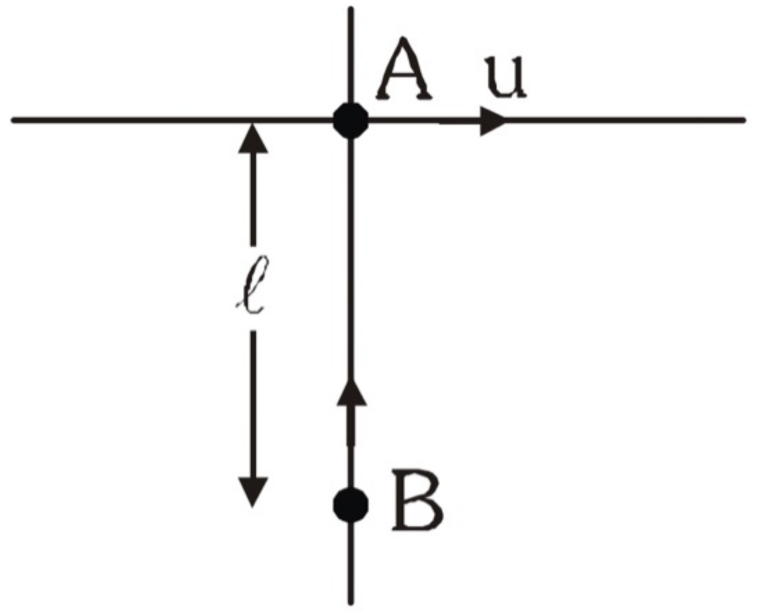
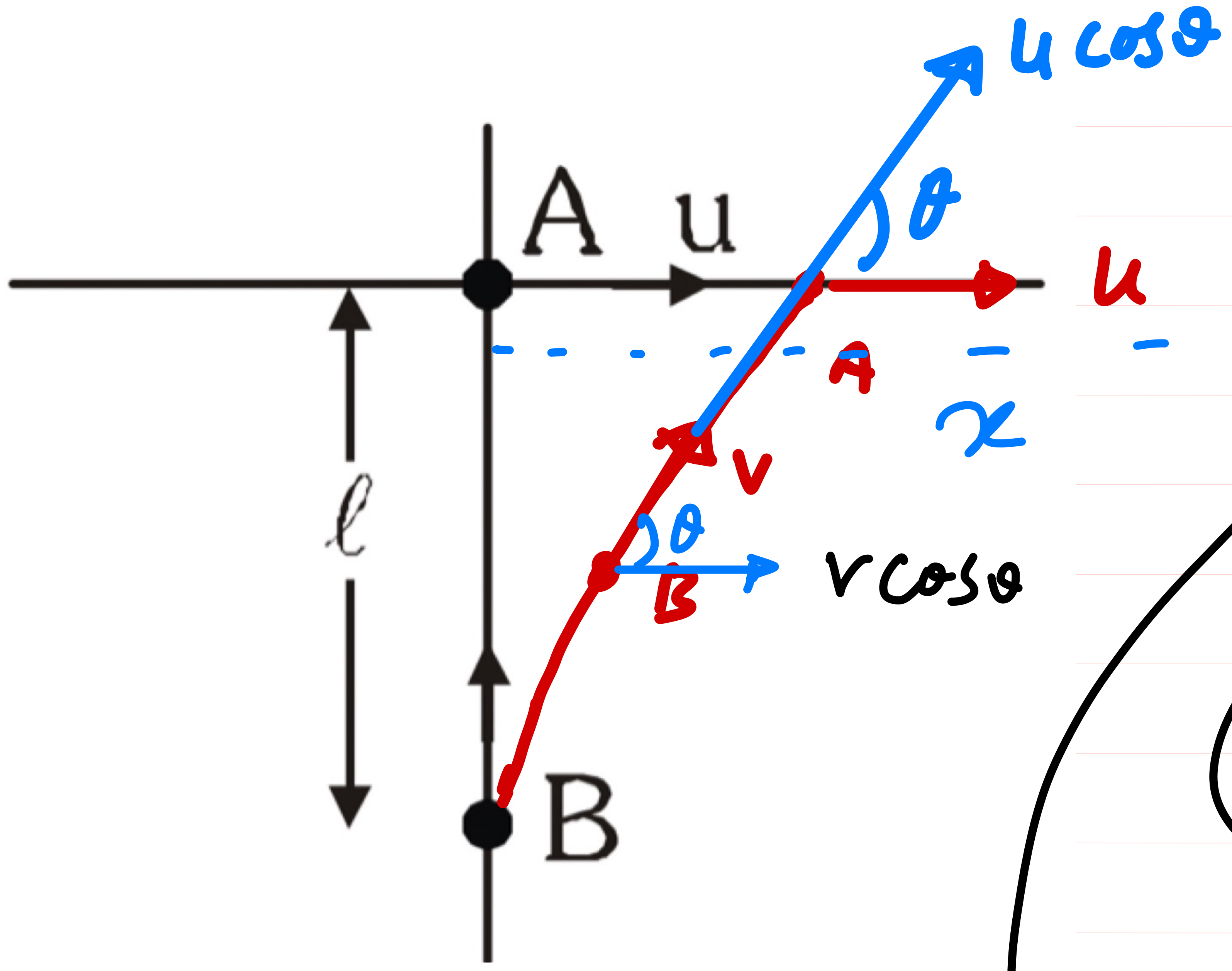
ALLEN CAREER INSTITUTE KOTA (RAJASTHAN) path to success

***Illustration 7*.***

'A' moves with constant velocity u along the 'x' axis. B always has velocity towards A. After how much time will B meet A if B moves with constant speed v. What distance will be travelled by A and B.




$(V_{BA})_{\parallel} = v - u\cos\theta$ { ||del to line joining them}

let they are meeting at point P

FOR – A

$x = ut$ —(1)

FOR B $\quad x = \int_0^t v\cos\theta \, dt$

$x = v\int_0^t \cos\theta \, dt$

$\frac{u}{v}t = \int_0^t \cos\theta \, dt$ —(2)

$-\frac{dl}{dt} = v - u\cos\theta$

$-\int_l^0 dl = v\int_0^t dt - u\int_0^t \cos\theta \, dt$

$l = vt - u\int_0^t \cos\theta \, dt$

$l = vt - u \times \frac{u}{v} t$

$l = \frac{v^2 - u^2}{v} t$

$\boxed{t = \frac{vl}{v^2 - u^2}}$

Ans

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distance by A $\quad x = u \times t$

$$\boxed{x = \frac{uvl}{v^2 - u^2}}$$

distance by B $\quad x' = vt$

$$\boxed{x' = \frac{v^2 l}{v^2 - u^2}}$$

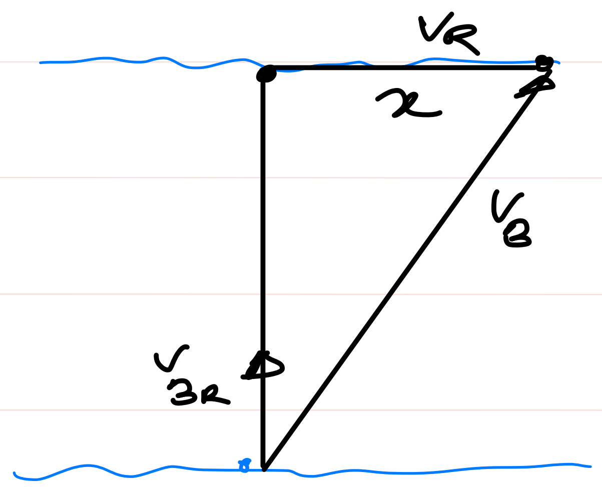
**6.** A man wishes to cross a river in a boat. If he crosses the river in minimum time he takes 10 minutes with a drift of 120 m. If he crosses the river taking shortest route, he takes 12.5 minutes, find velocity of the boat with respect to water

(A) 20 m/min (B) 12 m/min (C) 10 m/min (D) 8 m/min

$t_{min} = \frac{d}{V_{BR}} = 10 \Rightarrow d = 10 \times V_{BR}$ —(1)

$x = V_R \cdot t_{min}$

$120 = V_R \times 10 \Rightarrow \boxed{V_R = 12 \text{ m/min}}$




For shortest route mean $x = 0$

$\left\{ \sin\theta = \frac{V_R}{V_{BR}} \right\}$ $\quad t = \frac{d}{V_{BR}\cos\theta}$

$t = \frac{d}{\cancel{V_{BR}}\frac{\sqrt{V_{BR}^2 - V_R^2}}{\cancel{V_{BR}}}} \Rightarrow t = \frac{d}{\sqrt{V_{BR}^2 - V_R^2}} = 12.5$

$\Rightarrow 12.5 = \frac{10 \times V_{BR}}{\sqrt{V_{BR}^2 - 12^2}}$

$V_{BR}^2 - 12^2 = \left(\frac{100}{125} \times V_{BR}\right)^2$

$V_{BR}^2 - \frac{16}{25}V_{BR}^2 = 12^2$

$\frac{9}{25}V_{BR}^2 = 12^2$

$\frac{3}{5}V_{BR} = 12 \Rightarrow \boxed{V_{BR} = 20 \text{ m/min}}$

**7*.** For four particles A, B, C & D, the velocities of one with respect to other are given as $\vec{V}_{DC}$ is 20 m/s towards north, $\vec{V}_{BC}$ is 20 m/s towards east and $\vec{V}_{BA}$ is 20 m/s towards south. Then $\vec{V}_{DA}$ is

(A) 20 m/s towards north
(B) 20 m/s towards south
(C) 20 m/s towards east
(D) 20 m/s towards west

$\vec{V}_{DC} = 20\hat{j}$

$\vec{V}_D - \vec{V}_C = 20\hat{j}$ — (1)

$\vec{V}_{BC} = 20\hat{i}$

$\vec{V}_B - \vec{V}_C = 20\hat{i}$ — (2)

$\vec{V}_{BA} = 20(-\hat{j})$

$\vec{V}_B - \vec{V}_A = -20\hat{j}$ — (3)

$\vec{V}_{DA} = ?$

$\vec{V}_D - \vec{V}_A$

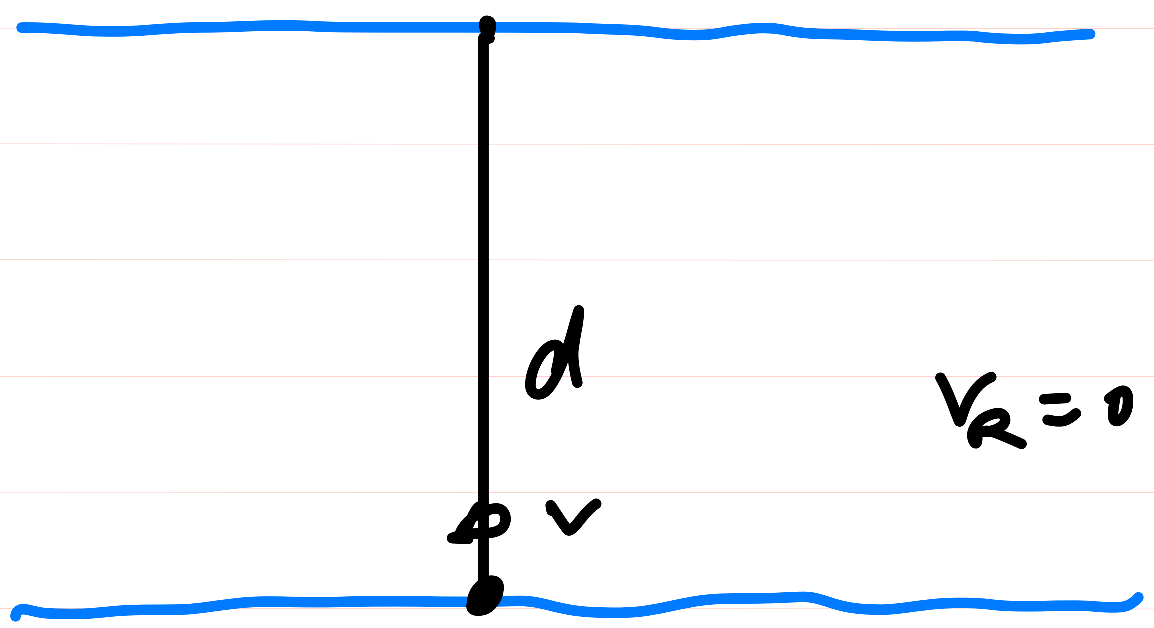
(1) − (2) + (3)

$\vec{V}_D - \cancel{\vec{V}_C} - \cancel{\vec{V}_B} + \cancel{\vec{V}_C} + \cancel{\vec{V}_B} - \vec{V}_A = \cancel{20\hat{j}} - 20\hat{i} - \cancel{20\hat{j}}$

$\vec{V}_D - \vec{V}_A = -20\hat{i}$

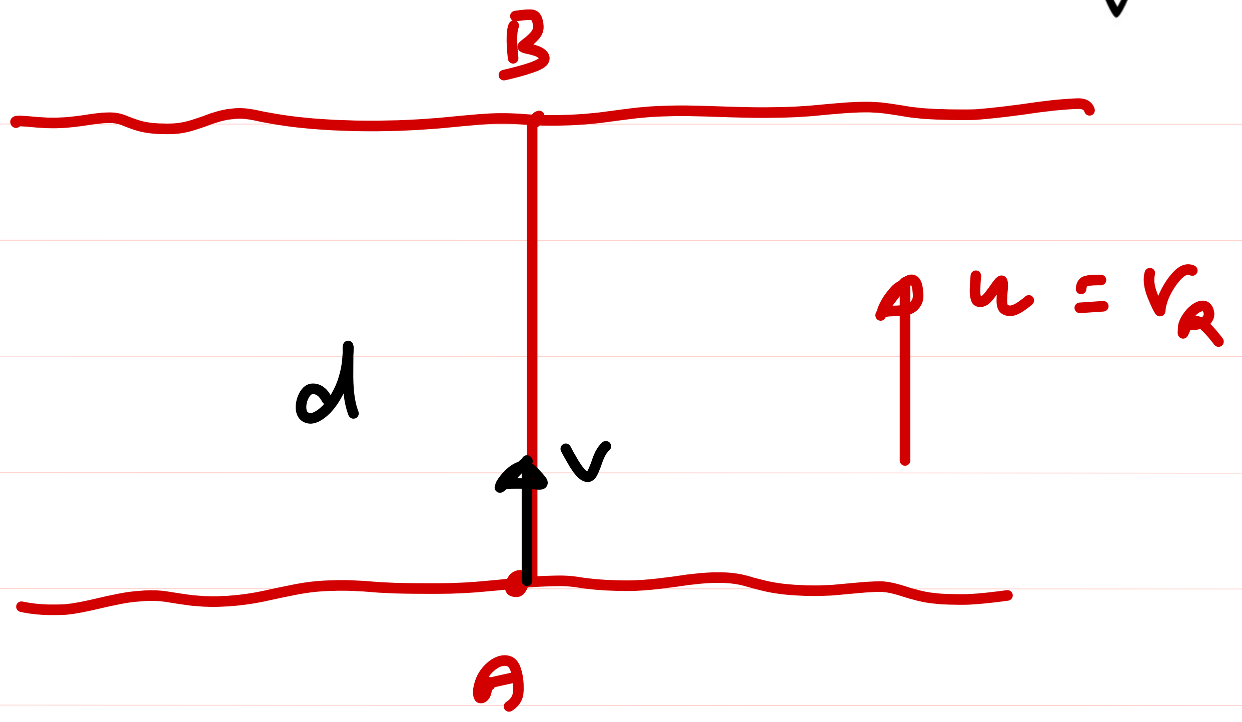
**12.** A boat can go across a lake and return in time $T_0$ at a speed V. On a rough day there is uniform current at speed u to help the onward journey and impede the return journey. If the time taken to go across and return on the rough day be T, then $T/T_0 =$

(A) $1 - \frac{u^2}{V^2}$ (B) $\frac{1}{1-\frac{u^2}{V^2}}$ (C) $1 + \frac{u^2}{V^2}$ (D) $\frac{1}{1+\frac{u^2}{V^2}}$




$V_R = 0$

$T_0 = \frac{2d}{v}$ —(1)




$d = (v+u)\, t_{AB}$

$d = (v-u)\, t_{BA}$

$\frac{d}{v+u} + \frac{d}{v-u} = t_{AB} + t_{BA} = T$

$\frac{2dv}{v^2-u^2} = T$ —(2)

$\frac{(2)}{(1)}$ $\frac{T}{T_0} = \frac{\cancel{2d}\, v}{v^2-u^2} \times \frac{v}{\cancel{2d}}$

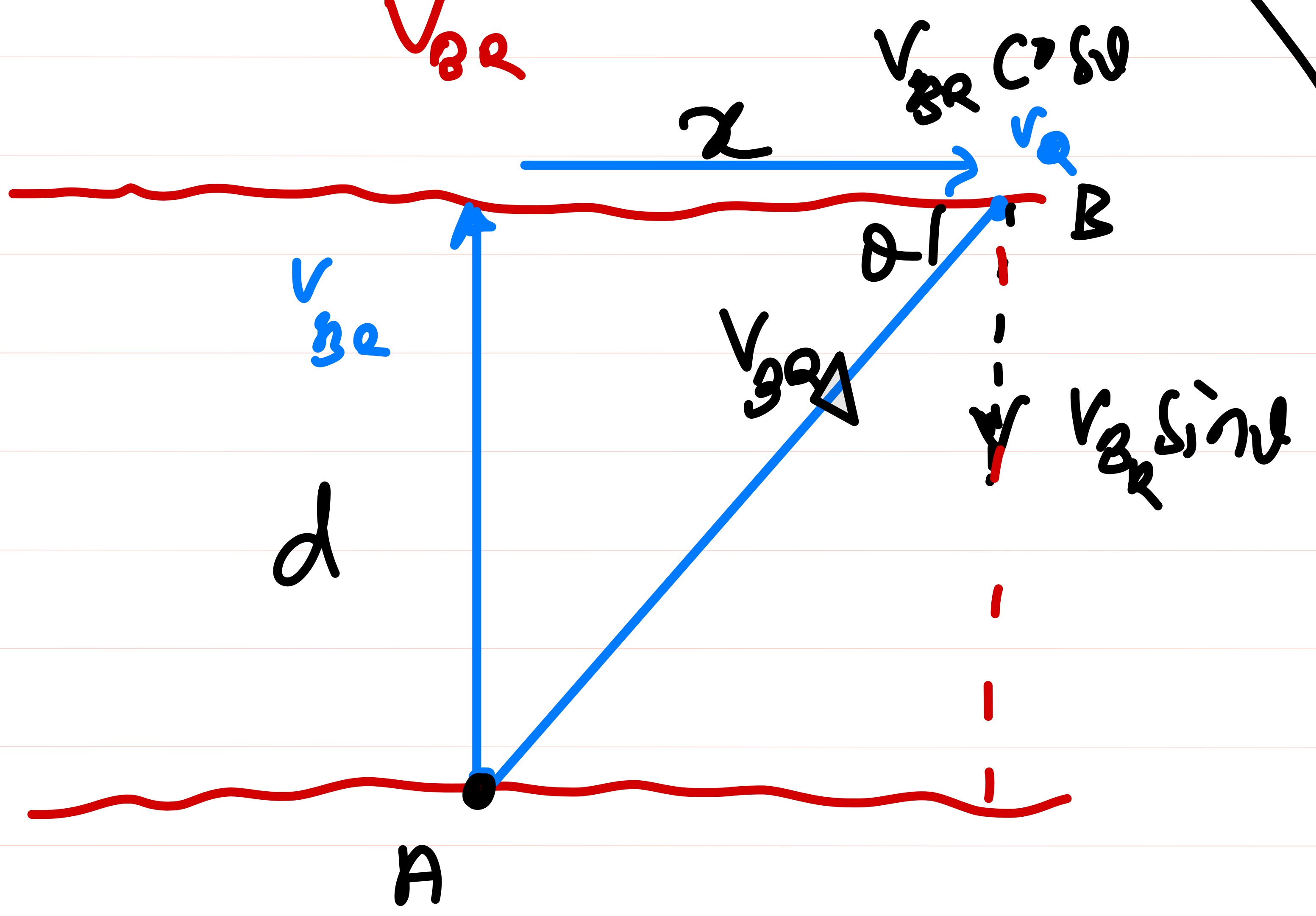
$= \frac{v^2}{v^2-u^2} = \frac{1}{1-u^2/v^2}$

**18.** A swimmer crosses a river with minimum possible time 10 second. And when he reaches the other end starts swimming in the direction towards the point from where he started swimming. Keeping the direction fixed the swimmer crosses the river in 15 sec. The ratio of speed of swimmer with respect to water and the speed of river flow is (Assume constant speed of river & swimmer) :

(A) $\frac{3}{2}$ (B) $\frac{9}{4}$ (C) $\frac{2}{\sqrt{5}}$ (D) $\frac{\sqrt{5}}{2}$

$10 = \frac{d}{V_{BR}}$ — (1)

$x = V_R \cdot 10$ — (2)

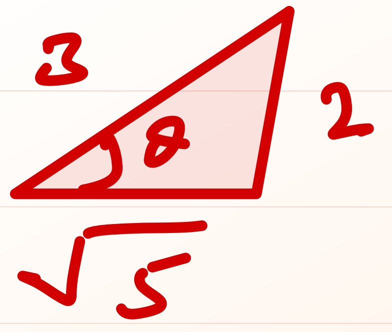



$x = (V_{BR}\cos\theta - V_R) \times 15$ — (3)

$d = V_{BR}\sin\theta \times 15$

$10\cancel{V_{BR}} = \cancel{V_{BR}}\sin\theta \times 15$

$\boxed{\sin\theta = \frac{2}{3}}$




$\tan\theta = \frac{V_{BR}}{V_R} = \frac{2}{\sqrt{5}}$

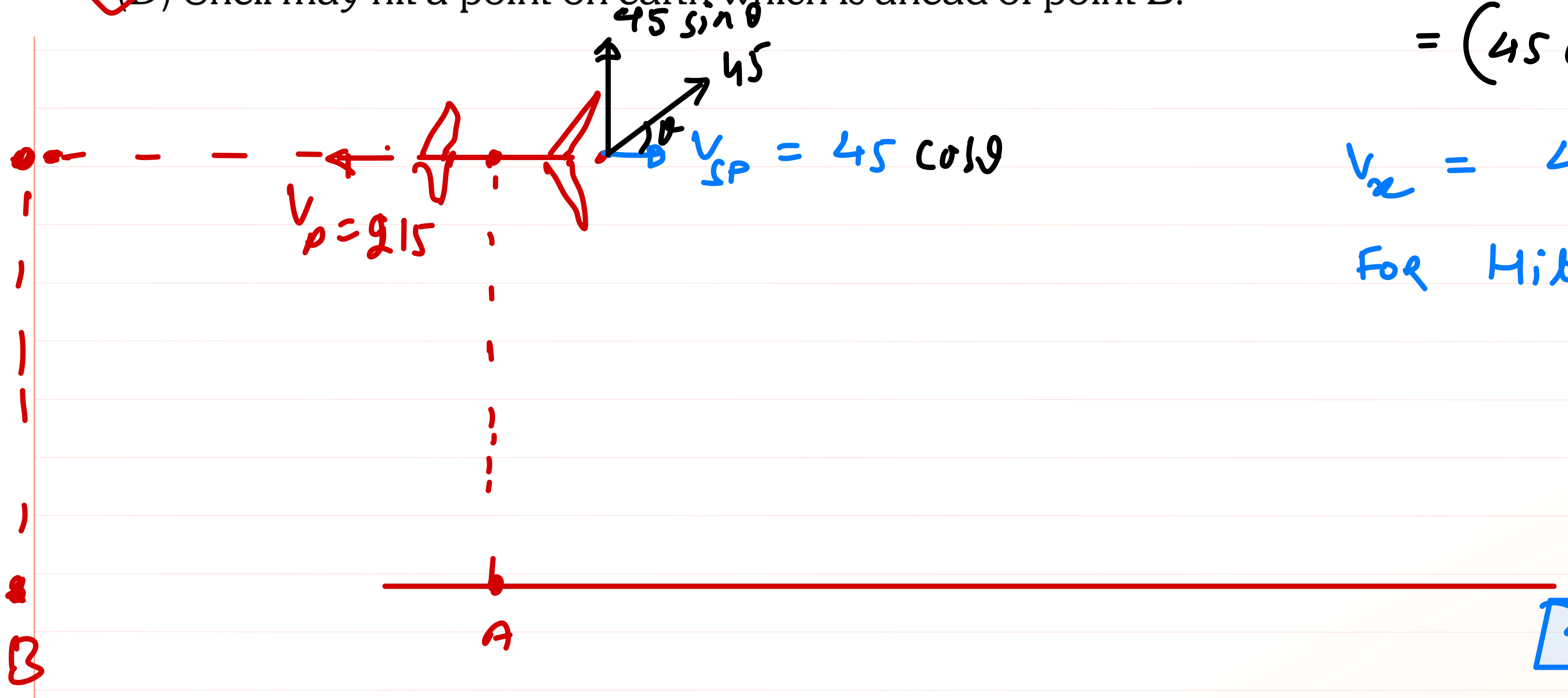
**6.** Consider a shell that has a muzzle velocity of 45 $ms^{-1}$ fired from the tail gun of an airplane moving horizontally with a velocity of 215 $ms^{-1}$. The tail gun can be directed at any angle with the vertical in the plane of motion of the airplane. The shell is fired when the plane is above point A on ground, and the plane is above point B on ground when the shell hits the ground. (Assume for simplicity that the Earth is flat)

(A) Shell may hit the ground at point A.

(B) Shell may hit the ground at point B.

(C) Shell may hit a point on earth which is behind point A.

(D) Shell may hit a point on earth which is ahead of point B.




$$\vec{V}_s = \vec{V}_{sp} + \vec{V}_p$$

$$= (45\cos\theta\,\hat{i} - 215\,\hat{i} + 45\sin\theta\,\hat{j})$$

$V_x = 45\cos\theta - 215$ $\quad V_y = 45\sin\theta$

For Hitting A $\quad x = 0$

$V_x = 0$

$45\cos\theta - 215 = 0$

$\cos\theta = \frac{215}{45}$

$\boxed{x \neq 0}$

Never Hit