$x \rightarrow \frac{d}{dt} = v \rightarrow \frac{d}{dt} = a$

Finding displacement & distance from velocity function :-

1. Velocity as function of time
2. Velocity as function of position

$$x \xrightarrow{d/dt} v \xrightarrow{d/dt} a$$

$$x \xleftarrow{\int} v \xleftarrow{\int} a$$

① $v = f(t)$

$$\frac{dx}{dt} = f(t)$$

$$\int_{x_i}^{x_f} dx = \int_{t_i}^{t_f} f(t)\, dt$$

$$\boxed{x_f - x_i = s = \int_{t_1}^{t_2} v\, dt}$$

② Given $v = f(x)$

$\because v = \frac{dx}{dt}$

$$\frac{dx}{dt} = f(x)$$

$$\boxed{\int_{x_i}^{x_f} \frac{dx}{f(x)} = \int_{t_i}^{t_f} dt}$$

5. The velocity of a particle traveling in a straight line is given by $v(t)=5-6e^{-t/2}$ m/s, where time t is in seconds and $t \geq 0$. If the particle is observed at $x=7$m at the instant t = 0, its position x is expressed as function of time $x(t)=kt+le^{-t/2}+m$. Find numerical value of $\frac{k+m}{\ell}$.

$\because \quad v = 5-6e^{-t/2}$

$$\frac{dx}{dt} = 5-6e^{-t/2}$$

$$\int_7^x dx = 5\int_0^t dt - 6\int_0^t e^{-t/2}\,dt$$

$$x-7 = \left[5t - 6\frac{e^{-t/2}}{\left(-\frac{1}{2}\right)}\right]_0^t$$

$$x-7 = 5t + 12\left(e^{-t/2} - e^{-0}\right]$$

$$x-7 = 5t + 12e^{-t/2} - 12$$

$$\boxed{x = 5t + 12e^{-t/2} - 5}$$

$K = 5, \quad \ell = 12 \quad m = -5$

$$\frac{K+m}{\ell} = \frac{5-5}{12} = 0 \text{ Ans}$$

18. A particle has a velocity of v = 8 – 2t ms^{-1} and moves in a straight line. It is at origin at t = 0. When will it pass through the origin again.

19. A particle has a velocity of v = 10 – 2t ms^{-1} and moves in a straight line. Find the distance traveled in 10 s

(18) Given

$V = 8-2t \quad t=0,\ x_i=0,\ x_f=0$

$$\frac{dx}{dt} = 8-2t$$

$$\int_0^0 dx = \int_0^t (8-2t)\,dt$$

$$0 = \int_0^t 8\,dt - 2\int_0^t t\,dt$$

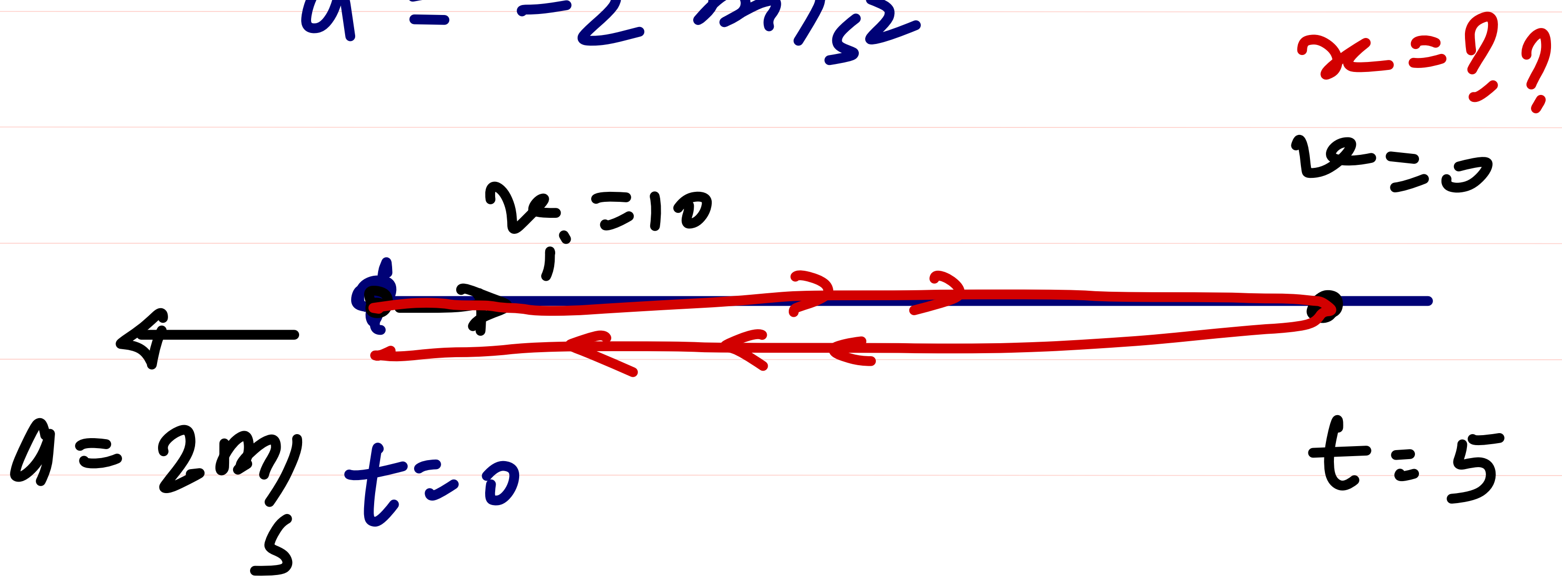
$$0 = 8t - 2\frac{t^2}{2}$$

$$0 = 8t - t^2 \Rightarrow \boxed{t = 8\ \text{sec}}$$

Ans

(19) $v = 10-2t$

$a = -2\ m/s^2$



$$\therefore \frac{dx}{dt} = 10-2t$$

$$\int_0^x dx = 10\int_0^5 dt - 2\int_0^5 t\,dt$$

$$x = 10\times5 - \cancel{2}\times\frac{5^2}{\cancel{2}}$$

$$x = 50-25$$

$$\boxed{x = 25\,m}$$

$d = 2x$

$$\boxed{d = 50\,m}$$

Based $v = f(x)$

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23. Velocity of a particle varies with position as per the equation $v = \frac{1}{x}$. At t = 0 the position is 2 m. Find the position at t = 1 s.

$$v = \frac{1}{x}$$

$$\frac{dx}{dt} = \frac{1}{x}$$

$$\int_2^x x\,dx = \int_0^1 dt$$

$$\left[\frac{x^2}{2}\right]_2^x = [t]_0^1$$

$$\frac{x^2 - 2^2}{2} = 1$$

$$x^2 - 4 = 2$$

$$\boxed{x = \sqrt{6}\text{ m}}$$ Ans

Ex Velocity - position Relⁿ is $v = x+2$ Find position at $t = 2$ sec if at $t = 0$ sec $x = 0$ m

$$v = x+2$$

$$\frac{dx}{dt} = x+2$$

$$\int_0^x \frac{dx}{x+2} = \int_0^2 dt$$

$$\left[\ln(x+2)\right]_0^x = 2$$

$$\ln(x+2) - \ln(2) = 2$$

$$\ln\left(\frac{x+2}{2}\right) = 2$$

$$\frac{x+2}{2} = e^2$$

$$x+2 = 2e^2$$

$$x = 2e^2 - 2$$

$$\boxed{x = 2(e^2-1)}$$ Ans

$a = dv/dt = v\,dv/dx$

Finding velocity from acceleration function :→

1. Acceleration as function of time
2. Acceleration as function of position
3. Acceleration as function of velocity

① $a = f(t)$

$$\frac{dv}{dt} = f(t)$$

$$\boxed{\int_{v_1}^{v_2} dv = \int_{t_1}^{t_2} f(t)\,dt}$$

(2) $\boxed{a = f(x) = v\frac{dv}{dx}}$

$$\frac{dv}{dt} = f(x)$$

$$\because v = \frac{dx}{dt}$$

$$dt = \frac{dx}{v}$$

$$\frac{dv}{dx/v} = f(x)$$

$$\boxed{\int_{v_1}^{v_2} v\,dv = \int_{x_1}^{x_2} f(x)\,dx}$$

③ $a = f(v)$

$$\frac{dv}{dt} = f(v)$$

$$\boxed{\int_{v_1}^{v_2} \frac{dv}{f(v)} = \int_{t_1}^{t_2} dt}$$

Ex-1 The motion of a body is given by the equation

$$a = \frac{dV(t)}{dt} = 6.0 - 3V(t)$$

where $V(t)$ is the speed (in ms^{-1}) at time t (in second), If the body was at rest at $t = 0$, the magnitude of the initial acceleration is

(a) $3\ ms^{-2}$ (b) $6\ ms^{-2}$ ✓
(c) $9\ ms^{-2}$ (d) zero

Ex-2 In Q.40, the speed of the body varies with time as

(a) $V(t) = (1 - e^{-3t})$

(b) $V(t) = 2\,(1 - e^{-3t})$ ✓

(c) $V(t) = \frac{2}{3}\left(1 - e^{\frac{-3t}{2}}\right)$

(d) $V(t) = \frac{3}{2}\left(1 - e^{\frac{-2t}{3}}\right)$

Ex-3 In Q.40, the speed of the body when the acceleration is half the initial value is

(a) $1\ ms^{-1}$ ✓ (b) $2\ ms^{-1}$
(c) $3\ ms^{-1}$ (d) $4\ ms^{-1}$

① $a = 6 - 3v$ Given $t=0$ $v_i = 0$

$a_i = 6 - 3\times 0$

$\boxed{a_{in} = 6\ m/s^2}$ Ans

② $\frac{dv}{dt} = 6 - 3v$

$\int_0^v \frac{dv}{6-3v} = \int_0^t dt$

$\left[\frac{\ln(6-3v)}{-3}\right]_0^v = t$

$\frac{\ln(6-3v) - \ln(6-3\times 0)}{-3} = t$

$\ln\left(\frac{6-3v}{6}\right) = -3t$

$\frac{6-3v}{6} = e^{-3t}$

$6 - 3v = 6e^{-3t}$

$6 - 6e^{-3t} = 3v$

$\boxed{v = 2(1 - e^{-3t})}$ Ans

③ $a = 6 - 3v$

$\because a = \frac{a_i}{2} = \frac{6}{2} = 3$

$3 = 6 - 3v$

$3v = 6 - 3$

$\boxed{v = 1\ m/s}$ Ans

Case $a = f(x)$ Yaad !! $\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1}(x) + C$

Illustration 9*. Acceleration of a particle moving along the x-axis is defined by the law $a = -4x$, where a is in m/s^2 and x is in meters. At the instant t = 0, the particle passes the origin with a velocity of 2 m/s moving in the positive x-direction.

(a) Find its velocity v as function of its position coordinates.

(b) Find its position x as function of time t.

(c) Find the maximum distance it can go away from the origin.

$a = -4x \qquad t = 0 \quad x = 0, \; v_i = 2\,m/s$

(a) $v\frac{dv}{dx} = -4x$

$$\int_2^v v\,dv = \int_0^x -4x\,dx$$

$$\left[\frac{v^2}{2}\right]_2^v = -4\left[\frac{x^2}{2}\right]_0^x$$

$$\frac{v^2 - 2^2}{2} = -2(x^2 - 0^2)$$

$$v^2 - 2^2 = -4x^2$$

$$v^2 = -4x^2 + 4$$

$$\boxed{v = 2\sqrt{1-x^2}} \quad \text{Ans}$$

(b) $v = 2\sqrt{1-x^2}$

$$\frac{dx}{dt} = 2\sqrt{1-x^2}$$

$$\int_0^x \frac{dx}{\sqrt{1-x^2}} = \int_0^t 2\,dt$$

$$\left[\sin^{-1}(x)\right]_0^x = 2t$$

$$\sin^{-1}(x) - \sin^{-1}(0) = 2t \quad \text{Ans}$$

$$\sin^{-1}(x) = 2t \Rightarrow \boxed{x = \sin(2t)}$$

(c) $\because \sin(2t) \le 1$

$$x \le 1$$

$$\boxed{x_{max} = 1\,m} \quad \text{Ans}$$

Ex

A particle is moving along the x-axis with an acceleration $a = 2x$ where a is in ms^{-2} and x is in metre. If the particle starts from rest at $x = 1$ m, find its velocity when it reaches the position $x = 3$ m.

Given

$a = 2x$

$t = 0$

$u_i = 0$

$x_i = 1m$

Find

$v = ??$

at $x_f = 3m$

$v\frac{dv}{dx} = 2x$

$\int_0^v v\,dv = \int_1^3 2x\,dx$

$\frac{v^2}{\cancel{2}} = 2\left[\frac{x^2}{\cancel{2}}\right]_1^3$

$v^2 = 2(3^2 - 1^2)$

$= 2[9-1]$

$= 2\times 8$

$v^2 = 16$

$\boxed{v = 4 m/s}$

$e^{-\infty} = 0 \quad e^{0} = 1$

24. A particle is given velocity of 5 m/s and its acceleration is $a = -2v$, where v is its velocity at any time t. Find the velocity v at any time t. Also find the total distance travelled.

25. A particle starts and has acceleration $a = 5 - 2v$, where v is its velocity at any time t. Find the velocity v at any time. Also find the terminal velocity.

24

$v_i = 5\, m/s$

$a = -2v$

$\frac{dv}{dt} = -2v$

$\int_5^v \frac{dv}{v} = -2\int_0^t dt$

$\ln(v/5) = -2t$

$\frac{v}{5} = e^{-2t}$

$\boxed{v = 5e^{-2t}}$

total distance

$\frac{dx}{dt} = 5e^{-2t}$

$\int_0^d dx = 5\int_0^\infty e^{-2t}\, dt$

$d = 5\left[\frac{e^{-2t}}{-2}\right]_0^\infty$

$d = 5\left[\frac{e^{-\infty} - e^{-0}}{-2}\right]$

$d = 5\left[\frac{0-1}{-2}\right] = \frac{5}{2}\, m$

25

$a = 5 - 2v$

$\frac{dv}{dt} = 5 - 2v$

$\int_0^v \frac{dv}{5-2v} = \int_0^t dt$

$\frac{\ln\left(\frac{5-2v}{5}\right)}{-2} = t$

$\ln\left(\frac{5-2v}{5}\right) = -2t$

$\frac{5-2v}{5} = e^{-2t}$

$5 - 2v = 5e^{-2t}$

$\boxed{v = \frac{5}{2}(1 - e^{-2t})}$

Ans

TERMINAL velocity

Velocity at instant where Net acc. is equal to $= 0$

$a = 5 - 2v$

$0 = 5 - 2v_T$

$\boxed{v_T = \frac{5}{2} = 2.5\, m/s}$

Ans

Ex

A particle of mass m moving with initial velocity u enters a medium at time $t = 0$. The medium offers a resistive force $F = kv$ wher k is a constant of the medium and v is the instantaneous velocity. The velocity of the particle varies with time t as

(a) $v = u + \dfrac{kt}{m}$ (b) $v = u - \dfrac{kt}{m}$

(c) $v = u\,e^{-kt/m}$ (d) $v = u\,e^{kt/m}$

$F = -kv$

$ma = -kv$

$a = -\dfrac{k}{m} v$

$\dfrac{dv}{dt} = -\dfrac{k}{m} v$

$\displaystyle\int_u^v \dfrac{dv}{v} = \int_0^t -\dfrac{k}{m}\,dt$

$[\ln v]_u^v = -\dfrac{k}{m}[t]_0^t$

$\ln(v) - \ln(u) = -\dfrac{k}{m} t$

$\ln\left(\dfrac{v}{u}\right) = -k/m\, t$

$\dfrac{v}{u} = e^{-k/m\, t}$

$\boxed{v = u e^{-k/m\, t}}$

H.W

BB #1

B.B #3

Race # 8

complete