STAT 6046 Tutorial Week 4

By Isaac Pan

The Australian National University

Mid-semester exam information

- Exam time:
- 18th April, 9:30 am

- Covered material:
- Lecture Notes Week 1-5

Today’s plan

- Brief review of course material
- Go through selective tutorial questions

Recap

The accumulated value of 1 from time 0 to time t under compound interest:

$$S(t) = \left(1 + \frac{i^{(m)}}{m}\right)^{mt} = (1+i)^t = v^{-t} = (1-d)^{-t} = \left(1 - \frac{d^{(m)}}{m}\right)^{-mt} = e^{\delta t}$$

The present value at time 0 of 1 payable at time t under compound interest is:

$$S(0) = \left(1 + \frac{i^{(m)}}{m}\right)^{-mt} = (1+i)^{-t} = v^{t} = (1-d)^{t} = \left(1 - \frac{d^{(m)}}{m}\right)^{mt} = e^{-\delta t}$$

$$\boxed{1 + x + x^2 + x^3 + \ldots + x^k = \frac{1 - x^{k+1}}{1-x} = \frac{x^{k+1} - 1}{x - 1}}$$

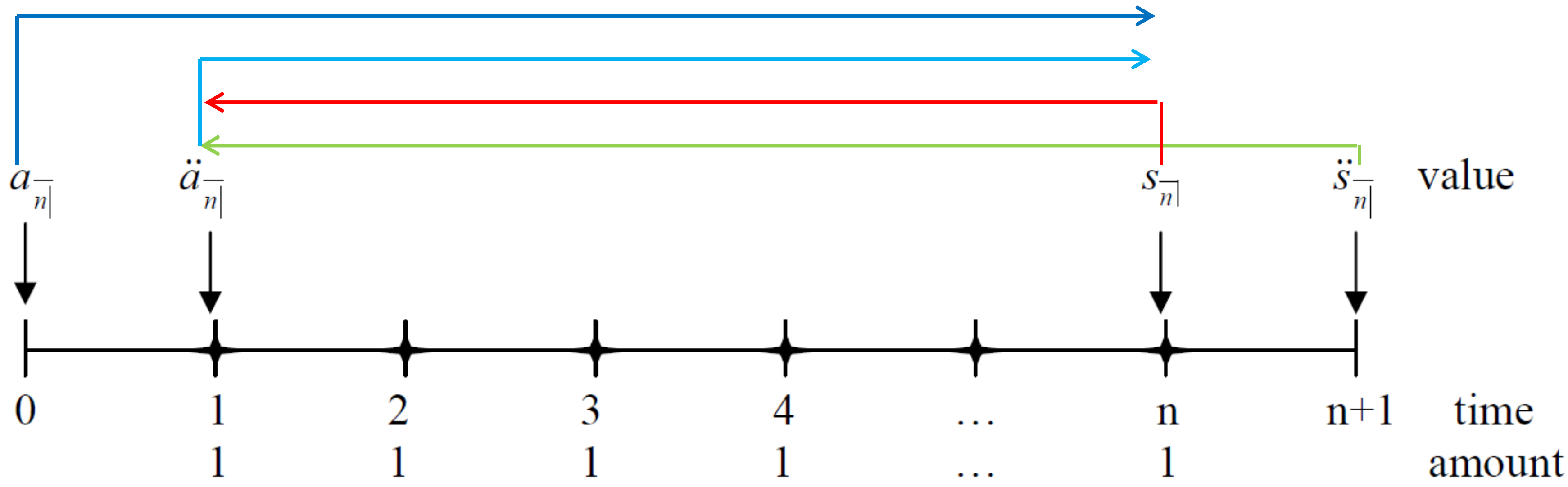
Australian National University

Annuity: Immediate VS due

- In the case of accumulated value, when the annuity is valued *at the time of the final payment* this is referred to as an ***immediate*** annuity.
- In the case of present value, an ***immediate*** annuity refers to an annuity valued *one payment period before the first payment*.

- An annuity payable in advance (ie. payments at the beginning of each period) is called an ***annuity due***.

Annuity: Immediate VS due



$$s_{\overline{n}|} = \sum_{t=0}^{n-1}(1+i)^t = \frac{(1+i)^n-1}{i}$$

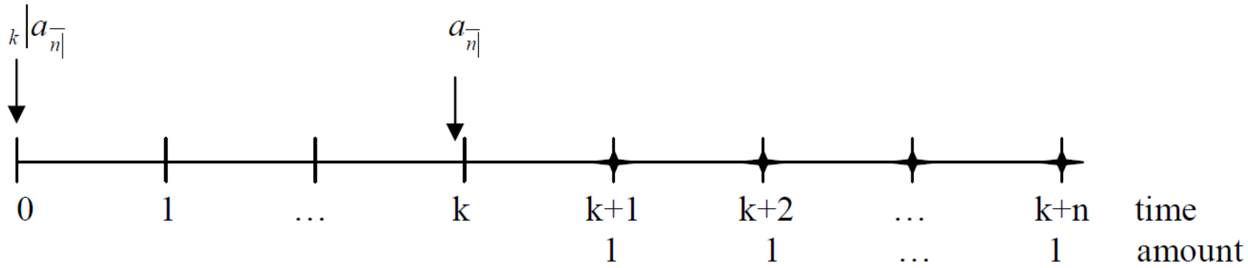
$$\ddot{s}_{\overline{n}|} = \frac{(1+i)^n-1}{d} = \frac{i}{d}s_{\overline{n}|}$$

$$a_{\overline{n}|} = s_{\overline{n}|}\cdot v^n = \frac{1-v^n}{i}$$

$$\ddot{a}_{\overline{n}|} = \frac{1-v^n}{d} = \frac{i}{d}a_{\overline{n}|}$$

Deferred Annuity

- If an annuity is to be valued more than 1 unit of time before commencement of the stream of payments, we call this a ***deferred annuity***.
- The value at time 0 of a series of *n* payments, each of amount 1, commencing at time $k+1$.



$$\boxed{{}_{k|}a_{\overline{n}|} = v^{k} \cdot a_{\overline{n}|} = a_{\overline{n+k}|} - a_{\overline{k}|}}$$

Different interests in annuity

- Accumulated value:

The accumulated value of an annuity at the time of the final payment can be found by:

(a) Finding the accumulated value of the first n payments at the time of the n^{th} payment:

$$s_{\overline{n}|i}$$

(b) Accumulating the result of part (a) for an additional k periods at compound rate j:

$$s_{\overline{n}|i}(1+j)^k$$

(c) Finding the accumulated value of the final k payments at compound rate j:

$$s_{\overline{k}|j}$$

(d) Adding the results of (b) and (c) together to get the final accumulated amount:

$$s_{\overline{n}|i}(1+j)^k + s_{\overline{k}|j}$$

- Present value:

$$\left(s_{\overline{n}|i}(1+j)^k + s_{\overline{k}|j}\right)v_j^k v_i^n = s_{\overline{n}|i}v_i^n + s_{\overline{k}|j}v_j^k v_i^n = a_{\overline{n}|i} + v_i^n \cdot a_{\overline{k}|j}$$

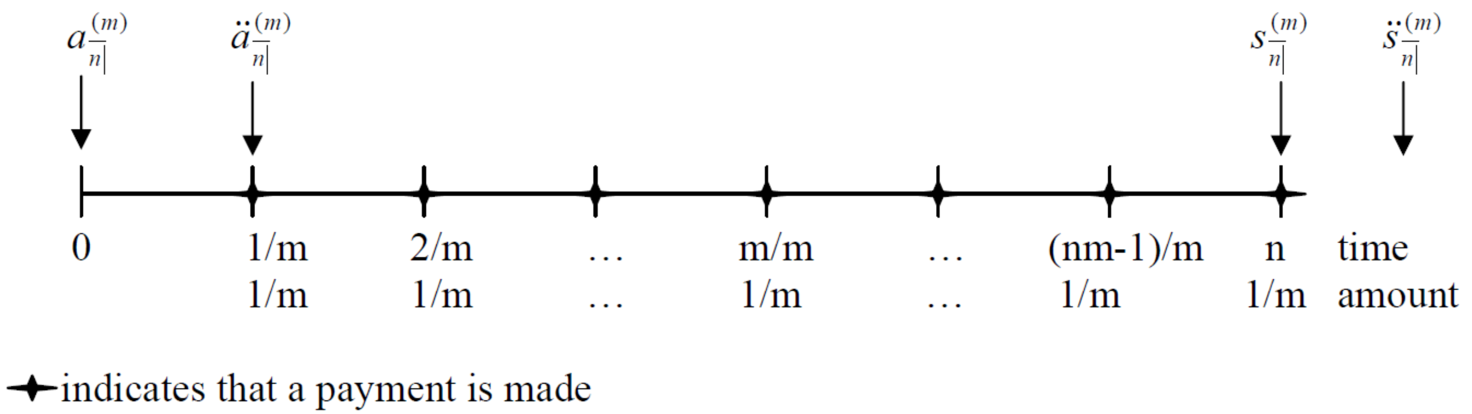
Australian National University

Annuities payable more frequently than annually

- We can do this by working in units of time consistent with the timing of annuity payments.
- This involves converting the interest rate quoted to an equivalent effective interest rate consistent with the timing of the annuity payments. We then can use the annuity formula introduced above (where *n* represents the number of payments).

Annuities payable more frequently than annually

- Another way to solve this type of question is to formulate the solution in terms of annuities payable *m* times per annum.



Annuities payable more frequently than annually

$$a_{\overline{n}|}^{(m)} = \frac{1-v^n}{i^{(m)}} = \frac{i}{i^{(m)}} a_{\overline{n}|}$$

$$\ddot{a}_{\overline{n}|}^{(m)} = \frac{1-v^n}{d^{(m)}} = \frac{i}{d^{(m)}} a_{\overline{n}|}$$

$$s_{\overline{n}|}^{(m)} = \frac{(1+i)^n - 1}{i^{(m)}} = \frac{i}{i^{(m)}} s_{\overline{n}|}$$

$$\ddot{s}_{\overline{n}|}^{(m)} = \frac{(1+i)^n - 1}{d^{(m)}} = \frac{i}{d^{(m)}} s_{\overline{n}|}$$

- Note: Payment is no longer P, instead use *annual payment* P*m!