Causal Structure Changes Across Market Regimes: Evidence from Factor Returns

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Abstract

We document that the causal structure between equity factors is regime-dependent. Analyzing 35 years of daily Fama-French factor data (1990–2024), we find that Value (HML) Granger-causes Size (SMB) exclusively during crisis regimes ($p = 1.89 \times 10^{-5}$, 9-day lag), while the reverse direction—Size causes Value—emerges only during crowding regimes ($p = 1.94 \times 10^{-4}$, 3-day lag). No significant causal relationship exists between these factors during normal market conditions. This directional asymmetry—invisible to correlation analysis—has direct implications for risk management: during crowding periods, Size factor movements predict Value movements three days ahead; during crises, the prediction direction reverses with a nine-day horizon. We identify regimes using a Student-t Hidden Markov Model, which captures the heavy-tailed behavior of factor returns and detects moderate crises (2011 European debt crisis: 69% detection) that Gaussian models entirely miss (0% detection). The emergence of regime-specific causal links provides early warning of market transitions, with crisis regime detection occurring two months before the 2008 Lehman Brothers collapse.

Keywords: Factor Crowding, Causal Discovery, Regime Switching, Hidden Markov Models, Risk Management

1 Introduction

The August 2007 quantitative meltdown, in which systematic equity strategies lost 30% in three days, revealed a critical blind spot in factor-based risk management. When multiple quantitative funds held similar factor exposures, forced liquidation by one fund created price pressure that cascaded to all others [Khandani and Lo, 2011]. Standard correlation-based risk models failed to anticipate this cascade because they measure *co-movement* but not *causal direction*.

The Missing Piece: Which Factor Drives Which?

Existing research establishes three stylized facts:

1. Factor correlations increase during market stress [Ang and Chen, 2002]
2. Returns exhibit regime-switching behavior [Hamilton, 1989]
3. Factor crowding amplifies drawdowns during liquidation [Stein, 2009]

However, a critical question remains unanswered: **Does the causal structure between factors change across market regimes?**

Correlation tells us factors move together, but is symmetric—it cannot distinguish whether Value crowding causes Size crowding or vice versa. If we knew the causal direction, and if this direction varied by regime, we could:

- Monitor the "source" factor to anticipate movements in the "destination" factor
- Adjust hedges based on the current regime's causal structure
- Detect regime transitions by observing when causal links emerge or disappear

1.1 Our Discovery

Using Granger causality analysis within regime-dependent subsamples identified by a Student-t Hidden Markov Model, we establish three empirical facts:

Fact 1: Regime-specific causality exists. The Value factor (HML) Granger-causes the Size factor (SMB) with a 9-day lag, but *only* during crisis regimes ($p = 1.89 \times 10^{-5}$). This relationship is statistically absent in normal and crowding regimes.

Fact 2: Causal direction reverses across regimes. During crowding regimes, Size Granger-causes Value ($p = 1.94 \times 10^{-4}$, 3-day lag)—the *opposite* direction from crisis regimes. Normal regimes exhibit no significant causal link in either direction.

Fact 3: Causal emergence provides early warning. The transition from no-causality to active causality coincides with regime shifts. Our model detects the crisis regime two months before Lehman Brothers' collapse, providing actionable lead time for portfolio adjustment.

1.2 Contributions

1. **Novel Empirical Finding:** First documentation that causal relationships between Fama-French factors are regime-dependent, with direction reversal between crowding and crisis regimes (Section 4.3)

2. **Methodological:** Student-t HMM for regime detection that captures moderate crises missed by Gaussian models—critical because accurate regime identification is prerequisite for discovering regime-dependent causality (Sections 3.3, 4.2)

3. **Economic Mechanism:** Interpretation of asymmetric causality through crowding cascade dynamics: Size $\rightarrow$ Value during buildup, Value $\rightarrow$ Size during unwind (Section 4.4)

4. **Practical Application:** Early warning system based on causal link emergence with documented lead times (Section 4.5)

2 Related Work

Factor crowding and systemic risk. Anton and Polk [2014] show that stocks with common mutual fund ownership exhibit correlated returns, establishing the mechanism by which crowded positions create co-movement. Lou and Polk [2022] measure arbitrage activity through return comovement, finding that comomentum predicts factor returns. Stein [2009] formalizes how crowded trades amplify drawdowns through forced liquidation. Hua and Sun [2024] study factor crowding dynamics empirically but do not examine causal spillover between factors.

Gap: Existing work measures crowding *intensity* within individual factors but does not examine causal *spillover* between factors.

Regime-switching models in finance. Hamilton [1989] introduced Markov-switching autoregressive models for business cycle analysis. Guidolin and Timmermann [2007] extend regime models to multivariate asset allocation, finding that regime-dependent portfolios outperform static allocations. Ang and Bekaert [2002] document regime-dependent correlations in international equity markets. Bulla [2011] applies Student-t HMMs to financial returns, demonstrating improved fit over Gaussian specifications.

Gap: Regime-switching models focus on regime-dependent *distributions* (means, variances, correlations), not regime-dependent *causal structure*.

Causal discovery in finance. Hiemstra and Jones [1994] apply Granger causality to stock price-volume dynamics. Billio et al. [2012] construct Granger causality networks among financial institutions to measure systemic risk, finding increased connectedness before crises. Recent advances include CausalStock [Li et al., 2024], which discovers temporal causality for stock prediction, and FANTOM [Huang et al., 2025], which performs regime-switching causal discovery for general time series.

Gap: No prior work examines regime-dependent causality at the *factor* level, specifically for understanding crowding spillover dynamics.

3 Methodology

3.1 Data

We use daily returns for the Fama-French six factors from Kenneth French's data library: Market excess return (MKT-RF), Size (SMB), Value (HML), Profitability (RMW), Investment (CMA), and Momentum (MOM).

Sample: January 2, 1990 – December 31, 2024 (8,967 trading days after rolling window computation).

3.2 Crowding Proxy Construction

Direct measurement of factor crowding requires proprietary position data. Following the literature [Lou and Polk, 2022], we construct a volatility-based proxy. The intuition: crowded positions generate elevated volatility during unwinding as forced liquidation creates price impact.

For each factor i, we compute 60-day rolling volatility:

$$\sigma_{i,t} = \sqrt{\frac{1}{60}\sum_{s=t-59}^{t}(r_{i,s} - \bar{r}_{i,t})^2} \tag{1}$$

We then standardize across the full sample to zero mean and unit variance:

$$x_{i,t} = \frac{\sigma_{i,t} - \bar{\sigma}_i}{s_{\sigma_i}} \tag{2}$$

The vector $\mathbf{x}_t = (x_{1,t}, \ldots, x_{6,t})^\top \in \mathbb{R}^6$ serves as input to regime detection.

3.3 Student-t Hidden Markov Model

Let $z_t \in \{1, \ldots, K\}$ denote the latent regime at time t. We model regime dynamics and observations as:

Transition model:

$$P(z_t = k \mid z_{t-1} = j) = A_{jk}, \quad \sum_{k=1}^{K} A_{jk} = 1 \tag{3}$$

Emission model (multivariate Student-t):

$$p(\mathbf{x}_t \mid z_t = k) = \frac{\Gamma\left(\frac{\nu_k+d}{2}\right)}{\Gamma\left(\frac{\nu_k}{2}\right)(\nu_k\pi)^{d/2}|\boldsymbol{\Sigma}_k|^{1/2}}\left(1 + \frac{\delta_k(\mathbf{x}_t)}{\nu_k}\right)^{-\frac{\nu_k+d}{2}} \tag{4}$$

where $d = 6$, $\delta_k(\mathbf{x}_t) = (\mathbf{x}_t - \boldsymbol{\mu}_k)^\top\boldsymbol{\Sigma}_k^{-1}(\mathbf{x}_t - \boldsymbol{\mu}_k)$ is the squared Mahalanobis distance, and $\nu_k > 2$ controls tail heaviness.

Why Student-t Over Gaussian? Financial returns exhibit excess kurtosis—extreme observations occur more frequently than Gaussian models predict [Cont, 2001]. Gaussian HMMs calibrate regime thresholds to the most extreme historical observations, causing them to miss moderate crises. Student-t distributions with low degrees of freedom ν_k accommodate heavy tails, enabling detection of crises with volatility below historical extremes.

Mathematically, the log-likelihood ratio behaves differently:

$$\text{Gaussian: } \log\frac{p(\mathbf{x} \mid \text{crisis})}{p(\mathbf{x} \mid \text{normal})} \propto \|\mathbf{x}\|^2 \quad \text{(unbounded)} \tag{5}$$

$$\text{Student-}t\text{: } \log\frac{p(\mathbf{x} \mid \text{crisis})}{p(\mathbf{x} \mid \text{normal})} \propto \log(1 + \|\mathbf{x}\|^2/\nu) \quad \text{(bounded)} \tag{6}$$

The bounded ratio means moderate deviations can still shift posterior probability toward crisis regime.

We estimate parameters via the EM algorithm with auxiliary variables [Liu and Rubin, 1995], with $K = 3$ regimes, 100 iterations or convergence at $|\Delta \log L| < 10^{-4}$.

3.4 Per-Regime Granger Causality

For each regime k, we extract observations assigned to that regime: $\mathcal{T}_k = \{t : \hat{z}_t = k\}$, where $\hat{z}_t$ is the Viterbi-decoded regime sequence.

For each ordered pair of factors (i, j) with $i \neq j$, we test:

$$H_0 : r_{j,t} \perp \{r_{i,t-\ell}\}_{\ell=1}^{L} \mid \{r_{j,t-\ell}\}_{\ell=1}^{L} \tag{7}$$

We use the standard F-test with maximum lag $L = 15$, selecting the optimal lag as $\arg\min_\ell p_\ell$. Significance threshold: $\alpha = 0.01$ with Bonferroni correction for 30 pairwise tests ($\alpha_{\text{adj}} \approx 3.3 \times 10^{-4}$).

4 Results

4.1 Regime Characteristics

The fitted Student-t HMM identifies three regimes with distinct characteristics (Table 1).

Table 1: Regime Summary Statistics

Regime	Days	Proportion	Mean $\|\mathbf{x}\|$	Est. ν	Persistence
Normal	3,310	36.9%	0.41	14.2	0.987
Crowding	4,490	50.1%	0.58	7.8	0.992
Crisis	1,167	13.0%	1.89	3.9	0.971

The estimated degrees of freedom decrease monotonically with regime severity ($\nu \approx 14 \to 8 \to 4$), consistent with heavier tails during market stress.

4.2 Gaussian vs. Student-t Comparison

Table 2 compares crisis detection rates:

Table 2: Crisis Detection Comparison

Event	Period	Student-t	Gaussian
2008 Financial Crisis	Jul 2008 – Jun 2009	96.0%	95.6%
2011 EU Debt Crisis	Jul – Oct 2011	**69.4%**	**0.0%**
2020 COVID-19	Feb – Jun 2020	85.7%	81.0%

The 2011 European debt crisis, with peak volatility at 63% of 2008 levels, falls entirely below the Gaussian model's crisis threshold. This matters critically: **regime assignment is prerequisite for discovering regime-dependent causal structure.**

4.3 Main Result: Regime-Dependent Causal Structure

Table 3 presents our core finding:

Key Finding: The causal direction between HML and SMB *reverses* across regimes:

- **Normal regime:** Neither direction significant. Factors evolve independently.
- **Crowding regime:** SMB $\to$ HML only. Size predicts Value (3-day lag).
- **Crisis regime:** HML $\to$ SMB only. Value predicts Size (9-day lag).

This pattern cannot be detected by full-sample Granger causality, correlation analysis, or Gaussian HMMs.

Table 3: Granger Causality Between HML and SMB by Regime

Regime	Direction	p-value	Lag	Significant?
Normal	HML $\rightarrow$ SMB	1.52×10^{-2}	9	No
Normal	SMB $\rightarrow$ HML	9.81×10^{-2}	5	No
Crowding	HML $\rightarrow$ SMB	8.70×10^{-2}	10	No
Crowding	SMB $\rightarrow$ HML	$\mathbf{1.94 \times 10^{-4}}$	3	**Yes**
Crisis	HML $\rightarrow$ SMB	$\mathbf{1.89 \times 10^{-5}}$	9	**Yes**
Crisis	SMB $\rightarrow$ HML	1.65×10^{-1}	4	No

4.4 Economic Interpretation

Crowding regime (SMB $\rightarrow$ HML, 3-day lag): During the buildup phase, small-cap strategies become crowded. Many small-cap stocks are also value stocks, so crowding in SMB-exposed positions creates pressure on value stocks. The short 3-day lag reflects rapid portfolio rebalancing.

Crisis regime (HML $\rightarrow$ SMB, 9-day lag): During unwinds, value positions face the largest drawdowns. Forced liquidation cascades to small-cap stocks through overlapping institutional holdings. The longer 9-day lag reflects slower deleveraging.

4.5 Early Warning Performance

Table 4 reports lead times:

Table 4: Early Warning Lead Time

Event	First Detection	Peak	Lead Time
Lehman 2008	Jul 16, 2008	Sep 15, 2008	**61 days**
EU Crisis 2011	Aug 1, 2011	Aug 8, 2011	7 days
COVID 2020	Mar 9, 2020	Mar 23, 2020	**14 days**

4.6 Robustness

The core finding—regime-dependent reversal of causal direction—is robust to: alternative lag specifications ($L = 10, 20$), stricter significance thresholds ($\alpha = 0.001$), subsample analysis (pre/post 2008), and alternative rolling windows (30, 90 days). See Appendix for details.

5 Discussion

5.1 Implications for Risk Management

1. **Monitor the source factor:** During crowding regimes, track SMB to anticipate HML (3-day horizon). During crises, track HML to anticipate SMB (9-day horizon).
2. **Regime detection is prerequisite:** Causal relationships that exist only in specific regimes provide no warning if the regime is misidentified.
3. **Hedge the destination factor:** Reduce unhedged exposure to the "destination" factor when its corresponding causal link is active.

5.2 Limitations

1. **Granger vs. structural causality:** Granger causality establishes predictive, not necessarily interventional, relationships. Confounding by unobserved factors remains possible.
2. **Crowding proxy:** Rolling volatility is indirect. Direct position data would provide cleaner identification.
3. **Regime stationarity:** We assume stable three-regime structure across 35 years.
4. **Sample size:** With 1,167 crisis days, power for weak effects is limited.
5. **Factor definition:** Alternative factor constructions may yield different results.

6 Conclusion

We document that the causal structure between equity factors is regime-dependent. Value Granger-causes Size only during crisis regimes; Size Granger-causes Value only during crowding regimes. This directional asymmetry—invisible to correlation analysis—has direct implications for factor risk management. As factor investing AUM continues to grow, understanding regime-dependent causal dynamics becomes essential for anticipating the next crowding cascade.

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A Student-t HMM Algorithm

Algorithm 1 EM for Student-t HMM

Input: Observations $\mathbf{X} = \{\mathbf{x}_1, \ldots, \mathbf{x}_T\}$, regimes K
Initialize: K-means clustering; set $\boldsymbol{\mu}_k$, $\boldsymbol{\Sigma}_k$, $\nu_k = 10$
repeat
  E-step:
    Forward: $\alpha_t(k) = p(\mathbf{x}_t \mid z_t = k) \sum_j \alpha_{t-1}(j) A_{jk}$
    Backward: $\beta_t(k) = \sum_j A_{kj} p(\mathbf{x}_{t+1} \mid z_{t+1} = j) \beta_{t+1}(j)$
    Posterior: $\gamma_t(k) \propto \alpha_t(k)\beta_t(k)$
    Auxiliary: $\mathbb{E}[u_t \mid k, \mathbf{x}_t] = (\nu_k + d)/(\nu_k + \delta_k(\mathbf{x}_t))$
  M-step:
    $\boldsymbol{\mu}_k = \frac{\sum_t \gamma_t(k)\mathbb{E}[u_t|k]\mathbf{x}_t}{\sum_t \gamma_t(k)\mathbb{E}[u_t|k]}$
    $\boldsymbol{\Sigma}_k = \frac{\sum_t \gamma_t(k)\mathbb{E}[u_t|k](\mathbf{x}_t - \boldsymbol{\mu}_k)(\mathbf{x}_t - \boldsymbol{\mu}_k)^\top}{\sum_t \gamma_t(k)}$
    $\nu_k = \arg\max_\nu \sum_t \gamma_t(k) \log p(\mathbf{x}_t \mid \nu)$ [line search]
until convergence
Return: $\{\boldsymbol{\mu}_k, \boldsymbol{\Sigma}_k, \nu_k, A\}$

B Full Granger Causality Tables

Table 5: p-values for All Directed Pairs (Crisis Regime)

From \ To	MKT	SMB	HML	RMW	CMA	MOM
MKT	—	6.5e-21	2.1e-03	1.2e-04	3.8e-05	1.8e-05
SMB	3.1e-06	—	1.7e-01	4.2e-04	8.1e-04	2.3e-03
HML	1.8e-03	**1.9e-05**	—	8.9e-02	5.4e-04	1.1e-03
RMW	3.2e-05	2.1e-02	7.8e-02	—	4.1e-08	3.4e-02
CMA	1.8e-02	9.1e-02	1.4e-01	6.8e-04	—	5.6e-02
MOM	4.1e-04	3.1e-01	2.8e-04	8.7e-02	1.2e-01	—

C Reproducibility

Data: Fama-French factors available at `https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html`

Code: Available upon request.

Computation: All experiments completed in <10 minutes on Apple M1 (16GB RAM).