EXAMPLAR

Index

FUNDAMENTAL OF MATHEMATICS

True/False

1. Consider the following statements :

S_1 : $x = \sqrt{\log_{11} 7}$ and $y = \sqrt{\log_7 11}$, then $e^{y\,\ell n 7 - x\,\ell n 11}$ is equal to 1.

S_2 : For real values of x : $x^2 - |x| = 0$ has 3 distinct solutions.

State, in order, whether S_1, S_2 are true or false

(A) T F (B) F F (C) F T (D) T T

2. Consider the following statements :

S_1 : $\log_{10} \alpha$, $\log_3 \alpha$, $\log_e \alpha$, $\log_2 \alpha$ ($\alpha > 1$) are in increasing order

S_2 : $\log 1 + \log 2 + \log 3 = \log (1 \times 2 \times 3)$

State, in order, whether S_1, S_2 are true or false

(A) T F (B) F F (C) F T (D) T T

Fill in the blanks

3. The solution set of the equation $x^{\log_a x} = \left(a^{\pi}\right)^{\log_a^3 x}$ is____________, (where $a > 0$ & $a \neq 1$) .

4. The value of b satisfying $\log_{\sqrt{8}} b = 3\frac{1}{3}$ is ____________.

***5.** Solution set of the equation, $\log_{10}^2 x + \log_{10} x^2 = \log_{10}^2 2 - 1$ is ____________ .

***6.** The expression $\sqrt{\log_{0.5}^2 8}$ has the value equal to ____________.

7. The expression $(0.05)^{\log_{\sqrt{20}}(0.\bar{1})}$ is a perfect square of the natural number ____________.

(where $0.\bar{1}$ denotes 0.111111 .... ∞)

8. If $\log_7 2 = m$, then $\log_{49} 28$ in terms of m has the value equal to ____________.

9. Let p be the integral part of $\log_3 108$ and q be the integral part of $\log_5 375$ then $p + q - pq$ has the value equal to ____________.

***10.** $\log(a + b) = \log ab$ $(a, b > 0)$, then $\dfrac{(a^3-1)(b^3-1)-1}{ba(a+b)}$ is____________.

Assertion & Reason

These questions contains, Statement I (assertion) and Statement II (reason).

(A) Statement-I is true, Statement-II is true ; Statement-II is correct explanation for Statement-I.
(B) Statement-I is true, Statement-II is true ; Statement-II is NOT a correct explanation for statement-I.
(C) Statement-I is true, Statement-II is false.
(D) Statement-I is false, Statement-II is true.

11. ***Statement-I :*** If $a = y^2$, $b = z^2$, $c = x^2$, then $8\log_a x^3 . \log_b y^3 . \log_c z^3 = 27$

Becuase

Statement-II : $\log_b a . \log_c b = \log_c a$, also $\log_b a = \dfrac{1}{\log_a b}$

12. ***Statement-I :*** If $\log_{(\log_5 x)} 5 = 2$, then $x = 5^{\sqrt{5}}$

Becuase

Statement-II : $\log_x a = b$, if $a > 0$, then $x = a^{1/b}$

***13.** Statement-I : The equation $\log_{\frac{1}{2+|x|}}(5+x^2) = \log_{(3+x^2)}(15+\sqrt{x})$ has real solutions.

Becuase

Statement-II : $\log_{1/b}a = -\log_b a$ (where a,b > 0 and $b \neq 1$) and if number and base both are greater than unity then the logarithm is positive.

14. ***Statement-I :*** $|x + 1| = |x| + 1$ Possible only when $x \in [0, \infty]$

Becuase

Statement-II : $|x_1 + x_2| = |x_1| + |x_2| \Rightarrow x_1x_2 \geq 0$.

15. ***Statement-I :*** $2^{2^n} + 1$ is divisible by 2 but not by 4, $n \in N$

Becuase

Statement-II : $2^{2^n} + 1$ is an odd number., $n \in N$

Single Choice Correct

16. The sum of the series : $\frac{1}{(1\times 2)} + \frac{1}{(2\times 3)} + \frac{1}{(3\times 4)} + \ldots\ldots.. + \frac{1}{(100\times 101)}$ is equal to

(A) 200/101 (B) 100/101 (C) 50/101 (D) 25/101

***17.** If $\frac{(x-1)(x+1)(x-2)^2(e^{3x}-1)}{x(3-x)^3(x-1)^2(x+3)} \leq 0$, the complete solution set of values of x is

(A) $(-\infty, -3) \cup (3, \infty)$ (B) $(-\infty, -3) \cup [-1, 1) \cup (3, \infty) \cup \{2\} - \{0\}$

(C) $(-\infty, -3) \cup (-1, 1) \cup (3, \infty)$ (D) none of these

***18.** The solution of $2^x + 2^{|x|} \geq 2\sqrt{2}$ is

(A) $\left((-\infty, \log_2(\sqrt{2}+1)\right)$ (B) $(0, \infty)$

(C) $\left(\frac{1}{2}, \ \log_2(\sqrt{2}-1)\right)$ (D) $\left(-\infty, \ \log_2(\sqrt{2}-1)\right] \cup \left[\frac{1}{2}, \ \infty\right)$

***19.** Number of positive integral solutions of the equation $\frac{1}{x}+\frac{2}{y}=\frac{1}{4}$

(A) 4 (B) 6 (C) 8 (D) 10

20. If $x = -\frac{1}{2}+\frac{\sqrt{3}}{2}i$ where $i = \sqrt{-1}$. Find the value of the expression $P(x)=x^4+2x^3+3x^2+2x+2$

(A) –1 (B) 0 (C) 2 (D) None of these

***21.** If a, b, c are integers such that

$|a-b|^{19} + |c-a|^{19} = 1$, find the value of $|c-a| + |a-b| + |b-c|$

(A) 2 (B) 3 (C) 1 (D) 4

22. The minimum value of the expression

$y = |x-1| + |x-2| + |x-3| + |x-4| + |x-5| + |x-6| + |x-7| + |x-8|$ is

(A) 12 (B) 14 (C) 16 (D) 18

23. Given that log (2) = 0.3010....., number of digits in the number 2000^{2000} is

(A) 6601 (B) 6602 (C) 6603 (D) 6604

Multiple Choice Correct

24. The equation $18^{4x-3} = (54\sqrt{2})^{3x-4}$ has a solution which is

(A) an even integer (B) a prime number (C) coprime with 5 (D) none of these

***25.** If x_1 and x_2 are the solution of the equation $x^{3\log_{10}^3 x-\frac{2}{3}\log_{10} x} = 100\sqrt[3]{10}$ then -

(A) $x_1x_2 = 1$ (B) $x_1 . x_2 = x_1 + x_2$ (C) $\log_{x_2} x_1 = -1$ (D) $\log (x_1 . x_2) = 0$

***26.** The number $N = \dfrac{1+2\log_3 2}{(1+\log_3 2)^2} + \log_6^2 2$ when simplified reduces to -

(A) a prime number
(B) an irrational number
(C) a real which is less than $\log_3\pi$
(D) a real which is greater than $\log_7 6$

***27.** If positive p, q, r satisfy pqr = 1, then for equation

$$\frac{2px}{pq+p+1}+\frac{2qx}{qr+q+1}+\frac{2rx}{rp+r+1}=1$$

x equals :

(A) p + q + r
(B) 1
(C) Independent of p, q & r
(D) $\frac{1}{2}$

***28.** If $x=\log_2\left[(2+1)(2^2+1)(2^4+1)\ldots(2^{2^{10}}+1)+1\right]$. Then "x' is divisible by

(A) 128 (B) 1024 (C) 4096 (D) 8192

29. Which of the following numbers is/are positive?

(A) $ln\,2 + ln\,3 + ln\,0.161$
(B) $ln\,3 + ln(1.4) - \frac{1}{2}\,ln\,16$
(C) $1 + 2\log_{10}3 - 3\log_{10}5 + \log_{10}5$
(D) $\log_{\frac{1}{2}}\left(\frac{1}{3}\right)-\log_{\frac{1}{3}}\left(\frac{1}{2}\right)$

30. Let $a = (\log_3 \pi)(\log_2 3)(\log_\pi 2)$, $b = \dfrac{\log 576}{3\log 2+\log 3}$ the base of the logarithm being 10,

$c = 2\big($ sum of the solution of the equation $(3)^{4x} - (3)^{(2x+\log_3(12))} + 27 = 0\big)$ and $d = 7^{(\log_7 2\ +\ \log_7 3)}$

then $(a+b+c \div d)$ simplifies to

(A) rational which is not natural.
(B) natural but not prime.
(C) irrational
(D) even but not composite

31. If $2x^3-5x^2+x+2 = (x-2)(ax^2-bx-1)$, then

(A) a = 2 (B) b = 1 (C) a = 1 (D) b = 2

***32.** If S is the set of all real x such that $\dfrac{(2x-1)}{(2x^3+3x^2+x)}$ is positive, then S contains:

(A) $\left(-\infty,-\frac{3}{2}\right)$ (B) $\left(-\frac{3}{2},-\frac{1}{4}\right)$ (C) $\left(-\frac{1}{4},\frac{1}{2}\right)$ (D) $\left(+\frac{1}{2},3\right)$

33. Given that $\frac{1}{\log_7 2}+\frac{1}{\log_9 4}=x$, then which of the following will divide $(4)^x$?

(A) 3 (B) 7 (C) 9 (D) 21

34. If $(k)^{\log_2 5}=16,$ then the value of $(k)^{(\log_2 5)^2}$ is divisible by :

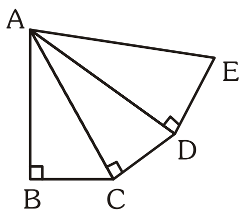
(A) 4 (B) 5 (C) 16 (D) 25

***35.** Consider the number $N = 24^{25}$.
Also, $\log_{10} 2 \approx 0.3010$ & $\log_{10} 3 \approx 0.4771$.
Then,

(A) Last digit of N is 4
(B) Last digit of N is 6
(C) No of digits in N are 34
(D) No of digits in N are 35

Match the Column

36. Match the following column

	Column–I		Column–II
(A)	Number of integers satisfying $(x+3).(x-3) \le 0$	(P)	24
(B)	If $(n + 1)(n + 2)(n + 3)(n + 4)$ is always divisible by r for all $n \in I$, then maximum value of r must be necessarily	(Q)	12
(C)	If $a_n = \underbrace{999....9}_{n\text{ times}}$ then a_{24} is divisible by	(R)	7
(D)	In the following figure, ΔABC, ΔACD and ΔADE and right angled triangle. If $\frac{AB}{BC}=\frac{CD}{AC}=\frac{AD}{DE}=2$ then find the value of $\left[\left(\frac{AE}{BC}\right)^2\right]$, where $[\cdot]$ represent G.I.F. 	(S)	31

***37.** Consider the number $N = 123x\ 43y$ where x & y are digits such that $0 \le x \le 9$ & $0 \le y \le 9$. Now answer the following :

	Column - I		Column - II
(A)	If N is divisible by 2, then the maximum value of y is	(P)	2
(B)	If N is divisible by 3, then the minimum value of x + y is	(Q)	4
(C)	If N is divisible by 11, then y – x can be	(R)	8
(D)	If N is divisibly by 8, then y can be	(S)	5

38. Find x :

	Column - I		Column - II
(A)	$\dfrac{(3^x-1)(x-1)^3}{x(x-2)^5} \le 0$	(P)	$x \in (-\infty, 0) \cup [1, 2)$
(B)	$(x-1)^3 (x-2)^5 \ge 0$	(Q)	$x \in [1, 2)$
(C)	$\dfrac{(x-1)^3}{(x-2)^5} \le 0$	(R)	$x \in (-\infty, 1] \cup [2, \infty)$
(D)	$x^2(x-1)^3 (x-2)^5 \le 0$	(S)	$x \in [1, 2] \cup \{0\}$

Comprehension

*Comprehension–1

Set of all real values of x satisfying the inequality $4(\ell n\, x)^3 - 8(\ell n\, x)^2 - 11(\ell n\, x) + 15 \le 0$ is $(a, e^{-b}] \cup [e^c, e^d]$ and $\ell = a^2 + b^2 + c^2 + d^2$.

39. If [.] represents greatest integer function, then [ℓ] is equal to
(A) 5 (B) 7 (C) 8 (D) 9

40. If sum of all prime numbers less than ℓ is m, then m is equal to
(A) 5 (B) 10 (C) 17 (D) None of these

41. If [.] is greatest integer function and m is as obtained in the above question, then $m^{\log_{[\ell]} c}$ is,
(A) a rational number (B) a prime number
(C) a composite number (D) an irrational number

Comprehension–2

Consider the number N = 8 7 a 2 7 9 3 1 b, where b is a digit at unit's place and a is a digit at ten lakh's place. Answer the following questions.

42. The greatest value of b for which N is divisible by 8 is
(A) 0 (B) 2 (C) 4 (D) 6

43. The least value of a for which N is divisible by 12 is
(A) 0 (B) 2 (C) 4 (D) 6

44. Number of values of a + b for which N is divisible by 11 is
(A) 0 (B) 1 (C) 2 (D) 3

*Comprehension–3

In comparison of two numbers, logarithm of smaller number is smaller, if base of the logarithm is greater than one. Logarithm of smaller number is larger, if base of logarithm is in between zero and one. For example $\log_2 4$ is smaller than $\log_2 8$ and $\log_{1/2} 4$ is larger than $\log_{1/2} 8$.

On the basis of above information, answer the following questions :

45. Identify the correct order :

(A) $\log_2 6 < \log_3 8 < \log_3 6 < \log_4 6$ (B) $\log_2 6 > \log_3 8 > \log_3 6 > \log_4 6$
(C) $\log_3 8 > \log_2 6 > \log_3 6 > \log_4 6$ (D) $\log_3 8 > \log_4 6 > \log_3 6 > \log_2 6$

46. $\log_{\frac{1}{20}} 40$ is-

(A) greater than one
(B) smaller than one
(C) greater than zero and smaller than one
(D) none of these

47. $\log_{\frac{2}{3}} \frac{5}{6}$ is-

(A) less than zero
(B) greater than zero and less than one
(C) greater than one
(D) none of these

Comprehesion-4

Consider the number N = $2^{\log_4 2^{100}}$.
Now answer the following questions with exactly one appropriate alternative.

48. N is :
(A) an irrational number
(B) a rational which is not an integer.
(C) a prime number
(D) a natural number

49. Number of digits in N before a decimal starts is
[Given $\log_{10}2 = 0.3010$]
(A) 14 (B) 15 (C) 16 (D) 17

50. Units place in the integral part of N has the digit
(A) 2 (B) 4 (C) 6 (D) none

Comprehesion-5

In inequalities of the form
$|x| > a$ & $|x| < a$ where a is positive. The solution can be represented directly as
$x \in (-\infty, -a) \cup (a, \infty)$ & $x \in (-a, a)$, respectively.

51. For the inequation $|||x| - 2| - 1| < 3$, set of all possible values of x is :
(A) $(-6, -2) \cup (2, 6)$ (B) $(-6, 2)$ (C) $(3, 6)$ (D) $(-6, 6)$

52. For the inequation $||x^2 - 1| - 2| > 3$, set of all possible values of x is :
(A) $(1, 2)$ (B) $(-2, -1)$ (C) $(-1, 1)$ (D) None of these

53. Number of integers satisfying the inequality $|||x| - 1| - 2| \le 1$
(A) 7 (B) 6 (C) 5 (D) 4

*Comprehesion-6

If min(a, b) implies the minimum of a & b, & maximum (a, b) implies the maximum of a & b. where a & b are real numbers.
Also, we can represent :

$$\min(a,b) = \frac{a+b-|a-b|}{2} \quad \text{and} \quad \max(a,b) = \frac{a+b+|a-b|}{2}$$

using this information, solve the problems given below :

54. Solution(s) of the equation $\min(x^2, 5 - x^2) = 1$ are :
(A) $\{\pm1, \pm2\}$ (B) $\{\pm1\}$ (C) $\{\pm2\}$ (D) No solution

55. Solution(s) of the equation $\max(2 - x^2, x^2 - 3) = 1$ are :
(A) $\{\pm1\}$ (B) $\{\pm1, \pm2\}$ (C) $\{\pm2\}$ (D) None of these

56. Number of integral solution of the inequality $\max(|x|, 4-|x|) \le 3$
(A) 6 (B) 4 (C) 2 (D) 1

*Comprehesion-7

Two circles C_1 and C_2 are externally separated. The distance between the centres of C_1 and C_2 is 5 units and their radii are 1 and 2 respectively

57. The length of internal /transverse common tangents is :
(A) 5 (B) $2\sqrt{6}$ (C) 4 (D) $3\sqrt{2}$

58. The length of external /direct common tangents is :
(A) 5 (B) $2\sqrt{6}$ (C) 4 (D) $3\sqrt{2}$

59. The mininum distance between the two circles is :
(A) 2 (B) 3 (C) 4 (D) 1

*Comprehesion-8

P(x) is a polynomial of degree 5 with leading coefficient unity such that
$P(1) = 1, P(2) = 4, P(3) = 9, P(4) = 16, P(5) = 25.$

60. $P(6) =$
(A) 156 (B) 120 (C) 36 (D) 126

61. The constant term in the expression of P(x) is
(A) 110 (B) –100 (C) –120 (D) 24

62. $P(-1) =$
(A) –120 (B) –721 (C) –719 (D) None of these

Subjetive Questions

63. Find the value of $49^A + 5^B$ where $A = 1 - \log_7 2$ & $B = -\log_5 4$.

***64.** Find the value of the expression $\dfrac{2}{\log_4 (2000)^6} + \dfrac{3}{\log_5 (2000)^6}$

***65.** Solve the system of equations :
$\log_a x \log_a (xyz) = 48$
$\log_a y \log_a (xyz) = 12,\ a > 0,\ a \ne 1$
$\log_a z \log_a (xyz) = 84$

***66.** Compute the following :

(a) $\dfrac{81^{\frac{1}{\log_5 9}} + 3^{\frac{3}{\log_{\sqrt{6}} 3}}}{409} \cdot \left(\left(\sqrt{7}\right)^{\frac{2}{\log_{25} 7}} - (125)^{\log_{25} 6} \right)$ (b) $5^{\log_{1/5}\left(\frac{1}{2}\right)} + \log_{\sqrt{2}} \dfrac{4}{\sqrt{7}+\sqrt{3}} + \log_{1/2} \dfrac{1}{10+2\sqrt{21}}$

(c) $4^{5\log_{4\sqrt{2}}\left(3-\sqrt{6}\right) - 6\log_8\left(\sqrt{3}-\sqrt{2}\right)}$

67. Solve for x :
(a) $5^{\log x} + 5x^{\log 5} = 3\ (a > 0)$; where base of log is a (b) $\log_x 2 . \log_{2x} 2 = \log_{4x} 2$

***68.** Solve for x :
(a) $\log_{x+1}(x^2+x-6)^2 = 4$
(b) $x + \log_{10}(1 + 2^x) = x.\log_{10}5 + \log_{10}6$

69. (a) Given : $\log_{10} 34.56 = 1.5386$, find $\log_{10} 3.456$; $\log_{10}0.3456$ & $\log_{10}0.003456$.
(b) Find the number of positive integers which have the characteristic 3, when the base of the logarithm is 7.

70. Find the antilogarithm of 0.75, if the base of the logarithm is 2401.

***71.** Find all real numbers x which satisfy the equation, $2\log_2 \log_2 x + \log_{1/2} \log_2 (2\sqrt{2}x) = 1$.

***72.** Solve for x : $\log_{3/4} \log_8 (x^2 + 7) + \log_{1/2} \log_{1/4} (x^2 + 7)^{-1} = -2$.

***73.** Find the square root of 7 + 24 i

***74.** $\log^2_{10} x + \log_{10}x^2 = \log^2_{10} 3 - 1$

75. Prove that $\dfrac{\log_a N}{\log_{ab} N} = 1 + \log_a b$ & indicate the permissible values of the letters :

***76.** Prove the identity ; $\log_a N.\log_b N + \log_b N.\log_c N + \log_c N.\log_a N = \dfrac{\log_a N.\log_b N.\log_c N}{\log_{abc} N}$

***77.** Prove that $a^x - b^y = 0$ where $x = \sqrt{\log_a b}$ & $y = \sqrt{\log_b a}$, a >0 , b > 0 & a , b $\neq$ 1.

78. If $a = \log_{12} 18$ & $b = \log_{24} 54$ then find the value of ab + 5 (a – b).

79. If $\dfrac{\log_a N}{\log_c N} = \dfrac{\log_a N - \log_b N}{\log_b N - \log_c N}$ where N > 0 & N $\neq$ 1, a, b, c > 0 & not equal to 1, then prove that $b^2 = ac$.

***80.** If $\log_b a.\log_c a + \log_a b.\log_c b + \log_a c.\log_b c = 3$ (where a, b, c are different positive real numbers $\neq$ 1), then find the value of abc.

***81.** Solve the system of the equations $(ax)^{\log a} = (by)^{\log b}$; $b^{\log x} = a^{\log y}$ where a > 0, b > 0 and $a \neq b$, $ab \neq 1$

***82.** Solve for x : $\log_5 120 + (x - 3) - 2.\log_5(1-5^{x-3}) = -\log_5(0.2 - 5^{x-4})$.

***83.** Find the real solutions to the system of equations
$\log_{10}(2000xy) - \log_{10}x . \log_{10}y = 4$
$\log_{10}(2yz) - \log_{10}y . \log_{10}z = 1$
and $\log_{10}(zx) - \log_{10}z . \log_{10}x = 0$

***84.** Find x satisfying the equation $\log^2\left(1+\dfrac{4}{x}\right) + \log^2\left(1-\dfrac{4}{x+4}\right) = 2\log^2\left(\dfrac{2}{x-1}-1\right)$.

***85.** Solve for x : $\log^2(4-x) + \log(4-x).\log\left(x+\dfrac{1}{2}\right) - 2\log^2\left(x+\dfrac{1}{2}\right) = 0$.

***86.** Solve the following equation for x & y : $\log_{100}|x+y| = \dfrac{1}{2}$, $\log_{10}y - \log_{10}|x| = \log_{100}4$.

ANSWERS

- ***True and False***

1. (D) **2.** (D)

- ***Fill in the Blanks***

3. $x \in \{1, a^{1/\pi}\}$ **4.** 32 **5.** $\frac{1}{5}, \frac{1}{20}$ **6.** 3

7. 9 **8.** $\frac{1}{2} + m$ **9.** – 5 **10.** 3

- ***Assertion–Reason***

11. (A) **12.** (A) **13.** (D) **14.** (A) **15.** (D)

- ***Single Choice Correct***

16. (B) **17.** (B) **18.** (D) **19.** (B) **20.** (D)
21. (A) **22.** (C) **23.** (C)

- ***Multiple Choice Correct***

24. (AC) **25.** (ACD) **26.** (CD) **27.** (CD) **28.** (AB)
29. (BCD) **30.** (A) **31.** (AB) **32.** (AD) **33.** (ABCD)
34. (BD) **35.** (AD)

- ***Match the Column***

36. (A) $\rightarrow$ (r), (B) $\rightarrow$ (p), (C) $\rightarrow$ (r), (D) $\rightarrow$ (s)
37. (A) R; (B) P; (C) R; (D) P
38. (A) - (Q); (B) - (R); (C) - (Q); (D) - (S)

- ***Comprehension***

Comprehension –1	**39.**	(D)	**40.**	(C)	**41.**	(A)
Comprehension –2	**42.**	(B)	**43.**	(A)	**44.**	(C)
Comprehension –3	**45.**	(B)	**46.**	(B)	**47.**	(B)
Comprehension –4	**48.**	(D)	**49.**	(C)	**50.**	(B)
Comprehension –5	**51.**	(D)	**52.**	(D)	**53.**	(A)
Comprehension –6	**54.**	(A)	**55.**	(B)	**56.**	(A)
Comprehension –7	**57.**	(C)	**58.**	(B)	**59.**	(A)
Comprehension –8	**60.**	(A)	**61.**	(C)	**62.**	(C)

- ***Subjective Questions***

63. $\frac{25}{2}$ **64.** $\frac{1}{6}$ **65.** $(a^4, a, a^7), (\frac{1}{a^4}, \frac{1}{a}, \frac{1}{a^7})$

66. (a) 1, (b) 6, (c) 9 **67.** (a) $x = 2^{-\log_5 a}$ (b) $x = 2^{\sqrt{2}}$ or $2^{-\sqrt{2}}$

68. (a) x = 1, (b) x = 1 **69.** (a) 0.5386, $\bar{1}.5386$, $\bar{3}.5386$ (b) 2058

70. 343 **71.** x = 8 **72.** x = 3 or –3

73. $\pm (4 + 3i)$

74. $x = \dfrac{3}{10}$ or $x = \dfrac{1}{30}$

78. 1

80. $abc = 1$

81. $x = 1/a$ and $y = 1/b$

82. $x = 1$

83. $x = 1, y = 5, z = 1$ or $x = 100, y = 20, z = 100$

84. $x = \sqrt{2}$ or $\sqrt{6}$

85. $\left\{0, \dfrac{7}{4}, \dfrac{3+\sqrt{24}}{2}\right\}$

86. $\{-10, 20\}, \{10/3, 20/3\}$

TRIGO RATIO & IDENTITIES

True / False

1. If A + B + C = π, then cos2A + cos2B + cos2C + 4cosA cosB cosC is positive.

2. $(\tan 20° \tan 40° \tan 80°)^2$ is a prime number.

3. ....$\sin^8\theta \le \sin^6\theta \le \sin^4\theta \le \sin^2\theta \le 1$ also ....$\cos^8\theta \le \cos^6\theta \le \cos^4\theta \le \cos^2\theta \le 1$.

Fill in the blanks

4. If tan α = 2 and $\alpha \in \left(\pi, \frac{3\pi}{2}\right)$ then the value of the expression $\frac{\cos\alpha}{\sin^3\alpha + \cos^3\alpha}$ is equal to ....

5. The expression $\frac{\sin^4 t + \cos^4 t - 1}{\sin^6 t + \cos^6 t - 1}$ when simplified reduces to ....

6. Exact value of tan 200° (cot 10° – tan 10°) is ....

7. $96\sqrt{3}\ \sin\frac{\pi}{48}\ \cos\frac{\pi}{48}\ \cos\frac{\pi}{24}\ \cos\frac{\pi}{12}\ \cos\frac{\pi}{6}$ has the value = ...

***8.** If $[1 - \sin(\pi + \alpha) + \cos(\pi + \alpha)]^2 + \left[1 - \sin\left(\frac{3\pi}{2} + \alpha\right) + \cos\left(\frac{3\pi}{2} - \alpha\right)\right]^2$ = a + b sin 2α then the value of 'a' & 'b' are................. & .... respectively.

***9.** The least value of the expression $\frac{\cot 2x - \tan 2x}{1 + \sin\left(\frac{5\pi}{2} - 8x\right)}$ for $0 < x < \frac{\pi}{8}$ is.................

Assertion & Reason

These questions contains, Statement I (assertion) and Statement II (reason).

(A) Statement-I is true, Statement-II is true ; Statement-II is correct explanation for Statement-I.

(B) Statement-I is true, Statement-II is true ; Statement-II is NOT a correct explanation for Statement-I.

(C) Statement-I is true, Statement-II is false.

(D) Statement-I is false, Statement-II is true.

***10.** **Statement-I–** $\sec^2\theta = \frac{4xy}{(x+y)^2}$ is possible for all real values of x and y only when x = y ≠ 0.

Because

Statement-II– $t^2 \ge 0\ \forall\ t \in R$

11. **Statement-I–** If A is obtuse angle in ΔABC, then tan B tanC < 1

Because

Statement-II– In ΔABC, $\tan A = \frac{\tan B + \tan C}{\tan B \tan C - 1}$

***12.** **Statement-I–** $\cos^3\alpha + \cos^3\left(\alpha + \frac{2\pi}{3}\right) + \cos^3\left(\alpha + \frac{4\pi}{3}\right) = 3\cos\alpha\cos\left(\alpha + \frac{2\pi}{3}\right)\cos\left(\alpha + \frac{4\pi}{3}\right)$

Because

Statement-II– If a + b + c = 0 $\Leftrightarrow$ $a^3 + b^3 + c^3 = 3abc$

Single Choice Correct

***13.** Exact value of $\cos 20° + 2\sin^2 55° - \sqrt{2}\sin 65°$ is -

(A) 1 (B) $\frac{1}{\sqrt{2}}$ (C) $\sqrt{2}$ (D) zero

***14.** $\frac{1}{\cos 290°} + \frac{1}{\sqrt{3}\sin 250°} =$

(A) $\frac{2\sqrt{3}}{3}$ (B) $\frac{4\sqrt{3}}{3}$ (C) $\sqrt{3}$ (D) None of these

15. If θ is internal angle of n sided regular polygon, then $\sin\theta$ is equal to -

(A) $\sin\frac{\pi}{n}$ (B) $\sin\frac{2\pi}{n}$ (C) $\sin\frac{\pi}{2n}$ (D) $\sin\frac{n}{\pi}$

16. If $\tan\theta = \sqrt{\frac{a}{b}}$ where a, b are positive reals then the value of $\sin\theta\sec^7\theta + \cos\theta\,\mathrm{cosec}^7\theta$ is -

(A) $\frac{(a+b)^3(a^4+b^4)}{(ab)^{7/2}}$ (B) $\frac{(a+b)^3(a^4-b^4)}{(ab)^{7/2}}$ (C) $\frac{(a+b)^3(b^4-a^4)}{(ab)^{7/2}}$ (D) $-\frac{(a+b)^3(a^4+b^4)}{(ab)^{7/2}}$

17. If $A + B + C = \pi$ & $\sin\left(A+\frac{C}{2}\right) = k\sin\frac{C}{2}$, then $\tan\frac{A}{2}\tan\frac{B}{2} =$

(A) $\frac{k-1}{k+1}$ (B) $\frac{k+1}{k-1}$ (C) $\frac{k}{k+1}$ (D) $\frac{k+1}{k}$

***18.** $l = \left(\frac{\cot^2 x.\cos^2 x}{\cot^2 x - \cos^2 x}\right)^2$ and $m = a^{\log_{\sqrt{a}}\left[2\cos\frac{y}{2}\right]}$, at $y = 4\pi$, then $l^2 + m^2$ is equal to -

(A) 4 (B) 16 (C) 17 (D) none of these

19. $\frac{\sin^3\theta - \cos^3\theta}{\sin\theta - \cos\theta} - \frac{\cos\theta}{\sqrt{1+\cot^2\theta}} - 2\tan\theta\cot\theta = -1$ if -

(A) $\theta \in \left(0, \frac{\pi}{2}\right)$ (B) $\theta \in \left(\frac{\pi}{2}, \pi\right)$ (C) $\theta \in \left(\pi, \frac{3\pi}{2}\right)$ (D) $\theta \in \left(\frac{3\pi}{2}, 2\pi\right)$

***20.** In a triangle ABC, angle A is greater than angle B. If the measures of angles A and B satisfy the equation $2\tan x - k(1 + \tan^2 x) = 0$, where $k \in (0, 1)$, then the measure of the angle C is -

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$ (D) $\frac{5\pi}{12}$ (D) $\frac{\pi}{2}$

***21.** If A and B are acute positive angles satisfying the equations $3\sin^2 A + 2\sin^2 B = 1$ and $3\sin 2A - 2\sin 2B = 0$ then A + 2B is-

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{2}$ (C) $\frac{2\pi}{3}$ (D) none

***22.** $2\sin 11° 15'$ is equal to –

(A) $\sqrt{2-\sqrt{2+\sqrt{2}}}$ (B) $\sqrt{2-\sqrt{2-\sqrt{2}}}$ (C) $\sqrt{\frac{2+\sqrt{2-\sqrt{2}}}{2}}$ (D) $\sqrt{\frac{2+\sqrt{2+\sqrt{2}}}{2}}$

23. If $60° + \alpha$ & $60° - \alpha$ are the roots of $\sin^2 x + b\sin x + c = 0$, then -
(A) $4b^2 + 3 = 12c$ (B) $4b + 3 = 12c$ (C) $4b^2 - 3 = -12c$ (D) $4b^2 - 3 = 12c$

24. If $x^2 + y^2 = 9$ & $4a^2 + 9b^2 = 16$, then maximum value of $4a^2x^2 + 9b^2y^2 - 12abxy$ is -
(A) 81 (B) 100 (C) 121 (D) 144

***25.** Let A,B,C are 3 angles such that cosA + cosB + cosC = 0
and if cosA cosB cosC = λ(cos3A + cos3B + cos3C), then λ is equal to -

(A) $\frac{1}{3}$ (B) $\frac{1}{6}$ (C) $\frac{1}{9}$ (D) $\frac{1}{12}$

***26.** Let n be an odd integer. If $\sin n\theta = \sum_{r=0}^{n} b_r \sin^r \theta$, for every value of θ, then -

(A) $b_0 = 1,\ b_1 = 3$ (B) $b_0 = 0,\ b_1 = n$

(C) $b_0 = -1,\ b_1 = n$ (D) $b_0 = 0,\ b_1 = n^2 - 3n + 3$

27. Value of $\tan 75° - \tan 15° - \sqrt{3}.\tan 75°.\tan 15°$ is

(A) $\sqrt{3}$ (B) 3 (C) 2 (D) None of these

***28.** If $\frac{3\pi}{4} < \alpha < \pi$, $\sqrt{2\cot\alpha + \frac{1}{\sin^2\alpha}}$ is equal to

(A) 1+ cot α (B) –1–cot α (C) 1 – cot α (D) – 1 + cot α

29. $\frac{\cos\theta + \cos 3\theta}{\sin\theta - \sin 3\theta}$ is equal to

(A) 1 + $\cot^2\theta$ (B) cot θ (C) – cot θ (D) – tan θ

30. Value of $\frac{\sin^2 20° + \cos^4 20°}{\sin^4 20° + \cos^2 20°}$ is

(A) 1 (B) 2 (C) $\frac{1}{2}$ (D) none of these

***31.** $\frac{\left(\sin^2\alpha\sin^2\beta + \sin^2\alpha\cos^2\beta + \cos^2\alpha\right) + \left(\sin^2(2\pi+\alpha) + \cos^2(6\pi-\alpha) + 1\right)}{\left(\sin^2\left(\frac{\pi}{3} - 4\pi\right) + \cos^2\left(8\pi + \frac{\pi}{3}\right) + 2\right)}$ is equal to

(A) 1 (B) 2 (C) $\frac{1}{2}$ (D) none of these

32. The value of $\frac{\cos 81°}{\sin 3° \sin 57° \sin 63°} =$

(A) 1 (B) 2 (C) 4 (D) none of these

Multiple Choice Correct

33. If cos(A – B) = 3/5, and tanA tanB = 2, then -

(A) $\cos A \cos B = \frac{1}{5}$ (B) $\sin A \sin B = \frac{-2}{5}$

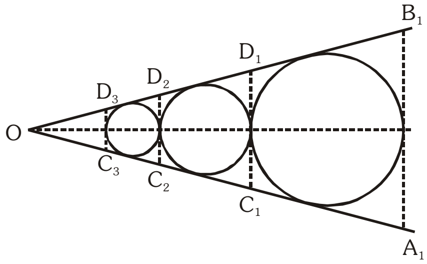
(C) $\cos(A+B) = \frac{-1}{5}$ (D) None of these

***34.** If $A + B = \frac{\pi}{3}$ and cosA + cosB = 1, then -

(A) cos(A – B) = 1/3 (B) | cosA – cosB | = $\sqrt{\frac{2}{3}}$

(C) $\cos(A-B) = -\frac{1}{3}$ (D) $|\cos A - \cos B| = \frac{1}{2\sqrt{3}}$

***35.** If $\angle B_1OA_1 = 60°$ & radius of biggest circle is r. According to figure trapezium $A_1B_1D_1C_1$, $C_1D_1D_2C_2$, $C_2D_2D_3C_3$........ and so on are obtained. Sum of areas of all the trapezium is -



(A) $\frac{r^2}{2\sqrt{3}}$ (B) $\frac{9r^2}{2\sqrt{3}}$ (C) $\frac{9r^2}{\sqrt{3}}$ (D) $\frac{r^2}{9\sqrt{3}}$

***36.** Factors of $\cos4\theta - \cos4\phi$ are -

(A) $(\cos\theta + \cos\phi)$ (B) $(\cos\theta - \cos\phi)$
(C) $(\cos\theta + \sin\phi)$ (D) $(\cos\theta - \sin\phi)$

37. For the equation $\sin 3\theta + \cos 3\theta = 1 - \sin 2\theta$ -

(A) $\tan\theta = 1$ is possible (B) $\cos\theta = 0$ is possible
(C) $\tan\frac{\theta}{2} = -1$ is possible (D) $\cos\frac{\theta}{2} = 0$ is possible

38. $f(x) = \frac{\sin x}{\sqrt{1+\tan^2 x}} + \frac{\cos x}{\sqrt{1+\cot^2 x}}$ is constant in which of following interval -

(A) $\left(0, \frac{\pi}{2}\right)$ (B) $\left(\frac{\pi}{2}, \pi\right)$ (C) $\left(\pi, \frac{3\pi}{2}\right)$ (D) $\left(\frac{3\pi}{2}, 2\pi\right)$

39.** For a positive integer n, let $f_n(\theta) = \left(\tan\frac{\theta}{2}\right)(1+\sec\theta)(1+\sec 2\theta)(1+\sec 4\theta)....(1+\sec 2^n\theta)$. Then ***[JEE 99, 3M]

(A) $f_2\left(\frac{\pi}{16}\right) = 1$ (B) $f_3\left(\frac{\pi}{32}\right) = 1$ (C) $f_4\left(\frac{\pi}{64}\right) = 1$ (D) $f_5\left(\frac{\pi}{128}\right) = 1$

40. If $\sin\theta + \sqrt{\sin\theta + \sqrt{\sin\theta + \sqrt{\sin\theta + ...\infty}}} = \sec^4\alpha$, then $\sin\theta$ is equal to -

(A) $\sec^2\alpha \tan^2\alpha$ (B) $2\frac{(1-\cos 2\alpha)}{(1+\cos 2\alpha)^2}$ (C) $2\frac{(1+\cos 2\alpha)}{(1-\cos 2\alpha)^2}$ (D) $\cot^2\alpha\ \mathrm{cosec}^2\alpha$

***41.** If 2tan10° + tan50° = 2x, tan20° + tan50° = 2y, 2tan10° + tan70° = 2w and tan20° + tan70° = 2z, then which of the following is/are true -

(A) z > w > y > x (B) w = x + y (C) 2y = z (D) z + x = w + y

***42.** If $(3 - 4\sin^2 1)(3 - 4\sin^2 3)(3 - 4\sin^2 3^2)$ ...... $(3 - 4\sin^2(3^{n-1}))$ = sina/sinb, where $n \in N$ & a, b are integers in radian, then the digit at the unit place of (a + b) may be-

(A) 4 (B) 0 (C) 8 (D) 2

43. The expression $\left(\frac{\cos A + \cos B}{\sin A - \sin B}\right)^m + \left(\frac{\sin A + \sin B}{\cos A - \cos B}\right)^m$ where $m \in N$, has the value -

(A) $2\cot^m\left(\frac{A-B}{2}\right)$, if m is odd (B) 0, if m is odd
(C) $2\cot^m\left(\frac{A-B}{2}\right)$, if m is even (D) 0, if m is even

***44.** The value of $\tan 3x - \tan 2x - \tan x$ is not equal to :

(A) $\tan x \tan 2x \tan 3x$ (B) $\tan 2x \cot x \cot 3x$

(C) $\cot x \cot 2x \cot 3x$ (D) $\tan x \tan 2x$

45. The equation $\sin^6 x + \cos^6 x = a^2$ has real solution, if

(A) $a \in [-1, 1]$ (B) $a \in \left[-1, -\frac{1}{2}\right]$ (C) $a \in \left[-\frac{1}{2}, \frac{1}{2}\right]$ (D) $a \in \left[\frac{1}{2}, 1\right]$

46. Let $a = \cos x + \cos\left(x + \frac{2\pi}{3}\right) + \cos\left(x + \frac{4\pi}{3}\right)$ and $b = \sin x + \sin\left(x + \frac{2\pi}{3}\right) + \sin\left(x + \frac{4\pi}{3}\right)$ then which one of the following hold(s) good?

(A) $a = 2b$ (B) $b = 2a$ (C) $a + b = 0$ (D) $a \neq b$

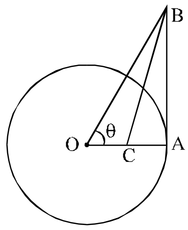
47. The value of x satisfying the equation $\cos(\ln x) = 0$, is

(A) $e^{\pi/2}$ (B) $e^{-(2009)\pi/2}$ (C) $e^{1000\pi}$ (D) $e^{-3\pi/2}$

***48.** A circle centred at 'O' has radius 1 and contains the point A. Segment AB is tangent to the circle at A and $\angle AOB = \theta$. If point C lies on OA and BC bisects the angle ABO then OC equals

(A) $\sec\theta(\sec\theta - \tan\theta)$ (B) $\dfrac{\cos^2\theta}{1+\sin\theta}$

(C) $\dfrac{1}{1+\sin\theta}$ (D) $\dfrac{1-\sin\theta}{\cos^2\theta}$



49. Which one of the following trigonometric statement does holds good?

(A) $\tan\left(\frac{\pi}{4} + x\right) = \cot\left(\frac{\pi}{4} - x\right)$ (B) $\tan\left(\frac{\pi}{4} + x\right) = \dfrac{1+\tan x}{1-\tan x}$

(C) $\tan\left(\frac{\pi}{4} + x\right) = \sec 2x + \tan 2x$ (D) $\tan\left(\frac{\pi}{4} + x\right) = \dfrac{\cos 2x}{1+\sin 2x}$

50. If $\sqrt{\dfrac{1-\sin A}{1+\sin A}} + \dfrac{\sin A}{\cos A} = \dfrac{1}{\cos A}$, for all permissible values of A, then A can belong to

(A) First Quadrant (B) Second Quadrant (C) Third Quadrant (D) Fourth Quadrant

Match the Column

Following question contains statements given in two columns, which have to be matched. The statements in ***Column-I*** are labelled as A, B, C and D while the statements in ***Column-II*** are labelled as p, q, r and s. Any given statement in ***Column-I*** can have correct matching with ***ONE*** statement in ***Column-II***.

***51.**

	Column-I		Column-II
(A)	$\operatorname{cosec} 10° - \sqrt{3}\sec 10° =$	(p)	$-\frac{1}{2}$
(B)	$4\cos 20° - \sqrt{3}\cot 20° =$	(q)	-1
(C)	$\dfrac{2\cos 40° - \cos 20°}{\sin 20°} =$	(r)	$\sqrt{3}$
(D)	$2\sqrt{2}\sin 10°\left[\dfrac{\sec 5°}{2} + \dfrac{\cos 40°}{\sin 5°} - 2\sin 35°\right] =$	(s)	4

52. Match the following column :

	Column I		**Column II**
(A)	$A+B+C=\frac{\pi}{2}$	(p)	$\tan A+\tan B+\tan C=\tan A.\tan B.\tan C$
(B)	$A+B+C=\pi$	(q)	$\tan A\tan B+\tan B\tan C+\tan C\tan A=1$
(C)	$A+B=\frac{\pi}{2}$	(r)	$(\tan A+1)(\tan B+1)=2$
(D)	$A+B=\frac{\pi}{4}$	(s)	$\tan A\,.\,\tan B=1$
		(t)	$\tan\frac{A}{2}\tan\frac{B}{2}+\tan\frac{B}{2}\tan\frac{C}{2}+\tan\frac{C}{2}\tan\frac{A}{2}=1$

53. Match the following column :

	Column-I	**Column-II**
(A)	The expression $[\cos^2(\alpha+\beta)+\cos^2(\alpha-\beta)-\cos 2\alpha\cos 2\beta]$, is	(p) independent of α
(B)	The expression $\cos^2\alpha+\cos^2(\alpha+\beta)-2\cos\alpha\cdot\cos\beta\cdot\cos(\alpha+\beta)$, is	(q) independent of β
(C)	The expression $\frac{\sin(\alpha-\beta)\sin(\alpha+\beta)}{1-\tan^2\alpha\cot^2\beta}$, is	(r) independent of α and β
(D)	The expression $2\sin^2\beta+4\cos(\alpha+\beta)\sin\alpha\cdot\sin\beta+\cos2(\alpha+\beta)$, is	(s) dependent on α and β.

54. Match the following column :

	Column- I		**Column-II**
(A)	$x^{\frac{3}{4}(\log_3 x)^2+\log_3 x-\frac{5}{4}}=\sqrt{3}$	(p)	number of solutions = 2
(B)	$\log_{\sqrt{3}}\left(\sin x+2\sqrt{2}\cos x\right)\geq 2,\ -2\pi\leq x\leq 2\pi$	(q)	number of solutions = 3
(C)	$\log_{0.2}\frac{x+2}{x}\leq 1$	(r)	number of solution = 1
(D)	$\log_2(x+5)=6-x$	(s)	number of solution = 0
		(t)	infinite number of solutions

Comprehension

Comprehension - 1

If θ increases from 0 to $\frac{\pi}{2}$ then value of $\sin\theta$, $\tan\theta$ and $\sec\theta$ increases while $\cos\theta$, $\cot\theta$ and $\text{cosec}\theta$ decreases.

The following pairs $(\sin\theta, \cos\theta)$, $(\tan\theta, \sec\theta)$ and $(\sec\theta, \text{cosec}\theta)$ have the same value at $\theta = \frac{\pi}{4}$

55. If A = 1125° then cosA – sinA will be

(A) positive (B) negative (C) zero (D) √3

56. If A = 295° then secA + cosecA will be

(A) positive (B) negative (C) zero (D) not defined

57. If $A = \frac{23\pi}{8}$ then cotA – tanA will be-

(A) positive (B) negative (C) zero (D) two

Comprehension - 2

Let $E = \sqrt{1+\sin A} + \sqrt{1-\sin A}$, match the following values of E :

58. If $0 \le A \le 90°$

(A) $2\sin\frac{A}{2}$ (B) $2\cos\frac{A}{2}$ (C) $-2\sin\frac{A}{2}$ (D) $-2\cos\frac{A}{2}$

59. If $90° \le A \le 180°$

(A) $2\sin\frac{A}{2}$ (B) $2\cos\frac{A}{2}$ (C) $-2\sin\frac{A}{2}$ (D) $-2\cos\frac{A}{2}$

60. If $360° \le A \le 450°$

(A) $2\sin\frac{A}{2}$ (B) $2\cos\frac{A}{2}$ (C) $-2\sin\frac{A}{2}$ (D) $-2\cos\frac{A}{2}$

Subjetive Questions

61. If $A+B+C=\pi$ prove that $\cot\left(\frac{A}{2}\right)+\cot\left(\frac{B}{2}\right)+\cot\left(\frac{C}{2}\right)=\cot\left(\frac{A}{2}\right)\times\cot\left(\frac{B}{2}\right)\times\cot\left(\frac{C}{2}\right)$ **(JEE 2000)**

***62.** Let A, B, C be three angles such that $A = \frac{\pi}{4}$ and tan B tan C = P. Find all possible values of P such that A, B, C are angles of a triangle. **(JEE 1997)**

63. Express $\sin 3A + \sin 3B + \sin 3C$ as the product of three trigonometrical ratios, where A,B,C are the angles of a triangle. If the given expression is zero, then at least one angle of the triangle is 60^0. **(JEE 1962)**

***64.** Let $A_1, A_2 ----A_n$ be the vertices of an n-sided polygon such that $\frac{1}{A_1A_2} = \frac{1}{A_1A_3} + \frac{1}{A_1A_4}$. Find the value of n. **(JEE 1994)**

***65.** Determine the smallest positive value of x (in degrees) for which $\tan(x+100^0) = \tan(x+50^0)\tan x \tan(x-50^0)$ **(JEE 1993)**

66. Prove that $\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\frac{5\pi}{8}\right)\left(1+\cos\frac{7\pi}{8}\right)=\frac{1}{8}$ **(JEE 1984)**

67. For all $\theta \in \left[0,\frac{\pi}{2}\right]$, show that $\cos(\sin\theta) \geq \sin(\cos\theta)$. **(JEE 1981)**

***68.** If $\operatorname{cosec}\theta - \sin\theta = m$; $\sec\theta - \cos\theta = n$, eliminate θ. **(JEE 1970)**

69. Prove that $\frac{\sin 2A+\sin 2B+\sin 2C}{\cos A+\cos B+\cos C-1}=8\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$ **(JEE 1977)**

***70.** If $xy + yz + zx = 1$ show that $\frac{x}{1-x^2}+\frac{y}{1-y^2}+\frac{z}{1-z^2}=\frac{4xyz}{(1-x^2)(1-y^2)(1-z^2)}$ **(JEE 1971)**

***71.** Find the value of $\tan^6 20° - 33\tan^4 20° + 27\tan^2 20° - 1$

72. Prove that in an acute angled triangle ABC, the least values of $\Sigma\sec A$ and $\Sigma\tan^2 A$ are 6 and 9 respectively.

73. If $A+B+C = \pi$; prove that $\tan^2\frac{A}{2} + \tan^2\frac{B}{2} + \tan^2\frac{C}{2} \geq 1$.

***74.** Prove that the triangle ABC is equilateral iff, $\cot A + \cot B + \cot C = \sqrt{3}$.

75. Prove that in triangle ABC, the least values of $\Sigma\operatorname{cosec}\left(\frac{A}{2}\right)$ and $\Sigma\sec^2\left(\frac{A}{2}\right)$ are 6 and 4 respectively.

76. Prove that from the equality $\frac{\sin^4\alpha}{a}+\frac{\cos^4\alpha}{b}=\frac{1}{a+b}$ follows the relation ;

$$\frac{\sin^8\alpha}{a^3}+\frac{\cos^8\alpha}{b^3}=\frac{1}{(a+b)^3}$$

***77.** If $\cos\theta = \frac{\cos\alpha - e}{1-e\cos\alpha}$, prove that $\tan\frac{\theta}{2} = \pm\sqrt{\frac{1+e}{1-e}}\tan\frac{\alpha}{2}$.

78. Let A_1, A_2, A_3 .... A_n are the vertices of a regular n sided polygon inscribed in a circle of radius R. If $(A_1A_2)^2 + (A_1A_3)^2 + \ldots\ldots\ldots + (A_1A_n)^2 = 14R^2$, find the number of sides in the polygon.

79. Let $k = 1°$, then prove that $\sum_{n=0}^{88}\frac{1}{\cos nk \cdot \cos(n+1)k}=\frac{\cos k}{\sin^2 k}$

80. Let $\theta \in R$ and $\sum_{k=2}^{\infty}\sin(2^k\theta) = a$. Find the value $\sum_{k=0}^{\infty}\left[\cot^3(2^k\theta)-\cot(2^k\theta)\right]\sin^4(2^k\theta)$ in terms of 'a'.

81. If $X = \sin\left(\theta+\frac{7\pi}{12}\right)+\sin\left(\theta-\frac{\pi}{12}\right)+\sin\left(\theta+\frac{3\pi}{12}\right)$, $Y = \cos\left(\theta+\frac{7\pi}{12}\right)+\cos\left(\theta-\frac{\pi}{12}\right)+\cos\left(\theta+\frac{3\pi}{12}\right)$ then prove that $\frac{X}{Y}-\frac{Y}{X} = 2\tan 2\theta$.

***82.** Prove that ; $\operatorname{cosec} x \cdot \operatorname{cosec} 2x \cdot \sin 4x \cdot \cos 6x \cdot \operatorname{cosec} 10x$

$$= \frac{\cos 3x}{\sin 2x \sin 4x} + \frac{\cos 5x}{\sin 4x \sin 6x} + \frac{\cos 7x}{\sin 6x \sin 8x} + \frac{\cos 9x}{\sin 8x \sin 10x}.$$

83. If $\tan\alpha = p/q$ where $\alpha = 6\beta$, α being an acute angle, prove that ;

$$\frac{1}{2}(p \operatorname{cosec} 2\beta - q \sec 2\beta) = \sqrt{p^2+q^2}$$

84. If $\tan\left(\frac{\pi}{4}+\frac{y}{2}\right) = \tan^3\left(\frac{\pi}{4}+\frac{x}{2}\right)$ prove that $\sin y = \sin x\left[\frac{3+\sin^2 x}{1+3\sin^2 x}\right]$

85. The value of $\frac{\sin^2 2\theta}{\cos^2\theta}+\frac{\sin^2 4\theta}{\cos^2 2\theta}+\frac{\sin^2 8\theta}{\cos^2 4\theta}$ (where $\theta=\frac{\pi}{7}$) is equal to :

***86.** If $P = \cos\frac{\pi}{19} + \cos\frac{3\pi}{19} + \cos\frac{5\pi}{19} + \ldots\ldots + \cos\frac{17\pi}{19}$ and

$Q = \cos\frac{2\pi}{21} + \cos\frac{4\pi}{21} + \cos\frac{6\pi}{21} + \ldots.. + \cos\frac{20\pi}{21}$, then find P – Q

87. If $\alpha = \frac{\pi}{7}$ then find the value of $\left(\frac{1}{\cos\alpha}+\frac{2\cos\alpha}{\cos 2\alpha}\right)$.

88. If $\alpha + \beta = \gamma$, prove that $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1 + 2\cos\alpha\cos\beta\cos\gamma$.

89. If $\tan^2 x \cdot \tan^2 y = \frac{a-b}{a+b}$ then prove that $(a - b\cos 2x)(a - b\cos 2y)$ is independent of x and y.

90. If $A + B + C = \pi$ and $\cot\theta = \cot A + \cot B + \cot C$, show that,
$\sin(A-\theta) \cdot \sin(B-\theta) \cdot \sin(C-\theta) = \sin^3\theta$.

ANSWERS

- ***True and False***

1. False **2.** True **3.** True

- ***Fill in the Blanks***

4. $\frac{5}{9}$ **5.** $\frac{2}{3}$ **6.** 2 **7.** 9 **8.** $a = 4$ & $b = -2$

9. 2

- ***Assertion–Reason***

10. (B) **11.** (A) **12.** (C)

- ***Single Choice Correct***

13.	(A)	**14.**	(B)	**15.**	(B)	**16.**	(A)	**17.**	(A)
18.	(C)	**19.**	(B)	**20.**	(D)	**21.**	(B)	**22.**	(A)
23.	(D)	**24.**	(D)	**25.**	(D)	**26.**	(B)	**27.**	(A)
28.	(B)	**29.**	(C)	**30.**	(A)	**31.**	(A)	**32.**	(C)

- ***Multiple Choice Correct***

33.	(AC)	**34.**	(BC)	**35.**	(C)	**36.**	(ABCD)	**37.**	(ABC)
38.	(BD)	**39.**	(ABCD)	**40.**	(AB)	**41.**	(ABCD)	**42.**	(ABCD)
43.	(BC)	**44.**	(BCD)	**45.**	(BD)	**46.**	(ABC)	**47.**	(ABD)
48.	(ACD)	**49.**	(ABC)	**50.**	(AD)				

- ***Match the Column***

51. (A) $\to$(s), (B) $\to$ (q), (C) $\to$ (r), (D) $\to$ (s)
52. (A) $\to$ q, (B) $\to$ p,t, (C) $\to$ s, (D) $\to$ r
53. (A) $\to$ p, q, r; (B) $\to$ p; (C) $\to$ s; (D) $\to$ q
54. (A) $\to$ (q), (B) $\to$ (p), (C) $\to$ (t), (D) $\to$ (r)

- ***Comprehension***

Comprehension-1 : **55.** (C) **56.** (A) **57.** (B)

Comprehension-2 : **58.** (B) **59.** (A) **60.** (D)

- ***Subjective Questions***

62. $\left(-\infty,\ 3-2\sqrt{2}\right]\cup\left[3+2\sqrt{2},\ \infty\right)$

63. $-4\cos\left(\frac{3A}{2}\right)\cos\left(\frac{3B}{2}\right)\cos\left(\frac{3C}{2}\right)$

64. 7

65. $x = 30^\circ$

68. $\left(m^2n\right)^{\frac{2}{3}}+\left(mn^2\right)^{\frac{2}{3}}=1$

71. 2

78. $n = 7$

80. a/4

85. 7

86. 1

87. 4

TRIGONOMETRIC EQUATION

True / False

1. For all θ in $\left[0, \frac{\pi}{2}\right]$, $\cos(\sin\theta) > \sin(\cos\theta)$.

2. Number of solutions of the equation $\cos(x^2) = 2^{|x|}$ is two.

3. There exists a value of θ between 0 and 2π which satisfies the equation $\sin^4\theta - 2\sin^2\theta - 1 = 0$.

4. Equation $e^{\sin x} - e^{-\sin x} = 4$ has no real solution.

5. If $2\cos^2 x + 4\cos x = 3\sin^2 x$ then $\cos x = \frac{-2+\sqrt{19}}{5}$

Fill in the Blanks

6. Number of values of θ in [0, 2π] for which vectors $\vec{v}_1 = (2\cos\theta)\hat{i} - (\cos\theta)\hat{j} + \hat{k}$ and $\vec{v}_2 = (\cos\theta)\hat{i} + 5\hat{j} + 2\hat{k}$ are perpendicular is ....

7. The solution set of the system of equations, $x + y = \frac{2\pi}{3}$, $\cos x + \cos y = \frac{3}{2}$, where x & y are real, is ....

8. If $\operatorname{cosec}\theta + \cot\theta = \frac{1}{2}$, then θ lies in .... quadrant.

9. Number of solutions of the equation $\sin 5\theta \cos 3\theta = \sin 9\theta \cos 7\theta$ in $\left[0, \frac{\pi}{4}\right]$ is ....

***10.** The number of pairs (x, y) satisfying the equation $\sin x + \sin y = \sin(x + y)$ and $|x| + |y| = 1$ is ....

***11.** If $\cos 2\theta + 9\sin 2\theta - 6\sin\theta + 54\cos\theta = 1$ then the value of $100\tan^2\theta + 9\tan\theta$ is equal to ....

12. General value of θ satistying the equation $\tan^2\theta + \sec 2\theta = 1$ is .... **[JEE 1996]**

13. The real roots of the equation $\cos^7 x + \sin^4 x = 1$ in the interval (–π, π) are .... and ...... **[JEE 1997]**

Assertion & Reason

These questions contains, Statement-I (assertion) and Statement-II (reason).
(A) Statement-I is true, Statement-II is true ; Statement-II is correct explanation for Statement-I.
(B) Statement-I is true, Statement-II is true ; Statement-II is NOT a correct explanation for Statement-I.
(C) Statement-I is true, Statement-II is false.
(D) Statement-I is false, Statement-II is true.

14. ***Statement-I –*** For any real value of $\theta \neq (2n+1)\pi$ or $(2n+1)\pi/2$, $n \in I$, the value of the expression $y = \frac{\cos^2\theta - 1}{\cos^2\theta + \cos\theta}$ is $y \leq 0$ or $y \geq 2$ (either less than or equal to zero or greater than or equal to two)
Because
Statement-II – $\sec\theta \in (-\infty, -1] \cup [1, \infty)$ for all real values of θ.

15. ***Statement-I –*** The equation $\sqrt{3}\cos x - \sin x = 2$ has exactly one solution in [0, 2π].
Because
Statement-II – For equations of type $a\cos\theta + b\sin\theta = c$ to have real solutions in [0, 2π], $|c| \leq \sqrt{a^2 + b^2}$ should hold true.

Single Choice Correct

***16.** The solutions of the equation sinx + 3sin2x + sin3x = cosx + 3cos2x + cos3x in the interval $0 \le x \le 2\pi$, are

(A) $\frac{\pi}{8}, \frac{5\pi}{8}, \frac{2\pi}{3}$ (B) $\frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}, \frac{13\pi}{8}$ (C) $\frac{4\pi}{3}, \frac{9\pi}{3}, \frac{2\pi}{3}, \frac{13\pi}{8}$ (D) $\frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{3}, \frac{4\pi}{3}$

***17.** If $\theta_1, \theta_2, \theta_3, \theta_4$ are the roots of the equation $\sin(\theta + \alpha) = k \sin 2\theta$, no two of which differ by a multiple of 2π, then $\theta_1 + \theta_2 + \theta_3 + \theta_4$ is equal to -

(A) $2n\pi, n \in Z$ (B) $(2n + 1)\pi, n \in Z$ (C) $n\pi, n \in Z$ (D) none of these

18. The number of integral values of k for which the equation 7cosx + 5sinx = 2k + 1 has a solution is ***[JEE 2002 (Screening), 3]***

(A) 4 (B) 8 (C) 10 (D) 12

19. $\cos(\alpha - \beta) = 1$ and $\cos(\alpha + \beta) = 1/e$, where $\alpha, \beta \in [-\pi, \pi]$, numbers of pairs of α, β which satisfy both the equations is ***[JEE 2005 (Screening)]***

(A) 0 (B) 1 (C) 2 (D) 4

20. The value of θ lying between $\theta = 0$ and $\theta = \frac{\pi}{2}$ and satisfying the equation.

$$\begin{vmatrix} 1+\sin^2\theta & \cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & 1+\cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & \cos^2\theta & 1+4\sin 4\theta \end{vmatrix} = 0$$; are ***[JEE 1988]***

(A) $\frac{7\pi}{24}$ (B) $\frac{5\pi}{24}$ (C) $\frac{11\pi}{24}$ (D) $\frac{\pi}{24}$

21. The general solution of sinx – 3sin 2x + sin 3x = cos x – 3 cos 2x + cos 3x is ***[JEE 1989]***

(A) $n\pi + \frac{\pi}{8}$ (B) $\frac{n\pi}{2} + \frac{\pi}{8}$ (C) $(-1)^n \frac{n\pi}{2} + \frac{\pi}{8}$ (D) $2n\pi + \cos^{-1}\left(\frac{2}{3}\right)$

22. The equation $(\cos p - 1)x^2 + (\cos p)x + \sin p = 0$, in the variable x, has real roots. Then p can take any value in the interval. ***[JEE 1990]***

(A) $(0, 2\pi)$ (B) $(-\pi, 0)$ (C) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (D) $(0, \pi)$

23. In a triangle ABC, angle A is greater than angle B. If the measures of angles A and B satisfy the equation $3\sin x - 4\sin^3 x - k = 0$, $0 < k < 1$, then the measure of angle C is ***[JEE 1990]***

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$ (C) $\frac{2\pi}{3}$ (D) $\frac{5\pi}{6}$

24. If $2\sin^2 x + 3\sin x - 2 > 0$ and $x^2 - x - 2 < 0$ (x is measured in radians). Then x lies in the interval ***[JEE 1994]***

(A) $\left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$ (B) $\left(-1, \frac{5\pi}{6}\right)$ (C) $(-1, 2)$ (D) $\left(\frac{\pi}{6}, 2\right)$

25. The number of values of x in the interval $[0, 5\pi]$ satisfying the equation $3\sin^2 x - 7\sin x + 2 = 0$ is ***[JEE 1998]***

(A) 0 (B) 5 (C) 6 (D) 10

26. The number of distinct real roots of

$$\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$$

in the interval $-\frac{\pi}{4} \le x \le \frac{\pi}{4}$ is ***[JEE 2001]***

(A) 0 (B) 2 (C) 1 (D) 3

27. The number of values of k for which the equation 7 cos x + 5 sin x = 2k + 1 has a solution is ***[JEE 2002]***

(A) 4 (B) 8 (C) 10 (D) 12

Multiple Choice Correct

***28.** Set of values of 'α' in [0, 2π] for which $\log_{\left(x+\frac{1}{x}\right)}(2\sin\alpha - 1) \le 0$, is -

(A) $\left[\frac{\pi}{6}, \frac{5\pi}{6}\right]$ (B) $\left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$ (C) $\left(0, \frac{\pi}{6}\right) \cup \left(\frac{5\pi}{6}, \pi\right)$ (D) $\left(\frac{5\pi}{6}, \frac{7\pi}{6}\right)$

***29.** The value(s) of θ, which satisfy $3 - 2\cos\theta - 4\sin\theta - \cos2\theta + \sin2\theta = 0$ is/are -

(A) $\theta = 2n\pi$; $n \in I$ (B) $2n\pi + \frac{\pi}{2}$; $n \in I$ (C) $2n\pi - \frac{\pi}{2}$; $n \in I$ (D) $n\pi$; $n \in I$

***30.** Given that A, B are positive acute angles and $\sqrt{3}\sin2A = \sin2B$ & $\sqrt{3}\sin^2A + \sin^2B = \frac{\sqrt{3}-1}{2}$, then A or B may take the value(s) -

(A) 15° (B) 30° (C) 45° (D) 75°

***31.** If $\left(\frac{1-a\sin x}{1+a\sin x}\right)\sqrt{\frac{1+2a\sin x}{1-2a\sin x}} = 1$, where $a \in R$ then -

(A) $x \in \phi$ (B) $x \in R\ \forall\ a$

(C) $a = 0,\ x \in R$ (D) $a \in R,\ x \in n\pi$, where $n \in I$

32. The general solution of the following equation : $2(\sin x - \cos2x) - \sin2x(1 + 2\sin x) + 2\cos x = 0$ is/are -

(A) $x = 2n\pi$; $n \in I$ (B) $n\pi + (-1)^n\left(-\frac{\pi}{2}\right)$; $n \in I$

(C) $x = n\pi + (-1)^n\frac{\pi}{6}$; $n \in I$ (D) $x = n\pi + (-1)^n\frac{\pi}{4}$; $n \in I$

33. The value(s) of θ, which satisfy the equation : $2\cos^3 3\theta + 3\cos3\theta + 4 = 3\sin^2 3\theta$ is/are -

(A) $\frac{2n\pi}{3} + \frac{2\pi}{9}, n \in I$ (B) $\frac{2n\pi}{3} - \frac{2\pi}{9}, n \in I$ (C) $\frac{2n\pi}{5} + \frac{2\pi}{5}, n \in I$ (D) $\frac{2n\pi}{5} - \frac{2\pi}{5}, n \in I$

Match the Column

34. If α and β are the roots of the equation, $a\cos\theta + b\sin\theta = c$ then match the entries of ***column-I*** with the entries of ***column-II***.

Column-I	Column-II
(A) $\sin\alpha + \sin\beta =$	(p) $\frac{2b}{a+c}$
(B) $\sin\alpha \cdot \sin\beta =$	(q) $\frac{c-a}{c+a}$
(C) $\tan\frac{\alpha}{2} + \tan\frac{\beta}{2} =$	(r) $\frac{2bc}{a^2+b^2}$
(D) $\tan\frac{\alpha}{2} \cdot \tan\frac{\beta}{2} =$	(s) $\frac{c^2-a^2}{a^2+b^2}$

35. Solve the equation for 'x' given in **Column-I** and match with the entries of **Column-II**

Column-I		**Column-II**	
(A)	$\cos 3x \cdot \cos^3 x + \sin 3x \cdot \sin^3 x = 0$	(p)	$n\pi \pm \frac{\pi}{3}$
(B)	$\sin 3\alpha = 4 \sin\alpha \sin(x+\alpha) \sin(x-\alpha)$ where α is a constant $\neq n\pi$.	(q)	$n\pi + \frac{\pi}{4}$, $n \in I$
(C)	$\lvert 2\tan x - 1\rvert + \lvert 2\cot x - 1\rvert = 2$.	(r)	$\frac{n\pi}{4} + \frac{\pi}{8}$, $n \in I$
(D)	$\sin^{10}x + \cos^{10}x = \frac{29}{16}\cos^4 2x$.	(s)	$\frac{n\pi}{2} \pm \frac{\pi}{4}$

36. $\frac{\sin 3\alpha}{\cos^2\alpha}$ is **[JEE 1992]**

Column - I		**Column - II**	
(A)	positive	(p)	$\left(\frac{13\pi}{48}, \frac{14\pi}{48}\right)$
(B)	negative	(q)	$\left(\frac{14\pi}{48}, \frac{18\pi}{48}\right)$
		(r)	$\left(\frac{18\pi}{48}, \frac{23\pi}{48}\right)$
		(s)	$\left(0, \frac{\pi}{2}\right)$

Comprehension

*Comprehension – 1

Let S_1 be the set of all those solutions of the equation $(1 + a)\cos\theta \cos(2\theta - b) = (1 + a\cos 2\theta)\cos(\theta - b)$ which are independent of a and b and S_2 be the set of all such solutions which are dependent on a and b.

On the basis of above information, answer the following questions

37. The sets S_1 and S_2 are given by -
(A) $\{n\pi, n \in Z\}$ and $\{m\pi + (-1)^m \sin^{-1}(a \sin b), m \in Z\}$
(B) $\left\{\frac{n\pi}{2}, n \in Z\right\}$ and $\{m\pi + (-1)^m \sin^{-1}(a \sin b), m \in Z\}$
(C) $\left\{\frac{n\pi}{2}, n \in Z\right\}$ and $\{m\pi + (-1)^m \sin^{-1}((a/2)\sin b), m \in Z\}$
(D) none of these

38. Condition that should be imposed on a and b such that S_2 is non-empty -
(A) $\left|\frac{a}{2}\sin b\right| < 1$ (B) $\left|\frac{a}{2}\sin b\right| \le 1$ (C) $|a \sin b| \le 1$ (D) none of these

39. All the permissible values of b, if a = 0 and S_2 is a subset of $(0, \pi)$ is -
(A) $b \in (-n\pi, 2n\pi)$; $n \in Z$ (B) $b \in (-n\pi, 2\pi - n\pi)$; $n \in Z$
(C) $b \in (-n\pi, n\pi)$; $n \in Z$ (D) none of these

Comprehension – 2

To solve a trigonometric inequation of the type $\sin x \ge a$ where $|a| \le 1$, we take a hill of length 2π in the sine curve and write the solution within that hill. For the general solution, we add $2n\pi$. For instance, to solve $\sin x \ge -\frac{1}{2}$, we take the hill $\left[-\frac{\pi}{2}, \frac{3\pi}{2}\right]$ over which solution is $-\frac{\pi}{6} < x < \frac{7\pi}{6}$. The general solution is $2n\pi - \frac{\pi}{6} < x < 2n\pi + \frac{7\pi}{6}$, n is any integer. Again to solve an inequation of the type $\sin x \le a$, where $|a| \le 1$, we take a hollow of length 2π in the sine curve. (since on a hill, $\sin x \le a$ is satisfied over two intervals). Similarly $\cos x \ge a$ or $\cos x \le a$, $|a| \le 1$ are solved.

40. Solution to the inequation $\sin^6 x + \cos^6 x < \frac{7}{16}$ must be :

(A) $n\pi + \frac{\pi}{3} < x < n\pi + \frac{\pi}{2}$
(B) $2n\pi + \frac{\pi}{3} < x < 2n\pi + \frac{\pi}{2}$
(C) $\frac{n\pi}{2} + \frac{\pi}{6} < x < \frac{n\pi}{2} + \frac{\pi}{3}$
(D) none of these

41. Solution to inequality cos 2x + 5 cos x + 3 ≥ 0 over [– π, π] is :

(A) [– π, π]
(B) $\left[\frac{-5\pi}{6}, \frac{5\pi}{6}\right]$
(C) [0, π]
(D) $\left[\frac{-2\pi}{3}, \frac{2\pi}{3}\right]$

42. Over [– π, π], the solution of $2\sin^2\left(x + \frac{\pi}{4}\right) + \sqrt{3}\cos 2x \geq 0$ is :

(A) [– π, π]
(B) $\left[\frac{-5\pi}{6}, \frac{5\pi}{6}\right]$
(C) [0, π]
(D) $\left[-\pi, \frac{-7\pi}{12}\right] \cup \left[-\frac{\pi}{4}, \frac{5\pi}{12}\right] \cup \left[\frac{3\pi}{4}, \pi\right]$

Subjetive Questions

43. Solve for x, $(-\pi \leq x \leq \pi)$, the equation 2(cos x + cos 2x) + sin 2x (1 + 2 cos x) = 2 sin x **[JEE 1978]**

44. Consider the system of linear equations in x, y, z:

$(\sin 3\theta)x - y + z = 0$

$(\cos 2\theta)x + 4y + 3z = 0$

$2x + 7y + 7z = 0$

Find the values of θ for which this system has non trivial solutions. **[JEE 1986]**

45. If $\exp\{(\sin^2 x + \sin^4 x + \sin^6 x + \ldots \text{ inf.}) \log_e 2\}$ satisfies the equation $x^2 - 9x + 8 = 0$,

find the value of $\frac{\cos x}{\cos x + \sin x}$, $0 < x < \frac{\pi}{2}$ **[JEE 1991]**

46. Let λ and α be real. Find the set of all values of λ for which the system of linear equation

$\lambda x + (\sin\alpha)y + (\cos\alpha)z = 0$

$x + (\cos\alpha)y + (\sin\alpha)z = 0$

$-x + (\sin\alpha)y - (\cos\alpha)z = 0$

has a non-trivial solution. For λ = 1, find all values of α. **[JEE 1993]**

47. Determine the smallest positive value of x (in degrees) for which tan(x + 100°) = tan(x + 50°)tan x tan (x – 50°) **[JEE 1993]**

48. Find the smallest positive number p for which the equation cos (p sin x) = sin(p cos x) has a solution x ∈ [0, 2π]. **[JEE 1995]**

49. Find all values of θ in the inverval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ satisfying the equation $(1 - \tan\theta)(1 + \tan\theta)\sec^2\theta + 2\tan^2\theta = 0$ **[JEE 1996]**

50. Find the value of $t \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ so that $2\sin t = \frac{5x^2 - 2x + 1}{3x^2 - 2x - 1}$, $x \in R - \left\{1, \frac{-1}{3}\right\}$ **[JEE 2005]**

51. Solve the equation for x, $5^{1/2} + 5^{1/2 + \log_5(\sin x)} = 15^{1/2 + \log_{15}\cos x}$.

52. Solve the equality: $2\sin 11x + \cos 3x + \sqrt{3}\sin 3x = 0$.

53. Find all value of θ, between 0 & π, which satisfy the equation; $\cos\theta.\cos 2\theta.\cos 3\theta = \frac{1}{4}$.

54. Find the general solution of the equation, $2 + \tan x.\cot\frac{x}{2} + \cot x.\tan\frac{x}{2} = 0$

55. Solve for x, the equation $\sqrt{13 - 18\tan x} = 6\tan x - 3$, where $-2\pi < x < 2\pi$.

56. Determine the smallest positive value of x which satisfy the equation, $\sqrt{1 + \sin 2x} - \sqrt{2}\cos 3x = 0$.

57. Solve the equation : $2\sin\left(3x + \frac{\pi}{4}\right) = \sqrt{1 + 8\sin 2x.\cos^2 2x}$.

58. Find the number of principal solution of the equation, $\sin x - \sin 3x + \sin 5x = \cos x - \cos 3x + \cos 5x$.

59. Find the general solution of the trigonometric equation $3^{\left(1/2 + \log_3(\cos x + \sin x)\right)} - 2^{\log_2(\cos x - \sin x)} = \sqrt{2}$.

60. Find all values of θ between $0°$ & $180°$ satisfying the equation : $\cos 6\theta + \cos 4\theta + \cos 2\theta + 1 = 0$.

***61.** Solve for x : $\sin 3\alpha = 4\sin\alpha\sin(x + \alpha)\sin(x - \alpha)$ where α is a constant $\neq n\pi$, $n \in I$.

62. Find all values of θ, between 0 & π, which satisfy the equation $\cos\theta\cos 2\theta\cos 3\theta = 1/4$.

63. Find the general solution of the trigonometric equation :

$$\sqrt{16\cos^4 x - 8\cos^2 x + 1} + \sqrt{16\cos^4 x - 24\cos^2 x + 9} = 2.$$

64. Find the principal solution of the trigonometric equation :

$$\sqrt{\cot 3x + \sin^2 x - \frac{1}{4}} + \sqrt{\sqrt{3}\cos x + \sin x - 2} = \sin\frac{3x}{2} - \frac{\sqrt{2}}{2}.$$

65. Solve : $2\sin\left(3x + \frac{\pi}{4}\right) = \sqrt{1 + 8\sin 2x.\cos^2 2x}$.

66. Solve for x, $(-\pi \le x \le \pi)$ the equation : $2(\cos x + \cos 2x) + \sin 2x(1 + 2\cos x) = 2\sin x$.

***67.** Find the smallest positive value of x and y satisfying the equations : $x - y = \frac{\pi}{4}$ & $\cot x + \cot y = 2$.

***68.** Find the value(s) of k for which the equation $\sin x + \cos(k + x) + \cos(k - x) = 2$ has real solutions.

69. Solve : $\tan\theta + \tan\left(\theta + \frac{\pi}{3}\right) + \tan\left(\theta + \frac{2\pi}{3}\right) = 3$.

70. Solve : $\sin 2\theta = \cos 3\theta$, $0° \le \theta \le 360°$.

***71.** Find all values of θ satisfying the equation $\sin 7\theta = \sin\theta + \sin 3\theta$, where $0 \le \theta \le \pi$.

ANSWERS

- ***True and False***

1.	True	**2.**	False	**3.**	False	**4.**	True	**5.**	True

- ***Fill in the Blanks***

6.	2	**7.**	ϕ	**8.**	II quadrant	**9.**	5	**10.**	6

11. 8181 **12.** $m\pi,\ n\pi \pm \frac{\pi}{3}$, m, n $\in$ I **13.** $0, \frac{\pi}{2}, -\frac{\pi}{2}$

- ***Assertion–Reason***

14. (D) **15.** (B)

- ***Single Choice Correct***

16.	(B)	**17.**	(B)	**18.**	(B)	**19.**	(D)	**20.**	(C)
21.	(B)	**22.**	(D)	**23.**	(C)	**24.**	(D)	**25.**	(C)
26.	(C)	**27.**	(B)						

- ***Multiple Choice Correct***

28.	(B)	**29.**	(AB)	**30.**	(AB)	**31.**	(CD)	**32.**	(ABC)
33.	(AB)								

- ***Match the Column***

34. (A) $\rightarrow$ (r), (B) $\rightarrow$ (s), (C) $\rightarrow$ (p), (D) $\rightarrow$ (q)

35. (A) $\rightarrow$ (s), (B) $\rightarrow$ (p), (C) $\rightarrow$ (q), (D) $\rightarrow$ (r)

36. (A) $\rightarrow$ (r), (B) $\rightarrow$ (p)

- ***Comprehension***

Comprehension – 1 **37.** (D) **38.** (C) **39.** (B)

Comprehension – 2 **40.** (C) **41.** (D) **42.** (D)

- ***Subjective Questions***

43. $\pm\pi, \pm\frac{\pi}{3}, -\frac{\pi}{2}$

44. $\theta = n\pi$ or $\theta = m\pi + (-1)^m \frac{\pi}{6}$, n, m $\in$ I

45. $\frac{(\sqrt{3}-1)}{2}$

46. $\lambda = \sin 2\alpha + \cos 2\alpha$; $\alpha = m\pi$ or $\alpha = k\pi + \frac{\pi}{4}$ (m, k $\in$ I)

47. 30^o **48.** $\frac{\sqrt{2}\pi}{4}$ **49.** $\pm\frac{\pi}{3}$ **50.** $\left[-\frac{\pi}{2}, \frac{\pi}{10}\right] \cup \left[\frac{3\pi}{10}, \frac{\pi}{2}\right]$

51. $x = 2n\pi + \frac{\pi}{6}$, n $\in$ I

52. $x = \frac{n\pi}{7} - \frac{\pi}{84}$ or $x = \frac{n\pi}{4} + \frac{7\pi}{48}$, n $\in$ I

53. $\frac{\pi}{8}, \frac{\pi}{3}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{2\pi}{3}, \frac{7\pi}{8}$

54. $x = 2n\pi \pm \frac{2\pi}{3}, n \in I$

55. $\alpha - 2\pi$; $\alpha - \pi$, α , $\alpha + \pi$ where $\tan\alpha = \frac{2}{3}$

56. $x = \frac{\pi}{16}$

57. $x = 2n\pi + \frac{\pi}{12}$ or $2n\pi + \frac{17\pi}{12}$; $n \in I$

58. 10 solutions

59. $x = 2n\pi + \frac{\pi}{12}$

60. 30°, 45°, 90°, 135°, 150°

61. $n\pi \pm \frac{\pi}{3}, n \in I$

62. $\frac{\pi}{8}, \frac{\pi}{3}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{2\pi}{3}, \frac{7\pi}{8}$

63. $x \in \left[n\pi + \frac{\pi}{6}, n\pi + \frac{\pi}{3}\right] \cup \left[n\pi + \frac{2\pi}{3}, n\pi + \frac{5\pi}{6}\right], n \in I$

64. $x = \pi/6$ only

65. $x = 2n\pi + \frac{\pi}{12}$ or $2n\pi + \frac{17\pi}{12}$; $n \in I$

66. $\left\{-\pi, -\frac{\pi}{2}, -\frac{\pi}{3}, \frac{\pi}{3}, \pi\right\}$

67. $x = \frac{5\pi}{12}, y = \frac{\pi}{6}$

68. $n\pi - \frac{\pi}{6} \le k \le n\pi + \frac{\pi}{6}$, $n \in I$

69. $\theta = (4n+1)\frac{\pi}{12}$; $n \in I$

70. θ = 18°, 90°, 162°, 234°, 270°, 306°

71. $0, \frac{\pi}{12}, \frac{\pi}{3}, \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{2\pi}{3}, \frac{11\pi}{12}, \pi$

QUADRATIC EQUATION & EXPRESSION

True / False

1. If a, b, c $\in$ Q, then roots of $ax^2 + 2(a + b)x - (3a + 2b) = 0$ are rational.

2. The necessary and sufficient condition for which a fixed number 'd' lies between the roots of quadratic equation $f(x) = ax^2 + bx + c = 0$; (a, b, c $\in$ R), is $f(d) < 0$.

***3.** If $0 < p < \pi$ then the quadratic equation, $(\cos p - 1)x^2 + x\cos p + \sin p = 0$ has real roots.

4. The necessary and sufficient condition for the quadratic function $f(x) = ax^2 + bx + c$, to take both positive and negative values is, $b^2 > 4ac$, where a, b, c $\in$ R & $a \neq 0$.

Fill in the Blanks

5. If $a + b + c = 0$ & $a^2 + b^2 + c^2 = 1$ then the value of $a^4 + b^4 + c^4$ is ....

6. If $x^2 - 4x + 5 - \sin y = 0$, $y \in (0, 2\pi)$ then x = ... & y = ...

7. If α, β be the roots of the equation $ax^2 + bx + c = 0$ then the value of $\frac{a\alpha^2}{b\alpha + c} + \frac{a\beta^2}{b\beta + c}$ is equal to ....

Assertion & Reason

These questions contains, Statement-I (assertion) and Statement-II (reason).

(A) Statement-I is true, Statement-II is true ; Statement-II is correct explanation for Statement-I.

(B) Statement-I is true, Statement-II is true ; Statement-II is NOT a correct explanation for statement-I

(C) Statement-I is true, Statement-II is false

(D) Statement-I is false, Statement-II is true

8. ***Statement-I –*** If roots of the equation $x^2 - bx + c = 0$ are two consecutive integers, then $b^2 - 4c = 1$.

Because

Statement-II – If a, b, c are odd integer then the roots of the equation $4abc\,x^2 + (b^2 - 4ac)x - b = 0$ are real and distinct.

9. ***Statement-I –*** If equation $ax^2 + bx + c = 0$; (a, b, c $\in$ R) and $2x^2 + 3x + 4 = 0$ have a common root, then a : b : c = 2 : 3 : 4.

Because

Statement-II – If p + iq is one root of a quadratic equation with real coefficients then p – iq will be the other root ; p, q $\in$ R, $i = \sqrt{-1}$

10.** ***Statement-I – If $a + b + c > 0$ and $a < 0 < b < c$, then the roots of the equation $a(x - b)(x - c) + b(x - c)(x - a) + c(x - a)(x - b) = 0$ are of both negative.

Because

Statement-II – If both roots are negative, then sum of roots < 0 and product of roots > 0

11. ***Statement-I –*** Let $(a_1, a_2, a_3, a_4, a_5)$ denote a re-arrangement of (1, –4, 6, 7, –10). Then the equation $a_1x^4 + a_2x^3 + a_3x^2 + a_4x + a_5 = 0$ has at least two real roots.

Because

Statement-II – If $ax^2 + bx + c = 0$ and $a + b + c = 0$, (i.e. in a polynomial the sum of coefficients is zero) then x = 1 is root of $ax^2 + bx + c = 0$.

Single Choice Correct

***12.** Let a and b be two distinct roots of the equation $x^3 + 3x^2 - 1 = 0$. The equation which has (ab) as its root is equal to

(A) $x^3 - 3x - 1 = 0$
(B) $x^3 - 3x^2 + 1 = 0$
(C) $x^3 + x^2 - 3x + 1 = 0$
(D) $x^3 + x^2 + 3x - 1 = 0$

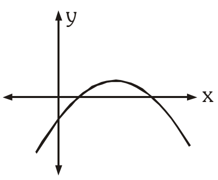
***13.** If two roots of the equation $(x - 1)(2x^2 - 3x + 4) = 0$ coincide with roots of the equation $x^3 + (a + 1)x^2 + (a + b)x + b = 0$ where $a, b \in R$ then $2(a + b)$ equals

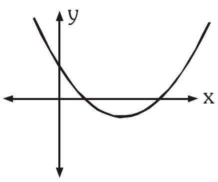
(A) 4
(B) 2
(C) 1
(D) 0

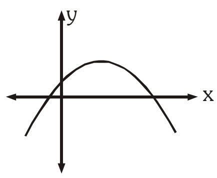
14. A quadratic equation with rational coefficients one of whose roots is $\tan\left(\frac{\pi}{12}\right)$ is -

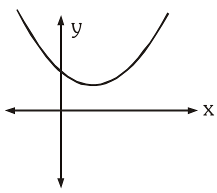
(A) $x^2 - 2x + 1 = 0$
(B) $x^2 - 2x + 4 = 0$
(C) $x^2 - 4x + 1 = 0$
(D) $x^2 - 4x - 1 = 0$

***15.** Graph of the function $f(x) = Ax^2 - BX + C$, where
$A = (\sec\theta - \cos\theta)(\text{cosec}\theta - \sin\theta)(\tan\theta + \cot\theta)$,
$B = (\sin\theta + \text{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2 - (\tan^2\theta + \cot^2\theta)$ &
$C = 12$, is represented by

(A) 

(B) 

(C) 

(D) 

***16.** If the quadratic equation $ax^2 + bx + 6 = 0$ does not have two distinct real roots, then the least value of $2a + b$ is -

(A) 2
(B) -3
(C) -6
(D) 1

17. For every $x \in R$, the polynomial $x^8 - x^5 + x^2 - x + 1$ is -

(A) positive
(B) never positive
(C) positive as well as negative
(D) negative

***18.** Three roots of the equation, $x^4 - px^3 + qx^2 - rx + s = 0$ are tanA, tanB & tanC where A, B, C are the angles of a triangle. The fourth root of the biquadratic is -

(A) $\frac{p-r}{1-q+s}$
(B) $\frac{p-r}{1+q-s}$
(C) $\frac{p+r}{1-q+s}$
(D) $\frac{p+r}{1+q-s}$

***19.** If the equation $a_nx^n + a_{n-1}x^{n-1} + \ldots + a_1x = 0$, $a_1 \neq 0$, $n \geq 2$, has a positive root $x = \alpha$, then the equation $na_nx^{n-1} + (n - 1)a_{n-1}x^{n-2} + \ldots + a_1 = 0$ has a positive root, which is- **[AIEEE-2005]**

(A) equal to α
(B) greater than or equal to α
(C) smaller than α
(D) greater than α

20. If α and β are the roots of the equation $x^2 - x + 1 = 0$, then $\alpha^{2009} + \beta^{2009} =$ **[AIEEE-2010]**

(A) -2
(B) -1
(C) 1
(D) 2

***21.** The set of all real numbers x for which $x^2 - |x + 2| + x > 0$, is **[JEE 2002 (screening), 3]**

(A) $(-\infty, -2) \cup (2, \infty)$
(B) $(-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$
(C) $(-\infty, -1) \cup (1, \infty)$
(D) $(\sqrt{2}, \infty)$

22.** ***(a) If one root of the equation $x^2 + px + q = 0$ is the square of the other, then

(A) $p^3 + q^2 - q(3p + 1) = 0$
(B) $p^3 + q^2 + q(1 + 3p) = 0$
(C) $p^3 + q^2 + q(3p - 1) = 0$
(D) $p^3 + q^2 + q(1 - 3p) = 0$

(b) If $x^2 + 2ax + 10 - 3a > 0$ for all $x \in R$, then **[JEE 2004 (Screening)]**

(A) $-5 < a < 2$ (B) $a < -5$ (C) $a > 5$ (D) $2 < a < 5$

23. If the quadratic polynomial $P(x) = (p - 3)x^2 - 2px + 3p - 6$ ranges from $[0, \infty)$ for every $x \in R$, then the value of p can be

(A) $\frac{3}{2}$ (B) 4 (C) 6 (D) 7

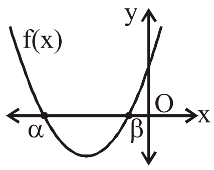
***24.** If the roots of the quadratic equation $ax^2 + bx + c = 0$ are $\frac{k+1}{k}$ and $\frac{k+2}{k+1}$, then $\left(\frac{a}{a+b+c}\right)^2$ equals

(A) k^2 (B) $(k + 1)^2$ (C) $(k + 2)^2$ (D) $k^2 (k + 1)^2$

***25.** If min. $(2x^2 - ax + 2) >$ max. $(b - 1 + 2x - x^2)$ then roots of the equation $2x^2 + ax + (2 - b) = 0$, are

(A) positive and distinct
(B) negative and distinct
(C) opposite in sign
(D) imaginary

26. The following figure shows the graph of $f(x) = ax^2 - bx + c$.
Then which one of the following is correct?



(A) $\frac{b}{c} > 0$
(B) a and c are of opposite sign
(C) a and b are of same sign
(D) None

***27.** The number of integral values of α for which the inequality $x^2 - 2(4\alpha - 1)x + 15\alpha^2 > 2\alpha + 7$ is true for every $x \in R$, is

(A) 0 (B) 1 (C) 2 (D) 3

***28.** If p and q are the roots of the quadratic equation $x^2 - (\alpha - 2)x - \alpha = 1$ $(\alpha \in R)$, then the minimum value of $(p^2 + q^2)$ is equal to

(A) 2 (B) 3 (C) 5 (D) 6

29. The product of all values of x which make the following statement true
$(\log_3 x)(\log_5 9) - \log_x 25 + \log_3 2 = \log_3 54$, is

(A) $\sqrt{5}$ (B) 5 (C) $5\sqrt{5}$ (D) 25

Multiple Choice Correct

30. The set of values of 'a' for which the inequality $(x-3a)(x-a-3)<0$ is satisfied for all x in the interval $1 \le x \le 3$

(A) (1/3, 3) (B) (0, 1/3) (C) (–2, 0) (D) (–2, 3)

***31.** The correct statement is / are -

(A) If x_1 & x_2 are roots of the equation $2x^2-6x-b=0$ $(b>0)$, then $\frac{x_1}{x_2}+\frac{x_2}{x_1}<-2$

(B) Equation $ax^2+bx+c=0$ has real roots if $a<0, c>0$ and $b \in R$

(C) If $P(x)=ax^2+bx+c$ and $Q(x)=-ax^2+bx+c$, where $ac \neq 0$ and $a, b, c \in R$, then P(x).Q(x) has at least two real roots.

(D) None of these

***32.** If $\alpha_1<\alpha_2<\alpha_3<\alpha_4<\alpha_5<\alpha_6$, then the equation $(x-\alpha_1)(x-\alpha_3)(x-\alpha_5)+3(x-\alpha_2)(x-\alpha_4)(x-\alpha_6)=0$ has -

(A) three real roots (B) no real root in $(-\infty, \alpha_1)$

(C) one real root in (α_1, α_2) (D) no real root in (α_5, α_6)

33.** Let $a \in \mathbb{R}$ and let $f: \mathbb{R} \to \mathbb{R}$ be given by $f(x)=x^5-5x+a$. Then ***[JEE Adv. 2014]

(A) f(x) has three real roots if $a>4$ (B) f(x) has only one real root if $a>4$

(C) f(x) has three real roots if $a<-4$ (D) f(x) has three real roots if $-4<a<4$

34. For $x \in R$, the expression $\frac{x^2+34x-71}{x^2+2x-7}$ can not lie between,

(A) (5, 7) (B) (12, 19) (C) (1, 4) (D) (8, 9)

***35.** In which of the following inequalities, the set of all real values of x is same as the set of all real values of k for which the equation $kx^2-4x+k=0$ has real roots and satisfying $1-k \le 0$?

(A) $0 \le \log_2 x \le 1$ (B) $x^2-3x+2 \le 0$

(C) $\sin(\pi x) \le 0$ in [0, 2] (D) $|x-1| \le 1$

36. If the vertex of the parabola $y=3x^2-12x+9$ is (a, b), then the parabola whose vertex is (b, a), is(are)

(A) $y = x^2+6x+11$ (B) $y = x^2-7x+3$

(C) $y = -2x^2-12x-16$ (D) $y = -2x^2+16x-13$

***37.** Let x and y be 2 real numbers which satisfy the equations

$(\tan^2 x-\sec^2 y) = \frac{5a}{6}-3$ and $(-\sec^2 x+\tan^2 y) = a^2$, then the value of a can be equal to

(A) $\frac{2}{3}$ (B) $\frac{-2}{3}$ (C) $\frac{3}{2}$ (D) $\frac{-3}{2}$

***38.** Let a, b and c be real numbers. Which of the following statement(s) about the equation $(x-a)(x-b)=c$ is/are incorrect?

(A) If $c>0$, then roots are always real. (B) If $c>0$, then roots are always non-real.

(C) If $c<0$, then roots are always real. (D) If $c<0$, then roots are always non-real.

***39.** If quadratic equation $x^2 + 2(a + 2b)x + (2a + b - 1) = 0$ has unequal real roots for all $b \in R$ then the possible values of a can be equal to

(A) 5 (B) – 1 (C) – 10 (D) 3

***40.** If all values of x which satisfies the inequality $\log_{1/3}(x^2 + 2px + p^2 + 1) \geq 0$ also satisfy the inequality $kx^2 + kx - k^2 \leq 0$ for all real values of k, then all possible values of p lies in the interval

(A) $[-1, 1]$ (B) $[0, 1]$ (C) $[0, 2]$ (D) $[-2, 0]$

Match the Column

***41.** The expression $y = ax^2 + bx + c$ $(a, b, c \in R$ and $a \neq 0)$ represents a parabola which cuts the x-axis at the points which are roots of the equation $ax^2 + bx + c = 0$. Column-II contains values which correspond to the nature of roots mentioned in column-I.

	Column-I	**Column-II**
(A)	For $a = 1, c = 4$, if both roots are greater than 2 then b can be equal to	(P) 4
(B)	For $a = -1, b = 5$, if roots lie on either side of -1 then c can be equal to	(Q) 8
(C)	For $b = 6, c = 1$, if one root is less than -1 and the other root greater than $\frac{-1}{2}$ then a can be equal to	(R) 10
		(S) no real value

Comprehension

*Comprehension – 1

Let $(a + \sqrt{b})^{Q(x)} + (a - \sqrt{b})^{Q(x) - 2\lambda} = A$, where $\lambda \in N$, $A \in R$ and $a^2 - b = 1$

$\therefore$ $(a + \sqrt{b})(a - \sqrt{b}) = 1 \Rightarrow (a + \sqrt{b}) = (a - \sqrt{b})^{-1}$ and $(a - \sqrt{b}) = (a + \sqrt{b})^{-1}$

ie, $(a \pm \sqrt{b}) = \left(a+\sqrt{b}\right)^{\pm 1}$ or $\left(a-\sqrt{b}\right)^{\mp 1}$

By substituting $(a+\sqrt{b})^{Q(x)}$ as t in the equation we get a quadratic in t.

Also $a + ar + ar^2 .... \infty = \dfrac{a}{1-r}$ where $-1 < r < 1$

On the basis of above information, answer the following questions

42. Solution of $(2+\sqrt{3})^{x^2-2x+1} + (2-\sqrt{3})^{x^2-2x-1} = \dfrac{4}{2-\sqrt{3}}$ are-

(A) $1 \pm \sqrt{3}, 1$ (B) $1 \pm \sqrt{2}, 1$ (C) $1 \pm \sqrt{3}, 2$ (D) $1 \pm \sqrt{2}, 2$

43. The number of real solutions of the equation $(15 + 4\sqrt{14})^t + (15 - 4\sqrt{14})^t = 30$ are - where $t = x^2 - 2|x|$

(A)0 (B) 2 (C) 4 (D) 6

44. If $\left(\sqrt{(49+20\sqrt{6})}\right)^{\sqrt{a\sqrt{a\sqrt{a.....\infty}}}} + (5-2\sqrt{6})^{x^2+x-3-\sqrt{x\sqrt{x\sqrt{x....\infty}}}} = 10$ where $a = x^2 - 3$, then x is -

(A) $-\sqrt{2}$ (B) $\sqrt{2}$ (C) -2 (D) 2

*Comprehension – 2

For $a, b \in R - \{0\}$, let $f(x) = ax^2 + bx + a$ satisfies $f\left(x + \frac{7}{4}\right) = f\left(\frac{7}{4} - x\right)\ \forall\ x \in R$.

Also the equation $f(x) = 7x + a$ has only one real and distinct solution.

45. The value of $(a + b)$ is equal to
(A) 4 (B) 5 (C) 6 (D) 7

46. The minimum value of f(x) in $\left[0, \frac{3}{2}\right]$ is equal to
(A) $\frac{-33}{8}$ (B) 0 (C) 4 (D) -2

*Comprehension – 3

Consider a rational function $f(x) = \frac{x^2 - 3x - 4}{x^2 - 3x + 4}$ and a quadratic function

$g(x) = x^2 - (b + 1)x + b - 1$, where b is a parameter.

47. The sum of integers in the range of $f(x)$, is
(A) -5 (B) -6 (C) -9 (D) -10

48. If both roots of the equation $g(x) = 0$ are greater than -1, then b lies in the interval
(A) $(-\infty, -2)$ (B) $\left(-\infty, \frac{-1}{4}\right)$ (C) $(-2, \infty)$ (D) $\left(\frac{-1}{2}, \infty\right)$

49. The largest natural number b satisfying $g(x) > -2\ \forall\ x \in R$, is
(A) 1 (B) 2 (C) 3 (D) 4

*Comprehension – 4

Consider two quadratic trinomials $f(x) = x^2 - 2ax + a^2 - 1$ and
$g(x) = (4b - b^2 - 5)x^2 - (2b - 1)x + 3b$, where $a, b \in R$.

50. The values of a for which both roots of the equation $f(x) = 0$ are greater than -2 but less than 4, lie in the interval
(A) $-\infty < a < -3$ (B) $-2 < a < 0$ (C) $-1 < a < 3$ (D) $5 < a < \infty$

51. If roots of the quadratic equation $g(x) = 0$ lie on either side of unity, then number of integral values of b is equal to
(A) 1 (B) 2 (C) 3 (D) 4

52. If $f(x) < 0\ \forall\ x \in [0, 1]$, then a lie in the interval
(A) $-1 < a < 1$ (B) $0 < a < 2$ (C) $0 < a < 1$ (D) $a > 3$

*Comprehension – 5

Consider the expression $g(x) = \sin^2 x - (b + 1)\sin x + 3(b - 2)$ where b is a real parameter.

53. Number of integral values of b for which the equation $g(x) = 0$ has exactly one root in the interval $[0, \pi]$ are
(A) 0 (B) 1 (C) 2 (D) 3

54. If the equation $g(x) = 0$ have two distinct roots in $(0, \pi)$ then b lie in the interval
(A) (0, 3) (B) (1, 3) (C) (2, 3) (D) (0, 2)

55. If g(x) is non-negative for all real x, then b lie in the interval

(A) $[1, \infty)$ (B) $(-\infty, 1]$ (C) $[-1, 1]$ (D) $[3, \infty)$

Subjetive Questions

56. Let a, b, c be real numbers with $a \neq 0$ and let α, β be the roots of the equation $ax^2 + bx + c = 0$. Express the roots of $a^3x^2 + abcx + c^3 = 0$ in terms of α, β. ***[JEE 2001, Mains, 5 out of 100]***

57.** If $x^2 + (a-b)x + (1-a-b) = 0$ where $a, b \in R$ then find the values of 'a' for which equation has unequal real roots for all values of 'b'. ***[JEE 2003, Mains-4 out of 60]

58.** Find the range of values of *t* for which $2\sin t = \frac{1-2x+5x^2}{3x^2-2x-1}$, $t \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. ***[JEE 2005(Mains), 2]

***59.** Let M be the minimum value of $f(\theta) = (3\cos^2\theta + \sin^2\theta)(\sec^2\theta + 3\,\text{cosec}^2\theta)$, for permissible real values of θ and P denotes the product of all real solutions of the equation

$\frac{(x-1)(50-10x)}{x^2-5x} = x^2 - 8x + 7$. Find (P M).

***60.** If the range of values of a for which the roots of the equation $x^2 - 2x - a^2 + 1 = 0$ lie between the roots of the equation $x^2 - 2(a+1)x + a(a-1) = 0$ is (p, q), find the value of $\left(q + \frac{1}{p^2}\right)$.

***61.** Let x_1 and x_2 be the real roots of the equation $x^2 - kx + (k^2 + 7k + 15) = 0$. What is the maximum value of $(x_1^2 + x_2^2)$?

***62.** If $1 - \log_x 2 + \log_{x^2} 9 - \log_{x^3} 64 < 0$, then range of x is (a, b). Find the minimum value of (a + 9b).

***63.** If α, β are roots of the equation $2x^2 + 6x + b = 0$ where $b < 0$, then find the least integral value of $\left(\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}\right)$.

***64.** Suppose that a, b, c, d are rationals which satisfy $a + b + c + d = 10$, $(a+b)(c+d) = 16$, $(a+c)(b+d) = 21$ and $(a+d)(b+c) = 24$, then find the value of $(a^2 + b^2 + c^2 + d^2)$.

***65.** If sum of maximum and minimum value of $y = \log_2(x^4 + x^2 + 1) - \log_2(x^4 + x^3 + 2x^2 + x + 1)$ can be expressed in form $((\log_2 m) - n)$, where m and 2 are coprime then compute (m + n).

***66.** If all the solutions of the inequality $x^2 - 6ax + 5a^2 \le 0$ are also the solutions of inequality $x^2 - 14x + 40 \le 0$ then find the number of possible integral values of a.

***67.** Let a, b, c, d be distinct real numbers and a and b are the roots of quadratic equation $x^2 - 2cx - 5d = 0$. If c and d are the roots of the quadratic equation $x^2 - 2ax - 5b = 0$ then find the numerical values of a + b + c + d.

***68.** Find the range of values of a, such that $f(x) = \dfrac{ax^2 + 2(a+1)x + 9a + 4}{x^2 - 8x + 32}$ is always negative.

69. If the quadratic equations $x^2 + bx + ca = 0$ & $x^2 + cx + ab = 0$ (where $a \neq 0$) have a common root, prove that the equation containing their other roots is $x^2 + ax + bc = 0$.

70. If $a < b < c < d$ then prove that the roots of the equation ; $(x - a)(x - c) + 2(x - b)(x - d) = 0$ are real & distinct.

***71.** Two roots of a biquadratic $x^4 - 18x^3 + kx^2 + 200x - 1984 = 0$ have their product equal to (–32). Find the value of k.

72. If one root of the quadratic equation $ax^2 + bx + c = 0$ is equal to the n^{th} power of the other, then show that $(ac^n)^{1/(n+1)} + (a^n c)^{1/(n+1)} + b = 0$.

***73.** Let $P(x) = x^2 + bx + c$, where b and c are integer. If P(x) is a factor of both $x^4 + 6x^2 + 25$ and $3x^4 + 4x^2 + 28x + 5$, find the value of P(1).

74. Find the true set of values of p for which the equation : $p.2^{\cos^2 x} + p.2^{-\cos^2 x} - 2 = 0$ has real roots.

***75.** If the coefficients of the quadratic equation $ax^2 + bx + c = 0$ are odd integers then prove that the roots of the equation cannot be rational number.

76. If the three equations $x^2 + ax + 12 = 0$, $x^2 + bx + 15 = 0$ and $x^2 + (a + b)x + 36 = 0$ have a common positive root, find a and b and the roots of the equations.

***77.** If the quadratic equation $ax^2 + bx + c = 0$ has real roots, of opposite sign in the interval (–2,2) then prove that $1 + \dfrac{c}{4a} - \left|\dfrac{b}{2a}\right| > 0$.

78. Show that the function $z = 2x^2 + 2xy + y^2 - 2x + 2y + 2$ is not smaller than -3.

***79.** For $a \leq 0$, determine all real roots of the equation $x^2 - 2a|x - a| - 3a^2 = 0$.

***80.** The equation $x^n + px^2 + qx + r = 0$, where $n \geq 5$ & $r \neq 0$ has roots $\alpha_1, \alpha_2, \alpha_3 \dots\dots\dots \alpha_n$.

Denoting $\sum\limits_{i=1}^{n} \alpha_i^k$ by S_k.

(a) Calculate S_2 & deduce that the roots cannot all be real.

(b) Prove that $S_n + pS_2 + qS_1 + nr = 0$ & hence find the value of S_n.

***81.** Find the number of integral values of a so that the inequation $x^2 - 2(a+1)x + 3(a-3)(a+1) < 0$ is satisfied by atleast one $x \in R^+$.

ANSWERS

- ***True and False***

1. True **2.** False **3.** True **4.** True

- ***Fill in the Blanks***

5. 1/2 **6.** $x = 2$ & $y = \pi/2$ **7.** –2

- ***Assertion–Reason***

8. (B) **9.** (A) **10.** (D) **11.** (A)

- ***Single Choice Correct***

12. (A) **13.** (C) **14.** (C) **15.** (B) **16.** (B)
17. (A) **18.** (A) **19.** (C) **20.** (C) **21.** (B)
22. ***(a)*** (D) ***(b)*** (A) **23.** (C) **24.** (D) **25.** (D)
26. (D) **27.** (B) **28.** (C) **29.** (C)

- ***Multiple Choice Correct***

30. (B) **31.** (ABC) **32.** (ABC) **33.** (B), (D) **34.** (AD)
35. (AB) **36.** (AC) **37.** (AD) **38.** (BCD) **39.** (BC)
40. (ABC)

- ***Match the Column***

41. (A) S (B) Q, R (C) P

- ***Comprehension***

Comprehension – 1 : **42.** (B) **43.** (C) **44.** (D)
Comprehension – 2 : **45.** (B) **46.** (D)
Comprehension – 3 : **47.** (B) **48.** (D) **49.** (B)
Comprehension – 4 : **50.** (C) **51.** (B) **52.** (C)
Comprehension – 5 : **53.** (B) **54.** (ABC) **55.** AD

- ***Subjective Questions***

56. $\gamma = \alpha^2\beta$ and $\delta = \alpha\beta^2$ or $\gamma = \alpha\beta^2$ and $\delta = \alpha^2\beta$ **57.** $a > 1$
58. $\left[-\frac{\pi}{2}, -\frac{\pi}{10}\right] \cup \left[\frac{3\pi}{10}, \frac{\pi}{2}\right]$
59. 24 **60.** 17 **61.** 18 **62.** 25 **63.** 10
64. 39 **65.** 5 **66.** 0 **67.** 30 **68.** $a \in \left(-\infty, -\frac{1}{2}\right)$
71. $k = 86$ **73.** $P(1) = 4$ **74.** [4/5, 1]
76. $a = -7$, $b = -8$; (3, 4); (3, 5) and (3, 12) **79.** $x = (1 - \sqrt{2})\,a$ or $(\sqrt{6} - 1)\,a$
80. (a) $S_2 = 0$, (b) $S_n = -nr$ **81.** 5

Fill in the blanks

1. The sum of n terms of two A.P.'s are in the ratio of $(n + 7) : (3n + 11)$. The ratio of their 9th term is ________.

2. The sum of the first nineteen terms of an A.P. a_1, a_2, a_3 ..... if it is known that $a_4+a_8+a_{12}+a_{16}=224$, is ________.

***3.** If $x \in R$ and the numbers $(5^{1+x} + 5^{1-x})$, $a/2$, $(25^x + 25^{-x})$ form an A.P. then 'a' must lie in the interval ________.

4. If $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots\ldots\ldots$ upto $\infty = \frac{\pi^2}{6}$, then $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots = $ ____________.

***5.** When 9^{th} term of an A.P. is divided by its 2^{nd} term the quotient is 5 & when 13^{th} term is divided by the 6^{th} term, the quotient is 2 and remainder is 5. The first term and the common difference of the A.P. are ____________ & ________ respectively.

6. The sum to infinity of the series $\frac{1}{1} + \frac{1}{1+2} + \frac{1}{1+2+3} + \ldots\ldots$ is equal to ______________.

Assertion & Reason

These questions contains, Statement-I (assertion) and Statement-II (reason).

(A) Statement-I is true, Statement-II is true ; Statement-II is correct explanation for Statement-I.

(B) Statement-I is true, Statement-II is true ; Statement-II is NOT a correct explanation for Statement-I.

(C) Statement-I is true, Statement-II is false.

(D) Statement-I is false, Statement-II is true.

7.** ***Statement-I– If a, b, c are three distinct positive number in H.P., then $\left(\frac{a+b}{2a-b}\right)+\left(\frac{c+b}{2c-b}\right) > 4$

Because

Statement-II– Sum of any number and it's reciprocal is always greater than or equal to 2.

8. ***Statement-I–*** If $x^2y^3 = 6(x, y > 0)$, then the least value of $3x + 4y$ is 10

Because

Statement-II– If $m_1, m_2 \in N$, $a_1, a_2 > 0$ then $\frac{m_1a_1 + m_2a_2}{m_1+m_2} \geq (a_1^{m_1}a_2^{m_2})^{\frac{1}{m_1+m_2}}$ and equality holds when $a_1 = a_2$.

9.** ***Statement-I– For $n \in N$, $2^n > 1 + n\left(\sqrt{(2^{n-1})}\right)$

Because

Statement-II– G.M. > H.M. and (AM) (HM) = $(GM)^2$

10.** ***Statement-I– If a, b, c are three positive numbers in G.P., then $\left(\frac{a+b+c}{3}\right).\left(\frac{3abc}{ab+bc+ca}\right) = \left(\sqrt[3]{abc}\right)^2$

Because

Statement-II– (A.M.) (H.M.) = $(G.M.)^2$ is true for any set of positive numbers.

11. ***Statement-I–*** n^{th} term (T_n) of the sequence (1, 6, 18, 40, 75, 126,....) is $an^3 + bn^2 + cn + d$, and $6a + 2b - d$ is $= 4$.

Because

Statement-II– If the second successive differences (Differences of the differences) of a series are in A.P., then T_n is a cubic polynomial in n.

12.** ***Statement-I– The format of n^{th} term (T_n) of the sequence ($\ell n2$, $\ell n4$, $\ell n32$, $\ell n1024$.......) is $an^2 + bn + c$.

Because

Statement-II– If the second successive differences between the consecutive terms of the given sequence are in G.P., then $T_n = a + bn + cr^{n-1}$, where a, b, c are constants and r is common ratio of G.P.

Single Choice Correct

***13.** If ln (a + c) , ln (c – a), ln (a – 2b + c) are in A.P., then :
(A) a, b, c are in A.P. (B) a^2, b^2, c^2 are in A.P (C) a, b, c are in G.P. (D) a, b, c are in H.P.

***14.** If $\frac{1}{a}+\frac{1}{a-2b}+\frac{1}{c}+\frac{1}{c-2b}=0$ and a, b, c are not in A.P., then -
(A) a, b, c are in G.P. (B) a, $\frac{b}{2}$, c are in A.P.
(C) a, $\frac{b}{2}$, c are in H.P. (D) a, 2b, c are in H.P.

15. The arithmetic mean of the nine numbers in the given set {9, 99, 999, ... 999999999} is a 9 digit number N, all whose digits are distinct. The number N does not contain the digit
(A) 0 (B) 2 (C) 5 (D) 9

16. If for an A.P. a_1 , a_2 , a_3 ,.... , a_n ,....
$a_1 + a_3 + a_5 = -12$ and $a_1 a_2 a_3 = 8$
then the value of $a_2 + a_4 + a_6$ equals
(A) – 12 (B) – 16 (C) – 18 (D) – 21

17. Consider the A.P. a_1 , a_2 ,..... , a_n ,....
the G.P. b_1 , b_2 ,....., b_n ,.....
such that $a_1 = b_1 = 1$; $a_9 = b_9$ and $\sum_{r=1}^{9} a_r = 369$ then
(A) $b_6 = 27$ (B) $b_7 = 27$ (C) $b_8 = 81$ (D) $b_9 = 18$

***18.** Let p, q, r $\in R^+$ and 27 pqr $\geq (p + q + r)^3$ and 3p + 4q + 5r = 12 then $p^3 + q^4 + r^5$ is equal to -
(A) 2 (B) 6 (C) 3 (D) None of these

***19.** If a_1, a_2,.................$a_n \in R^+$ and $a_1.a_2....... a_n = 1$ then the least value of
$(1+a_1+a_1^2)(1+a_2+a_2^2).....(1+a_n+a_n^2)$ is -
(A) 3^n (B) $n3^n$ (C) 3^{3n} (D) data inadequate

20. Consider an infinite geometric series with first term 'a' and common ratio r. If the sum is 4 and the second term is 3/4, then - ***[JEE 2000, Screening, 1+1M out of 35]***
(A) $a=\frac{7}{4}$, $r=\frac{3}{7}$ (B) $a = 2$, $r=\frac{3}{8}$ (C) $a=\frac{3}{2}$, $r=\frac{1}{2}$ (D) $a = 3$, $r=\frac{1}{4}$

21. If a, b, c, d are positive real numbers such that a + b + c + d = 2, then M = (a + b) (c + d) satisfies the relation -
(A) $0 \leq M \leq 1$ (B) $1 \leq M \leq 2$ (C) $2 \leq M \leq 3$ (D) $3 \leq M \leq 4$

22.** Let α ,β be the roots of $x^2 - x + p = 0$ and γ, δ be the roots of $x^2 - 4x + q = 0$. If α, β, γ, δ are in G.P., then the integer values of p and q respectively, are - ***[JEE 2001 Screening 1+1+1M out of 35]
(A) –2, –32 (B) –2, 3 (C) –6, 3 (D) –6, –32

23. If the sum of the first 2n terms of the A.P. 2, 5, 8 .... is equal to the sum of the first n terms of the A.P. 57, 59, 61, ... then n equals -
(A) 10 (B) 12 (C) 11 (D) 13

24. Let the positive numbers a, b, c, d be in A.P. Then abc, abd, acd, bcd are
(A) not in A.P./G.P./H.P. (B) in A.P. (C) in G.P. (D) in H.P.

***25.** Suppose a, b, c are in A.P. and a^2, b^2, c^2 are in G.P. If a < b < c and $a+b+c=\frac{3}{2}$, then the value of a is **[JEE 2002 (Screening), 3M]**

(A) $\frac{1}{2\sqrt{2}}$ (B) $\frac{1}{2\sqrt{3}}$ (C) $\frac{1}{2}-\frac{1}{\sqrt{3}}$ (D) $\frac{1}{2}-\frac{1}{\sqrt{2}}$

26. The first term of an infinite geometric progression is x and its sum is 5. Then - **[JEE 2004]**
(A) $0 \le x \le 10$ (B) $0 < x < 10$ (C) $-10 < x < 0$ (D) $x > 10$

27. Let $a_1, a_2, ..., a_{10}$ be in A.P. & $h_1, h_2, ...h_{10}$ be in H.P. . If $a_1 = h_1 = 2$ & $a_{10} = h_{10} = 3$ then a_4h_7 is -
(A) 2 (B) 3 (C) 5 (D) 6

Multiple Choice Correct

***28.** Let α, β, γ be the roots of the equation $x^3 + 3ax^2 + 3bx + c = 0$. If α, β, γ are in H.P. then β is equal to -
(A) – c/b (B) c/b (C) – a (D) a

***29.** If (1 + 3 + 5 +...+ a) + (1 + 3 + 5 +...+ b) = (1 + 3 + 5 + ... + c), where each set of parentheses contains the sum of consecutive odd integers as shown such that - (i) a + b + c = 21, (ii) a > 6
If G = Max{a, b, c} and L = Min{a, b, c}, then -
(A) G – L = 4 (B) b – a = 2 (C) G – L = 7 (D) a – b = 2

30. If a, b and c are distinct positive real numbers and $a^2 + b^2 + c^2 = 1$, then ab + bc + ca is -
(A) equal to 1 (B) less than 1 (C) greater than 1 (D) any real number

***31.** Let $a_1, a_2, a_3,........$ and $b_1, b_2, b_3,........$ be arithmetic progression such that $a_1 = 25$, $b_1 = 75$ and $a_{100} + b_{100} = 100$, then -
(A) The common difference in progression 'a_i' is equal but opposite in sign to the common difference in progression 'b_j'.
(B) $a_n + b_n = 100$ for any n.
(C) $(a_1 + b_1), (a_2 + b_2), (a_3 + b_3), ...$ are in A.P.
(D) $\sum_{r=1}^{100}(a_r + b_r) = 10^4$

32. Let a, x, b be in A.P. ; a, y, b be in G.P. and a, z, b be in H.P. If x = y + 2 and a = 5z then -
(A) $y^2 = xz$ (B) x > y > z (C) a = 9, b = 1 (D) $a = \frac{9}{4}, b = \frac{1}{4}$

33. If first and $(2n - 1)^{th}$ terms of an A.P., G.P. and H.P. are equal and their n^{th} terms are a, b, c respectively, then -
(A) a + c = 2b (B) $a \ge b \ge c$ (C) a + c = b (D) $b^2 = ac$

34. If sum of n terms of a sequence is given by $S_n = 3n^2 - 5n + 7$ & t_r represents its r^{th} term, then -
(A) $t_7 = 34$ (B) $t_2 = 7$ (C) $t_{10} = 34$ (D) $t_8 = 40$

35. Indicate the correct alternative(s), for $0 < \phi < \frac{\pi}{2}$, if $x = \sum_{n=0}^{\infty}\cos^{2n}\phi$, $y = \sum_{n=0}^{\infty}\sin^{2n}\phi$ and $z = \sum_{n=0}^{\infty}\cos^{2n}\phi\sin^{2n}\phi$, then -
(A) xyz = xz + y (B) xyz = xy + z (C) xyz = x + y + z (D) xyz = yz + x

***36.** If 10 harmonic means H_1, H_2, H_3 .... H_{10} are inserted between 7 and $-\frac{1}{3}$, then -

(A) $H_1 = -7$ (B) $H_2 = \frac{3}{7}$ (C) $H_1 = -\frac{1}{7}$ (D) $H_{10} = -\frac{7}{19}$

37. If t_n be the n^{th} term of the series 1 + 3 + 7 + 15 + ..., then -

(A) $t_5 + 1 = 32$ (B) $t_7 = 2^7 + 1$ (C) $t_{10} = 2^{10} - 1$ (D) $t_{100} = 2^{50} + 1$

38. a, b, c are the first three terms of geometric series. If the H.M. of *a* and *b* is 12 and that of b and c is 36 then which of the following hold(s) good?

(A) Sum of the first term and common ratio of the G.P. is 11.

(B) Sum of the first five terms of the G.P. is 948.

(C) If the value of the the first term and common ratio of the given G.P. is taken as the first term and common difference of an A.P. then its 8^{th} term is 29.

(D) The number 648 is one of the term of the G.P.

***39.** If the roots of the equation, $x^3 + px^2 + qx - 1 = 0$ form an increasing G.P. where p and q are real, then

(A) $p+q = 0$

(B) $p \in (-3, \infty)$

(C) one of the roots is unity

(D) one root is smaller than 1 and one root is greater than 1

40. If the triplets log a, log b, log c and (log a – log 2b), (log 2b – log 3c), (log 3c – log a) are in arithmetic progression then

(A) $18(a + b + c)^2 = 18(a^2 + b^2 + c^2) + ab$ (B) a, b, c are in G.P.

(C) a, 2b, 3c are in H.P. (D) a, b, c can be the lengths of the sides of a triangle

(Assume all logarithmic terms to be defined)

Match the Column

41.

	Column-I		Column-II
(A)	If a_i's are in A.P. and $a_1 + a_3 + a_4 + a_5 + a_7 = 20$, a_4 is equal to	(p)	21
(B)	Sum of an infinite G.P. is 6 and it's first term is 3. then harmonic mean of first and third terms of G.P. is	(q)	4
(C)	If roots of the equation $x^3 - ax^2 + bx + 27 = 0$, are in G.P. with common ratio 2, then a + b is equal to	(r)	24
(D)	If the roots of $x^4 - 8x^3 + ax^2 + bx + 16 = 0$ are positive real numbers then a is	(s)	6/5

Comprehension

Comprehension–1

There are 4n + 1 terms in a sequence of which first 2n + 1 are in Arithmetic Progression and last 2n + 1 are in Geometric Progression the common difference of Arithmetic Progression is 2 and common ratio of Geometric Progression is 1/2. The middle term of the Arithmetic Progression is equal to middle term of Geometric Progression. Let middle term of the sequence is T_m and T_m is the sum of infinite Geometric Progression whose sum of first two terms is $\left(\frac{5}{4}\right)^2$ n and ratio of these terms is $\frac{9}{16}$.

On the basis of above information, answer the following questions

42. Number of terms in the given sequence is equal to -
(A) 9 (B) 17 (C) 13 (D) none

43. Middle term of the given sequence, i.e. T_m is equal to -
(A) 16/7 (B) 32/7 (C) 48/7 (D) 16/9

44. First term of given sequence is equal to -
(A) –8/7, –20/7 (B) –36/7 (C) 36/7 (D) 48/7

45. Middle term of given A. P. is equal to -
(A) 6/7 (B) 10/7 (C) 78/7 (D) 11

46. Sum of the terms of given A. P. is equal to -
(A) 6/7 (B) 7 (C) 3 (D) 6

*Comprehension–2

If $a_i > 0$, i = 1, 2, 3, ... n and $m_1, m_2, m_3, \ldots, m_n$ be positive rational numbers, then

$$\left(\frac{m_1a_1+m_2a_2+\ldots+m_na_n}{m_1+m_2+\ldots+m_n}\right) \geq \left(a_1^{m_1}a_2^{m_2}\ldots a_n^{m_n}\right)^{1/(m_1+m_2+\ldots m_n)} \geq \frac{(m_1+m_2+\ldots+m_n)}{\frac{m_1}{a_1}+\frac{m_2}{a_2}+\ldots+\frac{m_n}{a_n}}$$

is called weighted mean theorem

where $A^* = \dfrac{m_1a_1+m_2a_2+\ldots+m_na_n}{m_1+m_2+\ldots+m_n}$ = Weighted arithmetic mean

$G^* = \left(a_1^{m_1}a_2^{m_2}\ldots a_n^{m_n}\right)^{1/(m_1+m_2+\ldots+m_n)}$ = Weighted geometric mean

and $H^* = \dfrac{m_1+m_2+\ldots+m_n}{\frac{m_1}{a_1}+\frac{m_2}{a_2}+\ldots\frac{m_n}{a_n}}$ = Weighted harmonic mean

i.e., $A^* \geq G^* \geq H^*$

Now, let a + b + c = 5(a, b, c > 0) and $x^2y^3 = 243(x > 0, y > 0)$

On the basis of above information, answer the following questions

***47.** The greatest value of ab^3c is -
(A) 3 (B) 9 (C) 27 (D) 81

***48.** Which statement is correct -
(A) $\frac{1}{5} \geq \frac{1}{\frac{1}{a}+\frac{3}{b}+\frac{1}{c}}$ (B) $\frac{1}{25} \geq \frac{1}{\frac{1}{a}+\frac{9}{b}+\frac{1}{c}}$ (C) $\frac{1}{5} \geq \frac{1}{\frac{1}{a}+\frac{9}{b}+\frac{1}{c}}$ (D) $\frac{1}{25} \geq \frac{1}{\frac{1}{a}+\frac{6}{b}+\frac{1}{c}}$

***49.** The least value of $x^2 + 3y + 1$ is -
(A) 15 (B) greater than 15 (C) 3 (D) less than 15

***50.** Which statement is correct -

(A) $\frac{2x+3y}{5} \geq 3 \geq \frac{5}{\frac{3}{x}+\frac{2}{y}}$ (B) $\frac{2x+3y}{5} \geq 3 \geq \frac{5xy}{3x+2y}$

(C) $\frac{2x+3y}{5} \geq 3 \geq \frac{5xy}{3x+4y}$ (D) $\frac{2x+3y}{5} \geq 3 \geq \frac{5xy}{2x+3y}$

Subjetive Questions

***51.** Prove that : $(ab + xy)(ax + by) \geq 4abxy$ where $a, b, x, y \in R^+$

***52.** If $a, b, c \in R^+$ & $a + b + c = 1$; then show that $(1 - a)(1 - b)(1 - c) \geq 8abc$

***53.** If a, b, c are sides of a scalene triangle then show that $(a + b + c)^3 > 27 (a + b - c)(b + c - a)(c + a - b)$

***54.** For positive number a, b, c show that $\frac{bc}{a}+\frac{ac}{b}+\frac{ab}{c} \geq a+b+c$

55. The odd positive numbers are written in the form of a triangle

$$\begin{array}{c} 1 \\ 3 \quad 5 \\ 7 \quad 9 \quad 11 \\ 13 \quad 15 \quad 17 \quad 19 \\ \dots\dots\dots \\ \dots\dots\dots\dots \end{array}$$

find the sum of terms in n^{th} row.

56. In a G.P., the ratio of the sum of the first eleven terms to the sum of the last eleven terms is 1/8 and the ratio of the sum of all the terms without the first nine to the sum of all the terms without the last nine is 2 . Find the number of terms in the G.P.

57. Find the nth term and the sum to 'n' terms of the series :
(a) $1 + 5 + 13 + 29 + 61 + \dots\dots$ (b) $6 + 13 + 22 + 33 + \dots\dots.$

***58.** If a, b, c are three positive real number then prove that :
$$\frac{a}{b+c}+\frac{b}{a+c}+\frac{c}{a+b} \geq \frac{3}{2}$$

59. The fourth power of the common difference of an arithmetic progression with integer entries is added to the product of any four consecutive terms of it. Prove that the resulting sum is the square of an integer. ***[JEE 2000, Mains, 4M out of 100]***

60. Let a_1, a_2........... be positive real numbers in G.P.. For each n, let A_n, G_n, H_n, be respectively, the arithmetic mean, geometric mean and harmonic mean of a_1, a_2, a_3,........... a_n. Find an expression for the G.M. of G_1, G_2,.............G_n in terms of A_1, A_2, ...A_n , H_1, H_2,..........H_n ***[JEE 2001 (Mains) ; 5M]***

61. Let a, b be positive real numbers. If a, A_1, A_2, b are in A.P. ; a, G_1, G_2, b are in G.P. and a, H_1, H_2, b are in H.P., show that $\frac{G_1G_2}{H_1H_2} = \frac{A_1+A_2}{H_1+H_2} = \frac{(2a+b)(a+2b)}{9ab}$. ***[JEE 2002, Mains, 5M out of 60]***

62.** If total number of runs scored in n matches is $\left(\frac{n+1}{4}\right)(2^{n+1} - n - 2)$ where $n > 1$, and the runs scored in the k^{th} match are given by $k.\ 2^{n+1-k}$, where $1 \leq k \leq n$. Find n. ***[JEE-05, Mains-2M out of 60]

***63.** If the sum to first *n* terms of a series, the r^{th} term of which is given by $(2r + 1)2^r$ can be expressed as $R(n \cdot 2^n) + S \cdot 2^n + T$, then find the value of $(R + S + T)$.

64. If a, b, c are in A.P., a^2, b^2, c^2 are in H.P., then prove that either a = b = c or a, b, $-\frac{c}{2}$ form a G.P.

***65.** If a, b, c are positive real numbers, then prove that $[(1 + a)(1 + b)(1 + c)]^7 > 7^7 a^4 b^4 c^4$.

66. In $\{a_n\}$, $a_1 = 4$ and $a_{n+1} = \sqrt{\frac{3+a_n}{2}}$. Let $b_n = |a_{n+1} - a_n|$, $n \in \mathbb{N}$ and $S_n = \sum_{k=1}^{n} b_k$. Prove that $S_n < \frac{5}{2}$.

67. Let $S_n \sum_{k=1}^{n} \frac{k}{(k+1)!}$. Find the value of $\frac{1-s_{2001}}{1-s_{2002}}$

68. In a A.P. & an H.P. have the same first term, the same last term & the same number of terms; prove that the product of the r^{th} term from the beginning in one series & the r^{th} term from the end in the other is independent of r.

69. Sum the following series to n terms and to infinity :

(a) $\frac{1}{1.4.7} + \frac{1}{4.7.10} + \frac{1}{7.10.13} + \ldots\ldots\ldots$ *(b) $\sum_{r=1}^{n} r(r+1)(r+2)(r+3)$ (c) $\sum_{r=1}^{n} \frac{1}{4r^2-1}$

***70.** Find the value of the sum $\sum_{r=1}^{n}\sum_{s=1}^{n} \delta_{rs} 2^r 3^s$ where δ_{rs} is zero if $r \neq s$ & δ_{rs} is one if r = s.

71. Find the sum $\sum_{i=1}^{n}\sum_{j=1}^{i}\sum_{k=1}^{j} 1$.

72. If there be 'm' A.P's beginning with unity whose common difference is 1 , 2 , 3 .... m. Show that the sum of their n^{th} terms is (m/2) (mn – m + n + 1).

***73.** If a, b, c are in H.P., b, c, d are in G.P. & c, d, e are in A.P., then Show that $e = ab^2/(2a-b)^2$.

74. The value of x + y + z is 15, if a , x , y , z , b are in A.P. while the value of ; (1/x)+(1/y)+(1/z) is 5/3 if a , x , y , z , b are in H.P. Find a & b.

75. Prove that the sum of the infinite series $\frac{1.3}{2} + \frac{3.5}{2^2} + \frac{5.7}{2^3} + \frac{7.9}{2^4} + \ldots\ldots\ldots \infty = 23$.

***76.** If a , b, c be in G.P. & $\log_c a$, $\log_b c$, $\log_a b$ be in A.P., then show that the common difference of the A.P. must be 3/2.

77. Find the sum to n terms :

(a) $\frac{1}{x+1} + \frac{2x}{(x+1)(x+2)} + \frac{3x^2}{(x+1)(x+2)(x+3)} + \ldots\ldots.$

(b) $\frac{a_1}{1+a_1} + \frac{a_2}{(1+a_1)(1+a_2)} + \frac{a_3}{(1+a_1)(1+a_2)(1+a_3)} + \ldots\ldots.$

78. In $\{a_n\}$, $a_n \geq 0$, $a_1 = 1$, $S_n = \frac{1}{2}\left(a_n + \frac{1}{a_n}\right)$. Find $\left|\sum_{k=1}^{100} \frac{1}{S_k}\right|$.

79. ***(i)*** a, b are given positive integers with a < b. Let $M(a,b) = \frac{\sum_{k=a}^{b} \sqrt{k^2+3k+3}}{b-a+1}$. Evaluate [M(a, b)];

(ii) Evaluate $N(a,b) = \frac{\left[\sum_{k=a}^{b} \sqrt{k^2+3k+3}\right]}{b-a+1}$, (where [.] = Greatest Integer Function).

ANSWERS

- ***Fill in the Blanks***

1. 12 : 31 **2.** 1064 **3.** $[12, \infty)$ **4.** $\pi^2/8$ **5.** a = 3 d = 4

6. 2

- ***Assertion–Reason***

7. (C) **8.** (A) **9.** (C) **10.** (C) **11.** (A)
12. (B)

- ***Single Choice Correct***

13. (D) **14.** (D) **15.** (A) **16.** (D) **17.** (B)
18. (C) **19.** (A) **20.** (D) **21.** (A) **22.** (A)
23. (C) **24.** (D) **25.** (D) **26.** (B) **27.** (D)

- ***Multiple Choice Correct***

28. (A) **29.** (AD) **30.** (B) **31.** (ABCD) **32.** (ABC)
33. (BD) **34.** (AD) **35.** (BC) **36.** (AD **37.** (AC)
38. (ACD) **39.** (ACD) **40.** (BD)

- ***Match the Column***

41. (A) → (q), (B) → (s), (C) → (p), (D) → (r)

- ***Comprehension***

Comprehension – 1 : **42.** (C) **43.** (C) **44.** (B) **45.** (A) **46.** (D)

Comprehension – 2 : **47.** (C) **48.** (C) **49.** (B) **50.** (B)

- ***Subjective Questions***

55. n^3 **56.** 38

57. (a) $2^{n+1} - 3; 2^{n+2} - 4 - 3n$; (b) $n^2 + 4n + 1$; $\frac{1}{6}n(n+1)(2n+13)+n$

60. $\left[(A_1, A_2, \ldots\ldots\ldots A_n)(H_1, H_2, \ldots\ldots\ldots H_n)\right]^{\frac{1}{2n}}$ **62.** 7

63. 4 **67.** 2003

69. (a) $\frac{1}{24} - \frac{1}{6(3n+1)(3n+4)}, \frac{1}{24}$ (b) $\frac{n(n+1)(n+2)(n+3)(n+4)}{5}$ (c) $\frac{n}{2n+1}, \frac{1}{2}$

70. $\frac{6}{5}(6^n - 1)$ **71.** [n(n+1)(n+2)]/6

74. a = 1, b = 9 or b = 1, a = 9

77. (a) $1 - \frac{x^n}{(x+1)(x+2)\ldots\ldots\ldots(x+n)}$ (b) $1 - \frac{1}{(1+a_1)(1+a_2)\ldots\ldots\ldots(1+a_n)}$

78. 18

79. (i) $\left[\frac{a+b+3}{2}\right]$ (ii) $\frac{a+b+2}{2}$

Assertion & Reason

These questions contain, Statement-I (assertion) and Statement-II (reason).

(A) Statement-I is true, Statement-II is true ; Statement-II is correct explanation for Statement-I.

(B) Statement-I is true, Statement-II is true ; Statement-II is NOT a correct explanation for Statement-I.

(C) Statement-I is true, Statement-II is false.

(D) Statement-I is false, Statement-II is true.

***1.** Let $L_1 : a_1x + b_1y + c_1 = 0$, $L_2 : a_2x + b_2y + c_2 = 0$ and $L_3 : a_3x + b_3y + c_3 = 0$.

Statement-I – If L_1, L_2 and L_3 are three concurrent lines, then $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$.

Because

Statement-II – If $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$, then the lines L_1, L_2 and L_3 must be concurrent.

2. **Statement-I –** The diagonals of the parallelogram whose sides are $\ell x + my + n = 0$, $\ell x + my + n' = 0$, $mx + \ell y + n = 0$, $mx + \ell y + n' = 0$ are perpendicular.

Because

Statement-II – If the perpedicular distances between parallel sides of a parallelogram are equal, then it is a rhombus.

3. **Statement-I –** The equation $2x^2 + 3xy - 2y^2 + 5x - 5y + 3 = 0$ represents a pair of perpendicular straight lines.

Because

Statement-II – A pair of lines given by $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ are perpendicular, if $a + b = 0$

4. **Statement-I –** The joint equation of lines $2y = x+1$ and $2y = -(x+1)$ is $4y^2 = -(x + 1)^2$.

Because

Statement-II – The joint equation of two lines satisfy every point lying on any one of the line.

Single Choice Correct

5. Given the four lines with the equations

$x + 2y - 3 = 0$, $3x + 4y - 7 = 0$

$2x + 3y - 4 = 0$, $4x + 5y - 6 = 0$

then **[JEE 1980]**

(A) they are all concurrent (B) they are the sides of a quadrilateral

(C) only three lines are concurrent (D) none of the above

***6.** The co-ordinates of a point P on the line $2x - y + 5 = 0$ such that $|PA - PB|$ is maximum where A is $(4, -2)$ and B is $(2, -4)$ will be -

(A) (11, 27) (B) (–11, –17) (C) (–11, 17) (D) (0, 5)

7. The line $x + 3y - 2 = 0$ bisects the angle between a pair of straight lines of which one has equation $x - 7y + 5 = 0$. The equation of the other line is -

(A) $3x + 3y - 1 = 0$ (B) $x - 3y + 2 = 0$ (C) $5x + 5y - 3 = 0$ (D) none

8. A ray of light passing through the point A (1, 2) is reflected at a point B on the x-axis line mirror and then passes through (5, 3). Then the equation of AB is -

(A) $5x + 4y = 13$ (B) $5x - 4y = -3$ (C) $4x + 5y = 14$ (D) None of these

9. Let the algebraic sum of the perpendicular distances from the points (3, 0), (0, 3) & (2, 2) to a variable straight line be zero, then the line passes through a fixed point whose co-ordinates are-

(A) (3, 2) (B) (2, 3) (C) $\left(\frac{3}{5}, \frac{3}{5}\right)$ (D) $\left(\frac{5}{3}, \frac{5}{3}\right)$

***10.** The image of the pair of lines respresented by $ax^2 + 2h\,xy + by^2 = 0$ by the line mirror $y = 0$ is :
(A) $ax^2 - 2hxy + by^2 = 0$ (B) $bx^2 - 2h\,xy + ay^2 = 0$ (C) $ax^2 - 2h\,xy - by^2 = 0$ (D) None of these

11. The pair of straight lines $x^2 - 4xy + y^2 = 0$ together with the line $x + y + 4\sqrt{6} = 0$ form a triangle which is :
(A) right angled but not isosscles (B) right isosceles
(C) scalene (D) equilateral

12. Distance between two lines respresented by the line pair, $x^2 - 4xy + 4y^2 + x - 2y - 6 = 0$ is -

(A) $\frac{1}{\sqrt{5}}$ (B) $\sqrt{5}$ (C) $2\sqrt{5}$ (D) none of these

13. The circumcentre of the triangle formed by the lines, $xy + 2x + 2y + 4 = 0$ and $x + y + 2 = 0$ is -
(A) (–1, –1) (B) (–2, –2) (C) (0, 0) (D) (–1, –2)

***14.** Area of the rhombus bounded by the four lines, $ax \pm by \pm c = 0$ is -

(A) $\frac{c^2}{2ab}$ (B) $\frac{2c^2}{ab}$ (C) $\frac{ab}{4c^2}$ (D) None of these

***15.** If the lines $ax + y + 1 = 0$, $x + by + 1 = 0$ & $x + y + c = 0$ where a, b & c are distinct real numbers different from 1 are concurrent, then the value of $\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} =$
(A) 4 (B) 3 (C) 2 (D) 1

***16.** The area enclosed by $2|x| + 3|y| \le 6$ is -
(A) 3 sq. units (B) 4 sq. units (C) 12 sq. units (D) 24 sq. units

***17.** The point (4, 1) undergoes the following three transformations successively -
(i) Reflection about the line $y = x$
(ii) Translation through a distance 2 units along the positive directions of x-axis.
(iii)Rotation through an angle $\pi/4$ about the origin.
The final position of the point is given by the coordinates :

(A) $\left(\frac{7}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$ (B) $\left(\frac{7}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ (C) $\left(-\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$ (D) none of these

***18.** Given the family of lines, $a(3x + 4y + 6) + b(x + y + 2) = 0$. The line of the family situated at the greatest distance from the point P (2,3) has equation -
(A) $4x + 3y + 8 = 0$ (B) $5x - 3y - 10 = 0$ (C) $15x + 8y + 30 = 0$ (D) none

19.** Let PQR be a right angled isosceles triangle, right angled at P (2, 1). If the equation of the line QR is $2x + y = 3$, then the equation representing the pair of lines PQ and PR is - ***[JEE 1999]
(A) $3x^2 - 3y^2 + 8xy + 20x + 10y + 25 = 0$ (B) $3x^2 - 3y^2 + 8xy - 20x - 10y + 25 = 0$
(C) $3x^2 - 3y^2 + 8xy + 10x + 15y + 20 = 0$ (D) $3x^2 - 3y^2 - 8xy - 10x - 15y - 20 = 0$

20. Area of the parallelogram formed by the lines $y = mx$, $y = mx + 1$, $y = nx$ and $y = nx + 1$ equals

(A) $\frac{|m+n|}{(m-n)^2}$ (B) $\frac{2}{|m+n|}$ (C) $\frac{1}{|m+n|}$ (D) $\frac{1}{|m-n|}$

21. The number of integer values of m, for which the x co-ordinate of the point of intersection of the lines $3x + 4y = 9$ and $y = mx + 1$ is also an integer, is
(A) 2 (B) 0 (C) 4 (D) 1
[JEE 2001 (Screening)]

22. Let $P = (-1, 0)$, $Q = (0, 0)$ and $R = (3, 3\sqrt{3})$ be three points. Then the equation of the bisector of the angle PQR is ***[JEE 2002 (Screening)]***

(A) $\frac{\sqrt{3}}{2}x + y = 0$ (B) $x + \sqrt{3}y = 0$ (C) $\sqrt{3}x + y = 0$ (D) $x + \frac{\sqrt{3}}{2}y = 0$

23. A straight line through the origin O meets the parallel lines $4x + 2y = 9$ and $2x + y + 6 = 0$ at points P and Q respectively. Then the point O divides the segment PQ in the ratio ***[JEE 2002 (Screening)]***
(A) 1 : 2 (B) 3 : 4 (C) 2 : 1 (D) 4 : 3

24.** The area bounded by the curves $y = |x| - 1$ and $y = -|x| + 1$ is ***[JEE 2002 (Screening)]

(A) 1 (B) 2 (C) $2\sqrt{2}$ (D) 4

***25.** The co-ordinates of the vertices P, Q, R & S of square PQRS inscribed in the triangle ABC with vertices $A \equiv (0, 0)$, B (3, 0) & $C \equiv (2, 1)$ given that two of its vertices P, Q are on the side AB are respectively :

(A) $\left(\frac{1}{4}, 0\right), \left(\frac{3}{8}, 0\right), \left(\frac{3}{8}, \frac{1}{8}\right) \& \left(\frac{1}{4}, \frac{1}{8}\right)$ (B) $\left(\frac{1}{2}, 0\right), \left(\frac{3}{4}, 0\right), \left(\frac{3}{4}, \frac{1}{4}\right) \& \left(\frac{1}{2}, \frac{1}{4}\right)$

(C) $(1, 0), \left(\frac{3}{2}, 0\right), \left(\frac{3}{2}, \frac{1}{2}\right) \& \left(1, \frac{1}{2}\right)$ (D) $\left(\frac{3}{2}, 0\right), \left(\frac{9}{4}, 0\right), \left(\frac{9}{4}, \frac{3}{4}\right) \& \left(\frac{3}{2}, \frac{3}{4}\right)$

26. The number of possible straight lines, passing through (2, 3) and forming a triangle with coordinate axes, whose area is 12 sq. units, is -
(A) one (B) two (C) three (D) four

27. The line $3x + 2y = 6$ will divide the quadrilateral formed by the lines $x + y = 5$, $y - 2x = 8$, $3y + 2x = 0$ & $4y - x = 0$ in -
(A) two quadrilaterals (B) one pentagon and one triangle
(C) two triangles (D) one triangle and one quadrilateral

28. Three vertices of triangle ABC are A(–1, 11), B(–9, –8) and C(15, –2). The equation of angle bisector of angle A is -
(A) $4x - y = 7$ (B) $4x + y = 7$ (C) $x + 4y = 7$ (D) $x - 4y = 7$

Multiple Choice Correct

***29.** Equation of two equal sides of a triangle are the lines $7x + 3y - 20 = 0$ and $3x + 7y - 20 = 0$ and the third side passes through the point (–3, 3), then the equation of the third side can be -

(A) $x + y = 0$ (B) $x - y + 6 = 0$ (C) $x + 3 = 0$ (D) $y = 3$

***30.** The line $x + y = p$ meets the axis of x and y at A and B respectively. A triangle APQ is inscribed in the triangle OAB, O being the origin, with right angle at Q. P and Q lie respectively on OB and AB. If the area of the triangle APQ is 3/8th of the area of the triangle OAB, then $\frac{AQ}{BQ}$ is equal to -
(A) 2 (B) 2/3 (C) 1/3 (D) 3

31. Lines, $L_1 : x + \sqrt{3}y = 2$, and $L_2 : ax + by = 1$, meet at P and enclose an angle of 45° between them. Line $L_3 : y = \sqrt{3}x$, also passes through P then -
(A) $a^2 + b^2 = 1$ (B) $a^2 + b^2 = 2$ (C) $a^2 + b^2 = 3$ (D) $a^2 + b^2 = 4$

***32.** A triangle is formed by the lines $2x - 3y - 6 = 0$; $3x - y + 3 = 0$ and $3x + 4y - 12 = 0$. If the points $P(\alpha,0)$ and $Q\ (0,\beta)$ always lie on or inside the ΔABC, then range of α & β -

(A) $\alpha \in [-1, 2]$ (B) $\alpha \in [-1, 3]$ (C) $\beta \in [-3, 4]$ (D) $\beta \in [-2, 3]$

33. Let $A \equiv (3, 2)$ and $B \equiv (5, 1)$. ABP is an equilateral triangle is constructed on the side of AB remote from the origin then the orthocentre of triangle ABP is -

(A) $\left(4-\frac{1}{2}\sqrt{3}, \frac{3}{2}-\sqrt{3}\right)$ (B) $\left(4+\frac{1}{2}\sqrt{3}, \frac{3}{2}+\sqrt{3}\right)$

(C) $\left(4-\frac{1}{6}\sqrt{3}, \frac{3}{2}-\frac{1}{3}\sqrt{3}\right)$ (D) $\left(4+\frac{1}{6}\sqrt{3}, \frac{3}{2}+\frac{1}{3}\sqrt{3}\right)$

Match the Column

***34.**

	Column-I		Column-II
(A)	If line $2x - by + 1 = 0$ intersects the curve $2x^2 - by^2 + (2b - 1)xy - x - by = 0$ at points A & B and AB subtends a right angle at origin, then value of $b + b^2$ is equal to	(p)	1
(B)	A line passes through point (3, 4) and the point of intersection of the lines $4x + 3y = 12$ and $3x + 4y = 12$ and length of intercepts on the co-ordinate axes are a and b, then ab is equal to	(q)	5
(C)	A light ray emerging from the point source placed at P(2, 3) is reflected at a point 'Q' on the y-axis and then passes through the point R(5, 10). If co-ordinates of Q are (a, b), then a + b is	(r)	4

Comprehension

*Comprehension – 1

A locus is the curve traced out by a point which moves under certain geomatrical conditions:
To find the locus of a point first we assume the co-ordinates of the moving point as (h,k) and then try to find a relation between h and k with the help of the given conditions of the problem. If there is any variable involved in the process then we eliminate them. At last we replace h by x and k by y and get the locus of the point which will be an equation in x and y.

On the basis of above information, answer the following questions

35. Locus of centroid of the triangle whose vertices are $(a\cos t, a\sin t)$, $(b\sin t, -b\cos t)$ and (1, 0) where t is a parameter is -

(A) $(3x - 1)^2 + (3y)^2 = a^2 - b^2$ (B) $(3x - 1)^2 + (3y)^2 = a^2 + b^2$
(C) $(3x + 1)^2 + (3y)^2 = a^2 + b^2$ (D) $(3x + 1)^2 + (3y)^2 = a^2 - b^2$

36. A variable line cuts x-axis at A, y-axis at B where OA = a, OB = b (O as origin) such that $a^2 + b^2 = 1$ then the locus of circumcentre of Δ OAB is -

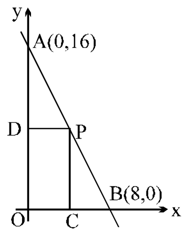
(A) $x^2 + y^2 = 4$ (B) $x^2 + y^2 = 1/4$ (C) $x^2 - y^2 = 4$ (D) $x^2 - y^2 = 1/4$

37. The locus of the point of intersection of the lines $x\cos\alpha + y\sin\alpha = a$ and $x\sin\alpha - y\cos\alpha = b$ where α is variable is -

(A) $x^2 + y^2 = a^2 + b^2$ (B) $x^2 + y^2 = a^2 - b^2$ (C) $x^2 - y^2 = a^2 - b^2$ (D) $x^2 - y^2 = a^2 + b^2$

***Comprehension –2**

In the diagram, a line is drawn through the points A(0, 16) and B(8, 0). Point P is chosen in the first quadrant on the line through A and B. Points C and D are chosen on the x and y-axis respectively, so that PDOC is a rectangle.



38. Perpendicular distance of the line AB from the point (2, 2) is

(A) $\sqrt{4}$ (B) $\sqrt{10}$ (C) $\sqrt{20}$ (D) $\sqrt{50}$

39. Sum of the coordinates of the point P if PDOC is a square is

(A) $\frac{32}{3}$ (B) $\frac{16}{3}$ (C) 16 (D) 11

40. Number of possible ordered pair(s) of all positions of the point P on AB so that the area of the rectangle PDOC is 30 sq. units, is

(A) three (B) two (C) one (D) zero

Comprehension –3

Consider a Δ ABC whose sides BC, CA and AB are represented by the straight lines $x - 2y + 5 = 0$, $x + y + 2 = 0$ and $8x - y - 20 = 0$ respectively.

41. The area of Δ ABC equals

(A) $\frac{41}{2}$ (B) $\frac{43}{2}$ (C) $\frac{45}{2}$ (D) $\frac{47}{2}$

42. If AD be the median of the Δ ABC then the equation of the straight line passing through (2, –1) and parallel to AD is

(A) $4x - 3y - 11 = 0$ (B) $13x - 4y - 30 = 0$ (C) $4x + 13y + 5 = 0$ (D) $13x + 4y - 22 = 0$

43. The orthocentre of the Δ ABC is

(A) (–3, 1) (B) $\left(-\frac{1}{3}, \frac{2}{3}\right)$

(C) (–2, 4) (D) $\left(-\frac{2}{3}, \frac{4}{3}\right)$

Comprehension –4

An equilateral triangle ABC has its centroid at the origin and the base BC lies along the line $x + y = 1$

44. Area of the equilateral ΔABC is

(A) $\frac{3\sqrt{3}}{2}$ (B) $\frac{3\sqrt{3}}{4}$ (C) $\frac{3\sqrt{2}}{2}$ (D) $\frac{2\sqrt{3}}{4}$

45. Gradient of the other two lines are

(A) $\sqrt{3}, \sqrt{2}$ (B) $\sqrt{3}, \frac{1}{\sqrt{3}}$ (C) $\sqrt{2}+1, \sqrt{2}-1$ (D) $2+\sqrt{3}, 2-\sqrt{3}$

***Comprehension –5**

Consider a family of lines $(4a + 3)x - (a + 1)y - (2a + 1) = 0$ where $a \in R$

46. The locus of the foot of the perpendicular from the origin on each member of this family, is
(A) $(2x - 1)^2 + 4(y + 1)^2 = 5$ (B) $(2x - 1)^2 + (y + 1)^2 = 5$
(C) $(2x + 1)^2 + 4(y - 1)^2 = 5$ (D) $(2x - 1)^2 + 4(y - 1)^2 = 5$

47. A member of this family with positive gradient making an angle of $\pi/4$ with the line $3x - 4y = 2$, is
(A) $7x - y - 5 = 0$ (B) $4x - 3y + 2 = 0$ (C) $x + 7y = 15$ (D) $5x - 3y - 4 = 0$

48. Minimum area of the triangle which a member of this family with negative gradient can make with the positive semi axes, is
(A) 8 (B) 6 (C) 4 (D) 2

Comprehension –6

Let $M\left(2, \frac{13}{8}\right)$ is the circumcentre of ΔPQR whose sides PQ and PR are represented by the straight lines $4x - 3y = 0$ and $4x + y = 16$ respectively.

49. The orthocentre of ΔPQR is
(A) $\left(\frac{7}{3}, \frac{4}{3}\right)$ (B) $\left(\frac{4}{3}, \frac{7}{3}\right)$ (C) $\left(3, \frac{3}{4}\right)$ (D) $\left(\frac{3}{4}, 3\right)$

50. If A, B and C are the midpoint of the sides PQ, QR and PR of ΔPQR respectively, then the area of ΔABC equals
(A) 1 (B) 2 (C) 3 (D) 4

51. If PB be the median of the ΔPQR, then the equation of the straight line passing through N(–2, 3) and perpendicular to PB is
(A) $4x + y + 5 = 0$ (B) $x - 4y + 14 = 0$ (C) $4x - y + 11 = 0$ (D) $x + 4y - 10 = 0$

Subjetive Questions

***52.** The vertices of a triangle are A $(x_1, x_1 \tan \theta_1)$, B $(x_2, x_2 \tan \theta_2)$ & C $(x_3, x_3 \tan \theta_3)$. If the circumcentre O of the triangle ABC is at the origin & H $(\bar{x}, \bar{y})$ be its orthocentre, then show that $\frac{\bar{x}}{\bar{y}} = \frac{\cos\theta_1 + \cos\theta_2 + \cos\theta_3}{\sin\theta_1 + \sin\theta_2 + \sin\theta_3}$.

***53.** The ends A, B of a straight line segment of constant length 'c' slide upon the fixed rectangular axes OX & OY respectively. If the rectangle OAPB be completed then show that the locus of the foot of the perpendicular drawn from P to AB is $x^{2/3} + y^{2/3} = c^{2/3}$.

54. Determie the ratio in which the point P(3, 5) divides the join of A(1, 3) & B(7, 9). Find the harmonic conjugate of P w.r.t. A & B.

55.** A straight line L through the origin meets the line $x + y = 1$ and $x + y = 3$ at P and Q respectively. Through P and Q two straight lines L_1 and L_2 are drawn, parallel to $2x - y = 5$ and $3x + y = 5$ respectively. Lines L_1 and L_2 intersect at R. Show that the locus of R, as L varies, is a straight line. ***[JEE 2002 (Mains)]

56.** A straight line L with negative slope passes through the point (8,2) and cuts the positive coordinates axes at points P and Q. Find the absolute minimum value of OP + OQ, as L varies, where O is the origin. ***[JEE 2002 Mains, 5]

57.** The area bounded by the angle bisectors of the lines $x^2 - y^2 + 2y = 1$ and the line $x + y = 3$, is ***[JEE 2004 (Screening)]

58.** The area of the triangle formed by the intersection of a line parallel to x-axis and passing through P(h,k) with the lines $y = x$ and $x + y = 2$ is $4h^2$. Find the locus of the point P. ***[JEE 2005, Mains, 2]

59.** A rectangle PQRS has its side PQ parallel to the line $y = mx$ and vertices P,Q and S on the lines $y = a$, $x = b$ and $x = -b$, respectively. Find the locus of the vertex R. ***[IIT-JEE 1996]

60.** A variable straight line of slope 4 intersects the hyperbola $xy=1$ at two points. Find the locus of the point which divides the line segment between these two points in the ratio 1 : 2. ***[IIT-JEE 1997]

61.** A line cuts the x-axis at A(7, 0) and the y-axis at B(0, –5). A variable line PQ is drawn perpendicular to AB cutting the x-axis in P and the y-axis in Q. If AQ and BP intersect at R, find the locus of R. ***[IIT-JEE 1990]

***62.** If a, b, c are all different and the points $\left(\frac{r^3}{r-1}, \frac{r^2-3}{r-1}\right)$ where r = a, b, c are collinear, then prove that $3(a + b + c) = ab + bc + ca - abc$.

63.** Straight lines $3x + 4y = 5$ and $4x - 3y = 15$ intersect at the point A. Points B and C are chosen on these two lines such that AB = AC. Determine the possible equations of the line BC passing through the point (1, 2) ***[IIT-JEE 1990]

64. A variable line, drawn through the point of intersection of the straight lines $\frac{x}{a}+\frac{y}{b}=1$ & $\frac{x}{b}+\frac{y}{a}=1$, meets the coordinate axes in A & B. Show that the locus of the mid point of AB is the curve $2xy(a + b) = ab(x + y)$.

***65.** In a triangle ABC, D is a point on BC such that $\frac{BD}{DC} = \frac{AB}{AC}$. The equation of the line AD is $2x + 3y + 4 = 0$ & the equation of the line AB is $3x + 2y + 1 = 0$. Find the equation of the line AC.

66. Find the co-ordinates of the incentre of the triangle formed by the line $x + y + 1 = 0$; $x - y + 3 = 0$ & $7x - y + 3 = 0$. Also find the centre of the circle escribed to $7x - y + 3 = 0$.

67.** Lines $L_1 \equiv ax + by + c = 0$ and $L_2 \equiv \ell x + my + n = 0$ intersect at the point P and makes an angle θ with each other. Find the equation of a line L different from L_2 which passes through P and makes the same angle θ with L_1 ***[IIT-JEE 1988]

***68.** A triangle is formed by the lines whose equations are AB : $x + y - 5 = 0$, BC : $x + 7y - 7 = 0$ and CA : $7x + y - 14 = 0$. Find the bisector of the interior angle at B and the exterior angle at C. Determine the nature of the interior angle at A and find the equation of the bisector.

69. The distance of a point (x_1, y_1) from each of two straight lines which passes through the origin of co-ordinates is δ ; find the combined equation of these straight lines.

***70.** Equation of a line is given by $y + 2at = t(x - at^2)$, t being the parameter. Find the locus of the point intersection of the lines which are at right angles.

***71.** Find the equation of the straight lines passing through (–2, – 7) & having an intercept of length 3 between the straight lines $4x + 3y = 12$, $4x + 3y = 3$.

***72.** Determine all values of α for which the point (α, α^2) lies inside the triangle formed by the lines
$2x + 3y - 1 = 0$; $x + 2y - 3 = 0$; $5x - 6y - 1 = 0$.

73. Find the co-ordinates of the orthocentre of the triangle, the equations of whose sides are $x + y = 1$, $2x + 3y = 6$, $4x - y + 4 = 0$, without finding the co-ordinates of its vertices.

74.** Let ABC be a triangle with AB = AC. If D is the midpoint of BC, E is the foot of the perpendicular drawn from D to AC and F the mid-point of DE, prove that AF is perpendicular to BE. ***[IIT-JEE 1989]

75. Find the condition that the diagonals of the parallelogram formed by the lines
$ax + by + c = 0$; $ax + by + c' = 0$; $a'x + b'y + c = 0$ & $a'x + b'y + c' = 0$ are at right angles. Also find the equation to the diagonals of the parallelogram.

ANSWERS

- ***Assertion–Reason***

1.	C	**2.**	A	**3.**	D	**4.**	D

- ***Single Choice Correct***

5.	C	**6.**	B	**7.**	C	**8.**	A	**9.**	D
10.	A	**11.**	D	**12.**	B	**13.**	A	**14.**	B
15.	D	**16.**	C	**17.**	C	**18.**	A	**19.**	B
20.	D	**21.**	A	**22.**	C	**23.**	B	**24.**	B
25.	D	**26.**	C	**27.**	A	**28.**	B		

- ***Multiple Choice Correct***

29.	(AB)	**30.**	(D)	**31.**	(B)	**32.**	(BD)	**33.**	(D)

- ***Match the Column***

34. (A) $\to$ (r); (B) $\to$ (p); (C) $\to$ (q);

- ***Comprehension***

Comprehension – 1	**35.**	B	**36.**	B	**37.**	A
Comprehension – 2	**38.**	C	**39.**	A	**40.**	B
Comprehension – 3	**41.**	C	**42.**	D	**43.**	B
Comprehension – 4	**44.**	***A***	**45.**	***D***		
Comprehension – 5	**46.**	D	**47.**	A	**48.**	C
Comprehension – 6	**49.**	C	**50.**	B	**51.**	D

- ***Subjective Questions***

54. 1 : 2 ; Q (–5, –3) **55.** $x - 3y + 5 = 0$; **56.** 18 **57.** 2

58. $y = 2x + 1, y = -2x + 1$ **59.** $(m^2 - 1)x - my + b(m^2 + 1) + am = 0$

60. $16x^2 + y^2 + 10xy = 2$ **61.** $x^2 + y^2 - 7x + 5y = 0$

63. $x - 7y + 13 = 0$ and $7x + y - 9 = 0$ **65.** $9x + 46y + 83 = 0$

66. (– 1, 1) ; (4, 1) **67.** $2(al + bm)(ax + by + c) - (a^2 + b^2)(lx + my + n) = 0$

68. $3x + 6y - 16 = 0$; $8x + 8y - 21 = 0$; acute angle bisector, $12x + 6y - 39 = 0$

69. $(y_1^2 - d^2)x^2 - 2x_1y_1xy + (x_1^2 - d^2)y^2 = 0$ **70.** $y^2 = a(x - 3a)$

71. $7x + 24y + 182 = 0$ or $x = -2$ **72.** $-\frac{3}{2} < a < -1 \cup \frac{1}{2} < a < 1$

73. $\left(\frac{3}{7}, \frac{22}{7}\right)$

75. $a^2 + b^2 = a'^2 + b'^2$; $(a + a')x + (b + b')y + (c + c') = 0$; $(a - a')x + (b - b')y = 0$

Assertion & Reason

These questions contains, Statement-I (assertion) and Statement-II (reason).

(A) Statement-I is true, Statement-II is true ; Statement-II is correct explanation for Statement-I.

(B) Statement-I is true, Statement-II is true ; Statement-II is NOT a correct explanation for statement-I.

(C) Statement-I is true, Statement-II is false.

(D) Statement-I is false, Statement-II is true.

1. Consider two circles $C_1 \equiv x^2 + y^2 + 2x + 2y - 6 = 0$ & $C_2 \equiv x^2 + y^2 + 2x + 2y - 2 = 0$.

Statement-I– Two tangents are drawn from a point on the circle C_1 to the circle C_2, then tangents always perpendicular.

Because

Statement-II– C_1 is the director circle of C_2.

(A) A (B) B (C) C (D) D

2. ***Statement-I–*** The line $(x - 3)\cos\theta + (y - 3)\sin\theta = 1$ touches a circle $(x - 3)^2 + (y - 3)^2 = 1$ for all values of θ.

Because

Statement-II– $x\cos\theta + y\sin\theta = a$ is a tangent of circle $x^2 + y^2 = a^2$ for all values of θ.

(A) A (B) B (C) C (D) D

***3.** Consider the circles $C_1 \equiv x^2 + y^2 - 6x - 4y + 9 = 0$ and $C_2 \equiv x^2 + y^2 - 8x - 6y + 23 = 0$.

Statement-I– Circle C_1 bisects the circumference of the circle C_2.

Because

Statement-II– Centre of C_1 lie on C_2.

(A) A (B) B (C) C (D) D

4. ***Statement-I–*** Circles $x^2 + y^2 = 4$ and $x^2 + y^2 - 8x + 7 = 0$ intersect each other at two distinct points

Because

Statement-II– Circles with centres C_1 and C_2 and radii r_1 and r_2 intersect at two distinct points, if $|C_1C_2| < r_1 + r_2$

(A) A (B) B (C) C (D) D

Single Choice Correct

5. The locus of the centers of the circles which cut the circles $x^2 + y^2 + 4x - 6y + 9 = 0$ and $x^2 + y^2 - 5x + 4y - 2 = 0$ orthogonally is -

(A) $9x + 10y - 7 = 0$ (B) $x - y + 2 = 0$

(C) $9x - 10y + 11 = 0$ (D) $9x + 10y + 7 = 0$

***6.** If $\left(a,\frac{1}{a}\right)$, $\left(b,\frac{1}{b}\right)$, $\left(c,\frac{1}{c}\right)$ & $\left(d,\frac{1}{d}\right)$ are four distinct points on a circle of radius 4 units then, abcd =

(A) 4 (B) 1/4 (C) 1 (D) 16

7. What is the length of shortest path by which one can go from (–2, 0) to (2, 0) without entering the interior of circle, $x^2 + y^2 = 1$?

(A) $2\sqrt{3}$ (B) $\sqrt{3}+\frac{2\pi}{3}$ (C) $2\sqrt{3}+\frac{\pi}{3}$ (D) none of these

8. The distance between the chords of contact of tangents to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ from the origin and from the point (g,f) is -

(A) $\sqrt{g^2+f^2}$ (B) $\frac{\sqrt{g^2+f^2-c}}{2}$ (C) $\frac{g^2+f^2-c}{2\sqrt{g^2+f^2}}$ (D) $\frac{\sqrt{g^2+f^2+c}}{2\sqrt{g^2+f^2}}$

***9.** AB is a diameter of a circle. CD is a chord parallel to AB and 2CD = AB. The tangent at B meets the line AC produced at E then AE is equal to -

(A) AB (B) $\sqrt{2}$AB (C) $2\sqrt{2}$AB (D) 2AB

***10.** The locus of the mid points of the chords of the circle $x^2 + y^2 - ax - by = 0$ which subtend a right angle at $\left(\frac{a}{2},\frac{b}{2}\right)$ is -

(A) $ax + by = 0$ (B) $ax + by = a^2 + b^2$

(C) $x^2+y^2-ax-by+\frac{a^2+b^2}{8}=0$ (D) $x^2+y^2-ax-by-\frac{a^2+b^2}{8}=0$

***11.** Number of points (x, y) having integral coordinates satisfying the condition $x^2 + y^2 < 25$ is -

(A) 69 (B) 80 (C) 81 (D) 77

12. The common chord of two intersecting circles C_1 and C_2 can be seen from their centres at the angles of 90° & 60° respectively. If the distance between their centres is equal to $\sqrt{3}$ + 1 then the radii of C_1 and C_2 are -

(A) $\sqrt{3}$ and 3 (B) $\sqrt{2}$ and $2\sqrt{2}$ (C) $\sqrt{2}$ and 2 (D) $2\sqrt{2}$ and 4

***13.** In a right triangle ABC, right angled at A, on the leg AC as diameter, a semicircle is described. The chord joining A with the point of intersection D of the hypotenuse and the semicircle, then the length AC equals to -

(A) $\frac{AB.AD}{\sqrt{AB^2+AD^2}}$ (B) $\frac{AB.AD}{AB+AD}$ (C) $\sqrt{AB.AD}$ (D) $\frac{AB.AD}{\sqrt{AB^2-AD^2}}$

14. A circle touches a straight line $\ell x + my + n = 0$ and cuts the circle $x^2 + y^2 = 9$ orthogonally. The locus of centres of such circles is -

(A) $(\ell x + my + n)^2 = (\ell^2 + m^2)(x^2 + y^2 - 9)$ (B) $(\ell x + my - n)^2 = (\ell^2 + m^2)(x^2 + y^2 - 9)$

(C) $(\ell x + my + n)^2 = (\ell^2 + m^2)(x^2 + y^2 + 9)$ (D) none of these

15.** Let PQ and RS be tangents at the extremities of the diameter PR of a circle of radius r. If PS and RQ intersect at a point X on the circumference of the circle then 2r equals ***[JEE 2001 (Screening) 1]

(A) $\sqrt{PQ\cdot RS}$ (B) $\frac{PQ+RS}{2}$ (C) $\frac{2PQ\cdot RS}{PQ+RS}$ (D) $\sqrt{\frac{(PQ)^2+(RS)^2}{2}}$

16. If the tangent at the point P on the circle $x^2 + y^2 + 6x + 6y = 2$ meets the straight line $5x - 2y + 6 = 0$ at a point Q on the y-axis, then the length of PQ is ***[JEE 2002 (Scr), 3]***

(A) 4 (B) $2\sqrt{5}$ (C) 5 (D) $3\sqrt{5}$

17.** If a > 2b > 0 then the positive value of m for which $y = mx - b\sqrt{1+m^2}$ is a common tangent to $x^2 + y^2 = b^2$ and $(x - a)^2 + y^2 = b^2$ is ***[JEE 2002 (Scr), 3]

(A) $\frac{2b}{\sqrt{a^2-4b^2}}$ (B) $\frac{\sqrt{a^2-4b^2}}{2b}$ (C) $\frac{2b}{a-2b}$ (D) $\frac{b}{a-2b}$

18. The radius of the circle, having centre at (2, 1), whose one of the chord is a diameter of the circle $x^2 + y^2 - 2x - 6y + 6 = 0$ ***[JEE 2004 (Scr)]***

(A) 1 (B) 2 (C) 3 (D) $\sqrt{3}$

19.** A circle is given by $x^2 + (y - 1)^2 = 1$, another circle C touches it externally and also the x-axis, then the locus of its centre is ***[JEE 2005 (Scr)]

(A) $\{(x, y) : x^2 = 4y\} \cup \{(x, y) : y \le 0\}$ (B) $\{(x, y) : x^2 + (y - 1)^2 = 4\} \cup \{x, y) : y \le 0\}$

(C) $\{(x, y) : x^2 = y\} \cup \{(0, y) : y \le 0\}$ (D) $\{(x, y) : x^2 = 4y\} \cup \{(0, y) : y \le 0\}$

20. The number of common tangents of the circles $x^2 + y^2 - 2x - 1 = 0$ and $x^2 + y^2 - 2y - 7 = 0$ -
(A) 1 (B) 3 (C) 2 (D) 4

21. If the circle $x^2 + y^2 = 9$ touches the circle $x^2 + y^2 + 6y + c = 0$, then c is equal to -
(A) –27 (B) 36 (C) –36 (D) 27

22. If the two circles, $x^2 + y^2 + 2g_1x + 2f_1y = 0$ and $x^2 + y^2 + 2g_2x + 2f_2y = 0$ touches each other, then -
(A) $f_1g_1 = f_2g_2$ (B) $\frac{f_1}{g_1} = \frac{f_2}{g_2}$ (C) $f_1f_2 = g_1g_2$ (D) none

23. The tangent from the point of intersection of the lines $2x - 3y + 1 = 0$ and $3x - 2y - 1 = 0$ to the circle $x^2 + y^2 + 2x - 4y = 0$ is -
(A) $x + 2y = 0,\ x - 2y + 1 = 0$ (B) $2x - y - 1 = 0$
(C) $y = x,\ y = 3x - 2$ (D) $2x + y + 1 = 0$

Multiple Choice Correct

***24.** If the circle $C_1 : x^2 + y^2 = 16$ intersects another circle C_2 of radius 5 in such a manner that the common chord is of maximum length 8 has a slope equal to $\frac{3}{4}$, then coordinates of centre of C_2 are -
(A) $\left(\frac{9}{5}, -\frac{12}{5}\right)$ (B) $\left(-\frac{9}{5}, \frac{12}{5}\right)$ (C) $\left(\frac{9}{5}, \frac{12}{5}\right)$ (D) $\left(-\frac{9}{5}, -\frac{12}{5}\right)$

25. For the circles $S_1 \equiv x^2 + y^2 - 4x - 6y - 12 = 0$ and $S_2 \equiv x^2 + y^2 + 6x + 4y - 12 = 0$ and the line $L \equiv x + y = 0$
(A) L is common tangent of S_1 and S_2
(B) L is common chord of S_1 and S_2
(C) L is radical axis of S_1 and S_2
(D) L is perpendicular to the line joining the centre of S_1 & S_2

***26.** Circles are drawn touching the co-ordinate axis and having radius 2, then -
(A) centre of these circles lie on the pair of lines $y^2 - x^2 = 0$
(B) centre of these circles lie only on the line $y = x$
(C) Area of the quadrilateral whose vertices are centre of these circles is 16 sq.unit
(D) Area of the circle touching these four circles internally is $4\pi(3 + 2\sqrt{2})$

***27.** Tangents are drawn to the circle $x^2 + y^2 = 1$ at the points where it is met by the circles, $x^2 + y^2 - (\lambda + 6)x + (8 - 2\lambda)y - 3 = 0$, λ being the variable. The locus of the point of intersection of these tangents is -
(A) $2x - y + 10 = 0$ (B) $x + 2y - 10 = 0$ (C) $x - 2y + 10 = 0$ (D) $2x + y - 10 = 0$

***28.** 3 circle of radii 1, 2 and 3 and centres at A, B and C respectively, touch each other. Another circle whose centre is P touches all these 3 circles externally and has radius r. Also $\angle PAB = \theta$ & $\angle PAC = \alpha$ -
(A) $\cos\theta = \frac{3 - r}{3(1 + r)}$ (B) $\cos\alpha = \frac{2 - r}{2(1 + r)}$ (C) $r = \frac{6}{23}$ (D) $r = \frac{6}{\sqrt{23}}$

29. (6,0) , (0,6) and (7,7) are the vertices of a triangle. The circle inscribed in the triangle has the equation -
(A) $x^2 + y^2 - 9x + 9y + 36 = 0$ (B) $x^2 + y^2 - 9x - 9y + 36 = 0$
(C) $x^2 + y^2 + 9x - 9y + 36 = 0$ (D) $x^2 + y^2 - 9x - 9y - 36 = 0$

***30.** Tangents are drawn to the circle $x^2 + y^2 = 50$ from a point 'P' lying on the x-axis. These tangents meet the y-axis at points 'P_1' and 'P_2'. Possible co-ordinates of 'P' so that area of triangle PP_1P_2 is minimum is/are -
(A) (10, 0) (B) $(10\sqrt{2}, 0)$ (C) (–10, 0) (D) $(-10\sqrt{2}, 0)$

Comprehension

Comprehension – 1

Let A ≡ (–3, 0) and B ≡ (3, 0) be two fixed points a-nd P moves on a plane such that PA = nPB (n > 0).

On the basis of above information, answer the following questions

31. If $n \neq 1$, then locus of a point P is -
(A) a straight line (B) a circle (C) a parabola (D) an ellipse

32. If n = 1, then the locus of a point P is -
(A) a straight line (B) a circle (C) a parabola (D) a hyperbola

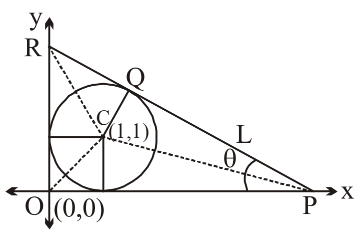
33. If 0 < n < 1, then -
(A) A lies inside the circle and B lies outside the circle
(B) A lies outside the circle and B lies inside the circle
(C) both A and B lies on the circle
(D) both A and B lies inside the circle

34. If n > 1, then -
(A) A lies inside the circle and B lies outside the circle
(B) A lies outside the circle and B lies inside the circle
(C) both A and B lies on the circle
(D) both A and B lies inside the circle

35. If locus of P is a circle, then the circle -
(A) passes through A and B
(B) never passes through A and B
(C) passes through A but does not pass through B
(D) passes through B but does not pass through A

*Comprehension – 2

In the diagram as shown, a circle is drawn with centre C(1, 1) and radius 1 and a line L. The line L is tangential to the circle at Q. Further L meet the y-axis at R and the x-axis at P in such a way that the angle OPQ equals θ where $0 < \theta < \frac{\pi}{2}$.



36. The coordinates of Q are
(A) $(1 + \cos\theta, 1 + \sin\theta)$ (B) $(\sin\theta, \cos\theta)$
(C) $(1 + \sin\theta, \cos\theta)$ (D) $(1 + \sin\theta, 1 + \cos\theta)$

37. Equation of the line PR is
(A) $x\cos\theta + y\sin\theta = \sin\theta + \cos\theta + 1$ (B) $x\sin\theta + y\cos\theta = \cos\theta + \sin\theta - 1$
(C) $x\sin\theta + y\cos\theta = \cos\theta + \sin\theta + 1$ (D) $x\tan\theta + y = 1 + \cot\left(\frac{\theta}{2}\right)$

38. If the area bounded by the circle, the x-axis and PQ is A(θ), then $A\left(\frac{\pi}{4}\right)$ equals

(A) $\sqrt{2}+1-\frac{3\pi}{8}$ (B) $\sqrt{2}-1+\frac{3\pi}{8}$ (C) $\sqrt{2}+1+\frac{\pi}{8}$ (D) $\sqrt{2}-1+\frac{\pi}{8}$

***Comprehension – 3**

Consider the circle S: $x^2 + y^2 - 4x - 1 = 0$ and the line L : $y = 3x - 1$. If the line L cuts the circle at A and B then

39. Length of the chord AB equal

(A) $2\sqrt{5}$ (B) $\sqrt{5}$ (C) $5\sqrt{2}$ (D) $\sqrt{10}$

40. The angle subtended by the chord AB in the minor arc of S is

(A) $\frac{3\pi}{4}$ (B) $\frac{5\pi}{6}$ (C) $\frac{2\pi}{3}$ (D) $\frac{\pi}{4}$

41. Acute angle between the line L and the circle S is

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{6}$

***Comprehension – 4**

Let A, B, C be three sets of real numbers (x, y) defined as

A : $\{(x, y): \quad y \ge 1\}$

B : $\{(x, y): \ x^2 + y^2 - 4x - 2y - 4 = 0\}$

C : $\{(x, y): \ x + y = \sqrt{2}\}$

42. Number of elements in the $A \cap B \cap C$ is

(A) 0 (B) 1 (C) 2 (D) infinite

43. $(x + 1)^2 + (y - 1)^2 + (x - 5)^2 + (y - 1)^2$ has the value equal to

(A) 16 (B) 25 (C) 36 (D) 49

44. If the locus of the point of intersection of the pair of perpendicular tangents to the circle B is the curve S then the area enclosed between B and S is

(A) 6π (B) 8π (C) 9π (D) 18π

***Comprehension – 5**

Let the variable line 'L' represented by equation $x + ay + 2 = 0$, $a \in R$ be tangents to varying circle 'S' whose equation is represented by $x^2 + y^2 = r^2$, $r \in R$.

45. The maximum possible area of circle S is $k\pi$, then k =

(A) 2 (B) 4 (C) 6 (D) 8

46. If area of circle circumscribing the triangle formed by two lines L and the line joining their points of contact is $k\pi$, then k =

(A) 1 (B) 2 (C) 4 (D) 8

47. Let angle between two lines 'L' lies in the range $\left[\frac{\pi}{3}, \frac{2\pi}{3}\right]$ then radius of circle 'S' lies in the range [a, b], then the value of $a^4 + b^4 =$

(A) 3 (B) 5 (C) 8 (D) 10

***Comprehension – 6**

Consider a circle $x^2 + y^2 = 4$ and a point P(4, 2). θ denotes the angle enclosed by the tangents from P on the circle and A, B are the points of contact of the tangents from P on the circle.

48. The value of θ lies in the interval

(A) (0, 15°) (B) (15°, 30°) (C) 30°, 45°) (D) (45°, 60°)

49. The intercept made by a tangent on the x-axis is

(A) 9/4 (B) 10/4 (C) 11/4 (D) 12/4

50. Locus of the middle points of the portion of the tangent to the circle terminated by the coordinate axes is

(A) $x^{-2} + y^{-2} = 1^{-2}$ (B) $x^{-2} + y^{-2} = 2^{-2}$ (C) $x^{-2} + y^{-2} = 3^{-2}$ (D) $x^{-2} - y^{-2} = 4^{-2}$

Subjetive Questions

51. Find the equation of the circle which passes through the points of intersection of circles $x^2 + y^2 - 2x - 6y + 6=0$ and $x^2 + y^2 + 2x - 6y + 6 = 0$ and intersects the circle $x^2 + y^2 + 4x + 6y + 4 =0$ orthogonally. ***[REE 2001 (Mains), 3]***

52.** Tangents TP and TQ are drawn from a point T to the circle $x^2 + y^2 = a^2$. If the point T lies on the line $px + qy = r$, find the locus of centre of the circumcircle of triangle TPQ. ***[REE 2001 (Mains), 5]

53.** Line $2x + 3y + 1 = 0$ is a tangent to a circle at $(1, -1)$. This circle is orthogonal to a circle which is drawn having diameter as a line segment with end points $(0, -1)$ and $(-2, 3)$. Find equation of circle. ***[JEE 2004, 4]

54. If the line $x \sin\alpha - y + a \sec\alpha = 0$ touches the circle with radius 'a' and centre at the origin then find the most general values of 'α' and sum of the values of 'α' lying in $[0, 100\pi]$.

***55.** Let a circle be given by $2x(x - a) + y(2y - b) = 0$, $(a \neq 0, b \neq 0)$. Find the condition on a & b if two chords, each bisected by the x-axis, can be drawn to the circle from the point $\left(a, \frac{b}{2}\right)$.

56. Find the equation of a line with gradient 1 such that the two circles $x^2 + y^2 = 4$ and $x^2+y^2-10x-14y+65=0$ intercept equal length on it.

***57.** Find the equations of straight lines which pass through the intersection of the lines $x - 2y - 5 = 0$, $7x + y = 50$ & divide the circumference of the circle $x^2 + y^2 = 100$ into two arcs whose lengths are in the ratio $2 : 1$.

58. Find the locus of the middle points of portions of the tangents to the circle $x^2 + y^2 = a^2$ terminated by the coordinate axes.

***59.** Show that the equation of a straight line meeting the circle $x^2 + y^2 = a^2$ in two points at equal distances 'd' from a point (x_1, y_1) on its circumference is $xx_1 + yy_1 - a^2 + \frac{d^2}{2} = 0$.

***60.** A point moving around circle $(x + 4)^2 + (y + 2)^2 = 25$ with centre C broke away from it either at the point A or point B on the circle and move along a tangent to the circle passing through the point D(3, –3).
Find the following :
(a) Equation of the tangents at A and B.
(b) Coordinates of the points A and B.
(c) Angle ADB and the maximum and minimum distances of the point D from the circle.
(d) Area of quadrilateral ADBC and the ΔDAB.
(e) Equation of the circle circumscribing the ΔDAB and also the intercepts made by the this circle on the coordinates axes.

61. Show that the equation $x^2 + y^2 - 2x - 2\lambda y - 8 = 0$ represents, for different values of λ, a system of circles passing through two fixed points A, B on the x–axis, and find the equation of that circle of the system the tangents to which at A & B meet on the line $x + 2y + 5 = 0$.

***62.** Through a fixed point (h, k) secants are drawn to the circle $x^2 + y^2 = r^2$. Show that the locus of the mid-points of the secants intercepted by the circle is $x^2 + y^2 = hx + ky$.

63. A triangle has two of its sides along the coordinate axes, its third side touches the circle $x^2 + y^2 - 2ax - 2ay + a^2 = 0$. Prove that the locus of the circumcentre of the triangle is : $a^2 - 2a(x + y) + 2xy = 0$.

64. Find the equations to the four common tangents to the circles $x^2 + y^2 = 25$ and $(x - 12)^2 + y^2 = 9$.

65. Show that the locus of the centres of a circle which cuts two given circles orthogonally is a straight line & hence deduce the locus of the centre of the circles which cut the circles $x^2 + y^2 + 4x - 6y + 9 = 0$ & $x^2 + y^2 - 5x + 4y + 2 = 0$ orthogonally.

***66.** Find the equation of the circle inscribed in a triangle formed by the lines $3x + 4y = 12$; $5x + 12y = 4$ & $8y = 15x + 10$ without finding the vertices of the triangle.

***67.** Find the intervals of values of 'a' for which the line $y + x = 0$ bisects two chords drawn from a point $\left(\frac{1+\sqrt{2}a}{2}, \frac{1-\sqrt{2}a}{2}\right)$ to the circle $2x^2 + 2y^2 - \left(1+\sqrt{2}a\right)x - \left(1-\sqrt{2}a\right)y = 0$. **[JEE 1996]**

***68.** Find the equations of the circles passing through $(-4, 3)$ and touching the lines $x + y = 2$ and $x - y = 2$.

***69.** Find the radius of the circle

$$(x\cos\alpha + y\sin\alpha - a)^2 + (x\sin\alpha - y\cos\alpha - b)^2 = k^2$$

and if α varies, find the locus of its centre.

70.** Let $2x^2 + y^2 - 3xy = 0$ be the equation of a pair of tangents drawn from the origin 'O' to a circle of radius 3 with centre in the first quadrant. If A is one of the points of contact, find the length of OA. ***[JEE 2001 (Mains), 5]

***71.** Consider a curve $ax^2 + 2hxy + by^2 = 1$ and a point P not on the curve. A line is drawn from the point P intersects the curve at points Q & R. If the product $PQ \cdot PR$ is independent of the slope of the line, then show that the curve is a circle.

72. Find the equation of a circle which is co-axial with circles $2x^2 + 2y^2 - 2x + 6y - 3 = 0$ & $x^2+y^2+4x + 2y+1 = 0$. It is given that the centre of the circle to be determined lies on the radical axis of these two circles.

***73.** If $3\ell^2 + 6\ell + 1 - 6m^2 = 0$, then find the equation of the circle for which $\ell x + my + 1 = 0$ is a tangent.

74. Circle are drawn which are orthogonal to both the circles $S \equiv x^2 + y^2 - 16 = 0$ and $S' \equiv x^2 + y^2 - 8x - 12y + 16 = 0$. If tangents are drawn from the centre of the variable circles to S. Then find the locus of the mid point of the chord of contact of these tangents.

***75.** Show that the locus of the point the tangents from which to the circle $x^2 + y^2 - a^2 = 0$ include a constant angle α is $(x^2 + y^2 - 2a^2)^2 \tan^2\alpha = 4a^2(x^2 + y^2 - a^2)$.

76. Find the locus of the mid point of the chord of a circle $x^2 + y^2 = 4$ such that the segment intercepted by the chord on the curve $x^2 - 2x - 2y = 0$ subtends a right angle at the origin.

***77.** Prove that the length of the common chord of the two circles $x^2 + y^2 = a^2$ and $(x - c)^2 + y^2 = b^2$ is $\frac{1}{c}\sqrt{(a+b+c)(a-b+c)(a+b-c)(-a+b+c)}$, where $a, b, c > 0$.

78. Find the equation of the circles passing through the point $(2, 8)$, touching the lines $4x - 3y - 24 = 0$ & $4x + 3y - 42 = 0$ & having x coordinate of the centre of the circle less than or equal to 8.

***79.** Lines $5x + 12y - 10 = 0$ & $5x - 12y - 40 = 0$ touch a circle C_1 of diameter 6. If the centre of C_1 lies in the first quadrant, find the equation of the circle C_2 which is concentric with C_1 & cuts intercepts of length 8 on these lines.

***80.** A circle touches the line $y = x$ at a point P such that $OP = 4\sqrt{2}$, where O is the origin. The circle contains the point $(-10,2)$ in its interior and the length of its chord on the line $x + y = 0$ is $6\sqrt{2}$. Determine the equation of the circle. **[JEE 1990]**

81. $A(-a, 0)$; $B(a, 0)$ are fixed points. C is a point which divides internally AB in a constant ratio $\tan\alpha$. If AC & CB subtend equal angles at P, prove that the equation of the locus of P is $x^2 + y^2 + 2ax\sec 2\alpha + a^2 = 0$.

82. A variable circle passes through the point A (a, b) & touches the x-axis ; show that the locus of the other end of the diameter through A is $(x - a)^2 = 4by$.

83. The foot of the perpendicular from the origin to a variable tangent of the circle $x^2 + y^2 - 2x = 0$ is N. Find the equation of the locus of N.

***84.** The line $\ell x + my + n = 0$ intersects the curve $ax^2 + 2hxy + by^2 = 1$ at the point P and Q. The circle on PQ as diameter passes through the origin. Prove that $n^2(a + b) = \ell^2 + m^2$.

***85.** Find the equation of the circle which cuts each of the circles $x^2 + y^2 = 4$, $x^2 + y^2 - 6x - 8y + 10 = 0$ & $x^2 + y^2 + 2x - 4y - 2 = 0$ at the extremities of a diameter.

ANSWERS

- ***Assertion–Reason***

1. A **2.** A **3.** B **4.** C

- ***Single Choice Correct***

5.	C	**6.**	C	**7.**	C	**8.**	C	**9.**	D
10.	C	**11.**	A	**12.**	C	**13.**	D	**14.**	A
15.	A	**16.**	C	**17.**	A	**18.**	C	**19.**	D
20.	A	**21.**	A	**22.**	B	**23.**	B		

- ***Multiple Choice Correct***

24.	(AB)	**25.**	(BCD)	**26.**	(ACD)	**27.**	(A)	**28.**	(ABC)
29.	(B)	**30.**	(AC)						

- ***Comprehension***

Comprehension – 1	**31.**	B	**32.**	A	**33.**	A	**34.**	B	**35.**	B
Comprehension – 2	**36.**	D	**37.**	C	**38.**	A				
Comprehension – 3	**39.**	D	**40.**	A	**41.**	C				
Comprehension – 4	**42.**	B	**43.**	C	**44.**	C				
Comprehension – 5	**45.**	B	**46.**	A	**47.**	D				
Comprehension – 6	**48.**	D	**49.**	A	**50.**	A				

- ***Subjective Questions***

51. $x^2 + y^2 + 14x - 6y + 6 = 0$; **52.** $2px + 2qy = r$

53. $2x^2 + 2y^2 - 10x - 5y + 1 = 0$ **54.** $\alpha = n\pi, 5050\pi$ **55.** $a^2 > 2b^2$

56. $2x - 2y - 3 = 0$ **57.** $4x - 3y - 25 = 0$ or $3x + 4y - 25 = 0$

58. $a^2(x^2 + y^2) = 4x^2y^2$

60. (a) $3x - 4y = 21$; $4x + 3y = 3$; (b) A(0, 1) and B(–1, –6); (c) $90°$, $5(\sqrt{2} \pm 1)$ units;

(d) 12.5 sq. units; (e) $x^2 + y^2 + x + 5y - 6 = 0$, x intercept 5; y intercept 7

61. $x^2 + y^2 - 2x - 6y - 8 = 0$

64. $2x - \sqrt{5}\,y - 15 = 0$, $2x + \sqrt{5}\,y - 15 = 0$, $x - \sqrt{35}\,y - 30 = 0$, $x + \sqrt{35}\,y - 30 = 0$

65. $9x - 10y + 7 = 0$ **66.** $x^2 + y^2 - 2x - 2y + 1 = 0$ **67.** $a \in (-\infty, -2) \cup (2, \infty)$

68. $x^2 + y^2 + 2(10 \pm \sqrt{54})x + 55 \pm 8\sqrt{54} = 0$ **69.** radius = k, Locus : $x^2 + y^2 = a^2 + b^2$.

70. $OA = 3(3 + \sqrt{10})$ **72.** $4x^2 + 4y^2 + 6x + 10y - 1 = 0$ **73.** $x^2 + y^2 - 6x + 3 = 0$

74. $x^2 + y^2 - 4x - 6y = 0$ **75.** $x^2 + y^2 - 2x - 2y = 0$

78. centre (2,3), r = 5; centre $\left(-\frac{182}{9}, 3\right)$, $r = \frac{205}{9}$ **79.** $x^2 + y^2 - 10x - 4y + 4 = 0$

80. $(x - 9)^2 + (y - 1)^2 = 50$ **83.** $(x^2 + y^2 - x)^2 = x^2 + y^2$

85. $x^2 + y^2 - 4x - 6y - 4 = 0$

PARABOLA

Assertion & Reason

These questions contain, Statement-I (assertion) and Statement-II (reason).

(A) Statement-I is true, Statement-II is true ; Statement-II is correct explanation for Statement-I.

(B) Statement-I is true, Statement-II is true ; Statement-II is NOT a correct explanation for Statement-I.

(C) Statement-I is true, Statement-II is false.

(D) Statement-I is false, Statement-II is true.

1. ***Statement-I –*** If normal at the ends of double ordinate x = 4 of parabola $y^2 = 4x$ meet the curve again at P and P' respectively, then PP' = 12 unit.

Because

Statement-II – If normal at t_1 of $y^2 = 4ax$ meets the parabola again at t_2, then $t_1^2 = 2 + t_1 t_2$.

2. ***Statement-I –*** The lines from the vertex to the two extremities of a focal chord of the parabola $y^2 = 4ax$ are at an angle of $\frac{\pi}{2}$.

Because

Statement-II – If extremities of focal chord of parabola are $(at_1^2, 2at_1)$ and $(at_2^2, 2at_2)$, then $t_1t_2 = -1$

3. ***Statement-I –*** If P_1Q_1 and P_2Q_2 are two focal chords of the parabola $y^2 = 4ax$, then the locus of point of intersection of chords P_1P_2 and Q_1Q_2 is directrix of the parabola. Here P_1P_2 and Q_1Q_2 are not parallel.

Because

Statement-II – The locus of point of intersection of perpendicular tangents of parabola is directrix of parabola.

Single Choice Correct

4. From the focus of the parabola $y^2 = 8x$ as centre, a circle is described so that a common chord of the curves is equidistant from the vertex and focus of the parabola. The equation of the circle is -

(A) $(x - 2)^2 + y^2 = 3$ (B) $(x - 2)^2 + y^2 = 9$ (C) $(x + 2)^2 + y^2 = 9$ (D) $x^2 + y^2 - 4x = 0$

5. The triangle PQR of area 'A' is inscribed in the parabola $y^2 = 4ax$ such that the vertex P lies at the vertex of the parabola and the base QR is a focal chord. The modulus of the difference of the ordinates of the points Q and R is

(A) $\frac{A}{2a}$ (B) $\frac{A}{a}$ (C) $\frac{2A}{a}$ (D) $\frac{4A}{a}$

6. Point P lies on $y^2 = 4ax$ & N is foot of perpendicular from P on its axis. A straight line is drawn parallel to the axis to bisect NP and meets the curve in Q. NQ meets the tangent at the vertex in a point T such that AT = k NP, then the value of k is : (where A is the vertex)

(A) 3/2 (B) 2/3 (C) 1 (D) none

7. T is a point on the tangent to a parabola $y^2 = 4ax$ at its point P. TL and TN are the perpendiculars on the focal radius SP and the directrix of the parabola respectively. Then -

(A) SL = 2 (TN) (B) 3 (SL) = 2 (TN) (C) SL = TN (D) 2 (SL) = 3 (TN)

8. If the tangent at the point P (x_1,y_1) to the parabola $y^2 = 4ax$ meets the parabola $y^2 = 4a(x + b)$ at Q & R, then the mid point of QR is -

(A) $(x_1 + b, y_1 + b)$ (B) $(x_1 - b, y_1 - b)$ (C) (x_1, y_1) (D) $(x_1 + b, y_1)$

Multiple Choice Correct

9. The tangent and normal at P (t), for all real positive t, to the parabola $y^2 = 4ax$ meet the axis of the parabola in T and G respectively, then the angle at which the tangent at P to the parabola is inclined to the tangent at P to the circle through the points P, T and G is -

(A) $\cot^{-1}t$ (B) $\cot^{-1}t^2$ (C) $\tan^{-1}t$ (D) $\sin^{-1}\left(\frac{t}{\sqrt{1+t^2}}\right)$

10. Through the vertex O of the parabola, $y^2 = 4ax$ two chords OP and OQ are drawn and the circles on OP and OQ as diameter intersect in R. If θ_1, θ_2 and ϕ are the angles made with the axis by the tangent at P and Q on the parabola and by OR then the value of $\cot\theta_1 + \cot\theta_2 =$

(A) $-2\tan\phi$ (B) $-2\tan(\pi - \phi)$ (C) 0 (D) $2\cot\phi$

11. Tangent to the parabola $y^2 = 4ax$ at point P meets the tangent at vertex A, at point B and the axis of parabola at T. Q is any point on this tangent and N is the foot of perpendicular from Q on SP, where S is focus. M is the foot of perpendicular from Q on the directrix then -

(A) B bisects PT (B) B trisects PT (C) QM = SN (D) QM = 2SN

12. If two distinct chords of a parabola $x^2 = 4ay$ passing through (2a, a) are bisected on the line $x + y = 1$, then length of latus rectum can be -

(A) 2 (B) 1 (C) 4 (D) 5

13. For parabola $y^2 = 4ax$ consider three points A, B, C lying on it. If the centroid of ΔABC is (h_1, k_1) & centroid of triangle formed by the point of intersection of tangents at A, B, C has coordinates (h_2, k_2), then which of the following is always true -

(A) $2k_1 = k_2$ (B) $k_1 = k_2$ (C) $k_1^2 = \frac{4a}{3}(h_1 + 2h_2)$ (D) $k_1^2 = \frac{4a}{3}(2h_1 + h_2)$

Match the Column

14.

	Column-I		Column-II
(A)	The normal chord at a point t on the parabola $y^2 = 4x$ subtends a right angle at the vertex, then t^2 is	(p)	4
(B)	The area of the triangle inscribed in the curve $y^2 = 4x$. If the parameter of vertices are 1, 2 and 4 is	(q)	2
(C)	The number of distinct normal possible from $\left(\frac{11}{4}, \frac{1}{4}\right)$ to the parabola $y^2 = 4x$ is	(r)	3
(D)	The normal at (a, 2a) on $y^2 = 4ax$ meets the curve again at $(at^2, 2at)$, then the value of $\lvert t - 1\rvert$ is	(s)	6

Comprehension

Observe the following facts for a parabola :

(i) Axis of the parabola is the only line which can be the perpendicular bisector of the two chords of the parabola.

(ii) If AB and CD are two parallel chords of the parabola and the normals at A and B intersect at P and the normals at C and D intersect at Q, then PQ is a normal to the parabola.

Let a parabola is passing through (0, 1), (–1, 3), (3, 3) & (2, 1)

On the basis of above information, answer the following questions

15. The vertex of the parabola is -

(A) $\left(1, \frac{1}{3}\right)$ (B) $\left(\frac{1}{3}, 1\right)$ (C) (1, 3) (D) (3, 1)

16. The directrix of the parabola is -

(A) $y - \frac{1}{24} = 0$ (B) $y + \frac{1}{2} = 0$ (C) $y + \frac{1}{24} = 0$ (D) $y + \frac{1}{12} = 0$

17. For the parabola $y^2 = 4x$, AB and CD are any two parallel chords having slope 1. C_1 is a circle passing through O, A and B and C_2 is a circle passing through O, C and D, where O is origin. C_1 and C_2 intersect at -

(A) (4, –4) (B) (–4, 4) (C) (4, 4) (D) (–4, –4)

Subjetive Questions

18. If the end points $P(t_1)$ and $Q(t_2)$ of a chord of a parabola $y^2 = 4ax$ satisfy the relation $t_1t_2 = k$ (constant) then prove that the chord always passes through a fixed point. Find that point also ?

19. O is the vertex of the parabola $y^2 = 4ax$ & L is the upper end of the latus rectum. If LH is drawn perpendicular to OL meeting OX in H , prove that the length of the double ordinate through H is $4a\sqrt{5}$.

20. Two perpendicular chords are drawn from the origin 'O' to the parabola $y = x^2$, which meet the parabola at P and Q. Rectangle POQR is completed. Find the locus of vertex R.

21. Prove that the locus of the middle points of all tangents drawn from points on the directrix to the parabola $y^2 = 4ax$ is $y^2(2x + a) = a(3x + a)^2$.

22. Two tangents to the parabola $y^2 = 8x$ meet the tangent at its vertex in the points P & Q. If PQ = 4 units, prove that the locus of the point of the intersection of the two tangents is $y^2 = 8 (x + 2)$.

23. Show that the normals at the points (4a , 4a) & at the upper end of the latus rectum of the parabola $y^2 = 4ax$ intersect on the same parabola.

24. Show that the locus of a point, such that two of the three normals drawn from it to the parabola $y^2 = 4ax$ are perpendicular is $y^2 = a(x - 3a)$.

25. Prove that the locus of the middle point of portion of a normal to $y^2 = 4ax$ intercepted between the curve & the axis is another parabola. Find the vertex & the latus rectum of the second parabola.

26. P & Q are the points of contact of the tangents drawn from the point T to the parabola $y^2 = 4ax$. If PQ be the normal to the parabola at P, prove that TP is bisected by the directrix.

27. The normal at a point P to the parabola $y^2 = 4ax$ meets its axis at G. Q is another point on the parabola such that QG is perpendicular to the axis of the parabola. Prove that $QG^2 - PG^2$ = constant.

28. If from the vertex of a parabola a pair of chords be drawn at right angles to one another, & with these chords as adjacent sides a rectangle be constructed , then find the locus of the outer corner of the rectangle.

29. Two perpendicular straight lines through the focus of the parabola $y^2 = 4ax$ meet its directrix in T & T′ respectively. Show that the tangents to the parabola parallel to the perpendicular lines intersect in the mid point of T T'.

30. Find the condition on 'a' & 'b' so that the two tangents drawn to the parabola $y^2 = 4ax$ from a point are normals to the parabola $x^2 = 4by$.

31. TP & TQ are tangents to the parabola and the normals at P & Q meet at a point R on the curve. Prove that the centre of the circle circumscribing the triangle TPQ lies on the parabola $2y^2 = a(x - a)$.

32. Let S is the focus of the parabola $y^2 = 4ax$ and X the foot of the directrix, PP' is a double ordinate of the curve and PX meets the curve again in Q. Prove that P'Q passes through focus.

33. Prove that on the axis of any parabola $y^2 = 4ax$ there is a certain point K which has the property that , if a chord PQ of the parabola be drawn through it , then $\frac{1}{(PK)^2} + \frac{1}{(QK)^2}$ is same for all positions of the chord. Find also the coordinates of the point K.

34. If (x_1, y_1), (x_2, y_2) and (x_3, y_3) be three points on the parabola $y^2 = 4ax$ and the normals at these points meet in a point, then prove that $\frac{x_1 - x_2}{y_3} + \frac{x_2 - x_3}{y_1} + \frac{x_3 - x_1}{y_2} = 0$

35. A variable chord joining points $P(t_1)$ and $Q(t_2)$ of the parabola $y^2 = 4ax$ subtends a right angle at a fixed point t_0 of the curve. Show that it passes through a fixed point. Also find the co-ordinates of the fixed point.

36. Show that a circle circumscribing the triangle formed by three co-normal points passes through the vertex of the parabola and its equation is, $2(x^2+y^2) - 2(h+2a)x - ky = 0$, where (h, k) is the point from where three concurrent normals are drawn.

37. A ray of light is coming along the line $y = b$ from the positive direction of x-axis & strikes a concave mirror whose intersection with the xy-plane is a parabola $y^2 = 4ax$. Find the equation of the reflected ray & show that it passes through the focus of the parabola. Both a & b are positive. ***[REE 95]***

ANSWERS

- ***Assertion–Reason***

1. C **2.** D **3.** B

- ***Single Choice Correct***

4. B **5.** C **6.** B **7.** C **8.** C

- ***Multiple Choice Correct***

9. CD **10.** A **11.** AC **12.** AB **13.** BC

- ***Match the Column***

14. (A) $\rightarrow$ (q); (B) $\rightarrow$ (s); (C) $\rightarrow$ (q); (D) $\rightarrow$ (p)

- ***Comprehension***

15. A **16.** C **17.** A

- ***Subjective Questions***

18. $(-ak, 0)$ **20.** $x^2 = y - 2$ **25.** $(a, 0)$; a **28.** $y^2 = 4a(x - 8a)$ **30.** $a^2 > 8b^2$

33. $(2a, 0)$ **35.** $[a(t_0^2 + 4), -2at_0]$ **37.** $4abx + (4a^2 - b^2)y - 4a^2b = 0$

ELLIPSE

Fill in the Blanks

1. The co-ordinates of the mid - point of the variable chord $y = \frac{1}{2}(x + c)$ of the ellipse $4x^2 + 9y^2 = 36$ are ________

2. A triangle ABC right angled at 'A' moves so that it always circumscribes the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. The locus of the point 'A' is ________.

3. Atmost ________ normals can be drawn from a point, to an ellipse.

4. Atmost ________ tangents can be drawn from a point, to an ellipse.

Assertion & Reason

These questions contain, Statement-I (assertion) and Statement-II (reason).

(A) Statement-I is true, Statement-II is true ; Statement-II is correct explanation for Statement-I.

(B) Statement-I is true, Statement-II is true ; Statement-II is NOT a correct explanation for Statement-I.

(C) Statement-I is true, Statement-II is false.

(D) Statement-I is false, Statement-II is true.

5. **Statement-I –** Tangent drawn at a point $P\left(\frac{4\sqrt{5}}{3}, 2\right)$ on the ellipse $9x^2 + 16y^2 = 144$ intersects a straight line $x = \frac{16}{\sqrt{7}}$ at M, then PM subtends a right angle at $(-\sqrt{7}, 0)$

Because

Statement-II – The portion of the tangent to an ellipse between the point of contact and the directrix subtends a right angle at the corresponding focus.

6. ***Statement-I –*** Feet of perpendicular drawn from foci of an ellipse $4x^2 + y^2 = 16$ on the line $2\sqrt{3}x + y = 8$ lie on a circle $x^2 + y^2 = 16$.

Because

Statement-II – If perpendicular are drawn from foci of an ellipse to its any tangent then feet of these perpendicular lie on director circle of the ellipse.

7. ***Statement-I –*** Any chord of the ellipse $x^2 + y^2 + xy = 1$ through (0, 0) is bisected at (0, 0)

Because

Statement-II – The centre of an ellipse is a point through which every chord is bisected.

8. ***Statement-I –*** If $P\left(\frac{3\sqrt{3}}{2}, 1\right)$ is a point on the ellipse $4x^2 + 9y^2 = 36$. Circle drawn AP as diameter touches another circle $x^2 + y^2 = 9$, where $A \equiv (-\sqrt{5}, 0)$

Because

Statement-II – Circle drawn with focal radius as diameter touches the auxilliary circle.

Single Choice Correct

9. PQ is a double ordinate of the ellipse $x^2 + 9y^2 = 9$, the normal at P meets the diameter through Q at R, then the locus of the mid point of PR is -

(A) a circle (B) a parabola (C) an ellipse (D) a hyperbola

Multiple Choice Correct

10. Q is a point on the auxiliary circle of an ellipse. P is the corresponding point on ellipse. N is the foot of perpendicular from focus S, to the tangent of auxiliary circle at Q. Then -

(A) SP = SN (B) SP = PQ (C) PN = SP (D) NQ = SP

11. The line, $lx + my + n = 0$ will cut the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ in points whose eccentric angles differ by $\pi/2$ if -

(A) $x^2 l^2 + b^2 n^2 = 2m^2$ (B) $a^2 m^2 + b^2 l = 2n^2$

(C) $a^2 l^2 + b^2 m^2 = 2n^2$ (D) $a^2 n^2 + b^2 m^2 = 2l$

12. The normal at a variable point P on an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ of eccentricity e meets the axes of the ellipse in Q and R then the locus of the mid-point of QR is a conic with an eccentricity e' such that -

(A) e' is independent of e (B) e' = 1

(C) e' = e (D) e' = 1/e

13. The length of the normal (terminated by the major axis) at a point of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is -

(A) $\frac{b}{a}(r + r_1)$ (B) $\frac{b}{a}|r - r_1|$ (C) $\frac{b}{a}\sqrt{rr_1}$ (D) independent of r, r_1

where r and r_1 are the focal distance of the point.

14. If P is a point of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, whose foci are S and S'. Let $\angle PSS' = \alpha$ and $\angle PS'S = \beta$, then -

(A) PS + PS' = 2a, if a > b

(B) PS + PS' = 2b, if a < b

(C) $\tan\frac{\alpha}{2}\tan\frac{\beta}{2}=\frac{1-e}{1+e}$

(D) $\tan\frac{\alpha}{2}\tan\frac{\beta}{2}=\frac{\sqrt{a^2-b^2}}{b^2}[a-\sqrt{a^2-b^2}]$ when a >b

Subjetive Questions

15. Find the condition so that the line $px + qy = r$ intersects the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ in points whose eccentric angles differ by $\frac{\pi}{4}$.

16. The tangent at the point α on a standard ellipse meets the auxiliary circle in two points which subtends a right angle at the centre. Show that the eccentricity of the ellipse is $(1 + \sin^2\alpha)^{-1/2}$.

17. ABC is an isosceles triangle with its base BC twice its altitude. A point P moves within the triangle such that the square of its distance from BC is half the area of rectangle contained by its distances from the two sides . Show that the locus of P is an ellipse with eccentricity $\sqrt{\frac{2}{3}}$ passing through B & C.

18. 'O' is the origin & also the centre of two concentric circles having radii of the inner & the outer circle as 'a' & 'b' respectively . A line OPQ is drawn to cut the inner circle in P & the outer circle in Q. PR is drawn parallel to the y-axis & QR is drawn parallel to the x– axis . Prove that the locus of R is an ellipse touching the two circles. If the foci of this ellipse lie on the inner circle , find the ratio of inner : outer radii & find also the eccentricity of the ellipse.

19. The tangent and normal to the ellipse $x^2 + 4y^2 = 4$ at a point $P(\theta)$ on it meet the major axis in Q and R respectively. If QR = 2, show that the eccentric angle θ of P is given by $\cos\theta = \pm (2/3)$.

20. If the normal at a point P on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ of semi axes a , b & centre C cuts the major & minor axes at G & g , show that $a^2.(CG)^2 + b^2.(Cg)^2 = (a^2 - b^2)^2$. Also prove that $CG = e^2CN$, where PN is the ordinate of P. (N is foot of perpendicular from P on its major axis.)

21. The tangent at any point P of a circle $x^2 + y^2 = a^2$ meets the tangent at a fixed point A (a, 0) in T and T is joined to B, the other end of the diameter through A . Prove that the locus of the intersection of AP and BT is an ellipse whose eccentricity is $\frac{1}{\sqrt{2}}$.

22. The tangent at $P\left(4\cos\theta, \frac{16}{\sqrt{11}}\sin\theta\right)$ to the ellipse $16x^2 + 11y^2 = 256$ is also a tangent to the circle $x^2 + y^2 - 2x - 15 = 0$. Find θ. Find also the equation to the common tangent.

23. Common tangents are drawn to the parabola $y^2 = 4x$ & the ellipse $3x^2 + 8y^2 = 48$ touching the parabola at A & B and the ellipse at C & D . Find the area of the quadrilateral .

24. Find the equation of the largest circle with centre (1, 0) that can be inscribed in the ellipse $x^2 + 4y^2 = 16$.

25. Prove that the length of the focal chord of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ which is inclined to the major axis at angle θ is $\frac{2ab^2}{a^2\sin^2\theta + b^2\cos^2\theta}$.

26. The tangent at a point P on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ intersects the major axis in T & N is the foot of the perpendicular from P to the same axis. Show that the circle on NT as diameter intersects the auxiliary circle orthogonally.

27. The tangents from (x_1, y_1) to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ intersect at right angles . Show that the normals at the points of contact meet on the line $\frac{y}{y_1}=\frac{x}{x_1}$.

28. If the normals at the points P, Q, R with eccentric angles α, β, γ on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ are concurrent, then show that $\begin{vmatrix} \sin\alpha & \cos\alpha & \sin 2\alpha \\ \sin\beta & \cos\beta & \sin 2\beta \\ \sin\gamma & \cos\gamma & \sin 2\gamma \end{vmatrix} = 0$.

29. Let d be the perpendicular distance from the centre of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ to the tangent drawn at a point P on the ellipse. If F_1 & F_2 are the two foci of the ellipse, then show that $(PF_1 - PF_2)^2 = 4a^2\left[1-\frac{b^2}{d^2}\right]$.

30. Consider the family of circles, $x^2 + y^2 = r^2$, $2 < r < 5$. If in the first quadrant, the common tangent to a circle of the family and the ellipse $4x^2 + 25y^2 = 100$ meets the co-ordinate axes at A & B , then find the equation of the locus of the mid–point of AB. ***[JEE 99]***

ANSWERS

- ***Fill in the Blanks***

1. $-\frac{9}{25}c, \frac{8}{25}c$ **2.** $x^2 + y^2 = a^2 + b^2$, director circle **3.** 4 **4.** 2

- ***Assertion–Reason***

5. D **6.** C **7.** A **8.** A

- ***Single Choice Correct***

9. C

- ***Multiple Choice Correct***

10. A **11.** C **12.** C **13.** C **14.** ABC

- ***Subjective Questions***

16. $a^2p^2 + b^2q^2 = r^2\sec^2\frac{\pi}{8} = (4 - 2\sqrt{2})\,r^2$ **18.** $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$

22. $\theta = \frac{\pi}{3}$ or $\frac{5\pi}{3}$; $4x \pm \sqrt{33}\,y = 32$ **23.** $55\sqrt{2}$ sq. units

24. $(x - 1)^2 + y^2 = \frac{11}{3}$ **30.** $25y^2 + 4x^2 = 4x^2y^2$

Assertion & Reason

These questions contain, Statement-I (assertion) and Statement-II (reason).

(A) Statement-I is true, Statement-II is true ; Statement-II is correct explanation for Statement-I.

(B) Statement-I is true, Statement-II is true ; Statement-II is NOT a correct explanation for Statement-I.

(C) Statement-I is true, Statement-II is false.

(D) Statement-I is false, Statement-II is true.

1. ***Statement-I –*** Ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ and $12x^2 - 4y^2 = 27$ intersect each other at right angle.

Because

Statement-II – Whenever confocal ellipse & hyperbola intersect, they intersect each other orthogonally.

2. ***Statement-I –*** $\frac{5}{3}$ and $\frac{5}{4}$ are the eccentricities of two hyperbola which are conjugate to each other.

Because

Statement-II – If e and e_1 are the eccentricities of two conjugate hyperbolas than $ee_1 > 1$.

3. ***Statement-I –*** A bullet is fired and hit a target. An observer in the same plane heard two sounds the crack of the riffle and the thud of the ball striking the target at the same instant, then locus of the observer is hyperbola where velocity of sound is smaller than velocity of bullet.

Because

Statement-II – If difference of distances of a point 'P' from the two fixed points is constant and less than the distance between the fixed points then locus of 'P' is a hyperbola.

4. ***Statement-I –*** If a circle $S = 0$ intersects a hyperbola $xy = 4$ at four points. Three of them are (2, 2)(4, 1) and (6, 2/3) then co-ordinates of the fourth point are (1/4, 16).

Because

Statement-II – If a circle $S = 0$ intersects a hyperbola $xy = c^2$ at t_1, t_2, t_3, t_4 then $t_1 . t_2 . t_3 . t_4 = 1$.

Single Choice Correct

5. The ellipse $4x^2 + 9y^2 = 36$ and the hyperbola $4x^2 - y^2 = 4$ have the same foci and they intersect at right angles then the equation of the circle through the points of intersection of two conics is -

(A) $x^2 + y^2 = 5$ (B) $\sqrt{5}(x^2 + y^2) - 3x - 4y = 0$

(C) $\sqrt{5}(x^2 + y^2) + 3x + 4y = 0$ (D) $x^2 + y^2 = 25$

6. If the normal to the rectangular hyperbola $xy = c^2$ at the point 't' meets the curve again at 't_1' then t^3t_1 has the value equal to -

(A) 1 (B) –1 (C) 0 (D) none

7. Let P $(a\sec\theta, b\tan\theta)$ and Q $(a\sec\phi, b\tan\phi)$, where $\theta+\phi=\frac{\pi}{2}$, be two points on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$.

If (h,k) is the point of intersection of the normals at P & Q, then k is equal to - ***[JEE 99]***

(A) $\frac{a^2+b^2}{a}$ (B) $-\left(\frac{a^2+b^2}{a}\right)$ (C) $\frac{a^2+b^2}{b}$ (D) $-\left(\frac{a^2+b^2}{b}\right)$

Multiple Choice Correct

8. The tangent to the hyperbola $xy = c^2$ at the point P intersects the x-axis at T and the y-axis at T'. The normal to the hyperbola at P intersects the x-axis at N and the y-axis at N'. The areas of the triangles PNT and PN'T' are Δ and Δ' respectively, then $\frac{1}{\Delta} + \frac{1}{\Delta'}$ is -

(A) equal to 1 (B) depends on t (C) depends on c (D) equal to 2

9. From any point on the hyperbola $H_1 : (x^2/a^2) - (y^2/b^2) = 1$ tangents are drawn to the hyperbola $H_2 : (x^2/a^2) - (y^2/b^2) = 2$. The area cut-off by the chord of contact on the asymptotes of H_2 is equal to -

(A) ab/2 (B) ab (C) 2 ab (D) 4 ab

10. The tangent at P on the hyperbola $(x^2/a^2) - (y^2/b^2) = 1$ meets the asymptote $\frac{x}{a} - \frac{y}{b} = 0$ at Q. If the locus of the mid point of PQ has the equation $(x^2/a^2) - (y^2/b^2) = k$, then k has the value equal to -

(A) 1/2 (B) 2 (C) 3/4 (D) 4/3

11. If a real circle will pass through the points of intersection of hyperbola $x^2 - y^2 = a^2$ & parabola $y = x^2$, then -

(A) $a \in (-1, 1)$ (B) $a \in \left[-\frac{1}{2}, \frac{1}{2}\right] - \{0\}$

(C) area of circle $= \pi - \pi a^2$; $a \in \left(-\frac{1}{2}, \frac{1}{2}\right) - \{0\}$ (D) area of circle $= \pi - 4\pi a^2$

12. If least numerical value of slope of line which is tangent to hyperbola $\frac{x^2}{a^2} - \frac{y^2}{(a^3 + a^2 + a)^2} = 1$ is $\frac{3}{4}$, $a \in R_0$ is obtained at $a = k$. For this value of 'a', which of the following is/are true -

(A) $a = -\frac{1}{2}$ (B) $a = \frac{1}{2}$ (C) $LR = \frac{9}{16}$ (D) $e = \frac{5}{4}$

13. Let an incident ray $L_1 = 0$ gets reflected at point A(–2, 3) on hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ & passes through focus S(2, 0), then -

(A) equation of incident ray is $x + 2 = 0$ (B) equation of reflected ray is $3x + 4y = 6$

(C) eccentricity, $e = 2$ (D) length of latus rectum = 6

Match the Column

Following question contains statements given in two columns, which have to be matched. The statements in ***Column-I*** are labelled as A, B, C and D while the statements in ***Column-II*** are labelled as p, q, r and s. Any given statement in ***Column-I*** can have correct matching with ***ONE OR MORE*** statement(s) in ***Column-II***.

14.

	Column - I		Column - II
(A)	A tangent drawn to hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at $p\left(\frac{\pi}{6}\right)$ forms a triangle of area $3a^2$ square units, with coordinate axes, then the square of its eccentricity is equal to	(p)	17
(B)	If the eccentricity of the hyperbola $x^2 - y^2 \sec^2\theta = 5$ is $\sqrt{3}$ times the eccentricity of the ellipse $x^2\sec^2\theta + y^2 = 25$ then smallest positive value of θ is $\frac{6\pi}{p}$, value of 'p' is	(q)	8
(C)	For the hyperbola $\frac{x^2}{3} - y^2 = 3$, angle between its asymptotes is $\frac{\ell\pi}{24}$ then value of 'ℓ' is	(r)	16
(D)	For the hyperbola $xy = 8$ any tangent of it at P meets co-ordinate axes at Q and R then area of triangle CQR where 'c' is centre of the hyperbola is	(s)	24

Comprehension

If we rotate the axes of the rectangular hyperbola $x^2 - y^2 = a^2$ through an angle $\pi/4$ in the clockwise direction then the equation $x^2 - y^2 = a^2$ reduces to $xy = \frac{a^2}{2} = \left(\frac{a}{\sqrt{2}}\right)^2 = c^2$ (say). Since $x = ct,\ y = \frac{c}{t}$ satisfies $xy = c^2$.

$\therefore$ $(x, y) = \left(ct, \frac{c}{t}\right)$ $(t \neq c)$ is called a 't' point on the rectangular hyperbola.

On the basis of above information, answer the following questions

15. If t_1 and t_2 are the roots of the equation $x^2 - 4x + 2 = 0$, then the point of intersection of tangents at 't_1' and 't_2' on $xy = c^2$ is -

(A) $\left(\frac{c}{2}, 2c\right)$ (B) $\left(2c, \frac{c}{2}\right)$ (C) $\left(\frac{c}{2}, c\right)$ (D) $\left(c, \frac{c}{2}\right)$

16. If e_1 and e_2 are the eccentricities of the hyperbolas $xy = 9$ and $x^2 - y^2 = 25$, then (e_1, e_2) lie on a circle C_1 with centre origin then the (radius)^2 of the director circle of C_1 is -

(A) 2 (B) 4 (C) 8 (D) 16

17. If the normal at the point 't_1' to the rectangular hyperbola $xy = c^2$ meets it again at the point 't_2' then the value of t_1t_2 is -

(A) $-t_1^{-1}$ (B) $-t_1^{-2}$ (C) $-t_1^{-3}$ (D) $-t_1^{-4}$

Subjetive Questions

18. If C is the centre of a hyperbola $x^2/a^2 - y^2/b^2 = 1$, S, S′ its foci and P a point on it.
Prove that SP. S′P = $CP^2 - a^2 + b^2$.

19. If the normal at a point P to the hyperbola $x^2/a^2 - y^2/b^2 = 1$ meets the x – axis at G, show that SG = e. SP, S being the focus of the hyperbola.

20. The tangents and normal at a point on $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ cut the y-axis at A & B. Prove that the circle on AB as diameter passes through the foci of the hyperbola.

21. Show that the locus of the middle points of normal chords of the rectangular hyperbola $x^2 - y^2 = a^2$ is $(y^2 - x^2)^3 = 4a^2x^2y^2$.

22. Find the equation of the standard hyperbola passing through the point $(-\sqrt{3}, 3)$ and having the asymptotes as straight lines $x\sqrt{5} \pm y = 0$.

23. If θ_1 & θ_2 are the parameters of the extremities of a chord through (ae, 0) of a hyperbola $x^2/a^2 - y^2/b^2 = 1$, then show that $\tan\frac{\theta_1}{2} . \tan\frac{\theta_2}{2} + \frac{e-1}{e+1} = 0$.

24. If the tangent at the point (h, k) to the hyperbola $x^2/a^2 - y^2/b^2 = 1$ cuts the auxiliary circle in points whose ordinates are y_1 and y_2 then prove that $1/y_1 + 1/y_2 = 2/k$.

25. Tangents are drawn from the point (α, β) to the hyperbola $3x^2 - 2y^2 = 6$ and are inclined at angles θ & ϕ to the x –axis. If $\tan\theta . \tan\phi = 2$, prove that $\beta^2 = 2\alpha^2 - 7$.

26. The perpendicular from the centre upon the normal on any point of the hyperbola $x^2/a^2 - y^2/b^2 = 1$ meets at R . Find the locus of R.

27. If the normal to the hyperbola $x^2/a^2 - y^2/b^2 = 1$ at the point P meets the transverse axis in G & the conjugate axis in g & CF be perpendicular to the normal from the centre C, then prove that $|PF \cdot PG| = b^2$ & $PF \cdot Pg = a^2$ where a & b are the semi transverse & semi-conjugate axes of the hyperbola.

28. If a rectangular hyperbola have the equation, $xy = c^2$, prove that the locus of the middle points of the chords of constant length $2d$ is $(x^2 + y^2)(xy - c^2) = d^2 xy$.

29. Prove that the locus of the middle point of the chord of contact of tangents from any point of the circle $x^2 + y^2 = r^2$ to the hyperbola $x^2/a^2 - y^2/b^2 = 1$ is given by the equation $(x^2/a^2 - y^2/b^2)^2 = (x^2 + y^2) / r^2$.

30. Find the equations of the tangents to the hyperbola $x^2 - 9y^2 = 9$ that are drawn from (3, 2). Find the area of the triangle that these tangents form with their chord of contact.

31. A tangent to the parabola $x^2 = 4ay$ meets the hyperbola $xy = k^2$ in two points P & Q. Prove that the middle point of PQ lies on a parabola.

32. Given the base of a triangle and the ratio of the tangent of half the base angles. Show that the vertex moves on a hyperbola whose foci are the extremities of the base.

ANSWERS

- ***Assertion–Reason***

1. A **2.** B **3.** A **4.** D

- ***Single Choice Correct***

5. A **6.** B **7.** D

- ***Multiple Choice Correct***

8. C **9.** D **10.** C **11.** BC **12.** ACD
13. ABCD

- ***Match the Column***

14. (A) $\rightarrow$ (p); (B) $\rightarrow$ (s); (C) $\rightarrow$ (q, r); (D) $\rightarrow$ (r)

- ***Comprehension***

15. D **16.** C **17.** B

- ***Subjective Questions***

22. $5x^2 - y^2 = 6$
26. $(x^2 + y^2)^2 (a^2y^2 - b^2x^2) = x^2y^2 (a^2 + b^2)^2$
30. $y = \frac{5}{12}x + \frac{3}{4}$; $x - 3 = 0$; 8 sq. unit

PERMUTATION & COMBINATION

True/False

1. The product of any r consecutive natural numbers is always divisible by r! **[JEE 1985]**

Fill in the blank

2. The number of n digit numbers which consists of the digits 1 and 2 only if each digit is to be used at least once, is equal to 510 then n is ....

3. Let $d_1, d_2, ..., d_k$ be all the factors of a positive integer 'N' including 1 and n. If $d_1 + d_2 + d_3 + ... + d_k = 72$, then $\frac{1}{d_1}+\frac{1}{d_2}+....+\frac{1}{d_k}$ is equal to __________ .

4. The sides AB, BC and CA of a triangle ABC have 3, 4 and 5 interior points respectively on them. The number of triangles that can be constructed using these interior points as vertices is _______ . **[JEE 1984]**

5. Let A be a set of n distinct elements. The the total number of distinct functions from A to A is ______ and out of these are _________ onto functions. **[JEE 1985]**

6. There are four balls of different colours and four boxes of the same colours as those of the balls. The number of ways in which the balls could be placed one each in a box, such that a ball does not go to a box of its own colour, is ________. **[JEE 1992]**

7. Let n and k be positive integers such that $n \ge \frac{k(k+1)}{2}$. The number of solutions $(x_1, x_2, ..., x_k)$, $x_1 \ge 1$, $x_2 \ge 2$, ..., $x_k \ge k$, all integers, satisfying $x_1 + x_2 + ... + x_k = n$, is __________. **[JEE 1996]**

Assertion & Reason

These questions contain, Statement I (assertion) and Statement II (reason).

(A) Statement-I is true, Statement-II is true ; Statement-II is correct explanation for Statement-I.

(B) Statement-I is true, Statement-II is true ; Statement-II is NOT a correct explanation for statement-I.

(C) Statement-I is true, Statement-II is false.

(D) Statement-I is false, Statement-II is true.

8. ***Statement-I –*** If a polygon has 45 diagonals, then its number of sides is 10.

Because

Statement-II – Number of ways of selecting 2 points from n non collinear points is ${}^nC_2$.

9. ***Statement-I –*** The expression $n!(20 - n)!$ is minimum where $n = 10$

Because

Statement-II – ${}^{2p}C_r$ is maximum where $r = p$, where p is a constant.

10. ***Statement-I –*** The number of non negative integral solutions of $x_1 + x_2 + x_3 + ... + x_n = r$ is ${}^{r+n-1}C_r$.

Because

Statement-II – The number of ways in which n identical things can be distributed among r students is ${}^{n+r-1}C_n$.

11. ***Statement-I –*** If a, b, c are positive integers such that $a + b + c \le 8$, then the number of possible values of the ordered triplets (a, b, c) is 56.

Because

Statement-II – The number of ways in which n distinct things can be distributed among r girls such that each get at least one is ${}^{n-1}C_{r-1}$.

12. ***Statement-I –*** Number of terms in the expansion of $(x_1 + x_2 + x_3 + ... + x_{11})^6 = {}^{16}C_6$.

Because

Statement-II – Number of ways of distributing n identical things among r persons when each person get zero or more things = ${}^{n+r-1}C_n$

13. ***Statement-I –*** Number of ways in which 400 different things can be distributed between Ramu & Shamu so that each receives 200 things > Number of ways in which 400 different things can be distributed between Sita & Geeta. So that Sita receives 238 things & Geeta receives 162 things.

Because

Statement-II – Number of ways in which (m + n) different things can be distributed between two receivers such that one receives m and other receives n is equal to ${}^{m+n}C_m$, for any two non-negative integers m and n.

14. Statement-I – The number of positive integral solutions of the equation xyzw = 770 is 2^8.

Because

Statement-II – The number of ways of selection of atleast one thing from n things of which 'p' are alike of one kind, q are alike of 2nd kind and rest of the things are different is $(p + 1)(q + 1)\, 2^{n-(p+q)} - 1$.

Single Choice Correct

15. A set contains (2n + 1) elements. The number of subset of the set which contain at most n elements is : -

(A) 2^n (B) 2^{n+1} (C) 2^{n-1} (D) 2^{2n}

16. There are n identical red balls & m identical green balls. The number of different linear arrangements consisting of "n red balls but not necessarily all the green balls" is ${}^xC_y$ then -

(A) x = m + n, y = m (B) x = m + n + 1, y = m

(C) x = m + n + 1, y = m + 1 (D) x = m + n, y = n

17. The sum of the digits in the unit's place of all numbers formed with the help of 3, 4, 5, 6 taken all at a time is

(A) 18 (B) 432 (C) 108 (D) 144

18. An old man while dialing a 7 digit telephone number remembers that the first four digits consists of one 1's, one 2's and two 3's. He also remembers that the fifth digit is either a 4 or 5 while has no memorising of the sixth digit, he remembers that the seventh digit is 9 minus the sixth digit. Maximum number of distinct trials he has to try to make sure that he dials the correct telephone number, is

(A) 360 (B) 240 (C) 216 (D) None of these

19. How many different nine digit numbers can be formed from the number 223355888 by rearranging its digits so that the odd digits occupy even positions ? ***[JEE 2000]***

(A) 16 (B) 36 (C) 60 (D) 180

20. ***(a)*** Let T_n denote the number of triangles which can be formed using the vertices of a regular polygon of 'n' sides. If $T_{n+1} - T_n = 21$, then 'n' equals - ***[JEE 2001]***

(A) 5 (B) 7 (C) 6 (D) 4

(b) Let E = {1, 2, 3, 4} and F = {1, 2}. Then the number of onto functions from E to F is -

(A) 14 (B) 16 (C) 12 (D) 8

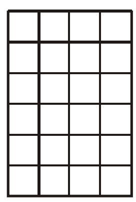
21. The number of arrangements of the letters of the word BANANA in which two 'N's do not appear adjacently is - ***[JEE 2002]***

(A) 40 (B) 60 (C) 80 (D) 100

22. Number of points with integral co-ordinates that lie inside a triangle whose co-ordinates are (0, 0), (0, 21) and (21, 0) ***[JEE 2003]***

(A) 210 (B) 190 (C) 220 (D) none of these

23. A rectangle with sides 2m – 1 and 2n – 1 is divided into squares of unit length by drawingparallel lines as shown then number of rectangles possible with odd side lengths is - ***[JEE 2005]***



(A) $(m + n + 1)^2$ (B) 4^{m+n-1}

(C) $m^2 n^2$ (D) mn (m + 1) (n + 1)

24. Number of divisors of the form $4n + 2 (n \geq 0)$ of integers 240 is **[JEE 1998]**
(A) 4 (B) 8 (C) 10 (D) 3

25. If $a_n = \sum_{r=0}^{n} \frac{1}{{}^nC_r}$, then $\sum_{r=0}^{n} \frac{r}{{}^nC_r}$ equals **[JEE 1998]**
(A) $(n - 1)a_n$ (B) na_n (C) $\frac{1}{2}na_n$ (D) None of these

26. An n-digit number is a positive number with exactly n digits. Nine hundred distinct n-digit numbers are to be formed using only the three digits 2, 5 and 7. The smallest value of n for which this is possible is **[JEE 1998]**
(A) 6 (B) 7 (C) 8 (D) 9

27. In a college of 300 students, every student reads 5 newspapers and every newspaper is read by 60 students. The number of newspapers is **[JEE 1998]**
(A) at least 30 (B) at most 20 (C) exactly 25 (D) none of these

28. How many different nine digit number can be formed from the number 223355888 by rearranging its digits so that the odd digits occupy even positions? **[JEE 2000]**
(A) 16 (B) 36 (C) 60 (D) 180

29. Ten different letters of an alphabet are given. Words with five letters are formed from these given letters. Then the number of words which have at least one letter repeated is **[JEE 1980]**
(A) 69760 (B) 30240 (C) 99748 (D) None of these

30. The value of the expression ${}^{47}C_4 + \sum_{j=1}^{5} {}^{(52-j)}C_3$ is equal to **[JEE 1980]**
(A) ${}^{47}C_5$ (B) ${}^{52}C_5$ (C) ${}^{52}C_4$ (D) None of these

31. Eight chairs are numbered 1 to 8. Two women and three men wish to occupy one chair each. First the women choose the chairs from amongst the chairs marked 1 to 4; and then the men select the chairs from amongst the remaining. The number of possible arrangements is **[JEE 1982]**
(A) ${}^6C_3 \times {}^4C_2$ (B) ${}^4P_2 \times {}^4P_3$ (C) ${}^4C_2 + {}^4P_3$ (D) none of theseff

32. A five digit number divisible by 3 is to be formed by using the numerals 0, 1, 2, 3, 4 and 5, without repetition. The total number of ways this can be done is **[JEE 1989]**
(A) 216 (B) 240 (C) 600 (D) 3125

Multiple Choice Correct

***33.** The number of ways of choosing a committee of 2 women & 3 men from 5 women & 6 men, if Mr. A refuses to serve on the committee if Mr. B is a member & Mr. B can only serve, if Miss C is the member of the committee, is -
(A) 60 (B) 84 (C) 124 (D) none of these

34. Sum of all the numbers that can be formed using all the digits 2, 3, 3, 4, 4, 4 is -
(A) 22222200 (B) 11111100 (C) 55555500 (D) 20333280

35. Let P_n denotes the number of ways in which three people can be selected out of 'n' people sitting in a row, if no two of them are consecutive. If $P_{n+1} - P_n = 15$ then the value of 'n' is -
(A) 7 (B) 8 (C) 9 (D) 10

36. All the 7 digit numbers containing each of the digits 1, 2, 3, 4, 5, 6, 7 exactly once and not divisible by 5 are arranged in the increasing order. Then -
(A) 1800th number in the list is 3124567
(B) 1897th number in the list is 4213567
(C) 1994th number in the list is 4312567
(D) 2001th number in the list is 4315726

37. If P(n, n) denotes the number of permutations of n different things taken all at a time then P(n, n) is also identical to :-

(A) n.P(n – 1, n – 1) (B) P(n, n – 1) (C) r! . P(n, n – r) (D) (n – r) . P(n, r)

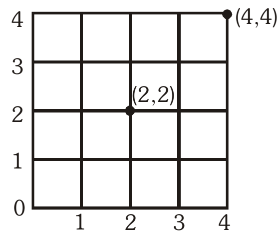
where $0 \le r \le n$

Match the Column

38. Consider all the different words that can be formed using the letters of the word HAVANA, taken 4 at a time.

	Column-I		Column-II
(A)	Number of such words in which all the 4 letters are different	(p)	36
(B)	Number of such words in which there are 2 alike letters & 2 different letters.	(q)	42
(C)	Number of such words in which A's never appear together	(r)	37
(D)	If all such 4 letters words are written, by the rule of dictionary then the rank of the word HANA	(s)	24

39.

	Column-I		Column-II
(A)	${}^{24}C_2 + {}^{23}C_2 + {}^{22}C_2 + {}^{21}C_2 + {}^{20}C_2 + {}^{20}C_3$ is equal to	(p)	102
(B)	In the adjoining figure number of progressive ways to reach from (0,0) to (4, 4) passing through point (2, 2) are (particle can move on horizontal or vertical line) 	(q)	2300
(C)	The number of 4 digit numbers that can be made with the digits 1, 2, 3, 4, 3, 2	(r)	82
(D)	If $\left\{\frac{500!}{14^k}\right\} = 0,$ then the maximum natural value of k is equal to (where {.} is fractional part function)	(s)	36

40. Match the column :

	Column-I		Column-II
(A)	Number of increasing permutations of m symbols are there from the n set numbers $\{a_1, a_2, ..., a_n\}$ where the order among the numbers is given by $a_1 < a_2 < a_3 < ... a_{n-1} < a_n$ is	(p)	n^m
(B)	There are m men and n monkeys. Number of ways in which every monkey has a master, if a man can have any number of monkeys	(q)	${}^mC_n$
(C)	Number of ways in which n red balls and (m – 1) green balls can be arranged in a line, so that no two red balls are together, is (balls of the same colour are alike)	(r)	${}^nC_m$
(D)	Number of ways in which 'm' different toys can be distributed in 'n' children if every child may receive any number of toys, is	(s)	m^n

Comprehension

Comprehension – 1

S = {0, 2, 4, 6, 8}. A natural number is said to be divisible by 2 if the digit at the unit place is an even number. The number is divisible by 5, if the number at the unit place is 0 or 5. If four numbers are selected from S and a four digit number ABCD is formed.

On the basis of above information, answer the following questions

41. The number of such numbers which are even (all digits are different) is
(A) 60 (B) 96 (C) 120 (D) 204

42. The number of such numbers which are even (all digits are not different) is
(A) 404 (B) 500 (C) 380 (D) none of these

43. The number of such numbers which are divisible by two and five (all digits are not different) is
(A) 125 (B) 76 (C) 65 (D) 100

Comprehension - 2

16 players $P_1, P_2, P_3, \ldots, P_{16}$ take part in a tennis tournament. Lower suffix player is better than any higher suffix player. These players are to be divided into 4 groups each comprising of 4 players and the best from each group is selected for semifinals.

44. Number of ways in which 16 players can be divided into four equal groups, is $K\prod_{r=1}^{8}(2r-1)$, where K equals

(A) $\frac{35}{27}$ (B) $\frac{35}{24}$ (C) $\frac{35}{52}$ (D) $\frac{35}{6}$

45. Number of ways in which they can be divided into equal groups if the player P_1, P_2, P_3 and P_4 are in different groups, is

(A) $\frac{(11)!}{36}$ (B) $\frac{(11)!}{72}$ (C) $\frac{(11)!}{108}$ (D) $\frac{(11)!}{216}$

46. Number of ways in which these 16 players can be divided into four equal groups, such that when the best player is selected from each group, P_6 is one among them, is $(m)\frac{12!}{(4!)^3}$. The value of 'm' is

(A) 36 (B) 24 (C) 18 (D) 20

Comprehension – 3

Twelve persons are to be arranged around two round tables such that one table can accommodate seven persons and another five persons only. Answer the following questions.

47. Number of ways in which these 12 persons can be arraged is
(A) ${}^{12}C_5 7!5!$ (B) $6!4!$ (C) ${}^{12}C_5 6!4!$ (D) None of these

48. Number of ways of arrangement if two particular persons A and B want to be together and consecutive is
(A) ${}^{10}C_7\, 6!\, 3!2! + {}^{10}C_5\, 4!5!2!$ (B) ${}^{10}C_5\, 6!\, 3! + {}^{10}C_7\, 4!5!$
(C) ${}^{10}C_7\, 6!\, 2! + {}^{10}C_5\, 5!2!$ (D) None of these

Comprehension – 4

Consider the letters of the word 'MATHEMATICS'.

49. Possible number of words taking all letters at a time such that in each word both M's are together and both T's are together but both A's are not together is

(A) $\frac{11!}{2!2!2!}-\frac{10!}{2!2!}$ (B) $7!\,{}^8C_2$ (C) $\frac{6!4!}{2!2!}$ (D) $\frac{9!}{2!2!2!}$

50. Possible number of words in which no two vowels are together is

(A) $7!\ {}^{8}C_4 \dfrac{4!}{2!}$ (B) $\dfrac{7!}{2!}\ {}^{8}C_4 \dfrac{4!}{2!}$ (C) $\dfrac{7!}{2!2!}\ {}^{8}C_4 \dfrac{4!}{2!}$ (D) $\dfrac{7!}{2!2!2!}\ {}^{8}C_4 \dfrac{4!}{2!}$

Comprehension – 5

Five balls are to be placed in three boxes. Each box should hold atleast one of the five balls so that no box remains empty.

51. Number of ways if balls are different but boxes are identical is

(A) 30 (B) 25 (C) 21 (D) 35

52. Number of ways if balls and boxes are identical is

(A) 3 (B) 1 (C) 2 (D) None of these

53. Number of ways if balls as well as boxes are identical but boxes are kept in a row is

(A) 10 (B) 15 (C) 20 (D) 6

Subjetive Questions

54. Using permutation or otherwise prove that $\dfrac{(n^2)!}{(n!)^n}$ is an integer, where n is a positive integer. **[JEE 2004]**

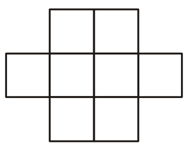
55. A crew of an eight oar boat has to be chosen out of 11 men five of whom can row on stroke side only, four on the bow side only and the remaining two on either side. How many different selections can be made?

56. There are n straight lines in a plane, no 2 of which are parallel & no 3 pass through the same point. Their points of intersection are joined. Show that the number of fresh lines introduced is $\dfrac{n(n-1)(n-2)(n-3)}{8}$.

57. There are 20 books on Algebra & Calculus in our library. Prove that the greatest number of selections each of which consists of 5 books on each topic is possible only when there are 10 books on each topic in the library.

58. Find the number of ways to invite one of the three friends for dinner on 6 successive nights such that no friend is invited more than 3 times.

59. Six X's have to be placed in the squares of figure, such that each row contains at least one X. In how many different ways can this be done? **[JEE 1978]**



60. m men and n women are to be seated in a row so that no two momen sit together. If m > n, then show that the number of ways in which this can be done is $\dfrac{m!(m+1)!}{(m-n+1)!}$. **[JEE 1983]**

61. A man has 7 relatives, 4 of them ladies and 3 gentlemen; his wife also has 7 relatives, 3 of them are ladies and 4 gentlemen. In how many ways can they invite a dinner party of 3 ladies and 3 gentlemen so that there are 3 of the man's relatives and 3 of the wife's relatives? **[JEE 1985]**

62. A student is allowed to select at most n books from a collection of (2n + 1) books. If the total number of ways in which he can select at least one book is 63, find the value of n. **[JEE 1987]**

63. Eighteen guests have to be seated, half on each side of a long table. Four particular guests desire to sit on one particular side and three others on the other side. Determine the number of ways in which the sitting arrangements can be made. **[JEE 1991]**

64. In how many ways three girls and nine boys can be seated in two vans each having numbered seats, 3 in front and 4 at the back? How many seating arrangements are possible if 3 girls sit together in a back row on adjacent seats? Now, if all seating arrangements are equally likely, what is the probability of 3 girls sitting together in a back row on adjacent seats? ***[JEE 1996]***

65. In how many ways can a team of 6 horses be selected out of a stud of 16, so that there shall always be 3 out of ABC A'B'C', but never AA', BB' or CC' together ?

66. Prove that if each of m points in one straight line be joined to each of n in another by straight lines terminated by the points, then excluding the given points, the lines will intersect $\frac{1}{4}mn(m-1)(n-1)$ times.

67. A shop sells 6 different flavours of ice-creams. In how many ways can a customer choose 4 ice-cream cones if
(a) they are all of different flavours ?
(b) they are non necessarily of different flavours ?
(c) they contain only 3 different flavours ?
(d) they contain only 2 or 3 different flavours ?

68. Lattice paths are paths consisting of one unit steps in the positive horizontal or positive vertical directions. Let distinct lattice paths from the point (–1, 0) to the point (3, 5), if number of shortest path are λ. Let μ be the sum of all digits of λ. Then find the value of μ.

69. If the number of ordered pairs (S, T) of subsets of {1, 2, 3, 4, 5, 6} are such that $S \cup T$ contains exactly three elements is

70. There are counters available in 7 different colours. Counters are all alike except colour and they are atleast ten of each colour. Find the number of ways in which an arrangement of 10 counters can be made. How many of these will have counters of each colour ?

71. How many integral solutions are there for the equation; $x + y + z + w = 29$ when $x > 0$, $y > 1$, $z > 2$ & $w \geq 0$?

72. A party of 10 consists of 2 Americans, 2 Britishmen, 2 Chinese & 4 men of other nationalities (all different). Find the number of ways in which they can stand in a row so that no two men of the same nationality are next to one another. Find also the number of ways in which they can sit at a round table.

73. Find the number of words each consisting of 3 consonants & 3 vowels that can be formed from the letters of the word "CIRCUMFERENCE". In how many of these 'C's will be together.

74. Find the number of ways in which the number 30 can be partitioned into three unequal parts, each part being a natural number. What this number would be if equal parts are also included.

75. Prove by combinatorial argument that :
(a) ${}^{n+1}C_r = {}^nC_r + {}^nC_{r-1}$
(b) ${}^{n+m}C_r = {}^nC_0 \cdot {}^mC_r + {}^nC_1 \cdot {}^mC_{r-1} + {}^nC_2 \cdot {}^mC_{r-2} + \ldots + {}^nC_r \cdot {}^mC_0$

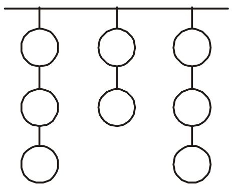
76. How many 6 digits odd numbers greater than 60,0000 can be formed from the digits 5,6,7,8,9.0 if
(a) repetitions are not allowed ? (b) repetitions are allowed ?

77. All the 7 digit numbers containing each of the digits 1, 2, 3, 4, 5, 6, 7 exactly once and not divisible by 5 are arranged in the increasing order. Find the $(2004)^{th}$ number in this list.

78. A firm of Chartered Accountants in Bombay has to send 10 clerks to 5 different companies, two clerks in each. Two of the companies are in Bombay and the others are outside. Two of the clerks prefer to work in Bombay while three others prefer to work outside. In how many ways can the assignment be made if the preferences are to be satisfied ?

79. There are 5 white, 4 yellow, 3 green, 2 blue & 1 red ball. The balls are all identical except colour. These are to be arranged in a line at 5 places. Find the number of distinct arrangements.

80. 8 clay targets are arranged as shown. In how many ways can they be shot (one at a time) if no target can be shot until the target(s) below it have been shot.



ANSWERS

- ***True and False***

1. True

- ***Fill in the Blanks***

2.	9	**3.**	$\frac{72}{n}$	**4.**	205	**5.**	n^n, n!	**6.**	9

7. ${}^mC_{k-1}$ where $m = \frac{1}{2}(2n - k^2 + k - 2)$

- ***Assertion–Reason***

8.	D	**9.**	A	**10.**	A	**11.**	C	**12.**	A
13.	C	**14.**	B						

- ***Single Choice Correct***

15.	(D)	**16.**	(B)	**17.**	(C)	**18.**	(B)	**19.**	(C)
20.	a(B) b(A)	**21.**	(A)	**22.**	(B)	**23.**	(C)	**24.**	(A)
25.	(C)	**26.**	(B)	**27.**	(C)	**28.**	(C)	**29.**	(A)
30.	(C)	**31.**	(D)	**32.**	(A)				

- ***Multiple Choice Correct***

33.	(C)	**34.**	(A)	**35.**	(B)	**36.**	(BD)	**37.**	(ABC)

- ***Match the Column***

38. (A) $\to$ (s), (B) $\to$ (p), (C) $\to$ (q), (D) $\to$ (r)

39. (A)$\to$(q), (B)$\to$(s), (C)$\to$(p), (D)$\to$(r)

40. (A)$\to$(r), (B)$\to$(s), (C)$\to$(q), (D)$\to$(p)

- ***Comprehension***

Comprehension – 1	**41.** B	**42.** A	**43.** B
Comprehension – 3	**44.** A	**45.** C	**46.** D
Comprehension – 4	**47.** C	**48.** A	
Comprehension – 5	**49.** B	**50.** C	
Comprehension – 5	**51.** B	**52.** C	**53.** D

- ***Subjective Questions***

55.	145	**58.**	510	**59.**	26	**60.**	Pro
61.	485	**62.**	3	**63.**	${}^{11}C_5\,(9!)^2$	**64.**	7(13!), $\frac{1}{91}$
65.	960	**67.**	***(a)*** 15; ***(b)*** 126; ***(c)*** 60; ***(d)*** 105			**68.**	9
69.	540	**70.**	$7^{10}; \left(\frac{49}{6}\right)10!$	**71.**	2600	**72.**	(47)8!; (244)6!
73.	22100, 52	**74.**	61, 75	**76.**	***(a)*** 240; ***(b)*** 15552		
77.	4316527	**78.**	5400	**79.**	2111	**80.**	560

Fill in the Blanks

1. The greatest binomial coefficient in the expansion of $(a + b)^n$ is ________ given that the sum of all the coefficients is equal to 4096.

2. The number 7^{1995} when divided by 100 leaves the remainder ________.

3. The algebraically greatest term in the expansion of $(a - 2x)^9$ when a = 1 and x = 1/4 is ________:

4. If $(1 + x + x^2 + ... + x^p)^n = a_0 + a_1x + a_2x^2 + ... + a_{np}x^{np}$ then $a_1 + 2a_2 + 3a_3 + ... + npa_{np}$ = ________.

5. If $(1 + x)(1 + x + x^2)(1 + x + x^2 + x^3) ...... (1 + x + x^2 + x^3 + ... + x^n) \equiv a_0 + a_1x + a_2x^2 + a_3x^3 + ... + a_mx^m$ then $\sum_{r=0}^{m} a_r$ has the value equal to ________ .

6. If the 6^{th} term in the expansion of the binomial $\left[\frac{1}{x^{8/3}} + x^2 \log_{10} x\right]^8$ is 5600, then x = ________ .

7. $(1 + x)(1 + x + x^2)(1 + x + x^2 + x^3) ...... (1 + x + x^2 + ... + x^{100})$ when written in the ascending power of x then the highest exponent of x is ________ .

8. The two consecutive terms in the expansion of $(3+2x)^{74}$ whose coefficients are equal is ________

9. The coefficient of $a^8b^4c^9d^9$ in $(abc + abd + acd + bcd)^{10}$ is

10. In the expansion of $(5^{1/2} + 7^{1/8})^{1024}$, the number of integral terms is

11. The value of ${}^{15}C_0^2 - {}^{15}C_1^2 + {}^{15}C_2^2 .... {}^{15}C_{15}^2$ is

12. If $(1 + ax)^n = 1 + 8x + 24x^2 + ...$, then a = ——— and n = ————. ***[JEE 1983]***

13. If $(1 + x)^n = C_0 + C_1x + C_2x^2 + ... + C_nx^n$, then sum of the products of the C_i's taken two at a time, represented by $\sum\sum_{0\le i< j\le n} C_iC_j$ is equal to _______.

Assertion & Reason

These questions contains, Statement I (assertion) and Statement II (reason).

(A) Statement-I is true, Statement-II is true ; Statement-II is correct explanation for Statement-I.

(B) Statement-I is true, Statement-II is true ; Statement-II is NOT a correct explanation for statement-I

(C) Statement-I is true, Statement-II is false

(D) Statement-I is false, Statement-II is true

14. ***Statement-I –*** If $q = \frac{1}{3}$ and p + q = 1, then $\sum_{r=0}^{15} r\ {}^{15}C_r p^r q^{15-r} = 15 \times \frac{1}{3} = 5$

Because

Statement-II – If p + q = 1 , 0 < p < 1, then $\sum_{r=0}^{n} r\ {}^{n}C_r p^r q^{n-r} = np$

15. ***Statement-I*** – The greatest value of ${}^{40}C_0 \cdot {}^{60}C_r + {}^{40}C_1 \cdot {}^{60}C_{r-1} \ldots\ldots {}^{40}C_{40} \cdot {}^{60}C_{r-40}$ is ${}^{100}C_{50}$

Because

Statement-II – The greatest value of ${}^{2n}C_r$, (where r is constant) occurs at r = n.

16. ***Statement-I*** – If $x = {}^{n}C_{n-1} + {}^{n+1}C_{n-1} + {}^{n+2}C_{n-1} + \ldots\ldots\ldots + {}^{2n}C_{n-1}$, then $\frac{x+1}{2n+1}$ is integer.

Because

Statement-II – ${}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r$ and ${}^{n}C_r$ is divisible by n if n and r are co-prime.

17. ***Statement-I :***If n is even, then ${}^{2n}C_1 + {}^{2n}C_3 + {}^{2n}C_5 + \ldots\ldots\ldots + {}^{2n}C_{n-1} = 2^{2n-1}$.

Statement-II : ${}^{2n}C_1 + {}^{2n}C_3 + {}^{2n}C_5 + \ldots\ldots + {}^{2n}C_{2n-1} = 2^{2n-1}$.

18. ***Statement-I*** – Coefficient of $ab^8c^3d^2$ in the expansion of $(a + b + c + d)^{14}$ is 180180

Because

Statement-II – General term in the expansion of $(a_1 + a_2 + a_3 + \ldots\ldots + a_m)^n$

$$= \sum \frac{n!}{n_1!n_2!n_3!\ldots.n_m!} a_1^{n_1} a_2^{n_2} \ldots a_m^{n_m}, \text{ where } n_1 + n_2 + n_3 + \ldots + n_m = n.$$

Single Choice Correct

19. Value of $\sum_{r=1}^{n}\left(\sum_{m=0}^{r} {}^{n}C_r \cdot {}^{r}C_m\right)$ is equal to ;

(A) $2^n - 1$ (B) $3^n - 1$ (C) $3^n - 2^n$ (D) None of these

20. $\lim_{n\to\infty} \frac{\sum\sum_{1\le i<j\le n}(i+1)(j+1)}{n^4}$ is equal to :

(A) $\frac{1}{4}$ (B) $\frac{1}{6}$ (C) $\frac{1}{8}$ (D) None of these

21. For $2 \le r \le n$, $\binom{n}{r} + 2\binom{n}{r-1} + \binom{n}{r-2} =$ ***[JEE 2000]***

(A) $\binom{n+1}{r-1}$ (B) $2\binom{n+1}{r+1}$ (C) $2\binom{n+2}{r}$ (D) $\binom{n+2}{r}$

22. In the binomial expansion of $(a - b)^n$, $n \ge 5$, the sum of the 5^{th} and 6^{th} terms is zero, Then $\frac{a}{b}$ equals -

(A) $\frac{n-5}{6}$ (B) $\frac{n-4}{5}$ (C) $\frac{5}{n-4}$ (D) $\frac{6}{n-5}$

23. The sum $\sum_{i=0}^{m}\binom{10}{i}\binom{20}{m-i}$ (where $\binom{p}{q} = 0$ if $p < q$) is maximum when m is - ***[JEE 2002]***

(A) 5 (B) 10 (C) 15 (D) 20

24. Coefficient of t^{24} in the expansion of $(1 + t^2)^{12} (1 + t^{12}) (1 + t^{24})$ is - ***[JEE 2003]***

(A) ${}^{12}C_6 + 2$ (B) ${}^{12}C_6 + 1$ (C) ${}^{12}C_6$ (D) none

25. If $n, r \in N$ and ${}^{n-1}C_r = (k^2 - 3)\,({}^{n}C_{r+1})$, then k lies in the interval - ***[JEE 2004]***

(A) $\left[-\sqrt{3}, \sqrt{3}\right]$ (B) $(2, \infty)$ (C) $\left[-\sqrt{3}, \infty\right]$ (D) $\left(\sqrt{3}, 2\right]$

26. The value of $\binom{30}{0}\binom{30}{10}-\binom{30}{1}\binom{30}{11}+\binom{30}{2}\binom{30}{12}..........+\binom{30}{20}\binom{30}{30}$, is where $\binom{n}{r}={}^nC_r$

[JEE 2005]

(A) ${}^{30}C_{10}$ (B) ${}^{60}C_{20}$ (C) ${}^{31}C_{11}$ or ${}^{31}C_{10}$ (D) ${}^{30}C_{11}$

27. If C_r stands for ${}^nC_r$ then the sum of the series $\dfrac{2\left(\frac{n}{2}\right)!\left(\frac{n}{2}\right)!}{n!}\left[C_0^2-2C_1^2+3C_2^2-...+(-1)^n(n+1)C_n^2\right]$ where n is an even positive integer is equal to **[JEE 1986]**

(A) 0 (B) $(-1)^{n/2}(n+1)$ (C) $(-1)^n(n+2)$ (D) None of these

28. Number of terms free from radical sign in the expansion of $(1+3^{1/3}+7^{1/7})^{10}$ is -

(A) 4 (B) 5 (C) 6 (D) 8

Multiple Choice Correct

***29.** In the expansion of $(1+x)^n(1+y)^n(1+z)^n$, the sum of the co-efficients of the terms of degree 'r' is -

(A) ${}^{n^3}C_r$ (B) ${}^nC_{r^3}$ (C) ${}^{3n}C_r$ (D) $3\,.\,{}^{2n}C_r$

***30.** The value of $\sum\limits_{\substack{r=0\\ r\le s}}^{s}\sum\limits_{s=1}^{n}{}^nC_s\,{}^sC_r$ is -

(A) 3^n-1 (B) 3^n+1 (C) 3^n (D) $3(3^n-1)$

***31.** The sum of the series $(1^2+1).1!+(2^2+1).2!+(3^2+1).3!+.....+(n^2+1).n!$ is -

(A) $(n+1).(n+2)!$ (B) $n.(n+1)!$ (C) $(n+1).(n+1)!$ (D) none of these

Comprehension

Comprehension – 1

If n is positive integer and if $(1+4x+4x^2)^n=\sum\limits_{r=0}^{2n}a_rx^r$, where a_i's are $(i=0,1,2,3,.....,2n)$ real numbers.

On the basis of above information, answer the following questions :

32. The value of $2\sum\limits_{r=0}^{n}a_{2r}$ is -

(A) 9^n-1 (B) 9^n+1 (C) 9^n-2 (D) 9^n+2

33. The value of $2\sum\limits_{r=1}^{n}a_{2r-1}$ is -

(A) 9^n-1 (B) 9^n+1 (C) 9^n-2 (D) 9^n+2

34. The value of a_{2n-1} is -

(A) 2^{2n} (B) $(n-1).2^{2n}$ (C) $n.2^{2n}$ (D) $(n+1).2^{2n}$

35. The value of a_2 is -

(A) $8n$ (B) $8n^2-4$ (C) $8n^2-4n$ (D) $8n-4$

Comprehension – 2

An equation $a_0 + a_1x + a_2x^2 + ... + a_{99}x^{99} + x^{100} = 0$ has roots ${}^{99}C_0, {}^{99}C_1, {}^{99}C_2 .... {}^{99}C_{99}$

36. The value of $\frac{a_{98}}{a_{99}}$ is

(A) $\frac{{}^{198}C_{99} - 2^{198}}{2^{100}}$ (B) $\frac{2^{198} + {}^{198}C_{99}}{2^{100}}$ (C) $2^{99} - {}^{99}C_{49}$ (D) None of these

37. The value of $\sum_{i=0}^{99} \left({}^{99}C_i\right)^2$ is

(A) $2a_{98} - a_{99}^2$ (B) $a_{99}^2 - a_{98}$ (C) $a_{99}^2 - 2a_{98}$ (D) None of these

Subjetive Questions

***38.** Prove that $\sum_{k=0}^{n} {}^nC_k \sin Kx. \cos(n - K)x = 2^{n-1} \sin nx$.

39. If $\sum_{r=0}^{2n} a_r (x-2)^r = \sum_{r=0}^{2n} b_r (x-3)^r$ & $a_k = 1$ for all $k \geq n$, then show that $b_n = {}^{2n+1}C_{n+1}$

40. Prove the following :

(a) $C_0^2 - C_1^2 + C_2^2 - C_3^2 + ... + (-1)^n C_n^2 = \begin{cases} 0 & \text{if } n \text{ is odd} \\ (-1)^{n/2}\ {}^nC_{n/2} & \text{if } n \text{ is even} \end{cases}$

(b) $1.C_0^2 + 3.C_1^2 + 5.C_2^2 + ... + (2n+1)C_n^2 = \frac{(n+1)(2n)!}{n!n!}$

41. (a) Find the index n of the binomial $\left(\frac{x}{5} + \frac{2}{5}\right)^n$ if the 9^{th} term of the expansion has numerically the greatest coefficient ($n \in N$).

(b) For which positive values of x is the fourth term in the expansion of $(5 + 3x)^{10}$ is the greatest.

42. If $a_0, a_1, a_2, ...$ be the coefficients in the expansion of $(1 + x + x^2)^n$ in ascending powers of x, then prove that :

(a) $a_0a_1 - a_1a_2 + a_2a_3 - .... = 0$

(b) $a_0a_2 - a_1a_3 + a_2a_4 - .... + a_{2n-2}\, a_{2n} = a_{n+1}$ or a_{n-1}

(c) $E_1 = E_2 = E_3 = 3^{n-1}$; where $E_1 = a_0 + a_3 + a_6 + ...$; $E_2 = a_1 + a_4 + a_7 + ...$ & $E_3 = a_2 + a_5 + a_8 + ...$

43. Prove that : $1^2 . C_0 + 2^2 C_1 + 3^2 . C_2 + 4^2 . C_3 . + ... + (n+1)^2 . C_n = 2^{n-2}(n+1)(n+4)$.

44. In the polynomial $(x - 1)(x^2 - 2)(x^3 - 3) ... (x^{11} - 11)$, the coefficient of x^{60} is

45. Let $1 + \sum_{f=1}^{10}\left(3^r \cdot {}^{10}C_r + r \cdot {}^{10}C_r\right) = 2^{10}\left(\alpha \cdot 4^5 + \beta\right)$ where $\alpha, \beta \in N$ and $f(x) = x^2 - 2x - k^2 + 1$. If α, β lies between the roots of $f(x) = 0$. Then find the smallest positive integral value of k.

46. Prove that : $\sum_{j=0}^{\infty}\binom{p}{j}\binom{q}{n+j}=\binom{p+q}{p+n}$, p, q $\in$ N; p, q are constants.

47. If $(1 + x + x^2)^{3n+1} = a_0 + a_1x + a_2x^2 + \ldots + a_{6n+2}\,x^{6n+2}$ then find $\sum_{r=0}^{2n}\left(a_{3r}-\left(\frac{a_{3r+1}+a_{3r+2}}{2}\right)\right)$.

48. Prove that : $\frac{C_0}{2}+\frac{C_1}{3}+\frac{C_2}{4}+\frac{C_3}{5}+\ldots\ldots+\frac{C_n}{n+2}=\frac{1+n.\ \ 2^{n+1}}{(n+1)(n+2)}$

49. Prove that : $\binom{n}{r}\binom{r}{k}=\binom{n}{k}\binom{n-k}{r-k}$

50. Prove that : $\sum_{r=0}^{25}(-1)^r\binom{30}{r}\binom{30}{25-r}=0$

51. Prove that : $\sum_{r=0}^{n-2}\binom{n-1}{r}\binom{n}{r+2}=\binom{2n-1}{n-2}$

Prove the following (here $C_r = {}^nC_r$) (Q. 13 to 20) :

52. $C_0C_1 + C_1C_2 + C_2C_3 + \ldots\ldots + C_{n-1}C_n = \frac{(2n)!}{(n+1)!(n-1)!}$

53. $C_0C_r + C_1C_{r+1} + C_2C_{r+2} + \ldots\ldots + C_{n-r}C_n = \frac{2n!}{(n-r)!(n+r)!}$

54. $C_0^2 + C_1^2 + C_2^2 + \ldots\ldots + C_n^2 = \frac{(2n)!}{n!n!}$

55. $C_0 - C_1 + C_2 - C_3 + \ldots\ldots + (-1)^r.C_r = \frac{(-1)^r(n-1)!}{r!.(n-r-1)!}$

56. $C_1 + 2C_2 + 3C_3 + \ldots\ldots + n.\ C_n = n.\ 2^{n-1}$

57. $C_0 + 2C_1 + 3C_2 + \ldots\ldots + (n+1)\ C_n = (n+2)\ 2^{n-1}$

58. $C_0+\frac{C_1}{2}+\frac{C_2}{3}+\ldots\ldots+\frac{C_n}{n+1}=\frac{2^{n+1}-1}{n+1}$

59. $\frac{C_1}{C_0}+\frac{2C_2}{C_1}+\frac{3C_3}{C_2}+\ldots\ldots+\frac{n.C_n}{C_{n-1}}=\frac{n(n+1)}{2}$

60. Prove the identity $\frac{1}{^{2n+1}C_r}+\frac{1}{^{2n+1}C_{r+1}}=\frac{2n+2}{2n+1}\frac{1}{^{2n}C_r}$.

61. If $(1+x)^{15}=C_0+C_1.x+C_2.x^2+....+C_{15}.x^{15}$ and $C_2+2C_3+3C_4+....+14C_{15}=a2^b+c$, then find $a+b+c$.

62. Evaluate : $2^{15}\binom{30}{0}\binom{30}{15}-2^{14}\binom{30}{1}\binom{29}{14}+2^{13}\binom{30}{2}\binom{28}{13}......-\binom{30}{15}\binom{15}{0}$

63. Prove that : $\frac{1}{2}\,{}^nC_1-\frac{2}{3}\,{}^nC_2+\frac{3}{4}\,{}^nC_3-\frac{4}{5}\,{}^nC_4+.....+\frac{(-1)^{n+1}n}{n+1}.{}^nC_n=\frac{1}{n+1}$

64. Prove that : $({}^{2n}C_1)^2+2.({}^{2n}C_2)^2+3.({}^{2n}C_3)^2+.....+2n.({}^{2n}C_{2n})^2=\frac{(4n-1)!}{[(2n-1)!]^2}$

65. For any positive integers m, n (with $n \ge m$), let $\binom{n}{m}={}^nC_m$. Prove that :

$$\binom{n}{m}+\binom{n-1}{m}+\binom{n-2}{m}+........+\binom{m}{m}=\binom{n+1}{m+1}$$

Hence or otherwise prove that, **[JEE 2000]**

$$\binom{n}{m}+2\binom{n-1}{m}+3\binom{n-2}{m}+........+(n-m+1)\binom{m}{m}=\binom{n+2}{m+2}.$$

66. If n and k are positive integers, prove that **[JEE 2003]**

$$2^k\binom{n}{0}\binom{n}{k}-2^{k-1}\binom{n}{1}\binom{n-1}{k-1}+2^{k-2}\binom{n}{2}\binom{n-2}{k-2}.....(-1)^k\binom{n}{k}\binom{n-k}{0}=\binom{n}{k}$$

67. Find the sum of the series : $\sum_{r=0}^{n}(-1)^r({}^nC_r)\left[\frac{1}{2^r}+\frac{3^r}{2^{2r}}+\frac{7^r}{2^{3r}}+\frac{15^r}{2^{4r}}+....\text{upto m terms}\right]$ **[JEE 1985]**

68. If $\sum_{r=0}^{2n}a_r(x-2)^r=\sum_{r=0}^{2n}b_r(x-3)^r$ and $a_k=1\ \forall k\ge n$, show that $b_n={}^{2n+1}C_{n+1}$. **[JEE 1992]**

69. Let n be a positive integer $(1+x+x^2)^n=a_0+a_1x+...+a_{2n}x^{2n}$ show that

$a_0^2-a_1^2+a_2^2-....+a_{2n}^2=a_n$ **[JEE 1994]**

ANSWERS

- ***Fill in the Blanks***

1. ${}^{12}C_6$ **2.** 43 **3.** 3^{rd} term **4.** $\frac{np}{2}(p+1)^n$ **5.** $(n + 1)!$

6. $x = 10$ **7.** 5050 **8.** 30^{th} and 31^{st} term terms **9.** 2520

10. 129 **11.** 0 **12.** $a = 2, n = 4$ **13.** $2^{2n-1} - \frac{(2n)!}{2(n!)^2}$

- ***Assertion–Reason***

14. D **15.** C **16.** A **17.** D **18.** C

- ***Single Choice Correct***

19. (B) **20.** (C) **21.** (D) **22.** (B) **23.** (C)
24. (A) **25.** (D) **26.** (A) **27.** (D) **28.** (C)

- ***Multiple Choice Correct***

29. (C) **30.** (A) **31.** (B)

- ***Comprehension***

Comprehension – 1 **32.** B **33.** A **34.** C **35.** C
Comprehension – 2 **36.** A **37.** C

- ***Subjective Questions***

41. ***(a)*** $n = 12$ ***(b)*** $\frac{5}{8} < x < \frac{20}{21}$ **44.** 1 **45.** 5 **47.** 0

61. 28 **62.** $\binom{30}{15}$ **67.** $\frac{2^{nm}-1}{2^{nm}\left(2^n-1\right)}$

SOLUTION OF TRIANGLE

True / False

1. If external angle bisector of any angle of triangle ABC is parallel to the opposite base then triangle is isosceles.

2. Sides of the pedal triangle of any acute or obtuse angle triangle are given by Rsin2A, Rsin2B and Rsin2C.

3. In the triangle ABC, the altitudes p_1, p_2, p_3 are in AP, then a, b, c are in HP.

4. In a triangle ABC, if $a^4 - 2(b^2 + c^2)a^2 + b^4 + b^2c^2 + c^4 = 0$, then $\angle A$ is 60° or 120°

5. In a ΔABC, if $\cos 3A + \cos 3B + \cos 3C = 1$, then the triangle is obtuse angled.

6. In a circle of radius 'r', chords of lengths a and b cms, subtend angles q and 3 q respectively at the centre, then $r = a.\sqrt{\frac{a}{2a-b}}$ cm.

7. In any ΔABC, $\frac{b-c}{r_1} + \frac{c-a}{r_2} + \frac{a-b}{r_3} = 0$

Fill in the Blanks

8. In a ΔABC, tan A : tan B : tan C = 1 : 2 : 3. Hence sinA : sinB : sinC = ____________.

9. In triangle ABC, if a = 2, b = 3 and $\tan A = \sqrt{\frac{3}{5}}$ then the two possible values of the side c are $K_1\sqrt{10}$ and $K_2\sqrt{10}$ then K_1 and K_2 are equal to ___________ and _____________.

10. If f, g and h are the lengths of the perpendiculars from the circumcentre on the sides a, b and c of a triangle ABC respectively then $\frac{a}{f} + \frac{b}{g} + \frac{c}{h} = K\frac{abc}{fgh}$ where K has the value equal to ______________.

11. If in a ΔABC,

$$\frac{2\cos A}{a} + \frac{\cos B}{b} + \frac{2\cos C}{c} = \frac{a}{bc} + \frac{b}{ca}$$, then value of the angle A is _______ .

12. The equation $ax^2 + bx + c = 0$, where a, b, c are the sides of a triangle ABC and the equation $x^2 + \sqrt{2}x + 1 = 0$ have a common root, then angle C is _______.

13. If the radius of the circumcircle of an isosceles triangle PQR is equal to PQ = PR, then the angle P is _______.

14. If the angles of a triangle are 30° and 45° and included side is $(\sqrt{3}+1)$ cm, then the area of the triangle is ...... **[JEE 1988]**

15. ABC is a isosceles triangle inscribed in a circle of radius r. If AB = AC and h is the altitude from A to BC, then the triangle ABC has perimeter $P = 2\left(\sqrt{2hr - h^2} + \sqrt{2hr}\right)$ and area A = ... also $\lim\limits_{x \to 0} \frac{A}{p^3}$ = ... **[JEE 1989]**

16. In a triangle ABC, AD is the altitude from A.

Given b > c, $\angle C = 23°$ and $AD = \frac{abc}{b^2 + c^2}$ then $\angle B$ = ... **[JEE 1994]**

17. A circle is inscribed in an equilateral triangle of side a. The area of any square inscribed in this circle is ....

18. In a trianlge ABC, a : b : c = 4 : 5 : 6. The ratio of the radius of the circumcircle to that of the incircle is .....

[JEE 1996]

19. In a triangle ABC, $\angle A = 90°$, and AD is the altitude, Complete the relation $\frac{BD}{BA} = \frac{AB}{DB} - \left(\frac{.......}{AB \times BD}\right)$

20. ABC is a triangle with $\angle B$ greater then $\angle C$, D and E are points on BC and AE is the bisector of $\angle A$. Complete the relation $\angle DAE = \left(\frac{1}{2}\right)[....... - C]$ ***[JEE 1980]***

Assertion & Reason

These questions contains, Statement I (assertion) and Statement II (reason).

(A) Statement-I is true, Statement-II is true ; Statement-II is correct explanation for Statement-I.
(B) Statement-I is true, Statement-II is true ; Statement-II is NOT a correct explanation for statement-I.
(C) Statement-I is true, Statement-II is false.
(D) Statement-I is false, Statement-II is true.

21. ***Statement-I –*** If two sides of a triangle are 4 and 5, then its area lies in (0, 10]

Because

Statement-II – Area of a triangle $= \frac{1}{2}$ ab sinC and sinC $\in$ (0, 1]

(A) A (B) B (C) C (D) D

22. ***Statement-I –*** Perimeter of a regular pentagon inscribed in a circle with centre O and radius a cm equals $10a\sin 36°$ cm

Because

Statement-II – Perimeter of a regular polygon inscribed in a circle with centre O and radius a cm equals $(3n - 5)\ \sin\left(\frac{360°}{2n}\right)$ cm, then it is n sided, where $n \geq 3$

(A) A (B) B (C) C (D) D

23. ***Statement-I –*** The statement that circumradius and inradius of a triangle are 12 and 8 respectively can not be correct.

Because

Statement-II – Circumradius $\geq$ 2 (inradius)

(A) A (B) B (C) C (D) D

24. ***Statement-I –*** In any triangle ABC, the minimum value of $\frac{r_1 + r_2 + r_3}{r}$ is 9

Because

Statement-II – For any three numbers AM $\geq$ GM

(A) A (B) B (C) C (D) D

25. ***Statement-I –*** Area of triangle having sides greater than 9 can be smaller than area of triangle having sides less than 3.

Because

Statement-II – Sine of an angle of triangle can take any value in (0, 1]

(A) A (B) B (C) C (D) D

Single Choice Correct

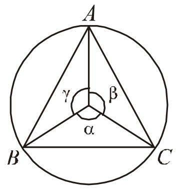
26. The area of cyclic quadrilateral ABCD is $\frac{3\sqrt{3}}{4}$. The radius of the circle circumscribing it is 1. If AB = 1, BD = $\sqrt{3}$ then BC . CD is equal to :

(A) 2 (B) $3-\frac{1}{\sqrt{3}}$ (C) $3\sqrt{3}+1$ (D) None of these

27. The sides of a triangle inscribed in a given circle subtend angle α, β and γ at the centre. The minimum value of the arithmetic mean of $\cos\left(\alpha+\frac{\pi}{2}\right)$, $\cos\left(\beta+\frac{\pi}{2}\right)$ and $\cos\left(\gamma+\frac{\pi}{2}\right)$ is equal to

(A) 0 (B) $\frac{1}{\sqrt{2}}$

(C) –1 (D) $-\frac{\sqrt{3}}{2}$



***28.** In a Δ ABC, $\angle C = 60^\circ$ & $\angle A = 75^\circ$. If D is a point on AC such that the area of the Δ BAD is $\sqrt{3}$ times the area of the Δ BCD, then the $\angle ABD$ =

(A) 60° (B) 30° (C) 90° (D) none of these

***29.** In a triangle ABC, right angled at B, the inradius is -

(A) $\frac{AB+BC-AC}{2}$ (B) $\frac{AB+AC-BC}{2}$ (C) $\frac{AB+BC+AC}{2}$ (D) none

***30.** If the orthocentre and circumcentre of a triangle ABC be at equal distances from the side BC and lie on the same side of BC then tanB tanC has the value equal to -

(A) 3 (B) $\frac{1}{3}$ (C) – 3 (D) $-\frac{1}{3}$

***31.** In ΔABC, if $r : r_1 : R = 2 : 12 : 5$, where all symbols have their usual meaning, then -

(A) ΔABC is an acute angled triangle
(B) ΔABC is an obtuse angled triangle
(C) ΔABC is right angled which is not isosceles
(D) ΔABC is isosceles which is not right angled

32. If α, β, γ are lengths of the altitudes of a triangle ABC then $\frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{\gamma^2}$ is equal to :

(A) $\frac{\cot A+\cot B+\cot C}{\Delta}$ (B) $\frac{\Delta}{\cot A+\cot B+\cot C}$

(C) $\Delta(\cot A+\cot B+\cot C)$ (D) None of these

33. If the radius of the circumcircle of a triangle is 12 and that of the incircle is 4, then the square of the sum of radii of the escribed circle must be

(A) 2704 (B) 2500 (C) 2601 (D) 2116

34. AB is a diameter of a circle and C is any point on the circumference of the circle. Then **[JEE 1983]**

(A) The area of the ΔABC is maximum when it is isosceles.
(B) The area of the ΔABC is minimum when it is isosceles.
(C) The perimeter of the ΔABC is minimum when it is isosceles.
(D) None of these

35. If a, b and c are distinct positive numbers then the expression **[JEE 1986]**
$(b + c - a)(c + a - b)(a + b - c) - abc$ is
(A) positive (B) negative (C) non-positive (D) non-negative
(E) none of these

36. If p_1, p_2, p_3 are the altitudes of a triangle from its vertices A, B, C and Δ, the area of the triangle ABC, then $\frac{1}{p_1}+\frac{1}{p_2}-\frac{1}{p_3}$ is equal to -
(A) $\frac{s}{\Delta}$ (B) $\frac{s-c}{\Delta}$ (C) $\frac{s-b}{\Delta}$ (D) $\frac{s-a}{\Delta}$

37. In a triangle ABC, $\angle B = \pi/3$ and $\angle C = \pi/4$. Let D divide BC intermally in the ratio 1 : 3 then $\frac{\sin\angle BAD}{\sin\angle CAD}$ equals
(A) $\frac{1}{\sqrt{6}}$ (B) $\frac{1}{3}$ (C) $\frac{1}{\sqrt{3}}$ (D) $\sqrt{\frac{2}{3}}$

38. If in a triangle PQR; sin P, sin Q, sin R are in A.P. then **[JEE 1998]**
(A) the altitudes are in A.P. (B) the altitudes are in H.P.
(C) the medians are in G.P. (D) the medians are in A.P.

Multiple Choice Correct

39. In a ΔABC, $A = \frac{\pi}{3}$ and b : c = 2 : 3. If $\tan\alpha = \frac{\sqrt{3}}{5}$, $0 < \alpha < \frac{\pi}{2}$, then -
(A) $B = 60° + \alpha$ (B) $C = 60° + \alpha$ (C) $B = 60° - \alpha$ (D) $C = 60° - \alpha$

40. In a triangle ABC, points D and E are taken on sides BC such that DB = DE= EC. If $\angle ADE = \angle AED = \theta$, then
(A) $\tan\theta = 3\tan B$ (B) $\tan\theta = 3\tan C$ (C) $\tan A = \frac{6\tan\theta}{\tan^2\theta - 9}$ (D) $9\cot^2\frac{A}{2} = \tan^2\theta$

41. In a ΔABC, AD is the bisector of the angle A meeting BC at D. If I is the incentre of the triangle, then AI : DI is equal to -
(A) (sinB + sinC) : sinA (B) (cosB + cosC) : cosA
(C) $\cos\left(\frac{B-C}{2}\right) : \cos\left(\frac{B+C}{2}\right)$ (D) $\sin\left(\frac{B-C}{2}\right) : \sin\left(\frac{B+C}{2}\right)$

42. D, E, F are the foot of the perpendiculars from vertices A, B, C to sides BC, CA, AB respectively, and H is the orthocentre of acute angled triangle ABC; where a, b, c are the sides of triangle ABC, then
(A) H is the incentre of triangle DEF
(B) A, B, C are excentres of triangle DEF
(C) Perimeter of ΔDEF is acosA + bcosB + c cosC
(D) Circumradius of triangle DEF is $\frac{R}{2}$, where R is circumradius of ΔABC.

43. In triangle ABC, cosA + 2cosB + cosC = 2, then -
(A) $\tan\frac{A}{2}\tan\frac{C}{2} = 3$ (B) $\cot\frac{A}{2}\cot\frac{C}{2} = 3$
(C) $\cot\frac{A}{2} + \cot\frac{C}{2} = 2\cot\frac{B}{2}$ (D) $\tan\frac{A}{2}\tan\frac{C}{2} = 0$

44. In a triangle, the lengths of the two larger sides are 10 and 9 respectively. If the angles are in A.P. then the length of the third side can be **[JEE 1987]**
(A) $5-\sqrt{6}$ (B) $3\sqrt{3}$ (C) 5 (D) $5+\sqrt{6}$

Comprehension

Let A_n be the area that is outside a n-sided regular polygon and inside it's circumscribing circle. Also B_n is the area inside the polygon and outside the circle inscribed in the polygon. Let R be the radius of the circle circumscribing n-sided polygon.

On the basis of above information, answer the following questions

45. If n = 6 then A_n is equal to-

(A) $R^2\left(\frac{\pi-\sqrt{3}}{2}\right)$ (B) $R^2\left(\frac{2\pi-6\sqrt{3}}{2}\right)$ (C) $R^2\left(\pi-\sqrt{3}\right)$ (D) $R^2\left(\frac{2\pi-3\sqrt{3}}{2}\right)$

46. If n = 4 then B_n is equal to -

(A) $R^2\frac{(4-\pi)}{2}$ (B) $R^2\frac{(4-\pi\sqrt{2})}{2}$ (C) $R^2\frac{(4\sqrt{2}-\pi)}{2}$ (D) none of these

47. $\frac{A_n}{B_n}$ is equal to $\left(\theta=\frac{\pi}{n}\right)$ -

(A) $\frac{2\theta-\sin 2\theta}{\sin 2\theta-\theta\cos^2\theta}$ (B) $\frac{2\theta-\sin\theta}{\sin 2\theta-\theta\cos^2\theta}$ (C) $\frac{\theta-\cos\theta\sin\theta}{\cos\theta\,(\sin\theta-\theta\cos\theta)}$ (D) none of these

Subjetive Questions

48. A quadrilateral ABCD is inscribed in a circle S and A, B, C, D are the points of contacts with S of another quadrilateral which is circumscribed about S. If this quadrilateral is also cyclic, prove that $(AB)^2 + (CD)^2 = (BC)^2 + (AD)^2$. ***[JEE 1978]***

49. If two sides of a triangle and the included angle are given by $a=\left(1+\sqrt{3}\right)$cm, b = 2 cm, C = 60°, find the other two angles and the third side. ***[JEE 1979]***

50. If a circle is inscribed in a right angled triangle ABC with right angle at B, show that the diameter of the circle is equal to AB + BC – AC. ***[JEE 1979]***

51. ABC is a trianlge with AB = AC, D is any point on the side BC. E and F are the points on the sides AB and AC respectively, such that DE is parallel to AC, and DF is parallel to AB.
Prove that DF + FA + AE + ED = AB + AC. ***[JEE 1980]***

52. Let the angles A, B, C of a triangle ABC be in A.P. and let b : c = $\sqrt{3}:\sqrt{2}$. Find the angle A. ***[JEE 1981]***

53. For a triangle ABC it is given that cos A + cos B + cos C = $\frac{3}{2}$, prove that the triangle is equilateral. ***[JEE 1984]***

54. With usual notation, if in a triangle ABC $\frac{b+c}{11}=\frac{c+a}{12}=\frac{a+b}{13}$, then prove that $\frac{\cos A}{7}=\frac{\cos B}{19}=\frac{\cos C}{25}$

55. In a triangle ABC, the median to the side BC is of length $\frac{1}{\sqrt{11-6\sqrt{3}}}$ and it divides the angle A into angles of 30° and 45°. Find the length of the side BC. ***[JEE 1985]***

56. If in a triangle ABC, cos A cos B + sin A sin B sin C = 1. Show that a : b : c = 1 : 1 : $\sqrt{2}$ ***[JEE 1986]***

57. ABC is a triangle such that sin (2A + B) = sin (C – A) = – sin (B + 2C) = 1/2. If A, B and C are in arithmetic progression, determine the values of A, B and C. ***[JEE 1990]***

58. The sides of a triangle are three consecutive natural numbers and its largest angle is twice the smallest one. Determine the sides of triangle. ***[JEE 1991]***

59. In a triangle of base a, the ratio of the other two sides is $r(<1)$ show that the altitude of the triangle is less than or equal to $ar/(1-r^2)$. ***[JEE 1991]***

60. Let $A_1, A_2, \ldots\ldots A_n$ be the vertices of an n-sided regular polygon such that $\dfrac{1}{A_1A_2} = \dfrac{1}{A_1A_3} + \dfrac{1}{A_1A_4}$, find the value of n. ***[JEE 1994]***

61. Consider the following statements concerning a triangle ABC.
(i) the sides a, b, c and area Δ are rational
(ii) a, tan (B/2), tan (C/2) are rational
(iii) a, sin A, sin B, sin C are rational.
Prove that (i) $\Rightarrow$ (ii) $\Rightarrow$ (iii) $\Rightarrow$ (i)

62. Let A, B, C be three angles such that $A = \pi/4$ and tan B tan C = p. Find all possible values of p such that A, B, C are the angles of a triangle. ***[JEE 1997]***

63. If in a triangle of base 'a', the ratio of the other two sides is r (< 1) . Show that the altitude of the triangle is less than or equal to $\dfrac{ar}{1-r^2}$.

64. If Δ is the area of a triangle with side lengths a, b, c, then show that: $\Delta \le \dfrac{1}{4}\sqrt{(a+b+c)abc}$

Also show that equality occurs in the above inequality if and only if a = b = c. ***[JEE 2001]***

65. If I_n is the area of n sided regular polygon inscribed in a circle of unit radius and O_n be the area of the polygon circumscribing the given circle, prove that $I_n = \dfrac{O_n}{2}\left(1+\sqrt{1-\left(\dfrac{2I_n}{n}\right)^2}\right)$. ***[JEE 2003, Mains, 4 out of 60]***

66. If p_1, p_2, p_3 are the altitudes of a triangle from the vertices A, B, C & Δ denotes the area of the triangle, prove that $\dfrac{1}{p_1}+\dfrac{1}{p_2}-\dfrac{1}{p_3} = \dfrac{2ab}{(a+b+c)\Delta}\cos^2\dfrac{C}{2}$.

****67.*** For any triangle ABC, if B = 3C , show that $\cos C = \sqrt{\dfrac{b+c}{4c}}$ & $\sin\dfrac{A}{2} = \dfrac{b-c}{2c}$.

****68.*** ABC is a triangle. D is the mid point of BC. If AD is perpendicular to AC, then prove that $\cos A \,.\, \cos C = \dfrac{2(c^2-a^2)}{3ac}$.

****69.*** Let 1 < m < 3. In a triangle ABC , if 2 b = (m+1) a & $\cos A = \dfrac{1}{2}\sqrt{\dfrac{(m-1)(m+3)}{m}}$ prove that there are two values to the third side, one of which is m times the other.

70. Consider a ΔDEF, the pedal triangle of the ΔABC such that A–F–B and B–D–C are collinear . If H is the incentre of ΔDEF and R_1, R_2, R_3 are the circumradii of the quadarilaterals AFHE; BDHF and CEHD respectively, then prove that $\sum R_1 = R + r$ where R is the circumradius and r is the inradius of ΔABC.

71. DEF is the triangle formed by joining the points of contact of the incircle with the sides of the triangle ABC, prove that

(a) its sides are $2r\cos\frac{A}{2}$, $2r\cos\frac{B}{2}$ and $2r\cos\frac{C}{2}$ where r is the radius of incircle of ΔABC.

(b) its angles are $\frac{\pi}{2}-\frac{A}{2}$, $\frac{\pi}{2}-\frac{B}{2}$ and $\frac{\pi}{2}-\frac{C}{2}$

(c) its area is $\frac{r^2 s}{2R}$ where 's' is the semiperimeter and R is the circumradius of the ΔABC.

72. If the bisector of angle C of triangle ABC meets AB in D & the circumcircle in E prove that, $\frac{CE}{DE}=\frac{(a+b)^2}{c^2}$.

73. If sides a, b, c, of the triangle ABC are in A.P., then prove that

$\sin^2\frac{A}{2}\operatorname{cosec}2A;\ \sin^2\frac{B}{2}\operatorname{cosec}2B;\ \sin^2\frac{C}{2}\operatorname{cosec}2C$ are in H.P.

74. Sides a, b, c of the triangle ABC are in H.P., then prove that
cosecA (cosecA + cot A) ; cosec B (cosecB + cotB) & cosecC (cosecC + cot C) are in A.P.

75. In a Δ ABC, GA,GB,GC makes angles α, β, γ with each other where G is the centroid to the Δ ABC then show that, $\cot A+\cot B+\cot C+\cot\alpha+\cot\beta+\cot\gamma=0$.

76. In a triangle ABC, the median to the side BC is of length $\frac{1}{\sqrt{11-6\sqrt{3}}}$ & it divides the angle A into angles of $30°$ & $45°$. Find the length of the side BC.

77. Prove that in a triangle $\frac{bc}{r_1}+\frac{ca}{r_2}+\frac{ab}{r_3}=2R\left[\left(\frac{a}{b}+\frac{b}{a}\right)+\left(\frac{b}{c}+\frac{c}{b}\right)+\left(\frac{c}{a}+\frac{a}{c}\right)-3\right]$.

78. In a triangle the angles A, B, C are in A.P. Show that $2\cos\frac{A-C}{2}=\frac{a+c}{\sqrt{a^2-ac+c^2}}$.

79. In a scalene triangle ABC the altitudes AD & CF are dropped form the vertices A & C to the sides BC & AB. The area of ΔABC is known to be equal to 18, the area of triangle BDF is equal to 2 and length of segment DF is equal to $2\sqrt{2}$. Find the radius of the circle circumscribing ΔABC.

80. With reference to a given circle, A_1 and B_1 are the areas of the inscribed and circumscribed regular polygons of n sides, A_2 and B_2 are corresponding quantities for regular polygons of 2n sides : Prove that
(a) A_2 is a geometric mean between A_1 and B_1
(b) B_2 is a harmonic mean between A_2 and B_1

81. Let a, b, c be the sides of a trianlge & Δ its area. Prove that $a^2+b^2+c^2\geq 4\sqrt{3}\Delta$, and find when does the equality hold ?

82. In ΔABC show that

$$\frac{1}{4}r^2s^2\left(\frac{1}{r}-\frac{1}{r_1}\right)\left(\frac{1}{r}-\frac{1}{r_2}\right)\left(\frac{1}{r}-\frac{1}{r_3}\right)=\frac{r+r_1+r_2-r_3}{4\cos C}=R$$

ANSWERS

- ***True and False***

1.	True	***2.***	False	***3.***	True	***4.***	True	***5.***	True
6.	False	***7.***	True						

- ***Fill in the Blanks***

8. $\sqrt{5} : 2\sqrt{2} : 3$ ***9.*** 1 and 1/2 ***10.*** $\frac{1}{4}$ ***11.*** 90°

12. $\frac{\pi}{4}$ ***13.*** $\frac{2\pi}{3}$ ***14.*** $\frac{\left(\sqrt{3}+1\right)}{2}\ cm^2$

15. $h\sqrt{2hr-h^2}\ ;\ \frac{r}{128}$ ***16.*** 113° ***17.*** $\frac{a^2}{6}$ units ***18.*** $\frac{16}{7}$

19. AD^2 ***20.*** ***B***

- ***Assertion–Reason***

21.	A	***22.***	C	***23.***	A	***24.***	C	***25.***	A

- ***Single Choice Correct***

26.	A	***27.***	D	***28.***	B	***29.***	A	***30.***	A
31.	C	***32.***	A	***33.***	A	***34.***	A	***35.***	B
36.	B	***37.***	A	***38.***	B				

- ***Multiple Choice Correct***

39.	BC	***40.***	ABCD	***41.***	AC	***42.***	ABCD	***43.***	BC
44.	(AD)								

- ***Comprehension***

45.	D	***46.***	A	***47.***	C

- ***Subjective Questions***

49. $45^\circ, 75^\circ, \sqrt{6}$ ***52.*** 75° ***55.*** 2 ***57.*** $A = 45^\circ, B = 60^\circ, C = 75^\circ$

58. 4, 5, 6 ***60.*** 7 ***62.*** $p \in (-\infty, 0) \cup \left[3+2\sqrt{2}, \infty\right)$

76. $a = 2$ ***79.*** 9/2 units ***81.*** $b = c\ \&\ A = 60^\circ$

Important Notes

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