GENERAL FORM OF VECTOR :→

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

A_x = Component of $\vec{A}$ along x-axis

A_y = " " " " " " y- "

A_z = " " " " " " z- "

Magnitude of $\vec{A}$

$$|\vec{A}| = \sqrt{(A_x)^2 + (A_y)^2 + (A_z)^2}$$

Dirn of $\vec{A}$ = Unit vector Along $\vec{A}$:→

$$\left\{\hat{A} = \frac{\vec{A}}{|\vec{A}|} = \frac{A_x\hat{i} + A_y\hat{j} + A_z\hat{k}}{\sqrt{A_x^2 + A_y^2 + A_z^2}}\right\}$$

Ex If $\vec{A} = 2\hat{i} + 2\hat{j} - \hat{k}$ Find

(i) $|\vec{A}|$

(2) Dirn of $\vec{A}$

(i) $|\vec{A}| = \sqrt{(2)^2 + (2)^2 + (-1)^2} = 3$ Ans

(ii) $\hat{A} = \frac{2\hat{i} + 2\hat{j} - \hat{k}}{3}$

$$\hat{A} = \frac{2}{3}\hat{i} + \frac{2}{3}\hat{j} - \frac{1}{3}\hat{k}$$ Ans

Addition / Subtraction :

Case (I) Vector's given in general form

let $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

$\vec{A}+\vec{B} = (A_x+B_x)\hat{i} + (A_y+B_y)\hat{j} + (A_z+B_z)\hat{k}$

$\vec{A}-\vec{B} = (A_x-B_x)\hat{i} + (A_y-B_y)\hat{j} + (A_z-B_z)\hat{k}$

Ex $\vec{A} = 2\hat{i} - \hat{j} + 2\hat{k}$

$\vec{B} = 3\hat{i} + \hat{j} - 2\hat{k}$

Find (i) $\vec{A}+\vec{B}$

(ii) $\vec{A}-\vec{B}$

(iii) unit vector along $(\vec{A}-\vec{B})$

(i) $\vec{A}+\vec{B} = (2+3)\hat{i} + (-1+1)\hat{j} + (2-2)\hat{k}$

$= 5\hat{i} + 0\hat{j} + 0\hat{k}$

$\boxed{\vec{A}+\vec{B} = 5\hat{i}}$ Ans

(ii) $\vec{A}-\vec{B} = (2-3)\hat{i} + (-1-1)\hat{j} + (2-(-2))\hat{k}$

$\boxed{\vec{A}-\vec{B} = -\hat{i} - 2\hat{j} + 4\hat{k}}$ Ans

(iii) $\vec{A}-\vec{B} = \vec{C}$ let

$\vec{C} = -\hat{i} - 2\hat{j} + 4\hat{k}$

$\hat{C} = \dfrac{\vec{C}}{C} = \dfrac{-\hat{i} - 2\hat{j} + 4\hat{k}}{\sqrt{(-1)^2 + (-2)^2 + (4)^2}} = \dfrac{-\hat{i} - 2\hat{j} + 4\hat{k}}{\sqrt{21}}$

Ans

Ex $\vec{A} = 2\hat{i} - \hat{j} + 2\hat{k}$ Find

(i) $4\vec{A}$

(ii) If $\vec{B} = \hat{i} + 2\hat{j} - 2\hat{k}$ then

(a) $\vec{A} + 2\vec{B}$

(b) $|\vec{A}| + |\vec{B}|$

(c) $\dfrac{|\vec{A}|}{|\vec{B}|}$

Sol (i) $4\vec{A}$

$= 4(2\hat{i} - \hat{j} + 2\hat{k})$

$= 8\hat{i} - 4\hat{j} + 8\hat{k}$

NOTE (1) If we multiply with +ve constant only magnitude changes

(2) If multiplied with -ve constant both dir^n and magnitude changes.

(ii) (a) $\vec{A} + 2\vec{B}$

$(2\hat{i} - \hat{j} + 2\hat{k}) + 2(\hat{i} + 2\hat{j} - 2\hat{k})$

$= 4\hat{i} + 3\hat{j} - 2\hat{k}$ Ans

(b) $|\vec{A}| + |\vec{B}|$

$= \sqrt{(2)^2 + (-1)^2 + (2)^2} + \sqrt{(1)^2 + (2)^2 + (-2)^2}$

$= 3 + 3 = 6$ Ans

(c) $\dfrac{|\vec{A}|}{|\vec{B}|} = \dfrac{3}{3} = 1$ Ans

Ex If a vector is multiplied by k then

(1) Both Dir^n & magnitude must change

✓(2) Both Dir^n . . . . may change

(3) only Dir^n changes

(4) only magnitude changes

$\vec{A} = 2\hat{i}$

$k = -4$ | $k = +4$

$\vec{A} = -8\hat{i}$ | $\vec{A} = 8\hat{i}$

Dir^n $-ve\ x$ | Dir$^n \to +ve\ x$

Ex If $0.8\hat{i} + 0.6\hat{j} + c\hat{k}$ is a unit vector find value of c

✓(I) 0

(II) -1.2

(III) 0.86

(V) None

let $\vec{A} = 0.8\hat{i} + 0.6\hat{j} + c\hat{k}$

$|\vec{A}| = 1$

$\sqrt{(0.8)^2 + (0.6)^2 + c^2} = 1$

$0.64 + 0.36 + c^2 = 1$

$1 + c^2 = 1$

$c^2 = 0$

$\boxed{c = 0}$

Case (ii) When magnitudes & Angle b/w vectors are Given.

Given $|\vec{A}| = A$ | Find $\vec{A}+\vec{B}$ OR $\vec{A}-\vec{B}$

$|\vec{B}| = B$

θ = Angle b/w them

How we add / subtract

Law of vector Addition

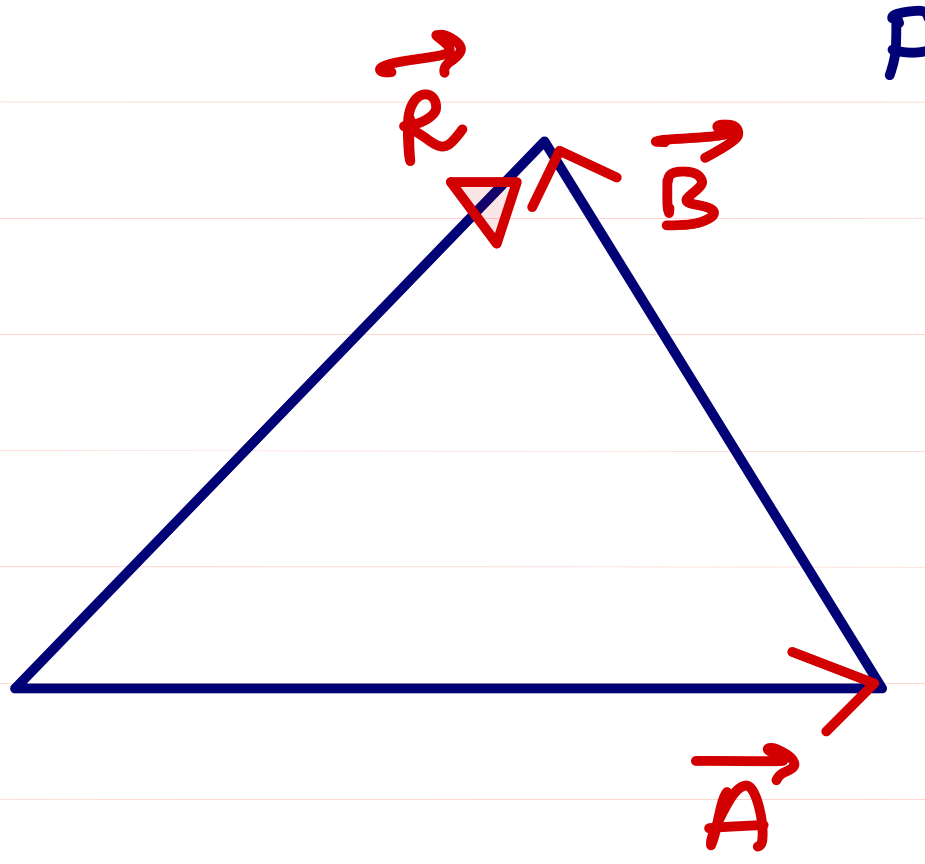
(1) Triangle law of vector Add.

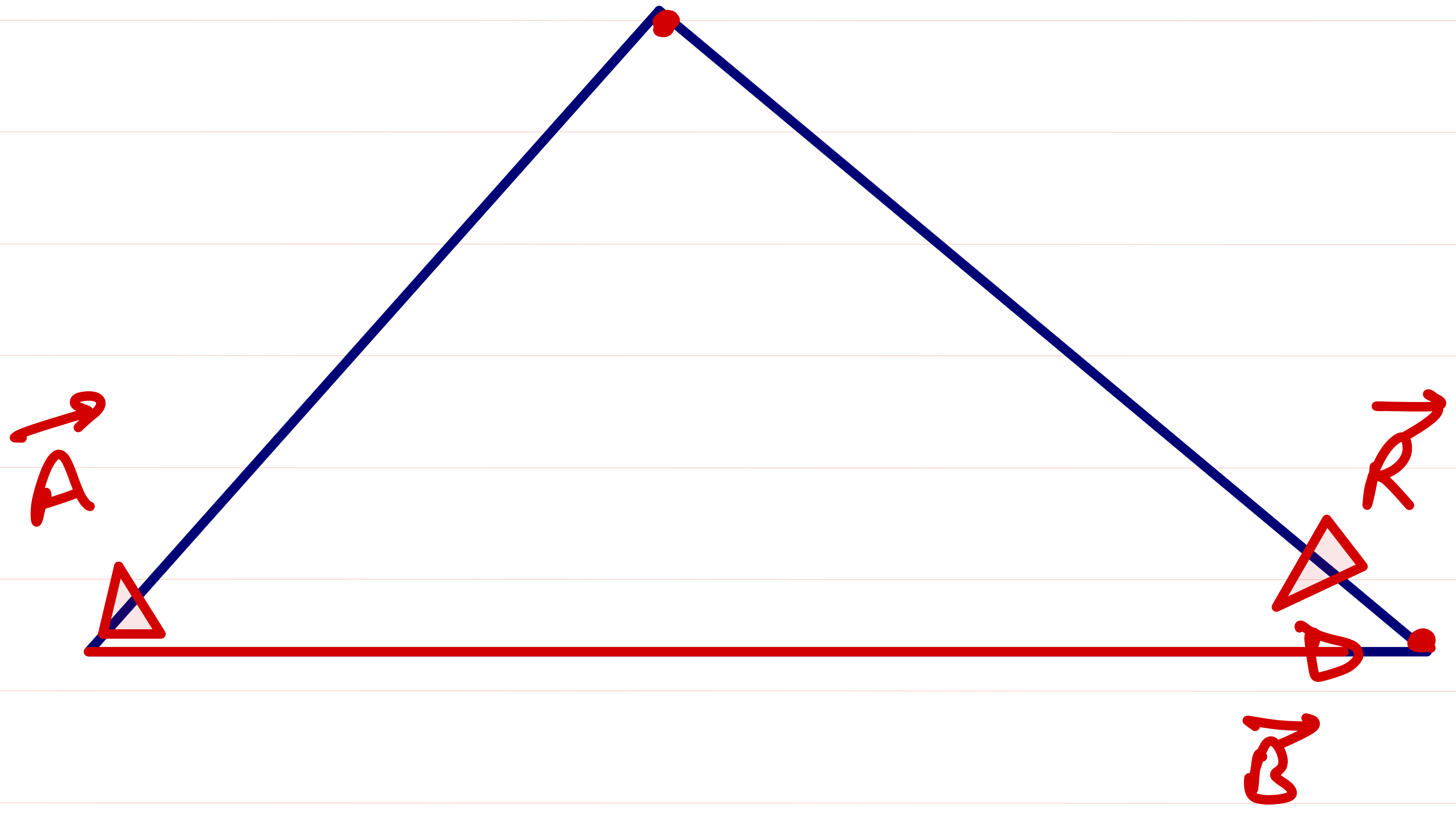
(ii) Parallelogram law of vector Add.

(1) Δ^{le} law of vector Addition :→

Find $\vec{A}+\vec{B}$

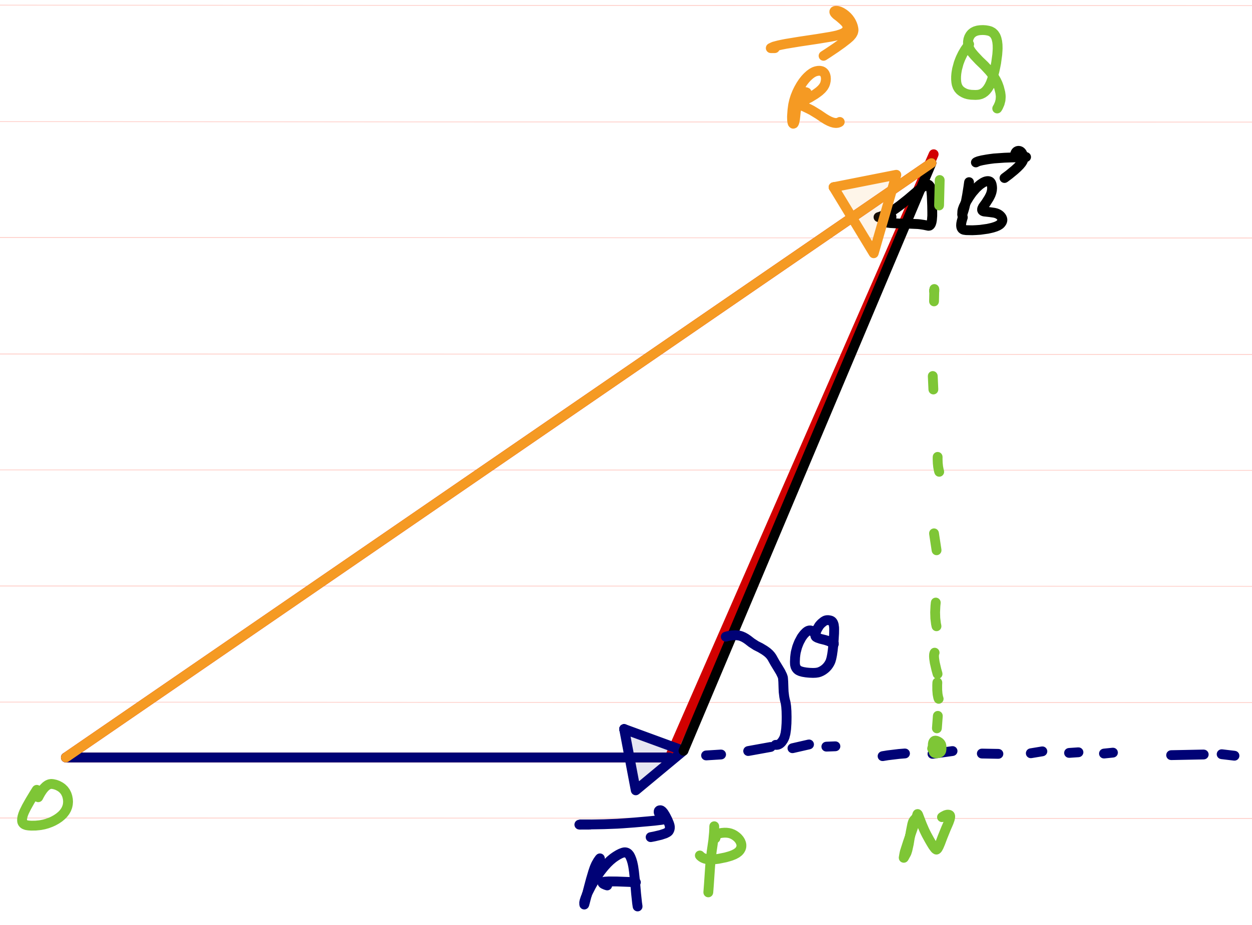
let $\vec{A}+\vec{B} = \vec{R}$





$\vec{R} = \vec{A}+\vec{B}$

Magnitude of $\vec{R}$



$PQ = |\vec{B}| = B$

$OP = |\vec{A}| = A$

$OQ = |\vec{R}| = R$

ΔPNQ

$\sin\theta = \frac{NQ}{PQ} \Rightarrow NQ = PQ\sin\theta \Rightarrow NQ = B\sin\theta$

$\cos\theta = \frac{PN}{PQ} \Rightarrow PN = PQ\cos\theta \Rightarrow PN = B\cos\theta$

ΔONQ

$OQ^2 = ON^2 + NQ^2$

$OQ^2 = (OP+PN)^2 + (NQ)^2$

Putting value

$$R^2 = (A + B\cos\theta)^2 + (B\sin\theta)^2$$

$$R^2 = A^2 + B^2\cos^2\theta + 2AB\cos\theta + B^2\sin^2\theta$$

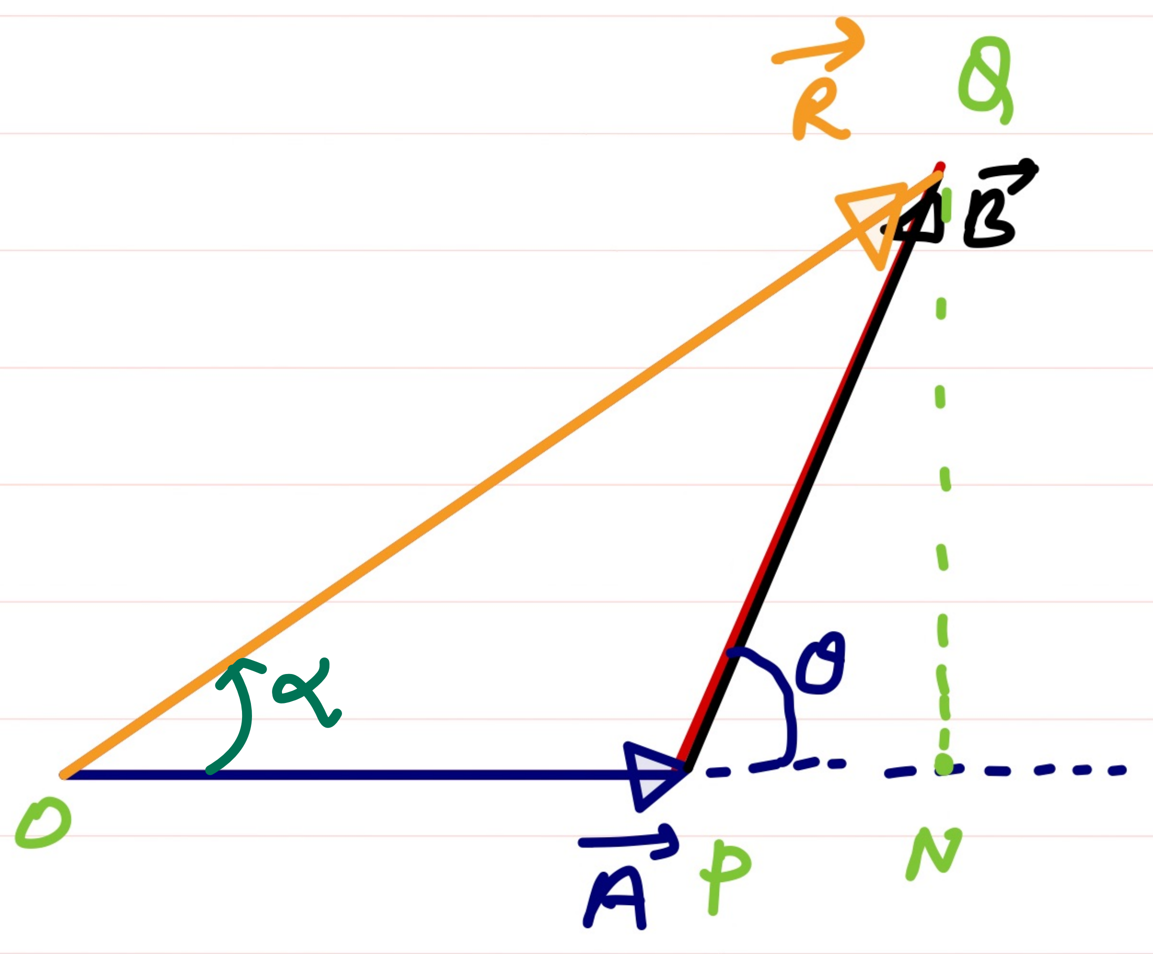
$$= A^2 + B^2(\cos^2\theta + \sin^2\theta) + 2AB\cos\theta$$

$$R^2 = A^2 + B^2 + 2AB\cos\theta$$

$$\boxed{R = \sqrt{A^2 + B^2 + 2AB\cos\theta}}$$

Yaad Rakhe!!

Dir^n of $\vec{R}$:→



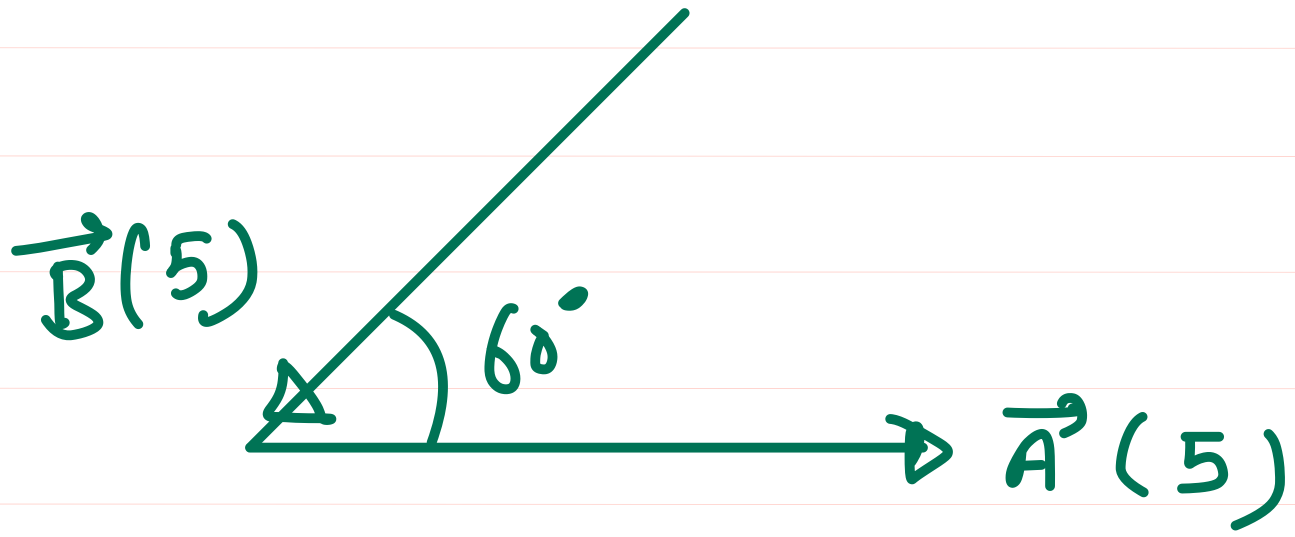
let $\vec{R}$ makes α angle from $\vec{A}$ & β from $\vec{B}$

ΔONQ

$$\boxed{\tan\alpha = \frac{NQ}{ON} = \frac{B\sin\theta}{A + B\cos\theta}}$$

$$\boxed{\tan\beta = \frac{A\sin\theta}{B + A\cos\theta}}$$

Ex



Find $\vec{A}+\vec{B}$

Given $|\vec{A}|=5$

$|\vec{B}|=5$

Angle $\theta = 120^\circ$

let $\vec{R} = \vec{A}+\vec{B}$

$$R = \sqrt{A^2+B^2+2AB\cdot\cos\theta}$$

$$= \sqrt{5^2+5^2+2\cdot5^2\cdot\cos(120)}$$

$$= \sqrt{5^2+5^2+\cancel{2}\times5^2\left(-\frac{1}{\cancel{2}}\right)}$$

$$= \sqrt{5^2+5^2-5^2}$$

$\boxed{R = 5 \text{ unit}}$

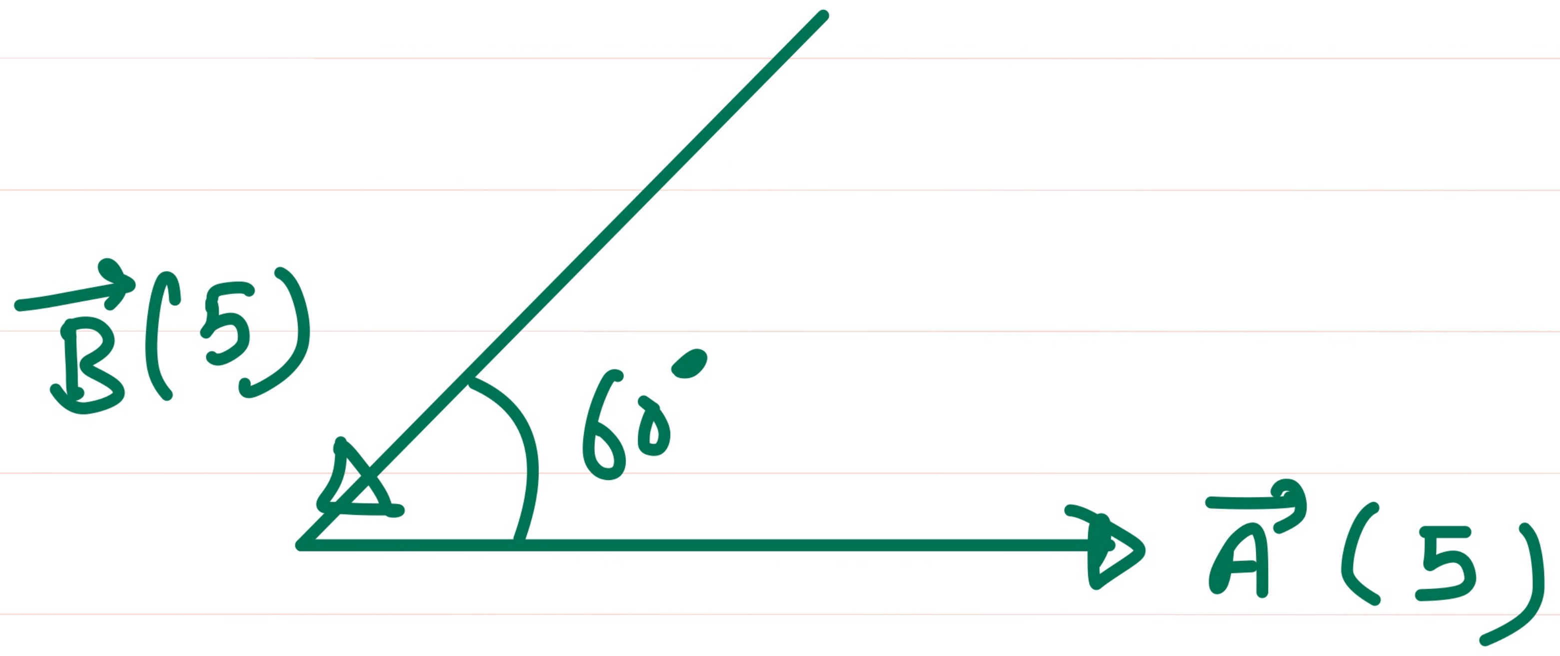
let α be Angle from $\vec{A}$

$$\tan\alpha = \frac{B\sin\theta}{A+B\cos\theta}$$

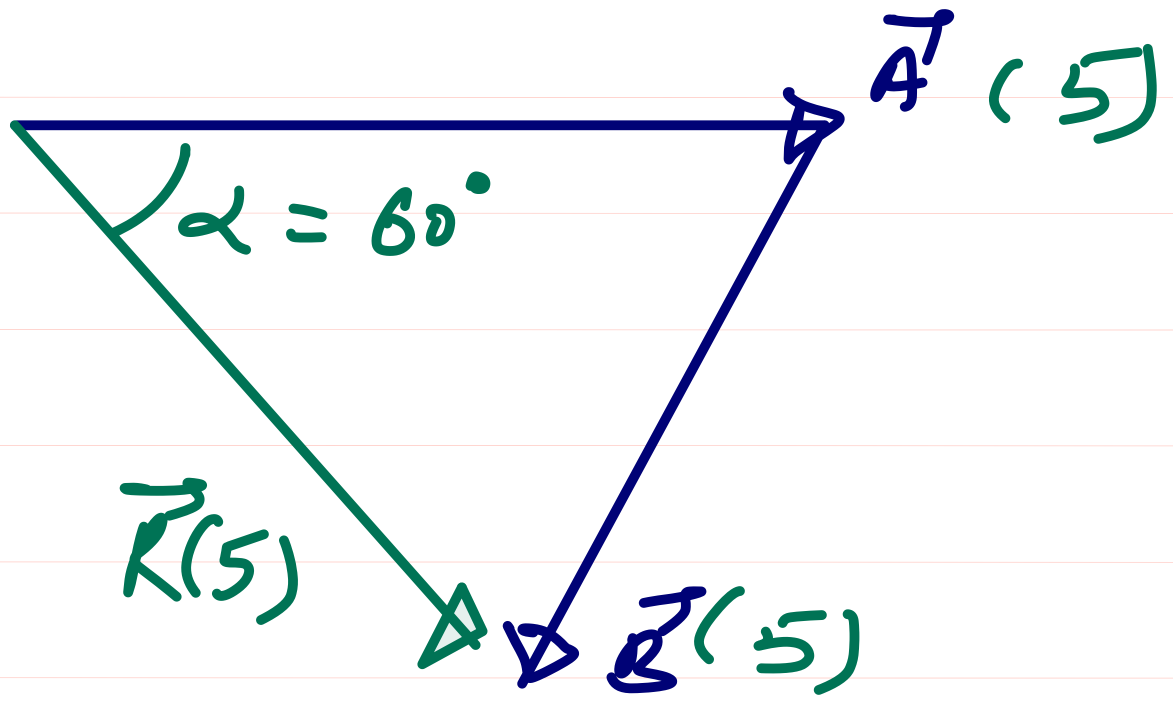
$$= \frac{5\sin(120)}{5+5\cos(120)} = \frac{\sin(120)}{1+\cos 120}$$

$$= \frac{\sqrt{3}/2}{\left(1-\frac{1}{2}\right)} = \frac{\sqrt{3}}{2\times\left(+\frac{1}{2}\right)} = \sqrt{3}$$

$\tan\alpha = \sqrt{3} \Rightarrow \boxed{\alpha = 60^\circ}$



$\Rightarrow$



<u>**Ans**</u> $\vec{R} = 5$ unit, $\frac{\pi}{3}$ from $\vec{A}$