Linear Model with higher order features

Our error analysis of the toy problem suggested that a straight line was perhaps not the best fit

- positive errors in the extremes
- negative errors in the center

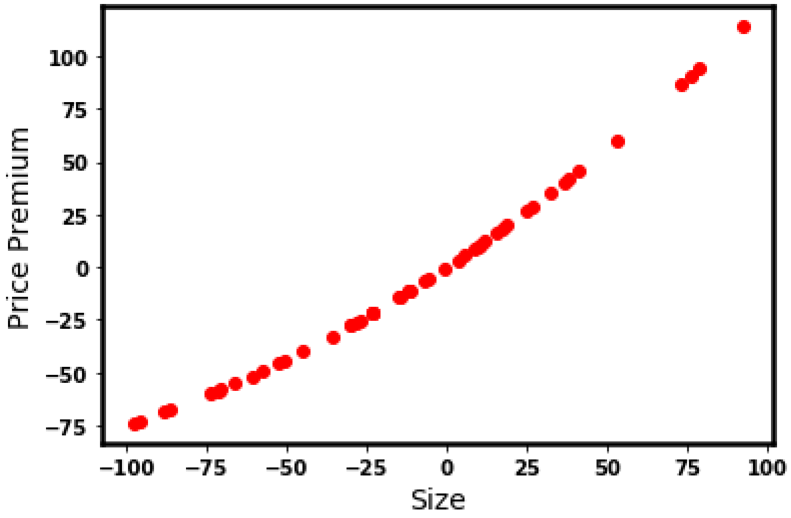
Perhaps a "curve" would be a better hypothesis ? What if our data is not linear ?

Here's what the first dataset looked like

```
In [4]: (xlabel, ylabel) = ("Size", "Price Premium")

        # I will give you the data via a function (so I can easily alter the data in sub
        sequent examples)
        v1, a1 = 1, .005
        lin = recipe_helper.Recipe_Helper(v = v1, a = a1)
        X_lin, y_lin = lin.gen_data(num=50)
```

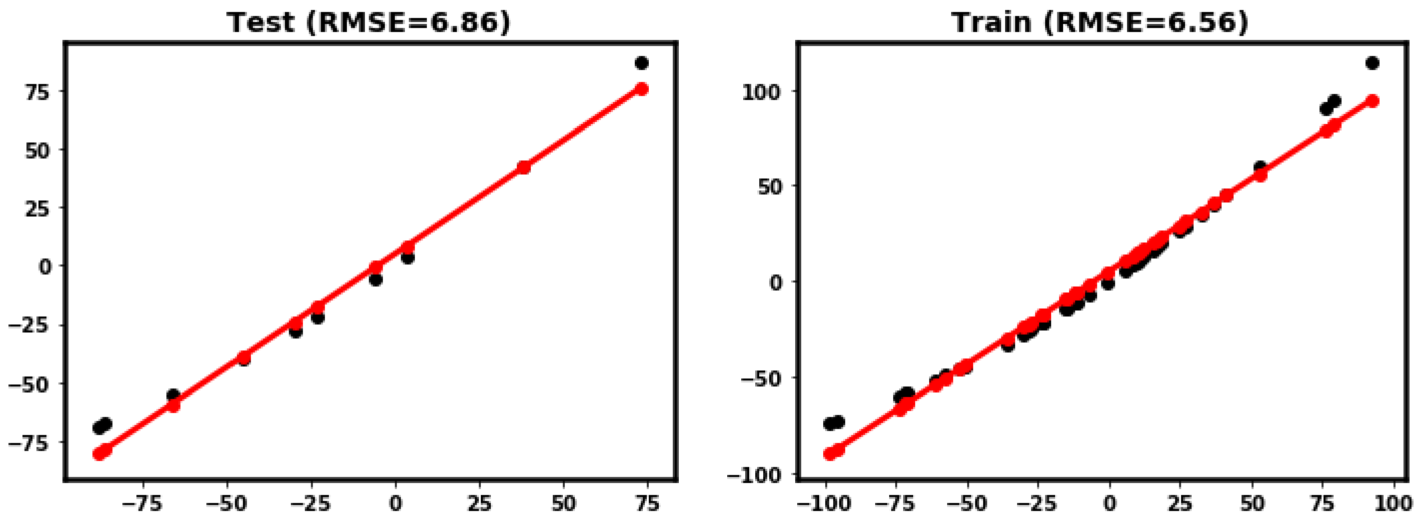
In [5]:

```
X_orig, y_orig = X_lin, y_lin

_ = lin.gen_plot(X_orig, y_orig, xlabel, ylabel)
```



In [6]:
```
_= lin.run_regress(X_orig, y_orig)
```

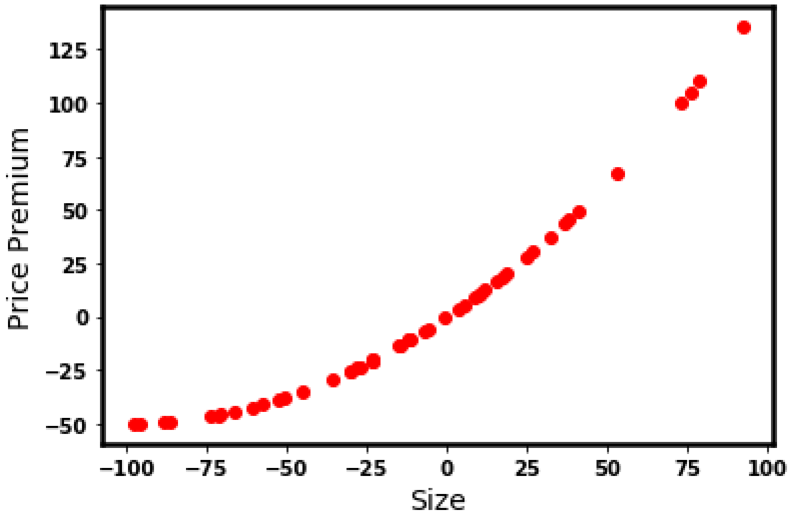
```
Coefficients:
 [4.93224426] [[0.96836946]]


R-squared (test): 0.98
Root Mean squared error (test): 6.86


R-squared (train): 0.98
Root Mean squared error (train): 6.56
```



We will make our point by creating a similar dataset (the "curvy" dataset) that **exaggerates the curvature**.

In [7]:

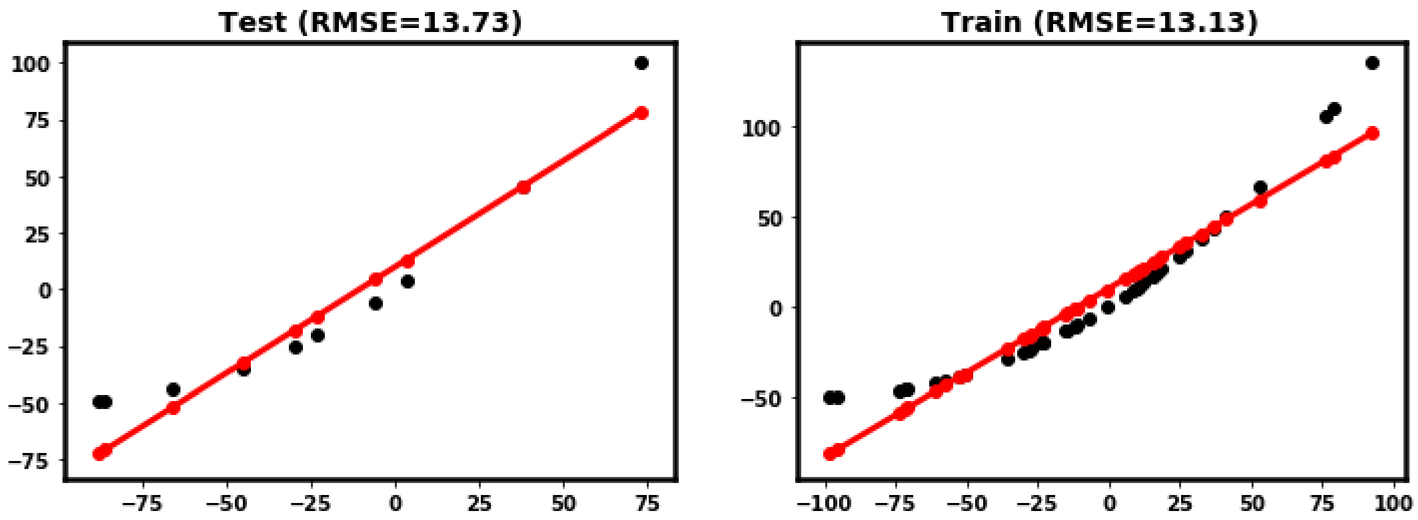
```
v2, a2 = v1, a1*2
curv = recipe_helper.Recipe_Helper(v = v2, a = a2)
X_curve, y_curve = curv.gen_data(num=50)
_= curv.gen_plot(X_curve,y_curve, xlabel, ylabel)
```



In [8]:

```
_= curv.run_regress(X_curve, y_curve)
```

```
Coefficients:
 [9.86448852] [[0.93673892]]


R-squared (test): 0.91
Root Mean squared error (test): 13.73


R-squared (train): 0.91
Root Mean squared error (train): 13.13
```



Compared to the original, the "curvy" data set has a lot more curvature

- the R^2 is still over 90%
- but the Performance Metric (RMSE) is twice as big

Curvature in a linear model

Our (first-order) linear model was

$$\mathbf{y} = \Theta_0 + \Theta_1 \mathbf{x}$$

We can create a *second order* linear model by adding a feature x^2:

$$\mathbf{y} = \Theta_0 + \Theta_1 \mathbf{x} + \Theta_2 \mathbf{x}^2$$

$\mathbf{y}$ is a second order polynomial, whose plot is a curve

- but it is linear in features $\mathbf{x}, \mathbf{x}^2$

In other words, we are performing feature iteration

- in this case: adding the missing feature $\mathbf{x}^2$

Let's modify $\mathbf{x^{(i)}}$ from a vector of length 1:

$$\mathbf{x^{(i)}} = (\mathbf{x}_1^{\mathbf{(i)}})$$

to a vector of length 2:

$$\mathbf{x^{(i)}} = (\mathbf{x}_1^{\mathbf{(i)}}, \mathbf{x}_1^{\mathbf{(i)}\,2})$$

by adding a squared term to the vector $\mathbf{x^{(i)}}$, for each i.

The modified $\mathbf{X}'$ becomes:

$$\mathbf{X} = \begin{pmatrix} 1 & \mathbf{x}_1^{(1)} & (\mathbf{x}_1^{(1)})^2 \\ 1 & \mathbf{x}_1^{(2)} & (\mathbf{x}_1^{(2)})^2 \\ \vdots & \vdots & \\ 1 & \mathbf{x}_1^{(m)} & (\mathbf{x}_1^{(m)})^2 \end{pmatrix}$$

Note that this modified $\mathbf{X}'$ fits perfectly within our Linear hypothesis

$$\hat{\mathbf{y}} = \mathbf{X}'\Theta$$

The requirement is that the model be linear in its *features*, **not** that the features be linear !

What we have done is added a second feature, that just so happens to be related to the first.

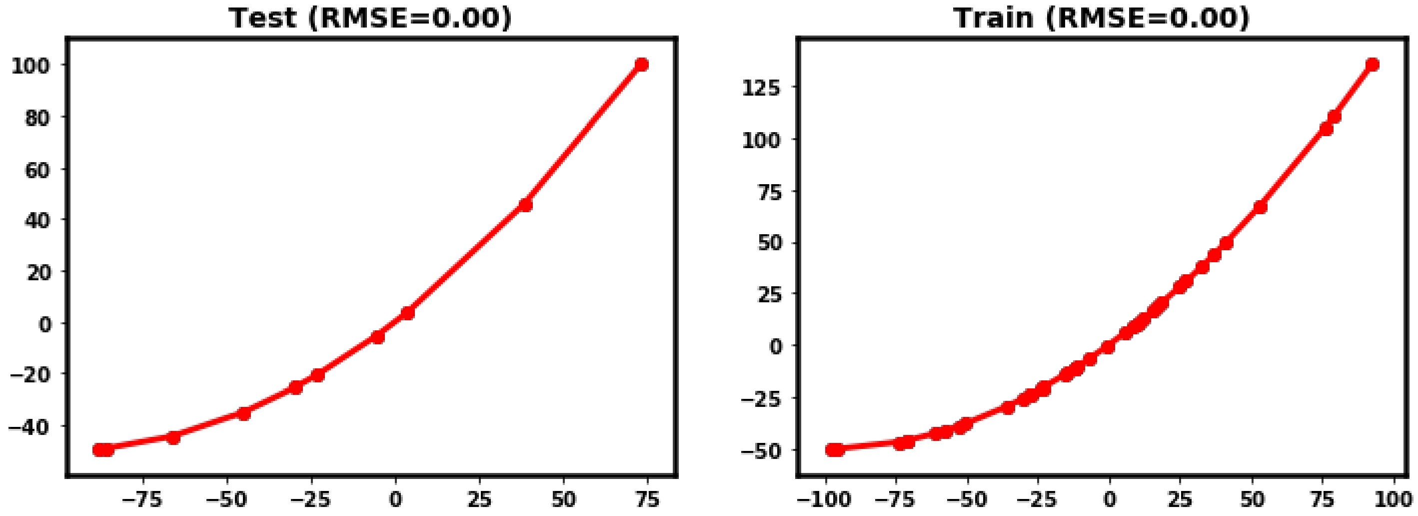
We can now run our linear model with the modified feature vectors

A word about our module

- we add the $\mathbf{x}^2$ column by setting optional parameter `run_transform` to `True`

In [9]:
```
_= curv.run_regress(X_curve, y_curve, run_transforms=True)
```

```
Coefficients:
 [-3.55271368e-15] [[1.    0.005]]


R-squared (test): 1.00
Root Mean squared error (test): 0.00


R-squared (train): 1.00
Root Mean squared error (train): 0.00
```



Perfect fit !

TIP

- Don't stop just because you scored 91%. And don't give up if the score was awful.
- Examining the errors (residuals) reveals a lot about how to improve your model.
    - Where was the fit good ? Where was it bad ?
    - Is there a pattern to the badly fit observations that points to a missing feature ?

One of the real arts of ML is diagnosing model deficiencies and knowing how to improve them.

We will have a separate module on this topic.

In [10]:

```python
print("Done !")
```

```
Done !
```