Lecture 9

Microeconomic Origins of Aggregate Fluctuations

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EC417

This term

Part I: Shocking theory of the business cycle

- Introduction to business cycles ✓
- Real Business Cycle (RBC) Model ✓
- New Keynesian DSGE Models ✓

Part II: Perspectives on business cycles and steady states

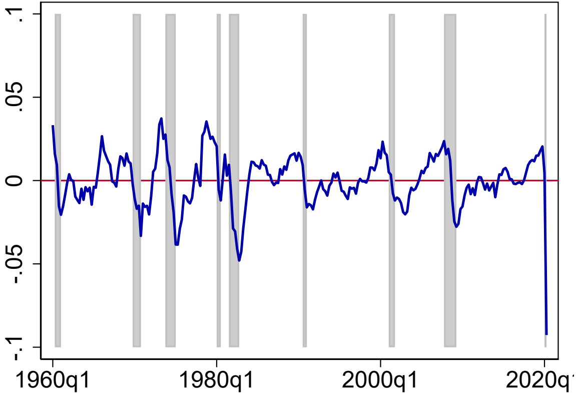
- Heterogeneity versus homogeneity and the effect of policy ✓
- Endogenous growth and persistent effects of recessions ✓
- Aggregate shocks? Firm-heterogeneity and the business cycle $\Leftarrow$

DSGE Paradigm

Key features:

1. **Representative** household makes optimal intertemporal decisions ✓
2. Business cycles are **transitory** deviations from the long-term trend ✓
3. Macroeconomic fluctuations are driven by **aggregate** shocks $\Leftarrow$
4. The source of fluctuations are **shocks** (random disturbances)

Business cycles: macro level



Real Gross Domestic Product (log deviations from HP Trend) for the U.S. 1958-2011

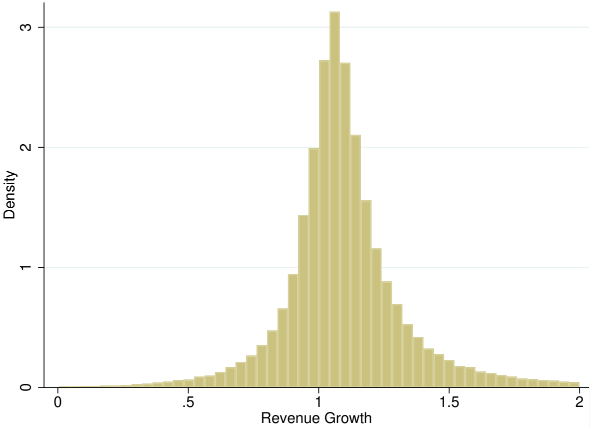
Source: FRED

Macroeconomic shocks?

So far, we've used **aggregate shocks** as the source of business cycles

- Macro-economy: made up of hundreds of sectors, millions of firms
- Is there really such a thing as an *aggregate shock*?
- Could shocks to individual firms or sectors drive business cycles?
- E.g.: Decisions by firm's departments/managers, issues with shipments, inventories, strikes, lightning strike, earthquake, .. ?

Macroeconomic shocks?



Growth in revenue (py_{it} / py_{it-1})

Source: Compustat data based on annual report from U.S. listed firms

Firm size distribution

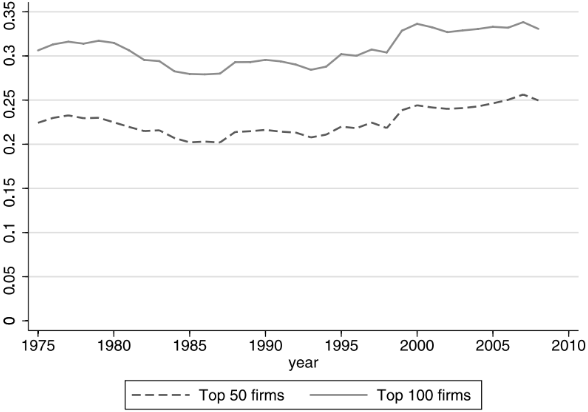
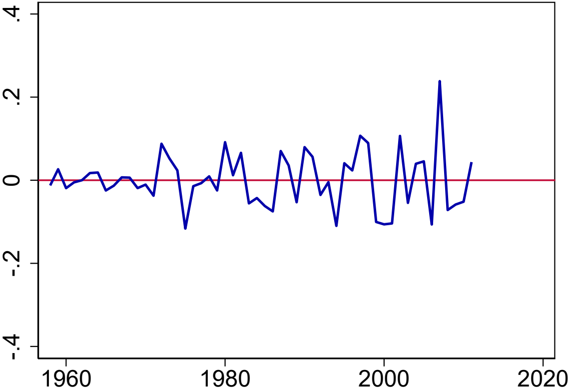


FIGURE 1.—Sum of the sales of the top 50 and 100 non-oil firms in Compustat, as a fraction of GDP. Hulten's theorem (Appendix B) motivates the use of sales rather than value added.

Source: Gabaix (2011)

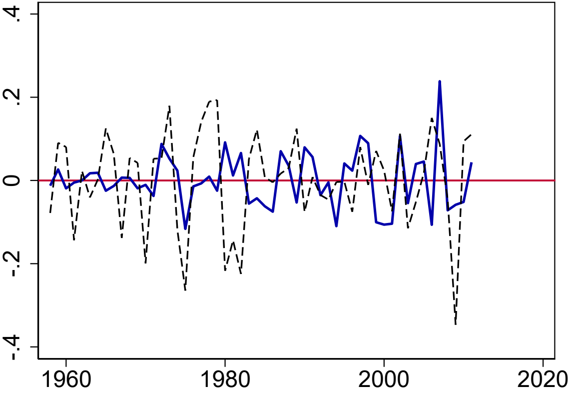
Business cycles: NAICS 325611 (Soap)



Real Value Added (log deviations from HP Trend) for the U.S. 1958-2011

Source: NBER Manufacturing Database (2016)

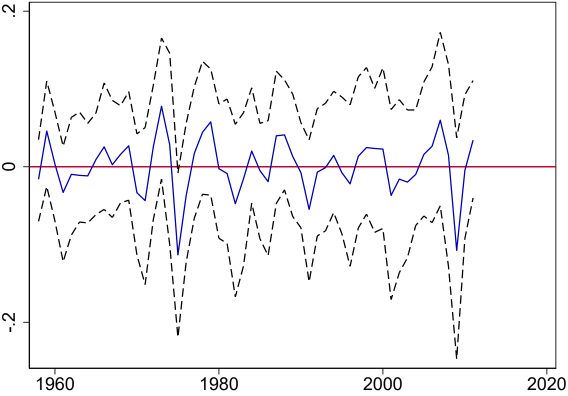
Business cycles: NAICS 336111 (Automobiles)



Real Value Added (log deviations from HP Trend) for the U.S. 1958-2011

Source: NBER Manufacturing Database (2016)

Business cycles: 6-digit NAICS



Real Value Added (log deviations from HP Trend) for the U.S. 1958-2011

Dashes: 10th/90th Percentile of Output Gap. Solid: Median. Source: NBER

Questions for today

- How to measure the effect of firm heterogeneity on the economy?
- Can micro shocks explain relevant portion of aggregate fluctuations?
- What amplifies the effect of microeconomic shocks?

References

Gabaix (2011), *Granular Origins of Aggregate Fluctuations*, American Economic Review (excl. sec 2.4 formal proof, 2.5, 3)

Carvalho & Tahbaz-Salehi (2019), *Production Networks: A Primer*, Annual Review of Economics (section 2.3.2. only)

Today

- Lucas' diversification argument
- Granular shocks: Gabaix (2011)
- Hulten's Theorem
- Empirical evidence

Today

- **Lucas' diversification argument**
- Granular shocks: Gabaix (2011)
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Lucas' (1977) diversification argument

Consider the following economy

- The economy is composed of a set K production entities
  - Factories, firms, sectors, .. $\Rightarrow$ some sub-aggregate level
- Factors are supplied inelastically, only productivity shocks
- Entity $k \in K$ is subject to idiosyncratic (i.i.d.) output a_{kt}
- Abstract from elastically supplied inputs, nominal rigidities, etc.
- They are aggregated to GDP Y_t by adding them up:

$$Y_t = \sum_{k \in K} a_{k,t}$$

Lucas' diversification argument

What's the GDP volatility in this economy?

- Say idiosyncratic shocks are as follows:

$$\frac{\Delta a_{k,t}}{a_{k,t-1}} = \frac{a_{k,t} - a_{k,t-1}}{a_{k,t-1}} = \sigma_k \varepsilon_{k,t}$$

- where $\varepsilon_{k,t}$ is mean 0, var 1. GDP growth:

$$\frac{\Delta Y_t}{Y_{t-1}} = \frac{1}{Y_{t-1}} \sum_{k \in K} \Delta a_{k,t} = \sum_{k \in K} \frac{a_{k,t-1}}{Y_{t-1}} \sigma_k \varepsilon_{k,t}$$

- Shocks are independent/uncorrelated, hence $\sigma_Y = \sqrt{\text{var} \frac{\Delta Y_t}{Y_{t-\mathbf{1}}}}$ is:

$$\sigma_Y = \left(\sum_{k \in K} \sigma_k^2 \left[\frac{a_{k,t-1}}{Y_{t-1}} \right]^2 \right)^{0.5}$$

Lucas' diversification argument

Say all firms/production entities have same volatility and same size:

$$
\begin{aligned}
\sigma_Y = \left(\sum_{k \in K} \sigma_k^2 \left[\frac{a_{k,t-1}}{Y_{t-1}} \right]^2 \right)^{0.5} &= \left(\sum_{k \in K} \sigma_k^2 \left[\frac{1}{N} \right]^2 \right)^{0.5} \\
&= \frac{\sigma}{\sqrt{N}}
\end{aligned}
$$

where $N = \mid K \mid$ (the number of firms).

Empirically: N is large, hence effect of shocks is small

$\Rightarrow$ Idiosyncratic shocks cannot explain macroeconomic fluctuations

- Diversification: individual shocks average out to 0

Example

- Across 473 NAICS 6-digit manufacturing industries, avg. $\sigma = 0.09$:

$$\widehat{\sigma}_Y = 0.09/\sqrt{473} = 0.0041$$

  Manufacturing GDP volatility: 3%

- Firm level: 5.5 million firms across economy, $\sigma = 0.12$ (Axtell 2001):

$$\widehat{\sigma}_Y = 0.12/\sqrt{5.5 \times 10^6} = 0.00005$$

  Even worse!

- Conclusion: micro shocks do not explain aggregate fluctuations

Today

- Lucas' diversification argument
- **Granular shocks: Gabaix (2011)**
- Hulten's Theorem
- Empirical evidence

Microeconomic shocks

In most models, microeconomic shocks have no aggregate effects

- Assume a very large number of identical (small) firms
- Idiosyncratic shocks cancel out by law of large numbers
- In practice: some firms are very large
  (e.g. Nokia in 2003: 26% of Finnish private-sector GDP)
- Can shocks to large firms explain business cycles?
- Gabaix (2011): **Granular shocks**
  - Large shocks to small entities (grains), not small macro shocks

Generalizing the diversification result (Gabaix 2011)

What is the effect of heterogeneous firm size on the diversification result?

Allow for differences in firm size, but same volatility:

$$
\begin{aligned}
\sigma_Y &= \left(\sum_{k \in K} \sigma_k^2 \left[\frac{a_{k,t-1}}{Y_{t-1}} \right]^2 \right)^{0.5} \\
&= \sigma \left(\sum_{k \in K} \left[\frac{a_{k,t-1}}{Y_{t-1}} \right]^2 \right)^{0.5} \\
&= \sigma h
\end{aligned}
$$

where h is the square root of the Herfindahl Index.

- The Herfindahl index is a famous measure of **market concentration**
- Herfindahl in NK-DSGE: 0. Herfindahl with monopolist: 1.
- Used (e.g.) to evaluate effect of mergers

Generalizing the diversification result (Gabaix 2011)

Herfindahl determines volatility. What happens with Herfindahl as N increases?

$$
\begin{aligned}
h &= \left(\sum_{k \in K} \left[\frac{a_{k,t-1}}{Y_{t-1}} \right]^2 \right)^{0.5} \\
\sqrt{N} h &= \left(N \sum_{k \in K} \left[\frac{a_{k,t-1}}{Y_{t-1}} \right]^2 \right)^{0.5} \\
&= \frac{N}{Y_{t-1}} \left(\frac{1}{N} \sum_{k \in K} \left[a_{k,t-1} \right]^2 \right)^{0.5} \\
&= \frac{\left(\frac{1}{N} \sum_{k \in K} a_{k,t-1}^2 \right)^{0.5}}{\frac{1}{N} \sum_{k \in K} a_{k,t-1}} \quad \underset{L.L.N}{\rightarrow} \quad \frac{\left[\mathbb{E}(a^2) \right]^{0.5}}{\mathbb{E}(a)}
\end{aligned}
$$

Generalizing the diversification result (Gabaix 2011)

$$\sigma_Y = \sigma h$$

Looking across economies with different N, how does volatility change?

$$\sigma_Y \sim \frac{\left[\mathbb{E}(a^2)\right]^{0.5}}{\mathbb{E}(a)} \frac{\sigma}{\sqrt{N}}$$

- Notation: $X_N \sim a_N Y$ means that $X_N / a_N \to Y$
- Effect of idiosyncratic shocks still diminishes at rate $1/\sqrt{N}$
- Hence: volatility of GDP still diminishes rapidly (distribution of volatility is **degenerate**)

However: formula requires that firm distribution has a second moment

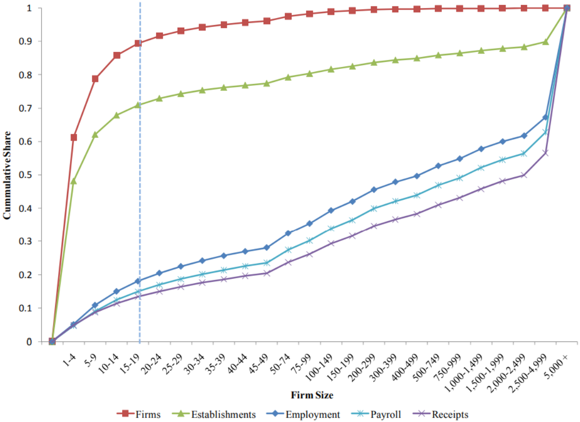
- Time to look at the firm size distribution

Firm size distribution

Data on the distribution of firm-sizes is difficult to obtain

- Needs data with good coverage of active firms: confidential
- Recent years: countries opened access to comprehensive tax data
- Main facts:
  - Many small firms account for small share of employment
  - Large firms are more important, right tail is fat

Firm size distribution



Source: Hurst and Pugsley (2011) using U.S. Census data

Pareto distribution

Many economic distributions are well-described by the Pareto distribution

- Power law distribution: log-log relation is linear
- Example: quality of patents, wealth, size of firms

The CDF of the Pareto distribution:

$$\overline{F}(x) = 1 - \left(\frac{x}{x_m}\right)^{-\delta} \quad \text{for } x \geq x_m$$

- δ is the tail parameter
- x_m is the scale parameter
- Key property: distribution admits finite moments of order $< \delta$

Moments of the Pareto distribution

The probability density function is:

$$f(x) = \frac{\delta}{x_m}\left(\frac{x}{x_m}\right)^{-1-\delta}$$

Hence, the k-th moment of the Pareto distribution is:

$$\begin{aligned} E(x^k) &= \int_{x_m}^{\infty} x^k \frac{\delta}{x_m}\left(\frac{x}{x_m}\right)^{-1-\delta} dx \\ &= \left[\frac{\delta}{x_m^{\delta}}\frac{x^{k-\delta}}{k-\delta}\right]_{x_m}^{\infty} \end{aligned}$$

- Fat tailed distribution: $\delta \leq 2$, no finite second moment

What's the firm-size distribution's δ?

The density function of the Pareto distribution:

$$f(x) = \frac{\delta}{x_m} \left(\frac{x}{x_m} \right)^{-1-\delta}$$

.. implies a log-log relationship between x and $f(x)$:

$$\begin{aligned} \log f(x) &= \log(\delta) + \log(x_m^{\delta}) - (1+\delta) \log x \\ &= \alpha + \beta \log x \end{aligned}$$

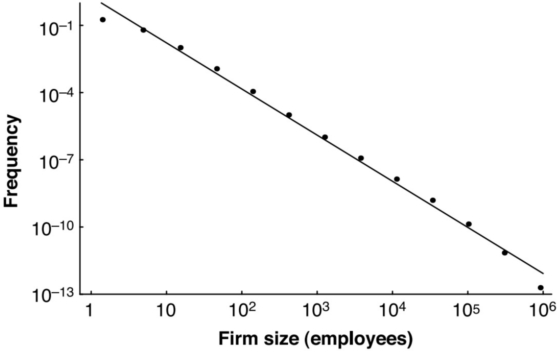
To estimate this on firms, particularly difficult to find good data

- Need data on size (employment, sales, ..) for the universe of firms
- Confidential data, usually obtained from Census or tax office
- Can't export results on individual firms: binned regression

Firm size distribution

$$\log f(x) = \alpha + \beta \log x$$

x: employment bin. $f(x)$: fraction of sample with x employees



Firm size distribution follows a power law

Source: Axtell (2001, *Science*)

OLS regression: $\beta = -2.059$ such that $\delta = 1.059$, $R^2 = 0.998$

Firm size distribution

- Pareto distribution with tail parameter of just above 1

$$\overline{F}(x) = 1 - \left(\frac{x}{x_m}\right)^{-1.05}$$

- If tail parameter ≤ 2: does **not** admit a second moment!
- Hence the generalized diversification result doesn't apply
- Fat tail of firm-size distribution means shocks don't 'cancel out'
- Gabaix then derives rate of decay of volatility for fat-tailed firm dist.
  - Cover heuristic proof on next 3 slides. Not examinable.

Volatility with fat-tailed firm distribution

We derived GDP volatility without making firm-size dist. assumptions:

$$\sigma_Y = \sigma h$$

Back to the relationship between the Herfindahl and firm-count N:

$$h = \frac{\frac{1}{N}\left(\sum_{k \in K} a_{k,t-1}^2\right)^{0.5}}{\frac{1}{N}\sum_{k \in K} a_{k,t-1}}$$

We can still use the law of large numbers (if $\delta > 1$) for the denominator:

$$h \sim \frac{\frac{1}{N}\left(\sum_{k \in K} a_{k,t-1}^2\right)^{0.5}}{\mathbb{E}[a]}$$

Volatility with fat-tailed firm distribution

We can also get an expression for the numerator. Assume:

$$P(a > x) \approx x^{-\delta} \quad \text{(simplify by normalizing } x_m = 1\text{)}$$

Then the size of the ith largest firm out of N is approximately:

$$a_{i,N} = \left(\frac{i}{N}\right)^{-\frac{1}{\delta}}$$

Derivation (heuristic):

$$\begin{aligned} P(a > x) = x^{-\delta} \Rightarrow P(a^{-\delta} > x) &= P(a > x^{-1/\delta}) \\ &= x \\ \mathbb{E}[a_{i,N}^{-\delta}] = \frac{i}{N+1} \Rightarrow a_{i,N}^{-\delta} &\approx \frac{i}{N+1} \end{aligned}$$

Volatility with fat-tailed firm distribution

Putting this into the square root of the Herfindahl:

$$
\begin{aligned}
h &\sim \frac{\frac{1}{N}\left(\sum_{k\in K} a_{k,t-1}^2\right)^{0.5}}{\mathbb{E}[a]} \\
h_N &\sim \frac{\frac{1}{N}\left(\sum_{i=1}^{N}\left(\frac{i}{N}\right)^{-\frac{2}{\delta}}\right)^{0.5}}{\mathbb{E}[a]} \\
h_N &\sim \frac{N^{-1+1/\delta}\left(\sum_{i=1}^{N}(i)^{-\frac{2}{\delta}}\right)^{0.5}}{\mathbb{E}[a]} \\
&= C N^{-(1-1/\delta)}
\end{aligned}
$$

- For $1 < \delta < 2$: the summation converges. With finite variance it diverges, returns original result.

Volatility with fat-tailed firm distribution

Putting this in the Herfindahl:

$$h_N \sim \frac{N^{-1+1/\delta}\overbrace{\left(\sum_{i=1}^{N} i^{-2/\delta}\right)^{1/2}}^{\text{converges}}}{\mathbb{E}[a]} = CN^{-(1-1/\delta)}$$

- Conclusion: Herfindahl and GDP-volatility decline at rate $(1-1/\delta)$ in N
- That is much slower than rate $\sqrt{N}$ for δ close to 1
- For $\delta = 1.05$, decay is $N^{0.05}$
    - **Economy with 10 million firms will have GDP growth volatility that is about half is high as volatility with 10 firms.**

Formal proof: appendix of Gabaix' paper

What we learn from this?

- Main lesson: if volatility comes from micro fluctuations, we have

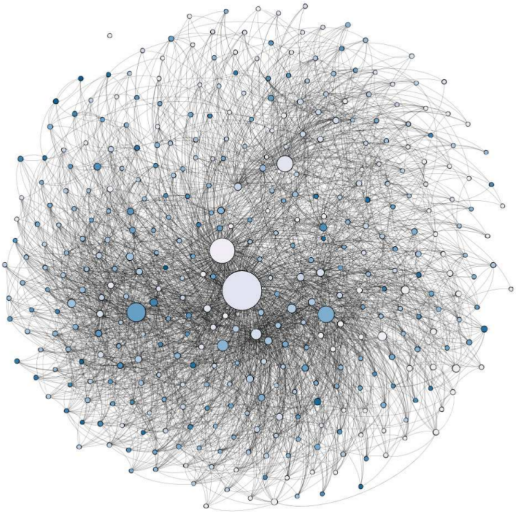
$$\sigma_Y = \sigma h$$

- With fat-tailed firm-size distribution, h decays slowly in N
- **Don't need the math!** Just need to observe the Herfindahl index
- Next: we assumed that GDP was sum of micro outputs. Can shocks from other firms spill over?

Today

- Lucas' diversification argument
- Granular shocks: Gabaix (2011)
- **Hulten's Theorem**
- Empirical evidence

Micro to macro



Input-Output links for the U.S. economy (Grassi 2017)

From Micro to Macro

So far, we've defined GDP as the sum of i.i.d. outputs a.

- Macro: typically think of an aggregate production function

$$GDP = Agg.\ Shock\ \times (Agg.\ Capital, Agg.\ Labor)$$

- Micro: there are many "production recipes"

$$Output_i = Shock_i \times (Capital_i, Labor_i, Intermediate\ Inputs_i)$$

- This introduces a source of **shock propagation** for micro shocks
  - Firm i's output affects firm j's output too
- How can you measure the impact of these linkages?

Hulten's Theorem

Hulten (1978):

- Consider an efficient economy (welfare theorems apply)
  - No distortions from price stickiness, market power, etc.
- Then under minimimal assumptions, the following holds:

$$\frac{\text{d log } GDP}{\text{d log } z_i} = \lambda_i$$

- Here λ_i is a firm/sector i's Domar weight:

$$\lambda_i = \frac{p_i y_i}{GDP}$$

Derivation - follows Carvalho and Tabbaz - Salebi (2019)

Consider the following very general setup:

- The economy consists of n competitive firms/industries
- Each industry produces using:
  - n intermediate inputs (other firms' output)
  - m primary factors (e.g. types of labor)
- Production function with constant returns to scale, arbitrary input-output:

$$y_i = z_i f(x_{i1}, ..., x_{in}; l_{i1}, ..., l_{im})$$

- Representative household has preferences over all goods:

$$u(c_1, .., c_n) \quad \text{(homogeneous of degree 1)}$$

- Endowed with h_k unites of primary good k

Social planner

Social planner chooses consumption, factor and intermediate input usage:

$$
\begin{aligned}
U &= \max_{c_i, l_{ik}, x_{ij}} u(c_1, .., c_n) \\
&\quad \text{s.t.} \quad c_i + \sum_{j=1}^{n} x_{ji} = z_i f(x_{i1}, ..., x_{in}; l_{i1}, ..., l_{im}) \quad i = 1, ..., n \\
&\quad \sum_{i=1}^{n} l_{ik} = h_k \quad k = 1, ..., m
\end{aligned}
$$

Welfare theorems: planners problem equals decentralized equilibrium

Social planner

Constrained optimization problem:

$$\mathcal{L} = u(c_1, ..,) - \sum_i^n \eta_i \left(c_i + \sum_{j=1}^n x_{ji} - z_i f(x_{i1}, ..., x_{in}; l_{i1}, ..., l_{im}) \right) - \sum_{i=1}^m \omega_j \left(\sum_{i=1}^n l_{ik} - h_k \right)$$

First order conditions for consumption of good i:

$$\frac{\partial u}{\partial c_i} = \eta_i$$

How does utility change when good i has higher productivity?

- Envelope theorem for the social planner:

$$\frac{\mathrm{d}\ U}{\mathrm{d}\ z_i} = \frac{\partial \mathcal{L}}{\partial z_i} = \eta_i f(x_{i1}, ..., x_{in}; l_{i1}, ..., l_{im}) = \frac{\eta_i y_i}{z_i}$$

- Hence, in the neighborhood of the optimal allocation:

$$\frac{\mathrm{d}\ \log\ U}{\mathrm{d}\ \log\ z_i} = \frac{\eta_i y_i}{U}$$

Household

Now consider the problem from the household side:

$$U = \max_{c_i, l_{ik}, x_{ij}} u(c_1, .., c_n) \quad \text{s.t.} \sum_{j=1}^{n} p_i c_i = \sum_{k=1}^{m} w_k h_k$$

Lagrangian:

$$\mathcal{L} = u(c_1, .., c_n) - \phi \left(\sum_{j=1}^{n} p_i c_i - \sum_{k=1}^{m} w_k h_k \right)$$

First order conditions of the decentralized problem:

$$\frac{\partial u}{\partial c_i} = \phi p_i = \eta_i \text{ (economy is efficient)}$$

Household

Euler's homogeneous function theorem for k-degree homogeneity:

$$x \nabla f(x) = kf(x)$$

- x: vector of variables, $\nabla f(x)$ is vector of partial derivatives of $f(x)$

Example for a Cobb-Douglas production function:

$$\begin{aligned} y = k^{\alpha} l^{1-\alpha} \quad &\rightarrow \quad y_k' = \alpha k^{\alpha-1} l^{1-\alpha}, \quad y_l' = (1-\alpha) k^{\alpha} l^{-\alpha} \\ &\rightarrow \quad k y_k' + l y_l' = y\alpha + y(1-\alpha) = y \end{aligned}$$

Our case: use Euler's homogeneous function theorem to relate GDP to utility

$$\frac{\partial u}{\partial c_i} = \phi p_i \quad \Rightarrow \quad U = \phi \sum_{i=1}^{n} p_i c_i = \phi GDP$$

Hulten's Theorem

Combine the results from the social planner and the household:

$$\frac{\text{d}\log U}{\text{d}\log z_i} = \underbrace{\frac{\eta_i y_i}{U}}_{SP} = \underbrace{\frac{\phi p_i y_i}{\phi GDP}}_{HH+Euler} = \frac{p_i y_i}{GDP}$$

Define an ideal price index $\mathcal{P}$ such that:

$$U = GDP/\mathcal{P}$$

Normalize the price index to 1 to get:

$$\frac{\text{d}\log GDP}{\text{d}\log z_i} = \frac{p_i y_i}{GDP} = \lambda_i$$

Implication: all you need to know for effect of shocks to i is sales share

- Microeconomic details of production structure are irrelevant

Hulten's Theorem

What's the intuition?

- Sales capture both consumption and use as intermediate input
- Makes it straightforward to analyze macro implications of micro (e.g. sector-level) shocks, even when economy has complex structure
- Powerful result: foundation for most published aggregate TFP statistics (e.g. BLS)
- Note: weights add up to more than one!
  - Volatility is larger than weighted average

Some remarks

Model was general, but:

- Envelope condition: around optimal allocation, no reallocation
    - This is a **first-order approximation**
    - Large shocks: maybe the approximation is imprecise? E.g. the Domar weights themselves could change because of shocks
- Assumed an efficient economy. Inefficient economy?
    - In reality: firms differ in market power, many frictions preventing the social planner from equaling decentralized outcome
- We ignore that Domar weights are endogenous to micro structure
    - See Carvalho (2009): size is endogenous to production network

Some remarks

If you are interested in this, two great recent papers:

- Large shocks and non-linearities:
  - Baquee and Farhi (2019), *The Macroeconomic Impact of Microeconomic Shocks: Beyond Hulten's Theorem* (ECTA)
- Aggregation in inefficient economies:
  - Baquee and Farhi (2019) *Productivity and Misallocation in General Equilibrium* (QJE)

Today

- Lucas' diversification argument
- Granular shocks: Gabaix (2011)
- Hulten's Theorem
- **Empirical evidence**

Combining Hulten's Theorem and Gabaix

From Hulten's theorem, in an economy with input-out linkages we have:

$$\frac{\mathrm{d}\ GDP}{GDP} = \sum_i \left(\frac{p_i y_i}{GDP} \mathrm{d} \log z_i \right)$$

Volatility of GDP growth:

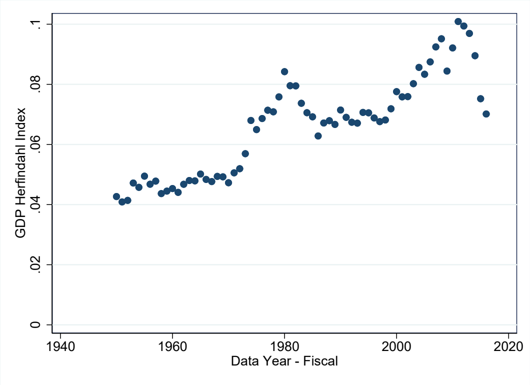
$$\mathrm{var}\ \frac{\mathrm{d}\ GDP}{GDP} = \mathrm{var}\ \left(\sum_i \frac{p_i y_i}{GDP} \mathrm{d} \log z_i \right)$$

$$\mathrm{var}\ \frac{\mathrm{d}\ GDP}{GDP} = \sum_i \left(\underbrace{\left[\frac{p_i y_i}{GDP} \right]^2}_{\text{sales Herfindahl}} \sigma^2 \right)$$

Hence:

$$\sigma_{GDP} = \sigma h_{py}$$

Data

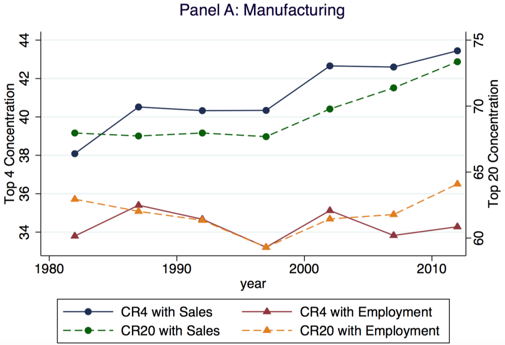


Herfindahl Index using Sales-to-GDP ratio

Compustat Data on universe of listed firms for the U.S.

Multiplied by $\sigma = 0.12$: volatility of about 1%

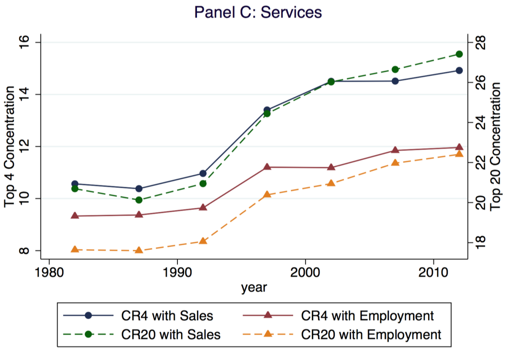
Recent trends: rising concentration



Fraction of sales and employment by top 4 or 20 firms.

Source: Autor et al (2019) based on U.S. Census

Recent trends: rising concentration



Fraction of sales and employment by top 4 or 20 firms.

Source: Autor et al (2019) based on U.S. Census

What have we done?

- Lucas' diversification result ✓
    - Macro effect of idiosyncratic shocks decays rapidly in N
    - Assumes equally-sized firms with equally sized shocks
- Gabaix (2011) ✓
    - Generalize the diversification result to heterogeneous firms
    - Macro effect of idiosyncratic shocks is Herfindahl $\times$ shock
    - Herfindahl decays slowly in number of firms if distribution is fat-tailed
- Hulten's theorem ✓
    - Domar weight is individual firms/sectors effect on aggregate output
    - Holds generally; very convenient result for empirical analysis

Congratulations!

Part I: Shocking theory of the business cycle

- Introduction to business cycles ✓
- Real Business Cycle (RBC) Model ✓
- New Keynesian DSGE Models ✓

Part II: Perspectives on business cycles and steady states

- Heterogeneity versus homogeneity and the effect of policy ✓
- Endogenous growth and persistent effects of recessions ✓
- Aggregate shocks? Firm-heterogeneity and the business cycle ✓