§3.5 – Revised Simplex Method

– Review of Chapter 3, based on the examples in the website (1 or 2 lec)

Last lecture: 3 Dec.

Eg. We will use the revised simplex method to solve the problem:

Maximize $z = x_1 + 2x_2 - x_3$ s.t.

$$x_1 + x_2 + x_3 \le 4$$
$$-x_1 + 2x_2 - 2x_3 \le 6$$
$$2x_1 + x_2 \le 5$$

$$x_1 \ge 0,\ x_2 \ge 0,\ x_3 \ge 0$$

In canonical form (x_4, x_5, x_6 are slacks)

Maximize $z = x_1 + 2x_2 - x_3$ s.t.

$$x_1 + x_2 + x_3 + x_4 = 4$$
$$-x_1 + 2x_2 - 2x_3 + x_5 = 6$$
$$2x_1 + x_2 + x_6 = 5$$

$$x_1 \ge 0, x_2 \ge 0, x_3 \ge 0, x_4 \ge 0, x_5 \ge 0, x_6 \ge 0$$

The basic data is preserved until the problem is solved.

$$A = \begin{array}{c} \begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ -1 & 2 & -2 & 0 & 1 & 0 \\ 2 & 1 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array} \qquad b = \begin{bmatrix} 4 \\ 6 \\ 5 \end{bmatrix}$$

$$c^T = [1 \ \ 2 \ \ -1 \ \ 0 \ \ 0 \ \ 0]$$

Tableau ① has basic variables $\{x_4, x_5, x_6\}$

$B^{-1} = \begin{bmatrix} 1&0&0\\0&1&0\\0&0&1 \end{bmatrix}$, $c_B^T = [0 \ 0 \ 0]$, $w_B^T = c_B^T B^{-1} = [0 \ 0 \ 0]$

Tableau ① objective row is $w_B^T A - c^T = \begin{array}{c} \begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \end{array} \\ [-1 \ \ -2 \ \ 1 \ \ 0 \ \ 0 \ \ 0] \end{array}$, so x_2 enters

To find θ-ratios:

$B^{-1}A_2 = \begin{bmatrix} 1\\2\\1 \end{bmatrix}$, $B^{-1}b = \begin{bmatrix} 4\\6\\5 \end{bmatrix}$, and θ-ratios are

$\frac{4}{1}$ x_4

$\frac{6}{2}$ $\leftarrow x_5$ exits

$\frac{5}{1}$ x_6

Tableau ② has basic variables $\{x_4, x_2, x_6\}$

To find B^{-1} for tableau ②:

$$\begin{array}{c} x_4 \\ x_5 \\ x_6 \end{array} \left[\begin{array}{c|ccc} \overbrace{\begin{matrix}1\\2\\1\end{matrix}}^{\text{pivotal column}} & \overbrace{\begin{matrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{matrix}}^{\text{old } B^{-1}} \end{array} \right] \simeq \left[\begin{array}{c|ccc} \begin{matrix}0\\1\\0\end{matrix} & \overbrace{\begin{matrix}1 & -\frac{1}{2} & 0\\0 & \frac{1}{2} & 0\\0 & -\frac{1}{2} & 1\end{matrix}}^{\text{new } B^{-1}} \end{array} \right]$$

$$c_B^T = \begin{array}{ccc} x_4 & x_2 & x_6 \\ \hline \end{array}$$
$$c_B^T = [0 \quad 2 \quad 0], \text{ so } w_B^T = c_B^T B^{-1} = [0 \quad 1 \quad 0]$$

Tableau ② objective row is

$$w_B^T A - c^T = \begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \\ \end{array}$$
$$w_B^T A - c^T = [-1 \quad 2 \quad -2 \quad 0 \quad 1 \quad 0]$$
$$-[1 \quad 2 \quad -1 \quad 0 \quad 0 \quad 0 \quad 0]$$
$$= [-2 \quad 0 \quad -1 \quad 0 \quad 1 \quad 0] \leftarrow x_1 \text{ enters}$$

For θ-ratios

$$B^{-1}A_1 = \begin{bmatrix} \frac{3}{2} \\ -\frac{1}{2} \\ \frac{5}{2} \end{bmatrix} \qquad B^{-1}b = \begin{array}{c|c} \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} & \begin{array}{c} x_4 \\ x_2 \\ x_6 \end{array} \end{array} \qquad \begin{array}{c} \theta\text{-ratios} \\ \frac{1}{3/2} \leftarrow x_4 \text{ exits} \\ \cancel{\frac{3}{-1/2}} \\ \frac{2}{5/2} \end{array}$$

Tableau ③ has basic variables $\{x_1, x_2, x_6\}$

$$\text{New } B^{-1} = \left[\begin{array}{c|ccc} \frac{3}{2} & 1 & -\frac{1}{2} & 0 \\ -\frac{1}{2} & 0 & \frac{1}{2} & 0 \\ \frac{5}{2} & 0 & -\frac{1}{2} & 0 \end{array}\right] \sim \left[\begin{array}{c|ccc} 1 & \frac{2}{3} & -\frac{1}{3} & 0 \\ 0 & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & -\frac{5}{3} & \frac{1}{3} & 1 \end{array}\right]$$

pivotal column; old B^{-1}; new B^{-1}