Code: 7938

DEPARTMENT OF MATHEMATICS
University of Toronto
MAT 135Y
Term-Test #1
Wednesday, November 4, 2009
Time allowed: 90 minutes

Please PRINT in INK or BALL-POINT PEN:
(Please PRINT full name and UNDERLINE surname):

NAME OF STUDENT: _____

STUDENT NO.: _____

SIGNATURE OF STUDENT
(in INK or BALL-POINT PEN): _____

TUTORIAL CODE (e.g. M4A, R5D, etc.): _____

TUTORIAL TIME (e.g. T4, R5, F3, etc.): _____

NAME OF YOUR T.A.: _____

NOTE:

1. Before you start, check that this test has 12 pages. There are NO blank pages.
2. This test has two parts:
   PART A [50 marks]: 10 multiple choice questions
   PART B [50 marks]: 7 written questions
   Answers to both PART A and PART B are to be given in this booklet. No computer cards will be used.
3. No aids allowed.
   No calculators!

DO NOT TEAR OUT ANY PAGES.

FOR MARKERS ONLY	
QUESTION	MARK
PART A	/ 50
B1	/ 6
B2	/ 8
B3	/ 8
B4	/ 6
B5	/ 7
B6	/ 7
B7	/ 8
TOTAL	/ 100

PART A *[50 marks]*

Please read carefully:
PART A consists of 10 multiple-choice questions, each of which has exactly one correct answer. Indicate your answer to each question by **completely filling in the appropriate circle with a dark pencil.**
MARKING SCHEME: 5 marks for a correct answer, 0 for no answer or a wrong answer. You are not required to justify your answers in PART A. Note that for PART A, **only your final answers (as indicated by the circles you darken) count; your computations and answers indicated elsewhere will NOT count.**
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1. Find the value of $\lim_{x \to 3} \dfrac{x^2 + 2x - 15}{x^2 - 5x + 6}$.

   Ⓐ 9

   ● 8

   Ⓒ 10

   Ⓓ 7

   Ⓔ 6

2. Find the value of $\lim_{x \to -4} \dfrac{\frac{1}{4} + \frac{1}{x}}{4 + x}$.

   Ⓐ undefined

   Ⓑ $\dfrac{1}{4}$

   ● $-\dfrac{1}{16}$

   Ⓓ $-\dfrac{1}{4}$

   Ⓔ 0

3. Find the value of $\lim_{x\to 0^-}\left(\frac{1-\cos x}{2x}+\frac{\sin 5x}{\sin 4x}\right)^2$.

Ⓐ $\frac{5}{4}$

Ⓑ $-\infty$

Ⓒ $\frac{1}{2}$

● $\frac{25}{16}$

Ⓔ $+\infty$

4. Find the value of $\lim_{x\to\infty}\frac{x^3-x^2+2}{4+x-6x^3}$.

● $-\frac{1}{6}$

Ⓑ $-\frac{1}{3}$

Ⓒ $+\infty$

Ⓓ $\frac{1}{2}$

Ⓔ $\frac{1}{4}$

5. The line tangent to the curve $y=3x^3-2x^2+5$ at the point $(1,\,6)$ will intersect the x-axis at the point

Ⓐ $(\frac{1}{4},\,0)$

Ⓑ $(-\frac{2}{3},\,0)$

Ⓒ $(\frac{1}{7},\,0)$

Ⓓ $(-\frac{3}{2},\,0)$

● $(-\frac{1}{5},\,0)$

6. Let $f(x) = \begin{cases} cx^2 - 2x & \text{if } x \leq 2 \\ x^3 - cx & \text{if } x > 2. \end{cases}$

Find the value of the constant c so that f is continuous everywhere.

● 2

Ⓑ 3

Ⓒ 5

Ⓓ 4

Ⓔ 1

7. If $2x^3 - xy + y^3 = 8$, find the value of $\dfrac{dy}{dx}$ at the point where $x = 1$, $y = 2$.

Ⓐ $\dfrac{7}{11}$

Ⓑ $-\dfrac{2}{9}$

● $-\dfrac{4}{11}$

Ⓓ $\dfrac{4}{5}$

Ⓔ $\dfrac{3}{10}$

8. If $f(x) = \sin x$, then $f'''(\frac{7\pi}{12}) =$

Ⓐ $\frac{1}{4}(\sqrt{3}-1)$

● $\frac{1}{4}(\sqrt{6}-\sqrt{2})$

Ⓒ $\frac{1}{4}(\sqrt{6}-\sqrt{3})$

Ⓓ $\frac{1}{4}(\sqrt{3}-\sqrt{2})$

Ⓔ $\frac{1}{4}(\sqrt{2}-1)$

9. Consider the following 3 statements:

I. $\arcsin\left(\sin\left(\frac{3\pi}{5}\right)\right) = \frac{3\pi}{5}$.

II. $\arcsin\left(\frac{1}{4}\right) + \arccos\left(\frac{1}{4}\right) = \frac{\pi}{2}$.

III. $\lim_{x\to\infty} \frac{\arctan x}{\arctan(2x)} = 1$.

Which of the 3 statement(s) is (or are) correct?

Ⓐ I and II only.

Ⓑ I only.

Ⓒ II only.

● II and III only.

Ⓔ I and III only.

10. Let $f(x) = xe^{-x}$. Find $f^{(99)}(49)$, i.e., find the value of the 99th derivative of $f(x)$ at $x = 49$.

Ⓐ $49e^{-49}$

Ⓑ $51e^{-49}$

Ⓒ $-50e^{-49}$

Ⓓ $-49e^{-49}$

● $50e^{-49}$

PART B *[50 marks]*

Please read carefully:
Present your complete solutions to the following questions in the spaces provided, in a neat and logical fashion, showing all your computations and justifications. Any answer in PART B without proper justification may receive very little or no credit. Use the back of each page for rough work only. If you must continue your formal solution on the back of a page, you should indicate clearly, in LARGE letters, "SOLUTION CONTINUED ON THE BACK OF PAGE _____". In this case, you may get credit for what you write on the back of that page, but you may also be penalized for mistakes on the back of that page.
MARKS FOR EACH QUESTION ARE INDICATED BY [ ].
DO NOT TEAR OUT ANY PAGES.

1. Let $f(x) = \sqrt{x}$. Find $f'(x)$ from first principles (i.e. by using only the definition of the derivative).

[6]

2. Use any suitable method to find $\frac{dy}{dx}$ for each of the following. There is no need to simplify your final answers for this question.

   (a) $y = \dfrac{\sec x}{1+x^2}$, $-\dfrac{\pi}{2} < x < \dfrac{\pi}{2}$.

[4]

   (b) $y = 3^{\sqrt{x}}$, $x > 0$.

[4]

3. For this question, simplify your final answers as much as possible.

   (a) If $f(x) = (1+x^2)\arctan x$, find $f'(1)$.

[4]

   (b) If $f(x) = \ln(\ln x)$, find $f'(e)$.

[4]

4. If $f(x) = \dfrac{4x-1}{2x+3}$, find a formula for the inverse function $f^{-1}(x)$.

[6]

5. Let $f(x) = e^{2x}$. If $g(x) = f^{-1}(x)$, i.e., if g is the inverse function of f, find the value of $g'(e^4)$.

[7]

6. Find the line passing through the point $(0, -18)$ and tangent to the curve $y = x^3 - 2$ at some point.

[7]

7. (Note: This is a hard problem. Very little or no credit will be given unless your solution is completely correct.)

Find the value of $\lim_{x\to\infty}\{(x^{10}+3x^9)^{\frac{1}{10}}-x\}$. Remember to fully justify your answer.

[8]

Hint: Limit can be re-written as

$$\lim_{x\to\infty}\frac{\left(1+\frac{3}{x}\right)^{\frac{1}{10}}-1}{1/x}$$

Now, try to write the limit as a derivative.

Ans. $\frac{3}{10}$