

**ARITHMETIC PROGRESSION - I**

- Show that the sequence  $\log a, \log(ab), \log(ab^2), \log(ab^3), \dots$  is an A.P. Find its  $n$ th term.
- Which term of the sequence 4, 9, 14, 19, ..... is 124 ?
- Which term of the sequence  $20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}, \dots$  is the first negative term ?
- If  $m$  times the  $m$ th term of an A.P. is equal to  $n$  times its  $n$ th term, show that the  $(m + n)$ th term of the A.P. is zero.
- If the  $p$ th term of an A.P. is  $q$  and the  $q$ th term is  $p$ , prove that its  $n$ th term is  $(p + q - n)$ .
- If  $a_1, a_2, a_3, \dots, a_n$  be an A.P. of non-zero terms, prove that  $\frac{1}{a_1 a_2} + \frac{1}{a_2 a_3} + \dots + \frac{1}{a_{n-1} a_n} = \frac{n-1}{a_1 a_n}$ .
- The sum of three numbers in A.P. is  $-3$ , and their product is 8. Find the numbers.
- Divide 32 into four parts which are in A.P. such that the product of extremes is to the product of means is 7 : 15.
- Find the sum of the series :  $5 + 13 + 21 + \dots + 181$ .
- Find the sum of all three digit natural numbers, which are divisible by 7.
- Find the sum of first 20 terms of an A.P., in which 3rd term is 7 and 7th term is two more than thrice of its 3rd term.
- If  $S_n$  denotes the sum of first  $n$  terms of A.P. and  $\frac{S_{3n} - S_{n-1}}{S_{2n} - S_{2n-1}} = 31$ , then  $n$  is equal to
- Find the number of terms in the series  $20, 19\frac{1}{3}, 18\frac{2}{3}, \dots$  of which the sum is 300, explain the double answer.
- The sum of the first  $p, q, r$  terms of an A.P. are  $a, b, c$  respectively. Show that  $\frac{a}{p}(q-r) + \frac{b}{q}(r-p) + \frac{c}{r}(p-q) = 0$ .
- The ratio of the sum of  $n$  terms of two A.P.'s is  $(7n + 1) : (4n + 27)$ . Find the ratio of their  $m$ th terms.
- If  $a, b, c$  are in A.P., prove that the following are also in A.P.  
(i)  $b + c, c + a, a + b$  (ii)  $a\left(\frac{1}{b} + \frac{1}{c}\right), b\left(\frac{1}{c} + \frac{1}{a}\right), c\left(\frac{1}{a} + \frac{1}{b}\right)$
- If  $a^2, b^2, c^2$  are in A.P., then prove that  $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$  is also in A.P.
- If  $\log_{10} 2, \log_{10}(2^x - 1)$  and  $\log_{10}(2^x + 3)$  are in A.P., then find the value of  $x$ .
- If  $S_n$  denotes the sum of  $n$  terms of A.P., then find  $S_{n+3} - 3S_{n+2} + 3S_{n+1} - S_n$  is equal to
- The digits of a positive integer, having three digits, are in A.P. and their sum is 15. The number obtained by reversing the digits is 594 less than the original number. Find the number.
- The least value of 'a' for which  $5^{1+x} + 5^{1-x}, a/2, 25^x + 25^{-x}$  are three consecutive terms of an AP is  
(A) 1 (B) 5 (C) 12 (D) None of these
- If  $p, q, r$  in A.P. and are positive, the roots of the quadratic equation  $px^2 + qx + r = 0$  are all real for  
(A)  $\left|\frac{r}{p} - 7\right| \geq 4\sqrt{3}$  (B)  $\left|\frac{p}{r} - 7\right| < 4\sqrt{3}$  (C) all  $p$  and  $r$  (D) No.  $p$  and  $r$
- Sum of first hundred numbers common to the two A.P.'s 12, 15, 18, ... and 17, 21, 25 ..., is  
(A) 56100 (B) 65100 (C) 61500 (D) none of these

**ARITHMETIC PROGRESSION - II**

- If  $S_r$  denotes the sum of  $r$  terms of an AP and  $\frac{S_a}{a^2} = \frac{S_b}{b^2} = c$  then  $S_c$  is  
(A)  $c^3$  (B)  $c/ab$  (C)  $abc$  (D)  $a + b + c$
- If  $a_r > 0$ ,  $r \in \mathbb{N}$  and  $a_1, a_2, a_3, \dots, a_{2n}$  are in AP then  $\frac{a_1 + a_{2n}}{\sqrt{a_1} + \sqrt{a_{2n}}} + \frac{a_2 + a_{2n-1}}{\sqrt{a_2} + \sqrt{a_{2n-1}}} + \frac{a_3 + a_{2n-2}}{\sqrt{a_3} + \sqrt{a_{2n-2}}} + \dots + \frac{a_n + a_{n+1}}{\sqrt{a_n} + \sqrt{a_{n+1}}}$  is equal to  
(A)  $n - 1$  (B)  $\frac{n(a_1 + a_{2n})}{\sqrt{a_1} + \sqrt{a_{n+1}}}$  (C)  $\frac{n - 1}{\sqrt{a_1} + \sqrt{a_{n+1}}}$  (D) none of these
- If  $a_1, a_2, a_3, \dots, a_{2n+1}$  are in AP then  $\frac{a_{2n+1} - a_1}{a_{2n+1} + a_1} + \frac{a_{2n} - a_2}{a_{2n} + a_2} + \dots + \frac{a_{n+2} - a_n}{a_{n+2} + a_n}$  is equal to  
(A)  $\frac{n(n+1)}{2} \cdot \frac{a_2 - a_1}{a_{n+1}}$  (B)  $\frac{n(n+1)}{2} \cdot \frac{a_2 - a_1}{a_n}$  (C)  $(n+1)(a_2 - a_1)$  (D) none of these
- Let  $a_1, a_2, a_3, \dots$  be terms of an A.P. If  $\frac{a_1 + a_2 + \dots + a_p}{a_1 + a_2 + \dots + a_q} = \frac{p^2}{q^2}$ ,  $p \neq q$ , then  $\frac{a_6}{a_{21}}$  equals  
(A)  $41/11$  (B)  $7/2$  (C)  $2/7$  (D)  $11/41$
- Let  $\{a_n\}$  ( $n \geq 1$ ) be a sequence such that  $a_1 = 1$ , and  $3a_{n+1} - 3a_n = 1$  for all  $n \geq 1$ . Then  $a_{2002}$  is equal to  
(A) 666 (B) 667 (C) 668 (D) 669
- If 4<sup>th</sup> term of an AP is 64 and its 54<sup>th</sup> term is  $-61$ , then its common difference is  
(A)  $5/2$  (B)  $-5/2$  (C)  $3/50$  (D)  $-3/50$
- The 19<sup>th</sup> term from the end of the series  $2 + 6 + 10 + \dots + 86$  is  
(A) 6 (B) 18 (C) 14 (D) 10
- If  $n^{\text{th}}$  term of an AP is  $1/3 (2n + 1)$ , then the sum of its 19 terms is  
(A) 131 (B) 132 (C) 133 (D) 134
- If the ratio of the sum of  $n$  terms of two AP's is  $2n : (n + 1)$ , then ratio of their 8<sup>th</sup> terms is  
(A)  $15 : 8$  (B)  $8 : 13$  (C)  $n : (n - 1)$  (D)  $5 : 17$
- The sum of  $n$  terms of an AP is  $3n^2 + 5n$ . Then number of term when  $n^{\text{th}}$  term equals 164 is  
(A) 13 (B) 21 (C) 27 (D) 29
- If the  $m^{\text{th}}$  term of an A.P. is  $\frac{1}{n}$  and the  $n^{\text{th}}$  term is  $\frac{1}{m}$  then sum to  $mn$  terms is  
(A)  $\frac{mn+1}{2}$  (B)  $\frac{mn-1}{2}$  (C)  $\frac{mn+1}{3}$  (D)  $\frac{mn-1}{3}$
- If  $a, b, c$  be the 1<sup>st</sup>, 3<sup>rd</sup> and  $n^{\text{th}}$  terms respectively of an A.P., then sum to  $n$  terms is  
(A)  $\frac{c+a}{2} + \frac{c^2-a^2}{b-a}$  (B)  $\frac{c+a}{2} - \frac{c^2-a^2}{b-a}$  (C)  $\frac{c+a}{2} + \frac{c^2+a^2}{b-a}$  (D)  $\frac{c+a}{2} + \frac{c^2+a^2}{b+a}$
- If  $a_1, a_2, a_3, \dots$  are in A.P. and  $a_1 + a_4 + a_7 + \dots + a_{16} = 147$  then  $a_1 + a_6 + a_{11} + a_{16}$  is equal to  
(A) 96 (B) 98 (C) 100 (D) None of these
- If  $a_1, a_2, a_3, \dots$  is an A.P. such that  $a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{24} = 225$  then  $a_1 + a_2 + a_3 + \dots + a_{23} + a_{24}$  is equal to  
(A) 909 (B) 75 (C) 750 (D) 900
- The sum of all even positive integers less than 200 which are not divisible by 6 is  
(A) 6534 (B) 6354 (C) 6543 (D) 6454

16. If  $x, y, z$  are in AP,  $a$  is AM between  $x$  and  $y$  and  $b$  is AM between  $y$  and  $z$ ; then AM between  $a$  and  $b$  will be  
(A)  $\frac{1}{3}(x + y + z)$  (B)  $z$  (C)  $x$  (D)  $y$
17. If  $n$  AM's are inserted between 1 and 31 and ratio of 7<sup>th</sup> and  $(n-1)$ <sup>th</sup> A.M. is 5 : 9, then  $n$  equals  
(A) 12 (B) 13 (C) 14 (D) None of these
18. Three numbers are in A.P., If their sum is 33 and their product is 792, then the smallest of these numbers is  
(A) 14 (B) 11 (C) 8 (D) 4
19. If the angles of a quadrilateral are in A.P. whose common difference is  $10^\circ$ , then the angles of the quadrilateral are  
(A)  $65^\circ, 85^\circ, 95^\circ, 105^\circ$  (B)  $75^\circ, 85^\circ, 95^\circ, 105^\circ$  (C)  $65^\circ, 75^\circ, 85^\circ, 95^\circ$  (D)  $65^\circ, 95^\circ, 105^\circ, 115^\circ$
20. 20 is divided into four parts which are in A.P., such that the product of the first and fourth is to the product of the second and third is 2 : 3, then the four parts are  
(A) 2, 4, 6, 8 (B) 3, 5, 7, 9 (C) 4, 6, 8, 10 (D) 6, 10, 17, 12
21. Insert three arithmetic means between 3 and 19.

**GEOMETRIC PROGRESSION - I**

- If the  $p^{\text{th}}$ ,  $q^{\text{th}}$ ,  $r^{\text{th}}$  terms of a G.P. be  $a$ ,  $b$ ,  $c$  respectively, then prove that  $a^{q-r}b^{r-p}c^{p-q} = 1$ .
- The fifth term of a G.P. is 81, and the second term is 24; find the series.
- Find the sum of the series :  $3, -4, \frac{16}{3}, \dots$  to  $2n$  terms.
- The sum of the first 6 terms of a G.P. is 9 times the sum of the first 3 terms; find the common ratio.
- The sum of a G.P. whose common ratio is 3 is 728, and the last term is 486; find the first term.
- In a G.P. the first term is 7, the last term 448, and the sum 889; find the common ratio.
- The sum of infinite number of terms of a G.P. is 4 and the sum of their cubes is 192; find the series.
- The sum of three numbers in G.P. is 38, and their product is 1728; find them.
- The continued product of three numbers in G.P. is 216, and the sum of the products of them in pairs is 156; find the numbers.
- The sum of three numbers in G.P. is 70; if the two extremes be multiplied each by 4, and the mean by 5, the products are in A.P., find the numbers.
- If the  $p^{\text{th}}$ ,  $q^{\text{th}}$ ,  $r^{\text{th}}$ ,  $s^{\text{th}}$  terms of an A.P. are in G.P., show that  $p - q$ ,  $q - r$ ,  $r - s$  are in G.P.
- The sum of first three terms of a G.P. is to the sum of the first six terms as 125 : 152. Find the common ratio of the G.P.
- Sum the series : (a)  $\left(x + \frac{1}{x}\right)^2 + \left(x^2 + \frac{1}{x^2}\right)^2 + \left(x^3 + \frac{1}{x^3}\right)^2 + \dots + \left(x^n + \frac{1}{x^n}\right)^2$   
(b)  $1 + (1 + x) + (1 + x + x^2) + (1 + x + x^2 + x^3) + \dots$  to  $n$  terms.
- Find the sum of  $n$  terms of the following series  
(a)  $.7 + .77 + .777 + \dots$   
(b)  $6 + 66 + 666 + \dots$
- (a) Find the value of  $.1\dot{2}\dot{3}$  regarding it as geometric series.  
(b) Find the value of  $.4\dot{2}\dot{3}$ .

**GEOMETRIC PROGRESSION - II**

- If  $p^{\text{th}}$ ,  $q^{\text{th}}$  and  $r^{\text{th}}$  terms of an A.P. are in G.P., then the common ratio of G.P. is  
(A)  $\frac{q-r}{p-q}$  (B)  $\frac{r-q}{p-q}$  (C)  $\frac{q-r}{q-p}$  (D)  $\frac{q-p}{q-r}$
- If the roots of cubic equation  $ax^3 + bx^2 + cx + d = 0$  are in G.P., then  
(A)  $c^3a = b^3d$  (B)  $ca^3 = bd^3$  (C)  $a^3b = c^3d$  (D)  $ab^3 = cd^3$
- If  $\frac{p+q.5^x}{p-q.5^x} = \frac{q+r.5^x}{q-r.5^x} = \frac{r+s.5^x}{r-s.5^x}$  then  $p$ ,  $q$ ,  $r$ ,  $s$  are in  
A) A.P. B) G.P. C) H.P. D) none of these
- If the sum of the series  $\sum_{n=0}^{\infty} r^n$ ,  $|r| < 1$ , is  $S$ , then sum of the series  $\sum_{n=0}^{\infty} r^{2n}$  is  
(A)  $S^2$  (B)  $\frac{2S}{S^2-1}$  (C)  $\frac{S^2}{2S+1}$  (D)  $\frac{S^2}{2S-1}$

5. If  $S$  denotes the sum of infinity and  $S_n$  the sum of  $n$  terms of the series  $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots$  such that  $S - S_n < \frac{1}{1000}$ , then the least value of  $n$  is  
(A) 11 (B) 9 (C) 10 (D) 8
6. If  $a, b, c$  are in G.P. then the equations  $ax^2 + 2bx + c = 0$  and  $dx^2 + 2ex + f = 0$  have a common root if  $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$  are in  
(A) A.P. (B) G.P. (C) H.P. (D) None of these
7. A certain number is inserted between the number 3 and the unknown number so that the three numbers form an A.P. If the middle term is diminished by 6 then the number are in G.P. The unknown number can be  
(A) 3 (B) 15 (C) 18 (D) 27
8. Let the numbers  $a_1, a_2, a_3, \dots, a_n$  constitute a geometric progression. If  $S = a_1 + a_2 + \dots + a_n$ ,  $T = \frac{1}{a_1} + \frac{1}{a_2} + \dots + \frac{1}{a_n}$  and  $P = a_1 a_2 a_3 \dots a_n$  then  $P^2$  is equal to  
(A)  $\left(\frac{S}{T}\right)^n$  (B)  $\left(\frac{T}{S}\right)^n$  (C)  $\left(\frac{2S}{T}\right)^n$  (D)  $\left(\frac{2T}{S}\right)^n$
9. Let  $\alpha, \beta$  be the roots of  $x^2 - x + p = 0$  and  $\gamma, \delta$  be the roots of  $x^2 - 4x + q = 0$ . If  $\alpha, \beta, \gamma, \delta$  are in G.P., then the integral values of  $p$  and  $q$  respectively, are  
(A)  $-2, -32$  (B)  $-2, 3$  (C)  $-6, 3$  (D)  $-6, -32$
10.  $a, b, c, d$  are in increasing G.P. If the AM between  $a$  and  $b$  is 6 and the AM between  $c$  and  $d$  is 54., then the AM of  $a$  and  $d$  is  
(A) 15 (B) 48 (C) 44 (D) 42
11. Insert 3 geometric means between  $\frac{9}{4}$  and  $\frac{4}{9}$ .
12. If the arithmetic mean between  $a$  and  $b$  is twice as great as the geometric mean, show that  $a : b = 2 + \sqrt{3} : 2 - \sqrt{3}$ .
13. If  $a, b, c, d$  be in G.P. Prove that  
(a)  $(a^2 + ac + c^2)(b^2 + bd + d^2) = (ab + bc + cd)^2$ .  
(b)  $(a^2 + b^2 + c^2)(b^2 + c^2 + d^2) = (ab + bc + cd)^2$ .
14. If  $a, b, c, d$  be in G.P. ( $a \neq b \neq c \neq d$ ). Prove that  
(a)  $(a - d)^2 = (b - c)^2 + (c - a)^2 + (d - b)^2$   
(b)  $a^2 - b^2, b^2 - c^2, c^2 - d^2$  are in G.P.
15. (a) If one geometric mean  $G$  and two arithmetic means  $p$  and  $q$  be inserted between any two given numbers, then show that  $G^2 = (2p - q)(2q - p)$ .  
(b) If one arithmetic mean  $A$  and two geometric means  $p$  and  $q$  be inserted between any two given numbers, then show that  $p^3 + q^3 = 2Apq$ .
16. Find the  $\prod_{i=1}^3 G_i$  (Geometric means) inserted between ' $a$ ' and ' $b$ ' which satisfy the equation  $(G_1 + 2)^4 + (G_2 - 4)^2 + |G_3 + 8| = 0$ . Also find  $ab =$

**HARMONIC PROGRESSION - I**

- Find the fourth term in the following series :  $2, 2\frac{1}{2}, 3\frac{1}{3}, \dots$
- Find the fourth term in the following series :  $2, 2\frac{1}{2}, 3, \dots$
- If the  $p^{\text{th}}, q^{\text{th}}, r^{\text{th}}$  terms of a H.P. be  $a, b, c$  respectively, prove that  $(q - r)bc + (r - p)ca + (p - q)ab = 0$ .
- If the  $m^{\text{th}}$  term of a H.P. be equal to  $n$ , and the  $n^{\text{th}}$  term be equal to  $m$ , prove that the  $(m + n)^{\text{th}}$  term is equal to  $\frac{mn}{m+n}$ .
- If  $a, b, c$  be in H.P., prove that
 

(A)  $\frac{1}{b-a} + \frac{1}{b-c} = \frac{2}{b}$

(B)  $\frac{b+a}{b-a} + \frac{b+c}{b-c} = 2$

(C)  $\left(\frac{1}{a} + \frac{1}{b} - \frac{1}{c}\right)\left(\frac{1}{b} + \frac{1}{c} - \frac{1}{a}\right) = \frac{4}{ac} - \frac{3}{b^2}$
- If  $\frac{1}{b-a} + \frac{1}{b-c} = \frac{1}{a} + \frac{1}{c}$ , then prove that  $a, b, c$  are in H.P. unless  $b = a + c$ .
- If  $a, b, c, d$  are in H.P., then show that  $ab + bc + cd = 3ad$ .
- (A) Solve the equation  $6x^3 - 11x^2 + 6x - 1 = 0$  if its roots are in harmonic progression.  
(B) If the roots of  $10x^3 - cx^2 - 54x - 27 = 0$  are in harmonic progression, then find  $c$  and all the roots.
- If  $a, b, c$  are in G.P. and  $a - b, c - a$  and  $b - c$  are in H.P. then prove that  $a + 4b + c$  is equal to 0.
- (A) If  $\frac{1}{a(b+c)}, \frac{1}{b(c+a)}, \frac{1}{c(a+b)}$  be in H.P. then  $a, b, c$  are also in H.P.  
(B) If  $b + c, c + a, a + b$  are in H.P. then prove that  $\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$  are in A.P.  
(C) If  $a, b, c$  be in H.P. prove that  $\frac{a}{b+c-a}, \frac{b}{c+a-b}, \frac{c}{a+b-c}$  are in H.P.
- Let  $a_1, a_2, \dots, a_{10}$  be in A.P. and  $h_1, h_2, \dots, h_{10}$  be in H.P. If  $a_1 = h_1 = 2$  and  $a_{10} = h_{10} = 3$  then  $a_4 h_7$  is  
(A) 2 (B) 3 (C) 5 (D) 6
- If  $x > 1, y > 1, z > 1$  are in G.P., then  $\frac{1}{1+\ln x}, \frac{1}{1+\ln y}, \frac{1}{1+\ln z}$  are in  
(A) A.P. (B) H.P. (C) G.P. (D) None of these
- If  $a, b, c, d$  are in H. P., then  $ab + bc + cd$  is equal to  
(A)  $3ad$  (B)  $(a+b)(c+d)$  (C)  $3ac$  (D)  $(a+c)(b+d)$

**HARMONIC PROGRESSION - II**

- The value of  $n$  for which  $\frac{a^{n+1} + b^{n+1}}{a^n + b^n}$  is the harmonic mean of  $a$  and  $b$ , is equal to  
(A)  $-1$  (B)  $0$  (C)  $1/2$  (D)  $1$

2. The harmonic mean of the roots of the equation  $(5 + \sqrt{2})x^2 - (4 + \sqrt{5})x + 8 + 2\sqrt{5} = 0$  is  
(A) 2 (B) 4 (C) 6 (D) 8
3. If  $2(y - a)$  is the H.M. between  $y - x$  and  $y - z$ , then  $x - a$ ,  $y - a$ ,  $z - a$  are in  
(A) A.P. (B) G.P. (C) H.P. (D) none of these
4. If  $a, b, c$  are in H.P., then  $a^2(b - c)^2$ ,  $\frac{b^2}{4}(c - a)^2$ ,  $c^2(a - b)^2$  are in  
(A) H.P. (B) G.P. (C) A.P. (D) All of the above
5. If  $m$  is a root of the equation  $(1 - ab)x^2 - (a^2 + b^2)x - (1 + ab) = 0$  and  $m$  harmonic means are inserted between  $a$  and  $b$ , then the difference between the last and the first of the means equals  
(A)  $b - a$  (B)  $ab(b - a)$  (C)  $a(b - a)$  (D)  $ab(a - b)$
6. If positive number  $a, b, c$  are in A.P. and  $a^2, b^2, c^2$  are in H.P., then  
(A)  $a = b = c$  (B)  $2b = a + c$  (C)  $b^2 = \sqrt{\frac{ac}{8}}$  (D) none of these
7. If the  $p$ th term of an H.P. is  $qr$  and the  $q$ th term is  $rp$ , then the  $r$ th term of the H.P. is  
(A)  $pqr$  (B) 1 (C)  $pq$  (D)  $pqr^2$
8. If  $x, y, z$  are in A.P.,  $a, b, c$  are in H.P. and  $ax, by, cz$  are in G.P., then  $\frac{x}{z} + \frac{z}{x}$  is equal to  
(A)  $\frac{a}{c} - \frac{c}{a}$  (B)  $\frac{a}{c} + \frac{c}{a}$  (C)  $\frac{b}{a} + \frac{a}{b}$  (D)  $\frac{b}{c} - \frac{c}{b}$
9. If the first two terms of an H.P. are  $2/5$  and  $12/13$  respectively, then the largest term is  
(A) 5th term (B) 6th term (C) 10th term (D) none of these.
10. If  $H_1, H_2, \dots, H_n$  be  $n$  H.M.s between  $a$  and  $b$ , then  $\frac{H_1 + a}{H_1 - a} + \frac{H_n + b}{H_n - b}$  is equal to  
(A)  $n$  (B)  $2n$  (C)  $3n$  (D)  $4n$
11. If three numbers are in HP then the numbers obtained by subtracting half of the middle number from each of them are in  
(A) AP (B) GP (C) HP (D) none of these
12. If  $HM : GM = 4 : 5$  for two positive numbers then the ratio of the numbers is  
(A)  $4 : 1$  (B)  $3 : 2$  (C)  $3 : 4$  (D)  $2 :$
13. Insert two harmonic means between 5 and 11.
14. If 12 and  $9\frac{3}{5}$  are the geometric and harmonic means, respectively between two numbers, find them.
15. If between any two quantities there be inserted two arithmetic means  $A_1, A_2$ ; two geometric means  $G_1, G_2$ ; and two harmonic means  $H_1, H_2$ ; show that  $G_1G_2 : H_1H_2 = A_1 + A_2 : H_1 + H_2$ .
16. (A) If  $A$  be the A.M. and  $H$  the H.M. between two numbers  $a$  and  $b$ , then  $\frac{a-A}{a-H} \times \frac{b-A}{b-H} = \frac{A}{H}$ .  
(B) If 9 arithmetic and harmonic means be inserted between 2 and 3, prove that  $A + 6/H = 5$  where  $A$  is any of the A.M.'s and  $H$  the corresponding H.M.
17. Find  $\sum_{i=1}^{100} \frac{1}{H_i}$ . If  $H_1, H_2, \dots, H_{100}$  are HMs between 1 and  $1/100$

**RACE # 26**

**SEQUENCE & SERIES**

**MATHEMATICS**

- If  $t_n$  denotes the  $n$ th term of the series  $2 + 3 + 6 + 11 + 18 + \dots$  then  $t_{50}$  is  
(A)  $49^2 - 1$  (B)  $49^2$  (C)  $50^2 + 1$  (D)  $49^2 + 2$
- Let  $x = 1 + 3a + 6a^2 + 10a^3 + \dots$   $|a| < 1$ ,  $y = 1 + 4b + 10b^2 + 20b^3 + \dots$   $|b| < 1$ .  
Then  $S = 1 + 3(ab) + 5(ab)^2 + \dots$  in terms of  $x$  and  $y$  is  
(A)  $\frac{1 + (1 - x^{-1/3})(1 - y^{-1/4})}{\{1 - (1 - x^{-1/3})(1 - y^{-1/4})\}^2}$  (B)  $\frac{1 + (1 + x^{-1/3})(1 + y^{-1/4})}{\{1 - (1 + x^{-1/3})(1 + y^{-1/4})\}^2}$   
(C)  $\frac{1 + (1 - x^{-1/3})(1 - y^{-1/4})}{\{1 + (1 - x^{-1/3})(1 - y^{-1/4})\}^2}$  (D) None of these
- If  $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots$  upto  $\infty = \frac{\pi^2}{6}$  then,  $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots \infty =$   
(A)  $\frac{\pi^2}{6}$  (B)  $\frac{\pi^2}{8}$  (C)  $\frac{\pi^2}{4}$  (D)  $\pi^2$
- The sum of infinite terms of the series  $\frac{1}{1+1^2+1^4} + \frac{2}{1+2^2+2^4} + \frac{3}{1+3^2+3^4} + \dots \infty$  is :  
(A)  $\frac{1}{2}$  (B)  $\frac{1}{3}$  (C) 1 (D)  $\frac{1}{4}$
- The sum of the series  $1.3^2 + 2.5^2 + 3.7^2 + \dots$  upto 20 terms is  
(A) 188090 (B) 180890 (C) 189820 (D) None of these
- The sum of series  $\frac{3}{1^2} + \frac{5}{1^2+2^2} + \frac{7}{1^2+2^2+3^2} + \frac{9}{1^2+2^2+3^2+4^2} + \dots$  to  $n$  terms equals  
(A)  $\frac{6n}{n+1}$  (B)  $\frac{6n}{n^2+1}$  (C)  $\frac{n+1}{n^2+1}$  (D) None of these
- Sum to infinite of the series  $1 + \frac{2}{5} + \frac{3}{5^2} + \frac{4}{5^3} + \dots$  is  
(A)  $5/4$  (B)  $6/5$  (C)  $25/16$  (D)  $16/9$
- The sum of the infinite series  $\frac{1}{10} + \frac{2}{10^2} + \frac{3}{10^3} + \dots$  is equal to :  
(A)  $\frac{1}{9}$  (B)  $\frac{10}{81}$  (C)  $\frac{1}{8}$  (D)  $\frac{17}{72}$
- The sum of the series,  $1 + 2 \cdot \left(1 + \frac{1}{n}\right) + 3 \cdot \left(1 + \frac{1}{n}\right)^2 + \dots \infty$  is (where  $|n| > 1$ ).  
(A)  $n^2$  (B)  $n(n+1)$  (C)  $n \left(1 + \frac{1}{n}\right)^2$  (D)  $(n+1)(n+2)$
- Sum of infinite terms of the series  $\left[ \frac{1}{5} - \frac{2}{7^2} + \frac{3}{5^3} - \frac{4}{7^4} + \dots \right]$  is  
(A)  $\frac{211}{1152}$  (B)  $\frac{220}{1811}$  (C)  $\frac{2}{311}$  (D) None of these.



11. If the sum to infinity of the series  $1 + 4x + 7x^2 + 10x^3 + \dots$  is  $\frac{35}{16}$  then find x.
- (A)  $\frac{1}{5}$  (B)  $\frac{19}{7}$  (C)  $\frac{15}{12}$  (D) None of these
12. The sum of  $0.2 + 0.004 + 0.00006 + 0.0000008 + \dots$  to  $\infty$  is
- (A)  $\frac{200}{891}$  (B)  $\frac{2000}{9801}$  (C)  $\frac{1000}{9801}$  (D) None of these
13. The sum of the series  $\frac{1}{\log_2 4} + \frac{1}{\log_4 4} + \frac{1}{\log_8 4} + \dots + \frac{1}{\log_{2^n} 4}$  is
- (A)  $\frac{n(n+1)}{2}$  (B)  $\frac{n(n+1)+(2n+1)}{12}$  (C)  $\frac{1}{n(n+1)}$  (D)  $\frac{n(n+1)}{4}$
14. The sum to infinity of the series  $\frac{1}{2.4} + \frac{1}{4.6} + \frac{1}{6.8} + \frac{1}{8.10} + \dots$  is
- (A)  $1/4$  (B)  $1/8$  (C)  $1/2$  (D)  $1/16$
15. The sum of  $\frac{3}{1.2} \cdot \frac{1}{2} + \frac{4}{2.3} \cdot \left(\frac{1}{2}\right)^2 + \frac{5}{3.4} \cdot \left(\frac{1}{2}\right)^3 + \dots$  to n terms is equal to
- (A)  $1 - \frac{1}{(n+1)2^n}$  (B)  $1 - \frac{1}{n \cdot 2^{n-1}}$  (C)  $1 + \frac{1}{(n+1)2^n}$  (D) none of these
16. The value of  $\sum_{n=3}^{\infty} \frac{1}{n^5 - 5n^3 + 4n}$  is equal to
- (A)  $\frac{1}{120}$  (B)  $\frac{1}{96}$  (C)  $\frac{1}{24}$  (D)  $\frac{1}{144}$
17. The value of  $\sum_{r=1}^n \frac{1}{\sqrt{a+rx} + \sqrt{a+(r-1)x}}$  is
- (A)  $\frac{n}{\sqrt{a} - \sqrt{a+nx}}$  (B)  $\frac{\sqrt{a+nx} - \sqrt{a}}{x}$  (C)  $\frac{n(\sqrt{a+nx} - a)}{x}$  (D) None of these
18. If the sum  $\sum_{k=1}^{\infty} \frac{1}{(k+2)\sqrt{k} + k\sqrt{k+2}} = \frac{\sqrt{a} + \sqrt{b}}{\sqrt{c}}$  where a, b, c  $\in \mathbb{N}$  and lie in [1, 15], then a + b + c equals to
- (A) 6 (B) 8 (C) 10 (D) 11
19. The value of  $\frac{1}{1.3.5} + \frac{1}{3.5.7} + \frac{1}{5.7.9} + \frac{1}{9.11.13} + \dots$  equals
- (A)  $\frac{1}{12}$  (B)  $\frac{53}{249}$  (C)  $\frac{35}{429}$  (D)  $\frac{35}{249}$
20. The value of  $\sum_{k=1}^{\infty} \frac{6^k}{(3^k - 2^k)(3^{k+1} - 2^{k+1})}$  is
- (A) 1 (B) 2 (C) 3 (D) 4

**ANSWER KEY**

**RACE-23**

**ARITHMETIC PROGRESSION - I**

1.  $\log(ab^{n-1})$  2.  $25^{\text{th}}$  3.  $28^{\text{th}}$  7.  $-4, -1, 2 ; 2, -1, -4$  8.  $2, 6, 10, 14$   
9.  $2139$  10.  $70336$  11.  $740$  12.  $15$  13.  $25$  or  $36$  15.  $(14m - 6) : (8m + 23)$  18.  $x = \log_2 5$   
19.  $0$  20.  $852$  21. (C) 22. (A) 23. (C)

**ARITHMETIC PROGRESSION - II**

1. (A) 2. (B) 3. (A) 4. (D) 5. (C) 6. (B) 7. (C) 8. (C) 9. (A)  
10. (C) 11. (A) 12. (A) 13. (B) 14. (D) 15. (A) 16. (D) 17. (C) 18. (D)  
19. (B) 20. (A) 21.  $7, 11, 15$

**RACE-24**

**GEOMETRIC PROGRESSION - I**

2.  $16, 24, 36, \dots$  3.  $\frac{9}{7} \left( 1 - \left( \frac{4}{3} \right)^{2n} \right)$  4.  $2$  5.  $2$  6.  $2$  7.  $6, -3, 1\frac{1}{2}, \dots$   
8.  $8, 12, 18$  9.  $2, 6, 18$  10.  $40, 20, 10$  12.  $\frac{3}{5}$   
13. (a)  $\frac{x^{2n} - 1}{x^2 - 1} \left[ \frac{x^{2n+2} + 1}{x^{2n}} \right] + 2n$  (b)  $\frac{1}{(1-x)^2} [n(1-x) - x(1-x^n)]$   
15. (a)  $\frac{7n}{9} - \frac{7}{81} \left( 1 - \frac{1}{10^n} \right)$  (b)  $\frac{2}{27} [10^{n+1} - 9n - 10]$  15. (a)  $\frac{122}{990}$  (b)  $\frac{419}{990}$

**GEOMETRIC PROGRESSION - II**

1. (A) 2. (A) 3. (B) 4. (D) 5. (A) 6. (A) 7. (D) 8. (A) 9. (A)  
10. (D) 11.  $\frac{3}{2}, 1, \frac{2}{3}$  16.  $64, 16$

**RACE-25**

**HARMONIC PROGRESSION - I**

1.  $5$  2.  $3\frac{1}{2}$  8. (a)  $1, \frac{1}{2}, \frac{1}{3}$  (b)  $9, 3, -\frac{3}{2}, -\frac{3}{5}$  11. (D) 12. (B) 13. (A)

**HARMONIC PROGRESSION - II**

1. (A) 2. (B) 3. (B) 4. (D) 5. (B) 6. (A) 7. (C) 8. (B)  
9. (D) 10. (B) 11. (B) 12. (A) 13.  $6\frac{1}{9}, 7\frac{6}{7}$  14.  $6, 24$  17.  $5050$

**RACE-26**

1. (D) 2. (A) 3. (B) 4. (A) 5. (A) 6. (A) 7. (C) 8. (B)  
9. (A) 10. (A) 11. (A) 12. (B) 13. (D) 14. (A) 15. (A) 16. (B)  
17. (B) 18. (D) 19. (A) 20. (B)