Lecture 1

$a_0 = \frac{1}{3}$

half way between a_n & 1 (average of a_n & 1)

$a_{n+1} = \boxed{\frac{a_n+1}{2}}$ for all $n \in \mathbb{N}$ (including 0 in this course)

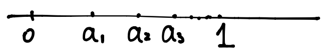
(e.g. $a_{0+1} = a_1 = \frac{a_0+1}{2}$

$a_{1+1} = \frac{a_1+1}{2}$

$a_2 = \frac{a_1+1}{2}$

$\Rightarrow$ induction

n	a_n
0	1/3
1	2/3
2	5/6
3	11/12



As $n \to \infty$, $a_n \to 1$

- For all $n \in \mathbb{N}$, a_{n+1} is half way between a_n and 1.
- Every term is less than 1

universal quantification

- For each natural number, let $P(n)$ be : $\underbrace{a_n < 1}_{\text{boolean (body of predicate)}}$

```
def P(n):
    return an<1
```

$P(236) : a_{236} < 1$

~~Let $Q(n)$ be $\forall n \in \mathbb{N}, a_n < 1$ $\quad Q(236): \forall 236 \in \mathbb{N}, a_{236} < 1$~~ Wrong

$\forall n \in \mathbb{N}$, $P(n)$ is defined as $a_n < 1$

$Q(n)$ is defined as $\forall n \in \mathbb{N}, a_n < 1$

```
def Q(n):
    result = True
    for n in range (0, length?):
        result = result
        and a[n]<1
    return result
```

$P(0)$ is True since $a_0 = 1/3 < 1$

If $n \in \mathbb{N}$ and $a_n < 1$ (*), then $a_{n+1} = \frac{a_n+1}{2}$ by def'n of sequence a

$= \frac{a_n}{2} + \frac{1}{2} < \frac{1}{2}$ (by *) $+ \frac{1}{2} = 1$

So if one term <1, the next term <1

So $a_{n+1} < 1$ [$P(n+1)$]

$\forall n \in \mathbb{N}$, $P(n) \Rightarrow P(n+1)$

We believe: $(P(0) \wedge \forall n \in \mathbb{N}, [P(n) \to P(n+1)]) \to \forall n \in \mathbb{N}, P(n)$

The principle of simple induction

$$\boxed{\begin{array}{l} \quad P(0) \\ \land\, P(0) \to P(1) \\ \land\, P(1) \to P(2) \\ \land\, P(2) \to P(3) \\ \land\, \cdots \end{array}} \to \boxed{\begin{array}{l} \ P(0) \\ \land\, P(1) \\ \land\, P(2) \\ \land\, P(3) \\ \land\, \cdots \end{array}}$$

Why is $a_5 < 1$?

$a_0 < 1$

$a_0 < 1 \to a_1 < 1$

So $a_1 < 1$

$a_1 < 1 \to a_2 < 1$

So $a_2 < 1$

$\cdots$

So $a_5 < 1$