University of Toronto
Department of Mathematics

Faculty of Arts and Science

MAT332H1F, Graph Theory

Final Examination, December 18, 2013

Instructor: Kasra Rafi
Duration: 3 hours

First	Last	Student Number

Instructions: No aids allowed. Write solutions on the space provided. To receive full credit you must show all your work. If you run out of room for an answer, continue on the back of the page. This exam has 8 questions, for a total of 100 points.

Problem #	Grade
1	
2	
3	
4	
5	
6	
7	
Bonus	
Total	

1. (24 points) Define the following terms and expressions:

   (a) k–edge-colourable

   (b) Triangulation

   (c) Dual of a planar graph

   (d) Subdivision

(e) Maximal matching

(f) Flow

(g) Forest

(h) Connected component

2. (20 points) Answer true of false. Justify your answer with an argument or a counter example.

(a) [ ] Every graph that is edge transitive is also vertex transitive.

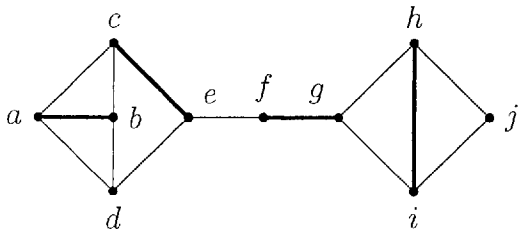
(b) [ ] Every subgraph of a bipartite graph is bipartite.

(c) [ ] All spanning trees of a graph G have the same number of edges.

(d) [ ] If G is planar, then all embeddings of G have the same number of faces.

3. (12 points) State and prove Hall's Theorem.

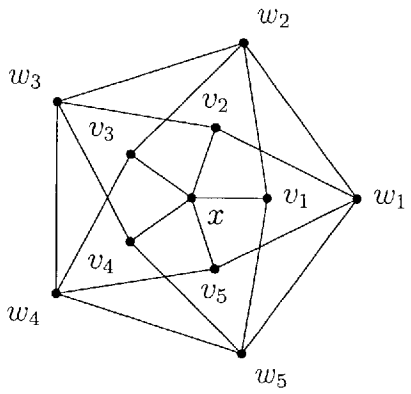
4. (12 points) Consider the following matching $M = \{ab, ce, fg, hi\}$

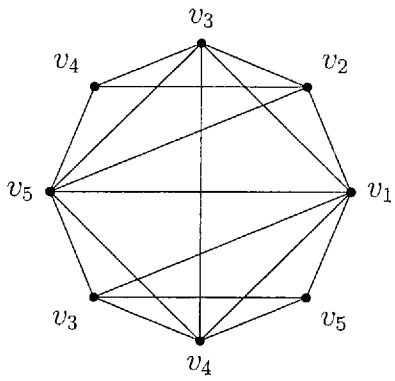


(a) Find an M–augmenting path and use it to improve the matching.

(b) Find a minimum covering and prove that it is a minimum covering.

5. (12 points) One of the following graphs is planar and one is not.





Give an embedding of the planar graph and prove the other one is not planar.

6. (10 points) Let G be a graph with 11 vertices. Show that either G or the complement of G is not planar. (Hint: Use Euler's formula.)

7. (10 points) Show that every even planar graph is 2–face-colourable. (Recall that a graph is even if the degree of every vertex is even.)

8. (Bonus Problem) Let S and T be maximal stable sets of a graph G. Show that $G[S \triangle T]$ has a perfect matching. (Recall that S is a stable set, or an independent set, of a graph G if G has no edges with both end points in S.)