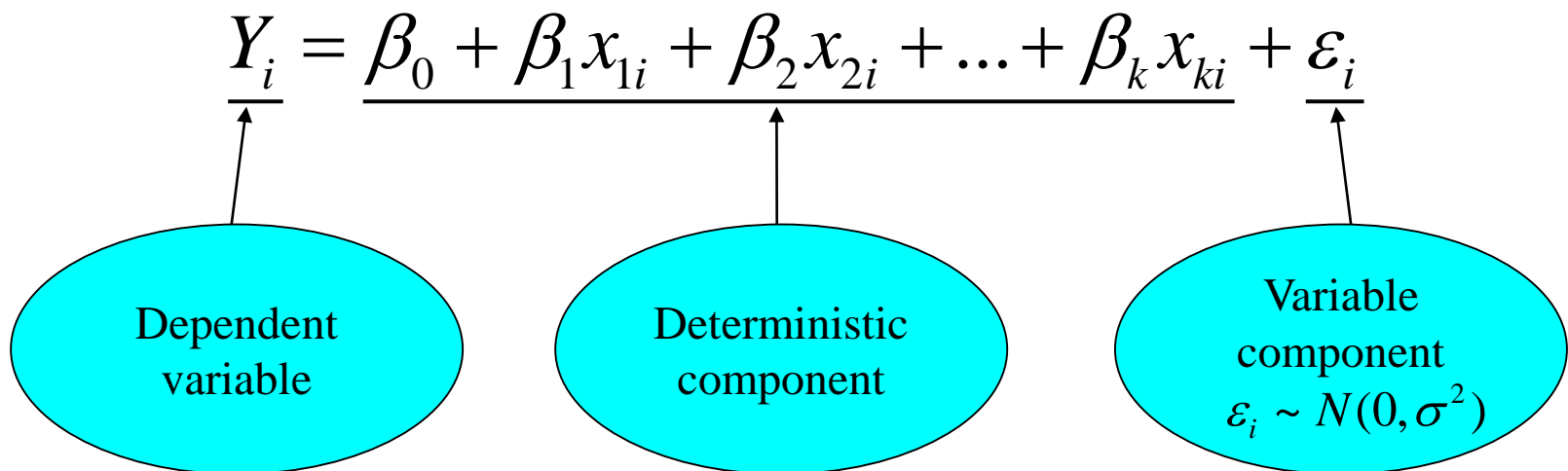


Linear Mixed-Effect Models

The standard linear model

- The standard linear model for individual i : takes the form:

$$\underbrace{Y_i}_{\text{Dependent variable}} = \underbrace{\beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \dots + \beta_k x_{ki}}_{\text{Deterministic component}} + \underbrace{\varepsilon_i}_{\text{Variable component } \varepsilon_i \sim N(0, \sigma^2)}$$


- or in matrix form: $Y = X\beta + \varepsilon$

The difference between the standard model and the mixed-effects model

- In this model all β 's are said to be fixed effects – with the only random element of the model being the error term.
- A mixed-effect model introduces additional random elements to the model.

The Linear Mixed-Effects Model

- The linear mixed-effects model for individual j in group i takes the form:

$$Y_{ij} = \beta_0 + \beta_1 x_{1,ij} + \dots + \beta_k x_{k,ij} + b_1 z_{1,ij} + \dots + b_m z_{m,ij} + \varepsilon_{ij}$$

Dependent
variable

Deterministic
component

Random effect
component group i
 $i = 1 \dots m.$

Variable
component
observation ij

- or in matrix form:

$$Y_i = X_i \beta + Z_i b + \varepsilon_i$$

The Linear Mixed-Effects Model

$$\underline{Y_{ij}} = \underline{\beta_0 + \beta_1 x_{1,ij} + \dots + \beta_k x_{k,ij}} + \underline{b_1 z_{1,ij} + \dots + b_m z_{m,ij}} + \underline{\varepsilon_{ij}}$$

- β 's are fixed effect coefficients
- b 's are random effect coefficients for groups $1 \dots m$.
- ε is residual error for observation ij

The error terms

- b_i and ε_{ij} may be correlated although for our purposes we will assume they are independent and distributed as:

$$b_i \sim N(0, \psi^2)$$

$$\varepsilon_{ij} \sim N(0, \sigma^2)$$

- R estimates the parameters for a mixed-effects model using restricted maximum likelihood (REML).

When are these models used?

- These models are generally used when multiple observations are taken from one individual; e.g. a longitudinal study or a study where the effect of multiple factors is trialed on the same individual. In this case the group i is one individual with j representing the observations from that individual.