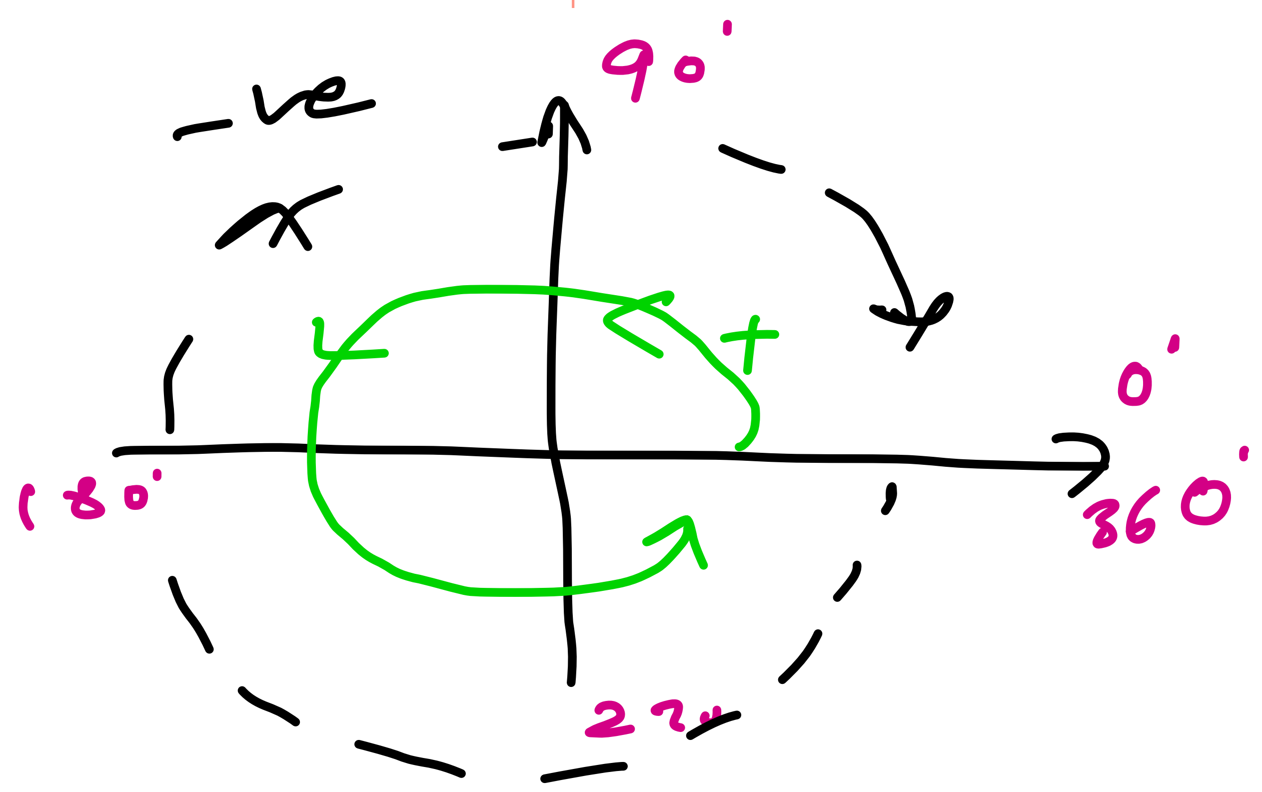
Trigonometric Ratios and Identities

Lecture - 1 & 2

INTRODUCTION :

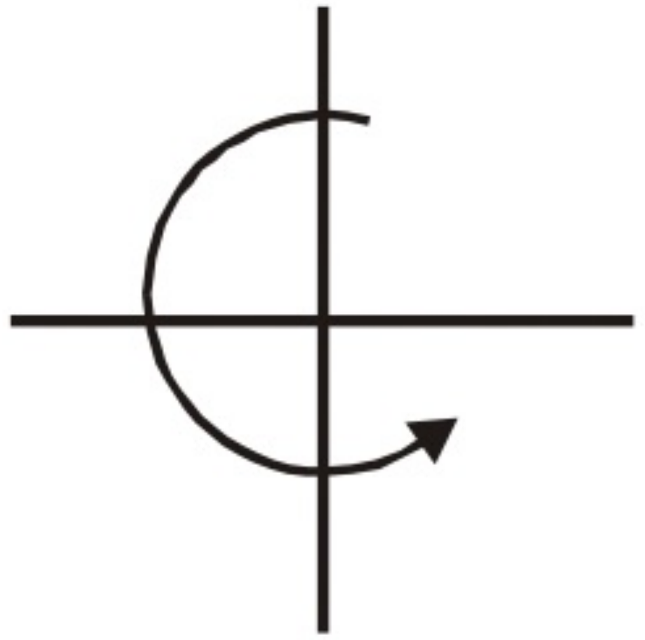
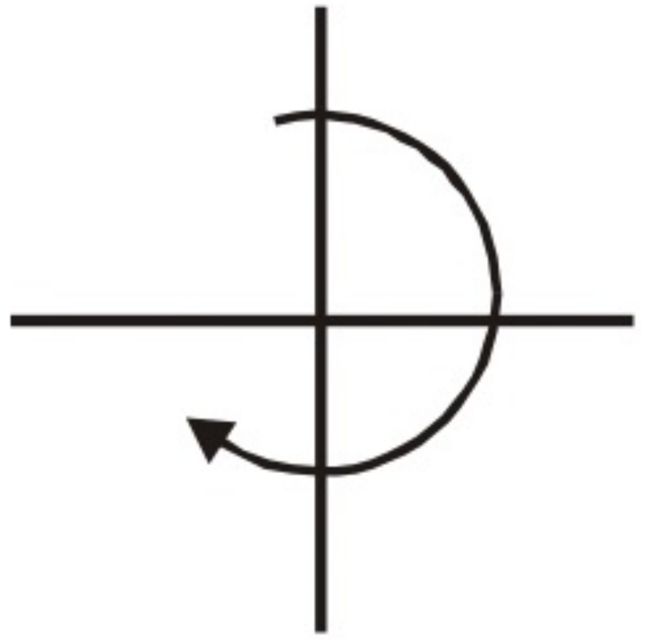


Trigonometry (Greek Word)

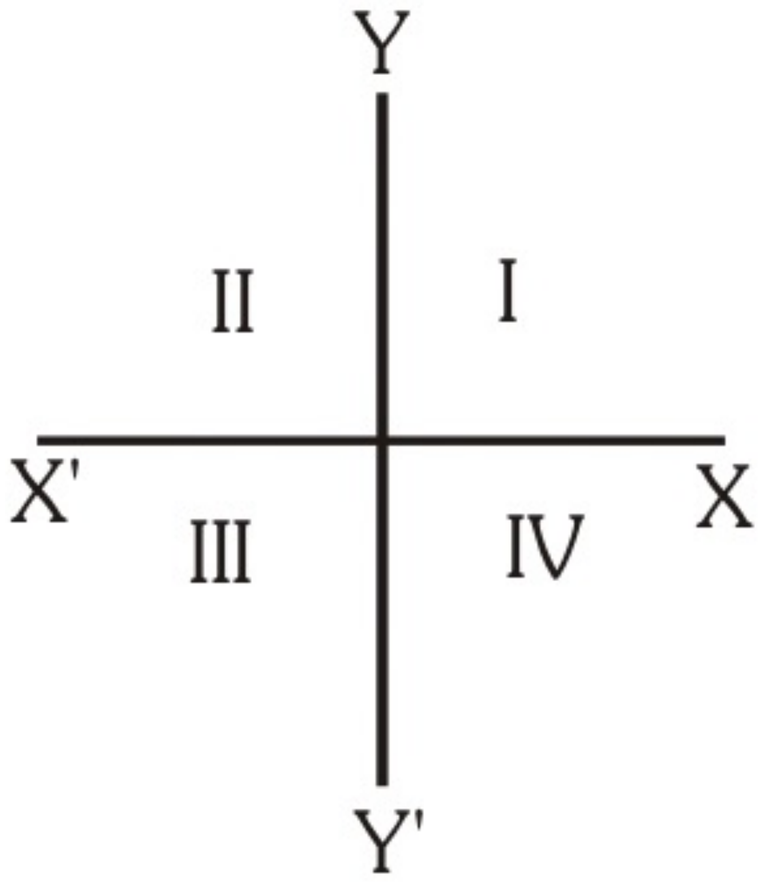
Tri	Gonos	Metron
(Three)	(Sides)	(Measure)

Trigonometry is the branch of mathematics in which we study about triangle. Basically there are six parameters in a triangle (Three sides & three angles).

. MEASUREMENT OF ANGLE :

Sign convention :  Anticlockwise direction will indicate +ve angle ,  Clockwise turning will indicate –ve angle

If α is the angle measured from positive x-axis in anticlockwise direction.

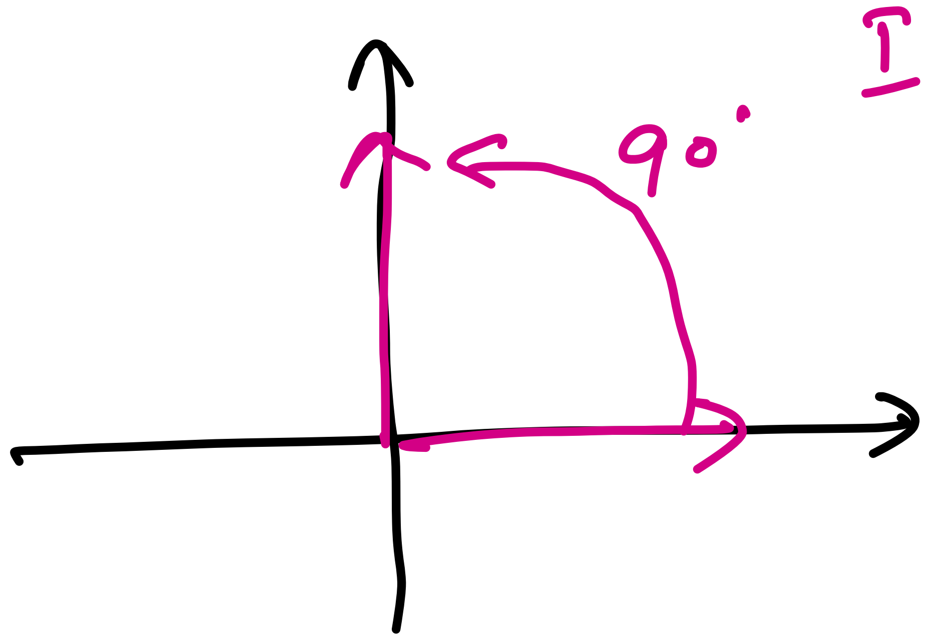


$0° < \alpha < 90°$ $\quad$ I[st] quadrant

$90° < \alpha < 180°$ $\quad$ II[nd] quadrant

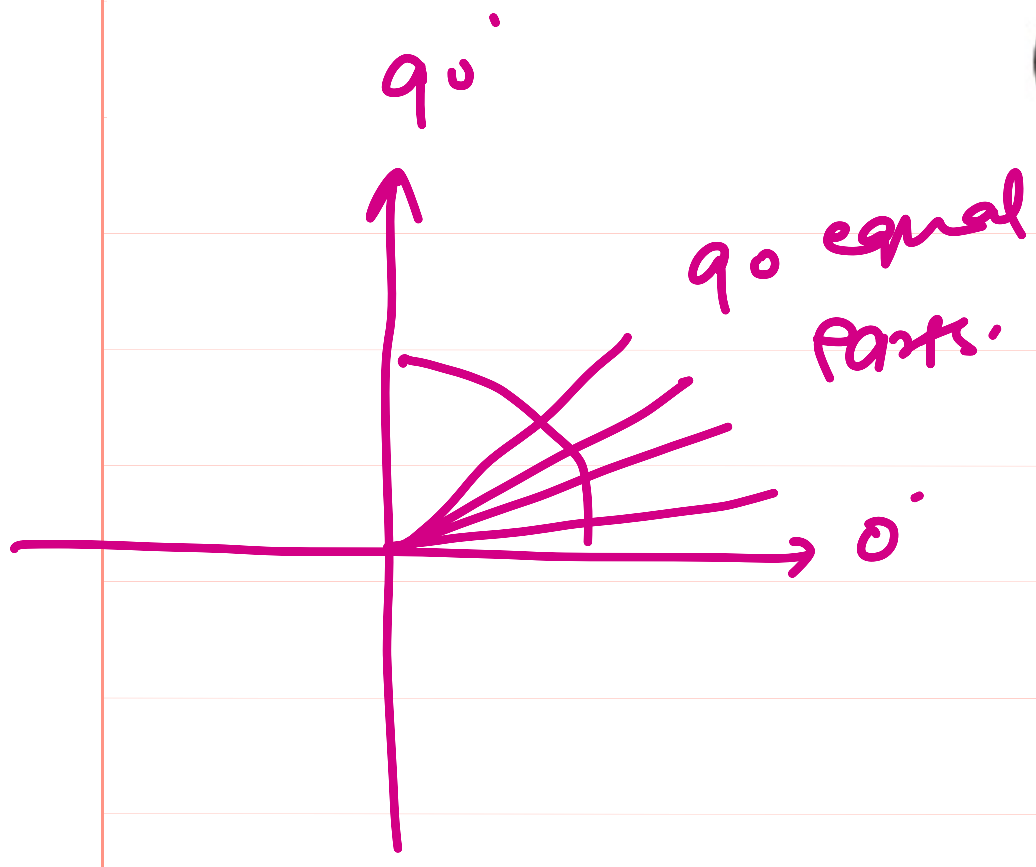
$180° < \alpha < 270°$ $\quad$ III[rd] quadrant

$270° < \alpha < 360°$ $\quad$ IV[th] quadrant



System of Measurement of angle

Degree (English) (Sexagesimal system)	Radian (Circular system)	French (Grade system) (Not in used) → 100 equal pa



2π Rad = 360 Deg.

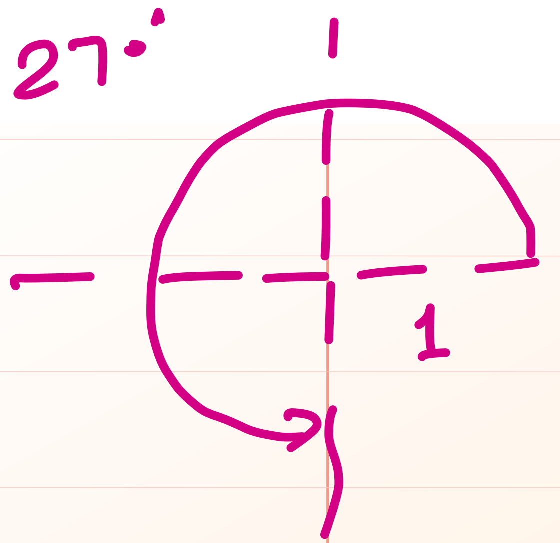
π Rad = 180 Deg.

Circumference

$= 2\pi(r)$

$= 2\pi$

$\frac{3}{4}$ of $2\pi = \frac{3}{4}(2\pi) = \frac{3}{2}\pi$



$\frac{3}{4}$ part of circumference

Degree system : When we draw a line perpendicular to another and then divide the angle between them in 90 equal part. Then each part will be equal to one degree. $1° = 60'$ minutes & $1' = 60''$ seconds

Hence $90° = 1$ right angle

or $180° = 2$ right angle.

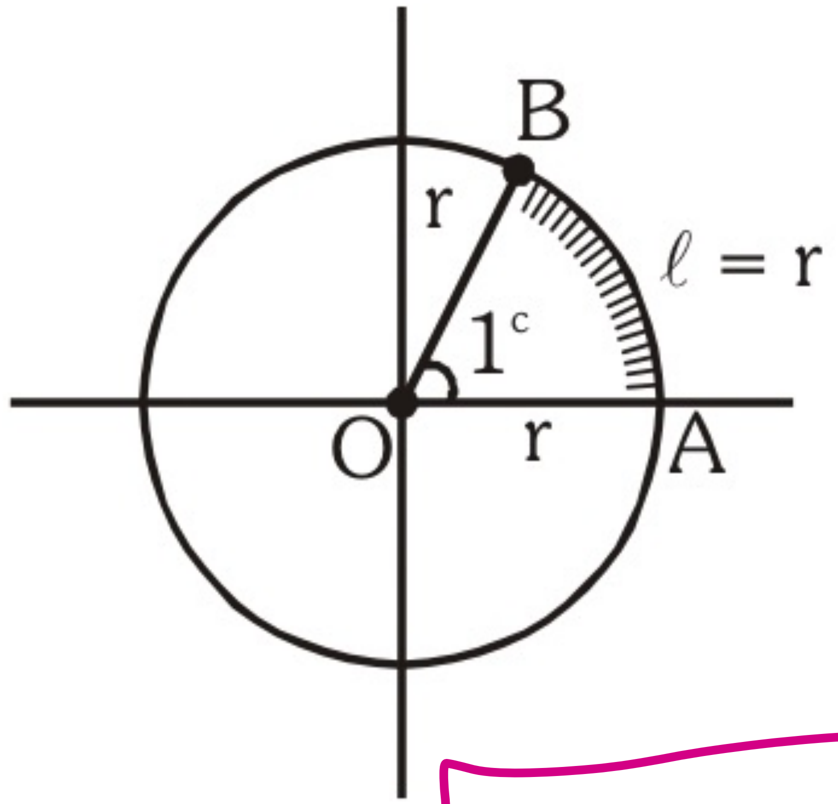
1 right angle $= 90°$

$1° = 60'$

$1' = 60''$

Grade system : One right angle $= 100^g$ grades, $1^g = 100'$ minutes, $1' = 100''$ seconds

Radian : If the length of arc (ℓ) is equal to r (radius), then the angle subtended by it at the centre of the circle is equal to 1 radian. (1^c)



Commonly, Angle $= \dfrac{\text{arc}}{\text{radius}}$, i.e. $\boldsymbol{\ell = r\theta}$, where θ is the angle subtended at the centre by circular arc of length ℓ.

$\pi^c = 180$ Dg.

$1^c = \dfrac{180}{\pi}$ Deg.

$= \dfrac{180}{3.14}$ Deg.

$= 57. - -$

$\boxed{\text{Angle} = \dfrac{\text{Arc}}{\text{Radius}}}$

Radian

If $\ell = r$ then $\theta = 1^c$

If $\ell = 2r$ then $\theta = 2^c$

If $\ell = 3r$ then $\theta = 3^c$

⋮

If $\ell = kr$ then $\theta = k^c$

⋮

If $\ell = \pi r$ then $\theta = \pi^c$

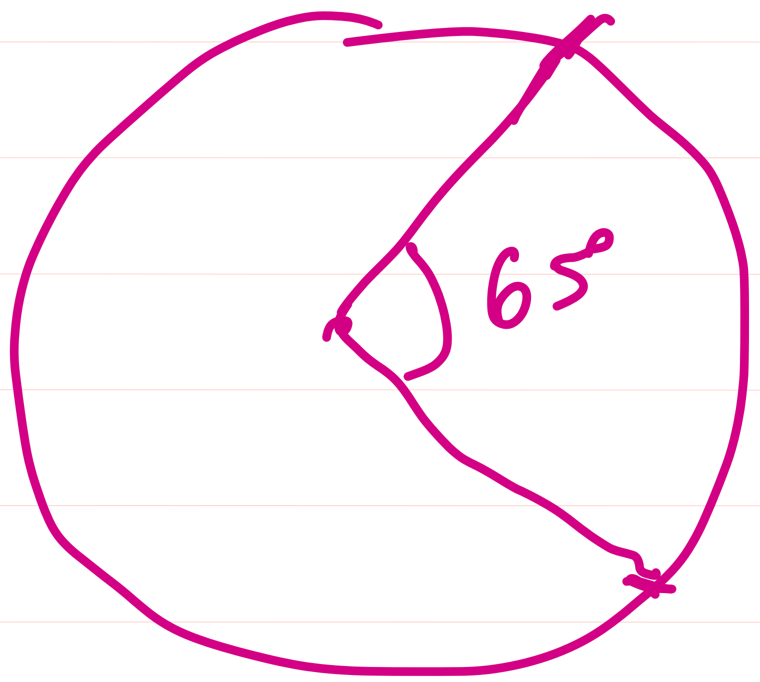
Hence $\pi^c = 180^\circ$ or $\dfrac{\pi^c}{2} = 90^\circ \Rightarrow$ 1 right angle $= \dfrac{\pi}{2}$ radians

$$\Rightarrow \quad 1^\circ = \frac{\pi^c}{180} = 0.01746 \text{ rad} \qquad \& \qquad 1^c = \frac{180^\circ}{\pi} \approx \frac{180}{22}\times 7 \approx \frac{630}{11} = 57.2^\circ$$

Relation in three systems : $\dfrac{D}{90} = \dfrac{G}{100} = \dfrac{R}{\pi/2}$

100 Pa

$$\frac{D}{90} = \frac{G}{100} = \frac{R}{\pi/2}$$



65°

π Rad = 180° Deg.

0, $(\pi/6)$ Rad, $\pi/4$ Rad., $\pi/3$, $\pi/2$, π, $3\pi/2$, 2π, $2\pi/3$, $5\pi/6$, $\pi/12$, $\pi/10$, $5\pi/3$, $-\frac{14\pi}{45}$

E(1) Convert into radians 0, 30°, 45°, 60°, 90°, 180°, 270°, 360°, 120°, 150°, 15°, 18°, 300°, – 56°.

$180^\circ = \pi^c$

$1^\circ = \left(\frac{\pi}{180}\right)^c$

$30^\circ = \left[\left(\frac{\pi}{180}\right)(30)\right]^c = \left(\frac{\pi}{6}\right)^c$

$45^\circ = \left(\frac{\pi}{180}\cdot 45\right)^c = \left(\frac{\pi}{4}\right)^c$

$-56\,\frac{\pi}{180} = -\frac{14\pi}{45}$

$\pi^c = 180^\circ$

$1^c = \frac{180}{\pi}^\circ$

$-3^c = \frac{180}{\pi}(-3) = -\left(\frac{540}{3.14}\right)^c$

E(2) Convert into degrees $\frac{9\pi}{5}, -\frac{5\pi}{6}, -3$

-150°

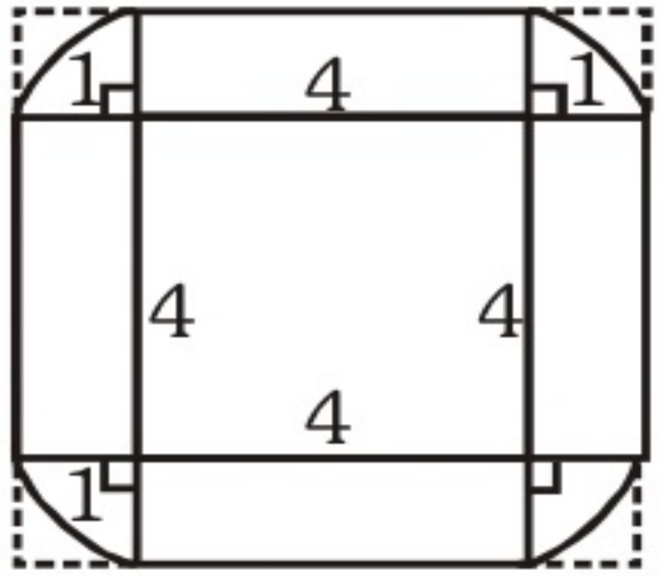
324°

E(3) Mark the angles and find quadrant $1^c, 2^c, 3^c, 4^c, 5^c, 6^c, 7^c$.

I II II III IV IV Ist

E(4) In a circle of diameter 40 cm and chord 20 cm. Find length of minor arc made by the chord.

E(5) Consider a square of side 4 cm. Now if an insect runs at a distance of 1cm from the sides of the square. How much distance will it travel in one round.



E(6) Find the angle between hour & minute hand of a clock when time is 3 : 30. **[Ans.** 75°]

E(7) Find interior angle in degree & radians of the regular polygons :

(i) Pentangon

(ii) Octagon

(4) $r = 20$ cm

$$\text{Arc} = \frac{2\pi r\theta}{360^\circ} = \frac{2\pi(20)\,60^\circ}{360^\circ}$$

$$= \frac{20\pi}{3}\ \text{cm.}$$

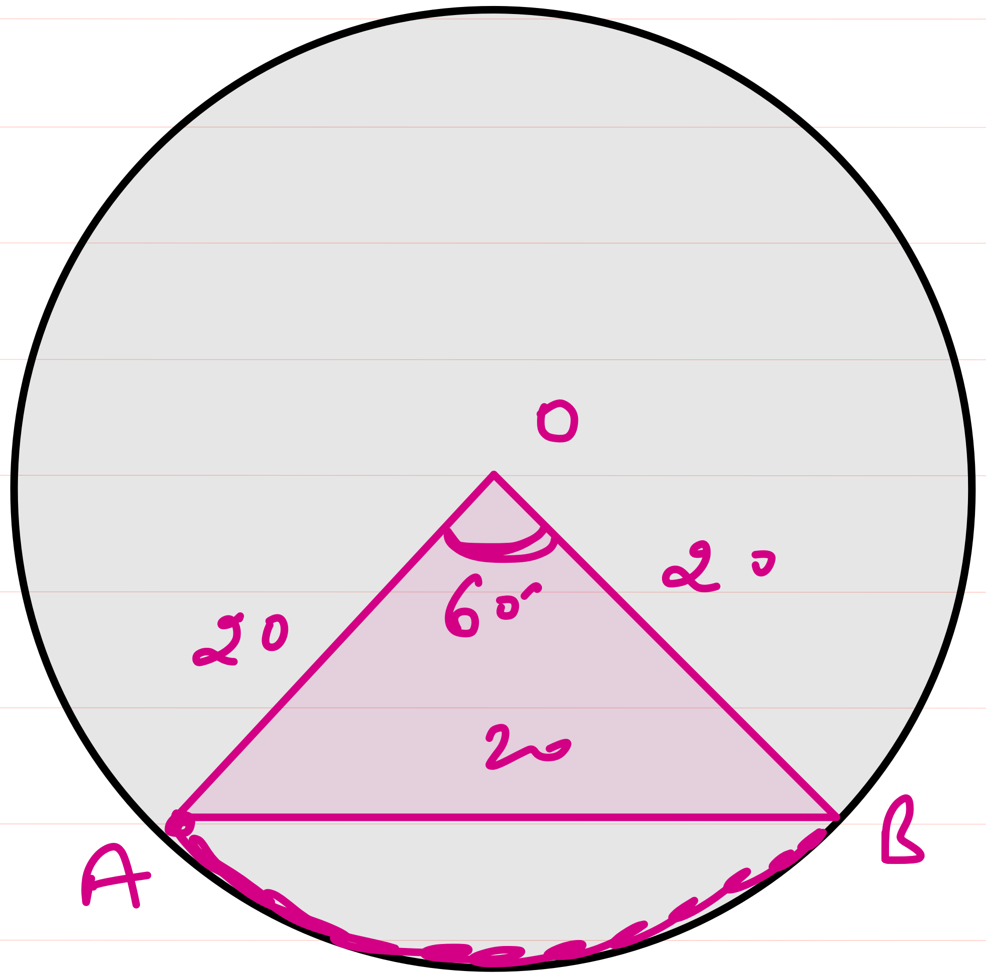
$$\text{Angle} = \frac{\text{Arc}}{\text{Radius}}$$

$$\frac{60\pi}{180} = \frac{\text{Arc}}{20}$$

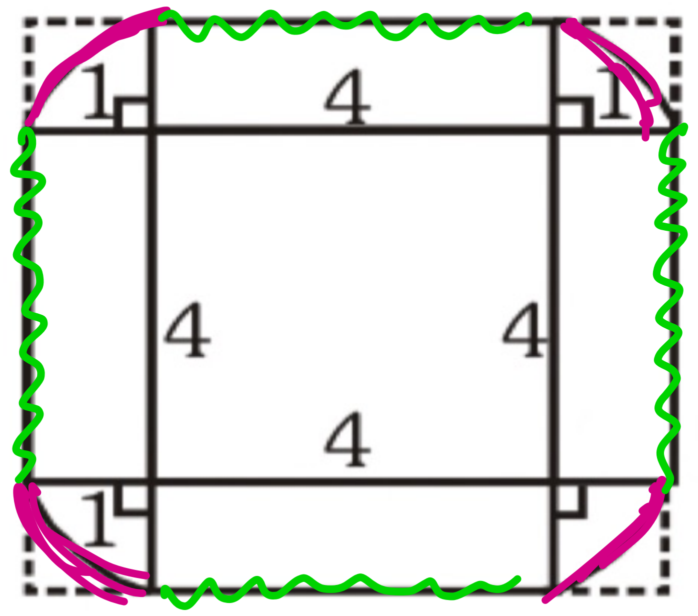
$$\frac{\pi}{3} = \frac{\text{Arc}}{20}$$

$$\boxed{\text{Arc} = \frac{20\pi}{3}}$$

Answer



E(5) Consider a square of side 4 cm. Now if an insect runs at a distance of 1cm from the sides of the square. How much distance will it travel in one round.

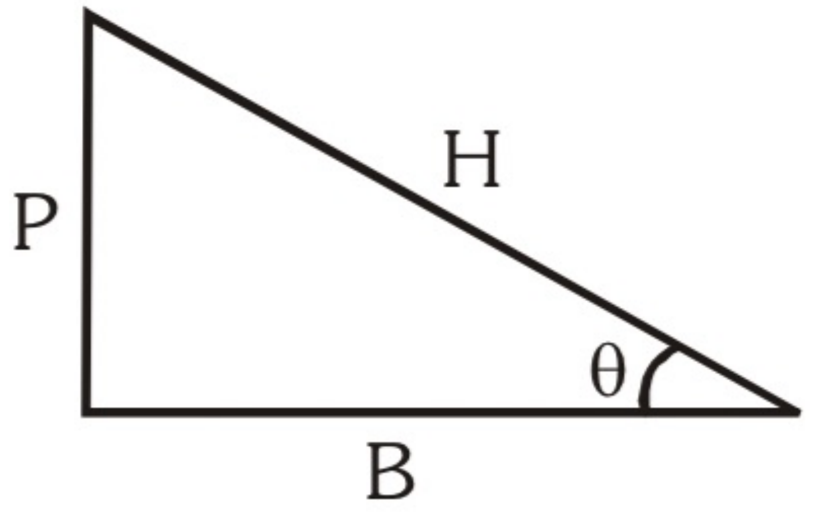


$$\text{Distance} = 4(4) + 4\left(\frac{2\pi(1)}{4}\right)$$

$$\text{Distance} = 16 + 2\pi$$

TRIGONOMETRIC RATIOS [for $0° < \theta < 90°$] :

On basis of ratio of any two sides in a right angled triangle, six trigonometric ratios are defined as sine, cosine, tangent and their reciprocals resp. as cosecant, secant and cotangent.



Commonly $\sin\theta = \frac{P}{H}$; $\cos\theta = \frac{B}{H}$; $\tan\theta = \frac{P}{B}$

$P^2 + B^2 = H^2$

$\frac{P^2}{H^2} + \frac{B^2}{H^2} = 1$

$\boxed{\sin^2\theta + \cos^2\theta = 1}$

$\frac{P^2}{B^2} + 1 = \frac{H^2}{B^2}$

$\boxed{\tan^2\theta + 1 = \sec^2\theta}$

$1 + \frac{B^2}{P^2} = \frac{H^2}{P^2}$

$\boxed{1 + \cot^2\theta = \text{cosec}^2\theta}$

Using pythagoras theorem,

$$P^2 + B^2 = H^2 \Rightarrow \left(\frac{P}{H}\right)^2 + \left(\frac{B}{H}\right)^2 = 1$$

Three basic identities :

(1) $\sin^2\theta + \cos^2\theta = 1$ (2) $\tan^2\theta + 1 = \sec^2\theta$ (3) $1 + \cot^2\theta = \text{cosec}^2\theta$

E(1) Prove that : (i) $\sin^4\theta + \cos^4\theta = 1 - 2\sin^2\theta\cos^2\theta$ (ii) $\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cos^2\theta$.

E(2) Prove that $\sqrt{\dfrac{1-\cos\theta}{1+\cos\theta}} = \text{cosec}\theta - \cot\theta$

E(3) Prove that $\sqrt{\sec^2 A + \text{cosec}^2 A} = \tan A + \cot A$, when $0° < A < 90°$

E(4) Prove that $(\text{cosec}A - \sin A)(\sec A - \cos A)(\tan A + \cot A) = 1$

E(5) Prove that $\dfrac{\cot A + \tan B}{\cot B + \tan A} = \cot A \tan B$

E(6) Prove that $\dfrac{\tan A + \sec A - 1}{\tan A - \sec A + 1} = \dfrac{1+\sin A}{\cos A}$

E(7) Prove that $\tan^2 A - \sin^2 A = \sin^4 A \sec^2 A$

D(8) Value of the expression $\text{cosec}^2A.\cot^2A - \sec^2A\tan^2A - (\cot^2A - \tan^2A)(\sec^2A.\text{cosec}^2A - 1)$ is -

(A) -1 (B) 0 (C) $\tan^2A + \cot^2A$ (D) $(\tan^2A - \cot^2A)^2$

[**Hint :** change sec/cosec in to tan/cot.]

E(9) In acute angle triangle if $\sec\theta + \tan\theta = 2$. Find $\sin\theta$. [**Ans.** $\dfrac{3}{5}$]

E(10) If $3\sec^4\theta + 8 = 10\sec^2\theta$. Find $\tan\theta$. $\theta \in (0, 90°)$ [**Ans.** $\left(1, \dfrac{1}{\sqrt{3}}\right)$]

D(11) If $\tan\theta = \dfrac{2x(x+1)}{2x+1}$. Find $\sin\theta$, $\cos\theta$. [**Ans.** $\dfrac{2x(x+1)}{2x^2+2x+1}, \dfrac{2x+1}{2x^2+2x+1}$]

E(1) Prove that : (i) $\sin^4\theta + \cos^4\theta = 1 - 2\sin^2\theta\cos^2\theta$ (ii) $\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cos^2\theta$.

E(2) Prove that $\sqrt{\dfrac{1-\cos\theta}{1+\cos\theta}} = \operatorname{cosec}\theta - \cot\theta$

(1) (i) LHS $= \sin^4\theta + \cos^4\theta$

$a^2+b^2 = (a+b)^2 - 2ab$

$= (\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta$

$= 1 - 2\sin^2\theta\cos^2\theta$

(ii) $\sin^6\theta + \cos^6\theta$

$a^3+b^3 = (a+b)(a^2+b^2-ab)$

$= \underbrace{(\sin^2\theta + \cos^2\theta)}(\sin^4\theta + \cos^4\theta - \sin^2\theta\cos^2\theta)$

$= 1\left((\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta - \sin^2\theta\cos^2\theta\right)$

$= 1 - 3\sin^2\theta\cos^2\theta$

E(3) Prove that $\sqrt{\sec^2 A + \operatorname{cosec}^2 A} = \tan A + \cot A$, when $0° < A < 90°$

$$\text{LHS} = \sqrt{\sec^2 A + \operatorname{cosec}^2 A}$$

$$= \sqrt{1 + \tan^2 A + 1 + \cot^2 A}$$

$$= \sqrt{\underbrace{2 + \tan^2 A + \cot^2 A}}$$

$$= \sqrt{(\tan A + \cot A)^2}$$

$$= \tan A + \cot A$$

~~D~~ (8) Value of the expression $\text{cosec}^2A.\cot^2A - \sec^2A\tan^2A - (\cot^2A - \tan^2A)(\sec^2A.\text{cosec}^2A - 1)$ is -

(A) -1 (B) 0 (C) $\tan^2A + \cot^2A$ (D) $(\tan^2A - \cot^2A)^2$

[**Hint :** change sec/cosec in to tan/cot.]

$$\text{cosec}^2A\ \cot^2A - \sec^2A\ \tan^2A - (\cot^2A - \tan^2A)(\text{cosec}^2A \cdot \sec^2A - 1)$$

$$= (1+\cot^2A)(\cot^2A) - (1+\tan^2A)\tan^2A - (\cot^2A - \tan^2A)((1+\cot^2A)(1+\tan^2A) - 1)$$

$$= \cot^2A + \cot^4A - \tan^2A - \tan^4A - (\cot^2A - \tan^2A)(\cancel{1} + \cot^2A + \tan^2A + \cot^2A\tan^2A \cancel{-1})$$

$$= \cancel{\cot^2A} + \cancel{\cot^4A} - \cancel{\tan^2A} - \cancel{\tan^4A} - (\cot^2A - \tan^2A)(\cot^2A + \tan^2A + 1)$$

$$= - \cancel{\cot^4A} + 1 - 1 + \cancel{\tan^4A} - \cancel{\cot^2A} + \cancel{\tan^2A}$$

$$= 0$$

Answer

E(5) Prove that $\dfrac{\cot A + \tan B}{\cot B + \tan A} = \cot A \tan B$

$\underline{LHS} = \dfrac{\cot A + \tan B}{\cot B + \tan A} = \dfrac{\dfrac{\cos A}{\sin A} + \dfrac{\sin B}{\cos B}}{\dfrac{\cos B}{\sin B} + \dfrac{\sin A}{\cos A}}$

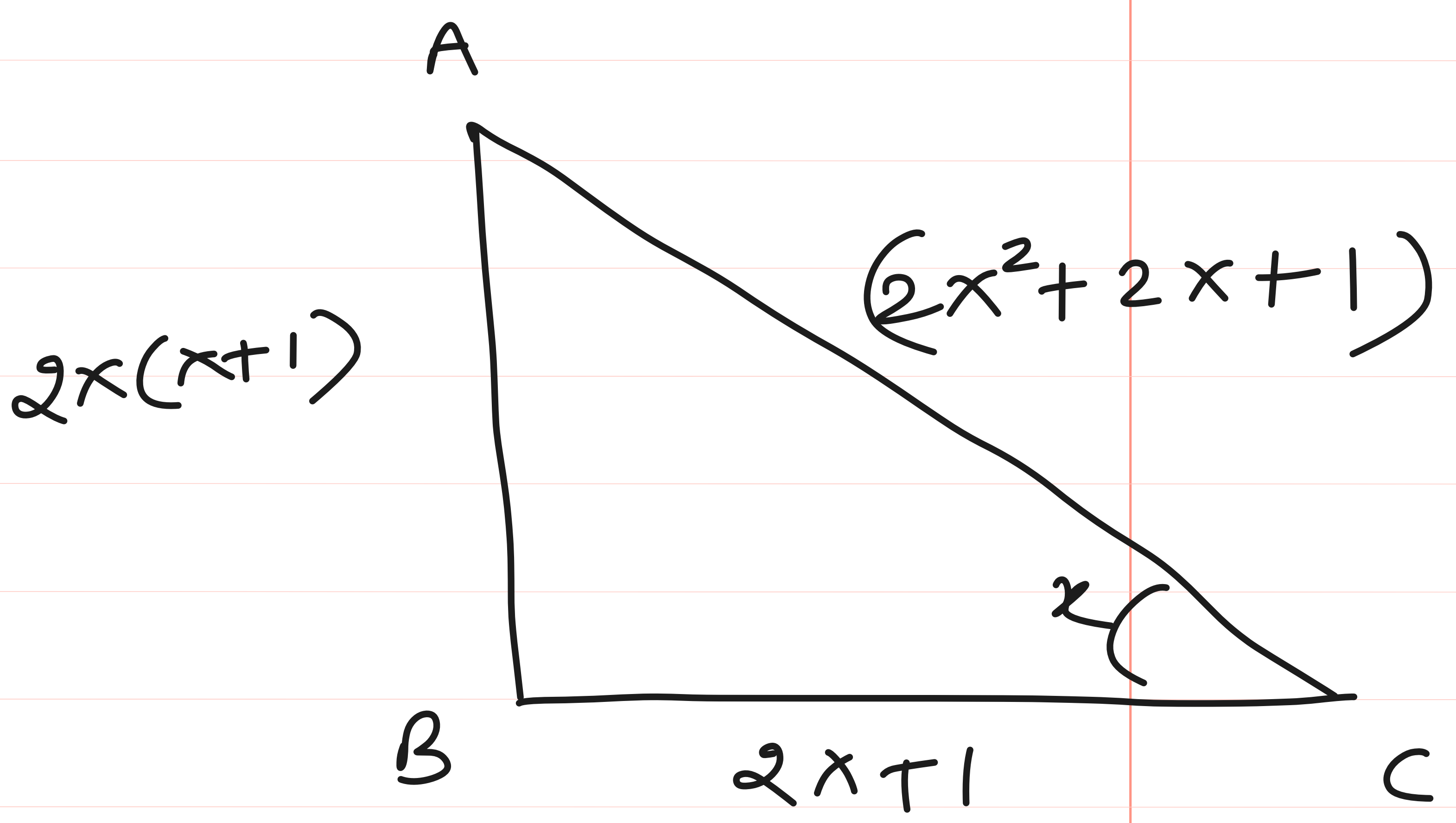
$= \dfrac{\left(\dfrac{\cancel{\cos A \cos B + \sin A \sin B}}{\sin A \cos B}\right)}{\left(\dfrac{\cancel{\cos A \cos B + \sin A \sin B}}{(\sin B \cos A)}\right)} = \dfrac{\sin B \cos A}{\sin A \cos B} = \cot A \tan B$

$= \underline{RHS.}$

(11) $\tan x = \dfrac{2x(x+1)}{(2x+1)}$

$\cos x = \dfrac{2x+1}{2x^2+2x+1}$

$\sin x = \dfrac{2x(x+1)}{2x^2+2x+1}$



$AC^2 = (2x(x+1))^2 + (2x+1)^2$

$= 4x^2(x^2+1+2x)$
$+ 4x^2+1+4x$

$= 4x^4 + 4x^2 + 8x^3$
$+ 4x^2 + 4x + 1$

$= 4x^4 + 8x^3 + 8x^2$
$+ 4x + 1$

$=$

$= \boxed{4x^4 + 8x^3 + 4x^2 + 4x^2 + 4x + 1}$

$= (2x^2 + 2x + 1)^2$

(6) LHS $\dfrac{\tan A + \sec A - 1}{\tan A - \sec A + 1} = \dfrac{(\tan A + \sec A) - (\sec^2 A - \tan^2 A)}{\tan A - \sec A + 1}$

$$= \frac{(\tan A + \sec A)\cancel{(1 - \sec A + \tan A)}}{\cancel{(\tan A - \sec A + 1)}}$$

$$= \frac{\sin A}{\cos A} + \frac{1}{\cos A} = \frac{\sin A + 1}{\cos A} = \text{RHS.}$$

Note :

D(i) Expressing trigonometrical ratio in terms of each other [for $0° < \theta < 90°$] :

	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\cot\theta$	$\sec\theta$	$\operatorname{cosec}\theta$
$\sin\theta$	$\sin\theta$	$\sqrt{1-\cos^2\theta}$	$\dfrac{\tan\theta}{\sqrt{1+\tan^2\theta}}$	$\dfrac{1}{\sqrt{1+\cot^2\theta}}$	$\dfrac{\sqrt{\sec^2\theta-1}}{\sec\theta}$	$\dfrac{1}{\operatorname{cosec}\theta}$
$\cos\theta$	$\sqrt{1-\sin^2\theta}$	$\cos\theta$	$\dfrac{1}{\sqrt{1+\tan^2\theta}}$	$\dfrac{\cot\theta}{\sqrt{1+\cot^2\theta}}$	$\dfrac{1}{\sec\theta}$	$\dfrac{\sqrt{\operatorname{cosec}^2\theta-1}}{\operatorname{cosec}\theta}$
$\tan\theta$	$\dfrac{\sin\theta}{\sqrt{1-\sin^2\theta}}$	$\dfrac{\sqrt{1-\cos^2\theta}}{\cos\theta}$	$\tan\theta$	$\dfrac{1}{\cot\theta}$	$\sqrt{\sec^2\theta-1}$	$\dfrac{1}{\sqrt{\operatorname{cosec}^2\theta-1}}$
$\cot\theta$	$\dfrac{\sqrt{1-\sin^2\theta}}{\sin\theta}$	$\dfrac{\cos\theta}{\sqrt{1-\cos^2\theta}}$	$\dfrac{1}{\tan\theta}$	$\cot\theta$	$\dfrac{1}{\sqrt{\sec^2\theta-1}}$	$\sqrt{\operatorname{cosec}^2\theta-1}$
$\sec\theta$	$\dfrac{1}{\sqrt{1-\sin^2\theta}}$	$\dfrac{1}{\cos\theta}$	$\sqrt{1+\tan^2\theta}$	$\dfrac{\sqrt{1+\cot^2\theta}}{\cot\theta}$	$\sec\theta$	$\dfrac{\operatorname{cosec}\theta}{\sqrt{\operatorname{cosec}^2\theta-1}}$
$\operatorname{cosec}\theta$	$\dfrac{1}{\sin\theta}$	$\dfrac{1}{\sqrt{1-\cos^2\theta}}$	$\dfrac{\sqrt{1+\tan^2\theta}}{\tan\theta}$	$\sqrt{1+\cot^2\theta}$	$\dfrac{\sec\theta}{\sqrt{\sec^2\theta-1}}$	$\operatorname{cosec}\theta$

$$\sin\theta = \frac{1}{\operatorname{cosec}\theta} = \frac{1}{\sqrt{\operatorname{cosec}^2\theta}}$$

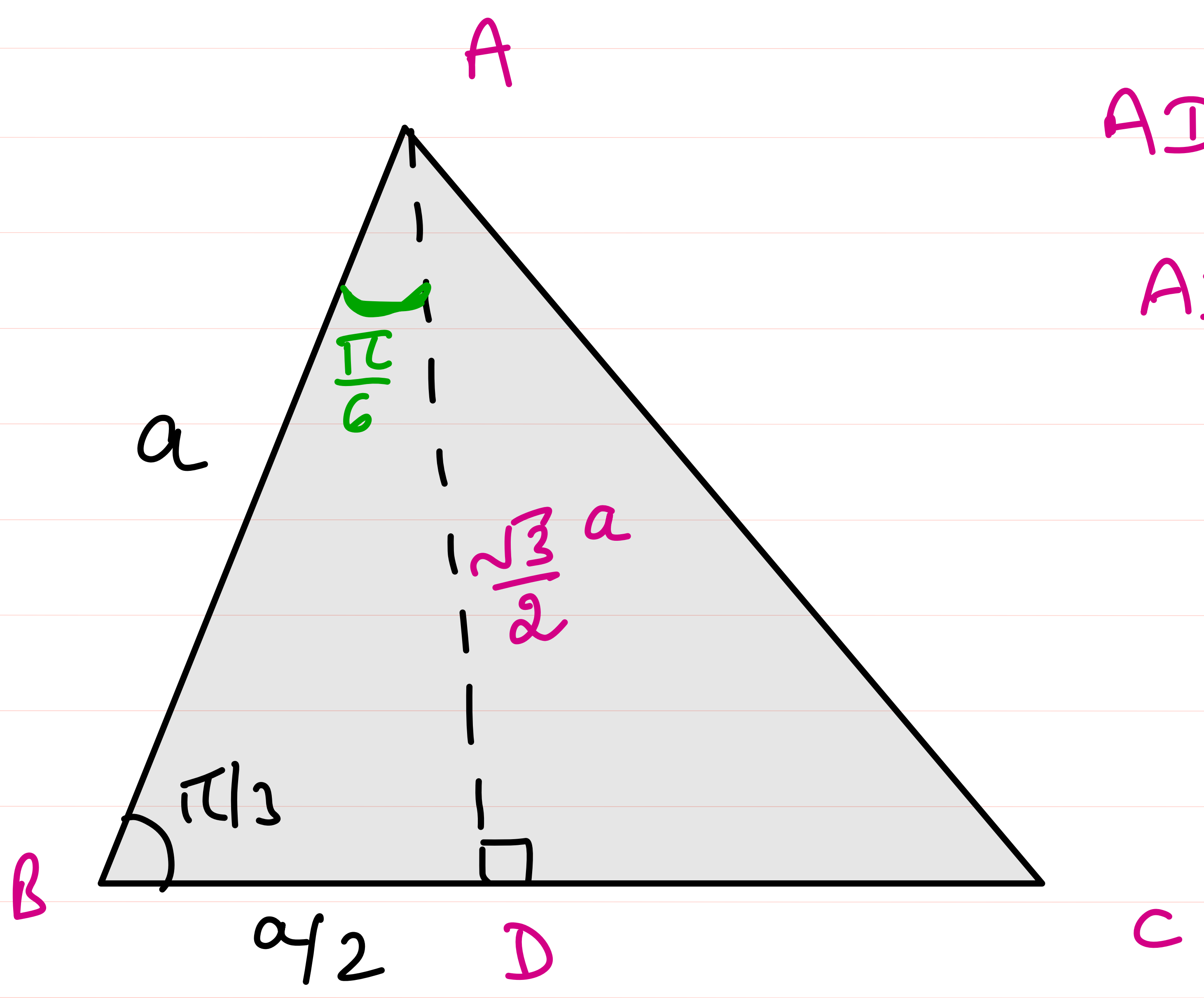
$$= \frac{1}{\sqrt{1+\cot^2\theta}} = \frac{\tan\theta}{\sqrt{1+\tan^2\theta}}$$

Angle $\pi/3$, $\pi/6$

$$\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}\,a}{2\,a} = \frac{\sqrt{3}}{2}$$

$$\cos\left(\frac{\pi}{3}\right) = \frac{a}{2\,(a)} = \frac{1}{2}$$

$$\tan\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}\,\cancel{a}}{\cancel{2}\,(\cancel{a}/\cancel{2})} = \sqrt{3}$$



$$AD^2 = a^2 - \frac{a^2}{4}$$

$$AD = \frac{\sqrt{3}}{2}\,a$$

$$\sin\left(\frac{\pi}{6}\right) = \frac{a}{2a} = \frac{1}{2}$$

$$\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}\,a}{2\,a} = \frac{\sqrt{3}}{2}$$

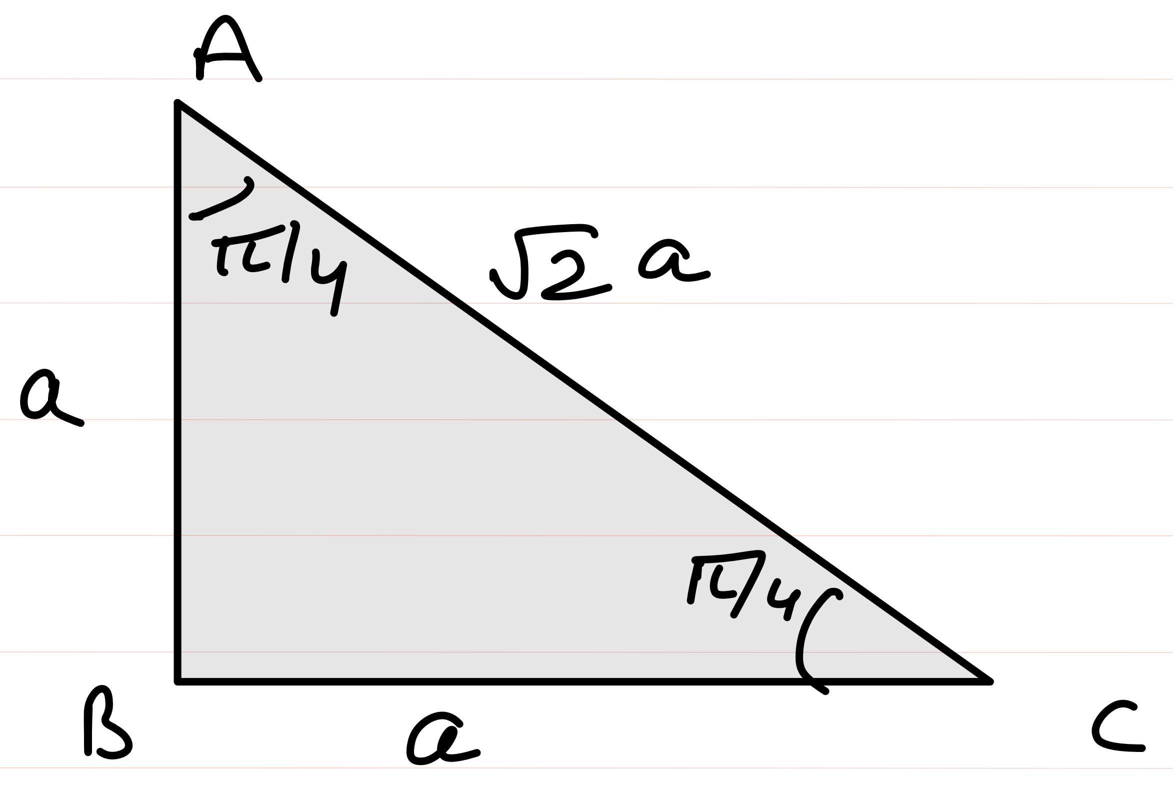
$$\tan\left(\frac{\pi}{6}\right) = \frac{a \quad 2}{2\,\sqrt{3}\,a} = \frac{1}{\sqrt{3}}$$

$\frac{\pi}{4}$

$\sin\left(\frac{\pi}{4}\right) = \frac{a}{\sqrt{2}a} = \frac{1}{\sqrt{2}}$

$\cos\left(\frac{\pi}{4}\right) = \frac{a}{\sqrt{2}a} = \frac{1}{\sqrt{2}}$

$\tan\left(\frac{\pi}{4}\right) = \frac{a}{a} = 1$

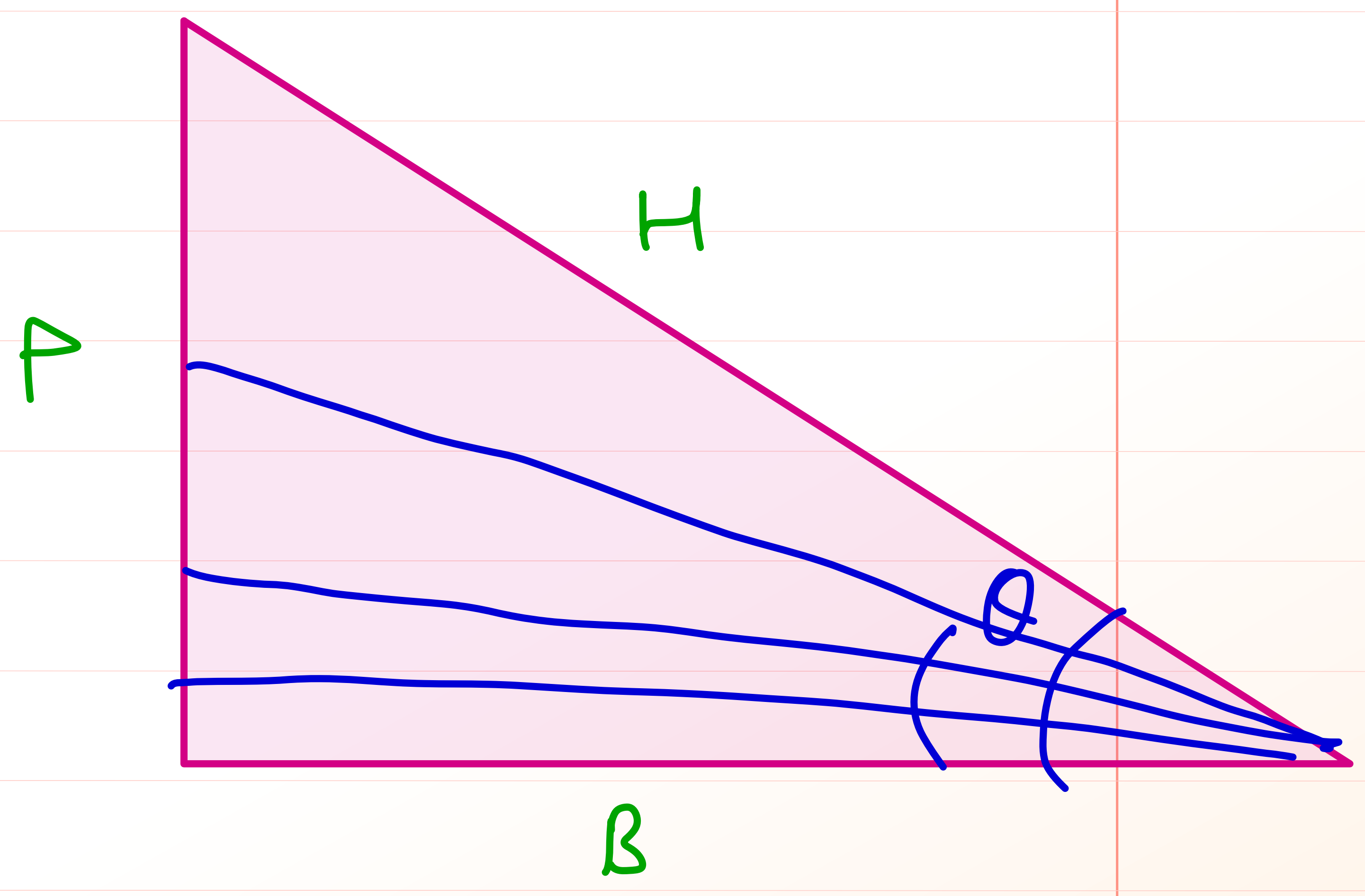


0°

$\sin 0^\circ = \frac{P}{H} = \frac{0}{H} = 0$

$\cos 0^\circ = \frac{B}{H} = \frac{K}{K} = 1$

$\tan 0 = \frac{P}{B} = \frac{0}{B} = 0$



$\theta \to 0$

when

$P \to 0$

$B \to B$

$H \to H$

$B \to H$

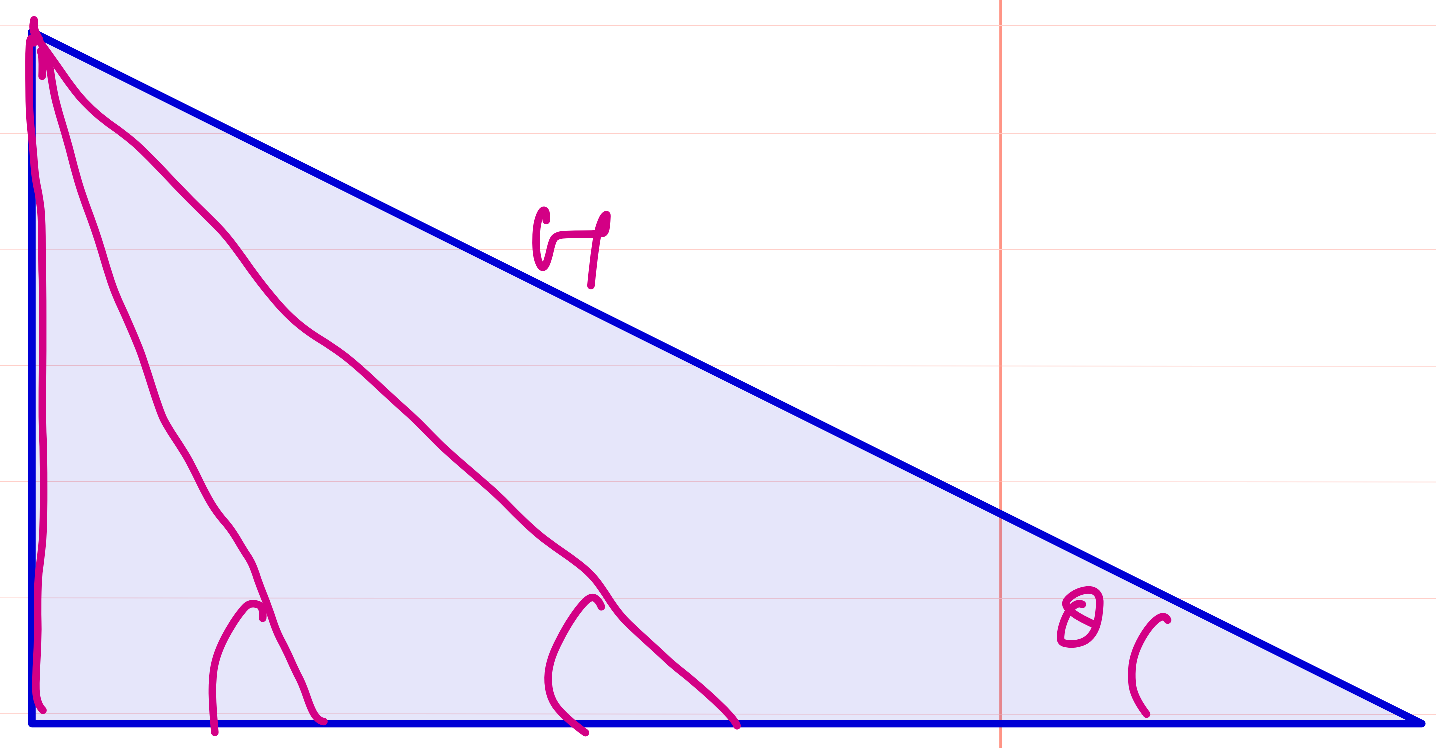
$\underline{\pi/2}$

$$\sin\frac{\pi}{2} = \frac{P}{H} = 1$$

$$\cos\frac{\pi}{2} = \frac{B}{H} = \frac{0}{H} = 0$$

$$\tan\frac{\pi}{2} = \frac{P}{B} = \frac{P}{0} = \infty \text{ (ND)}$$

$H = P$



$B \to 0$

$\theta \to 90^\circ \to \pi/2$

(ii) Values of T-Ratios of some standard angles :

Angles / T-ratio	$0°$	$30°$	$45°$	$60°$	$90°$
	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\sin\theta$	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0
$\tan\theta$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	N.D.
$\cot\theta$	N.D.	$\sqrt{3}$	1	$1/\sqrt{3}$	0
$\sec\theta$	1	$2/\sqrt{3}$	$\sqrt{2}$	2	N.D.
$\text{cosec}\theta$	N.D.	2	$\sqrt{2}$	$2/\sqrt{3}$	1

Ist quad

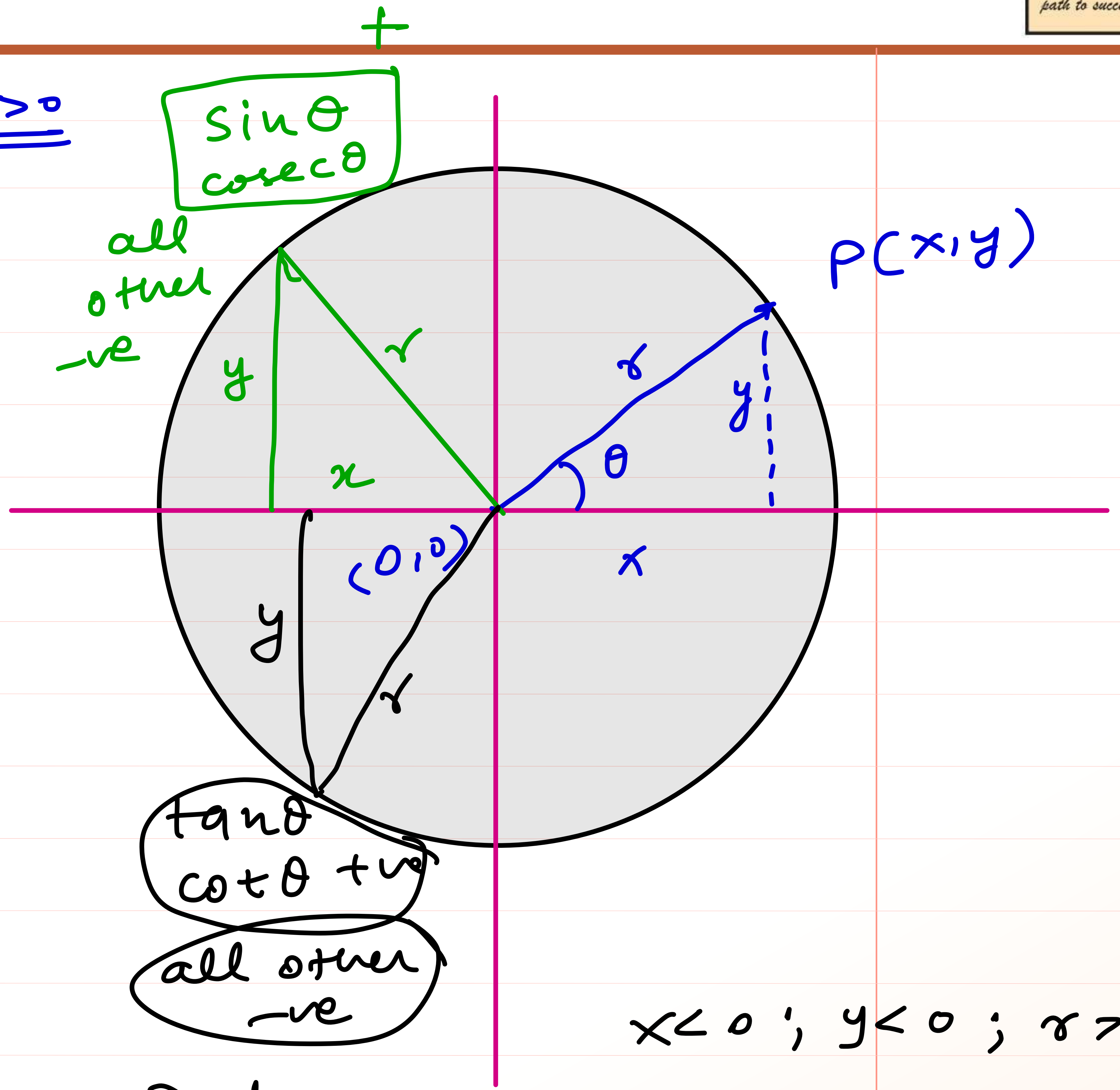
$x > 0,\ y > 0,\ r > 0$

$\underbrace{x > 0,\ y > 0}_{\text{Ist quad}}$

$\sin\theta = \dfrac{y}{r}$

$\cos\theta = \dfrac{x}{r}$ → all +ve

$\tan\theta = \dfrac{y}{x}$



IInd quad

$r > 0,\ y > 0,\ x < 0$

$\sin\theta = \dfrac{y}{r} \rightarrow +ve$

$\cos\theta = \dfrac{x}{r} \rightarrow -ve$

$\tan\theta = \dfrac{y}{x} \rightarrow -ve$

IIIrd

$x < 0;\ y < 0;\ r > 0$

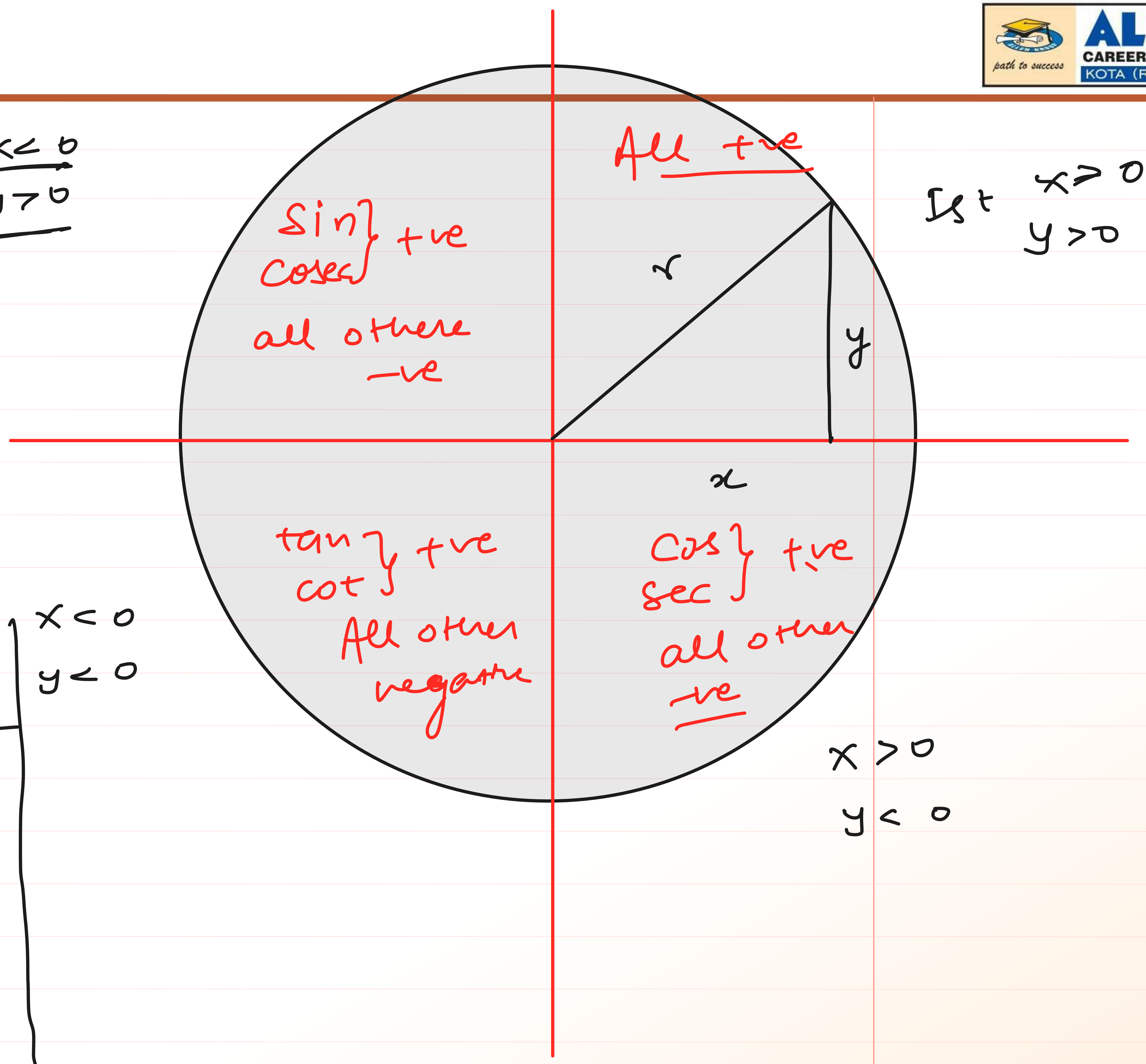
$\sin\theta = \dfrac{y}{r} \rightarrow -ve$

$\cos\theta = \dfrac{x}{r} \rightarrow -ve$

$\tan\theta = \dfrac{y}{x} \rightarrow +ve$

$r > 0$

$x < 0$
$y > 0$



	Ist	IInd	3rd	4th
$\sin\theta = \frac{y}{r}$	+	+	−	−
$\cos\theta = \frac{x}{r}$	+	−	−	+
$\tan\theta = \frac{y}{x}$	+	−	+	−

Rule 1

Vertical line $90^\circ, 270^\circ$

$90 \pm \theta, \quad 270 \pm \theta, \rightarrow$ change

$\sin \rightarrow \cos$
$\cos \rightarrow \sin$
$\tan \rightarrow \cot$
$\cot \rightarrow \tan$
$\sec \rightarrow \operatorname{cosec}$
$\operatorname{cosec} \rightarrow \sec$.

Rule 2

Horizontal line $0, 180^\circ, 360^\circ$

$0 \pm \theta, \quad 180 \pm \theta, \quad 360 \pm \theta$ No change

$90 \pm \theta$, $180 \pm \theta$, $0 \pm \theta$, $270 \pm \theta$, $360 \pm \theta$

$\sin(0-\theta) = -\sin\theta$ $\cos(0-\theta) = +\cos\theta$ $\tan(0-\theta) = -\tan\theta$	$\sin(90-\theta) = +\cos\theta$ $\cos(90-\theta) = +\sin\theta$ $\tan(90-\theta) = +\cot\theta$	$\sin(90+\theta) = +\cos\theta$ $\cos(90+\theta) = -\sin$ $\tan(90+\theta) = -\cot\theta$
$\sin(180-\theta) = +\sin\theta$ $\cos(180-\theta) = -\cos\theta$ $\tan(180-\theta) = -\tan\theta$	$\sin(180+\theta) = -\sin\theta$ $\cos(180+\theta) = -\cos\theta$ $\tan(180+\theta) = +\tan\theta$	$\sin(270-\theta) = -\cos\theta$ $\cos(270-\theta) = -\sin\theta$ $\tan(270-\theta) = +\cot\theta$
$\sin(270+\theta) = -\cos\theta$ $\cos(270+\theta) = +\sin\theta$ $\tan(270+\theta) = -\cot\theta$	$\sin(360-\theta) = -\sin\theta$ $\cos(360-\theta) = +\cos\theta$ $\tan(360-\theta) = -\tan\theta$	$\sin(360+\theta) = +\sin\theta$ $\cos(360+\theta) = +\cos\theta$ $\tan(360+\theta) = +\tan\theta$

$\cos(135) \rightarrow \underline{\cos(180-\boxed{45})}$

$-\cos 45 = -\frac{1}{\sqrt{2}}$

$\underline{\cos}(90+45)$

$-\sin 45 = -\frac{1}{\sqrt{2}}$

$\cos(\underline{360} - \underline{225})$

$= +\cos \underline{225}$

$= \cos(180+45)$

$= -\cos 45$

$= -\frac{1}{\sqrt{2}}$

$\cos(270-135)$

$= -\sin 135$

$= -\sin(\underline{90+45})$

$= -\cos 45$

$= -\frac{1}{\sqrt{2}}$

(i) $\sin(120) = \underline{\sin}(90+30)$

$= +\cos 30$

$= +\frac{\sqrt{3}}{2}$

$\sin(180-60)$

$= +\sin 60$

$= +\frac{\sqrt{3}}{2}$

(ii) $\cos(120^\circ) = \cos(90+30)$

$= -\sin 30 = -\frac{1}{2}$

(iii) $\tan(135^\circ) = \tan(90+45) = -\cot 45 = -1$

(iv) $\cos(150^\circ) = \cos(180-30) = -\cos 30 = -\frac{\sqrt{3}}{2}$

(iv) $\cos 10^\circ + \cos 20^\circ + \cos 30^\circ + \ldots\ldots + \cos 160^\circ + \cos 170^\circ = ?$

$\cancel{\cos 10} + \cancel{\cos 20} + \ldots + \cos 90^\circ \ldots\ldots + \cancel{\cos(180-20)} + \cancel{\cos(180-10)} = 0$

(vi) $\tan\frac{\pi}{11} + \tan\frac{2\pi}{11} + \tan\frac{4\pi}{11} + \tan\frac{7\pi}{11} + \tan\frac{9\pi}{11} + \tan\frac{10\pi}{11} = ?$

$\tan\left(\frac{\pi}{11}\right) + \tan\left(\frac{2\pi}{11}\right) + \tan\left(\frac{4\pi}{11}\right) + \tan\left(\pi - \frac{4\pi}{11}\right) + \tan\left(\pi - \frac{2\pi}{11}\right) + \tan\left(\pi - \frac{\pi}{11}\right)$

$\cancel{\tan\left(\frac{\pi}{11}\right)} + \cancel{\tan\frac{2\pi}{11}} + \cancel{\tan\frac{4\pi}{11}} - \cancel{\tan\left(\frac{4\pi}{11}\right)} - \cancel{\tan\left(\frac{2\pi}{11}\right)} - \cancel{\tan\left(\frac{\pi}{11}\right)} = 0$

$\frac{10\pi}{11} = \frac{11\pi - \pi}{11} = \frac{11\pi}{11} - \frac{\pi}{11} = \pi - \frac{\pi}{11}$

$\frac{7\pi}{11} = \frac{11\pi - 4\pi}{11} = \pi - \frac{4\pi}{11}$

(vii) $\sin(210^\circ) = \underline{\sin}(270-60) = -\cos 60 = -\frac{1}{2}$

(viii) $\cos(240^\circ) = \cos(270-30) = -\sin 30 = -\frac{1}{2}$

(ix) $\sin(315^\circ) = \sin(270+45) = -\cos 45 = -\frac{1}{\sqrt{2}}$

(x) $\cos(315^\circ) = \cos(360-45) = +\cos 45 = \frac{1}{\sqrt{2}}$

(xi) $\tan(330^\circ) = \tan(360-30) = -\tan 30 = -1/\sqrt{3}$

(xii) $\tan(-120^\circ) = -\tan(120) = -\underline{\tan(180-60)} = -[-\tan 60]$

$= +\sqrt{3}$

(xiii) $\sec(2001\pi)$

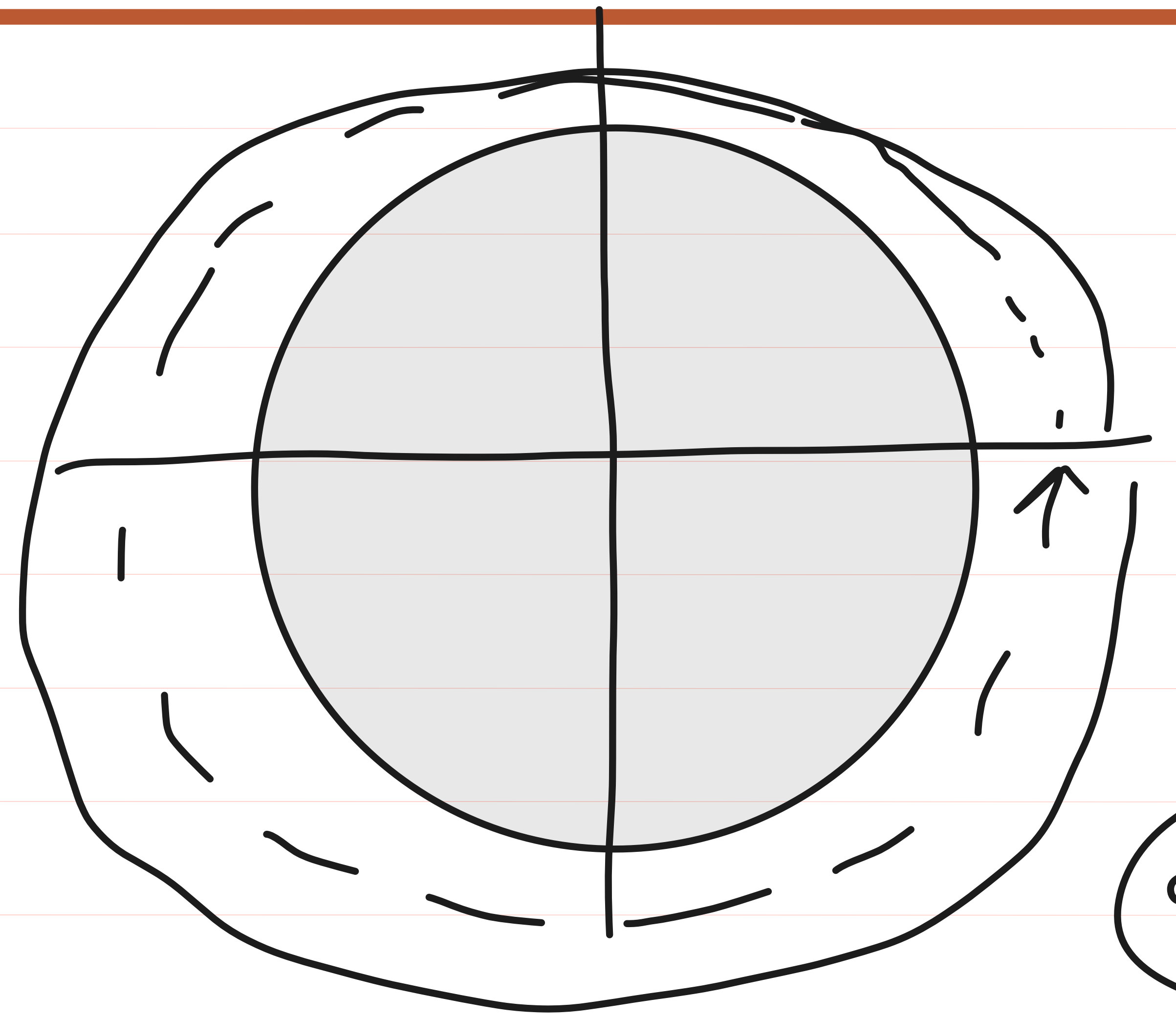
$= \sec(2000\pi + 1\pi)$

$= \sec(2\pi(1000) + \pi)$

$= \underline{\sec \pi}$

$= \sec(\pi + 0)$

$= -(1) = -1$



2π

4π

$2\pi n$

$\underline{n \in N} \rightarrow$ $\sin(2\pi n + \theta) = \underline{\sin\theta}$

$\cos(2\pi n + \theta) = \cos\theta$

$\tan(2\pi n + \theta) = \tan\theta$

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Q $\log[\cos(12346\pi)]$

$= \log(\cos 0) = \log 1 = 0$

$\cos(12346\pi)$

$\cos(2\pi(6173) + 0)$

$= \cos 0$

$\cos(2\pi + \theta) =$

$\cos(12345\pi)$

$= \cos(12344\pi + \pi)$

$= \cos(2\pi(6172) + \pi)$

$= \cos\pi$

$= -1$

$$\cos\left(\frac{12345\pi}{7}\right)$$

$$= \cos\left(\underbrace{1763\pi} + \frac{4\pi}{7}\right)$$

$$= \cos\left(\underbrace{1762\pi} + \left(\pi + \frac{4\pi}{7}\right)\right)$$

$$= \cos\left(\pi + \frac{4\pi}{7}\right)$$

$$= -\cos\frac{4\pi}{7}$$

Q $\tan\left(\frac{11\pi}{6}\right) = \tan\left(\frac{12\pi - \pi}{6}\right)$

$$= \tan\left(2\pi - \frac{\pi}{6}\right)$$

$$= -\tan\frac{\pi}{6} = \frac{-1}{\sqrt{3}}$$

Q $\cot\left(-\frac{5\pi}{6}\right) = -\cot\left(\frac{5\pi}{6}\right)$

$$= -\cot\left(\pi - \frac{\pi}{6}\right)$$

$$= +\cot\frac{\pi}{6} = \sqrt{3}$$