Assignment IX

1. The CMB tells us about:
   - Whether there was a beginning to the universe at all
   - The universe used to be much hotter and denser
   - Highly isotropic → leads to horizon problem and inflation
   - Anisotropies: exact shape of CMB power spectrum allows us to determine $\Omega_{m,0}$, $\Omega_{\Lambda,0}$, $\Omega_{b,0}$, etc. depending on a model of universe. Can also be used to constrain inflation mechanisms and test for validity of general relativity

2. a).
$$dF = \frac{L}{4\pi r^2} dN = \frac{L}{\cancel{4\pi r^2}} n \cancel{4\pi r^2} dr$$
$$= nL\,dr$$
$$F(r) = \int_0^{F(r)} dF = \int_0^{r} nL\,dr'$$ ← r' is a "dummy variable"
$$= nLr$$
$$F(\infty) = \lim_{r\to\infty} nLr = \underline{\underline{\infty}}$$

b). The flux from 1 star of luminosity L and distance r is given by
$$F = L/4\pi r^2$$
The number of stars at dist. r needed to cover the sky is $N = 4\pi / \pi R^2/r^2 = 4r^2/R^2$. Total flux is then
$$\Sigma F = NF = \frac{4r^2}{R^2} \frac{L}{4\pi r^2} = L/R^2\pi$$, <u>indep. of r</u>

c). $0.2 \text{ sq. deg.} = 6.09 \cdot 10^{-5}\text{ sr}$ $\quad (1\text{sr} = (\frac{\pi}{180})^2 \text{ sq. deg.})$
$$N = \frac{4\pi \text{ sr}}{6.09\cdot10^{-5}\text{ sr}} = \underline{2.06\cdot10^{5}}$$

d). In the normal case, Earth receives $F_{in} = \frac{L_{sun}}{4\pi r^2}$ flux from the Sun. The cross-section of Earth is πR_e^2, and its surface area is $4\pi R_e^2$. If Earth is in therm. equilibrium with surroundings, then:

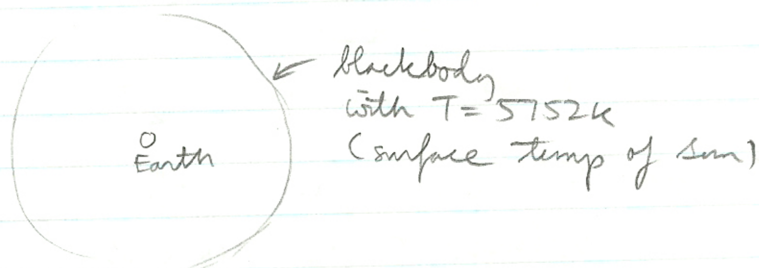
$$L_{in} = L_{out}$$
$$\pi R_e^2 F_{in} = 4\pi R_e^2 F_{out}$$
$$F_{in}/4 = F_{out} = \sigma T_{normal}^4 \qquad T_{normal} = 270\text{ K}$$

In our case, $F_{in} \rightarrow 2.06 \cdot 10^5 F_{in}$, so

$$F_{out} = \sigma T^4 = 2.06 \cdot 10^5 F_{in}/4 = 2.06 \cdot 10^5 \sigma T_{normal}^4$$
$$T = (2.06 \cdot 10^5)^{1/4} \cdot 270\text{K}$$
$$= \underline{\underline{5752\text{K}}}$$

Same as surface temperature of the Sun.

Cheap way:



Zeroth law of thermodynamics: $T_{Earth} = T_{black}$ if the two are in therm. eq., so $T_{Earth} = \underline{\underline{5752\text{K}}}$

e). From "cheap way", above, $\underline{\underline{2.7\text{K}}}$

3. a). $R_0/R_{CMB} = a_0/a_{CMB} = t_0^{2/3}/t_{CMB}^{2/3}$

$R_0 = R_{CMB}(t_0/t_{CMB})^{2/3}$

$R_{CMB} = ct_{CMB} = 3\cdot 10^5$ ly

$R_0 = 3\cdot 10^5 \text{ly}\cdot(13.7\cdot 10^9 \text{yrs}/3\cdot 10^5 \text{yrs})^{2/3}$

$= 3.83\cdot 10^8$ ly

(or $t_0 = 9.3$ Gyr from assign. 6; then

$R_0 = \underline{2.96\cdot 10^8 \text{ ly}}$)

b). $\rho_{clump} = (1+10^{-5})\rho(t_{CMB}) = (1+10^{-5})\rho_0\, a_0^3/a(t_{CMB})^3$

$= (1+10^{-5})\rho_0\, t_0^2/t_{CMB}^2$

$\rho_{clump}/\rho_0 = (1+10^{-5})\left(\frac{13.7\cdot 10^9 \text{yr}}{3\cdot 10^5 \text{yr}}\right)^2$

$= \underline{2.08\cdot 10^9}$

(or use 9.3 Gyr: $\rho_{clump}/\rho_0 = \underline{9.61\cdot 10^8}$)

Superclusters stopped expanding with the rest of the universe only recently; if they did earlier, they would be much more overdense.

c). $\rho_{c,0} = 3H_0^2/8\pi G = 9.21\cdot 10^{-27}$ kg/m³ (assign. 6)

$\rho_0 \approx 0.3\rho_{c,0} = 2.76\cdot 10^{-27}$ kg/m³

$M = \rho V$

$V = M/\rho \approx 10^{42}\text{kg}/2.76\cdot 10^{-27}\text{ kg/m}^3 = 3.62\cdot 10^{68}\text{ m}^3$

$= \underline{4.28\cdot 10^{20}\text{ ly}^3}$

$V = \frac{4}{3}\pi R^3$

$R = \sqrt[3]{\frac{3}{4\pi}V} = 4.67\cdot 10^6$ ly

Galaxy separation $= 2R = 9.34\cdot 10^6$ ly

9.34 Mly > 2.5 Mly, suggesting galaxies have been moving toward each other (expected from gravity).