Ingredients for the strong duality theorem

1) weak duality theorem optimal criterion (Theorem 36)

(2) B<sup>-1</sup>

3 (B or Co (reconstructing the objective row of a later tablean.

Eq. (A Simplex Optimization)

Given  $Z=3\pi_1+7\Lambda_2$  (maximize) (+0 $\Lambda_3$  +0 $\Lambda_4$ +0 $\Lambda_5$ )

The constraint part of a later tableau is

Tableau O objectiveness

To apply the optimality criterion, we eliminate the objective row coefficients of the variables. as in the 2-phase method.

Replace the tableau O objective row with

(To get  $\frac{|\pi_1|}{0}$   $\frac{\pi_2}{0}$   $\frac{\pi_3}{3}$   $\frac{\pi_4}{0}$   $\frac{\pi_5|}{0}$ , an opstimal tableau (4) in "A Simplex Optimization)

Here  $C_B = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$ , the coefficients of the basic variable  $\begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$  in the original objective function.

So  $C_B^T = [3,0,7]$  and the tableau @ objective row is [-3,7,0,0,0] tableau @ objective row objective row objective row

## Strong Dudity Theorem

(Notation) Let Im denote the mxm identity matrix. If A is a matrix having m rows,  $A_1, A_2, \dots$  denote the columns of A

Consider the problem:

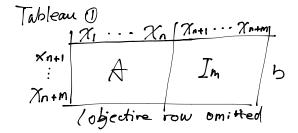
Maximize  $z = c^T x$  sit.

An  $\leq b$   $\chi > 0 \in \mathbb{R}^n$ 

where b>0 in Rm and A is mxn

If this problem has an optional solution, so does its dual. Moreover, the optimal objective values of the 2 problems are equal.

Proof of the duality theorem:



A later tableau

Table (P)

.7	$\times_1 \cdots \times_n$	No+1 ··· Xn+m	
X41 :	B-1A-	B	ВЪ
Xim			
-			

To constraint the objective row of tableau D start with the tableau 1 objective row:

$$\frac{\left|\begin{array}{c|c} x_1 & \cdots & x_n & x_{n+1} & \cdots & x_{n+m} \\ \hline -C^T & & & & \end{array}\right|}{0}$$

Then let  $CB = [CI_1 \cdots, Cim]$ and add CE[B'A|B''|B''b]to get  $|X_1 \cdots X_n|X_{n+1} \cdots X_{n+m}|$  |CEB'A - C'|CEB''Let w' = CEB''This is optimal provided CBB'A > C'' CBB'' > OER'''

That is,  $W_B^T$  is feasible for the dual problem  $A^Tw \ge C$ ,  $W_B \ge 0 \in \mathbb{R}^m$ And the objective values are equal  $C_B^T(B^Tb)$ =  $W^Tb$ =  $b^Tw$