Sequence and series

Lecture - 1

Sequence & series :-

(1) Arithmetic Progression

1, 4, 7, 10, - - - , 73

diff: 3 3 3

1, 2, 3, - - - - , 193

$7^{th} \rightarrow 7$

$17^{th} \rightarrow 17$

1, 4, 7, - - - - , 73

$$\underbrace{a_n = a + (n-1)d}$$

n	$P(n)$
$n=$ 1	2
2	6
3	12
4	20
5	30

$P(n) = an^2 + bn + c$

at $n = 1$

$P(1) = a + b + c$

$$\boxed{2 = a + b + c} \quad (1)$$

at $n = 2$

$P(2) = 4a + 2b + c$

$$\boxed{6 = 4a + 2b + c}$$

at $n = 3$

$$\boxed{12 = 9a + 3b + c}$$

$a = 1, b = 1, c = 0$

NOT an AP

$P(n)$		
2		
	4	
6		2
	6	
12		2
	8	
20		2
	10	
30		

Linear equation

quadratic equation

form $ax^2 + bx + c$

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Sequence :- A succession of terms which may be algebraic, real or complex numbers written according to a definite rule is called a sequence.

eg: (i) Sequence of prime numbers

$\{2, 3, 5, 7, 11, 13, ---\}$

(ii) $\{-1, 1, -1, 1, ---\}$

$T_n = \left\{\frac{n}{n^2+1}\right\}$

at $n = 1$, $T_n = \frac{1}{2}$

at $n = 2$; $T_n = \frac{2}{5}$

at $n = 3$; $T_n = \frac{3}{10}$

at $n = 4$; $T_n = \frac{4}{17}$

Sequence $\left\{\frac{1}{2}, \frac{2}{5}, \frac{3}{10}, \frac{4}{17}, ---\right\}$

Progression special case of sequence in which it is possible to express n^{th} term mathematically.

$$T_n = \left\{ \frac{n}{n^2+1} \right\}$$

at $n=1$, $T_1 = \frac{1}{2}$

at $n=2$; $T_2 = \frac{2}{5}$

at $n=3$; $T_3 = \frac{3}{10}$

at $n=4$; $T_4 = \frac{4}{17}$

$$T_n = \left\{ \frac{n^3-3}{2} \right\}$$

$T_1 =$

Series If we put sign of addition and subtraction between the terms of sequence, then it is called as series

$$S_{prime} = 2 + 3 + 5 + 7 + 11 + \cdots$$

$S_n \rightarrow$ sum of n terms

$S_{n-1} \rightarrow$ sum of $n-1$ terms

$a_n, T_n \rightarrow$ n^{th} term

$a_1 \rightarrow$ 1st term

Sequence

(i) AP

(ii) GP

(iii) HP

(iv) AGP

(v) Miscellaneous.

Arithmetic Progression :-

$$a, \quad a+d, \quad a+2d, \quad a+3d, \quad a+4d, \quad \text{-----}, \quad \underbrace{a+9d}_{10^{th}\ \text{term}}$$

$$\to \quad a+(n-1)d$$

$$\boxed{T_n = a + (n-1)d}$$

$$S_n = a + (a+d) + (a+2d) + (a+3d) + \text{------} + (a+(n-2)d) + (a+(n-1)d)$$

$$S_n = (a+(n-1)d) + (a+(n-2)d) + \text{----} + (a+d) + a$$

$$2S_n = (2a+(n-1)d) + (2a+(n-1)d) + \text{----} + (2a+(n-1)d)$$

$$2S_n = n(2a+(n-1)d)$$

$$\boxed{S_n = \frac{n}{2}[2a + (n-1)d]}$$

$$S_n = \frac{n}{2}[a + \underbrace{a + (n-1)d}]$$

$$\boxed{S_n = \frac{n}{2}[a + T_n]}$$

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AP $\quad T_n = a + (n-1)d \quad$ linear expression

Note (1) If $T_n = An + B$ then the series formed is an AP.

(2) (i) if $d > 0 \Rightarrow$ increasing AP

(ii) If $d = 0 \Rightarrow$ terms will remain same

(iii) if $d < 0 \Rightarrow$ decreasing AP.

Q ① $T_n = 2n+3$; is it an AP? Yes

② $2, 6, 10, \text{- - - -}, 86$; find 19^{th} term from end?

$86, 82, 78, \text{- - - -}, 10, 6, 2$ $\quad d = -4$

$T_{19} = 86 + (19-1)(-4) = 86 - 72 = 14$

Total number of terms $\quad 86 = 2 + (n-1)(4)$

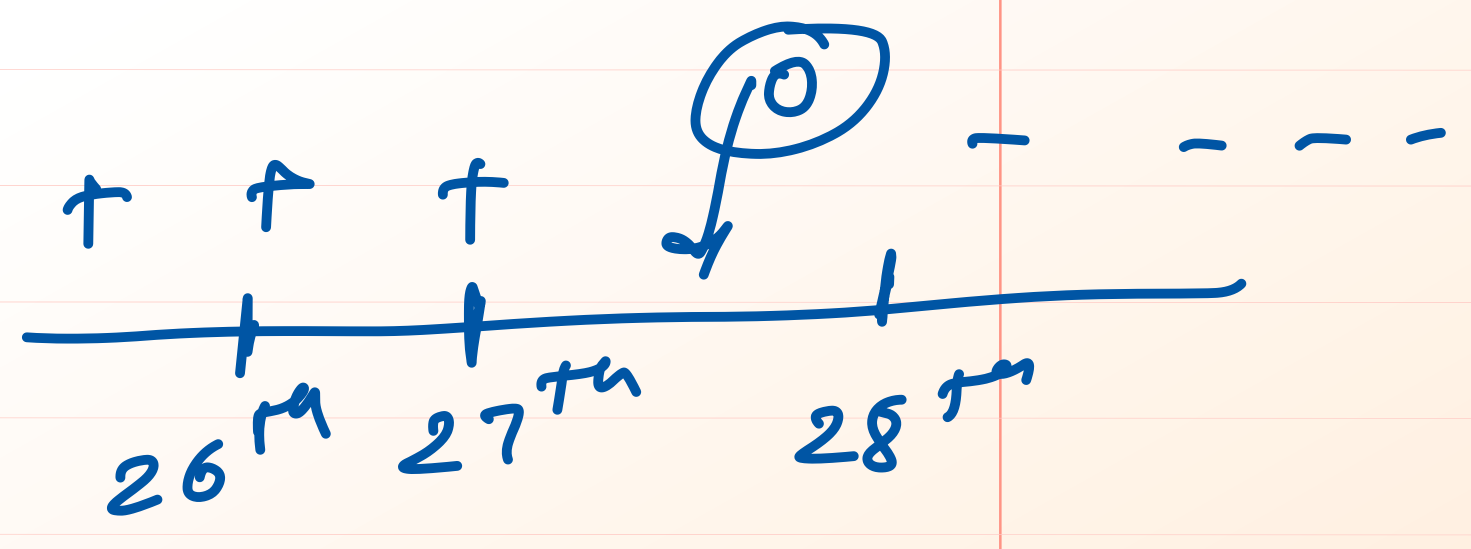
$n = 22$

③ $20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4} \dots$ find first negative term of AP.

$T_n > 20 + (n-1)\left(-\frac{3}{4}\right)$

$0 > 20 + (n-1)\left(-\frac{3}{4}\right)$

$n > 27.67$



first -ve terms $= 28$

Note In an AP summation of k^{th} term from beginning and k^{th} term from the last is always constant. which is equal to summation of first term and last term.

1, 4, 7, 10, 13, 16, 19, 22

23

23

23

23

$$\boxed{T_k + T_{n+1-k} = a + l = \text{constant}}$$

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Q If 6th term and 11th term of an AP are (respectively) 17 and 32. find 20th term

$T_6 = a+5d \Rightarrow a+5d = 17$

$T_{11} = a+10d \Rightarrow a+10d = 32$

$5d = 15$

$\boxed{d=3}$

$a = 17-5(3)$

$\boxed{a=2}$

$T_{20} = a+(19)d$

$= 2+19(3) = 2+57 = 59.$

Q In an A.P. $T_p = q$ and $T_q = p$;

find T_r?

$$T_p = a + (p-1)d = q$$

$$T_q = a + (q-1)d = p$$

$$d(p-q) = q-p$$

$$d = \frac{q-p}{p-q} \Rightarrow \boxed{d = -1}$$

$q-p$

$= -1(p-q)$

$$a + (p-1)(-1) = q$$

$$\boxed{a = q + p - 1}$$

$$T_r = a + (r-1)d$$

$$= (q+p-1) + (r-1)(-1)$$

$$= q + p - \cancel{1} - r + \cancel{1}$$

$$\boxed{T_r = q + p - r}$$

Q In an AP if $a_2+a_5-a_3=10$ and $a_2+a_9=17$ then find 1st term and common difference.

$$a_2+a_5-a_3=10$$

$$-(a+2d)$$

$$a+d+a+4d-a-2d=10$$

$$\boxed{a+3d=10}$$

$$a=10-3d$$

$$\boxed{a=13}$$

Answer

$$a_2+a_9=17$$

$$a+d+a+8d=17$$

$$\boxed{2a+9d=17}$$

$$2(10-3d)+9d=17$$

$$20-6d+9d=17$$

$$20-17=-3d$$

$$\boxed{d=-1}$$

Sequence and series

Lecture - 2

ALLEN CAREER INSTITUTE KOTA (RAJASTHAN) path to success

(1) If p^{th}, q^{th}, r^{th} term of an AP are respectively a, b, c then prove that $a(q-r) + b(r-p) + c(p-q) = 0$

(2) find the number of common terms in

$$5 + 10 + 15 + 20 + \text{----} + 200 ;$$

and

$$6 + 8 + 10 + 12 + \text{----} + 100 .$$

$\boxed{\text{Ans } 10}$

(3) If $S_n = 3n^2 + 5n$ of a sequence, find first term and common difference.

$\boxed{\text{Ans } 8, 6}$

(4) If sum of first $2n$ terms of AP $2, 5, 8, \text{----}$ is equal to sum of first n terms of AP. $57, 59, 61 \text{----}$ find the value of n

$\boxed{n = 11}$

(5) How many terms of the series $20 + 19\frac{1}{3} + 18\frac{2}{3} + \ldots$ must be taken so that sum is 300.

$\boxed{n = 25 \ \& \ 36}$

(6) The sum of n terms of two AP's are in the ratio of $7n+1 : 4n+27$. Find the ratio of their 11^{th} terms.

$\boxed{\text{Ans } 4/3}$

(7) In an AP if $T_m = \frac{1}{n}$ and $T_n = \frac{1}{m}$; then show that $S_{mn} = \frac{1}{2}(mn+1)$.

(8) In an AP. $S_p = q$ and $S_q = p$ then show that $S_{p+q} = -(p+q)$.

(1)

$T_p = [x + (p-1)d = a]\ (q-r)$

$T_q = [x + (q-1)d = b]\ (r-p)$

$T_r = [x + (r-1)d = c]\ (p-q)$

first term $= x$

diff $= d$

$x(q-r) + (p-1)d(q-r) = a(q-r)$

$x(r-p) + (q-1)d(r-p) = b(r-p)$

$x(p-q) + (r-1)d(p-q) = c(p-q)$

add

$0 = a(q-r) + b(r-p) + c(p-q)$

② common terms $10, 20, 30, 40, \text{-----}, 100$

Note :→

$S_1 = T_1$

$S_2 = \underbrace{T_1 + T_2}$ $\Rightarrow$ $T_2 = S_2 - S_1$

$S_3 = \underbrace{T_1 + T_2} + T_3 \Rightarrow T_3 = S_3 - (T_1 + T_2) = S_3 - S_2$

$T_3 = S_3 - S_2$

$S_4 = T_1 + T_2 + T_3 + T_4 \Rightarrow T_4 = S_4 - S_3$

$$\boxed{T_n = S_n - S_{n-1}}$$

$d = T_2 - T_1 = T_3 - T_2$

$d = T_n - T_{n-1}$

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3

$S_n = 3n^2 + 5n$

$T_n = S_n - S_{n-1}$

$S_1 = 8$

$S_2 = 22$

$T_2 = 22 - 8 = 14$

$d = 14 - 8 = 6$

$= (3n^2 + 5n) - [3(n-1)^2 + 5(n-1)]$

$= (3n^2 + 5n) - [3(n^2 - 2n + 1) + 5n - 5]$

$= (3n^2 + 5n) - [3n^2 - 6n + 3 + 5n - 5]$

$= (\cancel{3n^2} + \cancel{5n}) - \cancel{3n^2} + 6n - 3 - \cancel{5n} + 5$

$\boxed{T_n = 6n + 2}$

$T_1 = 8$

$T_2 = 14$

$T_3 = 20$

$T_4 = 26$

$T_5 = 32$

$\vdots$

$d = T_2 - T_1$

$d = 14 - 8 = 6$

(6)

$$\frac{S_{n_1}}{S_{n_2}} = \frac{7n+1}{4n+27}$$

$$\frac{\cancel{\frac{n}{2}}\left[2a_1 + (n-1)d_1\right]}{\cancel{\frac{n}{2}}\left[2a_2 + (n-1)d_2\right]} = \frac{7n+1}{4n+27}$$

$$\frac{T_{11_1}}{T_{11_2}} = \frac{a_1 + 10d_1}{a_2 + 10d_2}$$

$$\frac{2a_1 + (n-1)d_1}{2a_2 + (n-1)d_2} = \frac{7n+1}{4n+27}$$

$$\frac{n-1}{2} = 10$$

$$\boxed{n = 21}$$

$$\frac{\cancel{2}\left[a_1 + \left(\frac{n-1}{2}\right)d_1\right]}{\cancel{2}\left[a_2 + \left(\frac{n-1}{2}\right)d_2\right]} = \frac{7n+1}{4n+27}$$

$$\frac{a_1+10d_1}{a_2+10d_2} = \frac{7(21)+1}{4(21)+27} = \frac{148}{111} = \boxed{\frac{4}{3}}$$ Ans

(8)

$$S_p = q \Rightarrow \frac{p}{2}\left[2a + (p-1)d\right]$$

$$S_q = p = \frac{q}{2}\left[2a + (q-1)d\right]$$

$$2a + (p-1)d = \frac{2q}{p}$$

$$2a + (q-1)d = \frac{2p}{q}$$

$$d(p-1-q+1) = \frac{2q}{p} - \frac{2p}{q}$$

$$d(p-q) = \frac{2(q^2-p^2)}{pq} = \frac{2(q-p)(q+p)}{pq}$$

$$\boxed{d = \frac{-2(q+p)}{pq}}$$

$$2a + (p-1)(-2)\frac{(p+q)}{pq} = \frac{2q}{p}$$

$a =$

$$S_{p+q} = \frac{p+q}{2}\left[2a + (p+q-1)d\right]$$

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Properties of AP

(a) (i) Three terms in AP — $a-d, \quad a, \quad a+d$

(ii) four terms in AP — $a-3d, \quad a-d, \quad a+d, \quad a+3d$

(iii) Five terms in AP — $a-2d, \quad a-d, \quad a, \quad a+d, \quad a+2d$

(iv) Six terms in AP — $a-5d, \quad a-3d, \quad a-d, \quad a+d, \quad a+3d, \quad a+5d$

(b) If we pick the terms of an AP in a particular interval, then picked sequence is also an AP.

$\boxed{1}, \; 4, \; 7, \; \boxed{10}, \; 13, \; 16, \; \boxed{19}, \; 22, \; 25, \; \boxed{28}, \; 31$

$d = 3$

$d_f = \text{interval} \cdot d_i$

$d_f \rightarrow$ final c.d.

$d_i \rightarrow$ initial c.d.

(c) If $a_1, a_2, a_3, ----$ and $b_1, b_2, b_3, ----$ are two A.P. then $a_1 \pm b_1$, $a_2 \pm b_2$, $a_3 \pm b_3, ----$ is an A.P.

But

$a_1 b_1, a_2 b_2, a_3 b_3, ----$ and

$\frac{a_1}{b_1}, \frac{a_2}{b_2}, \frac{a_3}{b_3}$ ---- may or may not be an A.P.

1, 2, 3, 4, 5, - - - 10

1, 4, 7, 10, 13

Sum ↓

2, 6, 10, 14, 18, - - - - AP

Subtract ↓

0, -2, -4, -6, -8, - - - - AP

(d) (a) If each term of an AP is increased or decreased by the same number, then the resulting sequence is also an AP having the same common difference.

1, 2, 3, 4, 5, - - -

$\downarrow$ inc by 2

3, 4, 5, - - - - AP.

$\downarrow$ dec by 2

-1, 0, 1, 2, 3, - - - - AP.

multiply by 2

$\underbrace{2, 4, 6, 8, 10, - - - \text{ AP.}}$

(b) If each term of an AP is multiply or divided by the same non zero number (K), then the resulting sequence is also an AP. whose common difference is Kd ~~&~~ d/K respectively, where d is the c.d. of original AP.

$d=3 \rightarrow 1, 4, 7, 10, 13, \_\_ \_\_$

$\cancel{d}$ divide by 2

$d_1 = \frac{3}{2} \qquad \frac{1}{2}, \underline{2}, \frac{7}{2}, 5, \frac{13}{2}, \_\_\_\_$

$= \frac{d}{2}$

$a, a+d, a+2d, a+3d,$

$\underbrace{5a}, \underbrace{5a+5d}, \underbrace{5a+10d},$

$5000a, \ 5000a+5000d,$

$5000a+10000d$

$d=3 \longrightarrow 1, 4, 7, 10, 13, \_\_\_$

$d_1 = \underline{21} \longrightarrow 7, 28, 49, 70,$

$7 \times d$

$d_2 = 30 \qquad 10, 40, 70, 100$

$10(d)$

(Q) Any term of an AP (except the first and last) is equal to half of sum of terms which are equidistant from it.

$$\underline{T_3} = \frac{T_2 + T_4}{2} = \frac{T_1 + T_5}{2}$$

$$T_1 \quad T_2 \quad \underline{T_3} \quad T_4 \quad T_5$$

$$T_5 = \frac{T_4 + T_6}{2} = \frac{T_3 + T_7}{2} = \frac{T_2 + T_8}{2}$$

$$\boxed{T_r = \frac{T_{r-k} + T_{r+k}}{2} \; ; \quad k \le r}$$

Q The sum of first three terms of an AP is 27 and sum of their squares is 293. find first term and common difference.

three terms in AP $a-d,\ a,\ a+d$

$a-d+a+a+d = 27$

$\boxed{a = 9}$

$(a-d)^2 + a^2 + (a+d)^2 = 293$

$a^2+d^2-\cancel{2ad} + a^2 + a^2+d^2 + \cancel{2ad} = 293$

$3a^2 + 2d^2 = 293$

$3(81) + 2d^2 = 293$

$2d^2 = 50$

$d^2 = 25$

$\boxed{d = \pm 5}$

at $d=5$

Series $9-5,\ 9,\ 9+5$

$\Rightarrow 4, 9, 14$

at $d=-5$

Series $9+5,\ 9,\ 9-5$

$\Rightarrow 14, 9, 4$

Arithmetic Mean :-

When three quantities are in AP, then the middle one is called the arithmetic mean of the other two.

Note : If a, b, c are in AP $\qquad b = \dfrac{a+c}{2}$

or $2b = a+c$

Insertion of 'n' AM's between a & b :-

AP → $\underline{a}, A_1, A_2, A_3, \ldots\ldots, A_{n-1}, A_n, \underline{b}$

no. of terms $= \underline{n+2}$

$T_n = a + (n-1)d$

$b = a + (\underline{n+2} - 1)d$

$b - a = (n+1)d$

$$\boxed{d = \frac{b-a}{n+1}}$$

$A_1 = a + d = a + \left(\frac{b-a}{n+1}\right)$

$A_2 = a + 2d = a + 2\left(\frac{b-a}{n+1}\right)$

$A_3 = a + 3d = a + 3\left(\frac{b-a}{n+1}\right)$

$A_4 = a + 4d = a + 4\left(\frac{b-a}{n+1}\right)$

$\vdots$

$A_{n-1} = a + (n-1)d$

$= a + (n-1)\left(\frac{b-a}{n+1}\right)$

$A_n = a + nd = a + n\left(\frac{b-a}{n+1}\right)$

Q Insert 20 AM's between 4 & 67

$$\boxed{4, A_1, A_2, \text{---} A_{20}, 67}$$

$$d = \frac{67-4}{20+1} = \frac{63}{21} = 3$$

$$A_1 = a+d = 4+3 = 7$$

$$A_2 = a+2d = 4+2(3) = 10$$

$$A_3 = a+3d = 4+3(3) = 13$$

$$A_4 = a+4d = 4+4(3) = 16$$

$\vdots$

$$A_{19} = a+19d = 4+19(3) = 61$$

$$A_{20} = a+20d = 4+20(3) = 64$$

Self Attempt

1. If a, b, c are in A.P. then prove that

(a) b + c ; c + a; a + b are also in A.P.

(b) $(b + c)^2 - a^2$; $(c + a)^2 - b^2$; $(a + b)^2 - c^2$ are also in A.P.

2. Find four numbers in A.P. whose sum is 50 & in which the greatest number is four times the least.

[**Ans.** 5, 10, 15, 20]

3. If a^2, b^2, c^2 are in A.P. then prove that

(a) $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in A.P.

(b) $\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$ are in A.P.

4. If 101 means are inserted between 1 and 99 then find their sum. [**Ans.** 5050]

5. If $\frac{a^n + b^n}{a^{n-1} + b^{n-1}}$ is the A.M. between a and b, $(a \neq b)$, then find the value of n. [**Ans.** 1]

6. If p arithmetic means are inserted between 5 and 41 so that the ratio $\frac{A_3}{A_{p-1}} = \frac{2}{5}$, then find the value of p.

[**Ans.** p = 11]

7. If $\log_3 2$, $\log_3(2^x - 5)$ & $\log_3\left(2^x - \frac{7}{2}\right)$ are in A.P. determine x. [**Ans.** x = 3]

8. Solve the equation $\frac{x-1}{x} + \frac{x-2}{x} + \ldots\ldots. + \frac{1}{x} = 3$ [**Ans.** x = 7]

9. Find the condition that the roots of the equation $x^3 - px^2 + qx - r = 0$ may be in A.P. and hence solve the equation $x^3 - 12x^2 + 39x - 28 = 0$.

[**Ans.** $2p^3 - 9pq + 27r = 0$ (This is the required condition) : roots are 1, 4, 7]

10. If the first 3 terms of an increasing A.P. are the roots of the cubic $4x^3 - 24x^2 + 23x + 18 = 0$, then find S_n. [**Ans.** $(5n-7)\frac{n}{4}$]

Homework 27/05/2021

Race - 23 : Part - 1

Sequence and series

Lecture - 3

(4) $1, A_1, A_2, \ldots\ldots, A_{101}, 99$

$$A_1 = 1 + \frac{99-1}{101+1} = 1 + \frac{98}{102}$$

$$A_2 = 1 + 2\left(\frac{98}{102}\right)$$

$$A_3 = 1 + 3\left(\frac{98}{102}\right)$$

$$A_{51} = 1 + 51\left(\frac{98}{102}\right)$$

$$= 1 + \frac{51}{102} \cdot 98$$

$$= 1 + 49 = 50$$

$$1+99$$

$$= \underbrace{A_1 + A_{101}} = 100$$

$$= A_2 + A_{100} = 100$$

$$= A_3 + A_{99} = 100$$

$$= \underbrace{A_{50} + A_{52}} = 100$$

$$\underline{A_{51}}$$

$$\text{Sum} = 50(100) + A_{51}$$

$$= 5000 + 50 = \underline{5050}$$

Q $\quad a \,,\; \dfrac{a^n+b^n}{a^{n-1}+b^{n-1}} \,,\; b$

$$\frac{a+b}{2} = \frac{a^n+b^n}{a^{n-1}+b^{n-1}}$$

$$\frac{a^1+b^1}{a^0+b^0} = \frac{a^n+b^n}{a^{n-1}+b^{n-1}} \quad \boxed{n=1}$$

$$a^{n-1} = b^{n-1}$$

$$a^n = a^1 \cdot a^{n-1} =$$

$$(a+b)(a^{n-1}+b^{n-1}) = 2a^n+2b^n$$

$$a^n+b^n+b\cdot a^{n-1}+a\cdot b^{n-1} = 2a^n+2b^n$$

$$b\cdot a^{n-1}+a\cdot b^{n-1} = a^n+b^n$$

$$a^n+b^n-b\cdot a^{n-1}-a\cdot b^{n-1} = 0$$

$$a^{n-1}(a-b)+b^{n-1}(b-a) = 0$$

$$(a-b)(a^{n-1}-b^{n-1}) = 0$$

$$n-1=0$$

$$\boxed{n=1}$$

$$\cancel{b\cdot a^{n-1} = (ba)^{n-1}}$$

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Q $\quad 5, A_1, A_2, A_3 \qquad , 41$

$$d = \frac{41-5}{p+1}$$

$$A_3 = 5 + 3d$$

$$A_{p-1} = 5 + (p-1)d$$

8

$$\frac{x-1}{x} + \frac{x-2}{x} + \frac{x-3}{x} + \dots + \frac{1}{x} = 3$$

$$\frac{x-1}{x} + \frac{x-2}{x} + \frac{x-3}{x} + \dots + \frac{x-(x-1)}{x} = 3$$

$$x(x-1) - (1+2+3+4+\dots+x-1) = 3x$$

9

$$x^3 - \underline{px^2} + qx - r = 0$$

roots AP $\rightarrow$ $\alpha - d, \ \alpha, \ \alpha + d$

Sum of roots $\alpha - d + \alpha + \alpha + d = \frac{-(-p)}{1} = p$

$$3\alpha = p$$

$$\boxed{\alpha = p/3}$$

$$\left(\frac{p}{3}\right)^3 - p\left(\frac{p}{3}\right)^2 + q\left(\frac{p}{3}\right) - r = 0$$

$$ax^2+bx+c=0$$

$$\alpha+\beta=-\frac{b}{a}$$

$$\alpha\beta=\frac{c}{a}$$

$$ax^3+bx^2+cx+d=0$$

$$\alpha+\beta+\gamma=-\frac{b}{a}$$

$$\alpha\beta+\beta\gamma+\gamma\alpha=\frac{c}{a}$$

$$\alpha\beta\gamma=\frac{-d}{a}$$

Q $4x^3 - 24x^2 + 23x + 18 = 0,$

Roots $\alpha - d, \quad \alpha, \quad \alpha + d$

$\alpha - d + \alpha + \alpha + d = \frac{-(-24)}{4}$

$3\alpha = 6$

$\boxed{\alpha = 2}$

$(\alpha - d)\,\alpha\,(\alpha + d) = -\frac{18}{4}$

$2(2-d)(2+d) = -\frac{9}{2}$

$4 - d^2 = -\frac{9}{4}$

$d^2 = 4 + \frac{9}{4} \Rightarrow d^2 = \frac{25}{4} \Rightarrow \boxed{d = \pm\frac{5}{2}}$

Seq

$2-\frac{5}{2},\ 2,\ 2+\frac{5}{2},\ 2+5,\ \ldots$

$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

Geometric Progression :-

$$a,\ ar,\ ar^2,\ ar^3,\ \text{-----}\ a\cdot r^9,\ \text{-----},\ ar^{n-1}$$

$a\cdot r^9 \downarrow$ 10th term

$ar^{n-1} \downarrow$ nth term

$r \to$ common ratio

$a \to$ first term

$T_n \to$ nth term

$$\boxed{T_n = a\cdot r^{n-1}}$$

$$S = a + ar + ar^2 + ar^3 + \text{-----} + ar^{n-3} + ar^{n-2} + ar^{n-1}$$

multiply by r

$$rS = ar + ar^2 + ar^3 + ar^4 + \text{----} + ar^{n-2} + ar^{n-1} + ar^n$$

Subtract

$$S - rS = a - ar^n$$

$$S(1-r) = a(1-r^n) \Rightarrow \boxed{S = \frac{a(1-r^n)}{1-r}}$$

Q

$2,\ 4,\ 8,\ 16,\ 32,\ \ldots\ldots$

$\underbrace{2,\ 4}_{\times 2},\ \underbrace{4,\ 8}_{\times 2},\ \underbrace{8,\ 16}_{\times 2}$

$T_1 = a = \text{first term} = 2$

$r = \frac{T_2}{T_1} = \frac{4}{2} = 2$

$\frac{T_3}{T_2} = \frac{8}{4} = 2$

$= \frac{T_4}{T_3} = \frac{16}{8} = 2$

$T_n = a \cdot r^{n-1}$

$= 2 \cdot (2)^{n-1}$

$= 2^1 \cdot 2^{n-1} = 2^{1+n-1} = 2^n$

$S_n = \frac{a(r^n - 1)}{r-1}$ ✓

$= \frac{2(2^n - 1)}{2-1} = 2(2^n - 1)$

$$\boxed{S = \frac{a(1-r^n)}{1-r}}$$

$\downarrow$

$= \frac{a(r^n - 1)}{r-1}$

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Q GP $T_3 = 2$; $T_6 = -\frac{1}{4}$

$T_{10} = ?$

$T_3 \Rightarrow a \cdot r^2 = 2 \Rightarrow \underbrace{a = 2/r^2}$

$T_6 \Rightarrow \underline{a \cdot r}^5 = -\frac{1}{4}$

$$\frac{2}{r^2} \cdot r^5 = -\frac{1}{4}$$

$$2r^3 = -\frac{1}{4}$$

$$r^3 = -\frac{1}{8}$$

$$\boxed{r = -\frac{1}{2}}$$

$$a = \frac{2}{r^2} = \frac{2}{\left(-\frac{1}{2}\right)^2} = \frac{2(4)}{1} = 8$$

$$T_{10} = a \cdot r^9 = 8 \cdot \left(-\frac{1}{2}\right)^9 = -\frac{1}{2^6}$$

$$= -\frac{1}{64}$$

Q 2, 6, 18, 54, 162, - - -

find T_n & S_n.

$a = 2$; $r = 3$

$$T_n = 2 \cdot 3^{n-1}$$

$$S_n = \frac{2 \cdot (3^n - 1)}{3 - 1} = 3^n - 1$$

$$\cancel{2 \cdot 3^{n-1}} = \cancel{2^{n-1} \cdot 3^{n-1}}$$

$$\cancel{= 6^{n-1}}$$

Q $2, \frac{2}{3}, \frac{2}{9}, \frac{2}{27}, \frac{2}{81}, \text{-----}$ T_n, S_n

$a = 2; \quad r = \frac{1}{3}$

$T_n = 2 \cdot \left(\frac{1}{3}\right)^{n-1}$

$T_n = 2\left(3^{-1}\right)^{n-1} = 2 \cdot (3)^{1-n}$

$=$

$S_n = \frac{2\left(\left(\frac{1}{3}\right)^n - 1\right)}{\left(\frac{1}{3} - 1\right)} = \frac{2\left(\frac{1}{3^n} - 1\right)}{\left(-2/3\right)} = 3\left(1 - \frac{1}{3^n}\right)$

$= -3\left(\frac{1}{3^n} - 1\right)$

$-3\left(\frac{1}{3^n} - 1\right)$

$$S_\infty = a + \cancel{ar} + \cancel{ar^2} + \cancel{-----} \cdots\infty$$

$$r \cdot S_\infty = \cancel{ar} + \cancel{ar^2} + \cancel{ar^3} + \cdots\infty$$

$$S_\infty - r \cdot S_\infty = a$$

$$S_\infty(1-r) = a$$

$$\boxed{S_\infty = \frac{a}{1-r}}$$

<u>if</u> $|r| < 1 \Rightarrow 0 < |r| < 1$

$-1 < r < 1 \quad \neq 0$

Q The sum of infinite terms of a GP is 15 and the sum of their squares is 45. Find the series.

$a, ar, ar^2, \ldots \infty$

$a^2, a^2r^2, a^2r^4, \ldots \infty$

$S_\infty \Rightarrow \dfrac{a}{1-r} = 15$

$S_\infty \Rightarrow \dfrac{a^2}{1-r^2} = 45$

$a = 15(1-r)$

$\dfrac{a^2}{1-r^2} = 45$

$\dfrac{(15)^2(1-r)^{\cancel{2}}}{\cancel{(1-r)}(1+r)} = 45 \Rightarrow \dfrac{225(1-r)}{1+r} = 45$

$r = 2/3$

$a = 5$

Series $5, \dfrac{10}{3}, \dfrac{20}{9}, \ldots$

Homework

Race	23	Complete
Race	24	Part I

Sequence and series

Lecture - 4

Properties of GP: →

(i) In a GP product of k^{th} term from the beginning and k^{th} term from the end is always constant and is equal to the product of first and last term.

$$T_k \cdot T_{n-k+1} = a \cdot l = \text{Constant}$$

$a,\ ar,\ ar^2,\ \ldots,\ ar^{n-3},\ ar^{n-2},\ ar^{n-1}$

a^2r^{n-1} Product

a^2r^{n-1}

$1 \quad 3 \quad 9 \quad 27 \quad 81$

81

81

$1^2,\ 3^2,\ 9^2,\ 27^2,\ 81^2$

(ii) Three numbers in GP $\frac{a}{r}, a, ar$

four " $\frac{a}{r^3}, \frac{a}{r}, ar, ar^3$

five $\frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2$

Six $\frac{a}{r^5}, \frac{a}{r^3}, \frac{a}{r}, ar, ar^3, ar^5$

(iii) If each term of a GP is raised to the same power then resulting series is also a GP.

$\underline{a^2}, \underline{a^2r^2}, a^2r^4, a^2r^6, ---$

(iv) If each term of a GP be multiplied or divided by same non zero quantity, then the resulting sequence is also a GP.

(v) If $a_1, a_2, a_3 - - - -$ & $b_1, b_2, b_3 - - - -$ be two GPs of common ratios r_1 & r_2 respectively

$a_1 b_1, a_2 b_2, a_3 b_3 - - - - -$

CR $\rightarrow r_1 r_2$

CR $\rightarrow$ common ratio

$\frac{a_1}{b_1}, \frac{a_2}{b_2}, \frac{a_3}{b_3}, - - -$

CR $\rightarrow \frac{r_1}{r_2}$

$a \quad ar_1 \quad ar_1^2 \quad ar_1^3 \quad - -$

$b \quad br_2 \quad br_2^2 \quad - - -$

$ab, abr_1r_2, abr_1^2r_2^2, - -$

(vi) In a positive GP every term (except first) is equal to the square root of the product of its two terms which are equidistant from it.

$$T_r = \sqrt{T_{r-k} \cdot T_{r+k}} \quad ; \quad k < r$$

$$1, \underline{3}, 9, \underline{27}, \boxed{81}, \underline{243}, 729$$

(vii) If a_1, a_2, a_3 ---- GP, is a non zero, non negative terms then

$$\log a_1, \log a_2, \log a_3, \text{----} \log a_n \rightarrow \text{AP}$$

and vice versa.

Q If a_1, a_2, a_3 are in GP such that

$$a_1 + a_2 + a_3 = 13 \quad \text{and} \quad a_1^2 + a_2^2 + a_3^2 = 91$$

find a and r.

$a_1 \to a/r$

$a_2 \to a$

$a_3 \to ar$

$$\frac{a}{r} + a + ar = 13 \Rightarrow \frac{a}{r}(1+r+r^2) = 13$$

$$\frac{a}{r} = \frac{13}{(1+r+r^2)}$$

$$\left(\frac{a}{r}\right)^2 = \frac{169}{(1+r+r^2)^2}$$

$$\left(\frac{a}{r}\right)^2 + a^2 + (ar)^2 = 91$$

$$\frac{a^2}{r^2}(1+r^2+r^4) = 91$$

$$\frac{169}{(1+r+r^2)^2}(1+r^2+r^4) = 91$$

$$\frac{(169)(r^2+r+1)(r^2-r+1)}{(r^2+r+1)^2} = 91$$

$$r^4+r^2+1$$

$$= (r^2+r+1)(r^2-r+1)$$

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$$(169)\frac{\cancel{(r^2+r+1)}(r^2-r+1)}{(r^2+r+1)^{\cancel{2}}} = 91$$

$$\frac{a}{r} = \frac{13}{1+r+r^2}$$

$$\frac{a}{3} = \frac{13}{1+3+3^2} \Rightarrow a=3$$

Seq. 1, 3, 9

$r = 1/3$

$$3a = \frac{13}{1+\frac{1}{3}+\frac{1}{3^2}} \Rightarrow \boxed{a=3}$$

Seq. 9, 3, 1

$$\frac{r^2-r+1}{r^2+r+1} = \frac{\cancel{91}\,7}{\cancel{169}\,13}$$

$$13r^2 - 13r + 13 = 7r^2 + 7r + 7$$

$$6r^2 - 20r + 6 = 0$$

$$3r^2 - 10r + 3 = 0$$

$$(3r-1)(r-3) = 0$$

$$\boxed{r = \frac{1}{3}, 3}$$

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Q The sum of first three consecutive terms of a GP is 19 and their product is 216. find S_n & S_∞ (if exist)

$$a + ar + ar^2 = 19 \checkmark$$

$$a \cdot ar \cdot ar^2 = 216$$

$$a^3 r^3 = 216$$

$$ar = 6$$

$$\boxed{a = \frac{6}{r}}$$

$$\frac{6}{r} + 6 + 6r = 19$$

$$\frac{6 + 6r + 6r^2}{r} = 19$$

$$6r^2 + 6 - 13r = 0$$

$$6r^2 - 13r + 6 = 0$$

$$\boxed{r = \frac{3}{2}, \frac{2}{3}}$$

$$a = \frac{6}{(3/2)} = 4$$

$$a = \frac{6}{2/3} = 9$$

$a = 4,\ \boxed{r = 3/2}$

$$S_n = \frac{4\left(\left(\frac{3}{2}\right)^n - 1\right)}{\left(\frac{3}{2} - 1\right)}$$

$a = 9;\ r = 2/3$

$$S_n = \frac{9\left(\left(\frac{2}{3}\right)^n - 1\right)}{\left(\frac{2}{3} - 1\right)}$$

$$S_\infty = \frac{9}{1 - 2/3} = 27$$

$$\frac{a}{r},\ \underline{a},\ ar$$

$a = 6$ ✓

$r = 3/2$

1st $\to \dfrac{a}{r} = \dfrac{6}{(3/2)} = 4$

$r = 2/3$

1st $\to \dfrac{6}{2/3} = 9$

Geometrical Mean : $\rightarrow$ (GM)

a, b, c

If a, b, c are three positive numbers in GP, then b is called the GM between a and c.

$b^2 = ac$

$$GM = \sqrt{ac}$$

If a, b, c are in GP $\boxed{ac = b^2}$

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Insertion of n - GM between a & b :-

$a, G_1, G_2, G_3, \text{------}, G_{n-1}, G_n, b$

no. of terms $= n+2$

$T_{n+2} = a \cdot r^{n+2-1}$

$b = a \cdot r^{n+1}$

$\frac{b}{a} = r^{n+1}$

$r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$

$G_1 \rightarrow a \cdot r = a \cdot \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$

$G_2 \rightarrow a r^2 = a \cdot \left(\frac{b}{a}\right)^{\frac{2}{n+1}}$

$G_3 \rightarrow a r^3 = a \cdot \left(\frac{b}{a}\right)^{\frac{3}{n+1}}$

$\vdots$

$G_{n-1} = a \cdot r^{n-1} = a\left(\frac{b}{a}\right)^{\frac{n-1}{n+1}}$

$G_n = a \cdot r^n = a \cdot \left(\frac{b}{a}\right)^{\frac{n}{n+1}}$

$G_{n+1} =$

Note :- Product of all n GM's inserted between two numbers a & b in n^{th} power of single GM between a & b.

$$\prod_{k=1}^{n} G_k = \left(\sqrt{ab}\right)^n$$

$a, G_1\ G_2\ G_3 \text{---} G_{n-1}\ G_n\ b$

$ab = G_1 G_n$
$= G_2 \cdot G_{n-1}$
$= G_3 \cdot G_{n-2}$
$= G_4\ G_{n-3}$
$\vdots$

$n/2$

$(ab)^{n/2} = G_1 G_2 \text{---} G_n$

$3^{1+2+3+4+5} = 3^{15}$

$1, 3, 9, 27, 81, 243, 729$

$(729)^{5/2}$

$(3^6)^{5/2} = 3^{15}$

Q Insert 4 GM's between 5 & 160.

$$5, \quad G_1, \quad G_2, \quad G_3, \quad G_4, \quad 160$$

$$a \quad ar \quad ar^2 \quad ar^3 \quad ar^4 \quad ar^5$$

$ar^5 = 160$

$5r^5 = 160$

$r^5 = 32$

$\boxed{r = 2}$

$G_1 = 5(2) = 10$

$G_2 = 5(2)^2 = 20$

$G_3 = 5(2^3) = 40$

$G_4 = 5(2^4) = 80$

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Q If AM between a and b is 15 and GM between a & b is 9. find the number.

$$\frac{a+b}{2} = 15 \qquad \sqrt{ab} = 9$$

$$a+b = 30 \qquad ab = 81$$

$$(30-b)\,b = 81$$

$a = 27 \quad b = 3$

$a = 3 \quad b = 27$

Sequence and series

Lecture - 5

$0.2\dot{3}\dot{5} = 0.2353535\ldots$

$0.2\dot{3}\dot{5} = \underline{0}.\underline{2}\,\underline{35}\,35\,35\,35\ldots$

$= 0.2 + \underline{0.035} + \underline{0.00035} + 0.0000035 + \ldots$

$= 0.2 + \underbrace{(35\times10^{-3}) + (35\times10^{-5}) + (35\cdot10^{-7}) + \ldots}$

$= 0.2 + \dfrac{(35\times10^{-3})}{1-10^{-2}} = 0.2 + \dfrac{35\times10^{-3}}{1-0.01}$

$= 0.2 + \dfrac{35}{1000(0.99)}$

$= \dfrac{99}{99}\left(\dfrac{2}{10}\right) + \dfrac{35}{990} = \dfrac{198+35}{990}$

$= \dfrac{233}{990}$

$\underbrace{0.23}53535$

0.2
$+0.035$
$+0.00035$
$+0.0000035$

Q

$$7+77+777+\text{-----}$$

$$7\left[1+11+111+1111+\text{----}\right]$$

$$=\frac{7}{9}\left[9+99+999+9999+\text{----}\right]$$

$$=\frac{7}{9}\left[(10-1)+(100-1)+(1000-1)+(10000-1)+\text{-----}\right]$$

$$=\frac{7}{9}\left[(10+100+1000+10000+\text{----})-(1+1+1+1\text{---})\right]$$

$$=\frac{7}{9}\left[\frac{10\cdot(10^n-1)}{10-1}-n\right]$$

$$=\frac{7}{9}\left[\frac{10}{9}(10^n-1)-n\right]=\frac{70}{81}(10^n-1)-\frac{7}{9}n$$

Harmonic Progression :—

$a, b, c \rightarrow AP$

$\frac{1}{a}, \frac{1}{b}, \frac{1}{c} \rightarrow HP.$

A non-zero sequence is said to be an HP if the reciprocal of its terms are in AP.

If $a_1, a_2, a_3, ---$ are in HP.

then $\frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}, ----$ are in AP.

Standard HP $\quad \frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d}, \frac{1}{a+3d}, ---\frac{1}{a+(n-1)d}$

Q (i) Reciprocal of every AP is HP $\quad$ false

(ii) Reciprocal of every HP is AP $\quad$ True

Note

(i) There is no general formula for finding the sum of n terms of HP.

(ii) If a, b, c are in HP then $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in AP.

$$\frac{2}{b} = \frac{1}{a} + \frac{1}{c}$$

$$\frac{2}{b} = \frac{c+a}{ca}$$

$$\boxed{b = \frac{2ac}{a+c}}$$

Q The m^{th} term of an HP is n and n^{th} term is m. Prove that p^{th} term is $\frac{mn}{p}$

m^{th} term of HP $\Rightarrow \frac{1}{a+(m-1)d} = n$

$$a + (m-1)d = \frac{1}{n}$$

n^{th} term

$$a + (n-1)d = \frac{1}{m}$$

$$(m-n)d = -\frac{1}{m} + \frac{1}{n}$$

$$\boxed{d = \frac{1}{mn}}$$

$$a = \frac{1}{n} - (m-1)\frac{1}{mn} = \boxed{a = \frac{1}{mn}}$$

$$a_p = \frac{1}{a+(p-1)d}$$

$$= \frac{1}{\frac{1}{mn} + \frac{p-1}{mn}} = \boxed{\frac{mn}{p}}$$

Ans

Q If m^{th} term of an HP is n and n^{th} term is equal to m, then prove that $(m+n)^{th}$ term is $\frac{mn}{m+n}$

$$T_m = \frac{1}{a+(m-1)d} = n \Rightarrow a+(m-1)d = \frac{1}{n}$$

$$T_n = \frac{1}{a+(n-1)d} = m \Rightarrow a+(n-1)d = \frac{1}{m}$$

$$a = \frac{1}{mn}, \quad d = \frac{1}{mn}$$

$$T_{m+n} = \frac{1}{a+(m+n-1)d} = \frac{mn}{(m+n)}$$

Q If $a_1, a_2, a_3, ---- a_n$ are in H.P.

then prove that $a_1a_2 + a_2a_3 + a_3a_4 + ---- + a_{n-1}a_n$

$= (n-1)a_1a_n.$

$a_1, a_2, a_3, --- \text{ HP}$

$\frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}, --- \text{ AP}.$

LHS $= a_1a_2 + a_2a_3 + a_3a_4 + --- + a_{n-1}a_n$

$= \frac{1}{a}\cdot\frac{1}{a+d} + \frac{1}{a+d}\cdot\frac{1}{a+2d} + --- + \frac{1}{a+(n-2)d}\cdot\frac{1}{a+(n-1)d}$

$= \frac{1}{d}\left[\frac{(a+d)-a}{(a+d)a} + \frac{(a+2d)-(a+d)}{(a+d)(a+2d)} + \frac{(a+3d)-(a+2d)}{(a+2d)(a+3d)} + --- + \frac{(a+(n-1)d)-(a+(n-2)d)}{(a+(n-2)d)(a+(n-1)d)}\right]$

$= \frac{1}{d}\left[\frac{1}{a} - \cancel{\frac{1}{a+d}} + \cancel{\frac{1}{a+d}} - \cancel{\frac{1}{a+2d}} + \cancel{\frac{1}{a+2d}} - \cancel{\frac{1}{a+3d}} + --- + \cancel{\frac{1}{a+(n-2)d}} - \frac{1}{a+(n-1)d}\right]$

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$$\frac{1}{5\cdot(3)}$$

$$=\frac{1}{2}\left(\frac{(5-3)}{(5-3)}\right)$$

$$=\frac{1}{d}\left[\frac{1}{a}-\frac{1}{a+(n-1)d}\right]$$

$$=\frac{1}{d}\left[\frac{\cancel{a}+(n-1)d-\cancel{a}}{a(a+(n-1)d)}\right]$$

$$=\frac{1}{\cancel{d}}\,\frac{(n-1)\cancel{d}}{a(a+(n-1)d)}$$

$$=(n-1)\left(\frac{1}{a}\right)\cdot\frac{1}{(a+(n-1)d)}$$

$$=(n-1)(a_1)\cdot(a_n)$$

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Harmonic Mean :-

If a, b, c are in HP then middle term is called the Harmonic mean between them.

Hence if H is Harmonic mean (HM) between a and b then a, H, b are in HP.

$$\boxed{H = \frac{2ab}{a+b}}$$

Insertion of n HM's between a and b :-

$a, H_1, H_2, H_3, ----, H_{n-1}, H_n, b \longrightarrow HP$

$\frac{1}{a}, \frac{1}{H_1}, \frac{1}{H_2}, \frac{1}{H_3}, ----, \frac{1}{H_{n-1}}, \frac{1}{H_n}, \frac{1}{b} \longrightarrow AP.$

$\frac{1}{H_1} = \frac{1}{a} + d = \frac{1}{a} + \frac{\left(\frac{1}{b} - \frac{1}{a}\right)}{n+1}$

$\frac{1}{H_1} = \frac{1}{a} + \left(\frac{\frac{1}{b} - \frac{1}{a}}{n+1}\right)$

$H_1 = \frac{1}{\frac{1}{a} + \frac{\left(\frac{1}{b} - \frac{1}{a}\right)}{n+1}}$

$\frac{1}{H_2} = \frac{1}{a} + 2d = \frac{1}{a} + \frac{2\left(\frac{1}{b} - \frac{1}{a}\right)}{n+1}$

$\frac{1}{b} = \frac{1}{a} + (n+2-1)d$

$\frac{1}{b} - \frac{1}{a} = (n+1)\, d$

$d = \frac{\left(\frac{1}{b} - \frac{1}{a}\right)}{n+1}$

$\frac{1}{H_3} = \frac{1}{a} + 3\,\frac{\left(\frac{1}{b} - \frac{1}{a}\right)}{n+1}$

$\frac{1}{H_n} = \frac{1}{a} + n\left(\frac{\frac{1}{b} - \frac{1}{a}}{n+1}\right)$

Q If b is HM between a & c then prove that $\dfrac{1}{b-a}+\dfrac{1}{b-c}=\dfrac{1}{a}+\dfrac{1}{c}$

LHS $\quad \dfrac{1}{b-a}+\dfrac{1}{b-c}$ $\qquad b=\dfrac{2ac}{a+c}$

$$=\frac{1}{\frac{2ac}{a+c}-a}+\frac{1}{\frac{2ac}{a+c}-c} \Rightarrow \frac{a+c}{2ac-a^2-ca}+\frac{a+c}{2ac-c^2-ac}$$

$$=\frac{a+c}{ac-a^2}+\frac{a+c}{ac-c^2}$$

$$=(a+c)\left[\frac{1}{a(c-a)}+\frac{1}{c(a-c)}\right]=\frac{(a+c)}{(c-a)}\left(\frac{1}{a}-\frac{1}{c}\right)$$

$$=\frac{(a+c)}{(c-a)}\frac{(c-a)}{ac}=\frac{a+c}{ac}=\frac{1}{c}+\frac{1}{a}$$

Race -23 complete (submit 02/06/2021) →

Race -24 complete (submit 04/06/2021)

Race-25 complete (submit 06/06/2021)

Sequence and series

Lecture - 6

AM, GM, HM of n positive real numbers :-

* $AM = \dfrac{a_1 + a_2 + a_3 + a_4 + \cdots + a_n}{n} = \dfrac{\sum_{k=1}^{n} a_k}{n}$

* $GM = [a_1 \cdot a_2 \cdot a_3 \cdots a_n]^{1/n}$

$= \left[\prod_{k=1}^{n} a_k\right]^{1/n}$

* $HM = \dfrac{n}{\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3} + \cdots + \frac{1}{a_n}}$

$= \left[\dfrac{n}{\sum_{k=1}^{n} \frac{1}{a_k}}\right]$

Four terms (a, b, c, d)

$GM = (abcd)^{1/4}$

Two terms (a, c)

$HM = \dfrac{2}{\frac{1}{a} + \frac{1}{c}}$

Three terms

$HM = \dfrac{3}{\frac{1}{a} + \frac{1}{b} + \frac{1}{c}}$

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* RMS (Root mean square)

for n real numbers $a_1, a_2, a_3, ---a_n$

$$RMS = \sqrt{\frac{a_1^2 + a_2^2 + a_3^2 + ----+ a_{n-1}^2 + a_n^2}{n}}$$

$$= \sqrt{\frac{\left[\sum_{K=1}^{n} (a_K^2)\right]}{n}}$$

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Properties related with AM, GM, HM : $\rightarrow$

(a) **Applicable for only two positive real numbers: —**

If a and b are +ve real nos.

$AM \rightarrow A$
$GM \rightarrow G$
$HM \rightarrow H$

$$A = \frac{a+b}{2}$$

$$G = \sqrt{ab}$$

$$H = \frac{2}{\frac{1}{a}+\frac{1}{b}} = \frac{2ab}{a+b}$$

$$A \cdot H = \frac{\cancel{a+b}}{2} \cdot \frac{2ab}{\cancel{a+b}} = ab$$

$$\boxed{A \cdot H = G^2} \quad \text{or} \quad \boxed{G^2 = A \cdot H}$$

A, G, H are in G.P.

(b) If $a_1, a_2, a_3, \ldots, a_n$ are n positive numbers

$AM \rightarrow A_n$

$GM \rightarrow G_n$

$HM \rightarrow H_n$

A_n, G_n, H_n are in G.P.

$$\boxed{G_n^2 = A_n \cdot H_n}$$

(c) Applicable for n positive numbers :$\rightarrow$

If $a_1, a_2, a_3, \ldots, a_n$ are n positive numbers and A_n, G_n, H_n are AM, GM and HM respectively then

$$\boxed{R.M.S \geq A_n \geq G_n \geq H_n}$$

Equality holds if $a_1 = a_2 = a_3 = \ldots = a_{n-1} = a_n$

$$AM = \frac{a+b}{2}$$

$$\underbrace{AM \geq GM} \geq HM$$

$$a > b$$

$$GM = \sqrt{ab}$$

$$a-b > 0$$

$$b-a < 0$$

To Prove $AM \geq GM$

$$AM - GM = \frac{a+b}{2} - \sqrt{ab}$$

$$= \frac{a+b-2\sqrt{ab}}{2} = \frac{(\sqrt{a}-\sqrt{b})^2}{2}$$

$$AM - GM \geq 0$$

$$\boxed{AM \geq GM}$$

To Prove $GM \geq HM$

$$GM - HM = \sqrt{ab} - \frac{2ab}{a+b}$$

$$= \sqrt{ab} - \frac{2\sqrt{ab} \cdot \sqrt{ab}}{a+b}$$

$$= \sqrt{ab}\left[1 - \frac{2\sqrt{ab}}{a+b}\right]$$

$$= \sqrt{ab}\left[\frac{a+b-2\sqrt{ab}}{a+b}\right]$$

$$= \frac{\sqrt{ab}}{a+b}\left(\sqrt{a}-\sqrt{b}\right)^2$$

$$GM - HM \geq 0$$

$$\boxed{GM \geq HM}$$

Q 1 If $x>0$, $y>0$, $z>0$, then prove that

$$(x+y)(y+z)(z+x) \geq 8xyz$$

for x, y AM $\geq$ GM

$$\frac{x+y}{2} \geq \sqrt{xy} \Rightarrow (x+y) \geq 2\sqrt{xy} \quad \text{—— (1)}$$

for y, z AM $\geq$ GM $(y+z) \geq 2\sqrt{yz}$ —— (2)

for z, x AM $\geq$ GM $(z+x) \geq 2\sqrt{zx}$ —— (3)

Multiply eqn (1) (2) & (3)

$$(x+y)(y+z)(z+x) \geq 8\sqrt{xy}\cdot\sqrt{yz}\cdot\sqrt{zx}$$

$$\boxed{(x+y)(y+z)(z+x) \geq 8xyz}$$

Q If a, b, c, d are four +ve distinct real numbers $s = a+b+c+d$ then prove that

$$(s-a)(s-b)(s-c)(s-d) > 81abcd$$

$$(b+c+d)(a+c+d)(a+b+d)(a+b+c) > 81abcd$$

$(s-a) = b+c+d$

$(s-b) = a+c+d$

$(s-c) = a+b+d$

$(s-d) = a+b+c$

$$\frac{b+c+d}{3} > \sqrt[3]{bcd}$$

$$\frac{(a+c+d)}{3} > \sqrt[3]{acd}$$

$$\frac{(a+b+d)}{3} > \sqrt[3]{abd}$$

$$\left(\frac{a+b+c}{3}\right) > \sqrt[3]{abc}$$

Q If $A^x = G^y = H^z$, where A, G and H are AM, GM and HM respectively (between two quantities) then prove that x, y, z are in HP.

$$A^x = G^y = H^z = K$$

$$A = K^{1/x} \; ; \quad G = K^{1/y} \; ; \quad H = K^{1/z}$$

$$G^2 = AH$$

$$\left(K^{2/y}\right) = K^{1/x} \cdot K^{1/z}$$

$$K^{\frac{2}{y}} = K^{\frac{1}{x} + \frac{1}{z}}$$

$$\boxed{\frac{2}{y} = \frac{1}{x} + \frac{1}{z}}$$

$\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ are in AP.

x, y, z are in HP.

Cauchy Schwarz Inequality : $\rightarrow$

If $a_1, a_2, a_3, \ldots\ldots a_n$ and $b_1, b_2, b_3 \ldots\ldots, b_n$ be any real numbers such that

$$(a_1 b_1 + a_2 b_2 + a_3 b_3 + \ldots\ldots + a_n b_n)^2 \le (a_1^2 + a_2^2 + \ldots + a_n^2)(b_1^2 + b_2^2 + \ldots + b_n^2)$$

Equality holds if

$$\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3} = \ldots = \frac{a_n}{b_n}$$

Q If a, b, c, d are four positive real numbers such that $abcd = 1$, prove that

$$(1+a)(1+b)(1+c)(1+d) \geq 16$$

$$\frac{1+a}{2} \geq \sqrt{1.a} \Rightarrow 1+a \geq 2\sqrt{a}$$

$$\frac{1+b}{2} \geq \sqrt{b} \Rightarrow 1+b \geq 2\sqrt{b}$$

$$1+c \geq 2\sqrt{c}$$

$$1+d \geq 2\sqrt{d}$$

$$(1+a)(1+b)(1+c)(1+d) \geq 16\sqrt{abcd}$$

$$\boxed{(1+a)(1+b)(1+c)(1+d) \geq 16}$$

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Q If x, y, z are positive real numbers such that $x^2+y^2+z^2 = 27$ then show that $x^3+y^3+z^3 \geq 81$.

from Cauchy Schwarz inequality

$$(a_1a_2+b_1b_2+c_1c_2)^2 \leq (a_1^2+a_2^2+a_3^2)(b_1^2+b_2^2+b_3^2)$$

$a_1 \to x^{3/2}$

$b_1 \to x^{1/2}$

$$(x^2+y^2+z^2)^2 \leq (x^3+y^3+z^3)(x+y+z)$$

$$(x^2+y^2+z^2)^4 \leq (x^3+y^3+z^3)^2(x+y+z)^2$$

$$(27)^4 \leq (x^3+y^3+z^3)^2(81)$$

$$\boxed{(x^3+y^3+z^3) \geq 3^4}$$

$$(x+y+z)^2 = x^2+y^2+z^2 + 2(xy+yz+zx)$$

$$(x+y+z)^2 \leq x^2+y^2+z^2 + x^2+y^2+y^2+z^2 + z^2+x^2$$

$$(x+y+z)^2 \leq 3(x^2+y^2+z^2)$$

$$(x+y+z)^2 \leq 3(27)$$

$$(x+y+z)^2 \leq 81$$

$$\frac{x^2+y^2}{2} \geq \sqrt{x^2y^2}$$

$$x^2+y^2 \geq 2xy$$

$$\frac{x^2+y^2}{2} \geq \sqrt{x^2y^2}$$

$$\frac{x^2+y^2}{2} \geq xy$$

$$x^2+y^2 \geq 2xy$$

Sequence and series

Lecture -7

AGP Arithmatic Geometric Progression :-

$$S = \underline{(a)} + (a+d)r + (a+2d)r^2 + (a+3d)r^3 + \cdots + (a+(n-1)d)r^{n-1}$$

$$S = a + (a+d)r + (a+2d)r^2 + (a+3d)r^3 + \cdots\cdots + (a+(n-1)d)r^{n-1}$$

$$r \cdot S = ar + (a+d)r^2 + (a+2d)r^3 + \cdots + (a+(n-2)d)r^{n-1} + \underline{(a+(n-1)d)r^n}$$

$$S(1-r) = a + \underbrace{dr + dr^2 + dr^3 + \cdots + dr^{n-1}} - (a+(n-1)d \cdot r^n$$

$$S(1-r) = a + \frac{dr \cdot (r^{n-1}-1)}{r-1} - [a+(n-1)d]r^n$$

$$S(1-r) = a + \frac{dr \cdot (1-r^{n-1})}{1-r} - (a+(n-1)d)r^n$$

$$S(1-r) = a + \frac{dr\cdot(1-r^{n-1})}{1-r} - (a+(n-1)d)r^n$$

$$S = \frac{a}{1-r} + \frac{d(r-r^n)}{(1-r)^2} - \frac{(a+(n-1)d)r^n}{(1-r)}$$

$$S = \frac{1}{1-r}\left(a-(a+(n-1)d).r^n\right) + \frac{d(r-r^n)}{(1-r)^2} \qquad r \neq 1$$

for Infinite terms

$$S_\infty = a + (a+d)r + (a+2d)r^2 + (a+3d)r^3 + \cdots$$

$$r S_\infty = ar + (a+d)r^2 + (a+2d)r^3 + \cdots$$

$$S_\infty(1-r) = a + \underbrace{dr + dr^2 + dr^3 + dr^4 + \cdots}$$

$$S_\infty(1-r) = a + \frac{dr}{1-r} \Rightarrow \boxed{S_\infty = \frac{a}{1-r} + \frac{dr}{(1-r)^2}} \qquad 0<|r|<1$$

(1) If $|x| < 1$ then $S_\infty = ?$

$$S = 1 + 2x + 3x^2 + 4x^3 + 5x^4 + \text{-------------}$$

multiply by x

$$xS = x + 2x^2 + 3x^3 + 4x^4 + \text{-----}$$

$$S(1-x) = 1 + \underbrace{x + x^2 + x^3 + x^4 + \text{-----}}$$

$$S(1-x) = \frac{1}{1-x}$$

$$\boxed{S = \frac{1}{(1-x)^2}}$$

$$S(1-x) = 1 + \frac{x}{1-x}$$

$$= \frac{1}{1-x}$$

$$S_\infty = \frac{a}{1-r} + \frac{dr}{(1-r)^2}$$

$a = 1$; $d = 1$

$r = x$

$$S_\infty = \frac{1}{1-x} + \frac{x}{(1-x)^2}$$

$$= \frac{1-x+x}{(1-x)^2}$$

$$S_\infty = \frac{1}{(1-x)^2}$$

Q. If $|x| < 1$, then $S_\infty = ?$

$$S_\infty = 1 + 3x + 6x^2 + 10x^3 + \text{--------}$$

$$x \cdot S_\infty = x + 3x^2 + 6x^3 + \text{------}$$

$$S_\infty(1-x) = 1 + 2x + 3x^2 + 4x^3 + \text{------}$$

$$x \cdot S_\infty(1-x) = x + 2x^2 + 3x^3 + \text{------}$$

$$S_\infty\left[(1-x) - x(1-x)\right] = 1 + x + x^2 + x^3 + \text{-----}$$

$$S_\infty(1-x)(1-x) = \frac{1}{1-x}$$

$$S_\infty(1-x)^2 = \frac{1}{1-x}$$

$$\boxed{S_\infty = \frac{1}{(1-x)^3}}$$

Q 3 If the sum to infinity of the series

$3 + 5r + 7r^2 + \text{-----}$ is $\frac{44}{9}$, then find r.

$S = 3 + 5r + 7r^2 + 9r^3 \text{----}$

$rS = 3r + 5r^2 + 7r^3 + \text{---}$

$S(1-r) = 3 + 2r + 2r^2 + 2r^3 + \text{---}$

$S(1-r) = 3 + \frac{2r}{1-r}$

$S = \frac{3}{1-r} + \frac{2r}{(1-r)^2}$

$\frac{44}{9} = \frac{3(1-r) + 2r}{(1-r)^2}$

$\frac{44}{9} = \frac{3-r}{(1-r)^2}$

$44(1 + r^2 - 2r) = 27 - 9r$

$44r^2 - 79r + 17 = 0$

$44r^2 - 11r - 68r + 17 = 0$

$(4r-1)(11r-17) = 0$

$\boxed{r = 1/4,\ 17/11}$

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Q 4 find S_n & S_∞

$$S = 1 + \frac{4}{5} + \frac{7}{5^2} + \frac{10}{5^3} + \text{------} + \frac{(3n-2)}{5^{n-1}}$$

$$\frac{1}{5}S = \frac{1}{5} + \frac{4}{5^2} + \frac{7}{5^3} + \text{----} + \frac{3n-5}{5^{n-1}} + \frac{3n-2}{5^n}$$

$$S - \frac{1}{5}S = 1 + \underbrace{\frac{3}{5} + \frac{3}{5^2} + \frac{3}{5^3} + \text{------}}_{(n-1)\text{ terms.}} - \frac{3n-2}{5^n}$$

$$\frac{4}{5}S = 1 + \frac{\frac{3}{5}\left(1-\left(\frac{1}{5}\right)^{n-1}\right)}{\left(1-\frac{1}{5}\right)} - \frac{3n-2}{5^n}$$

$$\frac{4}{5}S = 1 + \frac{3}{4}\left(1-\frac{1}{5^{n-1}}\right) - \frac{3n-2}{5^n}$$

$$S = \frac{5}{4} - \frac{3}{4}\cdot\frac{5}{4}\left(1-\frac{1}{5^{n-1}}\right) - \frac{3n-2}{5^n}$$

$$S = \frac{35}{16} - \frac{15}{16}\left(1-\frac{1}{5^{n-1}}\right) - \frac{3n-2}{5^n}$$

$=$

$T_A = 1+(n-1)3$

$= 3n-2$

$T_{GP} = \left(\frac{1}{5}\right)^{n-1}$

Q

$$S = 1 + \frac{4}{5} + \frac{7}{5^2} + \frac{10}{5^3} + \cdots + \frac{3n-2}{5^{n-1}}$$

$$\frac{1}{5}S = \frac{1}{5} + \frac{4}{5^2} + \frac{7}{5^3} + \cdots + \frac{3n-5}{5^{n-1}} + \frac{3n-2}{5^n}$$

$$\frac{4}{5}S = 1 + \underbrace{\frac{3}{5} + \frac{3}{5^2} + \frac{3}{5^3} + \cdots + \frac{3}{5^{n-1}}} - \frac{3n-2}{5^n}$$

$1, 4, 7, 10, \ldots$

$T_n = 1 + (n-1)3$

$= 1 + 3n - 3$

$= 3n - 2$

$$\frac{4}{5}S = 1 + \frac{3/5\left(1 - \left(\frac{1}{5}\right)^{n-1}\right)}{\left(1 - \frac{1}{5}\right)} - \frac{3n-2}{5^n}$$

$$= 1 + \frac{3/\cancel{5}}{4/\cancel{5}}\left(1 - \frac{1}{5^{n-1}}\right) - \frac{3n-2}{5 \cdot 5^{n-1}}$$

$$\frac{4}{5}S = 1 + \frac{3}{4} - \frac{3}{4} \cdot \underbrace{\frac{1}{5^{n-1}}} - \underbrace{\frac{3n-2}{5 \cdot 5^{n-1}}}$$

$$\frac{4}{5}S = \frac{7}{4} + \frac{1}{5^{n-1}}\left(-\frac{3}{4} - \frac{3n-2}{5}\right)$$

$$\frac{4}{5}S = \frac{7}{4} + \frac{1}{5^{n-1}}\left(\frac{-15 - 12n + 8}{20}\right)$$

$$\frac{4}{5}S = \frac{5}{4} \cdot \frac{7}{4} - \frac{(12n+7)}{\cancel{20}_4 \cdot 5^{n-1}}\left(\frac{5}{4}\right)$$

$$\boxed{S = \frac{35}{16} - \frac{1}{16 \cdot 5^{n-1}}(12n+7)}$$

$$\boxed{S_\infty = 35/16}$$

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Q find S_n & S_∞

(i) $\frac{3}{5} + \frac{5}{15} + \frac{7}{45} + \frac{9}{135} + ----$

$S_n = \frac{3}{5}\left[2 - \frac{n+2}{3^n}\right]$

Q $A = \sqrt{2} \cdot \sqrt{\sqrt{4}} \cdot \sqrt{\sqrt{\sqrt{8}}} \sqrt{\sqrt{\sqrt{\sqrt{16}}}} ------$

$A = 2^{1/2} \cdot 2^{2/4} \cdot 2^{3/8} \cdot 2^{4/16}\, 2^{5/32} ----$

$A = 2^{\frac{1}{2} + \frac{2}{4} + \frac{3}{8} + \frac{4}{16} + \frac{5}{32} + ---}$

$\sqrt{\sqrt{4}}$

$(4^{1/2})^{1/2}$

$4^{1/4}$

$\sqrt{\sqrt{8}} = \sqrt{8^{1/2}}$

$8^{1/4}$

$S = \frac{1}{2} + \frac{2}{4} + \frac{3}{8} + \frac{4}{16} + \frac{5}{32} + ---$

$\frac{1}{2}S = \frac{1}{4} + \frac{2}{8} + \frac{3}{16} + \frac{4}{32} + ----$

$\boxed{\begin{aligned} A &= 2^S \\ &= 2^2 = 4 \end{aligned}}$

$\frac{S}{2} = \underbrace{\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + ----}$ $\Rightarrow \frac{S}{2} = \frac{1/2}{1 - 1/2} \Rightarrow \boxed{S = 2}$

Sequence and series

Lecture -8

Special sequences:-

(i) $$\sum_{r=1}^{n} a_r = a_1 + a_2 + a_3 + a_4 + \cdots + a_n$$

$$\sum_{r=1}^{n} b_r = b_1 + b_2 + b_3 + b_4 + \cdots + b_n$$

$$\sum_{r=1}^{n} (a_r \pm b_r) = (a_1 \pm b_1) + (a_2 \pm b_2) + (a_3 \pm b_3) + \cdots + (a_n \pm b_n)$$

$$= (a_1 + a_2 + \cdots + a_n) \pm (b_1 + b_2 + \cdots + b_n)$$

(i) $$\boxed{\sum_{r=1}^{n} (a_r \pm b_r) = \sum_{r=1}^{n} a_r \pm \sum_{r=1}^{n} b_r}$$

$$\sum_{r=1}^{10} a_r = a_1 + a_2 + a_3 + \cdots + a_{10}$$

$$\sum_{r=1}^{10} \frac{1}{a_r + 3} = \frac{1}{a_1+3} + \frac{1}{a_2+3} + \frac{1}{a_3+3} + \cdots + \frac{1}{a_{10}+3}$$

(ii) $$\sum_{r=1}^{n} k \cdot a_r = ka_1 + ka_2 + ka_3 + ka_4 + ka_5 + \cdots + ka_n$$

$$= k\left(\underline{a_1} + \underline{a_2} + \underline{a_3} + \cdots + a_n\right)$$

$$\boxed{\sum_{r=1}^{n} k \cdot a_r = k \cdot \sum_{r=1}^{n} a_r}$$

~~(ii)~~ $\cancel{\underbrace{\sum_{r=1}^{n} a_r \cdot b_r}_{\downarrow} \neq \underbrace{\sum_{r=1}^{n} a_r}\cdot \underbrace{\sum_{r=1}^{n} b_r}}$

$(a_1 b_1 + a_2 b_2 + a_3 b_3 + a_4 b_4 + \cdots + a_n b_n)$ ✓

$\cancel{\underbrace{(a_1 + a_2 + a_3 + \cdots + a_n)}\underbrace{(b_1 + b_2 + b_3 + \cdots b_n)}}$

(iii) $\displaystyle\sum_{r=1}^{n} (\underline{k}) = \underline{k} + \underline{k} + \underline{k} + \underline{k} + \underline{k} + \underline{k} \cdots + \underline{k}$

$= nk$

$$\boxed{\sum_{r=1}^{n} K = nK}$$

where $K \in$ constant.

$$\sum_{r=1}^{3} a_r = a_1 + a_2 + a_3$$

$$\sum_{r=1}^{3} k = k + k + k = 3k$$

$$\boxed{\sum_{r=1}^{n} \left(\frac{a_r}{b_r}\right)} \neq \boxed{\frac{\sum_{r=1}^{n} (a_r)}{\sum_{r=1}^{n} (b_r)}}$$

$$\underbrace{\frac{a_1}{b_1} + \frac{a_2}{b_2} + \frac{a_3}{b_3} + \dots + \frac{a_n}{b_n}} = \frac{a_1 + a_2 + a_3 + \dots + a_n}{b_1 + b_2 + b_3 + \dots + b_n}$$

Type 1

(Sum of first n. natural nos.) —

(i) $$\sum_{r=1}^{n} r = \underbrace{1+2+3+\,-----\,+(n-1)+n}$$

$$= \frac{n}{2}\left[2(1)+(n-1)1\right]$$

$$= \frac{n}{2}(2+n-1) = \frac{n(n+1)}{2}$$

$$\boxed{\sum_{r=1}^{n} r = \frac{n(n+1)}{2}}$$

(Sum of squares of first n natural nos:)-

(ii) $$\sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$$

$$n^3 - (n-1)^3 = n^3 - (n^3 - 1 - 3n(n-1))$$

$$n^3 - (n-1)^3 = \cancel{n^3} - \cancel{n^3} + 1 + 3n^2 - 3n$$

$$n^3 - (n-1)^3 = 3n^2 - 3n + 1$$

at $n=1$ $\quad 1^3 - 0^3 = 3(1^2) - 3(1) + 1$

at $n=2$ $\quad 2^3 - 1^3 = 3(2^2) - 3(2) + 1$

at $n=3$ $\quad 3^3 - 2^3 = 3(3^2) - 3(3) + 1$

at $n=4$ $\quad 4^3 - 3^3 = 3(4^2) - 3(4) + 1$

⋮

at $n=n$ $\quad n^3 - (n-1)^3 = 3(n^2) - 3(n) + 1$

<u>add</u> $\quad n^3 - 0^3 = 3(1^2 + 2^2 + 3^2 + 4^2 + \dots + n^2) - 3(1 + 2 + \dots + n) + n$

$$n^3 = 3\left(\sum_{r=1}^{n} r^2\right) - \underbrace{3\left(\frac{n(n+1)}{2}\right) \quad + n}$$

$$n^3 + \underbrace{\frac{3n(n+1)}{2}}\ \underline{-n} = 3\sum_{r=1}^{n}(r^2)$$

$$\frac{n}{3}\left[n^2 + \frac{3}{2}(n+1) - 1\right] = \sum_{r=1}^{n}(r^2)$$

$$\frac{n}{3}\left[\frac{2n^2+3n+3-2}{2}\right] = \sum_{r=1}^{n}(r^2)$$

$$\frac{n}{6}\underbrace{(2n^2+3n+1)} = \sum_{r=1}^{n}(r^2)$$

$$\frac{n(2n+1)(n+1)}{6} = \sum_{r=1}^{n}(r^2)$$

$$\boxed{\sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}}$$

(Sum of the cubes of first n natural nos.)

(iii) $\sum_{r=1}^{n} r^3 = \left(\frac{n(n+1)}{2}\right)^2$

$$n^4 - (n-1)^4 = n^4 - (n^4 - 4n^3 + 6n^2 - 4n + 1)$$

$$n^4 - (n-1)^4 = 4n^3 - 6n^2 + 4n - 1$$

at $n=1$ $\quad 1^4 - 0^4 = 4(1^3) - 6(1^2) + 4(1) - 1$

at $n=2$ $\quad 2^4 - 1^4 = 4(2^3) - 6(2^2) + 4(2) - 1$

at $n=3$ $\quad 3^4 - 2^4 = 4(3^3) - 6(3^2) + 4(3) - 1$

at $n=4$ $\quad 4^4 - 3^4 = 4(4^3) - 6(4^2) + 4(4) - 1$

$\vdots$

at $n=n$ $\quad n^4 - (n-1)^4 = 4(n^3) - 6(n^2) + 4(n) - 1$

add $\quad n^4 - 0^4 = 4(1^3 + 2^3 + 3^3 + \cdots + n^3) - 6(1^2 + 2^2 + \cdots + n^2)$

$+4(1 + 2 + 3 + \cdots + n) - (n)$

$$n^4 - 0^4 = 4(1^3 + 2^3 + 3^3 + \cdots + n^3) - 6(1^2 + 2^2 + \cdots + n^2) + 4(1 + 2 + 3 + \cdots + n) - (n)$$

$$n^4 = \underbrace{4\sum_{r=1}^{n} r^3} - 6\sum_{r=1}^{n} r^2 + 4\sum_{r=1}^{n} r - (n)$$

$$4\sum_{r=1}^{n} r^3 = n^4 + 6\sum_{r=1}^{n} r^2 - 4\sum_{r=1}^{n}(r) + n$$

$$= n^4 + \cancel{6}\,\frac{n(n+1)(2n+1)}{\cancel{6}} - \overset{2}{\cancel{4}}\left(\frac{n(n+1)}{\cancel{2}}\right) + n$$

$$= n^4 + n(n+1)(2n+1) - 2n(n+1) + n$$

$$= n\left[n^3 + (n+1)(2n+1) - 2(n+1) + 1\right]$$

$$= n\left[n^3 + 2n^2 + \cancel{2n} + n + \cancel{1} - \cancel{2n} - \cancel{2} + \cancel{1}\right]$$

$$4\sum r^3 = n(n)(n^2 + 2n + 1)$$

$$4\sum r^3 = n^2(n+1)^2$$

$$\sum r^3 = \frac{n^2(n+1)^2}{4}$$

$$\sum r^3 = \left(\frac{n(n+1)}{2}\right)^2$$

$$\boxed{\sum_{r=1}^{n} r^3 = \left(\frac{n(n+1)}{2}\right)^2} = \left[\sum_{r=1}^{n}(r)\right]^2$$

Note: If r^{th} term of a sequence is T_r, then the sum of n terms of sequence is given by $S_n = \sum_{r=1}^{n} T_r$.

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Q (1) find $\sum_{r=1}^{24}(25r) - \sum_{r=1}^{24}(r^2)$

$$= 25(1+2+3+\cdots+24) - (1^2+2^2+3^2+\cdots+24^2)$$

$$= 25\left(\frac{\overset{12}{\cancel{(24)}}(24+1)}{\cancel{2}}\right) - \left(\frac{\overset{4}{\cancel{24}}(24+1)(2(24)+1)}{\cancel{6}}\right)$$

$$= 25(25)(12) - 4(25)(49)$$

$$= 25(4)[75-49]$$

$$= 100[26] = 2600$$

Q $\sum_{n=1}^{10} n^2(n-1)$

$$= \sum_{n=1}^{10} (n^3 - n^2)$$

$$= \sum_{n=1}^{10} n^3 - \sum_{n=1}^{10} n^2$$

$$= \left(\frac{n(n+1)}{2}\right)^2 - \frac{n(n+1)(2n+1)}{6}$$

$$= \left(\frac{10\ (11)}{2}\right)^2 - \frac{10(11)(21)}{6}$$

Q $\sum_{i=1}^{n}\sum_{j=1}^{i}\sum_{k=1}^{j}(1)$

$$= \sum_{i=1}^{n}\sum_{j=1}^{i}[\underbrace{1+1+1+ \text{----} +1}_{j \text{ times}}]$$

$$= \sum_{i=1}^{n}\underbrace{\sum_{j=1}^{i}(j)}$$

$$= \sum_{i=1}^{n}[1+2+3+ \text{-----} +i]$$

$$= \sum_{i=1}^{n}\left(\frac{i(i+1)}{2}\right)$$

$$= \sum_{i=1}^{n}\left(\frac{i^2}{2}+\frac{i}{2}\right)$$

$$= \frac{1}{2}\cdot\frac{n(n+1)(2n+1)}{6} + \frac{1}{2}\frac{n(n+1)}{2}$$

$$= \frac{n(n+1)}{4}\left[\frac{2n+1}{3}+1\right]$$

$$= \frac{n(n+1)}{4}\ \frac{(2n+1+3)}{3}$$

$$= \frac{n(n+1)\ (2n+4)}{12}$$

$$= \frac{n(n+1)\ (n+2)}{6}$$

Q find sum $3^2 + 7^2 + 11^2 + \text{- - - -}$ (sum upto n terms)

$$T_n = (4n-1)^2$$

$3, 7, 11, \text{- - -}$

$T_n = 3 + (n-1)4 = (4n-1)$

$$T_n = 16n^2 + 1 - 8n$$

$$S_n = \sum_{r=1}^{n} T_r$$

$$= \sum \left(16n^2 + 1 - 8n\right)$$

$$= \cancel{16}^{\,8} \frac{n(n+1)(2n+1)}{\cancel{6}_{\,3}} + n - \cancel{8}^{\,4} \frac{n(n+1)}{\cancel{2}}$$

$$= n\left[\frac{8}{3}(n+1)(2n+1) + 1 - 4(n+1)\right]$$

$$= \frac{n}{3}\left[8(2n^2 + 3n + 1) + 3 - 12n - 12\right]$$

$$\boxed{S_n = \frac{n}{3}\left[16n^2 + 12n - 1\right]}$$

Homework

Race 25 Complete

Race 26 2, 3, 5, 7, 8, 9, 10, 11, 12, 13

Ex-1 Ques 1 to 15

Ex-2 Ques 1 to 5

Sequence and series

Lecture -9

Type-2 [using method of difference] →

If $T_1, T_2, T_3, \ldots\ldots\ldots$ are the terms of a sequence then the terms

$T_2 - T_1, T_3 - T_2, T_4 - T_3 \ldots\ldots\ldots$

some times are in A.P. and some times in G.P. for such series we first compute their n^{th} term and then compute the sum to n terms, using sigma notation.

Note :

(i) If k^{th} order difference between consecutive terms is constant then r^{th} term is polynomial of degree k in r.

e.g. (1) $1, 3, 5, 7, \ldots\ldots$

I^{st} order difference $d_1 = 2$

$\Rightarrow \quad T_r = ar + b$

(2) $3, 7, 14, 24 \ldots\ldots$

I^{st} order difference $d_1 : 4, 7, 10$

II^{st} order difference $d_2 = 3$

$\Rightarrow \quad T_r = ar^2 + br + c$

Now find the value a, b, c from given sequence

(ii) If k^{th} order difference are in G.P. then r^{th} term of the sequence is given by

$T_r = a(CR)^r$ + a polynomial of degree $(k-1)$ in 'n'

e.g. (1) $1, 3, 7, 15 \ldots\ldots$

I^{st} order difference $d_1 : 2, 4, 8\ldots$

$\Rightarrow \quad T_r = a(2)^r + b$

(2) $1 \quad 2 \quad 5 \quad 12 \quad 27$

I^{st} order difference $d_1 : 1, 3, 7, 15$

II^{st} order difference $d_2 = 2,4,8$

$\Rightarrow \quad T_r = a(2^r) + bn + c$

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① $S = 6 + 13 + 22 + 33 + \text{------}$ upto n terms

$S = 6 + 13 + 22 + 33 + \text{------} T_{n-1} + T_n$

$S = \quad 6 + 13 + 22 + \text{----} T_{n-2} + T_{n-1} + T_n$

Subtract

$0 = 6 + 7 + 9 + 11 + \text{-----} + (T_n - T_{n-1}) - T_n$

$T_n = \underbrace{6 + 7 + 9 + 11 + \text{----} + (T_n - T_{n-1})}_{n \text{ terms}}$

$T_n = 6 + \frac{n-1}{2}\left[2(7) + (n-1-1)2\right]$

$= 6 + \frac{n-1}{2}\left[14 + 2n - 4\right] = 6 + (n-1)(n+5)$

$= 6 + n^2 + 4n - 5 = n^2 + 4n + 1$

$\boxed{T_n = n^2 + 4n + 1}$ ✓

$S = T_1 + T_2 + T_3 + T_4$

$S = \quad T_1 + T_2 + T_3 + T_4$

$S - S = T_1 + (T_2 - T_1) + (T_3 - T_2) + (T_4 - T_3) - T_4$

$0 = \underbrace{T_1 + (T_2 - T_1) + (T_3 - T_2)}$
$+ \underbrace{(T_4 - T_3)}$ $+T_4$

$T_4 =$

$$S_n = \Sigma (n^2+4n+1)$$

$$= \underline{\Sigma n^2} + \underline{4\Sigma n} + \underline{\Sigma 1}$$

$$= \underbrace{\frac{n(n+1)(2n+1)}{6}} + \underbrace{4\,\frac{n(n+1)}{2}} + \underline{n}$$

$$= n\left[\frac{(n+1)}{6}(2n+1) + 2(n+1) + 1\right]$$

$$= \frac{n}{6}\left[2n^2+3n+1 + 12(n+1)+6\right]$$

$$= \frac{n}{6}\left[2n^2+15n+19\right]$$

$$\underbrace{\frac{n(n+1)(2n+13)}{6} + n}$$

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Q

$$S = 3 + 8 + 15 + 24 + \text{------} + T_{n-1} + T_n$$

$$S = 3 + 8 + 15 + \text{----} + T_{n-2} + T_{n-1} + T_n$$

$$0 = \underbrace{3 + 5 + 7 + 9 + \text{-----------} + (T_{n-1} - T_{n-2}) + (T_n - T_{n-1})} - T_n$$

$$T_n = \frac{n}{2}[2(3) + (n-1)2] = \frac{n}{2}[6 + 2n - 2] = n(n+2) = n^2 + 2n$$

$$S_n = \Sigma T_n = \Sigma n^2 + \Sigma 2n$$

$$= \Sigma n^2 + 2\Sigma n = \frac{n(n+1)(2n+1)}{6} + \cancel{2}\frac{n(n+1)}{\cancel{2}}$$

$$= n(n+1)\left[\frac{2n+1}{6} + 1\right]$$

$$= \frac{n(n+1)}{6}(2n+7)$$

Q

$$S = 5 + 7 + 13 + 31 + 85 + - - - - - - -$$

$$S = \quad 5 + 7 + 13 + 31 + - - - -$$

$$T_n = 5 + \underbrace{2 + 6 + 18 + 54 + - - - -}_{n-1 \text{ terms}}$$

$$= 5 + \frac{\cancel{2} \cdot (3^{n-1} - 1)}{\cancel{3-1}}$$

$$S = \frac{a(r^n - 1)}{(r-1)}$$

$$= 5 + 3^{n-1} - 1 = 3^{n-1} + 4$$

$$S_n = \sum_{n=1}^{n} (3^{n-1} + 4)$$

$$= \underbrace{3^0 + 3^1 + 3^2 + 3^3 + - - - + 3^{n-1}} + 4n$$

$$= \frac{3^n - 1}{3-1} + 4n = \frac{3^n - 1}{2} + 4n.$$

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Q $S = 2+5+14+41+122+\text{---------}$ upto n terms.

$S = \quad 2+5+14+41+\text{--}$

$T_n = 2+\underbrace{3+9+27+81+\text{---}}_{(n-1)\text{ terms}}$

$$= 2+\frac{3(3^{n-1}-1)}{(3-1)} = 2+\frac{3}{2}(3^{n-1}-1)$$

$$= \frac{1}{2}(4+3^n-3) = \frac{3^n+1}{2}$$

$$S_n = \Sigma\frac{3^n+1}{2} = \frac{1}{2}\Sigma 3^n + \Sigma\frac{1}{2}$$

$$= \frac{1}{2}\left[3^1+3^2+3^3+3^4+\text{----}\right]+\frac{n}{2}$$

$$= \frac{1}{2}\left[\frac{3\cdot(3^n-1)}{3-1}\right]+\frac{n}{2} = \frac{1}{4}(3^{n+1}-3)+\frac{n}{2}$$

Type-3 Splitting the n^{th} term as a difference of two

Q $S_n = \frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \frac{1}{4\cdot 5} + \cdots + \frac{1}{n(n+1)}$

$S_n = \frac{2-1}{1\cdot 2} + \frac{3-2}{2\cdot 3} + \frac{4-3}{3\cdot 4} + \cdots + \frac{(n+1)-n}{n(n+1)}$

$S_n = \frac{\cancel{2}}{1\cdot\cancel{2}} - \frac{1}{1\cdot 2} + \frac{\cancel{3}}{2\cdot\cancel{3}} - \frac{2}{2\cdot 3} + \frac{4}{3\cdot 4} - \frac{3}{3\cdot 4} + \cdots + \frac{(n+1)}{n(n+1)} - \frac{n}{n(1+n)}$

$= 1 - \cancel{\frac{1}{2}} + \cancel{\frac{1}{2}} - \cancel{\frac{1}{3}} + \cancel{\frac{1}{3}} - \cancel{\frac{1}{4}} + \cdots + \cancel{\frac{1}{n}} - \frac{1}{n+1}$

$= 1 - \frac{1}{n+1} = \frac{n+1-1}{n+1} = \frac{n}{n+1}$

$\boxed{S_n = \frac{n}{n+1}}$

$\boxed{S_\infty = 1}$

ALLEN CAREER INSTITUTE KOTA (RAJASTHAN) path to success

Q

$$S = \sum_{r=1}^{n} \frac{1}{4r^2-1} = \sum_{r=1}^{n} \frac{1}{(2r+1)(2r-1)}$$

$$= \sum_{r=1}^{n} \frac{(2r+1)-(2r-1)}{2(2r+1)(2r-1)} = \frac{1}{2}\sum_{r=1}^{n}\left[\frac{2r+1}{(2r+1)(2r-1)} - \frac{(2r-1)}{(2r+1)(2r-1)}\right]$$

$$= \frac{1}{2}\sum_{r=1}^{n}\left\{\underbrace{\left(\frac{1}{2r-1}\right) - \left(\frac{1}{2r+1}\right)}\right\}$$

$$T_1 = \frac{1}{2}\left[\frac{1}{1} - \cancel{\frac{1}{3}}\right]$$

$$T_2 = \frac{1}{2}\left[\cancel{\frac{1}{3}} - \cancel{\frac{1}{5}}\right]$$

$$T_3 = \frac{1}{2}\left[\cancel{\frac{1}{5}} - \cancel{\frac{1}{7}}\right]$$

$$T_n = \frac{1}{2}\left[\cancel{\frac{1}{2n-1}} - \frac{1}{2n+1}\right]$$

$$S = \frac{1}{2}\left[1 - \frac{1}{2n+1}\right]$$

$$= \frac{1}{2}\left[\frac{2n+1-1}{2n+1}\right] = \frac{n}{2n+1}$$

ALLEN CAREER INSTITUTE KOTA (RAJASTHAN) path to success

Q

$$S = \frac{1}{1\cdot 4} + \frac{1}{4\cdot 7} + \frac{1}{7\cdot 10} + \frac{1}{10\cdot 13} + \cdots + \frac{1}{(3n-2)(3n+1)}$$

$$S = \frac{1}{3}\left[\frac{4-1}{1\cdot 4} + \frac{7-4}{4\cdot 7} + \frac{10-7}{7\cdot 10} + \frac{13-10}{10\cdot 13} + \cdots \frac{(3n+1)-(3n-2)}{(3n+1)(3n-2)}\right]$$

$$= \frac{1}{3}\left[\frac{4}{1\cdot 4} - \frac{1}{1\cdot 4} + \frac{7}{4\cdot 7} - \frac{4}{4\cdot 7} + \frac{10}{7\cdot 10} - \frac{7}{7\cdot 10} + \cdots \frac{1}{3n-2} - \frac{1}{3n+1}\right]$$

$$= \frac{1}{3}\left[1 - \cancel{\frac{1}{4}} + \cancel{\frac{1}{4}} - \cancel{\frac{1}{7}} + \cancel{\frac{1}{7}} \cdots + \frac{1}{3n-2} - \frac{1}{3n+1}\right]$$

$$= \frac{1}{3}\left[1 - \frac{1}{3n+1}\right] = \frac{1}{3}\left[\frac{3n+1-1}{3n+1}\right]$$

$$\boxed{S = \frac{n}{3n+1}}$$

$1, 4, 7, 10, \ldots$

$T_{n_1} = 1 + (n-1)3$

$= 3n-2$

$4, 7, 10, 13, \ldots$

$T_{n_2} = 4 + (n-1)3$

$= 3n+1$

$$T_n = \frac{1}{(3n-2)(3n+1)} = \frac{1}{9n^2-3n-2}$$

$$\cancel{S_n = \frac{1}{\Sigma 9n^2 - 3\Sigma n - \Sigma 2}}$$

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Type - 4

① $1\cdot 2 + 2\cdot 3 + 3\cdot 4 + ---- +$ upto n terms.

$T_n = n(n+1)$

$S_n = \Sigma(n^2+n)$

$= \dfrac{n(n+1)(2n+1)}{6} + \dfrac{n(n+1)}{2}$

$1, 2, 3, 4, ---$

$T_{n_1} = 1 + (n-1)1$

$= 1+n-1 = n$

$2, 3, 4, --$

$T_{n_2} = 2 + (n-1)1$

$= n+1$

<u>Q</u> $1\cdot2\cdot3 + 2\cdot3\cdot4 + 3\cdot4\cdot5 + \text{-----}.$

Q $1\cdot5\cdot9 + 2\cdot6\cdot10 + 3\cdot7\cdot11 + \text{-----}$ upto n terms.

$1, 2, 3, 4, -\ -$

$T_{n_1} = n$

$5, 6, 7, -\ -$

$T_{n_2} = 5 + (n-1)1$

$= n+4$

$9, 10, 11, -\ -$

$T_{n_3} = 9 + (n-1)1$

$= n+8$

$T_n = n(n+4)(n+8)$

$= n(n^2 + 12n + 32)$

$T_n = n^3 + 12n^2 + 32n$

$S_n = \Sigma n^3 + 12\,\Sigma n^2 + 32\,\Sigma n$

V_n Method :-

(1) $1\cdot2\cdot3 + 2\cdot3\cdot4 + 3\cdot4\cdot5 + \text{-----}$

$$T_n = n(n+1)(n+2)$$

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Q $1\cdot4\cdot7\cdot10 + 4\cdot7\cdot10\cdot13 + 7\cdot10\cdot13\cdot16 + \cdots$

$T_n = (3n-2)(3n+1)(3n+4)(3n+7)$

$V_n = (3n-2)(3n+1)(3n+4)(3n+7)(3n+10)$

$V_{n-1} = (3n-5)(3n-2)(3n+1)(3n+4)(3n+7)$

$V_n = T_n(3n+10)$

$V_{n-1} = T_n(3n-5)$

$V_n - V_{n-1} = T_n(3n+10-3n+5)$

$$\boxed{V_n - V_{n-1} = T_n(15)}$$

$T_n = \frac{1}{15}(V_n - V_{n-1})$

$1, 4, 7, \ldots$

$T_{n_1} = 1 + (n-1)3$

$= 3n-2$

$4, 7, 10, \ldots$

$T_{n_2} = 3n+1$

$$T_n = \frac{1}{15}(V_n - V_{n-1})$$

$$\left[\begin{aligned}
T_1 &= \frac{1}{15}(\cancel{V_1} - V_0)\\
T_2 &= \frac{1}{15}(\cancel{V_2} - \cancel{V_1})\\
T_3 &= \frac{1}{15}(\cancel{V_3} - \cancel{V_2})\\
T_4 &= \frac{1}{15}(\cancel{V_4} - \cancel{V_3})\\
&\vdots\\
T_{n-1} &= \frac{1}{15}(\cancel{V_{n-1}} - \cancel{V_{n-2}})\\
T_n &= \frac{1}{15}(V_n - \cancel{V_{n-1}})
\end{aligned}\right]$$

$$S = T_1 + T_2 + T_3 + \cdots + T_n$$

$$S = \frac{1}{15}(V_n - V_0)$$

$$= \frac{1}{15}\left[(3n-2)(3n+1)(3n+4)(3n+7)(3n+10) + 560\right)$$

$$\boxed{S = \frac{1}{15}\left[(3n-2)(3n+1)(3n+4)(3n+7)(3n+10) + 560\right]}$$

$$V_n = (3n-2)(3n+1)(3n+4)(3n+7)(3n+10)$$

$$= (-2)(1)(4)(7)(10) = -560$$

Q

$1\cdot3\cdot5\cdot7 + 3\cdot5\cdot7\cdot9 + 5\cdot7\cdot9\cdot11 + \cdots\cdots$

$T_n = (2n-1)(2n+1)(2n+3)(2n+5)$

$V_n = \underline{(2n-1)(2n+1)(2n+3)(2n+5)}(2n+7) = (2n+7)T_n$

$V_{n-1} = (2n-3)\underline{(2n-1)(2n+1)(2n+3)(2n+5)} = (2n-3)T_n$

Subtract

$V_n - V_{n-1} = [2n+7-(2n-3)]T_n$

$V_n - V_{n-1} = 10T_n$

$T_n = \frac{1}{10}[V_n - V_{n-1}]$

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$$T_n = \frac{1}{10}\left[V_n - V_{n-1}\right]$$

$$V_n = (2n-1)(2n+1)(2n+3)(2n+5)(2n+7)$$

$$V_0 = (-1)(1)(3)(5)(7) = -105$$

$$T_1 = \frac{1}{10}\left[\cancel{V_1} - V_0\right]$$

$$T_2 = \frac{1}{10}\left[\cancel{V_2} - \cancel{V_1}\right]$$

$$T_3 = \frac{1}{10}\left[\cancel{V_3} - \cancel{V_2}\right]$$

$$\vdots$$

$$T_n = \frac{1}{10}\left[V_n - \cancel{V_{n-1}}\right]$$

add $\quad S = \frac{1}{10}\left[V_n - V_0\right]$

$$S = \frac{1}{10}\left[(2n-1)(2n+1)(2n+3)(2n+5)(2n+7) + 105\right]$$

Q $\frac{1}{1\cdot3\cdot5\cdot7\cdot9} + \frac{1}{3\cdot5\cdot7\cdot9\cdot11} + \frac{1}{5\cdot7\cdot9\cdot11\cdot13} + \text{------}$ find S_n & S_∞.

$$T_n = \frac{1}{(2n-1)(2n+1)(2n+3)(2n+5)(2n+7)}$$

$$V_n = \frac{1}{(2n+1)(2n+3)(2n+5)(2n+7)}\cdot\frac{(2n-1)}{(2n-1)} = (2n-1)\cdot T_n$$

$$V_{n-1} = \frac{1}{(2n-1)(2n+1)(2n+3)(2n+5)}\cdot\frac{(2n+7)}{(2n+7)} = (2n+7)\,T_n$$

$$V_n - V_{n-1} = T_n(2n-1-2n-7)$$

$$T_n = -\frac{1}{8}\left(V_n - V_{n-1}\right)$$

$$T_n = -\frac{1}{8}\left(V_n - V_{n-1}\right)$$

$$T_1 = -\frac{1}{8}\left(\cancel{V_1} - V_0\right)$$

$$T_2 = -\frac{1}{8}\left(\cancel{V_2} - \cancel{V_1}\right)$$

$$T_3 = -\frac{1}{8}\left(\cancel{V_3} - \cancel{V_2}\right)$$

$$T_4 = -\frac{1}{8}\left(\cancel{V_4} - \cancel{V_3}\right)$$

$$\vdots$$

$$T_n = -\frac{1}{8}\left(V_n - \cancel{V_{n-1}}\right)$$

Sum $S = -\frac{1}{8}\left(V_n - V_0\right)$

$$S_n = -\frac{1}{8}\left[\frac{1}{(2n+1)(2n+3)(2n+5)(2n+7)} - \frac{1}{105}\right]$$

$$V_n = \frac{1}{(2n+1)(2n+3)(2n+5)(2n+7)}$$

$$V_0 = \frac{1}{105}$$

$$S_\infty = -\frac{1}{8}\left(\frac{1}{\infty} - \frac{1}{105}\right)$$

$$= -\frac{1}{8}\left(0 - \frac{1}{105}\right)$$

$$= \frac{1}{840}$$

Q find S_n & S_∞

$$\frac{1}{1\cdot2\cdot3\cdot4}+\frac{1}{2\cdot3\cdot4\cdot5}+\frac{1}{3\cdot4\cdot5\cdot6}+\frac{1}{4\cdot5\cdot6\cdot7}+\cdots\cdots$$

$$T_n=\frac{1}{n(n+1)(n+2)(n+3)}$$

$$V_n=\frac{1}{(n+1)(n+2)(n+3)}=n\cdot T_n \qquad V_0=\frac{1}{6}$$

$$V_{n-1}=\frac{1}{n(n+1)(n+2)}=(n+3)\,T_n$$

$$V_n-V_{n-1}=(n-n-3)\,T_n$$

$$T_n=-\frac{1}{3}\left[V_n-V_{n-1}\right]$$

$$T_n = -\frac{1}{3}(v_n - v_{n-1})$$

$$T_1 = -\frac{1}{3}(v_1 - v_0)$$

$$T_2 = -\frac{1}{3}(v_2 - v_1)$$

$$T_3 = -\frac{1}{3}(v_3 - v_2)$$

$$T_4 = -\frac{1}{3}(v_4 - v_3)$$

$$\vdots$$

$$T_n = -\frac{1}{3}(v_n - v_{n-1})$$

$$S = -\frac{1}{3}(v_n - v_0)$$

$$= -\frac{1}{3}\left(\frac{1}{(n+1)(n+2)(n+3)} - \frac{1}{6}\right)$$

$$S = \frac{1}{18} - \frac{1}{3(n+1)(n+2)(n+3)}$$

$$S_\infty = \frac{1}{18} - 0 = \frac{1}{18}$$

$$S_\infty = \frac{1}{18}$$

Q find S_n

$$\frac{3}{1^2}+\frac{5}{1^2+2^2}+\frac{7}{1^2+2^2+3^2}+\text{-----}$$

$$T_n=\frac{2n+1}{1^2+2^2+3^2+\text{----}+n^2}=\frac{\cancel{(2n+1)}\,6}{n(n+1)\cancel{(2n+1)}}$$

$$T_n=\frac{6}{n(n+1)}=6\left[\frac{1}{n}-\frac{1}{n+1}\right]$$

$$T_1=6\left[1-\cancel{\frac{1}{2}}\right]$$

$$T_2=6\left[\cancel{\frac{1}{2}}-\cancel{\frac{1}{3}}\right]$$

$$T_3=6\left[\cancel{\frac{1}{3}}-\cancel{\frac{1}{4}}\right]$$

$$T_n=6\left[\cancel{\frac{1}{n}}-\frac{1}{n+1}\right]$$

$$S=6\left[1-\frac{1}{n+1}\right]=\frac{6n}{n+1}$$

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Q find S_n $\quad \frac{4}{1\cdot 2\cdot 3} + \frac{5}{2\cdot 3\cdot 4} + \frac{6}{3\cdot 4\cdot 5} + \cdots$

$4, 5, 6, \cdots$

$T_n = 4 + (n-1)1$

$= n+3$

$$T_n = \frac{n+3}{n(n+1)(n+2)} = \frac{\cancel{(n+2)}}{n(n+1)\cancel{(n+2)}} + \frac{1}{n(n+1)(n+2)}$$

$$T_n = \frac{1}{n(n+1)} + \frac{1}{n(n+1)(n+2)}$$

$$T_n = \left[\frac{1}{n} - \frac{1}{n+1}\right] + S_n$$

$$S = \left(1 - \frac{1}{n+1}\right) - \frac{1}{2}\left[\frac{1}{(n+1)(n+2)} - \frac{1}{2}\right]$$

$$= \left(1 + \frac{1}{4}\right) - \frac{1}{n+1} - \frac{1}{2(n+1)(n+2)}$$

$$= \frac{5}{4} - \frac{2n+4+1}{2(n+1)(n+2)}$$

$$T_n = \frac{1}{n(n+1)(n+2)}$$

$$V_n = \frac{1}{(n+1)(n+2)} = nT_n$$

$$V_{n-1} = \frac{1}{n(n+1)} = (n+2)T_n$$

$$T_n = -\frac{1}{2}(V_n - V_{n-1})$$

$$S_n = -\frac{1}{2}(V_n - V_0)$$

$$= -\frac{1}{2}\left[\frac{1}{(n+1)(n+2)} - \frac{1}{2}\right]$$

$$T_n = \frac{1}{(n+1)(n+2)} + \frac{3}{n(n+1)(n+2)}$$

$$\frac{1}{n+1} - \frac{1}{n+2}$$

$$S_n = \left(\frac{1}{2} - \frac{1}{n+2}\right) + 3\left(-\frac{1}{2}\right)\left[\frac{1}{(n+1)(n+2)} - \frac{1}{2}\right]$$

$$T_1 = \frac{1}{2} - \cancel{\frac{1}{3}}$$

$$T_2 = \cancel{\frac{1}{3}} - \cancel{\frac{1}{4}}$$

$$\vdots$$

$$S = \frac{1}{2} - \frac{1}{n+2}$$