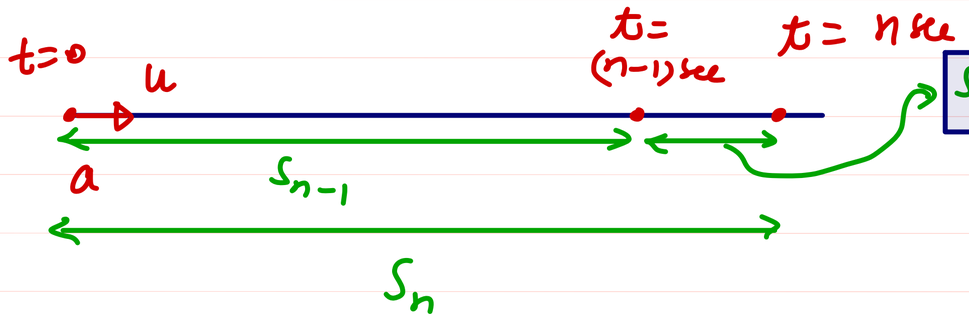
ALLEN CAREER INSTITUTE KOTA (RAJASTHAN) path to success

Displacement in nth second :→



$$\boxed{S_{nth} = S_n - S_{n-1}}$$

$$S_{nth} = \left(un + \frac{1}{2}an^2\right) - \left[u(n-1) + \frac{1}{2}a(n-1)^2\right]$$

$$\boxed{S_{nth} = u + \frac{a}{2}(2n-1)}$$

EX A body moving in a straight line covers a distance of 14 m in the 5th second and 20 m in the 8th second. How much distance will it cover in the 15th second?

$$14 = u + \frac{a}{2}[2\times5-1]$$

$$14 = u + \frac{9}{2}a \quad — (1)$$

$$20 = u + \frac{a}{2}[2\times8-1]$$

$$20 = u + \frac{15}{2}a \quad — (2)$$

from (2) − (1)

$$20-14 = \frac{15a}{2} - \frac{9a}{2}$$

$$6 = \frac{6a}{2} \Rightarrow \boxed{a = 2\,m/s^2}$$

from eq (1)

$$14 = u + \frac{9}{2}\times 2$$

$$\boxed{u = 5\,m/s^2}$$

$$S_{15th} = u + \frac{a}{2}[2\times15-1]$$

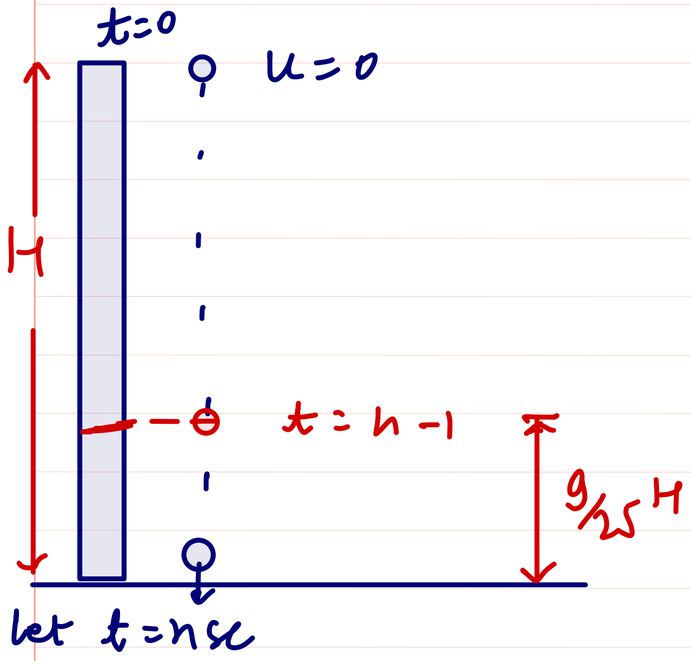
$$= 5 + \frac{2}{2}[30-1]$$

$$\boxed{S_{15th} = 34\,m}$$ Ans

Ex A ball is dropped from the top of a tower. In the last second of its fall, the ball covers a distance 9/25 times the height of the tower. If $g = 10\ ms^{-2}$, the height of the tower is

(a) 75 m (b) 100 m

(c) 125 m (d) 150 m



using

$$S_{nth} = u + \frac{a}{2}(2n-1)$$

$$-\frac{9}{25}H = 0 + -\frac{g}{2}(2n-1)$$

$$\frac{9}{25}H = 5(2n-1) \quad \text{—(1)}$$

from Top to Ground

$$-H = 0 \times n - \frac{g}{2}n^2$$

$$H = 5n^2 \quad \text{—(2)}$$

Put H in Eq (1)

$$\frac{9}{25} \times 5n^2 = 5(2n-1)$$

$$n = 5,\ \frac{5}{9}\ \text{sec}$$

Put n in Eq (2)

$$H = 5 \times 5^2$$

$$\boxed{H = 125\,m}$$ Ans

Ex A body, moving in a straight line with an initial velocity of 5 ms^{-1} and a constant acceleration, covers a distance of 30 m in the 3rd second. How much distance will it cover in the next 2 seconds?

(a) 70 m (b) 80 m

(c) 90 m (d) 100 m

Using

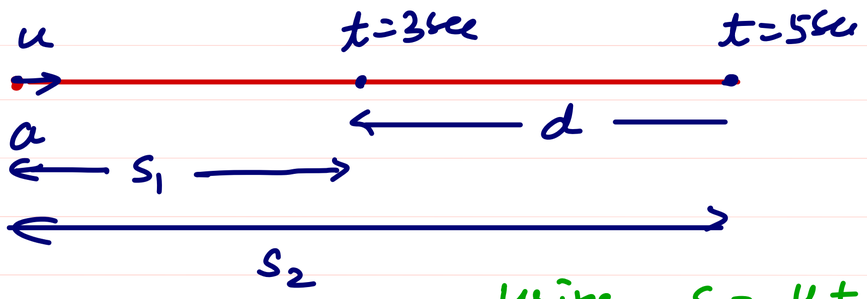
$$S_{nth} = u + \frac{a}{2}(2n-1)$$

$$30 = 5 + \frac{a}{2}[2\times 3-1]$$

$$30 = 5 + \frac{5}{2}a$$

$$6 = 1 + \frac{a}{2}$$

$$\boxed{a = 10\ m/s^2}$$



using $s = ut + \frac{1}{2}at^2$

$$d = s_2 - s_1$$

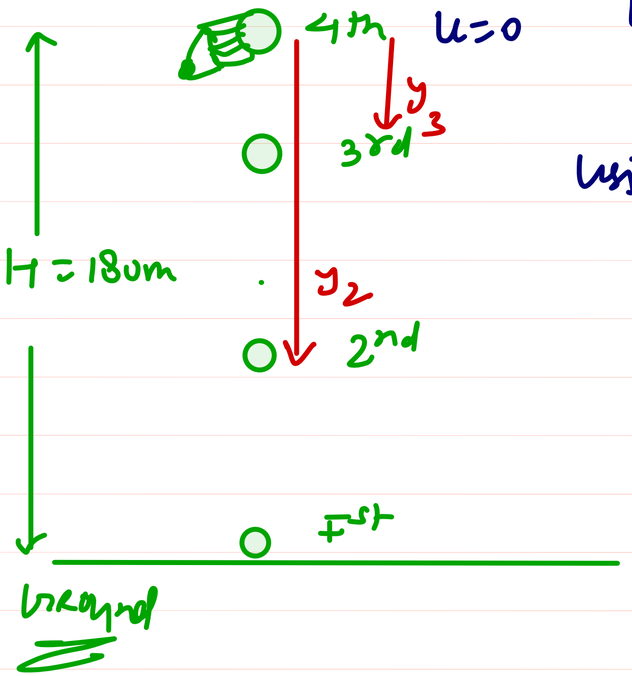
$$= \left(5\times 5 + \frac{1}{2}\times 10\times 5^2\right) - \left(5\times 3 + \frac{1}{2}\times 10\times 3^2\right)$$

$$= 5[5-3] + \frac{10}{2}[5^2 - 3^2]$$

$$\boxed{d = 90\ m}$$

5. A man holds four balls 180 m above the ground and drops them at regular intervals of time so that when the first ball hits ground, the fourth ball is just leaving his hand. At this time, the second and third balls from the ground are at the positions

(A) 160 m and 100 m respectively
(B) 80 m and 20 m respectively
(C) 20 m and 80 respectively
(D) 100 m and 160 m respectively



Let t be time interval b/w 2 balls

total time $= 3t$

using $S = ut + \frac{1}{2}at^2$

$-H = 0 \times 3t - \frac{g}{2}(3t)^2$

$180 = \frac{10}{2} \times 9t^2 \Rightarrow \boxed{t = 2sec}$

$y_3 = 0 \times 2 - \frac{1}{2} \times 10 \times 2^2 = -20m$

$y_2 = 0 \times 4 - \frac{1}{2} \times 10 \times 4^2 = -80m$

From ground

distance of $y_2 = 180 - 80$
$= 100m$

distance of $y_3 = 180 - 20$
$= 160m$

Paragraph for Question 5 and 6

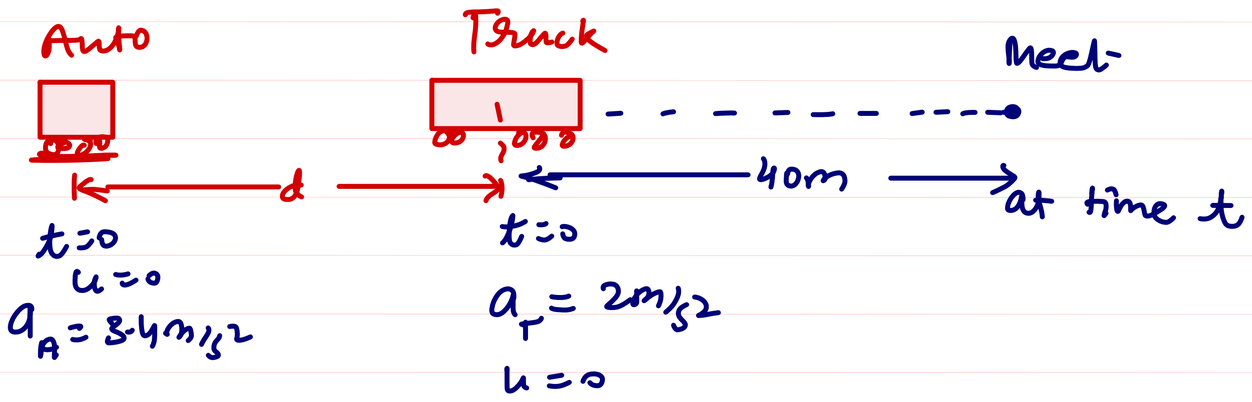
An auto-mobile and a truck start from rest at the same instant, with the automobile initially at some distance behind the truck. The truck has a constant acceleration of 2.0 m/s^2 and the automobile has acceleration 3.4 m/s^2. The automobile overtakes the truck after the truck has moved 40 m.

6. How much time does, it take for the automobile to overtake the truck ?

(A) 20 s (B) $\sqrt{10}$ s (C) 2 s (D) $2\sqrt{10}$ s

7. How far was the automobile behind the truck initially?

(A) 28 m (B) 26 m (C) 38 m (D) 18 m



Using $S = ut + \frac{1}{2}at^2$

For Truck

$40 = 0 \times t + \frac{1}{2} \times 2 t^2$

$t^2 = 40 \Rightarrow \boxed{t = 2\sqrt{10} \text{ sec}}$

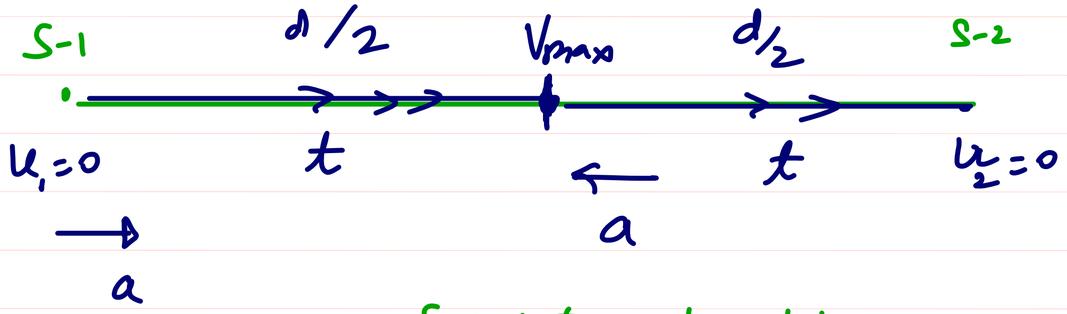
For Auto

$d + 40 = 0 \times t + \frac{1}{2} \times 3.4 \times t^2$

$d + 40 = \frac{1}{2} \times 3.4 \times 40 \Rightarrow \boxed{d = 28 \text{ m}}$

12. The greatest acceleration or deceleration that a train may have is a. The minimum time in which the train may reach from one station to the other separated by a distance d is

(A) $\sqrt{\frac{d}{a}}$ (B) $\sqrt{\frac{2d}{a}}$ (C) $\frac{1}{2}\sqrt{\frac{d}{a}}$ (D) $2\sqrt{\frac{d}{a}}$



$S = ut + \frac{1}{2}at^2$

$\frac{d}{2} = 0 \times t + \frac{1}{2}at^2$

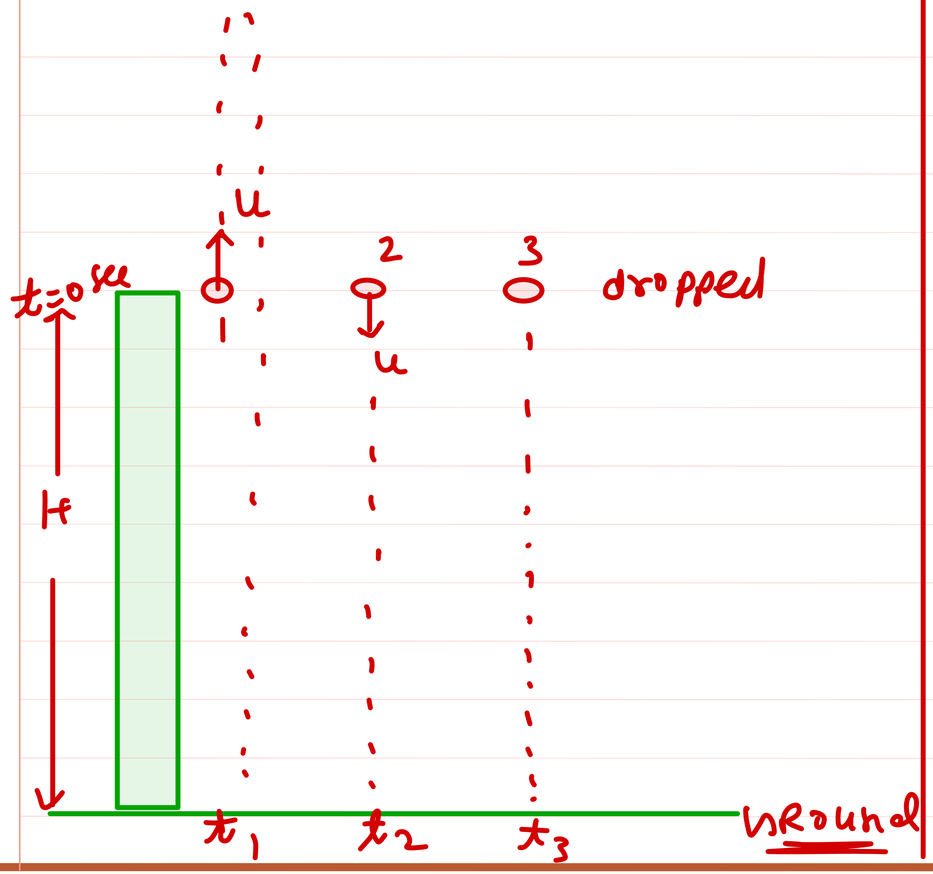
$t = \sqrt{\frac{d}{a}}$

total time $= 2t$

$= 2\sqrt{d/a}$

13. From the top of a tower, a stone is thrown up and reaches the ground in time t_1. A second stone is thrown down with the same speed and reaches the ground in time t_2. A third stone is released from rest and reaches the ground in time t_3.

(A) $t_3 = \frac{1}{2}(t_1 + t_2)$ (B) $t_3 = \sqrt{t_1 t_2}$ (C) $\frac{1}{t_3} = \frac{1}{t_2} - \frac{1}{t_1}$ (D) $t_3^2 = t_1^2 - t_2^2$



Using $s = ut + \frac{1}{2}at^2$

FOR-1 $-H = ut_1 - \frac{g}{2}t_1^2$ — (1)

FOR-2 $-H = -ut_2 - \frac{g}{2}t_2^2$ — (2)

FOR-3 $-H = 0t_3 - \frac{g}{2}t_3^2$ — (3)

From Eq (1) and (2)

$ut_1 - \frac{g}{2}t_1^2 = -ut_2 - \frac{g}{2}t_2^2$

$u(t_1 + t_2) = \frac{g}{2}(t_1^2 - t_2^2)$

$u = \frac{g}{2}(t_1 - t_2)$ — (4)

put H & u from Eq (3) & (4) in Eq (1)

$-\frac{g}{2}t_3^2 = \frac{g}{2}(t_1 - t_2)t_1 - \frac{g}{2}t_1^2$

$-t_3^2 = +t_1^2 - t_1t_2 - t_1^2$

$-t_3^2 = -t_1t_2$

$t_3 = \sqrt{t_1 t_2}$ Ans

Race #8

1. A particle is moving eastward with a velocity of 5m/s In 10 s, the velocity changes to 5m/s northward. Find the average acceleration in this time.

(A) zero

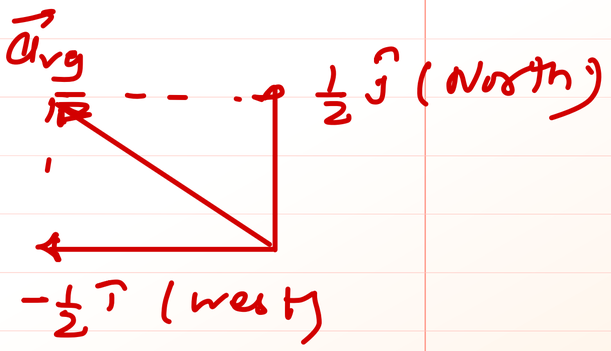
(B) $\frac{1}{\sqrt{2}}$ m/s^2 towards north-west

(C) $\frac{1}{\sqrt{2}}$ m/s^2 towards north-east

(D) $\frac{1}{2}$ m/s^2 towards north-west

$$\vec{a}_{vg} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t} = \frac{5\hat{j} - 5\hat{i}}{10} = \frac{1}{2}(\hat{j} - \hat{i})$$

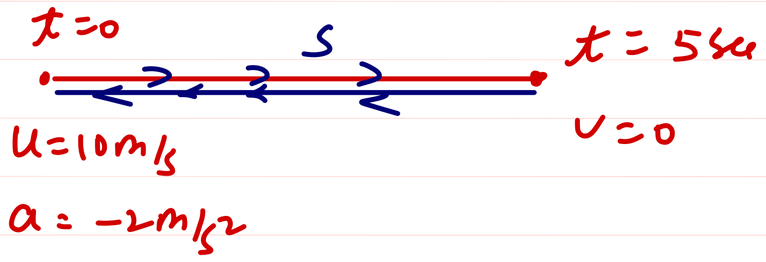
$$|\vec{a}_{vg}| = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}} \text{ m/s}^2$$



19. A particle has a velocity of $v = 10 - 2t$ ms^{-1} and moves in a straight line.Find the distance traveled in 10 s

20. A particle has an acceleration $a = 10 - 5t$ ms^{-2} and moves in a straight line initially at rest (a) Find the velocity after 4 s (b) Find the distance traveled in 6 s (c) draw the v-t graph.

(19)



$d = 2s$

$= 2[10 \times 5 - \frac{1}{2} \times 2 \cdot 5^2$

$= 2 \times (50 - 25)$

$\boxed{d = 50\,m}$ Ans

(20)

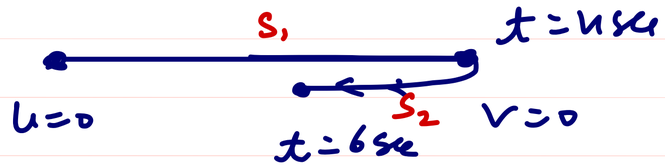
$a = 10 - 5t$

$\int_0^v dv = \int_0^t 10\,dt - \int_0^t 5t\,dt$

$\boxed{v = 10t - \frac{5t^2}{2}}$

$v = 0$

$10t = \frac{5t^2}{2} \Rightarrow \boxed{t = 4\,sec}$



$d = s_1 + s_2$

$= \int_0^4 v\,dt + \left| \int_4^6 v\,dt \right|$

$= \left(\int_0^4 10t\,dt - \int_0^4 \frac{5}{2} t^2\,dt\right) + \left| \int_4^6 10t\,dt - \frac{5}{2}\int_4^6 t^2\,dt \right|$

$= \left[10\frac{t^2}{2}\right]_0^4 - \left[\frac{5}{2}\frac{t^3}{3}\right]_0^4 + \left| \left(10\frac{t^2}{2} - \frac{5}{2}\frac{t^3}{3}\right)_4^6 \right|$

24. A particle is given velocity of 5 m/s and its acceleration is $a = -2v$, where v is its velocity at any time t. Find the velocity v at any time t. Also find the total distance travelled.

$$a = -2v$$

$$\frac{dv}{dt} = -2v$$

$$\int_5^v \frac{dv}{v} = -\int_0^t 2\,dt$$

$$\ln\left(\frac{v}{5}\right) = -2t$$

$$\frac{v}{5} = e^{-2t}$$

$$\boxed{v = 5e^{-2t}}$$

$$\int_0^d dx = \int_0^\infty v\,dt$$

$$d = 5\int_0^\infty e^{-2t}\,dt$$

$$d = 5\left[\frac{e^{-2t}}{-2}\right]_0^\infty$$

$$\boxed{d = \frac{5}{2}\,m}$$

21. A particle has an acceleration $a = 4\sqrt{x}$ ms^{-2} and moves in a straight line with zero velocity at x = 0. Find the velocity of the particle at x = 1.

$$a = 4\sqrt{x}$$

$$v\frac{dv}{dx} = 4\sqrt{x}$$

$$\int_0^v v\,dv = 4\int_0^1 \sqrt{x}\,dx$$

$$\frac{v^2}{2} = 4\left|\frac{x^{3/2}}{3/2}\right|_0^1$$

$$v^2 = 8 \times \frac{2}{3} \times 1$$

$$\boxed{v = \frac{4}{\sqrt{3}}\ m/s}$$ Ans

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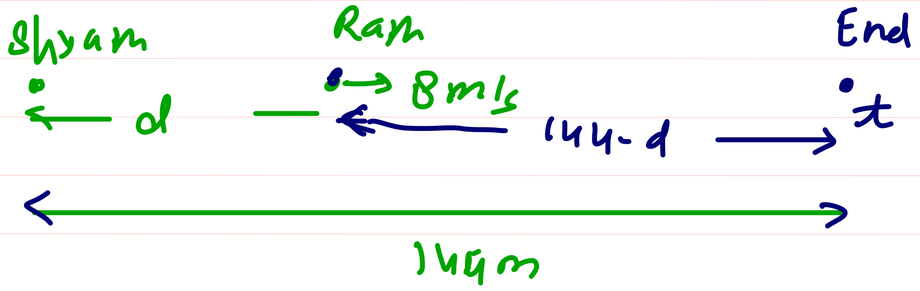
3. A particle starts with a velocity of 2 ms^{-1} and moves in a straight line with an acceleration = (– 0.1) m/s^2. The time when its displacement is 15 m is/are

(A) 20 s (B) 30 s (C) 10s (D) 40 s

$s = ut + \frac{1}{2}at^2 \Rightarrow 15 = 2 \times t - \frac{1}{20}t^2 \Rightarrow \boxed{t = 10\,sec, 30\,sec}$

8. Two runners Ram and Shyam in a 144 m race start from the same place, but one runner gives the other a little advantage. The first runner, Ram, starts right away and runs at a constant velocity of 8.0 m/s. The second runner, Shyam, waits two seconds and then runs at a velocity of 9.0 m/s. How much is the separation between them (in m) when the race is just about to finish?



$(144 - d) = 8 \times t$ —(1)

$144 = 9 \times (t - 2)$ —(2)

B.B # 2

1*. A stone is dropped into a well in which the level of water is h below the top of the well. If v is velocity of sound, the time T after which the splash is heard is given by :

(A) $T = 2h/v$

(B) $T = \sqrt{\frac{2h}{g}} + \frac{h}{v}$

(C) $T = \sqrt{\frac{2h}{g}} + \frac{h}{2v}$

(D) $T = \sqrt{\frac{h}{2g}} + \frac{2h}{v}$

$h = \frac{1}{2} g t_1^2 \Rightarrow t_1 = \sqrt{\frac{2h}{g}}$

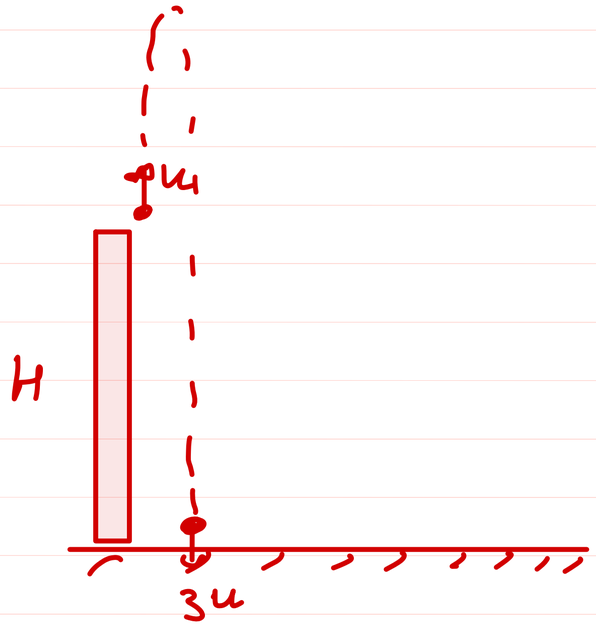
$t_{up} = \frac{h}{v}$

$t_{total} = t_1 + t_{up}$

$= \sqrt{\frac{2h}{g}} + \frac{h}{v}$

2. A stone is thrown vertically upward with an initial velocity u from the top of a tower, reaches the ground with a velocity 3u. The height of the tower is :

(A) $\frac{3u^2}{g}$ (B) $\frac{4u^2}{g}$ (C) $\frac{6u^2}{g}$ (D) $\frac{9u^2}{g}$



$$v^2 = u^2 + 2as$$

$$(3u)^2 = u^2 + 2(-g)(-H)$$

$$8u^2 = 2gH$$

$$\boxed{H = \frac{4u^2}{g}}$$

6. A ball is dropped from a tower. In the last second of its motion it travels a distance of 15m. Find the height of the tower. [take g = 10 m/sec^2]

$15 = \frac{1}{2} \times 10 (2n-1)$ $\qquad S_{nth} = u + \frac{1}{2}a(2n-1)$

$3 = 2n-1 \Rightarrow \boxed{n = 2sec} = t$

$h = \frac{1}{2} g t^2$

$h = \frac{1}{2} \times 10 \times 2^2 \Rightarrow \boxed{h = 20m}$ Ans

Illustration 8. The acceleration of a particle moving along the x-direction is given by equation a = (3–2t) m/s^2.

At the instants :

t = 0 and t = 6 s, it occupies the same position.

(a) Find the initial velocity v_o.

(b) What will be the velocity at t = 2 s?

$a = 3 - 2t$

$\int_{v_0}^{v} dv = 3\int_0^t dt - 2\int_0^t t\,dt$

$\boxed{v - v_0 = 3t - 2\frac{t^2}{2}}$

$v - v_0 = 3t - t^2$

$v = v_0 + 3t - t^2$

$\int_{x_0}^{x_0} dx = \int_0^6 v_0\,dt + 3\int_0^6 t\,dt - \int_0^6 t^2\,dt$

$0 = v_0 (t)_0^6 + 3\left[\frac{t^2}{2}\right]_0^6 - \left[\frac{t^3}{3}\right]_0^6$

$0 = 6v_0 + \frac{3}{2} 6^2 - \frac{1}{3} 6^3$

$0 = v_0 + 9 - 12$

$\boxed{v_0 = 3 m/s}$ Ans

BB #3

1. The acceleration of a particle is increasing linearly with time t as bt. The particle starts from the origin with an initial velocity v_0 The distance travelled by the particle in time t will be

(A) $v_0t+\frac{1}{3}bt^2$ (B) $v_0t+\frac{1}{3}bt^3$ (C) $v_0t+\frac{1}{6}bt^3$ (d) $v_0t+\frac{1}{2}bt^3$

$a = bt$

$\frac{dv}{dt} = bt$

$\int_{v_0}^{v} dv = b\int_0^t t\,dt$

$v - v_0 = \frac{bt^2}{2}$

$v = v_0 + \frac{b}{2}t^2$

$\frac{dx}{dt} = v_0 + \frac{b}{2}t^2$

$\int_0^d dx = v_0\int_0^t dt + \frac{b}{2}\int_0^t t^2\,dt$

$d = v_0 t + \frac{b}{2}\cdot\frac{t^3}{3}$

$$\boxed{d = v_0 t + \frac{b}{6}t^3}$$

4. The displacement of a particle is given by $y = a + bt + ct^2 - dt^4$. The initial velocity and acceleration are respectively

(A) $b, -4d$ (B) $-b, 2c$ (C) $b, 2c$ (D) $2c, -4d$

$v = \frac{dy}{dt} = 0 + b + 2ct - 4dt^3$

$a = 0 + 2c - 12dt^2$

at $t = 0$ $v = b + 2c \times 0 - 4d \times 0^3$

$\boxed{v = b}$

$a = 2c - 12d \times 0^2$

$\boxed{a = 2c}$