(iii*) $y = 4e^{x^2-2x}$

$$\frac{dy}{dx} = 4 \cdot e^{(x^2-2x)} \cdot \{2x - 2 \cdot 1\}$$

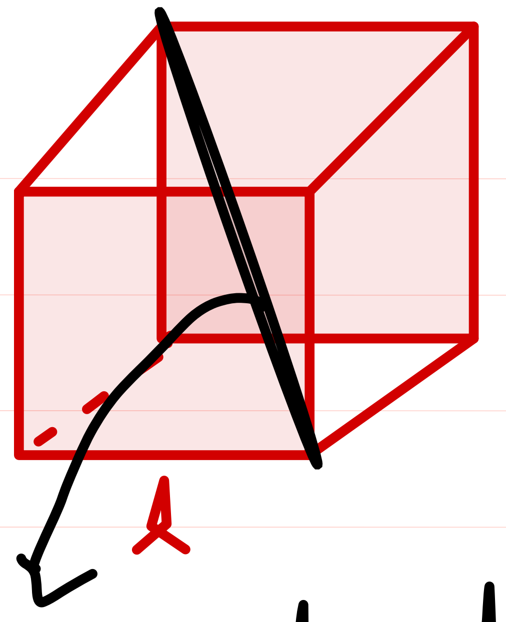
$$= 4e^{(x^2-2x)} \cdot 2(x-1)$$

$$= 8(x-1)e^{(x^2-2x)} \quad \underline{\underline{\text{Ans}}}$$

***Illustration 32*.*** If surface area of a cube is changing at a rate of 5 $m^2$/s, find the rate of change of body diagonal at the moment when side length is 1 m.

(A) 5 m/s (B) $5\sqrt{3}$ m/s (C) $\frac{5}{2}\sqrt{3}$ m/s ✓(D) $\frac{5}{4\sqrt{3}}$ m/s

Given $\frac{dA}{dt} = 5\ m^2/sec$




$A = 6l^2$

$\frac{dA}{dl} = 6\{2l\}$

$\left(\frac{dA}{dt}\right) = 12l \cdot \frac{dl}{dt}$

$\frac{5}{12l} = \frac{dl}{dt}$

$S = \sqrt{3}\,l$

$\frac{ds}{dl} = \sqrt{3}$

$\frac{ds}{dt} = \sqrt{3}\,\frac{dl}{dt}$

$\frac{ds}{dt} = \sqrt{3} \times \frac{5}{12l}$

$l = 1\,m$

$\frac{ds}{dt} = \frac{\sqrt{3}\times 5}{12}$

$= \frac{5}{4\times\sqrt{3}}$

**4.** Evaluate $\int \frac{2+e^x}{e^x}\,dx$

$$\Rightarrow \int\left(\frac{2}{e^x}+1\right)dx = \int 2e^{-x}\,dx + \int 1\,dx$$

$$= 2\,\frac{(e^{-x})}{-1} + x + C$$

$$= -\frac{2}{e^x} + x + C \quad \text{Ans}$$

**5*.** Evaluate $\int_1^4 \frac{e^{\sqrt{x}}\,dx}{\sqrt{x}}$

**6.** Evaluate $\int_0^1 (e^x + x^e)\,dx$

$$\int_0^1 e^x\,dx + \int_0^1 x^e\,dx$$

$$\left[e^x\right]_0^1 + \left[\frac{x^{e+1}}{e+1}\right]_0^1$$

$$\left[e^1 - e^0\right] + \frac{1}{e+1}\left[1^{e+1} - 0^{e+1}\right]$$

$$(e-1) + \frac{1}{e+1}[1-0]$$

$$= (e-1) + \frac{1}{e+1}$$

**5*.** Evaluate $\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$

let $\sqrt{x} = t$

$x^{1/2} = t$

$\frac{dt}{dx} = \frac{1}{2} x^{\frac{1}{2}-1}$

$\frac{dt}{dx} = \frac{1}{2\sqrt{x}}$

$2dt = \frac{dx}{\sqrt{x}}$

$= \int e^t\, 2dt$

$= 2\, e^t$

$= 2\left[e^{\sqrt{x}}\right]_1^4$

$= 2\left\{e^{\sqrt{4}} - e^{\sqrt{1}}\right\}$

$= 2\left\{e^2 - e\right\}$

$= 2e\left\{e-1\right\}$ Ans

B.B.F 2

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**8.** In each of the following find y' for the given value of x:

(i) $y = 2\sqrt{x}\,(3x-2)$, $x = 4$

(ii) $y = \dfrac{x+1}{\sqrt{x}}$, $x = \dfrac{1}{4}$

(1) $y = 2\sqrt{x}\,(3x-2)$

$= 6\sqrt{x}\cdot x - 4\sqrt{x}$

$y = 6x^{3/2} - 4x^{1/2}$

$\dfrac{dy}{dx} = 6\left\{\dfrac{3}{2}x^{3/2-1}\right] - 4\left[\dfrac{1}{2}x^{1/2-1}\right]$

$= 6\times\dfrac{3}{2}x^{1/2} - \dfrac{4}{2}\times x^{-1/2}$

$\dfrac{dy}{dx} = 9\sqrt{x} - \dfrac{2}{\sqrt{x}}$

$\dfrac{dy}{dx} = 9\sqrt{4} - \dfrac{2}{\sqrt{4}}$

$= 18 - \dfrac{2}{2}$

$= 18 - 1$

$= 17$ Ans

$y = a^x$

$\log y = x\log a$

$\dfrac{1}{y}\cdot\dfrac{dy}{dx} = 1\cdot\log a + 0$

$\dfrac{dy}{dx} = y\log a$

$= a^x\log a$

## Average value of a continuous function in an interval

SL AL

Average value of a function $\boxed{y = f(x),}$ over an interval $\boxed{a \le x \le b}$ is given by

$$y_{av} = \frac{\int_a^b y\,dx}{\int_a^b dx} = \frac{\int_a^b y\,dx}{b-a}$$

$$\Rightarrow \boxed{y_{avg} = \frac{1}{b-a}\int_a^b f(x)\,dx}$$

**Ex.** Kinetic energy of a particle executing S.H.M. is $K = \frac{1}{2}m\omega^2(a^2 - x^2)$ calculate average value of kinetic energy from x = 0 to x = a

$$K_{avg} = \frac{1}{a-0}\int_0^a \frac{1}{2}m\omega^2(a^2-x^2)\,dx$$

$$= \frac{1}{2}\frac{m\omega^2}{a}\left[\int_0^a a^2 dx - \int_0^a x^2 dx\right]$$

$$K_{avg} = \frac{m\omega^2}{2a}\left[a^2[x]_0^a - \left[\frac{x^3}{3}\right]_0^a\right]$$

$$= \frac{m\omega^2}{2a}\left\{a^3 - \frac{a^3}{3}\right\}$$

$$= \frac{m\omega^2}{\cancel{2a}}\left\{\frac{\cancel{2}a^{\cancel{3}\,2}}{3}\right\}$$

$$K_{avg} = \frac{m\omega^2 a^2}{3}$$ Ans

***Illustration 36.*** 37,38

Ex $y = \sin(x)$ Find Avg of $y$ b/w $x = \frac{\pi}{4}$ to $x = \frac{\pi}{2}$

$$y_{avg} = \frac{1}{\left(\frac{\pi}{2} - \frac{\pi}{4}\right)} \int_{\pi/4}^{\pi/2} \sin(x)\, dx$$

$$= \frac{4}{\pi} \int_{\pi/4}^{\pi/2} \sin x \, dx$$

$$= \frac{4}{\pi} \left[-\cos x\right]_{\pi/4}^{\pi/2}$$

$$y_{avg} = \frac{4}{\pi} \left[-\cos\left(\frac{\pi}{2}\right) + \cos\left(\frac{\pi}{4}\right)\right]$$

$$= \frac{4}{\pi} \left[-0 + \frac{1}{\sqrt{2}}\right]$$

$$= \frac{4}{\pi} \times \frac{1}{\sqrt{2}} = \frac{2\sqrt{2}}{\pi} \quad \underline{\underline{\text{Ans}}}$$

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**Ex** Q $y = 2x$ Find Avg of $y$ from $x = -1$ to $x = 1$

$$y_{avg} = \frac{1}{1-(-1)} \int_{-1}^{1} (2x)\, dx$$

$$= \frac{1}{2} \cdot 2 \int_{-1}^{1} x\, dx$$

$$= \left[\frac{x^2}{2}\right]_{-1}^{1}$$

$$= \frac{1^2}{2} - \frac{(-1)^2}{2}$$

$$= \frac{1}{2} - \frac{1}{2} = 0$$

**Ex.** $y = e^x$ Find avg of $y$ from $x = 0$ to $2$

$$y_{avg} = \frac{1}{2-0} \int_{0}^{2} e^x\, dx$$

$$= \frac{1}{2} \left[e^x\right]_0^2$$

$$= \frac{1}{2}\left[e^2 - e^0\right]$$

$$= \frac{e^2-1}{2} \quad \text{Ans}$$

Ex If $y = \cos x$ Find Avg of $y$ from $x = \frac{\pi}{3}$ to $x = \frac{\pi}{2}$

$$y_{avg} = \frac{1}{\left(\frac{\pi}{2} - \frac{\pi}{3}\right)} \int_{\pi/3}^{\pi/2} \cos x \, dx$$

$$= \frac{6}{\pi} \left[ \sin x \right]_{\pi/3}^{\pi/2}$$

$$= \frac{6}{\pi} \left[ \sin(\pi/2) - \sin(\pi/3) \right]$$

$$= \frac{6}{\pi} \left[ 1 - \frac{\sqrt{3}}{2} \right]$$

$$= \frac{3}{\pi} \left[ 2 - \sqrt{3} \right]$$ Ans

H.W
Ex # 1 & 2
Module

## SIGNIFICANT FIGURES :

The *significant figures* (SF) in a measurement are the figures or digits that are known with certainity plus one that is uncertain.

Significant figures in a measured value of a physical quantity tell the number of digits in which we have confidence. Larger the number of significant figures obtained in a measurement, greater is its accuracy and vice versa.

**Rules to find out the number of significant figures :**

**I Rule** : All the non-zero digits are significant e.g. 1984 has 4 SF.

**II Rule** : All the zeros between two non-zero digits are significant. e.g. 10806 has 5 SF.

**III Rule** : All the zeros to the left of first non-zero digit are not significant. e.g.00108 has 3 SF.

**IV Rule** : If the number is less than 1, zeros on the right of the decimal point but to the left of the first non-zero digit are not significant. e.g. 0.002308 has 4 SF.

length of wire = 2.235 cm → SF = 4 Ans

**I Rule** : All the non-zero digits are significant e.g. 1984 has 4 SF.

17568 = S.F = 5

1857 = S.F = 4

**II Rule** : All the zeros between two non-zero digits are significant. e.g. 10806 has 5 SF.

2002 S.F = 4

1809 S.F = 4

790805 S.F = 6

**III Rule :** All the zeros to the left of first non-zero digit are not significant. e.g.00108 has 3 SF.

Ex 005 , S.F = 1 05202 , S.F = 4

Ex 00502 , S.F = 3

**IV Rule :** If the number is less than 1, zeros on the right of the decimal point but to the left of the first non-zero digit are not significant. e.g. 0.002308 has 4 SF.

0.002 S.F = 1 , 0.0305 S.F = 3

0.00208 S.F = 3 ,