March 11-th

Recall,

FToA: If $p(z) = a_n z^n + \cdots + a_1 z + a_0$ with a_i real then $p(z)$ has at most n complex distinct roots.

Claim: If p and q both have degree n and $p=q$ at more than n points then $p=q$ everywhere $p(x)=q(x)$ $p(x)-q(x)=0$ $(p-q)(x)$

Note: If p and q agree at $n+1$ points then $p-q$ is a degree at most n poly.
So we get more roots than degree
So $p-q=0 \Rightarrow p=q$

Check: $(x-1)^2$ is x^2-2x+1 for $x=-1,0,1$

LHS $=(-1)^2=1$	RHS $=1$
0	0
4	4

$p(x) = (x-\alpha)(x-\beta)$
$= x^2 - (\alpha+\beta)x + \alpha\beta$

$\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

Let α, β be the roots of $x^2-10x+13$
$\alpha\beta = 13$ $\alpha^2+\beta^2 = 100-2\times 13 = 74$
$\alpha+\beta = 10$

Find the sum of the squares of the roots of $x^2-10x+13$

$$x = \frac{10 \pm \sqrt{10^2-4\times 13}}{2}$$

x^3-7x^2+6x+5 Find the sum of the roots

$p(x) = (x-\alpha)(x-\beta)(x-\gamma)$
$= \cdots - (\alpha+\beta+\gamma)x + \cdots$

Def'n: A polynomial $f(x_1, \cdots, x_n)$ is symmetric if $f(x_2, x_1, \cdots, x_n) = f(x_1, x_2, \cdots, x_n)$

Thm: If f is a polynomial then any symmetric polynomial of its roots, is a func of root.

Claim: If $\underline{p(z)=0}$ then $p(\bar{z})=0$ ← real coefficients
$a+bi$ $= a-bi$

If $r \in \mathbb{R}$, $\bar{r} = r$

If $a, b \in \mathbb{C}$, $\overline{ab} = \bar{a} \cdot \bar{b}$
$\overline{a+b} = \bar{a} + \bar{b}$

Solve z^2+iz+i for z

$$z=\left\{\frac{i\pm\sqrt{(i)^2-4i}}{2}\right\}=\left\{\frac{-i\pm\sqrt{-1-4i}}{2}\right\}$$

$$\sqrt{-1-4i}$$

$$=\left\{\frac{-i\pm(a+bi)}{2},\ \frac{-i\pm(c+di)}{2}\right\}$$

$$\sqrt[4]{17}\,e^{i\frac{\theta}{2}}=a+bi$$

$$\sqrt[4]{17}\,e^{i(\frac{\theta}{2}-\pi)}=c+di$$

$\boxed{Q3}$!!!

Chapter 10 Sets & Cardinality

Def'n: A map $f: X \longrightarrow Y$ is injective/one-to-one if $f(x)=f(y) \Rightarrow x=y$

$|X|<|Y|$ if $X \xrightarrow{\text{inj.}} Y$

A map is surjective (onto) if for all $y\in Y$ there is $x\in X$ $f(x)=y$

Bijective if 1-1 & onto

Def'n: size/cardinality is the smallest set which S admits a bijective map to.

Claim: the size of $\mathbb{N}$ and $\mathbb{Q}$ is the same

$$\begin{array}{cccc}
\frac{1}{3} & \frac{2}{3} & \frac{3}{3} & \frac{4}{3} \\
\frac{1}{2} & \frac{2}{2} & \frac{3}{2} & \frac{4}{2} \\
\frac{1}{1} & \frac{2}{1} & \frac{3}{1} & \frac{4}{1}
\end{array}$$