MAT135H1S Calculus I(A)

Solution to even-numbered problems in Section 2.2 and 2.3

(Section 2.2, Q36)

$$\lim_{x\to 2^-} \frac{x^2-2x}{x^2-4x+4} = \lim_{x\to 2^-} \frac{x(x-2)}{(x-2)^2} = \lim_{x\to 2^-} \frac{x}{x-2} = -\infty,$$

since the numerator is positive and the denominator approaches 0 through negative values as $x \to 2^-$.

(Section 2.3, Q20)

$$\lim_{t\to 1} \frac{t^4-1}{t^3-1} = \lim_{t\to 1} \frac{(t-1)(t^3+t^2+t+1)}{(t-1)(t^2+t+1)} = \lim_{t\to 1} \frac{t^3+t^2+t+1}{t^2+t+1} = \frac{4}{3}$$

(Section 2.3, Q32)

$$\lim_{h\to 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h} = \lim_{h\to 0} \frac{\frac{x^2-(x+h)^2}{x^2(x+h)^2}}{h} = \lim_{h\to 0} \frac{-2xh-h^2}{x^2(x+h)^2 h} = \lim_{h\to 0} \frac{h(-2x-h)}{x^2(x+h)^2 h}$$

$$= \lim_{h\to 0} \frac{-2x-h}{x^2(x+h)^2} = \frac{-2x}{x^2(x+0)^2} = -\frac{2}{x^3}$$

(Section 2.3, Q42)

$$\text{Note that } |x+6| = \begin{cases} x+6 & \text{if } x \geq -6, \\ -x-6 & \text{if } x < -6. \end{cases}$$

Therefore, we have

$$\lim_{x\to -6^+} \frac{2x+12}{|x+6|} = \lim_{x\to -6^+} \frac{2x+12}{x+6} = \lim_{x\to -6^+} \frac{2(x+6)}{x+6} = \lim_{x\to -6^+} 2 = 2$$

and

$$\lim_{x\to -6^-} \frac{2x+12}{|x+6|} = \lim_{x\to -6^-} \frac{2x+12}{-(x+6)} = \lim_{x\to -6^-} \frac{2(x+6)}{-(x+6)} = \lim_{x\to -6^-} -2 = -2$$

Since the left and right limits are not equal, $\displaystyle\lim_{x\to -6} \frac{2x+12}{|x+6|}$ does not exist.

(Section 2.3, Q58)

Given that $\displaystyle\lim_{x\to 0} \frac{f(x)}{x^2} = 5$.

(a) $\displaystyle\lim_{x\to 0} f(x) = \lim_{x\to 0} \left(\frac{f(x)}{x^2} \cdot x^2\right) = \left(\lim_{x\to 0} \frac{f(x)}{x^2}\right)\left(\lim_{x\to 0} x^2\right) = (5)(0) = 0$

(b) $\displaystyle\lim_{x\to 0} \frac{f(x)}{x} = \lim_{x\to 0} \left(\frac{f(x)}{x^2} \cdot x\right) = \left(\lim_{x\to 0} \frac{f(x)}{x^2}\right)\left(\lim_{x\to 0} x\right) = (5)(0) = 0$