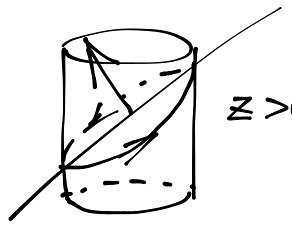
Use Stoke's thm to calculate

$\int_C [(x-z)dx + (x+y)dy + (y+z)dz]$. C is the ellipse

where the plane $z=y$ intersects the cylinder $x^2+y^2=1$ oriented counterclockwise as viewed from above

$$\int_{\partial S} \vec{F}\, d\vec{x} = \int_S (\nabla \times \vec{F})\, \vec{n}\, dA$$

$$\nabla \times \vec{F} = \begin{pmatrix} i & j & k \\ \partial_1 & \partial_2 & \partial_3 \\ x-z & x+y & y+z \end{pmatrix} = i - j + k$$



$z>0 \Rightarrow$ for normal vector

plane : $y - z = 0$

$\vec{n} \cdot dA = (-j + k)\, dx\, dy$

$$\int_S \vec{F} \cdot \vec{n}\, dA = \int_D \vec{F}\Big|_{\vec{G}(x,y)} \left| \frac{\partial \vec{G}}{\partial x} \times \frac{\partial \vec{G}}{\partial y} \right| dx\, dy$$

$$\int_S \nabla \times \vec{F} \cdot \vec{n}\, dA = 2 \cdot \int_D dx\, dy = 2\pi$$

↑ This is not a "typical" Stoke's thm problem

e.g. Stoke's : Calculate $\int_C y\,dx + y^2 dy + (x+2z)dz$. C is the curve of intersection of the sphere $x^2+y^2+z^2=a^2$ and the plane $y+z=a$ oriented counterclockwise as viewed from above.

$$\nabla \times \vec{F} = \begin{pmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ y & y^2 & x+2z \end{pmatrix} = 0i - j - k$$

$(0,1,1)$ $\qquad (\nabla \times \vec{F}) \cdot \vec{n}\, dA = (j + k)\, dx\, dy = -2\, dx\, dy$

$= -2 \int dA$

sub $z = a - y$ into $x^2+y^2+z^2 = a^2$

$\Rightarrow x^2 + y^2 + (a-y)^2 = a^2$ (an ellipse on xy-plane

$x^2 + 2(y^2 - ay + a^2/4) = a^2/2$

$\Rightarrow x^2 + 2(y - \frac{a}{2})^2 = \frac{a^2}{2}$

$\Rightarrow \frac{2x^2}{(a^2/2)} + \frac{(y - a/2)^2}{a^2/4} = 1$

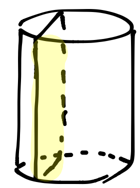
$A = a/\sqrt{2}$ Area $= \pi AB = \pi a^2/2\sqrt{2}$

$B = a/2$

• ① Find normal vector
② Find the domain (generally, it's a projection)

Given any unvertical plane parallel to x-axis, C is curve of intersection P with $x^2+y^2=a^2$

$$\int_C (yz-y)\,dx+(xz+x)\,dy=2\pi a^2$$



$z=by+c$

$$\text{Now find } C \begin{pmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ yz-y & xz+x & 0 \end{pmatrix}$$

$(-x\,i+y\,j+2\,k)=\vec{F}$

$\vec{n}=(0,-b,1)$

$$\int_C \vec{F}\cdot\vec{n}\,dA=\int_D(-by+2)\,dx\,dy=2\pi a^2$$

D: from 0 to a