Advanced Macroeconomics Part II

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London School of Economics

Winter Term 2023

Advanced Macroeconomics Part II

Aims of the course:

- Introduce you to modern business cycle theory at graduate level
- Give both a rigorous and intuitive understanding of the models
- Be critical about assumptions, empirical evidence, future research

Rough Plan

Part I: Shocking theory of the business cycle (weeks 1-6)

- Introduction to business cycles
- Real Business Cycle (RBC) Model
- New Keynesian DSGE Models

Part II: Perspectives on business cycles and steady states (weeks 7-10)

- Endogenous growth and persistent effects of recessions
- Aggregate shocks? Firm-heterogeneity and the business cycle
- Interesting steady states: firms, productivity, market power

Organization

- **Lectures**: Wednesdays 09.00 - 11.00, weekly
- **Classes:** Mondays 1hr-long, taught by me, weekly
- **Problem sets:** Handout out on after lecture, due 8.am class days
  - Subset is marked to gauge your progress
  - Working in teams is encouraged, hand in jointly
- **Exam**: ST, same format as last year, problem sets good practice
- **Office hour:** Friday 8.45-9.45, book via Student Hub
- **Textbook**: see syllabus

Feedback welcome!

Source: Borui Zhu (MSc EME 2021)

Feedback welcome!

New course, new circumstances

Link to anonymous Google Form

Or drop me an email: m.c.de-ridder @ lse.ac.uk

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Lecture 1

Introduction to Business Cycles

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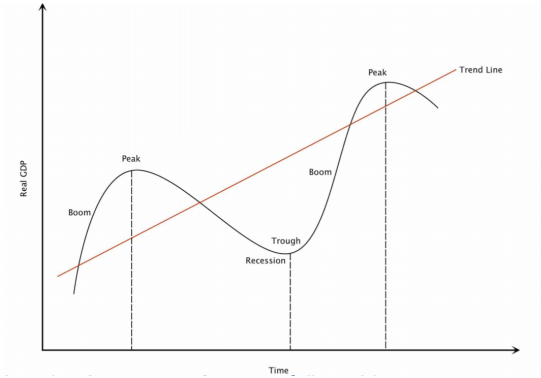
Macroeconomics

Conceptual division between two sub-fields

- Long-run growth: analyze the determinants of the trend growth
- Short-run business cycles: analyze deviations from the trend



Business cycles

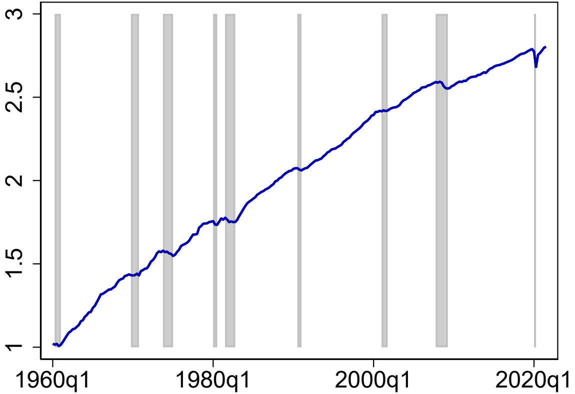
What's the business cycle?

- Informally: deviations of macroeconomic aggregates from trend

Why care? **Recession** episodes:

- Higher unemployment, lower consumption, investment and growth
- Negative effects on **welfare** (including mental and physical health)
- Potential role for **policy** to mitigate negative effects

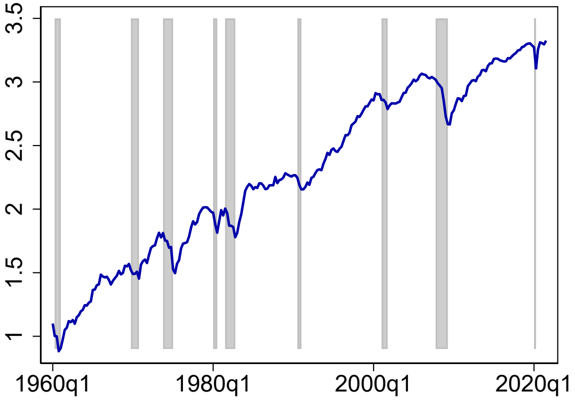
Gross domestic product



Real Gross Domestic Product (log) for the U.S. 1960-2022 (1960 =1)

Source: FRED

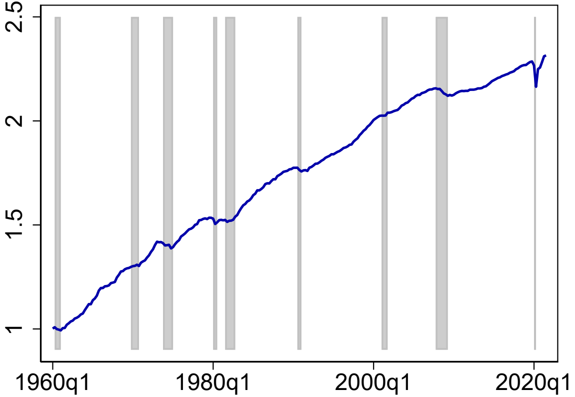
Investment



Real Gross Investment (log) for the U.S. 1960-2022 (1960 =1)

Source: FRED

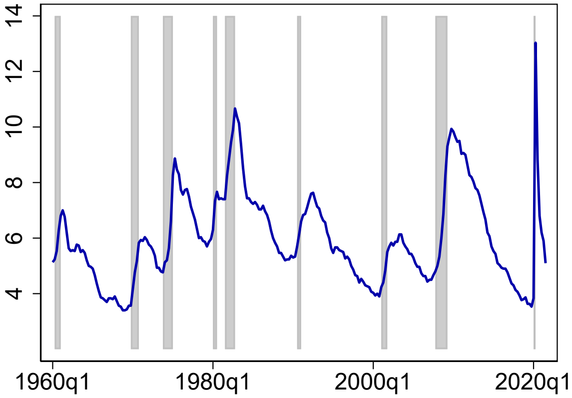
Consumption



Private Consumption (log) for the U.S. 1960-2022 (1960 =1)

Source: FRED

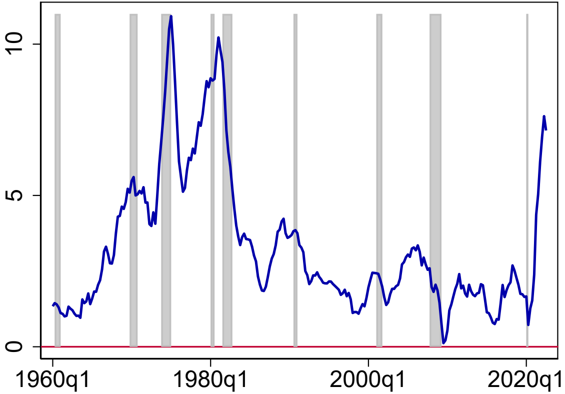
Unemployment



Unemployment Rate for the U.S. 1960-2022

Source: FRED

Prices



GDP inflation for the U.S. 1960-2022

Source: FRED

Plan for today

- Analyzing the business cycle **empirically**
- Introduce **Dynamic Stochastic General Equilibrium** models
- Introduce **productivity shocks**
- Set up the **Real Business Cycle** model

Analyzing the business cycle

First step: establish the empirical patterns that we want to explain

A macroeconomic time series y_t consists of a trend y_t^g and cycle y_t^c s.t.

$$y_t = y_t^g + y_t^c$$

Objective:

- To analyze business cycles, must **identify** them in the data
- Requires a separation of y_t^g from y_t^c

Separating trend from cycle

Various options. Most common:

1. **Deterministic time trend**
   - Assume that $y_t = \alpha t + y_t^c$ or $y_t = h^n(t) + y_t^c$
   - Take first differences:
     $$\Delta y_t = \alpha + \Delta y_t^c$$
     or remove the trend using a nth-order polynomial regression:
     $$y_t = \alpha + h^n(t) + y_t^c$$
     where y_t^c is the residual from a least squares regression

2. **Time filters**
   - Hodrick Prescott (**HP**) filter
   - **Bandpass** filter (e.g. Baxter and King 1999)

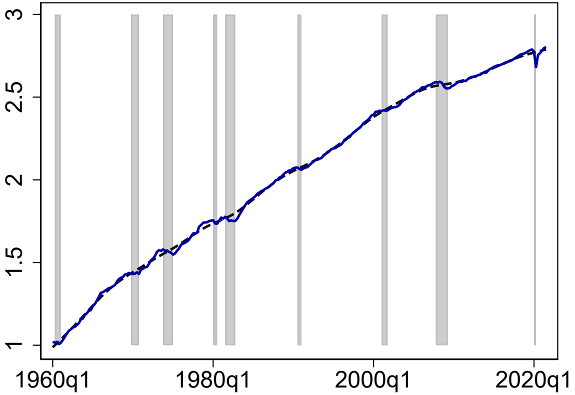
Hodrick and Prescott filter

Most frequently used tool:

$$\{y_t^g\} = \underset{\{y_t^g\}}{\arg\min} \sum_{t=t_0}^{t_0+T} \left((y_t - y_t^g)^2 + \lambda(\Delta y_{t+1}^g - \Delta y_t^g)^2 \right)$$

- Penalty for distance from original series (the cyclical component)
- Penalty for volatility in growth of the trend component
- Smoothing parameter determines relative weight
  - Least squares linear time trend as $\lambda \to \infty$

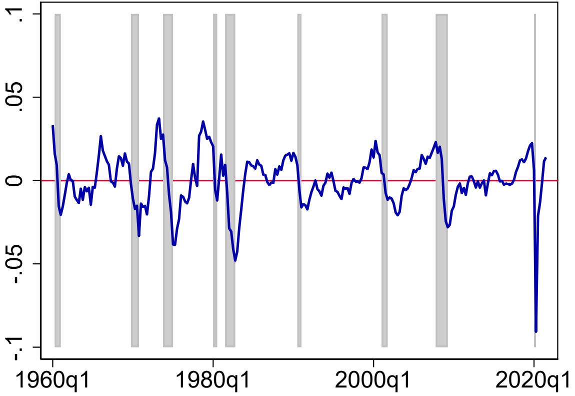
Output - HP Filter



Real Gross Domestic Product for the U.S. 1960-2022

Black-dashed: HP trend. Blue-solid: original series. Source: FRED

Output - cyclical component



Real Gross Domestic Product (log deviations from HP Trend) for the U.S. 1960-2022

Source: FRED

Hodrick and Prescott filter

Choice of λ is important but a bit arbitrary

- Assuming y_t^c and $\Delta^2 y^g$ are i.i.d. and normal with var σ_c^2 and $\sigma_{\Delta^2 g}^2$

$$\text{optimal } \lambda = \sigma_c^2 / \sigma_{\Delta^2 g}^2$$

- Convention: 1600 for quarterly data (see Hodrick and Prescott 1997)
- *"With this value, the implied trend path for the logarithm of real GNP is close to the one that students of the business cycle and growth would draw through a time plot of the series"*
  - Adjustment to different frequencies (N = frequency per year):

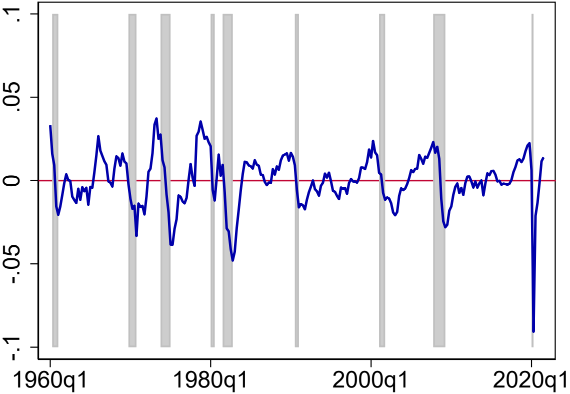
$$\lambda_N = \lambda_4 \left(N/4\right)^4 \Rightarrow \lambda_1 = 6.25, \lambda_{12} = 129,600$$

  see Uhlig and Ravn (2002)

Business cycle facts

1. **Irregularity**
2. **Comovements**
3. **Relative volatility**
4. **Persistence**

Output - cyclical component



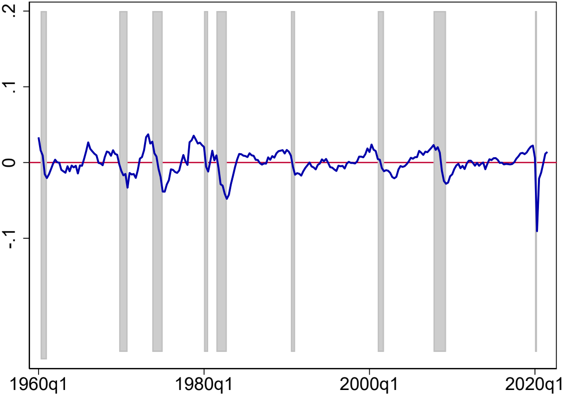
Real Gross Domestic Product (log deviations from HP Trend) for the U.S. 1960-2022

Source: FRED

Business cycle facts

1. **Irregularity** unevenly spaced over time, unequal duration
2. **Comovements**
3. **Relative volatility**
4. **Persistence**

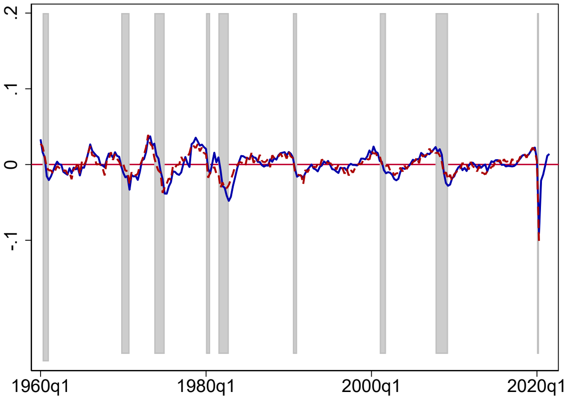
Output, Consumption, Investment - comovement



GDP, Consumption, Investment (log deviations from HP Trend) for the U.S. 1960-2022

Source: FRED

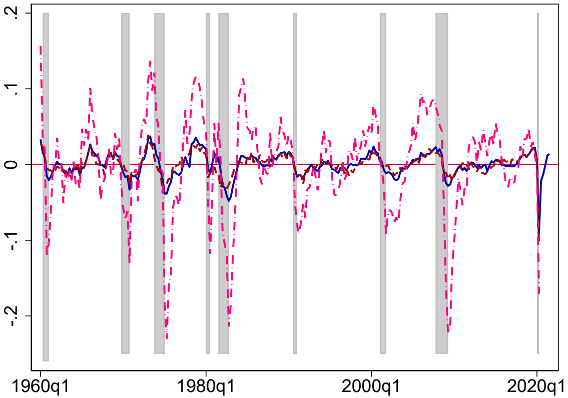
Output, Consumption, Investment - comovement



GDP, **Consumption**, Investment (log deviations from HP Trend) for the U.S. 1960-2022

Source: FRED

Output, Consumption, Investment - comovement



GDP, Consumption, **Investment** (log deviations from HP Trend) for the U.S. 1960-2022

Source: FRED

Business cycle facts

1. **Irregularity** unevenly spaced over time, unequal duration
2. **Comovements** consumption, investment, employment: procyclical
3. **Relative volatility**
4. **Persistence**

Business cycle facts

1. **Irregularity** unevenly spaced over time, unequal duration
2. **Comovements** consumption, investment, employment: procyclical
3. **Relative volatility** investment (and durable consumption): much more volatile than consumption and output
4. **Persistence**

Persistence

	Output	Consumption	Investment	Unemployment
1 Lag	0.80	0.77	0.83	0.78
2 Lags	0.61	0.58	0.62	0.55
3 Lags	0.41	0.43	0.40	0.39
4 Lags	0.23	0.25	0.19	0.23
5 Lags	0.03	0.09	0.00	0.08

Autocorrelations corr(x, $L^n x$) (log deviations from HP Trend) for the U.S. 1960-2019

Source: FRED

Business cycle facts

1. **Irregularity** unevenly spaced over time, unequal duration
2. **Comovements** consumption, investment, employment: procyclical
3. **Relative volatility** investment (and durable consumption): much more volatile than consumption and output
4. **Persistence** deviations from trends have high first-order autocorrelation; deviations are not 'one-off'

Plan for today

- Analyzing the business cycle **empirically** ✓
- Introduce **Dynamic Stochastic General Equilibrium** models
- Introduce **productivity shocks**
- Set up the **Real Business Cycle** model

DSGE models

Dynamic **S**tochastic **G**eneral **E**quilibrium Model

- The workhorse framework of modern business cycle analysis
- Builds on the neoclassical growth model
  - Micro-founded behavior, dynamic optimization, general equilibrium
- Difference: business cycle fluctuations
  - Aggregate variables fluctuate away from the steady state in short run
  - Source of variation is **shocks** to one or more **aggregate** variables
  - Aim: explain irregularity, comovements, relative volatility, persistence
    - Doesn't explain existence of business cycles themselves!

DSGE models

Main distinction

- Real Business Cycle (RBC) models
  - Source of aggregate fluctuations: shocks to **real** variables
  - No nominal frictions: prices set freely, no role for money or demand
- New Keynesian models (NK-DSGE)
  - Source of fluctuations: productivity, **demand**, **monetary policy**
  - **Frictions** in price/wage setting $\rightarrow$ nominal shocks have real effects

Real Business Cycles

Neoclassical growth model with short-term aggregate fluctuations

- Aggregate fluctuations are driven by shocks to **productivity**
- Households choose consumption and **labor** (leisure)
- Efficiency: fluctuations in GDP and employment are **best response**

Became popular in the 1980s, 2000s: still serve as a benchmark

- First large-scale attempt to model microfounded business cycles
- Response to the **Lucas Critique**, oil crises

Plan for today

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Productivity shocks

Measurement assumes:

- $Y_t = Z_t F(K_t, L_t)$ where F is differentiable, 1st-degree homogeneous
- $L = L(L_1, L_2, ...)$ and $K = K(K_1, K_2, ...)$
- Firms maximize $Z_t F(K_t, L_t) - w_t L_t - r_t K_t$ and 0 economic profits

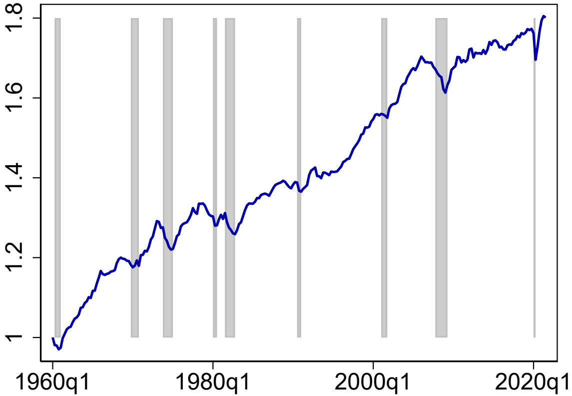
Hence:

- Growth:
  $\dot{Y}_t / Y_t = \dot{Z}_t / Z_t + \left(F'_K(K_t, L_t)\dot{K}_t + F'_L(K_t, L_t)\dot{L}_t \right) / F(K_t, L_t)$
- Firm optimization:
  $w_t / Z_t = F'_L(K_t, L_t) \Rightarrow F'_L(K_t, L_t)/F(K_t, L_t) = w_t / Y_t$
- Insert derivatives to get:

$$\frac{\dot{Y}_t}{Y_t} = \frac{\dot{Z}_t}{Z_t} + \frac{r_t K_t}{Y_t}\frac{\dot{K}_t}{K_t} + \frac{w_t L_t}{Y_t}\frac{\dot{L}_t}{L_t}$$

Productivity shocks

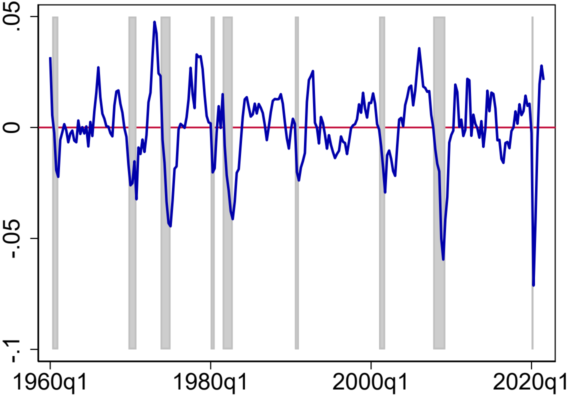
TFP is Solow residual: $\frac{\dot{Z}_t}{Z_t} = \frac{\dot{Y}_t}{Y_t} - \left(1 - \frac{w_t L_t}{Y_t}\right) \frac{\dot{K}_t}{K_t} - \frac{w_t L_t}{Y_t} \frac{\dot{L}_t}{L_t}$



Real TFP for the U.S. 1960-2022 (log, 1960 =1)

Source: Fernald (FRBSF)

Productivity shocks



Real TFP for the U.S. 1960-2022 - Deviations from HP Trend

Source: Fernald (FRBSF)

Productivity shocks

	GDP	Consum.	Invest.	Unempl.	Product.
GDP	1				
Consumption	0.89	1			
Investment	0.88	0.70	1		
Unemployment	-0.88	-0.82	-0.74	1	
Productivity	**0.79**	**0.74**	**0.76**	**-0.56**	**1**

Correlation matrix for the U.S. 1960-2019 - Deviations from HP Trend

Source: Fred, Fernald (FRBSF)

Plan for today

- Analyzing the business cycle **empirically** ✓
- Introduce **Dynamic Stochastic General Equilibrium** models ✓
- Introduce **productivity shocks** ✓
- Set up the **Real Business Cycle** model

Baseline Model

Ingredients:

- Representative household:
  Dynamic opt of consumption, labor, savings, rational expectations
- Representative firm:
  Static (or dynamic) opt. of capita and labor inputs, pay dividends
- No frictions (for now):
  Investments, labor, prices and wages have no adjustment costs
- Total factor productivity is subject to exogenous shocks

Households

Utility function:

$$\max \mathrm{E}_t \sum_{s=t}^{\infty} \beta^{s-t} U(C_s, L_s)$$

- Utility is additively separable
- Per-period utility is strictly increasing and concave in consumption
- Per-period disutility of labor is strictly increasing and concave

$\Rightarrow$ looks familiar from the neoclassical growth model

Important: household maximizes *expected* utility.

$$\mathrm{E}_t[X_{t+s}] = \mathrm{E}\left[X_{t+s} \mid \Omega_t\right]$$

- where Ω_t is information set at time t
- Rational expectations: formed in model-consistent way

Technology

Production function $F(.)$ is neoclassical:

$$Y_t = Z_t F(K_t, L_t)$$

- Output is sold competitively
- Concave in both arguments, twice differentiable
- Homogeneous of degree 1
- $F(0, h) = 0 \; \forall \; h$
- Inada: $\lim_{k \to 0} F_k(k, h) = \infty$, $\lim_{k \to \infty} F_k(k, h) = 0$ for $h > 0$
- Z_t follows some pre-specified stationary **stochastic** process

Households - example

Representative households maximize:

$$\max_{C_s, L_s, I_s} \mathrm{E}_t \sum_{s=t}^{\infty} \beta^{s-t} \left(\ln\ C_s - \frac{(L_s)^{1+1/\eta}}{1+1/\eta} \right)$$

$$\text{s.t. } I_s + C_s = r_s K_s + W_s L_s$$

$$K_{s+1} = I_s + (1-\delta) K_s$$

with initial $K_t > 0$

Households - example

Lagrangian (problem set: recursively):

$$\mathfrak{L} = \mathrm{E}_t \sum_{s=t}^{\infty} \beta^{s-t} \left(\ln C_s - \frac{(L_s)^{1+1/\eta}}{1+1/\eta} - \lambda_s \left(K_{s+1} - (1 + r_s - \delta) K_s - W_s L_s + C_s \right) \right)$$

First order conditions:

$$\frac{\partial \mathfrak{L}}{\partial C_t} = C_t^{-1} - \lambda_t = 0 \Rightarrow \lambda_t = C_t^{-1}$$

$$\frac{\partial \mathfrak{L}}{\partial K_{t+1}} = -\lambda_t + \mathrm{E}_t \left[\lambda_{t+1} (1 + r_{t+1} - \delta) \beta \right] = 0 \Rightarrow \lambda_t = \mathrm{E}_t \left[\lambda_{t+1} (1 + r_{t+1} - \delta) \right] \beta$$

New:

$$\frac{\partial \mathfrak{L}}{\partial L_t} = -(L)_t^{1/\eta} + \lambda_t W_t = 0 \Rightarrow \lambda_t = (L)_t^{1/\eta} W_t^{-1}$$

Households - example

Euler equation:

$$\frac{1}{C_t} = \beta \mathrm{E}_t \left[(1 + r_{t+1} - \delta) \frac{1}{C_{t+1}} \right]$$

Intratemporal labor-consumption choice:

$$L_t = \left(\frac{W_t}{C_t} \right)^{\eta}$$

- η: **Frish** elasticity of labor supply

Initial and terminal conditions: $K_0 > 0$ and TVC $\lim_{s\to\infty} \beta^s C_s^{-1} K_{s+1} = 0$

Firms - example

Representative firms maximize

$$\pi_t = Y_t - W_t L_t - r_t K_t$$

where

$$Y_t = Z_t K_t^{\alpha} L_t^{1-\alpha}$$

with the stochastic productivity process

$$Z_t = Z_{t-1}^{\rho} \exp\left(\sigma \epsilon_t\right)$$

where ϵ_{t+s} is i.i.d. with mean 0 and $0 < \rho < 1$

- This is the **productivity shock**

Firms - example

FOCs:

$$\begin{aligned}\frac{\partial \pi_t}{\partial L_t} &= (1-\alpha) Z_t L_t^{-\alpha} K_t^{\alpha} - W_t = 0 \\ \frac{\partial \pi_t}{\partial K_t} &= \alpha Z_t L_t^{1-\alpha} K_t^{\alpha-1} - r_t = 0\end{aligned}$$

Interpretation?

- What happens when a negative productivity shock hits?

Competitive equilibrium

Definition: sequence for the combination of quantities and prices $\{C_t, L_t, K_t, I_t, Y_t, Z_t\}$, $\{W_t, r_t\}$ such that

- Households solve utility maximization problem
- Firms choose profit-maximizing labor and capital
- Technology constraints:
  capital accumulation, production function, productivity process
- Budget constraint holds (or resource constraint $Y_t = C_t + I_t$)

$\Rightarrow$ 8 variables, 8 conditions

(Alternatively, can separately write L^s and K^s)

- $L_t^s = L_t$, $K_t^s = K_t$: 10 variables, 10 equations

Competitive equilibrium

$$
\begin{aligned}
C_t^{-1} &= \beta \mathrm{E}_t \left[(1 + r_{t+1} - \delta) C_{t+1}^{-1} \right] \\
L_t &= (W_t / C_t)^{\eta} \\
W_t &= Z_t (1-\alpha) K_t^{\alpha} L^{-\alpha} \\
r_t &= Z_t \alpha K_t^{\alpha - 1} L_t^{1-\alpha} \\
Y_t &= Z_t K_t^{\alpha} L_t^{1-\alpha} \\
Y_t &= C_t + I_t \\
K_{t+1} &= (1-\delta) K_t + I_t \\
Z_t &= Z_{t-1}^{\rho} \exp\left(\sigma \epsilon_t \right)
\end{aligned}
$$

Solution

Did we solve the model yet? No!

- Model is solved when **endogenous variables** are expressed as function of **exogenous variables**
- In other words: we need to find the **policy functions**
- System of non-linear difference equations $\Rightarrow$ hard to solve
  - Next lecture: a new **solution method** to help us out

Note: RBC model with growth

So far the productivity process was stationary:

$$Z_{t+s} = Z_{t+s-1}^{\rho} \exp\left(\sigma \epsilon_{t+s}\right)$$

such that

$$\lim_{s \to \infty} E_t(Z_{t+s}) = 1$$

In practice, productivity has a positive trend. E.g.:

$$Y_t = Z_t K_t^{\alpha} (X_t L_t)^{1-\alpha}$$

$$X_t = \bar{X}(1+g)^t$$

Note: RBC model with growth

Along the deterministic **balanced growth path**, simple to show that:

- Y, C, K, I, W grow at constant rate g
- r is constant

Can interpret all variables in the model as **detrended**; $V_t = \frac{\hat{V}_t}{X_t}$

Driven by utility function:

- Income and substitution effect of ΔW cancel

$$L_t \quad = \quad \left(\frac{W_t}{C_t}\right)^{\eta} = \left(\frac{\bar{W}(1+g)^t}{\bar{C}(1+g)^t}\right)^{\eta}$$

What have we done?

- Empirical analysis of the business cycle ✓
    - Separating cycle and trend with the **HP filter**
    - Features: irregularity, comovements, relative volatility, persistence
- Introduction to DSGE models ✓
    - Measuring total factor productivity (shocks)
- Setup of a Real Business Cycle model ✓
    - Derivation of first order conditions
    - Definition of the rational expectations competitive equilibrium

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