MAT 237, Quiz 5

NAME: STUDENT ID NUMBER :

Check your tutorial:

| ○ TUT5101 | ○ TUT5102 | ○ TUT5103 |
|---|---|---|
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Part A: (4 marks) What are the three representations of a smooth curve in space $\mathbb{R}^3$, and what does it mean for $S \subset \mathbb{R}^3$ to be a smooth curve near a point $\mathbf{a}$? Which condition on the parametric representation guarantees that it can be converted to the graph representation?

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i) graph : $y = f(x)$ & $z = g(x)$ [or any other permutations of the variables & $f, g \in C'$]
ii) locus : $F(x,y,z) = 0 = G(x,y,z)$, $F, G \in C'$
iii) parametric : $\vec{f}(t) = (x(t), y(t), z(t))$, $\vec{f} \in C'$

1 $S$ is a smooth curve if $\exists$ a nbd $N$ of $\vec{a}$, st. $S \cap N$ is the graph of a $C'$ curve (i)

$\boxed{\text{Regularity Condition is } \vec{f}'(t_0) \neq \vec{0}}$ 1

Part B: (2 marks) Is the parametric representation $\mathbf{f}(t) = (t(t^2-1), t(t-1), \sin(t\pi))$ a smooth curve near the point $(0,2,0)$? Explain why.

$\vec{f}(t) = (0,2,0) \Rightarrow t = -1$ and this is the only solution

1 $\vec{f}'(-1) = (2, -3, \pi) \neq 0$ implies the conversion from (iii) to (i) is possible.

1 Yes. Smooth curve near $(0,2,0)$

Part C: (4 marks) Prove, using the IFT that under the regularity assumption (as in part A), the parametric representation (iii) of a curve $\mathbf{f}(t) = (x(t), y(t), z(t))$ can be locally converted to the graph representation (i).

0.5

If $\vec{f}'(t_0) \neq 0$ Then one of the components of $\vec{f}'(t)$ must be non-zero. Say the first. Consider the function $F(x,t) = x - f_1(t)$.

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Let $x_0 = f_1(t_0)$, then $F(x_0, t_0) = 0$ and $\partial_t F(x_0, t_0) = -f_1'(t_0) \neq 0$

by IFT, one can find a (unique) way of representing $t$ as a function of $x$ in a nbd of $x_0$. so that $t = \varphi(x)$, $\varphi \in C'$ 0.5

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Then $y = f_2(t) = f_2(\varphi(x))$ & $z = f_3(t) = f_3(\varphi(x))$ and $f_2, f_3 \in C'$ which is the graph representation of a curve in $\mathbb{R}^3$.