Lecture 10

$Q_c(x)=x^2+c$

2-cycles $q_-=\dfrac{-1-\sqrt{-3-4c}}{2}$, $q_+=\dfrac{-1+\sqrt{-3-4c}}{2}$

And $Q_c'(q_-)Q_c'(q_+)=4(c+1)$

So $|Q_c'(q_-)Q_c'(q_+)|=4|c+1|<1$

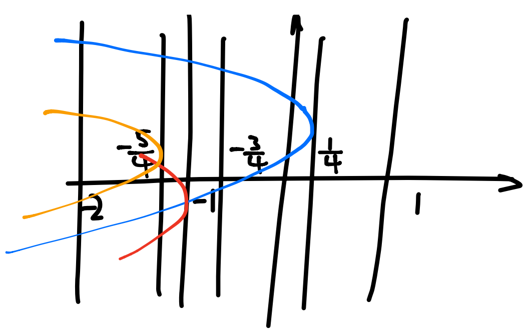
$\Rightarrow |c+1|<\frac{1}{4}$

$\Rightarrow -\frac{1}{4}<c+1<\frac{1}{4}$

$\Rightarrow -\frac{5}{4}<c<-\frac{3}{4}$

The 2-cycle q_-, q_+ is
- Attracting if $c\in(-5/4,-3/4)$
- Neutral if $c=-3/4$
- Repelling if $c<-5/4$

We can summarize this with a graph



$P_-=\dfrac{1-\sqrt{1-4c}}{2}$

$P_+=\dfrac{1+\sqrt{1-4c}}{2}$

§ 6.2 Saddle-node (fold) Bifurcation

Def'n: A one parameter family of functions F_λ have a saddle-node (or tangent) bifurcation in the open interval I at the parameter λ_0 if $\exists\ \varepsilon>0$ s.t.

(i) F_{λ_0} has one fixed pt in I & it's neutral.

(ii). For all λ in one half of the interval $(\lambda_0-\varepsilon,\lambda_0+\varepsilon)$, F_λ has no fixed pts.

(iii) For all λ in the other half of $(\lambda_0-\varepsilon,\lambda_0+\varepsilon)$, F_λ has two fixed pts in I, one attracting & one repelling.

Note: Periodic pts can also have a tangent bifurcation. Apply the def'n to F_λ^n for an n-cycle.

Example:
The quadratic function $Q_c=x^2+c$ has a tangent bifurcation at $c=1/4$

(i) $Q_{\frac{1}{4}}$ has one neutral fixed pt $\quad P=\frac{1}{2}$

(ii) For $c>\frac{1}{4}$, no fixed pts.

(iii) For $-3/4<c<1/4$, it has an attracting (P_-) & a repelling (P_+) fixed pts.

In the def'n, we can use $\varepsilon=1$ & $I=\mathbb{R}$

Example: let $E_\lambda(x)=e^x+\lambda$ be called the exponential family, it has a tangent bifurcation as $\lambda=-1$.