THE AUSTRALIAN NATIONAL UNIVERSITY

RESEARCH SCHOOL OF FINANCE, ACTUARIAL STUDIES AND APPLIED STATISTICS

Final Examination, Semester 2, 2015

STAT2032/6046
FINANCIAL MATHEMATICS

Study Time: 15 minutes
Writing Time: 3 hours

Permitted materials:
Paper Based Dictionary without annotations
Non-programmable calculator

INSTRUCTIONS:

1. This exam paper comprises a total of 6 pages. Please ensure your paper has the correct number of pages.
2. The exam includes a total of 10 questions. The questions are of unequal value, with marks indicated for each question.
3. STAT20323 students must attempt to answer the first 9 questions. STAT6046 students must attempt to answer all question.
4. Include all workings for each question, as marks will not be awarded for answers that do not include workings.
5. Ensure you include your student number on your answer book.

Total Marks = 90 (STAT2032) and 100 (STAT6046)

This exam counts towards 70% of your final assessment.

Question 1 **[10 Marks]**

The force of interest $\delta(t)$ is a function of time and at any time *t*, measured in years, is given by the formula

$$\delta(t) = \begin{cases} 0.04 & 0 < t \le 5 \\ 0.008t & 5 < t \le 10 \\ 0.005t + 0.0003t^2 & 10 < t \end{cases}$$

a. Calculate the present value of $200 due at time $t = 8$. **(3 marks)**

$$\begin{aligned} PV &= 200 \times \exp\left(-\int_0^8 \delta(t)\,dt\right) \\ &= 200 \times \exp\left(-\int_0^5 0.04dt - \int_5^8 0.008tdt\right) \\ &= 200 \times \exp\left(-0.04t\Big|_0^5 - 0.004t^2\Big|_5^8\right) \\ &= 200 \times \exp(-0.20 - 0.256 + 0.1) \\ &= \$140.09 \end{aligned}$$

b. Calculate the average effective annual rate of interest over the first 12 years. **(4 marks)**

Let *i* be the average annual rate of interest.

$$\begin{aligned} (1+i)^{12} &= \exp\left(\int_0^{12} \delta(t)\,dt\right) \\ &= \exp\left(\int_0^5 0.04dt + \int_5^{10} 0.008tdt + \int_5^{10} \left(0.005t + 0.0003t^2\right)dt\right) \\ &= \exp\left(0.04t\Big|_0^5 + 0.004t^2\Big|_5^{10} + \left(0.0025t^2 + 0.0001t^3\right)\Big|_{10}^{12}\right) \\ &= \exp(0.20 + 0.4 - 0.1 + 0.36 + 0.1728 - 0.25 - 0.1) \\ &= \exp(0.6828) \\ &= 1.979412 \\ i &= \sqrt[12]{1.979412} \\ &= 0.05855 \\ &= 5.86\% \end{aligned}$$

c. Calculate the accumulated value at time $t = 7$ of \$150 paid at time $t = 1$.

(3 marks)

The accumulated value is:

$$
\begin{aligned}
AV &= 150 \times \exp\left(\int_1^7 \delta(t)\,dt\right) \\
&= 150 \times \exp\left(\int_1^5 0.04dt + \int_5^7 0.008tdt\right) \\
&= 150 \times \exp\left(0.04t\Big|_1^5 + 0.004t^2\Big|_5^7\right) \\
&= 150 \times \exp(0.20 - 0.04 + 0.196 - 0.1) \\
&= \$193.76
\end{aligned}
$$

Question 2 **[10 marks]**

Alexander is considering investing in a project that requires an initial outlay of \$500,000 at outset. This is followed by five further yearly payments at the end of each year for the next 5 years. The first payment is \$100,000 and each subsequent payment increases by \$10,000.

This project is expected to provide a continuous income at a rate of \$80,000 in the first year and then increases at the rate of 4% p.a. each year. This income is received for 25 years.

It is assumed that a further investment of \$300,000 is required at the end of 15 years and that the project can be sold to another investor for \$700,000 at the end of 25 years.

a. Calculate the net present value of the project at a rate of 10% p.a. effective.
(8 marks)

First evaluate the present value of all the outlays (interest rate is 10% p.a. effective):

$$\begin{aligned}
PV(Outlay) &= 500000 + 90000a_{\overline{5|}} + 10000(Ia)_{\overline{5|}} + 300000v^{15} \\
&= 500000 + 90000 \times 3.7908 + 10000 \times 10.6526 + 300000 \times 0.23939 \\
&= 500000 + 341172 + 106526 + 71817 \\
&= 1{,}019{,}514.31
\end{aligned}$$

The present value of all the income:

$$\begin{aligned}
PV(Income) &= 700000v^{25} + 80000\left(v + (1.04)v^2 + \ldots + (1.04)^{24} v^{25}\right) \\
&= 700000v^{25} + 80000v\left(1 + (1.04)v + \ldots + (1.04)^{24} v^{24}\right) \\
&= 700000v^{25} + 80000v\left(1 + v_k + \ldots + v_k^{24}\right) \qquad v_k = 1.04v = 0.945455 \\
&= 700000v^{25} + 80000v\ddot{a}_{\overline{25|}k} \\
&= 700000 \times 0.092296 + 80000 \times 1.1^{-1} \times \left(\frac{1 - v_k^{25}}{1 - v_k}\right) \\
&= 64607.199 + 1{,}005{,}271.97 \\
&= 1{,}069{,}879.17
\end{aligned}$$

Alternatively using continuous payments for income:

$$\begin{aligned}
PV(Income) &= 700000v^{25} + 80000\left(\bar{a}_{\overline{1|}} + (1.04)v\bar{a}_{\overline{1|}} + \ldots + (1.04)^{24} v^{24}\bar{a}_{\overline{1|}}\right) \\
&= 700000v^{25} + 80000\bar{a}_{\overline{1|}}\left(1 + v_k + \ldots + v_k^{24}\right) \qquad v_k = 1.04v = 0.945455 \\
&= 700000v^{25} + 80000\bar{a}_{\overline{1|}}\ddot{a}_{\overline{25|}k} = 646{,}071.99 + 1{,}054{,}737.25 \\
&= 1{,}119{,}344.45
\end{aligned}$$

Thus the NPV is:

$$\begin{aligned} NPV &= PV(Income) - PV(Outlay) \\ &= 1{,}069{,}879.17 - 1{,}019{,}514.31 \\ &= \$50{,}364.86 \end{aligned}$$

Alternative NPV is:

$$\begin{aligned} NPV &= PV(Income) - PV(Outlay) \\ &= 1{,}119{,}344.45 - 1{,}019{,}514.31 \\ &= \$99{,}830.14 \end{aligned}$$

b. Without doing any further calculations, explain how the net present value would alter if the interest rate is greater than 10% p.a. effective. (2 marks)

If interest rate is greater than 10% then both the PVs will decrease. But the PV for the income will decrease more as it is spread out much further than the outlay. Hence the NPV will be lower.

Question 3 **[12 marks]**

A loan of $30,000 is to be repaid by a level monthly payment for a period of 20 years and calculated on the basis of an effective interest rate of 12% p.a.

a. Find the monthly repayment for this loan. **(4 marks)**

Let K be the level monthly repayment.

$$\begin{aligned} 30000 &= 12Ka_{\overline{20}|}^{(12)} \\ &= 12K\frac{i}{i^{(12)}}a_{\overline{20}|} \\ &= 12K \times 1.053875 \times 7.4694 \\ K &= \frac{30000}{94.4623} \\ &= 317.59 \end{aligned}$$

b. What is the interest paid in the 15th year of the loan? **(4 marks)**

The interest paid in the 15th year of the loan: $=12K-\left(OB_{14}-OB_{15}\right)$

$$\begin{aligned} OB_{14} &= 12Ka_{\overline{6}|}^{(12)} \\ &= 12\left(317.59\right)\frac{i}{i^{(12)}}a_{\overline{6}|} \\ &= 12 \times 317.59 \times 1.053875 \times 4.1114 \\ &= 16{,}512.97293 \end{aligned}$$

$$\begin{aligned} OB_{15} &= 12Ka_{\overline{5}|}^{(12)} \\ &= 12\left(317.59\right)\frac{i}{i^{(12)}}a_{\overline{5}|} \\ &= 12 \times 317.59 \times 1.053875 \times 3.6048 \\ &= 14{,}478.27135 \end{aligned}$$

$$\begin{aligned} Interest &= 12K-\left(OB_{14}-OB_{15}\right) \\ &= 3811.065694-\left(16512.97293-14478.27135\right) \\ &= 3811.065694-1776.364114 \\ &= \$1{,}776.36 \end{aligned}$$

At the end of the 15th year the terms of the loan are renegotiated and the remaining balance will be repaid within the next 5 years at an interest rate of 9% p.a. convertible monthly.

c. Find the new monthly repayment that will be required to pay off the loan.
(4 marks)

Let M be the new monthly level repayment

$$\begin{aligned} OB_{15} &= 12Ma^{(12)}_{\overline{5}|9\%} \\ &= 12M\,\frac{i}{i^{(12)}}\,a_{\overline{5}|9\%} \\ 14478.27135 &= 12M \times 1.040608 \times 3.8897 \\ M &= \frac{14478.27135}{48.571835} \\ &= \$300.54 \end{aligned}$$

Question 4 **[10 marks]**

In a particular accumulation fund, the unit price of the fund from 1 April 2008 to 1 April 2014 is given below:

Year	**2008**	**2009**	**2010**	**2011**	**2012**	**2013**	**2014**
Price of each unit on 1 April	**1.86**	**2.11**	**2.55**	**2.49**	**2.88**	**3.18**	**3.52**

a. **Find the time weighted rate of return for the fund over the period 1 April 2008 to 1 April 2014.** **(2 marks)**

$$TWRR = \left(\frac{3.52}{1.86}\right)^{1/6} - 1$$
$$= 0.112171$$
$$= 11.2171\%$$

b. **Sam buys 200 units on 1 April in each of the years 2008 to 2010. If he sells all the units on 1 April 2012 then find the holding period yield for this investment.** **(4 marks)**

Year	**2008**	**2009**	**2010**	**2012**
Investment	372	422	510	1728
Price per unit	1.86	2.11	2.55	2.88
No. of units	200	200	200	600

Thus the equation of value for this transaction where *i* is the holding period yield is:

$$1728 = 372(1+i)^4 + 422(1+i)^3 + 510(1+i)^2$$

Using linear interpolation on $f(i)=372(1+i)^4+422(1+i)^3+510(1+i)^2-1728$ we can solve for *i*.

$$i_1=10\% \to f(i)=-4.57$$
$$i_2=11\% \to f(i)=42.23$$
$$i_3=15\% \to f(i)=238.91$$

$$\to i=10.10\%$$

c. Tony buys units worth \$500 on 1 April of each of the years 2010 to 2013 and then sells them all on 1 April 2014. Find the yield achieved by Tony on his investment. (4 marks)

Year	2010	2011	2012	2013	2014
Investment	500	500	500	500	2561.59
Price per unit	2.55	2.49	2.88	3.18	3.52
No. of units	196.08	200.80	173.61	157.23	727.73

Thus the equation of value for this transaction where *i* is the holding period yield is:

$$2561.59=500s_{\overline{4}|i}$$

Using linear interpolation on $f(i)=s_{\overline{4}|i}=\dfrac{2561.5936}{500}=5.123187$ we can solve for *i*.

$$i_1=10\% \to f(i)=-9.0436$$
$$i_2=11\% \to f(i)=52.3071$$
$$i_3=15\% \to f(i)=309.5970$$

$$\to i=10.15\%$$

Question 5 **[10 marks]**

Tom buys a $10,000 bond with a maturity in 10 years and a redemption value of $12,000. The bond pays quarterly coupons at a rate of 8% p.a. Tom pays income tax at the rate 20% but pays no capital gains tax.

a. If Tom wishes to achieve a net yield of 10% p.a. effective find the price paid for the bond. **(5 marks)**

Let P be the price paid by Tom

$$\begin{aligned} P &= Fr\left(1-t_I\right)a_{\overline{10}|}^{(4)} + Cv^{10} \\ &= Fr\left(1-t_I\right)\frac{i}{i^{(4)}}a_{\overline{10}|} + Cv^{10} \\ &= 800\left(1-0.2\right)1.036756\times 6.144567 + 12000\times 0.385543 \\ &= \$8,703.58 \end{aligned}$$

At the end of 5 years, just after receiving the coupon, Tom sells this bond to Jane. Jane is subject to no income tax but a capital gains tax of 30%. Assume that Jane holds the bond until maturity.

b. If Jane is able to achieve a holding period yield of 12% p.a. convertible quarterly on this bond then find the price that Jane paid for this bond. **(5 marks)**

Let S be the sale price of the bond. Since the yield is higher than the modified coupon rate a capital gains tax will apply as price will be less than the redemption amount.

$$\begin{aligned} S &= Fra_{\overline{20}|3\%} + \left(C - t_G\left(C-S\right)\right)v_{3\%}^{20} \\ S\left(1-t_G v_{3\%}^{20}\right) &= Fra_{\overline{20}|3\%} + C\left(1-t_G\right)v_{3\%}^{20} \\ S &= \frac{200\times 14.877475 + 12000\left(1-0.3\right)0.553676}{\left(1-0.3\times 0.553676\right)} \\ S &= \frac{2975.4950 + 4650.8763}{0.833897} \\ S &= \frac{7626.3713}{0.833897} \\ S &= \$9,145.46 \end{aligned}$$

Question 6 **[10 marks]**

A business is looking at accessing a loan of $5,000,000 to invest in an exciting new project. The 10-year loan charges interest at 6% p.a. convertible monthly has the following repayment terms:

- **No payments in the first 2 years.**
- **At the start of the 3rd year, all the interest accrued up till that point is paid back to the lender.**
- **A semi-annual payment is paid in the next 4 years, which comprises of only the interest accrued until the time of the repayment.**
- **In the last 4 years of the loan, monthly payments are made at the end of each month, starting at $11,000 in the first month and then increasing by $1,000 each month.**
- **At the end of 10 years the business expects to sell the project at $5,000,000.**

a. **Without doing any calculation, state the outstanding balance on the loan at the end of 6 years? Briefly explain your answer.** **(3 marks)**

The outstanding balance at the end of year 6 is $5,000,000. Since over this period the interest accrued is always paid back the outstanding loan is always $5,000,000.

b. What is the present value of this profit/loss if the business has a yield requirement of 12% p.a. effective? (7 marks)

We first evaluate the loan outstanding at the end of 10 years. The difference between this and the sale price gives us the profit which is then discounted at the RDR of 12% p.a. effective.

$$\begin{aligned}\text{Loan Outstanding} &= 5000000(1.005)^{48} - 10000 s_{\overline{48}|j} - 1000(Is)_{\overline{48}|j} \\ &= 5000000 \times 1.2705 - 10000 \times 54.0978322 - 1000 \times 1273.6643 \\ &= 6352445.81 - 540978.322 - 1273664.276 \\ &= 4{,}537{,}803.2073\end{aligned}$$

The profit at the end of the 10th year is $\text{Profit} = 5{,}000{,}000 - 4{,}537{,}803.2073 = 462{,}196.7927$

The net present value of the profit is given as:

$$\begin{aligned}NPV &= 462{,}196.7927(1.12)^{-10} \\ &= 148{,}814.9973 \\ &= 148{,}815.00\end{aligned}$$

Question 7 **[6 marks]**

An investor entered into a short forward contract for a security 5 years ago and is due to mature in another 7 years' time. Five years ago the price of the security was $95 and is now $145. The risk free rate of interest is 4% p.a. effective throughout the 12 year period. The security is to pay a total of 6 dividends, the first payment being $5 at the time of issue of the forward contract and then every subsequent dividend to increase by $5 paid every 2 years.

What is the value of this short forward contract today?

First we evaluate the forward price at issue.

$$F_0 = (S_0 - PV_I)(1+i)^{12}$$

$$\begin{aligned} PV_I &= 5(I\ddot{a})_{\overline{6}|j} && \text{where } j = (1+i)^2 - 1 = 8.16\% \\ &= 5\left(\frac{\ddot{a}_{\overline{6}|} - 6v^6}{d_j}\right) && \text{where } d_j = \frac{j}{1+j} = 7.5444\% \\ &= 5\left(\frac{4.9759 - 6\times 0.624597}{0.075444}\right) \\ &= 5\times 16.281619 \\ &= 81.408096 \end{aligned}$$

$$\begin{aligned} F_0 &= (95 - 81.408096)(1.04)^{12} \\ &= 21.7611 \end{aligned}$$

The forward price at the current date.

$$\begin{aligned} F_5 &= (S_5 - PV_5)(1+i)^7 = (145 - 66.11349)(1.04)^7 \\ &= 103.809264 \end{aligned}$$

Where

$$\begin{aligned} PV_5 &= \left(15\ddot{a}_{\overline{3}|j} + 5(I\ddot{a})_{\overline{3}|j}\right)(1+i)^{-1} = \left(15\ddot{a}_{\overline{3}|j} + 5\left(\frac{\ddot{a}_{\overline{3}|j} - 3v_j^3}{d_j}\right)\right)(1+i)^{-1} \\ &= \left(15\times 2.77936 + 5\left(\frac{2.77936 - 3\times 0.790315}{0.075444}\right)\right)\times 0.961538 \\ &= 66.11349 \end{aligned}$$

Value of a short forward contract:

$$
\begin{aligned}
V_S &= \left(F_0 - F_5\right)\left(1+i\right)^{-(12-5)} \\
&= \left(21.7611 - 103.809264\right)\left(1.04\right)^{-7} \\
&= -\$62.35
\end{aligned}
$$

Question 8 **[8 marks]**

Consider the following term structure of spot rates that apply over the next 4 years.

Time t	*Spot Rate %p.a. effective*
0.5	**6.00%**
1.0	**6.50%**
1.5	**6.75%**
2.0	**7.00%**
2.5	**6.75%**
3.0	**6.50%**
3.5	**6.25%**
4.0	**6.00%**

a. **A payment of \$50 is made at each of the times t = 0.5, 1, 2 and 3. What is the accumulated value of these payments at time t = 4?** **(4 marks)**

One approach is to first find the PV of all payments and then accumulate to $t = 4$.

$$\begin{aligned} PV &= 50\left[(1+s_{0.5})^{-0.5} + (1+s_1)^{-1} + (1+s_2)^{-2} + (1+s_3)^{-3}\right] \\ &= 50\left[(1.06)^{-0.5} + (1.065)^{-1} + (1.07)^{-2} + (1.065)^{-3}\right] \\ &= 50[0.9713 + 0.9390 + 0.8734 + 0.8278] \\ &= 180.5770 \end{aligned}$$

Accumulated value at $t = 4$.

$$\begin{aligned} AV &= 180.5770(1+s_4)^4 \\ &= 180.5770(1.06)^4 \\ &= 227.97 \end{aligned}$$

b. **Evaluate the price of a \$1,000 3-year annual coupon paying bond issued at time *t* = 1. The coupons are paid at 7.5% p.a. and the redemption value is 1.2 times the face value. (4 marks)**

Using the present value approach as before.

$$\begin{aligned}
PV_0 &= 75\left[(1+s_2)^{-2}+(1+s_3)^{-3}+(1+s_4)^{-4}\right]+1200(1+s_4)^{-4} \\
&= 75\left[(1.07)^{-2}+(1.065)^{-3}+(1.06)^{-4}\right]+1200(1.06)^{-4} \\
&= 75[0.8734+0.8278+0.7921]+1200\times 0.7921 \\
&= 1137.5160
\end{aligned}$$

Price at $t = 1$

$$\begin{aligned}
\mathrm{P} &= PV_0(1+s_1)^1 \\
&= 1137.5160\times 1.065 \\
&= 1211.45
\end{aligned}$$

Question 9 **[14 marks]**

An insurance company has a liability of $2,000,000 due in 12 years' time. In order to pay this liability the insurance company decides to invest in the following 2 bonds:

- **A 15 year zero coupon bond redeemable at par.**
- **A 10-year zero coupon bond redeemable at par.**

Assume that the insurance company can earn interest at a force of interest of 5% p.a.

a. Calculate the nominal value of the 2 bonds such that duration of both assets and liabilities are matched. **(8 marks)**

$$\begin{aligned} PV(Liab) &= 2000000v^{12} \\ &= 2000000 \times 0.548812 \\ &= 1097623.27 \end{aligned}$$

Let A be the face value of the 15 year zero coupon bond and B the face value of the 10 year zero coupon bond.

$$\begin{aligned} PV(Asset) &= Av^{15} + Bv^{10} \\ &= 0.472367A + 0.606531B = PV(Liab) \\ &= 1097623.27 \end{aligned}$$

Duration of the liabilities $\tau_l = 12$

Duration of the assets

$$\tau_a = \frac{15Av^{15} + 10Bv^{10}}{1097623.27} = \tau_l$$
$$7.085498A + 6.065307B = 13171479.27$$

Solving the above equations to solve for A and B the face value of the bonds.

$$A = 929,467.39$$
$$B = 1,085,804.90$$

b. Calculate the convexity of both the assets and liabilities and comment on whether the portfolio is immunised against small changes in interest rates.
(6 marks)

Convexity of the liabilities is:

$$\begin{aligned} c_l &= \frac{12\times 13\times v^{14}\times 2000000}{2000000v^{12}} \\ &= 12\times 13v^2 \\ &= 156v^2 \\ &= 141.15 \end{aligned}$$

Convexity of the assets is:

$$\begin{aligned} c_a &= \frac{15\times 16\times v^{17}\times 929467.39+10\times 11\times v^{12}\times 1085804.90}{2000000v^{12}} \\ &= \frac{95344378.33+65549260.10}{1097623.27} \\ &= 146.58 \end{aligned}$$

Since convexity of assets is larger than that of the liabilities, all three conditions of immunisation are satisfies and hence the portfolio is immunised against change in interest rates.

Question 10 **[10 marks]**

An amount of $50,000 is invested in a 3-year project which pays interest at the end of each year. The interest in the first year will be one of 6% or 8% with equal probability. In the second year the interest rate will be 7% with a probability 0.75 and 10% with a probability of 0.25. In the final year, the interest rates will be 5% with probability 0.4 or 9% with probability of 0.6.

a. **What is the expected accumulated value at the end of 3 years?** **(4 marks)**

First we evaluate the expected values in each of the 3 years.

$$\begin{aligned} E(i_1) &= 0.06 \times 0.5 + 0.08 \times 0.5 = 0.07 \\ E(i_2) &= 0.07 \times 0.75 + 0.10 \times 0.25 = 0.0775 \\ E(i_3) &= 0.05 \times 0.4 + 0.09 \times 0.6 = 0.074 \end{aligned}$$

The accumulated value of $50,000 at the end of year 3 is:

$$\begin{aligned} AV &= 50000 E(1+i_1) E(1+i_2) E(1+i_3) \\ &= 50000(1.07)(1.0775)(1.074) \\ &= 61{,}912.07 \end{aligned}$$

b. **What is the standard deviation of the accumulated value at the end of 3 years?** **(6 marks)**

Evaluating first the squared expectations

$$E\left[(1+i_1)^2\right] = 1.06^2 \times 0.5 + 1.08^2 \times 0.5 = 1.145$$
$$E\left[(1+i_2)^2\right] = 1.07^2 \times 0.75 + 1.10^2 \times 0.25 = 1.1612$$
$$E\left[(1+i_3)^2\right] = 1.05^2 \times 0.4 + 1.09^2 \times 0.6 = 1.1539$$

The standard deviation of the accumulated value

$$\begin{aligned} Var(AV) &= 50000^2\left[E\left[(1+i_1)^2\right]E\left[(1+i_2)^2\right]E\left[(1+i_3)^2\right] - \left[E(AV)\right]^2\right] \\ &= 50000^2\left[1.145 \times 1.1612 \times 1.1539 - 1.2382^2\right] \\ &= 50000^2\left[1.5341 - 1.5332\right] \\ &= 2168344.75 \end{aligned}$$

$$\begin{aligned} SD &= \sqrt{2168344.75} \\ &= \$1,472.53 \end{aligned}$$

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c. What is the exact probability that the accumulated value at the end of 3 years will be greater than $62,500? (5 marks)

First we construct the joint probability distribution.

Alternative	Year 1	Year 2	Year 3	Overall	Probability
1	6%	7%	5%	19.09%	0.150
2			9%	23.63%	0.225
3		10%	5%	22.43%	0.050
4			9%	27.09%	0.075
5	8%	7%	5%	21.34%	0.150
6			9%	25.96%	0.225
7		10%	5%	24.74%	0.005
8			9%	29.49%	0.075

If accumulated value is greater than $62,500 then the accumulation factor over the 3-year period is 1.25 or an overall growth of 25%.

From the table above there are only 3 alternatives which provide more than 25% overall return.

$$\Pr(AV > 62500) = 0.075 + 0.225 + 0.075 = 0.375$$

Thus the probability that the AV is greater than $62,500 is 37.5%.

<u>Alternative (using normal approximation):</u>

Using the mean and standard deviation from the previous parts and using the normal approximation we get $AV \sim N(\mu = 61912.07, \sigma = 1472.53)$. Thus the required probability:

$$\begin{aligned}\Pr(AV > 62500) &\approx \Pr\left(Z > \frac{AV-\mu}{\sigma}\right)\\ &= \Pr\left(Z > \frac{62500 - 61912.07}{1472.53}\right)\\ &= \Pr(Z > 0.3993)\\ &= 34.5\%\end{aligned}$$

(End of Examination)