CSC343 Fall 2015 (L5101/L2501) – Week 10
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In-class Exercises: Functional Dependencies

Suppose we have a relation R with attributes $ABCD$

1. **What an FD means.** Suppose the functional dependency $BC \to D$ holds in R. Create an instance of R that violates this FD.

   Solution:

   In order to violate this FD, we need two tuples with the same value for B and the same value for C (both!), yet different values for D.

A	B	C	D
1	3	6	4
2	3	6	5

2. **Equivalent sets of FDs.**

   (a) Are the sets $A \to BC$ and $A \to B, A \to C$ equivalent? If yes, explain why. If your answer is no, construct an instance of R that satisfies one set of FDs but not the other.

   Solution:

   These are equivalent — there is no instance of the relation that satisfies one but not the other. This can be proven, as follows:

   - Assume that $A \to BC$.
     - Under this assumption, $A^+ = ABC$.
     - Therefore $A \to B$, and $A \to C$.
   - Assume that $A \to B$, and $A \to C$.
     - Under this assumption, $A^+ = ABC$.
     - Therefore $A \to BC$.
   - Therefore each set of FDs follows from the other. They are equivalent.

   (b) Are the sets $PQ \to R$ and $P \to Q, P \to R$ equivalent? If yes, explain why. If no, construct an instance of R that satisfies one set of FDs but not the other.

   Solution:

   These are *not* equivalent, as demonstrated by this instance that satisfies $PQ \to R$ but not $P \to Q, P \to R$:

P	Q	R
1	2	4
3	2	5

   In fact we can always "split the RHS" of an FD.

3. **Does an FD follow from a set of FDs?** Suppose we have a relation on attributes $ABCDEF$ with these FDs:

   $$AC \to F, \quad CEF \to B, \quad C \to D, \quad DC \to A$$

   (a) Does it follow that $C \to F$?

   (b) Does it follow that $ACD \to B$?

   Solution:

   We use the closure test to check whether an FD follow from a set of FDs.
   $C^+ = CDAF$. Therefore, $C \to F$ does follow.
   $ACD^+ = ACDF$. Therefore, $ACD \to B$ does not follow.

4. **Projecting a set of FDs onto a subset of the attributes.** Suppose we have a relation on attributes $ABCDE$ with these FDs:

$$A \to C, \quad C \to E, \quad E \to BD$$

Project the FDs onto attributes ABC:

Solution:

To project onto a set of attributes, we systematically consider every possible LHS of an FD that might hold on those attributes.

- $A^+ = ACEBD$, therefore $A \to BC$. (It also functionally determines DE, but these are not in our set of attributes. And it functionally determines itself, but we don't need to write down dependencies that are tautologies.)
- $B^+ = B$. This yields no FDs for our set of attributes.
- $C^+ = CEBD$, therfore $C \to B$.
- We don't need to consider any supersets of A. A already determines all of our attributes ABC, so supersets of A will be only yield FDs that already follow from $A \to BC$.
- The only remaining subset of the attributes ABC to consider is BC. $BC^+ = BCED$. This yields no FDs for our set of attributes.
- So the projection of the FDs onto ABC is: $\{A \to BC, \quad C \to B\}$.

5. **Minimal Basis of FDs.** Suppose we have a relation on attributes $ABCDEG$ with these FDs:

$$A \to B, \quad ABCD \to E, \quad G \to A, \quad G \to B$$

Find the minimal basis (aka minimal cover) for these FDs:

Solution:

(Solution guide available in Week 10 slides for Section L5101.)