Design Theory for Relational Databases

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Originally based on slides by Jeff Ullman

Introduction

- There are always many different schemas for a given set of data.
- E.g., you could combine or divide tables.
- How do you pick a schema? Which is better? What does "better" mean?
- Fortunately, there are some principles to guide us.

Database Design Theory

- It allows us to improve a schema systematically.
- General idea:
  - Express constraints on the relationships between attributes
  - Use these to decompose the relations
- Ultimately, get a schema that is in a "normal form" that guarantees good properties, such as no *anomalies*.
- "Normal" in the sense of conforming to a standard.
- The process of converting a schema to a normal form is called normalization.

Agenda

- Functional Dependencies (FD)
- Closure
- FD Projection
- Minimal Cover
- Normalization: BCNF, 3NF

Part I: Functional Dependency Theory

A poorly designed table

part	manufacturer	manAddress	seller	sellerAddress	price
1983	Hammers 'R Us	99 Pinecrest	ABC	1229 Bloor W	5.59
8624	Lee Valley	102 Vaughn	ABC	1229 Bloor W	23.99
9141	Hammers 'R Us	99 Pinecrest	ABC	1229 Bloor W	12.50
1983	Hammers 'R Us	99 Pinecrest	Walmart	5289 St Clair W	4.99

- In any domain, there are relationships between attribute values.
- Perhaps:
  - Every part has 1 manufacturer
  - Every manufacture has 1 address
  - Every seller has 1 address
- If so, this table will have redundant data.

Principle: Avoid redundancy

Redundant data can lead to anomalies.

part	manufacturer	manAddress	seller	sellerAddress	price
1983	Hammers ‘R Us	99 Pinecrest	ABC	1229 Bloor W	5.59
8624	Lee Valley	102 Vaughn	ABC	1229 Bloor W	23.99
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1983	Hammers ‘R Us	99 Pinecrest	Walmart	5289 St Clair W	4.99

- Update anomaly: if Hammers ‘R Us moves and we update only one tuple, the data is inconsistent.
- Deletion anomaly: If ABC stops selling part 8624 and Lee Valley makes only that one part, we lose track of its address.

Definition of FD

- Suppose R is a relation, and X and Y are subsets of the attributes of R.
- *X* -> *Y* asserts that:
  - If two tuples agree on all the attributes in set *X*, they must also agree on all the attributes in set *Y*.
- We say that "*X* -> *Y* holds in R", or "*X* functionally determines *Y*."
- An FD constrains what can go in a relation.

More formally…

$A \rightarrow B$ means:

$\forall$ tuples t_1, t_2,

$(t_1[A] = t_2[A]) \Rightarrow (t_1[B] = t_2[B])$

Or equivalently:

$\neg \exists$ tuples t_1, t_2 such that

$(t_1[A] = t_2[A]) \wedge (t_1[B] \neq t_2[B])$

Generalization to multiple attributes

$A_1 A_2 \dots A_m \rightarrow B_1 B_2 \dots B_n$ means:

$\forall$ tuples t_1, t_2,

$$(t_1[A_1] = t_2[A_1] \wedge \dots \wedge t_1[A_m] = t_2[A_m]) \Rightarrow$$
$$(t_1[B_1] = t_2[B_1] \wedge \dots \wedge t_1[B_n] = t_2[B_n])$$

Or equivalently:

$\neg \exists$ tuples t_1, t_2 such that

$$(t_1[A_1] = t_2[A_1] \wedge \dots \wedge t_1[A_m] = t_2[A_m]) \wedge$$
$$\neg (t_1[B_1] = t_2[B_1] \wedge \dots \wedge t_1[B_n] = t_2[B_n])$$

Why “functional dependency”?

- “dependency” because the value of *Y* depends on the value of *X*.
- “functional” because there is a function that takes a value for X and gives a *unique* value for Y.
- (It’s not a typical function; just a lookup.)

Equivalent sets of FDs

- When we write a set of FDs, we mean that all of them hold.
- We can very often rewrite sets of FDs in equivalent ways.
- When we say S_1 is equivalent to S_2 we mean that:
  - S_1 holds in a relation iff S_2 does.

FD - Exercises

- 1. Create an instance of R that violates $BC \rightarrow D$
  - Any tuple with equal values of B,C but unequal D values!
- 2.a) Is the FD $A \rightarrow BC$ equivalent to the two FDs $A \rightarrow B$, $A \rightarrow C$?
  - Yes!
- 2.b) Is the FD $PQ \rightarrow R$ equivalent to the two FDs $P \rightarrow Q$, $P \rightarrow R$?
  - No.
  - Example..

```
 P  |  Q  |  R
---------------
 2  |  1  |  4
 2  |  3  |  5
```

Splitting rules for FDs

- Can we split the RHS of an FD and get multiple, equivalent FDs?

- Can we split the LHS of an FD and get multiple, equivalent FDs?

Coincidence or FD?

- An FD is an assertion about *every* instance of the relation.
- You can't know it holds just by looking at one instance.
- You must use knowledge of the domain to determine whether an FD holds.

FDs are closely related to keys

- Suppose K is a set of attributes for relation R.
- Our old definition of superkey:

  a set of attributes for which no two rows can have the same values.
- A claim about FDs:

  K is a superkey for *R* iff
  K functionally determines all of *R*.

FDs are a *generalization* of keys

- Superkey:
  X -> $\boxed{R}$ All attributes
- Functional dependency:
  X -> Y
- A superkey must include *all* the attributes of the relation on the RHS.
- An FD can have just a subset of them.

Inferring FDs

- Given a set of FDs, we can often infer further FDs.
- This will come in handy when we apply FDs to the problem of database design.
- Big task: given a set of FDs, infer *every* other FD that must also hold.
- Simpler task: given a set of FDs, infer whether *a given* FD must also hold.

Examples

- If *A -> B* and *B -> C* hold, must *A -> C* hold?

Another way of asking: Does the FD A->C **follow from** these FDs?

- If *A -> H, C -> F,* and *F G -> A D* hold, must *F A -> D* hold? must *C G -> F H* hold?

- Aside: we are not generating new FDs, but testing specific possible FD(s).

Prove an FD (LHS->RHS) **follows** Method 1: using first principles

- You can prove it by referring back to
  - The FDs that you know hold, and
  - Apply FD inference rules (axioms)

- But the Closure Test is easier!

Prove an FD (LHS->RHS) follows
Method 2: using the Closure Test

- Assume you know the values of the LHS attributes, and figure out:
  everything else that is determined.

<e.g. restaurant name>

➔ *everything I can learn about it…?*

- If the result includes the RHS attributes, then you know that LHS -> RHS holds

- This is called the closure test.

Y *is a set of attributes,* ***S*** *is a set of* ***FDs****.*
Return the closure of ***Y*** *under* ***S****.*

Attribute_closure(Y, S):

Initialize Y^+ to Y

Repeat until no more changes occur:

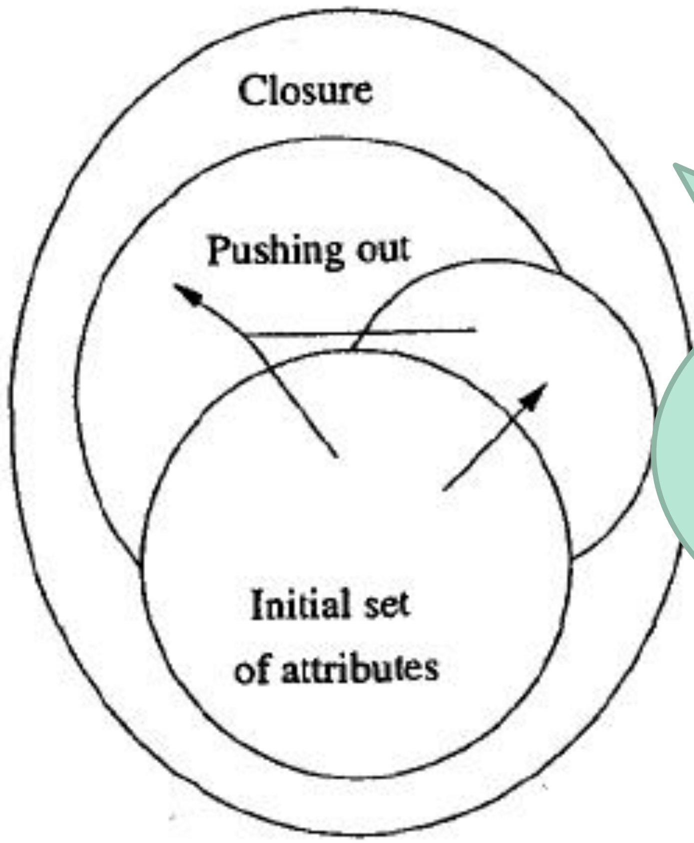
If there is an FD LHS ->RHS in S such that LHS is in Y^+:

Add RHS to Y^+

Return Y^+

If LHS is in Y^+ and LHS -> RHS holds, we can add RHS to Y^+

Visualizing attribute closure



We can use closures to answer the **follows from** question!

If LHS is in Y^+ and LHS -> RHS holds, we can add RHS to Y^+

S is a set of FDs; LHS ->RHS is a single FD.
Return true iff LHS ->RHS follows from S.

FD_follows(S, LHS ->RHS):

Y+ = Attribute_closure(LHS, S)
return (RHS is in Y^+)

Exercise

- Suppose we have a relation on attributes ABCDEF, with FDs:

  $AC \rightarrow F$, $CEF \rightarrow B$, $C \rightarrow D$, $DC \rightarrow A$

a) Does it follow that $C \rightarrow F$?

Ans: $C^+ = ..?$

$C^+ = CDAF$

Then, $C \rightarrow F$ follows

C^+

C	D	A	F

b) Does it follow that $ACD \rightarrow B$?

Ans: $ACD^+ = ..?$

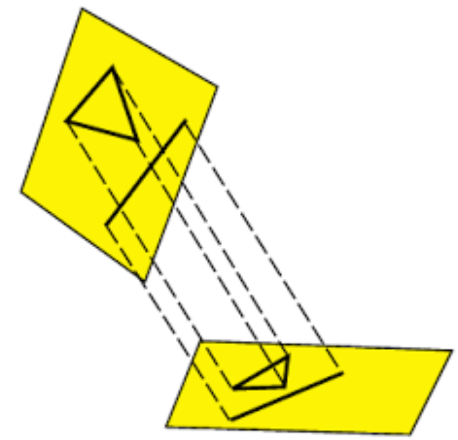
$ACD^+ = ACDF$

Then, $ACD \rightarrow B$ does not follow

ACD^+

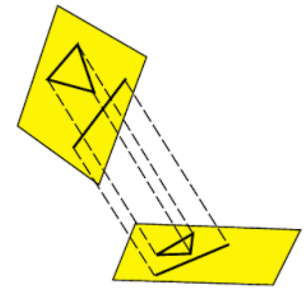
A	C	D	F

Projecting FDs



- Later, we will learn how to normalize a schema by decomposing relations. (This is the whole point of this theory.)
- We will need to be aware of what FDs hold in the new, smaller, relations.
- In other words, we must project our FDs onto the attributes of our new relations.

Exercise - Projecting FDs



- Suppose we have a relation on attributes ABCDE with FDs: A → C, C → E, E → BD

Project the FDs onto attributes ABC:

- ✓ To project onto a set of attributes, we systematically consider every possible LHS of an FD that might hold on those attributes.

A^+ =

B^+ =

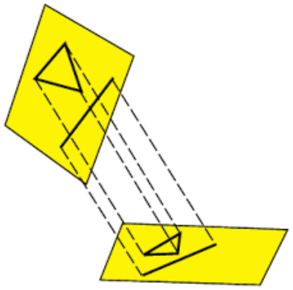
C^+ =

AB^+ =

AC^+ =

BC^+ =

Exercise - Projecting FDs



- Suppose we have a relation on attributes ABCDE with FDs: $A \rightarrow C$, $C \rightarrow E$, $E \rightarrow BD$

Project the FDs onto attributes ABC:

✓ To project onto a set of attributes, we systematically consider every possible LHS of an FD that might hold on those attributes.

$A^+ =$ ACEBD

therefore $A \rightarrow BC$.

$B^+ =$ B. Yields no FDs for our set of attributes.

$C^+ =$ CEBD, therefore $C \rightarrow B$.

✓ We don't need to consider any supersets of **A**. A already determines all of our attributes ABC, so supersets of A will be only yield FDs that already *follow* from $A \rightarrow BC$.

✓ The only superset left is **BC**:

$BC^+ =$ BCED. This yields no FDs for our set of attributes.

- So the projection of the FDs onto ABC is: $\{A \rightarrow BC, C \rightarrow B\}$

S is a set of FDs; L is a set of attributes.

Return the projection of S onto L:

all FDs that follow from S and involve only attributes from L.

Project(S, L):

Initialize T to {}.

For each subset X of L:

Compute X^+ *Close X and see what we get.*

For every attribute A in X^+:.

If A is in L: *X -> A is only relevant if A is in L (we know X is).*

add X -> A to T.

Return T.

A few speed-ups

- No need to add X -> A if A is in X itself. It's a trivial FD.
- These subsets of X won't yield anything, so no need to compute their closures:
  - the empty set
  - the set of all attributes
- Neither are big savings, but …

A big speed-up

- If we find X^+ = all attributes, we can ignore any *superset* of X.
  - It can only give use "weaker" FDs (with more on the LHS).
- This is a big time saver!

Projection is expensive

- Even with these speed-ups, projection is still expensive.

- Suppose R_1 has n attributes.
  How many subsets of R_1 are there?
  (A set of n elements has $\mathbf{2^n}$ *subsets)*

Minimal Basis (aka Minimal Cover)

- We saw earlier that we can very often rewrite sets of FDs in equivalent ways.
- Example: $S_1 = \{A\text{->}BC\}$ is equivalent to $S_2 = \{A\text{->}B, A\text{->}C\}$.
- Given a set of FDs S, we may want to find a minimal basis: A set of FDs that is equivalent, but has
  1. No redundant FDs
  2. No unnecessary attributes on the LHS

S is a set of FDs. Return a minimal basis for S.

Minimal_basis(S):

Repeat until no more changes result:

1. Remove FDs that are implied by the rest.
2. For each FD with 2$^+$ attributes on the left:

If you can remove one attribute from the LHS and get an FD that follows from the rest:

Do so! (It's a stronger FD.)

Example – Minimal Basis

- What is the minimal cover of these FDs in ABCDEG:

$$A \to B,\ ABCD \to E,\ G \to A,\ G \to B$$

Answer:

(1) Can any of the FDs be implied from another FD? (if so, drop)

***Systematic approach:**

- Calculate the closure of the LHS in each FD, using the rest of FDs; Can we reach the RHS using the other FDs?

  1. $A \to B$

     A^+ under {2,3,4} ? = …

  2. $ABCD \to E$

     $ABCD^+$ under {1,3,4} ? = …

  3. $G \to A$

     G^+ under {1,2,4} ? = …

  4. $G \to B$

     G^+ under {1,2,3} ? = GAB

- Drop FD#4.

Example – Minimal Basis

- What is the minimal cover of these FDs in ABCDEG:

$A \rightarrow B$, $ABCD \rightarrow E$, $G \rightarrow A$, $\cancel{G \rightarrow B}$

Answer:

(2) Check if any LHS can be reduced (any attributes can be removed?). If so, drop the extra attributes.

1. $A \rightarrow B$
2. **$ABCD \rightarrow E$**

   Start with removing 1 attribute.. (ACD+, ABC+, BCD+, ..and so on)

   ACD+ = ABCD

3. $G \rightarrow A$

Example – Minimal Basis

- What is the minimal cover of these FDs in ABCDEG:

$A \rightarrow B$, $ABCD \rightarrow E$, $G \rightarrow A$, $\cancel{G \rightarrow B}$

Answer:

(2) Check if any LHS can be reduced (any attributes can be removed?). If so, drop the extra attributes.

1. $A \rightarrow B$

~~**2. ABCD → E**~~

Start with removing 1 attribute.. (ACD+, ABC+, BCD+, ..and so on)

ACD+ = ABCD

2. ACD → E

3. $G \rightarrow A$

- Result: Minimal basis is $A \rightarrow B$, $ACD \rightarrow E$, $G \rightarrow A$

Please check course website for more detailed examples

Some comments on computing a minimal basis

- Often there are multiple possible results, depending on the order in which you consider the possible simplifications.

- After you identify a **redundant** FD, you must **not** use it when computing any subsequent closures (as you consider whether other FDs are redundant).

… and some that are less intuitive

- When you are computing closures to decide whether the LHS of an FD

  $$a_1 a_2 \ldots a_m \text{->} b_1 b_2 \ldots b_n$$

  can be simplified, continue to use that FD.

- When you have tried to eliminate each FD and to reduce each LHS, you must go back and try again.