Trigonometric Ratios and Identities

Lecture - 3

cosec θ → csc θ

(10) $\sqrt{\dfrac{1-\sin A}{1+\sin A}} + \dfrac{\sin A}{\cos A} = \dfrac{1}{\cos A}$

$\sqrt{\dfrac{1-\sin A}{1+\sin A} \cdot \dfrac{1-\sin A}{1-\sin A}} + \dfrac{\sin A}{\cos A}$

$\sqrt{\dfrac{(1-\sin A)^2}{1-\sin^2 A}} + \dfrac{\sin A}{\cos A}$

$\sin\theta \le 1 \Rightarrow 0 \le 1-\sin\theta$

$1-\sin\theta \ge 0$

$\sqrt{\cos^2 A} = |\cos A|$

$=$

$\dfrac{(1-\sin A)}{|\cos A|} + \dfrac{\sin A}{\cos A}$

$-1 \le \sin\theta \le 1$

$-1 \le \cos\theta \le 1$

$\tan\theta \in (-\infty, \infty)$

$\operatorname{cosec}\theta \in (-\infty, -1] \cup [1, \infty)$

$\sec\theta \in (-\infty, -1] \cup [1, \infty)$

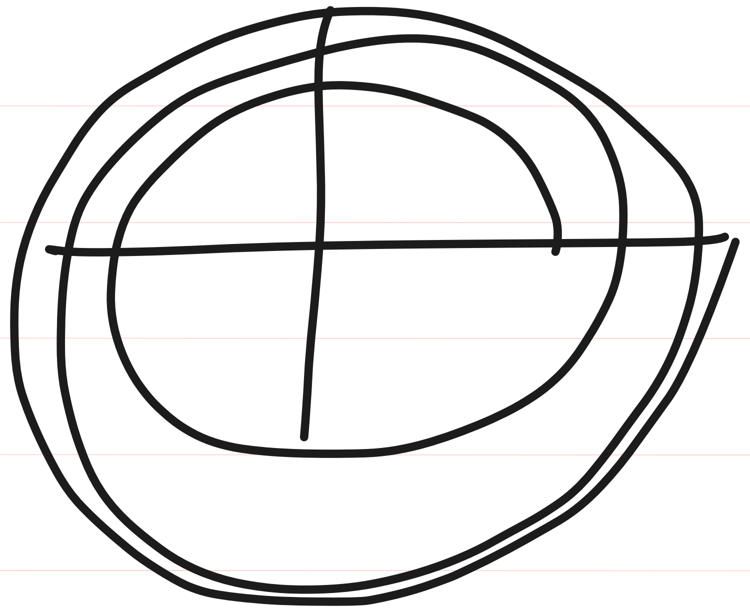
$\cot\theta \in (-\infty, \infty)$

(18) $\sin\left((4k+1)\frac{\pi}{2}-\theta\right)$

$= \sin\left(2k\pi + \frac{\pi}{2} - \theta\right)$

$= \sin\left(\frac{\pi}{2} - \theta\right)$

$= \cos\theta$



E(1) $\tan\dfrac{3\pi}{4} = \tan\left(\pi - \dfrac{\pi}{4}\right) = -\tan\dfrac{\pi}{4} = -1$

$\dfrac{3\pi}{4} = \dfrac{4\pi - \pi}{4} = \dfrac{4\pi}{4} - \dfrac{\pi}{4} = \pi - \dfrac{\pi}{4}$

E(2) $\cos\left(\dfrac{11\pi}{6}\right) = \dfrac{\sqrt{3}}{2}$ $\quad \cos\left(\dfrac{12\pi - \pi}{6}\right) = \cos\left(2\pi - \dfrac{\pi}{6}\right) = \cos\dfrac{\pi}{6} = \dfrac{\sqrt{3}}{2}$

E(3) $\text{cosec}\left(\dfrac{4\pi}{3}\right) = \cancel{\dfrac{\;}{\;}}$ $= \text{cosec}\left(\pi + \dfrac{\pi}{3}\right) = -\text{cosec}\dfrac{\pi}{3} = \dfrac{-2}{\sqrt{3}}$

E(4) If $A = \sin\theta + \sin\left(\dfrac{\pi}{2} + \theta\right)$, $B = \cos\theta + \cos\left(\dfrac{\pi}{2} + \theta\right)$ then prove that $A^2 + B^2 = 2$

(4) $A = \sin\theta + \cos\theta$

$B = \cos\theta - \sin\theta$

$A^2 + B^2 = \underline{\sin^2\theta + \cos^2\theta} + \cancel{2\sin\theta\cos\theta}$
$+ \underline{\cos^2\theta + \sin^2\theta} - \cancel{2\sin\theta\cos\theta}$

$A^2 + B^2 = 2$

E(5) $\sec(2001\,\pi) = -1$

E(6) $\tan(11\pi) = 0$

E(7) $\tan\left(\dfrac{11\pi}{6}\right) = -\dfrac{1}{\sqrt{3}}$

E(8) $\cot\left(-\dfrac{5\pi}{6}\right) = \sqrt{3}$

E(9) $\log(\cos(12346\pi)) = 0$

E(10) $\sin\left(\theta - \dfrac{\pi}{2}\right) = -\cos\theta$

E(11) No. of solⁿ of the eqⁿ. $(3\sin\theta - 1)(\sin\theta - 2) = 0$ in $(0, \pi)$ $\quad$ [**Sol.** $\sin\theta = \dfrac{1}{3} \Rightarrow 2$ solutions]

E(12) $\log_2 \text{cosec}\left(\dfrac{3\pi}{4}\right) = \dfrac{1}{2}$

$\text{cosec}\,\dfrac{3\pi}{4} = \text{cosec}\left(\pi - \dfrac{\pi}{4}\right) = +\,\text{cosec}\left(\dfrac{\pi}{4}\right) = \sqrt{2}$

$\log_2(\sqrt{2}) = \dfrac{1}{2}\log_2 2 = \dfrac{1}{2}$ $\quad$ Ans

E(5) $\sec(2001\pi) = -1$

E(6) $\tan(11\pi) = 0$

E(7) $\tan\left(\frac{11\pi}{6}\right) = -\frac{1}{\sqrt{3}}$

E(8) $\cot\left(-\frac{5\pi}{6}\right) = \sqrt{3}$

E(9) $\log(\cos(12346\pi)) = 0$

E(10) $\sin\left(\theta - \frac{\pi}{2}\right) = -\cos\theta$

(5) $\sec(2001\pi) = \sec(2000\pi + \pi)$

$= \sec(2(1000)\pi + \pi)$

$= \underbrace{\sec \pi}$

$= \sec(\pi + 0)$

$= -\sec 0$

$= -1$

(8) $\cot\left(-\frac{5\pi}{6}\right)$

$= -\cot\left(\frac{5\pi}{6}\right)$

$= -\cot\left(\pi - \frac{\pi}{6}\right)$

$= +\cot\left(\frac{\pi}{6}\right) = +\sqrt{3}$

(10) $\sin\left(-\left(\frac{\pi}{2} - \theta\right)\right)$

$= -\sin\left(\frac{\pi}{2} - \theta\right)$

$= -\cos\theta$

E(13) If $2^n = \text{cosec}\left(\frac{5\pi}{6}\right)$, then n is

13) $2^n = \text{cosec}\left(\pi - \frac{\pi}{6}\right) \Rightarrow 2^n = \text{cosec}\left(\frac{\pi}{6}\right) = 2$

$n = 1$

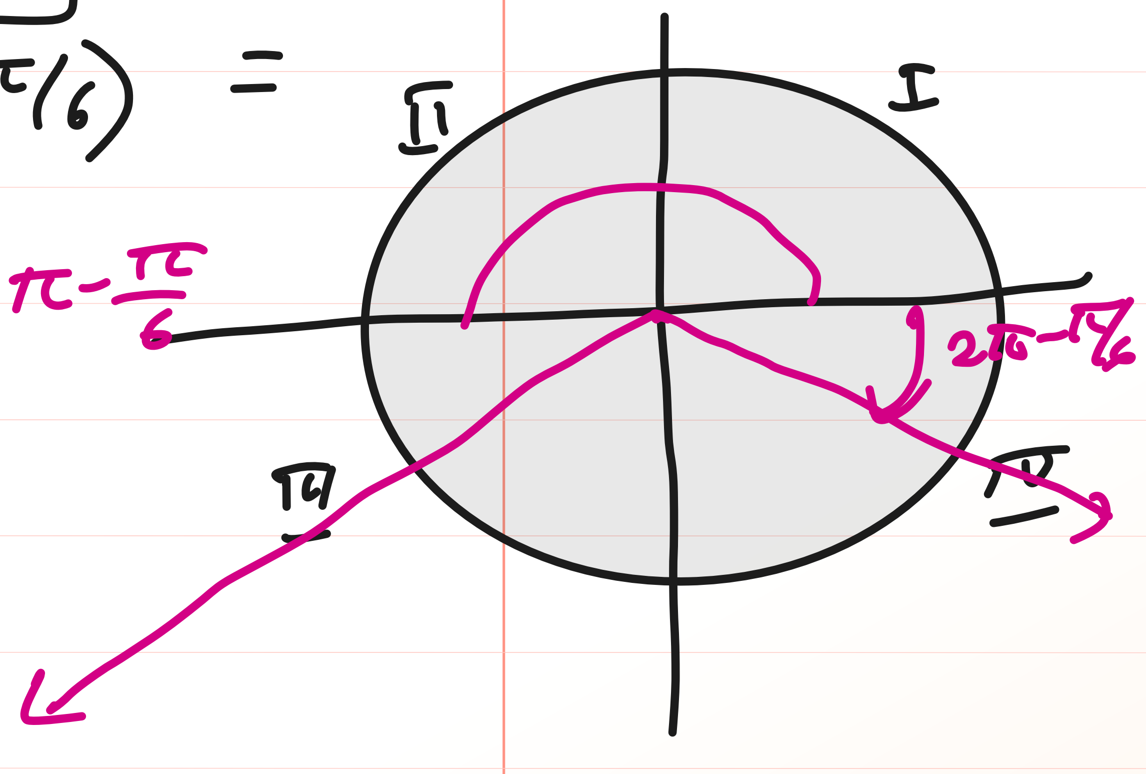
E(14) If $\sin\theta = -\frac{1}{2}$ then find the two possible values of θ in $(0, 2\pi)$

14) $\sin\theta = -\frac{1}{2}$

$\sin\left(\pi + \pi/6\right) =$

$\pi + \frac{\pi}{6},\ 2\pi - \frac{\pi}{6}$

$\sin \pi/6 \rightarrow \frac{1}{2}$



E(15) If $\cos\theta = -1$ then no. of sol^n of equation in $(-2\pi, 2\pi)$ is

E(16) If $\sin\theta + \cos\theta = 0$ & θ lies in fourth quadrant find θ.

E(17) Find the value of θ satisfying both $\sin\theta = -\frac{1}{2}$ & $\tan\theta = \frac{1}{\sqrt{3}}$ in $[0, 2\pi]$

E(18) (i) $\cos 1 - \sin 1 > 0$. T/F **[Ans.** F]

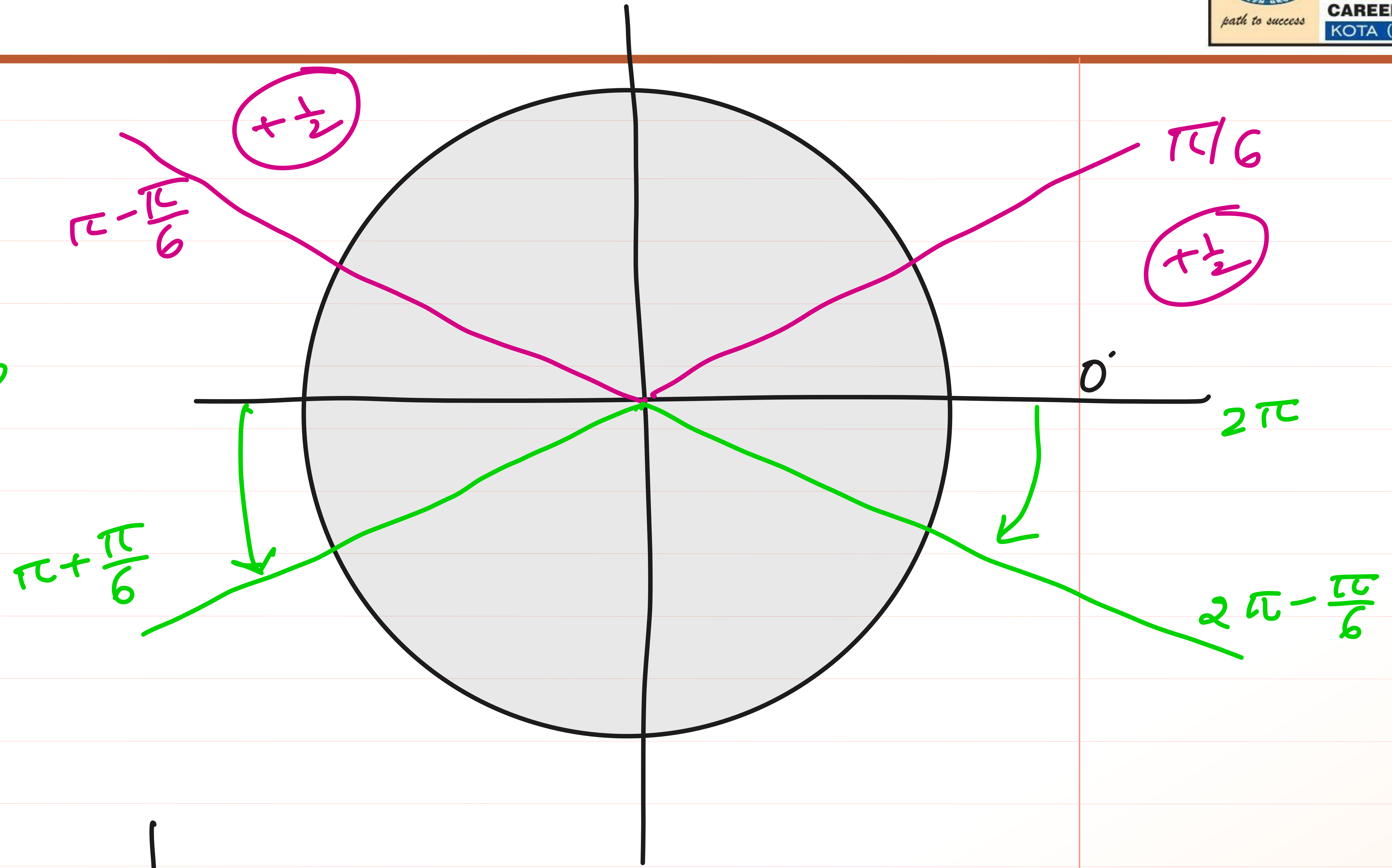
(ii) Find the sign of $\sin 140° + \cos 140°$ **[Ans.** –ve]

E(19) $\frac{\sin 4}{\sin 6}$ is negative. T/F **[Ans.** F]

Q $\sin\theta = -\frac{1}{2}$

$\theta = \pi + \frac{\pi}{6} = \frac{7\pi}{6} = 210^\circ$

$\theta = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6} = 330^\circ$



15 $\cos(\pi) = -1$

$\cos(\pi - 0) =$

$\cos(-\pi) = \cancel{(-)}\ \cos\pi = -1$

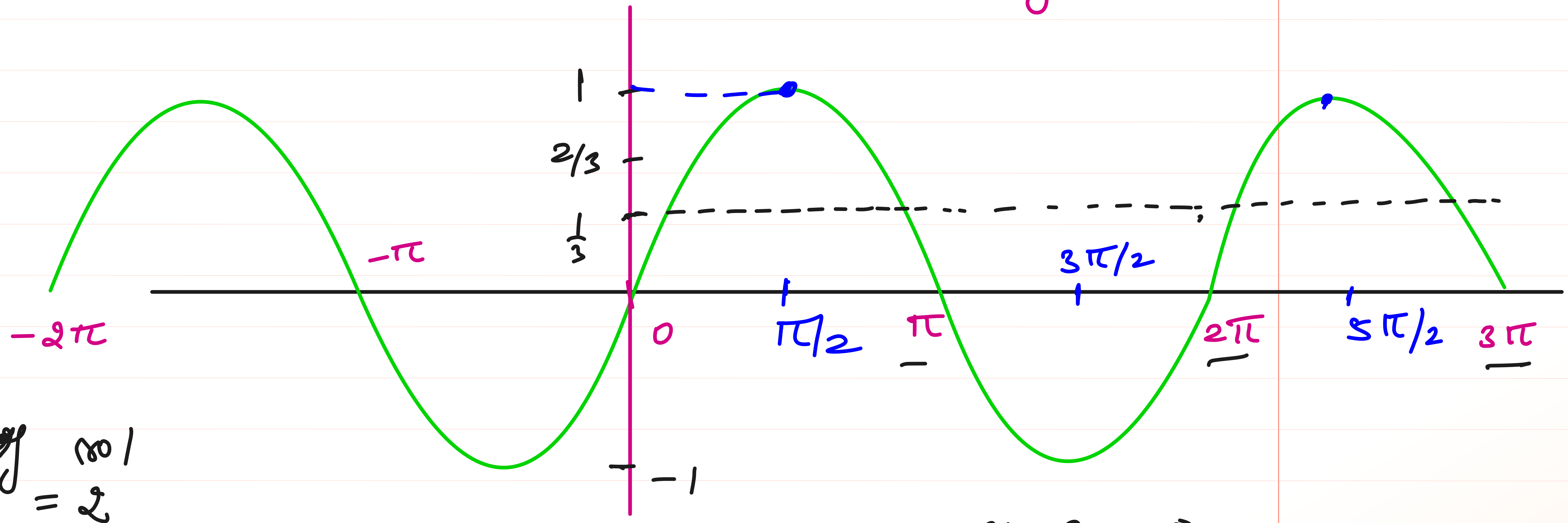
$x \in \{-\pi, \pi\}$

$\cos(\pi) = \cos(\pi + 0) = -\cos 0 = -1$

$\cos(-\pi) = +\cos(\pi) = -1$

$\sin\theta = \frac{1}{3}$

$y = \sin x$



$[0, \pi] \rightarrow$ No. of sol $= 2$

$[0, 2\pi] \Rightarrow$ No. of sol $= 2$

$[0, 3\pi] \rightarrow$ No. of sol $= 4$

$[-2\pi, 3\pi] \rightarrow$ No. of sol $= 6$

$\sin\theta = -1$

$\theta \in \left\{\frac{3\pi}{2}, \frac{7\pi}{2}, \frac{11\pi}{2}, \ldots\right\}$

$\theta \in \left\{(4n-1)\frac{\pi}{2}, n \in z\right\}$

$\sin\theta = 0$

$\theta = \{n\pi, n \in \text{Integer}\}$

$\sin\theta = 1$

$\theta \in \left\{\frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \ldots\right\}$

$\theta = \left\{(4n+1)\frac{\pi}{2}, n \in z\right\}$