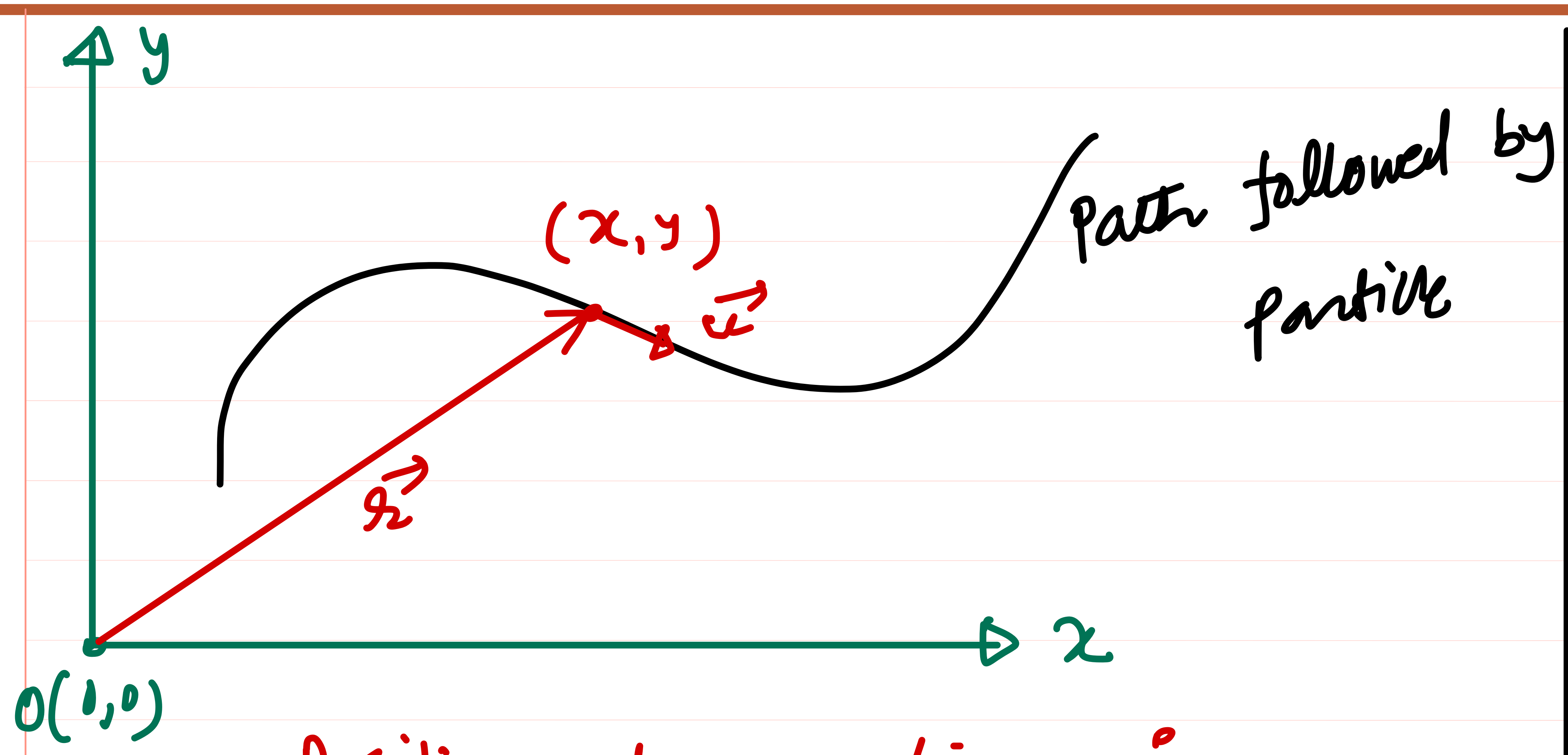


Position at any time is

$$\boxed{\vec{r} = x\hat{i} + y\hat{j}}$$

Velocity at any time t

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j}$$

$$\boxed{\vec{v} = v_x\hat{i} + v_y\hat{j}}$$

$$\boxed{\text{Speed} = |\vec{v}| = \sqrt{v_x^2 + v_y^2}}$$

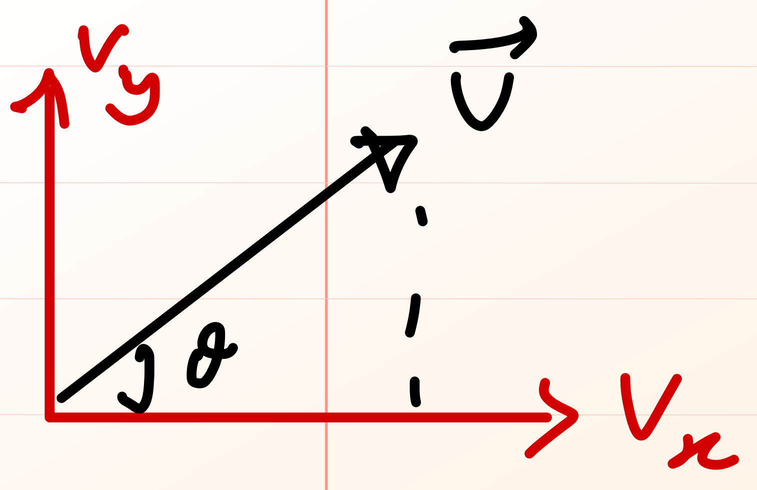
Acceleration of particle

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j}$$

$$\boxed{\vec{a} = a_x\hat{i} + a_y\hat{j}}$$

velocity makes angle from x-axis

$$\boxed{\tan\theta = \frac{V_y}{V_x}}$$

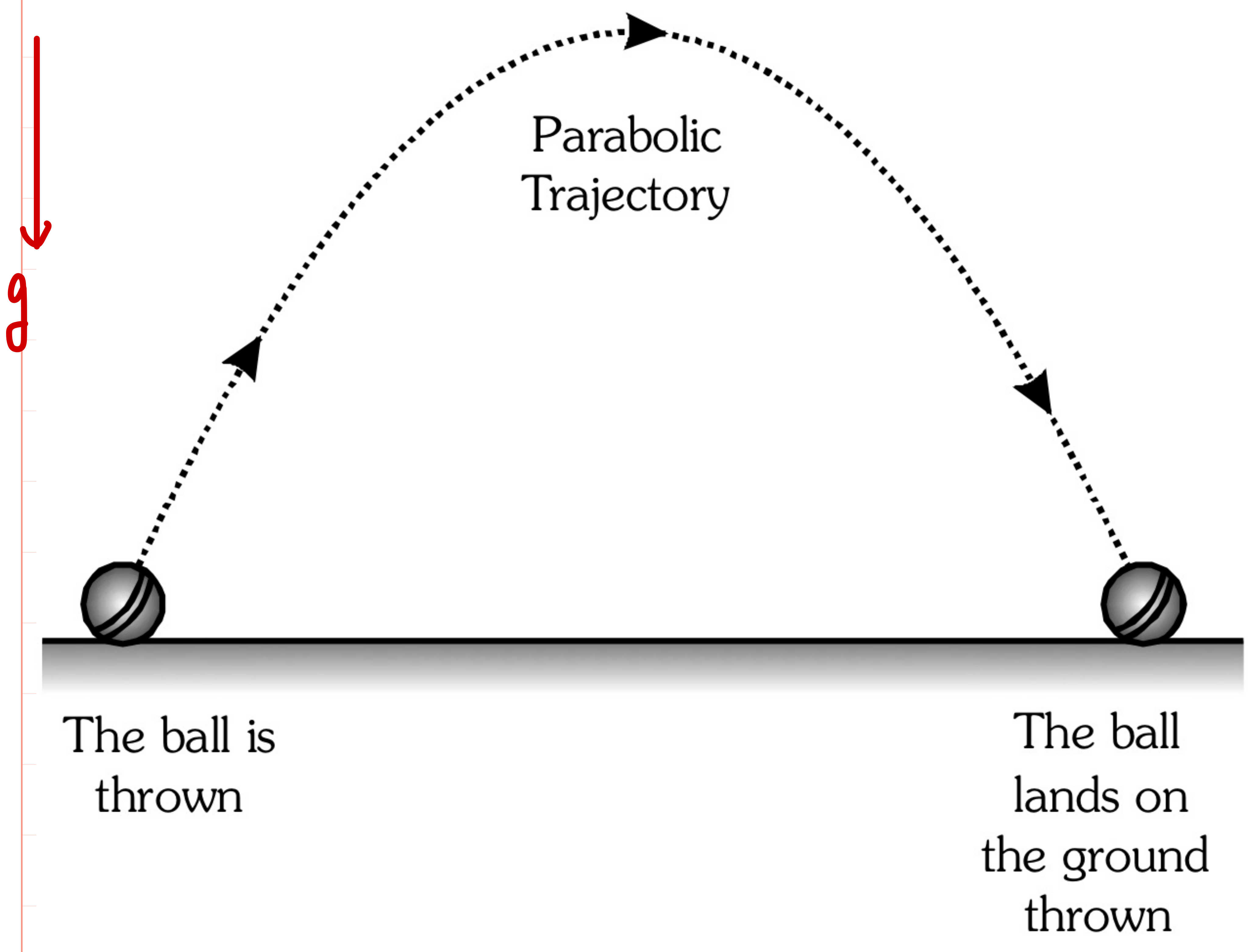


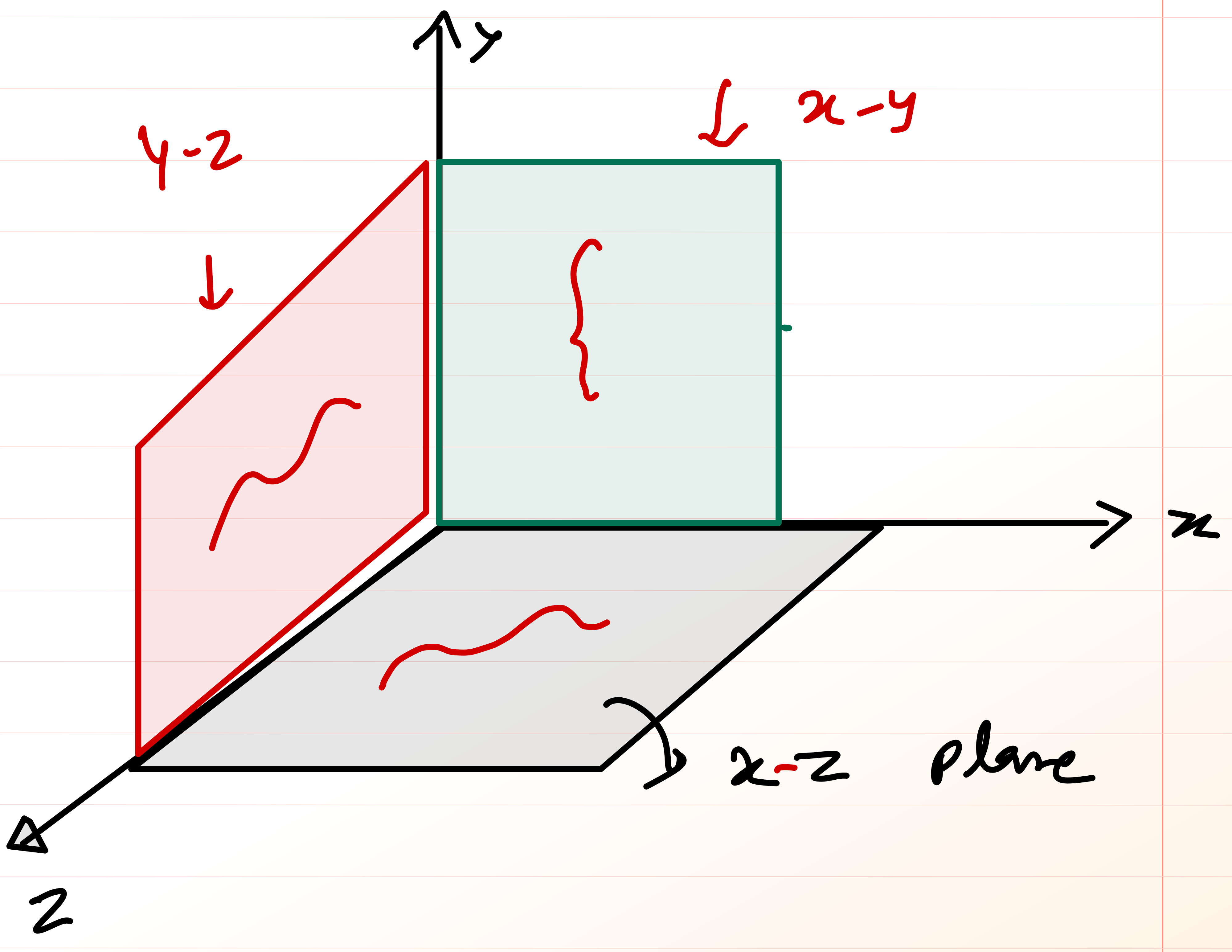
Eaxample of 2-D

PROJECTILE MOTION :→

An object projected by an external force when continues to move by its own inertia is known as projectile and its motion as projectile motion.

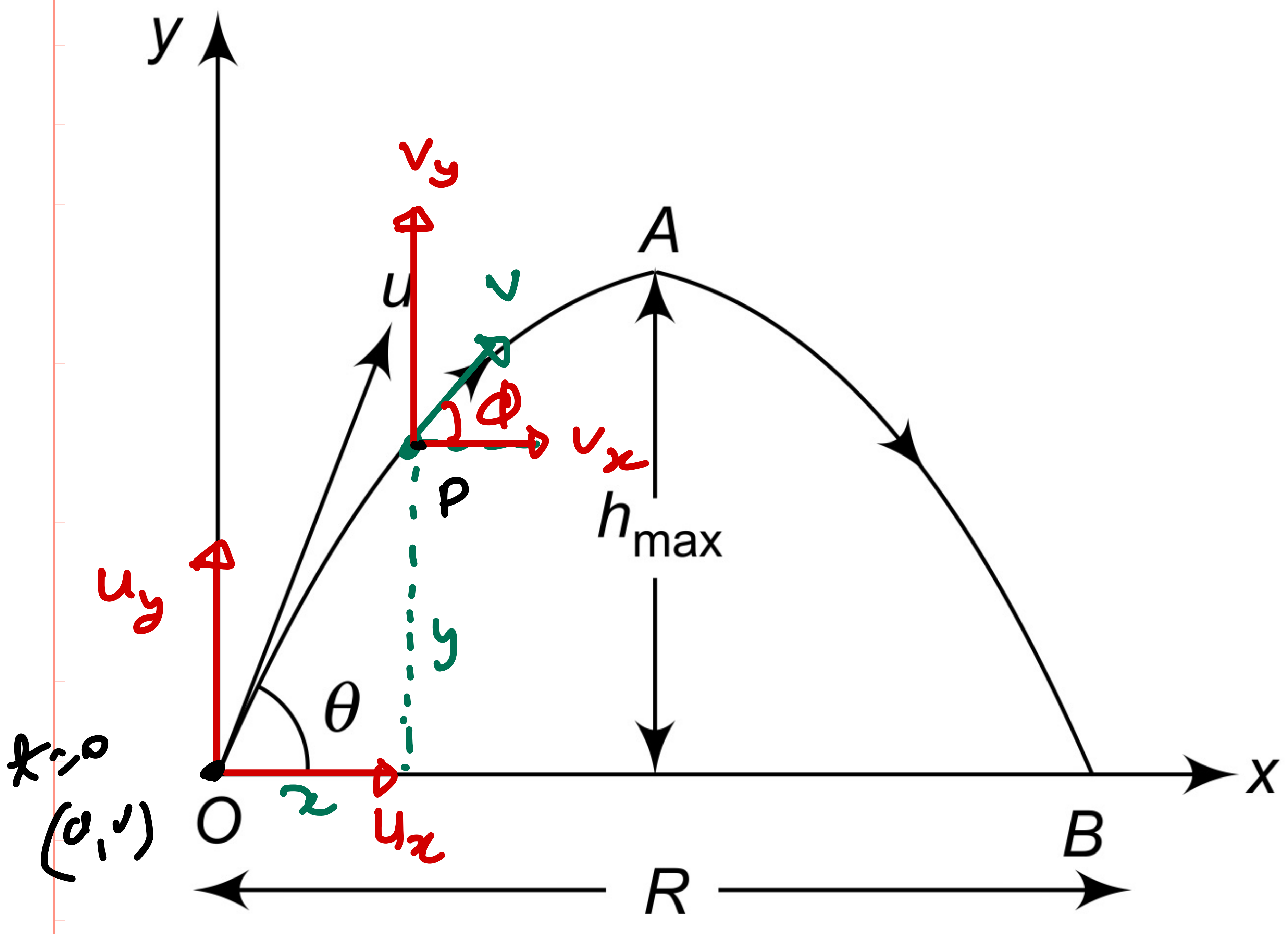
A football kicked by a player, an arrow shot by an archer, water sprinkling out a water–fountain, an athlete in long jump or high jump, a bullet or an artillery shell fired from a gun are some Illustrations of projectile motion.





Case -1

A body projected from the ground with a velocity u at an angle θ with the horizontal. (Fig. 4.2)



Along x-dirn velocity remains constant

$$u\cos\theta = v\cos\phi$$

Along x-axis	Along y-axis
$t = 0$ sec	
$\boxed{x = 0,\quad u_x = u\cos\theta,\quad a_x = 0}$	$\boxed{y = 0,\quad u_y = u\sin\theta,\quad a_y = -g}$

At time t (at point P)

Position vector $\vec{r} = x\hat{i} + y\hat{j}$

Along x-axis	Along y-axis
$v_x = u_x + a_x t$	$v_y = u_y + a_y t$
$\boxed{v_x = u\cos\theta} + 0t$	$\boxed{v_y = u\sin\theta - gt}$
$x = u_x t + \frac{1}{2}a_x t^2$	$y = u_y t + \frac{1}{2}a_y t^2$
$\boxed{x = u\cos\theta\, t}$	$\boxed{y = u\sin\theta\, t - \frac{1}{2}gt^2}$

From y-dirn we find time of flight and maximum Height

Net velocity at any time t

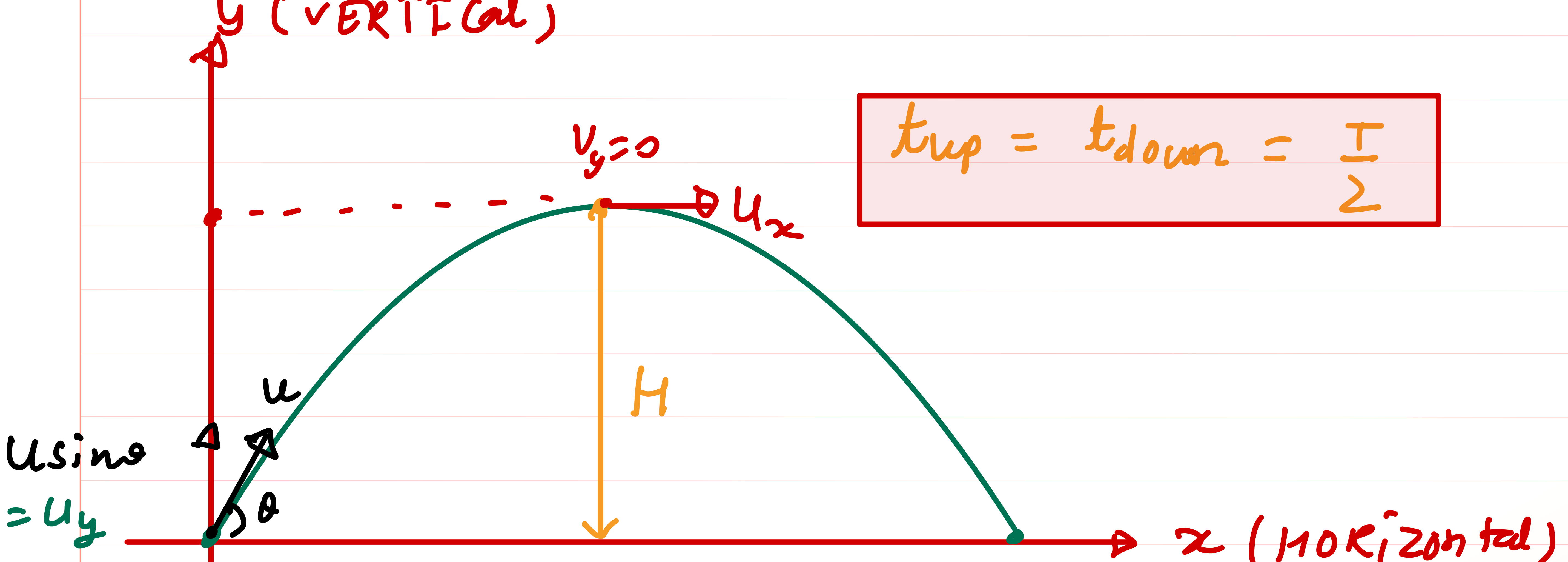
$$\vec{v} = v_x\hat{i} + v_y\hat{j}$$

$$\boxed{\vec{V} = u\cos\theta\,\hat{i} + (u\sin\theta - gt)\hat{j}}$$

Speed $= |\vec{v}|$

1. Position at any time
2. velocity " " "
3. Speed " " "
4. time of flight
5. maximum Height
6. Equation of path (Trajectory)
7. Horizontal Range

④ Time of flight :→ (T)



$$\boxed{t_{up} = t_{down} = \frac{T}{2}}$$

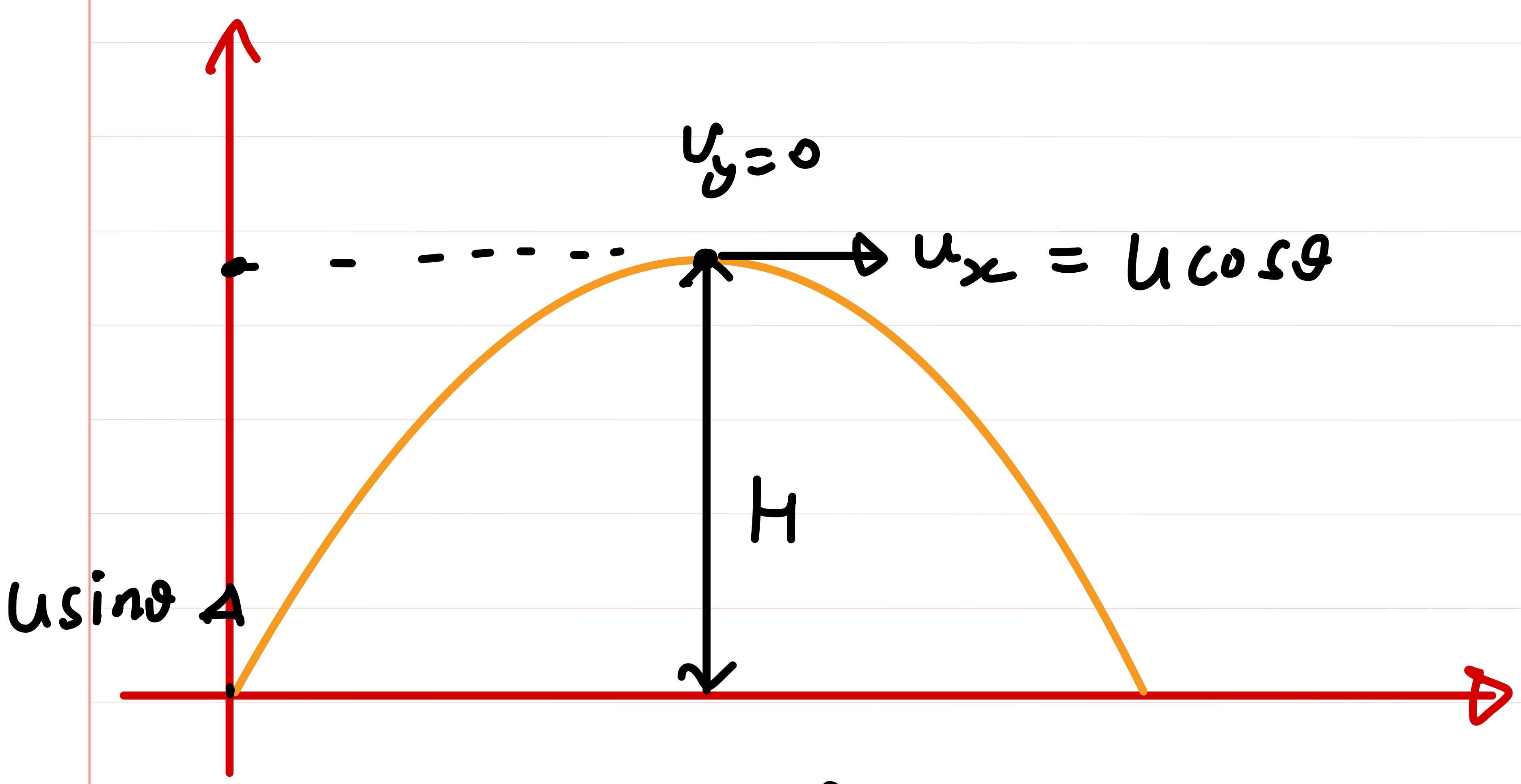
From y-dirn

$$S = u_y t + \frac{1}{2}a_y t^2$$

$$0 = u\sin\theta\, T - \frac{1}{2}gT^2$$

$$\boxed{T = \frac{2u\sin\theta}{g} = \frac{2u_y}{g}}$$

(5) Maximum Height (H) :→

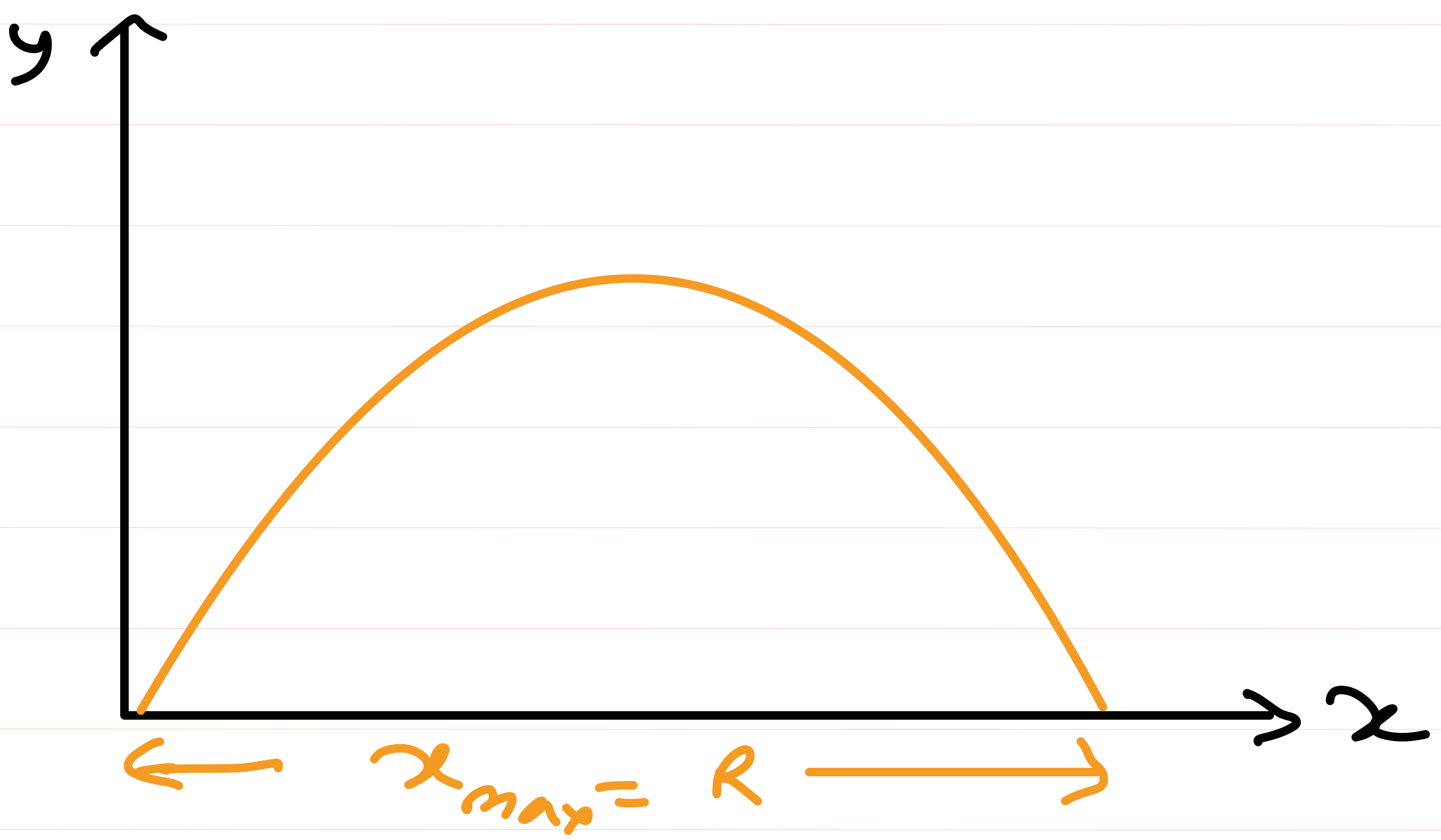


$$v_y^2 = u_y^2 + 2a_y S_y$$

$$0^2 = (u\sin\theta)^2 + 2(-g)H$$

$$\boxed{H = \frac{(u\sin\theta)^2}{2g} = \frac{u_y^2}{2g}}$$

(6) Range (Maximum Horizontal distance) (R)



$$x = u_x t + \frac{1}{2}\cdot 0\, t^2$$

at $t = T$

$x_{max} = R$

$$R = u_x \cdot T$$

$$R = u_x \cdot \frac{2u_y}{g}$$

$$\boxed{R = \frac{2u_x u_y}{g}}$$

$$R = \frac{2\, u\cos\theta \cdot u\sin\theta}{g}$$

$$\boxed{R = \frac{u^2 \sin 2\theta}{g}}$$

7 Equation of Trajectory (Path equation)

→ Rel^n b/w x-y

$\because \quad x = u_x t = u\cos\theta\, t \quad$ —①

$y = u\sin\theta\, t - \frac{1}{2} g t^2 \quad$ —②

Put value of t from eq-① into eq ②

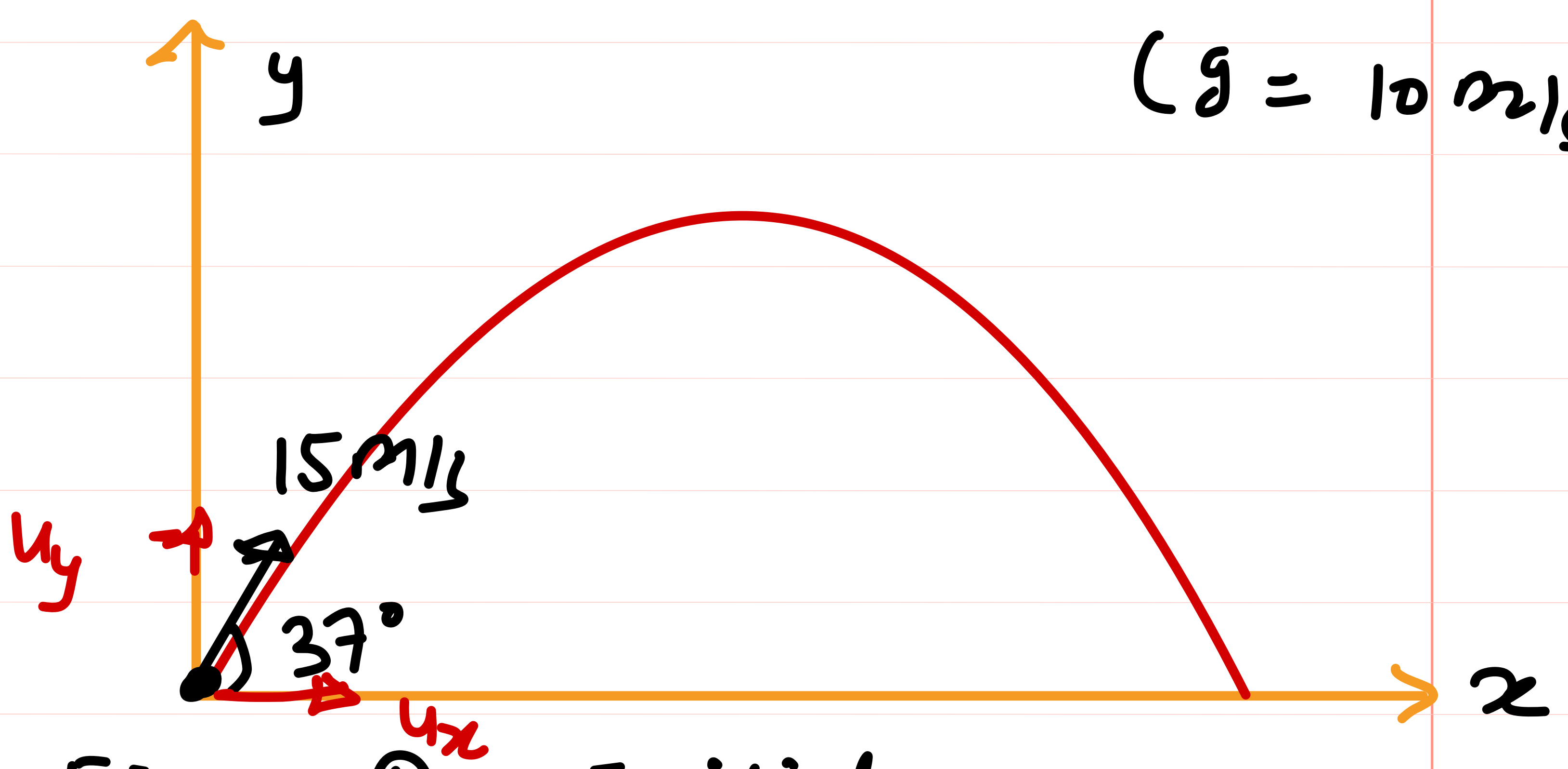
$$y = u\sin\theta \cdot \frac{x}{u\cos\theta} - \frac{1}{2} g \left(\frac{x}{u\cos\theta}\right)^2$$

$$\boxed{y = x\tan\theta - \frac{g x^2}{2u^2\cos^2\theta}}$$

OR

$$\boxed{y = x\tan\theta\left[1 - \frac{x}{R}\right]}$$

Ex A Particle projected as shown in fig ($g = 10\ m/s^2$)



Find ① Initial x and y-comp. of velocity

$u_x = 15\cos 37 = 15 \times \frac{4}{5} = 12\ m/s$

$u_y = 15\sin 37 = 15 \times \frac{3}{5} = 9\ m/s$

② position at any time t

$x = u_x t = 12t$

$y = u_y t - \frac{1}{2} g t^2 = 9t - 5t^2$

$$\boxed{\vec{r} = x\hat{i} + y\hat{j} = 12t\hat{i} + (9t - 5t^2)\hat{j}}$$

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(3) velocity at any time t

$$\vec{v} = u_x\hat{i} + (u_y - gt)\hat{j}$$

$$\boxed{\vec{v} = 12\hat{i} + (9 - 10t)\hat{j}}$$

(4) Time of flight

$$T = \frac{2u\sin\theta}{g} = \frac{2u_y}{g} = \frac{2\times 9}{g}$$

$$\boxed{T = 1.8 \text{ sec}}$$

(5) Maximum Height

$$H = \frac{(u\sin\theta)^2}{2g} = \frac{u_y^2}{2g}$$

$$\boxed{H = \frac{9^2}{2\times 10} = \frac{81}{20}\ m}$$

(6) Range

$$R = \frac{2u_x u_y}{g} = \frac{2\times 12\times 9}{10} \Rightarrow \boxed{R = 21.6m}$$

(7) Equation of Trajectory

$$y = x\tan\theta\left[1 - \frac{x}{R}\right]$$

$$y = x\tan(37)\left(1 - \frac{x}{21.6}\right)$$

$$\boxed{y = \frac{3x}{4} - \frac{3x^2}{4\times 21.6}}$$

Parabolic Path

Ex A projectile is projected with initial velocity $3\hat{i}+4\hat{j}$ from origin find

(1) Velocity at $\frac{1}{2}$ sec

(2) Position at 0.4 sec

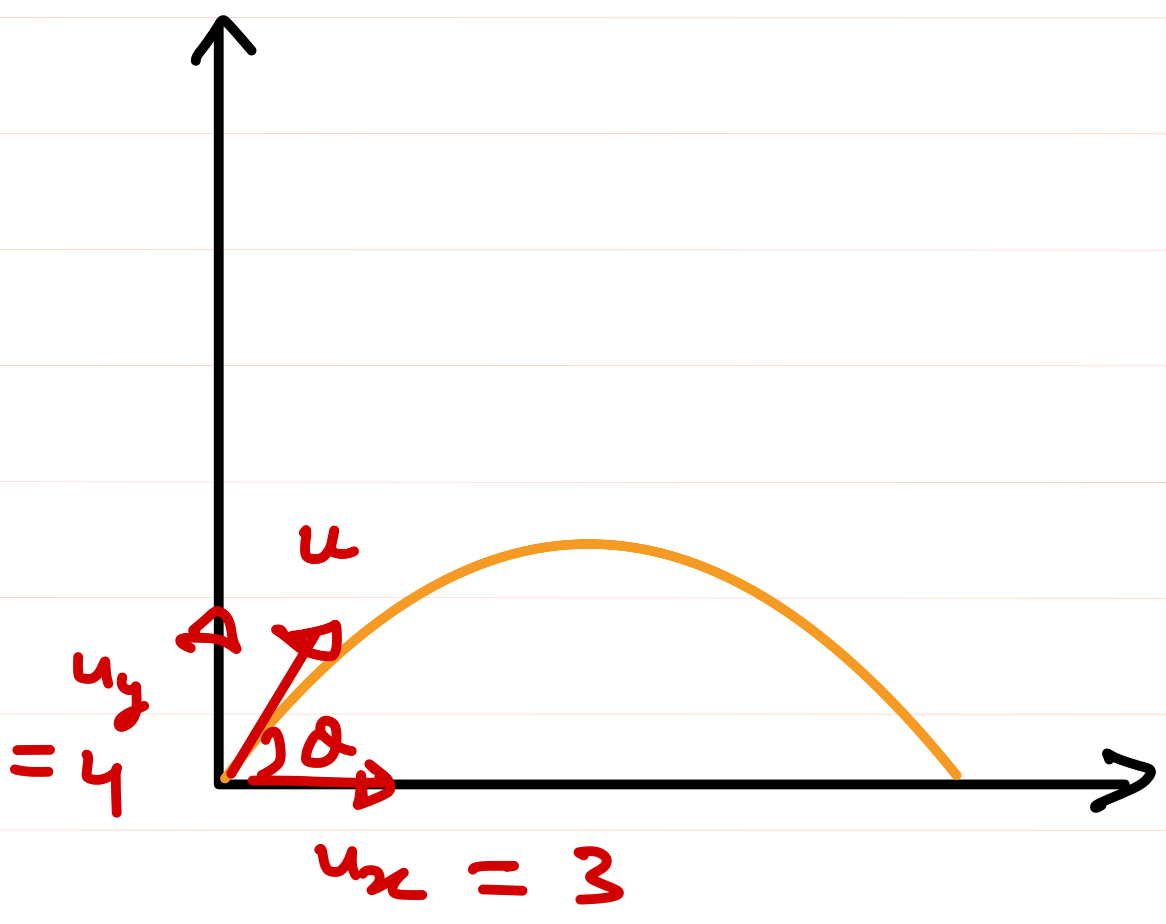
(3) Time of flight

(4) Maximum height

(5) Range

(6) Equation of Trajectory

(7) Angle of projection



Sol (1)

$\vec{v} = \vec{u} + \vec{a}\,t$

$= (3\hat{i}+4\hat{j}) + (-g\hat{j})\,t$

$$\boxed{\vec{v} = 3\hat{i} + (4-10t)\hat{j}}$$

at $t = \frac{1}{2}$: $\vec{v} = 3\hat{i} + (4 - 10\times\frac{1}{2})\hat{j}$

$$\boxed{\vec{v} = 3\hat{i} - \hat{j}}$$ Ans

(2) $\vec{r} = x\hat{i} + y\hat{j}$

$x = u_x t = 3\times 0.4 = 1.2\ m$

$y = u_y t - \frac{1}{2}\times g t^2$

$= 4\times 0.4 - 5\times(0.4)^2$

$= 1.6 - 0.8$

$y = 0.8\ m$

$$\boxed{\vec{r} = 1.2\hat{i} + 0.8\hat{j}}$$

(3) $T = \frac{2u_y}{g} = \frac{2\times 4}{10} = 0.8$ sec

(4) $H = \frac{u_y^2}{2g} = \frac{(4)^2}{2g} = \frac{16}{2\times 10} = 0.8\ m$

(5) $R = \frac{2u_x u_y}{g} = \frac{2\times 3\times 4}{10} = 2.4\ m$

(6) $y = x\tan\theta\left(1 - \frac{x}{R}\right)$

$y = x\times\frac{4}{3}\left[1 - \frac{x}{2.4}\right]$

$$\boxed{y = \frac{4}{3}x - \frac{4x^2}{3\times 2.4}}$$

(7) $\tan\theta = \frac{u_y}{u_x} = \frac{4}{3} \Rightarrow \theta = 53^\circ$