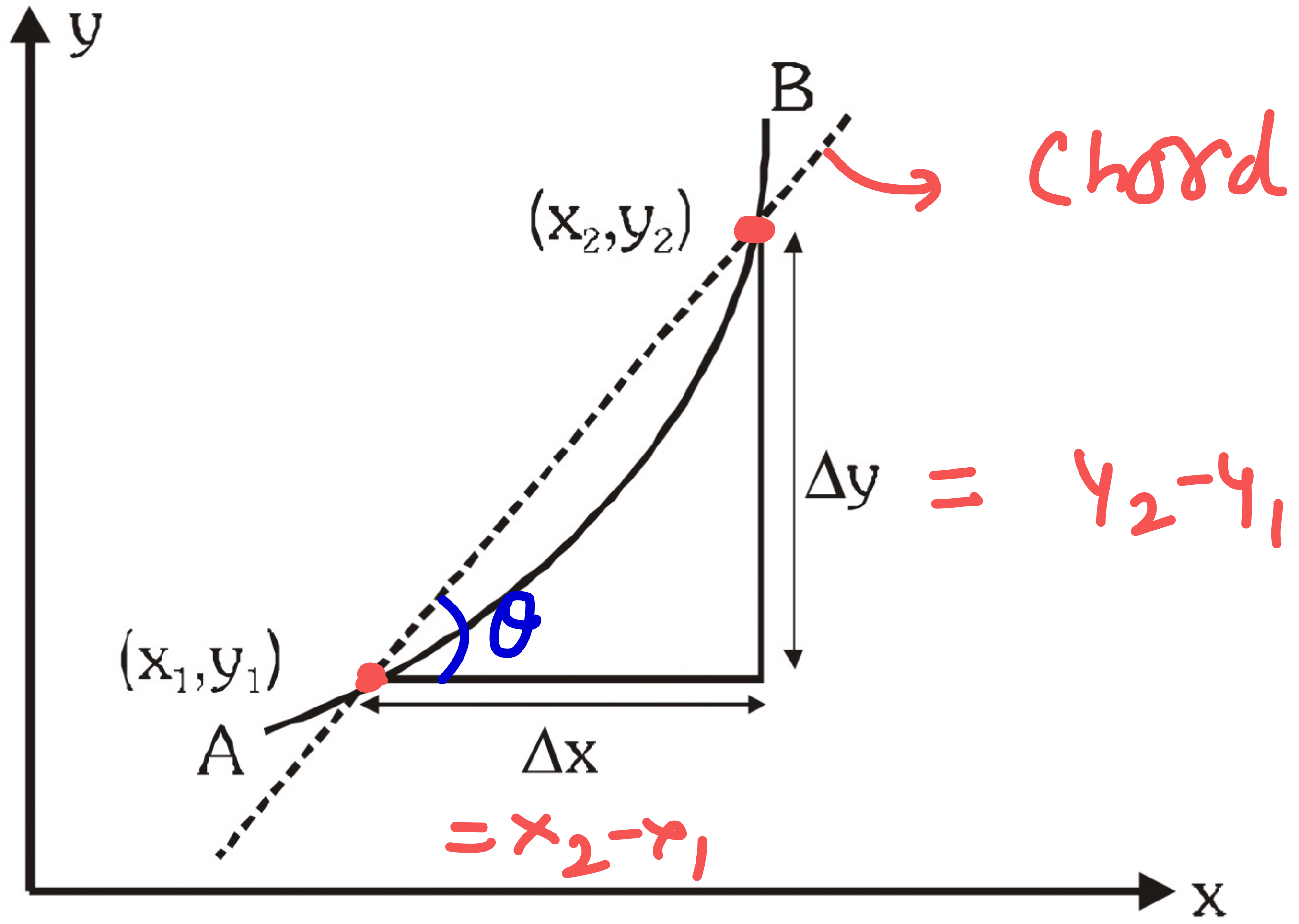
Differentiation

Average rate of change

Let a function y = f(x) be plotted as shown in figure. Average rate of change in y w.r.t. x in interval $[x_1, x_2]$ is

$$\boxed{\text{Average rate of change} = \frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}}$$

= slope of chord AB.



$y = f(x)$

↳ y is a function of x

y ↑ Depe. Variable

x ↑ Ind variable

$\tan(\theta)$ = slope of line AB

or

= slope of chord AB

$\tan(\theta) = \frac{\Delta y}{\Delta x}$ = Avg rate of change of y w.r.t x

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(iii) $\tan\frac{\pi}{10} + \tan\frac{3\pi}{10} + \tan\frac{7\pi}{10} + \tan\frac{9\pi}{10}$

$\tan\left(\frac{\pi}{10}\right) + \tan\left(\pi - \frac{7\pi}{10}\right) + \tan\left(\frac{7\pi}{10}\right) + \tan\left(\pi - \frac{\pi}{10}\right)$

$\cancel{\tan\left(\frac{\pi}{10}\right)} - \cancel{\tan\left(\frac{7\pi}{10}\right)} + \cancel{\tan\left(\frac{7\pi}{10}\right)} - \cancel{\tan\left(\frac{\pi}{10}\right)}$

$= 0$

(iv) $\frac{15+(3\cos\theta+4\sin\theta)}{15-(3\cos\theta+4\sin\theta)}$

max $= \frac{15+5}{15-5} = \frac{20}{10} = 2$

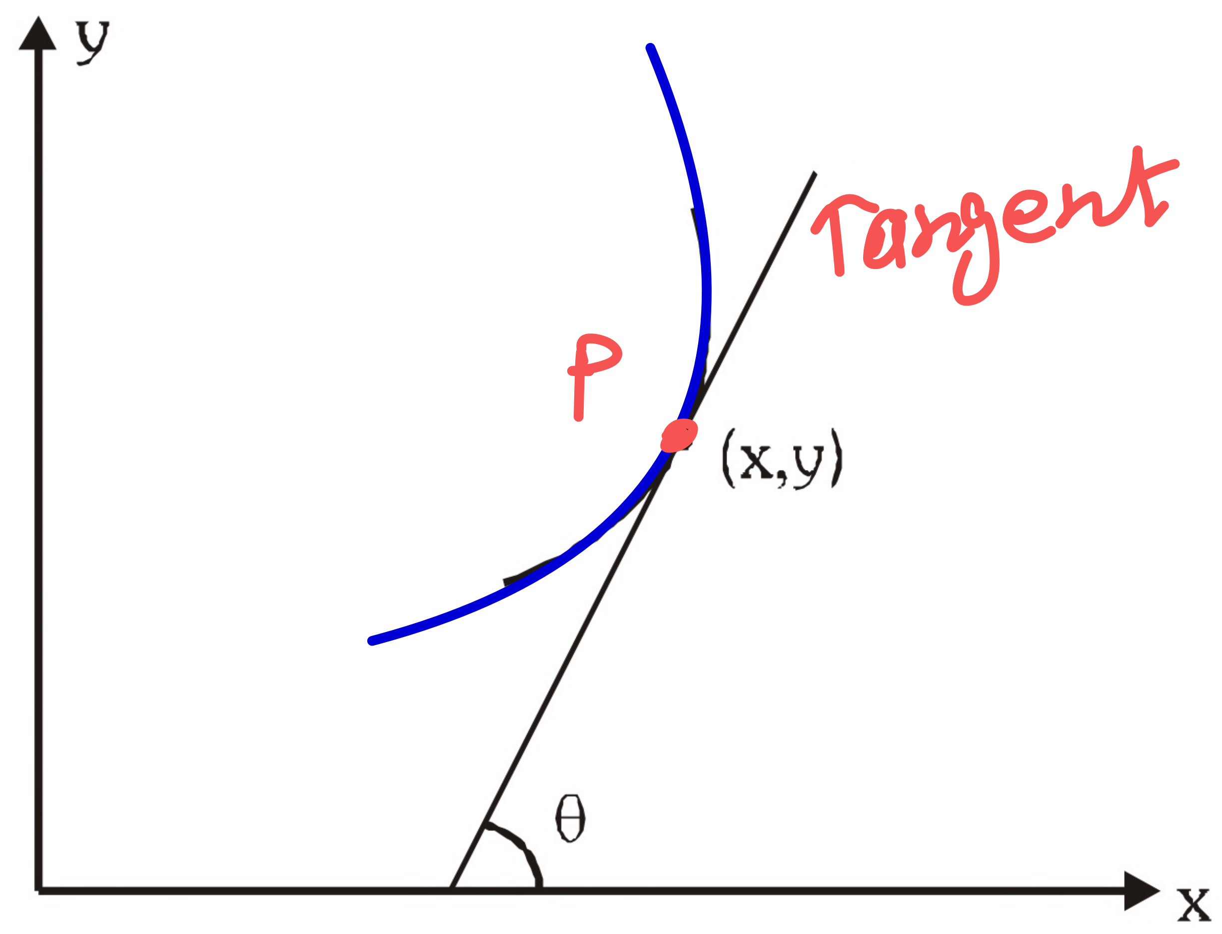
mini $= \frac{15-5}{15+5}$

$= \frac{10}{20}$

$= \frac{1}{2}$

Instantaneous rate of change

It is defined as the rate of change in y with x at a particular value of x. It is measured graphically by the slope of the tangent drawn to the y-x graph at the point (x,y) and algebraically by the first derivative of function y =f(x).

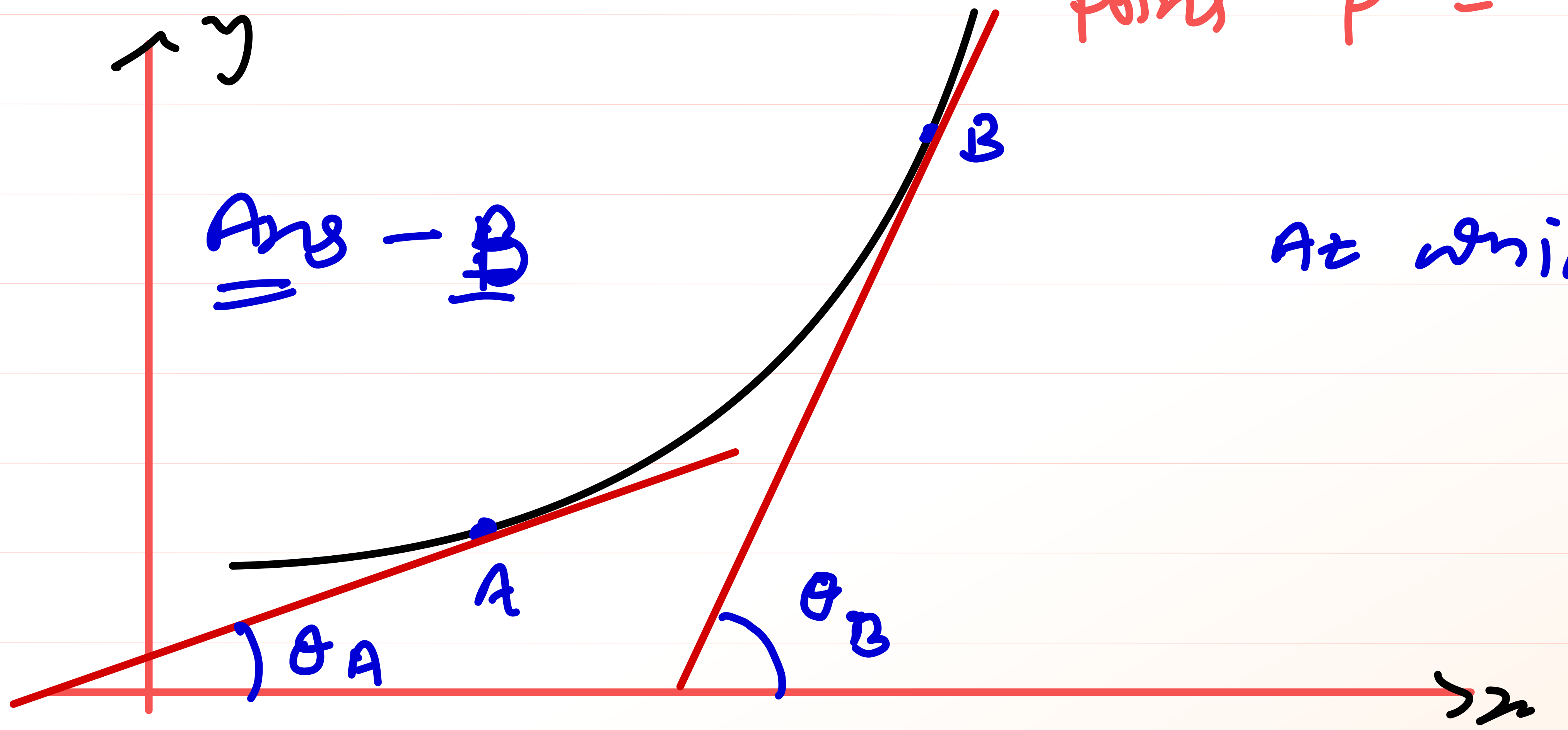


$$\boxed{\text{Instantaneous rate of change} = \frac{dy}{dx} = \text{slope of tangent} = \tan\theta}$$

$\frac{d(y)}{dx} \Rightarrow$ diff. of y w.r.t x

Rate of change of y w.r.t x at point P = Instantaneous rate of change

$\frac{d(y)}{dx} = \tan\theta$



Ans – B

At which point inst. rate of change is greater

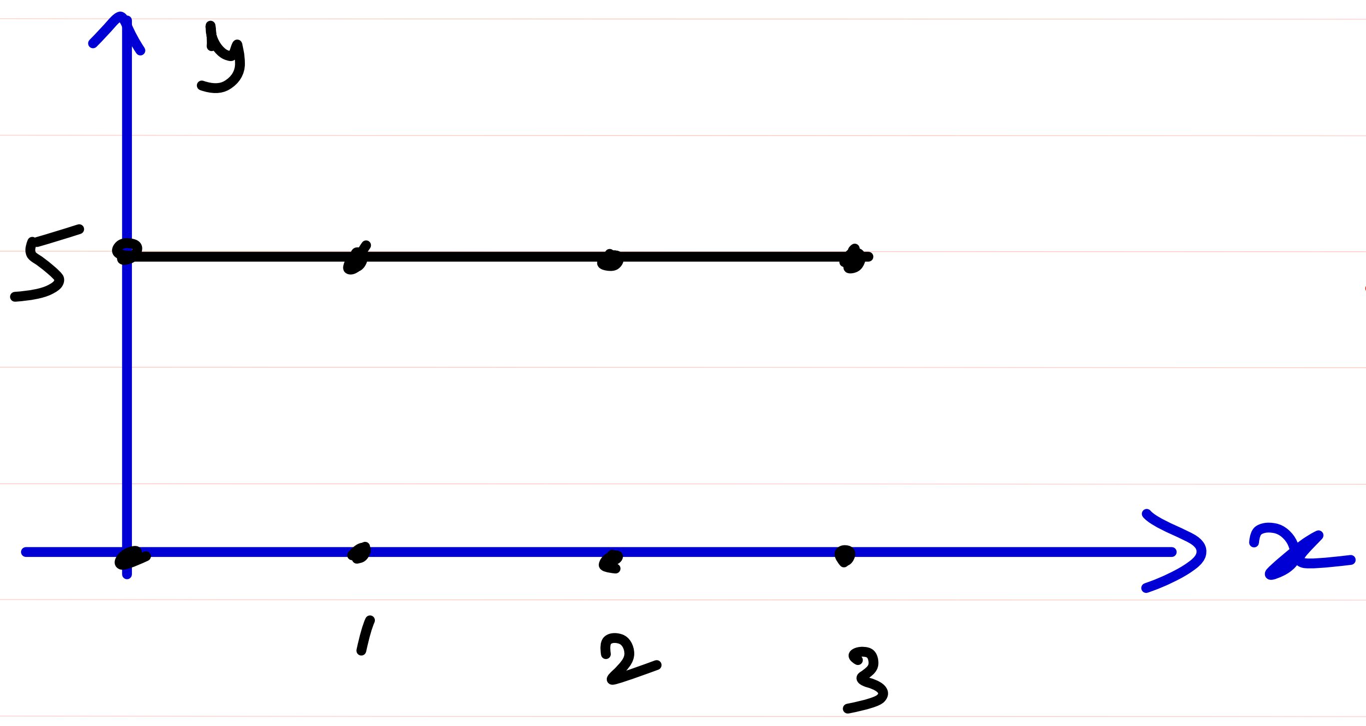
$\theta_B > \theta_A$

$\tan(\theta_B) > \tan(\theta_A)$

First Derivatives of Commonly used Functions

1. $y = \text{constant} \Rightarrow \dfrac{dy}{dx} = 0$
2. $y = e^x \Rightarrow \dfrac{dy}{dx} = e^x$ $\quad (2.71)^x$
3. $y = \sin x \Rightarrow \dfrac{dy}{dx} = \cos x$
4. $y = \tan x \Rightarrow \dfrac{dy}{dx} = \sec^2 x$
5. $y = x^n \Rightarrow \dfrac{dy}{dx} = nx^{n-1}$
6. $y = \ln x \Rightarrow \dfrac{dy}{dx} = \dfrac{1}{x}$
7. $y = \cos x \Rightarrow \dfrac{dy}{dx} = -\sin x$
8. $y = \cot x \Rightarrow \dfrac{dy}{dx} = -\operatorname{cosec}^2 x$

Ex $y = 5$



$\tan(0) = 0$

$\dfrac{dy}{dx} = 0$

$y = x^n$ $\quad n \in R$ (+ve, −ve)

$$\boxed{\frac{dy}{dx} = n x^{n-1}}$$

$y = \ln(x) = \log_e(x)$

$\dfrac{dy}{dx} = \dfrac{1}{x}$

Diff. following functions

(1) $y = 100$

(2) $y = \sin 30^\circ$

(3) $y = e^x$

(4) $y = \cos(x)$

(5) $y = \log(x)$

(6) $y = x^{-5}$

$\frac{dy}{dx} = (-5) \cdot x^{-5-1}$

$= -5x^{-6}$ Ans

(1) $\frac{dy}{dx} = 0$

(2) $\frac{dy}{dx} = 0$

$y = \sin 30$

$y = \frac{1}{2}$

$\frac{dy}{dx} = 0$

(3) $y = e^x$

$\frac{dy}{dx} = e^x$

(4) $y = \cos(x)$

$\frac{dy}{dx} = -\sin x$

(5) $y = \log x$

$\frac{dy}{dx} = \frac{1}{x}$

→ $x = \log(t)$

$\frac{dx}{dt} = \frac{1}{t}$

→ $x = e^t$

$\frac{dx}{dt} = e^t$

→ $x = \cos(t)$

$\frac{dx}{dt} = -\sin(t)$

Method of Differentiation or Rules of Differentiation

(i) Function multiplied by a constant i.e., $\boxed{y = kf(x) \Rightarrow \frac{dy}{dx} = kf'(x)}$

Ex $y = 2\sin(x)$

$\frac{dy}{dx} = 2\{\cos x\}$ Ans

Ex $y = 10e^x$

$\frac{dy}{dx} = 10e^x$ Ans

Ex $y = 10\ln(x)$

$\frac{dy}{dx} = \frac{10}{x}$ Ans

Ex $y = 3x^2$

$\frac{dy}{dx} = 3\{2 \cdot x^{2-1}\}$

$= 6x$

Ex $y = \frac{1}{x^3}$

$y = x^{-3}$

$\frac{dy}{dx} = (-3)\, x^{-3-1}$

$= -3x^{-4}$

$= -\frac{3}{x^4}$

Illustration 22.

Sum or Subtraction of Two functions

$$y = f(x) \pm g(x) \Rightarrow \frac{dy}{dx} = f'(x) \pm g'(x)$$

Ex

$y = \sin x + 2$

$\frac{dy}{dx} = \cos x + 0$

$\frac{dy}{dx} = \cos x$

Ex

$y = e^x + \ln(x)$

$\frac{dy}{dx} = e^x + \frac{1}{x}$

Ex

$y = 2x^2 + 1/x^3$

$\frac{dy}{dx} = 4x - 3x^{-4}$

$= 4x - \frac{3}{x^4}$ Ans

Ex

$y = \sin x + \cos x$

$\frac{dy}{dx} = \cos x - \sin(x)$ Ans

Illustration 23.

Ex If velocity of a particle changes according time by following Eq.

$V = t^2 - 2t$ Find time rate of velocity at $t = 3$ sec

$$\frac{dv}{dt} = 2t - 2$$

at $t = 3$

$$\left(\frac{dv}{dt}\right)_{t=3} = 2 \times 3 - 2$$

$$= 6 - 2$$

$$= 4 \left(\frac{m}{sec^2}\right)$$ Ans

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Product of two functions : Product rule

$$y = f(x)\,.\,g(x) \Rightarrow \frac{dy}{dx} = f(x)\cdot g'(x) + f'(x)\cdot g(x)$$

Ex $y = \underset{\text{I}}{2}\cdot\underset{\text{II}}{\sin x}$

$$\frac{dy}{dx} = 2\cdot\{\cos x\} + \{0\}\cdot\sin x$$

$$= 2\cos x + 0$$

Ex $y = \underset{\text{I}}{\cos x}\cdot\underset{\text{II}}{e^{x}}$

$$\frac{dy}{dx} = \cos x\cdot\{e^{x}\} + \{-\sin x\}\cdot e^{x}$$

$$\frac{dy}{dx} = e^{x}\{\cos x - \sin x\}$$

Ex $y = \dfrac{\ln(x)}{x^2}$

$$y = \underset{\text{I}}{x^{-2}}\cdot\underset{\text{II}}{\ln(x)}$$

$$\frac{dy}{dx} = x^{-2}\left\{\frac{1}{x}\right\} + \{-2x^{-3}\}\ln(x)$$

$$= \frac{1}{x^3} - \frac{2}{x^3}\ln(x)$$

$$= \frac{1}{x^3}\left[1 - 2\ln(x)\right] \quad \text{Ans}$$

Illustration 24.

Division of Two Functions : Quotient Rule

$$y=\frac{f(x)}{g(x)}\Rightarrow\frac{dy}{dx}=\frac{f'(x)g(x)-f(x)g'(x)}{[g(x)]^2}$$

Ex $y = \tan(x)$

$y = \dfrac{\sin(x)}{\cos(x)}$

$= \dfrac{\cos x \cdot \{\cos x\} - \sin x\{-\sin x\}}{\cos^2 x}$

$= \dfrac{\cos^2 x + \sin^2 x}{\cos^2 x}$

$= \dfrac{1}{\cos^2 x} = \sec^2 x$

Ex $y = \cot x$ $\quad y = \sec x$

$y = \dfrac{\cos x}{\sin x}$

$= \dfrac{\sin x\{-\sin x\} - \cos x\{\cos x\}}{\sin^2 x}$

$= \dfrac{-\sin^2 x - \cos^2 x}{\sin^2 x} = -\dfrac{1}{\sin^2 x} = -\operatorname{cosec}^2 x$

Ex $y = \sec x = \dfrac{1}{\cos x}$

$\dfrac{dy}{dx} = \dfrac{\cos x \cdot 0 - 1\{-\sin x\}}{\cos^2 x} = \dfrac{\sin x}{\cos^2 x}$

$= \dfrac{\sin x}{\cos x} \cdot \dfrac{1}{\cos x} = \tan x \cdot \sec x$

Illustration 25.

Ex Area of object changes according Eq.

$A = \pi r^2$ Find rate of change of Area at $r = 3$

$$\frac{dA}{dr} = 2\pi r$$

$$= 2\pi(3)$$

$$\frac{dA}{dr} = 6\pi$$

H.W

Race #4

Q- 1 to 6

module

Illu 22 to 25