NAME (PRINT): ______________________________
Last/Surname First /Given Name

STUDENT #: ____________________ **SIGNATURE:** ____________________

UNIVERSITY OF TORONTO MISSISSAUGA
APRIL 2013 FINAL EXAMINATION
STA305H5S
Experimental Design
Alison Weir
Duration - 3 hours
Aids Allowed: Calculators; 2 pages of double-sided Letter (8-1/2 x 11) sheet;
Aids Provided: Statistical Tables

The University of Toronto Mississauga and you, as a student, share a commitment to academic integrity. You are reminded that you may be charged with an academic offence for possessing any unauthorized aids during the writing of an exam, including but not limited to any electronic devices with storage, such as cell phones, pagers, personal digital assistants (PDAs), iPods, and MP3 players. Unauthorized calculators and notes are also not permitted. Do not have any of these items in your possession in the area of your desk. Please turn the electronics off and put all unauthorized aids with your belongings at the front of the room before the examination begins. If any of these items are kept with you during the writing of your exam, you may be charged with an academic offence. A typical penalty may cause you to fail the course.

Please note, you ***CANNOT*** *petition to* ***re-write*** *an examination once the exam has begun.*

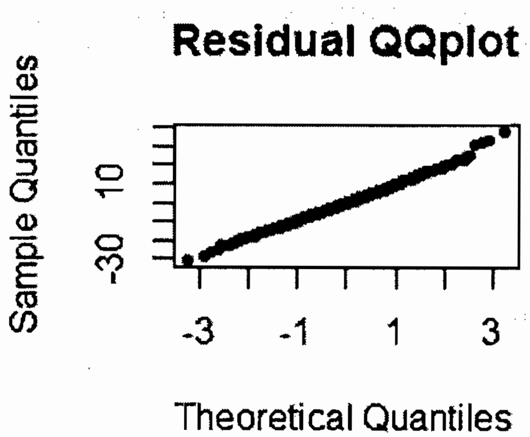
Instructions

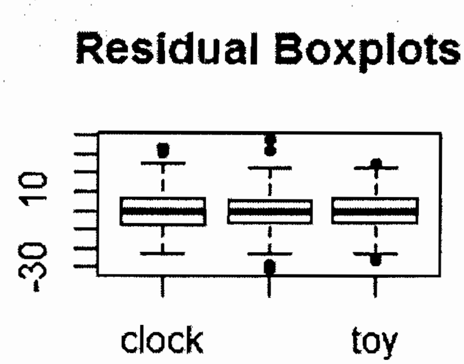
1. This exam has 16 pages, please be sure you have all 16 pages.
2. If you change an answer, be sure to clearly delete the answer you don't want to be marked. If you give two answers to any question, no marks will be awarded. This applies even if one of the answers is correct.
3. If you're asked to explain something, a detailed statistical answer is required.
4. If you're asked to explain something in plain English, your answer should not contain any statistical jargon. You will earn no marks if statistical terminology is included in such an answer.
5. Final answers should be correct to four decimal places. This means you'll need to carry more decimal places through the calculations.

Best Wishes!

1. *(12 marks)* A study was conducted to investigate the mean lifetimes of a particular brand of AAA batteries under constant use in a specific device. One production run of 800 AAA high-current-drain alkaline batteries was used in the study. 200 batteries were used in clocks, 300 batteries were used in toy cars, and 300 batteries were used in flashlights. The response is battery lifetime (in hours) and the factor is device (clock, toy, flashlight).

```
> model<-lm(lifetime~device,data=battery)

> summary(model)
Coefficients:
                  Estimate Std. Error t value Pr(>|t|)
(Intercept)        32.4970     0.6873  47.283  < 2e-16 ***
deviceflashlight    2.9923     0.8873   3.372 0.000781 ***
devicetoy           1.7613     0.8873   1.985 0.047475 *
---
Residual standard error: 9.72 on 797 degrees of freedom
Multiple R-squared: 0.01407,    Adjusted R-squared: 0.0116
F-statistic: 5.688 on 2 and 797 DF,  p-value: 0.003525

> anova(model)
           Df Sum Sq Mean Sq F value   Pr(>F)
device      2   1075  537.35   5.688 0.003525 **
Residuals 797  75293   94.47

> TukeyHSD(aov(model))
  Tukey multiple comparisons of means
    95% family-wise confidence level
                     diff        lwr       upr     p adj
flashlight-clock  2.992333  0.9089255 5.0757411 0.0022506
toy-clock         1.761333 -0.3220745 3.8447411 0.1165261
toy-flashlight   -1.231000 -3.0944566 0.6324566 0.2677179

> tapply(battery$lifetime,battery$device,mean)
     clock flashlight        toy
  32.49700   35.48933   34.25833

> tapply(battery$lifetime,battery$device,var)
     clock flashlight        toy
 107.35296   93.64878   86.71956
```





a. What is the factor effects model? Be sure to include ranges for all subscripts.

b. What distributional assumptions are needed to validate hypothesis testing?

c. Are there significant differences between the three mean lifetimes? Answer *yes* or *no*, and give the p-value that justifies your answer.

d. Briefly comment on the residual qq plot. Regardless of your comments in this part, for the remainder of this question assume the plot is acceptable.

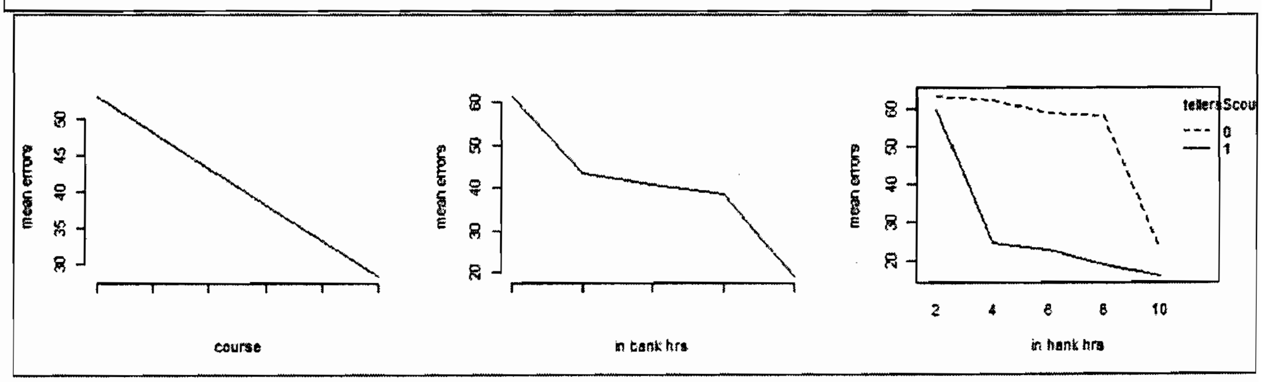
e. Briefly comment on the residual boxplots. Regardless of your comments in this part, for the remainder of this question assume these plots are acceptable.

f. Summarize the conclusions of the Tukey analysis.

g. Test the linear contrast that contrasts the mean lifetimes of AAA batteries in clocks and toy cars against the mean life time of AAA batteries in flashlights. In your answer (i) define the contrast symbolically, (ii) give a point estimate of the contrast, (iii) state the null and alternative hypothesis, (iv) calculate the test statistic, (v) state the distribution of the test statistic under the null hypothesis, (vi) find an approximate p-value for the test, and (vii) state your conclusion.

2. *(14 marks)* The vice president of a national bank decided to investigate the benefits of teller training. Bank staff assembled data on a random selection of 100 new tellers. The following independent variables were recorded for each of the selected tellers: if they took a two-week training course (*course* yes = 1, no = 0), the number of weeks of one-on-one in branch training (*inbank* 2, 4, 6, 8, or 10). The response was a measure of each teller's errors: the larger of the sum of their overages or the sum of their shortages over a one-week period (in dollars).

```
> modell<-lm(errors~factor(course)*factor(inbank),data=tellers)
> anova(modell)
                              Df  Sum Sq Mean Sq F value    Pr(>F)
factor(course)                 1 15265.8 15265.8 118.673 < 2.2e-16 ***
factor(inbank)                 4 18168.7  4542.2  35.310 < 2.2e-16 ***
factor(course):factor(inbank)  4  6376.4  1594.1  12.392 4.523e-08 ***
Residuals                     90 11577.4   128.6

> model2<-lm(errors~factor(course)*inbank,data=tellers)
> anova(model2)
                      Df  Sum Sq Mean Sq F value    Pr(>F)
factor(course)         1 15265.8 15265.8 73.0113 1.974e-13 ***
inbank                 1 16020.5 16020.5 76.6206 7.067e-14 ***
factor(course):inbank  1    29.6    29.6  0.1413    0.7078
Residuals             96 20072.5   209.1

> plot(tapply(tellers$errors,tellers$course,mean),type="l",xlab="course",
         + ylab="mean errors")
> plot(tapply(tellers$errors,tellers$inbank,mean),type="l",
         + xlab="in bank hrs",ylab="mean errors")
> interaction.plot(tellers$inbank,tellers$course,tellers$errors,
         + xlab="in hank hrs",ylab="mean errors")

> tapply(tellers$errors,tellers$course,mean)
      0       1
53.0628 28.3518

> tapply(tellers$errors,tellers$inbank,mean)
      2       4       6       8      10
61.5935 43.3445 40.8145 38.5365 19.2475

> tapply(tellers$errors,interaction(tellers$course,tellers$inbank),mean)
   0.2    1.2    0.4    1.4    0.6    1.6    0.8    1.8   0.10   1.10
63.462 59.725 62.243 24.446 58.715 22.914 58.277 18.796 22.617 15.878
```



a. Two models have been fit to the data. What is the difference between the two models? A conceptual answer is required here - lots of ideas and no numbers.

b. Which model should be used? Explain why.

Please answer parts (c)-(i) of this question using the model you selected in part (b). You will not lose marks in parts (c)-(i) if your answer to part (b) was wrong.

c. Is there a significant course-by-inbank interaction effect? Answer yes or no and give the p-value that justifies your answer.

d. Describe, in plain English, the nature of the interaction. Marks *will* be deducted if you use any statistical terminology.

e. Is there a significant course main effect? Answer yes or no and give the p-value that justifies your answer.

f. Describe, in plain English, the nature of the main effect for course. Marks *will* be deducted if you use any statistical terminology.

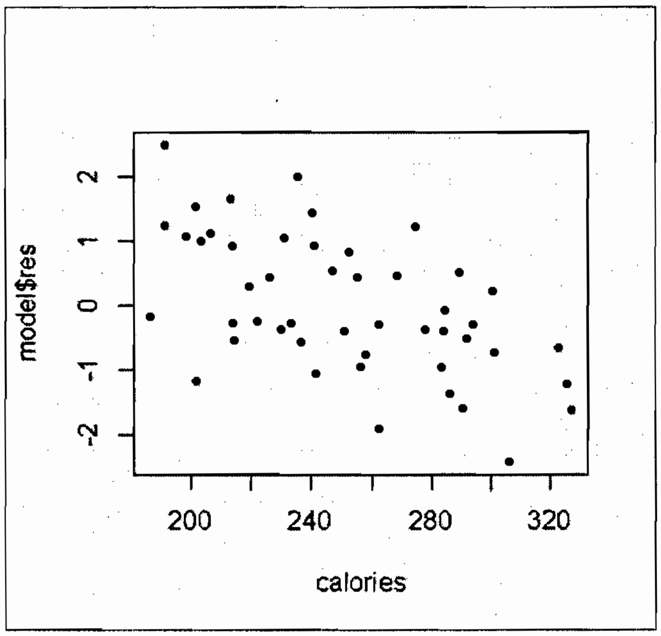
g. Is there a significant inbank main effect? Answer yes or no and give the p-value that justifies your answer.

h. Describe, in plain English, the nature of the main effect for in bank training time. Marks *will* be deducted if you use any statistical terminology.

i. What training program would you recommend? Comment on the course and length of in bank component of your selected program.

3. *(14 marks)* A researcher studied the sodium content in beer by selecting at random six brands, one from each of six randomly selected microbreweries. The researcher then chose eight 12-ounce cans or bottles of each selected brand at random from retail outlets in the area and measured the sodium content (in milligrams) of each can or bottle. (I found these beer names in a Huffington Post article called The Most Ridiculous Beer Names– they're real ☺. The data is not real.)

```
> tapply(beer$sodium,beer$brand,mean)
ArrogantBastard      KiltLifter      MooseDrool SeriouslyBadElf     SpicyFishWife       YellowSnow
       23.61851        27.41002        25.82032        26.48243          24.90880         25.55207
> tapply(beer$sodium,beer$brand,var)
ArrogantBastard      KiltLifter      MooseDrool SeriouslyBadElf     SpicyFishWife       YellowSnow
      1.8110847       1.3643812       0.9412613       0.8483096         1.6727765        1.0493141

> model<-lm(sodium~brand,data=beer)

> anova(model)
          Df Sum Sq Mean Sq F value    Pr(>F)
brand      5 68.029 13.6058   10.62 1.223e-06 ***
Residuals 42 53.810  1.2812

> plot(beer$calories,model$res,pch=20)
```



a. State the factor effects model that was used. Be sure to include ranges for all subscripts.

b. What distributional assumptions are necessary for hypothesis testing?

c. What are the hypotheses being tested in the ANOVA table?

d. What is your conclusion to the test in part (c)? Give the p-value that justifies your answer.

e. What is a point estimate of the variance of sodium content within one brand of beer?

f. What is a point estimate of the variance of sodium content between different brands of beer?

g. Give a point estimate of the proportion of variance in sodium content that is attributable to the different brands.

h. Suggest a sensible block factor for this analysis. Justify your selection.

i. Look at the plot of the ANOVA residuals against the calories in each of the 48 beers. Does it appear that calories would be a useful addition to the model? Explain your answer.

4. *(7 marks)* A nutrition scientist conducted an experiment to evaluate the effects of four vitamin supplements on the weight gain of laboratory rats. Since caloric intake will differ among rats and influence weight gain the investigator measured the caloric intake of each animal. For each animal the investigator recorded: weight gain (grams), caloric intake (calories/10), and dietary vitamin supplement (1, 2, 3, 4).

```
> model1<-lm(weight~calories*factor(diet),data=vitamin)
> model2<-lm(weight~calories+factor(diet),data=vitamin)
> model3<-lm(weight~factor(diet),data=vitamin)
> model4<-lm(weight~calories,data=vitamin)

> anova(model1)
Response: weight
                      Df  Sum Sq Mean Sq F value  Pr(>F)
calories               1  391.13  391.13  4.4297 0.05707 .
factor(diet)           3 1501.05  500.35  5.6667 0.01182 *
calories:factor(diet)  3  119.06   39.69  0.4495 0.72229
Residuals             12 1059.56   88.30
---
> anova(model2)
Response: weight
             Df  Sum Sq Mean Sq F value   Pr(>F)
calories      1  391.13  391.13  4.9778 0.041361 *
factor(diet)  3 1501.05  500.35  6.3678 0.005352 **
Residuals    15 1178.62   78.57
---
> anova(model3)
Response: weight
             Df  Sum Sq Mean Sq F value Pr(>F)
factor(diet)  3  802.00  267.33  1.8853 0.1728
Residuals    16 2268.80  141.80

> anova(model4)
Response: weight
          Df  Sum Sq Mean Sq F value Pr(>F)
calories   1  391.13  391.13  2.6273 0.1224
Residuals 18 2679.67  148.87
```

a. Is the interaction of calories and diet necessary in the model? Answer yes or no, and give the observed F statistic that justifies your answer. For the remainder of this question assume the interaction is not necessary.

b. Can calories be dropped from the additive model containing calories and diet? Answer yes or no and give the p-value that supports your answer.

c. Can diet be dropped from the additive model containing calories and diet? Answer yes or no and give the p-value that supports your answer.

d. Four models have been presented in the R output. Which of these models would you select? Why?

e. Describe, geometrically, the model you selected in part (d). A sketch is acceptable.

5. *(6 marks)* The ANOVA table for a two-way crossed design with fixed factors is given below.

Source	DF	SS	MS	F
A	2	70.191	35.095	23.88
B	1	290.683	290.683	197.80
Interaction	2	18.160	9.080	6.18
Error	6	8.817	1.470	
Total	11	387.851		

a. A naive statistician constructed the above ANOVA table. Factors A and B are both fixed, but Factor A should be nested within Factor B. What is the correct sum of squares for assessing the effect of Factor A?

b. A naive statistician constructed the above ANOVA table. Factor A is random and Factor B is fixed. What is the correct observed F statistic for assessing the effect of Factor B?

6. *(9 marks)* A two way balanced (n_{ij}=2) crossed, fixed factor, experiment was conducted and the following treatment means were calculated.

	B_1	B_2	B_3
A_1	3	6	12
A_2	2	4	8
A_3	1	6	13
A_4	7	9	11

a. What is $\overline{y}_{23\cdot}$?

b. What is $\overline{y}_{\cdot 3\cdot}$?

c. Estimate $\mu_{\cdot\cdot}$?

d. Estimate β_3.

e. Estimate$(\alpha\beta)_{23}$

7. *(4 marks)* Which surgical procedure is best? Two surgical procedures are being compared. Patients are randomly assigned to one, or the other, of the two treatments. Five different surgical teams are used. To prevent possible confounding of treatment and surgical team, each team is trained in both procedures, and each team performs equal numbers of surgery of each of the two types.

   a. Name, or clearly describe, the two factors in this experiment.

   b. For each of the factors: is it fixed or random?

   c. Are the two factors crossed? Or are they nested? If there is nesting, which factor is nested within the other?

8. *(4 marks)* Do interest rates on new car loans vary from city to city? To investigate this question, nine car models were selected and a dealership for each model was randomly selected in each of six selected urban centres in Canada. The new car interest rate was recorded for each of the 90 dealerships.

   a. Name, or clearly describe, the two factors in this experiment.

   b. For each of the factors: is it fixed or random?

   c. Are the two factors crossed? Or are they nested? If there is nesting, which factor is nested within the other?

9. *(20 marks)* Give a clear and concise answer to each of the following.

   a. 300 subjects are available for an experiment with three treatments. The subjects will be randomly assigned to the treatments; they can be assigned evenly (i.e., 100/100/100), or unevenly (e.g., 75/120/105). Which would you do – split evenly or unevenly? Describe the advantages and disadvantages of each approach, in statistical and in practical terms.

   b. A medical researcher assigns a placebo to the healthiest patients in a study of the efficacy of two new drugs. Describe the advantages and disadvantages of this approach, both in statistical and in practical terms.

   c. A two-factor designed experiment, with interaction, is a linear model. What do we mean by the word linear? Your answer should be a mathematical explanation.

d. What is the purpose of a concomitant variable in an ANCOVA design? Describe, statistically, the chain of events that is expected when a useful concomitant variable is introduced to an experiment.

e. A two factor design has factor B nested within factor A. The "B within A" effect is significant. Describe, in detail, the next step of the analysis (it's not diagnostic assessment). There should be some equations in your answer.

Total Marks: 90