Australian National University

RESEARCH SCHOOL OF FINANCE, ACTUARIAL STUDIES AND STATISTICS

First Semester Mid-Semester Examination (2018)

Survival Models/Biostatistics
(STAT 3032/4072/7042/8003)

Writing period:	*1.5 hours duration*
Study period:	*15 minutes duration*
Exam Conditions:	*Central Examination*
	Students must return the examination paper at the end of the examination
	This examination paper is not available to the ANU Library archives
Permitted materials:	*Non-programmable calculator, dictionary,*
	one A4 sized sheet of paper with notes on both sides
Total marks:	*50 marks*

INSTRUCTIONS TO CANDIDATES:

- *Students should attempt all questions.*
- *To ensure full marks show all the steps in working out your solutions. Marks may be deducted for failure to show appropriate calculations or formulae.*
- *All questions are to be completed in the script book provided.*
- *All answers should be rounded to 4 decimal places.*
- ***Based on the answer provided, your marks of question 2 could be negative.***

Question 1 [4 marks]

In 1729 de Moivre hypothesized the following force of mortality for an individual at age x (e.g. x is the future lifetime of an individual aged 0):

$$\mu_x = (m - x)^{-1}, \quad 0 \leq x < m.$$

Assuming this force of mortality holds,

(a) [**3 marks**] Calculate $S(x)$ the probability that an individual aged 0 survives to age x.

(b) [**1 mark**] Further explain in de Moivres law, why is x restricted to be in the range $0 \leq x < m$.

Question 2 [10 marks]

(For each part, you will gain 2 marks for a correct answer, be penalized 2 marks for an incorrect answer, and score 0 if no answer is given.) Answer each question "TRUE" or "FALSE". In each case, write the whole word. It is **not** acceptable to write only "T" or "F" and answers presented in this form **will be graded incorrect**.

(a) [**2 marks**] ${}_6p_{34} \cdot p_{33} \cdot (1 - q_{32})$ is equal to ${}_7p_{32}$.

(b) [**2 marks**] If the force of mortality function μ_x is assumed to follow Makehams law, this means that for each one year increment in age, μ_x increases by a constant scale.

(c) [**2 marks**] Gompertz Law is appropriate for modelling the force of mortality for humans over the age range 0 to 40 years.

(d) [**2 marks**] Treating censored observations as times of death can result in underestimating the survival function (e.g. $\hat{S}(t)$ is smaller than the true value $S(t)$ for some t).

(e) [**2 marks**] The complete expected future lifetime e_x^0 must be greater than or equal to the curtate expectation of life e_x.

Question 3 [7 marks]

For a particular population it is shown that $l_x = 50 - 0.5x$, $0 \leq x \leq 100$. Using this information about the number of lives aged x exact, calculate the following:

(a) [**2 marks**] the force of mortality at age 30.

(b) [**2 marks**] the complete expectation of life at age 30.

(c) [**3 marks**] the average age of individuals who die between ages 60 and 65.

Question 4 [12 marks]

Data are available from a small study on claim incidence. A subset of policy-holders all aged 50 with no previous claims history is monitored. The data, times to claim (in months), are given in the table below; the * indicates that an observation was censored.

$$2,\ 3^*,\ 3^*,\ 8,\ 12,\ 14^*,\ 16,\ 21$$

(a) [**5 marks**] Calculate the Kaplan-Meier estimate of the survivor function $S(t)$ for these policyholders. You should also provide standard errors for your estimated function.

(b) [**2 marks**] Roughly plot your estimates of the survivor function, you should label all the survival functions and times at death.

(c) [**2 marks**] Estimate $S(4)$ and explain why the estimates of $S(4)$ and $S(5)$ are the same.

(d) [**3 marks**] Provide an estimate of the mean time to claim for policyholders.

Question 5 [7 marks]

The Uniform Distribution of Deaths (UDD) assumes that the pdf of lifetime T_x follows a uniform distribution for $0 < t < 1$.

(a) [**2 marks**] Show that UDD implies that ${}_sq_x = s \cdot q_x$, where $0 < s < 1$.

(b) [**2 marks**] Demonstrate that for any $0 \leq a < b$, ${}_{b-a}q_{x+a} = 1 - \frac{{}_bp_x}{{}_ap_x}$.

(c) [**3 marks**] Prove that for $0 \leq a < b \leq 1$, we have ${}_{b-a}q_{x+a} = \dfrac{(b-a)q_x}{1 - a \cdot q_x}$ under UDD and use it to calculate ${}_{0.1}q_{x+0.5}$, given that ${}_{0.8}q_x = 0.2$.

Question 6 [10 marks]

The lifetimes of a certain species of insect, denoted x, are believed to follow a Pareto distribution. The density of the Pareto distribution is given by:

$$\frac{\theta\lambda^\theta}{x^{\theta+1}}, \theta > 0, \lambda > 0, x \geq \lambda.$$

A sample of six insects had the following survival times: 4, 5.5, 6.5, 7, 8, 11. It is known that $\lambda = 1$ for this species of insect.

(a) [**3 marks**] Compute the maximum likelihood estimate of θ

(b) [**3 marks**] Compute an approximate 95% confidence interval for θ. Comment on the appropriateness of your confidence interval.

(c) [**4 marks**] Estimate the probability that an insect will survive for more than 2 days. Provide a standard error for your estimate.

END OF EXAMINATION