$X \sim Bin(n, p)$, let $Y_1, \ldots, Y_n$ be Bernoulli r.v's with prob. of success p.

Then $X = \sum_{i=1}^{n} Y_i$

$$E(X) = E\left[\sum_{i=1}^{n} Y_i\right] = \sum_{i=1}^{n} E[Y_i] = np$$

Ex.

$X \sim$ Neg. Binomial (r, p)

X_1 be # of trials until 1^{st} success

X_2 be # of trials between 1^{st} and 2^{nd} success

$\vdots$

X_r be # between $(r-1)^{th}$ and r^{th}

$$X = \sum_{i=1}^{r} X_i, \quad X_i \sim Geom(p)$$

$$E(X) = E\left[\sum_{i=1}^{r} X_i\right] = \sum_{i=1}^{r} E[X_i] = \frac{r}{p}$$

Ex. $X \geq 0$, c - const

Prove: $E(X) \geq E(X \cdot I_{\{X \geq c\}})$

Soln: $I_{\{X \geq c\}} + I_{\{X < c\}} = 1$

$$E(X) = E(X \cdot 1) = E(X I_{\{X \geq c\}} + X I_{\{X < c\}})$$

$$= E[X I_{\{X \geq c\}}] + \underbrace{E[X \underbrace{I_{\{X < c\}}}]}_{\geq 0}$$

$$\geq E[X \cdot I_{\{X \geq c\}}]$$

∎