Ex -3 According to the quantum theory, the energy E of a photon of frequency ν is given by

$$E = h\nu$$

where h is Planck's constant. What is the dimensional formula for h?

(a) $M L^2 T^{-2}$ ~~(b)~~ $M L^2 T^{-1}$
(c) $M L^2 T$ (d) $M L^2 T^2$

Freq. $(\nu) = \frac{1}{\text{time}}$

$h = \frac{E}{\nu}$

$= E \times \text{time}$

$= [M^1 L^2 T^{-2}][T]$

$[h] = [M^1 L^2 T^{-1}]$

Ex-4

The volume V of water passing any point of a uniform tube during t seconds is related to the cross-sectional area A of the tube and velocity u of water by the relation

$$V \propto A^{\alpha} u^{\beta} t^{\gamma}$$

which one of the following will be true?

(a) $\alpha = \beta = \gamma$ (b) $\alpha \neq \beta = \gamma$

(c) $\alpha = \beta \neq \gamma$ (d) $\alpha \neq \beta \neq \gamma$

$V \propto A^{\alpha} u^{\beta} t^{\gamma}$

$[M^0 L^3 T^0] = [M^0 L^2 T^0]^{\alpha} [M^0 L^1 T^{-1}]^{\beta} [M^0 L^0 T^1]^{\gamma}$

$[M^0 L^3 T^0] = L^{2\alpha+\beta} \; T^{-\beta+\gamma}$

$2\alpha + \beta = 3$ — (1) $\qquad -\beta + \gamma = 0$ — (2)

$\boxed{\alpha = \dfrac{3-\beta}{2}}$ $\qquad \boxed{\beta = \gamma}$

$\beta = 1 = \gamma = \alpha$

$\beta = 2 = \gamma$

$\alpha = \dfrac{1}{2}$

Ex-5

The frequency n of vibrations of uniform string of length l and stretched with a force F is given by

$$n = \frac{p}{2l}\sqrt{\frac{F}{m}}$$

where p is the number of segments of the vibrating string and m is a constant of the string. What are the dimensions of m?

(a) $M L^{-1} T^{-1}$ (b) $M L^{-3} T^{0}$

(c) $M L^{-2} T^{0}$ (d) $M L^{-1} T^{0}$

Freq. $= \frac{1}{\text{time}} = \frac{1}{t}$

Squaring both sides

$$n^2 = \frac{p^2}{4l^2} \cdot \frac{F}{m}$$

$$m = K \frac{F}{l^2 . n^2} \qquad \left(K = \frac{p^2}{4}\right)$$

$$m = K \frac{F}{l^2} \times t^2$$

$$[m] = \frac{M^1 L^1 T^{-2}}{L^2} \times T^2$$

$$[m] = [M^1 L^{-1} T^0]$$

Race #2

9. The velocity v of waves produced in water depends on their wavelength λ, the density of water ρ, and acceleration due to gravity g. The square of velocity is proportional to

length

(A) $\lambda^{-1}g^{-1}\rho^{-1}$ (B) λg (C) $\lambda\rho g$ (D) $\lambda^2 g^{-2}\rho^{-1}$

10. If area (A), velocity (v) and density (ρ) are taken as the fundamental units, what is the dimensional formula for force

(A) $Av^2\rho$ (B) $A^2v\rho$ (C) $Av\rho^2$ (D) $Av\rho$

$K = \frac{F}{x} = \frac{M^1L^1T^{-2}}{L^1}$

$[K] = [M^1L^0T^{-2}]$

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$v \propto \lambda^a \rho^b g^c$

$[M^0L^1T^{-1}] = [L]^a [ML^{-3}]^b [M^0L^1T^{-2}]^c$

$[M^0L^1T^{-1}] = [M^b L^{a-3b+c} T^{-2c}]$

$b = 0$

$a - 3b + c = 1$

$-2c = -1$

$c = 1/2$

$a - 3\times 0 + \frac{1}{2} = 1$

$a = \frac{1}{2}$

$v \propto \lambda^{1/2}\rho^0 g^{1/2}$

$\boxed{v^2 \propto \lambda g}$ Ans

10

$F \propto A^a v^b \rho^c$

$[M^1L^1T^{-2}] = [L^2]^a [LT^{-1}]^b [M^1L^{-3}]^c$

$= [M^c L^{2a+b-3c} T^{-b}]$

$\boxed{c = 1}$

$2a + b - 3c = 1 \Rightarrow 2a + 2 - 3 = 1$

$-b = -2$

$\boxed{b = 2}$

$\boxed{a = 1}$

$F \propto A v^2 \rho$

4. In a particular system of unit, if the unit of mass becomes twice & that of time becomes half, then 8 Joules will be written as.... units of work.

(A) 16 (B) 1 (C) 4 (D) 64

New system
xyz

Unit of mass = (2 kg)

Unit of time = $\left(\frac{sec}{2}\right)$

$E = 8\,J = 8\ kg\,\frac{m^2}{sec^2}$

$E' = 8\left(\frac{2kg}{2}\right)\cdot\frac{m^2}{\left(\frac{sec}{2}\right)^2 \times 4}$

$= \frac{8}{8}\left\{\frac{(2kg)\cdot m^2}{(sec/2)^2}\right\}$

$E' = 1$ unit

$\underline{\underline{8\,J}} = 1$ unit In New system

3. If the units of mass, length and time are doubled, unit of angular momentum will be -

(A) Doubled (B) Tripled

(C) Quadrupled (D) 8 times the original value

AM = LM x dis

= mass x velo x dis

$= \text{mass} \times \dfrac{\text{dis}^2}{\text{time}}$

$= \cancel{2}\,\text{mass} \times \dfrac{(2\,\text{dist})^2}{\cancel{2}\,\text{time}}$

$= 4\ \dfrac{\text{mass} \times (\text{dist})^2}{\text{time}}$

6. The frequency of oscillation of an object of mass m suspended by means of spring of force constant K is given by f = Cm^xK^y, where C is a dimensionless constant. The value of x and y are -

(A) $x = \frac{1}{2}, y = \frac{1}{2}$ (B) $x = -\frac{1}{2}, y = \frac{1}{2}$ (C) $x = \frac{1}{2}, y = -\frac{1}{2}$ (D) $x = -\frac{1}{2}, y = -\frac{1}{2}$

$K = \frac{F}{x} = [M^1 L^0 T^{-2}]$

$[T^{-1}] = [M^1]^x [M^1 L^0 T^{-2}]^y$

$[M^0 L^0 T^{-1}] = [M^{x+y} L^0 T^{-2y}]$

$x + y = 0 \qquad -1 = -2y$

$\boxed{x = -\frac{1}{2}} \qquad \boxed{y = \frac{1}{2}}$

2. The density of a liquid is 1000 kg m^{-3}. Its value in CGS system -

(A) 1 (B) 0.1 (C) 10 (D) 100

$\rho = 1000\ \text{kg}/\text{m}^3$

$= \dfrac{1000\ (10^3\ \text{gm})}{(10^2\ \text{cm})^3}$

$= \dfrac{1000 \times 10^3}{10^6}\ \text{gm}/\text{cm}^3$

$= 1$

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5. If the speed of light (c), acceleration due to gravity (g) and pressure (p) are taken as fundamental units, the dimensions of gravitational constant (G) are -

(A) c^0gp^{-3} (B) $c^2g^3p^{-2}$ (C) $c^0g^2p^{-1}$ (D) $c^2g^2p^{-2}$

$$G \propto c^a g^b p^c$$

$$[M^{-1}L^3T^{-2}] = [L^1T^{-1}]^a [L^1T^{-2}]^b [M^1L^{-1}T^{-2}]^c$$

$$M^{-1}L^3T^{-2} = M^c L^{a+b-c} T^{-a-2b-2c}$$

$$c = -1 \quad \text{—} (1)$$

$$a+b-c = 3 \quad \text{—} (2)$$

$$-a-2b-2c = -2$$

$$a+2b+2c = 2 \quad \text{—} (3)$$

$$F = G\frac{m_1m_2}{r^2}$$

$$[G] = \left[\frac{F \cdot r^2}{m_1m_2}\right] = \frac{[M^1L^1T^{-2} \times L^2]}{[M^2]}$$

$$= [M^{-1}L^3T^{-2}]$$

7. The velocity of a body which has fallen under gravity varies as $g^a h^b$, where g is acceleration due to gravity and h is the height. The values of a and b are -

(A) a = 1, b = 1/2 (B) a = b = 1 (C) a = 1/2, b = 1 (D) a = 1/2 ; b = 1/2

$v \propto g^a h^b$

$[M^0 L^1 T^{-1}] = [M^0 L^1 T^{-2}]^a [M^0 L^1 T^0]^b$

$[M^0 L^1 T^{-1}] = [M^0 L^{a+b} T^{-2a}]$

$a + b = 1$ $\qquad -2a = -1$

$a = \frac{1}{2}$

$b = \frac{1}{2}$

Ex-6 The mass m of the heaviest stone that can be moved by the water flowing in a river depends on v, the speed of water, density (d) of water and the acceleration due to gravity (g). Then m is proportional to

(a) v^2 (b) v^4

(c) v^6 ✓ (d) v^8

$m \propto v^a d^b g^c$

$[M^1 L^0 T^0] = (L^1 T^{-1})^a [M^1 L^{-3}]^b [L T^{-2}]^c$

$[M^1 L^0 T^0] = M^b L^{a-3b+c} T^{-a-2c}$

$\boxed{b=1}$

$a - 3b + c = 0$ $\qquad -a - 2c = 0$

$a + c = 3$ ① $\qquad a + 2c = 0$ ②

$a + 2c = 0$

$a + c = 3$

$-\quad -\quad -$

$\boxed{c = -3}$

$\boxed{a = 6}$

$m \propto v^6 d^1 g^{-3}$

$\boxed{m \propto v^6}$

$\boxed{m \propto d^1}$

$\boxed{m \propto g^{-3}}$

Limitations of this method :

(i) This method can be used only if the dependency is of multiplication type. The formula containing exponential, trignometrical and logarithmic functions can't be derived using this method. Formulae containing more than one term which are added or subtracted like $s = ut + at^2/2$ also can't be derived.

(ii) The relation derived from this method gives no information about the dimensionless constants.

(iii) In a given equation correctness of numerical value or dimensionless function can not be checked.

(iv) We cannot derive an equation which depends more than three physical quantities.

(v) We cannot find out wheather physical quantity is scalar or a vector.