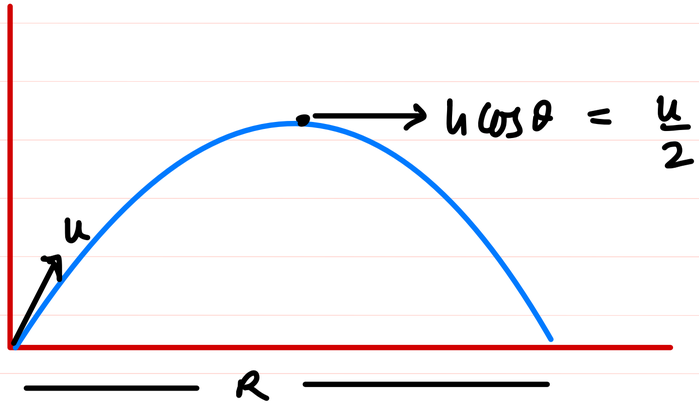
3. The speed of projectile at its highest point is observed to be half of its speed of projection u. Its range on horizontal plane is

(A) $\frac{3u^2}{g}$ (B) $\frac{\sqrt{3}}{2}\frac{u^2}{g}$ (C) $\frac{3}{2}\frac{u^2}{g}$ (D) $\frac{u^2}{3g}$



$u\cos\theta = \frac{u}{2} \Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = 60^\circ$

$R = \frac{u^2 \sin 2\theta}{g} = \frac{u^2}{g} \times 2\sin 60 \cos 60$

$= \frac{u^2}{g} \times 2 \times \frac{\sqrt{3}}{2} \times \frac{1}{2}$

$R = \frac{\sqrt{3}}{2}\frac{u^2}{g}$

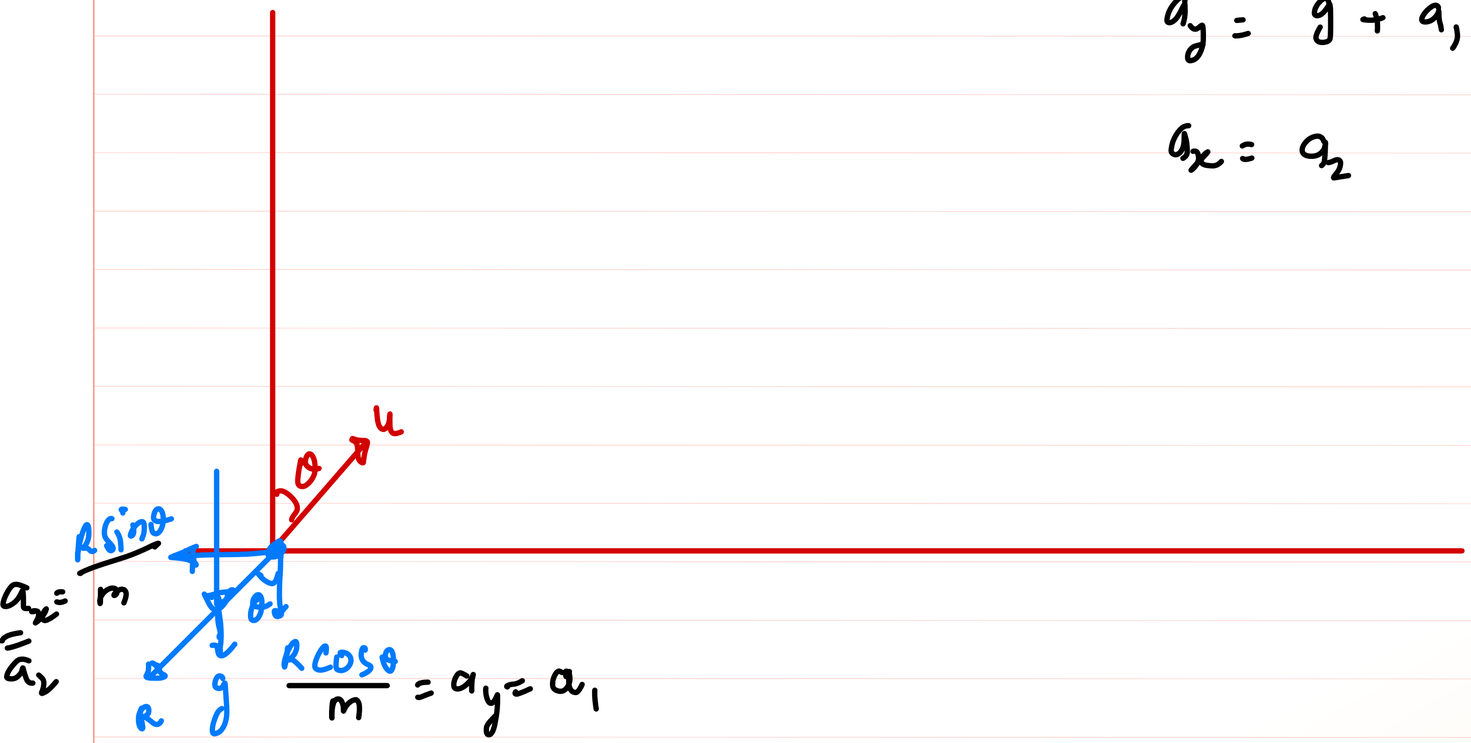
12. A projectile is projected from a point on the horizontal ground, at an angle with the vertical. If the air exerts a constant resistive force,

(A) the path of projectile will be a parabola

(B) at the highest point, the velocity is horizontal.

(C) the time for ascent equals the time for descent.

(D) the total mechanical energy of the projectile is not conserved.



$a_y = g + a_1$

$a_x = a_2$

$x = u\sin\theta\, t - \frac{1}{2}a_2 t^2$ — ①

$y = u\cos\theta\, t - \frac{1}{2}(g+a_1)t^2$ — ②

$\frac{a_2}{2}t^2 - u\sin\theta\, t + x = 0$

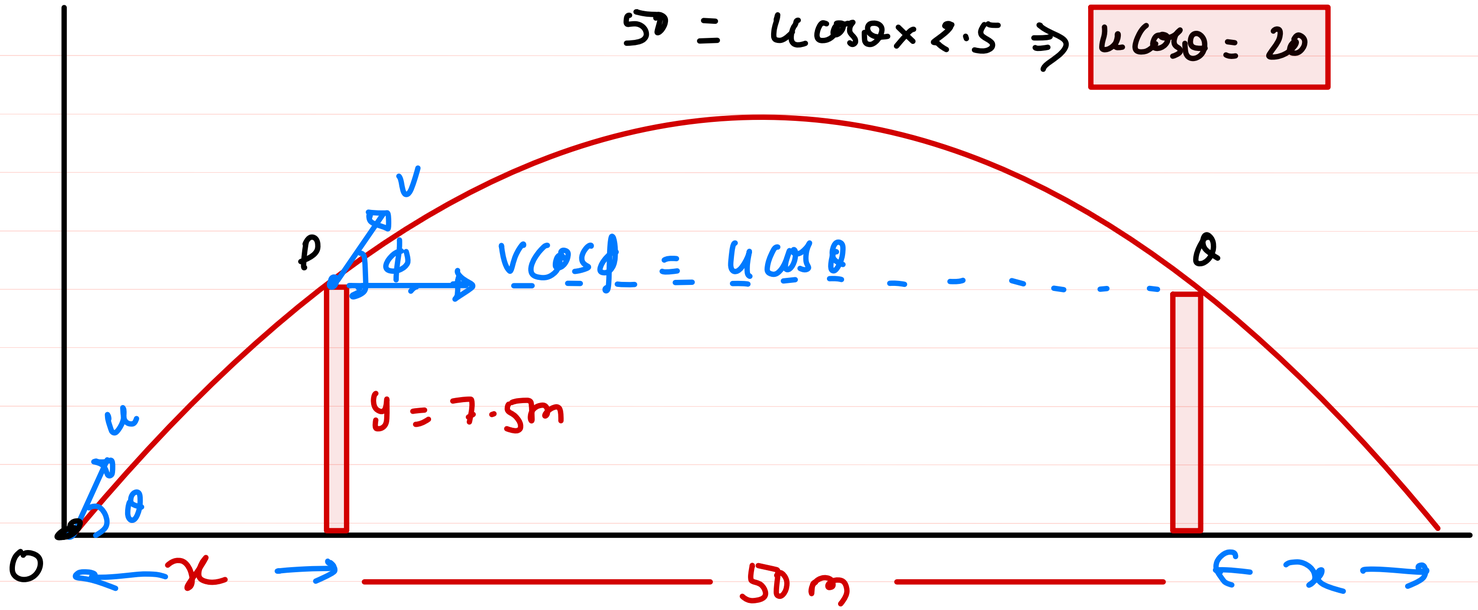
$t = \dfrac{u\sin\theta \pm \sqrt{(u\sin\theta)^2 - 2a_2 x}}{a_2}$

Put t in Eq ②

$y = \dfrac{u\cos\theta}{a_2}\left[u\sin\theta \pm \sqrt{(u\sin\theta)^2 - 2a_2 x}\right]$

14. An object is projected so that it must clear two obstacles each 7.5 m height, which are situated 50 m from each other. If the time of passing between the obstacles is 2.5 sec, calculate the complete range of projection and the initial velocity of the projection. ($g = 10 m/s^2$).

$50 = u\cos\theta \times 2.5 \Rightarrow \boxed{u\cos\theta = 20}$



$\because \quad V\cos\phi = u\cos\theta$ — ①

$t_{PQ} = \dfrac{2V\sin\phi}{g}$

$\dfrac{2.5 \times 10}{2} = V\sin\phi$

$12.5 = V\sin\phi$ — ②

Squaring and addition

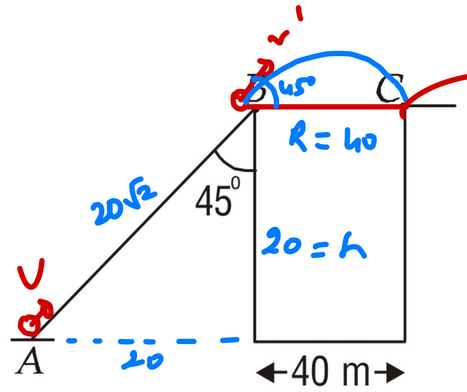
$V^2\cos^2\phi + V^2\sin^2\phi = (u\cos\theta)^2 + (12.5)^2$

$V^2 = (u\cos\theta)^2 + (12.5)^2$ — ③

Speed Rel^n

$V^2 = u^2 - 2g \times 7.5$ — ④

2. A body is projected up a smooth inclined plane with velocity V from the point A as shown in the figure. The angle of inclination is 45° and the top is connected to a well of diameter 40 m. If the body just manages to cross the well, what is the value of V ? Length of inclined plane is $20\sqrt{2}$m.



$$R = \frac{v'^2 \sin(2\times 45)}{g}$$

$$40 \times 10 = v'^2 \quad \text{— (2)}$$

(A) 40 ms^{-1} (B) $40\sqrt{2}$ ms^{-1} (C) 20 ms^{-1} ✓(D) $20\sqrt{2}$ ms^{-1}

Speed Relⁿ

$$v'^2 = v^2 - 2g \times 20$$

$$v'^2 = v^2 - 400 \quad \text{— (1)}$$

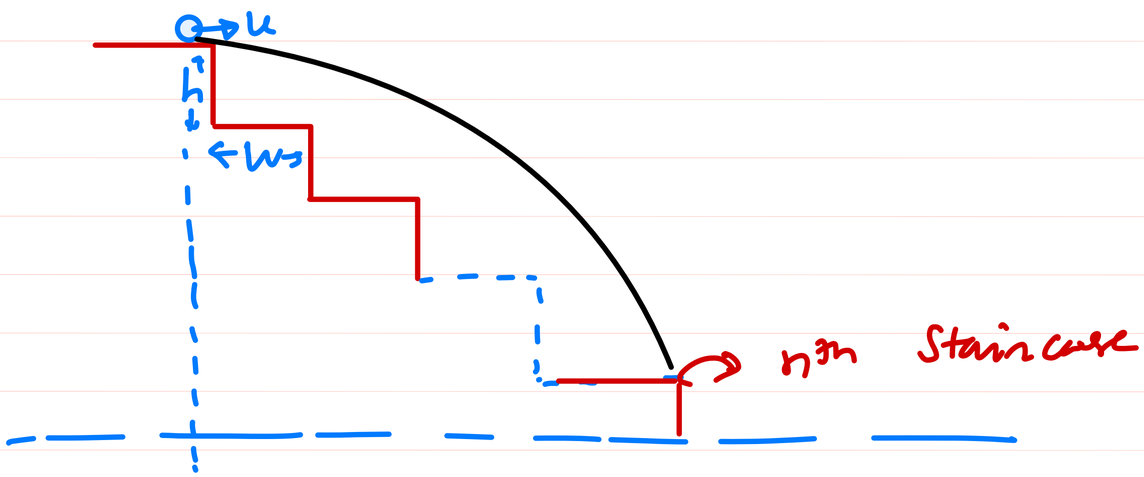
$$400 = v^2 - 400$$

$$v^2 = 800$$

$$v = 20\sqrt{2}\ m/s$$

4. A ball rolls off the top of a staircase with a horizontal velocity u ms^{-1}. If the steps are h m high and w m wide the ball will hit the edge of the n^{th} step if

(A) $n = \frac{gw^2}{2hu^2}$ (B) $n = \frac{2hu^2}{gw^2}$ (C) $n = \frac{2u^2}{gw^2h}$ (D) $n = \frac{2hw^2u^2}{g}$



$\because H = nh$

$\because R = nw = u\sqrt{\frac{2H}{g}}$

$nw = u\sqrt{\frac{2nh}{g}}$

$n^2w^2 = u^2 \times \frac{2nh}{g}$

$n = \frac{2u^2h}{gw^2}$

6. Select the correct alternative(s)

(A) In a projectile motion, H/R ratio is equal to (1/4) tanθ

(B) For angles of projection, which exceed or fall short of 45° by equal amounts, the ranges are equal.

(C) In projectile motion, velocity at initial and final points are same.

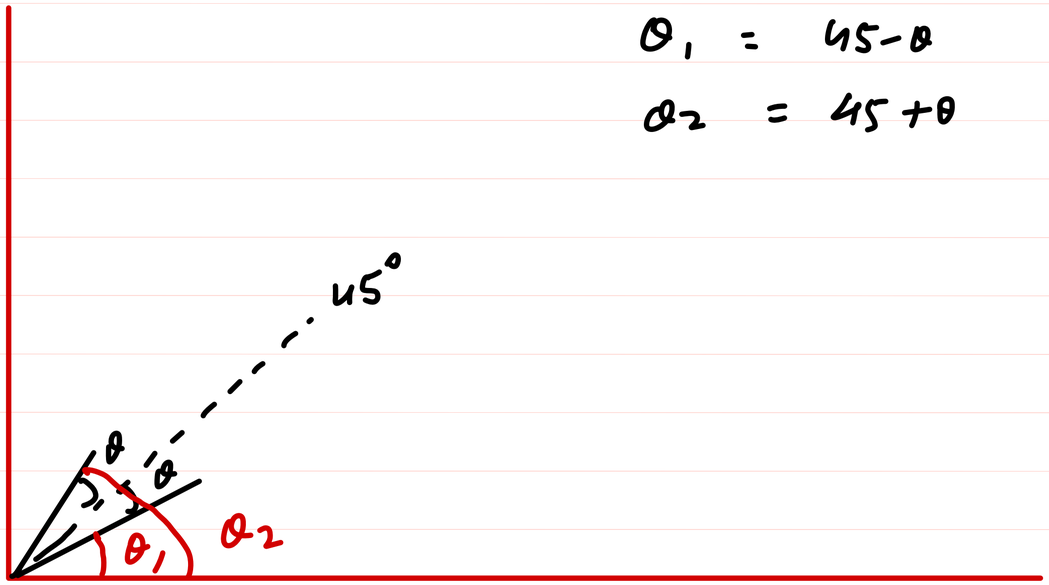
(D) None of these

$\theta_1 = 45 - \theta$

$\theta_2 = 45 + \theta$

If $\theta_1 + \theta_2 = 90°$

then $R_1 = R_2$



7. **Assertion** : When a body is projected at an angle α with vertical and then for the same angle with horizontal, the range is same

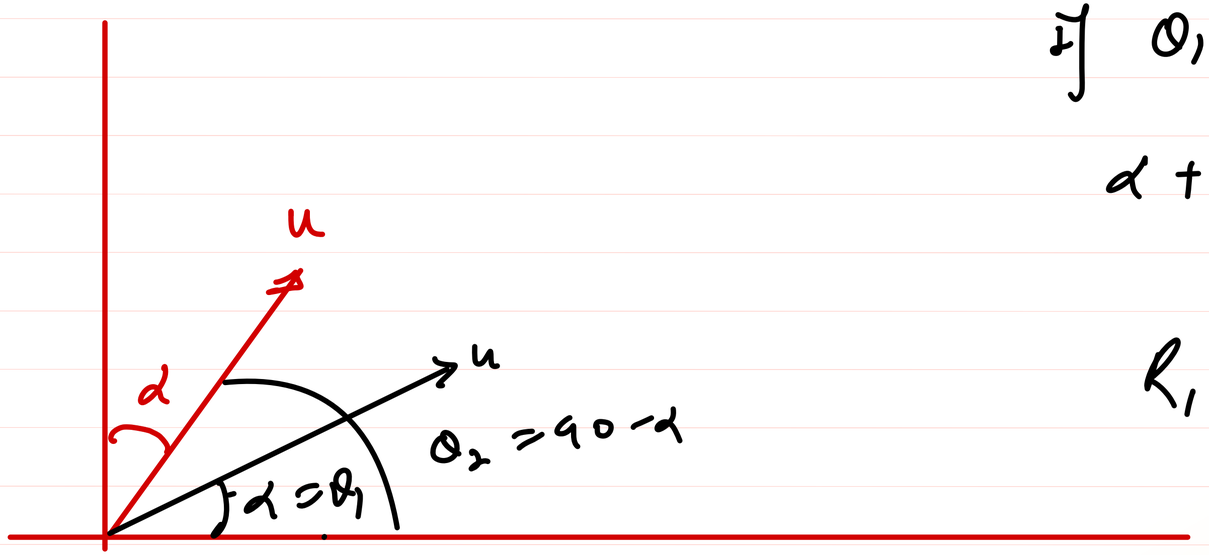
Reason : For oblique projection range, $R = \frac{u^2 \sin^2 \alpha}{2g}$ with usual notations.

(A) If both assertion and reason are true and reason is a correct explanation of the assertion.

(B) If both assertion and reason are true but the reason is not a correct explanation of assertion.

(C) If assertion is true but reason is false.

(D) Both assertion and reason are false.



If $\theta_1 + \theta_2 = 90$

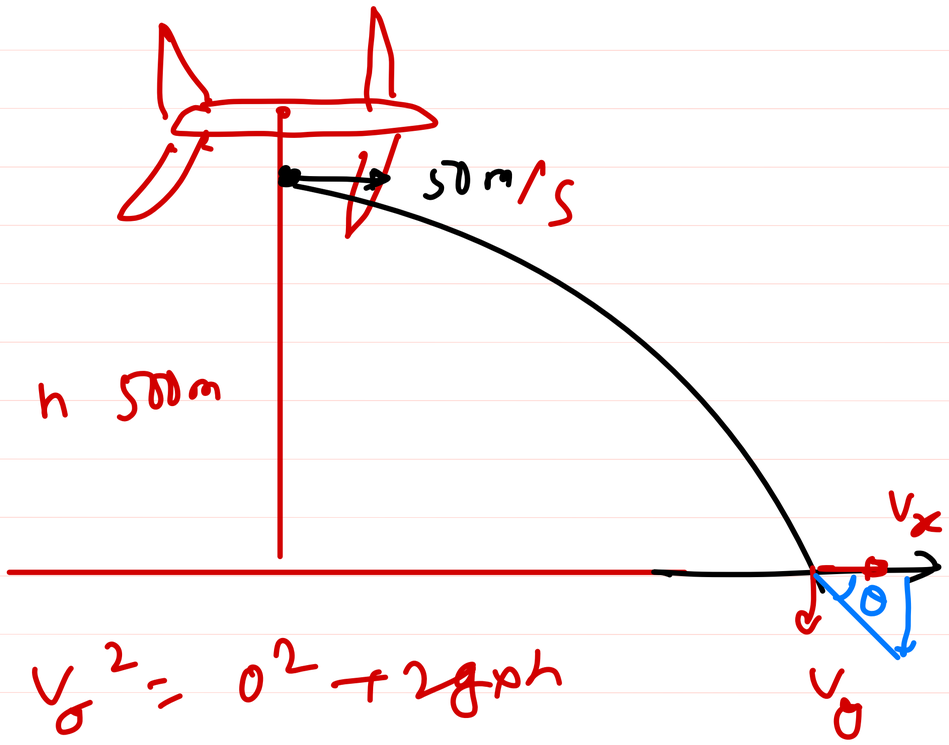
$\alpha + 90 - \alpha = 90$

$90 = 90$

$R_1 = R_2$

11. A food packet is dropped from the plane moving horizontally with velocity 50 ms^{-1} and at a height of 500 m. Find the angle with horizontal which the velocity vector makes at the time when it reaches the ground. Neglect air resistance

(A) $\tan^{-1}(-2)$ (B) $\tan^{-1}(1/2)$ (C) $-45°$ (D) 53°



$V_0^2 = 0^2 + 2gxh$

$V_y = \sqrt{20 \times 500}$

$= 100$ m/s

$\tan\theta = \frac{V_y}{V_x}$

$= \frac{100}{50} = 2$

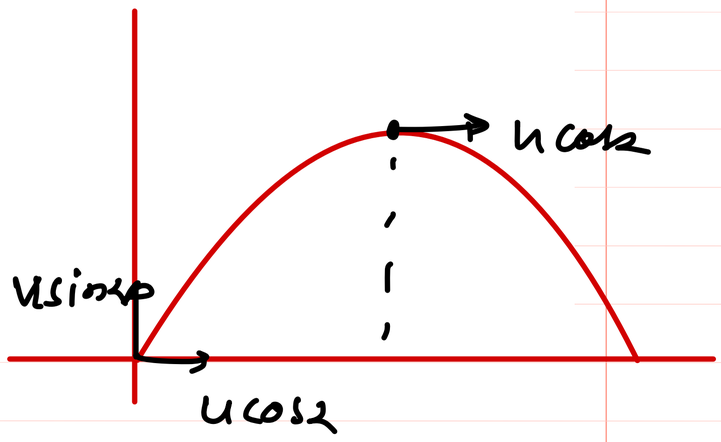
13. A particle is projected with a velocity u, at an angle α, with the horizontal. At what time its vertical component of velocity becomes half of its net speed at the highest point ?

(A) $\frac{u}{2g}$

(B) $\frac{u}{2g}(\sin\alpha - \cos\alpha)$

(C) $\frac{u}{2g}(2\cos\alpha - \sin\alpha)$

(D) $\frac{u}{2g}(2\sin\alpha - \cos\alpha)$



$V_y = u\sin\alpha - gt$

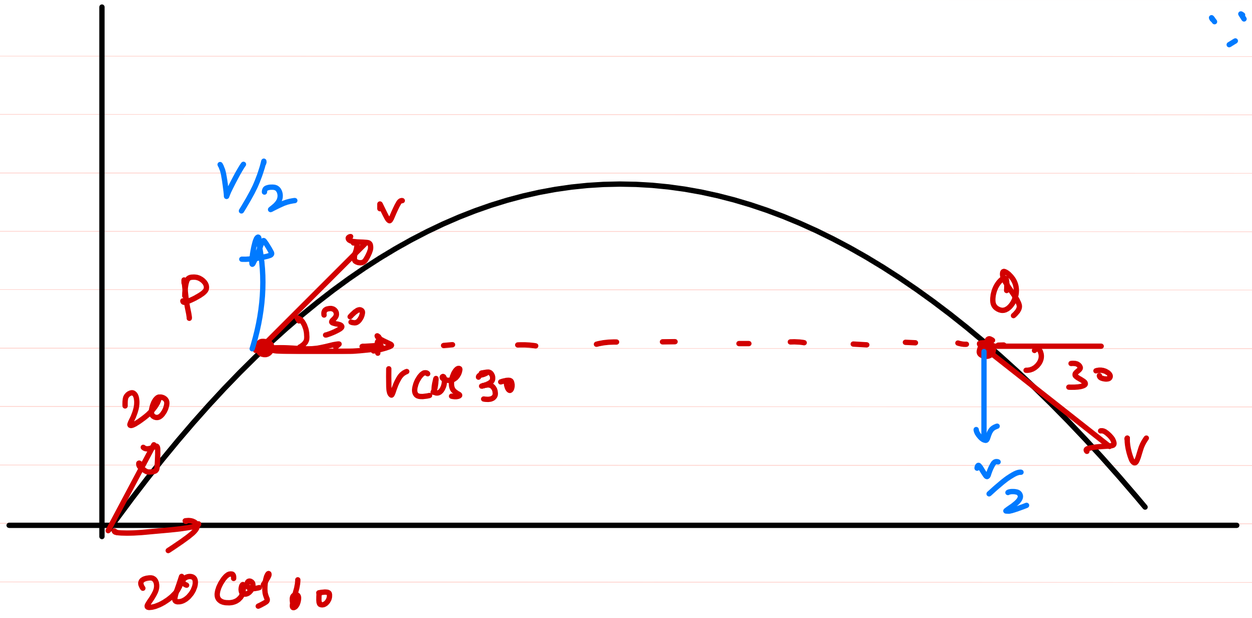
$\because \quad V_y = \frac{u\cos\alpha}{2} = u\sin\alpha - gt$

$gt = u\sin\alpha - \frac{u\cos\alpha}{2}$

$t = \frac{u}{2g}\{2\sin\alpha - \cos\alpha\}$

14. A body is thrown with velocity 20 m/s at an angle of 60° with the horizontal. Find the time gap between the two positions of body where velocity of body makes an angle of 30° with horizontal

(A) 1.15 sec (B) 0.95 sec (C) 1 sec. (D) 1.5 sec.



$\because v_y = u_y + a_y t$

$-\frac{v}{2} = \frac{v}{2} - g t$

$-v = -g t$

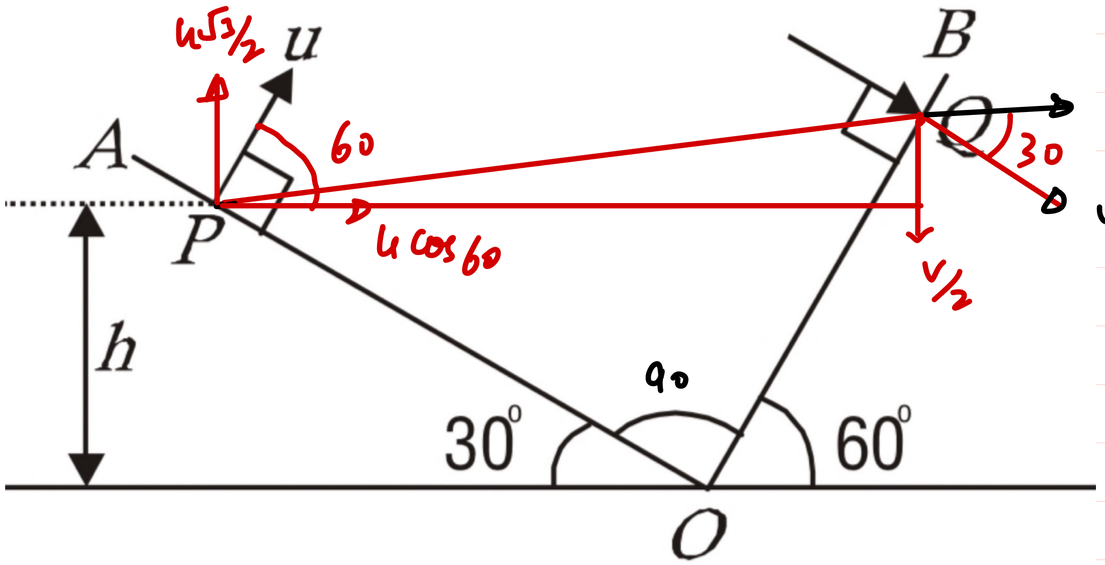
$t = \frac{v}{g} = \frac{20}{\sqrt{3} \times 10}$

$t = \frac{2}{\sqrt{3}}$

$20 \cos 60 = V \cos 30$

$20 \times \frac{1}{2} = V \frac{\sqrt{3}}{2} \Rightarrow \boxed{V = \frac{20}{\sqrt{3}}}$

Two inclined planes OA and OB having inclination of 30° and 60° with the horizontal respectively, intersect each other at O as shown in figure. A particle is projected from point P with velocity $u = 10\sqrt{3}$ m/s along a direction perpendicular to plane OA. If the particle strikes plane OB perpendicularly at Q, calculate



$u\cos 60$

$v\cos 30$

(2) From y-dirn

$$-\frac{v}{2} = \frac{u\sqrt{3}}{2} - 10t$$

$$-5 = 10\sqrt{3}\times\frac{\sqrt{3}}{2} - 10t$$

$$-5 = 15 - 10t$$

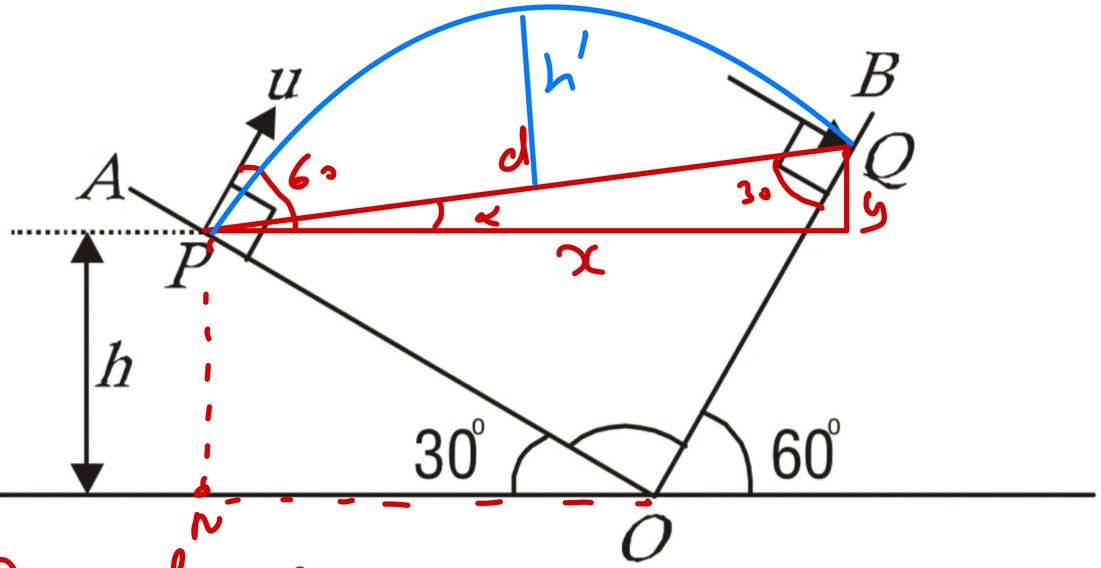
$$-20 = -10t$$

$$\boxed{t = 2\text{ sec}}$$

(1) $\because u\cos 60 = v\cos 30$

$$\frac{u}{2} = v\frac{\sqrt{3}}{2}$$

$$\boxed{v = \frac{u}{\sqrt{3}} = \frac{10\sqrt{3}}{\sqrt{3}} = 10\text{ m/s}}$$



④ Δ^l QOP

$$\sin 30 = \frac{PQ}{d} \Rightarrow PQ = 10\,m$$

Δ^l PNO

$$\sin 30 = \frac{h}{PO} \Rightarrow h = PO \sin 30$$

$$= 10 \times \frac{1}{2}$$

$$\boxed{h = 5\,m}$$

③ $x = u\cos 60 \times t$

$$= 10\sqrt{3} \times \frac{1}{2} \times 2$$

$$\boxed{x = 10\sqrt{3}\ m}$$

Speed Relⁿ

$$v^2 = u^2 - 2gy$$

$$10^2 = (10\sqrt{3})^2 - 2\times 10\, y$$

$$100 = 300 - 20y$$

$$20y = 200$$

$$\boxed{y = 10}$$

$$\because d = \sqrt{x^2 + y^2}$$

$$= \sqrt{10^2 + (10\sqrt{3})^2}$$

$$\boxed{d = 20\,m} \quad \text{Ans}$$