$\frac{\pi}{2} = 90°$ vertical Angle line

$5\frac{\pi}{2}, 9\frac{\pi}{2}$

HoriZontal line

$0°$ $360° = 2\pi, 4\pi, 6\pi$

$180° = \pi, 3\pi, 5\pi$

$270° = \frac{3\pi}{2}$ rad, $7\frac{\pi}{2}, 11\frac{\pi}{2}$

NOTE → ①

In HoriZontal Angle line if we add or Subtract angle θ $(0° < \theta < 90°)$ then function will remain same

② In vertical Angle line if we add or Subtract angle θ $(0° < \theta < 90°)$ then function will change as following

$\sin \rightleftarrows \cos$ $\tan \rightleftarrows \cot$ $\sec \rightleftarrows \cosec$

Find values of following function

① Sin (120)

② Cot (300°)

③ Sec (400°)

② Sec (400°)

Sec (360 + 40°)

+ Sec (40°) Ans

Sec (450 - 50)

Cosec (50°) Ans

① Sin (120°)

Sin (90 + 30°)

+ Cos (30°)

$\frac{\sqrt{3}}{2}$ Ans

② Cot (300°)

Cot (360 - 60°)   or   Cot (270 + 30°)

= - Cot 60°

= $-\frac{1}{\sqrt{3}}$

= - tan 30

= $-\frac{1}{\sqrt{3}}$

Sin (180° - 60°)

= + Sin (60°)

= $\frac{\sqrt{3}}{2}$ Ans

Find the value of following functions

① $\sin\left(\dfrac{7\pi}{4}\right)$

② $\cot(585°)$

③ $\cos\left(\dfrac{11\pi}{3}\right)$

④ $\tan(750°)$

---

① $\sin\left(\dfrac{7\pi}{4}\right)$

$\sin\left(2\pi - \dfrac{\pi}{4}\right)$

$= -\sin\left(\dfrac{\pi}{4}\right)$

$= -\dfrac{1}{\sqrt{2}}$ __Ans__

② $\cot(585°)$

$= \cot(540° + 45°)$

$= \cot(3 \times 180° + 45°)$

$= + \cot 45°$

$= 1$

---

③ $\cos\left(\dfrac{11\pi}{3}\right) = \cos\left(4\pi - \dfrac{\pi}{3}\right)$

$= + \cos\left(\dfrac{\pi}{3}\right)$

$= \dfrac{1}{2}$ Ans

④ $\tan(750°)$

$\tan(720° + 30°)$

$= + \tan 30°$

$= \dfrac{1}{\sqrt{3}}$ __Ans__

$\cos(11 \times 60°)$

$\cos(660°)$

$\cos(720 - 60°)$

$+ \cos(60°)$

$+ \dfrac{1}{2}$ __Ans__

Ex

① $\sin\left(\frac{9\pi}{6}\right)$

② $\tan\left(\frac{7\pi}{4}\right)$

③ $\operatorname{cosec}(135°)$

④ $\cot(220°)$

---

① $\sin\left(\frac{9\pi}{6}\right)$

$= \sin\left(\frac{3\pi}{2}\right)$

$= \sin(270°)$

$= \sin(270° + 0)$

$= -\cos(0°)$

$= -1$ __Ans__

② $\tan\left(\frac{7\pi}{4}\right)$

$\tan\left(8\pi - \frac{\pi}{4}\right)$

$-\tan\frac{\pi}{4}$

$= -1$

---

③ $\operatorname{cosec}(135°)$

$\operatorname{cosec}(90 + 45°)$

$= +\sec 45°$

$= \sqrt{2}$

④ $\cot(220°)$

$= \cot(180 + 40)$

$= \cot(40°)$ __Ans__

$= \cot(90 - 50)$

$= \tan(50°)$ __Ans__

Addition/Subtraction Formulae for Trigonometrical Ratios

- $\sin(A+B) = \sin A \cos B + \cos A \sin B$
- $\sin(A-B) = \sin A \cos B - \cos A \sin B$
- $\cos(A+B) = \cos A \cos B - \sin A \sin B$
- $\cos(A-B) = \cos A \cos B + \sin A \sin B$

$$\tan(A \pm B) = \left(\frac{\tan A \pm \tan B}{1 \mp \tan(A)\tan(B)} \right)$$

- If $A = B$

$$\sin(A+A) = \sin(A)\cos(A) + \cos A \sin(A)$$

$$\boxed{\sin(2A) = 2\sin A \cos(A)}$$

$$\cos(A+A) = \cos A \cos A - \sin(A)\sin(A)$$

$$\boxed{\cos(2A) = \cos^2 A - \sin^2 A}$$

$$\sin^2 A + \cos^2 A = 1$$

$$\boxed{\begin{array}{l} \cos(2A) = 2\cos^2 A - 1 \\[2mm] \cos(2A) = 1 - 2\sin^2 A \end{array}}$$

H.W.
Illustration #13,14

Find values of following functions

① sin (15°)

② cos (15°)

③ tan (15°)

④ sin (74°)

⑤ cos (106°)

④ sin(74°)

$\sin(2 \times 37°)$

$= 2 \sin(37°) \cos(37°)$

$= 2 \times \dfrac{3}{5} \times \dfrac{4}{5}$

$= \dfrac{24}{25}$ Ans

③ tan(15°)

tan(45 - 30°)

$= \dfrac{\tan 45 - \tan 30}{1 + \tan 45 \tan 30}$

$= \dfrac{1 - \dfrac{1}{\sqrt{3}}}{1 + 1 \times \dfrac{1}{\sqrt{3}}}$

$= \dfrac{\sqrt{3} - 1}{\sqrt{3} + 1}$ Ans

① $\sin(15) = \sin(45 - 30°)$

$= \sin 45 \cos 30 - \cos 45 \sin 30$

$= \dfrac{1}{\sqrt{2}} \times \dfrac{\sqrt{3}}{2} - \dfrac{1}{\sqrt{2}} \times \dfrac{1}{2}$

$= \dfrac{\sqrt{3} - 1}{2\sqrt{2}}$ Ans

② $\cos(15°) = \cos(45 - 30°)$

$= \cos 45 \cos 30 + \sin 45 \sin 30$

$= \dfrac{1}{\sqrt{2}} \times \dfrac{\sqrt{3}}{2} + \dfrac{1}{\sqrt{2}} \times \dfrac{1}{2}$

$= \dfrac{\sqrt{3} + 1}{2\sqrt{2}}$ Ans

⑤ $\cos(106°)$

$\cos(2 \times 53°)$

$= \cos^2 53 - \sin^2 53$

$= \left(\dfrac{3}{5}\right)^2 - \left(\dfrac{4}{5}\right)^2$

$= \dfrac{9}{25} - \dfrac{16}{25}$

$= \dfrac{-7}{25}$ Ans

<u>Ex</u> ① $\sin(75)$

② $\cos(75°)$

① $\sin(75°)$

$\sin(45° + 30°)$

$= \sin(45)\cos 30 + \cos 45 \sin 30$

$= \dfrac{1}{\sqrt{2}} \times \dfrac{\sqrt{3}}{2} + \dfrac{1}{\sqrt{2}} \times \dfrac{1}{2}$

$= \dfrac{\sqrt{3}+1}{2\sqrt{2}}$

---

② $\cos(75°)$

(M-1)

$\cos(90 - 15°)$

$= + \sin 15°$

$= \dfrac{\sqrt{3}-1}{2\sqrt{2}}$

(M-2)

$\cos(75) = \cos(45 + 30)$

$\cos 45 \cos 30 - \sin 45 \sin 30$

$= \dfrac{\sqrt{3}-1}{2\sqrt{2}}$ Ans

Maximum and Minimum Values of Some useful Trigonometric Functions (Range of Some Trigo functions)

- $-1 \le \sin\theta \le 1$
- $-1 \le \cos\theta \le 1$

$-\sqrt{a^2+b^2} \le a\cos\theta + b\sin\theta \le \sqrt{a^2+b^2}$

which of the following functions are True/false
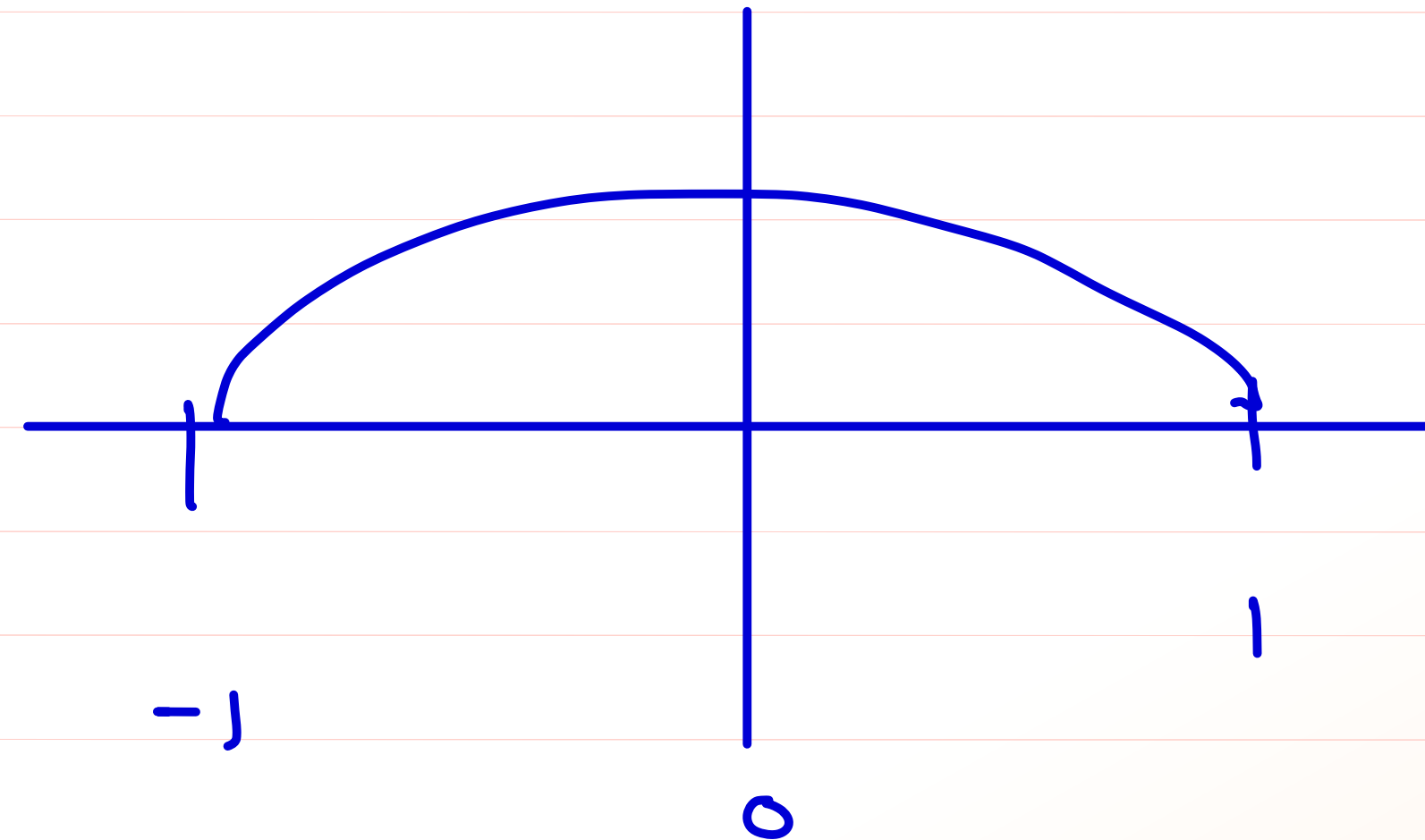
① $\sin\theta > 4$ (F)

② $\sin\theta < -\frac{3}{2} = -1.5$ (F)

③ $\cos\theta > \frac{3}{2} = 1.5$ (F)

④ $\cos\theta > \frac{4}{5} = 0.8$ (T)

⑤ $\sin\theta < -\frac{4}{6} = -\frac{2}{3}$ (T)

$= -0.66$ (T)

Ex

$\sin\theta = \frac{4}{3}$

Find θ

not possible

$\sin(\theta) = \frac{4}{3} > 1$

$-1 \quad 0 \quad 1$

H.W.
Illustration #16,17

Ex Find maximum & minimum value of $3\sin\theta + 4\cos\theta$

$$-\sqrt{3^2 + 4^2} \leq 3\sin\theta + 4\cos\theta \leq \sqrt{3^2 + 4^2}$$

$$-5 \leq \; \leq 3\sin\theta + 4\cos\theta \leq +5$$

range of $3\sin\theta + 4\cos\theta$ is $[-5, 5]$

Ex Which of the value are correct for $5\sin\theta + 12\cos\theta$

(1) -20

(ii) ✓ $+10$

(iii) $+20$

(iv) ✓ -8

Range $\sqrt{5^2 + 12^2} = 13$

$[-13, 13]$