Test **STA257** **Time**:3hrs

Instructions: The test is out of 100 and each question is worth 7. Your maximum grade is 100. See the end for some useful information. Please, at most 1 question/page in your booklets. No aids allowed.

1. Let Y be $binomial(16, 1/4)$. Evaluate $Var(Y)$. Note: You must show your work.
2. Let X be a $uniform((0,1))$ rv. Set $Y = -3log(X)$. Calculate the pdf of Y.
3. Four cards are selected from a deck of 52 cards. Calculate the probability that they are all spades. Note: There are 13 spades in a deck of 52 cards.
4. (4 marks) Let $Y \geq 0$ and suppose $E(Y) = 0$. Show $P(Y = 0) = 1$. (3 marks) Use this to conclude, for a rv X with mean 2 and$SD(X) = 0$, that $X \stackrel{wp1}{=} 2$.
5. Let A and B be independent events. Show that A^c and B^c are also independent.
6. Let $P(A) = P(B) = P(C) = 1$. Show $P(ABC) = 1$.
7. Let X_1, X_2, X_3 be independent Poisson rv's with means $1, 2, 3$, respectively. Show that $X_1 + X_2 + X_3 \sim Poisson(6)$.
8. Let $X \sim Poisson(2)$ be independent of $Y \sim Poisson(4)$. Set $W = X + Y$. Calculate $P(X = k|W = 2)$ for $k = 0, 1, 2$.
9. Let $X_1 \sim binomial(10, p)$, $X_2 \sim binomial(4, p)$, X_3 be independent rv's.. If $X_1 + X_2 + X_3 \sim binomial(20, p)$ show $X_3 \sim binomial(6, p)$.
10. Toss a fair coin. If H obtains you select a chip from Hat#1. Otherwise you select one from Hat#2. Hat#1 contains 3 red chips and 4 black chips while Hat#2 contains 5 reds and 2 black chips. Let A={a red chip is selected}. Calculate $P(H|A)$.
11. Let $X \sim geometric(1/2)$. Calculate $P(X > 1)$ and $Var(X)$.
12. Let $Z_1, Z_2, \ldots$ be *iid* $Bernoulli(1/2)$ and let $S_n = Z_1 + \cdots + Z_n$. Let T denote the smallest n such that $S_n = 3$. Calculate $Var(T)$.
13. A rv X has pgf given by $G(s) = E(s^X) = .1s + .5s^4 + .4s^{25}$. Calculate $E(\sqrt{X})$.
14. Four people each roll a fair 6 sided die. Calculate P(at least two of the dice show the same number of dots).
15. Let X be a random variable with range the positive integers. Show $E(X) = \sum_{k=0}^{\infty} P(X > k)$.

Information

$X = c$, $wp1$ or $X \overset{wp1}{=} c$ both mean $P(X = c) = 1$.

rv=random variable, pdf=probability density function, $SD(X)$ refers to the standard deviation of X, iid means independent with the same distribution, pgf=probability generating function, A^c refers to the complement of A

A $Bernoulli(p)$ rv can only take on 1 or 0 with probabilities p and $q = 1-p$, respectively.

The $geometric(p)$ probabilities are $q^{k-1}p, k = 1, 2, ...$

$1 + x + x^2 + \cdots = 1/(1-x) for |x| < 1$

The $Poisson(\lambda)$ probabilities are $e^{-\lambda}\lambda^k/k!$

The $multinomial(N; p_1, \ldots, p_k)$ probabilities are $\frac{N!}{(i_1!)...(i_k!)} p_1^{i_1} \cdots p_k^{i_k}, i_1 + \cdots + i_k = N$. Here $p_1 + \cdots + p_k = 1$. $k = 2$ yields the binomial which may also be thought of as a sum of k iid $Bernoulli(p)$ rv's.

A $uniform((0,1))$ rv has pdf $f(x) = 1$ for $0 < x < 1$ and is 0 otherwise.

The indicator rv of an event A is denoted by I_A or $I(A)$. This is a function from the sample space to rhe reals with range $\{0, 1\}$.

A sequence A_n, $n = 0, 1, \ldots$ is said to be increasing if $A_1 \subset A_2 \subset \cdots$ and is decreasing if $A_1 \supset A_2 \supset \cdots$.

We say $A_n \to A$ if $I(A_n) \to I(A)$. In the increasing case we write $A_n \uparrow A$. In the decreasing case we write $A_n \downarrow A$.