Nonparametric Counterfactual Predictions

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July, 2021

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    - E.g. #1: Old CGE: GTAP model [13,000 structural parameters]
    - E.g. #2: New CGE: EK model [1 key parameter]

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    - Eg.1: Old CGE: GTAP model [13,000 structural parameters]
    - E.g.2: New CGE: EK model [1 key parameter]
- Strong functional form assumptions may hinder the credibility of counterfactual predictions. Parametric assumptions on distribution of firm heterogeneity restrict aggregate predictions of the model

- Adao R., Costinot, A., Donalson, D., 2017, “Nonparametric Counterfactual Predictions in Neoclassical Models of International Trade”, American Economic Review, 107(3): 633-689.

- Adao, R., Arkolakis, C., Ganapati, S., 2021, “Not-parametric Gravity: Measuring the Macroeconomic Implications of Firm Heterogeneity”, Working Paper.

- Adao R., Costinot, A., Donalson, D., 2017, "Nonparametric Counterfactual Predictions in Neoclassical Models of International Trade", American Economic Review, 107(3): 633-689.

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    - Arkolakis, Costinot, and Rodriguez-Clare (2012): welfare gains
    - Head and Ries (2001): trade costs

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4. Empirical application: What was the impact of China's integration into the world economy in the past two decades?
   - Departures from CES modeled in the spirit of BLP (1995)

Related Literature

- **GE Theory and Trade:**
  - Taylor (1938); Rader (1972); Mas-Colell (1991); Meade (1952); Helpman (1976); Wilson (1980); Neary and Schweinberger (1986)
- **IO and Trade:**
  - Berry, Levinsohn and Pakes (1995); Nevo (2011); Berry, Gandhi and Haile (2013); Berry and Haile (2014)
- **Bridge within Trade:**
  - *Neoclassical:* Dixit and Norman (1980); Bowen, Leamer, and Sveikauskas (1987); Deardorff and Staiger (1988); Trefler (1993, 1995); Davis and Weinstein (2001); Burstein and Vogel (2011)
  - *Gravity:* Eaton and Kortum (2002); Anderson and van Wincoop (2003); handbook chapters of Costinot and Rodriguez-Clare (2013) and Head and Mayer (2013)

Outline of the paper

1. Introduction
2. Neoclassical trade models as factor exchange models
3. Counterfactual and welfare analysis
4. Identification
5. Estimation
6. Application: China's Integration in the World Economy

Neoclassical Trade Model

- $i = 1, ..., I$ countries
- $k = 1, ..., K$ goods
- $n = 1, ..., N$ factors
- Goods consumed in country i:

$$\boldsymbol{q_i} \equiv \{q_{ji}^k\}$$

- Factors used in country i to produce good k for country j:

$$\boldsymbol{l_{ij}^k} \equiv \{l_{ij}^{nk}\}$$

Neoclassical Trade Model

- Preferences:

$$u_i = u_i(\boldsymbol{q_i})$$

- Technology:

$$q_{ij}^k = f_{ij}^k(\boldsymbol{l_{ij}^k})$$

- Factor endowments:

$$\nu_i^n \; > 0$$

Competitive Equilibrium

A $\boldsymbol{q} \equiv \{\boldsymbol{q_i}\}$, $\boldsymbol{l} \equiv \{\boldsymbol{l_i}\}$, $\boldsymbol{p} \equiv \{\boldsymbol{p_i}\}$, and $\boldsymbol{w} \equiv \{\boldsymbol{w_i}\}$ such that:

1. Consumers maximize their utility:

$$\boldsymbol{q_i} \in \operatorname{argmax}_{\tilde{\boldsymbol{q}}_{\boldsymbol{i}}} u_i(\tilde{\boldsymbol{q}}_{\boldsymbol{i}})$$

$$\sum_{j,k} p_{ji}^k \tilde{q}_{ji}^k \leq \sum_n w_i^n \nu_i^n \text{ for all } i;$$

2. Firms maximize their profits:

$$\boldsymbol{l_{ij}^k} \in \operatorname{argmax}_{\tilde{\boldsymbol{l}}_{\boldsymbol{ij}}^{\boldsymbol{k}}} \{p_{ij}^k f_{ij}^k(\tilde{\boldsymbol{l}}_{\boldsymbol{ij}}^{\boldsymbol{k}}) - \sum_n w_i^n \tilde{l}_{ij}^{nk}\} \text{ for all } i,\ j, \text{ and } k;$$

3. Goods markets clear:

$$q_{ij}^k = f_{ij}^k(\boldsymbol{l_{ij}^k}) \text{ for all } i,\ j, \text{ and } k;$$

4. Factors markets clear:

$$\sum_{j,k} l_{ij}^{nk} = \nu_i^n \text{ for all } i \text{ and } n.$$

Reduced Exchange Model

- Fictitious endowment economy in which consumers directly exchange factor services
  - Taylor (1938), Rader (1972), Wilson (1980), Mas-Colell (1991)

- *Reduced preferences* over primary factors of production:

$$U_i(\boldsymbol{L_i}) \equiv \max_{\tilde{\boldsymbol{q}}_{\boldsymbol{i}}, \tilde{\boldsymbol{l}}_{\boldsymbol{i}}} u_i(\tilde{\boldsymbol{q}}_{\boldsymbol{i}})$$
$$\tilde{q}_{ji}^k \leq f_{ji}^k(\tilde{\boldsymbol{l}}_{\boldsymbol{ji}}^{\boldsymbol{k}}) \text{ for all } j \text{ and } k,$$
$$\sum_k \tilde{l}_{ji}^{nk} \leq L_{ji}^n \text{ for all } j \text{ and } n,$$

Reduced Equilibrium

Corresponds to $\boldsymbol{L} \equiv \{\boldsymbol{L_i}\}$ and $\boldsymbol{w} \equiv \{\boldsymbol{w_i}\}$ such that:

1. Consumers maximize their reduced utility:

$$\boldsymbol{L_i} \in \operatorname{argmax}_{\tilde{\boldsymbol{L}}_i} U_i(\tilde{\boldsymbol{L}}_i)$$
$$\sum_{j,n} w_j^n \tilde{L}_{ji}^n \leq \sum_n w_i^n \nu_i^n \text{ for all } i;$$

2. Factor markets clear:

$$\sum_j L_{ij}^n = \nu_i^n \text{ for all } i \text{ and } n.$$

Equivalence

- **Proposition 1**: *For any competitive equilibrium,* $(\boldsymbol{q}, \boldsymbol{l}, \boldsymbol{p}, \boldsymbol{w})$, *there exists a reduced equilibrium,* $(\boldsymbol{L}, \boldsymbol{w})$, *with:*
  1. *the same factor prices,* $\boldsymbol{w}$;
  2. *the same factor content of trade,* $L_{ji}^{n} = \sum_k l_{ji}^{nk}$ *for all* i, j, *and* n;
  3. *the same welfare levels,* $U_i(\boldsymbol{L_i}) = u_i(\boldsymbol{q_i})$ *for all* i.

  Conversely, for any reduced equilibrium, $(\boldsymbol{L}, \boldsymbol{w})$, *there exists a competitive equilibrium,* $(\boldsymbol{q}, \boldsymbol{l}, \boldsymbol{p}, \boldsymbol{w})$, *such that 1-3 hold.*

Equivalence

- **Comments:**
  - Proof is similar to First and Second Welfare Theorems.
  - Key implication of Prop. 1: If one is interested in the factor content of trade, factor prices and/or welfare, then one can always study a RE instead of a CE. One doesn't need *direct* knowledge of primitives u and f but only of how these *indirectly* shape U.

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Reduced Counterfactuals

- Suppose that the reduced utility function over primary factors in this economy can be parametrized as

$$U_i(\boldsymbol{L_i}) \equiv \bar{U}_i(\{L^n_{ji}/\tau^n_{ji}\}),$$

where $\tau^n_{ji} > 0$ are exogenous preference shocks

- **Counterfactual question:** *What are the effects of a change from $(\boldsymbol{\tau}, \boldsymbol{\nu})$ to $(\boldsymbol{\tau'}, \boldsymbol{\nu'})$ on trade flows, factor prices, and welfare?*

Reduced Factor Demand System

- Start from factor demand = solution of reduced UMP:

$$\boldsymbol{L_i}(\boldsymbol{w}, y_i | \boldsymbol{\tau_i})$$

- Compute associated expenditure shares:

$$\boldsymbol{\chi_i}(\boldsymbol{w}, y_i | \boldsymbol{\tau_i}) \equiv \{\{x_{ji}^n\} | x_{ji}^n = w_j^n L_{ji}^n / y_i \text{ for some } \boldsymbol{L_i} \in \boldsymbol{L_i}(\boldsymbol{w}, y_i | \boldsymbol{\tau_i})\}$$

- Rearrange in terms of *effective factor prices*, $\boldsymbol{\omega_i} \equiv \{w_j^n \tau_{ji}^n\}$:

$$\boldsymbol{\chi_i}(\boldsymbol{w}, y_i | \boldsymbol{\tau_i}) \equiv \boldsymbol{\chi_i}(\boldsymbol{\omega_i}, y_i)$$

Reduced Equilibrium

- RE:

$$\boldsymbol{x_i} \in \boldsymbol{\chi_i}(\boldsymbol{\omega_i}, y_i), \text{ for all } i,$$
$$\sum_j x_{ij}^n y_j = y_i^n, \text{ for all } i \text{ and } n$$

Reduced Equilibrium

- RE:

$$\boldsymbol{x_i} \in \boldsymbol{\chi_i}(\boldsymbol{\omega_i}, y_i), \text{ for all } i,$$
$$\sum_j x_{ij}^n y_j = y_i^n, \text{ for all } i \text{ and } n$$

- **Gravity model:** Reduced factor demand system is CES

$$\chi_{ji}(\boldsymbol{\omega_i}, y_i) = \frac{(\omega_{ji})^\epsilon}{\sum_l (\omega_{li})^\epsilon}, \text{ for all } j \text{ and } i$$

Exact Hat Algebra

- Start from the counterfactual equilibrium:

$$\boldsymbol{x_i'} \in \boldsymbol{\chi_i}(\boldsymbol{\omega_i'}, y_i') \text{ for all } i,$$
$$\sum_j (x_{ij}^n)' y_j' = (y_i^n)', \text{ for all } i \text{ and } n.$$

Exact Hat Algebra

- Start from the counterfactual equilibrium:

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$$\sum_j (x_{ij}^n)' y_j' = (y_i^n)', \text{ for all } i \text{ and } n.$$

- Rearrange in terms of proportional changes:

$$\{\hat{x}_{ji}^n x_{ji}^n\} \in \boldsymbol{\chi_i}(\{\hat{w}_j^n \hat{\tau}_{ji}^n \omega_{ji}^n\}, \sum_n \hat{w}_i^n \hat{\nu}_i^n y_i^n) \text{ for all } i,$$
$$\sum_j \hat{x}_{ij}^n x_{ij}^n (\sum_n \hat{w}_j^n \hat{\nu}_j^n y_j^n) = \hat{w}_i^n \hat{\nu}_i^n y_i^n, \text{ for all } i \text{ and } n.$$

Counterfactual Trade Flows and Factor Prices

- Wlog, pick location of preference shocks so that effective factor prices in the initial equilibrium are equal to one in all countries,

$$\omega_{ji}^{n} = 1, \text{ for all } i, j, \text{ and } n.$$

Counterfactual Trade Flows and Factor Prices

- **Proposition 2** *Under A1, proportional changes in expenditure shares and factor prices,* $\hat{\boldsymbol{x}}$ *and* $\hat{\boldsymbol{w}}$*, caused by proportional changes in preferences and endowments,* $\hat{\boldsymbol{\tau}}$ *and* $\hat{\boldsymbol{\nu}}$*, solve*

$$\{\hat{x}_{ji}^n x_{ji}^n\} \in \boldsymbol{\chi_i}(\{\hat{w}_j^n \hat{\tau}_{ji}^n \omega_{ji}^n\}, \sum_n \hat{w}_i^n \hat{\nu}_i^n y_i^n) \ \forall \ i,$$

$$\sum_j \hat{x}_{ij}^n x_{ij}^n (\sum_n \hat{w}_j^n \hat{\nu}_j^n y_j^n) = \hat{w}_i^n \hat{\nu}_i^n y_i^n \ \forall \ i \text{ and } n.$$

Welfare

- Equivalent variation for country i associated with change from $(\boldsymbol{\tau}, \boldsymbol{\nu})$ to $(\boldsymbol{\tau'}, \boldsymbol{\nu'})$, expressed as fraction of initial income:

$$\Delta W_i = (e_i(\boldsymbol{\omega_i}, U_i')) - y_i)/y_i,$$

with U_i' = counterfactual utility and e_i = expenditure function,

$$e_i(\boldsymbol{\omega_i}, U_i') \equiv \min_{\tilde{\boldsymbol{L}}_i} \sum \omega_{ji}^n L_{ji}^n$$
$$\bar{U}_i(\tilde{\boldsymbol{L}}_i) \geq U_i'.$$

Integrating Below Factor Demand Curves

- To go from $\boldsymbol{\chi_i}$ to ΔW_i, solve system of ODEs
- For any selection $\{x_{ji}^n(\boldsymbol{\omega}, y)\} \in \boldsymbol{\chi_i}(\boldsymbol{\omega}, y)$, Envelope Theorem:

$$\frac{d \ln e_i(\boldsymbol{\omega}, U_i')}{d \ln \omega_j^n} = x_{ji}^n(\boldsymbol{\omega}, e_i(\boldsymbol{\omega}, U_i')) \text{ for all } j \text{ and } n. \tag{1}$$

- Budget balance in the counterfactual equilibrium

$$e_i(\boldsymbol{\omega_i'}, U_i') = y_i'. \tag{2}$$

Counterfactual Welfare Changes

- **Proposition 3** *Under A1, equivalent variation associated with change from* $(\boldsymbol{\tau}, \boldsymbol{\nu})$ *to* $(\boldsymbol{\tau}', \boldsymbol{\nu}')$ *is*

$$\Delta W_i = (e(\boldsymbol{\omega}_i, U_i') - y_i)/y_i,$$

where $e(\cdot, U_i')$ *is the unique solution of (1) and (2).*

Application to Neoclassical Trade Models

- Suppose that technology in neoclassical trade model satisfies:

$$f_{ij}^{k}(\boldsymbol{l_{ij}^{k}}) \equiv \bar{f}_{ij}^{k}(\{l_{ij}^{nk}/\tau_{ij}^{n}\}), \text{ for all } i, j, \text{ and } k,$$

- Reduced utility function over primary factors of production:

$$\begin{aligned} U_i(\boldsymbol{L_i}) \equiv \max\nolimits_{\boldsymbol{\tilde{q}_i}, \boldsymbol{\tilde{l}_i}} & \, u_i(\boldsymbol{\tilde{q}_i}) \\ & \tilde{q}_{ji}^{k} \leq \bar{f}_{ji}^{k}(\{\tilde{l}_{ji}^{nk}/\tau_{ji}^{n}\}) \text{ for all } j \text{ and } k, \\ & \sum_{k} \tilde{l}_{ji}^{nk} \leq L_{ji}^{n} \text{ for all } j \text{ and } n. \end{aligned}$$

- Change of variable: $U_i(L_i) \equiv \bar{U}_i(\{L_{ji}^{n}/\tau_{ji}^{n}\}) \Rightarrow$ factor-augmenting productivity shocks in CE = preference shocks in RE

Taking Stock

- Propositions 2 and 3 provide a system of equations that can be used for counterfactual and welfare analysis in RF economy.
  - Proposition 1 $\Rightarrow$ same system can be used in neoclassical economy.

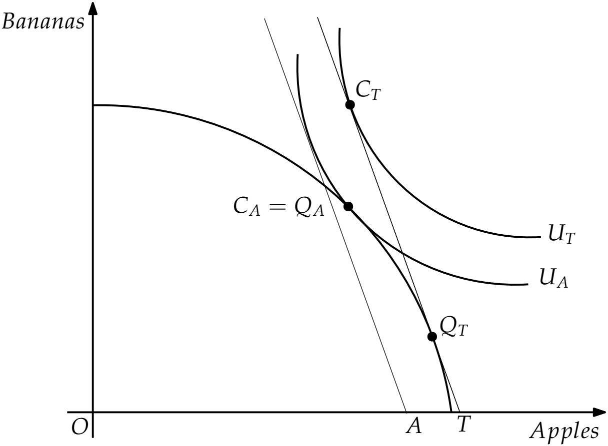
- Given data on expenditure shares and factor payments, $\{x_{ji}^n, y_i^n\}$, if one knows factor demand system, $\boldsymbol{\chi_i}$, then one can compute counterfactual factor prices, aggregate trade flows, and welfare.

Valuation of the Gains from Trade

- Two equilibria: Trade (T) and Autarky (A)
- Prices: p_T and p_A
- Utility: U_T and U_A
- Gains from Trade (GT) = welfare cost of autarky = money that country would be willing to pay to avoid going from T to A
- Expressed as a fraction of initial GDP:

$$GT = 1 - \frac{e(p_T, U_A)}{e(p_T, U_T)}$$

Back to The Textbook Approach



Can We Just Scale It Up?

- In practice, countries produce and consumer MANY goods
  - US has positive exports in 8,500 HS-10 digit product categories
  - plenty of product differentiation even within these categories

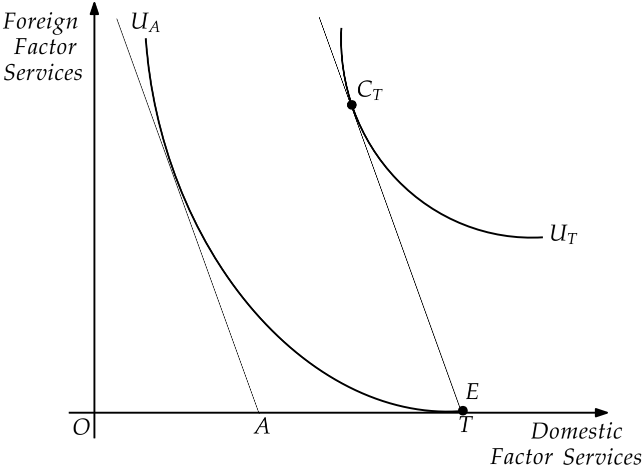
- Strategy to estimate GT:
  - Estimate production sets and indifference curves around the world
  - Compute counterfactual autarky equilibrium
  - Solve for p_A and U_A
  - Use previous formula

- Scaling up the textbook approach requires A LOT of information

The Factor Approach

- We can apply ACD's approach to valuation of GT
  - Instead of estimating production and demand functions around the world ...
  - ... we need to estimate reduced factor demand = demand for factor services embodied in goods purchased around the world

The Factor Approach



Parallel with New Good Problem

- Parallel between valuation of GT and "new good" problem in IO
- In order to evaluate the welfare gains from the introduction of a new product (e.g. Apple Cinnamon Cheerios, minivan), we can:
  - Estimate the demand for such products
  - Determine the reservation price at which demand would be zero
  - Measure consumer surplus by looking at the area under the (compensated) demand curve
- We can follow a similar strategy to measure GT:
  - foreign factor services are just like new products that appear when trade is free, but disappear under autarky

From Factor Demand to GT

- Recall definition of expenditure function:

$$e(p, U) = \min_{\{c_i\}} \{ \sum_i p_i c_i | u(\{c_i\}) \geq U \}$$

- Assume one domestic factor (numeraire) and one foreign factor (p)
- Envelope Theorem (Shepard's Lemma in this context) implies:

$$de(p, U) = q_F dp$$

$$\Longleftrightarrow d \ln e(p, U) = \frac{p q_F}{e(p, U)} d \ln p = \lambda_F(\ln p, U) d \ln p$$

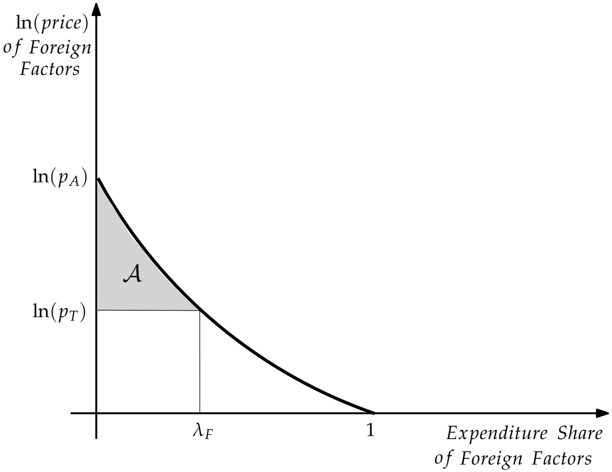
- Integrating between $\ln p_T$ and $\ln p_A$ for $U = U_A$:

$$\ln e(p_A, U_A) - \ln e(p_T, U_A) = \int_{\ln p_T}^{\ln p_A} \lambda_F(x, U_A) dx \equiv \mathcal{A}$$

- Noting that $e(p_A, U_A) = e(p_T, U_T)$

$$GT = 1 - \exp(-\mathcal{A})$$

Integrating Below the (Compensated) Demand Curve



CES Example

- Suppose that factor demand is CES, as in ACR

$$\lambda_F(\ln p, U) = \frac{\exp(-\varepsilon \ln p)}{1 + \exp(-\varepsilon \ln p)}$$

- This leads to

$$\mathcal{A} = \int_{\ln p_T}^{\infty} \frac{\exp(-\varepsilon x)}{1 + \exp(-\varepsilon x)} dx = \frac{\ln(1 + p_T^{-\varepsilon})}{\varepsilon}$$

- Since CES demand system is invertible, we can also express relative price of foreign factor services as a function of initial expenditure share

$$\lambda_F = \frac{p_T^{-\varepsilon}}{1 + p_T^{-\varepsilon}} \Longleftrightarrow 1 + p_T^{-\varepsilon} = \frac{1}{1 - \lambda_F}$$

- Combining theprevious expressions, we get

$$GT = 1 - \exp\left(\frac{\ln(1 - \lambda_F)}{\varepsilon}\right) = 1 - \lambda_D^{1/\varepsilon}$$

Take-Away From the Previous Formula

- CES/ACR formula captures the 2 key issues for valuation of GT:
  1. How large are imports of factor services in the current trade equilibrium?
  2. How elastic is the demand for these imported services along the path from trade to autarky?

- **Basic idea:** If we do not trade much or if the factor services that we import are close substitutes to domestic ones, then small GT

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Econometric Model

- Unobservable

$$\omega_{i,t} \equiv \left\{\omega^n_{ji,t}\right\},\ \omega^n_{ji,t_0} = 1, \quad \text{for all } i, j, \text{ and } n$$

- Observables
  - factor expenditure share $\quad \mathbf{x}_{i,t} \equiv \left\{x^n_{ji,t}\right\}$
  - factor payments $\quad \mathbf{y}_{i,t} \equiv \left\{y^n_{i,t}\right\}$
  - trade cost shifter $\quad \mathbf{z}^{\tau}_{i,t} \equiv \left\{\left(z^{\tau}\right)^n_{ji,t}\right\}$
  - income shifter $\quad z^y_{i,t}$

- Relation between trade cost shock and trade cost shifter

$$\ln \tau^n_{ji,t} = \ln\left(z^{\tau}\right)^n_{ji,t} + \varphi^n_{ji} + \tilde{\xi}^n_{j,t} + \eta^n_{ji,t}$$

$$\ln \omega^n_{ji,t} = \ln\left(z^{\tau}\right)^n_{ji,t} + \varphi^n_{ji} + \xi^n_{j,t} + \eta^n_{ji,t}\ , \text{ for all } i, j, n, \text{ and } t$$

Assumptions for identification

ASSUMPTION A1 (Exogeneity): $E\left[\eta^n_{ji,t} \mid \mathbf{z}_t\right] = 0$, with $\mathbf{z}_t \equiv \left\{\mathbf{z}^\tau_{l,t}, z^y_{l,t}\right\}$ the vector of all instruments in period t.

ASSUMPTION A2 (Completeness): For any importer pair (i_1, i_2), and any function $g\left(\mathbf{x}_{i_1,t}, y_{i_1,t}, \mathbf{x}_{i_2,t}, y_{i_2,t}\right)$ with finite expectation, $E\left[g\left(\mathbf{x}_{i_1,t}, y_{i_1,t}, \mathbf{x}_{i_2,t}, y_{i_2,t}\right) \mid \mathbf{z}_t\right] = 0$ implies $g\left(\mathbf{x}_{i_1,t}, y_{i_1,t}, \mathbf{x}_{i_2,t}, y_{i_2,t}\right) = 0$

ASSUMPTION A3 (Invertibility): In any country i, for any observed expenditure shares, $\mathbf{x} > \mathbf{0}$, and any observed income level, $y > 0$, there exists a unique vector of relative effective factor prices, $(\chi_i)^{-1}(\mathbf{x}, y)$, such that all $\boldsymbol{\omega}_i$ satisfying $\mathbf{x} \in \mathbf{X}_i(\boldsymbol{\omega}_i, y)$ also satisfy $\omega^n_{ji}/\omega^1_{1i} = \left(\chi^n_{ji}\right)^{-1}(\mathbf{x}, y)$

Identification of Invertible Demand Systems

$$ln\omega^n_{ji,t} = ln(z^\tau)^n_{ji,t} + \varphi^n_{j,i} + \xi^n_{j,t} + \eta^n_{ji,t} \quad \Rightarrow$$

$$\eta^n_{ji,t} = ln(x^n_{ji})^{-1}(x_{i,t}, y_{i.t}) - ln(z^\tau)^n_{ji,t} - \varphi^n_{j,i} + \xi^n_{j,t} \quad (1)$$

$$\eta^1_{1i,t} = ln(x^1_{1i})^{-1}(x_{i,t}, y_{i.t}) - ln(z^\tau)^1_{1i,t} - \varphi^1_{1,i} + \xi^1_{1,t} \quad (2)$$

$$\Downarrow$$

$$\Delta\eta^n_{ji,t} = ln(x^n_{ji})^{-1}(x_{i,t}, y_{i,t}) - \Delta ln(z^\tau)^n_{ji,t} - \Delta\varphi^n_{j,i} + \Delta\xi^n_{j,t} \quad \Rightarrow$$

$$\Delta\eta^n_{ji_1,t} - \Delta\eta^n_{ji_2,t} = ln(x^n_{ji_1})^{-1}(x_{i_1,t}, y_{i_1,t}) - ln(x^n_{ji_2})^{-1}(x_{i_2,t}, y_{i_2,t})$$

$$-(\Delta ln(z^\tau)^n_{ji_1,t} - \Delta ln(z^\tau)^n_{ji_2,t}) - (\Delta\varphi^n_{j,i_1} - \Delta\varphi^n_{j,i_2})$$

Identification of Invertible Demand Systems

Under Assumption A1, this leads to the following moment condition:

$$E\left[\ln\left(\chi^n_{ji_1}\right)^{-1}(\mathbf{x}_{i_1,t}, y_{i_1,t}) - \ln\left(\chi^n_{ji_2}\right)^{-1}(\mathbf{x}_{i_2,t}, y_{i_2,t}) - \zeta^n_{ji_1 i_2} \mid \mathbf{z}_t\right]$$

$$= \Delta \ln\left(z^\tau\right)^n_{ji_1,t} - \Delta \ln\left(z^\tau\right)^n_{ji_2,t}$$

Suppose exist $\left(\left(\chi^n_{ji_1}\right)^{-1}, \left(\chi^n_{ji_2}\right)^{-1}, \zeta^n_{ji_1 i_2}\right)$ and $\left(\left(\tilde{\chi}^n_{ji_1}\right)^{-1}, \left(\tilde{\chi}^n_{ji_2}\right)^{-1}\ \tilde{\zeta}^n_{ji_1 i_2}\right)$, then:

$$E\left[\ln\left(\chi^n_{ji_1}\right)^{-1}(\mathbf{x}_{i_1,t}, y_{i_1,t}) - \ln\left(\tilde{\chi}^n_{ji_1}\right)^{-1}(\mathbf{x}_{i_1,t}, y_{i_1,t}) - \ln\left(\chi^n_{ji_2}\right)^{-1}(\mathbf{x}_{i_2,t}, y_{i_2,t})\right.$$

$$\left. + \ln\left(\tilde{\chi}^n_{ji_2}\right)^{-1}(\mathbf{x}_{i_2,t}, y_{i_2,t}) - \zeta^n_{ji_1 i_2} + \tilde{\zeta}^n_{ji_1 i_2} \mid \mathbf{z}_t\right] = 0$$

Under Assumption A2, this requires:

$$\ln\left(\chi^n_{ji_1}\right)^{-1}(\mathbf{x}_{i_1,t}, y_{i_1,t}) - \ln\left(\tilde{\chi}^n_{ji_1}\right)^{-1}(\mathbf{x}_{i_1,t}, y_{i_1,t})$$

$$= \ln\left(\chi^n_{ji_2}\right)^{-1}(\mathbf{x}_{i_2,t}, y_{i_2,t}) - \ln\left(\tilde{\chi}^n_{ji_2}\right)^{-1}(\mathbf{x}_{i_2,t}, y_{i_2,t}) + \zeta^n_{ji_1 i_2} - \tilde{\zeta}^n_{ji_1 i_2}$$

Proposition 4: Suppose that Assumptions A1-A3 hold. Then factor demand and relative effective factor prices are identified.

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Assumptions on Factor Demand System

1. Preferences are homothetic
2. All goods have the same factor intensity in each country
3. Cross-country differences in factor demand can be reduced to differences in time-varving effective factor prices and time-invariant shifters $\chi_i(\omega_{i,t}) = \chi(\{\mu_{ji}\omega_{ji,t}\})$
4. Function form featured by mixed CES :

$$\chi_{ji}(\boldsymbol{\omega}_{i,t}) = \int \frac{(\kappa_j)^{\sigma_\alpha \alpha} (\mu_{ji}\omega_{ji,t})^{-(\bar{\epsilon}\cdot\epsilon^{\sigma_\epsilon})}}{\sum_{l=1}^{N} (\kappa_l)^{\sigma_\alpha \alpha} (\mu_{li}\omega_{li,t})^{-(\bar{\epsilon}\cdot\epsilon^{\sigma_\epsilon})}} dF(\alpha, \epsilon)$$

Features of the Factor Demand System

- It nests the case of CES demand
- It is invertible
- It captures flexibility and parsimoniously a number of natural features of demand substitution patterns through the structural paramete σ_α and σ_ϵ

$$\frac{\partial \ln\left(\frac{\chi_{ji}(\omega_{i,t})}{\chi_{ri}(\omega_{i,t})}\right)}{\partial \ln\left(\frac{\omega_{i,t}}{\omega_{r,t}}\right)} = \int \left(\bar{\epsilon} \cdot \epsilon^{\sigma_\epsilon}\right) \left(\frac{x_{ji,t}(\alpha,\epsilon)}{\chi_{ji(\omega_{i,t})}} - \frac{x_{ri,t}(\alpha,\epsilon)}{\chi_{ri(\omega_{i,t})}}\right) x_{li,t}(\alpha,\epsilon)\, dF(\alpha,\epsilon)$$

Estimation Procedure

Given: $\boldsymbol{\chi_i}\left(\boldsymbol{\omega_{i,t}}\right) = \boldsymbol{\chi}\left(\left\{\boldsymbol{\mu_{ji}\omega_{ji,t}}\right\}\right)$

$$\Delta\eta_{ji,t} - \Delta\eta_{j1,t} = \ln\left(\chi_{ji}\right)^{-1}\left(\mathbf{x}_{i,t}\right) - \ln\left(\chi_{j1}\right)^{-1}\left(\mathbf{x}_{1,t}\right)$$

$$-\left(\Delta\ln\left(z^{\tau}\right)_{ji,t} - \Delta\ln\left(z^{\tau}\right)_{j1,t}\right) - \left(\Delta\varphi_{ji} - \Delta\varphi_{j1}\right)$$

$$\Downarrow$$

$$\Delta\eta_{ji,t} - \Delta\eta_{j1,t} = \ln\chi_j^{-1}\left(\mathbf{x}_{i,t}\right) - \ln\chi_j^{-1}\left(\mathbf{x}_{1,t}\right)$$

$$-\left(\Delta\ln\left(z^{\tau}\right)_{ji,t} - \Delta\ln\left(z^{\tau}\right)_{j1,t}\right) + \zeta_{ji}$$

with $\zeta_{ji} \equiv -\left(\Delta\varphi_{ji} - \Delta\varphi_{j1}\right) - \left(\Delta\ln\mu_{ji} - \Delta\ln\mu_{j1}\right)$

Estimation Procedure

$$e_{ji,t}(\boldsymbol{\theta}) \equiv \ln \chi_j^{-1}\left(\mathbf{x}_{i,t}\right) - \ln \chi_j^{-1}\left(\mathbf{x}_{1,t}\right) - \left(\Delta \ln\left(z^{\tau}\right)_{ji,t} - \Delta \ln\left(z^{\tau}\right)_{j1,t}\right) + \zeta_{ji}$$

$$E\left(\left(\Delta\eta_{ji,t} - \Delta\eta_{j1,t}\right)\mathbf{Z}_{ji,t}\right) = 0$$

- Construct a consistent GMM estimator of by solving for

$$\hat{\boldsymbol{\theta}} = \arg\min_{\boldsymbol{\theta}} \mathbf{e}(\boldsymbol{\theta})'\mathbf{Z}\Phi\mathbf{Z}'\mathbf{e}(\boldsymbol{\theta})$$

Estimation Results

TABLE 2—GMM ESTIMATES OF MIXED CES DEMAND

	$\bar{\epsilon}$	σ_α	σ_ϵ
Panel A. CES			
	−5.955		
	(0.950)		
Panel B. Mixed CES (restricted heterogeneity)			
	−6.115	2.075	
	(0.918)	(0.817)	
Panel C. Mixed CES (unrestricted heterogeneity)			
	−6.116	2.063	0.003
	(0.948)	(0.916)	(0.248)

- Adao R., Costinot, A., Donalson, D., 2017, “Nonparametric Counterfactual Predictions in Neoclassical Models of International Trade”, American Economic Review, 107(3): 633-689.

- Adao, R., Arkolakis, C., Ganapati, S., 2021, “Not-parametric Gravity: Measuring the Macroeconomic Implications of Firm Heterogeneity”, Working Paper.

What are the aggregate implications of firm heterogeneity?

- The study of firm heterogeneity has transformed the trade field
  - Cornerstone observation: Correlation between firm attributes and trade performance
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- This paper: Firm heterogeneity **without parametric restrictions**
  - Theoretically and empirically characterize role of firm heterogeneity for aggregate outcomes
  - Nonparametric counterfactuals (& inversion of fundamentals) and semiparametric estimation

Methodology: Nonparametric Counterfactuals/Semiparametric Estimation

- Start with workhorse monopolistic competition model with
  - i) CES preferences ii) multiple sources of **flexible** heterogeneity (extend Melitz)

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Outline

- **Workhorse model of firm heterogeneity**
- Semiparametric gravity equations for firm exports
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- Empirical results
- Quantifying the Gains from Trade

Workhorse model of firm heterogeneity: Setup

- N locations (denote i the origin j the destination)
- Monopolistic competitive firms
  - Firms are unique world monopolists, each producing one variety ω
  - Linear production function and iceberg shipping. Fixed cost of selling to each market
- Consumers
  - CES Preferences

Firm Revenue and Cost

- Firm ω's demand is

$$R_{ij}(\omega) = \underbrace{\bar{b}_{ij} b_{ij}(\omega)}_{\text{Firm taste shifter}} \underbrace{(p_{ij}(\omega))^{1-\sigma}}_{\text{Firm price}} \left[E_j P_j^{\sigma-1}\right]$$

where E_j is spending and P_j is CES price index over available varieties, Ω_{ij}

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where E_j is spending and P_j is CES price index over available varieties, Ω_{ij}

- The cost of firm ω from i to sell q units in j

$$C_{ij}(q,\omega) = \underbrace{\frac{\tau_{ij}(\omega)}{a_i(\omega)} \frac{\bar{\tau}_{ij}}{\bar{a}_i} w_i}_{\text{Firm variable cost in } j} q + \underbrace{f_{ij}(\omega) \bar{f}_{ij} w_i}_{\text{Firm fixed cost in } j}$$

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Firm-specific revenue and entry potentials

- In monopolistic competition with CES, constant markup. Revenue:

$$R_{ij}(\omega) = \underbrace{\left[b_{ij}(\omega) \left(\frac{\tau_{ij}(\omega)}{a_i(\omega)} \right)^{1-\sigma} \right]}_{\text{Revenue potential, } r_{ij}(\omega)} \underbrace{\left[\left(\frac{\sigma}{\sigma - 1} \frac{\bar{\tau}_{ij}}{\bar{a}_i} \right)^{1-\sigma} \bar{b}_{ij} \right]}_{\text{Bilateral shifter,} \bar{r}_{ij}} \left[\left(\frac{w_i}{P_j} \right)^{1-\sigma} E_j \right]$$

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- Firm ω of i enters j (i.e., $\omega \in \Omega_{ij}$) if, and only if, $\pi_{ij}(\omega) \geq 0$. So,

$$\underbrace{\frac{r_{ij}(\omega)}{f_{ij}(\omega)}}_{\text{Entry potential, } e_{ij}(\omega)} \geq \underbrace{\left[\frac{\sigma \bar{f}_{ij}}{\bar{r}_{ij}} \right]}_{\text{Bilateral entry shifter,} \bar{e}_{ij}} \left[\frac{w_i^{\sigma}}{P_j^{\sigma-1} E_j} \right]$$

General Equilibrium

- Firms hire $\bar{F}_i$ workers to independently draw $v_i(\omega) \equiv \{b_{ij}(\omega), \tau_{ij}(\omega), f_{ij}(\omega), a_i(\omega)\}_j$:

$$v_i(\omega) \sim G_i(v)$$

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$$v_i(\omega) \sim G_i(v)$$

- **Equilibrium:** $\{w_i, N_i, P_i, \{\Omega_{ij}\}_j\}_i$ satisfying **(i) CES demand, (ii) export decision**,
  - **iii) Free Entry:** N_i firms enter with an expected profit of zero,

$$w_i \bar{F}_i = \sum_j E\left[\max\{\pi_{ij}(\omega);\ 0\}\right]$$

  - **iv) Market Clearing:** from trade balance,

$$E_i = w_i \bar{L}_i = \sum_j \int R_{ij}(\omega) d\omega$$

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Distributions of revenue and entry potentials

- Without loss of generality, we can think of firms as

$$r_{ij}(\omega) \sim H^r_{ij}(r|e) \quad \text{and} \quad e_{ij}(\omega) \sim H^e_{ij}(e)$$

- **Assumption 1:** $H^e_{ij}(e)$ is continuous and strictly increasing in $\mathbb{R}_+$ with $\lim_{e\to\infty} H^e_{ij}(e) = 1$

- Generalizes (practically) all existing cases in the literature

Gravity Equations: extensive and intensive margin of firm exports

- **Extensive margin of firm-level exports:**

$$\ln \bar{\epsilon}_{ij}(n_{ij}) = \ln(\sigma \bar{f}_{ij}/\bar{r}_{ij}) + \ln w_i^{\sigma} - \ln E_j P_j^{\sigma-1}$$

  - $\bar{\epsilon}_{ij}(n) \equiv (H_{ij}^{e})^{-1}(1-n)$ is *cost-to-sales ratio* supporting entry in j of n of i firms
  - Slope of $\bar{\epsilon}_{ij}(n)$ controls dispersion in entry potential: $\varepsilon_{ij}(n_{ij}) = \frac{\partial \ln \bar{\epsilon}_{ij}(n_{ij})}{\partial \ln n} < 0$

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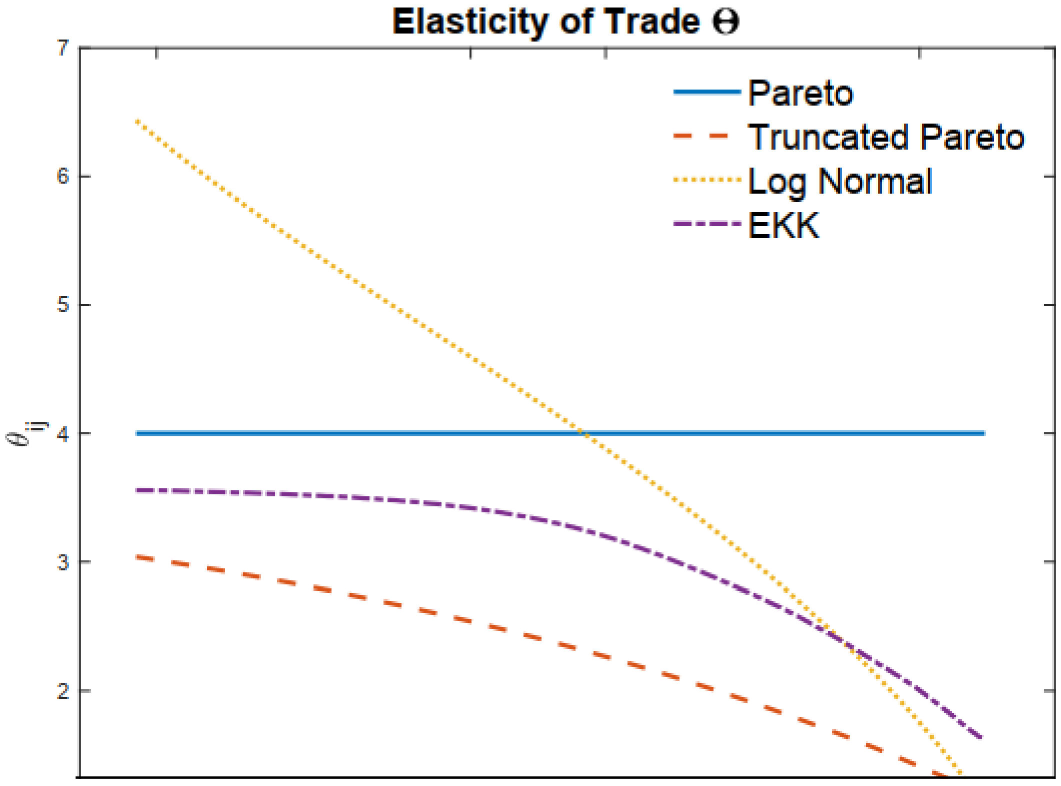
- **Intensive margin of firm level exports:**

$$\ln \bar{x}_{ij} - \ln \bar{\rho}_{ij}(n_{ij}) = \ln \bar{r}_{ij} + \ln w_i^{1-\sigma} + \ln E_j P_j^{\sigma-1}$$

  - $\bar{x}_{ij}$ is average sales of firms from i in j , $\bar{\rho}_{ij}(n) \equiv \frac{1}{n}\int_0^n E[r|e = \bar{\epsilon}_{ij}(n)]\, dn$ is the *avg. revenue potential* if n of i firms enter j
  - Slope of $\bar{\rho}_{ij}(n)$ controls difference between marginal and incumbent firms: $\varrho_{ij}(n_{ij}) = \frac{\partial \ln \bar{\rho}_{ij}(n_{ij})}{\partial \ln n}$

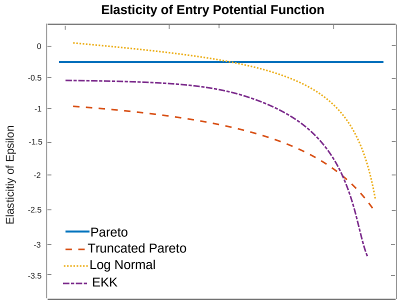
Firm heterogeneity distribution $\implies$ Trade elasticity varies with n_{ij}

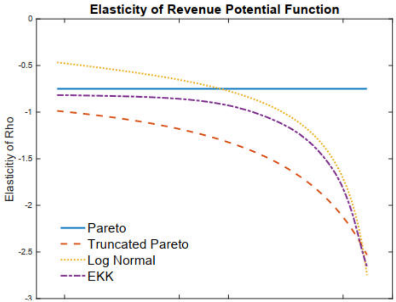
$$\theta_{ij}(n_{ij}) \equiv -\frac{\partial \ln X_{ij}}{\partial \ln \bar{\tau}_{ij}} = (\sigma - 1)\left(1 - \frac{1 + \varrho_{ij}(n_{ij})}{\varepsilon_{ij}(n_{ij})}\right)$$



Margins of the Trade Elasticity Function

$$\frac{\partial \ln n_{ij}}{\partial \ln \bar{\tau}_{ij}} \propto \left(\frac{\partial \ln \bar{\epsilon}_{ij}}{\partial \ln n}\right)^{-1} \qquad \frac{\partial \ln \bar{x}_{ij}}{\partial \ln \bar{\tau}_{ij}} \propto \frac{\partial \ln \bar{\rho}_{ij}}{\partial \ln n}$$





- **Decreasing** elasticity of $\bar{\epsilon}_{ij}(n)$: Entry is **less sensitive** to shocks when n_{ij} **is high**

Sufficient Statistics of Firm Heterogeneity

- **Lemma 1.** Based on the above definitions we can re-state $(w_i, N_i, P_i, \{X_{ij}, n_{ij}\}_j)$ in general equilibrium as a function of the shifters $\left(\bar{T}_i, \bar{F}_i, \bar{L}_i, \bar{f}_{ij}, \bar{\tau}_{ij}\right)$ and the elasticity functions $\sigma, \bar{\rho}_{ij}(n), \bar{\epsilon}_{ij}(n)$.

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  - Intuition: All outcomes in Melitz '03 and generalizations can be written as a function of bilateral entry cutoffs. We establish a mapping between the entry cutoff and n_{ij}
- **Takeaway 1:** All dimensions of heterogeneity can be folded into our two elasticity functions $(\bar{\rho}_{ij}(n), \bar{\epsilon}_{ij}(n))$
- Looking ahead: we will exploit Takeaway 1 to

  Characterize model counterfactuals using $(\bar{\rho}_{ij}(n), \bar{\epsilon}_{ij}(n))$

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Nonparametric Counterfactuals

- We now aim to use the characterization above to conduct counterfactuals without parametric assumptions on the distribution of firm heterogeneity

- Let us fix some terminology
  - $\left(\bar{T}_i, \bar{F}_i, \bar{L}_i, \bar{f}_{ij}, \bar{\tau}_{ij}\right)$ are "economic fundamentals" (or shifters)
  - $(\sigma, \bar{\rho}_{ij}(n), \bar{\epsilon}_{ij}(n))$ are "elasticities"
  - $(w_i, P_i, N_i, X_{ij}, n_{ij})$ are "economic outcomes" (wage, price index, entry, bilateral trade/ export share)
  - Denote with a hat a change in a variable from its initial value e.g. $\hat{w}_i \equiv w_i / w_i^0$

Counterfactual Responses to Changes in Fundamentals

- **Prop 1.** Consider any change in the economic fundamentals $\left(\bar{T}_i, \bar{F}_i, \bar{L}_i, \bar{f}_{ij}, \bar{\tau}_{ij}\right)$. Given (i) $\boldsymbol{X}^0 \equiv \{X_{ij}^0\}$ and $\boldsymbol{n}^0 \equiv \{n_{ij}^0\}$, (ii) the elasticities $\{\sigma, \bar{\rho}_{ij}(n), \bar{\epsilon}_{ij}(n)\}$, and we can compute $\left\{\hat{w}_i, \hat{P}_i, \hat{N}_i, \{\hat{n}_{ij}, \hat{X}_{ij}\}_j\right\}_i$. GE system
  - Generalizes the "sufficient statistics" result of Arkolakis Costinot Rodriguez-Clare '12 beyond class of constant-elasticity gravity
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- Multiple dimensions of heterogeneity matter only through extensive and intensive margin
    - It is all about these elasticity functions!

Aggregate Implications: Is all About Shape of the Elasticity Functions!

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- **Prop 2.** Let $Y_i \equiv \{w_i, P_i, N_i, \{X_{ij}\}_j\}$
    1. The elasticity of elements of Y_i to changes in trade costs is a function of $\left(\sigma, \boldsymbol{\theta}(\boldsymbol{n}^0), \boldsymbol{X}^0\right)$,

$$\frac{d \ln Y_i}{d \ln \bar{\tau}_{od}} = \Psi_{i,od}\left(\sigma, \boldsymbol{\theta}(\boldsymbol{n}^0), \boldsymbol{X}^0\right)$$

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- A synthesis of the gains from trade debate!
  - For **small changes**: firm heterogeneity **only matters through** $\boldsymbol{\theta}(\boldsymbol{n}^0)$ (a la ACR)

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- A synthesis of the gains from trade debate!
  - For **small changes**: firm heterogeneity **only matters through $\boldsymbol{\theta}(\boldsymbol{n}^0)$** (a la ACR)
  - For **large changes**: Need to compute change in $\theta_{ij}(n_{ij}^0)$ due to change in n_{ij}, so also need to know $\varepsilon_{ij}(n_{ij}^0)$
    - Heterogeneity plays a role (Melitz Redding '15, Head Mayer Thoenig '14)
    - If elasticities constant: back to ACR

Firm Heterogeneity Matters=Variable Elasticities

- **Takeaway 2:**
  Firm heterogeneity only matters for counterfactual responses through σ and $(\bar{\rho}_{ij}(n), \bar{\epsilon}_{ij}(n))$. For small shocks, $(\bar{\rho}_{ij}(n), \bar{\epsilon}_{ij}(n))$ matter only through their combined effect in $\bar{\theta}_{ij}(n)$. In addition, when elasticities are constant, $\bar{\rho}_{ij}(n) = n^{\varrho_{ij}}$ and $\bar{\epsilon}_{ij}(n) = n^{\varepsilon_{ij}}$, *the bilateral trade elasticities constant and aggregate trade elasticities θ_{ij} are sufficient to compute counterfactual responses to shocks*

Firm Heterogeneity Matters=Variable Elasticities

- **Takeaway 2:**
  Firm heterogeneity only matters for counterfactual responses through σ and $(\bar{\rho}_{ij}(n), \bar{\epsilon}_{ij}(n))$. For small shocks, $(\bar{\rho}_{ij}(n), \bar{\epsilon}_{ij}(n))$ matter only through their combined effect in $\bar{\theta}_{ij}(n)$. In addition, when elasticities are constant, $\bar{\rho}_{ij}(n) = n^{\varrho_{ij}}$ and $\bar{\epsilon}_{ij}(n) = n^{\varepsilon_{ij}}$, *the bilateral trade elasticities constant and aggregate trade elasticities θ_{ij} are sufficient to compute counterfactual responses to shocks*

- Thus, heterogeneity only matters when elasticities vary *and* shocks are large

Do we Still Have Sufficient Statistics for Welfare Changes?

- Gains of reallocating resources from low to high entry potential firms (i.e., $\downarrow n_{ii}$)

$$\ln\left(\frac{\hat{w}_i}{\hat{P}_i}\right) = \frac{1}{\sigma - 1}\ln\left(\frac{\bar{\epsilon}_{ii}(n_{ii}\hat{n}_{ii})}{\bar{\epsilon}_{ii}(n_{ii})}\right)$$

  - Measurable change in productivity cutoff in Melitz '03

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- Gains from consuming foreign varieties ($\downarrow$ domestic spending share x_{ii}):

$$d\ln\frac{w_i}{P_i} = -\frac{1}{\theta_{ii}(n_{ii})} d\ln\left(x_{ii}/N_i\right)$$

  - Similar formula in ACR '12 and Melitz-Redding '15, but here the trade elasticity is a function of n_{ij}.
  - We need to know **correlation between $\theta_{ii}(n_{ii})$ and $d\ln(x_{ii}/N_i)$.**

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- **Takeaway 3:** Nonparametric sufficient statistics with σ, $\varepsilon_{ii}(n)$, and $\theta_{ii}(n)$.

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- **Takeaway 3:** Nonparametric sufficient statistics with σ, $\varepsilon_{ii}(n)$, and $\theta_{ii}(n)$.

- Conclusion: Takeaways 2–3 constitute a synthesis of the gains from trade debate

Extensions

- **Multiple-Sectors/Factors/Input-Output:** as in Costinot and Rodriguez-Clare '14
  - Sector-specific semiparametric gravity equations of firm exports
- **Zeros in bilateral flows**: as in Helpman-Melitz-Rubinstein '08:
  - Extensive margin gravity equation has a censoring structure
- **Import tariffs**: Need to keep track of tariff revenue
- **Multi-product firms**: Bernard-Redding-Schott '11, Arkolakis-Ganapati-Muendler '20
  - Another semiparametric gravity equation for average number of products
- **Non-CES preferences**: generalizing Arkolakis et al. '19, Matsuyama-Uschev '17
  - Generalized gravity equations implied by similar inversion argument

Concluding Remarks

- Distribution of firm fundamentals determines elasticity of extensive and intensive margins of firm exports as functions of exporter firm share
- **Nonparametric counterfactuals:** Two elasticity functions are sufficient to compute impact of trade shocks on aggregate outcomes
- **Semiparametric estimation:** Flexibly estimate these functions using semiparametric gravity equations of firm exports
- The non-constant elasticities imply an average change in grains from trade of 15%. Gains are larger for countries with higher firm export shares.

Entry & revenue potential functions $\implies$ General Equilibrium, $\{w_i, P_i, N_i\}$

- **Bilateral trade outcomes:**

$$\bar{\epsilon}_{ij}(n_{ij}) = \frac{\sigma \bar{f}_{ij}}{\bar{r}_{ij}} \left(\frac{w_i}{P_j}\right)^{\sigma} \frac{P_j}{w_j L_j} \quad \text{and} \quad \frac{\bar{x}_{ij}}{\bar{\rho}_{ij}(n_{ij})} = \bar{r}_{ij} \left(\frac{w_i}{P_j}\right)^{1-\sigma} (w_j \bar{L}_j)$$

- **CES price index:**

$$P_j^{1-\sigma} = \sum_i (N_i n_{ij}) \left(\bar{r}_{ij} w_i^{1-\sigma} \bar{\rho}_{ij}(n_{ij})\right)$$

- **Free Entry:**

$$N_i = \left[\sigma \frac{\bar{F}_i}{\bar{L}_i} + \sum_j \frac{n_{ij} \bar{x}_{ij}}{w_i \bar{L}_i} \frac{\int_0^{n_{ij}} \frac{\rho_{ij}(n)}{\bar{\epsilon}_{ij}(n)} \, dn}{\int_0^{n_{ij}} \frac{\rho_{ij}(n)}{\bar{\epsilon}_{ij}(n_{ij})} \, dn}\right]^{-1}$$

- **Market Clearing:**

$$w_i \bar{L}_i = \sum_j N_i n_{ij} \bar{x}_{ij}$$

Return

Entry & revenue potential functions $\implies$ General Equilibrium, $\{w_i, P_i, N_i\}$

- **Bilateral trade outcomes:**

$$\frac{\bar{\epsilon}_{ij}(n_{ij}\hat{n}_{ij})}{\bar{\epsilon}_{ij}(n_{ij})} = \frac{1}{\hat{\bar{r}}_{ij}} \left(\frac{\hat{w}_i}{\hat{P}_j}\right)^{\sigma} \frac{\hat{P}_j}{\hat{w}_j} \quad \text{and} \quad \hat{\bar{x}}_{ij} = \hat{\bar{r}}_{ij} \frac{\bar{\rho}_{ij}(n_{ij}\hat{n}_{ij})}{\bar{\rho}_{ij}(n_{ij})} \left(\frac{\hat{w}_i}{\hat{P}_j}\right)^{1-\sigma} (\hat{w}_j)$$

- **CES price index:**

$$\hat{P}_j^{1-\sigma} = \sum_i x_{ij} \hat{\bar{r}}_{ij} (\hat{w}_i)^{1-\sigma} \left(\hat{n}_{ij}\hat{N}_i\right) \frac{\bar{\rho}_{ij}(n_{ij}\hat{n}_{ij})}{\bar{\rho}_{ij}(n_{ij})}$$

- **Free Entry:**

$$\hat{N}_i = \left[1 + \sum_j y_{ij} \frac{\bar{\epsilon}_{ij}(n_{ij})}{\int_0^{n_{ij}} \rho_{ij}(n)} \int_{n_{ij}}^{n_{ij}\hat{n}_{ij}} \frac{\rho_{ij}(n)}{\bar{\epsilon}_{ij}(n)} \, dn\right]^{-1}$$

- **Market Clearing:**

$$\hat{w}_i = \sum_j y_{ij} \left(\hat{N}_i \hat{n}_{ij} \hat{\bar{x}}_{ij}\right)$$

Return