Lecture 10

Heterogenous Firms: Hopenhayn Model

Macroeconomics EC417

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London School of Economics, Fall 2022

Heterogeneous Agent Models

1. Heterogeneous household models: Aiyagari-Bewley-Huggett
    - Stationary wealth and income distribution
    - Motivated by skewed wealth distribution, high MPCs, etc.

2. Heterogeneous firm models: Hopenhayn-Rogerson
    - Stationary employment and productivity distribution
    - Firm size (measured by employment) distrib. highly skewed
    - Firms do not perfectly adjust to productivity shocks

Motivation for Hopenhayn-Rogerson Model

- Productivity shocks are idiosyncratic to firms in the data
- "Business dynamism" varies across countries and industries:
  - Industries differ substantially in their entry/exit rates
  - US has higher turnover of firms and jobs than Europe
  - Ex: in U.S. manufacturing industries, 40% of firms disappear within 5 years, covering 30% of jobs
- Can policy differences explain patterns of business dynamism?
  - Firing costs (e.g. severance) may reduce business dynamism

Key Papers/Resources

1. Hopenhayn (1992): original industry model
2. Hopenhayn and Rogerson (1993): application to firing costs in GE (we focus on this one)
3. Coding resources:
   https://www.vfitoolkit.com/updates-blog/2020/entry-exit-example-based-on-hopenhayn-rogerson-1993/

Model Overview

1. Idiosyncratic uncertainty only: firm-specific productivity (or demand) shocks
2. Decreasing returns to scale imply an optimal size for each firm
3. Endogenous exit: productivity becomes too low
4. Endogenous entry: expected productivity sufficient to cover a fixed entry cost
5. Entry and exit rates equal and constant in equilibrium
6. Stationary firm size distribution

Firm Problem: Profits

$$\max_{n_t} p_t f(n_t, a_t) - n_t - p_t c_f - g(n_t, n_{t-1})$$

- n_t: number of workers today
- p_t: output price, firms take as given (wage rate normalized to 1)
- c_f: fixed operating cost, needed for exit
- $f(n_t, a_t)$ firm-level production function:

$$f(n_t, a_t) = a_t n_t^{1-\alpha}, \quad 1-\alpha \in [0,1]$$

- $g(n_t, n_{t-1})$: cost of adjusting labor: e.g. mandated severance

$$\text{Ex: } g(n_t, n_{t-1}) = \tau \cdot \max\{0, n_{t-1} - n_t\}$$

- a_t: stochastic productivity following AR(1) in logs:

$$\log(a_t) = \bar{a} + \rho \log(a_{t-1}) + \epsilon_t, \epsilon_t \sim N(0, \sigma_\epsilon^2), \bar{a} \geq 0, 0 \leq \rho < 1$$

Firm Problem: Timing for Incumbents

"Incumbent" firm with $n_{t-1} > 0$ and productivity a_{t-1} last period.

At the start of period t:

1. Exit decision: if exit, pay $g(0, n_{t-1})$, firm disappears
2. If continue, observe a_t and pay $p_t c_f$
3. Production stage: choose n_t

$$\max_{n_t} p_t f(n_t, a_t) - n_t - p_t c_f - g(n_t, n_{t-1})$$

Firm Problem: Timing for Potential Entrants

Mass of potential entrants with unknown productivities.

At the start of period t:

1. Pay entry cost $p_t c_e$
2. Observe productivity a_t drawn from distribution ν
3. Production stage: choose n_t

$$\max_{n_t} p_t f(n_t, a_t) - n_t - p_t c_f - g(n_t, 0)$$

Entry decision will depend on expected productivity.

Firm Bellman Equations

Value function of incumbents:

$$\begin{aligned} V(a,n;p) = \max_{n' \geq 0} \; & pa(n')^{1-\alpha} - n' - pc_f - g(n',n) \\ & + \beta \max\{E_a(V(a',n';p), -g(0,n')\} \end{aligned}$$

- "Guess and verify" that prices are constant over time in eqm
- Max operator captures exit decision at beginning of next period
- Policy functions for labor demand $N(a,n;p)$ and exit $X(a,n;p)$

Given $V(a,n;p)$, entrants' value (gross of entry cost) is:

$$V^e(p) = \int V(a,0;p)d\nu(a)$$

In equilibrium, free entry condition will hold:

$$V^e(p) = pc_e \tag{1}$$

Firm Problem: No Firing Costs

If $g(n_t, n_{t-1}) = 0$, firm's optimality implies:

$$\log n_t = \frac{1}{\alpha}(\log(1-\alpha) + \log(p) + \log a_t)$$

Firm size, measured by employment, is increasing in:

- The price of the firm's output
- The firm's productivity a_t

The firm will exit ($X(a, n; p) = 1$, or $n_t = 0$) when $a_t \leq a^*$ for some a^*

Households

Representative household (household members share income):

$$\max_{c_t, n_t} \sum_{t=0}^{\infty} \beta^t \left(\log(c_t) - A n_t \right)$$

subject to:

$$p_t c_t \leq n_t + \Pi_t + G_t$$

- Disutility of labor $A > 0$
- Π_t: profits of the firms
- G_t: government transfers tax receipts back to households
- Call the household's solution to labor supply problem L^s

Aggregate Objects

Let $\mu(a,n)$ the measure of firms at each point in state space (a,n), M the measure of entrants and $T(\mu,M;p) : \mu' = T(\mu,M;p)$:

$$\begin{aligned}
Y(\mu,M;p) &= \int [f(N(a,n;p),a) - pc_f] d\mu(a,n) \\
&\quad + M \int f(N(a,0;p),a) d\nu(a) && \text{[Output]} \\
L^d(\mu,M;p) &= \int N(a,n;p) d\mu(a,n) + M \int N(a,0;p) d\nu(a) && \text{[Lab. Demand]} \\
R(\mu,M;p) &= \int ([1 - X(a,n;p)] \int g(N(a',n';p),n') dF(a,a') \\
&\quad + X(a,n;p) g(0,n')) d\mu(a,n) && \text{[Adj. Costs]} \\
\Pi(\mu,M;p) &= pY(\mu,M;p) - L^d(\mu,M;p) - R(\mu,M;p) - Mpc_e && \text{[Profits]}
\end{aligned}$$

Stationary Equilibrium Definition

A stationary equilibrium of this model is $\{p^* \geq 0, M^* \geq 0, \mu^*\}$ such that the following conditions hold:

1. Labor market clearing:
   $L^s(p^*, \Pi(\mu^*, M^*; p^*) + R(\mu^*, M^*; p^*)) = L^d(\mu^*, M^*; p^*)$.
2. Stationary distribution: $T(\mu^*, M^*; p^*) = \mu^*$.
3. Free entry condition: $V^e(p^*) \leq p^* c_e$, with equality if $M^* > 0$.
4. Goods market clearing:

$$Y(\mu^*, M^*; p^*) = p^* c + M^* p^* c_e$$

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Possibility of multiple equilibria without entry (Hopenhayn (1992)).

- We will focus on stationary equilbrium with $M^* > 0$.
- If one such equilibrium exists, it is unique.

Unique Steady State with Positive Entry/Exit

Algorithm for finding the unique steady state has three steps:

1. Find the p^* such that $V^e(p^*) = c_e p^*$
2. Find μ^* as the fixed point of $T(\mu^*, M^*; p^*)$. Existence of a fixed point requires that expected exit age is finite (otherwise μ is growing over time) which requires a joint restriction on $N(a, n; p)$, $X(a, n; p)$, and $F(a, a')$ (see Hopenhayn (1992) for details).
3. Find M^* that clears the labor or goods market.

Role of Fixed Costs and Entry Costs

Some "comparative statics" (how parameters affect the steady state):

1. Proposition 1: an increase in the fixed cost c_f increases p, the average firm size, and the exit rate.
   - Intuition: holding price fixed, this causes small firms near a^* to want to exit, raising average firm size. At the old price no new firms would want to enter so the price must increase to induce more entry.
2. Proposition 2: an increase in entry cost c_e increases p and decreases the exit rate.
   - Intuition: at the old price, no firms will want to enter when c_e increases, so the price must rise, which reduces exit and partially offsets the decline in entry.

Numerical Solution: Sketch of Computational Algorithm

1. Discretize state space (a, n) with 20 points for a and 250 points for n (can have more nowadays!)
   - Maximum n is set to 5000 employees
   - a grid discretizes the productivity process into 20 points
   - Q: transition matrix for a with persistent productivity?
2. Guess a p^* and use value function iteration to find $V(a, n; p^*)$, $X(a, n; p^*)$, $N(a, n; p^*)$.
3. Check whether free entry condition 1 is satisfied. If no, return to step 2. If yes, proceed to step 4
4. Solve for the stationary distribution $\mu(a, n)$ associated with $M = 1$ mass of entrants.
5. Find the scale factor M^* that clears the goods market.

Data Source: Census of Manufactures

U.S. Census of Manufactures dataset: Link to CFM data for 2017

- Survey of U.S. manufacturing establishments and firms
- Variables include:
  - number of employees
  - sales
  - labor costs
  - input costs (materials, energy, etc.)
  - detailed industry codes and types of products produced
  - geographic information
- Can play around w/ synthetic micro-data for all U.S. industries here: Link to Synthetic Longitudinal Business Database

Calibrating the Model

Calibration rather than estimation: simulated method of moments. Period is 5 years (data = Census of Manufactures, 5 year freq.)

param.	interp	target
α	1 - labor share	labor share of 0.64
β	discount factor	$\beta = \frac{1}{1+r}$, ann. real interest rate 4%
ρ	prod. persistence	inferred from emp. growth regs
σ_ϵ	s.d. prod. shock	inferred from emp. growth regs.
c_f	fixed cost	avg. $\log(n)$ in the data
$\bar{a}$	mean prod.	avg. 5-year exit rate in the data
$\nu(a)$	prod. distrib.	size distrib. of firms ages 0-6 years
c_e	entry cost	set so that $V^e(p) = p^* c_e$
A	disutil of labor	emp. to pop. ratio of 0.6
$g(n_t, n_{t-1})$	adj. cost	no adj. cost in baseline

Sensitivity analysis: how much do variations of $\pm 10\%$ in individual parameters change the results (holding others fixed)?

Model Fit of the Data

All data moments computed at 5-year intervals using Census data:

Moment	Data	Model
Serial corr. in log emp	0.93	0.92
Variance in emp. growth rates	0.53	0.55
Mean employment	61.7	61.2
Exit rate	37%	39%
Share of firms with 1-19 emp.	0.74	0.52
Share of firms with 20-99 emp.	0.18	0.37
Share of firms with 100-499 emp.	0.08	0.10
Share of firms with 500+ emp.	0.01	0.01

Policy Experiments

Introduce firing costs of the form:

$$g(n_t, n_{t-1}) = \tau \cdot \max\{0, n_{t-1} - n_t\}$$

Interpretation with period length 5 years (and wages = 1):

- $\tau = 0.1$ is 6 months' wages per worker
- $\tau = 0.2$ is 1 years' wages per worker
- etc...

Effect of Firing Costs on Model Steady State

Effect of Changes in τ (Benchmark Model)

	$\tau = 0$	$\tau = .1$	$\tau = .2$
Price	1.00	1.026	1.048
Consumption (output)	100	97.5	95.4
Average productivity	100	99.2	97.9
Total employment	100	98.3	97.5
Utility-adjusted consumption	100	98.7	97.2
Average firm size	61.2	61.8	65.1
Layoff costs/wage bill	0	.026	.044
Job turnover rate	.30	.26	.22
Serial correlation in log(n)	.92	.94	.94
Variance in growth rates	.55	.45	.39

Interpreting the Results

1. Firms make fewer adjustments in response to shocks:
    - Higher serial correlation in $\log(n)$
    - Lower variance in growth rates
    - Lower job turnover (fraction of jobs destroyed each period)
2. Fewer jobs created overall (lower total employment)
3. Consumption falls, utility-adjusted consumption falls by less because leisure increased
    - Quite a large drop in utility as a result of the policy: 2.8%
4. Lower average productivity: firms farther from optimal size = "misallocation"

Consumption-Equivalent Welfare Measure

Let $U(\tau)$ be the lifetime utility of the household in steady state with adjustment cost τ, and define $c(\tau), n(\tau)$ the steady state levels of consumption and labor supply.

Recall the household's utility function:

$$U(0) = \sum_{t=0}^{\infty} \beta^t \left(\log(c(0)) - An(0)\right)$$

Consumption equivalent welfare ξ for a given τ solves:

$$U(0) = \sum_{t=0}^{\infty} \beta^t \left(\log(c(\tau)(1+\xi)) - An(\tau)\right)$$

In the $\tau = 0.2$ case, $\xi = 0.028$ or 2.8%. Consumption has to increase 2.8% each period to make households indifferent between the no tax case and the case with firing costs.

Limitations of Hopenhayn-Rogerson Model

1. No physical capital (for computational reasons)
2. No aggregate uncertainty
3. Goods are homogeneous
4. Households can perfectly share income
5. Take employment contracts as given–might respond to policies
6. Long run effects only–what about transition dynamics?
7. ...others?

Takeaways: Heterogeneous Agent Models

- Like HH wealth distribution, firm size distribution highly skewed
- Apply similar techniques (Bellman equations, value function iteration, calibration...) across variety of het. agent models
- Can use these models to study policy questions/welfare
    - Agents may disagree about policies: winners and losers
    - Ex: regulations that increase entry costs in this model
- Others?

Plan for Last Lecture: Review Session

1. Particular topics we should cover?