



Machine Learning for Time Series Forecasting

*Machine Learning ID Meet Up #3 Surabaya
March 10, 2018
DILo Surabaya*

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Novri SUHERMI

Master of Science, Applied Mathematics in Finance, Insurance, and Risk,
Sept 2014 – Sept 2016

Université Paris Diderot – Paris 7, Paris, France

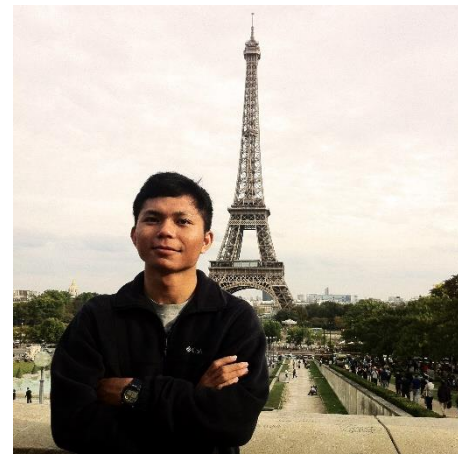
Master of Science, Statistics, September 2013 – September 2014

Institut Teknologi Sepuluh Nopember, Surabaya, Indonesia

Bachelor of Science, Statistics, September 2010 - March 2014

Institut Teknologi Sepuluh Nopember, Surabaya, Indonesia

September 2017 – present



Lecturer, Department of Statistics, ITS, September 2017 – present

Member of Statistics for Business and Industry Laboratory

Research area interest: Time Series, Forecasting, Machine Learning, Reliability, Computational Statistics

Data Analyst, GDP Labs & Kaskus, Jakarta, Indonesia, December 2016 – September 2017

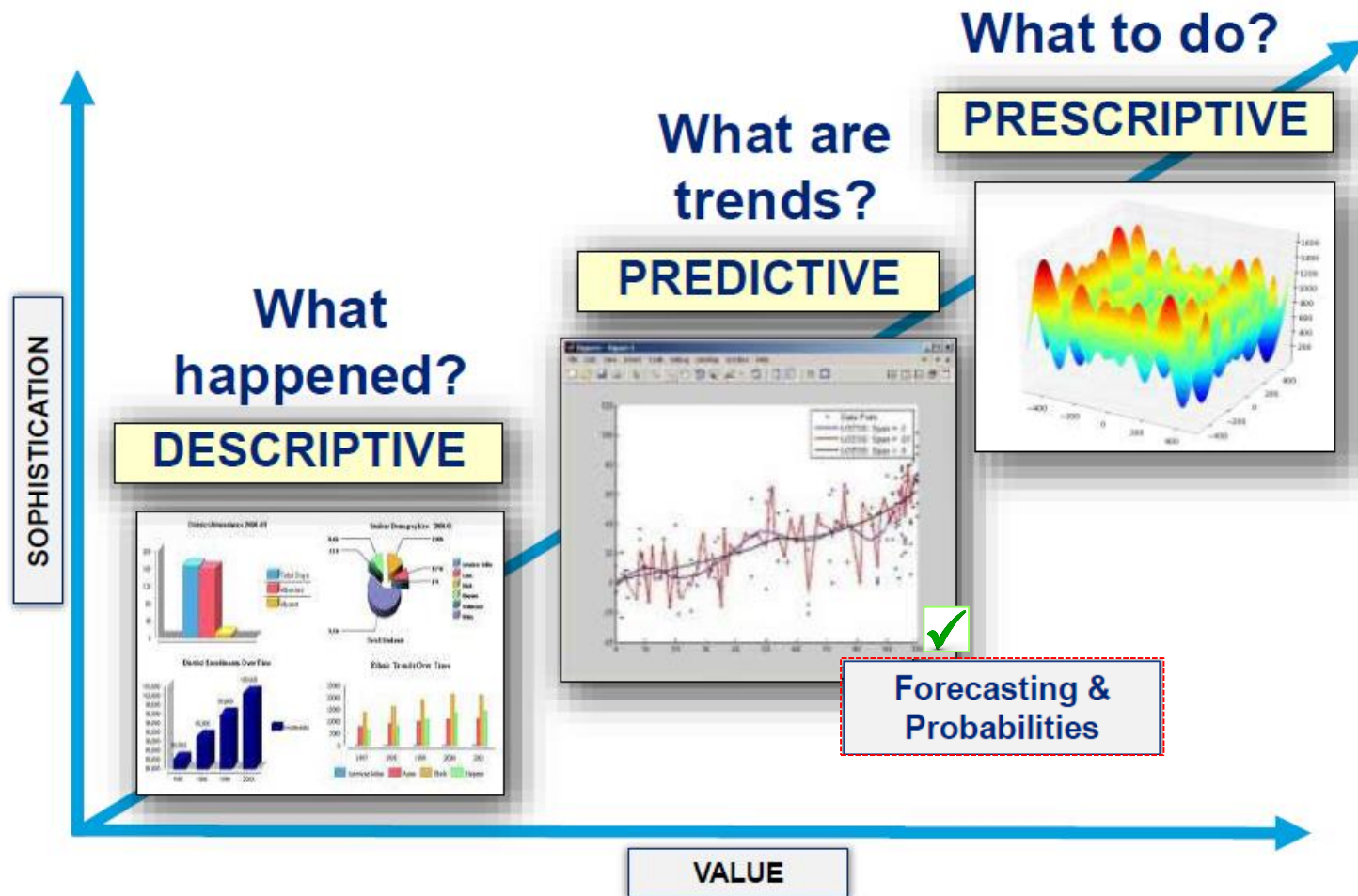
Research Assistant, Laboratoire d'Informatique, Université Paris Descartes, Paris, France, April 2016-October 2016

Developed a method to identify Near-infrared Spectroscopy (NIRS) data using time series management and analysis techniques.

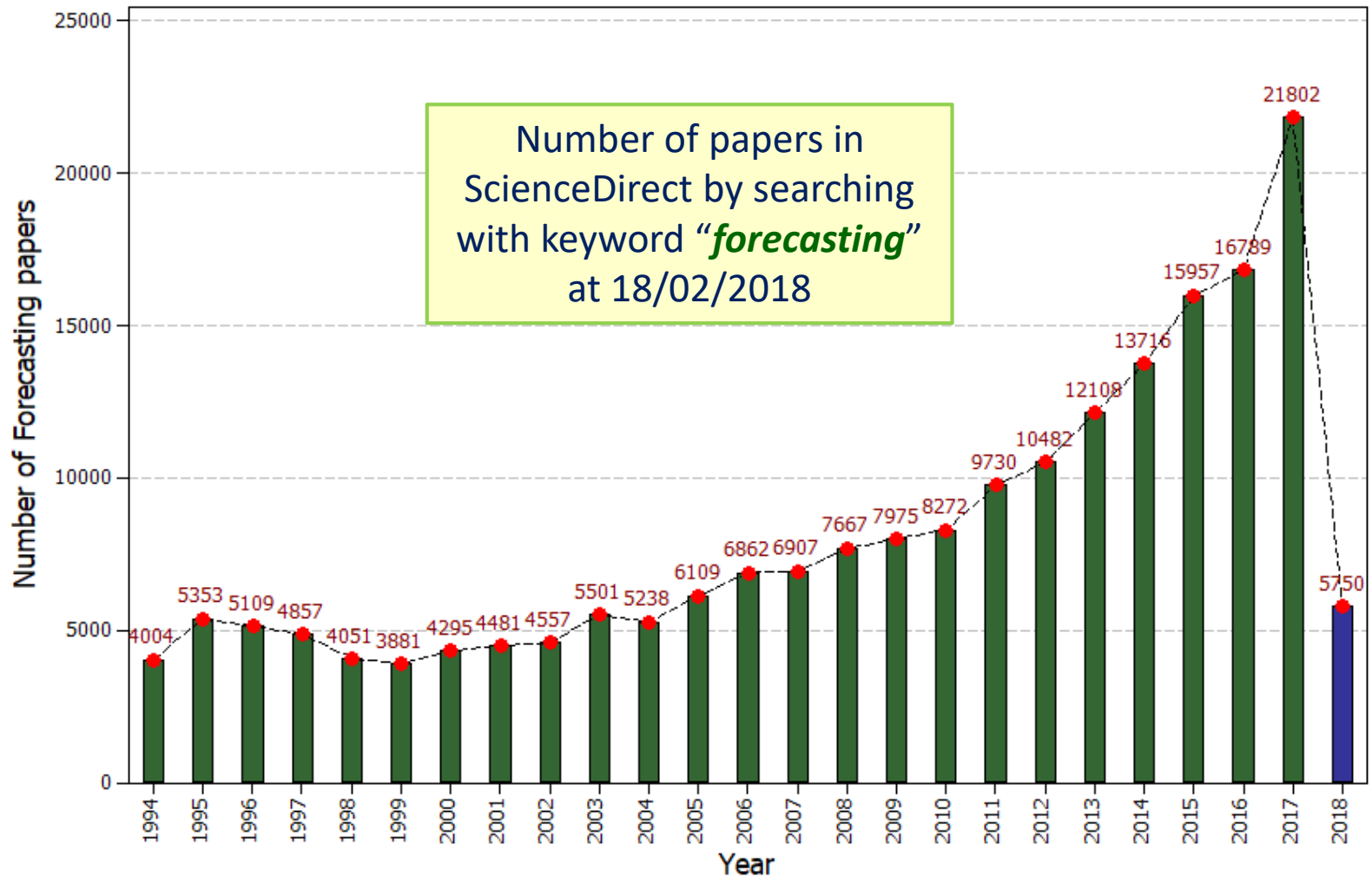
Research Assistant, Laboratoire Jean Kuntzmann, Grenoble-INP, Grenoble, France, April 2015-September 2015

Developed a program of exact monte carlo goodness-of-fit test for imperfect maintenance models using R with Rcpp package (Seamless R and C++ Integration).

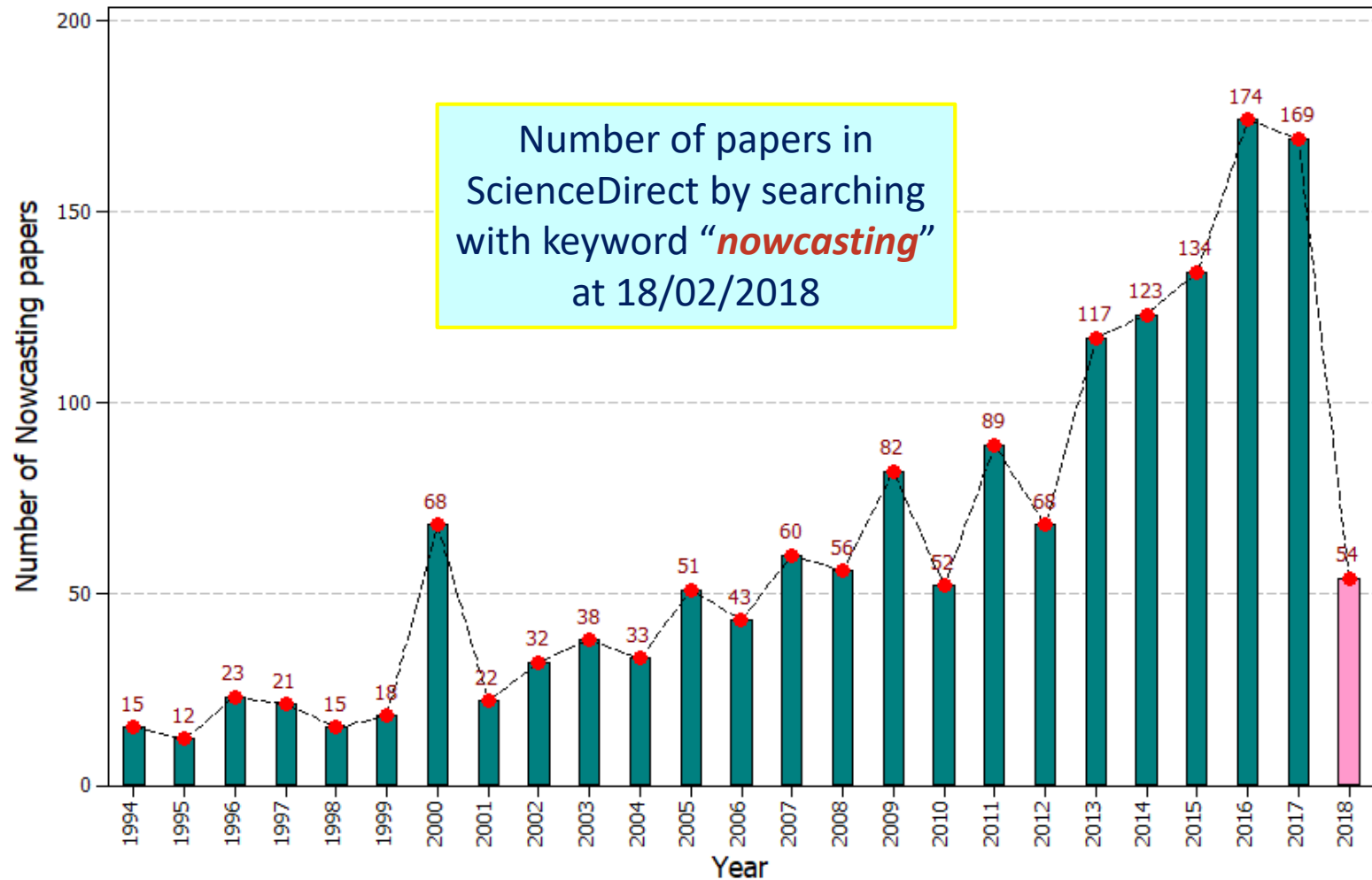
Motivation



Motivation



Motivation



Motivation

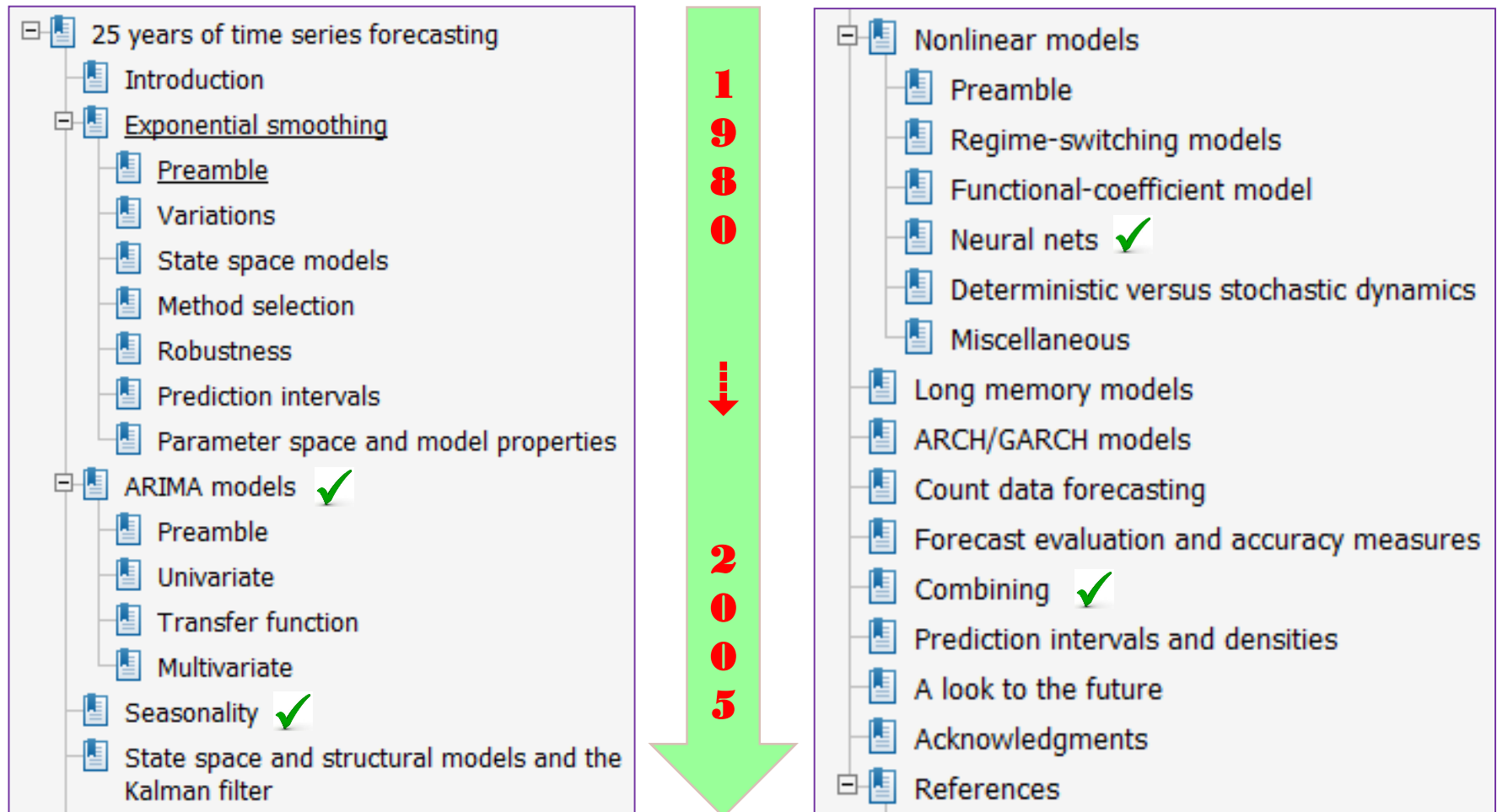
☞ *The M3-Competition*: results, conclusions and implications

- ➡ (1) Statistically sophisticated or complex methods do not necessarily provide more accurate forecasts than simpler ones.
- (2) The relative ranking of the performance of the various methods varies according to the accuracy measure being used.
- ➡ (3) The accuracy when various methods are being combined outperforms, on average, the individual methods being combined and does very well in comparison to other methods.
- (4) The accuracy of the various methods depends upon the length of the forecasting horizon involved.

Makridakis & Hibon (International Journal of Forecasting, 2000)

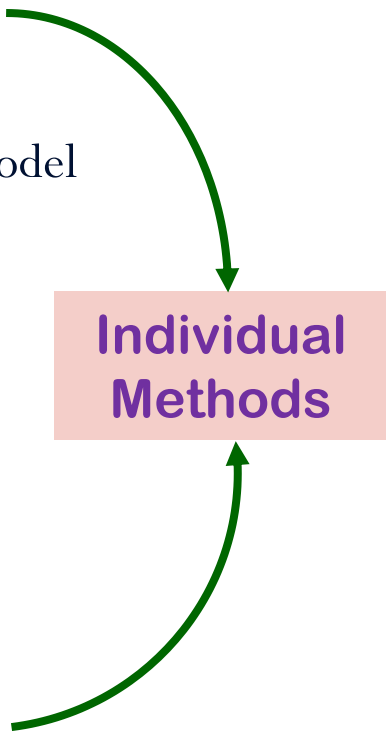
25 years of time series forecasting

De Gooijer & Hyndman (International Journal of Forecasting, 2006)



Forecasting Methods

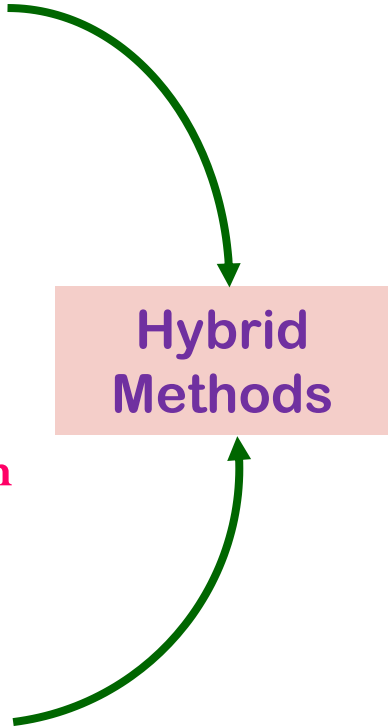
1. **Exponential Smoothing:** Holt-Winter's method, Holt-Winter-Taylor's method
2. **Decomposition Method**
3. **Trend Analysis & Time Series Regression**
4. **ARIMA & ARIMAX:** Intervention Analysis, Transfer Function model
5. **Neural Networks:** FFNN, RBFN, GRNN, RNN
6. **Adaptive Neuro Fuzzy Inference Systems (ANFIS)**
7. **Multiresolution Autoregressive (MAR)**
8. **Wavelet Neural Networks (WNN)**
9. **Support Vector Regression (SVR)**
10. **Fuzzy Time Series (FTS)**
11. **Quantile Regression** Autoregressive & ARIMAX
12. **Quantile Regression** Neural Networks (QRNN)
13. **Singular Spectrum Analysis (SSA)** - Linear Recurrence Relations (LRR)
14. **Deep Learning**



Individual
Methods

Forecasting Methods

1. Winter's model & ARIMA
2. Winter's model & Neural Networks
3. Winter's model & Fuzzy Time Series
4. Decomposition Method & ARIMA
5. Decomposition Method & Neural Networks
6. Time Series Regression & Neural Networks
7. Time Series Regression & ANFIS
8. Time Series Regression & Support Vector Regression
9. ARIMAX & Neural Networks
10. ARIMAX & ANFIS
11. ARIMAX & Support Vector Regression
12. ARIMAX & Quantile Regression Neural Networks
13. ARIMAX & Deep Learning



Hybrid
Methods

Forecasting Methods

1. **Decomposition** method & ARIMA
2. **Decomposition** method & Neural Networks
3. **Decomposition** method & Fuzzy Time Series
4. Wavelet Transform - MODWT & Autoregressive Model
5. Wavelet Transform - MODWT & Neural Networks
6. Wavelet Transform - MODWT & ANFIS
7. Singular Spectrum Analysis & Time Series Regression
8. Singular Spectrum Analysis & ARIMA
9. Singular Spectrum Analysis & Neural Networks
10. Singular Spectrum Analysis & ANFIS
11. Singular Spectrum Analysis & Support Vector Regression
12. Singular Spectrum Analysis & **Deep Learning**



Hybrid
Decomposition
Methods

The diagram illustrates a classification of forecasting methods. On the left, a list of 12 methods is provided. On the right, a light red rectangular box contains the text 'Hybrid Decomposition Methods'. Two curved red arrows originate from the list: one from the top three items (1-3) and another from the bottom three items (10-12), both pointing towards the box. This indicates that these specific methods are categorized as hybrid decomposition methods.

Forecasting Methods

☞ The **main** patterns and problems of time series forecasting

1. **TREND**: linear vs non linear
 2. **SEASONAL**: single vs multiple period
 3. **STATIONER**: linear vs non linear
 4. **CALENDAR VARIATION** effect
 5. **OUTLIER & INTERVENTION** analysis
 6. **ADDITIVE** vs **MULTIPLICATIVE** pattern
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Outlines

1. Time Series Regression (TSR) model

- ☞ Trend, Seasonal & Calendar Variation pattern
- ☞ Multiple Seasonal models.

2. ARIMA & ARIMAX Model

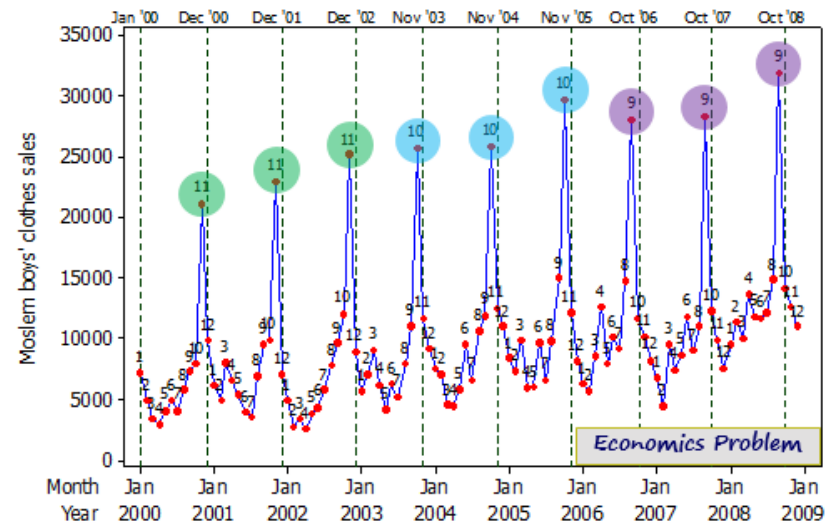
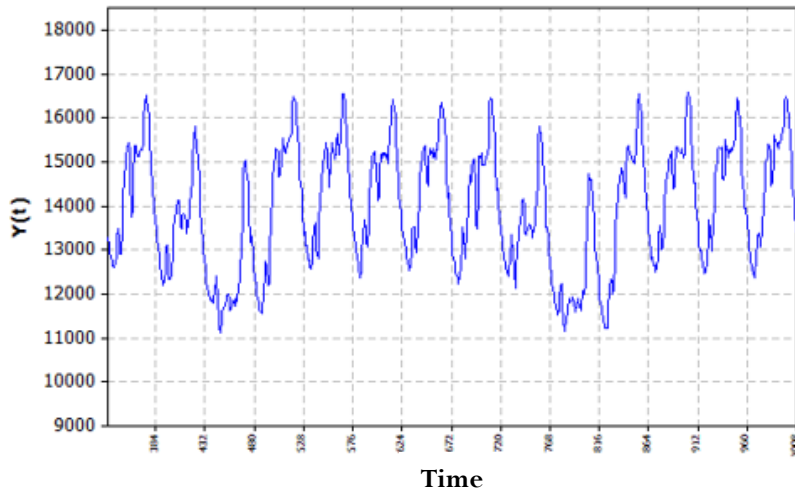
- ☞ Seasonal Model: Multiplicative, Additive, Subset
- ☞ Multiple Seasonal models.
- ☞ ARIMAX: Calendar Variation models.

3. Neural Network

- ☞ Feedforward Neural Network (FFNN)
 - ☞ Deep Feedforward Network.
-

Time Series Regression for Trend, Seasonal & Calendar Variation

Half-hourly of electricity load data



Introduction

- ✓ General time series “PATTERN”
 - ✗ Stationary
 - ✗ Trend: *linear* & *nonlinear*
 - ✗ Seasonal: *additive* & *multiplicative*
 - ✗ Cyclic
 - ✗ Calendar Variation
-

Introduction

cont'

- Two kinds of calendar variation effects:

- 1. Trading day effects

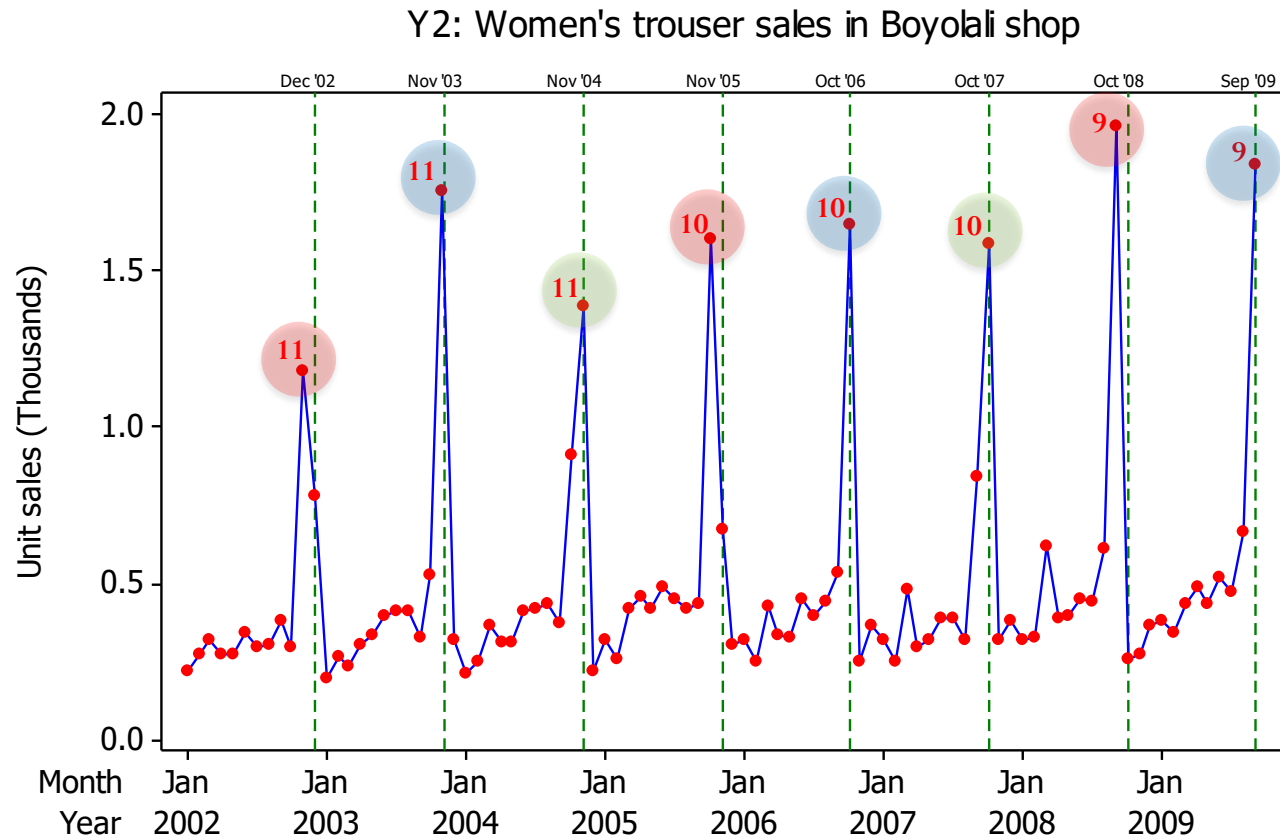
- The levels of economics or business activities may **change depending on the day of the week**. The composition of days of the week varies from month to month and year to year.

- 2. Holiday (traditional festivals) effects

- Some traditional festivals or holidays, such as Eid ul-Fitr, Easter, Chinese New Year, and Jewish Passover are set according to lunar calendars and the dates of such holidays may vary between two adjacent months in the Gregorian calendar from year to year.

Introduction

cont'



Introduction

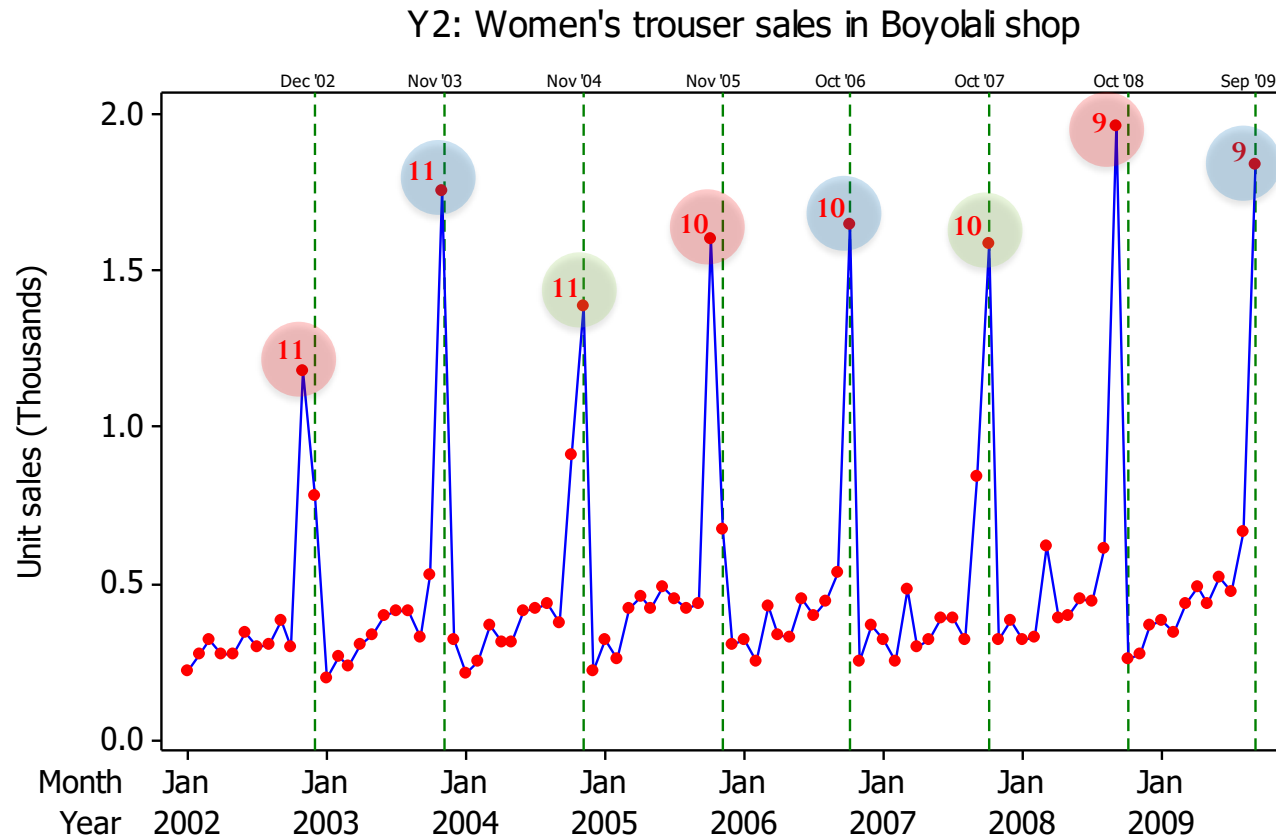
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Eid holidays for the period 2002 to 2011

Year	Date	Explanation
2002	06-07 December	There are 5 days before Eid in December
2003	25-26 November	There are 24 days before Eid in November
2004	14-15 November	There are 13 days before Eid in November
2005	03-04 November	There are 2 days before Eid in November
2006	23-24 October	There are 22 days before Eid in October
2007	12-13 October	There are 11 days before Eid in October
2008	01-02 October	There is 0 day before Eid in October
2009	21-22 September	There are 20 days before Eid in September
2010	10-11 September	There are 9 days before Eid in September
2011	30-31 August	There are 29 days before Eid in August

Introduction

cont'



Introduction

cont'

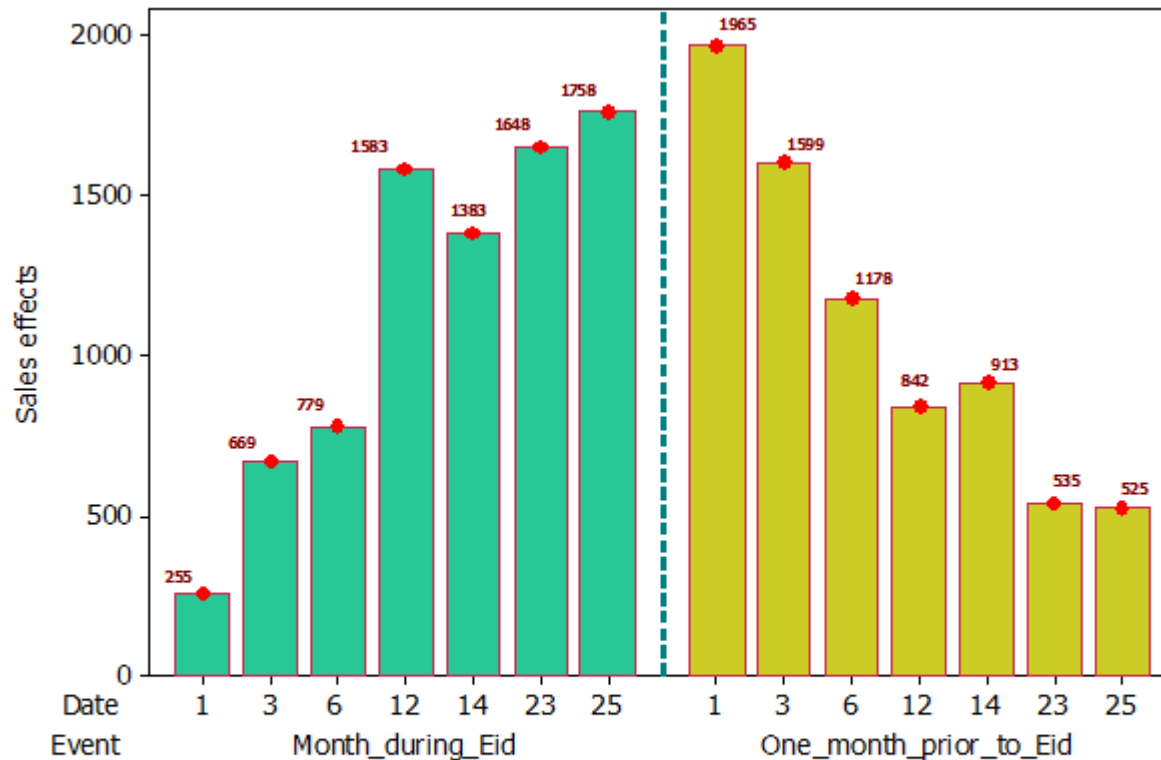


Fig. 2. Bar chart of Eid effects on the women's trouser sales in the month during and one month prior to the Eid celebration in Boyolali shop.

Modeling method

- Model for linear **trend**:

$$y_t = \beta_0 + \beta_1 t + w_t \quad \dots (1)$$

- Regression with dummy variable for **seasonal** pattern:

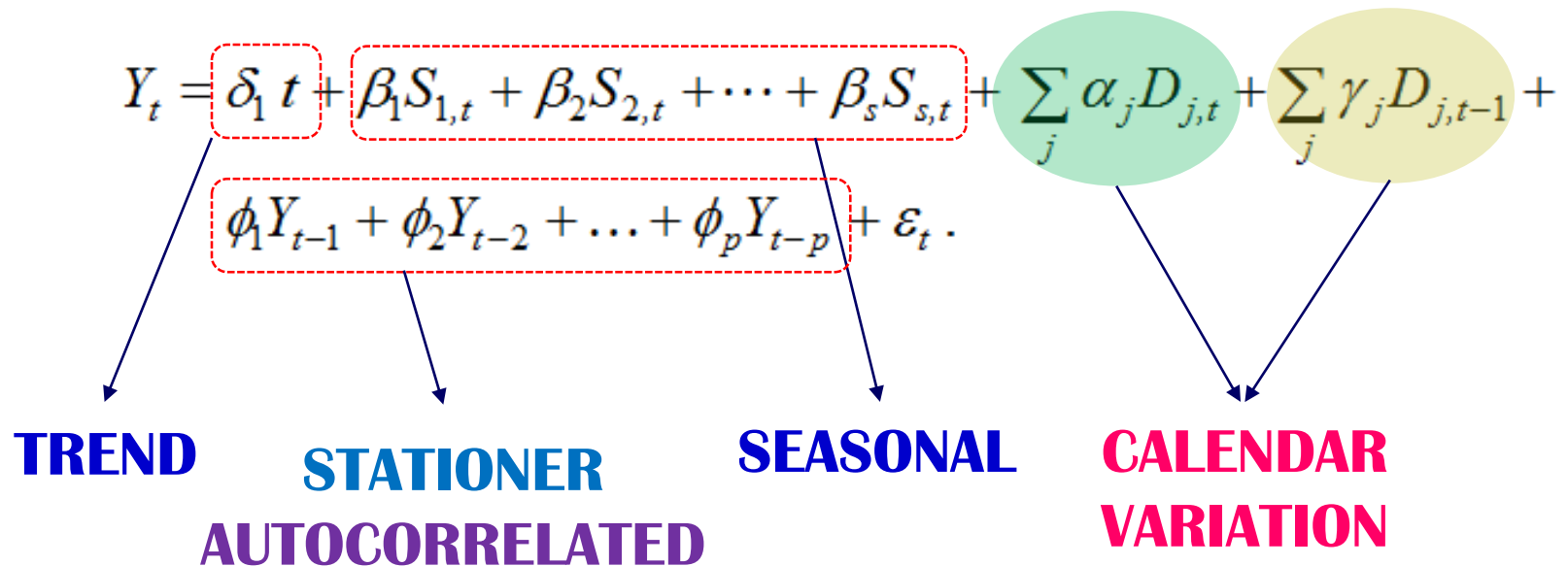
$$y_t = \beta_1 S_{1,t} + \beta_2 S_{2,t} + \dots + \beta_s S_{s,t} + w_t \quad \dots (2)$$

- Regression for **calendar variation** effects:

$$y_t = \beta_0 + \beta_1 V_{1,t} + \beta_2 V_{2,t} + \dots + \beta_p V_{p,t} + w_t \quad \dots (3)$$

The Proposed Model

- Time Series Regression Model:



The proposed procedure

- Step 1: Determination of **dummy variable** for calendar variation period.
- Step 2: Determination of **deterministic trend** and **seasonal** model.
- Step 3: **Simultaneous estimation** of **calendar effects** and other patterns.
- Step 4: **Diagnostic checks** on error model. If error is **not white noise**, add significant lags (**autoregressive order**) based on ACF and PACF plots of error model.
- Step 5: **Re-estimate** calendar effect, other pattern (trend, seasonal), and appropriate lags (autoregressive order) **simultaneously** for the model.
-



Simulation Data using R

□ TREND

$$T_t = \beta t$$

In this simulation, $\beta = 0.1$.

□ SEASONAL

$$M_t = \gamma_1 M_{1,t} + \gamma_2 M_{2,t} + \dots + \gamma_{12} M_{12,t}$$

The values for γ are:

$$M_t = 20M_{1,t} + 23M_{2,t} + 25M_{3,t} + 23M_{4,t} + 20M_{5,t} + 15M_{6,t} + \\ 10M_{7,t} + 7M_{8,t} + 5M_{9,t} + 7M_{10,t} + 10M_{11,t} + 15M_{12,t}$$



Simulation Data using R

□ CALENDAR VARIATION

$$V_t = \delta_1 V_{1,t} + \delta_2 V_{2,t} + \delta_3 V_{3,t} + \delta_4 V_{4,t} + \omega_1 V_{1,t-1} + \omega_2 V_{2,t-1} + \omega_3 V_{3,t-1} + \omega_4 V_{4,t-1}$$

where the value of δ and ω are:

$$V_t = 23V_{1,t} + 37V_{2,t} + 44V_{3,t} + 48V_{4,t} + 56V_{1,t-1} + 42V_{2,t-1} + 34V_{3,t-1} + 30V_{4,t-1}$$

□ NOISE

- Linear: AR(1) models

$$N_t = 0,7N_{t-1} + a_t, \text{ with } a_t \sim IIDN(0,1).$$

- Nonlinear: ESTAR(1) models

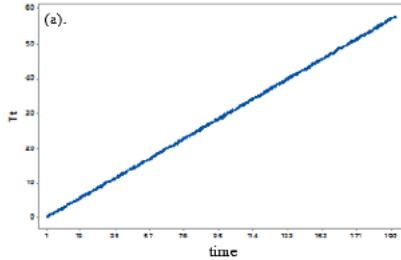
$$N_t = 6,5N_{t-1} \cdot \exp(-0,25N_{t-1}^2) + a_t, \text{ with } a_t \sim IIDN(0,1).$$



Dummy for Calendar Variation

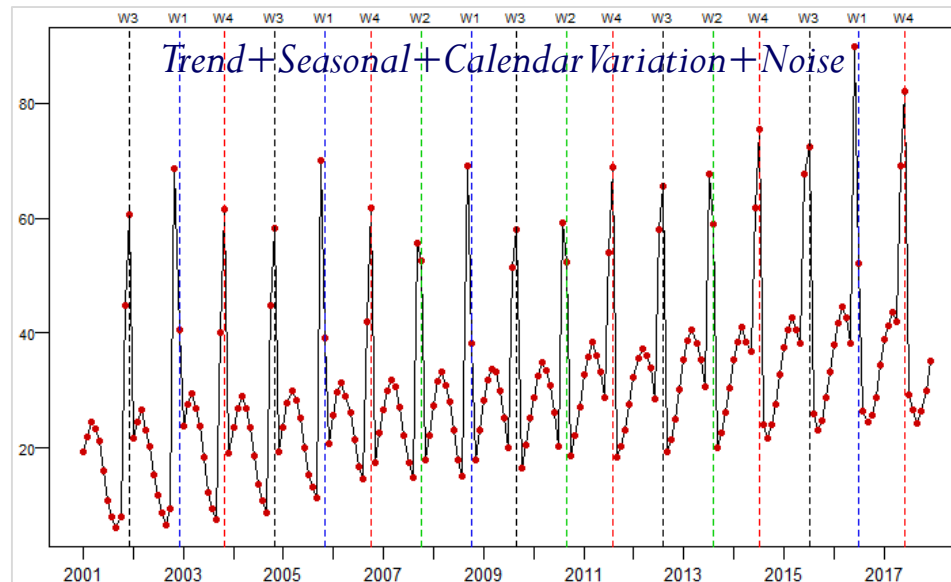
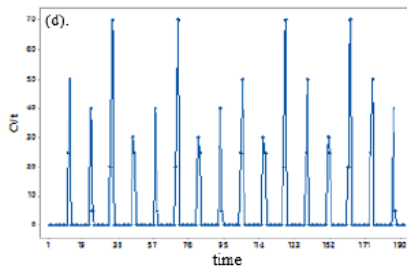
Year	Date of Eid ul-Fitr	Week of Eid ul-Fitr	Month of Eid ul-Fitr
2001	17-18	3 rd week	December
2002	06-07	1 st week	December
2003	25-26	4 th week	November
2004	13-14	2 nd week	November
2005	03-04	1 st week	November
2006	23-24	4 th week	October
2007	12-13	2 nd week	October
2008	01-02	1 st week	October
2009	20-21	3 rd week	September
2010	09-10	2 nd week	September
2011	30-31	4 th week	August
2012	18-19	3 rd week	August
2013	08-09	2 nd week	August
2014	28-29	4 th week	July
2015	19-20	3 rd week	July
2016	06-07	1 st week	July
2017	25-26	4 th week	June

Simulation data

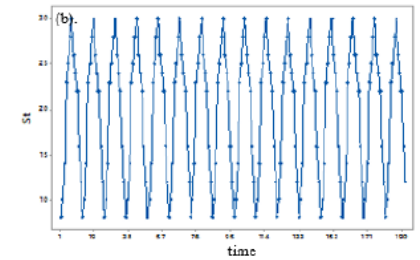


Trend

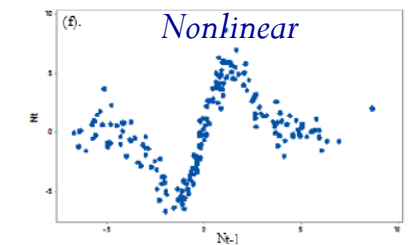
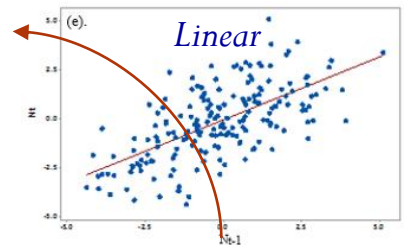
Calendar Variation



Scenario of Additive Pattern



1st Seasonal





Step 1

- Based on the time series plot, **two dummy variables** are used for evaluating calendar variation effect, i.e.
 - The months prior to Eid ul Fitr,
 $D_{j,t-1}$ = dummy variable for **ONE** month prior to Eid ul-Fitr celebration.
 - During the month of Eid ul-Fitr celebration,
 $D_{j,t}$ = dummy variable for **during** the month of Eid ul-Fitr celebration.
 - **j** = week of Eid ul-Fitr celebration
-



Step 2 - 5

- Model for **linear trend**:

$$y_t = \beta_0 + \beta_1 t + w_t$$

- Regression with dummy variable for **seasonal** pattern:

$$y_t = \beta_1 S_{1,t} + \beta_2 S_{2,t} + \dots + \beta_s S_{s,t} + w_t$$

- Regression for **calendar effects** and **other patterns**:

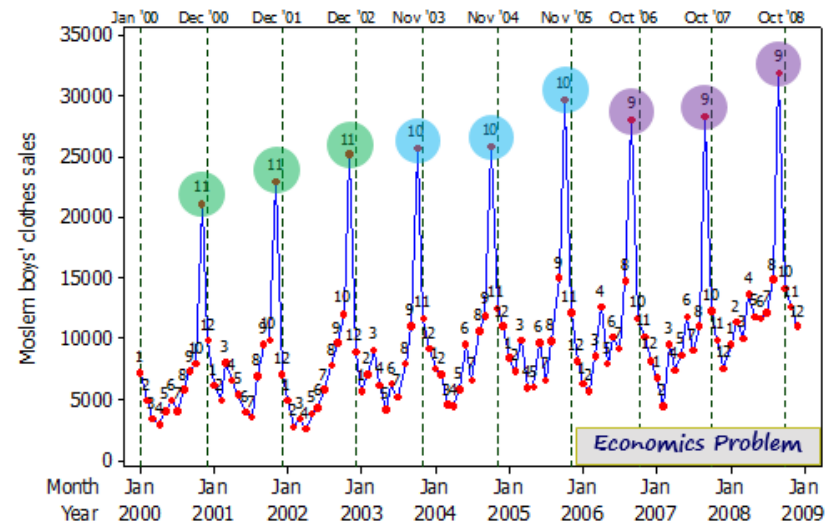
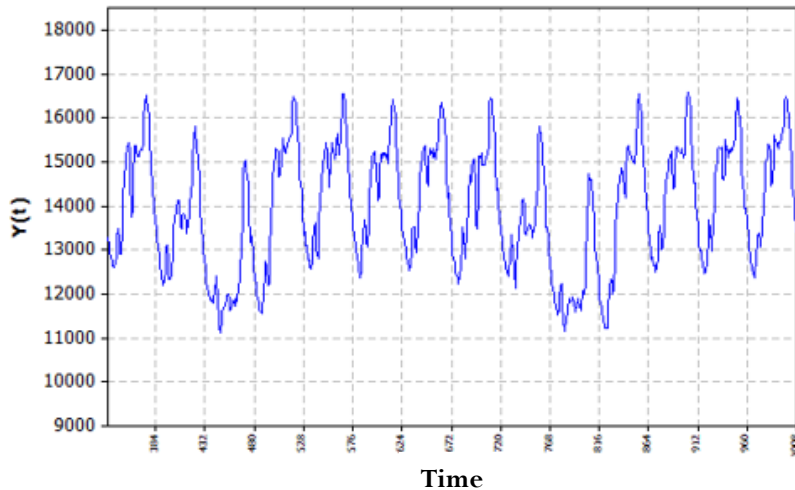
$$Y_t = \delta_1 t + \beta_1 S_{1,t} + \beta_2 S_{2,t} + \dots + \beta_s S_{s,t} + \sum_j \alpha_j D_{j,t} + \sum_j \gamma_j D_{j,t-1} + N_t.$$

- **Final Regression model is:**

$$Y_t = \delta_1 t + \beta_1 S_{1,t} + \beta_2 S_{2,t} + \dots + \beta_s S_{s,t} + \sum_j \alpha_j D_{j,t} + \sum_j \gamma_j D_{j,t-1} + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \dots + \phi_p Y_{t-p} + \varepsilon_t.$$

ARIMAX Model for Trend, Seasonal & Calendar Variation

Half-hourly of electricity load data



The Proposed ARIMAX Model

- The 1st proposed Model → **ARIMAX-1**: **stochastic TREND-SEASONAL**

$$Y_t = \underbrace{\sum_j \alpha_j D_{j,t}}_{\text{CALENDAR VARIATION}} + \underbrace{\sum_j \gamma_j D_{j,t-1}}_{\text{STATIONER AUTOCORRELATED}} + \underbrace{\frac{\theta_q(B)\Theta_Q(B^S)}{\phi_p(B)\Phi_P(B^S)(1-B)^d(1-B^S)^D}}_{\text{TREND SEASONAL}} \varepsilon_t$$

The diagram illustrates the ARIMAX-1 model equation. The equation is $Y_t = \sum_j \alpha_j D_{j,t} + \sum_j \gamma_j D_{j,t-1} + \frac{\theta_q(B)\Theta_Q(B^S)}{\phi_p(B)\Phi_P(B^S)(1-B)^d(1-B^S)^D} \varepsilon_t$. The first term, $\sum_j \alpha_j D_{j,t}$, is enclosed in a green circle and has an arrow pointing to the text 'CALENDAR VARIATION' in pink. The second term, $\sum_j \gamma_j D_{j,t-1}$, is enclosed in a yellow circle and has an arrow pointing to the text 'STATIONER AUTOCORRELATED' in blue and purple. The third term, the fraction, is enclosed in a red dashed box and has three arrows pointing to the text 'TREND SEASONAL' in blue.

The Proposed ARIMAX Model

- The 2nd proposed Model → **ARIMAX-2**: deterministic TREND-SEASONAL

$$Y_t = \underbrace{\delta_1 t}_{\text{TREND}} + \underbrace{\beta_1 M_{1,t} + \beta_2 M_{2,t} + \cdots + \beta_s M_{s,t}}_{\text{SEASONAL}} + \underbrace{\sum_j \alpha_j D_{j,t}}_{\text{CALENDAR VARIATION}} + \underbrace{\sum_j \gamma_j D_{j,t-1}}_{\text{CALENDAR VARIATION}} + \underbrace{\frac{\theta_q(B)}{\phi_p(B)} \varepsilon_t}_{\text{STATIONER Autocorrelated}}$$

The diagram illustrates the ARIMAX-2 model equation. The equation is: $Y_t = \delta_1 t + \beta_1 M_{1,t} + \beta_2 M_{2,t} + \cdots + \beta_s M_{s,t} + \sum_j \alpha_j D_{j,t} + \sum_j \gamma_j D_{j,t-1} + \frac{\theta_q(B)}{\phi_p(B)} \varepsilon_t$. Below the equation, four components are identified with arrows pointing to their respective parts in the equation: 1. **TREND** points to $\delta_1 t$. 2. **SEASONAL** points to the sum of seasonal terms $\beta_1 M_{1,t} + \beta_2 M_{2,t} + \cdots + \beta_s M_{s,t}$. 3. **CALENDAR VARIATION** (in red) points to both $\sum_j \alpha_j D_{j,t}$ and $\sum_j \gamma_j D_{j,t-1}$. 4. **STATIONER Autocorrelated** (in blue and purple) points to the moving average term $\frac{\theta_q(B)}{\phi_p(B)} \varepsilon_t$.

The Proposed ARIMAX Model

- The 3rd proposed Model → **ARIMAX-3**: deterministic TREND-SEASONAL, and stochastic TREND-SEASONAL

$$Y_t = \underbrace{\delta_1 t}_{\text{TREND}} + \underbrace{\beta_1 M_{1,t} + \beta_2 M_{2,t} + \dots + \beta_s M_{s,t}}_{\text{SEASONAL}} + \underbrace{\sum_j \alpha_j D_{j,t} + \sum_j \gamma_j D_{j,t-1}}_{\text{CALENDAR VARIATION}} + \underbrace{\frac{\theta_q(B)\Theta_Q(B^S)}{\phi_p(B)\Phi_P(B^S)(1-B)^d(1-B^S)^D} \varepsilon_t}_{\text{Stochastic Trend, Seasonal \& Stationer}}$$

The Proposed Procedure

Step 1: Determination of **dummy variable** for calendar variation period.

Step 2: Remove the calendar variation effect from the response by fitting

$$Y_t = \beta_0 + \sum_j \alpha_j D_{j,t} + \sum_j \gamma_j D_{j,t-1} + N_t$$

for model with **stochastic trend** and **seasonal** model, or fitting

$$Y_t = \delta_1 t + \beta_1 M_{1,t} + \beta_2 M_{2,t} + \cdots + \beta_s M_{s,t} + \sum_j \alpha_j D_{j,t} + \sum_j \gamma_j D_{j,t-1} + N_t$$

simultaneously for model with **deterministic trend** and **seasonal**, to obtain the error, N_t .

Step 3: Find the best ARIMA model of N_t using Box-Jenkins procedure.

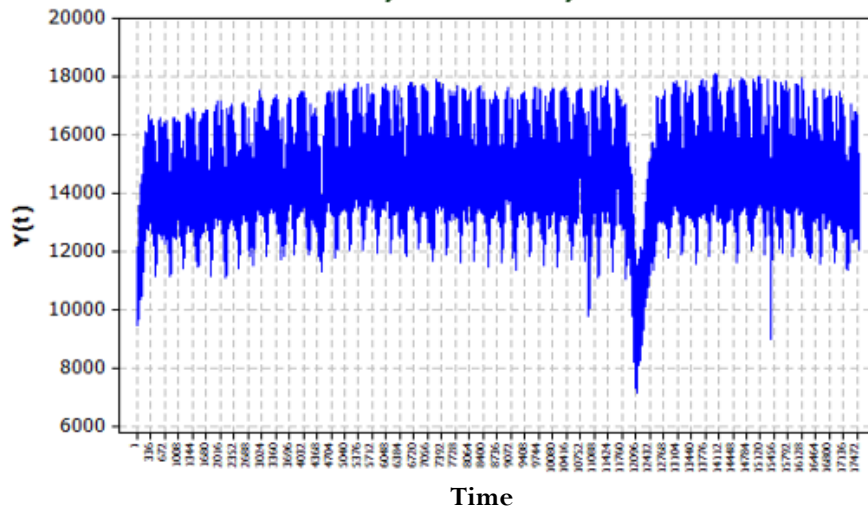
Step 4: Simultaneously fit the model from step 2 and 3. This model is the **calendar variation model based on ARIMAX method**.

Step 5: Test the significance of parameter and perform diagnostic check.

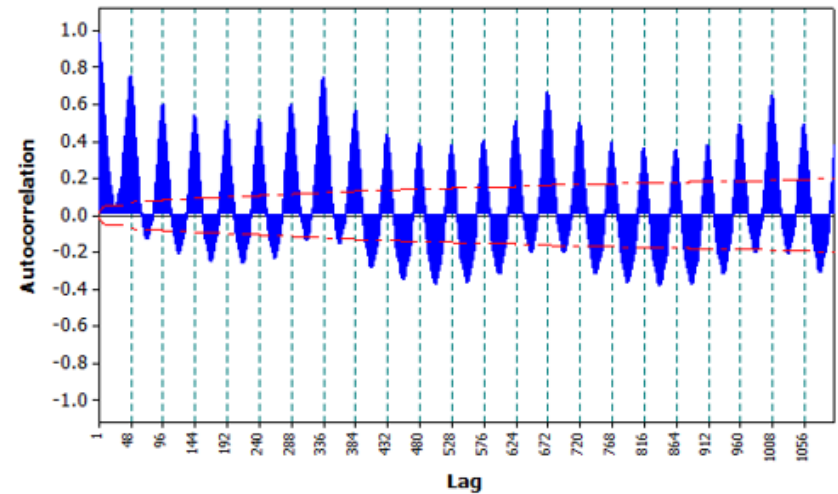
ARIMA model

Multiple Seasonal: multiplicative – subset

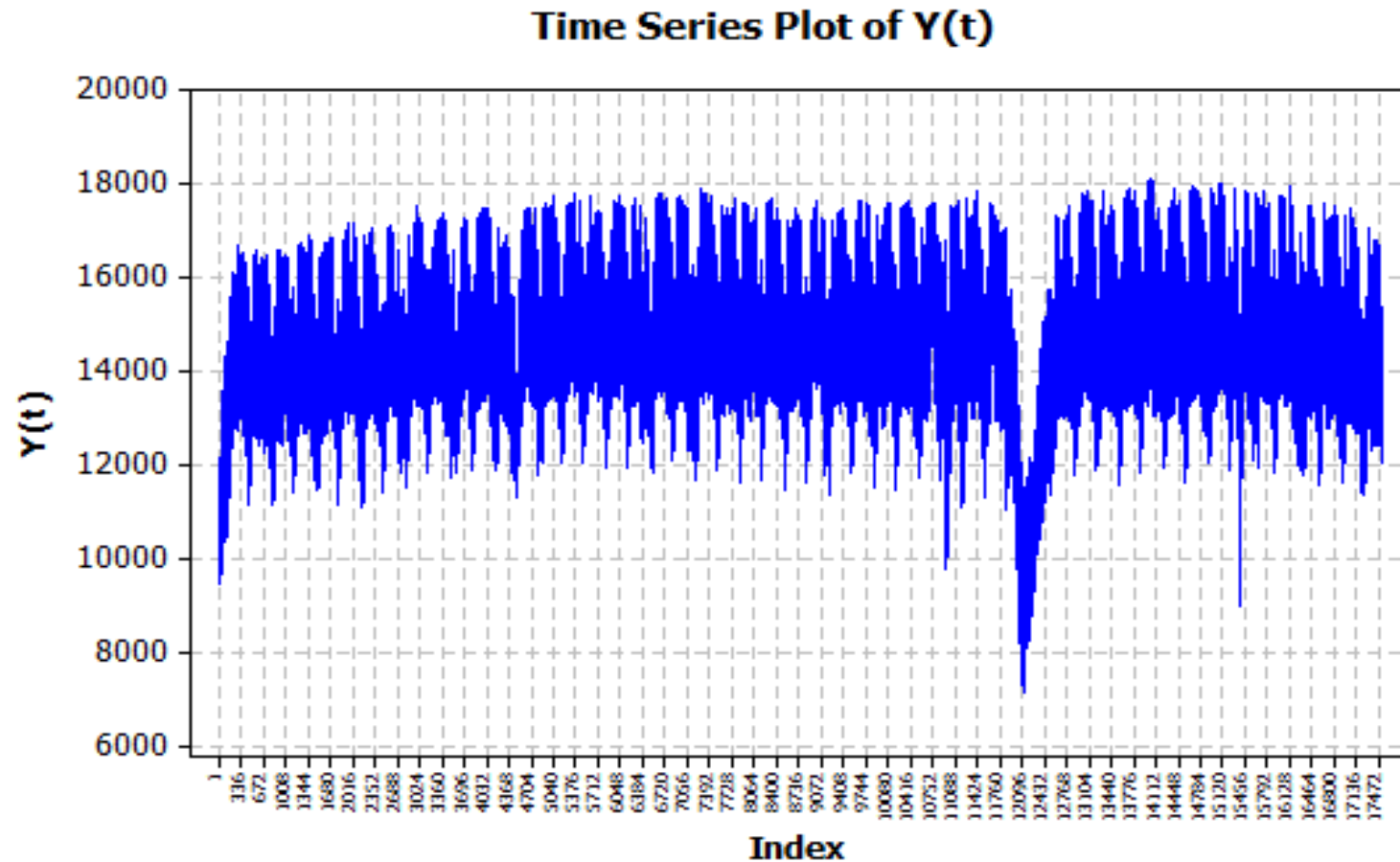
Half-hourly of electricity load data



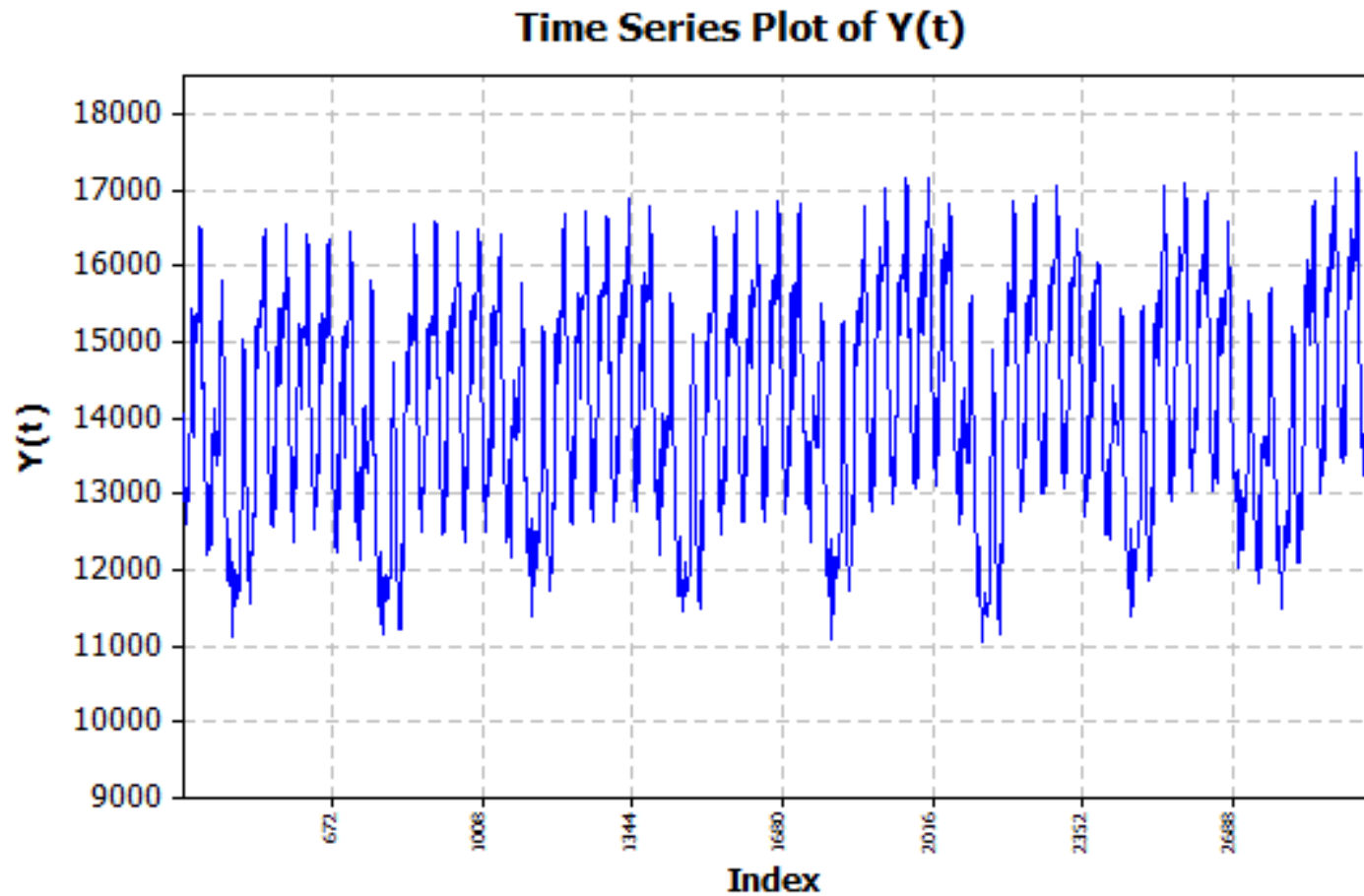
ACF for $Y(t)$



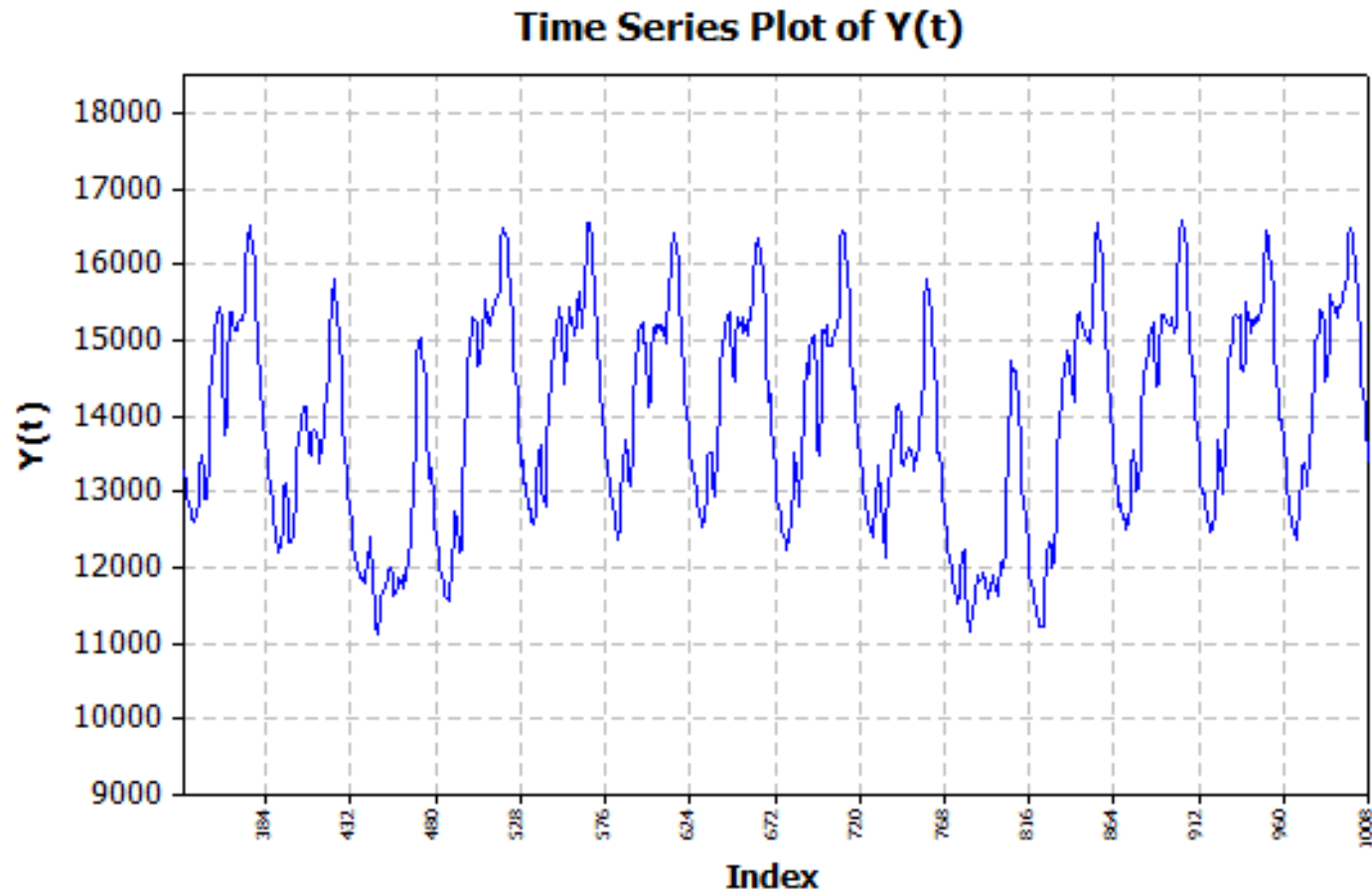
Problem: Prediction of half hourly load data



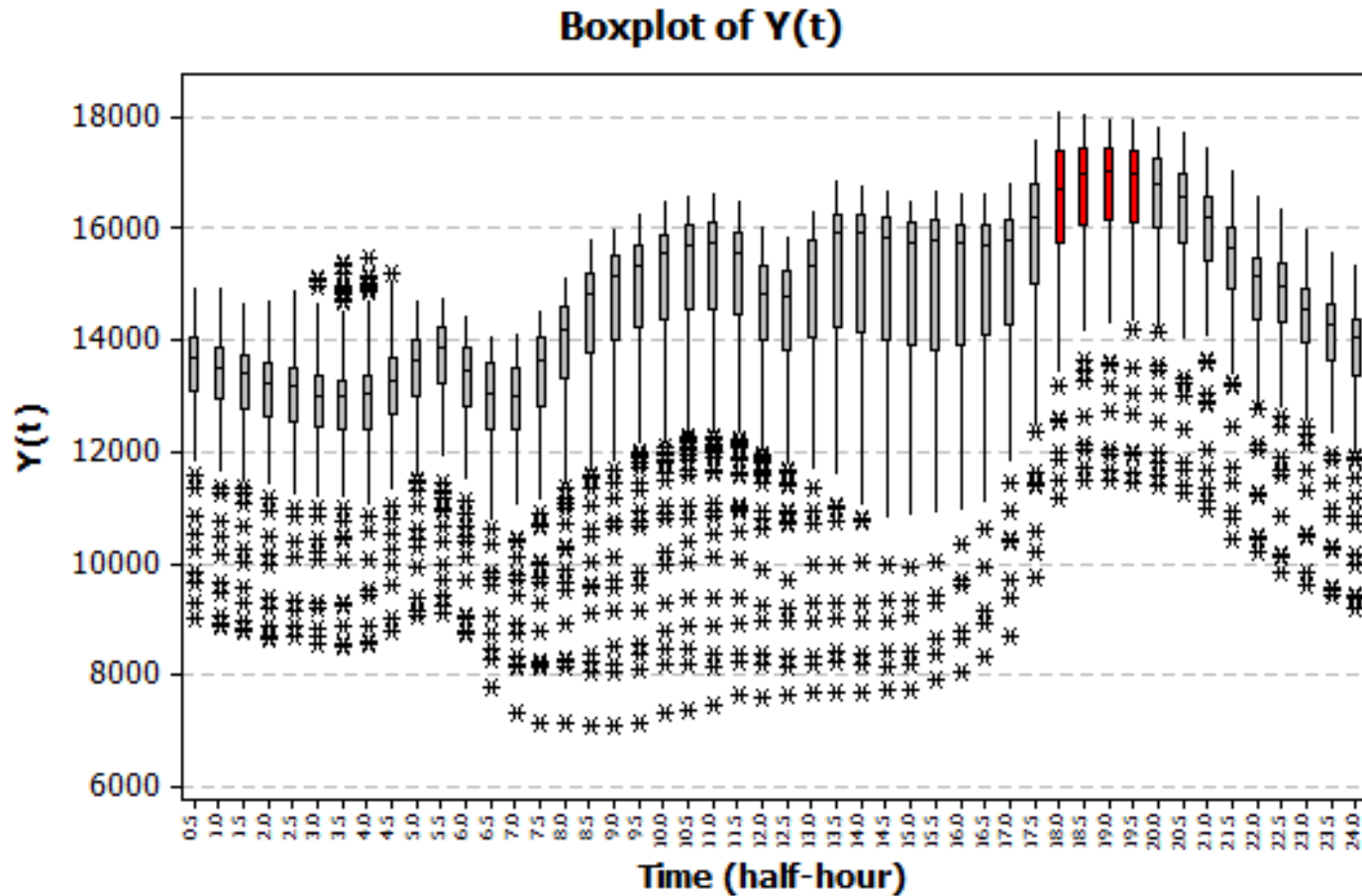
MINITAB Descriptive: half hourly load data



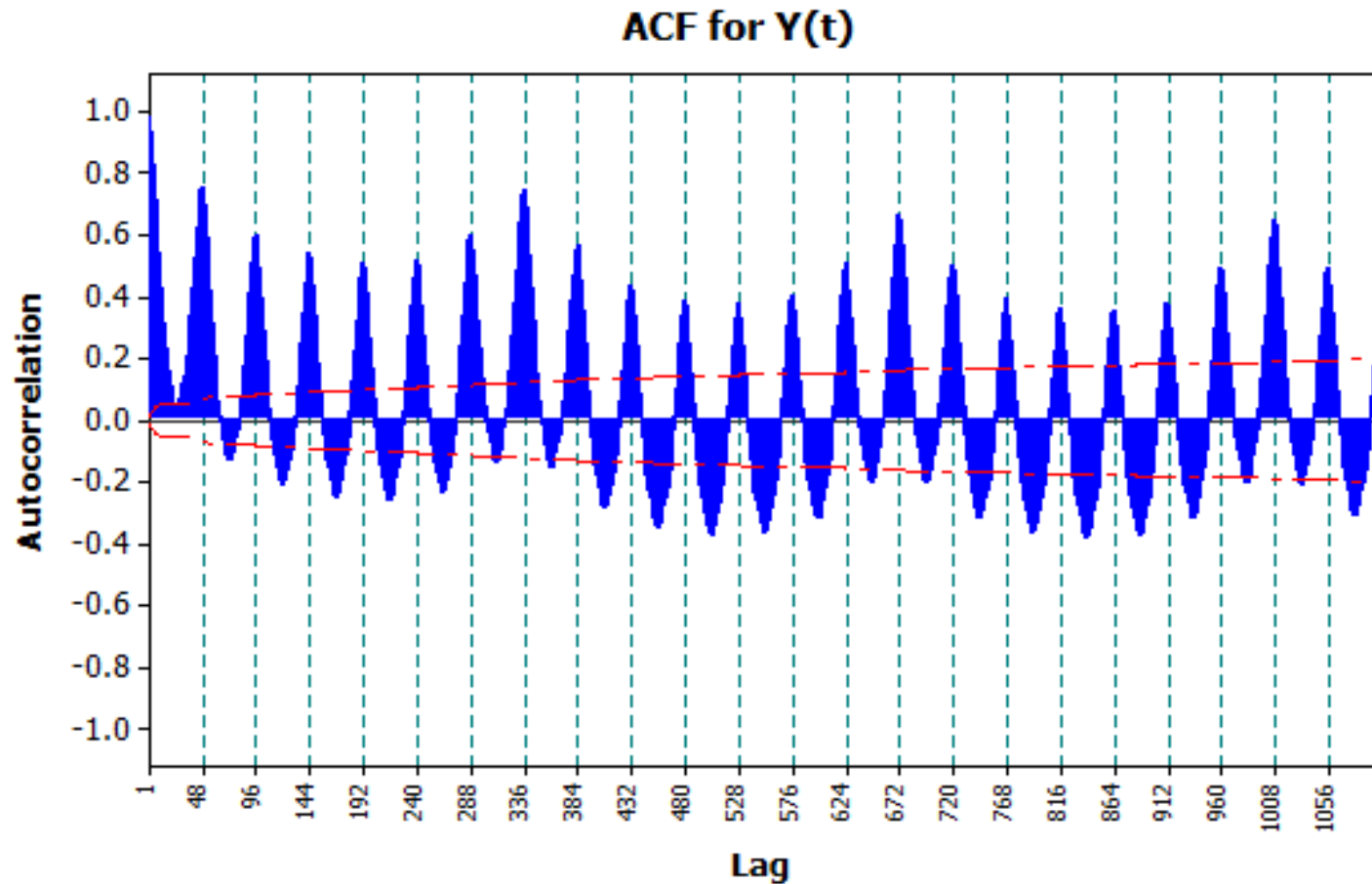
MINITAB Descriptive: half hourly load data



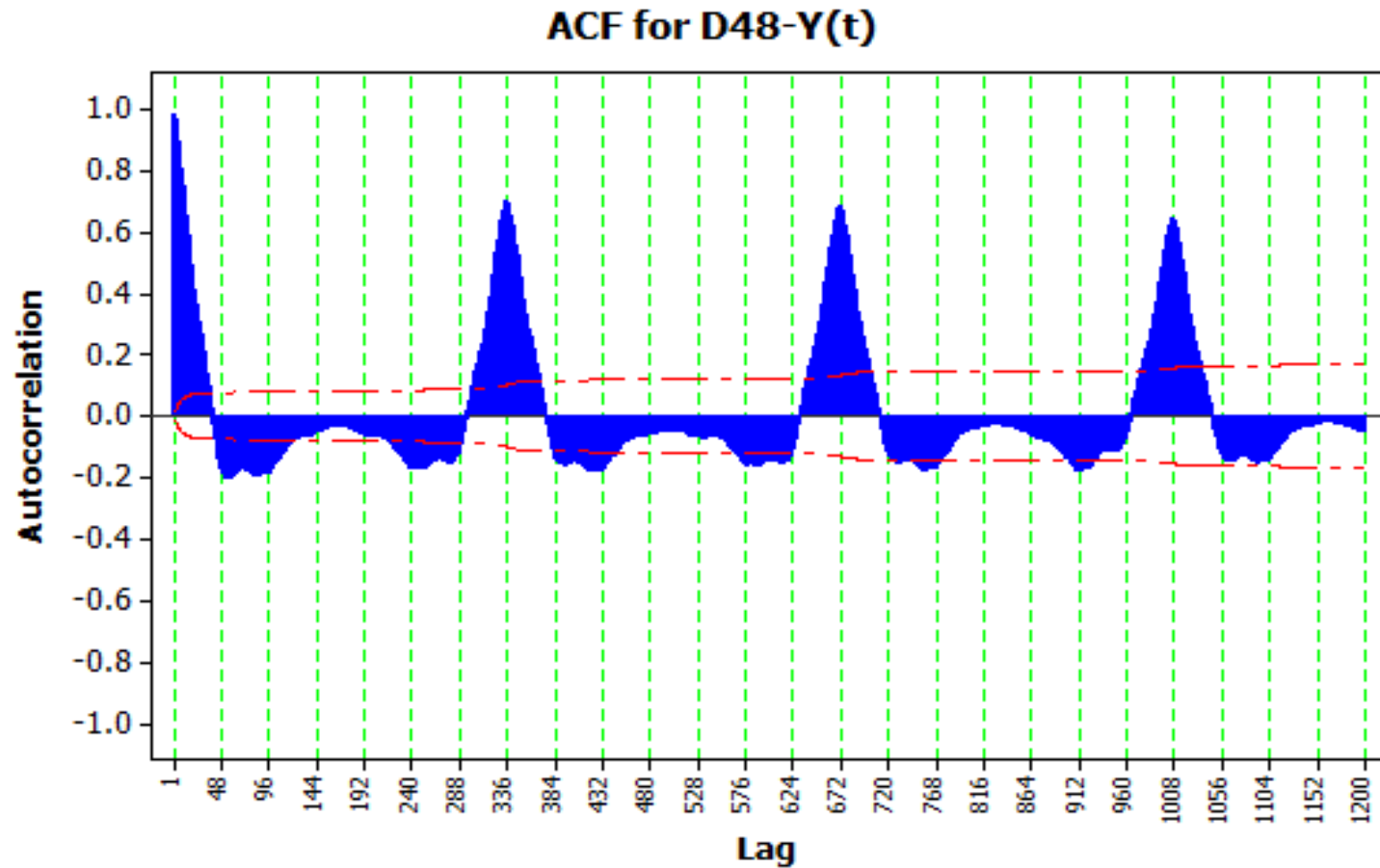
MINITAB Descriptive: half hourly load data



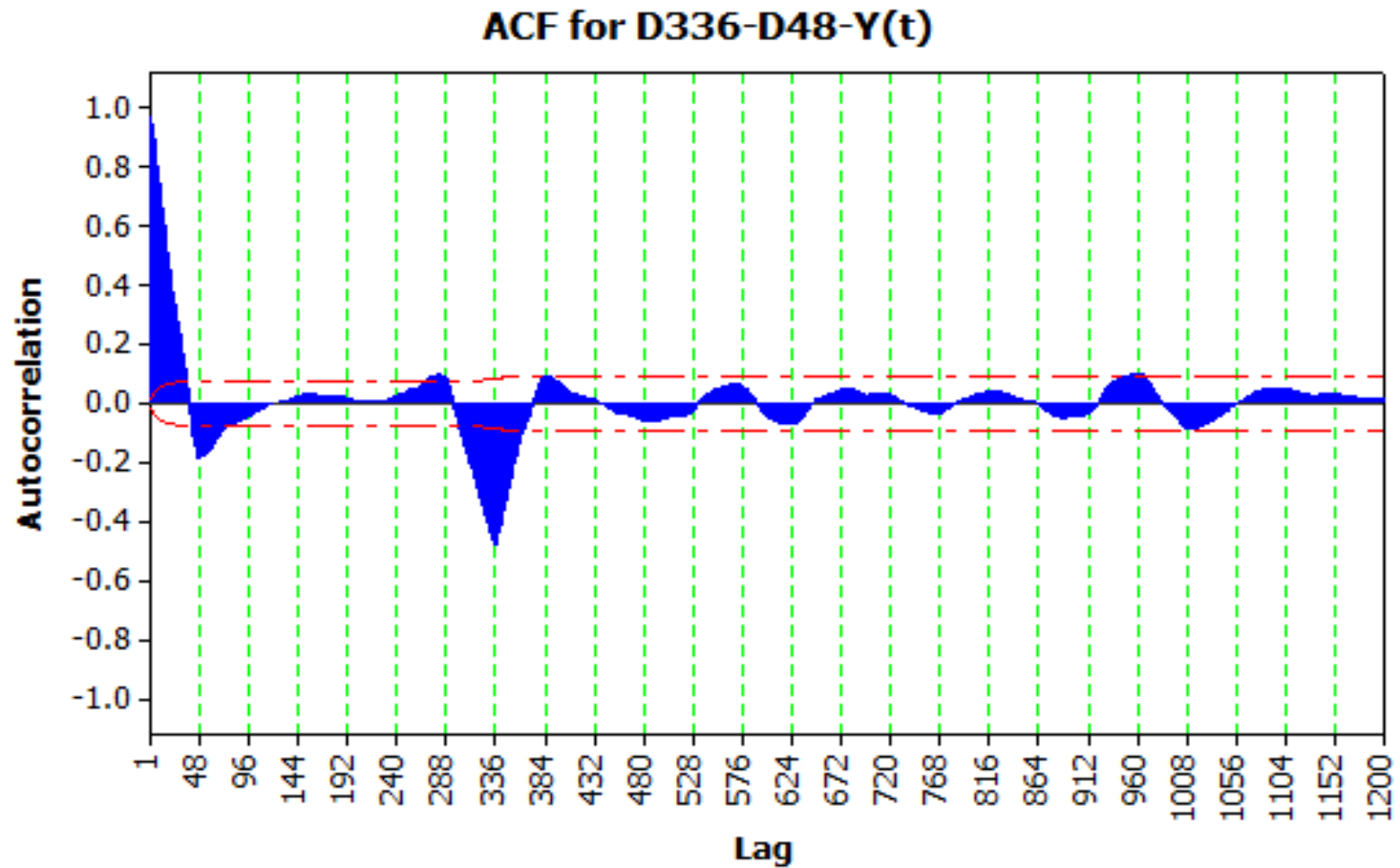
MINITAB Identification: half hourly load data



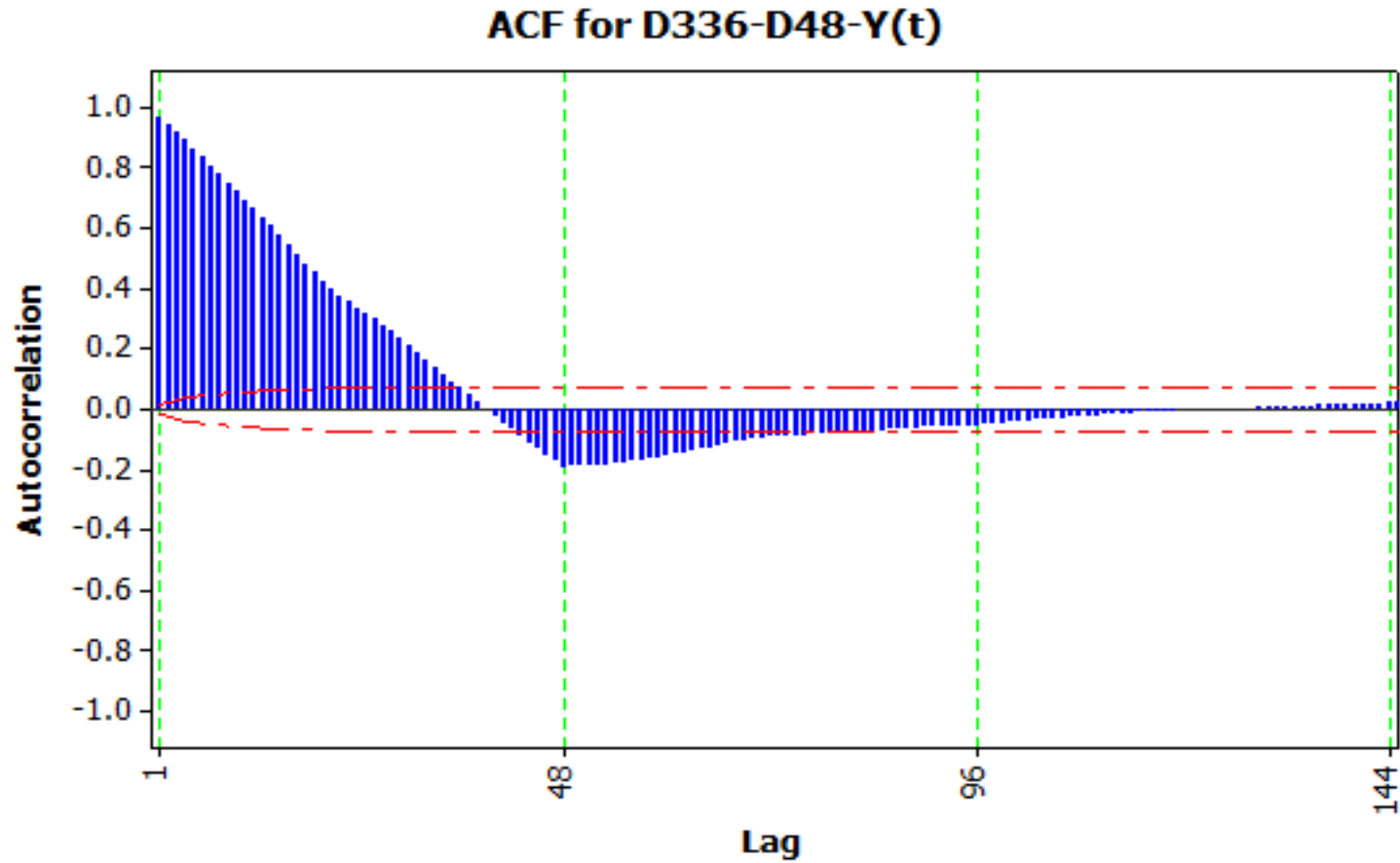
MINITAB Identification: half hourly load data



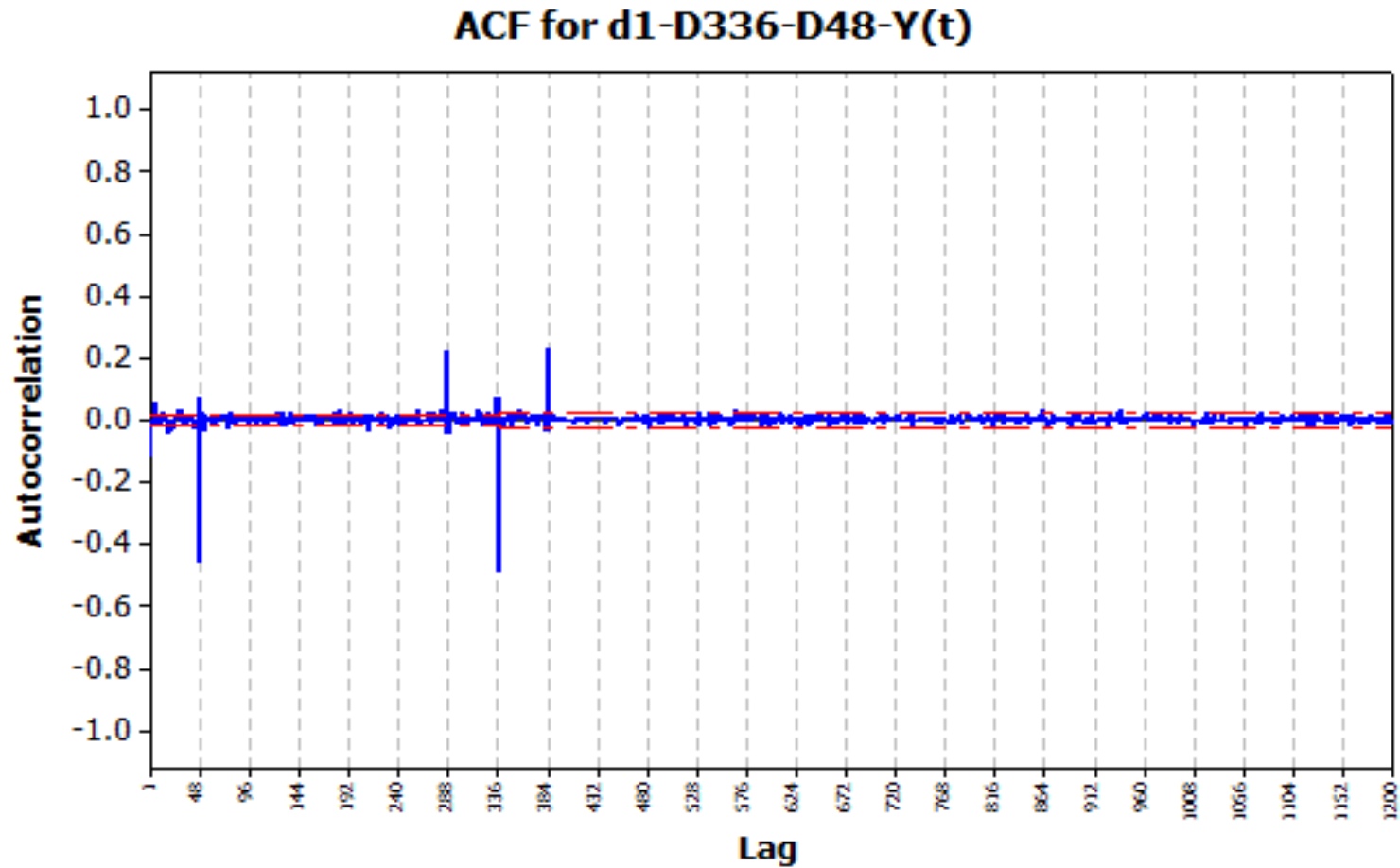
MINITAB Identification: half hourly load data



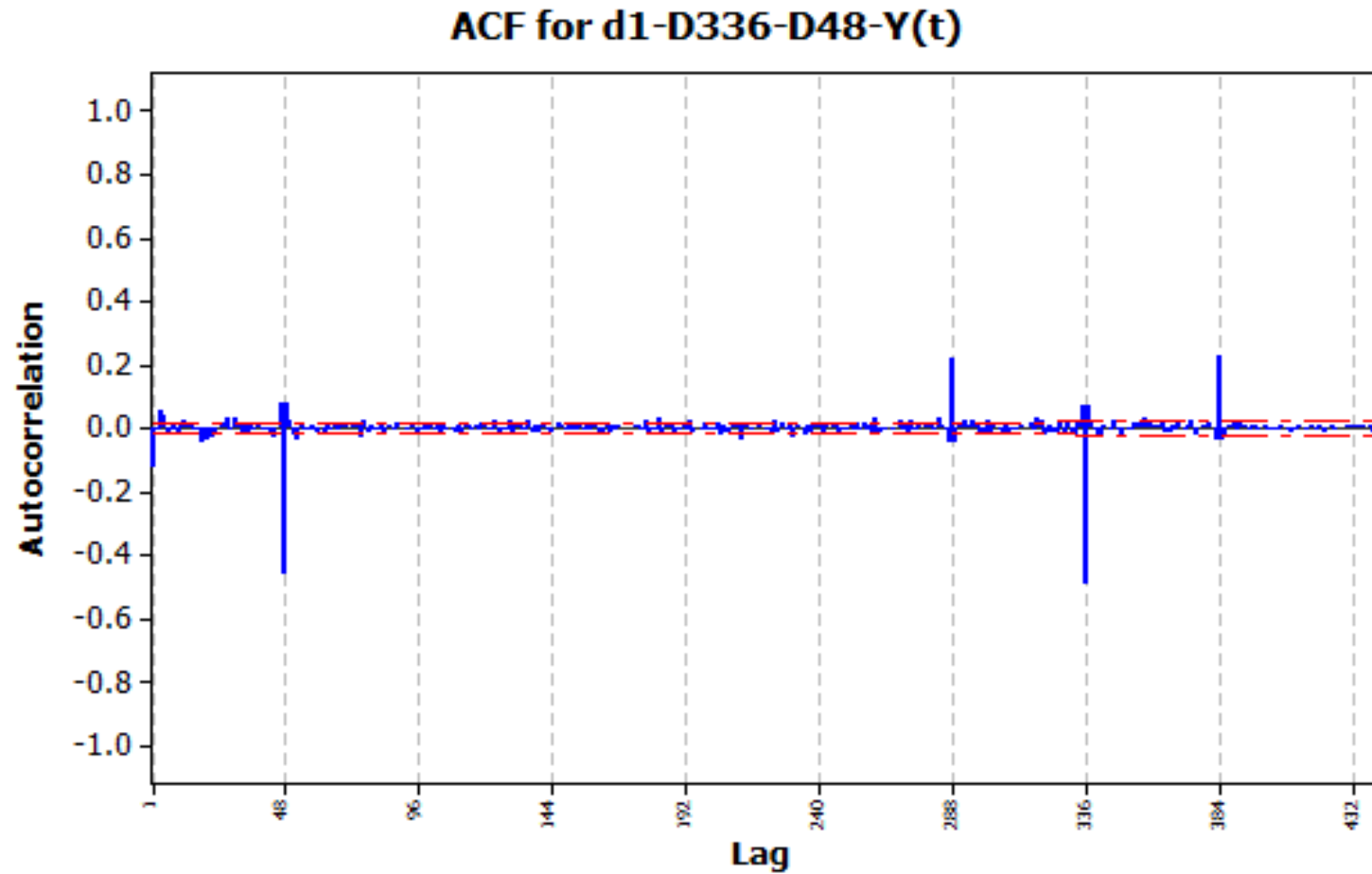
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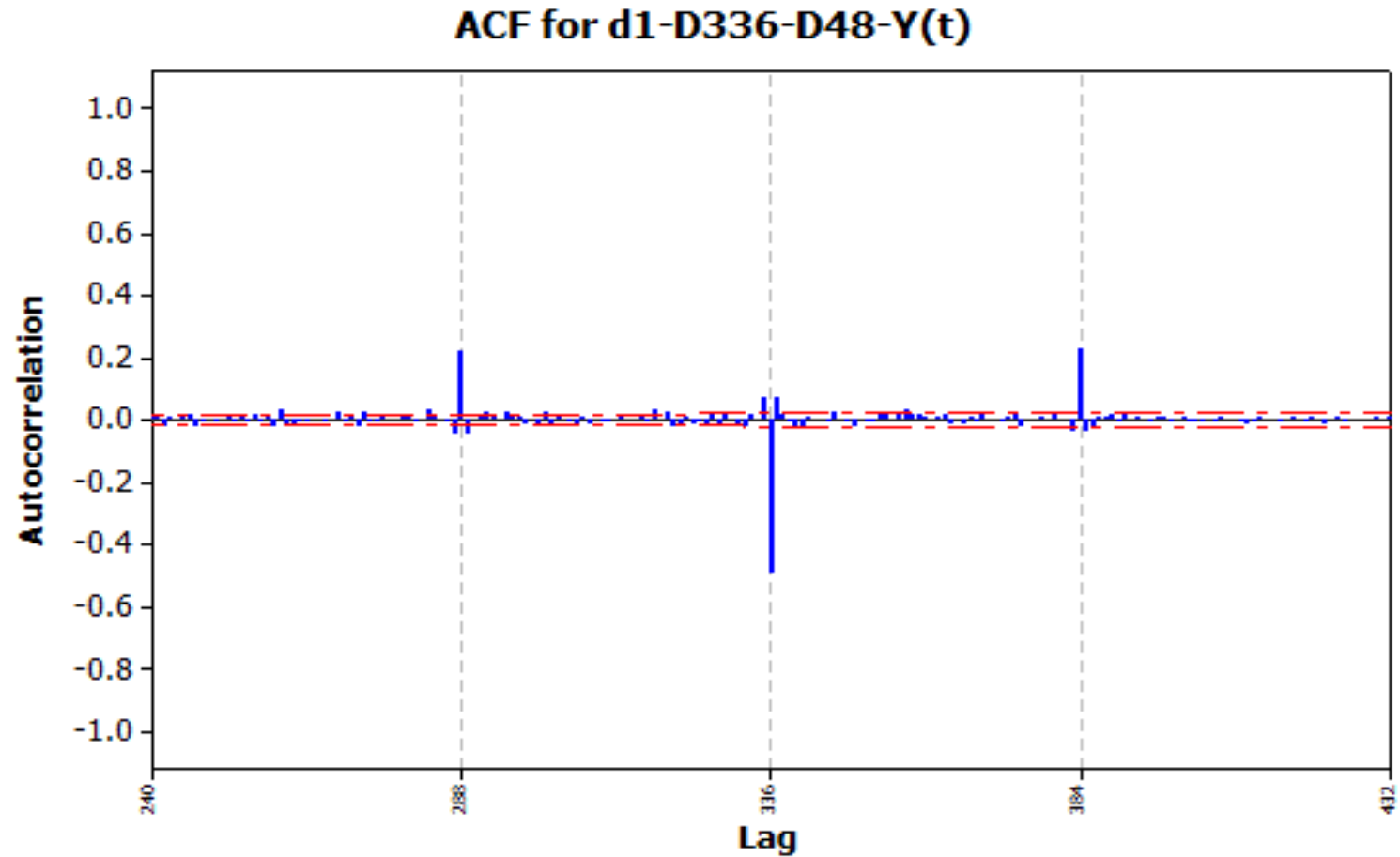
MINITAB Identification: half hourly load data



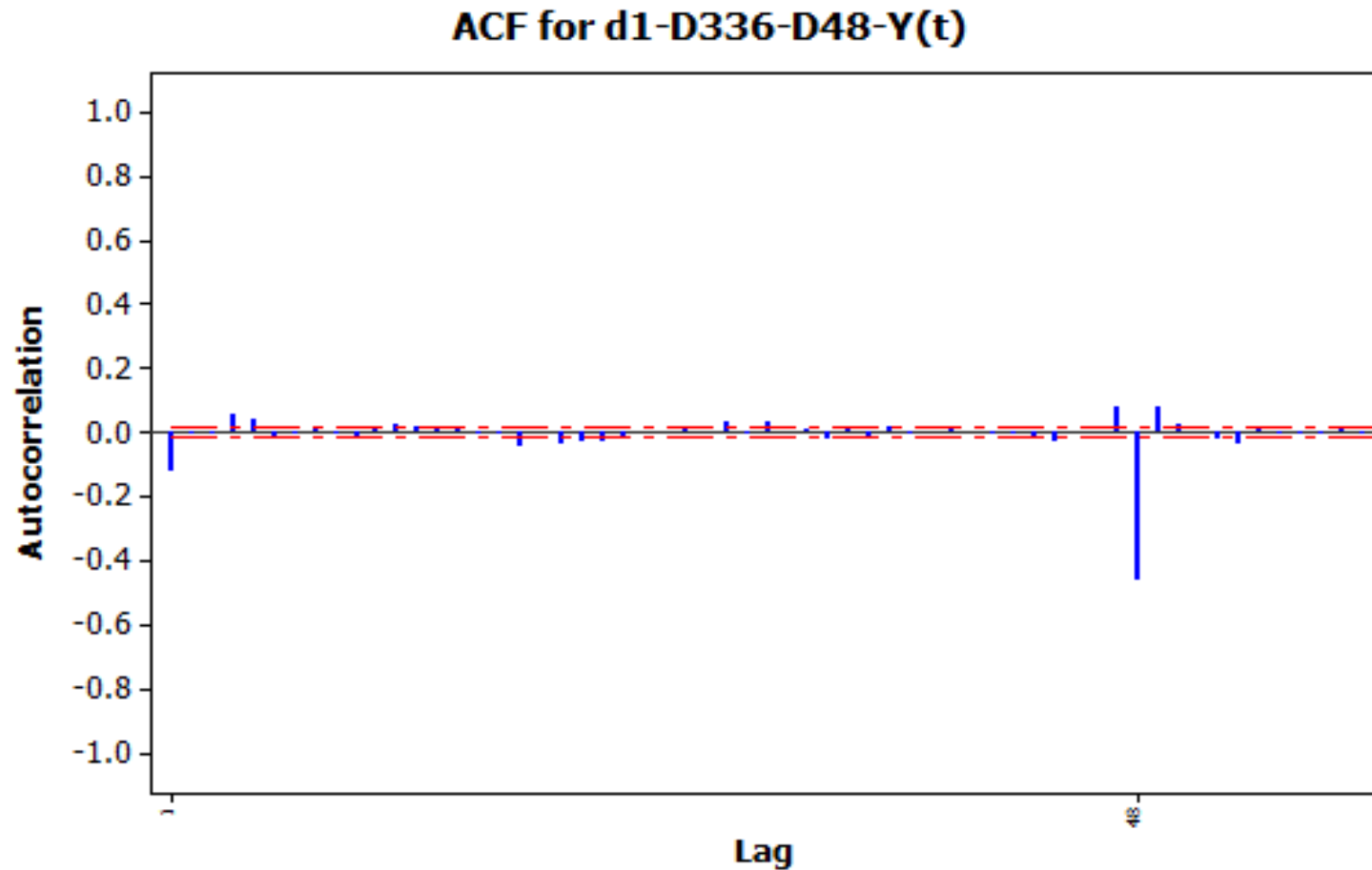
MINITAB Identification: half hourly load data



MINITAB Identification: half hourly load data



MINITAB Identification: half hourly load data



ARIMA, SARIMA, DSARIMA model

- ARIMA model

$$\phi_p(B)(1-B)^d Z_t = \theta_0 + \theta_q(B)a_t$$

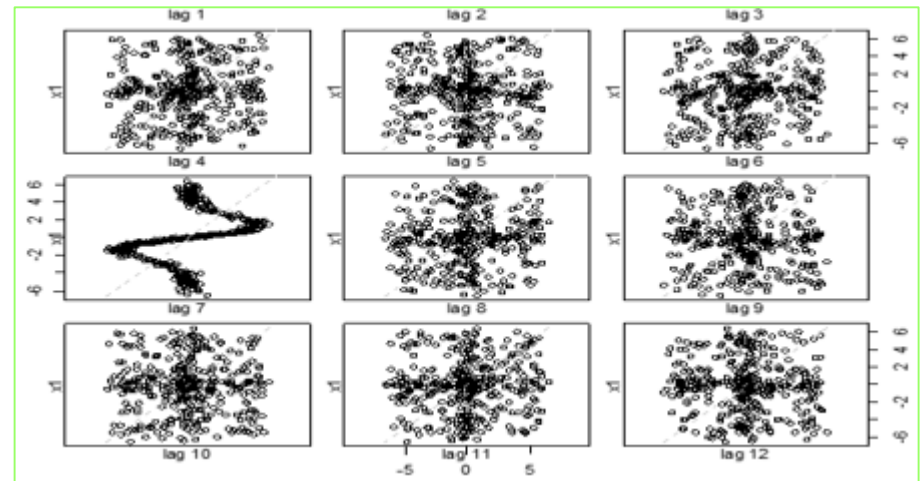
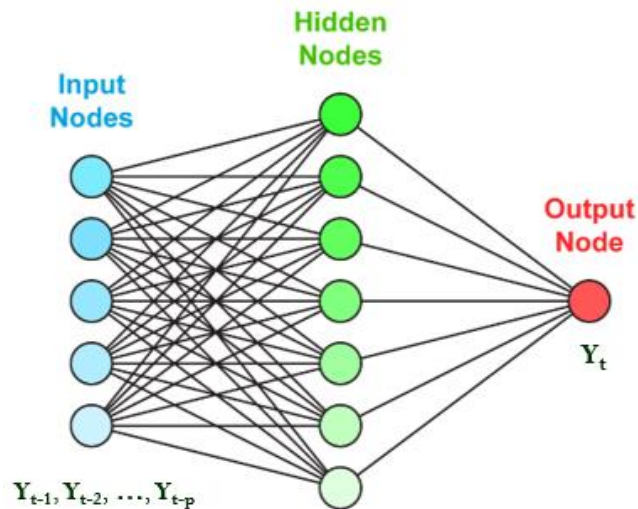
- SARIMA model

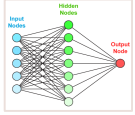
$$\phi_p(B)\Phi_P(B^s)(1-B)^d(1-B^s)^D Z_t = \theta_q(B)\Theta_Q(B^s)a_t$$

- DSARIMA model

$$\begin{aligned} \phi_p(B)\Phi_{P_1}(B^{s_1})\Phi_{P_2}(B^{s_2})(1-B)^d(1-B^{s_1})^{D_1}(1-B^{s_2})^{D_2} Z_t \\ = \theta_q(B)\Theta_{Q_1}(B^{s_1})\Theta_{Q_2}(B^{s_2})a_t \end{aligned}$$

Neural Network for Trend, Seasonal & Calendar Variation

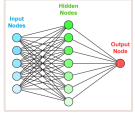




Outline

- Introduction: *Background, Motivation, Jargons, Goals.*
- Architecture of Neural Networks: *Supervised & Unsupervised networks*
- Model selection in Neural Networks: *Inputs, Number of hidden neurons, Activation function, Preprocessing method.*
- Application and Development: *Forecasting and Classification problems.*





Neural Networks – NN



- **Sven F. Crone:**

<http://www.sven-crone.com/presentations.htm>

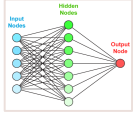
<http://www.neural-forecasting.com/>

- **Halbert L. White:**

<http://weber.ucsd.edu/~hwhite/>

- **Warren S. Sarle:**

<ftp://ftp.sas.com/pub/neural/FAQ.html>



General Background

⇒ During the last few decades,

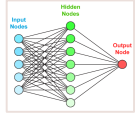
- ① modeling to explain nonlinear relationship between variables, and
- ② some procedures to detect this nonlinear relationship

have grown in a spectacular way and received a great deal of attention.

⇒ Granger, C.W.J. and Terasvirta, T., (1993)

⇒ Terasvirta, T., Tjostheim, D. and Granger, C.W.J., (1994)

⇒ Due to computational advances and increased computational power, nonparametric models that do not make assumptions about the parametric form of the functional relationship between the variables to be modelled have become more easily applicable.



Motivation of NN Research

- ➔ Today's research is largely motivated by the possibility of using NN model as an instrument to solve a wide variety of application problems such as:
 - pattern recognition (classification), signal processing, process control, and forecasting.
 - ➔ The use of the NN model in applied work is generally motivated by a mathematical result stating that under mild regularity conditions, a relatively simple NN model is capable of approximating any Borel-measurable function to any given degree of accuracy.
 - (see e.g. Hornik, Stichombe and White (1989, 1990), White (1990); Cybenko (1989))
-

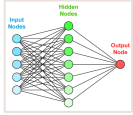


Chart of Neural Networks

<http://www.asimovinstitute.org/neural-network-zoo/>

A mostly complete chart of Neural Networks

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-  Backfed Input Cell
-  Input Cell
-  Noisy Input Cell
-  Hidden Cell
-  Probablistic Hidden Cell
-  Spiking Hidden Cell
-  Output Cell
-  Match Input Output Cell
-  Recurrent Cell
-  Memory Cell
-  Different Memory Cell
-  Kernel
-  Convolution or Pool

Perceptron (P)



Feed Forward (FF)



Radial Basis Network (RBF)



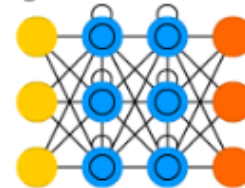
Deep Feed Forward (DFF)



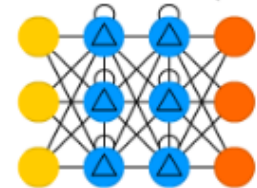
Recurrent Neural Network (RNN)



Long / Short Term Memory (LSTM)



Gated Recurrent Unit (GRU)



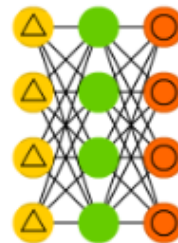
Auto Encoder (AE)



Variational AE (VAE)



Denosing AE (DAE)



Sparse AE (SAE)



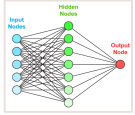


Chart of Neural Networks

<http://www.asimovinstitute.org/neural-network-zoo/>

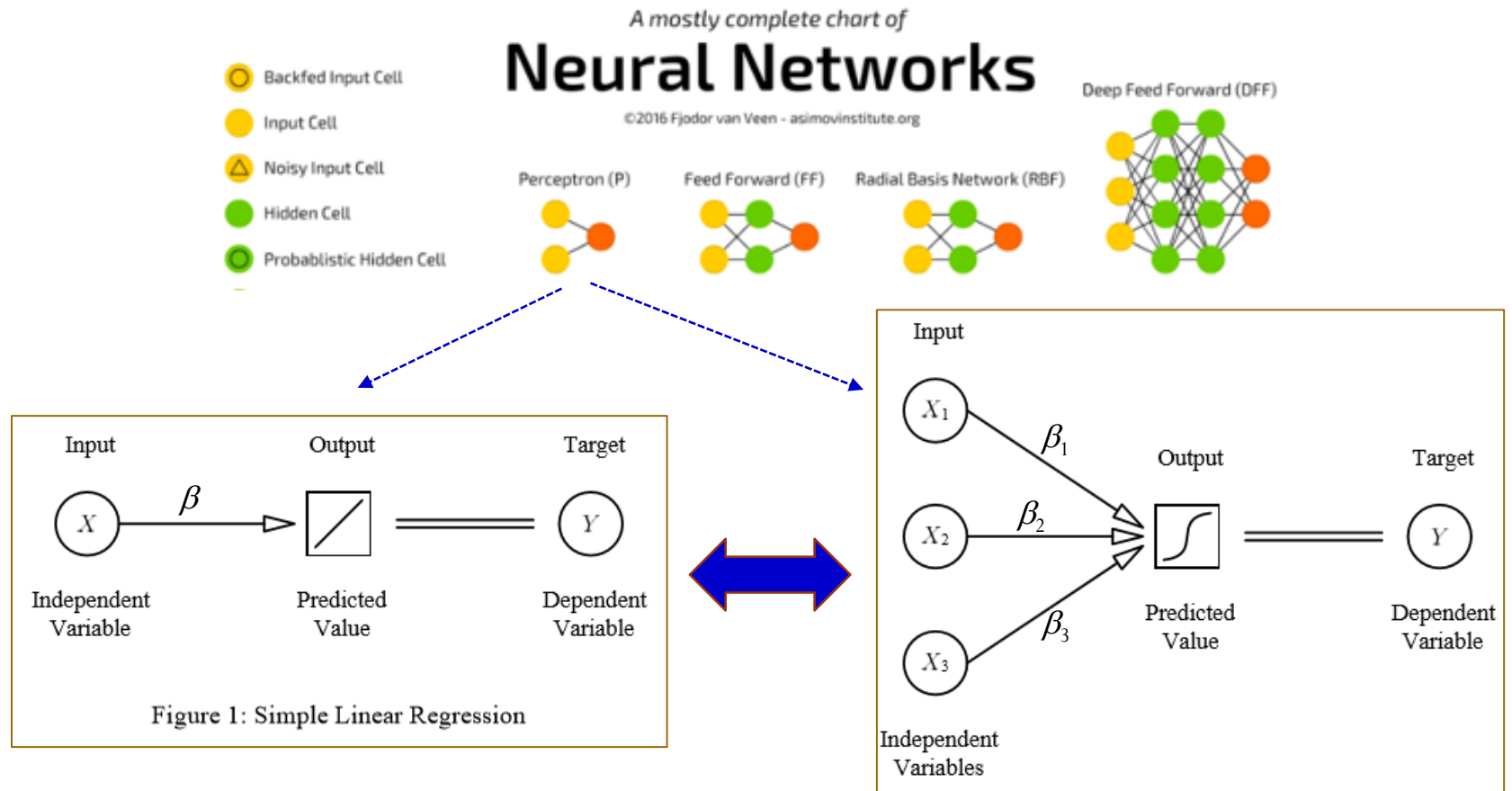
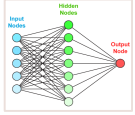


Figure 2: Simple Nonlinear Perceptron = Logistic Regression

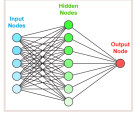
Source: Sarle (1994)



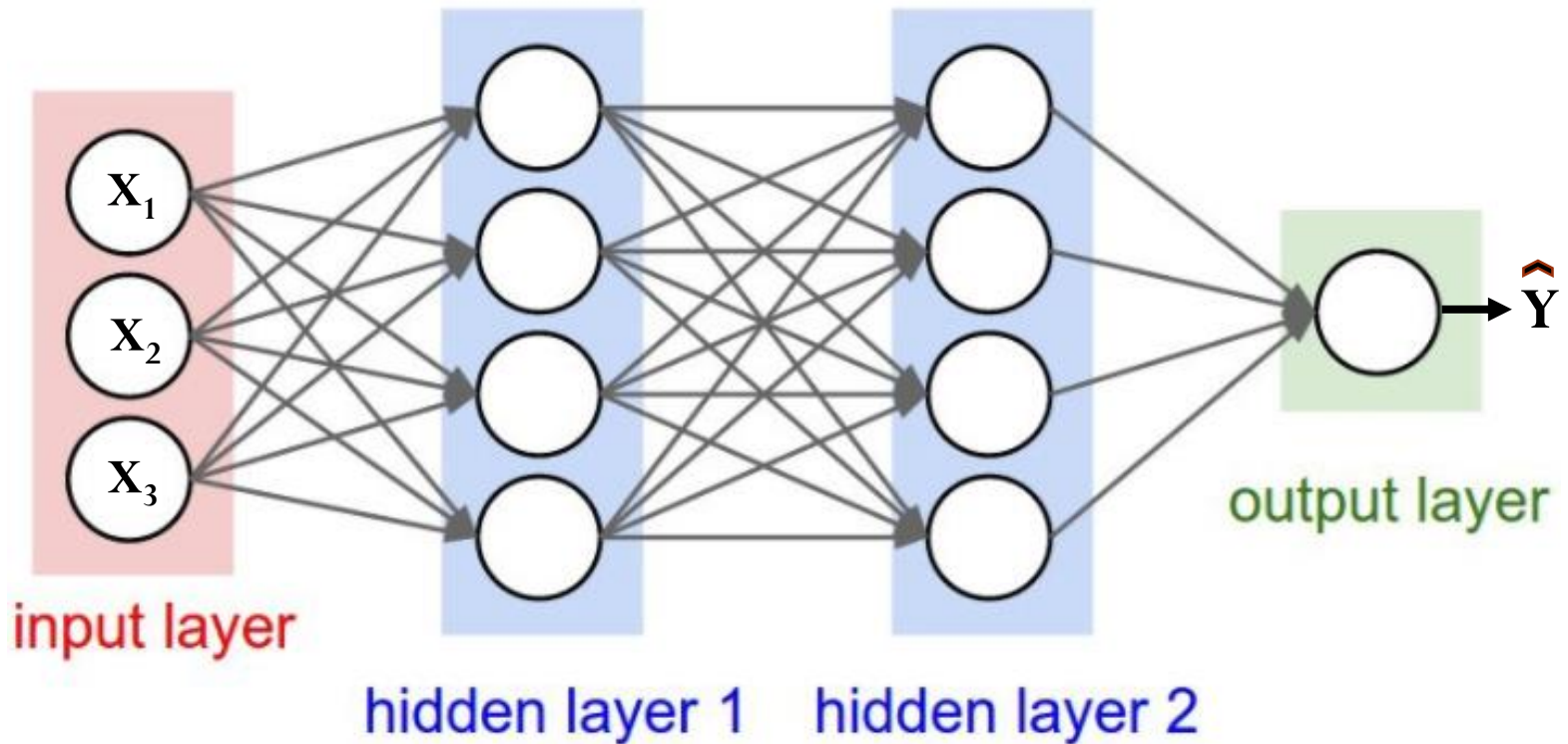


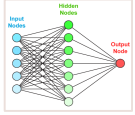
Feed Forward Neural Networks

- ➔ **Multilayer perceptron (MLP)**, also known as **feedforward neural networks (FFNN)**, is probably the **most commonly** used NN architecture in engineering application.
 - ➔ Typically, applications of NN for regression, time series modeling and classification (discriminant analysis) are **based on the FFNN architecture**.
-

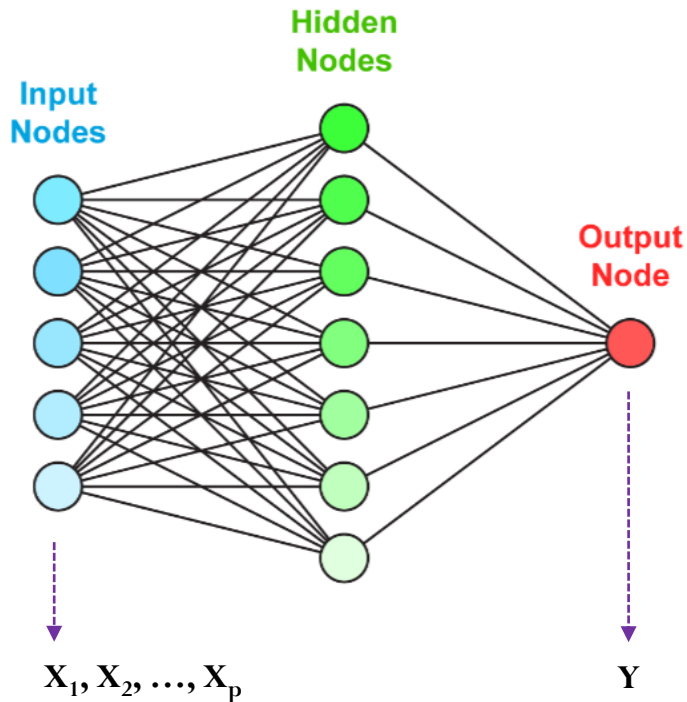


Deep Feedforward Network

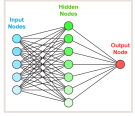




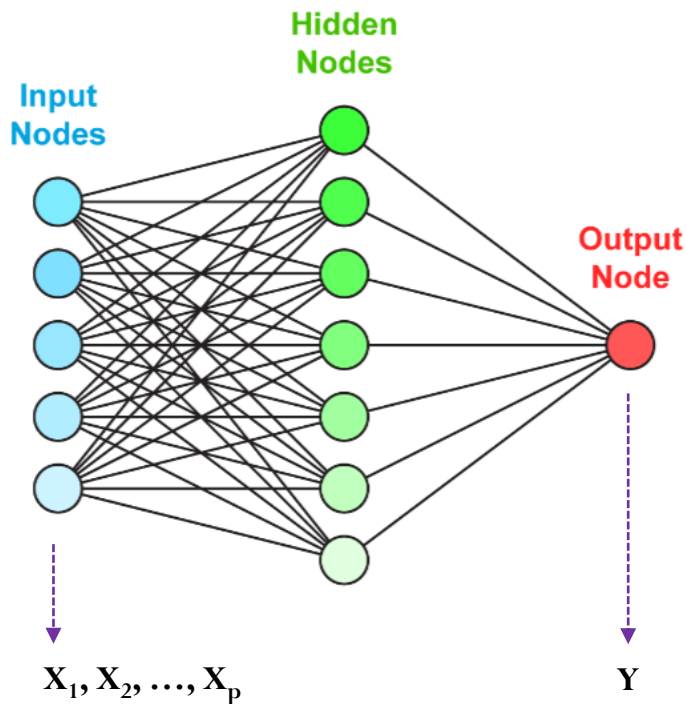
FFNN as Nonlinear regression



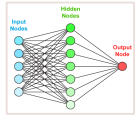
- FFNN includes estimated weights between the inputs and the hidden layer, and the hidden layer uses nonlinear activation functions such as the logistic function, the FFNN becomes genuinely nonlinear model, i.e., nonlinear in the parameters.
- ⇒ In this case, FFNN can be seen as nonlinear regression. FFNN can have multiple inputs and outputs (This figure is multiple inputs with single output), and this architecture is similar to multiple nonlinear regression.



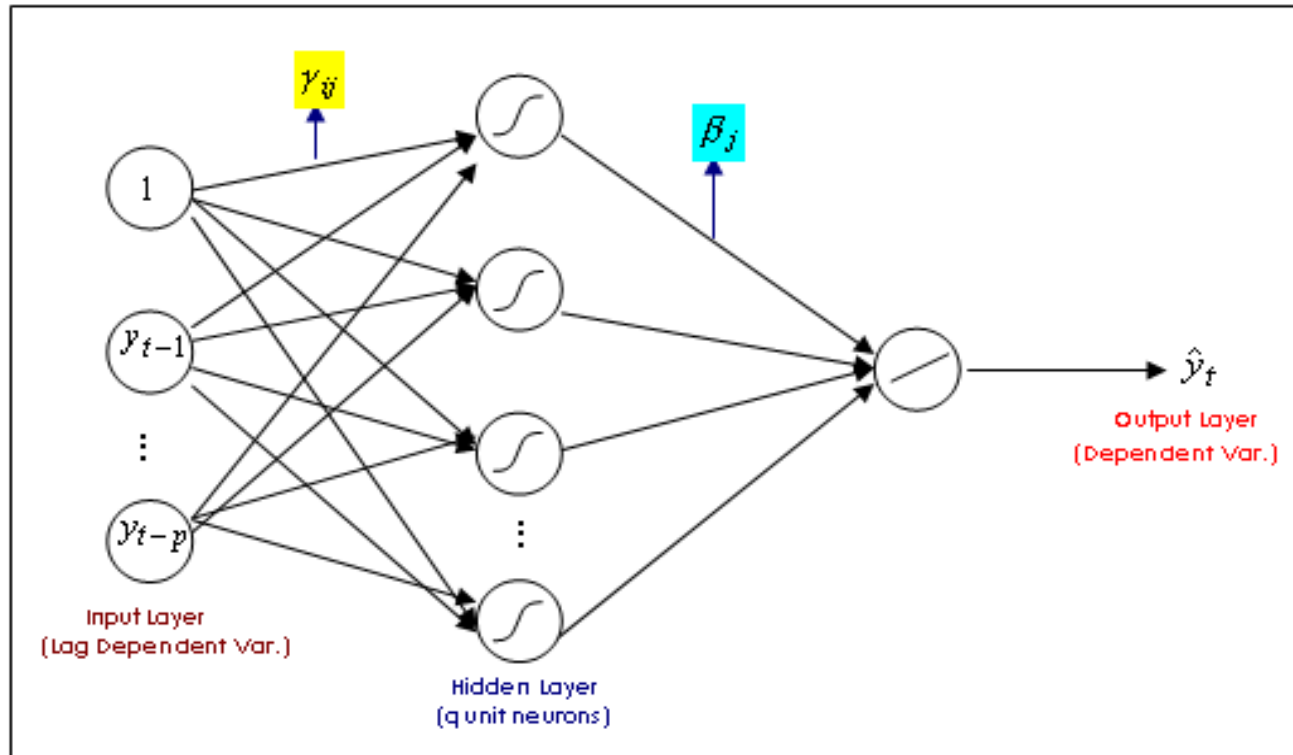
FFNN as Logistic Regression and Discriminant Analysis



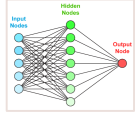
- FFNN with **nonmetric** data (**dichotomous** / **polychotomous**) in target values is identical to **logistic regression** and **nonlinear discriminant analysis**.
- ➔ In this case, FFNN often use a **multiple logistic function** to estimate the conditional probabilities of each class. A multiple logistic function is called a **softmax** activation function in the NN literature.



FFNN as Nonlinear AR(p) model

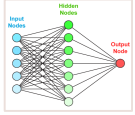


$$\Rightarrow y_t = \beta_0 + \sum_{j=1}^q \beta_j f \left(\sum_{i=1}^p \gamma_{ij} y_{t-i} + \gamma_{oj} \right) + \varepsilon_t$$

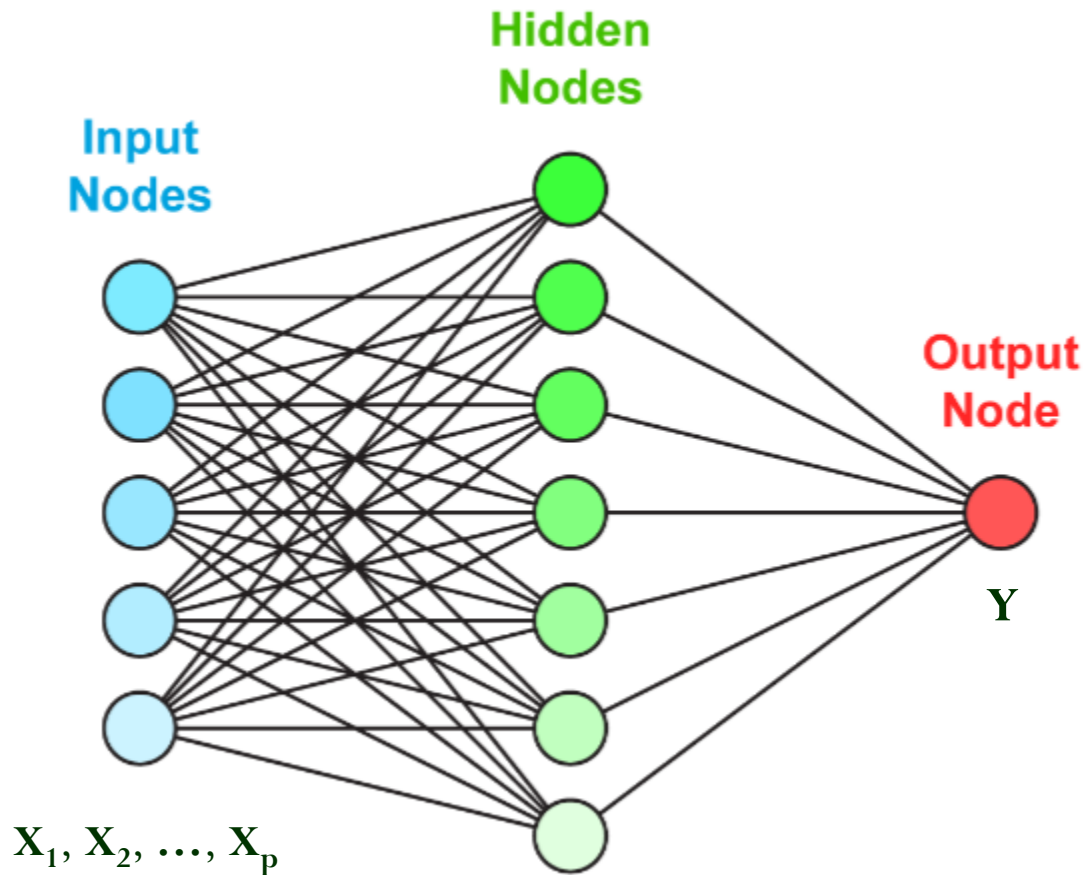


FFNN as Nonlinear AR(p) model

- ❖ Model building strategy that proposed by Terasvirta *et al.* (1994)
 1. Test Y_t for linearity, using linearity test (neglected nonlinearity).
 2. If linearity is rejected, consider a small number of alternative parametric models and/or nonparametric models.
 3. These models should be estimated in-sample and compared out-of-sample.



FFNN: the main problems !!!

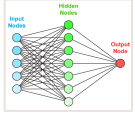


In **Classification & Regression**

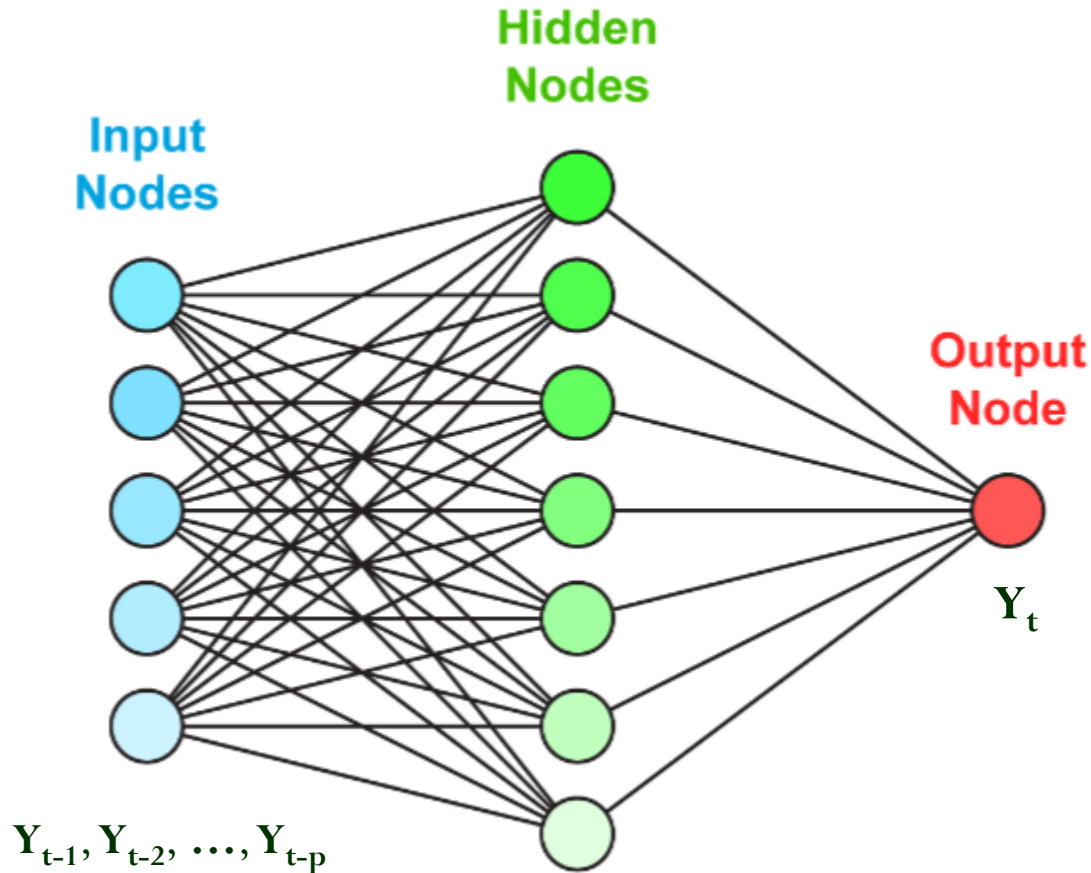


1. How many nodes (neurons) in hidden layer?
2. What is the best inputs (features selection)?
3. What is the best pre-processing method?
4. What is the best activation function in hidden and output layer?

Model selection in Neural Networks



FFNN: the main problems !!!

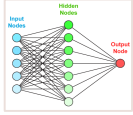


In Time Series Forecasting

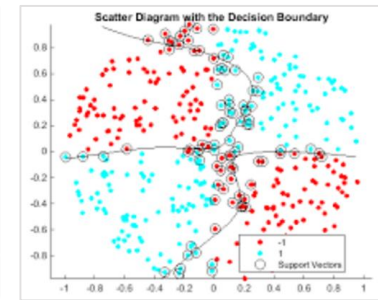
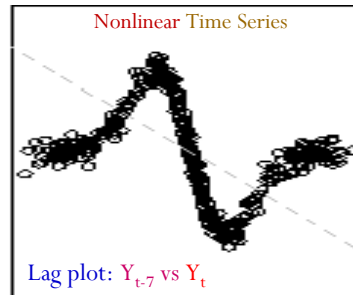
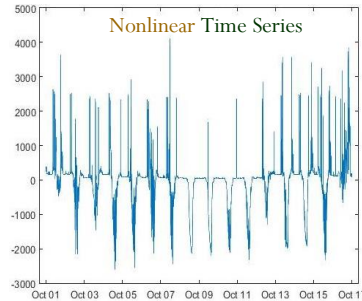
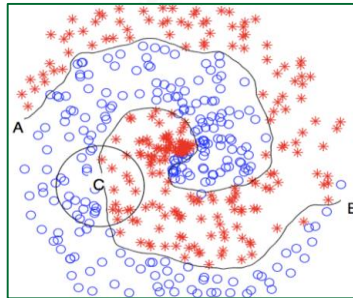
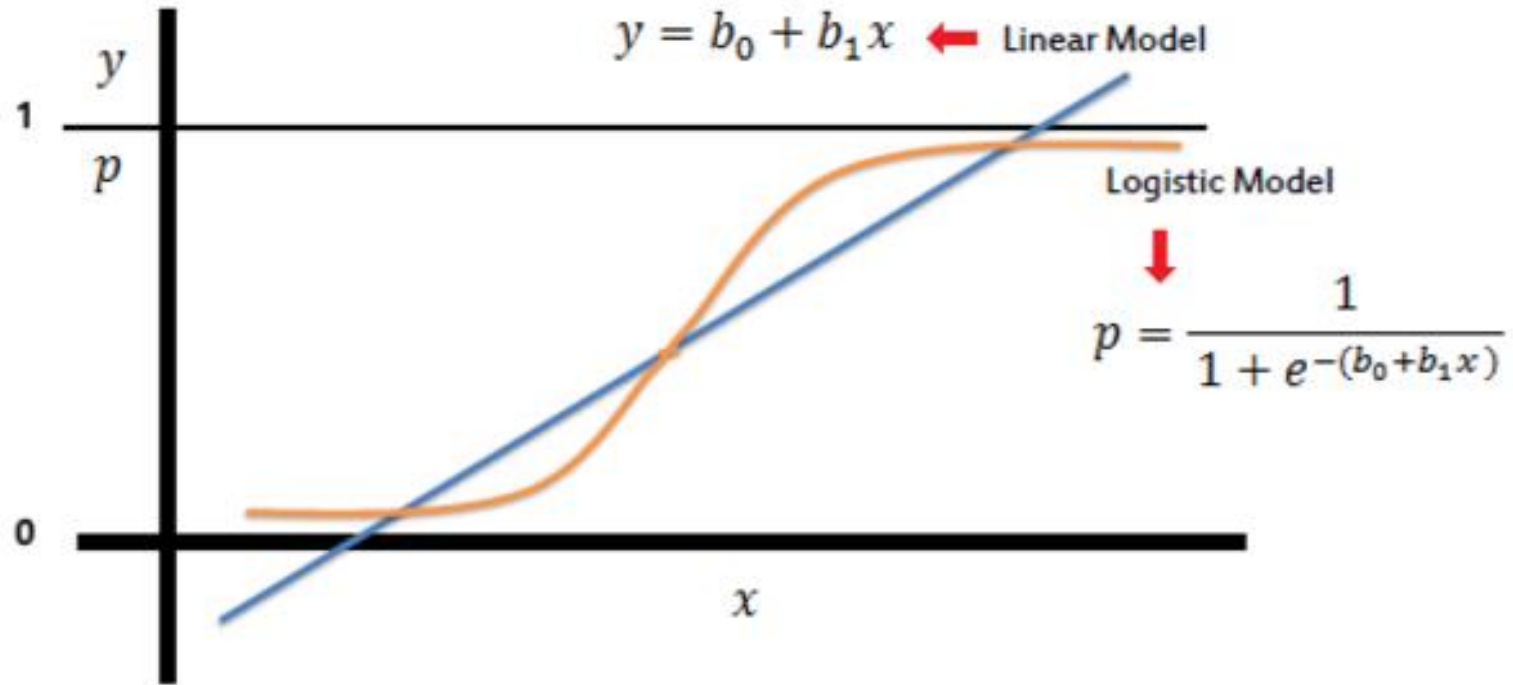


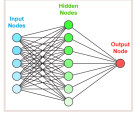
1. What is the best inputs (features selection)?
2. How many nodes (neurons) in hidden layer?
3. What is the best pre-processing method?
4. What is the best activation function in hidden and output layer?

Model selection in Neural Networks



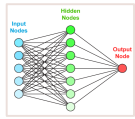
Nonlinear relationship Concept





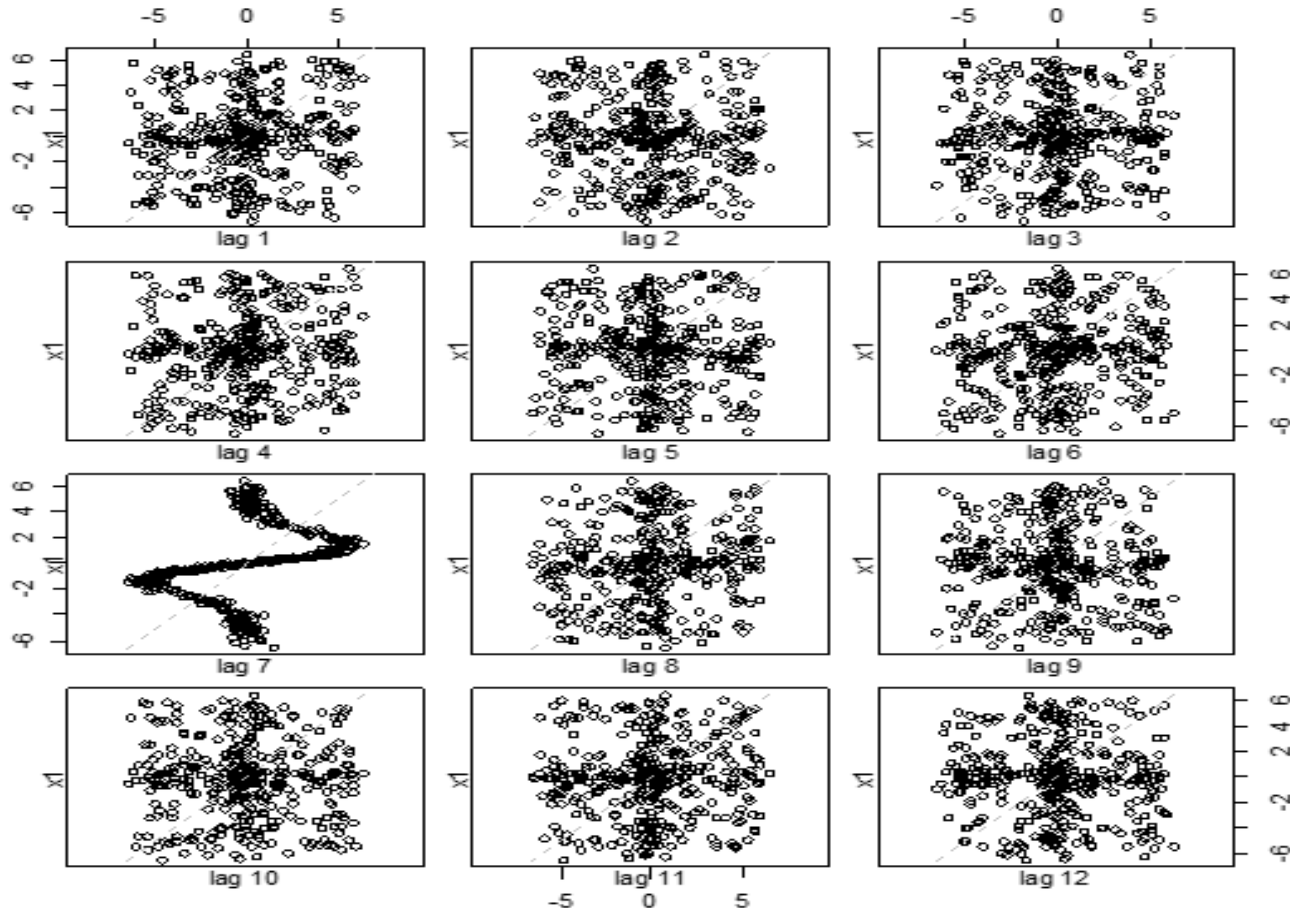
Model Selection in Neural Network

- In general, there are **two procedures** usually used to find **the best FFNN model** or **the optimal architecture**, those are “general-to-specific” or “top-down” and “specific-to-general” or “bottom-up” procedures.
- ⇒ “Top-down” procedure is started from complex model and then applies an algorithm to reduce number of parameters (number of input variables and unit nodes in hidden layer) by using some stopping criteria, whereas “bottom-up” procedure works from a simple model.

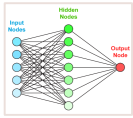


Application: NN for Time Series Forecasting

- Source: **simulation study** using **ESTAR(1)⁷** model

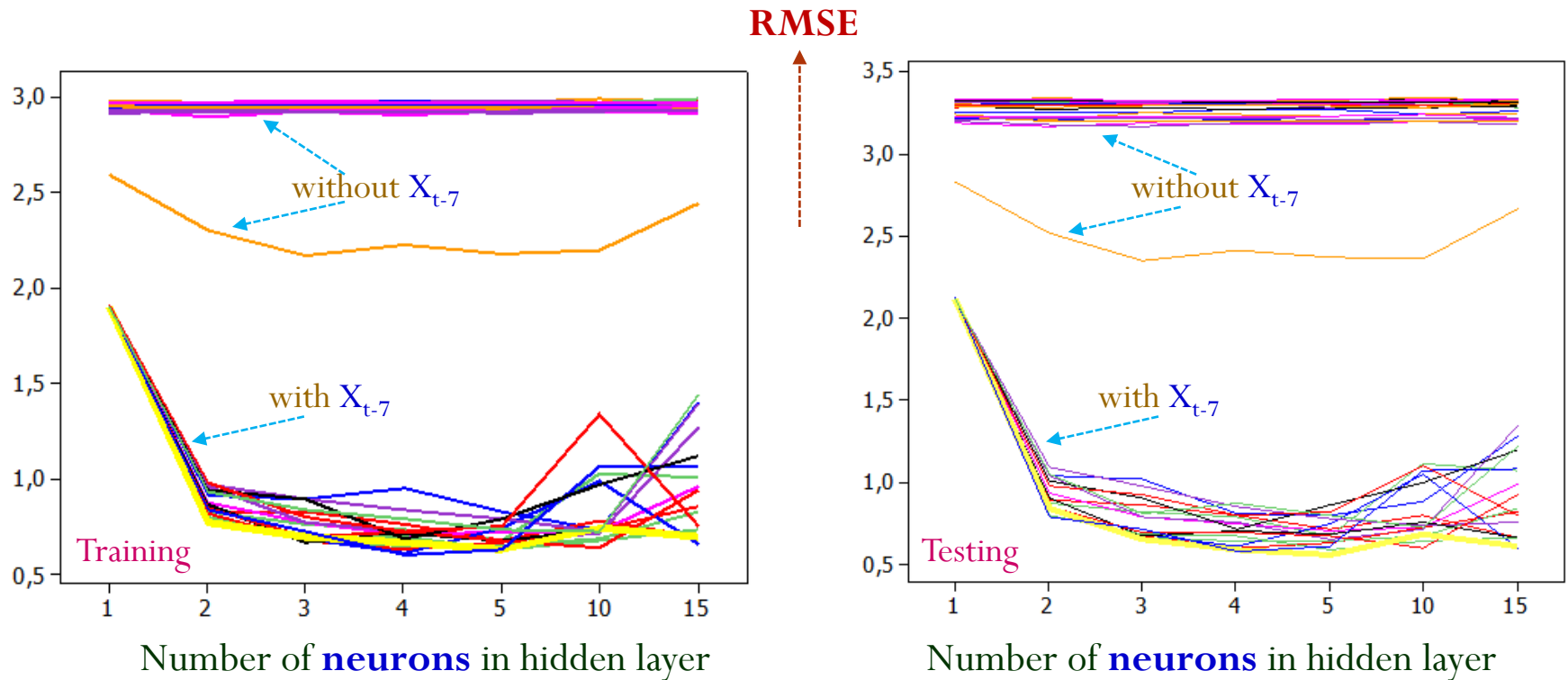


➤ Identification the appropriate lag inputs: use **LAG PLOT** in **R**

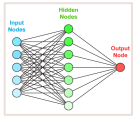


Application: NN for Time Series Forecasting

- Source: **simulation study** using **ESTAR(1)⁷** model



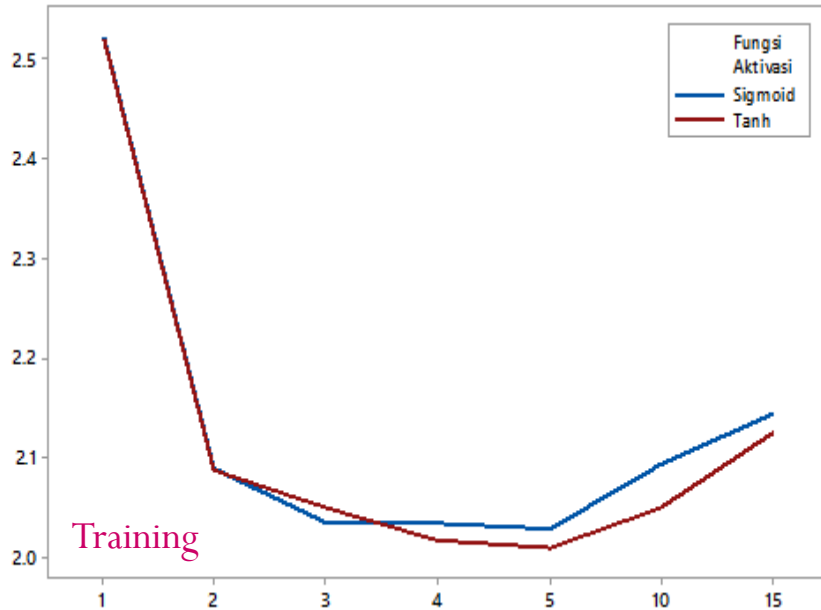
➡ The effect of **INPUTS** and number of **NEURONS** in hidden layer



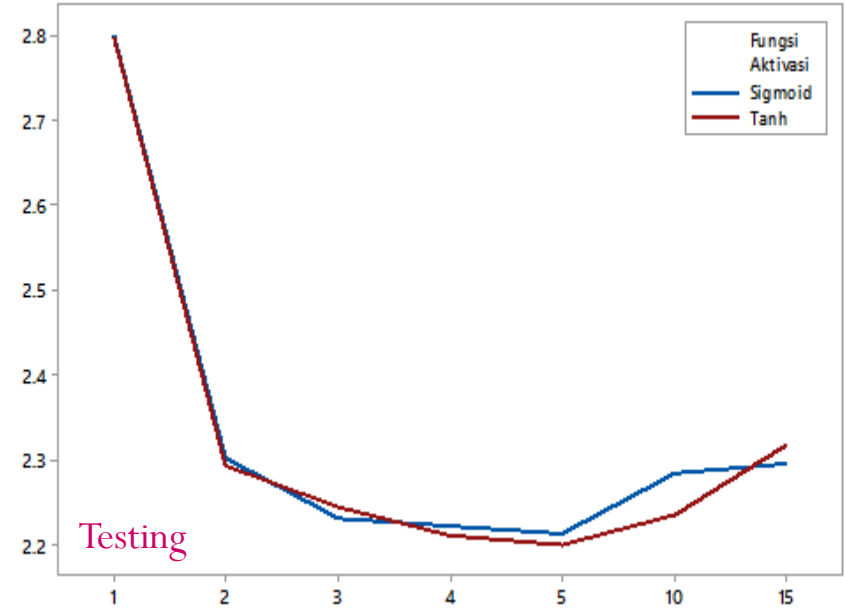
Application: NN for Time Series Forecasting

- Source: **simulation study** using **ESTAR(1)⁷** model

RMSE

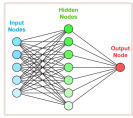


Number of **neurons** in hidden layer



Number of **neurons** in hidden layer

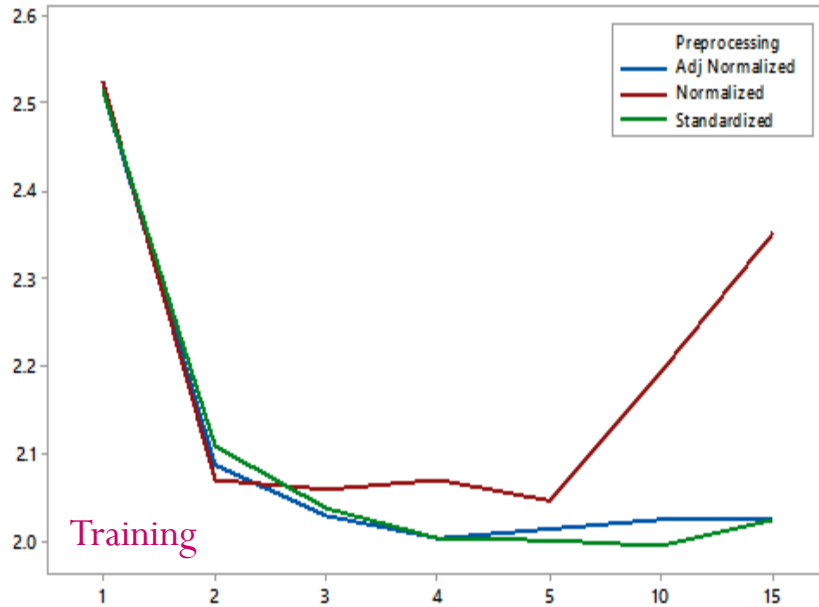
➡ The effect of **ACTIVATION** function and number of **NEURONS**



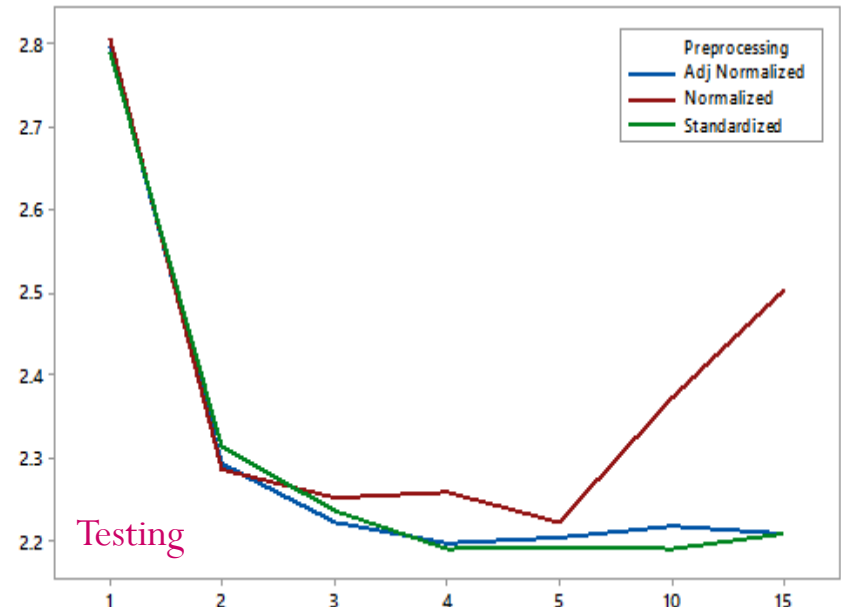
Application: NN for Time Series Forecasting

- Source: **simulation study** using **ESTAR(1)⁷** model

RMSE

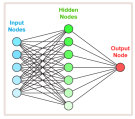


Number of **neurons** in hidden layer



Number of **neurons** in hidden layer

➡ The effect of PREPROCESSING method & number of NEURONS

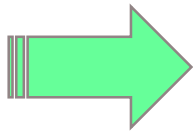


Application: NN for Time Series Forecasting

- Source: **simulation study** using **ESTAR(1)⁷** model

👉 *Summary* of the **results**:

- (1) More sophisticated or complex methods do not necessarily provide more accurate forecast than simpler ones.
- (2) The performance of the various NN methods for time series forecasting problem depends upon :



Inputs or **lag variables**,
Number of **neurons** in hidden layer,
Pre-processing method.