Answers to MCQs (Section A)

1. A 2. D 3. A 4. B 5. C

6. E 7. D

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8. B

Differentiate a few times, and the pattern suggests

$$f^{(n)}(x) = (-1)^n (x-n+2)e^{-x}$$

Therefore, $f^{(2012)}(-1) = -2011e$

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9. B $f(x) = x^8 + x^7 + |x|$

By the chain rule, $g'(-1) = f'(f(-1))\, f'(-1)$.

One checks that $f(-1) = (-1)^8 + (-1)^7 + |-1| = 1$.

Therefore, $g'(-1) = f'(1)\, f'(-1)$.

Now $f'(x) = \begin{cases} 8x^7 + 7x^6 + 1 & \text{if } x > 0 \\ 8x^7 + 7x^6 - 1 & \text{if } x < 0 \end{cases}$ (Note that we are using the fact that $\frac{d}{dx}(|x|) = \begin{cases} 1 & \text{if } x > 0 \\ -1 & \text{if } x < 0 \end{cases}$

Therefore, $f'(-1) = 8(-1)^7 + 7(-1)^6 - 1 = -2$

$f'(1) = 8(1)^7 + 7(1) + 1 = 16$

and $g'(-1) = (16)(-2) = -32$

10. E

$$\lim_{x\to 0^+} \frac{x+1-\sqrt{x^2+6x+1}}{x+3-\sqrt{x^2+3x+9}} = \lim_{x\to 0^+} \frac{x+1-\sqrt{x^2+6x+1}}{x+3-\sqrt{x^2+3x+9}} \cdot \frac{x+1+\sqrt{x^2+6x+1}}{x+1+\sqrt{x^2+6x+1}} \cdot \frac{x+3+\sqrt{x^2+3x+9}}{x+3+\sqrt{x^2+3x+9}}$$

$$= \lim_{x\to 0^+} \frac{(x+1)^2-(x^2+6x+1)}{(x+3)^2-(x^2+3x+9)} \cdot \frac{x+3+\sqrt{x^2+3x+9}}{x+1+\sqrt{x^2+6x+1}}$$

$$= \lim_{x\to 0^+} \frac{x^2+2x+1-(x^2+6x+1)}{x^2+6x+9-(x^2+3x+9)} \cdot \frac{x+3+\sqrt{x^2+3x+9}}{x+1+\sqrt{x^2+6x+1}}$$

$$= \lim_{x\to 0^+} \frac{-4\cancel{x}}{3\cancel{x}} \cdot \frac{x+3+\sqrt{x^2+3x+9}}{x+1+\sqrt{x^2+6x+1}}$$

$$= \lim_{x\to 0^+} \left(-\frac{4}{3}\right) \cdot \frac{x+3+\sqrt{x^2+3x+9}}{x+1+\sqrt{x^2+6x+1}}$$

$$= -\frac{4}{3} \cdot \frac{0+3+\sqrt{0+0+9}}{0+1+\sqrt{0+0+1}} = -4$$

(Partial) Solution to Section B.

1. $$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h}$$

The remainder of the solution to this problem can be found in the solution to (Section 2.3, Exercise 32).

2(a) $7(x^2+8)^6(2x)(5x-4)^9 + 9(x^2+8)^7(5x-4)^8(5)$

(b) $$\frac{(1+x^3)(2) - (2x-6)(3x^2)}{(1+x^3)^2}$$

(c) $\cos(\sin(\sin x))\cos(\sin x)\cos x$

3(a) $$f'(x) = \frac{1}{2}(3x^2+1)^{-\frac{1}{2}}(6x) = \frac{3x}{\sqrt{3x^2+1}}$$

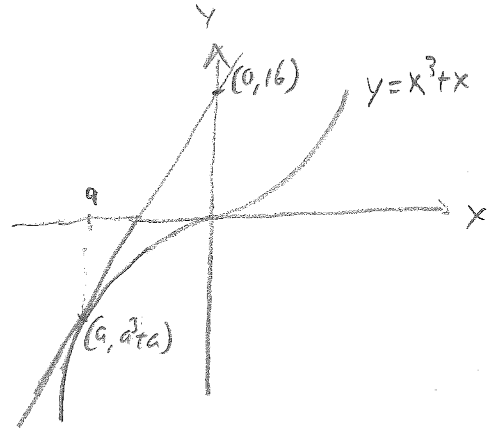
$$f'(1) = \frac{3(1)}{\sqrt{3(1)+1}} = \frac{3}{\sqrt{4}} = \frac{3}{2}$$

(b) $$f'(x) = -(e^{2x}-1)^{-2}(2e^{2x}) = -\frac{2e^{2x}}{(e^{2x}-1)^2}$$

$$f'(\ln 2) = -\frac{2e^{2\ln 2}}{(e^{2\ln 2}-1)^2} = -\frac{2e^{\ln 4}}{(e^{\ln 4}-1)^2} = -\frac{2(4)}{(4-1)^2} = -\frac{8}{9}$$

4. The point in question has the form (a, a^3+a) (since it lies on the curve $y = x^3+x$)

Let m be the slope of the line in question.



We can calculate m in two ways:

① $m = \dfrac{a^3+a-16}{a-0} = \dfrac{a^3+a-16}{a}$

② $m = \left.\dfrac{dy}{dx}\right|_{x=a} = 3a^2+1$

Equating these two, we obtain

$$\frac{a^3+a-16}{a} = 3a^2+1$$

$$a^3+a-16 = 3a^3+a$$

$$2a^3 = -16$$

$$a^3 = -8$$

$$a = -2.$$

Therefore, $m = 3(-2)^2+1 = 13.$

The line is $y-16 = 13(x-0)$

or $y = 13x+16.$

5a. $\lim_{x\to\infty} \dfrac{1+\sqrt{x}}{\sqrt{1+2x}} = \lim_{x\to\infty} \dfrac{1+\sqrt{x}}{\sqrt{1+2x}} \cdot \dfrac{1/\sqrt{x}}{1/\sqrt{x}}$

$= \lim_{x\to\infty} \dfrac{\frac{1}{\sqrt{x}}+1}{\sqrt{\frac{1}{x}+2}} = \dfrac{1}{\sqrt{2}}$

b. $\lim_{x\to-\infty} \dfrac{3e^{2x}+e^{x}+1}{e^{2x}-1} = \dfrac{0+0+1}{0-1}$ (because $\lim_{x\to-\infty} e^{x}=0$ and $\lim_{x\to-\infty} e^{2x}=0$)

$= -1$

$\lim_{x\to\infty} \dfrac{3e^{2x}+e^{x}+1}{e^{2x}-1} = \lim_{x\to\infty} \dfrac{3e^{2x}+e^{x}+1}{e^{2x}-1} \cdot \dfrac{1/e^{2x}}{1/e^{2x}}$

$= \lim_{x\to\infty} \dfrac{3+e^{-x}+e^{-2x}}{1-e^{-2x}}$

$= \dfrac{3+0+0}{1-0} = 3$