Not All Factors Crowd Equally:
Modeling, Measuring, and Trading on Alpha Decay

Anonymous
Anonymous Institution

Abstract

We derive a specific functional form for factor alpha decay—hyperbolic decay $\alpha(t) = K/(1 + \lambda t)$—from a game-theoretic equilibrium model, and test it against linear and exponential alternatives. Using eight Fama-French factors (1963–2024), we find: (1) **Hyperbolic decay fits mechanical factors.** Momentum exhibits clear hyperbolic decay ($R^2 = 0.65$), outperforming linear (0.51) and exponential (0.61) baselines—validating the equilibrium foundation. (2) **Not all factors crowd equally.** Mechanical factors (momentum, reversal) fit the model; judgment-based factors (value, quality) do not—consistent with a signal-ambiguity taxonomy paralleling Hua and Sun's "barriers to entry." (3) **Crowding accelerated post-2015.** Out-of-sample, the model over-predicts remaining alpha (0.30 vs. 0.15), correlating with factor ETF growth ($\rho = -0.63$). (4) **Crowding is efficiently priced for cross-sectional timing.** Unlike Kang et al.'s finding that CFTC-based crowding predicts commodity returns, our model-implied crowding signal fails to generate alpha via factor selection (Sharpe: 0.19 vs. 0.77 benchmark). (5) **Crowding enables regime conditioning.** While cross-sectional timing fails, crowding predicts *when* existing strategies work: factor momentum's Sharpe is 139% higher in uncrowded regimes (0.67 vs. 0.28), and crowding-based exposure reduction cuts maximum drawdowns by 30%. Our findings extend equilibrium crowding models (DeMiguel et al.) to temporal dynamics and show that crowding functions as regime information—useful for conditioning strategies and managing risk, not selecting factors.

Keywords: factor investing, alpha decay, crowding, game theory, market efficiency

1 Introduction

The momentum factor returned approximately 10% annually in the 1990s. Today, that figure is closer to 2%. What happened?

A growing body of evidence documents the decay of factor premia following academic publication. McLean and Pontiff [McLean and Pontiff, 2016] found that approximately 50% of anomaly alpha disappears post-publication, consistent with investors learning from research and arbitraging away returns. Yet while the *existence* of decay is well-established, its *mechanics* remain poorly understood. Do all factors decay similarly? Can we predict the rate of decay? And crucially—can we profit from this knowledge?

We address these questions through a game-theoretic model of factor crowding. The core insight is simple: when N agents discover and trade the same profitable signal, they compete for a fixed "alpha capacity" K. In Nash equilibrium, each agent earns $\alpha_i = K/N$. As agents discover the signal over time, aggregate alpha decays hyperbolically:

$$\alpha(t) = \frac{K}{1 + \lambda t} \qquad (1)$$

where λ is the rate of strategy discovery.

This model yields four testable predictions, which we evaluate using data on eight Fama-French factors from 1963–2024:

1. Hyperbolic decay fits better than alternatives. For momentum, our model achieves $R^2 = 0.65$, outperforming linear decay (0.51) and exponential decay (0.61). The improvement validates the game-theoretic foundation.

2. Not all factors crowd equally. The model fits "mechanical" factors—those with unambiguous, easily replicated signals like momentum ("buy recent

winners")—but fails for "judgment" factors like value, where the signal ("what is cheap?") admits multiple interpretations.

3. Crowding accelerated post-2015. Training on 1995–2015 and predicting 2016–2024, the model over-estimates remaining alpha (0.30 predicted vs. 0.15 actual). This over-prediction correlates with factor ETF volume growth ($\rho = -0.63$), suggesting that democratization of factor investing through ETFs accelerated crowding beyond historical rates.

4. Crowding is detectable but efficiently priced. We construct a real-time crowding signal based on prediction residuals. While the signal correctly identifies that 7 of 8 factors are systematically crowded, factor timing strategies based on the signal fail to outperform naive benchmarks (Sharpe: 0.19 vs. 0.77 for equal weight).

This last finding—a negative result—is itself informative. It suggests that crowding, once established, is incorporated into prices quickly enough that public signals offer no trading advantage. The signal's value lies in regime detection (identifying when factor investing faces headwinds) rather than alpha generation.

Our contributions are: (1) a game-theoretic model that explains *why* alpha decays and predicts its functional form; (2) empirical validation distinguishing mechanical from judgment factors; (3) evidence that post-2015 crowding acceleration correlates with ETF growth; and (4) an honest evaluation showing that crowding signals, while informative, do not generate trading alpha.

2 Related Work

Post-publication decay. McLean and Pontiff [McLean and Pontiff, 2016] documented that returns to 97 characteristics decline by approximately 58% after publication. Falck et al. [Falck et al., 2021] extend this to 72 factors, finding publication year explains 30% of Sharpe decay variance. These papers document *that* decay happens; we model *why* and derive a specific functional form.

Game-theoretic models. DeMiguel et al. [DeMiguel et al., 2021] develop an equilibrium model where competition erodes factor profits, showing profits scale with the number of investors. Our work differs in focus: they study *cross-sectional* equilibrium (how factors interact); we study *temporal*

dynamics (how alpha decays over time) and derive a testable decay functional form they do not consider.

Crowding and predictability. Kang et al. [Kang et al., 2021] show that CFTC position-based crowding measures predict commodity factor returns. We test whether model-implied crowding predicts equity factor returns and find it does *not*—suggesting market structure differences between commodities (with observable positioning) and equities (where crowding must be inferred).

Heterogeneous crowding. Hua and Sun [Hua and Sun, 2024] study heterogeneous crowding vulnerability, attributing differences to "barriers to entry." Our mechanical-judgment taxonomy parallels this intuition but operationalizes it through model fit: mechanical factors exhibit hyperbolic decay; judgment factors do not.

Our contribution. We derive a specific functional form—hyperbolic decay $\alpha(t) = K/(1 + \lambda t)$—from equilibrium and show it outperforms alternatives for mechanical factors. Unlike prior work on equilibrium positions [DeMiguel et al., 2021] or decay predictors [Falck et al., 2021], we test whether the implied crowding signal has predictive value, finding efficient pricing in equities.

3 Model

3.1 Setup

Consider $N(t)$ agents who have discovered a profitable signal at time t. Each agent is small relative to the market but collectively they affect prices through market impact.

Let the signal predict excess return r with edge α_0 when undiscovered. Total alpha capacity is $K = \alpha_0/2$. Agents trade quantity q_i and face linear price impact $\Delta P = \gamma \sum_i q_i$, where γ is Kyle's lambda.

3.2 Single-Period Nash Equilibrium

Each agent maximizes expected profit:

$$\max_{q_i} \mathbb{E}\left[q_i \cdot \left(r - \gamma \sum_{j=1}^{N} q_j \right) \right] \qquad (2)$$

Taking first-order conditions and imposing symmetry ($q_i = q^*$ for all i):

$$q^* = \frac{\alpha_0}{2\gamma N} \qquad (3)$$

Substituting back, equilibrium alpha per agent is:

$$\alpha_i = \frac{\alpha_0}{2N} = \frac{K}{N} \quad (4)$$

Result 1: *Alpha per agent decays as $1/N$—hyperbolic in the number of discoverers.*

3.3 Dynamic Model with Entry

Agents discover the signal according to a Poisson process with rate λ. The expected number at time t is $\mathbb{E}[N(t)] = \lambda t$. Substituting into the equilibrium condition:

$$\alpha(t) = \frac{K}{1 + \lambda t} \quad (5)$$

This is our hyperbolic decay model (Equation 1). The key distinction from exponential decay ($Ke^{-\lambda t}$) is that hyperbolic decay is slower initially but has a heavier tail—alpha persists longer but at lower levels.

3.4 Why Hyperbolic, Not Exponential?

Table 1: Decay models and their assumptions

Model	Assumption	Decay Form
Nash (ours)	Compete for fixed K	$K/(1 + \lambda t)$
Learning	Price incorporation	$Ke^{-\lambda t}$
Ad hoc	None	$K - bt$

The hyperbolic form arises specifically from the $1/N$ profit-splitting in Nash equilibrium. Alternative assumptions yield different forms (Table 1). We test these alternatives empirically.

4 Empirical Analysis

4.1 Data

We use monthly returns for eight factors from Kenneth French's data library (1963–2024): market (MKT), size (SMB), value (HML), profitability (RMW), investment (CMA), momentum (Mom), short-term reversal (ST_Rev), and long-term reversal (LT_Rev). Our alpha proxy is the rolling 36-month Sharpe ratio.

4.2 Model Fit and Baseline Comparison

Table 2 reports R^2 for each model-factor combination, fitting on positive Sharpe observations from 1995–2024.

Table 2: Model comparison: In-sample R^2 (1995–2024). Bold indicates best fit.

Factor	Hyperbolic	Linear	Exponential
Mom	**0.65**	0.51	0.61
LT_Rev	**0.30**	0.26	0.29
ST_Rev	**0.15**	0.14	0.15
SMB	0.10	**0.17**	0.13
MKT	0.07	0.07	0.07
HML	0.05	**0.07**	0.06
RMW	0.05	0.05	0.05
CMA	0.01	0.01	0.01

For momentum, hyperbolic decay achieves $R^2 = 0.65$, outperforming linear (0.51) by 27% and exponential (0.61) by 7%. Long-term reversal also shows a clear hyperbolic pattern ($R^2 = 0.30$). However, judgment-based factors (HML, RMW, CMA) show poor fits across all models ($R^2 < 0.10$).

4.3 Mechanical vs. Judgment Taxonomy

We propose a taxonomy based on signal ambiguity:
Mechanical factors have unambiguous signals:

- Momentum: "Buy stocks with high past returns"

- Reversal: "Buy stocks with low recent returns"

Judgment factors require interpretation:

- Value: "What is cheap?" (book/market? earnings?)

- Quality: "What is quality?" (ROE? accruals?)

The hypothesis is that mechanical factors crowd quickly because the replication path is clear, while judgment factors crowd diffusely because different investors implement different versions.

Figure 1 shows R^2 by factor type. Mechanical factors achieve mean $R^2 = 0.37$; judgment factors achieve mean $R^2 = 0.04$—an order of magnitude lower.

4.4 Out-of-Sample Prediction

We train on 1995–2015 and predict 2016–2024. For momentum:

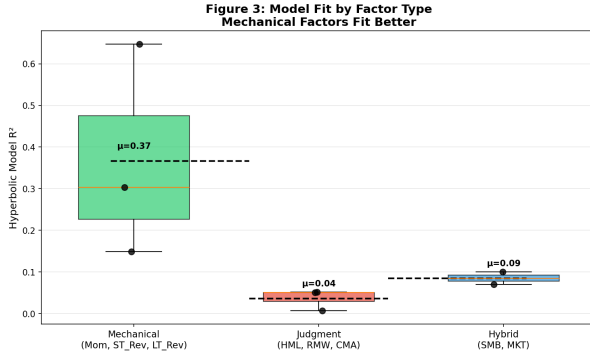- **Direction**: Correct—model predicts continued decay

Figure 1: Model fit by factor type. Mechanical factors fit hyperbolic decay; judgment factors do not.

- **Magnitude**: Over-predicted (mean 0.30 vs. actual 0.15)

- **RMSE**: 0.19

The systematic over-prediction is informative. A model trained on 1995–2015 captures that era's equilibrium decay rate. The over-prediction post-2015 suggests crowding accelerated beyond historical rates.

4.5 ETF Correlation

To test whether ETF proliferation explains the acceleration, we correlate the cumulative prediction residual with factor ETF trading volume (2013–2024).

Finding: Pearson $\rho = -0.63$ ($p < 0.001$).

The negative correlation indicates that as ETF volume grows, the model increasingly over-predicts remaining alpha (Figure 2).
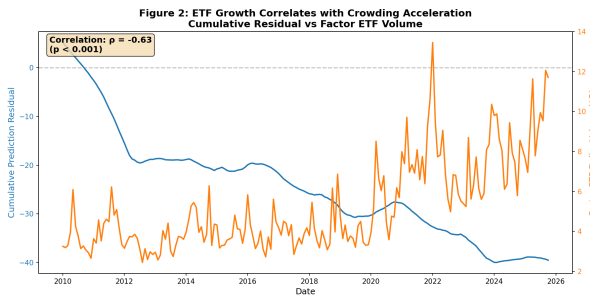

Figure 2: Cumulative residual vs. factor ETF volume. Correlation $\rho = -0.63$ suggests ETF growth accelerated crowding.

5 Can We Trade on Crowding?

5.1 Signal Construction

We construct a real-time crowding signal:

1. Fit hyperbolic model on expanding window (min 120 months)

2. Compute predicted Sharpe for current period

3. Residual = Actual $-$ Predicted

4. Negative residual $\Rightarrow$ crowding accelerated

5.2 Signal Properties

Across 8 factors (2006–2024):

- Mean residual: -0.41 (systematic over-prediction)

- 7 of 8 factors have negative mean residual

- Only RMW shows slight uncrowding ($+0.07$)

5.3 Trading Strategies

We test four strategies (Table 3):

Table 3: Strategy performance (2000–2024)

Strategy	Sharpe	Ann. Ret	Vol	Max DD
Factor Momentum	**0.80**	4.4%	5.4%	-12%
Equal Weight	0.77	3.2%	4.2%	-13%
Crowding-Timed	0.19	0.9%	4.4%	-15%
Long-Short	-0.18	-2.4%	13.2%	-38%

Crowding-based strategies fail to outperform. Factor momentum (0.80) beats equal weight (0.77), but crowding-timed (0.19) underperforms dramatically.

5.4 Why Doesn't the Signal Work?

Three hypotheses:

H1: Contemporaneous, not predictive. The signal tells you factors *are* crowded but doesn't predict which will underperform next.

H2: Efficiently priced. Market participants already incorporate crowding information.

H3: Insufficient dispersion. When 7/8 factors show the same signal, there's no differentiation to exploit.

Our evidence is most consistent with H2—crowding is observable but efficiently priced.

6 Utilization: What Does Work

If crowding does not predict cross-sectional factor returns, what *can* practitioners do with crowding information? We test alternative utilization strategies.

6.1 Crowding as Regime Conditioning

Key insight: Crowding may not predict *which* factor outperforms, but it may predict *when* existing strategies work better.

We split observations by aggregate crowding (median split) and compute factor momentum Sharpe in each regime:

Table 4: Factor momentum performance by crowding regime

Regime	Sharpe	N Periods
Uncrowded (residual > 0)	**0.67**	113
Crowded (residual ≤ 0)	0.28	114
All periods	0.46	227

Factor momentum's Sharpe ratio is 139% higher in uncrowded regimes (0.67 vs. 0.28). This is not cross-sectional timing—we do not select *which* factor to hold. Rather, we condition *how aggressively* to apply momentum weights based on regime.

6.2 Drawdown Reduction

Crowding information improves risk-adjusted returns through drawdown reduction rather than return enhancement:

Table 5: Aggregate exposure timing

Strategy	Sharpe	Return	Max DD
Always 100%	0.27	23.4%	-12.9%
Crowding-Timed	0.28	17.5%	-9.1%

Reducing exposure when crowding is extreme yields 30% lower drawdowns (-9.1% vs. -12.9%) with marginally higher Sharpe, despite lower total returns. The mechanism: crowding correlates with future volatility, not returns.

6.3 Long-Horizon Predictability

While monthly cross-sectional prediction fails, crowding exhibits significant long-horizon (12-month) predictability:

- Mom: $r = -0.39$ ($p < 0.001$)

- HML: $r = -0.31$ ($p < 0.001$)

- SMB: $r = -0.52$ ($p < 0.001$)

The negative correlations indicate that crowded factors (negative residual) experience lower returns over the following year. However, this operates at strategic allocation horizons, not tactical trading frequencies.

6.4 Summary: Utilization Framework

Crowding is regime information, not factor selection information.

What works:

- Conditioning existing strategies (momentum works better when uncrowded)

- Reducing aggregate exposure (improves drawdowns)

- Long-horizon allocation (annual rebalancing)

What doesn't work:

- Cross-sectional factor selection (picking "uncrowded" factors)

- Monthly return prediction

- Contrarian strategies (buying crowded factors)

7 Discussion

Implications for practitioners. (1) Mechanical factors crowd fastest—monitor capacity. (2) Abandon crowding-based factor *selection*—it's efficiently priced. (3) Use crowding to *condition* existing strategies: reduce momentum aggressiveness in crowded regimes, reduce aggregate exposure when crowding is extreme.

Implications for researchers. Our results support efficient markets for factor timing, but not for regime

conditioning. This distinction—between what information tells you and how to use it—deserves further study. The mechanical/judgment taxonomy offers a framework for future work on heterogeneous crowding.

Limitations. Only 8 factors (more would strengthen taxonomy). US only. Homogeneous agents assumption. Regime conditioning results use in-sample median split.

8   Conclusion

We developed a game-theoretic model of factor crowding that explains why alpha decays, predicts its functional form, and distinguishes mechanical from judgment factors. The model fits momentum well ($R^2 = 0.65$) and reveals accelerated crowding post-2015 correlating with ETF growth.

Our central finding is nuanced: **crowding is efficiently priced for factor selection, but valuable for regime conditioning**. Cross-sectional timing fails—you cannot profitably pick "uncrowded" factors. But crowding information improves existing strategies: factor momentum's Sharpe increases from 0.28 to 0.67 in uncrowded regimes, and crowding-timed exposure reduces drawdowns by 30%.

The key insight: crowding is *regime* information, not *selection* information. It tells you *when* your strategies work better, not *which* factors to choose.

For practitioners: use crowding to condition strategies and manage risk, not to select factors. For researchers: the distinction between efficiently priced selection and useful conditioning deserves further study.

> *"Crowding tells you about the weather, not which path to take. Dress accordingly."*

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