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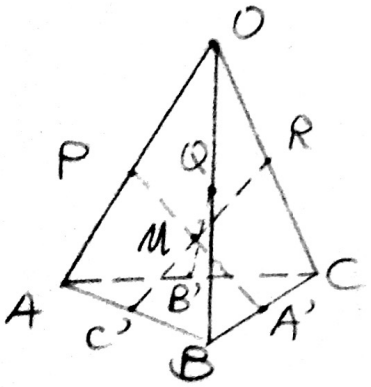
QUIZ 1, JANUARY 23, LECTURE 3

Name Rui Qiu Student number 999292509

Consider a tetrahedron. Join the middle point of each of its edges with the middle point of the opposite edge. Prove that the three segments you constructed pass through one point. In what proportion does this point divide each segment?



Name middle points P, Q, R and A', B', C' respectively.

want to prove segments PA', QB' and RC' pass through one point.

Proof:

Suppose PA' and QB' interseet at point M.

since $PQ \parallel \frac{1}{2}AB \cong B'A'$, $\angle PMQ = \angle A'MB'$

$\therefore \triangle PQM \cong \triangle A'B'M$

So M is the midpoint of PA' and QB'.

Similarly we can prove by the equality of $\triangle PRM$ and $\triangle C'MA'$, $\triangle RMQ$ and $\triangle C'B'M$ that the intersection of PA' & RC' is the middle point of PA' & RC', and the intersection of QB' and RC' is the middle point of PA' & RC' respectively.

Hence they pass through one point.

And as a result, M divides each segment into 2 equal parts (namely, 1:1).

What if they don't intersect at all?