Code:3137

DEPARTMENT OF MATHEMATICS
University of Toronto
MAT 135 H1F
Term-Test
Monday, October 24, 2011
Time allowed: 90 minutes

Please PRINT in INK or BALL-POINT PEN:
(Please PRINT full name and UNDERLINE surname):

NAME OF STUDENT:_____

STUDENT NO.:_____

SIGNATURE OF STUDENT:_____

TUTORIAL CODE(e.g.,M4A, R5D, etc.):_____

TUTORIAL TIME(e.g.,T4,R5,F3, etc.):_____

NAME OF YOUR TA:_____

NOTE: Before you start, check that this test has 13 pages. There are NO blank pages. This test has two parts:
PART A [50 marks]: 10 multiple choice questions
PART B [50 marks]: 7 written questions
Answers to both PART A and PART B are to be given in this booklet. No computer cards will be used. No aids allowed. No calculators!
DO NOT TEAR OUT ANY PAGES

FOR MARKERS ONLY	
QUESTION	MARK
PART A	/50
B1	/7
B2	/8
B3	/8
B4	/6
B5	/6
B6	/7
B7	/8
Total	/100

PART A *[50 marks]*

Please read carefully:

PART A consists of 10 multiple-choice questions, each of which has exactly one correct answer. Indicate your answer to each question by **completely filling in the appropriate circle with a dark pencil.**

MARKING SCHEME: 5 marks for a correct answer, 0 for no answer or a wrong answer. You are not required to justify your answers in PART A. Note that for PART A, **only your final answers (as indicated by the circles you darken) count; your computations and answers indicated elsewhere will NOT count.**

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1. Find the value of $\lim_{x \to 1} \dfrac{x-1}{\sqrt{x}-1}$.

   Ⓐ 2

   Ⓑ 3

   Ⓒ 4

   Ⓓ 5

   Ⓔ 1

2. Find the value of $\lim_{x \to 3} \dfrac{x^2+x-12}{x^2-x-6}$.

   Ⓐ Does not exist.

   Ⓑ 1

   Ⓒ $\frac{3}{4}$

   Ⓓ 0

   Ⓔ $\frac{7}{5}$

3. Find the value of $\lim_{x \to 0^-} \frac{\sin(2x)}{4x - 3x^4}$.

Ⓐ $-\infty$

Ⓑ $\frac{1}{2}$

Ⓒ 0

Ⓓ $+\infty$

Ⓔ $-\frac{2}{3}$

4. Let $f(x) = \frac{5x^3 - x^2 + 2}{3 + x - 4x^3}$. Find the horizontal asymptote for the graph of f.

Ⓐ the line $y = \frac{2}{3}$

Ⓑ f has no horizontal asymptotes at all.

Ⓒ the line $y = -\frac{1}{2}$

Ⓓ the line $y = \frac{5}{3}$

Ⓔ the line $y = -\frac{5}{4}$

5. Let

$$f(x) = \begin{cases} 2x^2 - 1 & \text{if } x < 2 \\ a & \text{if } x = 2 \\ x^3 - 2bx & \text{if } x > 2 \end{cases}$$

(where a and b are constants).
If f is continuous everywhere, find the value of the product ab.

Ⓐ $\frac{7}{3}$

Ⓑ $\frac{5}{4}$

Ⓒ $\frac{3}{4}$

Ⓓ $\frac{7}{5}$

Ⓔ $\frac{7}{4}$

6. The line perpendicular to the curve $y = x^2 - 3x + 5$ at the point $(1, 3)$ will intersect the y-axis at the point

Ⓐ $(0, 1)$

Ⓑ $(0, 3)$

Ⓒ $(0, 2)$

Ⓓ $(0, 0)$

Ⓔ $(0, 5)$

7. Let $f(x) = ax^2 + bx + c$, where a, b and c are constants. Suppose that the curve $y = f(x)$ passes through the point $(-1, -6)$ and is tangent to the line $y + 7x - 5 = 0$ at the point $(2, -9)$. Find the value of the sum $a + b + c$.

Ⓐ 0

Ⓑ -2

Ⓒ -5

Ⓓ -4

Ⓔ 6

8. Let $f(x) = x^3|x|$ for all x. Which one of the following 5 statements is true?

Ⓐ f''' is continuous but not differentiable at 0.

Ⓑ f is continuous but not differentiable at 0.

Ⓒ f is not continuous at 0.

Ⓓ f'' is continuous but not differentiable at 0.

Ⓔ f' is continuous but not differentiable at 0.

9. Find the value of $\lim_{x \to 0} \dfrac{\cos x - \sqrt{1 + \sin^2 x}}{x^2}$.

Ⓐ $-\frac{1}{2}$

Ⓑ Does not exist.

Ⓒ $\frac{1}{2}$

Ⓓ 0

Ⓔ -1

10. Let $f(x) = |\sin x|$ and $g(x) = |\cos x|$. If $h(x) = f(g(x))$, find $h'(\frac{3\pi}{4})$.

Ⓐ Does not exist.

Ⓑ $\frac{1}{\sqrt{2}} \cos(\frac{1}{\sqrt{2}})$

Ⓒ $-\frac{1}{\sqrt{2}} \sin(\frac{1}{\sqrt{2}})$

Ⓓ $\frac{1}{\sqrt{2}} \sin(\frac{1}{\sqrt{2}})$

Ⓔ $-\frac{1}{\sqrt{2}} \cos(\frac{1}{\sqrt{2}})$

PART B *[50 marks]*

Please read carefully:

Present your complete solutions to the following questions in the spaces provided, in a neat and logical fashion, showing all your computations and justifications. Any answer in PART B without proper justification may receive very little or no credit. Use the back of each page for rough work only. If you must continue your formal solution on the back of a page, you should indicate clearly, in LARGE letters, "SOLUTION CONTINUED ON THE BACK OF PAGE ____." In this case, you may get credit for what you write on the back of that page, but you may also be penalized for mistakes on the back of that page.

MARKS FOR EACH QUESTION ARE INDICATED BY [ ].

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1. Let $f(x) = \frac{1}{x}$. Find $f'(x)$ from first principles (i.e., by using only the definition of the derivative).

[7]

2. Use any suitable method to find $\frac{dy}{dx}$ for each of the following. There is no need to simplify your final answers for this question.

(a) $y = (x^3 + 1)\sin(5x)$.

[4]

(b) $y = \frac{1+e^{2x}}{2+\cos x}$.

[4]

3. For this question, simplify your final answers as much as possible.

(a) Find $f'(\frac{\pi}{4})$ if $f(x) = \sec x \tan x$.

[4]

(b) Find $f'(4)$ if $f(x) = \sqrt{2+\sqrt{x}}$.

[4]

4. Find the value of $\lim_{x\to\infty}(\sqrt{x^2-6x+1}-x)$.

[6]

5. Let $f(x)=\sqrt[3]{\frac{x+5}{x-2}}$. Find the inverse function $f^{-1}(x)$.

[6]

6. Let C be the curve $y = \frac{x^2}{4}$, for $x > 0$. Find the line passing through the point $(0, -4)$ and tangent to the curve C at some point.

[7]

7. (Note: This question will be marked very strictly. Very little credit will be given unless your solution is completely correct.)

Let $f(x) = \frac{\cos^2 x - \sin^2 x}{\sec x \csc x}$. Find $f^{(99)}(\frac{\pi}{12})$, i.e., the value of the 99th derivative of f at $x = \frac{\pi}{12}$. Simplify your final numerical answer as much as possible.

[8]