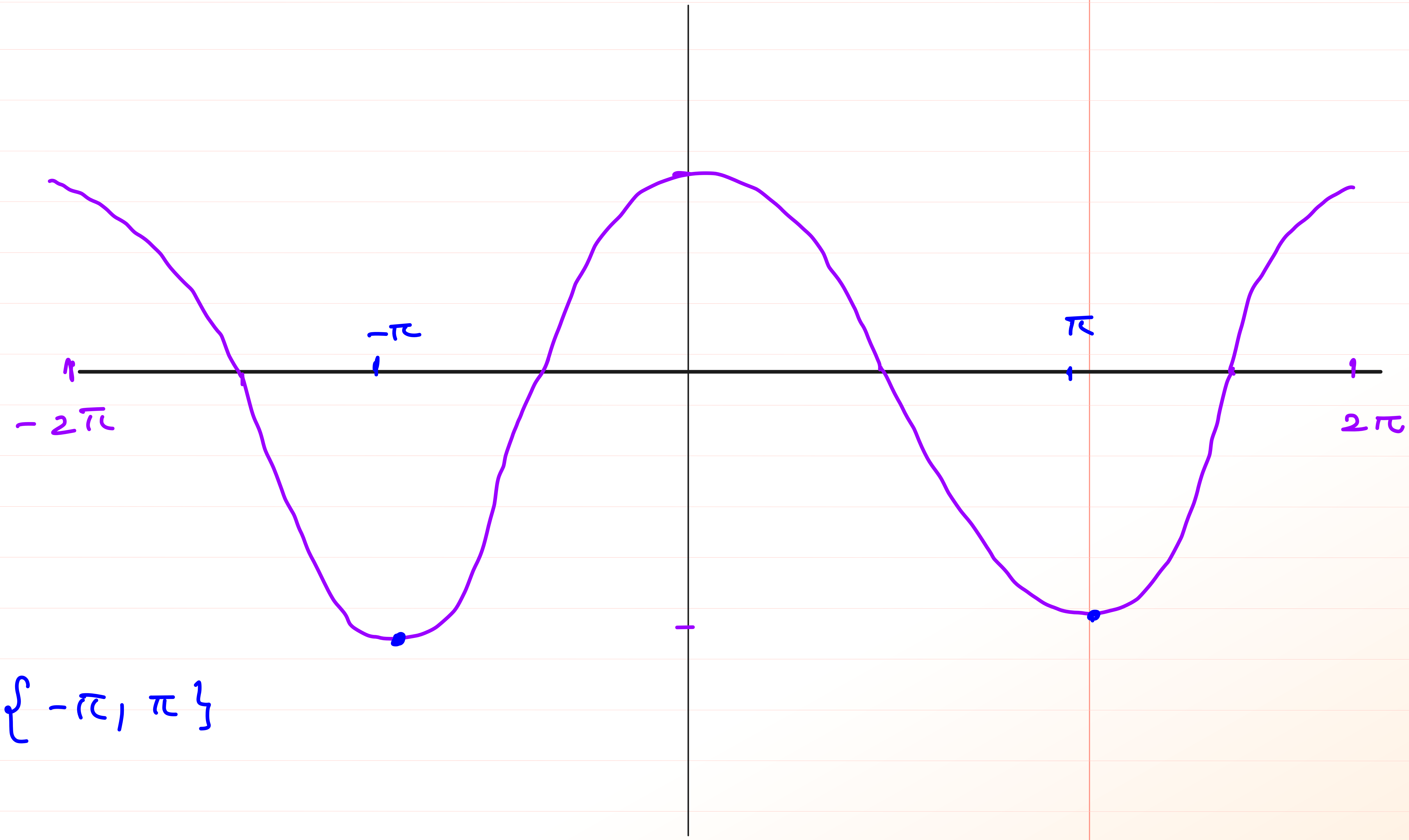
Trigonometric Ratios and Identities

Lecture - 5



$\theta \in \{-\pi, \pi\}$

TRIGONOMETRIC RATIO OF COMPOUND ANGLES :

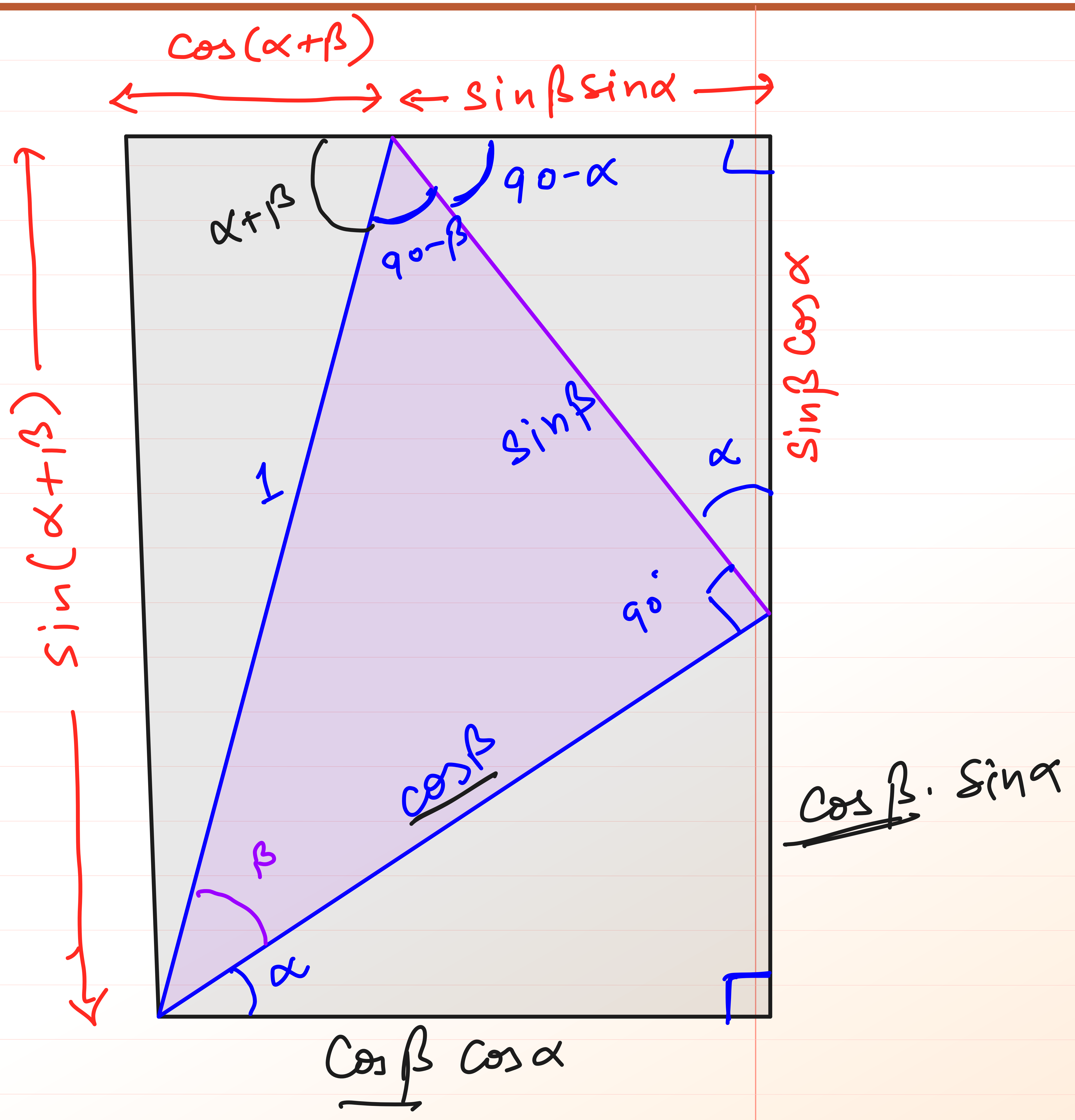
$\cos(\alpha+\beta) + \sin\beta\sin\alpha = \cos\beta\cos\alpha$

✓ $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

✓ $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$(\sin(\alpha+\beta))^2 = \sin^2(\alpha+\beta)$

$\sin(\alpha+\beta)\cdot\sin(\alpha+\beta)$



(i) $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

(ii) $\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

(iii) $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

(iv) $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

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(1) $\sin 99^\circ \cos 21^\circ + \cos 99 \sin 21^\circ = ?$

$= \sin(99+21) = \sin 120 = \sin(90+30) = \frac{\sqrt{3}}{2}$

(2) $\sin 15^\circ$

$= \sin(45-30)$

$= \sin 45 \cos 30 - \cos 45 \sin 30$

$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$

$= \frac{\sqrt{3} - 1}{2\sqrt{2}}$

(3) $\cos 15^\circ$

$= \cos(45-30)$

$= \cos 45 \cos 30 + \sin 45 \sin 30$

$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$

$= \frac{\sqrt{3}+1}{2\sqrt{2}}$

(4) $\sin 75 = \sin(90-15)$

$= \cos 15 = \frac{\sqrt{3}+1}{2\sqrt{2}}$

(5) $\cos 75 = \cos(90-15)$

$= \sin 15 = \frac{\sqrt{3}-1}{2\sqrt{2}}$

Formulae & Identities :

(a) $\sin(A + B) . \sin(A - B) = \sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A$

(b) $\cos^2 A - \sin^2 B = \cos(A + B) \cos(A - B) = \cos^2 B - \sin^2 A$

(c) **Formula to transform the product into sum or difference :**

(1) $2 \sin A \cos B = \sin(A + B) + \sin(A - B)$

(2) $2 \cos A \sin B = \sin(A + B) - \sin(A - B)$

(3) $2 \cos A \cos B = \cos(A + B) + \cos(A - B)$

(4) $2 \sin A \sin B = \cos(A - B) - \cos(A + B)$

(d) (1) $\sin\theta \sin(60° + \theta) \sin(60° - \theta) = \frac{1}{4}\sin 3\theta$

(2) $\cos\theta \cos(60° + \theta) \cos(60° - \theta) = \frac{1}{4}\cos 3\theta$

(3) $\tan\theta \tan(60° + \theta) \tan(60° - \theta) = \tan 3\theta$

(a) $\sin(A+B) . \sin(A-B)$

$= (\sin A \cos B + \cos A \sin B)(\sin A \cos B - \cos A \sin B)$

$= \sin^2 A \cos^2 B - \cos^2 A \sin^2 B$

$= \sin^2 A (1 - \sin^2 B) - (1 - \sin^2 A) \sin^2 B$

$= \sin^2 A - \sin^2 A \sin^2 B - \sin^2 B + \sin^2 A \sin^2 B$

$= \sin^2 A - \sin^2 B$

(c) **Formula to transform the product into sum or difference**

(1) $2\sin A\cos B = \sin(A+B) + \sin(A-B)$

(2) $2\cos A\sin B = \sin(A+B) - \sin(A-B)$

(3) $2\cos A\cos B = \cos(A+B) + \cos(A-B)$

(4) $2\sin A\sin B = \cos(A-B) - \cos(A+B)$

(A) $\sin(A+B) = \sin A\cos B + \cos A\sin B$

(B) $\sin(A-B) = \sin A\cos B - \cos A\sin B$

(C) $\cos(A+B) = \cos A\cos B - \sin A\sin B$

(D) $\cos(A-B) = \cos A\cos B + \sin A\sin B$

add (A) & (B) to get formula (1)

Subtract (B) from (A) to get (2)

add (C) & (D) to get (3)

Subtract (C) from (D) to get (4)

(d) (1) $\sin\theta\sin(60^\circ+\theta)\sin(60^\circ-\theta)=\frac{1}{4}\sin 3\theta$

(2) $\cos\theta\cos(60^\circ+\theta)\cos(60^\circ-\theta)=\frac{1}{4}\cos 3\theta$

(3) $\tan\theta\tan(60^\circ+\theta)\tan(60^\circ-\theta)=\tan 3\theta$

1

$$\sin\theta\,(\sin(60+\theta))\sin(60-\theta)$$

$$=\sin\theta\cdot(\sin^2 60-\sin^2\theta)$$

$$=\sin\theta\left(\frac{3}{4}-\sin^2\theta\right)$$

$$=\frac{1}{4}\sin\theta\,(3-4\sin^2\theta)$$

$$=\frac{1}{4}\cdot(3\sin\theta-4\sin^3\theta)$$

$$=\frac{1}{4}\cdot\sin 3\theta\,.$$

✓ (A) $\sin(A+B) = \sin A \cos B + \cos A \sin B$

(B) $\sin(A-B) = \sin A \cos B - \cos A \sin B$

✓ (C) $\cos(A+B) = \cos A \cos B - \sin A \sin B$

(D) $\cos(A-B) = \cos A \cos B + \sin A \sin B$

divide (A) by (C)

$$\frac{\sin(A+B)}{\cos(A+B)} = \frac{(\sin A \cos B + \cos A \sin B)/\cos A \cos B}{(\cos A \cos B - \sin A \sin B)/\cos A \cos B}$$

$$\boxed{\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}}$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

(d) Identities for converting sum to product :

Let $A + B = C$ & $A - B = D$

$\therefore \quad A = \dfrac{C+D}{2}, \quad B = \dfrac{C-D}{2}$

1. $\sin C + \sin D = 2\sin\left(\dfrac{C+D}{2}\right)\cos\left(\dfrac{C-D}{2}\right)$

2. $\sin C - \sin D = 2\cos\left(\dfrac{C+D}{2}\right)\sin\left(\dfrac{C-D}{2}\right)$

3. $\cos C + \cos D = 2\cos\left(\dfrac{C+D}{2}\right)\cos\left(\dfrac{C-D}{2}\right)$

4. $\cos C - \cos D = -2\sin\left(\dfrac{C+D}{2}\right)\sin\left(\dfrac{C-D}{2}\right)$

$= 2\sin\left(\dfrac{C+D}{2}\right)\sin\left(\dfrac{D-C}{2}\right)$

(1) $2\sin A\cos B = \sin(A+B) + \sin(A-B)$

(2) $2\cos A\sin B = \sin(A+B) - \sin(A-B)$

(3) $2\cos A\cos B = \cos(A+B) + \cos(A-B)$

(4) $2\sin A\sin B = \cos(A-B) - \cos(A+B)$

take $A+B = C$

$A-B = D$

$A = \dfrac{C+D}{2}$; $B = \dfrac{C-D}{2}$

$\sin C + \sin D = 2\sin\dfrac{C+D}{2}\cos\dfrac{C-D}{2}$

Trigonometric Ratios and Identities

Lecture - 6

E(1) $\dfrac{\sin 7\theta - \sin 5\theta}{\cos 7\theta + \cos 5\theta} = \tan\theta$

LHS $= \dfrac{2\cos 6\theta \sin\theta}{2\cos 6\theta \cos\theta} = \tan\theta$

E(2) $\dfrac{(\cos\theta - \cos 3\theta)(\sin 8\theta + \sin 2\theta)}{(\sin 5\theta - \sin\theta)(\cos 4\theta - \cos 6\theta)} = 1$

E(3) Prove that $\dfrac{\cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10}{\cos 5\theta + 5\cos 3\theta + 10\cos\theta} = 2\cos\theta$

E(4) $\dfrac{\sin A + 2\sin 3A + \sin 5A}{\sin 3A + 2\sin 5A + \sin 7A} = \dfrac{\sin 3A}{\sin 5A}$

LHS $= \dfrac{2\sin 3A\cos 2A + 2\sin 3A}{2\sin 5A\cos 2A + 2\sin 5A} = \dfrac{\sin 3A}{\sin 5A}$

E(5) If $\alpha = \dfrac{\pi}{19}$ Find the value of $\dfrac{\sin 23\alpha - \sin 3\alpha}{\sin 16\alpha + \sin 4\alpha}$

$= \dfrac{2\cos 13\alpha \sin 10\alpha}{2\sin 10\alpha \cos 6\alpha} = \dfrac{\cos 13\alpha}{\cos 6\alpha} = \dfrac{-\cos\frac{6\pi}{19}}{\cos\frac{6\pi}{19}}$

$= -1$

$\cos 13\alpha = \cos\dfrac{13\pi}{19}$

$= \cos\left(\dfrac{19\pi - 6\pi}{19}\right)$

$= \cos\left(\pi - \dfrac{6\pi}{19}\right) = -\cos\dfrac{6\pi}{19}$

E(7) Find the value $\dfrac{\sin 8\theta\cos\theta - \sin 6\theta\cos 3\theta}{\cos 2\theta\cos\theta - \sin 3\theta\sin 4\theta}$ given $\theta = 7.5°$

Ans $(2-\sqrt{3})$

E(8) If $\sin\alpha = \dfrac{15}{17}$, $\cos\beta = -\dfrac{5}{13}$, then find $\sin(\alpha - \beta)$.

E(3) Prove that $\dfrac{\cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10}{\cos 5\theta + 5\cos 3\theta + 10\cos\theta} = 2\cos\theta$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$= 1 - 2\sin^2\theta$$
$$= 2\cos^2\theta - 1$$
$$\cos 2\theta + 1 = 2\cos^2\theta$$

$$\text{LHS} = \frac{\cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10}{\cos 5\theta + 5\cos 3\theta + 10\cos\theta}$$

$$= \frac{\cos 6\theta + \cos 4\theta + 5\cos 4\theta + 5\cos 2\theta + 10\cos 2\theta + 10}{\cos 5\theta + 5\cos 3\theta + 10\cos\theta}$$

$$= \frac{\left(2\cos\frac{6\theta+4\theta}{2}\cos\frac{6\theta-4\theta}{2}\right) + 5\left(2\cos\frac{4\theta+2\theta}{2}\cos\frac{4\theta-2\theta}{2}\right) + 10(\cos 2\theta + 1)}{\cos 5\theta + 5\cos 3\theta + 10\cos\theta}$$

$$= \frac{2\cos 5\theta\cos\theta + 5(2\cos 3\theta\cos\theta) + 10(2\cos^2\theta)}{\cos 5\theta + 5\cos 3\theta + 10\cos\theta}$$

$$= \frac{2\cos\theta(\cos 5\theta + 5\cos 3\theta + 10\cos\theta)}{\cos 5\theta + 5\cos 3\theta + 10\cos\theta} = 2\cos\theta = \text{RHS}$$

Hence Proved

E(8) If $\underline{\sin\alpha = \dfrac{15}{17}}$, $\underline{\cos\beta} = -\dfrac{5}{13}$, then find $\sin(\alpha - \beta)$.

$\alpha \rightarrow$ Ist quad or 2nd quad.

$\beta \rightarrow$ 2nd quad or 3rd quad.

$\underline{C-I}$ $\alpha \rightarrow I$, $\beta \rightarrow$ 2nd

$\sin\alpha = \dfrac{15}{17}$; $\sin\beta = \dfrac{12}{13}$

$\cos\alpha = \dfrac{8}{17}$; $\cos\beta = -\dfrac{5}{13}$

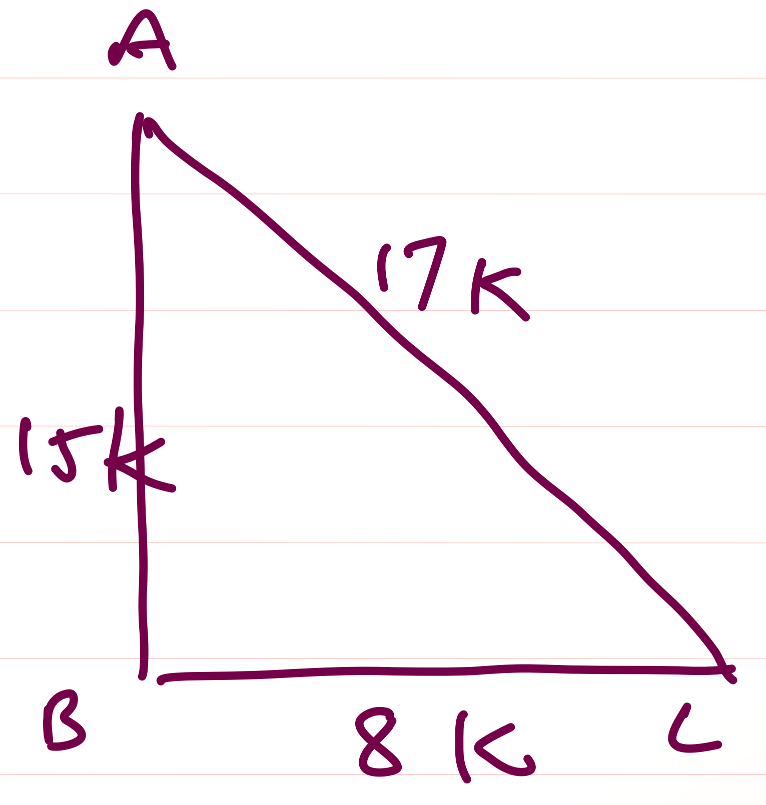
$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$= \dfrac{15}{17}\cdot\left(\dfrac{-5}{13}\right) - \dfrac{8}{17}\cdot\dfrac{12}{13}$

$\sin(\alpha-\beta) = \dfrac{-171}{221}$ ✓

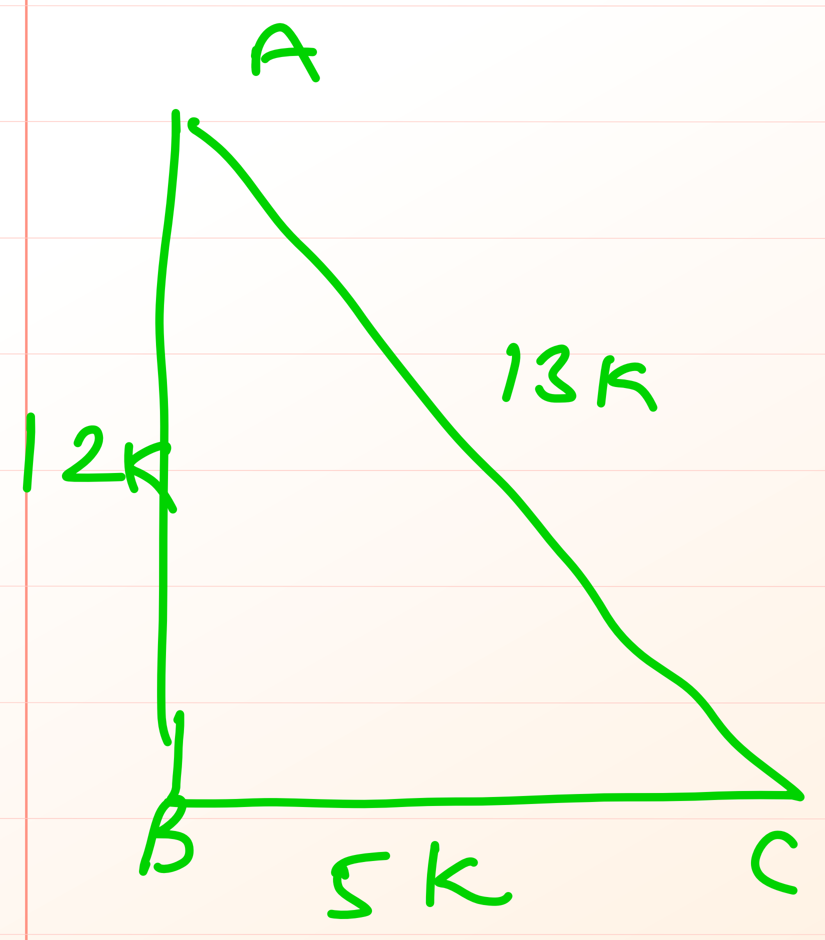
$\sin\alpha = \dfrac{15}{17}$

$\cos\alpha = \dfrac{8}{17}$



$\sin\beta = \dfrac{12}{13}$

$\cos\beta = \dfrac{-5}{13}$



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C II $\alpha \to I$ quad $\beta \to$ 3rd quad

$\sin\alpha = \frac{15}{17}$; $\sin\beta = -\frac{12}{13}$

$\cos\alpha = \frac{8}{17}$; $\cos\beta = -\frac{5}{13}$

$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$= \frac{15}{17}\cdot\left(-\frac{5}{13}\right) - \frac{8}{17}\cdot\left(\frac{-12}{13}\right) = \frac{21}{221}$

C III

$\alpha \to$ 2nd, $\beta \to$ 2nd qu

$\sin\alpha = +\frac{15}{17}$; $\sin\beta = +\frac{12}{13}$

$\cos\alpha = -\frac{8}{17}$; $\cos\beta = -\frac{5}{13}$

$\sin(\alpha-\beta) =$

C IV $\alpha \to$ 2nd ; $\beta \to$ 3rd qua

$\sin(\alpha-\beta) =$

$\sin\alpha = +\frac{15}{17}$; $\sin\beta = -\frac{12}{13}$

$\cos\alpha = -\frac{8}{17}$; $\cos\beta = -\frac{5}{13}$

(e) $$\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}$$

(f) Trigonometrical ratios of the sum of more than 2 angles :

$$\sin(A+B+C)=\sin(A+B)\cos C+\cos(A+B)\sin C$$

$$=[\sin A\cos B+\cos A\sin B]\cos C+[\cos A\cos B-\sin A\sin B]\sin C$$

$$=\sin A\cos B\cos C+\cos A\sin B\cos C+\cos A\cos B\sin C-\sin A\sin B\sin C$$

HW

$$\cos(A+B+C)=\cos(A+B)\cos C-\sin(A+B)\sin C$$

$$=(\cos A\cos B-\sin A\sin B)\cos C-(\sin A\cos B+\cos A\sin B)\sin C$$

$$=\cos A\cos B\cos C-\cos A\sin B\sin C-\sin A\cos B\sin C-\sin A\sin B\cos C.$$

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\sin(\underline{A}+\underline{B+C}) = \sin A\cos(B+C) + \cos A\sin(B+C)$$

$$= \sin A\cdot(\cos B\cos C - \sin B\sin C) + \cos A(\sin B\cos C + \cos B\sin C)$$

$$= \sin A\cos B\cos C - \sin A\sin B\sin C + \cos A\sin B\cos C + \cos A\cos B\sin C$$

$$\tan(A+B+C) = \frac{\tan A + \tan(B+C)}{1-\tan A\ \tan(B+C)}$$

$$= \frac{\tan A + \dfrac{\tan B+\tan C}{1-\tan B\tan C}}{1-\tan A\cdot\left(\dfrac{\tan B+\tan C}{1-\tan B\tan C}\right)}$$

$$= \frac{\tan A\,(1-\tan B\tan C) + \tan B+\tan C}{1\,(1-\tan B\tan C) - \tan A\,(\tan B+\tan C)}$$

$$\tan(A+B+C) = \frac{\tan A + \tan B + \tan C - \tan A\tan B\tan C}{1-(\tan A\tan B + \tan B\tan C + \tan C\tan A)}$$

$$\boxed{\tan(A+B+C) = \frac{S_1 - S_3}{1-S_2}}$$

$S_1 = \tan A + \tan B + \tan C$

$S_2 = \tan A\tan B + \tan B\tan C + \tan C\tan A$

$S_3 = \tan A\tan B\tan C$

$\tan(A+B+C+D)$

Also, $\tan(A + B + C) = \dfrac{\tan(A+B)+\tan C}{1-\tan(A+B)\tan C} = \dfrac{\dfrac{\tan A+\tan B}{1-\tan A\tan B}+\tan C}{1-\dfrac{\tan A+\tan B}{1-\tan A\tan B}\tan C}$

$$= \frac{\tan A+\tan B+\tan C-\tan A\tan B\tan C}{1-\tan B\tan C-\tan C\tan A-\tan A\tan B.} = \frac{s_1-s_3}{1-s_2}$$

$\tan A_1 + \tan A_2 + --- + \tan A_n$

$\tan A_1 \tan A_2 \tan A_3 + \tan A_1 \tan A_2 \tan A_4 + \tan A_1 \tan A_2 \tan A_5 + ---$

$$\tan(A_1+A_2+\ldots+A_n) = \frac{s_1-s_3+s_5-s_7+\ldots.}{1-s_2+s_4-s_6+\ldots.}$$

$= \sum_{i=1, i<j<k}^{n} \tan A_i \tan A_j \tan A_k$

where $s_1 = \tan A_1 + \tan A_2 + \ldots.. + \tan A_n$ = the sum of the tangent of the separate angles,

$s_2 = \tan A_1 \tan A_2 + \tan A_1 \tan A_3 + \ldots..$ = the sum of the tangents taken two at a time,

$s_3 = \tan A_1 \tan A_2 \tan A_3 + \tan A_2 \tan A_3 \tan A_4 + \ldots$

= the sum of the tangents taken three at a time, and so on.

(A) $\sin(A+B) = \sin A \cos B + \cos A \sin B$

(B) $\sin(A-B) = \sin A \cos B - \cos A \sin B$

(C) $\cos(A+B) = \cos A \cos B - \sin A \sin B$

(D) $\cos(A-B) = \cos A \cos B + \sin A \sin B$

(1) $\sin 2A = 2 \sin A \cos A$

(2) $\cos 2A = \cos^2 A - \sin^2 A$

$= 1 - 2\sin^2 A$

$= 2\cos^2 A - 1$

(3) $\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$

$\sin(A+A) = \sin A \cos A + \cos A \sin A$

$\sin 2A = 2 \sin A \cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$= \cos^2 A - (1 - \cos^2 A)$

$= \cos^2 A - 1 + \cos^2 A$

$\cos 2A = 2\cos^2 A - 1$

$= 1 - 2\sin^2 A$

$\tan(A+A) = \dfrac{\tan A + \tan A}{1 - \tan A \tan A} = \dfrac{2\tan A}{1-\tan^2 A}$

$$\sin 2A = 2\sin A\cos A = \frac{2\sin A\cos A}{1} = \frac{2\sin A\cos A}{\sin^2 A + \cos^2 A}$$

divide by $\cos^2 A$

$$\boxed{\sin 2A = \frac{2\tan A}{1+\tan^2 A}}$$

$$\cos 2A = \frac{\cos^2 A - \sin^2 A}{1} = \frac{\cos^2 A - \sin^2 A}{\cos^2 A + \sin^2 A} = \frac{1-\tan^2 A}{1+\tan^2 A}$$

$$\cos 2A = \frac{1-\tan^2 A}{1+\tan^2 A}$$

(A) $\sin(A+B) = \sin A \cos B + \cos A \sin B$

(B) $\sin(A-B) = \sin A \cos B - \cos A \sin B$

(C) $\cos(A+B) = \cos A \cos B - \sin A \sin B$

(D) $\cos(A-B) = \cos A \cos B + \sin A \sin B$

$B \to 2A$; $A \to A$

$$\sin(A+2A) = \sin A \cos 2A + \cos A \sin 2A$$

$$= \sin A(1-2\sin^2 A) + \cos A(2\sin A \cos A)$$

$$= \sin A - 2\sin^3 A + 2\sin A(1-\sin^2 A)$$

$$= \sin A - 2\sin^3 A + 2\sin A - 2\sin^3 A$$

$$\sin 3A = 3\sin A - 4\sin^3 A$$

$$\cos 3A = 4\cos^3 A - 3\cos A$$

T-RATIOS OF MULTIPLE & SUBMULTIPLE ANGLES :

Multiple angles are 2A, 3A, 4A etc & submultiple angles are $\frac{A}{2}, \frac{A}{4}, \frac{A}{8}$ etc.

$$\sin 2A = \sin(A + A) = 2\sin A \cos A = \frac{2\sin A\cos A}{\sin^2 A + \cos^2 A}$$

$$\Rightarrow \quad \sin 2A = \frac{2\tan A}{1 + \tan^2 A}$$

$$\cos(A + A) = \cos A \cos A - \sin A \,.\, \sin A$$

$$= \cos^2 A - \sin^2 A = \frac{\cos^2 A - \sin^2 A}{\cos^2 A + \sin^2 A}$$

$$\Rightarrow \quad \cos 2A = \frac{1 - \tan^2 A}{1 + \tan^2 A}$$

Also $\cos 2A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$

which gives $\sin A = \pm\sqrt{\frac{1 - \cos 2A}{2}}$ & $\cos A = \pm\sqrt{\frac{1 + \cos 2A}{2}}$

$$\tan 2A = \tan(A + A) = \frac{\tan A + \tan A}{1 - \tan^2 A} = \frac{2\tan A}{1 - \tan^2 A}$$

$$\cos 2A = 2\cos^2 A - 1$$

$$2A = y \Rightarrow A = \frac{y}{2}$$

$$\cos(y) = 2\cos^2\left(\frac{y}{2}\right) - 1$$

$$\cos y + 1 = 2\cos^2\left(\frac{y}{2}\right)$$

$$\pm\sqrt{\frac{\cos y + 1}{2}} = \cos\left(\frac{y}{2}\right)$$

$$\boxed{\cos\left(\frac{y}{2}\right) = \pm\sqrt{\frac{\cos y + 1}{2}}}$$

$$\cos 2A = 1 - 2\sin^2 A$$

$$\sin\left(\frac{y}{2}\right) = \pm\sqrt{\frac{1 - \cos y}{2}}$$

Trigonometric Ratios and Identities

Lecture - 7

E(1) Find the value of $-6 \sin 40° + 8 \sin^3 40°$ $[-\sqrt{3}]$

E(2) Find the value of $8 \sin^3 10° - 6\sin 10°$ (-1)

E(3) Let $f(\theta) = 16\cos^3 2\theta - 32\sin^3\theta - 12\cos 2\theta + 24\sin\theta$ and if $f\left(\frac{\pi}{30}\right) = 3\sqrt{a} - b, (a, b \in N)$, then value of $(a + b)$ is **[Ans. 6]**

E(4) P. T. $(4\cos^2 9° - 3)(4\cos^2 27° - 3) = \tan 9°$ → LHS

$$= \frac{\cos 9(4\cos^2 9 - 3)}{(\cos 9)}(4\cos^2 27 - 3)\frac{\cos 27}{\cos 27}$$

$$= \frac{(4\cos^3 9 - 3\cos 9)}{\cos 9} \cdot \frac{(4\cos^3 27 - 3\cos 27)}{(\cos 27)}$$

$$= \frac{\cancel{\cos 27}}{\cos 9} \cdot \frac{\cos 81}{\cancel{\cos 27}}$$

$$= \frac{\cos(90-9)}{\cos 9}$$

$$= \frac{\sin 9}{\cos 9} = \tan 9$$

E(5) Express cos 5A in terms of cos A.

E(6) P.T. $\tan 3A \cdot \tan 2A \cdot \tan A = \tan 3A - \tan 2A - \tan A$

E(7) Find the exact value of $\sqrt{3}$ cosec $20° - \sec 20°$.

E(8) Find the value of cosec $10° - \sqrt{3} \sec 10°$

E(9) Given $\sin\theta + \sin\phi = a$, $\cos\theta + \cos\phi = b$. Find the value of $\tan\left(\frac{\theta - \phi}{2}\right)$, $\sin(\theta + \phi)$, $\cos(\theta + \phi)$.

Hint (4)

$\cos 5A = \cos(2A + 3A)$

E(6) P.T. $\tan 3A \cdot \tan 2A \cdot \tan A = \tan 3A - \tan 2A - \tan A$

E(7) Find the exact value of $\sqrt{3}\ \operatorname{cosec} 20° - \sec 20°$.

E(8) Find the value of $\operatorname{cosec} 10° - \sqrt{3}\ \sec 10°$

6

$$\tan(A+2A) = \frac{\tan A + \tan 2A}{1 - \tan A \tan 2A}$$

$$(\tan 3A)(1 - \tan A \tan 2A) = \tan A + \tan 2A$$

$$\tan 3A - \tan 3A \tan A \tan 2A = \tan A + \tan 2A$$

$$\boxed{\tan 3A - \tan A - \tan 2A = \tan A \tan 2A \tan 3A}$$

Homework

Race 10 & 11 complete

Exercise 1 first 10 ques

B. B 1

E(3) Let $f(\theta) = 16\cos^3 2\theta - 32\sin^3\theta - 12\cos 2\theta + 24\sin\theta$ and if $f\left(\frac{\pi}{30}\right) = 3\sqrt{a} - b, (a, b \in N)$, then value of (a + b) is **[Ans. 6]**

$$f(\theta) = \underbrace{16\cos^3 2\theta}\ \underbrace{-32\sin^3\theta}\ \underbrace{-12\cos 2\theta}\ \underbrace{+24\sin\theta}$$

$$= 4\left[4\cos^3 2\theta - 3\cos 2\theta\right] + 8\left[-4\sin^3\theta + 3\sin\theta\right]$$

$$f(\theta) = 4\left[\cos 3(2\theta)\right] + 8\left[\sin 3\theta\right]$$

$$f(\theta) = 4\left[\cos 6\theta\right] + 8(\sin 3\theta)$$

$$f\left(\frac{\pi}{30}\right) = 4\cos\left(6\frac{\pi}{30}\right) + 8\sin\left(\frac{3\pi}{30}\right)$$

$$= 4\cos(36^\circ) + 8\sin 18^\circ$$

$$= 4\left(\frac{\sqrt{5}+1}{4}\right) + 8\left(\frac{\sqrt{5}-1}{4}\right) = \sqrt{5}+1 + 2\sqrt{5} - 2$$

$$= 3\sqrt{5} - 1$$

$$\cos(5A) = \cos(3A + 2A)$$

$$= \cos 3A \cos 2A - \sin 3A \sin 2A$$

$$= (4\cos^3 A - 3\cos A)(2\cos^2 A - 1) - (3\sin A - 4\sin^3 A)$$

$$(2\sin A \cos A)$$

$$= (8\cos^5 A - 6\cos^3 A - 4\cos^3 A + 3\cos A) - (3 - 4\sin^2 A)$$

$$(2\sin^2 A \cos A)$$

$$= (8\cos^5 A - 10\cos^3 A + 3\cos A) - (3 - 4(1 - \cos^2 A))$$

$$(2\cos A(1 - \cos^2 A))$$

$$= (8\cos^5 A - 10\cos^3 A + 3\cos A) - (3 - 4 + 4\cos^2 A)$$

$$(2\cos A - 2\cos^3 A)$$

$$= (8\cos^5 A - 10\cos^3 A + 3\cos A) - (-1 + 4\cos^2 A)(2\cos A - 2\cos^3 A)$$

$$= (8\cos^5 A - 10\cos^3 A + 3\cos A) - (-2\cos A + 2\cos^3 A + 8\cos^3 A - 8\cos^5 A)$$

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$$= (8\cos^5 A - 10\cos^3 A + 3\cos A) - (-2\cos A + 2\cos^3 A + 8\cos^3 A - 8\cos^5 A)$$

$$= \underline{8\cos^5 A} - \underline{10\cos^3 A} + 3\cos A + 2\cos A \underline{-10\cos^3 A} + \underline{8\cos^5 A}$$

$$\cos 5A = 16\cos^5 A - 20\cos^3 A + 5\cos A$$

$$\sin 5A = 16\sin^5 A - 20\sin^3 A + 5\sin A$$

(7) $\sqrt{3}\,\text{cosec}\,20^\circ - \sec 20^\circ = \dfrac{\sqrt{3}\,(2)}{(2)\sin 20} - \dfrac{1}{\cos 20}$

$$= \frac{(\sqrt{3}/2)}{\left(\frac{1}{2}\right)\sin 20^\circ} - \frac{1}{\cos 20^\circ}$$

$$= \frac{\sin 60^\circ}{\cos 60 \sin 20} - \frac{1}{\cos 20}$$

$$= \frac{\sin 60 \cos 20 - \cos 60 \sin 20}{\cos 60 \sin 20 \cos 20}$$

$$= \frac{(2)\sin(60-20)}{(2)\cos 60\,\underbrace{\sin 20 \cos 20}} = \frac{2\sin 40}{\cos 60 \sin(2(20))}$$

$$= \frac{2\sin 40}{\cos 60 \sin 40} = \frac{2}{\cos 60} = 4$$ Answer

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9) $$\boxed{\begin{aligned} \sin\theta + \sin\phi &= a \\ \cos\theta + \cos\phi &= b \end{aligned}}$$

$$\xrightarrow{\text{square}} \sin^2\theta + \sin^2\phi + 2\sin\theta\sin\phi = a^2$$

$$\xrightarrow{\text{square}} \cos^2\theta + \cos^2\phi + 2\cos\theta\cos\phi = b^2$$

add $\quad 1 + 1 + 2\sin\theta\sin\phi + 2\cos\theta\cos\phi = a^2+b^2$

$$2 + 2(\sin\theta\sin\phi + \cos\theta\cos\phi) = a^2+b^2$$

$$2(\cos(\theta-\phi)) = a^2+b^2-2$$

$$\cos(\theta-\phi) = \frac{a^2+b^2-2}{2}$$

$$\tan\left(\frac{\theta-\phi}{2}\right) = \frac{\sin\left(\frac{\theta-\phi}{2}\right)}{\cos\left(\frac{\theta-\phi}{2}\right)}$$

$$= \sqrt{\frac{1-\cos(\theta-\phi)}{1+\cos(\theta-\phi)}}$$

$$\tan\left(\frac{\theta-\phi}{2}\right) = \sqrt{\frac{1-\cos(\theta-\phi)}{1+\cos(\theta-\phi)}}$$

$$= \sqrt{\frac{1-\frac{a^2+b^2-2}{2}}{1+\frac{a^2+b^2-2}{2}}}$$

$$= \sqrt{\frac{(2-a^2-b^2+2)/\cancel{2}}{(2+a^2+b^2-2)/\cancel{2}}}$$

$$\tan\left(\frac{\theta-\phi}{2}\right) = \sqrt{\frac{4-a^2-b^2}{a^2+b^2}}$$

$\sin\theta + \sin\phi = a \Rightarrow 2\sin\left(\frac{\theta+\phi}{2}\right)\cos\left(\frac{\theta-\phi}{2}\right) = a$

$\cos\theta + \cos\phi = b \Rightarrow 2\cos\left(\frac{\theta+\phi}{2}\right)\cos\left(\frac{\theta-\phi}{2}\right) = b$

$\downarrow$ divide

$$\boxed{\tan\left(\frac{\theta+\phi}{2}\right) = \frac{a}{b}}$$

$$\sin(\theta+\phi) = \frac{2\tan\left(\frac{\theta+\phi}{2}\right)}{1+\tan^2\left(\frac{\theta+\phi}{2}\right)}$$

$$\cos(\theta+\phi) = \frac{1-\tan^2\left(\frac{\theta+\phi}{2}\right)}{1+\tan^2\left(\frac{\theta+\phi}{2}\right)}$$

$$\sin(\theta+\phi) = \frac{2ab}{a^2+b^2}$$

$$\cos(\theta+\phi) = \frac{b^2-a^2}{b^2+a^2}$$

T- ratios of some standard angles ! → $(18^\circ, 36^\circ, 72^\circ, 54^\circ)$

$$\sin 18^\circ = \sin\left(\frac{\pi}{10}\right) = \frac{\sqrt{5}-1}{4} = \cos 72^\circ = \cos\left(\frac{2\pi}{5}\right)$$

Let $\theta = 18^\circ$

$5\theta = 90^\circ$

$2\theta + 3\theta = 90^\circ$

$2\theta = 90 - 3\theta$

take sine to both sides

$\sin 2\theta = \sin(90 - 3\theta)$

$2\sin\theta\cos\theta = \cos 3\theta$

$2\sin\theta\cos\theta = 4\cos^3\theta - 3\cos\theta$

$2\sin\theta\cancel{\cos\theta} = \cancel{\cos\theta}(4\cos^2\theta - 3)$

$2\sin\theta = 4\cos^2\theta - 3$

$2\sin\theta = 4(1-\sin^2\theta) - 3$

$2\sin\theta = 4 - 4\sin^2\theta - 3$

$2\sin\theta = 1 - 4\sin^2\theta$

$4\sin^2\theta + 2\sin\theta - 1 = 0$

$$\sin\theta = \frac{-2 \pm \sqrt{4+16}}{2(4)}$$

$$\sin\theta = \frac{-2 \pm 2\sqrt{5}}{8}$$

$\boxed{\sin\theta = \frac{-1+\sqrt{5}}{4}}$ OR $\boxed{\frac{-1-\sqrt{5}}{4}}$

Answer   Rejected

$$\boxed{\sin(18^\circ) = \frac{\sqrt{5}-1}{4}}$$

$$\sin(90-72) = \frac{\sqrt{5}-1}{4}$$

$$\boxed{\cos 72 = \frac{\sqrt{5}-1}{4}}$$

$$\boxed{\cos 36 = \frac{\sqrt{5}+1}{4}}$$

$$\cos 36 = \cos(2(18))$$

$$= 1 - 2\sin^2 18$$

$$= 1 - 2\left(\frac{\sqrt{5}-1}{4}\right)^2$$

$$= 1 - \frac{\cancel{2}(5+1-2\sqrt{5})}{\cancel{16}\ 8}$$

$$\cos 36 = \frac{8-6+2\sqrt{5}}{8} = \frac{\sqrt{5}+1}{4}$$

$$\cos 18 = \sqrt{1-\sin^2 18}$$

$$= \sqrt{1-\left(\frac{\sqrt{5}-1}{4}\right)^2}$$

$$= \sqrt{1-\frac{5+1-2\sqrt{5}}{16}}$$

$$= \sqrt{\frac{16-6+2\sqrt{5}}{16}}$$

$$\boxed{\cos 18 = \frac{\sqrt{10+2\sqrt{5}}}{4}}$$

$$\cos(90-72) = \frac{\sqrt{10+2\sqrt{5}}}{4}$$

$$\boxed{\sin 72 = \frac{\sqrt{10+2\sqrt{5}}}{4}}$$

$$\boxed{\cos 36 = \frac{\sqrt{5}+1}{4}}$$

$$\cos(90-54) = \frac{\sqrt{5}+1}{4}$$

$$\boxed{\sin 54 = \frac{\sqrt{5}+1}{4}}$$

$$\sin 36 = \sqrt{1-\cos^2 36}$$

$$= \sqrt{1-\left(\frac{\sqrt{5}+1}{4}\right)^2}$$

$$= \sqrt{1-\frac{5+1+2\sqrt{5}}{16}}$$

$$= \sqrt{\frac{16-6-2\sqrt{5}}{16}}$$

$\sin(90-54) \leftarrow$

$$\boxed{\boxed{\sin 36} = \frac{\sqrt{10-2\sqrt{5}}}{4}}$$

$$\boxed{\cos 54 = \frac{\sqrt{10-2\sqrt{5}}}{4}}$$

① find $\cos^2 48 - \sin^2 12$

$\cos^2 A - \sin^2 B$

$= \cos(A+B)\cdot\cos(A-B)$

$= \cos(48+12)\cdot\cos(48-12)$

$= \cos 60 \cdot \cos 36$

$= \dfrac{\sqrt{5}+1}{8}$

② find $\sin(132^\circ)\ \sin(12^\circ)$

$= \frac{1}{2}\left[-\cos(144) + \underbrace{\cos(120)}\right]$

$= \frac{1}{2}\left[-\cos(180-36) + \cos 120\right]$

$\boxed{\dfrac{\sqrt{5}-1}{8}}$

Ans

ALLEN CAREER INSTITUTE KOTA (RAJASTHAN) path to success

(3) Prove that $\tan 6^\circ \tan 42^\circ \tan 66^\circ \tan 78^\circ = 1$

$$LHS = \tan 6^\circ \tan(60+6) \cdot \frac{\tan(60-6)}{\tan(60-6)} \cdot \boxed{\tan(60-18)\tan(60+18)} \frac{\tan 18}{\tan 18}$$

$$= \frac{\tan 18}{\tan 54} \cdot \frac{\tan 54}{\tan 18} = 1$$

formula

$$\tan\theta \tan(60-\theta)\tan(60+\theta) = \tan 3\theta$$

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Note : $\rightarrow$ If $A+B=45$

$A = 45 - B$

$\tan A = \tan(45 - B)$

$$\tan A = \frac{\tan 45 - \tan B}{1+\tan 45 \tan B}$$

$$\tan A = \frac{1-\tan B}{1+\tan B}$$

$\tan A + \tan A \tan B = 1 - \tan B$

$\tan A(1+\tan B) + \tan B = 1$

$+1 \qquad +1$

$\tan A(\tan B+1) + 1(\tan B+1) = 2$

$$\boxed{(1+\tan B)(1+\tan A)=2}$$

$$\left(1+\frac{1}{\cot B}\right)\left(1+\frac{1}{\cot A}\right) = 2$$

$(\cot B+1)(1+\cot A) = 2$

$\cot A \cot B$

$1+\cot B+\cot A+\cot A\cot B$

$= 2\cot A\cot B$

$1+\cot B+\cot A-\cot A\cot B = 0$

$-2 \qquad\qquad -2$

$\cot A(1-\cot B)$

$-1+\cot B+\cot A(1-\cot B) = -2$

$-1(1-\cot B)+\cot A(1-\cot B) = -2$

$(1-\cot B)(-1+\cot A) = -2$

$(1-\cot B)(1-\cot A) = 2$

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Q $\sin(22.5^\circ) = ?$

$\sin\left(\frac{45}{2}\right)$

$$\boxed{\cos(22.5^\circ) = \frac{\sqrt{2+\sqrt{2}}}{2} = \sin(67.5^\circ)}$$

$\cos 2\theta = 2\cos^2\theta - 1$

$\cos 2\theta = 1 - 2\sin^2\theta$

$\cos 45 = 1 - 2\sin^2\left(\frac{45}{2}\right)$ | $2\theta = 45$, $\theta = \frac{45}{2}$

$\frac{1}{\sqrt{2}} = 1 - 2\sin^2\left(\frac{45}{2}\right)$

$2\sin^2\left(\frac{45}{2}\right) = 1 - \frac{1}{\sqrt{2}}$

$2\sin^2\left(\frac{45}{2}\right) = \frac{\sqrt{2}-1}{\sqrt{2}}$

$\sin^2\left(\frac{45}{2}\right) = \frac{\sqrt{2}-1}{2\sqrt{2}}$

$\sin\left(\frac{45}{2}\right) = \sqrt{\frac{\sqrt{2}-1}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}}$

$$\cos(67.5^\circ) = \boxed{\sin(22.5^\circ) = \frac{\sqrt{2-\sqrt{2}}}{2}}$$

Trigonometric Identities in a triangle (conditional Identities):-

If A, B, C are angles of a triangle $\underline{A+B+C} = \pi$

$A+B+C = \pi$

$2A+2B+2C = 2\pi$

$2A+2B = 2\pi - 2C$

(i) $\sin(\underbrace{A+B}) = \sin(\pi - C) = \sin C$

$\sin(A+B) = \sin C$

(ii) $\cos(A+B) = \cos(\pi - C) = -\cos C$

$\cos(A+B) = -\cos C$

(iii) $\tan(A+B) = \tan(\pi - C) = -\tan C$

$\tan(A+B) = -\tan C$

(iv) $\sin(2A+2B) = \underline{\sin}(2\pi - 2C) = -\sin 2C$

$\sin(2A+2B) = -\sin 2C$

(v) $\cos(2A+2B) = \cos(2\pi-2C) = \cos 2C$

$$\boxed{\cos(2A+2B) = \cos 2C}$$

(vi) $\tan(2A+2B) = \tan(2\pi-2C) = -\tan 2C$

$$\boxed{\tan(2A+2B) = -\tan 2C}$$

(vii) $\sin\left(\frac{A+B}{2}\right) = \sin\left(\frac{\pi-C}{2}\right) = \sin\left(\frac{\pi}{2}-\frac{C}{2}\right) = \cos\frac{C}{2}$

$$\boxed{\sin\left(\frac{A+B}{2}\right) = \cos\frac{C}{2}}$$

(viii) $\cos\left(\frac{A+B}{2}\right) = \cos\left(\frac{\pi-C}{2}\right) = \sin\frac{C}{2}$

$$\boxed{\cos\frac{A+B}{2} = \sin\frac{C}{2}}$$

(ix) $\tan\left(\frac{A+B}{2}\right) = \tan\left(\frac{\pi-C}{2}\right) = \cot\frac{C}{2}$

$$\boxed{\tan\left(\frac{A+B}{2}\right) = \cot\frac{C}{2}}$$

(x) $\underbrace{\sin 2A + \sin 2B} + \sin 2C = 4\sin A \sin B \sin C$

$\underline{LHS} = 2\sin\left(\frac{2A+2B}{2}\right)\cos\left(\frac{2A-2B}{2}\right) + \sin(2C)$

$A+B+C=\pi$

$C = \pi-(A+B)$

$= 2\sin(A+B)\cos(A-B) + \sin 2C$

$= 2\sin(\pi - C)\cos(A-B) + \sin 2C$

$= 2\sin C\cos(A-B) + 2\sin C\cos C$

$= 2\sin C\left[\cos(A-B) + \cos C\right]$

$= 2\sin C\left[\cos(A-B) + \cos(\pi-(A+B))\right]$

$= 2\sin C\left[\underline{\cos(A-B)} - \cos(\underline{A+B})\right]$

$= 2\sin C\left[2\sin A\sin B\right] = 4\sin A\sin B\sin C = RHS.$

Hence Proved

(xi) $\cos 2A + \cos 2B + \cos 2C = -1 - 4 \cos A \cos B \cos C$ (HW)

(xii) $\sum \tan A = \prod \tan A$ $\quad A+B+C=\pi$

$\tan A + \tan B + \tan C = \tan A \tan B \tan C$

$A+B+C = \pi$

$A+B = \pi - C$

$\tan(A+B) = \tan(\pi - C)$

$$\frac{\tan A + \tan B}{1 - \tan A \tan B} = -\tan C$$

$\tan A + \tan B = -\tan C + \tan A \tan B \tan C$

$\tan A + \tan B + \tan C = \tan A \tan B \tan C$

$\sum \tan A = \prod \tan A$

(xiii) $\sum \cot A \cot B = 1$

(xiv) $\sum \tan \frac{A}{2} \tan \frac{B}{2} = 1$

(xv) $\sum \cot\left(\frac{A}{2}\right) = \prod \cot\left(\frac{A}{2}\right)$