MAT 237, Quiz 4

NAME: STUDENT ID NUMBER :

Check your tutorial:

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Part A: (2 marks) Let $f : \mathbb{R}^n \to \mathbb{R}$ be of class C^{k+1}. Precisely state the definition of the k-th order Taylor Polynomial $P_{\mathbf{a},k}(\mathbf{h})$ used in Taylor's Theorem in Several Variables.

$$P_{\vec{a},k}(\vec{h}) = \sum_{|\alpha| \le k} \frac{\partial^\alpha f(\vec{a}) \vec{h}^\alpha}{\alpha!}$$

Part B: (3 marks) Compute $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ when z is determined as a function of y and x by the following equation (and for $z \neq 0$):

$$x^3 + 3y + z^2 - cosz = 0$$

taking $\frac{\partial}{\partial x}$: $3x^2 + 2z\frac{\partial z}{\partial x} + \sin z \frac{\partial z}{\partial x} = 0 \Rightarrow \frac{\partial z}{\partial x} = \frac{-3x^2}{2z + \sin z}$

taking $\frac{\partial}{\partial y}$: $3 + 2z\frac{\partial z}{\partial y} + \sin z \frac{\partial z}{\partial y} = 0 \Rightarrow \frac{\partial z}{\partial o} = \frac{-3}{2z + \sin z}$

Part C: (5 marks) Prove the Multinomial Theorem. That is, prove that for any $\mathbf{x} = (x_1, x_2, \dots, x_n) \in \mathbb{R}^n$ and positive interger k that

$$(x_1 + x_2 + \cdots + x_n)^k = \Sigma_{|\alpha|=k} \frac{k!}{\alpha!} \mathbf{x}^\alpha$$

Hint: Try induction on n and use the binomial theorem $(x_1 + x_2)^k = \Sigma_{j=0}^k \frac{k!}{j!(k-j)!} x_1^j x_2^{k-j}$ as a basis.

See Thm 2.52 in Folland