16. $xy = c^2$, then $\frac{dy}{dx}$

(A) $\frac{x}{y}$ (B) $\frac{y}{x}$ (C) $-\frac{x}{y}$ (D) $-\frac{y}{x}$

$y = c^2 x^{-1}$

$\frac{dy}{dx} = c^2 \{-1 x^{-2}\}$

$\frac{dy}{dx} = \frac{-c^2}{x^2} = -\frac{xy}{x^2} = -\frac{y}{x}$

22. The maximum value of xy subject to $x + y = 8$, is :

(A) 8 (B) 16 (C) 20 (D) 24

Let $z = xy$

$z = x(8-x)$

$\frac{dz}{dx} = 1(8-x) + x(0-1)$

$= 8-x-x = 8-2x$

For $\frac{dz}{dx} = 0$

$8-2x = 0 \Rightarrow \boxed{x = 4}$

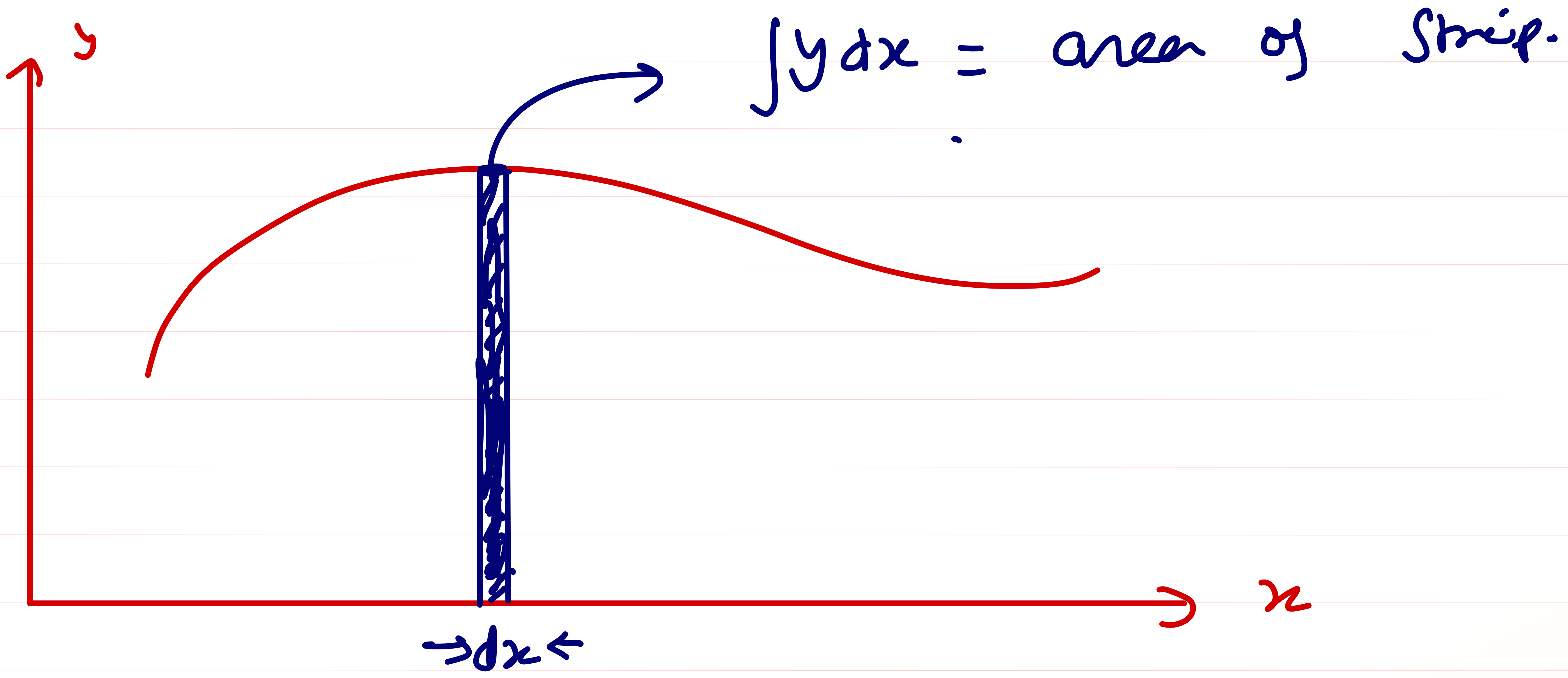
$x+y = 8 \Rightarrow \boxed{y = 4}$

$z_{max} = xy = 4 \times 4 = 16$ Ans

Integration is the reverse process of differentiation. By help of integration we can find a function whose derivative is known. Consider a function F(x) whose differentiation w.r.t. x is equal to f(x) then

$$\boxed{\int f(x)\,dx = F(x) + c}$$

here c is the constant of integration and this is called indefinite integration.



Few basic formulae of integration are :

Imp

① $\boxed{\int x^n dx = \frac{x^{n+1}}{n+1} + c}$

③ $\int \frac{1}{x} dx = \ln x + c$

② $\int \sin x dx = -\cos x + c$

④ $\int \cos x dx = \sin x + c$

$\int \sec^2 x dx = \tan x + c$

⑤ $\int e^x dx = e^x + c$

$$\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c$$

$$\int \frac{dx}{ax+b} = \frac{\ln(ax+b)}{a} + c$$

$$\int \sin(ax+b) dx = \frac{-\cos(ax+b)}{a} + c$$

$$\int \cos(ax+b) dx = \frac{\sin(ax+b)}{a} + c$$

$$\int \sec^2(ax+b) dx = \frac{\tan(ax+b)}{a} + c$$

$$\int e^{ax+b} dx = \frac{e^{ax+b}}{a} + c$$

$x \rightarrow (ax+b)$

Ex $y = x^2 + 2$

$\frac{dy}{dx} = 2x$

$\int dy = \int 2x\, dx$

$\int y^0 dy = 2 \frac{x^{1+1}}{1+1} + c$

$\frac{y^{0+1}}{0+1} = x^2 + c$

$\boxed{y = x^2 + c}$

Illustration 33.

Integrate following functions

① $\int x^{-5}\,dx = \dfrac{x^{-5+1}}{-5+1} + c = \dfrac{x^{-4}}{-4} + c = -\dfrac{1}{4x^4} + c$

② $\int \sin(x)\,dx = -\cos(x) + c$

③ $\int \frac{1}{x}\,dx = \ln(x) + c$

④ $\int e^x\,dx = e^x + c$

⑤ $\int \cos(x)\,dx = \sin(x) + c$

⑥ $\int \sin(2x+3)\,dx$

$= \frac{1}{2}\left[-\cos(2x+3)\right] + c$

⑦ $\int \dfrac{1}{2x+3}\,dx = \frac{1}{2}\ln(2x+3) + c$

⑧ $\int e^{(2x+3)}\,dx = \frac{1}{2} e^{(2x+3)} + c$

⑨ $\int \dfrac{4}{x^3}\,dx = 4\left[\dfrac{x^{-3+1}}{-3+1}\right] + c$

$= 4\left[\dfrac{x^{-2}}{-2}\right] + c = \dfrac{-2}{x^2} + c$

Ans

Ex $\int (2x+3)\,dx$

$= \int 2x\,dx + \int 3\,dx$

$= 2\int x\,dx + 3\int dx$

$= 2\frac{x^2}{2} + 3x + C$

$= x^2 + 3x + C$ Ans

$= \frac{(2x+3)^2}{2} \times \frac{1}{2} + C$

$= \frac{1}{4}\{4x^2 + 9 + 12x\} + C$

$= x^2 + \frac{9}{4} + 3x + C$

$= x^2 + 3x + C'$

$$\boxed{\begin{aligned}\int k\,f(x)\,dx &= k\,F(x) + C\\ \int f(kx)\,dx &= \frac{1}{k}F(kx) + C\end{aligned}}$$

Ex $\int (x^2 - 2x + 3)\,dx$

$\int x^2 dx - 2\int x\,dx + 3\int dx$

$= \frac{x^3}{3} - 2\,\frac{x^2}{2} + 3x + c$

$= \frac{x^3}{3} - x^2 + 3x + c$ Ans

Ex $\int (\sin(x) + x^2 - 2)\,dx$

$= \int \sin x\,dx + \int x^2 dx - 2\int dx$

$= -\cos x + \frac{x^3}{3} - 2x + c$

Ex $\int \left(\frac{1}{x} + 2x + e^x\right) dx$

$\int \frac{dx}{x} + 2\int x\,dx + \int e^x dx$

$= \ln(x) + 2\,\frac{x^2}{2} + e^x + c$

$= \ln(x) + x^2 + e^x + c$ Ans

Definite Integration

SL AL

When a function is integrated between a lower limit and an upper limit, it is called a definite integral.

Consider a function F(x) whose differentiation w.r.t. x is equal to f(x), in an interval $a \le x \le b$ then

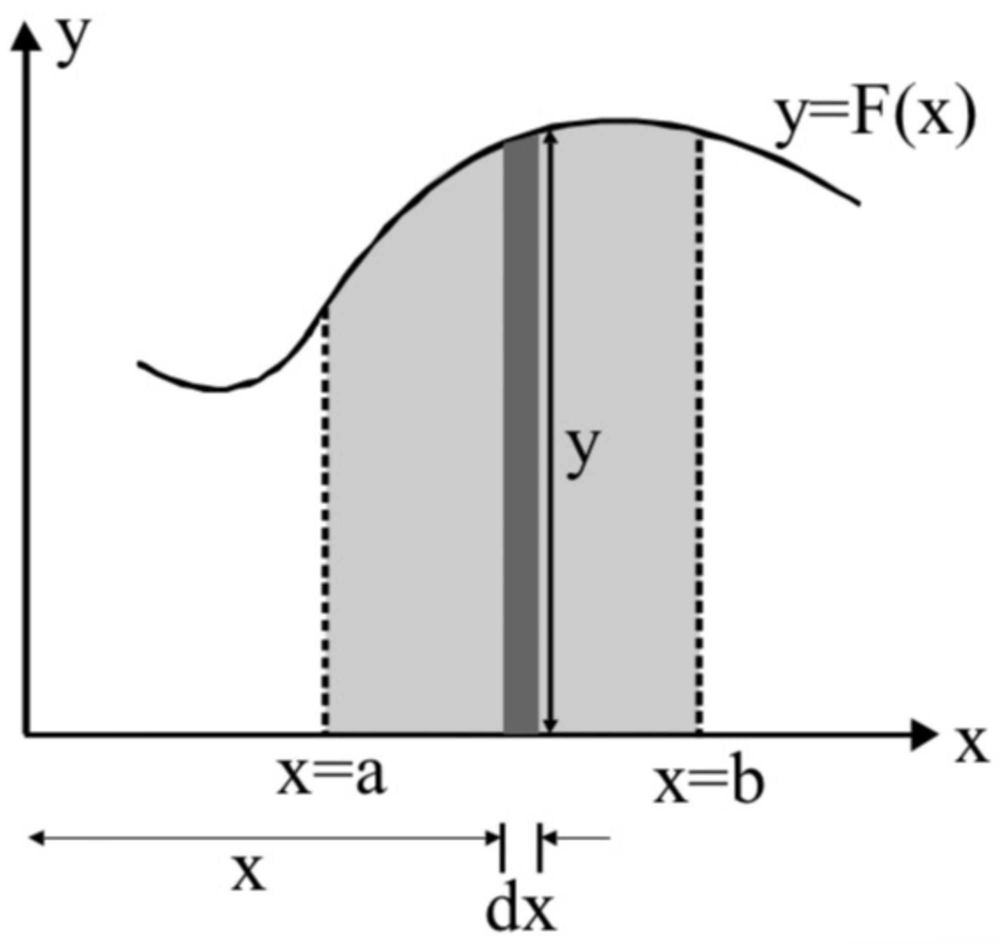
upper limit

$$\boxed{\int_a^b f(x)\,dx = F(b) - F(a)}$$

lower limit

Area Under a Curve and Definite Integration

SL AL



Area of small shown element = ydx = f(x) dx

If we sum up all areas between x=a and x= b then

$$\boxed{\int_a^b f(x)\,dx = \text{shaded area between curve and x-axis.}}$$

Illustration 34. ***Illustration 35*.***

Ex $\int_1^2 2x\,dx$

$= 2\int_1^2 x\,dx$

$= 2\left[\frac{x^2}{2}\right]_1^2$

$= 2^2 - 1^2$

$= 3$ Ans

$= 2\frac{x^2}{2} + c$

$= \left[x^2 + c\right]_1^2$

$= (2^2 + c) - (1^2 + c)$

$= 2^2 + c - 1^2 - c$

$= 2^2 - 1^2$

$= 3$

Ex $\int_{\pi/2}^{\pi/4} \sin(x)\,dx = \left[-\cos x\right]_{\pi/2}^{\pi/4}$

$= -\left[\cos\frac{\pi}{4} - \cos\frac{\pi}{2}\right]$

$= -\left[\frac{1}{\sqrt{2}} - 0\right]$

$= -\frac{1}{\sqrt{2}}$ Ans

H.W Race # 5

Q 1 - 23

BB # 3 → complete

Ex $\int_{-1}^{2}(2x+3x^2+2)\,dx$

$= 2\int_{-1}^{2}x\,dx + 3\int_{-1}^{2}x^2\,dx + 2\int_{-1}^{2}dx$

$= \cancel{2}\left[\frac{x^2}{\cancel{2}}\right]_{-1}^{2} + \cancel{3}\left[\frac{x^3}{\cancel{3}}\right]_{-1}^{2} + 2\left[x\right]_{-1}^{2}$

$= \left[x^2\right]_{-1}^{2} + \left[x^3\right]_{-1}^{2} + 2\left[x\right]_{-1}^{2}$

$= \left[4-1\right] + \left[2^3-(-1)^3\right] + 2\left[2-(-1)\right]$

$= 3 + (8+1) + 2\left[+3\right]$

$= 3+9+6 \Rightarrow 18$ Ans

$\left[\frac{\cancel{2}x^2}{\cancel{2}} + \frac{\cancel{3}x^3}{\cancel{3}} + 2x\right]_{-1}^{2}$

$\left(x^2+x^3+2x\right)_{-1}^{2}$

$= \left(2^2+2^3+2\times2\right)-\left((-1)^2+(-1)^3+2(-1)\right]$

$= 18$

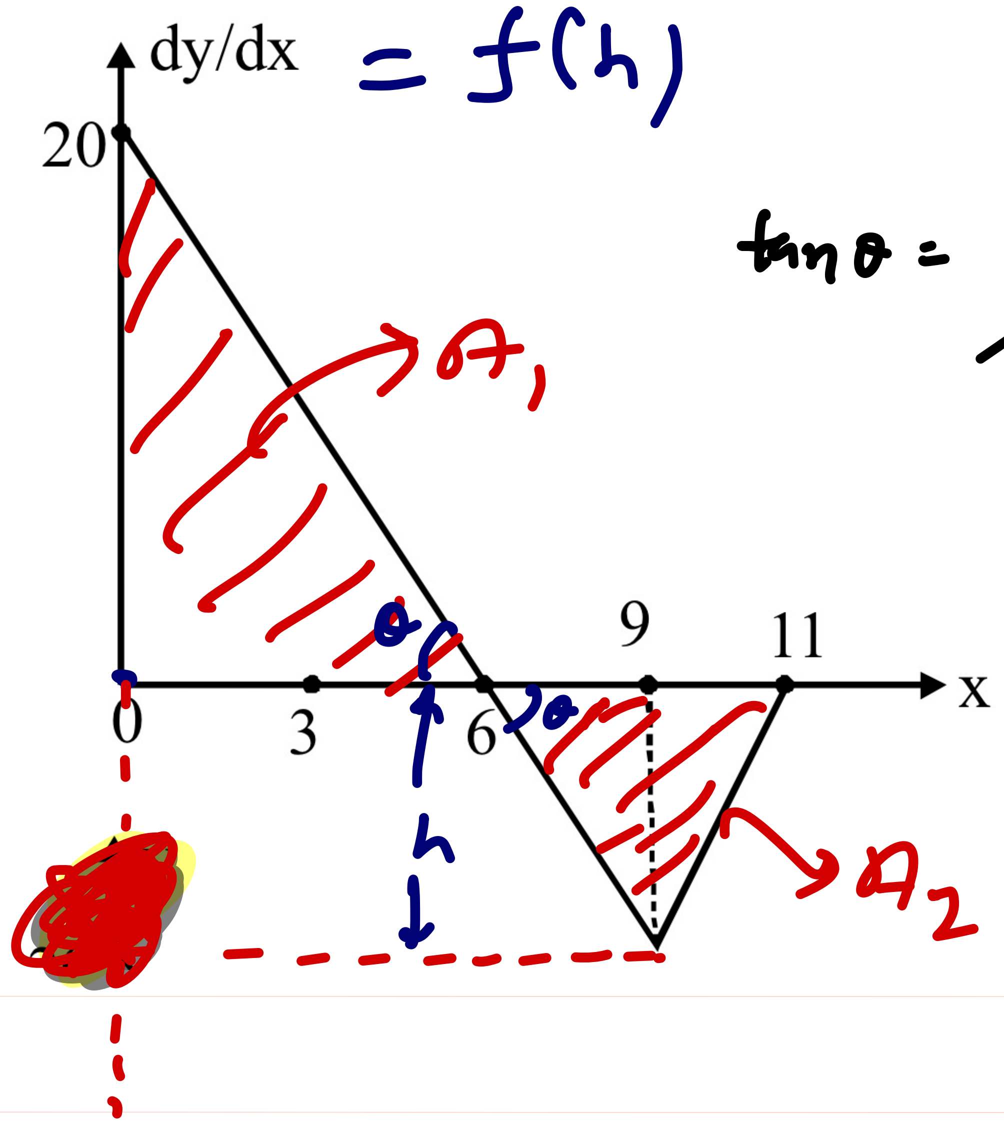
Illustration 35*. The following curve represents rate of change of a variable y w.r.t x. The change in the value of y when x changes from 0 to 11 is:

(A) 60

(B) 25

(C) 35

(D) 85



$= f(x)$

$\tan\theta = \frac{20}{6} = \frac{h}{3}$

$h = 10$

$$\int_{x=0}^{x=11} \left(\frac{dy}{dx}\right) dx = \text{Area of curve}$$

$= A_1 + A_2$

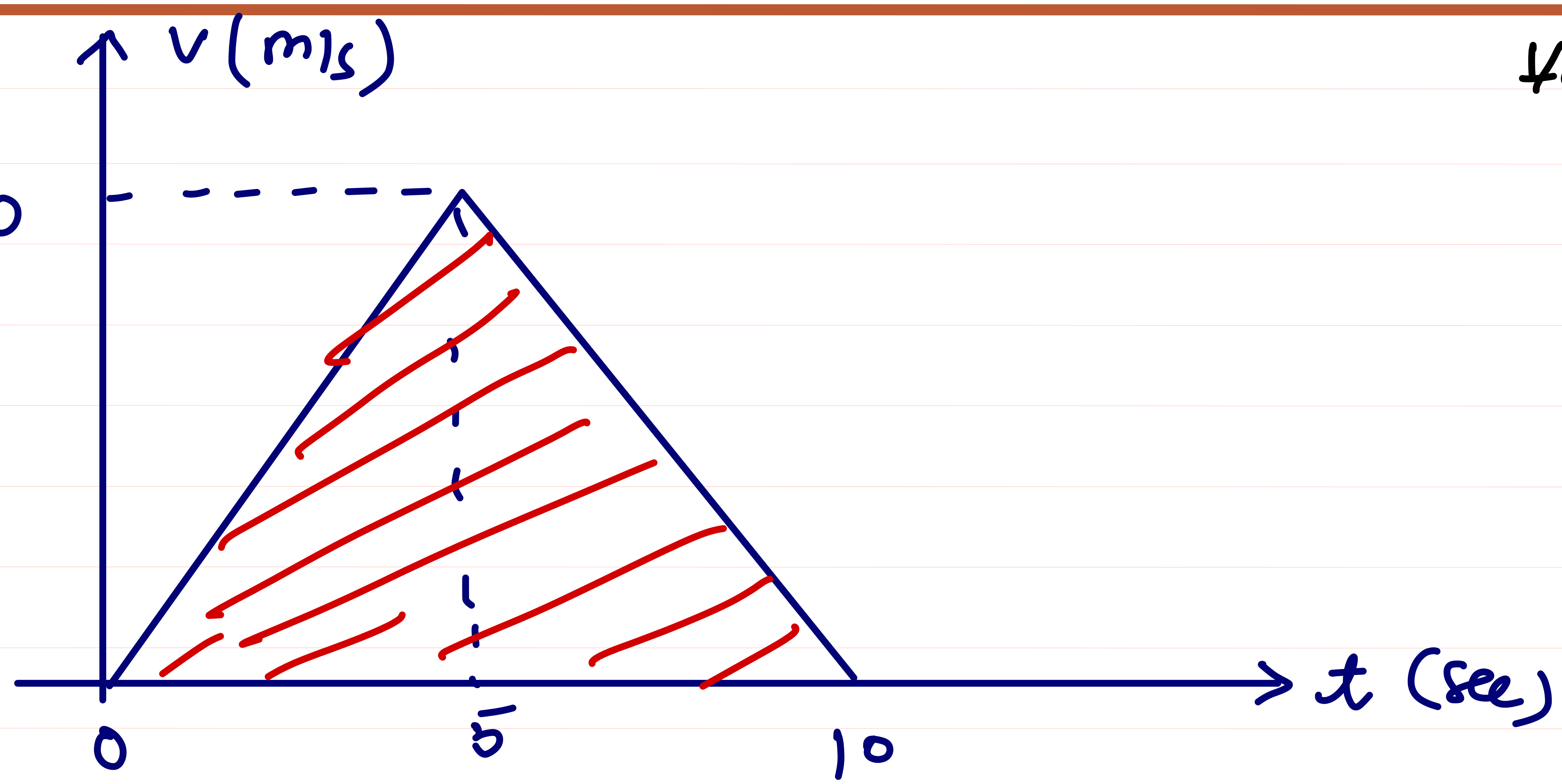
$= 60 - 25 = 35$

$A_1 = \frac{1}{2} \times 6 \times 20 = 60$

$A_2 = \frac{1}{2} \times (10) \times 5 = -25$

$$\int_{y_1}^{y_2} dy = (y)_{y_1}^{y_2} = (y_2 - y_1) = \Delta y = \text{Change of } y$$

Ex



Velocity - time curve of A particle is given. Find displacement of particle from $t=0$ to $t=10$ sec

$v = \frac{ds}{dt}$ = time rate of disp.

$\int_{s_1}^{s_2} ds = \int_0^{10} v\,dt$ = Area of v-t curve

Area = $\frac{1}{2} \times 10 \times 20$

$= 100$

$\boxed{s_2 - s_1 = 100\ m}$ Ans