

# **SCHOOL OF FINANCE AND APPLIED STATISTICS**

## **FINANCIAL MATHEMATICS (STAT 2032 / STAT 6046)**

### **TUTORIAL EXERCISES WEEK 10**

#### **Question 1**

A ten-year zero-coupon bond is issued on 1 February 2000 at a price of \$79 per \$100 nominal. On 1 February 2002 an investor entered into a forward contract to buy \$1000 nominal of the bond in 5 years' time. The price of the bond was \$83 per \$100 nominal on 1 February 2002 and \$92 per \$100 nominal on 1 February 2007.

Calculate the profit or loss made by the investor on 1 February 2007 if the risk-free force of interest was 3% pa.

#### **Question 2**

- (i) A fixed interest stock is redeemable at 106% (ie. 1.06 per unit nominal) in 15 years' time and pays coupons of 9% per annum payable half-yearly in arrears. What price should an investor pay per \$100 nominal to obtain a gross redemption yield of 9% per annum?
- (ii) Instead of purchasing the stock, the investor decides to agree a forward contract to buy the security in six years' time immediately after the coupon payment then due. Calculate the forward price based on a risk-free rate of return of 6% pa effective and no arbitrage. The current price of the stock is that calculated in part (i).

#### **Question 3**

On 30 June 2006 an investor wishes to enter a forward contract to buy 10,000 shares of Company ABC at 30 June 2016. The current share price is \$2.50 and the annual dividend payable at 31 December 2006 is expected to be \$0.08 per share.

If the risk free force of interest is 5% p.a. and dividends are expected to remain constant, calculate the value of the long forward contract at 30 June 2012 if the share price at that date is \$2.90 and dividends have remained constant as expected.

**Question 4**

If the  $n$  year spot rates can be approximated by the function  $s_n = 0.09 - 0.03e^{-0.1n}$ ,

Calculate the following quantities:

- a) the one-year forward rate at time 10 years (ie.  $f_{10,11}$ ).
- b) the price of \$100 nominal of a 10-year zero coupon bond redeemable at par.
- c) the 5 year spot rate in 20 years' time
- d) the price of \$100 nominal of a 10-year zero coupon bond redeemable at par purchased in 5 years' time

**Question 5**

Zero coupon bonds redeemable at par are available with the following prices for \$100 nominal:

Term	Price
4 years	\$79
5 years	\$74
6 years	\$69
7 years	\$64

Find the one-year forward rate of interest starting in 5 years' time implied by these prices.

**Question 6**

Find the price of a 3-year \$100 bond, redeemable at par, with annual coupons of 6% per annum, if the 3-year spot rate is 9% per annum and the following annual forward rates of interest apply:

$$s_1 = 10\% \text{ per annum}$$

$$f_{2,3} = 7\% \text{ per annum}$$

where  $f_{t,T}$  is the annual rate of interest agreed at time 0 for an investment made from time  $t$  until time  $T$ .

**Past Exam Question – 2005 Final Exam Q2(c)**

For discrete time periods ( $t = 0, 1, 2, 3, \dots$ ), the forward effective rate per annum can be calculated as follows:

$$f_{t,t+1} = \exp(0.07 + 0.001t^2) - 1$$

Calculate the following (on an effective per annum basis):

- i)  $s_3$  (2 marks)
- ii)  $f_{6,9}$  (2 marks)
- iii)  $f_{t,t+2}$  (2 marks)