Quantitative Trade/Spatial Economics: A Practitioner's Guide

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Miscellaneous issues

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1. Resources

- Dave Donaldson (slides, updated frequently)
- Thomas Chaney (slides with detailed notes! Good for classic models)
- Treb Allen (slides/book/notes)
- Elhanan Helpman (models)
- Handbook Chapters/Papers
  - Costinot and Rodriguez-Clare(2014)
  - Head and Mayer(2014)
  - Arkolakis, Costinot, and Rodriguez-Clare(2013)
- Data(CEPII, FREIT, UNComtrade, WITS…)
- B站：runahead；　　https://tingji.weebly.com/links.html

2. Exact Hat Algebra

- Exact Hat Algebra：直接计算equilibrium (on changes)
- 优点
  - 不用calibrate部分参数 (EK: technology, trade costs)
  - 不用计算pre/after这两个equilibrium（on levels）
  - Some restrictions no longer matter: theta>sigma-1.（可能是缺点。）
- 缺点
  - 没啥缺点，能用的时候就推荐用（本质上两种做法结果一样的）。
  - 因为没有计算，所以不知道没有calibrate的参数
  - 冲击的shock要给hat（不是缺点，其实只是特点）

均衡方程可以同时有多种形式的变量

Definition 2. *Let* $(\mathbf{w}, P)$ *be an equilibrium under tariff structure* τ *and let* $(\mathbf{w}', P')$ *be an equilibrium under tariff structure* τ'*. Define* $(\hat{\mathbf{w}}, \hat{P})$ *as an equilibrium under* τ' *relative to* τ*, where a variable with a hat "*$\hat{x}$*" represents the relative change of the variable, namely* $\hat{x} = x'/x$*. Using equations (2), (4), (6), (7), and (9) the equilibrium conditions in relative changes satisfy:*

Cost of the input bundles:

$$\hat{c}_n^j = \hat{w}_n^{\gamma_n^j} \prod_{k=1}^{J} \hat{P}_n^{k\ \gamma_n^{k,j}}. \tag{10}$$

Price index:

$$\hat{P}_n^j = \left[\sum_{i=1}^{N} \pi_{ni}^j [\hat{\kappa}_{ni}^j \hat{c}_i^j]^{-\theta^j} \right]^{-1/\theta^j}. \tag{11}$$

Bilateral trade shares:

$$\hat{\pi}_{ni}^j = \left[\frac{\hat{c}_i^j \hat{\kappa}_{ni}^j}{\hat{P}_n^j} \right]^{-\theta^j}. \tag{12}$$

Total expenditure in each country n and sector j:

$$X_n^{j'} = \sum_{k=1}^{J} \gamma_n^{j,k} \sum_{i=1}^{N} \frac{\pi_{in}^{k'}}{1+\tau_{in}^{k'}} X_i^{k'} + \alpha_n^j I_n'. \tag{13}$$

Exact hat algebra

- “exact” hat algebra，没有做过近似
- 与之相对的，宏观中“log linearization”是一阶展开
  - 哪个好啊？
- 但是国际贸易中也有求local counterfactual的，也是一阶展开（比如对df(x)，只要写成f’(x)dx，那就是一阶展开）——想想泰勒展开。
  - 比如说Caliendo and Parro (2015)中间关税效应分解
  - Allen notes

3. Trade Deficit

- 数据中是有trade deficit的，模型中是否保留（即D’=D~=0）
- 经济学意义：认为trade deficit对于counterfactual的shock来说是正交的，即invariant to counterfactual shocks.（这是个强假设，是这种静态模型的缺点。Trade surplus是“借钱”给外国，期待将来外国还回来，即通过外国实现储蓄从而跨时间调整本国消费的。）
- 更严重的，会产生两个实际问题(Ossa, 2016)
  - 可能找不到均衡
  - Numeraire的选择会产生实际区别（因为国家间的相对价格会发生变化）

Trade Deficit

- 解决办法
  - Ossa（2016），Caliendo and Parro（2015）：先解一个D’=0的counterfactual，然后在此基础上进行后续分析。（缺点： D’=0 的新均衡和实际数据会有偏离，详见Ossa 2016。）
  - 不使用D的level，使用share。（解决了第二个问题，但是没有解决第一个问题。）
  - Caliendo et al. (2018)：假设一个capital good/land/fixed factor，然后将其return从各地集中，然后按照一定比例再分配到各地。只要各地收支比例不一致，就会产生trade imbalance。（Balance of payments：经常账户和资本账户要保持平衡。）
  - 注意是解决办法，并不是“解释”！（modelling device）

4. Trade Elasticity

- Trade Elasiticity

$$\frac{\partial\left(\ln\left(X_{ij}/X_{jj}\right)\right)}{\partial \ln \tau_{ij}} = -\varepsilon$$

- Armington/Krugman: sigma-1 (sigma: elasticity of substitution in CES)
  - Only intensive margin works
- EK: theta (shape parameter of Frechet distribution)
- Melitz: theta (shape parameter of Pareto distribution)
- 把trade flow写出来就知道

4. Trade Elasticity

$$X_{ij} = \frac{a_{ij}\tau_{ij}^{1-\sigma}\left(\frac{w_i}{A_i}\right)^{1-\sigma}}{\sum_k a_{kj}\tau_{kj}^{1-\sigma}\left(\frac{w_k}{A_k}\right)^{1-\sigma}} Y_j$$

$$X_{ij} = \frac{T_i(w_i\tau_{ij})^{-\theta}}{\sum_k T_k(w_k\tau_{kj})^{-\theta}} Y_j$$

$$X_{ij} = \frac{\overline{\alpha_i}^{-\theta} N_i w_i^{-\theta-\mu\left[\frac{\theta}{\sigma-1}-1\right]} \xi_{ij}^{-\left[\frac{\theta}{\sigma-1}-1\right]} \tau_{ij}^{-\theta}}{\sum_{i'=1}^{n} \overline{\alpha_{i'}}^{-\theta} N_{i'} w_{i'}^{-\theta-\mu\left[\frac{\theta}{\sigma-1}-1\right]} \xi_{i'j}^{-\left[\frac{\theta}{\sigma-1}-1\right]} \tau_{i'j}^{-\theta}} Y_j.$$

但怎么estimate才是更大的问题。

5. Intensive margin vs Extensive margin

- Chaney（2008）
- Firm export（intensive margin）

$$x_{ij}(\varphi) = \begin{cases} \lambda_3 \times \left(\frac{Y_j}{Y}\right)^{(\sigma-1)/\gamma} \times \left(\frac{\theta_j}{w_i \tau_{ij}}\right)^{\sigma-1} \times \varphi^{\sigma-1}, \text{ if } \varphi \geq \overline{\varphi}_{ij} \\ 0 \text{ otherwise,} \end{cases}$$

- Export

$$X_{ij}^h = \mu_h \times \frac{Y_i \times Y_j}{Y} \times \left(\frac{w_i \tau_{ij}^h}{\theta_j^h}\right)^{-\gamma_h} \times (f_{ij}^h)^{-[\gamma_h/(\sigma_h-1)-1]}.$$

Intensive margin vs Extensive margin

- Margins

$$\zeta \equiv -\frac{d \ln X_{ij}}{d \ln \tau_{ij}} = \underbrace{(\sigma - 1)}_{\substack{\text{Intensive margin} \\ \text{Elasticity}}} + \underbrace{(\gamma - (\sigma - 1))}_{\substack{\text{Extensive margin} \\ \text{Elasticity}}} = \gamma .$$

$$\xi \equiv -\frac{d \ln X_{ij}}{d \ln f_{ij}} = \underbrace{0}_{\substack{\text{Intensive margin} \\ \text{Elasticity}}} + \underbrace{\frac{\gamma}{\sigma - 1} - 1}_{\substack{\text{Extensive margin} \\ \text{Elasticity}}} = \frac{\gamma}{\sigma - 1} - 1.$$

Intensive margin vs Extensive margin

- “The main prediction of the model is that the extensive margin and the intensive margin are affected in opposite directions by the elasticity of substitution. If the elasticity of substitution is high, then the impact of trade barriers is strong on the intensive margin and mild on the extensive margin. The reverse holds true when the elasticity of substitution is low.”

6. Long term vs. Short term

- Dekle, Eaton and Kortum (2008)
  - Margins: “In either case we allow adjustment to take the form of changes in the range of goods that countries exchange (the extensive margin) as well as changes in the amounts of each good traded (the intensive margin). But adjustment at the extensive margin may take time. Hence, to capture very short run effects we consider a case in which both the allocation of labor and the extensive margin are fixed.”
- Extension Margin Operative

$$\pi'_{ni} = \frac{T_i \left(c'_i d_{ni}\right)^{-\theta}}{\sum_{k=1}^{N} T_k \left(c'_k d_{nk}\right)^{-\theta}} = \frac{T_i \left(c_i d_{ni}\right)^{-\theta} \widehat{c_i}^{-\theta}}{\sum_{k=1}^{N} T_k \left(c_k d_{nk}\right)^{-\theta} \widehat{c_k}^{-\theta}} = \frac{\overline{\pi}_{ni} \widehat{c_i}^{-\theta}}{\sum_{k=1}^{N} \overline{\pi}_{nk} \widehat{c_k}^{-\theta}}.$$

- Extension Margin Inoperative

$$\left(\pi_{ni}^{SR}\right)' = \frac{\overline{\pi}_{ni} \widehat{c_i}^{-(\sigma-1)}}{\sum_{k=1}^{N} \overline{\pi}_{nk} \widehat{c_k}^{-(\sigma-1)}}.$$

Long term vs. Short term

- Dekle, Eaton and Kortum (2008)
  - Labor mobility

> We consider the two extremes of internal labor market mobility. In the mobile labor case, which we take as reflecting the long run, the wage equilibrates between sectors, so that $w_i^M = w_i^N = w_i$ with L_i^M and L_i^N determined endogenously. In the immobile labor case, which we take as reflecting the short tun, workers are tied to either manufacturing or nonmanufacturing. For this case we take L_i^M and L_i^N as given and solve for w_i^M and w_i^N separately.

- Melitz: free entry (firm mass)?

7. Numbers of Equations/Unknowns

- Equal number of equations/unknowns (minimum level)
  - 可以使用显函数表示的变量?
  - EK: One set of equations/unknowns

$$w_i L_i = \sum_n \frac{T_i (w_i d_{ni})^{-\theta}}{\sum_j T_j (w_j d_{nj})^{-\theta}} w_n L_n$$

- Usually:
  - Goods/factor markets clear
  - Trade balance
- More to come:
  - FOC/budget constraints
  - Household's budget constraint implies trade balance
- Walras's Rule: -1

8. Numeraire/Normalization

- Numeraire: w1 or world total output
  - Different interpretations
- Normalization:
  - Productivities (A) matter up to scale, within sector!!
  - Trade costs(tau): tau_ii=1? (iceberg? Reducing tau increase world output?)
- EK: shape parameter of Frechet distribution
  - One for each sector (must be the same for all countries, unlike Pareto distribution in Melitz model)

9. Functional Forms

- EK: Frechet; Melitz: Pareto
  - 为什么用？：为什么想用？为什么能用？
  - 想用：为了方便，有助于构建analytical solution
  - 能用：Approximation to reality
    - 直接证据：firm productivity就是Pareto的？（Luttmer, 2007）
    - 间接证据：human talent是Frechet的？无法直接观测，咋办？
- Shape matters: tail
- Parameters matter!

10. Universal Gravity

- Gravity models长得都很像： Universal Gravity (Allen, Arkolakis and Takahashi, 2020)

- 各种models异同点
  - Armington model
  - EK model (Ricardian model, Jones model)
  - Krugman model (Armington assumption)
  - Melitz model
  - General neo-classical models （a la Adao et al, 2019）

10. Universal Gravity

- Xij和Pij
- Costinot and Rodriguez-Clare(2014)

$$P_{ij} = \underbrace{\tau_{ij} c_i^p}_{\text{Intensive Margin}} \times \underbrace{\left(\left(\frac{E_j}{c_{ij}^x} \right)^{\frac{\delta}{1-\sigma}} \frac{\tau_{ij} c_i^p}{P_j} \right)^{\eta}}_{\text{Extensive Margin: Selection}} \times \underbrace{\left(\frac{R_i}{c_i^e} \right)^{\frac{\delta}{1-\sigma}}}_{\text{Extensive Margin: Entry}} \times \xi_{ij},$$

- One sector case:

$$P_{ij} = \tau_{ij} Y_i \left(\left(\frac{E_j}{c_{ij}^x} \right)^{\frac{\delta}{1-\sigma}} \frac{\tau_{ij} Y_i}{P_j} \right)^{\eta} \xi_{ij}.$$

Universal gravity

- Arkolakis, Costinot, and Rodriguez-Clare(2013)

$$c_{ij}(w_i, t_j, \omega) \equiv \tau_{ij} \cdot w_i \cdot \alpha_{ij}(\omega) \cdot t_j^{1/(1-\sigma)},$$

$$f_{ij}(w_i, w_j, t_j, \omega) \equiv \xi_{ij} \cdot h_{ij}(w_i, w_j) \cdot \phi_{ij}(\omega) \cdot m_{ij}(t_j),$$

R1: *For any country* j, $\sum_{i=1}^{n} X_{ij} = \sum_{i=1}^{n} X_{ji}$.

R2: *For any country* j, Π_j / R_j *is constant.*

R3: *The import demand system is such that for any importer* j *and any pair of exporters* $i \neq j$ *and* $i' \neq j$, $\varepsilon_j^{ii'} = \varepsilon < 0$ *if* $i = i'$, *and zero otherwise.*

R3′: *The import demand system is such that for any exporter* i *and importer* j,

$$X_{ij} = \frac{\chi_{ij} \cdot N_i \cdot (w_i \tau_{ij})^{\varepsilon} \cdot Y_j}{\sum_{i'=1}^{n} \chi_{i'j} \cdot N_{i'} \cdot (w_{i'} \tau_{i'j})^{\varepsilon}},$$

where χ_{ij} *is a function of, and only of, structural parameters distinct from* $\boldsymbol{\tau}$.

- Arkolakis, Costinot, and Rodriguez-Clare(2013)

COROLLARY 1: *Suppose that Restrictions R1–R3 hold. Then the change in real income associated with moving to autarky in country j can be computed as*

$$\widehat{W}_j^A = \lambda_{jj}^{-1/\varepsilon}. \tag{10}$$

PROPOSITION 2: *Suppose that Restrictions R1–R3′ hold. Then the percentage change in real income associated with any change in variable trade costs in country j can be computed using equation (9) combined with*

$$\widehat{\lambda}_{jj} = \frac{1}{\sum_{i=1}^{n} \lambda_{ij} (\hat{w}_i \hat{\tau}_{ij})^{\varepsilon}}, \tag{11}$$

where $\hat{w}_j = 1$ *by choice of numeraire, and* $\{\hat{w}_i\}_{i \neq j}$ *are implicitly given by the solution of*

$$\hat{w}_i = \sum_{j'=1}^{n} \frac{\lambda_{ij'} \hat{w}_{j'} Y_{j'} (\hat{w}_i \hat{\tau}_{ij'})^{\varepsilon}}{Y_i \sum_{i'=1}^{n} \lambda_{i'j'} (\hat{w}_{i'} \hat{\tau}_{i'j'})^{\varepsilon}}. \tag{12}$$

- Allen, Arkolakis and Takahashi （2020）

Condition 1. The bilateral price is equal to the product of the output price and a bilateral scalar:

$$p_{ij} = p_i \tau_{ij}, \tag{8}$$

where, as above, $\{\tau_{ij}\}_{i,j \in S} \in \overline{\mathbb{R}}_{++}$ are referred to as *trade frictions*.[9]

Condition 2. (CES Aggregate Demand). There exists an exogenous (negative of the) *demand elasticity* $\phi \in \mathbb{R}$ such that the expenditure in location $j \in S$ can be written as:

$$E_j = \left(\sum_{i \in S} p_{ij}^{-\phi} \right)^{-\frac{1}{\phi}} W_j, \tag{9}$$

where W_j is the real expenditure and the associated price index is $P_j \equiv \left(\sum_{i \in S} p_{ij}^{-\phi} \right)^{-\frac{1}{\phi}}$.

- Allen, Arkolakis and Takahashi （2020）

Condition 3. (CES Aggregate Supply) There exists exogenous *supply shifters* $\{\bar{c}_i\} \in \mathbb{R}^N_{++}$, an exogenous *aggregate supply elasticity* $\psi \in \mathbb{R}$, and an endogenous scalar $\kappa > 0$ such that output in each location $i \in S$ can be written as: (11)

$$Q_i = \kappa \bar{c}_i \left(\frac{p_i}{P_i} \right)^{\psi}. \tag{11}$$

Condition 4. (Output market clearing). For all $i \in S$, $Q_i = \sum_{j \in S} \tau_{ij} Q_{ij}$.

- Allen, Arkolakis and Takahashi （2020）

Condition 5. (Exogenous deficits). For all $i \in S$, $E_i = \Xi \xi_i p_i Q_i$, where ξ_i is exogenous expenditure-output ratio for location i up to constant and Ξ is an endogenous scalar that ensures the world market clearing condition holds:

$$\Xi = \frac{\sum_i p_i Q_i}{\sum_i \xi_i p_i Q_i}. \tag{12}$$

Condition 6. World income equals to one:

$$\sum_i Y_i = 1. \tag{13}$$

Universal gravity

- Allen, Arkolakis and Takahashi （2020）
  - 解的存在性、唯一性
  - Local counterfactual
  - Applicable to many models (quantitative trade/spatial)

11. Quantitative Trade/Spatial vs. CGE

- Costinot and Rodriguez-Clair (2014)
  - First, new quantitative trade models have more appealing micro-theoretical foundations.
  - Second, recent quantitative papers offer a tighter connection between theory and data. Instead of relying on off-the-shelf elasticities, today's researchers try to use their own model to estimate the key structural parameters necessary for counterfactual analysis.
  - Third, new quantitative trade models put more emphasis on transparency and less emphasis on realism. The idea is to construct middle-sized models that are rich enough to speak to first-order features of the data, like the role of country size and geography, yet parsimonious enough so that one can credibly identify its key parameters and understand how their magnitude affects counterfactual analysis.

12. Structural vs. Reduced-form

- Structural Analysis
- Pro:
  - Micro-founded model (Lucas Critique): “in a 1976 paper, Lucas drove to the point that this simple notion invalidated policy advice based on conclusions drawn from large-scale macroeconometric models. Because the parameters of those models were not structural, i.e. not policy-invariant, they would necessarily change whenever policy (the rules of the game) was changed.”
  - Not local effect
  - General equilibrium effect
- Con: rest on (many) assumptions (u=u(C)? CD? CES? Competitive market?)
  - Not sure how correct they are…(但依然有机会对模型整体validity做test！)
  - Reduced-form analysis actually uses (even crazier?) assumptions: linearity/error term (Recall medal winners of mistakes in empirical international trade..)
    - 当然可以用来定性
  - Attempts to reduce assumptions: Rodrigo Adão

Structural vs. Reduced-form

- Structural Analysis: what-if question/post-policy evaluation
  - What-if scenarios
  - post-policy evaluation
- Structural Analysis: 看数字（定量）
- Reduced-form Analysis：经常是看星星（定性）？
  - R2? Quantitative implications?
  - Values of other related economic variables/coefficients?

13 Quantitative models: General comments

- 目的是定量
  - 木星风暴——大红斑直径最大时可以超过40000公里，甚至接近50000公里。而南北的直径，基本稳定在12000-14000公里之间。肆虐长达3个多世纪。
  - 如何解释？

Quantitative models: General comments

- 如何定量
  - 要求定的对!
    - 机制被广泛认可（但这种经常大佬就做了。。）
    - Identification as model？ （来自于模型以外的经济学家“智慧” 和共识）
  - 要求定的准!
    - 在人力可及的范围内尽可能准确（how?）
    - Connect heavily with (new!) microdata （对比各种宏观puzzle）
    - 很多的robustness
  - 要求科学性、标准性，结果可积累、可比较

14 Quantitative models: Practice

- 主要工作量：写model，估参数，编程序
  - 写model是最简单的。。（本质上常用的只有两个model）
    - 基础Model只需要5分钟
    - 写新model需要一辈子？：EK先是研究innovation和growth，发现innovation是pareto的话那么productivity是frechet的，然后把这个放到了trade里面
    - AA：homogeneous Hammerstein equation of the second kind???
  - 各种细节会花掉主要的时间（20页3天，2段话3个月）
- 初学者：
  - 得试一下，replicate经典论文
  - Get your hands dirty！ = 撸起袖子加油干