Instantaneous Velocity and speed

Instantaneous velocity OR velocity = time rate of position / displacement

$$\boxed{\vec{v} = \frac{d\vec{r}}{dt} \text{ or } \frac{d\vec{s}}{dt}}$$

$\Rightarrow$ slope of $\vec{r}-t$ curve OR $\vec{s}-t$ curve

Instantaneous speed OR speed = | velocity |

= time rate of distance

$$\boxed{v = |\vec{v}|}$$

Instantaneous Acceleration :$\Rightarrow$

= time rate of velocity

$$\boxed{\vec{a} = \frac{d\vec{v}}{dt} = \text{slope of } \vec{v}-t \text{ curve}}$$

$$\boxed{\vec{a} = \frac{d^2\vec{r}}{dt^2}}$$

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Ex A particle moving along x - axis whose position time reln is given

$x = t^2 - 2t$

Find (i) displacement in initial 2 sec.

(ii) distance in initial 2 sec

(iii) velocity and speed at $t = 2$ sec

(iv) acceleration at $t = 2$ sec

(v) Avg velocity in initial 2 sec

(vi) Avg speed " " "

(vii) Avg Acceleration " " " "

(i) $\vec{S} = \vec{x}_f - \vec{x}_i$

$S = x(2) - x(0)$

$= (2^2 - 2\times 2) - (0^2 - 2\times 0)$

$S = 4 - 4$

$\boxed{S = 0\,m}$ Ans

(iii) $v = \frac{dx}{dt} = 2t - 2$

Speed $= |v| = |2t - 2|$

at $t = 2$ sec

$v = 2\times 2 - 2 = 2\,m/s$

$|v| = \text{speed} = |2\times 2 - 2| = 2\,m/s$ Ans

NOTE :→ Avg of Constant always equal to that constant

(iv) $a = \dfrac{dv}{dt}$

$\because v = 2t - 2$

$a = 2\times1 - 0$

$\boxed{a = 2\,m/s^2}$

this is constant

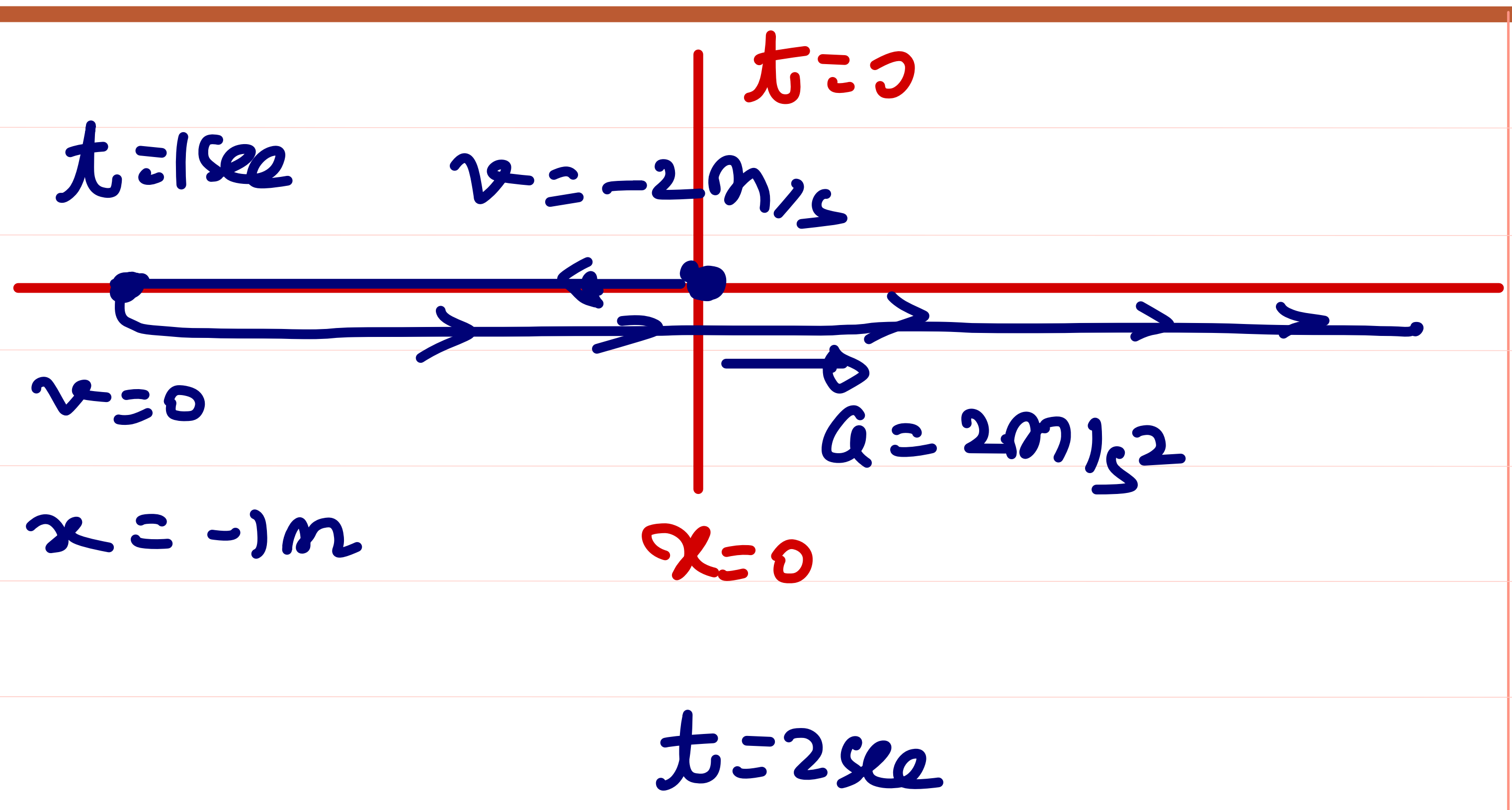
(v) Avg velocity $= \dfrac{\Delta x}{\Delta t} = \dfrac{x(2) - x(0)}{2-0}$

$= \dfrac{0}{2} = 0\ m/s$

(vii) avg Acc. $= \dfrac{\Delta v}{\Delta t} = \dfrac{v(2) - v(0)}{2-0}$

$= \dfrac{(2\times2-2) - (2\times0-2)}{2-0}$

$= \dfrac{4-2+2}{2} = \dfrac{4}{2} = 2\,m/s^2$



(ii) distance = 2m

(vi) Avg Speed $= \dfrac{d}{\Delta t} = \dfrac{2}{2} = 1\,m/s$

$x = t^2 - 2t$

$v = 2t - 2$

$a = 2\,m/s^2$

NOTE

Type of motion

If $\vec{v}$ = constant Uniform motion $\boxed{\vec{a}=0}$

If $\begin{matrix}\vec{v} > 0 \\ \vec{a} > 0\end{matrix}$ OR $\left.\begin{matrix}\vec{v} < 0 \\ \vec{a} < 0\end{matrix}\right)$ Motion is accelerating ($\vec{v}$ & $\vec{a}$ both are in same Dirn)

If $\begin{matrix}\vec{v} > 0 \\ \vec{a} < 0\end{matrix}$ OR $\left.\begin{matrix}\vec{v} < 0 \\ \vec{a} > 0\end{matrix}\right)$ Motion is Retarding OR deaccelerating (Initial both $\vec{v}$ & $\vec{a}$ are in opposite Dirn)

4. A particle moves such that its position x varies with time according to relation $x = 2t - t^2$, where x is in metres and time in seconds. The incorrect statement about the particle is

(A) Velocity of the particle in interval $t = 0$ to $t = 2$ sec is in positive x-direction.

(B) Speed of the particle is 1 m/s at $t = \frac{3}{2}$ s.

(C) Displacement travelled in the interval $t = 0$ to $t = 2$s is zero.

(D) Its speed first increases then decreases.

Sol

$x = 2t - t^2$

$v = \frac{dx}{dt} = 2 - 2t \Rightarrow \boxed{v = 2(1-t)}$

$a = 0 - 2$

$\boxed{a = -2\ m/s^2}$

(A) $v > 0$ in $0 < t < 1$

$v < 0$ in $t > 1$

(B) $|v| = |2(1-t)|$

at $t = 3/2$

$|v| = |2(1 - 3/2)|$

$= |-1|$

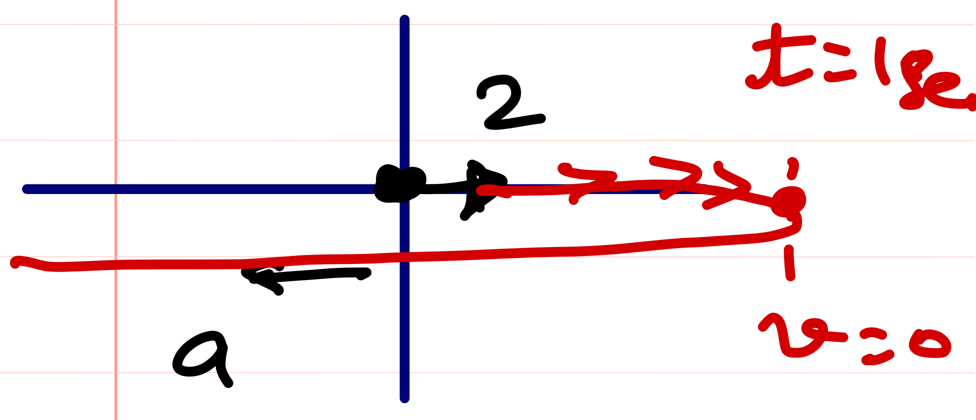
$= 1\ m/s$

(C) $s = x(2) - x(0)$

$= (2\times2 - 2^2) - (2\times0 - 0^2)$

$s = 0m$

at $t=0$



$x=0$

$v = 2\ m/s$

$a = -2\ m/s^2$

Speed 1st decreases then increases

6. A particle is moving in a straight line according to equation $x = \frac{t^3}{3} - \frac{5}{2}t^2 + 6t$. The time interval in which velocity i.e. instantaneous rate of change of position w.r.t. time is negative is

(A) $0 < t < 3$ (B) $0 < t < 2$ (C) $2 < t < 3$ (D) $t > 3$ and $t < 2$

$$x = \frac{t^3}{3} - \frac{5}{2}t^2 + 6t$$

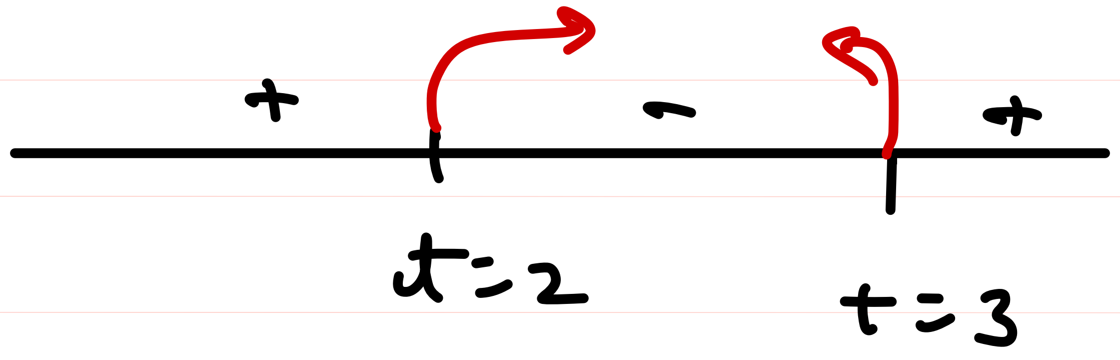
$$v = \frac{dx}{dt} = \frac{\cancel{3}t^2}{\cancel{3}} - \frac{5}{\cancel{2}}\cdot\cancel{2}t + 6$$

$$= t^2 - 5t + 6$$

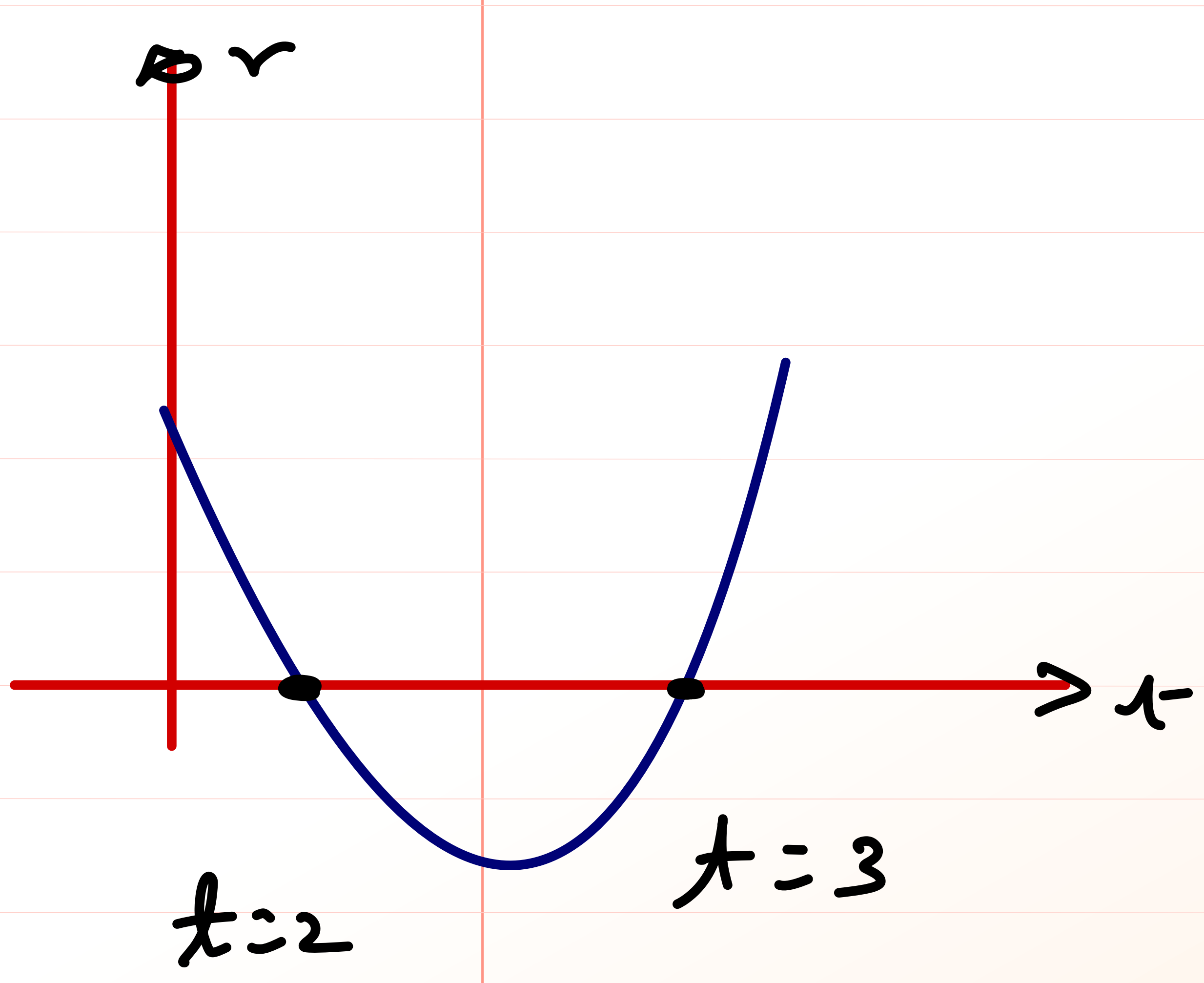
$$= t^2 - 3t - 2t + 6$$

$$= t(t-3) - 2(t-3)$$

$$v = (t-2)(t-3) < 0$$



$2 < t < 3$



7. The position of

(A) Velocity will

(B) Acceleration

(C) Acceleration

(D) None of the

Sol

$x = t(t-1)(t-2)$

$= (t^2-t)(t-2)$

$= t^3 - t^2 - 2t^2 + 2t$

$\boxed{x = t^3 - 3t^2 + 2t}$

$v = \frac{dx}{dt} = 3t^2 - 6t + 2$

$\boxed{v = 3t^2 - 6t + 2}$

$\boxed{a = 6t - 6}$

(A) $v = 0$

$3t^2 - 6t + 2 = 0$

$t = \frac{+6 \pm \sqrt{(-6)^2 - 4\times 2\times 3}}{2\times 3}$

$= \frac{6 \pm \sqrt{12}}{6}$

$= 1 \pm \sqrt{\frac{12}{36}}$

$= 1 \pm \frac{1}{\sqrt{3}}$

$\boxed{t_1 = 1 - \frac{1}{\sqrt{3}} \text{ OR } t_2 = 1 + \frac{1}{\sqrt{3}}}$

(B) $a = 6t - 6 = 6(t-1)$

$t < 1 \quad a < 0$

$t > 1 \quad a > 0$

straight line motion

15. A point moves rectilinearly. Its position x at time t is given by, $x^2 = t^2 + 1$. Its acceleration at time t is:

(A) $\frac{1}{x^3}$ (B) $\frac{1}{x} - \frac{1}{x^2}$ (C) $-\frac{t}{x^2}$ (D) none of these

$x^2 = t^2 + 1$

$2x \cdot \frac{dx}{dt} = 2t + 0$

$\frac{dx}{dt} = \frac{t}{x}$

$xv = t$

diff again

Product Rule

$x \cdot \frac{dv}{dt} + v\frac{dx}{dt} = 1$

$x \cdot a + v \cdot v = 1$

$xa + v^2 = 1$

$xa = 1 - v^2$

$xa = 1 - \frac{t^2}{x^2}$

$a = \frac{x^2 - t^2}{x^3}$

$\boxed{a = \frac{1}{x^3}}$

Ex

If Position of a particle is given by
$x = (4t^2 - 8t)$, then which of the following is true?

(a) Acceleration is zero at $t = 0$

(b) Velocity is zero at $t = 0$

(c) Velocity is zero at $t = 1s$

(d) Velocity and acceleration will never be zero

$$\frac{dx}{dt} = 8t - 8$$

$$\boxed{v = 8t - 8}$$

$t = 1 \quad v = 0$

$$\frac{dv}{dt} = 8$$

$$\boxed{a = 8\ m/s^2}$$

Ex

A particle moves along a straight line OX. At a time *t* (in second) the distance x (in metre) of the particle from O is given by $x = 40 + 12t - t^3$. How long would the particle travel before coming to rest?

(a) 24m (b) 40m

(c) 56m (d) 16m ✓

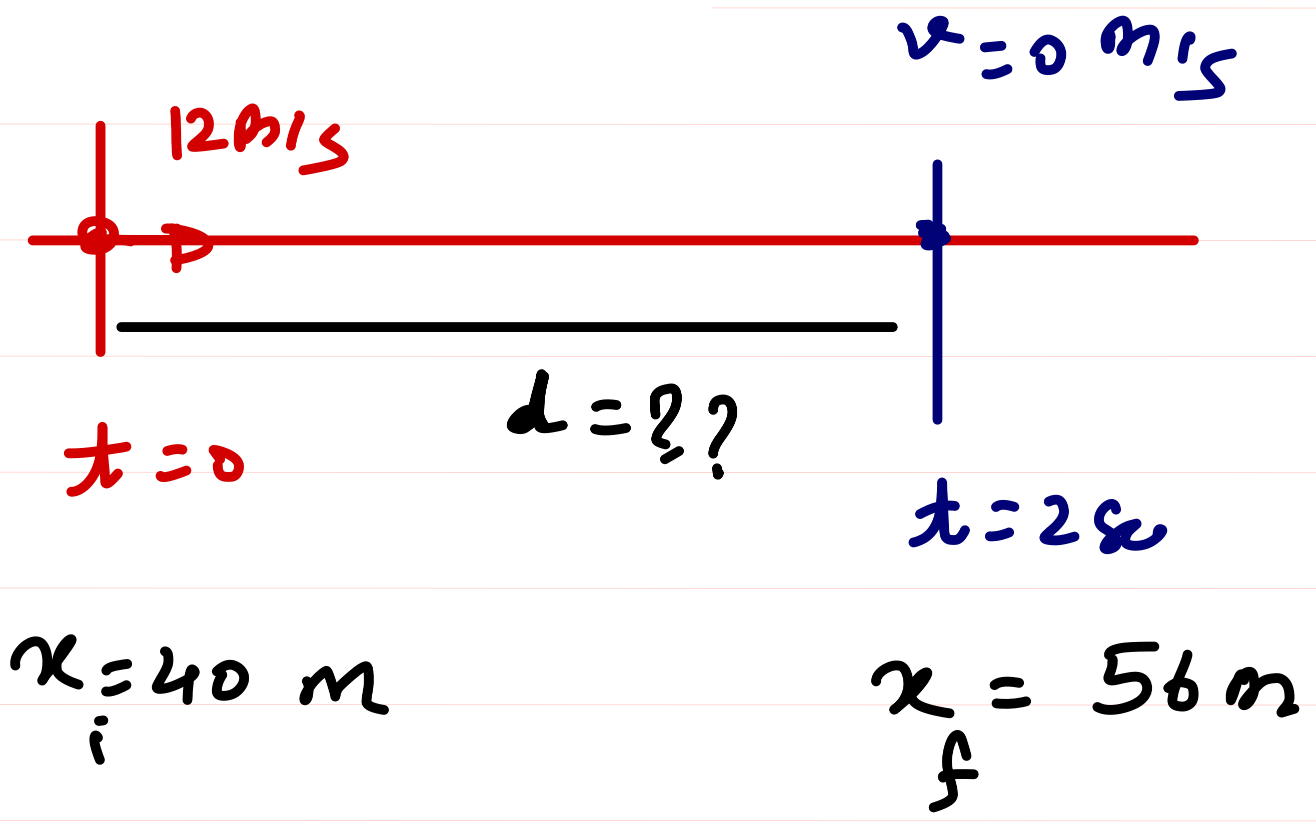
$$\frac{dx}{dt} = 0 + 12 - 3t^2$$

$$v = 12 - 3t^2$$

$$v = 0$$

$$12 - 3t^2 = 0$$

$$\boxed{t = 2\,sec}$$



$$d = x_f - x_i$$

$$= 56 - 40$$

$$= 16\,m$$

Ex

A particle moves a distance x in time t according to equation $x = (t + 5)^{-1}$. The acceleration of particle is proportional to

(a) $(\text{velocity})^{3/2}$ (b) $(\text{distance})^{2}$

(c) $(\text{distance})^{-2}$ (d) $(\text{velocity})^{2/3}$

speed speed

$$x = (t+5)^{-1}$$

$$\frac{dx}{dt} = -1\,(t+5)^{-2}\,(1+0)$$

$$\boxed{v = -\left(\frac{1}{t+5}\right)^{2}}$$

$$v = -(t+5)^{-2}$$

$$a = +2\,(t+5)^{-3}\cdot(1+0)$$

$$a = \frac{2}{(t+5)^{3}}$$

$$a = 2\left(\frac{1}{(t+5)^{2}}\right)^{3/2}$$

$$a = 2\,(-v)^{3/2}$$

$$a \propto (\text{velocity})^{3/2}$$

H.W BB#1 Q-1 to 5

Illu- 1,2,3,4,5