1. If t_n denotes the nth term of the series $2 + 3 + 6 + 11 + 18 + \ldots$ then t_{50} is

(A) $49^2 - 1$ (B) 49^2 (C) $50^2 + 1$ (D) $49^2 + 2$

Sol^n:

$$S = 2 + 3 + 6 + 11 + 18 + \ldots + t_{n-1} + t_n$$

$$S = 2 + 3 + 6 + 11 + \ldots + t_{n-1} + t_n$$

$$0 = 2 + 1 + 3 + 5 + 7 + \ldots + (t_n - t_{n-1}) - t_n$$

$$t_n = 2 + \underbrace{1 + 3 + 5 + 7 + \ldots + (t_n - t_{n-1})}_{(n-1)\text{ terms}}$$

$$t_n = 2 + \frac{(n-1)}{\cancel{2}}\left(2 \times 1 + (n-2) \times \cancel{2}\right)$$

$$t_n = 2 + (n-1)^2$$

$$t_{50} = (49)^2 + 2$$

2. Let $x = 1 + 3a + 6a^2 + 10a^3 + \ldots\ldots\ldots\ldots\ |a| < 1,\ \ y = 1 + 4b + 10b^2 + 20b^3 + \ldots\ldots\ldots\ldots\ |b| < 1.$
Then $S = 1 + 3\,(ab) + 5\,(ab)^2 + \ldots\ldots\ldots$in terms of x and y is

(A) $\dfrac{1+(1-x^{-1/3})(1-y^{-1/4})}{\{1-(1-x^{-1/3})(1-y^{-1/4})\}^2}$ ✓

(B) $\dfrac{1+(1+x^{-1/3})(1+y^{-1/4})}{\{1-(1+x^{-1/3})(1+y^{-1/4})\}^2}$

(C) $\dfrac{1+(1-x^{-1/3})(1-y^{-1/4})}{\{1+(1-x^{-1/3})(1-y^{-1/4})\}^2}$

(D) None of these

Solⁿ:

$$x = 1 + 3a + 6a^2 + 10a^3 + \ldots \infty$$
$$-ax = \quad -a - 3a^2 - 6a^3 - \ldots$$

$$(1-a)x = \underbrace{1 + 2a + 3a^2 + 4a^3 + \ldots \infty}_{S'}$$

$$S' = 1 + 2a + 3a^2 + 4a^3 + \ldots \infty$$
$$-aS' = \quad -a - 2a^2 - 3a^3 - \ldots \infty$$

$$(1-a)S' = 1 + a + a^2 + \ldots$$
$$S' = \frac{1}{(1-a)^2}$$

$$(1-a)x = \frac{1}{(1-a)^2} \Rightarrow \boxed{x = \frac{1}{(1-a)^3}}$$

$$y = 1 + 4b + 10b^2 + 20b^3 + \ldots \infty$$
$$by = \quad b + 4b^2 + 10b^3 + \ldots$$

$$(1-b)y = \underbrace{1 + 3b + 6b^2 + 10b^3}_{S'} + \ldots$$

$$S' = \frac{1}{(1-b)^3}$$

$$y = \frac{1}{(1-b)^4} \qquad x = \frac{1}{(1-a)^3}$$

$$S = 1 + 3(ab) + 5(ab)^2 + \cdots \infty$$

$$abS = \quad ab + 3(ab)^2 + \cdots \infty$$

$$(1-ab)S = 1 + 2ab + 2(ab)^2 + \cdots \infty$$

$$(1-ab)S = 1 + \frac{2ab}{(1-ab)}$$

$$S = \frac{1}{(1-ab)} + \frac{2ab}{(1-ab)^2}$$

$$S = \frac{1+ab}{(1-ab)^2}$$

$$S = \frac{1 + (1-x^{-3})(1-y^{-4})}{\left(1 - (1-x^{-3})(1-y^{-4})\right)^2}$$

3. If $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+$ ....upto $\infty = \frac{\pi^2}{6}$ then, $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+....\infty =$

(A) $\frac{\pi^2}{6}$ (B) $\frac{\pi^2}{8}$ (C) $\frac{\pi^2}{4}$ (D) π^2

$$\Rightarrow \quad \frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+--- = S$$

$$\frac{\pi^2}{6} = \left(\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+--\right)+\left(\frac{1}{2^2}+\frac{1}{4^2}+\frac{1}{6^2}+----\right)$$

$$\frac{\pi^2}{6} = S + \frac{1}{4}\left(\underbrace{\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+---}_{\pi^2/6}\right)$$

$$\frac{\pi^2}{6}-\frac{\pi^2}{24} = S \quad \Rightarrow \quad S = \frac{3\pi^2}{24} = \frac{\pi^2}{8}$$

4. The sum of infinite terms of the series $\frac{1}{1+1^2+1^4}+\frac{2}{1+2^2+2^4}+\frac{3}{1+3^2+3^4}+...\infty$ is :

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) 1 (D) $\frac{1}{4}$

Sol^n:

$$T_r = \frac{r}{1+r^2+r^4}$$

$$S = \sum_{r=1}^{n} T_r \Rightarrow S = \sum_{r=1}^{n} \frac{r}{r^4+r^2+1}$$

$$S = \frac{1}{2}\sum_{r=1}^{n} \frac{(r^2+r+1)-(r^2-r+1)}{(r^2-r+1)(r^2+r+1)}$$

$$S = \frac{1}{2} \sum_{r=1}^{n} \frac{1}{(r^2-r+1)} - \frac{1}{(r^2+r+1)}$$

$$S = \frac{1}{2} \left[\frac{1}{1} - \cancel{\frac{1}{3}} + \cancel{\frac{1}{3}} - \cancel{\frac{1}{7}} + \cancel{\frac{1}{7}} - \cancel{\frac{1}{13}} \; \cdots \; + \cancel{\frac{1}{(n^2-n+1)}} - \frac{1}{(n^2+n+1)} \right]$$

$$S = \frac{1}{2} \left[1 - \frac{1}{(n^2+n+1)} \right]$$

$$S_\infty = \frac{1}{2} \left[1 - \cancel{\frac{1}{(n^2+n+1)}}^{\,\to 0} \right] \Rightarrow \frac{1}{2}.$$

5. The sum of the series $1.3^2 + 2.\ 5^2 + 3.7^2 + \dots$ upto 20 terms is

~~(A)~~ 188090 (B) 180890 (C) 189820 (D) None of these

Soln:

$$T_n = n \times (2n+1)^2$$

$$T_n = (4n^3 + 4n^2 + n)$$

$$S = \sum_{n=1}^{20} (4n^3 + 4n^2 + n)$$

$$S = 4\left(\frac{n(n+1)}{2}\right)^2 + \frac{4 \cdot n \cdot (n+1)(2n+1)}{6} + \frac{n(n+1)}{2}$$

put $n = 20$

$$S = 4\left(\frac{\cancel{20}^{10} \times 21}{\cancel{2}}\right)^2 + \frac{4 \times 20 \times 21 \times 41}{6} + \frac{20 \times 21}{2}$$

$$S = 176400 + 11480 + 210 = 188090$$

6. The sum of series $\frac{3}{1^2}+\frac{5}{1^2+2^2}+\frac{7}{1^2+2^2+3^2}+\frac{9}{1^2+2^2+3^2+4^2}+\ldots$ to n terms equals

~~(A)~~ $\frac{6n}{n+1}$ (B) $\frac{6n}{n^2+1}$ (C) $\frac{n+1}{n^2+1}$ (D) None of these

Sol^n!

$$T_n = \frac{(2n+1)}{1^2+2^2+\ldots+n^2}$$

$$T_n = \frac{\cancel{(2n+1)}}{\frac{n(n+1)\cancel{(2n+1)}}{6}}$$

$$T_n = \frac{6}{n(n+1)}$$

$$S_n = \sum_{n=1}^{n} \frac{6}{n(n+1)} = 6\sum_{n=1}^{n} \frac{(n+1)-n}{n(n+1)}$$

$$S_n = 6\left[\sum_{n=1}^{n} \frac{1}{n} - \frac{1}{n+1}\right]$$

$$S_n = 6\left[\frac{1}{1} - \cancel{\frac{1}{2}} + \cancel{\frac{1}{2}} - \cancel{\frac{1}{3}} + \ldots + \cancel{\frac{1}{n}} - \frac{1}{(n+1)}\right]$$

$$\Rightarrow S_n = 6\left[\frac{n}{n+1}\right]$$

$$S_n = \frac{6n}{n+1}$$

7. Sum to infinite of the series $1+\frac{2}{5}+\frac{3}{5^2}+\frac{4}{5^3}+\ldots$ is

(A) 5/4 (B) 6/5 (C) 25/16 (D) 16/9

Solⁿ:

$$S = 1+\frac{2}{5}+\frac{3}{5^2}+\frac{4}{5^3}+\ldots\infty$$

$$\frac{S}{5} = \frac{1}{5}+\frac{2}{5^2}+\frac{3}{5^3}+\ldots\infty$$

$$\frac{4S}{5} = 1+\frac{1}{5}+\frac{1}{5^2}+\frac{1}{5^3}+\ldots\infty$$

$$\frac{4S}{5} = \frac{1}{1-1/5}$$

$$\frac{4S}{5} = \frac{5}{4}$$

$$S_\infty = \frac{25}{16}$$

8. The sum of the infinite series $\frac{1}{10}+\frac{2}{10^2}+\frac{3}{10^3}+\ldots$ is equal to :

(A) $\frac{1}{9}$ (B) $\frac{10}{81}$ (C) $\frac{1}{8}$ (D) $\frac{17}{72}$

Solⁿ:

$$S = \frac{1}{10}+\frac{2}{10^2}+\frac{3}{10^3}+\ldots\infty$$

$$\frac{S}{10} = \frac{1}{10^2}+\frac{2}{10^3}+\ldots\infty$$

$$\frac{9S}{10} = \frac{1}{10}+\frac{1}{10^2}+\frac{1}{10^3}+\ldots\infty$$

$$9S = \frac{1}{1-1/10}$$

$$\boxed{S = \frac{10}{81}}$$

9. The sum of the series, $1+2\cdot\left(1+\frac{1}{n}\right)+3\cdot\left(1+\frac{1}{n}\right)^2+\ldots \infty$ is (where $|n| > 1$).

~~(A) n^2~~ (B) $n(n+1)$ (C) $n\left(1+\frac{1}{n}\right)^2$ (D) $(n+1)(n+2)$

Sol^n:

$$S = 1 + 2\left(1+\frac{1}{n}\right) + 3\left(1+\frac{1}{n}\right)^2 + \ldots \infty$$

$$\left(1+\frac{1}{n}\right)S = \left(1+1/n\right) + 2\left(1+\frac{1}{n}\right)^2 + \ldots \infty$$

$$-\frac{S}{n} = 1 + \left(1+1/n\right) + \left(1+1/n\right)^2 + \ldots \infty$$

$$-\frac{S}{n} = \frac{1}{1-1-1/n}$$

$$\boxed{S = n^2}$$

10. Sum of infinite terms of the series $\left[\frac{1}{5}-\frac{2}{7^2}+\frac{3}{5^3}-\frac{4}{7^4}+\ldots\ldots\right]$ is

~~(A) $\frac{211}{1152}$~~ (B) $\frac{220}{1811}$ (C) $\frac{2}{311}$ (D) None of these.

Sol^n:

$$S = \left[\underbrace{\left(\frac{1}{5}+\frac{3}{5^3}+\ldots\right)}_{S_1} - \underbrace{\left(\frac{2}{7^2}+\frac{4}{7^4}+\ldots\infty\right)}_{S_2}\right]$$

$$S_1 = \frac{1}{5} + \frac{3}{5^3} + \frac{5}{5^5} + \ldots \infty$$

$$\frac{S_1}{25} = \frac{1}{5^3} + \frac{3}{5^5} + \ldots \infty$$

$$\frac{24S_1}{25} = \frac{1}{5} + \frac{2}{5^3} + \frac{2}{5^5} + \ldots \infty$$

$$\frac{24S_1}{5} = 1 + \frac{2}{5^2} + \frac{2}{5^4} + \cdots \infty$$

$$\frac{24S_1}{5} = 1 + \frac{2/5^2}{1-\frac{1}{25}}$$

$$\frac{24S_1}{5} = 1 + \frac{1}{12} \qquad S_1 = \frac{65}{24\times 12}$$

$$S_2 = \frac{2}{7^2} + \frac{\cancel{4}}{7^4} + \cdots \infty$$

$$\frac{S_2}{49} = \frac{2}{7^4} + \cdots \infty$$

$$\frac{48S_2}{49} = \frac{2}{49} + \frac{2}{7^4} + \cdots$$

$$\cancel{24}48\, S_2 = \cancel{2}\left(1 + \frac{1}{7^2} + \cdots \infty\right)$$

$$24S_2 = \frac{49}{48} \Rightarrow S_2 = \frac{49}{24\times 48}$$

$$S_1 - S_2 = \frac{65}{24\times 12} - \frac{49}{24\times 4\times 12}$$

$$\frac{(65\times 4 - 49)}{24\times 12\times 4} = \frac{211}{1152}$$

11. If the sum to infinity of the series $1 + 4x + 7x^2 + 10x^3 + \ldots$ is $\frac{35}{16}$ then find x.

~~(A)~~ $\frac{1}{5}$ (B) $\frac{19}{7}$ (C) $\frac{15}{12}$ (D) None of these

Solⁿ:

$$S = 1 + 4x + 7x^2 + 10x^3 + \text{----}$$

$$xS = x + 4x^2 + 7x^3 + \text{----}$$

$$(1-x)S = 1 + 3x + 3x^2 + 3x^3 + \text{---}$$

$$(1-x)S = 1 + \frac{3x}{(1-x)}$$

$$(1-x)^2 \times \frac{35}{16} = 1 + 2x$$

$$35x^2 - 70x + 35 = 32x + 16$$

$$\Rightarrow \quad 35x^2 - 102x + 19 = 0$$

$$35x^2 - 7x - 95x + 19 = 0$$

$$(7x - 19)(5x - 1) = 0$$

$$\cancel{x = 19/7} \qquad x = 1/5 \checkmark$$

12. The sum of 0.2 + 0.004 + 0.00006 + 0.0000008 + ... to ∞ is

(A) $\frac{200}{891}$ (B) $\frac{2000}{9801}$ (C) $\frac{1000}{9801}$ (D) None of these

Sol^n:

$$S = \frac{2}{10} + \frac{4}{10^3} + \frac{6}{10^5} + \frac{8}{10^7} + \cdots$$

$$\frac{S}{100} = \frac{2}{10^3} + \frac{4}{10^5} + \frac{6}{10^7} + \cdots$$

$$\frac{99S}{100} = \frac{2}{10} + \frac{2}{10^3} + \frac{2}{10^5} + \cdots$$

$$\frac{99S}{10} = 2 + \frac{2}{10^2} + \frac{2}{10^4} + \cdots$$

$$\frac{99S}{10} = \frac{2}{1 - \frac{1}{100}}$$

$$S = \frac{2000}{9801}$$

13. The sum of the series $\frac{1}{\log_2 4} + \frac{1}{\log_4 4} + \frac{1}{\log_8 4} + \dots + \frac{1}{\log_{2^n} 4}$ is

(A) $\frac{n(n+1)}{2}$ (B) $\frac{n(n+1)+(2n+1)}{12}$ (C) $\frac{1}{n(n+1)}$ (D) $\frac{n(n+1)}{4}$

Sol^n:

$$S = \log_4 2 + \log_4 4 + \log_4 8 + \cdots + \log_4 2^n$$

$$S = \frac{1}{2} + 1 + \frac{3}{2} + \cdots + \frac{n}{2}$$

$$S = \frac{n}{2}\left(\frac{1}{2} + \frac{n}{2}\right)$$

14. The sum to infinity of the series $\frac{1}{2.4}+\frac{1}{4.6}+\frac{1}{6.8}+\frac{1}{8.10}+$ ....is

(A) 1/4 (B) 1/8 (C) 1/2 (D) 1/16

Sol^n:

$$T_n = \frac{1}{2n \times (2n+2)}$$

$$S = \frac{1}{2}\sum_{n=1}^{T_n} \frac{(2n+2)-2n}{2n \times (2n+2)}$$

$$S = \frac{1}{2}\sum_{n=1}^{n} \frac{1}{2n} - \frac{1}{2n+2}$$

$$S = \frac{1}{2}\left[\frac{1}{2} - \cancel{\frac{1}{4}} + \cancel{\frac{1}{4}} - \cancel{\frac{1}{6}} + \cdots + \cancel{\frac{1}{2n}} - \frac{1}{2n+2}\right]$$

$$S_\infty = \frac{1}{2}\left[\frac{1}{2} - \cancel{\frac{1}{2n+2}}^{\,0}\right] = \frac{1}{4}$$

15. The sum of $\frac{3}{1.2}\cdot\frac{1}{2}+\frac{4}{2\cdot 3}\cdot\left(\frac{1}{2}\right)^2+\frac{5}{3\cdot 4}\left(\frac{1}{2}\right)^3+\ldots$ to n terms is equal to

(A) $1-\frac{1}{(n+1)2^n}$ (B) $1-\frac{1}{n\cdot 2^{n-1}}$ (C) $1+\frac{1}{(n+1)2^n}$ (D) none of these

Sol^n:

$$T_n = \frac{(n+2)}{n(n+1)} \times \left(\frac{1}{2}\right)^n$$

$$T_n = \left(\frac{(2n+2)-n}{n(n+1)}\right)\left(\frac{1}{2}\right)^n$$

$$T_n = \left(\frac{2}{n} - \frac{1}{(n+1)}\right) \times \frac{1}{2^n}$$

$$T_n = \left(\frac{1}{n \times 2^{n-1}} - \frac{1}{(n+1)2^n}\right)$$

$$S_n = \sum_{n=1}^{n} T_n \Rightarrow S_n = \left[\frac{1}{1\times 1} - \cancel{\frac{1}{2\times 2}} + \cancel{\frac{1}{2\times 2}} - \cancel{\frac{1}{3\times 2^2}} + \cancel{\frac{1}{3\times 2^2}} - \cancel{\frac{1}{4\times 2^3}} \;\vdots\; + \cancel{\frac{1}{n\times 2^{n-1}}} - \frac{1}{(n+1)2^n}\right]$$

$$\boxed{S_n = 1 - \frac{1}{(n+1)2^n}}$$

Ans

16. The value of $\sum_{n=3}^{\infty} \frac{1}{n^5 - 5n^3 + 4n}$ is equal to

(A) $\frac{1}{120}$ ~~(B)~~ $\frac{1}{96}$ (C) $\frac{1}{24}$ (D) $\frac{1}{144}$

Sol^n:

$$\sum_{n=3}^{\infty} \frac{1}{n(n^4 - 5n^2 + 4)}$$

$$= \sum_{n=3}^{\infty} \frac{1}{n(n^2-4)(n^2-1)}$$

$$= \frac{1}{4} \sum_{n=3}^{\infty} \frac{(n+2)-(n-2)}{(n-2)(n-1)n(n+1)(n+2)}$$

$$= \frac{1}{4} \sum_{n=3}^{\infty} \frac{1}{(n-2)(n-1)n(n+1)} - \frac{1}{(n-1)(n)(n+1)(n+2)}$$

$$= \frac{1}{4}\left[\frac{1}{1.2.3.4} - \cancel{\frac{1}{2.3.4.5}} + \cancel{\frac{1}{2.3.4.5}} - \cancel{\frac{1}{3.4.5.6}} \right.$$

$$\vdots$$

$$\left. + \cancel{\frac{1}{(n-2)(n-1)n(n+1)}} - \frac{1}{(n-1).n(n+1)(n+2)}\right]$$

$\to 0$ $n \to \infty$.

$$= \frac{1}{4}\left(\frac{1}{24}\right) = \frac{1}{96}$$

17. The value of $\sum_{r=1}^{n} \frac{1}{\sqrt{a+rx}+\sqrt{a+(r-1)x}}$ is

(A) $\frac{n}{\sqrt{a}-\sqrt{a+nx}}$ ~~(B)~~ $\frac{\sqrt{a+nx}-\sqrt{a}}{x}$ (C) $\frac{n(\sqrt{a+nx}-a)}{x}$ (D) None of these

Sol^n:

$$\sum_{r=1}^{n} \frac{\left(\sqrt{a+rx}-\sqrt{a+(r-1)x}\right)}{(a+rx)-(a+(r-1)x)}$$

$$\frac{1}{x}\sum_{r=1}^{n} \sqrt{a+rx}-\sqrt{a+(r-1)x}$$

$$\Rightarrow \frac{1}{x}\left[\begin{array}{l} \cancel{\sqrt{a+x}} - \sqrt{a} \\ + \cancel{\sqrt{a+2x}} - \cancel{\sqrt{a+x}} \\ + \cancel{\sqrt{a+3x}} - \cancel{\sqrt{a+2x}} \\ \vdots \\ + \sqrt{a+nx} - \cancel{\sqrt{a+(n-1)x}} \end{array}\right]$$

$$= \frac{1}{x}\left[\sqrt{a+nx}-\sqrt{a}\right]$$

$$= \frac{1}{x}\frac{(a+nx-a)}{(\sqrt{a+nx}+\sqrt{a})} = n \times \frac{1}{(\sqrt{a+nx}+\sqrt{a})}$$

18. If the sum $\sum_{k=1}^{\infty}\frac{1}{(k+2)\sqrt{k}+k\sqrt{k+2}}=\frac{\sqrt{a}+\sqrt{b}}{\sqrt{c}}$ where a, b, c $\in$ N and lie in [1, 15], then a + b + c equals to

(A) 6 (B) 8 (C) 10 (D) 11

Sol^n:

$$S=\sum_{k=1}^{\infty}\frac{(k+2)\sqrt{k}-k\sqrt{k+2}}{(k+2)^2\times k-k^2(k+2)}$$

$$S=\frac{1}{2}\sum_{k=1}^{\infty}\frac{(k+2)\sqrt{k}-k\sqrt{k+2}}{k(k+2)}$$

$$S=\frac{1}{2}\sum_{k=1}^{\infty}\frac{1}{\sqrt{k}}-\frac{1}{\sqrt{k+2}}$$

$$S=\frac{1}{2}\Big[\frac{1}{1}-\cancel{\frac{1}{\sqrt{3}}}+\frac{1}{\sqrt{2}}-\cancel{\frac{1}{\sqrt{4}}}+\cancel{\frac{1}{\sqrt{3}}}-\cancel{\frac{1}{\sqrt{5}}}+\cancel{\frac{1}{\sqrt{4}}}-\cancel{\frac{1}{\sqrt{6}}}+\dots+\cancel{\frac{1}{\sqrt{k-1}}}-\cancel{\frac{1}{\sqrt{k+1}}}+\cancel{\frac{1}{\sqrt{k}}}-\cancel{\frac{1}{\sqrt{k+2}}}\Big]$$

$$S=\frac{1}{2}\left(1+\frac{1}{\sqrt{2}}\right)$$

$$S=\frac{\sqrt{2}+1}{\sqrt{8}}$$

$a+b+c=8+2+1$

$=11$

19. The value of $\frac{1}{1.3.5}+\frac{1}{3.5.7}+\frac{1}{5.7.9}+\frac{1}{9.11.13}+\ldots\infty$ equals

(A) $\cancel{\frac{1}{12}}$ (B) $\frac{53}{249}$ (C) $\frac{35}{429}$ (D) $\frac{35}{249}$

Sol^n:

$$T_n = \frac{1}{(2n-1)(2n+1)(2n+3)}$$

$$S = \frac{1}{4}\sum_{n=1}^{n} \frac{(2n+3)-(2n-1)}{(2n-1)(2n+1)(2n+3)}$$

$$S = \frac{1}{4}\sum_{n=1}^{n} \frac{1}{(2n-1)(2n+1)} - \frac{1}{(2n+1)(2n+3)}$$

$$S = \frac{1}{4}\left[\frac{1}{1\times 3} - \cancel{\frac{1}{3\times 5}} + \cancel{\frac{1}{3\times 5}} - \cancel{\frac{1}{5\times 7}} \right.$$

$$\vdots$$

$$\left. + \cancel{\frac{1}{(2n-1)(2n+1)}} - \cancel{\frac{1}{(2n+1)(2n+3)}} \right]$$

$\rightarrow 0$ as $n \rightarrow \infty$

$$S_\infty = \frac{1}{4}\times\frac{1}{3} = \frac{1}{12}$$

20. The value of $\sum_{k=1}^{\infty} \frac{6^k}{(3^k - 2^k)(3^{k+1} - 2^{k+1})}$ is

(A) 1 (B) 2 (C) 3 (D) 4

Sol^n:

$$\sum_{k=1}^{\infty} \frac{2^k}{(3^k - 2^k)} - \frac{2^{k+1}}{(3^{k+1} - 2^{k+1})}$$

$$= \left[\frac{2}{(3-2)} - \cancel{\frac{2^2}{(3^2-2^2)}} \right.$$

$$+ \cancel{\frac{2^2}{(3^2-2^2)}} - \cancel{\frac{2^3}{(3^3-2^3)}}$$

$$+$$
$$\vdots$$
$$+ \cancel{\frac{2^k}{(3^k-2^k)}} - \frac{2^{k+1}}{(3^{k+1}-2^{k+1})}$$

$$S = \left[2 - \left(\frac{1}{\left(\frac{3}{2}\right)^{k+1} - 1} \right) \right]$$

As $k \to \infty$ $\left(\frac{3}{2}\right)^{k+1} - 1 \longrightarrow \infty$

and $\frac{1}{\left(\frac{3}{2}\right)^{k+1} - 1} \longrightarrow 0$

$S_\infty = 2$