Name: SOLUTIONS

MAT 334H
SUMMER 2014
QUIZ 2

Problem	1	2	3	Total
Points	5	5	5	15
Score				

- Solve the following problems, and write up your solutions neatly, in black or blue ink, in the space provided.
- Please make sure your name is entered at the top of this page.
- This quiz contains 4 pages. Please ensure they are all there.
- Please do not tear out any pages.
- You have 30 minutes to complete this quiz.
- There are *no* aids allowed.

Good Luck!

(1). Determine whether the statement is True or False. Circle your answer. (No justification required.)

(a) The function $f(z) = (1 - \cos z)(z^3 - z^2)$ has a zero of order 5 at $z_0 = 0$. True $\boxed{\text{False}}$

(b) If f has a zero of order 3 at z_0, and g has a zero of order 2 at z_0, then $\frac{f^2}{g^3}$ has a removable singularity at z_0. $\boxed{\text{True}}$ False

(c) If $f(z) = \sum_{k=0}^{\infty} \frac{k}{k^2+1}(z - z_0)^k$, then $\text{Res}(\dfrac{f(z)}{(z - z_0)^3} : z_0) = \dfrac{3}{10}$. True $\boxed{\text{False}}$

(d) If $\lim\limits_{z \to z_0} \dfrac{1}{|f(z)|} = \infty$, then f has a removable singularity . $\boxed{\text{True}}$ False

(e) If $\lim\limits_{z \to z_0} (z - z_0)^5 f(z) = 5$, then z_0 is a pole of f. $\boxed{\text{True}}$ False

(3) Consider the function $f(z) = \dfrac{e^{z^2-4z+4}}{(z-2)^3}$.

(a) Find a power series for e^{z^2-4z+4} centred at $z_0 = 2$.

$$e^{z^2-4z+4} = e^{(z-2)^2} = \sum_{n=0}^{\infty} \frac{\left((z-2)^2\right)^n}{n!}$$

$$= \sum_{n=0}^{\infty} \frac{1}{n!} (z-2)^{2n}$$

(b) Compute Res$(f : 2)$.

$z_0 = 2$ is a pole of order 3.

So $\mathrm{Res}(f : 2) = C_{3-1} = C_2 = \frac{1}{4}$

(2) Let $f(z) = \dfrac{e^{\frac{1}{z+1}}(z^3-27)^3}{(z^4-81)^4}$.

(a) Find the zeroes of f, and determine their orders.

$e^{\frac{1}{z+1}} \neq 0$, so we need $z^3 - 27 = 0$, or $z^3 = 27$

$r^3 e^{i3\theta} = 27 e^{i\cdot 0}$

So $r = 3$, $3\theta = 2\pi k$

$\theta = 2\pi k/3 \quad k = 0, 1, 2.$

$z = 3$ isn't a zero of f since $(3^4-81)^4 = 0$ (← denom $= 0$) & has order 4

$z = 3e^{i2\pi/3}, 3e^{i4\pi/3}$ are zeros of order 3

(b) Find and classify each isolated singularity of f. If there are any poles, determine their orders.

$z = -1$ is an essential singularity.

$z^4 - 81 = 0$ has solutions $z = \pm 3, \pm 3i$

$z = 3$ is a pole of order $4 - 3 = 1$

$z = 3, \pm 3i$ are poles of order 4.