Esercizio

Dati i seguenti segnali

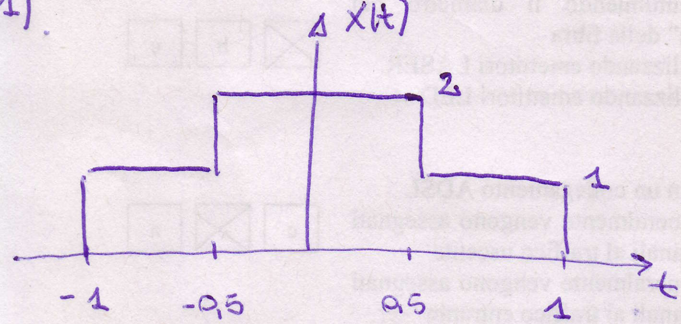
$$x(t) = \begin{cases} 2 & @\ |t| \le 0{,}5 \\ 1 & @\ 0{,}5 < |t| \le 1 \\ 0 & \text{altrove} \end{cases} \qquad h(t) = \begin{cases} 2 & @\ |t| \le 0{,}8 \\ 0 & \text{altrove} \end{cases}$$

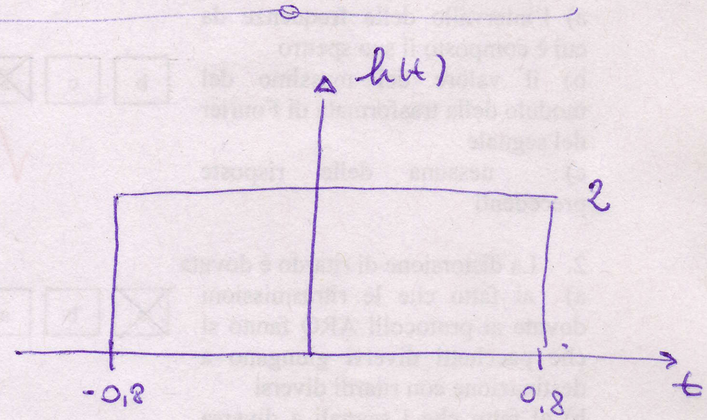
1) Calcolare la convoluzione $y(t) = (x * h)(t)$.

2) Calcolare lo spettro di $y(t)$.

---

1).



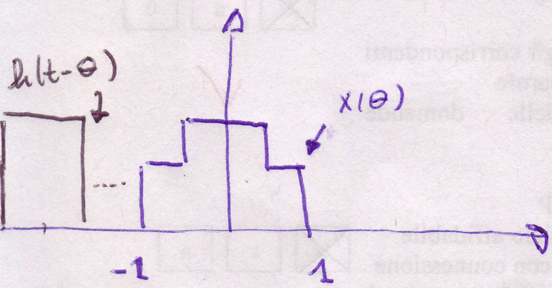


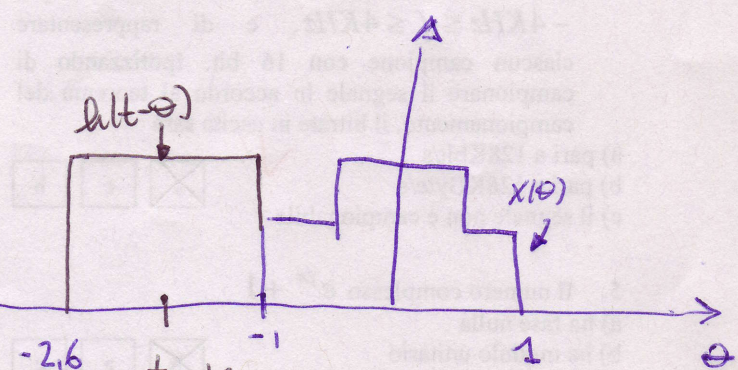
Ribaltiamo e trasliamo il segnale $h(t)$.

Si identificano i seguenti intervalli di interesse

CASO A

$\Rightarrow -\infty < t \le -1{,}8$





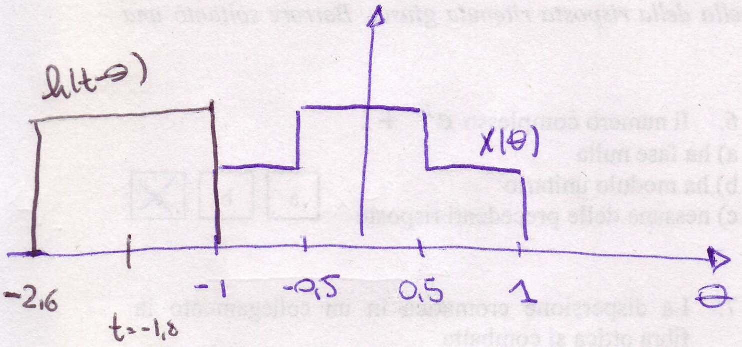
$t \cong -\infty$ $\qquad$ $t = -1{,}8$

→ Nell'intervallo $t \in (-\infty, -1{,}8]$ i segnali $x(\theta)$ e $h(t-\theta)$ non si sovrappongono, conseguentemente la convoluzione è nulla

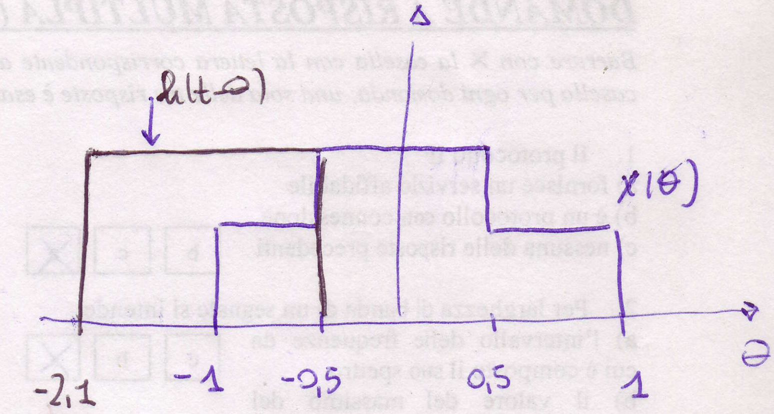
↳ $y(t) = 0$ @ $-\infty < t \leq -1{,}8$.

CASO B

⇒ $-1{,}8 < t \leq -1{,}3$



$t = -1{,}8$

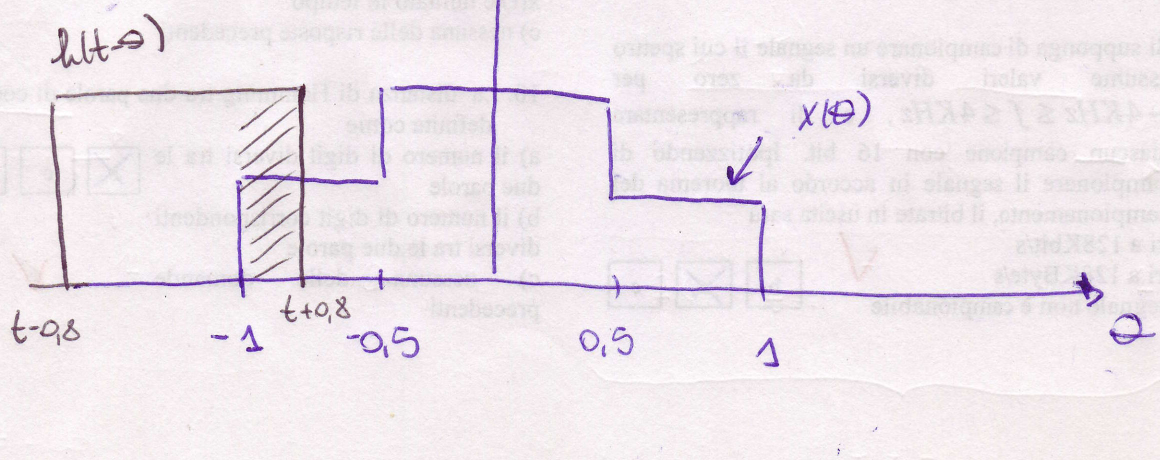


$t = -1.3$

↳ In questo intervallo il segnale $h(t-\theta)$ si sovrappone parzialmente a $x(\theta)$

L'operazione di integrazione va eseguita sull'intervallo nel quale i segnali si sovrappongono.

⇒ Valutazione degli estremi di integrazione

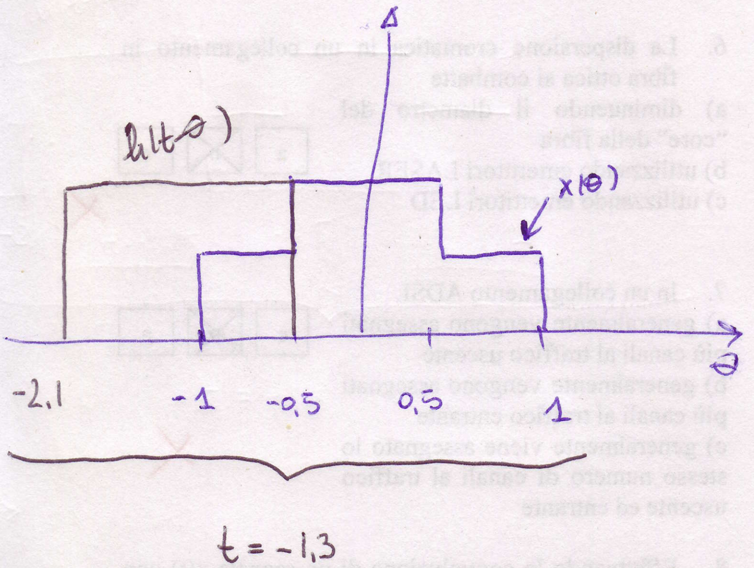


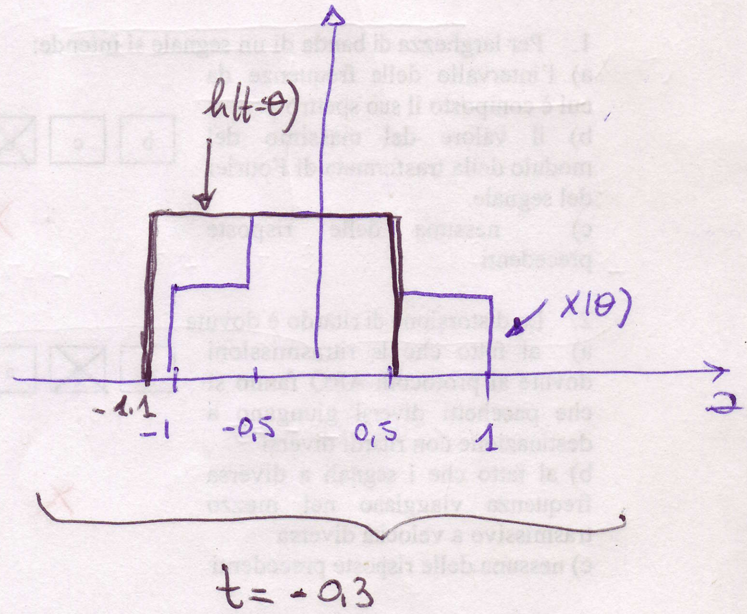
↪ l'intervallo su cui l'integrale ha valori non nulli inizia in $\theta=-1$ e termina in $\theta=t+0{,}8$ $\forall t \in [-1{,}8, -1{,}3]$

↪ $$y(t) = \int_{-1}^{t+0{,}8} 2\, d\theta = 2(t+1{,}8) \quad @ \ -1{,}8 < t \leq -1{,}3$$

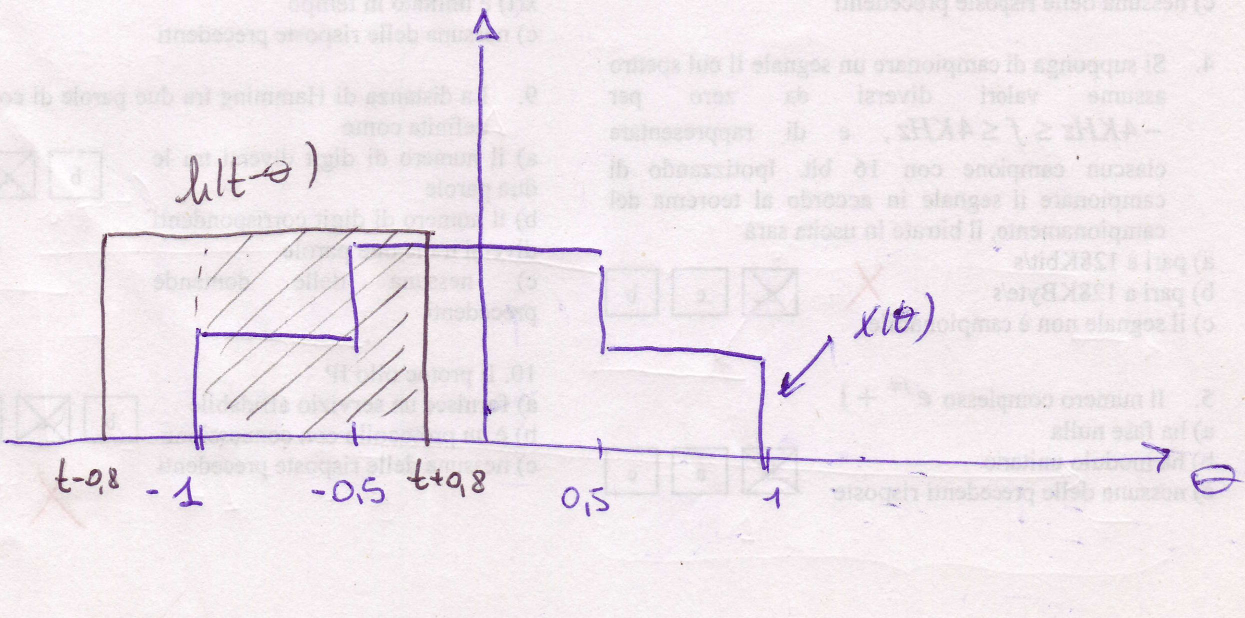
$\boxed{\text{CASO C}}$

$\Rightarrow -1{,}3 < t \leq -0{,}3$





↪ Valutazione degli estremi di integrazione



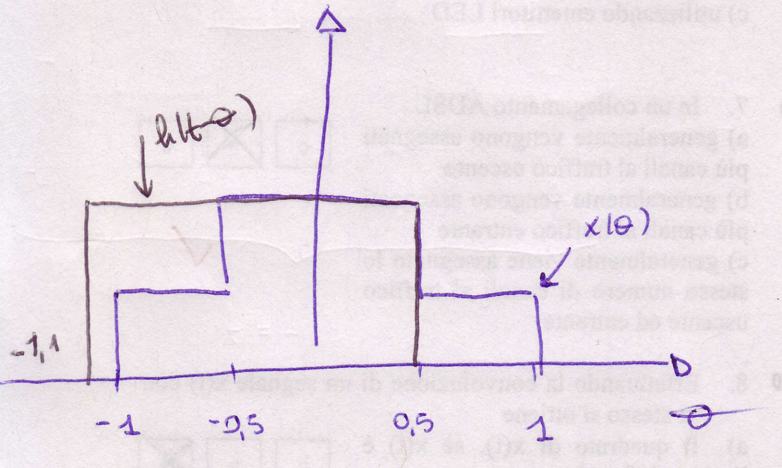
↪ l'integrale ha valori non nulli per $-1 \le \theta \le t+0{,}8$

↪ $$y(t) = \int_{-1}^{t+0{,}8} x(\theta)\, h(t-\theta)\, d\theta$$

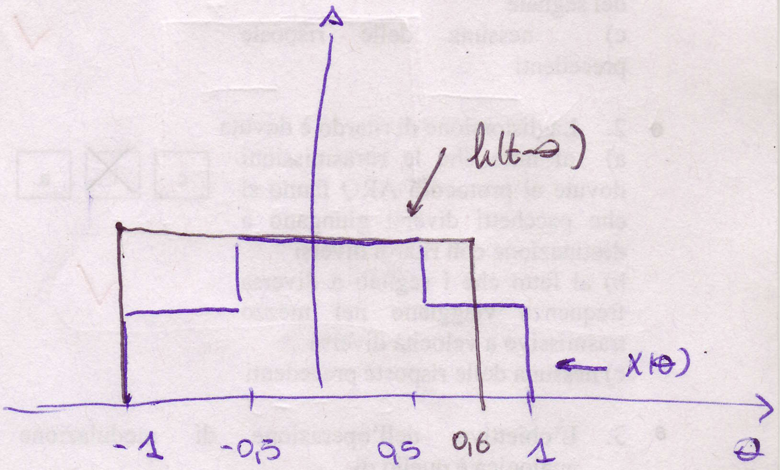
$$= \int_{-1}^{-0{,}5} 2\, d\theta + \int_{-0{,}5}^{t+0{,}8} 4\, d\theta = 1 + 4(t+1{,}3)$$

$\boxed{\text{CASO D}}$

$\Rightarrow -0{,}3 < t \le -0{,}2$

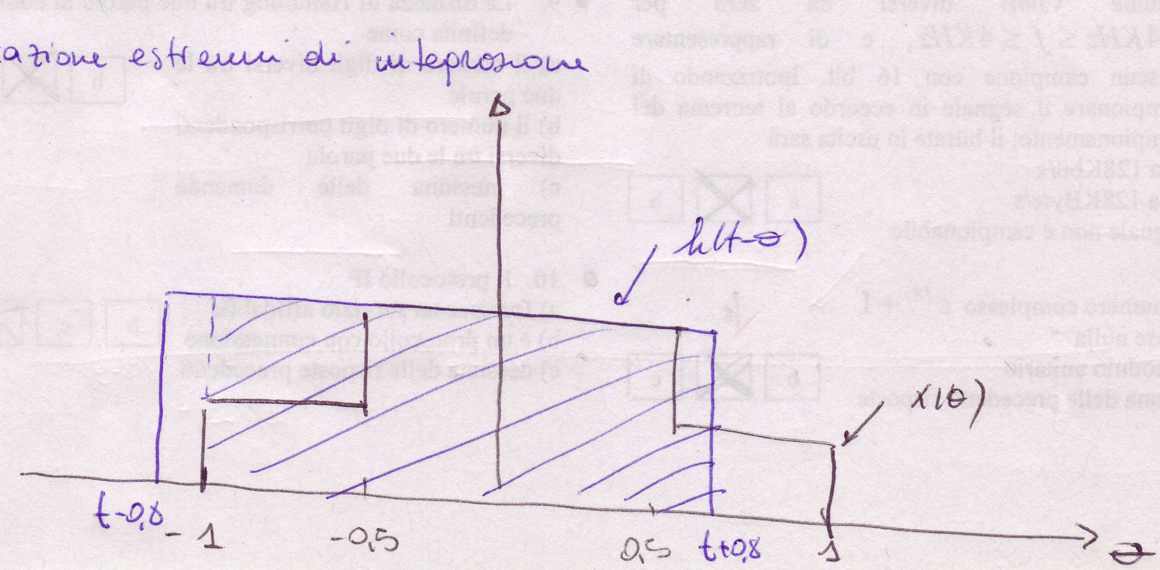


t = -0,3



t = -0,2

↪ Valutazione estremi di integrazione



$\Rightarrow$ estremi di integrazione $\theta = 1$

$\theta = t + 0{,}8$

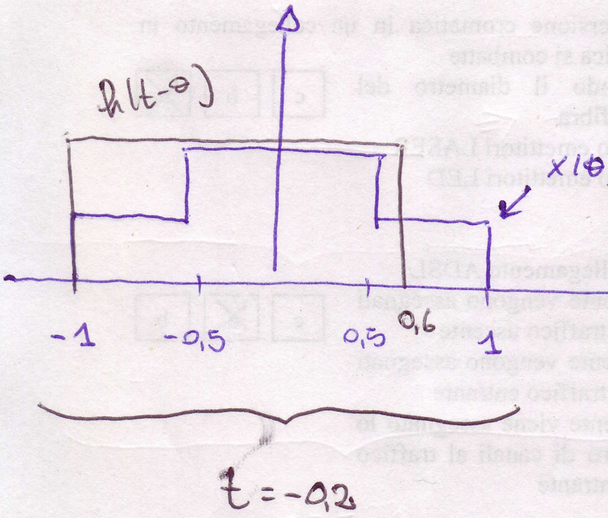
$$y(t) = \int_{-1}^{t+0,8} x(\theta)\, h(t-\theta)\, d\theta =$$

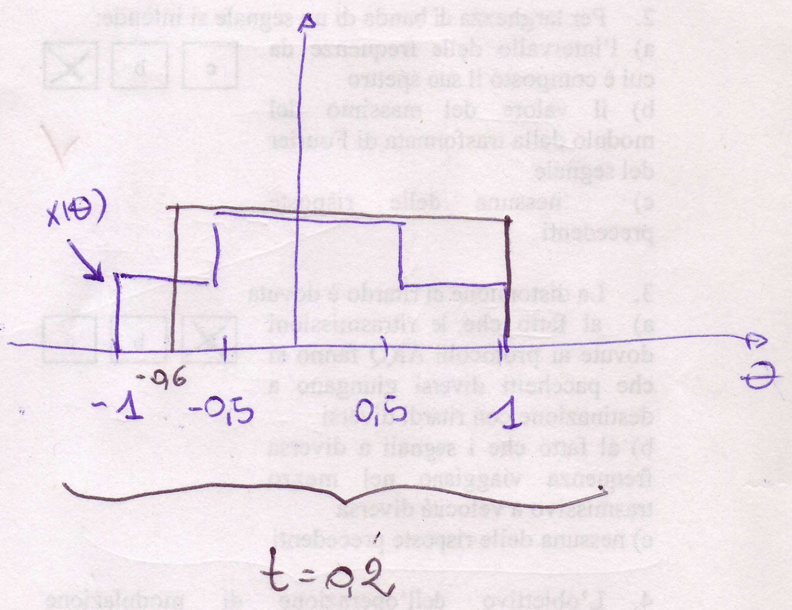
$$\int_{-1}^{-0,5} 2\,d\theta + \int_{-0,5}^{0,5} 4\,d\theta + \int_{0,5}^{t+0,8} 2\,d\theta = 1 + 4 + 2(t+0{,}3)$$

$$= 5 + 2(t+0{,}3)$$

CASO E

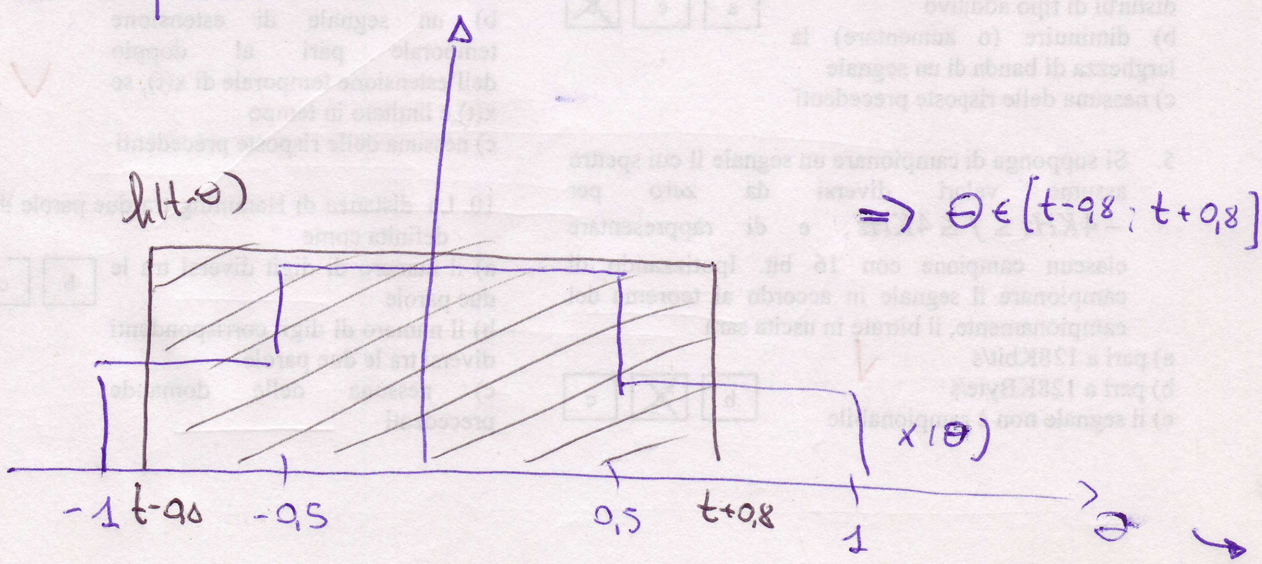
$\Rightarrow -0{,}2 < t \leq 0{,}2$





↳ Estremi di integrazione

$\Rightarrow \theta \in [t-0{,}8 \,;\, t+0{,}8]$



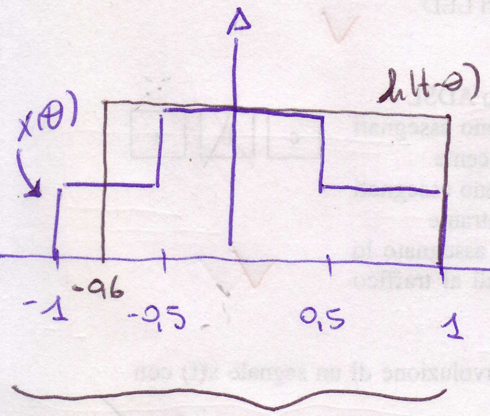
$$y(t) = \int_{t-0.8}^{t+0.8} x(\theta)\, h(t-\theta)\, d\theta =$$

$$\int_{t-0.8}^{-0.5} 2\, d\theta + \int_{-0.5}^{0.5} 4\, d\theta + \int_{0.5}^{t+0.8} 2\, d\theta = 2(0.3-t) + 4 + 2(t+0.3)$$

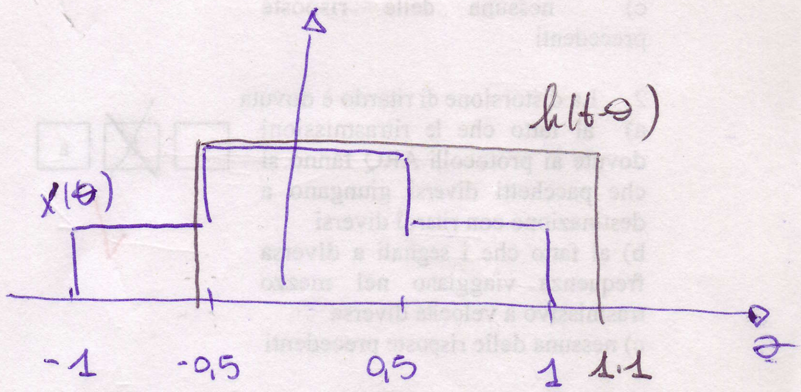
$$= 4 + 2(0.3+0.3) = 5.2$$

$\boxed{\text{CASO F}}$

$\Rightarrow 0.2 < t \leq 0.3$



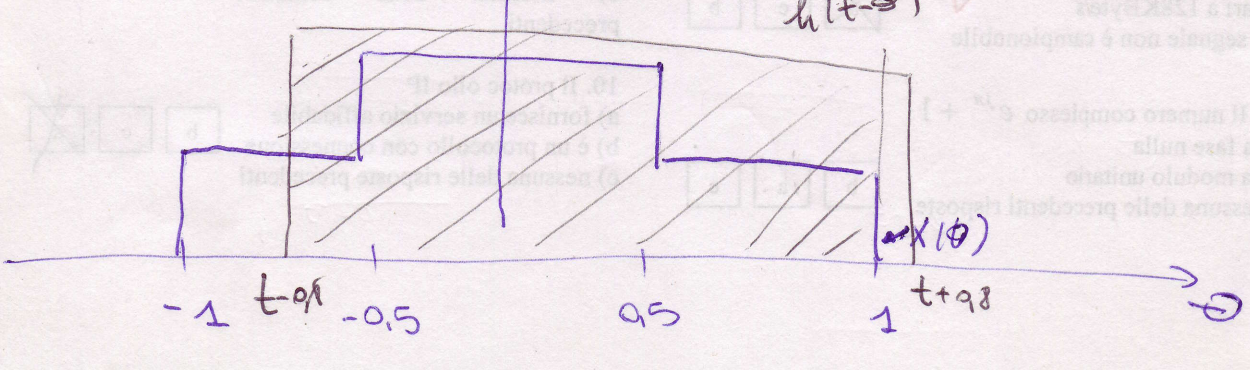
$t = 0.2$



$t = 0.3$

$\Rightarrow$ Valutazione degli estremi di integrazione

$\theta \in [t-0.8;\ 1]$



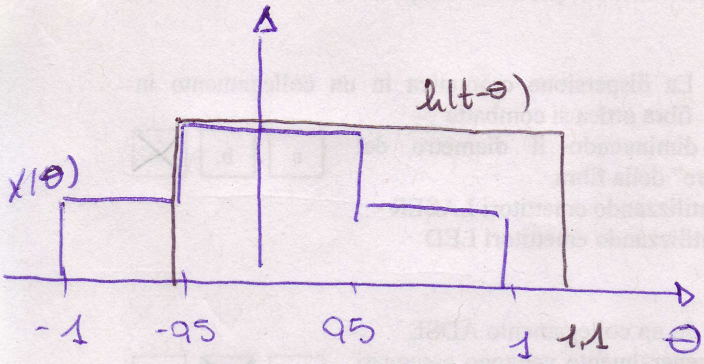
↳ $y(t) = \int_{t-0.8}^{1} x(\theta)\, h(t-\theta)\, d\theta =$

$$= \int_{t-0.8}^{-0.5} 2\,d\theta + \int_{-0.5}^{0.5} 4\,d\theta + \int_{0.5}^{1} 2\,d\theta = 2(0.3-t) + 4 + 1$$

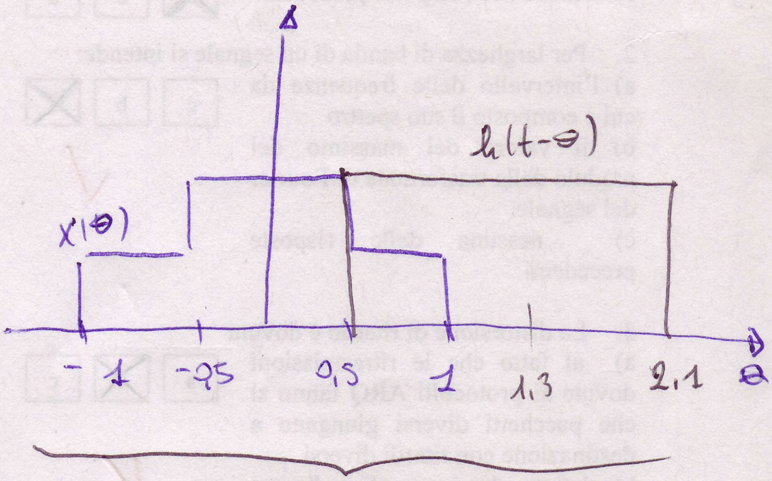
$$= 5 + 2(0.3-t)$$

$\boxed{\text{CASO 6}}$

$\Rightarrow 0.3 < t \leq 1.3$



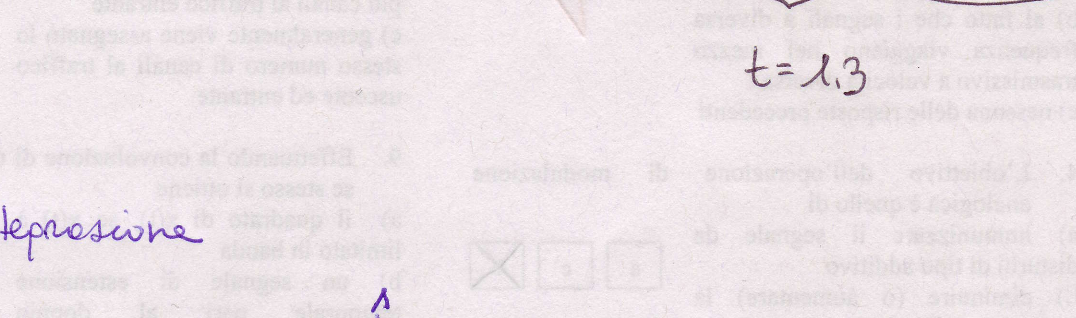
t=0.3



t=1.3

↳ Estremi di integrazione

$\Rightarrow \theta \in [t-0.8,\ 1]$



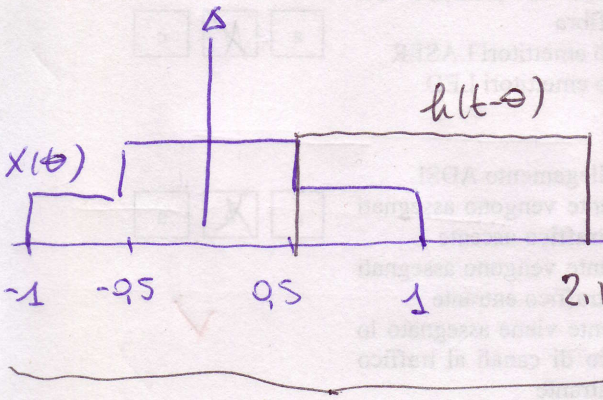
8

$$y(t) = \int_{t-0.8}^{1} x(\theta)\, h(t-\theta)\, d\theta$$

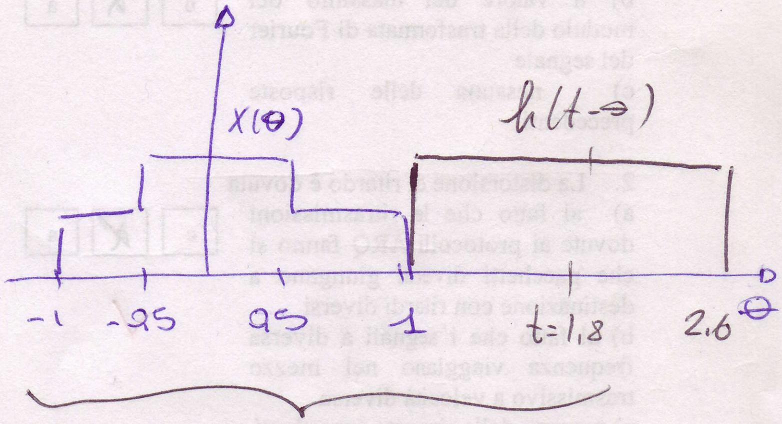
$$= \int_{t-0,8}^{0,5} 4\, d\theta + \int_{0,5}^{1} 2\, d\theta = 4(1,3-t) + 1$$

$\boxed{\text{CASO 4}}$

$\Rightarrow 1,3 < t \leq 1,8$

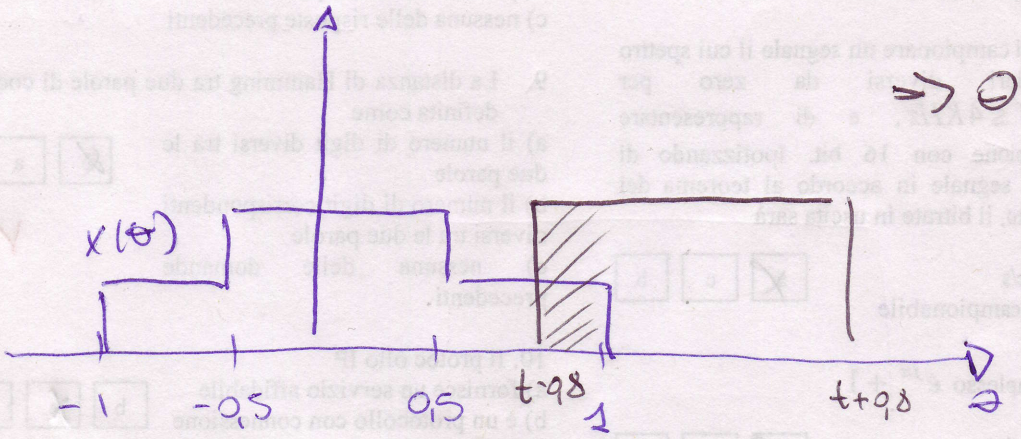


t=1,3



t=1,8

↳ Estremi di integrazione



$\Rightarrow \theta \in [t-0.8\ ,\ 1]$

$$y(t) = \int_{t-0{,}8}^{1} 2\,d\theta = 2\,(1{,}8-t)$$

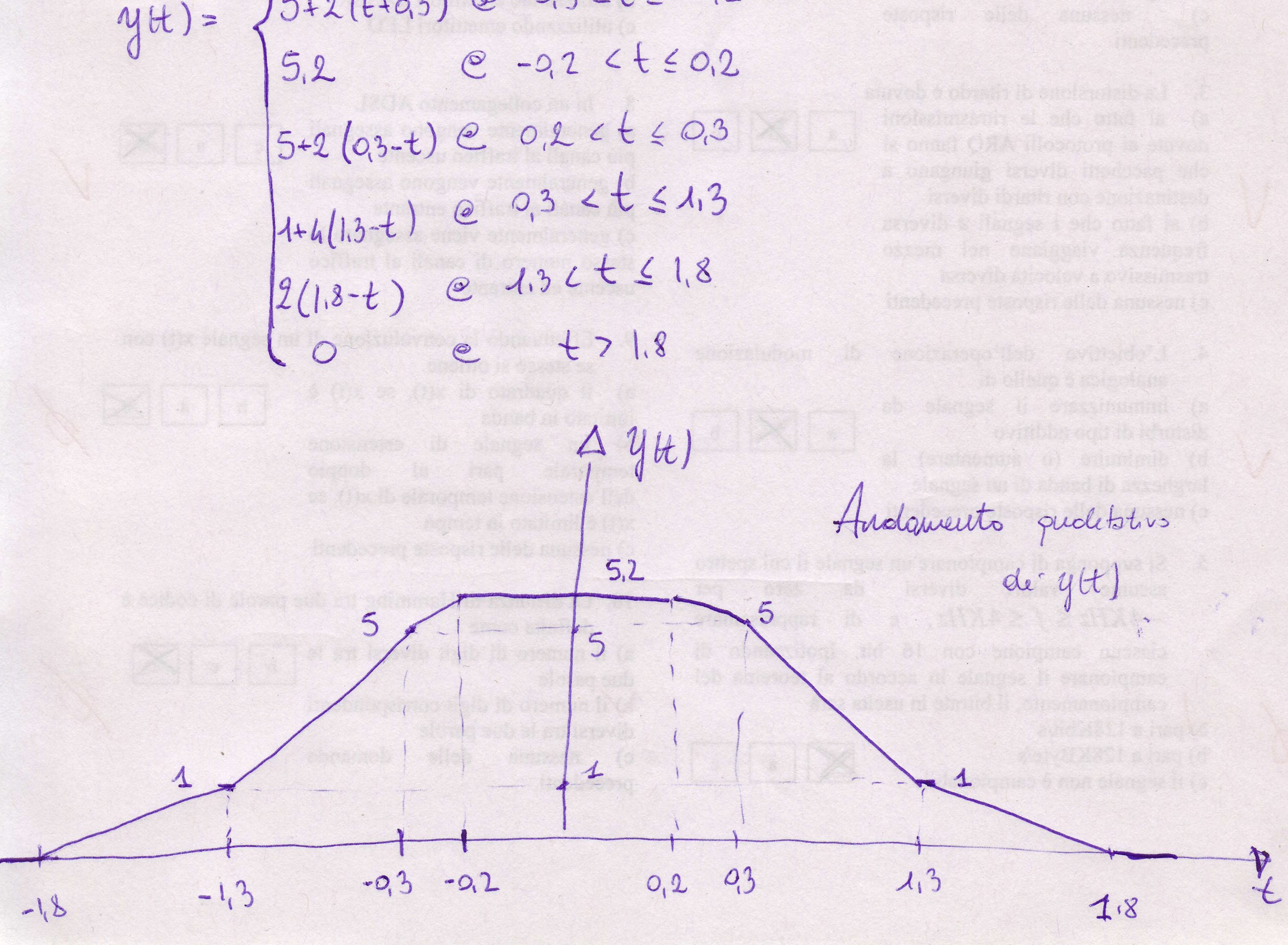
$\boxed{\text{CASO I}}$

$\Rightarrow \quad 1{,}8 < t < +\infty$

$\hookrightarrow \quad y(t) = 0$ (non vi è sovrapposizione)

Concludendo

$$y(t) = \begin{cases} 0 & \text{e} \quad t \leq -1{,}8 \\ 2(t+1{,}8) & \text{e} \quad -1{,}8 < t \leq -1{,}3 \\ 1+4(t+1{,}3) & \text{e} \quad -1{,}3 < t \leq -0{,}3 \\ 5+2(t+0{,}3) & \text{e} \quad -0{,}3 < t \leq -0{,}2 \\ 5{,}2 & \text{e} \quad -0{,}2 < t \leq 0{,}2 \\ 5+2(0{,}3-t) & \text{e} \quad 0{,}2 < t \leq 0{,}3 \\ 1+4(1{,}3-t) & \text{e} \quad 0{,}3 < t \leq 1{,}3 \\ 2(1{,}8-t) & \text{e} \quad 1{,}3 < t \leq 1{,}8 \\ 0 & \text{e} \quad t > 1{,}8 \end{cases}$$



2) SPETTRO di $y(t)$

Essendo $y(t) = (x * h)(t)$

il suo spettro $Y(f)$ sarà dato dal prodotto degli spettri di $x(t)$ e $h(t)$

$$Y(f) = X(f)H(f).$$

Possiamo riscrivere $x(t)$ come

$$x(t) = \text{rect}_2(t) + \text{rect}_1(t)$$

Ricordando che il segnale $A\,\text{rect}_\Delta(t)$

presenta la seguente trasformata di Fourier: $A\Delta\,\text{sinc}(\pi f \Delta)$,

per la proprietà di linearità abbiamo:

$$X(f) = 2\,\text{sinc}(2\pi f) + \text{sinc}(\pi f)$$

Per $H(f)$ invece abbiamo che

$$h(t) = 2\,\text{rect}_{1.6}(t) \Rightarrow H(f) = 3.2\,\text{sinc}(1.6\pi f)$$

$$\Rightarrow Y(f) = \left(2\,\text{sinc}(2\pi f) + \text{sinc}(\pi f)\right) \cdot 3.2\,\text{sinc}(1.6\pi f)$$