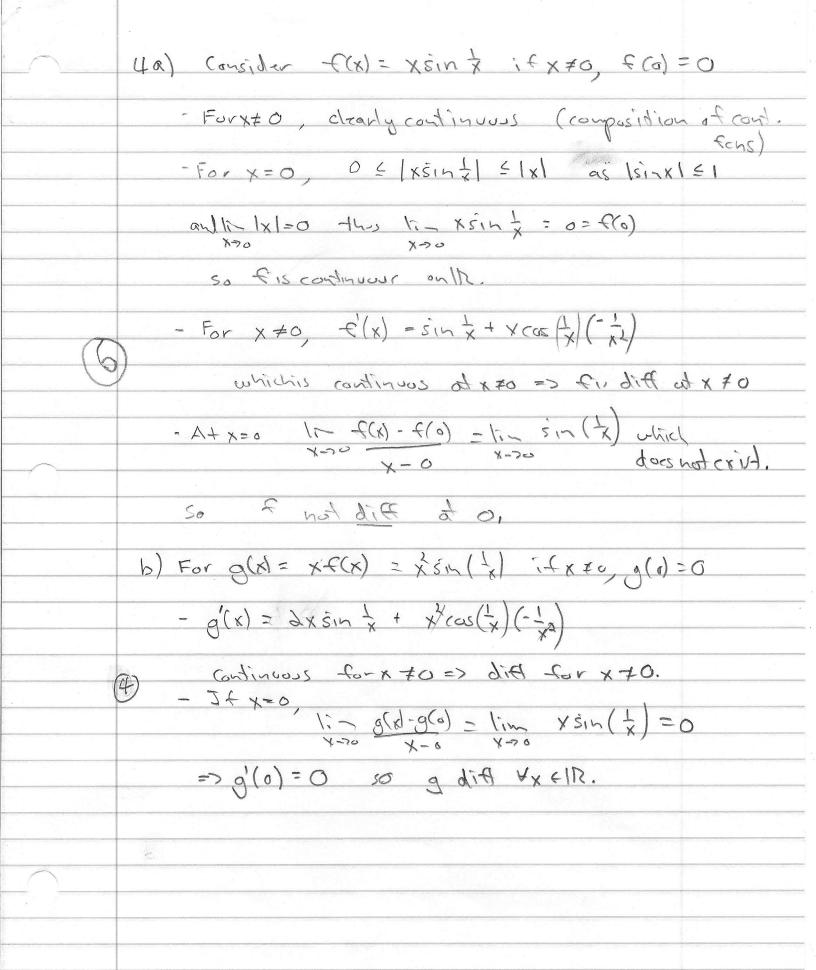
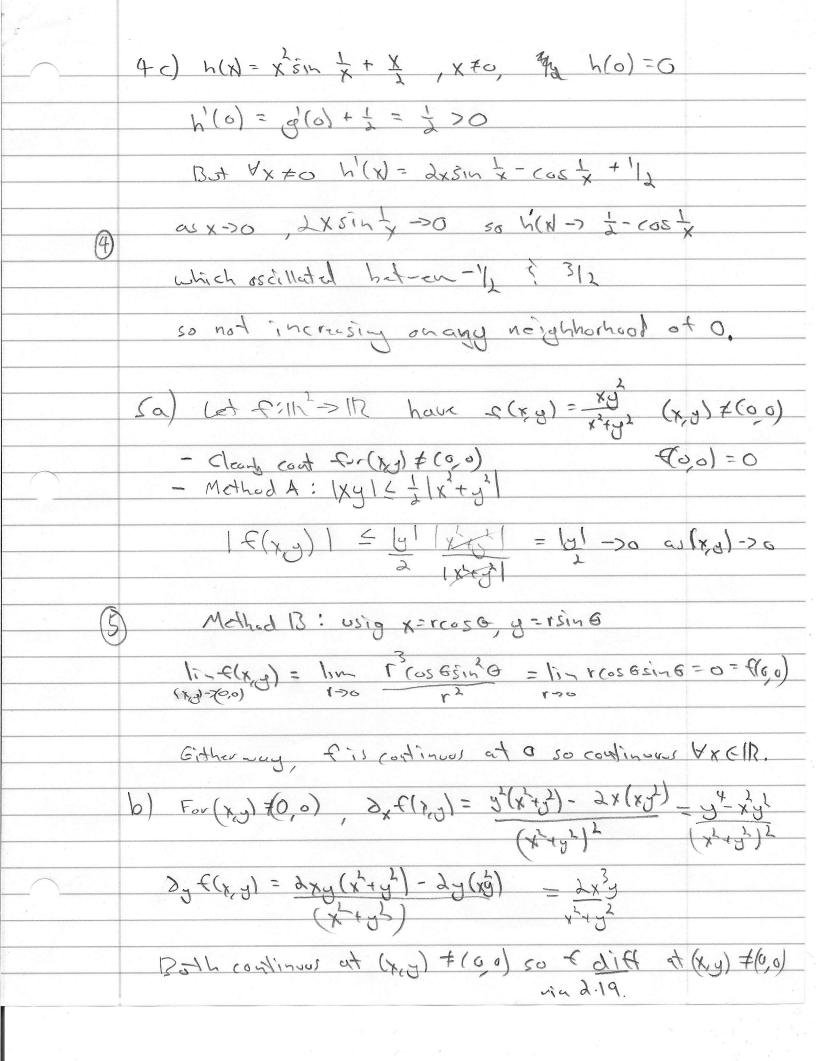
## S2 Problem Set 2 Soldions

Ia) For f(x) = Tx x Ec, 13 VIX-Tyl & (TX+Jy) intriagle => 15x-5y/1x-5y/ \(\tau\)\[\ta\]\]\[\ta\] => 15x-5y12 = 15x-5y1 = |x-y1 (B) => | Tx - 571 & 2.1x-4/1 => f(x) is Hölder (ontinuos with )= 1/2, c=1 Exercise 1 => Tx is continues b) Consider  $f(x) = \{\ln(x) \mid 0 < x \leq 1/L\}$ Suppose f(x) was Hölder Continuous forsone >>0. at x =0. 1e = C>0 so | 0 - 1 | < C| x| => 1 < |x//nx . But lin |x//nx = 0 => € ". Not Holder Continuous as property doesn't hold at x =0 However, f(x) is continued at 0 as lix lax =0 and flatis uniformly continues as it is continued on a compact internal via 1.33.

	a) Let f: [0, 0) -> IR be continuous i lim f(x) = L Let 870 exists.
	Let 870 exists.
	That is, I M 70 such that
	H(x)-L/( & when x 7, M. (If your prefer x 7M, thin use x 7 M+1 instead)
	Then $[0, \infty) = [0, M] \cup (M, \infty)$
	As [O,M] is congact, Theorem 1.33=> f(x) is
	uniformly continuous on [a,M].
	1c 4870 7870 st.  f(x) - f(y)   < & when 1x-y   < 8.
	Choose $x, y \in [0, \infty)$ . Clearly if $x, y \in [0, M]$ or $x, y \in (M, \infty)$
9	we get 1+(x)-+(y)   L & when 1x-y1 < 8.
	Suppose x = [0, M], y + (M, D), 1x-y1 < S.
	Three If(x)-f(y) =  f(x)-f(m) +  f(m)-f(y)
	4 \( \frac{\x}{2} = \xi
	Fon As 1x-M   5   x-y   ( w x, y, M >0, x = M & y )
	1 J 3 M
	Have fis uniformly continuous on Lo, s)

3) Basis: n=1: for +(x)=x, +(x+h)=x+h al E(h)=0 hence Un E(h)=0 Assure (xh) = nxn-1. Let +(x) = xh+1  $=(x+h)[x_{0}+hx_{0}]$   $=(x+h)[x_{0}+hx_{0}]$ Ed. like E(h) = 0, Aus via induction hypothems. so E(x+h) = xh+1 + hxh + nxhh + nxh-1h + (x+h) E(h) = xh+1 + (+1)xh + Ex(h) where Ex (h) = nx h-1 h + (x+h) E(h) has In E, (h) =0. · (xx) = (n+1) x Theres It follows via induction -Hilroy





At (x, y) = (00), Dx f(0,0) = 1/2 f(x,0) - f(0,0) = 1/2 0-0 = 0 dyf(0,0) = In f(0,0) - fin 0-0-0 : P((00) = (00) Now 1 - (x,y) - - (0,0) - V+(0,0) (x,y) = lim xy = lim [cos 65in 6] [ros 65in 6] [ros 65in 6] which does not exist as 6 not determind. : f not differentiable at (00) () Let u = (4, 4) he avail vector in |u|= 14, 14, = 1 Duf(0,0) = lim f(ta, ta,)-f(0,0)-lim ta,u, - 4,u, ++>0 +30 +3(1,2+u) 1 12-412 => all direction deviatives existat (0,0).