INTRODUCTORY MATHEMATICAL STATISTICS (STAT2001/6039)

Tutorial 3

Problem 1

Consider a diagnostic test for a disease which is 90% accurate, in the sense that when this test is applied to a large number of people:

90% of those people who *have* the disease test *positive*, and
90% of those people who *do not have* the disease test *negative*.

Suppose that 1% of the population has the disease in question.

A person is randomly chosen from the population and given the test.

(a) What is the probability that this person tests positive?

(b) What is the probability that this person tests positive and has the disease?

(c) What is the probability that this person has the disease if they test positive?

(d) We learn that the selected person has lost their appetite and is constantly thirsty. It is known that 30% of people with these symptoms have the disease in question. Repeat (c) in the light of this information.

Problem 2

(a) Consider any two events A and B. Show that

$$P(AB) \geq P(A) + P(B) - 1.$$

(b) Consider any three events A, B and C. Show that

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(AB) - P(AC) - P(BC) + P(ABC).$$

Problem 3

Five cards are drawn, without replacement, from a standard pack of 52 cards.

(A standard pack consists of four suits, where each suit consists of one ace and 12 other cards.)

Find the probability that the drawn cards will contain:

(a) three cards of any one suit and two of another

(b) at least one ace.

Problem 4

(a) From 7 different consonants and 5 different vowels, how many different words can be formed consisting of 4 consonants and 3 vowels?

(No letter may be used more than once.)

(b) How many different salads can be made from lettuce, escarole, endive, watercress and chicory?

(By "salad", we mean a mixture of one or more of the above ingredients, in equal proportions; for example, a mixture of 50% lettuce and 50% endive.)