ARITHMETIC PROGRESSION - I

1. Show that the sequence log a, log(ab), log(ab^2), log (ab^3),....... is an A.P. Find its nth term.

2. Which term of the sequence 4, 9, 14, 19,............. is 124 ?

3. Which term of the sequence 20, $19\frac{1}{4}$, $18\frac{1}{2}$, $17\frac{3}{4}$,........ is the first negative term ?

① $\log a, \log(ab), \log(ab^2), \log(ab^3)$ - - - - -

$= \log a, \log a + \log b, \log a + 2\log b, \log a + 3\log b,$ - - -

It is an AP with common difference log b.

$$\boxed{T_n = \log a + (n-1)\log b}$$

② $T_n = 4 + (n-1)5$

$124 = 4 + (n-1)5$

$$\boxed{n = 25}$$

③ $20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}$, - - - - - -

$a_1 = 20$; $d = \frac{77}{4} - 20 = -\frac{3}{4}$

$T_n = 20 + (n-1)\left(-\frac{3}{4}\right)$

$0 > 20 + (n-1)\left(-\frac{3}{4}\right)$

$-3n + 3 + 80 < 0$

$3n > 83$

$n > \frac{83}{3} \Rightarrow \boxed{n = 28}$

4. If m times the mth term of an A.P. is equal to n times its nth term, show that the $(m + n)^{th}$ term of the A.P. is zero.

5. If the pth term of an A.P. is q and the qth term is p, prove that its nth term is $(p + q - n)$.

④ $m \cdot T_m = n \cdot T_n$

$m(a + (m-1)d) = n(a + (n-1)d)$

$a(m-n) + d(m^2 - m - n^2 + n) = 0$

$a(m-n) + d(m^2 - n^2 - (m-n)) = 0$

$a(m-n) + d((m+n)(m-n) - (m-n)) = 0$

$(m-n)[a + (m+n-1)d] = 0$

$\therefore \quad T_{m+n} = a + (m+n-1)d = 0$

⑤ $T_p = q \Rightarrow a + (p-1)d = q$

$T_q = p \Rightarrow a + (q-1)d = p$

$(p-q)d = q - p \Rightarrow \boxed{d = -1}$

$a = q - (p-1)d$

$= q - (p-1)(-1)$

$\boxed{a = q + p - 1}$

$T_{p+q-n} = a + (p+q-n-1)(-1)$

$= q + p - 1 - p - q + n + 1 = n$

6. If $a_1, a_2, a_3, \ldots, a_n$ be an A.P. of non-zero terms, prove that $\frac{1}{a_1a_2}+\frac{1}{a_2a_3}+\ldots+\frac{1}{a_{n-1}a_n}=\frac{n-1}{a_1a_n}$.

7. The sum of three numbers in A.P. is –3, and their product is 8. Find the numbers.

(6) $$\frac{1}{a_1 \cdot a_2}+\frac{1}{a_2 \cdot a_3}+\frac{1}{a_3 \cdot a_4}+\frac{1}{a_4 \cdot a_5}+ \cdots + \frac{1}{a_{n-1} \cdot a_n}$$

$$= \frac{1}{d}\left[\frac{a_2-a_1}{a_1a_2}+\frac{a_3-a_2}{a_2 \cdot a_3}+\frac{a_4-a_3}{a_3 \cdot a_4}+ \cdots +\frac{a_n-a_{n-1}}{a_{n-1} \cdot a_n}\right]$$

$$= \frac{1}{d}\left[\frac{1}{a_1}-\frac{1}{a_2}+\frac{1}{a_2}-\frac{1}{a_3}+\frac{1}{a_3}-\frac{1}{a_4}+ \cdots +\frac{1}{a_{n-1}}-\frac{1}{a_n}\right]$$

$$= \frac{1}{d}\left[\frac{1}{a_1}-\frac{1}{a_n}\right] = \frac{1}{d}\left[\frac{a_n-a_1}{a_n \cdot a_1}\right]$$

$$= \frac{1}{d}\left[\frac{a+(n-1)d-a}{a_n \cdot a_1}\right] = \frac{1}{d}\left[\frac{(n-1)d}{a_na_1}\right] = \frac{n-1}{a_na_1}$$

(7) Let the numbers are $a-d,\ a,\ a+d$

$$a-d+a+a+d=-3 \Rightarrow \boxed{a=-1}$$

$$(a-d) \cdot a \cdot (a+d) = 8$$

$$(-1-d)(-1)(-1+d)=8 \Rightarrow (d+1)(d-1)=8$$

$$\boxed{d = \pm 3}$$

Numbers $= -1-3,\ -1,\ -1+3 \Rightarrow \boxed{-4, -1, 2}$

or $-1+3,\ -1,\ -1-3 \Rightarrow \boxed{2, -1, -4}$

8. Divide 32 into four parts which are in A.P. such that the product of extremes is to the product of means is 7 : 15.

9. Find the sum of the series : 5 + 13 + 21 + ... + 181.

10. Find the sum of all three digit natural numbers, which are divisible by 7.

(8) Let the parts are $a-3d,\ a-d,\ a+d,\ a+3d$

$a-3d + a-d + a+d + a+3d = 32 \Rightarrow \boxed{a=8}$

product of extremes $= (a-3d)(a+3d) = 8^2-9d^2$

product of means $= (a-d)(a+d) = 8^2-d^2$

$$\frac{8^2-9d^2}{8^2-d^2} = \frac{7}{15} \Rightarrow 15(64-9d^2) = 7(64-d^2)$$

$$960 - 135d^2 = 448 - 7d^2$$

$$960 - 448 = 128d^2$$

$$d^2 = \frac{512}{128} \Rightarrow d = \pm 2$$

Numbers are $8-(6), (8-2), (8+2), 8+2(3)$

$\Rightarrow 2, 6, 10, 14$ or $14, 10, 6, 2$

(9) $\text{Sum} = \frac{n}{2}[2a+(n-1)d]$

$= \frac{23}{2}[186] = 23(93) = 2139$

$181 = 5+(n-1)8$

$176 = (n-1)8$

$(n = 23)$

(10) Three digit number div by 7

105, 112, ----- , 994

$S_n = \frac{128}{2}[105+994] = 64 \times 1099$

$= 70336$

$994 = 105 + (n-1)7$

$\frac{889}{7} = n-1$

$127 = n-1$

$n = 128$

11. Find the sum of first 20 terms of an A.P., in which 3rd term is 7 and 7th term is two more than thrice of its 3rd term.

12. If S_n denotes the sum of first n terms of A.P. and $\frac{S_{3n}-S_{n-1}}{S_{2n}-S_{2n-1}}=31$, then n is equal to

(11) $T_3 = 7 \Rightarrow a+2d=7 \Rightarrow \boxed{a = 7-2d}$

$T_7 = 3T_3+2 \Rightarrow a+6d = 3(a+2d)+2$

$a+6d = 3a+6d+2$

$\boxed{a=-1}$

$\because a = 7-2d \Rightarrow 2d = 7-a \Rightarrow 2d=8$

$\boxed{d=4}$

$S_{20} = \frac{20}{2}[2(-1)+(20-1)4] = 740$

(12) $\frac{S_{3n}-S_{n-1}}{S_{2n}-S_{2n-1}} = 31 \Rightarrow \frac{\frac{3n}{\cancel{2}}[2a+(3n-1)d]-\frac{n-1}{\cancel{2}}[2a+(n-2)d]}{\frac{2n}{\cancel{2}}[2a+(2n-1)d]-\frac{2n-1}{\cancel{2}}[2a+(2n-2)d]} = 31$

$$\frac{2a(3n-n+1)+d(3n(3n-1)-(n-1)(n-2))}{2a(2n-2n+1)+d(2n(2n-1)-(2n-1)(2n-2))} = 31$$

$$\frac{2a(2n+1)+d(8n^2-2)}{2a+d(4n^2-2n-4n^2+6n-2)} = 31$$

$$\frac{2a(2n+1)+2d(2n+1)(2n-1)}{2a+2(2n-1)d} = 31$$

$$\frac{2(2n+1)\cancel{(a+(2n-1)d)}}{2\cancel{(a+(2n-1)d)}} = 31 \Rightarrow 2n+1 = 31$$

$\boxed{n=15}$

13. Find the number of terms in the series 20, $19\frac{1}{3}$, $18\frac{2}{3}$,.... of which the sum is 300, explain the double answer.

14. The sum of the first p, q, r terms of an A.P. are a, b, c respectively. Show that $\frac{a}{p}(q-r)+\frac{b}{q}(r-p)+\frac{c}{r}(p-q)=0$.

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$$S = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$300 = \frac{n}{2}\left[2(20) + (n-1)\left(-\frac{2}{3}\right)\right]$$

$$300 = n\left[20 + (n-1)\left(-\frac{1}{3}\right)\right]$$

$$900 = n\left[60 - n + 1\right] \Rightarrow n^2 - 61n + 900 = 0$$

$$(n-25)(n-36) = 0$$

$$\boxed{n = 25} \quad \boxed{n = 36}$$

14

$$S_p = \frac{p}{2}\left[2a_1 + (p-1)d\right] = a$$

$$\left[a_1 + \left(\frac{p-1}{2}\right)d = \frac{a}{p}\right] \times (q-r)$$

similarly

$$\left[a_1 + \left(\frac{q-1}{2}\right)d = \frac{b}{q}\right] (r-p)$$

&

$$\left[a_1 + \left(\frac{r-1}{2}\right)d = \frac{c}{r}\right] (p-q)$$

$$0 = \frac{a}{p}(q-r) + \frac{b}{q}(r-p) + \frac{c}{r}(p-q)$$

15. The ratio of the sum of n terms of two A.P.'s is (7n + 1) : (4n + 27). Find the ratio of their m[th] terms.

16. If a, b, c are in A.P., prove that the following are also in A.P.

(i) b + c, c + a, a + b

(ii) $a\left(\frac{1}{b}+\frac{1}{c}\right), b\left(\frac{1}{c}+\frac{1}{a}\right), c\left(\frac{1}{a}+\frac{1}{b}\right)$

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$$\frac{S_n}{S_n'} = \frac{\cancel{\frac{n}{2}}[2a_1+(n-1)d_1]}{\cancel{\frac{n}{2}}[2a_2+(n-1)d_2]} = \frac{7n+1}{4n+27}$$

$$\frac{T_m}{T_m'} = \frac{a_1+(m-1)d_1}{a_2+(m-1)d_2}$$

$$\frac{a_1+\left(\frac{n-1}{2}\right)d_1}{a_2+\left(\frac{n-1}{2}\right)d_2} = \frac{7n+1}{4n+27}$$

$$\frac{n-1}{2} = m-1$$

$$n = 2m-1$$

$$\frac{T_m}{T_m'} = \frac{7(2m-1)+1}{4(2m-1)+27}$$

$$\frac{T_m}{T_m'} = \frac{14m-6}{8m+23}$$

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(i) $b+c,\ c+a,\ a+b$

$T_2 - T_1 = T_3 - T_2 \Rightarrow c+a-b-c = a+b-c-a$

$a-b = b-c$

$$\boxed{a+c=2b}$$

(ii) $a\left(\frac{1}{b}+\frac{1}{c}\right), b\left(\frac{1}{c}+\frac{1}{a}\right), c\left(\frac{1}{a}+\frac{1}{b}\right)$

add 1 to each term

$a\left(\frac{1}{b}+\frac{1}{c}\right)+1,\ b\left(\frac{1}{c}+\frac{1}{a}\right)+1,\ c\left(\frac{1}{a}+\frac{1}{b}\right)+1$

$a\left(\frac{1}{b}+\frac{1}{c}\right)+\frac{a}{a},\ b\left(\frac{1}{c}+\frac{1}{a}\right)+\frac{b}{b},\ c\left(\frac{1}{a}+\frac{1}{b}\right)+\frac{c}{c}$

$a\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right),\ b\left(\frac{1}{c}+\frac{1}{a}+\frac{1}{b}\right),\ c\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$

$a, b, c \to$ AP.

17. If a^2, b^2, c^2 are in A.P., then prove that $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ is also in A.P.

18. If $\log_{10}2$, $\log_{10}(2^x - 1)$ and $\log_{10}(2^x + 3)$ are in A.P., then find the value of x.

(17)

$$b^2 - a^2 = c^2 - b^2$$

$$(b-a)(b+a) = (c-b)(c+b)$$

$$\frac{b-a}{(c+a)(b+c)} = \frac{c-b}{(b+a)(c+a)}$$

$$\frac{b-a+c-c}{(c+a)(b+c)} = \frac{(c-b+a-a)}{(b+a)(c+a)}$$

$$\frac{(b+c)-(a+c)}{(b+c)(c+a)} = \frac{(c+a)-(a+b)}{(a+b)(c+a)}$$

$$\frac{1}{c+a} - \frac{1}{b+c} = \frac{1}{a+b} - \frac{1}{c+a}$$

$\therefore$ $\frac{1}{a+b}, \frac{1}{b+c}, \frac{1}{c+a}$ are in AP.

(18)

$$2\log_{10}(2^x - 1) = \log_{10}2 + \log_{10}(2^x + 3)$$

$$(2^x - 1)^2 = 2(2^x + 3)$$

Let $2^x = t$

$$(t-1)^2 = 2(t+3) \Rightarrow t^2 - 2t + 1 = 2t + 6$$

$$t^2 - 4t - 5 = 0$$

$$(t-5)(t+1) = 0$$

$$t = 5, \quad t = -1$$

$$2^x = 5 \Rightarrow \boxed{x = \log_2 5}$$

$2^x = -1$ not possible

19. If S_n denotes the sum of n terms of A.P., then find $S_{n+3} - 3S_{n+2} + 3S_{n+1} - S_n$ is equal to

20. The digits of a positive integer, having three digits, are in A.P. and their sum is 15. The number obtained by reversing the digits is 594 less than the original number. Find the number.

19. $S_{n+3} - 3S_{n+2} + 3S_{n+1} - S_n$

$= S_{n+3} - S_{n+2} - 2S_{n+2} + 2S_{n+1} + S_{n+1} - S_n$

$= (S_{n+3} - S_{n+2}) - 2(S_{n+2} - S_{n+1}) + (S_{n+1} - S_n)$

$= T_{n+3} - 2T_{n+2} + T_{n+1}$

$= T_{n+3} - T_{n+2} - T_{n+2} + T_{n+1}$

$= (T_{n+3} - T_{n+2}) - (T_{n+2} - T_{n+1})$

$= d - d = 0$

20. Let the digits are $a-d, a, a+d$

$a-d+a+a+d = 15 \Rightarrow \boxed{a=5}$

Number $= 100(a-d) + 10a + a+d = 111a - 99d$

Reversed number $= 100(a+d) + 10a + a-d = 111a + 99d$

$111a + 99d = 111a - 99d - 594$

$198d = -594 \Rightarrow \boxed{d=-3}$

digits $5-(-3), 5, 5-3 \Rightarrow 8, 5, 2$

Required number $= 852$

21. The least value of 'a' for which $5^{1+x} + 5^{1-x}$, a/2, $25^x + 25^{-x}$ are three consecutive terms of an AP is

(A) 1 (B) 5 (C) 12 ✓ (D) None of these

22. If p,q, r in A.P. and are positive, the roots of the quadratic equation $px^2 + qx + r = 0$ are all real for

✓ (A) $\left|\frac{r}{p}-7\right| \geq 4\sqrt{3}$ (B) $\left|\frac{p}{r}-7\right| < 4\sqrt{3}$ (C) all p and r (D) No. p and r

(21)

$$2\left(\frac{a}{2}\right) = 5^{1+x} + 5^{1-x} + 25^{x} + 25^{-x}$$

$$a = 5\cdot 5^{x} + 5\cdot 5^{-x} + 5^{2x} + 5^{-2x}$$

$$= 5\left(5^{x} + \frac{1}{5^{x}}\right) + 5^{2x} + 5^{-2x}$$

$\because$ $5^{x} + \frac{1}{5^{x}} \geq 2$

$$\therefore\ a\Big|_{min} = 5(2) + 2 \Rightarrow a\Big|_{min} = 12$$

(22)

$px^2 + qx + r = 0$

$D \geq 0$

$q^2 - 4pr \geq 0$

$\because$ $p, q, r \longrightarrow AP$

$2q = p + r$

$q = \frac{p+r}{2}$

$$\left(\frac{p+r}{2}\right)^2 - 4pr \geq 0$$

$$\frac{p^2 + r^2 + 2pr}{4} - 4pr \geq 0$$

$$p^2 + r^2 - 14pr \geq 0$$

divide by r^2

$$\frac{p^2}{r^2} + 1 - 14\frac{p}{r} + 49 \geq 0 + 49$$

$$\left(\frac{p}{r} - 7\right)^2 \geq 48 \Rightarrow \boxed{\left|\frac{p}{r} - 7\right| \geq 4\sqrt{3}}$$

23. Sum of first hundred numbers common to the two A.P.'s 12, 15, 18,... and 17, 21, 25 ..., is

(A) 56100 (B) 65100 ✓(C) 61500 (D) none of these

(23) ⇒ 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45 ----

⇒ 17, 21, 25, 29, 33, 37, 41, 45, 49, 53, ------

common terms ⇒ 21, 33, 45, -----

$$S_{100} = \frac{100}{2}\left[2(21) + (100-1)(12)\right]$$

$$= 50\left[42 + 99(12)\right] = 61500$$

ARITHMETIC PROGRESSION - II

1. If S_r denotes the sum of r terms of an AP and $\frac{S_a}{a^2} = \frac{S_b}{b^2} = c$ then S_c is

(A) c^3 (B) c/ab (C) abc (D) a + b + c

1 $S_a = a^2c \Rightarrow \frac{a}{2}[2a_1 + (a-1)d] = a^2c$

$2a_1 + (a-1)d = 2ac$ —— (1)

$S_b = b^2c \Rightarrow \frac{b}{2}[2a_1 + (b-1)d] = b^2c$

$2a_1 + (b-1)d = 2bc$ —— (2)

Subtract

$d(a-b) = 2ac - 2bc$

$d(a-b) = 2c(a-b)$

$\boxed{d = 2c}$

$2a_1 + (a-1)2c = 2ac \Rightarrow \boxed{a_1 = c}$

$S_c = \frac{c}{2}[2a_1 + (c-1)d] = \frac{c}{\cancel{2}}[2c + (c-1)2c] = c^3$

2. If $a_r > 0$, $r \in N$ and $a_1, a_2, a_3 \ldots, a_{2n}$ are in AP then $\frac{a_1+a_{2n}}{\sqrt{a_1}+\sqrt{a_2}}+\frac{a_2+a_{2n-1}}{\sqrt{a_2}+\sqrt{a_3}}+\frac{a_3+a_{2n-2}}{\sqrt{a_3}+\sqrt{a_4}}+\ldots.+\frac{a_n+a_{n+1}}{\sqrt{a_n}+\sqrt{a_{n+1}}}$ is equal to

(A) $n-1$ (B) $\frac{n(a_1+a_{2n})}{\sqrt{a_1}+\sqrt{a_{n+1}}}$ (C) $\frac{n-1}{\sqrt{a_1}+\sqrt{a_{n+1}}}$ (D) none of these

$$\frac{a_1+a_{2n}}{\sqrt{a_1}+\sqrt{a_2}}+\frac{a_2+a_{2n-1}}{\sqrt{a_2}+\sqrt{a_3}}+\frac{a_3+a_{2n-2}}{\sqrt{a_3}+\sqrt{a_4}}+\text{-----}+\frac{a_n+a_{n+1}}{\sqrt{a_n}+\sqrt{a_{n+1}}}$$

$$=\frac{a+a+(2n-1)d}{\sqrt{a_1}+\sqrt{a_2}}+\frac{a+d+a+(2n-2)d}{\sqrt{a_2}+\sqrt{a_3}}+\frac{a+2d+a+(2n-3)d}{\sqrt{a_3}+\sqrt{a_4}}$$

$$+\text{----}\ \frac{a+(n-1)d+a+nd}{\sqrt{a_n}+\sqrt{a_{n+1}}}$$

$$=[2a+(2n-1)d]\left[\frac{1}{(\sqrt{a_1}+\sqrt{a_2})}\frac{(\sqrt{a_2}-\sqrt{a_1})}{(\sqrt{a_2}-\sqrt{a_1})}+\frac{1}{(\sqrt{a_2}+\sqrt{a_3})}\frac{(\sqrt{a_3}-\sqrt{a_2})}{(\sqrt{a_3}-\sqrt{a_2})}+\frac{1}{\sqrt{a_3}+\sqrt{a_4}}+\text{----}+\frac{1}{\sqrt{a_n}+\sqrt{a_{n+1}}}\right]$$

$$=[2a+(2n-1)d]\left[\frac{\sqrt{a_2}-\sqrt{a_1}}{d}+\frac{\sqrt{a_3}-\sqrt{a_2}}{d}+\frac{\sqrt{a_4}-\sqrt{a_3}}{d}+\text{----}+\frac{\sqrt{a_{n+1}}-\sqrt{a_n}}{d}\right]$$

$$=\frac{(2a+(2n-1)d)}{d}\left[\sqrt{a_{n+1}}-\sqrt{a_1}\right]\frac{\sqrt{a_{n+1}}+\sqrt{a_1}}{\sqrt{a_{n+1}}+\sqrt{a_1}}$$

$$=\frac{(2a+(2n-1)d)}{d}\left(\frac{a_{n+1}-a_1}{\sqrt{a_{n+1}}+\sqrt{a_1}}\right)=\frac{(2a+(2n-1)d)}{d}\ \frac{d\cdot n}{\sqrt{a_{n+1}}+\sqrt{a_1}}$$

$$=\frac{n(a_1+a_{2n})}{\sqrt{a_{n+1}}+\sqrt{a_n}}$$

3. If $a_1, a_2, a_3, \ldots a_{2n+1}$ are in AP then $\frac{a_{2n+1}-a_1}{a_{2n+1}+a_1}+\frac{a_{2n}-a_2}{a_{2n}+a_2}+\ldots+\frac{a_{n+2}-a_n}{a_{n+2}+a_n}$ is equal to

✓ (A) $\frac{n(n+1)}{2}\cdot\frac{a_2-a_1}{a_{n+1}}$ (B) $\frac{n(n+1)}{2}\cdot\frac{a_2-a_1}{a_n}$ (C) $(n+1)(a_2-a_1)$ (D) none of these

③ $$\frac{a_{2n+1}-a_1}{a_{2n+1}+a_1}+\frac{a_{2n}-a_2}{a_{2n}+a_2}+\text{----}+\frac{a_{n+2}-a_n}{a_{n+2}+a_n}$$

$$=\frac{a+2nd-a}{K}+\frac{a+(2n-1)d-a-d}{K}+\ldots+\frac{a+(n+1)d-a-(n-1)d}{K}$$

$K=a_{2n+1}+a_1$
$=a_{2n}+a_2$
$=a_{2n-1}+a_3$

$$=\frac{d}{K}\left[2n+(2n-2)+(2n-4)+(2n-6)+\text{----}+4+2\right]$$

$$=\frac{2d}{K}\left[n+(n-1)+(n-2)+\text{----}+3+2+1\right]$$

$$=\frac{2d}{K}\left[1+2+3+\text{------}+(n-2)+(n-1)+n\right]$$

$$=\frac{2d}{K}\cdot\frac{n(n+1)}{2}=\frac{n(n+1)}{K}\cdot d=\frac{n(n+1)}{a_{2n+1}+a_1}\cdot(a_2-a_1)$$

$$=\frac{n(n+1)}{a+2nd+a}(a_2-a_1)=\frac{n(n+1)}{2(a+nd)}(a_2-a_1)$$

$$=\frac{n(n+1)}{2a_{n+1}}(a_2-a_1)$$

4. Let $a_1, a_2, a_3, \ldots\ldots$ be terms of an A.P. If $\dfrac{a_1 + a_2 + \ldots. + a_p}{a_1 + a_2 + \ldots.. + a_q} = \dfrac{p^2}{q^2}$, $p \neq q$, then $\dfrac{a_6}{a_{21}}$ equals

(A) 41/11 (B) 7/2 (C) 2/7 (D) 11/41 ✓

5. Let $\{a_n\}$ $(n \geq 1)$ be a sequence such that $a_1 = 1$, and $3a_{n+1} - 3a_n = 1$ for all $n \geq 1$. Then a_{2002} is equal to

(A) 666 (B) 667 (C) 668 ✓ (D) 669

(4)

$$\frac{a_1 + a_2 + \ldots\ldots + a_p}{a_1 + a_2 + \ldots\ldots + a_q} = \frac{p^2}{q^2}$$

$$\frac{\frac{p}{2}\left[2a + (p-1)d\right]}{\frac{q}{2}\left[2a + (q-1)d\right]} = \frac{p^2}{q^2} \Rightarrow \frac{a + \left(\frac{p-1}{2}\right)d}{a + \left(\frac{q-1}{2}\right)d} = \frac{p}{q}$$

$$\frac{a_6}{a_{21}} = \frac{a+5d}{a+20d} \qquad \therefore \quad \frac{p-1}{2} = 5 \Rightarrow p = 11$$

$$\frac{q-1}{2} = 20 \Rightarrow q = 41$$

$$\therefore \quad \frac{a_6}{a_{21}} = \frac{11}{41}$$

(5) $a_1 = 1$ $\qquad 3a_{n+1} - 3a_n = 1$

$$3a_2 - 3a_1 = 1 \Rightarrow a_2 - a_1 = \frac{1}{3}$$

$$\therefore \quad d = 1/3$$

$$a_{2002} = a_1 + 2001d$$

$$= 1 + (2001)\frac{1}{3} = 1 + 667 = 668$$

6. If 4th term of an AP is 64 and its 54th term is – 61, then its common difference is

(A) 5/2 (B) – 5/2 (C) 3/50 (D) – 3/50

7. The 19th term from the end of the series 2 + 6 + 10 + ...+ 86 is

(A) 6 (B) 18 (C) 14 (D) 10

8. If nth term of an AP is 1/3 (2n + 1) , then the sum of its 19 terms is

(A) 131 (B) 132 (C) 133 (D) 134

⑥ $a_4 = 64 \Rightarrow a + 3d = 64$

$a_{54} = -61 \Rightarrow a + 53d = -61$

$50d = -125$

$d = \frac{-125}{50} = -\frac{5}{2}$

$a + 3a = 64$

$a = 64 + 3\left(\frac{5}{2}\right)$

$= 64 + \frac{15}{2}$

$= \frac{143}{2}$

⑦ Series $\rightarrow$ $2 + 6 + 10 + \text{----} + 82 + 86$

Reversed $\rightarrow$ $86 + 82 + \text{-----} + 10 + 6 + 2$

$T_{19} = 86 + (19-1)(-4) = 14$

⑧ $T_n = \frac{1}{3}(2n+1)$

$S = \frac{1}{3}(3) + \frac{1}{3}(5) + \frac{1}{3}(7) + \frac{1}{3}(9) + \text{----}$

$S_{19} = \frac{19}{2}\left[2(1) + (19-1)\frac{2}{3}\right] = 19(7) = 133$

9. If the ratio of the sum of n terms of two AP's is 2n : (n + 1), then ratio of their 8^{th} terms is

(A) 15 : 8 (B) 8 : 13 (C) n : (n– 1) (D) 5 : 17

10. The sum of n terms of an AP is $3n^2 + 5n$. Then number of term when n^{th} term equals 164 is

(A) 13 (B) 21 (C) 27 (D) 29

11. If the m^{th} term of an A.P. is $\frac{1}{n}$ and the n^{th} term is $\frac{1}{m}$ then sum to mn terms is

(A) $\frac{mn+1}{2}$ (B) $\frac{mn-1}{2}$ (C) $\frac{mn+1}{3}$ (D) $\frac{mn-1}{3}$

(9)
$$\frac{S_n}{S_n'} = \frac{2n}{n+1} \Rightarrow \frac{\frac{n}{2}[2a_1 + (n-1)d_1]}{\frac{n}{2}[2a_2 + (n-1)d_2]} = \frac{2n}{n+1}$$

$$\frac{a_8}{a_8'} = \frac{a_1 + 7d_1}{a_2 + 7d_2}$$

$$\frac{a_1 + \left(\frac{n-1}{2}\right)d_1}{a_2 + \left(\frac{n-1}{2}\right)d_2} = \frac{2n}{n+1}$$

$$\frac{n-1}{2} = 7 \Rightarrow n = 15$$

$$= \frac{30}{16} = \frac{15}{8}$$

(10)
$$S_n = 3n^2 + 5n$$

$$T_n = S_n - S_{n-1} = (3n^2 + 5n) - (3(n-1)^2 + 5(n-1))$$

$$= (3n^2 + 5n) - (3n^2 + 3 - 6n + 5n - 5)$$

$$T_n = 6n + 2 = 164$$

$$n = \frac{162}{6} \Rightarrow \boxed{n = 27}$$

(11)
$$T_m = \frac{1}{n} \Rightarrow a + (m-1)d = \frac{1}{n}$$

$$T_n = \frac{1}{m} \Rightarrow a + (n-1)d = \frac{1}{m}$$

$$\Rightarrow (m-n)d = \frac{1}{n} - \frac{1}{m}$$

$$d = \frac{1}{mn}$$

$$a + (m-1)\frac{1}{mn} = \frac{1}{n} \Rightarrow a = \frac{1}{mn}$$

$$S_{mn} = \frac{mn}{2}[2a + (mn-1)d] = \frac{mn}{2}\left[\frac{2}{mn} + (mn-1)\frac{1}{mn}\right] = \frac{mn+1}{2}$$

12. If a,b,c be the 1^{st}, 3^{rd} and n^{th} terms respectively of an A.P., then sum to n terms is

(A) $\frac{c+a}{2}+\frac{c^2-a^2}{b-a}$ (B) $\frac{c+a}{2}-\frac{c^2-a^2}{b-a}$ (C) $\frac{c+a}{2}+\frac{c^2+a^2}{b-a}$ (D) $\frac{c+a}{2}+\frac{c^2+a^2}{b+a}$

(12) $T_1 = a$

$T_3 = b = a + 2d \Rightarrow d = \frac{b-a}{2}$

$T_n = c = a + (n-1)d \Rightarrow \frac{c-a}{d} = n-1$

$n-1 = \frac{2(c-a)}{(b-a)}$

$n = \frac{2c-2a+b-a}{b-a}$

$n = \frac{2c+b-3a}{b-a}$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{n}{2}[a+c]$ ← (first + last term)

$= \frac{(2c+b-3a)(a+c)}{2(b-a)}$

$= \frac{(2c-2a+b-a)(a+c)}{2(b-a)}$

$= \frac{[2(c-a)+(b-a)](a+c)}{2(b-a)}$

$= \frac{\cancel{2}(c-a)(c+a)}{\cancel{2}(b-a)} + \frac{(b-a)(a+c)}{2(b-a)}$

$= \frac{c^2-a^2}{b-a} + \frac{a+c}{2}$

13. If $a_1, a_2, a_3,....$ are in A.P. and $a_1 + a_4 + a_7 + ... + a_{16} = 147$ then $a_1 + a_6 + a_{11} + a_{16}$ is equal to

(A) 96 (B) 98 (C) 100 (D) None of these

14. If $a_1, a_2, a_3, ...$ is an A.P. such that $a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{24} = 225$ then $a_1 + a_2 + a_3 + ... + a_{23} + a_{24}$ is equal to

(A) 909 (B) 75 (C) 750 (D) 900

13

$$a_1 + a_4 + a_7 + ---- + a_{16} = 147$$

$$a + a+3d + a+6d + a+9d + a+12d + a+15d = 147$$

$$6a + 45d = 147$$

div by 3 $\quad 2a + 15d = 49$

$$a_1 + a_6 + a_{11} + a_{16} = a + a+5d + a+10d + a+15d$$

$$= 4a + 30d$$

$$= 2(2a+15d) = 2(49)$$

$$= 98$$

14

$$a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{24} = 225$$

$$\underline{a} + \underline{a}+4d + \underline{a}+9d + \underline{a}+14d + \underline{a}+19d + \underline{a}+23d = 225$$

$$6a + 69d = 225 \Rightarrow \boxed{2a + 23d = 75}$$

$$a_1 + a_2 + a_3 + ---- + a_{23} + a_{24} =$$

$$= \frac{24}{2}[2a + (24-1)d] = 12(2a + 23d)$$

$$= 12(75) = 900$$

15. The sum of all even positive integers less then 200 which are not divisible by 6 is

(A) 6534 (B) 6354 (C) 6543 (D) 6454

16. If x, y, z are in AP, a is AM between x and y and b is AM between y and z; then AM between a and b will be

(A) $\frac{1}{3}(x + y + z)$ (B) z (C) x (D) y

15 $(2+4+6+8+ \ldots + 198) - (6+12+18+ \ldots +198)$

$= \frac{99}{2}(2+198) - \frac{33}{2}(6+198)$

$= 9900 - 33(102) = 6534$

16 $x, y, z \rightarrow AP$ $a = \frac{x+y}{2}$; $b = \frac{y+z}{2}$

$2y = x+z$

$AM = \frac{a+b}{2} = \frac{\frac{x+y}{2} + \frac{y+z}{2}}{2}$

$= \frac{x+2y+z}{4} = \frac{2y+2y}{4} = y$

17. If n AM's are inserted between 1 and 31 and ratio of 7th and (n–1)th A.M. is 5 : 9, then n equals

(A) 12 (B) 13 (C) 14 (D) None of these

18. Three numbers are in A.P., If their sum is 33 and their product is 792, then the smallest of these numbers is

(A) 14 (B) 11 (C) 8 (D) 4

(17) $1, A_1, A_2, A_3, \ldots, A_{n-1}, A_n, 31$

$A_7 = a+7d = 1+7\left(\frac{30}{n+1}\right) = \frac{n+211}{n+1}$ | $d = \frac{31-1}{n+1} = \frac{30}{n+1}$

$A_{n-1} = a+(n-1)d = 1+(n-1)\frac{30}{n+1} = \frac{31n-29}{n+1}$

$\frac{A_7}{A_{n-1}} = \frac{n+211}{31n-29} = \frac{5}{9}$

$9n+1899 = 155n-145$

$1899+145 = 146n$

$\boxed{n=14}$

(18) Three nos in AP $\rightarrow a-d, a, a+d$

$a-d+a+a+d = 33 \Rightarrow \boxed{a=11}$

$(a-d)a(a+d) = 792$

$(11-d)(11)(11+d) = 792$

$121-d^2 = 72$

$d^2 = 121-72 = 49$

$\boxed{d = \pm 7}$

Smallest number $= 11-7 = 4$

19. If the angles of a quadrilateral are in A.P. whose common difference is 10º, then the angles of the quadrilateral are

(A) 65º, 85º, 95º, 105º (B) 75º, 85º, 95º, 105º (C) 65º, 75º, 85º, 95º (D) 65º, 95º, 105º, 115º

20. 20 is divided into four parts which are in A.P., such that the product of the first and fourth is to the product of the second and third is 2 : 3, then the four parts are

(A) 2, 4, 6, 8 (B) 3, 5, 7, 9 (C) 4, 6, 8, 10 (D) 6, 10, 17, 12

19 Angles → $a-3d,\ a-d,\ a+d,\ a+3d$

Common diff $= 2d = 10^\circ \Rightarrow d = 5$

Sum of angles $= 360^\circ$

$a-3d + a-d + a+d + a+3d = 360$

$a = 90^\circ$

Angle 1 $= 90-3(5) = 75$

Angle 2 $= 90-5 = 85$

Angle 3 $= 90+5 = 95$

Angle 4 $= 90+3(5) = 105$

20 $a-3d + a-d + a+d + a+3d = 20 \Rightarrow \boxed{a=5}$

$$\frac{(a-3d)(a+3d)}{(a-d)(a+d)} = \frac{2}{3} \Rightarrow$$

$$\frac{a^2-9d^2}{a^2-d^2} = \frac{2}{3} \Rightarrow \frac{25-9d^2}{25-d^2} = \frac{2}{3}$$

$75-27d^2 = 50-2d^2$

$25d^2 = 25 \Rightarrow \boxed{d = \pm 1}$

Numbers ⇒ $5-3,\ 5-1,\ 5+1,\ 5+3$

2, 4, 6, 8

21. Insert three arithmetic means between 3 and 19.

$3, A_1, A_2, A_3 \quad 19$

$$d = \frac{b-a}{n+1} = \frac{19-3}{3+1} = \frac{16}{4} = 4$$

$$A_1 = a + d = 3 + 4 = 7$$

$$A_2 = a + 2d = 3 + 2(4) = 11$$

$$A_3 = a + 3d = 3 + 3(4) = 15$$

$$\boxed{7, 11, 15}$$