Race - 15

1. $16\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}\cos\frac{16\pi}{15}$

(A) 0 (B) –1 (C) –2 (D) 1

$$\frac{8\times\left(2\times\sin\frac{2\pi}{15}\cos\frac{2\pi}{15}\right)\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}\cos\frac{16\pi}{15}}{\sin\frac{2\pi}{15}}$$

$$=\frac{4\times\left(2\times\sin\frac{4\pi}{15}\cos\frac{4\pi}{15}\right)\cos\frac{8\pi}{15}\cos\frac{16\pi}{15}}{\sin\frac{2\pi}{15}}$$

$$=\frac{2\times2\sin\frac{8\pi}{15}\cos\frac{8\pi}{15}\cos\frac{16\pi}{15}}{\sin\frac{2\pi}{15}}$$

$$=\frac{2\sin\frac{16\pi}{15}\cos\frac{16\pi}{15}}{\sin 2\pi/15}$$

$$= \frac{\sin \frac{32\pi}{15}}{\sin \frac{2\pi}{15}} = \frac{\sin\left(2\pi + \frac{2\pi}{15}\right)}{\sin\left(\frac{2\pi}{15}\right)}$$

$$= \frac{\sin \frac{2\pi}{15}}{\sin \frac{2\pi}{15}} = 1 \text{ Q}$$

2. The value of $64\sqrt{3}\sin\frac{\pi}{48}\cos\frac{\pi}{48}\cos\frac{\pi}{24}\cos\frac{\pi}{12}\cos\frac{\pi}{6}$ is

(A) 8 (B) 6 ✓ (C) 4 (D) 12

$$= 32\sqrt{3}\left(2\sin\frac{\pi}{48}\cos\frac{\pi}{48}\right)\cos\frac{\pi}{24}\cos\frac{\pi}{12}\cos\frac{\pi}{6}$$

$$= 16\sqrt{3} \times 2 \cdot \sin\frac{\pi}{24} \times \cos\frac{\pi}{24}\cos\frac{\pi}{12}\cos\frac{\pi}{6}$$

$$= 8\sqrt{3} \times 2 \cdot \sin\frac{\pi}{12}\cos\frac{\pi}{12} \cdot \cos\frac{\pi}{6}$$

$$= 8\sqrt{3} \cdot \sin\frac{\pi}{6}\cos\frac{\pi}{6} = \overset{2}{\cancel{8}}\sqrt{3} \cdot \frac{1}{\cancel{2}} \times \frac{\sqrt{3}}{\cancel{2}}$$

$$= 6 \text{ Q}$$

3. If $P = \text{cosec}\frac{\pi}{8} + \text{cosec}\frac{2\pi}{8} + \text{cosec}\frac{3\pi}{8} + \text{cosec}\frac{13\pi}{8} + \text{cosec}\frac{14\pi}{8} + \text{cosec}\frac{15\pi}{8}$ & $Q = 8\sin\frac{\pi}{18}\sin\frac{5\pi}{18}\sin\frac{7\pi}{18}$, then value of P + Q is

(A) 0 (B) 1 (C) 2 (D) 3

$$P = \text{cosec}\frac{\pi}{8} + \text{cosec}\frac{2\pi}{8} + \text{cosec}\frac{3\pi}{8} + \text{cosec}\left(2\pi - \frac{3\pi}{8}\right) + \text{cosec}\left(2\pi - \frac{2\pi}{8}\right) + \text{cosec}\left(2\pi - \frac{\pi}{8}\right)$$

$$P = \text{cosec}\frac{\pi}{8} + \text{cosec}\frac{2\pi}{8} + \text{cosec}\frac{3\pi}{8} - \text{cosec}\frac{3\pi}{8} - \text{cosec}\frac{2\pi}{8} - \text{cosec}\frac{\pi}{8}$$

$$P = 0$$

$$Q = 8\sin\frac{\pi}{18}\sin\frac{5\pi}{18}\sin\frac{7\pi}{18}$$

$$Q = 8\sin 10^\circ \sin 50^\circ \sin 70^\circ$$

$$= 4(2\sin 10^\circ \sin 50^\circ)\sin 70^\circ$$

$$= 4(\cos 40 - \cos 60)\sin 70$$

$$= 4\cos 40 \cdot \sin 70 - 2\sin 70$$

$$= 2(\sin 110 + \sin 30) - 2\sin 70$$

$$= \cancel{2\sin(180-70)} + 2\times\frac{1}{2} - \cancel{2\sin 70}$$

$$= 1$$

$$\therefore P+Q = 1$$

4. Let $f(\theta) = \sum_{r=1}^{9}(\sin(2r-1)\theta + \cos 2r\theta)$ and $\sin\frac{\pi}{18} = a$, then $f\left(\frac{\pi}{18}\right)$ is equal to

(A) $\frac{1+a}{a}$ (B) $\frac{a}{1+a}$ (C) $\frac{1-2a}{1+a}$ (D) $\frac{1-a}{a}$

$$f(\theta) = \sin\theta + \cos 2\theta + \sin 3\theta + \cos 4\theta + \sin 5\theta + \cos 6\theta \dots \sin 17\theta + \cos 18\theta$$

$$\therefore f(\theta) = (\sin\theta + \sin 3\theta \dots \sin 17\theta) + (\cos 2\theta + \cos 4\theta \dots \cos 18\theta)$$

$$f(\theta) = \frac{\sin \frac{9\cdot(2\theta)}{2}}{\sin \frac{2\theta}{2}} \sin\left(\frac{\theta + 17\theta}{2}\right)$$

$$+ \frac{\sin 9\left(\frac{2\theta}{2}\right)}{\sin\left(\frac{2\theta}{2}\right)} \cos\left(\frac{2\theta + 18\theta}{2}\right)$$

$$f(\theta) = \frac{\sin 9\theta}{\sin\theta}\left[\sin 9\theta + \cos 10\theta\right]$$

$$f\left(\frac{\pi}{18}\right) = \frac{\sin \pi/2}{\sin \pi/18}\left[\sin\frac{\pi}{2} + \cos\left(\frac{\pi}{2} + \frac{\pi}{18}\right)\right]$$

$$= \frac{1}{a}\left[1 - \sin\frac{\pi}{18}\right]$$

$$= \frac{1-a}{a} \quad A$$

5. In the interval $\left(\frac{31\pi}{4}, \frac{33\pi}{4}\right)$, the expression $\frac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta}$ is

(A) always positive (B) always negative (C) some times positive (D) can't say

$$\frac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta} \times \frac{\sin\theta+\cos\theta}{\sin\theta+\cos\theta}$$

$$= \frac{1+\sin 2\theta}{-\cos 2\theta}$$

Now

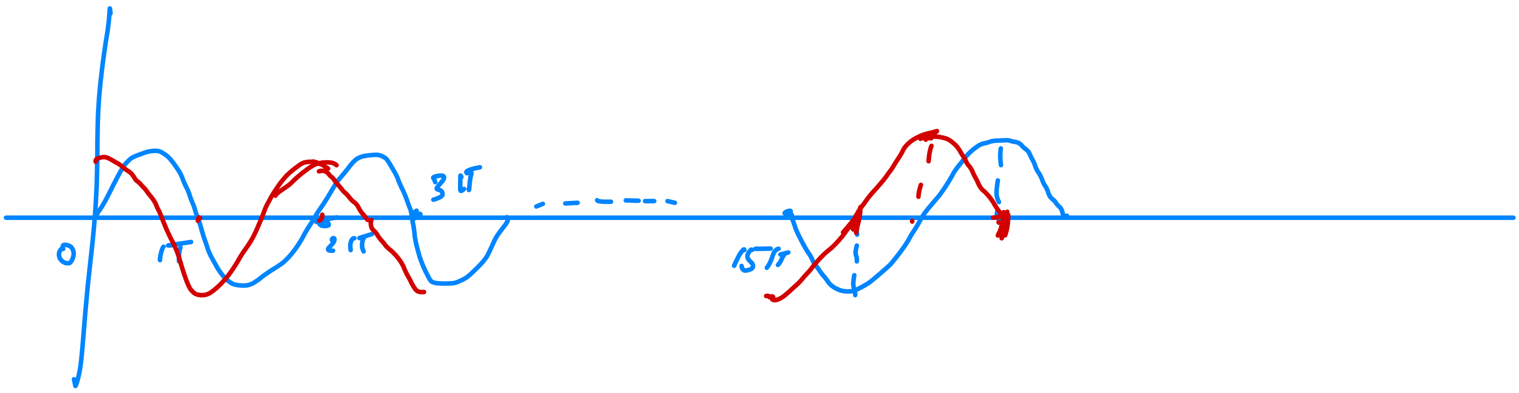
$$\theta \in \left(\frac{31\pi}{4}, \frac{33\pi}{4}\right)$$

$$2\theta \in \left(\frac{31\pi}{2}, \frac{33\pi}{2}\right)$$

$$2\theta \in \left(15\pi + \frac{\pi}{2}, 16\pi + \frac{\pi}{2}\right)$$

$$-1 < \sin 2\theta < 1$$

$$0 < 1+\sin 2\theta < 2$$



$$0 < \cos 2\theta < 1$$

$\therefore$ it is $-$ve

6. The exact value of $\dfrac{60\sin 82^\circ \sin 51^\circ \sin 47^\circ}{\sin 16^\circ + \sin 78^\circ + \sin 86^\circ}$ is -

(A) 12 (B) 15 (C) 16 (D) 20

$$= \frac{60\sin(90-8)\sin(90-39)\sin(90-43)}{\sin 16 + \sin 78 + \sin 86}$$

$$= \frac{60\cos 8\cos 39\cos 43}{\sin 16 + \sin 78 + \sin 86} \quad \text{as } 16+78+86=180$$

$$\therefore = \frac{60\cos 8\,\cancel{\cos 39\cos 43}}{4\cancel{\cos 8\cos 39}\cos 43} = 15$$

7. $\cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{3\pi}{15}\cos\frac{4\pi}{15}\cos\frac{5\pi}{15}\cos\frac{6\pi}{15}\cos\frac{7\pi}{15} =$

~~•(A)~~ $\frac{1}{128}$ ~~•(B)~~ $e^{-7\log 2}$ ~~•(C)~~ 2^{-7} (D) 1551

$$= \frac{2\sin\frac{\pi}{15}\cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cdots\cos\frac{7\pi}{15}}{2\sin\frac{\pi}{15}}$$

$$= \frac{2\sin\frac{2\pi}{15}\cos\frac{2\pi}{15}\cos\frac{3\pi}{15}\cdots\cos\frac{7\pi}{15}}{2^2\sin\frac{\pi}{15}}$$

$$= \frac{2\sin\frac{4\pi}{15}\cos\frac{4\pi}{15}\cos\frac{3\pi}{15}\cos\frac{5\pi}{15}\cos\frac{6\pi}{15}\cos\frac{7\pi}{15}}{2^3\sin\frac{\pi}{15}}$$

$$= \frac{\sin\frac{8\pi}{15}\cos\frac{3\pi}{15}\cos\frac{5\pi}{15}\cos\frac{6\pi}{15}\cos\frac{7\pi}{15}}{2^3\sin\frac{\pi}{15}}$$

$$\cos\frac{7\pi}{15} = \cos\left(\pi - \frac{8\pi}{15}\right) = -\cos\frac{8\pi}{15}$$

$$\therefore \ = \frac{-2\sin\frac{8\pi}{15}\cos\frac{8\pi}{15}\cos\frac{3\pi}{15}\cos\frac{5\pi}{15}\cos\frac{6\pi}{15}}{2\cdot 2^3\sin\frac{\pi}{15}}$$

$$= \frac{-\sin\frac{16\pi}{15}\cos\frac{3\pi}{15}\cos\frac{\pi}{3}\cos\frac{6\pi}{15}}{2^4\sin\frac{\pi}{15}}$$

$$= \frac{-\left(-\cancel{\sin\frac{\pi}{15}}\right)\cos\frac{3\pi}{15}\cos\frac{\pi}{3}\cos\frac{6\pi}{15}}{2^4\cancel{\sin\frac{\pi}{15}}}$$

$$= \frac{1}{2^4}\cos\frac{3\pi}{15}\times\frac{1}{2}\cos\frac{6\pi}{15}$$

$$= \frac{1}{2^5}\cdot\frac{2\sin\frac{3\pi}{15}\cdot\cos\frac{3\pi}{15}\cos\frac{6\pi}{15}}{2\sin\frac{3\pi}{15}}$$

$$= \frac{1}{2^6} \cdot \frac{2 \sin \frac{6\pi}{15} \cdot \cos \frac{6\pi}{15}}{2 \sin \frac{3\pi}{15}}$$

$$= \frac{1}{2^7} \, \frac{\sin \frac{12\pi}{15}}{\sin \frac{3\pi}{15}}$$

$$= \frac{1}{2^7} \, \frac{\cancel{\sin\left(\pi - \frac{3\pi}{15}\right)}}{\cancel{\sin \frac{3\pi}{15}}} = \frac{1}{128}$$

$$= 2^{-7}$$

$$\text{or} \quad e^{-7 \ln 2} = e^{\ln 2^{-7}} = 2^{-7}$$

8. In ΔABC, $\tan B + \tan C = 5$ and $\tan A \tan C = 3$, then

(A) ΔABC is acute angled triangle

(B) ΔABC is obtuse angled triangle

(C) sum of all possible values of $\tan A$ is 10

(D) sum of all possible values of $\tan A$ is 9

$$\tan A + \tan B + \tan C = \tan A \tan B \tan C$$

$$\tan A + 5 = 3 \tan B$$

$$\tan A + 5 = 3(5 - \tan C)$$

$$\tan A + 5 = 3\left(5 - \frac{3}{\tan A}\right)$$

$$\tan^2 A + 5\tan A = 15 \tan A - 9$$

$$\tan^2 A - 10 \tan A + 9 = 0$$

Sum of roots $= 10$ (C)

$$\tan A = 1, 9$$

If $\tan A = 1$, $\tan B = 2$, $\tan C = 3$

If $\tan A = 9$, $\tan B = 14/3$, $\tan C = 1/3$

$\therefore$ acute angle for both

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9. **Column-I** **Column-II**

(A) If the value of (tan 18°)(sin 36°)(cos 54°)(tan 72°)(tan 108°) × (cos 126°)(sin 144°)(tan162°)(cos180°) is k $\sin^2 18°$, then 'k' has the value equal to

(B) If $\sin^3 x \cos 3x + \cos^3 x \sin 3x = \frac{3}{8}$, then the value of sin 4x is

(C) For all permissible values of x, the maximum value of the $f(x) = \frac{5\sin^3 x \cos x}{\tan^2 x + 1}$, is

(P) $\frac{1}{2}$

(Q) $\frac{3}{4}$

(R) $\frac{5}{4}$

(S) $\frac{5}{8}$

$= \tan 18° \sin 36° \cos 54° \tan 72° \tan 108°$

$\cos 126° \sin 144° \tan 162° \cos 180°$

$\tan 18° \sin 36° \cos(90° - 36°) \tan(90° - 18°)$

$\tan(90 + 18) \cos(90 + 36°) \sin(180 - 36)$

$\tan(180 - 18)(-1)$

$= \cancel{\tan} 18 \cdot \sin 36 \cdot \sin 36 \cdot \cancel{\cot} 18 \cdot (-\cancel{\cot} 18)$

$(-\sin 36) \sin 36 \cdot \cancel{\tan} 18$

$= 4\sin^4 36° = (\sin^2 36°)^2$

$= \left(\frac{10-2\sqrt5}{16}\right)^2 = \frac{(2\sqrt5)^2}{4^2}\left[\frac{\sqrt5-1}{4}\right]^2$

$= \frac{4\times5}{4\times4} \cdot \sin^2 18$

$\therefore K = 5/4$ (a)

B) $4\sin^3 x\cos 3x + 4\cos^3 x\sin 3x = \frac{3}{2}$

$(3\sin x - \sin 3x)\cos 3x + (\cos 3x + 3\cos x)\sin 3x$

$= \frac{3}{2}$

$3\left[\sin x\cos 3x + \cos x\cdot\sin 3x\right]$

$- \cancel{\sin 3x\cos 3x} + \cancel{\cos 3x\sin 3x} = \frac{3}{2}$

$$3\left[\sin(x+3x)\right] = \frac{3}{2}$$

$$\therefore \quad \sin 4x = \frac{1}{2} \quad \mathcal{L}$$

c) $$\frac{5\sin^3 x \cos x}{\sec^2 x}$$

$$= 5\sin^3 x \cos^3 x$$

$$= \frac{5}{8}(2\sin x \cos x)^3$$

$$= \frac{5}{8}\sin^3 2x$$

$$\max = \frac{5}{8}$$

10.	Column-I	Column-II
(A)	If $2^{2013} - 2^{2012} - 2^{2011} + 2^{2010} = k.2^{2010}$ then k form pythagorean triplet with	(P) 3
(B)	If $N = \dfrac{1}{2\sin 10°} - 2\sin 70°$; then antilog$_5$N is twin prime with	(Q) 4
(C)	The value of $\dfrac{\sqrt{1-\sin\frac{\pi}{5}}}{\sqrt{1+\sin\frac{\pi}{5}}} + \dfrac{2\sin\frac{\pi}{10}}{\sin\frac{\pi}{10}+\cos\frac{\pi}{10}}$ is relatively prime with	(R) 5
(D)	If $x = \sqrt[3]{\sqrt{108}+10} - \sqrt[3]{\sqrt{108}-10}$ then $x^3 + 6x$ is divisible by	(S) 6
		(T) 7

A) $2^{2010}\left[2^3 - 2^2 - 2 + 1\right] = K\, 2^{2010}$

$K = 8 - 4 - 2 + 1$

$K = 3$

It forms pythagoras triplet with 4 & 5

B) $N = \dfrac{1}{2\sin 10} - 2\sin 70$

$= \dfrac{1 - 4\sin 10 \sin(70)}{2\sin 10}$

$$= \frac{1 - 2\left[\cos(60) - \cos 80\right]}{2 \sin 10}$$

$$= \frac{1 - 2\left[\frac{1}{2} - \cos(90 - 10)\right]}{2 \sin 10}$$

$$= \frac{2 \sin 10}{2 \sin 10} = 1$$

$$\text{antilog}_5 1 = L$$

$$\log_5 L = 1$$

$$L = 5$$

$\therefore$ P, T

c) $$\frac{\sqrt{1-\sin\frac{\pi}{5}}}{\sqrt{1+\sin\frac{\pi}{5}}} + \frac{2\sin\frac{\pi}{10}}{\sin\frac{\pi}{10}+\cos\frac{\pi}{10}}$$

Rationalize

$$\frac{\left(1-\sin \pi/5\right)}{\cos \pi/5} + \frac{2\sin \pi/10}{\left(\sin \pi/_{10}+\cos \pi/_{10}\right)} \times \frac{\left(\sin \pi/_{10}-\cos \pi/_{10}\right)}{\left(\sin \pi/_{10}-\cos \pi/_{10}\right)}$$

$$= \frac{\left(1-\sin \pi/5\right)}{\cos \pi/5} + \frac{2\sin^2 \pi/10 - 2\sin\frac{\pi}{10}\cos\frac{\pi}{10}}{-\cos \pi/5}$$

$$= \frac{1-\cancel{\sin \pi/5} + \cancel{\sin \pi/5} - 2\sin^2 \pi/10}{\cos \pi/5}$$

$$= \frac{1-2\sin^2 \pi/10}{\cos \pi/5} = \frac{\cos \pi/5}{\cos \pi/5}$$

$$= 1$$

1 is relative prime with all

D)

$$x^3 = (\sqrt{108}+10) - (\sqrt{108}-10)$$

$$- 3\sqrt[3]{(\sqrt{108}+10)(\sqrt{108}-10)} \cdot x$$

$$x^3 = 20 - 6x$$

$$x^3 + 6x = 20$$

is div by 4, 5

11 $\cos 0 + \cos 2\theta + \cos 4\theta + \cos 6\theta \ldots \cos 10\theta$

$$= \frac{\sin\left(6 \cdot \frac{(2\theta)}{2}\right)}{\sin \frac{2\theta}{2}} \cdot \cos\left(\frac{0+10\theta}{2}\right)$$

$$= \frac{\sin 6\theta}{\sin \theta} \cos 5\theta$$

$m = 5, \quad n = 6$

$m + n = 11$

12. Let $f(\theta) = \cot\left(\frac{\theta}{2}\right)(\sec\theta - 1)(1 + \sec 2\theta)(\sec 4\theta - 1)$ and $f\left(\frac{\pi}{16}\right) = a - \sqrt{b}$ (where a & b are coprime numbers), then the value of (5a – b) is

$$\frac{\cos\theta/2}{\sin\theta/2}\left(\frac{1-\cos\theta}{\cos\theta}\right)\left(\frac{\cos 2\theta + 1}{\cos 2\theta}\right)\left(\frac{1-\cos 4\theta}{\cos 4\theta}\right)$$

$$\frac{\cos\theta/2}{\cancel{\sin\theta/2}}\left(\frac{2\sin^{\cancel{2}}\theta/2}{\cancel{\cos\theta}}\right)\left(\frac{2\cos^{\cancel{2}}\theta}{\cos 2\theta}\right)(\sec 4\theta - 1)$$

$$f(\theta) = \frac{\cos\frac{\theta}{2} \cdot 2\sin\theta/2 \cdot 2\cos\theta \cdot (\sec 4\theta - 1)}{\cos 2\theta}$$

$$f(\theta) = \frac{\sin 2\theta}{\cos 2\theta}(\sec 4\theta - 1)$$

$f(\theta) = \tan 2\theta(\sec 4\theta - 1)$

$f\left(\frac{\pi}{16}\right) = \tan\frac{\pi}{8}\left(\sec\frac{\pi}{4} - 1\right)$

$= (\sqrt{2}-1)(\sqrt{2}-1)$

$= 2+1-2\sqrt{2}$

$= 3-2\sqrt{2} = 1-\sqrt{8}$

$a=3, \quad b=8$

$5a-b = 7$ Q

13. Number of integers in the range of $\dfrac{\sin 3x - \sin 2x}{\sin x}$ is

$$\frac{3\sin x - 4\sin^3 x - 2\sin x\cos x}{\sin x}$$

$$y = \frac{\cancel{\sin x}\,[3 - 4\sin^2 x - 2\cos x]}{\cancel{\sin x}}$$

$$y = 3 - 4 + 4\cos^2 x - 2\cos x$$

$$= 4\cos^2 x - 2\cos x - 1$$

$$y = 4\left(\cos x - \frac{1}{4}\right)^2 - \frac{5}{4}$$

$$y_{min} = \frac{1}{4} - \frac{5}{4} = -1$$

$$y_{max} = 4\left(\frac{25}{16}\right) - \frac{5}{4} = 5$$

but for $\cos x = -1$, $\sin x = 0$ so 5 is not included

$\therefore$ Range $\in [-1, 5)$ $\therefore$ Integers $= 6$

14. If $\theta \neq (2n+1)\frac{\pi}{2}, n \in I$ where the minimum value of $\tan^2\theta - \sec\theta + 2$ is k, then 4k is equal to

$$f(\theta) = \sec^2\theta - 1 - \sec\theta + 2$$

$$= \sec^2\theta - \sec\theta + 1$$

$$= \left(\sec\theta - \frac{1}{2}\right)^2 + 1 - \frac{1}{4}$$

$$y = \left(\sec\theta - \frac{1}{2}\right)^2 + \frac{3}{4}$$

for y_{min} $\sec\theta - \frac{1}{2} = 0$ but $\sec\theta \neq \frac{1}{2}$

$\therefore\ y_{min}$ is when $\sec\theta = 0$

$\therefore\ K = \frac{1}{4} + \frac{3}{4} = 1 \quad \therefore\ 4K = 4$ Ans

5. If A,B,C are the angles (in radian) of triangle ABC, such that

$$\cos(A-B)\sin C + \cos^2(A-B).\ \sin^2 C + \cos^3(A-B)\sin^3 C = 3,$$

then the value of $\frac{4}{\pi}(A+2B+3C)$ is

$\cos(A-B)\sin C = 1$

$\cos(A-B) = 1$ & $\sin C = 1$

$A = B$ & $C = \pi/2$

$\frac{4}{\pi}\left(\frac{\pi}{4} + \frac{2\pi}{4} + \frac{3\pi}{2}\right)$

$= 4\left(\frac{1+2+6}{4}\right) = 9$ Ans