

ABOUT OUR FINAL

To prepare for the final exam solve again the problems from the exercises you should have solved before the quizzes and the midterm. Solve the problems that appeared on past midterms and exams.

Repeat the following material from our syllabus (revisited before the exam):

Affine geometry

Theorems of Ceva and Menelaus. Three heights, three medians and three bisectors in a triangle are concurrent.

Center of mass and its properties.

Affine transformations, quantities invariant under affine transformations.

Convex geometry

Convex hulls.

Simple polyhedra and their h-vectors. Dehn-Sommerville duality. Euler's formula for 3-dimensional convex polyhedra.

Helly's theorem.

Extreme problems in geometry

Use of reflections for minimizing lengths of broken lines. Triangle of minimal perimeter inscribed in a given triangle. A point which minimizes the sum of the distances from three given points.

Isoperimetric problem.

Optical properties of conic sections. Billiard trajectories in an elliptic billiard.

Inversions and Mobius transformations

Properties of inversion: angle preservation, invariance of circles and lines as well as spheres and planes.

Existence of inversions mapping a pair of non-intersecting circles into a pair of concentric circles.

Stereographic projection of a sphere onto a plane and its properties (as an application of inversion).

Mobius transformation is either fractional linear or composition of conjugation and fractional-linear transformation.

Compositions of inversions are Mobius transformations. Mobius transformation mapping three distinct points to three distinct points.

Mobius transformations preserving a circle.

Ruler and Compass constructions

What is possible to construct using ruler and compass.

Impossibility of constructing roots of irreducible cubic equations over \mathbb{Q} .

Projective geometry

Projections. Desargues's theorem. Cross ratio of four collinear points and of four concurrent lines on a plane.

Projective transformation of a line are fractional-linear.

Coordinate formulas of projective transformations on a line and on a plane.

Projective transformations between two lines in a plane.

Cross ratio of four points on a conic section. Its direct and dual descriptions.

Theorems of Pascal and Brianchon, including degenerate cases (Pappus theorem and its dual statement).

General duality principle. Homogeneous coordinates.

Spherical geometry

Law of sines for spherical triangle.

Cheva's theorem for spherical triangle

Law of cosines and triangular inequality for spherical triangle.

Areas of spherical triangle.