Lecture 3

Analyzing and Evaluating Real Business Cycle Models

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EC417

This term

Part I: Shocking theory of the business cycle (weeks 1-6)

- Introduction to business cycles ✓
- Real Business Cycle (RBC) Model $\Leftarrow$
- New Keynesian DSGE Models

Part II: Perspectives on business cycles and steady states (weeks 7-10)

- Persistent effects of recessions
- Aggregate shocks? Firm-heterogeneity and the business cycle
- Interesting steady states: firms, productivity, market power

References

Gali, Jordi (1999) "Technology, Employment, and the Business Cycle: Do Technology Shocks Explain Aggregate Fluctuations?", *American Economic Review*, March

King, Robert and Rebelo, Sergio (1999) "Resuscitating Real Business Cycles", *Handbook of Macroeconomics*, Volume 1B, Chapter 14

Prescott, Edward (1986) "Theory Ahead of Business Cycle Measurement", *Federal Reserve Bank of Minneapolis Quarterly Review*, Fall

Summers, Lawrence (1986) "Some Skeptical Observations on Real Business Cycle Theory", *Federal Reserve Bank of Minneapolis Quarterly Review*, Fall

Last week

- Tool: local approximation (a.k.a perturbation) ✓
- Solve the model using Matlab plug-in Dynare ✓
- Tool: Log-linearization ✓
- **Determinacy and existence: Blanchard Kahn conditions**

Blanchard - Kahn conditions

So far: found the (approximate) policy functions using software

- Didn't think about existence, uniqueness of a solution
- Our approach 'worked' because of the parameters I picked

Blanchard Kahn (1980) conditions determine for linear(ized) models

- Whether solutions exist and are unique
- Both under the condition of stability
- Slides based on notes from Wouter Den Haan

Existence, uniqueness

Consider the following simple model:

$$y_{t+1} = \rho y_t$$

- How many solutions does the above model have?
- Does it depend on the value of ρ?

Existence, uniqueness

Consider the following simple model:

$$y_{t+1} = \rho y_t$$

$$y_1 \text{ is given}$$

- How many solutions does the above model have?
- Does it depend on the value of ρ?

Existence, uniqueness

Consider the following simple model:

$$y_{t+1} = \rho y_t$$

$$y_t \text{ cannot explode}$$

$$\text{i.e. } \lim_{j \to \infty} \mathbb{E}_t y_{t+j} < \infty$$

- How many solutions does the above model have?
- Does it depend on the value of ρ?
  - $\rho > 1$: unique solution
  - $\rho \leq 1$: many solutions

Back to DSGE models

State space representation of a (log) linearized RBC model:

$$Ay_{t+1} + By_t = C\epsilon_{t+1}$$

- y_t is an $n \times 1$ vector of variables
- n_y elements are not pre-determined ($n - n_y$ state variables)
- Blanchard Kahn: look at A, B, C, check if it's a stable/unique solution

Simple RBC model

Linearized model:

$$k_{t+1} = a_1 k_t + a_2 k_{t-1} + a_3 z_{t+1} + a_4 z_t + e_{\mathbb{E},t+1}$$

$$z_{t+1} = \rho z_t + e_{z,t+1}$$

$$k_0 \text{ given}$$

- Allowing for persistent productivity: z_t is a state variable
- Capital is 'end of period' capital (k_t chosen at time t)

State space representation:

$$\begin{bmatrix} 1 & 0 & -a_3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} k_{t+1} \\ k_t \\ z_{t+1} \end{bmatrix} + \begin{bmatrix} -a_1 & -a_2 & -a_4 \\ -1 & 0 & 0 \\ 0 & 0 & -\rho \end{bmatrix} \begin{bmatrix} k_t \\ k_{t-1} \\ z_t \end{bmatrix} = \begin{bmatrix} e_{\mathbb{E},t+1} \\ 0 \\ e_{z,t+1} \end{bmatrix}$$

Back to DSGE models

State-space representation of a (log) linearized RBC model:

$$Ay_{t+1} + By_t = C\epsilon_{t+1}$$

- y_t is an $n \times 1$ vector of variables
- n_y elements are not pre-determined ($n - n_y$ state variables)

Solving the model:

$$\begin{aligned} y_{t+1} &= -A^{-1}By_t + A^{-1}C\epsilon_{t+1} \\ &= Dy_t + G\epsilon_{t+1} \end{aligned}$$

Blanchard Kahn conditions:

- **Define** h as number of eigenvalues of D with absolute value above 1
- Model has stable, unique solution if $h = n_y$

Decomposing the policy function

Eigenvalue-eigenvector decomposition of D:

$$D = P \Lambda P^{-1}$$

- Λ is a **diagonal** matrix with the eigenvalues of D
- P: matrix of the associated eigenvectors
- let $P^{-1} = (p_1, p_2, ..., p_n)'$ with p_i being an $1 \times n$ vector

Path of the economy

Iterating the state-space system:

$$\begin{aligned} y_{t+1} &= Dy_t + G\epsilon_{t+1} \\ &= D^t y_1 + \sum_{j=1}^{t} D^{t-j} G\epsilon_{j+1} \end{aligned}$$

Using the property $D^n = P\Lambda^n P^{-1}$

$$\begin{aligned} y_{t+1} &= P\Lambda^t P^{-1} y_1 + \sum_{j=1}^{t} P\Lambda^{t-j} P^{-1} G\epsilon_{j+1} \\ P^{-1} y_{t+1} &= \Lambda^t P^{-1} y_1 + \sum_{j=1}^{t} \Lambda^{t-j} P^{-1} G\epsilon_{j+1} \end{aligned}$$

Path of the economy

Row by row (we can do this because Λ is diagonal):

$$
\begin{aligned}
P^{-1} y_{t+1} &= \Lambda^t P^{-1} y_1 + \sum_{j=1}^{t} \Lambda^{t-j} P^{-1} G \epsilon_{j+1} \\
p_i y_{t+1} &= {\lambda_i}^t p_i y_1 + \sum_{j=1}^{t} {\lambda_i}^{t-j} p_i G \epsilon_{j+1}
\end{aligned}
$$

Note that:

- y_t is $n \times 1$ and p_i is $1 \times n$. Hence: $p_i y_{t+1}$, $p_i y_{t+1}$ are scalars
- Same holds for $p_i G \epsilon_j$ as G is $n \times n$
- $\tilde{\lambda}_i$ is a row of zeros except for λ_i

Blanchard Kahn conditions

What do we learn from this?

$$p_i y_{t+1} = {\lambda_i}^t p_i y_1 + \sum_{j=1}^{t} {\lambda_i}^{t-j} p_i G \epsilon_j$$

Economy is explosive for $\mid \lambda_i \mid > 1$ unless $p_i y_1 = 0$ and $p_i G \epsilon_j = 0 \; \forall j$

- Predetermined variable? Problem because y_1 is given..
- Choice variables can adjust! So the conditions:

$$p_i y_1 = 0 \text{ and } p_i G \epsilon_j = 0$$

.. pin down the policy functions for choice variables

Our simple model again

In the simple RBC model, we have:

$$y_t = \begin{bmatrix} k_{t+1} \\ k_t \\ z_t \end{bmatrix} \quad \Rightarrow \quad y_1 = \begin{bmatrix} k_1 \\ k_0 \\ z_1 \end{bmatrix}$$

- $\mid \lambda_1 \mid > 1$, $\mid \lambda_2 \mid < 1$, $\mid \lambda_3 \mid = \rho < 1$,
- $p_i y_1 = 0$ pins down k_1 as a function of k_0, z_1

What conditions do we require?

Let h be the number of eigenvalues outside the unit circle

- $h = n_y$: **unique solution**
  - h non-explosiveness restrictions
  - discipline n_y choice variable policy rules

- $h < n_y$: **indeterminacy** (multiple equilibria)
  - can choose arbitrary $n_y - h$ initial values s.t. solution is stable

- $h > n_y$: **no solution**
  - $h - n_y$ non-explosiveness conditions apply to state-variables
  - they would have to be functions of remaining variables
  - cannot be the case, they are exogenous or predetermined

Policy functions

If the condition holds, isn't this a simple way to obtain policy functions?

Problem: to go from ..

$$Ay_{t+1} + By_t = C\epsilon_t$$

.. to this:

$$y_{t+1} = -A^{-1}By_t + A^{-1}C\epsilon_t$$

you need invertibility of $A \Rightarrow$ often not the case.

- Alternative: Schur decomposition (Klein 2000)
- That's what Dynare uses

Today

- Calibration
- Quantitative evaluation
- Mechanisms and criticisms

Business cycle facts

1. **Irregularity** unevenly spaced over time, unequal duration
2. **Comovements** consumption, investment, employment: procyclical
3. **Relative volatility** investment (and durable consumption): much more volatile than consumption and output
4. **Persistence** deviations from trends have high first-order autocorrelation; deviations are not 'one-off'

Our baseline framework

Model with log utility, Cobb Douglas production function:

$$\begin{aligned}
C_t^{-1} &= \beta \mathrm{E}_t \left[(1 + r_{t+1} - \delta) C_{t+1}^{-1} \right] \\
L_t &= (W_t / C_t)^{\eta} \\
W_t &= Z_t (1-\alpha) K_t^{\alpha} L_t^{-\alpha} \\
r_t &= Z_t \alpha K_t^{\alpha-1} L_t^{1-\alpha} \\
Y_t &= Z_t K_t^{\alpha} L_t^{1-\alpha} \\
Y_t &= C_t + I_t \\
K_{t+1} &= (1-\delta) K_t + I_t \\
Z_t &= Z_{t-1}^{\rho} \exp(\epsilon_t)
\end{aligned}$$

Our baseline framework

Steady state:

$$
\begin{aligned}
\bar{Z} &= 1 \\
\bar{Y}/\bar{K} &= \bar{K}^{\alpha-1}\bar{L}^{1-\alpha} \text{ and } \bar{Y}/\bar{L} = \bar{K}^{\alpha}\bar{L}^{-\alpha} \\
\bar{K} &= \left(\frac{\alpha}{\beta^{-1}-(1-\delta)}\right)^{\frac{1}{1-\alpha}}\bar{L} \\
\bar{C} &= (1-\alpha)\bar{L}^{-1/\eta}\bar{K}^{\alpha}\bar{L}^{-\alpha} \\
\bar{I} &= \delta\bar{K} \\
\bar{Y} &= \bar{C}+\bar{I}
\end{aligned}
$$

Today

- **Calibration**
- Quantitative evaluation
- Mechanisms and criticisms

Parameterizing a model

Macroeconomists assign parameters in two main ways

1. **Calibration**
   - Choose 'reasonable' parameters to target selected real-world features
2. Structural estimation
   - Statistical techniques: find parameters that satisfy objective function
   - Matching moments (SMM, indirect inference), Bayesian estimation
   - Problem: invalid when model is misspecified

Nowadays: often a **combination** of calibration and structural estimation

Calibration

Approach:

- Choose parameters such that:
  - **Steady state** matches main long-run features of economy
  - Or: evidence from **microeconometric** studies
- Evaluate the models by looking at untargetted moments
  - E.g. can model explain second moments of business cycle variables
  - Note: this drives applied microeconomists mad :)

Following King and Rebelo (1999)

- Timing: each period is a *quarter*, matches frequency of GDP data
- β: discount rate
  - Moment: return to capital (use avg. S&P return 1948-1986: 6.5%)

$$\beta^{-1} = (1 + r) = 1.065^{-\frac{1}{4}} \approx 0.988$$

  - Derivation: Euler equation in the steady state $C = \beta(1 + r)C$

Following King and Rebelo (1999)

- α: elasticity of output with respect to capital
  - Moment: historical labor share in GDP

$$1 - \alpha = \frac{WL}{Y} \approx 0.67 \Rightarrow \alpha = 0.33$$

  - Derivation: firm's first order condition gives $W = MPL = (1-\alpha)\frac{Y}{L}$

- δ: depreciation rate
  - Obtained from national account data: use 10% per year,

$$\delta = 1 - (0.9)^{\frac{1}{4}} = 0.026$$

  - Note: same δ from NIPA data on inv./capital using s.s. $\delta = \frac{I}{K}$

Productivity process

- ρ: persistence of the productivity process
  - Take productivity from the Fernald series (see lecture 1), 1960-2020
  - Remove trend from productivity using the HP filter (see lecture 1)
  - Estimate an AR(1) productivity process:

$$z_t = \rho z_{t-1} + \varepsilon_t$$

  - Result: $\rho = 0.81$ and the st. dev. of $\varepsilon = 0.009$
    - Replication file: lec3_analysis.do and tfp_fernald.csv on Moodle

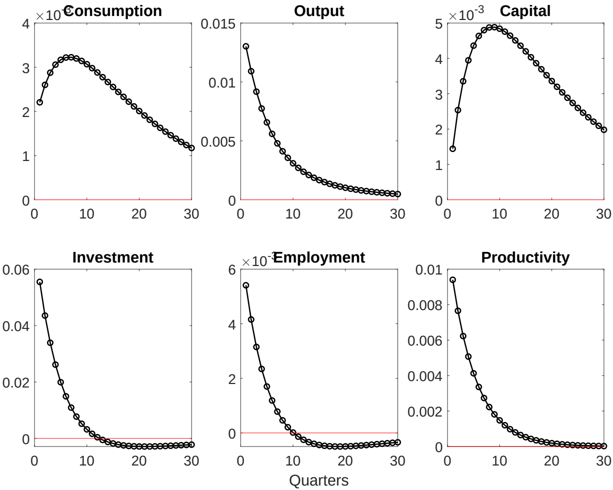
Tricky one

- η: elasticity of labor supply
  - Labor supply is hard to measure from macro data
  - Micro estimates from 0-0.5
  - RBC models: assumed much higher (e.g. $\eta > 1$). More on this later.

Calibration

Parameter	Description	Value
β	Discount factor	0.988
δ	Capital depreciation rate	0.026
α	Capital share in production	0.330
η	Frisch elasticity of labor supply	1.000
ρ	Persistence of productivity	0.813

Impulse responses



Impulse responses to one standard dev. productivity shock (in log dev. from steady state)

Today

- Calibration
- **Quantitative evaluation**
- Mechanisms and criticisms

Business cycle facts

1. **Irregularity** unevenly spaced over time, unequal duration ✓
2. **Comovements** consumption, investment, employment: procyclical
3. **Relative volatility** investment (and durable consumption): much more volatile than consumption and output
4. **Persistence** deviations from trends have high first-order autocorrelation; deviations are not 'one-off'

Cyclicality, Relative Variance

	Relative SD (model)	Relative SD (data)	Corr(Y_t, X_t) (model)	Corr(Y_t, X_t) (data)
Output	1.00	1.00	1.00	1.00
Consumption	0.58	0.90	0.77	0.88
Investment	3.49	4.19	0.91	0.88
Employment	0.33	0.71	0.83	0.78
Productivity	0.65	1.10	0.99	0.79

Model: Calibrated RBC model (quantitative_RBC.mod).

Data: U.S. 1960-2020, log-deviations from HP trend

Business cycle facts

1. **Irregularity** unevenly spaced over time, unequal duration ✓
2. **Comovements** consumption, investment, employment: procyclical✓
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Persistence

	Output		Consumption		Investment		Employment	
	Data	Model	Data	Model	Data	Model	Data	Model
1 Lag	0.87	0.87	0.81	0.99	0.83	0.80	0.83	0.78
2 Lags	0.70	0.75	0.73	0.97	0.64	0.61	0.69	0.59
3 Lags	0.48	0.64	0.50	0.95	0.41	0.46	0.53	0.43
4 Lags	0.27	0.56	0.37	0.92	0.20	0.34	0.44	0.31
5 Lags	0.07	0.51	0.10	0.89	0.00	0.27	0.19	0.23

Autocorrelations corr(x, $L^n x$) (log deviations from HP Trend) for the U.S. 1960-2020

Source: FRED

Business cycle facts

1. **Irregularity** unevenly spaced over time, unequal duration ✓
2. **Comovements** consumption, investment, employment: procyclical✓
3. **Relative volatility** investment (and durable consumption): much more volatile than consumption and output ✓- ish
4. **Persistence** deviations from trends have high first-order autocorrelation; deviations are not 'one-off' ✓

Calibration

Chetty (2012): Frisch elasticity should at most be calibrated to 0.75.

Parameter	Description	Value
β	Discount factor	0.988
δ	Capital depreciation rate	0.026
α	Capital share in production	0.330
η	Frisch elasticity of labor supply	$1 \Rightarrow 0.75$
ρ	Persistence of productivity	0.813

Cyclicality, Relative Variance

	Relative SD (model)	Relative SD (data)	Corr(Y_t, X_t) (model)	Corr(Y_t, X_t) (data)
Output	1.00	1.00	1.00	1.00
Consumption	0.58 $\Rightarrow$ 0.59	0.90	0.77 $\Rightarrow$ 0.77	0.88
Investment	3.49 $\Rightarrow$ 3.46	4.19	0.91 $\Rightarrow$ 0.91	0.88
Employment	0.33 $\Rightarrow$ **0.28**	0.71	0.83 $\Rightarrow$ 0.83	0.78
Productivity	0.65 $\Rightarrow$ 0.68	1.10	0.99 $\Rightarrow$ 0.98	0.79

Model: Calibrated RBC model (quantitative_RBC.mod).

Data: U.S. 1960-2020, log-deviations from HP trend

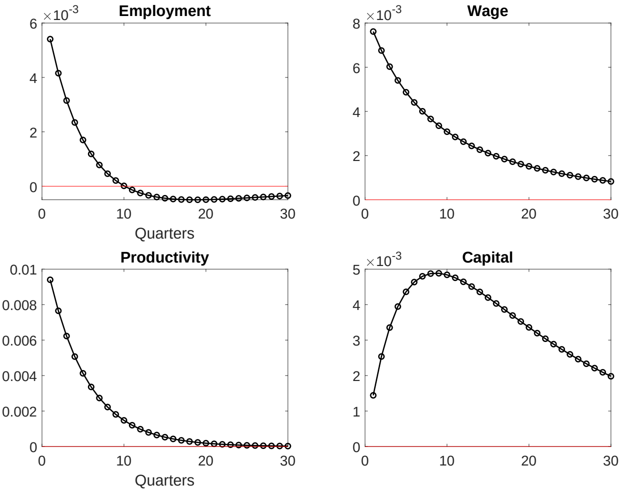
Today

- Calibration
- Quantitative evaluation
- **Mechanisms and criticisms**

Overview of criticisms

1. Transmission mechanism: employment, wages, productivity
2. Persistence and propagation of shocks
3. What are technology shocks?
4. Limited role for policy

Transmission mechanism



Impulse responses to one standard dev. productivity shock (in log dev. from steady state)

Transmission mechanism

Key role for **wages**:

- Stimulate employment after positive shock, dissuade employment after negative shock ('voluntary unemployment')
- But how do wages respond to productivity shock?

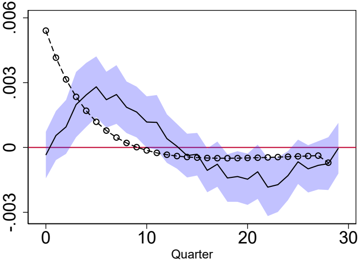
	Relative SD (model)	Relative SD (data)	Corr(W_t, X_t) (model)	Corr(W_t, X_t) (data)
Wage	1.00	1.00	1.00	1.00
Output	1.34	2.14	0.97	**0.16**
Consumption	0.78	1.92	0.90	**0.13**
Investment	4.68	8.96	0.78	**0.11**
Employment	0.45	1.51	0.67	**0.24**
Productivity	0.87	2.35	0.91	**0.04**

Model: Calibrated RBC model (quantitative_RBC.mod).
Data: U.S. 1960-2020, log-deviations from HP trend

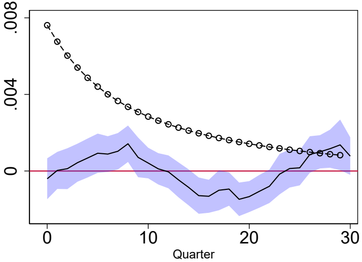
Transmission Mechanism

Quick and dirty regression (Jorda's local projection method):

$$
\begin{aligned}
z_t &= \alpha + \rho f(z_{t-1}) + \varepsilon_t \\
x_{t+h} &= \gamma^h + \beta^h \varepsilon_t + \nu^h_{t+h} \text{ for } h = 0, 1, .., 30
\end{aligned}
$$



(a) Employment



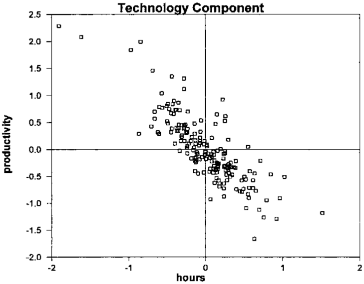
(b) Wages

Productivity shock: residual from AR(1) regression for detrended TFP.

Solid lines: Data: U.S. from 1960-2020. Circled lines: Dynare IRFs from RBC model.

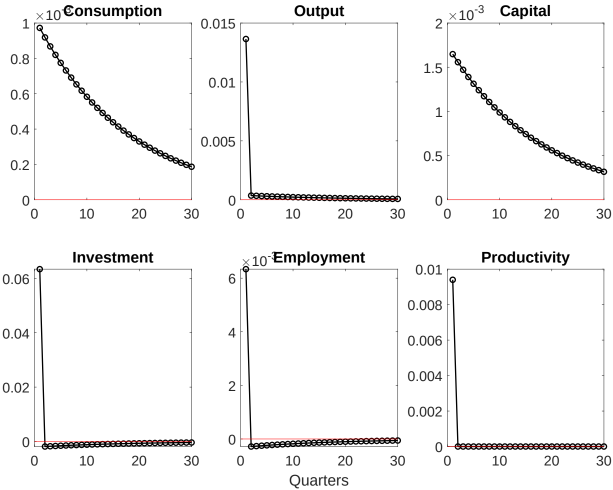
Formal analysis: Gali (1999, AER)

- Identify exogenous shocks in data using a structural VAR
- Result: pure technology shocks **negatively** affect **hours** worked
- Explanation: **sticky prices** (see next class)



Correlation between growth rate in hours and growth rate in productivity (Gali 1999)

Persistence



Impulse responses to one standard dev. productivity shock (in log dev. from st. st.). $\rho = 0$

Persistence

Most persistent in endogenous variables comes from **persistent shock**

- Additional channel: higher capital stock raises output, consumption
  - Households have transitory income boom $\Rightarrow$ increase savings
  - After boom: disinvest (consume savings), more leisure
- Additional channel: intertemporal substitution
  - Productivity raises wages temporarily $\Rightarrow$ boom in employment
  - More leisure in subsequent periods: **negative** autocorrelation

What are technology shocks?

All fluctuations in the RBC model come from shocks to **productivity**

Conceptual issue:

- What is a **negative** technology shock? See Summers (1986)
- Maybe oil shocks of 1973 and 1979? Coronavirus?
- Intuitively: temporarily ‘forget’ how to produce?

What are technology shocks?

Practical issue: **measurement**

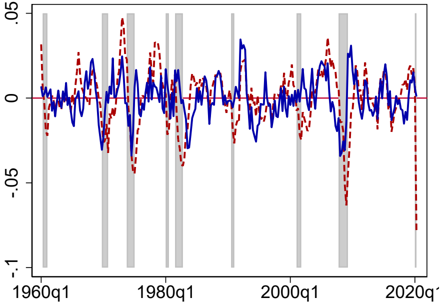
$$\frac{\dot{Z}_t}{Z_t} = \frac{\dot{Y}_t}{Y_t} - \left(1 - \frac{w_t L_t}{Y}\right) \frac{\dot{K}_t}{K_t} - \frac{w_t L_t}{Y} \frac{\dot{L}_t}{L_t}$$

- Productivity is measured as a **residual** (Solow residual)
- Is it really technology?
  - Evans (1992): Productivity shocks correlate with lagged changes in **money supply**, government spending
- Does not account for changes in factor **utilization**
  - Reduced capital utilization? Same capital
  - Think of the TFP of airlines right now!

Measurement

Basu, Fernald, Kimball (2006): control for changes in factor utilization

$$\frac{\dot{Z}_t}{Z_t} = \frac{\dot{Y}_t}{Y_t} - \left(1 - \frac{wL}{Y}\right)\frac{\dot{K}_t}{K_t} - \frac{wL}{Y}\frac{\dot{L}_t}{L_t} - \frac{\dot{U}_t}{U_t}$$



Path of productivity: Solow residual (red-dashed) and utilization adjusted (blue-solid)

Source: Fernald productivity series, FRBSF

Measurement

	Productivity (Solow R.)	Productivity (Util. Adj.)	Utilization
GDP	0.79	-0.01	0.78
Consumption	0.73	0.01	0.72
Investment	0.76	-0.11	0.83
Employment	0.45	-0.19	0.74

Correlation matrix for the U.S. 1960-2020 - Deviations from HP Trend

Source: Fred, Fernald productivity series (FRBSF)

Measurement

	Δ Productivity (Solow R.)	ΔProductivity (Util. Adj.)	ΔUtilization
ΔGDP	0.84	0.17	0.70
ΔConsumption	0.64	0.16	0.50
ΔInvestment	0.63	0.03	0.62
ΔEmployment	0.45	-0.08	0.54

Correlation matrix for the U.S. 1960-2020 - Deviations from HP Trend

Source: Fred, Fernald productivity series (FRBSF)

Conclusion:

- Utilization, not technology, drives link between productivity and business cycle

Limited role for policy

There is no central bank

- Introducing a central bank wouldn't matter: money is superneutral
- Empirically: monetary policy affects real outcomes
  (e.g. Christiano et al. 1999; Romer and Romer 2004; Ramey 2016)
- Next week: introduce **sticky prices** to change this

We've not discussed fiscal policy yet

Introduce fiscal policy

Say a government spends some (exogenous) G_t

- **Keynesian** fiscal spending: G_t is "wasted"
- Government finances spending with lump-sum tax
  - Representative consumer must pay $T_t = G_t$
  - **More on this in the problem set**

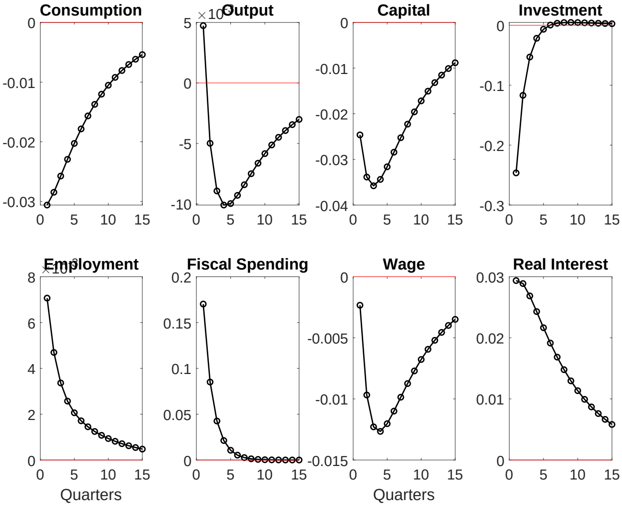
Assume government spending has AR(1) shocks:

$$g_t = \rho_g g_{t-1} + \epsilon_t^g$$

Competitive equilibrium log-linearized

$$
\begin{aligned}
\mathrm{E}_t\left[\widehat{c}_{t+1}\right] - \widehat{c}_t &= (1-\beta[1-\delta])\mathrm{E}_t\left[(\widehat{y}_{t+1} - \widehat{k}_{t+1})\right] \\
\widehat{c}_t + \eta^{-1}\widehat{l}_t &= \widehat{y}_t - \widehat{l}_t \\
\widehat{y}_t &= \widehat{z}_t + \alpha\widehat{k}_t + (1-\alpha)\widehat{l}_t \\
\widehat{k_{t+1}} &= (1-\delta)\widehat{k}_t + \delta\widehat{i}_t \\
\widehat{z}_t &= \rho\widehat{z}_{t-1} + \epsilon_t \\
\widehat{y}_t &= \widehat{c}_t\left(\frac{C}{Y}\right) + \widehat{i}_t\left(\frac{I}{Y}\right) + \widehat{g}_t\left(\frac{G}{Y}\right) \\
\widehat{g}_t &= \rho_g\widehat{g}_{t-1} + \epsilon_t^g
\end{aligned}
$$

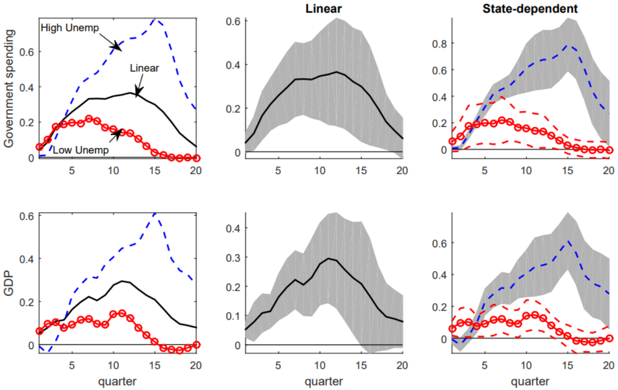
Impulse responses (baseline_RBC_linear_fiscalpolicy.mod)



Impulse responses to 1% of steady state GDP fiscal spending shock (in % dev. from st. st.).

Impulse responses in the data

Figure 5. Government spending and GDP responses to a news shock: Considering slack states



Impulse responses to 1% of GDP fiscal spending shock (Ramey and Zubairy 2018)

What have we done?

- Calibration
- Quantitative evaluation
  - Autoregressive properties, cyclicality of variables, volatility
  - Sensitivity to calibration, estimates of labor supply elasticity
- Mechanisms and criticisms
  - Transmission: response of employment and wages to technology
  - (Mis)measurement of technology shocks
  - Introduction to fiscal policy in RBC models

▸ Feedback welcome! Link to anonymous Google Form