Whiteboard Notes: Lecture 25/7/2017 2pm

KEY EQUATION BAYESIAN INFERENCE:

POSTERIOR $\propto$ PRIOR x LIKELIHOOD

$$p(\theta|y) \propto \underbrace{p(\theta) \times p(y|\theta)}$$

update prior beliefs on θ with ~~da~~ information from data collected.

THINKING LIKE A BAYESIAN - Example 1:

Let $\tilde{y}_{MU}$ be the number of goals that Man U scores in next game

→ Let $\tilde{y}_C$ be similarly defined for Chelsea

We are interested in the probability $Pr(\tilde{y}_{MU} - \tilde{y}_C > 2)$

→ We can collect historical data on y_{MU} and y_C.

→ that is, no. of goals scored by Man U and chelsea in the past 'n' games ('n' = sample size).

→ What sampling distribution assumption could we assume for y? ⇒ POISSON.

we know that if $Y \sim Pois(\lambda)$ $p(y) = \frac{e^{-\lambda}\lambda^y}{y!}$

→ We need to estimate 'λ' (rate parameter).

→ In a Bayesian context, we allow 'λ' to be random

→ factors that may affect 'λ' - home/away game
team injuries
team skill.

→ Then we are interested in

$Pr(\tilde{y}_{MU} - \tilde{y}_C > 2 | y_{MU}, y_C)$ } posterior predictive probability.

λ can be continuously updated as more matches are...

$$p(\tilde{y}_{MU} - \tilde{y}_C > 2 \mid y_{MU}, y_C)$$

$$= \iint p(\tilde{y}_{MU} - \tilde{y}_C > 2, \lambda_{MU}, \lambda_C \mid y_{MU}, y_C)\, d\lambda_{MU}\, d\lambda_C$$

Rule of marginal probability

$$= \iint p(\tilde{y}_{MU} - \tilde{y}_C > 2 \mid \lambda_{MU}, \lambda_C)\, P(\lambda_{MU}, \lambda_C \mid y_{MU}, y_C)\, d\lambda_{MU}\, d\lambda_C$$

(posterior distn)

Allow for uncertainty in true values of λ_{MU}, λ_C in estimating the probability of interest.

We can approximate the above integral using Monte Carlo simulation.

For complicated functions, the Bayesian approach does not require us to derive and work ~~with~~ analytically with complicated likelihood function -> in this case $\ell(\tilde{y}_{MU} - \tilde{y}_C \mid \lambda_{MU}, \lambda_C)$.

difference b/w 2 independent Poisson r.v's is a complicated distribution

THINKING LIKE A BAYESIAN - EXAMPLE 2

- Eg. $y_1 = y_2 = 0$. → which of Hospital 1 or 2 is better?
  → is it reasonable to assume the death rate will remain zero in the future?
- A Bayesian approach allows for a non-zero probability that the true death rate $\neq 0$ even though the observed number of deaths is zero. → more realistic.

↳ Bayesian approach deals better with extreme/sparse data outcomes, in this case $y = 0$.