4. Conversion of units from one system to another:

$1\,m = 100\,cm$

$1\,kg = 1000\,gm$

$1\,m = 3.2\,ft$

MAGNITUDE

Magnitude of physical quantity = (numerical value) x (unit)

Magnitude of a physical quantity is always constant. It is independent of the type of unit.

$$n_1 u_1 = n_2 u_2 = \text{constant}$$

Ex-1 Dyne is the unit of force in the c.g.s. system and newton is the unit of force in the SI system. Convert 1 dyne into newton.

Given
C.G.S

Req.
SI

$n_1 = 1 \quad U_1 = \text{dyne}$

$n_2 = ? \quad U_2 = \text{Newton}$

Physical Quantity = Force

$$n_1 u_1 = n_2 u_2$$

$$n_2 = n_1 \frac{u_1}{u_2}$$

$$n_2 = n_1 \frac{[gm \cdot cm\ \cancel{sec^{-2}}]}{[kg\ m\ \cancel{sec^{-2}}]}$$

$$= 1 \frac{[\cancel{gm}\ \cancel{cm}]}{[1000\,\cancel{gm}\ 100\,\cancel{cm}]}$$

$$n_2 = \frac{1}{10^5}$$

$$n_2 = 10^{-5}\,N$$

$1\,\text{dyne} = 10^{-5}\,N$

$$\boxed{1\,N = 10^5\,\text{dyne}}$$

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Method

$F = 1 \text{ dyne}$

$= 1 \; \frac{gm \cdot cm}{sec^2}$

$= 1 \times 10^{-3} kg \cdot \frac{10^{-2} m}{sec^2}$

$= 10^{-5} \left(\frac{kg \cdot m}{sec^2}\right)$

$F = 10^{-5} N$ Ans

$1 \text{ dyne} = 10^{-5} N$

$M^1 L^1 T^{-2}$

General

$N_1 u_1 = N_2 u_2$

$N_2 = N_1 \left[\frac{u_1}{u_2}\right]$

$[M^a L^b T^c]$ = Dim of Ph. quant.

$N_2 = N_1 \frac{[M_1^a L_1^b T_1^c]}{[M_2^a L_2^b T_2^c]}$

$$\boxed{N_2 = N_1 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c}$$

$N_2 = 1 \left[\frac{gm}{kg}\right]^1 \left[\frac{cm}{m}\right]^1 \left[\frac{sec}{sec}\right]^{-2}$

$= 1 \left[\frac{10^{-3} \cancel{kg}}{\cancel{kg}}\right]^1 \left[\frac{10^{-2} \cancel{m}}{\cancel{m}}\right]^1$

$N_2 = 10^{-5}$

$$\boxed{1 \text{ dyne} = 10^{-5} N}$$

Ex-2 Convert 72 kmh^{-1} into ms^{-1} by using the method of dimensions.

$72 \frac{km}{hr}$

$= 72 \times \frac{1000\,m}{60 \times 60\,sec}$

$= 20 \frac{m}{s}$ Ans

$72 \frac{km}{hr} = 20\, m/sec$

$\boxed{1 \frac{km}{hr} = \frac{5}{18} \frac{m}{s}}$

$[V] = [M^0 L^1 T^{-1}]$

$a = 0 \quad b = 1 \quad c = -1$

$N_2 = 72 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$

$= 72 \left[\frac{M_1}{M_2}\right]^0 \left[\frac{km}{m}\right]^1 \left[\frac{hr}{sec}\right]^{-1}$

$= 72 [1] \left[\frac{1000\cancel{m}}{\cancel{m}}\right]^1 \left[\frac{3600\,\cancel{sec}}{\cancel{sec}}\right]^{-1}$

$= 72 \times 1000 [3600]^{-1}$

$= \frac{72 \times 1000}{3600}$

$= 20$

$\boxed{72 \frac{km}{hr} = 20\, m/s}$

Ex-3 If the units of force, energy and velocity are 10 N, 100 J and 5 ms^{-1}, find the units of length, mass and time.

Unit of Force = 10 N

Unit of Energy = 100 J

Unit of velocity = 5 m/s

Energy = Force × length

100 J = 10 N × Length

$\boxed{\text{length} = 10\,\text{m}}$ Ans

velocity = $\frac{\text{displ}}{\text{time}}$

5 m/s = $\frac{10\,\text{m}}{\text{time}}$ ⇒ $\boxed{\text{time} = 2\,\text{sec}}$

Force = mass × $\frac{\text{velocity}}{\text{time}}$

10 N = mass × $\frac{5\,\text{m/s}}{2\,\text{sec}}$

$\boxed{\text{mass} = 4\,\text{kg}}$

Ex-4 The accelaration due to gravity is 9.8 m s^{-2}. Give its value in ft s^{-2}

M-1

$9.8\ m/sec^2$

$= 9.8 \times \frac{3.2\ ft}{sec^2}$

$\simeq 32 \left(\frac{ft}{sec^2}\right)$

M-2

$N_2 = N_1 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$

$[a] = [M^0 L^1 T^{-2}]$

$= 9.8 \left[\frac{Kg}{P}\right]^0 \left[\frac{m}{ft}\right]^1 \left[\frac{sec}{sec}\right]^{-2}$

$= 9.8 \left[\frac{3.2\ ft}{ft}\right]^1 [1]^{-2}$

$= 9.8 \times 3.2$

$N_2 \simeq 32$

$$\boxed{9.8\ m/s^2 = 32\ \frac{ft}{sec^2}}$$

Ex-5 The value of Gravitational constant G in MKS system is 6.67×10^{-11} N-m^2/kg^2 . What will be its value in CGS

M1

$$6.67 \times 10^{-11} \frac{N \cdot m^2}{kg^2}$$

$$= 6.67 \times 10^{-11} \left(\frac{10^5 dyne \cdot (100\, cm)^2}{(1000\, gm)^2} \right)$$

$$= 6.67 \times 10^{-11} \left[\frac{10^5 \times 10^4}{10^6} \frac{dyne \cdot cm^2}{gm^2} \right]$$

$$= 6.67 \times 10^{-8} \left[\frac{dyne \cdot cm^2}{gm^2} \right]$$

Ans

M-2

$$N_2 = 6.67 \times 10^{-11} \left[\frac{M_1}{M_2} \right]^a \left[\frac{L_1}{L_2} \right]^b \left[\frac{T_1}{T_2} \right]^c$$

Unit of G $= \frac{m^3}{kg \cdot sec^2}$ Dim $[M^{-1} L^3 T^{-2}]$

$$N_2 = 6.67 \times 10^{-11} \left[\frac{kg}{gm} \right]^{-1} \left[\frac{m}{cm} \right]^3 \left[\frac{sec}{sec} \right]^{-2}$$

$$= 6.67 \times 10^{-11} \left[\frac{1000\, \cancel{gm}}{\cancel{gm}} \right]^{-1} \left[\frac{100\, \cancel{cm}}{\cancel{cm}} \right]^3 [1]^{-2}$$

$$= \frac{6.67 \times 10^{-11}}{\cancel{1000}} \times \cancel{100} \times 100 \times 100$$

$$\boxed{N_2 = 6.67 \times 10^{-8}}$$ Ans

NOTE

1. Trigonometric function (sin, cos, tan, cot etc) are dimensionless. The arguments of these functions are also dimensionless
2. Exponential functions are dimensionless. Their exponents are also dimensionless

$\sin(\theta)$ $\theta =$ Dimensionless

$e^{(2x+3)} \rightarrow$ Dimensionless

$\cos(2t+3) \rightarrow$ Dimensionless

Ex-1 When a plane wave travels in a medium, the displacement y of a particle located at x at time t is given by

$$y = a\sin(bt + cx)$$

where a, b and c are constants. Find the dimensions of $\frac{b}{c}$.

$[a] = [y] = [M^0 L^1 T^0]$

$[bt + cx] = [M^0 L^0 T^0]$

$[b] \propto [\frac{1}{t}] = [M^0 L^0 T^{-1}]$

$[c] \propto \frac{1}{[x]} = [M^0 L^{-1} T^0]$

$[\frac{b}{c}] = [\frac{M^0 L^0 T^{-1}}{M^0 L^{-1} T^0}]$

$= [M^0 L^1 T^{-1}]$ Ans

Ex-2 In the expression

$$P = \frac{a^2}{b} e^{-ax}$$

P is pressure, x is a distance and a and b are constants. Find the dimensional formula for b.

$[a] \propto \frac{1}{[x]}$

$$\boxed{[a] = [M^0 L^{-1} T^0]}$$

$[P] = \left(\frac{a^2}{b}\right)$

$b = \left(\frac{a^2}{P}\right)$

$= \frac{[M^0 L^{-2} T^0]}{[M^1 L^{-1} T^{-2}]}$

$$\boxed{[b] = [M^{-1} L^{-1} T^2]}$$

Race #1

Ex-3

17. The velocity v of a particles is given in terms of time t by the equation. $\mathbf{v} = at + \frac{b}{t+c}$. The dimension of a, b and c are

(A) L^2, T, L T^2 (B) LT^2, LT, L ~~(C)~~ LT^{-2}, L, T (D) L, LT, T^2

$[at] = [v]$

$[a] = \left[\frac{v}{t}\right]$

$= \left[\frac{L}{t^2}\right]$

$[a] = [M^0 L^1 T^{-2}]$

$\left[\frac{b}{t+c}\right] = [v]$

$[t] = [c] = [M^0 L^0 T^1]$

$[b] = [v][t+c]$

$= [v][t]$ or $[v][c]$

$= \left[\frac{L}{t}\right][t]$

$= [L] = [M^0 L^1 T^0]$

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$$\text{Power} = \text{Force} \times \text{Velocity}$$

$$(\text{Power})' = \left(\frac{\text{Force}}{2}\right) \times \left(\frac{\text{Velocity}}{2}\right)$$

$$(\text{Power})' = \frac{\text{Power}}{4}$$

$$\text{Energy} = \text{Force} \times \text{Length}$$

$$E' = 4F \times 4L$$

$$E' = 16\,(FL)$$

$$\boxed{E' = 16\ \text{Energy}}$$