LN 11.2. 

Ex: $\tilde{i} = \begin{cases} 0.06, & prob = 0.25 \\ 0.07, & prob = 0.15 \\ 0.08, & prob = 0.60 \end{cases}$ $\Rightarrow$ $\tilde{i}^2 = \begin{cases} 0.06^2, & prob = 0.25 \\ 0.07^2, & prob = 0.15 \\ 0.08^2, & prob = 0.60 \end{cases}$

$$E[\tilde{i}] = 0.06 \times 0.25 + 0.07 \cdot 0.15 + 0.08 \times 0.60$$
$$= 0.0735.$$

$$Var[\tilde{i}] = E[\tilde{i}^2] - (E[\tilde{i}])^2$$
$$= \left(\underline{0.06^2 \times 0.25 + 0.07^2 \times 0.15 + 0.08^2 \times 0.60}\right) - (0.0735)^2$$
$$= 0.00007275.$$

$$SD[\tilde{i}] = \sqrt{Var[\tilde{i}]} = 0.008529.$$

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$\Rightarrow$ $\underbrace{\tilde{S}(n)}_{}$ $\rightarrow$ time.

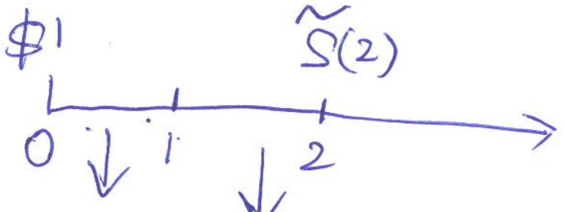

①. $n = 1$, $\tilde{i}_1$: $E[\tilde{i}_1]$, $Var[\tilde{i}_1]$

$$\boxed{\tilde{S}(1) = 1 + \tilde{i}_1}$$

$$E[\tilde{S}(1)] = E[1 + \tilde{i}_1] = 1 + \underline{E[\tilde{i}_1]} =$$

$$Var[\tilde{S}(1)] = Var[1 + \tilde{i}_1] = Var[\tilde{i}_1]$$

② $n = 2$.




$\tilde{i}_1 \perp\!\!\!\perp \tilde{i}_2 \Rightarrow 1+\tilde{i}_1 \perp\!\!\!\perp 1+\tilde{i}_2$

$$\boxed{\tilde{S}(2) = (1+\tilde{i}_1)(1+\tilde{i}_2)}$$

$$E[\tilde{S}(2)] = E[(1+\tilde{i}_1)(1+\tilde{i}_2)] = E[(1+\tilde{i}_1)] \cdot E[(1+\tilde{i}_2)]$$

$$Var[\tilde{S}(2)] = \boxed{E[\tilde{S}(2)^2]} - (E[\tilde{S}(2)])^2$$

$$= \boxed{E[(1+\tilde{i}_1)^2(1+\tilde{i}_2)^2]} - (E[1+\tilde{i}_1] \cdot E[1+\tilde{i}_2])^2$$

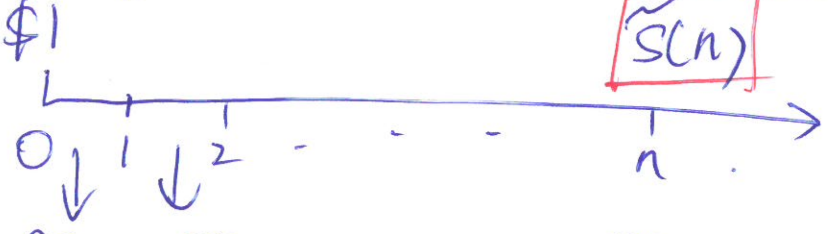
$$= E[(1+\tilde{i}_1)^2] \cdot E[(1+\tilde{i}_2)^2] - (E[1+\tilde{i}_1] \cdot E[1+\tilde{i}_2])^2$$

Assume $\tilde{i}_1$ & $\tilde{i}_2$ : i.i.d $E[\tilde{i}]$ & $Var[\tilde{i}]$

$$\Rightarrow E[\tilde{S}(2)] = (E[1+\tilde{i}])^2$$

$$Var[\tilde{S}(2)] = (E[(1+\tilde{i})^2])^2 - (E[1+\tilde{i}])^4$$

③ $\boxed{n}$




$\tilde{i}_1 \perp\!\!\!\perp \tilde{i}_2 \cdots \perp\!\!\!\perp \tilde{i}_n$

$$\tilde{S}(n) = (1+\tilde{i}_1) \times (1+\tilde{i}_2) * \cdots * (1+\tilde{i}_n)$$

$$\Rightarrow E[\tilde{S}(n)] = E[1+\tilde{i}_1] \times E[1+\tilde{i}_2] * \cdots * E[1+\tilde{i}_n]$$

$$Var[\tilde{S}(n)] = \boxed{E[\tilde{S}(n)^2]} - \left(E[S(n)]\right)^2 .$$

$$= \boxed{\left\{E[(1+\tilde{i}_1)^2] \cdot E[(1+\tilde{i}_2)^2] \cdots E[(1+\tilde{i}_n)^2]\right\}}$$

$$- \left\{E[1+\tilde{i}_1] \cdot E[1+\tilde{i}_2] \cdots E[1+\tilde{i}_n]\right\}^2$$

Assume: $\tilde{i}_1, \tilde{i}_2 \cdots \tilde{i}_n$, i.i.d. $E[\tilde{i}]$, $Var[\tilde{i}]$.

$$\Rightarrow \boxed{\begin{aligned} E[\tilde{S}(n)] &= \left(E[1+\tilde{i}]\right)^n . \\ E[\tilde{S}(n)^2] &= \left(E[(1+\tilde{i})^2]\right)^n . \\ Var[\tilde{S}(n)] &= E[\tilde{S}(n)^2] - \left(E[\tilde{S}(n)]\right)^2 \\ &= \left(E[(1+\tilde{i})^2]\right)^n - \left(E[1+\tilde{i}]\right)^{2n} . \end{aligned}}$$

Ex:


$$\tilde{i} = \begin{cases} 0.06 , & prob = 0.25 \\ 0.07 , & prob = 0.15 \\ 0.08 , & prob = 0.60 . \end{cases}$$

Q: $E[\ \ ]$ & $Var[\ \ ]$ of $\boxed{50 \cdot \tilde{S}(20)}$

Sol. $50\,\boxed{E[\tilde{S}(20)]} = E[50\cdot\tilde{S}(20)]$

$$E[\tilde{S}(20)] = \left(\boxed{E[(1+\tilde{i})]}\right)^{20}.$$

$$\boxed{\begin{aligned} E[1+\tilde{i}] &= 1+E[\tilde{i}] = 1+0.0735 \\ &= 1.0735. \end{aligned}}$$

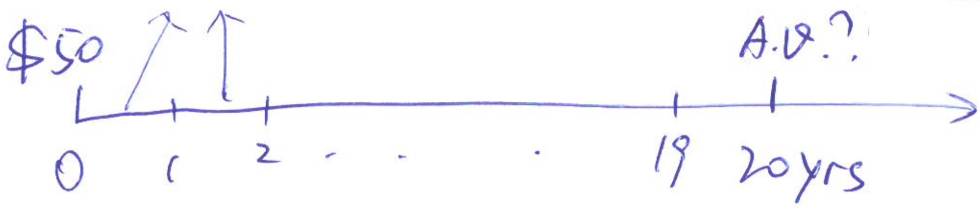
$$\Rightarrow\quad E[50\cdot\tilde{S}(20)] = 50\cdot E[\tilde{S}(20)] = 50\cdot(1.0735)^{20}$$
$$= \$\ 206.54.$$

$$\begin{aligned} Var[50\cdot\tilde{S}(20)] &= 50^2\cdot Var[\tilde{S}(20)] \\ &= 2500\cdot Var[\tilde{S}(20)] \\ &= 2500\left\{\left(\boxed{E[(1+\tilde{i})^2]}\right)^{20} - \left(\boxed{\overset{1.0735}{E(1+\tilde{i})}}\right)^{40}\right\} \end{aligned}$$

$$\boxed{\begin{aligned} E[(1+\tilde{i})^2] &= 1.06^2\times 0.25 + 1.07^2\times 0.15 + 1.08^2\times 0.6 \\ &= 1.152475 \end{aligned}}$$

$$Var[50\cdot\tilde{S}(20)] = 53.89.$$

Ex: $\tilde{i} \sim$ Uni~~variate~~form $[0.08, 0.12]$.




$$E[50\tilde{S}(20)] = 50\,E[\tilde{S}(20)]$$

$$= 50\cdot\left(\boxed{E[1+\tilde{i}]}\right)^{20}$$

$$\Downarrow$$

$$\boxed{E[1+\tilde{i}] = 1+E[\tilde{i}] = 1+\frac{0.08+0.12}{2} = 1.1}$$

$$\Rightarrow E[50\tilde{S}(20)] = 50\cdot(1.1)^{20} = \$336.38$$

$$Var[50\cdot\tilde{S}(20)] = 50^2\cdot\left[\left(\boxed{E[(1+\tilde{i})^2]}\right)^{20} - \left(\boxed{E[1+\tilde{i}]}\right)^{40}\right]$$

$$\Downarrow$$

$$\boxed{E[(1+\tilde{i})^2]} = \int_{0.08}^{0.12}(1+\tilde{i})^2\cdot f(\tilde{i})\,d\tilde{i}$$

$$= \int_{0.08}^{0.12}\frac{(1+\tilde{i})^2}{0.12-0.08}\,d\tilde{i}$$

$$= 25\int_{0.08}^{0.12}(1+\tilde{i})^2\,d\tilde{i}$$

$$= 25\left[\frac{(1+\tilde{i})^3}{3}\right]_{0.08}^{0.12}$$

$$= 25\times\frac{(1+0.12)^3-(1+0.08)^3}{3}$$

$$= 1.210133$$

$$\boxed{E[h(x)] = \int_{-\infty}^{\infty} h(x)\cdot f(x)\,dx}$$

$$\boxed{Var[(1+\tilde{i})]} + \left(E[1+\tilde{i}]\right)^2$$

$$= \frac{(0.12-0.08)^2}{12} + 1.1^2$$

$$=$$

$$\Rightarrow Var[50 \cdot \hat{S}(20)] = 2500 \cdot [1.210133^{20} - 1.1^{40}]$$

$$= 249$$

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