Trigonometric equations
CL04

Type-4 Solving equations with the use of boundness of the function sin x or cos x

① $\sin^4 x = 1 + \cos^6 y$

$-1 \le \sin x \le 1$

$0 \le \sin^4 x \le 1$

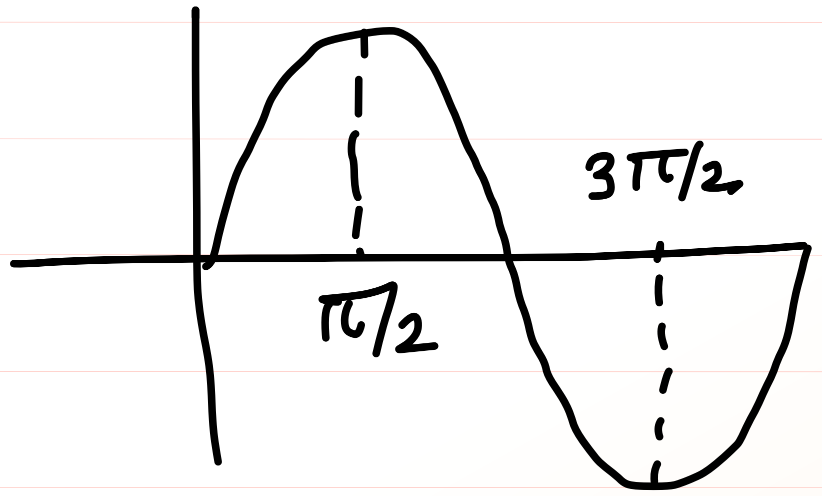
$-1 \le \cos y \le 1$

$0 \le \cos^6 y \le 1$

$1 \le 1 + \cos^6 y \le 2$

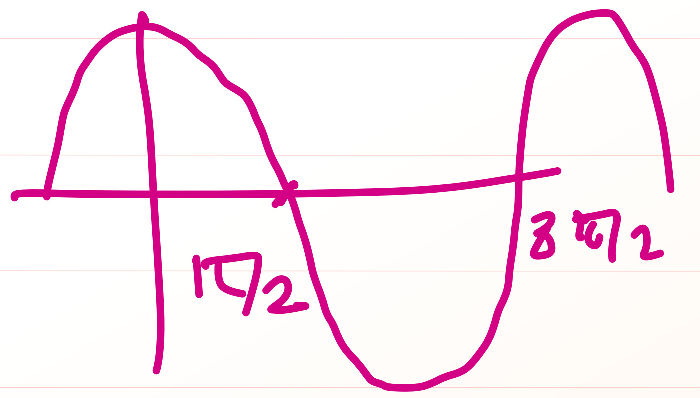
$\sin^4 x = 1$

$\sin x = \pm 1$



$\boxed{x \in (2n+1)\frac{\pi}{2},\ n \in Z}$

$\cos^6 y = 0$



$\boxed{y \in (2m+1)\frac{\pi}{2},\ m \in Z}$

(2) $\cos x + \cos 2x + \cos 3x = 3$

$\cos x = 1$ $\quad\quad$ $\cos 2x = 1$ $\quad\quad$ $\cos 3x = 1$

$x = 2n\pi$ $\quad\quad$ $2x = 2n\pi$ $\quad\quad$ $3x = 2n\pi$ $\quad\quad$ $n \in Z.$

$x = \frac{2n\pi}{1}$ $\quad\quad$ $x = \frac{n\pi}{1}$ $\quad\quad$ $x = \frac{2n\pi}{3}$ $\quad\quad$ $n \in Z$

$$x = \frac{2n\pi}{1} \quad ; \; n \in Z$$

$$③ \quad \sin x\left(\cos\frac{x}{4} - 2\sin x\right) + \left(1 + \sin\frac{x}{4} - 2\cos x\right)\cos x = 0$$

$$\underbrace{\sin x \cdot \cos\frac{x}{4}} - 2\sin^2 x + \cos x + \underbrace{\sin\frac{x}{4}\cos x} - 2\cos^2 x = 0$$

$$\sin\left(x + \frac{x}{4}\right) - 2\left(\sin^2 x + \cos^2 x\right) + \cos x = 0$$

$$\sin\left(\frac{5x}{4}\right) + \cos x = 2$$

$$\sin\left(\frac{5x}{4}\right) = 1 \qquad \cos x = 1$$

$$\frac{5x}{4} = (4n+1)\frac{\pi}{2} \qquad \boxed{x = 2m\pi \; ; \; m \in Z} \qquad x \in \{0, 2\pi, 4\pi.$$

$$\boxed{x = (4n+1)\frac{2\pi}{5} \; ; n \in Z}$$

common solutions $\{ -- -6\pi, \underline{2\pi}, \underline{10\pi} --- \}$

$$x \in = \frac{2\pi}{5}, 2\pi, \frac{18\pi}{5}, \frac{26\pi}{5}, \frac{34\pi}{5}, \frac{42\pi}{5}, 10\pi$$

$$= (2 + (p-1)8)\pi \quad p \in z$$

$$= (8p-6)\pi \; ; p \in Z$$

④ Solve for x and y

$$1-2x-x^2 = \tan^2(x+y) + \cot^2(x+y)$$

$$-(x^2+2x)+1 = \tan^2(x+y) + \cot^2(x+y)$$

$$-(x^2+2x+1)+1+1 = \tan^2(x+y) + \cot^2(x+y)$$

$$\underbrace{-(x+1)^2} + 2 = \tan^2(x+y) + \cot^2(x+y)$$

$$(x+1) = 0$$

$$\boxed{x=-1}$$

$$\tan^2(x+y) = 1$$

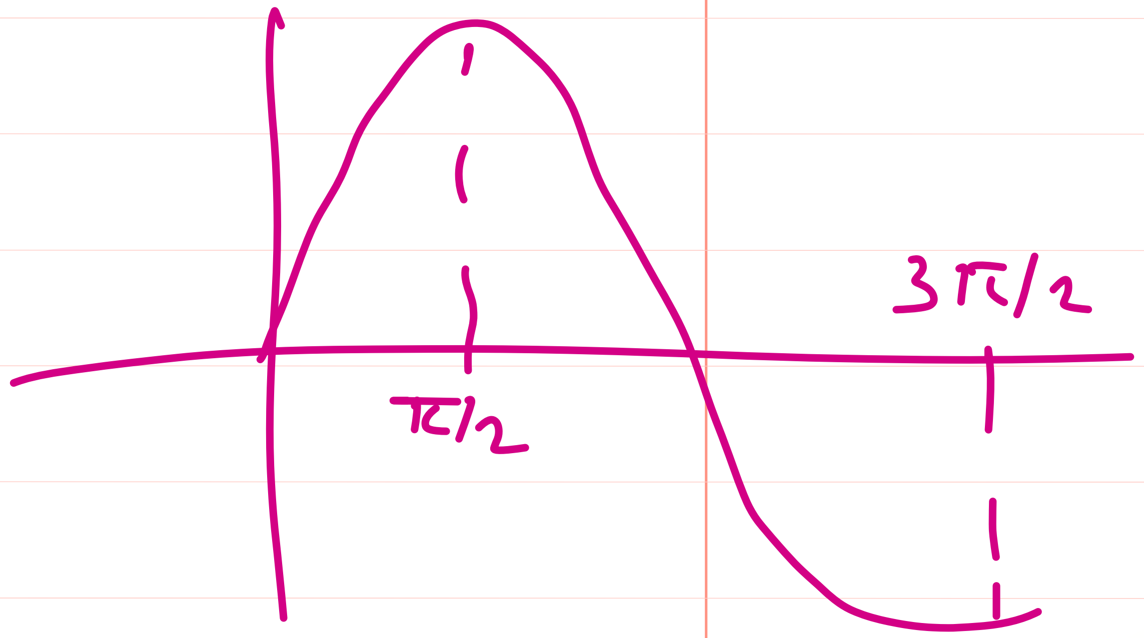
$$(x+y) = n\pi \pm \frac{\pi}{4}$$

$$y-1 = n\pi \pm \frac{\pi}{4}$$

$$y = n\pi \pm \frac{\pi}{4} + 1$$

Type 5 Solution of trigo equations of the form $f(x) = \sqrt{\phi(x)}$

① $\sqrt{1-\cos x} = \sin x$



$1 - \cos x = 1 - \cos^2 x$

$\cos x = 0, 1$

$\cos x = 0$

$x = (2n+1)\frac{\pi}{2}$

$x = \left\{ \ldots, \cancel{-\frac{\pi}{2}}, \frac{\pi}{2}, \cancel{\frac{3\pi}{2}}, \frac{5\pi}{2}, \ldots \right\}$

$x = (4n+1)\frac{\pi}{2} \quad \forall\, n \in Z.$

$\cos x = 1$

$x = 2n\pi$

$x = \{-2\pi, 0, 2\pi, \ldots\}$

$x = 2n\pi \quad \forall\, n \in Z$

(2)

$$2^{\frac{1}{\sin^2 x}} \cdot \sqrt{y^2-2y+2} \le 2$$

$$2^{\operatorname{cosec}^2 x} \cdot \underbrace{\sqrt{y^2-2y+2}} \le \underbrace{2}$$

$$2^{\operatorname{cosec}^2 x} \sqrt{y^2-2y+1+1} \le 2$$

$$\underbrace{\underbrace{2^{\operatorname{cosec}^2 x}}_{\ge 2} \underbrace{\sqrt{(y-1)^2+1}}_{\ge 1}}_{\ge 2} \le 2$$

$= 2$

$(y-1)^2 = 0$

$\boxed{y=1}$

$2^{\operatorname{cosec}^2 x} = 2$

$\operatorname{cosec}^2 x = 1$

$\sin^2 x = 1$

$x \in (2n+1)\frac{\pi}{2}$

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$$2\sin\left(3x+\frac{\pi}{4}\right) = \sqrt{1+8\sin 2x\cos^2 2x}$$

$$4\sin^2\left(3x+\frac{\pi}{4}\right) = 1+\underbrace{8\sin 2x\cdot\cos 2x}\cdot\cos 2x$$

$$4\sin^2\left(3x+\frac{\pi}{4}\right) = 1+4\cdot(\sin 4x)\cdot\cos 2x$$

$$2\left[1-\cos 2\left(3x+\frac{\pi}{4}\right)\right] = 1+4\sin 4x\cos 2x$$

$$2\left[1-\underbrace{\cos\left(\frac{\pi}{2}+6x\right)}\right] = 1+2\left(\underbrace{2\sin 4x\cos 2x}\right)$$

$$2+\cancel{2\sin(6x)} = 1+2\cdot(\cancel{\sin 6x}+\sin(2x))$$

$$\sin 2x = \frac{1}{2}$$

$$2x = n\pi + (-1)^n\frac{\pi}{6}$$

$$x = \frac{n\pi}{2} + (-1)^n\frac{\pi}{12}$$

$\cos 2x = 1-2\sin^2 x$

$2\sin^2 x = 1-\cos 2x$

$$x = \frac{n\pi}{2} + (-1)^n \frac{\pi}{12} \qquad [0, 2\pi]$$

$$n = 0 \qquad x = \frac{\pi}{12}$$

$$2\sin\left(3x+\frac{\pi}{4}\right) = 2\sin\left(3\cdot\frac{\pi}{12}+\frac{\pi}{4}\right) = 2\sin\frac{\pi}{2} = 2 = \underline{\text{tve}}$$

$$n = 1\ ; \qquad x = \frac{\pi}{2} - \frac{\pi}{12} = \frac{5\pi}{12}$$

$$2\sin\left(3x+\frac{\pi}{4}\right) = 2\sin\left(3\left(\frac{5\pi}{12}\right)+\frac{\pi}{4}\right) = 2\sin\left(\frac{3\pi}{2}\right) = 2(-1) = -2$$

$$n = 2\ ; \qquad x = \frac{2\pi}{2} + \frac{\pi}{12} = \frac{13\pi}{12}$$

$$2\sin\left(3x+\frac{\pi}{4}\right) = 2\sin\left(3\left(\frac{13\pi}{12}\right)+\frac{\pi}{4}\right) = 2\sin\left(\frac{7\pi}{2}\right) = 2(-1) = -2$$

$$n = 3\ ; \qquad x = \frac{3\pi}{2} - \frac{\pi}{12} = \frac{17\pi}{12}$$

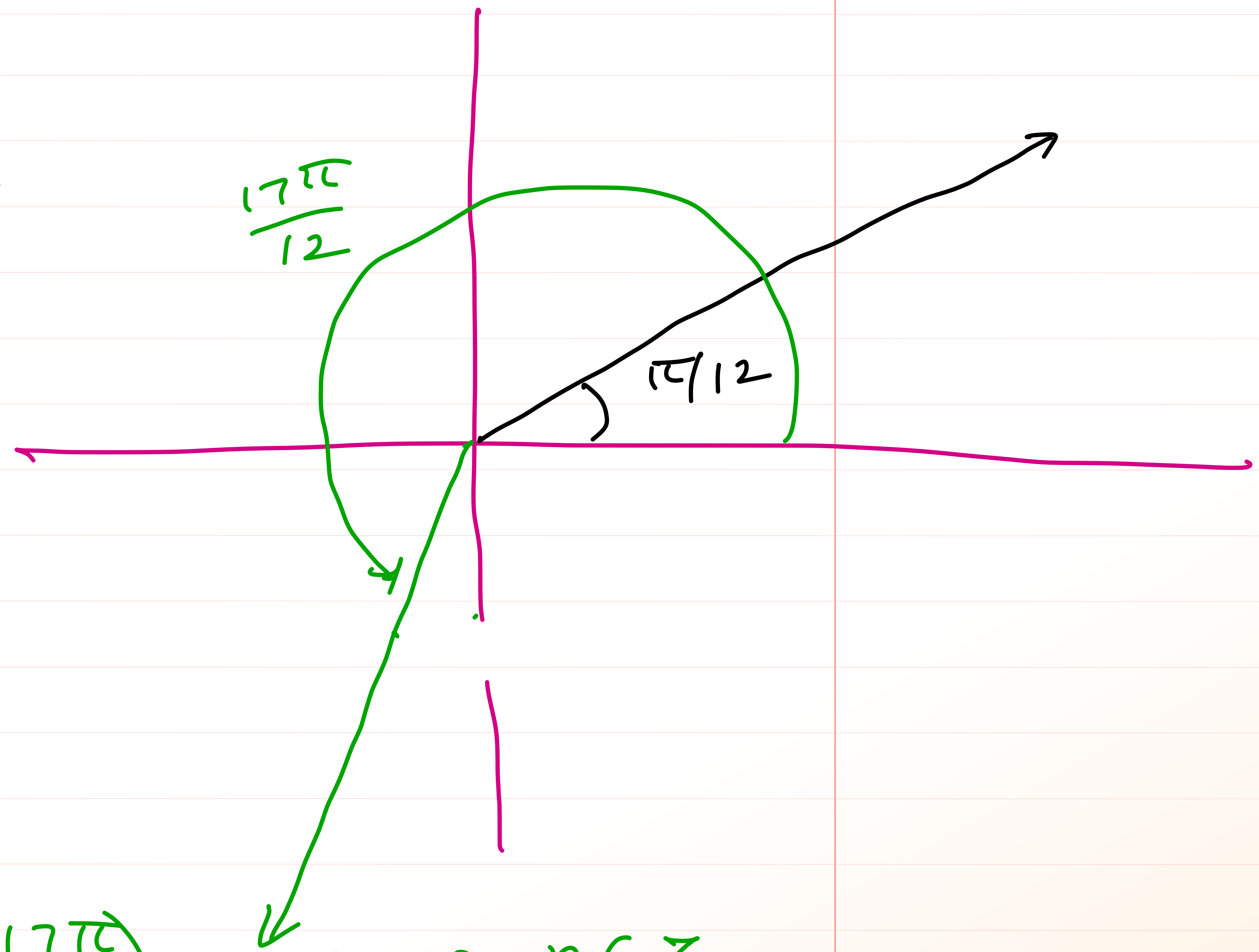
$$2\sin\left(3x+\frac{\pi}{4}\right) = 2\sin\left(3\left(\frac{17\pi}{12}\right)+\frac{\pi}{4}\right) = 2\sin\left(\frac{9\pi}{2}\right) = +2$$

$x = \frac{\pi}{12}$

$x = 2n\pi + \frac{\pi}{12} \quad \forall\, n \in Z.$

$x = \frac{17\pi}{12}$

$x = 2m\pi + \frac{17\pi}{12} \quad \forall\, m \in Z.$



$x = \left(2n\pi + \frac{\pi}{12}\right) \cup \left(2m\pi + \frac{17\pi}{12}\right) \quad \forall\, m, n \in Z.$

System of Trigonometric equations : →

(1) $\cos x \cdot \cos y = \frac{3}{4}$ and $\sin x \cdot \sin y = \frac{1}{4}$

add

$\cos x \cos y + \sin x \sin y = 1$

$\cos(x-y) = 1 \Rightarrow x-y = 2n\pi \quad \forall\, n \in \mathbb{Z}$ —— (1)

Subtract

$\cos x \cos y - \sin x \sin y = \frac{1}{2}$

$\cos(x+y) = \cos\frac{\pi}{3} \Rightarrow x+y = 2m\pi \pm \frac{\pi}{3}$

$x - y = 2n\pi$ —— (1)

$x + y = 2m\pi \pm \frac{\pi}{3}$

$2x = 2n\pi + 2m\pi \pm \frac{\pi}{3} \Rightarrow \boxed{x = (n+m)\pi \pm \frac{\pi}{6}}$

$\boxed{y = (m-n)\pi \pm \frac{\pi}{6}}$

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(2) $x+y=\frac{2\pi}{3}$; $\frac{\sin x}{\sin y}=2$

$\sin x = 2\sin y$

$\sin x = 2\sin\left(\frac{2\pi}{3}-x\right)$

$\sin x = 2\left[\sin\left(\frac{2\pi}{3}\right)\cos x - \cos\left(\frac{2\pi}{3}\right)\sin x\right]$

$\sin x = 2\left[\frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x\right]$

$\cancel{\sin x} = \sqrt{3}\cos x + \cancel{\sin x}$

$\cos x = 0$

$$\boxed{x=(2n+1)\frac{\pi}{2}\ ;\ n\in Z}$$

$y=\frac{2\pi}{3}-x$

$=\frac{2\pi}{3}-n\pi-\frac{\pi}{2}$

$$\boxed{y=\frac{\pi}{6}-n\pi}$$

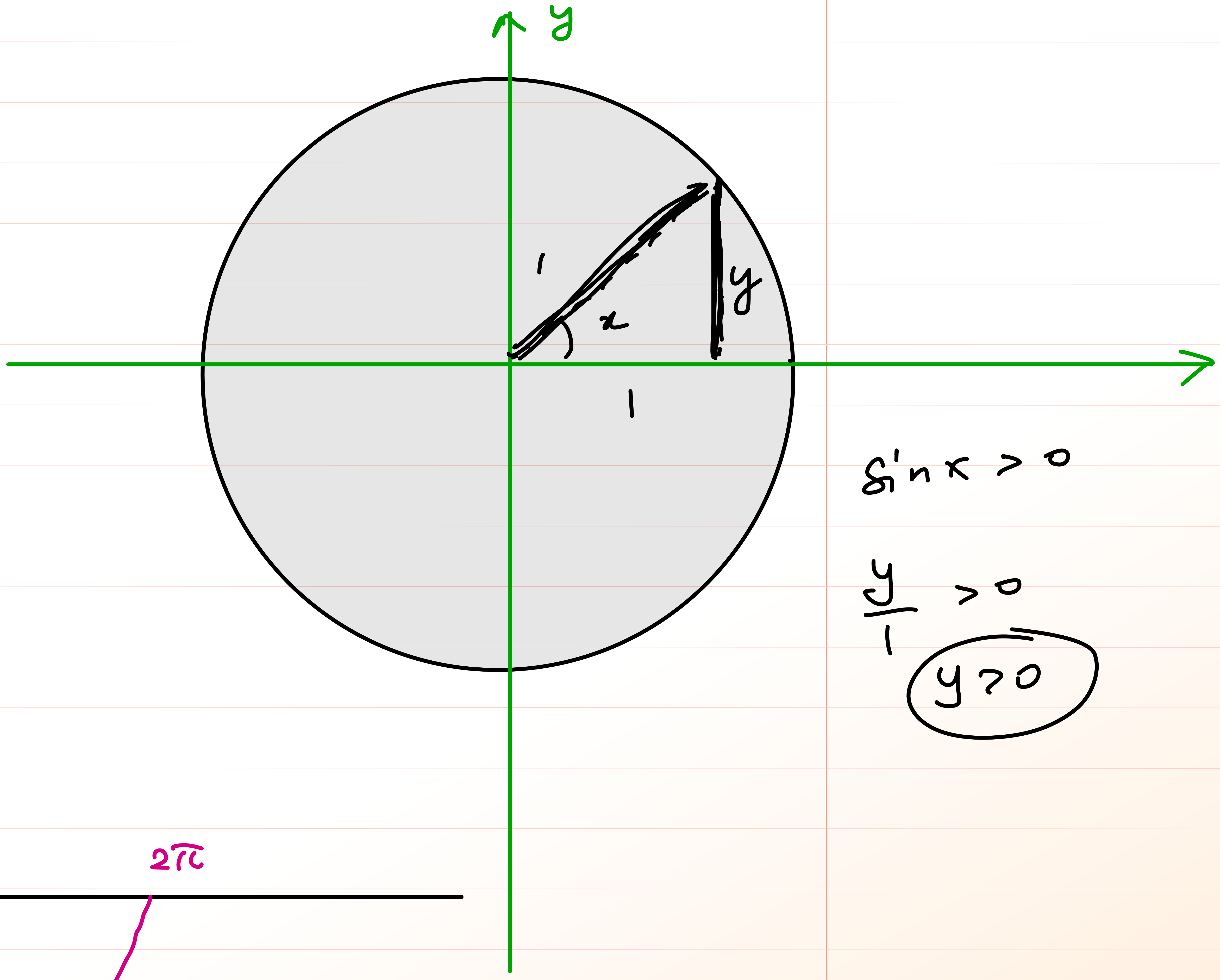
Trigonometric Inequalities and system of inequality :→

(1) $\sin x > 0$

M-I

$y > 0$

$\boxed{x \in (0, \pi)}$



$\sin x > 0$

$\frac{y}{1} > 0$

$y > 0$

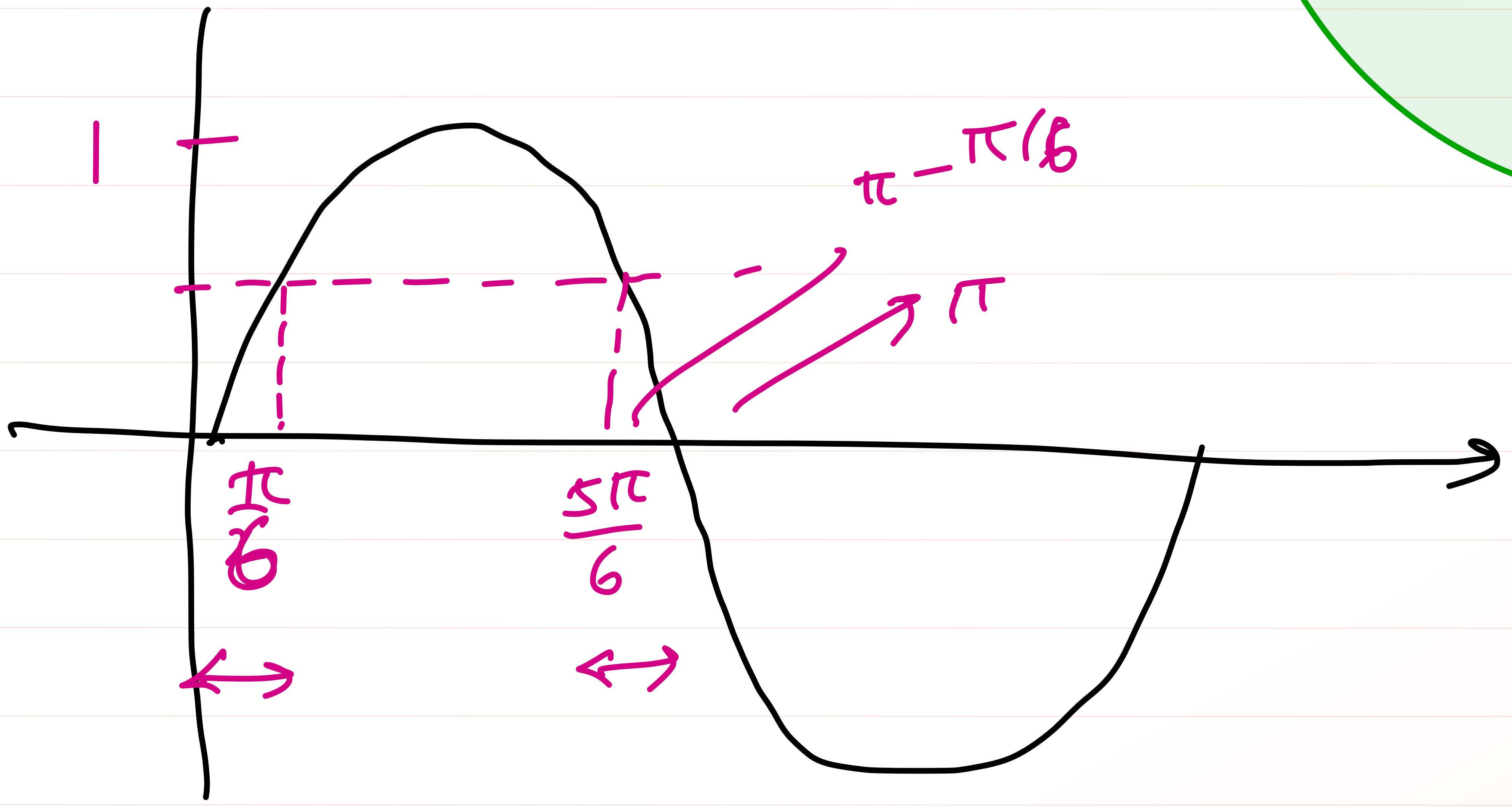
M-II

$x \in (0, \pi)$

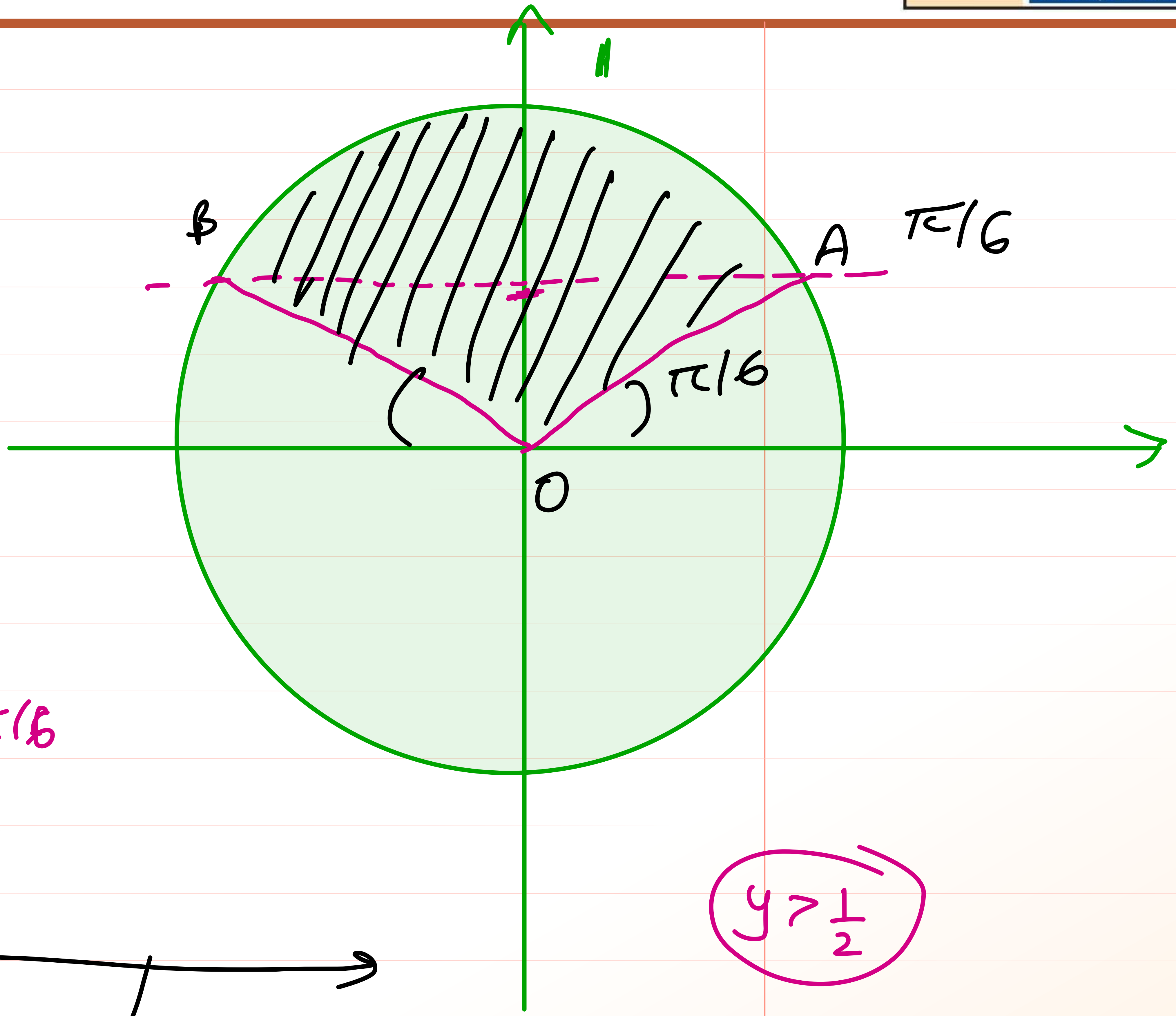
2 $\sin x > \frac{1}{2}$

MI

$$\boxed{x \in \left(\frac{\pi}{6}, \frac{5\pi}{6}\right)}$$



$$\boxed{x \in \left(\frac{\pi}{6}, \frac{5\pi}{6}\right)}$$

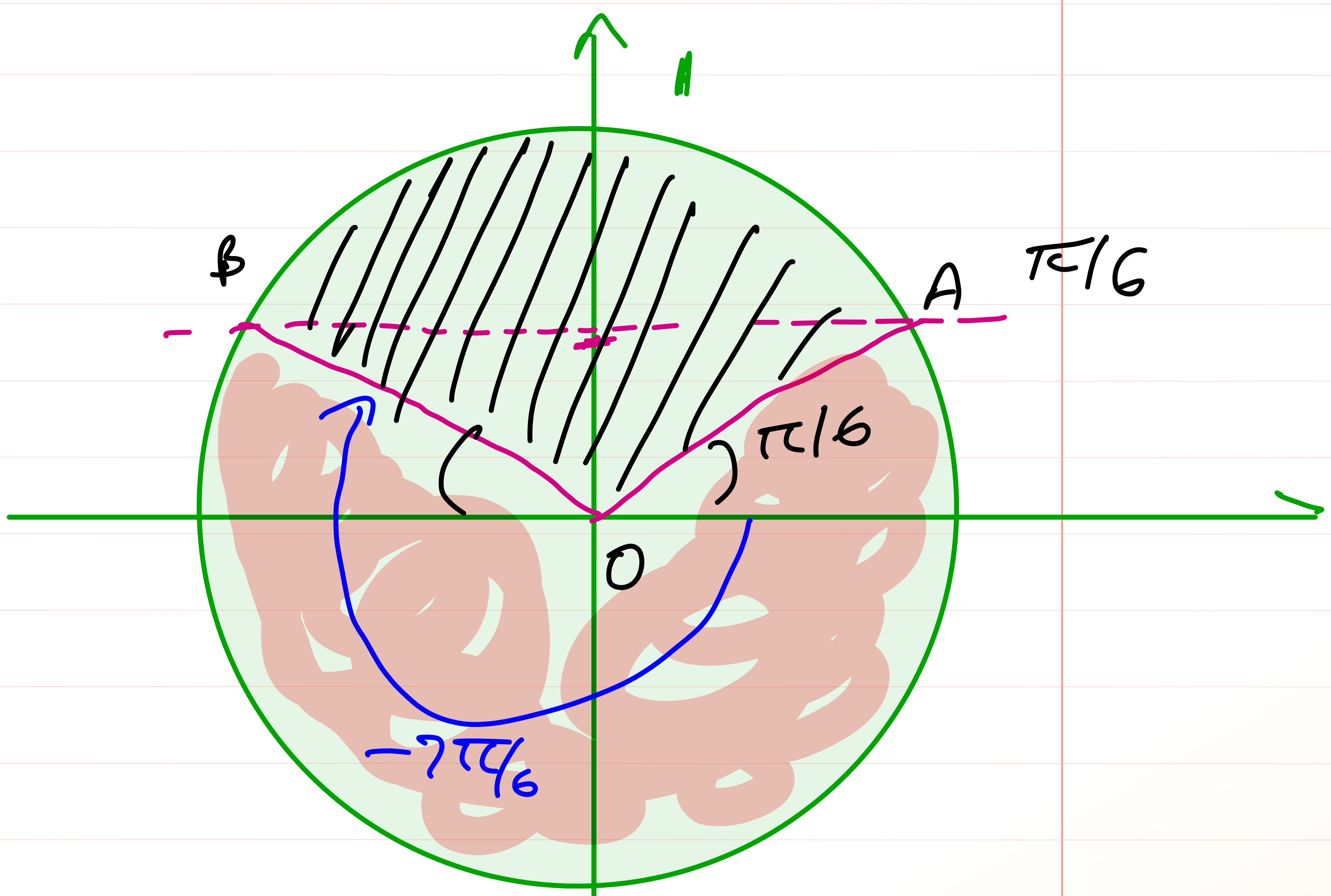


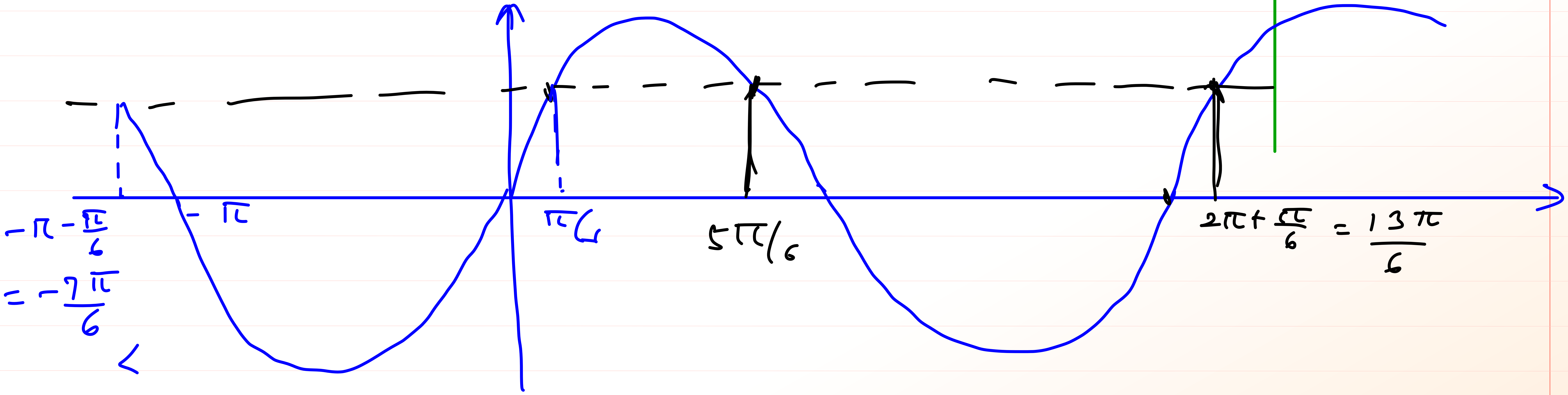
$y > \frac{1}{2}$

$\sin x \le \frac{1}{2}$

M I

$x \in \left[-\frac{7\pi}{6}, \frac{\pi}{6}\right]$





Q $\log_2\left(\sin\frac{x}{2}\right) < -1$

$0 < \sin\frac{x}{2} < 2^{-1}$

$0 < \sin\frac{x}{2} < \frac{1}{2}$

$0 < \frac{x}{2} < \frac{\pi}{6}$ or $\frac{5\pi}{6} < \frac{x}{2} < \pi$

$$\boxed{0 < x < \pi/3 \quad \text{or} \quad \frac{5\pi}{3} < x < 2\pi}$$