

Principle of Homogeneity of dimension

Consider a simple equation,

$$A + B = C$$
.

If this is an equation of physics, i.e. if A, B and C are physical quantities, then this equation says that one physical quantity A, when added to another physical quantity B, gives a third physical quantity C. This equation will have no meaning in physics if the nature (i.e. the dimensions) of the quantities on the left-hand side of the equation is not the same as the nature of the quantity on the right-hand side. For example, if A is a length, B must also be a length and the result of addition of A and B must express a length. In other words, the dimensions of both sides of a physical equation must be identical. This is called the principle of homogeneity of dimensions.

According to this

Each and Every term in Physical Equation must have Some Dimensions"

NOTE: Same Dinensions Com be added or Subparted while Diff. Dimensions

Com be Divide & multiply

[4] = LB] = Lc2

If
$$AB = C$$
 Here $B2 + B3$
then $CAB = C$



USES OF DIMENSIONAL ANALYSIS

There are four important uses of dimensional equations:

- 1. Checking the correctness of an equation.
- 2. Derivation of the relationship between the physical quantities involved in any phenomenon.
- 3. Finding the dimensions of constants or variables in an equation.
- 4. Conversion of units from one system to another.

1.checking the correctness of an equation

To check the dimensional correctness of a given physical relation:

If in a given relation, the terms on both the sides have the same dimensions, then the relation is dimensionally correct. This is known as the *principle of homogeneity of dimensions*.

Ey-) Check the accuracy of the relation $T = 2\pi \sqrt{\frac{L}{g}}$ for a simple pendulum using dimensional analysis.

$$T = 2\pi \sqrt{\frac{L}{g}}$$
 L= length of Pendulum

 $g = Acc$. due to gravity

$$\begin{bmatrix} \sqrt{2} \\ \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \end{bmatrix}$$



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$$S = ut + 1 at^2$$
 $S = displacement$

$$\begin{bmatrix} 1 & 1 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} M^{\circ} L' T^{-2} \cdot T^{2} \end{bmatrix}$$
$$= \begin{bmatrix} M^{\circ} L' T^{\circ} \end{bmatrix}$$

Hence

$$(S) = [ut] = [at^2]$$

$$S_{nm} = u + \frac{1}{2}a(2n-1)$$
 Disp. in n^{m} second.



2. To derive relationship between different physical quantities:

Ex-1

The time period (t) of a simple pendulum may depend upon m the mass of the bob, l the length of the string and g the acceleration due to gravity. Find the dependence of t on m, l and g.

$$C = -\frac{1}{2}$$

Put

$$t \prec \sqrt{\frac{1}{g}} \Rightarrow t = \kappa \sqrt{\frac{g}{g}}$$



Find relationship between speed of sound in a medium (v), the elastic constant (E) and the density of the medium (ρ).

ve & Eagb

$$a_{1} + b_{1} = 0$$
 $- 0$ $+ a_{1} - 3b_{1} = 1 - 0$ $-1 = -2a \Rightarrow a_{1} = 1$

Area

[shess] =
$$\left[\frac{M'L'T^{-2}}{3}\right] = \left[\frac{M'L'T^{-2}}{3}\right]$$

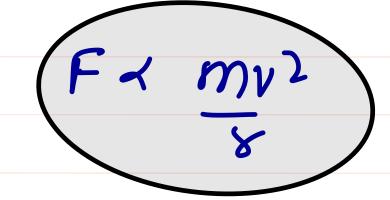
Strain => Dimension less quantity

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Centripetal force (F) on a body of mass (m) moving with uniform speed (v) in a circle of radius (r) depends upor m, v and r. Derive a formula for the centripetal force using theory of dimensions.

$$Q = 0$$



$$F = K \frac{mv^2}{8}$$

3. Finding the dimensions of constants and variables in an equation:

Ex-

The distance x travelled by a body varies with time t as

$$x = at + bt^2$$
, where a and b are constants.

Find the dimensions of a and b.

$$[n] = (at) \Rightarrow (a) = (\frac{2}{t}) = (M^{\circ}L^{\dagger}r^{-1})$$

 $[n] = (5t^{2}) \Rightarrow [b] = (\frac{2}{t^{2}}) = [M^{\circ}L^{\dagger}r^{-2}]$



The pressure P, volume V and temperature T of a gas are related as

$$\left(P + \frac{a}{V^2}\right)(V - b) = cT$$

where a, b, and c are constants. Find the dimensions

of
$$\frac{a}{b}$$
.

of
$$\frac{a}{b}$$
.
$$\left(P + \frac{9}{V^{2}}\right) \qquad \left(P\right) = \left(\frac{9}{V^{2}}\right)$$

$$\therefore [a] = (Pv^2)$$

$$\left[\frac{PV^2}{V}\right] = \left[\frac{PV}{V}\right]$$

$$= \left[\frac{M^{1} L^{1} - 2}{L^{2}} \times L^{3} \right]$$

$$\begin{bmatrix} a \\ - \end{bmatrix} = \begin{bmatrix} M' L^2 T - 2 \end{bmatrix}$$
Ans

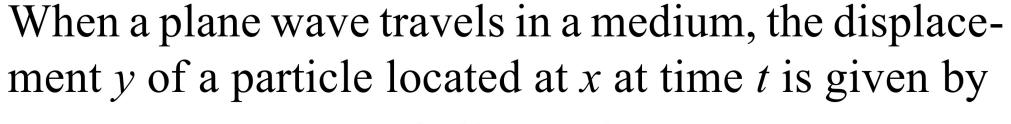
$$\begin{bmatrix} C \end{bmatrix} = \begin{bmatrix} PV \\ T \end{bmatrix}$$

$$\begin{bmatrix} C \end{bmatrix} = \begin{bmatrix} M^{1} L^{2} \Gamma^{-2} K^{-1} \end{bmatrix}$$



NOTE >

- 1. Trigonometric function (sin, cos, tan, cot etc) are dimensionless. The arguments of these functions are also dimensionless
- 2. Exponential functions are dimensionless. Their exponents are also dimensionless



$$y = a \sin(bt + cx)$$

where a, b and c are constants. Find the dimensions

of
$$\frac{b}{c}$$
.

Sin(8)
$$O = Dimensim less$$

$$e^{(2x+3)} \quad Dimensim less$$

$$Cos((2++3)) \quad Dimensionless$$

$$\frac{b}{c} = \frac{m^{\circ} L^{-1} \Gamma^{\circ}}{m^{\circ} L^{-1} \Gamma^{\circ}}$$

$$= L M^{\circ} L^{1} \Gamma^{-1} J$$

$$= 2 M^{\circ} L^{1} \Gamma^{-1} J$$



In the expression

$$P = \frac{a^2}{b} e^{-ax}$$

P is pressure, x is a distance and a and b are constants. Find the dimensional formula for b.

$$(P) = \left(\frac{4^2}{6}\right)$$

$$b = \left(\frac{q^2}{p}\right)$$

$$= \left[M^{\circ} L^{-1} T^{\circ}\right]$$

$$= \left[M^{\dagger} L^{-1} T^{-2}\right]$$