University of Toronto

MAT237Y1Y PROBLEM SET 7

DUE: End of tutorial, Thursday August 1st, no exceptions

Instructions:

1. Please clearly identify your **Tutorial** (TUT5101 with Boris Lishak, TUT5102 with James Mracek, or TUT5103 with Nan Wu), your full **Name** and your **Student Number** at the top of your problem set. If you do not indicate these, you may not have your problem sets returned.

2. Please write clearly and present polished, perfected solutions; all rough work and initial drafts should be done separately. You are being graded not just for the correctness of the final solution, but for your skill at presenting a cogent argument.

3. This problem set contains practice exercises that are complementary to the textbook material and as such they are designed to help with reflection on the subject. It is extremely important that each student have a chance to individually think about the problems.

4. By participating in this problem set you agree that you are aware of the university regulations regarding plagiarism. You are encouraged to discuss the problem set and to help one another with the ideas and concepts, but may not copy from each others' solutions. The markers are carefully monitoring your solutions, and any incidence of plagiarism will be dealt with according to the university regulations.

Problems:

1. Carefully read all of optional reading 4.4 and do questions 19 and 21.

2. We define the improper integral on the entire plane $\mathbb{R}^2$ as

$$\iint_{\mathbb{R}^2} f(x,y)dA = \lim_{a\to\infty} \iint_{D_\alpha} f(x,y)dA,$$

where D_a is the disc of radius a centered at the origin.

a) Prove, using polar coordinate that

$$\iint_{\mathbb{R}^2} e^{-(x^2+y^2)}dA = \pi.$$

b) Another way of defining the improper integral (as in part a) is

$$\iint_{\mathbb{R}^2} f(x,y)dA = \lim_{a\to\infty} \iint_{S_a} f(x,y)dA,$$

where S_a is the square of radius sides $2a$ centered at the origin. Show that, using this definition,

$$\iint_{\mathbb{R}^2} f(x)g(x)dA = \left(\int_{-\infty}^{+\infty} f(x)dx\right)\left(\int_{-\infty}^{+\infty} g(y)dy\right).$$

Then apply this fact to conclude that

$$\left(\int_{-\infty}^{+\infty} e^{-x^2}dx\right)\left(\int_{-\infty}^{+\infty} e^{-y^2}dy\right) = \pi.$$

c) Conclude from part (b) that

$$\int_{-\infty}^{+\infty} e^{-\frac{x^2}{2\sigma^2}}dx = \sqrt{2\pi}\sigma.$$

3. Read Section 4.5 example 2,3 and do the following quesitns:

a) Evaluate

$$\frac{d}{dx}\int_0^{\frac{\pi}{2x}} x\sin(xt)dt.$$

b) It can be proved that the results of Theorem 4.47 holds for improper integrals, which is defined as

$$\int_0^{\infty} f(x)dx = \lim_{t\to\infty}\int_0^t f(x)dx.$$

First calculate

$$\int_0^{\infty} e^{-ax}dx$$

for any $a > 0$. Then differentiate both side (three times with respect to a) to obtain a value for

$$\int_0^{\infty} x^3e^{-ax}dx.$$