CSC165H1 S - Exercise 3

Yizhou Sheng, Student# 999362602

Rui Qiu, Student# 999292509

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Question 1:

Solution:

P	Q	R	$P \Rightarrow Q$	$Q \Rightarrow R$	$P \Rightarrow R$	$P \wedge Q$	$Q \wedge R$	(a) $P \Rightarrow (Q \Rightarrow R)$	(b) $Q \Rightarrow (P \Rightarrow R)$	(c) $(P \Rightarrow Q) \wedge (P \Rightarrow R)$	(d) $(P \wedge Q) \Rightarrow R$	(e) $P \Rightarrow (Q \wedge R)$
T	T	T	T	T	T	T	T	T	T	T	T	T
T	T	F	T	F	F	T	F	F	F	F	F	F
T	F	T	F	T	T	F	F	T	T	F	T	F
T	F	F	F	T	F	F	F	T	T	F	T	F
F	T	T	T	T	T	F	T	T	T	T	T	T
F	T	F	T	F	T	F	F	T	T	T	T	T
F	F	T	T	T	T	F	F	T	T	T	T	T
F	F	F	T	T	T	F	F	T	T	T	T	T

According to the truth tables, (a)(b)and(d) are equivalent; (c)and(e) are equivalent.

Question 2:

Proof:

$(P \Rightarrow Q) \vee (P \Rightarrow R)$

$\Leftrightarrow (\neg P \vee Q) \vee (\neg P \vee R)$......(Implication)

$\Leftrightarrow (\neg P \vee \neg P) \vee (Q \vee R)$......(Associativity)

$\Leftrightarrow \neg P \vee (Q \vee R)$......(Idempotency)

$\Leftrightarrow P \Rightarrow (Q \vee R)$......(Implication)

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Question 3:

Proof:

$(P \Rightarrow Q) \vee (Q \Rightarrow R)$

$\Leftrightarrow (\neg P \vee Q) \vee (\neg Q \vee R)$ ......(Implication)

$\Leftrightarrow (\neg P \vee (Q \vee \neg Q)) \vee R$ ......(Associativity)

$\Leftrightarrow \neg P \vee R \vee (Q \vee \neg Q)$ ......(Commutativity)

$\Leftrightarrow Q \vee \neg Q$ ......(Absorption)

$\because Q \vee \neg Q$ is always true no matter what the domain is.

$\therefore (P \Rightarrow Q) \vee (Q \Rightarrow R)$ is a tautology.

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Question 4:

Proof:

$P \wedge (P \vee Q)$

$\Leftrightarrow (P \wedge (Q \vee \neg Q)) \wedge (P \vee Q)$ ......(Identity)

$\Leftrightarrow P \wedge ((Q \vee \neg Q)) \wedge (P \vee Q))$ ......(Associativity)

$\Leftrightarrow P \wedge ((Q \vee \neg Q)) \wedge (Q \vee P))$ ......(Commutativity)

$\Leftrightarrow P \wedge (Q \vee (\neg Q \wedge P))$ ......(Distributivity)

$\Leftrightarrow P \wedge (Q \vee (P \wedge \neg Q))$ ......(Commutativity)

$\Leftrightarrow (P \wedge Q) \vee (P \wedge (P \wedge \neg Q))$ ......(Distributivity)

$\Leftrightarrow (P \wedge Q) \vee ((P \wedge P) \wedge \neg Q)$ ......(Associativity)

$\Leftrightarrow (P \wedge Q) \vee (P \wedge \neg Q)$ ......(Idempotency)

$\Leftrightarrow P \wedge (Q \vee \neg Q)$ ......(Distributivity)

$\Leftrightarrow P$ ......(Identity)

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Question 5:

<u>Solution</u>:

Since W represents the statement that "William cheated", X represents the statement that "Xavier cheated", Y represents the statement that "Youssef cheated", Z represents the statement "Zachary cheated".
Translate their words to logical expressions:

$S(William): X \Rightarrow Z$

$S(Xavier): W \wedge \neg Z$

$S(Youssef): \neg Y \wedge (W \vee Z)$

$S(Zachary): \neg W \Rightarrow Y$

(a) If each student is telling the truth:
From S(Xavier) we know that William cheated but Zachary did not.

From S(William), we know that $(X \Rightarrow Z) \Leftrightarrow (\neg Z \Leftrightarrow \neg X)$, so Xavier did not cheat.

$(W \vee Z)$ is true; to make S(Youssef) true, $\neg Y$ must be true, which means Youssef did not cheat.

In conclusion, if each student is telling the truth, then William cheated.

(b) ①Assume X is false, which means Xavier is telling the truth:
Then William cheated but Zachary did not.

$\neg S(William) \Leftrightarrow \neg(X \Rightarrow Z) \Leftrightarrow \neg(\neg X \vee Z) \Leftrightarrow (X \wedge \neg Z)$

Since Zachary did not cheat, $\neg Z$ is true. $\neg S(William)$ is true, so X is true,, which contradicts the assumption.

②Assume X is true, which means Xavier cheated and he is lying.
Then William did not cheat or Zachary cheated. (W is false or Z true)
Assume Z is true:

$\neg S(Zachary) \Leftrightarrow \neg(\neg W \Rightarrow Y) \Leftrightarrow \neg(W \vee Y) \Leftrightarrow (\neg W \wedge \neg Y)$ is true.

So $\neg W$ is true and $\neg Y$ is true, which means William did not cheat and Youssef did not cheat either.

W false means $(X \Rightarrow Z)$, does not contradict the assumption.

Y false means $(\neg Y \wedge (W \vee Z))$, at least one of William and Zachary cheated, actually it is Zachary, also does not contradict the assumption:

$$S(Youssef) \Leftrightarrow (\neg Y \wedge (W \vee Z)) \Leftrightarrow (True \vee \neg(False \vee True)) \Leftrightarrow True$$

Thus we have: W false, X true, Y false, Z true.

In conclusion, Xavier and Zachary cheated.