ECON 33620 **Greg Kaplan**
Introduction to Heterogeneous Agent Macroeconomics **University of Chicago**
Spring 2022 gkaplan@uchicago.edu

Problem Set 3
Due: 11:59pm Tuesday 17 May 2022

Instructions

You may work on this assignment in groups, but you may also work alone if desired. However, **every student must submit their own assignment** and note clearly on their submission the names of the other members of their group. If you do not credit the other members of your group, you will be penalized. Please submit your assignment via canvas. The TA is responsible for grading the problem sets. Therefore, all questions related to the problem sets, including requests for extensions, should be initially directed to the TA. Late problem sets will not be accepted without prior authorization from the TA or me. If you anticipate handing in your problem set late, you must inform your TA with a valid explanation before the deadline.

Please **do not** include print outs of all your code. You may use any programming language that you wish. Please pay attention to the units on of your figures and make sure they are meaningful. Even though you have two weeks to complete the problem set, I strongly suggest getting started on it immediately.

Part 1: Exercises

1. Time is discrete and indexed by $t \in \{0, 1, ..., \infty\}$. The economy is populated by a continuum of measure one of infinitely-lived households indexed by i on $[0, 1]$. Preferences for individual i are given by

$$E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_{it}\right)$$

where $\beta \in (0, 1)$ is the discount factor, $c_{it} \geq 0$ denotes consumption of household i at time t, $u' > 0$ and $u'' < 0$. Each individual has a stochastic endowment of efficiency units of labor $z_t \in \{z_H, z_L\}$. The shocks follow a Markov process with transition probabilities $\pi_{z,z'} = \Pr\left(z_{t+1} = z' \mid z_t = z\right)$. Shocks are IID across individuals. Households can trade a risk-free asset a_{it} in zero-net supply whose interest rate is denoted by r, and they can accumulate debt up to the exogenous borrowing limit $-b$.

   (a) Define a stationary recursive competitive equilibrium (RCE) for this economy, and draw a diagram to illustrate how the equilibrium interest rate r is determined

(b) Suppose that $b = 0$, and that $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$. What is so special about this case? Prove that the equilibrium interest rate is lower than $\frac{1}{\beta} - 1$.

(c) Explain how the highest possible equilibrium risk-free rate changes with the ratio $\frac{z_H}{z_L}$, with the persistence parameter $\pi_{HH} = \Pr(z_{t+1} = z_H \mid z_t = z_H)$, and with risk aversion γ. Provide some intuition for your answer.

2. Consider a stationary economy populated by a continuum of measure one of infinitely lived, ex-ante identical agents with preferences over sequences of consumption and leisure given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_{it}) + e^{\varphi_{it}} v(1 - h_{it}) \right]$$

with

$$u', v' > 0, \; u'', v'' < 0$$

Agents face individual shocks to their preference for leisure φ_{it}, which follow the stochastic process $\varphi_{it} = \rho \varphi_{it-1} + \eta_{it}$ with $\eta_{it} \sim N\left(0, \sigma_\eta^2\right)$. Agents can save but cannot borrow. Production takes place through the aggregate technology

$$C_t + K_{t+1} + (1 - \delta) K_t = Z K_t^\alpha H_t^{1-\alpha}$$

where C_t, K_t and H_t are, respectively, aggregate consumption, aggregate capital, and aggregate hours at time t. Labor and asset markets are competitive and clear, every period, with prices w_t and r_t, respectively. The government taxes capital income at a proportional rate τ . Tax revenues are returned to agents as tax-exempt lump-sum transfers b. Households can evade taxes by deciding in every period the fraction of capital income ϕ_{it} to declare in their tax return, i.e., the fraction of capital income on which they pay taxes. Let x_{it} be the total undeclared taxes at time t. The government, knowing that agents may have evaded taxes at time $t-1$, at time t can monitor and perfectly verify the past period individual tax returns. Let π be the probability that, at time t, the time $t-1$ tax return of a household is subject to monitoring. The household finds out whether her $t-1$ period tax return is monitored at the beginning of period t, i.e., before consumption decisions are taken. In the event the household is caught, at time t the tax agency gives her a fine equal to $f(x_{it-1})$, where x_{it-1} is the tax amount due from the past period, with $f(0) = 0$ and $f'(x_{it-1}) > 1$.

(a) Write down the problem of the household in recursive form, explicitly stating the individual state variables.

(b) Write down the individual first-order necessary condition that characterizes the optimal tax evasion choice.

(c) Define a stationary recursive competitive equilibrium (RCE) for this economy.

Part 2: Coding

1. Consider the consumption-savings model that you solved on your computers in Problem set 2. Fix the discount factor $\beta = 0.97$, the coeffecient of relative risk aversion $\gamma = 2$, and set the tax rate $\tau = 0$ and lump sum transfer $T = 0$.

   (a) Compute the aggregate quantity of savings in the stationary distribution when the interest rate $r = -2\%$. Are aggregate savings positive, negative or zero? Explain the intuition.

   (b) Produce a plot that shows how the aggregate wealth in the economy varies as a function of the interest rate r, for values of the interest rate in $[-2\%, 3\%]$

   (c) Now assume that households can borrow up to an amount equal to one quarter of average annual labor income in the economy. Produce the analogous plot for this version of the model. Explain the intution for why the two plots look the way that they do.

2. Consider the following version of the general equilibrium heterogeneous agent model. Households are infinitely lived and solve a standard consumption-savings problem.

$$V(k, z) = \max_{c,k'} \frac{c^{1-\gamma} - 1}{1-\gamma} + \beta \sum_{z'} V(k', z') \Pr(z'|z)$$

subject to

$$c + k' = (1+r) k + (1-\tau) zw + T$$
$$k' \geq 0$$

where z follows a discrete state Markov chain with Markov transition matrix P. Let the stationary distribution of P be denoted by the vector π. Each household's labor income is given by the product of the aggregate wage rate per efficiency unit of labor, w, and their own number of efficiency units, z.
There is a representative firm with Cobb-Douglas production function

$$Y = AK^{\alpha} N^{1-\alpha}$$

where A is aggregate TFP. The firm rents capital directly from households and pays a rental rate r per unit of capital. The firm incurs depreciation at rate δ while it rents the capital. The labor input N is the total number of efficiency units of labor that the firm hires, at a rate of w per efficiency unit. Thus the firm solves the static problem

$$\max_{K,N} AK^{\alpha} N^{1-\alpha} - (r+\delta) K - wN$$

The asset market clearing condition says that the total savings of households must equal the capital used by the firm in production, while the labor market clearing condition says that the demand for efficiency units of labor by the firm must equal the total number of efficiency units of labor supplied by households

$$\int_k \sum_z k\lambda(k,z)\,dk = K$$

$$\sum_z z\pi(z) = N$$

where $\lambda(k,z)$ is the stationary distribution of households across asset and productivity states.The government budget constraint says that the total amount of tax revenue collected by the government must be equal to the total amount of transfers paid out

$$\tau w \sum_z z\pi(z) = T$$

Use the following parameter values for the household problem: $\gamma = 1.5$, $\beta = 0.95$, $\tau = 0.15$. Use the following process for the efficiency units of labor : $n = 5$, $z = [0.4039; 0.6094; 0.9194; 1.3873; 2.0932]'$, and a matrix P with elements:

0.9413366	0.0573403	0.0013098	0.0000133	0.0000001
0.0143351	0.9419915	0.0430152	0.0006550	0.0000033
0.0002183	0.0286768	0.9422098	0.0286768	0.0002183
0.0000033	0.0006550	0.0430152	0.9419915	0.0143351
0.0000001	0.0000133	0.0013098	0.0573403	0.9413366

Set TFP $A = 1$, the capital share $\alpha = \frac{1}{3}$ and the depreciation rate $\delta = 20\%$.

(a) For a given a capital-labor ratio, $\frac{K}{N}$, use the FOC of the firm problem together with the production function, to derive the values of the interest rate r and wage rate w, and use the government budget constraint to derive the implied value of the lump-sum transfer T. Note that there is a one-to-one mapping between the capital-labor ratio and the equilibrium values of (r, w, T).

(b) Using the results in part (a), write some code to solve for the mean level of household savings $E[k] = \int_k \sum_z k\lambda(k,z)\,dk$ for a given value of the capital-labor ratio. Because of the mapping defined in part (a), this is a simple modification of the code that you produced in problem set 2.

(c) Using your code from part (b), construct a plot of mean household savings relative to the labor input, $\frac{E[k]}{N}$, versus the interest rate. On the same graph, draw a plot of total capital demanded by the firm relative to the labor input, $\frac{K}{N}$, versus the interest rate. You can do this by repeating parts (1) and (2) for a few carefully chosen values of $\frac{K}{N}$.

(d) By iterating on the capital-labor ratio or by solving a non-linear equation, find the equilibrium values of the interest rate r, the wage rate per efficiency unit of labor w and lump-sum transfer T. You can use the plot from part (c) to come up with a good starting guess for your iterations.

(e) Compute the distribution of wealth in the stationary equilibrium of this model, and plot a histogram.

(f) For each of the following two experiments:

   i. Compute the equilibrium interest rate r, wage rate per efficiency unit of labor w and lump-sum transfer T in the new stationary equilibrium.

   ii. Explain the intuition for how each of (r, w, T) are affected by the change.

   iii. Explain also how and why total wealth, and wealth inequality, are affected by the change.

   Experiment 1: An increase in the tax rate from 0.15 to 0.30.
   Experiment 2: An increase in aggregate productivity from $A = 1$ to $A = 1.05$.