Solution

prepared by Zhenhua Lin

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1 Question 7 (The method of moment estimation)

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> ## R code for Question 7 (The method of moment estimation)
>
> # solve the Yule-Walker equation to estimate phi
> G = matrix(c(1,-0.78,-0.78,1),2,2,byrow=T)
> b = matrix(c(-0.78,0.64),2,1)
> phi = solve(G,b)
> phi

            [,1]
[1,] -0.71705822
[2,]  0.08069459

> # roots
> r = polyroot(c(1,-phi[1],-phi[2]))
> abs(r)

[1]  1.225559 10.111635

> # Ljung and Box test
> rho = c(0.030, -0.072, 0.013, 0.020, -0.131, 0.036, 0.057, -0.063, 0.019, 0.054)
> n = 200
> m = 5
> LB = sum(rho[1:m]^2/(n-1:m))*n*(n+2)
> LB

[1] 4.912973

> 1-pchisq(LB,df=m-2)

[1] 0.1782817

> m = 10
> LB = sum(rho[1:m]^2/(n-1:m))*n*(n+2)
> LB

[1] 7.394499

> 1-pchisq(LB,df=m-2)

[1] 0.4947275

>
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a) No. The partial ACF sharply drops off after lag 2, which indicates a AR(2) model.

b) By matrix-form Yule-Walker equations, $\rho_2\phi = \rho_2$, where $\rho_2 = [\gamma(0), \rho(1); \rho(1), \rho(0)]$ is a 2×2 matrix and $\rho_2 = (\rho(1), \rho(2))$. Note that we use ACF $\rho(\cdot)$ in the equation here. It is equivalent to using the autovariance function $\gamma(\cdot)$. Now, $\rho(0) = 1$, and using the data provided in the problem, we have $\hat{\rho}(1) = -0.78$ and $\hat{\rho}(2) = 0.64$, thus,

$$\hat{\phi} = \begin{pmatrix} 1 & -0.78 \\ -0.78 & 1 \end{pmatrix}^{-1} \begin{pmatrix} -0.78 \\ 0.64 \end{pmatrix} = \begin{pmatrix} -0.7170582226762 \\ 0.0806945863125638 \end{pmatrix}$$

c) Yes, since the roots of $1 - \hat{\phi}_1 B - \hat{\phi}_2 B^2 = 0$ are outside the unit circle.

d) Just follow the formula in Silde 70 in Lecture 2. Also see the R code above. The result indicates no evidence of lack of adequacy.

2 **Forecast, Q2:** This is a AR(1) process with mean $\mu = 9$.

c) Yes, because the root of $1 - 0.6B = 0$ is $B = 5/3 > 1$.

d) This is a causal AR(1) process. By Yule-Walker equation, or (3.67) in textbook, the forecasting equation for one-step-ahead is $\hat{X}_{101} = 9 + 0.6(X_{100} - 9) = 9 + 0.6(8.9 - 9) = 8.94$. In the tutorial, I showed that $\hat{X}_{102} = 9 + 0.6(\hat{X}_{101} - 9)$. This can be generalized to $\hat{X}_t(h) = 9 + 0.6(\hat{X}_t(h+1) - 9)$. Thus, we can compute the forecasting $\hat{X}_{102} = 9 + 0.6 * (8.96 - 9) = 8.976$ and $\hat{X}_{103} = 9 + 0.6 * (8.976 - 9) = 8.9856$. To compute the forecast limits, we use the integrated form: $\hat{X}_t(l) = \sum_0^\infty \psi_i[a_{t+l-i}]$. Since $\psi(B) = \theta(B)/\phi(B)$, and in this problem, $\theta(B) = 1$ and $\phi(B) = 1 - 0.6B$, we have $\psi(B) = \sum_0^\infty 0.6^j B^j$. Thus, $\psi_j = 0.6^j$. Now use the formula $\sqrt{(1 + \sum_{j=1}^{l-1} \psi_j^2)\sigma^2}$ in Slide 33 of Lecture 3, we have the standard deviation of $\hat{X}_t(l)$ to be $\sqrt{(1 + \sum_{j=1}^{l-1} 0.6^{2j})}$, as $\sigma = 1$ in this problem. So, the standard deviation of $\hat{X}_{101}$ is 1, the standard deviation of $\hat{X}_{102}$ is $\sqrt{(1 + 0.6^2)} = 1.16619$, and similarly, we can compute the standard deviation of other forecast. Then, the 95% limit for $\hat{X}_{101}$ is 8.94 ± 1.96, and for $\hat{X}_{102}$ is $8.976 \pm 1.96 * 1.16619$.

e) The updating forecast formula is given in Slide 32 of lecture 3: $\hat{X}_{t+1}(l) = \hat{X}_t(l+1) + \psi_l a_{t+1}$, with $a_t = X_t - \hat{X}_{t-1}(1)$. Thus, $a_{101} = 8.8 - 8.94 = -0.14$. This gives $\hat{X}_{101}(1) = 8.94 + 0.6 * (-0.14) = 8.856$, $\hat{X}_{101}(2) = \hat{X}_{100}(3) = 8.9856 + 0.6^2(-0.14) = 8.9352$, etc.