Production Network (3)

duanyuwan@cufe.edu.cn

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Paper

Antras, P. and de Gortari, A. On the Geography of Global Value Chains, 2020, Econometrica

Contribution

- Develop a general-equilibrium model of GVCs with a general geography of trade costs across countries
- Characterize the optimality of a centrality-downstreamness nexus
- Develop tools to solve the model in high-dimensional environments
- Show how to map our model to world Input-Output tables
- Structurally estimate the model and perform counterfactuals

Model: A Multi-Stage Ricardian Model

- Ricardian dierences in technology across stages and countries
- There are J countries where consumers derive utility from consuming a final good
- The good is produced combining N stages that need to be performed sequentially (stage N = assembly)
- At each stage $n > 1$, production combines (equipped) labor with the good finished up to the previous stage $n - 1$
- The wage rate varies across countries and is denoted by w_i in i
- Countries differ in their geography $J \times J$ matrix of iceberg trade cost coecients τ_{ij}
- Technology features constant returns to scale and market structure is perfectly competitive

Household

- Preferences.

$$U\left(y_i^N(z)\right) = \left(\int_0^1 \left(y_i^N(z)\right)^{(\sigma-1)/\sigma} dz\right)^{\sigma/(\sigma-1)}, \sigma > 1. \tag{1}$$

where $y_i^N(z)$ is the amount of the final output of variety z available in country i.

Production

- The lead firm optimally choose location $l(n) \in \mathcal{J}$ for each stage l in each value chain
- A production path $\boldsymbol{l} = l(1), l(2), \ldots l(N) \in \mathcal{J}^{\mathcal{N}}$
- For producer $l(n)$ in stage $n \in \mathcal{N}$

$$y^n_{l(n)} = \left(\frac{\alpha_n \left(\gamma_{l(n)} L^n_{l(n)} \right)^{\gamma_{l(n)}} \left((1 - \gamma_{l(n)}) m^n_{l(n)} \right)^{1-\gamma_{l(n)}}}{a^n_{l(n)}} \right)^{\alpha_n} \left((1 - \alpha_n) x^{n-1}_{l(n-1)} \right)^{1-\alpha_n}$$

with $\alpha_N = 1$;

- Unit cost of input bundle in stage n:

$$c^n_{l(n)} = (w_{l(n)})^{\gamma_{l(n)}} (P_{l(n)})^{1-\gamma_{l(n)}}$$

- Pice of final output in stage n :

$$p^n_{l(n)}(\boldsymbol{l}) = \left(a^n_{l(n)} c_{l(n)} \right)^{\alpha_n} \left(p^{n-1}_{l(n-1)}(\boldsymbol{l}) \tau_{l(n-1)l(n)} \right)^{1-\alpha_n}, \textit{ for all } n \in \mathcal{N}. \qquad (2)$$

Production

- Denoting by $p_j^F(\boldsymbol{l})$ the price paid by consumers in j produced following the path $\boldsymbol{l}$
- The problem of a lead firm choosing the location of production of all stages $n \in \mathcal{N}$ that minimize the $p_j^F(\boldsymbol{l})$.
- Using and iterating eq:(4), this problem reduces to:

$$\boldsymbol{l}^j = \underset{\boldsymbol{l} \in \mathcal{J}^N}{arg\ min} \left\{ \prod_{n=1}^{N} \left(a_{l(n)}^n c_{l(n)}^n \right)^{\alpha_n \beta_n} \times \prod_{n=1}^{N-1} \left(\tau_{l(n)l(n+1)} \right)^{\beta_n} \times \tau_{l(N)j} \right\} \tag{3}$$

where

$$\beta_n \equiv \prod_{m=n+1}^{N} (1 - \alpha_m) \tag{4}$$

Production

- If $\tau_{ij} = \tau$ for all i and j, eq (3) reduced to a sequence of N independent cost-minimization problems in which the optimal location of stage n is simply given by $l^j(n) = arg\ min_i\{a_i^n c_i^n\}$. Thus is independent of the country of consumtion j.
- The trade-cost elaticity of the unit cost of serving consumers in country j inceases along the value chain.
  - i.e. $\beta_1 < \beta_2 < ... < \beta_N = 1$
  - Because the costs of transporting goods have been modeled to be proportional to the gross value, instead of value added. Thus, cost increase as the value of the good rises along the value chain.
  - Firm will be more concerned about reducing trade costs in downstream stages.

Production

- The overall productivity of a given chain $\boldsymbol{l} = \{l(1), l(2), ..., l(N)\} \in \mathcal{J}^N$ is characterized by

$$Pr\left(\prod_{n=1}^{N}\left(a^n_{l(n)}(z)\right)^{\alpha_n\beta_n} > a\right) = exp\left\{-a^{\theta}\prod_{n=1}^{N}\left(T_{l(n)}\right)^{\alpha_n\beta_n}\right\}, \quad (5)$$

  i.e. assuming that $\prod_{n=1}^{N}\left(a^n_{l(n)}(z)\right)^{\alpha_n\beta_n}$ is distributed Frechet with a shape parameter θ, and a location parameter that is a function of technology in all countries in the chain, as captured by $\prod_{n=1}^{N}\left(T_{l(n)}\right)^{\alpha_n\beta_n}$.

- The distribution of $p^F_j(\boldsymbol{l}, z)$ (the consumption price in country j for a good z produced following the path $\boldsymbol{l}$

$$Pr\left(p^F_j(\boldsymbol{l}, z) \geq p\right) = exp\left\{-p^{\theta}\times\prod_{n=1}^{N}\left(\left(c^n_{l(n)}\right)^{-\theta} T_{l(n)}\right)^{\alpha_n\beta_n}\times\prod_{n=1}^{N-1}\left(\tau_{l(n)l(n+1)}\right)^{-\theta\beta_n}(\tau_{l(N}\right. \quad (6)$$

Production

- The probability of a given GVC $\boldsymbol{l}$ being the cost-minimizing production path for serving consumers in j is given by

$$\pi_{\boldsymbol{l}j} = \frac{\prod_{n=1}^{N-1} \left((T_{l(n)})^{\alpha_n} ((c^n_{l(n)})^{\alpha_n} \tau_{l(n)l(n+1)})^{-\theta} \right)^{\beta_n} \times (T_{l(N)})^{\alpha_N} \left(c^N_{l(N)}{}^{\alpha_N} \tau_{l(N)j} \right)^{-\theta}}{\Theta_j}, \tag{7}$$

  where $\Theta_j$is the sume of the numerator over all possible paths.
- $\pi_{\boldsymbol{l}j}$ corresponds to the share of GVCs ending in j for which $\boldsymbol{l}$ is the cost-minimizing production path.
- When $N = 1$

$$\pi_{\boldsymbol{l}(N)j} = \frac{T_{l(N)} \left(c^N_{l(N)} \tau_{l(N)j} \right)^{-\theta}}{\Theta_j}, \tag{8}$$

  as in Eaton and Kortum (2002)

Trade and Price

- The distribution of final-good prices $p_j^F(\boldsymbol{I}, z)$ paid by consumers in j satisfies

$$Pr\left(p_j^F(\boldsymbol{I}, z) \leq p\right) = 1 - exp\left\{-\Theta_j p^{\theta}\right\}. \tag{9}$$

- Following EK, the exact ideal price index P_j in country j:

$$P_j = \kappa(\Theta_j)^{-1/\theta}, \tag{10}$$

- where $\kappa = \left[\Gamma\left(\frac{\theta+1-\sigma}{\theta}\right)\right]^{\frac{1}{1-\sigma}}$ and Γ is the gamma function. Assume $\sigma - 1 < \theta$ to obtain a well-defined Γ function in κ.

General equilibrium

- The share of that spending by j going to GVCs in which country i is in position n:

$$Pr(\Lambda^n_{k\to i}, j) = \sum_{\boldsymbol{l}\in\Lambda^n_i} \pi_{\boldsymbol{l}j}, where\ \Lambda^n_i = \{l \in J^N | l(n) = i\}$$

- Final good share from country i to country j:

$$\pi^F_{ij} = \sum_{\boldsymbol{l}\in\Lambda^N_i} \pi_{\boldsymbol{l}(N)j}, where\ \Lambda^n_i = \{l \in J^N | l(n) = i\}$$

- wage:

$$\frac{1}{\gamma_i} w_i L_i = \sum_{j\in\mathcal{J}} \sum_{n\in\mathcal{N}} \alpha_n \beta_n Pr(\Lambda^n_{k\to i}, j) \frac{1}{\gamma_j}(w_j L_j)$$

- Trade in intermediate goods (from i to k)

$$\chi_{ki} = \sum_{j\in\mathcal{J}} \sum_{n=1}^{N-1} \beta_n Pr(\Lambda^n_{k\to i}, j) \frac{1}{\gamma_j}(w_j L_j)$$

General equilibrium

- When N=1, we have $\alpha_N \beta_N = 1$ and $Pr(\Lambda^n_{k \to i}, j) = \pi_{ij} = \frac{T_i (c_i \tau_{ij})^{-\theta}}{\prod_{k=1}^{J} T_k (c_k \tau_{kj})^{-\theta}}$.
- The equilibruium reduces to that in EK model.

Gains from Trade

- Consider a 'purely-domestic' value chain that performs all stages in a given country j to serve consumers in the same country j. It captures a share of country j 's spending equal to

$$\pi_{\boldsymbol{jj}} = Pr(j, j, ..., j) = \frac{(\tau_{jj})^{-\theta(1+\sum_{n=1}^{N-1}\beta_n)} \times \left(c_j\right)^{-\theta} T_j}{\Theta_j}$$

- Welfare

$$\frac{w_j}{P_j} = (\kappa(\tau_{jj})^{\sum_{n=1}^{N-1}\beta_n})^{-1/\gamma} \left(\frac{T_j}{\pi_{\boldsymbol{jj}}}\right)^{1/\theta\gamma}$$

- Under antarky $\pi_{\boldsymbol{jj}} = 1$, so the real income gains from trade relative to autarky are $\left(\frac{T_j}{\pi_{jj}}\right)^{1/\theta\gamma} - 1$
- The share $\pi_{\boldsymbol{jj}}$ is lower than π_{jj}^F , and thus the gains from trade here are larger than EK.

The Centrality-Downstreamness Nexus

- Define the average upstreamness $U(i,j)$ of production of a given country i in value chains that seek to serve consumers in country j :

$$U(i,j) = \sum_{n=1}^{N} (N - n + 1) \frac{Pr(i = l(n); j)}{\sum_{n'=1}^{N} Pr(i = l(n'); j)}$$

- Suppose we can decompose $\tau_{ij} = (\rho_i \rho_j)^{-1}$. Then

Theorem

The more central a country i is (i.e., the higher is ρ_i), the lower is the average upstreamness $U(i,j)$ of this country in global value chains leading to consumers in any country j.

Extending to Match IO Data

- Incorperate Caliendo and Parro (2015) model: Multi-sector with roundabout production.
- H industries indexed by $h \in \mathcal{H}$.
- Household

$$U\left(y_i^{h,N}(z)\right) = \prod_{h=1}^{H} \left(\zeta_i^h \left(\int_0^1 \left(y_i^{h,N}(z)\right)^{(\sigma^h-1)/\sigma^h} dz\right)^{\sigma^h/(\sigma^h-1)}\right)^{\zeta_i^h} \tag{11}$$

Extending to Match IO Data

- n-stage price of an industry h intermediate input z produced for industry h'through a supply chain $\mathbf{l}$:

$$p_{l(n)}^{n,h,h'}(\boldsymbol{l},z) = \left(a_{l(n)}^{n,h,h'} c_{l(n)}^{n,h}\right)^{\alpha_n^{h,X}} \left(p_{l(n-1)}^{n-1,h,h'}(\boldsymbol{l},z)\tau_{l(n-1)l(n)}^{h}\right)^{1-\alpha_n^{h,X}}, \textit{ for all } n \in \mathcal{N}. \tag{12}$$

- Unit cost of input bundle:

$$c_j^{n,h} = (w_j)^{\gamma_j^h} \prod_{h=1}^{H} (P_j^{h,h'})^{\gamma_j^{h,h'}}, \tag{13}$$

- Productivity:

$$Pr\left(\prod_{n=1}^{N} \left(a_{l(n)}^{n,h,h'}\right)^{\alpha_n^{h,X}\beta_n^{h,X}} > a\right) = exp\left\{-a^{\theta^h} \prod_{n=1}^{N} \left(T_{l(n)}^{h,h'}\right)^{\alpha_n^{h,X}\beta_n^{h,X}}\right\}, \tag{14}$$

Extending to Match IO Data

- The probability of a given GVC $\boldsymbol{l}$ being the cost-minimizing production path for serving industry h' in j is given by

$$\pi_{\boldsymbol{l}j}^{h,h'} = \frac{\prod_{n=1}^{N-1}\left((T_{l(n)}^{h,h'})^{\alpha_n^{h,X}}((c_{l(n)}^{n,h})^{\alpha_n^{h,X}}\tau_{l(n)l(n+1)}^{h})^{-\theta^h}\right)^{\beta_n^{h,X}}\left((T_{l(N)}^{h,h'})^{\alpha_N^{h,X}}((c_{l(N)}^{N,h})^{\alpha_N^{h,X}}\tau_{l(N)j}^{h}\right)^{-\theta^h}}{\Theta_j^{h,h'}} \tag{15}$$

Welfare gains

- Welfare changes:

$$\hat{W}_j = \prod_{h \in H} (\hat{\pi}_{\mathbf{I}j}^{h,F})^{-\frac{1}{\theta^h}\zeta_i^h} \prod_{h \in H} \prod_{h' \in H} \prod_{h'' \in H} (\hat{\pi}_{\mathbf{I}j}^{h,h'})^{-\frac{1}{\theta^h}\gamma_j^{h,h'}\delta_i^{h',h''}\zeta_i^{h''}}$$

- ζ_i^h gross value of industry h finished output that need to be produced, to supply one dollar of aggregate final consumption.
- $\sum_{h' \in H} \sum_{h'' \in H} \gamma_j^{h,h'} \delta_i^{h',h''} \zeta_i^{h''}$: gross value of industry h finished intermediate inputs that need to be produced for industry h'a cross all stages of the supply chain, to satisfy one dolalr of aggregate final consumption.
- Same T for intermediate goods and final goods, reducing to Caliendo and Parro (2015)
- One industry and a common VA share accross country, reducing to EK (2002)

Other related

- Acemonglu (2019, AER);
- Endogeous production network (Acemonglu (2018, Eco); Lim (2018))
- Firm level: Bernard et al. (2017); Boehm et al. (2019, Tohoku Earthquake); Carvalho et al. (2016, earthquake);
- Theoritical paper: Baqaee and Farhi (2018, 2019, 2020).