DERIVATIONS
NATURAL DEDUCTION
Part 2 Solutions

3.11 E1 Which inference rule if any justifies the following arguments?
(S, ADJ, ADD, MTP, BC, CB or none)

a) $R \to (P \vee \sim S)$ $(P \vee \sim S) \to R$ ——— $\therefore (P \vee \sim S) \leftrightarrow R$ CD	b) $(P \vee Q) \wedge (S \to T)$ ——— $\therefore S \to T$ sl	c) $(P \to (S \to \sim Q))$ ——— $\therefore \sim P \vee (P \to (S \to \sim Q))$ add	d) $P \to R$ S ——— $\therefore P \to R \wedge S$ none
e) $Q \vee \sim(S \to P)$ $(S \to P)$ ——— $\therefore Q$ none (2 steps: pr2 dn, pr1 mtp)	f) $(V \leftrightarrow Z) \vee (\sim W \wedge Y)$ $\sim(\sim W \wedge Y)$ ——— $\therefore V \leftrightarrow Z$ mtp	g) $S \to \sim R$ $S \leftarrow \sim R$ ——— $\therefore S \leftrightarrow \sim R$ none $\leftarrow$ is not a symbol of SL	h) P $P \wedge R$ ——— $\therefore P \wedge R \wedge P$ adj

3.11 EG1 Let's try out the new rules

$P \wedge Q$. $(R \vee P) \to \sim S$. $S \vee T$. $(V \to W) \leftrightarrow Q$. $W \to V$. $\therefore T \wedge (V \leftrightarrow W)$

1	Show $T \wedge (V \leftrightarrow W)$		First line is always a show line. Show conclusion! The conclusion is a conjunction. Thus, we probably want to derive each conjunct separately (T and $V \leftrightarrow W$) and then use ADJ to join them and achieve our goal.
2	P	pr1 S	Premise 1 is a conjunction. We can use S to free up the conjuncts – in this instance, to infer P.
3	Q	pr1 S	We can use S again to free up the other conjunct – this time to infer Q.
4	$R \vee P$	2 ADD	The antecedent of premise 2 is $(R \vee P)$ – we can easily derive that from line 2 using ADD. I make sure I'm putting the two disjuncts in the order that they appear in premise 2.
5	$\sim S$	4 pr2 MP	Now I can use MP with lines 4 and the second premise to get ~S.
6	T	5 pr3 MTP	Premise 3 is a disjunction and line 5 is the negation of one of the disjuncts. Thus, I can use MTP to derive the other disjunct, T. That will give me one half of my goal –the first conjunct.

Now I just need to derive the other half of my goal – $(V \leftrightarrow W)$.

$P \wedge Q$. $(R \vee P) \rightarrow \sim S$. $S \vee T$. $(V \rightarrow W) \leftrightarrow Q$. $W \rightarrow V$. $\therefore T \wedge (V \leftrightarrow W)$

1	Show $T \wedge (V \leftrightarrow W)$		My goal is to get both conjuncts: T and $V \leftrightarrow W$. Then I'll use ADJ.
2	P	pr1 SL	Here I am using the alternate justification for S.
3	Q	pr1 SR	Here I am using the alternate justification for S.
4	$R \vee P$	2 ADD	
5	$\sim S$	4 pr2 MP	
6	T	5 pr3 MTP	Now I have half my goal. Now I need the other half: $V \leftrightarrow W$ If I had both $V \rightarrow W$ and $W \rightarrow V$ then I could achieve my goal by using CB. I already have $W \rightarrow V$ – that's the last premise. So all I really want is $V \rightarrow W$.
7	$Q \rightarrow (V \rightarrow W)$	pr4 BC	Premise 4 is a biconditional. One side is the same as line 3. The other side is the sentences that I want: $V \rightarrow W$. By using BC on premise 4, I can turn it into a usable conditional. I could derive either $Q \rightarrow (V \rightarrow W)$ OR $(V \rightarrow W) \rightarrow Q$ with BC. I make sure that I derive the former so that Q is the antecedent.
8	$V \rightarrow W$	3 7 MP	Now I can use MP with lines 3 and 7 to derive $V \rightarrow W$.
9	$V \leftrightarrow W$	pr5 8 CB	Now I've got both $V \rightarrow W$ and $W \rightarrow V$, I can turn it into the biconditional I want using CB. I make sure that I put V on the left of the $\leftrightarrow$.
10	$T \wedge (V \leftrightarrow W)$	6 9 ADJ	I've got both parts of my goal, line 6 and line 9. Now I just use ADJ to get the sentence that I want. I make sure that I put the two conjuncts in the right order. It's all but complete! I just need to write DD, box and cancel.

Here's the completed derivation. It uses each of the new rules!

$P \wedge Q$. $(R \vee P) \rightarrow {\sim}S$. $S \vee T$. $(V \rightarrow W) \leftrightarrow Q$. $W \rightarrow V$. $\therefore T \wedge (V \leftrightarrow W)$

1	~~Show~~ $T \wedge (V \leftrightarrow W)$		
2	P	pr1 SL	
3	Q	pr1 SR	← Simplification
4	$R \vee P$	2 ADD	← Addition
5	${\sim}S$	4 pr2 MP	
6	T	5 pr3 MTP	← Modus Tollendo Ponens
7	$Q \rightarrow (V \rightarrow W)$	pr4 BC	← Biconditional to Conditional
8	$V \rightarrow W$	3 7 MP	
9	$V \leftrightarrow W$	pr5 8 CB	← Conditional to Biconditional
10	$T \wedge (V \leftrightarrow W)$	6 9 ADJ	← Adjunction
11		10 DD	

3.11 EG2 Let's try another:

$S \rightarrow (W \wedge (S \rightarrow P))$. $V \wedge S$. $X \leftrightarrow (V \wedge W)$. $(X \vee Q) \rightarrow (P \rightarrow S)$. ${\sim}(P \leftrightarrow S) \vee (Z \wedge R)$. $\therefore Z \wedge R$

1	~~Show~~ $Z \wedge R$	
2	S	PR2 S
3	$W \wedge (S \rightarrow P)$	2 PR1 MP
4	$S \rightarrow P$	3 S
5	P	2 4 MP
6	V	PR2 S
7	W	3 S
8	$V \wedge W$	6 7 ADJ
9	$(V \wedge W) \rightarrow X$	PR3 BC
10	X	8 9 MP
11	$X \vee Q$	10 ADD
12	$P \rightarrow S$	11 PR4 MP
13	$P \leftrightarrow S$	4 12 CB
14	${\sim}{\sim}(P \leftrightarrow S)$	13 DN
15	$Z \wedge R$	PR5 14 MTP DD

Of course the new rules also work with CD and ID.

3.11 EG3 Let's try this one:

$(P \vee W) \rightarrow \sim R. \quad \sim Q \vee S. \quad (S \rightarrow R) \wedge (S \leftrightarrow \sim Q). \quad \therefore (P \leftrightarrow Q)$

1	~~Show~~ $P \leftrightarrow Q$	
2	~~Show~~ $P \rightarrow Q$	Show conditional (so it can be used for CB)
3	P	ASS CD
4	$P \vee W$	3 ADD
5	$\sim R$	4 PR1 MP
6	$S \rightarrow R$	PR3 S (or PR3 SL)
7	$\sim S$	5 6 MT
8	$S \leftrightarrow \sim Q$	PR3 S (or PR3 SR)
9	$\sim Q \rightarrow S$	8 BC
10	$\sim\sim Q$	7 9 MT
11	Q	10 DN CD
12	~~Show~~ $Q \rightarrow P$	Show conditional (so it can be used for CB)
13	Q	ASS CD
14	~~Show~~ P	Show cons. (Show consequent of 12)
15	$\sim P$	Ass ID
16	$\sim\sim Q$	13 DN
17	S	16 PR2 ADJ
18	$S \leftrightarrow \sim Q$	PR3 S
19	$S \rightarrow \sim Q$	18 BC
20	$\sim Q$	17 19 MP 16 ID
21		14 CD
22	$P \leftrightarrow Q$	2 12 CB DD

3.11 EG4 Let's try this one:

$(P \wedge {\sim}R) \rightarrow T$. $(S \leftrightarrow T) \wedge {\sim}(P \wedge S)$. ${\sim}P \vee {\sim}R$ $\therefore Z \vee (P \rightarrow W)$

1	~~Show~~ $Z \vee (P \rightarrow W)$	
2	~~Show~~ $P \rightarrow W$	show disjunct (for ADD)
3	P	ASS CD
4	~~Show~~ W	show cons.
5	${\sim}W$	ASS ID
6	${\sim}{\sim}P$	3 DN
7	${\sim}R$	6 PR3 MTP
8	$P \wedge {\sim}R$	3 7 ADJ
9	T	8 PR1 MP
10	$S \leftrightarrow T$	PR2 SL
11	$T \rightarrow S$	10 BC
12	S	11 9 MP
13	$P \wedge S$	3 12 ADJ
14	${\sim}(P \wedge S)$	PR2 SR 13 ID
15		4 CD
16	$Z \vee (P \rightarrow W)$	2 ADD DD

3.11 E1

(a) I realized, as I lay in bed thinking, that we are not responsible for what we do. This is because either determinism or indeterminism must be true. Provided that determinism is true, we cannot do other than we do. If so, we are but puppets on strings – our actions are not free. If indeterminism is true, then human actions are random, and hence not free. If our actions are not free, it must be conceded that we are not responsible for what we do.

$P \vee Q$. $P \rightarrow \sim R$. $\sim R \rightarrow \sim S$. $Q \rightarrow T$. $T \rightarrow \sim S$. $\sim S \rightarrow \sim U$ $\therefore \sim U$

3.11E1A: $P \vee Q$. $P \rightarrow \sim R$. $\sim R \rightarrow \sim S$. $Q \rightarrow T$. $T \rightarrow \sim S$. $\sim S \rightarrow \sim U$. $\therefore \sim U$

1	~~Show~~ ~U	"SHOW CONC"
2	~~U	ASS ID
3	~~S	2 PR6 MT
4	~T	3 PR5 MT
5	~Q	4 PR4 MT
6	P	5 PR1 MTP
7	~R	6 PR2 MP
8	~S	7 PR3 MP
9		3 8 ID

(b) In our world, there are conscious experiences. Yet, there is a logically possible world physically identical to ours, and in that world there are no conscious experiences. If there are conscious experiences in our world, but not in a physically identical world, then facts about consciousness are further facts about our world, over and above the physical facts. If this is so, not all facts are physical facts. It follows, then, that materialism is false. For, in virtue of the meaning of materialism, materialism is true only if all facts are physical facts.
(Based on David Chalmers, The Conscious Mind: In Search of a Fundamental Theory. Oxford: Oxford University Press, 1996. 123-129.)

P. Q. $P \wedge Q \rightarrow R$. $R \rightarrow \sim S$. $T \rightarrow S$. $\therefore \sim T$

3.11E1B: P. Q. $P \wedge Q \rightarrow R$. $R \rightarrow \sim S$. $T \rightarrow S$. $\therefore \sim T$

1	~~Show~~ ~T	"SHOW CONC"
2	P∧Q	PR1 PR2 ADJ
3	R	2 PR3 MP
4	~S	3 PR4 MP
5	~T	4 PR5 MT
6		5 DD

(c) Next we must consider what virtue is. Since things that are found in the soul are of three kinds —passions, faculties, states of character — virtue must be one of these. We are not called good or bad on the ground of our passions, but are so called on the ground of our virtues. And if we are called good or bad on the grounds of the one, but not the other, then virtues cannot be passions. Likewise, virtues are faculties only if we are called good or bad on the grounds of our faculties as we are so called on the grounds of our virtues. If we have the faculties by nature (which we do) but we are not made good or bad by nature (which we are not) then we cannot be called good or bad on the grounds of our faculties. And since this shows that the virtues are neither passions nor faculties, all that remains is that they should be states of character.

(Based on Aristotle, Nicomachean Ethics, Book 1, Ch. V)

$P \vee Q \vee R$. $\sim S$. U. $U \wedge \sim S \rightarrow \sim P$. $Q \rightarrow T$. $V \wedge \sim W \rightarrow \sim T$. V. $\sim W$. $\therefore R$

3.11E1C: $P\vee Q\vee R$. $\sim S$. U. $U\wedge\sim S\rightarrow\sim P$. $Q\rightarrow T$. $V\wedge\sim W\rightarrow\sim T$. V. $\sim W$. $\therefore R$

1	~~Show~~ R	"SHOW CONC"
2	$\sim R$	ASS ID
3	$U\wedge\sim S$	PR2 PR3 ADJ
4	$\sim P$	3 PR4 MP
5	$P\vee Q$	2 PR1 MTP
6	Q	4 5 MTP
7	T	6 PR5 MP
8	$V\wedge\sim W$	PR7 PR8 ADJ
9	$\sim T$	8 PR6 MP
10		7 9 ID

3.11 E2

(a) I have to pay Protagoras only if I win my first case. Either I won't take any cases and will not have to pay, or Protagoras will sue me. Of, course, if he does that then I will defend myself. But if I defend myself and win this case, then I won't have to pay him (by judgment of the court.) If I don't win then I won't have won my first case. Thus, I will not have to pay Protagoras.

$R \to P. \quad (\sim T \wedge \sim R) \vee S. \quad S \to U. \quad (U \wedge Q) \to \sim R. \quad \sim Q \to \sim P. \quad \therefore \sim R.$

1	~~Show~~ ~R	"show conc"
2	R	ass id
3	P	2 pr1 mp
4	~~P	3 dn
5	~~Q	4 pr5 mt
6	Q	5 dn
7	~~Show~~ U	
8	~U	ass id
9	~S	pr3 8 mt
10	~T∧~R	pr2 9 mtp
11	~R	10 sr
12	R	2 r
13		11 12 id
14	U∧Q	6 7 adj
15	~R	14 pr4 mp
16		2 15 id

(b) Euathlus must pay me, since he has to pay me if he wins his first case. I am suing Protagoras and he is defending himself. And if he defends himself, then if he wins this case then he will have won his first case. But if I am suing Protagoras and Euathlus doesn't win, then he must pay me (by judgment of the court).

$P \to R. \quad S \wedge U. \quad U \to (Q \to P). \quad (S \wedge \sim Q) \to R. \quad \therefore R$

1	~~Show~~ R	"SHOW CONC"
2	~R	ASS ID
3	~P	PR1 2 MT
4	U	PR2 SR
5	Q→P	4 PR3 MP
6	~Q	3 5 MT
7	S	PR2 SL
8	S∧~Q	7 6 ADJ
9	R	8 PR4 MP
10		2 9 ID

(c) Euathlus has to pay Protagoras if and only if he wins his first case; and he wins his first case only if he wins this case. But if we make a ruling and Euathlus wins this case then he doesn't have to pay Protagoras (by our judgment). On the other hand, if Euathlus doesn't have to pay Protagoras if he doesn't win this case, then he does have to pay Protagoras if we make a ruling. If we do not make a ruling then we shall adjourn the court for an indefinite length of time. Therefore, we shall do exactly that.

$(P \leftrightarrow R) \wedge (P \rightarrow Q)$. $V \wedge Q \rightarrow \sim R$. $(\sim Q \rightarrow \sim R) \rightarrow (V \rightarrow R)$. $\sim V \rightarrow W$. $\therefore$ W

1	~~Show~~ W	"show conc"
2	$\sim W$	ass id
3	V	pr4 2 mt dn
4	~~Show~~ Q	
5	$\sim Q$	ass id
6	$P \rightarrow Q$	pr1 sr
7	$\sim P$	5 6 mt
8	$P \leftrightarrow R$	pr1 sl
9	$\sim R$	8 bc 7 mt
10	~~Show~~ $\sim Q \rightarrow \sim R$	
11	$\sim Q$	ass cd
12		9 r cd
13	$V \rightarrow R$	pr3 10 mp
14	$\sim V$	9 13 mt
15	V	3 r
16		14 15 id
17	$V \wedge Q$	3 4 adj
18	$\sim R$	17 pr2 mp
19	~~Show~~ $\sim Q \rightarrow \sim R$	
20	$\sim Q$	ass cd
21		18 r cd
22	$V \rightarrow R$	19 pr3 mp
23	R	3 22 mp
24		23 18 id

Another way to do this one, using derived rules.

3.11 E2 c: $(P \leftrightarrow R) \wedge (P \to Q)$. $V \wedge Q \to {\sim}R$. $({\sim}Q \to {\sim}R) \to (V \to R)$. ${\sim}V \to W$. $\therefore$ W

1	~~Show~~ W	"show conc"
2	${\sim}W$	ass id
3	${\sim}{\sim}V$	2 pr4 mt
4	V	3 dn
5	~~Show~~ ${\sim}Q$	
6	Q	ass id
7	${\sim}R$	4 6 adj pr2 mp
8	$Q \vee {\sim}R$	7 add
9	${\sim}Q \to {\sim}R$	8 cdj
10	$V \to R$	9 pr3 mp
11	R	4 10 mp
12		7 11 id
13	${\sim}P$	pr1 sr 5 mt
14	${\sim}R$	pr1 sl bc 13 mt
15	$V \wedge {\sim}R$	4 14 adj
16	${\sim}(V \to R)$	15 nc
17	${\sim}({\sim}Q \to {\sim}R)$	16 pr3 mt
18	${\sim}Q \wedge {\sim}{\sim}R$	17 nc
19	${\sim}{\sim}R$	18 sr
20		14 19 id

3.12 Theorems with ∧, ∨ and ↔

We can also use the new rules to derive theorems:

Theorems are sentences derivable from the empty set – from nothing. In the last unit, we looked at 23 numbered theorems – but of course there are an infinite number of sentences that can be derived from the empty set (and thus are theorems). In the exercises, you may have derived other theorems.

With the new symbols we have about 100 new numbered theorems. We will derive a few of the new theorems – but it is a good exercise to do more on your own. And once you have derived a theorem (proven it to be true), you are entitled to use it in any other derivation. That makes your efforts pay off and saves you time later!

3.12 EG1 Let's try one:

T27 $((P \wedge Q) \rightarrow R) \leftrightarrow (P \rightarrow (Q \rightarrow R))$

1	~~Show~~ (P∧Q→R)↔(P→(Q→R))	"Show conc"
2	~~Show~~ (P∧Q→R)→(P→(Q→R))	"show cond"
3	P∧Q→R	ass cd
4	~~Show~~ P→(Q→R)	"show cons"
5	P	ass cd
6	~~Show~~ Q→R	"show cons"
7	Q	ass cd
8	P∧Q	5 7 adj
9	R	8 3 mp
10		9 cd
11		6 cd
12		4 cd
13	~~Show~~ (P→(Q→R))→(P∧Q→R)	"show cond"
14	P→(Q→R)	ass cd
15	~~Show~~ P∧Q→R	"show cons"
16	P∧Q	ass cd
17	P	16 sl
18	Q→R	14 17 mp
19	Q	16 sr
20	R	18 19 mp
21		20 cd
22		15 cd
23	(P∧Q→R)↔(P→(Q→R))	2 13 cb
24		23 dd

3.12 EG2

Let's try another one:

T 63 $P \wedge Q \leftrightarrow \sim(\sim P \vee \sim Q)$

1	~~Show~~ P∧Q↔~(~P∨~Q)	"show conc"
2	~~Show~~ P∧Q→~(~P∨~Q)	"show cond"
3	P∧Q	ass cd
4	~~Show~~ ~(~P∨~Q)	"show cons"
5	~P∨~Q	ass id
6	~~P	3 sl dn
7	~Q	5 6 mtp
8	Q	3 sr
9		7 8 id
10		4 cd
11	~~Show~~ ~(~P∨~Q)→P∧Q	"show cond"
12	~(~P∨~Q)	ass cd
13	~~Show~~ P	
14	~P	ass id
15	~P∨~Q	14 add
16	~(~P∨~Q)	12 r
17		15 16 id
18	~~Show~~ Q	
19	~Q	ass id
20	~P∨~Q	19 add
21	~(~P∨~Q)	12 r
22		20 21 id
23	P∧Q	13 18 adj
24		23 cd
25	P∧Q↔~(~P∨~Q)	2 11 cb
26		25 dd

3.12 EG

T61. $P \wedge (Q \vee R) \leftrightarrow (P \wedge Q) \vee (P \wedge R)$

1	Show $P \wedge (Q \vee R) \leftrightarrow (P \wedge Q) \vee (P \wedge R)$	
2	~~Show~~ $P \wedge (Q \vee R) \rightarrow (P \wedge Q) \vee (P \wedge R)$	
3	$P \wedge (Q \vee R)$	ass cd
4	~~Show~~ $(P \wedge Q) \vee (P \wedge R)$	show consequent
5	$\sim[(P \wedge Q) \vee (P \wedge R)]$	ass id
6	P	3 sl
7	~~Show~~ $\sim R$	
8	R	ass id
9	$P \wedge R$	6 9 adj
10	$(P \wedge Q) \vee (P \wedge R)$	9 add
11	$\sim[(P \wedge Q) \vee (P \wedge R)]$	5 r 10 id
12	$Q \vee R$	3 sr
13	Q	7 12 mtp
14	$P \wedge Q$	6 13 adj
15	$(P \wedge Q) \vee (P \wedge R)$	14 add
16	$\sim[(P \wedge Q) \vee (P \wedge R)]$	5 r 15 id
17		4 cd
18	~~Show~~ $(P \wedge Q) \vee (P \wedge R) \rightarrow P \wedge (Q \vee R)$	
19	$(P \wedge Q) \vee (P \wedge R)$	ass cd
20	~~Show~~ $P \wedge (Q \vee R)$	show cons
21	$\sim (P \wedge (Q \vee R))$	ass id
22	~~Show~~ $\sim(P \wedge Q)$	
23	$P \wedge Q$	ass id
24	Q	23 sr
25	$Q \vee R$	24 add
26	P	23 sl
27	$P \wedge (Q \vee R)$	25 26 adj
28	$\sim (P \wedge (Q \vee R))$	21 r 27 id
29	$P \wedge R$	22 19 mtp
30	R	29 sr
31	$Q \vee R$	30 add
32	P	29 sl
33	$P \wedge (Q \vee R)$	31 32 adj
34	$\sim (P \wedge (Q \vee R))$	21 r 33 id
35		20 cd
35		2 18 cb dd

3.12 E1 Try the following using the suggested theorems as rules

T27	$(P \wedge Q \to R) \leftrightarrow (P \to (Q \to R))$
T29	$(P \to Q \wedge R) \leftrightarrow (P \to Q) \wedge (P \to R)$
T24	$P \wedge Q \leftrightarrow Q \wedge P$
T61	$P \wedge (Q \vee R) \leftrightarrow (P \wedge Q) \vee (P \wedge R)$
T62	$P \vee (Q \wedge R) \leftrightarrow (P \vee Q) \wedge (P \vee R)$

a) $(S \wedge T) \to (V \wedge W)$. $\therefore S \to (T \to V)$

3.12 E1 RT: $(S\wedge T)\to(V\wedge W)$. $\therefore S\to(T\to V)$

1	~~Show~~ $S\to(T\to V)$	"show conc"
2	$(S\wedge T\to V)\wedge(S\wedge T\to W)$	pr1 rt29
3	$S \wedge T\to V$	2 sl
4	$S\to(T\to V)$	3 rt27
5		4 dd

b) $\sim(T \wedge U)$. $(S \vee U) \wedge (S \vee T)$. $\therefore S$

3.12 E1b RT: $\sim(T\wedge U)$. $(S\vee U)\wedge(S\vee T)$. $\therefore S$

1	~~Show~~ S	"show conc"
2	$(S\vee T)\wedge(S\vee U)$	pr2 rt24
3	$S\vee(T\wedge U)$	2 rt62
4	S	3 pr1 mtp
5		4 dd

c) $R \wedge (S \vee T)$. $R \to (T \to U)$. $\therefore \sim(R \wedge S) \to U$

3.12 E1 c RT: $R\wedge(S\vee T)$. $R\to(T\to U)$. $\therefore\sim(R\wedge S)\to U$

1	~~Show~~ $\sim(R\wedge S)\to U$	"show conc"
2	$\sim(R\wedge S)$	ass cd
3	$(R\wedge S)\vee(R\wedge T)$	pr1 rt61
4	$R\wedge T\to U$	pr2 rt27
5	$R\wedge T$	2 3 mtp
6	U	4 5 mp
7		6 cd

3.13 E2 Use the derived rules in the following derivations.

(a) ~(P ∨ Q). R → Q. ~(~R ↔ S). ∴~S

3.13 E2 a): ~(P ∨ Q). R→Q. ~(~R↔S). ∴ ~S

1	~~Show~~ ~S	"show conc"
2	~P∧~Q	pr1 dm
3	~P	2 sl
4	~Q	2 sr
5	~R	4 pr2 mt
6	~R↔~S	pr3 nb
7	~R→~S	6 bc
8	~S	5 7 mp
9		8 dd

(b) W → ~(S ∧ T). ~S → ~Z. ~T → ~Z. ∴ (~W ∨ ~Z)

3.13 E2b: W→~(S∧T). ~S→~Z. ~T→~Z. ∴ (~W ∨ ~Z)

1	~~Show~~ ~W∨~Z	"show conc"
2	~(~W∨~Z)	ass id
3	W∧Z	2 dm
4	W	3 sl
5	~(S∧T)	4 pr1 mp
6	~S∨~T	5 dm
7	~~Z	3 sr dn
8	~~T	7 pr3 mt
9	~S	8 6 mtp
10	~Z	pr2 9 mp
11		7 10 id

(c) ~(R → S). ~(T ∨ W ↔ R). V → T. ~V → (S ∨ W). ∴P

3.13 E2c: ~(R→S). ~(T∨W↔R). V→T. ~V →(S∨W). ∴ P

1	~~Show~~ P	"show conc"
2	~P	ass id
3	R∧~S	pr1 nc
4	R	3 sl
5	~S	3 sr
6	T∨W↔~R	pr2 nb
7	T∨W→~R	6 bc
8	~(T∨W)	4 dn 7 mt
9	~T∧~W	8 dm
10	~T	9 sl
11	~W	9 sr
12	~S∧~W	5 11 adj
13	~(S∨W)	12 dm
14	~~V	13 pr4 mt
15	T	14 dn pr3 mp
16		10 15 id

3.14 E1

Try the following derivations using derived rules, abbreviated proofs and/or theorems as rules:

a) V ∧ W → Y. ~ W → Z. (Y ∨ Z) → ~U. ∴U → ~V

3.14 E1a: V ∧ W → Y. ~W → Z. (Y ∨ Z) → ~U. ∴ U → ~V

1	~~Show~~ U→~V	"show conc"
2	U	ass cd
3	~(Y∨Z)	2 dn pr3 mt
4	~Y∧~Z	3 dm
5	~(V∧W)	4 sl pr1 mt
6	~V∨~W	5 dm
7	~~W	4 sr pr2 mt
8	~V	6 7 mtp
9		8 cd

b) $P \vee Q$. $Q \rightarrow S$. $U \vee \sim S$. $P \vee S \rightarrow R$. $R \rightarrow U$. $\therefore U$

3.14 E1b: $P \vee Q$. $Q \rightarrow S$. $U \vee \sim S$. $P \vee S \rightarrow R$. $R \rightarrow U$. $\therefore U$

1	~~Show~~ U	"show conc"
2	~U	ass id
3	~R	pr5 2 mt
4	~(P∨S)	pr4 3 mt
5	~P∧~S	4 dm
6	~Q	5 sr pr2 mt
7	P	6 pr1 mtp
8	P∨S	7 add
9		4 8 id

c) $R \wedge (S \vee T)$. $R \rightarrow (T \rightarrow U)$. $\therefore \sim(R \wedge S) \rightarrow U$

3.14 E1c: $R \wedge (S \vee T)$. $R \rightarrow (T \rightarrow U)$. $\therefore \sim(R \wedge S) \rightarrow U$

1	~~Show~~ ~(R∧S)→U	"show conc"
2	(R∧S)∨(R∧T)	pr1 rt61
3	~(R∧S)→R∧T	2 cdj
4	R∧T→U	pr2 rt27
5	~(R∧S)→U	3 4 rt26
6		5 dd