Exercise

(a) $f(t) = F'(t) = +0.02e^{-0.02t}$

(b) $u(t) = \dfrac{f(t)}{S(t)} = \dfrac{0.02e^{-0.02t}}{e^{-0.02t}} = 0.02$

(c) $P(T>10) = S(10) = e^{-0.02 \times 10}$

$= e^{-0.2}$

(d) $P(10<T<25 / T>10)$ $\neq$

Note:

$10/15\ 8\ x$

$= P((10<T<25)$

$= \dfrac{P((10<T<25)}{P(T>10)}$

$= \dfrac{S(10) - S(25)}{S(10)} = \dfrac{e^{-0.2} - e^{-0.5}}{e^{-0.2}} = 0.2592$

(e) $E(T) = \int_0^{\infty} t \cdot f(t)\,dt = \int_0^{\infty} 0.02te^{-0.02t}\,dt$

$= \left[-t \cdot e^{-0.02t}\right]_0^{\infty} + \int_0^{\infty} e^{-0.02t}\,dt$

integral by parts

$\int u\,dv = uv - \int v\,du$

$u = t$

$dv = e^{-0.02t} \cdot 0.02\, dt \quad \Rightarrow v = -e^{-0.02t}$

$uv = -te^{-0.02t}$ $\quad \int_0^\infty v\,du = -\int_0^\infty e^{-0.02t}\,dt$

continue

$= \left[-\frac{1}{0.02} e^{-0.02t} \right]_0^\infty$ $\quad$ (Note: $\lim_{t\to\infty} \frac{t}{e^{0.02t}} = 0$)

$= 0 - (-50) = 50$

(f) $\quad p(T < t) = 0.1$ $\quad$ the number of years lived by this person

e.g. $\quad 1, 2, 3 \cdots \cdots 100$

$\cdots\cdots 10 < 11$

$1 - e^{-0.02t} = 0.1$

$\Rightarrow t = -50 \ln 0.9 = 5.27$