Code: 4801

DEPARTMENT OF MATHEMATICS
University of Toronto

MAT 135Y
Term-Test #1
Tuesday, November 6, 2007
Time allowed: 90 minutes

Please PRINT in INK or BALL-POINT PEN:
(Please PRINT full name and UNDERLINE surname):

NAME OF STUDENT: _____

STUDENT NO.: _____

SIGNATURE OF STUDENT
(in INK or BALL-POINT PEN): _____

TUTORIAL CODE (e.g. M4A, R5D, etc.): _____

TUTORIAL TIME (e.g. T4, R5, F3, etc.): _____

NAME OF YOUR T.A.: _____

NOTE:

1. Before you start, check that this test has 13 pages. There are NO blank pages.
2. This test has two parts:
   PART A [50 marks]: 10 multiple choice questions
   PART B [50 marks]: 7 written questions
   Answers to both PART A and PART B are to be given in this booklet. No computer cards will be used.
3. No aids allowed.
   No calculators!

DO NOT TEAR OUT ANY PAGES.

FOR MARKERS ONLY	
QUESTION	MARK
PART A	/50
B1	/6
B2	/8
B3	/8
B4	/6
B5	/7
B6	/7
B7	/8
TOTAL	/100

PART A *[50 marks]*

Please read carefully:
PART A consists of 10 multiple-choice questions, each of which has exactly one correct answer. Indicate your answer to each question by **completely filling in the appropriate circle with a dark pencil.**
MARKING SCHEME: 5 marks for a correct answer, 0 for no answer or a wrong answer. You are not required to justify your answers in PART A. Note that for PART A, **only your final answers (as indicated by the circles you darken) count; your computations and answers indicated elsewhere will NOT count.**
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1. If $f(x) = \dfrac{4x-1}{2x+3}$, then $f^{-1}(x) =$

   ● $\dfrac{3x+1}{4-2x}$

   Ⓑ $\dfrac{4-2x}{3x+1}$

   Ⓒ $\dfrac{3x+1}{2x-4}$

   Ⓓ $\dfrac{3x+4}{2x-1}$

   Ⓔ $\dfrac{2x-4}{3x+1}$

2. Find the value of $\lim\limits_{x\to 1} \dfrac{x^2+x-2}{x^2+4x-5}$.

   Ⓐ 0

   Ⓑ $-\infty$

   Ⓒ $+\infty$

   ● $\dfrac{1}{2}$

   Ⓔ $-\dfrac{1}{2}$

3. Let

$$f(x) = \begin{cases} cx^3 + 3x & \text{if } x < 3 \\ x^2 - cx & \text{if } x \geq 3. \end{cases}$$

Find the value of the constant c so that f is continuous everywhere.

Ⓐ 3

● 0

Ⓒ 4

Ⓓ 1

Ⓔ 2

4. A ball is being thrown into the air. Its height above ground t seconds after it is thrown is $(40t - 16t^2)$ feet. Find the velocity of the ball when it is 16 feet above ground on its way up.

Ⓐ 26 ft/sec

Ⓑ 30 ft/sec

● 24 ft/sec

Ⓓ 20 ft/sec

Ⓔ 16 ft/sec

5. Find the value of $\lim_{x \to -\infty} \dfrac{5x - 1}{\sqrt{4x^2 + 9} - 3x}$.

Ⓐ $-\dfrac{5}{3}$

● -1

Ⓒ 0

Ⓓ -5

Ⓔ undefined

6. Find the value of $\lim_{x \to 0} \dfrac{x^2 \sec x}{\sec x - 1}$

Ⓐ $+\infty$

Ⓑ $-\infty$

Ⓒ $\dfrac{1}{2}$

Ⓓ 0

● 2

7. Suppose that $xy + e^y = e$. Find the value of $\dfrac{d^2y}{dx^2}$ at the point where $x = 0$.

Ⓐ $\dfrac{2}{e^2}$

Ⓑ not determinable due to insufficient data

Ⓒ $\dfrac{e^2}{2}$

● $\dfrac{1}{e^2}$

Ⓔ $\dfrac{1}{2e^2}$

8. If $y = \arctan\left(\sqrt{\dfrac{1+x}{1-x}}\right)$, then $\dfrac{dy}{dx} =$

Ⓐ $\dfrac{x}{2\sqrt{1-x^2}}$

Ⓑ $\sqrt{\dfrac{x}{1-x^2}}$

Ⓒ $\dfrac{-x}{2\sqrt{1-x^2}}$

● $\dfrac{1}{2\sqrt{1-x^2}}$

Ⓔ $\dfrac{-1}{2\sqrt{1-x^2}}$

9. Let $f(x) = \lim\limits_{t \to x} \dfrac{\csc t - \csc x}{t - x}$. Find the value of $f'\left(\dfrac{\pi}{4}\right)$.

Ⓐ $4\sqrt{2}$

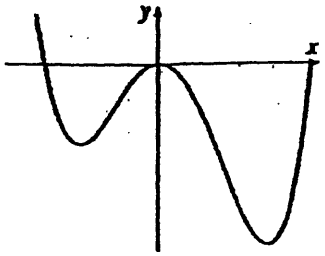
Ⓑ $5\sqrt{2}$

Ⓒ $2\sqrt{2}$

● $3\sqrt{2}$

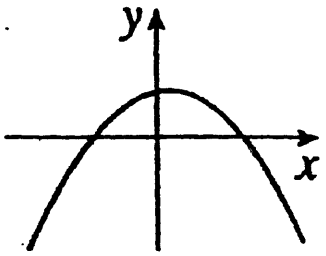
Ⓔ $\sqrt{2}$

10.

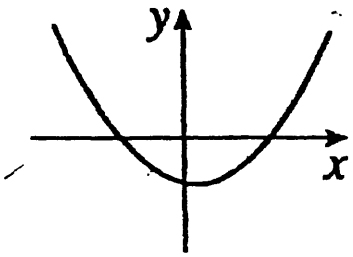


Given the graph of $y = f(x)$ shown above, select a graph which best represents the graph of $y = f'(x)$.

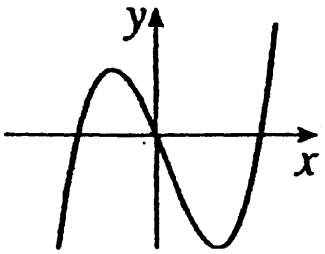
Ⓐ



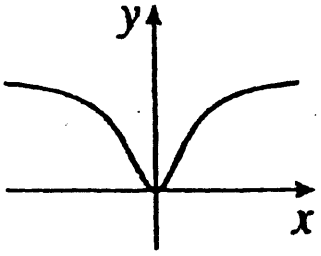
Ⓑ



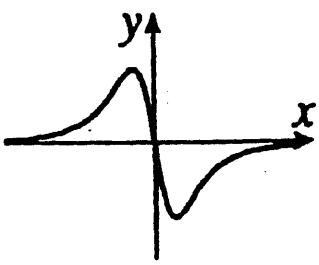
●



Ⓓ



Ⓔ



PART B *[50 marks]*

Please read carefully:
Present your complete solutions to the following questions in the spaces provided, in a neat and logical fashion, showing all your computations and justifications. Any answer in PART B without proper justification may receive very little or no credit. Use the back of each page for rough work only. If you must continue your formal solution on the back of a page, you should indicate clearly, in LARGE letters, "SOLUTION CONTINUED ON THE BACK OF PAGE _____". In this case, you may get credit for what you write on the back of that page, but you may also be penalized for mistakes on the back of that page.
MARKS FOR EACH QUESTION ARE INDICATED BY [ ].
DO NOT TEAR OUT ANY PAGES.

1. Let $f(x) = x^3$. Find $f'(x)$ from first principles (i.e. by using only the definition of the derivative).

[6]

2. Use any suitable method to find $\frac{dy}{dx}$ for each of the following. There is no need to simplify your final answers for this question.

   (a) $y = (1 + x^2) \arcsin x$.

[4]

   (b) $y = \dfrac{1 + e^{2x}}{2 + x^4}$.

[4]

3. For this question, simplify your final answers as much as possible.

(a) If $f(x) = \ln|5 - x^2|$, find $f'(2)$.

[4]

(b) If $f(x) = 2^{\sqrt{x}}$, find $f'(16)$.

[4]

4. Suppose that $y = f(x)$ satisfies $y^2 + xy = x^3 + 6x^2 - 1$. Find the value of $\frac{dy}{dx}$ at the point where $x = 1$, $y = 2$.

[6]

5. Let $f(x) = x^x$, for $x > 0$. For what value of x does the graph of f have a horizontal tangent?

[7]

Ans. $\frac{1}{e}$

6. Find the line passing through the point $(0, -8)$ and tangent to the curve $y = 4x^2 + 1$ (where $x \geq 0$) at some point.

[7]

7. (Note: This is not an easy question and will be marked very strictly.)

   Find the values of the constants a and b such that $\lim_{x \to 0} \dfrac{(ax+b)^{1/3} - 2}{x} = \dfrac{5}{12}$.

   NOTE: You are not allowed to use L'Hospital's Rule for this question.

[8]

Ans. $a = 5$, $b = 8$.