International Economics I

Lecture 2: Quantitative Spatial Models

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- Worker ω residing in location n

$$U_n(\omega) = b_n(\omega)\left(\frac{C_n(\omega)}{\alpha}\right)^{\alpha}\left(\frac{H_{U_n}(\omega)}{1-\alpha}\right)^{1-\alpha}$$

  where $H_{U_n}(\omega)$ is the residential land use

- Good consumption

$$C_n = \left[\int_0^1 c_n(j)^{\rho}\, dj\right]^{\frac{1}{\rho}}$$

- Idiosyncratic amenity shocks $b_n(\omega)$ are drawn independently across locations and workers from a Fréchet distribution

$$G_n(b) = e^{-B_n b^{-\epsilon}}$$

- Idiosyncratic location-specific productivity $z(j)$ for each good j, which is independently drawn across goods and locations from a Fréchet distribution
$$F_i(z) = e^{-A_i z^{-\theta}}$$
- Perfect competition market
$$p_{ni}(j) = \frac{d_{ni} w_i}{z_i(j)}$$

- The share of expenditure of location n on goods produced in location i

$$\pi_{ni} = \frac{A_i \left(d_{ni} w_i\right)^{-\theta}}{\sum_s A_s \left(d_{ns} w_s\right)^{-\theta}}$$

- And price index in location n is

$$P_n^{-\theta} = \gamma^{-\theta} \left[\sum_s A_s \left(d_{ns} w_s\right)^{-\theta}\right] = \frac{\gamma^{-\theta} A_n w_n^{-\theta}}{\pi_{nn}}, \theta > \sigma - 1$$

- Indirect utility function

$$U_n(\omega) = \frac{b_n(\omega)\, v_n}{P_n^{\alpha} r_n^{1-\alpha}}$$

where v_n is the total income, i.e., $v_n L_n = \frac{w_n L_n}{\alpha}$

- $U_n(\omega)$ follows the distribution of $b_n(\omega)$

$$G_n(U) = e^{-\psi_n U^{-\epsilon}}$$

where $\psi_n \equiv B_n \left(v_n / P_n^{\alpha} r_n^{1-\alpha}\right)^{\epsilon}$

- Share of people in location n

$$\frac{L_n}{\bar{L}} = \frac{B_n \left(v_n / P_n^{\alpha} r_n^{1-\alpha} \right)^{\epsilon}}{\sum_k B_k \left(v_k / P_k^{\alpha} r_k^{1-\alpha} \right)^{\epsilon}}$$

- Expected utility

$$\bar{U} = \delta \left[\sum_k B_k \left(v_k / P_k^{\alpha} r_k^{1-\alpha} \right)^{\epsilon} \right]^{\frac{1}{\epsilon}}, \epsilon > 1$$

- Labor

$$w_n L_n = \sum_k \pi_{kn} w_k L_k$$

- Land

$$r_n = \frac{(1-\alpha) v_n L_n}{H_n} = \frac{1-\alpha}{\alpha} \frac{w_n L_n}{H_n}$$

- Market system
  - endogeneous variables $\{\mathbf{L}_n, \mathbf{w}_n, \mathbf{r}_n\}$
  - exogenous variables $\{\mathbf{H}_n, \mathbf{A}_n, \mathbf{B}_n, \mathbf{d}_{ni}\}$
  - parameters $\{\epsilon, \theta, \alpha, \rho\}$
- Equations

$$
\begin{aligned}
w_n L_n &= \sum_k \pi_{kn} w_k L_k \\
\pi_{ni} &= \frac{A_i \left(d_{ni} w_i\right)^{-\theta}}{\sum_s A_s \left(d_{ns} w_s\right)^{-\theta}} \\
\lambda_n &= \frac{L_n}{\bar{L}} = \frac{B_n \left(v_n / P_n^{\alpha} r_n^{1-\alpha}\right)^{\epsilon}}{\sum_k B_k \left(v_k / P_k^{\alpha} r_k^{1-\alpha}\right)^{\epsilon}} \\
r_n &= \frac{1-\alpha}{\alpha} \frac{w_n L_n}{H_n}
\end{aligned}
$$

- Symmetric trade costs

$$d_{ni} = \begin{cases} 1 & \text{if } n = i \\ D_n D_i D_{ni} & \text{if } n \neq i \end{cases}$$

- Unique solution

$$\begin{aligned} & L_n^{\tilde{\theta}\gamma_1} A_n^{-\tilde{\theta}} B_n^{-\frac{\tilde{\theta}(1+\theta)}{\alpha\epsilon}} H_n^{-\frac{\tilde{\theta}(1+\theta)(1-\alpha)}{\alpha}} \\ = \quad & \bar{W}^{-\theta}\gamma^{-\theta}\left[\sum_k d_{nk}^{-\theta} A_k^{\frac{\tilde{\theta}(1+\theta)}{\theta}} B_k^{\frac{\tilde{\theta}\theta}{\alpha\epsilon}} H_k^{-\frac{\tilde{\theta}\theta(1-\alpha)}{\alpha}} \left(L_k^{\tilde{\theta}\gamma_1}\right)^{\frac{\gamma_2}{\gamma_1}}\right] \end{aligned}$$

- Given
  - values of parameters $\{\epsilon, \theta, \alpha, \rho\}$ parametrization of d_{ni}
  - data of $\{\mathbf{L}_n, \mathbf{w}_n, \mathbf{H}_n\}$
- Solve uniquely $\{\mathbf{A}_n, \mathbf{B}_n\}$

- Dekle et al. (2007)'s approach

$$
\begin{aligned}
\hat{w}_n \hat{\lambda}_n Y_n &= \sum_k \hat{\pi}_{kn} \pi_{kn} \hat{w}_k \hat{\lambda}_k Y_k \\
\hat{\pi}_{ni} \pi_{ni} &= \frac{\pi_{ni} \hat{A}_i \left(\hat{d}_{ni} \hat{w}_i \right)^{-\theta}}{\sum_s \pi_{ns} \hat{A}_s \left(\hat{d}_{ns} \hat{w}_s \right)^{-\theta}} \\
\hat{\lambda}_n \lambda_n &= \frac{\hat{B}_n \hat{A}_n^{\frac{\alpha\epsilon}{\theta}} \hat{\pi}_{nn}^{-\frac{\alpha\epsilon}{\theta}} \hat{\lambda}_n^{-\epsilon(1-\alpha)} \lambda_n}{\sum_k \hat{B}_k \hat{A}_k^{\frac{\alpha\epsilon}{\theta}} \hat{\pi}_{kk}^{-\frac{\alpha\epsilon}{\theta}} \hat{\lambda}_k^{-\epsilon(1-\alpha)} \lambda_k}
\end{aligned}
$$

where $Y_n \equiv w_n L_n$

- Setup
  - $j \in J$ countries where labor is immobile across
  - $n \in N^i$ regions within country i where labor is mobile
- Equilibrium systems

$$
\begin{aligned}
w_n L_n &= \sum_{j \in J} \sum_{k \in N^j} \pi_{kn} w_k L_k \\
\pi_{ni} &= \frac{A_i \left(d_{ni} w_i\right)^{-\theta}}{\sum_{j \in J} \sum_{k \in N^j} A_k \left(d_{nk} w_k\right)^{-\theta}} \\
\frac{L_n}{\bar{L}^j} &= \frac{B_n \left(v_n / P_n^{\alpha} r_n^{1-\alpha}\right)^{\epsilon}}{\sum_{k \in N^j} B_k \left(v_k / P_k^{\alpha} r_k^{1-\alpha}\right)^{\epsilon}}
\end{aligned}
$$

- Consumption

$$C_n = \left[\sum_{i \in N} \int_0^{M_i} c_{ni}(j)^{\rho} \, dj \right]^{\frac{1}{\rho}}$$

- Production

$$l_i(j) = F + \frac{x_i(j)}{A_i}$$

$\Rightarrow$

$$\begin{aligned} p_{ni}(j) &= \frac{\sigma}{\sigma - 1} \frac{d_{ni} w_i}{A_i} \\ l_i(j) &= \bar{l} = \sigma F \end{aligned}$$

- Labor market clearing

$$M_i = \frac{L_i}{\sigma F}$$