April 5th

Last time

$p(x) = a_3x^3 + a_2x^2 + a_1x + a_0$

$a_i \in \mathbb{Q}$

then $p(x)=0$ has a constructible root $\Leftrightarrow$ it has a rational root

$\Rightarrow \sqrt[3]{2}$ is not constructible root $x^3-2=0$

Sps $\sqrt[3]{2}$ is constructible $\Rightarrow x^3-2=0$ has a rational root $\frac{p}{q}$, $(p,q)=1$

$\Rightarrow p \mid 2, q \mid 1$. $p=\pm1$, $p=\pm2$, $q=\pm1$, $p/q = \pm1, \pm2 \rightarrow$ none of these are roots of $x^3-2=0$

$\sqrt[3]{5}, \sqrt[3]{7}$ are not constructible

$\sqrt[6]{2}$ is also not constructible

if $\sqrt[6]{2}$ is constructible $\Rightarrow \sqrt[6]{2}\cdot\sqrt[6]{2} = \sqrt[3]{2}$ is also constructible and it's not.

$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$, angle $\theta = 20^\circ \Rightarrow 3\theta = 60^\circ$ is constructible but $\cos 60^\circ = \frac{1}{2}$

if $\theta = 20^\circ$ is a constructible angle $\Rightarrow x = \cos 20^\circ$ is a constructible number

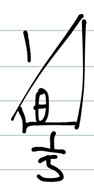
$4\cos^3 20^\circ - 3\cos 20^\circ = \cos 60^\circ = \frac{1}{2}$

$4x^3 - 3x = \frac{1}{2} \Rightarrow 8x^3 - 6x - 1 = 0$ this has no rational solutions using rational root theorem

$\Rightarrow 20^\circ$ is not constructible angle.

~~$\sqrt[5]{2}$ is this constructible?~~

let θ be an angle with $\cos\theta = \frac{1}{5}$, can it be trisected?



if we can $\theta = 3\alpha$, $\alpha = \frac{\theta}{3}$

$\frac{1}{5} = 4\cos^3\alpha - 3\cos\alpha$

$\frac{1}{5} = 4x^3 - 3x$ if α is constructible angle, $\Rightarrow \cos\alpha$ is a constructible number

$\Rightarrow 20x^3 - 15x - 1 = 0$ — has a constructible root $\Rightarrow$ has a rational root.

$x = p/q$, $(p,q)=1$

$\Rightarrow p \mid -1$, $q \mid 20$, $p = \pm1$, $q = \pm1, \pm2, \pm4, \pm5, \pm10, \pm20$

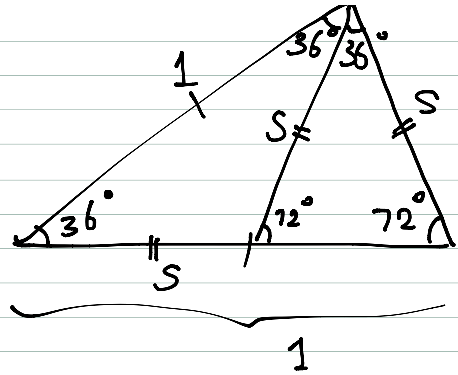
$p/q = \pm1, \pm\frac{1}{2}, \pm\frac{1}{4}, \pm\frac{1}{5}, \pm\frac{1}{10}, \pm\frac{1}{20}$

Check that none of these solve $20x^3 - 15x - 1 = 0$

20° is not constructible $\Rightarrow 1^\circ, 2^\circ$ angles are not constructible

3°-angle? constructible!

We'll construct 36° angle



Similar triangles

$$\frac{1}{S} = \frac{S}{1-S}$$

$$S^2 = 1 - S$$

$$S^2 + S - 1 = 0$$

$$S = \frac{\sqrt{5}-1}{2}$$

constructible

So $36°$ is constructible

$30°$ is constructible

$36° - 30° = 6°$ is constructible

bisect it we have $3°$ constructible.

$4°$ not constructible
$5°$ not constructible
$7°$ not constructible if it is $\Rightarrow 7° - 3° \times 2 = 1°$ would be also constructible.
$\Rightarrow 7° - 6° = 1°$ (or, we can say this)

$n°$ angle
n - natural number

if $3 \mid n \longrightarrow$ it's constructible
if $3 \nmid n$, $n \equiv 1 \bmod 3$
$n \equiv 2 \bmod 3$
$n = 3k+1$ or $3k+2$ these are not constructible angles.
if we could construct an angle of $(3k+1)°$ $\Rightarrow$ then we could also construct $(3k+1) - 3k = 1°$ angle
— not constructible $\Rightarrow$ $n°$ angle is constructible $\Longleftrightarrow 3 \mid n$

angle of $4.5°$? Yes, constructible

$\cos\theta = \frac{\pi}{6}$ is this angle constructible?
No.
b/c $\frac{\pi}{6}$ is not a constructible number
b/c π is transcendental
b/c constructible numbers are algebraic

Final on April 22nd. Monday