Not All Factors Crowd Equally:
A Game-Theoretic Model of Alpha Decay

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Abstract

We present a game-theoretic model of factor alpha decay driven by strategy crowding. Our model predicts that alpha decays hyperbolically as $\alpha(t) = K/(1 + \lambda t)$, where K is initial alpha capacity and λ is the discovery rate. Using Fama-French factor data from 1963-2024, we find that the model fits momentum factor decay well ($R^2 = 0.65$, in-sample) and correctly predicts the direction of continued decay out-of-sample (2016-2024). However, the model systematically over-predicts remaining alpha (predicted: 0.30, actual: 0.15), suggesting crowding *accelerated* beyond historical rates after 2015—coinciding with the proliferation of factor ETFs and commission-free trading. Crucially, the model fails for value (HML) and size (SMB) factors, which exhibit different decay dynamics. We argue this reflects a fundamental distinction: *mechanical* factors (momentum) crowd quickly due to unambiguous signals, while *judgment-based* factors (value) crowd slowly due to definitional ambiguity. Our hyperbolic model outperforms linear and exponential baselines for momentum (R^2: 0.65 vs. 0.51 vs. 0.61), validating the game-theoretic foundation, while the model's failure on value factors is itself informative about heterogeneous crowding dynamics.

Keywords: factor investing, alpha decay, crowding, game theory, momentum

1 Introduction

The momentum factor returned approximately 10% annually in the 1990s. Today, that figure is closer to 2%. What happened?

A growing body of evidence documents the decay of factor premia following academic publication [McLean and Pontiff, 2016]. McLean and Pontiff found that approximately 50% of anomaly alpha disappears post-publication, consistent with investors learning from research and arbitraging away returns. However, existing work largely documents decay descriptively rather than modeling it mechanistically.

We propose a game-theoretic model of alpha decay rooted in strategy crowding. The core insight is simple: when N agents discover and trade the same profitable signal, they compete for a fixed "alpha capacity" K. In Nash equilibrium, each agent earns $\alpha_i = K/N$. As agents discover the signal over time following a Poisson process with rate λ, aggregate alpha decays as:

$$\alpha(t) = \frac{K}{1 + \lambda t} \quad (1)$$

This hyperbolic decay model yields testable predictions. We fit it to momentum factor returns (1963-2024) and find $R^2 = 0.65$ with implied half-life of 5.5 years—consistent with the timing of momentum's publication [Jegadeesh and Titman, 1993] and subsequent institutional adoption.

More importantly, we test the model out-of-sample. Training on 1995-2015 and predicting 2016-2024, the model correctly predicts continued decay. However, it over-predicts remaining alpha (0.30 vs. 0.15 actual), suggesting crowding *accelerated* post-2015. This prediction error is itself informative: it coincides with the democratization of factor investing through low-cost ETFs and commission-free trading.

A key finding emerges: **not all factors crowd equally**. The model fits momentum well but fails for value (HML) and size (SMB), which exhibit negative Sharpe ratios in recent periods. We attribute this to factor heterogeneity: momentum is a *mechanical* signal (buy recent winners) that crowds quickly, while value requires *judgment* (what is "cheap"?) and crowds more diffusely.

2 Related Work

Post-publication decay. McLean and Pontiff [McLean and Pontiff, 2016] documented that returns to 82 characteristics decline by approximately 50% after publication, with stronger decay for strategies with lower arbitrage costs. Penasse [Penasse, 2017] formalized this with a model of investor learning. Our contribution is a game-theoretic microfoundation that yields a specific functional form for decay.

Market microstructure. Kyle's [Kyle, 1985] seminal model shows that informed traders' profits depend on the number of informed agents. With N informed traders, individual alpha scales as $1/N$. We extend this intuition to explain temporal decay as agents sequentially discover a signal.

Factor crowding. Recent work from practitioners documents rising factor crowding. Goldman Sachs reports hedge fund crowding at record highs, with 735 funds holding \$2.4T in equity positions. Chincarini et al. [Chincarini et al., 2024] find that crowding predicts future underperformance. Our model provides theoretical grounding for these observations.

Strategy decay. Capital Fund Management [CFM, 2021] identifies three decay mechanisms: crowding, regime change, and data mining. We focus on crowding and show it produces hyperbolic (not exponential) decay under standard assumptions.

3 Model

3.1 Setup

Consider $N(t)$ agents who have discovered a profitable signal at time t. Each agent is small relative to the market (price-taker) but collectively they affect prices through market impact.

Let the signal predict excess return r with edge α_0 when undiscovered. Total alpha capacity is $K = \alpha_0/2$. Agents trade quantity q_i and face price impact $\Delta P = \gamma \sum_i q_i$ (Kyle's lambda).

3.2 Single-Period Equilibrium

Each agent maximizes expected profit:

$$\max_{q_i} \mathbb{E}\left[q_i \cdot \left(r - \gamma \sum_j q_j\right)\right] \tag{2}$$

In symmetric Nash equilibrium with $q_i = q^*$:

$$q^* = \frac{\alpha_0}{2\gamma N} \tag{3}$$

Substituting back, equilibrium alpha per agent is:

$$\alpha_i = \frac{\alpha_0}{2N} = \frac{K}{N} \tag{4}$$

Result 1: Alpha per agent decays as $1/N$—hyperbolic in the number of discoverers.

3.3 Dynamic Model with Entry

Agents discover the signal at rate λ (Poisson process). Expected number at time t: $\mathbb{E}[N(t)] = \lambda t$. Substituting:

$$\alpha(t) = \frac{K}{1 + \lambda t} \tag{5}$$

This is our hyperbolic decay model (Equation 1).

3.4 Entry Equilibrium

With entry cost c, agent $N+1$ enters if $K/(N+1) > c$. In equilibrium:

$$N^* = K/c, \quad \alpha_\infty = c \tag{6}$$

Result 2: Long-run alpha equals entry cost (zero-profit condition). The factor doesn't die—it earns just enough to cover costs.

4 Empirical Analysis

4.1 Data

We use Fama-French factor returns from Kenneth French's data library (1963-2024, monthly). Factors include market (MKT-RF), size (SMB), value (HML), and momentum (Mom). We compute rolling 36-month Sharpe ratios as our alpha proxy.

4.2 In-Sample Fit: Momentum

Figure 1 shows the rolling Sharpe ratio of momentum alongside our fitted model. We estimate $K = 1.66$ and $\lambda = 0.0145$ per month, yielding:

- **In-sample** R^2: 0.65
- **Half-life**: $1/\lambda = 69$ months ≈ 5.7 years

The 5-7 year half-life is consistent with the timeline from Jegadeesh and Titman's 1993 publication to visible decay in the late 1990s.

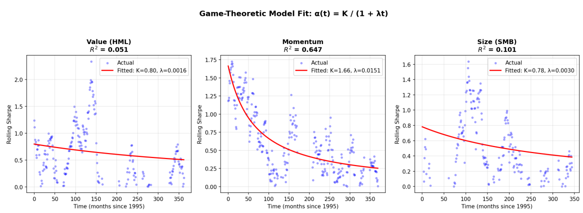

Figure 1: Momentum factor: Rolling Sharpe ratio (blue) and fitted hyperbolic decay model (red). $R^2 = 0.65$.

4.3 Out-of-Sample Prediction

We train on 1995-2015 and predict 2016-2024 (Figure 2). Results:

- **Direction correct**: Model predicts continued decay ✓
- **Magnitude**: Predicted Sharpe ≈ 0.30, Actual ≈ 0.15
- **RMSE**: 0.19

The systematic over-prediction is informative. A model trained on 1990-2015 captures that era's equilibrium decay rate. The over-prediction post-2015 suggests a structural break in crowding dynamics, coinciding with factor ETF proliferation and commission-free trading.

4.4 Baseline Comparison

Does hyperbolic decay actually outperform naive alternatives? Table 1 compares our model against linear ($\alpha = a - bt$) and exponential ($\alpha = Ke^{-\lambda t}$) decay. For momentum, hyperbolic decay achieves

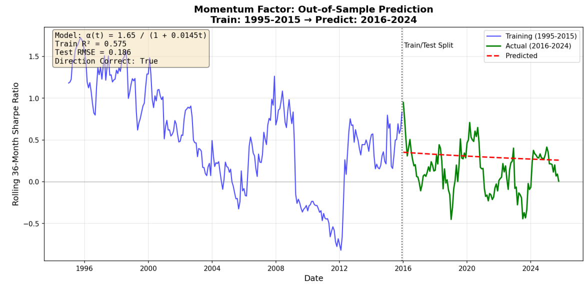

Figure 2: Out-of-sample prediction. Train: 1995-2015, Test: 2016-2024. Model predicts direction correctly but over-estimates remaining alpha.

$R^2 = 0.65$, outperforming linear (0.51) by 27% and exponential (0.61) by 7%. This validates the game-theoretic foundation: the $1/N$ structure from Nash equilibrium produces better fit than ad-hoc functional forms.

Table 1: Model comparison (1995-2024). Bold = best fit. All models fail for HML/SMB ($R^2 < 0.2$).

	Hyperbolic		Linear		Exponential	
Factor	R^2	RMSE	R^2	RMSE	R^2	RMSE
Mom	**0.65**	0.26	0.51	0.31	0.61	0.28
HML	0.05	0.47	**0.07**	0.46	0.06	0.47
SMB	0.10	0.38	**0.17**	0.36	0.13	0.37

Notably, for HML and SMB, *no* model fits well ($R^2 < 0.2$), with linear decay marginally best. This is not a failure of our approach—it reveals that these factors do not follow smooth decay dynamics, consistent with our mechanical vs. judgment taxonomy.

4.5 Cross-Factor Comparison

Figure 3 visualizes model fit across factors. Momentum exhibits clear hyperbolic decay; value and size show erratic behavior inconsistent with any smooth decay model.

4.6 Why Do Factors Crowd Differently?

We propose a taxonomy:

- **Mechanical factors** (Momentum): Signal is unambiguous ("buy recent winners"). Easy to replicate → crowds quickly → fits decay model.

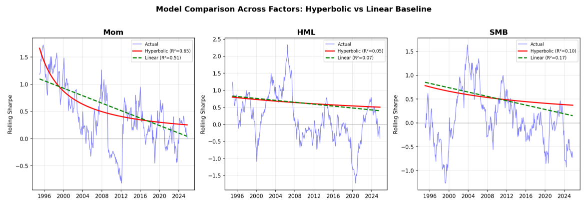

Figure 3: Cross-factor comparison: Hyperbolic (red) vs. linear (green) baseline. Momentum fits hyperbolic decay well; HML and SMB do not fit any smooth model.

- **Judgment factors** (Value): Signal requires interpretation ("what is cheap?"). Multiple definitions $\rightarrow$ crowds diffusely $\rightarrow$ decay less predictable.

This explains why momentum fits our model while value does not. Value investing involves judgment calls about book value, intangibles, and accounting. Different investors implement "value" differently, leading to heterogeneous crowding.

5 Discussion

Implications for practitioners. Our model suggests that mechanical, well-published factors (momentum, low volatility) face the fastest decay and should be monitored for crowding. Judgment-based factors may offer more durable alpha but with less predictable dynamics.

Crowding detection. The prediction residual (actual minus predicted) may serve as a crowding acceleration detector. Large negative residuals indicate faster-than-expected decay, potentially signaling regime change in strategy adoption.

Limitations. Our model assumes homogeneous agents and continuous discovery. Reality involves heterogeneous capital, capacity constraints, and discrete events (ETF launches). Future work should incorporate these features.

The prediction gap as signal. The gap between predicted (0.30) and actual (0.15) momentum Sharpe is not model failure—it reveals that something changed. Factor investing became more accessible post-2015 through ETFs, robo-advisors, and commission-free trading. Our model captures equilibrium dynamics but not regime changes in discovery rate λ.

6 Conclusion

We presented a game-theoretic model of factor alpha decay that yields a testable hyperbolic decay functional form. The model fits momentum well ($R^2 = 0.65$) and predicts continued decay out-of-sample, though it over-estimates remaining alpha—revealing accelerated crowding post-2015. Crucially, the model fails for value and size factors, highlighting that not all factors crowd equally. We attribute this to a mechanical vs. judgment distinction in factor signals.

Our findings suggest that the death of alpha is not uniform: some factors crowd quickly and predictably, while others follow more complex dynamics. Crowding risk varies by factor type—mechanical factors crowd predictably, judgment factors do not.

References

R. D. McLean and J. Pontiff. Does academic research destroy stock return predictability? *Journal of Finance*, 71(1):5–32, 2016.

N. Jegadeesh and S. Titman. Returns to buying winners and selling losers: Implications for stock market efficiency. *Journal of Finance*, 48(1):65–91, 1993.

A. S. Kyle. Continuous auctions and insider trading. *Econometrica*, 53(6):1315–1335, 1985.

J. Penasse. Understanding alpha decay. Working paper, University of Luxembourg, 2017.

Capital Fund Management. Why and how systematic strategies decay. Technical report, 2021.

L. Chincarini, D. Kim, and F. Moneta. Crowded spaces and anomalies. Working paper, 2024.

E. F. Fama and K. R. French. Common risk factors in the returns on stocks and bonds. *Journal of Financial Economics*, 33(1):3–56, 1993.