Unit 3 Part 2: Derivations Exercises

Derivations with AND, OR and BICONDITIONAL

.

SECTION 1: use only MP, MT, R, DN, ADD, S, ADJ, MTP, BC and CB

1. $(P \wedge Q) \wedge (R \wedge S)$. $\therefore$ $(P \wedge S) \wedge Q$

1	~~Show~~ $(P \wedge S) \wedge Q$		show conc
2	$P \wedge Q$	pr1 S	or SL - must use S on the main '$\wedge$'
3	$R \wedge S$	pr1 S	or SR
4	P	2 S	
5	Q	2 S	
6	S	3 S	
7	$P \wedge S$	4 6 ADJ	must adjoin $P \wedge S$ first
8	$(P \wedge S) \wedge Q$	5 7 ADJ	
9		8 dd	

3. $P \leftrightarrow Q$. $Q \leftrightarrow R$. $\therefore$ $R \leftrightarrow P$

1	~~Show~~ $R \leftrightarrow P$		show conc
2	~~Show~~ $R \rightarrow P$		show one direction of the $\leftrightarrow$
3	R	ass cd	show line 2 is $\rightarrow$ Ass ant., show cons.
4	$R \rightarrow Q$	pr2 BC	
5	Q	3 4 mp	
6	$Q \rightarrow P$	pr1 BC	
7	P	5 6 mp, cd	now 2 is shown and available to use
8	~~Show~~ $P \rightarrow R$		show the other direction of the $\leftrightarrow$
9	P	ass cd	show line 8 is $\rightarrow$. Ass ant, show cons.
10	$P \rightarrow Q$	pr1 BC	
11	Q	9 10 mp	
12	$Q \rightarrow R$	pr2 BC	
13	R	11 12 mp, cd	now 8 is shown and available to use
14	$R \leftrightarrow P$	2 8 CB dd	the derivation is direct (but used two conditional subderivations).

4. $R \vee \sim S$. $S \wedge \sim T$. $W \leftrightarrow T$. $\therefore$ $R \wedge \sim W$

1	~~Show~~ $R \wedge \sim W$		show conc: goal R, ~W
2	S	pr2 s	
3	$\sim\sim S$	2 dn	
4	R	3 pr1 mtp	
5	$\sim T$	pr2 s	
6	$W \rightarrow T$	pr3 bc	
7	$\sim W$	5 6 mt	
8	$R \wedge \sim W$	4 7 adj, dd	

5. $W \vee P$. $P \vee S \rightarrow Q$. $X \leftrightarrow {\sim}Q$. $\therefore {\sim}W \rightarrow {\sim}X$

1	~~Show~~ ${\sim}W \rightarrow Q$		show conc
2	${\sim}W$	ass cd	goal: Q
3	P	2 pr1 mtp	
4	$P \vee S$	3 add	Disjoin ‘P’ to ‘S’ so it matches ant. of pr2.
5	Q	4 pr2 mp	
6	$X \rightarrow {\sim}Q$	pr3 bc	
7	${\sim}{\sim}Q$	5 dn	
8	${\sim}X$	7 6 mt	
9		8 cd	

6. $({\sim}S \wedge T) \wedge (W \vee S)$. $\therefore ({\sim}T \leftrightarrow {\sim}X) \rightarrow (X \leftrightarrow W)$

1	~~Show~~ $({\sim}T \leftrightarrow {\sim}X) \rightarrow (X \leftrightarrow W)$		show conc
2	${\sim}T \leftrightarrow {\sim}X$	ass cd	assume ant, show cons. goals: X, W
3	~~Show~~ $X \leftrightarrow W$		Show line is a biconditional
4	~~Show~~ $X \rightarrow W$		Show Left side $\rightarrow$ Right Side
5	X	ass cd	
6	$W \vee S$	pr1 sr	
7	${\sim}S \wedge T$	pr1 sl	
8	${\sim}S$	7 sr	
9	W	6 8 mtp cd	
10	~~Show~~ $W \rightarrow X$		Now show Right Side $\rightarrow$ Left Side
9	W	ass cd	
10	${\sim}X \rightarrow {\sim}T$	2 BC	choose direction of $\rightarrow$
11	${\sim}S \wedge T$	pr1 sl	You can’t use line 7. It’s boxed!
12	T	11 sr	
13	${\sim}{\sim}T$	12 dn	
14	${\sim}{\sim}X$	10 13 mt	
15	X	14 dn cd	
16	$X \leftrightarrow W$	4 10 cb, dd	adjoin 7 & 11 to match cons. of 1
17		3 cd	12 is cons of 1, box and cancel.

7. $P \leftrightarrow (Q \vee R)$. $R \rightarrow S$. ${\sim}S \wedge P$. $W \vee Q \rightarrow R$. $\therefore T$

1	~~Show~~ T		show conc
2	${\sim}T$	ass id	premises must be inconsistent, since no T in them.
3	${\sim}S$	pr3 s	
4	${\sim}R$	pr2 3 mt	
5	${\sim}(W \vee Q)$	pr4 4 mt	
6	P	pr3 s	
7	$P \rightarrow (Q \vee R)$	pr1 bc	
8	$Q \vee R$	6 7 mp	
9	Q	4 8 mtp	
10	$W \vee Q$	9 add	
11		5 10 id	

8. $P \vee (Q \vee R)$. $Q \leftrightarrow (X \wedge Z)$. $(W \rightarrow Z) \rightarrow S$ $\therefore$ $\sim P \rightarrow (\sim R \rightarrow S)$

1	~~Show~~ $\sim P \rightarrow (\sim R \rightarrow S)$		show conc
2	$\sim P$	ass cd	assume ant. of 1, show cons.
3	~~show~~ $\sim R \rightarrow S$		
4	$\sim R$	ass cd	assume ant of 3, new goal: cons. S
5	$Q \vee R$	2 pr1 mtp	line 2 is available to use with pr1
6	Q	4 5 mtp	
7	$Q \rightarrow X \wedge Z$	pr2 BC	choose direction of $\rightarrow$ to match line 6
8	$X \wedge Z$	6 7 mp	
9	~~Show~~ $W \rightarrow Z$		show the antecent of Pr3.
10	W	ass cd	
11	Z	8 sr, cd	
12	S	9 pr3 mp, cd	cd complete, box back to 4 & cancel 3
13		3 cd	3 is cons. of 1, cd done, box & cancel.

9. $W \leftrightarrow Q$. $(\sim Q \vee S) \wedge W$. $S \rightarrow T \wedge U$. $\therefore U \vee \sim T$

1	~~Show~~ $U \vee \sim T$		show conc
2	W	pr2 s	
3	$W \rightarrow Q$	pr1 bc	
4	Q	2 3 mp	
5	$\sim\sim Q$	4 dn	
6	$\sim Q \vee S$	pr2 s	
7	S	5 6 mtp	
8	$T \wedge U$	pr3 7 mp	
9	U	8 s	
10	$U \vee \sim T$	9 add, dd	

NOTE: out of order to save space.

12 . $\therefore$ $(T \wedge S) \rightarrow (((T \vee W) \rightarrow \sim S) \rightarrow \sim W)$

1	~~Show~~ $(T \wedge S) \rightarrow (((T \vee W) \rightarrow \sim S) \rightarrow \sim W)$		show conc
2	$T \wedge S$	ass id	goal: consequent of 1
3	~~Show~~ $((T \vee W) \rightarrow \sim S) \rightarrow \sim W$		show consequent of 1
4	$T \vee W \rightarrow \sim S$	ass id	
5	~~Show~~ $\sim W$		goal: consequent of 3
6	W	ass id	
7	$T \vee W$	6 add	
8	$\sim S$	7 4 mp	
9	S	2 s, 8 id	
10		5 cd	
11		3 cd	

10. $(\sim P \vee R) \wedge (\sim Q \rightarrow \sim R)$. $Q \leftrightarrow (S \wedge W)$. $S \vee R \rightarrow T$. $T \wedge W \rightarrow P$. $\therefore P \leftrightarrow Q$

1	~~Show~~ $P \leftrightarrow Q$		show conc
2	~~Show~~ $P \rightarrow Q$		show one direction of $\leftrightarrow$
3	P	ass cd	Show line 2 is $\rightarrow$ assume antecedent.
4	$\sim P \vee R$	pr1 s	
5	$\sim\sim P$	3 dn	
6	R	4 5 mtp	
7	$\sim Q \rightarrow \sim R$	pr1 s	
8	$\sim\sim R$	6 dn	
9	$\sim\sim Q$	7 8 mt	
10	Q	9 dn, cd	
11	~~Show~~ $Q \rightarrow P$		show other direction of $\leftrightarrow$
12	Q	ass cd	show line 11 is $\rightarrow$ assume antecedent
13	$Q \rightarrow (S \wedge W)$	pr2 bc	choose direction of $\rightarrow$ to work with 12
14	$S \wedge W$	12 13 mp	
15	S	14 s	
16	$S \vee R$	15 add	
17	T	16 pr3 mp	
18	W	14 s	
19	$T \wedge W$	17 18 adj	
20	P	19 pr4 mp, cd	
21	$P \leftrightarrow Q$	2 11 cb, dd	

11. $\sim(P \wedge Q)$. $\sim P \rightarrow T$. $\sim T \rightarrow Q$. $\therefore$ T

1	~~Show~~ T		show conc
2	$\sim T$	ass id	assume the opposite of show line 1. Goal: contradiction
3	Q	2 pr3 mp	
4	$\sim\sim P$	2 pr2 mt	
5	P	4 dn	
6	$P \wedge Q$	3 5 adj	
7	$\sim(P \wedge Q)$	pr1, 6 id	

13. $P \vee Q \rightarrow R$. $\sim T \wedge (P \rightarrow S)$. $S \leftrightarrow \sim R$. $\sim P \rightarrow T$. $\therefore$ W

1	~~Show~~ W	
2	$\sim W$	ass id
3	$\sim T$	pr2 s
4	$\sim\sim P$	3 pr4 mt
5	P	4 dn
6	$P \rightarrow S$	pr2 s
7	S	5 6 mp
8	$P \vee Q$	5 add
9	R	8 pr1 mp
10	$S \rightarrow \sim R$	pr3 bc
11	$\sim\sim R$	9 dn
12	$\sim S$	10 11 mt, 7 id

14. $\sim(P \rightarrow Q)$. $P \leftrightarrow R$. $Q \vee S$. $\therefore$ $R \wedge S$

1	~~Show~~ $R \wedge S$		show conc
2	~~Show~~ P		You need P, and can get it from Pr1.
3	$\sim P$	ass id	
4	~~Show~~ $P \rightarrow Q$		
5	P	ass cd	
6	$\sim P$	3 r	5 and 6 contradict each other, completing
7		5 6 id	the requirements for ID. Box and cancel.
8	$\sim(P \rightarrow Q)$	pr1, 4 id	
9	~~Show~~ $\sim Q$		You need ~Q, and can get it from Pr1
10	Q	ass id	
11	~~Show~~ $P \rightarrow Q$		
12	P	ass cd	
13	Q	10 r	
14	$\sim(P \rightarrow Q)$		
15	S	9 pr3 mtp	
16	$P \rightarrow R$	pr2 bc	
17	R	2 16 mp	
18	$R \wedge S$	15 17 adj, dd	

15. $\sim(P \wedge R)$. $S \rightarrow R$. $\sim P \rightarrow \sim S$. $\therefore$ $T \vee \sim S$

1	~~Show~~ $T \vee \sim S$		show conc
2	$\sim(T \vee \sim S)$	ass id	Can't show conc. directly, so use id.
3	~~Show~~ S		2 is a negated $\vee$: gives you either disjunct,
4	$\sim S$	ass id	unnegated. Here, S would be useful. You
5	$T \vee \sim S$	4 add	need it, can show it, so show it!
6	$\sim(T \vee \sim S)$	2 r, 5 id	
7	R	3 pr2 mp	
8	$\sim\sim S$	3 dn	
9	$\sim\sim P$	8 pr3 mt	
10	P	9 dn	
11	$P \wedge R$	7 10 adj	
12	$\sim(P \wedge R)$	pr1, 11 id	

18. $\therefore$ $\sim S \vee Q \rightarrow ((Q \rightarrow \sim(P \rightarrow T)) \rightarrow \sim(T \wedge S))$

1	~~Show~~ $\sim S \vee Q \rightarrow ((Q \rightarrow \sim(P \rightarrow T)) \rightarrow \sim(T \wedge S))$		show conc
2	$\sim S \vee Q$	ass cd	1 is $\rightarrow$, assume ant.
3	~~Show~~ $(Q \rightarrow \sim(P \rightarrow T)) \rightarrow \sim(T \wedge S)$		show consequent of 1
4	$Q \rightarrow \sim(P \rightarrow T)$	ass cd	3 is $\rightarrow$, assume ant
5	~~Show~~ $\sim(T \wedge S)$		show consequent of 3
6	$T \wedge S$	ass id	
7	S	6 s	
8	$\sim\sim S$	7 dn	
9	Q	2 8 mtp	
10	$\sim(P \rightarrow T)$	4 9 mp	
11	~~Show~~ $P \rightarrow T$		
12	P	ass cd	
13	T	6 s, cd	
14		11 10 id	
15		5 cd	
16		3 cd	

19. $\therefore$ $\sim(P \rightarrow Q) \rightarrow (\sim(R \vee S) \rightarrow \sim(S \vee \sim P))$

1	~~Show~~ $\sim(P \rightarrow Q) \rightarrow (\sim(R \vee S) \rightarrow \sim(S \vee \sim P))$		show conc
2	$\sim(P \rightarrow Q)$	ass cd	1 is $\rightarrow$, assume ant.
3	Show $\sim(R \vee S) \rightarrow \sim(S \vee \sim P)$		show consequent of 1
4	$\sim(R \vee S)$	ass cd	3 is $\rightarrow$, assume ant
5	~~Show~~ $\sim(S \vee \sim P)$		show consequent of 3
6	$S \vee \sim P$	ass id	
7	~~Show~~ $\sim S$		
8	S	ass id	
9	$R \vee S$	8 add	
10	$\sim(R \vee S)$	4 r, 9 id	
11	$\sim P$	6 7 mtp	
12	~~Show~~ $P \rightarrow Q$		
13	P	ass cd	
14	$\sim P$	11 r, 13 id	
15	$\sim(P \rightarrow Q)$	2 r, 12 id	
16		5 cd	
17		3 cd	

T33 and T49 justify Separation of Cases (Derived rule: SC)

T33 $\therefore (P \to Q) \wedge (\sim P \to Q) \to Q$ (Separation of cases, special case)

1	~~Show~~ $(P \to Q) \wedge (\sim P \to Q) \to Q$		show conc
2	$(P \to Q) \wedge (\sim P \to Q)$	ass cd	
3	Show Q		
4	~Q	ass id	
5	$P \to Q$	2 sl	
6	$\sim P \to Q$	2 sr	
7	~P	4 5 mt	
8	~~P	4 6 mt	
9		7 8 id	
10			

T49 $\therefore (P \vee Q) \wedge (P \to R) \wedge (Q \to R) \to R$ (Separation of cases)

1	~~Show~~ $(P \vee Q) \wedge (P \to R) \wedge (Q \to R) \to R$		show conc
2	$(P \vee Q) \wedge (P \to R) \wedge (Q \to R)$	ass cd	ass ant of 1
3	~~Show~~ R		show cons.
4	~R	ass id	
5	$(P \vee Q) \wedge (P \to R)$	2 sl	The $\wedge$ on the right is the main connective in informal notation.
6	$Q \to R$	2 sr	
7	$P \vee Q$	4 sl	
8	$P \to R$	4 sr	
9	~P	4 8 mt	
10	Q	7 9 mtp	
11	R	6 10 mp	
12		4 11 id	4 is neg. of 11, id done
13		3 cd	3 is cons. of 2, cd done

T40 justifies Negation of Conditional (Derived rule: NC)

T40 $\therefore \sim(P \rightarrow Q) \leftrightarrow P \wedge \sim Q$ (Negation of conditional)

1	~~Show~~ $\sim(P \rightarrow Q) \leftrightarrow P \wedge \sim Q$		show conc
2	~~Show~~ $\sim(P \rightarrow Q) \rightarrow P \wedge \sim Q$		show LS $\rightarrow$ RS
3	$\sim(P \rightarrow Q)$	ass cd	
4	~~Show~~ P		We need P & know a negated
5	$\sim P$	ass id	conditional gives us the ant. Show it!
6	~~Show~~ $P \rightarrow Q$		Show this to contradict 3 for id
7	P	ass cd	assume ant of 6
8	~~Show~~ Q		show cons
9	$\sim Q$	ass id	we have contradiction already! 5 & 7
10	P	7 r	
11	$\sim P$	5 r, 10 id	11 is negation of 10, id done.
12		8 cd	
13	$\sim(P \rightarrow Q)$	3 r	reiterate to get below ass. for 4
14		13 6 id	13 is negation of 6, id done
15	~~Show~~ $\sim Q$		We need it & can show it!
16	Q	ass id	
17	~~Show~~ $P \rightarrow Q$		Show this to contradict 3 for id.
18	P	ass cd	
19	Q	16 r, cd	19 is cons. of 17, cd done
20	$\sim(P \rightarrow Q)$	3 r	reiterate to get below ass. for 15
21		17 20	20 is negation of 17, id done
22		3 cd	3 is cons. of 2, cd done
23	~~Show~~ $P \wedge \sim Q \rightarrow \sim(P \rightarrow Q)$		show RS $\rightarrow$ LS
24	$P \wedge \sim Q$	ass cd	assume ant of 23
25	~~Show~~ $\sim(P \rightarrow Q)$		show cons.
26	$P \rightarrow Q$	ass id	
27	P	24 s	
28	Q	26 27 mp	
29	$\sim Q$	24 s	
30		28 29 id	29 is negation of 28, id done
31			
32	$\sim(P \rightarrow Q) \leftrightarrow P \wedge \sim Q$	2 23 cb	We've shown both directions of $\leftrightarrow$
33		32 dd	

T46 justifies Conditional as Disjunction (Derived rule: CDJ)

T46 $\therefore (P \to Q) \leftrightarrow \sim P \vee Q$ (Conditional as disjunction)

1	~~Show~~ $(P \to Q) \leftrightarrow \sim P \vee Q$		show conc
2	~~Show~~ $(P \to Q) \to \sim P \vee Q$		show LS → RS
3	$P \to Q$	ass cd	assume ant. of 2
4	~~Show~~ $\sim P \vee Q$		show cons
5	$\sim(\sim P \vee Q)$	ass id	
6	~~Show~~ P		Why show P? We can use it with 3
7	$\sim P$	ass id	to get Q, which will give us ~P ∨ Q.
8	$\sim P \vee Q$	7 add	We can show it easily with ID.
9	$\sim(\sim P \vee Q)$	5 r	
10		8 9 id	9 is neg. of 8, id done, box & cancel
11	Q	6 3 mp	
12	$\sim P \vee Q$	11 add	
13		12 5 id	5 is neg. of 12, id done, box & cancel
14		4 cd	4 is cons. of 3, cd done.
15	~~Show~~ $\sim P \vee Q \to (P \to Q)$		show RS → LS
16	$\sim P \vee Q$	ass cd	assume ant of 15
17	~~show~~ $P \to Q$		show cons
18	P	ass cd	17 is conditional, assume ant, goal: Q
19	$\sim\sim P$	18 dn	
20	Q	16 19 mtp	
21		20 cd	20 is cons. of 17, cd done.
22		17 cd	17 is cons. of 15, cd done
23	$(P \to Q) \leftrightarrow \sim P \vee Q$	2 15 cb	We've shown both directions of ↔
24		23 dd	

T63, T64, T65, T66 together justify De Morgan's Law (derived rule: DM)

T63 $\therefore P \wedge Q \leftrightarrow \sim(\sim P \vee \sim Q)$ (De Morgan's Law)

1	~~Show~~ $P \wedge Q \leftrightarrow \sim(\sim P \vee \sim Q)$		show conc
2	~~Show~~ $P \wedge Q \rightarrow \sim(\sim P \vee \sim Q)$		show LS → RS
3	$P \wedge Q$	ass cd	assume ant. of 2
4	~~Show~~ $\sim(\sim P \vee \sim Q)$		show cons
5	$\sim P \vee \sim Q$	ass id	assume the opposite of 4
6	P	3 sl	
7	~~P	6 dn	
8	~Q	5 7 mtp	
9	Q	3 sr	
10		8 9 id	8 contradicts 9. ID done. Box/cancel
11		4 cd	4 is cons. of 2, cd done. Box/cancel
12	~~Show~~ $\sim(\sim P \vee \sim Q) \rightarrow P \wedge Q$		show RS → LS
13	$\sim(\sim P \vee \sim Q)$	ass cd	assume ant of 12
14	~~Show~~ P		Why show P rather than $P \wedge Q$? It is
15	~P	ass id	easy to show P and Q separately
16	$\sim P \vee \sim Q$	15 add	with id!
17	$\sim(\sim P \vee \sim Q)$	13 r	reiterate 13 to get it under ass. for id
18		16 17 id	16 is neg of 17, ID done. Box/cancel
19	~~Show~~ Q		
20	~Q		
21	$\sim P \vee \sim Q$		
22	$\sim(\sim P \vee \sim Q)$		
23		21 22 id	21 is neg of 22, id done. Box/cancel
24	$P \wedge Q$	14 19 adj	
25		24 cd	24 is cons of 12, cd done. Box/cancel
26	$P \wedge Q \leftrightarrow \sim(\sim P \vee \sim Q)$	2 12 cb	We've shown both directions of ↔
27		26 dd	

Now it should be easy to prove T66:

T66 $\therefore \sim(P \vee Q) \leftrightarrow \sim P \wedge \sim Q$ (De Morgan's Law)

T64 $\therefore P \vee Q \leftrightarrow \sim(\sim P \wedge \sim Q)$ (De Morgan's Law)

1	~~Show~~ $P \vee Q \leftrightarrow \sim(\sim P \wedge \sim Q)$		show conc
2	~~Show~~ $P \vee Q \rightarrow \sim(\sim P \wedge \sim Q)$		show LS $\rightarrow$ RS
3	$P \vee Q$	ass cd	assume ant. of 2
4	~~Show~~ $\sim(\sim P \wedge \sim Q)$		show cons
5	$\sim P \wedge \sim Q$	ass id	assume the opposite of 4
6	$\sim P$	5 sl	
7	Q	6 3 mtp	
8	$\sim\sim Q$	7 dn	
9	$\sim Q$	3 sr	
10		8 9 id	8 contradicts 9. ID done. Box/cancel
11		4 cd	4 is cons. of 2, cd done. Box/cancel
12	~~Show~~ $\sim(\sim P \wedge \sim Q) \rightarrow P \vee Q$		show RS $\rightarrow$ LS
13	$\sim(\sim P \wedge \sim Q)$	ass cd	assume ant of 12
14	~~Show~~ $P \vee Q$		
15	$\sim(P \vee Q)$	ass id	Now it is easy to show ~P and ~Q which will contradict line 13.
16	~~Show~~ $\sim P$		
17	P	ass id	
18	$P \vee Q$	17 add	
19	$\sim(P \vee Q)$	15 r 18 id	reiterate 15 to get it below ass for id 17
20	~~Show~~ $\sim Q$		
21	Q	ass id	
22	$P \vee Q$	21 add	
23	$\sim(P \vee Q)$	15 r 21 id	reiterate 15 to get it below ass for id 21
24	$\sim P \wedge \sim Q$	16 20 add	
25	$\sim(\sim P \wedge \sim Q)$	13 r	reiterate 13 to get it below ass for id 15
26		21 22 id	
27		14 cd	14 is cons of 12, cd done. Box/cancel.
28	$P \vee Q \leftrightarrow \sim(\sim P \wedge \sim Q)$	2 12 cb	We've shown both directions of $\leftrightarrow$
29		26 dd	

Now it should be easy to prove T65.

T65 $\therefore \sim(P \wedge Q) \leftrightarrow \sim P \vee \sim Q$ (De Morgan's Law)

T90 justifies Negation of Biconditional (Derived rule: NB)

T90 $\therefore \sim(P \leftrightarrow Q) \leftrightarrow (P \leftrightarrow \sim Q)$ (Negation of biconditional)

1	~~Show~~ $\sim(P \leftrightarrow Q) \leftrightarrow (P \leftrightarrow \sim Q)$		show conc
2	~~Show~~ $\sim(P \leftrightarrow Q) \rightarrow (P \leftrightarrow \sim Q)$		show LS → RS
3	$\sim(P \leftrightarrow Q)$	ass cd	assume ant. of 2
4	~~Show~~ $P \leftrightarrow \sim Q$		show cons
5	~~Show~~ $P \rightarrow \sim Q$		show LS → RS
6	P	ass cd	ass ant of 5
7	~~Show~~ $\sim Q$		show cons
8	Q	ass id	
9	~~Show~~ $P \rightarrow Q$		You need to show $P \leftrightarrow Q$ (opposite of 3)
10	P	ass cd	to get a contradiction. This is easy
11	Q	8 r, cd	because you already have P (6) & Q (8).
12	~~Show~~ $Q \rightarrow P$		
13	Q	ass cd	
14	P	6 r, cd	
15	$P \leftrightarrow Q$	9 12 cb	
16	$\sim(P \leftrightarrow Q)$	3 r, 15 id	Get 3 under assump. for 5. ID done.
17		7 cd	
18	~~Show~~ $\sim Q \rightarrow P$		
19	$\sim Q$	ass cd	
20	~~Show~~ P		
21	$\sim P$	ass id	
22	~~Show~~ $P \rightarrow Q$		Again, you need to show $P \leftrightarrow Q$.
23	P	ass cd	You have a contradiction (21, 23) So you
24	~~Show~~ Q		can show anything. Show Q because you
25	$\sim Q$	ass id	need it!
26	P	23 r	
27	$\sim P$	21 r, 26 id	
28		24 cd	
29	~~Show~~ $Q \rightarrow P$		
30	Q	ass cd	You have a contradiction (19, 30) so if you
31	$\sim Q$	19 r	want, show P (like 24-27)
32		30 31 id	Or just use ID now!
33	$P \leftrightarrow Q$	22 29 bc	
34	$\sim(P \leftrightarrow Q)$	3 r	Get 3 under assump. for 18. ID done.
35		20 cd	
36	$P \leftrightarrow \sim Q$	5 18 cb	
37		4 cd	4 is cons. of 2, cd done. Box/cancel
38	~~Show~~ $(P \leftrightarrow \sim Q) \rightarrow \sim(P \leftrightarrow Q)$		show RS → LS

CONTINUED ON NEXT PAGE (You would think there would be a faster way to prove this theorem! Good thing for that derived rule!)

T90 cont.

38	~~Show~~ $(P \leftrightarrow \sim Q) \rightarrow \sim(P \leftrightarrow Q)$		show RS $\rightarrow$ LS
39	$P \leftrightarrow \sim Q$	ass cd	assume ant of 12
40	~~Show~~ $\sim(P \leftrightarrow Q)$		
41	$P \leftrightarrow Q$	ass id	
42	~~Show~~ $\sim P$		
43	P	ass id	
44	$P \rightarrow Q$	39 bc	
45	$P \rightarrow \sim Q$	41 bc	
46	Q	43 44 mp	
47	$\sim Q$	43 45 mp	
48		46 47 id	
49	$\sim Q \rightarrow P$	39 bc	
50	$Q \rightarrow P$	41 bc	
51	$\sim\sim Q$	42 49 mt	
52	$\sim Q$	42 50 mt, 51 id	
53		40 cd	
54	$(P \leftrightarrow \sim Q) \leftrightarrow \sim(P \leftrightarrow Q)$	2 38 cb	We've shown both directions of $\leftrightarrow$
55		54 dd	

SECTION 3: REMEMBER, YOU CAN'T USE THE DERIVED RULES (DM, NC, ADJ, SC AND NB) UNTIL AFTER YOU HAVE PROVEN THE RELATED THEOREMS!

21. $S \to W. \quad T \vee (\sim S \to R) \therefore (W \vee T) \vee R$

1	~~Show~~ $(W \vee T) \vee R$	
2	$\sim((W \vee T) \vee R)$	ASS ID
3	$\sim(W \vee T) \wedge \sim R$	2 DM
4	$\sim R$	3 S
5	$\sim(W \vee T)$	3 S
6	$\sim W \wedge \sim T$	5 DM
7	$\sim W$	6 S
8	$\sim T$	6 S
9	$\sim S$	PR1 7 MT
10	$\sim S \to R$	8 PR2 MTP
11	$\sim\sim S$	4 10 MT, 9 ID

22. $\sim(\sim P \to Q). \quad \sim(R \vee S \leftrightarrow Q). \quad R \to \sim T. \quad \sim S \vee \sim T. \quad \therefore \sim(T \vee P)$

1	~~Show~~ $\sim(T \vee P)$	
2	$\sim P \wedge \sim Q$	PR1 NC
3	$\sim P$	2 S
4	$\sim Q$	2 S
5	$R \vee S \leftrightarrow \sim Q$	2 NB
6	$\sim Q \to R \vee S$	5 BC
7	$R \vee S$	4 6 MP
8	$S \to \sim T$	PR4 CDJ
9	$\sim T$	7 PR3 8 SC
10	$\sim T \wedge \sim P$	3 9 ADJ
11	$\sim(T \vee P)$	10 DM DD

24. $P \vee Q. \quad \sim(P \wedge Q). \quad P \vee \sim Q. \quad \therefore \sim(\sim P \vee Q)$

1	~~Show~~ $\sim(\sim P \vee Q)$	
2	$\sim P \vee Q$	ASS ID
3	$P \to Q$	2 CDJ
4	$\sim P \to Q$	PR1 CDJ
5	Q	3 4 SC
6	$\sim\sim Q$	5 DN
7	P	6 PR3 MTP
8	$\sim P \vee \sim Q$	PR2 DM
9	$\sim P$	6 8 MTP, 7 ID

25. $\sim T \rightarrow \sim(S \rightarrow \sim R) \;\therefore\; (R \wedge S) \vee T$

1	~~Show~~ $(R \wedge S) \vee T$	
2	~~Show~~ $\sim(R \wedge S) \rightarrow T$	
3	$\sim(R \wedge S)$	ass cd
4	~~show~~ T	
5	$\sim T$	ass id
6	$\sim(S \rightarrow \sim R)$	pr1 5 mp
7	$S \wedge \sim\sim R$	6 nc
8	$\sim\sim R$	7 sr
9	R	8 dn
10	$R \wedge S$	7 sl 9 adj
11	$\sim(R \wedge S)$	3 r
12		4 cd
13	$(R \wedge S) \vee T$	2 cdj dd

26. $(R \leftrightarrow S) \vee Q. \quad \sim(Q \rightarrow \sim R) \rightarrow S. \quad \therefore\; R \rightarrow S$

1	~~Show~~ $R \rightarrow S$	
2	R	ass cd
3	~~Show~~ S	
4	$\sim S$	ass id
5	$Q \rightarrow \sim R$	4 pr2 mt dn
6	$\sim Q$	2 dn 5 mt
7	$R \leftrightarrow S$	pr1 6 mtp
8	S	7 bc 2 mp, 4 id
9		3 cd

27. $\therefore\ \sim(R \lor S) \land \sim(\sim P \to Q) \leftrightarrow \sim((P \lor Q) \lor (R \lor S))$

1	~~Show~~ $\sim(R \lor S) \land \sim(\sim P \to Q) \leftrightarrow \sim((P \lor Q) \lor (R \lor S))$	
2	~~Show~~ $\sim(R \lor S) \land \sim(\sim P \to Q) \to \sim((P \lor Q) \lor (R \lor S))$	
3	$\sim(R \lor S) \land \sim(\sim P \to Q)$	ass cd
4	$\sim(R \lor S)$	3 sl
5	$\sim(\sim P \to Q)$	3 sr
6	$\sim P \land \sim Q$	5 nc
7	$\sim(P \lor Q)$	6 dm
8	$\sim(P \lor Q) \land \sim(R \lor S)$	4 7 adj
9	$\sim((P \lor Q) \lor (R \lor S))$	8 dm cd
10	~~Show~~ $\sim((P \lor Q) \lor (R \lor S)) \to \sim(R \lor S) \land \sim(\sim P \to Q)$	
11	$\sim((P \lor Q) \lor (R \lor S))$	ass cd
12	$\sim(P \lor Q) \land \sim(R \lor S)$	11 dm
13	$\sim(P \lor Q)$	12 sl
14	$\sim P \land \sim Q$	13 dm
15	$\sim(\sim P \to Q)$	14 nc
16	$\sim(R \lor S)$	12 sr
17	$\sim(R \lor S) \land \sim(\sim P \to Q)$	16 15 adj
18	$\sim(R \lor S) \land \sim(\sim P \to Q) \leftrightarrow \sim((P \lor Q) \lor (R \lor S))$	2 10 cb dd

30. $\sim R \lor W.\quad X \land S \to T.\quad R \lor W.\quad \sim W \lor X.\quad \therefore\ S \to T \lor P$

1	~~Show~~ $S \to T \lor P$	
2	S	ass cd
3	$R \to W$	pr1 cdj
4	$\sim R \to W$	pr3 cdj
5	W	3 4 sc
6	$\sim\sim W$	5 dn
7	X	pr4 6 mtp
8	$X \land S$	2 7 adj
9	T	8 pr2 mp
10	$T \lor P$	9 add cd

32. $\sim(P \vee Q)$. $(R \rightarrow S) \rightarrow \sim(P \rightarrow T)$. $(S \vee Q) \vee W$. $\therefore R \vee W$

1	~~Show~~ $R \vee W$	
2	$\sim(R \vee W)$	ass id
3	$\sim R \wedge \sim W$	2 dm
4	$\sim P \wedge \sim Q$	pr1 dm
5	$\sim W$	3 s
6	$S \vee Q$	pr3 mtp
7	$\sim Q$	4 s
8	S	6 7 mtp
9	$\sim R \vee S$	8 add
10	$R \rightarrow S$	9 cdj
11	$\sim(P \rightarrow T)$	10 pr2 mp
12	$P \wedge \sim T$	11 nc
13	P	12 s
14	$\sim P$	4 s 13 id

34. $(S \rightarrow (\sim P \vee T)) \rightarrow W$. $(R \leftrightarrow W) \wedge \sim R$. $\sim(S \rightarrow T) \rightarrow Q$. $\therefore P \wedge Q$

1	~~Show~~ $P \wedge Q$	
2	$\sim R$	ass id
3	$R \leftrightarrow W$	2 dm
4	$W \rightarrow R$	3 bc
5	$\sim W$	2 4 mt
6	$\sim(S \rightarrow (\sim P \vee T))$	5 pr1 mt
7	$S \wedge \sim(\sim P \vee T)$	6 nc
8	S	7 s
9	$\sim\sim P \wedge \sim T$	7s, dm
10	$\sim T$	9 s
11	$S \wedge \sim T$	8 10 adj
12	$\sim(S \rightarrow T)$	11 nc
13	Q	12 pr3 mp
14	P	9 sl dn
15	$P \wedge Q$	13 14 adj