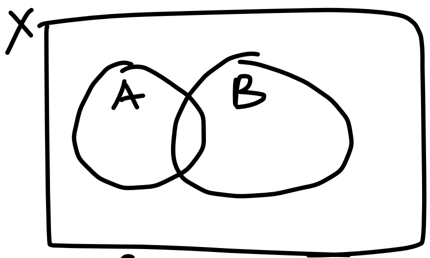
Prove $I(A \cup B) = I(A) + I(B) - I(A \cap B)$
$\quad\quad a \quad\quad\quad b \quad\quad c \quad\quad\quad d$



1. $(A \cup B)^c$ 2. $A \cap B$ 3. $A-B$ 4. $B-A$

1. $(A \cup B)^c$, $a=0, b=0, c=0, d=0$
2. $A \cap B$, $a=1, b=1, c=1, d=1$
3. $A-B$, $a=1, b=1, c=0, d=0$.
4. $B-A$, $a=1, b=0, c=1, d=0$

#47. $\boxed{A}$ 4R 3B 2G
$\boxed{B}$ 2R 3B 4G

Event X: pick 1 ball from $\boxed{A}$ to $\boxed{B}$, and $\boxed{R}$ is drawn from $\boxed{B}$.

$X = A_1 \cup A_2 \cup A_3$ A_i are mutually exclusive.

(a). $P(X) = P(X|A_1)P(A_1) + P(X|A_2)P(A_2) + P(X|A_3)P(A_3)$

(b). $P(A_1|X) = \frac{P(X|A_1)P(A_1)}{P(X)}$ (BAYES')

#57 A B C
$\boxed{\begin{matrix} G \\ G \end{matrix}}$ $\boxed{\begin{matrix} S \\ S \end{matrix}}$ $\boxed{\begin{matrix} G \\ S \end{matrix}}$ $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{2}{6}}{\frac{3}{6}} = \frac{2}{3}$

#71. (b) A, B, C mutually indept. Prove $A \cup B$ & C indep.

$$\begin{aligned} P((A \cup B) \cap C) &= P((A \cap C) \cup (B \cap C)) \\ &= P(A \cap C) + P(B \cap C) - P(A \cap B \cap C) \\ &= P(A)P(C) + P(B)P(C) - P(A)P(B)P(C) \\ &= [P(A) + P(B) - P(A)P(B)]P(C) \\ &= P(A \cup B)P(C) \quad \square \end{aligned}$$