Deliverable 1-a: Modelling and Deterministic Optimization

- **Sets**
  Warehouse: $w \in W = \{1, 2, 3\}$
  Neighboring warehouse: $q \in W = \{1, 2, 3\}$
  Stage $t \in T = \{1, 2, \ldots\}$
- **Variables**
  $x_{w,t}$: *At stage t, warehouse w can order an amount* $x_{w,t} \geq 0$ *of coffee from external suppliers*
  $z_{w,t}$: *storage level of w at t*
  $m_{w,t}$: *the missing amount*
  $y_{w,q,t}^{send}$: *At stage t, the amount of coffee is sent from warehouse w to the neighboring warehouse q*
  $y_{w,q,t}^{receive}$: *At stage t, the amount of coffee is received by the neighboring warehouse q*
- **Parameters**
  p: *the price of ordering coffee*
  $b_{w,t} \cdot m_{w,t}$: *the cost of failing to miss the demand*
  $e_{w,q}$: *the per-unit transportation cost*
  $C_w^{storgae}$: *each warehouse can store coffee up to a capacity limit*
  C_w^{transp}: *daily transportation limit*
  $D_{w,t}$: *demand*
  $z_{w,0} = 2$
- **Objective model**
  Min cost $= \sum_{w,t}\left(x_{w,t} \cdot p + \sum_q e_{w,q} \cdot y_{w,q,t}^{send} + b_{w,t} \cdot m_{w,t}\right)$
- **Constraints**
  1. Storage capacity: $z_{w,t} \leq C_w^{storgae}$,∀w,t
  2. Transportation capacity: $y_{w,q,t}^{send} \leq C_{w,q}^{transp}$,∀w,t,q
  3. Send Receive Balance: $y_{w,q,t}^{send} = y_{w,q,t}^{receive}$
  4. Self-limitation: $y_{w,q,t}^{send} = 0$ when w=q
  5. Warehouses must have the storage to send and cannot send the amount more than initial storage:

     $y_{w,q,t}^{send} \leq z_{w,t-1}$, ∀w,t,q

     $y_{w,q,1}^{send} \leq = z_{w,0}$, ∀ t=1,w ,q

  6. Inventory balance:

     $D_{w,t} = z_{w,t-1} - z_{w,t} + x_{w,t} + \sum_{w,q,t} y_{w,q,t}^{receive} - \sum_{w,q,t} y_{w,q,t}^{send} + m_{w,t}$,∀w,t,q

     $D_{w,1} = z_{w,0} - z_{w,1} + + \sum_{w,q,1} y_{w,q,1}^{receive} - \sum_{w,q,1} y_{w,q,1}^{send} + m_{w,t}$,∀ t=1,w ,q

  7. $x_{w,t}, z_{w,t}, m_{w,t}, y_{w,q,t}^{send}, y_{w,q,t}^{receive} \geq 0$,∀w,t,q