Trigonometric equations
CL03

(4) $2\sin^2 2x + 6\sin^2 x = 5$

$$2(1-\cos^2 2x) + 3(1-\cos 2x) = 5$$

$$-2\cos^2 2x - 3\cos 2x = 0$$

$$\cos 2x(-2\cos 2x - 3) = 0$$

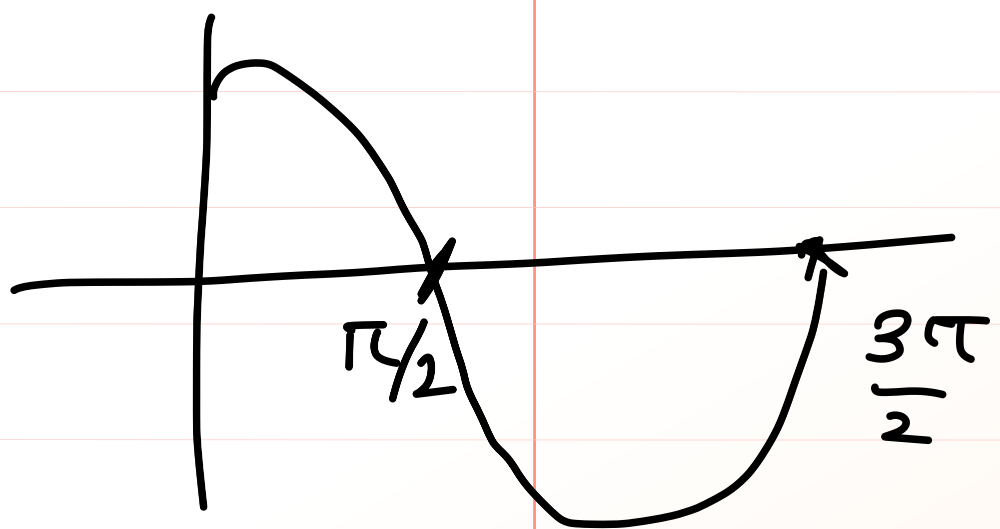
$\cos 2x = 0$ $\qquad$ $\cos 2x = -\frac{3}{2}$ (Rejected)

$$2x = (2n+1)\frac{\pi}{2} \quad \forall n \in \mathbb{Z}$$

$$\boxed{x = (2n+1)\frac{\pi}{4} \quad \forall n \in \mathbb{Z}.}$$

$1 - 2\sin^2 x = \cos 2x$

$2\sin^2 x = 1 - \cos 2x$



solving equations by Trigonometric formulae : →

① $\cos 3x + \underbrace{\sin 2x - \sin 4x}_{} = 0$

$\cos 3x + 2\cos 3x \sin(-x) = 0$

$\cos 3x\,(1 - 2\sin x) = 0$

$\cos 3x = 0$

$\cos 3x = 0$

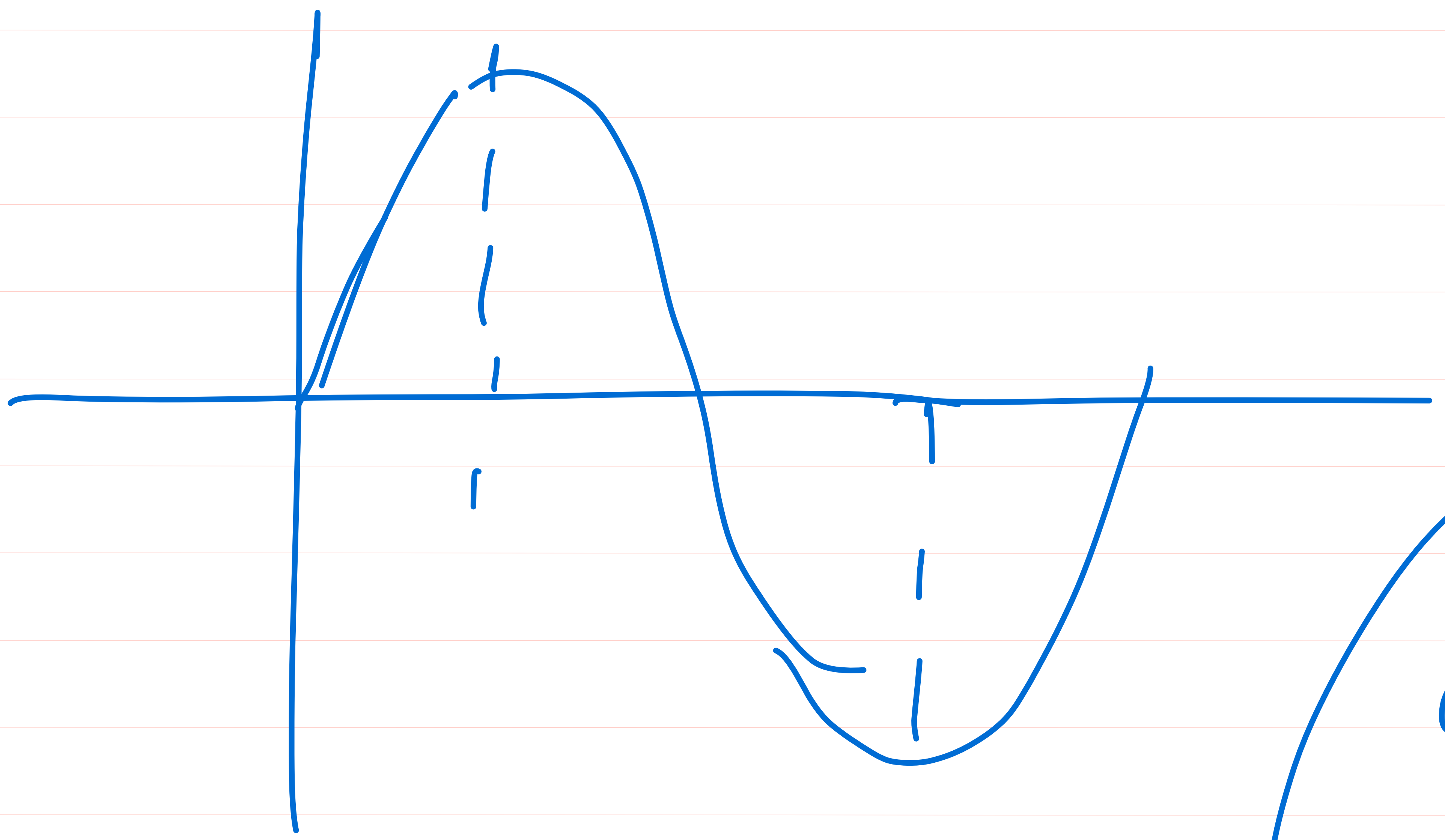
$3x = (2n+1)\frac{\pi}{2}$

$\boxed{x = (2n+1)\frac{\pi}{6}}$

$1 - 2\sin x = 0$

$\sin x = \frac{1}{2}$

$\boxed{x = m\pi + (-1)^m \cdot \frac{\pi}{6}}$ $\forall\, m, n \in Z$



slide 4, 5, 6, 7

$\sin x = 0, 1, -1$

$\cos x = 0, 1, -1$

$\tan x = 0$

$\cot x = 0$

Besides above values

use

$\sin \rightarrow n\pi + (-1)^n \alpha$

$\cos \rightarrow 2n\pi \pm \alpha$

$\tan \rightarrow n\pi + \alpha$

② find no. of solutions in $[0, \pi]$;

$$\sin 3\theta = 4\sin\theta \cdot \sin 2\theta \cdot \sin 4\theta$$

$$\sin 3\theta = 4\sin\theta \cdot \sin(3\theta-\theta)\sin(3\theta+\theta)$$

$$3\sin\theta - 4\sin^3\theta = 4\sin\theta \cdot [\sin^2 3\theta - \sin^2\theta]$$

$$3\sin\theta \cancel{-4\sin^3\theta} = 4\sin\theta \cdot \sin^2 3\theta \cancel{- 4\sin^3\theta}$$

$$3\sin\theta - 4\sin\theta \cdot \sin^2 3\theta = 0$$

$$\sin\theta(3 - 4\sin^2 3\theta) = 0$$

$$\sin\theta = 0$$

$$\theta = n\pi$$

$$\boxed{\theta \in \{0, \pi\}}$$

$$\sin^2 3\theta = \frac{3}{4} \Rightarrow \sin^2 3\theta = \sin^2\left(\frac{\pi}{3}\right)$$

$$3\theta = n\pi \pm \frac{\pi}{3}$$

$$\theta = \frac{\pi}{9}, \frac{2\pi}{9}, \frac{4\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}, \frac{8\pi}{9}$$

at $n=0$

$n=1$

$$3\theta = \pi \pm \frac{\pi}{3}$$

$$3\theta = \frac{4\pi}{3}, \frac{2\pi}{3}$$

$$\boxed{\theta = \frac{4\pi}{9}, \frac{2\pi}{9}}$$

$n=2$

$$3\theta = 2\pi \pm \frac{\pi}{3}$$

$$= \frac{7\pi}{3}, \frac{5\pi}{3}$$

$$\boxed{\theta = \frac{7\pi}{9}, \frac{5\pi}{9}}$$

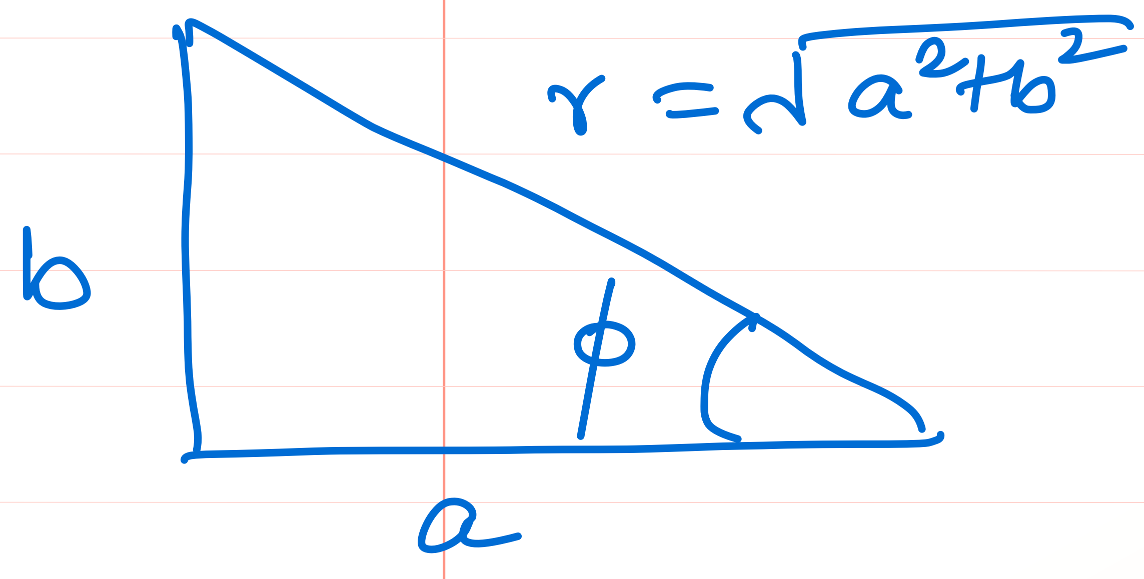
$n=3$

$$3\theta = 3\pi \pm \frac{\pi}{3}$$

$$\boxed{\theta = \frac{8\pi}{9}}$$

Type-3 Solving trigo equations by introducing auxiliary argument :→

$$\frac{a\cos\theta}{\sqrt{a^2+b^2}} + \frac{b\sin\theta}{\sqrt{a^2+b^2}} = \frac{c}{\sqrt{a^2+b^2}}$$



$$\cos\phi = \frac{a}{\sqrt{a^2+b^2}}$$

$$\sin\phi = \frac{b}{\sqrt{a^2+b^2}}$$

$$\cos\phi\cos\theta + \sin\phi\sin\theta = \frac{c}{\sqrt{a^2+b^2}}$$

$$\cos(\theta-\phi) = \frac{c}{\sqrt{a^2+b^2}}$$

$$-1 \le \frac{c}{\sqrt{a^2+b^2}} \le 1$$

$$\left|\frac{c}{\sqrt{a^2+b^2}}\right| \le 1 \Rightarrow \boxed{|c| \le \sqrt{a^2+b^2}}$$

Note :→ If $|c| > \sqrt{a^2+b^2}$ then the given equation has no real solution.

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(1) $1 \cdot \sin x + 1 \cos x = \sqrt{2}$

$a = 1; \quad b = 1$

$\sqrt{a^2+b^2} = \sqrt{1+1} = \sqrt{2}$

divide by $\sqrt{2}$ to both sides

$$\frac{1 \cdot \sin x}{\sqrt{2}} + \frac{1 \cdot \cos x}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{2}}$$

$$\sin\left(\frac{\pi}{4}\right) \cdot \sin x + \cos\left(\frac{\pi}{4}\right) \cdot \cos x = 1$$

$$\cos\left(x - \frac{\pi}{4}\right) = 1$$

$$x - \frac{\pi}{4} = 2n\pi$$

$$x = 2n\pi + \frac{\pi}{4} \quad \forall \; n \in \mathbb{Z}$$

(2) $\sqrt{3}\cos x + \sin x = 2$

$a = \sqrt{3}\,;\; b = 1$

$\sqrt{a^2+b^2} = \sqrt{3+1} = 2$

$\frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x = \frac{2}{2}$

$\cos\left(\frac{\pi}{6}\right)\cos x + \sin\left(\frac{\pi}{6}\right)\sin x = 1$

$\cos\left(x - \frac{\pi}{6}\right) = 1$

$x - \frac{\pi}{6} = 2n\pi$

$\boxed{x = 2n\pi + \frac{\pi}{6} \;\; \forall n \in Z}$

(3) $1\sin x + 1\cos x = 1.5$

$a = 1\,;\; b = 1$

$\sqrt{a^2+b^2} = \sqrt{2}$

$\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x = \frac{1.5}{\sqrt{2}}$

$\sin\frac{\pi}{4}\cdot\sin x + \cos\frac{\pi}{4}\cdot\cos x = \frac{1.5}{\sqrt{2}}$

$\cos\left(x - \frac{\pi}{4}\right) = \boxed{\frac{1.5}{\sqrt{2}}} \downarrow\; > 1$

No solution

$x \in \phi$

Q4 $4\cos x + 3\sin x = 5$

$a = 4;\quad b = 3$

$$\sqrt{a^2+b^2} = \sqrt{4^2+3^2} = 5$$

$$\frac{4}{5}\cos x + \frac{3}{5}\sin x = 1$$

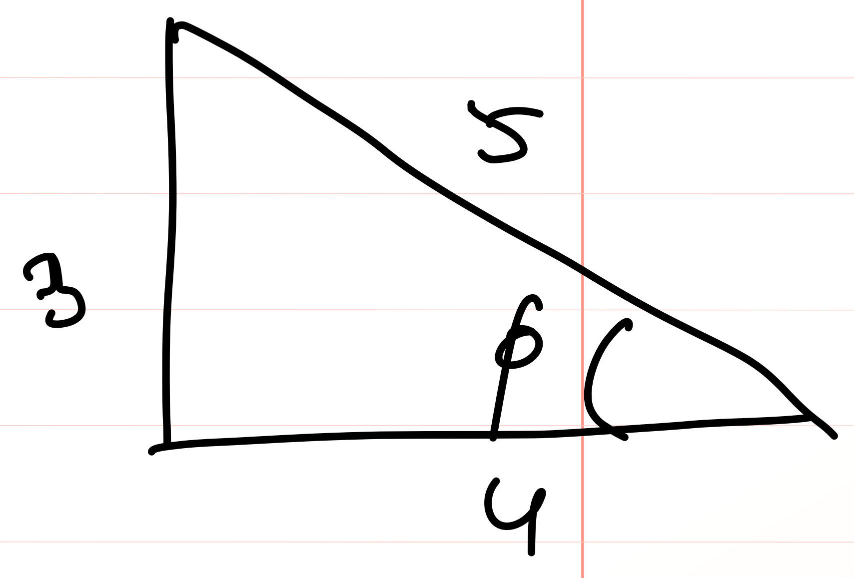
$$\cos\phi\cos x + \sin\phi\sin x = 1$$

$$\cos(x-\phi) = 1$$

$$x - \phi = 2n\pi$$

$$x = 2n\pi + \phi$$

$$\boxed{x = 2n\pi + \tan^{-1}\left(\frac{3}{4}\right)}$$



$$\cos\phi = \frac{4}{5}$$

$$\sin\phi = \frac{3}{5}$$

$$\tan\phi = \frac{3}{4}$$

$$\phi = \tan^{-1}\frac{3}{4}$$

(5) $1+\sin^3 x+\cos^3 x=\frac{3}{2}\sin 2x$

$a^3+b^3+c^3=3abc$

$1^3+(\sin x)^3+(\cos x)^3=\frac{3}{\cancel{2}}\cdot\cancel{2}\sin x\cos x\cdot 1$

$1^3+(\sin x)^3+(\cos x)^3=3\sin x\cos x(1)$

$1+\sin x+\cos x=0$ or $\boxed{1=\sin x=\cos x \quad \sin x=1}$

$\sin x+\cos x=-1$

$\frac{1}{\sqrt{2}}\sin x+\frac{1}{\sqrt{2}}\cos x=-\frac{1}{\sqrt{2}}$

$\cos\left(x-\frac{\pi}{4}\right)=\cos\left(\frac{3\pi}{4}\right)$

$x=2n\pi\pm\frac{3\pi}{4}+\frac{\pi}{4}$

+ sign $\boxed{x=2n\pi+\pi}$

− sign $\boxed{x=2n\pi-\frac{\pi}{2}}$