June 11 th

Continuing last week ...

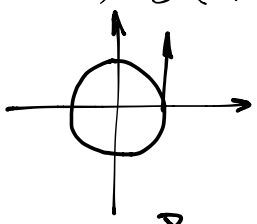
$x_i(t)$ is diff in t

1. a diff. curve in $\mathbb{R}^n$ is a differentiable function $f:[0,1] \to \mathbb{R}^n$, $t \to (x_1(t), \cdots x_n(t))$

ex 1:
$(\cos(2\pi t), \sin(2\pi t))$, $t \in [0,1]$

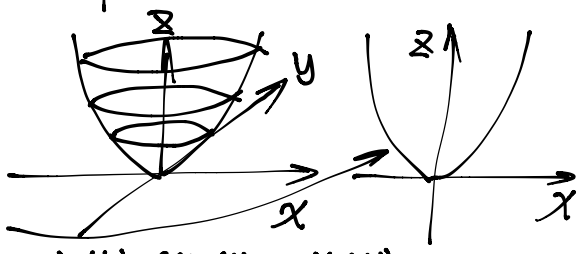
tangent vector at t_0 i.e. $(x_1(t_0), \cdots, x_n(t_0))$ is $(x_1'(t_0), \cdots, x_n'(t_0))$

$t=0$, $(-2\pi \sin 2\pi t, 2\pi \cos 2\pi t)$
$\searrow (0, 2\pi)$



2. a graph of diff. function in $\mathbb{R}^{n+1}$
$(x_1, x_2, \cdots, x_n, f(x_1, x_2, \cdots, x_n))$
assume $f: \mathbb{R}^n \to \mathbb{R}$

ex 2: (x, y, x^2+y^2), paraboloid:
$(t, 0, t^2)$



a curve on the graph $(x_1(t), x_2(t), \cdots, x_n(t), f(x_1(t), \cdots, x_n(t))$

3. line through point $(a_1, \cdots, a_n)$ is direction $(v_1, \cdots, v_n)$
$(a_1+v_1t, \cdots, a_n+v_nt)$

$u_1=(1,0,\cdots,0)$ ~~coordinate~~ curve on the graph at $(a_1, \cdots, a_n, f(a_1, \cdots, a_n))$
$u_2=(0,1,\cdots,0)$ $(a_1+v_1t, \cdots, a_n+v_nt, f(a_1+v_1t, \cdots, a_n+v_nt))$
$\vdots$
$u_n=(0,0,\cdots,1)$ $(v_1, v_2, \cdots, v_n, v_1\partial f_1(a_1)+\cdots+v_n\partial f_n(a_n))$

directional deri.

Special case is that
$(1, 0, \cdots, 0, \partial f_1(a_1))$
$(0, 1, \cdots, 0, \partial f_2(a_2))$
$\vdots$
$(0, 0, \cdots, 1, \partial f_n(a_n))$

Define the tangent space of the graph at point $(a_1, \cdots, a_n, f(a_1, \cdots, a_n))$ is subspace of $\mathbb{R}^{n+1}$ generated by $\{w_1, \cdots, w_n\}$
$w_i=(0, \cdots, 1, \cdots 0, \partial_i f(a_1, \cdots, a_n))$
$\downarrow$ ith

collection of tangent vectors of all curves on the graph through the point.

$(X_1(t) \cdots, X_n(t),\ f(X_1(t), \cdots, X_n(t))$
$(X_1'(t_0) \cdots, X_n'(t_0),\ \sum X_i'(t_0) \partial_i f(X_1(t_0), \cdots, X_n(t_0))$

$$X_i(t_0) = a_i$$

$f(x_1, x_2, x_3 \cdots, x_n) = 0$ it's a level set.
$(x_1, \cdots, x_n, g(x_1, \cdots, x_n))$
$f = x_{n+1} - g(x_1, \cdots, x_n) = 0$