Estimating Quantitative Model

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UIBE

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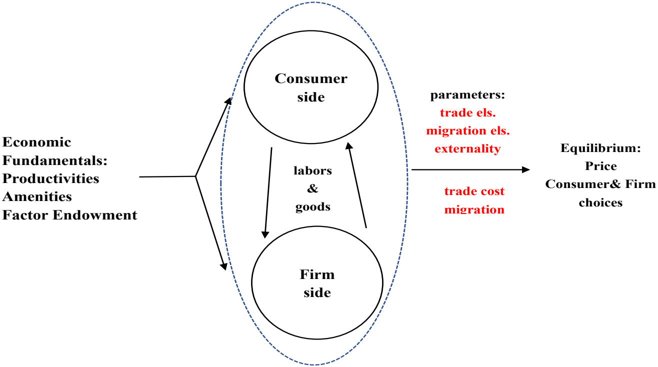
Road Map

- General introduction of estimation
- Structural trade gravity equation
  - Levchenko and Zhang (2016)
- Nested least square method
  - Fan(2019)
- Identification using quasi-experiment
  - Ahlfeldt,Redding,Sturm and Wolf (2015)

General introduction of estimation



General introduction of estimation

- Direct calibration
  - borrow from literature (eg. trade elasticity)
  - simple calculation(eg. Cobb-Douglas function share)
- Theoretic-model implied regression (Do not need to solve equilibrium)
  - structural gravity equation
  - regressors can be calculated from theoretic model
- Methods need to solve equilibrium
  - SMM, Indirect inference method, nested least square method
- Model inversion (backout econ fundamentals)
  - Model residuals (productivity, amenity,etc)

"The evolution of comparative advantage: Measurement and welfare implications"
Levchenko and Zhang,2016, JME

Overview

- This paper
  - builds multi-country, multi-sector EK model
  - estimates productivities at the sector level for 72 countries and 5 decades
  - examines how they evolve over time in both developed and developing countries
  - finds that comparative advantage has become weaker
  - finds that convergence in comparative advantage lowers trade and welfare

Model- Households

- final consumption
$$Y_{nt} = \left(\sum_{j=1}^{J} \omega_j^{\frac{1}{\eta}} \left(Y_{nt}^j \right)^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}\xi_{nt}} \left(Y_{nt}^{J+1} \right)^{1-\xi_{nt}}$$
where sector $J+1$ is untradable
- budget constrain $P_{nt}\left(C_{nt} + I_{nt}\right) = P_{nt} Y_{nt} = w_{nt} L_{nt} + r_{nt} K_{nt}$
- capital $K_{nt+1} = \left(1 - \delta_{nt}\right) K_{nt} + s_{nt} Y_{nt}$
where saving rate s_{nt} is assumed exogenous

Model- firms

- sectoral final goods $Q^j_{nt} = \left[\int_0^1 Q^j_{nt}(q)^{\frac{\varepsilon-1}{\varepsilon}} dq\right]^{\frac{\varepsilon}{\varepsilon-1}}$
- variety production

$$y^j_{nt}(q) = z^j_{nt}(q)\left(k^j_{nt}(q)^{1-\alpha_j} l^j_{nt}(q)^{\alpha_j}\right)^{\beta_j}\left(\prod_{j'=1}^{J+1} m^{j'j}_{nt}(q)^{\gamma'_j j}\right)^{1-\beta_j}$$

- idiosyncratic productivity shock $F^j_{nt}(z) = e^{-T^j_{nt} z^{-\theta}}$
- cost of input bundle $c^j_{nt} = \left(w^{\alpha_j}_{nt} r^{1-\alpha_j}_{nt}\right)^{\beta_j}\left(\prod_{j'=1}^{J+1}\left(p^{j'}_{nt}\right)^{\gamma_{j'j}}\right)^{1-\beta_j}$
- trade share $\frac{X^j_{nit}}{X^j_{nt}} = \pi^j_{nit} = \frac{T^j_{it}\left(c^j_{it} d^j_{nit}\right)^{-\theta}}{\Phi^j_{nt}}$
- price $P^j_{nt} = \Gamma(\Phi^j_{nt})^{-\frac{1}{\theta}}$

Direct Calibration

- Cobb-Douglas share α_j, β_j, $\gamma_{j'j}$, ξ (directly from data)
- fundamental L_{nt}, K_{nt}, s_{nt} (directly from data)
- other parameters θ, ε,η, ω_j (directly from literature)

Estimation

- step-1 estimate the fundamental productivity in the tradeable sectors relative to the U.S. and estimate trade cost
  - data: sectoral output and bilateral trade
- step-2 estimate the fundamental productivity in the tradeable sectors for the U.S.
  - data: US sectoral input and output
- step-3 estimate the fundamental productivity in the nontradeable sectors
  - data: PPP income per capita corss countries

step-1 Structural Regressable Equation

- divide the trade share

$$\frac{\pi^j_{nit}}{\pi^j_{nnt}} = \frac{X^j_{nit}}{X^j_{nnt}} = \frac{T^j_{it}\left(c^j_{it} d^j_{nit}\right)^{-\theta}}{T^j_{nt}\left(c^j_{nt}\right)^{-\theta}} \tag{1}$$

- take log of equation (1)

$$\ln\left(\frac{X^j_{\text{nit}}}{X^j_{nnt}}\right) = \ln\left(T^j_{it}\left(c^j_{it}\right)^{-\theta}\right) - \ln\left(T^j_{nt}\left(c^j_{nt}\right)^{-\theta}\right) - \theta \ln d^j_{\text{nit}} \tag{2}$$

- parameterize trade cost

$$\ln d^j_{\text{nit}} = d^j_{k,t} + b^j_{\text{nit}} + \mathrm{CU}^j_{\text{nit}} + \mathrm{RTA}^j_{\text{nit}} + ex^j_{it} + \nu^j_{nit} \tag{3}$$

step-1 Structural Regressable Equation

- plug (3) into (2) and rearrange the formula

$$
\ln\left(\frac{X_{nit}^{j}}{X_{nnt}^{j}}\right) = \underbrace{\ln\left(T_{it}^{j}\left(c_{it}^{j}\right)^{-\theta}\right) - \theta ex_{it}^{j}}_{\text{Exporter Fixed Effect}} \underbrace{- \ln\left(T_{nt}^{j}\left(c_{nt}^{j}\right)^{-\theta}\right)}_{\text{Importer Fixed Effect}} \\
\underbrace{-\theta d_{k,t}^{j} - \theta b_{\text{nit}}^{j} - \theta \mathrm{CU}_{\text{nit}}^{j} - \theta \mathrm{RTA}_{\text{nit}}^{j}}_{\text{Bilateral Observables}} \underbrace{-\theta \nu_{nit}^{j}}_{\text{Error Term}} \tag{4}
$$

step-1 Structural Interpretation of Importer Fixed Effect

- estimated importer FE with US as reference

$$S_{nt}^{j} = \frac{T_{nt}^{j}}{T_{ust}^{j}} \left(\frac{c_{nt}^{j}}{c_{ust}^{j}} \right)^{-\theta} \tag{5}$$

- to estimate $\frac{T_{nt}^{j}}{T_{ust}^{j}}$, need to calculate $\frac{c_{nt}^{j}}{c_{ust}^{j}}$, given estimated importer FE $\hat{S}_{nt}^{j}$

step-1 Calculate relative unit cost

- share of total spending going to home-produced goods

$$\pi^j_{nnt} = \frac{X^j_{nnt}}{X^j_{nt}} = T^j_{nt}\left(\frac{\Gamma c^j_{nt}}{p^j_{nt}}\right)^{-\theta}$$

- dividing by US counterpart

$$\frac{X^j_{nnt}/X^j_{nt}}{X^j_{us,us,t}/X^j_{ust}} = \frac{T^j_{nt}}{T^j_{ust}}\left(\frac{c^j_{nt}}{c^j_{ust}}\frac{p^j_{ust}}{p^j_{nt}}\right)^{-\theta} = S^j_{nt}\left(\frac{p^j_{ust}}{p^j_{nt}}\right)^{-\theta} \tag{6}$$

- inferred relative price

$$\frac{p^j_{nt}}{p^j_{ust}} = \left(\frac{X^j_{nnt}/X^j_{nt}}{X^j_{us,us,t}/X^j_{ust}}\frac{1}{S^j_{nt}}\right)^{\frac{1}{\theta}} \tag{7}$$

step-1 Backout relative technology

- cost of input bundles relative to the U.S.

$$\frac{c_{nt}^{j}}{c_{ust}}^{j} = \left(\frac{w_{nt}}{w_{ust}}\right)^{\alpha_j\beta_j} \left(\frac{r_{nt}}{r_{ust}}\right)^{\left(1-\alpha_j\right)\beta_j} \left(\prod_{j'=1}^{J}\left(\frac{p_{nt}^{j'}}{p_{ust}^{j'}}\right)^{\gamma_{j'j}}\right)^{1-\beta_j} \left(\frac{p_{nt}^{J+1}}{p_{ust}^{J+1}}\right)^{\gamma_{J+1,j}\left(1-\beta_j\right)} \tag{8}$$

- back out the relative technology parameter

$$\frac{T_{nt}^{j}}{T_{ust}^{j}} = S_{nt}^{j}\left(\frac{c_{nt}^{j}}{c_{ust}^{j}}\right)^{\theta} \tag{9}$$

step-2 U.S. Technology

- observed TFP for US tradable sector

$$\ln \Lambda^j_{ust} = \ln Z^j_{ust} - \beta_j \alpha_j \ln L^j_{ust} + \beta_j (1 - \alpha_j) \ln K^j_{ust} + (1 - \beta_j) \sum_{j'=1}^{J+1} \gamma_{j'j} \ln M^{j'j}_{ust} \tag{10}$$

- Ricardian selection (Finicelli et al., 2013)

$$\left(\Lambda^j_{ust}\right)^{\theta} = T^j_{ust} + \sum_{i \neq us} T^j_{it} \left(\frac{c^j_{it} d^j_{usit}}{c^j_{ust}}\right)^{-\theta} \tag{11}$$

- Correct selection bias

$$\begin{aligned} \left(\Lambda^j_{ust}\right)^{\theta} &= T^j_{ust} \left[1 + \sum_{i \neq us} \frac{T^j_{il}}{T^j_{ust}} \left(\frac{c^j_{it} d^j_{usit}}{c^j_{ust}}\right)^{-\theta}\right] \\ &= T^j_{ust} \left[1 + \sum_{i \neq us} S^j_{it} \left(d^j_{usit}\right)^{-\theta}\right] \end{aligned} \tag{12}$$

step-3 Nontradable Technology (model residual)

- match the observed PPP-adjusted income per capita
  - first, initial guess of $\{T_{nt}^{J+1}\}_{n=1}N$
  - second, solve model equilibrium and calculate income p.p.
  - third, update guess of $\{T_{nt}^{J+1}\}_{n=1}N$ until income p.p. in model converge to the observed counterpart (one-to-one mapping)

"Internal Geography, Labor Mobility, and the Distributional Impacts of Trade"
Fan, 2019, AEJ:Macro

Overview

- This paper
  - builds a spatial model with intranational trade and migration costs
  - estimates trade cost, migration cost, productivity, amenity, etc
  - quantifies the distributional impact of China's trade liberalization

Model- Wokers

- indirect utility by migrating from o to d

$$V_{o,d}^{e} = \frac{B_d^e W_d^e z_d(i)}{P_d d_{od}^e}$$

- idiosyncratic labor productivity shock

$$F(\mathbf{z}) = \exp\left(- \left(\sum_{d \in \mathbf{G}} z_d(i)^{-\epsilon_e} \right)^{1-\rho} \right)$$

- migration share

$$\pi_{od}^{e} = \frac{\left(\frac{v_d^e}{d_{od}}\right)^{\epsilon_e}}{\sum_{g \in \mathcal{G}} \left(\frac{v_g^e}{d_{og}}\right)^{\epsilon_e}}$$

- labor distribution

$$L_d^e = \sum_{o \in \mathcal{G}} l_{od}^e = \sum_{o \in \mathcal{G}} l_o^e \pi_{od}^e$$

Model- Production and trade

- sector $\{A, M, K, S\}$
- final consumption $C_d = \left(C_d^A\right)^{s_A} \left(C_d^M\right)^{s_M} \left(C_d^S\right)^{s_S}$
- sectoral final output

$$Q_d^s = \left[\int_{\omega \in \Omega_s} q_d^s(\omega)^{\frac{\sigma_s - 1}{\sigma_s}} d\omega\right]^{\frac{\sigma_s}{\sigma_s - 1}}$$

Model- Production and trade

- variety production

$$y_d^s(\omega) = t_d^s(\omega) l_d^s(\omega)^{\gamma_s^L}_{s' \in \{A,M,S\}} \prod_d^{ss'} (\omega)^{\gamma_s'}$$

- idiosyncratic productivity shock

$$F_d^S(t) = \exp\left(-\mathrm{T}^{\mathrm{s}}_d t^{-\theta}\right)$$

- trade share

$$\delta_{do}^s = \frac{T_o^s \left(c_o^s \tau_{do}\right)^{-\theta}}{\sum_{o'} T_{o'}^s \left(c_{o'}^s \tau_{do'}\right)^{-\theta}}$$

Model- Equipped Composite Labor Production

- equipped high-skill labor

$$E_d^{eh} = \left[\left(1 - \eta_d^h\right)^{\frac{1}{\rho_{kh}}} (K_d)^{\frac{\rho_{kh}-1}{\rho_{kh}}} + \left(\eta_d^h\right)^{\frac{1}{\rho_{kh}}} \left(E_d^h\right)^{\frac{\rho_{kh}-1}{\rho_{kn}}} \right]^{\frac{\rho_{kh}}{\rho_{kh}-1}}$$

- equipped composite Labor

$$E_d = \left[\left(1 - \eta_d^l\right)^{\frac{1}{\rho_{kh}}} \left(E_d^l\right)^{\frac{\rho_{kh}-1}{\rho_{kh}}} + \left(\eta_d^l\right)^{\frac{1}{\rho_{kk}}} \left(E_d^{eh}\right)^{\frac{\rho_{kh}-1}{\rho_{kh}}} \right]^{\frac{\rho_{kh}}{\rho_{kh}-1}}$$

Data for estimation

Calibrate the model to the Chinese economy around 2005

- regional productivity
  - data: wages and employment for high- and low-skill workers, sectoral output in all regions
- region-specific parameters in the equipped composite labor production function
  - data: shares of factors in equipped composite labor
- domestic migration costs
  - data: migration flows
- trade costs
  - data: domestic trade flows

Model parameters

Parameter	Description	Target/source	Value
Panel A. Parameters calibrated independently			
ρ	Worker productivity draws correlation	Correlation in wages for migrants	0.36
ϵ^h, ϵ^l	Worker productivity draws dispersion	Equation (17)	$\epsilon^h = 2.73/(1-\rho), \epsilon^l = 2.5/(1-\rho)$
θ	Elasticity of trade	Simonovska and Waugh (2014)	4
ρ_{kh}, ρ_{lkh}	Elasticities in equipped composite labor	Krusell et al. (2000)	$\rho_{kh} = 0.67, \rho_{lkh} = 1.67$
s_A, s_M, s_S	Sectoral shares in final consumption	Aggregate consumption share	$s_A = 0.22, s_M = 0.24$ $s_S = 0.53$
$\gamma_s^{s'}$	Input-output linkages	National input-output tables	Online Appendix B.4
Panel B. Parameters estimates/calibrated in equilibrium			
$\{d_{o,d}\}$	Migration costs	Migration flows	Table 3
$\{\tau_{o,d}\}$	Domestic trade costs	Domestic trade and city import/export	Table 4
$\{t_a, t_m, t_k\}$	Trade costs between ports and RoW	Sectoral international trade	Table 4
$\{\eta_d^h\}, \{\eta_d^e\}$	Equipped labor production function	Corresponding factor shares	—

Parameters Calibrated Independently

- variation of wage distribution of workers sharing the same migration origin and destination

$$\frac{\text{Variance}}{\text{Mean}^2} = \frac{\Gamma\left(1 - \frac{2}{\epsilon_e(1-\rho)}\right)}{\left(\Gamma\left(1 - \frac{1}{\epsilon_e(1-\rho)}\right)\right)^2} - 1$$

- dispersion in individual level wage within regions
  $\Rightarrow \widehat{\epsilon_e(1-\rho)}$

- explanatory power of individual FE in panel regression of migrants' wage
  $\Rightarrow$ correlation of individuals' productivity draws $\widehat{\rho}$

trade block estimation

- parameterize the domestic trade cost

$$\log\left(\tau_{o,d}\right) = \sum_{i=1}^{4} \gamma_i l_i' + \gamma_5 \times l_1' \times dist_{o,d} + \gamma_6 \times l_2' \times \text{dist}_{o,d} + \gamma_7 \times l_3' \times \text{ dist }_{o,d} + \gamma \times \text{ Cdist }_{o,d} + \epsilon_{o,d} \tag{13}$$

- international trade cost

$$\log\left(\tau_{\text{interior },RoW}\right) = \log\left(\tau_{\text{interior,port }}\right) + t_s$$

nested nonlinear least square procedure

- Given initial $\{\gamma\}$ in eq.(13), solve intl. trade cost $\{t_s\}$ and sectoral-level productivity $\{T_d^s\}$ (given labor distribution)
  - given initial $\{t_s\}$, solve $\{T_d^s\}$ to match region-sector output
  - update $\{t_s\}$ to match sectoral openness ratio
- update $\{\gamma\}$ in eq.(13) to minimize

$$\max_{\{\gamma\}} \sum_{p_i, p_j \in \mathcal{P}} \left(\log\left(X_{p_i,p_j}^{\text{data}}\right) - \log\left(\sum_{o \in p_i, d \in p_j} X_{o,d}^{\text{model}} \right) \right)^2 + \sum_{o \in \mathcal{G}, d = RoW \cup d \in \mathcal{G}, o = RoW} \left(\log\left(X_{o,d}^{\text{model}}\right) - \log\left(X_{o,d}^{\text{data}}\right) \right)^2$$

Domestic and international trade cost

	Coefficient	Standard error
Panel A. Domestic trade cost estimates		
1(Different cities, same province)	0.57	(0.13)
1(Different provinces, same region)	1.21	(0.10)
1(Different regions)	1.51	(0.07)
1(Sharing provincial border)	−0.06	(0.06)
1(Same province) × Distance	0.01	(0.12)
1(Different provinces, same region) × Distance	0.21	(0.10)
1(Different regions) × Distance	0.04	(0.03)
Cultural distance	0.20	(0.08)
Observations	1,580	
R^2	0.50	
	Trade/production	Trade costs
Panel B. International trade cost calibration: Targets and parameter values		
Agricultural industry	0.12	0.99
Manufacturing industry	0.36	0.80
Capital and equipment industry	0.46	0.72

Migration block

- parameterize migration cost

$$\ln\left(d_{o,d}^{e}\right)=\sum_{i=1}^{4}\beta_i^e I_i+\beta_5^e*I_1*\text{dist}_{0,d}+\beta_6^e*I_2*\text{dist}_{o,d}+\beta_7^e*I_3*\text{dist}_{0,d}+\beta_8^e*\text{Cdist}_{o,d}+\mu_{o,d} \tag{14}$$

- nested nonlinear least square procedure
  - Outer-loop: choose $\{\beta\}$ in eq.(14) to minimize

$$\min_{\{\beta^e\}}\sum_{p_i\in\mathbf{P},d\in G}\left(\log\left(\sum_{o\in p_i} l_o^e\pi_{od}^e\right)-\log\left(L_{p_i d}^{\text{data},\ e}\right)\right)^2$$

  - Inner-loop: choose $\{v_d^e\}$(amenity-adjusted real wages) to match workers distribution, given initial labor distribution (one-to-one mapping)

Back out amenity B_d

- amenity-adjusted real wages $v_d^e = \frac{B_d^e W_d^e}{P_d}$
    - v_d^e is calculated in migration block estimation
    - wage W_d is from data
    - regional price index P_d can be calculated using wage data and estimated trade cost

Model validation

NON-TARGETED MOMENTS

	Data	Model
Skill-premium change and distance to port:		
Coastal minus non-coastal province	≈ 5	4.3
Distance measure	≈ -1.9	-1.6
City growth and distance to port:		
City-level regression	-3.4	-4.8
Within province variation only	-7.9	-11.1

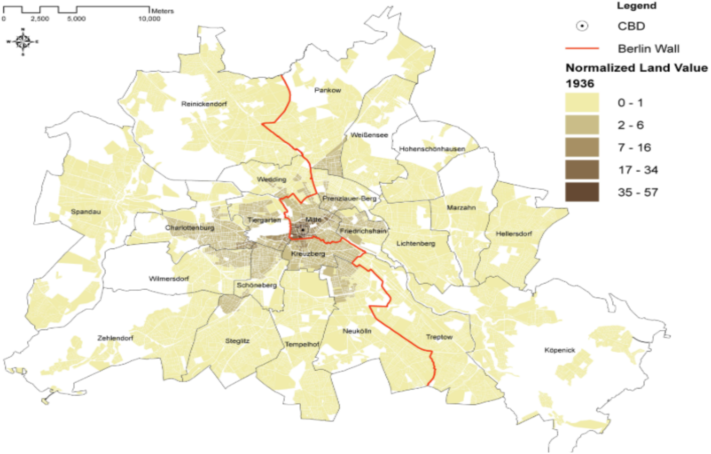
- model part is the change from autarky to benchmark
- upper panel compares the model predictions on the impacts of trade liberalization on skill premia to the empirical estimates from Han, Liu, and Zhang (2012)
- lower panel compares the model predictions on population growth to the empirical counterparts

"The economics of density: evidence from the Berlin Wall"
Ahlfeldt GM, Redding SJ, Sturm DM, Wolf N. 2015, Econometrica

Overview

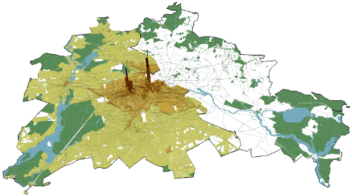
- This paper
  - paper develops a quantitative model of internal city structure that features agglomeration and dispersion forces and an arbitrary number of heterogeneous city blocks
  - structurally estimates agglomeration and dispersion forces using and exogenous variation from the cityąŕs division and reunification
  - finds substantial and highly localized production and residential externalities
  - shows that the model with the estimated agglomeration parameters can account both qualitatively and quantitatively for the observed changes in city structure

Land Values in Berlin in 1936

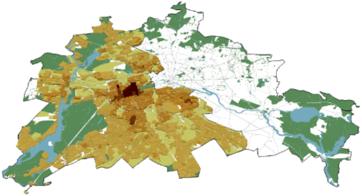


The Evolution of Land Prices in Berlin Over Time

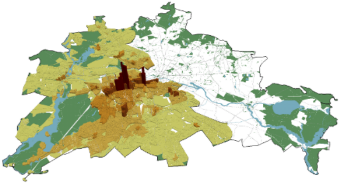
Panel B: West Berlin Land Prices 1936



Panel C: West Berlin Land Prices 1986



Panel E: West Berlin Land Prices 2006



DID estimation

- $\Delta \ln O_i = \alpha + \sum_{k=1}^{K} \mathbb{I}_{ik}\beta_k + \ln M_i\gamma + u_i$
  - $\Delta \ln O_i$: the change in an economic outcome of interest (floor prices, workplace employment, residence employment)
  - $\mathbb{I}_{ik}$: indicator variable for whether block i lies within a distance grid cell k from the pre-war CBD
  - M_i: time-invariant observable block characteristics
  - u_i: error term
  - Identification assumption: change in fundamentals are uncorrelated with distance to pre-war CBD

DID estimation

	(1) $\Delta \ln Q$	(2) $\Delta \ln Q$	(3) $\Delta \ln Q$	(4) $\Delta \ln Q$	(5) $\Delta \ln Q$	(6) $\Delta \ln$ EmpR	(7) $\Delta \ln$ EmpR	(8) $\Delta \ln$ EmpW	(9) $\Delta \ln$ EmpW
CBD 1	−0.800***	−0.567***	−0.524***	−0.503***	−0.565***	−1.332***	−0.975***	−0.691*	−0.639*
	(0.071)	(0.071)	(0.071)	(0.071)	(0.077)	(0.383)	(0.311)	(0.408)	(0.338)
CBD 2	−0.655***	−0.422***	−0.392***	−0.360***	−0.400***	−0.715**	−0.361	−1.253***	−1.367***
	(0.042)	(0.047)	(0.046)	(0.043)	(0.050)	(0.299)	(0.280)	(0.293)	(0.243)
CBD 3	−0.543***	−0.306***	−0.294***	−0.258***	−0.247***	−0.911***	−0.460**	−0.341	−0.471**
	(0.034)	(0.039)	(0.037)	(0.032)	(0.034)	(0.239)	(0.206)	(0.241)	(0.190)
CBD 4	−0.436***	−0.207***	−0.193***	−0.166***	−0.176***	−0.356**	−0.259	−0.512***	−0.521***
	(0.022)	(0.033)	(0.033)	(0.030)	(0.026)	(0.145)	(0.159)	(0.199)	(0.169)
CBD 5	−0.353***	−0.139***	−0.123***	−0.098***	−0.100***	−0.301***	−0.143	−0.436***	−0.340***
	(0.016)	(0.024)	(0.024)	(0.023)	(0.020)	(0.110)	(0.113)	(0.151)	(0.124)
CBD 6	−0.291***	−0.125***	−0.094***	−0.077***	−0.090***	−0.360***	−0.135	−0.280**	−0.142
	(0.018)	(0.019)	(0.017)	(0.016)	(0.016)	(0.100)	(0.089)	(0.130)	(0.116)
Inner Boundary 1–6			Yes	Yes	Yes		Yes		Yes
Outer Boundary 1–6			Yes	Yes	Yes		Yes		Yes
Kudamm 1–6				Yes	Yes		Yes		Yes
Block Characteristics					Yes		Yes		Yes
District Fixed Effects		Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	6,260	6,260	6,260	6,260	6,260	5,978	5,978	2,844	2,844
R^2	0.26	0.51	0.63	0.65	0.71	0.19	0.43	0.12	0.33

[a] Q denotes the price of floor space. EmpR denotes employment by residence. EmpW denotes employment by workplace. CBD1–CBD6 are six 500 m distance grid cells for distance from the pre-war CBD. Inner Boundary 1–6 are six 500 m grid cells for distance to the Inner Boundary between East and West Berlin. Outer Boundary 1–6 are six 500 m grid cells for distance to the outer boundary between West Berlin and East Germany. Kudamm 1–6 are six 500 m grid cells for distance to Breitscheid Platz on the Kurfürstendamm. The coefficients on the other distance grid cells are reported in Table A.2 of the Technical Data Appendix. Block characteristics include the log distance to schools, parks and water, the land area of the block, the share of the block's built-up area destroyed during the Second World War, indicators for residential, commercial and industrial land use, and indicators for whether a block includes a government building and urban regeneration policies post-reunification. Heteroscedasticity and Autocorrelation Consistent (HAC) standard errors in parentheses (Conley (1999)). * significant at 10%; ** significant at 5%; *** significant at 1%.

Theoretic Model

Workers

- utility function $C_{ijo} = \frac{B_i z_{ijo}}{d_{ij}} \left(\frac{c_{ijo}}{\beta}\right)^{\beta} \left(\frac{\ell_{ijo}}{1-\beta}\right)^{1-\beta}$
- idiosyncratic component $F\left(z_{ijo}\right) = e^{-T_i E_j z_{ijo}^{-\epsilon}}$
- indirect utility function $u_{ijo} = \frac{z_{ijo} B_i w_j Q_i^{\beta-1}}{d_{ij}}$
- commuting cost $d_{ij} = e^{\kappa \tau_{ij}}$

Workers

- the probability that a worker chooses to live in block i and work in block j

$$\pi_{ij} = \frac{T_i E_j \left(d_{ij} Q_i^{1-\beta}\right)^{-\epsilon} (B_i w_j)^{\epsilon}}{\sum_{r=1}^{S} \sum_{s=1}^{S} T_r E_s \left(d_{rs} Q_r^{1-\beta}\right)^{-\epsilon} (B_r w_s)^{\epsilon}} \equiv \frac{\Phi_{ij}}{\Phi}$$

- probability that a worker resides in block i

$$\pi_{Ri} = \sum_{j=1}^{S} \pi_{ij} = \frac{\sum_{j=1}^{S} \Phi_{ij}}{\Phi}$$

- probability that a worker works in block j

$$\pi_{Mj} = \sum_{i=1}^{S} \pi_{ij} = \frac{\sum_{i=1}^{S} \Phi_{ij}}{\Phi}$$

- Conditional on living in block i, the probability that a worker commutes to block j

$$\pi_{ij|i} = \frac{E_j \left(w_j / d_{ij}\right)^{\epsilon}}{\sum_{s=1}^{S} E_s \left(w_s / d_{is}\right)^{\epsilon}}$$

Workers

- commuting market clearing condition

$$H_{Mj} = \sum_{i=1}^{S} \frac{E_j \left(w_j / d_{ij}\right)^{\epsilon}}{\sum_{s=1}^{S} E_s \left(w_s / d_{is}\right)^{\epsilon}} H_{Ri}$$

- Expected worker income conditional on living in block i

$$\mathbb{E}\left[w_j \mid i\right] = \sum_{j=1}^{S} \frac{E_j \left(w_j / d_{ij}\right)^{\epsilon}}{\sum_{s=1}^{S} E_s \left(w_s / d_{is}\right)^{\epsilon}} w_j$$

- the reservation level of utility in the wider economy$\longrightarrow$ pin down total labor supply

$$\mathbb{E}[u] = \gamma \left[\sum_{r=1}^{S} \sum_{s=1}^{S} T_r E_s \left(d_{rs} Q_r^{1-\beta}\right)^{-\epsilon} \left(B_r w_s\right)^{\epsilon}\right]^{1/\epsilon} = \bar{U}$$

Production

- production function $y_j = A_j H_{Mj}^{\alpha} L_{Mj}^{1-\alpha}$
- employment demand in block j $H_{Mj} = \left(\frac{\alpha A_j}{w_j}\right)^{\frac{1}{1-\alpha}} L_{Mj}$
- commercial floor prices $q_j = (1-\alpha)\left(\frac{\alpha}{w_j}\right)^{\frac{\alpha}{1-\alpha}} A_j^{\frac{1}{1-\alpha}}$

Land Market

- The share of floor space used commercially (no-arbitrage)
  $\theta_i = 1 \quad \text{if} \quad q_i > \xi_i Q_i$
  $\theta_i \in [0,1] \quad \text{if} \quad q_i = \xi_i Q_i$
  $\theta_i = 0 \quad \text{if} \quad q_i < \xi_i Q_i$
  where ξ_i captures tax equivalent of land use regulations

- observed floor prices
  $$\mathbb{Q}_i = q_i, \quad q_i > \xi_i Q_i, \quad \theta_i = 1$$
  $$\mathbb{Q}_i = q_i, \quad q_i = \xi_i Q_i, \quad \theta_i \in [0,1],$$
  $$\mathbb{Q}_i = Q_i, \quad q_i < \xi_i Q_i, \quad \theta_i = 0$$

- supply of floor: $L_i = \varphi_i K_i^{(1-\mu)}$
  where K_i is land area

Land Market

- Residential land market clearing

$$\mathbb{E}\left[\ell_i\right] H_{Ri} = (1-\beta)\frac{\mathbb{E}\left[w_s \mid i\right] H_{Ri}}{Q_i} = \left(1-\theta_i\right) L_i$$

- Commercial land market clearing

$$\left(\frac{(1-\alpha)A_j}{q_j}\right)^{\frac{1}{\alpha}} H_{Mj} = \theta_j L_j$$

- land market clearing

$$\left(1-\theta_i\right) L_i + \theta_i L_i = L_i = \varphi_i K_i^{(1-\mu)}$$

Externality

- production externality

$$A_j = a_j \Upsilon_j^{\lambda}, \quad \Upsilon_j \equiv \sum_{s=1}^{S} e^{-\delta \tau_{js}} \left(\frac{H_{Ms}}{K_s} \right)$$

where $\left(\frac{H_{Ms}}{K_s} \right)$ is workplace employment density

- residential externality

$$B_i = b_i \Omega_i^{\eta}, \quad \Omega_i \equiv \sum_{r=1}^{S} e^{-\rho \tau_{ir}} \left(\frac{H_{Rr}}{K_r} \right)$$

where $\left(\frac{H_{Rr}}{K_r} \right)$ is residence employment density

fundamental composites

$$\tilde{A}_i = A_i E_i^{\alpha/\varepsilon}$$

$$\tilde{B}_i = B_i T_i^{1/\varepsilon} \zeta_{Ri}^{1-\beta}$$

$$\tilde{w}_i = w_i E_i^{1/\varepsilon}$$

$$\tilde{a}_i = a_i E_i^{\alpha/\varepsilon}$$

$$\tilde{b}_i = b_i T_i^{1/\varepsilon} \zeta_{Ri}^{1-\beta}$$

Backout composite fundamentals

- backout adjusted wage $\omega_{jt} = \tilde{w}_{jt}^{\varepsilon}$ using workplace/residence employment and commuting time, given ν ($\nu = \varepsilon\kappa$)

$$H_{Mjt} = \sum_{i=1}^{S} \frac{\omega_{jt}/e^{\nu\tau_{ijt}}}{\sum_{s=1}^{S} \omega_{st}/e^{\nu\tau_{ist}}} H_{Rit}$$

- backout adjusted productivity using floor price and adj. wage

$$\ln\left(\frac{\tilde{A}_{it}}{\bar{\tilde{A}}_t}\right) = (1-\alpha)\ln\left(\frac{\mathbb{Q}_{it}}{\bar{\mathbb{Q}}_t}\right) + \frac{\alpha}{\varepsilon}\ln\left(\frac{\omega_{it}}{\bar{\omega}_t}\right)$$

- backout adjusted amenity using residence employment, floor price and commuting market access

$$\ln\left(\frac{\tilde{B}_{it}}{\bar{\tilde{B}}_t}\right) = \frac{1}{\varepsilon}\ln\left(\frac{H_{Rit}}{\bar{H}_{Rt}}\right) + (1-\beta)\ln\left(\frac{\mathbb{Q}_{it}}{\bar{\mathbb{Q}}_t}\right) - \frac{1}{\varepsilon}\ln\left(\frac{W_{it}}{\bar{W}_t}\right)$$

where commuting market access
$W_{it} = \sum_{s=1}^{S} \omega_{st}/e^{\nu\tau_{ist}}, \quad \omega_{st} = \tilde{w}_{st}^{\varepsilon} = E_{st} w_{st}^{\varepsilon}$

Identification assumption

- backout change in exogenous fundamentals
  - exogenous productivity

$$\Delta \ln \left(\frac{\tilde{a}_{it}}{\bar{\tilde{a}}_t}\right) = (1-\alpha)\Delta \ln \left(\frac{\mathbb{Q}_{it}}{\bar{\mathbb{Q}}_t}\right)+\frac{\alpha}{\varepsilon}\Delta \ln \left(\frac{\omega_{it}}{\bar{\omega}_t}\right)-\lambda\Delta \ln \left(\frac{\Upsilon_{it}}{\bar{\Upsilon}_t}\right)$$

  - exogenous amenity

$$\begin{aligned}\Delta \ln \left(\frac{\tilde{b}_{it}}{\bar{\tilde{b}}_t}\right) =&\frac{1}{\varepsilon}\Delta \ln \left(\frac{H_{Rit}}{\bar{H}_{Rt}}\right) + (1-\beta)\Delta \ln \left(\frac{\mathbb{Q}_{it}}{\bar{\mathbb{Q}}_t}\right) \\ &- \frac{1}{\varepsilon}\Delta \ln \left(\frac{W_{it}}{\bar{W}_t}\right) - \eta\Delta \ln \left(\frac{\Omega_{it}}{\bar{\Omega}_t}\right)\end{aligned}$$

- Identification assumption:
  $\mathbb{E}\left[\mathbb{I}_k \times \Delta \ln \left(\tilde{a}_{it}/\bar{\tilde{a}}_t\right)\right] = 0$
  $\mathbb{E}\left[\mathbb{I}_k \times \Delta \ln \left(\tilde{b}_{it}/\bar{\tilde{b}}_t\right)\right] = 0$

GMM estimation

Moment condition

- number of workers commuting for less than 30 minutes (ν)

$$\mathbb{E}\left[\psi H_{Mj} - \sum_{i\in\aleph_j}^{S} \frac{\omega_j / e^{\nu\tau_{ij}}}{\sum_{s=1}^{S}\omega_s / e^{\nu\tau_{is}}} H_{Ri}\right] = 0$$

- variance of log adjusted wages (ϵ)

$$\mathbb{E}\left[(1/\varepsilon)^2 \ln(\omega_j)^2 - \sigma^2_{\ln w_i}\right] = 0$$

- exclusive restriction (λ,δ,η,ρ)

$$\mathbb{E}\left[\mathbb{I}_k \times \Delta \ln\left(\tilde{a}_{it} t \bar{\tilde{a}}_t\right)\right] = 0$$

$$\mathbb{E}\left[\mathbb{I}_k \times \Delta \ln\left(\tilde{b}_{it} / \bar{\tilde{b}}_t\right)\right] = 0$$

GMM estimation

The efficient GMM estimator solves

$$\hat{\mathbf{\Lambda}}_{\mathrm{GMM}} = \arg\min \left(\frac{1}{N} \sum_{i=1}^{N} m\left(\mathbf{X}_i, \mathbf{\Lambda}\right)' \right) \mathbb{W} \left(\frac{1}{N} \sum_{i=1}^{N} m\left(\mathbf{X}_i, \mathbf{\Lambda}\right) \right)$$

GMM estimation result

	(1) Division Efficient GMM	(2) Reunification Efficient GMM	(3) Division and Reunification Efficient GMM
Commuting Travel Time Elasticity ($\kappa\varepsilon$)	0.0951***	0.1011***	0.0987***
	(0.0016)	(0.0016)	(0.0016)
Commuting Heterogeneity (ε)	6.6190***	6.7620***	6.6941***
	(0.0939)	(0.1005)	(0.0934)
Productivity Elasticity (λ)	0.0793***	0.0496***	0.0710***
	(0.0064)	(0.0079)	(0.0054)
Productivity Decay (δ)	0.3585***	0.9246***	0.3617***
	(0.1030)	(0.3525)	(0.0782)
Residential Elasticity (η)	0.1548***	0.0757**	0.1553***
	(0.0092)	(0.0313)	(0.0083)
Residential Decay (ρ)	0.9094***	0.5531	0.7595***
	(0.2968)	(0.3979)	(0.1741)

[a]Generalized Method of Moments (GMM) estimates. Heteroscedasticity and Autocorrelation Consistent (HAC) standard errors in parentheses (Conley (1999)). * significant at 10%; ** significant at 5%; *** significant at 1%.

ν ε δ λ η ρ Monte Carlo

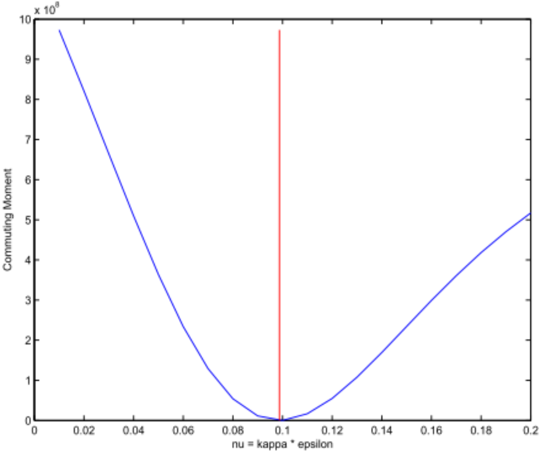
Externalities and commuting costs

	(1) Production Externalities $(1 \times e^{-\delta\tau})$	(2) Residential Externalities $(1 \times e^{-\rho\tau})$	(3) Utility After Commuting $(1 \times e^{-\kappa\tau})$
0 minutes	1.000	1.000	1.000
1 minute	0.696	0.468	0.985
2 minutes	0.485	0.219	0.971
3 minutes	0.338	0.102	0.957
5 minutes	0.164	0.022	0.929
7 minutes	0.079	0.005	0.902
10 minutes	0.027	0.001	0.863
15 minutes	0.004	0.000	0.802
20 minutes	0.001	0.000	0.745
30 minutes	0.000	0.000	0.642

Counterfactual treatment effects

	(1) $\Delta \ln QC$ 1936–1986	(2) $\Delta \ln QC$ 1936–1986	(3) $\Delta \ln QC$ 1936–1986	(4) $\Delta \ln QC$ 1936–1986	(5) $\Delta \ln QC$ 1986–2006	(6) $\Delta \ln QC$ 1986–2006	(7) $\Delta \ln QC$ 1986–2006
CBD 1	−0.836***	−0.613***	−0.467***	−0.821***	0.363***	1.160***	0.392***
	(0.052)	(0.032)	(0.060)	(0.051)	(0.041)	(0.052)	(0.043)
CBD 2	−0.560***	−0.397***	−0.364***	−0.624***	0.239***	0.779***	0.244***
	(0.034)	(0.025)	(0.019)	(0.029)	(0.028)	(0.044)	(0.027)
CBD 3	−0.455***	−0.312***	−0.336***	−0.530***	0.163***	0.594***	0.179***
	(0.036)	(0.030)	(0.030)	(0.036)	(0.031)	(0.045)	(0.031)
CBD 4	−0.423***	−0.284***	−0.340***	−0.517***	0.140***	0.445***	0.143***
	(0.026)	(0.019)	(0.022)	(0.031)	(0.021)	(0.042)	(0.021)
CBD 5	−0.418***	−0.265***	−0.351***	−0.512***	0.177***	0.403***	0.180***
	(0.032)	(0.022)	(0.027)	(0.039)	(0.032)	(0.038)	(0.032)
CBD 6	−0.349***	−0.222***	−0.304***	−0.430***	0.100***	0.334***	0.103***
	(0.025)	(0.016)	(0.022)	(0.029)	(0.024)	(0.034)	(0.023)
Counterfactuals	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Agglomeration Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	6,260	6,260	6,260	6,260	7,050	6,260	7,050
R^2	0.11	0.13	0.07	0.13	0.12	0.24	0.13

Commuting Moment Condition



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Wage Moment Condition

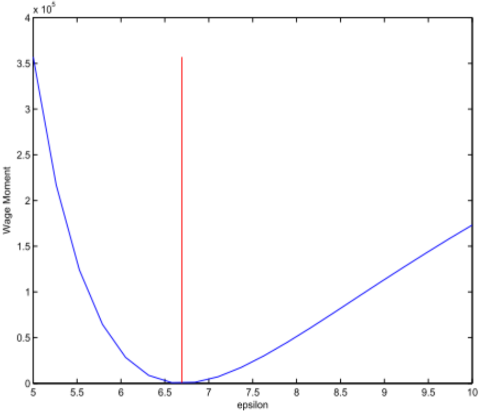

Figure A.4: Wage Moment Condition (Sum of Squared Deviations)

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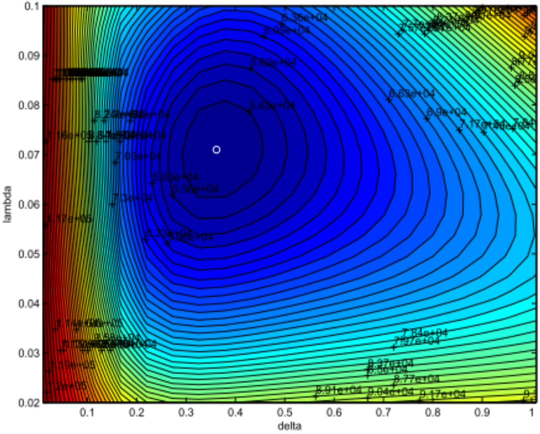
Production Fundamentals Moment Condition



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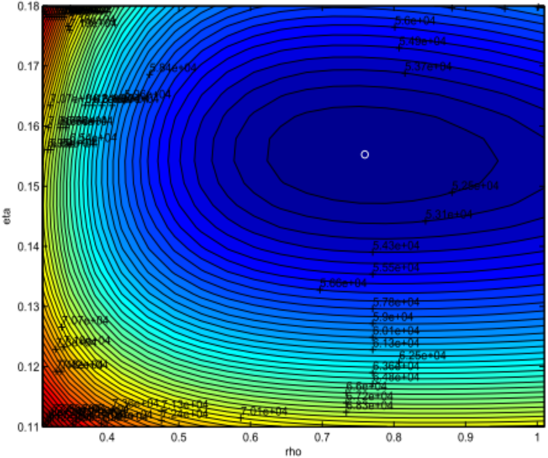
Residential Fundamentals Moment Condition



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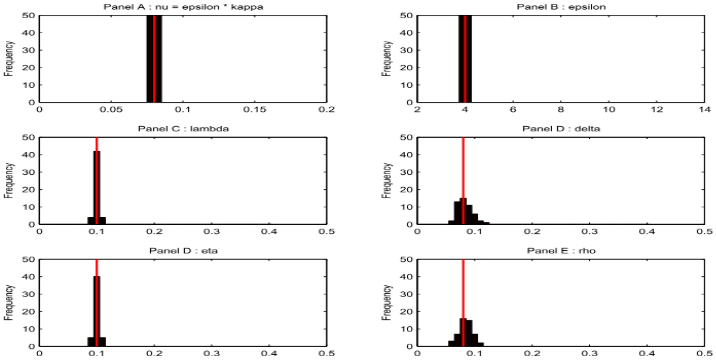
Monte Carlo Results



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