Department of Mathematics	**Problem Set #1**
University of Toronto	**Deadline: Tuesday October 4, 3:00 p.m.**
MAT332F, 2011	Assignment Posted/Revised: September 18, 2011

Read the following instructions carefully! In contains guidelines on handing in assignments for this course.

For solutions to all problem sets, please remember that when you are asked to find or calculate something you must justify that what you have found is correct and complete. You may use results from your lecture to help in your justification. You are learning to present your results in a clear and convincing manner. Thus, you will be graded on your presentation and justification; we are not simply verifying whether you know the answer.

Present your solutions with complete sentences. Pretend the marker does **not** know how to solve the question.

Required Information. The front page must include your name and student number. *Failure to put your name and/or your student number on your problem set will result in a zero on your assignment.* A cover page is not required as long as the necessary information is on the top of the first page.

Submitting your assignment. You must hand your assignment to your instructor before the beginning of lecture, or deposit the instructor's personal mailbox on the 6th floor of the Bahen Centre.

If you are unable to complete homework or if you miss a term test due to illness or other circumstances outside of your control, please contact your instructor immediately in order to receive special consideration. Note that special consideration will be given on an individual basis and will not be given automatically. In other words, you risk getting a mark of zero for missed work unless you contact the instructor promptly.

In the case of illness, medical documentation must be supplied on the standard University of Toronto Student Medical Certificate. You can also obtain a paper copy of this certificate from your college registrar or in your registration handbook. (A simple "note" from your doctor is not acceptable.)

Late submission. Late assignments will be accepted up to 25 hours after their deadlines with the following penalties.

Submission time	Penalty
by 3pm on Tuesday	none
by 10am on Wednesday	-10%
by 4pm on Wednesday	-25%

Note that lateness penalties will be computed as a percentage of the total marks on the assignment, not of the mark you obtain. Late assignments must be submitted directly into the instructor's personal mailbox on the 6th floor of Bahen Center (in the Math Department office), unless you require special consideration (see the section above for details). Please write the *exact* submission time on your assignment if you are submitting late.

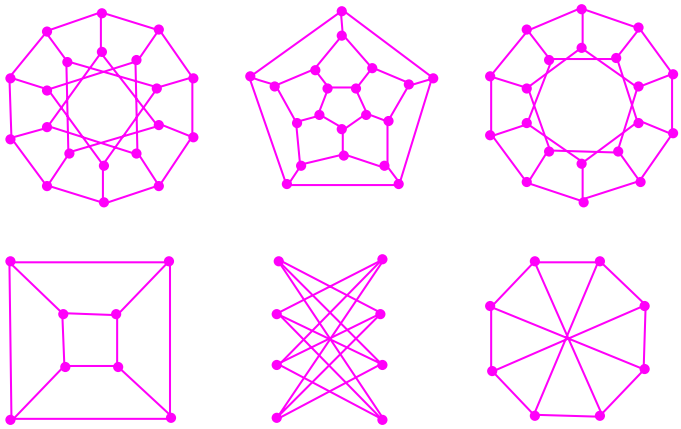
Policy on Plagiarism on Assignments. Plagiarism is a form of academic fraud and is treated very seriously by the Faculty. *The assignments you hand in must not contain anyone else's work or ideas without proper attribution.* A working definition of plagiarism suitable for this course may be found at `http://www.northwestern.edu/provost/students/integrity/plagiarism.html`.

In science, collaboration is the norm, and in this course student collaboration is permitted to an extent. Namely, you are permitted to abstractly discuss possible solutions to a problem with other students. However, a student is forbidden from guiding another student through a solution step by step.

You are permitted to submit a joint answer to a problem set question. If two students have contributed to the solution of the problem, please write both names and student numbers near the problem, and you may share the marks.

Core Problems.

(1) (a) Prove: Two graphs are isomorphic if and only if their complements are isomorphic. (*5pt*)
(b) Which of the following drawings represent isomorphic graphs, and which non-isomorphic graphs? Exhibit an explicit isomorphism, or prove that none can exist. (*15pt*)



(2) Prove that any two longest paths in a connected graph share a vertex in common. (*15pt*)

(3) Let S be a set of n points in a plane, such that the distance between any two points in S is at least one. Show that there cannot be more than $3n$ pairs of points in S of distance exactly 1. (*10pt*)

(4) Let G be a loopless graph each of whose vertices has valence ≥ 3. Prove that G has a cycle of even length. (Hint: Consider a maximal path.) (*15pt*)

(5) Prove that a graph G is bipartite if any only if every subgraph H of G has an independent set containing at least half of $V(H)$. (*15pt*)

(6) (a) Prove that a simple graph on n vertices with $n > 1$ contains at least two vertices of the same valence. (*10pt*)
(b) Give an example of a loopless graph with four vertices that all have different valences. (*5pt*)

(7) The Health Department conducted a survey on basic mathematical proficiency among medical doctors. The survey contained three questions. 70% of doctors got the first question wrong, 75% of doctors got the second question wrong, and 80% of doctors got the third question wrong. Prove that at least 25% of doctors got all three questions wrong. (*10pt*)

Bonus Problem.

(1) A *strongly regular graph with parameters* (n, k, λ, μ) is a k–regular graph G on n vertices such that:
- Any two adjacent vertices of G have λ common neighbours.
- Any two nonadjacent vertices of G have μ common neighbours.

Let G be a strongly regular graph with parameters (n, k, λ, μ). Prove:
(a) $\bar{G}$ is strongly regular.
(b) $k(k - \lambda - 1) = (n - k - 1)\mu$.
(c) $A^2 = kI + \lambda A + \mu(J - I - A)$,

where I denotes the $n \times n$ identity matrix, J denotes the $n \times n$ matrix whose entries are all 1, and A is the adjacency matrix for G. (12 *bonus points*)