Trigonometric equations
CL01 & CL02

09/07/2021

Three Types of solutions :→

(1) **Principal solution** :→ $0 \leq \theta \leq 2\pi$

(2) **Particular Solution** :→ Solutions lying in the given interval.

(3) **General Solution** :→ solution in the form of n.

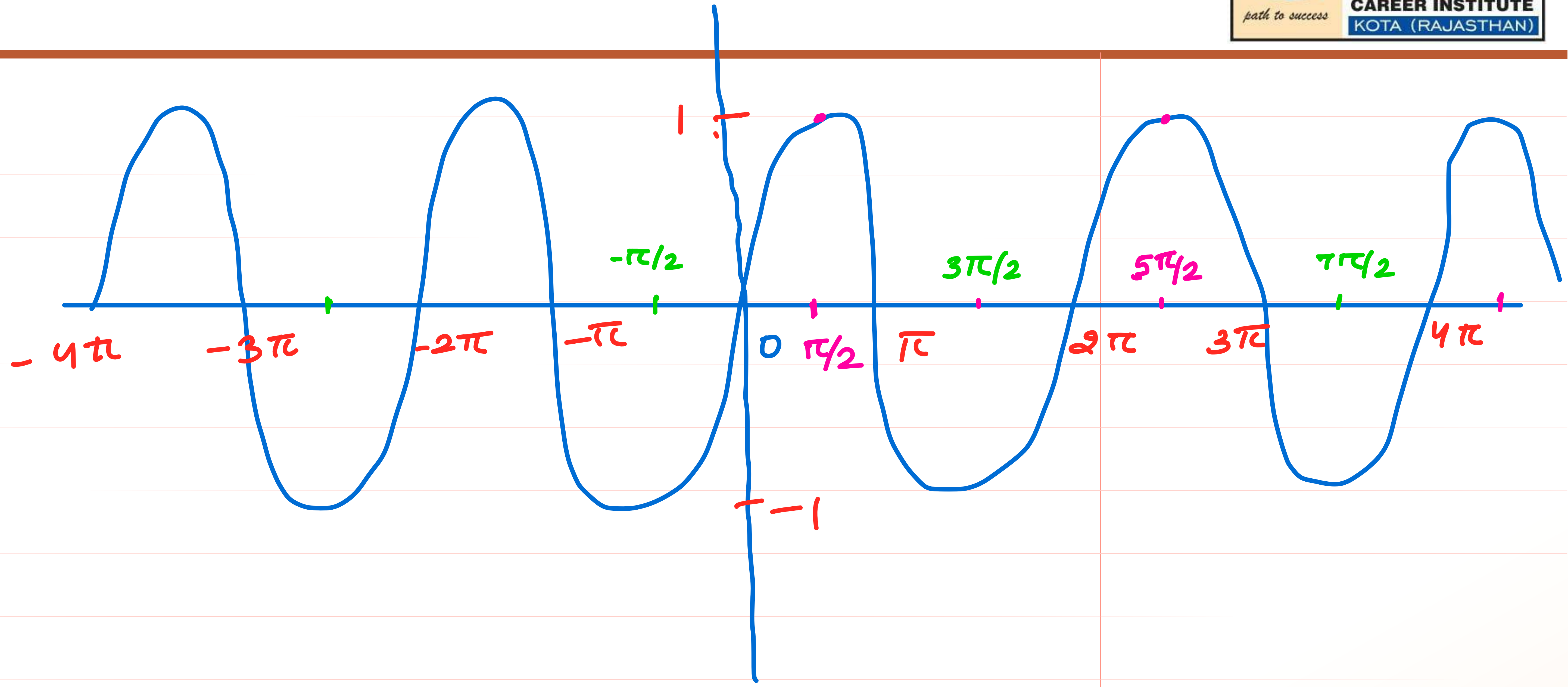
Type I :→

(1) $\sin\theta = 0$

$\theta = n\pi \qquad n \in \mathbb{Z}$

(2) $\sin\theta = 1$

$\theta = \frac{\pi}{2}, \underbrace{\frac{5\pi}{2}, \frac{9\pi}{2}, \ldots}$



$\theta = \frac{\pi}{2}(4n+1) \; ; \; n \in \mathbb{Z}$

OR

$\theta = \frac{\pi}{2}(4n-3) \; ; \; n \in \mathbb{Z}.$

$1, 5, 9, \ldots$

$T_n = 1 + (n-1)4$
$= 4n - 3$

$5, 9, \ldots$

$T_n = 5 + (n-1)4$
$= (4n+1)$

(3) $\sin\theta = -1$

$\theta = \frac{3\pi}{2}, \frac{7\pi}{2}, \frac{11\pi}{2}$

$\theta = \frac{\pi}{2}(4n-1)$; $n \in Z$

$3, 7, 11, ---$

$T_n = 3 + (n-1)4$

$T_n = 4n-1$

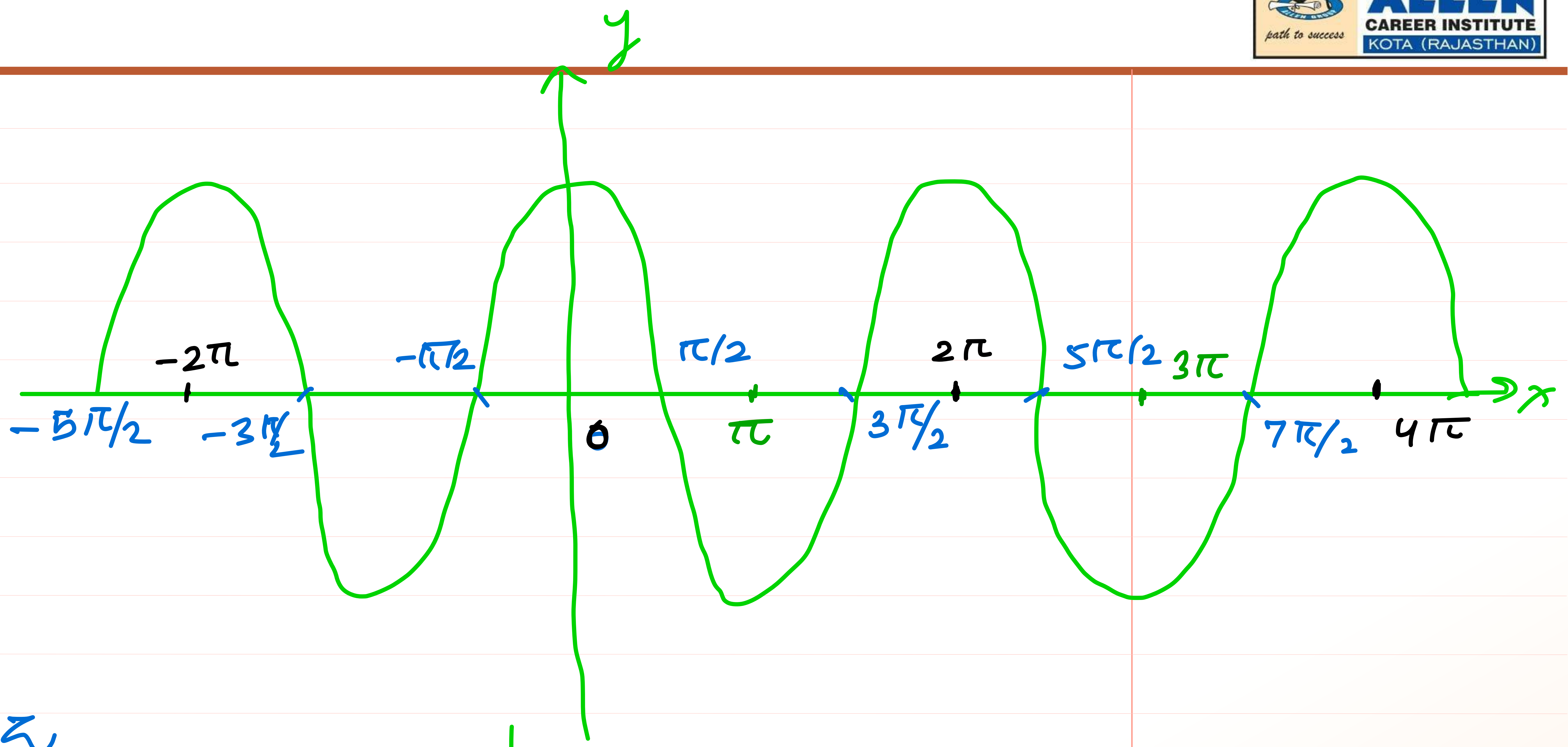
(4) $\cos\theta = 0$

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \ldots$

$\theta = (2n+1)\frac{\pi}{2}\ ;\ n \in \mathbb{Z}$

or

$\theta = (2n-1)\frac{\pi}{2}\ ;\ n \in \mathbb{Z}$



(5) $\cos\theta = 1$

$\theta = 0, 2\pi, 4\pi, 6\pi, 8\pi, \ldots$

$\theta = 2n\pi\ ;\ n \in \mathbb{Z}$

(6) $\cos\theta = -1$

$\theta = (2n+1)\pi\ ;\ n \in \mathbb{Z}$

or

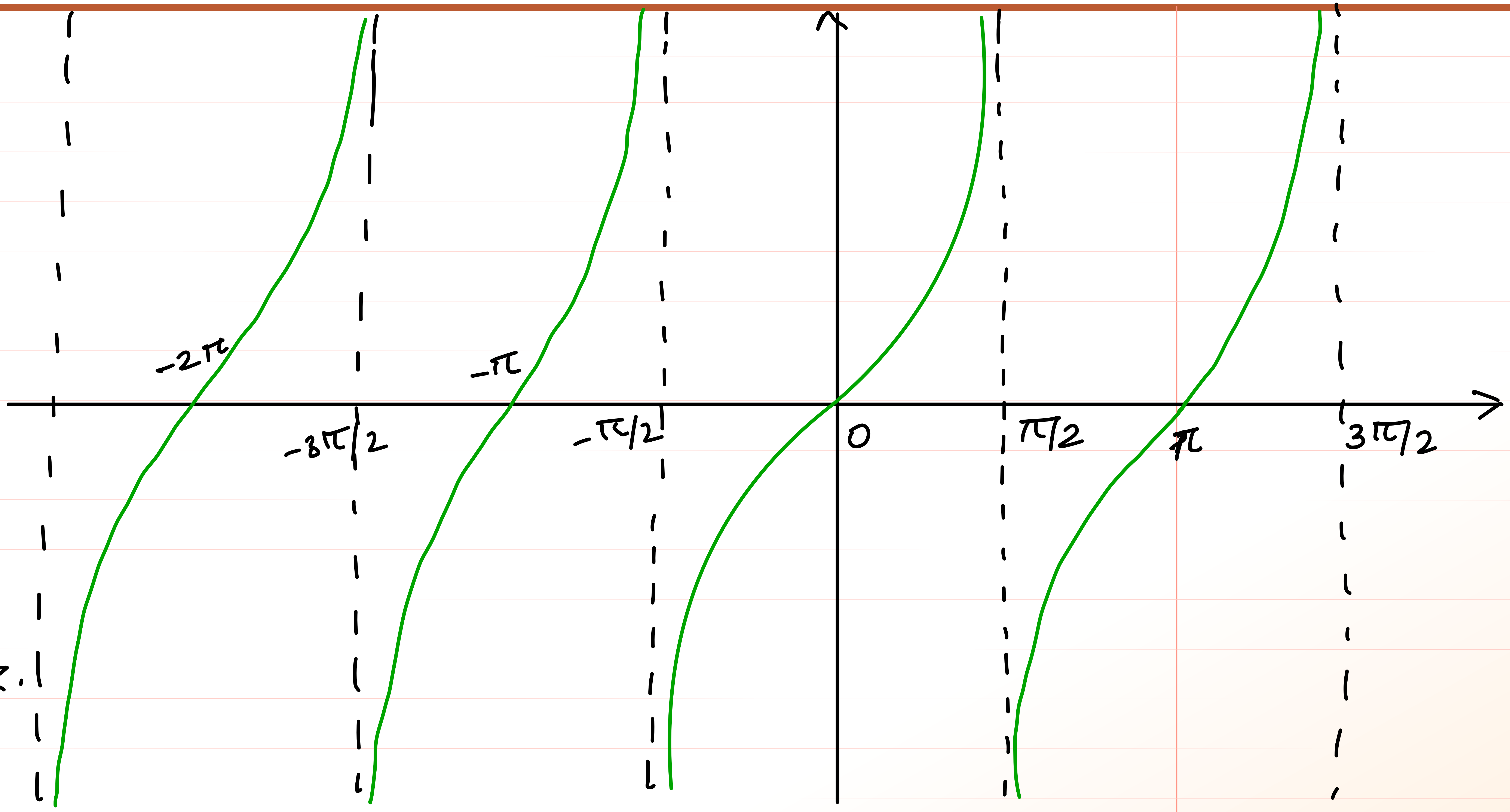
$\theta = (2n-1)\pi\ ;\ n \in \mathbb{Z}$

⑦ $\tan\theta = 0$

$\theta \in n\pi \,;\; n \in \mathbb{Z}$

⑧ $\cot\theta = 0$

$\theta \in (2n+1)\dfrac{\pi}{2} \,;\; n \in \mathbb{Z}.$



Type 2 ① If $\sin\theta = \sin\alpha \qquad \alpha \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$$\theta = n\pi + (-1)^n \alpha \quad ; n \in \mathbb{Z}$$

$$\sin\theta - \sin\alpha = 0$$

$$2\cos\left(\frac{\theta+\alpha}{2}\right) \cdot \sin\left(\frac{\theta-\alpha}{2}\right) = 0$$

$$\cos\left(\frac{\theta+\alpha}{2}\right) = 0$$

$$\frac{\theta+\alpha}{2} = (2m+1)\frac{\pi}{2} \quad ; m \in \mathbb{Z}$$

$$\theta + \alpha = (2m+1)\pi$$

$$\theta = (2m+1)\pi - \alpha$$

$$\theta = (2m+1)\pi + (-1)\alpha$$

$$\theta = (2m+1)\pi + (-1)^{2m+1} \cdot \alpha$$

$$\sin\left(\frac{\theta-\alpha}{2}\right) = 0$$

$$\frac{\theta-\alpha}{2} = m\pi$$

$$\theta - \alpha = 2m\pi$$

$$\theta = 2m\pi + \alpha$$

$$\theta = 2m\pi + (-1)^{2m} \cdot \alpha$$

$$\theta = n\pi + (-1)^n \alpha \qquad n \in \mathbb{Z}$$

(1) Solve $2\cos^2\theta + 3\sin\theta = 0$

$$2(1-\sin^2\theta) + 3\sin\theta = 0$$

$$2 - 2\sin^2\theta + 3\sin\theta = 0$$

$$2\sin^2\theta - 3\sin\theta - 2 = 0$$

$$2\sin^2\theta - 4\sin\theta + \sin\theta - 2 = 0$$

$$(2\sin\theta + 1)(\sin\theta - 2) = 0$$

$$\sin\theta = -\frac{1}{2} \qquad \boxed{\sin\theta = 2} \times$$

$$\sin\theta = \sin\left(-\frac{\pi}{6}\right)$$

$$\theta = n\pi + (-1)^n\left(-\frac{\pi}{6}\right)$$

$$\boxed{\theta = n\pi + (-1)^{n+1}\left(\frac{\pi}{6}\right)} \; ; \; n \in Z$$

(2)

$\cos\theta = \cos\alpha$ $\qquad \alpha \in [0, \pi]$

$\theta = 2n\pi \pm \alpha \qquad n \in Z$

$\cos\theta - \cos\alpha = 0$

$-2\sin\left(\frac{\theta+\alpha}{2}\right) \cdot \sin\left(\frac{-\alpha+\theta}{2}\right) = 0$

$\sin\left(\frac{\theta+\alpha}{2}\right) = 0 \qquad\qquad \sin\left(\frac{\theta-\alpha}{2}\right) = 0$

$\left(\frac{\theta+\alpha}{2}\right) = n\pi \qquad\qquad \frac{\theta-\alpha}{2} = n\pi$

$\theta+\alpha = 2n\pi \qquad\qquad \theta-\alpha = 2n\pi$

$\theta = 2n\pi - \alpha \qquad\qquad \theta = 2n\pi + \alpha$

$\boxed{\theta = 2n\pi \pm \alpha} \qquad n \in Z$

(3) $\tan\theta = \tan\alpha$ $\quad \alpha \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\theta = n\pi + \alpha \quad ; n \in \mathbb{Z}$

$$\tan\theta = \tan\alpha$$

$$\frac{\sin\theta}{\cos\theta} = \frac{\sin\alpha}{\cos\alpha}$$

$$\sin\theta\cos\alpha = \sin\alpha\cos\theta$$

$$\sin\theta\cos\alpha - \sin\alpha\cos\theta = 0$$

$$\sin(\theta - \alpha) = 0$$

$$\theta - \alpha = n\pi$$

$$\theta = n\pi + \alpha \qquad \underline{n \in \mathbb{Z}}.$$

Q

$$\tan 3\theta = -1$$

$$\tan 3\theta = \tan\left(-\frac{\pi}{4}\right)$$

$$3\theta = n\pi + \left(-\frac{\pi}{4}\right)$$

$$3\theta = n\pi - \frac{\pi}{4}$$

$$\boxed{\theta = \frac{n\pi}{3} - \frac{\pi}{12}} \quad n \in \mathbb{Z}.$$

Answer

Q

$$\sqrt{3}\sec 2\theta = 2$$

$$\cos 2\theta = \frac{\sqrt{3}}{2}$$

$$\cos 2\theta = \cos\left(\frac{\pi}{6}\right)$$

$$2\theta = 2n\pi \pm \frac{\pi}{6}$$

$$\theta = n\pi \pm \frac{\pi}{12} \quad ; \quad n \in Z$$

Type-3

$\sin^2\theta = \sin^2\alpha$ OR $\cos^2\theta = \cos^2\alpha$ OR $\tan^2\theta = \tan^2\alpha$

$\sin^2\theta - \sin^2\alpha = 0$

$\underbrace{\sin(\theta+\alpha)} \cdot \underbrace{\sin(\theta-\alpha)} = 0$

$\sin(\theta+\alpha) = 0$ OR $\sin(\theta-\alpha) = 0$

$\theta + \alpha = n\pi$ $\quad\quad$ $\theta - \alpha = n\pi$

$\theta = n\pi - \alpha$ OR $\theta = n\pi + \alpha$

$$\boxed{\theta = n\pi \pm \alpha \quad \forall\, n \in Z}$$

$\tan^2\theta = \tan^2\alpha$

$\tan\theta = \pm\tan\alpha$

$$\boxed{\theta = n\pi \pm \alpha}$$

$\tan^2\theta = \tan^2\alpha$

$(\tan\theta + \tan\alpha)(\tan\theta - \tan\alpha) = 0$

$\left(\frac{\sin\theta}{\cos\theta} + \frac{\sin\alpha}{\cos\alpha}\right)\left(\frac{\sin\theta}{\cos\theta} - \frac{\sin\alpha}{\cos\alpha}\right) = 0$

$\frac{\sin(\theta+\alpha)}{\cos\theta\cos\alpha} \cdot \frac{\sin(\theta-\alpha)}{\cos\theta\cos\alpha} = 0$

$\sin(\theta+\alpha) \cdot \sin(\theta-\alpha) = 0$

Q what is the most general value of θ which satisfy both equations

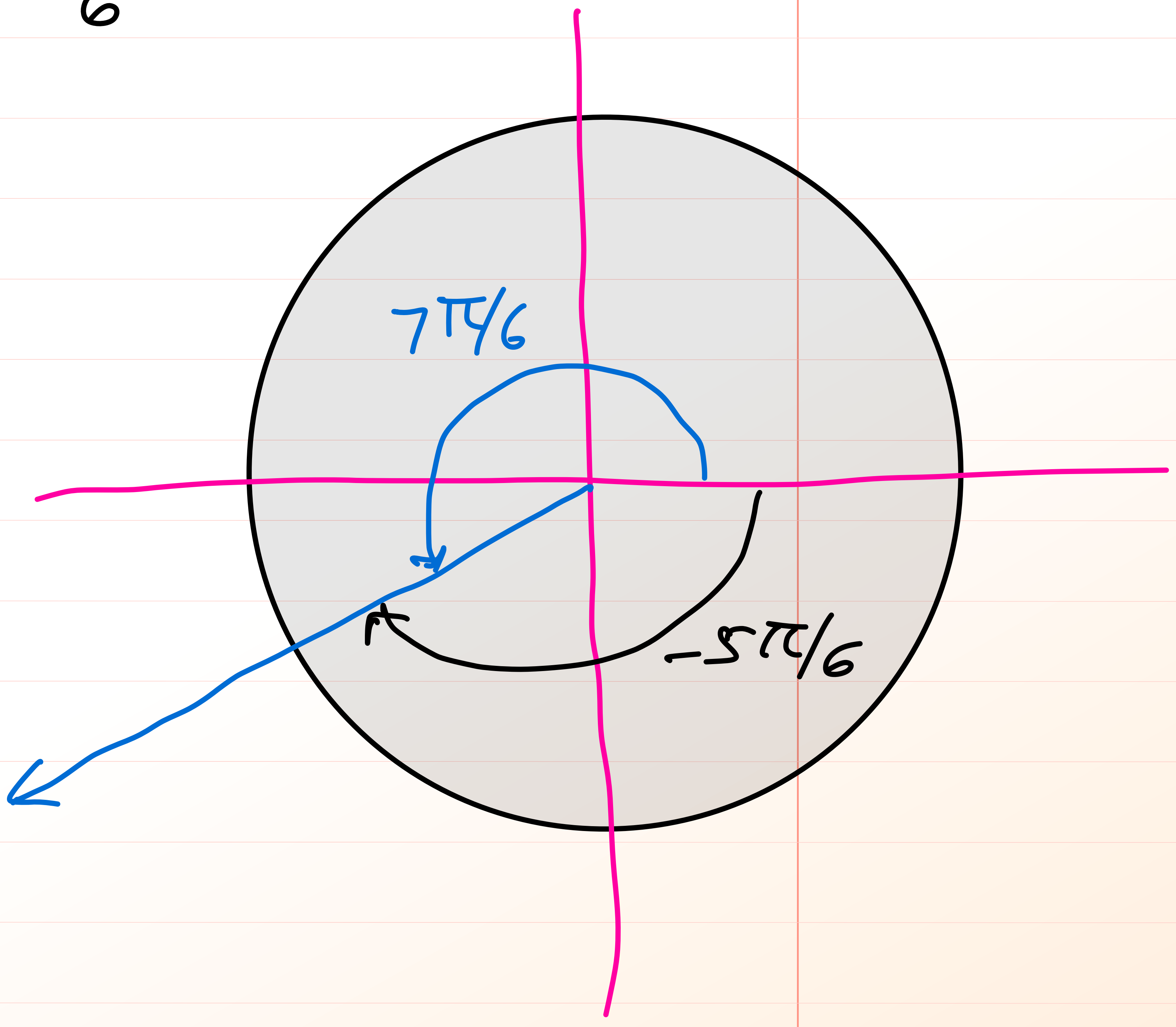
$$\sin\theta = -\frac{1}{2} \quad \text{and} \quad \tan\theta = \frac{1}{\sqrt{3}}$$

3rd quad

$$\theta = \frac{7\pi}{6} \qquad\qquad \theta = \frac{7\pi}{6}$$

$$\boxed{\theta = 2n\pi + \frac{7\pi}{6}} \quad \forall\, n \in Z$$

$$\boxed{\theta = 2n\pi - \frac{5\pi}{6}} \quad \forall\, n \in Z.$$



Q $2 + 7\tan^2\theta = \frac{13}{4}\sec^2\theta$

$$2 + 7\tan^2\theta = \frac{13}{4}(1 + \tan^2\theta)$$

$$8 + 28\tan^2\theta = 13 + 13\tan^2\theta$$

$$15\tan^2\theta = 5$$

$$\tan^2\theta = \frac{1}{3} = \left(\frac{1}{\sqrt{3}}\right)^2$$

$$\tan^2\theta = \tan^2(\pi/6)$$

$$\boxed{\theta = n\pi \pm \frac{\pi}{6}}$$

$$2\cos^2\theta + 7\sin^2\theta = \frac{13}{4\cancel{\cos^2\theta}} \cdot \cancel{\cos^2\theta}$$

$$8\cos^2\theta + 28\sin^2\theta = 13$$

$$8 - 8\sin^2\theta + 28\sin^2\theta = 13$$

$$20\sin^2\theta = 5$$

$$\sin^2\theta = \frac{1}{4} = \sin^2\frac{\pi}{6}$$

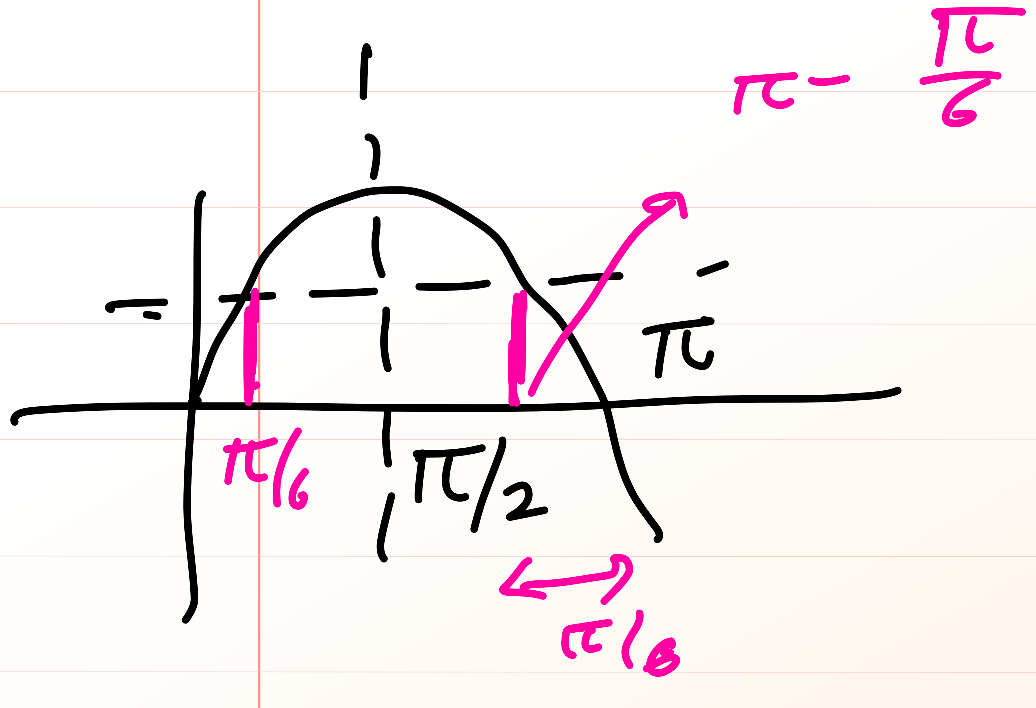
$$\boxed{\theta = n\pi \pm \frac{\pi}{6}}$$

Q $\sin^2 x + 2\tan^2 x + \frac{4}{\sqrt{3}}\tan x - \sin x + \frac{11}{12} = 0$

$$\left(\sin^2 x - \sin x\right) + 2\left(\tan^2 x + \frac{2}{\sqrt{3}}\tan x\right) + \frac{11}{12} = 0$$

$$\left(\sin^2 x - \sin x + \frac{1}{4}\right) + 2\left(\tan^2 x + \frac{2}{\sqrt{3}}\tan x + \frac{1}{3}\right) \underbrace{- \frac{2}{3} + \frac{11}{12} - \frac{1}{4}} = 0$$

$$\left(\sin x - \frac{1}{2}\right)^2 + 2\left(\tan x + \frac{1}{\sqrt{3}}\right)^2 = 0$$



$\sin x - \frac{1}{2} = 0$ and $\tan x + \frac{1}{\sqrt{3}} = 0$

$\sin x = \frac{1}{2}$ $\qquad$ $\tan x = -\frac{1}{\sqrt{3}}$

$x = \frac{5\pi}{6}$ $\qquad$ $x = \frac{5\pi}{6}$

$$x = 2n\pi + \frac{5\pi}{6} \quad \forall\ n \in Z$$

Types of Trigo eqn :→

(a) Type 1 : Solving by factorization :→

(i) $(2\sin x - \cos x)(1+\cos x) = \sin^2 x$

$(2\sin x - \cos x)(1+\cos x) = 1-\cos^2 x$

$(2\sin x - \cos x)(1+\cos x) - (1+\cos x)(1-\cos x) = 0$

$(1+\cos x)\cdot[2\sin x - \cancel{\cos x} - 1 + \cancel{\cos x}] = 0$

$\cos x = -1$ OR $\sin x = \frac{1}{2}$

$\boxed{x = (2m+1)\pi \ \forall m \in Z}$ $\qquad x = n\pi + (-1)^n \frac{\pi}{6} \ ; n \in Z$

Principal sol: $\theta \in \{\pi, \frac{\pi}{6}, \frac{5\pi}{6}\}$

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(2) $2\cos x \cdot \cos 2x = \cos x$

$\cos x\left[2\cos 2x - 1\right] = 0$

$\cos x = 0$ OR $2\cos 2x - 1 = 0$

$\boxed{x \in (2m+1)\frac{\pi}{2} \ \forall\ m \in Z}$

$\cos 2x = \frac{1}{2}$

$\cos 2x = \cos\left(\frac{\pi}{3}\right)$

$2x = 2n\pi \pm \frac{\pi}{3}$

$\boxed{x = n\pi \pm \frac{\pi}{6}}$

$2\cos 2x - 1 = 0$

$2(2\cos^2 x - 1) - 1 = 0$

$\cos^2 x = \frac{3}{4}$

$\cos^2 x = \cos^2 \frac{\pi}{6}$

$\boxed{x = n\pi \pm \frac{\pi}{6}}$

$2(1 - 2\sin^2 x) - 1 = 0$

$-4\sin^2 x + 1 = 0$

$\sin^2 x = \frac{1}{4}$

$\boxed{x = n\pi \pm \frac{\pi}{6}}$

Q

$$\cot x - \cos x = 1 - \cot x \cdot \cos x$$

$$\cot x - \cos x - 1 + \cot x \cos x = 0$$

$$\cot x (1 + \cos x) - 1(\cos x + 1) = 0$$

$$(\cos x + 1)(\cot x - 1) = 0$$

$\cot x = 1$ OR $\cos x = -1$

$$\boxed{x = n\pi + \frac{\pi}{4}}$$

Rejected

Trigonometric equations
CL03

(4) $2\sin^2 2x + 6\sin^2 x = 5$

$1 - 2\sin^2 x = \cos 2x$

$2\sin^2 x = 1 - \cos 2x$

$2(1-\cos^2 2x) + 3(1-\cos 2x) = 5$

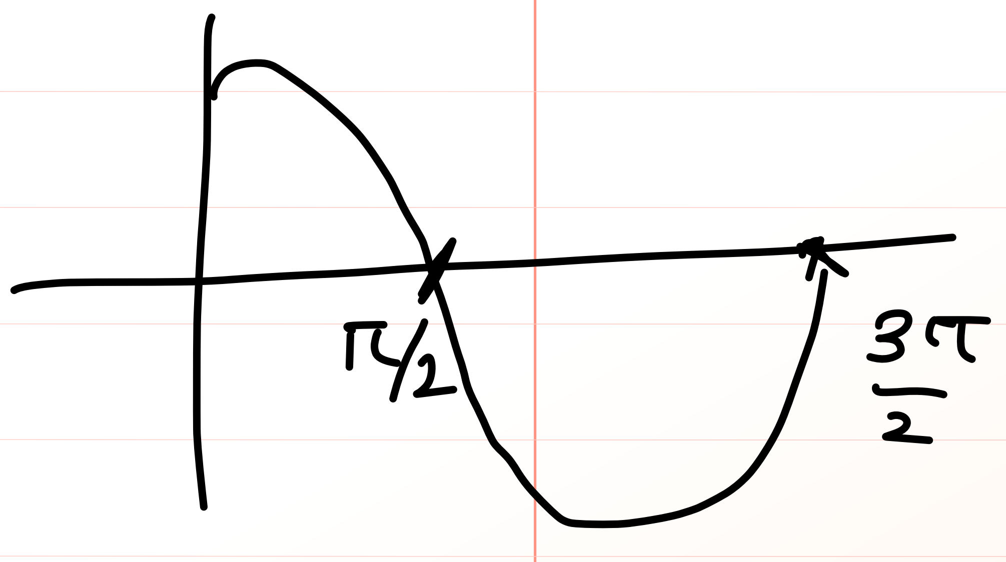
$-2\cos^2 2x - 3\cos 2x = 0$

$\cos 2x(-2\cos 2x - 3) = 0$

$\cos 2x = 0$ $\qquad$ $\cos 2x = -\frac{3}{2}$ (Rejected)

$2x = (2n+1)\frac{\pi}{2} \quad \forall\, n \in Z$

$$\boxed{x = (2n+1)\frac{\pi}{4} \quad \forall\, n \in Z.}$$



solving equations by Trigonometric formulae :→

(1) $\cos 3x + \underbrace{\sin 2x - \sin 4x} = 0$

$\cos 3x + 2\cos 3x \sin(-x) = 0$

$\cos 3x (1 - 2\sin x) = 0$

$\cos 3x = 0$

$\cos 3x = 0$

$3x = (2n+1)\frac{\pi}{2}$

$\boxed{x = (2n+1)\frac{\pi}{6}}$

$1 - 2\sin x = 0$

$\sin x = \frac{1}{2}$

$\boxed{x = m\pi + (-1)^m \cdot \frac{\pi}{6}}$ $\forall m, n \in Z$

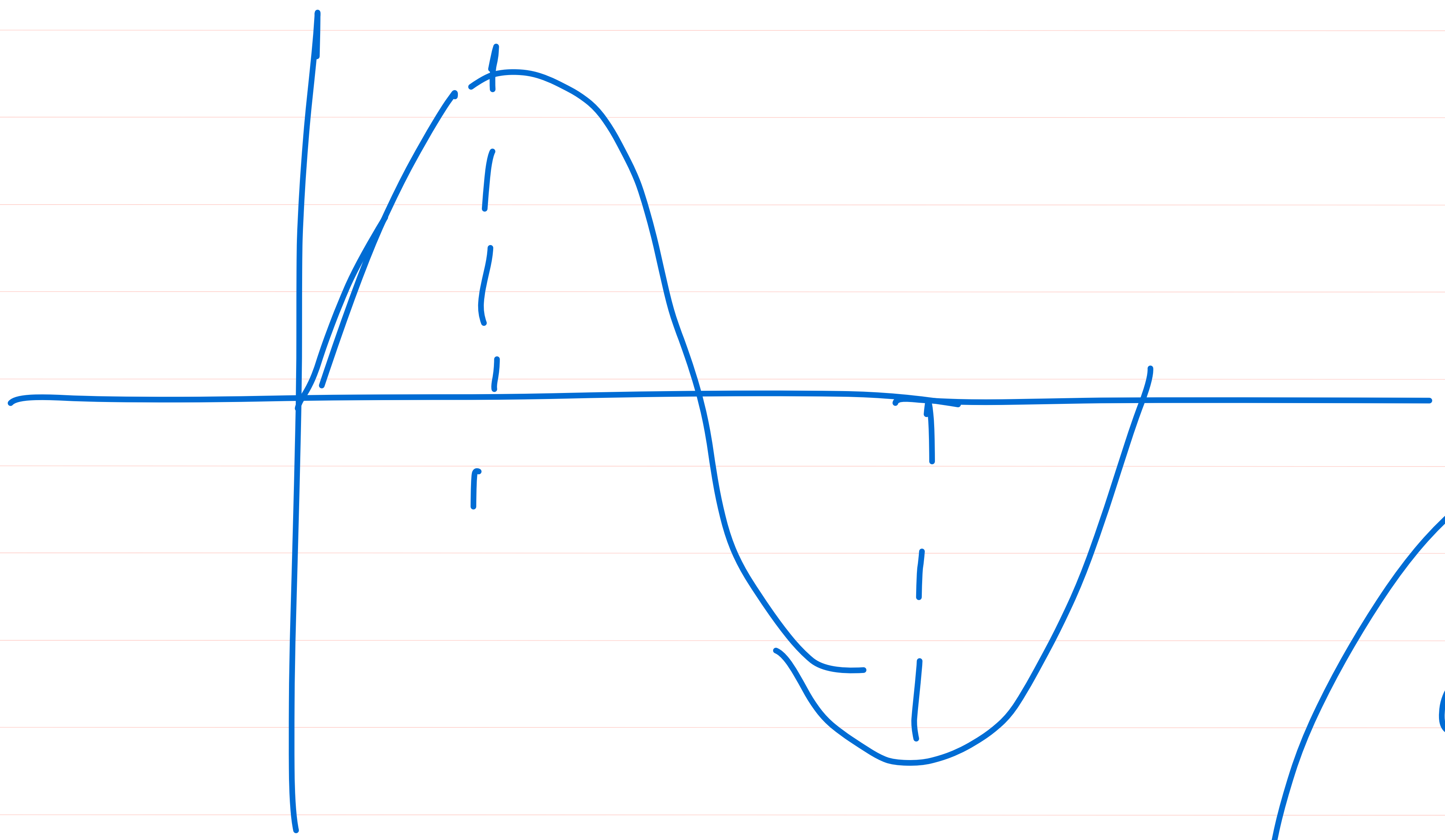
Slide 4, 5, 6, 7

$\sin x = 0, 1, -1$

$\cos x = 0, 1, -1$

$\tan x = 0$

$\cot x = 0$



use

$\sin \to n\pi + (-1)^n \alpha$

$\cos \to 2n\pi \pm \alpha$

$\tan \to n\pi + \alpha$

Besides above values

② find no. of solutions in $[0, \pi]$;

$$\sin 3\theta = 4\sin\theta \cdot \sin 2\theta \cdot \sin 4\theta$$

$$\sin 3\theta = 4\sin\theta \cdot \sin(3\theta - \theta)\sin(3\theta + \theta)$$

$$3\sin\theta - 4\sin^3\theta = 4\sin\theta \cdot [\sin^2 3\theta - \sin^2\theta]$$

$$3\sin\theta - \cancel{4\sin^3\theta} = 4\sin\theta \cdot \sin^2 3\theta - \cancel{4\sin^3\theta}$$

$$3\sin\theta - 4\sin\theta \cdot \sin^2 3\theta = 0$$

$$\sin\theta(3 - 4\sin^2 3\theta) = 0$$

$$\sin\theta = 0$$

$$\theta = n\pi$$

$$\boxed{\theta \in \{0, \pi\}}$$

$$\sin^2 3\theta = \frac{3}{4} \Rightarrow \sin^2 3\theta = \sin^2\left(\frac{\pi}{3}\right)$$

$$3\theta = n\pi \pm \frac{\pi}{3}$$

$$\theta = \frac{\pi}{9}, \frac{2\pi}{9}, \frac{4\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}, \frac{8\pi}{9}$$

at $n = 0$

$n = 1$

$$3\theta = \pi \pm \frac{\pi}{3}$$

$$3\theta = \frac{4\pi}{3}, \frac{2\pi}{3}$$

$$\boxed{\theta = \frac{4\pi}{9}, \frac{2\pi}{9}}$$

$n = 2$

$$3\theta = 2\pi \pm \frac{\pi}{3}$$

$$= \frac{7\pi}{3}, \frac{5\pi}{3}$$

$$\boxed{\theta = \frac{7\pi}{9}, \frac{5\pi}{9}}$$

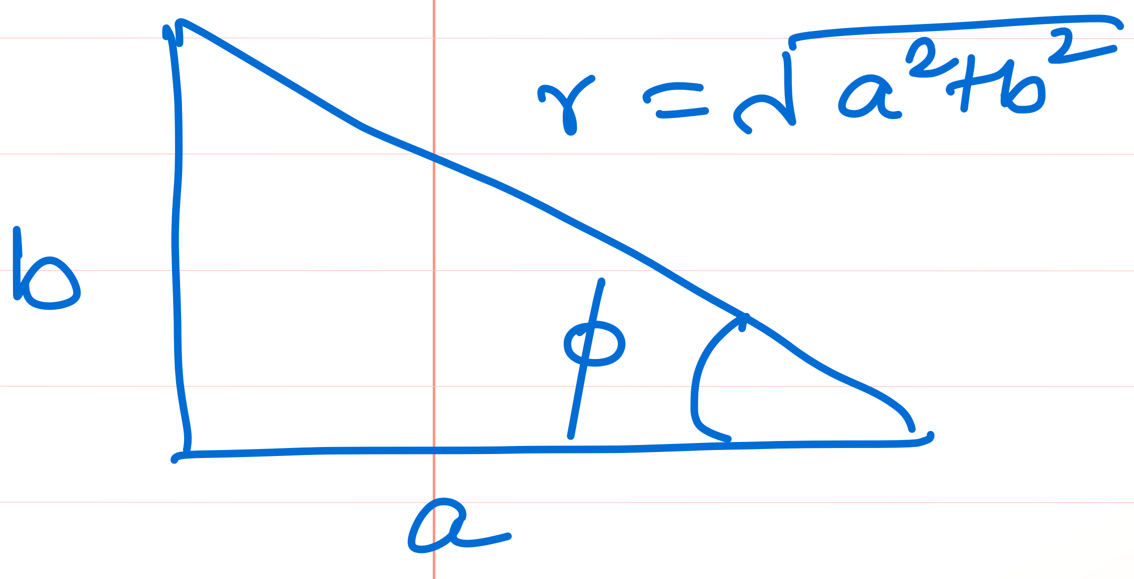
$n = 3$

$$3\theta = 3\pi \pm \frac{\pi}{3}$$

$$\boxed{\theta = \frac{8\pi}{9}}$$

Type-3 Solving trigo equations by introducing auxiliary argument :→

$$\frac{a\cos\theta}{\sqrt{a^2+b^2}} + \frac{b\sin\theta}{\sqrt{a^2+b^2}} = \frac{c}{\sqrt{a^2+b^2}}$$



$$\cos\phi = \frac{a}{\sqrt{a^2+b^2}}$$

$$\sin\phi = \frac{b}{\sqrt{a^2+b^2}}$$

$$\cos\phi\,\cos\theta + \sin\phi\,\sin\theta = \frac{c}{\sqrt{a^2+b^2}}$$

$$\cos(\theta-\phi) = \frac{c}{\sqrt{a^2+b^2}}$$

$$-1 \le \frac{c}{\sqrt{a^2+b^2}} \le 1$$

$$\left|\frac{c}{\sqrt{a^2+b^2}}\right| \le 1 \Rightarrow \boxed{|c| \le \sqrt{a^2+b^2}}$$

Note :→ If $|c| > \sqrt{a^2+b^2}$ then the given equation has no real solution.

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(1) $1 \cdot \sin x + 1 \cos x = \sqrt{2}$

$a = 1; \quad b = 1$

$\sqrt{a^2+b^2} = \sqrt{1+1} = \sqrt{2}$

divide by $\sqrt{2}$ to both sides

$$\frac{1 \cdot \sin x}{\sqrt{2}} + \frac{1 \cdot \cos x}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{2}}$$

$$\sin\left(\frac{\pi}{4}\right) \cdot \sin x + \cos\left(\frac{\pi}{4}\right) \cdot \cos x = 1$$

$$\cos\left(x - \frac{\pi}{4}\right) = 1$$

$$x - \frac{\pi}{4} = 2n\pi$$

$$x = 2n\pi + \frac{\pi}{4} \quad \forall \ n \in \mathbb{Z}$$

(2) $\sqrt{3}\cos x + \sin x = 2$

$a = \sqrt{3}\,;\ b = 1$

$\sqrt{a^2+b^2} = \sqrt{3+1} = 2$

$\frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x = \frac{2}{2}$

$\cos\left(\frac{\pi}{6}\right)\cos x + \sin\left(\frac{\pi}{6}\right)\sin x = 1$

$\cos\left(x - \frac{\pi}{6}\right) = 1$

$x - \frac{\pi}{6} = 2n\pi$

$\boxed{x = 2n\pi + \frac{\pi}{6}\ \forall n \in Z}$

(3) $1\sin x + 1\cos x = 1.5$

$a = 1\,;\ b = 1$

$\sqrt{a^2+b^2} = \sqrt{2}$

$\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x = \frac{1.5}{\sqrt{2}}$

$\sin\frac{\pi}{4}\cdot\sin x + \cos\frac{\pi}{4}\cdot\cos x = \frac{1.5}{\sqrt{2}}$

$\cos\left(x - \frac{\pi}{4}\right) = \boxed{\frac{1.5}{\sqrt{2}}}$

> 1

No solution

$x \in \phi$

Q4 $\quad 4\cos x + 3\sin x = 5$

$a = 4; \quad b = 3$

$$\sqrt{a^2+b^2} = \sqrt{4^2+3^2} = 5$$

$$\frac{4}{5}\cos x + \frac{3}{5}\sin x = 1$$

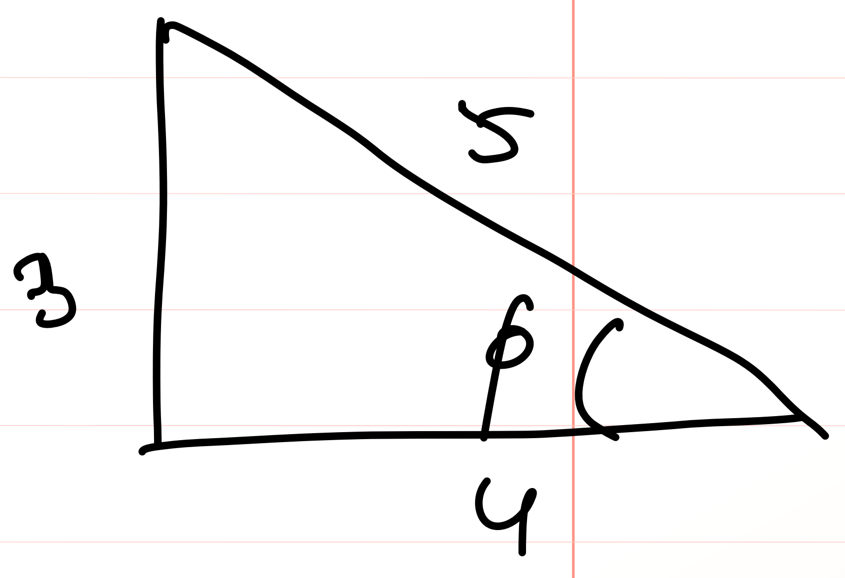
$$\cos\phi \cos x + \sin\phi \sin x = 1$$

$$\cos(x-\phi) = 1$$

$$x - \phi = 2n\pi$$

$$x = 2n\pi + \phi$$

$$\boxed{x = 2n\pi + \tan^{-1}\left(\frac{3}{4}\right)}$$



$$\cos\phi = \frac{4}{5}$$

$$\sin\phi = \frac{3}{5}$$

$$\tan\phi = \frac{3}{4}$$

$$\phi = \underbrace{\tan^{-1}\frac{3}{4}}$$

(5) $1+\sin^3 x+\cos^3 x=\frac{3}{2}\sin 2x$

$a^3+b^3+c^3=3abc$

$1^3+(\sin x)^3+(\cos x)^3=\frac{3}{\cancel{2}}\cdot\cancel{2}\sin x\cos x\cdot 1$

$1^3+(\sin x)^3+(\cos x)^3=3\sin x\cos x(1)$

$1+\sin x+\cos x=0$ or $\boxed{1=\sin x=\cos x \quad \sin x=1}$

$\sin x+\cos x=-1$

$\frac{1}{\sqrt{2}}\sin x+\frac{1}{\sqrt{2}}\cos x=-\frac{1}{\sqrt{2}}$

$\cos\left(x-\frac{\pi}{4}\right)=\cos\left(\frac{3\pi}{4}\right)$

$x=2n\pi\pm\frac{3\pi}{4}+\frac{\pi}{4}$

+ sign — sign

$\boxed{x=2n\pi+\pi}$ $\boxed{x=2n\pi-\frac{\pi}{2}}$

Trigonometric equations
CL04

Type-4 Solving equations with the use of boundness of the function sin x or cos x

① $\sin^4 x = 1 + \cos^6 y$

$-1 \le \sin x \le 1$

$0 \le \sin^4 x \le 1$

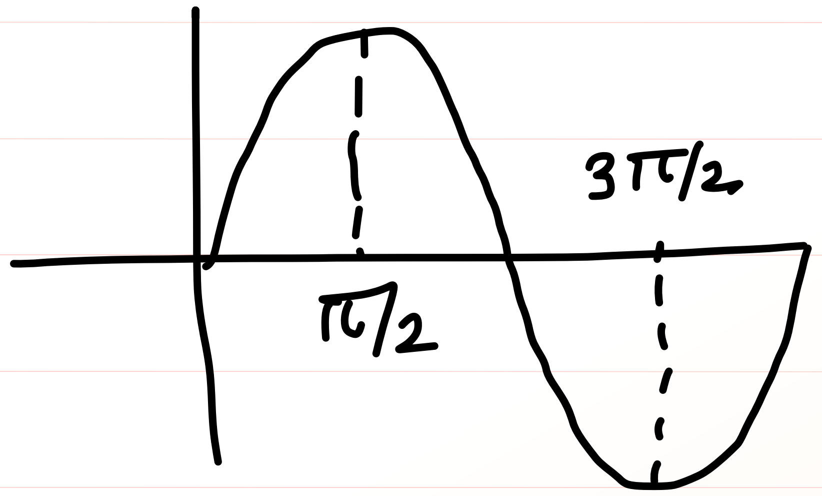
$-1 \le \cos y \le 1$

$0 \le \cos^6 y \le 1$

$1 \le 1 + \cos^6 y \le 2$

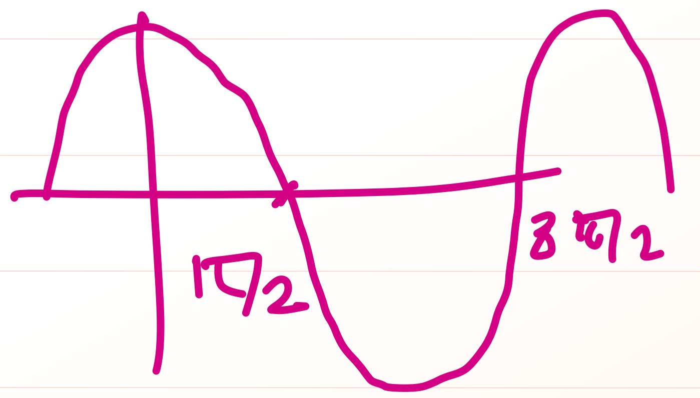
$\sin^4 x = 1$

$\sin x = \pm 1$



$$\boxed{x \in (2n+1)\frac{\pi}{2},\ n \in Z}$$

$\cos^6 y = 0$



$$\boxed{y \in (2m+1)\frac{\pi}{2},\ m \in Z}$$

② $\cos x + \cos 2x + \cos 3x = 3$

$\cos x = 1$ $\quad\quad$ $\cos 2x = 1$ $\quad\quad$ $\cos 3x = 1$

$x = 2n\pi$ $\quad\quad$ $2x = 2n\pi$ $\quad\quad$ $3x = 2n\pi$ $\quad\quad$ $n \in Z.$

$x = \frac{2n\pi}{1}$ $\quad\quad$ $x = \frac{n\pi}{1}$ $\quad\quad$ $x = \frac{2n\pi}{3}$ $\quad\quad$ $n \in Z$

$$x = \frac{2n\pi}{1} \;;\; n \in Z$$

(3) $\sin x\left(\cos\frac{x}{4} - 2\sin x\right) + \left(1 + \sin\frac{x}{4} - 2\cos x\right)\cos x = 0$

$\underbrace{\sin x \cdot \cos\frac{x}{4}} - 2\sin^2 x + \cos x + \underbrace{\sin\frac{x}{4}\cos x} - 2\cos^2 x = 0$

$\sin\left(x + \frac{x}{4}\right) - 2\left(\sin^2 x + \cos^2 x\right) + \cos x = 0$

$\sin\left(\frac{5x}{4}\right) + \cos x = 2$

$\sin\left(\frac{5x}{4}\right) = 1$ $\qquad$ $\cos x = 1$

$\frac{5x}{4} = (4n+1)\frac{\pi}{2}$ $\qquad$ $\boxed{x = 2m\pi \; ; m \in Z}$ $\qquad$ $x \in \{0, 2\pi, 4\pi,$

$\boxed{x = (4n+1)\frac{2\pi}{5} \; ; n \in Z}$ $\qquad$ common solutions $\{-- -6\pi, 2\pi, 10\pi ---\}$

$x \in = \frac{2\pi}{5}, 2\pi, \frac{18\pi}{5}, \frac{26\pi}{5}, \frac{34\pi}{5}, \frac{42\pi}{5}, 10\pi$

$= (2 + (p-1)8)\pi \quad p \in z$

$= (8p-6)\pi \; ; p \in Z$

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(4) Solve for x and y

$$1-2x-x^2 = \tan^2(x+y) + \cot^2(x+y)$$

$$-(x^2+2x)+1 = \tan^2(x+y) + \cot^2(x+y)$$

$$-(x^2+2x+1)+1+1 = \tan^2(x+y) + \cot^2(x+y)$$

$$\underbrace{-(x+1)^2}\ +2 = \tan^2(x+y) + \cot^2(x+y)$$

$$(x+1) = 0$$

$$\boxed{x=-1}$$

$$\tan^2(x+y) = 1$$

$$(x+y) = n\pi \pm \frac{\pi}{4}$$

$$y-1 = n\pi \pm \frac{\pi}{4}$$

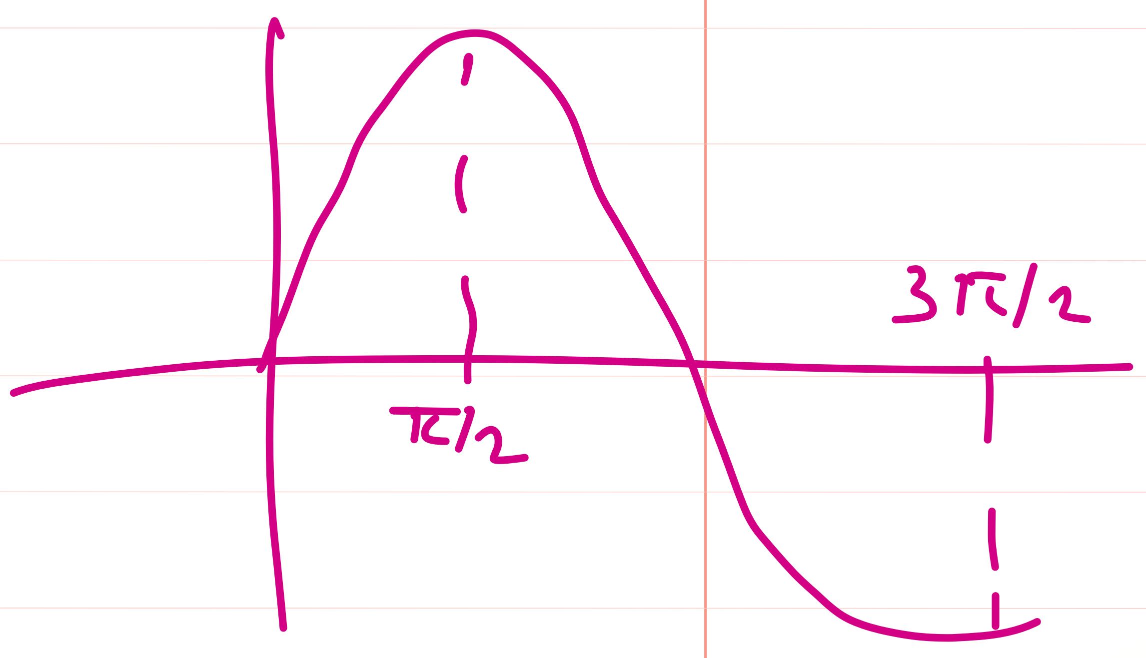
$$y = n\pi \pm \frac{\pi}{4} + 1$$

Type 5 Solution of trigo equations of the form $f(x) = \sqrt{\phi(x)}$

① $\sqrt{1-\cos x} = \sin x$ ✓

$1-\cos x = 1-\cos^2 x$

$\cos x = 0, 1$



$\cos x = 0$

$x = (2n+1)\frac{\pi}{2}$

$x = \{ \ldots, \cancel{-\frac{\pi}{2}}, \frac{\pi}{2}, \cancel{\frac{3\pi}{2}}, \frac{5\pi}{2}, \ldots \}$

$x = (4n+1)\frac{\pi}{2} \quad \forall\, n \in \mathbb{Z}.$

$\cos x = 1$

$x = 2n\pi$

$x = \{ -2\pi, 0, 2\pi, \ldots \}$

$x = 2n\pi \quad \forall\, n \in \mathbb{Z}$

(2)

$$2^{\frac{1}{\sin^2 x}} \cdot \sqrt{y^2-2y+2} \leq 2$$

$$2^{\operatorname{cosec}^2 x} \cdot \underbrace{\sqrt{y^2-2y+2}} \leq \underbrace{2}$$

$$2^{\operatorname{cosec}^2 x} \sqrt{y^2-2y+1+1} \leq 2$$

$$\underbrace{2^{\operatorname{cosec}^2 x}}_{\geq 2} \underbrace{\sqrt{(y-1)^2+1}}_{\geq 1} \leq 2$$

$$\geq 2$$

$$= 2$$

$$(y-1)^2 = 0$$

$$\boxed{y=1}$$

$$2^{\operatorname{cosec}^2 x} = 2$$

$$\operatorname{cosec}^2 x = 1$$

$$\sin^2 x = 1$$

$$x \in (2n+1)\frac{\pi}{2}$$

Q $2\sin\left(3x+\frac{\pi}{4}\right) = \sqrt{1+8\sin 2x\cos^2 2x}$

$$4\sin^2\left(3x+\frac{\pi}{4}\right) = 1+\underbrace{8\sin 2x\cdot\cos 2x}\cdot\cos 2x$$

$$4\sin^2\left(3x+\frac{\pi}{4}\right) = 1+4\cdot(\sin 4x)\cdot\cos 2x$$

$\cos 2x = 1-2\sin^2 x$

$\underline{2\sin^2 x} = 1-\cos 2x$

$$2\left[1-\cos 2\left(3x+\frac{\pi}{4}\right)\right] = 1+4\sin 4x\cos 2x$$

$$2\left[1-\underbrace{\cos\left(\frac{\pi}{2}+6x\right)}\right] = 1+2\left(\underbrace{2\sin 4x\cos 2x}\right)$$

$$2+2\cancel{\sin(6x)} = 1+2\cdot\left(\cancel{\sin 6x}+\sin(2x)\right)$$

$$\sin 2x = \frac{1}{2}$$

$$2x = n\pi+(-1)^n\frac{\pi}{6}$$

$$x = \frac{n\pi}{2}+(-1)^n\frac{\pi}{12}$$

$$x = \frac{n\pi}{2} + (-1)^n \frac{\pi}{12}$$

$[0, 2\pi]$

$n = 0 \qquad x = \frac{\pi}{12}$

$2\sin\left(3x + \frac{\pi}{4}\right) = 2\sin\left(3\cdot\frac{\pi}{12} + \frac{\pi}{4}\right) = 2\sin\frac{\pi}{2} = 2 = \underline{\text{tve}}$

$n = 1 \; ; \qquad x = \frac{\pi}{2} - \frac{\pi}{12} = \frac{5\pi}{12}$

$2\sin\left(3x + \frac{\pi}{4}\right) = 2\sin\left(3\left(\frac{5\pi}{12}\right) + \frac{\pi}{4}\right) = 2\sin\left(\frac{3\pi}{2}\right) = 2(-1) = -2$

$n = 2 \; ; \qquad x = \frac{2\pi}{2} + \frac{\pi}{12} = \frac{13\pi}{12}$

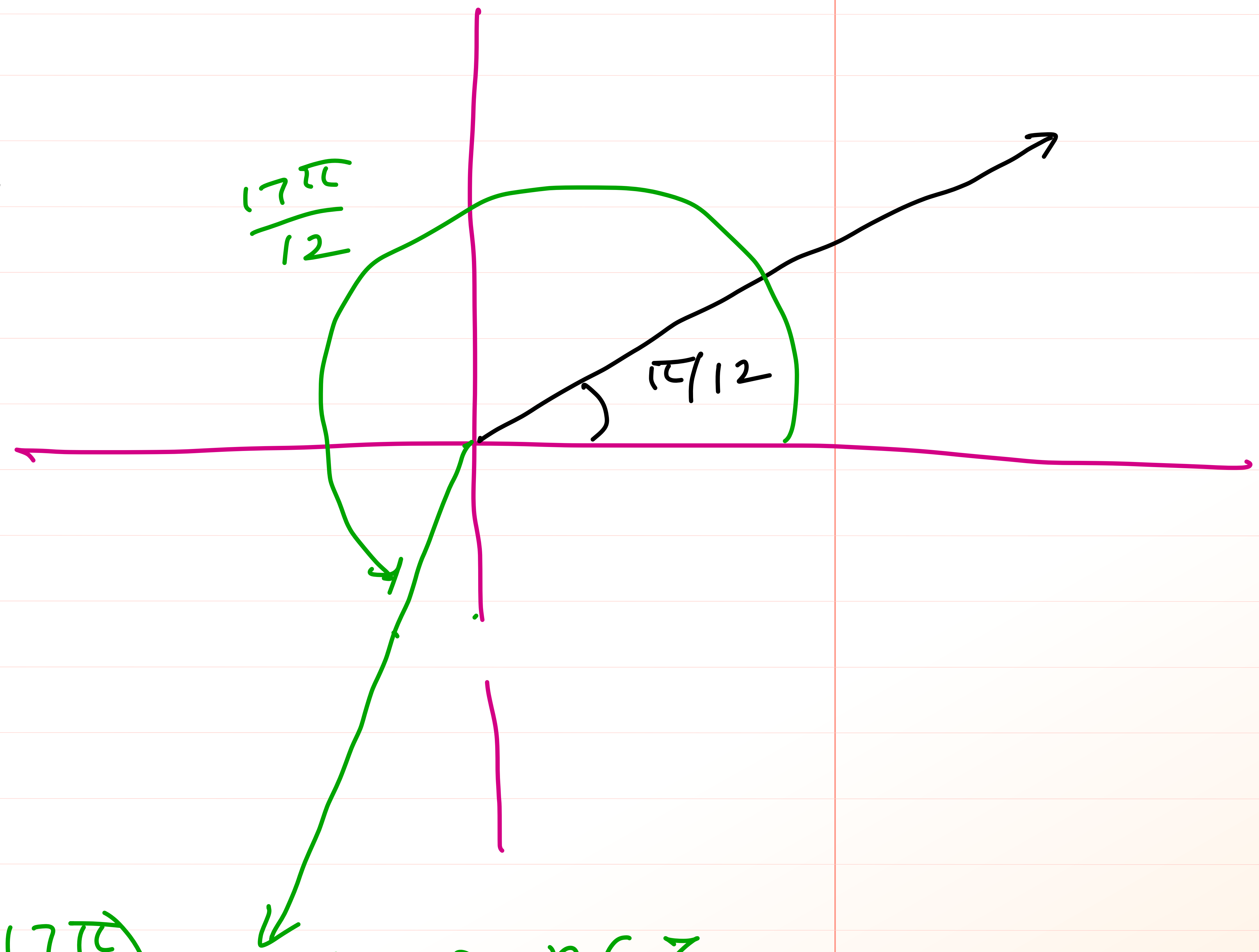
$2\sin\left(3x + \frac{\pi}{4}\right) = 2\sin\left(3\left(\frac{13\pi}{12}\right) + \frac{\pi}{4}\right) = 2\sin\left(\frac{7\pi}{2}\right) = 2(-1) = -2$

$n = 3 \; ; \qquad x = \frac{3\pi}{2} - \frac{\pi}{12} = \frac{17\pi}{12}$

$2\sin\left(3x + \frac{\pi}{4}\right) = 2\sin\left(3\left(\frac{17\pi}{12}\right) + \frac{\pi}{4}\right) = 2\sin\left(\frac{9\pi}{2}\right) = +2$

$x = \frac{\pi}{12}$

$x = 2n\pi + \frac{\pi}{12}$ ✓ $\forall\, n \in Z.$



$x = \frac{17\pi}{12}$

$x = 2m\pi + \frac{17\pi}{12}$ $\forall\, m \in Z.$

$x = \left(2n\pi + \frac{\pi}{12}\right) \cup \left(2m\pi + \frac{17\pi}{12}\right)$ $\forall\, m, n \in Z.$

System of Trigonometric equations ! →

(1) $\cos x \cdot \cos y = \frac{3}{4}$ and $\sin x \cdot \sin y = \frac{1}{4}$

add

$\cos x \cos y + \sin x \sin y = 1$

$\cos(x-y) = 1 \Rightarrow x-y = 2n\pi \quad \forall\, n \in Z$ —— (1)

Subtract

$\cos x \cos y - \sin x \sin y = \frac{1}{2}$

$\cos(x+y) = \cos\frac{\pi}{3} \Rightarrow x+y = 2m\pi \pm \frac{\pi}{3}$

$x-y = 2n\pi$ —— (1)

$x+y = 2m\pi \pm \frac{\pi}{3}$

$2x = 2n\pi + 2m\pi \pm \frac{\pi}{3} \Rightarrow \boxed{x = (n+m)\pi \pm \frac{\pi}{6}}$

$\boxed{y = (m-n)\pi \pm \frac{\pi}{6}}$

(2) $x+y=\frac{2\pi}{3}$; $\frac{\sin x}{\sin y}=2$

$\sin x = 2\sin y$

$\sin x = 2\sin\left(\frac{2\pi}{3}-x\right)$

$\sin x = 2\left[\sin\left(\frac{2\pi}{3}\right)\cos x - \cos\left(\frac{2\pi}{3}\right)\sin x\right]$

$\sin x = 2\left[\frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x\right]$

$\cancel{\sin x} = \sqrt{3}\cos x + \cancel{\sin x}$

$\cos x = 0$

$$\boxed{x = (2n+1)\frac{\pi}{2} \; ; \; n\in Z}$$

$y = \frac{2\pi}{3} - x$

$= \frac{2\pi}{3} - n\pi - \frac{\pi}{2}$

$$\boxed{y = \frac{\pi}{6} - n\pi}$$

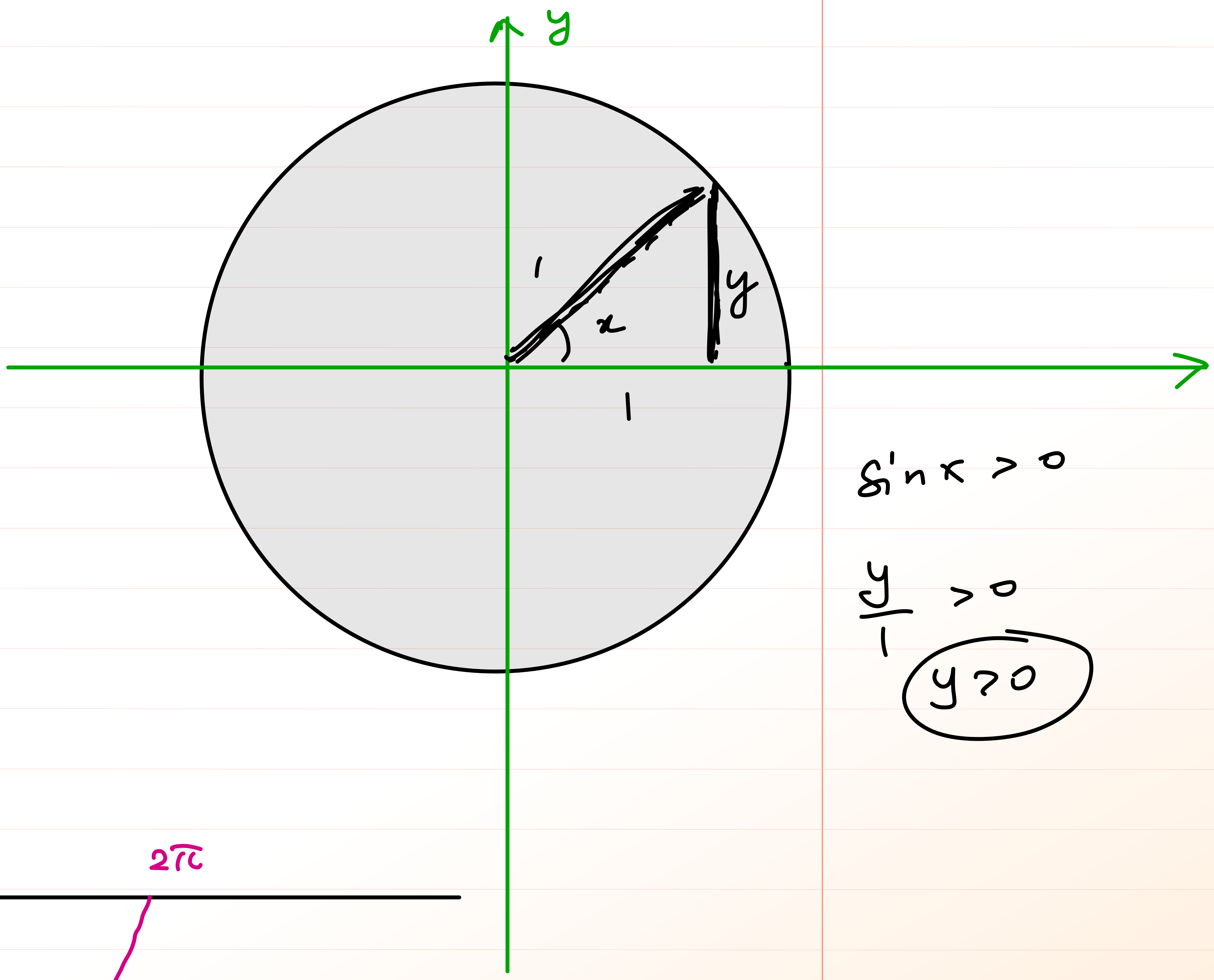
Trigonometric Inequalities and system of inequality :→

① $\sin x > 0$

M–I

$y > 0$

$\boxed{x \in (0, \pi)}$



$\sin x > 0$

$\frac{y}{1} > 0$

$y > 0$

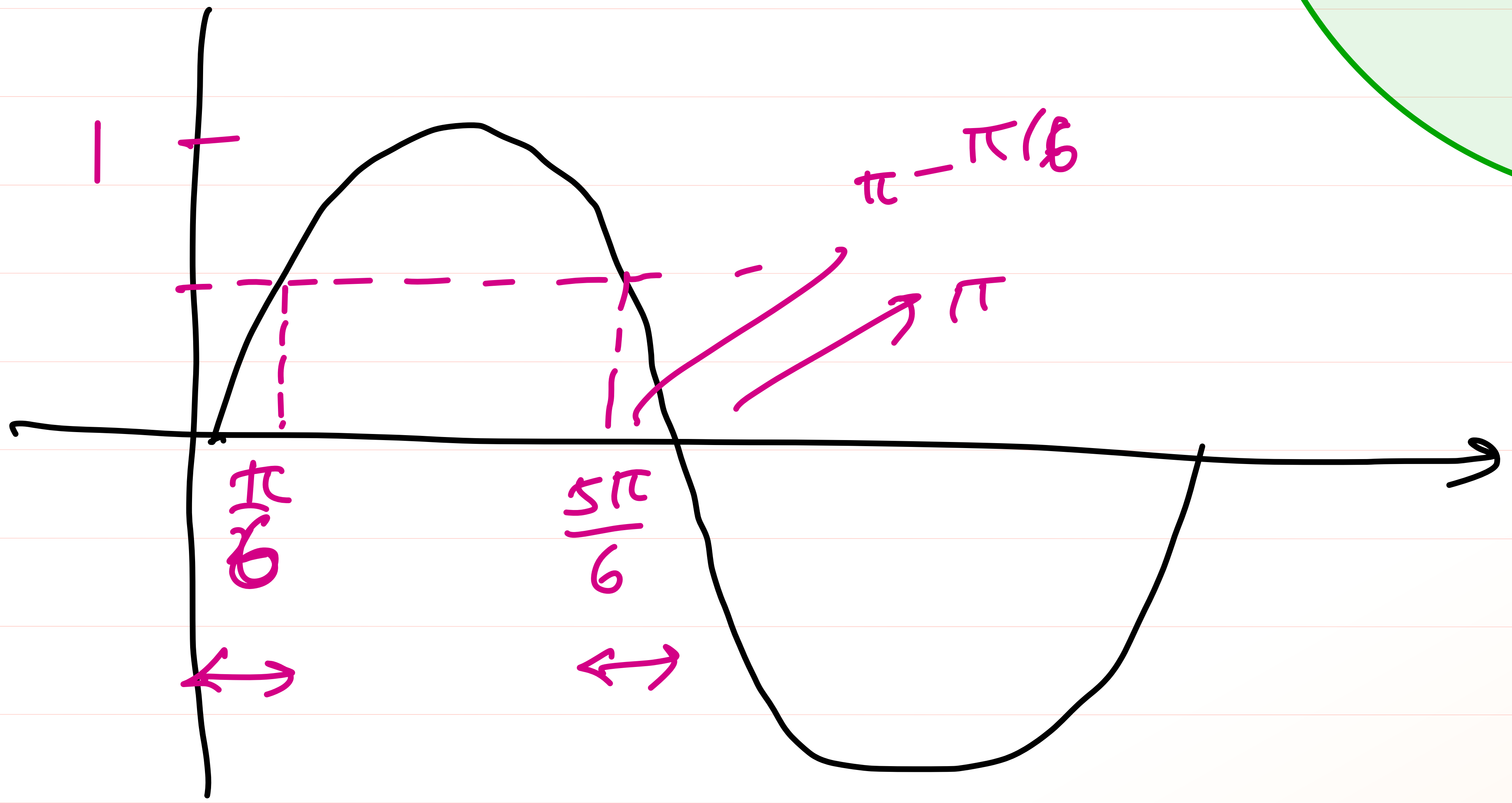
M–II

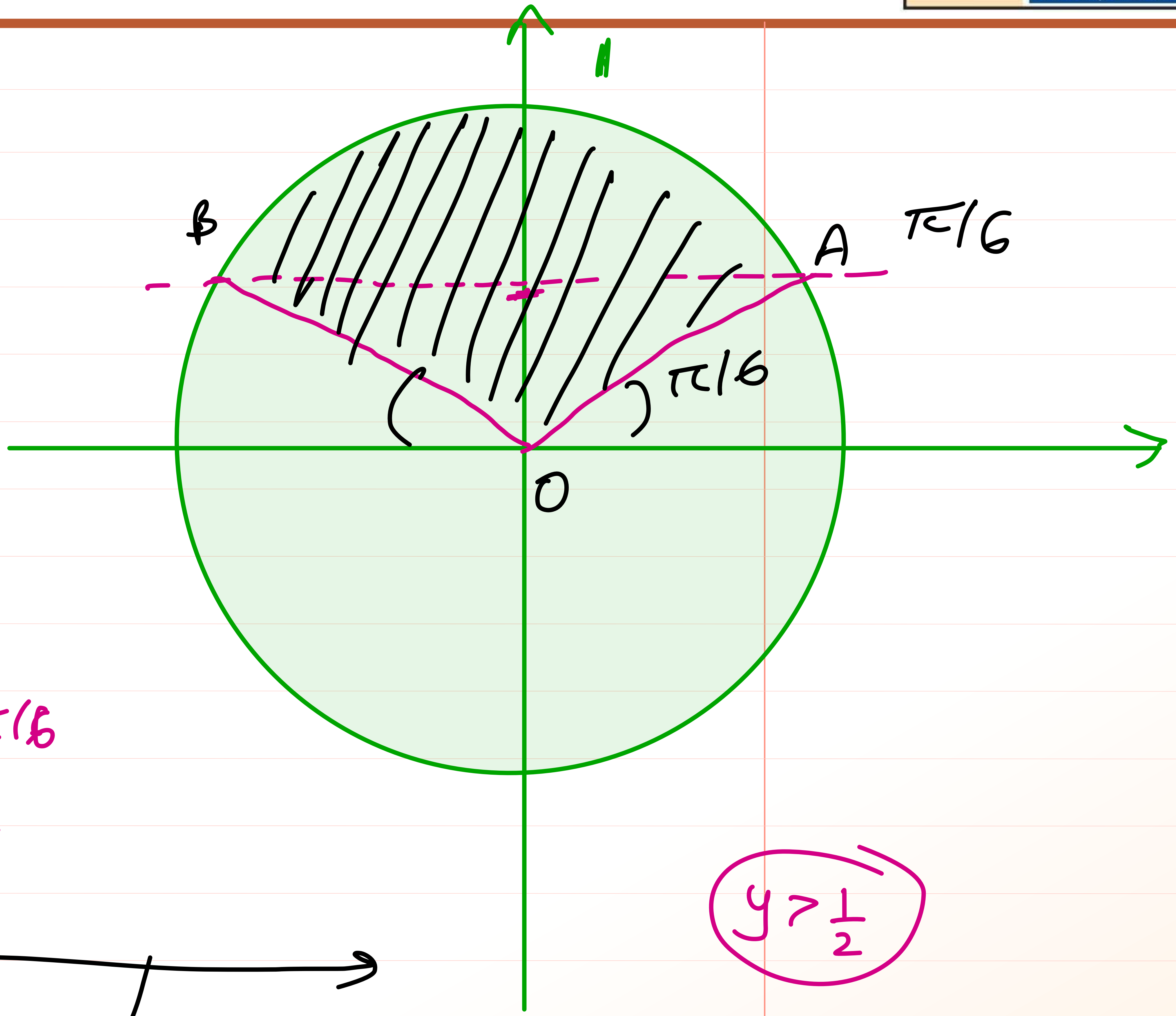
$x \in (0, \pi)$

② $\sin x > \frac{1}{2}$

MI

$$\boxed{x \in \left(\frac{\pi}{6}, \frac{5\pi}{6}\right)}$$





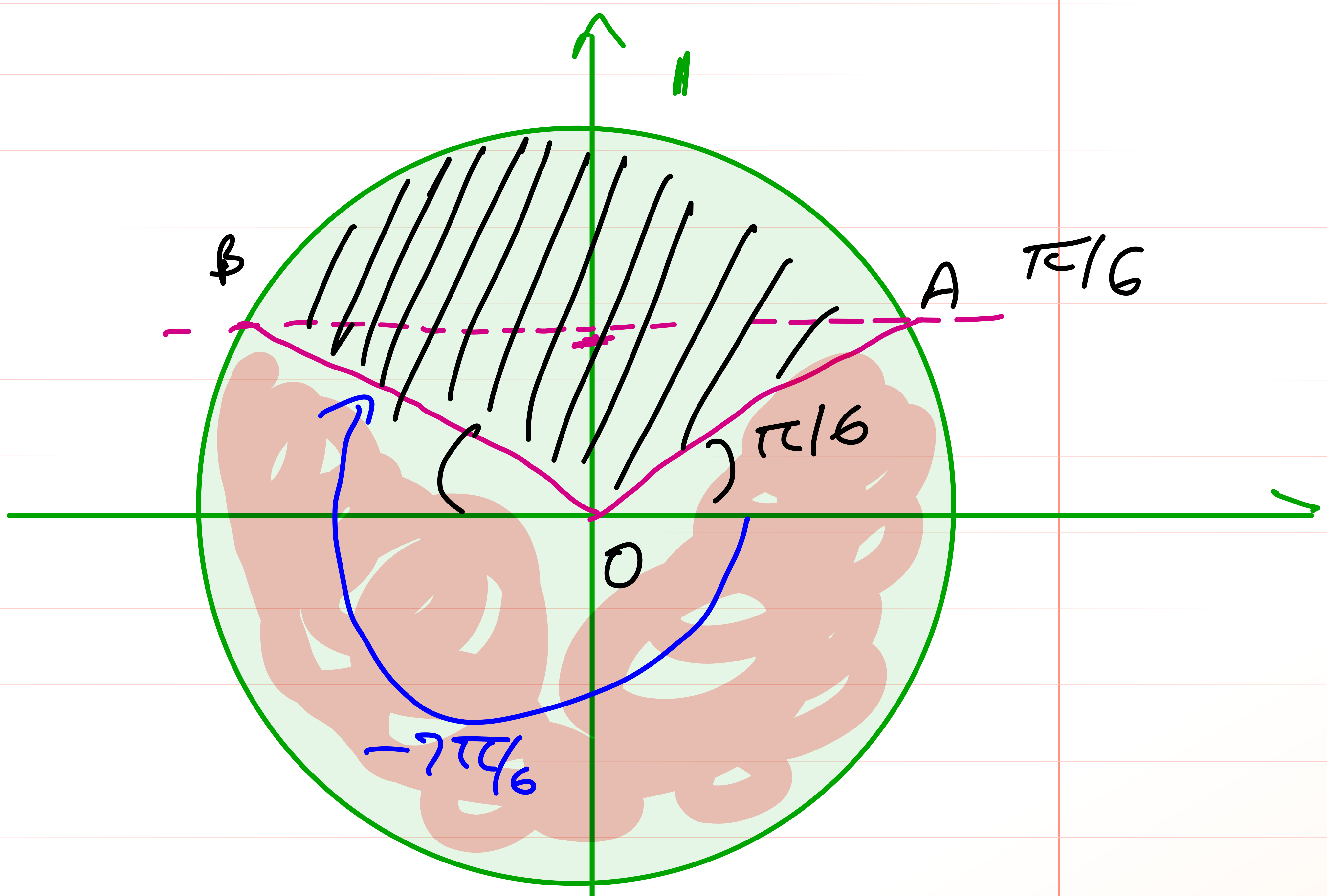
$y > \frac{1}{2}$

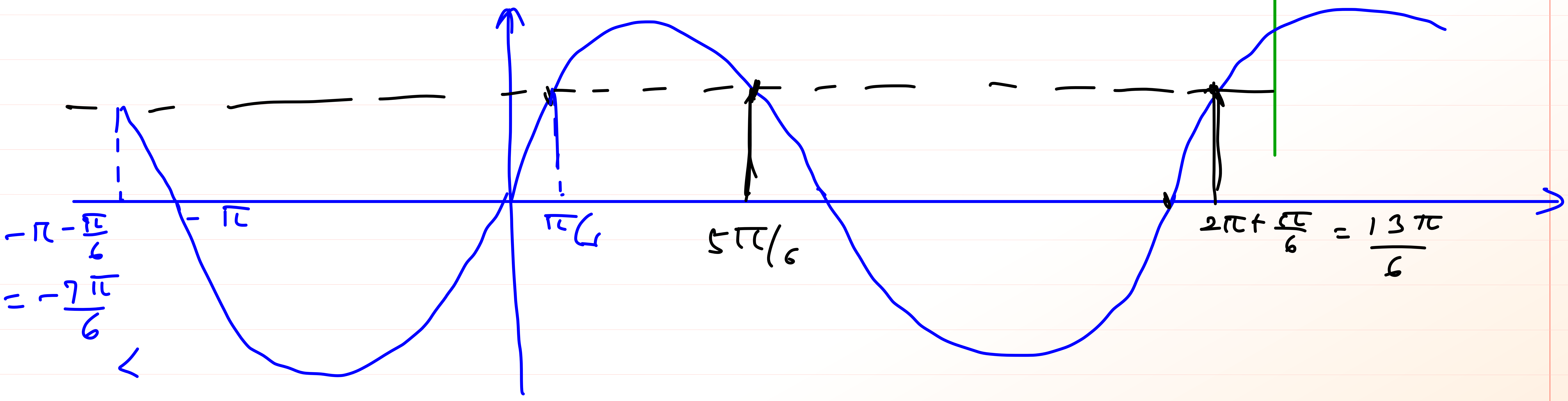
$$\boxed{x \in \left(\frac{\pi}{6}, \frac{5\pi}{6}\right)}$$

$\sin x \leq \frac{1}{2}$

MI

$x \in \left[-\frac{7\pi}{6}, \frac{\pi}{6}\right]$





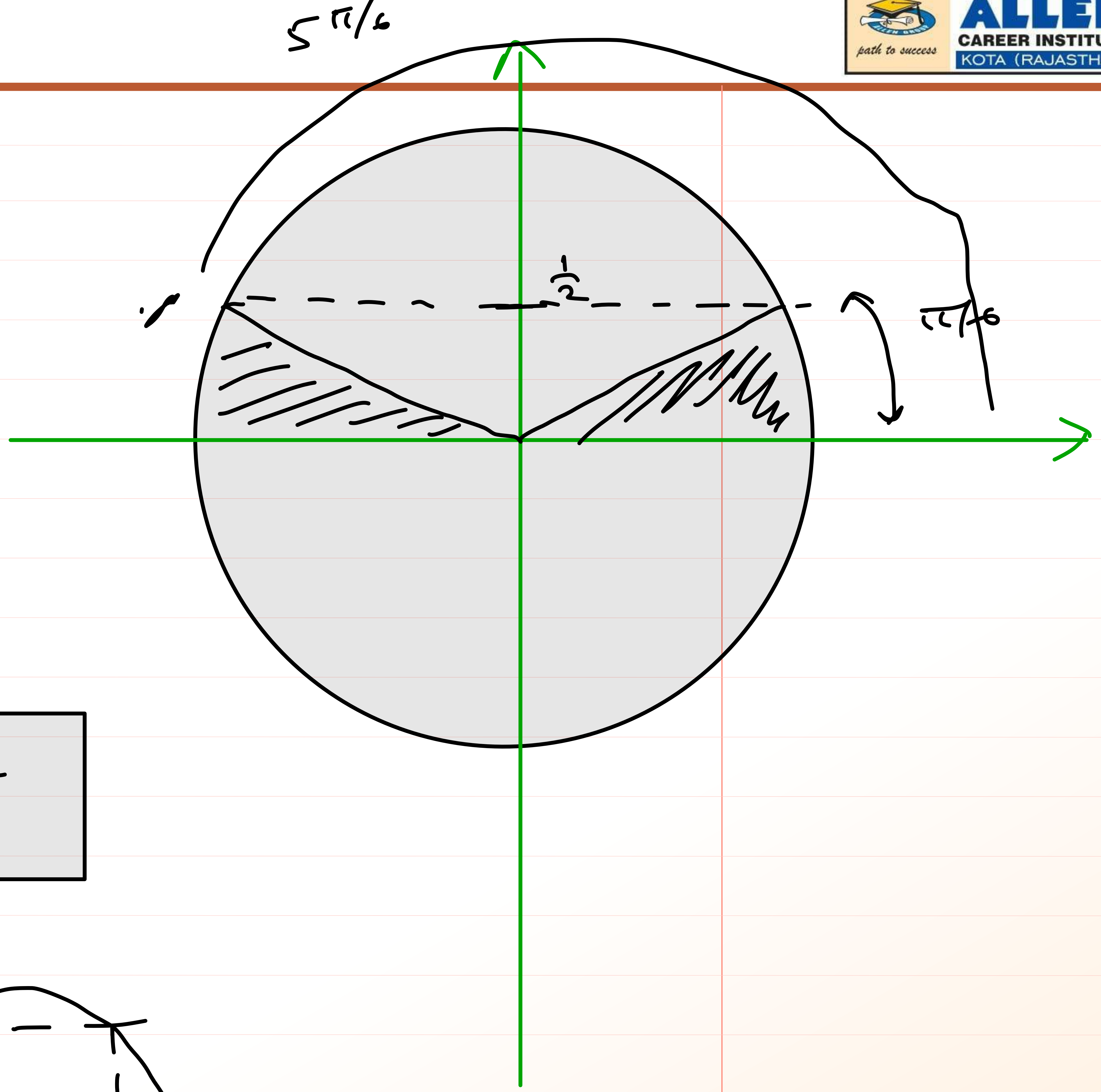
Q $\log_2\left(\sin\frac{x}{2}\right) < -1$

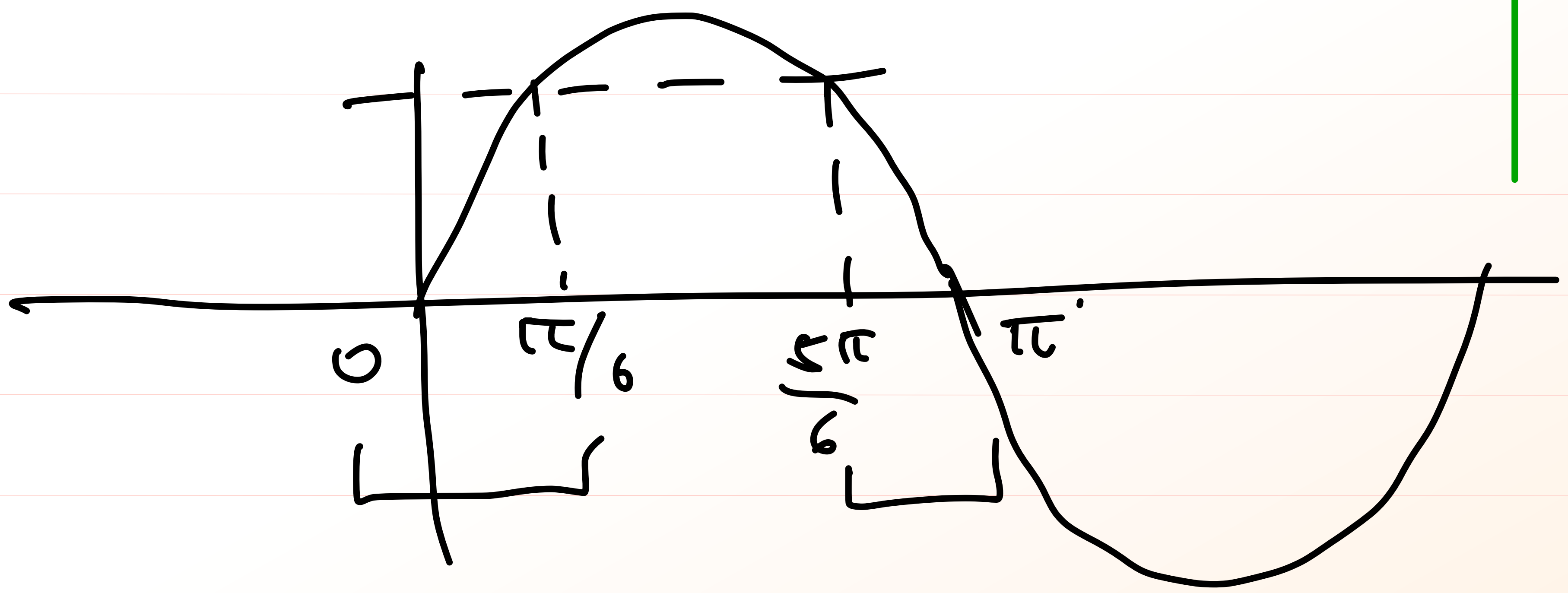
$0 < \sin\frac{x}{2} < 2^{-1}$

$0 < \sin\frac{x}{2} < \frac{1}{2}$

$0 < \frac{x}{2} < \frac{\pi}{6}$ or $\frac{5\pi}{6} < \frac{x}{2} < \pi$

$$\boxed{0 < x < \pi/3 \quad \text{or} \quad \frac{5\pi}{3} < x < 2\pi}$$

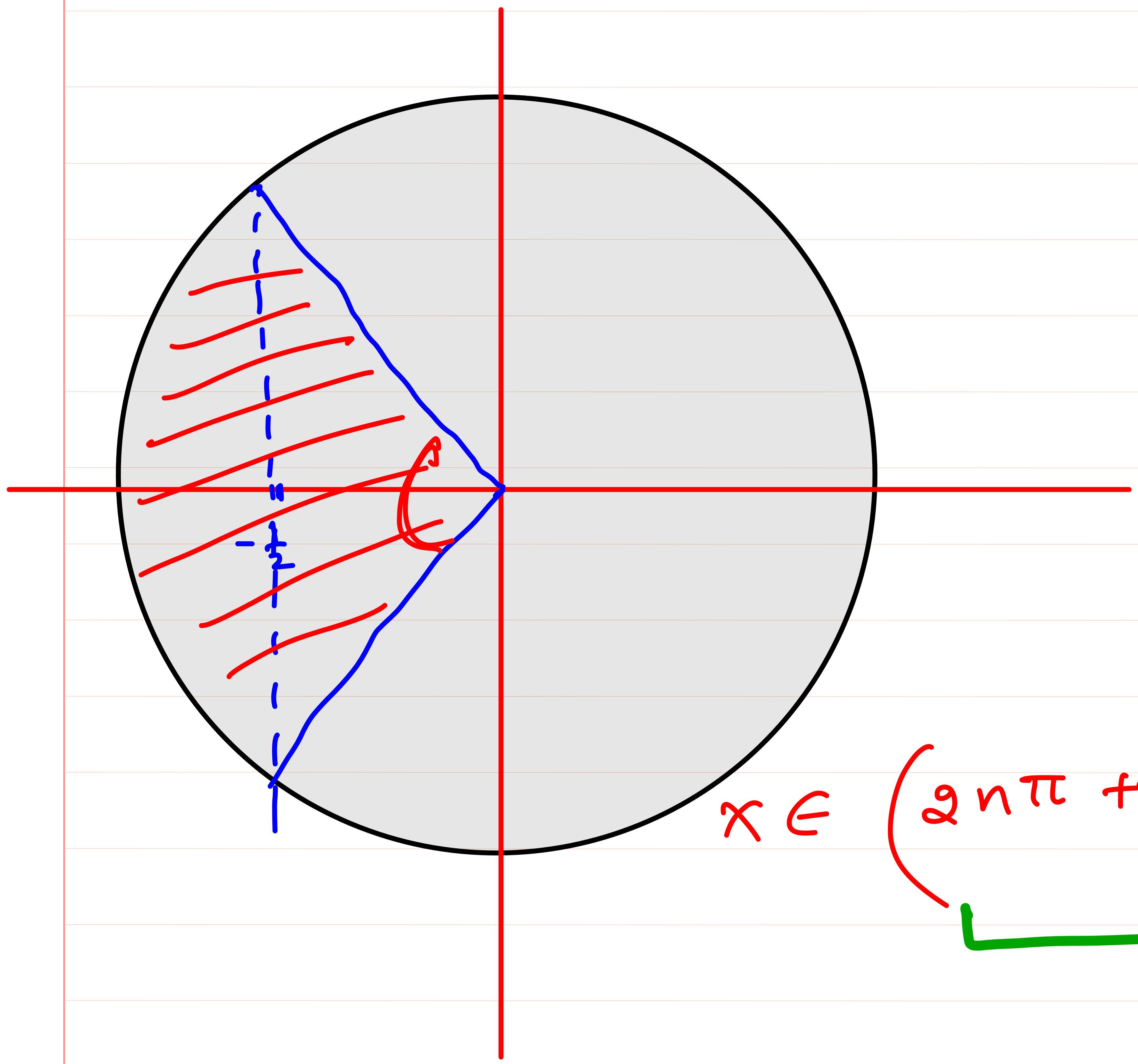




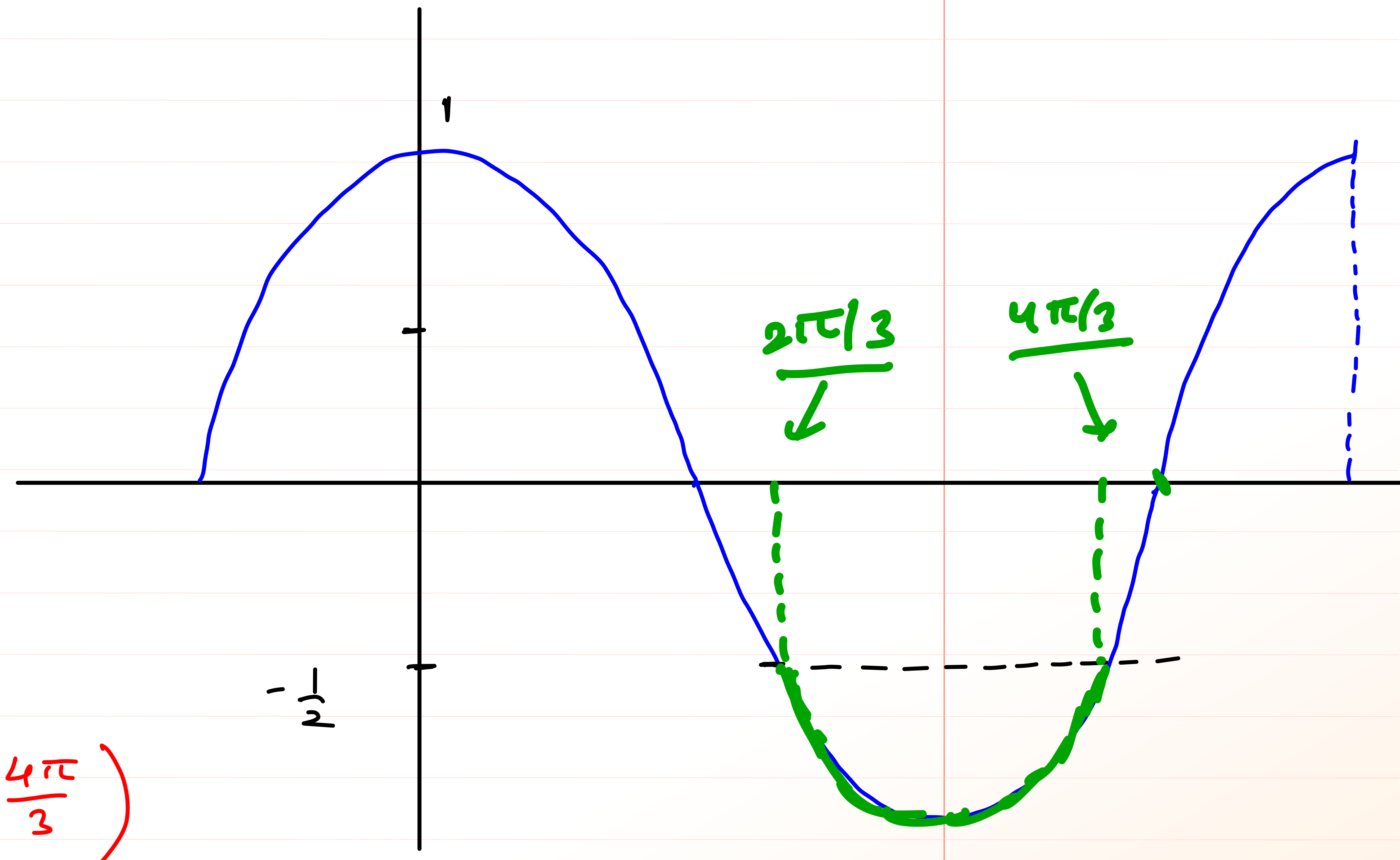
Race 15, 16, 17

Q $\cos x < -\frac{1}{2}$

axis $x < -\frac{1}{2}$



$x \in \underbrace{\left(2n\pi + \frac{2\pi}{3},\ 2n\pi + \frac{4\pi}{3}\right)}$



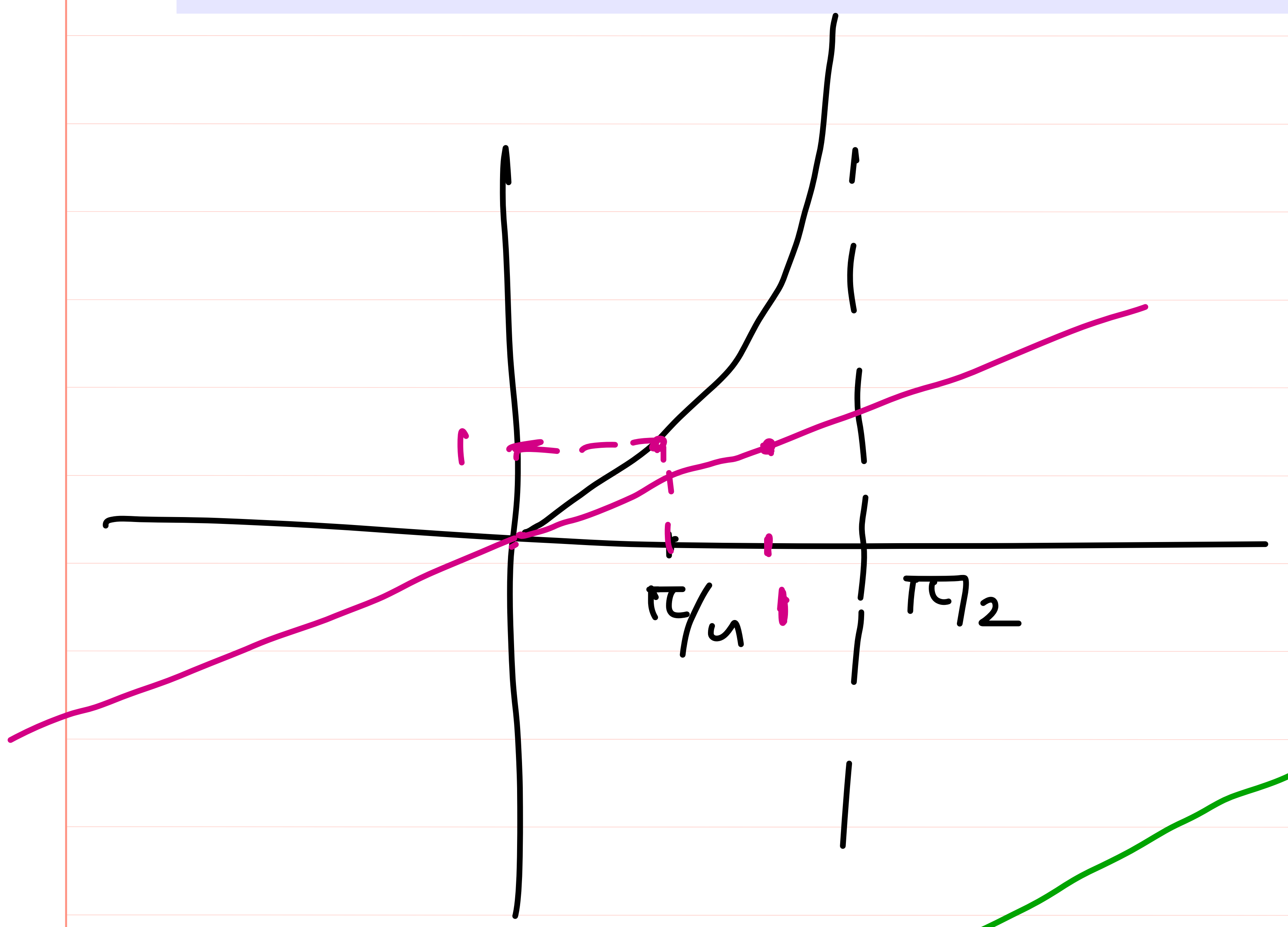
$\cos x = -\frac{1}{2} = -\cos\frac{\pi}{3}$

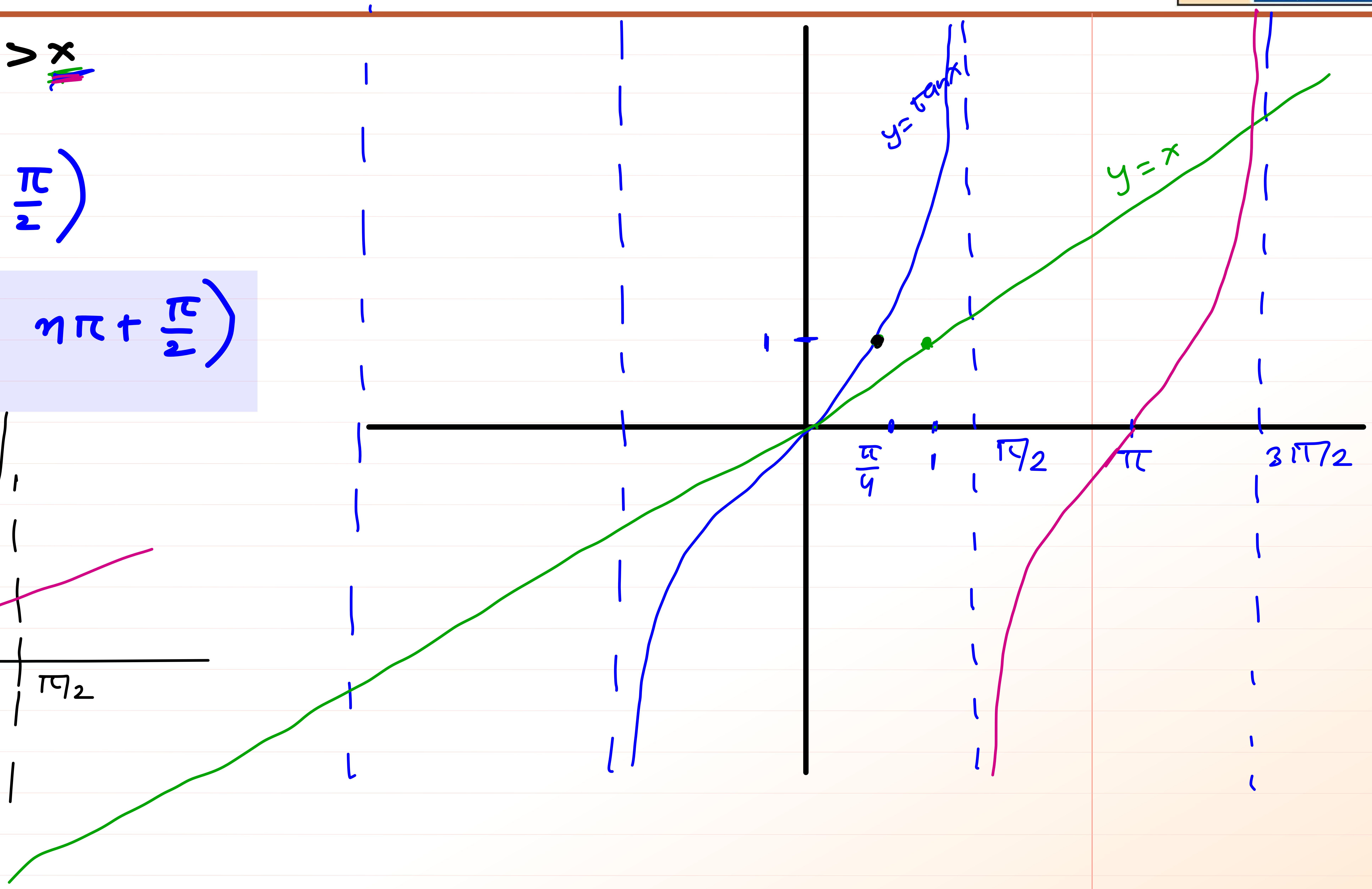
$\underbrace{x = \pi - \frac{\pi}{3}}$

Q $\tan x > x$

$x \in \left(0, \frac{\pi}{2}\right)$

$$\boxed{x \in \left(n\pi, n\pi + \frac{\pi}{2}\right)}$$





Q $\tan^2 x - (\sqrt{3}+1)\tan x + \sqrt{3} < 0$

$(\tan x - \sqrt{3})(\tan x - 1) < 0$

$\tan x \in (1, \sqrt{3})$

$x \in \left(\frac{\pi}{4}, \frac{\pi}{3}\right)$

$$\boxed{x \in \left(n\pi + \frac{\pi}{4}, n\pi + \frac{\pi}{3}\right)}$$

Q

$\sin 3x < \sin x$

$3\sin x - 4\sin^3 x < \sin x$

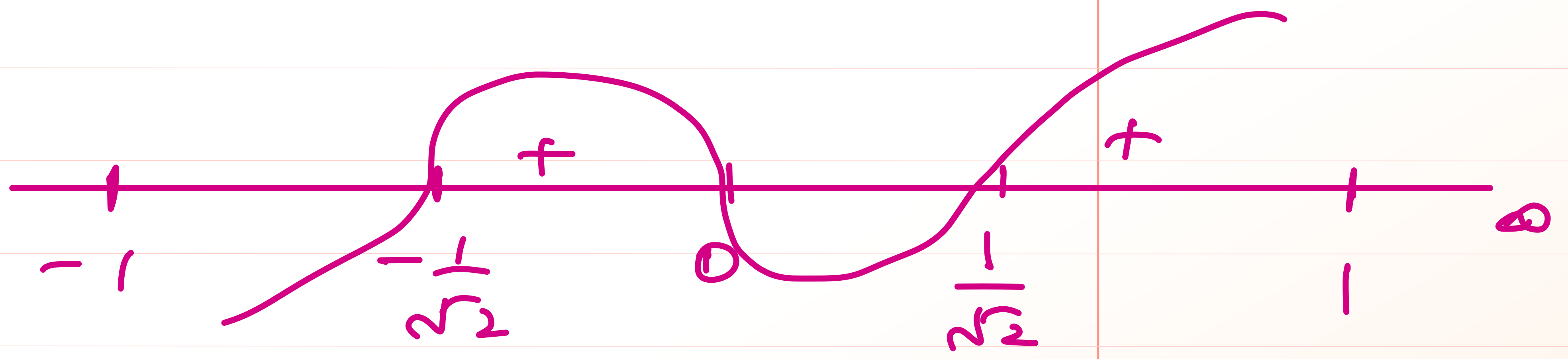
$-4\sin^3 x + 2\sin x < 0$

$2\sin^3 x - \sin x > 0$

$\sin x\,(2\sin^2 x - 1) > 0$

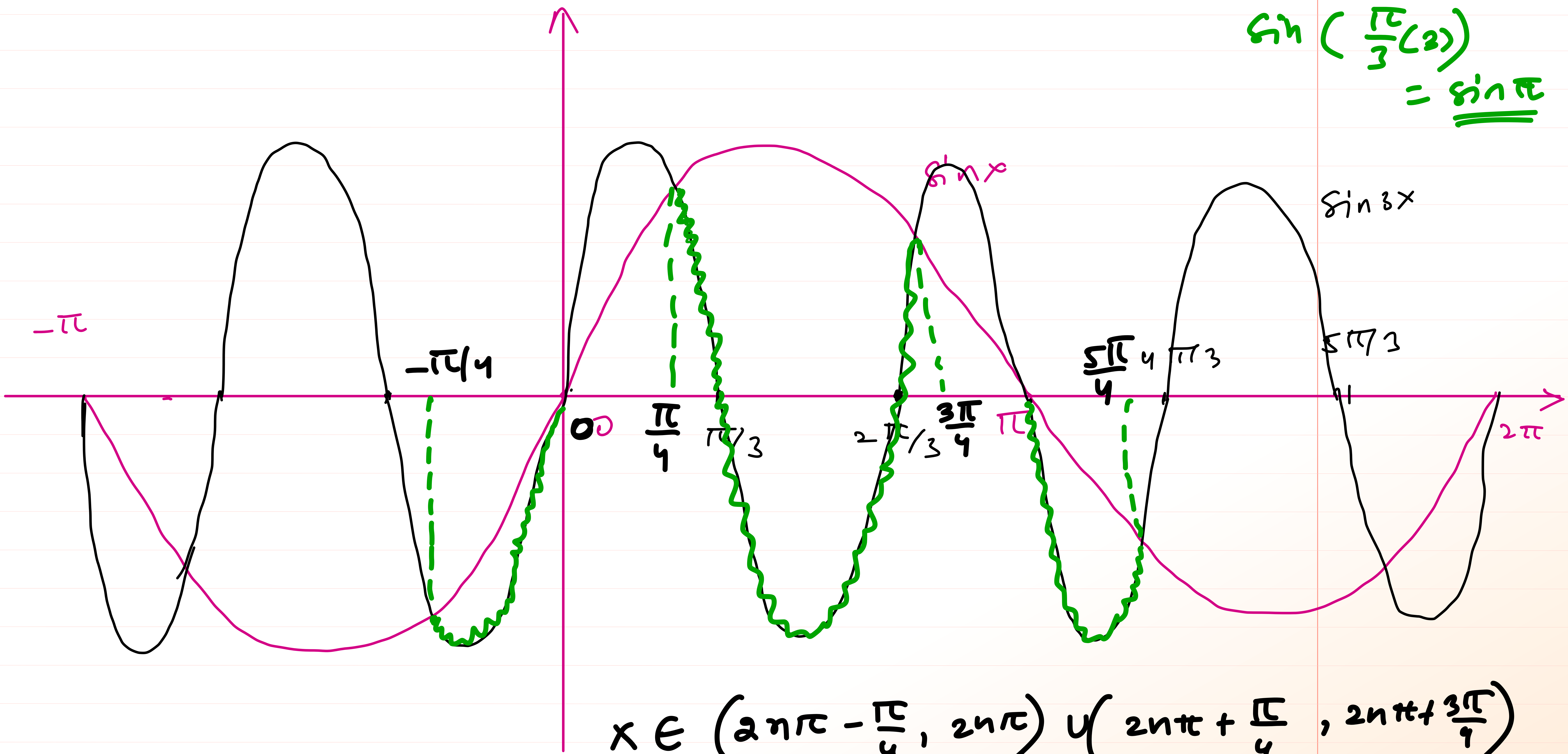
$\underline{\sin x}\,(\underline{\sqrt{2}\sin x + 1})(\underline{\sqrt{2}\sin x - 1}) > 0$

$$\boxed{\sin x \in \left(-\frac{1}{\sqrt{2}}, 0\right) \cup \left(\frac{1}{\sqrt{2}}, 1\right)}$$



$\sin\left(\frac{\pi}{3}(3)\right)$

$= \underline{\sin\pi}$



$x \in \left(2n\pi - \frac{\pi}{4},\ 2n\pi\right) \cup \left(2n\pi + \frac{\pi}{4},\ 2n\pi + \frac{3\pi}{4}\right)$

$\cup \left(2n\pi + \pi,\ 2n\pi + \frac{5\pi}{4}\right)$

Q find the number of solutions of $\sin x = \frac{x}{10}$