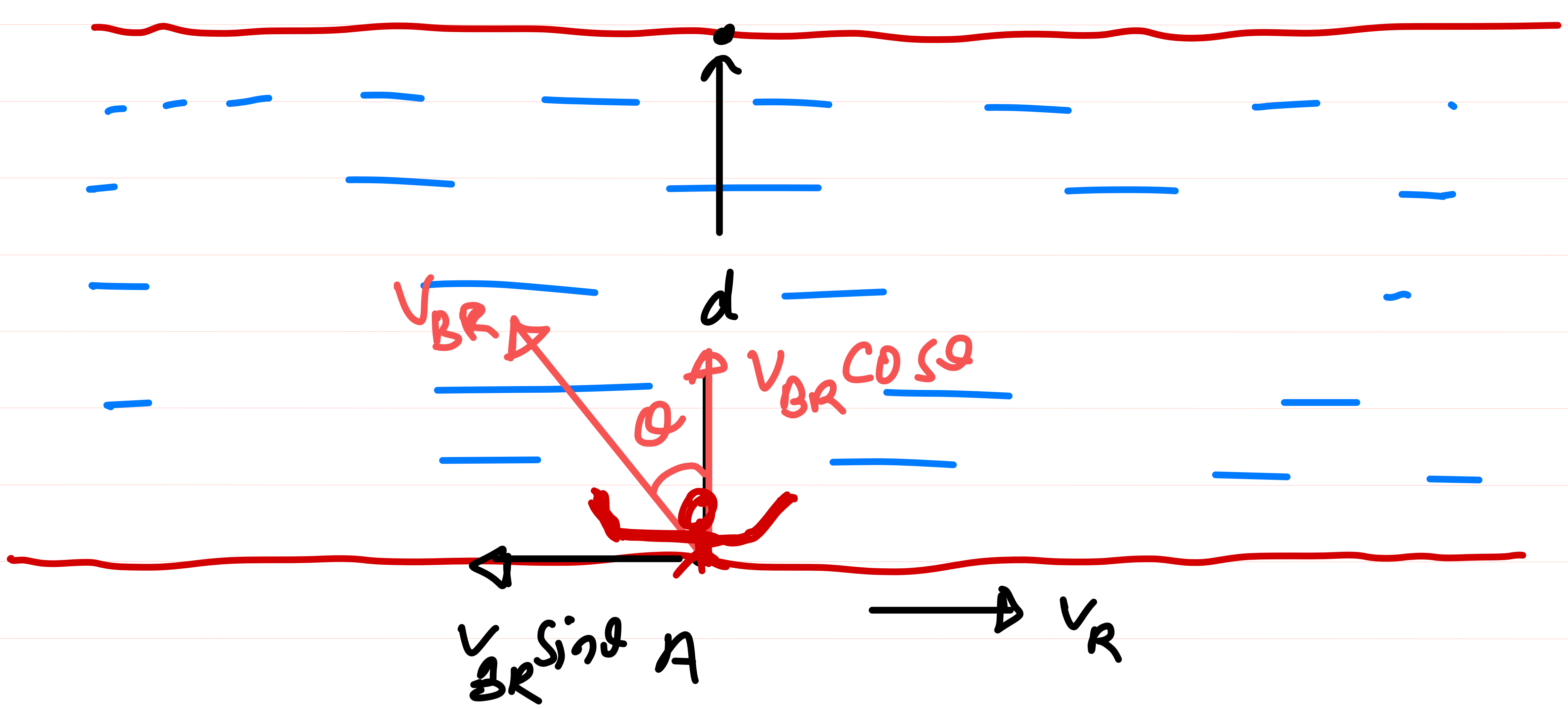
Case-② Motion at angle θ from vertical :→

$\because \vec{V}_B = \vec{V}_{BR} + \vec{V}_R$

$(V_B)_{\perp} = V_{BR}\cos\theta \qquad (V_B)_{\parallel} = V_{BR}\sin\theta - V_R$



Dirⁿ of flow (V_R)

① Time taken to cross River

$$\boxed{t = \frac{d}{V_{BR}\cos\theta}}$$

Case minimum time to cross River

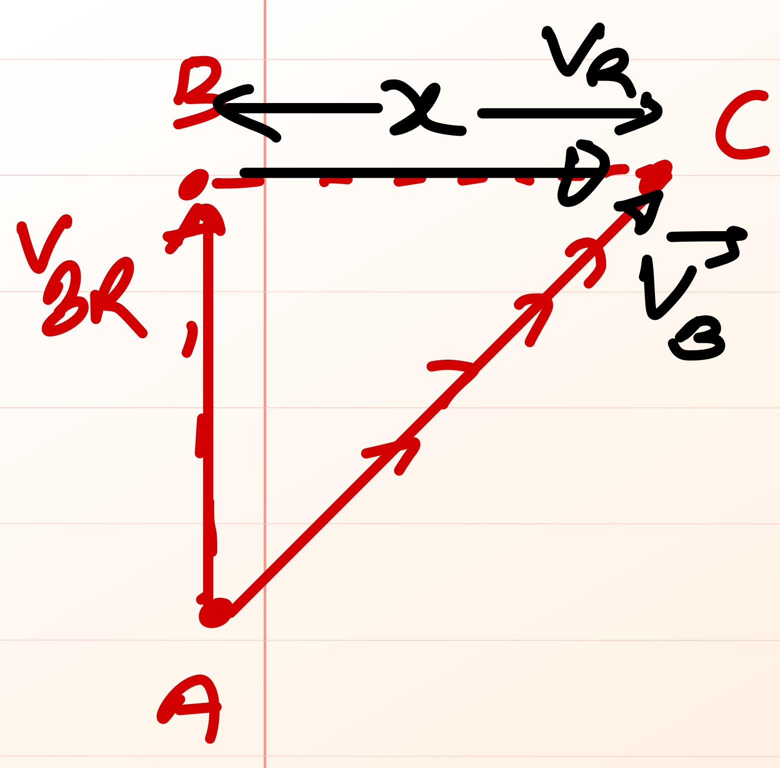
For $t \to \min$

$\cos\theta = 1$

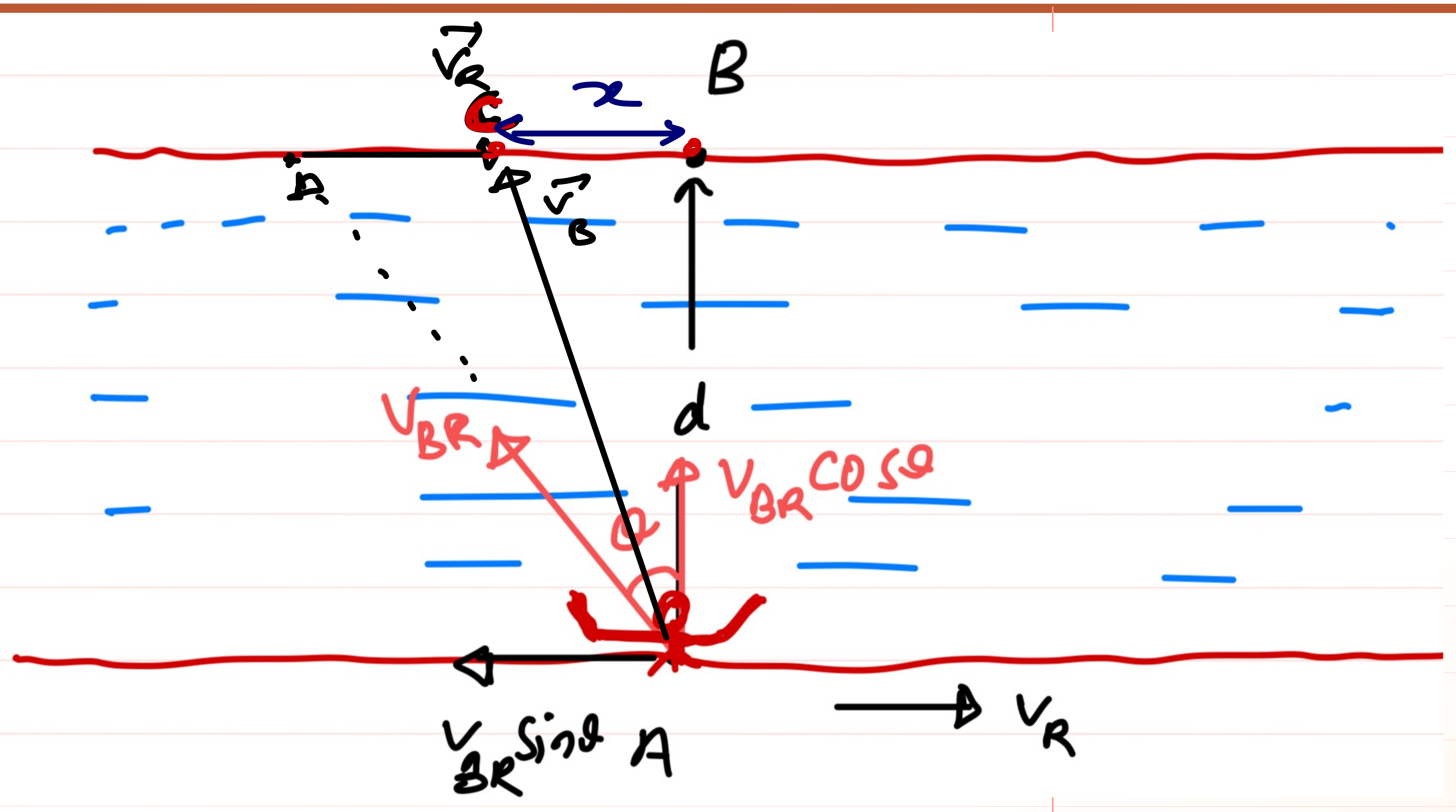
$\boxed{\theta = 0^\circ}$ From vertical

90° From flow

$$\boxed{t_{min} = \frac{d}{V_{BR}}}$$



Drift (x) in minimum time



$$x = (V_B)_{\parallel} \times t_{min}$$

$$(V_B)_{\parallel} = (V_{BR}\sin(0) - V_R)$$

$$= |0 - V_R|$$

$$\boxed{x = V_R \cdot \frac{d}{V_{BR}}}$$

② Drift (x) = distance along River Bank at opposite end

$$x = (V_B)_{\parallel} \cdot t$$

$$\boxed{x = (V_{BR}\sin\theta - V_R) \cdot \frac{d}{V_{BR}\cos\theta}}$$

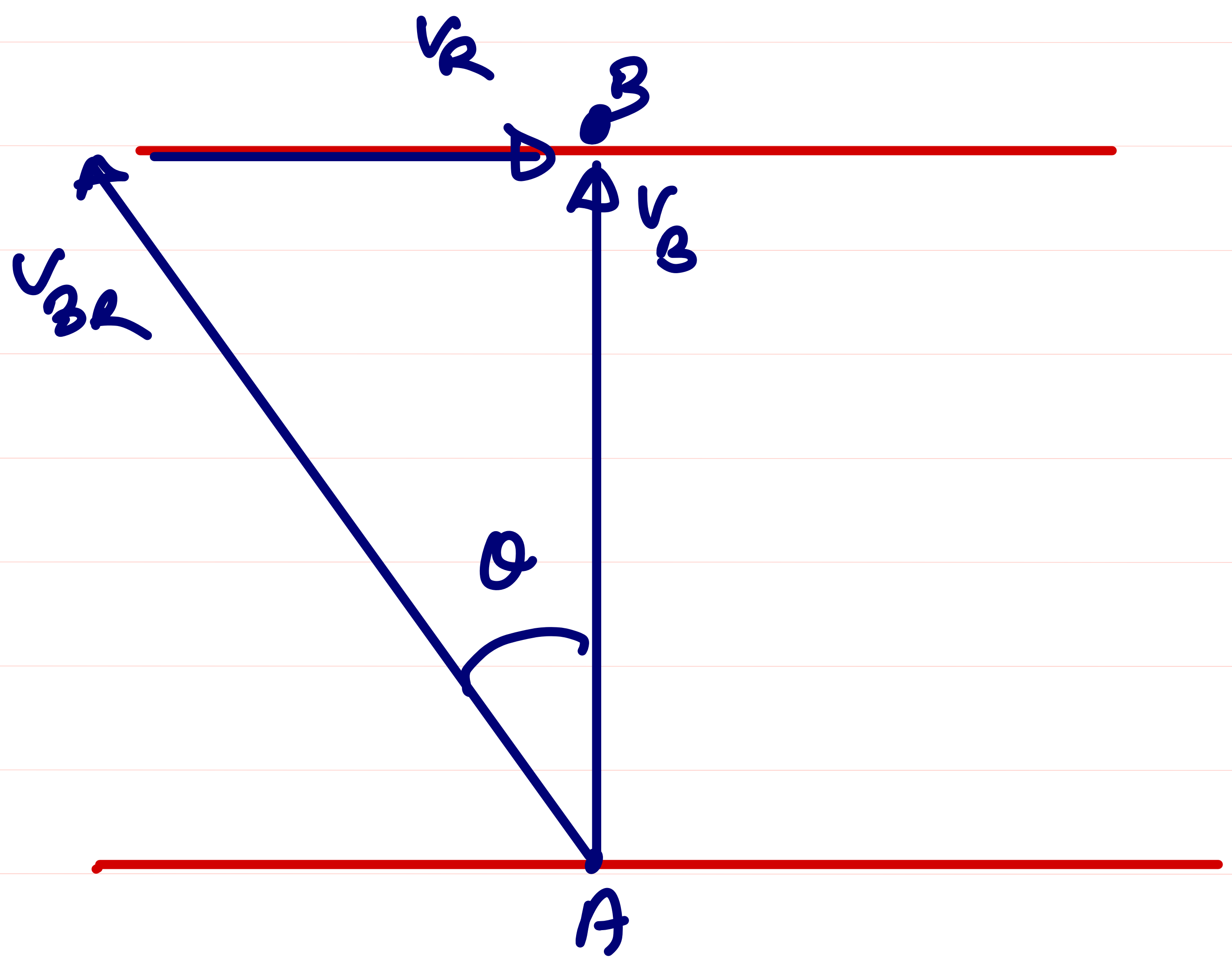
Case ① When $(V_{BR} > V_R)$

For $x \to$ minimum

$$\boxed{x = 0}$$

$$V_{BR}\sin\theta - V_R = 0$$

$$\boxed{\sin\theta = \frac{V_R}{V_{BR}}}$$



time to cross River

$$t = \frac{d}{V_{BR}\cos\theta}$$

$$= \frac{d}{\cancel{V_{BR}}\dfrac{\sqrt{V_{BR}^2 - V_R^2}}{\cancel{V_{BR}}}}$$

$$\boxed{t = \frac{d}{\sqrt{V_{BR}^2 - V_R^2}}}$$

Case ② When $(V_{BR} < V_R)$

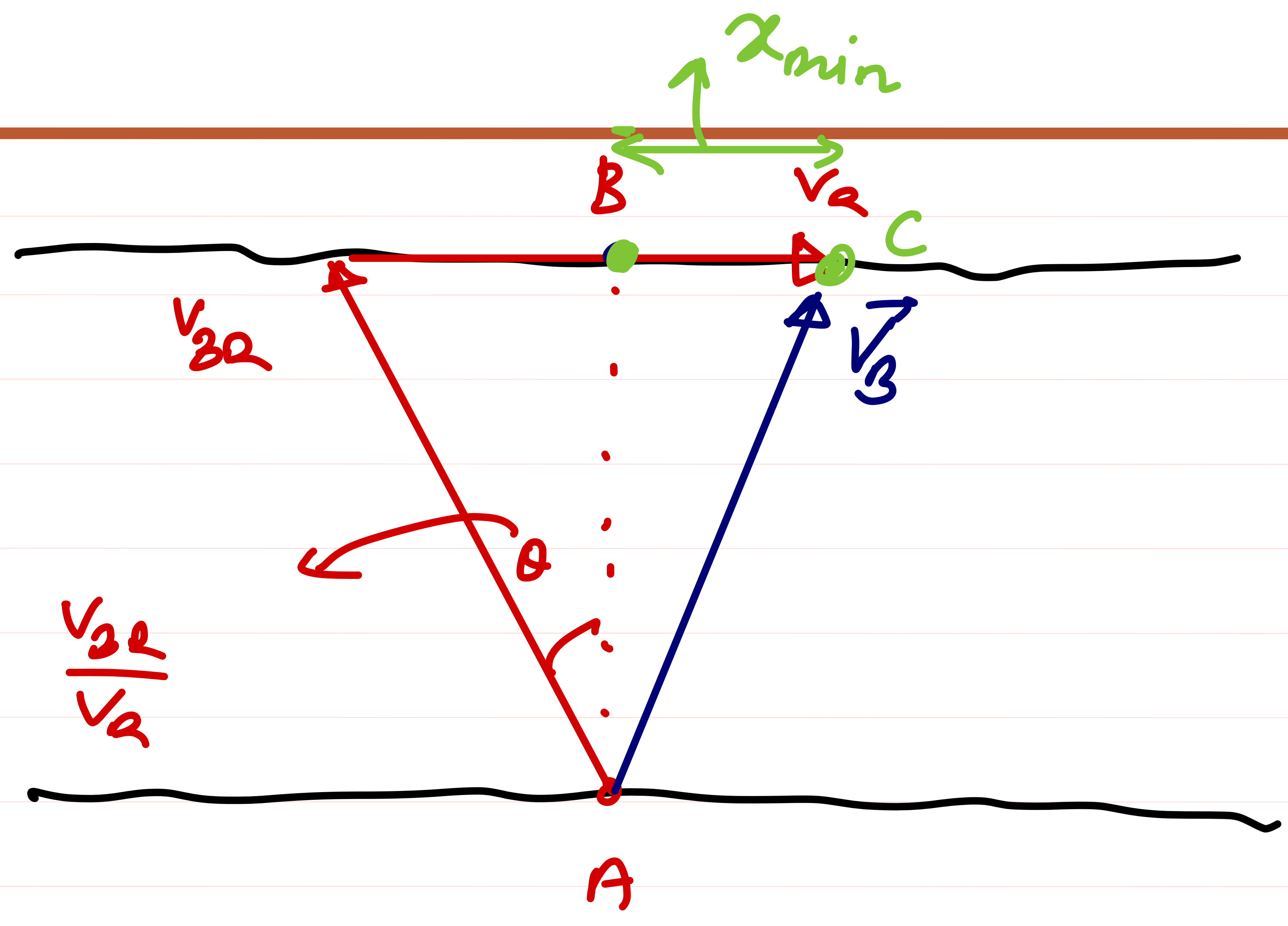
For $x \to$ minimum $\quad \dfrac{dx}{d\theta} = 0$

$$\boxed{\sin\theta = \frac{V_{BR}}{V_R}}$$

$$x_{min} = \left(V_{BR}\cdot\frac{V_{BR}}{V_R} - V_R\right)\frac{d}{V_{BR}\left(\dfrac{\sqrt{V_R^2 - V_{BR}^2}}{V_R}\right)}$$

$$x_{min} = \frac{V_{BR}^2 - V_R^2}{\cancel{V_R}} \times \frac{\cancel{V_R}\, d}{V_{BR}\left(\sqrt{V_R^2 - V_{BR}^2}\right)}$$

$$x_{min} = \left| -\frac{d}{v_{BR}}\sqrt{v_R^2 - v_{BR}^2} \right|$$



$$\sin\theta = \frac{v_{BR}}{v_R}$$

time To Cross

$$t = \frac{d}{v_{BR} \cdot \cos\theta}$$

$$\cos\theta = \frac{\sqrt{v_R^2 - v_{BR}^2}}{v_R}$$

$$\boxed{t = \frac{v_R \, d}{v_{BR}\sqrt{v_R^2 - v_{BR}^2}}}$$

Illustration 3*. A boat can be rowed at 5 m/s on still water. It is used to cross a 200 m wide river from south bank to the north bank. The river current has uniform velocity of 3 m/s due east.

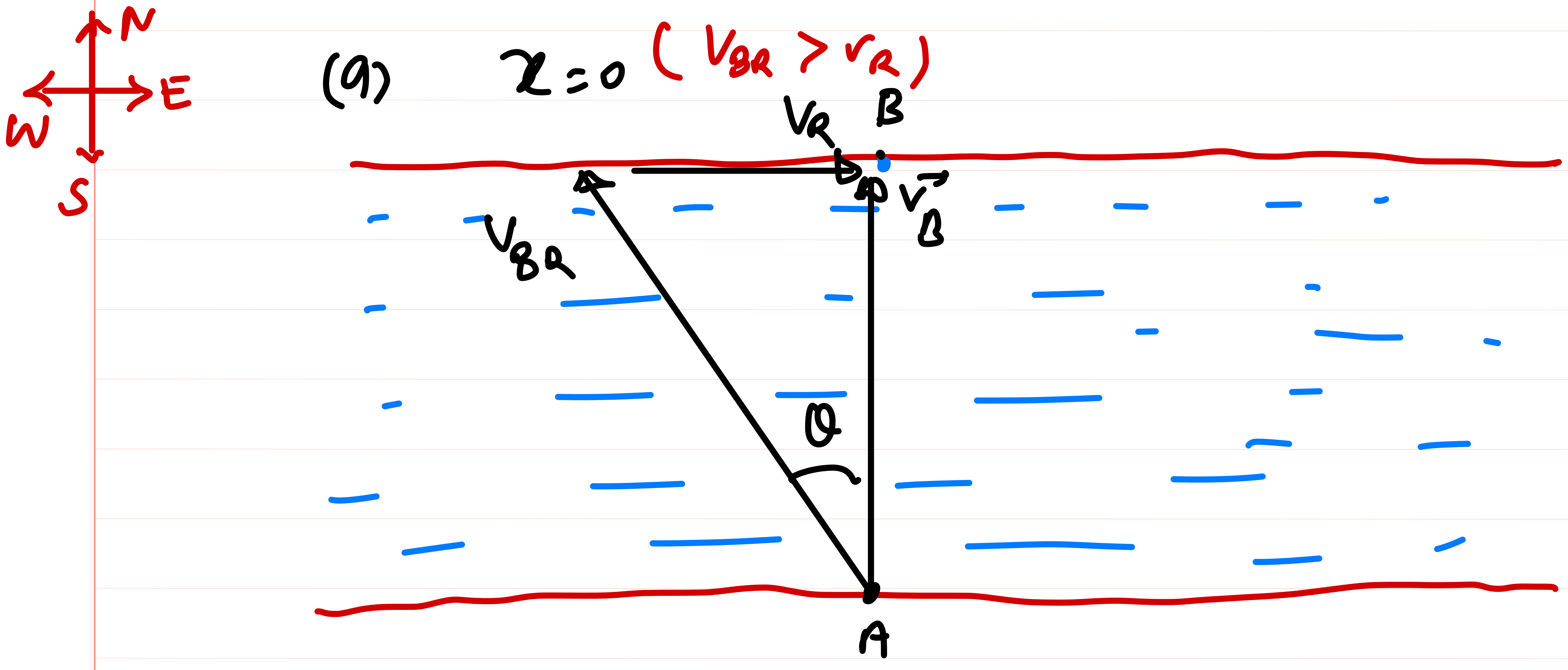
(a) In which direction must it be steered to cross the river perpendicular to current?

(b) How long will it take to cross the river in a direction perpendicular to the river flow?

(c) In which direction must the boat be steered to cross the river in minimum time? How far will it drift?

Given: $V_{BR} = 5\, m/s$, $d = 200\, m$, $V_R = 3\, m/s$

(a) $x = 0$ ($V_{BR} > V_R$)



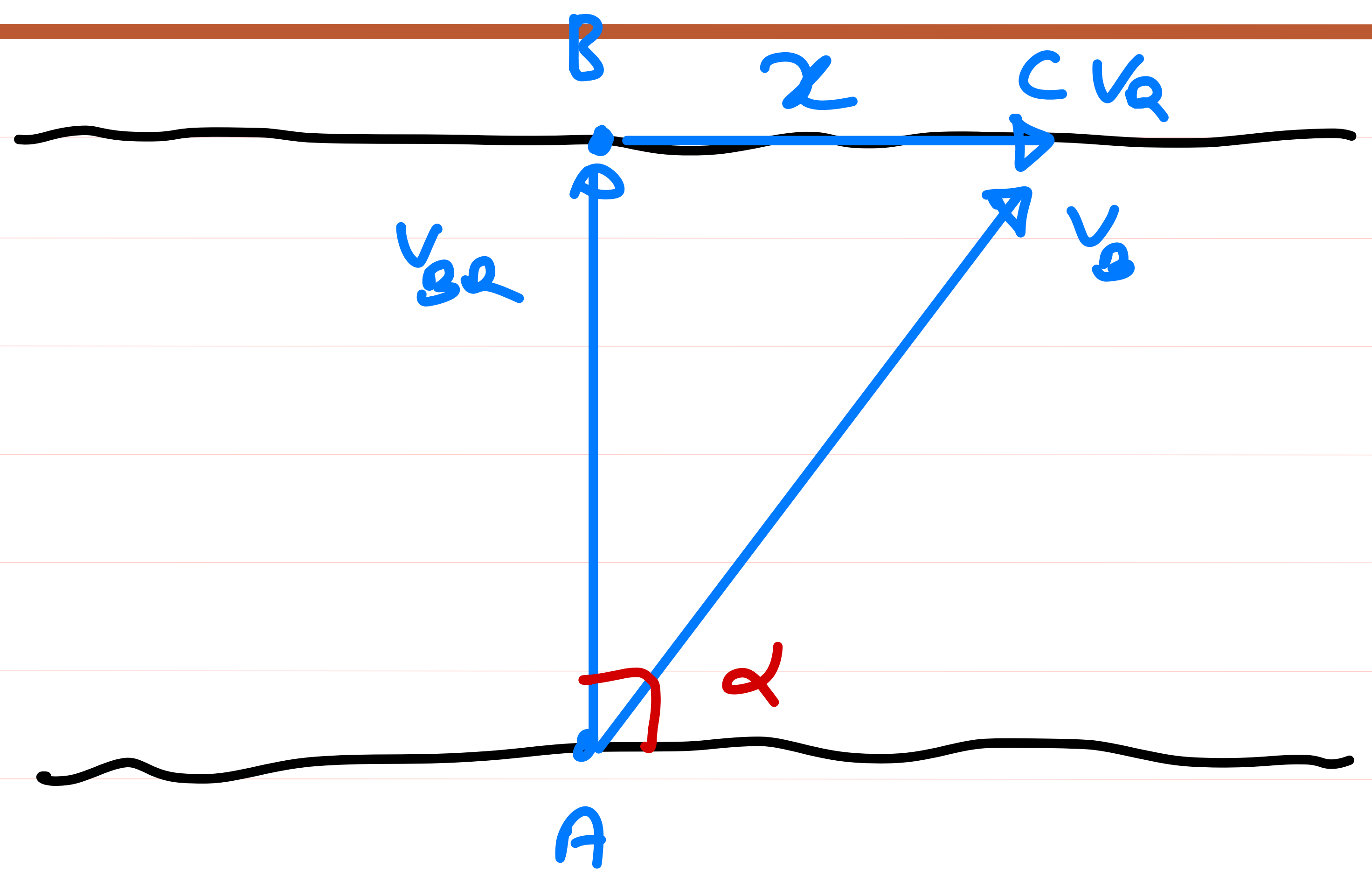
$$\sin\theta = \frac{V_R}{V_{BR}} = \frac{3}{5} \Rightarrow \theta = 37^\circ$$

$$\boxed{\theta = 37^\circ \text{ W of N}}$$

(b) $$t = \frac{d}{V_{BR}\cos\theta} = \frac{200}{5 \times \cos 37} = \frac{200}{5 \times \frac{4}{5}}$$

$$\boxed{t = 50\, sec}$$ Ans

(C)



$\theta = 0°$ from North

$\alpha = 90°$ from East

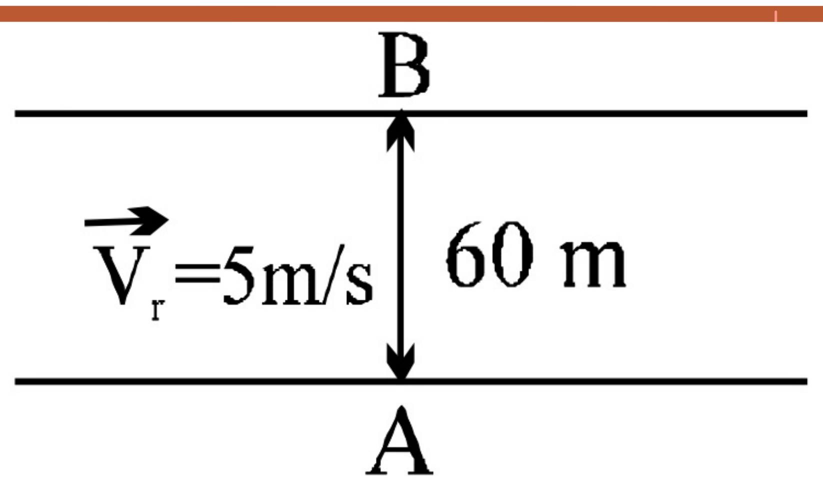
$$\boxed{t_{min} = \frac{d}{V_{BR}} = \frac{200}{5} = 40 \text{ sec}}$$ Ans

$x = V_R \cdot t_{min}$

$= 3 \times 40$

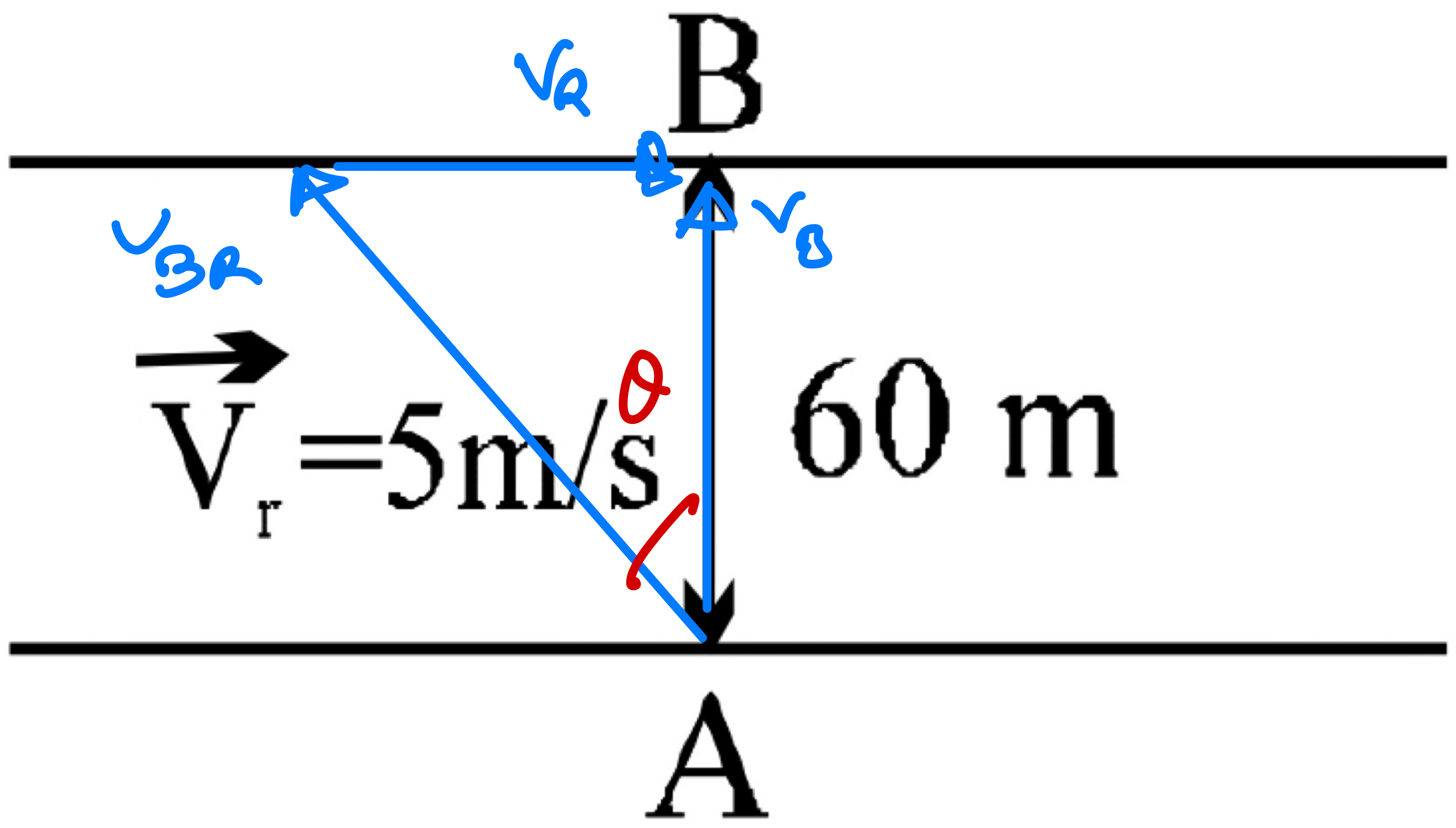
$$\boxed{x = 120 \text{ m}}$$ Ans

2. A man is crossing a river flowing with velocity of 5 m/s. He reaches a point directly across at a distance of 60 m in 5 sec. His velocity in still water should be



(A) 12 m/s (B) 13 m/s
(C) 5 m/s (D) 10 m/s

Given $V_R = 5$
$d = 60$
$t = 5$



$$V_{BR} = \sqrt{V_R^2 + V_B^2}$$

$$V_R = 5$$

$$V_B = \frac{d}{t} = \frac{60}{5} = 12$$

$$V_{BR} = \sqrt{5^2 + 12^2}$$

$$\boxed{V_{BR} = 13 \text{ m/s}}$$ Ans

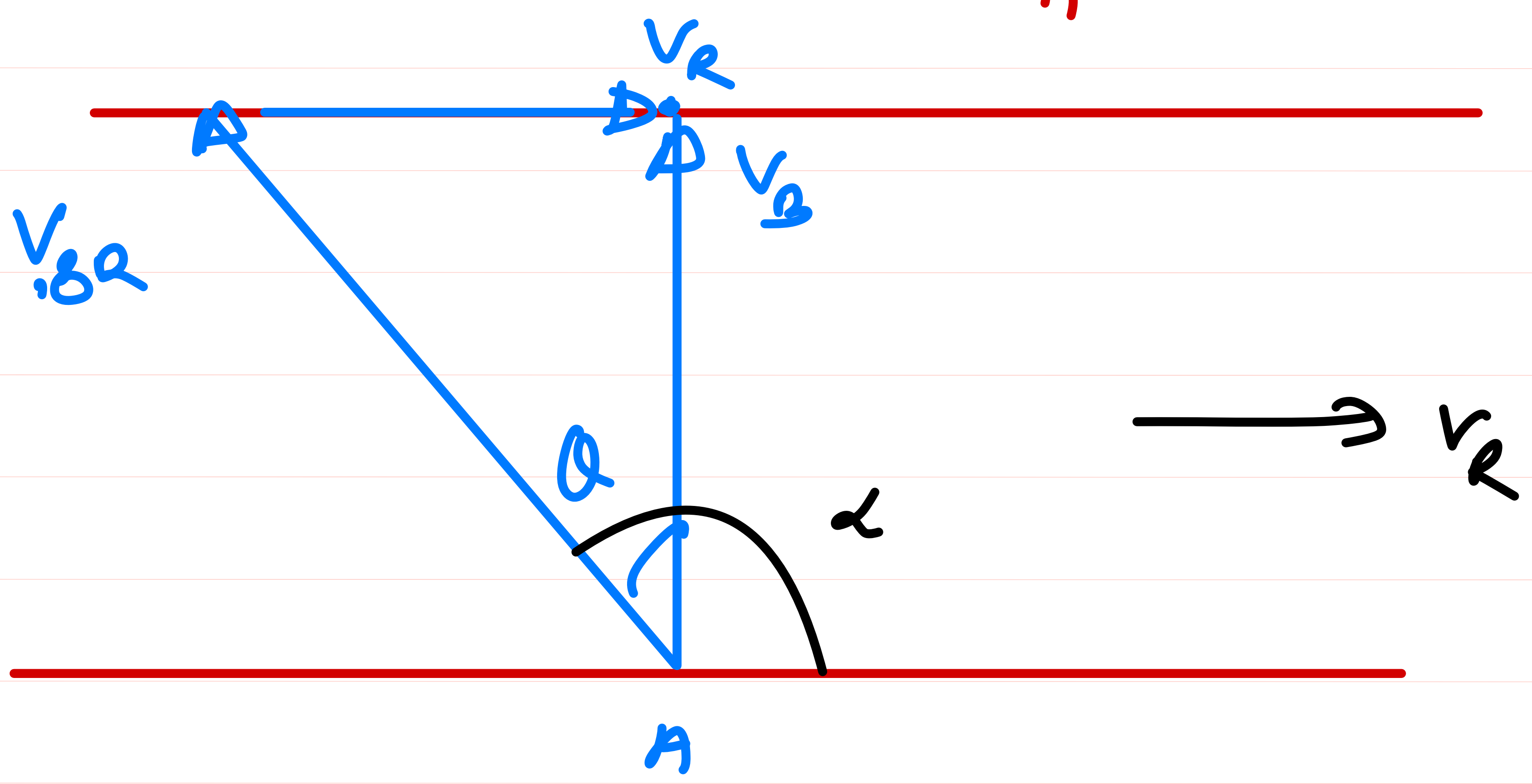
$$\sin\theta = \frac{V_R}{V_{BR}} = \frac{5}{13}$$

5. A man can swim in still water with a speed of 2 m/s. If he wants to cross a river of water current speed $\sqrt{3}$ m/s along shortest possible path, then in which direction should he swim?

(A) at an angle 120° to the water current
(B) at an angle 150° to the water current
(C) at an angle 90° to the water current
(D) none of these

$V_{BR} = 2$ m/s $V_R = \sqrt{3}$ m/s

$V_{BR} > V_R$ $x = 0$ (min)



$\sin\theta = \frac{V_R}{V_{BR}} = \frac{\sqrt{3}}{2}$

$\theta = 60°$

$\alpha = 90 + \theta$
$= 150°$ from flow

WIND AIRPLANE PROBLEMS

SL AL

This is very similar to boat river flow problems the only difference is that boat is replaced by a plane and river is replaced by wind.

Thus, velocity of aeroplane with respect to wind

$$\boxed{\vec{v}_{aw} = \vec{v}_a - \vec{v}_w \quad \text{or} \quad \vec{v}_a = \vec{v}_{aw} + \vec{v}_w}$$

where, $\vec{v}_a$ = velocity of aeroplane with respect to ground

and, $\vec{v}_w$ = velocity of wind with respect to ground

In general resultant velocity for object moving in any medium

$$\boxed{\underbrace{\vec{v}_{\text{object/ground}}}_{\text{Resultant velocity}} = \underbrace{\vec{v}_{\text{object/medium}}}_{\text{Velocity of object relative to medium}} + \underbrace{\vec{v}_{\text{medium/ground}}}_{\text{Velocity of medium}}}$$

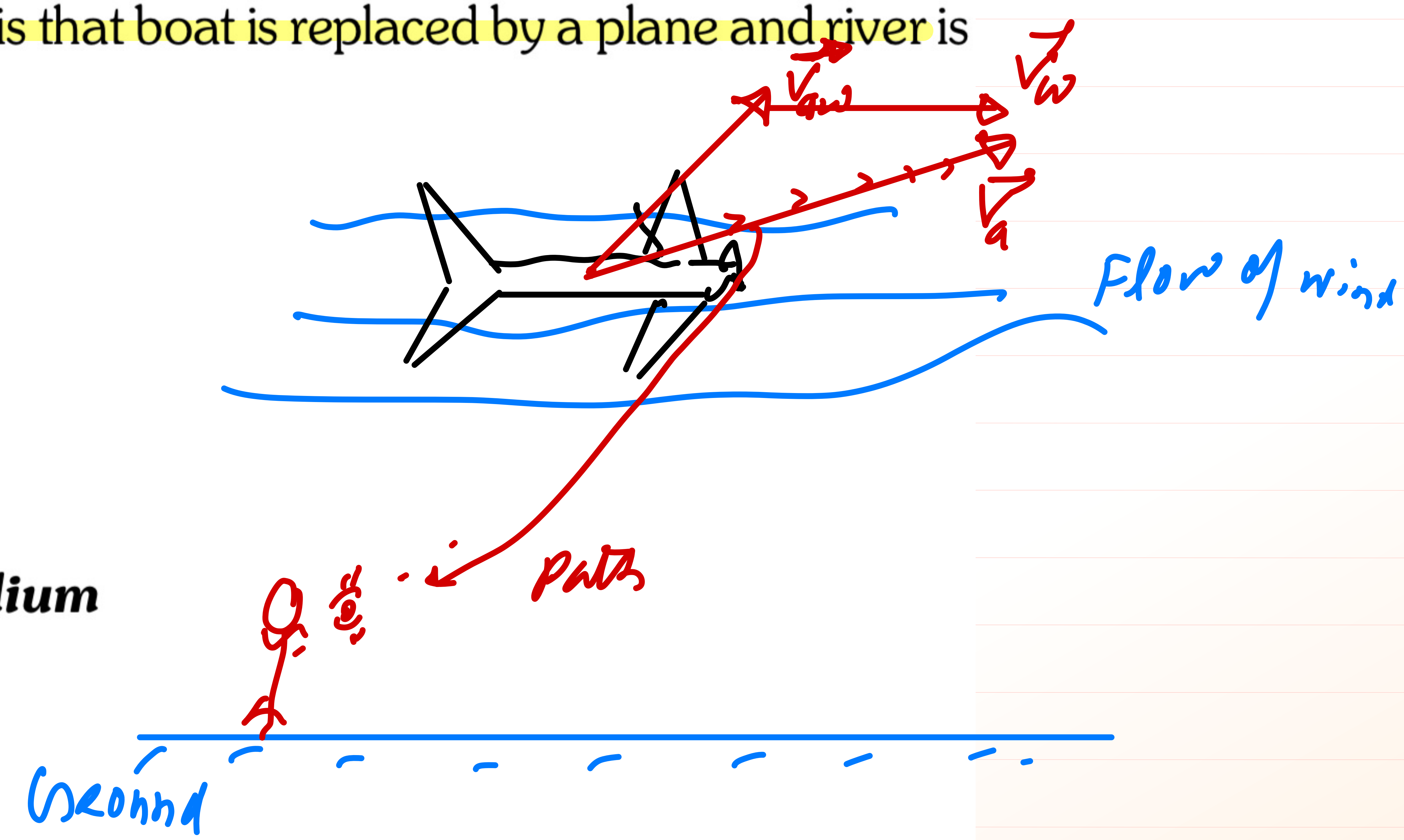
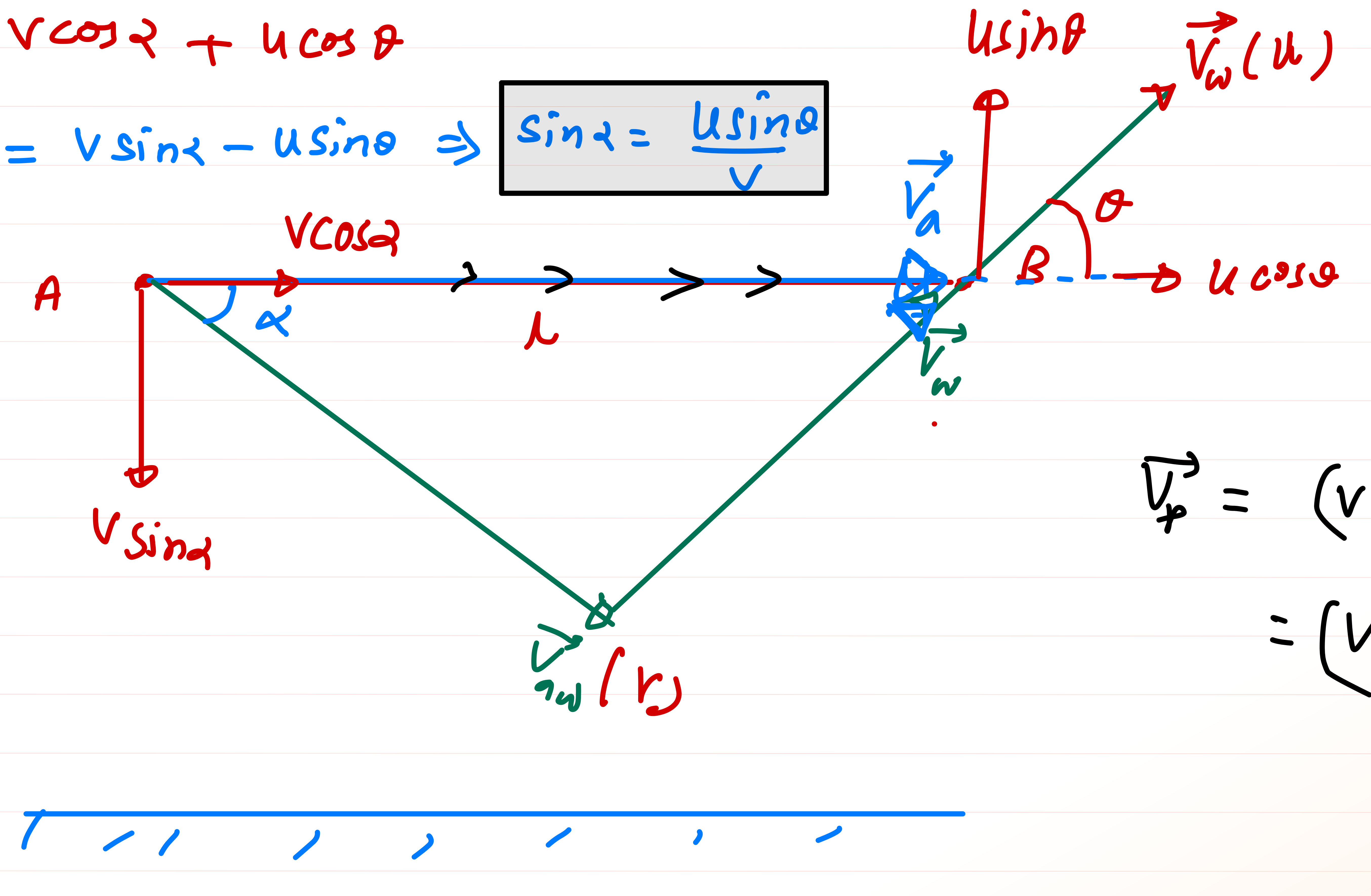


Illustration 5*. An aeroplane flies along a straight path A to B and returns back again. The distance between A and B is ℓ and the aeroplane maintains the constant speed v. There is a steady wind with a speed u at an angle θ with line AB. Determine the expression for the total time of the trip.

$(V_p)_{\parallel} = v\cos\alpha + u\cos\theta$

$(V_p)_{\perp} = 0 = v\sin\alpha - u\sin\theta \Rightarrow \boxed{\sin\alpha = \frac{u\sin\theta}{v}}$

$$\boxed{t_{AB} = \frac{\ell}{v\cos\alpha + u\cos\theta}}$$



$\vec{V}_p = (v\cos\alpha\,\hat{i} - v\sin\alpha\,\hat{j}) + (u\cos\theta)\hat{i} + u\sin\theta\,\hat{j}$

$= \underbrace{(v\cos\alpha + u\cos\theta)}_{(V_p)_{\parallel}}\hat{i} + \underbrace{(-v\sin\alpha + u\sin\theta)}_{(V_p)_{\perp}}\hat{j}$