PLEASE HAND IN

University of Toronto
Department of Mathematics
FACULTY OF ARTS AND SCIENCE
MAT224H1F
Linear Algebra II

Final Examination
December 2009

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Duration: 3 hours

Last Name: ______________________

Given Name: ______________________

Student Number: ______________________

No calculators or other aids are allowed.

FOR MARKER USE ONLY	
Question	Mark
1	/10
2	/10
3	/10
4	/10
5	/10
6	/10
7	/10
TOTAL	/70

[10] **1.** Let $V = P_2(\mathbb{R})$, with the inner product

$$< p(t), q(t) >= p(-1)q(-1) + p(0)q(0) + p(1)q(1)$$

for all $p(t), q(t) \in V$. Consider the subspace $W = \{p(t) \in V \mid p(1) + p(-1) = 0\}$ of V. Find an orthogonal basis for the orthogonal complement $W^{\perp}$ of W in V.

EXTRA PAGE FOR QUESTION 1 - please do not remove.

[10] **2.** Let T be the linear operator on $\mathbb{C}^2$ defined by

$$T(z_1, z_2) = (2z_1 + iz_2, (1-i)z_1).$$

(a) Find $T^*(3-i, 1+2i)$.

(a) Determine if T is self-adjoint, normal, or neither.

[10] **3.** Let W be a subspace of a vector space V. Prove that if $V = W \oplus W^{\perp}$ and T is the orthogonal projection onto W, then $T = T^*$.

[10] 4. Let $T\colon \mathbb{R}^3 \to \mathbb{R}^3$ be the linear operator that has the matrix

$$A = \begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$$

realtive to the standard basis of $\mathbb{R}^3$. Find the spectral decomposition of T.

EXTRA PAGE FOR QUESTION 4 - please do not remove

[10] **5.** Let $T\colon V \to V$ be a linear operator satisfying $T^2 = I_V$ (Note: I_V denotes the identity operator on V). Define

$$U_1 = \{v \in V \mid T(v) = v\} \qquad \text{and} \qquad U_2 = \{v \in V \mid T(v) = -v\}$$

(a) Show that U_1 and U_2 are T-invariant..

(b) Show that $V = U_1 \oplus U_2$. **Hint**: $v + T(v) \in U_1$ and $v - T(v) \in U_2$.

EXTRA PAGE FOR QUESTION 5 - please do not remove

[10] **6.** Let $V = W_1 + W_2$, where W_i are subspaces of V for $i = 1, 2$. Prove that $V = W_1 \oplus W_2$ if and only if $w_1 + w_2 = 0$, and $w_i \in W_i$ imply that each $w_i = 0$ for $i = 1, 2$.

[10] **7.** Let $T\colon \mathbb{C}^4 \to \mathbb{C}^4$ be the linear operator that has the matrix

$$A = \begin{pmatrix} 0 & 4 & 0 & i \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

realtive to the standard basis of $\mathbb{C}^4$. Find a basis of $\mathbb{C}^4$ such that the matrix of T relative to this basis is the Jordan canonical matrix J for T, and find a matrix P such that $P^{-1}AP = J$.

EXTRA PAGE FOR QUESTION 7 - please do not remove.