ECON 33620	**Greg Kaplan**
Introduction to Heterogeneous Agent Macroeconomics	**University of Chicago**
Spring 2022	gkaplan@uchicago.edu

Problem Set 2
Due: 11:59pm Thursday 28 April 2022

Instructions

You may work on this assignment in groups, but you may also work alone if desired. However, **every student must submit their own assignment** and note clearly on their submission the names of the other members of their group. If you do not credit the other members of your group, you will be penalized. Please submit your assignment via canvas. The TA is responsible for grading the problem sets. Therefore, all questions related to the problem sets, including requests for extensions, should be initially directed to the TA. Late problem sets will not be accepted without prior authorization from the TA or me. If you anticipate handing in your problem set late, you must inform your TA with a valid explanation before the deadline.

Please **do not** include print outs of all your code. You may use any programming language that you wish. Please pay attention to the units on of your figures and make sure they are meaningful. Even though you have two weeks to complete the problem set, I strongly suggest getting started on it immediately.

Part 1: Exercises

1. Consider the following infinite horizon consumption-savings problem. A household has preferences defined by

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t \log\left(c_t\right)$$

and faces the sequence of budget constraints

$$\begin{aligned} c_t + a_{t+1} &\leq (1+r)\, a_t + y_t \\ a_{t+1} &\geq 0 \end{aligned}$$

where a_t is wealth and y_t is labor income. Labor income is stochastic.

   (a) Assume that the labor income process is such that $\log y_t \sim N\left(\mu, \sigma^2\right)$ and is IID over time. Write down the Bellman equation for this problem using assets and income (a_t, y_t) as the state variable.

   (b) Find an alternative way of representing this same optimization problem in recursive form that requires only 1 rather than 2 state variables.

(c) Assume now that labor income follows an AR(1) in logs, i.e. $\log y_t = \rho \log y_{t-1} + \epsilon_t$ with $\epsilon_t \sim N(\mu, \sigma^2)$. Write down the Bellman equation for this problem. Can you pull the same trick as in (b) and represent the same problem with only a single state variable? Why or why not?

(d) Assume now that the household choose how many hours to work in addition to how much to consume. Its preferences are now

$$\mathbb{E}\sum_{t=0}^{\infty} \beta^t \left[\log(c_t) - \frac{1}{2}h_t^2\right]$$

and its budget constraint is now

$$\begin{aligned} c_t + a_{t+1} &\leq (1+r)\, a_t + w_t h_t \\ a_{t+1} &\geq 0 \end{aligned}$$

Assume that the wage process w_t is IID log-normal as in part (a). Write down the Bellman equation with the fewest possible state variables that corresponds to this optimization problem.

2. Consider the following model with labor supply.

$$V(a, w_i) = \max_{c,h,a'} \frac{c^{1-\gamma}-1}{1-\gamma} - \psi\frac{h^{1+\sigma}}{1+\sigma} + \beta\sum_{j=1}^{n} V(a', w_j)\, p_{ij}$$

subject to

$$\begin{aligned} c + a' &= (1+r)\, a + w_i h \\ h &\leq 1 \\ a' &\geq 0 \end{aligned}$$

Outline a version of the method of endogenous grid points that you could use to solve this model. You do not have to program it up, just clearly describe the steps that you would take. Be careful to consider how you would deal with the two constraints and the possible interaction between them.

Part 2: Coding

1. Consider the following inifinite horizon consumption-savings problem.

$$V(a, y_i) = \max_{c,a'} \frac{c^{1-\gamma}-1}{1-\gamma} + \beta\sum_{j=1}^{n} V(a', y_j)\, p_{ij}$$

subject to

$$\begin{aligned} c + a' &= (1+r)\, a + (1-\tau)\, y_i + T \\ a' &\geq 0 \end{aligned}$$

where $p_{ij} = \Pr(y_j|y_i)$ are the (i,j) th element of the Markov transition matrix P. The stationary distribution corresponding to this Markov chain is denoted by π with ith element π_i. There are $n = 5$ possible income states, which are given by the vector y. Income is subject to a proportional tax τ and households receive a lump sum transfer T. This tax policy is such that the government balances its budget constraint in every period, i.e.

$$\tau \sum_{i=1}^{n} y_i \pi_i = T$$

Use the following parameter values: $\gamma = 1.5$, $\beta = 0.975$, $r = 0.02$, $\tau = 0.15$, $T = 0.15$. Use the following income process: $n = 5$, $y = [0.4039; 0.6094; 0.9194; 1.3873; 2.0932]'$,

and a matrix P with elements: :
$$\begin{matrix} 0.9413366 & 0.0573403 & 0.0013098 & 0.0000133 & 0.0000001 \\ 0.0143351 & 0.9419915 & 0.0430152 & 0.0006550 & 0.0000033 \\ 0.0002183 & 0.0286768 & 0.9422098 & 0.0286768 & 0.0002183 \\ 0.0000033 & 0.0006550 & 0.0430152 & 0.9419915 & 0.0143351 \\ 0.0000001 & 0.0000133 & 0.0013098 & 0.0573403 & 0.9413366 \end{matrix}$$

For those of you who are interested, this income process was constructed by discretizing an AR(1) process for the log of income using the Rouwenhorst method

$$\log y_t = \rho \log y_{t-1} + \epsilon_t$$

with $\rho = 0.97$, $\sigma = 0.1$ and normalized so that $E[y] = 1$.

(a) Write some code to solve this model using either the Euler equation iteration or the method of endogenous grid points. Produce a plot with the value function and consumption policy functions as a function of assets for the highest and lowest income states.

(b) Compute the stationary distribution of wealth for this model and produce a histogram of the stationary distribution of assets.

(c) Compute the following statistics of the wealth distribution: (i) mean, (ii) median, (iii) fraction with zero wealth, (iv) 90th percentile, (v) 99th percentile.

(d) Compute the following measures of wealth inequality: (i) coefficient of variation, (ii) gini coefficient, (iii) 99-50 ratio, (iv) 90-50 ratio, (v) wealth share of top 10%, (vi) wealth share of top 1%. How does the model's predictions compare to the statistics you computed from the 2019 SCF in the first problem set.

2. Now consider the same model and calibration as in the previous question, except increase the tax rate τ from 0.15 to 0.30. Note that since the lump sum transfer T is set to balance the government budget constraint, and since mean gross labor income is normalized to 1, this means the lump sum transfer will also increase to 0.30. How does average wealth in the new economy compare with the old economy? How does wealth inequality in the new economy compare with the old economy? What is the

economic intuition?

Part 4: Data

In Problem Set 1 you computed statistics from the US wealth distribution in 2019. In this exercise you will compute analogous statistics in previous years and see how they have changed over time. The Survey of Consumer Finances is available every three years, going back to 1989. You can download comparable data for previous years here here. For convenience, I have posted Stata versions of these previous surveys on Canvas.

1. For each of the following measures of inequality, produce a plot that shows how it has changed over the last three decades:

   (a) coefficient of variation

   (b) variance of logs

   (c) gini coefficient

   (d) 99-50 ratio

   (e) 90-50 ratio

   (f) wealth share of top 10%

   (g) wealth share of top 1%

   Comment on anything interesting that you notice about these trends.