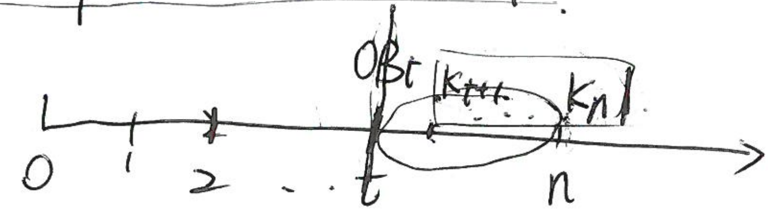
4N. 7.1

$OB_t <$ ① retrospective method ① / ② prospective method

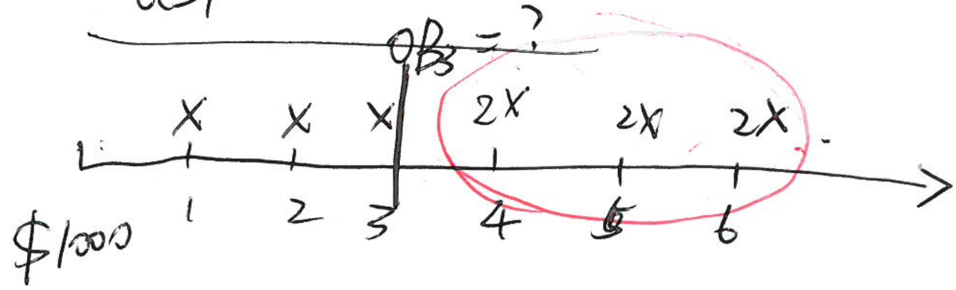
② Prospective Method



$$OB_t = v \cdot K_{t+1} + v^2 \cdot K_{t+2} + \cdots + v^{n-1} K_n$$

$$= \sum_{a=1}^{n-1} v^a K_{a+1}$$

Ex.



$X = 115.61 \Rightarrow 2X = 231.21,$

$i = 0.01 \Rightarrow v = \frac{1}{1+i}$

$$OB_3 = v \cdot 2X + v^2 \cdot 2X + v^3 2X$$

$$= 231.21 \left(1.01^{-1} + 1.01^{-2} + 1.01^{-3}\right)$$

$$\simeq 679.99$$

loan with level payment.

$$K_i = K$$

Loan Schedule.

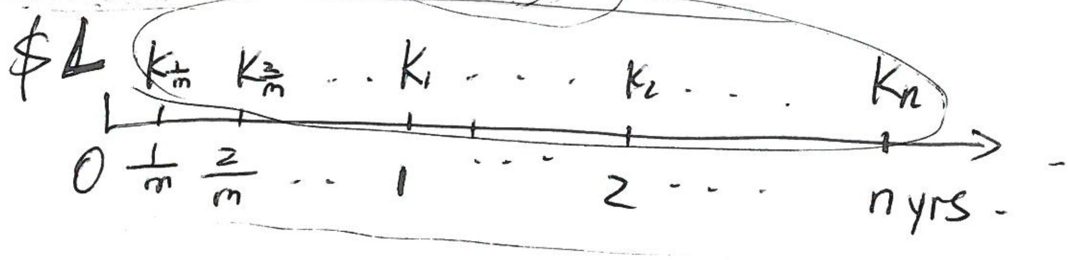
t	Payment	Interest Due	Principal Repaid	Outstanding Balance
0				$L = OB_0 = K a_{\overline{n}\rceil}$
1	K	$I_1 = i \cdot K a_{\overline{n}\rceil} = K(1-v^n)$	$PR_1 = K \cdot v^n$	$OB_1 = K a_{\overline{n}\rceil} - K \cdot v^n = K a_{\overline{n-1}\rceil}$
2	K	$I_2 = i \cdot K a_{\overline{n-1}\rceil} = K(1-v^{n-1})$	$PR_2 = K \cdot v^{n-1}$	$OB_2 = K a_{\overline{n-2}\rceil}$
t	K	$I_t = K(1-v^{n-t+1})$	$PR_t = K v^{n-t+1}$	$OB_t = K a_{\overline{n-t}\rceil}$
n.	K.	$I_n = K \cdot (1-v)$	$PR_n = K \cdot v.$	$OB_n = K(\underset{a_{\overline{1}\rceil}}{\underbrace{a_{\overline{1}\rceil}} } - v) = 0$

$$\sum_{t=1}^{n} I_t = K \cdot (n - a_{\overline{n}\rceil})$$

$$\sum_{t=1}^{n} PR_t = K a_{\overline{n}\rceil} = L.$$

Payments made mthly.

$a^{(m)}_{\overline{n}|i}$ | $a_{\overline{n\times m}|j}$



$n \times m$ payments.

$$L = OB_0 = K_{\frac{1}{m}} \cdot v^{\frac{1}{m}} + K_{\frac{2}{m}} \cdot v^{\frac{2}{m}} \cdots + K_n \cdot v^n$$

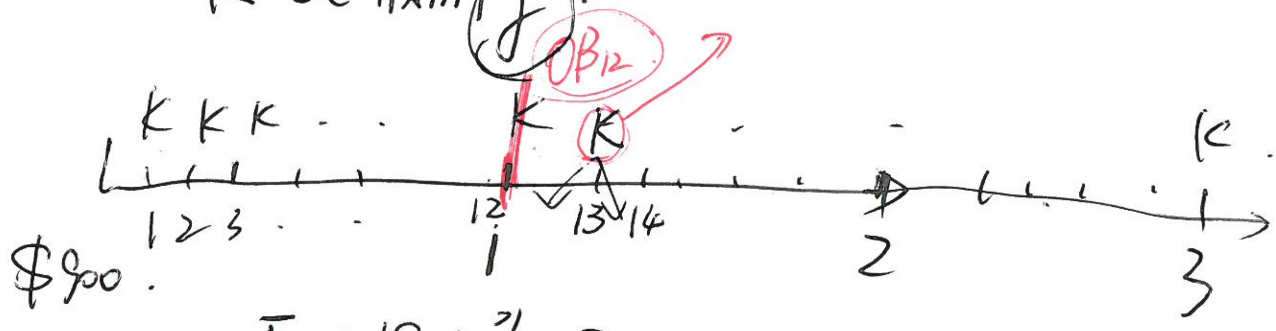
$$K_{\frac{1}{m}} = K = K \cdot v^{\frac{1}{m}} + K \cdot v^{\frac{2}{m}} + \cdots K v^n$$

$$= m \cdot K \cdot a^{(m)}_{\overline{n}|}$$

$$= K \, a_{\overline{n\times m}|j}$$

I13.

Ex:



$i = 18.5\%$ p.a.

$$900 = K \cdot 12 \cdot a^{(12)}_{\overline{3}|18.5\%} \Rightarrow K = 32.13$$

$$= K \cdot a_{\overline{3\times 12}|j} \qquad j = (1+i)^{\frac{1}{12}} - 1$$

Ex. $OB_{12} = K\, a_{\overline{n-t}|} = 32.13 \cdot a_{\overline{36-12}|j}$

$n = 3 \times 12$

$= 32.13 \times 12\, a^{(12)}_{\overline{2}|i}$

$= 649.25$

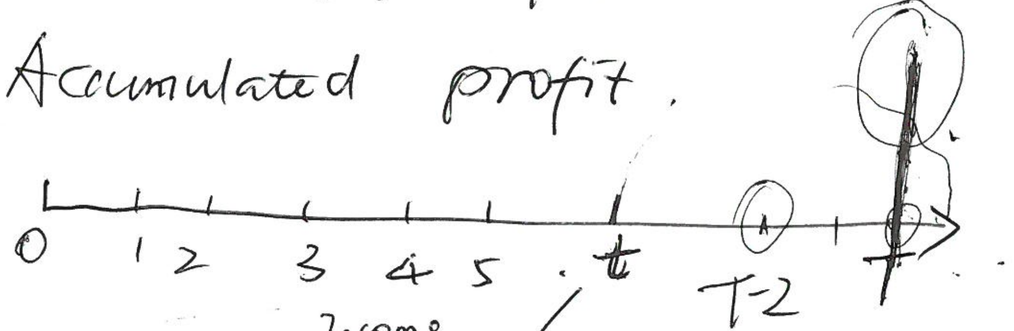
$I_{13} = OB_{12} \times j = 649.25 \times j = 9.25$

$j = (1+i)^{\frac{1}{12}} - 1$

Capital Budgeting

* Criteria:
1. Accumulated profit
2. NPV
3. IRR
4. DPB

① Accumulated profit



$C_t = I_t - O_t$ — I_t: Income, O_t: outgo; $\rho(t) = \rho_I(t) - \rho_O(t)$

C_t: Net Cash Flow at t

$\rho(t)$: continuous CF

$$AV(T) = \sum_t C_t \cdot (1+i)^{T-t} + \int_0^T \rho(t) \cdot (1+i)^{T-t}\, dt$$

$\downarrow$

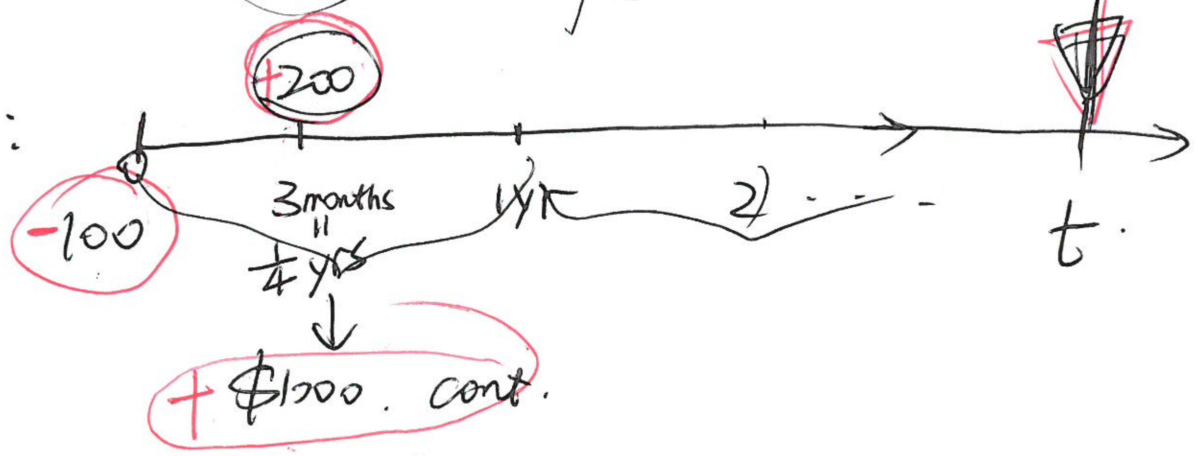
profit

Disadvatages

① different T

② $AV(\infty)$

Ex:



i p.a.

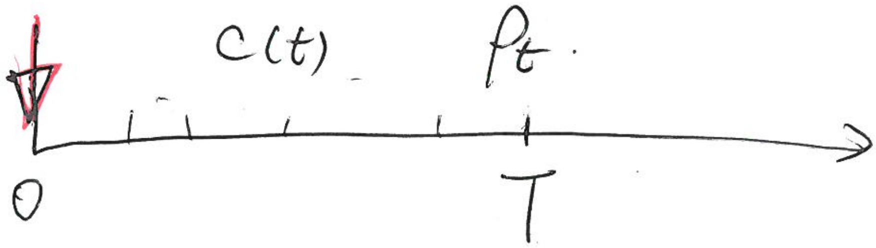
Sol: $AV(t) = (-100)(1+i)^t + 200 \cdot (1+i)^{t-\frac{1}{4}}$

$$+ \boxed{\int_0^t 1000 \cdot (1+i)^{t-s}\, ds}$$

$\bar{s}_{\overline{n}|} = \int_0^n (1+i)^{n-t}\, dt$

$=$

② NPV



$$NPV(T) = \sum_{T} C_t \cdot (1+i)^{-t} + \int_0^T \rho(t) \cdot (1+i)^{-t} dt$$

risk discount rate

$\boxed{d}$

$\boxed{v} = \frac{1}{1+i}$

Ex: $PV = -100 + 200 \cdot (1+i)^{-\frac{1}{4}} + \boxed{\int_0^\infty 1000 \cdot (1+i)^{-s} ds}$

$$= -100 + 200(1+i)^{-\frac{1}{4}} + 1000 \cdot \boxed{\bar{a}_{\overline{\infty}|}}$$

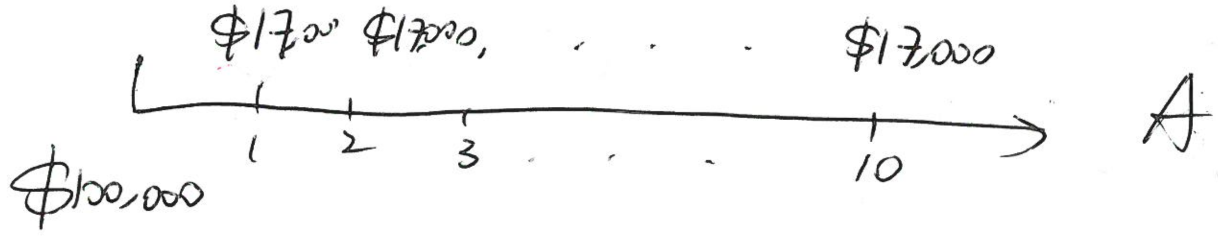
$$\boxed{\bar{a}_{\overline{n}|}} = \int_0^n v^s ds = \cdots = \boxed{\frac{1+v^n}{\ln(1+i)}} \xrightarrow{n \to \infty} \frac{1}{\ln(1+i)}$$

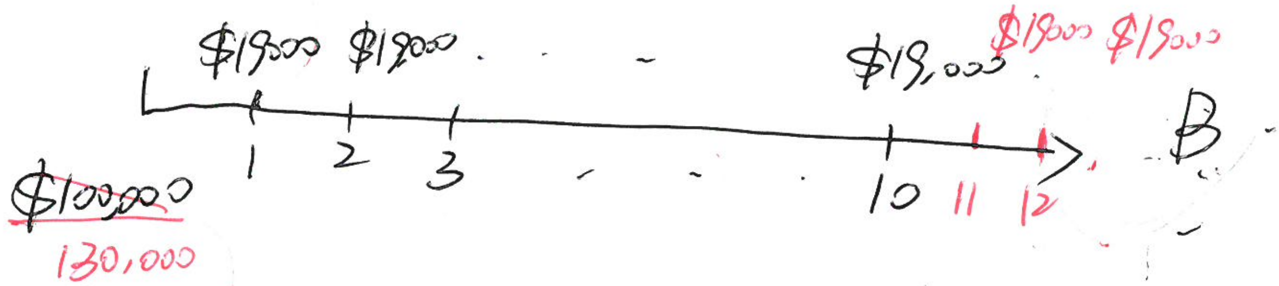
$$= -100 + 200(1+i)^{-\frac{1}{4}} + \frac{1000}{\ln(1+i)}$$

③. IRR = Internal Rate of Return = Yield Rates

NPV=0

Ex.





$11.0\% = IRR_A \Leftarrow 100{,}000 = 17{,}000 \cdot a_{\overline{10}|i}$

$13.8\% = IRR_B \Leftarrow 100{,}000 = 19{,}000 \cdot a_{\overline{10}|i}$

$11.0\% = IRR_A \Leftarrow 100{,}000 = 17{,}000\, a_{\overline{10}|i}$

$9.9\% = IRR_B \Leftarrow 130{,}000 = 19{,}000 \cdot a_{\overline{12}|i}$

$NPV = 0 \Rightarrow IRR = r_0$

* Investor lends or borrows money at r_1

① $\boxed{NPV(r_1) > 0}$.

② $NPV(r_0) = 0$. $NPV > 0$? $\Leftrightarrow$ profitable.

$\boxed{r_1 < r_0} \Rightarrow NPV > 0$.

③ $NPV_A(r_1) \;?\; NPV_B(r_1)$.

r_1: hurdle rate ; target rate.

Ex. ① 9% p.a.

$NPV_A(9\%) = \$9100$

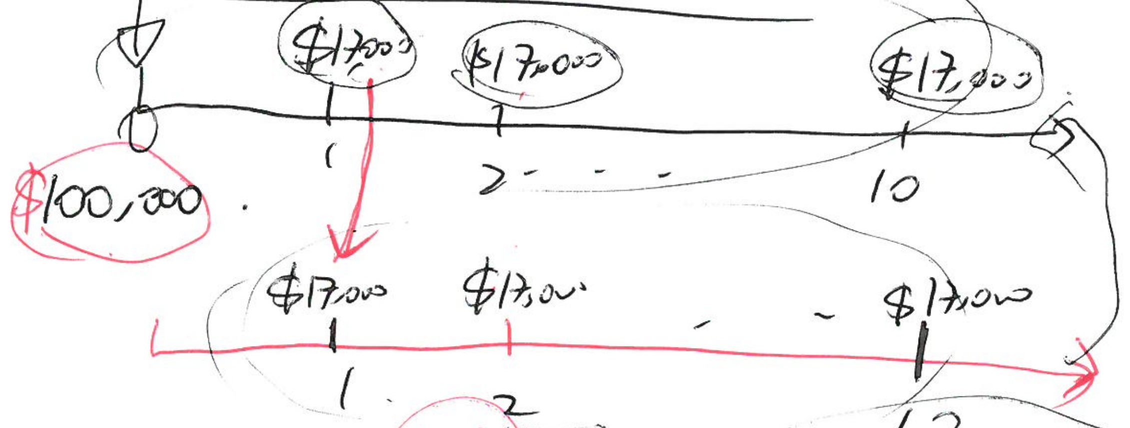
$NPV_B(9\%) = \$6054$

② 6% p.a.

$NPV_A(6\%) = \$25121$

$NPV_B(6\%) = \$29293$

Reinvestment Rate



8% p.a. reinvestment rate.

11%

$\Rightarrow$ IRR :

$NPV(i_0) = 0 \Rightarrow i_0$

$$NPV = -100{,}000 + 17{,}000 \cdot s_{\overline{10}|0.08} \times v_{i_0}^{10} = 0$$

$$\Rightarrow i_0 \cong 9.43\%$$

$$NPV = \boxed{-100{,}000 + 17{,}000 \cdot a_{\overline{10}|i_0} = 0 \Rightarrow i_0 = 11\%}$$

$$NPV = -100{,}000 + 17{,}000 \cdot s_{\overline{10}|11\%} \cdot v_{i_0}^{10} = 0$$

$$\Rightarrow i_0 = 11\%$$

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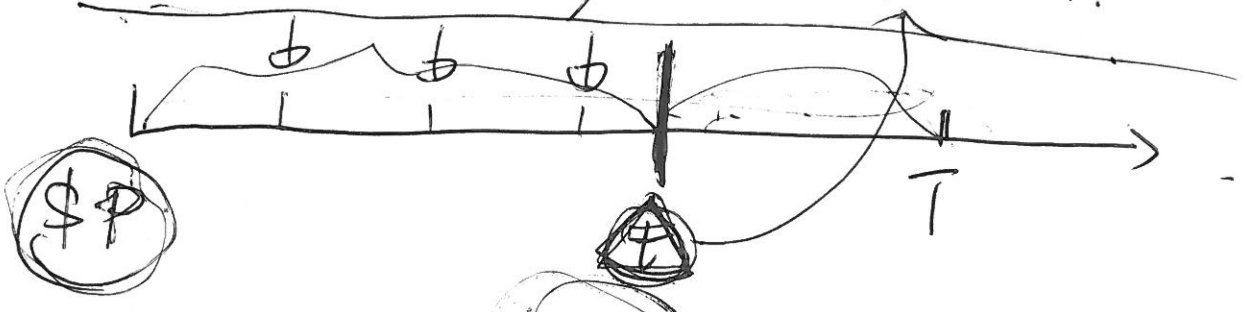
④ Discounted Payback Period.



$AV(t) = 0$

$$AV(t) = \sum_{h \le t} C_h \cdot (1+i)^{t-h} + \int_0^t \rho(s)(1+i)^{t-s}\,ds$$

$$\downarrow$$

$$NCF = (I_h - O_h)$$

$$\geq 0$$

Smallest t, $AV(t) \geq 0$

$$\downarrow$$

D.P.B.