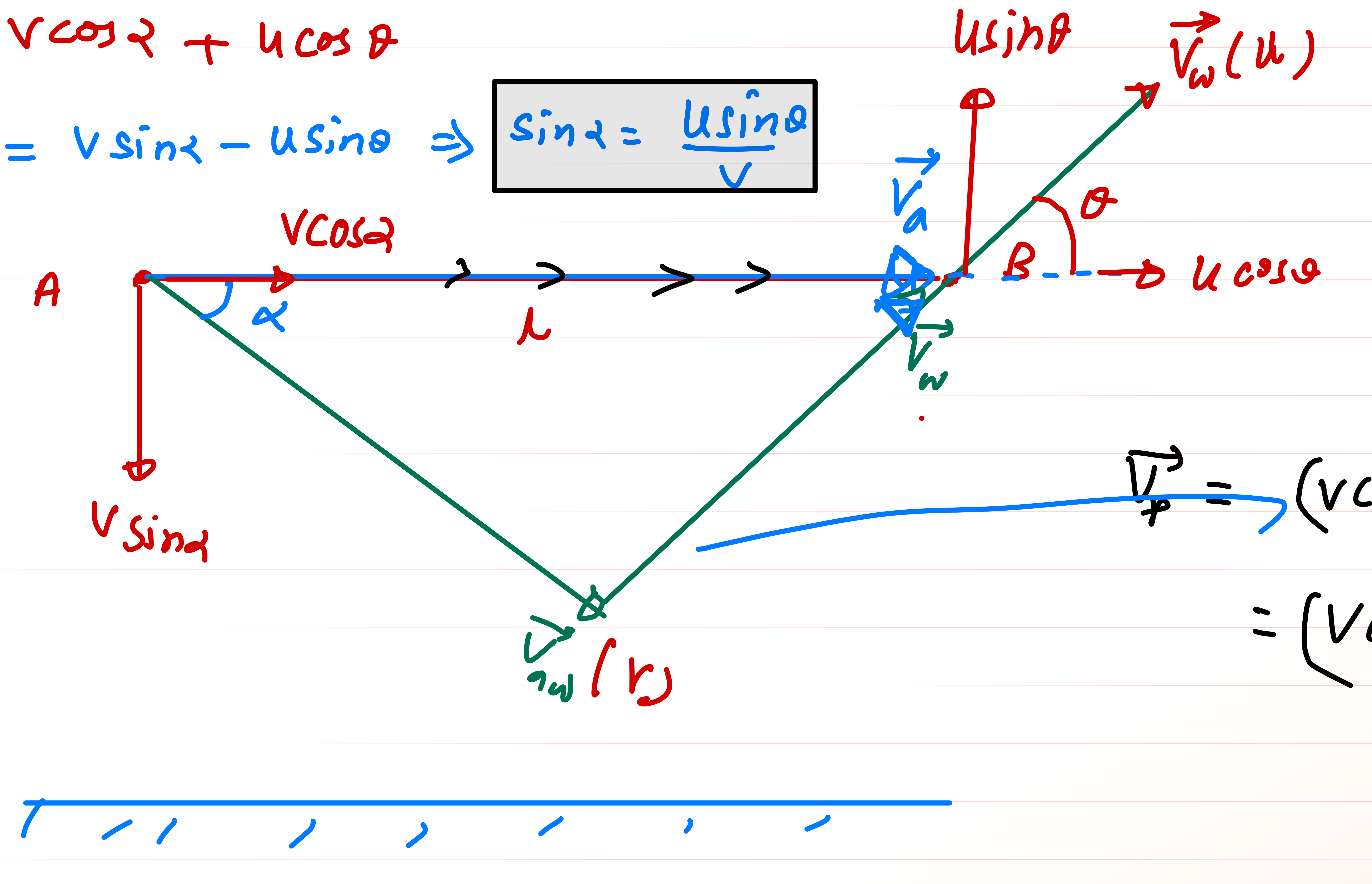
$\vec{V}_a = \vec{V}_{aw} + \vec{V}_w$

Illustration 5*. An aeroplane flies along a straight path A to B and returns back again. The distance between A and B is ℓ and the aeroplane maintains the constant speed v. There is a steady wind with a speed u at an angle θ with line AB. Determine the expression for the total time of the trip.

$(V_p)_{\parallel} = v\cos\alpha + u\cos\theta$

$(V_p)_{\perp} = 0 = v\sin\alpha - u\sin\theta \Rightarrow \boxed{\sin\alpha = \dfrac{u\sin\theta}{v}}$

$$\boxed{t_{AB} = \frac{\ell}{v\cos\alpha + u\cos\theta}}$$



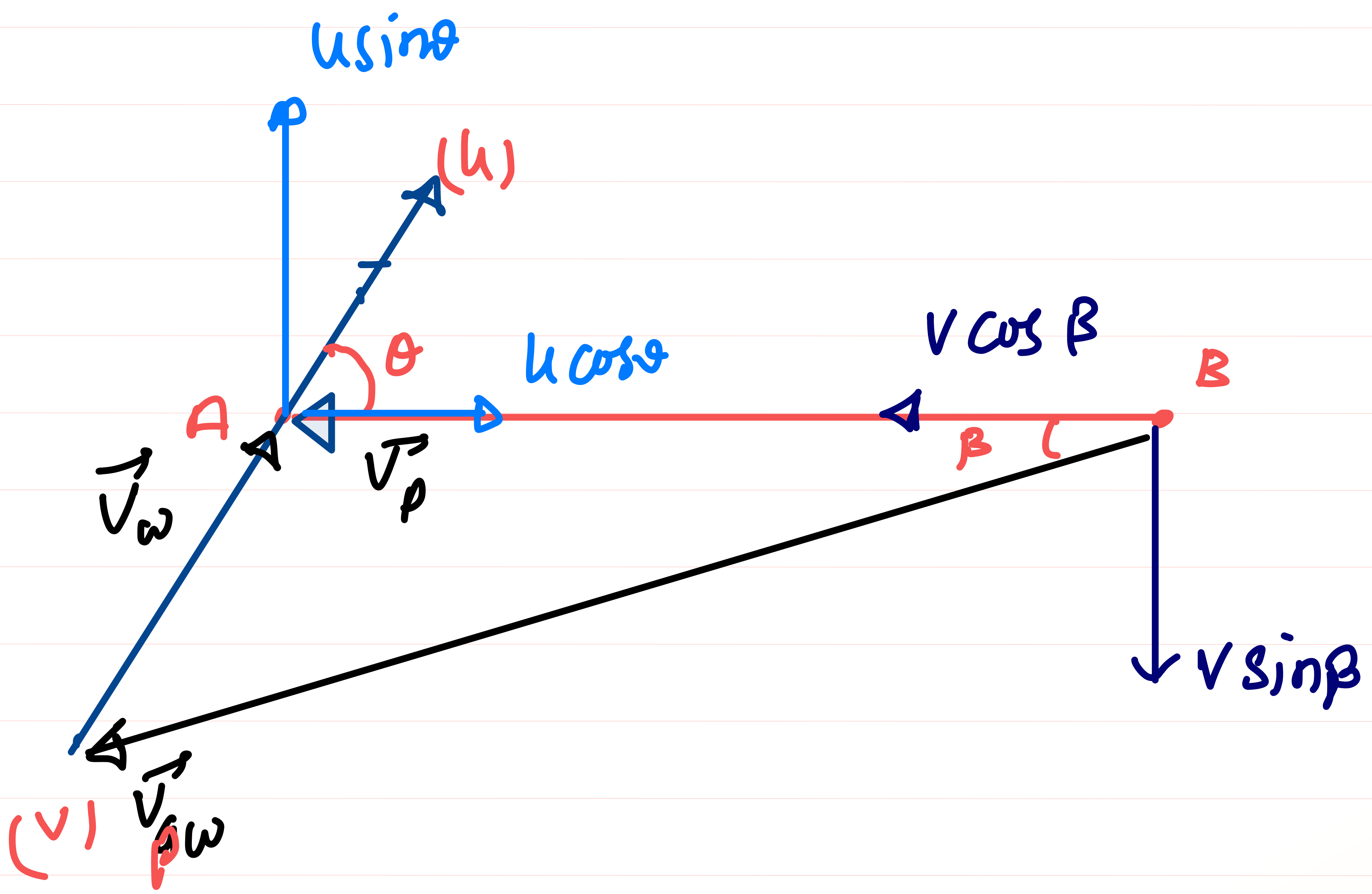
$\vec{V}_p = (v\cos\alpha\,\hat{i} - v\sin\alpha\,\hat{j}) + (u\cos\theta)\hat{i} + u\sin\theta\,\hat{j}$

$= \underbrace{(v\cos\alpha + u\cos\theta)}_{(V_p)_{\parallel}}\hat{i} + \underbrace{(-v\sin\alpha + u\sin\theta)}_{(V_p)_{\perp}}\hat{j}$

$$t_{AB} = \frac{l}{v\sqrt{1-\sin^2\alpha} + u\cos\theta}$$

$$= \frac{l}{v\sqrt{1-\left(\frac{u\sin\theta}{v}\right)^2} + u\cos\theta}$$

time taken from B to A :→



$$t_{BA} = \frac{l}{v\cos\beta - u\cos\theta}$$

$(V_p)_\perp$ to line AB = 0

$v\sin\beta = u\sin\theta$

$$\sin\beta = \frac{u\sin\theta}{v}$$

$$t_{BA} = \frac{l}{v\sqrt{1-\left(\frac{u\sin\theta}{v}\right)^2} - u\cos\theta}$$

total time $= t_{AB} + t_{BA}$

$$= \frac{l}{v\sqrt{1-\frac{u^2\sin^2\theta}{v^2}} + u\cos\theta} + \frac{l}{v\sqrt{1-\frac{u^2\sin^2\theta}{v^2}} - u\cos\theta}$$

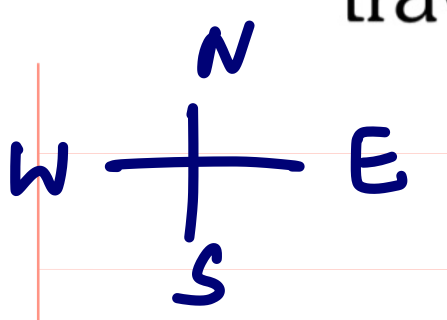
$$= l\left[\frac{2v\sqrt{1-u^2\sin^2\theta/v^2}}{v^2\left(1-\frac{u^2\sin^2\theta}{v^2}\right) - u^2\cos^2\theta}\right]$$

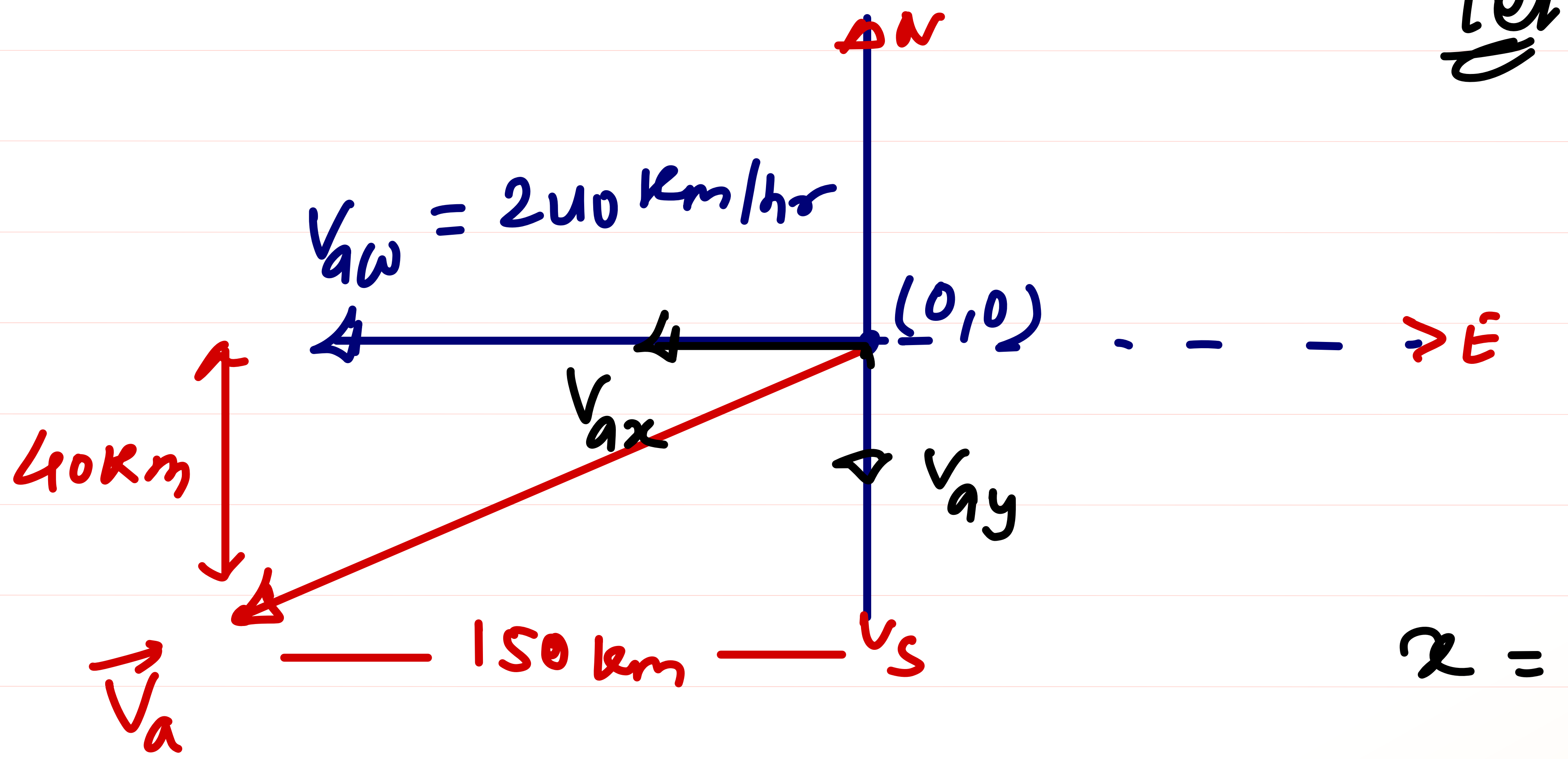
total time $= \dfrac{2lv\sqrt{1 - \dfrac{u^2\sin^2\theta}{v^2}}}{v^2 - u^2}$ Ans

7*. An airplane pilot sets a compass course due west and maintains an air speed of 240 km. hr^{-1}. After flying for $\frac{1}{2}$ hr, he finds himself over a town that is 150 km west and 40 km south of his starting point.

(a) Find the wind velocity, in magnitude and direction.

(b) If the wind velocity were 120 km. hr^{-1} due south, in what direction should the pilot set his course in order to travel due west ? Take the same air speed of 240 km. hr^{-1}.





let $\vec{V}_w = v_x\hat{i} + v_y\hat{j}$

$\vec{V}_a = \vec{V}_{aw} + \vec{V}_w$

$= 240(-\hat{i}) + v_x\hat{i} + v_y\hat{j}$

$\vec{V}_a = (v_x - 240)\hat{i} + v_y\hat{j}$

$x = V_{ax}\, t$

$-150 = (v_x - 240) \times \frac{1}{2}$

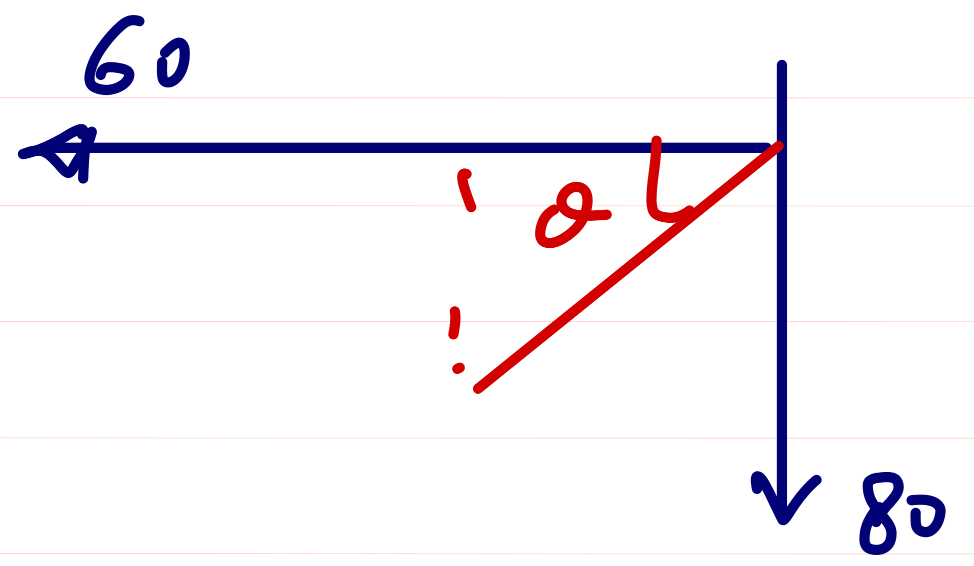
$\boxed{V_x = -60 \text{ Km/hr}}$

$y = V_{ay}\, t$

$-40 = v_y \times \frac{1}{2} \Rightarrow \boxed{v_y = -80}$

$\vec{V}_w = -60\hat{i} - 80\hat{j}$

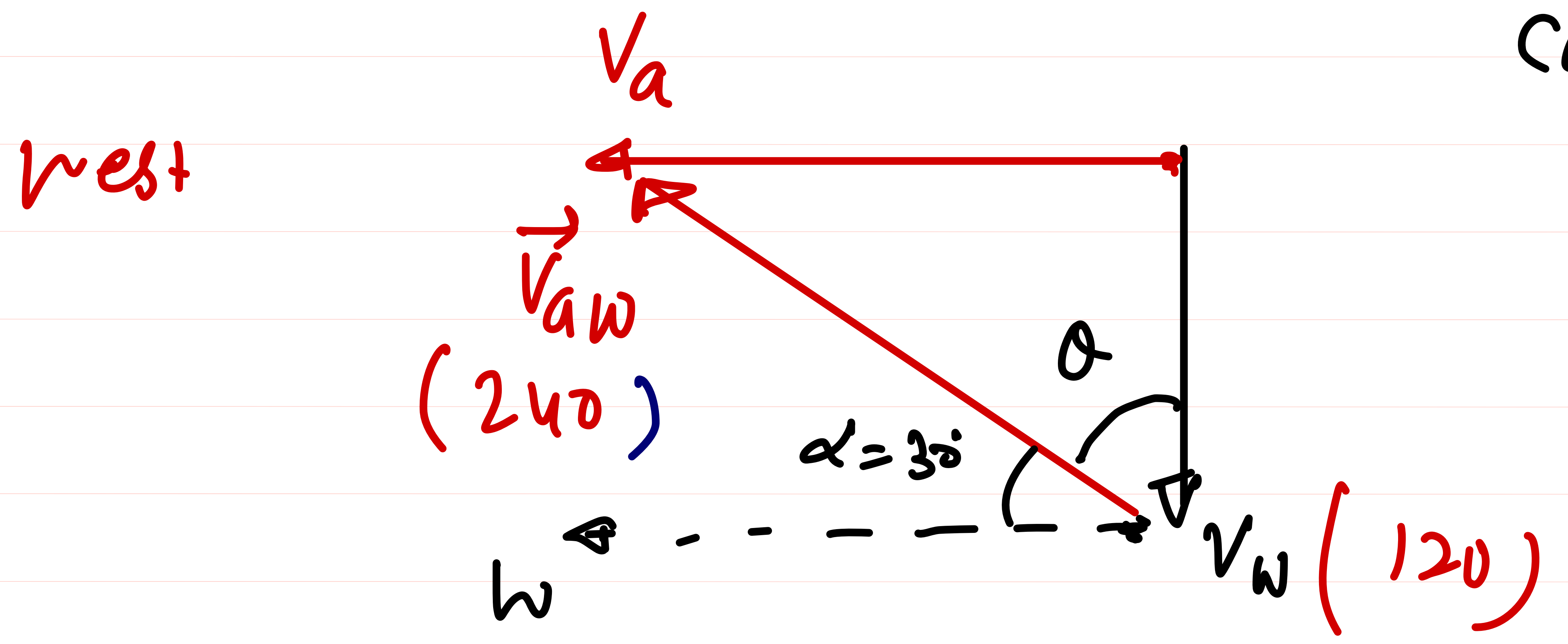
$V_w = 100 \text{ km/hr}$



$\tan\theta = \frac{80}{60} = \frac{4}{3} \Rightarrow \boxed{53^\circ = \theta}$

Ans $\boxed{V_w = 100 \text{ km/hr} \quad 53^\circ \text{ due S of W}}$ Ans

(b) If the wind velocity were 120 km. hr^{-1} due south, in what direction should the pilot set his course in order to travel due west ? Take the same air speed of 240 km. hr^{-1}.



$\cos\theta = \frac{120}{240} = \frac{1}{2} \Rightarrow \theta = 60^\circ$

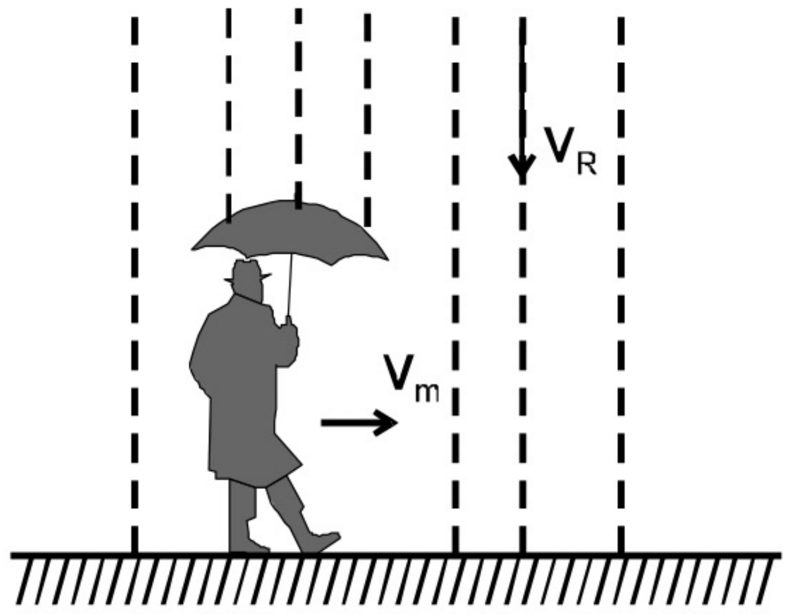
$\alpha = 30$ N of W

$\theta = 60^\circ$ W of N

RAIN PROBLEMS

SL AL

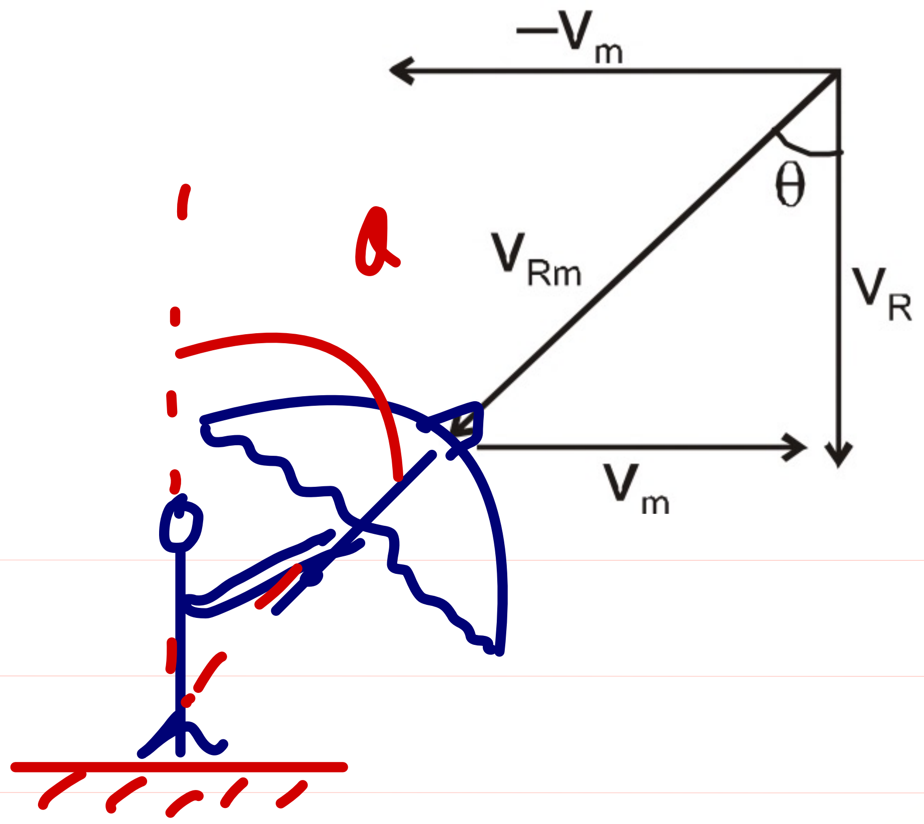
If rain is falling vertically with a velocity $\vec{v}_R$ and an observer is moving horizontally with velocity $\vec{v}_m$, the velocity of rain relative to observer will be :



$$\vec{v}_{Rm} = \vec{v}_R - \vec{v}_m$$

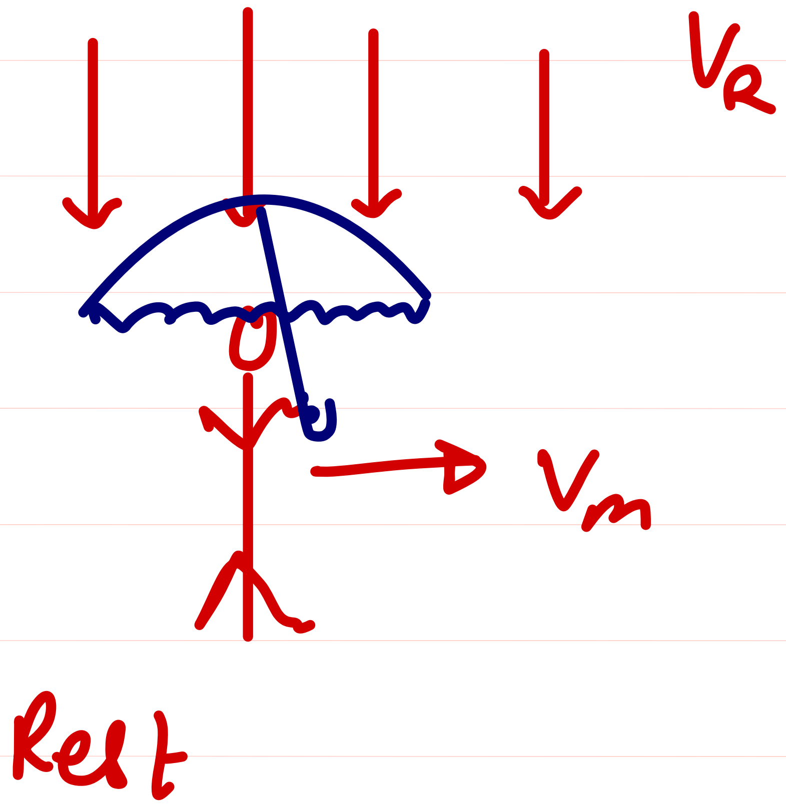
or $$v_{Rm} = \sqrt{v_R^2 + v_m^2}$$

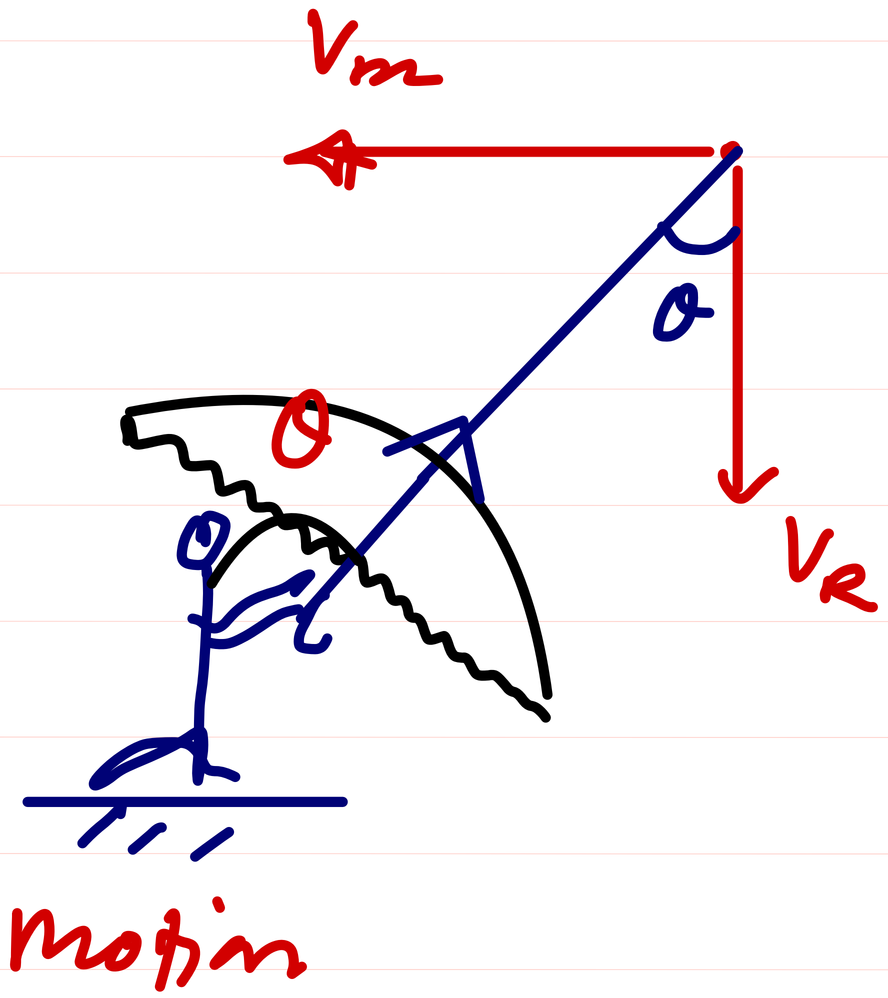
and direction $\boxed{\theta = \tan^{-1}\left(\frac{v_m}{v_R}\right)}$ with the vertical as shown in figure.



$\tan\theta = \frac{v_m}{v_R}$

Illustration 6. A man when standstill observes the rain falling vertically and when he walks at 4 km/h he has to hold his umbrella at an angle of 53° from the vertical. Find velocity of the raindrops.





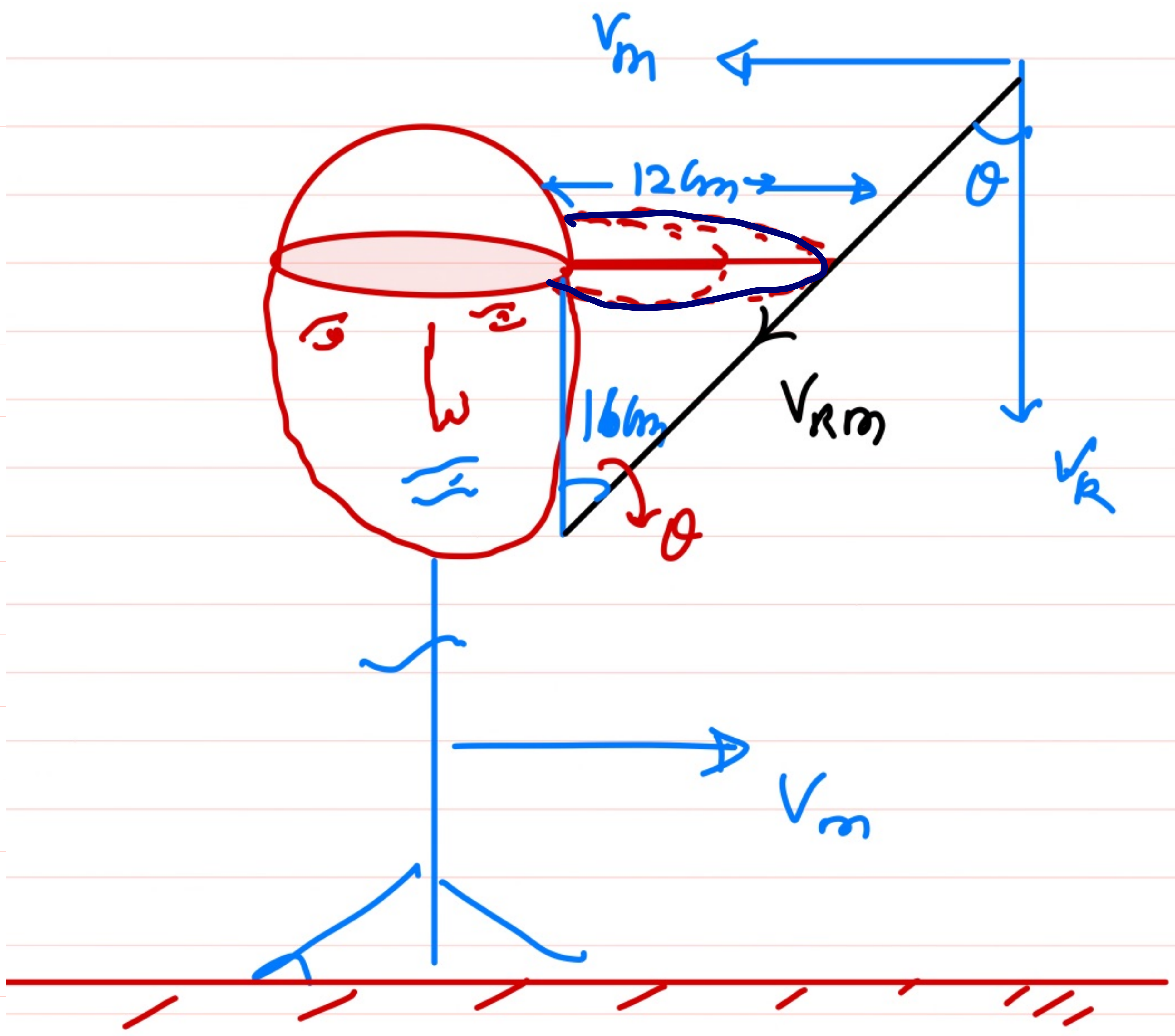
$$\tan\theta = \frac{V_m}{V_R}$$

$$\tan(53^\circ) = \frac{V_m}{V_R}$$

$$\frac{4}{3} = \frac{4}{V_R}$$

$$\boxed{V_R = 3\ \text{km/hr}}$$ Ans

Illustration 7. A man wearing a hat of extended length 12 cm is running in rain falling vertically downwards with speed 10 m/s. The maximum speed with which man can run, so that rain drops do not fall on his face (the length of his face below the extended part of the hat is 16 cm) will be :



$$\tan\theta = \frac{V_m}{V_R} = \frac{12}{16} = \frac{3}{4}$$

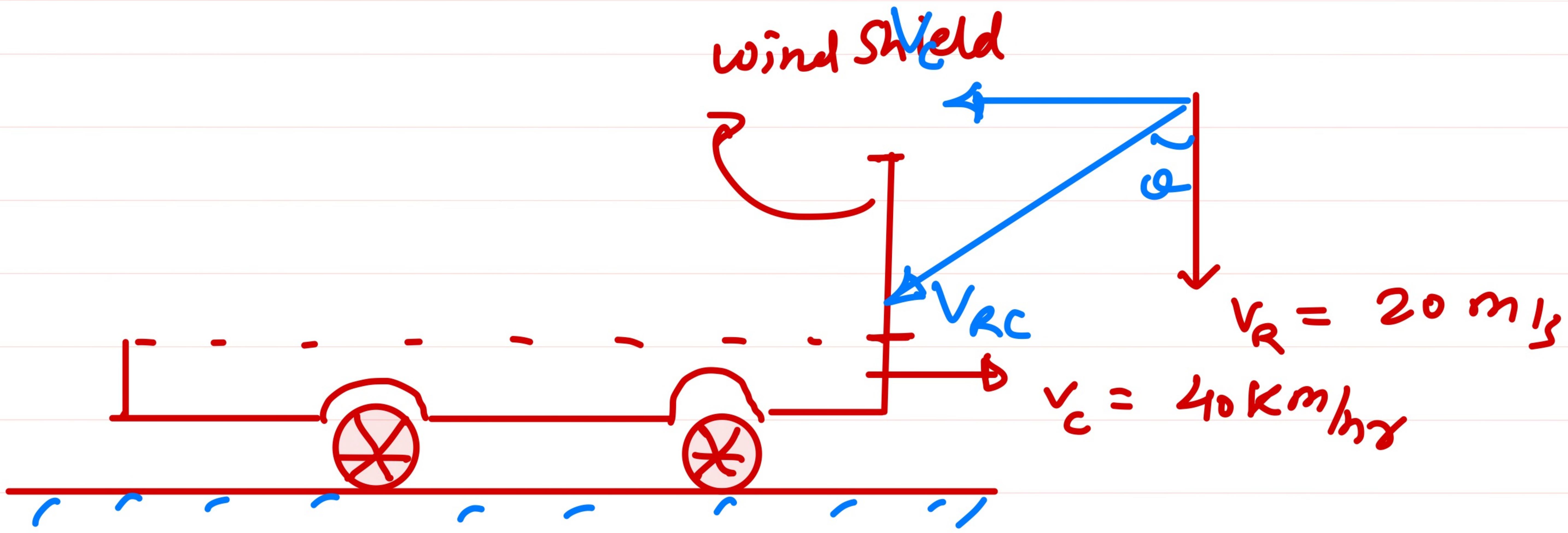
$$V_m = \frac{3}{4} \times 10$$

$$\boxed{V_m = 7.5\ m/s}$$

Ans

5. A car with a vertical wind shield moves along in a rain storm at the speed of 40 km/hr. The rain drops fall vertically with a terminal speed of 20 m/s. The angle with the vertical at which the rain drop strike the wind shield is -

(A) $\tan^{-1}(5/9)$ (B) $\tan^{-1}(9/5)$ (C) $\tan^{-1}(3/2)$ (D) $\tan^{-1}(3)$



$$\tan\theta = \frac{V_c}{V_R} = \frac{40}{20} \times \frac{5}{18} = \frac{10}{18}$$

$$\tan\theta = \frac{5}{9} \Rightarrow \boxed{\theta = \tan^{-1}(5/9)}$$ Ans

15. Rain is falling vertically with a speed of 20 ms^{-1} relative to air. A person is running in the rain with a velocity of 5 ms^{-1} and a wind is also blowing with a speed of 15 ms^{-1} (both towards east). Find the angle with the vertical at which the person should hold his umbrella so that he may not get drenched.

(A) $\tan^{-1}\sqrt{2}$ (B) $\tan^{-1}\left(\frac{1}{2}\right)$ (C) $\tan^{-1}(2)$ (D) 45°

$\vec{V}_{RW} = \vec{V}_R - \vec{V}_W$

$\vec{V}_{RW} = 20(-\hat{j})$

$\vec{V}_m = 5\hat{i}$

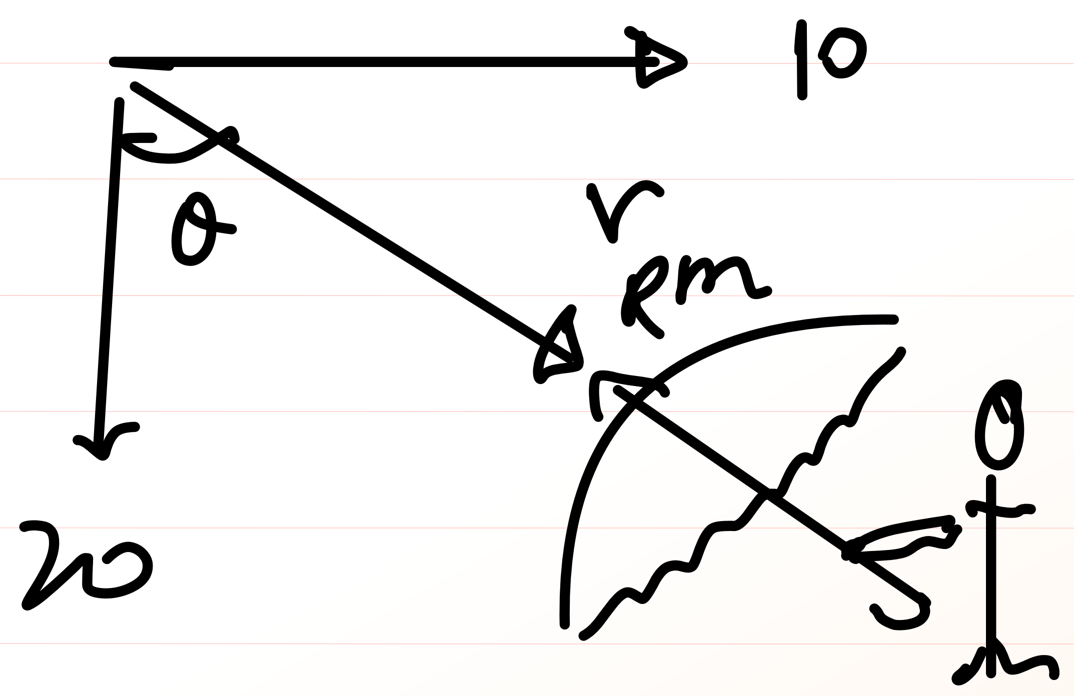
$\vec{V}_W = 15\hat{i}$

$\vec{V}_{Rm} = \vec{V}_R - \vec{V}_m$

$= (\vec{V}_{RW} + \vec{V}_W) - \vec{V}_m$

$= -20\hat{j} + 15\hat{i} - 5\hat{i}$

$= -20\hat{j} + 10\hat{i}$



$\tan\theta = \frac{10}{20} \Rightarrow \boxed{\theta = \tan^{-1}\left(\frac{1}{2}\right)}$

Ans

16. The velocity of a boat in still water is η times less than the velocity of flow of the river (η > 1). The angle with the stream direction at which the boat must move to minimise drifting is

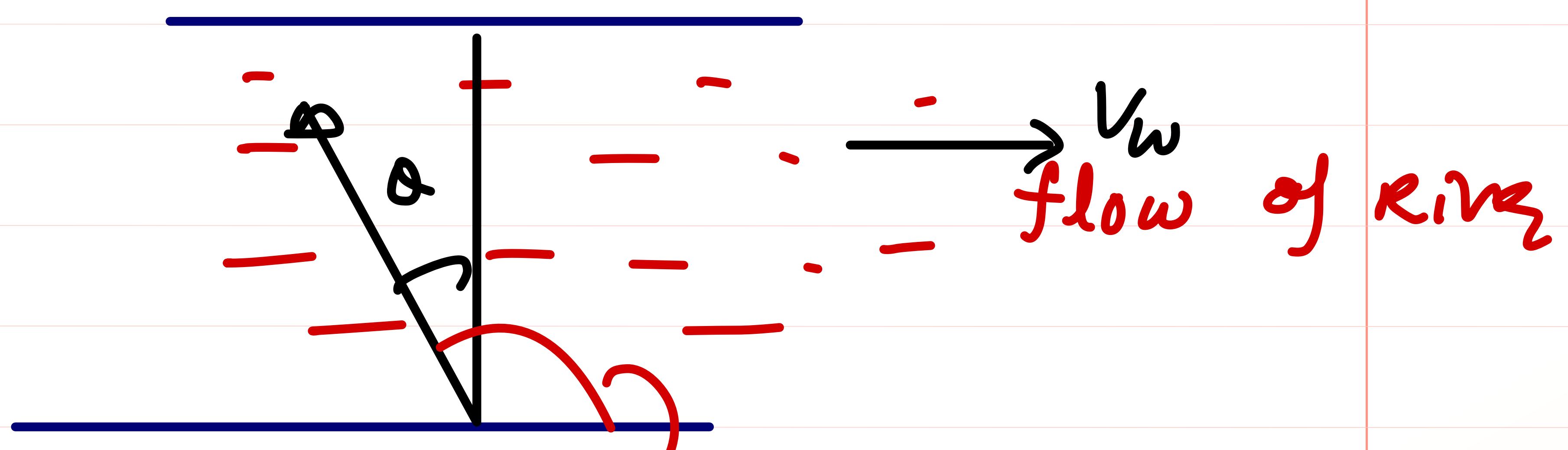
(A) $\sin^{-1}\left(\frac{1}{\eta}\right)$ (B) $\cot^{-1}\left(\frac{1}{\eta}\right)$ (C) $\frac{\pi}{2}+\sin^{-1}\left(\frac{1}{\eta}\right)$ (D) $\frac{\pi}{2}+\cot^{-1}\left(\frac{1}{\eta}\right)$

$$V_{BW} = \frac{V_W}{\eta}$$

$$V_W > V_{BW}$$

$$\sin\theta = \frac{V_{BW}}{V_W} = \frac{1}{\eta}$$

$$\theta = \sin^{-1}(1/\eta)$$

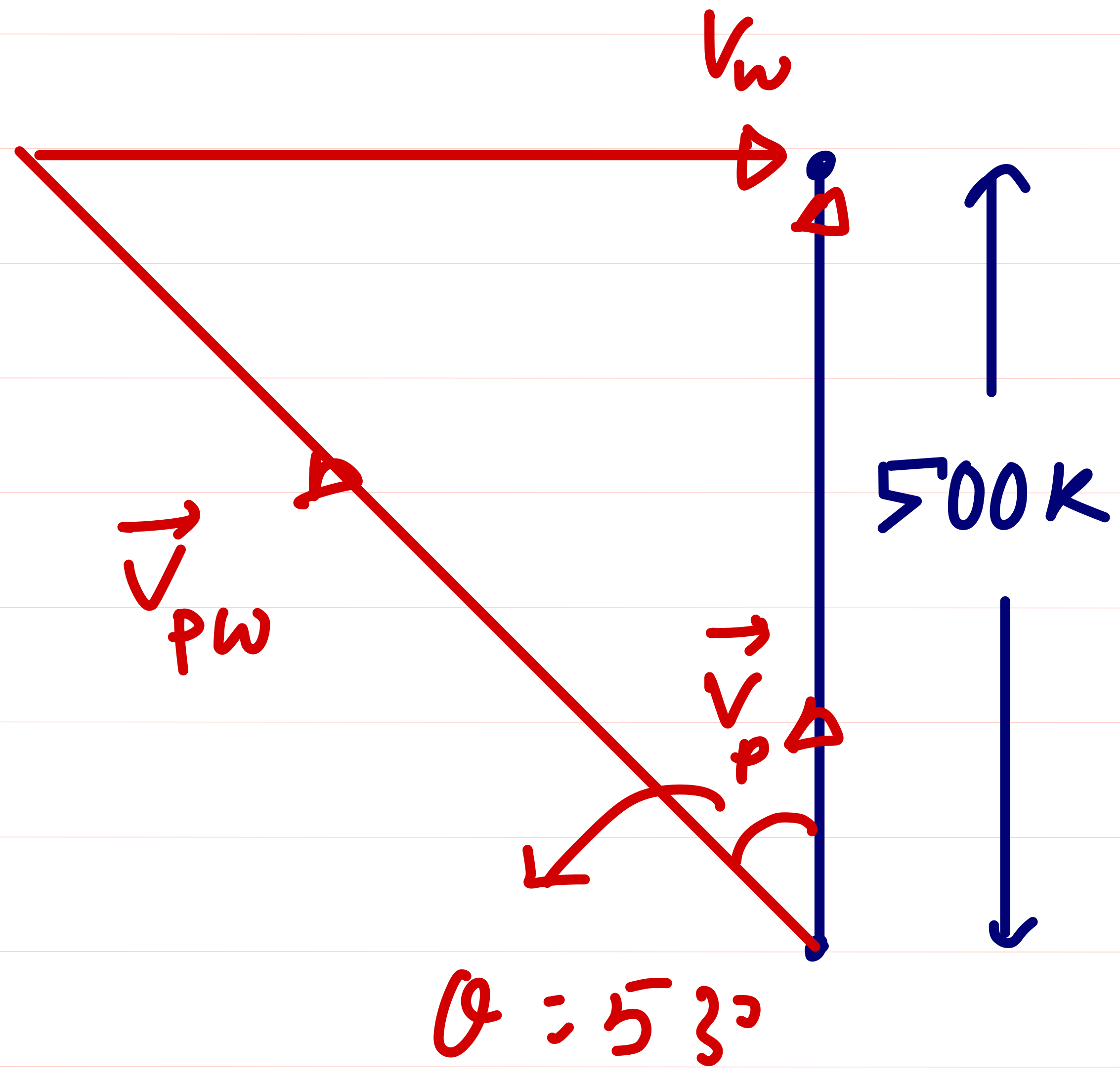


from flow = 90 + θ

$= \frac{\pi}{2} + \sin^{-1}(1/\eta)$

17. A pilot in a plane wants to go 500 km towards north. To reach straight to his desired position the pilot has to drive his plane 53° west of north in presence of wind, which is blowing in due east. The time taken by pilot to reach his destination is 10 hr. The velocity of wind is [take tan 37° = 3/4]

(A) 200/3 km/hr (B) 100/3 km/hr (C) 200 km/hr (D) 150 km/hr



$$\tan 53° = \frac{V_w}{V_p}$$

$$V_w = V_p \cdot \frac{4}{3}$$

$$V_p = \frac{d}{t} = \frac{500}{10} \text{ km/hr}$$

$$V_w = 50 \times \frac{4}{3} = \frac{200}{3} \text{ km/hr}$$