Trigonometric equations
CL01 & CL02

09/07/2021

Three Types of solutions :→

(1) Principal solution :→ $0 \leq \theta \leq 2\pi$

(2) Particular Solution :→ Solutions lying in the given interval.

(3) General Solution :→ solution in the form of n.

Type I : →

① $\sin\theta = 0$

$\theta = n\pi \qquad n \in \mathbb{Z}$

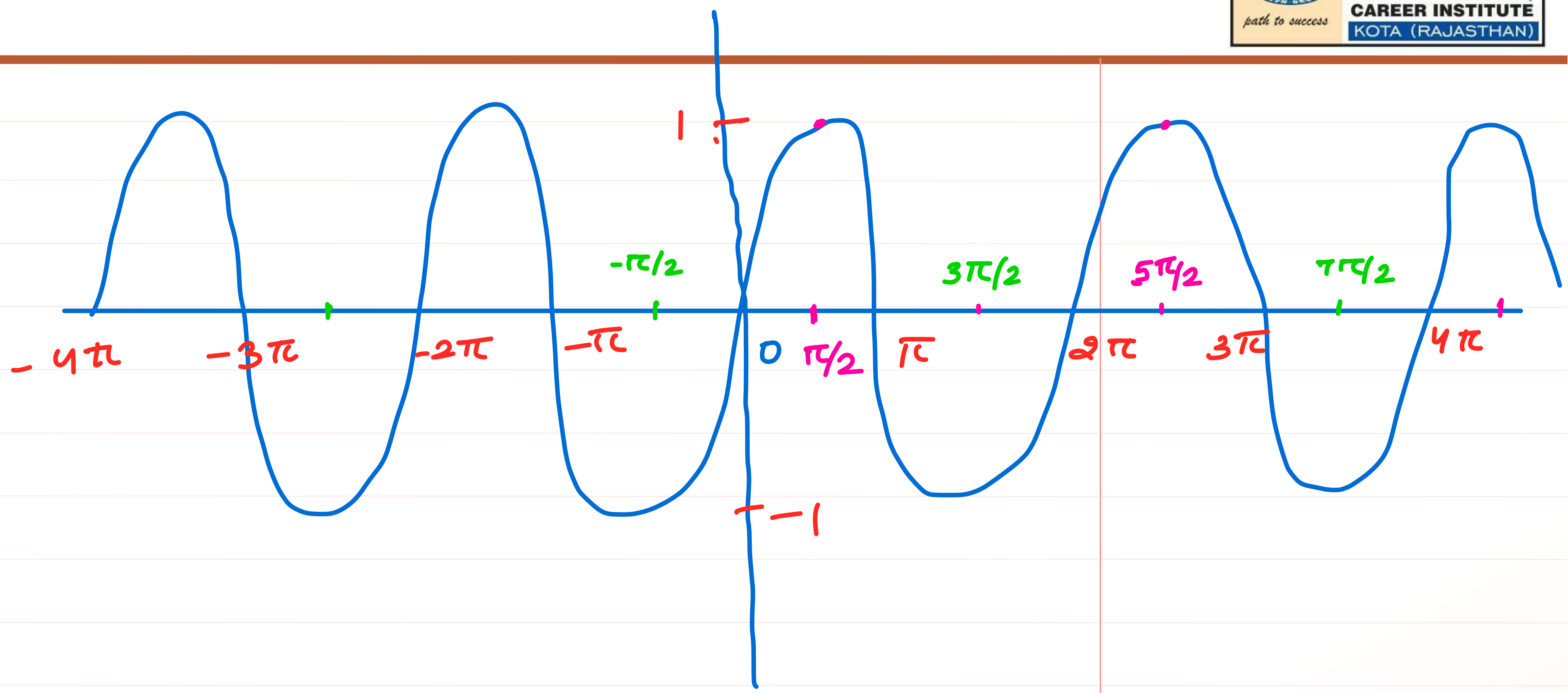
② $\sin\theta = 1$

$\theta = \frac{\pi}{2}, \underbrace{\frac{5\pi}{2}, \frac{9\pi}{2}, \ldots}$

$\theta = \frac{\pi}{2}(4n+1) \quad ; \quad n \in \mathbb{Z}$

OR

$\theta = \frac{\pi}{2}(4n-3) \quad ; \quad n \in \mathbb{Z}.$



$1, 5, 9, \ldots$

$T_n = 1 + (n-1)4$
$= 4n - 3$

$5, 9, \ldots$

$T_n = 5 + (n-1)4$
$= (4n+1)$

③ $\sin\theta = -1$

$\theta = \frac{3\pi}{2}, \frac{7\pi}{2}, \frac{11\pi}{2}$

$\theta = \frac{\pi}{2}(4n-1)\ ; \ n \in Z$

$3, 7, 11, ---$

$T_n = 3 + (n-1)4$

$T_n = 4n-1$

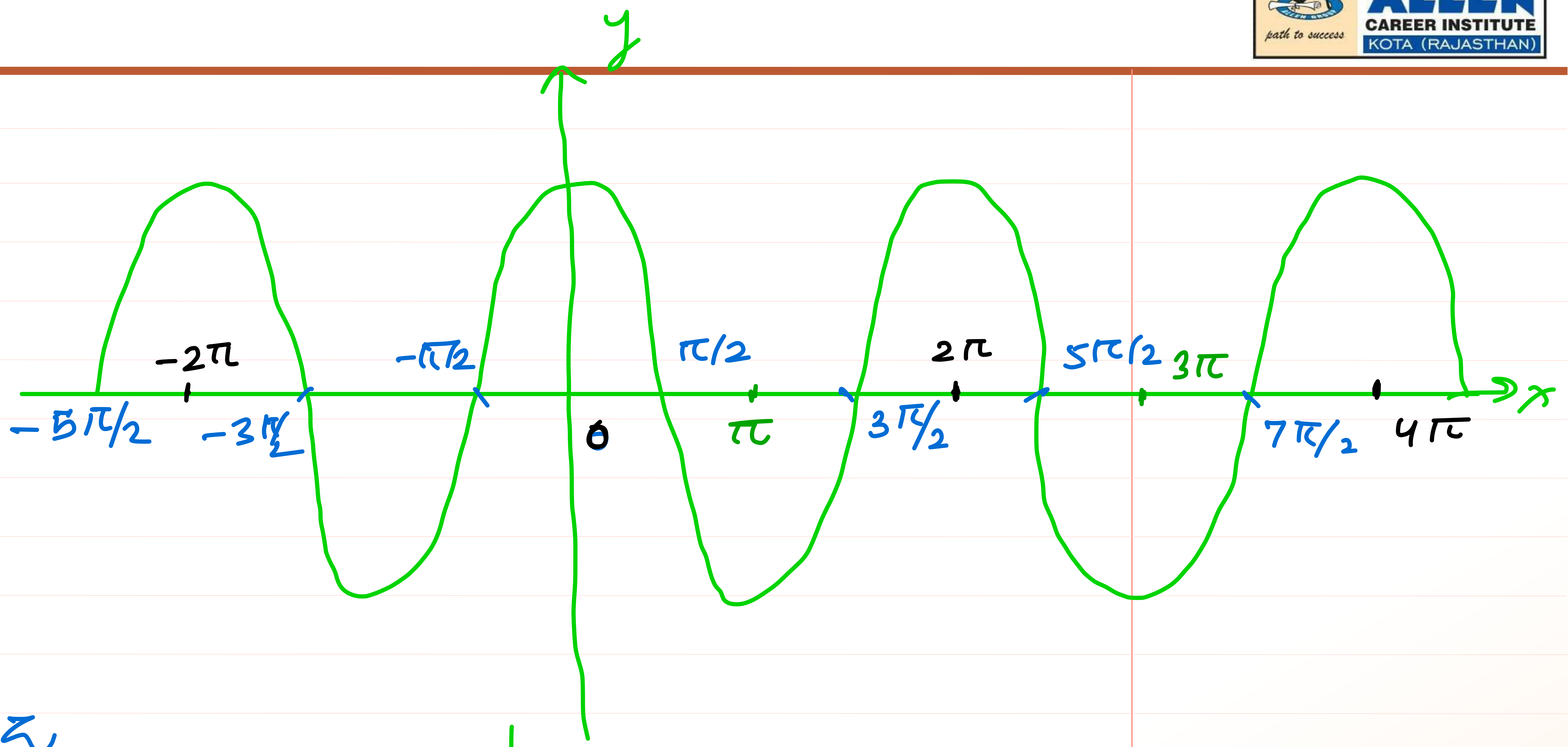
(4) $\cos\theta = 0$

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \ldots$

$\theta = (2n+1)\frac{\pi}{2}$; $n \in \mathbb{Z}$

or

$\theta = (2n-1)\frac{\pi}{2}$; $n \in \mathbb{Z}$



(5) $\cos\theta = 1$

$\theta = 0, 2\pi, 4\pi, 6\pi, 8\pi, \ldots$

$\theta = 2n\pi$; $n \in \mathbb{Z}$

(6) $\cos\theta = -1$

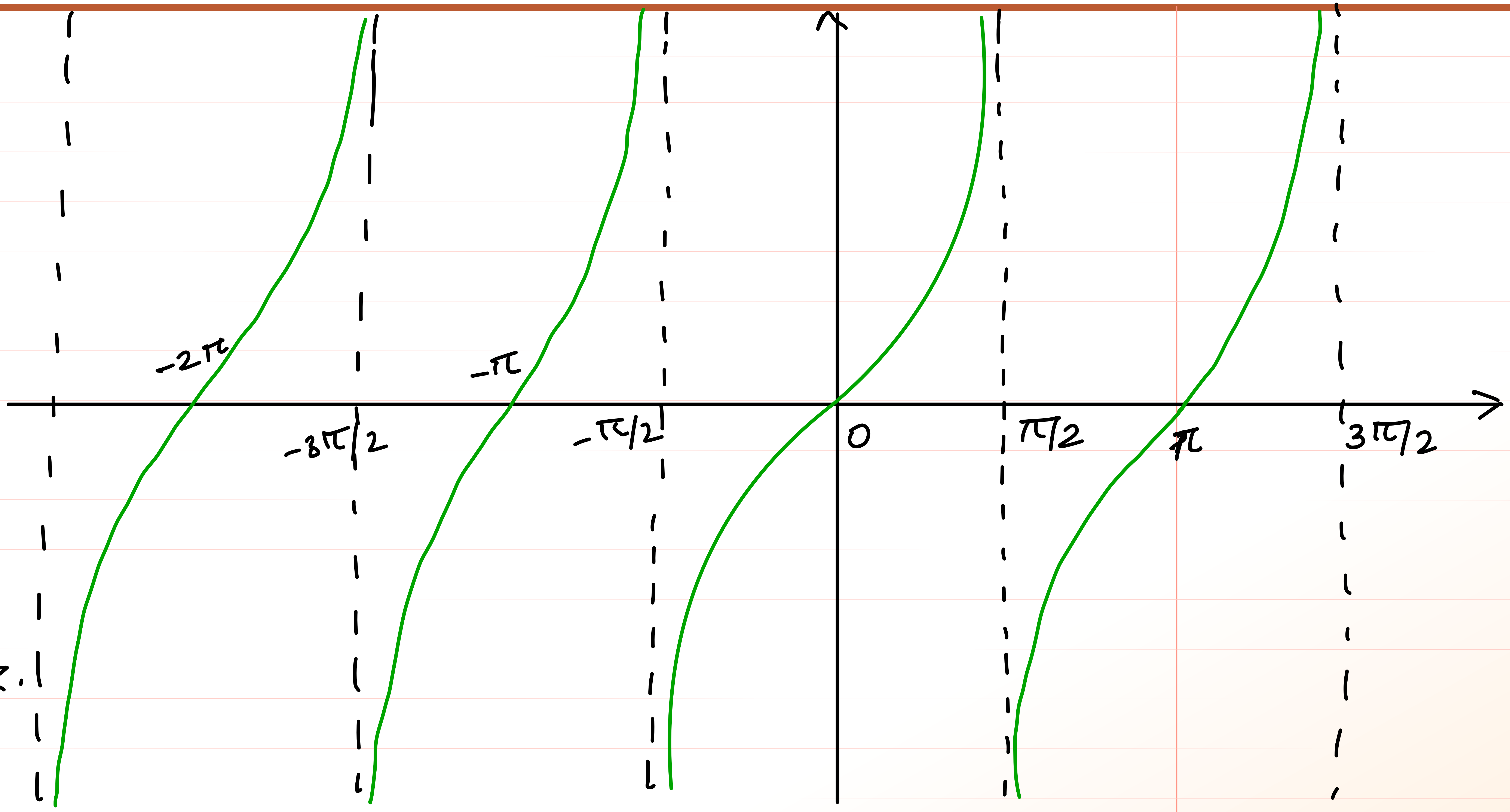
$\theta = (2n+1)\pi$; $n \in \mathbb{Z}$

or

$\theta = (2n-1)\pi$; $n \in \mathbb{Z}$

⑦ $\tan\theta = 0$

$\theta \in n\pi \,;\ n \in Z$



⑧ $\cot\theta = 0$

$\theta \in (2n+1)\frac{\pi}{2}\,;\ n \in z.$

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Type 2 ① If $\sin\theta = \sin\alpha$ $\alpha \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$$\theta = n\pi + (-1)^n \alpha \; ; n \in Z$$

$$\sin\theta - \sin\alpha = 0$$

$$2\cos\left(\frac{\theta+\alpha}{2}\right) \cdot \sin\left(\frac{\theta-\alpha}{2}\right) = 0$$

$$\cos\left(\frac{\theta+\alpha}{2}\right) = 0$$

$$\frac{\theta+\alpha}{2} = (2m+1)\frac{\pi}{2} \; ; m \in Z$$

$$\theta + \alpha = (2m+1)\pi$$

$$\theta = (2m+1)\pi - \alpha$$

$$\theta = (2m+1)\pi + (-1)\alpha$$

$$\theta = (2m+1)\pi + (-1)^{2m+1} \cdot \alpha$$

$$\sin\left(\frac{\theta-\alpha}{2}\right) = 0$$

$$\frac{\theta-\alpha}{2} = m\pi$$

$$\theta - \alpha = 2m\pi$$

$$\theta = 2m\pi + \alpha$$

$$\theta = 2m\pi + (-1)^{2m} \cdot \alpha$$

$$\theta = n\pi + (-1)^n \alpha \quad n \in Z$$

(i) Solve $2\cos^2\theta + 3\sin\theta = 0$

$$2(1-\sin^2\theta) + 3\sin\theta = 0$$

$$2 - 2\sin^2\theta + 3\sin\theta = 0$$

$$2\sin^2\theta - 3\sin\theta - 2 = 0$$

$$2\sin^2\theta - 4\sin\theta + \sin\theta - 2 = 0$$

$$(2\sin\theta + 1)(\sin\theta - 2) = 0$$

$$\sin\theta = -\frac{1}{2} \qquad \sin\theta = 2 \ \times$$

$$\sin\theta = \sin\left(-\frac{\pi}{6}\right)$$

$$\theta = n\pi + (-1)^n\left(-\frac{\pi}{6}\right)$$

$$\boxed{\theta = n\pi + (-1)^{n+1}\left(\frac{\pi}{6}\right)} ; \ n \in \mathbb{Z}$$

② $\cos\theta = \cos\alpha$ $\qquad \alpha \in [0, \pi]$

$\theta = 2n\pi \pm \alpha \qquad n \in Z$

$\cos\theta - \cos\alpha = 0$

$-2\sin\left(\frac{\theta+\alpha}{2}\right) \cdot \sin\left(\frac{\cancel{\alpha}+\theta}{2}\right) = 0$

$\sin\left(\frac{\theta+\alpha}{2}\right) = 0 \qquad\qquad \sin\left(\frac{\theta-\alpha}{2}\right) = 0$

$\left(\frac{\theta+\alpha}{2}\right) = n\pi \qquad\qquad \frac{\theta-\alpha}{2} = n\pi$

$\theta+\alpha = 2n\pi \qquad\qquad \theta - \alpha = 2n\pi$

$\theta = 2n\pi - \alpha \qquad\qquad \theta = 2n\pi + \alpha$

$\boxed{\theta = 2n\pi \pm \alpha} \qquad n \in Z$

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(3) $\tan\theta = \tan\alpha$ $\quad \alpha \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\theta = n\pi + \alpha \quad ; n \in \mathbb{Z}$

$\tan\theta = \tan\alpha$

$$\frac{\sin\theta}{\cos\theta} = \frac{\sin\alpha}{\cos\alpha}$$

$$\sin\theta\cos\alpha = \sin\alpha\cos\theta$$

$$\sin\theta\cos\alpha - \sin\alpha\cos\theta = 0$$

$$\sin(\theta - \alpha) = 0$$

$$\theta - \alpha = n\pi$$

$$\theta = n\pi + \alpha \qquad n \in \mathbb{Z}.$$

Q

$$\tan 3\theta = -1$$

$$\tan 3\theta = \tan\left(-\frac{\pi}{4}\right)$$

$$3\theta = n\pi + \left(-\frac{\pi}{4}\right)$$

$$3\theta = n\pi - \frac{\pi}{4}$$

$$\boxed{\theta = \frac{n\pi}{3} - \frac{\pi}{12}} \quad n \in \mathbb{Z}.$$

Answer

Q

$$\sqrt{3}\sec 2\theta = 2$$

$$\cos 2\theta = \frac{\sqrt{3}}{2}$$

$$\cos 2\theta = \cos\left(\frac{\pi}{6}\right)$$

$$2\theta = 2n\pi \pm \frac{\pi}{6}$$

$$\theta = n\pi \pm \frac{\pi}{12} \quad ; \quad n \in Z$$

Type-3 $\sin^2\theta = \sin^2\alpha$ OR $\cos^2\theta = \cos^2\alpha$ OR $\tan^2\theta = \tan^2\alpha$

$$\sin^2\theta - \sin^2\alpha = 0$$

$$\underbrace{\sin(\theta+\alpha)}\cdot\underbrace{\sin(\theta-\alpha)} = 0$$

$\sin(\theta+\alpha) = 0$ OR $\sin(\theta-\alpha) = 0$

$\theta + \alpha = n\pi$ $\qquad$ $\theta - \alpha = n\pi$

$\theta = n\pi - \alpha$ OR $\theta = n\pi + \alpha$

$$\boxed{\theta = n\pi \pm \alpha \quad \forall\ n \in Z}$$

$$\tan^2\theta = \tan^2\alpha$$

$$\tan\theta = \pm\tan\alpha$$

$$\boxed{\theta = n\pi \pm \alpha}$$

$$\tan^2\theta = \tan^2\alpha$$

$$(\tan\theta + \tan\alpha)(\tan\theta - \tan\alpha) = 0$$

$$\left(\frac{\sin\theta}{\cos\theta} + \frac{\sin\alpha}{\cos\alpha}\right)\left(\frac{\sin\theta}{\cos\theta} - \frac{\sin\alpha}{\cos\alpha}\right) = 0$$

$$\frac{\sin(\theta+\alpha)}{\cos\theta\cos\alpha}\cdot\frac{\sin(\theta-\alpha)}{\cos\theta\cos\alpha} = 0$$

$$\sin(\theta+\alpha)\cdot\sin(\theta-\alpha) = 0$$

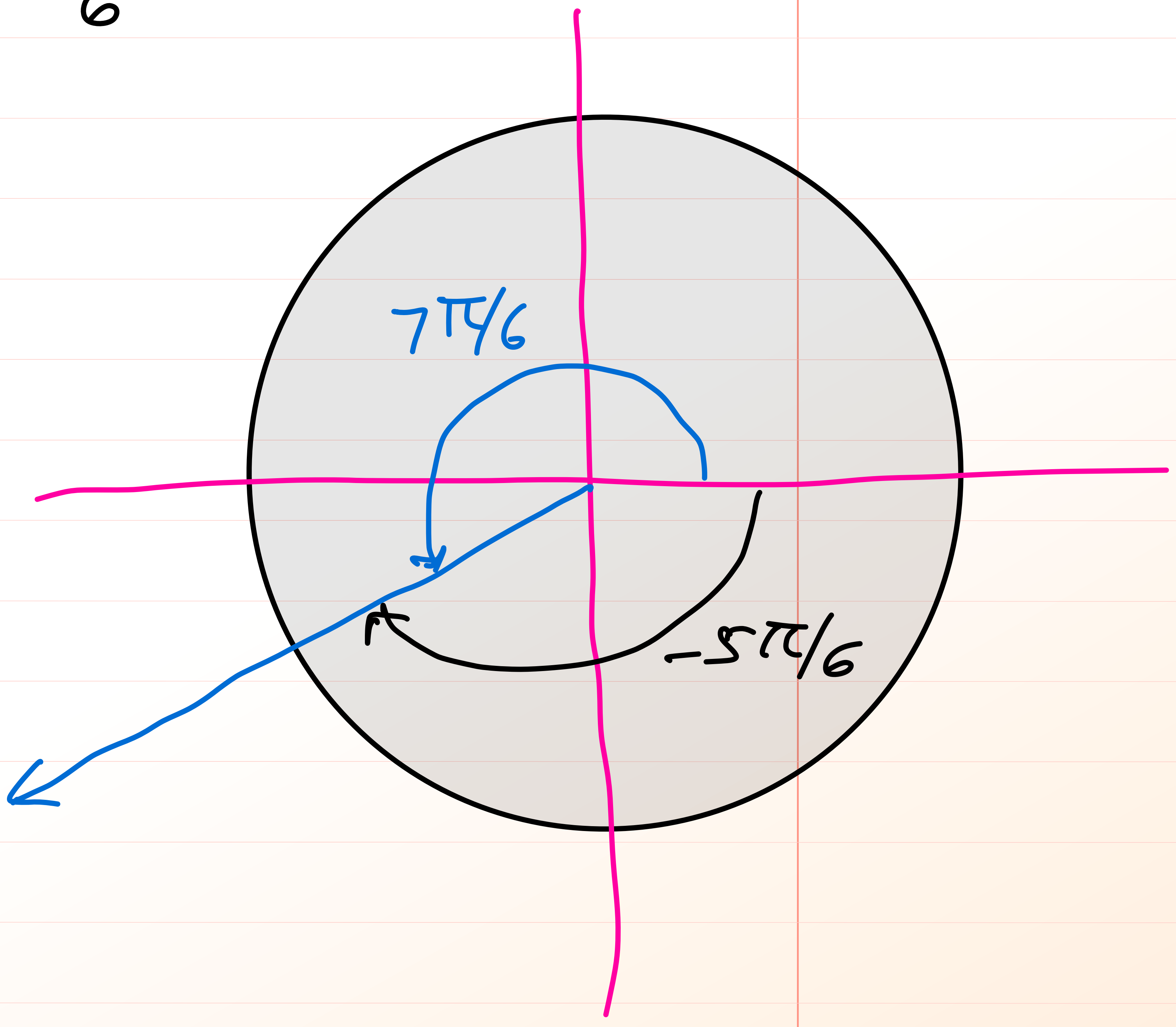
Q what is the most general value of θ which satisfy both equations

$\sin\theta = -\frac{1}{2}$ and $\tan\theta = \frac{1}{\sqrt{3}}$ 3rd quad

$\theta = \frac{7\pi}{6}$ $\theta = \frac{7\pi}{6}$

$$\boxed{\theta = 2n\pi + \frac{7\pi}{6}} \quad \forall n \in Z$$

$$\boxed{\theta = 2n\pi - \frac{5\pi}{6}} \quad \forall n \in Z.$$



Q $2+7\tan^2\theta = \frac{13}{4}\sec^2\theta$

$2+7\tan^2\theta = \frac{13}{4}(1+\tan^2\theta)$

$8+28\tan^2\theta = 13+13\tan^2\theta$

$15\tan^2\theta = 5$

$\tan^2\theta = \frac{1}{3} = \left(\frac{1}{\sqrt{3}}\right)^2$

$\tan^2\theta = \tan^2(\pi/6)$

$\theta = n\pi \pm \frac{\pi}{6}$

$2\cos^2\theta + 7\sin^2\theta = \frac{13}{4\cancel{\cos^2\theta}} \cdot \cancel{\cos^2\theta}$

$8\cos^2\theta + 28\sin^2\theta = 13$

$8-8\sin^2\theta + 28\sin^2\theta = 13$

$20\sin^2\theta = 5$

$\sin^2\theta = \frac{1}{4} = \sin^2\frac{\pi}{6}$

$\boxed{\theta = n\pi \pm \frac{\pi}{6}}$

Q. $\sin^2 x + 2\tan^2 x + \frac{4}{\sqrt{3}}\tan x - \sin x + \frac{11}{12} = 0$

$$\left(\sin^2 x - \sin x\right) + 2\left(\tan^2 x + \frac{2}{\sqrt{3}}\tan x\right) + \frac{11}{12} = 0$$

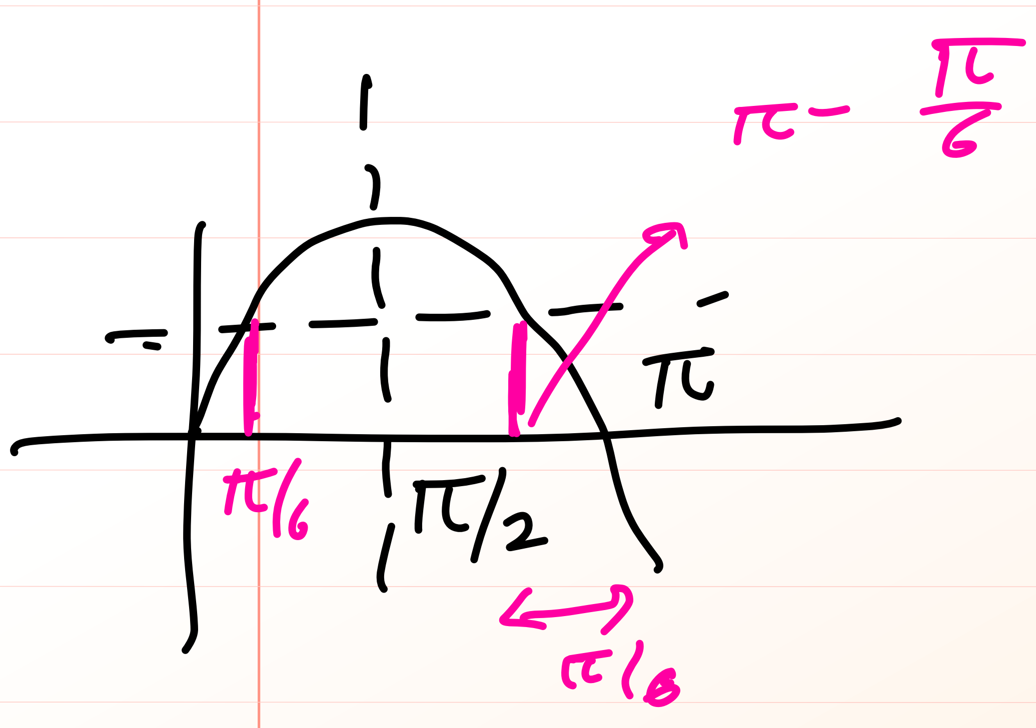
$$\left(\sin^2 x - \sin x + \frac{1}{4}\right) + 2\left(\tan^2 x + \frac{2}{\sqrt{3}}\tan x + \frac{1}{3}\right) \underbrace{- \frac{2}{3} + \frac{11}{12} - \frac{1}{4}}_{} = 0$$

$$\left(\sin x - \frac{1}{2}\right)^2 + 2\left(\tan x + \frac{1}{\sqrt{3}}\right)^2 = 0$$

$\sin x - \frac{1}{2} = 0$ and $\tan x + \frac{1}{\sqrt{3}} = 0$

$\sin x = \frac{1}{2}$ $\qquad$ $\tan x = -\frac{1}{\sqrt{3}}$

$x = \frac{5\pi}{6}$ $\qquad$ $x = \frac{5\pi}{6}$



$$\boxed{x = 2n\pi + \frac{5\pi}{6} \;\; \forall \; n \in \mathbb{Z}}$$

Types of Trigo eqn :→

(a) Type 1 : Solving by factorization :→

(i) $(2\sin x - \cos x)(1+\cos x) = \sin^2 x$

$(2\sin x - \cos x)(1+\cos x) = 1-\cos^2 x$

$(2\sin x - \cos x)(1+\cos x) - (1+\cos x)(1-\cos x) = 0$

$(1+\cos x)\cdot[2\sin x - \cancel{\cos x} - 1 + \cancel{\cos x}] = 0$

$\cos x = -1$ OR $\sin x = \frac{1}{2}$

$\boxed{x = (2m+1)\pi \ \forall m \in Z}$ $x = n\pi + (-1)^n \frac{\pi}{6}; n \in Z$

Principal sol: $\theta \in \{\pi, \frac{\pi}{6}, \frac{5\pi}{6}\}$

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(2) $2\cos x \cdot \cos 2x = \cos x$

$\cos x \left[2\cos 2x - 1 \right] = 0$

$\cos x = 0$ OR $2\cos 2x - 1 = 0$

$\boxed{x \in (2m+1)\frac{\pi}{2} \;\forall\; m \in Z}$

$\cos 2x = \frac{1}{2}$

$\cos 2x = \cos\left(\frac{\pi}{3}\right)$

$2x = 2n\pi \pm \frac{\pi}{3}$

$\boxed{x = n\pi \pm \frac{\pi}{6}}$

$2\cos 2x - 1 = 0$

$2(2\cos^2 x - 1) - 1 = 0$

$\cos^2 x = \frac{3}{4}$

$\cos^2 x = \cos^2 \frac{\pi}{6}$

$\boxed{x = n\pi \pm \frac{\pi}{6}}$

$2(1 - 2\sin^2 x) - 1 = 0$

$-4\sin^2 x + 1 = 0$

$\sin^2 x = \frac{1}{4}$

$\boxed{x = n\pi \pm \frac{\pi}{6}}$

Q

$$\cot x - \cos x = 1 - \cot x \cdot \cos x$$

$$\underline{\cot x} - \cos x - 1 + \underline{\cot x \cos x} = 0$$

$$\cot x(1 + \cos x) - 1(\cos x + 1) = 0$$

$$(\cos x + 1)(\cot x - 1) = 0$$

$\cot x = 1$ OR $\cos x = -1$

$$\boxed{x = n\pi + \frac{\pi}{4}}$$

Rejected