$$\rightarrow \quad c_i = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

$$\rightarrow \quad d_{ni} = \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix}$$

$$\rightarrow \quad c_i^T \;.*\; d_{ni} \qquad \underline{1}\ \underline{i}\ .*\ \underline{n}\ \underline{i} \rightarrow \underline{n}\ \underline{i}$$

$$\rightarrow \quad \begin{bmatrix} c_1 & c_2 \end{bmatrix} .* \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix}$$

$$= \begin{bmatrix} d_{11}c_1 & d_{12}c_2 \\ d_{21}c_1 & d_{22}c_2 \end{bmatrix}$$

sum along "i"

$$\rightarrow \quad \pi_{ni} = \begin{bmatrix} \dfrac{d_{11}c_1}{d_{11}c_1 + d_{12}c_2} & \dfrac{d_{12}c_2}{d_{11}c_1 + d_{12}c_2} \\[2ex] \dfrac{d_{21}c_1}{d_{21}c_1 + d_{22}c_2} & \dfrac{d_{22}c_2}{d_{21}c_1 + d_{22}c_2} \end{bmatrix}$$

$$= \begin{bmatrix} \pi_{11} & \pi_{12} \\ \pi_{21} & \pi_{22} \end{bmatrix}$$

$$\rightarrow \gamma_n^{kj} = \frac{\begin{bmatrix} \gamma_1^{11} & \gamma_1^{21} \\ \gamma_2^{11} & \gamma_2^{21} \end{bmatrix} \rightarrow [1,2,1]}{\begin{bmatrix} \gamma_1^{12} & \gamma_1^{22} \\ \gamma_2^{12} & \gamma_2^{22} \end{bmatrix}_{\substack{(n-k-j) \\ (N-J-J)}}}$$

→ sum γ_n^{kj} along k $(N-1-J)$, then permute $(N-1-J)$

$$\gamma_n^{kj}: \underline{n}\ \underline{k}\ \underline{j} \rightarrow \underline{n}\ \underline{1}\ \underline{j} \rightarrow \underline{n}\ \underline{j}\ \underline{1} \rightarrow \underline{n}\ \underline{j}$$

$$\begin{bmatrix} \gamma_1^{11} + \gamma_1^{21} \\ \gamma_2^{11} + \gamma_2^{21} \end{bmatrix} \begin{matrix} \rightarrow [1,1,1] \xrightarrow{\text{permute}} [1,1,1] \rightarrow [1,1] \\ \rightarrow [2,1,1] \rightarrow [2,1,1] \rightarrow [2,1] \end{matrix}$$

$$\begin{bmatrix} \gamma_1^{12} + \gamma_1^{22} \\ \gamma_2^{12} + \gamma_2^{22} \end{bmatrix} \begin{matrix} \rightarrow [1,1,2] \rightarrow [1,2,1] \rightarrow [1,2] \\ \rightarrow [2,1,2] \rightarrow [2,2,1] \rightarrow [2,2] \end{matrix}$$

$(N-1-J)$

$$\Rightarrow \begin{bmatrix} \gamma_1^{11} + \gamma_1^{21} & \gamma_1^{12} + \gamma_1^{22} \\ \gamma_2^{11} + \gamma_2^{21} & \gamma_2^{12} + \gamma_2^{22} \end{bmatrix}_{(N-J)}$$

→ then plus γ_n^j

$$\begin{bmatrix} (\gamma_1^{11} + \gamma_1^{21}) + \gamma_1^1 & (\gamma_1^{12} + \gamma_1^{22}) + \gamma_1^2 \\ (\gamma_2^{11} + \gamma_2^{21}) + \gamma_2^1 & (\gamma_2^{12} + \gamma_2^{22}) + \gamma_2^2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$c_i^j = \begin{bmatrix} c_1^1 & c_1^2 \\ c_2^1 & c_2^2 \end{bmatrix} \quad (i-j) \quad [N-J]$$

$$d_{ni}^j = \begin{array}{c} \begin{bmatrix} d_{11}^1 & d_{12}^1 \\ d_{21}^1 & d_{22}^1 \end{bmatrix} \\ \hdashline \begin{bmatrix} d_{11}^2 & d_{12}^2 \\ d_{21}^2 & d_{22}^2 \end{bmatrix} \end{array} \quad (n-i-j) \quad [N-N-J]$$

c_i^j : $\underline{i}$ $\underline{j}$ $\xrightarrow{\text{reshape}}$ $\underline{1}$ $\underline{i}$ $\underline{j}$

d_{ni}^j : $\underline{n}$ $\underline{i}$ $\underline{j}$

$$\begin{array}{c} \begin{bmatrix} c_1^1 & c_2^1 \end{bmatrix} .* \begin{bmatrix} d_{11}^1 & d_{12}^1 \\ d_{21}^1 & d_{22}^1 \end{bmatrix} \\ \hdashline \underset{[1-N-J]}{\begin{bmatrix} c_1^2 & c_2^2 \end{bmatrix}} .* \begin{bmatrix} d_{11}^2 & d_{12}^2 \\ d_{21}^2 & d_{22}^2 \end{bmatrix} \end{array} \quad [N-N-J]$$

calculate π_{ni}^j $\Rightarrow$

$$\begin{array}{c} \begin{bmatrix} \pi_{11}^1 & \pi_{12}^1 \\ \pi_{21}^1 & \pi_{22}^1 \end{bmatrix} \\ \hdashline \begin{bmatrix} \pi_{11}^2 & \pi_{12}^2 \\ \pi_{21}^2 & \pi_{22}^2 \end{bmatrix} \end{array}$$