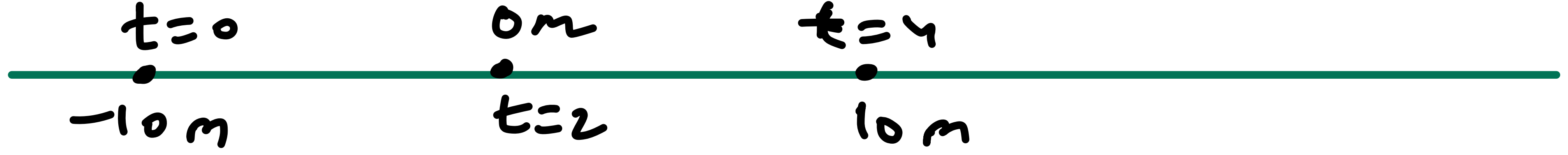


GRAPHICAL REPRESENTATION

1. *Displacement – time (x – t) graphs* (*Fig. 2.4*)
   - Fig. 2.4(a) : Body at rest
   - Fig. 2.4(b) : Body in uniform motion
   - Fig. 2.4(c) : Body subjected to acceleration ($a > 0$)
   - Fig. 2.4(d) : Body subjected to retardation ($a < 0$)
   - Fig. 2.4(e) : Body accelerating and then decelerating

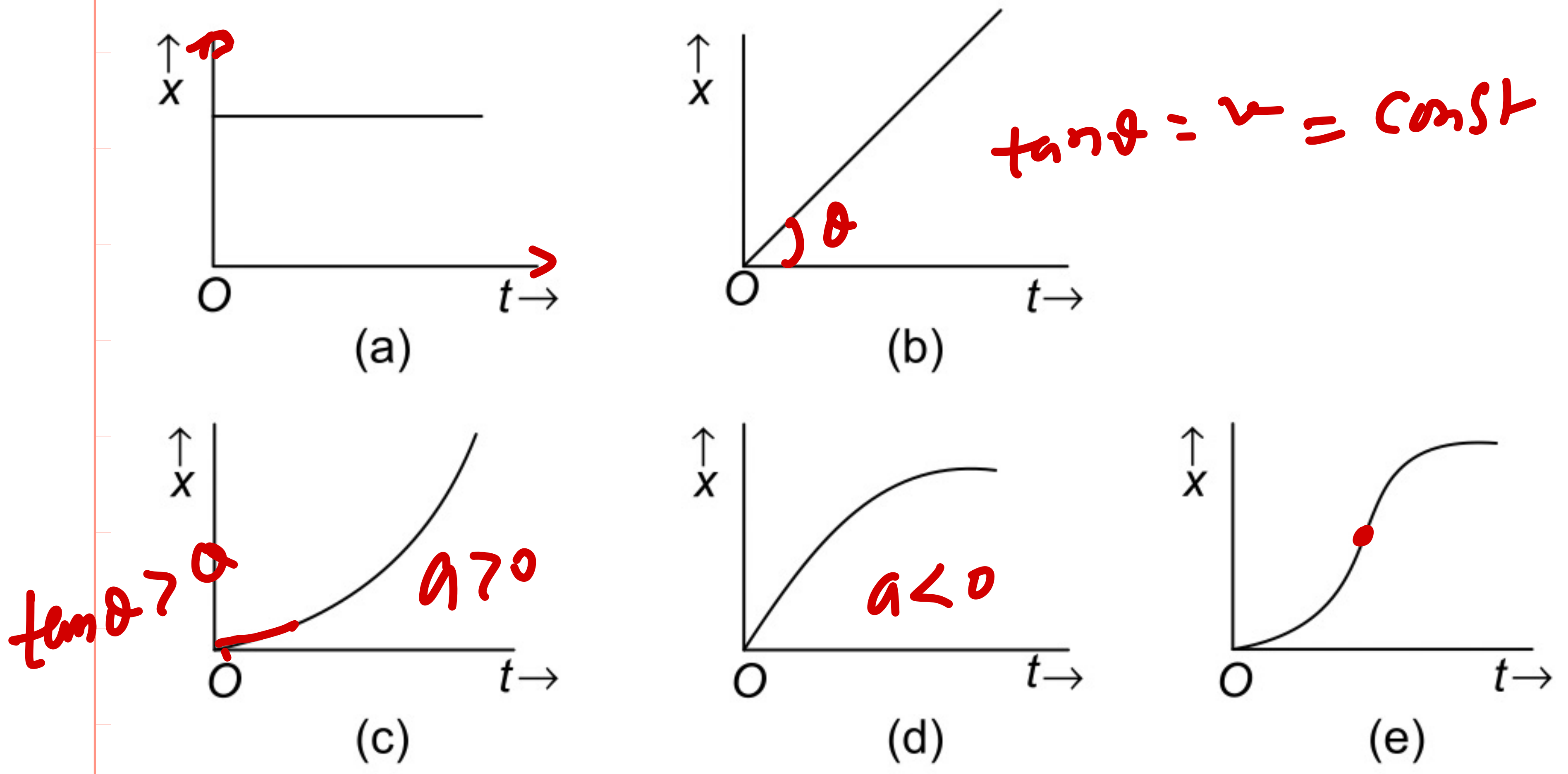
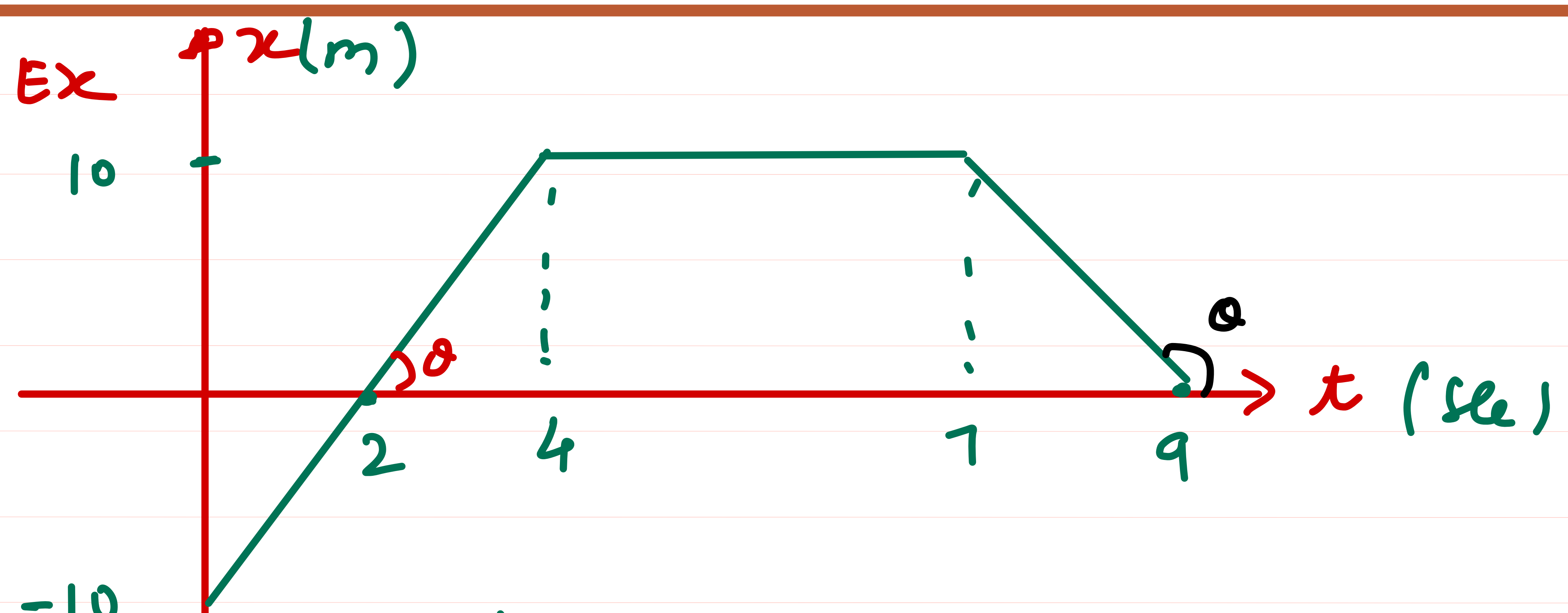


Fig. 2.4

NOTE

The slope of $x - t$ graph gives velocity for uniform motion [Fig. 2.4(b)]. For non-uniform motion [Fig. 2.4(c), (d) and (e)], the slope of the tangent to the curve at a point gives velocity at that instant.

Ex



(1) Velocity at t = 3, 5 and 8 sec

(2) total distance and displacement

Sol 0 to 4 sec

$v = \tan\theta = \frac{10}{2} = 5\ m/s$ Ans

4 to 7 sec

$v = \tan\theta = 0\ m/s$ Ans

7 to 9 sec

$v = \tan\theta = -\frac{10}{2} = -5\ m/s$ Ans

(2) $\boxed{d = 30\ m}$ Ans

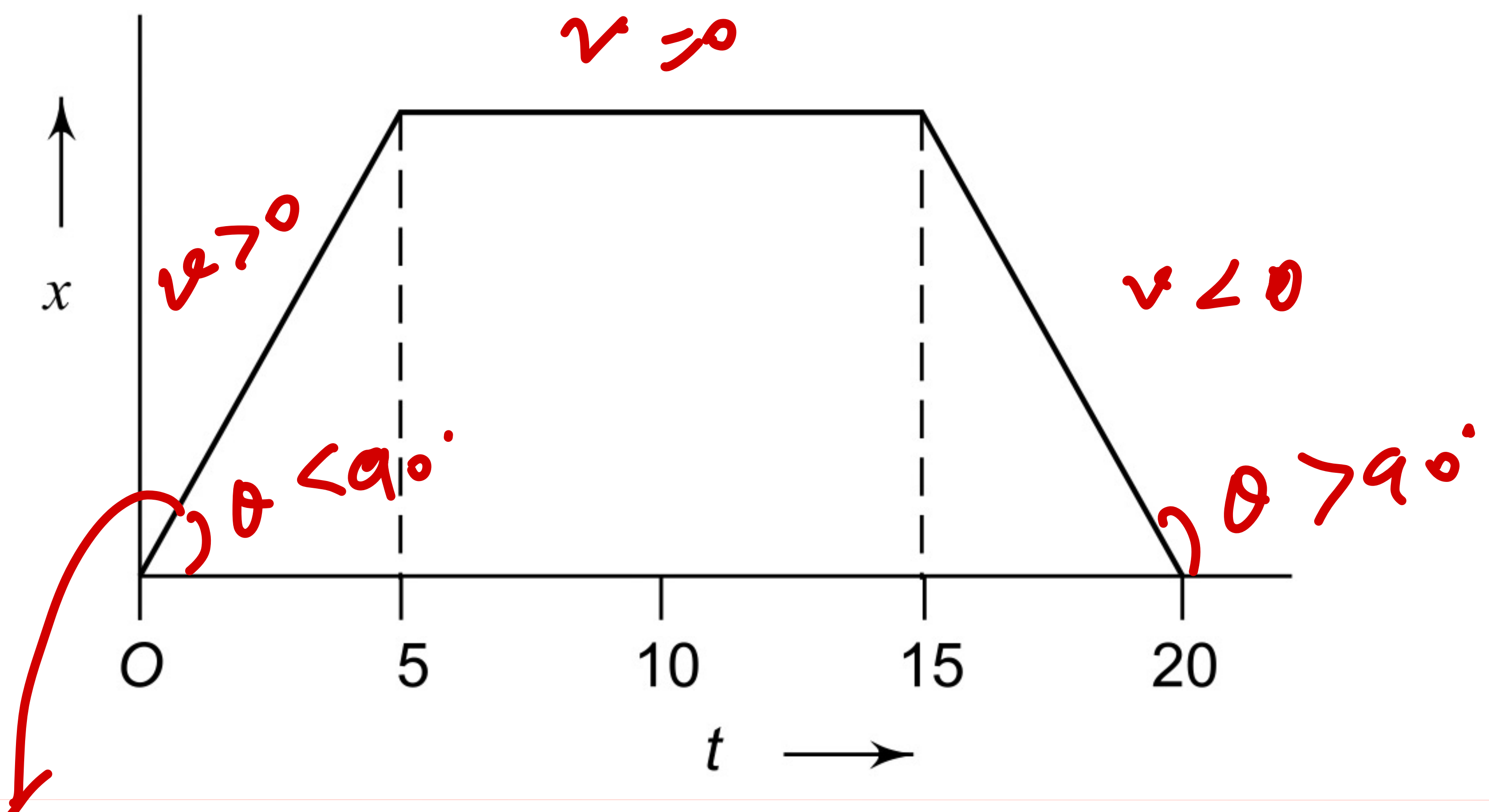
$s = x_f - x_i$

$= 0 - (-10)$

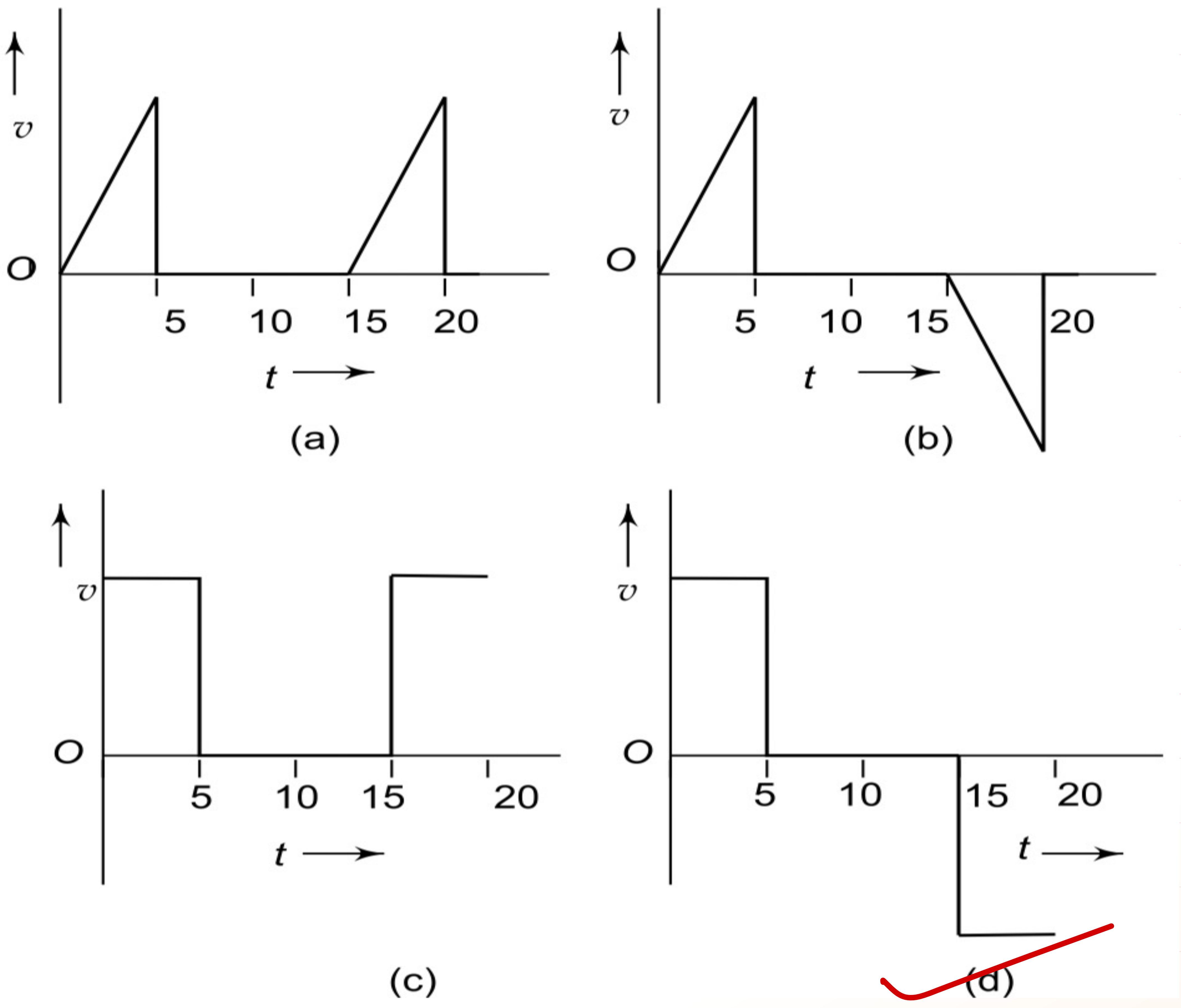
$\boxed{s = 10\ m}$

Ex

Figure 2.12 shows the displacement-time (x-t) graph of a body moving in a straight line. Which one of the graphs shown in Fig. 2.13 represents the velocity-time (v-t) graph of the motion of the body.



tanθ = constant



2. Velocity-time ($v - t$) graphs for uniformly accelerated motion (Fig. 2.5)

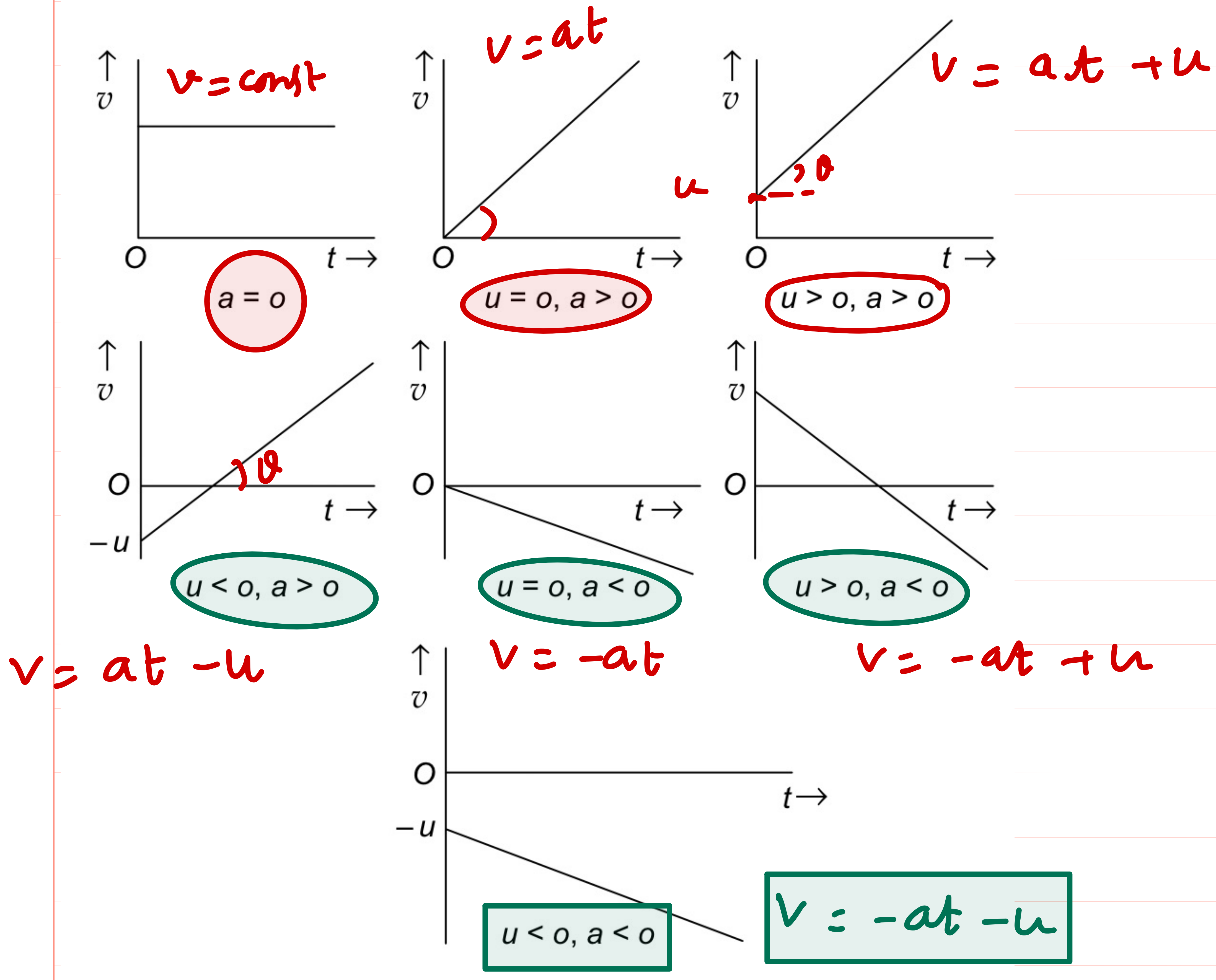
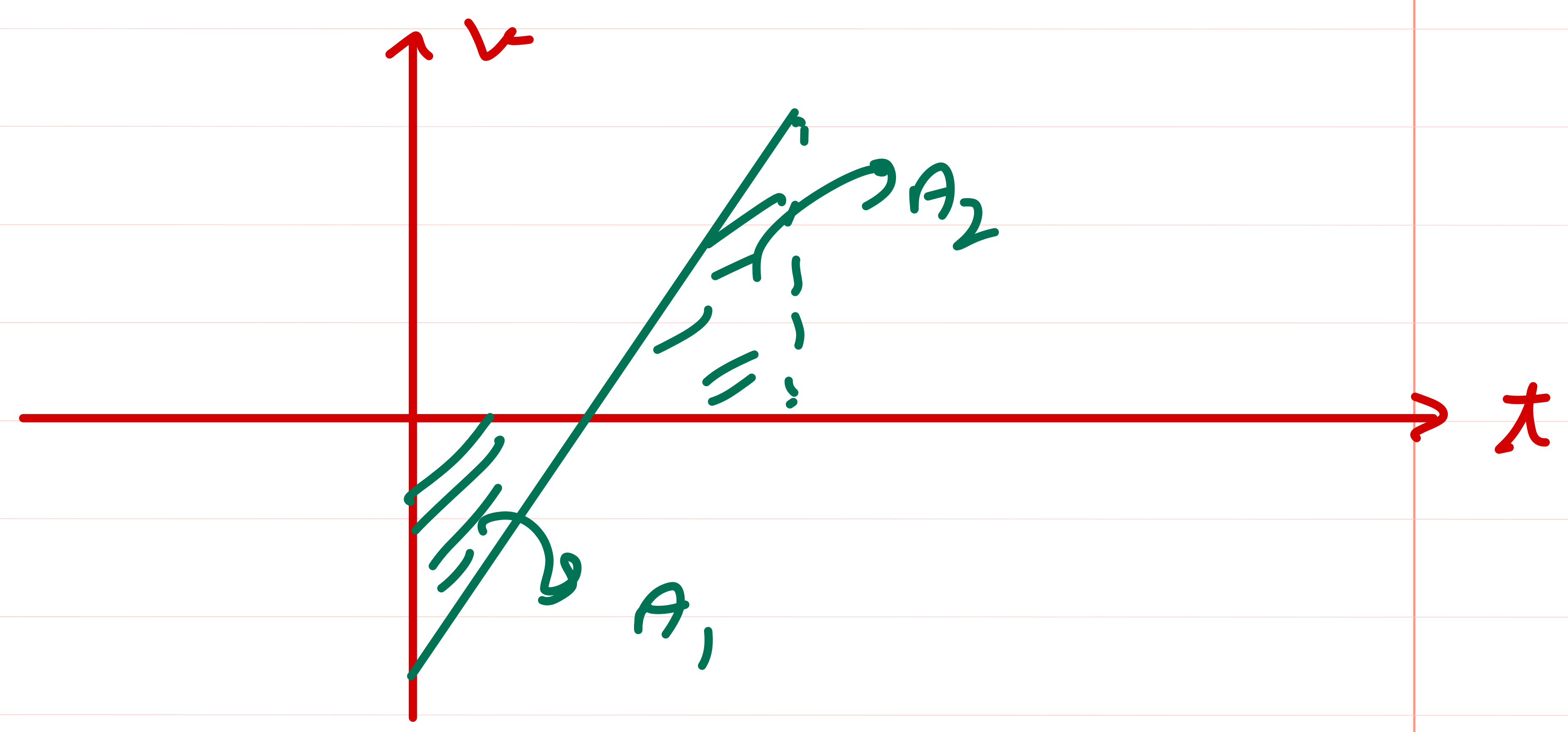


Fig. 2.5

NOTE

Acceleration = slope of ($v - t$) graph
Displacement = area under ($v - t$) graph

distance / displacement from v-t curve

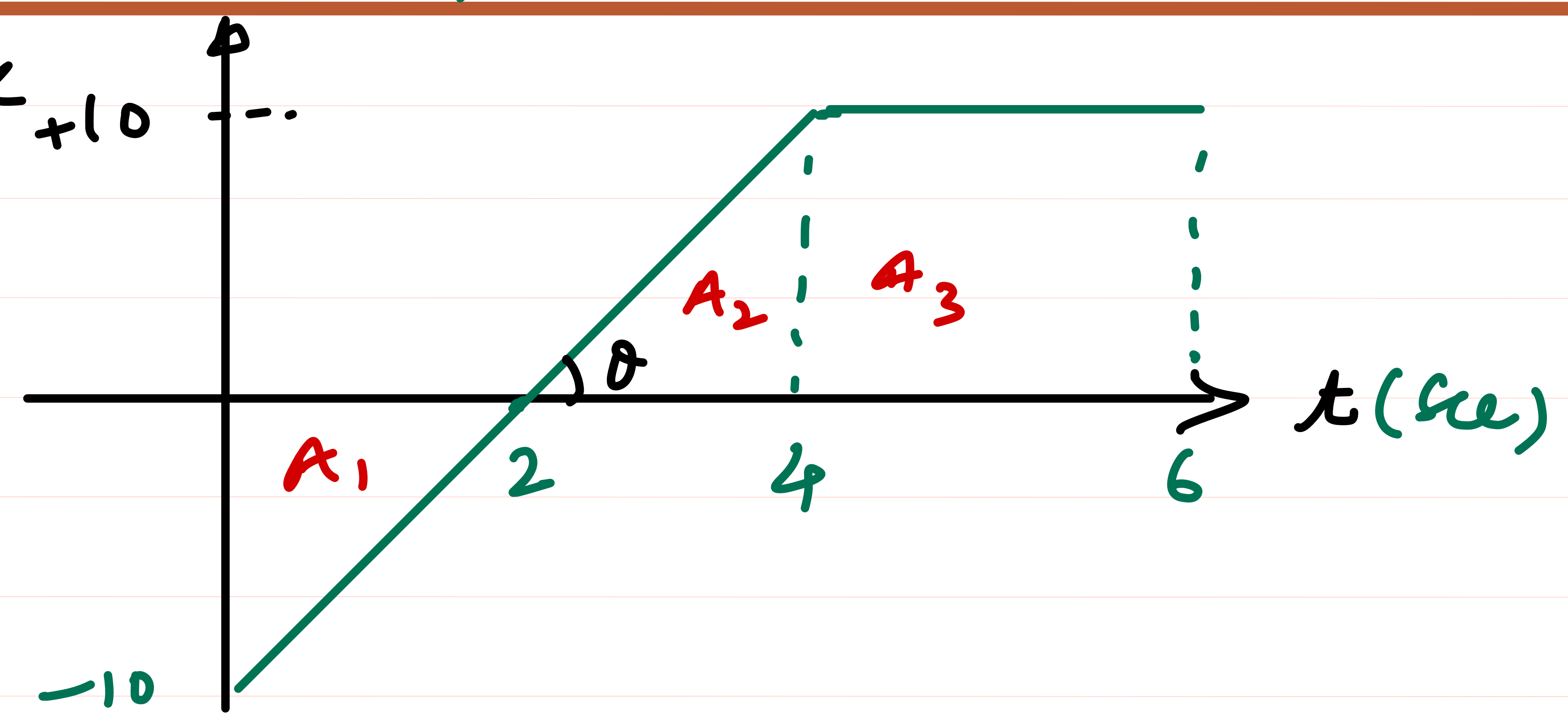


$d = A_1 + A_2$

$s = A_2 - A_1$

V-t area = distance or displace

Ex



Find ① acceleration at t = 3 sec and 5 sec

② b/w 0 to 6 sec distance and displacement

Sol ① t = 0 to 4 sec

$$\boxed{a = \tan\theta = \frac{10}{2} = 5\ m/s^2}$$ Ans

t > 4 sec

$a = \tan\theta = 0$

$$\boxed{a = 0\ m/s^2}$$ Ans

$A_1 = \frac{1}{2} \times 10 \times 2 = 10$

$A_2 = \frac{1}{2} \times 10 \times 2 = 10$

$A_3 = 2 \times 10 = 20$

$d = A_1 + A_2 + A_3$

$= 10 + 10 + 20$

$$\boxed{d = 40\ m}$$

$S = -A_1 + A_2 + A_3$

$$\boxed{S = 20\ m}$$

③ Avg velocity b/w t = 0 to 6

$V_{avg} = \frac{S}{\Delta t} = \frac{20}{6} = \frac{10}{3}\ m/s$

④ Avg speed b/w t = 0 to 6 sec

Avg speed $= \frac{d}{\Delta t} = \frac{40}{6} = \frac{20}{3}$

⑤ Avg acc b/w t = 0 to 6 sec

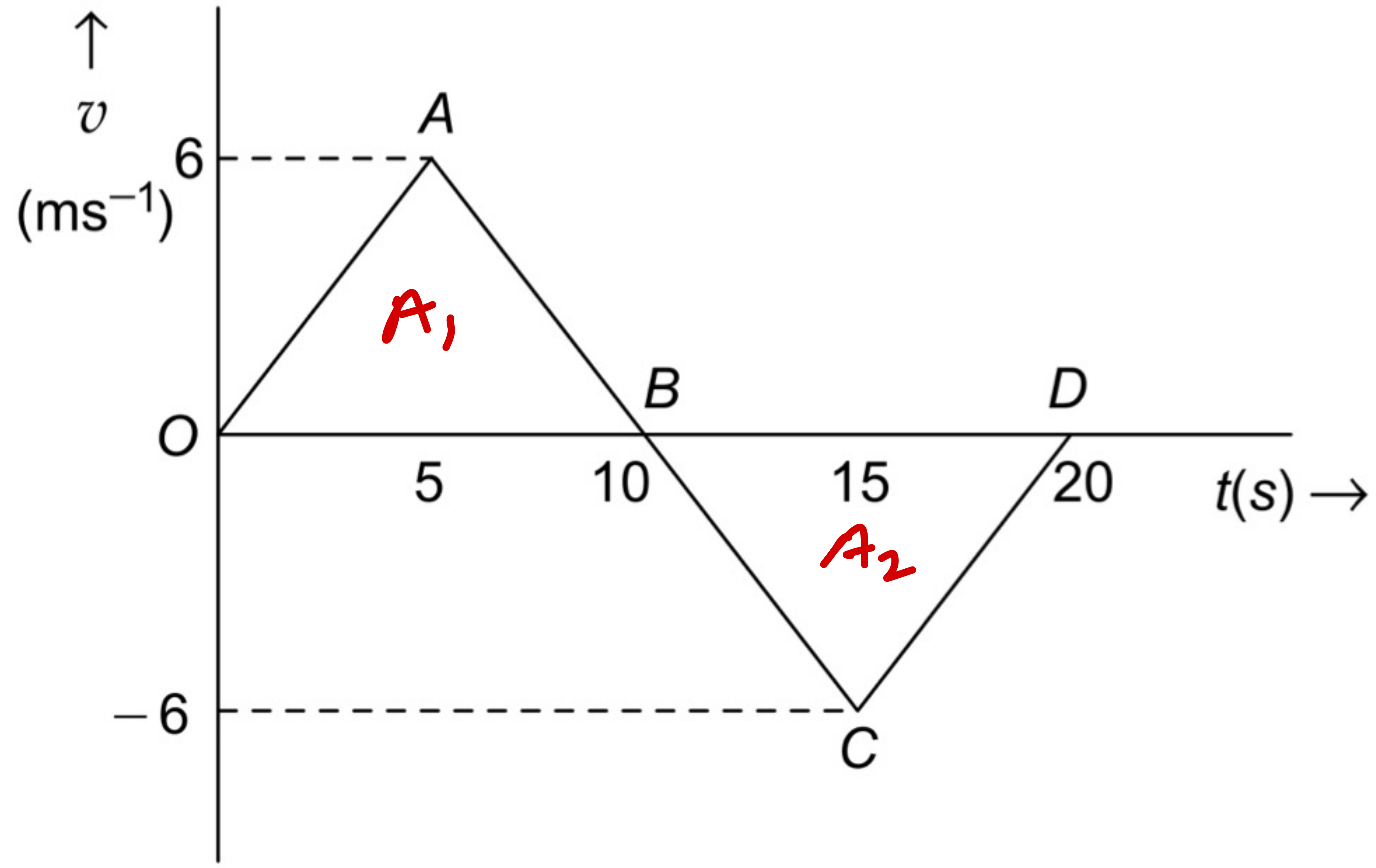
Avg acc $= \frac{v_f - v_i}{\Delta t}$

$= \frac{10 - (-10)}{6}$

$= \frac{20}{6} = \frac{10}{3}\ m/s^2$

Ex

Figure 2.8 shows the velocity – time graph of a body moving in a straight line. Find (a) the distance travelled by the body in 20 s, (b) the displacement of the body in 20 s and (c) the average velocity in the time interval $t = 0$ to $t = 20$ s.



$A_1 = \frac{1}{2} \times 10 \times 6 = 30$

$A_2 = \frac{1}{2} \times 10 \times 6 = 30$

① $d = A_1 + A_2 = 60\,m$

$S = A_1 - A_2 = 0\,m$

② Avg velocity $= \frac{S}{20} = 0\,m/s$

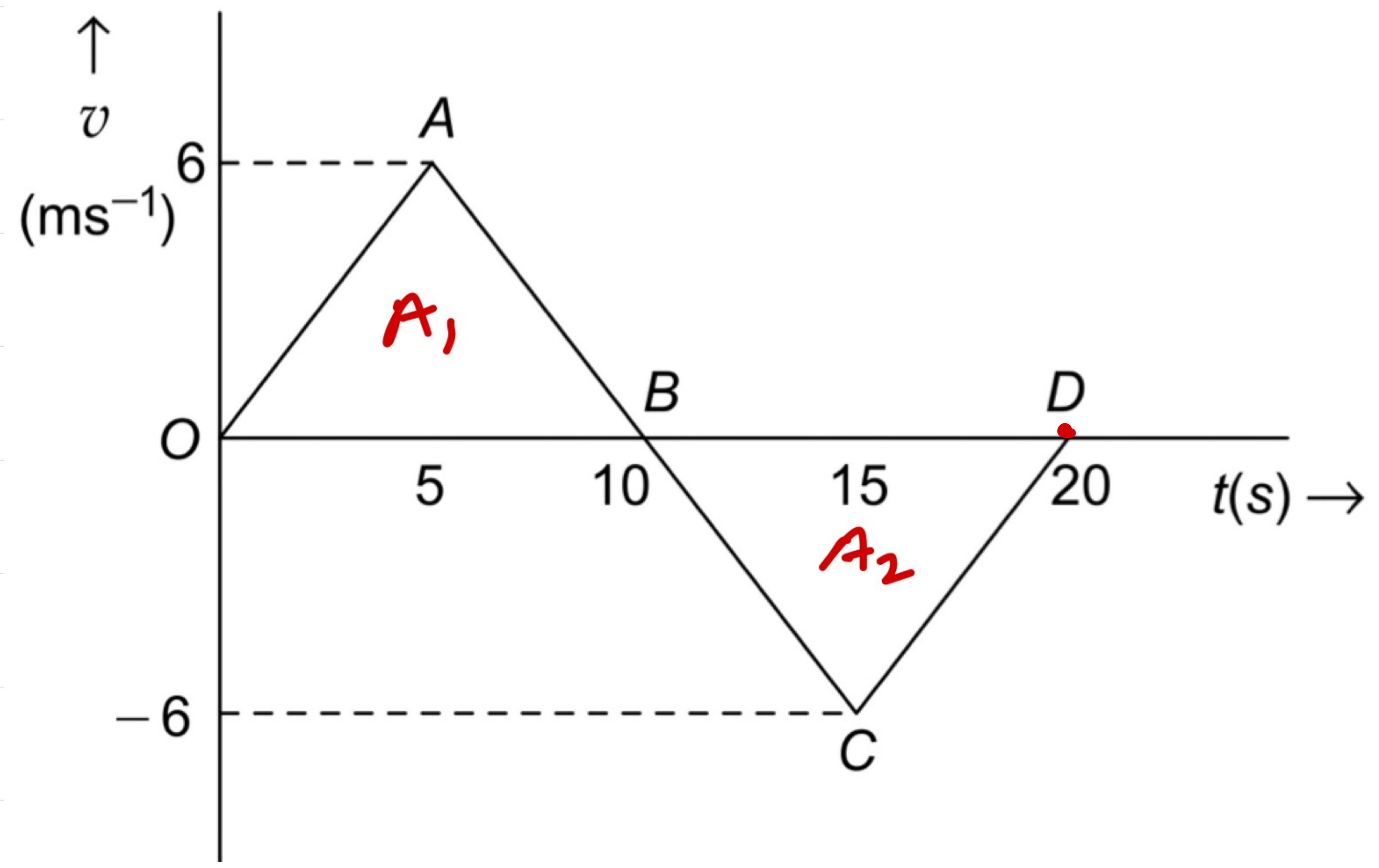
Avg speed $= \frac{60}{20} = 3\,m/s$

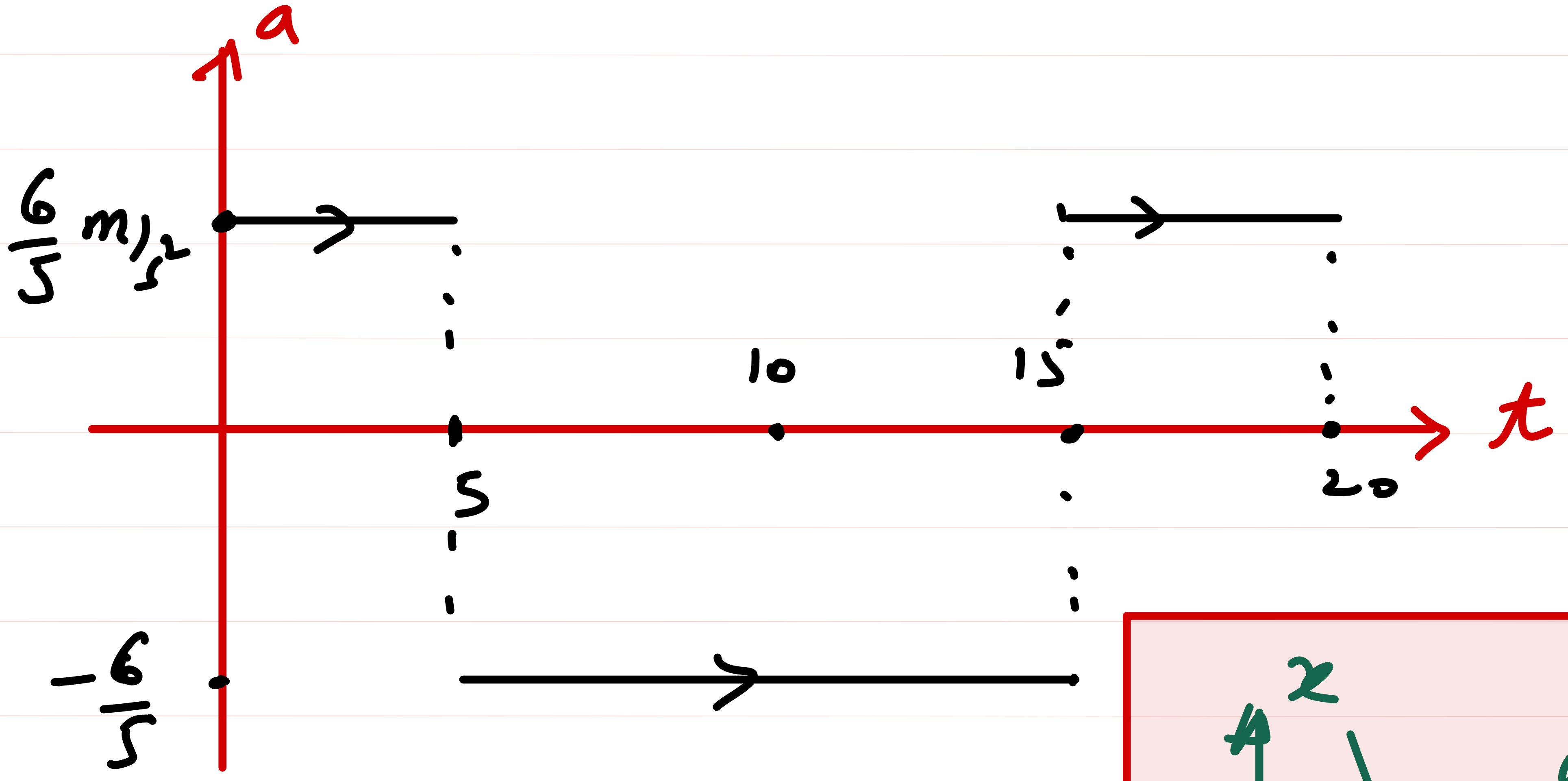
Avg Acc $= \frac{0-0}{20} = 0\,m/s^2$

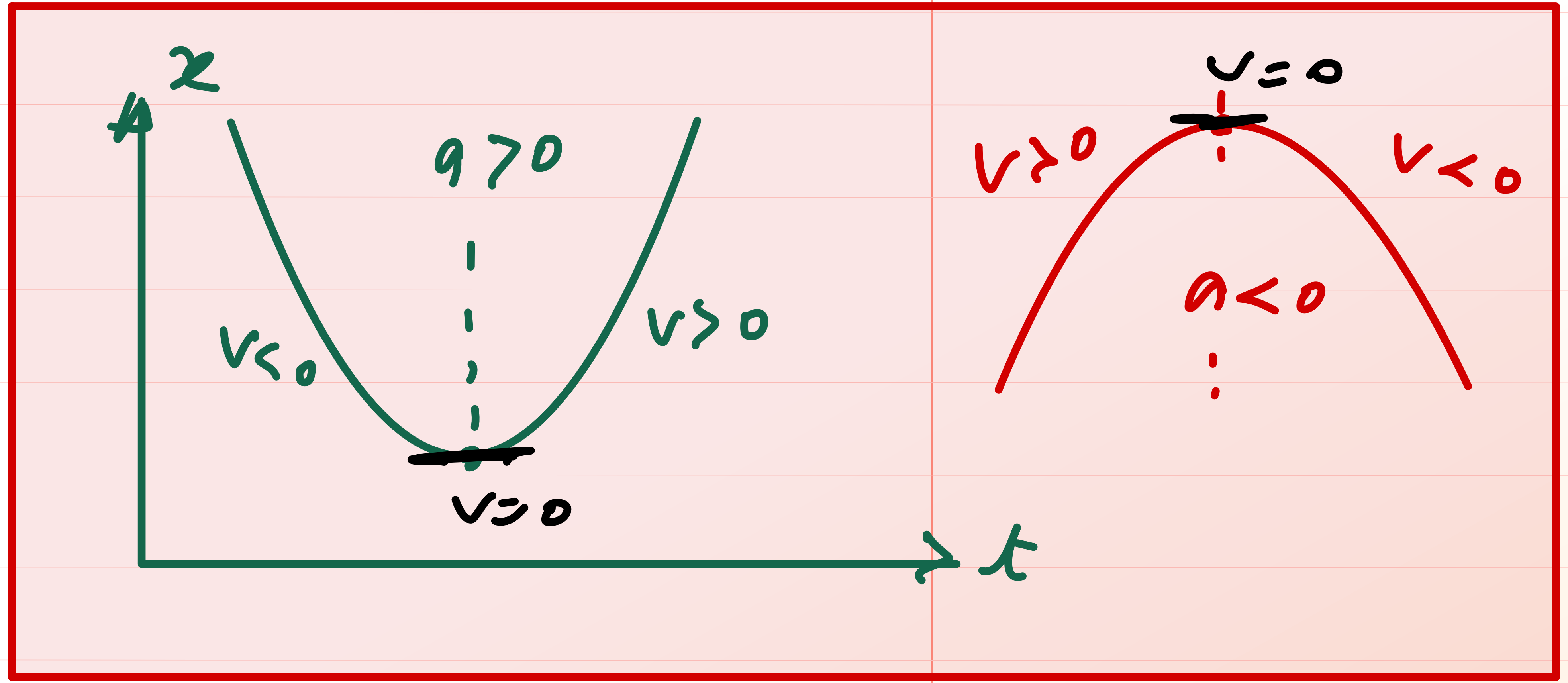
Avg Acc b/w $t=5$ to 15 sec

Avg Acc $= \frac{-6-6}{15-5} = -\frac{12}{10} = -1.2\,m/s^2$

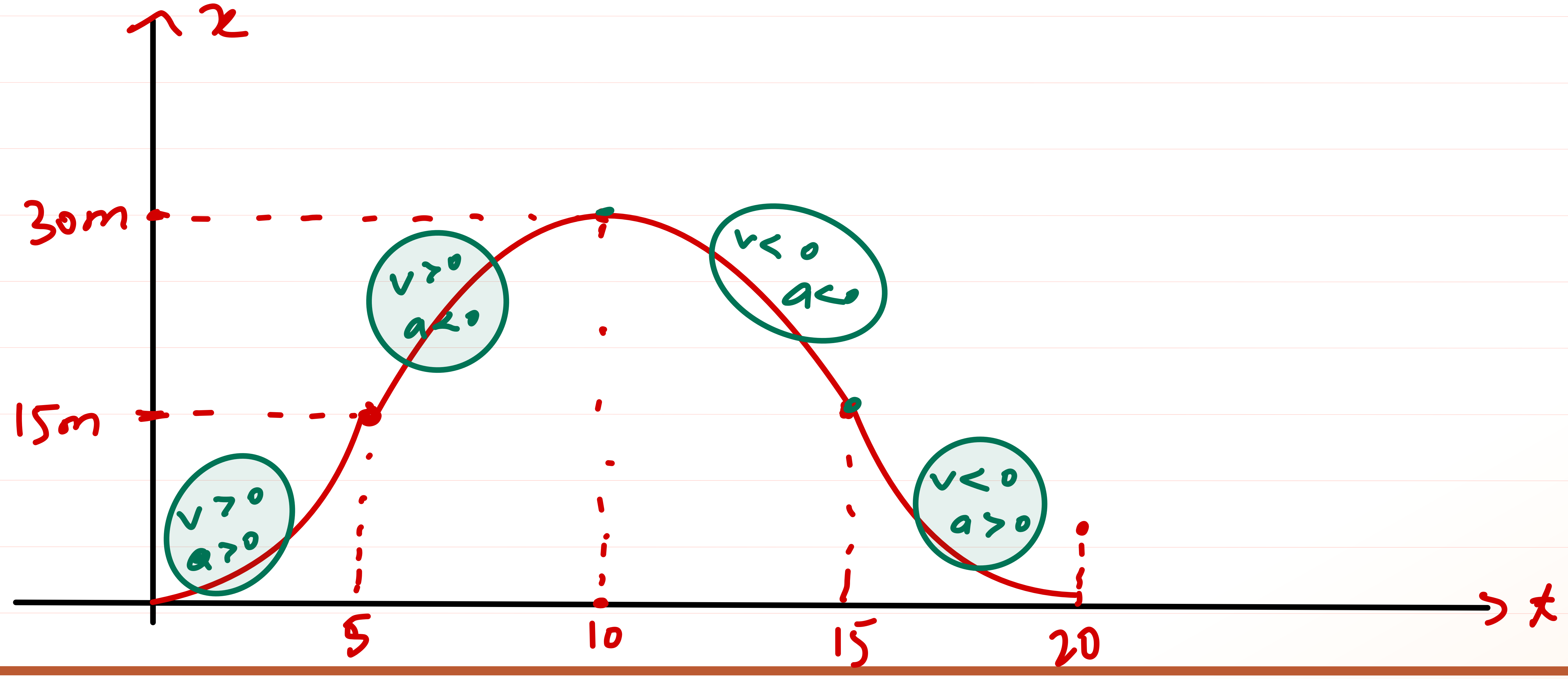
Eg From previous problem draw a – t curve







Ex Draw x – t curve



Yaad रखें

3. Acceleration – time ($a - t$) graph [Fig. 2.6]

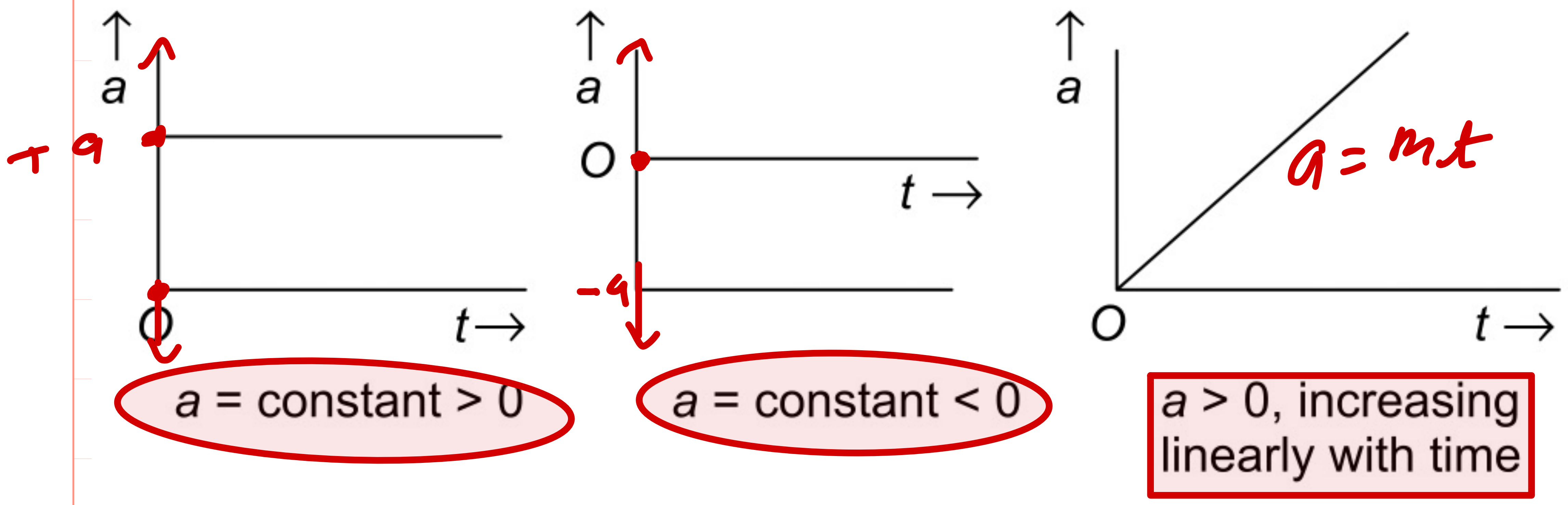


Fig. 2.6

4. ($a - t$), ($v - t$), ($x - t$) graphs for free fall [Fig 2.7]

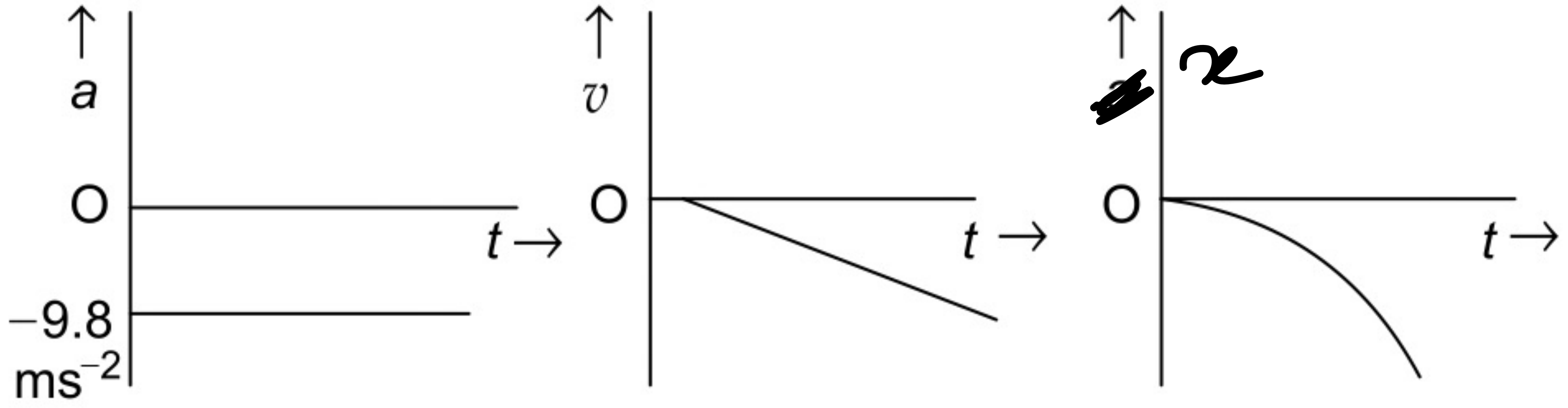
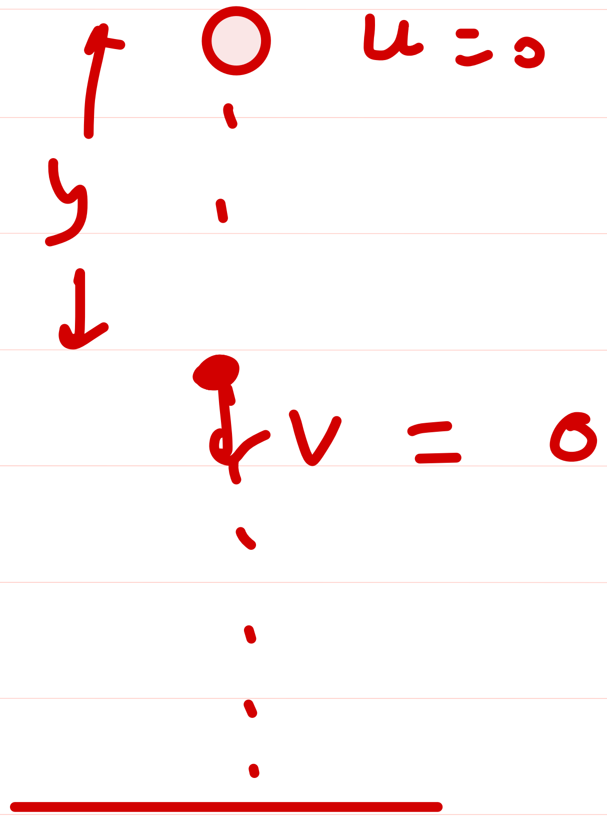
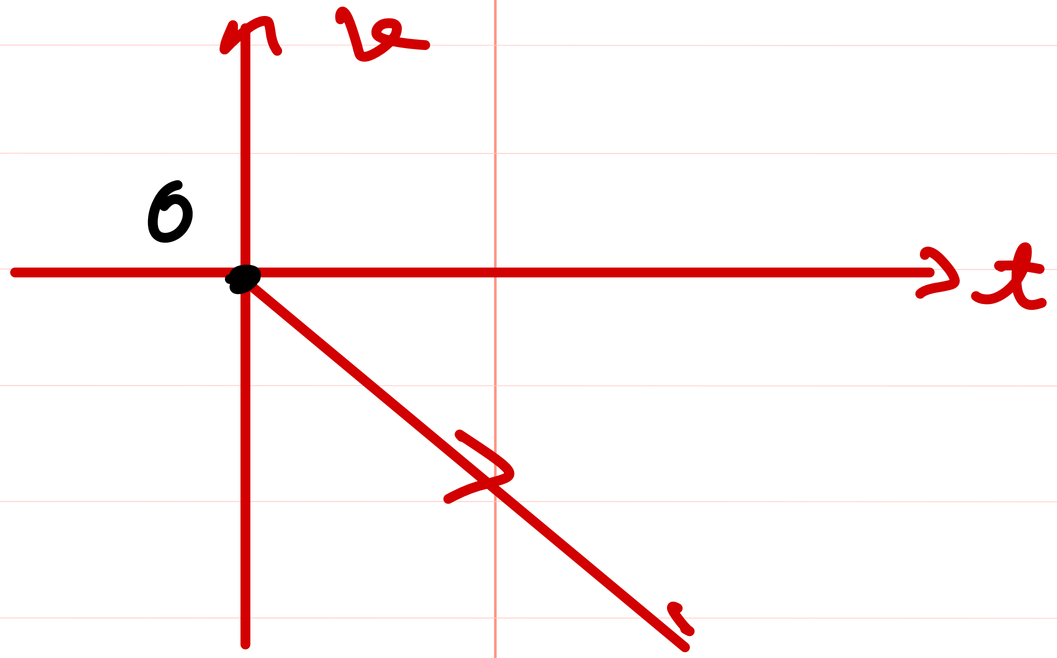


Fig. 2.7

4



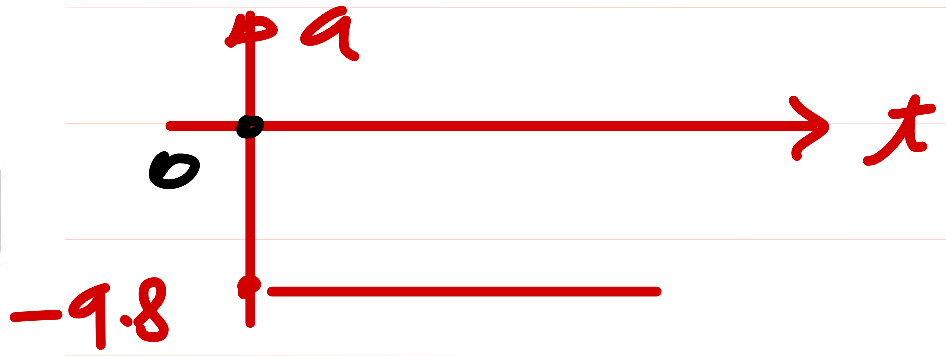
$v = 0 - gt$

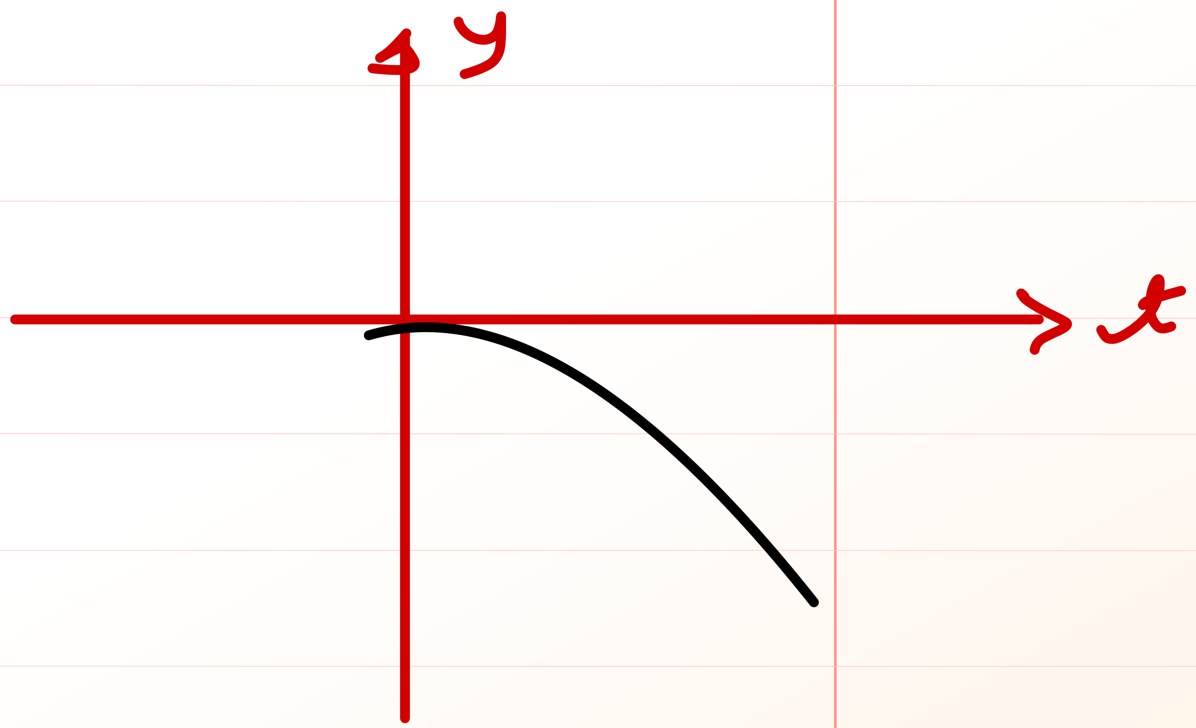


$y = 0 \cdot t - \frac{1}{2} g t^2$

$y = -\frac{1}{2} g t^2$

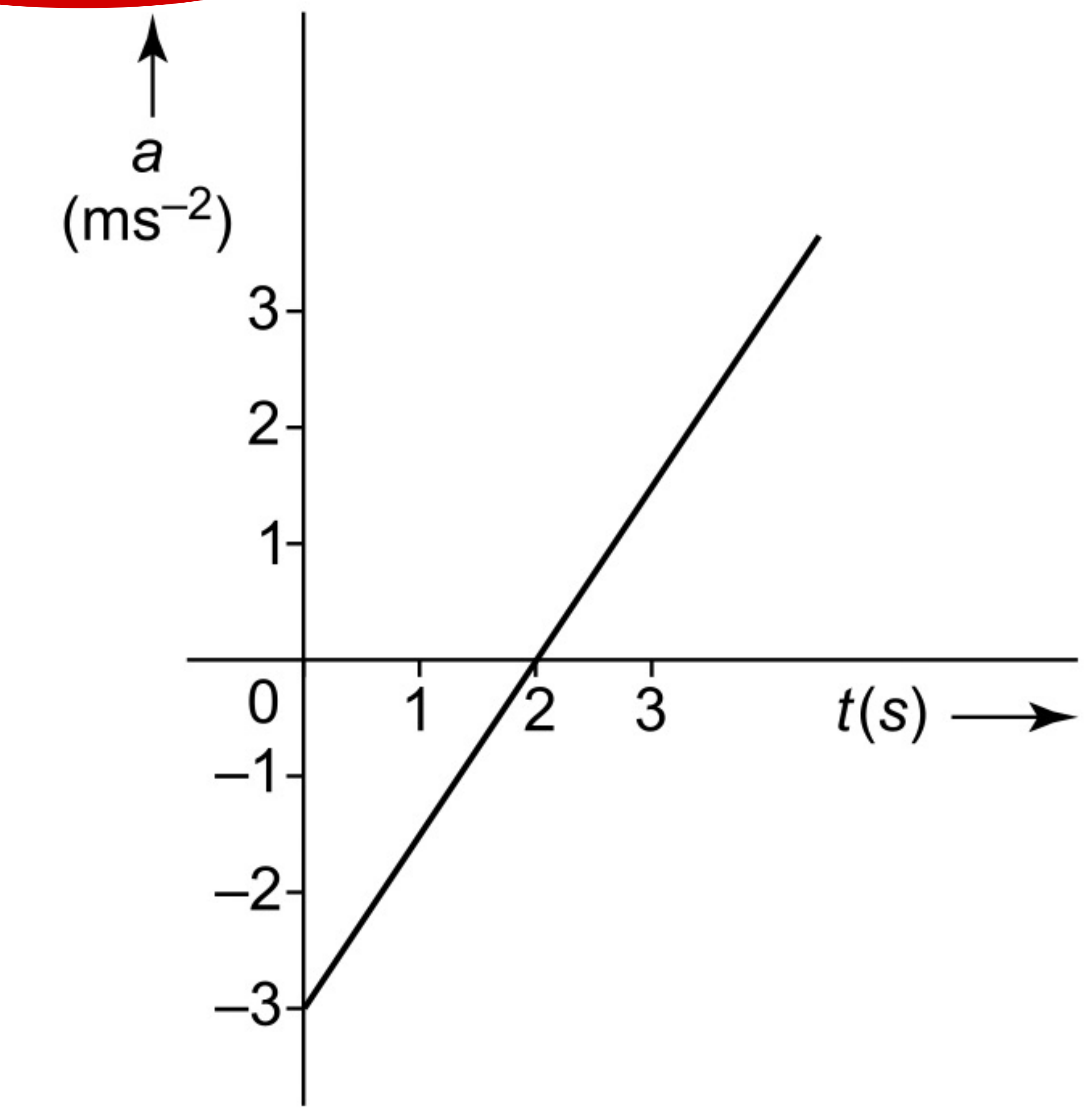
$a = -g$

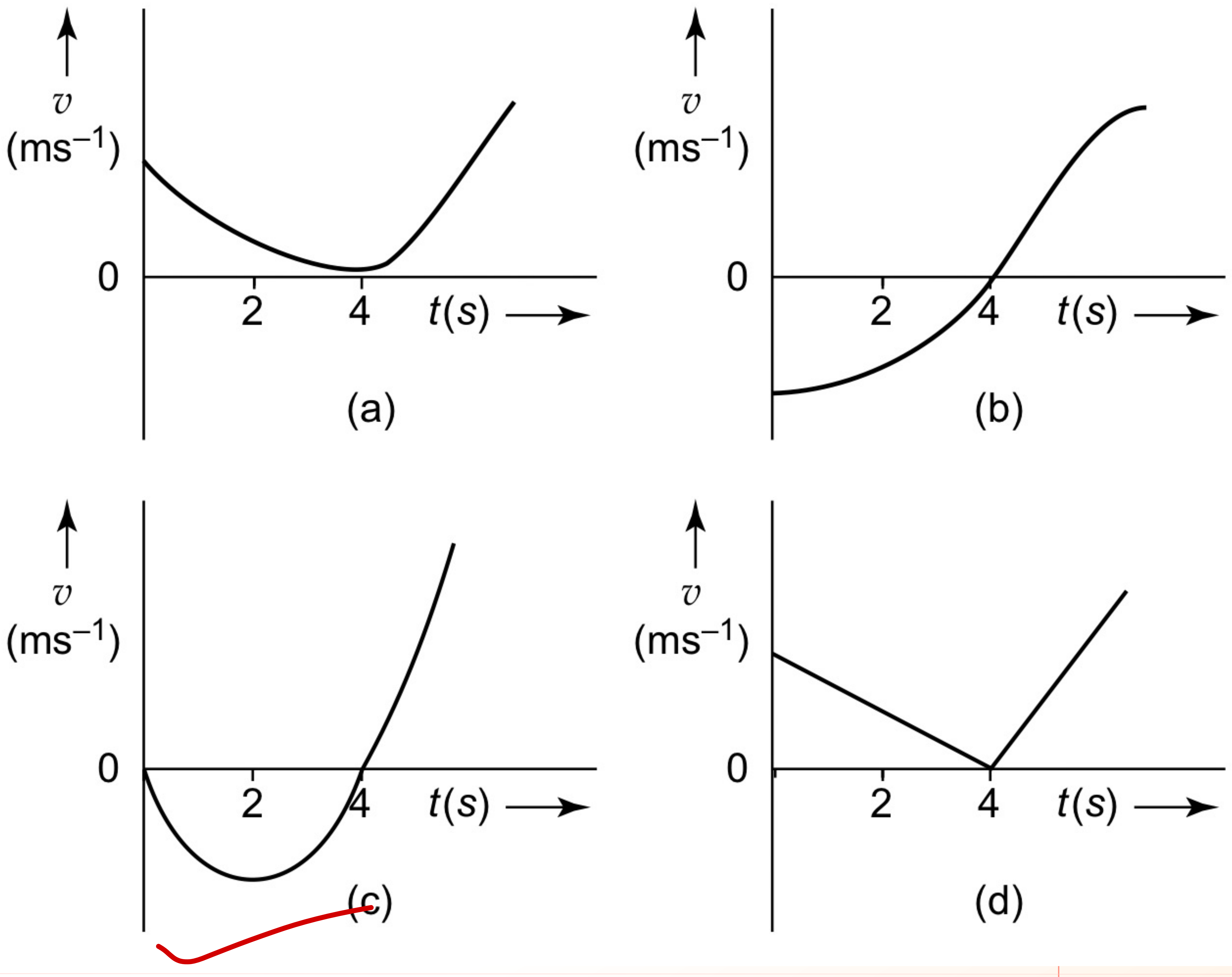




Ex

Figure 2.25 shows the acceleration – time ($a - t$) graph of a body moving in a straight line. Which graph in Fig. 2.26 shows the velocity – time ($v - t$) of the motion of the body? Assume that $x = 0$ and $v = 0$ at $t = 0$.

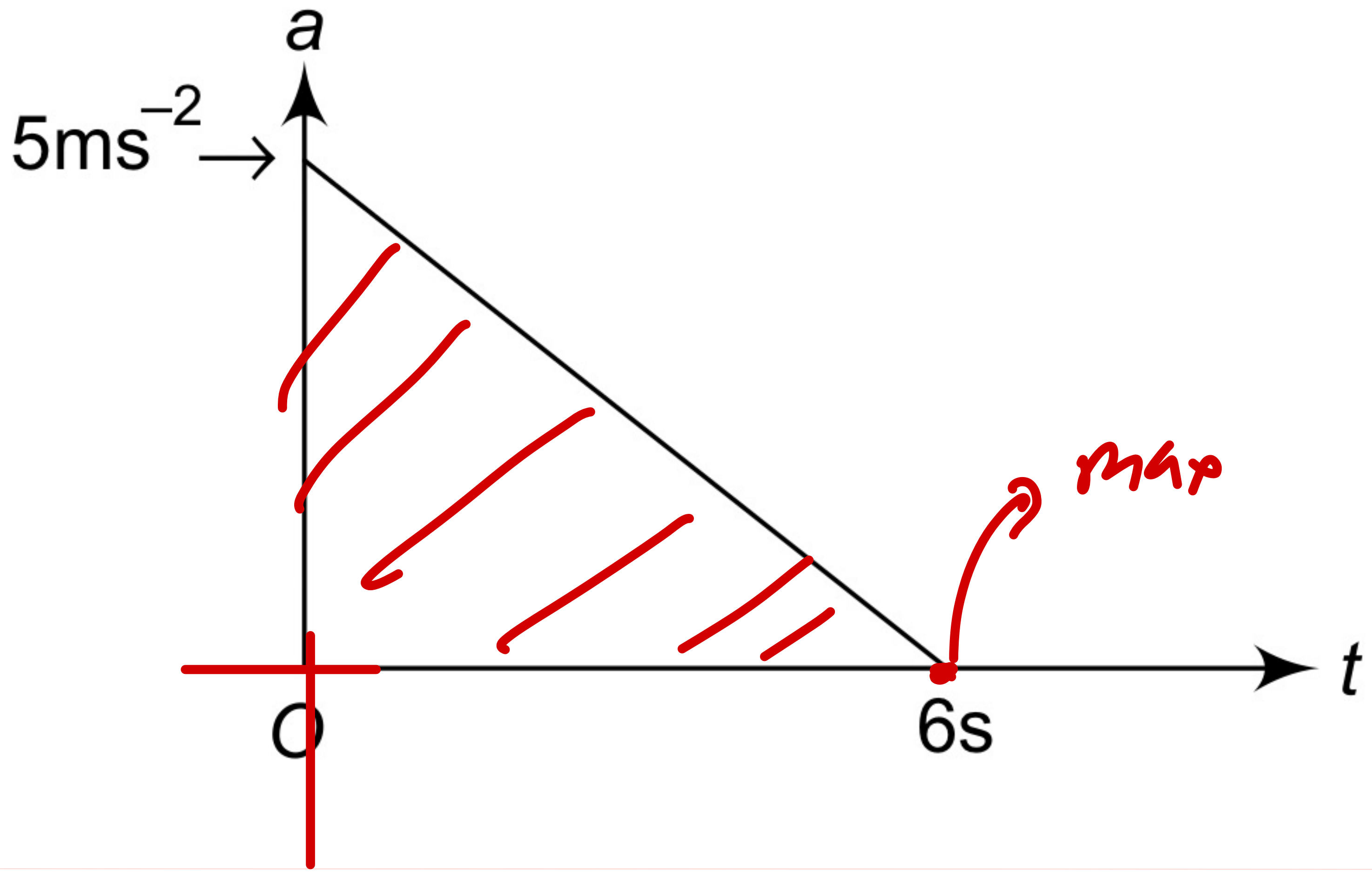




$a - t$ Area = Change in velocity

$a = dv/dt \Rightarrow \int dv = \int a\,dt =$ area

Ex A particle starts from rest at $x = 0$. Its acceleration at time $t = 0$ is 5 ms $^{-2}$ which varies with time as shown in Fig. 2.11. Find (a) the maximum speed of the particle and (b) its displacement in time interval from $t = 0$ to $t = 2$ s.



$V_{max} - 0 = \frac{1}{2} \times 6 \times 5 = 15$

$\boxed{V_{max} = 15 m/s}$ Ans

a - t Relⁿ

$a = -\frac{5}{6}t + 5$

$\int_0^v dv = -5/6 \int_0^t t\,dt + 5\int_0^t dt$

$v = -\frac{5}{6} \cdot \frac{t^2}{2} + 5t$

$v = -\frac{5}{12}t^2 + 5t$

$\int_0^x dx = \int_0^2 -\frac{5}{12} t^2\,dt + \int_0^2 5t\,dt$

$x = -\frac{5}{12}\left[\frac{t^3}{3}\right]_0^2 + 5\left[\frac{t^2}{2}\right]_0^2$

$= -\frac{5}{12} \times \frac{8}{3} + 5 \times 2 = -\frac{10}{9} + 10$

$\boxed{x = \frac{80}{9} m}$ Ans

Ex Figure 2.22 shows the variation of velocity (v) of a body with position (x) from the origin O. Which of the graphs shown in Fig. 2.23 correctly represents the variation of the acceleration (a) with position (x)?

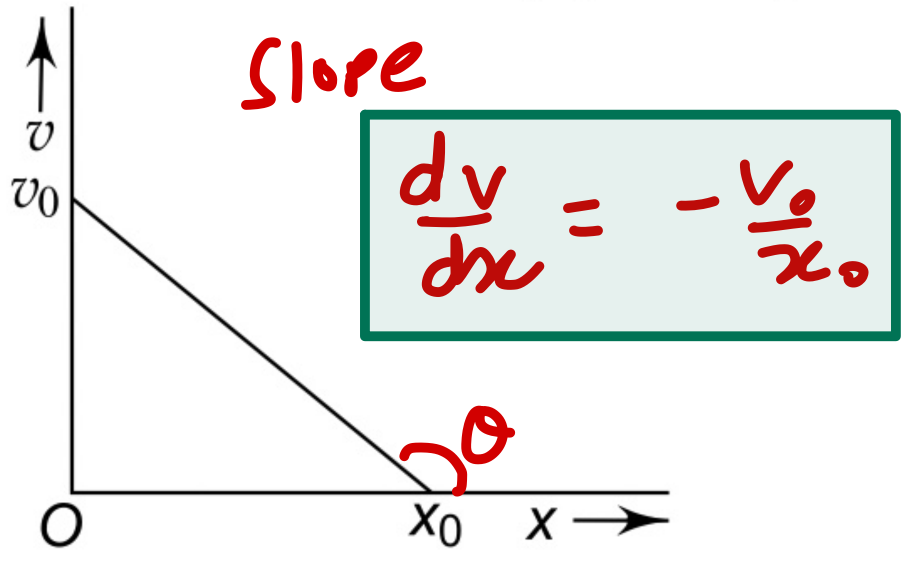
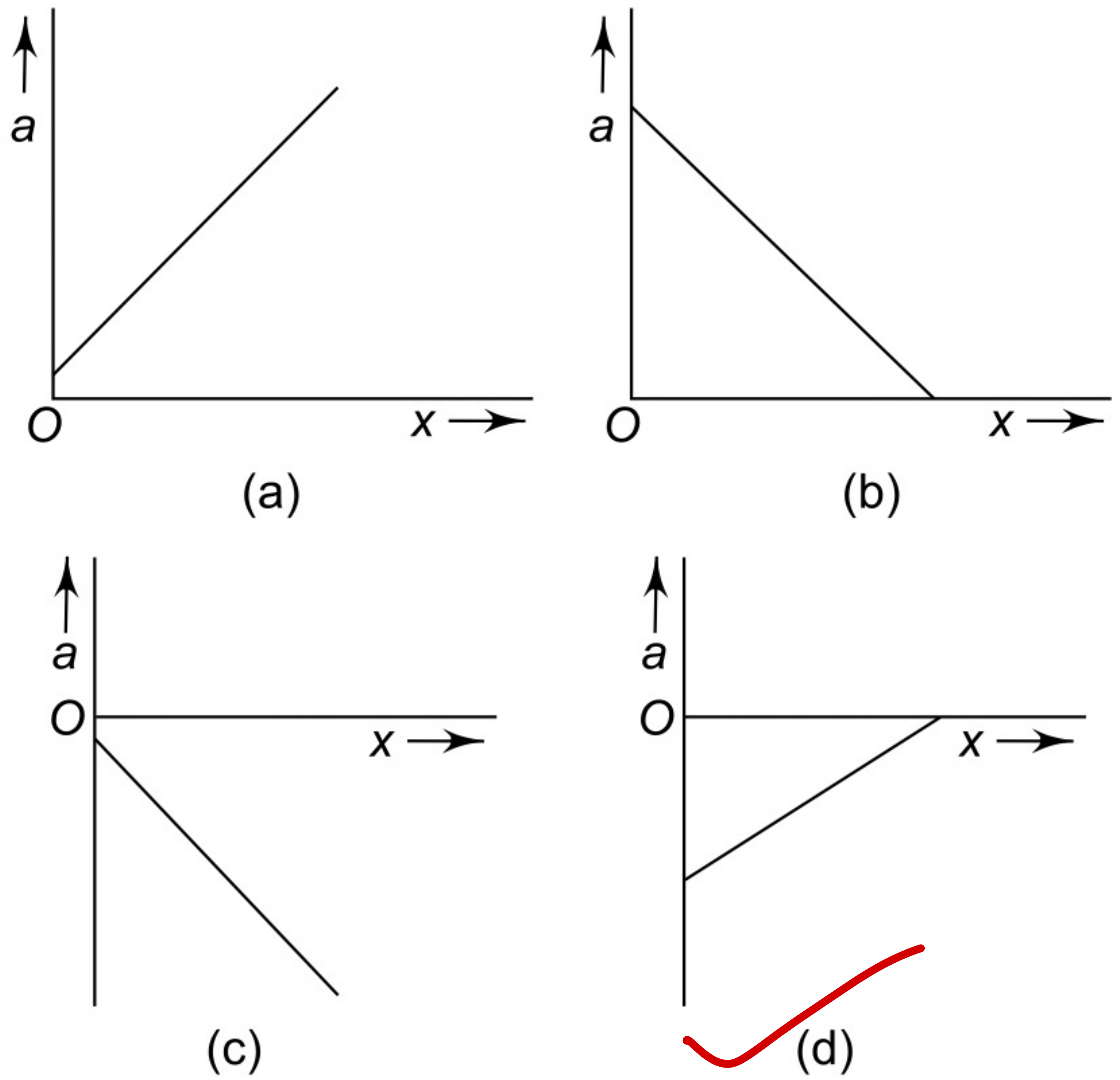


Fig. 2.22



Soln

$$a = v\frac{dv}{dx} \longrightarrow (1)$$

Reln V- x (From graph)

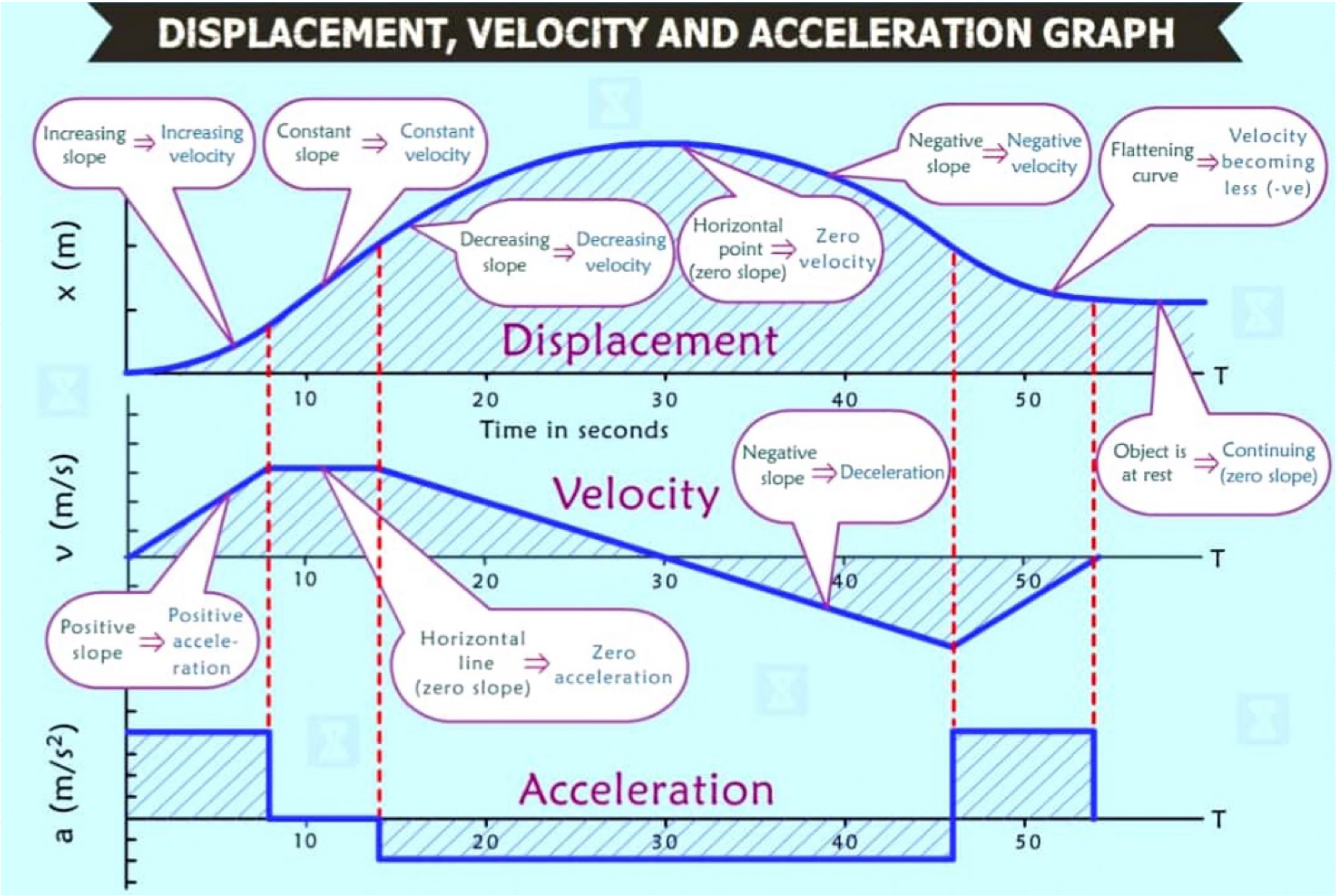
$$V = -\frac{v_0}{x_0}x + v_0$$

Put values of v and dv/dx into eq-(1)

$$a = \left(-\frac{v_0}{x_0}x + v_0\right)\left(-\frac{v_0}{x_0}\right)$$

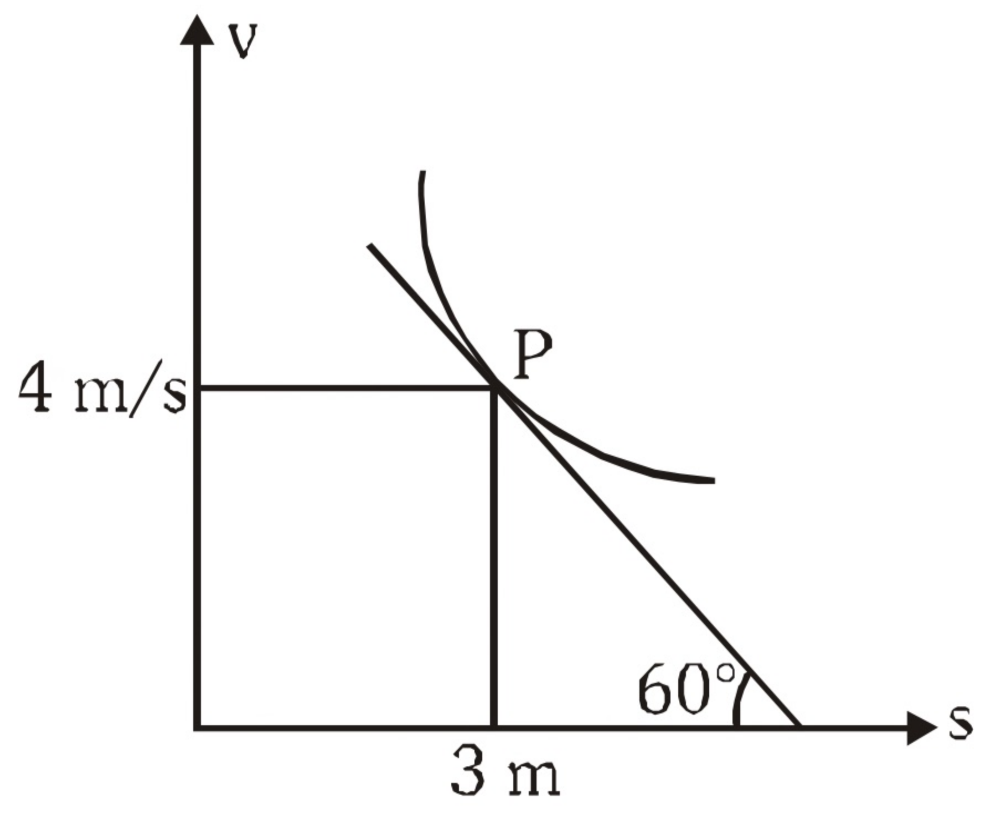
$$a = \underbrace{\left(\frac{v_0}{x_0}\right)^2}_{m}x - \underbrace{\frac{v_0^2}{x_0}}_{c}$$

$$\boxed{a = mx - c}$$



Only for acc. constant

7. A particle is moving along a straight line whose velocity-displacement graph is as shown in figure : A tangent is drawn at point P on the graph. At the point P



(A) the particle is speeding up

(B) numerical value of velocity and acceleration of the particle are equal

(C) numerical value of velocity is more than the numerical value of acceleration of the particle

(D) numerical value of acceleration is more than the numerical value of velocity of the particle

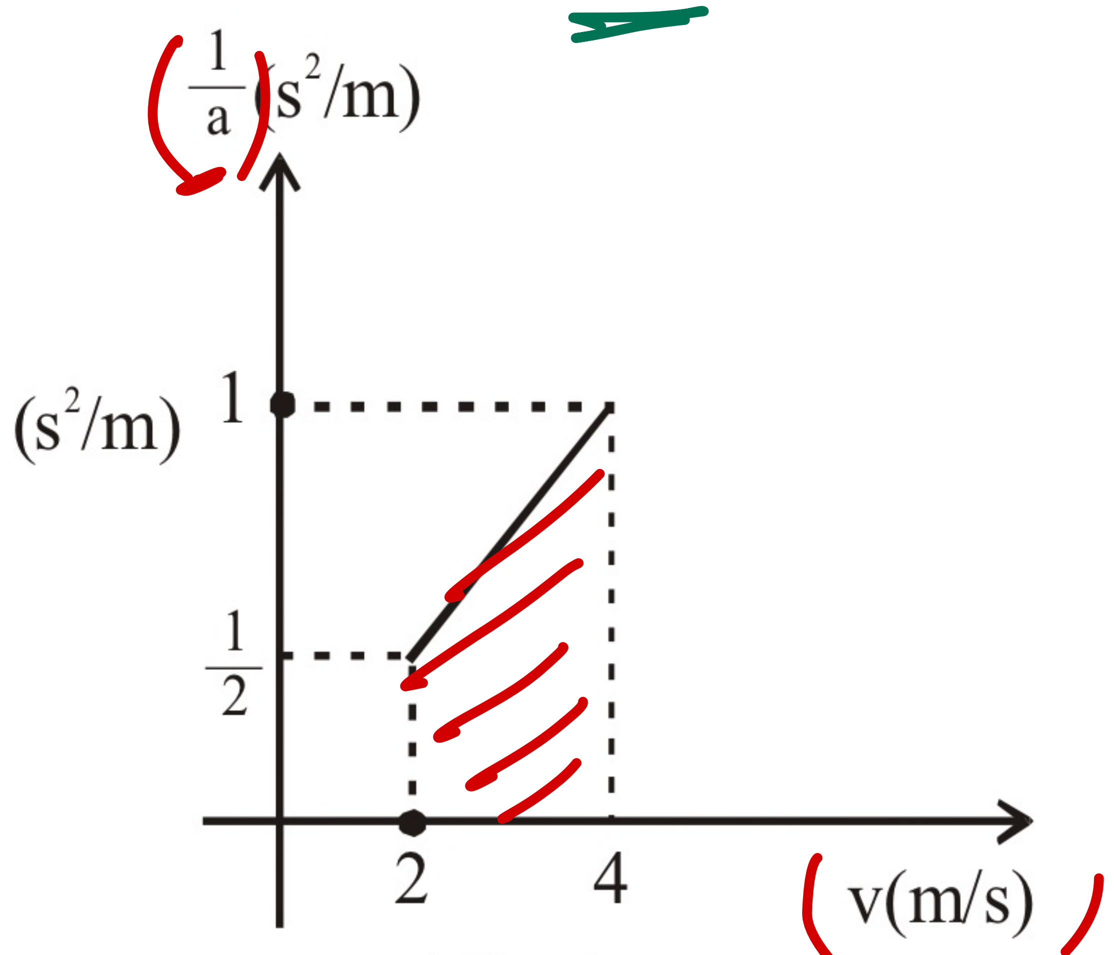
$a = v\frac{dv}{dx}$

$\left.\frac{dv}{dx}\right|_P = -\tan 60 = -\sqrt{3}$

$\boxed{v_P = 4 \text{ m/s}}$

$\boxed{a_P = -4\sqrt{3} \text{ m/s}^2}$

12. Given graph is $\frac{1}{\text{acceleration}}$ vs velocity graph. If the time interval during which velocity changes from 2m/s to 4m/s is given by Δt seconds. Then find the value of $2\Delta t$



(A) 3 (B) 4 (C) 5 (D) 6

$\because a = \frac{dv}{dt}$

$\int_0^{\Delta t} dt = \int_2^4 \left(\frac{1}{a}\right) \cdot dv$ = area of $\left(\frac{1}{a}\right)$ and v curve

$\Delta t = \frac{1}{2}\left(1+\frac{1}{2}\right)(4-2)$

$= \frac{1}{2} \times \frac{3}{2} \times 2$

$\Delta t = \frac{3}{2}$

Value of $2\Delta t = 2 \times \frac{3}{2} = 3$