5.8 indefinite integrals of vector derivatives or

anti derivatives of vector derivatives

Thm 5.60

vs

If $\int_C \mathbb{G}\cdot d\mathbf{x}$ is independent of path on S ← S is open connected ⇒ path connected 1.30

(i.e. it depends only on the initial & terminal pts)

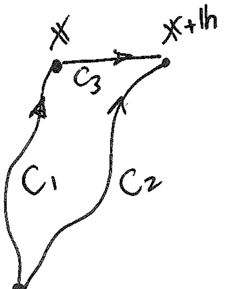
Then $\mathbb{G}=\nabla f$ for some f,

namely, fix $a\in S$, $f(\mathbf{x}) = $ "$\int_a^{\mathbf{x}} \mathbb{G}\cdot d\mathbf{x}$"

$= \int_C \mathbb{G}\cdot d\mathbf{x}$ any curve C that connects a to $\mathbf{x}$ ← such a path exists

Note: This definition is unique since $\mathbb{G}$ has indep of path property.

in the proof



$\int_{C_2} \mathbb{G}\cdot d\mathbf{x} = \int_{C_1} \mathbb{G}\cdot d\mathbf{x} + \int_{C_3} \mathbb{G}\cdot d\mathbf{x}$

independence of path

Thm 5.62

instead of $\mathbb{G}$ has indep of path we assume $\nabla\times\mathbb{G}=\mathbb{O}$

or in general ($\mathbb{R}^n$) $\left(\frac{\partial F_k}{\partial x_j} - \frac{\partial F_j}{\partial x_k}\right) = 0$

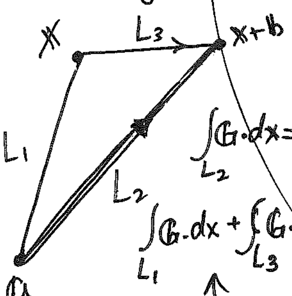
& S is convex

if S is convex & $a\in S$ & $\mathbf{x}\in S$ then a straight line $L(a,\mathbf{x})$ exists

straight line is unique among all curves

$f(\mathbf{x}) = \int_{L(a,\mathbf{x})} \mathbb{G}\cdot d\mathbf{x}$

in the proof



$\int_{L_2} \mathbb{G}\cdot d\mathbf{x} = \int_{L_1} \mathbb{G}\cdot d\mathbf{x} + \int_{L_3} \mathbb{G}\cdot d\mathbf{x}$

using Green's thm or Stokes thm b/c $\left(\frac{\partial F_k}{\partial x_j} - \frac{\partial F_j}{\partial x_k}\right)=0$ on the triangle . . .

Question: given $\mathbb{G}$ Solve $\nabla f = \mathbb{G}$

i.e find $f(\mathbf{x})$ st. grad $f = \mathbb{G}$

This is like indefinite integral or antiderivative

Note: 1. if $\mathbb{G}=\nabla f$ for some f

Then necessarily

$\int_C \nabla f\cdot d\mathbf{x} = f(\text{terminal pt}) - f(\text{initial pt})$ of C

or $\int_C \nabla f\cdot d\mathbf{x} = 0$ C ← Closed Curve

both mean independence of path in line integrals

Question (See Thm 5.60)

does $\int_C \mathbb{G}\cdot d\mathbf{x} = 0 \Rightarrow \mathbb{G}=\nabla f$ for some f ? C ← closed curve

Note 2 if $\mathbb{G}=\nabla f$ Then $\nabla\times\mathbb{G} = \nabla\times\nabla f = \mathbb{O}$

Question (See Thm 5.61)

does $\nabla\times\mathbb{G}=\mathbb{O}$ imply $\mathbb{G}=\nabla f$ for some f ?