UNIVERSITY OF TORONTO
Faculty of Arts and Science

EXAMINATION APRIL 2013

PHL 245 H1S
L0101 - Niko Scharer

Duration - 3 hours

Examination Aid: Sheet with rules (provided)

Last Name ______________________________

First Name ______________________________

Student Number__________________________

Answer **all** questions on the exam paper.

Use the supplied examination booklet for rough work OR if you need further space.

The exam consists of fourteen (14) pages. Pages 2-12 have questions on them.

The final two pages (13-14) are an aid sheet and may be detached from the rest of the exam.

1. A set of sentences {P, Q, R} is logically inconsistent. (In this question, P, Q and R can represent any sentence in Sentential Logic, and are not necessarily atomic sentences.) Which of the following arguments must be valid? Circle the correct answer. (3 pts.)

a)
P
$R \wedge T$
―――
$\therefore \sim Q$

Valid
Not necessarily valid

b)
$P \wedge S$
$\sim(Q \rightarrow \sim R)$
―――
$\therefore T$

Valid
Not necessarily valid.

c)
$\sim(P \vee S)$
$\sim Q$
―――
$\therefore R$

Valid
Not necessarily valid

d)
$P \leftrightarrow \sim S$
$Q \vee R$
―――
$\therefore \sim(P \wedge Q \wedge R)$

Valid
Not necessarily valid.

2. Table 1:

P	Q	R	ψ
T	T	T	F
T	T	F	F
T	F	T	T
T	F	F	F
F	T	T	F
F	T	F	F
F	F	T	T
F	F	F	F

a) Suppose Table 1 is the full truth-table for sentence ψ.
Provide one symbolic sentence that is logically equivalent to ψ. (1 pt.)

b) Provide a truth-value assignment which demonstrates that: $(\sim P \wedge Q) \rightarrow (R \leftrightarrow \psi)$ is not a tautology (where ψ stands for any sentence that has the full truth-table shown in Table 1). (2 pts.)

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3. Provide an English language interpretation that shows that the following set of three sentences is logically consistent. Your interpretation should specify the universe of discourse and a symbolization scheme. (4 pts.)

$\exists x(Gx \wedge \forall y{\sim}L(xy))$. $\forall x\exists y(Hy \wedge L(yx))$. ${\sim}\exists x\exists y(L(xy) \wedge L(yx))$.

4. Explain why the following sentence is logically true. (4 pts.)

$$\exists x\exists y({\sim}Bx \wedge G(xy)) \leftrightarrow {\sim}\forall z(\exists yG(zy) \rightarrow Bz)$$

5. Use this symbolization scheme to symbolize the following sentences: (36 pts. total)

A^1: *a* is a song.	B^1: *a* is popular.	C^1: *a* is a time	D^1: *a* is difficult to play.
G^1: *a* is a game.	H^1: *a* is a person.	J^2: *a* downloads *b*.	K^2: *a* can win *b*.
L^2: *a* likes *b*.	M^2: *a* plays *b*.	N^2: *a* is newer than *b*.	F^3: *a* sings *b* at *c*.
a^0: Amanda	b^1: the brother of *a*.		

a) Some songs and games are difficult to play. (2 pts.)

b) Although not all songs are popular, Amanda likes every popular song. (3 pts.)

c) Among games, only the ones that aren't difficult to play are popular. (3 pts.)

d) For a game to be popular, it is necessary that it is liked by people who cannot win it. (4 pts.)

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5 continued. Use this symbolization scheme to symbolize the following sentences:

A^1: *a* is a song.	B^1: *a* is popular.	C^1: *a* is a time	D^1: *a* is difficult to play.
G^1: *a* is a game.	H^1: *a* is a person.	J^2: *a* downloads *b*.	K^2: *a* can win *b*.
L^2: *a* likes *b*.	M^2: *a* plays *b*.	N^2: *a* is newer than *b*.	F^3: *a* sings *b* at *c*.
a^0: Amanda	b^1: the brother of *a*.		

e) On the assumption that not every song is liked by everyone who downloads it, there are songs that never get sung by anybody. (4 pts.)

f) People who only play games that they can win don't dislike games unless they are games that nobody likes. (4 pts.)

g) Only Amanda likes exactly those games that she downloads. (4 pts.)

5 continued. Use this symbolization scheme to symbolize the following sentences:

A^1: *a* is a song.	B^1: *a* is popular.	C^1: *a* is a time	D^1: *a* is difficult to play.
G^1: *a* is a game.	H^1: *a* is a person.	J^2: *a* downloads *b*.	K^2: *a* can win *b*.
L^2: *a* likes *b*.	M^2: *a* plays *b*.	N^2: *a* is newer than *b*.	F^3: *a* sings *b* at *c*.
a^0: Amanda	b^1: the brother of *a*.		

h) Neither Amanda nor Amanda's brother downloads the one song that everybody likes. (4 pts.)

i) Using the symbolization scheme above, provide an idiomatic English sentence that expresses: (4 pts.)

$$\exists x(Gx \wedge M(ax) \wedge \forall y(Gy \wedge M(ay) \wedge {\sim}x{=}y \rightarrow N(xy)) \wedge {\sim}\exists z(Hz \wedge K(zx)))$$

j) Using the symbolization scheme above, symbolize the following ambiguous sentence **three** logically distinct ways. For each, provide an English sentence that explains exactly what the symbolized sentence means: (4 pts.)

Someone is always singing a song.

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6. Provide a derivation that shows the following theorem is valid **using only the 10 basic rules from SL** (R, DN, MP, MT, ADJ, S, ADD, MTP, BC, CB) (9 pts.)

$$\therefore ((P \rightarrow Q) \wedge {\sim}(R \rightarrow S)) \rightarrow ((Q \vee {\sim}P) \wedge (W \vee {\sim}S))$$

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7. Provide a derivation that shows that this is a valid argument **using only the 10 basic rules from SL** (R, DN, MP, MT, ADJ, S, ADD, MTP, BC, CB) **and the 3 basic rules from PL** (UI, EG, EI) (9 pts.)

$\exists y(Hy \wedge \forall zL(yz)) \wedge \forall y{\sim}By.$ $\exists wCw \rightarrow \forall y({\sim}Hy \leftrightarrow Ay).$

$\therefore \forall x((Bx \vee Cx) \rightarrow \exists y({\sim}Ay \wedge L(yx)))$

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8. Provide a derivation to show that this is a valid argument (use any rules). (9 pts.):

$\exists x B(xa(x)) \rightarrow \forall x \exists y \sim(Fx \rightarrow Gy)$. $\quad \therefore \exists x \forall y B(a(x)a(y)) \rightarrow \sim\forall z(\sim Fz \vee Gz)$

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9. Show that the following is a valid argument (use any rules). (9 pts.):

$\forall x \forall y(A(xy) \rightarrow \sim L(yx)) \rightarrow \forall x \exists y \forall z F(xyz) \wedge \sim\exists y \sim H(yy)$. $\forall z(\exists w A(wz) \rightarrow B(zz))$.

$\exists x \forall y(F(xyy) \rightarrow \forall z \sim H(xz))$. $\therefore \sim\forall x(B(xx) \rightarrow \sim\exists z L(xz))$

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10. Use an abstract finite model to demonstrate that the following argument is invalid: (8 pts.)

i) provide a truth-functional expansion (to two individuals) for each sentence in this argument.

ii) define a finite model with a universe of two individuals that shows that the argument is invalid.

$\exists x(Ax \wedge \forall y H(xy))$. $\forall x \exists y(Bx \rightarrow H(xy))$. $\therefore \sim\forall x(Bx \rightarrow \sim H(xx))$

11. When you use UI in a derivation, you can instantiate a universally quantified sentence to any individual term, with one exception. What is that exception? Briefly explain why it is not allowed. Provide an example that supports your answer. (3 pts.)

12. Consider this sentence: $\exists x \forall y \sim x=y$ (3 pts.)

a) What does the sentence mean? (Translate the symbolic sentence into idiomatic English.)

b) Is the sentence is logically true, logically false or logically contingent? Explain.

=100 pts. total.

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AID SHEET: DERIVATION RULES (SEE BOTH SIDES)

Derivation Types:

Direct Derivation (DD)

Conditional Derivation (CD)

Indirect Derivation (ID)

Universal Derivation (UD) Restriction: the instantiating term cannot occur unbound in any previous line.

Basic Rules for Sentential Operators:

Modus Ponens (MP)

$$\frac{(\phi \to \psi) \quad \phi}{\psi}$$

Modus Tollens (MT)

$$\frac{(\phi \to \psi) \quad \sim\psi}{\sim\phi}$$

Double Negation (DN)

$$\frac{\phi}{\sim\sim\phi} \qquad \frac{\sim\sim\phi}{\phi}$$

Repetition (R)

$$\frac{\phi}{\phi}$$

Simplification (S)

$$\frac{\phi \wedge \psi}{\phi} \qquad \frac{\phi \wedge \psi}{\psi}$$

Adjunction (ADJ)

$$\frac{\phi \quad \psi}{\phi \wedge \psi}$$

Addition (ADD)

$$\frac{\phi}{\phi \vee \psi} \qquad \frac{\psi}{\phi \vee \psi}$$

Modus Tollendo Ponens (MTP)

$$\frac{\phi \vee \psi \quad \sim\phi}{\psi} \qquad \frac{\phi \vee \psi \quad \sim\psi}{\phi}$$

Biconditional-Conditional (BC)

$$\frac{\phi \leftrightarrow \psi}{\phi \to \psi} \qquad \frac{\phi \leftrightarrow \psi}{\psi \to \phi}$$

Conditional-Biconditional (CB)

$$\frac{\phi \to \psi \quad \psi \to \phi}{\phi \leftrightarrow \psi}$$

Derived Rules for Sentential Operators:

Negation of Conditional (NC)

$$\frac{\sim(\phi \rightarrow \psi)}{\phi \wedge \sim\psi} \qquad \frac{\phi \wedge \sim\psi}{\sim(\phi \rightarrow \psi)}$$

Conditional as Disjunction (CDJ)

$$\frac{\phi \rightarrow \psi}{\sim\phi \vee \psi} \qquad \frac{\sim\phi \vee \psi}{\phi \rightarrow \psi} \qquad \frac{\sim\phi \rightarrow \psi}{\phi \vee \psi} \qquad \frac{\phi \vee \psi}{\sim\phi \rightarrow \psi}$$

Separation of Cases (SC)

$$\begin{array}{l} \phi \vee \psi \\ \phi \rightarrow \chi \\ \psi \rightarrow \chi \\ \hline \chi \end{array} \qquad \begin{array}{l} \phi \rightarrow \chi \\ \sim\phi \rightarrow \chi \\ \hline \chi \end{array}$$

Negation of Biconditional (NB)

$$\frac{\sim(\phi \leftrightarrow \psi)}{\phi \leftrightarrow \sim\psi} \qquad \frac{\phi \leftrightarrow \sim\psi}{\sim(\phi \leftrightarrow \psi)}$$

De Morgan's (DM)

$$\frac{\sim(\phi \vee \psi)}{\sim\phi \wedge \sim\psi} \qquad \frac{\sim\phi \wedge \sim\psi}{\sim(\phi \vee \psi)} \qquad \frac{\sim(\phi \wedge \psi)}{\sim\phi \vee \sim\psi} \qquad \frac{\sim\phi \vee \sim\psi}{\sim(\phi \wedge \psi)}$$

$$\frac{\sim(\sim\phi \vee \sim\psi)}{\phi \wedge \psi} \qquad \frac{\phi \wedge \psi}{\sim(\sim\phi \vee \sim\psi)} \qquad \frac{\sim(\sim\phi \wedge \sim\psi)}{\phi \vee \psi} \qquad \frac{\phi \vee \psi}{\sim(\sim\phi \wedge \sim\psi)}$$

Derivation Rules for Predicate Logic:

Existential Generalization (EG)

$$\frac{\phi_\zeta}{\exists\alpha\phi_\alpha}$$

Universal Instantiation (UI)

$$\frac{\forall\alpha\phi_\alpha}{\phi_\zeta}$$

Restriction: ζ does not occur as a bound variable in ϕ_α

Existential Instantiation (EI)

$$\frac{\exists\alpha\phi_\alpha}{\phi_\zeta}$$

Restriction: ζ does not occur in any previous line or premise.

Quantifier Negation (QN)

$$\frac{\sim\forall\alpha\phi}{\exists\alpha\sim\phi} \qquad \frac{\sim\exists\alpha\,\phi}{\forall\alpha\sim\phi}$$

$$\frac{\exists\alpha\sim\phi}{\sim\forall\alpha\phi} \qquad \frac{\forall\alpha\sim\phi}{\sim\exists\alpha\,\phi}$$