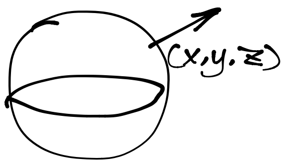
August 6th

P242

$F(x,y,z)=(x^2+y^2+z^2)(x\vec{i}+y\vec{j}+z\vec{k})$

S is sphere of radius a about origin.

Compute $\iint_S \vec{F}\cdot\vec{n}$ both directly & by DTHM

① 

$\dfrac{(x,y,z)}{\sqrt{x^2+y^2+z^2}}$ ← Vector field of unit outward normal vectors

$$\vec{F}\cdot\vec{n}=\frac{(x^2+y^2+z^2)\cdot(x^2+y^2+z^2)}{\sqrt{x^2+y^2+z^2}}=(x^2+y^2+z^2)^{3/2}=a^3$$

$$\iint_S \vec{F}\cdot\vec{n}\,dA=a^3\iint_S dA=4\pi a^5$$

② . . .

Prove $\iiint_R f\frac{\partial g}{\partial x}\,dV=-\iiint_R g\frac{\partial f}{\partial x}\,dV+\iint_{\partial R} fg\,n_x\,dA$

$\Longleftrightarrow \iiint_R\left(f\frac{\partial g}{\partial x}+g\frac{\partial f}{\partial x}\right)dV=\iint_{\partial R} fg\,n_x\,dA$

define $\vec{F}=(f\cdot g)\vec{i}$, then use divergence thm

$$\nabla(f\cdot g\vec{i})=\frac{\partial}{\partial x}(f\cdot g)=\frac{\partial f}{\partial x}g+f\frac{\partial g}{\partial x}$$