Determining the Rydberg constant using a spectrometer

Sam Brind

8926382

School of Physics and Astronomy

The University of Manchester

First Year Laboratory Report

April 2014

This experiment was performed in collaboration with *Helen Bennett*.

Abstract

The Rydberg constant, R, was determined by using a spectrometer to observe light from a hydrogen bulb passing through a diffraction grating. The final value for R was $(1.10 \pm 0.13) \times 10^7$ m^{-1}. The consistency of the diffraction angle measurements limited this result.

1. Introduction

Rydberg's constant is a fundamental value first postulated in work done by the physicist J.R. Rydberg, on spectral lines in 1890. The constant is useful as it allows the wavelengths and frequencies of emission spectra to be calculated easily for atoms such as hydrogen [1].

This experiment involved the use of a spectrometer to observe light being passed through a diffraction grating. The resulting angles of the diffraction pattern were measured and used to obtain the emission wavelengths. The Rydberg constant was determined using these wavelengths and known information on atomic emission.

2. Theory

Consider a diffraction grating, composed of N lines/mm, with light that is perpendicular to the grating incident upon it. The line spacing, a, is given by,

$$a = \frac{1}{N}. \qquad (1)$$

Figure 1 shows a close up view of the geometrical relation between the grating slits and the diffracted light.

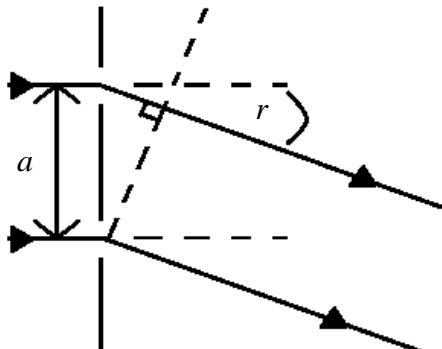
Figure 1.Close up view of a diffraction grating.

One of the light rays can be seen to travel a slightly longer path. The extra distance travelled is called the path difference. For these two light rays to reinforce each other and produce a maxima the path difference must be a whole number of wavelengths (i.e. $m\lambda$). It can be seen from the trigonometry of Fig.1 that,

$$m\lambda = a\sin(r) \qquad (2)$$

where λ is the wavelength of incident light, r is the angle of diffraction and m is the diffraction order.

If the incident light is produced from a hydrogen bulb then the diffraction pattern will consist of hydrogen's spectral series. These are lines at wavelengths that correspond to specific transitions between atomic energy levels. The relationship between this and the Rydberg constant is given by,

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \qquad (3)$$

where n_2 is the higher energy level and n_1 is the lower energy level [2].

The only transitions in the visible spectrum for hydrogen are within the Balmer series. All of these come from transitions down to the $n = 2$ state. Table 1 shows the exact transition details. [3]

Transition ($n_2 \rightarrow n_1$)	Wavelength (nm)	Colour
3→2	656.3	Red
4→2	486.1	Blue
5→2	434.1	Violet
6→2	410.2	Violet

Table 1. Properties of lines from the Balmer series that are visible.

3. Experimental Method
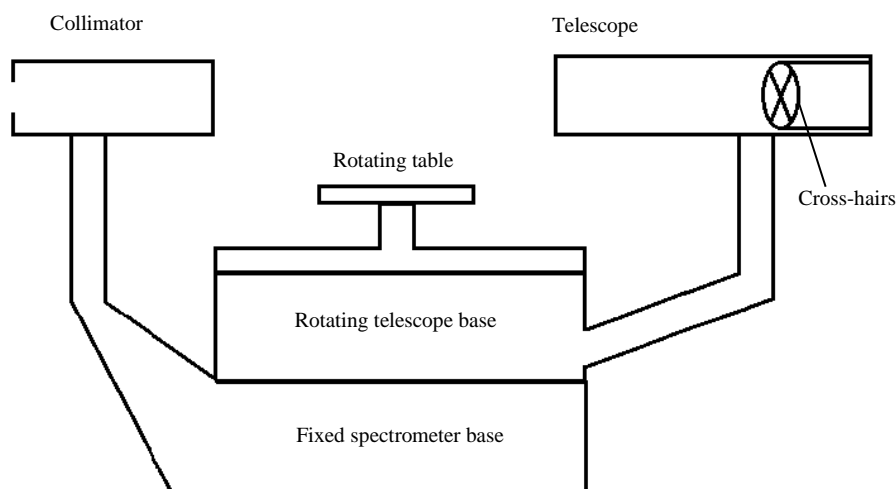
3.1. Diagram of Experiment

Figure 2. The spectrometer used in the experiment.

3.2. Method

The telescope was focused by imaging a distant object, making sure the crosshairs were in the focal plane for parallel light. A hydrogen bulb was then placed in line with the collimator to illuminate the slit. This light was then brought to a sharp focus by placing a condensing lens between the bulb and the slit. The apparatus was also checked by eye to be collinear and the table horizontally level.

A diffraction grating, with 200 ± 0.02 lines/mm, was mounted on the table and rotated until perpendicular to the incident light from the collimator. The angles of each coloured emission line and their orders were then measured. This was done by sweeping the telescope around from the central (0th order) line to the left and then repeated but to the right of the central line. The 2nd order was the furthest that could be seen clearly. Angles were measured from the spectrometer's two vernier scales separated by 180° on the spectrometer, V_1 and V_2. This was then used to calculate an average angle for each line, V_{avg}. The angle measurements were performed by two independent experimenters, giving two sets of data.

4. Experimental Results

4.1. Finding the wavelength of emission lines

Appendix A gives the recorded table of results. The angle of diffraction was calculated for each line. This is simply half the angle turned in going from the left line to the right line. The sine of the angle of diffraction was plotted against the line's order for each colour of spectral line, as shown in Fig.3. From (2) it can be seen this plot should be a linear relationship with a gradient of $\dfrac{\lambda}{a}$.
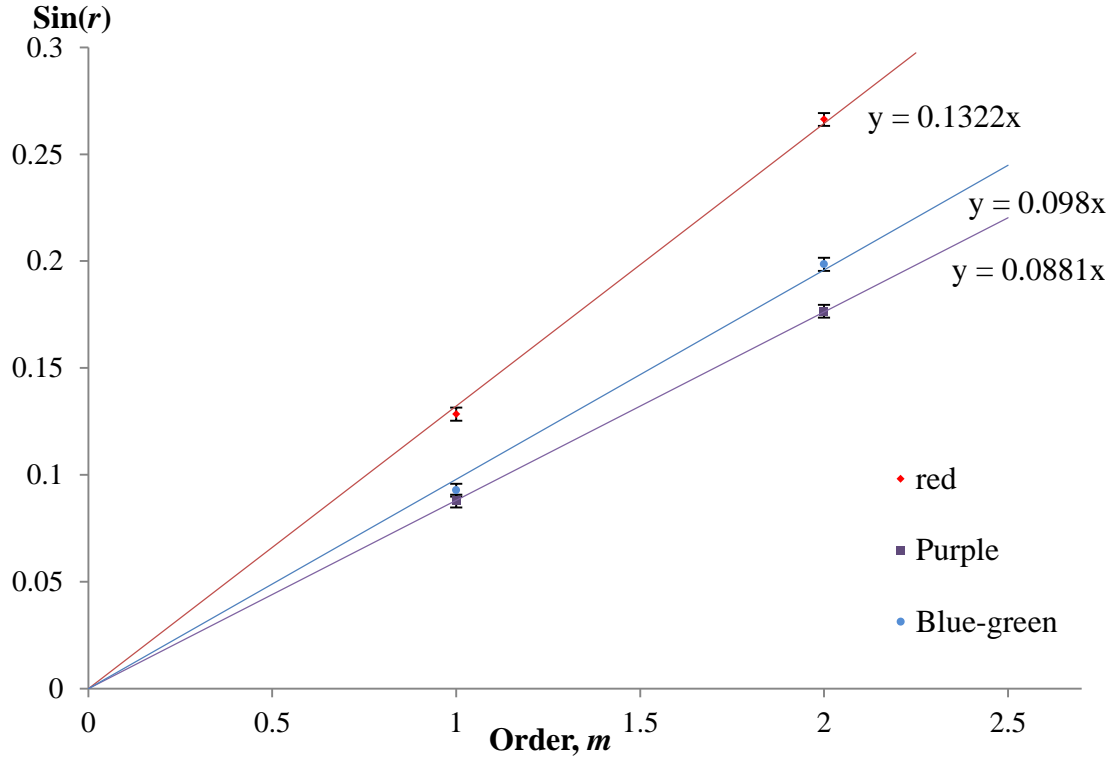
Figure 3. Linear fits of all 3 coloured emission lines from data set 1 plotted. All lines are forced through the origin to agree with the theory.

The uncertainty on r was determined by looking at the difference between the angles in data sets 1 and 2. This would give an indication of the consistency and therefore accuracy of our measurements. The differences were plotted as a scatter graph as shown in Fig.4.
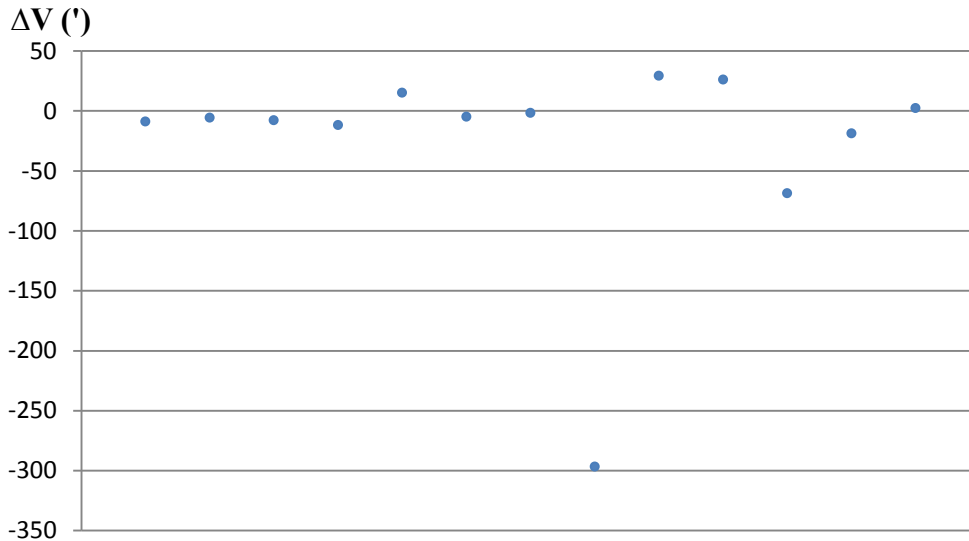

Figure 4. A scatter graph of the difference in data set angles plotted in Excel, where $\Delta V = V_{avg.\ 1} - V_{avg.\ 2}$.

Apart from a couple of outlying results, it can be seen that most angles lie within a variation of only a few arc minutes. From this observation, we assumed the absolute error on r to be $\pm$ 10'. This gives an uncertainty on $\sin(r)$ of ± 0.003. As the line of best fit has been forced through the origin, the error on the gradient is simply the error on $\sin(r)$.

The wavelength of each emission line was calculated by multiplying the gradient from Fig.3 by a, where $a = (5 \pm 0.0005) \times 10^{-6}$ m from use of (1) with the given value of N. The uncertainty on a is 0.01% and the uncertainty on the gradients vary from 2.3-3.4%. Therefore, the gradient uncertainty will dominate and the error on λ can be assumed to be the same percentage error as the gradient. The three wavelengths were therefore 661 ± 15 nm, 490 ± 15 nm and 441 ± 15 nm for red, blue-green and purple respectively.

4.2. Finding R and its associated uncertainty

Multiplying (3) by a and rearranging gives

$$\frac{\lambda}{a} = \frac{1}{aR\left(\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2}\right)} \quad . \tag{4}$$

The gradients of the lines from Fig.3 (i.e. $\dfrac{\lambda}{a}$) were then plotted against the factor

$\dfrac{1}{\left(\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2}\right)}$, where this factor was calculated using known values from Table 1.

From (4) it can be seen that there should be a linear relationship with a gradient of $\dfrac{1}{aR}$
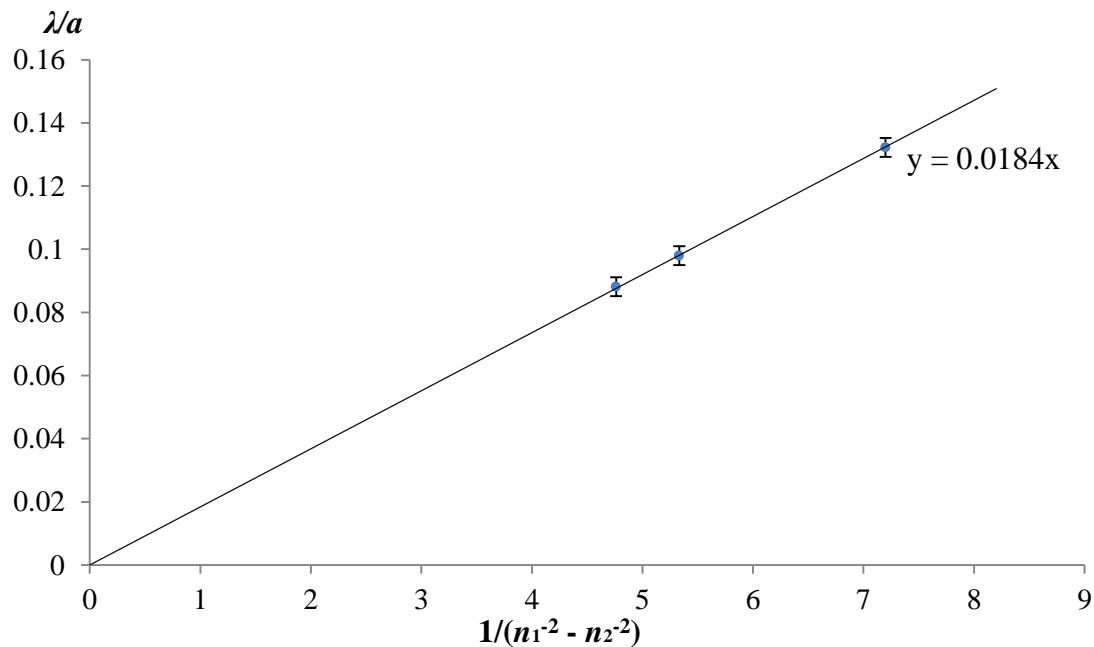
.

Figure 5. A linear fit of the gradients from Fig.3 plotted against the factor from energy level differences. Again, this has been forced through the origin to fit the theory.

The Rydberg constant was determined by multiplying the gradient from Fig.5 by a and taking the inverse of the result. The uncertainty on the gradient is still equal to the

uncertainty on $\sin(r)$ and this gives a percentage error on the gradient of 16.3%. As before, this will dominate over the error on a and therefore will equal the percentage error on R. R was calculated to be $(1.09 \pm 0.18) \times 10^7$ m^{-1} for data set 1.

This whole process was then performed with data set 2 and the value of R was calculated as $(1.12 \pm 0.19) \times 10^7$ m^{-1}. Data set 1 and set 2 are both independent as the measurements were taken by different people. A weighted mean of the two values was then calculated to give a final value of $(1.10 \pm 0.13) \times 10^7$ m^{-1}.

5. Conclusion

A chi-squared (χ_2) analysis was performed on all the linear fits. Chi-squared was found to be 2.01, 3.70 and 0.03 for the fits shown on Fig.3 of red, blue-green and purple. The number of degrees of freedom, N_{dof}, on all these lines is 1. This shows that the fits are good and the error assumptions were sensible, because $\chi_2 \approx N_{dof}$. The chi-squared for the fit on Fig.5 was calculated to be 0.11. This fit had 2 degrees of freedom, again indicating the linear fit was good.

Uncertainties could have occurred from the grating not being perfectly perpendicular to the light, judging the centre of emission lines and reading the vernier scale. The grating was rotated and set by eye and a more precise method could be used to ensure a perpendicular grating. Emission lines weren't perfectly sharp when viewed and this relies on judgement and the observer's eyesight to determine the centre, introducing human error. The scale also involved human judgement to read. It would also be useful to investigate whether there is a systematic error in the angle measurements as ΔV tends to be negative more frequently than it is positive (shown in Fig.4).

If another experiment was to be performed, the outlying results on Fig.4 should be investigated to determine whether they were anomalous. This experiment would also be further improved by being able to measure more orders. This could be achieved by using a grating with a greater value of a. Seeing further orders would allow more data points to be fitted, increasing the accuracy of linear fits and therefore providing better results.

References

[1]   Rydberg constant, Encyclopaedia Britannica, Accessed on 26/03/14, URL: http://www.britannica.com/EBchecked/topic/514511/Rydberg-constant

[2]   J. Clark, The atomic hydrogen emission spectrum, Chemguide, Accessed on 28/03/14, URL: http://www.chemguide.co.uk/atoms/properties/hspectrum.html

[3]   R. Nave, Hydrogen Spectrum, HyperPhysics, Accessed on 28/03/14, URL: http://hyperphysics.phy-astr.gsu.edu/Hbase/hyde.html

Length and Date

The number of words in this document is 1475

This document was printed on 02/04/2014 at 23:02.

Appendix A

Side	Order	Colour	V_1 (o , ')		V_2 (o , ')		$V_{avg.}$ (o , ')	
			1	2	1	2	1	2
Centre	0	Red	271,32	271,40	91,30	91,40	91,31	91,40
Left	1	Purple	276,40	276,45	96,36	96,43	96,38	96,44
		Blue-green	277,14	277,20	97,12	97,22	97,13	97,21
		Red	279,12	279,24	99,10	99,22	99,11	99,23
	2	Purple	281,40	281,05	101,40	101,45	101,40	101,25
		Blue-green	283,55	283,00	102,55	103,00	102,55	103,00
		Red	287,00	287,00	107,00	107,03	107,00	107,02
Right	1	Purple	266,30	276,30	86,36	86,32	86,34	91,31
		Blue-green	265,50	266,00	85,57	86,00	86,29	86,00
		Red	263,52	264,00	84,00	84,00	84,26	84,00
	2	Purple	261,20	261,31	81,20	82,27	81,20	82,29
		Blue-green	260,00	260,20	80,01	80,20	80,01	80,20
		Red	256,80	256,04	76,06	76,06	76,7	76,05

Table 2. Measured telescope angles (in degrees and arcminutes) for the 1st 2 orders of the Balmer series. Data sets 1 and 2 were measured by two independent experimenters. Note: The 2nd violet line, as indicated in table 1, is not recorded as the line was to faint to be seen.