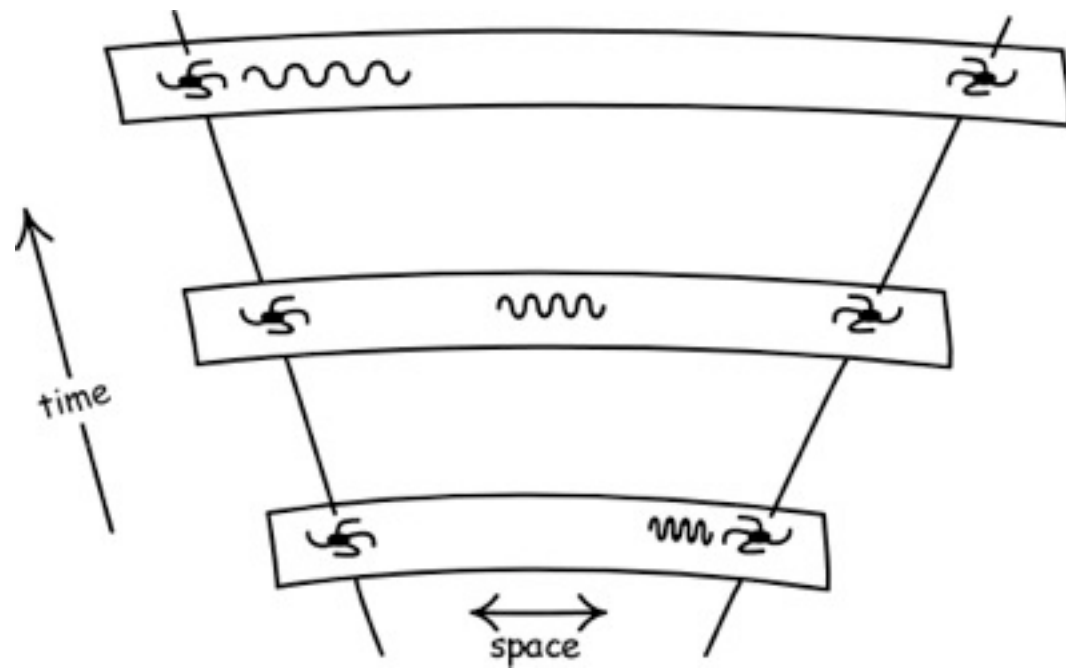


Cosmological Redshift:

light redshifted by **expansion**, not Doppler Effect



$a=a(t)$ is also called
the scale factor of
the universe.

past: photon emitted with λ_0
when universe size a_0

photon stretched as it propagate
toward us with speed of light

now: at reception, λ
universe size a

**redshift tells us how large
the universe was when
light was emitted.**

$$1 + z = \lambda / \lambda_0 = a / a_0$$

(where z is redshift)

Lecture 5: Expansion of the Universe (cont'd)

- Define redshift (usually denoted by 'z')



$$z = (\lambda - \lambda_0) / \lambda_0 = \Delta\lambda / \lambda_0,$$

where λ_0 = rest wavelength, and λ = observed wavelength

(Note that $\lambda / \lambda_0 = 1 + z$)

- if we interpret the recession velocity v as Doppler shift,
then: $v = c\Delta\lambda / \lambda_0 = cz$ (note: this only works for small z)

The Hubble Law: $cz = v = H_0 d,$

where H_0 = the Hubble constant, d = distance to the galaxy

(note, again, this is true only for small z)

A 'defective' analogy: fire-work

In a firework display, faster sparks runs further

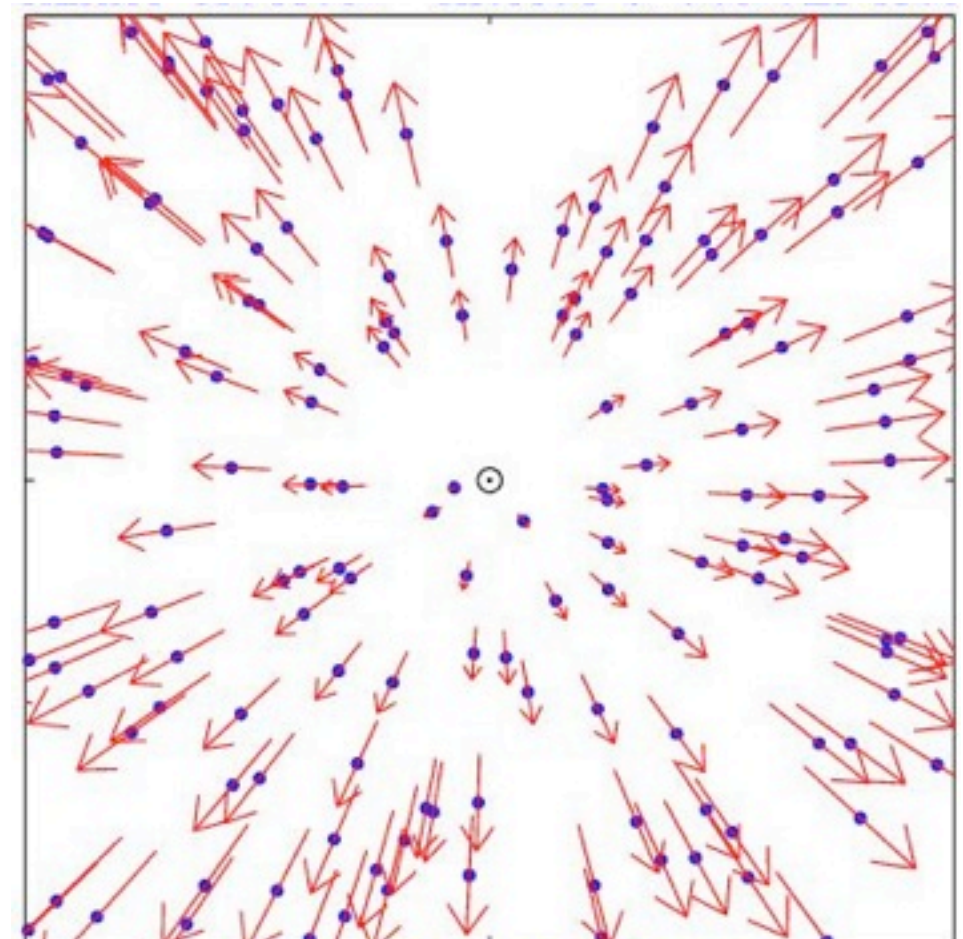
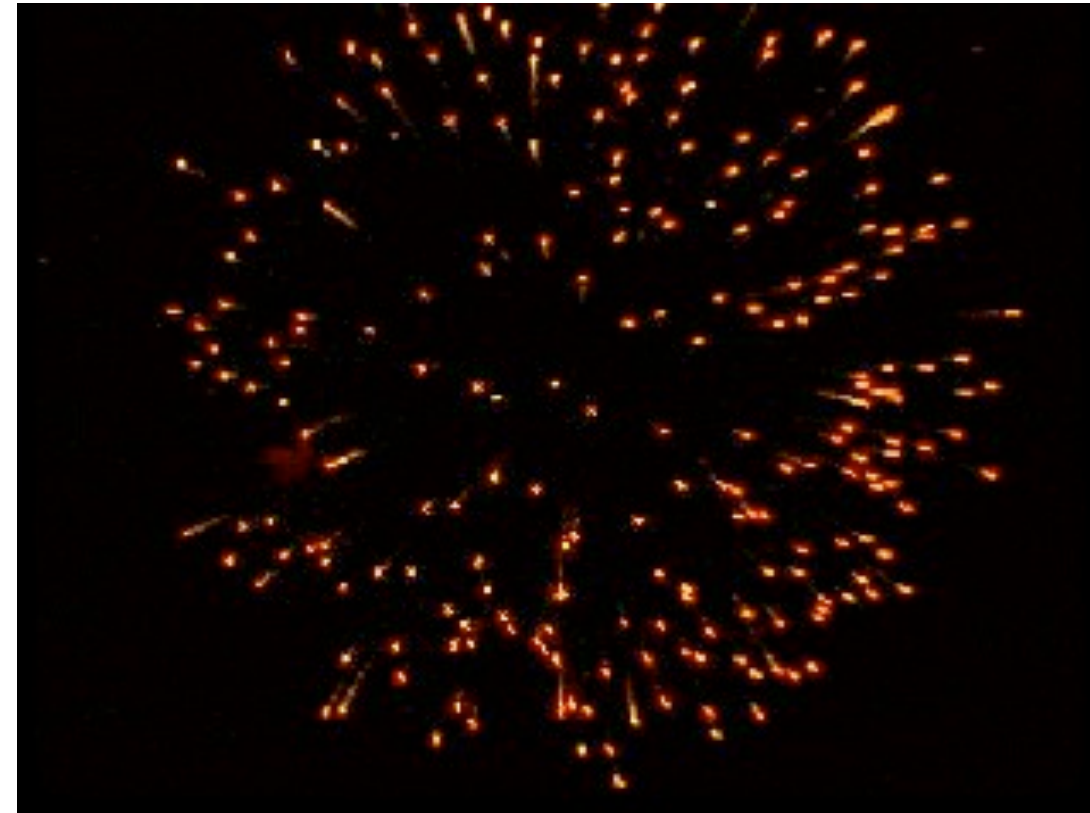
$$v = H * d \text{ (Hubble's law)}$$

viewed from **EVERY** spark,

$$v = H * d \text{ (Hubble's law)}$$

“defective”: sparks moving through space,
not space expansion

“defective”: going backward in time, there
is a special point in space where it all
started; this violates the cosmological
principle that the universe has to be
homogeneous and isotropic



can galaxies move apart faster than speed of light?

general relativity takes the galaxies as being *at rest* relative to one another (not moving away at all), while the space between them is expanding.

the distance between two objects, however, can be increasing faster than light because of the expansion of the universe.

But if so, can we see them?

all $z > 1.5$ objects are **currently** pulled apart from us faster than the speed of light. But in the past, they were closer to us and recedes with a slower velocity.

Observations routinely probe this regions.

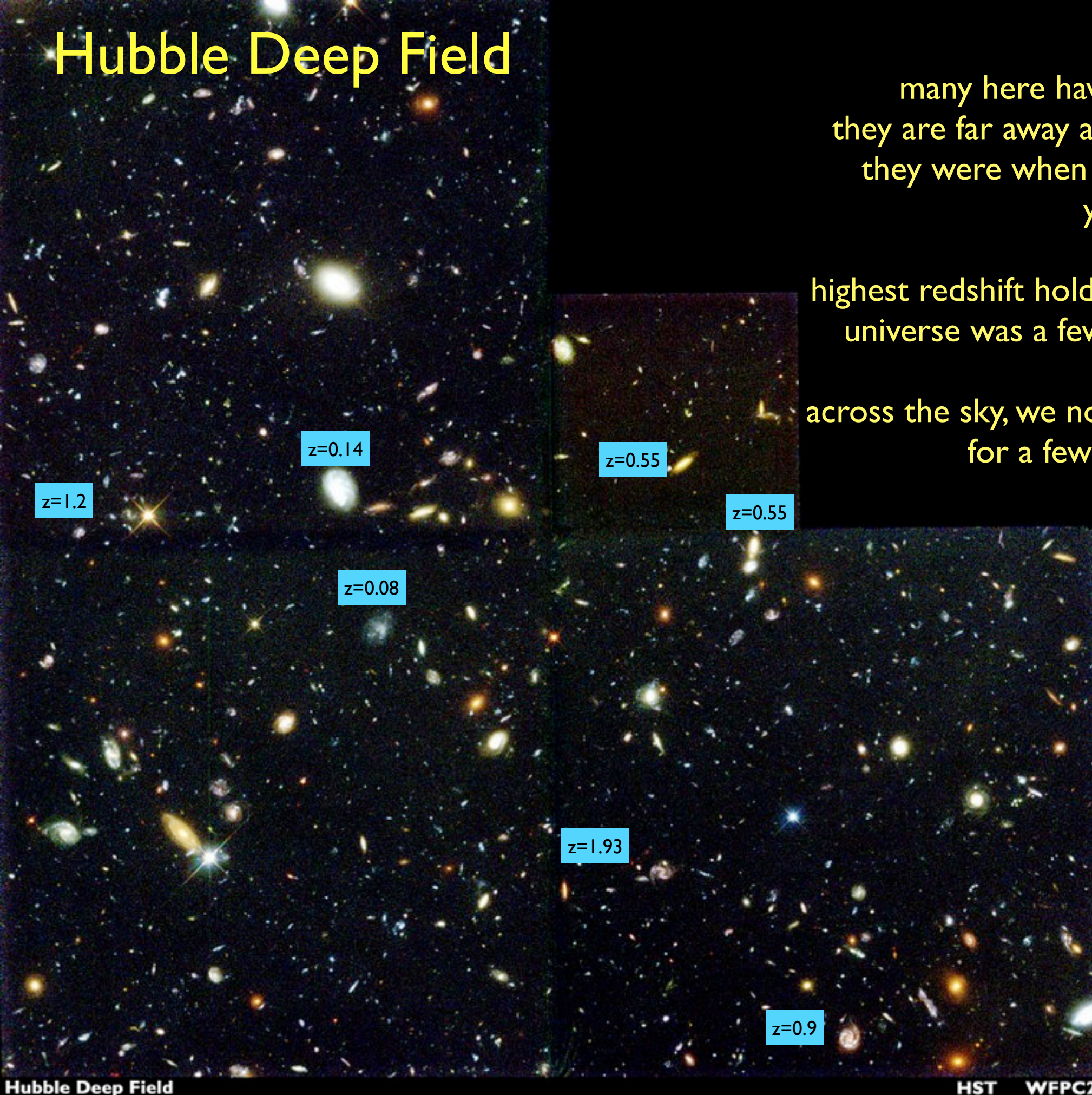
see: <http://curious.astro.cornell.edu/question.php?number=575>

Hubble Deep Field

many here have redshifts 0.5 - 1.2
they are far away and they are being seen as
they were when the universe was much
younger

highest redshift holder: $z=8.6$. light left it when
universe was a few percent its current age

across the sky, we now have measured redshifts
for a few million galaxies



The Hubble constant

Hubble 'constant'
because H is the same
everywhere in universe,
now.

. in an isotropic, homogenous universe,
expansion has to satisfy $v = H_0 * r$

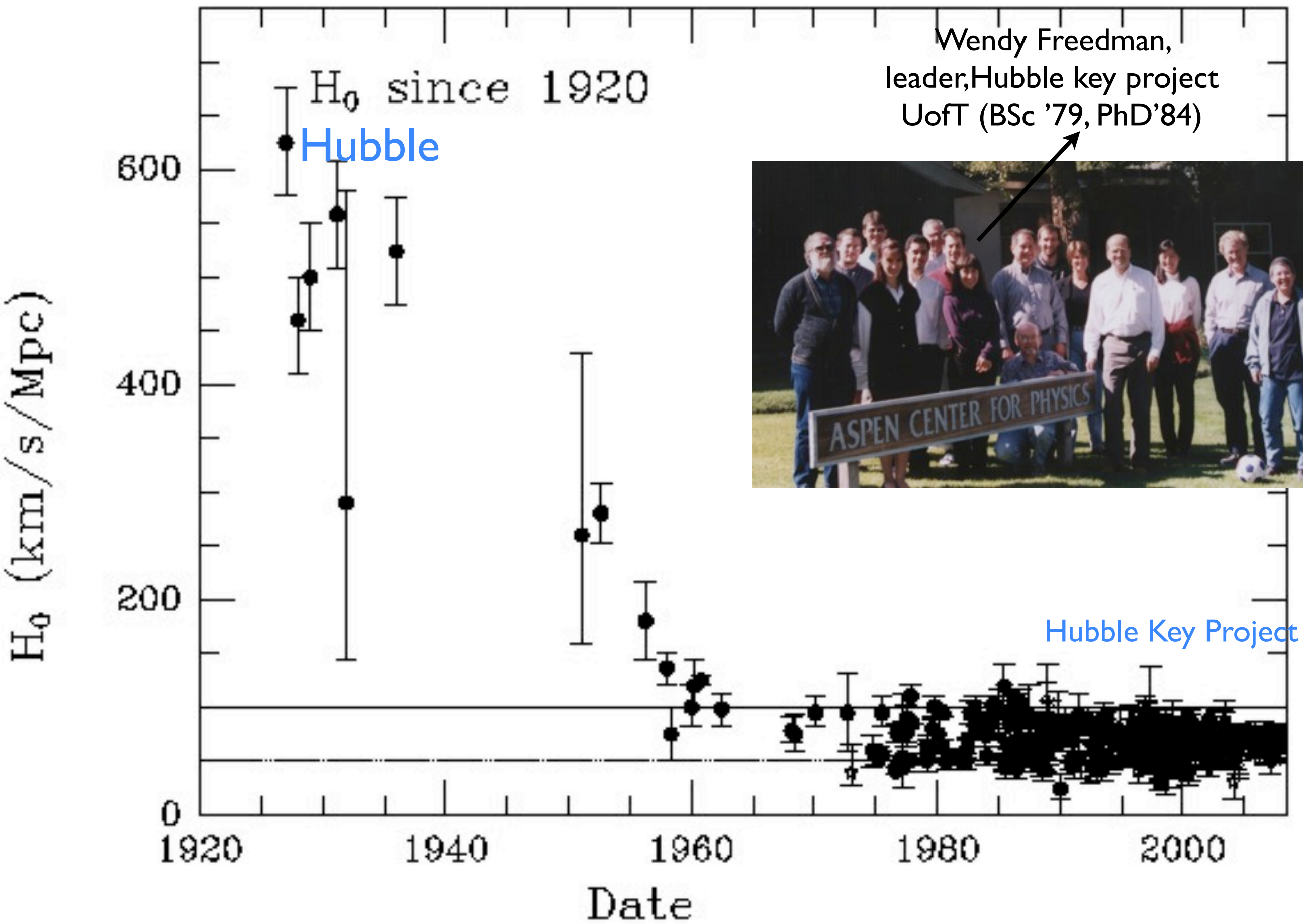
$H_0 = 0$: static; $H_0 < 0$: shrinking; $H_0 > 0$: expanding

. Astronomers determine H_0 using nearby
galaxies for which we have independent
distance measurements (Cepheid variables)

$$H_0 = c * z / r$$

. But once we know H_0 , we can use it to
calculate distances to further/fainter galaxies
for which we only have redshifts.

(Caveat: need to know cosmological model)



NGC 1365

$z=0.00545$ ($v = c*z = 1636$ km/s)

$r = 56.2 \pm 2.6$ million light years

~ 50 Cepheid
variables here

--> $H = c*z/r \sim 90$ km/s/Mpc
error: this is part of Virgo supercluster

Hubble constant is also related to the age of the universe.

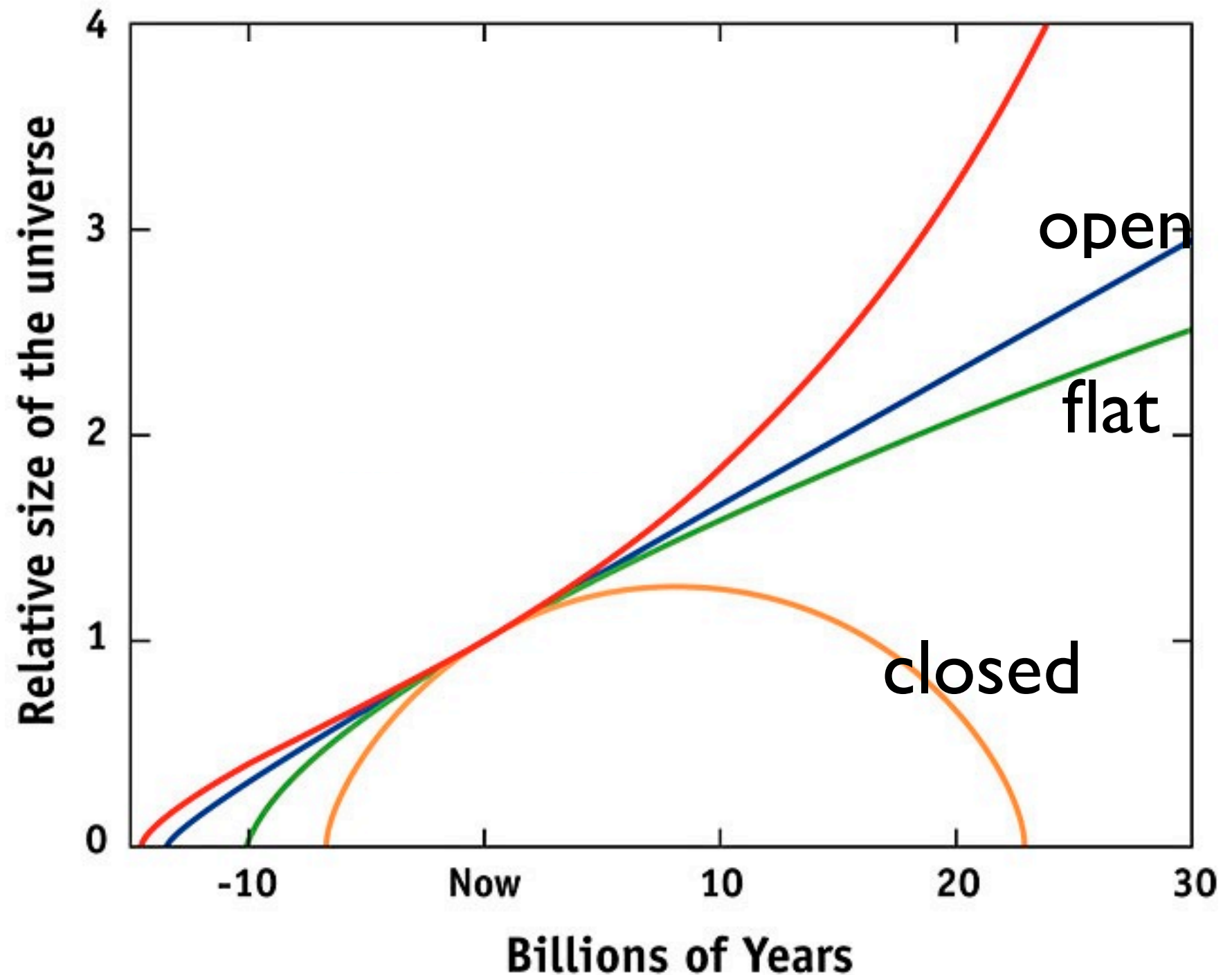
$$H_0 = 73 \pm 4 \text{ km/s/Mpc}$$

$$1/H_0 = 13.7 \text{ billion years}$$

- It has a value around 70 km/s/Mpc. Note the curious units!
 - $1/H_0$ has units of time. It's the “Hubble Time”, the approximate age of the Universe.
 - c/H_0 has units of distance. It's the “Hubble Distance”, the approximate size of the (visible) Universe.
- The ages and sizes of the Universe obtained from the Hubble constant are only approximate because the exact relation includes terms relating to Ω , the amount of matter in the Universe.

age of the universe ranges from
 $\frac{2}{3} 1/H_0$ (flat universe) to $1/H_0$ (empty universe)

redshift:
 $1 + z$
 $= a_{\text{now}}/a_{\text{past}}$



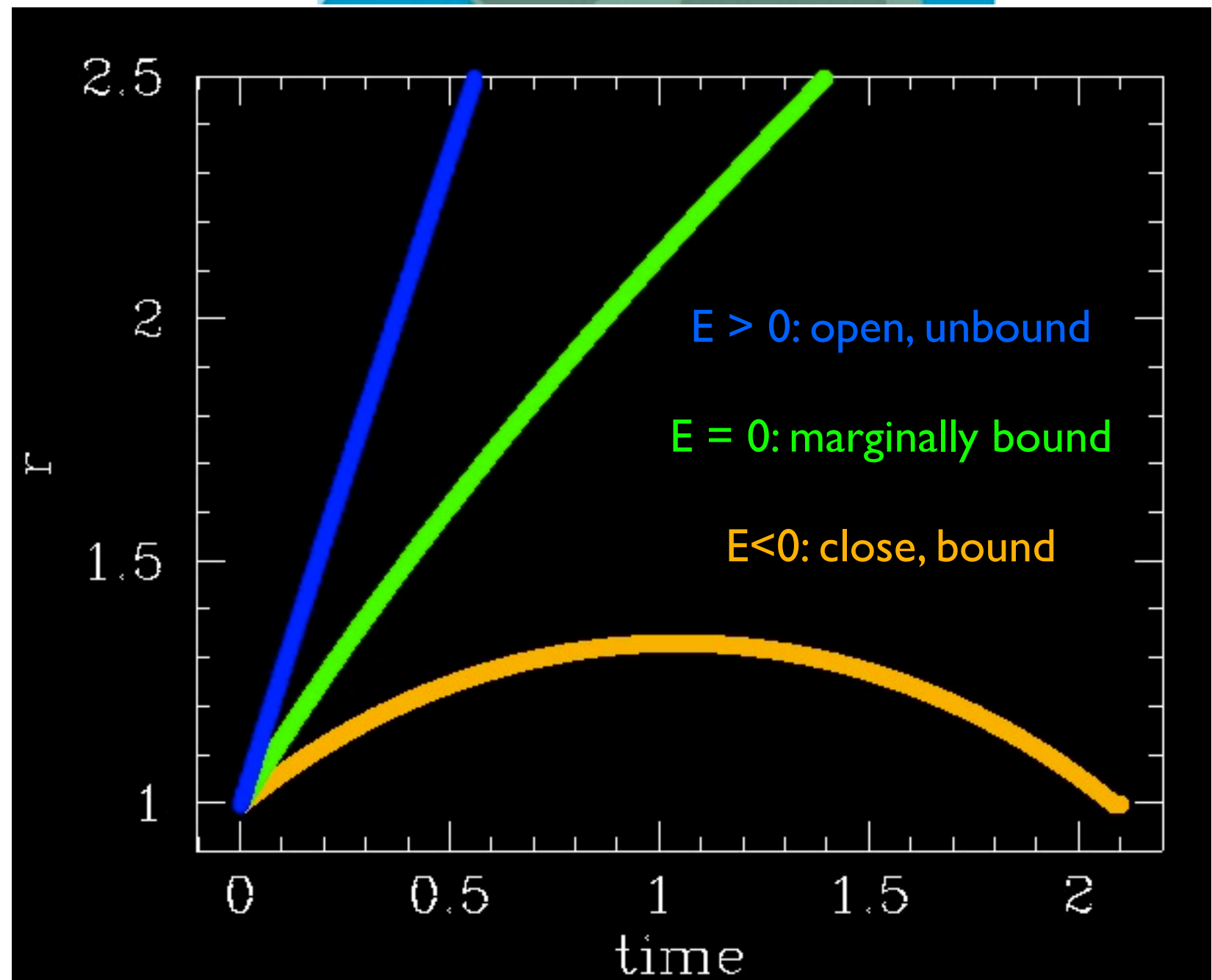
scale factor, $a(t)$, is not the size of the universe,
rather, it is the relative size at different times.
The universe can be infinite with a finite scale factor

Escape velocity of the Earth

launch spaceship (m)
upward with velocity v

$$E = \frac{1}{2} m v^2 - G M m / r$$

Hubble const = v/r
larger H_0 , more open



Escape velocity of the universe

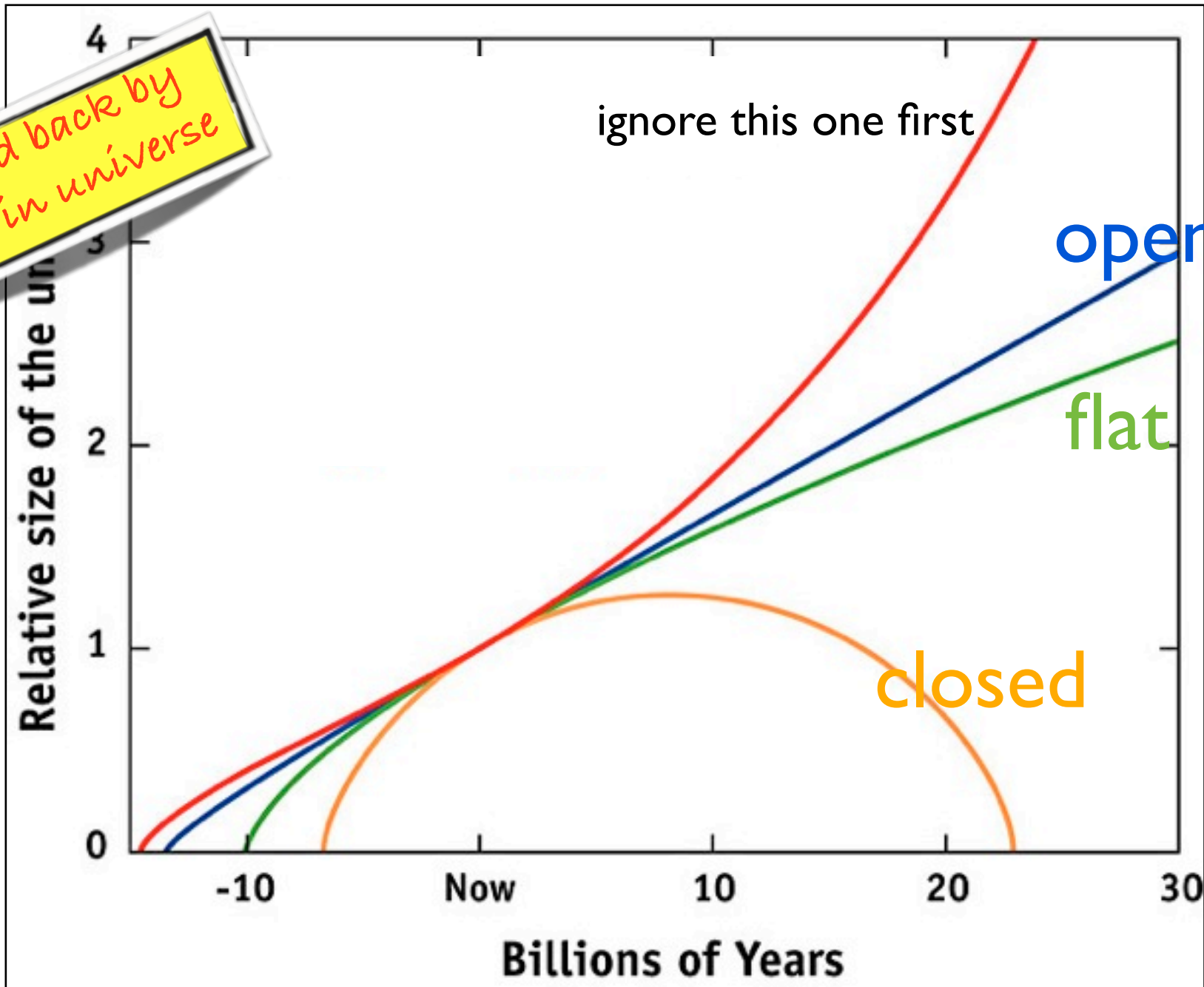
launch spaceship with velocity v toward

v : big kick

M : held back by mass in universe

$$E = \frac{1}{2} m v^2 - G M m / r$$

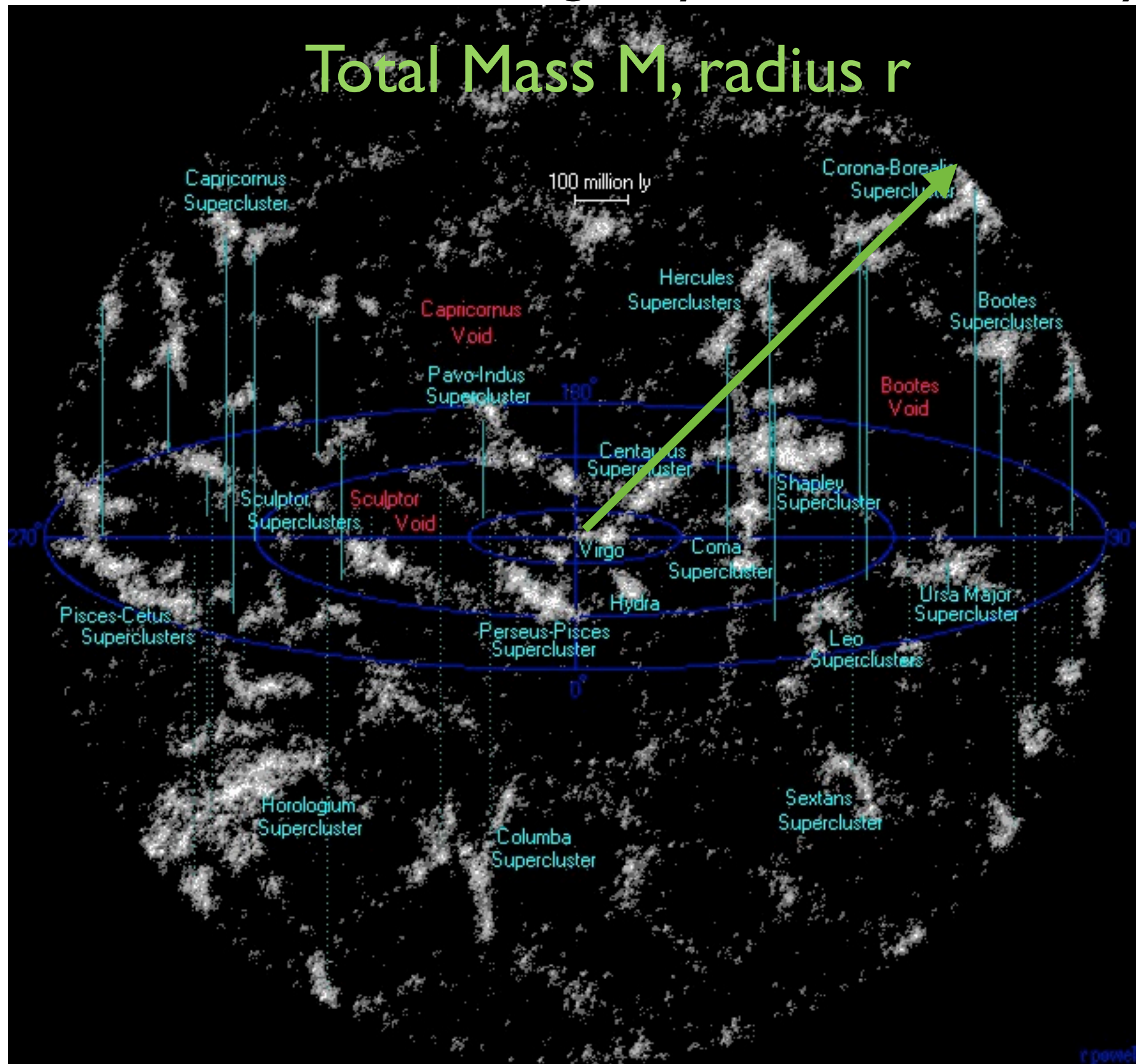
- $E > 0$: open, unbound
- $E = 0$: marginally bound
- $E < 0$: close, bound



$$\begin{aligned} \text{Hubble const} &= v/r \\ &= (dr/dt)/r = (da/dt)/a \end{aligned}$$

Hubble const. fixes the current expansion rate. But to know where the universe is going, need to know density.

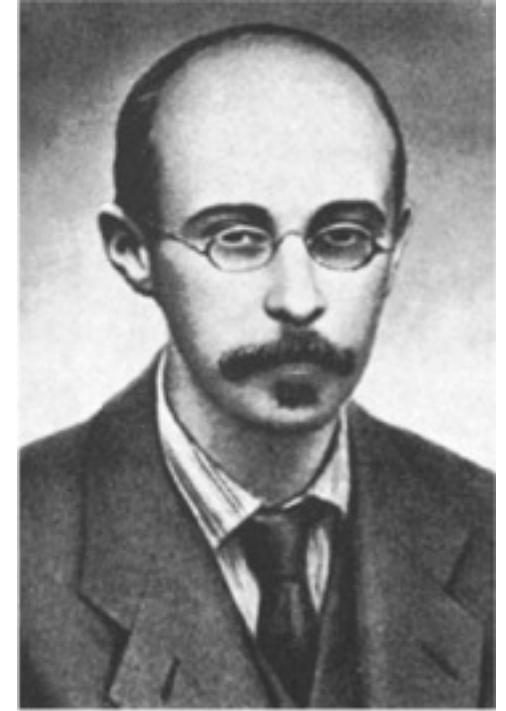
Consider the recession of a galaxy at distance r away:



(Birkhoff theorem: mass outside irrelevant as long as spherical)

Friedmann's equation for the evolution of the universe

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2}$$



A. Friedmann

Derived by Alexander Friedmann (1922)

Using Einstein's field equation (general relativity, 1915) by assuming that the universe is homogeneous and isotropic.

It describes how the scale factor of the universe evolves.

a is the scale factor of the universe

ρ is the density of the universe

$H = (da/dt)/a$ is the Hubble parameter

k is the curvature, $k=-1,0,+1$

Friedmann's equation for matter-dominated universe

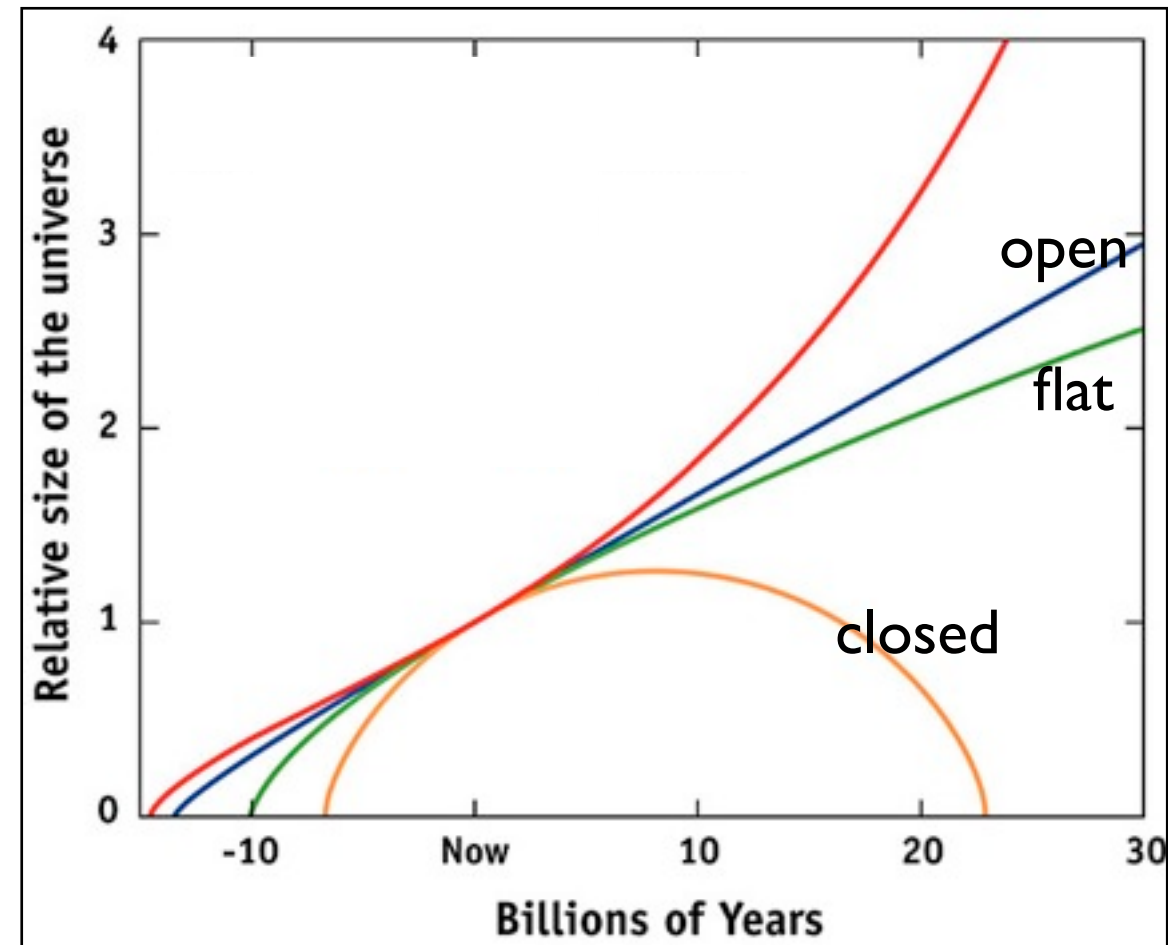
$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2}$$

$$E = \frac{1}{2} m v^2 - GMm/r$$

$$E = 0, \text{ flat, } k = 0$$

$$E > 0, \text{ open, } k = -1$$

$$E < 0, \text{ closed, } k = +1$$



- Expansion of the universe analogous to the motion of a rocket shot upward.
- H_0 fixes the current expansion rate. But need to know matter density to tell the future/past.
- In all cases, cosmic expansion slowed down with time as expected --- like the rocket.

Critical Density -- in a universe dominated by matter (no radiation, dark energy...), there exists a threshold density at which the universe is flat.

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2}$$

we define

$$\rho_{\text{crit}} \equiv \frac{3H^2}{8\pi G}$$

so Friedmann's equation turns into

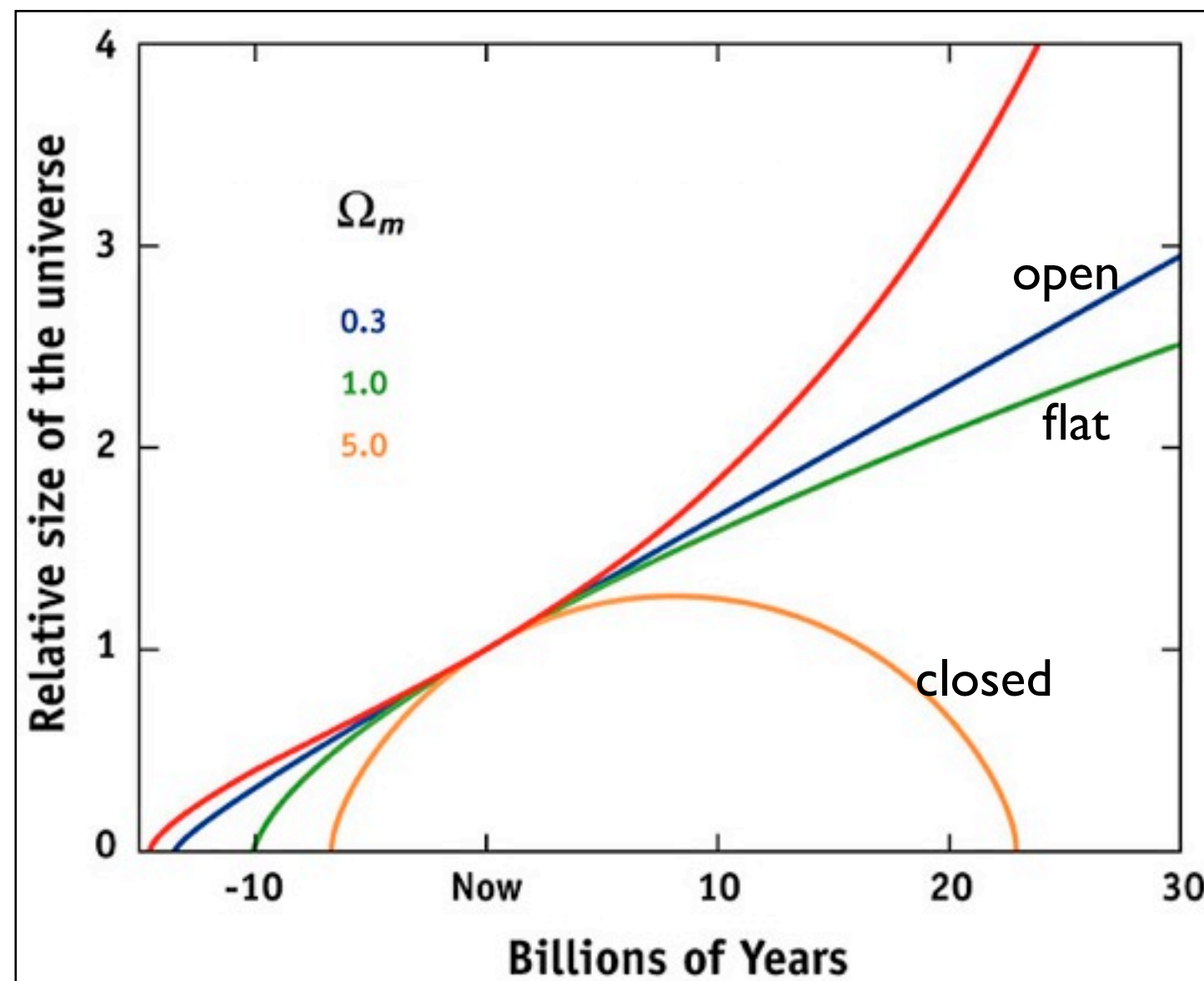
$$\Omega_m \equiv \frac{\rho}{\rho_{\text{crit}}} = 1 + \frac{kc^2}{a^2 H^2}$$

$\Omega_m < 1$ gravity loses, open
“big freeze”

$\Omega_m = 1$ flat

$\Omega_m > 1$ gravity wins, closed
“big crunch”

(ignoring dark energy for the moment)



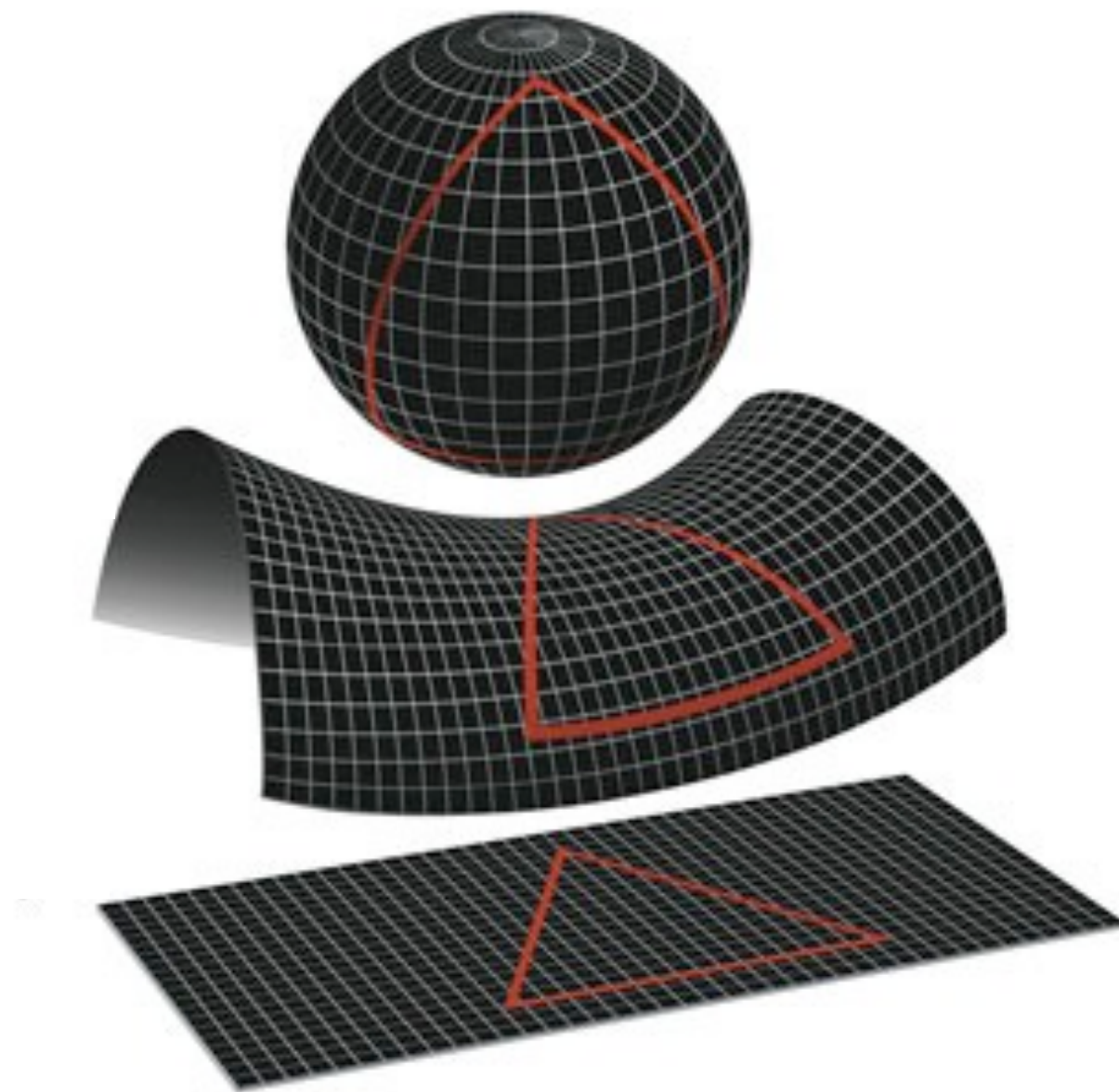
density also determines Shape of the universe

$$\Omega_m = \rho/\rho_{\text{crit}}$$

$\Omega_m > 1$, lots of matter,
matter curves space around
it (positive curvature)

$\Omega_m < 1$, not much matter,
expansion shapes the
universe (negative
curvature)

$\Omega_m = 1$, just the right
amount of matter, flat
geometry (zero curvature)



Which one is the correct answer for our universe?

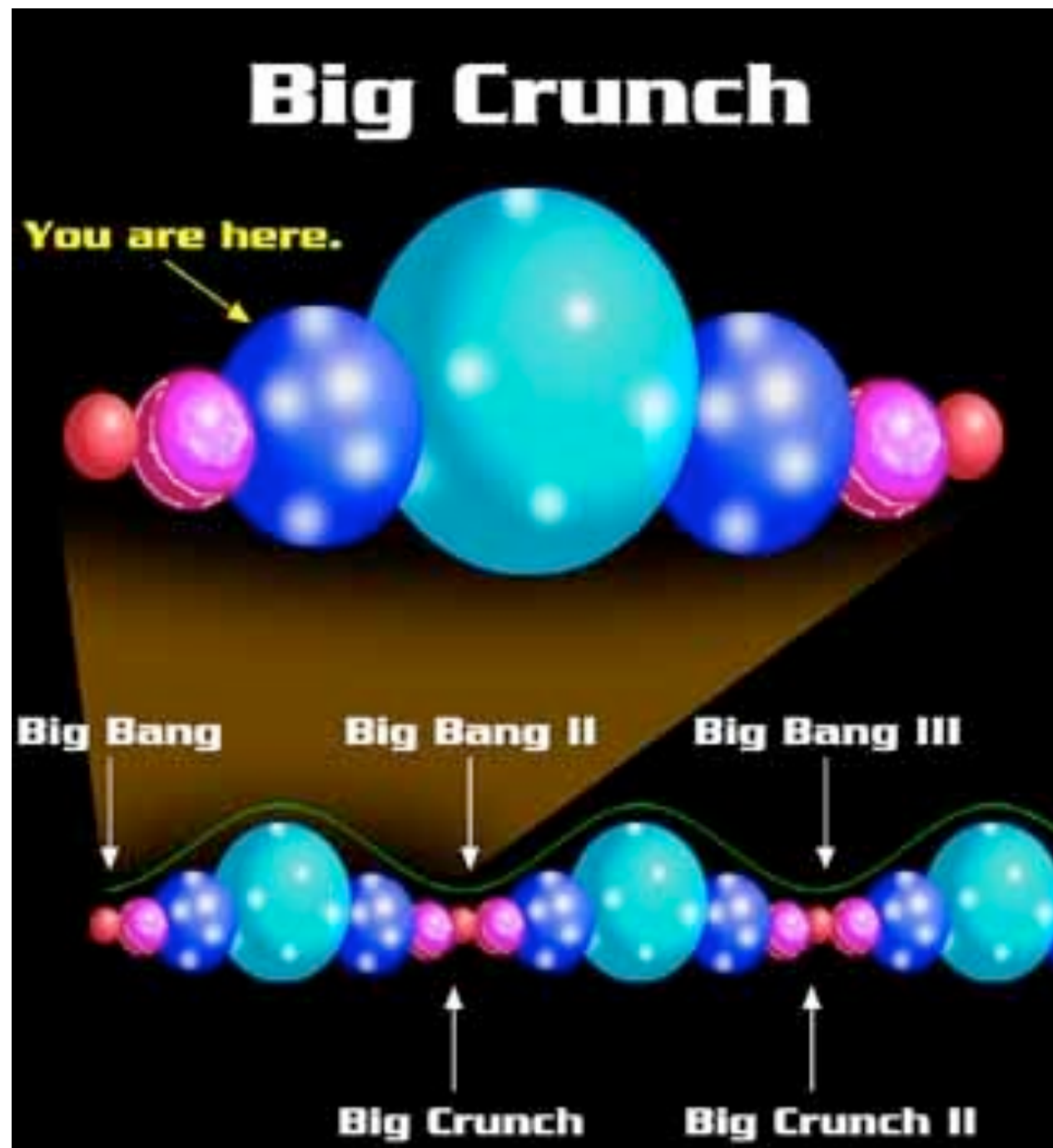
How does it feel to be living in a ... closed universe

to visualize, we imagine the universe sits on a 2-D balloon



- space and number of galaxies finite
- but no edge. nor centre.
- at some point, your horizon encompasses the whole universe
- light travels along geodesics (great circles), you can see your rear, given enough time.
- but universe will contract, $H < 0$, big crunch into a single blackhole.

Life after death? Big Bounce...



some suggest that a big crunch may be the seed for the next big bang, so on, and so on.... cyclical universe

How does it feel to be living in an ... open universe



- space and number of galaxies infinite.
- no edge. nor centre.
- light never returns to same point
- enlargement of your horizon $<$ expansion of space. finally, only one mega-galaxy left visible.
- with time, the universe cools, all stars run out of nuclear fuel and become dark objects (neutron stars, white dwarfs and blackholes), no more light/heat, cold death (“big freeze”)

Universe in the far future (Krauss & Scherrer, Sci.American, 2008)

COSMIC MILESTONES

10⁻³⁰ second

Cosmic inflation occurs

100 seconds

Deuterium and helium are created

400,000 years

Microwave background is released

8 billion years

Expansion begins to accelerate

13.7 billion years

Today

20 billion years

Milky Way and Andromeda collide

100 billion years

All other galaxies are invisible

1 trillion years

Primordial isotopes are lost or diluted

100 trillion years

Last star burns out

- future civilization won't discover expansion of the universe
- other galaxies are beyond or approaching horizon, with enormous redshifts
- cosmic microwave background too cold to be observed
- light element ratios lost finger print of big bang
- we live truly in an island universe where all vestiges of the big bang is erased... may come to the wrong idea about the origin of the universe

Show us the number.

Adding up the mass...

$\Omega_m \sim 0.3$, gravity loses, open

Fukugita & Peebles, Astrophysical Journal, 2004

Parameter	Components ^a	Totals ^a
Dark sector:		0.954 ± 0.003
Dark energy	0.72 ± 0.03	
Dark matter	0.23 ± 0.03	
Primeval gravitational waves	$\lesssim 10^{-10}$	
Primeval thermal remnants:		0.0010 ± 0.0005
Electromagnetic radiation	$10^{-4.3 \pm 0.0}$	
Neutrinos	$10^{-2.9 \pm 0.1}$	
Prestellar nuclear binding energy	$-10^{-4.1 \pm 0.0}$	
Baryon rest mass:		0.045 ± 0.003
Warm intergalactic plasma	0.040 ± 0.003	
Virialized regions of galaxies	0.024 ± 0.005	
Intergalactic	0.016 ± 0.005	
Intracluster plasma	0.0018 ± 0.0007	
Main-sequence stars: spheroids and bulges	0.0015 ± 0.0004	
Main-sequence stars: disks and irregulars	0.00055 ± 0.00014	
White dwarfs	0.00036 ± 0.00008	
Neutron stars	0.00005 ± 0.00002	
Black holes	0.00007 ± 0.00002	
Substellar objects	0.00014 ± 0.00007	
H I + He I	0.00062 ± 0.00010	
Molecular gas	0.00016 ± 0.00006	
Planets	10^{-6}	
Condensed matter	$10^{-5.6 \pm 0.3}$	
Sequestered in massive black holes	$10^{-5.4}(1 + \epsilon_n)$	

The real density doesn't have to be close to Critical Density

$$\Omega_m = \rho/\rho_{\text{crit}}$$

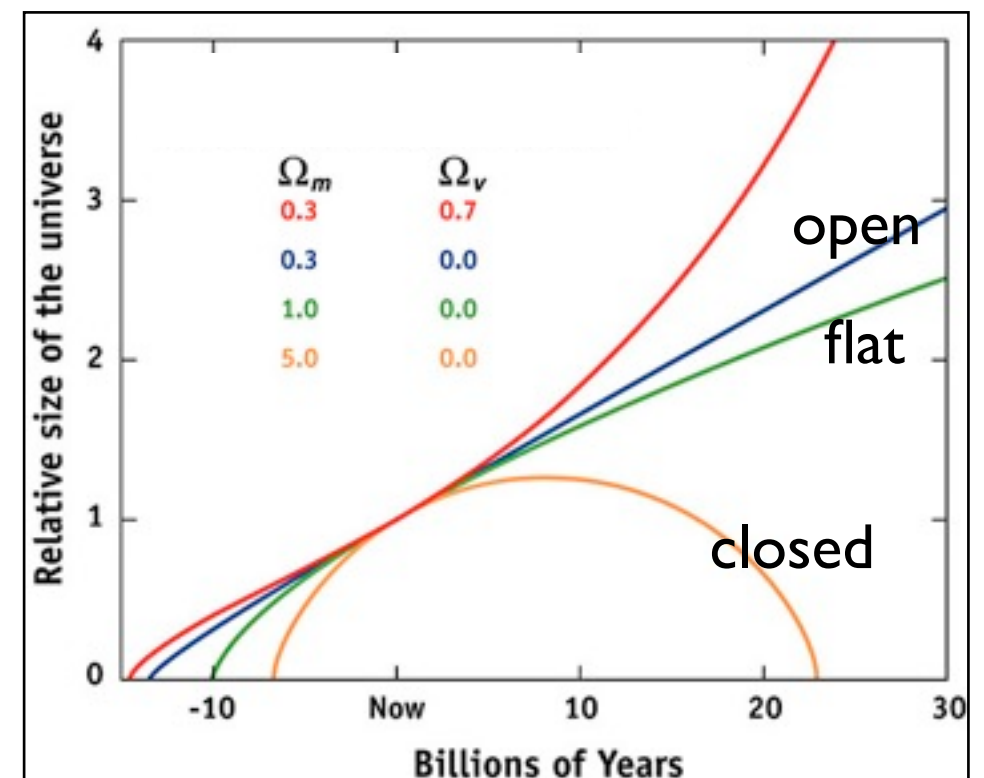
$\Omega_m \ll 1$ expansion too fast, prevent galaxies & stars to form

$\Omega_m \sim 1$, permitted range for life

$\Omega_m \gg 1$ universe re-collapse too fast before life develops

To have $\Omega_m \sim 0.3$ today, Ω_m will have to be within $1/10^{15}$ of unity at one second after big-bang

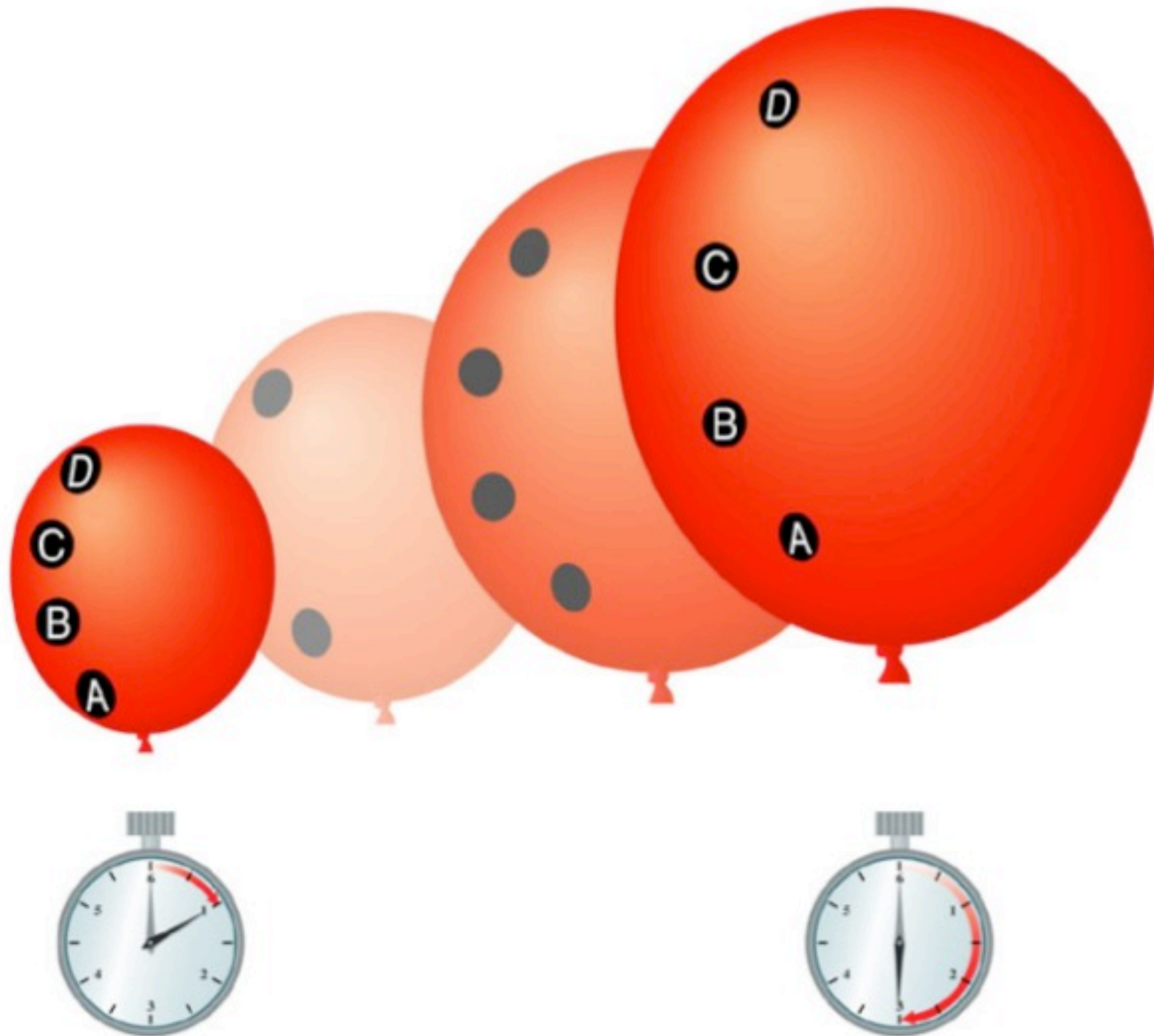
(Rees 'Just six numbers')



So many questions...

- If the Universe is expanding, what is it expanding into? And if it's expanding, doesn't it have to have an edge? What's beyond the edge?
- If the Universe is 13 billion years old, then what was happening before it was created?
- If the Universe is expanding away from us in all directions, doesn't this mean we're at the center of the Universe?
- If the Universe is expanding, does this mean that I am getting bigger?
- If the Universe was born in a gigantic explosion, where in space did the explosion occur?

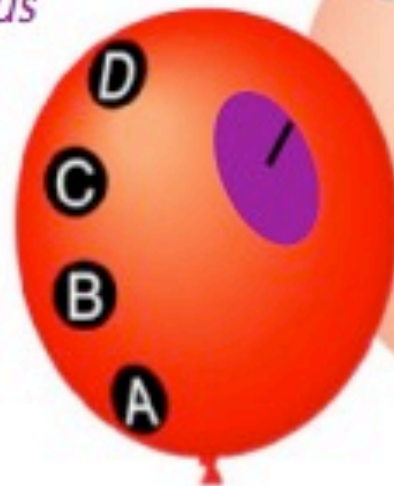
A 2D metaphor for the 3D expansion



Our Horizon

You can never see further out into space than how far light has had time to travel in the age of the Universe.

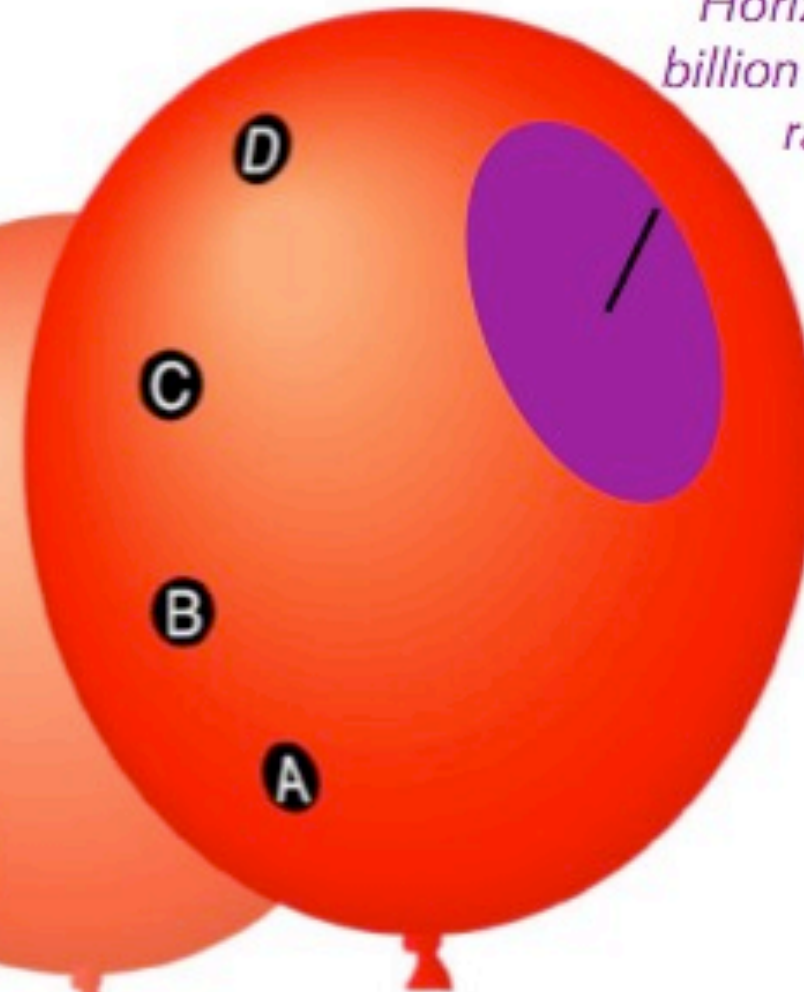
Horizon is 5 billion light years radius



5 billion years



Horizon is 13 billion light years radius



13 billion years

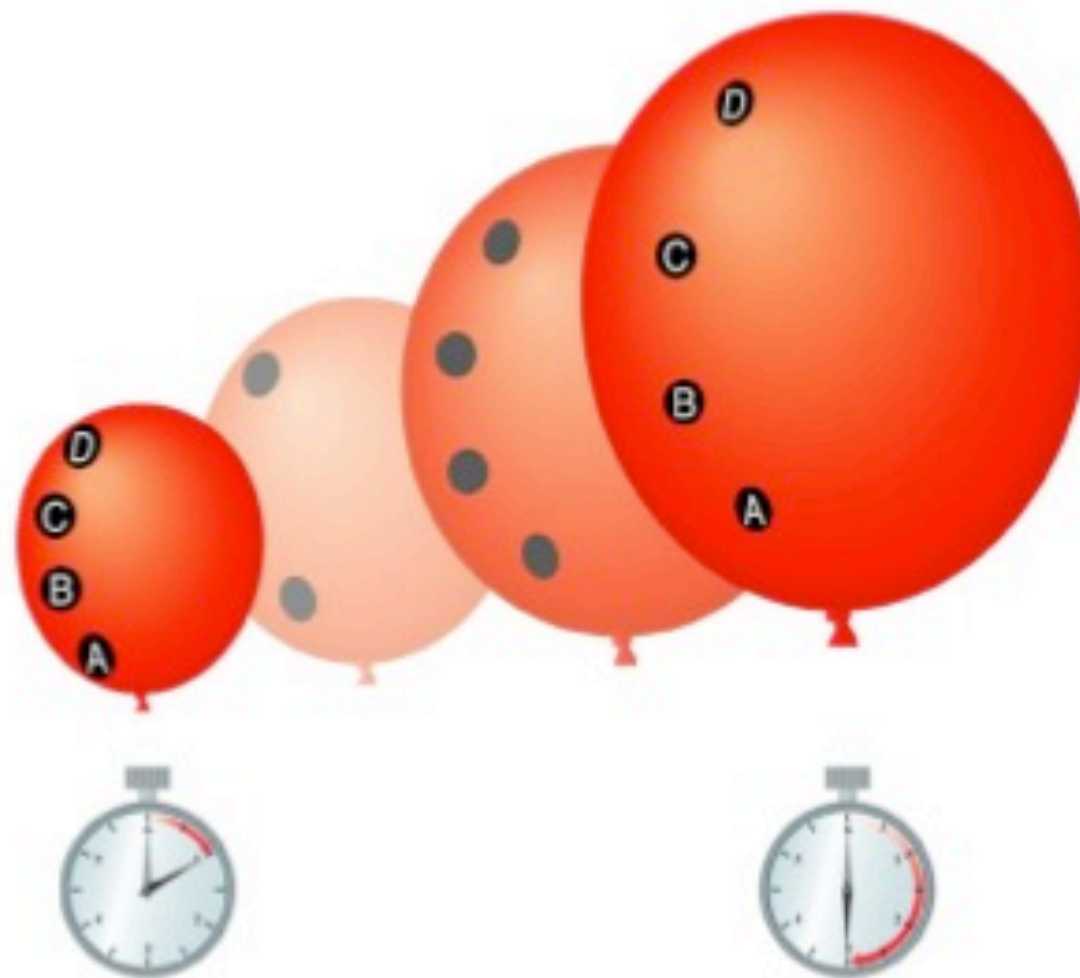


Important Note

- The metaphor of the balloon implies a closed Universe of finite size.
- An arguably somewhat better model in light of current observations is a infinitely big flat rubber sheet being stretched out. This is a Universe of infinite size (though again, the observable Universe inside the horizon is finite).
- We'll use the balloon model as it's easier to draw, and that's what's usually in textbooks. But the same ideas go for both pictures (balloon vs. infinite rubber sheet).

Q: If the Universe is expanding away from us in all directions, doesn't this mean we're at the center of the Universe?

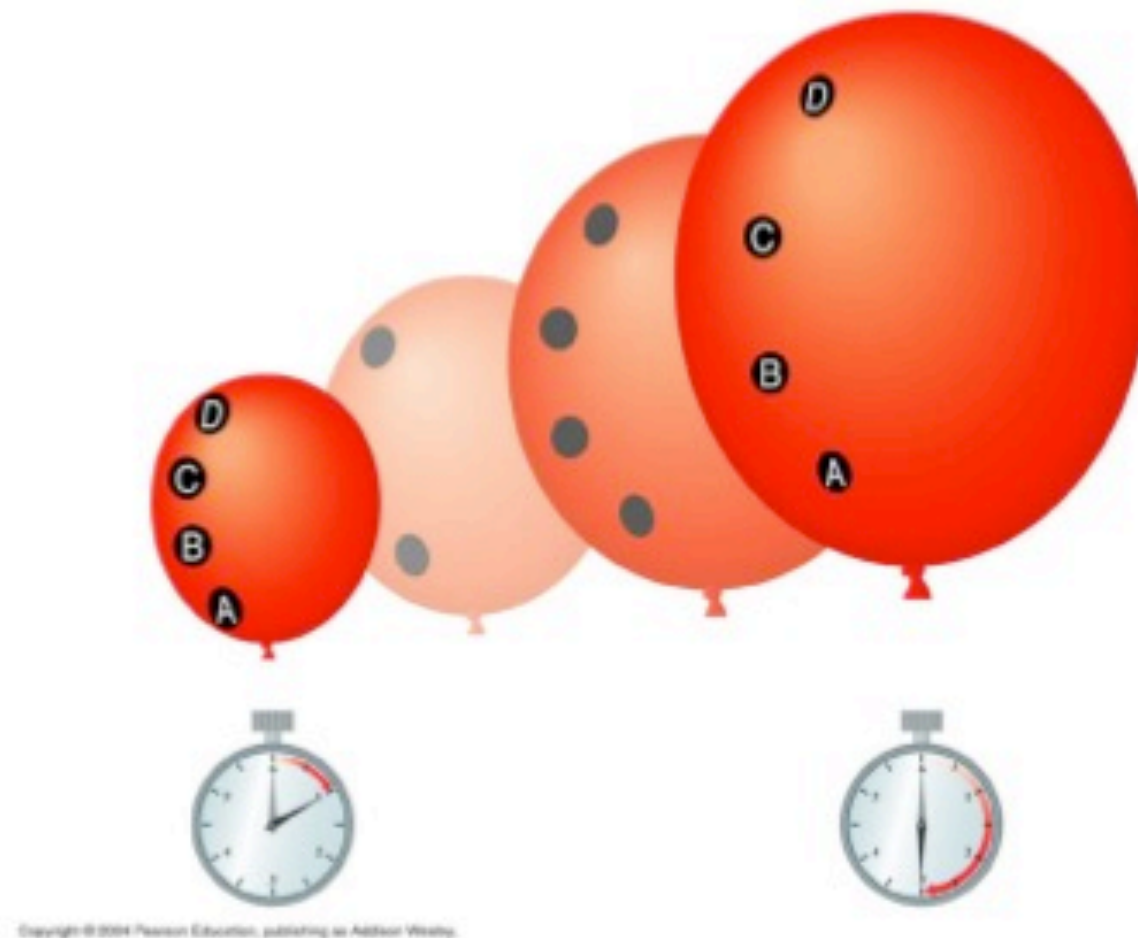
- No! Consider the balloon analogy again. Each object sees all the other objects expanding away from it. (Same goes for stretching out an infinite rubber sheet).



Copyright © 2004 Pearson Education, publishing as Addison Wesley

Q: If the Universe was born in a gigantic explosion, where in space did the explosion occur?

- Everywhere! And nowhere! Consider the balloon analogy again. Space is expanding everywhere. For beings that live on the 2D surface, the real “center of expansion” in the balloon analogy is off in a higher dimension (the 3D center of the balloon). And in the infinite rubber sheet model there is no center at all.



Copyright © 2004 Pearson Education, publishing as Addison Wesley.

**Q: If the Universe is expanding, what is it expanding into?
And if it's expanding, doesn't it have to have an edge?
What's beyond the edge?**

- Consider the balloon analogy. The denizens of the surface of the balloon wonder the same thing. The answer from their point of view is that the Universe expands into a higher dimension (namely, the 3rd dimension we inhabit). From their point of view there is no edge. If the geometry is spherical you can go round and come right back where you started. If it's not spherical (say it's flat) you would go on forever.
- Same thing goes for us!
- There is an effective edge to the Universe though, namely our astrophysical horizon, but it's not the same thing as a real physical "edge of the Universe".
- This begs consideration of a "multiverse" of Universes outside our horizon. But is this science?

Q: If the Universe is 13 billion years old, then what was happening before it was created?

- A1: The standard answer is that the question is meaningless since time was created along with the Universe. (i.e. consult the theologian of your choice – this question is theology and not science).
- A2: In my opinion an equally valid answer is that the question *is* scientific, but our state of knowledge is such that presently we don't have a clue. Perhaps physics will eventually reach the point where we understand enough to be able to say whether we live in a cyclical Universe that creates itself in cycles of Big Bang, Big Crunch, Big Bang, Big Crunch, etc.

Some other ideas...

Multiverse: our universe is but one of many universes that occur simultaneously, some matter, some anti... they may collide

avored by some inflation theorists

fundamental constants in our universe conducive to intelligence, other universes?

