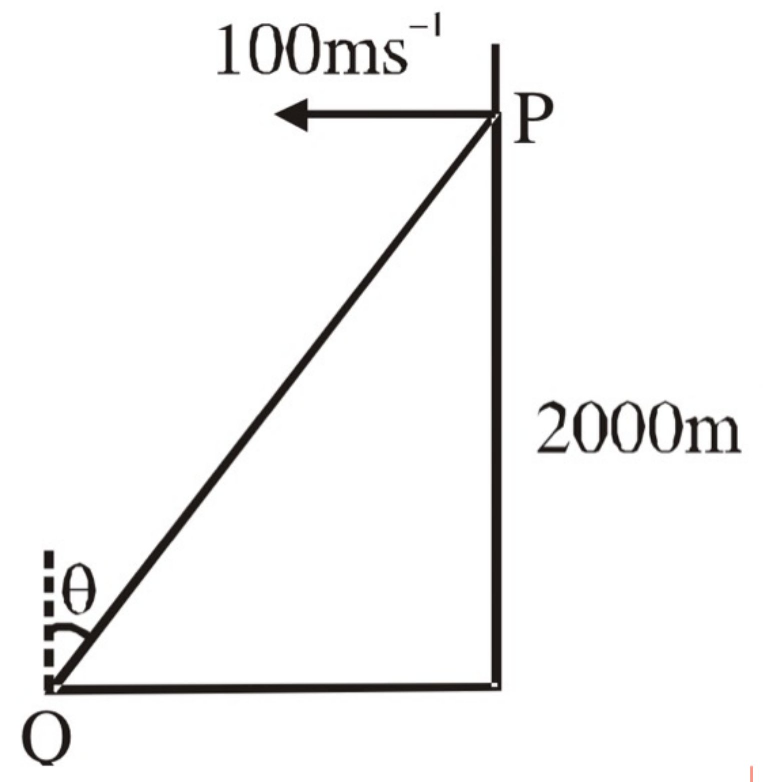
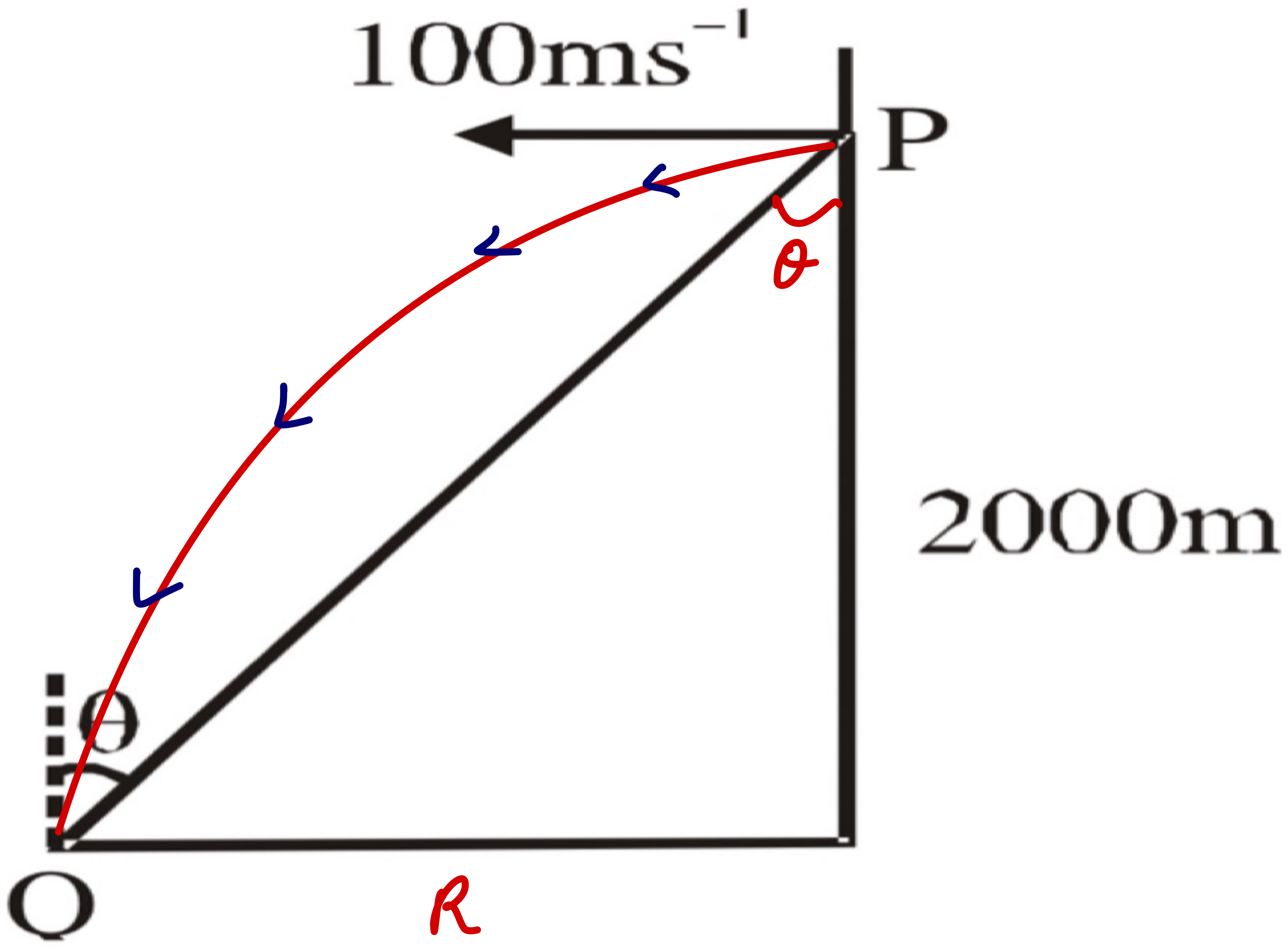
Illustration 3.

An aeroplane is travelling horizontally at a height of 2000 m from the ground. The aeroplane, when at a point P, drops a bomb to hit a stationary target Q on the ground. In order that the bomb hits the target, what angle θ must the line PQ make with the vertical ? [$g = 10 ms^{-2}$]



(A) 15° (B) 30°
(C) 90° (D) 45° ✓



$\tan\theta = \frac{R}{2000} = \frac{2000}{2000} = 1 \Rightarrow \tan\theta = 1$

$\theta = 45°$

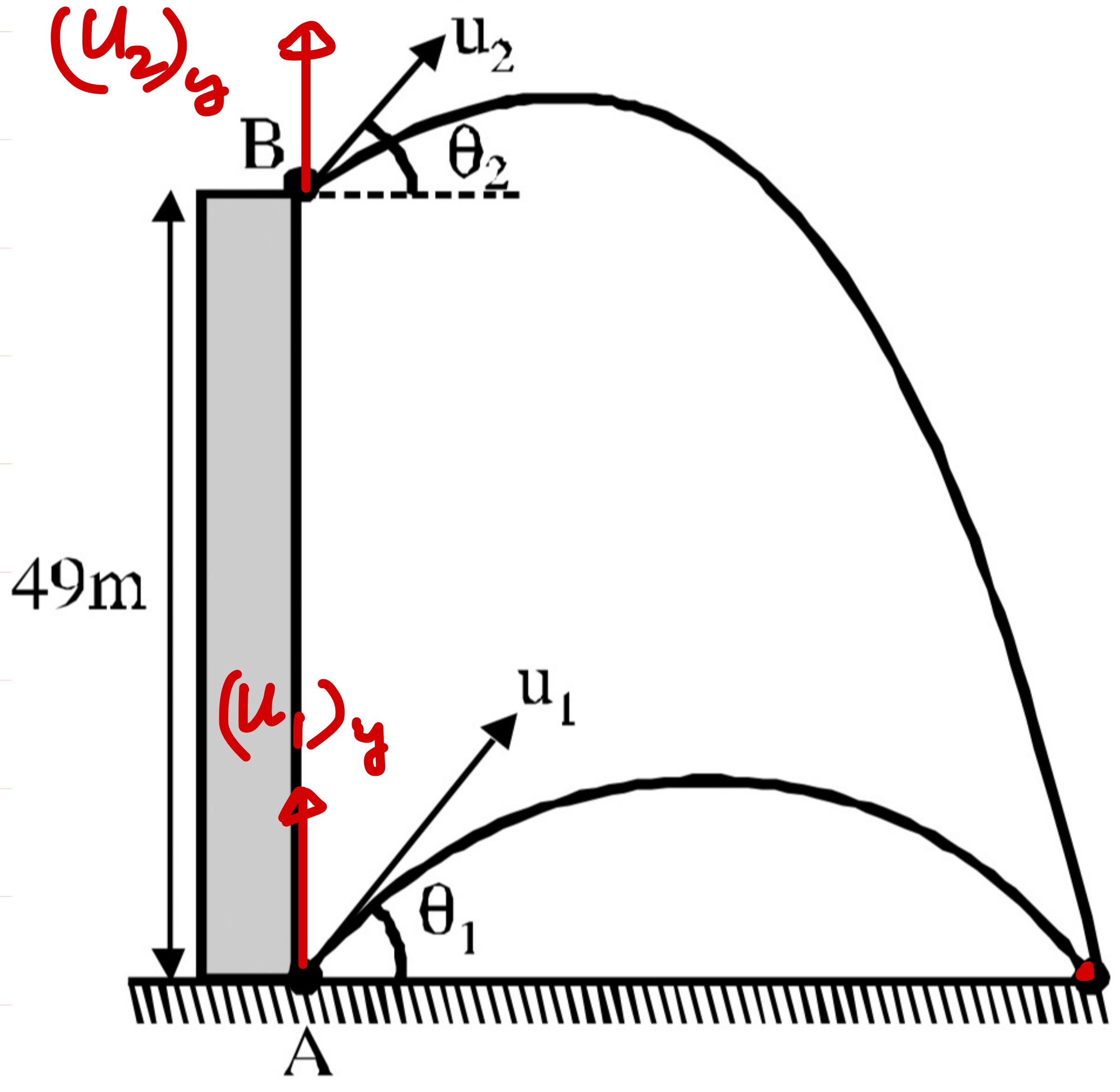
$R = u\sqrt{\frac{2h}{g}}$

$= 100\sqrt{\frac{2\times 2000}{10}}$

$R = 2000$

Illustration 8.

Two stones A and B are projected simultaneously as shown in figure. It has been observed that both the stones reach the ground at the same place after 7 sec of their projection. Determine difference in their vertical components of initial velocities in m/s. ($g = 9.8\ m/s^2$)



For - A

$$t_A = 7 = \frac{2(u_1)_y}{g} \Rightarrow (u_1)_y = \frac{7g}{2}$$

For - B

$$y = u_y t + \frac{1}{2} a_y t^2$$

$$-49 = (u_2)_y \cdot 7 - \frac{1}{2} g\, 7^2$$

$$-7 = (u_2)_y - \frac{7}{2} g$$

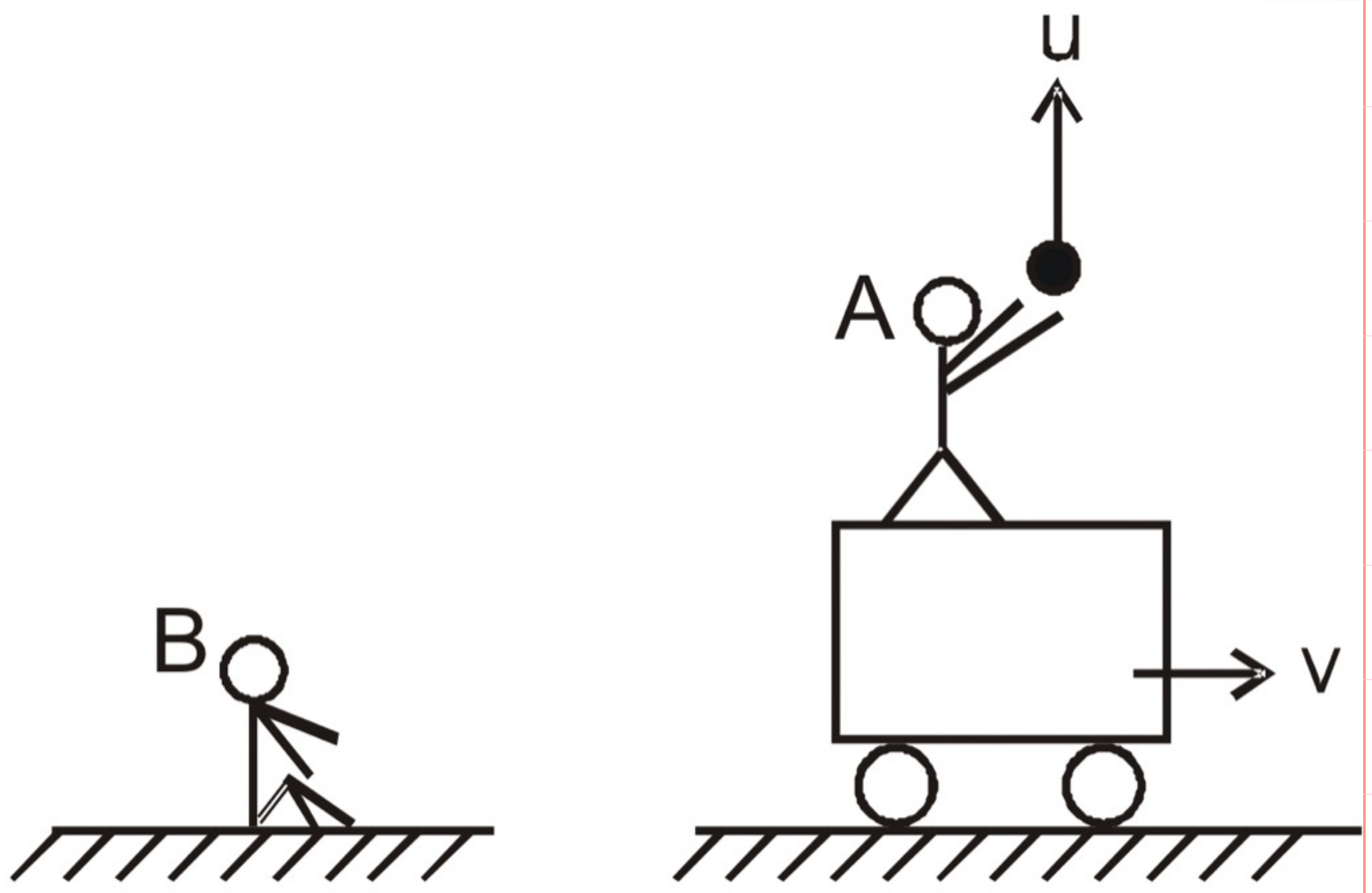
$$\boxed{(u_1)_y - (u_2)_y = 7}$$

$$\boxed{-7 = (u_2)_y - (u_1)_y}$$ Ans

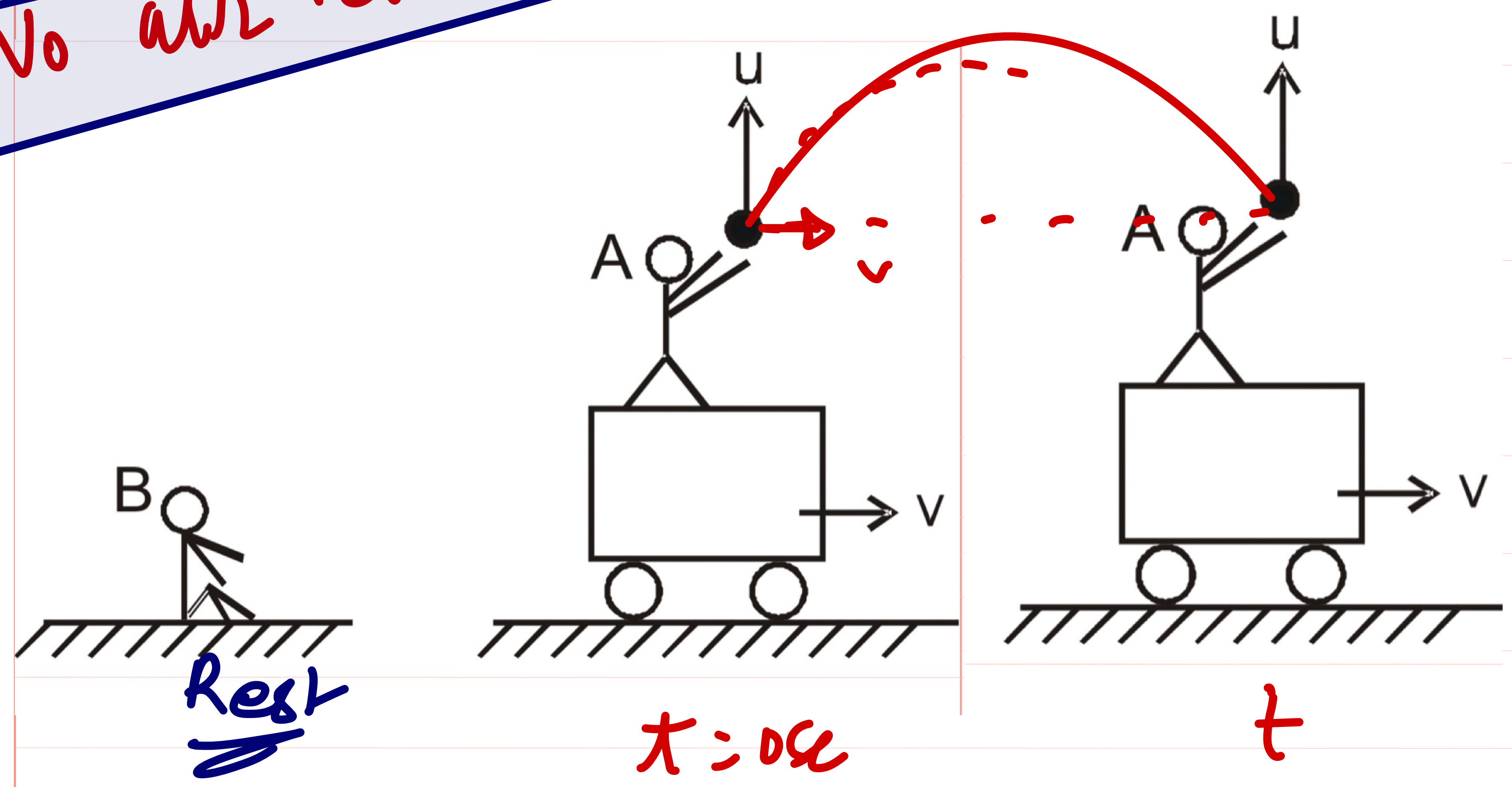
PROJECTION FROM A MOVING PLATFORM :→

Case (1) – When a ball is thrown upward from a truck moving with uniform speed, then observer A standing in the truck, will see the ball moving in straight vertical line (upward & downward).

The observer B sitting on road, will see the ball move in a parabolic path. The horizontal speed of the ball is equal to the speed of the truck.

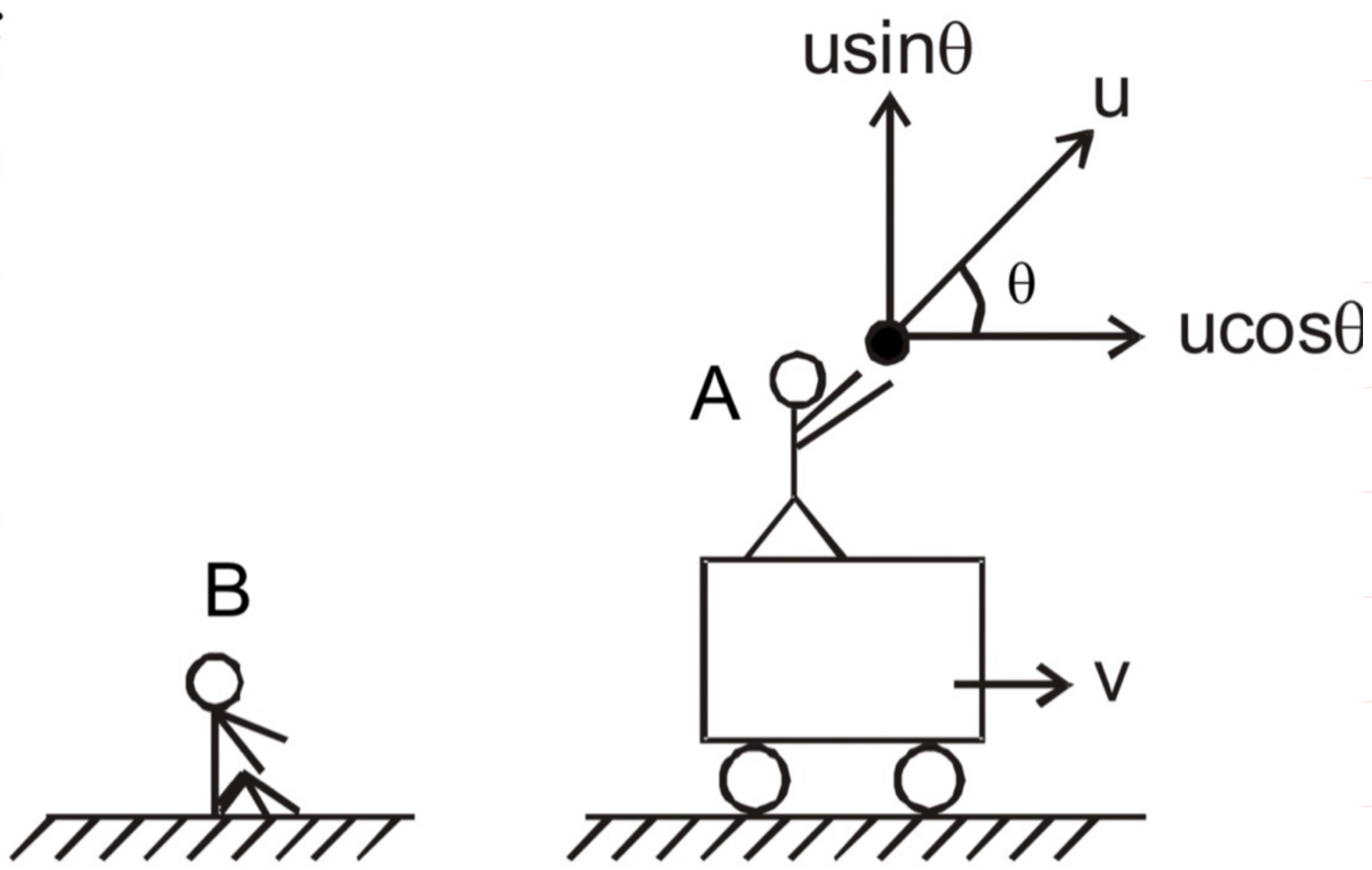


No air Resistance



Case (2) – When a ball is thrown at some angle 'θ' in the direction of motion of the truck, horizontal & vertical component of ball's velocity w.r.t. observer A standing on the truck, is $u\cos\theta$, and $u\sin\theta$ respectively.

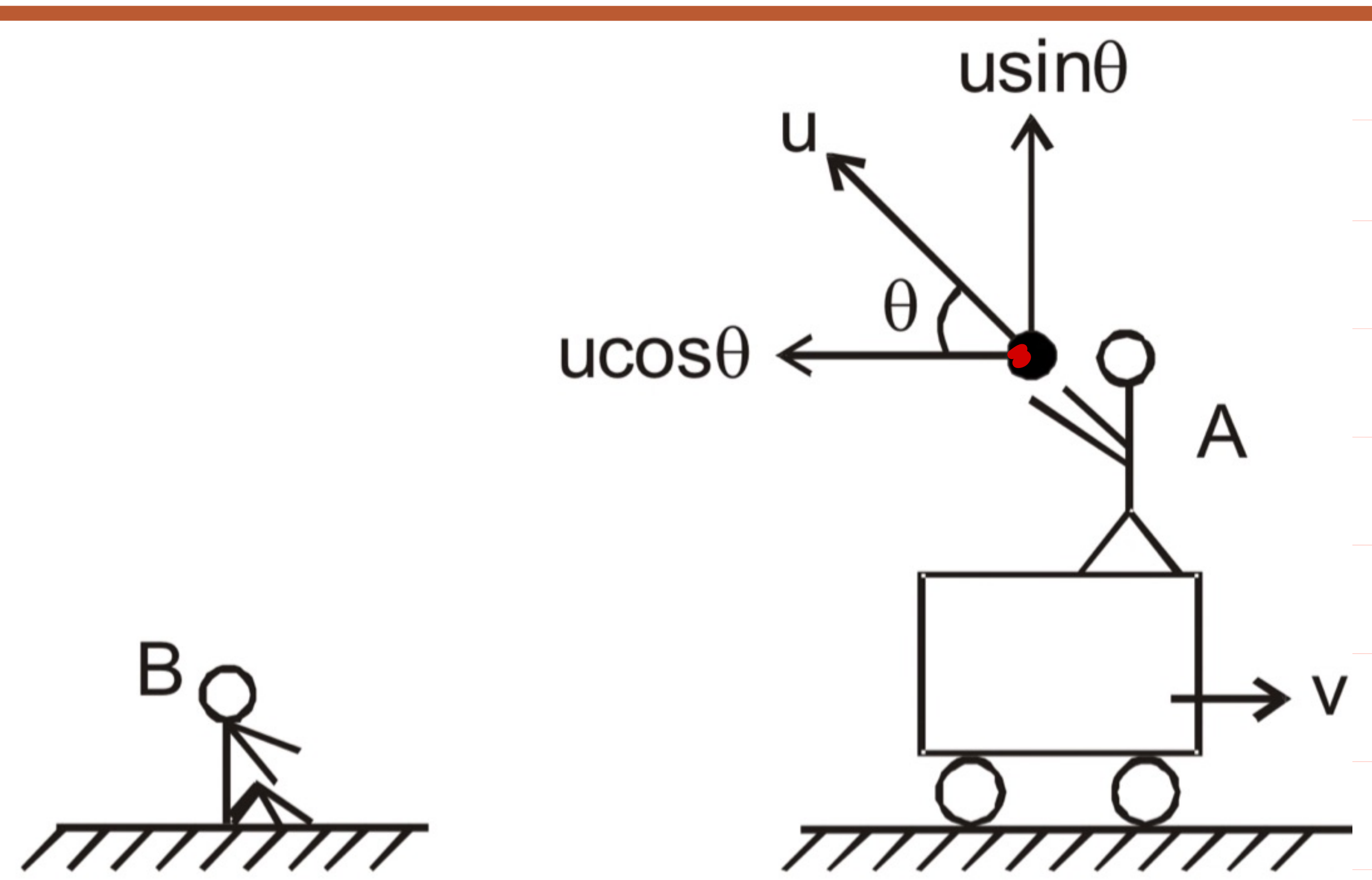
Horizontal & vertical component of ball's velocity w.r.t. observer B sitting on the ground, is $u_x = u\cos\theta + v$ and $u_y = u\sin\theta$ respectively.



Case (3) – When a ball is thrown at some angle 'θ' in the opposite direction of motion of the truck, horizontal & vertical component of ball's velocity w.r.t. observer A standing on the truck, is $u\cos\theta$, and $u\sin\theta$ respectively.

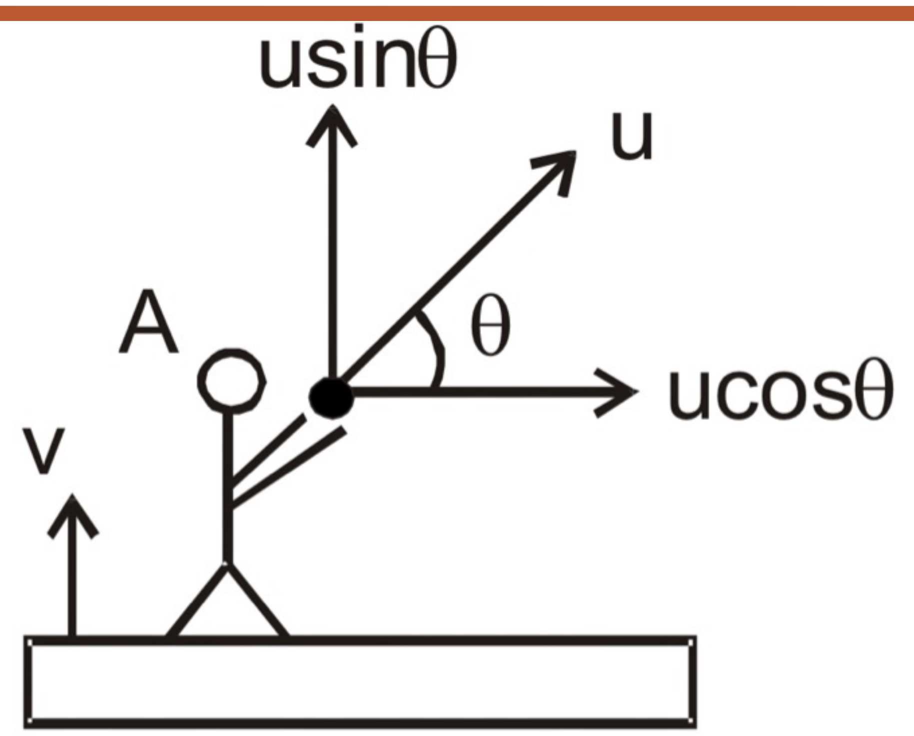
Horizontal & vertical component of ball's velocity w.r.t. observer B sitting on the ground, is

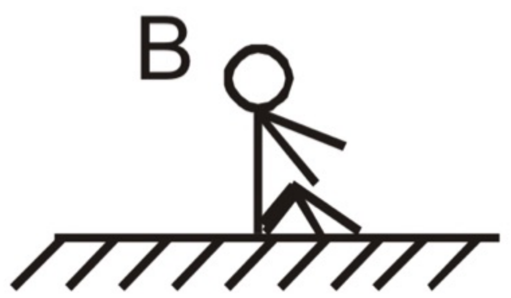
$\boxed{u_x = u\cos\theta - v \text{ and } u_y = u\sin\theta}$ respectively.



Case (4) – When a ball is thrown at some angle 'θ' from a platform moving with speed v upwards, horizontal & vertical component of ball's velocity w.r.t. observer A standing on the moving platform, is ucosθ and usinθ respectively. Horizontal & vertical component of ball's velocity w.r.t. observer B sitting on the ground, is

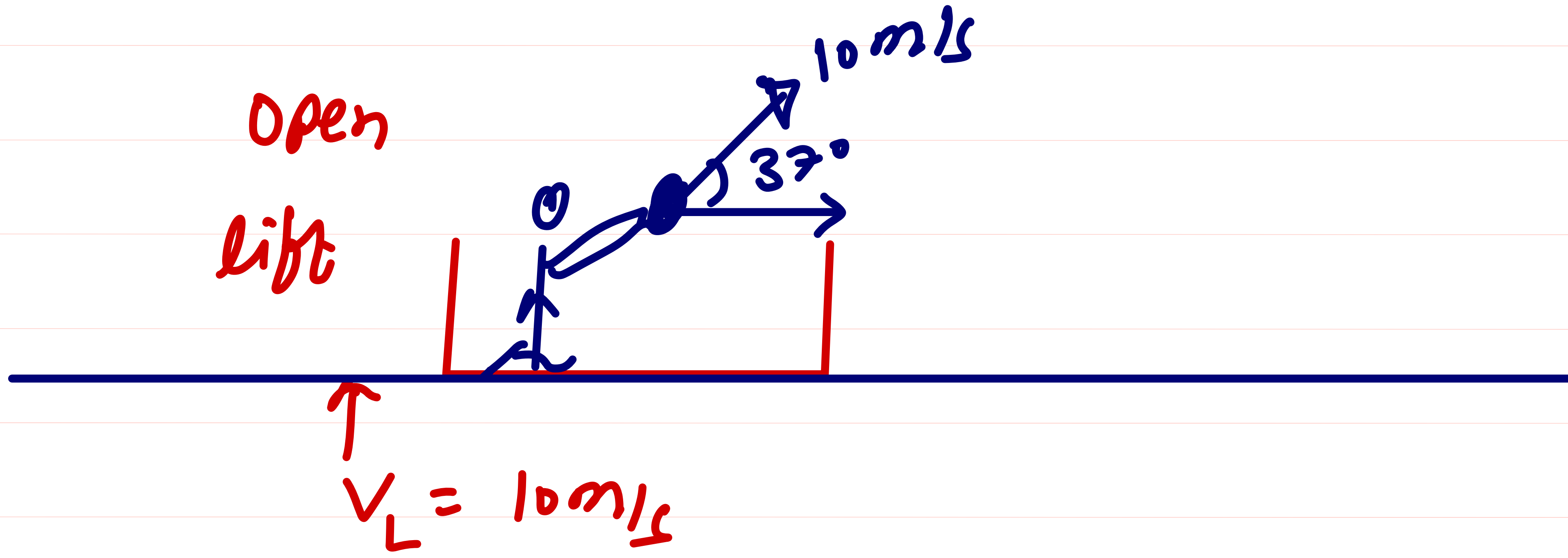
$\boxed{u_x = u\cos\theta \text{ and } u_y = u\sin\theta + v}$ respectively.

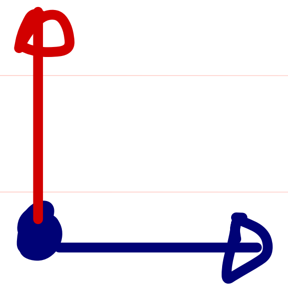




Ex

$u_y = 10\sin 37 + 10 = 10 \times \frac{3}{5} + 10 = 16$





$10\cos 37 = u_x = 8$

Find time of Flight and Horizontal Range of ball

$\boxed{T = \frac{2u_y}{g} = \frac{2 \times 16}{10} = 3.2 \text{ sec}}$ Ans

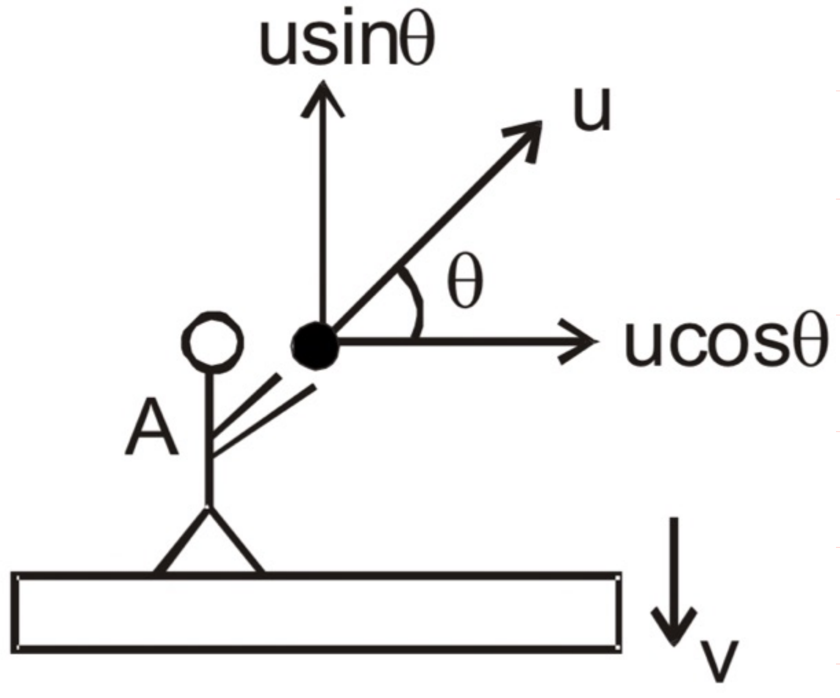
$R = u_x T$

$= 8 \times 3.2$

$\boxed{R = 25.6 \text{ m}}$ Ans

Case (5) – When a ball is thrown at some angle 'θ' from a platform moving with speed v downwards, horizontal & vertical component of ball's velocity w.r.t. observer A standing on the moving platform, is $u\cos\theta$ and $u\sin\theta$ respectively. Horizontal & vertical component of ball's velocity w.r.t. observer B sitting on the ground, is

$\boxed{u_x = u\cos\theta \text{ and } u_y = u\sin\theta - v}$ respectively.



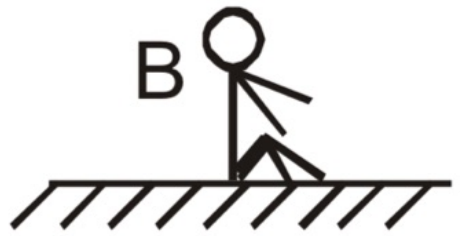
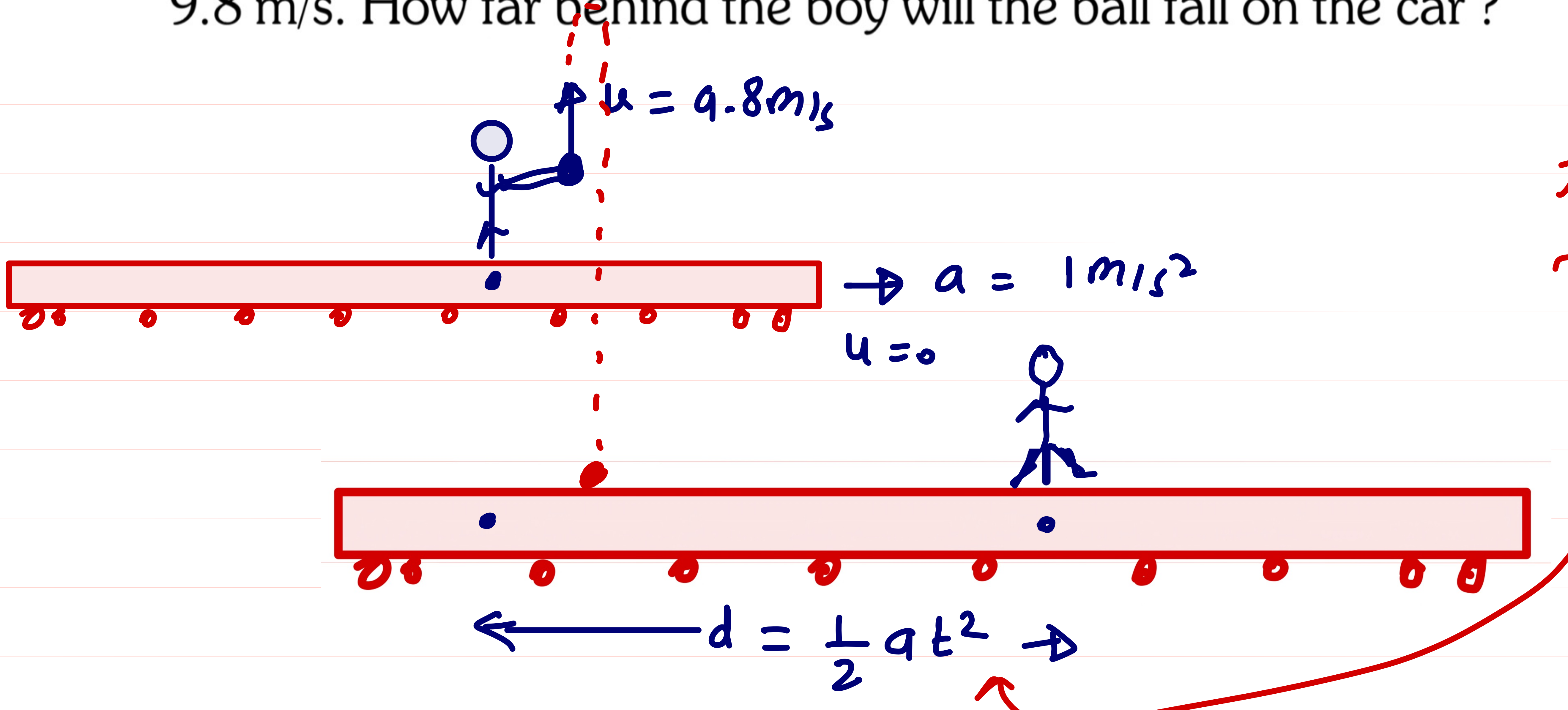


Illustration 14*. A boy standing on a long railroad car throws a ball straight upwards. The car is moving on the horizontal road with an acceleration of 1 m/s^2 and the projection speed in the vertical direction is 9.8 m/s. How far behind the boy will the ball fall on the car ?



$$t_3 = \frac{2u_y}{g} = \frac{2\times 9.8}{9.8} = 2 \text{ sec}$$

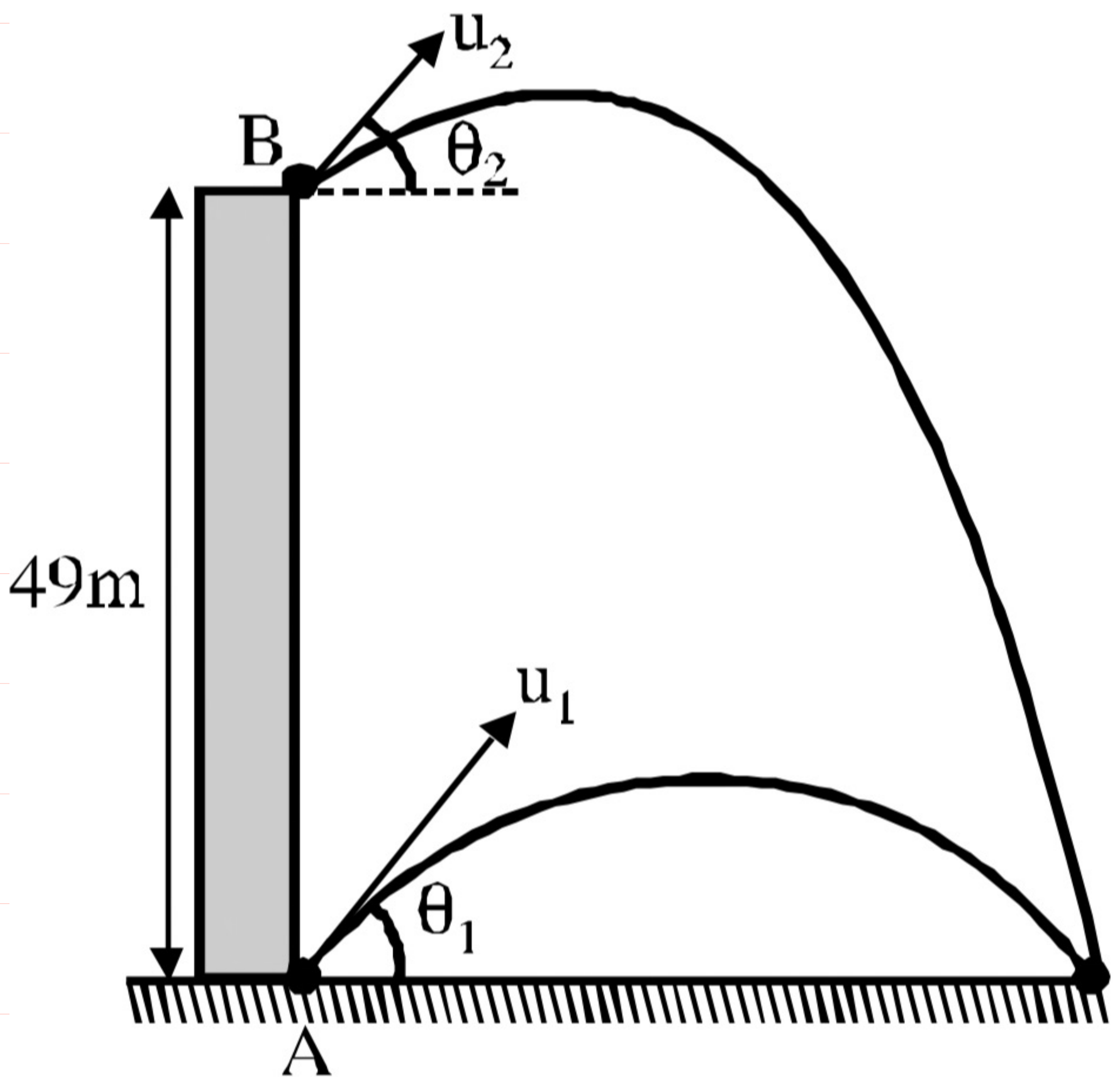
$$d = \frac{1}{2}\times 1\ 2^2$$

$$\boxed{d = 2m}$$

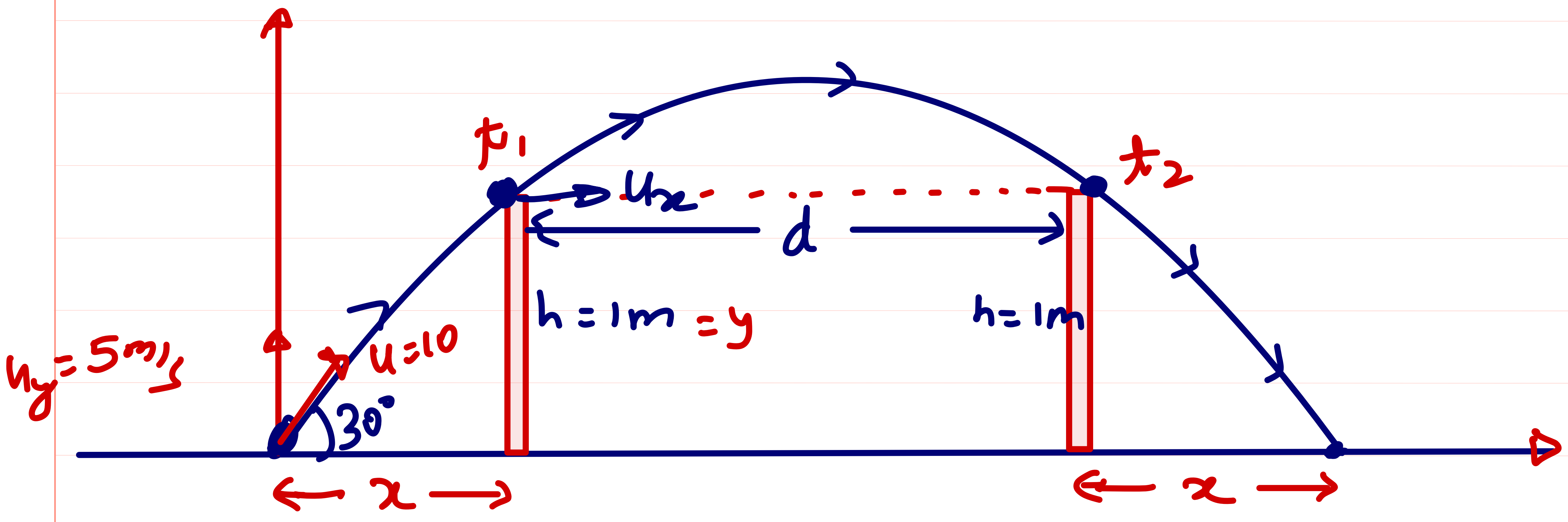
Ans

Illustration 8.

Two stones A and B are projected simultaneously as shown in figure. It has been observed that both the stones reach the ground at the same place after 7 sec of their projection. Determine difference in their vertical components of initial velocities in m/s. ($g = 9.8\ m/s^2$)



Ex A ball projected with a velocity of 10 ms^{-1} at an angle of 30° with the horizontal just clears two vertical poles, each of height 1.0 m. Find the separation between the poles. Take $g = 10$ ms^{-2}.



$u_x = 5\sqrt{3}$

$y = u_y t + \frac{1}{2} a_y t^2$

$1 = 5t - \frac{10}{2} t^2$

$1 = 5t - 5t^2$

$5t^2 - 5t + 1 = 0$

$t = \frac{5 \pm \sqrt{5^2 - 4\times 5}}{10}$

$t = \frac{5 \pm \sqrt{5}}{10}$

$t_1 = \frac{5 - \sqrt{5}}{10}$

$t_2 = \frac{5 + \sqrt{5}}{10}$

$\Delta t = t_2 - t_1$

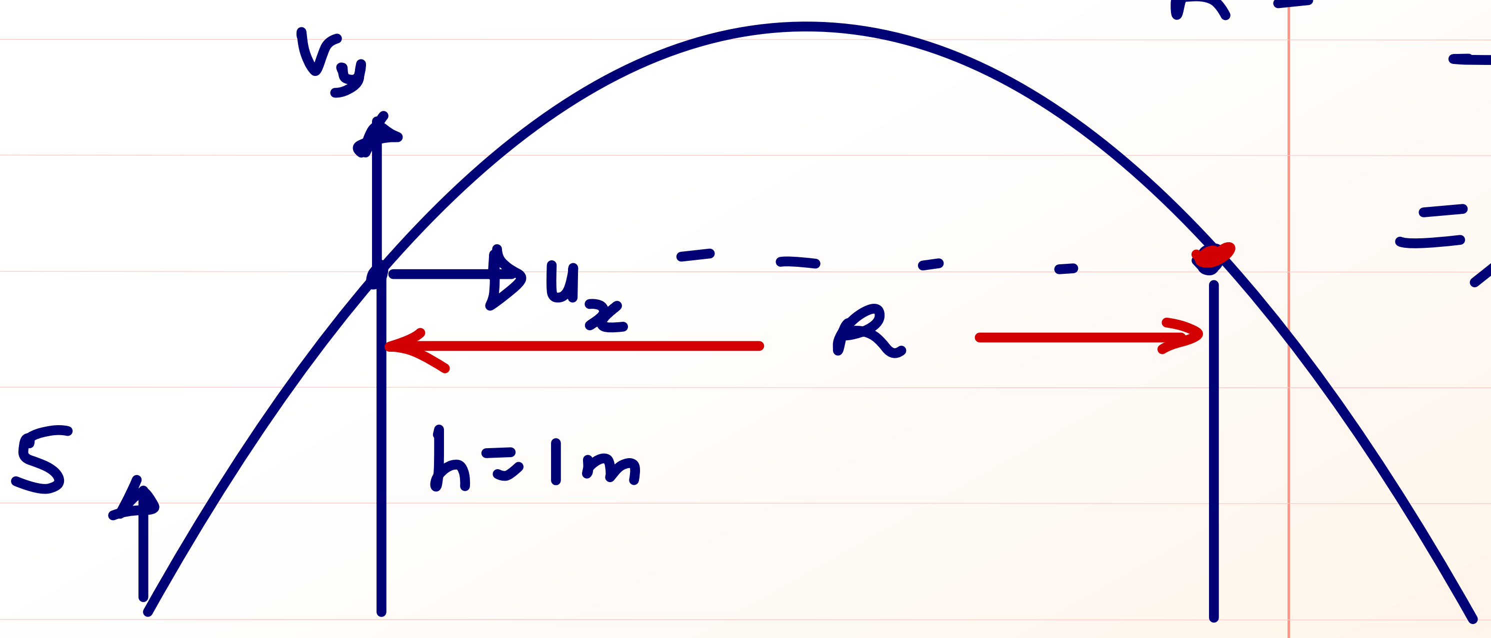
$= \frac{\sqrt{5}}{10} \times 2$

$d = u_x \cdot \Delta t$

$= 5\sqrt{3} \times \frac{1}{\sqrt{5}}$

$$\boxed{d = \sqrt{15}\ m}$$

M-2



$R = \frac{2u_x u_y}{g}$

$= \frac{\cancel{2}\times 5\sqrt{3}\times\sqrt{5}}{\cancel{10}}$

$= \sqrt{15}$

$v_y^2 = 5^2 - 2\times 10\times 1 = 5$

$v_y = \sqrt{5}$

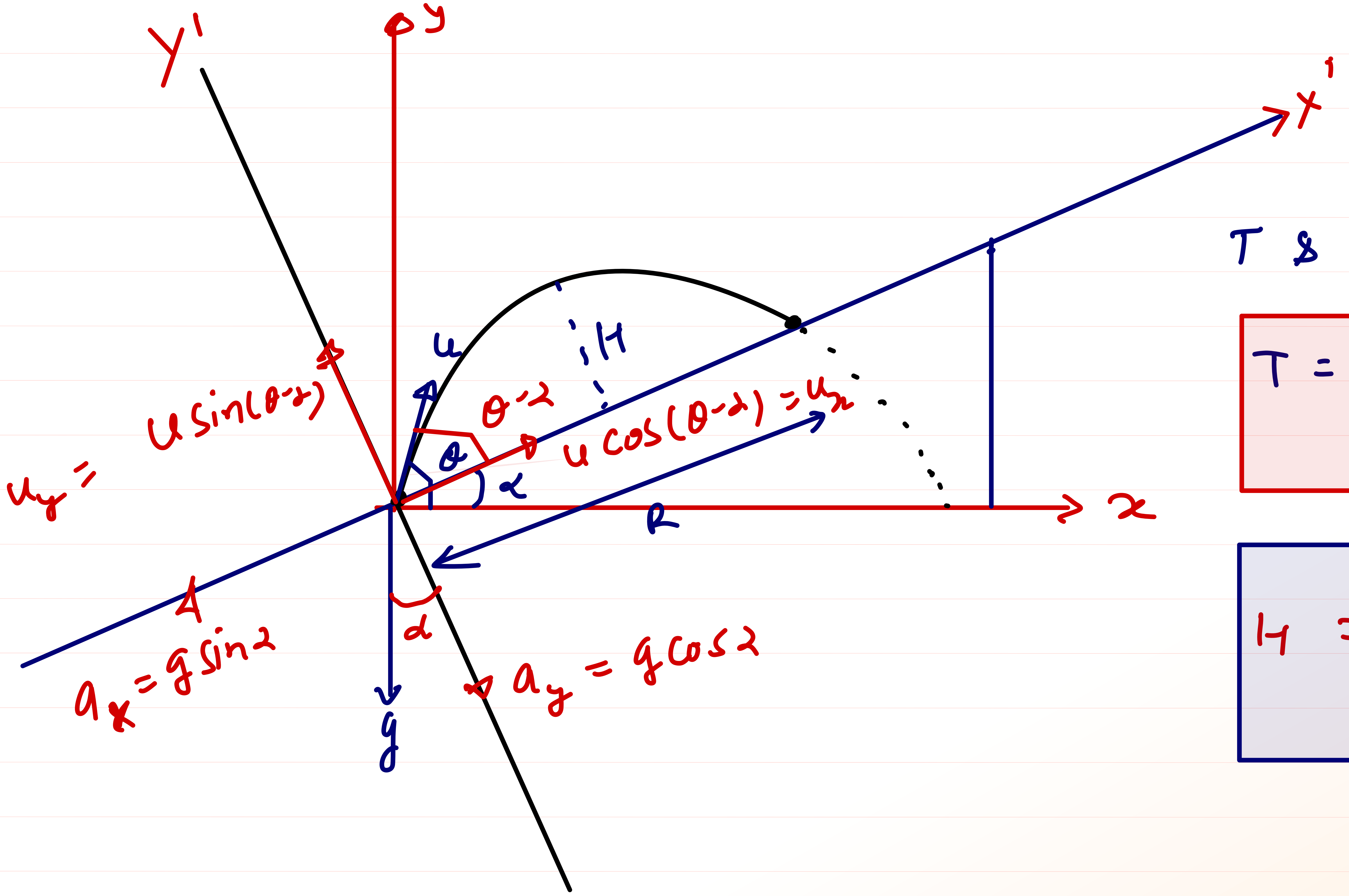
PROJECTILE MOTION ON AN INCLINED PLANE

Case -1

Up the Plane

A projectile is projected up the inclined plane from the point O with an initial velocity u at an angle θ with horizontal. The angle of inclination of the plane with horizontal is α [Fig.].

We will solve eq. of motion along x'-y' plane



T & H always find from y'-axis

$$\boxed{T = \frac{2u_y}{a_y} = \frac{2u\sin(\theta-\alpha)}{g\cos\alpha}}$$

$$\boxed{H = \frac{u_y^2}{2a_y} = \frac{u^2\sin^2(\theta-\alpha)}{2g\cos\alpha}}$$

Along x' - axis

$$R = u_x T + \frac{1}{2} a_x T^2$$

$$R = u\cos(\theta-\alpha)\cdot\frac{2u\sin(\theta-\alpha)}{g\cos\alpha} - \frac{1}{2} g\sin(\alpha)\left(\frac{2u\sin(\theta-\alpha)}{g\cos\alpha}\right)^2$$

$$= \frac{2u\sin(\theta-\alpha)}{g\cos\alpha}\left[u\cos(\theta-\alpha) - \frac{\cancel{g}\sin\alpha}{\cancel{2}}\cdot\frac{\cancel{2}u\sin(\theta-\alpha)}{\cancel{g}\cos\alpha}\right]$$

$$= \frac{2u^2}{g}\frac{\sin(\theta-\alpha)}{\cos\alpha}\left[\frac{\cos\alpha\cos(\theta-\alpha) - \sin\alpha\cdot\sin(\theta-\alpha)}{\cos\alpha}\right]$$

$$\boxed{R = \frac{2u^2}{g}\cdot\frac{\sin(\theta-\alpha)}{\cos^2\alpha}\cdot\cos\theta}$$