

Worth: 3%**Due:** By 12 noon on Tuesday 28 February.

Remember to write the the *full name* and *student number* of each member of your group prominently on your submission. Your submission must be a PDF file named **e3.pdf** and it must be handed-in using the MarkUs system. You may create the PDF file using a typesetting system (export to PDF) or by scanning in handwritten work to create a PDF file.

Each exercise may be completed in groups of 1 – 2 students who are in the **same** tutorial section.

Please read and understand the policy on Collaboration given on the Course Information Sheet. Then, to protect yourself, list on the front of your submission **every** source of information you used to complete this homework (other than your own lecture and tutorial notes, and materials available directly on the course webpage). For example, indicate clearly the **name** of every student with whom you had discussions, the **title** of every additional textbook you consulted, the **source** of every additional web document you used, etc.

For each question, please write up detailed answers carefully. Make sure that you use notation and terminology correctly, and that you explain and justify what you are doing. Marks **will** be deducted for incorrect or ambiguous use of notation and terminology, and for making incorrect, unjustified, ambiguous, or vague claims in your solutions.

1. (a) Write a *detailed structured proof* of the following statement:

If m and n are integers, with $m^2 - n^2$ odd, then $(m + n)^2$ is odd.

- (b) Write a *detailed structured proof* of the following statement:

If m and n are integers, with $(m + n)^2$ odd, then $m^2 - n^2$ is odd.

- (c) What conclusion follows from the conjunction of the statements from parts (a) and (b).

2. Without using derivatives, or formulas that you might know about the roots of quadratic, cubic or quartic equations, write a *detailed structured proof* to prove or disprove the following statement:

If x is a real number such that $x^4 + 2x^2 - 2x < 0$, then $0 < x < 1$.

3. Consider the following two functions that are defined for any $x \in \mathbb{R}$:

$\lfloor x \rfloor$ = the largest integer less than or equal to x ,

$\lceil x \rceil$ = the smallest integer greater than or equal to x .

- (a) Use a *detailed structured proof* to prove or disprove the statement $\forall x \in \mathbb{R}, \lceil -x \rceil = -\lfloor x \rfloor$.

- (b) Use a *detailed structured proof* to prove or disprove the statement $\forall x \in \mathbb{R}, \forall n \in \mathbb{N}, \lfloor n \cdot x \rfloor = n \cdot \lfloor x \rfloor$.