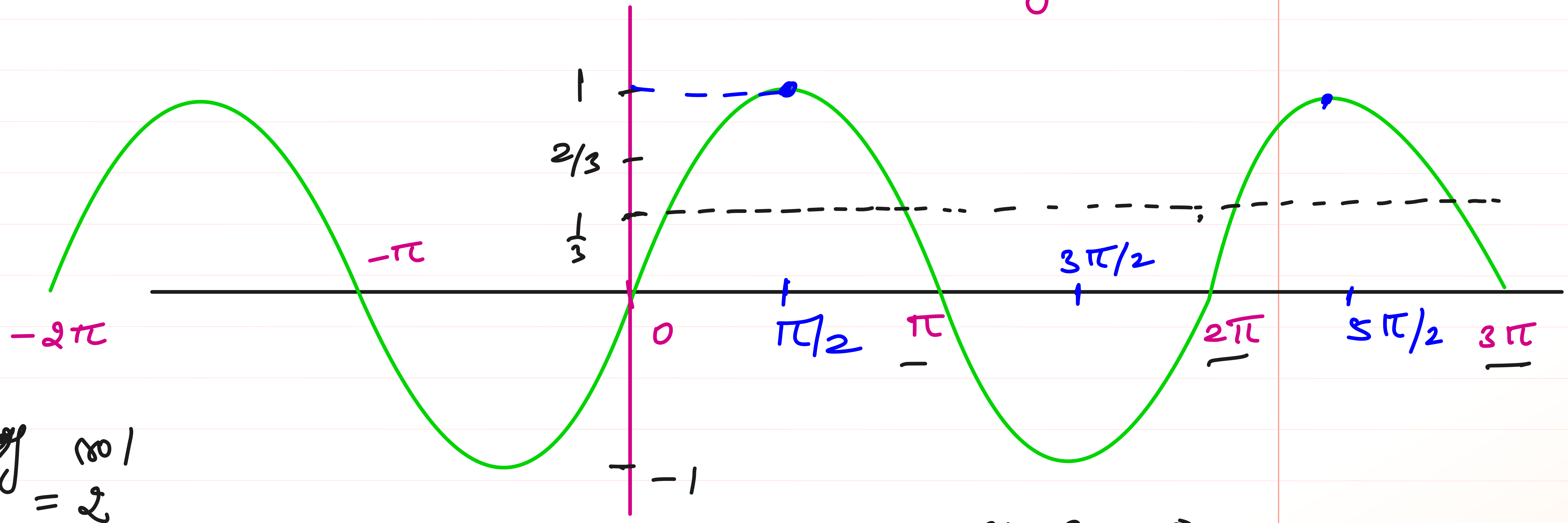
Trigonometric Ratios and Identities

Lecture - 4

$\sin\theta = \frac{1}{3}$

$y = \sin x$



$[0, \pi] \rightarrow$ No. of sol $= 2$

$[0, 2\pi] \Rightarrow$ No. of sol $= 2$

$[0, 3\pi] \rightarrow$ No. of sol $= 4$

$[-2\pi, 3\pi] \rightarrow$ No. of sol $= 6$

$\sin\theta = -1$

$\theta \in \left\{\frac{3\pi}{2}, \frac{7\pi}{2}, \frac{11\pi}{2}, \ldots\right\}$

$\theta \in \left\{(4n-1)\frac{\pi}{2}, n \in Z\right\}$

$\sin\theta = 0$

$\theta = \{n\pi, n \in \text{Integer}\}$

$\sin\theta = 1$

$\theta \in \left\{\frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \ldots\right\}$

$\theta = \left\{(4n+1)\frac{\pi}{2}, n \in Z\right\}$

Graphs of trigonometric ratios :

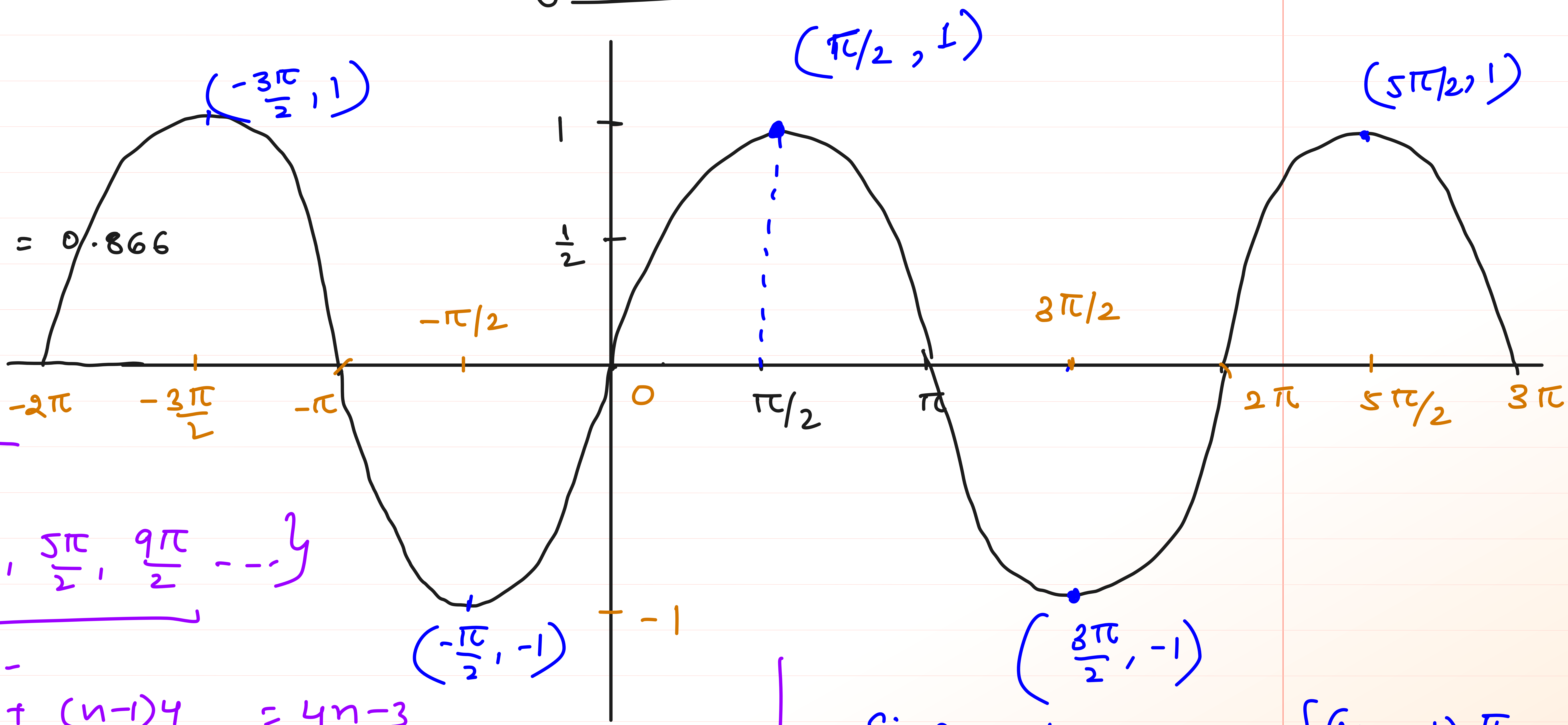
$y = \sin x$

$\sin 0 = 0$

$\sin \frac{\pi}{6} = \frac{1}{2}$

$\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2} = 0.866$

$\sin \frac{\pi}{2} = 1$



$\sin\theta = 1$

$\theta = \left\{ --- \frac{-3\pi}{2}, \underbrace{\frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}} --- \right\}$

1, 5, 9, - -

$T_n = 1 + (n-1)4 = 4n-3$

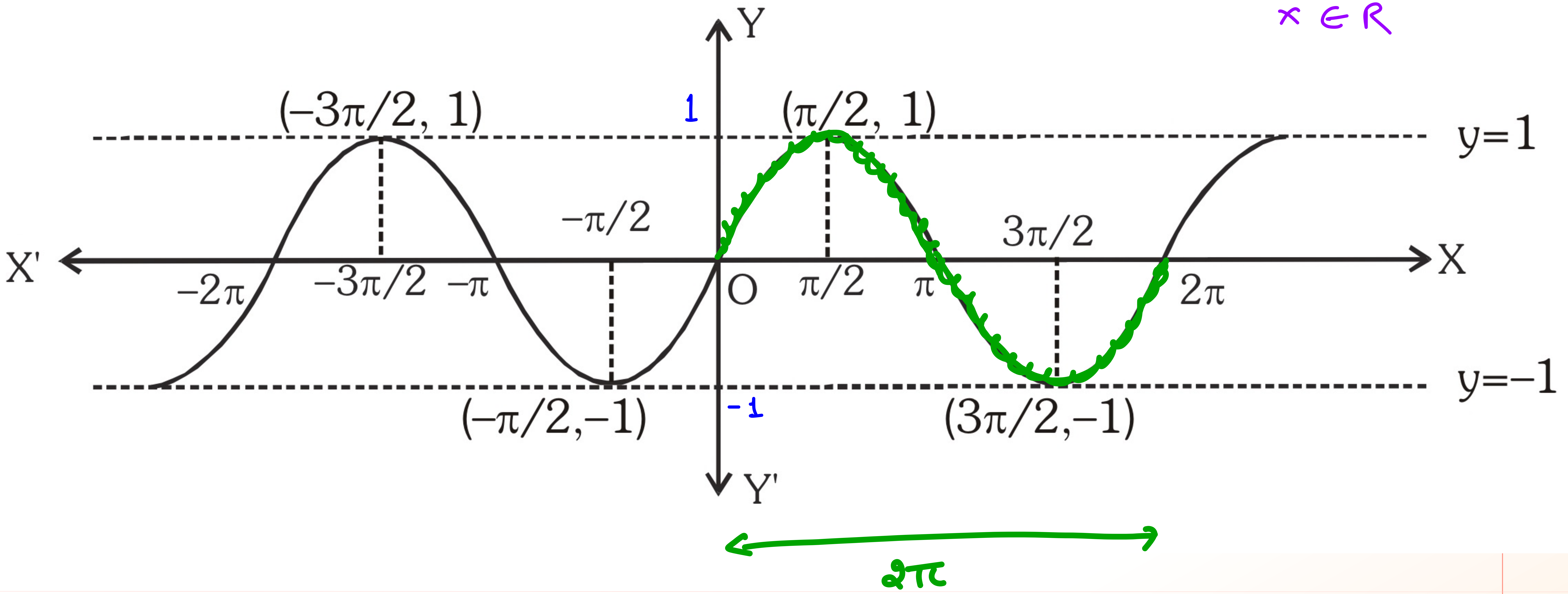
$\theta = \left\{ (4n-3)\frac{\pi}{2}, n \in Z \right\} \Rightarrow \left\{ (4n+1)\frac{\pi}{2}, n \in Z \right\}$

$\sin\theta = -1 \qquad \theta \in \left\{ (4n-1)\frac{\pi}{2}, n \in Z \right\}$

$\sin\theta = 0 \Rightarrow \theta \in \{ n\pi, n \in Z \}$

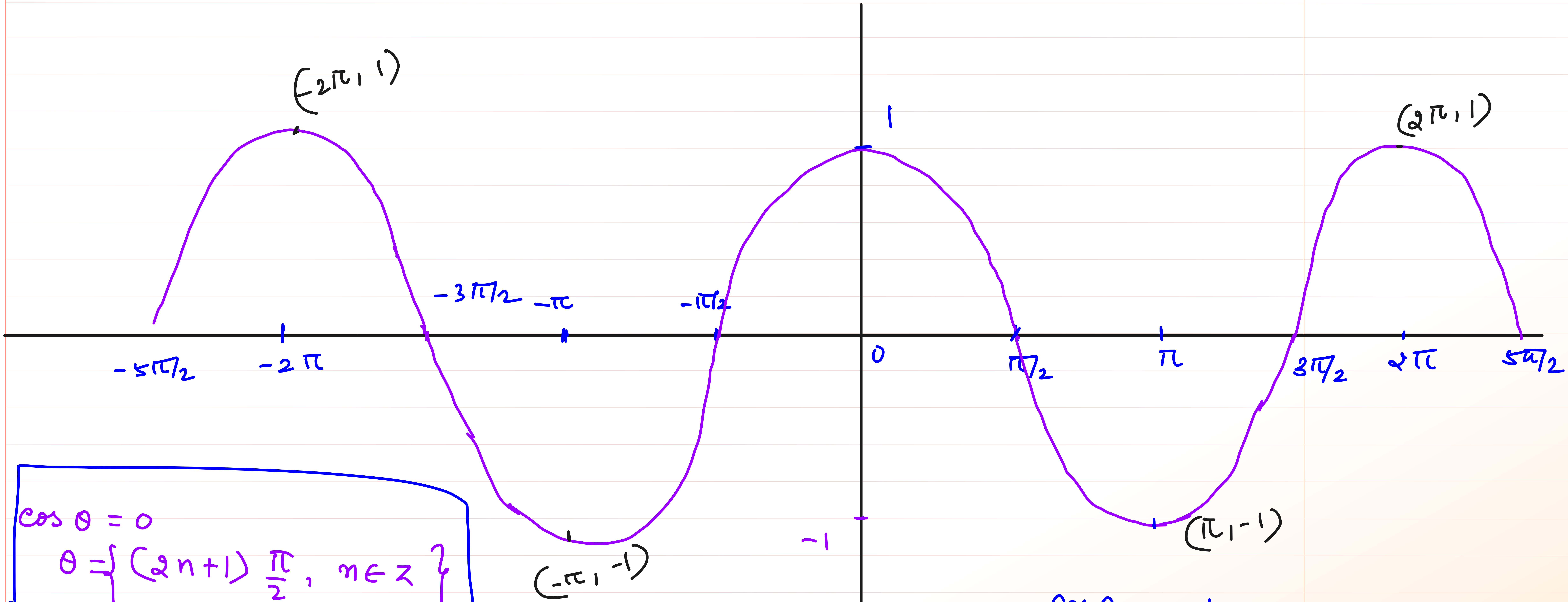
$y = \sin x,\ x \in R,\ y \in [-1, 1]$

$y = \sin x$
$y \in [-1, 1]$
$x \in R$



Period of $\{y = \sin x\} \Rightarrow 2\pi$

$y = \cos x$



$\cos\theta = 0$

$\theta = \{(2n+1)\frac{\pi}{2},\ n \in Z\}$

$\cos\theta = 1$

$\theta = \{2n\pi,\ n \in Z\}$

$\cos\theta = -1$

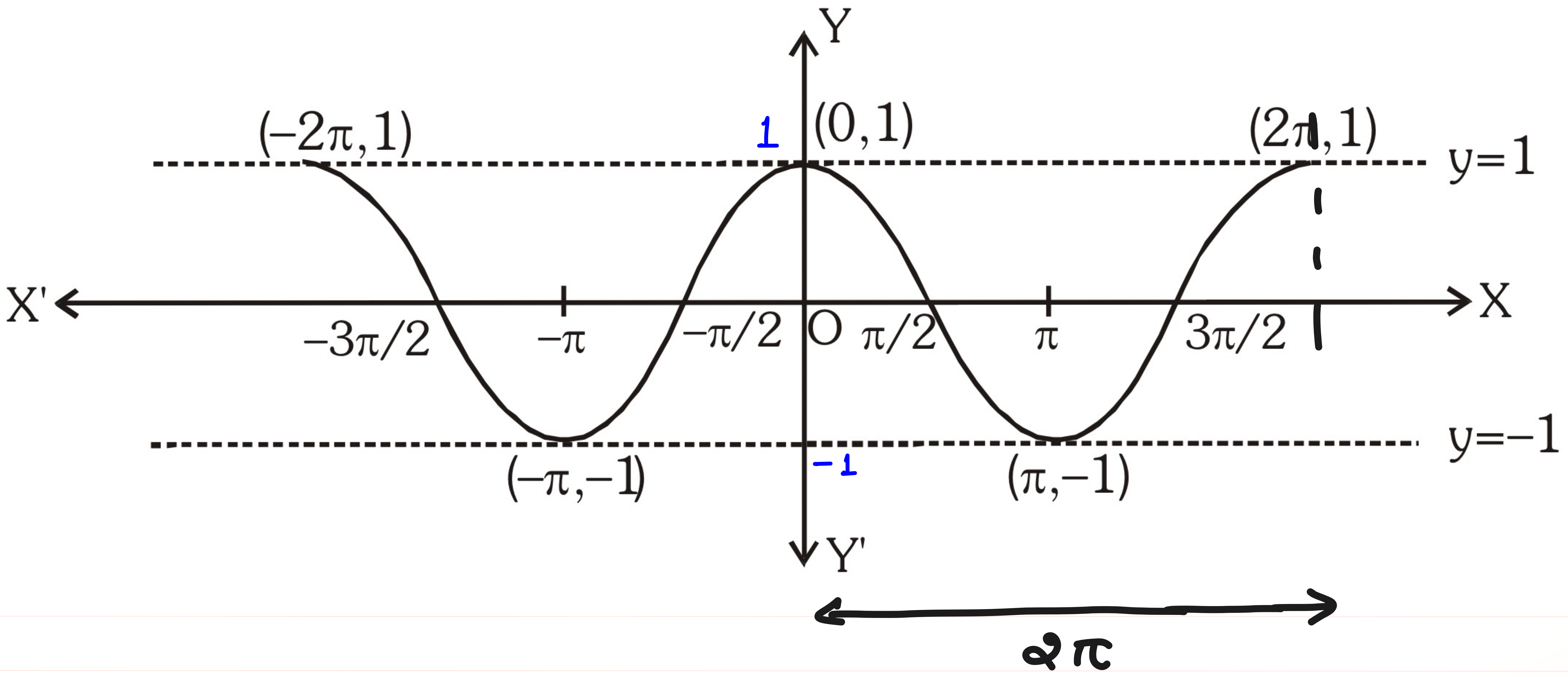
$\theta \in \{(2n+1)\pi,\ n \in Z\}$

$y = \cos x$

$y = \cos x,\ x \in R,\ y \in [-1,1]$

$y \in [-1, 1]$

$x \in R$

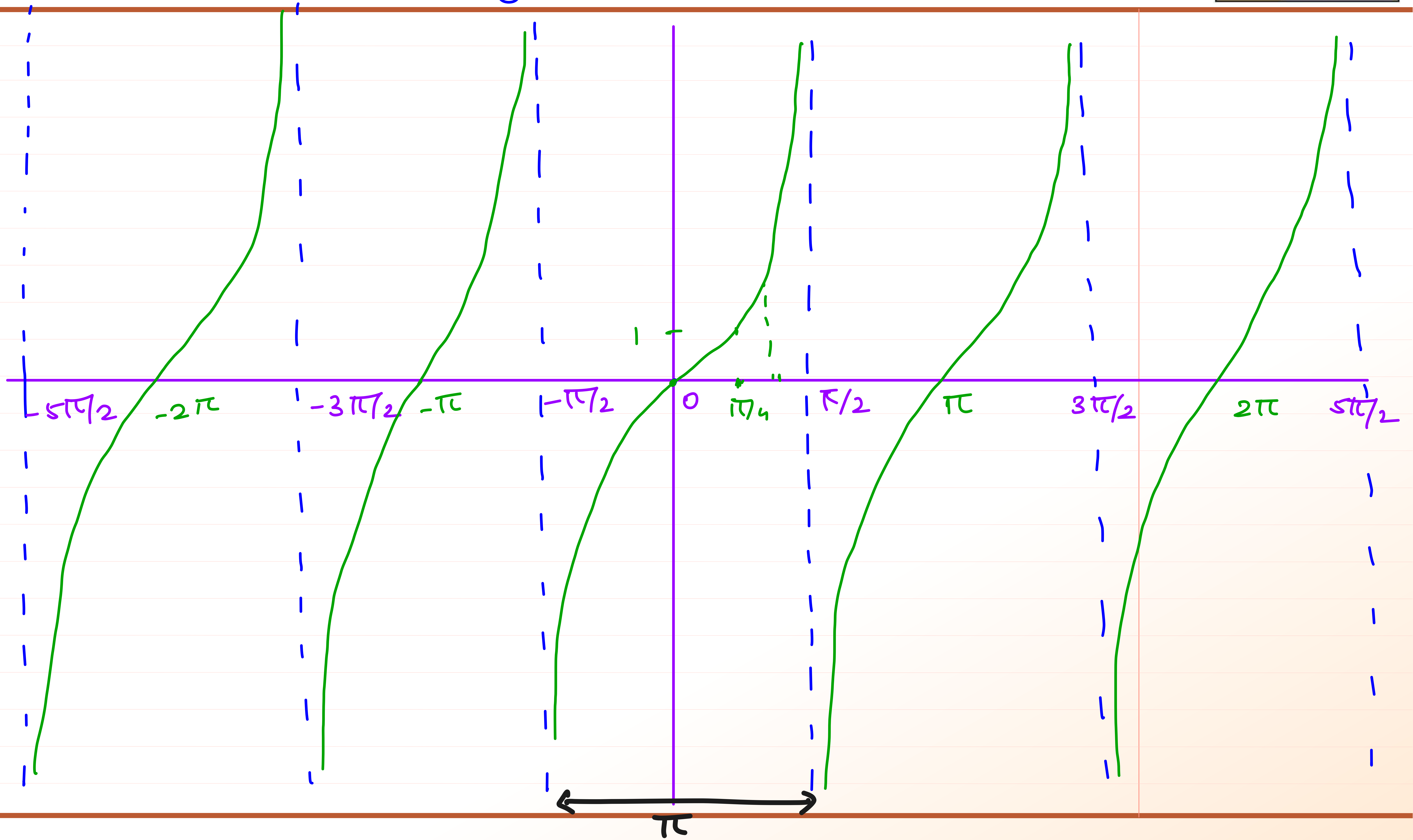


Periodic with period 2π

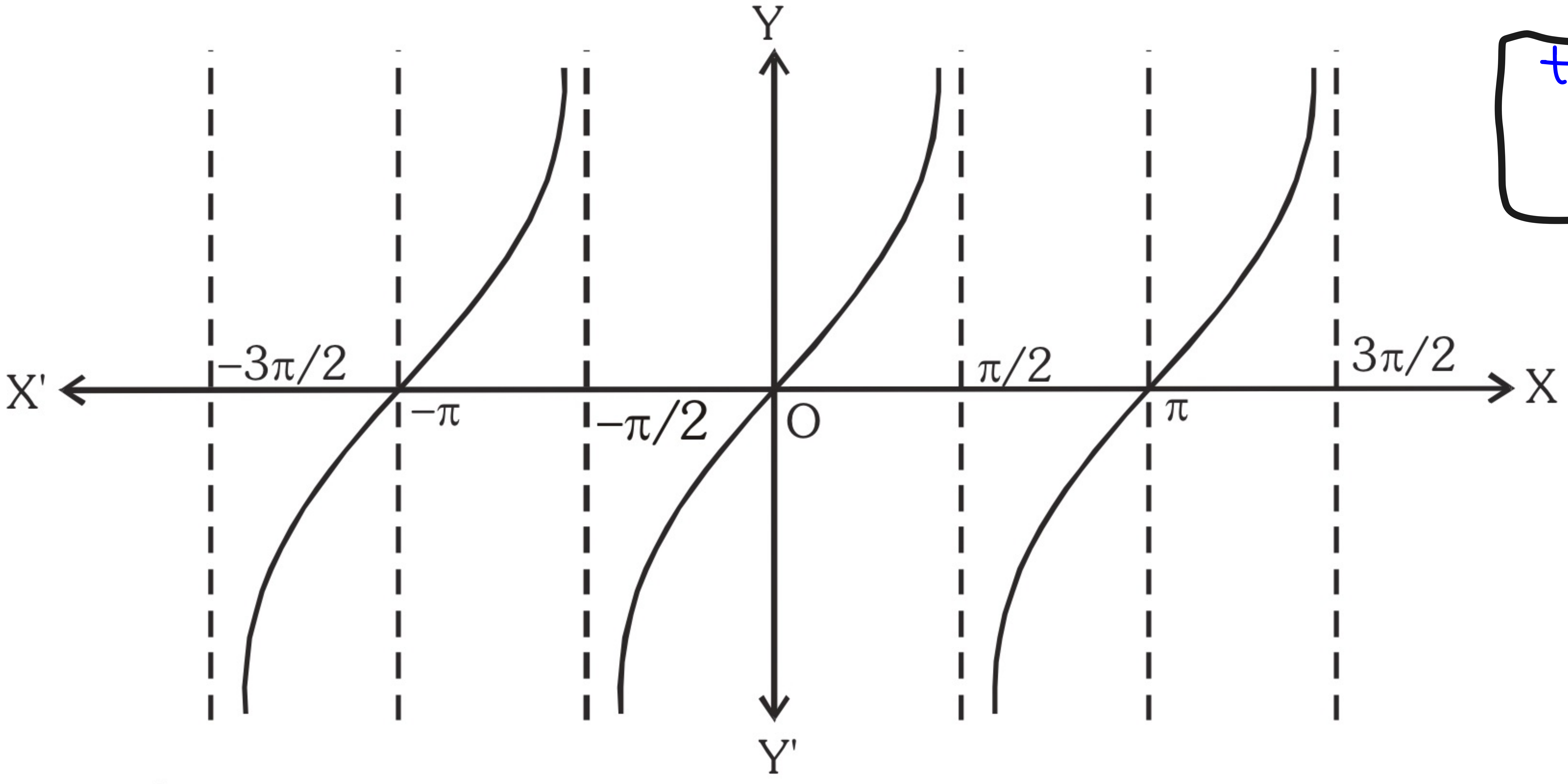
Period of $\cos x = 2\pi$

$y = \tan x$

$\frac{1}{x-5}$



$y = \tan x,\ x \neq \frac{\pi}{2}, \frac{3\pi}{2} \ldots\ldots \neq (2n-1)\frac{\pi}{2}\ :\ n \in I$



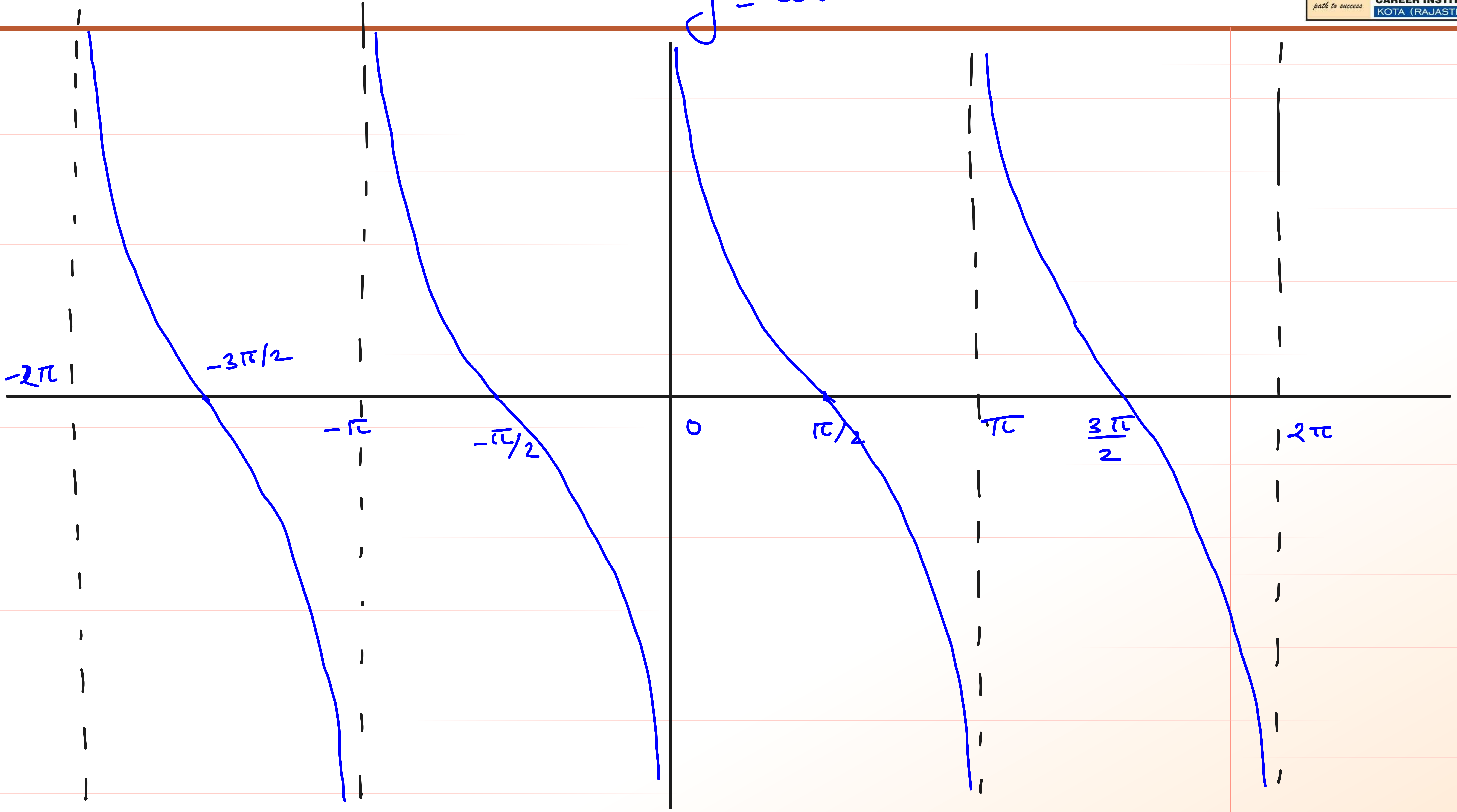
$x \in R - \{(2n-1)\frac{\pi}{2},\ n \in Z\}$

$y \in R$ or $y \in (-\infty, \infty)$

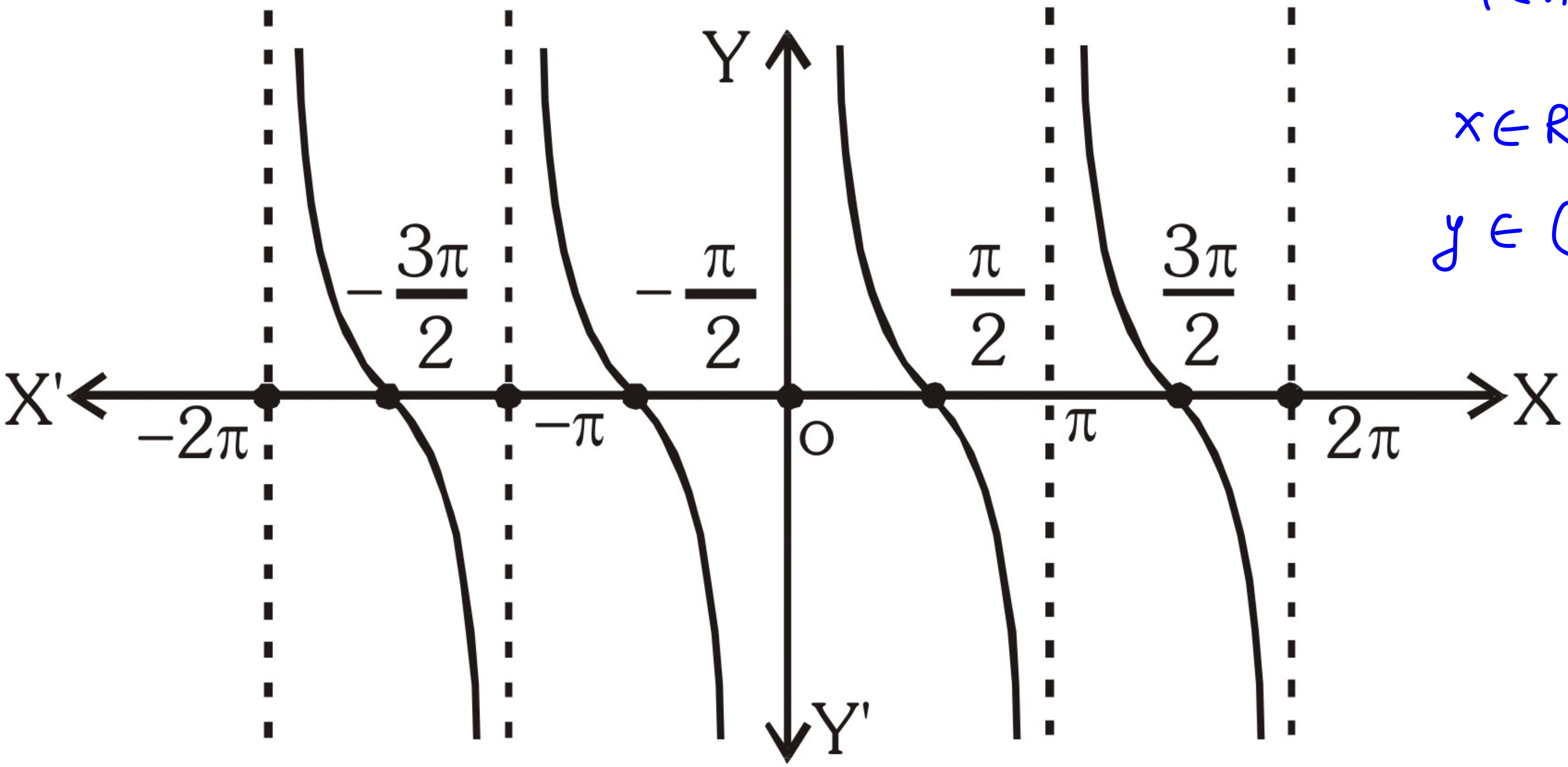
$\boxed{\begin{array}{l} \tan\theta = 0 \\ \theta \in \{n\pi,\ n \in Z\} \end{array}}$

$\boxed{\text{Periodic with period } \pi}$

$y = \cot x$



y = cotx

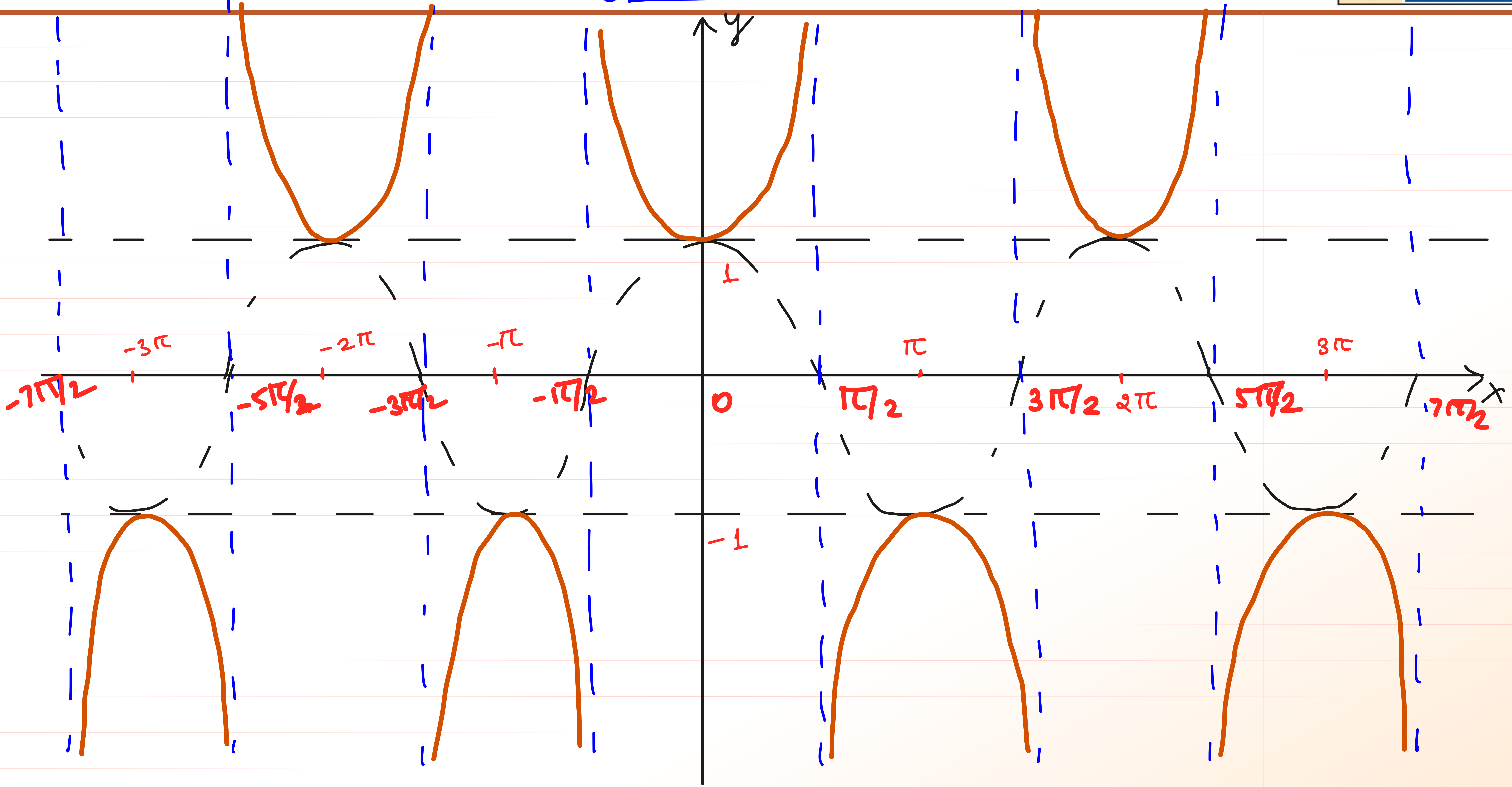


Period $= \pi$

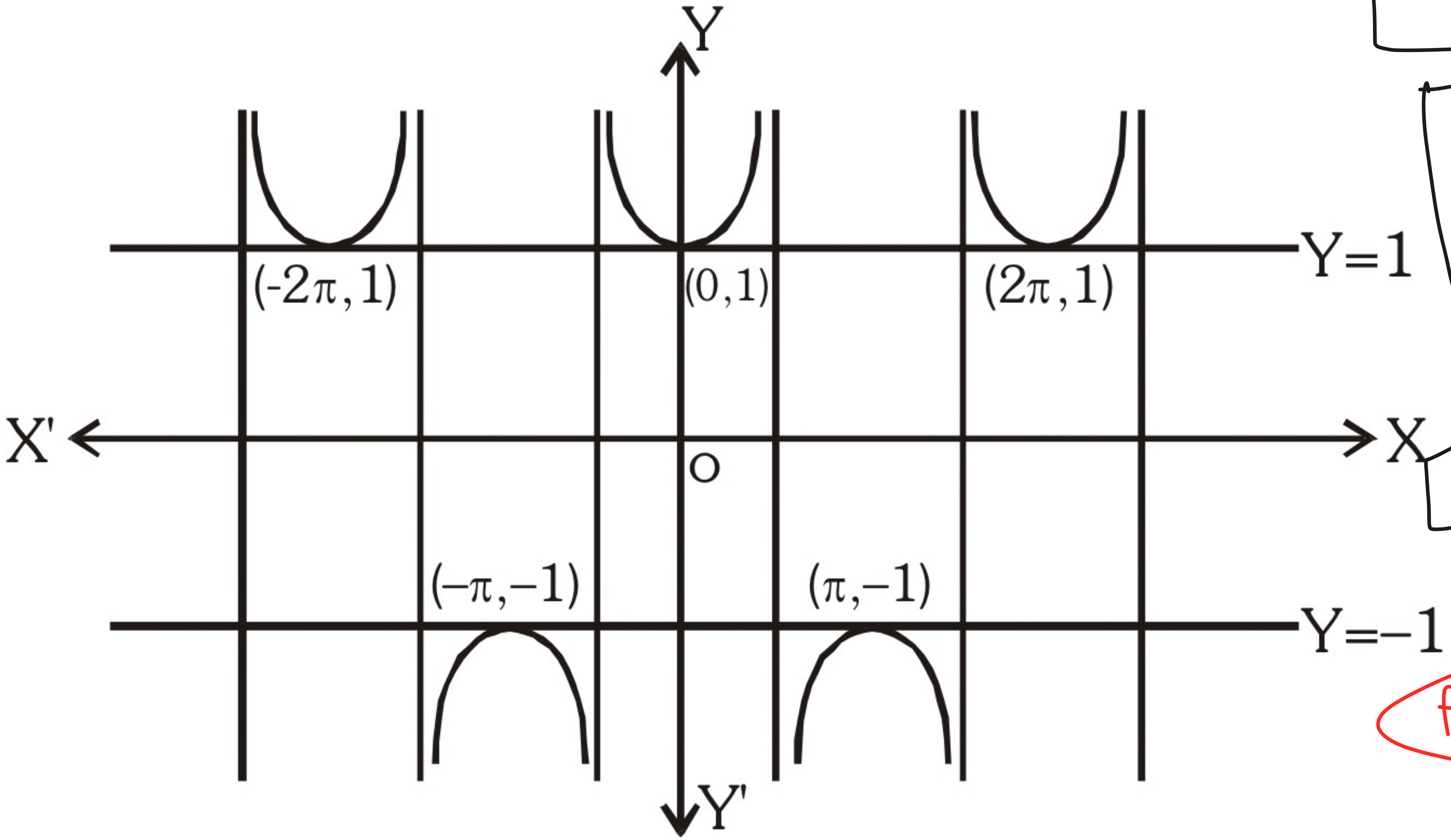
$x \in R - \{n\pi, n \in Z\}$

$y \in (-\infty, \infty)$

$y = \sec x$



y = secx



$\sec x = 1$

$x \in \{2n\pi, n \in Z\}$

$y = \sec x$

$x \in \{R - (2n+1)\frac{\pi}{2}, n \in Z\}$

$y \in (-\infty, -1] \cup [1, \infty)$

$y \in R - (-1, 1)$

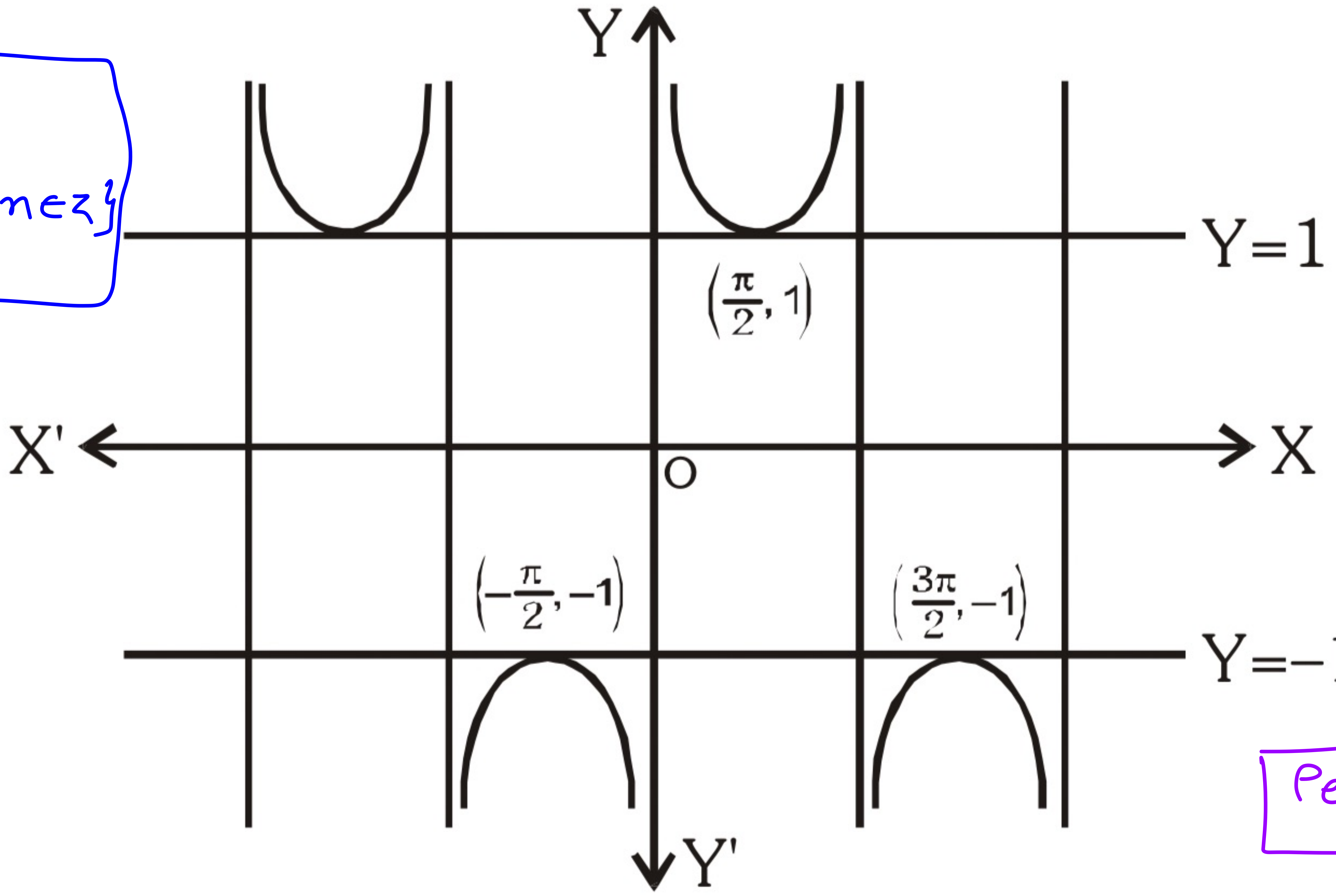
Period $= 2\pi$

$y = \text{cosec}\, x$

y = cosecx

$\text{Cosec } x = -1$

$x \in \{(4n-1)\frac{\pi}{2}, n \in Z\}$



$\text{Cosec } x = 1$

$x \in \{(4n+1)\frac{\pi}{2}, n \in Z\}$

$y = \text{cosec } x$

$x \in R - \{n\pi, n \in Z\}$

$\boxed{y \in (-\infty, -1] \cup [1, \infty)}$

$y \in R - (-1, 1)$

$\boxed{\text{Period} = 2\pi}$

(b) $-1 \leq \sin\theta \leq 1$

(c) $-1 \leq \cos\theta \leq 1$

(d) $\sin 0 = 0; \quad \sin\pi = 0; \;\; \sin 2\pi = 0$

$\sin(n\pi) = 0, \;\; n \in I$

i.e. sine of integral multiple of $\pi = 0$

Also $\tan(n\pi) = 0$ and $\sin\frac{\pi}{2} = \sin\frac{5\pi}{2} = \sin\left(2n\pi + \frac{\pi}{2}\right) = 1$ and $\sin\frac{3\pi}{2} = \sin\frac{7\pi}{2} = \sin\left(2n\pi - \frac{\pi}{2}\right) = -1$

(e) $\cos\frac{\pi}{2} = 0; \;\; \cos\frac{3\pi}{2} = 0; \; \cos\frac{5\pi}{2} = 0$

$\cos(2n-1)\frac{\pi}{2} = 0, \; n \in I$

i.e. cosine of odd integral multiple of $\frac{\pi}{2}$ is zero.

Also $\cot(2n-1)\frac{\pi}{2} = 0 : n \in I$

(f) $\cos 0 = 1; \quad \cos 2\pi = 1\ ; \cos 4\pi = 1$

$\cos 2m\pi = 1,\ m \in I$

i.e. cos of *even* multiple of $\pi = 1$

(g) $\cos\pi = -1;\ \cos 3\pi = -1;\ \cos 5\pi = -1$

$\cos(2m - 1)\pi$ = cos of odd multiple of $\pi = -1$

(h) $\sin(2n\pi + \theta) = \sin\theta,\ n \in I$

$\cos(2n\pi + \theta) = \cos\theta,\ n \in I$

$\tan(n\pi + \theta) = \tan\theta,\ n \in I$

E(21) Prove that : $\dfrac{\tan(90^\circ-\theta)\sec(180^\circ-\theta)\sin(-\theta)}{\sin(180^\circ+\theta)\cot(360^\circ+\theta)\operatorname{cosec}(90^\circ-\theta)}=-1$

E(22) If $\sec\theta=\sqrt{2},\ \dfrac{3\pi}{2}<\theta<2\pi$. Find $\dfrac{1+\tan\theta+\operatorname{cosec}\theta}{1+\cot\theta-\operatorname{cosec}\theta}$ **Ans.** (–1)

E(23) Prove that $\sin^2\dfrac{\pi}{18}+\sin^2\dfrac{\pi}{9}+\sin^2\dfrac{7\pi}{18}+\sin^2\dfrac{4\pi}{9}=2$

E(24) In a ΔABC prove that $\tan\dfrac{(A+B)}{2}=\cot\dfrac{C}{2}$

E(25) In a quadrilateral ABCD. Prove that sin(A + B) + sin(C +D) = 0

E(26) Express sin(–65°) into trignometric ratio of positive angle less than 45°