Ordinary Differential Equation

2 Term Tests 10.10 + 11.7

3 Quizzes 9.25 + 10.25 + 11.22

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Do hw !!! Eckhard Meinronken

What is a DE?

Eg. $F=ma$ Newton's equ's.

$X(t)$ position at time t

$V(t)=\frac{dx}{dt}$ velocity at time t.

$a(t)=\frac{d^2x}{dt^2}$ accelaration at time t.

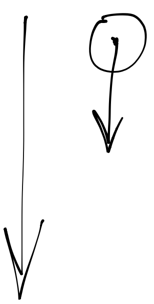
$=\frac{dV}{dt}$

since m=mass (constant)

$F(x,t)$ force (some function of x, and sometimes t.)

$\frac{d^2x}{dt^2}=\frac{1}{m}F(x,t)$ This is a 2nd order ODE.

Sol. $x=\varphi(t)$ satisfying this eq'n.



Here, $F=mg$

$\frac{d^2x}{dt^2}=g$ constant

$x=\frac{1}{2}gt^2+V(0)t+X(0)$

Some with friction

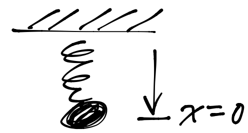
$m\frac{d^2x}{dt^2}=mg-r\frac{dx}{dt}$

more realistically, friction could be ~~complict~~ complicated func of velocity.

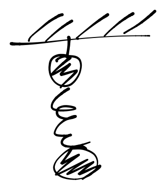
$-H(\frac{dx}{dt})$

particle attached to spring

$$m\frac{d^2x}{dt^2}=mg-kx$$



system of ODE's
(more dependent variable, more equation)



wave eqn (in electricdynamics).
u = field depending on time t and position x,y,z

$$u=u(x,y,z)$$
$$\frac{\partial^2 u}{\partial t^2}=\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}$$

this is a partial diff. eqn. (PDE)

An ordinary diff. equation is an equation of the form.

$F(t,y,y',\dots,y^{(n)})=0$ for y (a function of t)

t: indep. variable y: dep. variable.

Example:

1. $t^2\sin(y)+ey''+y''=0$
   2nd order ODE
2. $y'=y$ $(y'-y=0)$
   $y=C\cdot e^t$ (general solution)
3. $y'=e^t y$
   $y'=e^t y+C$

But: An equation $y(t+1)=2y(t)$ is not a DE.