$X \sim \text{Gamma}(\alpha, \lambda)$

$$f_X(x) = \frac{\lambda^\alpha x^{\alpha-1} e^{-\lambda x}}{\Gamma(\alpha)}, \quad x \geq 0$$

$$E(X) = \frac{\alpha}{\lambda}$$

$$E(X^2) = \int_0^\infty x^2 \frac{\lambda^\alpha x^{\alpha-1} e^{-\lambda x}}{\Gamma(\alpha)} dx$$

$$= \frac{\lambda^\alpha}{\Gamma(\alpha)} \int_0^\infty \frac{\lambda^{\alpha+2} x^{\alpha+1} e^{-\lambda x}}{\Gamma(\alpha+2)} dx \cdot \frac{\Gamma(\alpha+2)}{\lambda^{\alpha+2}}$$

$$= 1$$

$$= \frac{1}{\lambda^2} \frac{(\alpha+1)\alpha\Gamma(\alpha)}{\Gamma(\alpha)}$$

$$= \frac{\alpha^2}{\lambda^2} + \frac{\alpha}{\lambda^2}$$

$$\text{Var}(X) = \frac{\alpha^2}{\lambda^2} + \frac{\alpha}{\lambda^2} - \frac{\alpha^2}{\lambda^2} = \frac{\alpha}{\lambda^2}$$