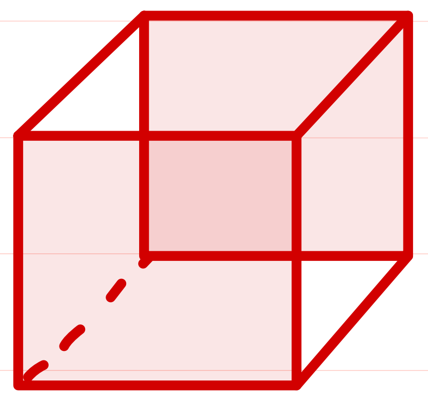
Ex. A thin copper wire of length L increase in length by 2% when heated from T_1 to T_2. If a copper cube having side 10 L is heated from T_1 to T_2 what will be the percentage change in

(i) area of one face of the cube and.

(ii) volume of the cube.

NOTE: Error in constant is equal to zero

Given $\frac{\Delta l}{l} = \frac{2}{100}$



$\leftarrow 10L \rightarrow$

(i) Area of one face $= (10L)^2$

$A = 100\,L^2$

$= \frac{\cancel{\Delta(100)}}{100} + 2\frac{\Delta L}{L}$ → 0

$\frac{\Delta A}{A} = \left(2\frac{\Delta l}{L}\right)$

% ERROR

$\left(\frac{\Delta A}{A} \times 100\right) = 2 \times 2$

$= 4\%$ Ans

(ii) volume $= (10L)^3$

$V = 1000\,L^3$

$\frac{\Delta V}{V} = \left(3\frac{\Delta L}{L}\right)$

$\left(\frac{\Delta V}{V} \times 100\right) = 3\left(\frac{\Delta L}{L} \times 100\right)$

$= 3 \times 2$

$= 6\%$

H.W

Question For Practice

Q1. Two rods have lengths measured as (1.8 ± 0.2)m and (2.3 ± 0.1)m. Calculate their combined length with error limits.

Q2. The original length of wire is (153.7 ± 0.6) cm . It is stretched to (155.3 ± 0.2) cm. Calculate the elongation in the wire with error limits.

Q3. In an experiment, values of two resistances are measured to be $r_1 = (5.0 \pm 0.2)$ ohm and $r_2 = (10.0 \pm 0.1)$ ohm. Find the values of total resistance in series with limits of percentage error.

Q4. The radius of a sphere is measured to be (2.1 ± 0.5) cm. Calculate its surface area with absolute error limits.

Q5. A physical quantity x is calculated from the relation $x = a^3b^2\sqrt{cd}$. Calculate percentage error in x, if a, b, c and d are measured respectively with an error of 1%, 3%, 4% and 2%.

Ans. 1. (4.1 ± 0.3) m   2. (1.6 ± 0.8) cm   3. $R_s = 15$ ohm $\pm 2\%$

4. (55.4 ± 26.4) cm^2 5. $\pm 12\%$

Q1. Two rods have lengths measured as (1.8 ± 0.2)m and (2.3 ± 0.1)m. Calculate their combined length with error limits.

$l_1 = (1.8 \pm 0.2)\,m \qquad l_2 = (2.3 \pm 0.1)\,m$

$l = l_1 + l_2 = l_m \pm \Delta l_m = (l_{1m} + l_{2m}) \pm (\Delta l_{1m} + \Delta l_{2m})$

$l = (1.8 + 2.3) \pm (0.2 + 0.1) = (4.1 \pm 0.3)\,m$

Q2. The original length of wire is (153.7 ± 0.6) cm . It is stretched to (155.3 ± 0.2) cm. Calculate the elongation in the wire with error limits.

$l_i = (153.7 \pm 0.6)\,cm \qquad l_f = (155.3 \pm 0.2)\,cm$

$\text{Elongation} = (l_f - l_i) = (155.3 - 153.7) \pm (0.2 + 0.6)$

$= [001.6 \pm 0.8]\,cm$

Q3. In an experiment, values of two resistances are measured to be $r_1 = (5.0 \pm 0.2)$ ohm and $r_2 = (10.0 \pm 0.1)$ ohm. Find the values of total resistance in series with limits of percentage error.

$R_s = (R_1 + R_2) = (5.0 + 10.0) \pm (0.1 + 0.2)$

$= (15.0 \pm 0.3)\,\Omega$ Ans

In terms of %

$\frac{0.3}{15} \times 100 = 2\%$

$(15.0\,\Omega \pm 2\%)$ Ans

Q4. The radius of a sphere is measured to be (2.1 ± 0.5) cm. Calculate its surface area with absolute error limits.

$r = (2.1 \pm 0.5)$ cm

Surface area $(A) = 4\pi r^2$

Find $A = A_m \pm \Delta A_m$

$A_m = 4\pi r_m^2$

$= 4 \times \frac{22}{7} \times 2.1 \times 2.1$ cm^2

$= 55.44$ cm^2

$\frac{\Delta A}{A} = 2\frac{\Delta r}{r}$

$\Delta A = 2\left(\frac{0.5}{\cancel{2.1}}\right) \times 4 \times \frac{22}{\cancel{7}} \times \cancel{2.1} \times \cancel{2.1}^{\,0.3}$

$= 1.2 \times 22$

$\Delta A = 26.4$ cm^2

$\boxed{A = (55.44 \pm 26.4)\text{ cm}^2}$

Ans

Q5. A physical quantity x is calculated from the relation $x = a^3b^2\sqrt{cd}$. Calculate percentage error in x, if a, b, c and d are measured respectively with an error of 1%, 3%, 4% and 2%.

$$\frac{\Delta x}{x} = 3\frac{\Delta a}{a} + 2\frac{\Delta b}{b} + \frac{1}{2}\frac{\Delta c}{c} + \frac{1}{2}\frac{\Delta D}{D}$$

$$= 3\times 1 + 2\times 3 + \frac{1}{2}\times 4 + \frac{1}{2}\times 2$$

$$\frac{\Delta x}{x} = 3 + 6 + 2 + 1$$

$$\boxed{\frac{\Delta x}{x}\% = 12\%}$$ Ans

Applications

1. For a simple pendulum $T = 2\pi\sqrt{\frac{\ell}{g}}$

$$\Rightarrow \quad \frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta \ell}{\ell}$$

If $g = \text{const}$ $\Rightarrow$

If $g \neq$ constant

$$T = 2\pi\, \ell^{1/2} g^{-1/2} \Rightarrow \boxed{\frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta \ell}{\ell} + \frac{1}{2}\frac{\Delta g}{g}}$$

2. For a sphere of radius r,

Surface area $A = 4\pi r^2 \Rightarrow \frac{\Delta A}{A} = \frac{2\Delta r}{r}$

Volume $V = \frac{4}{3}\pi r^3 \Rightarrow \frac{\Delta V}{V} = \frac{3\Delta r}{r}$

3. Acceleration due to gravity $g = \frac{GM}{R^2}$

$$\Rightarrow \quad \boxed{\frac{\Delta g}{g} = \frac{2\Delta R}{R} + \frac{\Delta M}{M}}$$

4. For resistances connected in series

$$\boxed{R_s = R_1 + R_2 \Rightarrow \frac{\Delta R_s}{R_s} = \frac{\Delta R_1 + \Delta R_2}{R_1 + R_2}}$$

5. For resistances connected in parallel

$$\boxed{\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2}} \Rightarrow -\frac{\Delta R_p}{R_p^2} = -\frac{\Delta R_1}{R_1^2} - \frac{\Delta R_2}{R_2^2}$$

$$\Rightarrow \quad \boxed{\frac{\Delta R_p}{R_p^2} = \frac{\Delta R_1}{R_1^2} + \frac{\Delta R_2}{R_2^2}}$$

6. Kinetic energy K and linear momentum p are related as

$$\boxed{K = \frac{p^2}{2m} \Rightarrow \frac{\Delta K}{K} = \frac{2\Delta p}{p}}$$

Ex $R_1 = (5 \pm 0.1)\,\Omega$ $R_2 = (5 \pm 0.2)\,\Omega$, Parallelly connected

Find Resultant Resistance In Error limits

$$\frac{1}{(R_p)_m} = \frac{1}{5} + \frac{1}{5} = \frac{2}{5} \Rightarrow \boxed{(R_p)_m = \frac{5}{2} = 2.5\,\Omega}$$

$$\frac{\Delta R_p}{(R_p)_m^2} = \frac{\Delta R_1}{R_1^2} + \frac{\Delta R_2}{R_2^2} = \frac{0.1}{(5)^2} + \frac{0.2}{(5)^2} = \frac{0.3}{(5)^2}$$

$$(\Delta R_p) = \frac{0.3}{(5)^{\cancel{2}}} \times \frac{\cancel{5}^2}{2^2} = \frac{0.3}{4} = 0.075$$

$$\boxed{R_p = (2.5 \pm 0.075)\,\Omega} \quad \text{Ans}$$

It is minimum value measured by an Instrument OR Absolute ERROR

Ex- The time period of a simple pendulum is given by $T = 2\pi\sqrt{L/g}$. The measured value of L is 20.0 cm using a scale of $\boxed{\text{least count}}$ 1 mm and time t for 100 oscillations is found to be 90 s using a watch of least count 1 s. Find the value of g (in m s^{-2}) up to appropriate significant figure, stating the uncertainity in the value of g.

$\Delta L = 1\text{mm} = 10^{-3}\ m$

$\Delta t = 1\text{sec}$

$T = 2\pi\sqrt{\frac{L}{g}}$ = time period in one osci.

let N be No. of oscillation

$t = NT$

$t = N \cdot 2\pi\sqrt{\frac{L}{g}}$

$t^2 = 4\pi^2 N^2 \frac{L}{g}$

$$\boxed{g = (4\pi^2 N^2) \cdot \frac{L}{t^2}}$$

$$\frac{\Delta g}{g} = \frac{\Delta L}{L} + 2\frac{\Delta t}{t}$$

$$\frac{\Delta g}{g} = \frac{10^{-3} m}{20 \times 10^{-2} m} + 2\frac{1}{90}$$

$$\frac{\Delta g}{g} = \left(\frac{1}{200} + \frac{1}{45}\right)$$

$$\Delta g = \left(\frac{1}{200} + \frac{1}{45}\right) g$$

$$g = 4\left(\frac{22}{7}\right)^2 (100)^2 \frac{20 \times 10^{-2}}{90 \times 90}$$

$$g = 9.74 \,(m/sec^2)$$

$$\Delta g = 0.26\ m/sec^2$$

$$g = (9.74 \pm 0.26)\ m/s^2$$

$$\boxed{g = (9.7 \pm 0.3)\ m/s^2}$$

Ex

In the measurement of a physical quantity $X = \frac{A^2 B}{C^{1/3} D^3}$. The percentage errors introduced in the measurements of the quantities A, B, C and D are 2%, 2%, 4% and 5% respectively. Then the minimum amount of percentage of error in the measurement of X is contributed by:

(a) A (b) B

(c) C (d) D

$$\frac{\Delta X}{X} = 2\frac{\Delta A}{A} + \frac{\Delta B}{B} + \frac{1}{3}\frac{\Delta C}{C} + 3\frac{\Delta D}{D}$$

$$= 2(2\%) + (2\%) + 1/3(4\%) + 3(5\%)$$

$$= 4\% + 2\% + \underline{\underline{4/3\%}} + 15\%$$

1. The error in the measurement of the radius of a sphere is 1%. The error in the measurement of the volume is

(a) 1% (b) 3%

(c) 5% (d) 8%

2. If the error in the measurement of the volume of a sphere is 6%, then the error in the measurement of its surface area will be

(a) 2% (b) 3%

(c) 4% (d) 7.5%

3. The moment of inertia of a body rotating about a given axis is 6.0 kg m^2 in the SI system. What is the value of the moment of inertia in a system of units in which the unit of length is 5 cm and the unit of mass is 10 g?

(a) 2.4×10^3 (b) 2.4×10^5

(c) 6.0×10^3 (d) 6.0×10^5

① $\frac{\Delta V}{V}\% = 3\frac{\Delta r}{r}\% = 3(1\%) = 3\%$

② $\frac{\Delta V}{V} = 3\frac{\Delta r}{r}$

$6 = 3\frac{\Delta r}{r}$

$2 = \frac{\Delta r}{r}$

$\frac{\Delta A}{A} = 2\frac{\Delta r}{r}$

$= 2 \times 2$

$= 4\%$

③ M.I $=$ mass $\times$ (radius)^2

$= (\text{kg} \cdot \text{m}^2) \times 6$

$= (1000\,\text{gm})(100\,\text{cm})^2 \times 6$

$= 10^3\,\text{gm} \cdot 10^4\,\text{cm}^2 \times 6$

$= 10^2 (10\,\text{gm}) \frac{10^4}{25} (5\,\text{cm})^2 \times 6$

$= 6 \times \frac{10^6}{25} \left[10\,\text{gm} \cdot (5\,\text{cm})^2\right] = 2.4 \times 10^5$

Method -2

$N_1 U_1 = N_2 U_2$ $[M^1 L^2 T^0]$

$N_2 = N_1 \left[\frac{U_1}{U_2}\right]$

$= 6 \left[\frac{kg}{10 gm}\right]^1 \left[\frac{m}{5 cm}\right]^2 \left[\frac{sec}{sec}\right]^0$

$= 6 \left[\frac{1000 \cancel{gm}}{10 \cancel{gm}}\right]^1 \left[\frac{100 \cancel{cm}}{5 \cancel{cm}}\right]^2$

$= 6 [10^2] [20]^2$

$= 6 \times 10^2 \times 4 \times 10^2$

$= 24 \times 10^4$

$\boxed{N_2 = 2.4 \times 10^5}$ Ans

Ex If energy E, velocity V and time T are chosen as the fundamental units, the dimensional formula for surface tension will be

(a) $E\ V^2T^{-2}$ (b) $E\ V^{-1}T^{-2}$

(c) $E\ V^{-2}T^{-2}$ ✓ (d) $E^2V^{-1}T^{-2}$

Tension (S) $= \frac{\text{Force}}{\text{length}} = \frac{M^1 L^1 T^{-2}}{L^1} = M^1 L^0 T^{-2}$

$S \propto E^a v^b T^c$

$(M^1 L^0 T^{-2}) = [M^1 L^2 T^{-2}]^a [M^0 L^1 T^{-1}]^b [T^1]^c$

$M^1 L^0 T^{-2} = M^a L^{2a+b} T^{-2a-b+c}$

$\boxed{a=1}$ $2a+b=0$ $-2a-b+c=-2$

$\boxed{b=-2}$ $-2+2+c=-2$

$\boxed{c=-2}$

$S \propto E\ V^{-2} T^{-2}$

Ex-

The amplitude of a damped oscillator of mass m varies with time t as

$$A = A_0 e^{(-at/m)}$$

The dimensions of a are

(a) ML^0T^{-1} (b) M^0LT^{-1}

(c) MLT^{-1} (d) $ML^{-1}T$

Ex-

A student measures the value of g with the help of a simple pendulum using the formula

$$g = \frac{4\pi^2 L}{T^2}$$

The errors in the measurements of L and T are ΔL and ΔT respectively. In which of the following cases is the error in the value of g the minimum?

(a) $\Delta L = 0.5$ cm, $\Delta T = 0.5$ s

(b) $\Delta L = 0.2$ cm, $\Delta T = 0.2$ s

(c) $\Delta L = 0.1$ cm, $\Delta T = 1.0$ s

(d) $\Delta L = 0.1$ cm, $\Delta T = 0.1$ s

① $\frac{at}{m} = M^0 L^0 T^0 \Rightarrow [a] = \frac{m}{t} = [M^1 L^0 T^{-1}]$

② $\frac{\Delta g}{g} = \frac{\Delta L}{L} + 2\frac{\Delta T}{T}$

For minimum, ΔL & ΔT should be minimum.