\left[\tilde{S}(10)^2\right]-\left(E\left[\tilde{S}(10)\right]\right)^2=1.21167^{10}-2.5937^2=0.0932$$

Question 15

Annual effective interest rates will be random in years 1 and 2, following a uniform distribution between 6% and 12%. In years 3 and 4 the rates will be uniformly distributed between 5% and 15%. Find $Var\left[\tilde{S}(4)\right]$.

Solution

$$Var\left[\tilde{S}(4)\right]=E\left[\tilde{S}(4)^2\right]-\left(E\left[\tilde{S}(4)\right]\right)^2$$

Let $\tilde{i}_t$ be the random interest rate for year *t*. Assuming that interest rates are independent:

$$\Rightarrow E\left[\tilde{S}(4)\right]=E\left[(1+\tilde{i}_1)(1+\tilde{i}_2)(1+\tilde{i}_3)(1+\tilde{i}_4)\right]$$

$$E\left[1+\tilde{i}_1\right]=E\left[1+\tilde{i}_2\right]=1+\frac{a+b}{2}=1+\frac{0.06+0.12}{2}=1.09$$

$$E\left[1+\tilde{i}_3\right]=E\left[1+\tilde{i}_4\right]=1+\frac{a+b}{2}=1+\frac{0.05+0.15}{2}=1.10$$

$$E\left[\tilde{S}(4)\right]=E\left[(1+\tilde{i}_1)(1+\tilde{i}_2)(1+\tilde{i}_3)(1+\tilde{i}_4)\right]=1.09^2 1.10^2=1.437601$$
$$E\left[\tilde{S}(4)^2\right]=E\left[(1+\tilde{i}_1)^2(1+\tilde{i}_2)^2(1+\tilde{i}_3)^2(1+\tilde{i}_4)^2\right]$$

For a uniform distribution, $f\left(\tilde{i}\right)=\frac{1}{b-a}$

$$E\left[\left(1+\tilde{i}_1\right)^2\right]=E\left[\left(1+\tilde{i}_2\right)^2\right]=\int_{0.06}^{0.12}(1+\tilde{i})^2 f(\tilde{i})d\tilde{i}=\int_{0.06}^{0.12}\frac{(1+\tilde{i})^2}{0.06}d\tilde{i}=\left.\frac{(1+\tilde{i})^3}{0.18}\right|_{0.06}^{0.12}$$
$$=\frac{(1.12)^3-(1.06)^3}{0.18}=1.1884$$

$$E\left[\left(1+\tilde{i}_3\right)^2\right]=E\left[\left(1+\tilde{i}_4\right)^2\right]=\int_{0.05}^{0.15}(1+\tilde{i})^2 f(\tilde{i})d\tilde{i}=\int_{0.05}^{0.15}\frac{(1+\tilde{i})^2}{0.1}d\tilde{i}=\left.\frac{(1+\tilde{i})^3}{0.3}\right|_{0.05}^{0.15}$$
$$=\frac{(1.15)^3-(1.05)^3}{0.3}=1.2108$$

$$E\left[\tilde{S}(4)^2\right]=E\left[(1+\tilde{i}_1)^2(1+\tilde{i}_2)^2(1+\tilde{i}_3)^2(1+\tilde{i}_4)^2\right]=1.1884^2 1.2108^2=2.070476$$

$$Var\left[\tilde{S}(4)\right]=E\left[\tilde{S}(4)^2\right]-\left(E\left[\tilde{S}(4)\right]\right)^2=2.070476-1.437601^2=0.0038$$

Question 16

Interest rates for the next 25 years are independently and identically distributed according to the following distribution:

$$\tilde{i}=\begin{cases}0.08 & p=0.3\\ 0.13 & p=0.7\end{cases}$$

Using the lognormal approximation for $\tilde{S}(25)$, find the approximate probability that the accumulation of \$500 for 25 years will be greater than \$8,000 and also the approximate probability that the accumulation will be greater than $E\left[500\tilde{S}(25)\right]$.

Solution

$$E\left[500\tilde{S}(25)\right]=500(0.3\times1.08+0.7\times1.13)^{25}=7{,}600$$

We need to find $\Pr\left[\tilde{S}(10)>8{,}000/500=16\right]$ and $\Pr\left[\tilde{S}(10)>7{,}600/500=15.2\right]$

The distribution of $\tilde{\delta}$ is:

$$\tilde{\delta}=\begin{cases}\ln(1.08) & prob=0.3\\ \ln(1.13) & prob=0.7\end{cases}$$

$$E\left[\tilde{\delta}\right]=0.3\ln(1.08)+0.7\ln(1.13)=0.108641$$
$$E\left[\tilde{\delta}^2\right]=0.3(\ln(1.08))^2+0.7(\ln(1.13))^2=0.012233$$
$$\Rightarrow Var\left[\tilde{\delta}\right]=0.012233-0.108641^2=0.000430133$$

So,

$$E\left[\ln\left[\tilde{S}(25)\right]\right]=25\cdot E\left[\tilde{\delta}\right]=2.716025$$
$$Var\left[\ln\left[\tilde{S}(25)\right]\right]=25\cdot Var\left[\tilde{\delta}\right]=0.010753$$

For accumulation greater than \$8,000, this is equivalent to:

$$\begin{aligned}\Pr\left[\tilde{S}(25)>16\right]&=\Pr\left[\ln\left(\tilde{S}(25)\right)>\ln(16)\right]\\&=\Pr\left[\frac{\ln\left(\tilde{S}(25)\right)-E\left[\ln\left[\tilde{S}(25)\right]\right]}{\sqrt{Var\left[\ln\left[\tilde{S}(25)\right]\right]}}>\frac{\ln(16)-E\left[\ln\left[\tilde{S}(25)\right]\right]}{\sqrt{Var\left[\ln\left[\tilde{S}(25)\right]\right]}}\right]\\&=\Pr\left[\tilde{Z}>\frac{\ln(16)-2.716025}{\sqrt{0.010753}}\right]\\&=\Pr\left[\tilde{Z}>0.545\right]\end{aligned}$$

where $\tilde{Z}$ is a standard normal variable with mean 0 and variance 1. Referring to tables for the standard normal distribution:

$$\Pr\left[\tilde{Z}>0.545\right]=0.293$$

For accumulation greater than $E\left[500\tilde{S}(25)\right]$, this is equivalent to:

$$
\begin{aligned}
\Pr\left[\tilde{S}(25) > 15.2\right] &= \Pr\left[\ln\left(\tilde{S}(25)\right) > \ln(15.2)\right] \\
&= \Pr\left[\frac{\ln\left(\tilde{S}(25)\right) - E\left[\ln\left[\tilde{S}(25)\right]\right]}{\sqrt{Var\left[\ln\left[\tilde{S}(25)\right]\right]}} > \frac{\ln(15.2) - E\left[\ln\left[\tilde{S}(25)\right]\right]}{\sqrt{Var\left[\ln\left[\tilde{S}(25)\right]\right]}}\right] \\
&= \Pr\left[\tilde{Z} > \frac{\ln(15.2) - 2.716025}{\sqrt{0.010753}}\right] \\
&= \Pr\left[\tilde{Z} > 0.051\right]
\end{aligned}
$$

where $\tilde{Z}$ is a standard normal variable with mean 0 and variance 1. Referring to tables for the standard normal distribution:

$$\Pr\left[\tilde{Z} > 0.051\right] = 0.480$$