Assignment V: week of Feb. 4th

This is the 5th of the 10 assignments. You are encouraged to work on this by coming to the help sessions (Thursday 12-1 MP202, Friday 1-2 at MP102) and **grouping** *up with a few other students. You do not have to hand this one in. Problem 4 is courtesy of Dr. Anne-Marie Weijmans, who will be doing a guest lecture next Wednesday. You can attempt this problem before her lecture. There will be no assignment next week (mid-term week).*

1. Angular resolution of your eyes. You need angular resolution to pull thread through a needle hole. Human eyes can see a hair as thin as 0.1 millimeter, held at a distance of 25 centimeters. Calculate the angle extended by such a hair,express it in unit of 'arcsecond' (1 arcsecond = 1/60 arcminute, 1 arcminute = 1/60 degree). This is the angular resolution of our eyes.

Angular resolution improves linearly with the diameter of the lens. Average pupils have a diameter of 5 millimeters. In contrast, the Keck telescope (largest telescope today, situated atop the Mauna Kea volcano in Hawaii) is a pupil with a diameter of 10 meters. What is the angular resolution of Keck (express in arcsec)? Armed with such a telescope, how far can you see a hair?

2. Event horizon of a blackhole. A blackhole is an object from which light can not escape. It has a physical size determined by its mass. This size is called the 'event horizon'. No information can be passed back from within the event horizon (hence the name). Here, we will estimate the size of event horizon for a solar-mass object, using only Newtonian physics and not assuming any knowledge of the general relativity.

Assume photon has a mass m_γ (as you will see its value doesn't enter the problem). When placed near a blackhole, it has to keep moving to prevent falling inward. The orbital speed can be obtained by balancing the centrifugal acceleration against the gravitational acceleration. Let this equality be,

$$m_\gamma \frac{v^2}{r} = G\frac{M_{\rm BH} m_\gamma}{r^2} \tag{1}$$

The required orbital speed v rises as the distance between the photon and the blackhole (r) decreases. Since nothing can move faster than c (the speed of light), this equation yields the minimum distance from the blackhole within which even a photon can't stay orbiting. Let's call this the event horizon of a blakhole. Determine r (in unit of kilometers) when $M_{\rm BH} = 1 M_\odot$, where $M_\odot$ is the mass of the Sun and is $\sim 2 \times 10^{30}$ kg. Use of general relativity gives a slightly different value of $r = 2.5$ km.

3. Jupiter is the most massive planet in our Solar System. It has 63 moons, of which Ganymede is the largest – it is actually the largest moon in the Solar System! It orbits Jupiter in 7.15 days, with a semi-major axis of 1.07×10^6 km. Use this information together with Kepler's Third Law to calculate the mass of Jupiter. How does this mass compare to that of the sun (2×10^{30} kg)?

4. Let's look at dark matter in elliptical galaxies. For most elliptical galaxies this is not an easy task, as they often lack the large cold gas discs that are used to find the dark haloes around spiral galaxies. NGC 2974 is an exception: this galaxy is surrounded by a ring of cold gas, that we can use for a dark matter study. The rotation curve of the ring is given in the figure.

- Given that NGC 2974 resides at a distance of 20.89 Mpc (Mpc: mega parsec, 1 Mpc= 10^6 pc)

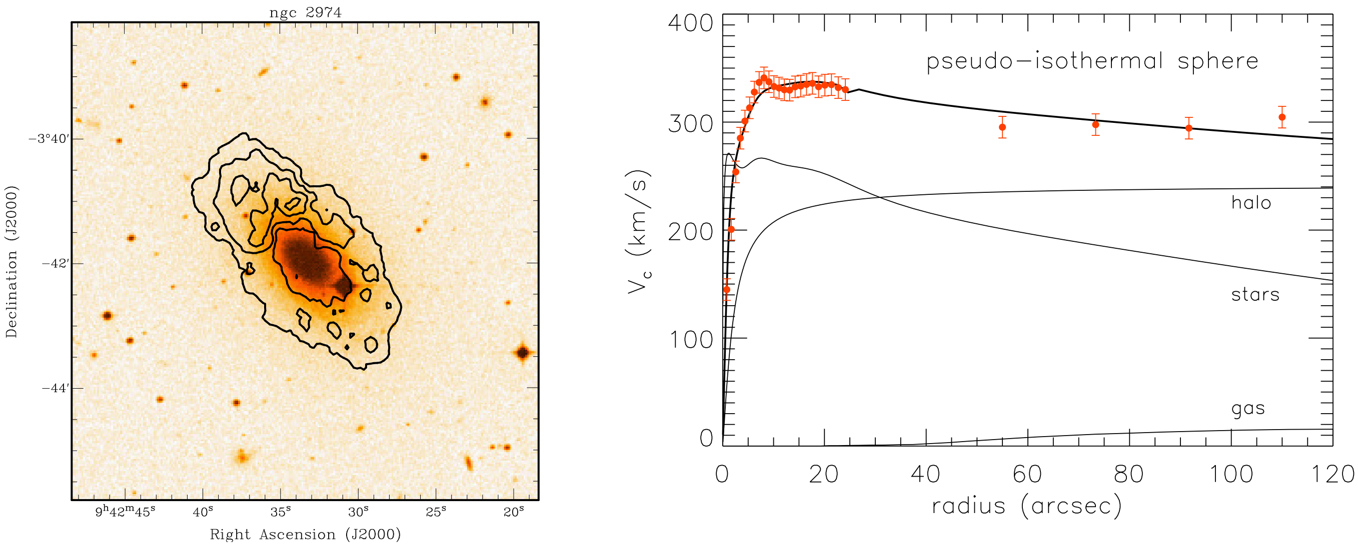

Figure 1: Left: NGC 2974 in visible light, with contours of the cold gas ring overlaid. Right: rotation curve of this galaxy. The rotation curve is reproduced with a small gas disc, a stellar component and a dark matter halo (parametrised by a so-called pseudo-isothermal sphere). From Weijmans et al. (2008).

from us, calculate the size of a patch in NGC 2974 that extends an angular size of 1 arcsecond (= 1/3600 degree) at Earth, expressed in units of kilo-parsec (kpc).
1 parsec $= 3.1 \times 10^{16}\,\mathrm{m} \sim 3$ *light years, this unit is often used in astronomy to indicate distances. Our sun is located at 8 kpc from the centre of the Milky Way.*

- The cold gas ring is situated a distance of 12 kpc from the center of NGC 2974. It exhibits a line-of-sight rotational velocity of 305 km/s. Assuming a spherical galaxy and that the measured velocity is the full rotational velocity, what is the total mass within this radius? How much solar masses ($M_\odot$) is this?

- To find the dark halo mass, we need to know how much of the total mass is in the stars and the gas (the luminous, baryonic matter). The enclosed luminosity of NGC 2974 within 12 kpc is 2.7×10^{10} $\mathrm{L}_\odot$ ($L_\odot$ stands for solar luminosity). Given a stellar mass-to-light ratio of 2.3 (meaning every $L_\odot$ of light needs $2.3 M_\odot$ of stellar mass), what is the stellar mass within that radius?

- Given your answers to the above questions, what is the fraction of dark matter over total matter in this galaxy, within 12 kpc? The gas mass in this system is $5.5 \times 10^8 M_\odot$, which is much smaller than both the stellar and dark mass, and can therefore be neglected. Note also that the halo continues beyond 12 kpc, so the total dark matter fraction in this galaxy will be higher.

Disclaimer: this galaxy is not really spherical and we made some simplifications in our model. A more detailed model and corresponding calculations are given in Weijmans et al. 2008, MNRAS, 383, 1343.