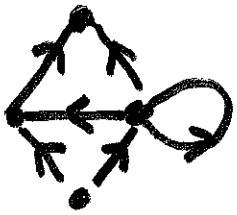
# Lecture 5 handout

(1.5)

A directed graph (digraph) is:



- A set $V$
- A set $E$
- An incidence function $\psi: E \to V \times V$

- $D$ —forget orientations→ $G(D)$ underlying graph

- $G$ ⟶ $D$ associated digraph

- $G$ —choose→ $D = \vec{G}$ orientation

A tournament is an oriented complete graph.

indeg = 2
outdeg = 3.

$D \to \overleftarrow{D}$ is the converse.

(reverse all arrows)

Anything true for $D$ is true for $\overleftarrow{D}$ mutatis mutandis.

## Some additional terms:


Strongly connected if any 2 vertices are connected by an oriented path.


(weakly) connected


strongly connected.


sink


source

---

Spanning subgraph

$H \subseteq G$ obtained only by edge delete.

Spanning subgraph that is a cycle is a Hamiltonian cycle.

" " " " " path " " " path

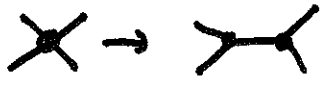
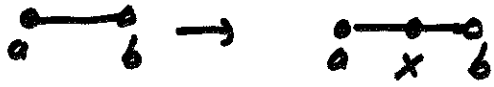
A complete graph has a Hamiltonian cycle. (How many?)

A tournament has a directed Hamiltonian path.

(Rédei's Theorem)

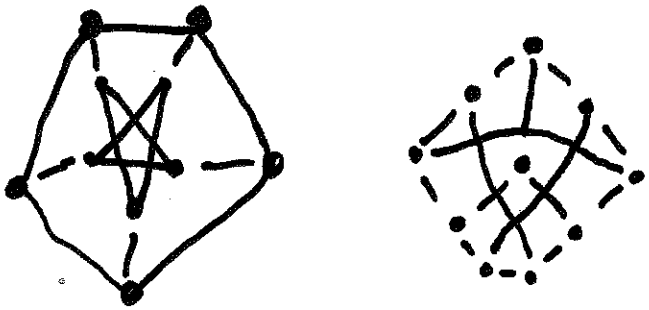
Proof: Induction

## Odds and ends

- A subgraph obtained by vertex delete is an induced subgraph.
- Vertex split  (not unique)
- Edge subdivide 


---

## Petersen's Graph



Girth (smallest cycle): 5

Proof: Check cases, G as union of two 5-cycles.

Circumference: (longest cycle): 9

Proof: Again, look at bridge edges, check cases.

---

Next time: Trees (Chapter 4)

4.1 - 4.3