Camera calibration

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This lecture is being livestreamed and recorded

(hopefully)

Learning objectives

After this lecture you should be able to:

- implement and use the direct linear transformation to calibrate a pinhole camera
- implement and use Zhang's method (2000) to calibrate a pinhole camera using checkerboards

Presentation topics

- Direct linear transformation
- Zhang's method (2000)
    - Reprojection error
- Non-linear calibration
- Practical remarks

Culmination of previous weeks

- Pinhole camera model
- Homogeneous coordinates
- Homographies
- Linear algorithms
- Calibration

Direct linear transformation

Direct linear transformation

Start from the projection equation

$$\boldsymbol{q}_i = \boldsymbol{\mathcal{P}}\boldsymbol{Q}_i$$

$$\begin{bmatrix} sx_i \\ sy_i \\ s \end{bmatrix} = \boldsymbol{\mathcal{P}} \begin{bmatrix} X_i \\ Y_i \\ Z_i \\ 1 \end{bmatrix}$$

then rearrange into the form $\boldsymbol{B}^{(i)}\text{flatten}(\boldsymbol{\mathcal{P}}^\top) = \boldsymbol{0}$.

I use the form explained in LN: 2.7 Camera Resection.

Direct linear transformation i

$$
\begin{aligned}
\boldsymbol{q}_i &= \boldsymbol{\mathcal{P}} \boldsymbol{Q}_i, \\
\mathbf{0} &= \boldsymbol{q}_i \times \boldsymbol{\mathcal{P}} \boldsymbol{Q}_i, \\
&= [\boldsymbol{q}_i]_\times \boldsymbol{\mathcal{P}} \boldsymbol{Q}_i, \\
&= \boldsymbol{B}^{(i)} \mathsf{flatten}(\boldsymbol{\mathcal{P}}^\mathsf{T}),
\end{aligned}
$$

where (continue to next slide) ...

Direct linear transformation ii

$$\mathbf{0} = \boldsymbol{B}^{(i)}\text{flatten}(\boldsymbol{\mathcal{P}}^\mathsf{T}),$$

$$\boldsymbol{B}^{(i)} = \begin{bmatrix} 0 & -X_i & X_iy_i & 0 & -Y_i & Y_iy_i & 0 & -Z_i & Z_iy_i & 0 & -1 & y_i \\ X_i & 0 & -X_ix_i & Y_i & 0 & -Y_ix_i & Z_i & 0 & -Z_ix_i & 1 & 0 & -x_i \\ -X_iy_i & X_ix_i & 0 & -Y_iy_i & Y_ix_i & 0 & -Z_iy_i & Z_ix_i & 0 & -y_i & x_i & 0 \end{bmatrix},$$

$$= \boldsymbol{Q}_i \otimes [\boldsymbol{q}_i/s]_\times$$

$$\text{flatten}(\boldsymbol{\mathcal{P}}^\mathsf{T}) = \begin{bmatrix}\mathscr{P}_{11} & \mathscr{P}_{21} & \mathscr{P}_{31} & \mathscr{P}_{12} & \mathscr{P}_{22} & \mathscr{P}_{32} & \mathscr{P}_{13} & \mathscr{P}_{23} & \mathscr{P}_{33} & \mathscr{P}_{14} & \mathscr{P}_{24} & \mathscr{P}_{34}\end{bmatrix}^\mathsf{T}$$

Direct linear transformation

Now let

$$\mathbf{0} = \boldsymbol{B}\ \mathsf{flatten}(\boldsymbol{\mathcal{P}}^\top),$$

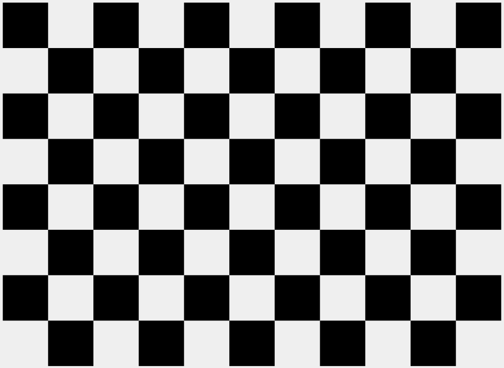
where

$$\boldsymbol{B} = \begin{bmatrix} \boldsymbol{B}^{(1)} \\ \boldsymbol{B}^{(2)} \\ \vdots \end{bmatrix},$$

and solve using SVD on $\boldsymbol{B}$.

Zhang's method (2000)

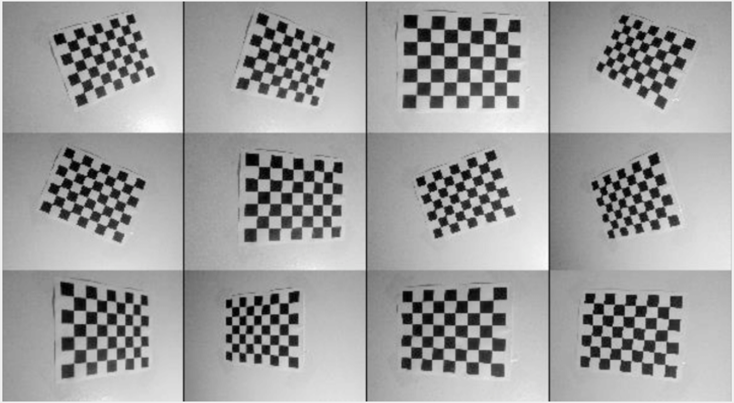
Using a checkerboard



www.calib.io | 8x11 | Checker Size: 15 mm.

Using a checkerboard

View the checkerboard in different poses:



Using a checkerboard

Problem:

Each view i has a different rotation $\boldsymbol{R}_i$ and translation $\boldsymbol{t}_i$

How do we find all $\mathcal{P}_i$?

Zhang's method

First, assume all checkerboard corners are in the $Z = 0$ plane:

$$\boldsymbol{Q}_j = \begin{bmatrix} X_j \\ Y_j \\ 0 \\ 1 \end{bmatrix}$$

Simplifying the projection equation

Let $\boldsymbol{r}_i^{(c)}$ is the c^{th} column of $\boldsymbol{R}_i$. Now projection is

$$\boldsymbol{q}_{ij} = \mathcal{P}_i \boldsymbol{Q}_j = \boldsymbol{K} \begin{bmatrix} \boldsymbol{r}_i^{(1)} & \boldsymbol{r}_i^{(2)} & \boldsymbol{r}_i^{(3)} & \boldsymbol{t}_i \end{bmatrix} \underbrace{\begin{bmatrix} X_j \\ Y_j \\ 0 \\ 1 \end{bmatrix}}_{\boldsymbol{Q}}$$

Simplifying the projection equation

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$$= \boldsymbol{K} \begin{bmatrix} \boldsymbol{r}_i^{(1)} & \boldsymbol{r}_i^{(2)} & \boldsymbol{t}_i \end{bmatrix} \underbrace{\begin{bmatrix} X_j \\ Y_j \\ 1 \end{bmatrix}}_{\tilde{\boldsymbol{Q}}_j}.$$

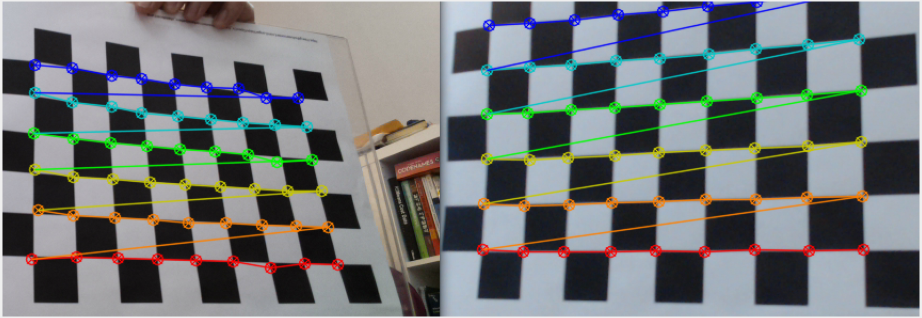
From projections to homographies

$$
\begin{aligned}
\boldsymbol{q}_{ij} &= \underbrace{\boldsymbol{K}\begin{bmatrix}\boldsymbol{r}_i^{(1)} & \boldsymbol{r}_i^{(2)} & \boldsymbol{t}_i\end{bmatrix}}_{\boldsymbol{H}_i} \underbrace{\begin{bmatrix} X_j \\ Y_j \\ 1 \end{bmatrix}}_{\tilde{\boldsymbol{Q}}_j} \\
&= \boldsymbol{H}_i \tilde{\boldsymbol{Q}}_j
\end{aligned}
$$

The homographies $\boldsymbol{H}_i$ can be determined from the plane-plane correspondence $\tilde{\boldsymbol{Q}}_j$ to $\boldsymbol{q}_{ij}$ (week 2).

Corner correspondences

Need to find the unique positions of corners $\boldsymbol{q}_{ij}$.



Find all H_i from corners $\tilde{Q}_j$ and projections q_{ij} with $q_{ij} = H_i \tilde{Q}_j$

For example, using SVD.

From homographies to the camera matrix

$$\boldsymbol{H}_i = \begin{bmatrix} \boldsymbol{h}_i^{(1)} & \boldsymbol{h}_i^{(2)} & \boldsymbol{h}_i^{(3)} \end{bmatrix} = \lambda_i \boldsymbol{K} \begin{bmatrix} \boldsymbol{r}_i^{(1)} & \boldsymbol{r}_i^{(2)} & \boldsymbol{t}_i \end{bmatrix}.$$

$\boldsymbol{r}_i^{(1)}$ and $\boldsymbol{r}_i^{(2)}$ are orthonormal, i.e.

$$\boldsymbol{r}_i^{(1)\mathsf{T}} \boldsymbol{r}_i^{(1)} = \boldsymbol{r}_i^{(2)\mathsf{T}} \boldsymbol{r}_i^{(2)} = 1,$$
$$\boldsymbol{r}_i^{(1)\mathsf{T}} \boldsymbol{r}_i^{(2)} = \boldsymbol{r}_i^{(2)\mathsf{T}} \boldsymbol{r}_i^{(1)} = 0.$$

From homographies to the camera matrix

Express $\boldsymbol{r}_i^{(\alpha)}$ using $\boldsymbol{h}_i^{(\alpha)}$:

$$\boldsymbol{h}_i^{(\alpha)} = \lambda_i \boldsymbol{K} \boldsymbol{r}_i^{(\alpha)}, \Leftrightarrow$$

$$\boldsymbol{K}^{-1} \boldsymbol{h}_i^{(\alpha)} = \lambda_i \boldsymbol{r}_i^{(\alpha)}.$$

From homographies to the camera matrix

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$$\boldsymbol{K}^{-1} \boldsymbol{h}_i^{(\alpha)} = \lambda_i \boldsymbol{r}_i^{(\alpha)}.$$

Now the constraints from the previous slide are:

$$\boldsymbol{h}_i^{(1)}{}^{\mathsf{T}} \boldsymbol{K}^{-\mathsf{T}} \boldsymbol{K}^{-1} \boldsymbol{h}_i^{(2)} = 0,$$

$$\boldsymbol{h}_i^{(1)}{}^{\mathsf{T}} \boldsymbol{K}^{-\mathsf{T}} \boldsymbol{K}^{-1} \boldsymbol{h}_i^{(1)} = \boldsymbol{h}_i^{(2)}{}^{\mathsf{T}} \boldsymbol{K}^{-\mathsf{T}} \boldsymbol{K}^{-1} \boldsymbol{h}_i^{(2)} = \lambda_i^2.$$

We have found constraints on the camera matrix! 🥳

Number of constraints

- Two constraints doesn't seem that impressive?

Number of constraints

- Two constraints doesn't seem that impressive?
- Homography has eight degrees of freedom
- Pose of checkerboard has six (3 rotation, 3 translation)
- A single homography can only fix two degrees of freedom of a camera matrix.

Define some new variables i

How to put into practice?

Define the matrix:

$$\boldsymbol{B} = \boldsymbol{K}^{-\mathsf{T}}\boldsymbol{K}^{-1} = \begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{12} & B_{22} & B_{23} \\ B_{13} & B_{23} & B_{33} \end{bmatrix},$$

$$\boldsymbol{b} = \begin{bmatrix} B_{11} & B_{12} & B_{22} & B_{13} & B_{23} & B_{33} \end{bmatrix}^{\mathsf{T}},$$

Define some new variables ii

Now define $\boldsymbol{v}_i^{(\alpha\beta)}$ such that

$$\boldsymbol{v}_i^{(\alpha\beta)}\boldsymbol{b} = \boldsymbol{h}_i^{(\alpha)}{}^T \boldsymbol{B}\, \boldsymbol{h}_i^{(\beta)}.$$

Then $\boldsymbol{v}_i^{(\alpha\beta)}$ must be an 1×6 vector given by

$$\begin{aligned}\boldsymbol{v}_i^{(\alpha\beta)} = [&H_i^{(1\alpha)}H_i^{(1\beta)}, \quad H_i^{(1\alpha)}H_i^{(2\beta)} + H_i^{(2\alpha}H_i^{(1\beta)}, \quad H_i^{(2\alpha)}H_i^{(2\beta)}, \quad \ldots \\ &H_i^{(3\alpha)}H_i^{(1\beta)} + H_i^{(1\alpha)}H_i^{(3\beta)}, \quad H_i^{(3\alpha)}H_i^{(2\beta)} + H_i^{(2\alpha)}H_i^{(3\beta)}, \quad H_i^{(3\alpha)}H_i^{(3\beta)}],\end{aligned}$$

Define some new variables iii

where $H_i^{(rc)}$ is the element in row r and column c of $\boldsymbol{H}_i$.

From homographies to the camera matrix

Recall our constraints:

$$\boldsymbol{h}_i^{(1)^\mathsf{T}} \boldsymbol{K}^{-\mathsf{T}} \boldsymbol{K}^{-1} \boldsymbol{h}_i^{(1)} = \boldsymbol{h}_i^{(2)^\mathsf{T}} \boldsymbol{K}^{-T} \boldsymbol{K}^{-1} \boldsymbol{h}_i^{(2)} \Leftrightarrow$$

$$\boldsymbol{h}_i^{(1)^\mathsf{T}} \boldsymbol{B}\, \boldsymbol{h}_i^{(1)} - \boldsymbol{h}_i^{(2)^\mathsf{T}} \boldsymbol{B}\, \boldsymbol{h}_i^{(2)} = 0 =$$

$$\left(\boldsymbol{v}_i^{(11)} - \boldsymbol{v}_i^{(22)}\right) \boldsymbol{b} = 0.$$

Now we can express our constraints in matrix-form

$$\begin{bmatrix} \boldsymbol{v}_i^{(12)} \\ \boldsymbol{v}_i^{(11)} - \boldsymbol{v}_i^{(22)} \end{bmatrix} \boldsymbol{b} = \boldsymbol{0}$$

From homographies to the camera matrix

For all checkerboard poses

$$\boldsymbol{V}\boldsymbol{b} = \begin{bmatrix} \boldsymbol{v}_1^{(12)} \\ \boldsymbol{v}_1^{(11)} - \boldsymbol{v}_1^{(22)} \\ \boldsymbol{v}_2^{(12)} \\ \boldsymbol{v}_2^{(11)} - \boldsymbol{v}_2^{(22)} \\ \vdots \end{bmatrix} \boldsymbol{b} = \boldsymbol{0}$$

When $\boldsymbol{b}$ is found then $\boldsymbol{K}$ can be determined using the formulas in Zhang's paper.

Find the camera matrix K through b using $Vb = 0$ and SVD, where V is built from the homographies H_i.

Reprojection error

Is it a good camera calibration? How to find out?

Reprojection error

Is it a good camera calibration? How to find out?

Reproject the points from the checkerboards to the camera, and compare to where we've seen them

$$\tilde{\boldsymbol{q}}_{ij} = \boldsymbol{K} \begin{bmatrix} \boldsymbol{R}_i & \boldsymbol{t}_i \end{bmatrix} \boldsymbol{Q}_j$$

$$\Pi(\tilde{\boldsymbol{q}}_{ij}) - \Pi(\boldsymbol{q}_{ij})$$

Reprojection error

We can now compute the reprojection error as the root mean squared error (RMSE)

$$\sqrt{\frac{1}{n}\sum_{i}\sum_{j}\left\|\Pi(\tilde{\boldsymbol{q}}_{ij}) - \Pi(\boldsymbol{q}_{ij})\right\|_2^2}$$

where n is the total number of points.

We have $\boldsymbol{K}$, but we are still missing something.

Reprojection error

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We have $\boldsymbol{K}$, but we are still missing something.

How do we recover $\boldsymbol{R}_i$ and $\boldsymbol{t}_i$?

From homographies to poses

Recall: $\boldsymbol{H}_i = \begin{bmatrix} \boldsymbol{h}_i^{(1)} & \boldsymbol{h}_i^{(2)} & \boldsymbol{h}_i^{(3)} \end{bmatrix} = \lambda_i \boldsymbol{K} \begin{bmatrix} \boldsymbol{r}_i^{(1)} & \boldsymbol{r}_i^{(2)} & \boldsymbol{t}_i \end{bmatrix}$

Now we can recover $\boldsymbol{R}_i$ and $\boldsymbol{t}_i$:

$$\begin{aligned} \boldsymbol{r}_i^{(1)} &= \tfrac{1}{\lambda_i} \boldsymbol{K}^{-1} \boldsymbol{h}_i^{(1)}, \\ \boldsymbol{r}_i^{(2)} &= \tfrac{1}{\lambda_i} \boldsymbol{K}^{-1} \boldsymbol{h}_i^{(2)}, \\ \boldsymbol{t}_i &= \tfrac{1}{\lambda_i} \boldsymbol{K}^{-1} \boldsymbol{h}_i^{(3)}, \\ \boldsymbol{r}_i^{(3)} &= \boldsymbol{r}_i^{(1)} \times \boldsymbol{r}_i^{(2)}, \end{aligned}$$

where $\lambda_i = \left\| \boldsymbol{K}^{-1} \boldsymbol{h}_i^{(1)} \right\|_2 = \left\| \boldsymbol{K}^{-1} \boldsymbol{h}_i^{(2)} \right\|_2$.

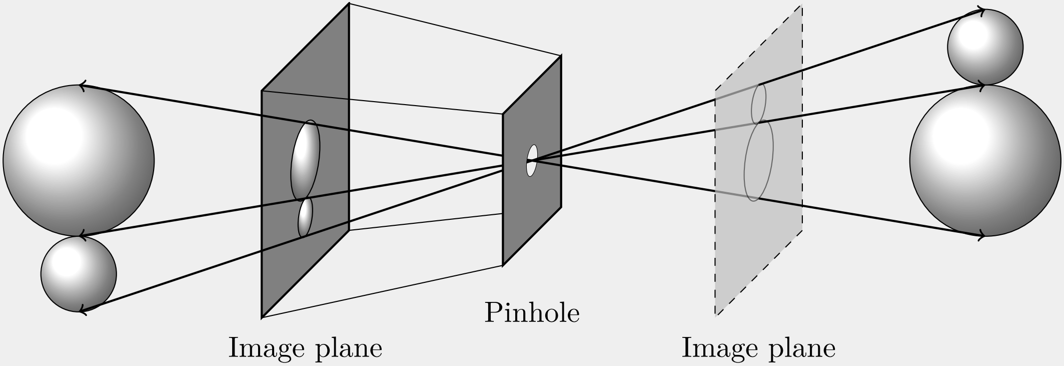
Homography and poses

$$\boldsymbol{t}_i = \frac{1}{\lambda_i}\boldsymbol{K}^{-1}\boldsymbol{h}_i^{(3)}$$

What happens if $t_{iz} < 0$? And what does that mean?

Can we ensure correctness?

Pinhole camera revisited



Points behind the camera also get projected to the image plane.
Multiply all 3D points by -1 and they still project to the same place.

Homography and poses

- If $t_{iz} < 0$ the checkerboard is behind the camera.
- How to get the correct solution?
- Estimate $\boldsymbol{R}_i$ and $\boldsymbol{t}_i$ again using $-\boldsymbol{H}_i$ (flipped sign).

Non-linear calibration

Least-squares method

With projection equation

$$\boldsymbol{q}_{ij} = \boldsymbol{K}\begin{bmatrix}\boldsymbol{R}_i & \boldsymbol{t}_i\end{bmatrix}\boldsymbol{Q}_j$$

then

$$E = \sum_{i,j} \left\| \text{dist}\Big(\pi\left(\mathbf{K}\begin{bmatrix}\mathbf{R}_i & \mathrm{t}_i\end{bmatrix}\boldsymbol{Q}_j\right)\Big) - \pi(\boldsymbol{q}_{ij}) \right\|^2$$

Solution: minimize E w.r.t. $\mathbf{K}$, $\mathbf{R}_i$, t_i and lens distortion.

Practical remarks

What images should we take?

- Our two constraints per image are based on $\boldsymbol{r}_i^{(1)}$ and $\boldsymbol{r}_i^{(2)}$.
- Two parallel checkerboards express the same constraints.

What images should we take?

- Our two constraints per image are based on $\boldsymbol{r}_i^{(1)}$ and $\boldsymbol{r}_i^{(2)}$.
- Two parallel checkerboards express the same constraints.
  - at least when we don't consider lens distortion
- Make sure to rotate the checkerboards so they are not parallel.
- Make the checkerboards take up as as much of the frame as possible!

How many images?

- Without lens distortion: In theory at least three.
- In practice: It depends...
  - Some people use 2000+ images for a single calibration[1]
  - Look at the reprojection error
  - Both of the training set and the validation set

Learning objectives

After this lecture you should be able to:

- implement and use the direct linear transformation to calibrate a pinhole camera
- implement and use Zhang's method (2000) to calibrate a pinhole camera using checkerboards

Last week's exercise

The goal of exercise 3.6-3.10 last week was to:

- Understand how to handle when no camera has $\boldsymbol{R} = \boldsymbol{I}$ and $\boldsymbol{t} = \boldsymbol{0}$.
- Easily find epipolar lines in image 1 from points in image 2.

I have uploaded a new `TwoImageDataCar.npy`, where both have nontrivial $\boldsymbol{R}$ and $\boldsymbol{t}$.

For exercise 3.10, you can use the transpose.

$$
\begin{aligned}
\boldsymbol{q}_1^\top \boldsymbol{F} \boldsymbol{q}_2 &= 0 \\
\boldsymbol{q}_2^\top \boldsymbol{F}^\top \boldsymbol{q}_1 &= 0
\end{aligned}
$$

If you have already solved the exercise, loading the new file should not take long

Exercise time!