STA305/1004 Homework 1

Due: In class at 11:10 on Wednesday, January 21, 2015.

1. A chemist has seven light objects to weigh on a balance pan scale. The standard deviation of each weighing is denoted by σ.

   In a 1935 paper Frank Yates suggested an improved technique by weighing all seven objects together, and also weighing them in groups of three. The groups are chosen so that each object is weighed four times altogether, twice with any other object and twice without it.

   Let $y_1, ..., y_8$ be the readings from the scale so that the equations for determining the unknown weights, $\beta_1, ..., \beta_7$, are

$$
\begin{aligned}
y_1 &= \beta_1 + \beta_2 + \beta_3 + \beta_4 + \beta_5 + \beta_6 + \beta_7 + \epsilon_1 \\
y_2 &= \beta_1 + \beta_2 + \beta_3 + \epsilon_2 \\
y_3 &= \beta_1 + \beta_4 + \beta_5 + \epsilon_3 \\
y_4 &= \beta_1 + \beta_6 + \beta_7 + \epsilon_4 \\
y_5 &= \beta_2 + \beta_4 + \beta_6 + \epsilon_5 \\
y_6 &= \beta_2 + \beta_5 + \beta_7 + \epsilon_6 \\
y_7 &= \beta_3 + \beta_4 + \beta_7 + \epsilon_7 \\
y_8 &= \beta_3 + \beta_5 + \beta_6 + \epsilon_8,
\end{aligned}
$$

   where the $\epsilon_i, i = 1, ..., 8$ are independent errors.

   In a 1949 paper Harold Hotelling suggested modifying Yates' procedure by placing in the other pan of the scale those of the objects not included in one of his weighings. In other words if the first three objects are to be weighed then the remaining four objects would be placed in the opposite pan.

   (a) Write Yates' procedure in matrix form $\mathbf{y} = X\beta + \epsilon$, where $\mathbf{y}' = (y_1, ..., y_8)$, $\beta' = (\beta_1, ..., \beta_8)$, $\epsilon' = (\epsilon_1, ..., \epsilon_8)$, and X is an 8×7 matrix. Find the least

squares estimate of β. (HINT: use the R code given in class to carry out the matrix multiplication)

(b) Write Hotellings procedure in matrix form $\mathbf{y} = X\beta + \epsilon$, where $\mathbf{y}' = (y_1, ..., y_8)$, $\beta' = (\beta_1, ..., \beta_8)$, $\epsilon' = (\epsilon_1, ..., \epsilon_8)$, and X is an 8×7 matrix. Find the least squares estimate of β.

(c) Find the variance of a weight using Yates' and Hotelling's procedures (you may use known results from regression analysis).

(d) If the chemist wanted estimates of the weights with the highest precision then which procedure (Yates or Hotelling) would you recommend that the chemist use to weigh objects? Explain your reasoning.

2. A student conducted a study of hot chicken wings and beer consumption at a bar in Minneapolis (Chichara, Hesterberg, 2011). She asked patrons at the bar to record their consumption of hot wings and beer over the course of several hours. She wanted to know if people who ate more hot chicken wings would then drink more beer and whether gender had an impact on hot wings or beer consumption.

   Answer the following questions using `Beerwings` data set using R.

   (a) Does beer consumption significantly increase as wing consumption increases? Conduct an appropriate data analysis to investigate this question. Make sure to check any assumptions that your analysis depends on.

   (b) Let μ_M denote the mean number of hot chicken wings consumed by males, μ_F the mean number of hot chicken wings consumed by females, and $H_0 : \mu_M = \mu_F$.

      i. If H_0 is true then how many differences in means are possible? In other words if the null hypothesis is true then how many ways are there to distribute the responses (chicken wing consumption) by gender?

      ii. Conduct a randomization test to evaluate if men consume more chicken wings than women. Create a histogram of the randomization distribution. What is the P-value of your test? **(Hand in your R code and output for this part)**

      iii. Is this study an experiment or observational study? Is it appropriate to make an inference about the population?

3. Use the `Beerwings` data in the previous question to conduct a two sample t-test. **(Hand in your R code and output for this part)**

   (a) Are the assumptions behind the t-test satisfied?

   (b) Does your answer agree with the randomization test? Explain.

4. Suppose that two drugs A and B are to be tested on 12 subjects' eyes. The drugs will be randomly assigned to the left eye or right eye based on the flip of a fair coin. If the coin toss is heads then a subject will receive drug A in their right eye. The coin was flipped 12 times and the following sequence of heads and tails was obtained:

T T H T H T T T H T T H

(a) Create a table that shows how the treatments will be allocated to the 12 subjects' left and right eyes?

(b) What is the probability of obtaining this treatment allocation?

(c) What type of experimental design has been used to assign treatments to subjects? Explain.