PROBLEM-SOLVING AND PROOFS: ASSIGNMENT 2
DUE FRIDAY, MARCH 17, 4PM.

Warm-up problems. These are completely optional.

(1) Prove that
$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2},$$
for all $n \in \mathbb{N}$.

(2) Prove that $2n - 8 < n^2 - 8n + 17$ for all $n \in \mathbb{N}$.

Problems to be handed in. Solve four of the following five problems. One of the four must be Problem (5).

(1) Find and prove a formula for
$$\sum_{i=1}^{n} \frac{1}{i(i+1)}.$$ 3.28

(2) Determine (with proof) the set of natural numbers for which the following inequalities hold.

(a) $3^{n+1} > n^4$. 3.49 (c) (d)
(b) $n^3 + (n+1)^3 > (n+2)^3$.

(3) Determine the set of positive real numbers x such that 3.48
$$x^n + n < x^{n+1}$$
for all $n = 1, 2, 3, \ldots$ correction: x^n+ x here 3.38

(4) Starting from 0, two players take turns adding 1, 2, or 3 to a single running total. The first player who brings the total to 1,000 or more wins. Prove that the second player has a winning strategy for this game.

(5) Recall that an L-tile is just a tile with three squares shaped like an L. We say a board admits an L-tiling if it is possible to completely cover it with L-tiles, such that each tile lies completely on the board, and no two tiles overlap. 3.58

(a) Prove that a $2^k \times 2^k$ chessboard with a single square in the lower left corner deleted admits an L-tiling, for any $k \in \mathbb{N}$.
(b) Prove that a $2^k \times 2^k$ chessboard with *any* single square deleted admits an L-tiling, for any $k \in \mathbb{N}$.