Student ID

Australian National University

THE AUSTRALIAN NATIONAL UNIVERSITY

RESEARCH SCHOOL OF FINANCE, ACTUARIAL STUDIES AND APPLIED STATISTICS

Mid-Semester Exam
Semester 1, 2016

STAT4027/STAT8027 Statistical Inference

Study Time: 15 minutes
Writing Time: 1 $\frac{1}{2}$ hours

Permitted materials:
A4 pages (Two sheets) with handwritten notes on both sides
Paper-based Dictionary, no approval required (must be clear of ALL annotations)
Calculator (Any - programmable or not)

Marks

Question 1	Question 2	Question 3	Question 4	Total

INSTRUCTIONS:

1.) This exam paper comprises a total of 22 pages. Please ensure your paper has the correct number of pages.

2.) The exam includes a total of 4 questions.

3.) After each question there are four blank pages to write your solutions. You may use both sides of each page to write your solutions.

4.) Each question appears on the following pages [marks are indicated]:

- Question 1 is on page 3 **[10 marks]**.
- Question 2 is on page 8 **[30 marks]**.
- Question 3 is on page 13 **[30 marks]**.
- Question 4 is on page 18 **[30 marks]**.

5.) Include all workings for each question, as marks will not be awarded for answers that do not include workings.

6.) Draw a $\boxed{\text{box}}$ around each final answer.

7.) Ensure you include your student number on this exam book.

8.) A table of probability distributions is provided with the exam.

Total Marks = 100

This exam is a redeemable exam. It will be worth either 20% or 0% of your final grade based on your final exam mark.

Question 1 **[10 marks]**: A researcher from the College of Medicine states: "I just fit a least-squares model to determine the effects of age and gender on blood pressure." Clearly discuss the appropriateness of this statement.

Question 2 **[30 marks]**: Let X and Y be independent random variables, where $X \sim \text{gamma}(\alpha = r, \beta = 1)$ and $Y \sim \text{gamma}(\alpha = s, \beta = 1)$. Consider the following random variables based on X and Y:

$$\begin{aligned} Z_1 &= X + Y \\ Z_2 &= \frac{X}{X + Y} \end{aligned}$$

- $Note: E[X] = r; \quad V[X] = r; \quad E[Y] = s; \quad V[Y] = s.$

a. **[10 marks]** Determine the distributions of Z_1 and Z_2.

b. **[5 marks]** Show that Z_1 and Z_2 are independent.

c. **[5 marks]** Determine the means and variances for Z_1 and Z_2.

d. **[10 marks]** Write pseudo-code to determine a direct or indirect computational method to generate random samples of Z_1 and Z_2. You may assume that you are able to generate standard uniform random variables [i.e. $U \sim \text{uniform}(0, 1)$]. Additionally, you may assume that r and s are positive integers.

Question 3 [**30 marks**]: An original method for generating random standard normal variables based on random uniform variables was through the following transformation:

$$\begin{aligned} X &= \sum_{i=1}^{12} U_i - 6 \\ U_i &\overset{\text{iid}}{\sim} \text{uniform}(0,1) \end{aligned}$$

a. [**7 marks**] What is the moment generating function (mgf) for X? Use the mgf to determine the $E[X]$.

b. [**3 marks**] Let $Z \sim \text{normal}(\mu = 0, \sigma^2 = 1)$. Compare the first two moments of X and Z.

c. [**10 marks**] Justify that X may be considered **approximately** normal$(\mu = 0, \sigma^2 = 1)$.

d. [**10 marks**] Can you think of any obvious ways in which the approximation fails?

Question 4 **[30 marks]**: Let $X_1, \ldots, X_n \overset{\text{iid}}{\sim} f(x|\theta) = \theta x^{\theta-1}$, where $0 \leq x \leq 1$ and $0 < \theta < \infty$.

a. **[8 marks]** Find the maximum likelihood estimator (MLE) of θ.

b. **[14 marks]** What are the mean and variance of the MLE? What happens to the variance as $n \to \infty$.

c. **[8 marks]** While a closed form solution for the MLE exists, write pseudo-code to perform a Newton-Raphson algorithm to find the MLE. For this particular problem, a friend states that you should have used Fisher scoring. Is your friend correct?

End Of Examination