UNIVERSITY OF TORONTO
Faculty of Arts and Science

EXAMINATION DECEMBER 2010

PHL 245 H1F
L0101 - Niko Scharer

PLEASE HAND IN

Duration - 3 hours

Examination Aid: Sheet with rules (provided)

Name ____________________________ Student Number ____________________________

Answer **all** questions (on the exam paper).

Use the supplied examination booklet for rough work OR if you need further space.

The exam consists of fourteen (14) pages. Pages 2-12 have questions on them.

The final two pages (13-14) are an aid sheet and may be detached from the rest of the exam.

1. An argument consists of two premises and a conclusion. On every interpretation that the conclusion is false, so are the premises. Does it follow that the argument is valid? Briefly explain. (2%)

2. A set of sentences {P, Q, R} is logically inconsistent. Which of the following arguments must be valid? Circle the correct answer. (2 %)

a)

$$\begin{array}{l} P \\ \sim Q \\ \hline \therefore R \end{array}$$

VALID NOT VALID

b)

$$\begin{array}{l} \sim(Q \rightarrow \sim R) \\ \hline \therefore \sim P \end{array}$$

VALID NOT VALID

c)

$$\begin{array}{l} S \vee R \\ \sim S \vee Q \\ \hline \therefore \sim(P \wedge Q \wedge R) \end{array}$$

VALID NOT VALID

d)

$$\begin{array}{l} S \wedge P \\ R \rightarrow \sim S \\ \hline \therefore \sim R \vee Q \end{array}$$

VALID NOT VALID

Name:______________________________ Student Number:______________________

3. Consider the following truth-table for the NEW symbol: $\ast$ (2%)

P	Q	P $\ast$ Q
T	T	F
T	F	F
F	T	F
F	F	T

a) Using the new symbol, and other logical connectives if necessary, symbolize: P unless Q.

b) What ordinary English expression can this new truth-functional connective ($\ast$) be used to symbolize (given its truth-table)?

4. Provide an interpretation that shows that the following argument is not valid.
Your interpretation should specify the universe of discourse and a symbolization scheme. (4%)

$\forall x \exists y(Ax \rightarrow L(xy))$. $\exists x(Ax \wedge \forall y(\sim Ay \rightarrow L(xy)))$. $\therefore \sim\forall x(Bx \rightarrow \exists yL(yx))$

5. Use this symbolization scheme to symbolize the following sentences: $(36\% = 9 \times 4\%)$

A^1: *a* is a ticket.	D^1: *a* is a day.	E^1: *a* is expensive.	H^1: *a* is a person
J^1: a is a movie.	K^1: a is a play.	M^1: *a* is a theatre	
B^2: *a* buys *b*.	C^2: *a* sees *b*.	F^2: *a* is a friend of *b*.	G^2: *a* goes to *b*.
I^2: *a* is shown at *b*	L^2: *a* likes *b*.	O^3: *a* watches *b* on *c*.	
a^0: Anne	b^0: Billy Elliot	d^0: The Odeon Theatre	c^1: the cousin of *a*.

a) Some theatres show plays only if not all the tickets are expensive.

b) It is necessary that a person buy a ticket in order to see things shown at the Odeon Theatre.

c) Anne only goes to movies that her friends like.

d) If there are days when nobody watches a play then nobody likes some plays.

Name:______________________________ Student Number:_______________________

5 continued. Use this symbolization scheme to symbolize the following sentences: (36 % = 9 × 4%)

A^1: *a* is a ticket.	D^1: *a* is a day.	E^1: *a* is expensive.	H^1: *a* is a person
J^1: a is a movie.	K^1: a is a play.	M^1: *a* is a theatre	
B^2: *a* buys *b*.	C^2: *a* sees *b*.	F^2: *a* is a friend of *b*.	G^2: *a* goes to *b*.
I^2: *a* is shown at *b*	L^2: *a* likes *b*.	O^3: *a* watches *b* on *c*.	
a^0: Anne	b^0: Billy Elliot	d^0: The Odeon Theatre	c^1: the cousin of *a*.

e) People who only like theatres that show movies don't go to plays unless their friends do.

f) Only Anne's cousin likes exactly the same movies that Anne likes.

g) Assuming that her friends don't watch Billy Elliot on the day that Anne does, Anne buys just one ticket but it is an expensive one.

5 continued. (36 % = 9 × 4%)

A^1: *a* is a ticket.	D^1: *a* is a day.	E^1: *a* is expensive.	H^1: *a* is a person
J^1: a is a movie.	K^1: a is a play.	M^1: *a* is a theatre	
B^2: *a* buys *b*.	C^2: *a* sees *b*.	F^2: *a* is a friend of *b*.	G^2: *a* goes to *b*.
I^2: *a* is shown at *b*	L^2: *a* likes *b*.	O^3: *a* watches *b* on *c*.	
a^0: Anne	b^0: Billy Elliot	d^0: The Odeon Theatre	c^1: the cousin of *a*.

g) Using the symbolization scheme above, provide an idiomatic English sentence that expresses:

$$\sim(\exists x(Jx \wedge L(ax)) \vee \exists y(Ky \wedge L(ay))) \rightarrow \forall x(Jx \rightarrow \sim C(ax) \vee \exists y(F(ya) \wedge C(yx)))$$

h) Using the symbolization scheme above, symbolize the following ambiguous sentence **three** logically distinct ways. For each, provide an English sentence that explains exactly what the symbolized sentence means:

A movie is watched by somebody every day.

Name:______________________________ Student Number:______________________

6. Provide a derivation that shows the following theorem is valid **using only the 10 basic rules from SL** (R, DN, MP, MT, ADJ, S, ADD, MTP, BC, CB) **and the 3 basic rules from PL** (UI, EG, EI) (9%)

$$\therefore\ (\exists x{\sim}(Ax \vee Gx) \wedge \forall y{\sim}(By \rightarrow Hy)) \rightarrow \exists x(Hx \leftrightarrow Ax)$$

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7. Provide a derivation that shows the following theorem is valid **using only the 10 basic rules from SL** (R, DN, MP, MT, ADJ, S, ADD, MTP, BC, CB) **and the 3 basic rules from PL** (UI, EG, EI) (9%)

$\forall x(Fx \rightarrow Hx).$ $\exists x(Fx \wedge \forall y B(xy)).$ $\exists x Hx \rightarrow \forall y \forall z(Ay \wedge B(zz) \rightarrow G(yz)).$

$\therefore \forall x(Ax \rightarrow \exists y(Hy \wedge G(xy)))$

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Name:______________________________ Student Number:______________________

8. Provide a derivation to show that this is a valid argument (use any rules). (9 %):

$\forall x(G(xx) \rightarrow \sim\exists z A(xz))$. $\forall x \forall y(B(xy) \rightarrow \sim A(yx)) \rightarrow \forall x \exists y \sim(Kx \rightarrow My)$.

$\therefore \forall w(\exists z B(zw) \rightarrow G(ww)) \rightarrow \sim\forall x(\sim Kx \vee Mx)$

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9. Show that the following is a valid argument (use any rules). (9%):

$\sim\forall x Mx \rightarrow \forall x \exists y \forall z H(xyz).$ $\exists x \forall y (H(xyy) \rightarrow \forall z \sim A(xz)).$

$\exists y \sim A(yy) \rightarrow \exists x \forall y G(b(x)b(y)).$ $\therefore\ \sim\exists x G(xb(x)) \rightarrow Ma$

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Name:______________________________ Student Number:_____________________

10. Use a model to demonstrate the invalidity of this argument (6 %):

    i) provide a truth-functional expansion (to two individuals) for each sentence in this argument with respect to the model

    ii) define a model with a universe of two individuals that shows that this argument is invalid.

    $\exists x\forall y(Bx \wedge C(xy))$. $\quad\quad$ $\forall x(Ax \rightarrow \exists y{\sim}C(xy))$. $\quad\quad$ $\therefore$ $\forall x(Ax \rightarrow {\sim}C(xx))$

11. Explain why the following is a contradiction (why it is false on any interpretation). (4 %)

    $\exists x\forall yL(xy) \wedge {\sim}\forall x\exists yL(yx)$

12. Is the material conditional a necessary logical connective in our system? Justify your answer with an explanation that considers the role of the material conditional in both symbolization and derivations. (4 %)

13. Our derivation system is complete in that every valid argument and theorem which can be expressed in our system can be proven valid with a derivation. Show, in an organized manner, that every valid argument and theorem can be proven valid with an INDIRECT derivation. (4 %)

Name:________________________________ Student Number:________________________

AID SHEET: DERIVATION RULES (SEE BOTH SIDES)

Derivation Types:

Direct Derivation (DD)

Conditional Derivation (CD)

Indirect Derivation (ID)

Universal Derivation (UD) Restriction: the instantiating term cannot occur unbound in any previous line.

Basic Rules for Sentential Operators:

Modus Ponens (MP)

$$\frac{(\phi \to \psi) \quad \phi}{\psi}$$

Modus Tollens (MT)

$$\frac{(\phi \to \psi) \quad \sim\psi}{\sim\phi}$$

Double Negation (DN)

$$\frac{\phi}{\sim\sim\phi} \qquad \frac{\sim\sim\phi}{\phi}$$

Repetition (R)

$$\frac{\phi}{\phi}$$

Simplification (S)

$$\frac{\phi \wedge \psi}{\phi} \qquad \frac{\phi \wedge \psi}{\psi}$$

Adjunction (ADJ)

$$\frac{\phi \quad \psi}{\phi \wedge \psi}$$

Addition (ADD)

$$\frac{\phi}{\phi \vee \psi} \qquad \frac{\psi}{\phi \vee \psi}$$

Modus Tollendo Ponens (MTP)

$$\frac{\phi \vee \psi \quad \sim\phi}{\psi} \qquad \frac{\phi \vee \psi \quad \sim\psi}{\phi}$$

Biconditional-Conditional (BC)

$$\frac{\phi \leftrightarrow \psi}{\phi \to \psi} \qquad \frac{\phi \leftrightarrow \psi}{\psi \to \phi}$$

Conditional-Biconditional (CB)

$$\frac{\phi \to \psi \quad \psi \to \phi}{\phi \leftrightarrow \psi}$$

Derived Rules for Sentential Operators:

Negation of Conditional (NC)

$$\frac{\sim(\phi \to \psi)}{\phi \wedge \sim\psi} \qquad \frac{\phi \wedge \sim\psi}{\sim(\phi \to \psi)}$$

Conditional as Disjunction (CDJ)

$$\frac{\phi \to \psi}{\sim\phi \vee \psi} \qquad \frac{\sim\phi \vee \psi}{\phi \to \psi} \qquad \frac{\sim\phi \to \psi}{\phi \vee \psi} \qquad \frac{\phi \vee \psi}{\sim\phi \to \psi}$$

Separation of Cases (SC)

$$\frac{\begin{array}{l}\phi \vee \psi \\ \phi \to \chi \\ \psi \to \chi\end{array}}{\chi} \qquad \frac{\begin{array}{l}\phi \to \chi \\ \sim\phi \to \chi\end{array}}{\chi}$$

Negation of Biconditional (NB)

$$\frac{\sim(\phi \leftrightarrow \psi)}{\phi \leftrightarrow \sim\psi} \qquad \frac{\phi \leftrightarrow \sim\psi}{\sim(\phi \leftrightarrow \psi)}$$

De Morgan's (DM)

$$\frac{\sim(\phi \vee \psi)}{\sim\phi \wedge \sim\psi} \qquad \frac{\sim\phi \wedge \sim\psi}{\sim(\phi \vee \psi)} \qquad \frac{\sim(\phi \wedge \psi)}{\sim\phi \vee \sim\psi} \qquad \frac{\sim\phi \vee \sim\psi}{\sim(\phi \wedge \psi)}$$

$$\frac{\sim(\sim\phi \vee \sim\psi)}{\phi \wedge \psi} \qquad \frac{\phi \wedge \psi}{\sim(\sim\phi \vee \sim\psi)} \qquad \frac{\sim(\sim\phi \wedge \sim\psi)}{\phi \vee \psi} \qquad \frac{\phi \vee \psi}{\sim(\sim\phi \wedge \sim\psi)}$$

Derivation Rules for Predicate Logic:

Existential Generalization (EG)

$$\frac{\phi_\zeta}{\exists\alpha\phi_\alpha}$$

Universal Instantiation (UI)

$$\frac{\forall\alpha\phi_\alpha}{\phi_\zeta}$$

Restriction: ζ does not occur as a bound variable in ϕ_α

Existential Instantiation (EI)

$$\frac{\exists\alpha\phi_\alpha}{\phi_\zeta}$$

Restriction: ζ does not occur in any previous line or premise.

Quantifier Negation (QN)

$$\frac{\sim\forall\alpha\phi}{\exists\alpha\sim\phi} \qquad \frac{\sim\exists\alpha\,\phi}{\forall\alpha\sim\phi}$$

$$\frac{\exists\alpha\sim\phi}{\sim\forall\alpha\phi} \qquad \frac{\forall\alpha\sim\phi}{\sim\exists\alpha\,\phi}$$