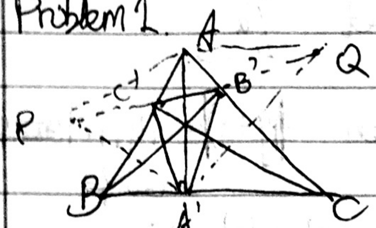
Problems of Lecture 4.

Problem 1.



$C'B' + B'A' + A'C' < 2AA'$

$< 2BB'$

$< 2CC'$

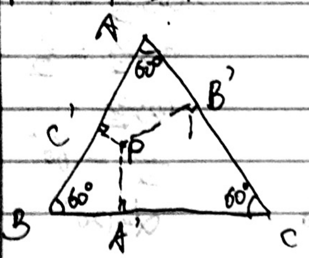
we wanna know this true.

WLOG say AA' is shortest among AA', BB' & CC'

in $\triangle PAQ$, $PA = QA = AA'$

$PA + QA = 2AA' > C'B' + B'A' + A'C' = PQ$ — Perimeter.

Problem 2.

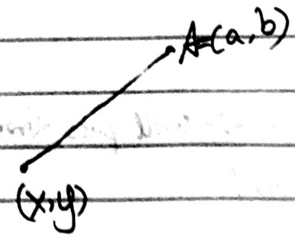


Prove: $\forall P$ in $\triangle ABC$, $PA' + PB' + PC'$ fixed.

note $\left.\begin{array}{l} \triangle APB = \frac{1}{2} AB \cdot PC' \\ \triangle BPC = \frac{1}{2} BC \cdot PA' \\ \triangle APC = \frac{1}{2} AC \cdot PB' \end{array}\right\}$ add up

$\underbrace{\triangle ABC}_{\text{fixed}} = \frac{1}{2}\, \underbrace{\text{side}}_{\text{fixed}} \times (\underbrace{PA' + PB' + PC'}_{\text{fixed}})$

Problem 3.



$\nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right)$

$= \frac{\partial}{\partial x}[(x-a)^2 + (y-b)^2]^{\frac{1}{2}}, \frac{\partial}{\partial y}[(x-a)^2 + (y-b)^2]^{\frac{1}{2}}$

$= \left[\frac{1}{2}[(x-a)^2 + (y-b)^2]^{-\frac{1}{2}} \cdot (2x-2a), \frac{1}{2}[(x-a)^2 + (y-b)^2]^{-\frac{1}{2}}(2y-2b)\right]$

$= [(x-a)^2 + (y-b)^2]^{-\frac{1}{2}} (x-a, y-b)$

↓ looking away from A

$\|\nabla f\| = \frac{1}{(x-a)^2 + (y-b)^2} \cdot [(x-a)^2 + (y-b)^2] = 1.$

↓ length 1.

Problem 4.

x, y, z on plane.

$x+y+z=0$

$\|x\|+\|y\|+\|z\|=1.$

Let $\vec{x}$ be the positive direction

$x=-(y+z)$, let 

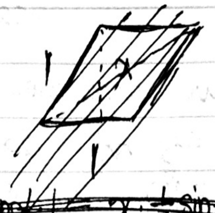
Then $|y+z| = |y||z| \cancel{\cos\alpha}$

$\cancel{= \cos\alpha}$

$\cancel{\cos\alpha = |x| - 1}$

$\cancel{\cos\alpha = 1}$

$\alpha =$



$\cancel{\sin\alpha \cdot 1 = x \cdot \frac{1}{2}\sin\frac{\alpha}{2}}$

$\cancel{2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} = x \cdot \frac{1}{2}\sin\frac{\alpha}{2}}$

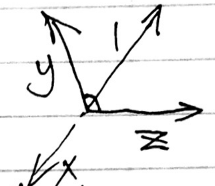
$\cancel{4\cos\frac{\alpha}{2} = x}$

$\cancel{\cos\frac{\alpha}{2} = \frac{1}{4}x}$

$\cancel{\text{and } x = 1}$

$\cancel{\cos\frac{\alpha}{2} = \frac{1}{4}}$

$\cancel{\cos\alpha}$



$\cos\frac{\alpha}{2} = \frac{1}{2}$

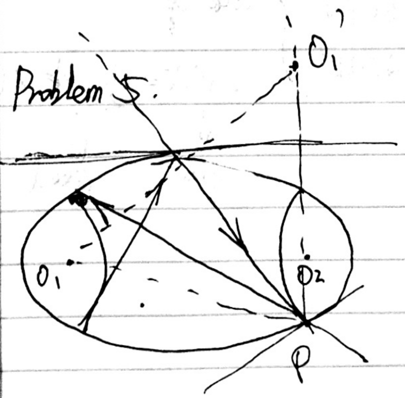
$2\cos\frac{\alpha}{2} = 1$

$\cos\frac{\alpha}{2} = \frac{1}{2}$

$\frac{\alpha}{2} = 60^\circ$

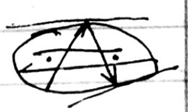
$\alpha = 120^\circ$

Problem 5.



$O_1'O_2 = O_1O_2 = PO_1 - PO_2$ fixed

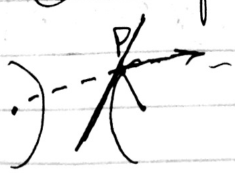
optical property: ray intersects a segment joining O_1, O_2



Know: In ellipse, if ray pass through 1 foci ⇒ will pass through one of the foci after every reflection!

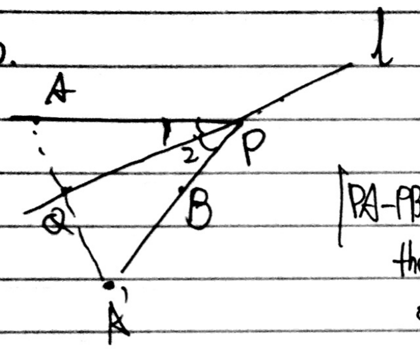
Know if initial does not pass through foci, then after infinitely many reflections, still doesn't pass $O_1 O_2$.

(small ellipse area inside) untouchable

Hyperbola  "looks like from another foci"

Problem 6.

1).



$|PA-PB|$ max

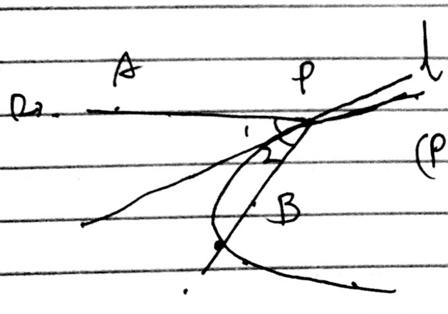
then ~~是~~ The ~~关~~ reflection point A' about l is on the line PB.

Therefore ·

$$\left.\begin{aligned} PA&=PA' \\ QA&=QA' \\ QP&=QP \end{aligned}\right\} \Rightarrow \triangle APQ \cong \triangle A'PQ$$

$\Rightarrow \angle 1 = \angle 2$.

2).



$(PA-PB)$ constant

hyperbola $\Rightarrow A, B$ ~~as~~ focus

Property. P tangent $\Rightarrow l$ bisector of $\angle APB$.