Goal-Based Agents: Solving Problems by Searching

Chapter 3, Sections 1-4

Problem solving agents

Form of **goal-based agent** that formulates the problem of **reaching a goal** in its environment, searches for a **sequence of actions** solving the problem, and executes it.

Assumptions about the task environment:

◇ static

◇ single agent

◇ deterministic

◇ fully observable

do not have to constantly interacting with environment

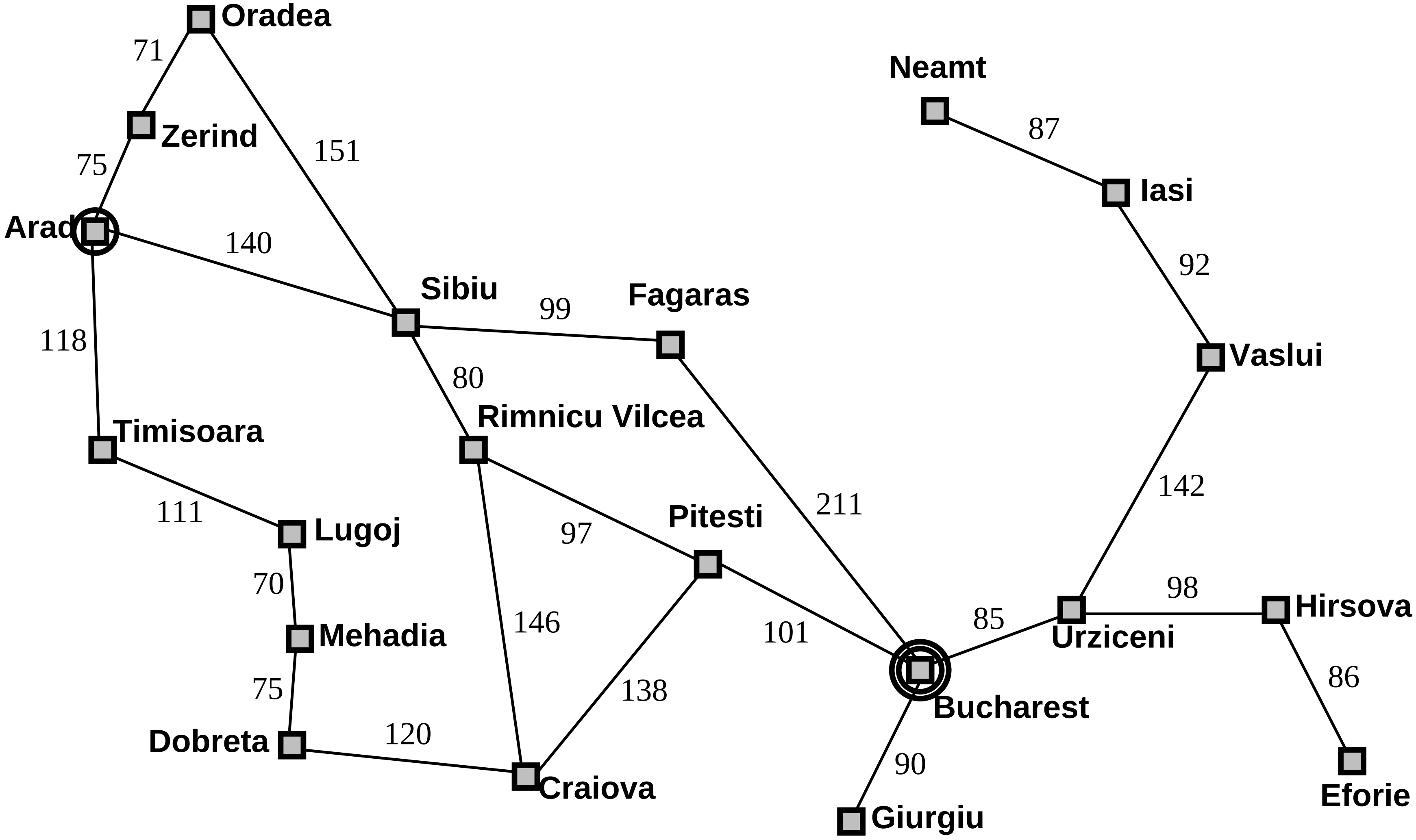
offline (or open-loop) problem solving is suitable under those assumptions; the entire sequence solution can be **executed "eyes closed."**

as you can surely succeed given the correct model.

Outline

- ♢ Problem formulation
- ♢ Problem formulation examples
- ♢ Tree search algorithm
- ♢ Uninformed search strategies

Example: Romania



Example: Romania

On holiday in Romania; currently in Arad.
Flight leaves tomorrow from Bucharest

Formulate problem:

initial state: in Arad
goal: be in Bucharest
states: various cities
actions: drive between cities

Search for a solution:

sequence of drive actions or equivalently (in this case)
sequence of cities, e.g., Arad, Sibiu, Fagaras, Bucharest

Problem formulation

A problem is defined by four items:

initial state e.g., "at Arad"

successor function $S(x)$ = set of action–state pairs

e.g., $S(Arad) = \{\langle \underset{\text{action}}{Arad \rightarrow Zerind}, \underset{\text{state}}{Zerind} \rangle, \ldots\}$

goal test, can be

explicit, e.g., x = "at Bucharest"

implicit, e.g., $HasAirport(x)$

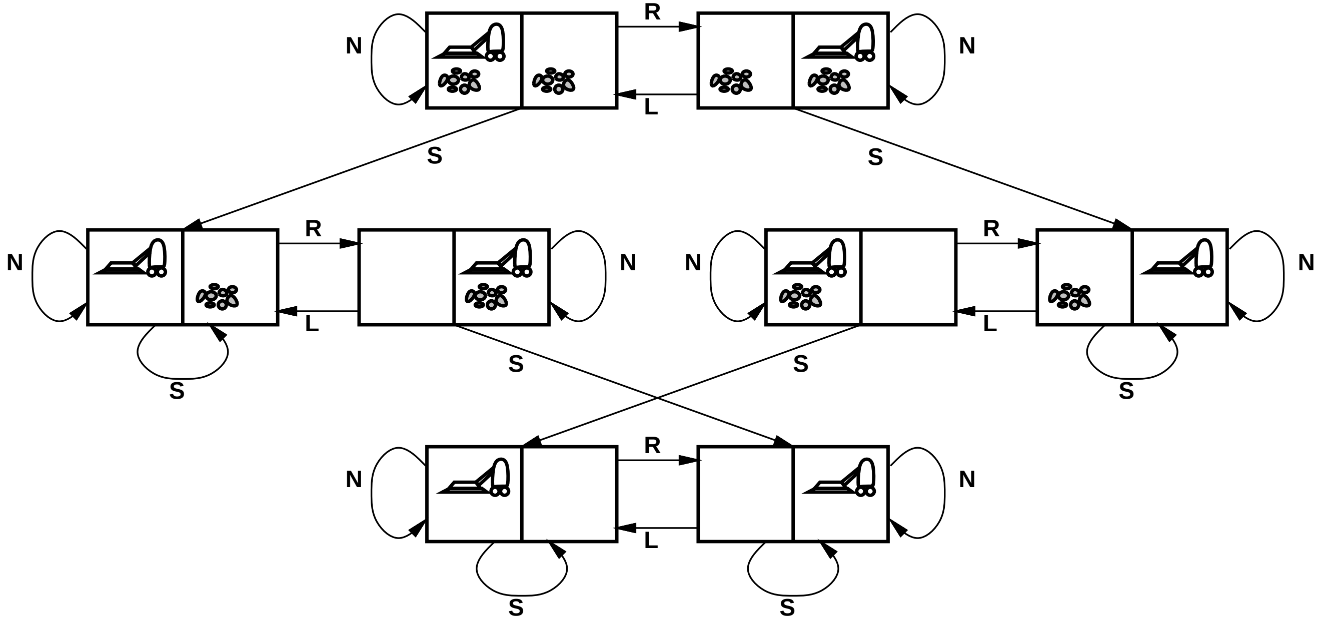
path cost (additive)

e.g., sum of distances, number of actions executed, etc.

$c(x, a, y)$ is the step cost, assumed to be ≥ 0

A solution is a sequence of actions
leading from the initial state to a goal state

Example: Vacuum world



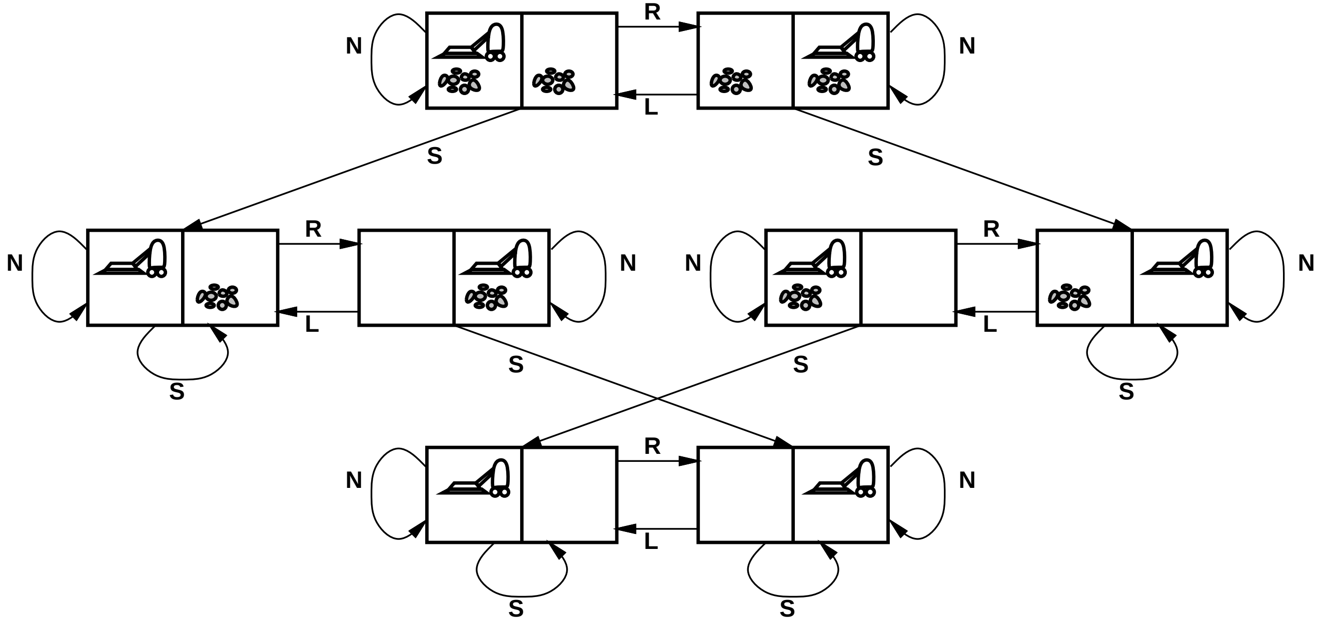
states??

actions??

goal test??

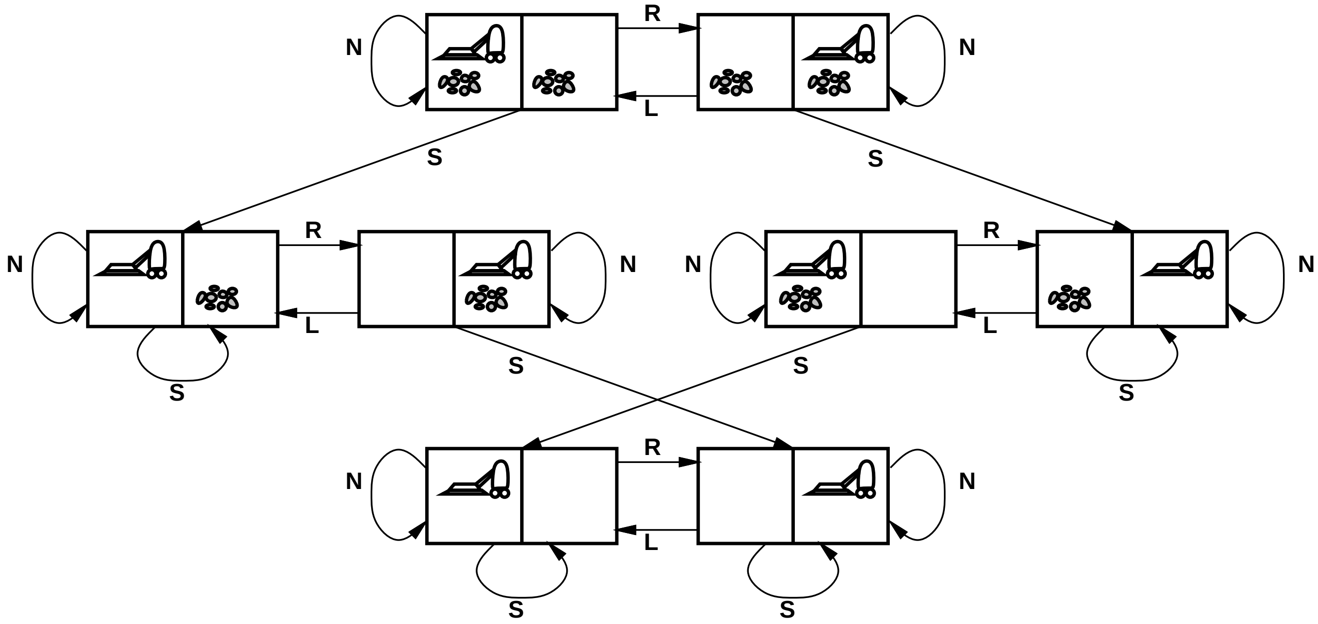
path cost??

Example: Vacuum world



states??: dirt presence in each room and robot location (ignore dirt amounts)
actions??
goal test??
path cost??

Example: Vacuum world



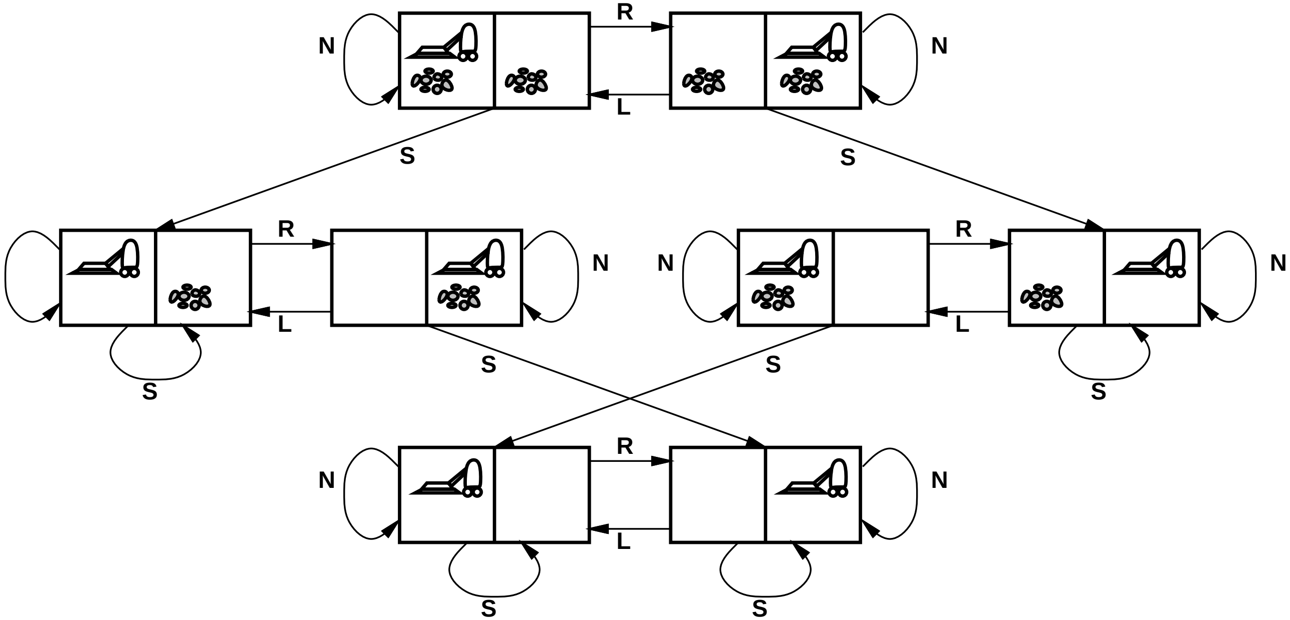
states??: dirt presence in each room and robot location (ignore dirt amounts)

actions??: $Left$, $Right$, $Suck$, $NoOp$

goal test??

path cost??

Example: Vacuum world



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actions??: $Left$, $Right$, $Suck$, $NoOp$

goal test??: $=$ no dirt

path cost??

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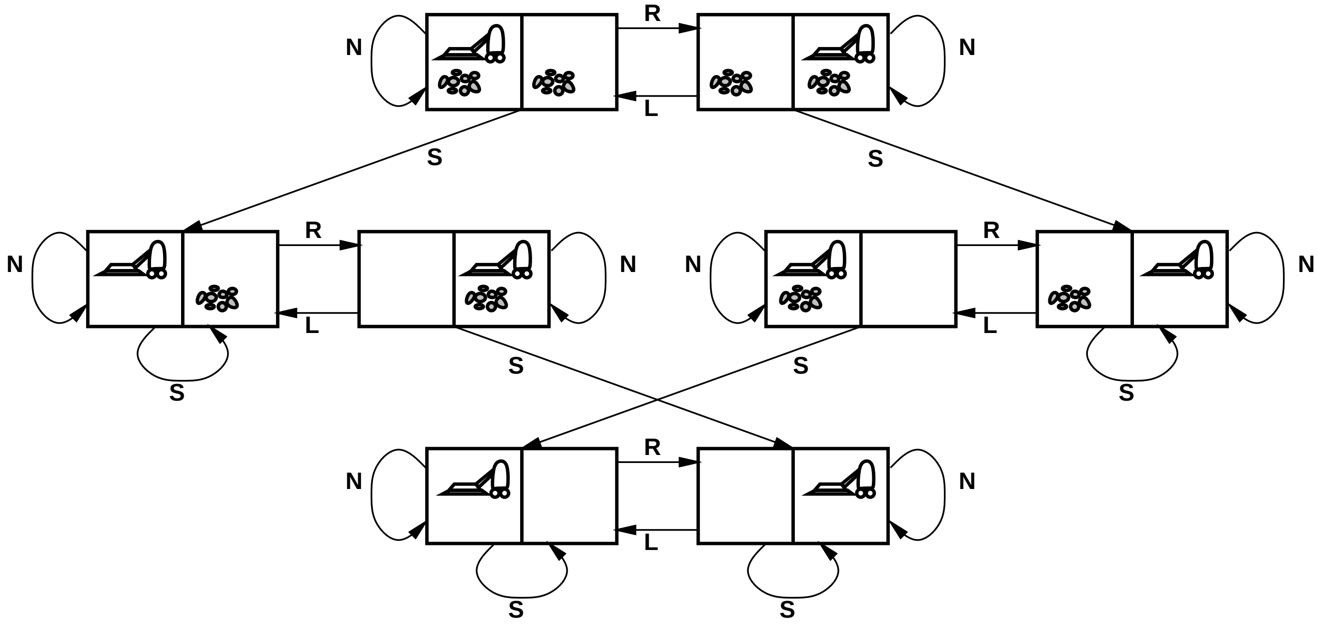


states??: dirt presence in each room and robot location (ignore dirt amounts)

actions??: $Left$, $Right$, $Suck$, $NoOp$

goal test??: $=$ no dirt

path cost??: 1 per action (0 for $NoOp$)

Selecting a state space

Real world is absurdly complex

⇒ state space must be **abstracted** for problem solving // extract only elements important for solving the problem.

(Abstract) state = set of real states

(Abstract) action = complex combination of real actions

e.g., "Arad → Zerind" represents a complex set of possible routes, detours, rest stops, etc.

For guaranteed realizability, **any** real state "in Arad" must get to some real state "in Zerind"

(Abstract) solution =

set of real paths that are solutions in the real world

Abstraction should be "easier" than the original problem!

Example: The 8-puzzle

7	2	4
5		6
8	3	1

Start State

1	2	3
4	5	6
7	8	

Goal State

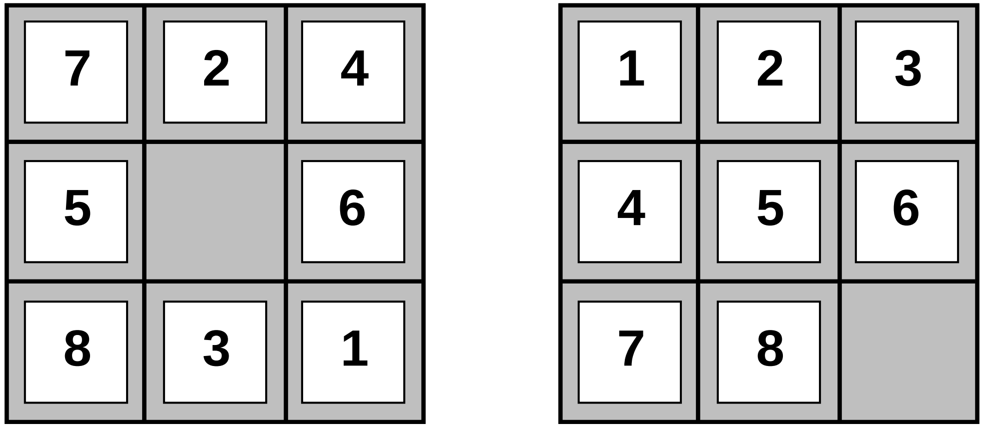
states??

actions?? // just consider the action of the blank space

goal test??

path cost??

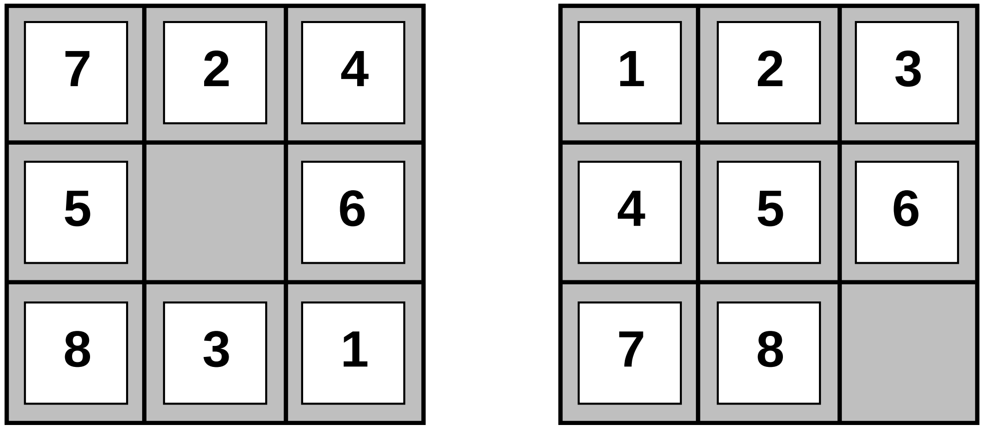
Example: The 8-puzzle



Start State **Goal State**

states??: integer locations of tiles (ignore intermediate positions)
actions??
goal test??
path cost??

Example: The 8-puzzle



Start State Goal State

states??: integer locations of tiles (ignore intermediate positions)
actions??: move blank left, right, up, down (ignore unjamming etc.)
goal test??
path cost??

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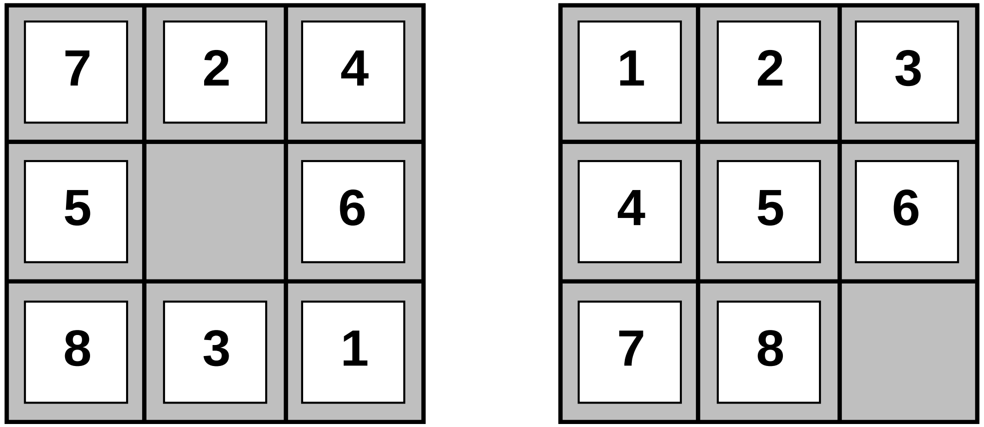
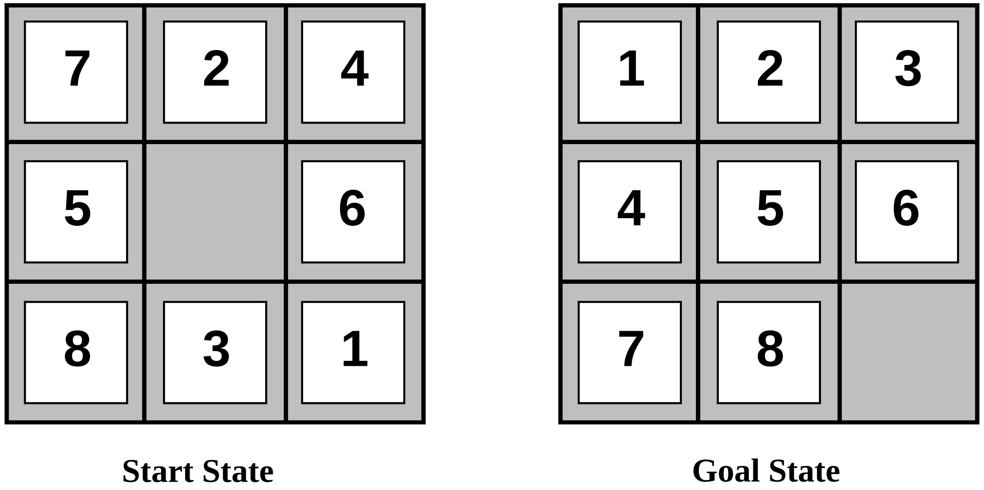


Start State Goal State

states??: integer locations of tiles (ignore intermediate positions)
actions??: move blank left, right, up, down (ignore unjamming etc.)
goal test??: = goal state (given)
path cost??

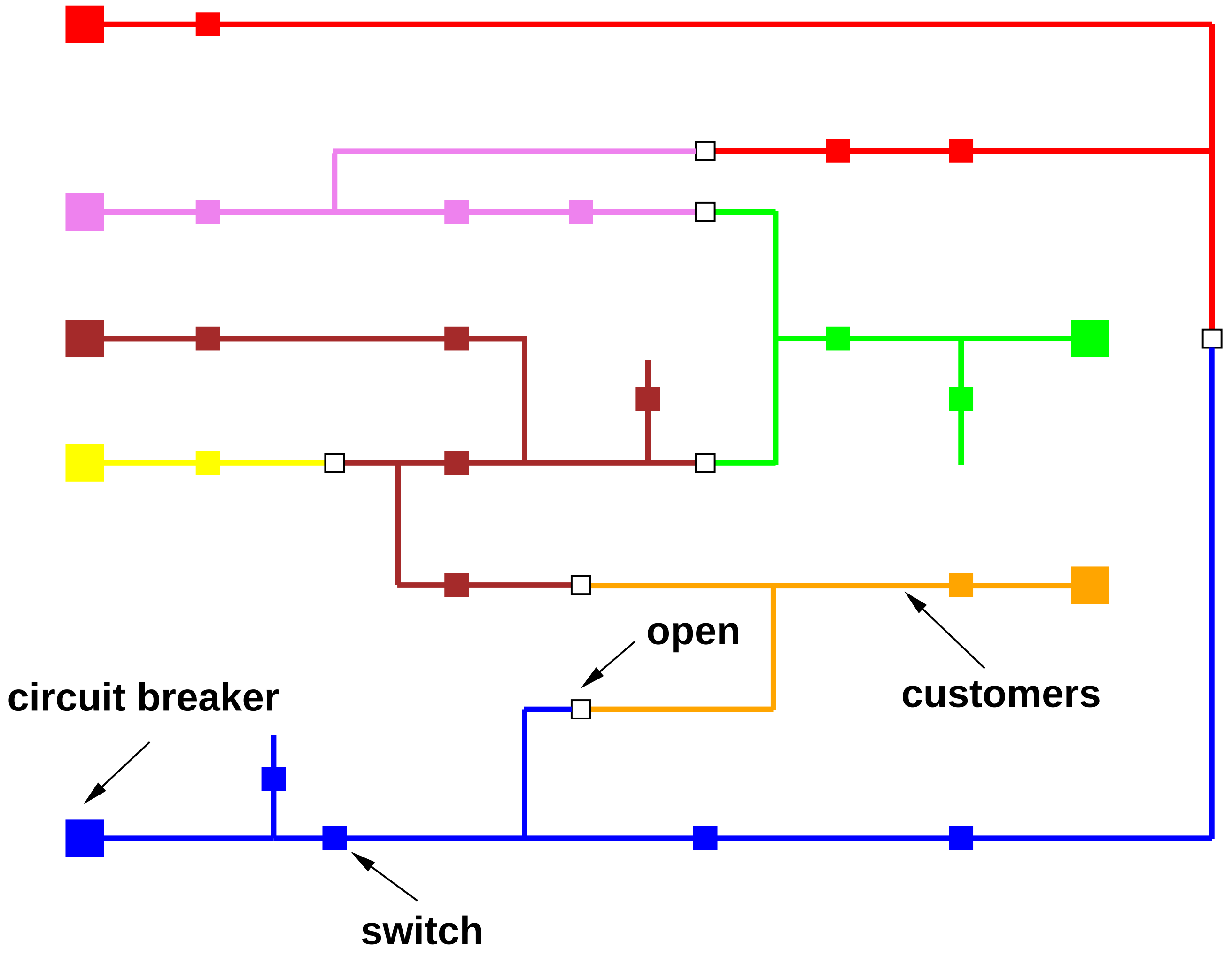
Example: The 8-puzzle



states??: integer locations of tiles (ignore intermediate positions)
actions??: move blank left, right, up, down (ignore unjamming etc.)
goal test??: = goal state (given)
path cost??: 1 per move

[Note: optimal solution of n-Puzzle family is NP-hard]

Example: power supply restoration



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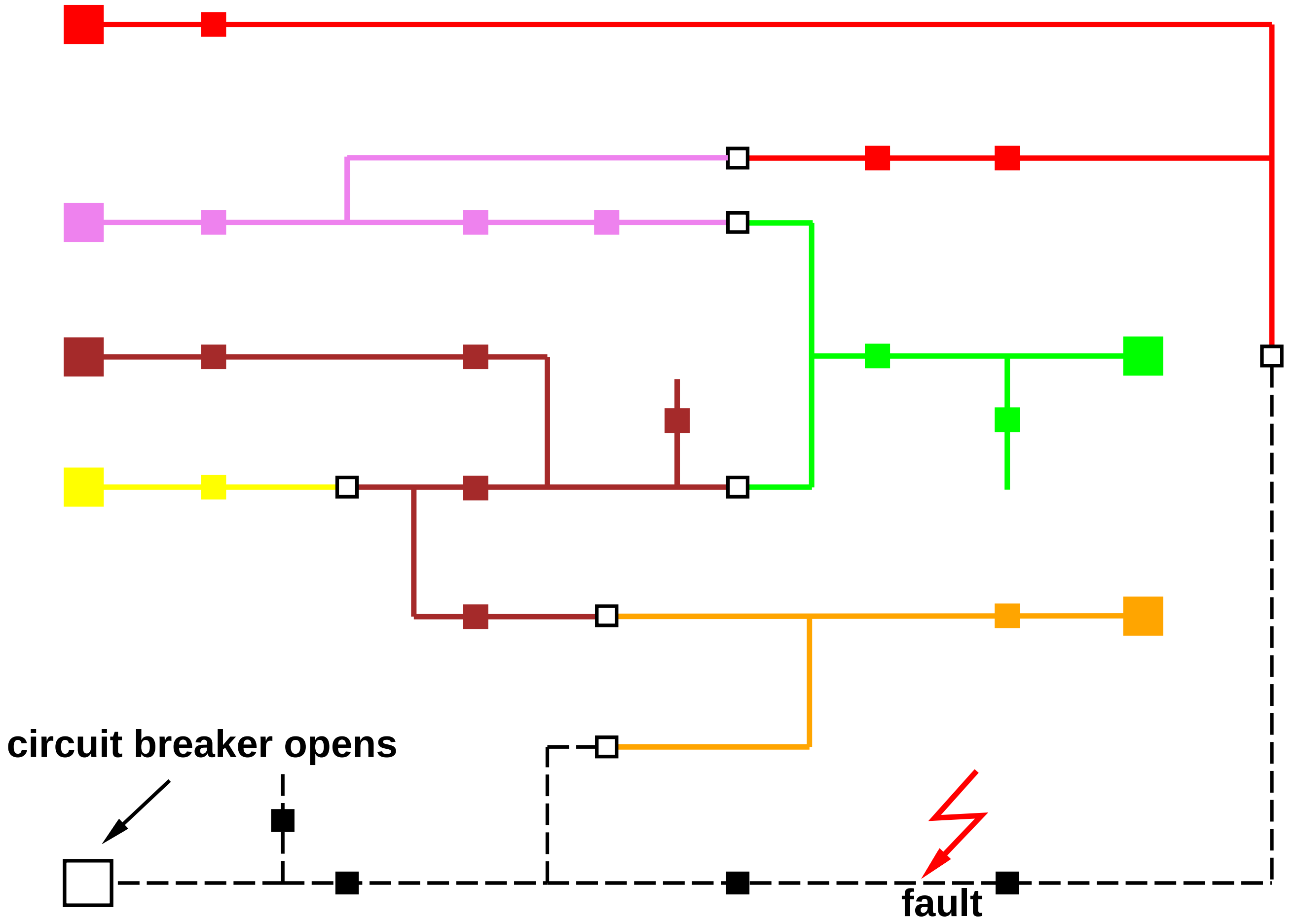

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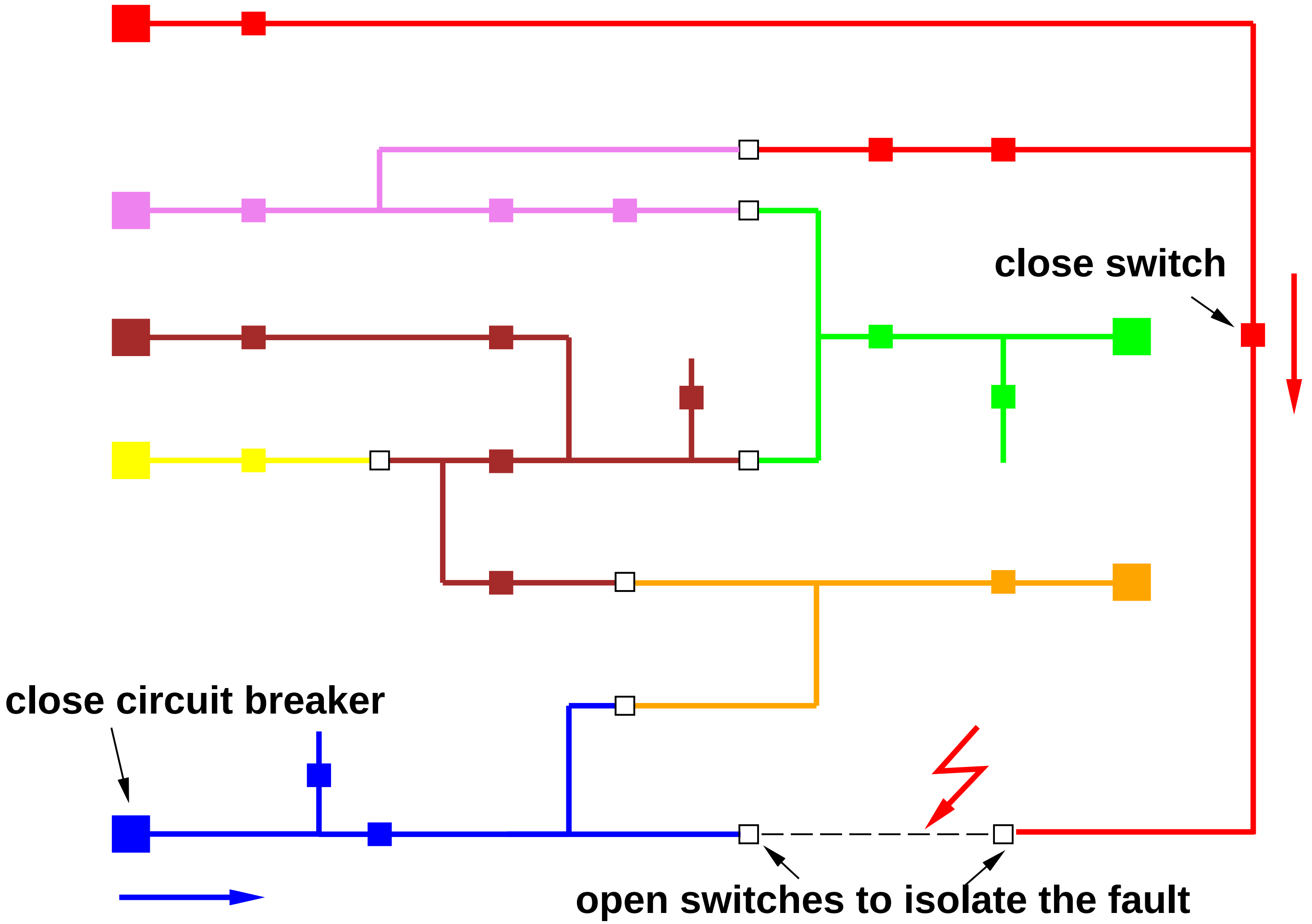

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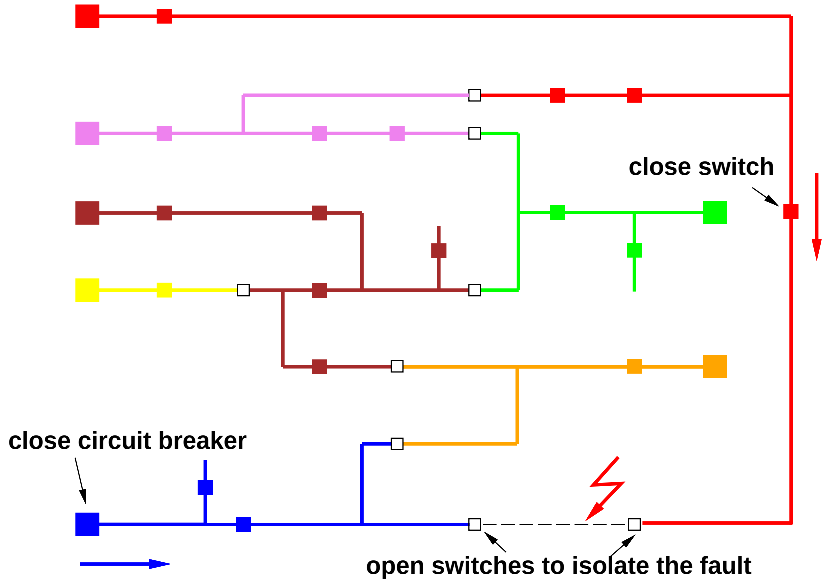


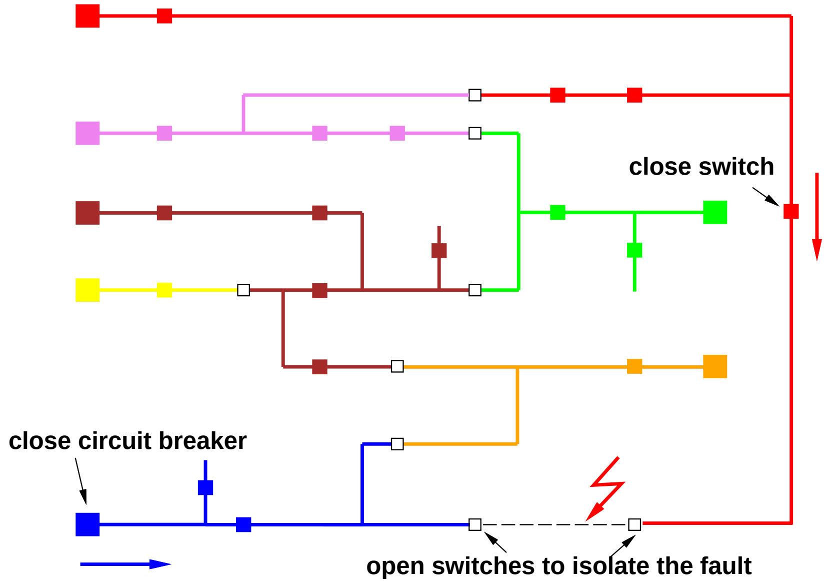
states??: connections, status faulty/non-faulty of the lines,
positions open/closed of the switches and circuit-breakers,

actions??: open/close a switch or a circuit-breaker

goal test??: resupply all non-faulty lines

path cost??: number of actions

Example: power supply restoration



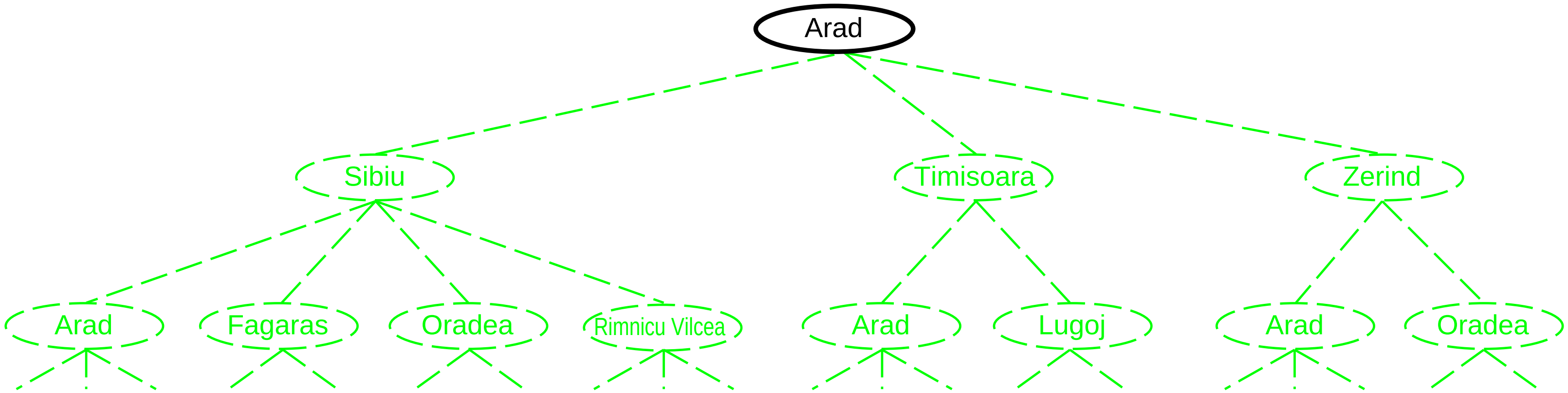
states??: connections, status faulty/non-faulty of the lines,
positions open/closed of the switches and circuit-breakers,
power consummed on each line, capacity of circuit-breakers and lines

actions??: open/close a switch or a circuit-breaker without exceeding capacity

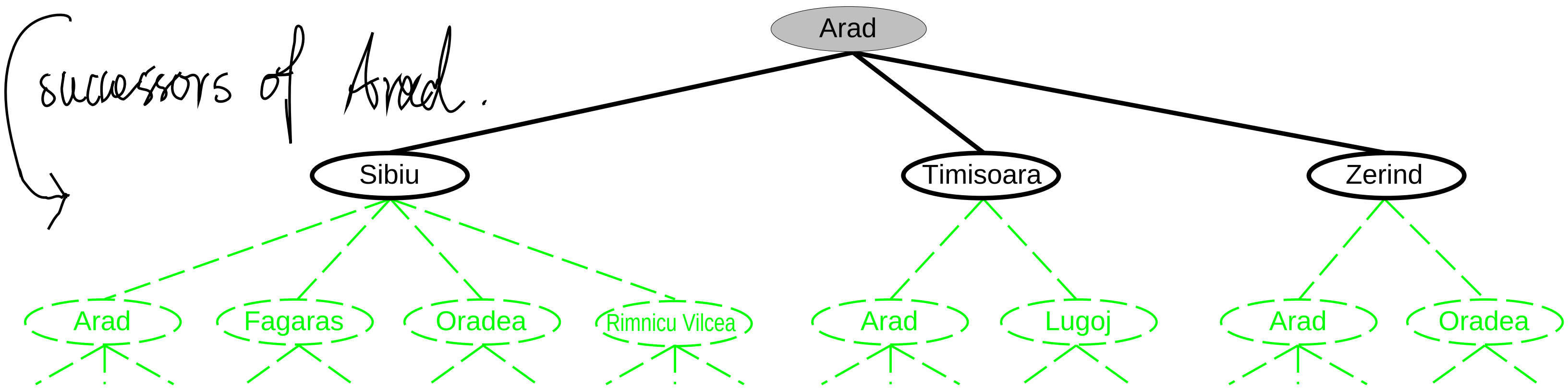
goal test??: resupply all non-faulty lines

path cost??: number of actions, power margins

Tree search example



Tree search example



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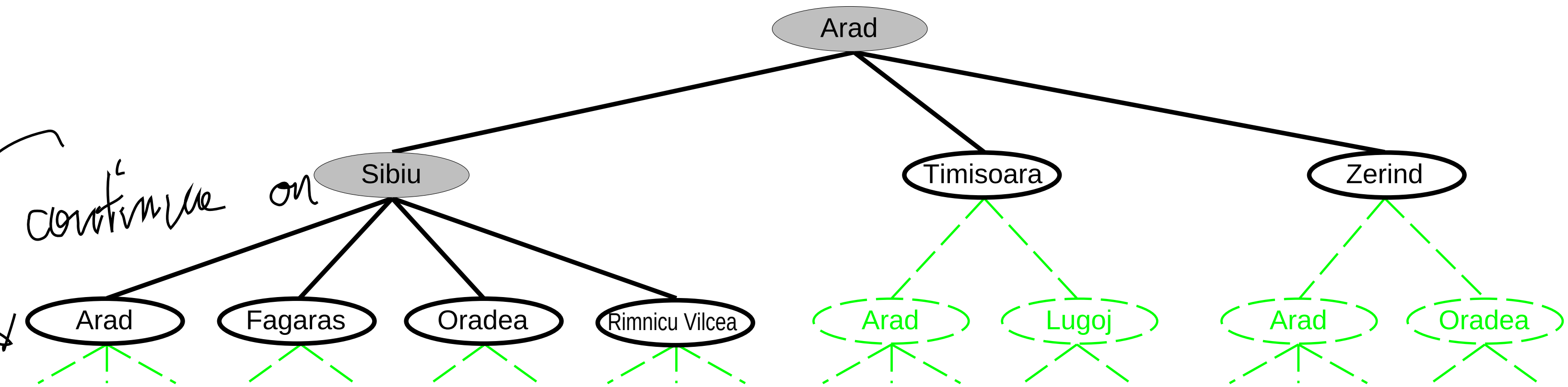


continue on

⋮

until we have a goal.

Tree search algorithm

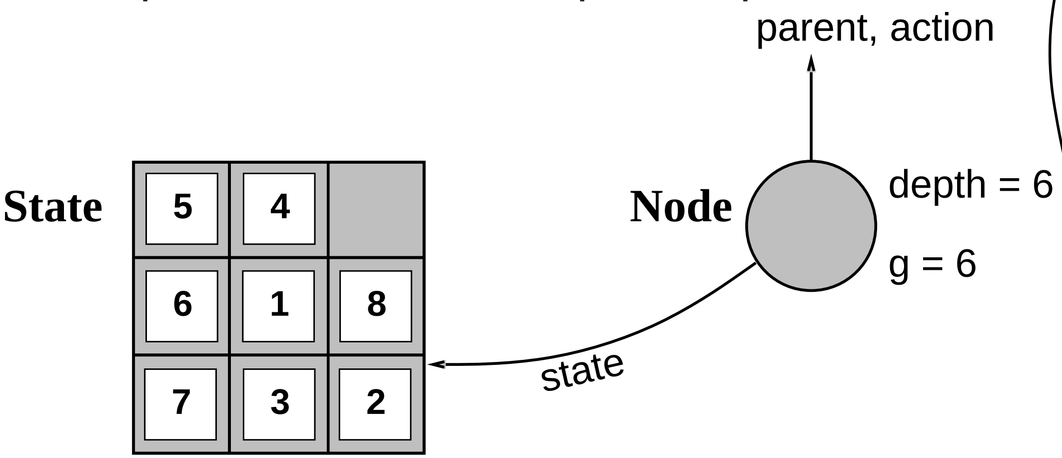
Basic idea:

offline, simulated exploration of state space
by generating successors of already-explored nodes
(a.k.a. expanding nodes)

```
function TREE-SEARCH(problem, strategy) returns a solution, or failure
   initialize the search tree using the initial state of problem
   loop do
       if there are no candidates for expansion on the frontier then return failure
       choose a frontier node for expansion according to strategy
       if the node contains a goal state then return the corresponding solution
       else expand the node and add the resulting nodes to the frontier of the tree
   end
```

Implementation: states vs. nodes

A state is a (representation of) a physical configuration
A node is a data structure constituting part of a search tree
includes state, parent, children, depth, path cost $g(n)$
States do not have parents, children, depth, or path cost!



State

Expand Current Nodes

Form New State w/ New Nodes

The EXPAND function creates new nodes, filling in the various fields and using the SUCCESSORFN of the problem to create the corresponding states.

Frontier implemented as priority queue of nodes ordered according to strategy

Implementation: general tree search

function TREE-SEARCH(*problem, frontier*) **returns** a solution, or failure
    frontier ← INSERT(MAKE-NODE(INITIAL-STATE[*problem*]), *frontier*)
    loop do
emp   **if** *frontier* is empty **then return** failure
        node ← REMOVE-FRONT(*frontier*) ≪ Clear the previous state
res   **if** GOAL-TEST(*problem*, STATE(*node*)) **then return** *node*
nxt   *frontier* ← INSERTALL(EXPAND(*node, problem*), *frontier*) ≪ insert the new state

---

function EXPAND(*node, problem*) **returns** a set of nodes
    successors ← the empty set
    for each *action, result* **in** SUCCESSOR-FN(*problem*, STATE[*node*]) **do**
        s ← a new NODE
        PARENT-NODE[*s*] ← *node*; ACTION[*s*] ← *action*; STATE[*s*] ← *result*
        PATH-COST[*s*] ← PATH-COST[*node*] + STEP-COST(STATE[*node*], *action*, *result*)
        DEPTH[*s*] ← DEPTH[*node*] + 1 // depth count for tree search
        add *s* to *successors*
    return *successors*

Uninformed search strategies

A strategy is defined by picking the **order of node expansion**

This is the order used for the **priority queue** implementing the frontier

distance/ measurement from side to side of something

Uninformed strategies use only the information available in the definition of the problem

Breadth-first search

Uniform-cost search

Depth-first search

Depth-limited search

Iterative deepening search

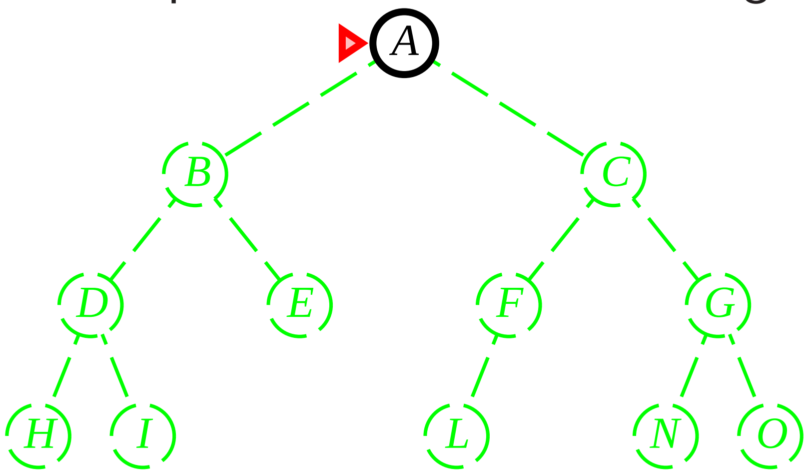
Breadth-first search

Expand shallowest unexpanded node

Implementation:

frontier is a FIFO queue, i.e., new successors go at end

"first in. first out"



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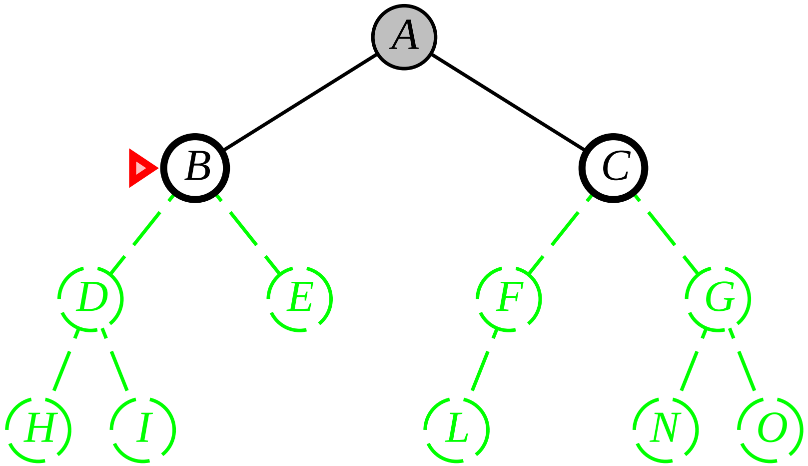

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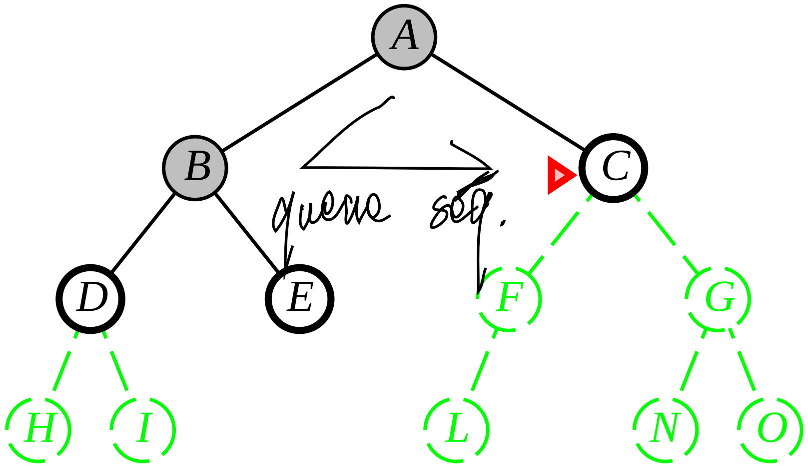

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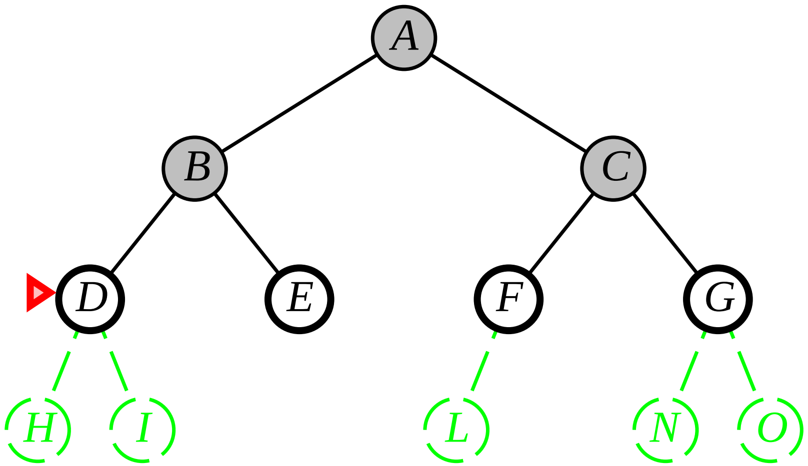

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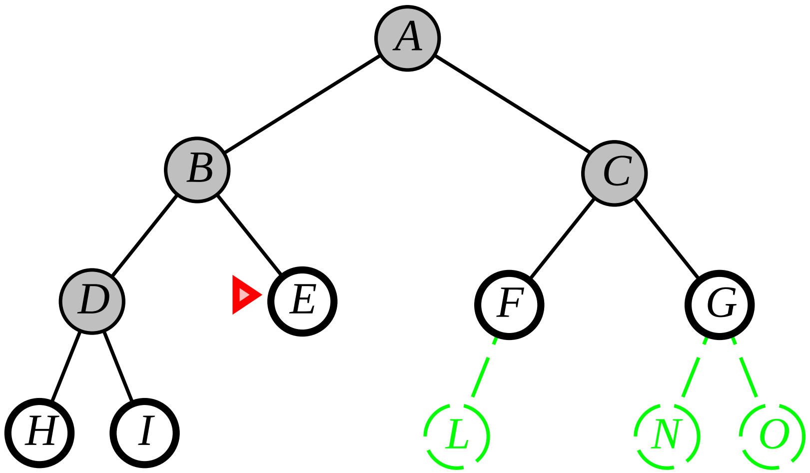

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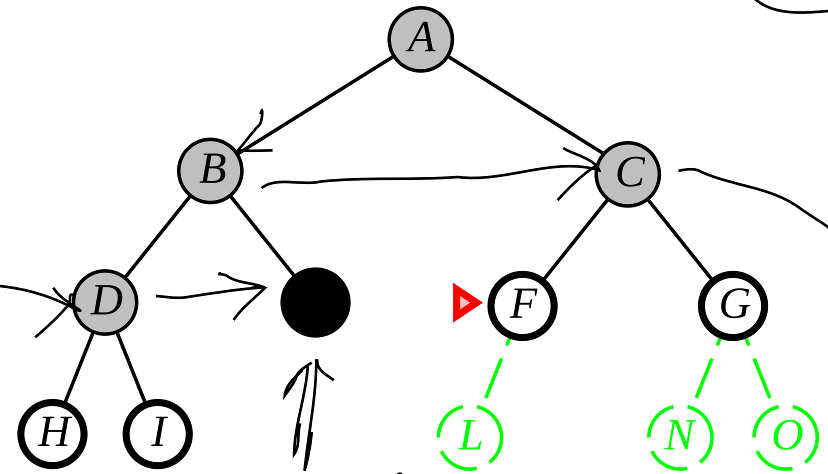

since no further expansions, investigate this point.

Breadth-first search

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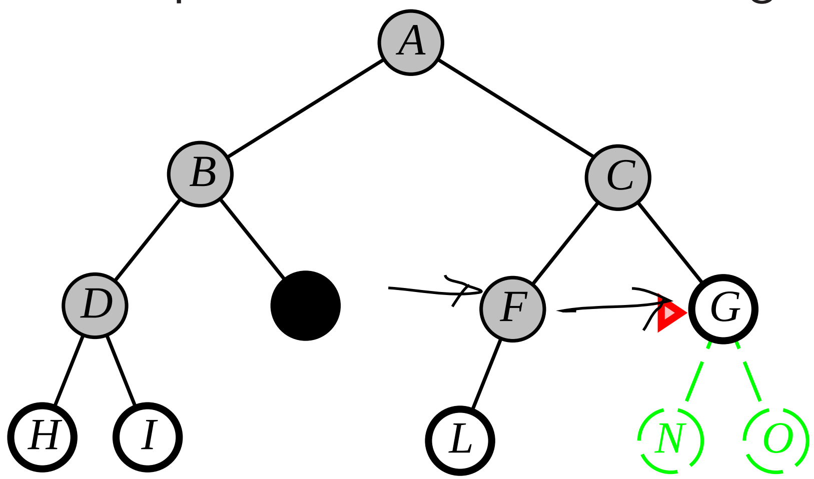


continue expansion

Search strategies

A strategy is defined by picking the **order of node expansion**

Strategies are evaluated along the following dimensions:

completeness—does it always find a solution if one exists?
solution optimality—does it always find a least-cost solution?
time complexity—number of nodes generated (or expanded)
space complexity—maximum number of nodes in memory

Time and space complexity are measured in terms of

b—maximum branching factor of the search tree
d—depth of the shallowest solution
m—maximum depth of the state space (may be ∞)

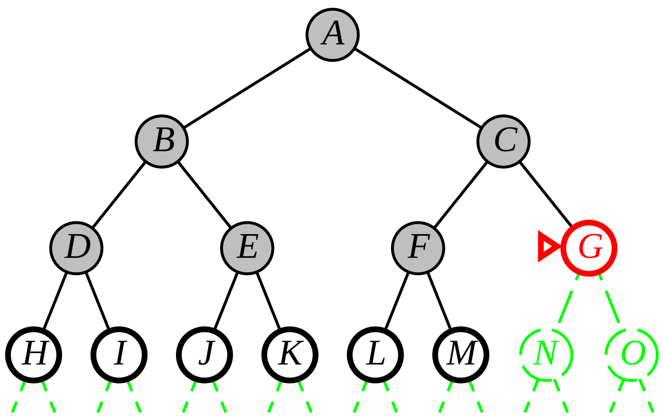
Properties of breadth-first search

Complete??

Properties of breadth-first search

Complete?? Yes (if b and d are finite)

Time??



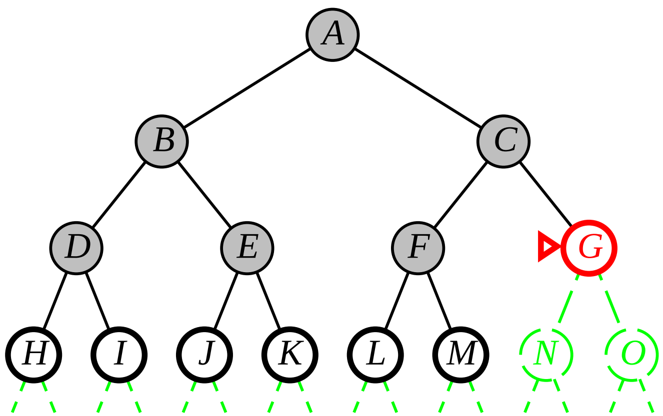
depth	*#nodes*
0	b^0
1	b^1
2=d	$b^2 = b^d$
3=d+1	$b^3 - b = b^{d+1} - b$

Properties of breadth-first search

Complete?? Yes (if b and d are finite)

Time?? $1 + b + b^2 + b^3 + \ldots + b^d + (b^{d+1} - b) = O(b^{d+1})$, i.e., exp. in d

Space??



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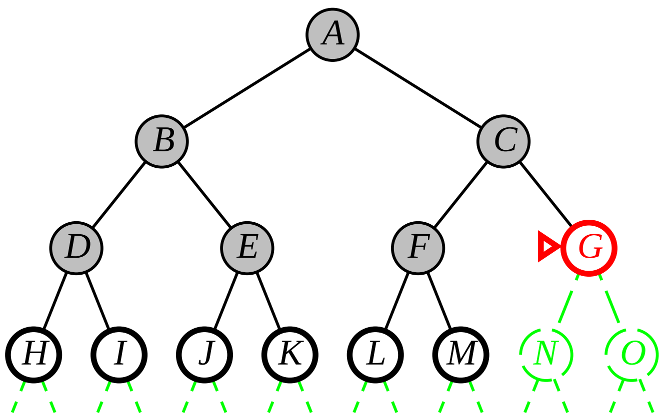
Complete?? Yes (if b and d are finite)

Time?? $1 + b + b^2 + b^3 + \ldots + b^d + (b^{d+1} - b) = O(b^{d+1})$, i.e., exp. in d

Space?? $O(b^{d+1})$

Optimal??

$b = 10$, 1 million node/sec, 1Kb/node, d=12 would take 13 days and 1 petabyte of memory.



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Complete?? Yes (if b and d are finite)

Time?? $1 + b + b^2 + b^3 + \ldots + b^d + (b^{d+1} - b) = O(b^{d+1})$, i.e., exp. in d

Space?? $O(b^{d+1})$

Optimal?? Yes if cost $=$ 1 per step; not optimal in general

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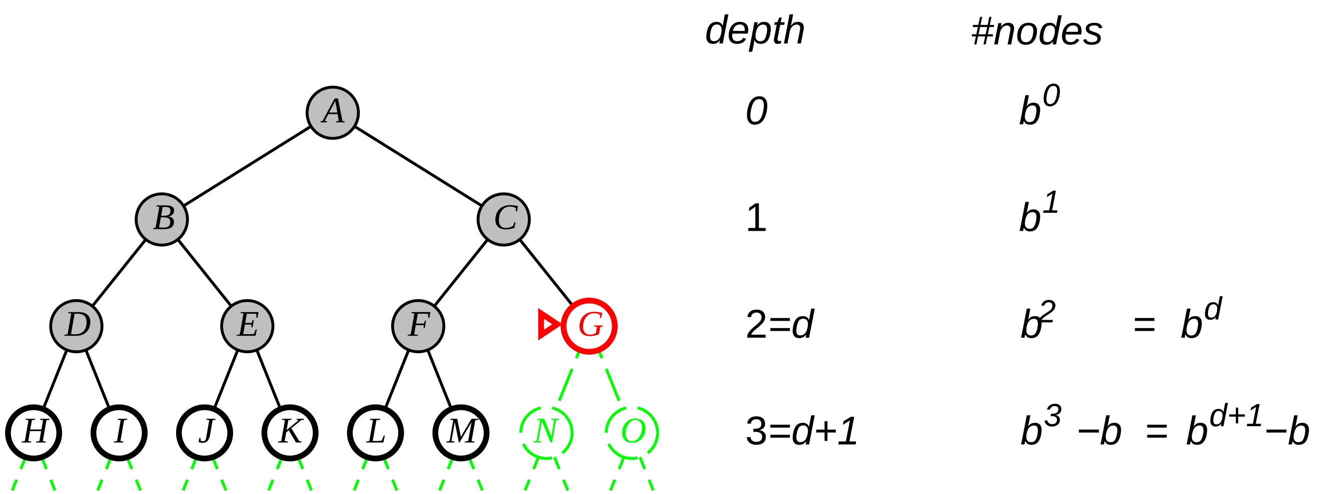


Uniform-cost search

Expand least-cost unexpanded node

Implementation:

$\mathit{frontier}$ = queue ordered by path cost, lowest first

Equivalent to breadth-first if step costs all equal

Complete?? Yes, if step cost $\geq \epsilon$

Time?? # of nodes with $g \leq C^*$, $O(b^{1+\lceil C^*/\epsilon \rceil})$
where C^* is the cost of the optimal solution

Space?? # of nodes with $g \leq C^*$, $O(b^{1+\lceil C^*/\epsilon \rceil})$

Optimal?? Yes—nodes expanded in increasing order of $g(n)$

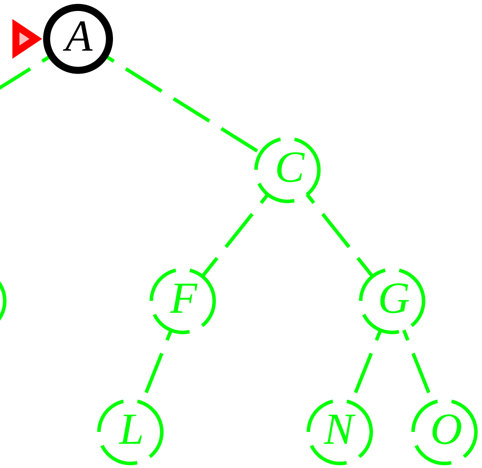
Depth-first search

Expand deepest unexpanded node

Implementation:

frontier = LIFO queue, i.e., put successors at front

"last in first out"



Depth-first search

Expand deepest unexpanded node

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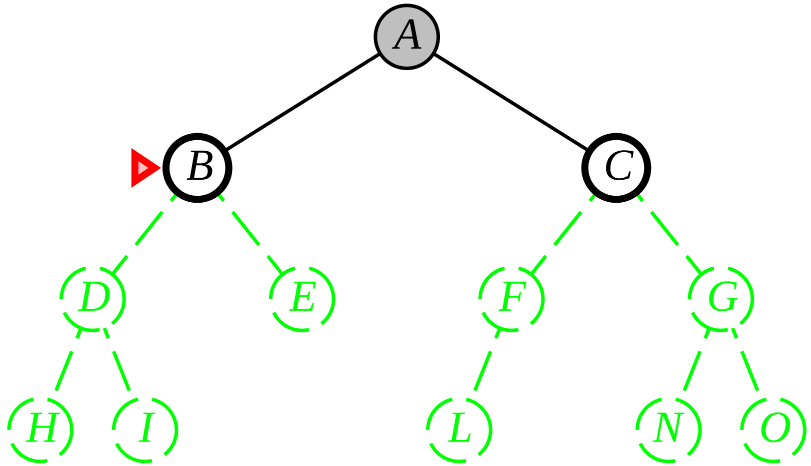

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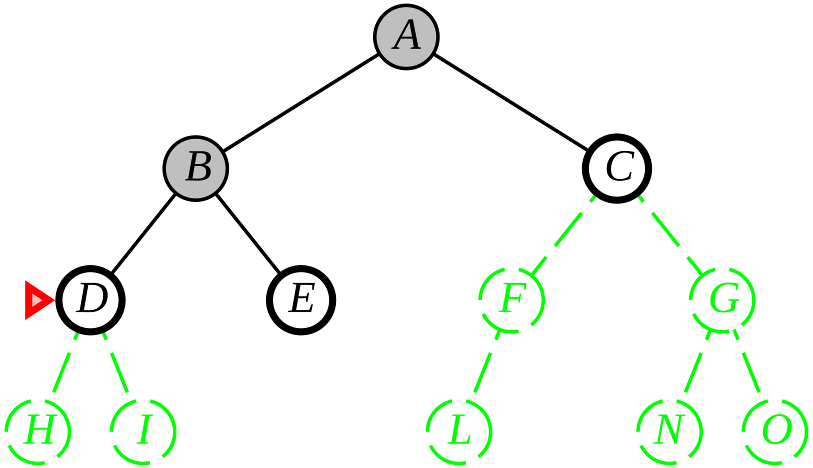

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Expand deepest unexpanded node

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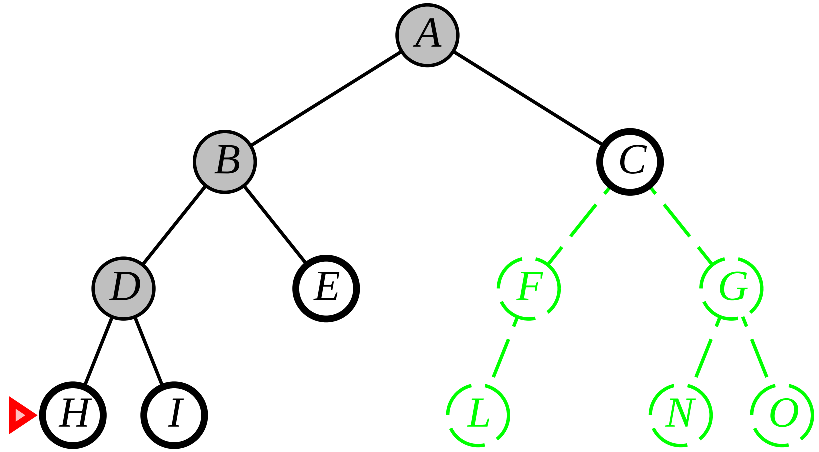

Depth-first search

Expand deepest unexpanded node

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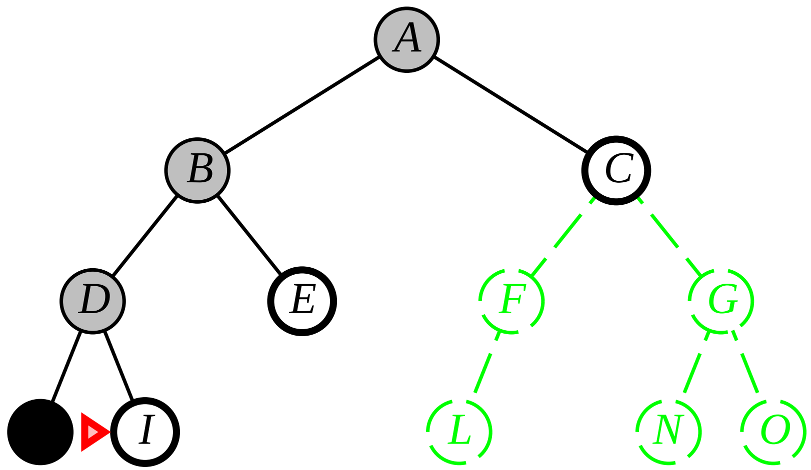

Depth-first search

Expand deepest unexpanded node

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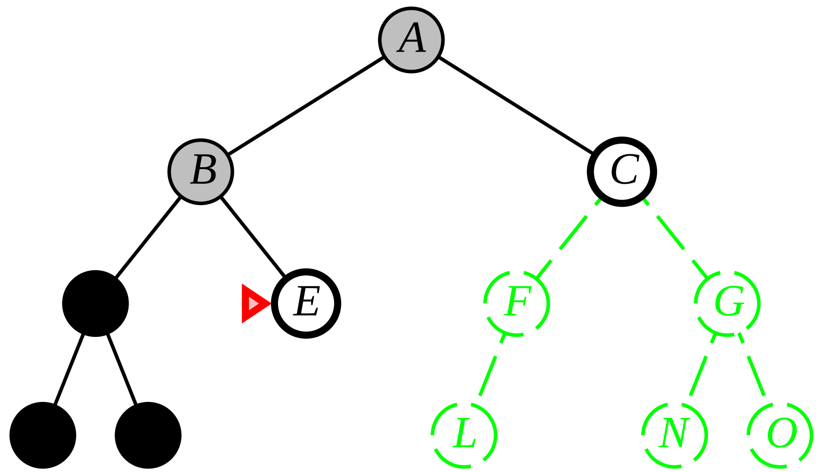

Depth-first search

Expand deepest unexpanded node

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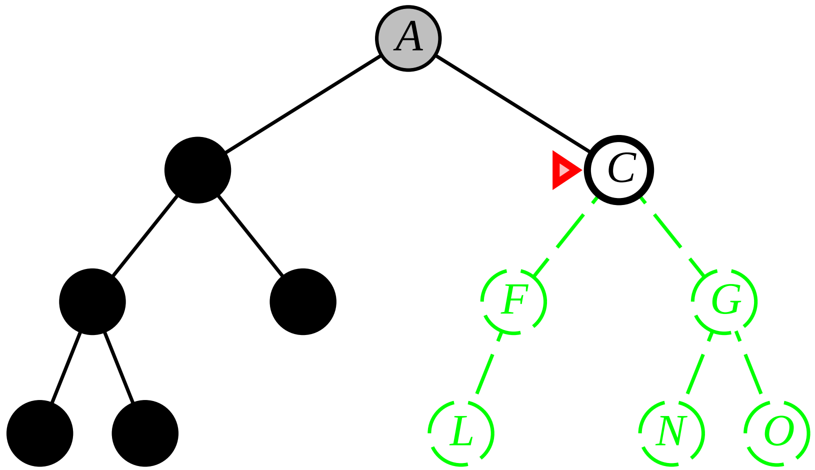

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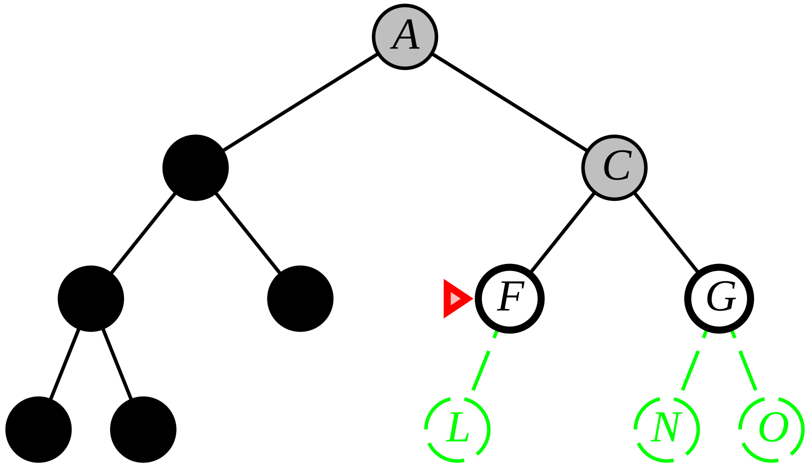

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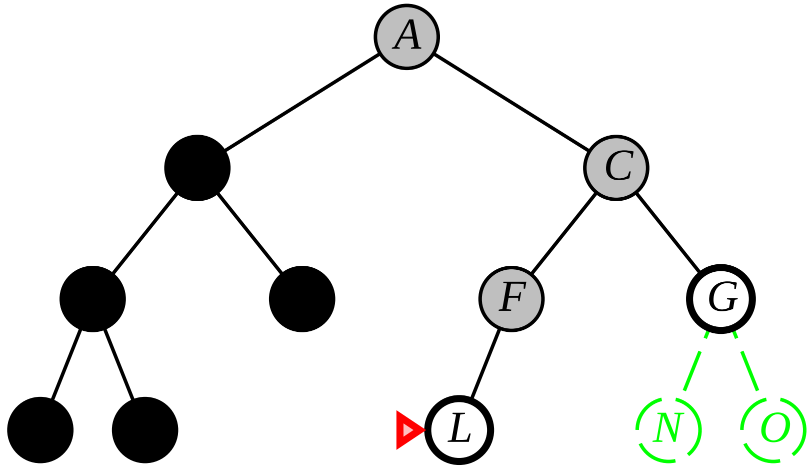

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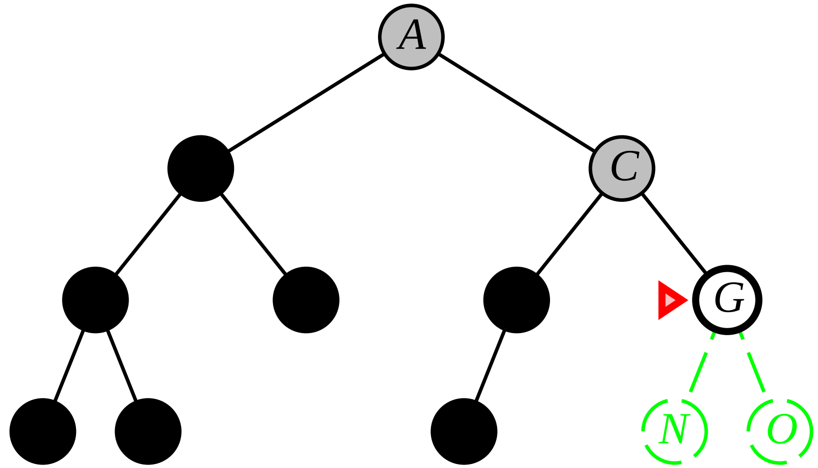


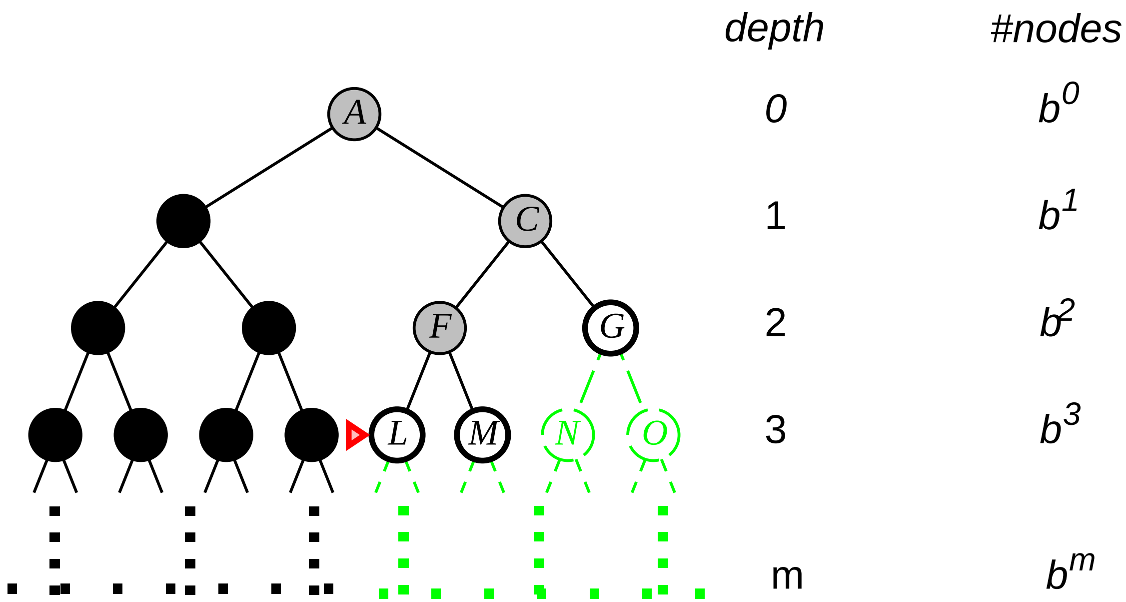
Properties of depth-first search

Complete??

Properties of depth-first search

Complete?? No: fails in infinite-depth spaces, spaces with loops
Modify to avoid repeated states along path
$\Rightarrow$ complete in finite spaces

Time??

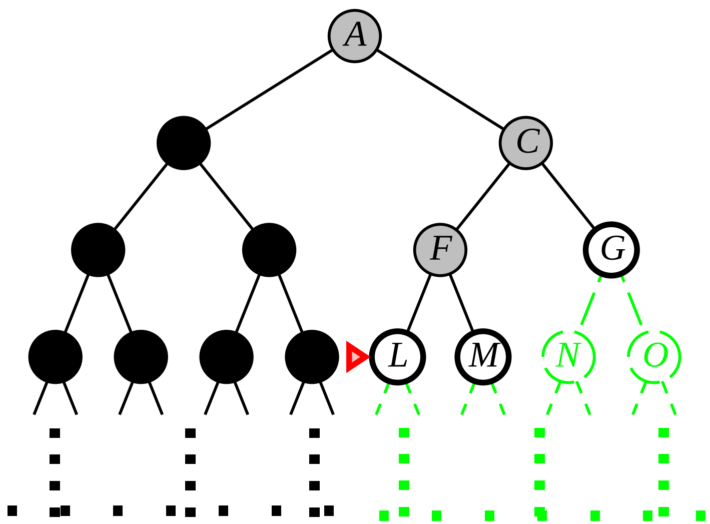


Properties of depth-first search

Complete?? No: fails in infinite-depth spaces, spaces with loops
Modify to avoid repeated states along path
$\Rightarrow$ complete in finite spaces

Time?? $O(b^m)$: terrible if m is much larger than d
but if solutions are dense, may be much faster than breadth-first

Space??



depth	*#nodes*
0	b^0
1	b^1
2	b^2
3	b^3
m	b^m

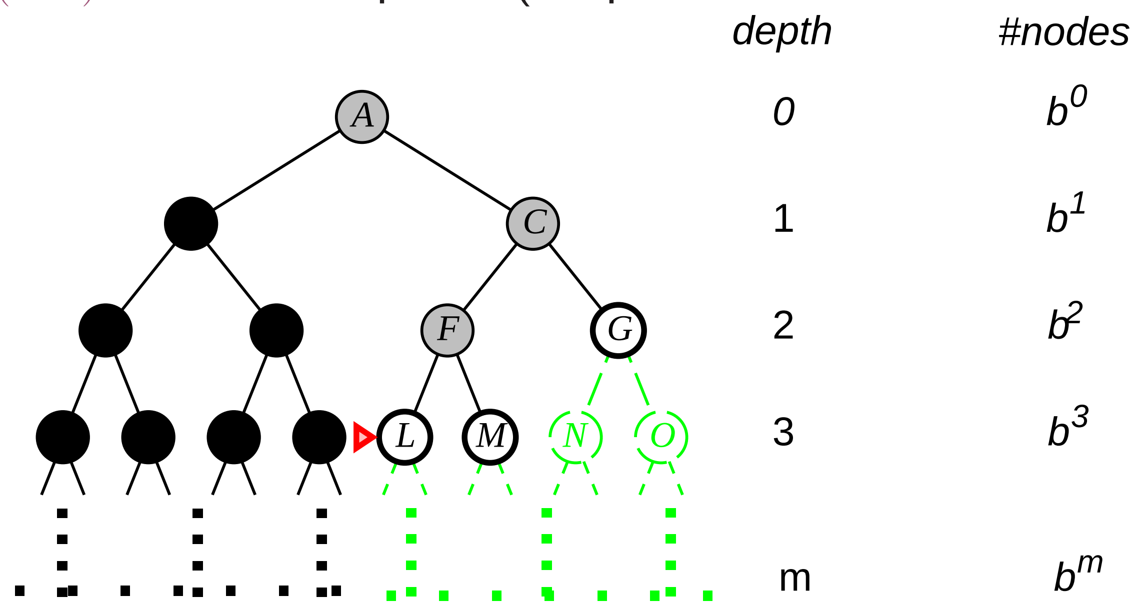
Properties of depth-first search

Complete?? No: fails in infinite-depth spaces, spaces with loops
Modify to avoid repeated states along path
$\Rightarrow$ complete in finite spaces

Time?? $O(b^m)$: terrible if m is much larger than d
but if solutions are dense, may be much faster than breadth-first

Space?? $O(bm)$, i.e., linear space! (deepest node+ancestors+their siblings)

Optimal??



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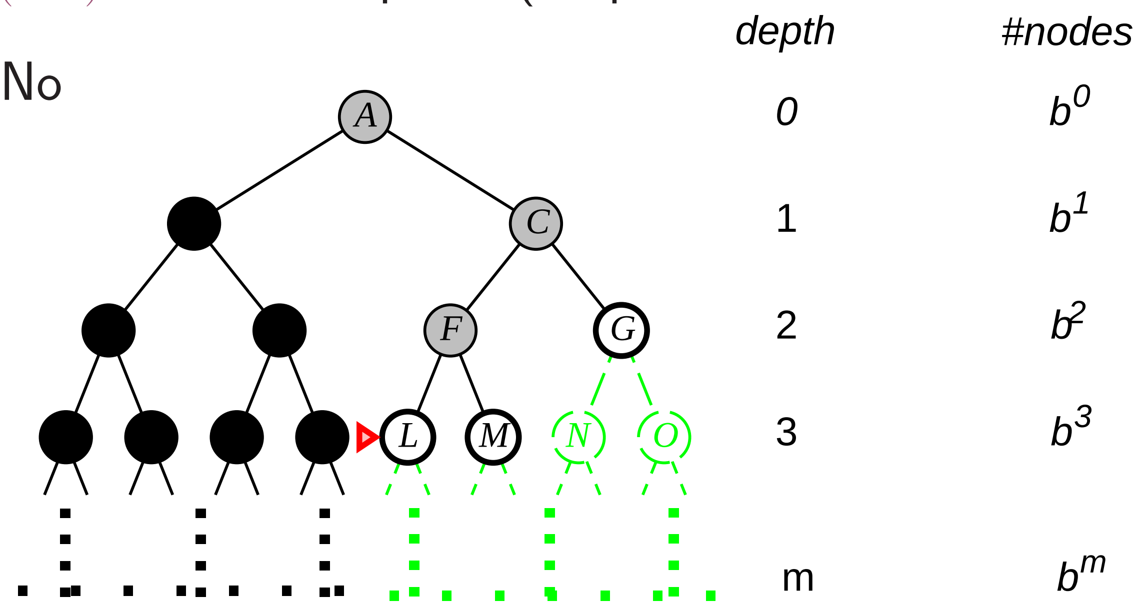
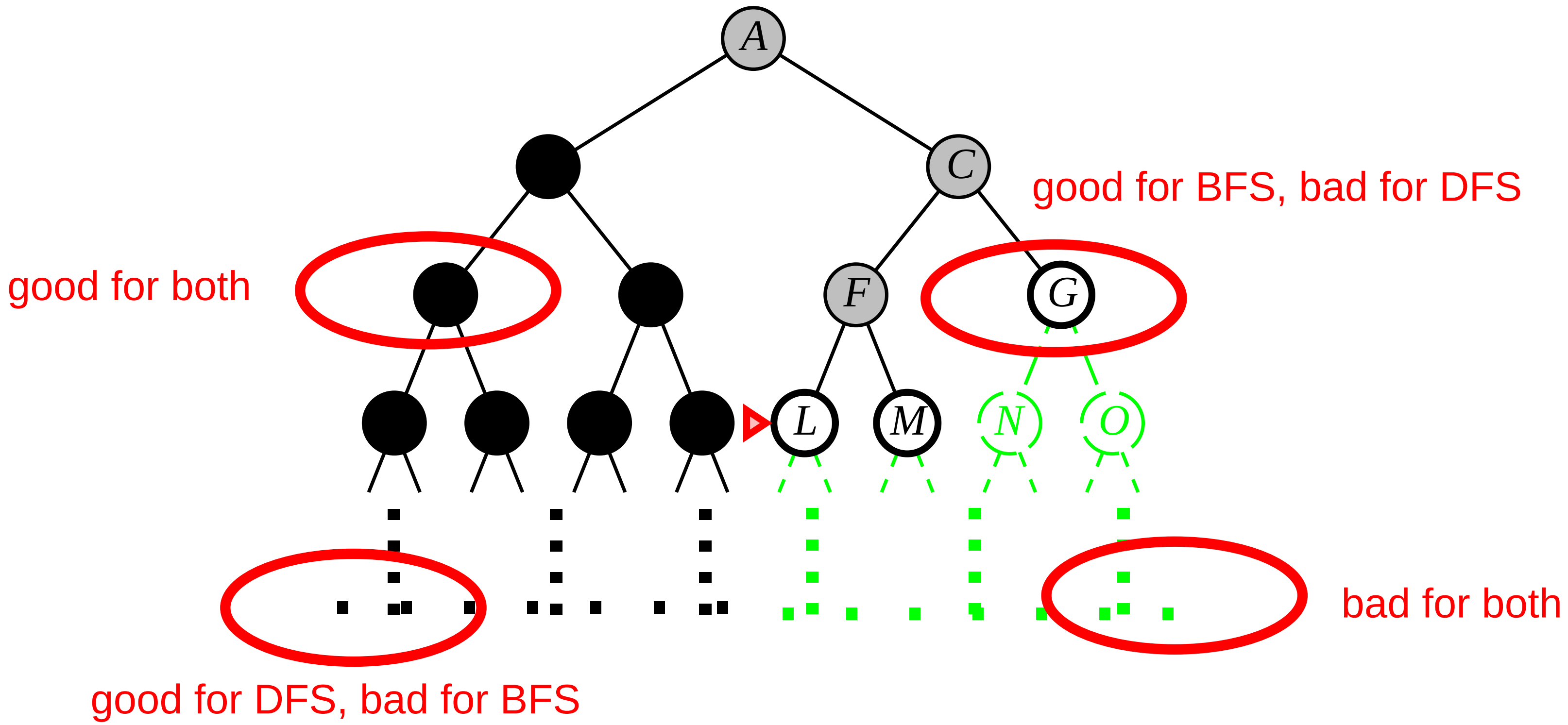


Breadth-first versus depth-first search

Use **breadth-first** search when there exists **short** solutions.

Use **depth-first** search when there exists **many** solutions.



Eternity II Puzzle

Use **breadth-first** search when there exists **short** solutions.

Use **depth-first** search when there exists **many** solutions.

2 million dollar prize! Few deep solutions

Iterative deepening search

Combines advantages of breadth-first and depth-first search:

completeness
returns shallowest solution
use linear amount of memory

Performs a **series of depth limited depth-first searches**

```
function ITERATIVE-DEEPENING-SEARCH(problem) returns solution/failure
    inputs: problem, a problem

    for depth ← 0 to ∞ do
        result ← DEPTH-LIMITED-SEARCH(problem, depth)
        if result ≠ cutoff then return result
    end
```

// Haskell Assignment Algorithm ?

Depth-limited search

= depth-first search with depth limit l,
i.e., nodes at depth l have no successors

Recursive implementation:

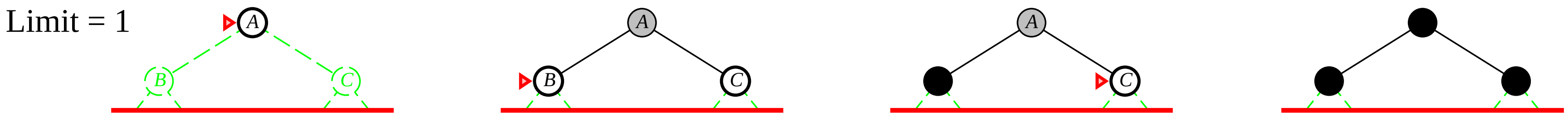
```
function DEPTH-LIMITED-SEARCH( problem, limit) returns soln/fail/cutoff
   RECURSIVE-DLS(MAKE-NODE(INITIAL-STATE[problem]), problem, limit)

function RECURSIVE-DLS(node, problem, limit) returns soln/fail/cutoff
   cutoff-occurred? ← false
   if GOAL-TEST(problem, STATE[node]) then return node
   else if DEPTH[node] = limit then return cutoff
   else for each successor in EXPAND(node, problem) do
       result ← RECURSIVE-DLS(successor, problem, limit)
       if result = cutoff then cutoff-occurred? ← true
       else if result ≠ failure then return result
   if cutoff-occurred? then return cutoff else return failure
```

cutoff: no solution within the depth limit, failure: the problem has no solution

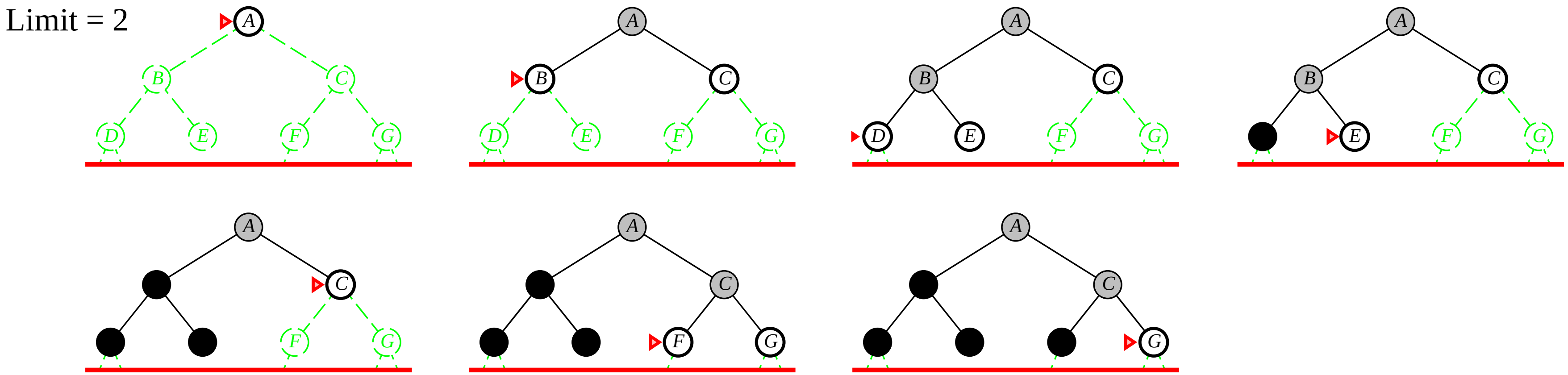
Iterative deepening search $l = 0$

Limit = 0

Iterative deepening search $l = 1$



Iterative deepening search $l = 2$



Properties of iterative deepening search

Complete??

Properties of iterative deepening search

Complete?? Yes (if b and d are finite)

Time??

Properties of iterative deepening search

Complete?? Yes (if b and d are finite)

Time?? $(d+1)b^0 + db^1 + (d-1)b^2 + \ldots + b^d = O(b^d)$

Space??

Properties of iterative deepening search

Complete?? Yes (if b and d are finite)

Time?? $(d+1)b^0 + db^1 + (d-1)b^2 + \ldots + b^d = O(b^d)$

Space?? $O(bd)$

Optimal??

Properties of iterative deepening search

Complete?? Yes (if b and d are finite)

Time?? $(d+1)b^0 + db^1 + (d-1)b^2 + \ldots + b^d = O(b^d)$

Space?? $O(bd)$

Optimal?? Yes, if step cost $= 1$
Can be modified to explore uniform-cost tree

Numerical comparison for $b = 10$ and $d = 5$, solution at far right leaf:

Time: IDS does better because other nodes at depth d are not expanded

$$
\begin{aligned}
N(\mathsf{IDS}) &= 6 + 50 + 400 + 3,000 + 20,000 + 100,000 = 123,456 \\
N(\mathsf{BFS}) &= 1 + 10 + 100 + 1,000 + 10,000 + 100,000 + 999,990 = 1,111,101
\end{aligned}
$$

BFS can be modified to apply goal test when a node is **generated**

Space: IDS does much better N(IDS)$=$ 50, N(BFS) $\simeq$ 1,000,000

Summary of algorithms

Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening
Complete?	Yes*	Yes*	No	Yes, if $l \geq d$	Yes*
Time	b^{d+1}	$b^{1+\lceil C^*/\epsilon \rceil}$	b^m	b^l	b^d
Space	b^{d+1}	$b^{1+\lceil C^*/\epsilon \rceil}$	bm	bl	bd
Optimal?	Yes*	Yes	No	No	Yes*

Summary

◇ **Problem solving agents**
formulate a problem, search off-line for a solution, execute it

◇ **Problem formulation**
initial state, successor function, goal test, path cost

◇ **Example problems**
traveling around romania, 8-puzzle, power supply restoration

◇ **Tree search algorithms**
build and explore a tree, strategy picks up the order of node expansion

◇ **Implementation**
select the first node on the frontier, test for goal, expand

◇ **(Uninformed) strategies** (breadth first, uniform cost, ...)
characterised by their completeness, optimality, complexity

◇ **Iterative deepening** complete, finds the shallowest solution
uses only linear space and no more time than uninformed strategies