## A Few Tips for the Final

Hi everyone,

Over the term weve noticed a few recurring issues that pop up in the homework. These issues are important to keep in mind on the final, as they can often make a short answer fatally flawed. To rectify them, here are a few tips for the final:

- **Understand how to use the scale factor**. We posted a Youtube video on this - feel free to e-mail us any questions you have about the content.
- **Remember the relationships between scale factor, temperature and redshift**. These are:

$$1 + z(t) \propto 1/a(t) \tag{1}$$

and

$$T(t) \propto 1/a(t) \tag{2}$$

which of course implies that $1 + z \propto T$.

- **Remember that mass is conserved in any comoving sphere[1] in the universe**. In other words:

$$\rho(t) \propto 1/a(t)^3 \tag{3}$$

- **Do not add 0 to the end of cosmological parameters unless you're referring to today**. You also can't simply remove 0s from cosmological parameters except in specific circumstances. $a_0/r_0 = a/r$ from Assignment 6 is one such exception.
- **Do not fail to differentiate/integrate when differentiation/integration is required**. Given a velocity $v$ and period of time $t$, we can calculate the distance travelled $d$. We might write $d = vt$, but if $v$ is a function of time, this doesn't pan out, and instead we **must integrate**:

$$d = \int_0^t v(t')dt' \tag{4}$$

[1] A comoving sphere is a sphere that expands with the universe, like the imaginary sphere with radius $R(t)$ used in the textbook, and in one of the Youtube videos.

Note that we added apostrophes after $t$ in the integrand. This is because it's formally illegal to say $d = \int_0^t v(t)dt$, since $t$ is being varied over the integral. $t'$ is called a "dummy variable" since we use it to indicate a dependency and that we're integrating over it, but it doesn't represent a single value.

- **Integration and differentiation must take into account dependencies**. We've often seen examples like the following

$$
\begin{aligned}
H &= \frac{1}{a}\frac{da}{dt} \\
\int H dt &= \int da/a \\
Ht &= ln(a) + C
\end{aligned}
$$

or (Assignment 9, question 2)

$$
\begin{aligned}
\frac{dF}{dN} &= \frac{L}{4\pi r^2} \\
\int dF &= \int dN \frac{L}{4\pi r^2} \\
F &= \frac{NL}{4\pi r^2}
\end{aligned}
$$

In the first case, the math is wrong because $H$ is a function of $t$, and **not** a constant. To solve for $H(t)$ we have to know more about the universe we're calculating for, and solve Friedmann's equation. The second case may be a bit less clear, but, from Assignment 9, we have $dN/dr = n4\pi r^2$, meaning there is a relationship between $dN$ and $dr$, and therefore $N$ and $r$. In the integral above, therefore, $L/4\pi r^2$ is **not** a constant.

- **An indefinite integral will result in a constant** $C$. An indefinite integral is one without integration limits. When an indefinite integral is performed, the resulting will contain a constant of integration $C$. This constant is usually needed to allow the expression to statisfy some initial condition we require. For example, let's say the velocity of an object $v(t) = t$ and that the object starts out at $x = 1$. If we want to solve for the position of the object as a function of time, $x(t)$, we integrate:

$$
\begin{aligned}
x(t) &= \int v(t)dt \\
&= \frac{1}{2}t^2 + C
\end{aligned}
$$

Since $x(t = 0) = 1$, $C = 1$, and we get $x(t) = \frac{1}{2}t^2 + 1$.

- **Check your answer!** Make sure that your answer has the correct units, and appears physically reasonable. Writing that the age of the universe is $10^{-20}$ s, or that the Earth is heated to a lower temperature when 200,000 Suns are shining on it instead of one, should make you do a double-take. If your answer looks insane, look to unit and unit conversion errors - they're usually the easiest to mess up.

Best wishes for the final exam,

Charles and Sergei