

Lecture 8

Endogenous Growth and Persistent Effects of Recessions

Maarten De Ridder

London School of Economics

Lent Term 2022

This term

Part I: Shocking theory of the business cycle

- Introduction to business cycles ✓
- Real Business Cycle (RBC) Model ✓
- New Keynesian DSGE Models ✓

Part II: Perspectives on business cycles and steady states

- Heterogeneity versus homogeneity and the effect of policy ✓
- Endogenous growth and persistent effects of recessions ⇐
- Aggregate shocks? Firm-heterogeneity and the business cycle

Background reading

Barlevy (2004), *The Cost of Business Cycles Under Endogenous Growth*,
American Economic Review

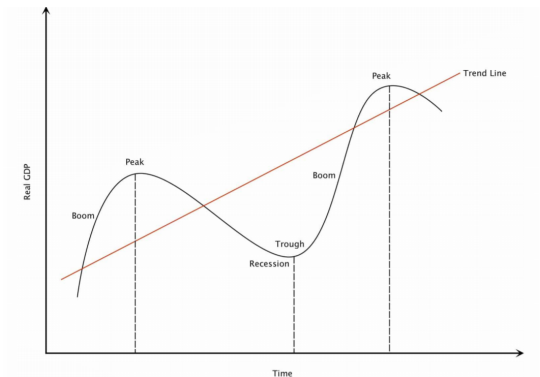
Acemoglu (2009), *Introduction to Modern Economic Growth*, Ch 13.1
(LSE library)

Comin (2009), *On the Integration of Growth and Business Cycles*,
Empirica

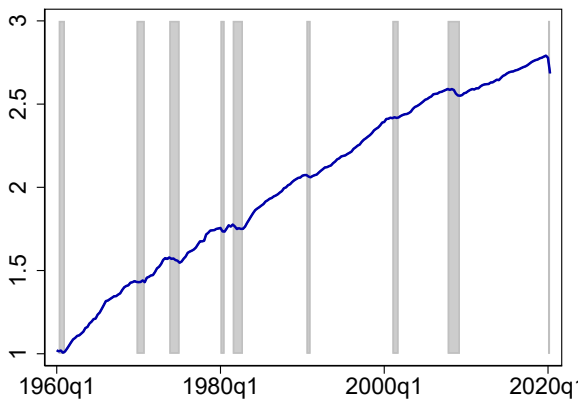
Recall: Macroeconomics

Conceptual division between two sub-fields

- Long-run growth: analyze the determinants of the trend growth
- Short-run business cycles: analyze deviations from the trend



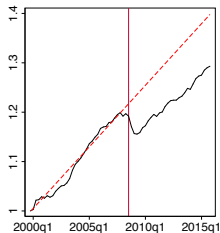
Business Cycles



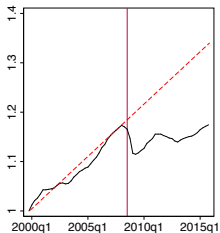
Real Gross Domestic Product (log) for the U.S. 1960-2020 (1960 =1)

Source: FRED

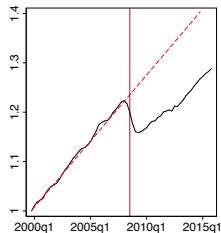
Persistent Business Cycles



(a) United States



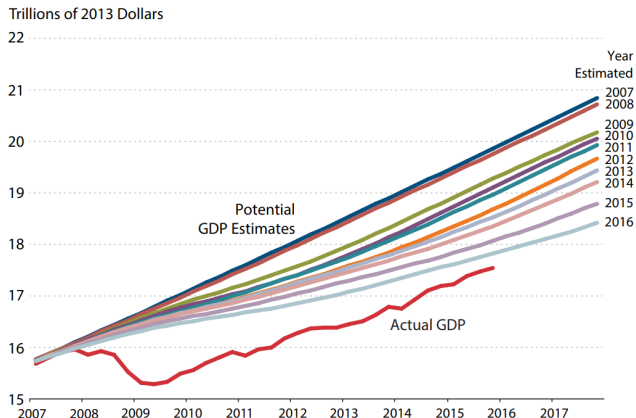
(b) Euro Area



(c) United Kingdom

Real Gross Domestic Product (log index, 2000 = 1). Source: OECD

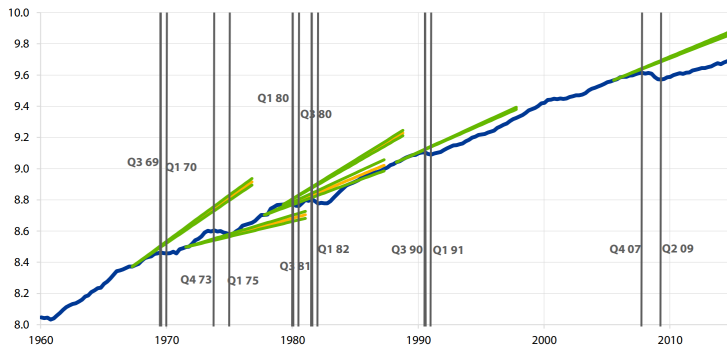
Persistent Business Cycles



Estimates of Potential Real Gross Domestic Product for the U.S.

Source: Summers (2017) based on Congressional Budget Office data

Persistent business cycles



Log Real Gross Domestic Product for the U.S.

Source: Summers (2015), ECB Forum on Central Banking

Persistent effect of large shocks

- Persistent effect of banking crises, currency crises, civil unrest
- Cross-country regressions, ARDL along:

$$g_{it} = \alpha_i + \sum_{j=1}^4 \beta_j g_{i,t-j} + \sum_{s=0}^4 \beta_s D_{i,t-s} + \varepsilon_{it}$$

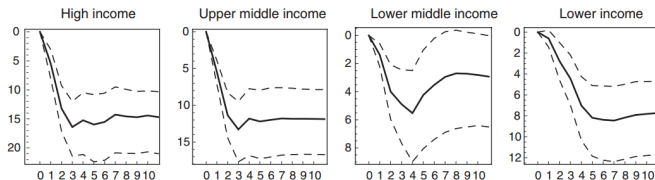


FIGURE 4. IMPULSE RESPONSES: BANKING CRISES

Impulse Responses Systemic Banking Crises on Real GDP

Source: Cerra and Saxena (2008), AER

Today

- Persistent recessions and the costs of business cycles
- Endogenous productivity growth
- Comin and Gertler (2006) Model

Today

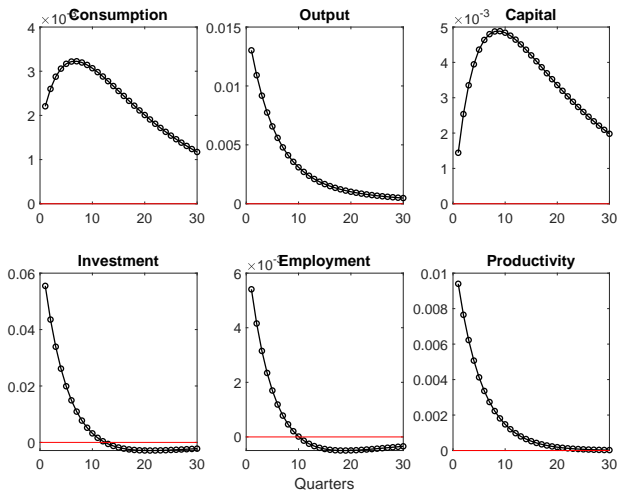
- **Persistent recessions and the costs of business cycles**
- Endogenous productivity growth
- Comin and Gertler (2006) Model

Persistence in DSGE models

Why does the economy return to the old steady state in the model?

- Neoclassical production function: diminishing returns to factors
- Hence: temporary decline in investment raises marginal product
- Policy: investments increase in marginal product of capital
- Eventual return to steady state capital and output

Impulse responses - lecture 3



Impulse responses to one standard dev. productivity shock (in log dev. from steady state)

Persistence in DSGE models

- Drop assumption of diminishing returns ('AK model'):

$$Y_t = \theta A_t K_t$$

$$A_t = A_{t-1}^\rho \exp(\epsilon_t)$$

- Representative household:

$$U = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\gamma}}{1-\gamma}$$

$$\text{s.t. } C_t + K_{t+1} = (r_t + 1 - \delta)K_t$$

- Firms rent capital from households on competitive markets

Persistence in DSGE models

Equilibrium conditions:

- First order conditions:

$$\begin{aligned}C_t^{-\gamma} &= \beta \mathbb{E}_t [(1 + r_{t+1} - \delta) C_{t+1}^{-\gamma}] \\ \theta A_t &= r_t\end{aligned}$$

- Resource constraint:

$$C_t + K_{t+1} = \theta A_t K_t + (1 - \delta) K_t$$

$$A_t = A_{t-1}^{\rho} \exp(\epsilon_t)$$

Steady state: balanced growth path with constant growth rate g ,

$$G = G^Y = G^K = (\beta(1 + \theta - \delta))^{1/\gamma}$$

where $G_t^X \equiv \frac{X_t}{X_{t-1}}$

\Rightarrow AK model: 1st generation **endogenous growth** model

Persistence in DSGE models

How do you solve this?

- Perturbation: need to log-linearize around the steady state
- Capital, consumption do **not** have steady state in levels
- Solution: log-linearize around the balanced growth path with:
 - Constant **growth** of consumption, capital
 - As well as constant capital/consumption ratio

Log-linearized equations

- Resource constraint and capital accumulation:

$$\widehat{g}_{t+1}^k = \frac{\theta}{G^k} \widehat{a}_t - \frac{C^*}{G^k} \widehat{c}_t^*$$

where c_t^* is the log consumption to capital ratio

- Euler equation written in terms of C_t^* :

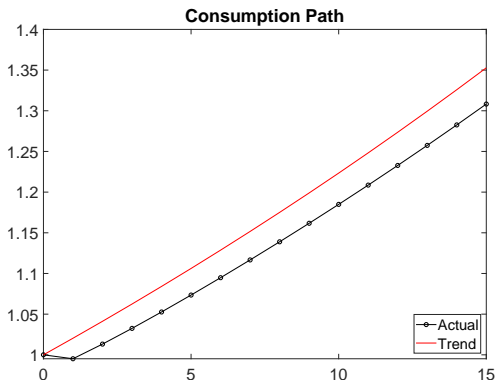
$$\begin{aligned} 1 &= \beta \mathbb{E}_t \left[(1 + \theta A_{t+1} - \delta) \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \right] \\ 1 &= \beta \mathbb{E}_t \left[(1 + \theta A_{t+1} - \delta) \left(\frac{C_{t+1}}{C_t} \frac{K_{t+1}}{K_{t+1}} \frac{K_t}{K_t} \right)^{-\gamma} \right] \\ 1 &= \beta \mathbb{E}_t \left[(1 + \theta A_{t+1} - \delta) \left(\frac{C_{t+1}^*}{C_t^*} \frac{K_{t+1}}{K_t} \right)^{-\gamma} \right] \end{aligned}$$

Log-linearized:

$$\mathbb{E}_t [\widehat{g}_{t+1}^{c^*}] + \widehat{g}_{t+1}^k = \left(\frac{\theta}{1 + \theta - \delta} \right) \frac{1}{\gamma} \mathbb{E}_t [\widehat{a}_{t+1}]$$

as well as definition $\widehat{g}_t^{c^*} = \widehat{c}_t^* - \widehat{c}_{t-1}^*$ and process $\widehat{a}_t = \rho \widehat{a}_{t-1} + \varepsilon_t$

Persistence in DSGE models



Path of consumption index (year 0 = 1) after 8% negative productivity shock

$\gamma = 4$, $\beta = 0.99$, $\rho = 0.5$, θ is such that $g = 1.02$. Code: Moodle.

Cost of business cycles

Lucas (1987): business cycles have **minimal costs**

- Assume a simple growth process for stochastic consumption:

$$C_t = \lambda^t(1 + \varepsilon_t)C_0$$

with $\lambda > 1$, $C_0 > 0$, ε_t i.i.d. disturbances with variance σ^2

- Utility:

$$U = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\gamma}}{1-\gamma}$$

- Risk-averse consumer would prefer certain income ($\varepsilon = 0 \quad \forall t$)
But how much?

Cost of business cycles

Find percentage of consumption household would give up for certainty:

$$\sum_{t=0}^{\infty} \beta^t \frac{[\lambda^t C_0]^{1-\gamma}}{1-\gamma} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{[\lambda^t (1 + \varepsilon_t)(1 + \chi) C_0]^{1-\gamma}}{1-\gamma}$$

Solve for χ (**problem set**) to get:

$$\chi \approx \frac{1}{2} \gamma \sigma^2$$

Derive σ^2 from variance of HP deviations of consumption. Lucas (2003):

$$\begin{aligned} \chi &= \frac{1}{2} \cdot 4 \cdot (0.022)^2 \\ &= 0.0003872 \\ &\approx 0.04\% \end{aligned}$$

Business cycles and growth

Lucas (1987) assumes that **volatility** σ^2 does not affect **growth rate** λ

- In other words: transitory shocks have transitory effects
- How does the model need to change to match the data?
- Introduce **endogenous growth**

Our Model

Representative household and firm:

$$U = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\gamma}}{1-\gamma}$$

$$Y_t = A_t K_t$$

Capital accumulation:

$$K_{t+1} = I_t + (1 - \delta)K_t$$

$$K_t = \left[\prod_{s=0}^t \left(\frac{I_s}{K_s} + 1 - \delta \right) \right] K_0$$

Path of consumption

Define

$$c_t = \frac{C_t}{Y_t} \quad i_t = \frac{I_t}{Y_t}$$

Path of consumption:

$$\begin{aligned} C_t &= c_t A_t K_t \\ &= \left[\prod_{s=0}^t \left(\frac{I_s}{K_s} + 1 - \delta \right) \right] K_0 c_t A_t \\ &= \left[\prod_{s=0}^t \lambda_s \right] (1 + \epsilon_t) C_0 \end{aligned}$$

Where:

- Growth in potential output: $\lambda_s = i_s A_s + 1 - \delta$
- Disturbances: $1 + \epsilon_t = \frac{c_t A_t}{c_0 A_0} \rightarrow$ deviation of consumption from trend

Cost of business cycles

Repeat the Lucas calculation:

$$\sum_{t=0}^{\infty} \beta^t \frac{[\lambda^t C_0]^{1-\gamma}}{1-\gamma} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{[(\prod_{s=0}^t \lambda_s) (1 + \epsilon_t)(1 + \chi) C_0]^{1-\gamma}}{1-\gamma}$$
$$1 + \chi = \left(\frac{\sum_{t=0}^{\infty} \beta^t \lambda^{t(1-\gamma)}}{\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [(\prod_{s=0}^t \lambda_s) (1 + \epsilon_t)]^{1-\gamma}} \right)^{\frac{1}{1-\gamma}}$$

Simulate path of consumption for large number of years (e.g. 200,000), set $\rho = 0$, calculate χ using the equation to get:

$$\chi \approx 2.3\%$$

\Rightarrow cost of the same shocks is two orders of magnitude larger when they have persistent effect

- Code: Moodle
- Note: we've increased b-cycle costs by making shocks more persistent

Cost of business cycles

Barlevy (2004), AER:

- AK model like the one presented here
- **Transitory** shocks to total factor productivity
- Additional assumption: **diminishing** returns to investments

$$K_{t+1} = \phi \left(\frac{I_t}{K_t} \right) K_t + (1 - \delta) K_t$$

$$\frac{\partial \phi \left(\frac{I_t}{K_t} \right)}{\partial I_t} > 0, \quad \frac{\partial^2 \phi \left(\frac{I_t}{K_t} \right)}{\partial I_t^2} < 0$$

- Results:
 1. Permanent reduction in consumption from transitory shock
 2. **Lower average growth** when volatility is higher

Summary

So far:

- Some recessions seem to have persistent effect on output
- If capital does not have diminishing returns, model can replicate this
- Large increase in predicted costs of business cycles

Empirical evidence: strong **diminishing returns** to capital

- Macro: capital share in national accounts implies $\alpha \approx 0.3 - 0.4$
- Firm-level production data: $\alpha \approx 0.2 - 0.45$
(e.g. Akerberg, Caves & Frazer (2015), ECTA)

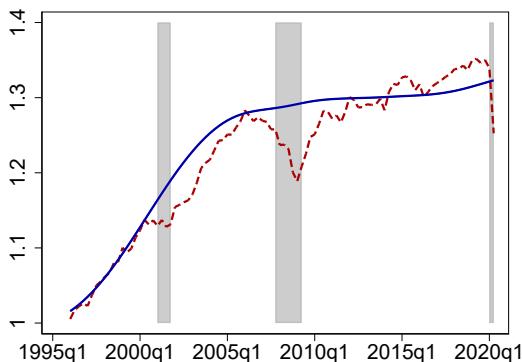
Solution: endogenous growth in **productivity**

Today

- Persistent recessions and the costs of business cycles
- **Endogenous productivity growth**
- Comin and Gertler (2006) Model

Endogenous productivity growth

$$\text{Total factor productivity: } \frac{\dot{A}_t}{A_t} = \frac{\dot{Y}_t}{Y_t} - \left(1 - \frac{w_t L_t}{Y_t}\right) \frac{\dot{K}_t}{K_t} - \frac{w_t L_t}{Y_t} \frac{\dot{L}_t}{L_t} - \frac{\dot{U}_t}{U_t}$$



Real TFP for the U.S. 1995-2020 (log, 1995 = 1)

Red-dashed: raw series. Blue-solid: utilization-adjusted. Source: Fernald (FRBSF)

Endogenous productivity growth

Key insights:

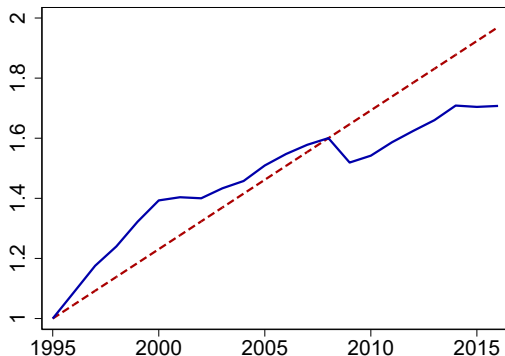
- Productivity growth comes from technology creation and adoption
- Neoclassical production fn: admits exponential growth through A
- Technology creation/adoption require particular investments: R&D

$$\frac{\dot{A}_t}{A_t} = g(RD_t, A_t, ..)$$

- These investments depend on expected profits, financing costs, etc

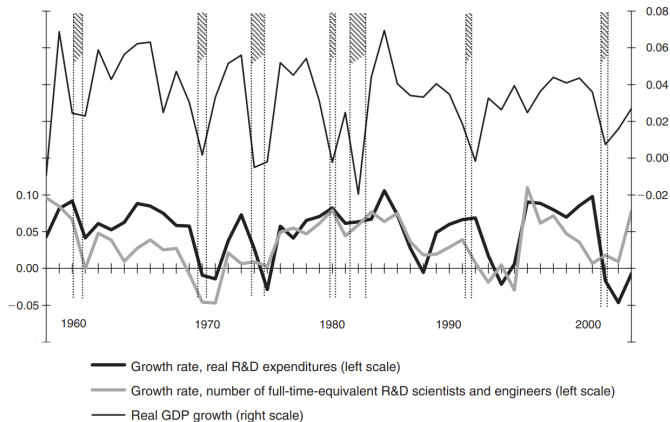
$$RD_t = h(r_t, A_t, \pi_t, \pi_{t+1}, ..)$$

Investment in Productivity



Intangible investments (R&D, employee training, software development) in the United States (1995-2016). Source: Intan-Invest Project (2020)

Procyclical R&D



Procyclicality of Research and Development in the United States

Source: Barlevy (2007), Figure 1

Endogenous technological change

Broadly divided into two groups:

1. Models of process innovation

- R&D expands the variety of technologies that used in production
- Seminal reference: Romer (1990)

2. Models of product innovation

- Invention of new or better goods
- Grossman and Helpman (1991), Aghion and Howitt (1992)

Aim of endogenous growth literature: understand **growth**

- Policies, incentives to maximize welfare on **balanced growth path**

Romer (1990) model: summary

- Innovation as generating new blueprints or *ideas* for production
- Three important features:
 1. Cost of research and development are paid as fixed upfront costs
 2. Ideas and technologies are *nonrival*:
 - Many firms can benefit from the same idea
 - Innovator needs a reward: introduce *monopolistic competition* (CES)
 3. Increasing returns to scale?

Preferences

- Time is discrete, representative household is infinitely lived
- Utility function:

$$\sum_{t=0}^{\infty} \left(\frac{1}{1+\rho} \right)^t \frac{C_t^{1-\gamma} - 1}{1-\gamma}$$

- Labor is supplied inelastically, measure of labor supply is L .
- Households hold a portfolio of all firms in the economy

Production

- Consumption good C_t is homogeneous
- Competitively produced with aggregate production function:

$$Y_t = \frac{1}{1-\beta} \left[\int_0^{A_t} x_{it}^{1-\beta} di \right] L^\beta$$

- x_{it} is the amount of input from technology type i at time t
- A_t : measure of intermediate-input technologies available at time t
- For given A_t , economy exhibits constant returns to scale

Resource Constraint

Resource constraint of the economy:

$$C_t + X_t + RD_t = Y_t$$

- RD_t : spending on **research and development**
 - Innovators spend RD_t to develop new intermediate input technologies
- X_t : spending on intermediate input production
 - Once invented, inputs can be produced at marginal cost $\Psi > 0$.

Innovation

- A continuum of innovators invests to generate new inputs
- Aggregate innovation:

$$A_{t+1} - A_t = \varphi RD_t$$

- Successful innovator receives perpetual patent to produce some x_{it}
- Note: no aggregate uncertainty
 - Individual projects by innovators can fail, but overall level of innovation is deterministic function of aggregate R&D RD_t

Optimization: final good sector

- Maximization by competitive final good producers:

$$\max_{L, x_{it}; i \in [0, A_t]} \frac{1}{1 - \beta} \left[\int_0^{A_t} x_{it}^{1-\beta} di \right] L^\beta - \int_0^{A_t} p_{it}^x x_{it} di - w_t L$$

- Demand for intermediate goods

$$x_{it} = (p_{it}^x)^{-\frac{1}{\beta}} L$$

- Note: doesn't depend on the wage rate or the number of technologies A_t

Optimization: technology monopolists

Problem of the owner of patent to produce i obtained at time 0

- Goal: maximize present value of profits

$$V_{i0} = \max_{p_{it}^x} \sum_{t=1}^{\infty} \prod_{s=1}^t \left(\frac{1}{1+r_s} \right) \pi_{it}$$

- Profits under constant marginal costs Ψ :

$$\pi_{it} = (p_{it}^x - \Psi)x_{it}$$

- Constrained by demand function $x_{it} = (p_{it}^x)^{-\frac{1}{\beta}} L$.

Optimization: technology monopolists

For all i at all t :

- Optimal price is markup over (constant) marginal cost:

$$p_{it}^x = \frac{\Psi}{1 - \beta} = 1 \text{ (normalization)}$$

- Hence profit is:

$$\begin{aligned}\pi_{it} &= (p_{it}^x - \Psi)(p_{it}^x)^{-\frac{1}{\beta}} L \\ &= \left(\frac{\Psi}{1 - \beta} \beta \right) \left(\frac{\Psi}{1 - \beta} \right)^{-\frac{1}{\beta}} L = \beta L\end{aligned}$$

- Output:

$$x_{it} = (p_{it}^x)^{-\frac{1}{\beta}} L = L$$

Equilibrium output

It follows that output has increasing returns to scale:

$$\begin{aligned} Y_t &= \frac{1}{1-\beta} \left[\int_0^{A_t} x_{it}^{1-\beta} di \right] L^\beta \\ &= \frac{1}{1-\beta} \left[\int_0^{A_t} L^{1-\beta} di \right] L^\beta \\ &= \frac{1}{1-\beta} A_t L \end{aligned}$$

Hence: a constant growth rate of A_t will generate constant GDP growth

Equilibrium interest rate

- Free entry condition: profits compensate for investment upon entry
- Recall:

$$A_{t+1} - A_t = \varphi R D_t$$

- Average cost of entry: $1/\varphi$. Benefit: V_t

$$V_t \varphi = 1$$

- Combine this with the definition of the value function:

$$\begin{aligned} V_0 \varphi &= \varphi \sum_{t=1}^{\infty} \left(\frac{1}{1+r} \right)^t \pi = 1 \\ &= \varphi \beta L \left(\frac{1}{1 - \frac{1}{1+r}} \right) \left(\frac{1}{1+r} \right) = 1 \\ \Rightarrow r_t &= r^* = \varphi \beta L \end{aligned}$$

Equilibrium growth

Household's first order condition for consumption:

$$\frac{C_{t+1}}{C_t} = ((1 + \rho)^{-1}(1 + r_t))^{1/\gamma}$$

Combined with the transversality condition. Note that:

$$\begin{aligned} G_c &= ((1 + \rho)^{-1}(1 + r_t))^{1/\gamma} \\ &= ((1 + \rho)^{-1}(1 + \varphi\beta L))^{1/\gamma} \end{aligned}$$

Straightforward to show that model admits a **balanced growth path equilibrium** where Y , C and A grow at constant rate

$$G = ((1 + \rho)^{-1}(1 + \varphi\beta L))^{1/\gamma}$$

(see Acemoglu)

Intuition

$$G = ((1 + \rho)^{-1}(1 + \varphi\beta L))^{1/\gamma}$$

- Growth **falls** in rate of impatience ρ
- Growth **increases** in research-effectiveness parameter φ
- Growth **increases** in population size L ('scale effect', see Jones, 1995)
- Growth **increases** in profitability parameter β
 - β raises incentive for research
 - Means interest rate must increase for free entry condition
 - Household responds to higher interest rate by lowering consumption
 - \Rightarrow More resources to growth

Cycles and growth

Aim of endogenous growth literature: understand growth

- Policies, incentives to maximize welfare along balanced growth path
- Does not consider effect of fluctuations on growth, welfare

Example: we've seen that profitability is positive for growth

- But profitability is **parameter-determined**: it does not vary over time
- Needed: a model where incentives for R&D are subject to **shocks**

Today

- Persistent recessions and the costs of business cycles
- Endogenous productivity growth
- **Comin and Gertler (2006) Model**

Comin and Gertler (2006, *AER*)

- Understand both growth and fluctuations, and how they interact
- Start with the Romer model of expanding varieties
- New layer: consider both innovation and technology adoption (match evidence)
- Study short and long-run effect of transitory real shocks (e.g. labor supply)

We'll study the main mechanism and intuition

Production

Final output is produced competitively along:

$$Y_t = (K_t^\alpha L_t^{1-\alpha})^\gamma M_t^{1-\gamma}$$

Materials M_t are produced by combining intermediate goods x_{it} :

$$M_t = \left(\int_0^{A_t} x_{it}^{1/\mu} di \right)^\mu$$

$\mu > 1$ is the markup charged by monopolist producer of intermediate i .

Innovation

Key assumption: innovation comes with **convex costs** between periods

- Innovators: equate marginal benefit of innovation to marginal costs
- Optimization: set net-present value of marginal innovation equal to innovation costs
- Net-present value of innovation comes from profits (procyclical)

Households

Shocks come through labor disutility of the household:

$$\mathbb{E}_t \sum_{i=0}^{\infty} \beta^{t+i} \left[\ln C_{t+i} - \mu_{t+i}^w \left(\frac{L_{t+i}^{1+1/\eta}}{1+1/\eta} \right) \right]$$

$$\text{s.t. } C_t + K_{t+1} = (R_t - \delta)K_t + w_t L_t$$

Note: these are “balanced growth preferences” (see lecture 1)

- Euler equation:

$$C_t^{-1} = \beta \mathbb{E}_t [(R_{t+1} - \delta) C_{t+1}^{-1}]$$

- Static consumption-labor decision:

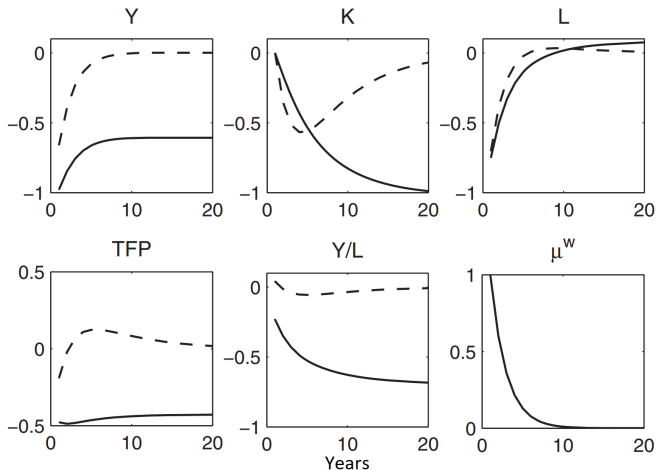
$$L_t = \left(\frac{w_t}{C_t} \frac{1}{\mu_t^w} \right)^{\eta}$$

Mechanism

Say there is a positive shock to μ_t^w :

- Contraction in labor supply
- Reduction in aggregate output (**recession**)
- Reduction in profitability of owning intermediate input
- Reduction in effort to adopt new intermediate input technology
- Reduction in R&D to develop new technologies

Impulse Responses



Impulse Response to Unit Shock to Wage Markup

Solid: full model. Dashed: no endogenous TFP. Source: Comin and Gertler (2006)

Comin and Gertler (2006)

The actual model contains much more

- Variable capital utilization with endogenous depreciation
- Endogenous markups and profits through entry and exit
- A goods-producing and capital-producing sector
- Careful calibration and comparison with data

Persistent business cycles are in fashion

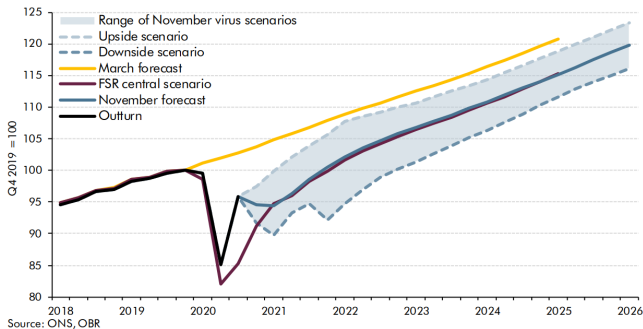
Embed the Comin and Gertler structure in a New Keynesian model:

- Anzoategui, Comin, Gertler, Martinez (2019), *AEJ: Macroeconomics*
- Ikeda and Kurozumi (2019), *Journal of Monetary Economics*

Endogenous growth in New Keynesian models:

- Benigno and Fornaro (2017), *Review of Economic Studies*
- Bianchi, Kung and Morales (2019), *Journal of Monetary Economics*
- Garga and Singh (2019), *Journal of Monetary Economics*
- Queralto and Moran (2018), *Journal of Monetary Economics*
- Queralto (2019), *Journal of Monetary Economics*

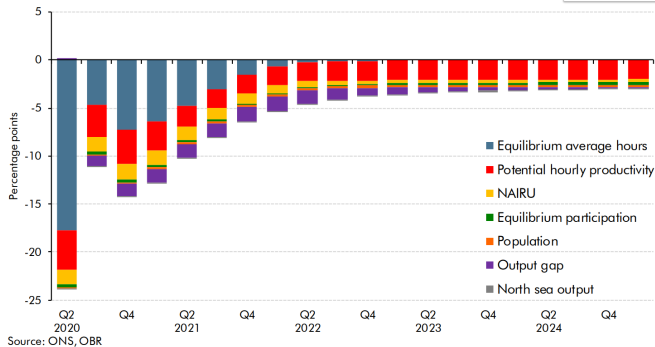
Lesson Learned?



GDP Forecast: November 2020 versus March 2020

Source: UK Office of Budget Responsibility

Lesson Learned?



Contributors to change in GDP Forecast: November 2020 versus March 2020

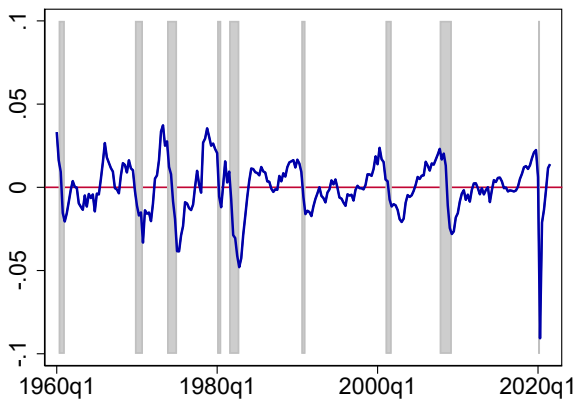
Source: UK Office of Budget Responsibility (OBR)

Lesson Learned?

*"We will continue our **V-shaped recovery** and launch a record-smashing economic boom" - Donald J Trump, October 2020*



Trumped



Real Gross Domestic Product (log deviations from HP Trend) for the U.S. 1960-2021

Source: FRED

What have we done?

- Evidence on persistent effect of recessions ✓
- First generation endogenous growth model, cost of business cycles ✓
- Second generation endogenous growth models ✓
- Growth models with business cycles ✓