

Code: 2895

UNIVERSITY OF TORONTO
Faculty of Arts and Science
APRIL 2011 EXAMINATIONS
MAT135Y1Y
Calculus I
Duration — 3 hours

APR 20 11 12 41

NAME

(Please PRINT full name
and UNDERLINE surname):

STUDENT NO:

SIGNATURE OF STUDENT

(in INK or BALL-POINT PEN):

This Exam has 2 Parts:

PART A: 8 questions (55 marks).

PART B: 18 multiple choice questions (45 marks).

Indicate your answer to each multiple-choice questions in PART B by completely filling in the appropriate circle in the ANSWER BOX on this front page. (Use a dark pencil!)

FOR MARKERS ONLY

A1	/6
A2	/7
A3	/7
A4	/7
A5	/7
A6	/5
A7	/8
A8	/8
B	/45
TOTAL	/100

NOTE:

1. Before you start, check that this test has has 21 pages.
2. No aids allowed.
NO CALCULATORS!
3. **DO NOT TEAR OUT THIS PAGE OR ANY OTHER PAGE.**
4. **COMPUTER CARDS AND ANSWER BOOKS WILL NOT BE USED.**

ANSWER BOX
FOR PART B

1. (A) (B) (C) (D) (E)
2. (A) (B) (C) (D) (E)
3. (A) (B) (C) (D) (E)
4. (A) (B) (C) (D) (E)
5. (A) (B) (C) (D) (E)
6. (A) (B) (C) (D) (E)
7. (A) (B) (C) (D) (E)
8. (A) (B) (C) (D) (E)
9. (A) (B) (C) (D) (E)
10. (A) (B) (C) (D) (E)
11. (A) (B) (C) (D) (E)
12. (A) (B) (C) (D) (E)
13. (A) (B) (C) (D) (E)
14. (A) (B) (C) (D) (E)
15. (A) (B) (C) (D) (E)
16. (A) (B) (C) (D) (E)
17. (A) (B) (C) (D) (E)
18. (A) (B) (C) (D) (E)

PART A [55 marks]

Answer all questions in PART A in the spaces provided. Show all your work for PART A. Any answer in PART A (except NO. 6) without proper justification may receive very little or no credit. Use the back of each page for rough work.

DO NOT TEAR OUT ANY PAGES.

1. [6 marks]

Find $\int x e^{\sqrt{2}x} dx$

2. [7 marks]

Find $\int \tan^3 x \sec x dx$

3. [7 marks]

Find $\int \frac{1}{x^2 \sqrt{25 - x^2}} dx$

4. [7 marks]

Let R be the region in the first quadrant bounded by the curves $y = 2x + 3$ and $y = 2x^3 + 3$. Find the volume of the solid generated by revolving the region R about the y -axis. Simplify your final answer as much as possible.

5. [7 marks]

Find the arc length of the curve $y = \frac{1}{3}x^{3/2}$, $0 \leq x \leq 5$. Simplify your final answer as much as possible.

6. [5 marks]

A vat holds 100 litres of pure water. At time 0, liquid starts to drain out of the tank at 1 litre/minute while salt water is being added at 1 litre/minute. The concentration of the salt in the water being added is 35 g/litre. If $x(t)$ is the amount of salt in the vat at time t and if the liquid is fully mixed before it drains, then $x(t)$ satisfies the differential equation

$$\frac{dx}{dt} = 35 - \frac{x(t)}{100}.$$

Decide which one of the following is the correct expression for $x(t)$. Circle one of the letters A , B , C , D , E , or F . You don't have to show your reasoning.

- A. $x(t) = 3500 - 3500e^{-t/100}$
- B. $x(t) = 3500 - 3400e^{-t/100}$
- C. $x(t) = 3500 - 3500e^{t/100}$
- D. $x(t) = 35 - 35^{-t/100}$
- E. $x(t) = 35 - 34e^{-t/100}$
- F. $x(t) = 35 - 35e^{t/100}$

Also, as t goes to infinity, what does the amount of salt in the vat tend to? In 20 words or less, explain why it tends to this value and not to some other value.

7. [8 marks]

Find the interval of convergence of the series $\sum_{n=1}^{\infty} \frac{(2x-3)^n}{3n-1}$. Remember to fully justify your answer.

8. [8 marks]

NOTE: This is a hard problem and will be marked extremely strictly. Very little or no credit will be given unless your solution is completely correct.

Let k be a constant so that $\int_0^1 kx^{25}(1-x)^{25}dx = 40$.

Find the value of $\int_0^1 kx^{25}(1-x)^{26}dx$.

PART B [45 marks]

18 multiple choice questions

PLEASE READ CAREFULLY: Each of the following multiple-choice questions has exactly one correct answer. Indicate your answer to each question by completely filling in the appropriate circle in the ANSWER BOX on the front page. Use a dark pencil.

MARKING SCHEME: $2\frac{1}{2}$ marks for a correct answer,
0 for no answer, a wrong answer or giving more than one answer.

You are not required to justify your answers in PART B.

NOTE: If there is any discrepancy between the circles you darken on these inside pages and those you darken on the front page, the circles you darken on the front page will be regarded as your final answers. Note that only the circles you darken will count. For Part B, your computations and answers (other than the circles you darken) will NOT count.

WARNING: If you darken the circles on these inside pages but do not darken the circles on the front page, you will still get credit for your correct answers, but there will be a **PENALTY** of minus 4 marks.

YOU MUST NOT TEAR OUT ANY PAGES OF THIS EXAM.

1. Find the value of $\lim_{x \rightarrow 0} \frac{e^x - 1 - x}{x^2}$

- (A) $\frac{1}{e}$
- (B) undefined
- (C) $-\frac{1}{2}$
- (D) $\frac{1}{2}$
- (E) 0

2. The slant (i.e. oblique) asymptote of the curve $y = \frac{x^2 + 3x + 7}{x + 2}$ is the line

- (A) $y = x + 3$
- (B) $y = x + \frac{3}{2}$
- (C) $y = x - 1$
- (D) $y = x$
- (E) $y = x + 1$

INDICATE YOUR ANSWERS ON THE FRONT PAGEPenalty for not doing so is MINUS 4 marks.

3. The graph of $y = \frac{x^4}{12} + \frac{x^3}{6} - 3x^2 - 4x + 2$ is concave down on the interval

Ⓐ $(-1, 4)$

Ⓑ $(-4, 1)$

Ⓒ $(-3, 2)$

Ⓓ $(0, \infty)$

Ⓕ $(-2, 3)$

4. The function $f(x) = 12x^5 - 15x^4 - 40x^3 + 18$ has a local max at $x =$

Ⓐ $\frac{1}{2}$

Ⓑ -1

Ⓒ 2

Ⓓ 0

Ⓕ -2

INDICATE YOUR ANSWERS ON THE FRONT PAGEPenalty for not doing so is MINUS 4 marks.

5. At noon, a rectangle has length 10 inches and width 30 inches. The length starts increasing at a constant rate of 3 inches/min, and the width starts decreasing at a constant rate of 2 inches/min. At what rate will the area of the rectangle be increasing at the moment when the area first reaches 500 square inches?
- Ⓐ 10 sq in/min.
 - Ⓑ 8 sq in/min.
 - Ⓒ 12 sq in/min.
 - Ⓓ 16 sq in/min.
 - Ⓕ 14 sq in/min.

INDICATE YOUR ANSWERS ON THE FRONT PAGEPenalty for not doing so is MINUS 4 marks.

6. The product of two positive numbers is 25. What is the smallest possible value of their sum?

- Ⓐ 10
- Ⓑ 7
- Ⓒ 8
- Ⓓ 9
- Ⓕ 11

7. The improper integral $\int_0^{\infty} \frac{\sqrt{\ln x}}{x} dx$

- Ⓐ converges to $\frac{1}{e}$
- Ⓑ converges to $e - 1$
- Ⓒ converges to $\ln 2$
- Ⓓ diverges
- Ⓕ converges to 1

INDICATE YOUR ANSWERS ON THE FRONT PAGEPenalty for not doing so is MINUS 4 marks.

8. The function $y = f(x)$ satisfies the differential equation $\frac{dy}{dx} = \frac{x+3}{e^{2y}}$.

If $f(0) = \frac{1}{2} \ln 2$, then $f(1) =$

- Ⓐ $\frac{1}{3} \ln 2$
- Ⓑ $\frac{1}{2} \ln 3$
- Ⓒ $\ln 3$
- Ⓓ $\sqrt{\ln 2}$
- Ⓕ $\ln 2$

9. Consider the predator-prey system $\frac{dR}{dt} = 3R - 4RW$, $\frac{dW}{dt} = -5W + 2RW$.
When the system is in equilibrium with $W \neq 0$, $R \neq 0$, then $RW =$

- Ⓐ $\frac{5}{2}$
- Ⓑ $\frac{14}{9}$
- Ⓒ $\frac{12}{5}$
- Ⓓ $\frac{13}{7}$
- Ⓕ $\frac{15}{8}$

INDICATE YOUR ANSWERS ON THE FRONT PAGEPenalty for not doing so is MINUS 4 marks.

10. Suppose that a population grows according to a logistic model, i.e., the growth is modeled by the differential equation $\frac{dP}{dt} = kP(1 - \frac{P}{K})$. Suppose that the initial population is 250 and the population grows to 625 after one year, with the carrying capacity K being equal to 2,500. What will be the population after 3 more years (i.e. 4 years from the very beginning)?

- Ⓐ 2,000
- Ⓑ 2,250
- Ⓒ 2,750
- Ⓓ 1,750
- Ⓕ 2,500

INDICATE YOUR ANSWERS ON THE FRONT PAGEPenalty for not doing so is MINUS 4 marks.

11. Consider the following three series:

I.
$$\sum_{n=1}^{\infty} \left(\frac{1}{\sqrt{n}} - \frac{1}{n^3} \right)$$

II.
$$\sum_{n=1}^{\infty} \frac{2 + (-1)^n}{5^n}$$

III.
$$\sum_{n=1}^{\infty} \frac{2^{-n} + 3^{-n}}{4^{-n} + 3^{-n}}$$

Decide which of the series converge (or converges).

- Ⓐ I and II only
- Ⓑ III only
- Ⓒ II only
- Ⓓ I, II, and III
- Ⓕ II and III only

INDICATE YOUR ANSWERS ON THE FRONT PAGEPenalty for not doing so is MINUS 4 marks.

12. Consider the following three series:

I.
$$\sum_{n=1}^{\infty} (-1)^n \frac{(n+2)!}{(n+45)^{45}}$$

II.
$$\sum_{n=1}^{\infty} (-1)^n \frac{(n+4)^2 \arctan(1+n)}{(n+5)^3 \arctan(1+n^2)}$$

Which one of the following statements is correct?

- Ⓐ I diverges and II converges conditionally.
- Ⓑ I diverges and II converges absolutely.
- Ⓒ I and II both converge conditionally.
- Ⓓ I converges conditionally and II converges absolutely.
- Ⓕ I and II both converge absolutely.

INDICATE YOUR ANSWERS ON THE FRONT PAGEPenalty for not doing so is MINUS 4 marks.13. Find the coefficient of x^3 of the Maclaurin series for $f(x) = e^{3x} \cos 2x$.

Ⓐ $\frac{5}{6}$

Ⓑ $-\frac{1}{3}$

Ⓒ $-\frac{5}{2}$

Ⓓ $-\frac{3}{2}$

Ⓕ $\frac{2}{3}$

INDICATE YOUR ANSWERS ON THE FRONT PAGEPenalty for not doing so is MINUS 4 marks.

14. $\int_0^1 \frac{x-1}{(x+1)(x^2+1)} dx =$

Ⓐ $\frac{\pi}{4} + \ln 2$

Ⓑ $-\frac{1}{2} \ln 2$

Ⓒ $\frac{\pi}{4}$

Ⓓ $-\ln 2$

Ⓕ $-\sqrt{2}$

15. Suppose that $\int_1^{x^3} f(t) dt = x^2 e^x$ for all x . Then $f(8) =$

Ⓐ $\frac{2}{3}e - 1$

Ⓑ $e^2 - e$

Ⓒ $\frac{e^2}{8}$

Ⓓ $\frac{e}{3}$

Ⓕ $\frac{2}{3}e^2$

INDICATE YOUR ANSWERS ON THE FRONT PAGEPenalty for not doing so is MINUS 4 marks.

16. $\int_0^{\frac{\pi}{2}} \frac{dx}{2 + \cos x} =$

Ⓐ $\frac{\pi}{3\sqrt{2}}$

Ⓑ $\frac{\pi}{3}$

Ⓒ $\frac{\pi}{3\sqrt{3}}$

Ⓓ $\frac{\pi}{\sqrt{3}}$

Ⓕ $\frac{\pi}{2\sqrt{3}}$

INDICATE YOUR ANSWERS ON THE FRONT PAGEPenalty for not doing so is MINUS 4 marks.17. Let $a_1 = \sqrt{5}$ Let $a_{n+1} = \sqrt{5 + a_n}$, for $n = 1, 2, 3, \dots$ Then the sequence $\{a_n\}$ will

- Ⓐ converge to $\frac{1}{2}(1 + \sqrt{23})$.
- Ⓑ converge to $10\sqrt{5}$.
- Ⓒ converge to $\frac{1}{2}(1 + 3\sqrt{5})$.
- Ⓓ converge to $\frac{1}{2}(1 + 5\sqrt{5})$.
- Ⓕ converge to $\frac{1}{2}(1 + \sqrt{21})$.

INDICATE YOUR ANSWERS ON THE FRONT PAGEPenalty for not doing so is MINUS 4 marks.18. Find the area of the region enclosed by the curves $y = x + 1$ and $y = |x| + |x - 1|$.

Ⓐ $\frac{3}{2}$

Ⓑ $\frac{4}{3}$

Ⓒ $\frac{5}{4}$

Ⓓ 1

Ⓕ $\frac{3}{4}$