FTC & Vector Calculus

definite integral on a region

effect of the anti derivative on the boundary

FTC: $\int_{[a\ b]} F'(x)\,dx = F(b) - F(a)$

Single integral of derivative; anti derivative

$[a\ b]$ oriented from a to b

net Change of F along/on $[a\ b]$

net Change of F on the boundary

boundary of $[a\ b]$ is $\{a, b\}$

Green's Theorem: $\iint_S \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA = \int_{\partial S} P\,dx + Q\,dy = \int_{\partial S} \mathbb{F} \cdot dx$

double; derivative; Single integral; anti derivative

S: region of plane

two dimversion of curl, it is a derivative, and measures twisting effect of $\mathbb{F} = \langle P, Q \rangle$ on spot, so this integral is sum total twist on S

∂S: boundary of S

total work of $\mathbb{F}$ along the boundary of S. $\mathbb{F}$ is anti derival of $\left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)$ in a sense.

anti derivative

Stoke's Theorem: $\iint_S \nabla \times \mathbb{F} \cdot n\,dA = \int_{\partial S} \mathbb{F} \cdot dx$

net flux of $\nabla \times \mathbb{F}$ across S, total twist on S

total work of $\mathbb{F}$ on the boundary

derivative; anti derivative

divergence Theorem: $\iiint_W \nabla \cdot \mathbb{F}\,dV = \iint_{\partial W} \mathbb{F} \cdot n\,dA$

W: region in $\mathbb{R}^3$

sum total of expansion/generation of $\mathbb{F}$ inside W

net flux of $\mathbb{F}$ across boundary of ∂W

anti derivative