(12)

$$\frac{S_{3n} - S_{n-1}}{S_{2n} - S_{2n-1}} = 31$$

$$\frac{\frac{3n}{2}\left[2a + (3n-1)d\right] - \frac{n-1}{2}\left[2a + (n-1-1)d\right]}{a + (2n-1)d} = 31$$

(22)

$p, q, r \rightarrow AP \Rightarrow \quad q = \frac{p+r}{2}$

$px^2 + qx + r = 0$

$D \geq 0 \qquad D = b^2 - 4ac$

$q^2 - 4pr \geq 0$

$\left(\frac{p+r}{2}\right)^2 - 4pr \geq 0$

$\frac{p^2 + r^2 + 2pr}{4} - 4pr \geq 0$

$p^2 + r^2 - 14pr \geq 0$

$\frac{p^2}{r^2} + 1 - 14\frac{p}{r} \geq 0$

$\frac{p^2}{r^2} - 14\left(\frac{p}{r}\right) + 49 \geq 49 - 1$

$\left(\frac{p}{r} - 7\right)^2 \geq 48 \Rightarrow \boxed{\left|\frac{p}{r} - 7\right| \geq 4\sqrt{3}}$

21

$$2\left(\frac{a}{2}\right) = 5^{1+x} + 5^{1-x} + 25^{x} + 25^{-x}$$

$$= \underbrace{5^{1} \cdot 5^{x} + 5^{1} \cdot 5^{-x}} + 5^{2x} + 5^{-2x}$$

$$a = 5\underbrace{\left(5^{x} + \frac{1}{5^{x}}\right)}_{\geq 2} + \underbrace{\left(5^{2x} + \frac{1}{5^{2x}}\right)}_{\geq 2}$$

$$a \geq 5(2) + 2 \Rightarrow \boxed{a \geq 12}$$

⑥ $$\frac{1}{a_1 a_2} + \frac{1}{a_2 a_3} + \cdots$$

$$\frac{1}{d}\left[\frac{a_2-a_1}{a_1 a_2} + \frac{a_3-a_2}{a_2 a_3} + \frac{a_4-a_3}{a_4 \cdot a_3} + \cdots\right]$$

$$\frac{1}{d}\left[\frac{a_2}{a_1 a_2} - \frac{a_1}{a_1 a_2} + \right.$$

(16) $a, b, c, \rightarrow AP \Rightarrow a \rightarrow a, \quad b \rightarrow a+d, \quad c \rightarrow a+2d$

$a\left(\frac{1}{b}+\frac{1}{c}\right), \quad b\left(\frac{1}{c}+\frac{1}{a}\right), \quad c\left(\frac{1}{a}+\frac{1}{b}\right),$ ✓

$a\left(\frac{1}{b}+\frac{1}{c}\right)$, $b\left(\frac{1}{c}+\frac{1}{a}\right)$, $c\left(\frac{1}{a}+\frac{1}{b}\right)$

add 1 with all term

$a\left(\frac{1}{b}+\frac{1}{c}\right)+1 \rightarrow a/a$, $b\left(\frac{1}{c}+\frac{1}{a}\right)+1 \rightarrow b/b$, $c\left(\frac{1}{a}+\frac{1}{b}\right)+1 \rightarrow \frac{c}{c}$

$a\left(\underbrace{\frac{1}{b}+\frac{1}{c}+\frac{1}{a}}\right)$, $b\left(\underbrace{\frac{1}{c}+\frac{1}{a}+\frac{1}{b}}\right)$, $c\left(\underbrace{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}}\right) \rightarrow$ A.P.

divide by $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}$

$a, b, c \rightarrow$ A.P.

18

$$2\log_{10}(2^x-1) = \log_{10}2 + \log_{10}(2^x+3)$$

$$(2^x-1)^2 = 2(2^x+3)$$

15

$$\frac{S_{n_1}}{S_{n_2}} = \frac{\cancel{\frac{n}{2}}[2a_1+(n-1)d_1]}{\cancel{\frac{n}{2}}[2a_2+(n-1)d_2]} = \frac{7n+1}{4n+27}$$

$$\frac{a_1+\left(\frac{n-1}{2}\right)d_1}{a_2+\left(\frac{n-1}{2}\right)d_2} = \frac{7n+1}{4n+27}$$

$$\frac{T_{m_1}}{T_{m_2}} = \frac{a_1+(m-1)d_1}{a_2+(m-1)d_2}$$

$$a_1 + \qquad = \frac{7(2m-1)+1}{4(2m-1)+27}$$

$$\frac{n-1}{2} = m-1$$

$$n-1 = 2m-2$$

$$n = \underline{2m-1}$$

Q 20

Number $\rightarrow$ abc $\rightarrow$ $100a + 10b + c$

digits $\rightarrow$ A.P.

$2b = a + c$

① $S_a = a^2c \Rightarrow$ $\frac{\cancel{a}}{2}\left[2a_1 + (a-1)d\right] = a^{\cancel{2}}c \Rightarrow$

$$\boxed{2a_1 + (a-1)d = 2ac}$$

$$2a_1 + (b-1)d = 2bc$$

$$(a-1-b+1)\,d = 2ac - 2bc$$

$$(a-b)d = 2c(a-b)$$

$$\boxed{d = 2c}$$

$$\cancel{2}a_1 + (a-1)(\cancel{2}c) = \cancel{2}ac$$

$$a_1 + a\cancel{c} - c = \cancel{a}\cancel{c}$$

$$\boxed{a_1 = c}$$

$$S_c = \frac{c}{2}\left[2a_1 + (c-1)d\right]$$

$$= \frac{c}{2}\left[2c + (c-1)2c\right]$$

$$= \frac{c}{\cancel{2}}\left[2\cancel{c} + \cancel{2}c^2 - \cancel{2c}\right]$$

$$= c^3$$

② $K = a_1 + a_{2n} = a_2 + a_{2n-1} = a_3 + a_{2n-2} = a_4 + a_{2n-3}$

$$\frac{K}{\quad} + \frac{K}{\quad} + \frac{K}{\quad} + - - -$$

$$K\left[\frac{1}{\sqrt{a_1}+\sqrt{a_2}} + \frac{1}{\sqrt{a_2}+\sqrt{a_3}} + \frac{1}{\sqrt{a_3}+\sqrt{a_4}} + - - - - - \frac{1}{\sqrt{a_{n+1}}+\sqrt{a_n}}\right]$$

$\sqrt{a_2}-\sqrt{a_1}$ $\quad$ $\sqrt{a_3}-\sqrt{a_2}$

$\sqrt{a_2}-\sqrt{a_1}$ $\quad$ $\sqrt{a_3}-\sqrt{a_2}$

$$= K\left[\frac{\sqrt{a_2}-\sqrt{a_1}}{a_2-a_1} + \frac{\sqrt{a_3}-\sqrt{a_2}}{a_3-a_2} + \frac{\sqrt{a_4}-\sqrt{a_3}}{a_4-a_3} + - - \right.$$

$$= \frac{K}{d}\left[\cancel{\sqrt{a_2}} - \underline{\sqrt{a_1}} + \cancel{\sqrt{a_3}} - \cancel{\sqrt{a_2}} + \cancel{\sqrt{a_4}} - \cancel{\sqrt{a_3}} + - - - + \cancel{\sqrt{a_n}} - \sqrt{a_{n-1}} + \underline{\sqrt{a_{n+1}}} - \cancel{\sqrt{a_n}}\right]$$

$$= \frac{K}{d}\left[\frac{\left(\sqrt{a_{n+1}}-\sqrt{a_1}\right)\left(\sqrt{a_{n+1}}+\sqrt{a_1}\right)}{\sqrt{a_{n+1}}+\sqrt{a_1}}\right] = \frac{K}{d}\left[\frac{a_{n+1}-a_1}{\sqrt{a_{n+1}}+\sqrt{a_1}}\right]$$

$$= \frac{K}{d}\left[\frac{\cancel{a}+(nd)-\cancel{a}}{\sqrt{a_{n+1}}+\sqrt{a_1}}\right] = \frac{nK}{\sqrt{a_{n+1}}+\sqrt{a_1}} = \frac{n(a_1+a_{2n})}{\sqrt{a_{n+1}}+\sqrt{a_n}}$$

(3) $\frac{1}{K}\left[(a_{2n+1}-a_1)+(a_{2n}-a_2)+(a_{2n-1}-a_3)+(a_{2n-2}-a_4)+\cdots+(a_{n+2}-a_n)\right]$

$= \frac{1}{K}\left[(\cancel{a_1}+2nd-\cancel{a_1}) + (\cancel{a_1}+(2n-1)d-(\cancel{a_1}+d)) + (a_1+(2n-2)d-(a_1+2d)) + \cdots + (a_1+(n+1)d-a_1+(n-1)d)\right]$

$= \frac{d}{K}\left[(2n)+(2n-2)+(2n-4)+(2n-6)+\cdots+2\right]$

$= \frac{2d}{K}\left[n+(n-1)+(n-2)+(n-3)+\cdots+1\right]$

$= \frac{2d}{K}\left[1+2+3+\cdots+(n-2)+(n-1)+n\right]$

$= \frac{2d}{K}\left[\frac{n}{2}(2+(n-1)1)\right] = \frac{\cancel{2}d}{K}\frac{n(n+1)}{\cancel{2}} = \frac{n(n+1)d}{K}$

$$= \frac{n(n+1)}{K} d$$

$$= \frac{n(n+1)}{2(a_{n+1})} \cdot (a_2 - a_1)$$

$$K = a_1 + a_{2n+1}$$

$$= a_1 + \underbrace{a_1 + (2n\,d)}_{}$$

$$= 2a_1 + 2nd$$

$$= 2(\underbrace{a_1 + nd}_{})$$

$$= 2(a_{n+1})$$

④

$$\frac{\cancel{\frac{p}{2}}\left[2a_1 + (p-1)d\right]}{\cancel{\frac{q}{2}}\left[2a_1 + (q-1)d\right]} = \frac{p^2}{q^2}$$

$$\frac{a_6}{a_{21}} = \frac{a_1 + 5d.}{a_1 + 20d}$$

$$\frac{a_1 + \left(\frac{p-1}{2}\right)d}{a_1 + \left(\frac{q-1}{2}\right)d} = \frac{p}{q}$$

$$\frac{p-1}{2} = 5 \qquad \frac{q-1}{2} = 20$$

(10)

$$T_n = S_n - S_{n-1}$$

$$= (3n^2+5n) - (3(n-1)^2 + 5(n-1))$$

$$= (\cancel{3n^2} + \cancel{5n}) - [\cancel{3n^2} + 3 - \underline{6n} + \cancel{5n} - 5]$$

$$T_n = 6n + 2 = 164$$

$$\boxed{n = 27}$$

(12)

$a \rightarrow$ 1st

$$T_3 = a + 2d = b \Rightarrow d = \frac{b-a}{2}$$

$$T_n = a + (n-1)d = c$$

15 $(2+4+6+8+ \cdots + 200) - (6+12+18+ \cdots + 198)$

17 $$\frac{A_7}{A_{n-1}} = \frac{a+7d}{a+(n-1)d} = \frac{5}{9}$$

$$= 1 +$$