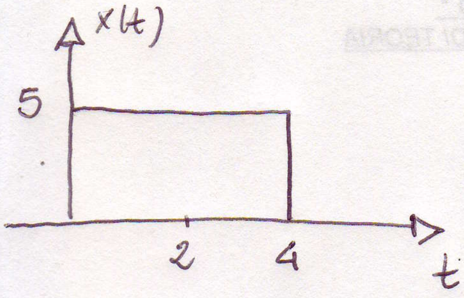
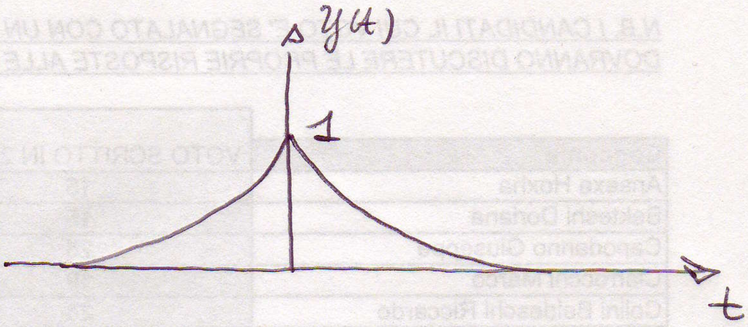
Esercizio

Calcolare la convoluzione $c(t)$ tra i seguenti segnali

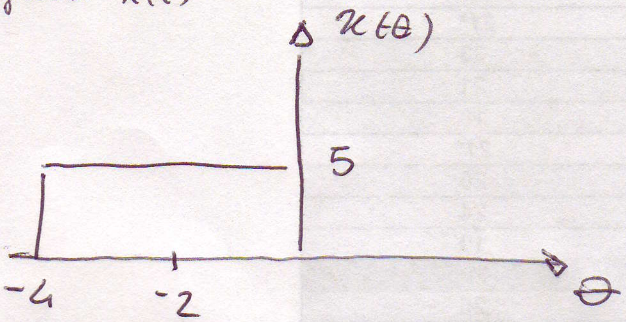
$x(t) = 5\,\mathrm{rect}_4(t-2)$ $\quad$ $y(t) = e^{-14|t|}$





=SOLUZIONE=

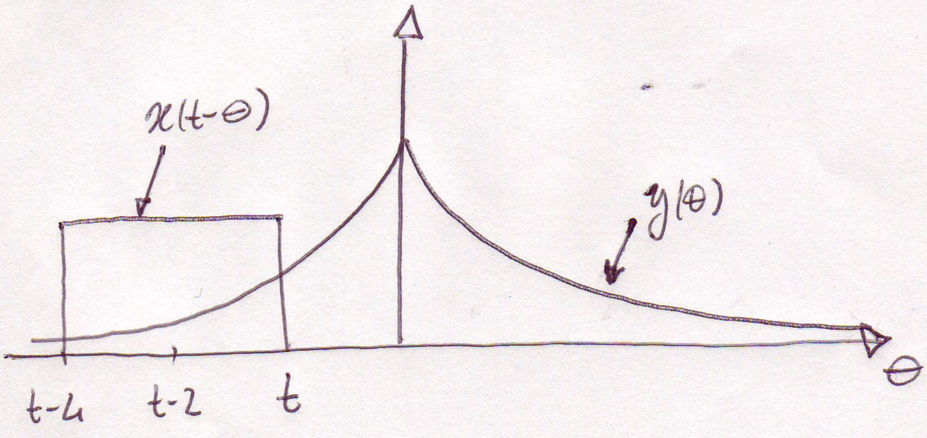
Ribalto il segnale $x(t)$



Per ogni possibile traslazione t imposta sul segnale $x(-\theta)$, quest'ultimo è sempre sovrapposto al segnale $y(\theta)$, avendo il segnale $y(\theta)$ estensione infinita.

Si identificano i seguenti intervalli di interesse

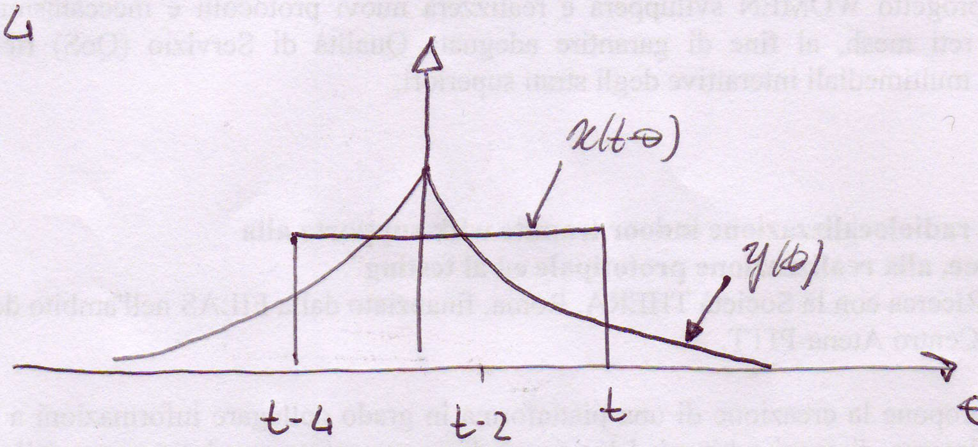
1) $t \leq 0$



$$c(t) = \int_{t-4}^{t} 5e^{-14|\theta|}\, d\theta = (t \leq 0) = \int_{t-4}^{t} 5e^{14\theta}\, d\theta$$

$$= \frac{5}{14} e^{14\theta} \Big|_{t-4}^{t} = \frac{5}{14} e^{14t}\left(1 - e^{-56}\right)$$

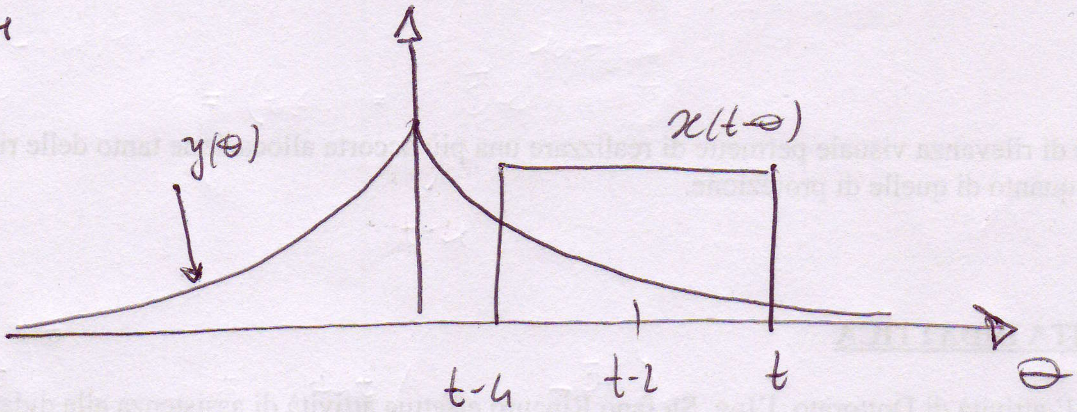
2) $0 \leq t < 4$



$$c(t) = \int_{t-4}^{t} 5e^{-14|\theta|}\, d\theta = \int_{t-4}^{0} 5e^{14\theta}\, d\theta + \int_{0}^{t} 5e^{-14\theta}\, d\theta$$

$$= \frac{5}{14} e^{14t} \Big|_{t-4}^{0} + \left[-\frac{5}{14} e^{-14\theta} \Big|_{0}^{t} \right] = \frac{5}{14}\left(2 - \left(e^{-14t} + e^{14(t-4)}\right)\right)$$

3) $t \geq 4$



$$c(t) = \int_{t-4}^{t} 5 e^{-14|\theta|} \, d\theta = \int_{t-4}^{t} 5 e^{-14\theta} \, d\theta$$

$$= -\frac{5}{14} e^{-14\theta} \Big|_{t-4}^{t} = \frac{5}{14} e^{-14t} \left(e^{56} - 1 \right)$$

Concludendo

$$c(t) = \begin{cases} \frac{5}{14} e^{14t} \left(1 - e^{-56}\right) & t \leq 0 \\ \frac{5}{14} \left(2 - \left(e^{-14t} + e^{14(t-4)}\right)\right) & 0 \leq t \leq 4 \\ \frac{5}{14} e^{-14t} \left(e^{56} - 1\right) & t \geq 4 \end{cases}$$