PREDICATE LOGIC DERIVATIONS FOR UNIT 6   ANSWERS for 1-18

1. $\exists x(Gx \wedge \sim Hx)$. $\forall x(Hx \leftrightarrow \sim Bx)$. $\therefore\ \forall x(Gx \rightarrow \exists y(Gy \wedge By))$

1	~~Show~~ $\forall x(Gx \rightarrow \exists y(Gy \wedge By))$		
2	~~Show~~ $Gx \rightarrow \exists y(Gy \wedge By)$		
3	Gx	ass cd	
4	$Gi \wedge \sim Hi$	pr1 ei	(make sure you use a new variable!)
5	$Hi \leftrightarrow \sim Bi$	pr2 ui	(make it match line 4!)
6	$\sim Bi \rightarrow Hi$	5 bc	
7	$\sim Hi$	4 sr	
8	$\sim\sim Bi$	6 7 mt	
9	Bi	8 dn	
10	Gi	4 sl	
11	$Gi \wedge Bi$	9 10 adj	
12	$\exists y(Gy \wedge By)$	11 eg	(make it match show line 1!)
13		12 cd	
14		2 ud	

2. $\exists xAx \rightarrow \exists xGx$. $\forall y(Jy \rightarrow Hy)$. $\sim\exists x(\sim Jx \vee Cx) \vee \forall xFx \therefore \forall x(Gx \rightarrow \sim Hx) \rightarrow \forall x(Ax \rightarrow Fx)$

1	~~Show~~ $\forall x(Gx \rightarrow \sim Hx) \rightarrow \forall x(Ax \rightarrow Fx)$		
2	$\forall x(Gx \rightarrow \sim Hx)$	ass cd	
3	~~show~~ $\forall x(Ax \rightarrow Fx)$		
4	~~show~~ $Ax \rightarrow Fx$		show instantiation of 3
5	Ax	ass cd	
6	$\exists xAx$	5 eg	make it match pr1
7	$\exists yGx$	6 pr1 mp	
8	Gi	7 ei	use a new variable
9	$Gi \rightarrow \sim Hi$	2 ui	make it match 8
10	$\sim Hi$	8 9 mp	
11	$Ji \rightarrow Hi$	pr2 ui	make it match 10
12	$\sim Ji$	10 11 mt	
13	$\sim Ji \vee Ci$	12 add	
14	$\exists x(\sim Jx \vee Cx)$	13 eg	make it match pr3
15	$\sim\sim\exists x(\sim Jx \vee Cx)$	14 dn	
16	$\forall xFx$	15 pr3 mtp	
17	Fx	16 ui cd	
18		4 ud	
19		3 cd	

3. $\exists x Gx.\quad \forall x(Gx \leftrightarrow \sim Dx).\quad \sim\forall y Dy \rightarrow \forall x(\sim Cx \rightarrow Ax)\quad \therefore \forall y(\sim Ay \rightarrow Cy)$

1	~~Show~~ $\forall y(\sim Ay \rightarrow Cy)$		
2	~~Show~~ $\sim Ay \rightarrow Cy$		show instantiation of 1
3	$\sim Ay$	ass cd	
4	Gi	pr1 ei	use EI asap. Use a new variable.
5	$Gi \leftrightarrow \sim Di$	pr2 ui	make it match 4
6	$Gi \rightarrow \sim Di$	5 bc	
7	$\sim Di$	4 6 mp	
8	~~Show~~ $\sim\forall y Dy$		show antecedent of pr3
9	$\forall y Dy$	ass id	
10	Di	9 ui	make it match 7
11	$\sim Di$	7 r 10 id	
12	$\forall x(\sim Cx \rightarrow Ax)$	8 pr3 mp	
13	$\sim Cy \rightarrow Ay$	12 ui	make it match 3
14	$\sim\sim Cy$	3 13 mt	
15	Cy	14 dn cd	
16		2 ud	

4. $\exists x(Bx \vee Cx).\quad \forall x(Fx \vee Hx).\quad \forall x Fx \rightarrow \forall x \sim Cx.\ \therefore \sim\exists z Hz \rightarrow \exists x Bx$

1	~~Show~~ $\sim\exists x Hx \rightarrow \exists x Bx$		
2	$\sim\exists z Hz$	ass cd	
3	$Bi \vee Ci$	ei	use EI as soon as possible.
4	~~Show~~ $\forall x Fx$		show antecedent of Pr3.
5	~~Show~~ Fx		show instantiation of 4
6	$Fx \vee Hx$	pr2 ui	make it match line 4
7	~~show~~ $\sim Hx$		
8	Hx	ass id	
9	$\exists z Hz$	8 eg	make it match line 2
10	$\sim\exists z Hz$	2 r	
11		9 10 id	
12	Fx	6 7 mtp dd	
15		5 ud	
16	$\forall x \sim Cx$	4 pr3 mp	
17	$\sim Ci$	15 ui	make it match line 3
18	Bi	3 17 mtp	
19	$\exists x Bx$	18 eg cd	make it match show line 1.

5. $\therefore (\forall x(Ax \rightarrow {\sim}Bx) \wedge \exists x(Bx \vee {\sim}Aa)) \rightarrow \exists x{\sim}(Ax \wedge Cx)$

1	~~Show~~ $(\forall x(Ax \rightarrow {\sim}Bx) \wedge \exists x(Bx \vee {\sim}Aa)) \rightarrow \exists x{\sim}(Ax \wedge Cx)$		
2	$\forall x(Ax \rightarrow {\sim}Bx) \wedge \exists x(Bx \vee {\sim}Aa)$	ass cd	
3	$\forall x(Ax \rightarrow {\sim}Bx)$	2 sl	
4	$\exists x(Bx \vee {\sim}Aa)$	2 sr	
5	~~Show~~ ${\sim}\exists x{\sim}(Ax \wedge Cx)$		show consequent
6	$\exists x{\sim}(Ax \wedge Cx)$	ass id	
7	$Bi \vee {\sim}Aa$	4 ei	Do ei asap. New variable.
8	~~show~~ ${\sim}(Aa \wedge Ca)$		show instantiation to match 7 (Aa)
9	$Aa \wedge Ca$	ass id	
10	Aa	9 sl	
11	Bi	10 dn 7 mtp	
12	$Ai \rightarrow {\sim}Bi$	3 ui	match 11
13	${\sim}Ai$	11 dn 12 mt	Since i doesn't have
14	${\sim}Ai \vee {\sim}Ci$	13 add	property A (line 13),
15	${\sim}(Ai \wedge Ci)$	14 dm	something (i) is not both A and C! You could also
16	$\exists x{\sim}(Ax \wedge Cx)$	15 eg	use a subderivation to derive line 16.
17	${\sim}\exists x{\sim}(Ax \wedge Cx)$	6 r 16 id	
18	$\exists x{\sim}(Ax \wedge Cx)$	8 eg 6 id	make it match 6
19		5 cd	

6. $\exists x(Ax \wedge Bx)$. $\exists y(Gy \vee Hy)$. $\exists xAx \rightarrow \forall y(By \rightarrow {\sim}Hy)$ $\therefore$ ${\sim}\exists xGx \rightarrow \exists x\exists y({\sim}Hx \wedge Hy)$

1	~~Show~~ ${\sim}\exists xGx \rightarrow \exists x\exists y({\sim}Hx \wedge Hy)$		
2	${\sim}\exists xGx$	ass cd	
3	$Ai \wedge Bi$	pr1 ei	Use EI asap! new variable
4	$Gk \vee Hk$	pr2 ei	Use EI asap! new variable
5	Ai	3 sl	
6	$\exists xAx$	5 eg	make it match pr2
7	$\forall y(By \rightarrow {\sim}Hy)$	6 7 mp	
8	$Bk \rightarrow {\sim}Hk$	7 ui	make it match 4
9	~~Show~~ ${\sim}Gk$		
10	Gk	ass id	
11	$\exists xGx$	10 eg	make it match 2
12	${\sim}\exists xGx$	2 r	
13		11 12 id	
14	Hk	9 4 mtp	
15	$Bi \rightarrow {\sim}Hi$	7 ui	make it match 3
16	Bi	3 s	
17	${\sim}Hi$	15 16 mp	
18	$Hk \wedge {\sim}Hi$	14 17 adj	
19	$\exists y({\sim}Hx \wedge Hy)$	18 eg	make it match show line
20	$\exists x\exists y({\sim}Hx \wedge Hy)$	19 eg	make it match show line
21		20 cd	

7. $\forall x(Ax \to (\forall y(By \to Cy) \to Dx))$. $\forall x(Dx \to (\forall z(Bz \to Ez) \to Fx))$. $\therefore \forall y(By \to (Cy \wedge Ey)) \to \forall x(Ax \to Fx)$

1	~~Show~~ $\forall y(By \to (Cy \wedge Ey)) \to \forall x(Ax \to Fx)$		
2	$\forall y(By \to (Cy \wedge Ey))$	ass cd	
3	~~show~~ $\forall x(Ax \to Fx)$		
4	~~Show~~ $Ax \to Fx$		show instantiation of showline 3
5	Ax	ass cd	
6	$Ax \to (\forall y(By \to Cy) \to Dx))$.	pr1 ui	make it match 5
7	$\forall y(By \to Cy) \to Dx$	5 6 mp	
8	~~Show~~ $\forall y(By \to Cy)$		show antecedent of 7
9	~~Show~~ $By \to Cy$		show instantiation of 8 for UD
10	By	ass cd	
11	$By \to (Cy \wedge Ey)$	2 ui	make it match 10
12	$Cy \wedge Ey$	10 11 mp	
13	Cy	12 s cd	
14		9 ud	
15	Dx	7 8 mp	
16	$Dx \to (\forall z(Bz \to Ez) \to Fx))$	pr2 ui	make it match 15
17	$\forall z(Bz \to Ez) \to Fx$	15 16 mp	
18	~~Show~~ $\forall z(Bz \to Ez)$		show antecedent of 17
19	~~Show~~ $Bz \to Ez$		show instantiation of 18 for UD
20	Bz	ass cd	
21	$Bz \to Cz \wedge Ez$	2 ui	
22	Ez	20 21 mp sr cd	
23		19 ud	
24	Fx	17 18 mp cd	
25		4 ud	
26		3 cd	

8. ∀x(Fx → ∀y(Gy ∨ Hx)) → ~∀xAx. ∴ ∀x(Fx → ∀zGz) → ∃x(Cx → ~Ax)

1	~~Show~~ ∀x(Fx → ∀xGx) → ∃x(Cx → Ax)		
2	∀x(Fx → ∀zGz)	ass cd	
3	~~show~~ ∀x(Fx → ∀y(Gy ∨ Hx))		show antecedent of pr1
4	~~Show~~ (Fx → ∀y(Gy ∨ Hx))		show instantiation of 3
5	Fx	ass cd	
6	~~show~~ ∀y(Gy ∨ Hx)		show consequent
7	~~Show~~ Gy ∨ Hx		show instantiation of 6
8	Fx → ∀zGz	2 ui	match 5
9	∀zGz	5 8 mp	
10	Gy	9 ui	match show line 7
11	Gy ∨ Hx	10 add dd	
12		7 ud	
13		6 cd	
14		4 ud	
15	~∀xAx	pr1 3 mp	
16	∃x~Ax	15 QN	
17	~Ai	16 ei	
18	Ci ∨ ~Ai	17 add	
19	Ci → ~Ai	18 cdj	
20	∃x(Cx → ~Ax)	19 eg	match show line 1
21		20 cd	

9. ∀xBa(b(x)) → (∃xFx ∨ Ga(e)). ∀x(Ba(x) ∧ Ca(x)). ∴ ∀x(~Gx → ~∃yFy) → ∃zGa(z)

1	~~Show~~ ∀x(Gx → ~∃yFy) → ∃zGa(z)		
2	∀x(~Gx → ~∃yFy)	ass cd	
3	~~Show~~ ∀xBa(b(x))		Show antecedent of pr1
4	~~show~~ Ba(b(x))		show instantiation of 3
5	Ba(b(x)) ∧ Ca(b(x))	pr2 ui	match show line 4: b(x)/ x
6	Ba(b(x))	5 sl dd	
7		4 ud	
8	∃xFx ∨ Ga(e)).	3 pr1 cd	
9	~~Show~~ Ga(e)		
10	~Ga(e)	ass id	
11	∃xFx	8 10 mtp	
12	Fi	11 ei	new variable
13	~Ga(e) → ~∃yFy	2 ui	match 10: a(e)/x
14	~∃yFy	10 11 mp	
15	∀x~Fx	14 qn	
16	~Fi	15 ui 12 id	
16	∃zGa(z)	9 eg cd	

10. $\exists x\sim(Fx \rightarrow \sim Gx) \rightarrow \exists x\sim Hx \therefore \forall x\exists y\sim(Fy \rightarrow \sim Gx) \rightarrow \sim\forall yHy$

1	~~Show~~ $\forall x\exists y\sim(Fy \rightarrow \sim Gx) \rightarrow \sim\forall yHy$		
2	$\forall x\exists y\sim(Fy \rightarrow \sim Gx)$	ass cd	Goal: $\exists x\sim(Fx \rightarrow \sim Gx)$ antecedent of pr1
3	$\exists y\sim(Fy \rightarrow \sim Gx)$	2 ui	there is nothing to match! so use any variable.
4	$\sim(Fi \rightarrow \sim Gx)$	3 ei	use new variable
5	$Fi \wedge \sim\sim Gx$	4 nc	problem… mismatch (i & x)
6	$\exists y\sim(Fy \rightarrow \sim Gi)$	2 ui	solution: UI again to match.
7	$\sim(Fk \rightarrow \sim Gi)$	6 ei	use a new variable
8	$Fk \wedge \sim\sim Gi$	7 nc	now Fi & ~~Gi match!
9	Fi	5 sl	
10	$\sim\sim Gi$	8 sr	
11	$Fi \wedge \sim\sim Gi$	9 10 adj	
12	$\sim(Fi \rightarrow \sim Gi)$	11 nc	
13	$\exists x\sim(Fx \rightarrow \sim Gx)$	12 eg	
14	$\exists x\sim Hx$	13 pr1 mp	
15	$\sim Hm$	14 ei	new variable
16	$\exists y\sim Hy$	15 eg	match show line 1
17	$\sim\forall yHy$	16 QN cd	

Try some of these using ONLY the basic rules (S, ADJ, ADD, MTP, MP, MT, BC, CB, DN, EG, EI, UI).

11. ∃xBx. ∀x(~Bx ∨ Cx). ∀y((Ay ∨ ~Dy)→ ~Cy). ∴∃xDx ∧ ∃y~Ay

1	~~Show~~ ∃xDx ∧ ∃y~Ay		
2	Ba	pr1 ei a/x	Deal with EI first since you need arbitrary term.
3	~Ba ∨ Ca	pr2 ui a/x	instantiate pr2 using a for x to match 2
4	(Aa ∨ ~Da) → ~Ca	pr3 ui a/y	instantiate pr3 using a for y to match 2 and 3
5	~~Ba	2 dn	
6	Ca	5 3 mtp	
7	~~Ca	6 dn	
8	~(Aa ∨ ~Da)	7 4 mt	If only you had dm, it would be easy to get ~Aa &
9	~~Show~~ ~Aa		~~Da. You need them & can get them… show them
10	Aa	ass id	
11	Aa ∨ ~Da	10 add	
12	~(Aa ∨ ~Da)	8 r, 11 id.	
13	~~Show~~ Da		
14	~Da	ass id	
15	Aa ∨ ~Da	14 add	
16	~(Aa ∨ ~Da)	8 r, 15 id	
17	∃xDx	13 eg	Generalize 13 to match conc.
18	∃y~Ay	9 eg	Generalize 9 to match conclusion.
19	∃xDx ∧ ∃y~Ay	17 18 adj, dd	

12. ∴(~Ba ∨ Ga) → (∀x~(Cx → Gx) → ∃x(Cx ∧ ~Bx))

1	~~Show~~ (~Ba ∨ Ga) → (∀x~(Cx → Gx) → ∃x(Cx ∧ ~Bx))		
2	~Ba ∨ Ga	ass cd	
3	~~Show~~ (∀x~(Cx → Gx) → ∃x(Cx ∧ ~Bx))		
4	∀x~(Cx → Gx)	ass cd	
5	~(Ca → Ga)	4 ui	match line 2
6	~~Show~~ ~Ga		
7	Ga	ass id	
8	~~Show~~ Ca → Ga		
9	Ca	ass cd	
10	Ga	7 r cd	
11	~(Ca → Ga)	5 r 8 id	
12	~Ba	6 2 mtp	
13	~~Show~~ Ca		
14	~Ca	ass id	
15	~~Show~~ Ca → Ga		
16	Ca	ass cd	
17	~Ca	14 r 16 id	
18	~(Ca → Ga)	5 r 15 id	
19	Ca ∧ ~Ba	12 13 adj	
20	∃x(Cx ∧ ~Bx)	19 eg	match show line 3
21		3 cd	

13. $\forall y(By \to \sim(Dy \to Ey))$. $\forall x(Dx \to \sim(Fx \wedge \sim Cx))$. $\forall x(Ex \vee Fx)$. $\therefore \forall x(\sim Bx \vee Cx)$

1	~~Show~~ $\forall x(\sim Bx \vee Cx)$		
2	~~Show~~ $\sim Bx \vee Cx$		show line1 is $\forall$, so show instantiation for UD
3	$\sim(\sim Bx \vee Cx)$	ass ID	Looks hard to derive this directly, so use ID!
4	~~Show~~ Bx		You need Bx and can get it (very easily if you had
5	$\sim Bx$	ass id	DM)… so show it!
6	$\sim Bx \vee Cx$	5 add	
7	$\sim(\sim Bx \vee Cx)$	3 r, 6 id	
8	$Bx \to \sim(Dx \to Ex)$	pr1 ui x/y	instantiate pr1 to match 4
9	$\sim(Dx \to Ex)$	4 8 mp	With NC you could get Dx and ~Ex. You need Dx and
10	~~Show~~ Dx		know you can get it… so show it!
11	$\sim Dx$	ass id	
12	~~Show~~ $Dx \to Ex$		Show this to get a contradiction with 9!
13	Dx	ass cd	Show line 12 is $\to$ so ass antecedent.
14	$\sim Dx$	11 r, id	(If you prefer, show Ex with ID then reiterate 13 & 11)
15	$\sim(Dx \to Ex)$	9 r, 13 id	
16	$Dx \to \sim(Fx \wedge \sim Cx)$	pr2 ui x/x	instantiate pr2 using x for x to match 10
17	$\sim(Fx \wedge \sim Cx)$	10 16 mp	
18	$Ex \vee Fx$	pr3 ui x/x	instantiate pr3 using x for x to match
19	~~Show~~ $\sim Ex$		You can get this from 9 (easy with NC!)
20	Ex	ass id	
21	~~Show~~ $Dx \to Ex$		
22	Dx	ass cd	
23	Ex	20 r, cd	
24	$\sim(Dx \to Ex)$	9 r, 21 id	
25	Fx	18 19 mtp	
26	~~Show~~ $\sim Cx$		You need this to contradict 17 (with 25) and can get it
27	Cx	ass id	with line 3 (DM), so show it!
28	$\sim Bx \vee Cx$	27 add	
29	$\sim(\sim Bx \vee Cx)$	3 r,28 id	
30	$Fx \wedge \sim Cx$	25 26 adj, 17 id	
31		2 ud	You've shown it for any arbitrary x, so it's true for all!

14. $\forall x(\sim Bx \rightarrow Cx)$. $Ba \leftrightarrow \forall y\sim(By \wedge Cy)$ $\therefore$ $\exists x(\sim Cx \leftrightarrow Bx)$

1	~~Show~~ $\exists x(\sim Cx \leftrightarrow Bx)$		
2	~~Show~~ $\sim Ca \rightarrow Ba$		show one conditional for the instantiation of show line $(\sim Ca \leftrightarrow Ba)$ Match pr2 $(Ba \leftrightarrow\ldots)$
3	$\sim Ca$	ass cd	
4	$\sim Ba \rightarrow Ca$	pr1 ui	match 3
5	$\sim\sim Ba$	3 4 mt	
6	Ba	5 dn cd	
7	~~Show~~ $Ba \rightarrow \sim Ca$		
8	Ba	ass cd	
9	$Ba \rightarrow \forall y\sim(By \wedge Cy)$	pr2 bc	
10	$\forall y\sim(By \wedge Cy)$	8 9 mp	
11	$\sim(Ba \wedge Ca)$	10 ui	match 8
12	~~Show~~ $\sim Ca$		
13	Ca	ass id	
14	$Ba \wedge Ca$	8 13 adj	
15	$\sim(Ba \wedge Ca)$	11 r 14 id	
16		12 cd	
17	$\sim Ca \leftrightarrow Ba$	2 7 cb	
18	$\exists x(\sim Cx \leftrightarrow Bx)$	17 eg	match show line 1

15. $\forall y(By \wedge Fy \rightarrow Cy)$. $\exists xFx \rightarrow \forall x(Bx \vee Ax)$. Fb. $\therefore$ $\forall x(\sim Ax \vee Bx) \rightarrow Cb$

1	~~Show~~ $\forall x(\sim Ax \vee Bx) \rightarrow Cb$		
2	$\forall x(\sim Ax \vee Bx)$	ass cd	
3	$\exists xFx$	pr3 eg	match pr2
4	$\forall x(Bx \vee Ax)$	pr2 3 mp	
5	$Bb \wedge Fb \rightarrow Cb$	pr1 ui	match show line 1
6	$Bb \vee Ab$	4 ui	
7	$\sim Ab \vee Bb$	2 ui	
8	~~Show~~ Bb		
9	$\sim Bb$	ass id	
10	Ab	6 9 mtp	
11	$\sim\sim Ab$	10 dn	
12	Bb	11 7 mtp	
13		9 12 id	
14	$Bb \wedge Fb$	8 pr3 adj	
15	Cb	5 15 mp cd	

16. $\forall x(\sim Cx \vee (Aa \leftrightarrow \sim Fx))$. $\forall x(\sim Fx \rightarrow (\sim Cx \rightarrow Ax))$ $\therefore \exists x(Ax \vee Fx)$

1	~~Show~~ $\exists x(Ax \vee Fx)$		
2	$\sim Ca \vee (Aa \leftrightarrow \sim Fa)$	pr1 ui	match a in pr1
3	$\sim Fa \rightarrow (\sim Ca \rightarrow Aa)$	pr2 ui	match 2
4	~~Show~~ $Aa \vee Fa$		
5	$\sim(Aa \vee Fa)$	ass id	
6	~~Show~~ $\sim Fa$		
7	Fa	ass id	
8	$Aa \vee Fa$	7 add	
9	$\sim(Aa \vee Fa)$	5 r 8 id	
10	$\sim Ca \rightarrow Aa$	3 6 mp	
11	~~Show~~ $\sim Aa$		
12	Aa	ass id	
13	$Aa \vee Fa$	12 add	
14	$\sim(Aa \vee Fa)$	5 r 13 id	
15	$\sim\sim Ca$	11 10 mt	
16	$Aa \leftrightarrow \sim Fa$	2 15 mtp	
17	$\sim Fa \rightarrow Aa$	16 bc	
18	Aa	6 17 mp	
19		11 18 id	
20	$\exists x(Ax \vee Fx)$	4 eg dd	match show line

17. $A(ab) \vee B(ba)$. $\forall x\forall y(B(xy) \rightarrow C(yx))$. $\forall w\forall z(C(wz) \leftrightarrow A(wz))$. $\forall x\sim(G(xx) \wedge C(xb))$.
$\therefore \sim\forall x\forall y(A(xy) \rightarrow G(xx))$

1	~~Show~~ $\sim\forall x\forall y(A(xy) \rightarrow G(xx))$		
2	$\forall x\forall y(A(xy) \rightarrow G(xx))$	ass id	
3	$\forall y(A(ay) \rightarrow G(aa))$	2 ui	match pr1
4	$A(ab) \rightarrow G(aa)$	3 ui	match pr1
5	$\forall y(B(by) \rightarrow C(yb))$	pr2 ui	match pr1
6	$B(ba) \rightarrow C(ab)$	5 ui	match pr1
7	$\sim(G(aa) \wedge C(ab))$	pr4 ui	match lines 4 and 6
8	$\forall z(C(az) \leftrightarrow A(az))$	pr3 ui	match lines 4 and 6
9	$C(ab) \leftrightarrow A(ab)$	8 ui	match lines 4 and 6
10	~~Show~~ $\sim A(ab)$		Begin by showing something simple.
11	$A(ab)$	ass id	Show negation of one of the disjuncts of pr1.
12	$G(aa)$	11 4 mp	Alternatively, show A(ab) or B(ba) – the
13	$A(ab) \rightarrow C(ab)$	9 bc	antecedents of 4 and 6.
14	$C(ab)$	11 13 mp	
15	$G(aa) \wedge C(ab)$	12 14 adj	
16	$\sim(G(aa) \wedge C(ab))$	7 r 15 id	
17	$C(ab) \rightarrow A(ab)$	9 bc	
18	$\sim C(ab)$	10 17 mt	
19	$\sim B(ba)$	6 18 mt	
20	$A(ab)$	19 pr1 mtp	
		10 id	

18. $\exists x \forall y \sim(B(xy) \vee F(yx))$. $\forall x \forall y(Gx \rightarrow B(xy))$. $\exists x \forall y(F(xy) \vee H(yy))$. $\therefore \exists x(\sim Gx \wedge H(xx))$

1	~~Show~~ $\exists x(Ax \vee Fx)$		
2	$\sim Ca \vee (Aa \leftrightarrow \sim Fa)$	pr1 ui	match a in pr1
3	$\sim Fa \rightarrow (\sim Ca \rightarrow Aa)$	pr2 ui	match 2
4	~~Show~~ $Aa \vee Fa$		
5	$\sim(Aa \vee Fa)$	ass id	
6	~~Show~~ $\sim Fa$		
7	Fa	ass id	
8	$Aa \vee Fa$	7 add	
9	$\sim(Aa \vee Fa)$	5 r 8 id	
10	$\sim Ca \rightarrow Aa$	3 6 mp	
11	~~Show~~ $\sim Aa$		
12	Aa	ass id	
13	$Aa \vee Fa$	12 add	
14	$\sim(Aa \vee Fa)$	5 r 13 id	
15	$\sim\sim Ca$	11 10 mt	
16	$Aa \leftrightarrow \sim Fa$	2 15 mtp	
17	$\sim Fa \rightarrow Aa$	16 bc	
18	Aa	6 17 mp	
19		11 18 id	
20	$\exists x(Ax \vee Fx)$	4 eg dd	match show line