

PLEASE HAND IN

UNIVERSITY OF TORONTO
Faculty of Arts and Science
AUGUST 2009 FINAL EXAMINATIONS
MAT135Y
Calculus I
FINAL EXAM, August 12, 2009.
Duration: 3 Hours

PLEASE HAND IN

NO AIDS ALLOWED.

Total: 102.5 Marks

Family Name:

(Please Print)

Given Name(s):

(Please Print)

Your Tutorial Section/TA's Name:

Student ID Number:

Print "yes" here if you are writing
this exam as a deferred exam
(ie you are not enrolled in the summer course):

READ INSTRUCTIONS ON PAGE 2 BEFORE YOU START!

Make sure that this booklet has 22 pages.

You may not use calculators, cell phones, MP3 players, PDAs, or any other unauthorized aid of any kind during the exam. Please read through the entire test before starting, and take note of how many points each question is worth. Carefully read the instructions to Part A and Part B, and note in particular that full justification to answers in Part A is required, while in Part B justifications are optional but will be considered for part marks if the result given is incorrect.

Partial credit will be given for partially correct work. Do not remove any pages from the exam booklet.

Anything you write on the back of any page will not count unless you write clearly in LARGE letters **CONTINUED ON BACK OF PAGE** — in which case anything you write on the back of that page will be graded.

FOR MARKER'S USE ONLY	
Problem 1:	/7
Problem 2:	/5
Problem 3:	/8
Problem 4:	/7
Problem 5:	/7
Problem 6:	/7
Problem 7:	/7
Problem 8:	/7
Problem 9:	/2.5
Problem 10:	/2.5
Problem 11:	/2.5
Problem 12:	/2.5
Problem 13:	/2.5
Problem 13:	/2.5
Problem 14:	/2.5
Problem 15:	/2.5
Problem 16:	/2.5
Problem 17:	/2.5
Problem 18:	/2.5
Problem 19:	/2.5
Problem 20:	/2.5
Problem 21:	/2.5
Problem 22:	/2.5
Problem 23:	/2.5
Problem 24:	/2.5
Problem 25:	/2.5
Problem 26:	/2.5
TOTAL:	/102.5

PART A

Answer all questions in Part A in the space provided. Show all your work for Part A. Any answer in Part A without proper justification may receive little or no credit. Use the back of each page for rough work. Marks for each question in Part A are indicated in brackets []. DO NOT TEAR OUT ANY PAGES.

- [7] 1. Sketch the graph of the function

$$f(x) = \frac{\ln |x|}{x}$$

indicating all

- intercepts
- asymptotes
- critical numbers
- inflection points

and drawing correctly all

- regions of increase/decrease
- regions of up/down concavity

(Extra page for Problem 1. DO NOT REMOVE!)

[5]

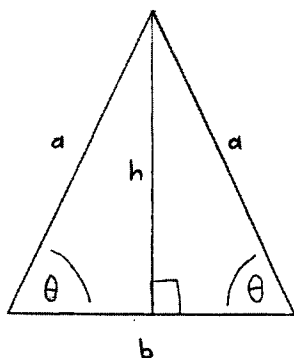
2. A helium balloon is released from the ground and floats upwards. Suppose you've figured out how long it will take the balloon to reach any given height. The needed time, t , (measured in seconds) is related to the height h , (measured in metres) by the equation

$$t = \frac{1}{300}(h^3 + 3h^2 + 3h).$$

What is the balloon's height when it is going up at a speed of 1 m/s?

[8]

3. Recall that an isosceles triangle is one which has two sides of the same length, or, equivalently, one which has two angles the same. (*See picture.*)



For a fixed perimeter P , which shape of isosceles triangle will have the maximum area? Justify your answer. (*Hint: can you write the perimeter and area in terms of h and θ in the diagram?*)

(Extra page for Problem 3. DO NOT REMOVE!)

4. Determine whether the following series are convergent or divergent. If convergent, are they absolutely convergent? Prove your answer.

[1.5] (i) $\sum_{i=1}^{\infty} \frac{2+n}{1+n}$

[1.5] (ii) $\sum_{i=1}^{\infty} \frac{\sin(2n)}{n^2}$

[2.5] (iii) $\sum_{i=1}^{\infty} (-1)^n \frac{1}{\ln n}$

[1.5] (iv) $\sum_{i=1}^{\infty} \frac{(-3)^n}{n!}$

- [7] 5. Evaluate $\int \frac{2x-1}{x^3+x} dx$

6. Find the solution $y(x)$ of the differential equation $x^3 y'(x) = y(x)^2$, subject to the initial condition $y(1) = 4$.

[7]

- [7] 7. Find the arc length of the curve $x = \frac{y^4}{8} + \frac{1}{4y^2}$, for $1 \leq y \leq 2$.

8. Find the volume of the solid of revolution obtained by rotating the area enclosed by the curve $x^2 + (y - 1)^2 = 1$ about the line $x = 2$.

[7]

PART B

Answer all questions in the space provided. Clearly state your answers to the specific questions asked. You do not need to justify your answers, and you will get full credit if your result is correct. However, if you choose to justify your answer, you may get part marks even if your result is not correct. Each question in Part B is worth 2.5 marks. **DO NOT TEAR OUT ANY PAGES.**

[2.5] 9. Find the following limit, if it exists: $\lim_{x \rightarrow 0} \frac{\sqrt{x+16}-4}{2x}$.

[2.5] 10. Find the following limit, if it exists: $\lim_{x \rightarrow 0} \frac{e^{x^2}-1}{x^2}$.

[2.5] 11. Find the following limit, if it exists: $\lim_{x \rightarrow 0} \frac{\tan^2(\pi x)}{\sin x \sin 2x}$

[2.5] 12. Evaluate $\frac{d}{dx} x^{4x}$

[2.5] 13. Evaluate $\frac{d}{dx} \sin(\cos(\ln x))$.

[2.5] 14. Find a number c such that the following function is continuous:

$$f(x) = \begin{cases} c^2x + 4, & \text{if } x \leq 1 \\ 4cx^3, & \text{if } x > 1 \end{cases}$$

[2.5]

15. A radioactive substance decays over time, such that after t years a block which started with mass of 1 kilogram will have mass of $m(t) = \pi^{-2t}$ kilograms. What is the half-life of this substance?

[2.5]

16. Evaluate: $\int_1^2 \frac{e^{1/x}}{x^2} dx$

[2.5] 17. Evaluate: $\int_1^2 x^2 \ln(x) dx$

[2.5] 18. Evaluate: $\int_{-2}^3 |x^2 - 2x - 3| dx$

[2.5] 19. Is $\int_1^3 \frac{1}{\sqrt{3-x}} dx$ convergent or divergent? Evaluate the integral if it is convergent.

[2.5] 20. Find the number a such that the line $x = a$ divides the area under the curve $y = \frac{1}{x^2}$ (for $1 \leq x \leq 5$) into two equal-sized pieces (not necessarily of the same shape).

21. The curve $y = 3 - x^2$, for $0 \leq x \leq 1$, is rotated about the y -axis. Find the area of the resulting surface.

[2.5]

22. Evaluate $\int \tan^3 x \sec^3 x dx$.

[2.5]

[2.5]

23. Some biologists are studying a predator-prey system modeled by the equations $\frac{dW}{dt} = 2W + \frac{RW}{3}$ and $\frac{dR}{dt} = -7R + 2RW$. Suppose the system is in equilibrium with the size of both populations strictly positive. Find RW .

[2.5]

24. Suppose the fish population in a fish hatchery is initially 500, and one day later it is 750. Assume the rate of growth of the population is proportional to the population itself, what will the population be after one more day?

[2.5] 25. Find the orthogonal trajectories of the family of curves $y = \frac{k}{x}$.

[2.5] 26. Determine the Maclaurin series for the function $f(x) = \sin^2 x$.

(Extra page if needed for your answers. DO NOT REMOVE!)