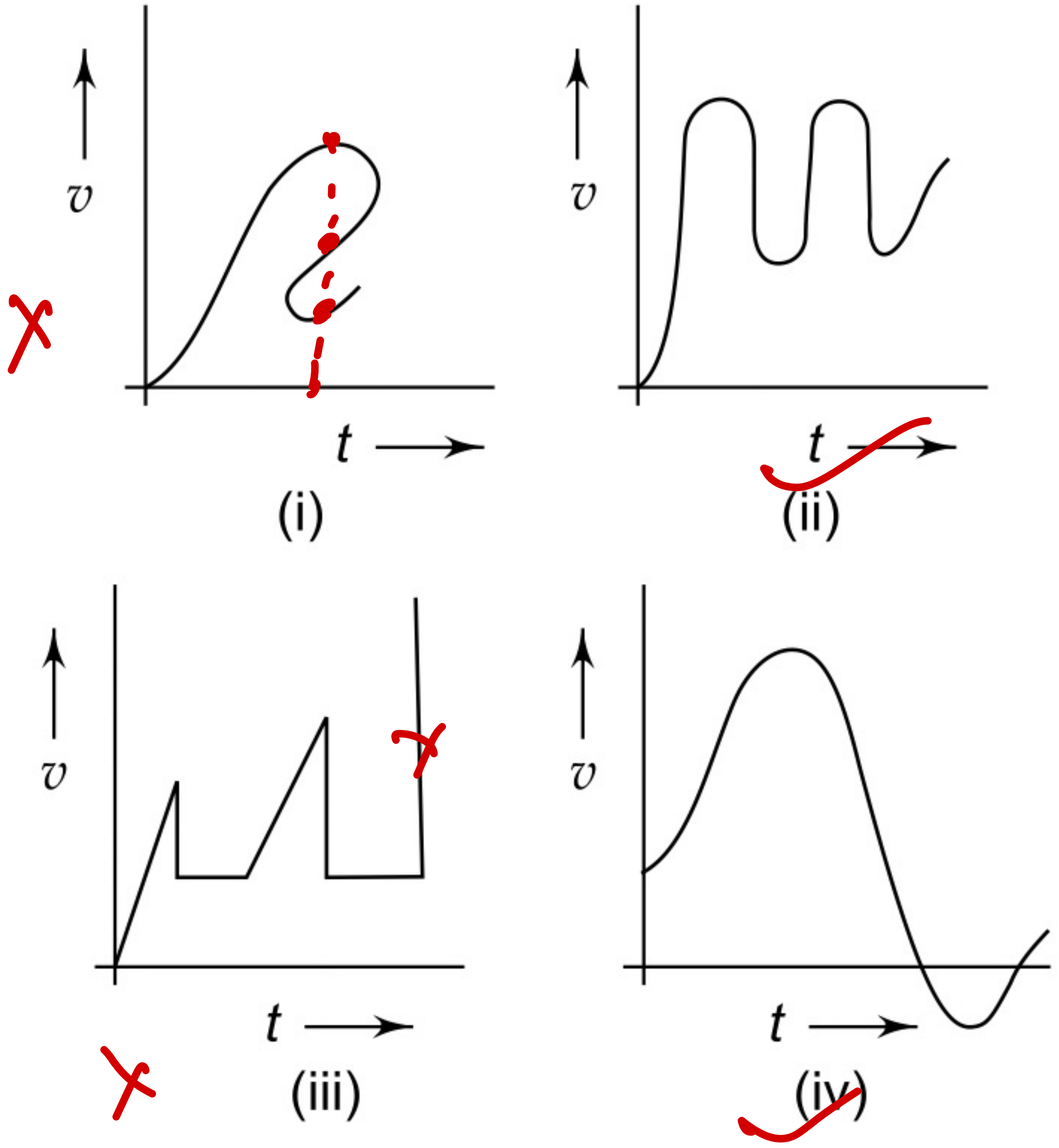
Ex Figure 2.18 shows the velocity–time ($v - t$) graphs for one dimensional motion. But only some of these can be realized in practice. These are

(a) (i), (ii) and (iv) only
(b) (i), (ii) and (iii) only
(c) (ii) and (iv) only
(d) all



Ex A ball is dropped vertically from a height h above the ground. It hits the ground and bounces up vertically to a height $h/2$. Neglecting subsequent motion and air resistance, its velocity v varies with the height h as (see Fig. 2.19)

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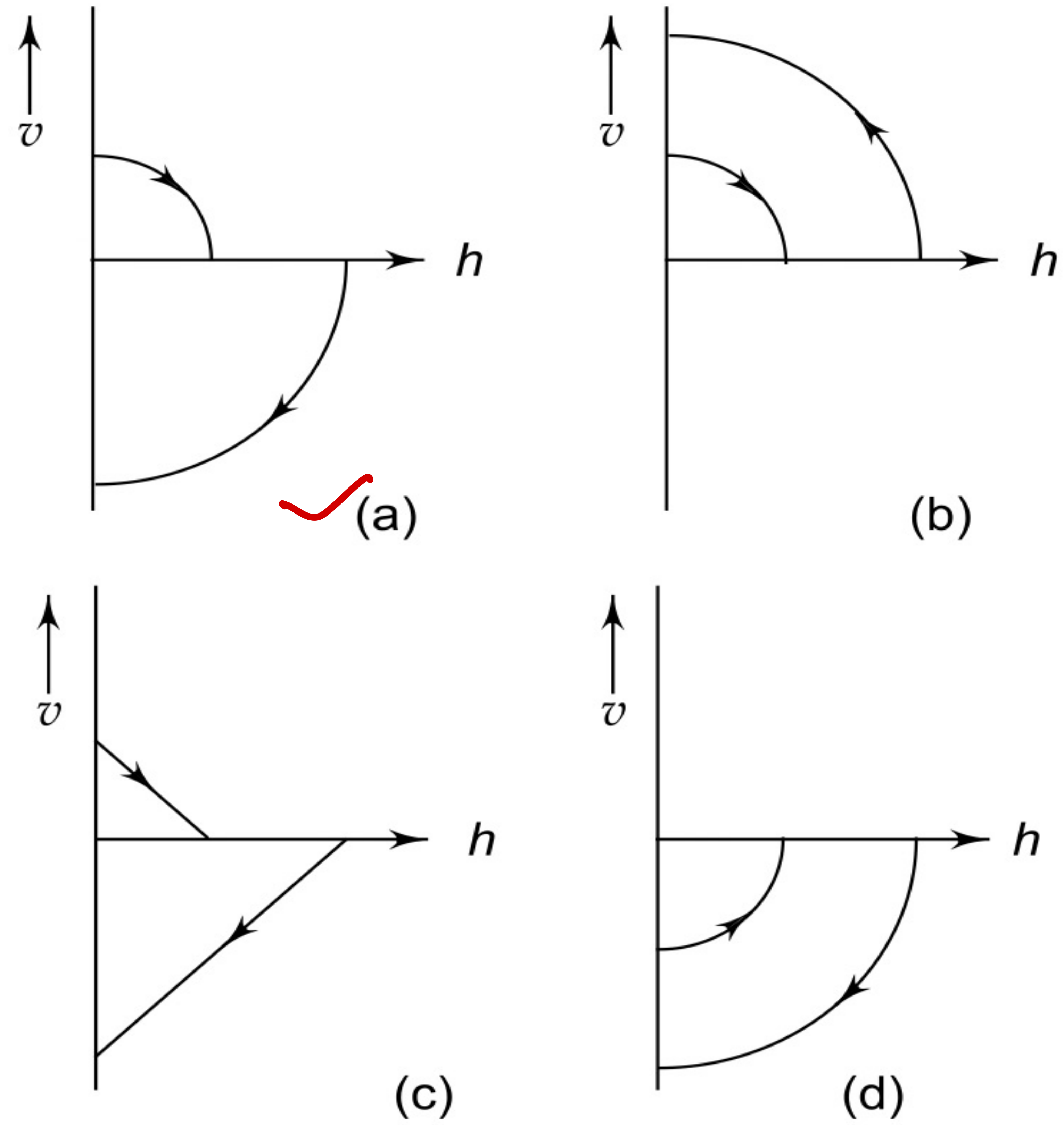
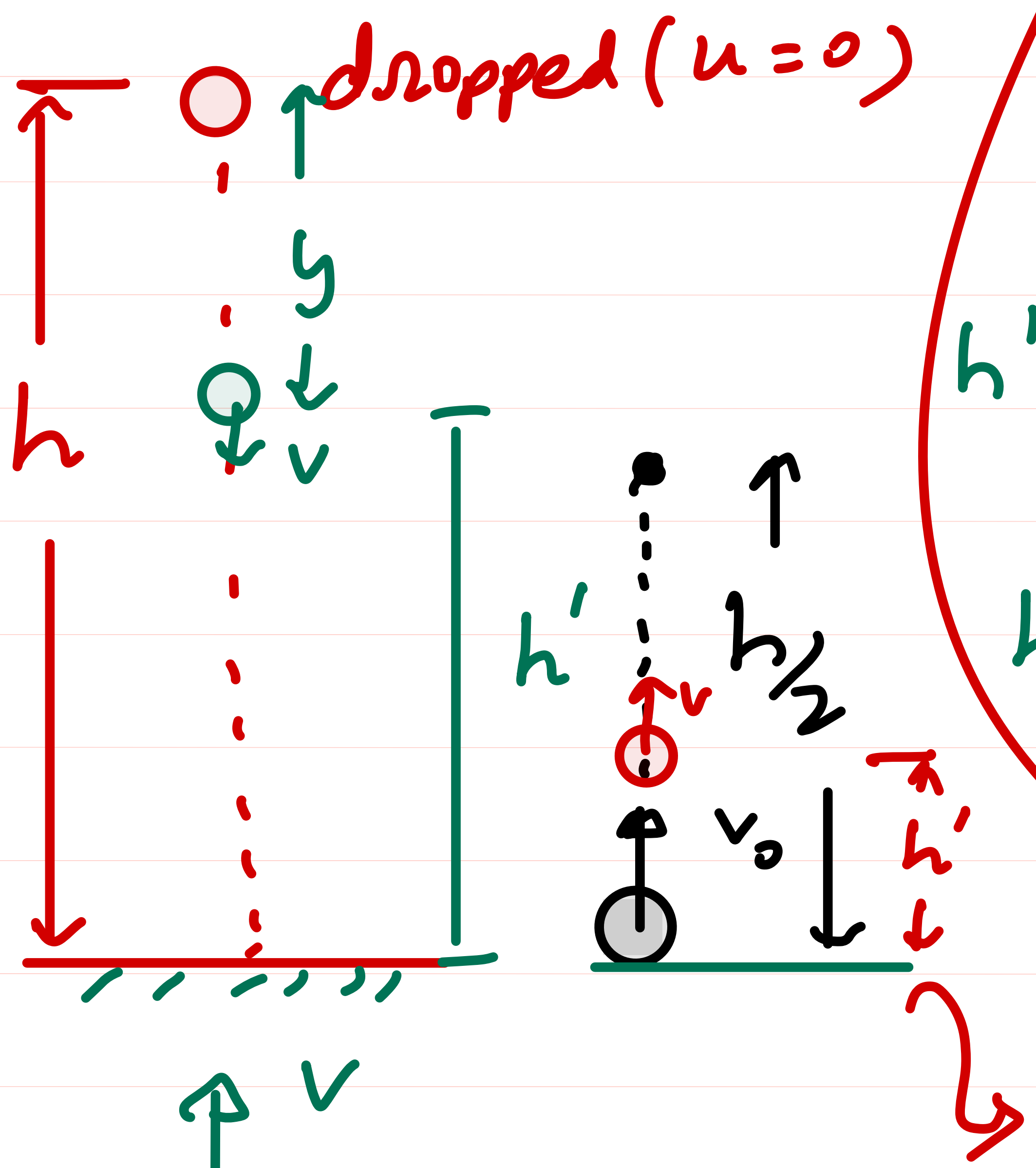




For downward motion

$h' = h - y$

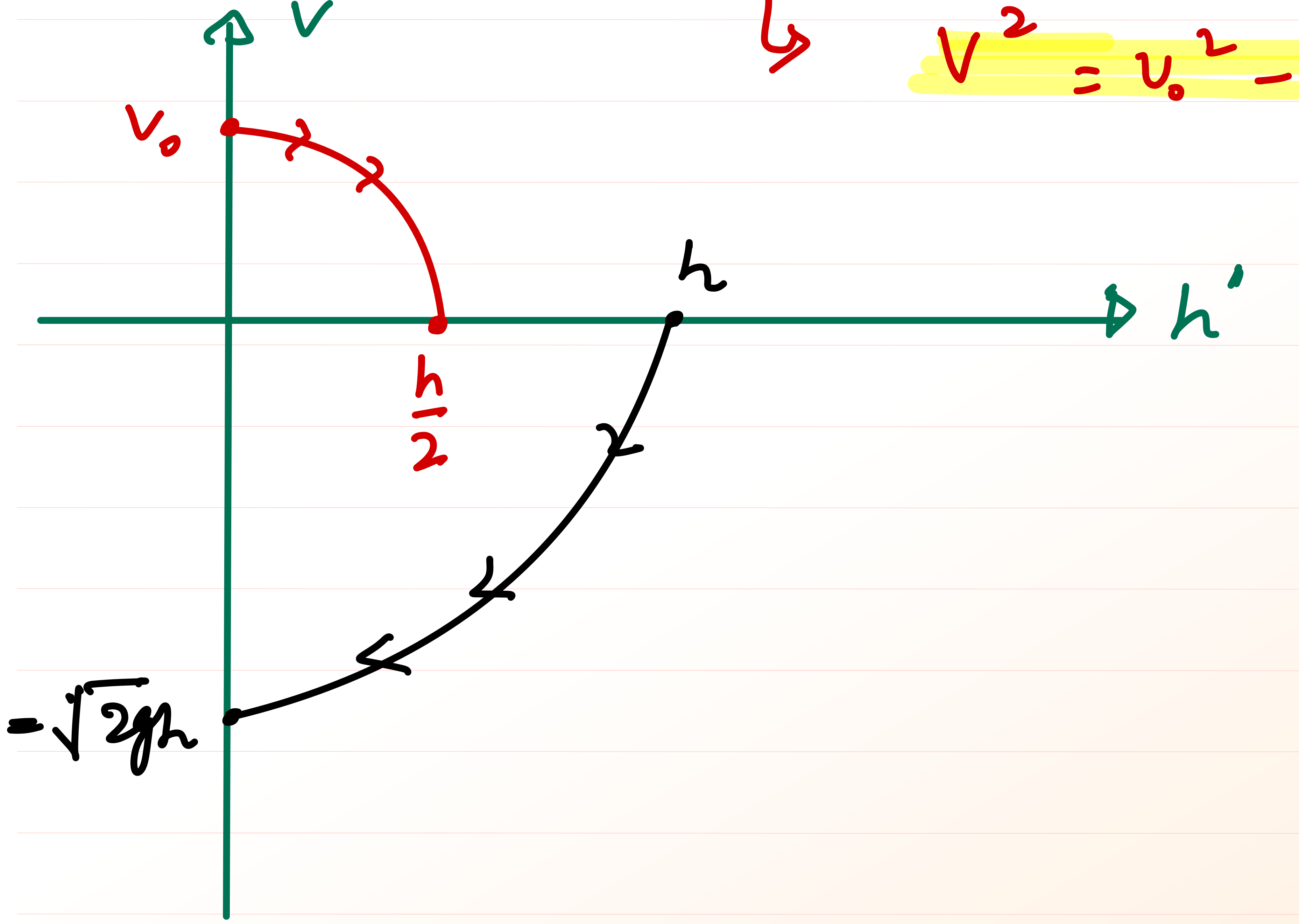
$v^2 = 0^2 - 2g(-y)$

$\frac{v^2}{2g} = y$

$h' = h - \frac{v^2}{2g} \Rightarrow \frac{v^2}{2g} = -h' + h$

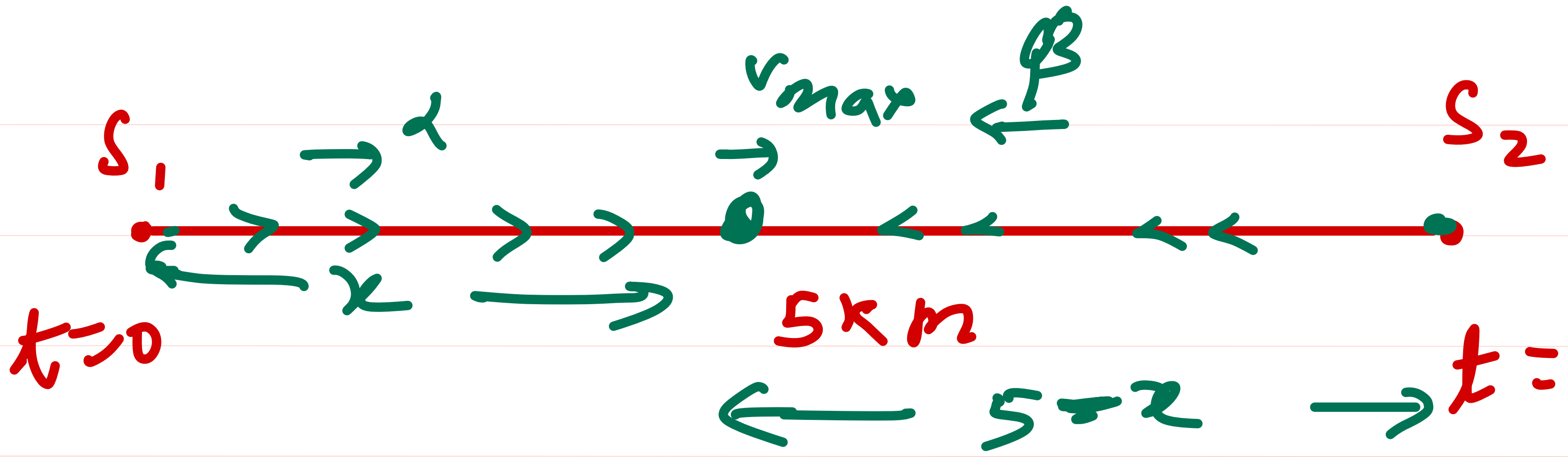
$v^2 = -2g h' + 2gh$

$V^2 = v_0^2 - 2g h'$ (For upward motion)



6. A train stopping at two stations 5 km apart takes 5 min on the journey from one of the station to the other. Assuming that its first accelerates with a uniform acceleration α and then that of uniform retardation β. if units of mass, length, and time are kg, km and min respectively then

(A) $\frac{1}{\alpha}+\frac{1}{\beta}=2$ (B) $\frac{1}{\alpha}+\frac{1}{\beta}=\frac{2}{5}$ (C) $\frac{1}{\alpha}+\frac{1}{\beta}=\frac{5}{2}$ (D) $\frac{1}{\alpha}+\frac{1}{\beta}=\frac{1}{2}$



$t = 5\text{ min} = 300\text{ sec}$

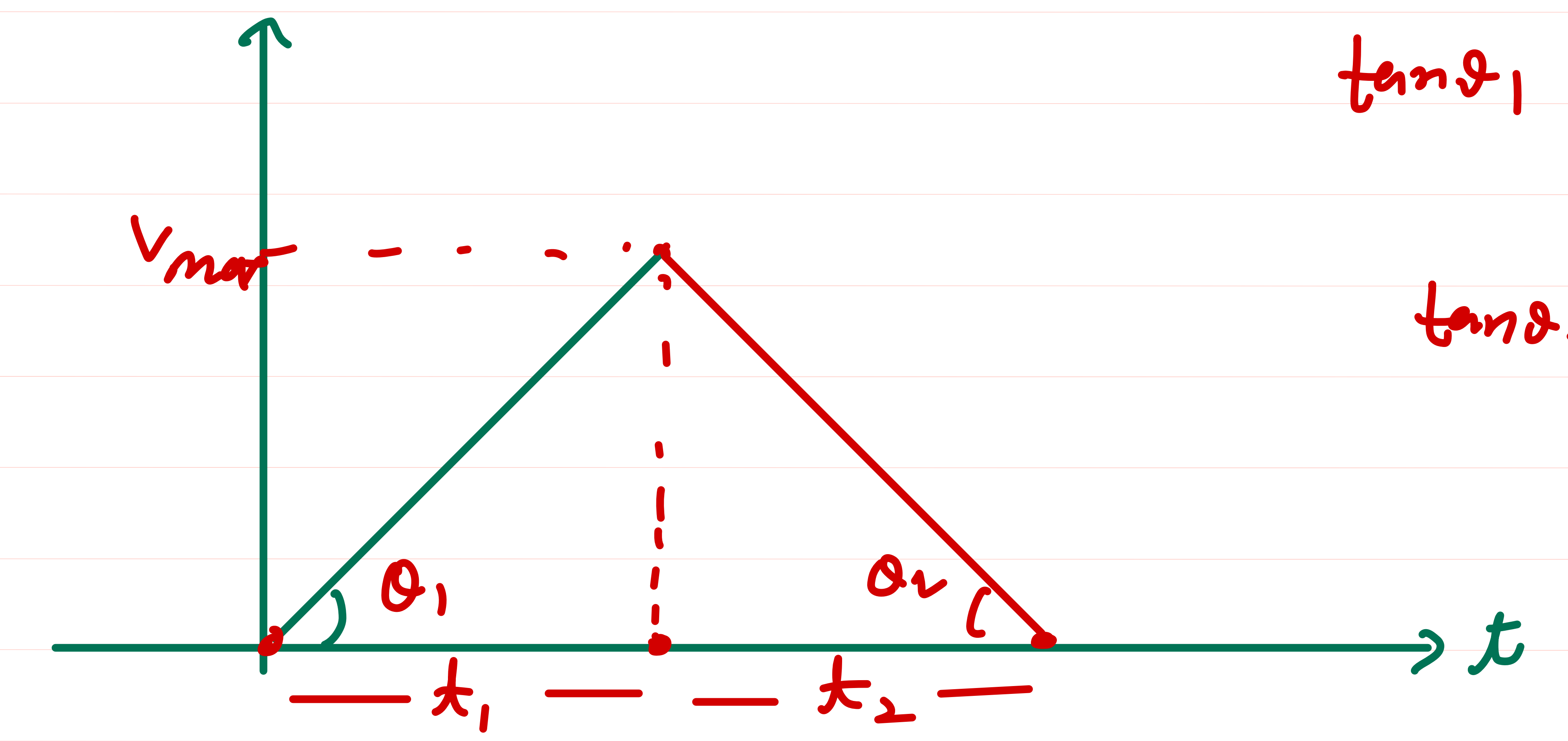
area $= 5 = \frac{1}{2}(t_1+t_2)V_{max}$

$5 = \frac{1}{2}\,5\,V_{max} \Rightarrow \boxed{V_{max}=2}$

$5 = 2\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)$

$\therefore \boxed{\frac{5}{2} = \frac{1}{\alpha}+\frac{1}{\beta}}$

v - t Rel^n



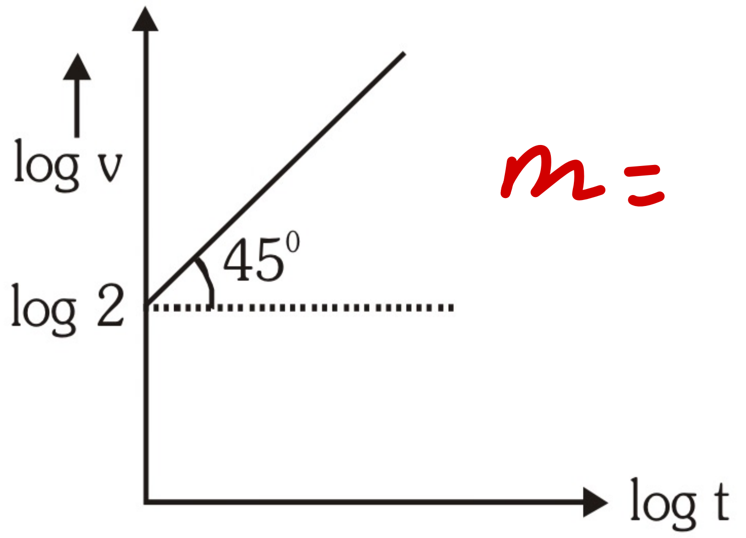
$\tan\theta_1 = \frac{V_{max}}{t_1} = \alpha \Rightarrow t_1 = \frac{V_{max}}{\alpha}$

$\tan\theta_2 = \frac{V_{max}}{t_2} = \beta \Rightarrow t_2 = \frac{V_{max}}{\beta}$

$t_1+t_2 = V_{max}\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)$

$\boxed{5 = V_{max}\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)}$ —(1)

9. Figure shows the plot of velocity versus time on a log–log scale. Assuming straight line motion and the particle to start from origin, the distance covered at the end of t = 3s is



$m = \tan 45 = 1$

(A) 9 m (B) 18 m (C) 10 m (D) Can't be determined

$\log v = \log 2 + m \cdot \log t$

$\log v = \log 2 + \log t$

$\log v = \log 2t$

$\boxed{v = 2t}$

$\frac{dx}{dt} = 2t$

$\int_0^d dx = 2\int_0^3 t\,dt$

$d = 2 \cdot \frac{t^2}{2} = 3^2 = 9m$

10. Acceleration of a particle is defined as $a = \left(75V^2 - 30V + 3\right)$ (m/s²), find constant speed achieved by the particle.

(A) 3 m/s (B) $\frac{1}{5}$ m/s

(C) 5 m/s (D) It will never achieve constant speed.

When $a = 0$ speed = const

$0 = 75v^2 - 30v + 3$

11. Velocity of an object depends on displacement as $V^{3/2} = K8(y)^{3/4}$, where V is velocity (in m/s), y is displacement (in meter) & K is constant, then acceleration in m/s² when y = 16

(A) $8\ K^{2/3}$ (B) 8 (C) $8K^{4/3}$ (D) $32\ K^{4/3}$

$a = v\left(\frac{dv}{dy}\right)$

$v^{3/2} = 8K\,y^{3/4}$

$v = (8K)^{2/3}\left(y^{3/4}\right)^{2/3} = 4K^{2/3}\,y^{1/2}$

$v = (8K)^{2/3}\cdot y^{1/2}$

$\frac{dv}{dy} = (8K)^{2/3}\,\frac{1}{2}\,y^{\frac{1}{2}-1} = \frac{2K^{2/3}}{\sqrt{y}}$

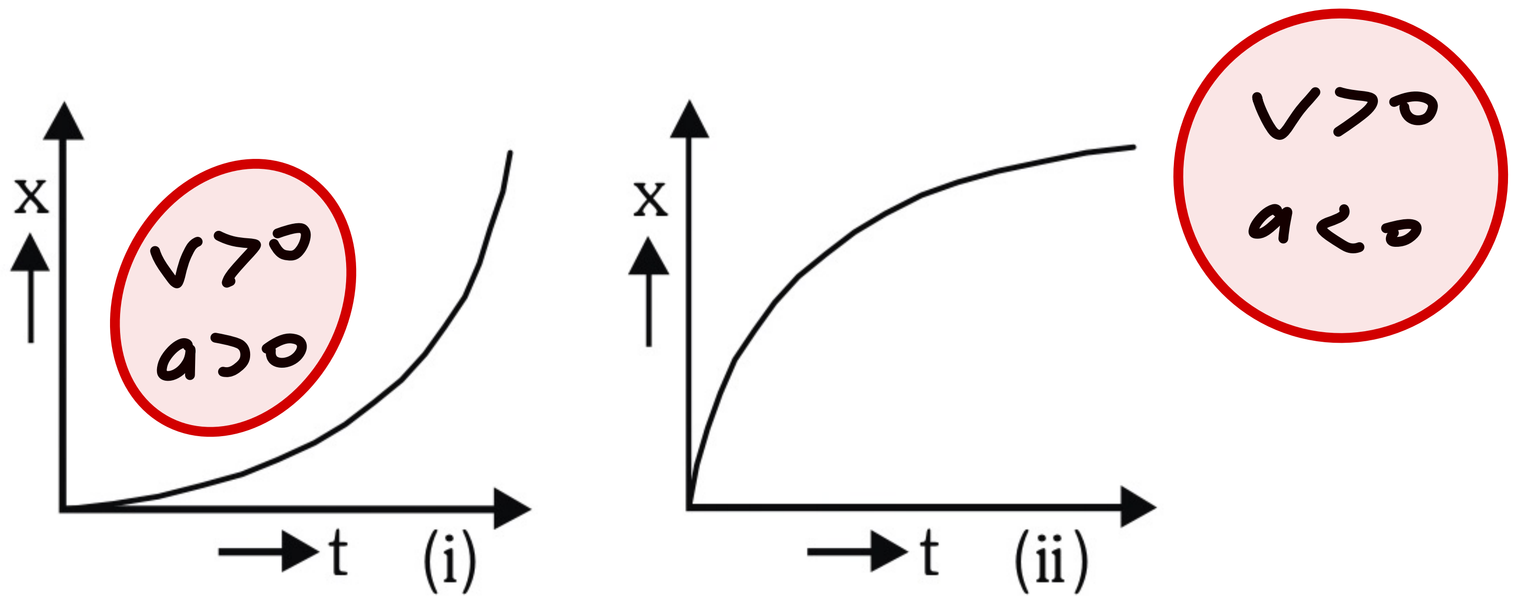
$a = \left(4K^{2/3}\cdot \cancel{y^{1/2}}\right)\cdot\left(\frac{2K^{2/3}}{\cancel{\sqrt{y}}}\right)$

$= 8K^{\frac{2}{3}+\frac{2}{3}}$

$\boxed{a = 8K^{4/3}}$

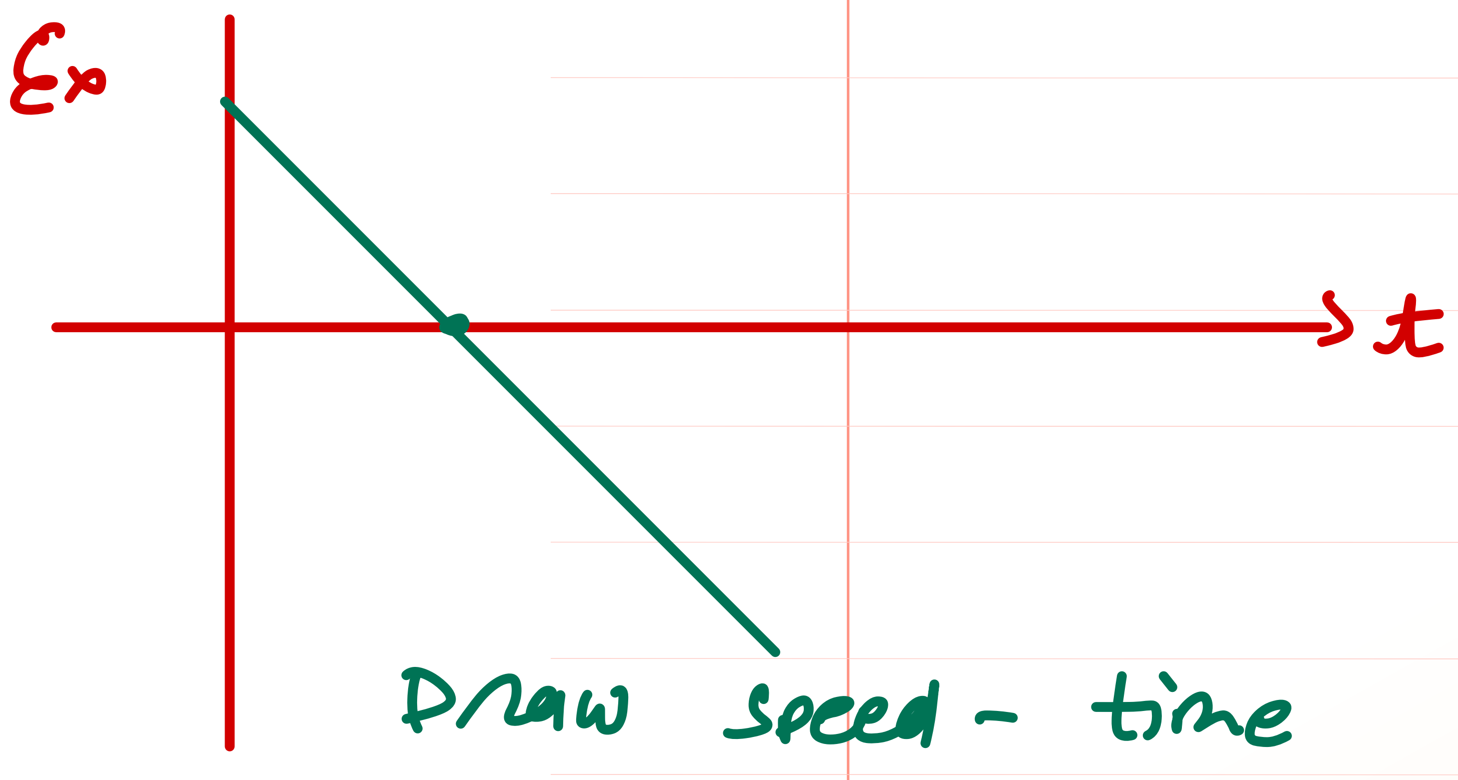
B·8 #4

4*. Figures (i) and (ii) below show the displacement-time graphs of two particles moving along the x-axis (curves are parabolic). We can say that



Velocity

Ex



(A) Both the particles are having a uniformly acceleration

(B) Both the particles are having a non uniformly acceleration

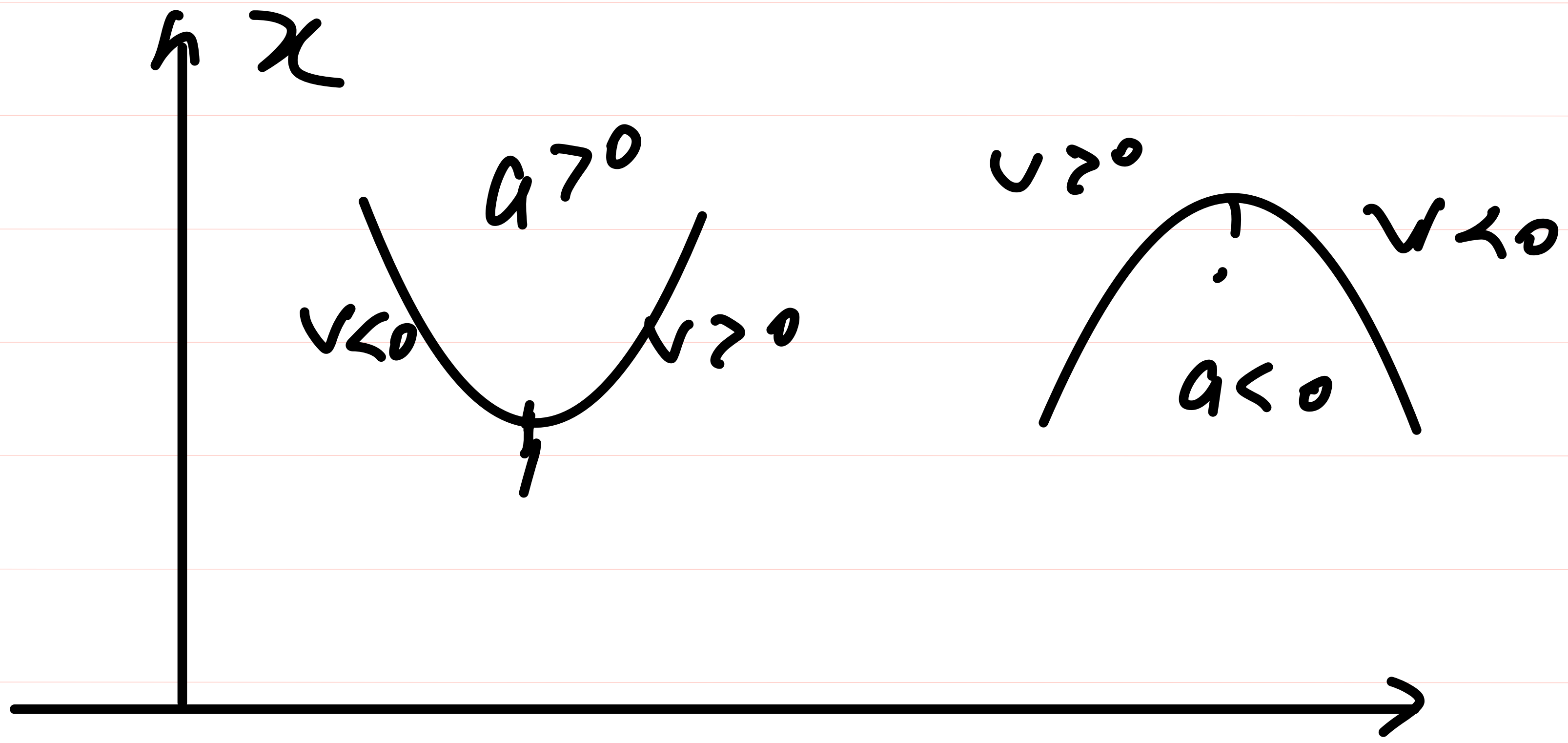
(C) Particle (i) is speeding up.

(D) Particle (ii) is slowing down.

Draw speed - time

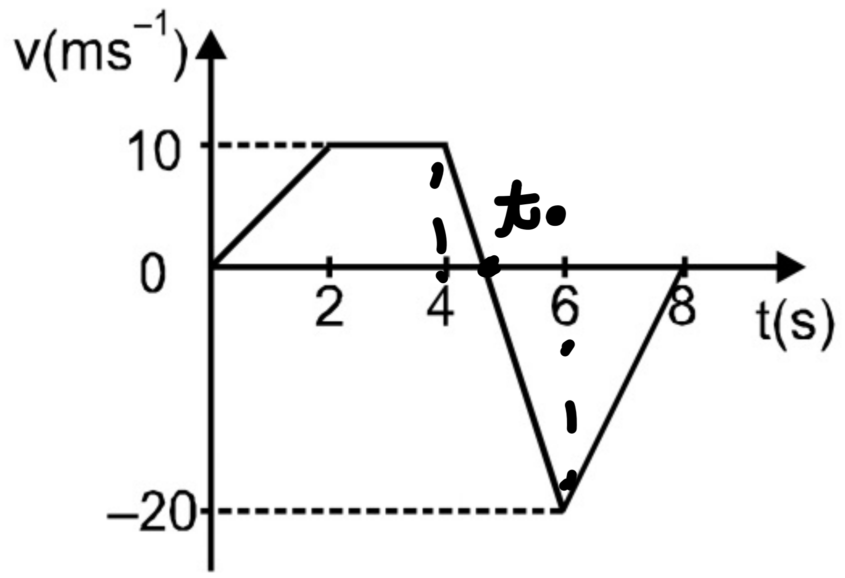
Speed

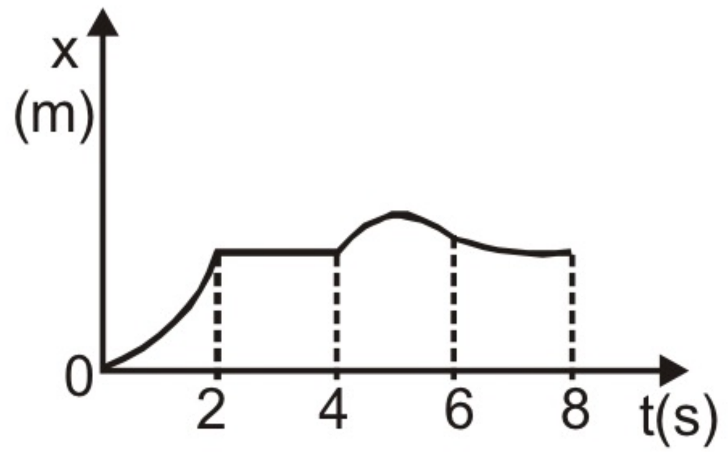
v(ms^{-1})

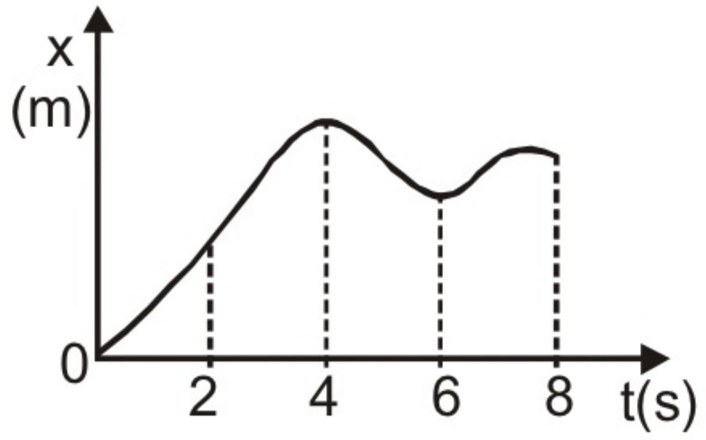


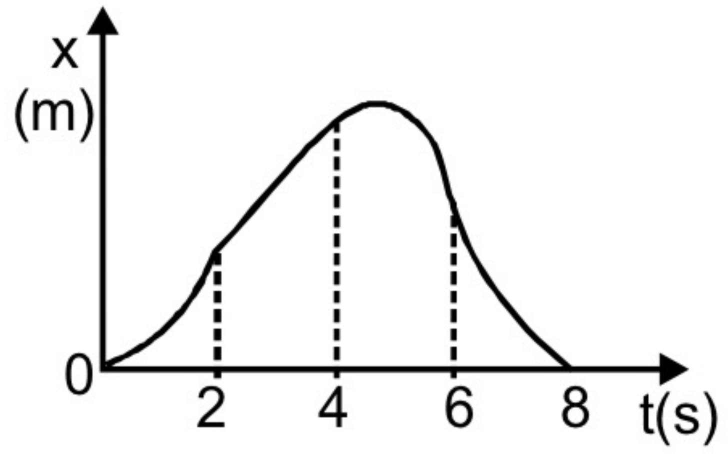
(D) Particle (ii) is slowing down.

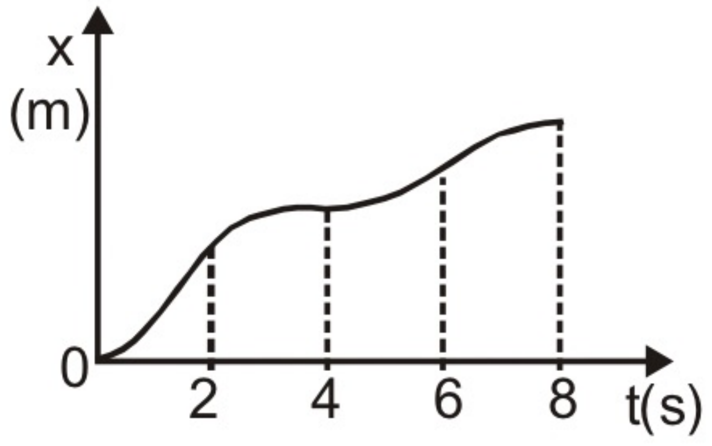
5*. The figure shows a velocity-time graph of a particle moving along a straight line, The correct displacement-time graph of the particle is shown as :



(A)


(B)


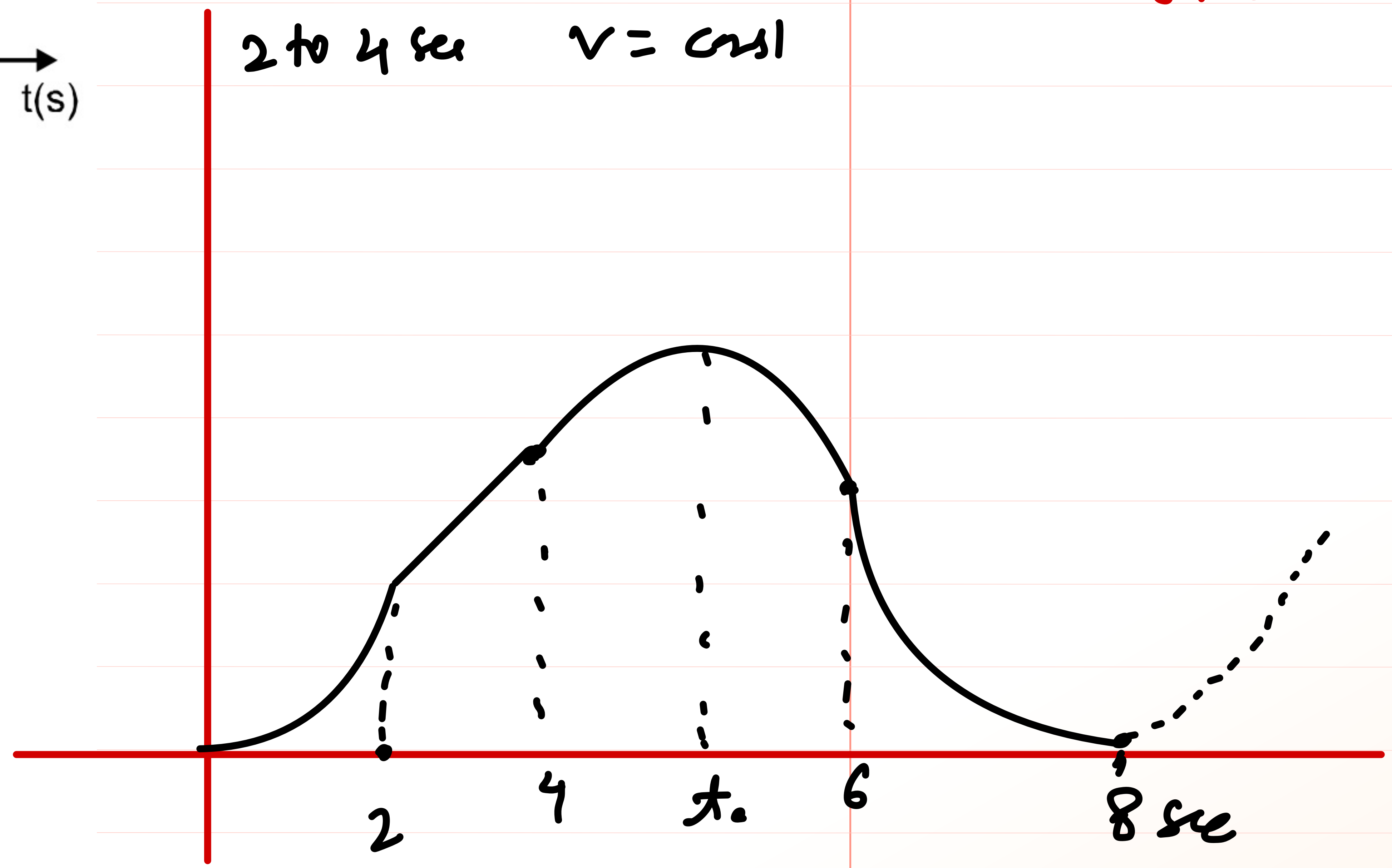
(C)


(D)


0 to 2 sec $a = \frac{10}{2} = 5 m/s^2 > 0$

$v > 0$

2 to 4 sec $v = $ const



4 to t_0

$v > 0$

$a < 0$

t_0 to 6

$v < 0$

$a < 0$

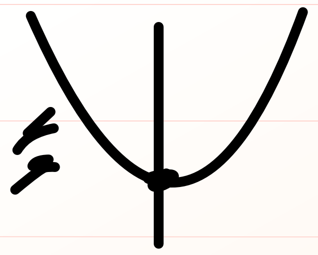
4 to 6 sec

$a < 0$

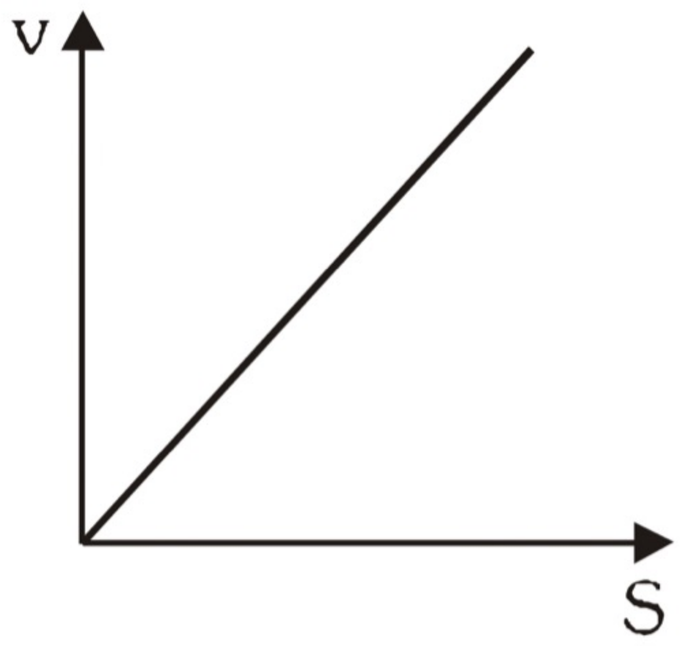
6 to 8 sec

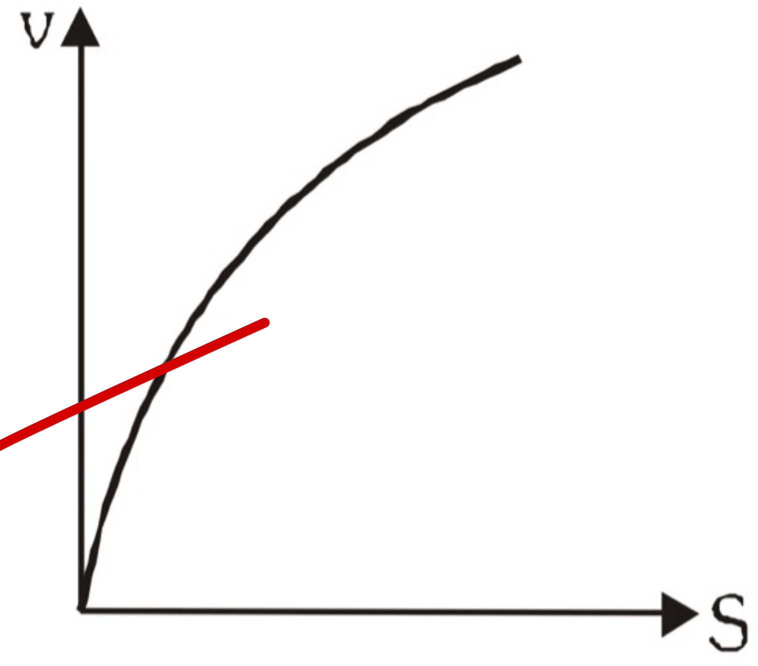
$v < 0$

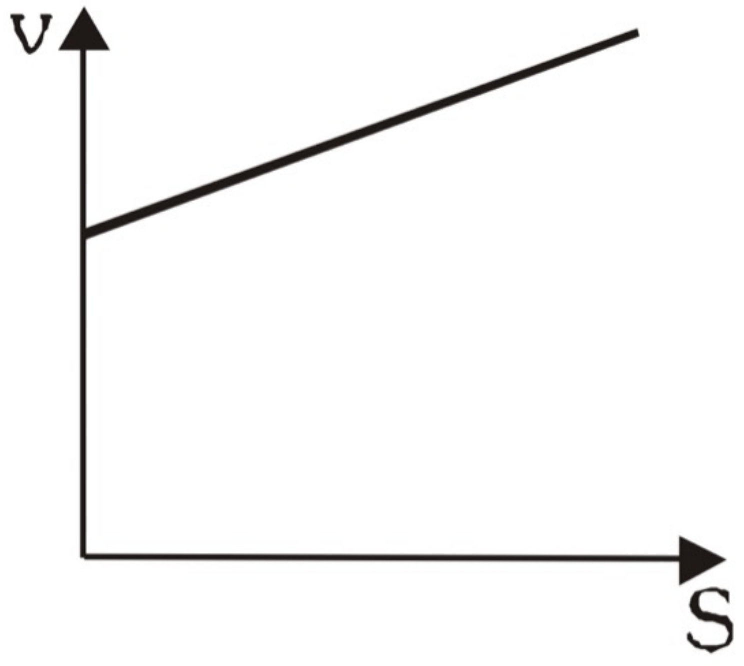
$a > 0$

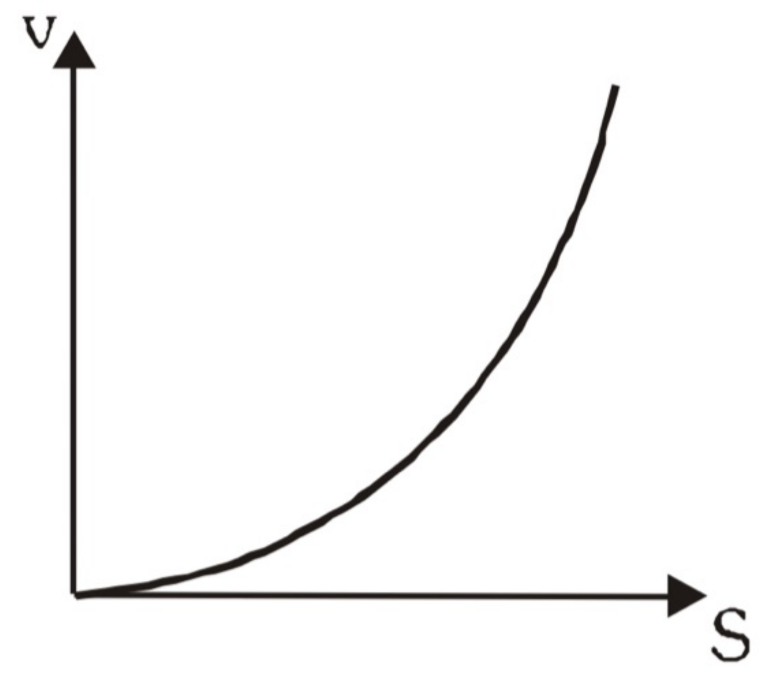


7*. A particle starts from rest and moves along a straight line with constant acceleration. The variation of velocity v with displacement S is :

(A) 

(B) 

(C) 

(D) 

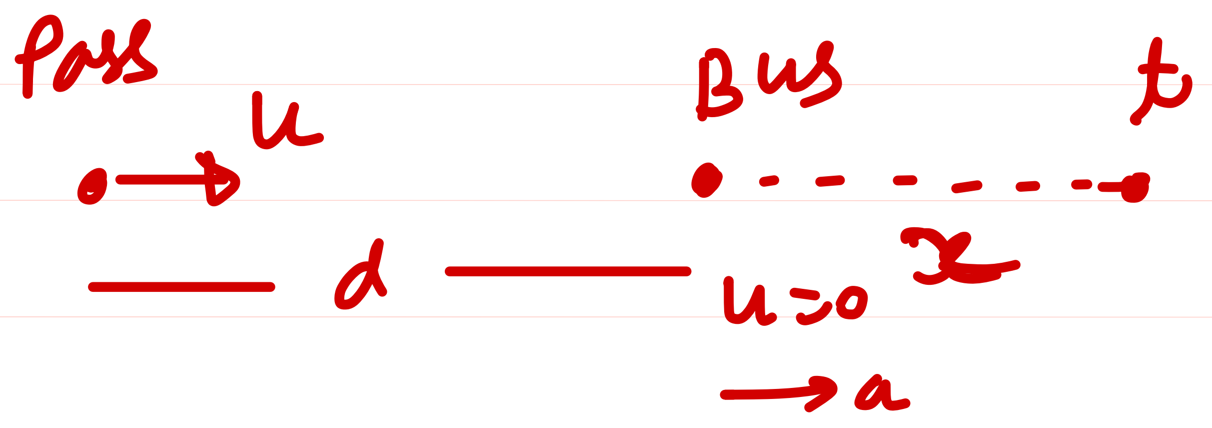
$$v^2 = 0^2 + 2as$$

$$v^2 = 2as$$

Parabolic

Illustration 5*.

A passenger is standing d distance away from a bus. The bus begins to move with constant acceleration a. To catch the bus, the passenger runs at a constant speed u towards the bus. What must be the minimum speed of the passenger so that he may catch the bus?



For Pass.

$$(d+x) = ut \quad \text{—(1)}$$

For Bus

$$x = \frac{1}{2}at^2 \quad \text{—(2)}$$

$$d + \frac{1}{2}at^2 = ut$$

$$\frac{a}{2}t^2 - ut + d = 0$$

$$b^2 - 4ac \geq 0 \quad \text{(For Real time)}$$

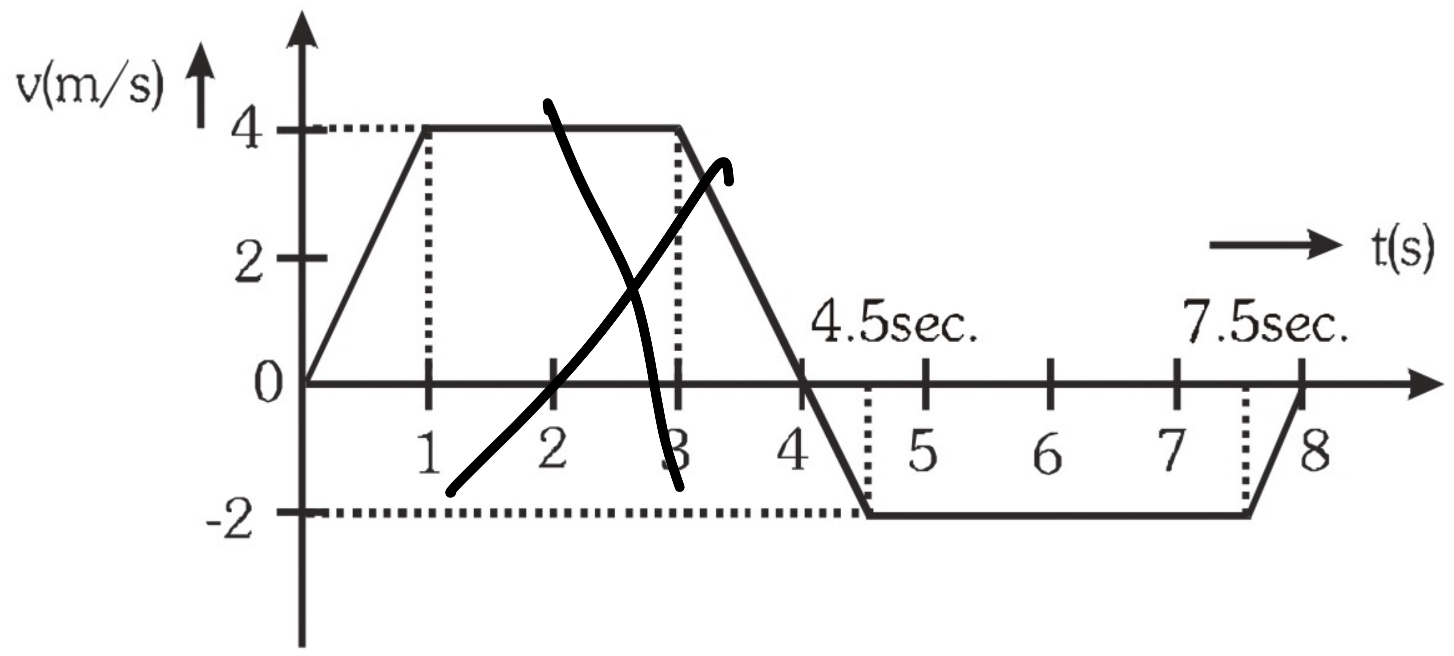
$$u^2 - 4\left(\frac{a}{2}\right)d \geq 0$$

$$u^2 - 2ad \geq 0$$

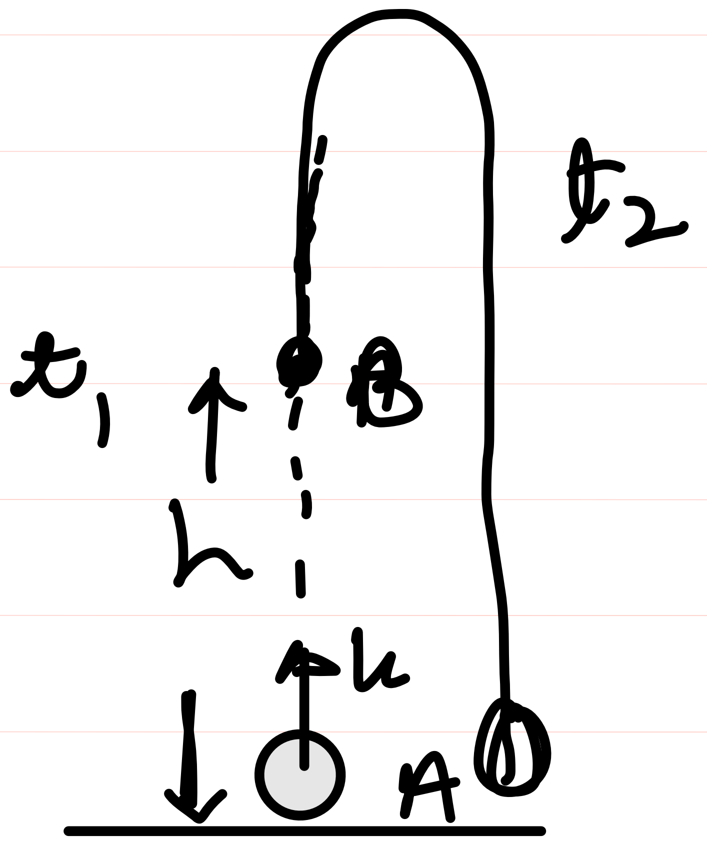
$$u^2 \geq 2ad$$

$$\boxed{u \geq \sqrt{2ad}}$$

8. A particle is projected vertically upwards from a point A on the ground. It takes t_1 time to reach a point B but it still continues to move up. If it takes further t_2 time to reach the ground from point B then height of point B from the ground is :



(A) $\frac{1}{2}g(t_1+t_2)^2$ (B) gt_1t_2 (C) $\frac{1}{8}g(t_1+t_2)^2$ (D) $\frac{1}{2}gt_1t_2$



Total time of Flight $= (t_1+t_2) = \frac{2u}{g} \Rightarrow u = \frac{g}{2}(t_1+t_2)$

$h = ut_1 - \frac{1}{2}gt_1^2$

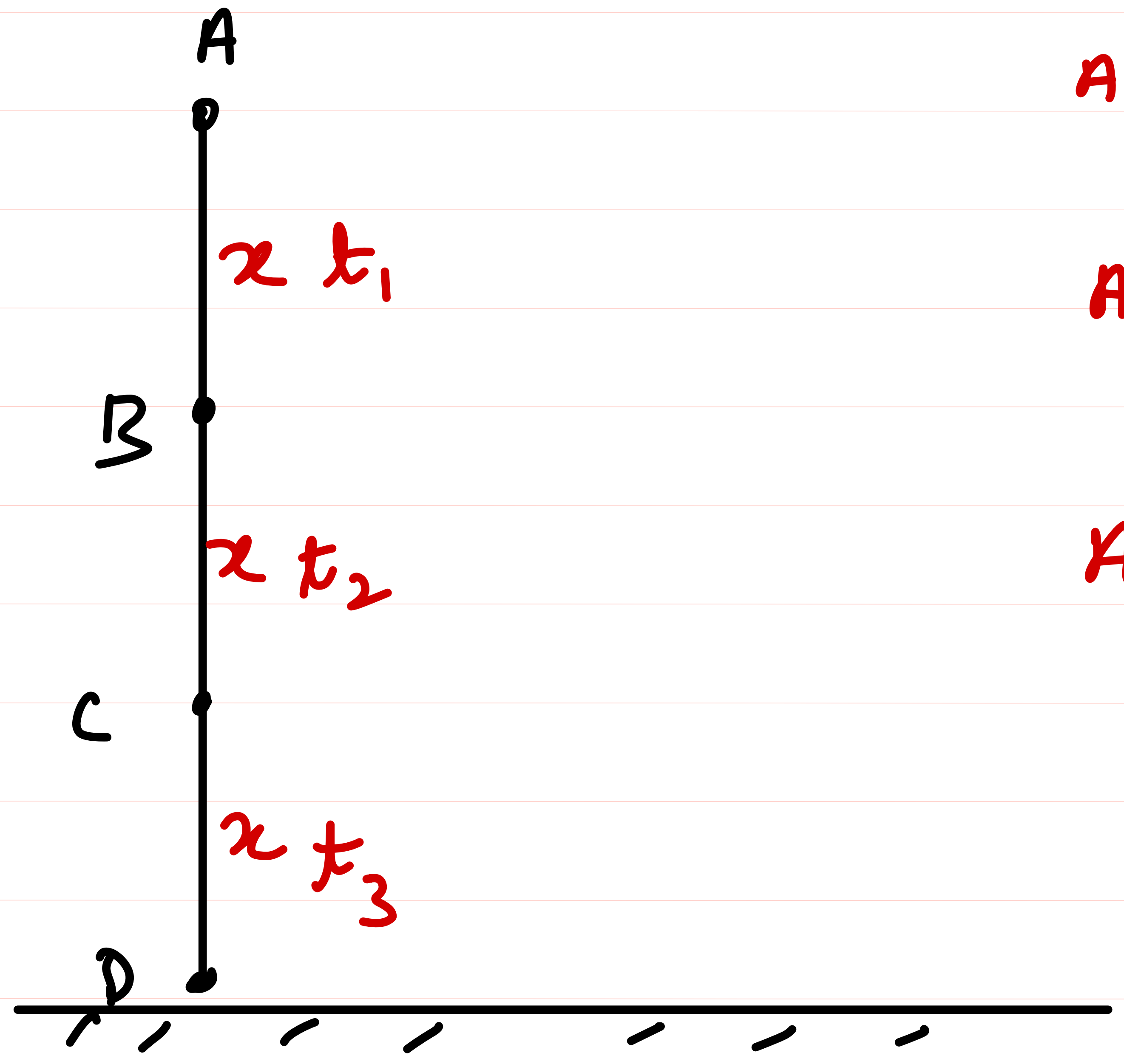
$h = \frac{g}{2}t_1(t_1+t_2) - \frac{1}{2}gt_1^2$

$h = \cancel{\frac{g}{2}t_1^2} + \frac{g}{2}t_1t_2 - \cancel{\frac{gt_1^2}{2}}$

5. A, B, C and D are points in a vertical line such that AB = BC = CD. If a body falls from rest from A, then the times of descend through AB, BC and CD are in the ratio :

(A) $1 : \sqrt{2} : \sqrt{3}$ (B) $\sqrt{2} : \sqrt{3} : 1$

(C) $\sqrt{3} : 1 : \sqrt{2}$ (D) $1 : (\sqrt{2} - 1) : (\sqrt{3} - \sqrt{2})$



A to B $\quad x = \frac{1}{2} g t_1^2 \Rightarrow t_1 = \sqrt{\frac{2x}{g}}$

A to C $\quad 2x = \frac{1}{2} g (t_1 + t_2)^2 \Rightarrow \sqrt{\frac{4x}{g}} = t_1 + t_2 \Rightarrow t_2 = \sqrt{\frac{4x}{g}} - \sqrt{\frac{2x}{g}}$

A to D $\quad 3x = \frac{1}{2} g (t_1 + t_2 + t_3)^2$

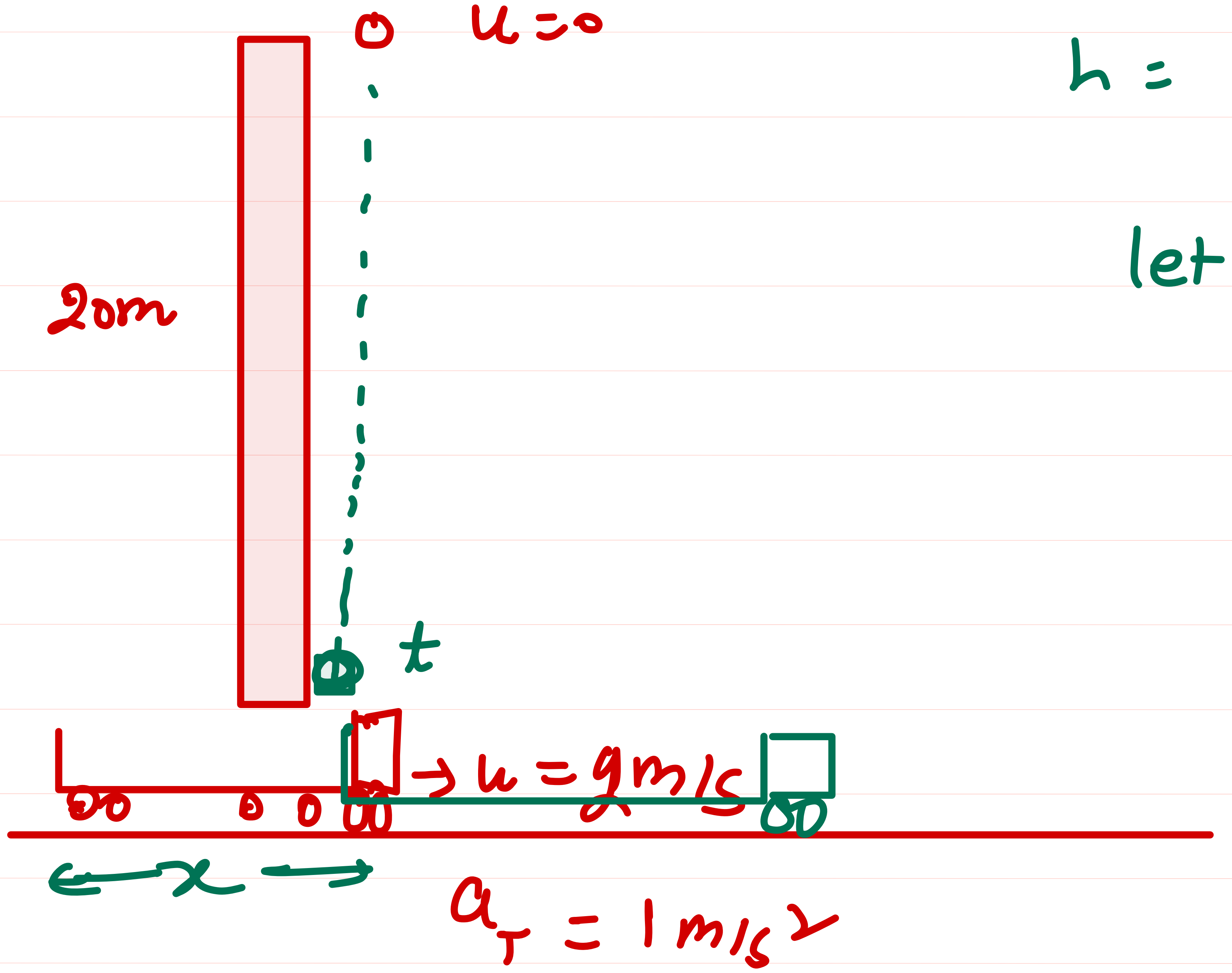
$$\sqrt{\frac{6x}{g}} = (t_1 + t_2) + t_3 \Rightarrow t_3 = \sqrt{\frac{6x}{g}} - \sqrt{\frac{4x}{g}}$$

$$\therefore t_1 : t_2 : t_3 = \sqrt{\frac{2x}{g}} : \left(\sqrt{\frac{4x}{g}} - \sqrt{\frac{2x}{g}}\right) : \left(\sqrt{\frac{6x}{g}} - \sqrt{\frac{4x}{g}}\right)$$

$$= 1 : (\sqrt{2} - 1) : (\sqrt{3} - \sqrt{2})$$

19*. A person drops a stone from a building of height 20 m. At the same instant the front end of a truck passes below the building moving with constant acceleration of 1 m/s^2 and velocity of 2 m/s at that instant. Length of the truck if the stone just misses to hit its rear part is :

(A) 6 m (B) 4 m (C) 5 m (D) 2 m



$h = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2h}{g}}$

let x be length of truck

$x = ut + \frac{1}{2}at^2$ (for Truck)

$x = 2 \times \sqrt{\frac{2h}{g}} + \frac{1}{2} 1 \left(\sqrt{\frac{2h}{g}}\right)^2$

$x = 2\sqrt{\frac{2\times 20}{10}} + \frac{1}{2}\left(\frac{2\times 20}{10}\right)$

$= 4 + \frac{1}{2} \times 4 = 6\,m$