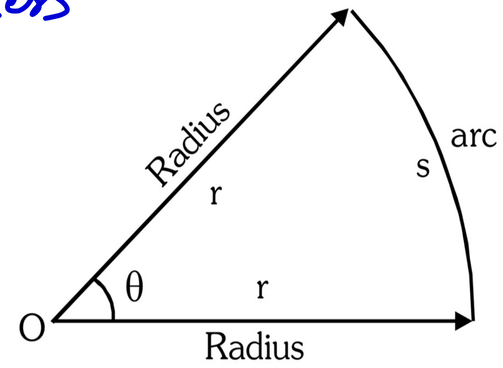
TRIGONOMETRY

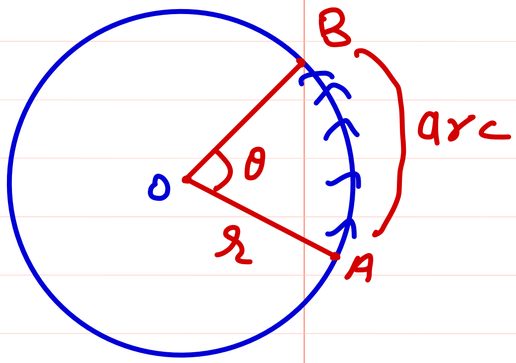
Angle: (Plane Angle)

it is measure of change in direction.

→ Dimensionless

$$\text{Angle } (\theta) = \frac{\text{Arc}(s)}{\text{Radius}(r)}$$





Angels measured in anticlockwise and clockwise direction are usually taken positive and negative respectively.

Supp. SI unit → Radian

θ = Ang. Seperation blw A B

θ = plane Angle

θ = Angular displacement.

System of Measurement of an Angle

Sexagesimal system

In this system, angle is measured in degrees.

In this system, 1 right angle = 90°, 1° = 60′ (arc minutes), 1′ = 60″ (arc seconds)

from A to B → ACW → θ → +ve

from B to A → C.W. → θ → −ve

$1' = \left(\frac{1}{60}\right)^\circ$ $\quad 1'' = \left(\frac{1}{60}\right)' = \left(\frac{1}{60}\times\frac{1}{60}\right)^\circ$

$\left\{\theta = \frac{\ell_{AB}}{r}\right\}$

Unit → Radian (rad)

Circular system

In this system, angle is measured in radian.

if arc = radius then $\theta = 1$ rad

Relation between degrees and radian

2π rad $= 360°$

π rad $= 180° \Rightarrow 1 \text{ rad} = \dfrac{180°}{\pi} = 57.3°$

$\pi = 3.14 = \dfrac{22}{7}$

π rad $= 180°$

$60° \rightarrow$ radian

$180° \longrightarrow \pi$ rad

$1° \longrightarrow \left(\dfrac{\pi}{180}\right)$ rad

$\dfrac{\pi}{180} \times 60 = \dfrac{\pi}{3}$ rad

(1) To convert from degree to radian multiply by $\dfrac{\pi}{180°}$

(2) To convert from radian to degree multiply by $\dfrac{180°}{\pi}$

H.W.

Illustration #7,8,9

Convert into radian

(1) 1 degree $\Rightarrow \frac{\pi}{180}$ rad

(2) $60^\circ \rightarrow \frac{\pi}{180} \times 60 = \frac{\pi}{3}$ rad

(3) $45^\circ \rightarrow \frac{\pi}{180} \times 45 = \frac{\pi}{4}$ rad

(4) $90^\circ \rightarrow \frac{\pi}{180} \times 90 \rightarrow \frac{\pi}{2}$ rad

(5) $120^\circ \rightarrow \frac{\pi}{180} \times 120 \rightarrow \frac{2\pi}{3}$ rad

(6) $135^\circ \rightarrow \frac{\pi}{180} \times 135 \rightarrow \frac{3\pi}{4}$ rad

(7) $150^\circ \rightarrow \frac{\pi}{180} \times 150 \rightarrow \frac{5\pi}{6}$ rad

(8) $270^\circ \rightarrow \frac{\pi}{180} \times 270 \rightarrow \frac{3\pi}{2}$ rad

convert radian into degree

① $\frac{5\pi}{4} = \frac{5}{4} \times 180 = 225^\circ$

② $\frac{7\pi}{2} = 630^\circ$

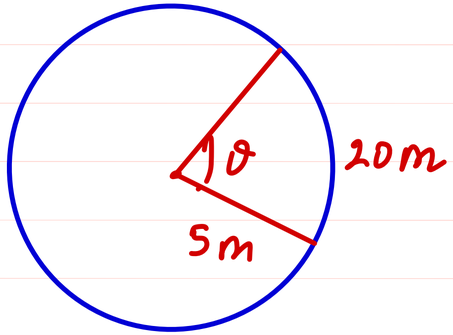
③ $\frac{2\pi}{6} = 60^\circ$

④ $\frac{3\pi}{4} = 135^\circ$

⑤ 3 rad $\Rightarrow 3 \times \frac{180}{\pi} = \left(\frac{540}{\pi}\right)^\circ$

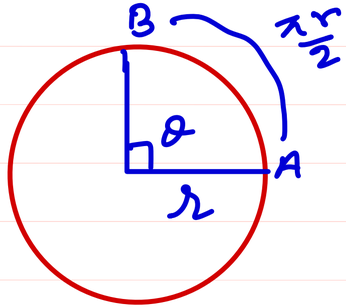
Ex Ans. following Qus.

(i)



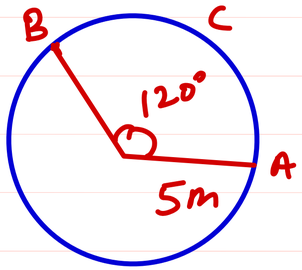
$\theta = ?$ $\Rightarrow \theta = \frac{20}{5} = 4$ rad

(ii)



$\theta = \frac{\frac{\pi r}{2}}{r} = \frac{\pi}{2}$ rad.

(iii)



Arc length (ACB)

$120° \times \frac{\pi}{180} = \frac{l_{ACB}}{5}$

$\frac{2\pi}{3} = \frac{l_{ACB}}{5} \Rightarrow l_{ACB} = \frac{10\pi}{3}$ m

TRIGONOMETRIC RATIOS (T-RATIOS)

Following ratios of the sides of a right angled triangle are known as trigonometrical ratios.

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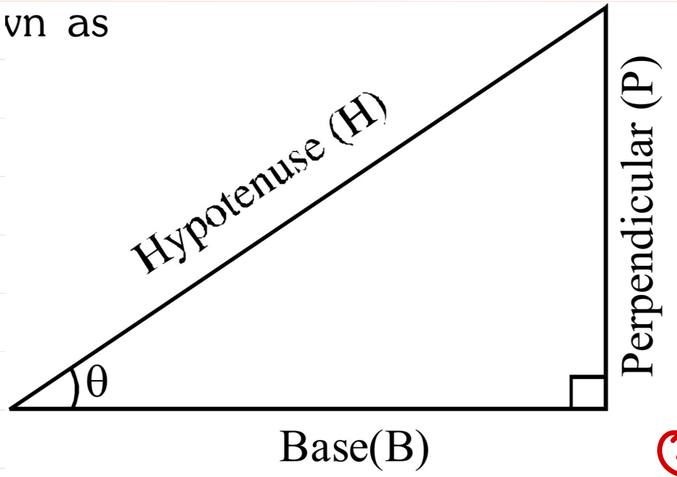


$$\frac{B^2 + P^2}{H^2} = \frac{H^2}{H^2}$$

$$\left(\frac{B}{H}\right)^2 + \left(\frac{P}{H}\right)^2 = 1 \Rightarrow \boxed{\cos^2\theta + \sin^2\theta = 1}$$

① $\cos\theta = \frac{B}{H}$

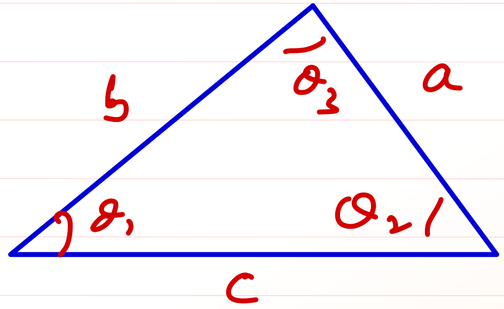
② $\sin\theta = \frac{P}{H}$

③ $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{P}{B}$

④ $\cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$

⑤ $\sec\theta = \frac{1}{\cos\theta}$

⑥ $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$



$\sin\theta_1 \propto a$

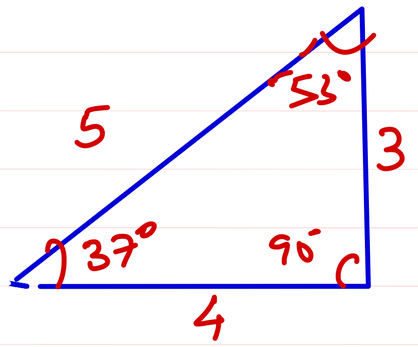
$\sin\theta_2 \propto b$

$\sin\theta_3 \propto c$

$\theta_1 + \theta_2 + \theta_3 = 180^\circ$

Trigonometric Identities :-

Angle(θ)	0°	30°	37°	45°	53°	60°	90°
sin θ	0	$\frac{1}{2}$	$\frac{3}{5}$	$\frac{1}{\sqrt{2}}$	$\frac{4}{5}$	$\frac{\sqrt{3}}{2}$	1
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{4}{5}$	$\frac{1}{\sqrt{2}}$	$\frac{3}{5}$	$\frac{1}{2}$	0
tan θ	0	$\frac{1}{\sqrt{3}}$	$\frac{3}{4}$	1	$\frac{4}{3}$	$\sqrt{3}$	∞



$3^2+4^2=5^2$

infinite or not defined

① $\sin^2\theta + \cos^2\theta = 1$

② $1 + \cot^2\theta = \operatorname{cosec}^2\theta$

③ $\tan^2\theta + 1 = \sec^2\theta$

Four Quadrants and ASTC Rule

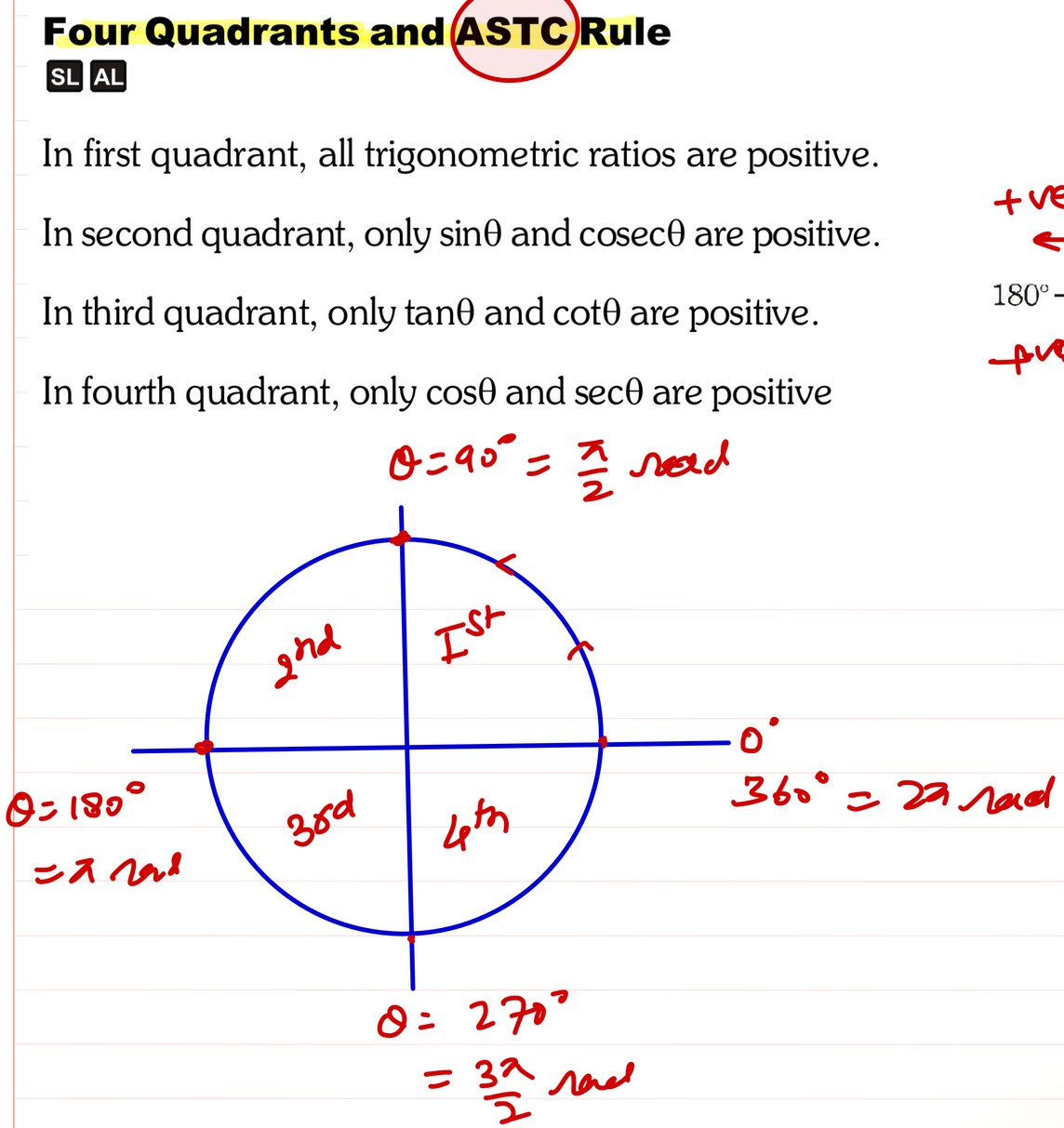
SL AL

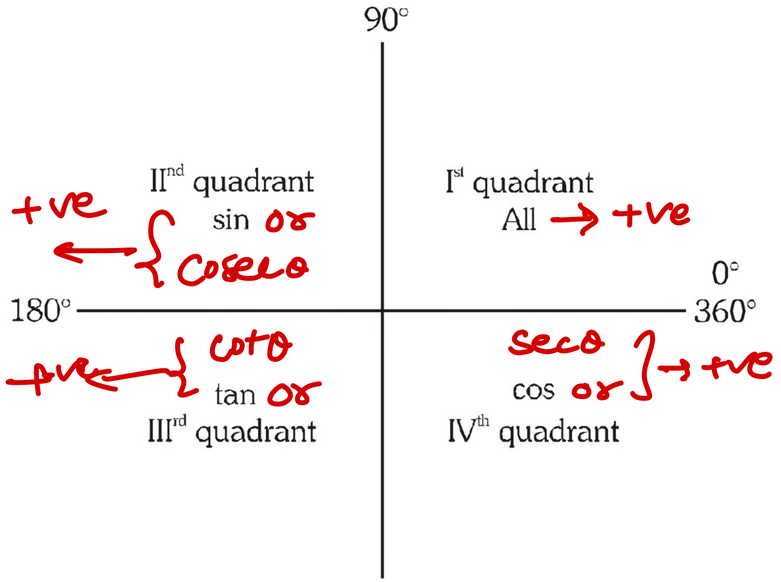
In first quadrant, all trigonometric ratios are positive.

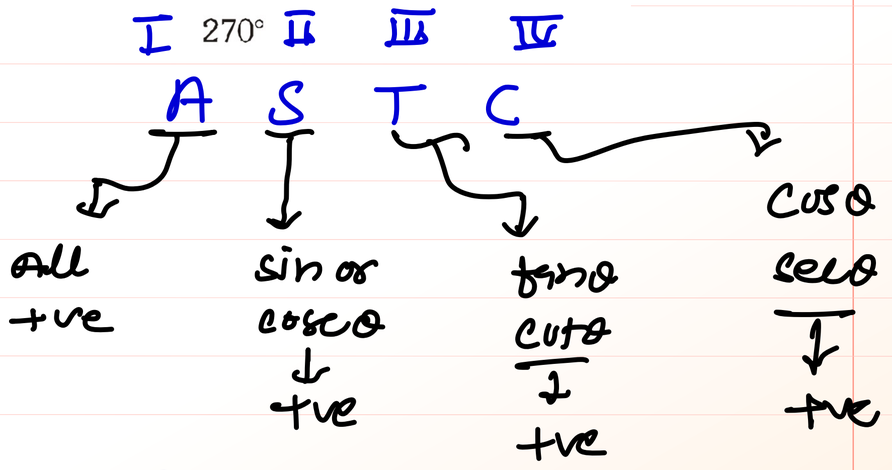
In second quadrant, only sinθ and cosecθ are positive.

In third quadrant, only tanθ and cotθ are positive.

In fourth quadrant, only cosθ and secθ are positive







① Sinθ
② Cosθ
③ tanθ
④ Cotθ
⑤ Secθ
⑥ Cosecθ

All Science Teachers are Criminal
Add Sugar To Coffee

Ex Which of the following one are T/F

(1) $\sin 65^\circ < 0 \rightarrow F$

(2) $\cot 275^\circ > 0 \rightarrow F$

(3) $\tan 210^\circ > 0 \rightarrow T$

(4) $\sec(290^\circ) > 0 \rightarrow T$

(5) $\operatorname{cosec}\left(7\frac{\pi}{3}\right) < 0 \rightarrow \operatorname{cosec}(7\times 60^\circ) = \operatorname{cosec}(420^\circ) = \operatorname{cosec}(360+60^\circ) > 0$
↳ F

(6) $\cos\left(\frac{5\pi}{4}\right) > 0 \rightarrow \cos(5\times 45^\circ) = \cos(225^\circ) < 0$
↳ F

Trigonometrical Ratios of General Angles (Reduction Formulae)

(i) Trigonometric function of an angle $2n\pi + \theta$ where $n=0, 1, 2, 3,....$ will remain same.

$\sin(2n\pi+\theta) = \sin\theta \quad \cos(2n\pi+\theta) = \cos\theta \quad \tan(2n\pi+\theta)=\tan\theta$

(ii) Trigonometric function of an angle $\left(\frac{n\pi}{2}+\theta\right)$ will remain same if n is even and sign of trigonometric function will be according to value of that function in quadrant.

$\sin(\pi-\theta) = +\sin\theta \quad \cos(\pi-\theta) = -\cos\theta \quad \tan(\pi-\theta) = -\tan\theta$

$\sin(\pi+\theta) = -\sin\theta \quad \cos(\pi+\theta) = -\cos\theta \quad \tan(\pi+\theta) = +\tan\theta$

$\sin(2\pi-\theta) = -\sin\theta \quad \cos(2\pi-\theta) = +\cos\theta \quad \tan(2\pi-\theta) = -\tan\theta$

$\sin\left(\frac{n\pi}{2}+\theta\right) = \sin\theta$ if n = even

$\cos\left(\frac{n\pi}{2}+\theta\right) = \cos\theta$ " "

$\tan\left(\frac{n\pi}{2}+\theta\right) = \tan\theta$ " "

(1)

Ex Find $\sin(750^\circ)$

$\sin(720^\circ + 30^\circ)$

$\sin(2\times 360^\circ + 30^\circ)$

$\sin(2\times 2\pi + 30^\circ)$

$\sin(4\pi + 30^\circ) = \sin(30^\circ) = \frac{1}{2}$ Ans

(iii) Trigonometric function of an angle $\left(\frac{n\pi}{2}+\theta\right)$ will be changed into co-function if n is odd and sign of trigonometric function will be according to value of that function in quadrant.

$$\sin\left(\frac{\pi}{2}+\theta\right)=+\cos\theta \qquad \cos\left(\frac{\pi}{2}+\theta\right)=-\sin\theta \qquad \tan\left(\frac{\pi}{2}+\theta\right)=-\cot\theta$$

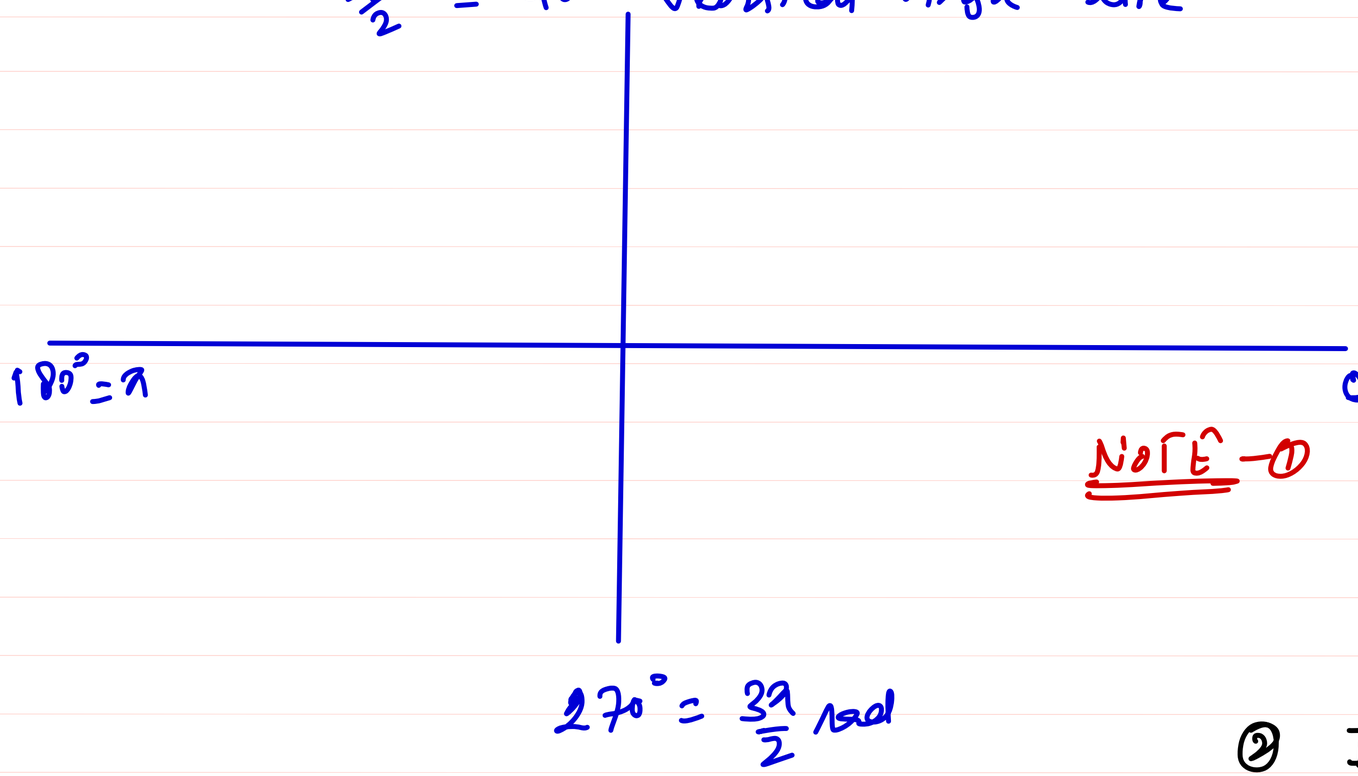
$$\sin\left(\frac{\pi}{2}-\theta\right)=+\cos\theta \qquad \cos\left(\frac{\pi}{2}-\theta\right)=+\sin\theta \qquad \tan\left(\frac{\pi}{2}-\theta\right)=+\cot\theta$$

(iv) Trigonometric function of an angle $-\theta$ (negative angles)

$\sin(-\theta)=-\sin\theta \quad \cos(-\theta)=+\cos\theta \qquad \tan(-\theta)=-\tan\theta$

H.W.

Illustration #10,11,12



NOTE - ①

In Horizontal angle line if we add or Subtract angle θ (0°<θ<90°) then function will remain same

② In vertical angle line if we add or Subtract angle θ (0°<θ<90°) then function will change as following

sin ⇌ cos tan ⇌ cot sec ⇌ cosec

Find values of following function

(1) $\sin(120)$

(2) $\cot(300^\circ)$

(3) $\sec(400^\circ)$

(1) $\sec(400^\circ)$

$\sec(360+40^\circ)$

$+\sec(40^\circ)$ Ans

$\sec(450-50)$

$\operatorname{cosec}(50^\circ)$ Ans

(1) $\sin(120^\circ)$

$\sin(90+30^\circ)$

$+\cos(30^\circ)$

$\frac{\sqrt{3}}{2}$ Ans

$\sin(180-60^\circ)$

$=+\sin(60^\circ)$

$=\frac{\sqrt{3}}{2}$ Ans

(2) $\cot(300^\circ)$

$\cot(360-60^\circ)$ or $\cot(270+30^\circ)$

$=-\cot 60^\circ$

$=-\frac{1}{\sqrt{3}}$

$=-\tan 30$

$=-\frac{1}{\sqrt{3}}$