Ex

Find time after starting where they will meet

Motion of A w.r.t B

$V_{AB} = V_A - V_B$

$= 10 - 5$

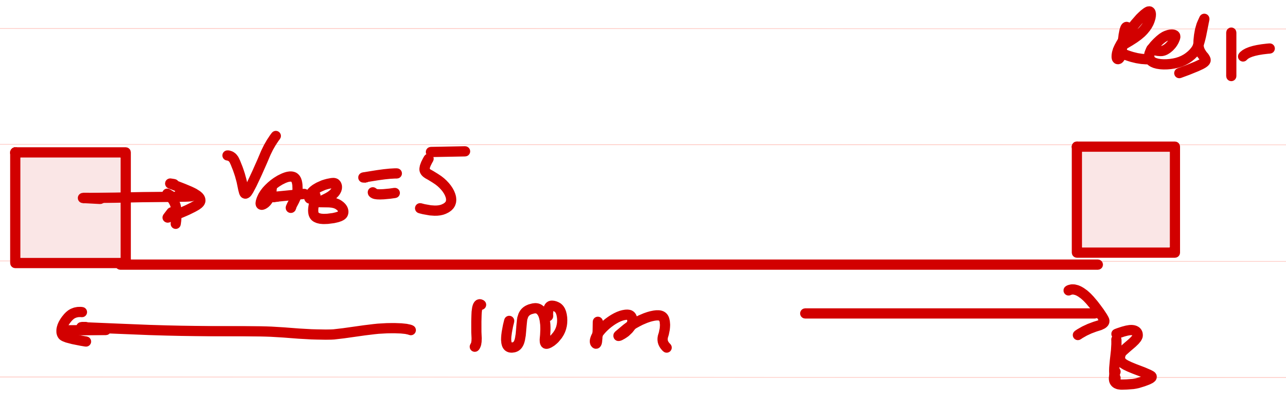
$= 5\ m/s$

$d_{AB} = 100m$

$a_{AB} = a_A - a_B$

$= 0 - 0$

$a_{AB} = 0$

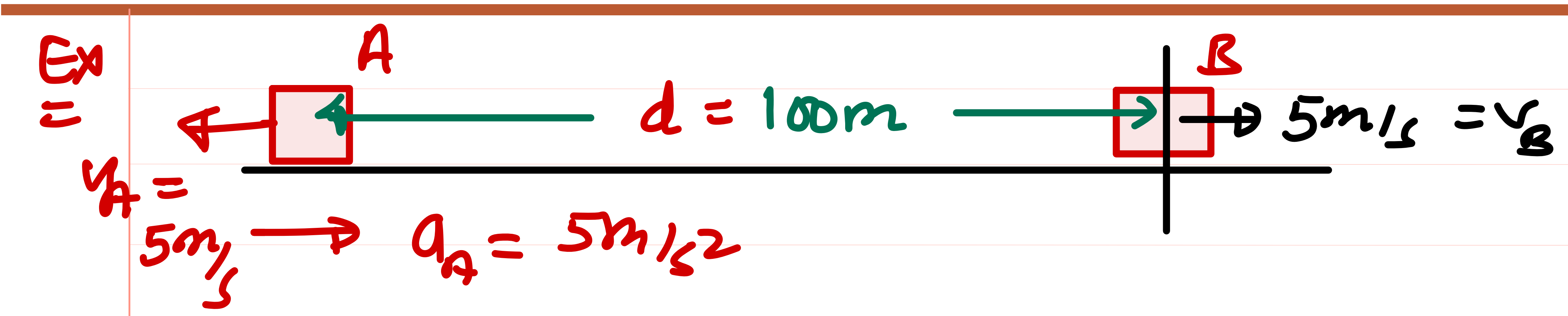


$d_{AB} = V_{AB} t + \frac{1}{2} a_{AB} t^2$

$100 = 5t + \frac{1}{2} \times 0\ t^2$

$\boxed{t = 20 sec}$

Ex



After what from starting A will catch B

Motion of A w.r.t B

$v_{AB} = v_A - v_B$

$= -5 - 5$

$v_{AB} = -10$ m/s

$a_{AB} = a_A - a_B$

$= 5 - 0$

$a_{AB} = 5$ m/s²

$d_{AB} = +100$ m

$$d_{AB} = u_{AB}t + \frac{1}{2}a_{AB}t^2$$

$$100 = -10t + \frac{1}{2}\times 5t^2$$

$$20 = -2t + \frac{t^2}{2}$$

$$40 = -4t + t^2$$

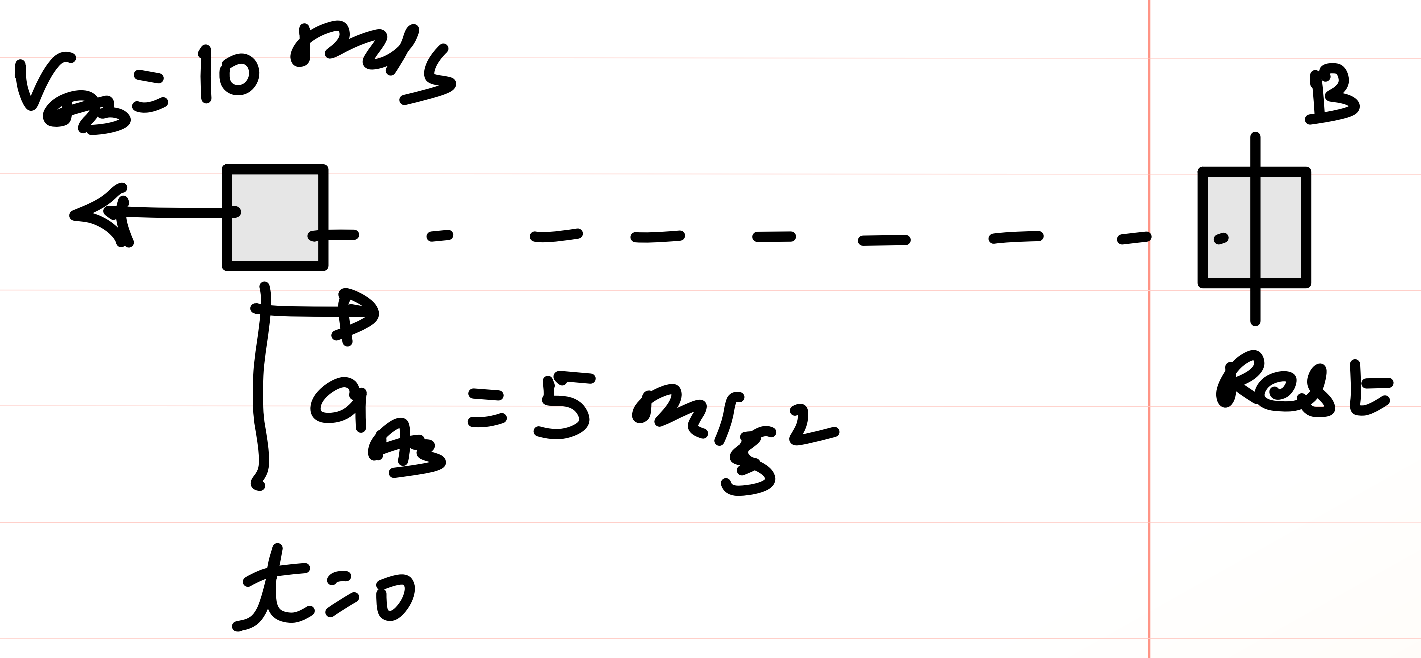
$$t^2 - 4t - 40 = 0$$

$$t = \frac{4 \pm \sqrt{4^2 + 4\times 40}}{2}$$

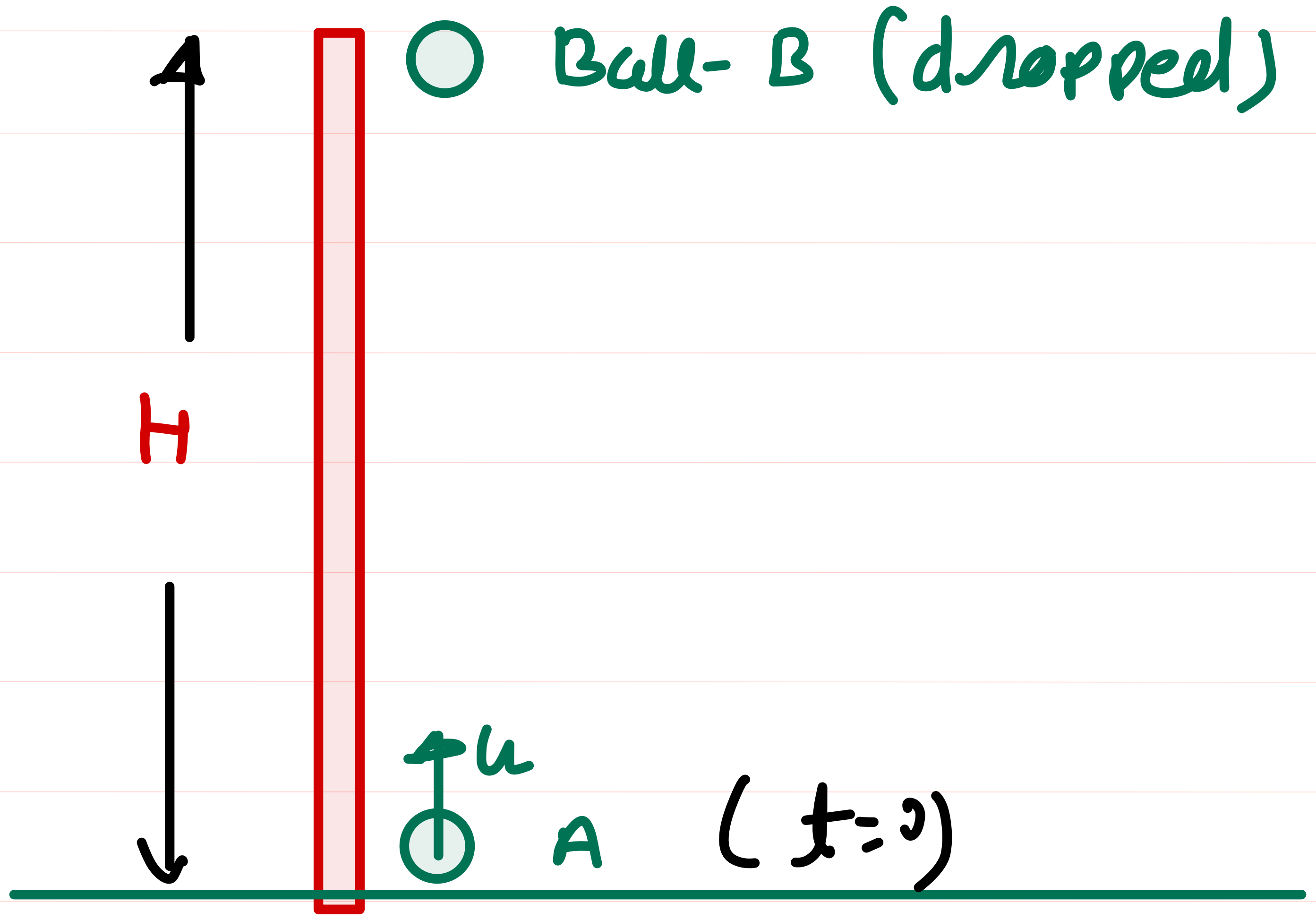
$$= \frac{4 \pm 4\sqrt{11}}{2}$$

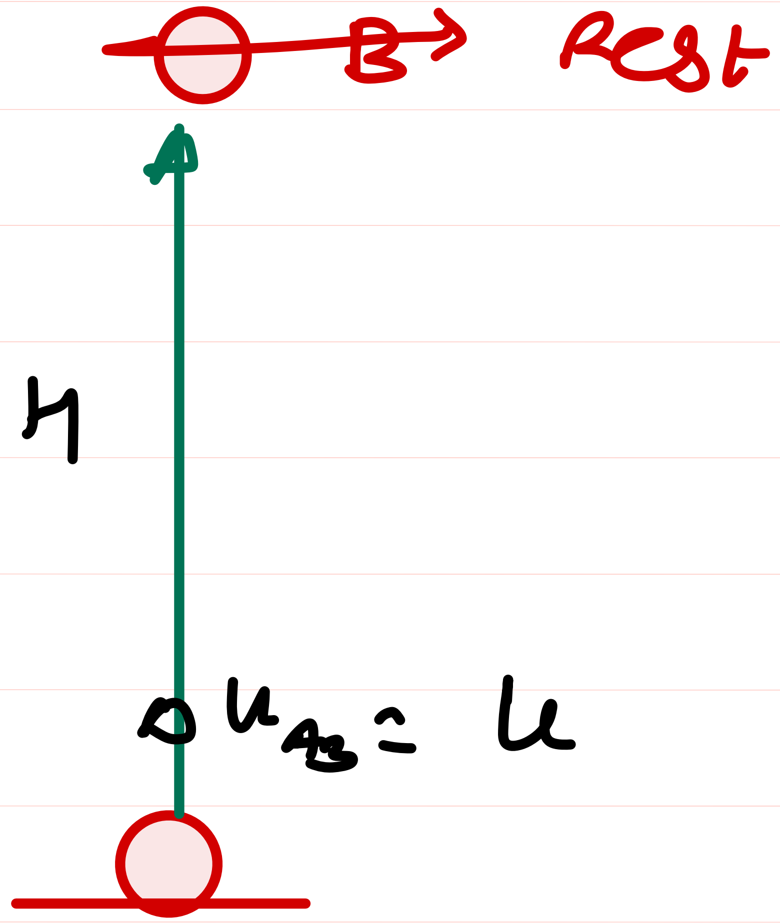
$$= 2 \pm 2\sqrt{11}$$

$$\boxed{t = 2(1+\sqrt{11})\text{ sec}}$$



Ex





$$d_{AB} = u_{AB}\, t + \frac{1}{2} a_{AB}\, t^2$$

$$H = u\, t + \frac{1}{2} \times 0\, t^2$$

$$\boxed{t = \frac{H}{u}}$$

After what time they will meet

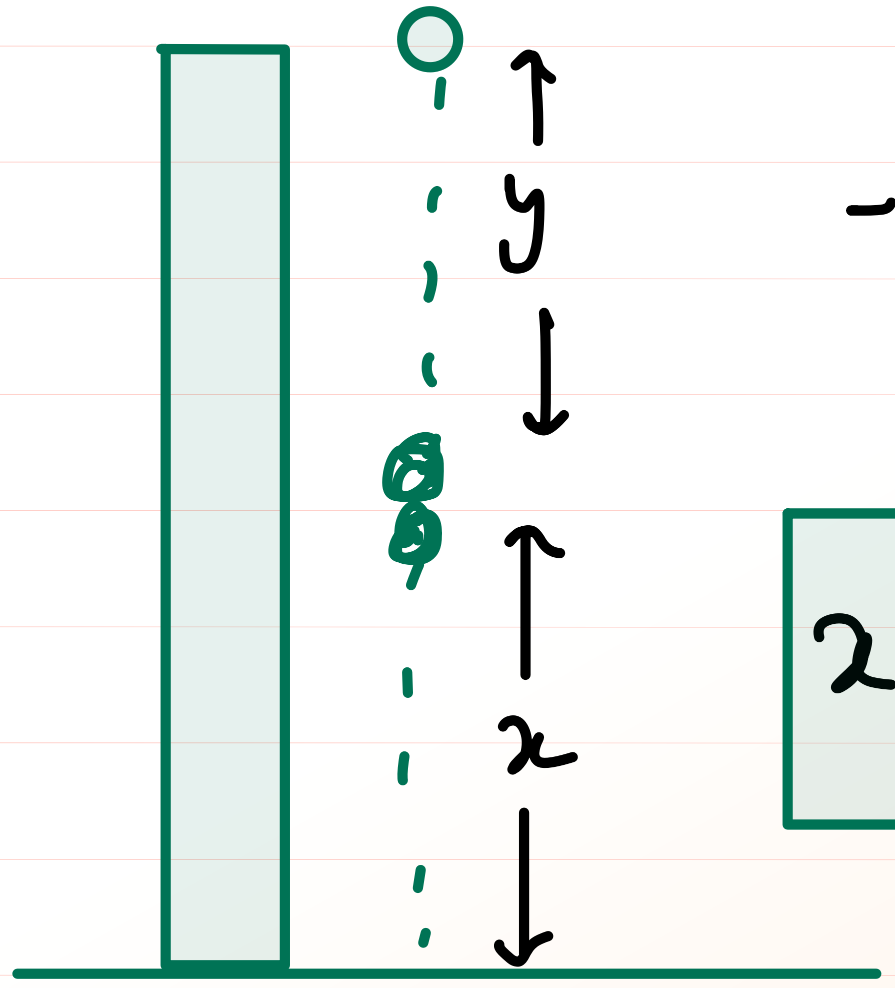
motion of A w.r.t

$V_{AB} = u - 0 = u$

$a_{AB} = (-g) - (-g) = 0$

$d_{AB} = H$

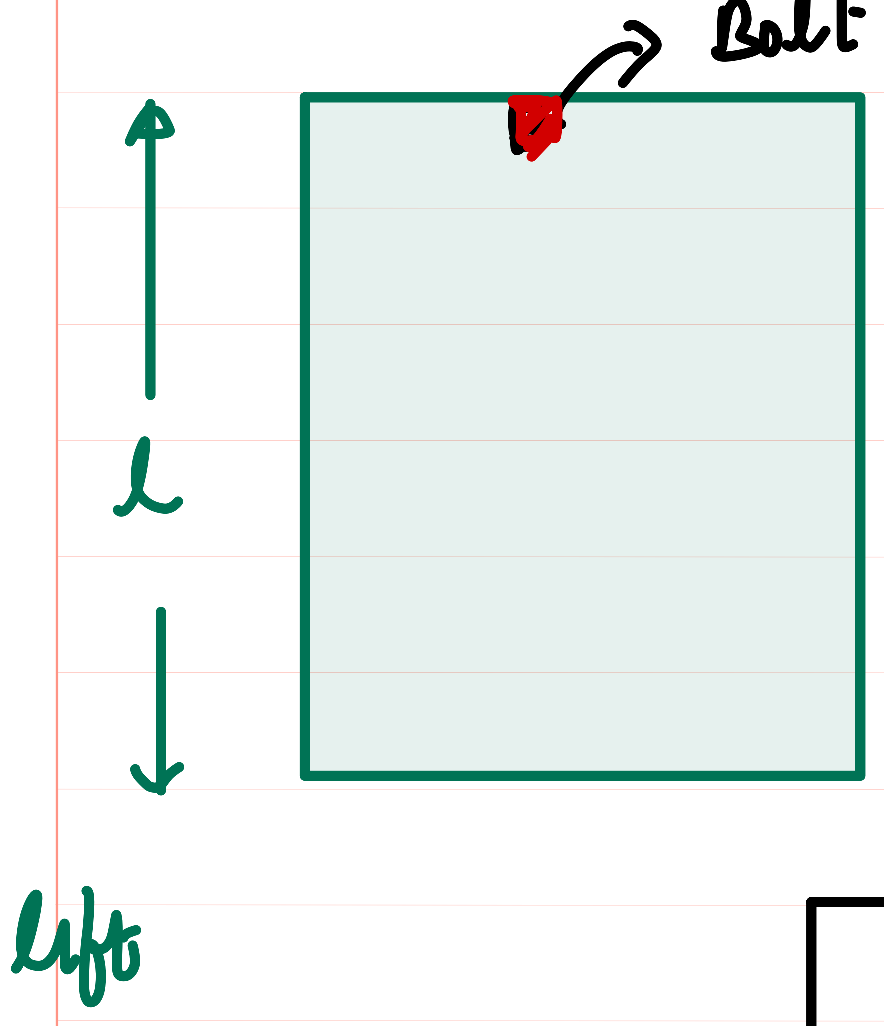
location from Ground where they meet



$$-y = 0\, t - \frac{1}{2} g t^2 \Rightarrow \boxed{y = \frac{g}{2}\left(\frac{H}{u}\right)^2}$$

$$\boxed{x = H - y = H - \frac{g}{2}\left(\frac{H}{u}\right)^2}$$

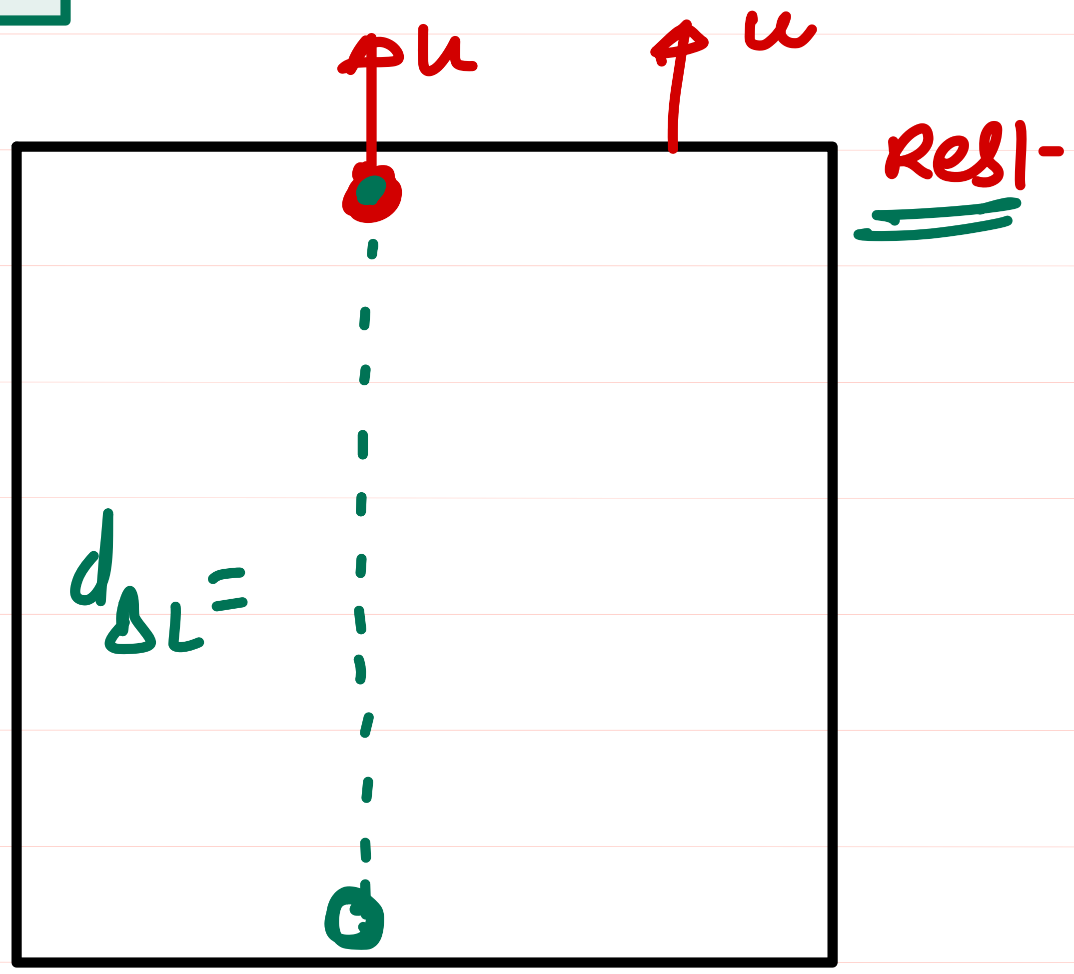
Ex



Bolt ↑ u moving up-ward

Bolt detached from lift Find time after with it Collides with floor of lift

We observe motion w.r.t lift



$V_{BL} = u - u = 0$

$a_{BL} = -g - 0$

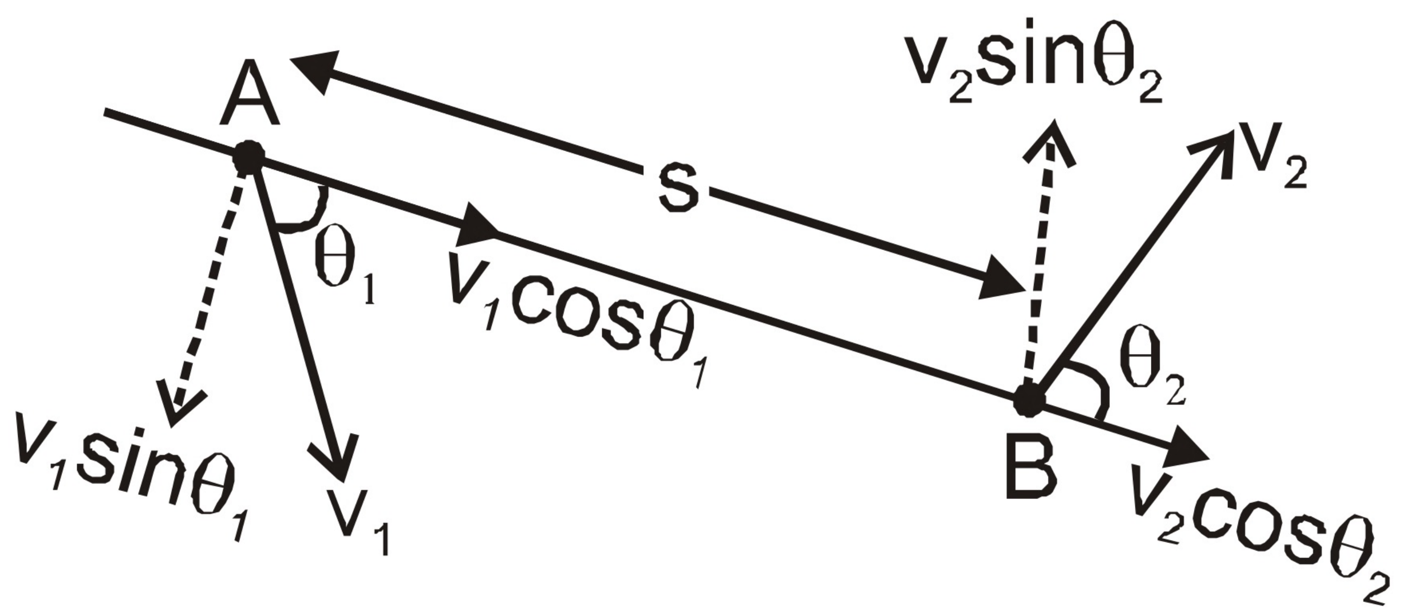
$a_{BL} = -g$

$d_{BL} = u_{BL}\,t + \frac{1}{2} a_{BL}\, t^2$

$-l = 0\,t + \frac{1}{2}(-g)\, t^2$

$$\boxed{t = \sqrt{\frac{2l}{g}}}$$

VELOCITY OF APPROACH / SEPARATION :→



It is the component of relative velocity of one particle w.r.t. another, along the line joining them.

$v_2\cos\theta_2 - v_1\cos\theta_1 = \dfrac{ds}{dt}$ rate of increase/decrease in separation distance

If the separation is decreasing, we say it is velocity of approach and if separation is increasing, then we say it is velocity of separation.

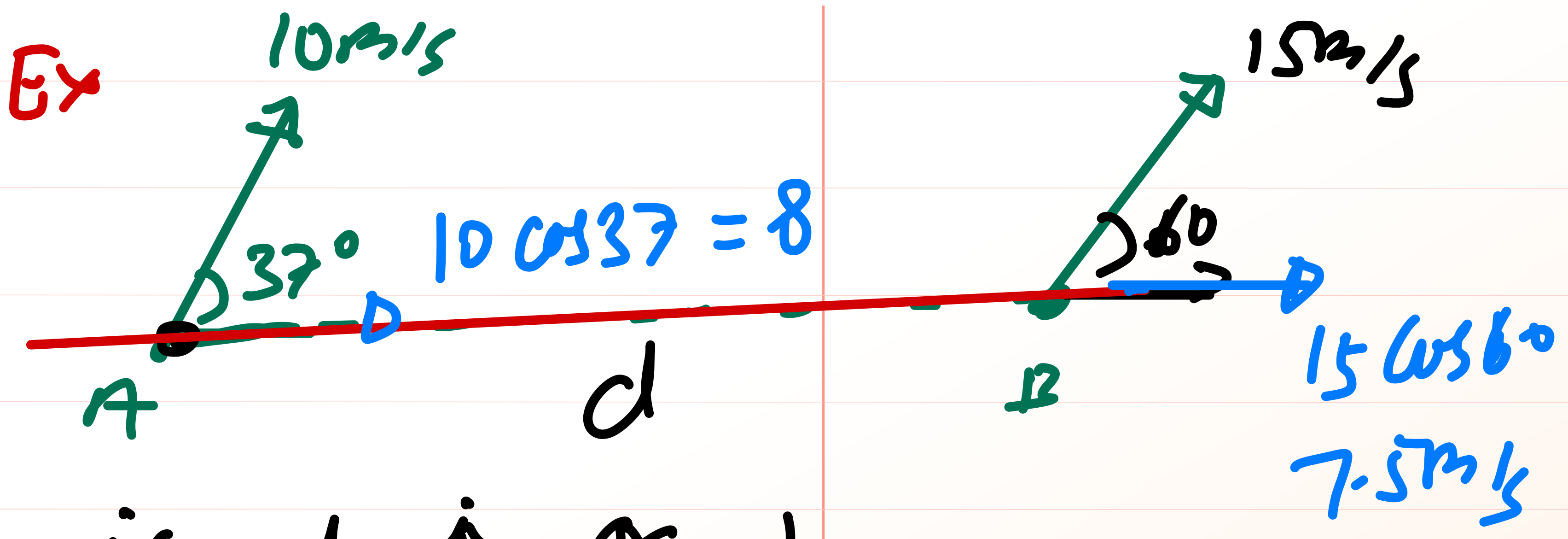
$(V_{AB})_{\parallel} = V_1\cos\theta_1 - V_2\cos\theta_2$

↳ Along line Joining them (A and B)

If $(V_{AB})_{\parallel} > 0$ $\quad (V_{AB})_{\parallel}$ = velocity of approach

If $(V_{AB})_{\parallel} < 0$ $\quad (V_{AB})_{\parallel}$ = velocity of Separation

Ex



is d ↑ or ↓

$(V_{AB})_{\parallel} = 8 - 7.5 = 0.5 > 0$

this is velocity of approach

Hence d ↓

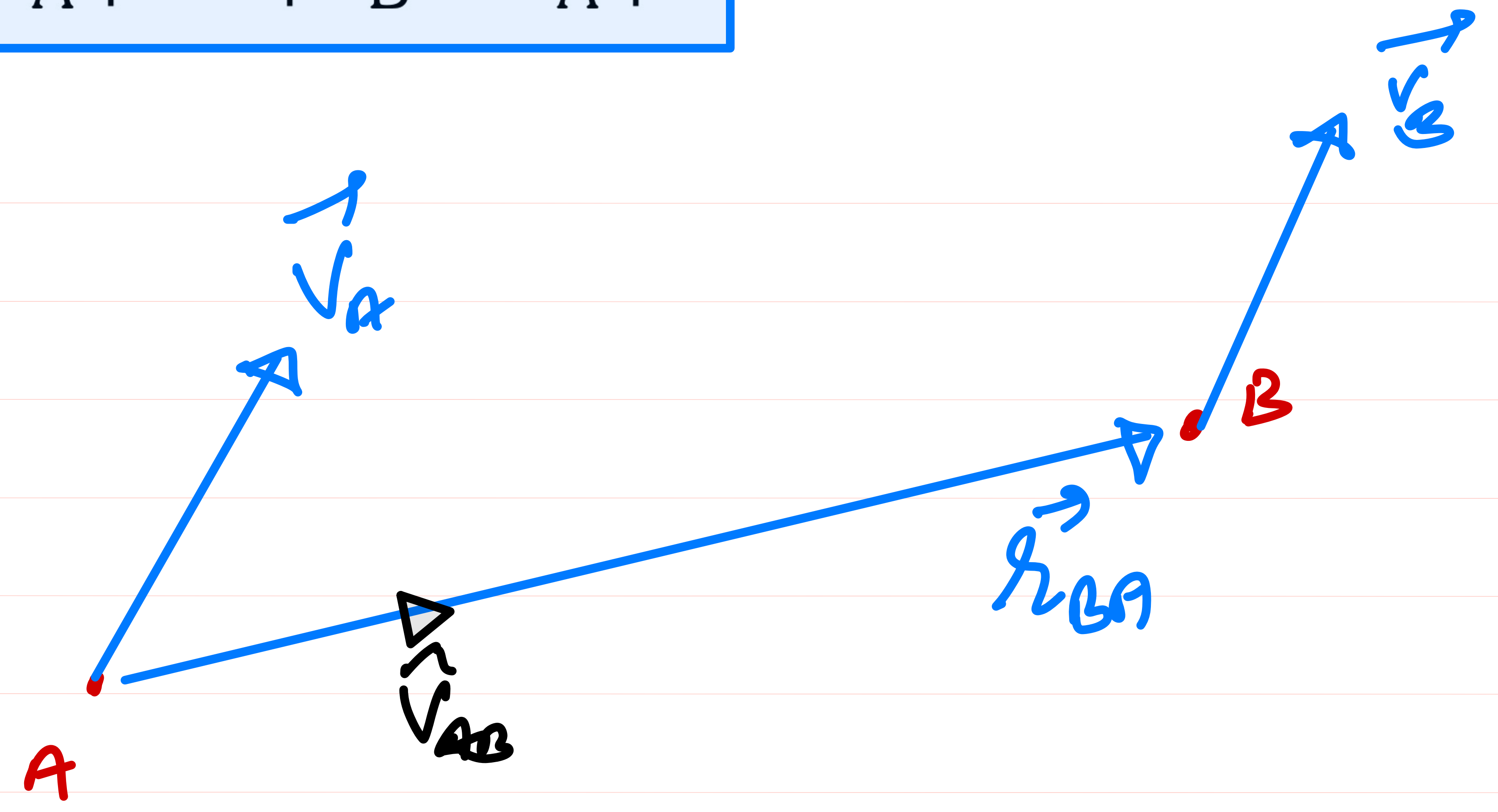
Ans

Condition to collide or to reach at the same point

When the relative velocity of one particle w.r.t. to other particle is directed towards each other then they will collide. (If there is a zero relative acceleration).

Two particles collide if $\vec{r}_{BA}$ and $\vec{v}_{AB}$ have same direction. For the same direction of these two vectors unit vectors in the direction of $\vec{r}_{BA}$ and $\vec{v}_{AB}$ must be equal.

$$\boxed{\frac{\vec{r}_B - \vec{r}_A}{|\vec{r}_B - \vec{r}_A|} = -\frac{\vec{v}_B - \vec{v}_A}{|\vec{v}_B - \vec{v}_A|}}$$



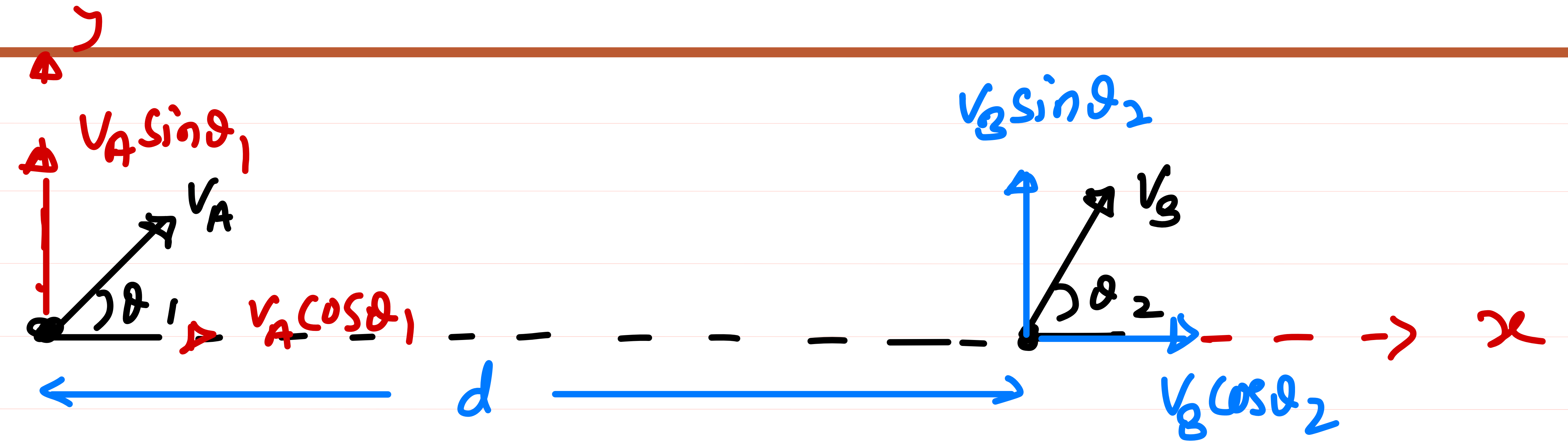
$\vec{V}_{AB}$

For collision

Dir^n of $\vec{V}_{AB}$ = Dir^n of $\vec{r}_{BA}$

$$\frac{\vec{V}_{AB}}{|\vec{V}_{AB}|} = \frac{\vec{r}_{BA}}{|\vec{r}_{BA}|}$$

$$\frac{\vec{V}_A - \vec{V}_B}{|\vec{V}_A - \vec{V}_B|} = \frac{\vec{r}_B - \vec{r}_A}{|\vec{r}_B - \vec{r}_A|}$$



Condition for collision

$$\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$$

$$= (V_A \cos\theta_1 \hat{i} + V_A \sin\theta_1 \hat{j}) - (V_B \cos\theta_2 \hat{i} + V_B \sin\theta_2 \hat{j})$$

$$\vec{V}_{AB} = (V_A \cos\theta_1 - V_B \cos\theta_2)\hat{i} + (V_A \sin\theta_1 - V_B \sin\theta_2)\hat{j}$$

for collision $\vec{V}_{AB}$ must be along x-axis

$$\therefore (V_{AB})_y = 0$$

$$\boxed{V_A \sin\theta_1 = V_B \sin\theta_2}$$

time after they will meet

$$t = \frac{d}{(V_{AB})_x}$$

$$\boxed{t = \frac{d}{V_A \cos\theta_1 - V_B \cos\theta_2}}$$

Ex

For what value of θ they will collide.

$$(V_A)_y = (V_B)_y$$

$$15\sin 37 = 5\sin\theta$$

$$\overset{3}{\cancel{15}} \times \frac{3}{5} = \cancel{5}\sin\theta$$

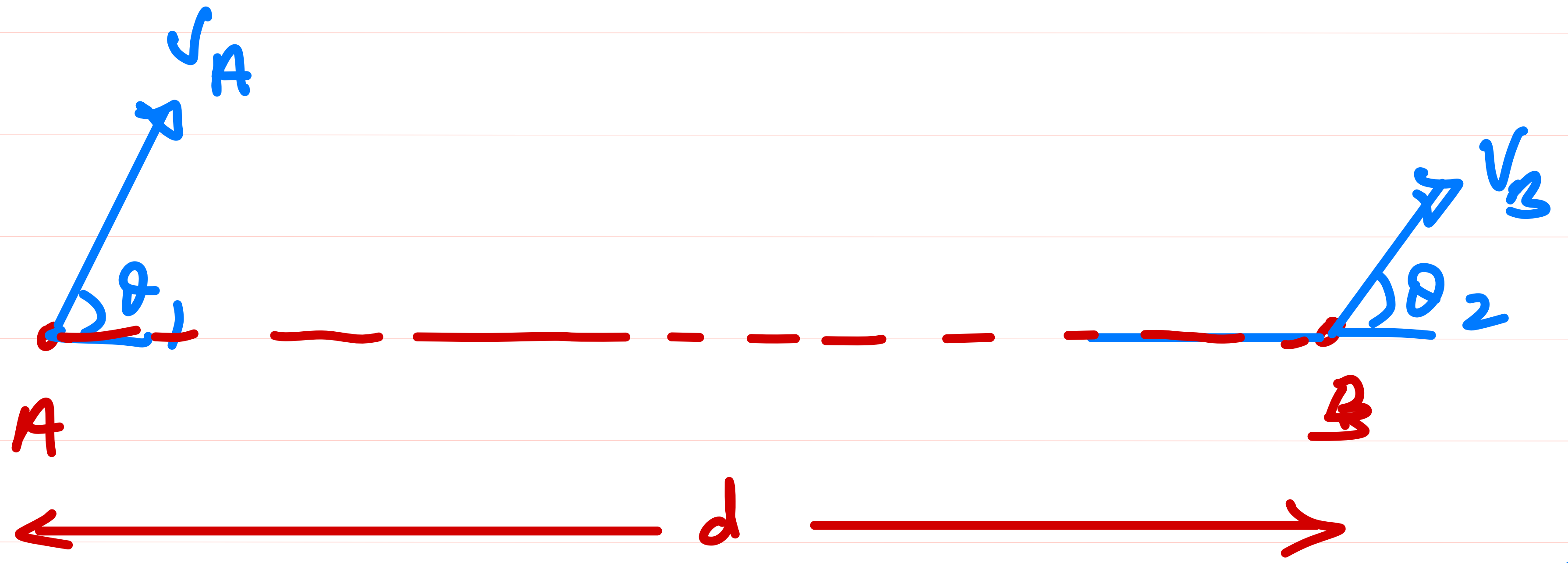
$$\frac{9}{5} = \sin\theta > 1$$

they will not collide

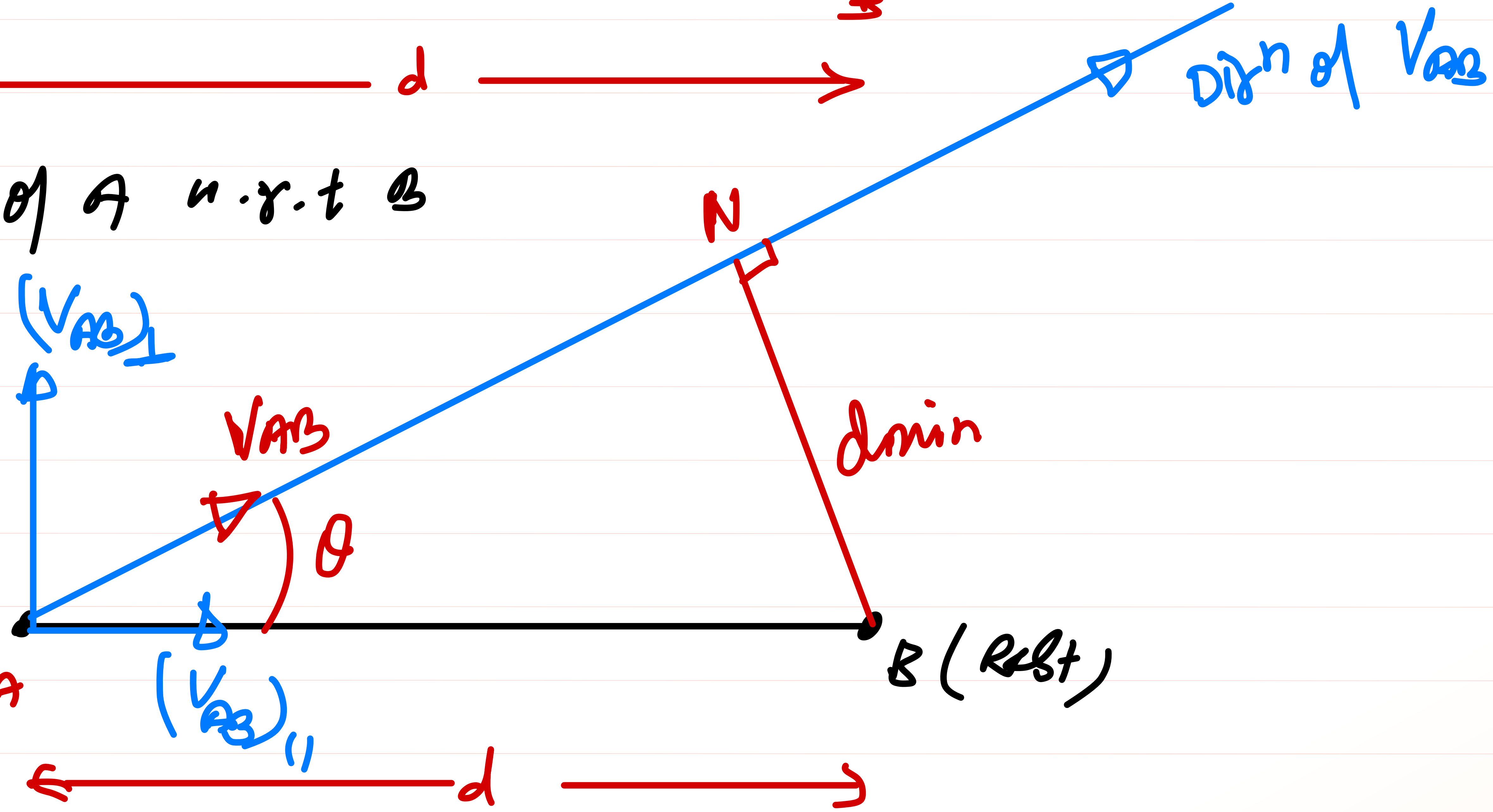
No possible value of θ

Minimum / Maximum distance between two particles :-

If $(v_A)_\perp \neq (v_B)_\perp$ then they will not collide



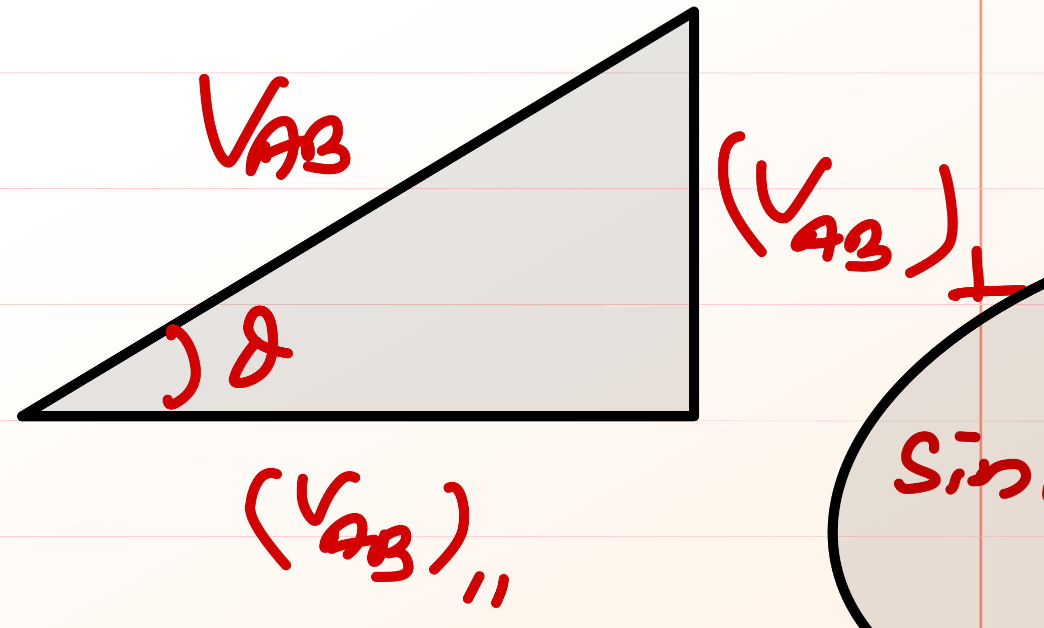
Motion of A w.r.t B



ΔANB

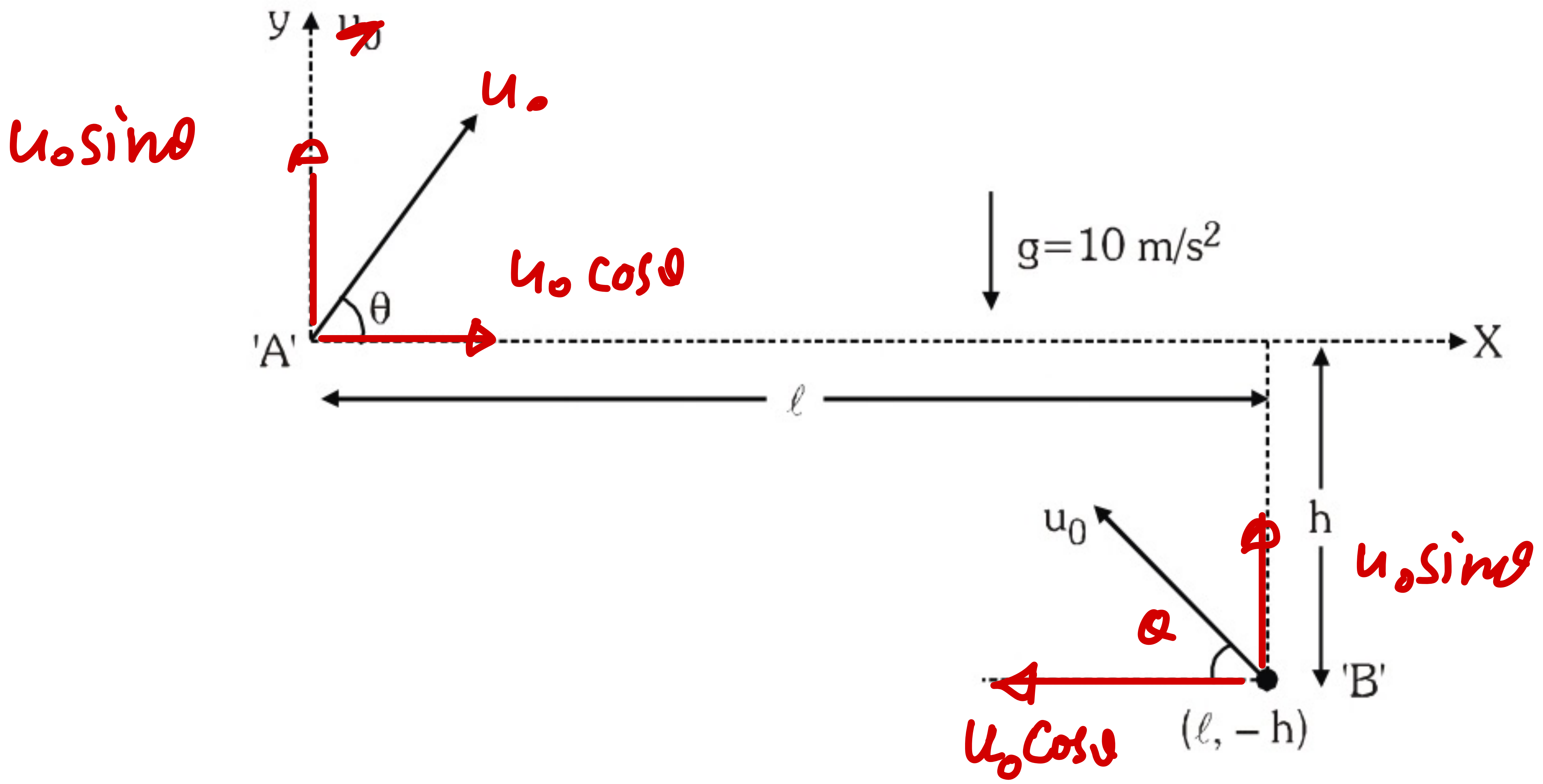
$$\sin\theta = \frac{d_{min}}{d} \Rightarrow \boxed{d_{min} = d\sin\theta}$$

$$\tan\theta = \frac{(V_{AB})_\perp}{(V_{AB})_{\parallel}}$$



$$\sin\theta = \frac{(V_{AB})_\perp}{V_{AB}}$$

Illustration 1*. Two particles 'A' and 'B' are projected in the vertical plane with same intial velocity u_0 from point (0, 0) and (ℓ, - h) towards each other as shown in figure at t = 0.



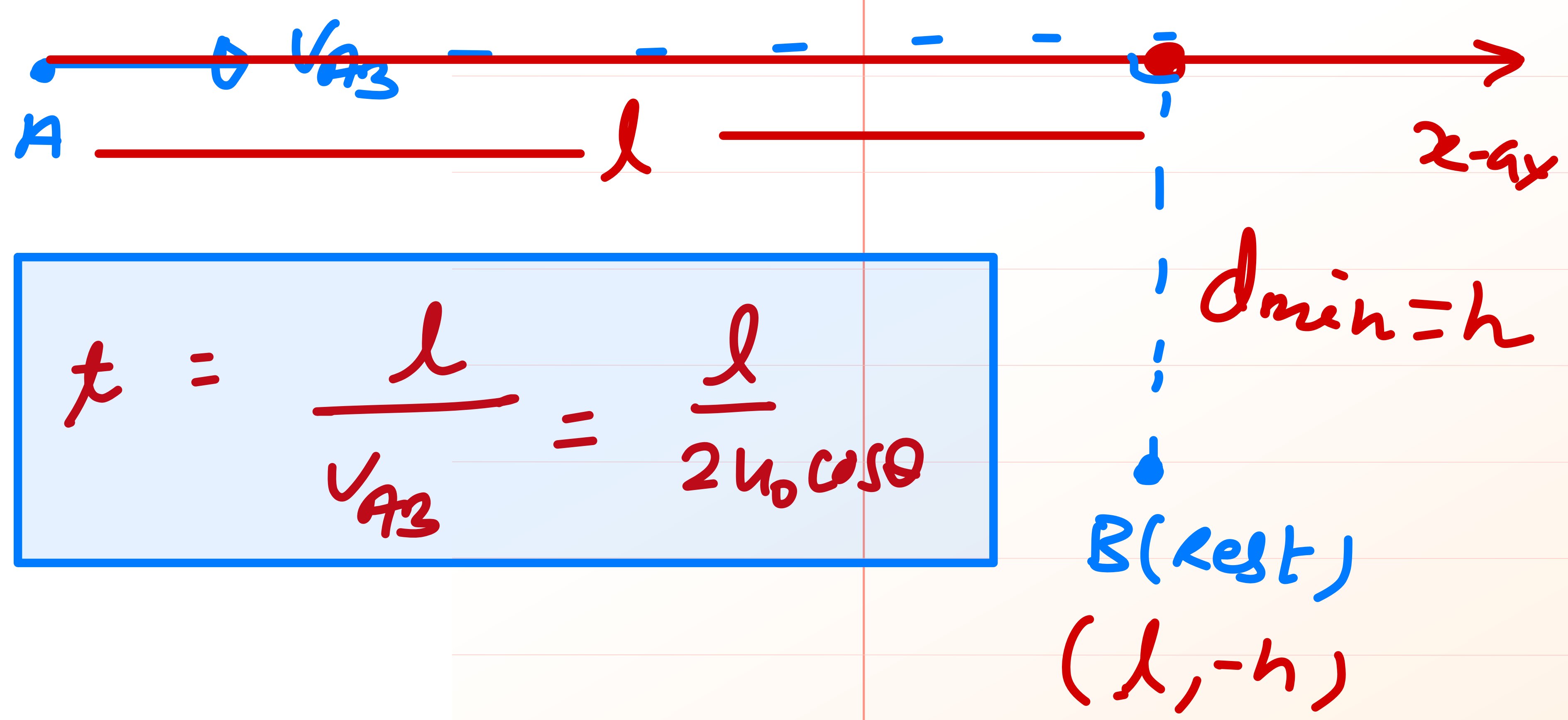
$\vec{V_A} = u_0\cos\theta\,\hat{i} + u_0\sin\theta\,\hat{j}$

$\vec{V_B} = -u_0\cos\theta\,\hat{i} + u_0\sin\theta\,\hat{j}$

$\vec{V_{AB}} = 2u_0\cos\theta\,\hat{i}$

$\vec{a_{AB}} = (-g\hat{j}) - (-g\hat{j})$

$\vec{a_{AB}} = 0$



$$\boxed{t = \frac{\ell}{V_{AB}} = \frac{\ell}{2u_0\cos\theta}}$$

(I) The path of particle 'A' with respect to particle 'B' will be :

(A) parabola
(B) straight line parallel to x - axis.
(C) straight line parallel to y-axis
(D) none of these.

(II) Minimum distance between particle A and B during motion will be :

(A) ℓ
(B) h
(C) $\sqrt{\ell^2 + b^2}$
(D) ℓ + h

(III) The time when separation between A and B is minimum is :

(A) $\dfrac{x}{u_0 \cos\ \theta}$
(B) $\sqrt{\dfrac{2h}{g}}$
(C) $\dfrac{\ell}{2u_0 \cos\ \theta}$
(D) $\dfrac{2\ell}{u_0 \cos\ \theta}$