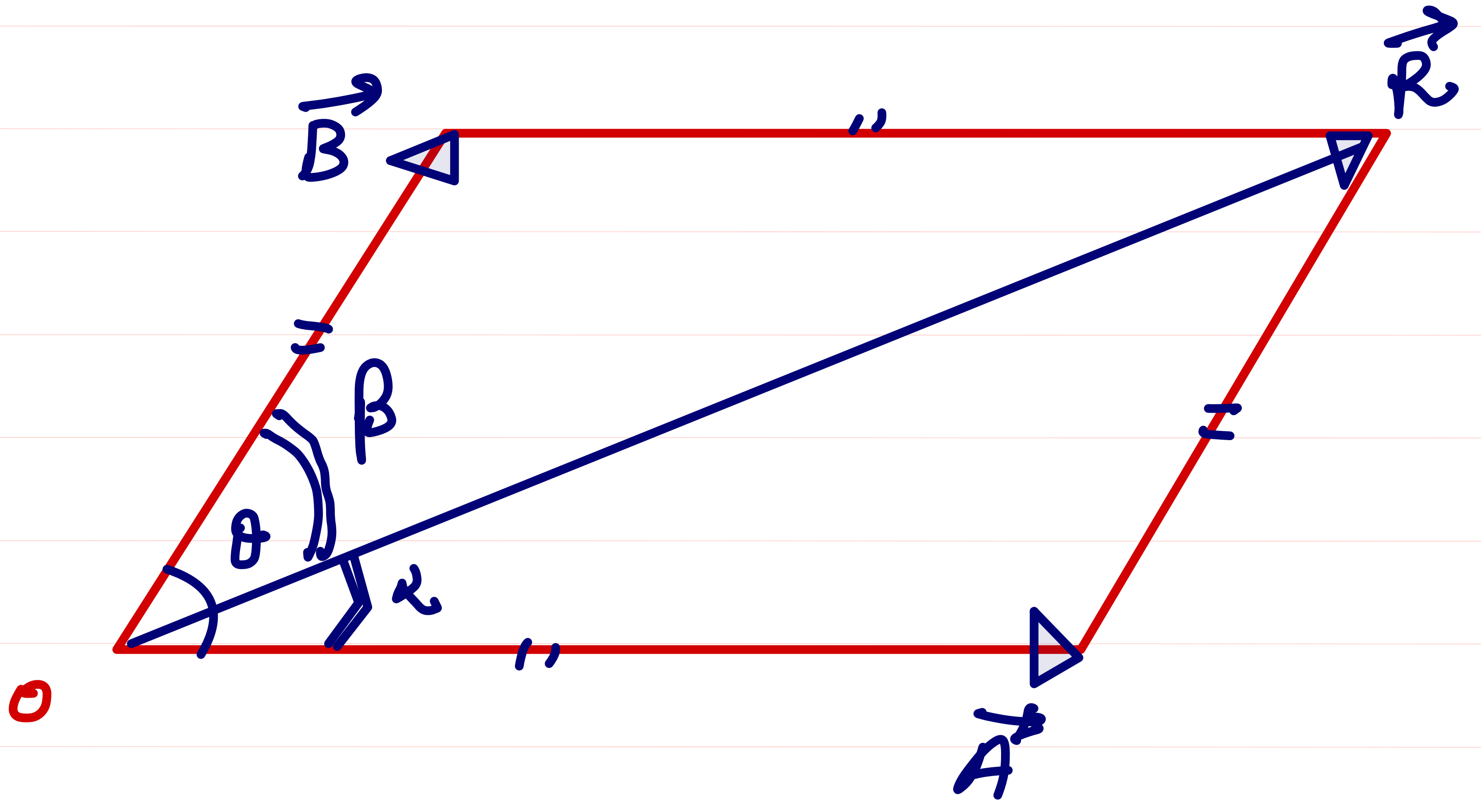
## Parallelogram Law :-

*If the two vectors are represented in magnitude and direction by the two adjacent sides of a parallelogram drawn from a point, then their resultant is represented in magnitude and direction by the diagonal of the parallelogram passing through that point.*




$$\boxed{R = \sqrt{A^2 + B^2 + 2AB\cos\theta}}$$

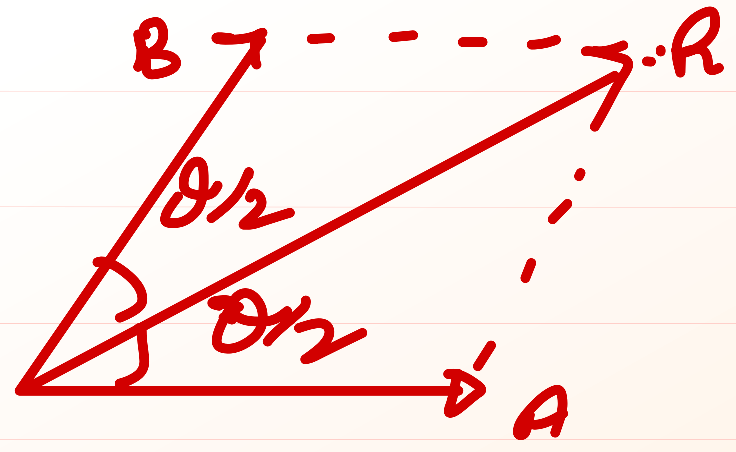
$$\boxed{\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta} \qquad \tan\beta = \frac{A\sin\theta}{B + A\cos\theta}}$$

$$\boxed{\vec{A} + \vec{B} = \vec{B} + \vec{A}}$$

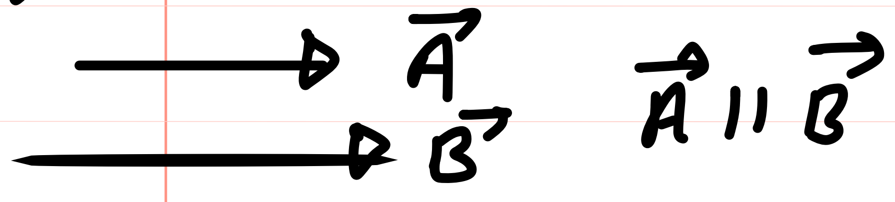
**Cases**

① If $A = B$

$$\boxed{R = 2A\cos(\theta/2) \qquad \alpha = \beta = \frac{\theta}{2}}$$




② If $\theta = 0°$, $\cos 0 = 1$




$\vec{A} \parallel \vec{B}$

$$\boxed{R_{max} = A + B}$$

③ If $\theta = 180°$, $\cos(180) = -1$

$$\boxed{R_{min} = |A - B|}$$

④ $\boxed{|A-B| \le R \le (A+B)}$

Range

**Ex** Two vectors having equal magnitude of 5 units, have an angle of 60° between them. Find the magnitude of their resultant vector and its angle from one of the vectors.

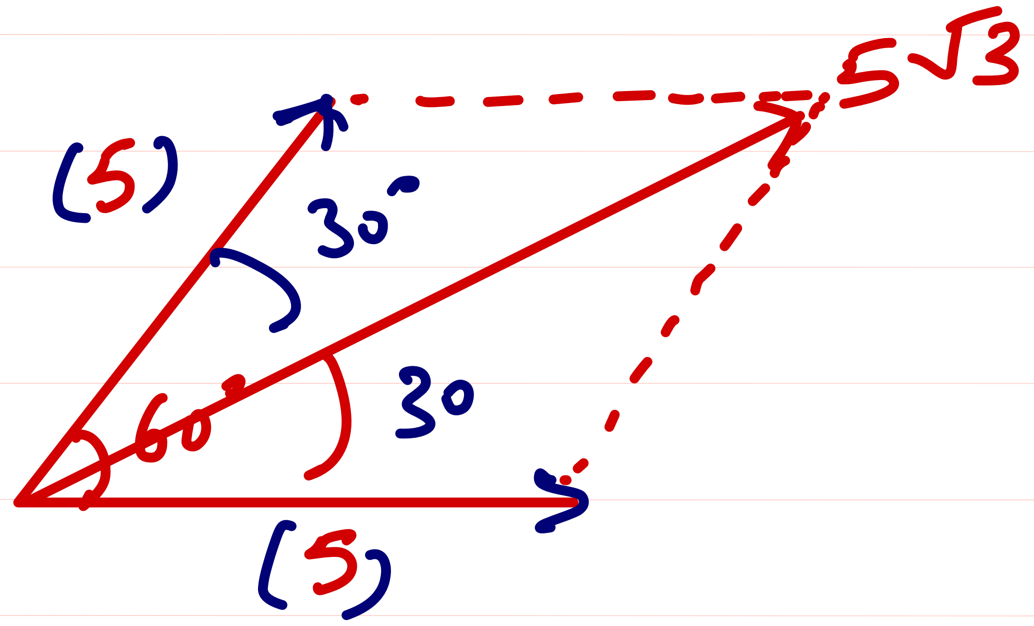
Given $A = B = 5$ $\quad \theta = 60^\circ$

$$R = 2A\cos\frac{\theta}{2}$$

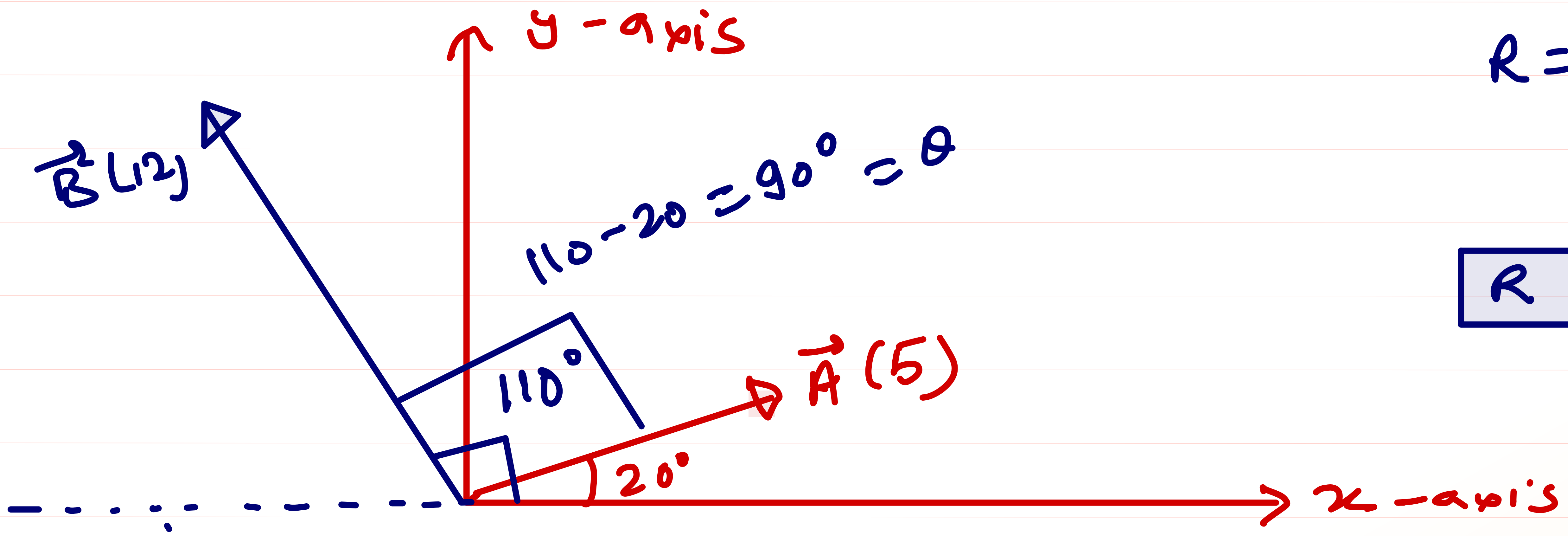
$$= 2\times5\cos\frac{60}{2}$$

$$= 2\times5\,\frac{\sqrt3}{2} = 5\sqrt3 \quad \text{Ans}$$

$$\alpha = \beta = \frac{\theta}{2} = 30^\circ$$




**Ex** A vector $\vec{A}$ and $\vec{B}$ make angles of 20° and 110° respectively with the X-axis. The magnitudes of these vectors are 5m and 12m respectively. Find their resultant vector.




$$R = \sqrt{A^2+B^2+2AB\cos 90}$$

$$= \sqrt{5^2+12^2}$$

$$\boxed{R = 13\ m}$$

$$\tan\alpha = \frac{B\sin\theta}{A+B\cos\theta}$$

$$\tan\alpha = \frac{12\sin(90)}{5+12\cos(90)}$$

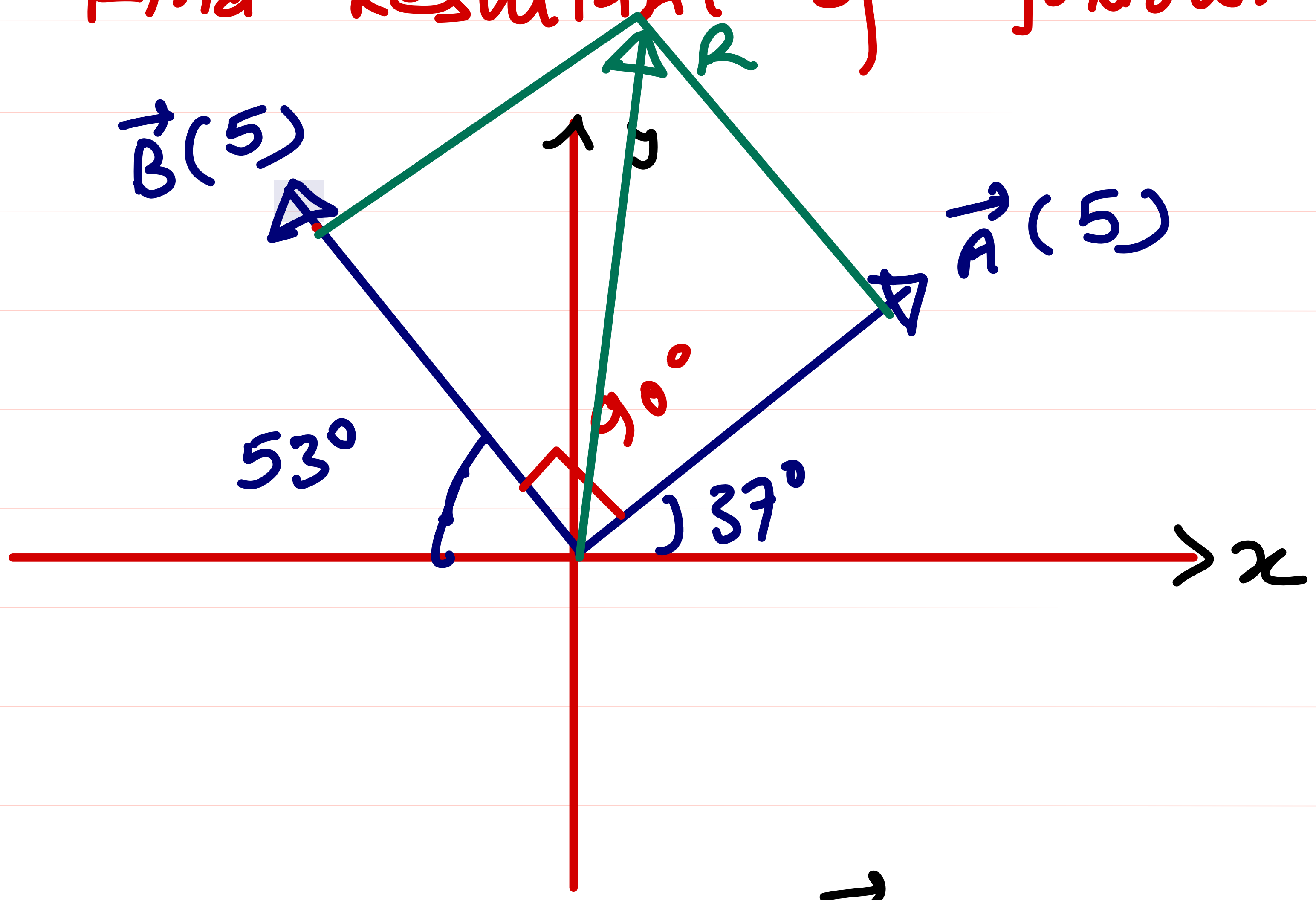
$$\tan\alpha = \frac{12}{5}$$

$$\boxed{\alpha = \tan^{-1}\left(\frac{12}{5}\right)}$$

From $\vec{A}$

Ex Find Resultant of following vectors in Given figures

(i)




$R = 2A\cos(\theta/2)$

$= 2\times5\cos\frac{90}{2}$

$= 2\times5\cos45$

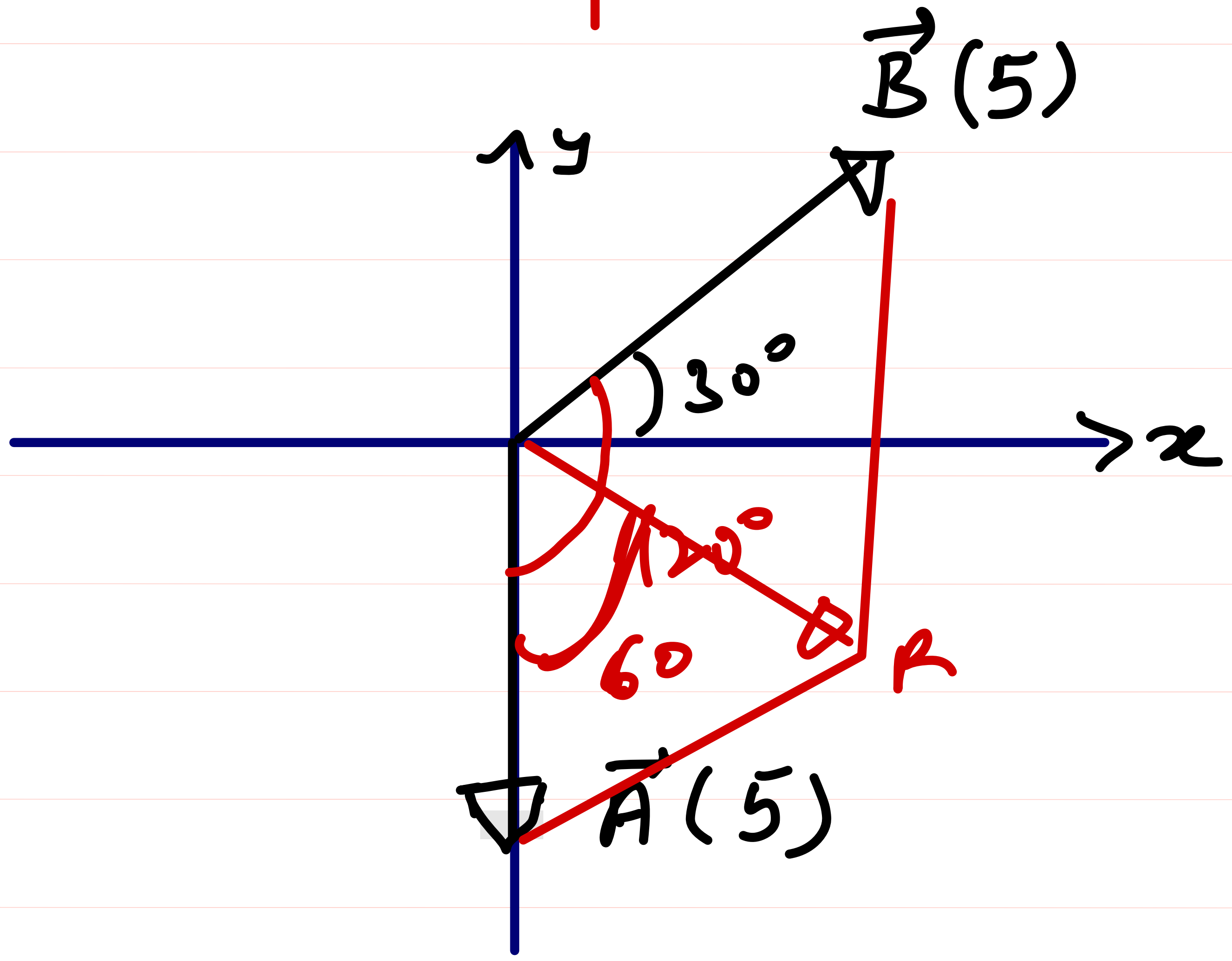
$R = 2\times5\times\frac{1}{\sqrt{2}}$

$\boxed{R = 5\sqrt{2}}$ Ans

$\alpha = \beta = \frac{\theta}{2} = \frac{90}{2} = 45^\circ$ from one of vector

$R = 5\sqrt{2}$, $(45^\circ + 37^\circ)$ from x-axis Ans

(ii)




$R = 2A\cos(\theta/2)$

$= 2\times5\cos\left(\frac{120}{2}\right)$

$= 2\times5\cos60$

$= 2\times5\times\frac{1}{2}$

$\boxed{R = 5}$

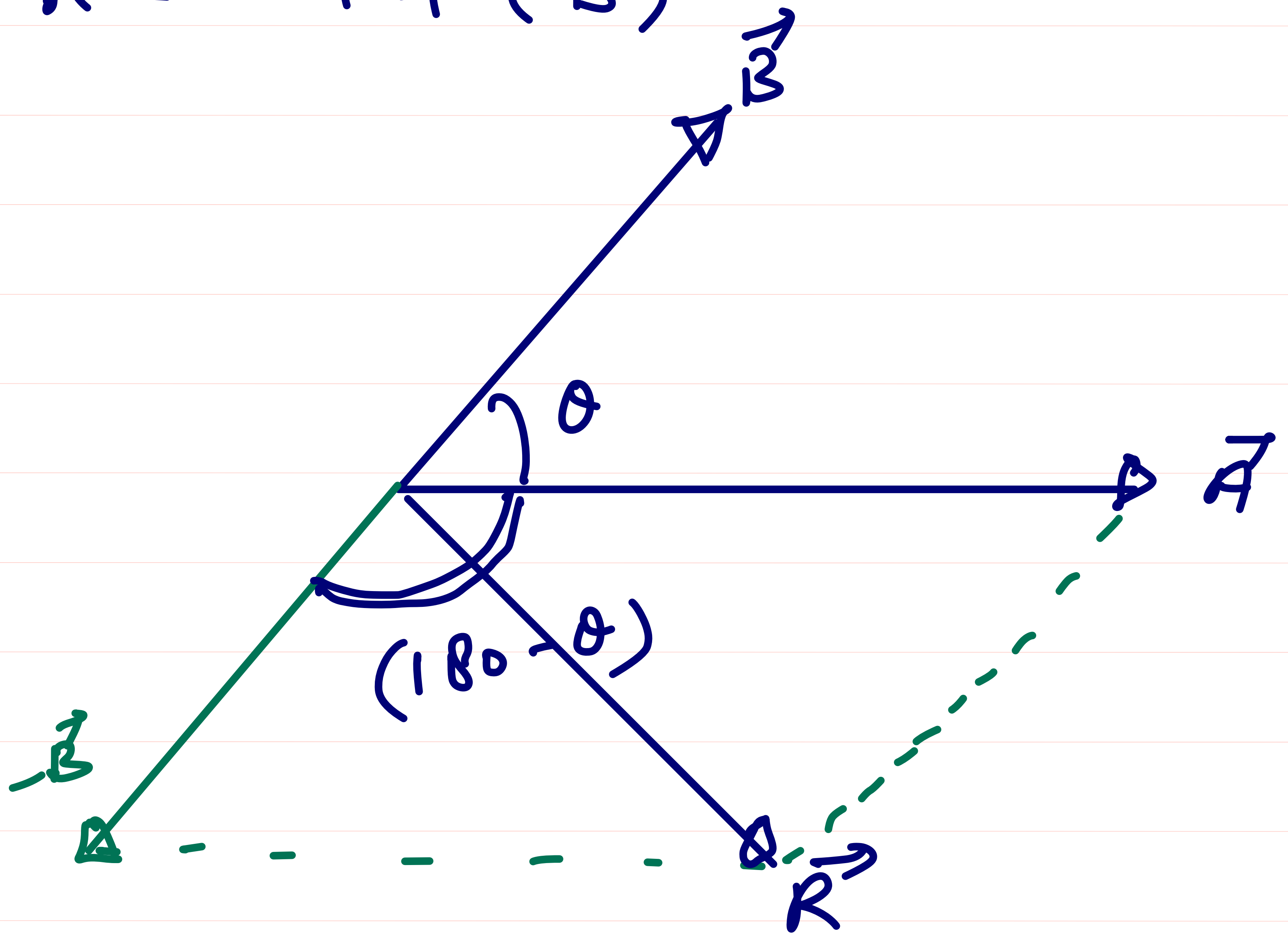
$\alpha = \beta = \frac{120}{2} = 60^\circ$ from one of the vector

$R = 5$, $30^\circ$ from +ve x-axis (c.w)

## Subtraction of Two vector's :→

$$\vec{R} = \vec{A} - \vec{B}$$

OR $$\vec{R} = \vec{A} + (-\vec{B})$$




$$R = \sqrt{A^2 + B^2 + 2AB\cos(180 - \theta)}$$

$$\boxed{R = \sqrt{A^2 + B^2 - 2AB\cos\theta}}$$

$$\tan\alpha = \frac{B\sin\theta}{A - B\cos\theta}$$

$$\tan\beta = \frac{A\sin\theta}{B - A\cos\theta}$$

Case If $A = B$

$$\boxed{R = 2A\sin(\theta/2)}$$

$$\boxed{\alpha = \beta = \left(\frac{180 - \theta}{2}\right) = \frac{\pi - \theta}{2}}$$

Range

$$\boxed{|A - B| \le R \le A + B}$$

**Ex** Which of the following values of Resultant of $\vec{A}(5)$ and $\vec{B}(3)$ are possible

(i) 3

(ii) 4

(iii) 4.5

(iv) 5

(v) 6

(vi) 8

(vii) 7.5

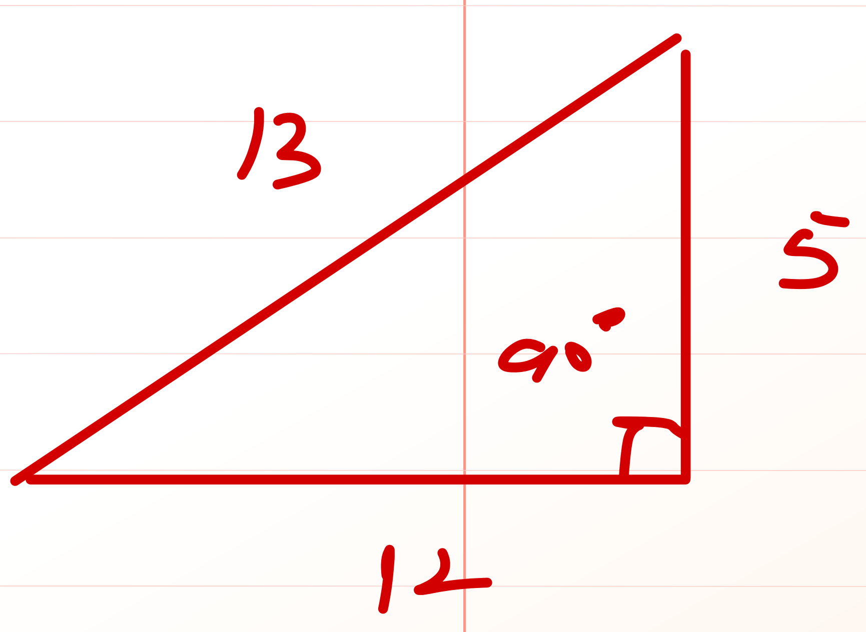
$$|5-3| \le R \le (5+3)$$

$$\boxed{2 \le R \le 8}$$

**Ex** The magnitudes of vectors **A**, **B** and **C** are respectively 12, 5 and 13 units and **A** + **B** = **C**. The angle between **A** and **B** is

(a) zero (b) $\pi$

(c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$




$$C^2 = A^2 + B^2 + 2AB\cos\theta$$

$$\cancel{13^2} = \cancel{5^2} + \cancel{12^2} + 2\times5\times12\cos\theta$$

$$0 = 2\times5\times12\cos\theta$$

$$\cos\theta = 0 \Rightarrow \theta = \frac{\pi}{2}$$

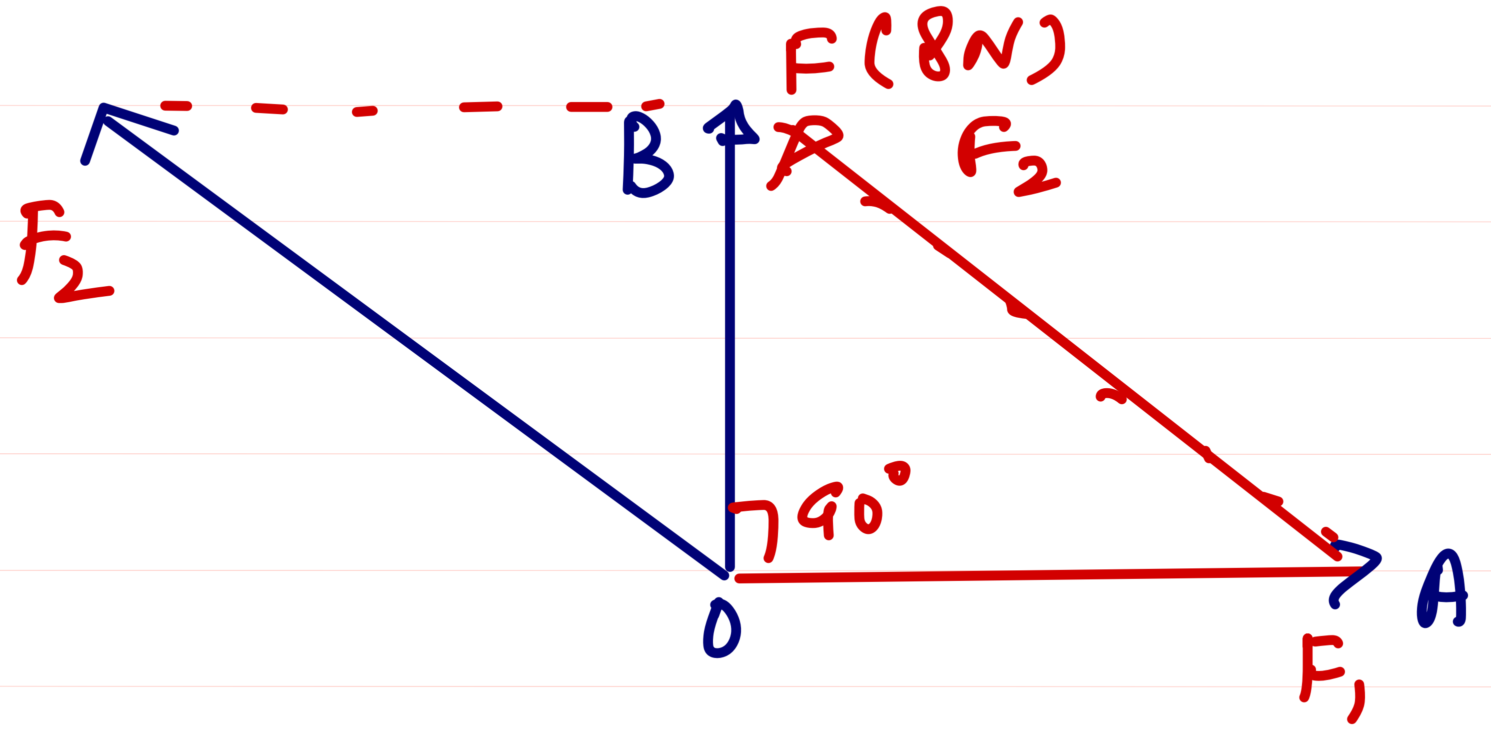
Ex

The sum of two forces acting at a point is 16 N. If the resultant force is 8 N and its direction is perpendicular to the smaller force, then the forces are

(a) 6 N and 10 N (b) 8 N and 8 N

(c) 4 N and 12 N (d) 2 N and 14 N

$F_1 + F_2 = 16\,N$ —(1)




From $\Delta OAB$

$F_2^2 = F^2 + F_1^2$

$F_2^2 - F_1^2 = F^2$

$(F_2 - F_1)(F_2 + F_1) = F^2$

$(F_2 - F_1) \times 16 = 8^2$

$(F_2 - F_1)\cancel{16} = \cancel{64}\ 4$

$F_2 - F_1 = 4$ —(2)

(1) + (2)

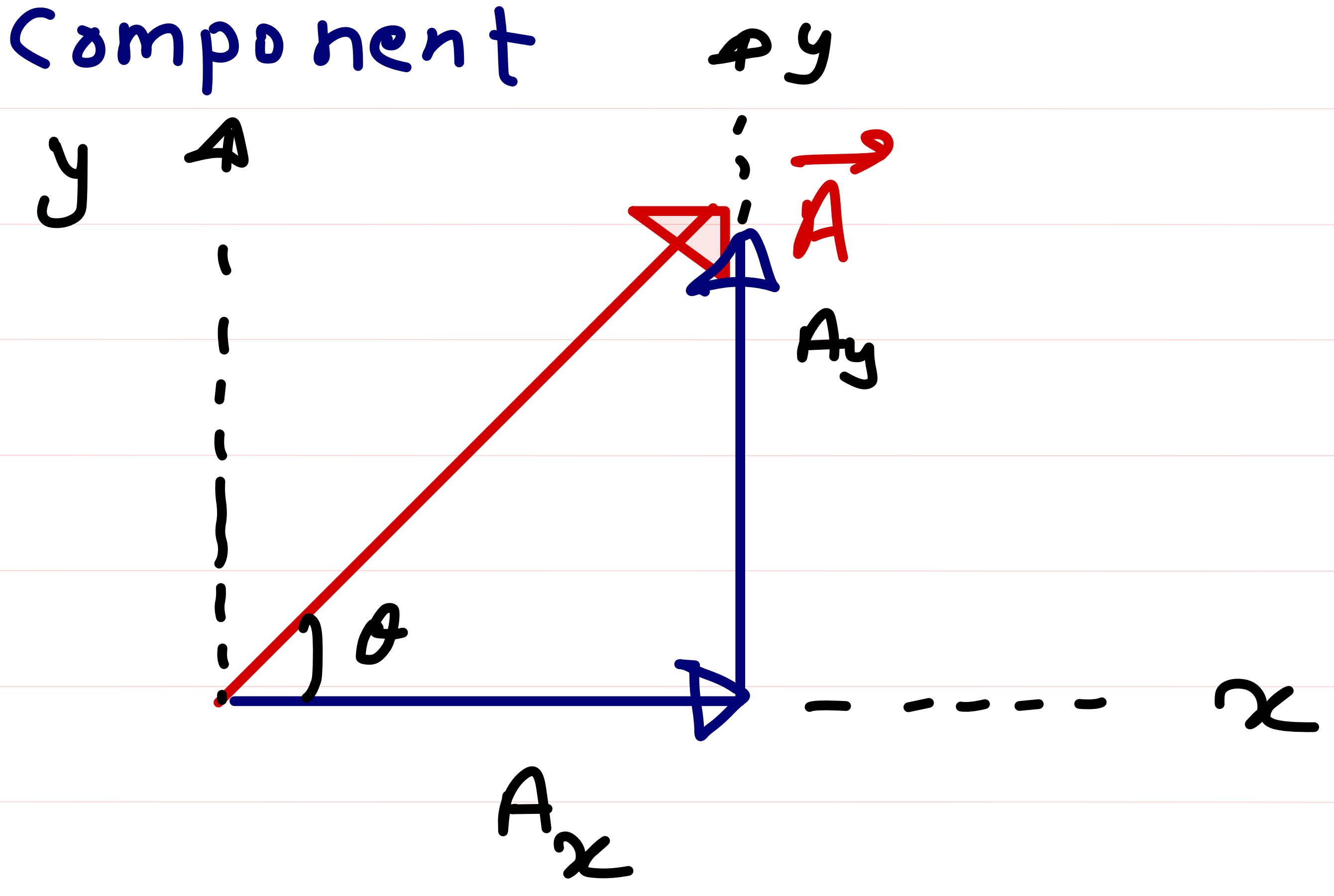
$2F_2 = 20$

$\boxed{F_2 = 10\,N}$

$\boxed{F_1 = 6\,N}$

# Component of A vector / Resolution of vector :→

## 2-D Component




$\cos\theta = \frac{A_x}{A} \Rightarrow$ $\boxed{A_x = A\cos\theta}$

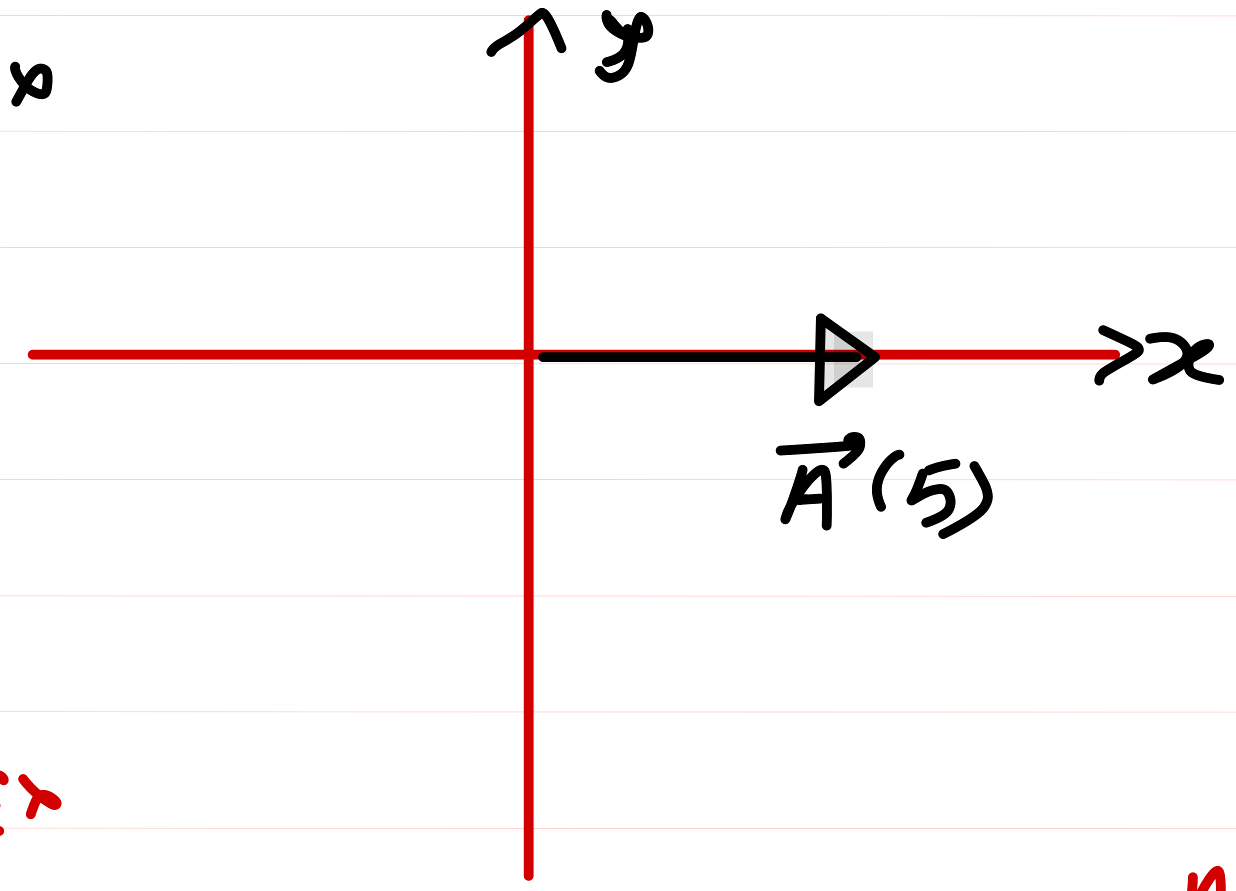
$\sin\theta = \frac{A_y}{A} \Rightarrow$ $\boxed{A_y = A\sin\theta}$

Here $A_x$ & $A_y$ are 2-D component of $\vec{A}$ vector

$\vec{A} = \vec{A}_x + \vec{A}_y$

$A = \sqrt{A_x^2 + A_y^2}$

Ex



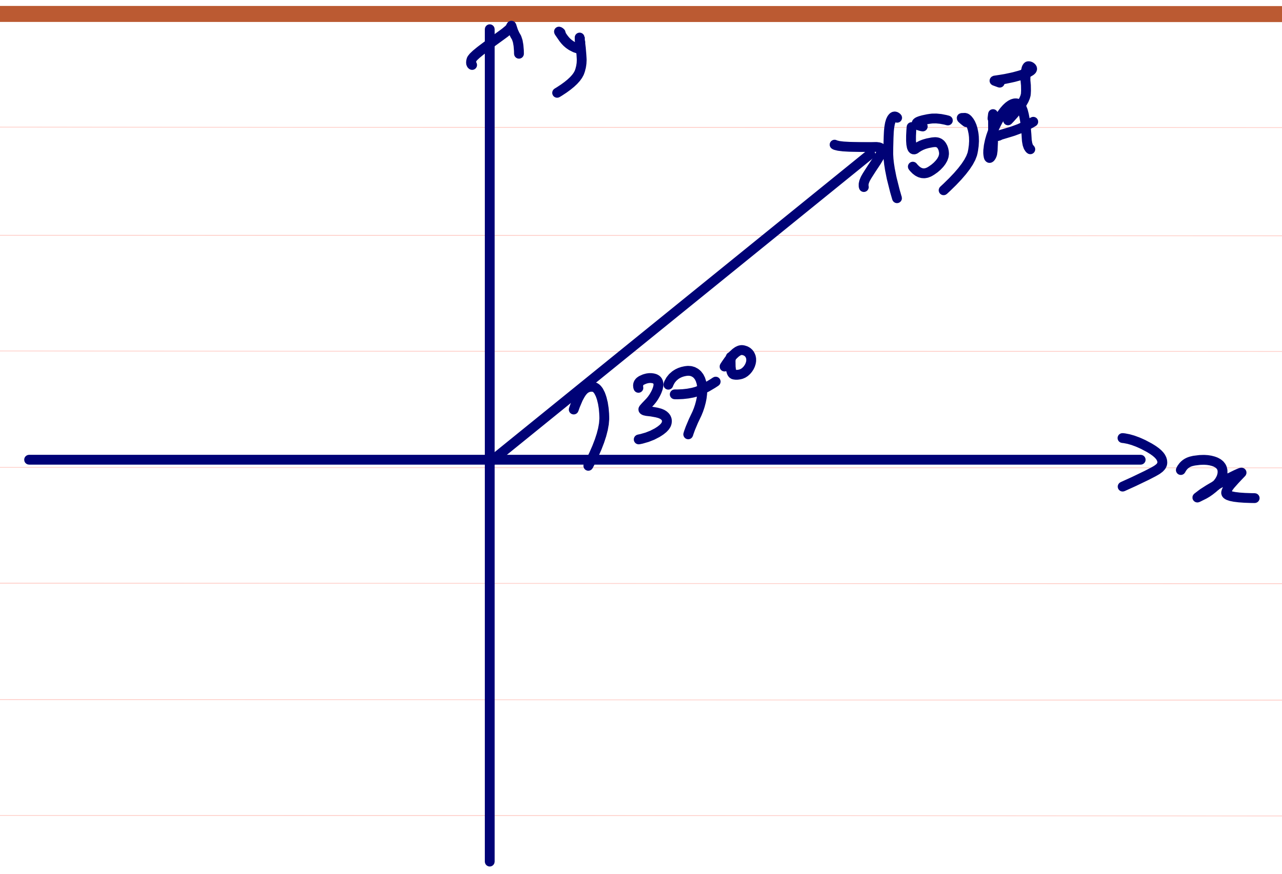

Find $A_x = 5\cos(0) = 5$

$A_y = 5\sin(0) = 0$

Ex

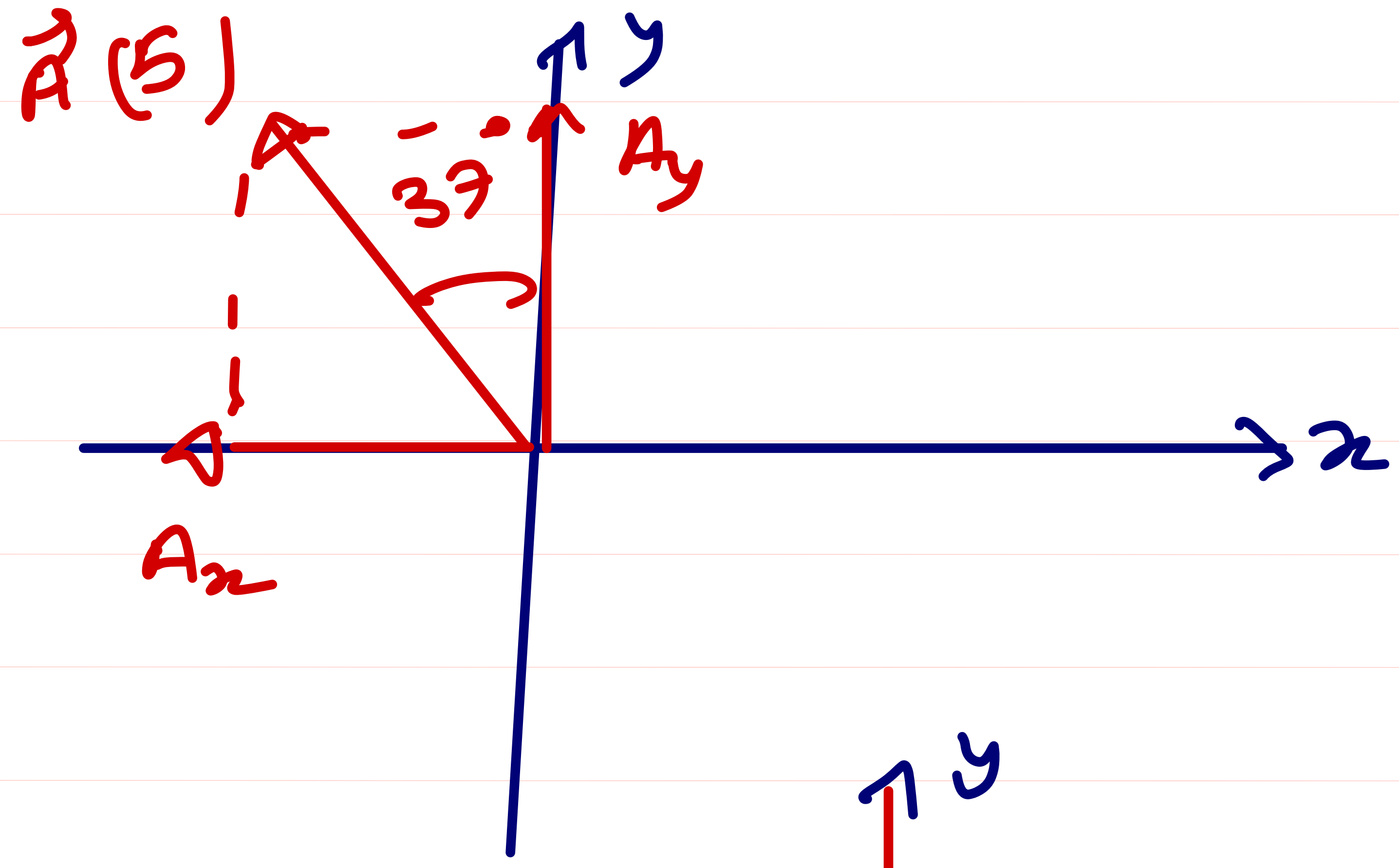

$A_x = 5\sin(0) = 0$

$A_y = 5\cos(0) = 5$



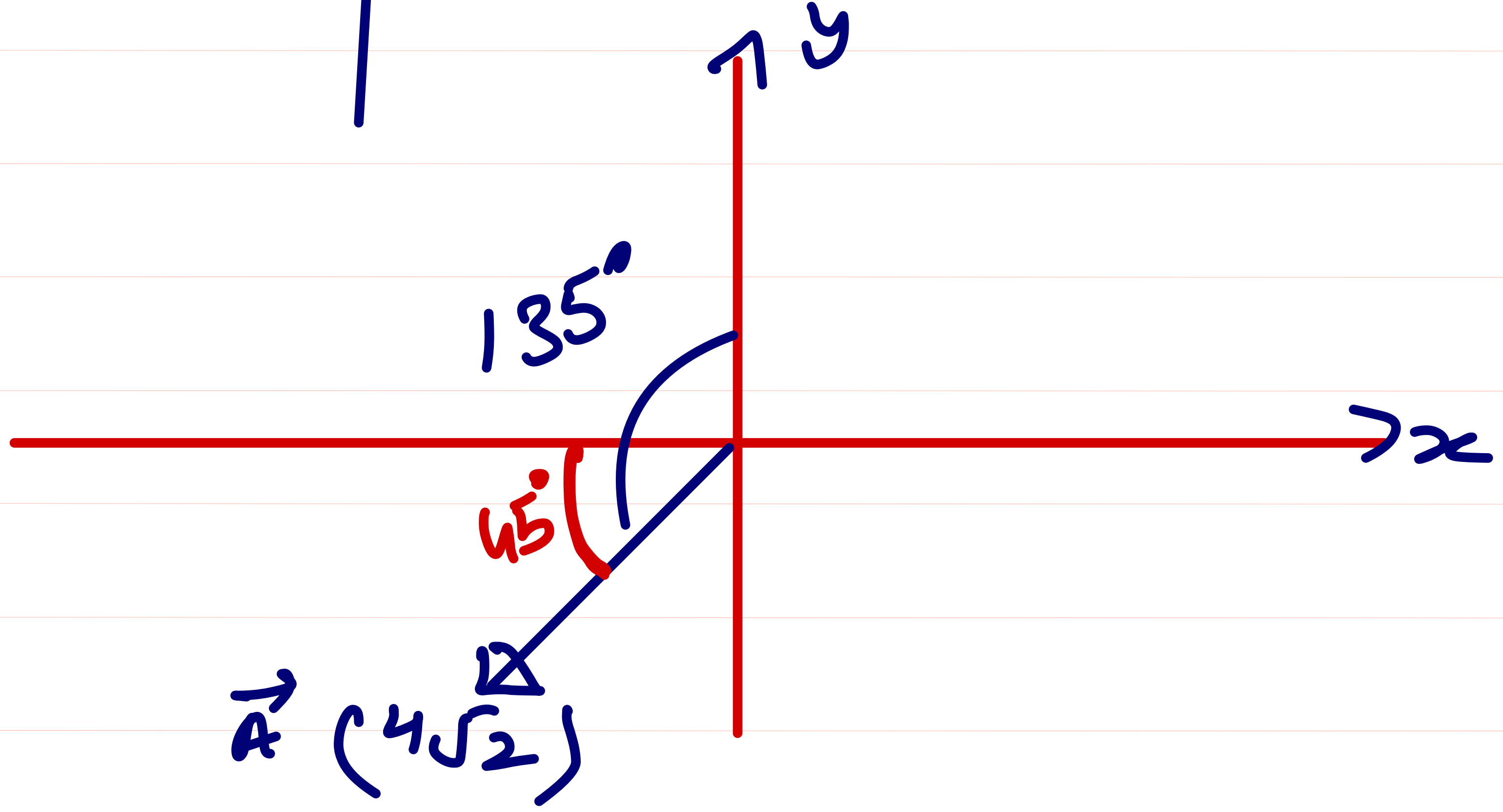

$A_x = A\cos\theta = 5\cos 37 = 5\times 4/5 = 4 \Rightarrow \vec{A}_x = 4\hat{i}$

$A_y = A\sin\theta = 5\sin 37 = 5\times 3/5 = 3 \Rightarrow \vec{A}_y = 3\hat{j}$




$A_y = 5\cos 37 = 5\times 4/5 = 4 \Rightarrow \vec{A}_y = 4\hat{j}$

$A_x = 5\sin 37 = 5\times 3/5 = 3 \Rightarrow \vec{A}_x = -3\hat{i}$




$A_x = 4\sqrt{2}\cos(45) = 4 \Rightarrow \vec{A}_x = -4\hat{i}$

$A_y = 4\sqrt{2}\sin(45) = 4 \Rightarrow \vec{A}_y = -4\hat{j}$

H.W

Ill - 38, 39

BB # 4

Q 1 to 10

Q # 3

Race # 6

Q - 1 to 10