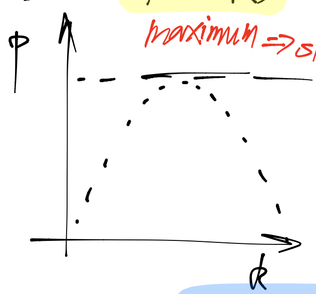
Ch. 2

#11 $\frac{P(X=k+1)}{P(X=k)} = \frac{\binom{n}{k+1} p^{k+1} q^{n-k-1}}{\binom{n}{k} p^k q^{n-k}} = \frac{(n-k)\cdot p}{(k+1)(1-p)} = 1$



maximum $\Rightarrow$ slope $=1$

$k=(n+1)p-1$

$\rightarrow k = \lceil (n+1)p \rceil$

#30. Poisson:

$$\boxed{p(X=k) = \frac{\lambda^k}{k!} e^{-\lambda}}$$

Suicide problem: $\lambda = 0.33/\text{month}$

$\lambda_1 = 0.33 \times 12/\text{year}$

$\lambda_2 = 0.33/4$ week

.

30. Suppose that in a city, the number of suicides can be approximated by a Poisson process with λ = .33 per month.

a. Find the probability of k suicides in a year for k = 0, 1, 2, ... What is the most probable number of suicides? b. What is the probability of two suicides in one week?

a). $k=0, k=1, k=2, \cdots$

b). $P(X=k) = \frac{\lambda_2^k}{k!} e^{-\lambda_2} = \frac{\lambda_2^2}{2!} e^{-\lambda_2} = \cdots$ (USE A Different λ !)

31. Phone calls are received at a certain residence as a Poisson process with parameter λ = 2 per hour.

a. If Diane takes a 10-min. shower, what is the probability that the phone rings during that time? b. How long can her shower be if she wishes the probability of receiving no phone calls to be at most .5?

a). $\lambda_1 = \frac{2}{6} = 0.33$ per 10minutes

b) $e^{-\frac{2}{n}} = 0.5$

$n = 2.89$

$T = 60/2.89 = 20.79$ min

$-\alpha x^{\beta}$

33. Let F(x) = 1 − exp(−αx β) for x ≥ 0, α > 0, β > 0, and F(x) = 0 for x < 0. Show that F is a cdf, and find the corresponding density

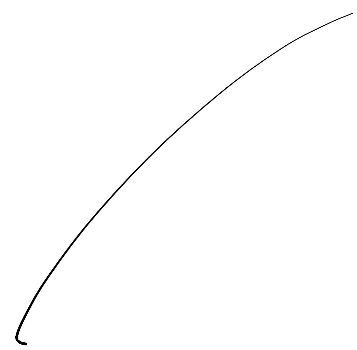
What makes a cdf?

① $\lim_{x \to -\infty} f(x) = 0$ ...

② $\lim_{x \to \infty} f(x) = 1$ ...

③ left-continuous ...

④ monotone ↗ $\frac{dF(x)}{dx} = \alpha\beta x^{\beta-1} \cdot \exp(-\alpha x^{\beta}) > 0$



cx^2

40. Suppose that X has the density function f (x) = cx 2 for 0 ≤ x ≤ 1 and f (x) = 0 otherwise.

a. Find c. b. Find the cdf. c. What is P(.1 ≤ X < .5)?

a) $f(x) = \begin{cases} cx^2, & 0 \le x < 1 \\ 0, & \text{else} \end{cases}$

f is a pdf.

$$F(x) = \begin{cases} \int_0^x cx^2 dx = \frac{c}{3}x^3 \Big|_0^x = \frac{c}{3}x^3 & 0 \le x < 1 \\ 0, & x \le 0 \\ 1, & x \ge 1 \end{cases}$$

$\Rightarrow \frac{c}{3} 1^3 = 1 \Rightarrow c = 3$

48. Show that the gamma density integrates to 1.

$$\int_0^\infty g(t)\,dt$$

$$= \frac{\lambda^\alpha}{\Gamma(\alpha)} \cdot \int_0^\infty t^{\alpha-1} e^{-\lambda t}\,dt$$

$$= \frac{1}{\Gamma(\alpha)} \int_0^\infty U(t)^{\alpha-1} e^{-\lambda t} \lambda\,dt = \frac{1}{\Gamma(\alpha)} \int_0^\infty (\lambda t)^{\alpha-1-\lambda t} e\, d(\lambda t)$$

note: $\lambda t = \theta$ $\qquad \frac{1}{\Gamma(\alpha)} \int_0^\infty \theta^{\alpha-1} e^{-\theta}\,d\theta$

#49. a) $\Gamma(1) = \int_0^\infty t^0 e^{-t}\,dt = \int_0^\infty e^{-t}\,dt = 1$

b). $\Gamma(z+1) = \int_0^\infty t^{z+1} e^{-t}\,dt = -t^{z+1} e^{-t}\Big|_0^\infty + \int_0^\infty z\, t^{z-1} e^{-t}\,dt$

$$= 0 + z\int_0^\infty t^{z-1} e^{-t}\,dt$$

c). induction

(odd integer)

d). $n = 2k+1$

$$\Gamma\left(\frac{2k+1}{2}\right) = \frac{2k-1}{2}\Gamma\left(\frac{2k-1}{2}\right)$$

$z = \frac{2k-1}{2}$ as $\Gamma(z+1) = z\Gamma(z)$ just proved in c).

$$= \frac{2k-1}{2} \cdot \frac{2k-3}{2} \cdots \frac{3}{2}\,\frac{1}{2} \cdot \Gamma\left(\frac{1}{2}\right)$$

$$= \frac{(2k-1)!!}{2 \cdot \frac{2k}{2}+1} = \frac{(2k-1)!}{2^{k+1} \cdot (k!\,2^k)}$$

52. Suppose that in a certain population, individuals' heights are approximately nor-mally distributed with parameters μ = 70 and σ = 3 in.

a. What proportion of the population is over 6 ft. tall? b. What is the distribution of heights if they are expressed in centimeters? In meters?

$X \sim N(\mu, \sigma^2)$

$\frac{X-\mu}{\sigma} \sim N(0,1)$