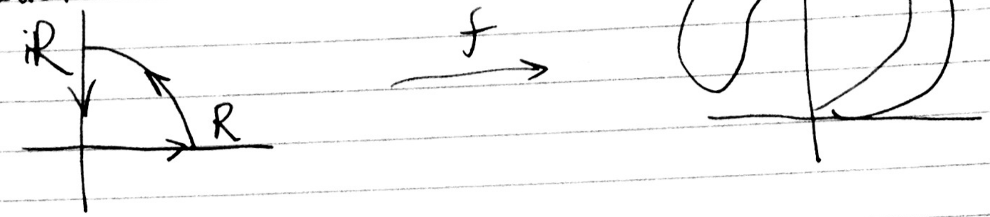
Homework 4 Rui Qiu #999292509

1. 3.1.4

$f(z)=z^2+iz+2+i$

Solution:



On the segment $0\le x\le R$, $f(x)=x^2+ix+2+i$ & $|f(x)|\ge|2+i|$

On the quater-circle, $z=Re^{it}$, $0\le t\le\frac{\pi}{2}$

$$f(Re^{it})=R^2e^{2it}\left(1+\frac{i}{Re^{it}}+\frac{2+i}{R^2e^{2it}}\right)=R^2e^{2it}(1+\xi)$$

which approaches R^2e^{2it} as $R\to\infty$

Thus $\arg f(Re^{it})$ is approximately $\arg(\cancel{R^{2it}}e^{2it})=2t$ for $R\to\infty$

so $\arg f(Re^{it})$ increases from 0 to π as t increases from 0 to $\frac{\pi}{2}$.

On the segment $z=iy$, $R\ge y\ge 0$

$f(iy)=-y^2-y+2+i$

$Re(f(iy))=\cancel{y^2}-y^2-y+2 \begin{cases}\ge 0 & \text{when } 0\le y\le 1\\ <0 & \text{when } y>1\end{cases}$

$Im(f(iy))=1>0$

Hence as y decreases from R to 0, $f(iy)$ lies in the 4th quadrant & then moves towards the point $w=2+i$

Consequently, z traverses the contour, $\arg f(z)$ increases exactly by 0π (from $2+i$ to $2+i$)

Then by the Argument Principle:

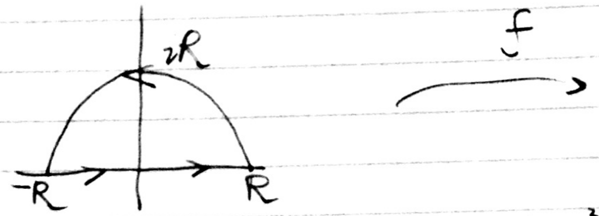
$$\frac{1}{2\pi}\cdot 0=0$$

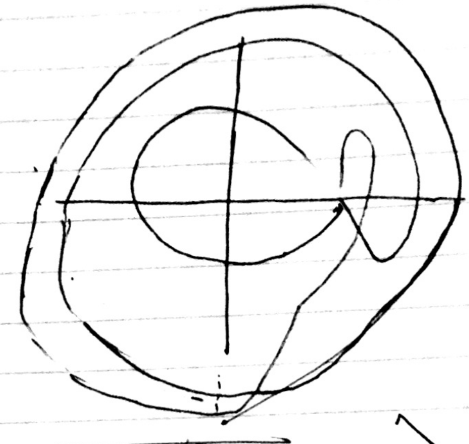
No zeros in the 1st quadrant.

HW4 Rui Qiu

2. 3.1.8

$f(z) = 2z^4 - 2iz^3 + z^2 + 2iz - 1$





Solution: $f(z) = 2z^4 + z^2 - 1 + 2iz(-z^2+1)$

$\cancel{\lim_{z\to\infty} \frac{-2z^3+2z}{2z^4+z^2-1} = \lim \frac{-6z^2+2}{8z^3+2z} = \lim \frac{-12z}{24z^2+2} = \lim \frac{-12}{48z}} = 0$

$\cancel{\lim_{z\to 0} \frac{-2z^3+2z}{2z^4+z^2-1} = \infty}$

~~So when z travels from -R to 0, the f(z) travels~~

As z increases from $-R$ to R

① $x \in [R, -1)$: first quadrant and $f(-1) = 2+1-1 = 2$

② $x \in (-1, -\frac{\sqrt{2}}{2})$: forth quadrant and $f(-\frac{\sqrt{2}}{2}) = -\frac{\sqrt{2}}{2}i$

③ $x \in (-\frac{\sqrt{2}}{2}, 0)$: third quadrant and $f(0) = -1$

④ $x \in (0, \frac{\sqrt{2}}{2})$: second quadrant and $f(\frac{\sqrt{2}}{2}) = \frac{\sqrt{2}}{2}i$

⑤ $x \in (\frac{\sqrt{2}}{2}, 1)$: first quadrant and $f(1) = 2$

⑥ $x \in (1, R)$: forth quadrant

So $\arg f(z)$ decreases 2π (increases -2π) as z goes along the segment.

On the curve, $f(Re^{i\theta}) = 2R^4 e^{i4\theta}\left(1 - \frac{i}{Re^{i\theta}} + \frac{1}{2R^2 e^{2i\theta}} + \frac{i}{R^3 e^{3i\theta}} - \frac{1}{2R^4 e^{i4\theta}}\right)$

$\to 2R^4 e^{i4\theta}$

as $R \to \infty$

so $\arg f(z)$ increases 4π on the curve

so $\frac{1}{2\pi} \cdot 4\pi = 2 = $ # of zeros in upper half-plane.

3.1.12

z^3-3z+1 in $1<|z|<2$

Solution: $p(z)=z^3-z+1$

On the circle $|z|=1$

$$|p(z)+3z|=|z^3-3z+1+3z|$$
$$\leq |z^3|+1$$
$$=2<3=|3z|$$

so $p(z)$ and $f(z)=3z$ have the same number of zeros within $|z|=1$, by Rouché's thm

i.e. $p(z)$ has 1 zero within $|z|=1$.

On the circle $|z|=2$

$|p(z)-z^3|\leq 3(2)+1=7<2^3=|z^3|$

So $p(z)$ and $f(z)=z^3$ have an equal number of zeros with the circle $|z|=2$.

i.e. $p(z)$ has 3 zeros within $|z|=2$

so 2 zeros lie in $1<|z|<2$. $(3-1=2)$

5. 3.3.4(c)

$(1,0,i)$ onto $(1,0,1+i)$

Sol: Plug in $\frac{1a+b}{1c+d}=1 \Rightarrow a+b=c+d$

$\frac{b}{d}=0 \Rightarrow b=0,\ d\neq 0$

$\frac{ai+b}{ci+d}=1+i \Rightarrow ci+d-c+di=ai+b$

$$\left.\begin{array}{l} a=c+d \\ (d-c)+(c+d)i=ai \end{array}\right\} \Rightarrow \begin{array}{l} c=d \\ a=2 \end{array}$$

So $T(z)=\frac{2tz}{tz+t}=\frac{2z}{z+1}$ where $z\neq -1$