Lecture 6

The Neoclassical Growth Model and the Data

Macroeconomics EC417

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London School of Economics, Fall 2022

Plan of Lecture: the growth model and the data

1. Steady states and the data
2. Choosing parameter values, calibration
3. Transition dynamics and the data

Steady States and the Data

- Recall: current version of growth model does not feature growth
  - introduce growth in next lecture
- Here: take model to data in way that remains valid with growth
- In model steady state, we have

$$\frac{k_t}{y_t} = \text{constant}$$

$$\frac{i_t}{y_t} = \text{constant}$$

$$\frac{w_t h_t}{y_t} = \text{constant} \quad \text{(labor share)}$$

$$\frac{R_t k_t}{y_t} = \text{constant} \quad \text{(capital share)}$$

$$r_t = \text{constant}$$

- In fact, in version without growth, all of $k_t, i_t, \ldots$ constant
- No longer true with growth, but ratios above remain constant

Steady States and the Data

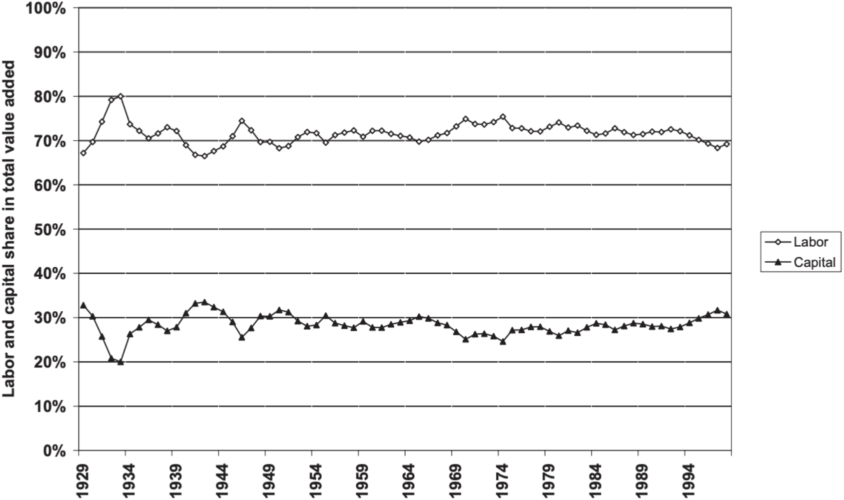
- For U.S. economy post 1950, over longer time periods, we observe

    1. k_t/y_t roughly constant
    2. i_t/y_t roughly constant
    3. $R_t k_t/y_t$ roughly constant
    4. $w_t h_t/y_t$ roughly constant

- These observations are known as the "Kaldor Facts" (after economist Nicholas Kaldor)

- Note: accuracy of "Kaldor Facts" recently questioned
    - ignore for now, revisit in a few slides

U.S. Capital and Labor Shares



Steady States and the Data

- Conclusion: for the U.S. economy post 1950, one could interpret the data as fluctuating around a steady state
- Note: accuracy of "Kaldor Facts" recently questioned (see e.g. Piketty, 2014; Karabarbounis and Neiman, 2014)
  - people have proposed a variety of "fixes" to growth model
  - ongoing debate, let's ignore this for now

Accuracy of Kaldor Facts?

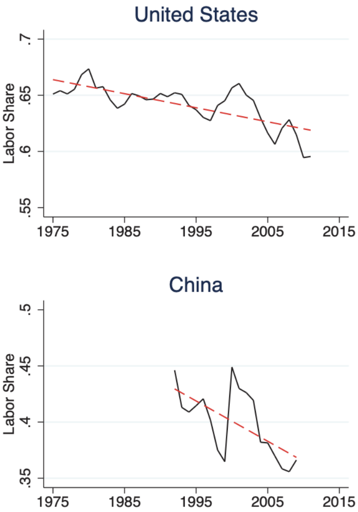


Figure: Labor shares from Karabarbounis and Neiman (2014).

Functional Forms

- If we want to use growth model to provide quantitative assessments, need to choose functional forms & parameter values
- Guidelines:
  1. parsimony
  2. choose functional forms in which parameters have clear economic interpretations
- Need to choose two functional forms

$$u(c_t) = \frac{c_t^{1-\sigma} - 1}{1-\sigma}$$

$$f(k_t) = Ak_t^{\alpha}$$

- Note: $\sigma = 1$ corresponds to log utility

$$\lim_{\sigma\to 1} \frac{c^{1-\sigma} - 1}{1-\sigma} = \lim_{\sigma\to 1} \frac{e^{(1-\sigma)\log c} - 1}{1-\sigma} = \frac{-\log c}{-1} \lim_{\sigma\to 1} e^{(1-\sigma)\log c} = \log c$$

Choosing Parameter Values

- set $A = 1$: just choice of units
- 4 parameters remain: $\delta, \beta, \sigma, \alpha$
- How to set their values?
- Literature often distinguishes (next slide: distinction is fuzzy)
  1. estimation
  2. calibration

Choosing Parameter Values

- Traditional distinction:
  - estimation e.g. via maximum likelihood: take all aspects of data seriously / weigh them equally
  - calibration: choose aspects of data model was most aimed to capture. Pick parameters to match specific moments of data.
- Nice discussion in Sargent (2021) "Learning from Lucas"
  `https://www.tandfonline.com/doi/full/10.1080/1350178X.2021.1993307`
  - "...likelihood ratio tests of yours are rejecting too many good models"
  - The method of ML provides a good estimator if a model is correctly specified. But if you regard your model only as an approximation to a better model, you should not use ML.
- Also see `http://www.tomsargent.com/research/SargentinterviewMD.pdf`

Choosing Parameter Values

Distinction between calibration and estimation is fuzzy

- Calibration can be viewed moment matching estimation (e.g. Hansen & Heckman, 1996)
- Informally, "calibration = GMM without standard errors"
- Perhaps more relevant distinction: full-information (e.g. MLE) vs limited-information methods (e.g. GMM, calibration)?

What about those standard errors? Shouldn't we take into account sampling uncertainty, e.g. in time-series averages of $k_t/y_t, i_t/y_t$ etc?

- Yes, absolutely (though we won't do this here).
- Ideally employ full moment matching / minimum distance estimation. But may not be possible due to data limitations.
- Interesting potential alternative: Cocci and Plagborg-Møller (2021) "Standard Errors for Calibrated Parameters"

  `https://scholar.princeton.edu/sites/default/files/calibration.pdf`

Application: Calibration of Growth Model

- Model designed to capture capital accumulation process
- So let's use moments that relate to this process

$$\frac{k_t}{y_t}, \quad \frac{i_t}{y_t}, \quad r_t$$

- Post 1950, U.S. looks like fluctuations around steady state (SS), can use average values in data to think about SS values
- Intrepreting 1 time period = 1 year, this gives something like

$$\frac{k}{y} \approx 3, \quad \frac{i}{y} \approx 0.25, \quad r \approx 0.02$$

- Choice of post 1950 matters, e.g. r much lower in recent years
  - point is to teach you approach, exact numbers less important
- 4 parameters $\delta, \beta, \sigma, \alpha$, but only 3 moments ("under-identified")
- Note: σ doesn't influence SS so cannot identify it from SS $\Rightarrow$ 3 moments are sufficient to identify δ, β, α

Application: Calibration of Growth Model

1. $r = 0.02$: in steady state

$$1 + r = \frac{1}{\beta} \Rightarrow \quad \beta = \frac{1}{1+r} = \frac{1}{1.02} \approx 0.98$$

2. in steady state $i = \delta k$

$$\delta = \frac{i}{k} = \frac{i/y}{k/y} = \frac{0.25}{3} \approx 0.08$$

3. in steady state

$$\alpha k^{\alpha - 1} = \alpha \frac{y}{k} = \frac{1}{\beta} - 1 + \delta$$

$$\alpha = \frac{k}{y}\left(\frac{1}{\beta} - 1 + \delta\right) = 3 \times (0.02 + 0.08) = 0.3$$

4. σ: range of estimates in literature is $[1, 2.5]$. Will use $\sigma \to 1$

Transition Dynamics and the Data

- Recall from Lecture 4: half-life for convergence to steady state

$$t_{1/2} = \frac{\ln(2)}{|\lambda_1|}, \quad \lambda_1 = \frac{\rho - \sqrt{\rho^2 - 4\frac{1}{\sigma} f''(k) c}}{2}$$
$$= \frac{\rho - \sqrt{\rho^2 + 4\frac{1-\alpha}{\sigma}\alpha \frac{y}{k}\left(\frac{y}{k} - \delta\right)}}{2}$$

- in steady state of continuous-time model: $r = \rho$

$$\lambda_1 = \frac{0.02 - \sqrt{(0.02)^2 + 4 \times 0.7 \times 0.3 \frac{1}{3}\left(\frac{1}{3} - 0.08\right)}}{2}$$
$$\approx -0.12$$
$$\Rightarrow t_{1/2} \approx \frac{\ln(2)}{0.12} \approx 5.7 \text{ years}$$

Transition Dynamics and the Data

- Given our parameter values, model converges to steady state very quickly
  - suggests that it is reasonable that we are around steady state from 1950 to present
  - if instead half-life were 100 years, things would be different
- Summary so far: growth model does a good job at capturing some salient features of U.S. economy post 1950
  - also true once we extend it to actually feature growth
  - also true for other developed countries (e.g. entire OECD)
- Can growth model also capture growth experience of poorer countries?

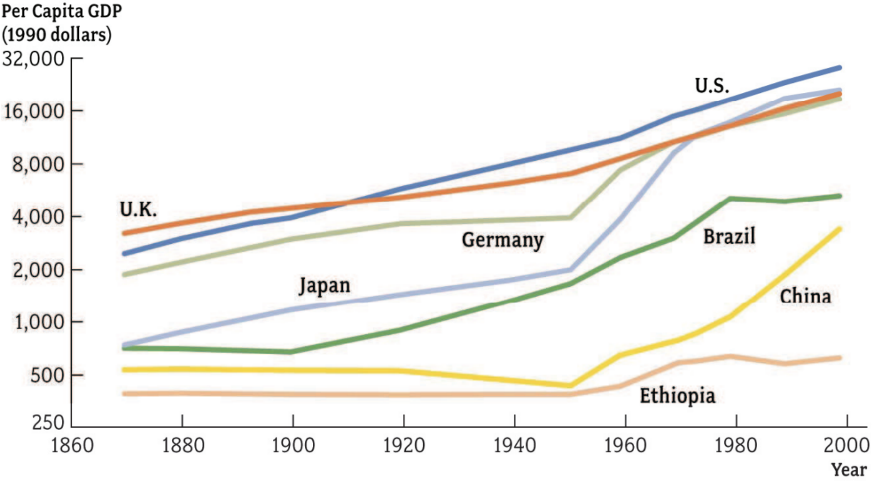
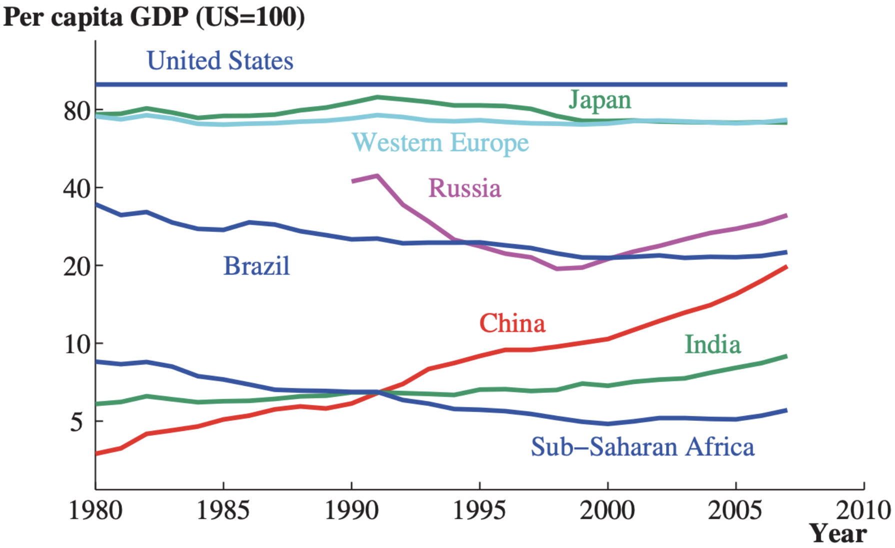


FIGURE 1.1 Per Capita GDP in Seven Countries, 1870–2000

Macroeconomics, Charles I. Jones



Per capita GDP (US=100)
United States
Japan
Western Europe
Russia
Brazil
China
India
Sub–Saharan Africa
80
40
20
10
5
1980
1985
1990
1995
2000
2005
2010
Year

East Asian Miracles

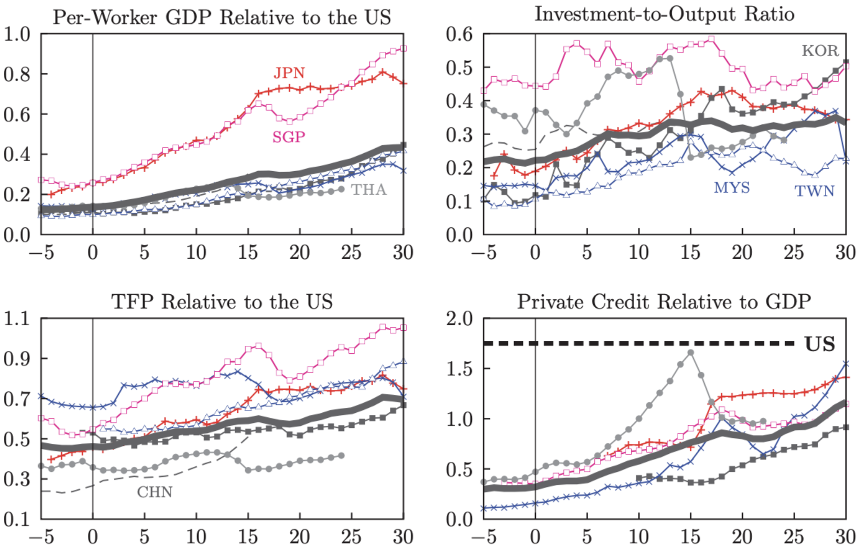


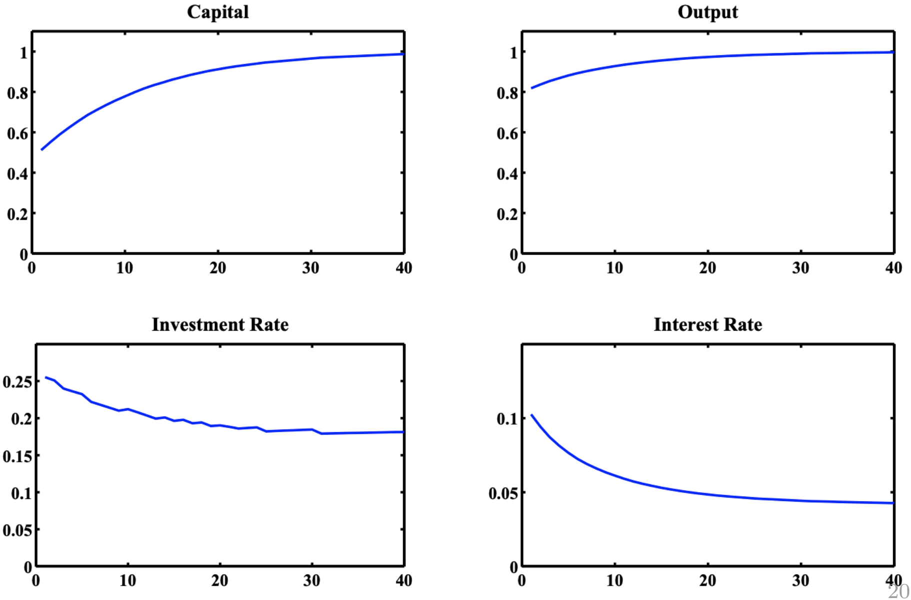
Fig. 1: Transitional Dynamics from the Economic Miracles

Development Dynamics in Data

1. Slow convergence
2. Rising investment-to-output ratio in early stages
3. Output growth partly explained by aggregate productivity (TFP) and reallocation dynamics

Can growth model capture these facts? No!

Neoclassical Transitions Get it Wrong



Neoclassical Transitions Get it Wrong

- King and Rebelo (1993): neoclassical growth model with constant (or no) exogenous TFP growth has no hope of explaining sustained growth as stemming from transitional dynamics.
- extremely counterfactual implications for the time path of the interest rate.
- According to their calculations for example, if the neoclassical growth model were to explain the postwar growth experience of Japan, the interest rate in 1950 should have been around 500 percent.

Sketch of King-Rebelo interest-rate calculation

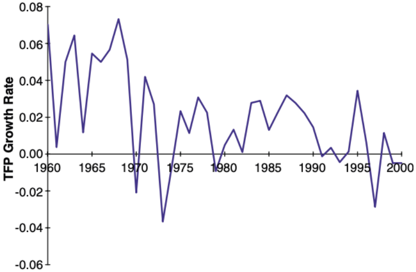
- Let's use calibrated parameter values $\rho = 0.02, \delta = 0.08, \alpha = 0.3$
- Eyeballing Fig.1 a few slides ago, Japanese output grew by $\approx 80\%$
- Suppose = all transitional dynamics to new steady state. Then
$$\frac{y_0}{y^*} = 0.2$$
- Have $r_t + \delta = \alpha k_t^{\alpha-1}$ and $r^* = \rho$
- With $y = k^\alpha$, can write MPK as $\alpha k^{\alpha-1} = \alpha y^{1-1/\alpha}$

$$\frac{r_0+\delta}{\rho+\delta} = \left(\frac{y_0}{y^*}\right)^{1-1/\alpha} \quad \Rightarrow \quad r_0 = \left(\frac{y_0}{y^*}\right)^{1-1/\alpha}(\rho+\delta) - \delta$$
$$= (0.2)^{1-1/0.3}(0.02+0.08) - 0.08$$
$$\approx 4.2 = 420\%!!!$$

- Logic: large y difference $\Rightarrow$ huge k difference $\Rightarrow$ huge MPK diff!
- Note $k_0/k^* = (y_0/y^*)^{1/\alpha} = 0.2^{1/0.3} \approx 0.0046 = 0.46\%!$

Needed: A Theory of TFP (Dynamics)

- In contrast, Chen, Imrohoroglu and Imrohoroglu (2006, 2007): the neoclassical growth model is, in fact, consistent with the Japanese postwar growth experience once one takes as given the time-varying TFP path measured in the data.
- TFP time path they feed into their model



- Growth model fails as model of development dynamics
- What we need instead: a theory of endogenous TFP dynamics
  - material for 2nd-year PhD course