About Midterm:

3. 10,8 . 4 from lot $Y \sim Bin(4, 0.1)$

$$E(C) = E(3Y^2 + Y + 2) = 3E(Y^2) + E(Y) + 2 \qquad E(AX+B) = AE(X) + B$$

$$E(Y^2) = Var(Y) + (E(Y))^2$$

5. $$E(Y^2) = \sum_{y=0}^{\infty} \frac{y^2 \lambda^y \cdot e^{-\lambda}}{y!} = \sum_{y=0}^{\infty} \frac{(y(y-1)+y)\lambda^y \cdot e^{-\lambda}}{y!} = \sum_{y=0}^{\infty} \frac{y(y-1)\lambda^y e^{-\lambda}}{y!} + \sum_{y=0}^{\infty} \frac{y\lambda^y e^{-\lambda}}{y!} = \sum_{y=0}^{\infty} \frac{\lambda^2 \lambda^{y-2} e^{-\lambda}}{(y-2)!}$$

$$+ \sum_{y=0}^{\infty} \frac{\lambda \lambda^{y-1} e^{-\lambda}}{(y-1)!} = \lambda^2 \underbrace{\sum_{z_1=0}^{\infty} \frac{\lambda^{z_1} e^{-\lambda}}{z_1!}}_{1} + \lambda \underbrace{\sum_{z_2=0}^{\infty} \frac{\lambda^{z_2} e^{-\lambda}}{z_2!}}_{1} = \lambda^2 + \lambda$$

$$Var(Y) = E(Y^2) - [E(Y)]^2 = \lambda$$

6. $$f_Y(y) = p[h^{-1}(y)] \cdot \underbrace{\left| \frac{d}{dy} h^{-1}(y) \right|}_{\text{"abs"}}$$

Note that, after calculation we get $0 < y < \infty$

so you should say:

$$f_Y(y) = \begin{cases} e^{-y}, & 0 < y < \infty \\ 0, & y \le 0 \end{cases}$$

7. a). $1 = \int_0^1 (cy^2 + y)dy = \frac{c}{3} + \frac{1}{2} \qquad c = \frac{3}{2}$

b). $$F_Y(y) = \begin{cases} \int_0^y (\quad)dt = \frac{t^3}{2} + \frac{t^2}{2} \Big|_0^y = \frac{y^3}{2} + \frac{y^2}{2}, & 0 \le y \le 1 \\ 0, & y < 0 \\ 1, & y > 0 \end{cases}$$

c). $P(0 \le y \le \frac{1}{2}) = F(\frac{1}{2}) - F(0) = (\frac{1}{16} + \frac{1}{8}) - (0+0) = \frac{3}{16}$

8. $E(Y)$, $Y = \Phi(X)$, $X = \Phi(Z)$, $Z \sim N(2, 4)$

$Z \sim N(2, 4)$, $X = -Z + 2$

$X \sim N(0, 4)$

$E(Y) = E(X^2) = Var(X) + [E(X)]^2 = 4 + 0 = 4$