STA305/1004 – Experimental Design Midterm Test
February 12, 2014
Time allowed: 90 minutes

Name: ____

Student Number: ______________________________________

There are 4 questions. Each question is worth 25 marks.

Instructions: Complete all questions in pen. Any questions completed in pencil will not be eligible to be remarked even if there was a marking error.

Aids allowed: You are allowed to bring in: one 8.5'x11' sheet with writing; and a calculator

1. A clinical scientist wishes to investigate if the mean survival time (months) of a new treatment (N) for a certain cancer is different compared to the standard treatment (S).

   Six patients with the disease were randomly assigned to the treatments by flipping a fair coin (a coin toss resulting in a head received treatment N). The Survival times are given in the table below:

Patient	Treatment	Survival time (months)
1	N	10
2	N	8
3	N	16
4	S	9
5	S	8
6	N	15

An analysis of the data using R is below

```
> x <- c(10,8,16,9,8,15)
> index <-combn(1:6,4) #Generates all combinations of the elements of 1:6 4 at a time
> index #print index
     [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10] [,11] [,12] [,13] [,14] [,15]
[1,]    1    1    1    1    1    1    1    1    1     1     2     2     2     2     3
[2,]    2    2    2    2    2    2    3    3    3     4     3     3     3     4     4
[3,]    3    3    3    4    4    5    4    4    5     5     4     4     5     5     5
[4,]    4    5    6    5    6    6    5    6    6     6     5     6     6     6     6
> N <- 15
> res <- numeric(N) # store the results
> for (i in 1:N)
+ {
+   print(x[index[,i]])
+   x[-index[,i]]
+   res[i] <- mean(x[index[,i]])-mean(x[-index[,i]])
+ }

#printout of x[index[,i]]
[1] 10  8 16  9
[1] 10  8 16  8
[1] 10  8 16 15
[1] 10  8  9  8
[1] 10  8  9 15
[1] 10  8  8 15
[1] 10 16  9  8
[1] 10 16  9 15
[1] 10 16  8 15
[1] 10  9  8 15
[1]  8 16  9  8
[1]  8 16  9 15
[1]  8 16  8 15
[1]  8  9  8 15
[1] 16  9  8 15


> res #print results
```

```
 [1] -0.75 -1.50  3.75 -6.75 -1.50 -2.25 -0.75  4.50  3.75 -1.50 -2.25  3.00  2.25 -3.00
3.00
> trtN <- c(10,8,16,15)
> trtS <- c(9,8)
> mean(trtN)
[1] 12.25
> mean(trtS)
[1] 8.5
```

(25 marks, 5 marks each)

a) State the null and alternative hypotheses of this experiment.

b) What is the probability that a patient will receive the new treatment? Does the observed data make you suspicious that the coin used was not fair? Explain.

c) List all the values of the randomization distribution of the mean difference.

d) What is the empirical cumulative distribution function of the randomization distribution?

e) Is there evidence that the difference in means is due to random chance? If you didn't find evidence state two possible reasons?

2. The following data are from a study on the pharmacological effects of nalbuphione (A) compared to morphine (B). The purpose of the study was to compare the effects of the two drugs on pupil diameter.

   Pupil diameter (mm) was obtained from 12 subjects after receiving nalbuphione in one eye and morphine in the other eye. Nalbuphione was administered to the left or right eye of each subject based on the toss of a fair coin. (**25 marks, 5 marks each)**

   Some of the raw data is in the following table.

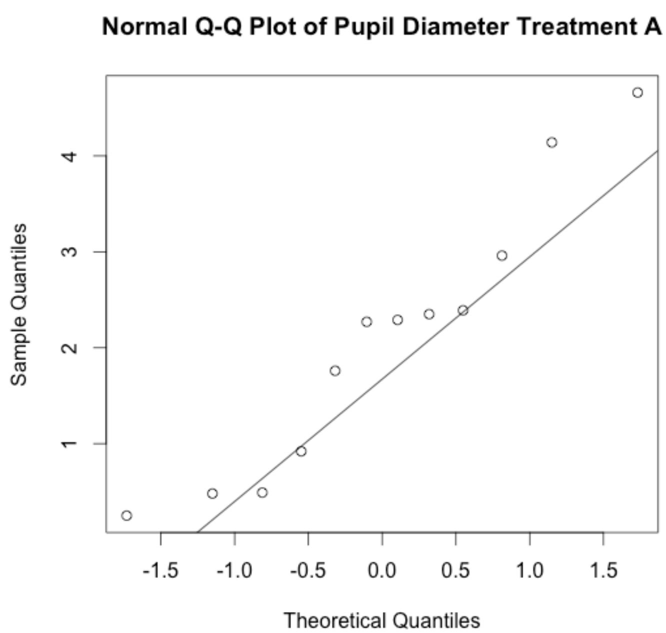
Subject	Treatment	Pupil diameter (mm)
1	A	0.49
2	A	2.40
3	A	1.76
…	…	…
12	A	2.29
1	B	0.50
2	B	1.91
3	B	0.99
…	…	…
12	B	1.96

An analysis of the data using R is below.

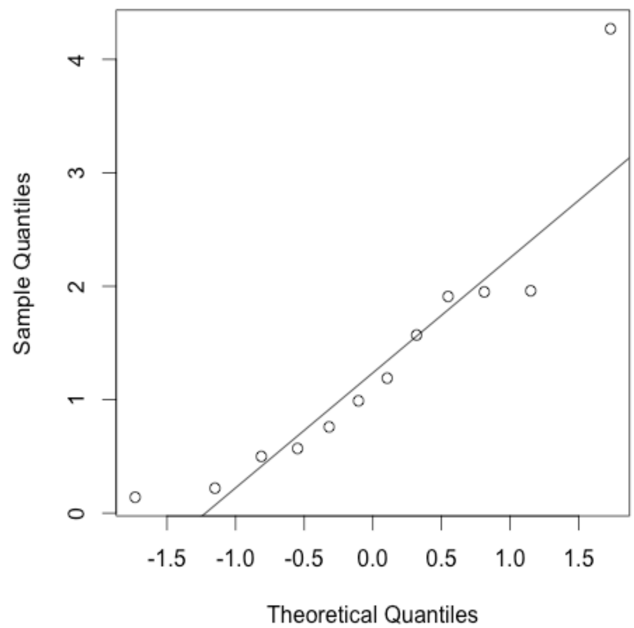
```
#Pupil Diameter Treatment A
> yA
 [1] 0.49 2.39 1.76 4.66 2.35 2.96 0.25 4.14 0.48 0.92 2.27 2.29

#Pupil Diameter Treatment B
> yB
 [1] 0.50 1.91 0.99 0.76 0.22 1.57 0.57 4.27 0.14 1.95 1.19 1.96

> mean(yA)
[1] 2.080944
> mean(yB)
[1] 1.334132
> sd(yA)
[1] 1.407883
> sd(yB)
[1] 1.135004
```

```
> t.test(yA,yB,var.equal=T)

      Two Sample t-test

data:  yA and yB
t = 1.4306, df = 22, p-value = 0.1666
alternative hypothesis: true difference in means is not equal to
0
95 percent confidence interval:
 -0.3358424  1.8294666
sample estimates:
mean of x mean of y
 2.080944  1.334132

> t.test(yA,yB,var.equal=T,paired=T)

      Paired t-test

data:  yA and yB
t = 1.9991, df = 11, p-value = 0.07092
alternative hypothesis: true difference in means is not equal to
0
95 percent confidence interval:
 -0.07543818  1.56906237
sample estimates:
mean of the differences
              0.7468121

> sd.ABpaired <- sd(yA-yB)
> sd.ABpaired # print SD of paired difference
[1] 1.294129
>
power.t.test(delta=0.5,sd=sd.ABpaired,power=0.8,type="paired",alt
ernative="two.sided")

     Paired t test power calculation

              n = 54.53218
          delta = 0.5
             sd = 1.294129
      sig.level = 0.05
          power = 0.8
    alternative = two.sided

NOTE: n is number of *pairs*, sd is std.dev. of *differences*
within pairs
```

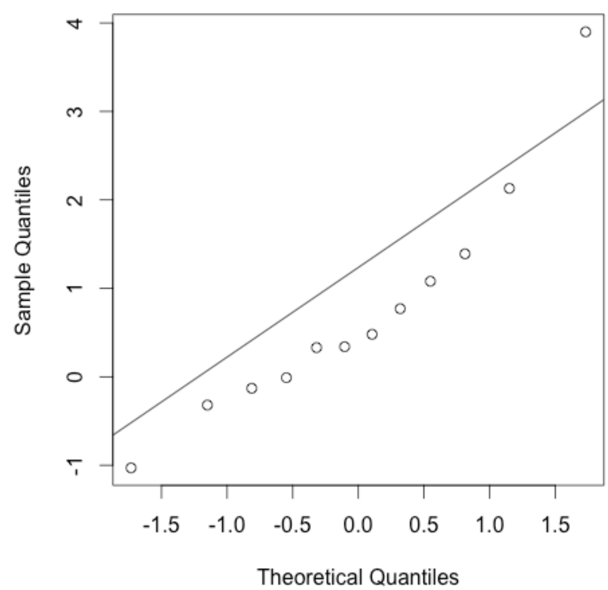
```
> sd.AB <- sqrt(var(yA)/12+var(yB)/12)
> sd.AB #print SD of yA-yB
[1] 0.5220447
>
power.t.test(delta=0.5,sd=sd.AB,power=0.8,type="two.sample",alter
native="two.sided")

     Two-sample t test power calculation

              n = 18.1249
          delta = 0.5
             sd = 0.5220447
      sig.level = 0.05
          power = 0.8
    alternative = two.sided

NOTE: n is number in *each* group
```



Normal Q-Q Plot of Pupil Diameter Treatment B



Normal Q-Q Plot of the Difference in Pupil Diamteter Treatment A-B



a) Describe the type of experimental design used in this experiment. What are the experimental units? What are the treatments?

b) Is there statistical evidence at the 10% significance level that the treatments are different? If there is evidence of a difference then which treatment resulted in a larger mean pupil diameter?

c) For your answer in part b) what statistical assumptions does your answer rely on?

d) Describe how you would conduct a randomization test to evaluate the evidence against the null hypothesis. Would you expect that the results would be the same as the t-test?

e) Suppose that another scientist wanted to design another similar study that will have at least 80% power to detect a difference of 0.5mm at the 5% significance level. How many patients should be assigned to each treatment group? Would you expect the number of patients required for 80% power to increase or decrease if the scientist changed her mind and instead wanted to detect a difference of 0.4mm? Explain.

3. Consider the ANOVA model $y_{it} = \mu + \tau_t + \epsilon_{it}$, where $i = 1, \ldots, n$, $t = 1, \ldots, v$, $\varepsilon_{it} \overset{iid}{\sim} N(0, \sigma^2)$, and $\sum_{t=1}^{v} \tau_t = 0$. **(25 marks)**

   a) Briefly explain what the parameters μ, τ_t, and ϵ_{it} represent. (3 marks)

   b) What is the ANOVA model if all the treatment means are equal? (2 marks)

c) Find the least squares estimates of τ_t, $t = 1, \ldots, v$. (10 marks)

d) Show that the mean square for error $MSE = \frac{\sum_{i=1}^{n}\sum_{t=1}^{v}\left(y_{it} - \overline{y}_{i.}\right)^2}{N-v}$, $N = nv$ is an unbiased estimator of σ^2 when all the treatment means are equal. (10 marks)

4. An experiment was conducted in order to determine whether pushing a certain pedestrian light button had an effect on how long a person had to wait before the pedestrian light showed "walk". The treatment factor of interest was the number of pushes of the button, and 13 observations were taken for each of 0, 1, 2, and, 3 pushes of the button. The waiting times for the "walk" sign for the first 32 observations in the order collected are shown below where the levels of the treatment factors are coded 0, 1, 2, 3.
(25 marks)

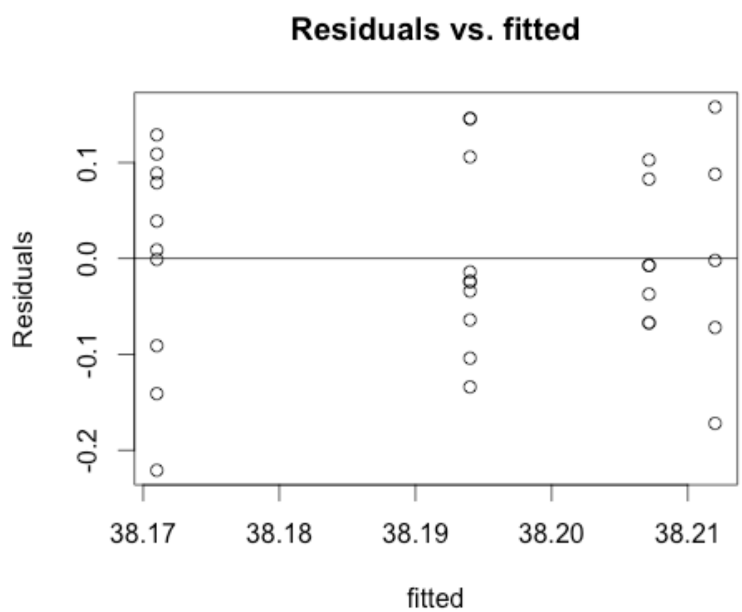
Times (in seconds) for the "walk" sign to appear in the pedestrian light experiment

Number of pushes			
0	1	2	3
38.14	38.28	38.17	38.14
38.20	38.17	38.13	38.30
38.31	38.08	38.16	38.21
38.14	38.25	38.30	38.04
38.29	38.18	38.34	38.37
38.17	38.03	38.34	
38.20	37.95	38.17	
	38.26	38.18	
	38.30	38.09	
	38.21	38.06	

R output

```
>
sapply(split(pedestrian.light$TIME,as.factor(pedestrian.light$PUSHES)),
mean)
       0        1        2        3
38.20714 38.17100 38.19400 38.21200
```

```
> ped.aov <- lm(TIME~as.factor(PUSHES),data=pedestrian.light)
> summary(ped.aov)

Call:
lm(formula = TIME ~ as.factor(PUSHES), data = pedestrian.light)

Residuals:
      Min        1Q    Median        3Q       Max
-0.221000 -0.067143 -0.007143  0.088250  0.158000

Coefficients:
                    Estimate Std. Error t value Pr(>|t|)
(Intercept)        38.207143   0.039509 967.042   <2e-16 ***
as.factor(PUSHES)1 -0.036143   0.051514  -0.702    0.489
as.factor(PUSHES)2 -0.013143   0.051514  -0.255    0.800
as.factor(PUSHES)3  0.004857   0.061208   0.079    0.937
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

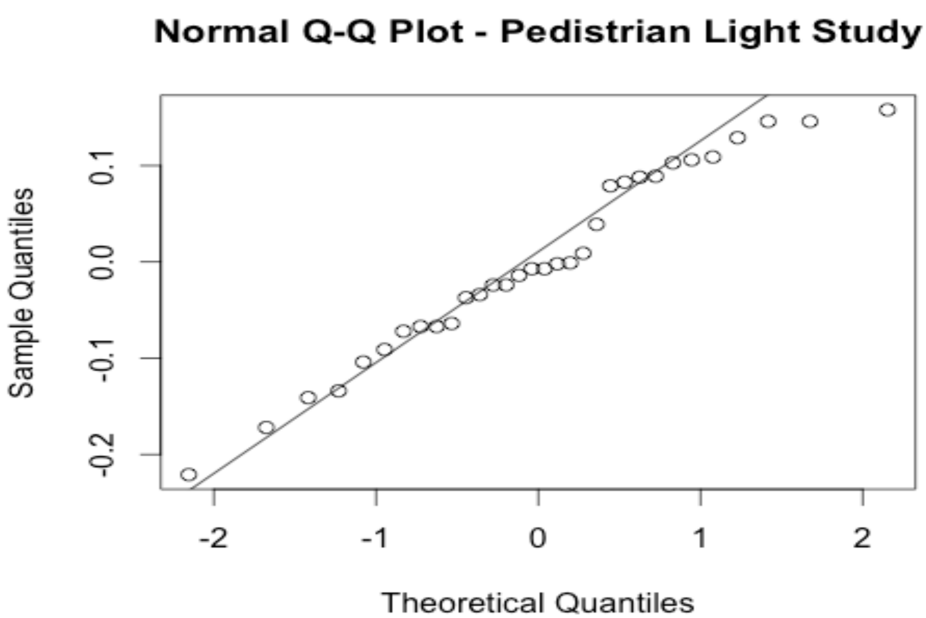


```
> TukeyHSD(aov(TIME~as.factor(PUSHES),data=pedestrian.light))
  Tukey multiple comparisons of means
    95% family-wise confidence level

Fit: aov(formula = TIME ~ as.factor(PUSHES), data = pedestrian.light)

$`as.factor(PUSHES)`
            diff        lwr       upr     p adj
1-0 -0.036142857 -0.1767916 0.1045059 0.8955744
2-0 -0.013142857 -0.1537916 0.1275059 0.9940396
3-0  0.004857143 -0.1622585 0.1719728 0.9998162
2-1  0.023000000 -0.1046367 0.1506367 0.9602303
3-1  0.041000000 -0.1153224 0.1973224 0.8898585
3-2  0.018000000 -0.1383224 0.1743224 0.9890012

>
pairwise.t.test(pedestrian.light$TIME,as.factor(pedestrian.light$PUSHES
),p.adj="bonf")

      Pairwise comparisons using t tests with pooled SD

data:  pedestrian.light$TIME and as.factor(pedestrian.light$PUSHES)

  0 1 2
1 1 - -
2 1 1 -
3 1 1 1

P value adjustment method: bonferroni

> #0 push vs. 1 push
>
t.test(pedestrian.light$TIME[push=="0"],pedestrian.light$TIME[push=="1"
],var.equal=T)

      Two Sample t-test

data:  pedestrian.light$TIME[push == "0"] and
pedestrian.light$TIME[push == "1"]
t = 0.7353, df = 15, p-value = 0.4735
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -0.06862718  0.14091289
sample estimates:
mean of x mean of y
 38.20714  38.17100
```

```
> #0 push vs. 2 push
>
t.test(pedestrian.light$TIME[push=="0"],pedestrian.light$TIME[push=="2"
],var.equal=T)

      Two Sample t-test

data:  pedestrian.light$TIME[push == "0"] and
pedestrian.light$TIME[push == "2"]
t = 0.3017, df = 15, p-value = 0.767
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -0.07969752  0.10598324
sample estimates:
mean of x mean of y
 38.20714  38.19400

> #0 push vs. 3 push
>
t.test(pedestrian.light$TIME[push=="0"],pedestrian.light$TIME[push=="3"
],var.equal=T)

      Two Sample t-test

data:  pedestrian.light$TIME[push == "0"] and
pedestrian.light$TIME[push == "3"]
t = -0.0849, df = 10, p-value = 0.934
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -0.1323028  0.1225885
sample estimates:
mean of x mean of y
 38.20714  38.21200
```

```
> #1 push vs. 2 push
>
t.test(pedestrian.light$TIME[push=="1"],pedestrian.light$TIME[push=="2"
],var.equal=T)

      Two Sample t-test

data:  pedestrian.light$TIME[push == "1"] and
pedestrian.light$TIME[push == "2"]
t = -0.4755, df = 18, p-value = 0.6401
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -0.12461216  0.07861216
sample estimates:
mean of x mean of y
   38.171    38.194

> #1 push vs. 3 push
>
t.test(pedestrian.light$TIME[push=="1"],pedestrian.light$TIME[push=="3"
],var.equal=T)


      Two Sample t-test

data:  pedestrian.light$TIME[push == "1"] and
pedestrian.light$TIME[push == "3"]
t = -0.6212, df = 13, p-value = 0.5452
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -0.1835881  0.1015881
sample estimates:
mean of x mean of y
   38.171    38.212
```

```
> #2 push vs. 3 push
>
t.test(pedestrian.light$TIME[push=="2"],pedestrian.light$TIME[push=="3"
],var.equal=T)

      Two Sample t-test

data:  pedestrian.light$TIME[push == "2"] and
pedestrian.light$TIME[push == "3"]
t = -0.2993, df = 13, p-value = 0.7694
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -0.1479209  0.1119209
sample estimates:
mean of x mean of y
   38.194    38.212
```



a) State the null and alternative hypotheses of the F test. Fill in the six values (??) of the ANOVA table below. Is there evidence against the null hypothesis (the appropriate F critical value – upper 5% point is 2.95)? **(10 marks)**

```
> anova(ped.aov)
Analysis of Variance Table

Response: TIME
                  Df   Sum Sq   Mean Sq F value Pr(>F)
as.factor(PUSHES) ?? 0.008047      ??     ??      ??
Residuals         ?? 0.305953      ??
```

b) State the assumptions of the F test. Are the assumptions satisfied? **(5 marks)**

c) If all possible pairs of mean times are compared using a type I error rate of 5% then are all pairwise confidence intervals 95% confidence intervals? Explain. **(5 marks)**

d) Does this experiment support the following statement: "pushing the button has a small effect on the time a person has to wait for the "walk" sign to appear". State if you agree or disagree and justify your answer. **(5 marks)**