삼각행렬 G의 역행렬 T

$$T = G^{-1}$$

$$G = \begin{pmatrix} g_{11} & & & 0 \\ g_{21} & g_{22} & & \\ \vdots & & \ddots & \\ g_{n1} & g_{n2} & \cdots & g_{nn} \end{pmatrix} \quad T = \begin{pmatrix} t_1 & t_2 & \cdots & t_n \end{pmatrix}$$

t_i : $(n \times 1)$ column vector

① $G\begin{pmatrix} t_{11} \\ \vdots \\ t_{n1} \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$

$g_{11} t_{11} = 1 \Rightarrow t_{11} = 1/g_{11}$

$g_{21} t_{11} + g_{22} t_{21} = 0 \Rightarrow t_{21} = -g_{21} t / g_{22}$

$g_{31} t_{11} + g_{32} t_{21} + g_{33} t_{31} = 0 \Rightarrow t_{31} = -(g_{31} t_{11} + g_{32} t_{21}) / g_{33}$

$\vdots$

$$\boxed{g_{n1} t_{11} + \cdots + g_{nn} t_{n1} = 0 \Rightarrow t_{n1} = -\sum_{k=1}^{n-1} g_{nk} t_{k1} / g_{nn}}$$

② $G\begin{pmatrix} t_{12} \\ \vdots \\ t_{n2} \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ \vdots \end{pmatrix}$

$g_{11} t_{12} = 0 \Rightarrow t_{12} = 0$

$\cancel{g_{21} t_{12}} + g_{22} t_{22} = 1 \Rightarrow t_{22} = 1/g_{22}$

$\cancel{g_{31} t_{12}} + g_{32} t_{22} + g_{33} t_{32} = 0 \Rightarrow t_{32} = -g_{32} t_{22} / g_{33}$

$\vdots$

$$\boxed{\cancel{g_{n1} t_{12}} + g_{n2} t_{22} + \cdots + g_{nn} t_{n2} = 0 \Rightarrow t_{n2} = -\sum_{k=2}^{n-1} g_{nk} t_{k2} / g_{nn}}$$

T의 j번째 컬럼에 대해서 일반화

$$\begin{cases} t_{1j} = t_{2j} = \cdots = t_{j-1,j} = 0 \\ t_{jj} = 1/g_{jj} \\ t_{ij} = -\sum_{k=j}^{i-1} g_{ik} t_{kj} / g_{ii} \end{cases}$$

$(i > j)$

T 역시 삼각행렬임.
(lower triangular)