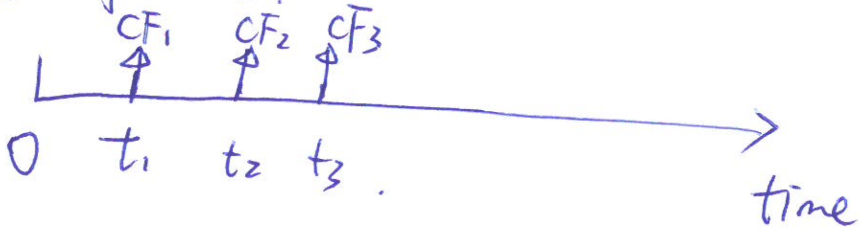
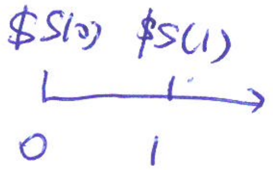
Cash flows.



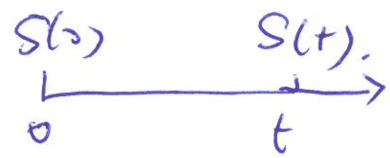
$$V_t(CF_1, CF_2 \ldots CF_T \ldots)$$

Interest rates.

effective interest rate.

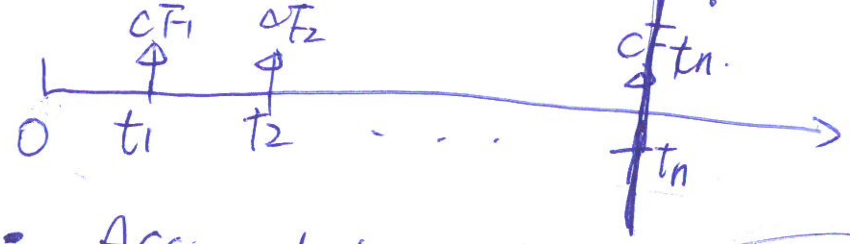


$$\frac{\$S(1) - S(0)}{S(0)}.$$



$$\begin{cases} \text{simple} & S(t) = S(0)\cdot(1 + \underline{i}\cdot t) \\ \text{compound} & S(t) = S(0)\cdot(1+i)^t. \end{cases}$$

Accumulated Value ?



- Accumulation factor $A(t_1, t_2) = \dfrac{S(t_2)}{S(t_1)}$

$$\Rightarrow \quad S(t_2) = A(t_1, t_2)\cdot S(t_1)$$

- The principle of consistency.

$$A(0, t_n) = A(0,t_1) \cdot A(t_1, t_2) \cdots A(t_{n-1}, t_n).$$

Present Values:

* present value factor / discount factor

$$X(1+i) = 1 \Rightarrow X = \frac{1}{1+i} = \boxed{v} \triangleq v_i$$

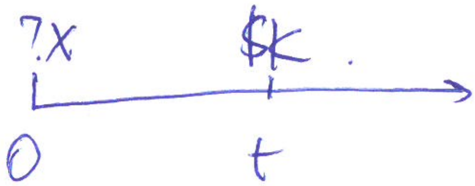
* 

$$\boxed{\frac{1}{A(0,t)} = v^t} = \left(\frac{1}{1+i}\right)^t.$$

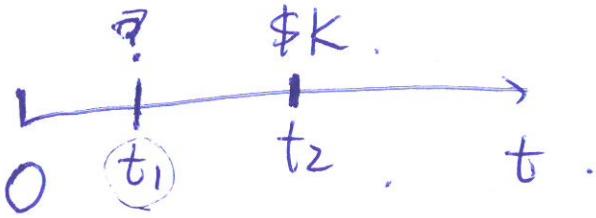
$$\$1 \times (1+i)^t = A(0,t).$$

$$\boxed{\begin{aligned} v^{-t} &= A(0,t). \\ A^{-1}(0,t) &= v^t. \end{aligned}}$$

Ex:



$$X = K\cdot\frac{1}{(1+i)^t} = K\cdot\frac{1}{A(0,t)} = K\cdot v^t$$



(i).

$$\boxed{\begin{aligned} PV_{t=t_1} &= K\cdot\left(\frac{1}{1+i}\right)^{t_2-t_1} \\ &= K\cdot v^{t_2-t_1} \end{aligned}}$$

Ex.

8% annual compound.

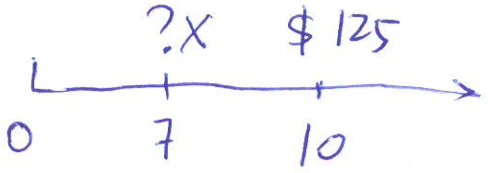
$$PV(0) = 10{,}000 \times (1+8\%)^{-8}$$

Ex $PV(4) = 10{,}000 \times (1+8\%)^{-4}$ $= 8-4$

Rounding

- Intermidrate steps: 5.
- overall results: .

Q7:



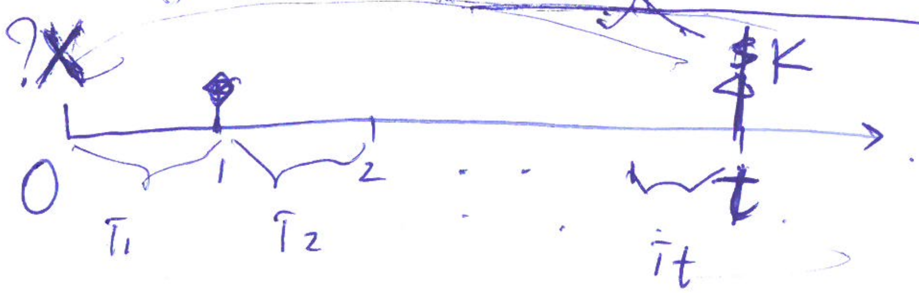
$$X = 125 \cdot (1+5\%)^{-3} \quad -(10-7)$$

$$= 125 \cdot v^3_{0.05}$$

$$= 107.98$$

---

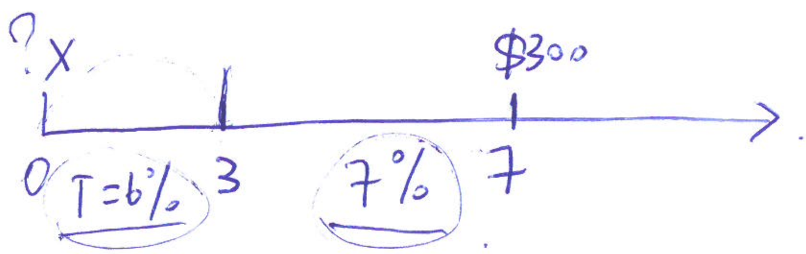
* Investing with different interest rates



$$X(1+i_1)(1+i_2)\cdots(1+i_t) = K$$

$$\Rightarrow \quad X = \frac{K}{(1+i_1)\cdots(1+i_t)}$$

Ex:



$$X(1+6\%)^3(1+7\%)^{(7-3)} = 300$$

$$\Rightarrow \quad X = \frac{300}{(1+6\%)^3 \cdot (1+7\%)^4}$$

* effective interest rates

frequency	compounding periods (1 year)
annual	1
semiannual	2
Quarterly	4
monthly	12
weekly	52
Daily	365

$100 ⟶ $? $= 100(1+12\%) = 112$

0 ⟶ 1 year

12% effective annual interest

$100 ⟶ $? $= 100(1+1\%)^{12} = 112.68$

0, i ⟶ 12 months

1% effective monthly interest

$$\boxed{100(1+i)^{12} = 112}$$

$$\Downarrow$$

$$i \cong 0.009489$$

equivalent effective interest rate j

$\underbrace{\text{effective}}_{\text{monthly}}$

to an effective interest rate i

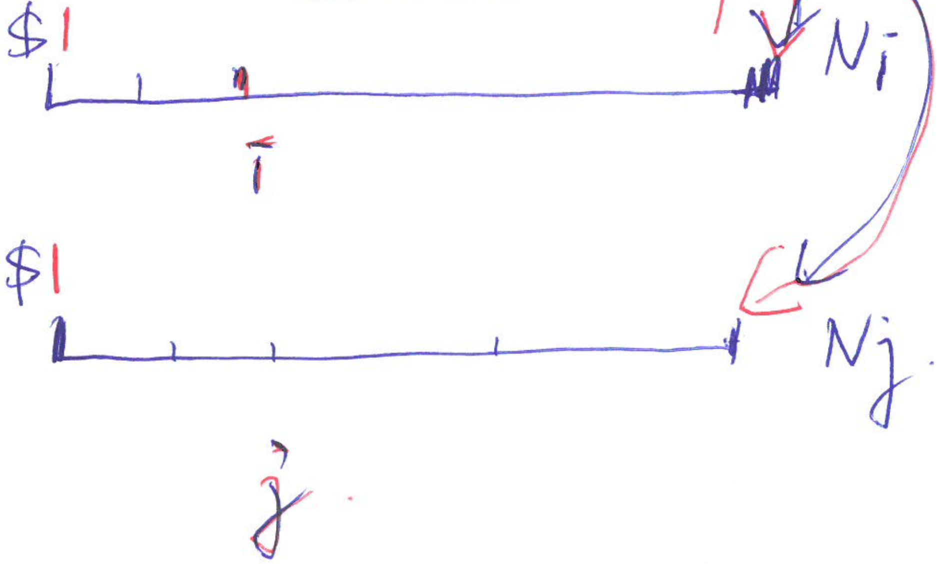
$\underbrace{\text{effective}}_{\text{annual}}$

compounding periods of j > compounding periods of i

$= 12 = \boxed{N_j}$; $= 1 = \boxed{N_i}$

$$(1+i) = (1+j)^{\frac{N_j}{N_i}}$$

$$\Longleftrightarrow \boxed{(1+i)^{N_i}} = \boxed{(1+j)^{N_j}}$$



Ex: effective monthly rate is 1%

$\Rightarrow$ . . . weekly rate :

$\boxed{\frac{52}{12}}$

$i = 1\%$.

$(1+1\%)^{12} = (1+j)^{52}$.

$\Rightarrow \quad j = (1+1\%)^{\frac{12}{52}} - 1$

$\Uparrow$

$\Rightarrow (1+1\%)^{1} = (1+j)^{\frac{52}{12}}$.