Trigonometric equations
CL01

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Three Types of solutions :→

(1) Principal solution :→ $0 \leq \theta \leq 2\pi$

(2) Particular solution :→ Solutions lying in the given interval.

(3) General solution :→ solution in the form of n.

Type I : →

① $\sin\theta = 0$

$\theta = n\pi \quad n \in \mathbb{Z}$

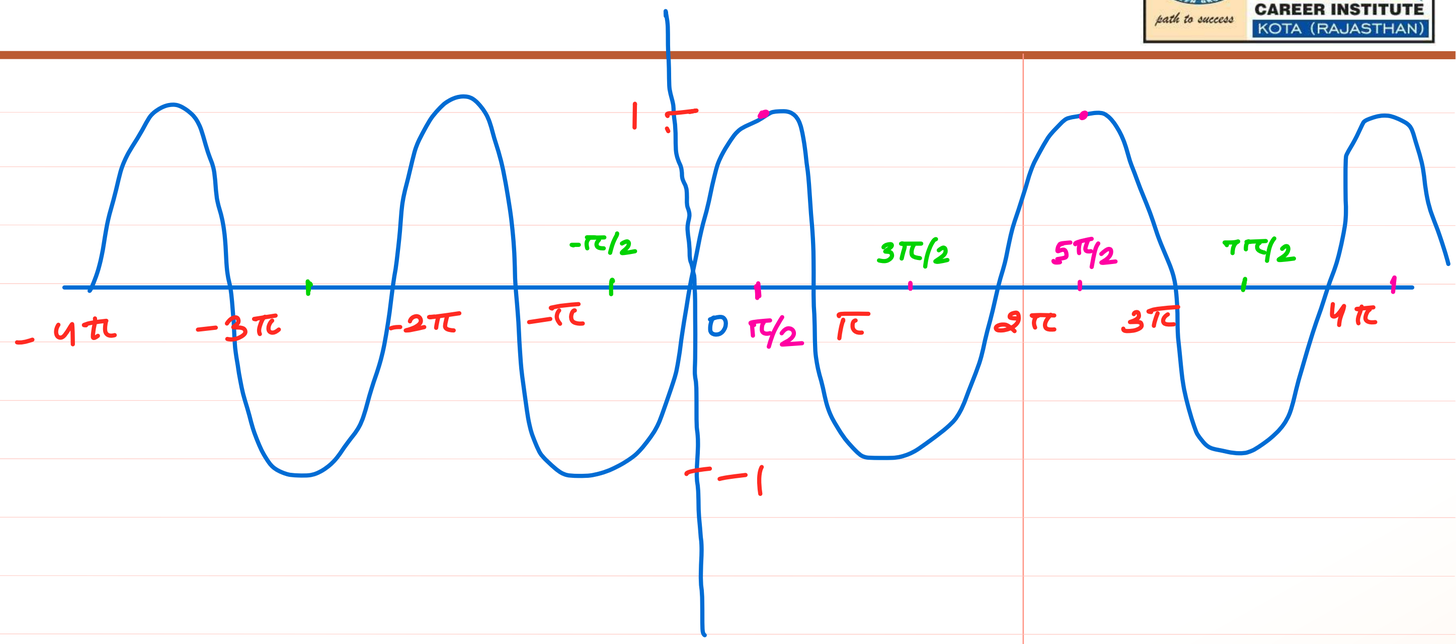
② $\sin\theta = 1$

$\theta = \frac{\pi}{2}, \underbrace{\frac{5\pi}{2}, \frac{9\pi}{2}, \ldots}$

$\theta = \frac{\pi}{2}(4n+1) \; ; \; n \in \mathbb{Z}$

OR

$\theta = \frac{\pi}{2}(4n-3) \; ; \; n \in \mathbb{Z}.$



1, 5, 9, ---

$T_n = 1 + (n-1)4$

$= 4n - 3$

5, 9, ---

$T_n = 5 + (n-1)4$

$= (4n+1)$

③ $\sin\theta = -1$

$\theta = \frac{3\pi}{2}, \frac{7\pi}{2}, \frac{11\pi}{2}$

$\theta = \frac{\pi}{2}(4n-1)\ ;\ n \in z$

$3, 7, 11, - - -$

$T_n = 3 + (n-1)4$

$T_n = 4n - 1$

(4) $\cos\theta = 0$

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \ldots$

$\theta = (2n+1)\frac{\pi}{2}$; $n \in Z$

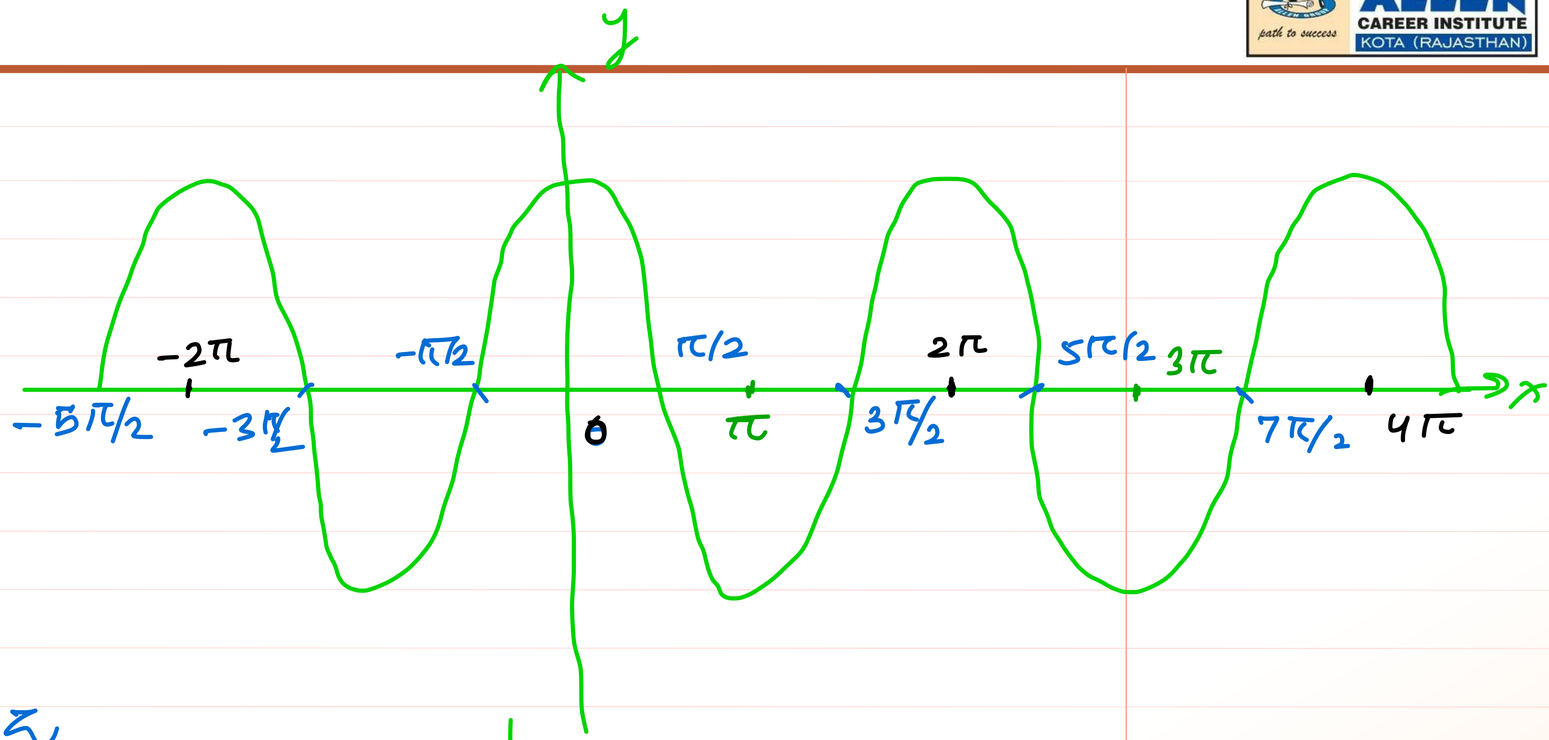
or

$\theta = (2n-1)\frac{\pi}{2}$; $n \in Z$

(5) $\cos\theta = 1$

$\theta = 0, 2\pi, 4\pi, 6\pi, 8\pi, \ldots$

$\theta = 2n\pi$; $n \in Z$



(6) $\cos\theta = -1$

$\theta = (2n+1)\pi$; $n \in Z$

or

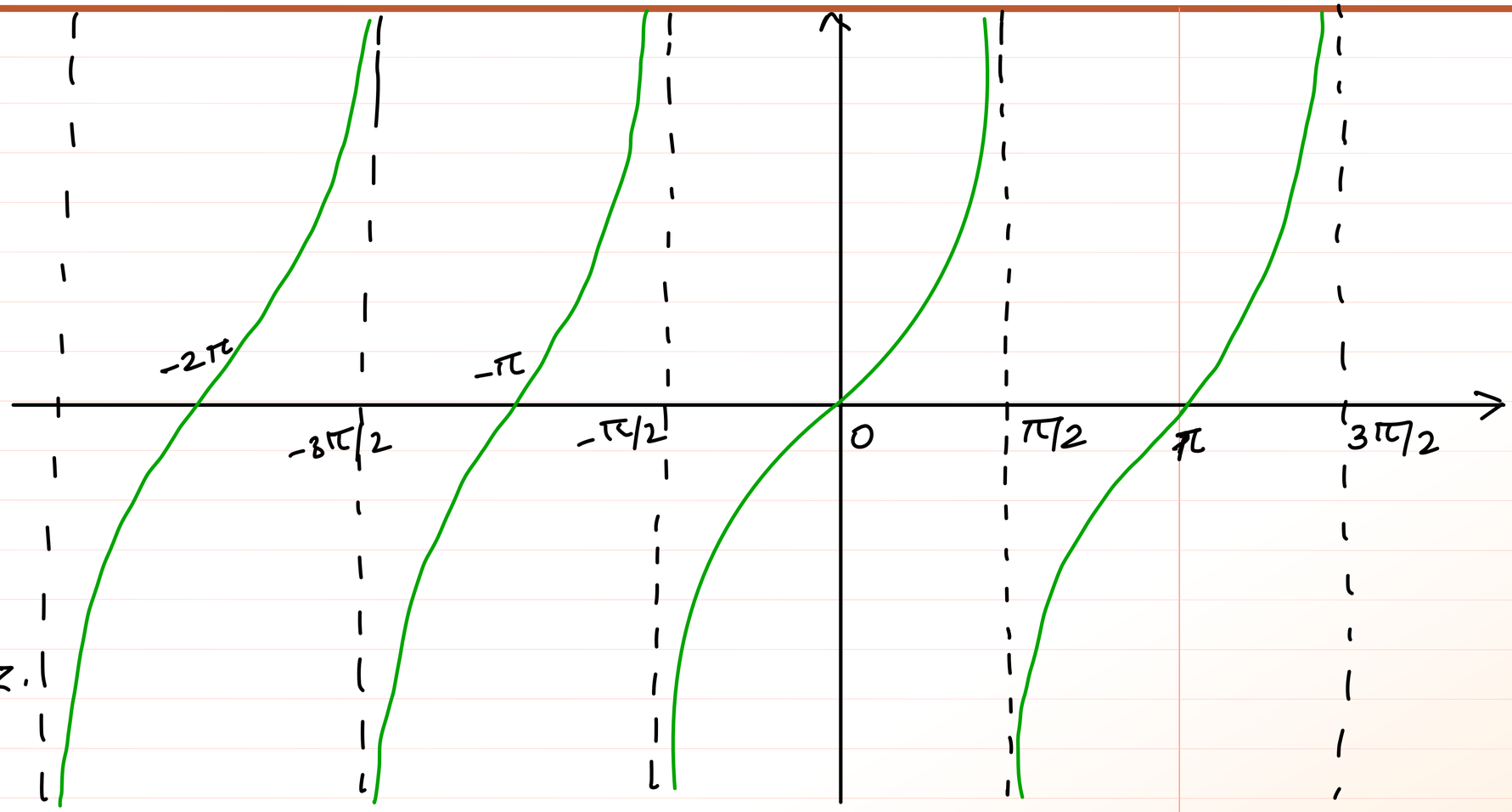
$\theta = (2n-1)\pi$; $n \in Z$

(7) $\tan\theta = 0$

$\theta \in n\pi\ ;\ n \in Z$

(8) $\cot\theta = 0$

$\theta \in (2n+1)\frac{\pi}{2}\ ;\ n \in Z.$



Type 2 (1) If $\sin\theta = \sin\alpha$ $\quad \alpha \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$$\theta = n\pi + (-1)^n \alpha \quad ; n \in Z$$

$\sin\theta - \sin\alpha = 0$

$2\cos\left(\frac{\theta+\alpha}{2}\right) \cdot \sin\left(\frac{\theta-\alpha}{2}\right) = 0$

$\cos\left(\frac{\theta+\alpha}{2}\right) = 0$

$\frac{\theta+\alpha}{2} = (2m+1)\frac{\pi}{2} \quad ; m \in Z$

$\theta + \alpha = (2m+1)\pi$

$\theta = (2m+1)\pi - \alpha$

$\theta = (2m+1)\pi + (-1)\alpha$

$\theta = (2m+1)\pi + (-1)^{2m+1} \cdot \alpha$

$\sin\left(\frac{\theta-\alpha}{2}\right) = 0$

$\frac{\theta-\alpha}{2} = m\pi$

$\theta - \alpha = 2m\pi$

$\theta = 2m\pi + \alpha$

$\theta = 2m\pi + (-1)^{2m} \cdot \alpha$

$\theta = n\pi + (-1)^n \alpha \quad n \in Z$

(1) Solve $2\cos^2\theta + 3\sin\theta = 0$

$2(1-\sin^2\theta) + 3\sin\theta = 0$

$2 - 2\sin^2\theta + 3\sin\theta = 0$

$2\sin^2\theta - 3\sin\theta - 2 = 0$

$2\sin^2\theta - 4\sin\theta + \sin\theta - 2 = 0$

$(2\sin\theta + 1)(\sin\theta - 2) = 0$

$\sin\theta = -\frac{1}{2}$ $\qquad$ $\sin\theta = 2$ ✗

$\sin\theta = \sin\left(-\frac{\pi}{6}\right)$

$\theta = n\pi + (-1)^n\left(-\frac{\pi}{6}\right)$

$$\theta = n\pi + (-1)^{n+1}\left(\frac{\pi}{6}\right) \quad ; \quad n \in \mathbb{Z}$$

(2) $\cos\theta = \cos\alpha$ $\quad \alpha \in [0, \pi]$

$\theta = 2n\pi \pm \alpha \quad n \in Z$

$\cos\theta - \cos\alpha = 0$

$-2\sin\left(\frac{\theta+\alpha}{2}\right).\sin\left(\frac{\alpha+\theta}{2}\right) = 0$

$\sin\left(\frac{\theta+\alpha}{2}\right) = 0$

$\left(\frac{\theta+\alpha}{2}\right) = n\pi$

$\theta + \alpha = 2n\pi$

$\theta = 2n\pi - \alpha$

$\sin\left(\frac{\theta-\alpha}{2}\right) = 0$

$\frac{\theta-\alpha}{2} = n\pi$

$\theta - \alpha = 2n\pi$

$\theta = 2n\pi + \alpha$

$\boxed{\theta = 2n\pi \pm \alpha} \quad n \in Z$

(3) $\tan\theta = \tan\alpha \qquad \alpha \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\theta = n\pi + \alpha \quad ; n \in \mathbb{Z}$

$\tan\theta = \tan\alpha$

$\frac{\sin\theta}{\cos\theta} = \frac{\sin\alpha}{\cos\alpha}$

$\sin\theta\cos\alpha = \sin\alpha\cos\theta$

$\sin\theta\cos\alpha - \sin\alpha\cos\theta = 0$

$\sin(\theta-\alpha) = 0$

$\theta - \alpha = n\pi$

$\theta = n\pi + \alpha$ $\qquad n \in \mathbb{Z}.$

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$$\tan 3\theta = -1$$

$$\tan 3\theta = \tan\left(-\frac{\pi}{4}\right)$$

$$3\theta = n\pi + \left(-\frac{\pi}{4}\right)$$

$$3\theta = n\pi - \frac{\pi}{4}$$

$$\boxed{\theta = \frac{n\pi}{3} - \frac{\pi}{12}} \quad n \in Z.$$

Answer

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$$\sqrt{3}\sec 2\theta = 2$$

$$\cos 2\theta = \frac{\sqrt{3}}{2}$$

$$\cos 2\theta = \cos\left(\frac{\pi}{6}\right)$$

$$2\theta = 2n\pi \pm \frac{\pi}{6}$$

$$\theta = n\pi \pm \frac{\pi}{12} \quad ; \quad n \in Z$$