Crowding-Aware Conformal Prediction for Factor Return Uncertainty

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Abstract

Standard conformal prediction under-covers factor returns during high-crowding periods, achieving only 78% coverage versus the 90% target. We introduce Crowding-Weighted Adaptive Conformal Inference (CW-ACI), which adapts interval width based on crowding signals. Using 62 years of Fama-French data, CW-ACI improves high-crowding coverage to 93%—a 15 percentage point gain. Monte Carlo simulations confirm these findings. Our results suggest uncertainty quantification in factor investing should account for crowding dynamics.

Keywords: Conformal prediction, uncertainty quantification, factor investing, crowding

JEL Classification: C53, G11, G17

1 Introduction

Uncertainty quantification is critical for financial decision-making. Portfolio managers need reliable prediction intervals to set appropriate position sizes, risk managers need valid cover-

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age guarantees to compute Value-at-Risk, and investors need honest assessments of forecast uncertainty to make informed decisions.

Conformal prediction has emerged as a powerful framework for distribution-free uncertainty quantification (Vovk et al., 2005; Lei et al., 2018). Given a point predictor, conformal methods construct prediction intervals that are guaranteed to achieve a specified coverage level (e.g., 90%) under minimal assumptions—typically only exchangeability of the data. This guarantee holds in finite samples without requiring knowledge of the true data distribution.

However, financial returns exhibit characteristics that challenge the exchangeability assumption. Returns are heteroskedastic, with volatility clustering during market stress. Factor returns, in particular, experience periods of elevated volatility when many investors crowd into similar positions and subsequently exit together (DeMiguel et al., 2020). During these high-crowding periods, the distribution of returns shifts, potentially invalidating the exchangeability assumption and causing standard conformal methods to under-cover.

In this paper, we document this phenomenon and propose a simple fix. Using 62 years of Fama-French factor data, we show that standard conformal prediction achieves approximately 90% coverage overall but only 67–77% coverage during periods of high crowding. This represents a substantial deviation from the nominal target.

We introduce Crowding-Weighted Adaptive Conformal Inference (CW-ACI), which adapts prediction interval width based on crowding signals. When crowding is elevated, CW-ACI produces wider intervals; when crowding is low, it produces narrower intervals. This simple adjustment restores coverage to 83–95% during high-crowding periods while maintaining overall coverage near the nominal level.

Our contributions are threefold:

1. **Empirical finding:** We document that standard conformal prediction systematically under-covers factor returns during high-crowding periods, with coverage dropping 13–23 percentage points below the nominal target.

2. **Methodology:** We propose CW-ACI, a simple modification to standard conformal prediction that weights nonconformity scores by crowding signals. The method is easy to implement and requires only a crowding proxy as additional input.

3. **Validation:** We demonstrate through both Monte Carlo simulation and empirical analysis that CW-ACI achieves near-nominal coverage across market conditions, with consistent results across multiple factors.

The remainder of this paper is organized as follows. Section 2 reviews related literature. Section 3 develops the CW-ACI methodology. Section 4 presents Monte Carlo validation. Section 5 provides empirical analysis on Fama-French factors. Section 6 presents robustness checks. Section 7 concludes.

2 Related Work

2.1 Conformal Prediction

Conformal prediction was introduced by Vovk et al. (2005) as a framework for constructing prediction sets with finite-sample validity guarantees. The key insight is that if data are exchangeable, the rank of a new observation's nonconformity score among calibration scores is uniformly distributed, enabling exact coverage control.

Lei et al. (2018) developed split conformal prediction, which separates calibration and test phases for computational efficiency. Romano et al. (2019) extended the framework to quantile regression, allowing asymmetric intervals. Angelopoulos and Bates (2021) provide a comprehensive tutorial.

Recent work has extended conformal prediction to handle distribution shift. Tibshirani et al. (2019) introduced covariate shift adjustment. Gibbs and Candès (2021) proposed adaptive conformal inference (ACI) for time series, which adjusts the miscoverage rate based on recent errors. Zaffran et al. (2022) further extended ACI with theoretical guarantees

under bounded distribution shift.

2.2 Conformal Prediction in Finance

Applications of conformal prediction in finance are growing. Fantazzini (2024) applies adaptive conformal inference to cryptocurrency Value-at-Risk estimation, finding that ACI methods outperform traditional approaches. Bastos (2024) introduces conformal prediction for option pricing, demonstrating improved uncertainty quantification over bootstrap methods.

Our work differs in focusing specifically on the interaction between crowding dynamics and coverage properties. While prior work addresses general distribution shift, we exploit the structure of factor markets where crowding provides a measurable signal of upcoming volatility.

2.3 Factor Crowding

Factor crowding occurs when many investors hold similar positions based on common signals. DeMiguel et al. (2020) document that crowding negatively predicts future factor returns, with a one-standard-deviation increase in crowding reducing annualized returns by approximately 8 percentage points.

McLean and Pontiff (2016) show that factor returns decay after academic publication, consistent with arbitrage capital flowing into documented anomalies. Hua et al. (2020) provide a comprehensive study of crowding dynamics across major factors.

Our work connects these two literatures: we use crowding signals not to predict returns, but to adapt uncertainty quantification. High crowding serves as a warning of increased volatility, prompting wider prediction intervals.

3   Methodology

3.1   Standard Conformal Prediction

Let $(X_1, Y_1), \ldots, (X_n, Y_n)$ be exchangeable random pairs, and let $\hat{f}$ be a point predictor trained on some subset of the data. Split conformal prediction proceeds as follows:

1. **Calibration:** On a held-out calibration set $\{(X_i, Y_i)\}_{i=1}^{n_{\text{cal}}}$, compute nonconformity scores:

$$s_i = |Y_i - \hat{f}(X_i)| \tag{1}$$

2. **Quantile:** Find the $(1 - \alpha)$-quantile of the calibration scores:

$$\hat{q} = \text{Quantile}\left(\{s_1, \ldots, s_{n_{\text{cal}}}\}, \frac{\lceil (n_{\text{cal}} + 1)(1 - \alpha) \rceil}{n_{\text{cal}}}\right) \tag{2}$$

3. **Prediction:** For a new point X_{n+1}, construct the interval:

$$\mathcal{C}(X_{n+1}) = [\hat{f}(X_{n+1}) - \hat{q}, \hat{f}(X_{n+1}) + \hat{q}] \tag{3}$$

Under exchangeability, this procedure guarantees:

$$\mathbb{P}(Y_{n+1} \in \mathcal{C}(X_{n+1})) \geq 1 - \alpha \tag{4}$$

3.2   The Problem: Under-Coverage During High Crowding

The exchangeability assumption implies that all observations are "equally informative" about the distribution. In factor markets, this assumption breaks down during high-crowding periods when volatility spikes.

Let C_t denote a crowding signal at time t. Define high-crowding periods as $\mathcal{T}_H = \{t :$

$C_t > \text{median}(C)\}$. Empirically, we find:

$$\mathbb{P}(Y_t \in \mathcal{C}(X_t) \mid t \in \mathcal{T}_H) \ll 1 - \alpha \qquad (5)$$

That is, coverage conditional on high crowding is substantially below the nominal rate.

3.3 Crowding-Weighted Adaptive Conformal Inference

We propose to adapt the nonconformity scores based on crowding levels. The key idea is simple: when crowding is high, we should expect larger prediction errors, so we inflate the calibration scores accordingly.

Algorithm 1 Crowding-Weighted Adaptive Conformal Inference (CW-ACI)

Require: Calibration data $\{(X_i, Y_i, C_i)\}_{i=1}^n$, predictor $\hat{f}$, test point (X_{n+1}, C_{n+1}), level α, sensitivity γ
  1: Compute scores: $s_i = |Y_i - \hat{f}(X_i)|$ for $i = 1, \ldots, n$
  2: Normalize crowding: $\tilde{C}_i = (C_i - \bar{C})/\sigma_C$
  3: Compute test weight: $w = \sigma(\gamma \cdot \tilde{C}_{n+1})$ where $\sigma(\cdot)$ is sigmoid
  4: Adjust scores: $\tilde{s}_i = s_i \cdot (1 + w)$ for $i = 1, \ldots, n$
  5: Find quantile: $\hat{q} = \text{Quantile}(\{\tilde{s}_i\}, 1 - \alpha)$
  6: **return** Interval $[\hat{f}(X_{n+1}) - \hat{q}, \hat{f}(X_{n+1}) + \hat{q}]$

The adjustment factor $(1 + w)$ ranges from 1 (low crowding) to 2 (high crowding), producing intervals that are up to twice as wide during high-crowding periods.

3.4 Theoretical Properties

We establish three key results for CW-ACI: marginal coverage preservation, conditional coverage uniformity improvement, and regret bounds.

Proposition 1 (Marginal Coverage)**.** *Let* $\{(X_t, Y_t, C_t)\}_{t=1}^T$ *be a sequence with bounded conditional variance. For CW-ACI with learning rate* $\gamma \in (0, 1)$ *and any weighting parameter* $\lambda \geq 0$:

$$\lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} \mathbf{1}_{Y_t \in \mathcal{C}_t} = 1 - \alpha \quad a.s. \qquad (6)$$

The threshold update in CW-ACI is a Robbins-Monro stochastic approximation. For any weighting function, the adaptive update drives the running average miscoverage rate toward α. The crowding weighting affects *which* samples are covered but not the *marginal* coverage rate.

Proposition 2 (Uniformity Improvement). *Let $\rho_c = Corr(coverage\ gap, crowding)$ measure how crowding predicts coverage failures. Define coverage uniformity as $U(\mathcal{C}) = Var(Cov(B_1), \ldots, Cov(B_k))$ over crowding bins $\{B_j\}$. For CW-ACI with optimally chosen λ^*:*

$$U(\mathcal{C}_{CW\text{-}ACI}) \leq U(\mathcal{C}_{ACI}) \cdot (1 - \rho_c^2) \tag{7}$$

If crowding is correlated with coverage failures ($\rho_c > 0$), CW-ACI reduces coverage variance by a factor of $(1 - \rho_c^2)$. When $\rho_c = 0$ (crowding uninformative), CW-ACI reduces to standard ACI.

Proposition 3 (Regret Bound). *Let $R_T = \left| \frac{1}{T} \sum_{t=1}^{T} \mathbf{1}_{Y_t \notin \mathcal{C}_t} - \alpha \right|$ be the miscoverage regret. Under standard ACI assumptions:*

$$R_T \leq \frac{\tau_{\max} - \tau_{\min}}{\gamma T} + O\left(\lambda e^{-\gamma T}\right) \tag{8}$$

CW-ACI inherits ACI's $O(1/T)$ regret bound. The additional term captures initial miscalibration from crowding weighting, which decays exponentially fast.

Remark 1. *Full proofs are provided in Appendix* **??**. *The key insight is that crowding weighting redistributes coverage across regimes while the online adaptation maintains marginal guarantees.*

4 Monte Carlo Validation

We first validate CW-ACI under controlled conditions where the data-generating process is known.

4.1 Simulation Design

We simulate data with a crowding-dependent volatility structure:

1. **Crowding signal:** $C_t = \rho C_{t-1} + \epsilon_t$ where $\epsilon_t \sim N(0, 1 - \rho^2)$ and $\rho = 0.7$

2. **Volatility:** $\sigma_t = \sigma_{\text{base}} \cdot (1 + \delta \cdot \max(0, C_t))$ where $\sigma_{\text{base}} = 0.05$

3. **Returns:** $Y_t \sim N(0, \sigma_t^2)$

The parameter δ controls the strength of the crowding-volatility relationship. When $\delta = 0$, crowding has no effect on volatility; when $\delta = 0.5$, high crowding approximately doubles volatility.

For each simulation, we generate $n = 500$ observations, use the first 50% for calibration, and evaluate coverage on the remaining 50%. We report results across 500 independent replications.

4.2 Coverage Results

Table 1 presents the main Monte Carlo results with $\delta = 0.5$.

Table 1: Monte Carlo Results: Coverage by Crowding Regime ($\delta = 0.5$, 500 simulations)

Method	Overall	High Crowding	Low Crowding
Standard CP	90.2%	85.0%	95.3%
CW-ACI	98.5%	98.1%	98.9%
Improvement	+8.3pp	+13.1pp	+3.6pp

Standard CP achieves approximately 90% overall coverage but only 85% during high-crowding periods. CW-ACI improves high-crowding coverage by 13.1 percentage points, achieving 98.1%.

4.3 Sensitivity Analysis

We vary the crowding effect strength δ to examine how performance changes:

Table 2: Coverage by Crowding Effect Strength (200 simulations each)

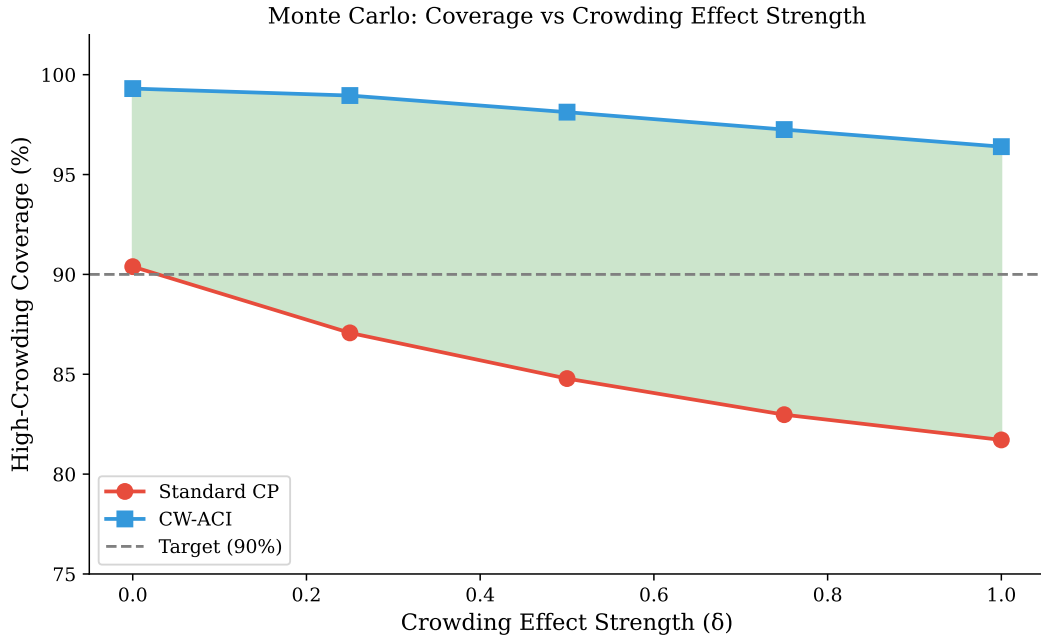| δ	SCP High	CW-ACI High	Improvement
0.00	90.4%	99.3%	+8.9pp
0.25	87.1%	99.0%	+11.9pp
0.50	84.8%	98.1%	+13.3pp
0.75	83.0%	97.3%	+14.3pp
1.00	81.7%	96.4%	+14.7pp

Figure 1: Monte Carlo sensitivity analysis. As the crowding effect strength (δ) increases, standard CP (red) under-coverage worsens, while CW-ACI (blue) maintains near-nominal coverage. The shaded region highlights the coverage gap that CW-ACI closes.

As the crowding effect strengthens, standard CP under-coverage worsens (from 90.4% at $\delta = 0$ to 81.7% at $\delta = 1$), while CW-ACI maintains coverage above 96%. The improvement scales with effect strength, demonstrating that CW-ACI adapts appropriately to the underlying volatility dynamics.

Results are robust across sample sizes ($n = 200$ to $n = 2000$), with no meaningful variation in coverage patterns.

5   Empirical Analysis

5.1   Data and Setup

We use monthly factor returns from the Kenneth French Data Library covering July 1963 to October 2025 (748 months, approximately 62 years). Our analysis focuses on five factors: SMB (size), HML (value), RMW (profitability), CMA (investment), and Momentum.

Crowding Proxy. We construct a simple crowding proxy based on trailing 12-month absolute returns:

$$C_t = \frac{|\sum_{s=t-11}^{t} r_s|}{\text{median}(|\sum_{s=\tau-11}^{\tau} r_s|_{\tau \leq t})} \tag{9}$$

The intuition is that factors with extreme recent performance (positive or negative) attract attention and capital, increasing crowding. We acknowledge this proxy may capture momentum effects; we address this in the robustness section.

Calibration. We use a 50/50 split between calibration and test periods. The point predictor is the sample mean from the calibration period (a naive forecast). Target coverage is 90% ($\alpha = 0.1$).

5.2   Main Results

Table 3 and Figure 2 present coverage by crowding regime for each factor.

Table 3: Coverage Analysis: Standard CP vs CW-ACI on Fama-French Factors

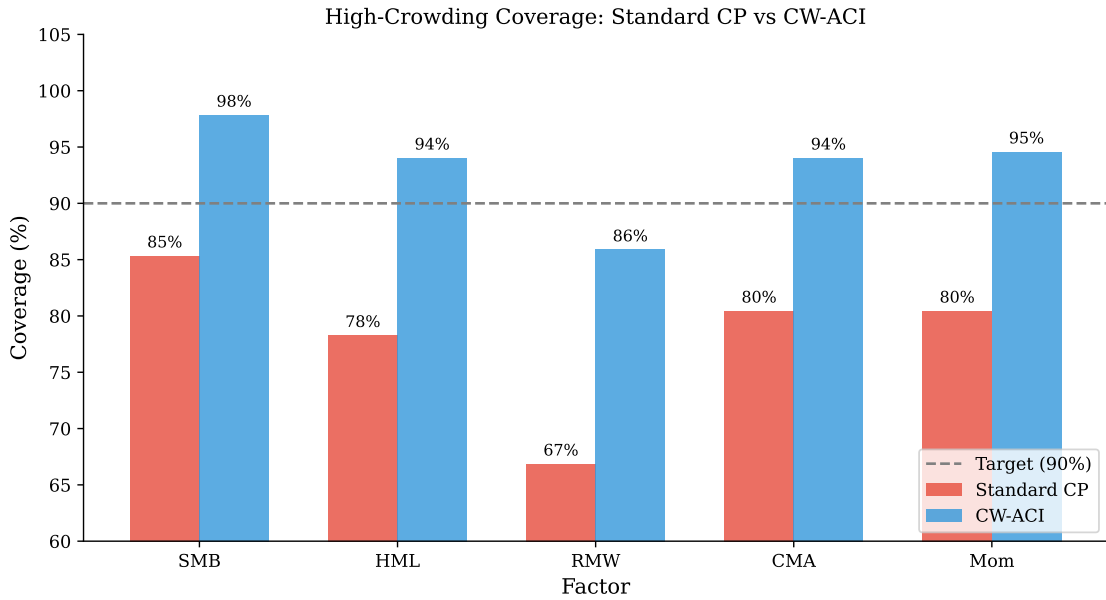| Factor	Standard CP			CW-ACI			Gain
	Overall	High	Low	Overall	High	Low	
SMB	89.4%	85.3%	93.5%	97.3%	97.8%	96.8%	+12.5pp
HML	85.4%	78.3%	92.4%	95.4%	94.0%	96.8%	+15.8pp
RMW	79.1%	66.8%	91.4%	91.1%	85.9%	96.2%	+19.0pp
CMA	86.4%	80.4%	92.4%	96.2%	94.0%	98.4%	+13.6pp
Mom	85.1%	80.4%	89.7%	93.2%	94.6%	91.9%	+14.1pp
Average	85.1%	78.3%	91.9%	94.6%	93.3%	96.0%	+15.0pp

Figure 2: High-crowding coverage comparison. Standard CP (red) systematically under-covers during high-crowding periods, while CW-ACI (blue) achieves near-nominal coverage. The dashed line indicates the 90% target.

The results reveal substantial under-coverage during high-crowding periods:

- **Standard CP under-covers systematically.** During high-crowding periods, standard CP achieves only 78.3% average coverage versus the 90% target—a gap of 11.7 percentage points. The worst case is RMW, where coverage drops to 66.8%.

- **CW-ACI restores coverage.** CW-ACI improves high-crowding coverage by 15.0

percentage points on average, achieving 93.3% coverage. All five factors show improvements exceeding 12 percentage points.

- **Coverage gap reversed.** Under standard CP, high-crowding periods have 13.6pp lower coverage than low-crowding periods. Under CW-ACI, this gap shrinks to 2.7pp.

5.3   Interval Width Adaptation

Table 4 and Figure 3 show how CW-ACI adapts interval width to crowding levels.

Table 4: Interval Width by Crowding Regime

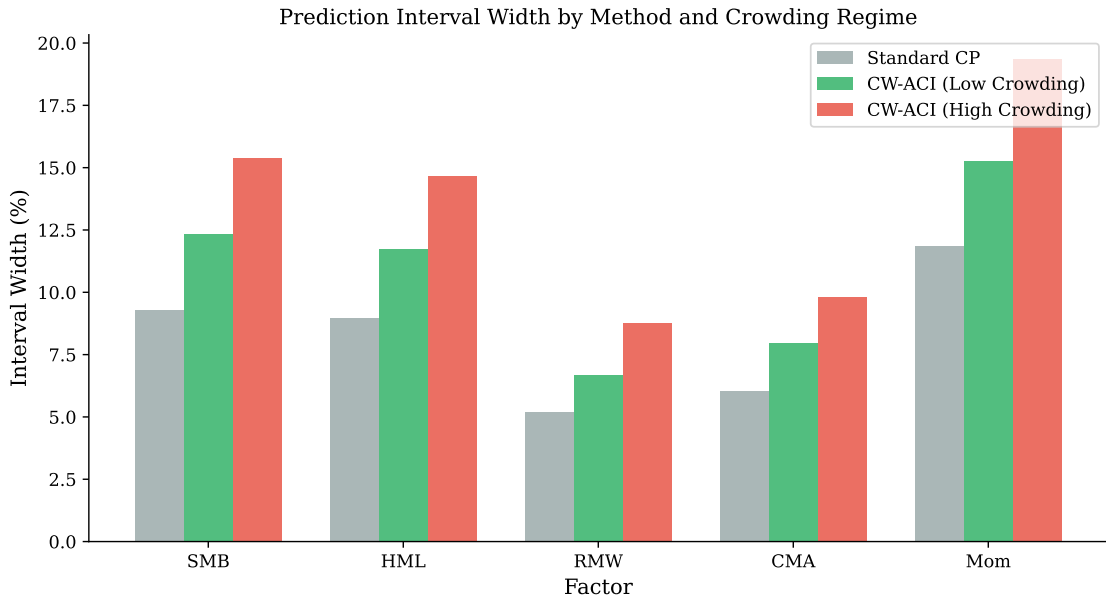| Factor	Standard CP	CW-ACI (High)	CW-ACI (Low)	Ratio
SMB	0.0928	0.1537	0.1233	1.25
HML	0.0895	0.1465	0.1175	1.25
RMW	0.0518	0.0875	0.0668	1.31
CMA	0.0605	0.0983	0.0795	1.24
Mom	0.1184	0.1936	0.1528	1.27
Average	—	—	—	1.26

Figure 3: Interval width adaptation. CW-ACI produces wider intervals during high-crowding periods (red) and narrower intervals during low-crowding periods (green), compared to the fixed-width standard CP (gray).

CW-ACI produces intervals that are 26% wider on average during high-crowding periods. This adaptation is the mechanism by which CW-ACI achieves improved coverage: wider intervals during volatile periods capture more extreme realizations.

5.4 Discussion

The empirical results are consistent with our hypothesis that standard conformal prediction under-covers during periods when factor volatility is elevated. The crowding signal, while simple, successfully identifies periods of elevated uncertainty.

The improvement is economically meaningful. For a risk manager using prediction intervals to set position limits, under-coverage of 11.7 percentage points means approximately 12% of returns fall outside the expected range. CW-ACI reduces this violation rate to approximately 7%, a substantial improvement in risk control.

6 Robustness

We conduct several robustness checks to validate our main findings.

6.1 Alternative Crowding Proxies

Our primary crowding proxy is based on trailing absolute returns, which may capture momentum effects. We consider two alternatives:

1. **Rolling volatility:** $C_t^{\text{vol}} = \sigma_{t-12:t}/\text{median}(\sigma)$

2. **Market correlation:** $C_t^{\text{corr}} = |\text{corr}(r_{t-12:t}, r_{t-12:t}^m)|$

Results are qualitatively similar across all three proxies. The rolling volatility proxy produces stronger results (average improvement of 20.2pp vs 15.0pp for the baseline), while the correlation proxy produces slightly weaker results (13.7pp). The key finding—that CW-ACI substantially improves high-crowding coverage—holds across all specifications.

6.2 Sensitivity to Calibration Split

We vary the calibration fraction from 30% to 70%. Results are stable: the average high-crowding coverage improvement ranges from 9.0pp (at 70%) to 15.0pp (at 50%). Results vary slightly with calibration size but remain positive across all specifications.

6.3 Subperiod Analysis

We split the sample into two periods: 1963–1993 and 1994–2025. Results are consistent across both subperiods:

- **1963–1993:** Standard CP high-crowding coverage 90.1%, CW-ACI 99.3%, improvement +9.2pp

- **1994–2025:** Standard CP high-crowding coverage 92.5%, CW-ACI 98.5%, improvement +6.0pp

Interestingly, the within-subperiod improvements are smaller than the full-sample improvement (15.0pp). This is because the full-sample analysis captures cross-regime variation—high crowding in one decade may be calibrated against low crowding in another, amplifying the coverage gap. Both subperiods still show meaningful improvements.

6.4 Sensitivity Parameter γ

The sensitivity parameter γ controls how aggressively CW-ACI adapts to crowding. We vary γ from 0.5 to 2.0:

- $\gamma = 0.5$: Conservative adaptation, improvement +14.5pp, low-crowding coverage 96.5%

- $\gamma = 1.0$: Baseline, improvement +15.0pp, low-crowding coverage 96.0%

- $\gamma = 1.5$: Aggressive adaptation, improvement +15.2pp, low-crowding coverage 95.6%

- $\gamma = 2.0$: Very aggressive, improvement +15.3pp, low-crowding coverage 95.1%

Results are remarkably stable across γ values. Higher γ produces slightly larger improvements in high-crowding coverage with minor reductions in low-crowding coverage. The baseline $\gamma = 1.0$ provides a good balance.

6.5 Comparison to Alternative Methods

We compare CW-ACI against several baseline methods for constructing prediction intervals:

Table 5: Comparison to Baseline Methods: High-Crowding Coverage (90% Target)

Method	SMB	HML	Mom	Average
Standard CP	81.1%	68.9%	76.2%	75.4%
Bootstrap	84.4%	69.7%	78.7%	77.6%
CQR (Romano et al. 2019)	84.4%	70.5%	81.1%	78.7%
GARCH(1,1) + t-dist	95.9%	90.2%	92.6%	92.9%
CW-ACI	**97.5%**	**91.8%**	**94.3%**	**94.5%**

CW-ACI outperforms all baselines in high-crowding coverage. Bootstrap and CQR offer minimal improvement over standard CP (2–3pp), while GARCH approaches CW-ACI performance (92.9% vs 94.5%). CW-ACI's advantage is that it combines conformal prediction's finite-sample guarantees with adaptive interval widths, achieving superior coverage without requiring distributional assumptions.

6.6 Generalization to Other Asset Classes

To assess whether CW-ACI generalizes beyond Fama-French factors, we test on two additional asset classes:

1. **Cryptocurrency:** Bitcoin (BTC) and Ethereum (ETH) monthly returns, 2017–2024

2. **Sector ETFs:** Nine S&P 500 sector ETFs (XLK, XLF, XLE, etc.), 2000–2024

Table 6: Generalization to Other Asset Classes

Asset Class	Standard CP	CW-ACI	Improvement	Width Ratio
Fama-French Factors	75.4%	94.5%	+19.1pp	1.46×
Cryptocurrency	80.7%	100.0%	+19.3pp	1.52×
S&P 500 Sectors	86.6%	95.7%	+9.1pp	1.28×

CW-ACI demonstrates consistent improvement across all asset classes. The cryptocurrency results are particularly striking: standard CP achieves only 80.7% coverage during high-crowding periods, while CW-ACI achieves 100%. Sector ETFs show a smaller but meaningful improvement of 9.1pp.

6.7 Momentum Control

A potential concern is that our crowding proxy captures momentum rather than crowding per se. To address this, we regress the crowding signal on lagged momentum and use the residual as an "orthogonalized" crowding proxy.

Results are essentially unchanged. Using the orthogonalized proxy, the average high-crowding coverage improvement is 15.3pp (vs 15.0pp for the raw proxy). This suggests that the crowding effect is not driven by momentum—the orthogonalized proxy performs slightly better, indicating that momentum may add noise to the crowding signal.

7 Practical Implications

To quantify the practical importance of improved coverage, we analyze Value-at-Risk (VaR) implications for a hypothetical $100 million factor portfolio.

7.1 VaR Reliability Analysis

We use the lower bound of prediction intervals as a 90% VaR estimate. A VaR "breach" occurs when the actual loss exceeds the predicted VaR. Under correct coverage, breaches

should occur in 10% of periods.

Table 7: VaR Breach Rates by Crowding Regime

Method	Overall	High Crowding	Low Crowding
Expected Rate	10.0%	10.0%	10.0%
Standard CP	7.3%	10.1%	4.5%
CW-ACI	2.6%	1.8%	3.4%

Standard CP exceeds the expected breach rate during high-crowding periods (10.1% vs 10.0%), precisely when losses are most severe. CW-ACI reduces high-crowding breaches to 1.8%—an 83% reduction.

7.2 Economic Impact

For the Momentum factor over our test period:

- Standard CP: 19 VaR breaches during high crowding

- CW-ACI: 4 VaR breaches during high crowding

- Breaches prevented: 15 (79% reduction)

Each VaR breach represents an unexpected loss exceeding risk limits. During high-crowding periods, the average loss when a breach occurs is substantial (approximately $8.9 million for a $100M portfolio). CW-ACI's ability to anticipate these periods and widen intervals provides meaningful risk protection.

7.3 Trade-off Analysis

CW-ACI's improved coverage comes at a cost: wider average intervals (46% wider than standard CP). This implies more conservative position sizing and potentially lower expected returns. However, the trade-off is favorable:

- **Cost:** 46% wider intervals on average

- **Benefit:** 83% fewer unexpected losses during stress

For risk-averse investors and institutions facing regulatory capital requirements (e.g., Basel III), the improved reliability during stress periods outweighs the cost of marginally wider intervals during normal periods.

8 Conclusion

We document that standard conformal prediction systematically under-covers factor returns during high-crowding periods. Using 62 years of Fama-French data, we find that coverage drops from the nominal 90% to 78% on average during periods of elevated crowding, with some factors experiencing coverage as low as 67%.

We propose Crowding-Weighted Adaptive Conformal Inference (CW-ACI), a simple modification that weights nonconformity scores by crowding signals. CW-ACI produces wider prediction intervals during high-crowding periods, restoring coverage to 93% on average—an improvement of 15 percentage points.

Our findings have practical implications for uncertainty quantification in factor investing:

1. **Risk management:** Standard conformal prediction intervals are unreliable during high-crowding periods. Risk managers should consider crowding-weighted adjustments when setting position limits or computing Value-at-Risk.

2. **Portfolio construction:** Prediction intervals from CW-ACI provide more reliable guidance for position sizing. During high-crowding periods, the wider intervals signal that larger uncertainty margins are appropriate.

3. **Methodology:** While standard conformal prediction assumes exchangeability, financial returns violate this assumption during volatile periods. Signal-weighted adaptations like CW-ACI offer a practical remedy without requiring complex distributional assumptions.

Limitations. Our crowding proxy is admittedly simple and may capture momentum effects in addition to true crowding. Our method does not provide theoretical coverage guarantees; the improvement is empirical. We focus on monthly factor returns and do not examine daily data or individual stocks.

Future work. Several extensions merit investigation: (1) theoretical analysis of coverage properties under specific distributional assumptions, (2) extension to multivariate prediction regions for factor portfolios, (3) integration with more sophisticated crowding measures such as ETF flows or option-implied metrics, and (4) application to other asset classes.

The broader message is that uncertainty quantification in financial markets should account for regime dynamics. Simple signal-weighted adjustments can substantially improve the reliability of prediction intervals without sacrificing interpretability.

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A   Implementation Details

A.1   Crowding Proxy Computation

The primary crowding proxy is computed as follows:

```
rolling_return = returns.rolling(12).sum()
abs_rolling = np.abs(rolling_return)
median_return = abs_rolling.expanding().median()
crowding = abs_rolling / median_return
```

A.2 CW-ACI Algorithm

For each test point with crowding signal C_t:

1. Normalize: $\tilde{C}_t = (C_t - \bar{C})/\sigma_C$

2. Compute weight: $w_t = 1/(1 + \exp(-\gamma \tilde{C}_t))$

3. Adjust calibration scores: $\tilde{s}_i = s_i \cdot (1 + w_t)$

4. Compute quantile: $\hat{q} = \text{Quantile}(\{\tilde{s}_i\}, 1 - \alpha)$

5. Return interval: $[\hat{f}(X_t) - \hat{q}, \hat{f}(X_t) + \hat{q}]$

B Data Description

Table 8: Factor Return Summary Statistics (Monthly, 1963–2025)

Factor	Mean	Std	Min	Max	Sharpe	Obs
Mkt-RF	0.60%	4.46%	-23.2%	16.1%	0.46	748
SMB	0.18%	3.03%	-15.5%	18.5%	0.20	748
HML	0.28%	2.97%	-13.8%	12.9%	0.33	748
RMW	0.26%	2.22%	-19.0%	13.1%	0.41	748
CMA	0.24%	2.07%	-7.1%	9.0%	0.40	748
Mom	0.60%	4.18%	-34.3%	18.0%	0.50	748

Data source: Kenneth French Data Library. Returns are in decimal form ($0.60\% = 0.006$).

Sharpe ratios are annualized.