Fundamentals of Maths

Lecture - 1

Number System :-

(1) No. sy

- → Rational
  - → $\frac{p}{q}$ form
  - → terminating decimal
  - → Non- terminating recurring decimal.
- → irrational → Non terminating non recurring.

Irr ← $\boxed{\pi} \neq \boxed{\frac{22}{7}}$ → Rational

π → ~~3.14~~ approx

e = Napier's constant

$e \approx 2.71$

→ Irrational no.

(1) Natural nos: $\Rightarrow$ $\{1, 2, 3, 4, \ldots\ldots\}$
(N)

(2) Whole no :- (W, N_0) $\{0, 1, 2, 3, \ldots\}$

(3) Integer (I, Z) :- $\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

(4) Positive int (I^+) :- $\{1, 2, 3, 4, \ldots\}$

(5) Negative int (I^-) : $\longrightarrow$ $\{\ldots -3, -2, -1\}$

(6) Non-negative int $\{0, 1, 2, 3, \ldots\}$

(7) Non-positive int $\{\ldots -3, -2, -1, 0\}$

(8) Even int :- $\{\ldots, -6, -4, -2, 0, 2, 4, 6, \ldots\}$

(9) odd int :- $\{\ldots -5, -3, -1, 1, 3, 5, \ldots\}$

Note:

(i) zero is neither positive nor negative.

(ii) zero is an even number

(iii) Positive mean greater than 0 (>0)

(iv) Non-negative mean (greater than or equal to 0)

(≥ 0)

Fraction $\left(\frac{p}{q}\right)$

$N^r \rightarrow$ Numerator

$D^r \rightarrow$ denominator

ⓐ Proper fraction $N^r < D^r$

$\frac{3}{5}, \frac{7}{9}, \frac{8}{15}, \frac{100}{107}$

$\frac{2}{3} = \boxed{0.6666}$

$\left(\frac{2}{3}\right)^2 = \frac{4}{9} = 0.4444$

$\frac{2}{3} = 0.6666$

$\frac{2+1}{3+1} = \frac{3}{4} = 0.75$

$\frac{3+1}{4+1} = \frac{4}{5} = 0.80$

(b) Improper fraction $N^r > D^r$

$$\frac{5}{3}, \frac{7}{4}, \frac{9}{2}, \ldots$$

$$\left(\frac{5}{3}\right)^2 = \frac{25}{9} = 2\ldots$$

$$\frac{5}{3} = \underbrace{1.6666}$$

$$\frac{5+1}{3+1} = \frac{6}{4} = 1.50$$

$$\frac{6+1}{4+1} = \frac{7}{5} = 1.2$$

(c) Continued fraction

$$2 + \cfrac{2}{2 + \cfrac{2}{2 + \cfrac{2}{\cdots \infty}}}$$

$$x = 2 + \cfrac{2}{\boxed{2 + \cfrac{2}{2 + \cfrac{2}{\cdots}}}} \Rightarrow x = 2 + \frac{2}{x}$$

$$x^2 = 2x + 2$$

$$x^2 - 2x - 2 = 0$$

3. **RATIONAL NUMBERS (Q, $\mathbb{Q}$) :**

All the numbers that can be represented in the form p/q, where p and q are integers and $q \neq 0$ and gcd(p, q) = 1, are called rational numbers. Integers, Fractions, Terminating decimal numbers, Non-terminating but repeating decimal numbers are all rational numbers.

$$Q = \left\{\frac{p}{q} : p, q \in I, q \neq 0 \text{ and } \gcd(p,q)=1\right\}$$

Note :

(i) Integers are rational numbers, but converse need not be true.

(ii) A rational number always exists between two distinct rational numbers, hence infinite rational numbers exist between two rational numbers.

IRRATIONAL NUMBERS (Q^C) :

There are real numbers which can not be expressed in p/q form. Non-Terminating non repeating decimal numbers are irrational number e.g. $\sqrt{2}, \sqrt{5}, \sqrt{3}, \sqrt[3]{10}$; e, π.

Note: (Rational $\rightarrow Q$; Irrational $\rightarrow Q^c$)

(i) Sum of a rational and an irrational number is irrational.

eg: $2+\sqrt{3}$; $3+\sqrt{7}$, - -

(ii) If $a \in Q$ and $b \notin Q$ then ab = rational if $a=0$

(iii) Sum, difference, product and quotient of two irrational numbers need not be an irrational number or we can say, result may be a rational number also.

(iv) Nos. π and e are known as transcendental numbers which comes under the category of irrational numbers.

(v) Number $\pi \approx \frac{22}{7}$; $e \approx 2.71$

Real Nos :-

The complete set of rational and irrational number is the set of real nos. $R = Q \cup Q^c$. The real nos. can be represented as a position of a point on the real number line.

Complex Numbers :-

Number in the form $a + ib$; $i = \sqrt{-1}$ $\boxed{i \rightarrow \text{iota}}$

where $a, b \in \underline{R}$

Complex numbers are usually denoted by z and set of complex numbers is denoted by

$$C = \{(x + iy) : x, y \in R ; i = \sqrt{-1}\}$$

Note : N C W C I C Q C R

Even Int No. div by 2 (represented by $2n$) ; $n \in \mathbb{Z}$.
$(0, 2, 4, 6, 8, ----)$ or unit digit $(0, 2, 4, 6, 8)$

Odd int No. not div by 2 (rep by $2n+1$ or $2n-1$) $n \in \mathbb{Z}$.
unit digit $(1, 3, 5, 7, 9)$.

Prime numbers : — Let p is a natural no., p is said to be prime if it has exactly two distinct positive integral factors. (namely 1 and itself).
eg: $2, 3, 5, 7, ----$

Composite Numbers :-

No. with more than two factors. $\{4, 6, 9, 15, ---\}$

Note:

(i) 1 is neither prime nor composite.

(ii) 2 is the only even prime no.

(iii) 4 is smallest composite no.

(iv) Natural nos. (except 1) ; which are not prime are composite numbers.

Coprime nos. / relatively prime nos:-

Two natural nos. (not necessarily prime) are coprime, if their HCF is 1.

(4, 9) ; (1, 2) ; (3, 4) - - -

Note:-

(i) Two distinct prime numbers are always coprime but converse need not be true.

(ii) Consecutive natural nos. are always coprime nos.

Twin Prime numbers:-

If the difference between two prime numbers is two, then the nos. are twin prime numbers.

eg: (3, 5) ; (5, 7) ; - - - (11, 13) ; (59, 61) ; - - -

Prime triplet :- $\{3, 5, 7\}$

Q find the integral solutions of $xy = 2x - y$

$$xy - 2x = -y$$

$$2x - xy = y$$

$$x = \frac{y}{2-y}$$

$$x = -1 + \frac{\boxed{2}}{2-y}$$

x	0	1	-3	-2
y	0	1	3	4

$2-y \,\overline{)\; y \;(}\; -1$

$\quad y - 2$

$\quad \underline{- \quad +}$

$\quad\quad 2$

$$x = -1 + \frac{2}{2-y}$$

$2-y \rightarrow 1, 2, -1, -2$

$2-y = -2$

$y = 4$

Note:

(i) Square of a real number is always non-negative (i.e. $x^2 \geq 0$)

(ii) Square root of a positive number is always positive. $\sqrt{4} = 2$

(iii) $\sqrt{x^2} = |x|$; $x \in R$

$x = \sqrt{4} \Rightarrow x = 2$ ✓

$\cancel{x = \pm 2}$

$x^2 = 4 \Rightarrow x = \pm\sqrt{4}$

$x = \pm 2$

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Algebraic identities :-

1) $(a+b)^2 = a^2 + 2ab + b^2$

2) $(a-b)^2 = a^2 - 2ab + b^2$

3) $a^2 - b^2 = (a+b)(a-b)$

4) $(a+b)^3 = \underline{a^3 + b^3} + 3ab(a+b)$

5) $(a-b)^3 = \underline{a^3 - b^3} - 3ab(a-b)$

6) $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

7) $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

8) $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

9) $a^2 + b^2 + c^2 - ab - bc - ca = \frac{1}{2}\left[(a-b)^2 + (b-c)^2 + (c-a)^2\right]$

Answers

(1) If $\dfrac{3+2\sqrt{2}}{3-\sqrt{2}} = a+b\sqrt{2}$; where a and b are rationals. find the values of a and b. — $a = 13/7$; $b = 9/7$

(2) If $x = \dfrac{1}{2+\sqrt{3}}$; find the value of $x^3 - x^2 - 12x + 4$. — $-1+\sqrt{3}$

(3) If $x = \dfrac{1}{2+\sqrt{3}}$; find the value of $x^3 - x^2 - 11x + 4$. — 1

(4) If $x = 3 - 2\sqrt{2}$; find $x^2 + \dfrac{1}{x^2}$. — 34

(5) If $x = 1 - \sqrt{2}$; find $\left(x - \dfrac{1}{x}\right)^3$. — 8

(6) If $x = \dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$ and $y = \dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$; find $x^2 + y^2$. — 98

(7) Rationalise the denominator of $\dfrac{1}{\sqrt{3}-\sqrt{2}-1}$ — $-\dfrac{1}{4}(\sqrt{6}+\sqrt{2}+2)$

(8) Rationalize the denominator of $\dfrac{12}{3+\sqrt{5}-2\sqrt{2}}$ — $1+\sqrt{5}+\sqrt{10}-\sqrt{2}$

Fundamentals of Maths

Lecture - 2

Algebraic identities :-

1) $(a+b)^2 = a^2 + 2ab + b^2$

2) $(a-b)^2 = a^2 - 2ab + b^2$

3) $a^2 - b^2 = (a+b)(a-b)$

4) $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

5) $(a-b)^3 = a^3 - b^3 - 3ab(a-b)$

6) $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

7) $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

8) $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

9) $a^2 + b^2 + c^2 - ab - bc - ca = \frac{1}{2}\left[(a-b)^2 + (b-c)^2 + (c-a)^2\right]$

$\frac{1}{2}(2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ac)$

$$\frac{1}{2}\left[\begin{array}{l} a^2 + b^2 - 2ab \\ b^2 + c^2 - 2bc \\ c^2 + a^2 - 2ac \end{array}\right]$$

$= \frac{1}{2}\left[(a-b)^2 + (b-c)^2 + (c-a)^2\right]$

$(a+b)^2(a+b)$

$a^2 + b^2 = (a+b)^2 - 2ab$

$a^2 + b^2 = (a-b)^2 + 2ab$

(10) $a^3+b^3+c^3-3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ac)$

$= \frac{1}{2}(a+b+c)\left[\underbrace{(a-b)^2+(b-c)^2+(c-a)^2}\right]$

if $a+b+c=0$

then $a^3+b^3+c^3-3abc=0$

$a^3+b^3+c^3 = 3abc$

≤ 0

$\underbrace{\leq}$

$<$ or $=$

(11) $(a+b+c)^3 = a^3+b^3+c^3+3(a+b)(b+c)(c+a)$

(12) $\underbrace{a^4+a^2+1} = a^4+2a^2+1-a^2$

$= (a^2+1)^2-a^2$

$= (a^2+1+a)(a^2+1-a)$

Note: (1) If $a,b,c \in R$, then $\underbrace{a^2+b^2+c^2-ab-bc-ca} \geq 0$

(2) If $a^2+b^2+c^2-ab-bc-ca=0$ or ≤ 0 then $a=b=c$

(7) $$\frac{1}{\sqrt{3}-\sqrt{2}-1} = \frac{1}{\sqrt{3}-(\sqrt{2}+1)} \cdot \frac{\sqrt{3}+(\sqrt{2}+1)}{\sqrt{3}+(\sqrt{2}+1)}$$

$$= \frac{\sqrt{3}+\sqrt{2}+1}{(\sqrt{3})^2-(\sqrt{2}+1)^2} = \frac{(\sqrt{3}+\sqrt{2}+1)}{3-(2+1+2\sqrt{2})}$$

$$= \frac{-(\sqrt{3}+\sqrt{2}+1)}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = -\frac{1}{4}(\sqrt{6}+2+\sqrt{2})$$

(4) $$x = 3-2\sqrt{2}$$

$$\frac{1}{x} = \frac{1}{3-2\sqrt{2}} \cdot \frac{3+2\sqrt{2}}{3+2\sqrt{2}} = \frac{3+2\sqrt{2}}{9-8} = 3+2\sqrt{2}$$

$$x^2+\frac{1}{x^2} = \left(x+\frac{1}{x}\right)^2 - 2\cdot x\cdot\frac{1}{x} = \left(x+\frac{1}{x}\right)^2 - 2 = (3-2\sqrt{2}+3+2\sqrt{2})^2 - 2$$

$$= 34$$

$$a^2-b^2 = (a+b)(a-b)$$

(2) $x = \frac{1}{2+\sqrt{3}} \cdot \frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{2-\sqrt{3}}{4-3} = 2-\sqrt{3}$

$x = 2-\sqrt{3} \Rightarrow x-2 = -\sqrt{3} \Rightarrow (x-2)^2 = 3$

$x^2+4-4x = 3$

$x^2-4x = -1$

$x^3 - x^2 - 12x + 4$

$x^3 (x^2-4x) - 16x + 4$

$x^2+1 = 4x$

(8) $\frac{12}{3+\sqrt{5}-2\sqrt{2}} \cdot \frac{(3+\sqrt{5})+(2\sqrt{2})}{(3+\sqrt{5})+2\sqrt{2}} = \frac{12(3+\sqrt{5}+2\sqrt{2})}{(3+\sqrt{5})^2-(2\sqrt{2})^2}$

$= \frac{12(3+\sqrt{5}+2\sqrt{2})}{9+5+6\sqrt{5}-8} = \frac{12(3+\sqrt{5}+2\sqrt{2})}{6+6\sqrt{5}}$

$= \frac{\cancel{12}^{2}(3+\sqrt{5}+2\sqrt{2})}{\cancel{6}(1+\sqrt{5})} = \frac{2(3+\sqrt{5}+2\sqrt{2})}{(\sqrt{5}+1)} \cdot \frac{(\sqrt{5}-1)}{(\sqrt{5}-1)}$

(9) Simplify

$$\frac{2+\sqrt{3}}{\sqrt{2}+\sqrt{2+\sqrt{3}}}+\frac{2-\sqrt{3}}{\sqrt{2}-\sqrt{2-\sqrt{3}}}$$

$$\frac{2+\sqrt{3}}{\sqrt{2}+\sqrt{2+\sqrt{3}}}\cdot\frac{\sqrt{2}-\boxed{\sqrt{2+\sqrt{3}}}}{\sqrt{2}-\sqrt{2+\sqrt{3}}}+\frac{2-\sqrt{3}}{\sqrt{2}-\sqrt{2-\sqrt{3}}}\cdot\frac{\sqrt{2}+\sqrt{2-\sqrt{3}}}{\sqrt{2}+\sqrt{2-\sqrt{3}}}\ ✓$$

$$\frac{(2+\sqrt{3})\left(\sqrt{2}-\left(\frac{\sqrt{3}}{\sqrt{2}}+\frac{1}{\sqrt{2}}\right)\right)}{2-(2+\sqrt{3})}+\frac{(2-\sqrt{3})\left(\sqrt{2}+\left(\frac{\sqrt{3}}{\sqrt{2}}-\frac{1}{\sqrt{2}}\right)\right)}{2-(2-\sqrt{3})}$$

$$\frac{(2+\sqrt{3})(2-\sqrt{3}-1)}{-\sqrt{3}\sqrt{2}}+\frac{(2-\sqrt{3})(2+\sqrt{3}-1)}{\sqrt{3}\sqrt{2}}$$

$$=\frac{1}{\sqrt{6}}\left[-(2+\sqrt{3})(1-\sqrt{3})+(2-\sqrt{3})(1+\sqrt{3})\right]$$

$$=\frac{1}{\sqrt{6}}\left[-\cancel{2}-\sqrt{3}+2\sqrt{3}+\cancel{3}+\cancel{2}-\sqrt{3}+2\sqrt{3}-\cancel{3}\right]=\frac{2\sqrt{3}}{\sqrt{6}}=\sqrt{2}$$

$$\boxed{\sqrt{2-\sqrt{3}}}=\boxed{\sqrt{a}-\sqrt{b}}$$

$$2-\sqrt{3}=a+b-2\sqrt{ab}$$

$$a+b=2$$

$$2\sqrt{ab}=\sqrt{3}$$

$$\sqrt{ab}=\frac{\sqrt{3}}{2}$$

$$ab=\frac{3}{4}$$

$$a=\frac{3}{2};\quad b=\frac{1}{2}$$

$$\sqrt{5+2\sqrt{6}} = \sqrt{a}+\sqrt{b}$$

$$5+2\sqrt{6} = a+b+2\sqrt{ab}$$

rational $\boxed{a+b=5}$

irrational $2\sqrt{ab} = 2\sqrt{6}$

$$\boxed{ab=6}$$

$$a=3,\ b=2$$

$$\sqrt{5+2\sqrt{6}} = \sqrt{3}+\sqrt{2}$$

$$\sqrt{5-2\sqrt{6}} = \sqrt{a}-\sqrt{b}$$

$$5-2\sqrt{6} = a+b-2\sqrt{ab}$$

$$a+b=5$$

$$ab=6$$

$a=3 \quad b=2$ ✓

$a=2 \quad b=3$ ✓

$$\sqrt{5-2\sqrt{6}} = \boxed{\cancel{\sqrt{2}-\sqrt{3}}}$$ ✓

$$= \sqrt{3}-\sqrt{2}$$

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(10) If $\sqrt{9+\sqrt{48}-\sqrt{32}-\sqrt{24}} = \sqrt{p} - \sqrt{q} + 2$; where $a, b \in N$; find $p+q$ $\boxed{=5}$

$-\sqrt{3}-\sqrt{4}+\sqrt{2}$ ✗

$\sqrt{3}+\sqrt{4}-\sqrt{2}$

$\sqrt{3}+2-\sqrt{2}$ ✓

ques'

$$\sqrt{(\sqrt{a}+\sqrt{b}+\sqrt{c})^2} = \sqrt{a+b+c+2\sqrt{ab}+2\sqrt{bc}+2\sqrt{ca}}$$

$a+b+c=9$ ✓

$2\sqrt{ab} = \sqrt{48}$

$\sqrt{ab} = \sqrt{12}$

$\boxed{ab=12}$ ✓

$\sqrt{\frac{48}{4}}$

$2\sqrt{bc} \Rightarrow -ve$

$2\sqrt{bc} = \sqrt{32}$

$\sqrt{bc} = \sqrt{8}$

$\boxed{bc=8}$ ✓

$\sqrt{b} \to -ve$

(−) OR

$\sqrt{c} \to -ve$

$2\sqrt{ca} = \sqrt{24}$ ✓ (−)

$\boxed{ca=6}$ ✓

a	b	c
3	4	2 ✓
$\sqrt{a} \to +ve$	$\sqrt{b} \to +ve$	$\sqrt{c} \to -ve.$
6	2	4 ✗
12	1	8 ✗

(11) If $x^3 + 8y^3 + x^3y^3 = 6x^2y^2$ where $x, y \in R - \{0\}$ and $x \neq 2y$; then find $\frac{2}{x} + \frac{1}{y}$.

(−1)

(12) If $x = 3 - \sqrt{8}$; then find $\frac{1}{\sqrt{x}} - \sqrt{x}$

(2)

(13) $$\frac{\left[\left(2^{\sqrt[8]{2}+1}\right)^{\sqrt[4]{2}+1}\right]^{(\sqrt{2}+1)}}{\left(2^{\sqrt[8]{2}-1}\right)} = 2^{(\sqrt[8]{2}+1)(\sqrt[4]{2}+1)(\sqrt{2}+1)} \cdot 2^{(-\sqrt[8]{2}+1)}$$

(1)

(1)

(14) The value of $\frac{a^3 + b^3 + c^3 - 3abc}{ab + bc + ca - a^2 - b^2 - c^2}$ is (where $a = -5$; $b = -6$; $c = 10$)

$$= \frac{(a+b+c)(a^2+b^2+c^2-ab-bc-ac)}{-(a^2+b^2+c^2-ab-bc-ca)} =$$

(15) The value of $\dfrac{2b(3a^2+b^2)}{(a+b)^3-(a-b)^3}$ is (wherever defined). (1)

(16) If $x^2+\frac{1}{x^2}=38$; then $x-\frac{1}{x}=?$ $\left(x-\frac{1}{x}\right)^2=\underbrace{x^2+\frac{1}{x^2}}_{=36}-2$ (± 6)

(17) The value of $\dfrac{(p-q)^3+(q-r)^3+(r-p)^3}{(p-q)(q-r)(r-p)}$ is (wherever defined) (3)

(17) $a \to p-q$; $b=q-r$; $c=r-p$

$a+b+c=0$

$a^3+b^3+c^3=3abc$

$\dfrac{3(p-q)(q-r)(r-p)}{(p-q)(q-r)(r-p)}$

$=3.$

(13)

$$\frac{\left[\left(2^{\sqrt[8]{2}+1}\right)^{\sqrt[4]{2}+1}\right]^{\sqrt{2}+1}}{2^{\left(\frac{1}{\sqrt[8]{2}-1}\right)}} = \frac{\left[2^{(\sqrt[8]{2}+1)\cdot\frac{\sqrt[8]{2}-1}{\sqrt[8]{2}-1}\,(\sqrt[4]{2}+1)}\right]^{\sqrt{2}+1}}{2^{\frac{1}{\sqrt[8]{2}-1}}}$$

$$(2^{1/8}+1)(2^{1/8}-1)$$

$$= (2^{1/8})^2-(1)^2$$

$$= 2^{1/4}-1$$

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$x^3 + 8y^3 + x^3y^3 = 6x^2y^2$

divide by x^3y^3

$\frac{1}{y^3} + \frac{8}{x^3} + 1 = \frac{6}{xy}$

$\left(\frac{1}{y}\right)^3 + \left(\frac{2}{x}\right)^3 + (1)^3 = 3 \cdot \left(\frac{1}{y}\right)\left(\frac{2}{x}\right)(1)$

$a^3 + b^3 + c^3 = 3abc$

when $a + b + c = 0$

$a \to \frac{1}{y}$

$b \to \frac{2}{x}$

$c = 1$

$\frac{1}{y} + \frac{2}{x} + 1 = 0$

$\boxed{\frac{1}{y} + \frac{2}{x} = -1}$

$x + 2y + xy = $

DIVISIBILITY RULES :

Divisible by	Remark.
2	Last digit of number is 0, 2, 4, 6 or 8
3	Sum of digits of number divisible by 3 (Remainder will be same when number is divided by 3 or sum of digits is divided by 3.)
4	Number formed by last two digits divisible by 4 (Remainder will be same whether we divide the number or its last two digits)
5	Last digit 0 or 5
6	Divisible by 2 and 3 simultaneously.
8	Number formed by last three digits is divisible by 8 (Remainder will be same whether we divide the number or its last three digits)
9	Sum of digits divisible by 9. (Remainder will be same when number is divided by 9 or sum of digit is divided by 9)
10	Last digit 0
11	(Sum of digits at even places) – (sum of digits at odd places) = 0 or divisible by 11

1259372

$\hookrightarrow$ div by 2 ✓

72 is div by 4 ✓

372 is not div by 8 ✗

Sum of digit by 3 ✓

Sum of digit by 9

1 2 5 9 3 7 2

$(1+5+3+2) - (2+9+7) = 11 - (18) = -7$

55429

$(4+9) - (2)$

$(5+4+9) - (5+2)$

LCM AND HCF :

(a) HCF is the highest common factor between any two or more numbers or algebraic expressions. When dealing only with numbers, it is also called "Greatest common divisor" (GCD).

(b) LCM is the lowest common multiple of two or more numbers or algebraic expressions.

(c) The product of HCF and LCM of two numbers (or expressions) is equal to the product of the numbers.

HCF

$21 = \boxed{3} \times 7$

$39 = \boxed{3} \times 13$

HCF = 3

LCM = $3 \times 7 \times 13$

$42 = \boxed{3 \times 2} \times 7$

$78 = \boxed{3 \times 2} \times 13$

HCF = 6

LCM = $6 \times 7 \times 13$

HCF $\xrightarrow{\text{divides}}$ all numbers (HCF has been taken)

Numbers $\boxed{a, b, c, d}$ $\longrightarrow$ LCM

LCM is divided by all the nos.

INDICES

Some useful Formulae

(i) $a^m \times a^n = a^{m+n}$

(ii) $a^m \div a^n = a^{m-n}$

(iii) $(a^m)^n = (a^n)^m = a^{mn}$

(iv) $\left(\frac{a}{b}\right)^{-\frac{m}{n}} = \left(\frac{b}{a}\right)^{\frac{m}{n}}$

(v) $a^m \div b^{-n} = a^m \times b^n$

(xv) $a + \sqrt{b} = c + \sqrt{d} \Rightarrow a = c$ and $b = d$ where a,c are rational numbers and $\sqrt{b}, \sqrt{d}$ are irrational numbers

(xix) $a^0 = 1,\ a \neq 0$

$a^m \cdot a^n = a^{m+n}$

$a^m / a^n = a^{m-n}$

$(a^m)^n = a^{m \cdot n}$

$a^{-m} = \frac{1}{a^m}$

$\left(\frac{a}{b}\right)^{-m/n} = \left(\frac{b}{a}\right)^{m/n}$

$a^m \div b^{-n}$

$= a^m \cdot b^n =$

POLYNOMIAL IN ONE VARIABLE

⇒ algebraic expression
⇒ exponent of variable must be whole no.

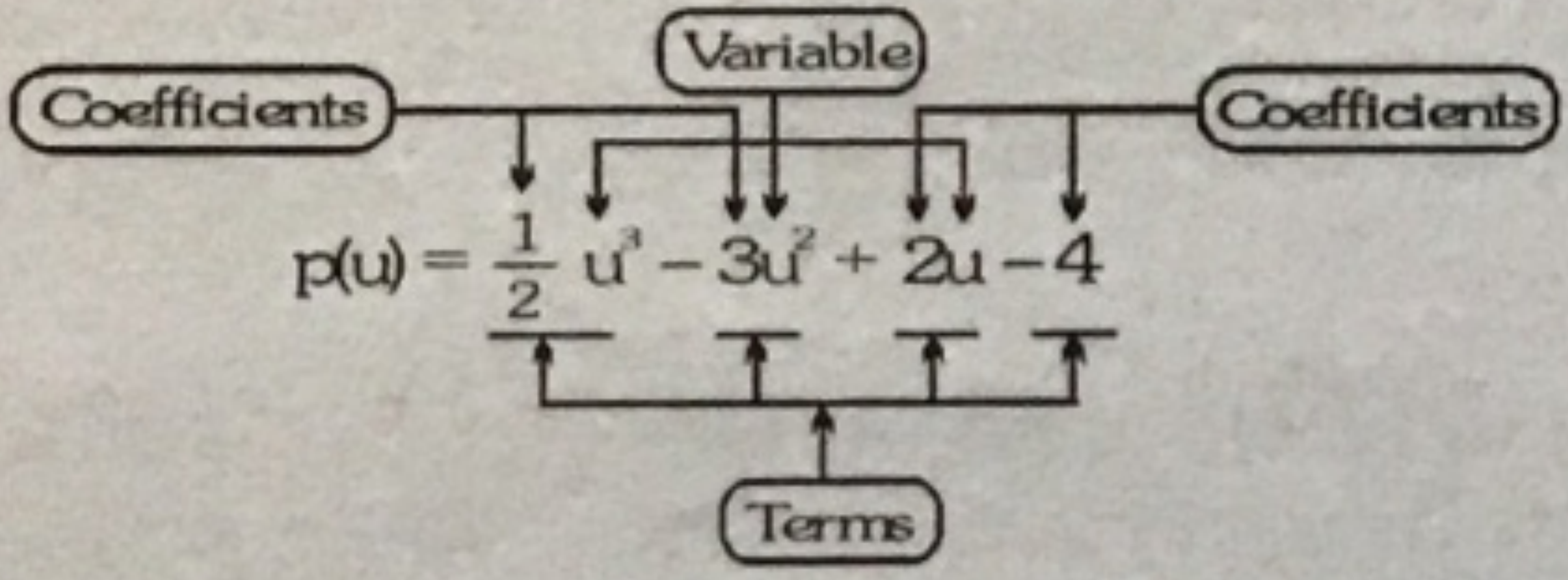
An algebraic expression of the form

$p(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + ... + a_1 x^1 + a_0 x^0$, where

(i) $a_n \neq 0$

(ii) power of x is whole number, is called a polynomial in one variable.

Hence, $a_n, a_{n-1}, a_{n-2}, ..., a_0$ are coefficients of $x^n, x^{n-1}, ..., x^0$ respectively and $a_n x^n, a_{n-1}x^{n-1}, a_{n-2}x^{n-2}, ...$ are terms of the polynomial. Here the term $a_n x^n$ is called the ***leading term*** and its coefficient a_n, the ***leading coefficient***.



DEGREE OF POLYNOMIALS

Degree of the polynomial in one variable is the largest exponent of the variable. For example, the degree of the polynomial $3x^7 - 4x^6 + x + 9$ is 7 and the degree of the polynomial $5x^6 - 4x^2 - 6$ is 6.

Polynomials classified by degree

Degree	Name	General form	Example
(undefined)	Zero polynomial	0	0
0	(Non-zero) constant polynomial	$a;\ (a \neq 0)$	1
1	Linear polynomial	$ax + b;\ (a \neq 0)$	$x + 1$
2	Quadratic polynomial	$ax^2 + bx + c;\ (a \neq 0)$	$x^2 + 1$
3	Cubic polynomial	$ax^3 + bx^2 + cx + d;\ (a \neq 0)$	$x^3 + 1$

Biquadratic (quartic) deg 4

quintic deg 5

Usually, a polynomial of degree n, for n greater than 3, is called a polynomial of degree n, although the phrases quartic polynomial and quintic polynomial are sometimes used.

ZEROS OF A POLYNOMIAL

In general, we can say $f(\alpha)$ is the value of the polynomial $f(x)$ at $x = \alpha$, where α is a real number. A real number α is zero of a polynomial $f(x)$ if the value of the polynomial $f(x)$ is zero at $x = \alpha$ i.e. $f(\alpha) = 0$.

OR

The value of the variable x, for which the polynomial $f(x)$ becomes zero is called zero of the polynomial.

ROOTS OF A POLYNOMIAL EQUATION

An expression f(x) = 0 is called a polynomial equation if f(x)is a polynomial of degree $n \geq 1$.

A real number α is a root of a polynomial f(x) = 0 if $f(\alpha) = 0$ i.e. α is a zero of the polynomial f(x).

REMAINDER THEOREM

Statement : Let p(x) be a polynomial of degree ≥ 1 and 'a' is any real number. If p(x) is divided by (x – a), then the remainder is p(a).

Factor theorem

Statement : Let f(x) be a polynomial of degree ≥ 1 and a be any real constant such that f(a) = 0, then (x – a) is a factor of f(x). Conversely, if (x – a) is a factor of f(x), then f(a) = 0.

dividend = divisor . quotient + Remainder

$$p(x) = \underbrace{(x-a)\ q(x)} + R$$

$x - a = 0$

$x = a$

at $x = a$

$$p(a) = 0 \cdot q(a) + R$$

$$\boxed{R = p(a)}$$

ALLEN CAREER INSTITUTE KOTA (RAJASTHAN)

$P(x) = x^2 - 3x + 2$

div by $\underline{x-5}$ Remainder $= P(5) = \underline{5}^2 - 3(5) + \underline{2}$

$=$

div by $\underline{x-2}$ ✓ Rem $= \underline{P(2) = 0}$

Homework 30/04/2021

Race 1 - Complete (exclude ques of log and iota)
BB 1 Complete

Fundamentals of Maths

Lecture - 3

ALLEN CAREER INSTITUTE KOTA (RAJASTHAN) path to success

1 (i) $\left[\sqrt[3]{64}\right]^{-1/2}$

$(64)^{1/3}$

$= \left[(64)^{\frac{1}{3}}\right]^{-1/2}$

$= (4)^{-1/2}$

$= \frac{1}{4^{1/2}} = \frac{1}{2}$

(ii) $\left[\frac{121}{169}\right]^{-3/2}$

$= \left(\left[\frac{121}{169}\right]^{1/2}\right)^{-3}$

$= \left(\frac{11}{13}\right)^{-3} = \left(\frac{13}{11}\right)^{3}$

$= \frac{2197}{1331}$

ALLEN CAREER INSTITUTE KOTA (RAJASTHAN) path to success

② (i) $(\sqrt{25})^{-7}(\sqrt{5})^{-5}$

$= 5^{(-7)} \cdot 5^{-5/2}$

$= 5^{-7-5/2}$

$= 5^{-19/2}$

$= \dfrac{1}{5^{19/2}}$

(ii) $\left(\frac{4}{5}\right)^{7} \div \left(\frac{5}{4}\right)^{-5}$

Ans $16/25$

③ If $a^x = b$; $\underbrace{b^y = c}$; $c^z = a$; prove that $xyz = 1$;

where a, b, c are distinct numbers.

$$a^x = b$$

take exponent y to both sides

$$a^{xy} = b^y$$

$$a^{xy} = c \qquad \because b^y = c$$

take exponenent z on both sides

$$a^{xyz} = c^z$$

$$a^{xyz} = a^1 \qquad \because c^z = a$$

on comparing exponents $\boxed{xyz = 1}$

④ If $\underline{x \in R^+}$; $a, b, c \in Q$, show that $\left(\frac{x^b}{x^c}\right)^a \cdot \left(\frac{x^c}{x^a}\right)^b \cdot \left(\frac{x^a}{x^b}\right)^c = 1$

Hint : Simplify

Apply $\frac{p^a}{p^b} = p^{a-b}$

$\underline{LHS} = (x^{b-c})^a (x^{c-a})^b (x^{a-b})^c$

$= x^0$

$= 1$

ALLEN CAREER INSTITUTE KOTA (RAJASTHAN) path to success

5. Solve for x

$$2^{2x+1} - 33\left(2^{x-1}\right) + 4 = 0$$

$$2^{2x} \cdot 2^{1} - 33 \cdot 2^{x} \cdot 2^{-1} + 4 = 0$$

Let $2^{x} = t \Rightarrow 2^{2x} = t^2$

$$t^2 \cdot (2) - 33t\left(\frac{1}{2}\right) + 4 = 0$$

$$4t^2 - 33t + 8 = 0$$

$$4t^2 - 32t - t + 8 = 0$$

$$4t(t-8) - 1(t-8) = 0$$

$$(4t-1)(t-8) = 0$$

$$\boxed{t = 1/4 ; 8}$$

$a^{m+n} = a^m \cdot a^n$

$2^{2x+1} = 2^{2x} \cdot 2^{1}$

$2^{x-1} = 2^{x} \cdot 2^{-1}$

$(2^x)^2 = (t)^2$

$2^{2x} = t^2$

when $t = \frac{1}{4}$

$2^x = \frac{1}{4} = \left(\frac{1}{2}\right)^2$

$2^x = 2^{-2}$

$\boxed{x = -2}$

$t = 8$

$2^x = 8 \Rightarrow \boxed{x = 3}$

$(2^x)^2 = t^2$

$2^{2x} = t^2$

Ans

$\boxed{x = -2, 3}$

6

$$64(9^x) - 84(12^x) + 27(16^x) = 0$$

$$64(3^{2x}) - 84(3^x \cdot 4^x) + 27(4^{2x}) = 0$$

divide by $3^x \cdot 4^x$

$$64\,\frac{3^{2x}}{3^x \cdot 4^x} - 84\left(\frac{3^x \cdot 4^x}{3^x \cdot 4^x}\right) + 27\left(\frac{4^{2x}}{3^x \cdot 4^x}\right) = 0$$

$$64 \cdot \left(\frac{3^x}{4^x}\right) - 84 + 27\left(\frac{4^x}{3^x}\right) = 0$$

$$64\left(\frac{3}{4}\right)^x - 84 + 27\left(\frac{4}{3}\right)^x = 0$$

let $\left(\frac{3}{4}\right)^x = t \Rightarrow \left(\frac{4}{3}\right)^x = \frac{1}{t}$

$$64t - 84 + \frac{27}{t} = 0$$

$\boxed{9} \to a$

nine

$x \to x$

$12 \to 3 \cdot (4)$

$12^x \to 3^x \cdot 4^x$

2^{2x}

$\frac{3^{2x}}{3^x} = 3^{2x-x} = 3^x$

$$64\left(\frac{3}{4}\right)^{2x} - 84\left(\frac{3}{4}\right)^x + 27 = 0$$

$$64t - 84 + \frac{27}{t} = 0$$

multiply by t to both sides

$$64t^2 - \underline{84}t + 27 = 0$$

$$\underline{64t^2} - \underline{48t} \; \underline{-36t} + \underline{27} = 0 \checkmark$$

$$16t(4t-3) - 9(4t-3) = 0$$

$$(4t-3)(16t-9) = 0$$

$$t = 3/4 \quad \& \quad t = 9/16$$

$$\left(\frac{3}{4}\right)^x = 3/4$$

$$\boxed{x = 1}$$

$$\left(\frac{3}{4}\right)^x = \left(\frac{9}{16}\right) = \left(\frac{3}{4}\right)^2$$

$$\boxed{x = 2}$$

If $ax^2 + bx + c = 0$

then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

64×27

$32 \;\&\; 54$

$16 \quad 108$

$48 \quad 36$

16×3

(7) $2^{x^2-6} \cdot 3^{x^2-6} = \frac{(6^{x-1})^4}{6^5}$, $x \in R$

$a^b \cdot c^b = (ac)^b$

$2^3 \cdot 3^3 = 216$

$= (6)^3$

$6^{x^2-6} = \frac{6^{4x-4}}{6^5}$

$6^{x^2-6} = 6^{4x-4-5}$

$x^2 - 6 = 4x - 9$ ✓ solve for x $\boxed{x = 1, 3}$ Ans

(3) $3 \cdot \underbrace{2^{x/2}} - 7 \cdot \underbrace{2^{x/4}} = 20 \;;\; x \in R$

$3t^2 - 7t = 20$

$t = 4, \cancel{-5/3}$

Solve for t and then for x.

$2^{x/4} = t$

$2^{x/2} = t^2$

~~square~~ $\cancel{2^{x/2} = t}$

$\cancel{2^x = t^2}$

$\cancel{2^{x/4} = t^{1/2} = \sqrt{t}}$

(9) $4^{x-\sqrt{x^2-5}} - 6\cdot 2^{x-\sqrt{x^2-5}} + 8 = 0 \;, \; x \in R$

Let $2^{x-\sqrt{x^2-5}} = t$

$t^2 - 6t + 8 = 0$

$(t-4)(t-2) = 0$

$t = 4$

$2^{x-\sqrt{x^2-5}} = 4$

$x - \sqrt{x^2-5} = 2$

$x-2 = \sqrt{x^2-5}$

$x^2 - 4x + 4 = x^2 - 5$

$\boxed{x = 9/4}$

$t = 2$

$2^{x-\sqrt{x^2-5}} = 2$

$x - \sqrt{x^2-5} = 1$

$x-1 = \sqrt{x^2-5}$

$x^2 - 2x + 1 = x^2 - 5$

$\boxed{x = 3}$

(10) find the remainder when $2x^4+x^3-2x^2+x+1$ divided by $2x-1$.

$2x-1=0 \Rightarrow x=1/2$

$$P(x)= 2x^4+x^3-2x^2+x+1$$

$$P\left(\tfrac{1}{2}\right) = 2\left(\tfrac{1}{2}\right)^4+\left(\tfrac{1}{2}\right)^3-2\left(\tfrac{1}{2}\right)^2+\tfrac{1}{2}+1 = \tfrac{5}{4}$$

(11) find remainder when x^3-ax^2+6x-a is divided by $\underline{x-a}$.

(12) If the polynomials ax^3+4x^2+3x-4 and x^3-4x+a leave the same remainder when divided by $x-3$, find the value of a.

$$a(3^3)+4(3^2)+3(3)-4 = (3)^3 - 4(3) + a$$

$$27a + \underline{36} + \underline{9} - 4 = \underline{27} - \underline{12} + a$$

$$26a = -26$$

$$\boxed{a = -1}$$

(13) for what value of k, $x-1$ is a factor of $P(x) = kx^2 - 3x + k$?

$x - 1 = 0$

factor th

$k(1)^2 - 3(1) + k = 0$

$\boxed{k = 3/2}$

ALLEN CAREER INSTITUTE KOTA (RAJASTHAN) path to success

(14) If $x = \sqrt{2+\underbrace{\sqrt{2+\sqrt{2+\ldots\infty}}}}$; find x.

$x = \sqrt{2+x}$

$x^2 = 2+x$

$x^2 - x - 2 = 0$

$(x+1)(x-2) = 0$

$x = 2$ & $\boxed{x = -1}$ ✗

final ans ⇒ $x = 2$

$x \in \{2\}$

$x = \sqrt{2+\sqrt{2+\sqrt{2+\ldots\infty}}}$

$x^2 = 2+\underbrace{\sqrt{2+\sqrt{2+\ldots\infty}}}$

$x^2 = 2+x$

Imaginary Numbers / complex Nos :- $(a+ib)$

$i = \sqrt{-1}$

$i^2 = -1$

$i^3 = \underline{i^2 \cdot i} = -i$

$i^4 = (i^2)^2 = 1$

$i^1 = i$

$i^2 = -1$

$i^3 = -i$

$i^4 = +1$

$i^5 = i^4 \cdot i = i$

$i^6 = i^4 \cdot i^2 = -1$

$i^7 = i^4 \cdot i^3 = -i$

$i^{27} = i^{24+3} = i^{24} \cdot i^3 = -i$

$\Rightarrow$

$$\left.\begin{aligned} i^{4n+1} &= i \\ i^{4n+2} &= i^2 = -1 \\ i^{4n+3} &= i^3 = -i \\ i^{4n} &= i^4 = 1 \end{aligned}\right] \rightarrow \text{where } n \in I$$

ALLEN CAREER INSTITUTE KOTA (RAJASTHAN) path to success

$$i^{59} + i^{60} + i^{61} + i^{62} = -i + 1 + i - 1 = 0$$

Algebraic operations on complex nos :-

$z_1 = a_1 + ib_1$; $z_2 = a_2 + ib_2$

addition $z_1 + z_2 = (a_1 + a_2) + i(b_1 + b_2)$

Subtraction $z_1 - z_2 = (a_1 - a_2) + i(b_1 - b_2)$

Product $z_1 \cdot z_2 = (a_1 + ib_1)(a_2 + ib_2)$

$$= a_1a_2 + ib_1a_2 + ib_2a_1 + i^2 b_1b_2$$

$$= (a_1a_2 - b_1b_2) + i(b_1a_2 + b_2a_1)$$

division

$$\frac{z_1}{z_2} = \frac{a_1+ib_1}{a_2+ib_2} \cdot \frac{a_2-ib_2}{(a_2-ib_2)}$$

$$= \frac{(a_1+ib_1)(a_2-ib_2)}{(a_2^2+b_2^2)}$$

$a_2^2 - i^2 b_2^2$

Intervals :-

Intervals are basically subsets of R. If there are two numbers $a, b \in R$ such that $a < b$, we can define four types of intervals as follows :

(a) <u>open interval</u> $(a, b) \Rightarrow \{x : a < x < b\}$ (i.e. end points are not included)

(b) <u>closed interval</u> $[a, b] \Rightarrow \{x : a \leq x \leq b\}$ (i.e. end points are included)

(c) <u>Semi-open or semi closed intervals</u> :- $(a, b] \Rightarrow \{x : a < x \leq b\}$

$[a, b) \Rightarrow \{x : a \leq x < b\}$

(d) Infinite intervals:-

(i) $(a, \infty) \Rightarrow \{x : x > a\}$

(ii) $[a, \infty) \Rightarrow \{x : x \geq a\}$

(iii) $(-\infty, b) \Rightarrow \{x : x < b\}$

(iv) $(-\infty, b] \Rightarrow \{x : x \leq b\}$

(v) $(-\infty, \infty) \Rightarrow \{x : x \in R\} \Rightarrow R$

Note:-

(i) for some particular values of x, we use symbol $\{\ \}$; i.e. if $x = 1, 2$; $x \in \{1, 2\}$

(ii) If there is no value of x then we write $x \in \phi$ (null set).

Homework 03/05/2021

Race 1 → complete
Race 2 → complete

Fundamentals
of
Maths

Lecture -4

ALLEN CAREER INSTITUTE KOTA (RAJASTHAN) path to success

① Let $A = \{1, 2, 4, 6\}$; $B = \{2, 4, 5, 7, 9\}$

(i) $A \cup B = \{1, 2, 4, 5, 6, 7, 9\}$ (ii) $A \cap B = \{2, 4\}$

$\cup \to$ Union

$\cap \to$ Intersection

② Let $A = [2, 5]$; $B = (-\infty, 4)$

(i) $A \cup B$ (ii) $A \cap B$

$\boxed{\sqrt{x^2} = |x|}$

$\sqrt{9} = |-3|$
$= 3$

③ Let $A = [-1, 3] \cup [7, 15]$ and $B = [1, 10)$

(i) $A \cup B$ (ii) $A \cap B$

④ Write true / false

(i) $4 \geq 4$ True

(ii) $3 \leq 5$ True

$3 < 5$ or $\cancel{3 > 5}$

(iii) $\sqrt{9} = \pm 3$ false

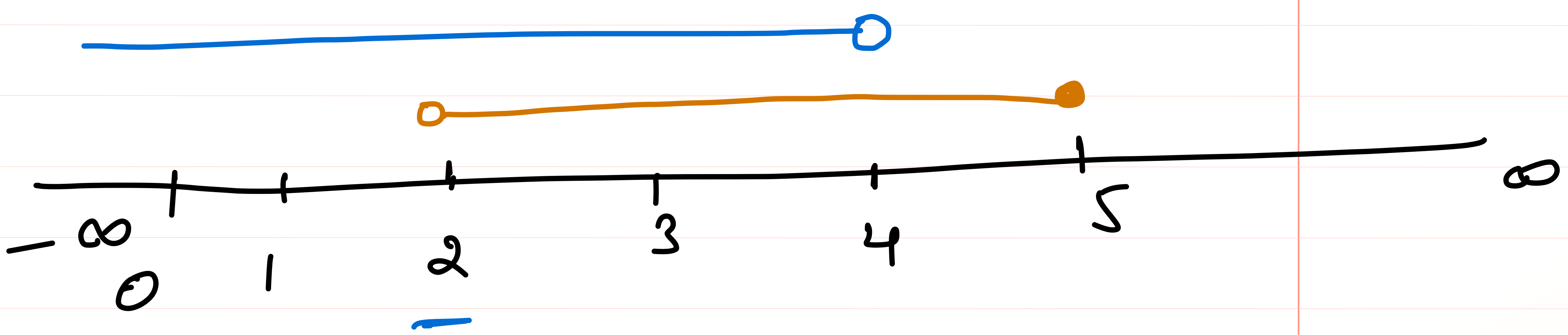
$+3$ only $+3$

(iv) If $x^2 = 25$ then $x = \pm 5$

True

② Let $\underline{A = [2,5]}$; $\underline{B = (-\infty, 4)}$

(ii) $\underline{A \cap B = (2,4)}$

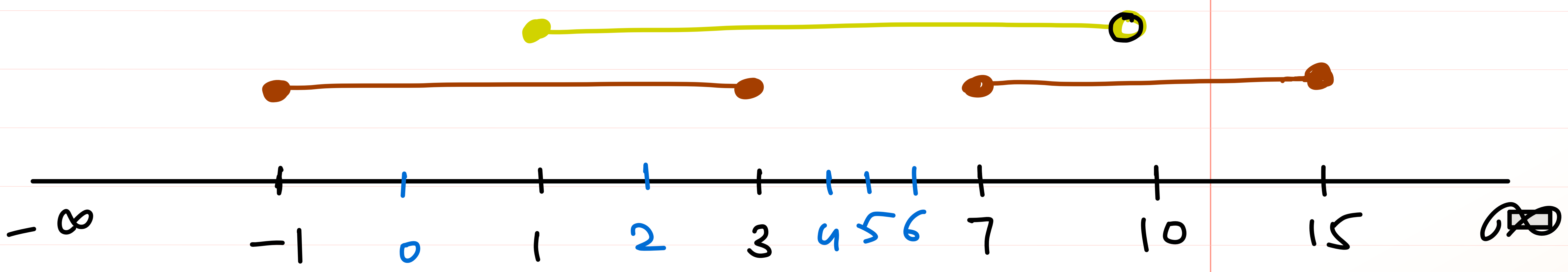


$\left[\begin{array}{llll} \text{open} \rightarrow \circ & \text{or} \rightarrow & ; & (\ \ ) \\ \text{close} \rightarrow \bullet & & ; & [\ \ ] \end{array}\right.$

ALLEN CAREER INSTITUTE KOTA (RAJASTHAN) path to success

(3) Let $A = [-1, 3] \cup [7, 15]$ and $B = [1, 10)$

$A \cap B = ?$



$$\boxed{A \cap B \in [1, 3] \cup [7, 10)}$$

Rational Algebraic inequalities :-

T-1

Q 1 $\underbrace{2x^2 - 3x + 4}$ > 0

$(x \quad\quad)(x \quad\quad\quad) > 0$

If coe of x^2 is positive and Discriminant $D = b^2 - 4ac$ is less than 0. then for $f(x) > 0$; $x \in R$.

(2) $x^2 - 3x + 2 < 0$

$(x-1)(x-2) < 0$

$\boxed{x \in (1, 2)}$ Ans

If < 0 then $-$ve sign

> 0 then $+$ve sign

$+ \quad - \quad +$

$-\infty$ $\quad -100 \quad 1 \quad 1.5 \quad 2 \quad 100 \quad \infty$

$(-100-1)(-100-2)$

$\downarrow \qquad \downarrow$

$-$ve $\quad -$ve

$+$ve

$(1.5-1)(1.5-2)$

$\downarrow \qquad \downarrow$

$+ \qquad -$

$-$

$(100-1)(100-2) < 0$

$\downarrow \qquad \downarrow$

$+ \qquad +$

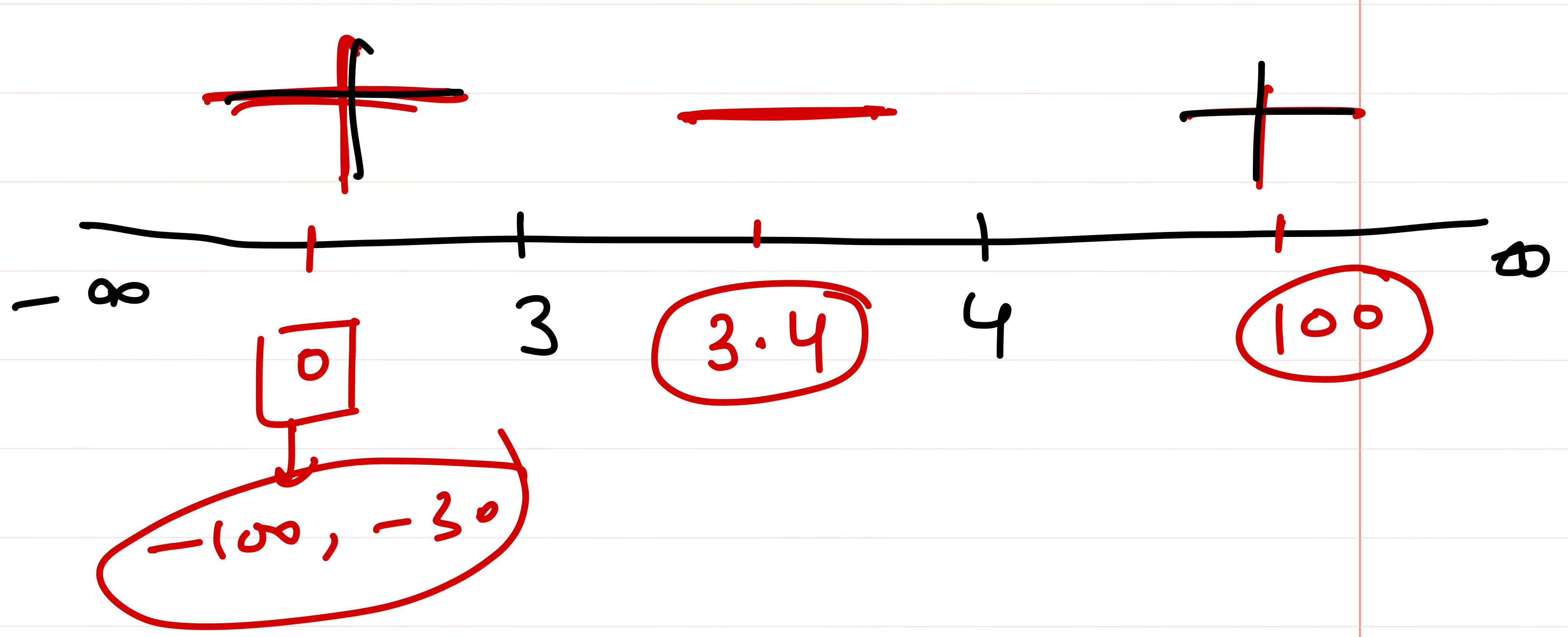
$+$

③ $x^2 - 7x + 12 \geq 0$

$(x-3)(x-4) \geq 0$

$\boxed{x \in (-\infty, 3] \cup [4, \infty)}$

$\leq$, $<$	$\rightarrow$	$-ve$
$\geq$, $>$	$\rightarrow$	$+ve$



at $x = 0$

$(x-3)(x-4)$

$(-)\ (-)$

$+ve$

at $x = 3.4$

$(3.4-3)(3.4-4)$

$+ \quad -$

$-$

at $x = 100$

$(100-3)(100-4)$

$+ \quad +$

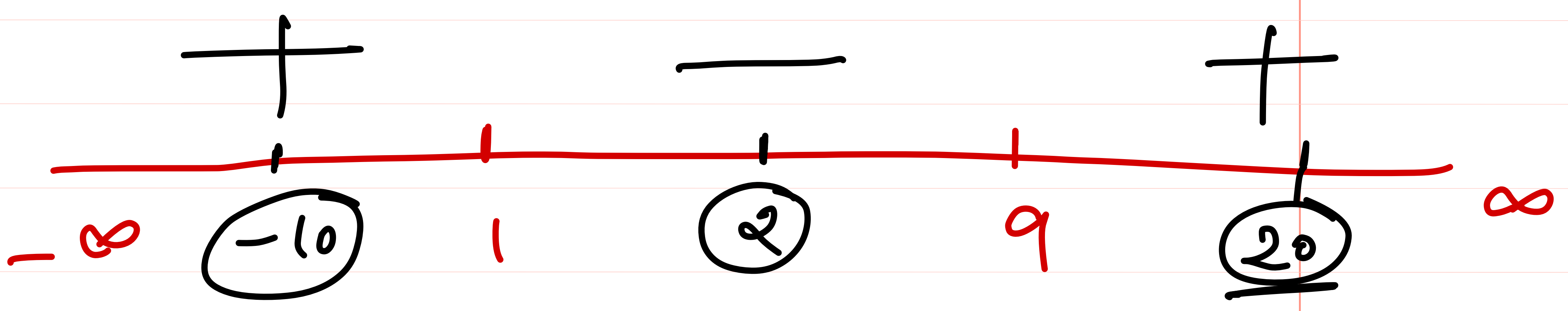
$+ve$

$\leq, < 0 \rightarrow -ve$

$\geq, > 0 \rightarrow +ve$

(4) $(x-1)(x-9) > 0$

$$\boxed{x \in (-\infty, 1) \cup (9, \infty)}$$



at $x = -10$ (LHS)

$(-10-1)(-10-9)$

$-ve \quad -ve$

$+ve$

at $x = 2$

$(2-1)(2-9)$

$+ve \quad -ve$

$-ve$

at $x = 20$

$(20-1)(20-9)$

$+ \quad +$

$+$

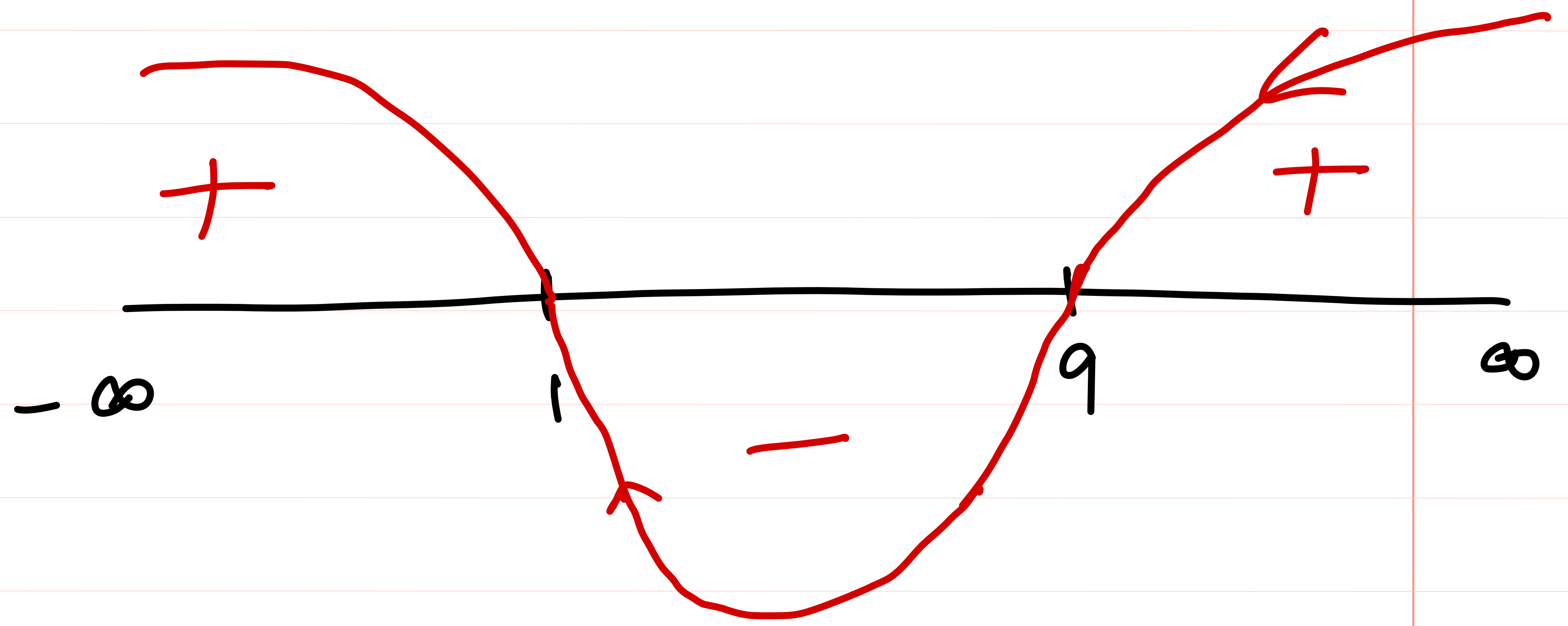
⑤ $(x-1)(x-9) > 0$

$\cancel{x}$ x

$\downarrow$

+ve

$$\boxed{x \in (-\infty, 1) \cup (9, \infty)}$$



$(x-9)^1$

$\downarrow$

odd power

$\downarrow$

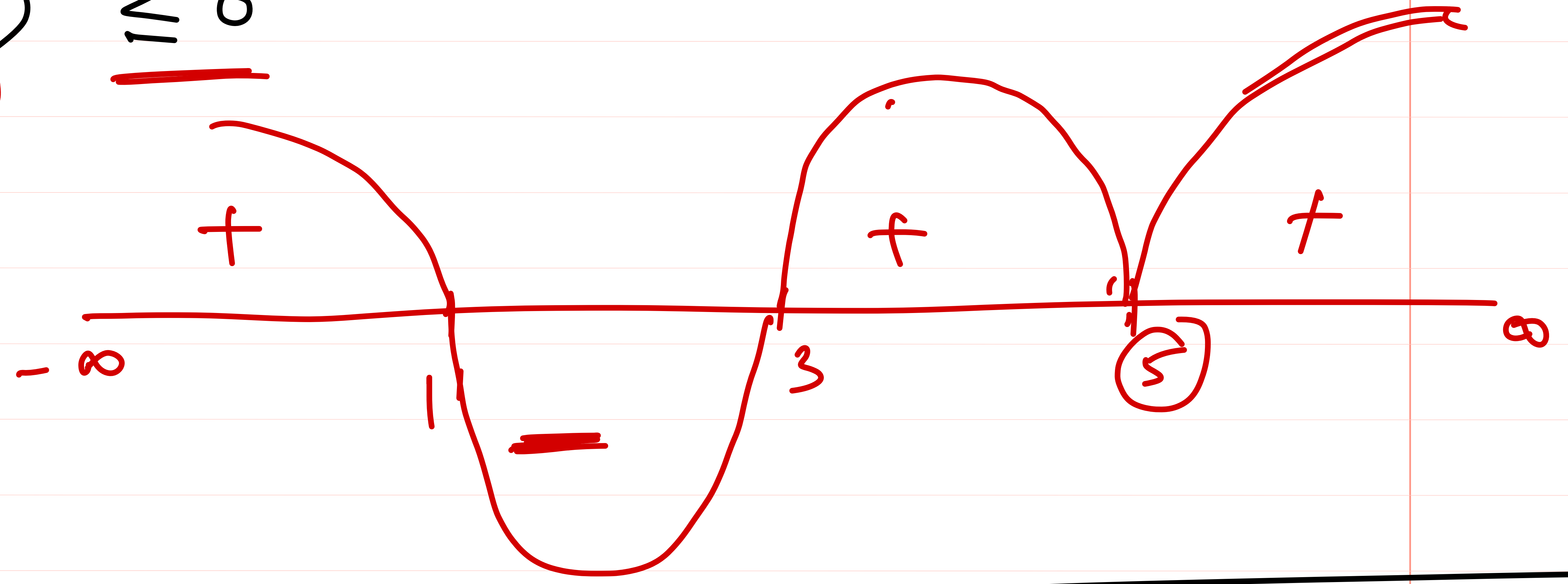
Cross

$(x-9)^2$

$\downarrow$ even pw

bounce

(4) $(x-5)^2 (x-3)^3 (x-1) \le 0$



$$\boxed{x \in [1,3] \cup \{5\}}$$

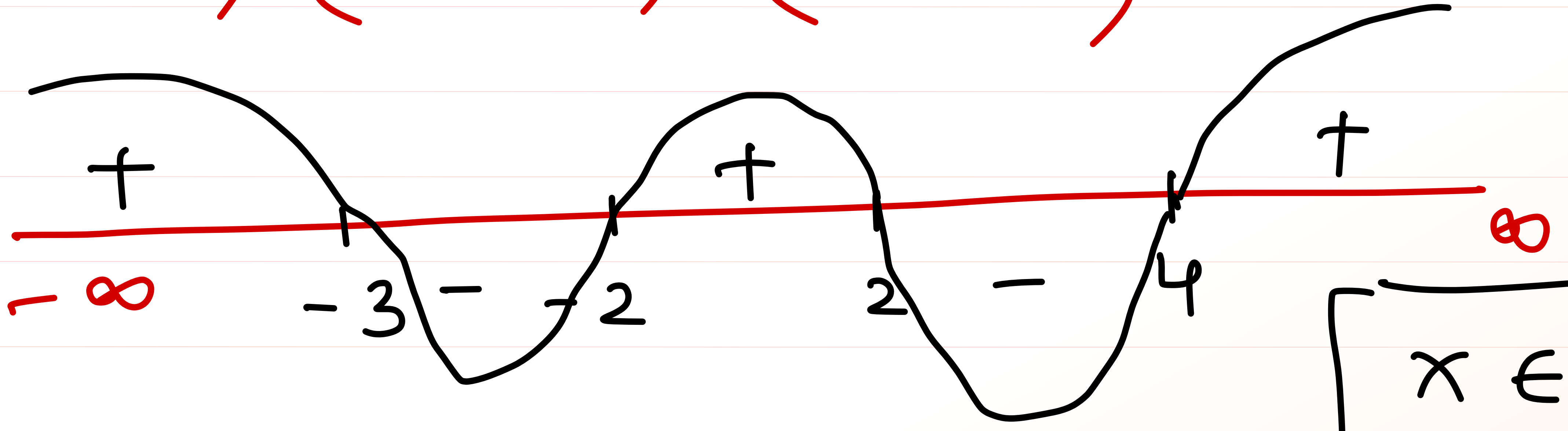
(5) $-(x^2+x-6)(x^2-2x-8) \le 0$

$(x^2+x-6)(x^2-2x-8) \ge 0$

$(x-2)(x+3)(x-4)(x+2) \ge 0$

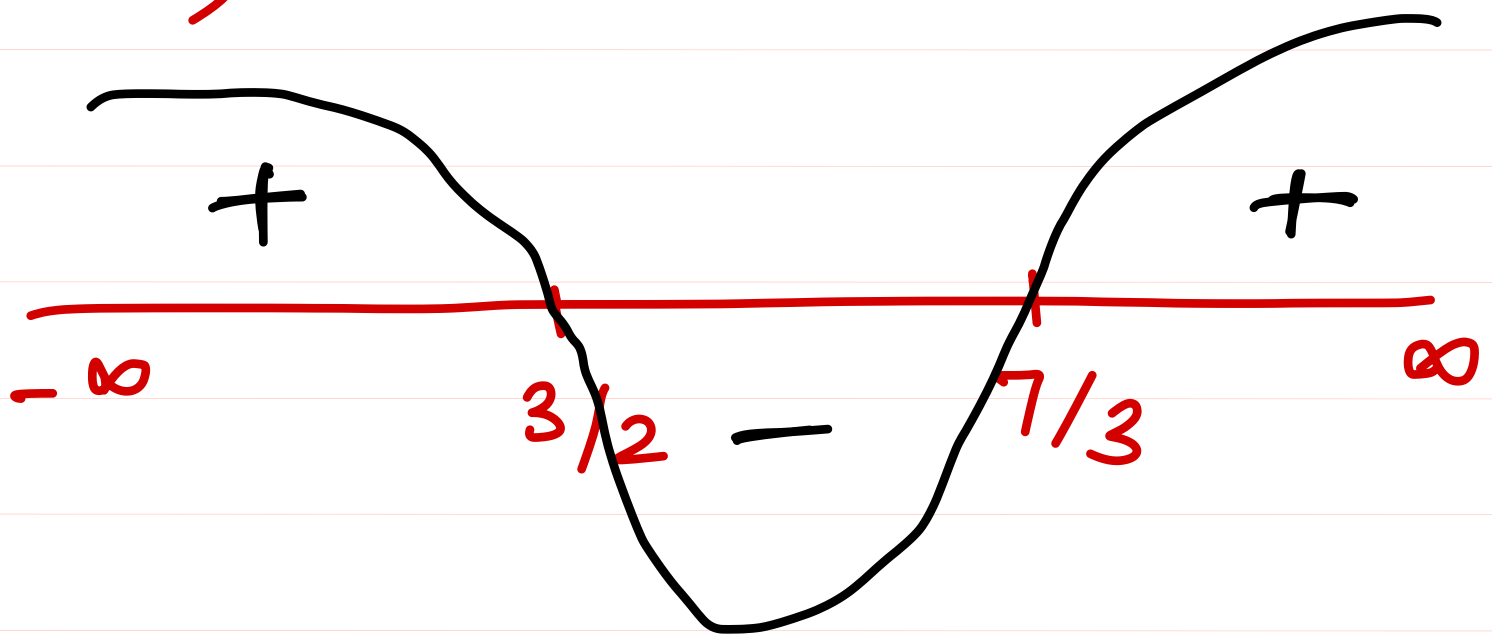
(when an inequation is multiplied with negative number then inequation sign reverses)

Wavy Curve Method



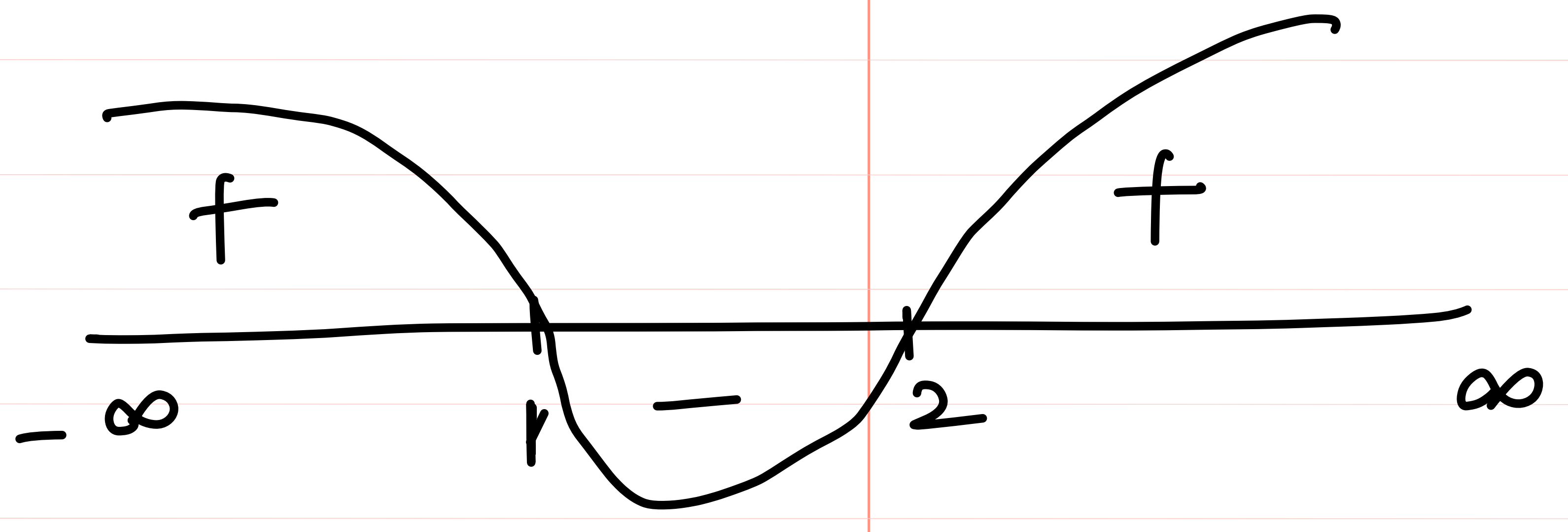
$$\boxed{x \in (-\infty,-3] \cup [-2,2] \cup [4,\infty)}$$

(6) $\dfrac{(2x-3)}{(3x-7)} > 0$



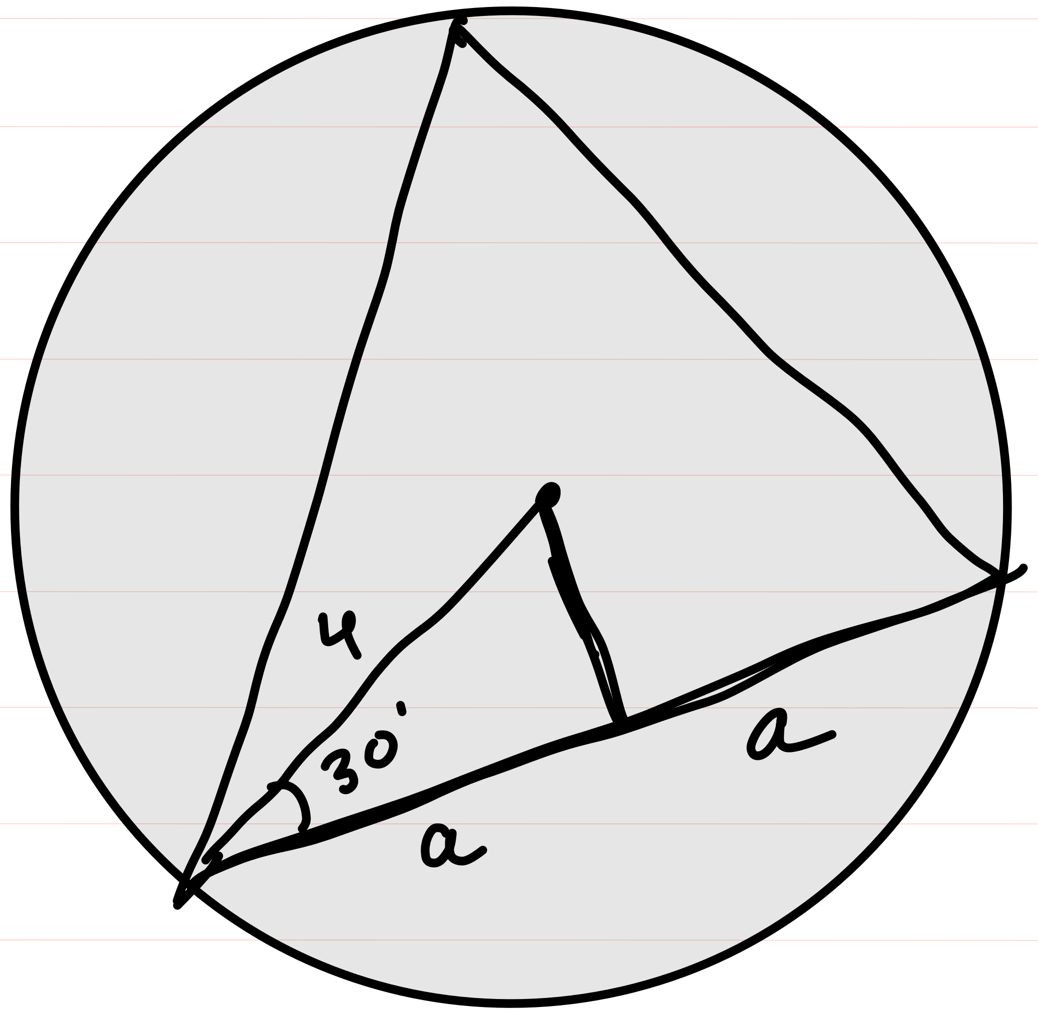
$$\boxed{x \in \left(-\infty, \frac{3}{2}\right) \cup \left(\frac{7}{3}, \infty\right)}$$

(7) $\dfrac{x-1}{x-2} \le 0$



$$x \in [1, 2)$$

(3)



$\cos 30 = a/4$

$a = 4 \frac{\sqrt{3}}{2} = 2\sqrt{3}$

Side length $= 2a$

$= 4\sqrt{3}$

Area $= \frac{\sqrt{3}}{4} (\text{Side})^2 =$

40

1 A Ractangle $= \underline{8r^2}$

Shaded area $= 8r^2 - (2\pi r^2)$

radius $= r$

$l = 4r$

$w = 2r$

(9) $a^4+a^2+1 = a^4+2a^2+1-a^2$

$= (a^2+1)^2 - a^2$

$= (a^2+1+a)(a^2+1-a)$

$a^2+1+a = 1$ OR $a^2+1-a = 1$

$a=0, -1$ $a=0, +1$

(14)

$$n+20 = a^2$$
$$n-20 = b^2$$

$$41 = a^2 - b^2$$

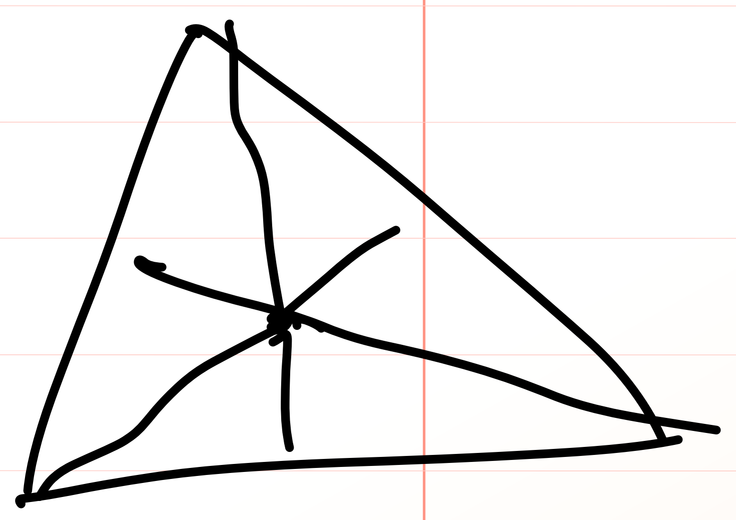
$$(a-b)(a+b) = 41$$

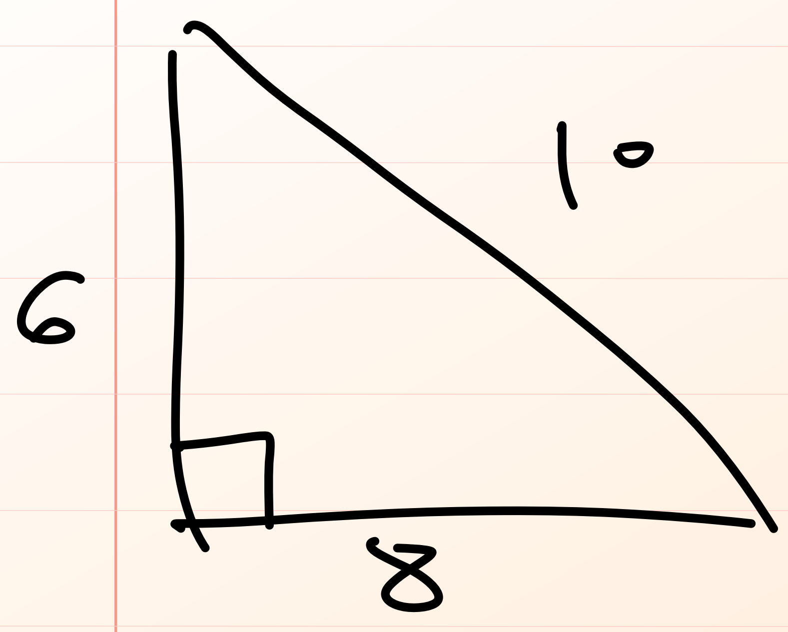
$$a-b = 1$$
$$a+b = 41$$

Area

$$\frac{1}{2} \cdot 8(x_2) + \frac{1}{2} 10(x_3) + \frac{1}{2}(6)(x_1) = \frac{1}{2} \cdot (6)(8)$$

$$4x_2 + 5x_3 + 3x_1 = 24$$





ALLEN CAREER INSTITUTE KOTA (RAJASTHAN) path to success

15

$$m^2 = n^2 + 2010$$

$$m^2 - n^2 = 2010$$

$$(m+n)(m-n) = 2010$$

$m+n = 2010$
$m-n = 1$

$m+n = 1005$
$m-n = 2$

$m+n = 402$
$m-n = 5$

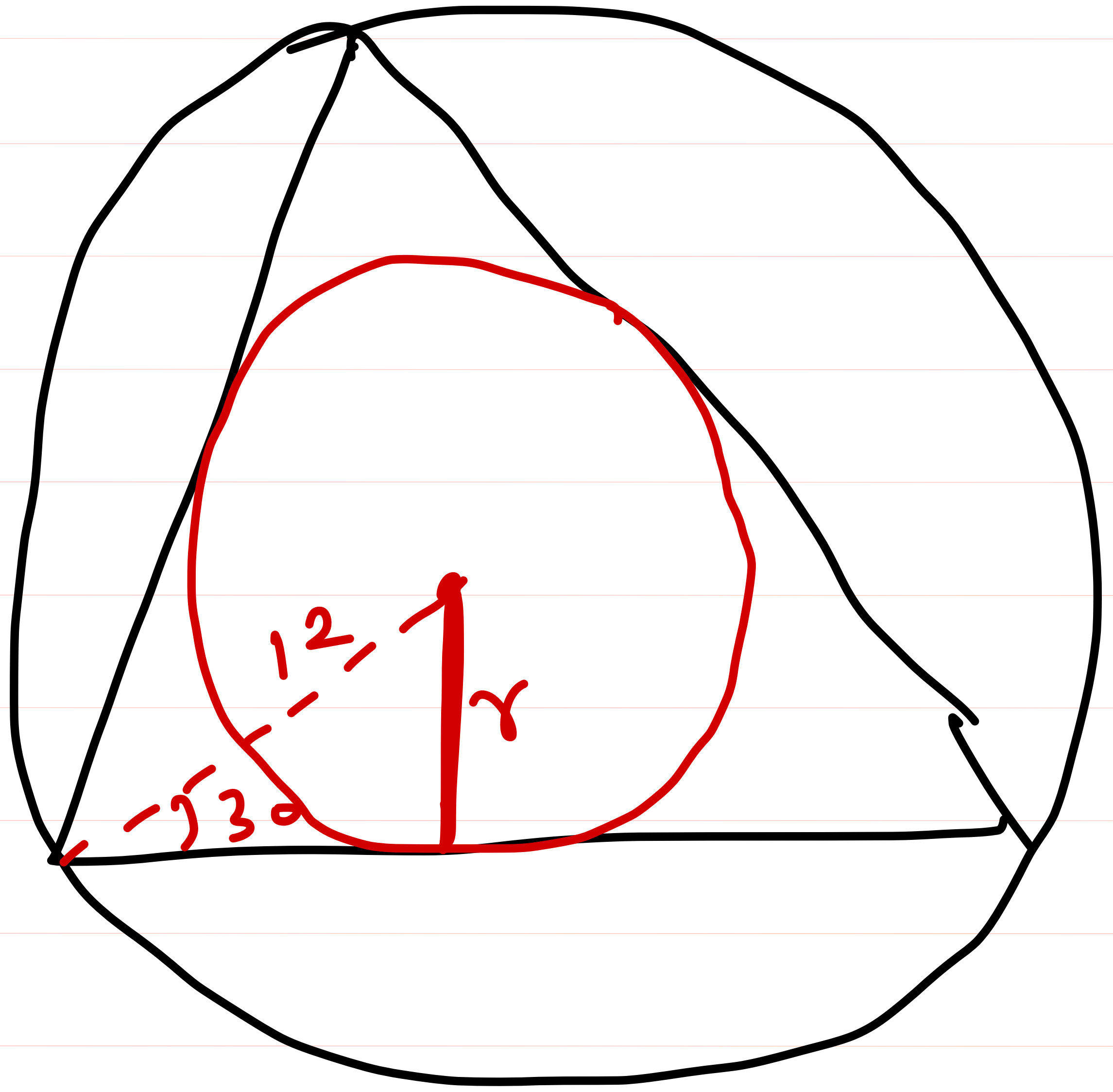
$m+n = 201$
$m-n = 10$

⑦

$$2\pi r = 24\pi$$

$$\underline{r = 12}$$

$$\text{Area} = \pi(6)^2 = \underline{36\pi}$$



$$\sin 30 = \frac{r}{12}$$

$$\boxed{r = 6}$$

Homework

Race - 3 Question 1 to Ques 15

Beginner Box → 2 Complete

Exercise 1 → Q 1, 2, 3, 7,

Fundamentals of Maths

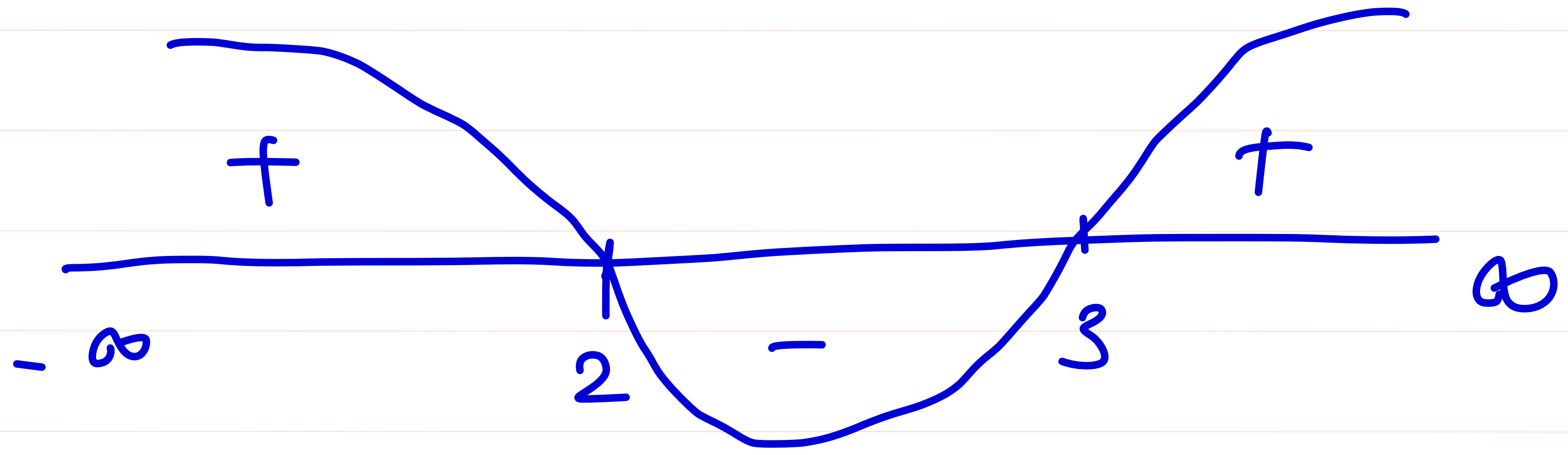
Lecture - 5

Q 1

$$\frac{x^2-5x+6}{x^2+x+1} < 0$$

$$\frac{(x-2)(x-3)}{x^2+x+1} < 0$$

Remove



$x \in (2, 3)$

$$\frac{x^2-5x+6}{-x^2-x-1} < 0 \Rightarrow \frac{x^2-5x+6}{x^2+x+1} > 0$$

$1x^2+x+1$

coe of x^2 → +ve

$D = b^2-4ac < 0$

$1^2-4(1)(1)$

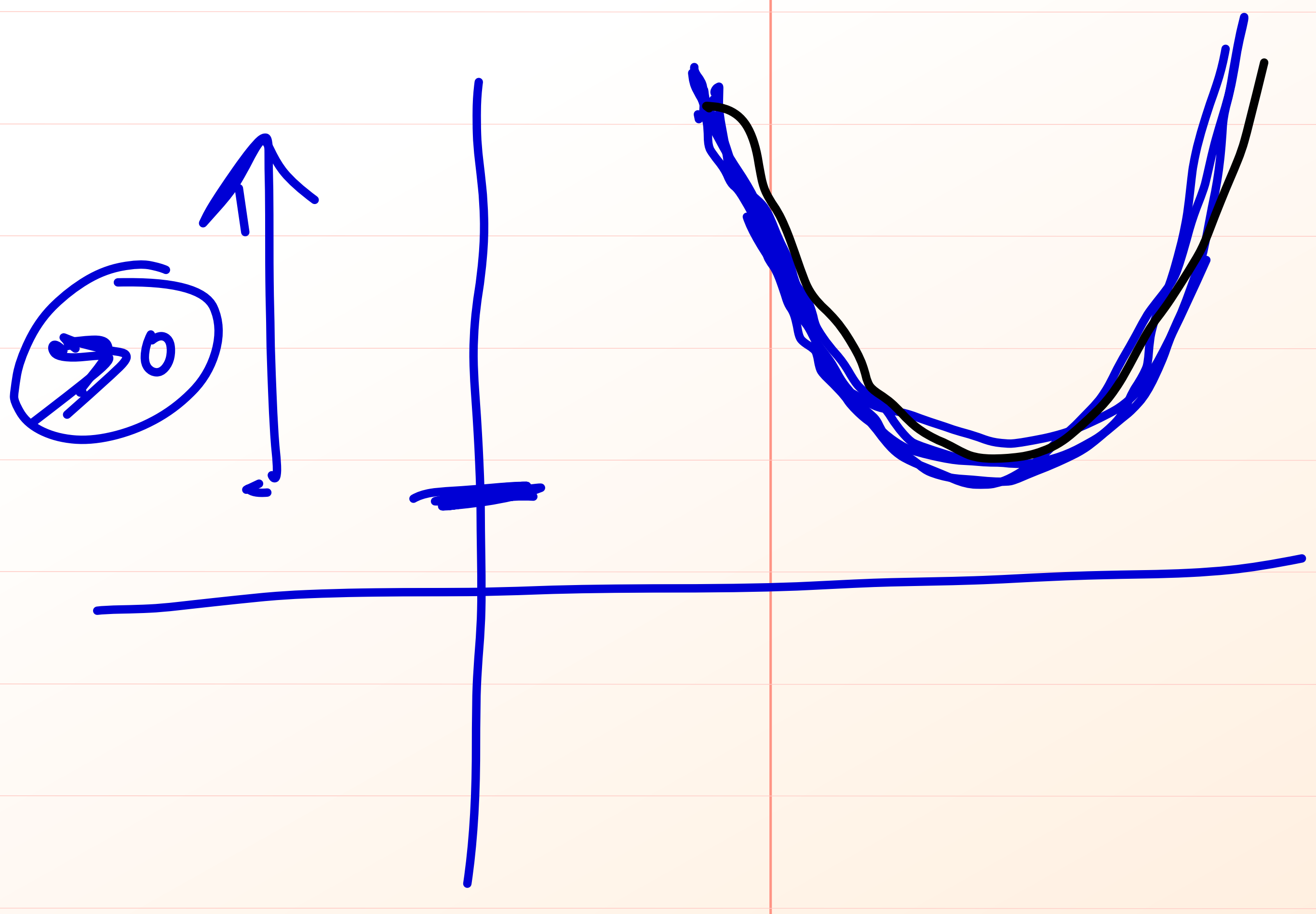
$\boxed{-3} < 0$

$\therefore \quad x^2+x+1 > 0$

$a \to$ coe of x^2

$b \to$ coe of x

$c \to$ constant



(2)

$$\frac{x+1}{x-1} \geq \frac{x+5}{x+1}$$

$$\frac{(x+1)}{(x+1)}\cdot\frac{x+1}{(x-1)} - \frac{x+5}{(x+1)}\cdot\frac{(x-1)}{(x-1)} \geq 0$$

$$\frac{(x+1)(x+1) - (x+5)(x-1)}{(x-1)(x+1)} \geq 0$$

$$\frac{\cancel{x^2}+1+2x-\cancel{x^2}-5x+x+5}{(x-1)(x+1)} \geq 0$$

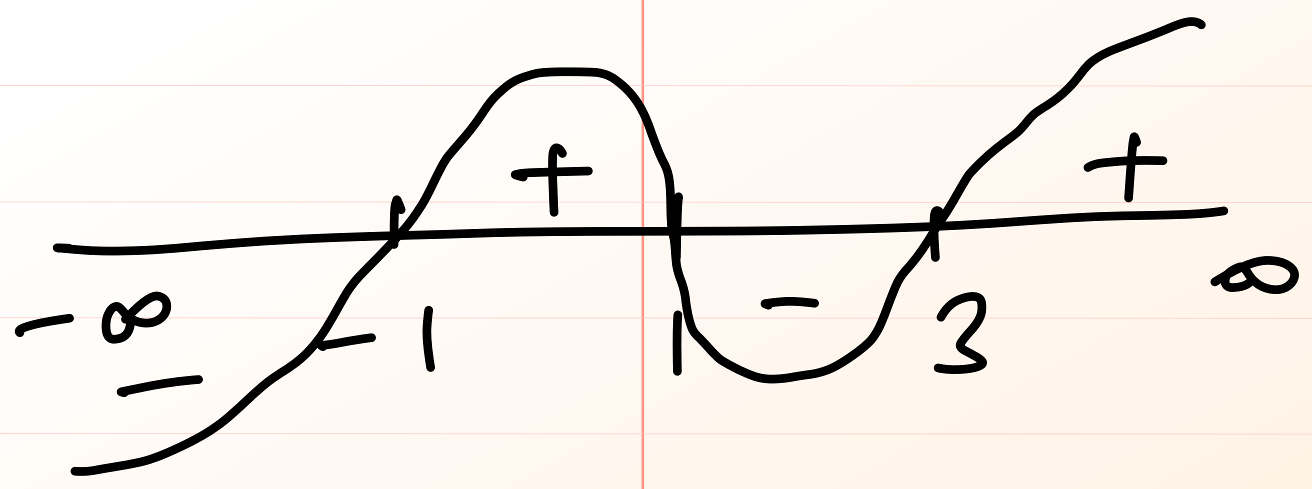
$$\frac{-2x+6}{(x-1)(x+1)} \geq 0$$

$$a \geq b$$

$$a-b \geq 0$$

$$\frac{2x-6}{(x-1)(x+1)} \leq 0$$

$$\frac{\cancel{2}(x-3)}{\cancel{2}(x-1)(x+1)} \leq \frac{0}{2}$$



$$\boxed{x \in (-\infty,-1) \cup (1,3]}$$

$16-36 \quad \underline{D<0}$

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③ $$\frac{2(x-4)}{(x-1)(x-7)} \geq \frac{1}{x-2}$$

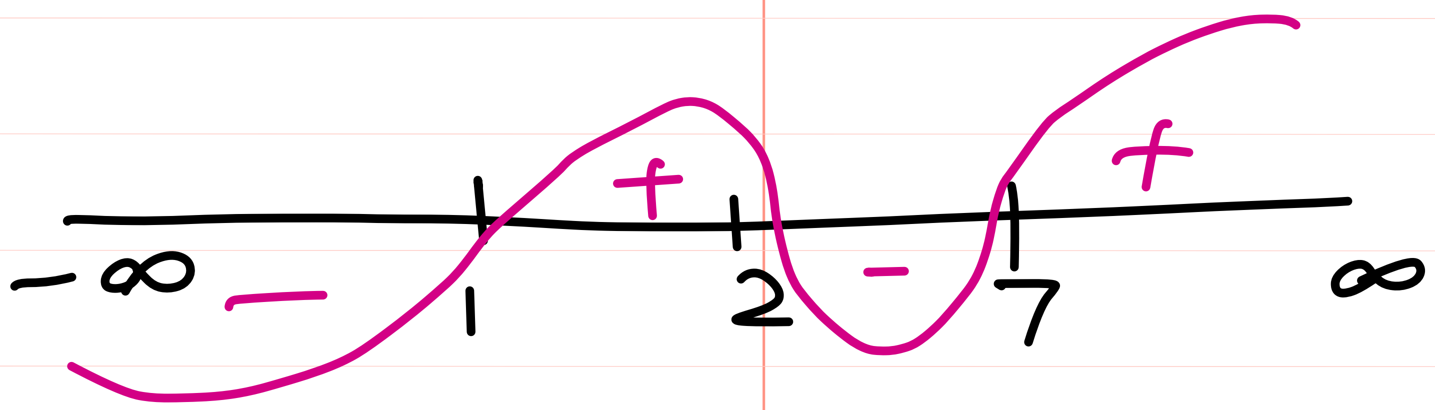
$$\frac{2(x-4)}{(x-1)(x-7)} - \frac{1}{x-2} \geq 0$$

$$\frac{2(x-4)(x-2) - (x-1)(x-7)}{(x-1)(x-7)(x-2)} \geq 0$$

$$\frac{2(x^2-6x+8) - (x^2-8x+7)}{(x-1)(x-7)(x-2)} \geq 0$$

$$\frac{x^2-4x+9}{(x-1)(x-7)(x-2)} \geq 0 \qquad x^2-4x+9 \rightarrow >0$$

$$\frac{1}{(x-1)(x-2)(x-7)} \geq 0$$

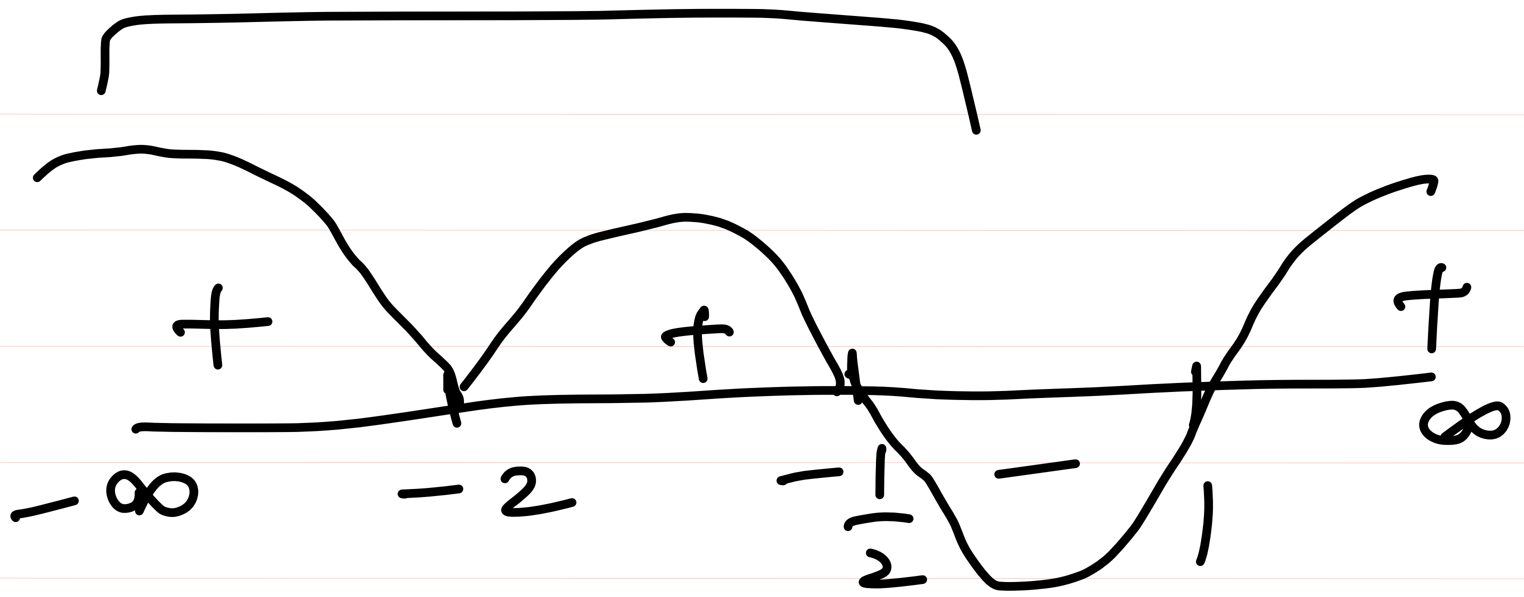


$$\boxed{x \in (1,2) \cup (7,\infty)}$$

(4) $$\frac{x^2+4x+4}{2x^2-x-1} > 0$$

$$2x^2-x-1$$
$$2x^2-2x+x-1$$
$$(2x+1)(x-1)$$

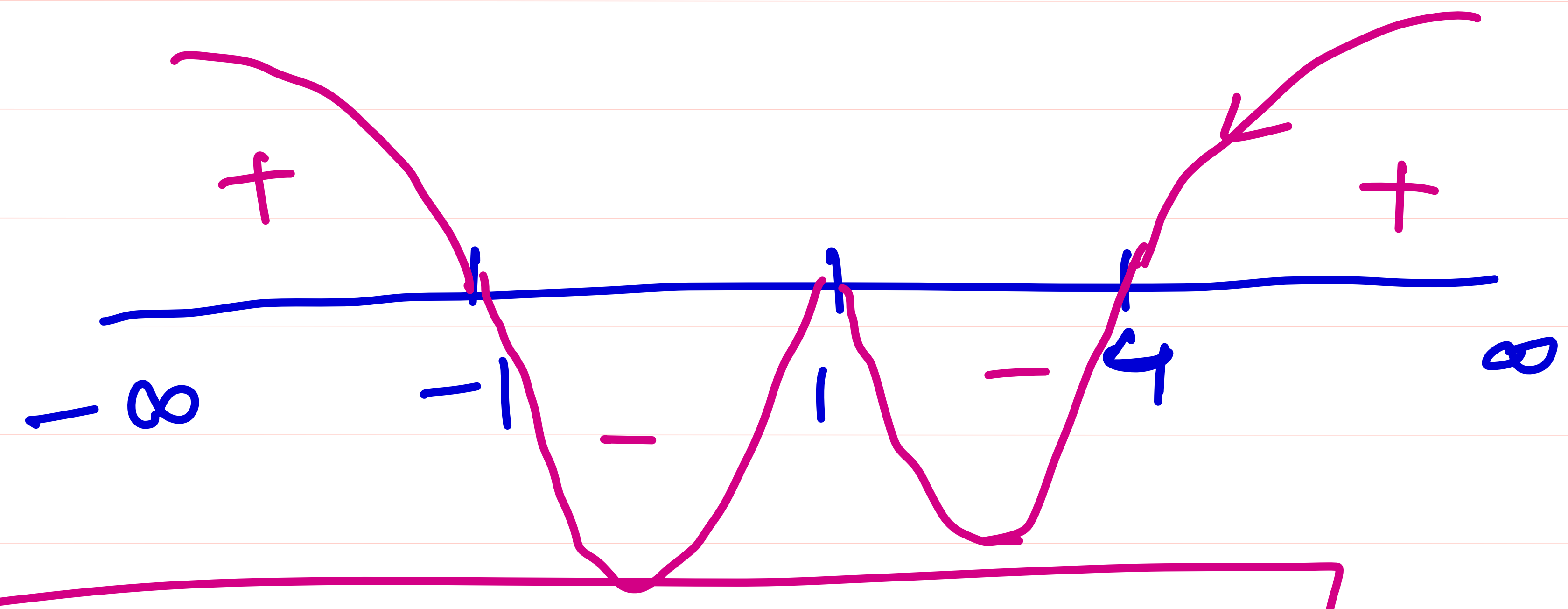
$$\frac{(x+2)^2}{(2x+1)(x-1)} > 0$$



$$\boxed{x \in (-\infty,-2) \cup \left(-2,-\tfrac{1}{2}\right) \cup (1,\infty)}$$

$$\boxed{x \in \left(-\infty,-\tfrac{1}{2}\right) \cup (1,\infty) - \{-2\}}$$

(5) $(x+1)^3 (x-1)^2 (x-4)^1 < 0$



$$\boxed{x \in (-1, 1) \cup (1, 4)}$$

OR

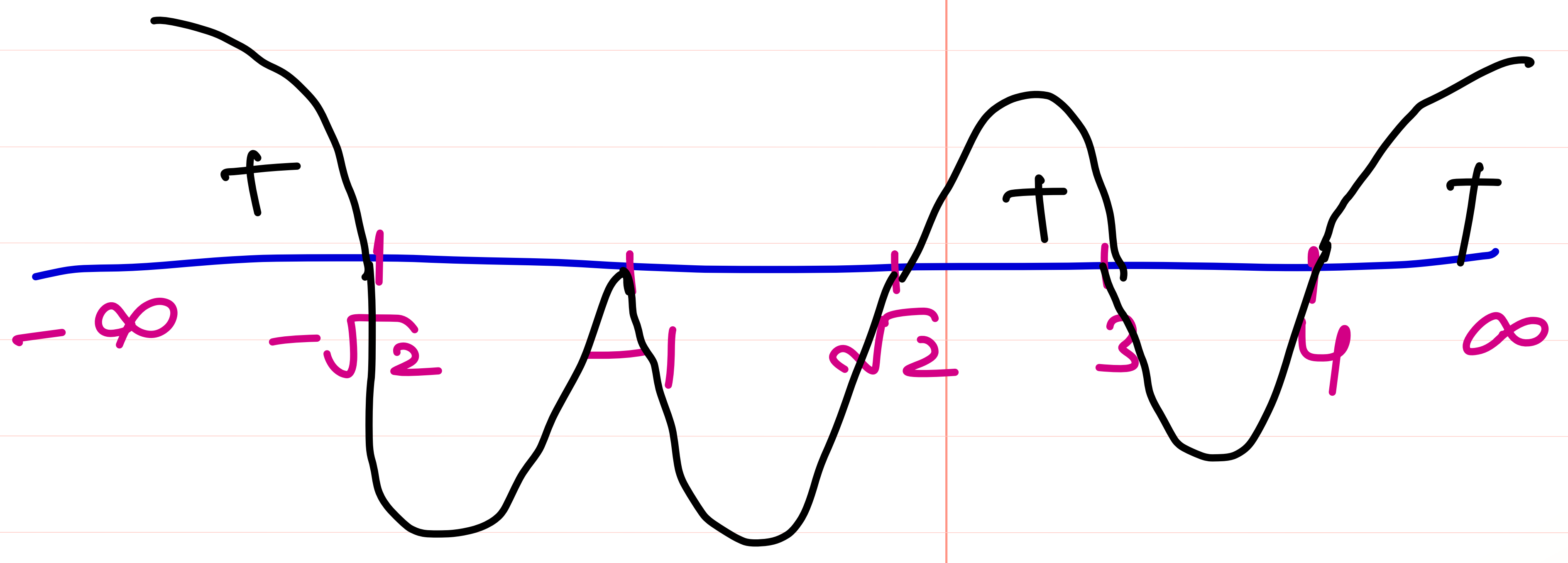
$$\boxed{x \in (-1, 4) - \{1\}}$$

(6) $$\frac{(2-x^2)\,(3-x)^3}{(x+1)\,(x^2-3x-4)} \ge 0$$

$$\frac{(x^2-2)\,(x-3)^3}{(x+1)\,(x^2-3x-4)} \ge 0$$

$$\frac{(x-\sqrt{2})\,(x+\sqrt{2})\,(x-3)^3}{(x+1)\,(x-4)\,(x+1)} \ge 0$$

$$\frac{(x-\sqrt{2})\,(x+\sqrt{2})\,(x-3)^3}{(x+1)^2\,(x-4)} \ge 0$$



$$\boxed{x \in (-\infty, -\sqrt{2}] \cup [\sqrt{2}, 3] \cup (4, \infty)}$$

(7) $$\frac{x^2-5x+12}{x^2-4x+5} > 3$$

$$\frac{x^2-5x+12}{x^2-4x+5} - 3 > 0$$

$$\frac{(x^2-5x+12) - 3(x^2-4x+5)}{x^2-4x+5} > 0$$

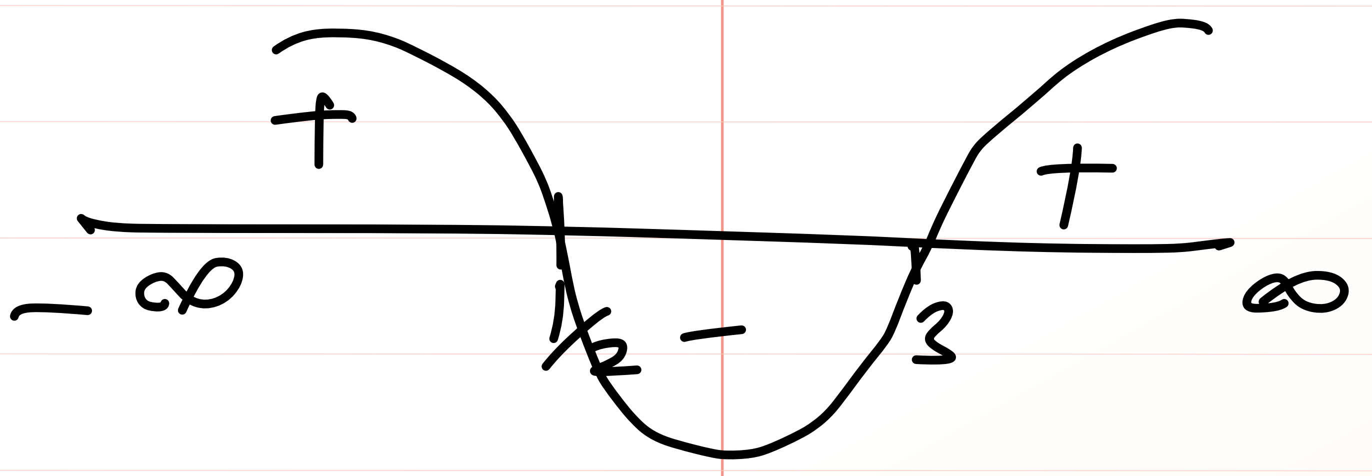
$$\frac{x^2-5x+12-3x^2+12x-15}{x^2-4x+5} > 0$$

$$\frac{-2x^2+7x-3}{x^2-4x+5} > 0$$

$$\frac{2x^2-7x+3}{x^2-4x+5} < 0$$

$$\frac{(2x-1)(x-3)}{x^2-4x+5} < 0$$

$$(2x-1)(x-3) < 0$$



$$\boxed{x \in \left(\tfrac{1}{2}, 3\right)}$$

(8) $$1 < \frac{3x^2-7x+8}{x^2+1} \le 2$$

$$1 < \frac{3x^2-7x+8}{x^2+1}$$

$$\frac{3x^2-7x+8}{x^2+1} > 1$$

$$\frac{3x^2-7x+8}{x^2+1} - 1 > 0$$

$$\frac{3x^2-7x+8-x^2-1}{x^2+1} > 0$$

$$\frac{2x^2-7x+7}{x^2+1} > 0$$

$$2x^2-7x+7 > 0$$

$$x \in R$$

(?) $$\frac{3x^2-7x+8}{x^2+1} \le 2$$

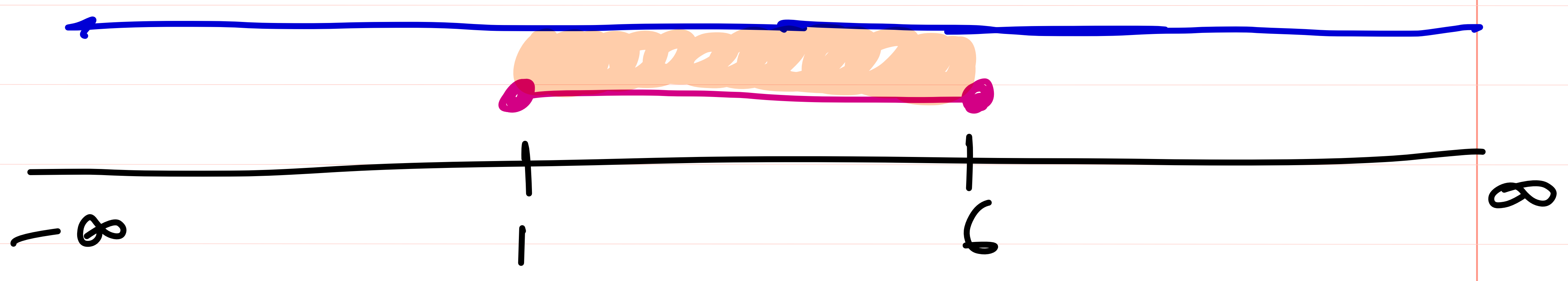
$$\frac{3x^2-7x+8-2x^2-2}{x^2+1} \le 0$$

$$\frac{x^2-7x+6}{x^2+1} \le 0$$

$$(x-1)(x-6) \le 0$$

$$\boxed{x \in [1, 6]}$$

$x \in \mathbb{R} \quad \cap \quad x \in [1, 6]$



$$\boxed{x \in [1, 6]}$$

(9) $(x^2+3x+1)(x^2+3x-3) \geq 5$

Let $x^2+3x = t$

$(t+1)(t-3) \geq 5$

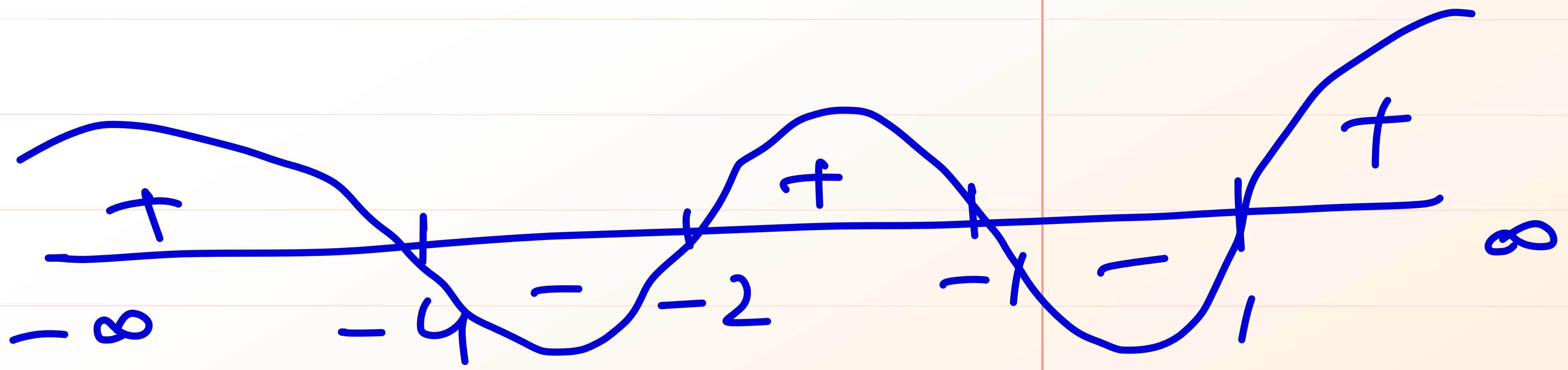
$t^2-2t-3 \geq 5$

$t^2-2t-8 \geq 0$

$(t-4)(t+2) \geq 0$

$(x^2+3x-4)(x^2+3x+2) \geq 0$

$(x+4)(x-1)(x+1)(x+2) \geq 0$



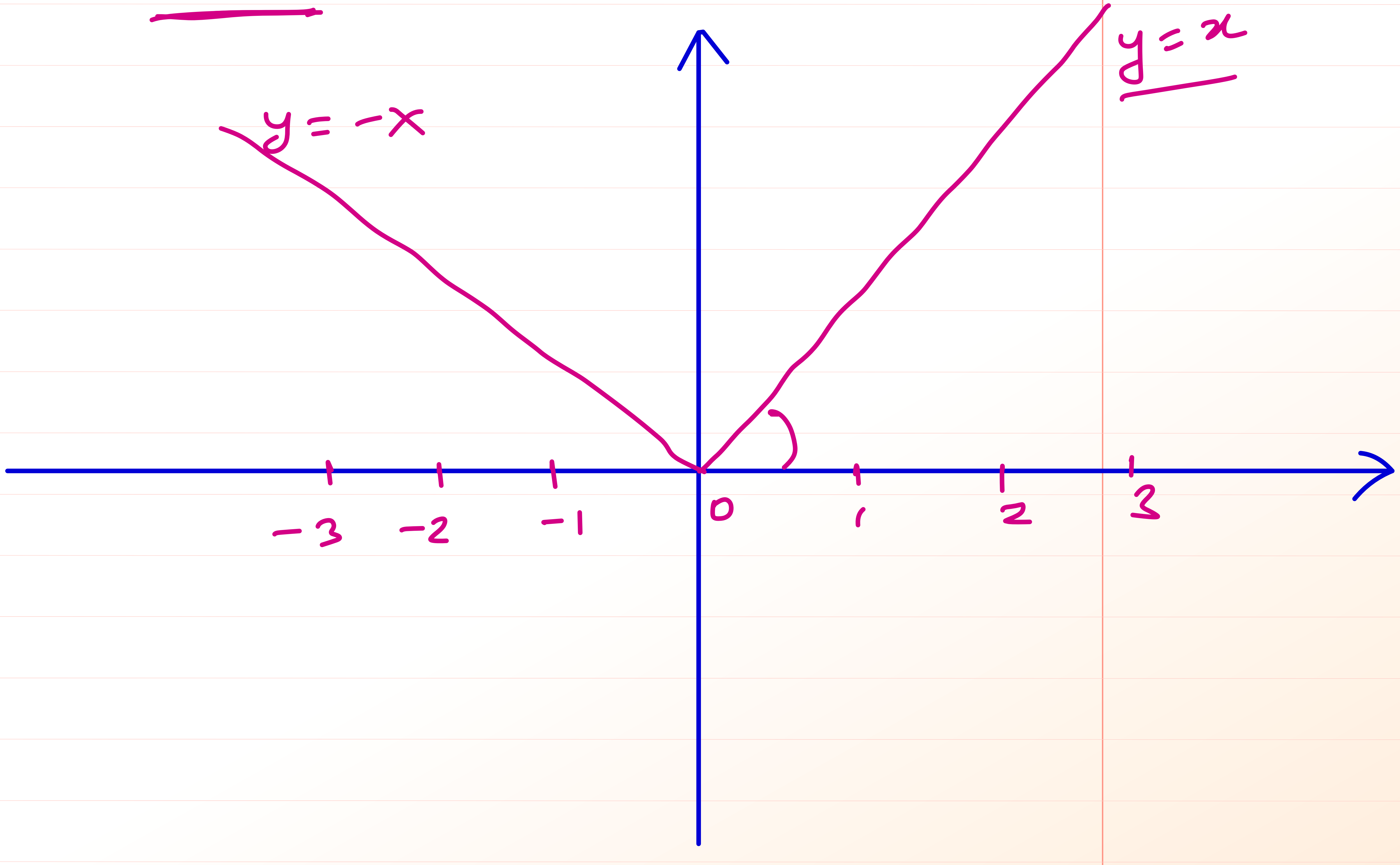
$$\boxed{x \in (-\infty, -4] \cup [-2, -1] \cup [1, \infty)}$$

Modulus function / Absolute value function :-

$$y = |x| = \begin{cases} -x & x < 0 \\ +x & x \geq 0 \end{cases}$$

$\sqrt{9} = +3$

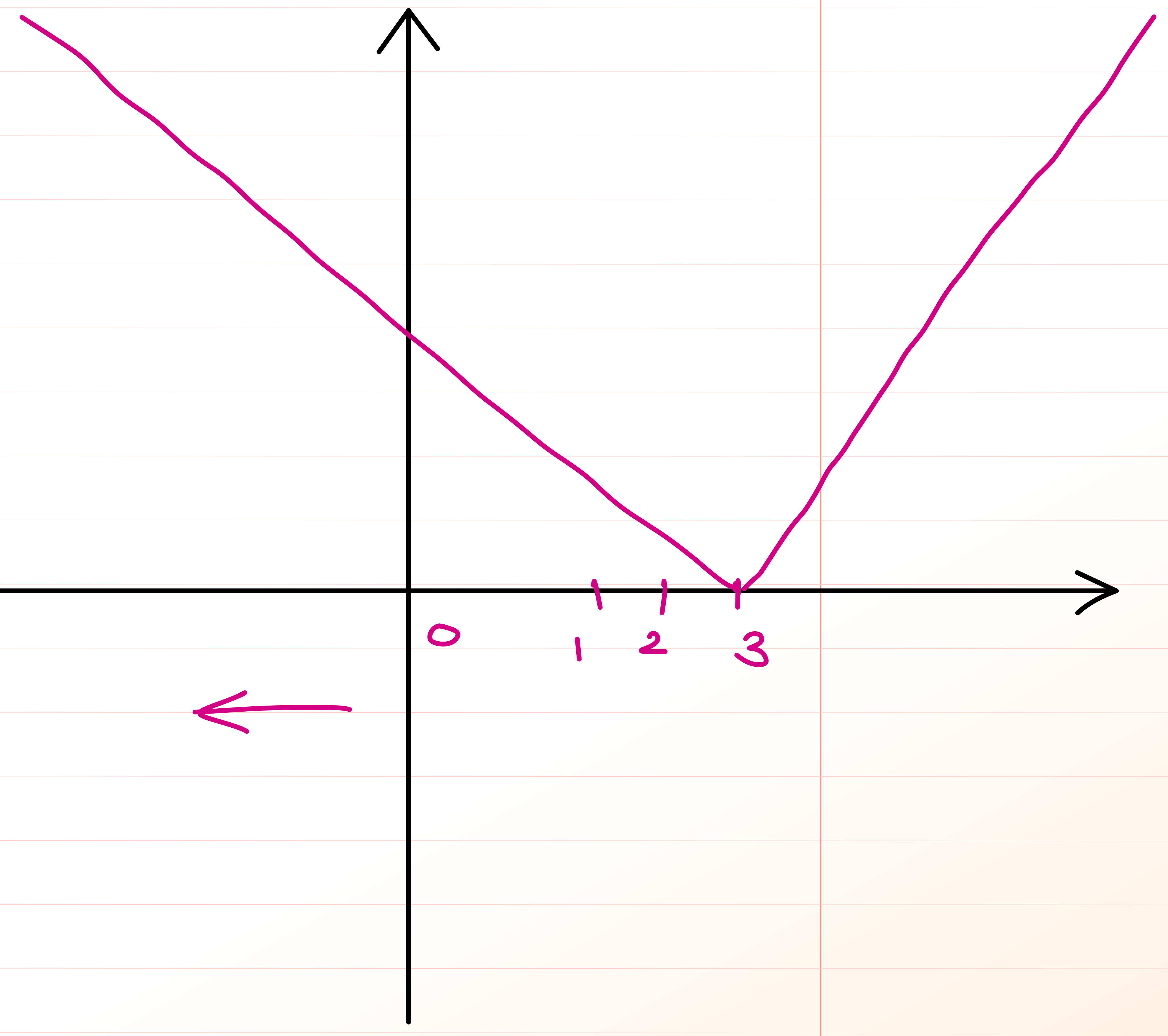
$\sqrt{x^2} = |x|$



$$y = |\underbrace{x-3}| =$$

$$= \begin{cases} \underline{-(x-3)} & x-3 < 0 \\ +(x-3) & x-3 \geq 0 \end{cases}$$

$$= \begin{cases} -(x-3) & \underline{x < 3} \\ +(x-3) & \underline{x \geq 3} \end{cases}$$



$$y = |2x - 7|$$

$$= \begin{cases} -(2x-7) & 2x-7 < 0 \\ +(2x-7) & 2x-7 \geq 0 \end{cases}$$

$$= \begin{cases} -(2x-7) & x < 7/2 \\ (2x-7) & x \geq 7/2 \end{cases}$$

Q $|3x-2| + x = 11$

$\underbrace{3x-2 < 0}$ $\Rightarrow$ $x < 2/3$

$-(3x-2) + x = 11$

$-3x + 2 + x = 11$

$\boxed{x = -9/2}$

$x = -9/2$

$3x-2 \geq 0 \Rightarrow \boxed{x \geq 2/3}$

$+(3x-2) + x = 11$

$4x = 13$

$\boxed{x = 13/4}$

$x = 13/4$

$\boxed{x \in \left\{-\frac{9}{2}, \frac{13}{4}\right\}}$

Homework

Race - 3 complete

Fundamentals
of
Maths

Lecture - 6

Modulus function :-

$$y = |x| = \begin{cases} -x & x < 0 \\ +x & x \geq 0 \end{cases}$$

$$|-3| = \begin{cases} -(-3) & -3 < 0 \\ = +3 \end{cases}$$

$$|+3| = \begin{cases} +3 & 3 > 0 \end{cases}$$

$$\sqrt{x^2} = |x|$$

$$y = |x-5| = \begin{cases} -(x-5) & x-5 < 0 \\ +(x-5) & x-5 \geq 0 \end{cases}$$

$$|x-5| = \begin{cases} -(x-5) & x < 5 \\ (x-5) & x \geq 5 \end{cases}$$

$$|5-x| = \begin{cases} -(5-x) & 5-x < 0 \\ +(5-x) & 5-x \geq 0 \end{cases}$$

$$= \begin{cases} x-5 & x > 5 \\ 5-x & x \leq 5 \end{cases}$$

$$|x| = \begin{cases} -x & x<0 \\ x & x \geq 0 \end{cases}$$

$$2+5 = 7$$

① $|x-2| = 5$

$\boxed{x-2<0}$ ✓ $\to x < 2$

$-(x-2) = 5$

$\boxed{x=-3}$ ✓ Accept

OR

$\boxed{x-2 \geq 0}$ ✓ $\Rightarrow x \geq 2$

$x-2 = 5$

$\boxed{x = 7}$ ✓ Accept

② $|x-2| = -5$

$x-2 < 0 \Rightarrow x < 2$

$-(x-2) = -5$

$\boxed{x=7}$ ✗

$x \in \phi$

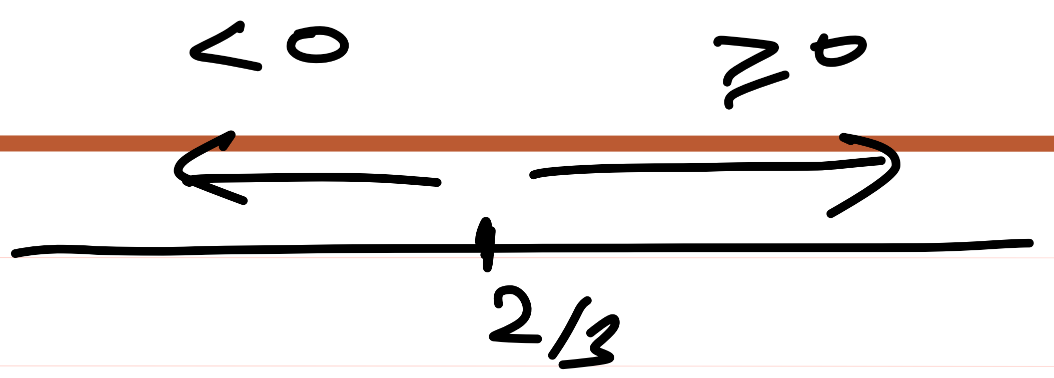
$x-2 \geq 0 \Rightarrow x \geq 2$

$x-2 = -5$

$\boxed{x=-3}$ ✗

No Solution

3. $|3x-2| + x = 11$

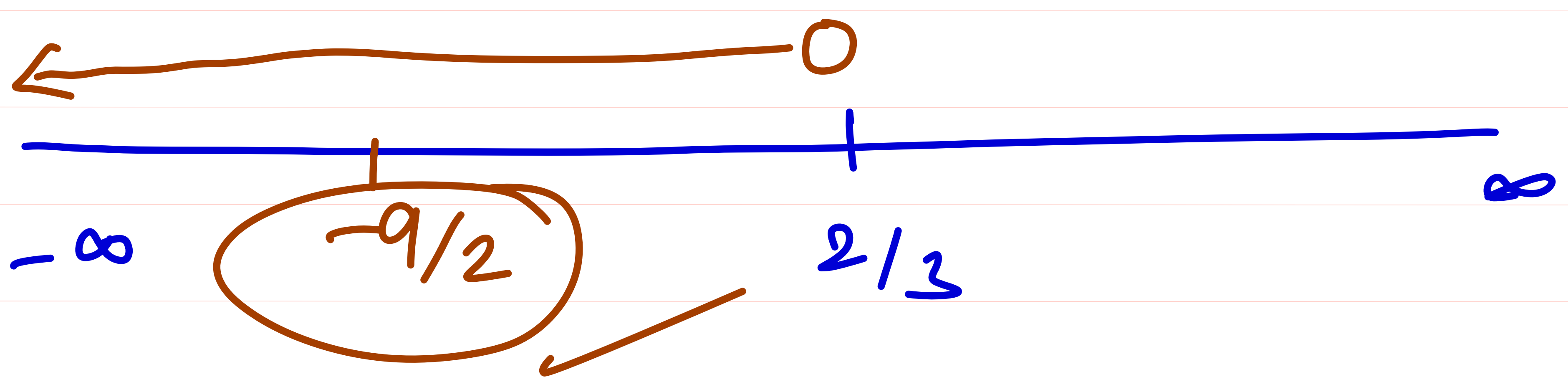


$3x-2 < 0 \Rightarrow x < 2/3$

$-(3x-2) + x = 11$

$-2x = 9$

$x = -9/2$



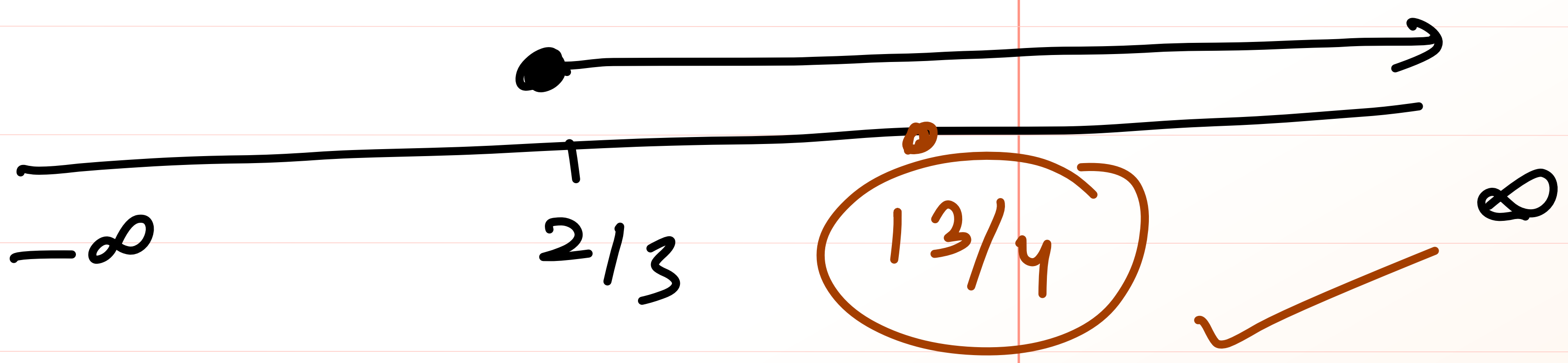
$3x-2 \geq 0$

$+(3x-2) + x = 11$

$\boxed{x = 13/4}$

$-3 < 0$

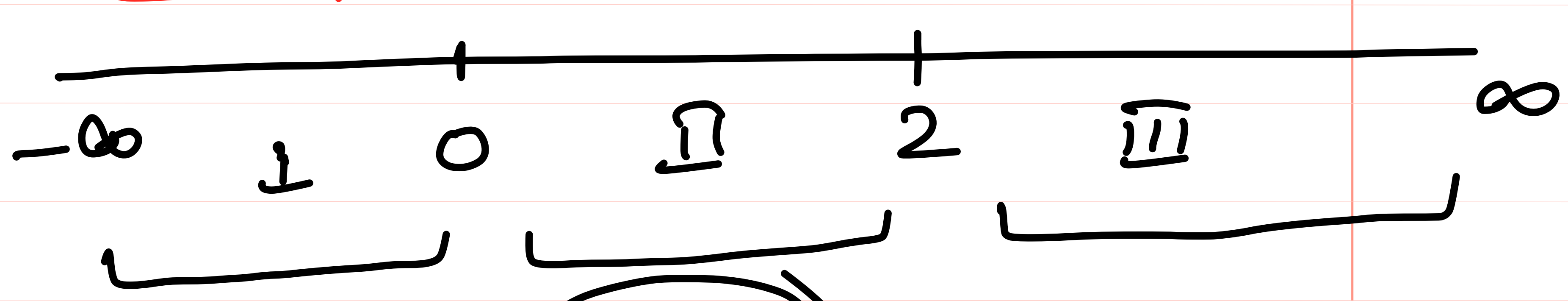
$\cancel{9 < 0}$



$$\boxed{x \in \left\{-\frac{9}{2}, \frac{13}{4}\right\}}$$

(4) $|x| - |x-2| = 2$

$\boxed{\begin{matrix} <2 \\ <0 \end{matrix}}$ $\quad \begin{matrix} <2 \\ \geq 0 \end{matrix} \quad \begin{matrix} \geq 2 \\ > 0 \end{matrix}$



$0 \to -$
$2 \to -$

$0 \to +$
$2 \to -$

Case I $\quad x < 0$

$$-(x) + (x-2) = 2$$

$$\boxed{-2 = 2} \quad \text{False}$$

$$x \in \phi$$

Case II $\quad 0 \leq x < 2$

$$+(x) + (x-2) = 2$$

$$\boxed{x = 2} \quad \text{No sol}$$

if $x < 2$
then $x < 0$

Case III : $\rightarrow$ $\quad \boxed{x \geq 2}$ ✓ $\longrightarrow$ Answer

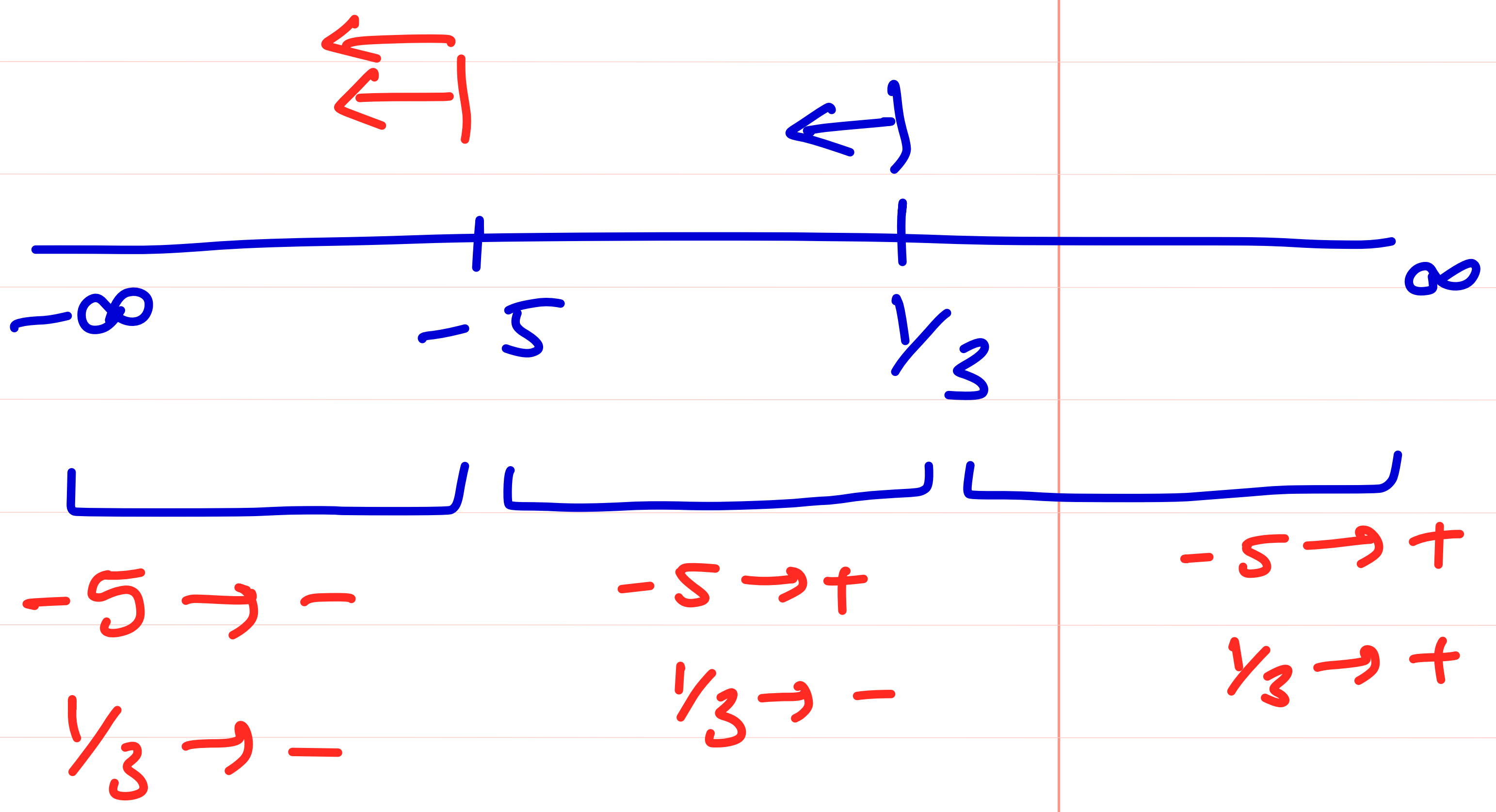
$$x - (x-2) = 2$$

$$2 = 2 \quad \text{True}$$

if $x < 0$
then $x < 2$

$$\boxed{x \in [2, \infty)}$$

(5) $|3x-1| - |x+5| = 0$



C I $\boxed{x < -5}$ ✓

$-(3x-1) + (x+5) = 0$

$-3x + 1 + x + 5 = 0$

$-2x = -6$

$\boxed{x = 3}$ ~~$\times$~~ Rejected

C II :- $-5 \le x < 1/3$

$-(3x-1) - (x+5) = 0$

$-4x + 1 - 5 = 0$

$\boxed{x = -1}$ ✓

Accepted

C III $x \ge 1/3$ ✓

$(3x-1) - (x+5) = 0$

$2x = 6$

$\boxed{x = 3}$ ✓ Accepted

$\boxed{x \in \{-1, 3\}}$

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(6) $$|x-3| + |x+2| - |x-4| = 3$$

Case I

$-\infty \quad -2 \quad 3 \quad 4 \quad \infty$

$-2 \to -$, $3 \to -$, $4 \to -$

$-2 \to +$, $3 \to -$, $4 \to -$

$-2 \to +$, $3 \to +$, $4 \to -$

$-2 \to +$, $+3 \to +$, $4 \to +$

Case II $\underbrace{-2 \le x < 3}$

$$-(x-3) + (x+2) + (x-4) = 3$$

$$\boxed{x=2}$$

Case III $\underbrace{3 \le x < 4}$

$$(x-3) + (x+2) + (x-4) = 3$$

$x = 8/3$ ✗

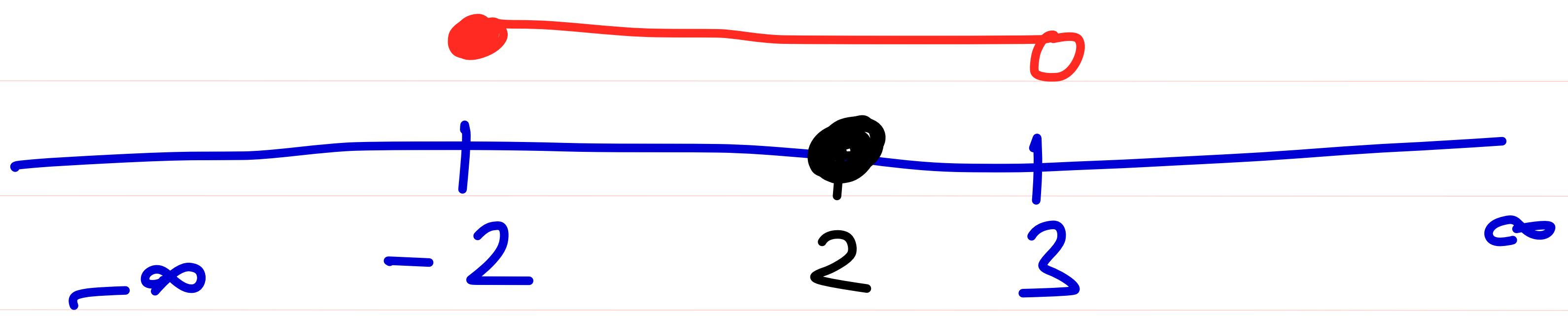
Case IV $x \ge 4$

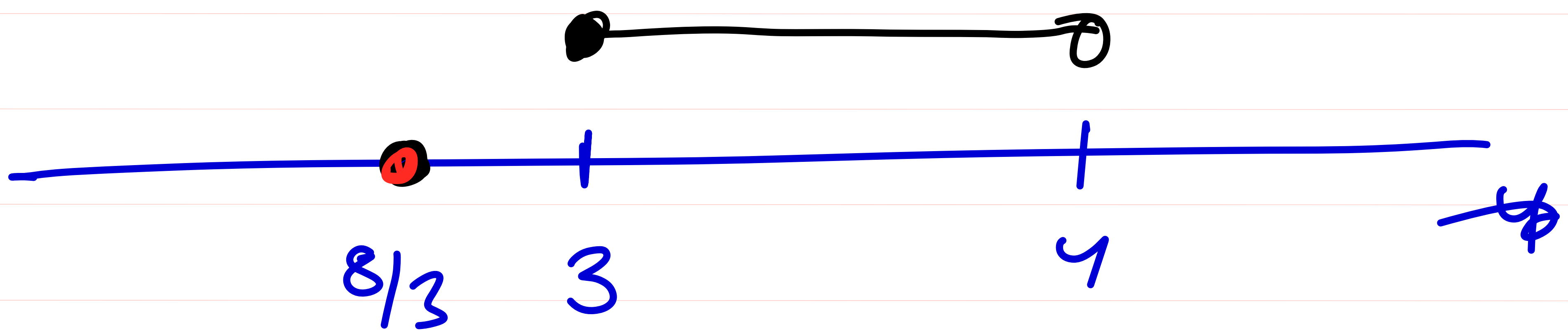
$$\boxed{x \in \{-6, 2\}}$$

Case II $\quad \underbrace{-2 \le x < 3}$

$$-(x-3) + (x+2) + (x-4) = 3$$

$$\boxed{x = 2}$$





int $\quad \phi$

ine

(1) $|5-2x| < 1$

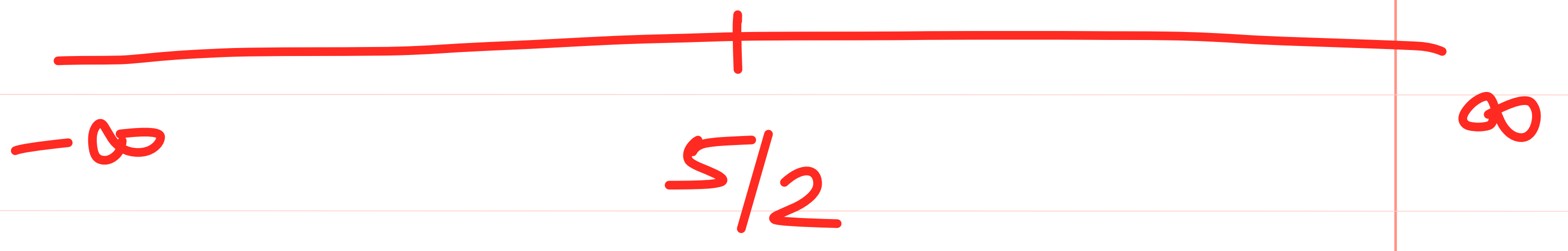
Case I $\quad \boxed{5-2x < 0} \Rightarrow \boxed{x > 5/2}$

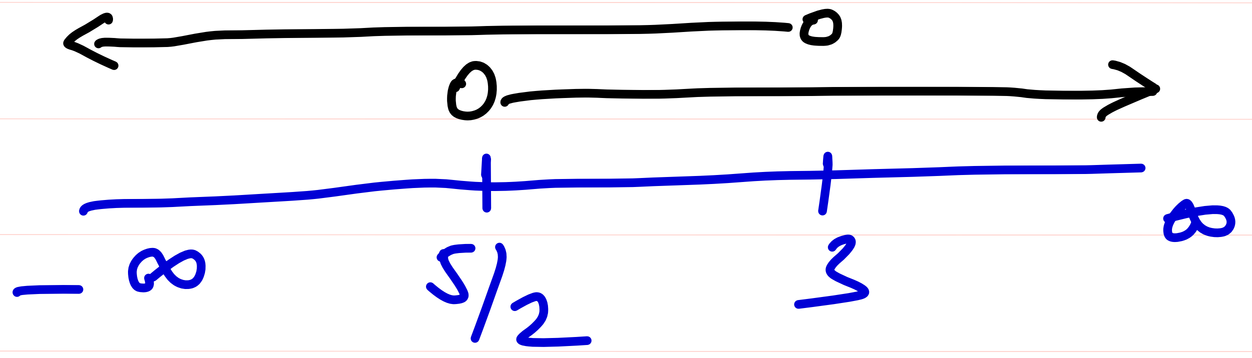
$-(5-2x) < 1$

$-5+2x < 1$

$\boxed{x < 3}$

$\boxed{x \in \left(\frac{5}{2}, 3\right)}$

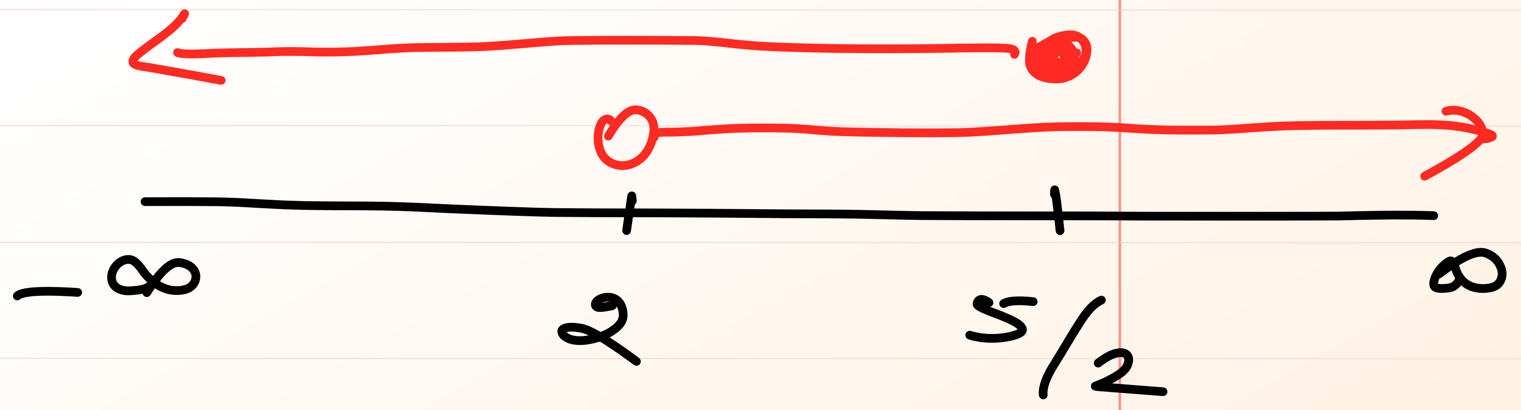




Case II $\quad 5-2x \geq 0 \Rightarrow x \leq 5/2$

$+(5-2x) < 1$

$-2x < -4 \Rightarrow x > 2$

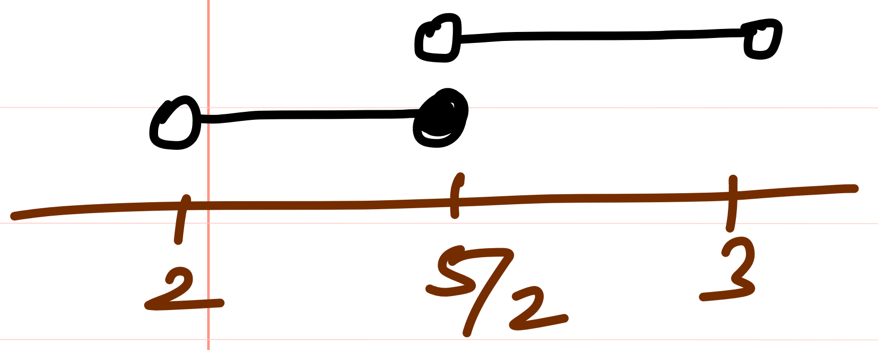


$\boxed{x \in (2, 5/2]}$

Case I $\quad x \in \left(\frac{5}{2}, 3\right)$

Case II $\quad x \in (2, 5/2]$

$\boxed{x \in (2, 3)}$



Case I → Case interval, Ans → intersection

∪

Case II → Case int, Ans. → intersection

∪

Case III → Case int, Ans → intersection

∪

Case IV → Case int, Ans → intersection

(2) $2|x+1| > x+4$

(3) $|x+2| - |x-1| < x - 3/2$

(4) $\dfrac{|x^2+1|}{|x-2|} \geq 0$

(5) $|x-2| \leq 0$

(6) $|x-1| \geq 0$

Fundamentals of Maths

Lecture - 7

Q $|x+2| - |x-1| < x - \frac{3}{2}$

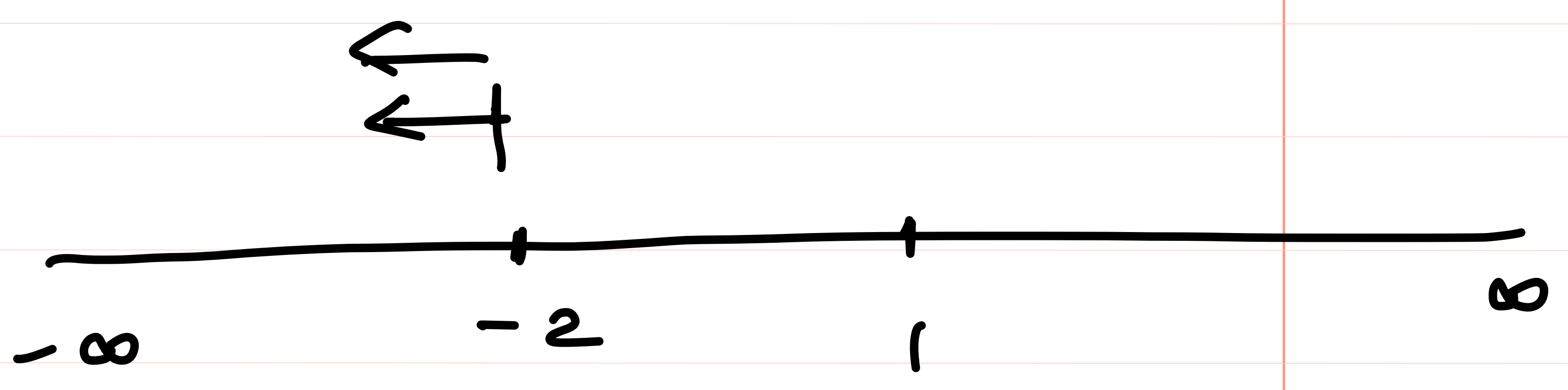
Case I $x < -2$

$$-(x+2) + (x-1) < x - \frac{3}{2}$$

$$-3 < x - \frac{3}{2}$$

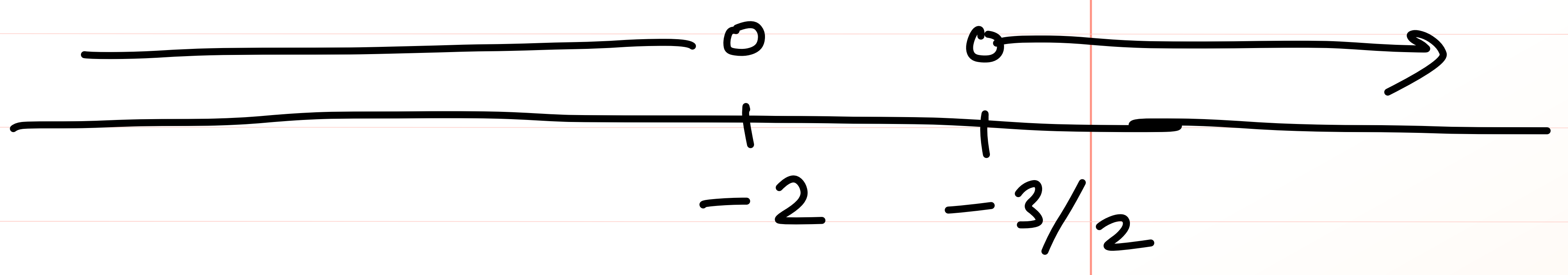
$$x > -3/2$$

$$x \in \phi$$



$-2 \to -$ $\quad$ $-2 \to +$

$+1 \to -$ $\quad$ $1 \to -$

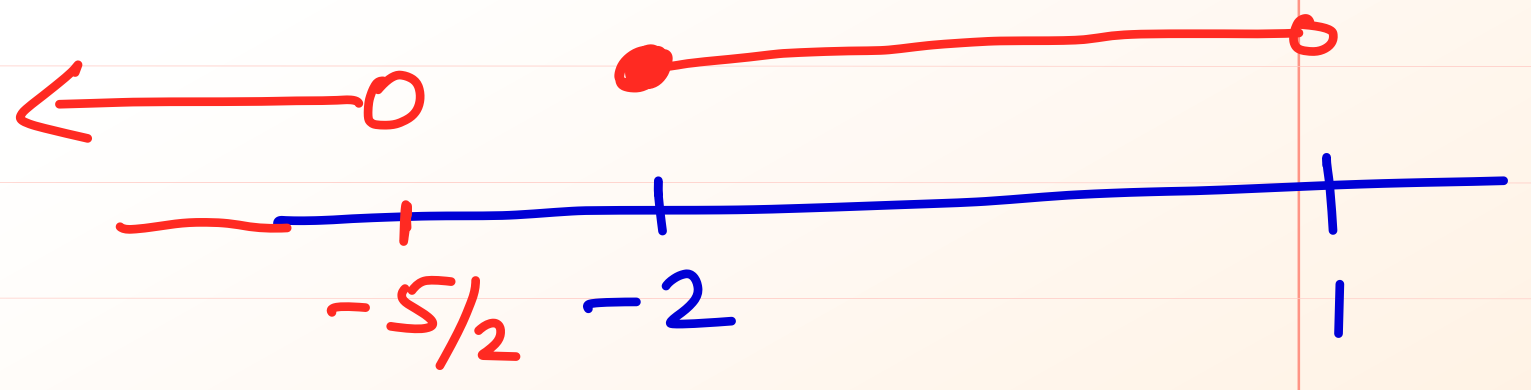


Case II $\boxed{-2 \le x < 1}$

$$+(x+2) + (x-1) < x - \frac{3}{2}$$

$$2x + 1 < x - \frac{3}{2}$$

$$\boxed{x < -5/2}$$

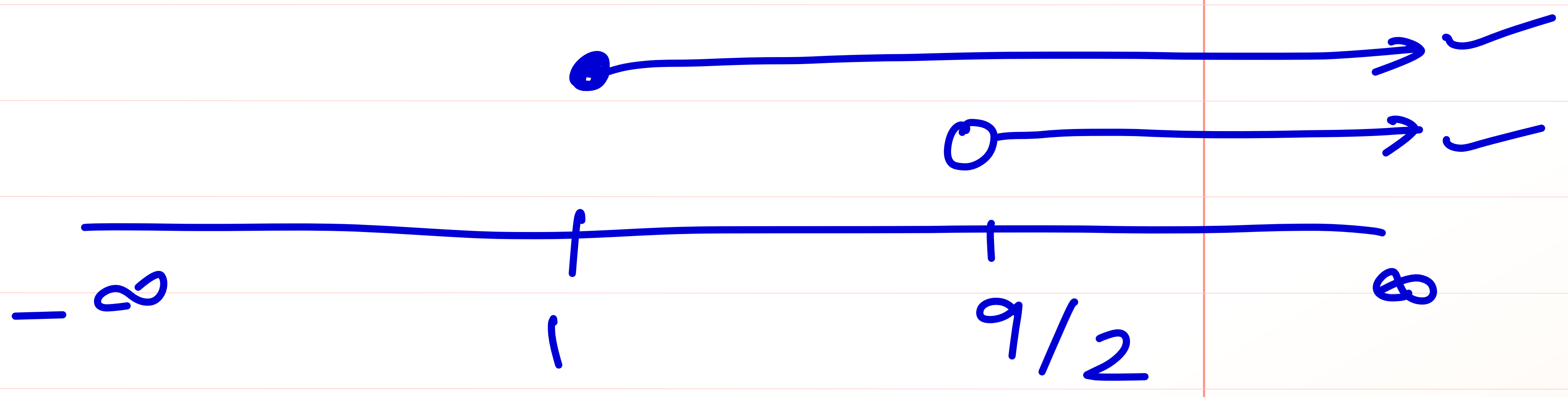


$x \in \phi$

Case II :- $\boxed{x \geq 1}$

$$(x+2) - (x-1) < x - 3/2$$

$$\boxed{x > 9/2}$$



$$\boxed{x \in \left(\frac{9}{2}, \infty\right)}$$

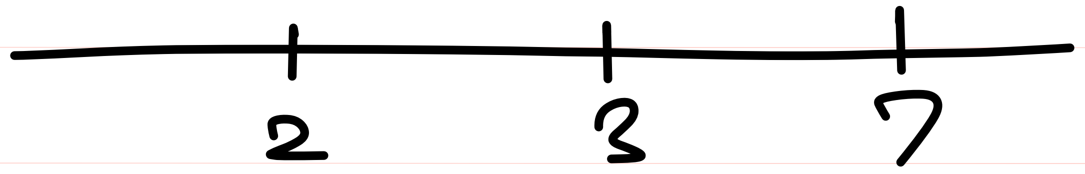
Q 2

$$\frac{x^2-5x+6}{|x|+7} < 0$$

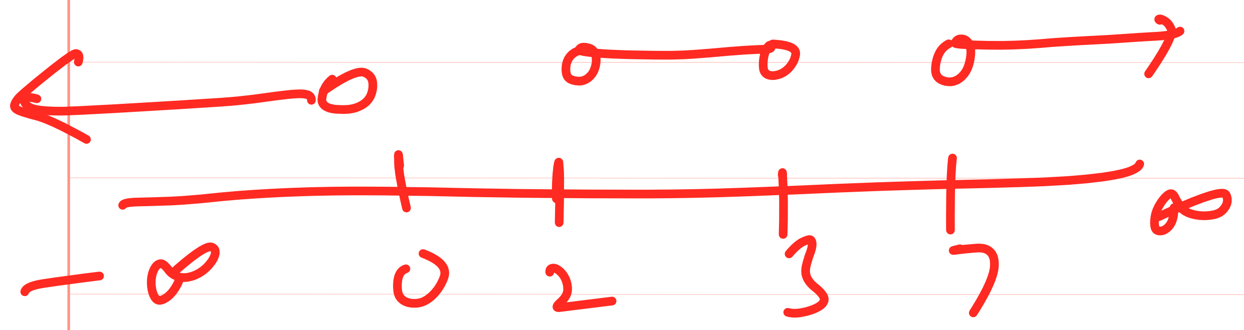
$\boxed{x < 0}$

$$\frac{x^2-5x+6}{-x+7} < 0$$

$$\frac{(x-2)(x-3)}{(x-7)} > 0$$



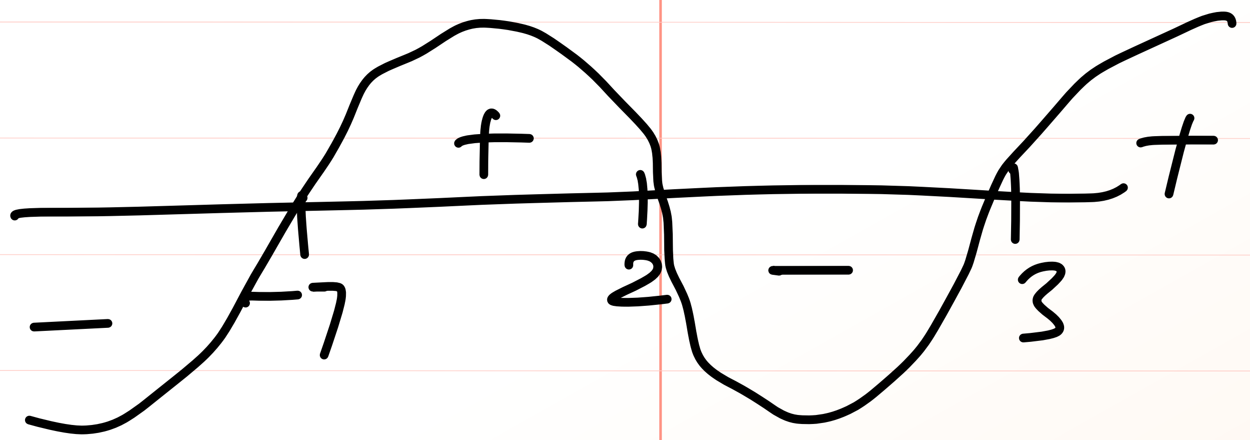
$x \in (2,3) \cup (7,\infty)$



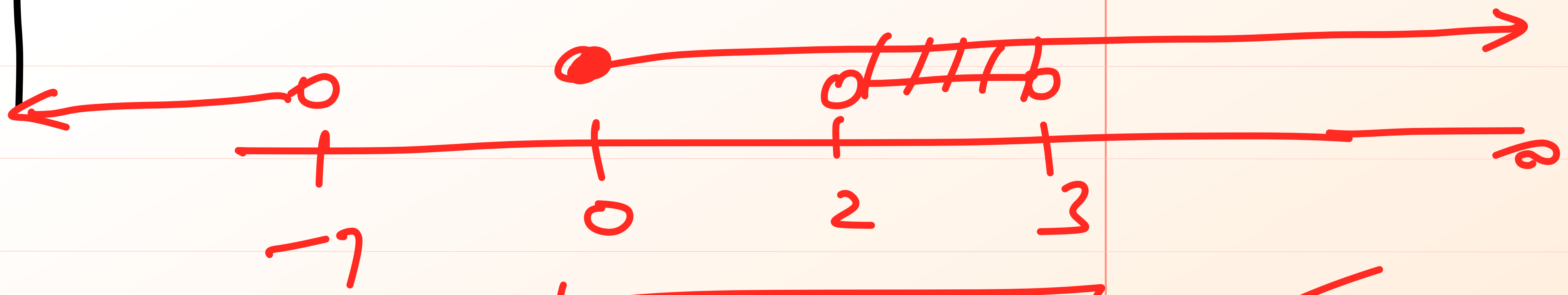
$x \in \phi$

$x \geq 0$

$$\frac{(x-2)(x-3)}{x+7} < 0$$



$x \in (-\infty,-7) \cup (2,3)$



$\boxed{x \in (2,3)}$

Q $\dfrac{|x+2| - x}{x} < 2$

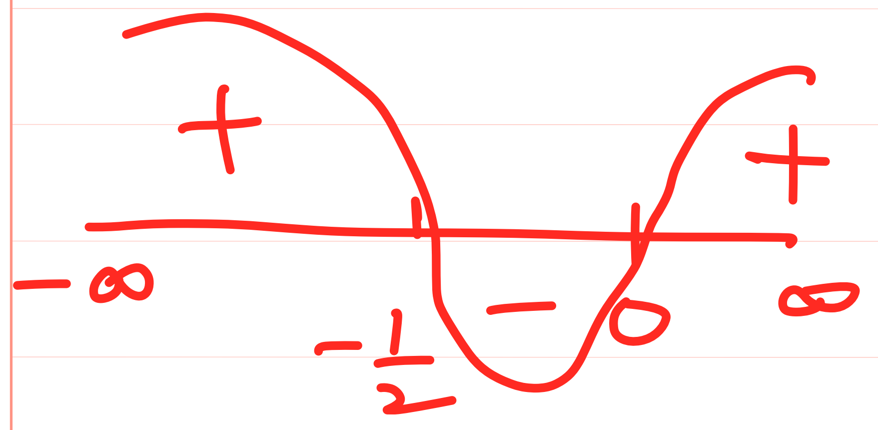
Case I $\quad x < -2$

$\dfrac{-(x+2) - x}{x} < 2$

$\dfrac{-x-2-x}{x} - 2 < 0$

$\dfrac{-2x-2-2x}{x} < 0$

$-\dfrac{(4x+2)}{x} < 0$

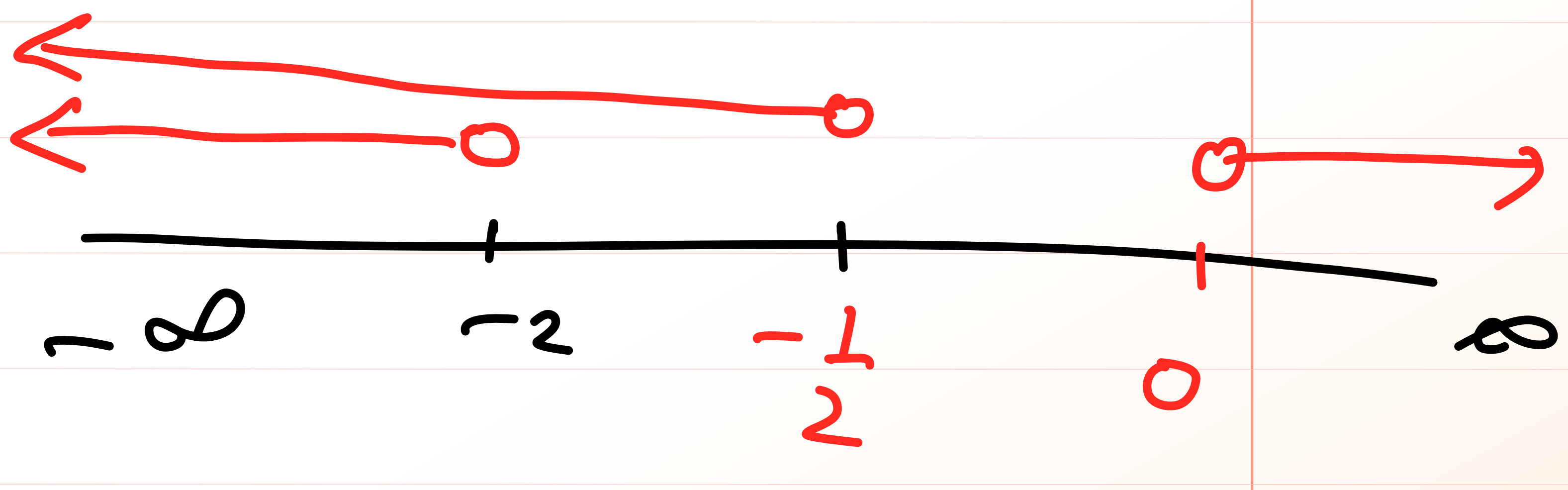


$\dfrac{4x+2}{x} > 0$

$$\boxed{x \in \left(\infty, -\tfrac{1}{2}\right) \cup (0, \infty)}$$

$|-3| = -(-3) \qquad -3<0$

$|x| = \begin{cases} -x & x<0 \\ +x & x \geq 0 \end{cases}$



$x \in (-\infty, -2)$

Case II

$x \geq -2$

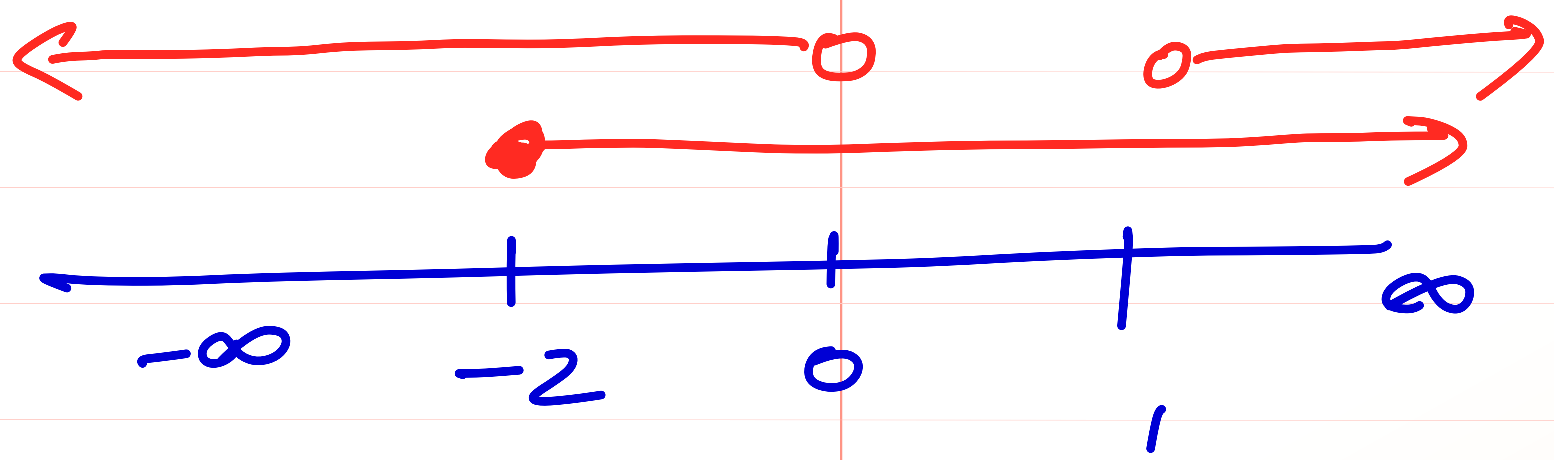
$$\frac{+(x+2)-x}{x} - 2 < 0$$

$$\frac{2-2x}{x} < 0$$

$$\frac{1-x}{x} < 0$$

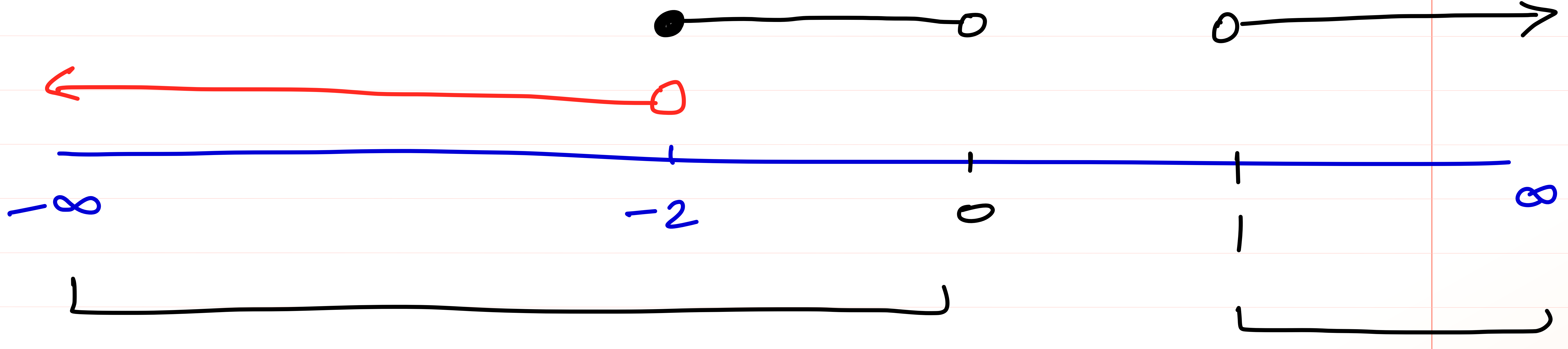
$$\frac{x-1}{x} > 0$$

$$\boxed{x \in (-\infty, 0) \cup (1, \infty)}$$



$$\boxed{x \in [-2, 0) \cup (1, \infty)}$$

$x \in (-\infty, -2) \quad \cup \quad x \in [-2, 0) \cup (1, \infty)$



from union

$\therefore$

$$\boxed{x \in (-\infty, 0) \cup (1, \infty)}$$

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Q $\dfrac{1}{|x|-3} < \dfrac{1}{2}$

C I $x<0$

$\dfrac{1}{-x-3} < \dfrac{1}{2}$

$\dfrac{1}{-x-3} - \dfrac{1}{2} < 0$

$\dfrac{2+x+3}{2(-x-3)} < 0$

$\dfrac{x+5}{(x+3)} > 0$

$\boxed{x \in (-\infty,-5) \cup (-3,\infty)}$ ✓

$-\infty \quad -5 \quad -3 \quad 0 \quad \infty$

$\Rightarrow \boxed{x \in (-\infty,-5) \cup (-3,0)}$

C II $x \geq 0$

$\dfrac{1}{x-3} - \dfrac{1}{2} < 0$

$\dfrac{2-x+3}{x-3} < 0$

$\dfrac{-x+5}{x-3} < 0$

$\dfrac{x-5}{x-3} > 0$

$-\infty \quad 0 \quad 3 \quad 5 \quad \infty$

✓ $\boxed{x \in (-\infty,3) \cup (5,\infty)}$

$\boxed{x \in [0,3) \cup (5,\infty)}$

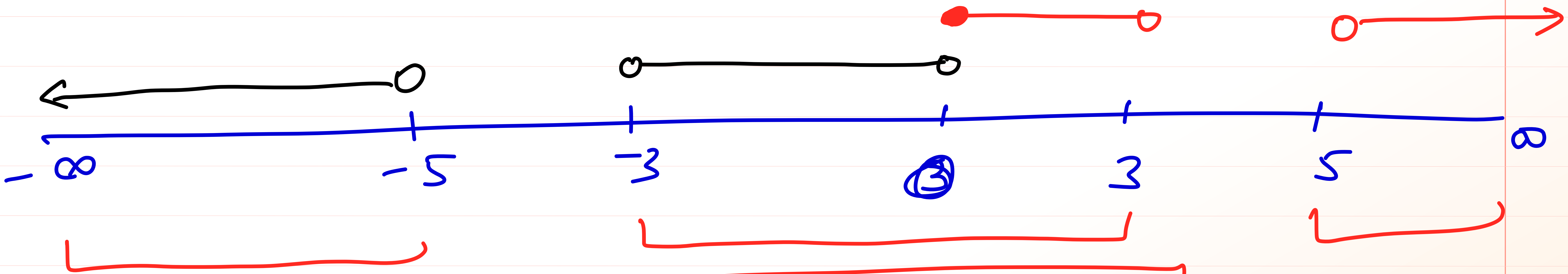
$\cup$

$\Rightarrow \boxed{x \in (-\infty,-5) \cup (-3,3) \cup (5,\infty)}$

$$\boxed{x \in [0, 3) \cup (5, \infty)}$$

$$\cup$$

$$\boxed{x \in (-\infty, -5) \cup (-3, 0)} \Rightarrow \boxed{x \in (-\infty, -5) \cup (-3, 3) \cup (5, \infty)}$$



$$\boxed{x \in (-\infty, -5) \cup (-3, 3) \cup (5, \infty)}$$

4) $\dfrac{|x^2+1|}{|x-2|} \geq 0$ $\qquad x^2+1>0$

$\dfrac{1}{|x-2|} \geq 0$ ✓

C I $\quad x<2$

$\dfrac{1}{-x+2} \geq 0$

$\dfrac{1}{x-2} \leq 0$

$\boxed{x \in (-\infty, +2)}$ ✓

$\cap$

$x \in (-\infty, +2)$

C II $\quad x \geq 2$

$\dfrac{1}{x-2} \geq 0$

$x > 2$

$\xrightarrow{\cap}$ $x \in (2, \infty)$

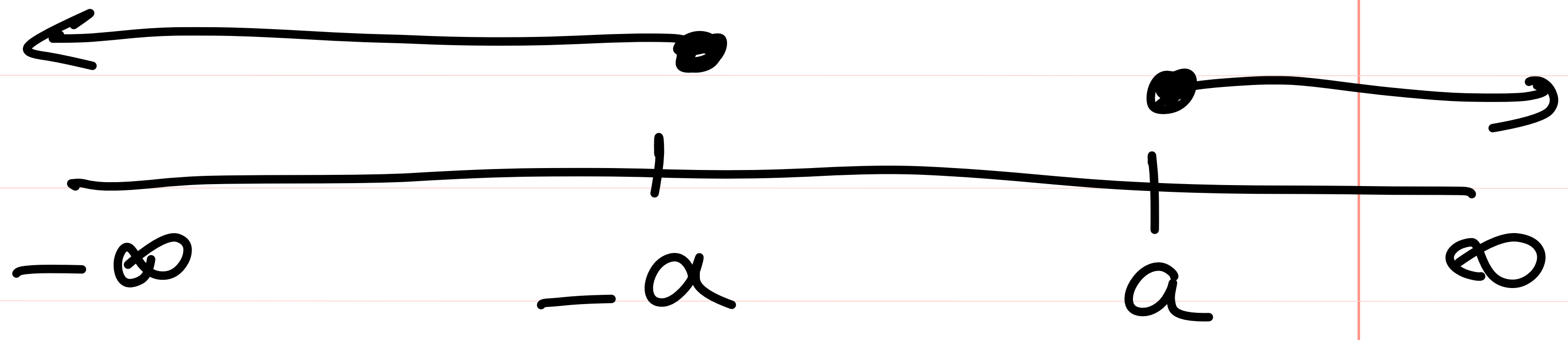
$\boxed{x \in (-\infty, 2) \cup (2, \infty)}$

Properties of modulus function :-

(i) $|x| \geq a \Rightarrow x \leq -a$ OR $x \geq a$

where a is positive

$x < 0$	$x \geq 0$
$-x \geq a$	$x \geq a$
$x \leq -a$	



Q

$$\left|\frac{2x-1}{x-1}\right| > 2$$

$$\underbrace{|x| > a}_{\Rightarrow} \quad \underbrace{x < -a}\ \text{OR}\ \underbrace{x > +a}$$

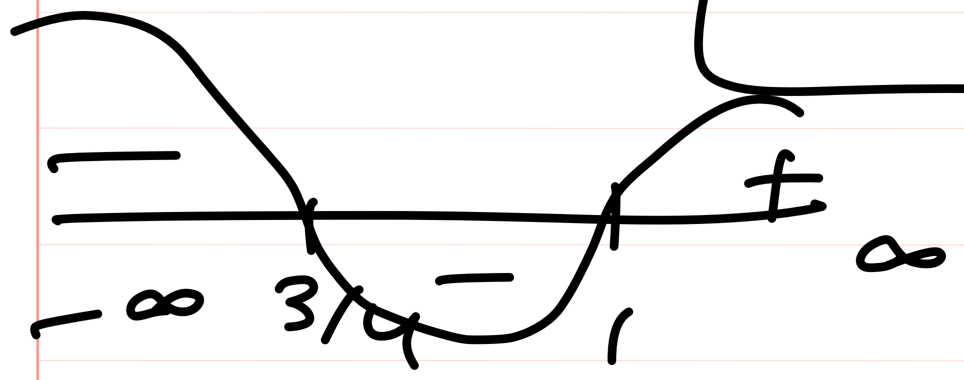
$$\frac{2x-1}{x-1} < -2$$

$$\frac{2x-1}{x-1} + 2 < 0$$

$$\frac{2x-1+2x-2}{x-1} < 0$$

$$\frac{4x-3}{x-1} < 0$$

$$\boxed{x \in \left(\frac{3}{4}, 1\right)}$$



$$\frac{2x-1}{x-1} > 2$$

$$\frac{2x-1}{x-1} - 2 > 0$$

$$\frac{2x-1-2x+2}{x-1} > 0$$

$$\frac{1}{x-1} > 0$$

$$\boxed{x \in (1, \infty)}$$

$\cup$

$$\boxed{x \in \left(\frac{3}{4}, 1\right) \cup (1, \infty)}$$

P-2 $\quad |x| \le a \Rightarrow -a \le x \le a \quad a \in +ve$

Q $\quad \left| \dfrac{x^2-3x-1}{x^2+x+1} \right| < 3$

$$-3 < \frac{x^2-3x-1}{x^2+x+1} < 3$$

$$-3 < \frac{x^2-3x-1}{x^2+x+1} \qquad \text{intersection} \qquad \frac{x^2-3x-1}{x^2+x+1} < 3$$

final ans.

P-3 $\quad |x| > |y| \Rightarrow \underbrace{x^2 > y^2}$

Q $\quad |\underbrace{x^2-1}| \le |\underbrace{2x-1}|$

$(x^2-1)^2 \le (2x-1)^2$

$(x^2-1)^2 - (2x-1)^2 \le 0$

$(x^2-1+2x-1)(x^2-1-2x+1) \le 0$

$(\underbrace{x^2+2x-2})(\underbrace{x^2-2x}) \le 0$

$(x-(-1+\sqrt{3}))(x-(-1-\sqrt{3}))\, x(x-2) \le 0$

x^2+2x-2

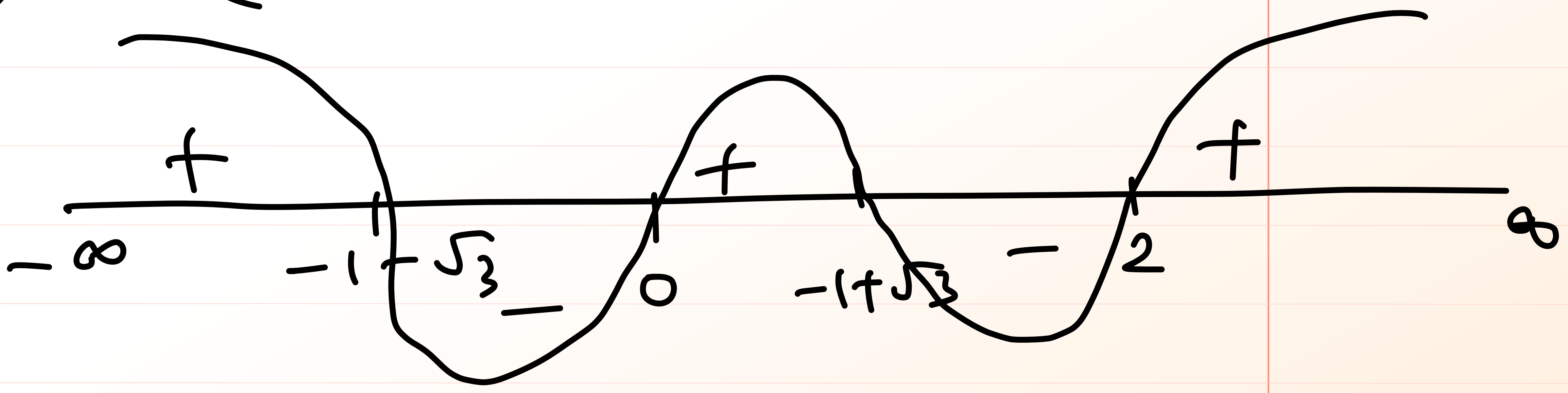
$x = \dfrac{-2 \pm \sqrt{4+8}}{2}$

$= \dfrac{-2 \pm 2\sqrt{3}}{2}$

$x = -1 \pm \sqrt{3}$

$x = -1+\sqrt{3}$

$-1-\sqrt{3}$



$x \in [-1-\sqrt{3}, 0] \cup [-1+\sqrt{3}, 2]$

P-4 if $|x+y| = |x| + |y|$ then $x \cdot y \geq 0$

Q $|x+1| + |x-1| = |2x|$

$|x+1| + |x-1| = |x+1+x-1|$

$(x-1)(x+1) \geq 0$

$$\boxed{x \in (-\infty, -1] \cup [1, \infty)}$$

P-5 if $|x-y| = |x| + |y| \Rightarrow x \cdot y \le 0$

Q $|2x-3| - |x^2-4x+3| = |x^2-2x|$

$|2x-3| = |x^2-2x| + |x^2-4x+3|$

$|(x^2-2x) - (x^2-4x+3)| = |x^2-2x| + |x^2-4x+3|$

$(x^2-2x)(x^2-4x+3) \le 0$

$x(x-2)(x-3)(x-1) \le 0$

$x \in [0,1] \cup [2,3]$

Ans

Fundamentals of Maths

Lecture - 8

Logarithms : →

If $\underline{a^x = N}$

then $\underline{\log_a N = x}$

$3^a = 27 \Rightarrow a = 3$

$3^a = 17 \Rightarrow \log$

Every positive real number N can be expressed in exponential form as $a^x = N$.

where a is also a positive real different than unity and is called the base and x is called the exponent.

$a^x = N$

$\log_a N = x \longrightarrow a^x = N$

(i) $a \neq 1$; ✓

(ii) $a > 0$ ✓

(iii) $N > 0$

$1^x = 1$

$(0.5)^x \longrightarrow +ve$

$(2)^x \longrightarrow +ve$

$(100)^x \longrightarrow +ve$

$\log_5 4 = \boxed{a}$

$5^a = 4$

$3^a = 17$

$\log_3 17 = a$

$4^a = 5$

$\log_4 5 = a$

log form | **Exponent form**

$$\log_{\boxed{2}} 3 = \underline{x} \Rightarrow \underbrace{2^x = 3} \longrightarrow \log_2 3 = x$$

$$\log_3 5 = q \Rightarrow 3^q = 5$$

$$\log_a b = x \Rightarrow a^x = b \quad \cancel{\text{or } b^x = a}$$

① Solve for x

ⓐ $\log_2 x = 0 \Rightarrow x = 2^0 \Rightarrow x = 1$

ⓑ $\log_3 (\log_2 x) = 1 \Rightarrow$

$3^1 = \log_2 x$

$\log_2 x = 3^1$

$x = 2^3$

$\boxed{x = 8}$

$\log_a N = x$

$\log_2 (\log_3 x) = 1$

$\log_3 x = 2^1$

$\log_3 x = 2$

$x = 3^2$

$x = 9$

Note: (1) For a given value of N, $\log_a N$ will give us a unique value.

(2) log of zero does not exist.

(3) Logarithms of negative reals are not defined in the system of real numbers.

Q 1 If $\log_2 (x-1)$ is meaningful, then find x.

$a = 2 \longrightarrow$

$(x-1) > 0$

$x > 1$

$$\boxed{x \in (1, \infty)}$$

$\log_a N = x$

$a > 0$

$a \neq 1$

$N > 0$

Q 2 If $\log_x (2x-1)$ is meaningful, then find x.

$x \neq 1$ ✓

$x > 0$ ✓

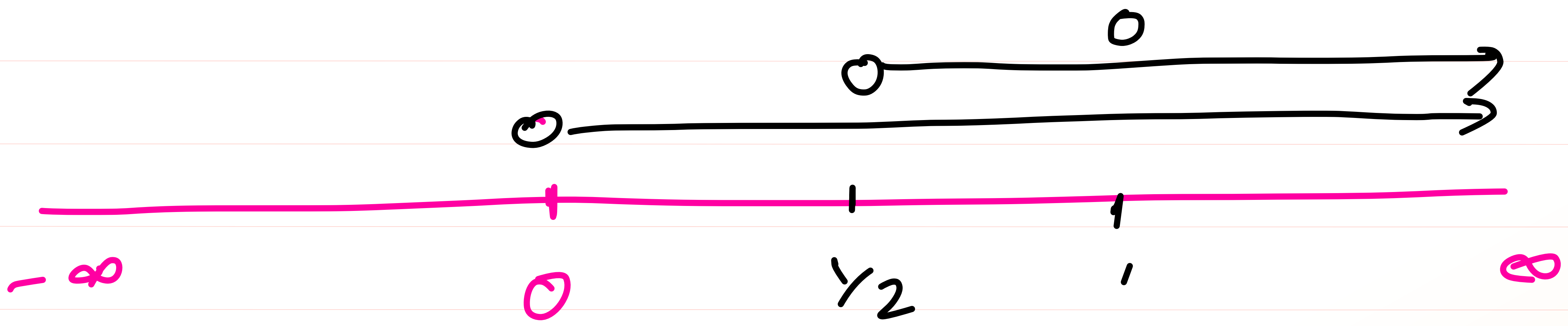
$2x-1 > 0$ ✓ $\Rightarrow$ $x > 1/2$

$\log_a N = x$

$a \neq 1$

$a > 0$

$N > 0$



$$\boxed{x \in \left(\frac{1}{2}, \infty\right) - \{1\}}$$

Fundamental identities :- $(N>0;\ N\neq 1)$

① $\boxed{\log_N N = 1}$

$N^1 = N$

$\log_N N = 1$

logarithm of a number of the same base is 1.

② $\boxed{\log_{1/N} N = -1}$

$\left(\frac{1}{N}\right)^{-1} = N$

$\log_{1/N} N = -1$

logarithm of a number to its reciprocal is -1.

③ $\boxed{\log_N 1 = 0}$

$N^0 = 1$

$\log_N 1 = 0$

logarithm of unity of any base is 0.

Examples

(1) (i) $\log_{\sin 30}(\cos 60) = \log_{(1/2)}\left(\frac{1}{2}\right) = 1$

(ii) $\log_{10}(\cos 0) = \log_{10}(1) = 0$

(iii) $\log_{1.5}(0.\bar{6}) = \log_{(3/2)}\left(\frac{2}{3}\right) = -1$

(iv) $\log_{(4/3)}(1.\bar{3}) = \log_{(4/3)}\left(\frac{4}{3}\right) = 1$

(v) $\log_2(\sin^2 x + \cos^2 x) = \log_2(1) = 0$

(vi) $\log_{0.125}(8) = \log_{(1/8)}(8) = -1$

$\boxed{\log_a a = 1}$

$0.\bar{6} = 0.666\ldots = 2/3$

$1.\bar{3} = 1.333\ldots$

$= 4/3$

(vii) $\log_{(2-\sqrt{3})}(2+\sqrt{3}) = \log_{\frac{1}{2+\sqrt{3}}}(2+\sqrt{3}) = -1$

$(2-\sqrt{3})(2+\sqrt{3}) = 1$

$2-\sqrt{3} = \frac{1}{2+\sqrt{3}}$

(2) (i) $\log_{\sqrt[3]{7}}(2401) = \log_{(7^{1/3})} 7^4 = \log_{(7^{1/3})}\left(7^{12/3}\right) = \log_{(7^{1/3})}\left(7^{1/3}\right)^{12}$

$= 12$

(ii) $\log_{5\sqrt{5}}(125) = \log_{(5^{3/2})}$

(iii) $\log_2\{\log_2\{\log_3(\log_3(27^3))\}\} =$

$\log_a (b)^m = m$

(1) $x = 2^{\log_2 3} \Rightarrow x = 3$

(2) $\underbrace{3^{\log_3 x}} = 5 \Rightarrow x = 5$

(3) $3^{\log_3 (x-1)} = 3 \Rightarrow x = 4$

(4) $7^{\log_7 (x^2-4x+5)} = x-1$

$x^2 - 4x + 5 = x - 1$

$x^2 - 5x + 6 = 0$

$(x-2)(x-3) = 0$

$\boxed{x = 2, 3}$ Answer

Property

$x = a^{\log_a N}$

$x = N$

$a^x = N$

$\underbrace{\log_a N} = x$

$a^{\log_a N} = a^x$

$\boxed{a^{\log_a N} = N}$

The principal properties of logarithm :- $(a>0,\ a\neq 1,\ N>0)$

(a) $\log_a(mn) = \log_a m + \log_a n$

$\log_a m = x \Rightarrow m = a^x$

$\log_a n = y \Rightarrow n = a^y$

multiply

$mn = a^x \cdot a^y$

$mn = a^{x+y}$

$\log_a(mn) = x+y$

$$\boxed{\log_a(mn) = \log_a m + \log_a n}$$

Ans

(b) $\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$

$\log_a m = x \Rightarrow a^x = m$

$\log_a n = y \Rightarrow a^y = n$

$\longrightarrow \frac{m}{n} = \frac{a^x}{a^y}$

$\frac{m}{n} = a^{x-y}$

$\frac{m}{n} = a^{x-y}$

$\log_a\left(\frac{m}{n}\right) = x-y$

$$\boxed{\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n}$$

(c) $\log_a (m^x) = x \cdot \log_a m$

$\log_a m = k$

$m = a^k$

take exponent x to both sides

$m^x = (a^k)^x$

$\underline{m^x} = \underline{a}^{\underline{kx}}$

$\log_a (m^x) = kx$

$$\boxed{\log_a (m^x) = x \log_a m}$$

Hence Proved

$\boxed{\log_a a = 1}$

Q $\log_{10} 25 + \log_{10} 4 = \log_{10}(25(4)) = \log_{10}(100) = \log_{10}(10^2) =$

$= 2\boxed{\log_{10} 10} = 2$

Q $\log_{30}(5) + \log_{30} 3 + \log_{30} 2 = \log_{30}(5(3)(2)) = \log_{30}(30) = 1$

Q $\log_5 100 - \log_5 4 = \log_5\left(\frac{100}{4}\right) = \log_5(25) = \log_5(5^2)$

$= 2\log_5 5 = 2$

Q $\log_2(2^3) = 3$

2 (i) $\log_{\sqrt[3]{7}}(2401) = \log_{(7^{1/3})} 7^4 = \log_{7^{1/3}} 7^{12/3} = (12)\underbrace{\log_{7^{1/3}}(7^{1/3})}$

$= 12$

(ii) $\log_{5\sqrt{5}}(125) = \log_{5^{3/2}}(5^3) = \log_{5^{3/2}}(5^{3/2})^2 = 2\underbrace{\log_{5^{3/2}} 5^{3/2}}$

$= 2$

(iii) $\log_2\{\log_2\{\log_3(\log_3 27^3)\}\} =$

$= \log_2\{\log_2 \log_3\{\underbrace{\log_3 (3^9)}\}\}$

$= \log_2\{\log_2 \log_3 9\}$

$5\sqrt{5}$

$= 5^1 . 5^{1/2}$

$= 5^{1+1/2}$

$= 5^{3/2}$

$= \log_2 \{\log_2 \underbrace{\log_3 (3^2)}\}$

$= \log_2\{\underbrace{\log_2 2}\} = \log_2(1) = 0$

Homework

Race - 5 Question 1 to 10

Fundamentals of Maths

Lecture - 9

Q Simplify

(i) $\underbrace{\log_2 10} + \underbrace{\log_2 5} - \log_2 25 + 1$

$= \underbrace{\log_2 10} + \underbrace{\log_2 5} + \underbrace{\log_2 2} - \log_2 25$

$= \log_2 ((10)(5)(2)) - \log_2 25 = \log_2 100 - \log_2 25 = \log_2 \left(\frac{100}{25}\right)$

$= \log_2 4$

$= 2$

(ii) $\log_5 \left(\frac{15}{4}\right) + \log_5 \left(\frac{16}{25}\right) + \log_5 \left(\frac{1}{12}\right)$

$= \log_5 \left(\frac{\cancel{15}^{\,5}}{\cancel{4}} \cdot \frac{\cancel{16}}{\cancel{25}_{\,5}} \cdot \frac{1}{\cancel{12}_{\,3}}\right)$

$= \log_5 \left(\frac{1}{5}\right) = \underbrace{\log_5 1} - \log_5 5 = -1$

(iii) $2\log\left(\frac{8}{45}\right) + 3\log\left(\frac{25}{8}\right) - 4\log\left(\frac{5}{6}\right) = K\log 2$; $K = ?$

LHS $= \log\left(\frac{8}{45}\right)^2 + \log\left(\frac{25}{8}\right)^3 - \log\left(\frac{5}{6}\right)^4$

$= \log\left[\left(\frac{8}{45}\right)^2 \cdot \left(\frac{25}{8}\right)^3 \cdot \left(\frac{6}{5}\right)^4\right]$

$= \log \frac{\cancel{8}\times\cancel{8}}{\cancel{45}\times\cancel{45}} \times \frac{\cancel{25}\times\cancel{25}\times\cancel{25}}{\cancel{8}\times\cancel{8}\times\cancel{8}} \cdot \frac{\cancel{6}\times\cancel{6}\times\cancel{6}\times 6}{\cancel{5}\times\cancel{5}\times\cancel{5}\times\cancel{5}}$

$= \log\left(\frac{6}{3}\right) = \log 2$

$\boxed{K = 1}$

$\log 3 - \log\left(\frac{1}{2}\right)$

$= \log\left(\frac{3}{(1/2)}\right)$

$= \log(3(2))$

$= \log 3(2)$

$\log 2$

$= \log(\sqrt{4})$

$= \log(4^{1/2})$

$= \frac{1}{2}\log 4$

Base changing Theorem :—

Can be stated as

" Quotient of the logarithm of two numbers is independent of their common base."

$$\underbrace{\log_a b}_{} = \frac{\log_m b}{\log_m a}$$

$m > 0$; $m \neq 1$

$b > 0$; $a > 0$

$a \neq 1$

$$\log_2 3 = \frac{\log_4 3}{\log_4 2}$$

$$\log_a b = \frac{\log_m b}{\log_m a}$$

$$\log_a b = \frac{\log_m b}{\log_m a}$$

$$\log_a b = x$$

$$b = a^x$$

Take log at base m to both sides

$$\log_m b = \log_m (a^x)$$

$$\log_m b = x \cdot \log_m a$$

$$x = \frac{\log_m b}{\log_m a} \Rightarrow \boxed{\log_a b = \frac{\log_m b}{\log_m a}}$$

Note :

(i) $\log_b a \cdot \log_a b = 1$

$\dfrac{\log_a a}{\cancel{\log_a b}} \cdot \cancel{\log_a b} = 1$

LHS: $\rightarrow \log_b a \cdot \log_a b = \dfrac{\cancel{\log_e a}}{\cancel{\log_e b}} \cdot \dfrac{\cancel{\log_e b}}{\cancel{\log_e a}} = 1$

(ii) $a^{\log_b c} = c^{\log_b a}$

Let LHS $a^{\log_b c} = x$

take log at base b to both sides

$\log_b \left(a^{\log_b c}\right) = \log_b x$

$\log_b c \cdot \log_b a = \log_b x$

$\log_b a \cdot \log_b c = \log_b x$

$\log_b \left(c^{\log_b a}\right) = \log_b x$

$x = c^{\log_b a}$

$$\boxed{a^{\log_b c} = c^{\log_b a}}$$

(iii) $\log_{(a^K)} m = \dfrac{1}{K} \log_a(m)$

LHS. $\log_{a^K} m = \dfrac{\log_e m}{\log_e (a^K)} = \dfrac{\log_e m}{K \cdot \log_e a}$

$= \dfrac{1}{K} \dfrac{\log_e m}{\log_e a} = \dfrac{1}{K} \cdot \log_a m$

(iv) The base of the log can be any positive number other than 1, but in normal practice, only two base are popular (these are 10 and e
$e = 2.718$ (approx))
logarithms of numbers to the base 10 are named as 'common logarithms'
and the logarithms of the numbers to the base e are called natural or Napierian logarithm. we will consider $\log x$ as $\log_e x$ or $\ln x$.

(V) Conversion of base e to base 10 & vice versa :-

$$\boxed{\underbrace{\log_e a}_{} = \underbrace{2.303 \log_{10} a}_{}}$$ or $$\log_{10} a = 0.434 \log_e a$$

$$\log_e a = \frac{\log_{10} a}{\underbrace{\log_{10} e}} = 2.303 \log_{10} a$$

$$\log_{10} a = \frac{\log_e a}{\underbrace{\log_e 10}} = \underline{0.434}\, \underline{\log_e a}$$

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① Simplify $\dfrac{1}{\log_3 2} + \dfrac{2}{\log_9 4} - \dfrac{3}{\log_{27} 8} =$

$\log_a b = \dfrac{1}{\log_b a}$

$= \log_2 3 + 2\log_4 9 - 3\log_8 27$

$= \log_2 3 + 2\log_{(2^2)}(3^2) - 3\log_{2^3}(3^3)$

$= \log_2 3 + \dfrac{2(\cancel{2})}{\cancel{2}}\log_2 3 - \dfrac{3(\cancel{3})}{\cancel{3}}\log_2 3 = 3\log_2 3 - 3\log_2 3$

$= 0$ Ans.

② $\dfrac{\log_3 12}{\log_{36} 3} - \dfrac{\log_3 4}{\log_{108} 3}$

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(2) $\dfrac{\log_3 12}{\log_{36} 3} - \dfrac{\log_3 4}{\log_{108} 3}$

$= \log_3 12 \cdot \log_3 36 - \log_3 4 \cdot \log_3 108$

$= \log_3 (2^2 \cdot (3)) \log_3 (3^2 \cdot 2^2) - \log_3 (2^2) \log_3 (2^2 \cdot 3^3)$

$= [\log_3 (2^2) + \log_3 3][\log_3 (3^2) + \log_3 (2^2)] - 2\log_3 2 [\log_3 2^2 + \log_3 3^3]$

$= (2\log_3 2 + 1)(2\log_3 3 + 2\log_3 2) - (2\log_3 2)[2\log_3 2 + 3\log_3 3]$

$= (2\log_3 2 + 1)(2 + 2\log_3 2) - 2\log_3 2\,[2\log_3 2 + 3]$

$= \cancel{4\log_3 2} + \cancel{4(\log_3 2)^2} + 2 + \cancel{2\log_3 2} - \cancel{4(\log_3 2)^2} - \cancel{6\log_3 2} = 2$

Q find $\log_{54} 168$ if $\log_7 12 = a$; $\log_{12} 24 = b$

$$\log_{54}(168) = \frac{\log 168}{\log 54} = \frac{\log 12 + \log 14}{3\log 3 + \log 2}$$

$$= \frac{\underline{2\log 2 + \log 3} + \log 2 + \log 7}{3\log 3 + \log 2}$$

$$= \frac{\log 12 + \log 2 + \log 7}{\frac{1}{\log 12}(3\log 3 + \log 2)} = \frac{1 + \frac{\log 2}{\log 12} + \frac{\log 7}{\log 12}}{\frac{1}{\log 12}(3\log 3 + \log 2)}$$

$$= \frac{b + \frac{1}{a}}{\left(\frac{3\log 3 + \log 2}{\log 12}\right)} = \frac{b + \frac{1}{a}}{\frac{3\log 3}{\log 12} + (b-1)}$$

$a = \log_7 12$

$$a = \frac{2\log 2 + \log 3}{\log 7}$$

$$b = \frac{\log 24}{\log 12}$$

$$= \frac{\log 2 + \log 12}{\log 12}$$

$$= 1 + \underbrace{\frac{\log 2}{\log 12}}$$

$$b = 1 + \frac{\log 2}{\log 12}$$

$$b - 1 = \frac{\log 2}{\log 12}$$

$$\log 12 = \left(\frac{\log 2}{b-1}\right)$$

$\cancel{2\log 12}$

$$(b-1)(2\log 2 + \log 3) = \log 2$$

divide by $\log 2$

$$\underline{(b-1)}\left(2 + \frac{\log 3}{\log 2}\right) = 1$$

$$\frac{3\log 3}{\log 12} = \frac{3\log 3}{2\log 2 + \log 3}$$

$$= \frac{3\log 3}{\log 2}(b-1)$$

$$= 3\underbrace{\log_2 3}\,(b-1)$$

Fundamentals of Maths

Lecture - 10

Q find the value of $\log_{54} 168$ if $\log_7 12 = a$; $\log_{12} 24 = b$.

$$\log_{54} 168 = \frac{\log 168}{\log 54}$$

$54 = 3^3 \cdot 2^1$

$$= \frac{\log 7 + \log 24}{3\log 3 + \log 2}$$

$\log_{12} 7 = \frac{1}{a}$

$$\frac{\log 7}{\log 12} = \log_{12} 7$$

divide by log12 in num & den

$$\log_{54} 168 = \frac{\log_{12} 7 + \log_{12} 24}{\left(\frac{3\log 3 + \log 2}{\log 12}\right)} = \frac{\frac{1}{a} + b}{\left(\frac{3\log 3 + \log 2}{2\log 2 + \log 3}\right)}$$

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$$\log_{54} 168 = \frac{\frac{1}{a} + b}{\left(\frac{3\log 3 + \log 2}{2\log 2 + \log 3}\right)}$$

$$= \frac{\left(\frac{1}{a} + b\right)}{\left(8 - 5b\right)}$$

$$= \frac{1 + ab}{a(8 - 5b)}$$

$$\frac{(3\log 3 + \log 2)/\log 2}{(2\log 2 + \log 3)/\log 2}$$

$$= \frac{3\frac{\log 3}{\log 2} + \frac{\log 2}{\log 2}}{2\frac{\log 2}{\log 2} + \frac{\log 3}{\log 2}}$$

$$= \frac{3\log_2 3 + 1}{2 + \log_2 3}$$

$$= \frac{3\left(\frac{3-2b}{b-1}\right) + 1}{2 + \left(\frac{3-2b}{b-1}\right)}$$

$$= \frac{(9 - 6b + b - 1)\cancel{(b-1)}}{(2b - 2 + 3 - 2b)\cancel{(b-1)}}$$

$$= \frac{8-5b}{1}$$

$$\log_2 3 = \boxed{\frac{3-2b}{b-1}}$$

$$\log_{12} 24 = b$$

$$b = \frac{\log 24}{\log 12}$$

$$b = \frac{3\log 2 + \log 3}{2\log 2 + \log 3}$$

divide by $\log 2$

$$b = \frac{3 + \log_2 3}{2 + \log_2 3}$$

$$b(2 + \log_2 3) = 3 + \log_2 3$$

$$2b + b\log_2 3 = 3 + \log_2 3$$

$$\log_2 3\,(b-1) = 3 - 2b$$

Logarithmic Equations :-

$$\log_2 x^2 \neq \log_2 x$$

③ $\log_2 (x^2-1) = 3$

④ $\log_2 (x+1) + \log_2 (x-1) = 3$

① $2^{\log_2 (x^2)} - 3x - 4 = 0$

$x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0$

$x = 4, -1$ ✓

② $2^{2\log_2 x} - 3x - 4 = 0$

$\Rightarrow \boxed{2^{\log_2 x^2}} - 3x - 4 = 0$

$x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0$

$\boxed{x = 4}$; $\boxed{\cancel{x = -1}}$ ✓

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(3) $\log_2(x^2-1) = 3$

$x^2-1 = 2^3$

$x^2-1 = 8$

$x^2 = 9$

$\boxed{x = 3, -3}$

(4) $\log_2(x+1) + \log_2(x-1) = 3$

$\log_2[(x+1)(x-1)] = 3$

$x^2-1 = 2^3$

$\boxed{x = 3, \cancel{-3}}$

$x = 3$ only

(5) $x^2 + 7^{\log_7 x} - 2 = 0$

$x^2 + x - 2 = 0$

$(x-1)(x+2) = 0$

$\boxed{x = 1, \cancel{-2}}$

(6) $\log_4\left[2\log_3\left(1+\log_2\left(1+3\log_2 x\right)\right)\right] = \frac{1}{2}$

$2\log_3\left(1+\log_2\left(1+3\log_2 x\right)\right) = 4^{1/2}$

$\cancel{2}\log_3\left(1+\log_2\left(1+3\log_2 x\right)\right) = \cancel{2}^1$

$\log_3\left[1+\log_2\left(1+3\log_2 x\right)\right] = 1$

$1+\log_2\left(1+3\log_2 x\right) = 3^1$

$\log_2\left(1+3\log_2 x\right) = 2$

$1+3\log_2 x = 2^2$

$3\log_2 x = 3$

$\log_2 x = 1$

$x = 2^1$

$\boxed{x = 2}$

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(7) $\log_2(9-2^x) = \boxed{10^{\log_{10}}}(3-x)$

$\log_2(9-2^x) = 3-x$

$9-2^x = 2^{3-x}$

$9-2^x = 2^3 \cdot 2^{-x}$

$9-2^x = \dfrac{8}{2^x}$

Let $2^x = t$

$9-t = \dfrac{8}{t}$

$9t - t^2 = 8$

$t^2 - 9t + 8 = 0$

$(t-8)(t-1) = 0$

$t = 8 \qquad t = 1$

$2^x = 8 \qquad 2^x = 1$

$2^x = 2^3 \qquad 2^x = 2^0$

$\cancel{\boxed{x=3}} \qquad \boxed{x=0}$

Characteristic and mantissa :-

For any given number N, logarithm can be expressed as $\log_a N$ = Integer + fraction (+ve)

[The integer part is called characteristic
The fraction part is called mantissa.

$\log_{10} 10 = 1 \Rightarrow \log_{10} 10^1$

$\log_{10} 100 = 2 \Rightarrow \log_{10}(10^2) = 2\underbrace{\log_{10} 10} = 2$

$\log_{10} 1000 = 3 \Rightarrow \log_{10} 10^3 = 3\log_{10} 10 = 3$

$\log_2 2 = 1$

$\log_2 4 = 2$

$\log_2 8 = 3$

$\log_2 16 = 4$

$\log_2 32 = 5$

$\log_{27}(755) = 2.---$

$\log_2 3 = \underbrace{1}_{\text{char}}.\underbrace{-----}_{\text{Mantissa}}$

$\log_2 15 = \underbrace{3}_{\text{character}}.\underbrace{-----}_{\text{mantissa}}$

$\log_2 33 = 5.----$

$\log_5(27) = \underbrace{2}_{c}.\underbrace{---}_{m.}$

$\underbrace{\log_{10} 2}$ $= 0.3010$; $\underbrace{\log_{10} 3}$ $= 0.4771$

Q No. of digits in $5^6 = ?$

$$x = 5^6$$

take log at base 10 on both sides

$$\log_{10} x = \log_{10}(5^6)$$

$$= 6(\log_{10} 5)$$

$$= 6\left[\log_{10}\left(\frac{10}{2}\right)\right]$$

$$= 6\left[\log_{10} 10 - \log_{10} 2\right] = 6\left[1 - 0.3010\right]$$

$$\log_{10} x = 6[0.6990] = \underbrace{4}_{C}.\underbrace{194}_{M}$$

$$\boxed{\begin{aligned}&\text{No. of digits}\\ &= \text{characteristic} + 1\\ &= 4 + 1 = 5\end{aligned}}$$

Q find no. of digits in 5^{12}

$x = 5^{12}$

$\log_{10} x = 12 \log_{10} 5$

$= 12 [0.6690]$

$= 8.\underbrace{388}$

char $\rightarrow$ 8

No. of digits $= 8+1 = 9$

Q find no. of digits in $3^{12} \cdot 2^{8}$

$x = 3^{12} \cdot 2^{8}$

$\log_{10} x = \log_{10}(3^{12} \cdot 2^{8})$

$= 12 \log 3 + 8 \log 2$

$= \underbrace{12(0.4771)} + \underbrace{8(0.3010)}$

$= 8.1332$

Char $\rightarrow$ 8

No. of digits $= 8+1 = 9$

$$\underbrace{-1.823} = -1 + 0.823 \quad (A)$$

$$\underbrace{-1 \underbrace{- 0.823}} \quad (B)$$

$$\underbrace{-1.823}$$

$1.\underbrace{823}$ → c, m

Note:

(i) The mantissa part of log of a number is always non-negative $(0 \le m < 1)$

(ii) If the characteristic of $\log_{10} N$ is n, then no. of digits in N is $|n+1|$.

(iii) If the characteristic of $\log_{10} N$ be $(-n)$ then there exist $(n-1)$ zeroes after decimal in N.

Q

$$\big|\,|x+1| + |x-5|\,\big| = 17$$

$$\underbrace{|x+1| + |x-5| = +17}$$

$$|x+1| + |x-5| = -17$$

Fundamentals of Maths

Lecture - 11

$\log_{10} 1 = 0$

$\log_{10}(0.1) = \log_{10}\left(\frac{1}{10}\right) = \underbrace{\log_{10} 1}_{} - \underbrace{\log_{10} 10}_{} = 0 - 1 = -1$

✓ $\log_{10}(0.1) = \log_{10}(10^{-1}) = -1$

✓ $\log_{10}(0.01) = \log_{10}(10^{-2}) = -2$

✓ $\log_{10}(0.001) = \log_{10}(10^{-3}) = -3$

$\log_{10}(\underbrace{0.0875}_{})$

$= -2 \rightarrow$ char.

0.01 0.02, 0.03, ---- 0.09, 0.10 ✓

0.001, 0.002, --- , 0.010

$\log_{10}(0.0875) = -1.324$

$= \bar{2}.676$

$\log_{10}(x) = -5.310$

$= \boxed{\underbrace{-5}_{-1}} \; \underbrace{-0.310}_{+1}$

~~Mant~~

$= \underbrace{-6} + 0.690$

$= \bar{6}.690$

$-1 + 0.324$

$\underbrace{-1 - 0.324}$

$-1 \quad -0.324$

$-1 \quad +1$

$\underline{-2} + \underline{0.676}$

$= \underbrace{\bar{2}.676}$

$= \underbrace{-2 + 0.676}$

$\cancel{-2.676}$

$\cancel{-2 - 0.676}$

$\Rightarrow \quad \log_{10} x = \boxed{-2.379} = \underline{-3} + 0.621$

$\boxed{\text{char}} = \underbrace{-3}$

mantissa $= \underbrace{0.621}$

$\boxed{\begin{matrix}-2 \\ -1\end{matrix}}$ → char

$\boxed{\begin{matrix}-0.379 \\ +1\end{matrix}}$ → M.

$\Rightarrow \quad \log_{10}(x) = -3.235$

char $= -4$

mantissa $= 0.765$

$\underbrace{-3} + 0.621$

$= \underbrace{\bar{3}.621}$

$\cancel{3 = \bar{3}\ 3\ 3\ 3}$

$\bar{2}.355$

$-2 + 0.355$

$\log_7 x = 5$

$x = 7^5$

$\sqrt[3]{3} = x$

$\log_{10}(\sqrt{3}) = \log x$

$\frac{1}{2}\log_{10}(3) = \log x$

$\log_{10} x = \frac{1}{2}(0.4771)$

$\log_{10} x = 0.23855$

$\boxed{x = 10^{0.2386}}$

$\boxed{x = 1.73}$

char $= 0$

Mantissa $= 0.2386$

Q find the number of zeroes after decimal and before first significant figure in $\left(\frac{9}{8}\right)^{-100}$

$$x = \left(\frac{9}{8}\right)^{-100}$$

2×0.4771

$= 0.9542$

$- \; 0.9030$

11

$$\log_{10} x = -100 \log_{10}\left(\frac{9}{8}\right)$$

$$= -100\left[\log_{10} 9 - \log_{10} 8\right]$$

$$= -100\left[2\log_{10} 3 - 3\log_{10} 2\right]$$

$$= -100\left[2(0.4771) - 3(0.3010)\right] = -100\,(0.0512)$$

$\log_{10} x = -6 + 0.88$

$= \bar{6}.88$

$$\log_{10} x = -5.12 = \bar{6}.88 \quad \text{char} = -6$$

No. of zeroes after decimal $= 6 - 1 = 5$ zeroes

Q find the number of zeroes after decimal and before first significant figure in 3^{-50}.

$$x = 3^{-50}$$

$$\log_{10} x = -50 \log 3$$

$$= -50\,(0.4771)$$

$$= -23.855$$

$$= \underbrace{-24}_{} + \underbrace{(1 - 0.855)}_{\rightarrow \text{man}}$$

-23.855

$= \underline{-23} \boxed{-0.855}$

$= -23 + \boxed{1 - 0.855}$
-1

$= -24 + 0.145$

$= \bar{24}.145$

$\text{char} = -24$

No. of zeroes after decimal $= \underline{24-1} = \underline{23}$

Antilogarithm : — The positive real number 'n' is called the antilogarithm of a number 'm'

if $\log n = m$. then $n = \text{antilog } m$

Q $\text{antilog}_{\left(\frac{1}{100}\right)}\left(-\frac{1}{2}\right) = \left(\frac{1}{100}\right)^{-1/2} = \left[\left(\frac{1}{100}\right)^{1/2}\right]^{-1} = \left[\frac{1}{10}\right]^{-1} = 10$

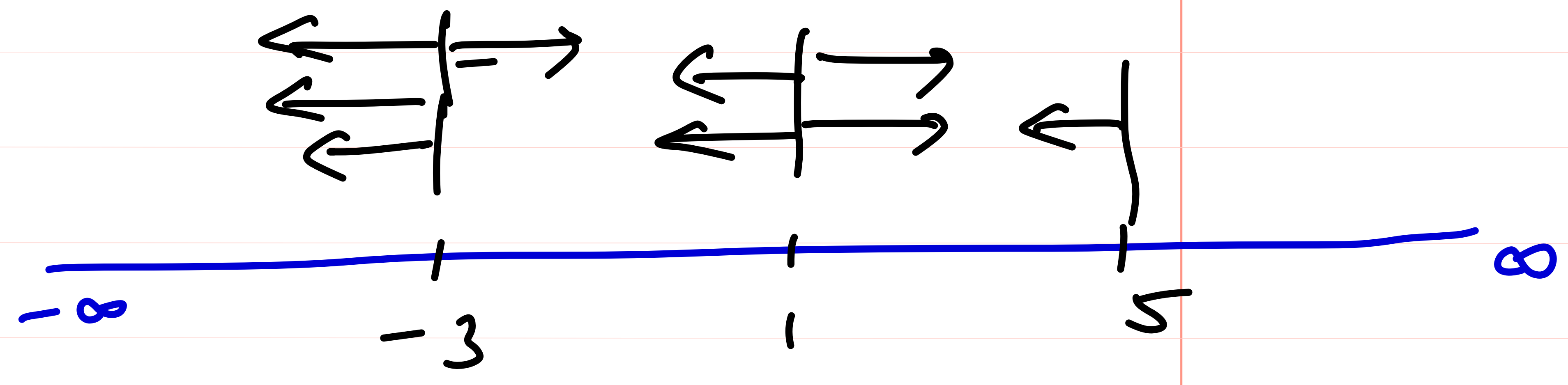
Q.

Q $\text{antilog}_8\left(\frac{2}{3}\right) = (8)^{2/3} = (2^3)^{2/3} = 2^{3\left(\frac{2}{3}\right)} = 2^2 = 4$

Q find the antilog of $\frac{2}{3}$ when base of log is (1331)

$(1331)^{2/3} =$

$$|x-1| + |x+3| + |x-5| = k$$

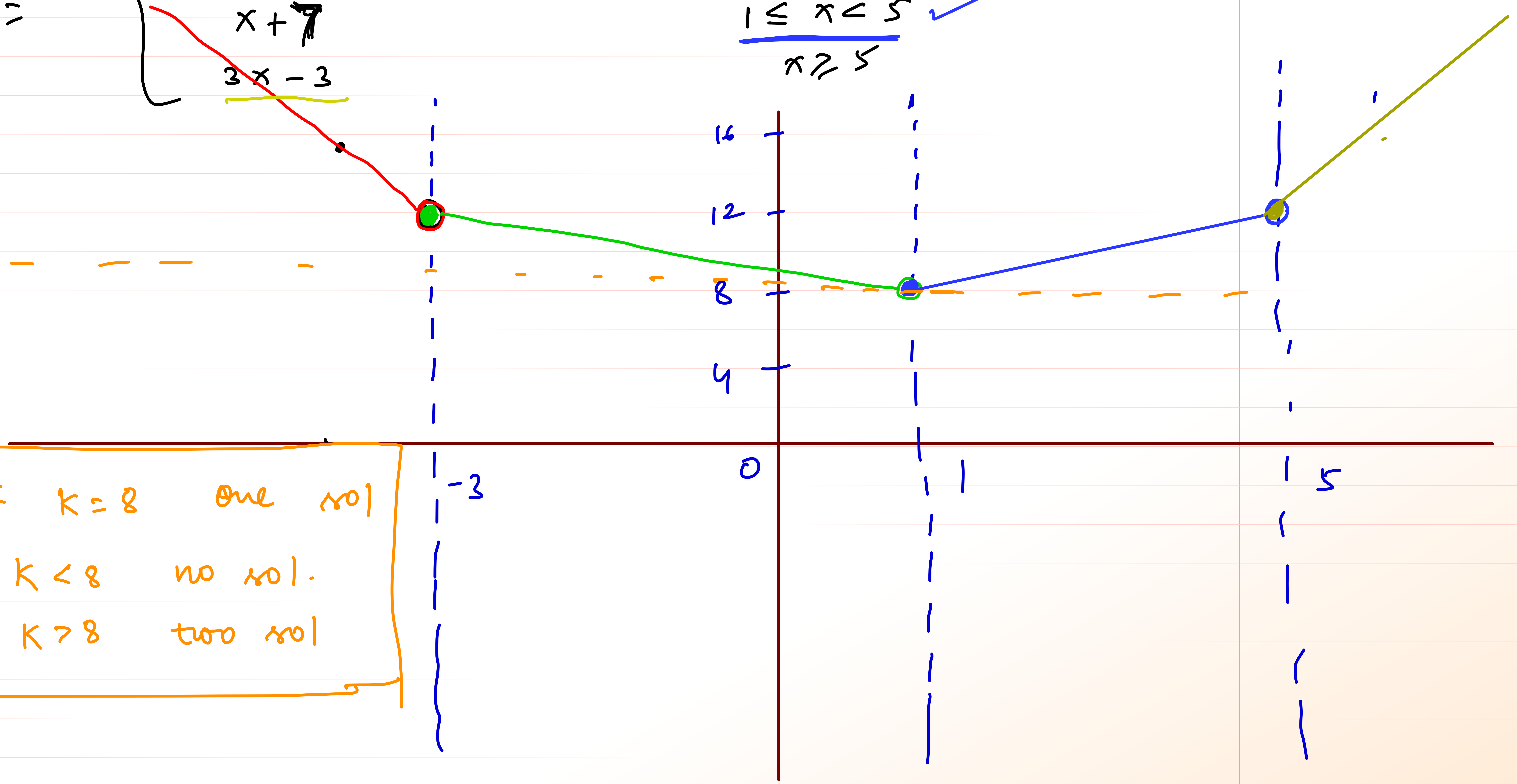


Case I $\quad x < -3$

$$k = \begin{cases} -(x-1)-(x+3)-(x-5) & x<-3 \\ -(x-1)+(x+3)-(x-5) & -3 \le x < 1 \\ +(x-1)+(x+3)-(x-5) & 1 \le x < 5 \\ +(x-1)+(x+3)+(x-5) & x \ge 5 \end{cases}$$

$$k = \begin{cases} -3x+3 & x<-3 \\ -x+9 & -3 \le x < 1 \\ x+7 & 1 \le x < 5 \\ 3x-3 & x \ge 5 \end{cases}$$

$$k = \begin{cases} -3x+3 & x < -3 \\ -x+9 & -3 \le x < 1 \\ x+7 & 1 \le x < 5 \\ 3x-3 & x \ge 5 \end{cases}$$



if $k = 8$ one sol

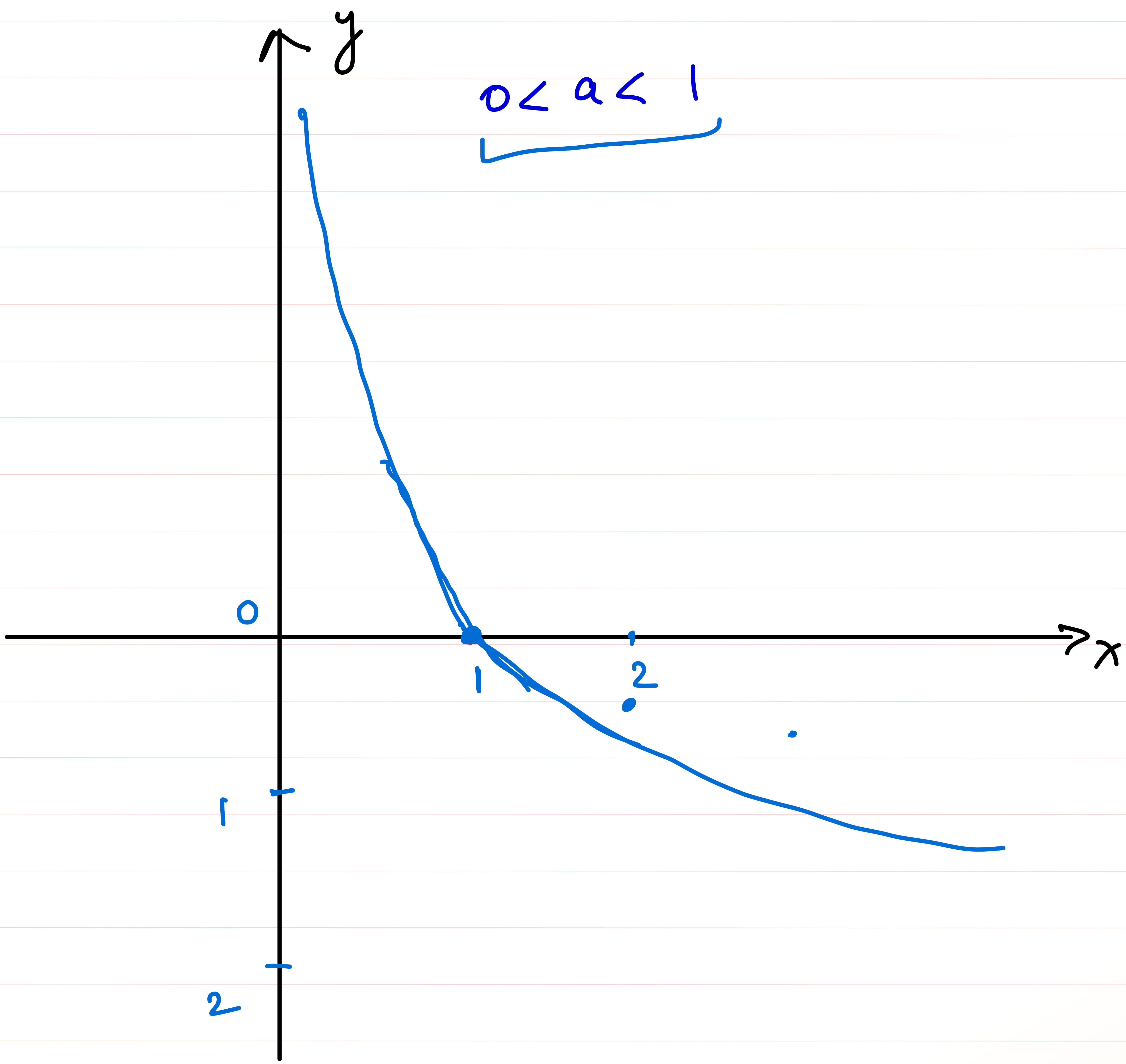
if $k < 8$ no sol.

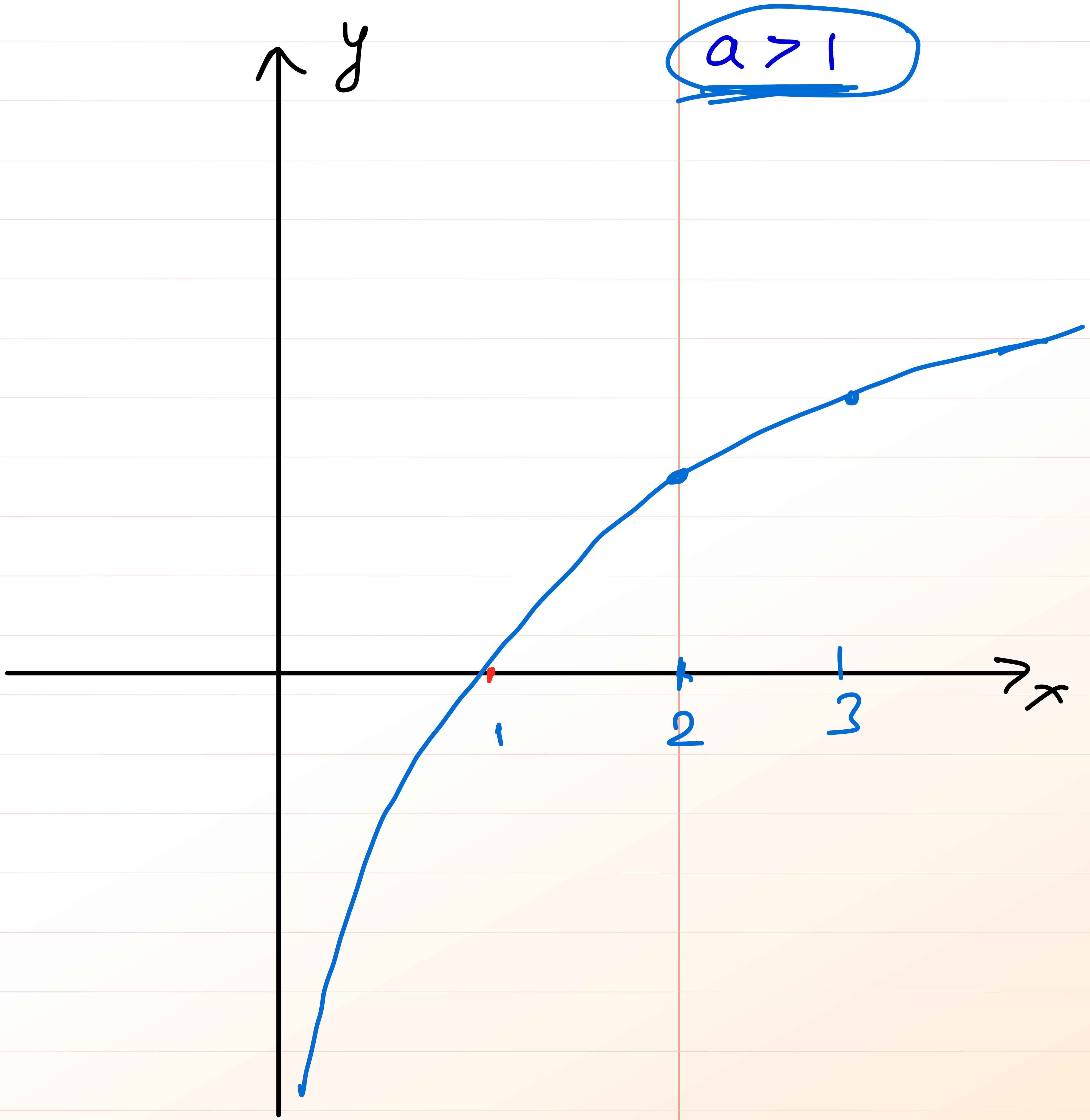
if $k > 8$ two sol

Fundamentals
of
Maths

Lecture - 12

Graph of $y = \log_a x$





$y = \log_a x = \dfrac{\log x}{\log a}$

$\underbrace{\log 0.5} = -ve$

$\underline{\log a} \rightarrow \begin{cases} -ve & 0 < a < 1 \\ +ve & a > 1 \end{cases}$

$y = \underbrace{\log_a x} \rightarrow \dfrac{\log x}{\boxed{\log a}} \rightarrow -ve$

$\underline{0 < a < 1}$ ✓

$y = \underline{\text{negative}}$ of $\boxed{\log x}$

$=$ neg. of $\log 2$ $\qquad \underline{x \rightarrow 2}$

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Points to remember : →

(i) If base of logarithm is greater than 1 then logarithm of greater number is greater.

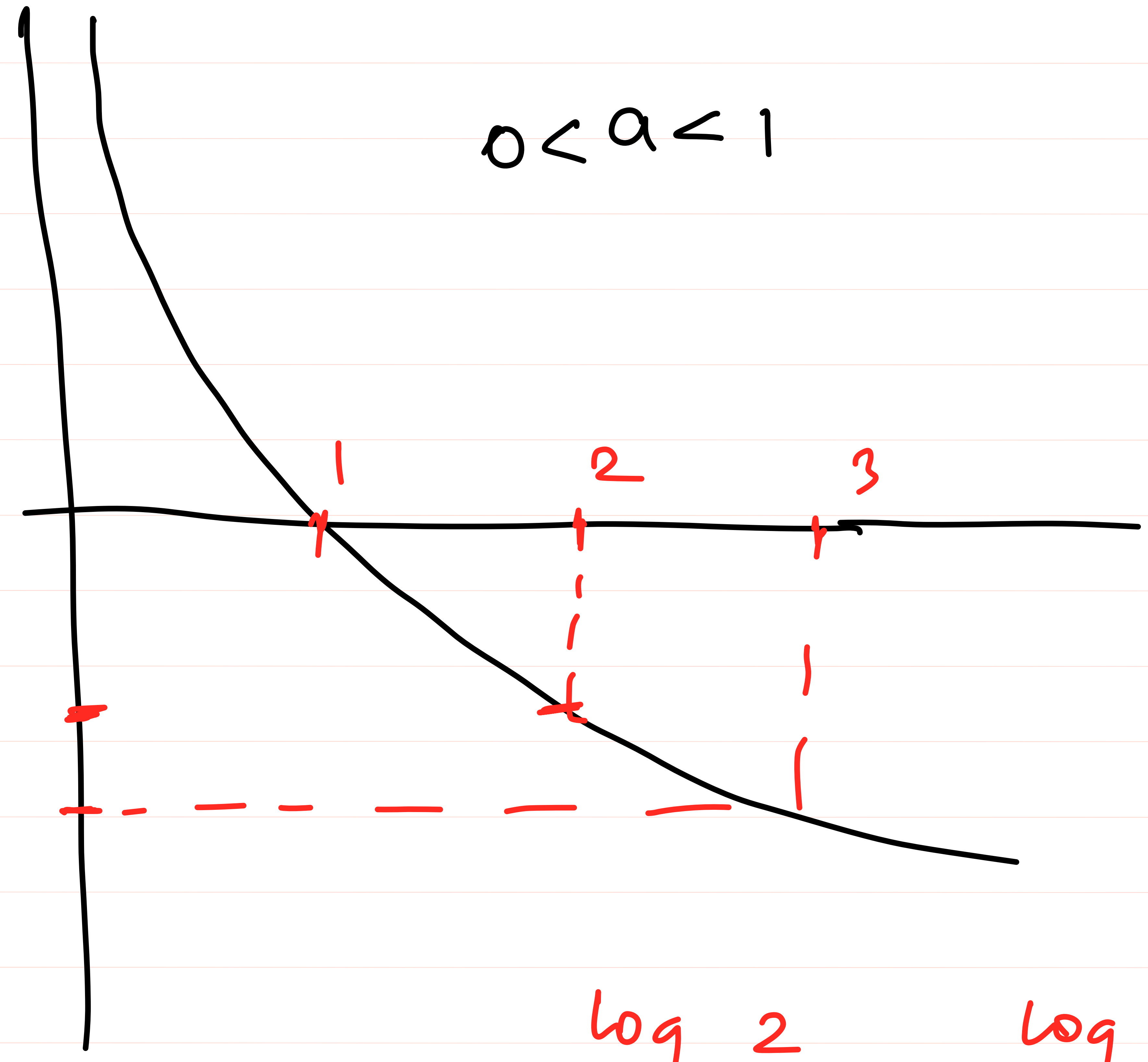
$\log_2 8 = 3$; $\log_2 16 = 4$; $\log_2 4 = 2$

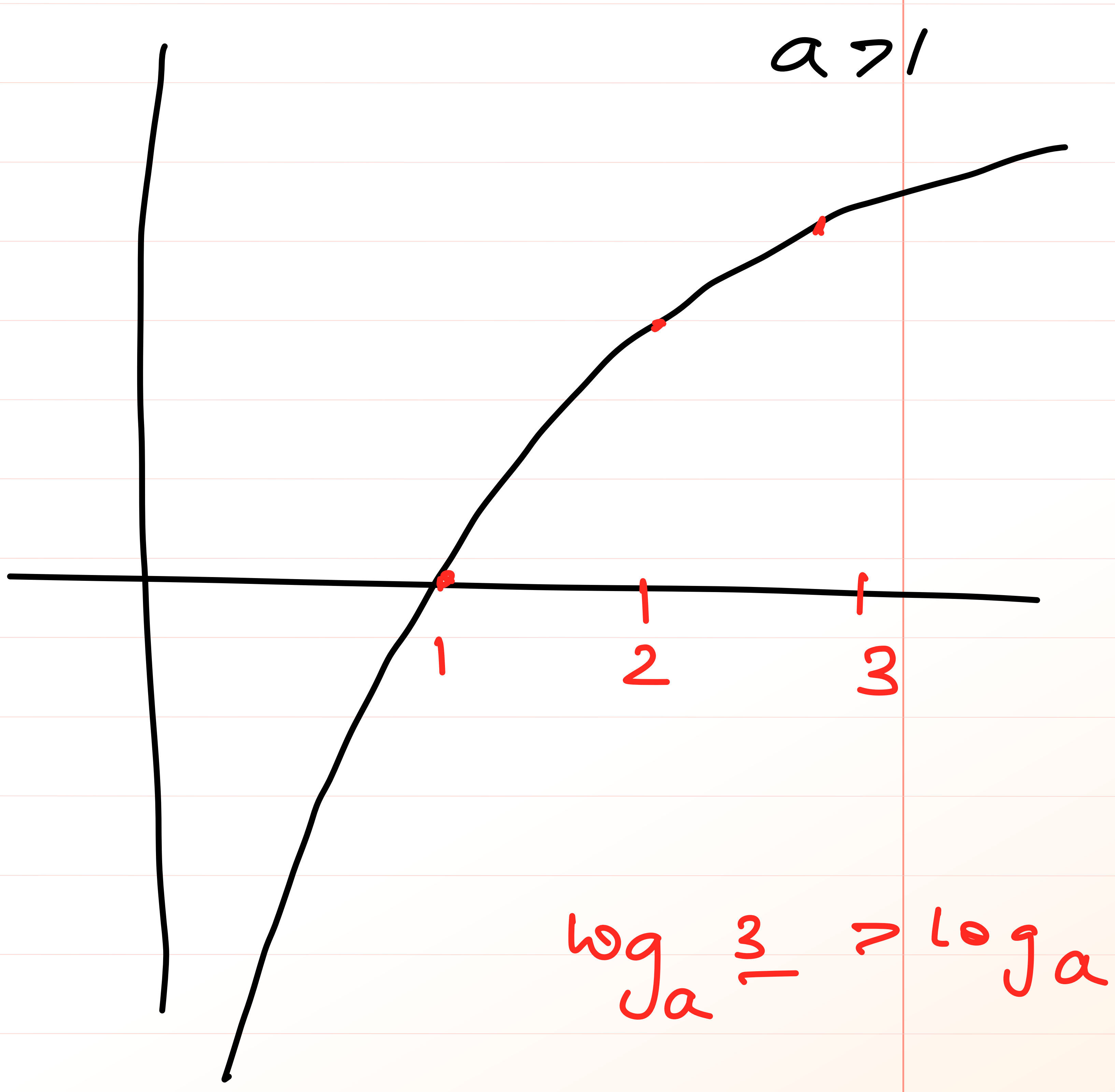
and if base of logarithm is $0 < a < 1$ then logarithm of greater no. is smaller.

$\log_{(1/2)}(2) = -1$; $\log_{1/2}(4) = -2$; $\log_{1/2}(8) = -3$

$$\frac{\log 8}{\log (1/2)} = \frac{\log 8}{\log 1 - \log 2} = \frac{3\log 2}{0 - \log 2} = -3$$

$$\log_a \boxed{x} > \log_a \boxed{y} = \left\{ \begin{array}{ll} x > y & a > 1 \\ x < y & 0 < a < 1 \end{array} \right]$$





$\log_{0<a<1} 2$ $\quad$ $\log_{0<a<1} 3$

$\log_a 3 > \log_a 2$

(ii) It must be noted that whenever the number and the base are on the same side of unity then logarithm of that number to the base is positive, <u>however</u> if the number and the base are located on different side of unity then logarithm of that number to that base is negative.

$$\log_{1/2}\left(\frac{1}{3}\right) = \frac{-1}{-1}\log_2 3 = +ve.$$

$$\log_{(1/2)}(3) = -ve$$

$$\log_2 3 = +ve$$

$$\log_3\left(\frac{1}{4}\right) = -ve.$$

(iii) $x + \frac{1}{x} \geq 2$ if x is a positive real number and $x + \frac{1}{x} \leq -2$; if x is a negative real no.

(iv) $n \geq 2$; $n \in N$

$\sqrt[n]{a} = a^{(1/n)}$ $\Rightarrow$ n^{th} root of a (a $\rightarrow$ non-negative no).

$\log_2 3 = 1.585$

$\log_3 3 = 1$

$\log_4 3 = 0.792$

Q 1

$a = \log_9 (7)$ or $b = \log_3 7$

$b > a$

Q-2 If $a = \log_3 5 \Rightarrow$

$b = \log_{17} 25$

$b = \dfrac{\frac{1}{2}}{(1/2)} \dfrac{\log 25}{\log 17}$

$= \dfrac{\log 5}{\log \sqrt{17}}$ =

$= \log_{\sqrt{17}} 5$

$b = \boxed{2} \log_{17} 5$

=

$a = \dfrac{2}{2} \cdot \dfrac{\log 5}{\log 3} = \dfrac{\log 5^2}{\log 3^2}$

$= \dfrac{\log 25}{\log 9} = \log_9 25$

$a > b$

Q 3 Prove that

$$\log_{\pi} 3 + \log_{3} \pi > 2$$

$$\boxed{\frac{\log 3}{\log \pi}} + \frac{\log \pi}{\log 3}$$

$$x = \frac{\log 3}{\log \pi}$$

$$\underbrace{x + \frac{1}{x}} > 2$$

$$\boxed{\frac{\log 3}{\log \pi}} + \boxed{\frac{1}{\left(\frac{\log 3}{\log \pi}\right)}} > 2$$

$$x + \frac{1}{x} \geq 2$$

$$x = 1$$

$$\boxed{\log_{\pi} 3 + \frac{1}{\log_{\pi} 3} > 2}$$

Hence Proved

Logarithmic Inequations :-

$$\log_a x > \log_a y \Rightarrow \begin{cases} x > y > 0 & a > 1 \\ 0 < x < y & 0 < a < 1 \end{cases}$$

$$\log_a x = p \qquad x = a^p$$

$$\log_a x > p \Rightarrow \begin{cases} x > a^p & a > 1 \\ 0 < x < a^p & 0 < a < 1 \end{cases}$$

Q 1 If $\log_2 x \in \underbrace{(2,3)}$ then $x \in (\_\_,\_\_)$

interval

$x \in (2^2, 2^3)$

$x \in (4, 8)$

$\log_a x = p$

$x = a^p$

Q-2 If $\log_5 x \in (0,1) \cup \underline{(2,3)}$; $x \in$?

$x \in (5^0, 5^1) \cup \underline{(5^2, 5^3)}$

$x \in (1,5) \cup \underline{(25, 125)}$

Q $\log_2 \left(\log_4 \log_5 x \right) > 0$

$\log_4 \log_5 x > 2^0$

$\log_4 \log_5 x > 1$

$\log_5 x > 4^1$

$\log_5 x > 4$

$x > 5^4$

$x > 625$

$x \in (625, \infty)$

$\log_a x > p$

$x > a^p$

Hints (Ex - 1)

Q - 1 No Hint.

Q - 2

Q - 3 $\cancel{(p-q)^3}$

$$= \frac{3\,\cancel{(p-q)}\,\cancel{(q-r)}\,\cancel{(r-p)}}{\cancel{(p-q)}\,\cancel{(q-r)}\,\cancel{(r-p)}}$$

$= 3$

$a \to p-q$ ✓

$b \to q-r$ ✓

$c \to r-p$ ✓

0

$a^3+b^3+c^3 = 3abc$

if $a+b+c=0$

Q-4 put $x = 4$ in $P(x)$ and $Q(x)$

and equate.

Q-5

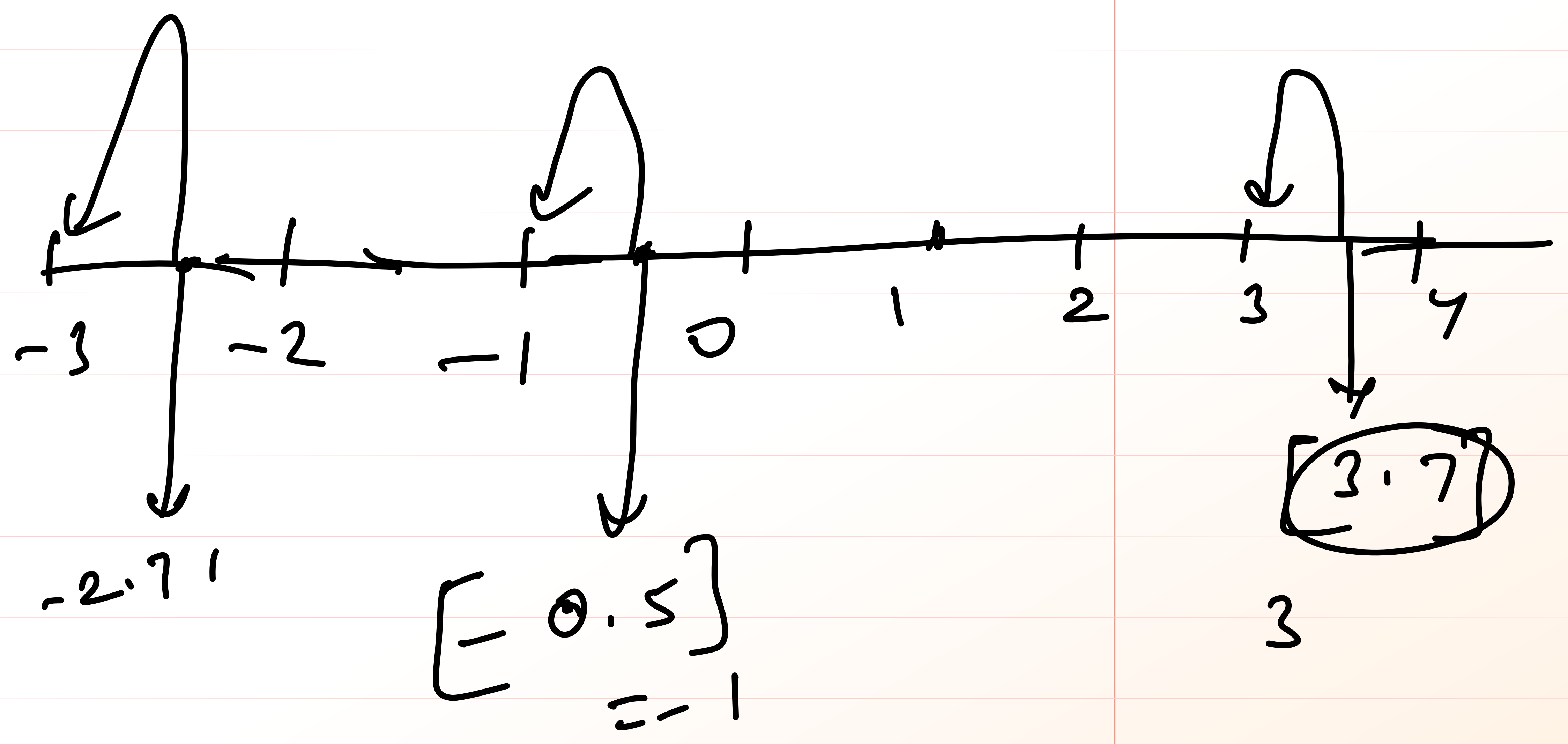
Q-6 $|x|^2 = x^2$

Q-7 $[\pi] - [-e]$

$= \underbrace{[3.14]} - [-2.71]$

$= 3 - (-3)$

$= 6$



(10) $60^a = 3$ $\qquad$ $60^b = 5$

(12) $a = \log_{60} 3$ $\qquad$ $b = \log_{60} 5$

$$\frac{1-a-b}{2(1-b)}$$

(12) Base change.

(13) $\boxed{3}^{\log_4 \boxed{5}} + 4^{\log_5 3} - 5^{\log_4 3} - 3^{\log_5 4}$

$5^{\log_4 3}$

$\Rightarrow$ $\boxed{a^{\log_c b} = b^{\log_c a}}$

(14) $B = \dfrac{12}{3+\sqrt{5}+\sqrt{8}} \cdot \dfrac{(3+\sqrt{5})-(\sqrt{8})}{(3+\sqrt{5})-(\sqrt{8})}$

$= \dfrac{12(3+\sqrt{5}-\sqrt{8})}{(9+5-6\sqrt{5}-8)} = \dfrac{12(3+\sqrt{5}-\sqrt{8})}{6(1-\sqrt{5})} \left(\cdot \dfrac{1+\sqrt{5}}{1+\sqrt{5}}\right)$

111

1111

$\boxed{11111}$

111111

$\boxed{1111111}$

3

div by 11

dont kn

3

111111 --- 11

91 times

7x13

$\boxed{1111111}$

11111

$\boxed{11111}$

Fundamentals of Maths

Lecture - 13

Q $\log_3 |(3-4x)| > 2$

$|3-4x| > 3^2$

$|3-4x| > 9$

$3-4x < -9$ or $3-4x > 9$

$3+9 < 4x$ $\cup$ $4x < -6$

$4x > 12$ $\cup$ $x < -3/2$

$\boxed{x > 3}$ $\cup$

$x \in (3, \infty)$ $\quad x \in \left(-\infty, -\frac{3}{2}\right)$

$$\boxed{x \in \left(-\infty, -\frac{3}{2}\right) \cup (3, \infty)}$$

$\log_a x > p$

$$= \begin{cases} x > a^p & a > 1 \\ 0 < x < a^p & 0 < a < 1 \end{cases}$$

$|x| > a$

$x < -a$ or $x > a$

Q $\log_{0.5}(x^2-5x+6) > -1$

$$\log_a x > p = \begin{cases} x > a^p & a > 1 \\ 0 < x < a^p & 0 < a < 1 \end{cases}$$

intersection

$$\boxed{0 < \underbrace{x^2-5x+6} < (0.5)^{-1}}$$

$x^2-5x+6 > 0$

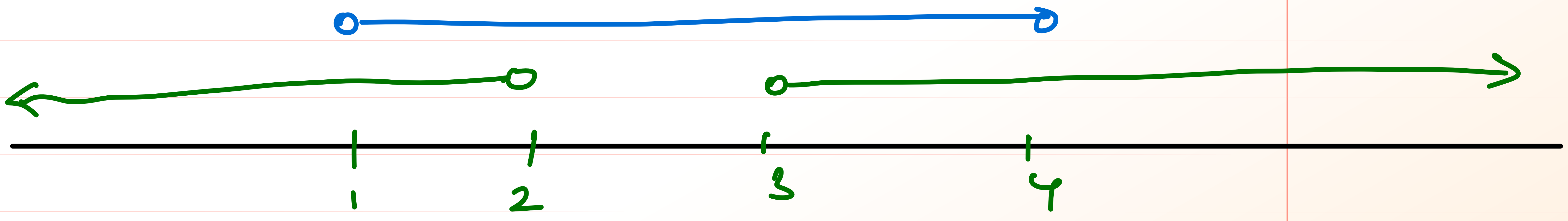
$\boxed{x \in (-\infty, 2) \cup (3, \infty)}$

$x^2-5x+6 < 2$

$x^2-5x+4 < 0$

$(x-4)(x-1) < 0$

$\boxed{x \in (1,4)}$



$\boxed{x \in (1,2) \cup (3,4)}$

$$\log_a x > \log_a y = \begin{cases} x > y > 0 & a > 1 \\ 0 < x < y & 0 < a < 1 \end{cases}$$

Q. $\log_{0.2}(x^2 - x - 2) > \log_{0.2}(-x^2 + 2x + 3)$

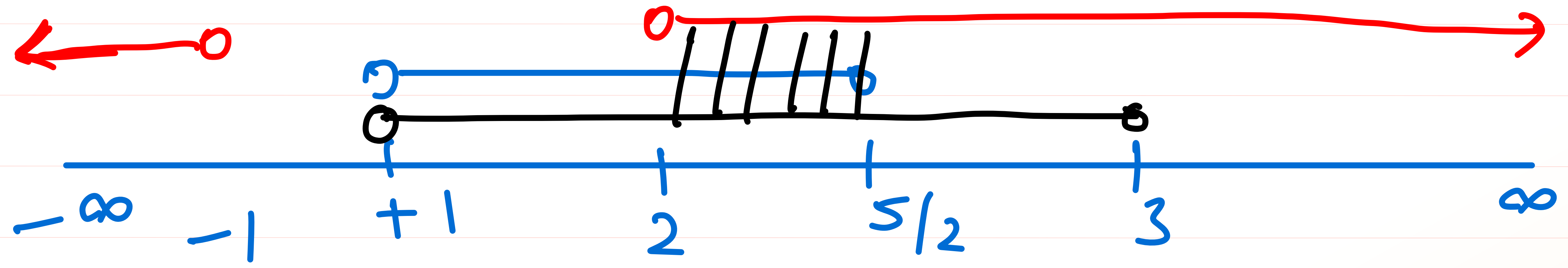
$$(x^2 - x - 2) < (-x^2 + 2x + 3)$$

$$2x^2 - 3x - 5 < 0$$

$$2x^2 + 2x - 5x - 5 < 0$$

$$(2x - 5)(x + 1) < 0$$

$$x \in \left(-1, \frac{5}{2}\right) \checkmark$$



$$x^2 - x - 2 > 0 \checkmark$$

$$(x - 2)(x + 1) > 0$$

$$x \in (-\infty, -1) \cup (2, \infty)$$

& $$-x^2 + 2x + 3 > 0 \checkmark$$

$$x^2 - 2x - 3 < 0$$

$$(x - 3)(x + 1) < 0$$

$$\boxed{x \in (-1, 3)}$$

$$\boxed{x \in \left(2, 5/2\right)}$$

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① $\dfrac{2}{2\log_2 x + 1} < \dfrac{1}{\log_2 x - 1}$ Ans $\left(0, \frac{1}{\sqrt{2}}\right) \cup (2, \infty)$

② $\log_x \left(2x - \frac{3}{4}\right) > 2$ $x \in \left(\frac{3}{8}, \frac{1}{2}\right) \cup \left(1, \frac{3}{2}\right)$

③ $\log_{0.5}\left(\log_6\left(\dfrac{x^2+x}{x+4}\right)\right) < 0$ $(-4, -3) \cup (8, \infty)$

④ $\log_{\left(\frac{x+6}{3}\right)}\left(\log_2\left(\dfrac{x-1}{x+2}\right)\right) > 0$ $(-6, -5) \cup (-3, -2)$

Exponential Equations / inequations :-

$$a^x > a^y \Rightarrow \begin{cases} x > y & a > 1 \\ x < y & 0 < a < 1 \end{cases}$$

$$a^{f(x)} > b \Rightarrow \begin{cases} f(x) > \log_a b & a > 1 \\ f(x) < \log_a b & 0 < a < 1 \end{cases}$$

$$a^{f(x)} = b$$

$$f(x) \log a = \log b$$

$$f(x) = \frac{\log b}{\log a}$$

$$= \log_a b$$

Q

$$\left(\frac{2}{5}\right)^{\frac{6-5x}{2+5x}} < \left(\frac{25}{4}\right)$$

$$\left(\frac{2}{5}\right)^{\frac{6-5x}{2+5x}} < \left(\frac{2}{5}\right)^{-2}$$

$0<a<1$ $\qquad$ $0<a<1$

$$\frac{6-5x}{2+5x} > -2$$

$$x \in (-\infty, -2) \cup \left(-\frac{2}{5}, \infty\right)$$

$$\left(\frac{5}{2}\right)^{\frac{-6+5x}{2+5x}} < \left(\frac{5}{2}\right)^{2}$$

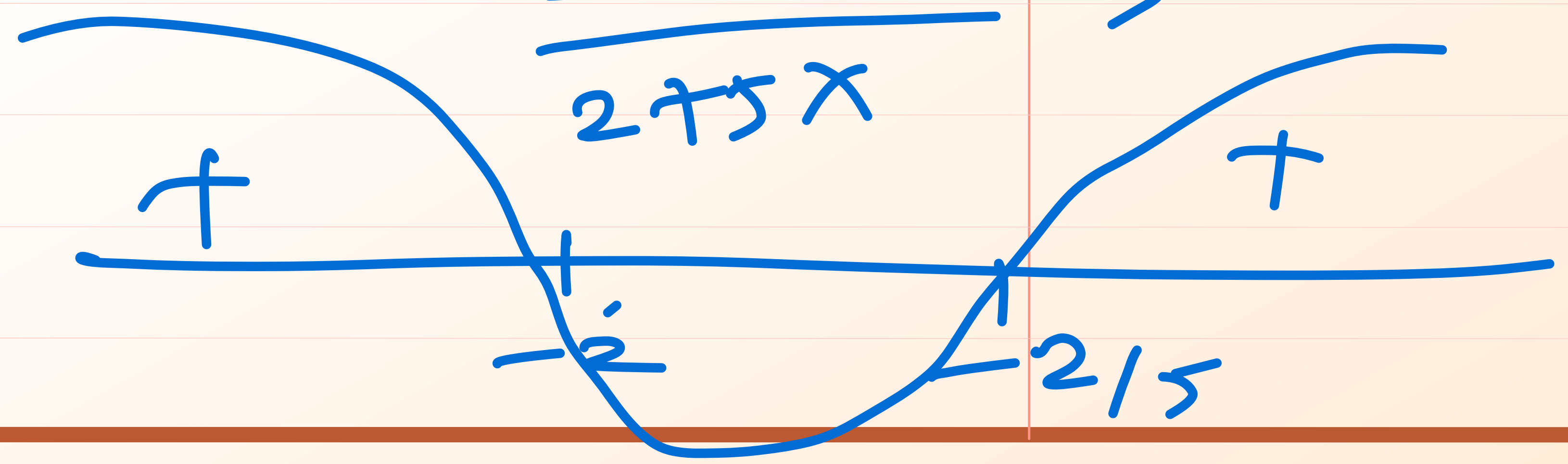
$a>1$ $\qquad$ $a>1$

$$\frac{-6+5x}{2+5x} < 2$$

$$\frac{6-5x}{2+5x} + 2 > 0$$

$$\frac{6-5x+4+10x}{2+5x} > 0$$

$$\frac{5x+10}{2+5x} > 0$$



Ratio and proportion :-

a, d → extremes
b, c → means

Property : Product of means = Product of extremes.

(1) Invertendo if $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{b}{a} = \frac{d}{c}$

(2) Alternando if $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a}{c} = \frac{b}{d}$

(3) Componendo if $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a+b}{b} = \frac{c+d}{d}$

(4) Dividendo if $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a-b}{b} = \frac{c-d}{d}$

(5) Componendo & dividendo $\frac{a+b}{a-b} = \frac{c+d}{c-d}$

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I 18

$$\frac{3x^4+x^2-2x-3}{3x^4-x^2+2x+3} = \frac{5x^4+2x^2-7x+3}{5x^4-2x^2+7x-3}$$

$$\frac{a}{b}=\frac{c}{d}$$

$$\frac{a+b}{a-b}=\frac{c+d}{c-d}$$

$$\frac{6x^4}{2x^2-4x-6} = \frac{10x^4}{4x^2-14x+6}$$

$$\frac{3x^4}{x^2-2x-3} - \frac{5x^4}{2x^2-7x+3} = 0$$

$$x^4\left[\frac{3}{x^2-2x-3} - \frac{5}{2x^2-7x+3}\right] = 0$$

$$x^4 = 0$$

$$\frac{5}{6}=\frac{10}{12}$$

$$\frac{5+6}{5-6}=\frac{10+12}{10-12}$$

$$(x-2)(x-3) = 0$$

$x=2 \quad x=3$

Fundamentals
of
Maths

Lecture - 14

Set Theory

The collection of well defined things is called set. well defined means a law by which we are able to find whether a given thing is contained in the given set or not.

eg: (i) $A = \{x : x \text{ is a prime no.}\}$ ✓R

$A = \{2, 3, 5, 7, 11, ----\}$ SB

(ii) $B = \{x : x \text{ is a natural no}\}$

$B = \{1, 2, 3, ---\}$

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Methods to write a set :-

(i) Roster form.

In this method a set is describing by listing elements, separated by commas and enclosed in curly brackets.

$A = \{a, e, i, o, u\}$

$B = \{1, 2, 3, 4, 5\}$

$C = \{2, 4, 6, 8, 10\}$

(ii) Set Builder Form :-

In this case we write down a property (or rule) which gives us all the elements of the set.

$A = \{x : x$ is a set of vowels$\}$

$B = \{x : x$ is a natural no. and $x \leq 5\}$

$B = \{x : x \leq 5, x \in N\}$

$B = \{x : x = 2n ; n \in N$ & $n \leq 5\}$

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Types of sets :-

(i) Null set (Denote 'ϕ')

A set having no element in it, is called an empty set or a null set or void set. it is denoted by 'ϕ'.

$$A = \{x : x \in N, \ 5 < x < 6\} \Rightarrow A = \phi$$

$$B = \{x : \sqrt{x} \in R ; \ x < 0 \}$$

$\phi \rightarrow$ Null set

$\{\ \} \rightarrow$ null set

$\{\phi\} \rightarrow$ Not a null set

$x \in \phi$ Nw

$x \in \{\ \}$ ✓ Nw

$x \in \{\phi\}$ ✓

* A set consisting of at least one element is called a non-empty set or non-void set.

(ii) Singleton set :- (single element)

A set consisting of a single element is called a singleton set.

eg: $\{0\}$, $\{1\}$, $\{5\}$, $\{500000\}$, $\{a\}$ ...

(iii) Finite set :- (finite no. of elements)

A set which has only finite number of elements is called a finite set.

eg: $\{1, 2, 3\}$, $\{3, 4, 5, 7, 9, 15\}$

Order of a finite set :- The number of elements in a finite set is called the order of the set (A) and is denoted by o(A) or n(A).
It is also called cardinal number of the set.

$$A = \{1, 2, 3, 4\}$$

$$n(A) = 4$$

$$B = \{1, 2, 3, \ldots, 100\}$$

$$n(B) = 100$$

Infinite set :- A set which has an infinite number of elements is called an infinite set.

$$A = \{1, 2, 3, \ldots\}$$

$$A = \{x : x \text{ is a natural number}\}$$

Equal sets :- Two sets A and B are said to be equal if every element of A is a member of B, and every element of B is a member of A.

If set A & B are equal then we write $A = B$.

If set A & B are not equal then we write $A \neq B$.

$$A = \{1, 2, 6, 7\} \quad ; \quad B = \{6, 1, 2, 7\}$$

$$A = B$$

Equivalent set :-

$\longleftrightarrow$

Two finite sets A & B are equivalent if their number of elements are same.

$$A = \{1, 2, 3, 4\} \quad ; \quad B = \{a, b, c, d\}$$

$$n(A) = n(B)$$

Note : Equal sets are always equivalent but equivalent sets are not always equal.

Subset or Superset :-

$\Rightarrow$ implies

$A \Rightarrow B$
A implies B

Let A and B be two sets if every element of A is an element of B then A is called a subset of B. and B is called the superset of A.

If A is a subset of B, we write $(A \subseteq B)$.

$A = \{1, 2, 3, 4\}$

$B = \{1, 2, 3, 4, 5, 6, 7\}$

A is a subset of B $\Rightarrow$ $A \subseteq B$

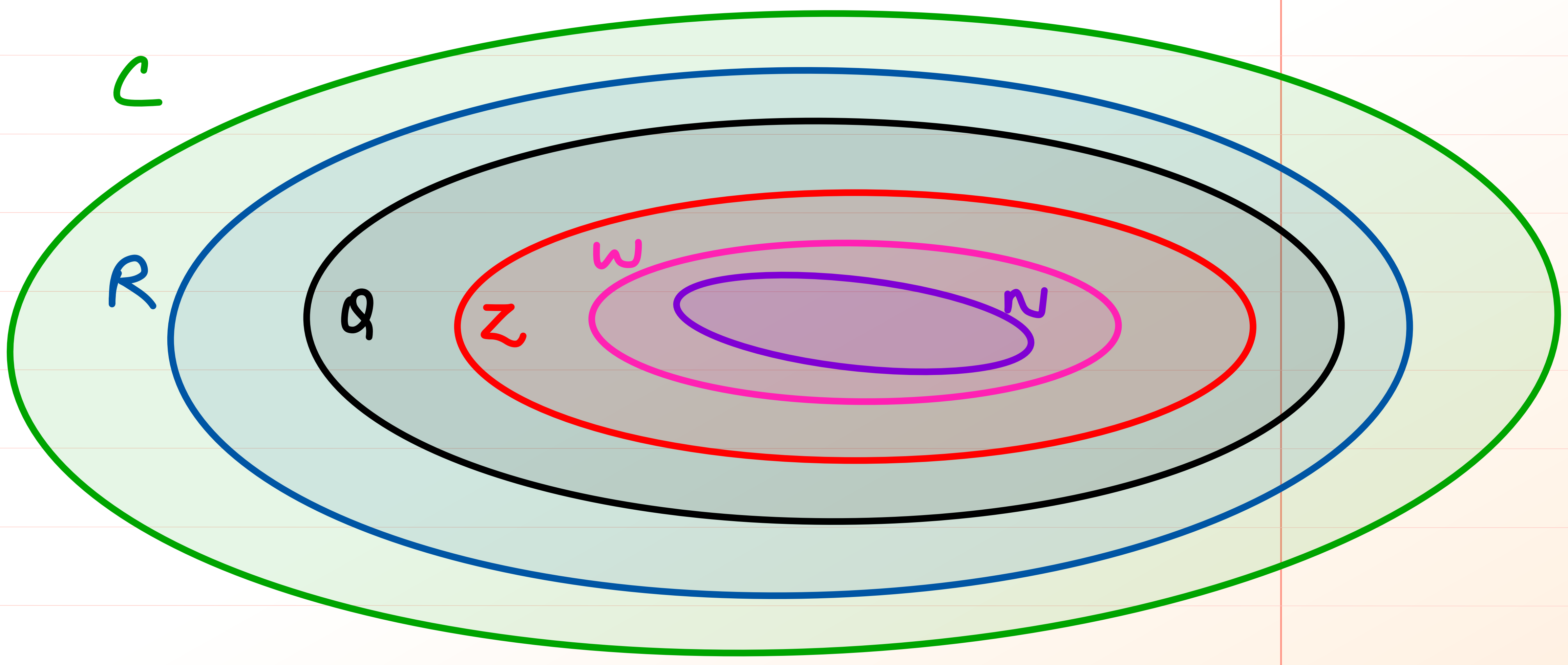
Proper subset :- If A is a subset of B and $A \neq B$ then A is a proper subset of B. we write $A \subset B$.

Note (1) Every set of is a subset of itself.

i.e. $A \subseteq A$ for all A.

(2) Empty set ϕ is a subset of every set.

(3) clearly $N \subset W \subset Z \subset Q \subset R \subset C$



(4) The total number of subsets of a finite set containing n elements is 2^n.

(5) Number of proper subsets of a set having n elements is $2^n - 1$.

(6) Empty set ϕ is a proper subset of every set except itself.

Q $A = \{1, 2, 3\}$

Subsets A = $\{1\}, \{2\}, \{3\}, \{1, 2\}, \{2, 3\}, \{3, 1\}, \{1, 2, 3\}, \phi$

$$\boxed{\text{No. of subsets of } A = 2^n \quad (n = \text{no. of elements in } A)}$$

Q $A = \{1, 2, 3, 4\}$

Subsets of A = $\{1\}$ $\{2\}$ $\{3\}$ $\{4\}$ $\{1, 2\}$ $\{1, 3\}$ $\{1, 4\}$
$\{2, 3\}$ $\{2, 4\}$ $\{3, 4\}$ $\{1, 2, 3\}$ $\{1, 3, 4\}$
$\{2, 3, 4\}$ $\{1, 2, 4\}$ $\{1, 2, 3, 4\}$ ϕ

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Universal set :→ A set consisting of all possible elements which occur in the discussion is called a universal set and it is denoted by U.

Note: All sets are contained in the universal set.

eg

$A = \{1, 2, 3\}$

$B = \{2, 4, 5, 6\}$

$C = \{1, 3, 5, 7\}$

$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ can be taken as universal set.

Power set :- Let A be any set, the set of all subsets of A is called the power set of A and it is denoted by $P(A)$.

eg $A = \{1, 2\}$

Subsets of A $= \phi, \{1\}, \{2\}, \{1,2\}$

Power set of A $P(A) = \{\phi, \{1\}, \{2\}, \{1,2\}\}$

eg $P(\phi) = \{\phi\} \Rightarrow$ subsets $\Rightarrow \phi, \{\phi\}$

$P(P(\phi)) = \{\phi, \{\phi\}\} \Rightarrow$ Subsets $\phi, \{\phi\}\ \{\{\phi\}\}$

$\{\phi, \{\phi\}\}$

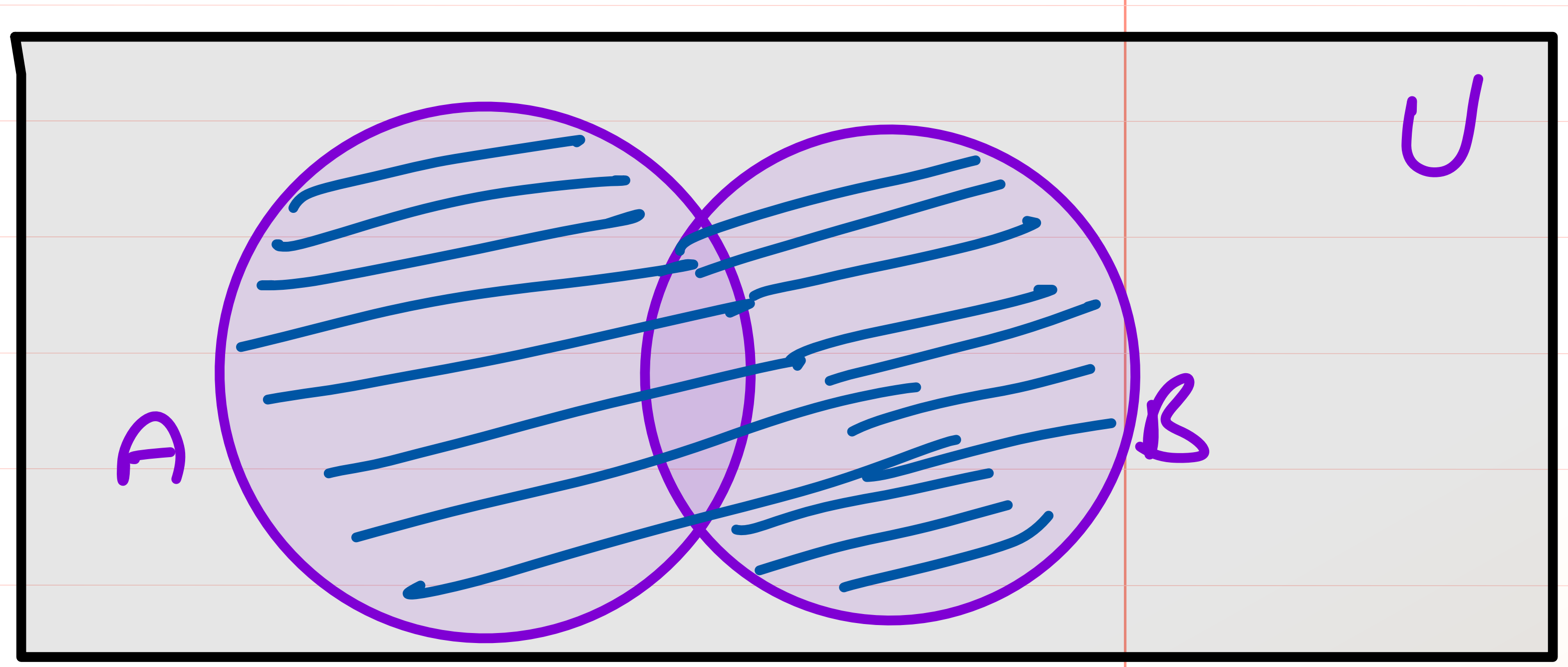
$P(P(P(\phi))) = \{\phi, \{\phi\}, \{\{\phi\}\}, \{\phi, \{\phi\}\}\}$

Union of sets :-

The union of two sets is represented by $A \cup B$ or $A + B$.

This set contains those elements which are in A or in B or in A and B both

$A \cup B = \{x : x \in A \text{ OR } x \in B\}$

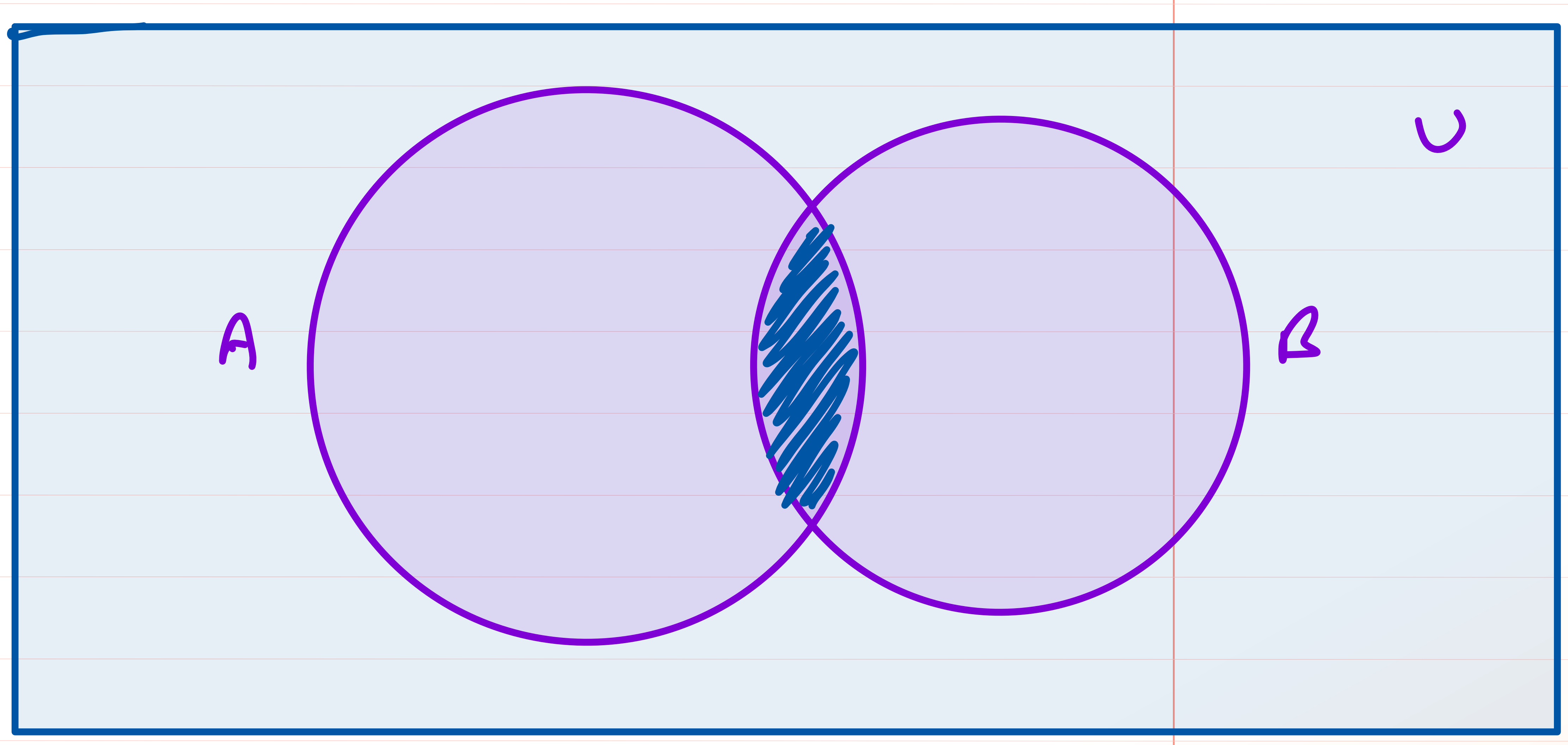


eg : $A = \{1, 2, 3, 4\}$; $B = \{2, 3, 9, 11\}$

$A \cup B = \{2, 3, 1, 4, 9, 11\}$

Intersection of sets :-

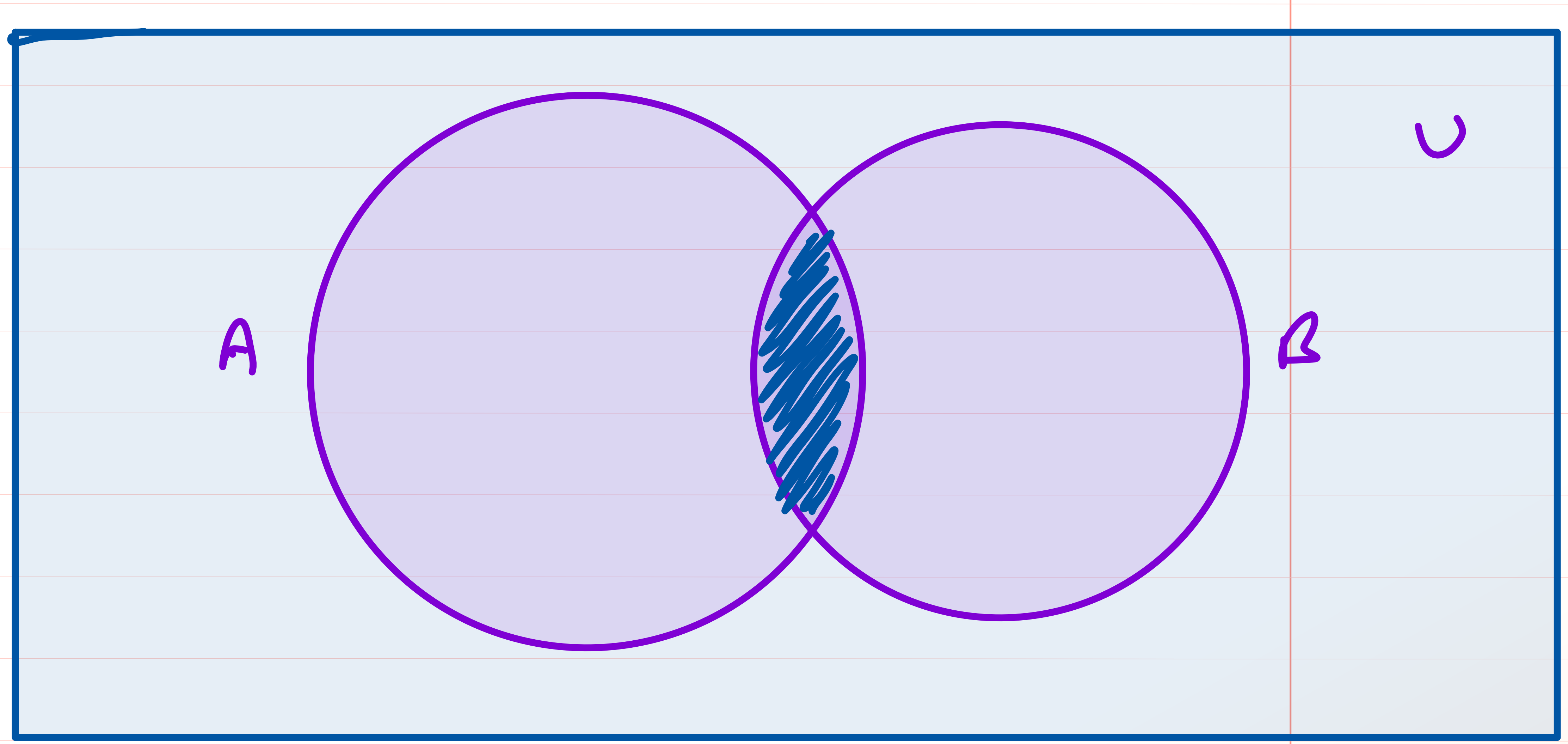
The intersection of two sets is represented by $A \cap B$ or AB.
It contains those all elements which are contained in both sets A & B both.



$$A \cap B = \{x : x \in A \text{ and } x \in B\}$$

eg : $A = \{1, \underline{2, 3}, 4\}$; $B = \{2, 3, 9, 11\}$

$A \cap B = \{2, 3\}$



Difference of two sets :-

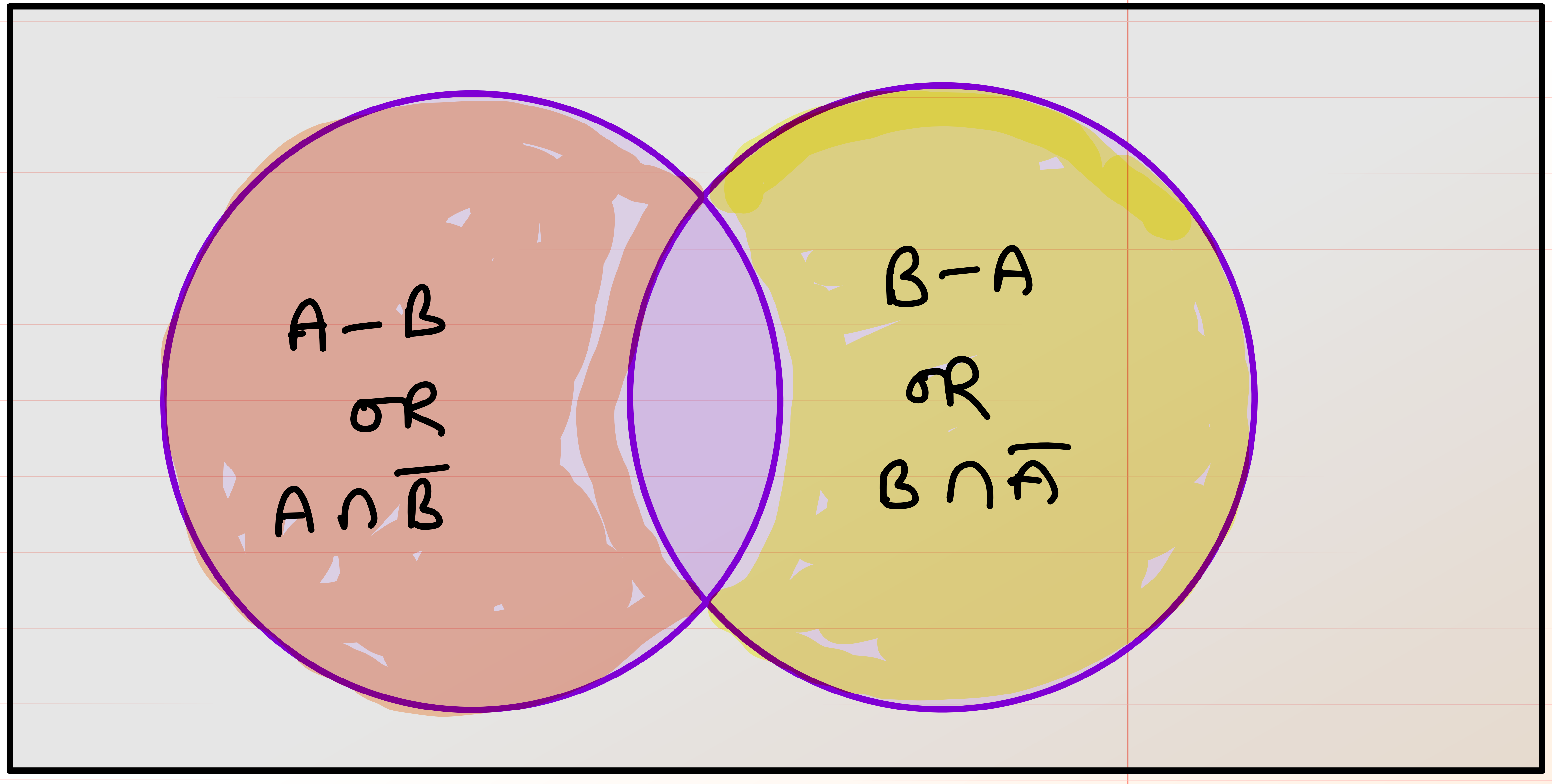
If A and B are two sets then A − B represents the set of those elements which are in A and not in B. In the same manner B − A represents the set of those elements which are in B and not in A.

$$A = \{1, 2, 3, 4\}$$

$$B = \{2, 3, 9, 11\}$$

$$A - B = \{1, 4\}$$

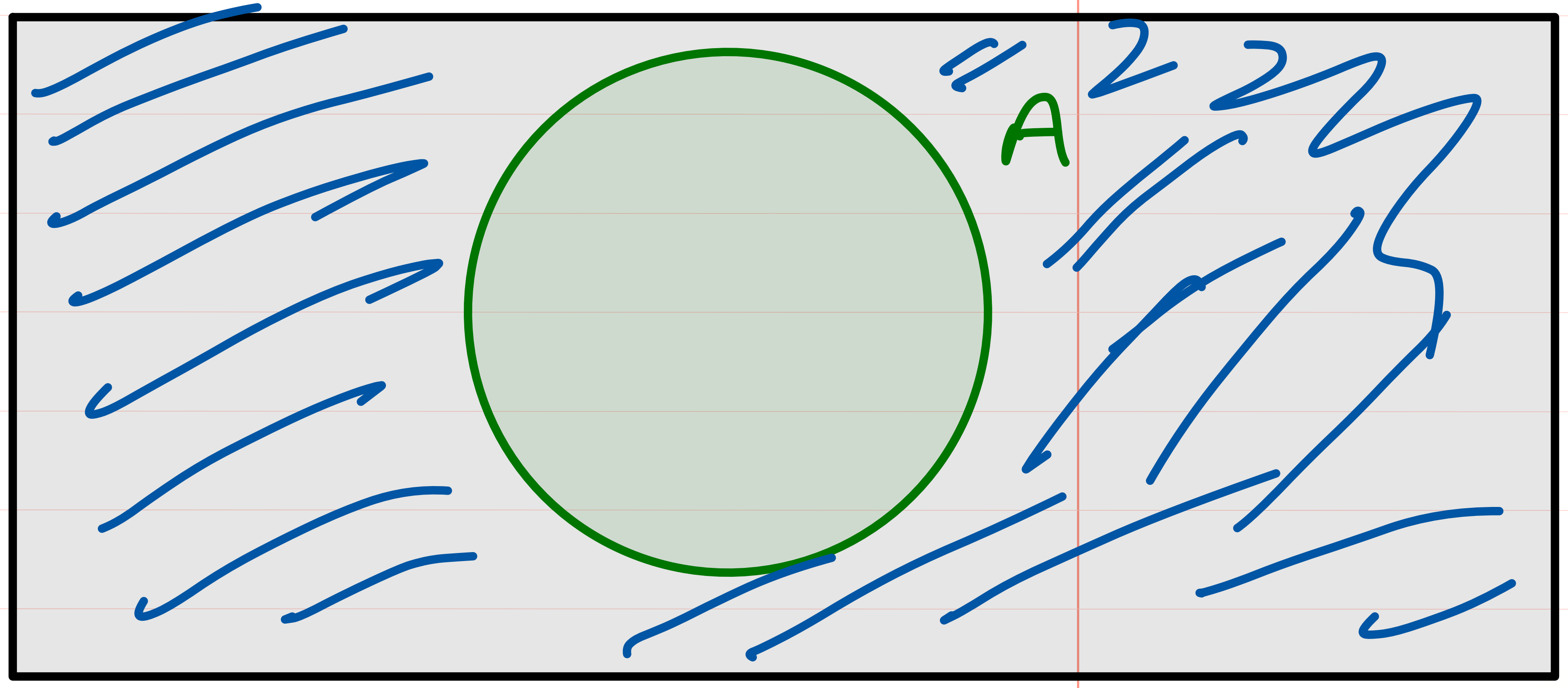
$$B - A = \{9, 11\}$$



Complement of a set :-

$A' = (U - A)$

$\bar{A} = U - A$



Eg $U = \{1, 2, 3, ---, 9, 10\}$

$A = \{1, 2, 3, 4\}$

$U - A = \{5, 6, 7, 8, 9, 10\}$

De-Morgan's Law :-

$A' = U - A$

(i) $(A \cup B)' = A' \cap B'$

(ii) $(A \cap B)' = A' \cup B'$

Eg $A = \{1, 2, 3, 4\}$ $U = \{1, 2, 3, 4, 5, \ldots 10\}$

$B = \{5, 6\}$

$(A \cup B)' = \{7, 8, 9, 10\}$

$A' = \{5, 6, 7, 8, 9, 10\}$

$B' = \{1, 2, 3, 4, \cancel{7}, 8, 9, 10\}$

$A' \cap B' = \{7, 8, 9, 10\}$

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(vi) $A - (B \cup C) = (A - B) \cap (A - C)$

$A - (B \cap C) = (A - B) \cup (A - C)$

(vii) Distributive Laws :-

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(viii) Commutative Laws :- $A \cup B = B \cup A$,

$A \cap B = B \cap A$

(ix) Associative Law :- $(A \cup B) \cup C = A \cup (B \cup C)$

$(A \cap B) \cap C = A \cap (B \cap C)$

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(x) $A \cap \phi = \phi$ $\quad$ $A \cap U = A$

$A \cup \phi = A$ $\quad$ $A \cup U = U$

(xi) $A \cap B \subseteq A$ $\quad$ $A \cap B \subseteq B$

(xii) $A \subseteq A \cup B$; $B \subseteq A \cup B$

(xiii) $A \subseteq B \Rightarrow A \cap B = A$

(xiv) $A \subseteq B \Rightarrow A \cup B = B$

$A = \{1, 2\}$

$B = \{1, 2, 3, 4\}$

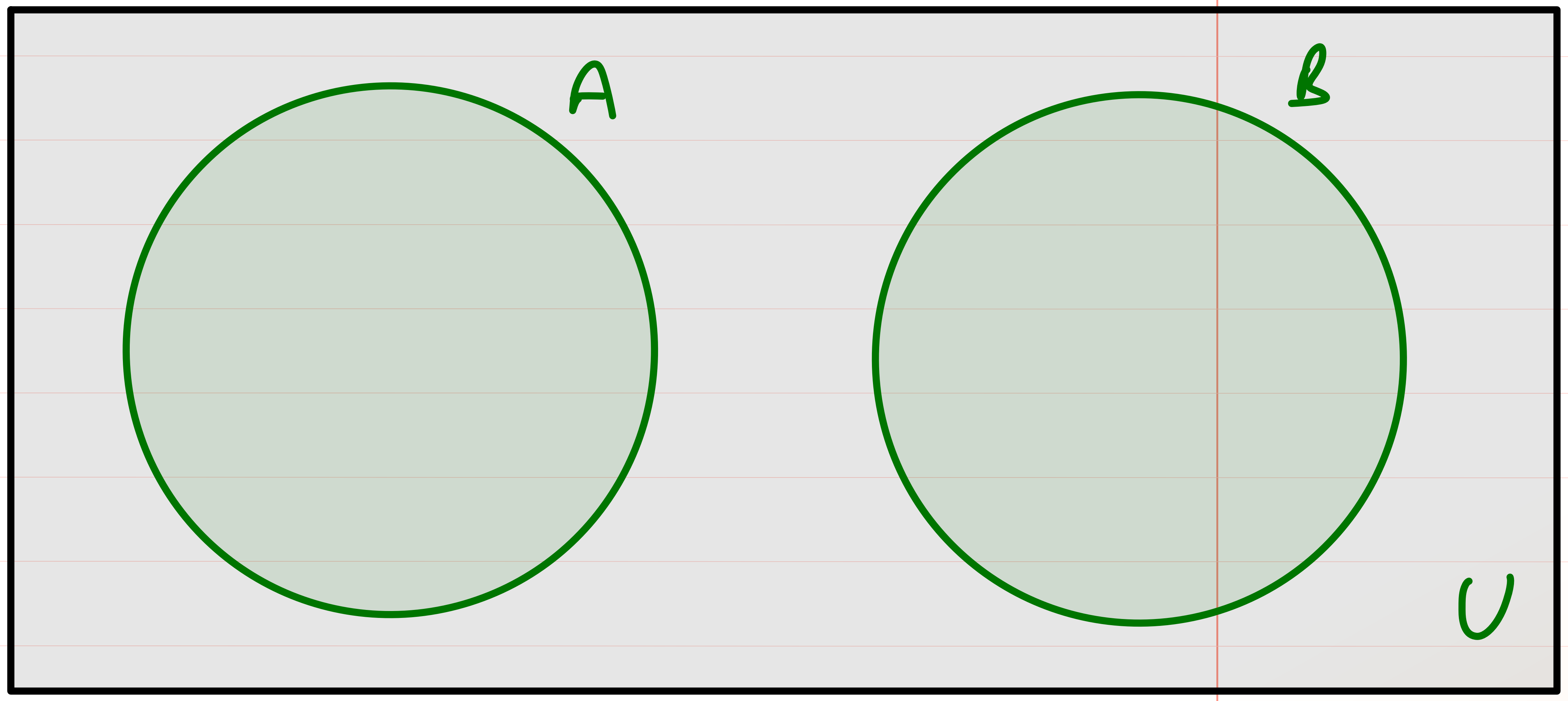
$A = \{2, 4\}$

$B = \{2, 4, 6, 8, 10\}$

Disjoint sets :- $A \cap B = \phi$

$A \cap B = \phi$

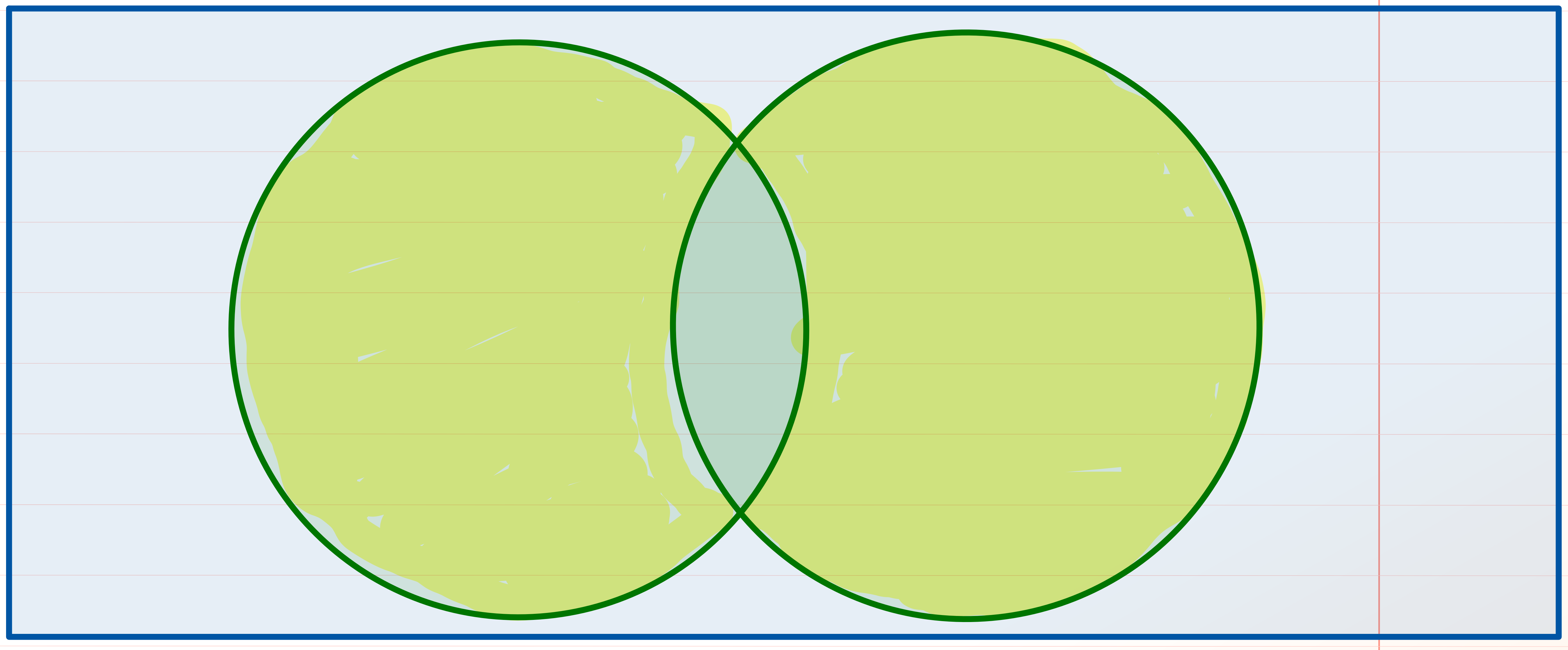
then A & B are disjoint.



Symmetric difference of sets :-

$$A \triangle B = (A-B) \cup (B-A)$$

$(A \cap B)'$



$A = \{1, 2, 3, 4\}$

$B = \{2, 3, 4, 5\}$

$A \triangle B = \{1, 5\}$

Note:-

(i) $(A')' = A$

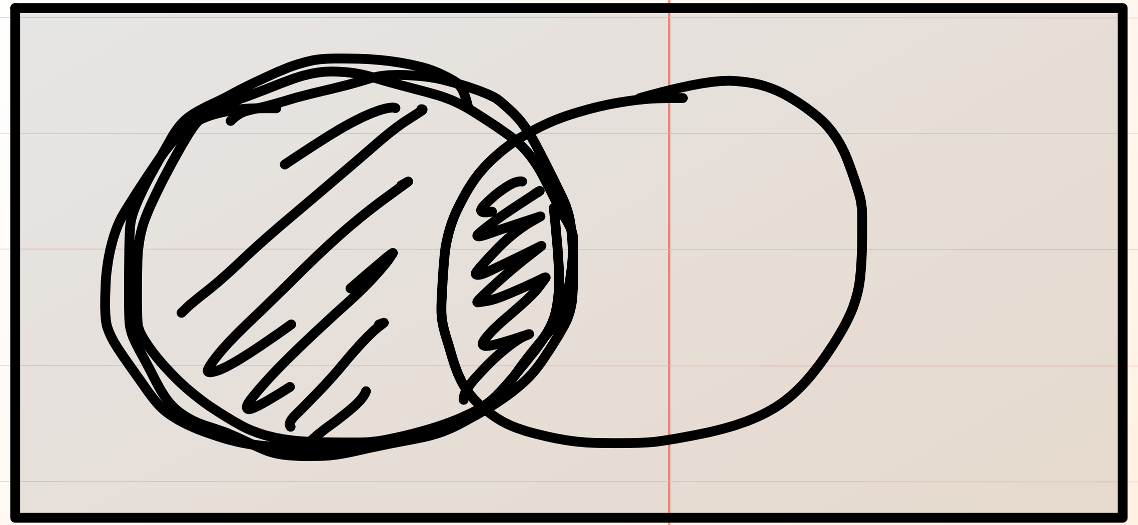
(ii) $A \subseteq B \Rightarrow B' \subseteq A'$

(iii) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

(iv) $n(A \cup B) = n(A) + n(B) \Rightarrow A, B$ are disjoint non-void sets.

(v) $n(A - B) = n(A) - n(A \cap B)$

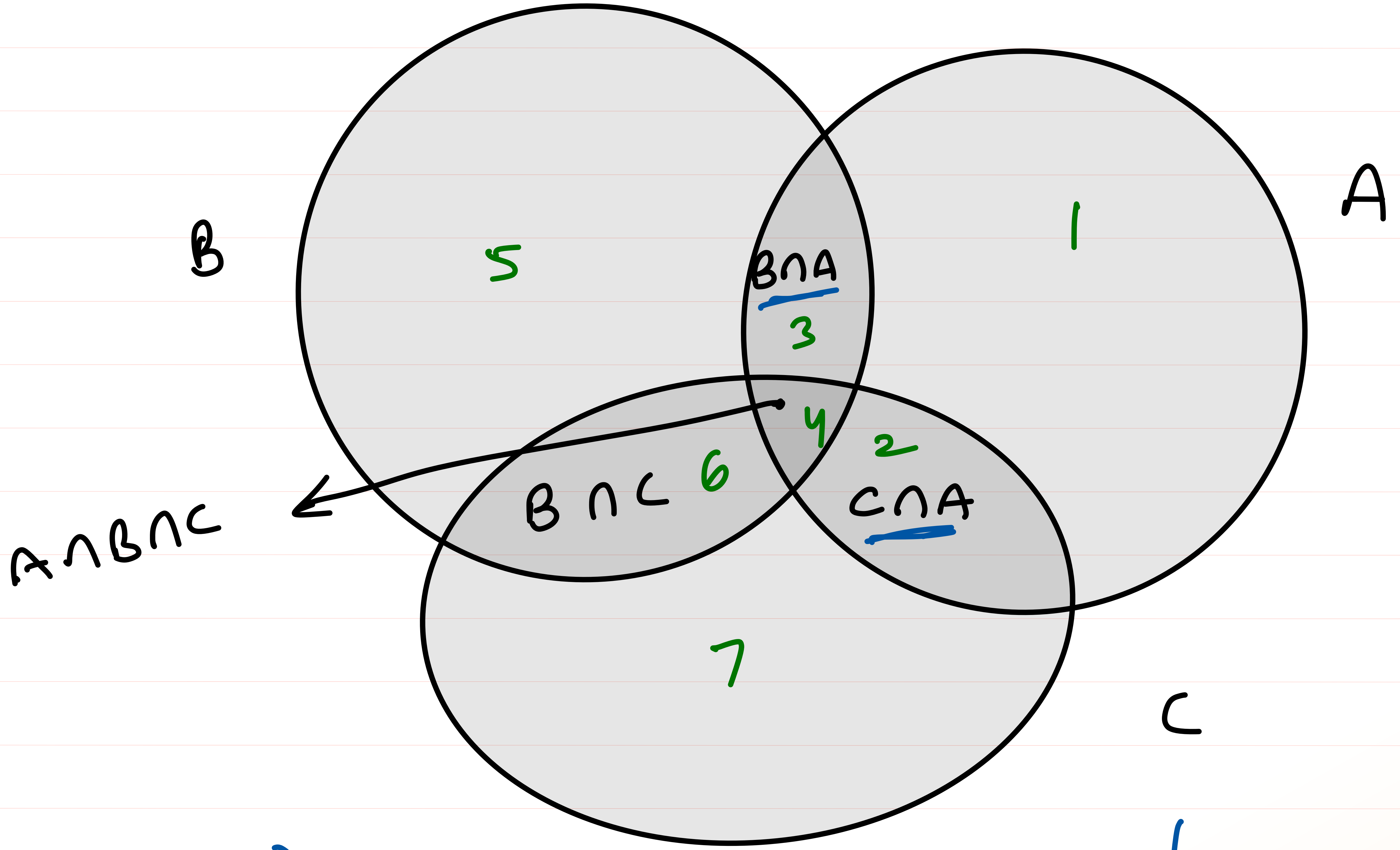
$n(A - B) + n(A \cap B) = n(A)$



(vi) $n(A \cup B \cup C) = n(A) + n(B) + n(C)$

$- n(A \cap B) - n(A \cap C) - n(B \cap C)$

$+ n(A \cap B \cap C)$.



$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C).$$

$n(A) =$

$n(A \cup B \cup C)$

$= n(A) + n(B)$

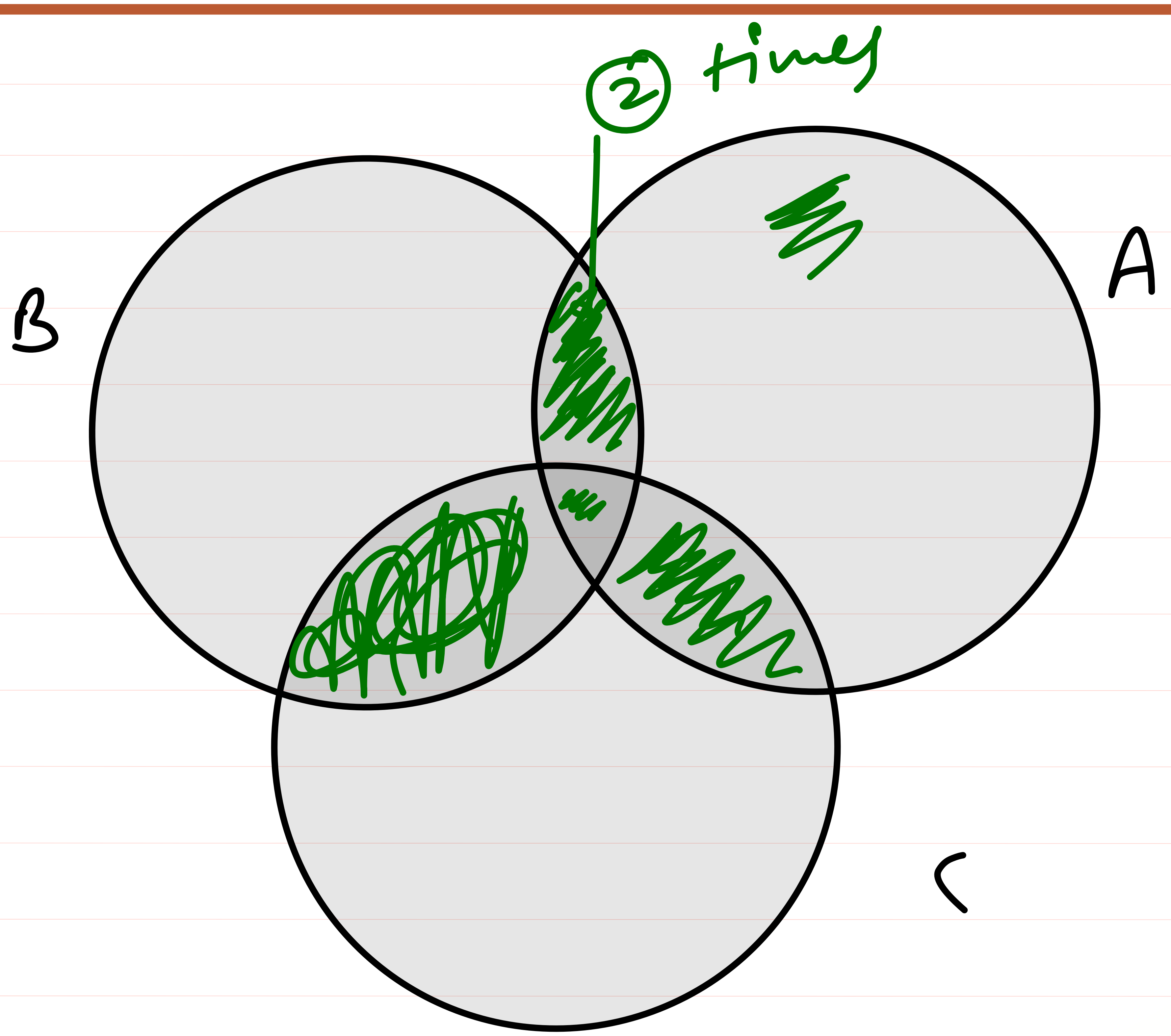
$+ n(C)$

$- n(A \cap B)$

$- n(B \cap C)$

$- n(C \cap A)$

$+ n(A \cap B \cap C)$.



Q Let A and B be two finite sets such that $n(A) = 20$; $n(B) = 28$; $n(A \cup B) = 36$

find (i) $n(A \cap B) = n(A) + n(B) - n(A \cup B)$

$= 20 + 28 - 36 = 12$

Q In a group of 60 people, 27 like cold drink, 42 like hot coffee and each person likes at least one of the two drinks. How many likes both?