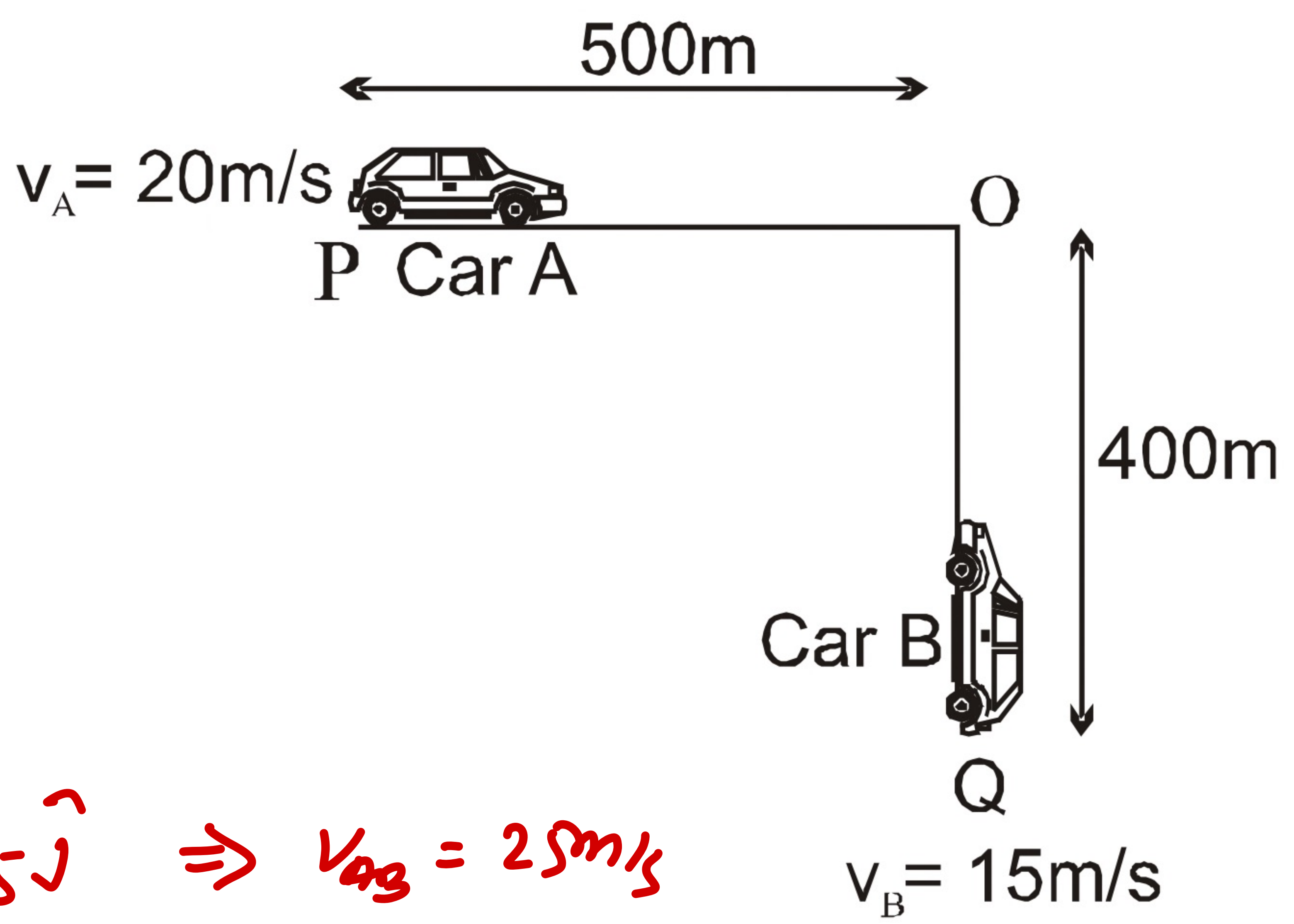
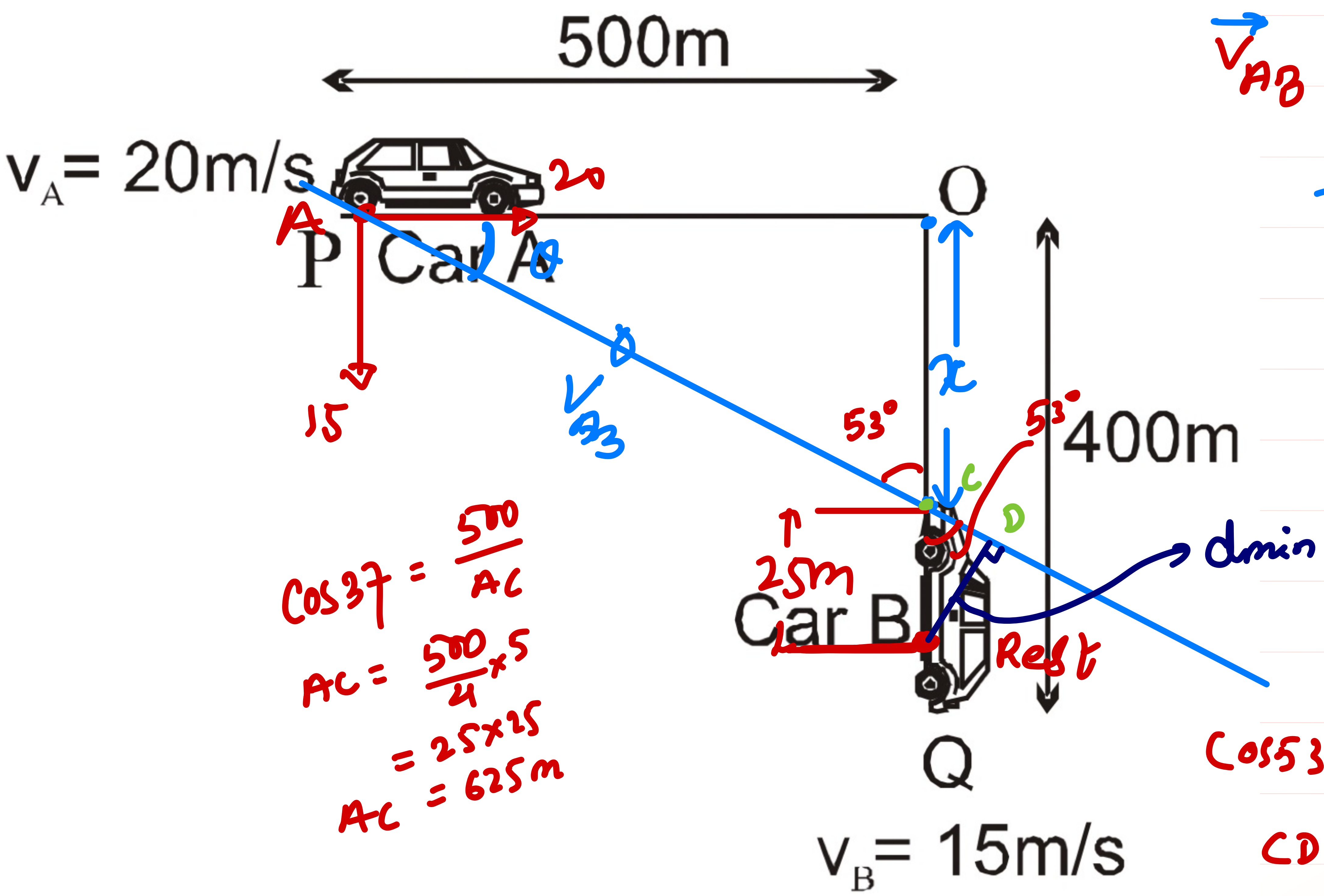
Illustration 2*. Two roads intersect at right angles. Car A is situated at P which is 500m from the intersection O on one of the roads. Car B is situated at Q which is 400m from the intersection on the other road. They start out at the same time and travel towards the intersection at 20m/s and 15m/s respectively. What is the minimum distance between them ?
How long do they take to reach it.





$\vec{V}_{AB} = 20\hat{i} - 15\hat{j} \Rightarrow V_{AB} = 25\,m/s$

$\tan\theta = \frac{15}{20} = \frac{3}{4} = \frac{x}{500} \Rightarrow x = \frac{3}{4} \times 500 = 375\,m$

$\cos 37 = \frac{500}{AC}$

$AC = \frac{500}{4} \times 5$

$= 25 \times 25$

$AC = 625\,m$

$\underline{\triangle CDB}$

$\sin 53 = \frac{d_{min}}{25}$

$\frac{4}{5} \times 25 = d_{min}$

$\boxed{d_{min} = 20\,m}$ Ans

$\cos 53 = \frac{CD}{25}$

$CD = 25 \times \frac{3}{5} = 15\,m$

$time = \frac{AC + CD}{V_{AB}}$

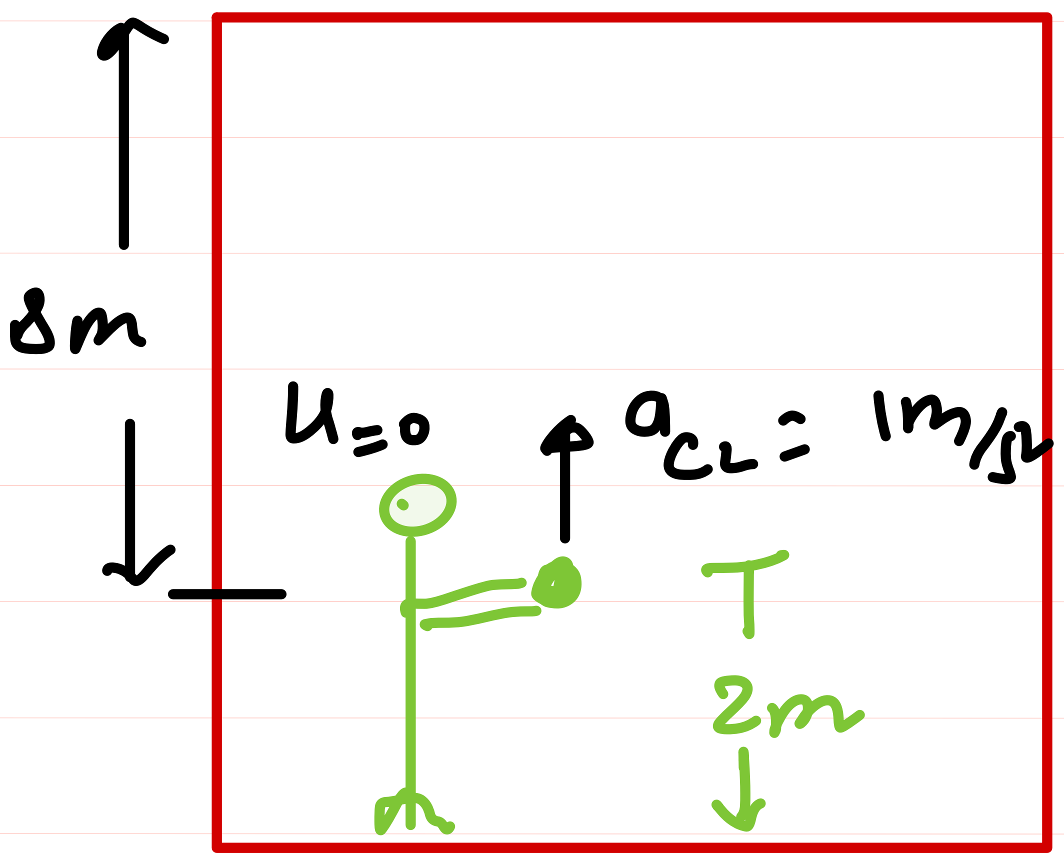
$= \frac{625 + 15}{25}$

$= 25 + \frac{3}{5}$

$\boxed{time = 25.6\ sec}$ Ans

3*. A coin is released inside a lift at a height of 2 m from the floor of the lift. The height of the lift is 10 m. The lift is moving with an acceleration of 11 m/s^2 downwards. The time after which the coin will strike with the lift is :

(A) 4 s (B) 2 s (C) $\frac{4}{\sqrt{21}}$ s (D) $\frac{2}{\sqrt{11}}$ s



$\downarrow a_L = 11 \text{ m/s}^2$

Motion of coin w.r.t Lift

$V_{CL} = V - V = 0$

$a_{CL} = (-g) - (-11) = -10 + 11$

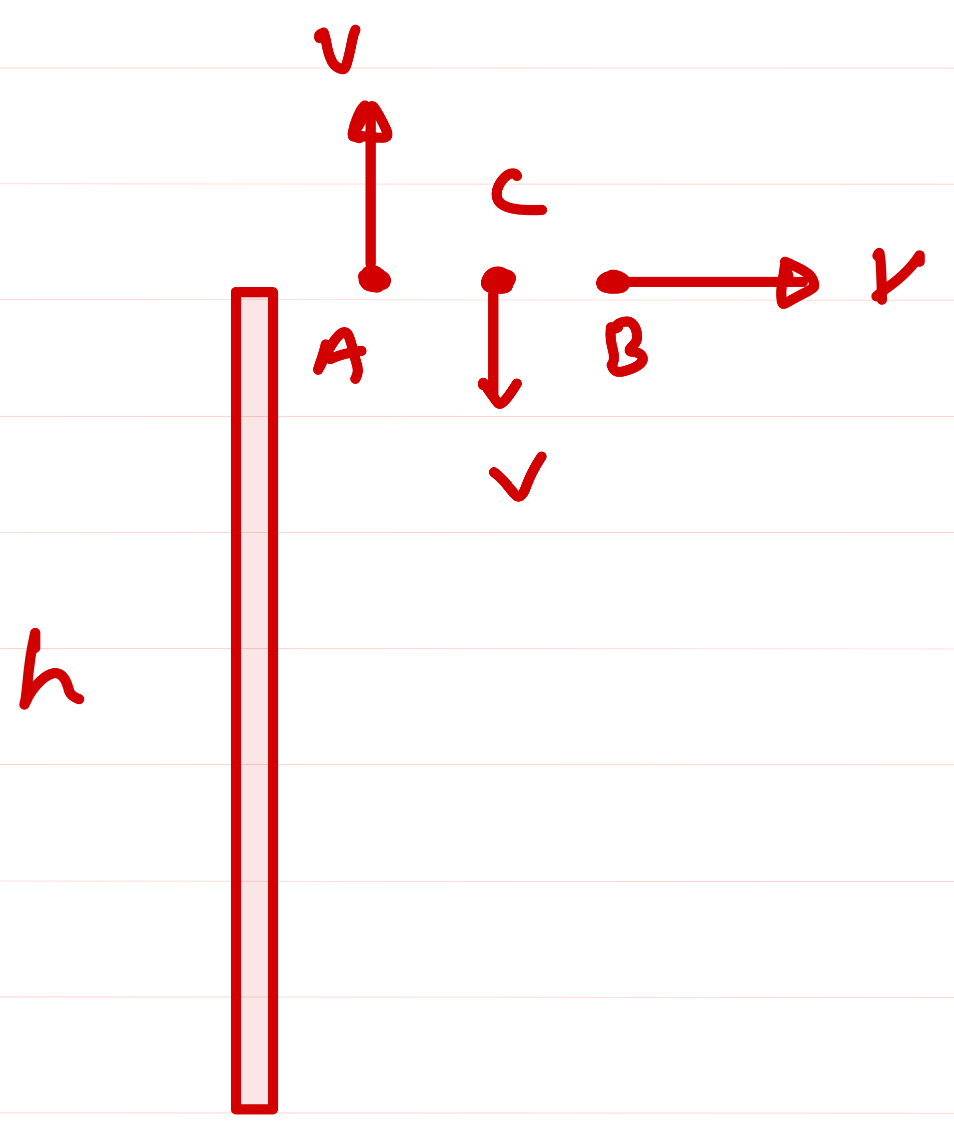
$= +1 \text{ m/s}^2$

$d_{CL} = 8 = 0 \cdot t + \frac{1}{2} \times a_{CL} \cdot t^2$

$8 = \frac{1}{2} \times 1 \; t^2 \Rightarrow \boxed{t = \sqrt{16} = 4 \text{ sec}}$ Ans

6*. Three stones A, B and C are simultaneously projected from same point with same speed. A is thrown upwards, B is thrown horizontally and C is thrown downwards from a building. When the distance between stone A and C becomes 10 m, then distance between A and B will be :

(A) 10 m (B) 5 m (C) $5\sqrt{2}$ m (D) $10\sqrt{2}$ m



Motion A w.r.t C

$V_{AC} = v - (-v) = 2v$

$a_{AC} = (-g) - (-g) = 0$

$d_{AC} = 2v \times t - 0$

$10 = 2v\,t$

$\boxed{t = \frac{5}{v}}$

Motion of A w.r.t B

$\vec{V}_{AB} = (v\hat{j}) - v\hat{i}$

$\vec{a}_{AB} = (-g) - (-g) = 0$

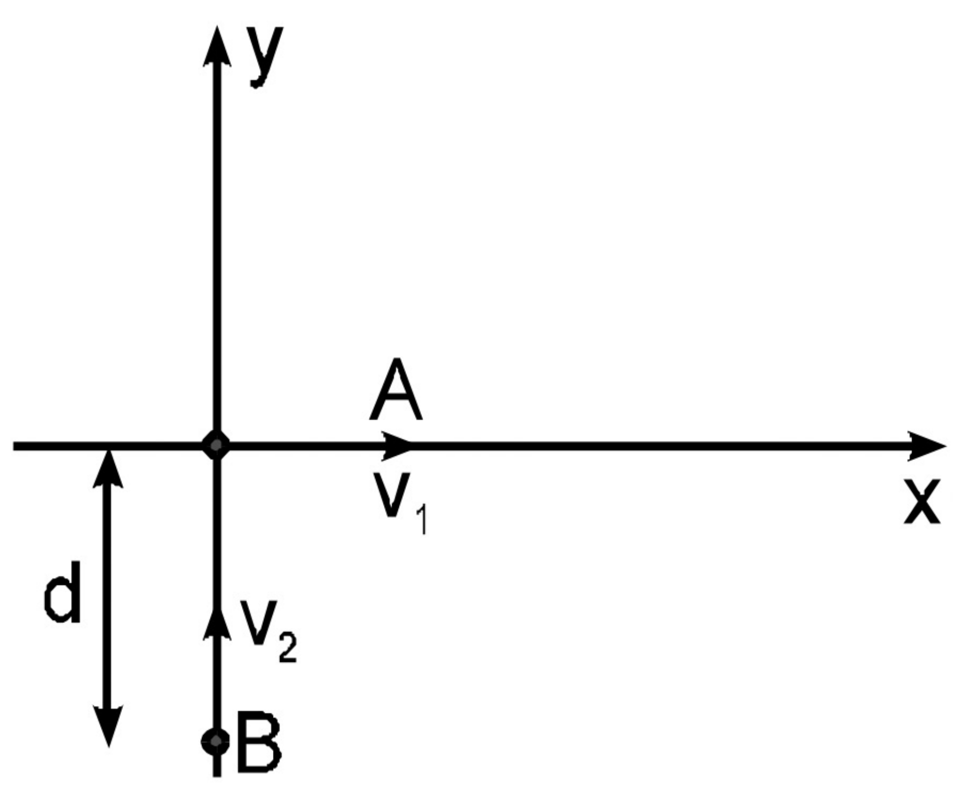
$V_{AB} = \sqrt{2}v$

$d_{AB} = \sqrt{2}v \times t$

$d_{AB} = \sqrt{2}v \times \frac{5}{v}$

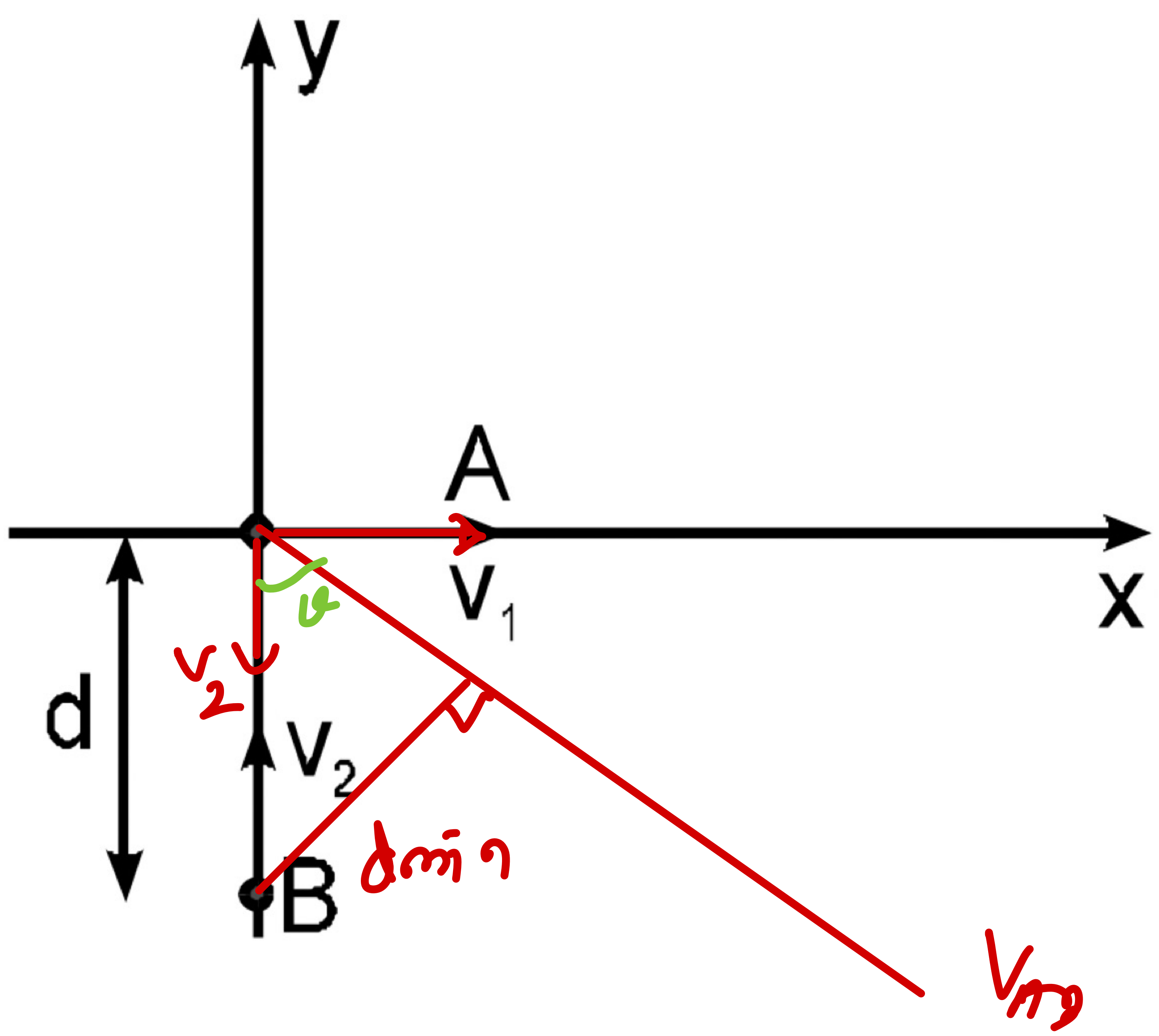
$\boxed{d_{AB} = 5\sqrt{2}m}$ Ans

7*. Two particles A and B move with velocities v_1 and v_2 respectively along the x & y axis. The initial separation between them is ' d ' as shown in the fig. Find the least distance between them during their motion.



(A) $\frac{d.v_1^2}{v_1^2+v_2^2}$ (B) $\frac{d.v_2^2}{v_1^2+v_2^2}$

(C) $\frac{d.v_1}{\sqrt{v_1^2+v_2^2}}$ (D) $\frac{d.v_2}{\sqrt{v_1^2+v_2^2}}$



$\vec{V}_{AB} = v_1\hat{i} - (v_2\hat{j})$

$\tan\theta = \frac{v_1}{v_2}$

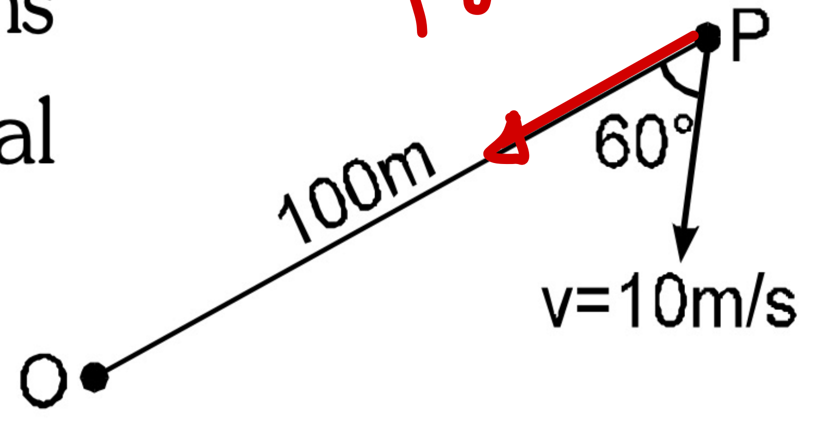
$\sin\theta = \frac{d_{min}}{d}$

$\boxed{d_{min} = \frac{d\,v_1}{\sqrt{v_1^2+v_2^2}}}$

Ans

8*. P is a point moving with constant speed 10 m/s such that its velocity vector always maintains an angle 60° with line OP as shown in figure (O is a fixed point in space). The initial distance between O and P is 100 m. After what time shall P reach O.

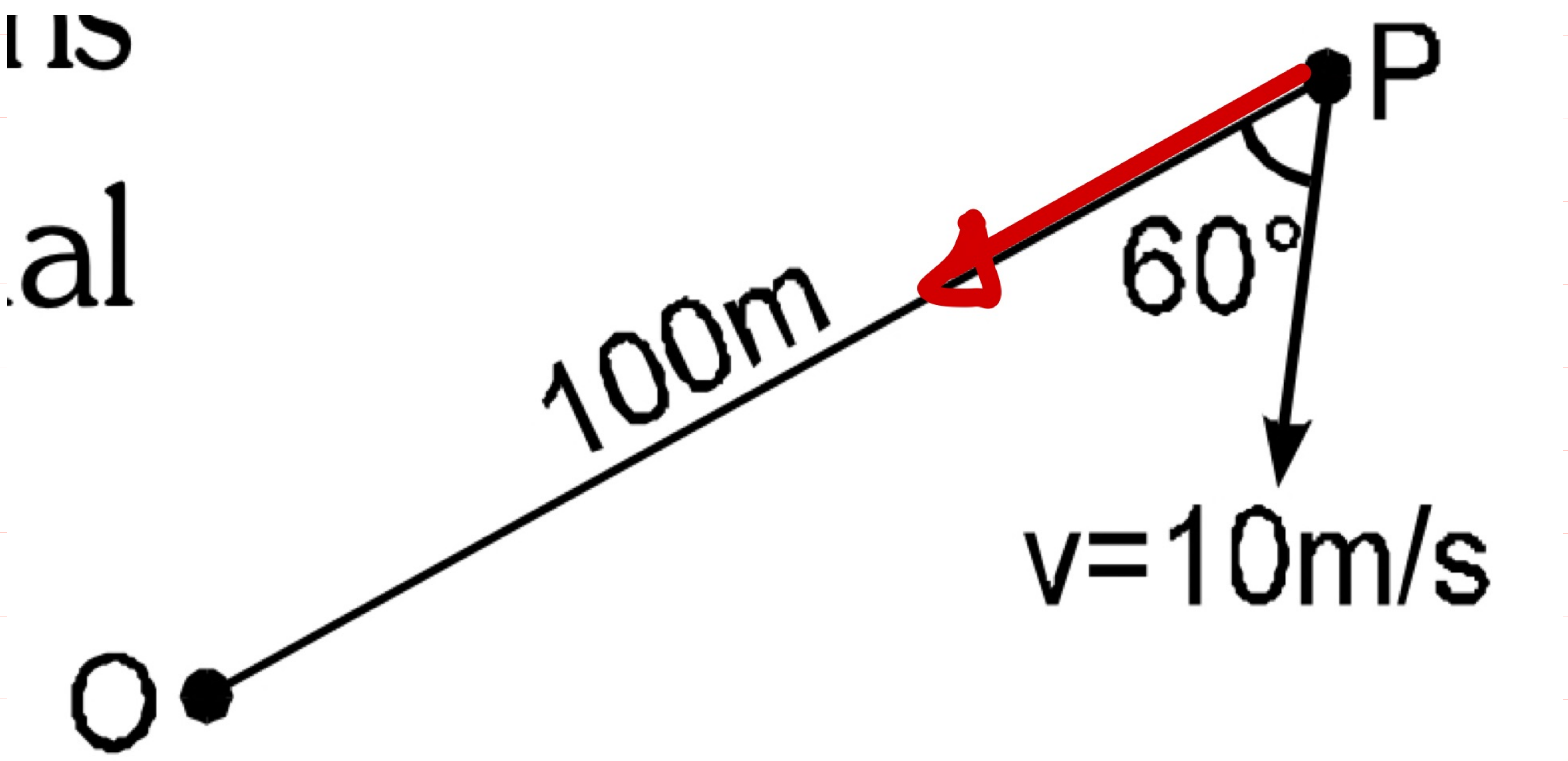
$10\cos 60 = 5\,m/s$



(A) 10 sec. (B) 15 sec.

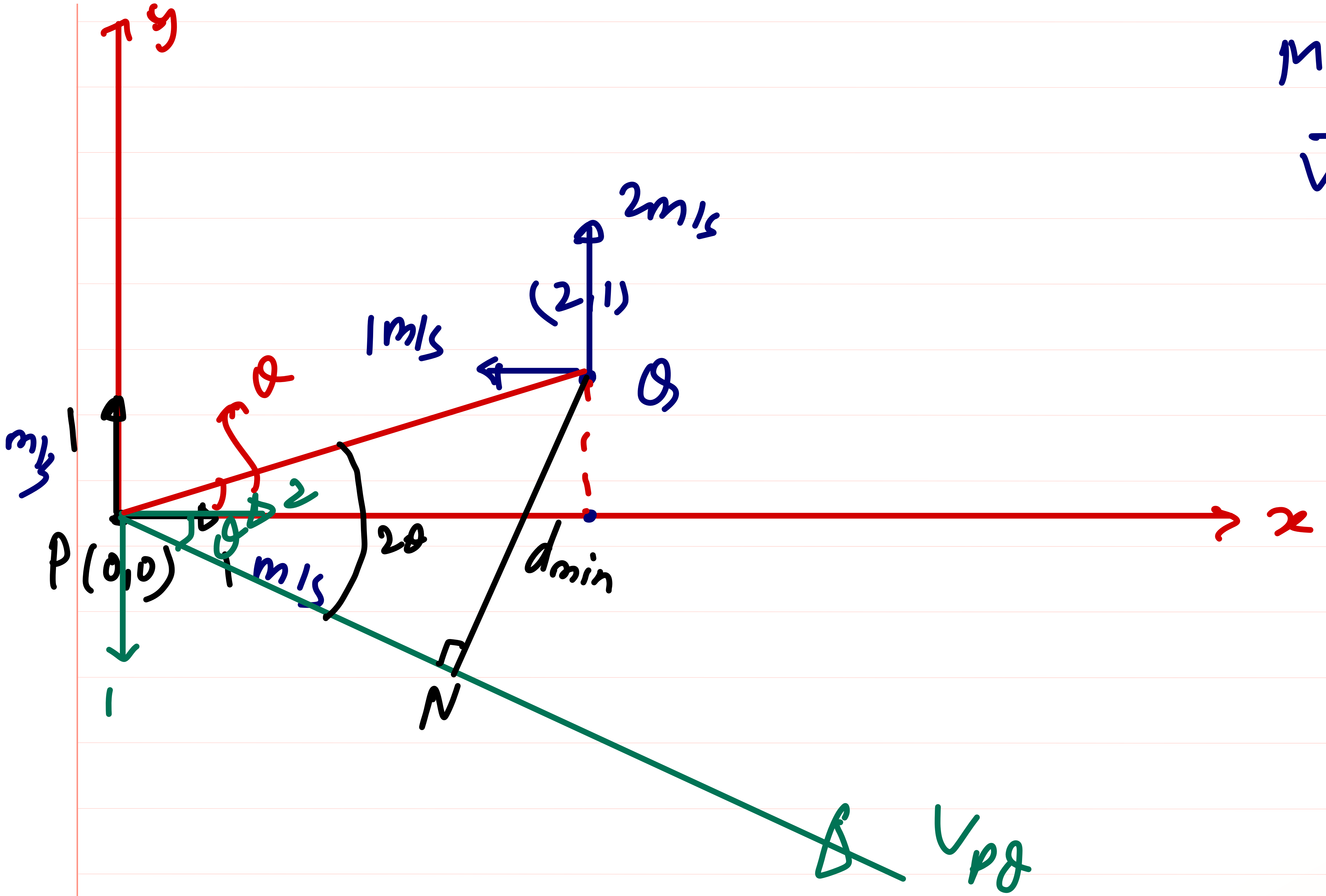
(C) 20 sec. (D) $20\sqrt{3}$ sec

$$t = \frac{d}{v_{\parallel}} = \frac{100}{5} = 20\,sec$$



8. Two particles P and Q are moving with velocities of $(\hat{i}+\hat{j})$ and $(-\hat{i}+2\hat{j})$ respectively. At time t = 0, P is at origin and Q is at a point with position vector $(2\hat{i}+\hat{j})$. Then the shortest distance between P & Q is :-

(A) $\frac{2\sqrt{5}}{5}$ (B) $\frac{4\sqrt{5}}{5}$ (C) $\sqrt{5}$ (D) $\frac{3\sqrt{5}}{5}$

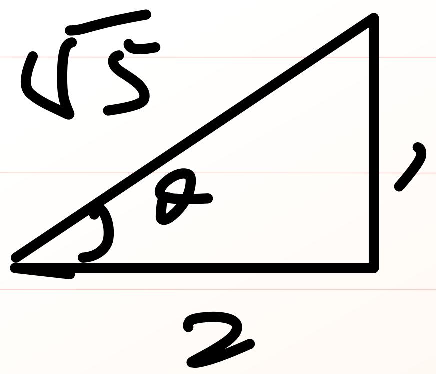


Motion of P w.r.t Q

$\vec{V}_{PQ} = (\hat{i}+\hat{j}) - (-\hat{i}+2\hat{j})$

$= (2\hat{i}-\hat{j})$

$\tan\theta = \frac{1}{2}$



ΔPNQ

$\sin 2\theta = \frac{d_{min}}{\sqrt{5} = PQ}$

$2\sin\theta\cos\theta = \frac{d_{min}}{\sqrt{5}}$

$2\times\frac{1}{\sqrt{5}}\times\frac{2}{\cancel{\sqrt{5}}} = \frac{d_{min}}{\cancel{\sqrt{5}}}$

$\boxed{\frac{4}{\sqrt{5}} = d_{min}}$ Ans

BB # 1

11. Two particles A and B moving in x-y plane are at origin at t = 0 sec. The initial velocity vectors of A and B are $\bar{u}_A = 8\hat{i}$ m/s and $\bar{u}_B = 8\hat{j}$ m/s. The acceleration of A and B are constant and are $\bar{a}_A = -2\hat{i}$ m/s^2 and $\bar{a}_B = -2\hat{j}$ m/s^2. Column I gives certain statements regarding particle A and B. Column II gives corresponding results. Match the statements in column I with corresponding results in Column II.

Columns I	***Column II***
S (A) The time (in seconds) at which velocity of A relative to B is zero	(p) $16\sqrt{2}$
P (B) The distance (in metres) between A and B when their relative velocity is zero.	(q) $8\sqrt{2}$
r (C) The time (in seconds) after t = 0 sec. at which A and B are at same position	(r) 8
q (D) The magnitude of relative velocity of A w.r. to and B at the instant when they are at same position.	(s) 4
	(t) 6 seconds

at time t

$\vec{V}_A = 8\hat{i} - 2\hat{i}t = (8-2t)\hat{i}$

$\vec{V}_B = 8\hat{j} - 2\hat{j}t = (8-2t)\hat{j}$

(A) $\vec{V}_{AB} = 0$

$\vec{V}_A = \vec{V}_B$

$(8-2t)\hat{i} = (8-2t)\hat{j}$

$8-2t=0 \Rightarrow \boxed{t = 4\,sec}$

(B) $\vec{d}_{AB} = \vec{V}_{AB}t + \frac{1}{2}\vec{a}_{AB}t^2$

$= 0.4 + \frac{1}{2}(-2\hat{i}+2\hat{j}) \times 4^2$

$\boxed{d_{AB} = \sqrt{2} \times 16\,m}$

(C) $d_{AB} = 0 = (8\hat{i} - 8\hat{j})t + \frac{1}{2}(-2\hat{i}+2\hat{j})t^2$

$0 = (8t - t^2)\hat{i} + (-8t + t^2)\hat{j}$

$8t - t^2 = 0 \Rightarrow \boxed{t = 8\,sec}$

(d) $\vec{V}_{AB} = (8-2t)\hat{i} - (8-2t)\hat{j}$

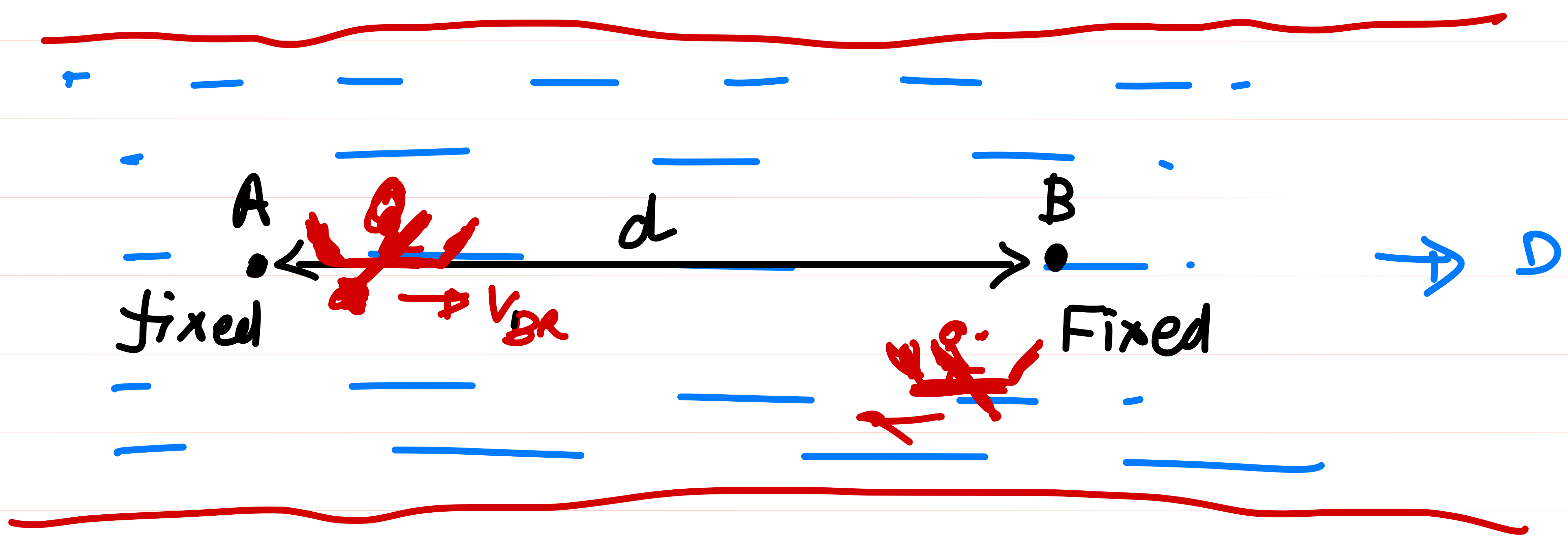
$V_{AB} = |-8\hat{i} + 8\hat{j}| = 8\sqrt{2}$ m/s

River-Boat Problems

Case ① Motion along / opposite to flow of River

V_r = velocity of river

V_{br} = velocity of boat w.r.t River
OR
velocity of boat in still water



→ Dir^n of flow of River

$\vec{V}_{BR} = \vec{V}_B - \vec{V}_R$

$\vec{V}_B = \vec{V}_{BR} + \vec{V}_R$

time taken from A to B (down Stream motion)

$$\boxed{t_1 = \frac{d}{V_B} = \frac{d}{V_{BR} + V_R}}$$

time taken from B to A (upstream motion)

$$\boxed{t_2 = \frac{d}{V_B} = \frac{d}{V_{BR} - V_R}}$$

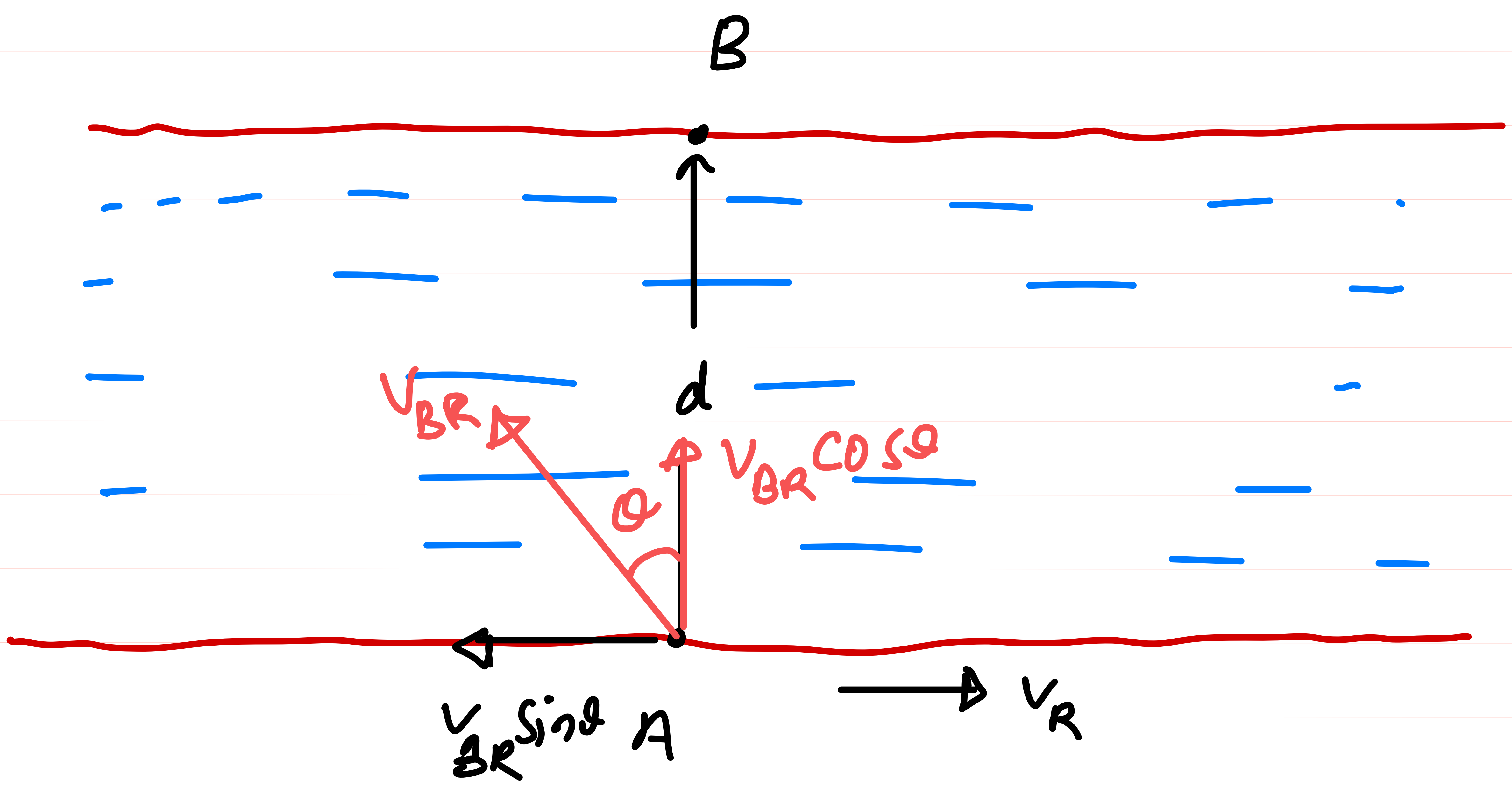
total time of a round trip

$= t_1 + t_2$

$$\boxed{t = \frac{2d\, V_{BR}}{V_{BR}^2 - V_R^2}}$$

Case-② Motion at angle θ from vertical :→

$\because \vec{V}_B = \vec{V}_{BR} + \vec{V}_R$



Dirn of flow (V_R)

① Time taken to cross River

$$\boxed{t = \frac{d}{V_{BR}\cos\theta}}$$

Case minimum time to cross River

For $t \to$ min

$\cos\theta = 1$

$\boxed{\theta = 0^\circ}$ From vertical

90° From flow

$$\boxed{t_{min} = \frac{d}{V_{BR}}}$$