Passive seismic imaging applied to synthetic data[a]

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ABSTRACT

It can be shown that for a 1-D Earth model illuminated by random plane waves from below, the cross-correlation of noise traces recorded at two points on the surface is the same as what would be recorded if one location contained a shot and the other a receiver. If this is true for real data, it could provide a way of building 'pseudo-reflection seismograms' from background noise, which could then be processed and used for imaging. This conjecture is tested on synthetic data from simple 1-D and point diffractor models, and in all cases, the kinematics of observed events appear to be correct. The signal to noise ratio was found to increase as $\sqrt{n}$, where n is the length of the time series. The number of incident plane waves does not directly affect the signal to noise ratio; however, each plane wave contributes only its own slowness to the common shot domain, so that if complete hyperbolas are to be imaged then upcoming waves must be incident from all angles.

INTRODUCTION

Conventional seismic reflection methodology relies on having a source of seismic waves at the surface and studying the reflections from impedance contrasts in the earth. Ambient noise present in the subsurface is also reflected by impedance contrasts in the same way. Therefore, do we need the source or can we determine earth structure simply by listening to the background noise for a long enough time ?

With 4-D seismic surveys aimed at monitoring fluid movements becoming more and more popular, a number of oil fields are having geophone arrays laid out permanently to reduce acquisition costs. These geophones will typically only record while a survey is being shot, and for most of their life they will be turned off. However, if the geophones are left recording while surveys are not being shot, we hypothesize the information contained in the background seismic energy could be used for imaging the subsurface between main surveys.

For this to be realized a technique has to be developed that: firstly, is able to extract the useful information from the background noise; and secondly, is able to do

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this quickly enough, ideally in real-time, so the huge amounts of raw data that would be recorded would not have to be stored.

With this in mind, a technique for creating 'pseudo-cmp gathers' out of background noise traces using cross-correlation is explored and tested on a variety of synthetic models.

CONJECTURE

By cross-correlating noise traces noise traces recorded at two locations on the surface, we can construct the wavefield that would be recorded at one of the locations if there was a source at the other. In this way we can create 'pseudo-reflection seismograms' that include such effects as NMO and DMO.

PROOF FOR A 1-D EARTH

The proof of this conjecture for a one dimensional earth is given as a problem set in Claerbout (1976). The outline of the derivation that follows uses the Z transform approach developed there, where Z is the unit delay operator $e^{-i\omega\Delta t}$.

Consider a plane-layered Earth model with the reflection seismology geometry shown in Figure 1. If the system is lossless then the energy flux through the top layer has to be equal to the flux through the half-space below. Therefore,

$$Y_1 \left\{ R\left(\frac{1}{Z}\right) R(Z) - \left[1 + R\left(\frac{1}{Z}\right)\right] [1 + R(Z)] \right\} = -Y_k E\left(\frac{1}{Z}\right) E(Z) \qquad (1)$$

where Y_1 is the impedance of the top layer and Y_k is the impedance of the half-space, or

$$1 + R\left(\frac{1}{Z}\right) + R(Z) = \frac{Y_k}{Y_1} E\left(\frac{1}{Z}\right) E(Z) \qquad (2)$$

Comparing the reflection seismology geometry with the earthquake seismology geometry shown in Figure 2 gives $E(Z) = X(Z)$ by reciprocity. Therefore

$$1 + R\left(\frac{1}{Z}\right) + R(Z) = (constant)\, X\left(\frac{1}{Z}\right) X(Z) \qquad (3)$$

Since $R(Z) = 0$ for times less than zero, this means the positive time part of the 'earthquake' seismogram's auto-correlation function equals the reflection seismogram.

This theorem can be extended to a two dimensional plane-layered Earth by considering slant stacks (Claerbout, 1985).

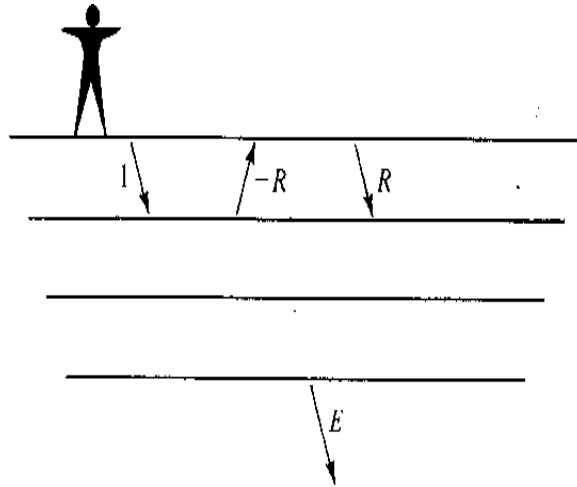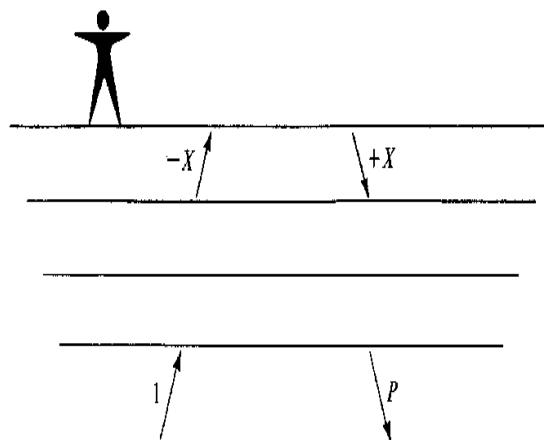Figure 1: Reflection seismology geometry - Claerbout (1979).

Figure 2: Earthquake seismology geometry - from Claerbout (1979). All the waves (1, X and P) could be multiplied by the same random signal, and this would not effect their spectra or auto-correlation functions.

SYNTHETIC DATA

Cole (1995) tested this conjecture on both synthetic and real data. However his field data was very noisy, and he did not draw any solid conclusions.

With this as a starting point, however, I continued modeling a single reflection, using a program based on the flow

```
loop over each plane wave {
     calculate a random slowness, p
     calculate the time delay due to a reflection, Δt
     loop over each frequency, ω {
         calculate a random amplitude
         loop over each spatial location, x {
             multiply each frequency by a factor (1 + re^{iωΔt}) e^{iωpx}
         }
     }
}
```

Having produced synthetics, it was then possible to go ahead and cross-correlate traces to try and create pseudo-reflection seismograms. Figure 3 is a pseudo-shot gather generated by cross-correlating one trace with every other. The center panel shows how the clarity of the signal was improved by applying a $\sqrt{-i\omega}$ filter. The black line which has been overlain in the left panel corresponds to the expected hyperbola which would be observed in a real shot gather, offset by 0.05 s so it does not obscure the data. Therefore the kinematics in this case appear to be consistent with the conjecture.

Limited angular bandwidth

Figure 4 shows how the hyperbolas build up from the individual plane waves. In the left panel there is only one plane wave present, instead of 200 as in Figure 3. The trace at zero spatial offset is the autocorrelation function, and it has two spike as expected: one at zero lag and one corresponding to the reflected event. Other traces, corresponding to cross-correlations, have the two same spikes, but these are offset in time due to the different receiver positions and the apparent velocity of the incident wave. In the right panel of Figure 4 there are 5 plane waves incident, and a hyperbola is beginning to form. Only one part of each plane wave adds coherently with the others, the rest of the energy is smeared out.

The corollary of this is that, in order to see a slope in the pseudo-reflection seismograms, we need incident plane waves with the relevant slowness. This will become an issue when looking at real data where the direction of incoming waves will not be spatially white.
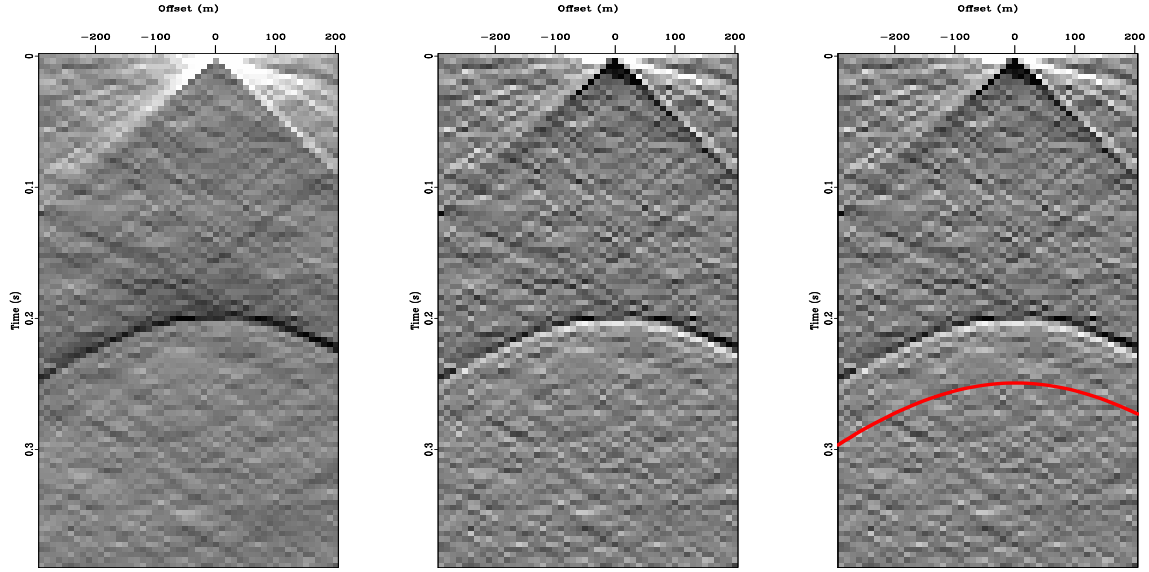
Figure 3: Pseudo-shot gather over model with single horizontal layer and 200 incoming plane waves. The left panel is raw cross-correlations, the center panel has a half differentiation filter applied and the right panel is labeled with the correct kinematics shifted by 0.05 s.
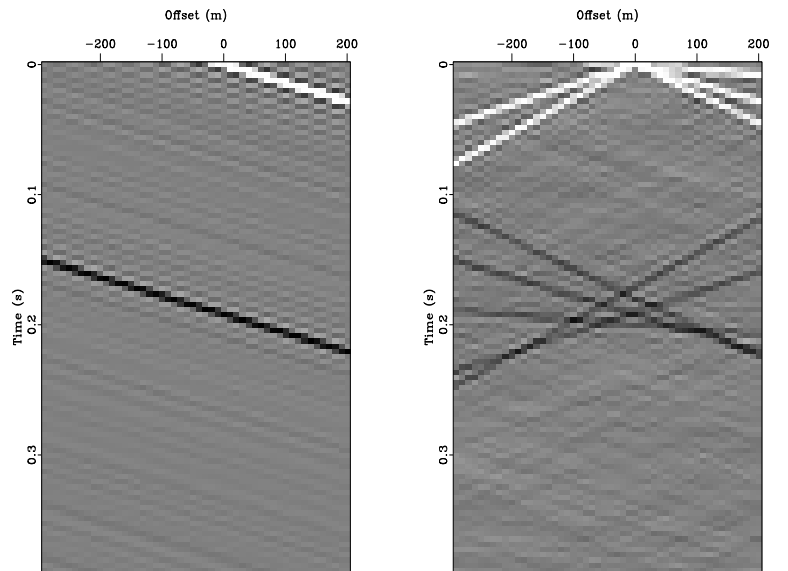

Figure 4: Pseudo-shot gather over model with single horizontal layer. Left panel has one incoming plane wave and right panel has five.

More complex Earth models - point diffractors and multiple layers

The horizontal bed code shown earlier was easy to adapt to test slightly more complicated models. The left panel of Figure 5 shows a pseudo-shot gather with a point diffractor in the subsurface. Again the kinematics are correct, as they are in the zero-offset section of the same model shown in the right panel of Figure 5. Notice the offset center of the hyperbola in the pseudo-shot gather.

Figure 5: Model with single point diffractor. Left panel shows common-shot gather, and right panel shows zero-offset section.
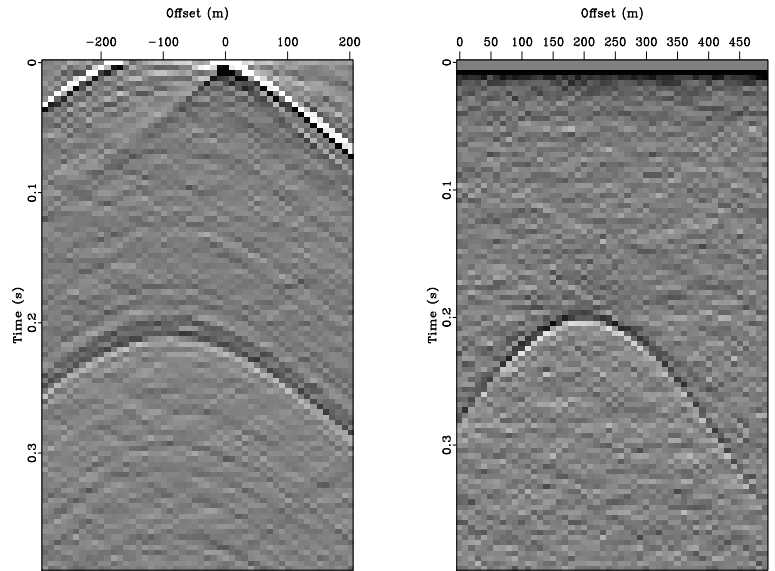
With a model of two horizontal layers, the results (Figure 6) seem to contain a spurious event at the time of the inter-bed multiple. This is not the multiple itself but comes from the correlation between the first layer and the second layer. If the inter-bed multiple was included in the modeling process, it would arrive at the same time but with opposite amplitude and so cancel this event out. Therefore this event is an artifact of the modeling technique and does not contradict the conjecture.

Noise suppression

So far the reflection coefficients have been very high (0.5) and still the images have been relatively noisy. If this technique is to be used for looking at real targets, reflection coefficient of an order of magnitude smaller will have to be imaged.

Tests showed that the signal to noise ratio decreased as the $\sqrt{n}$, where n is the number of time samples, and by increasing the length of the time series used I was able to clearly image reflection coefficients of 0.05, as shown in Figure 7.

A similar study, but this time comparing signal to noise ratio with the number of incident plane waves, was also conducted. Interestingly, it showed that more plane

Figure 6: Common shot gather for a model with two horizontal layers.
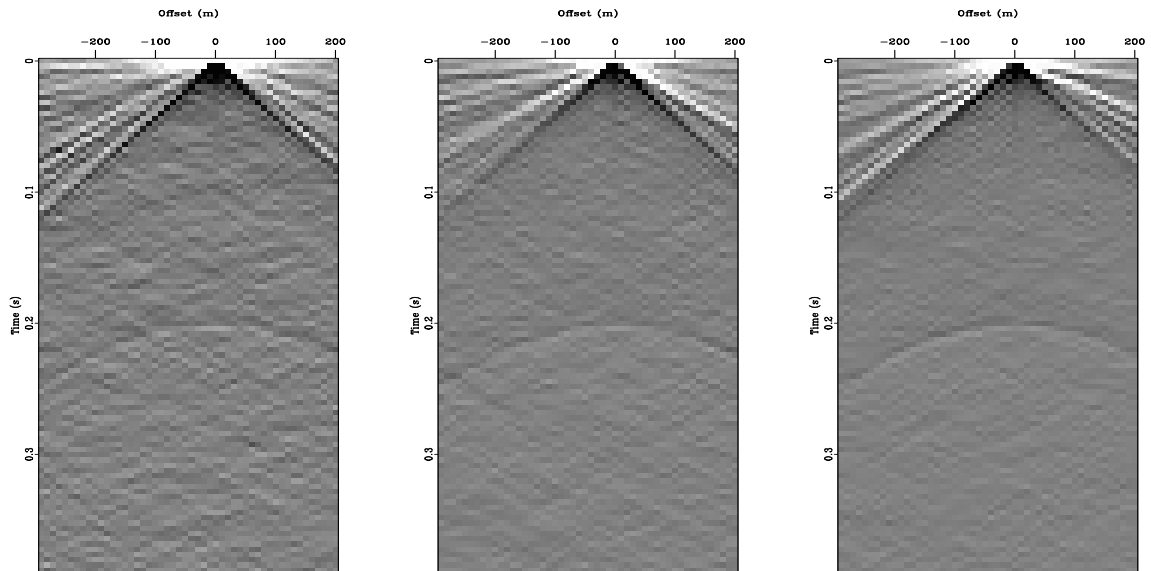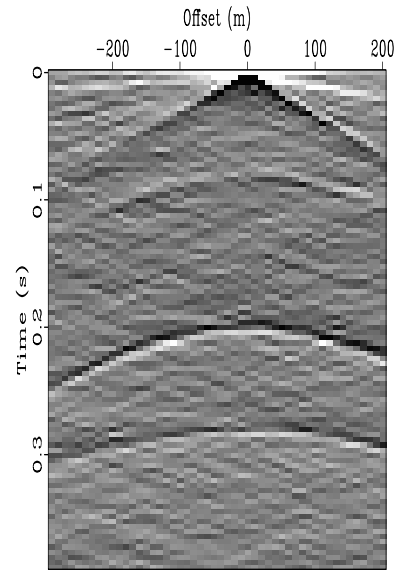

Figure 7: Common shot gathers for a model with a single horizontal layer with reflection coefficient 0.05. Left panel used 65,000 points in the time series, the center panel about 130,000 and the right panel about 260,000.

waves did not increase the amplitude of the observed signal to noise; however, more plane waves do improve the shape and definition of the hyperbola.

CONCLUSIONS AND FURTHER WORK

So far this method has worked well on a variety of simple Earth models that have been illuminated by plane waves coming from all angles in the subsurface.

Before the method is tested on real data, the following points need to be addressed:

1. A method of spatially pre-whitening upcoming waves has to be developed, so that all dips can be imaged even if the angular distribution of upcoming waves is uneven.

2. The conjecture should be tested with full wave-form modeling on more complicated Earth models (multiple dipping beds and velocities which vary both laterally and vertically, for example).

3. In order to avoid storing huge amounts of data, the cross-correlations should be done in the field in real time. Implementation of this will be difficult.

REFERENCES

Claerbout, J. F., 1976, Fundamentals of geophysical data processing: Blackwell.
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Cole, S., 1995, Passive seismic and drill-bit experiments using 2-D arrays: PhD thesis, Stanford University.