$$\boxed{R = \frac{u^2}{g\cos^2\alpha}\left[\sin(2\theta-\alpha) - \sin\alpha\right]}$$

Yaad Rakho

For maximum $\rightarrow R$

$$\sin(2\theta-\alpha) = 1$$

$$2\theta - \alpha = \frac{\pi}{2}$$

$$2\theta = \frac{\pi}{2} + \alpha$$

$$\boxed{\theta = \frac{\pi}{4} + \frac{\alpha}{2}}$$

$$\boxed{R_{max} = \frac{u^2}{g\cos^2\alpha}\left[1 - \sin\alpha\right]}$$

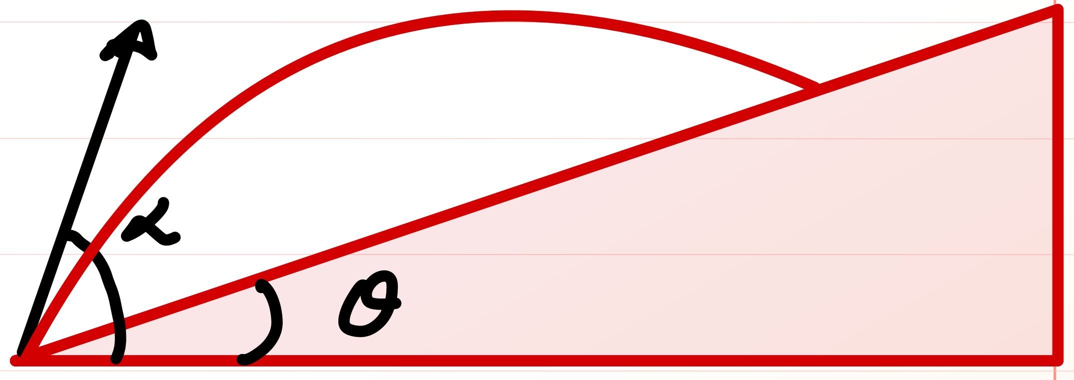
Yaad Rakho

1. A plane surface is inclined making an angle θ with the horizontal. From the bottom of this inclined plane, a bullet is fired with velocity v. The maximum possible range of the bullet on the inclined plane is

(A) $\dfrac{v^2}{g}$ (B) $\dfrac{v^2}{g(1+\sin\theta)}$ (C) $\dfrac{v^2}{g(1-\sin\theta)}$ (D) $\dfrac{v^2}{g(1+\cos\theta)}$

$\because \boxed{R_{max} = \dfrac{u^2}{g\cos^2\alpha}(1-\sin\alpha)}$ $\because \alpha = \theta$

$= \dfrac{u^2}{g}\dfrac{(1-\sin\alpha)}{(1-\sin^2\alpha)}$



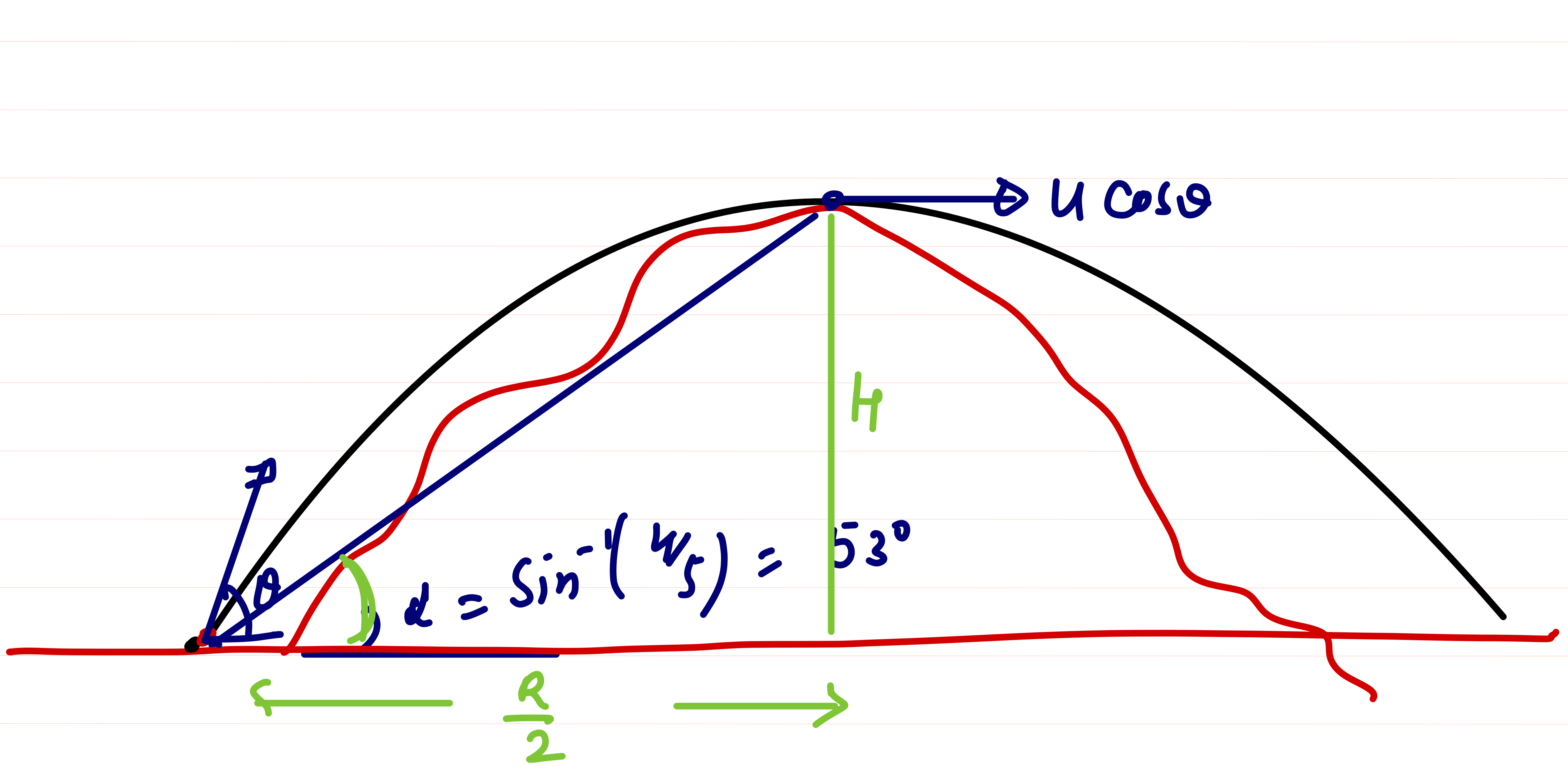
$= \dfrac{u^2}{g}\dfrac{\cancel{(1-\sin\alpha)}}{(1+\sin\alpha)\cancel{(1-\sin\alpha)}} = \dfrac{u^2}{g(1+\sin\alpha)}$

R_{max}

$\boxed{\alpha = \dfrac{\pi}{4} + \dfrac{\theta}{2}}$

7. A shot is fired at an angle 'θ' to the horizontal up a hill of inclination $\sin^{-1}\left(\frac{4}{5}\right)$ to the horizontal. If the shot strikes the hill horizontally, then the value of 'θ' is

(A) $\sin^{-1}\left(\frac{3}{\sqrt{73}}\right)$ (B) $\cos^{-1}\left(\frac{3}{\sqrt{73}}\right)$ (C) $\tan^{-1}\left(\frac{3}{\sqrt{73}}\right)$ (D) $\cot^{-1}\left(\frac{3}{\sqrt{73}}\right)$

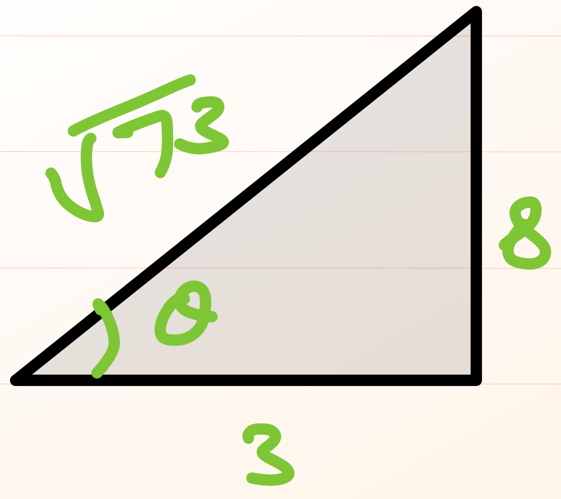


$$\tan 53 = \frac{H}{R/2} = \frac{4}{3} \Rightarrow \boxed{H = \frac{2R}{3}}$$

$$\frac{\cancel{u^2}\sin^2\theta}{2\cancel{g}} = \frac{2}{3}\,\frac{\cancel{u^2}\sin 2\theta}{\cancel{g}}$$

$$\frac{\sin^{\cancel{2}}\theta}{2} = \frac{2}{3}\times 2\cancel{\sin\theta}\cos\theta$$

$$\tan\theta = \frac{8}{3}$$

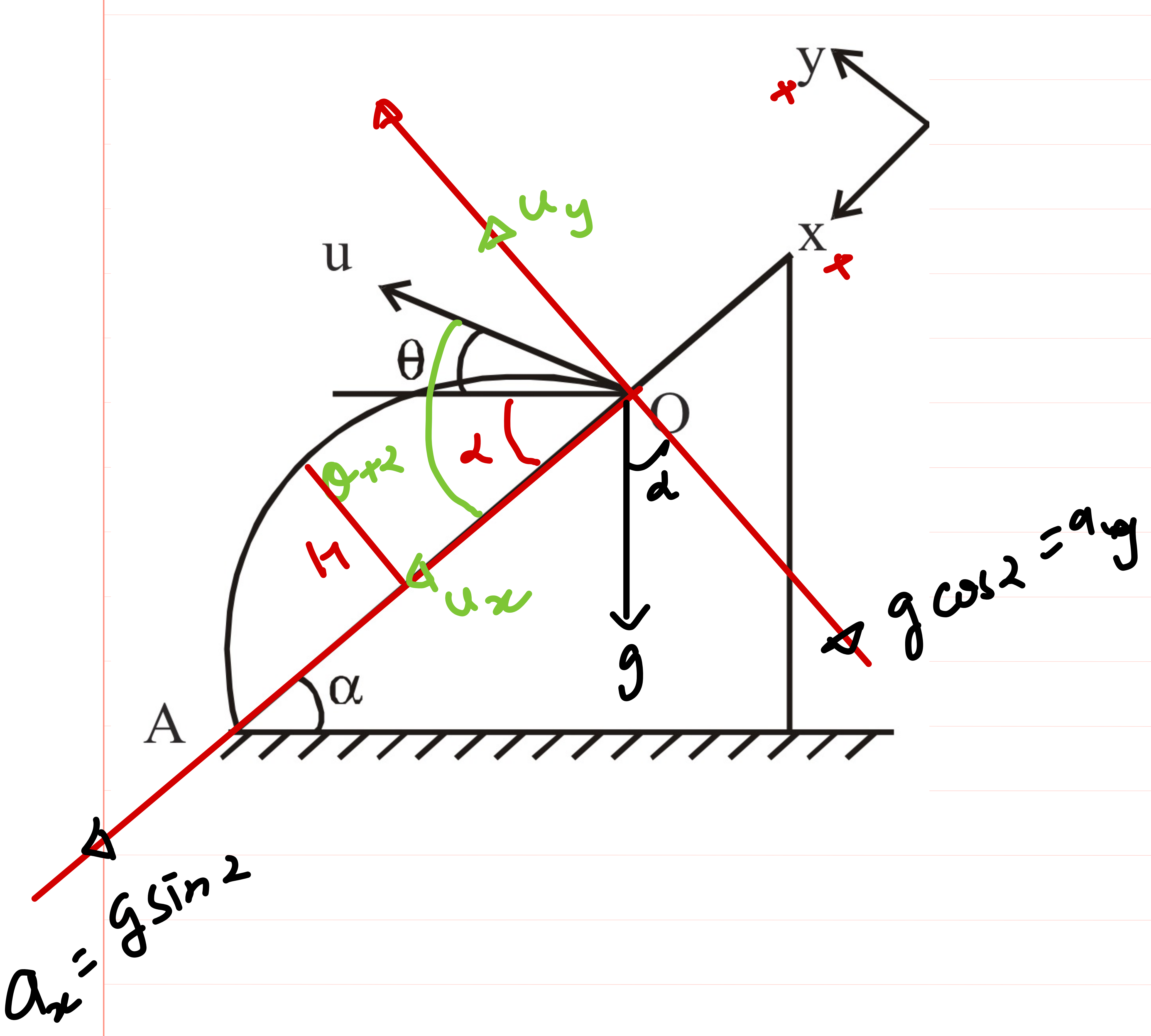


$$\cos\theta = \frac{3}{\sqrt{73}}$$

Case -2

Down the Plane

A projectile is projected down the plane from the point O with an initial velocity u at an angle θ with horizontal [Fig.]. The angle of inclination of plane with horizontal is α.



at $t=0$

Along x-axis

$u_x = u\cos(\theta+\alpha)$

$a_x = g\sin\alpha$

Along y-axis

$u_y = u\sin(\theta+\alpha)$

$a_y = -g\cos\alpha$

Time of Flight (T)

$$\boxed{T = \frac{2u_y}{a_y} = \frac{2u\sin(\theta+\alpha)}{g\cos\alpha}}$$

Maximum Height (H)

$$\boxed{H = \frac{u_y^2}{2a_y} = \frac{u^2\sin^2(\theta+\alpha)}{2g\cos\alpha}}$$

Range

$$x = u_x T + \frac{1}{2} a_x T^2$$

Yaad

$$\boxed{R = \frac{u^2}{g\cos^2\alpha}\left[\sin(2\theta+\alpha) + \sin\alpha\right]}$$

For maximum R

$$\sin(2\theta+\alpha) = 1$$

$$2\theta + \alpha = \frac{\pi}{2}$$

$$2\theta = \frac{\pi}{2} - \alpha$$

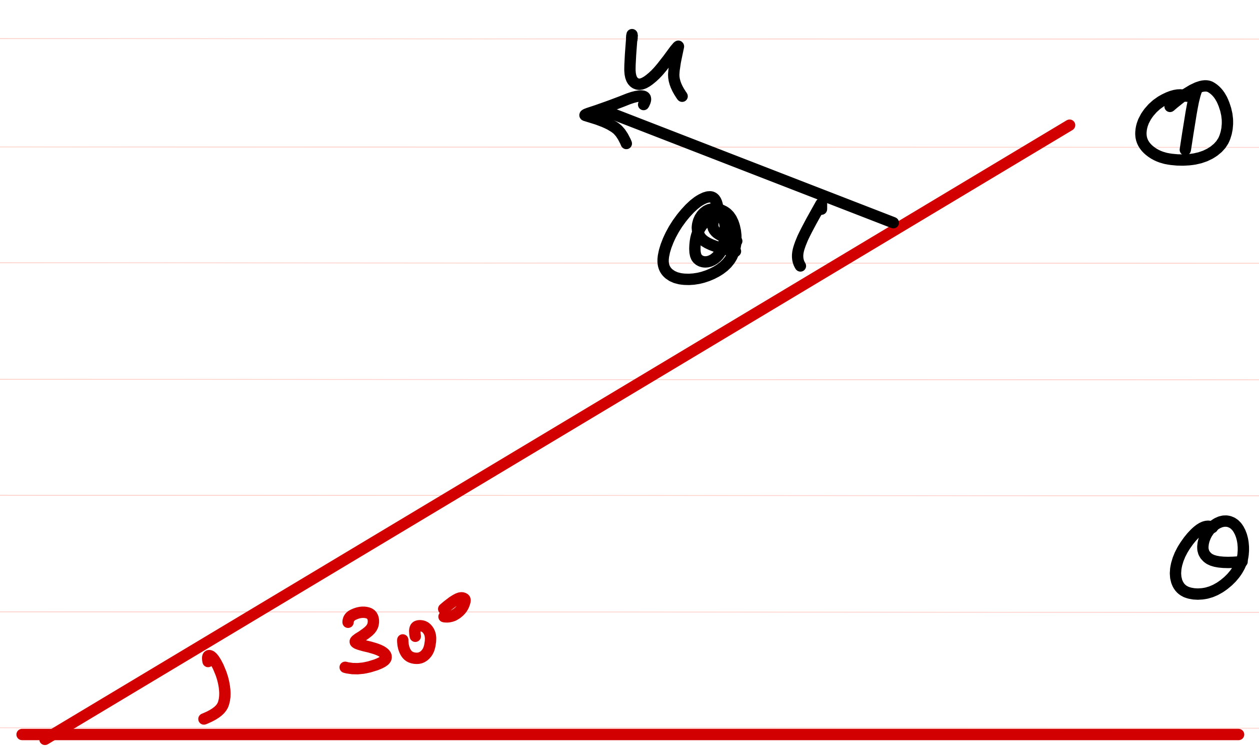
$$\boxed{\theta = \frac{\pi}{4} - \frac{\alpha}{2}}$$

Yaad

Yaad

$$\boxed{R_{max} = \frac{u^2}{g\cos^2\alpha}\left[1 + \sin\alpha\right]}$$

Ex



① For what θ R will be maximum

$$\theta = \frac{\pi}{4} - \frac{\alpha}{2}$$

$$= 45 - \frac{30}{2}$$

$$\boxed{\theta = 30°}$$

② What is R_{max}

$$R_{max} = \frac{u^2}{g\cos^2 30}\left[1 + \sin 30\right]$$

$$= \frac{u^2}{g\,\frac{3}{4}} \times \frac{3}{2} = \frac{2u^2}{g}$$

$$\boxed{R_{max} = \frac{2u^2}{g}}$$ Ans

8*. For a given velocity of projection from a point on the inclined plane, the maximum range down the plane is three times the maximum range up the incline. Then find the angle of inclination of the inclined plane.

$$(R_d)_{max} = 3(R_u)_{max}$$

$$\frac{\cancel{u^2}}{\cancel{g}\cos^2\alpha}(1+\sin\alpha) = 3\,\frac{\cancel{u^2}}{\cancel{g}\cos^2\alpha}[1-\sin\alpha]$$

$$1+\sin\alpha = 3-3\sin\alpha$$

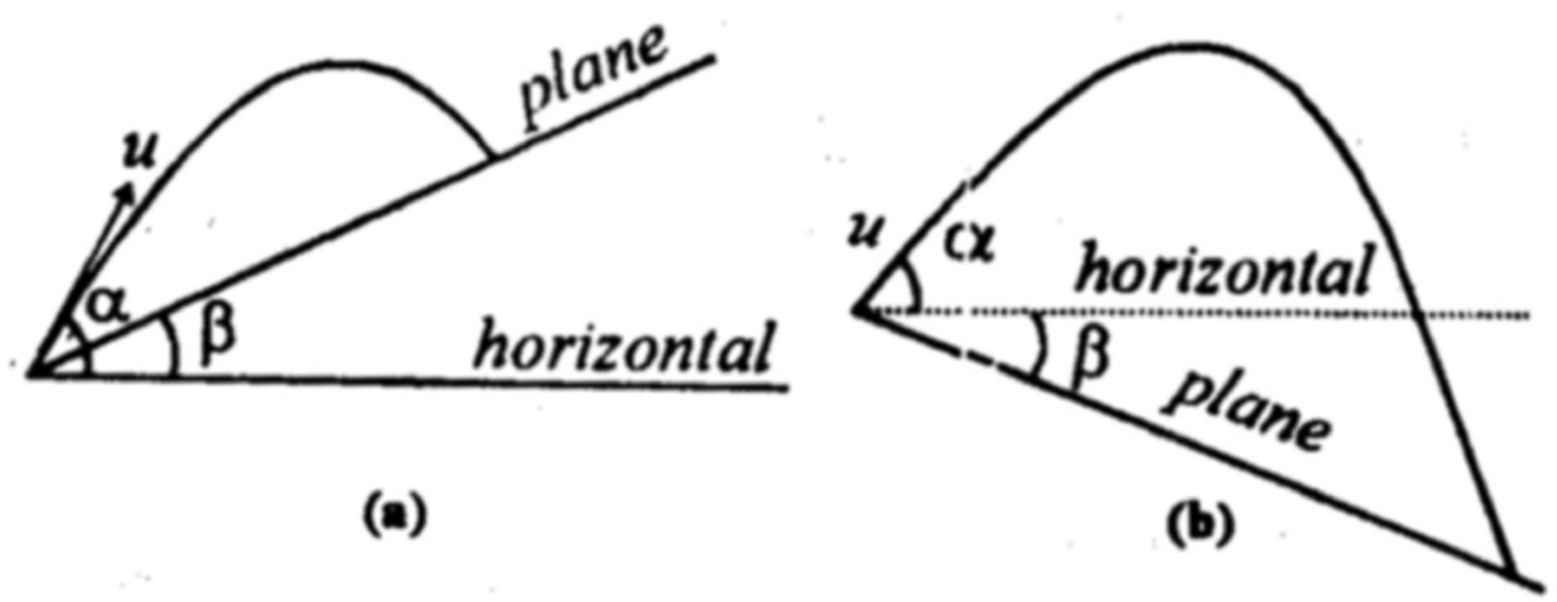
$$4\sin\alpha = 2$$

$$\sin\alpha = \frac{1}{2}$$

$$\boxed{\alpha = 30^\circ}\quad \underline{\underline{\text{Ans}}}$$

ALLEN CAREER INSTITUTE KOTA (RAJASTHAN)

9. A particle of projected at an angle 'α' to the horizon, up and down is a plane, inclined at an angle β to the horizontal.



If the ratio of time flights be 1 : 2, then the ratio $\dfrac{\tan\alpha}{\tan\beta}$ is equal to

(A) $\dfrac{2}{1}$ ~~(B)~~ $\dfrac{3}{1}$ (C) $\dfrac{4}{1}$ (D) $\dfrac{5}{3}$

$$\dfrac{t_u = \dfrac{2u\sin(\alpha-\beta)}{g\cos\alpha}}{t_d = \dfrac{2u\sin(\alpha+\beta)}{g\cos\alpha}} \Rightarrow \frac{\sin(\alpha-\beta)}{\sin(\alpha+\beta)} = \frac{1}{2}$$

$$2(\sin\alpha\cos\beta - \cos\alpha\sin\beta) = (\sin\alpha\cos\beta + \cos\alpha\sin\beta)$$

diving Both sides by $\cos\alpha\cos\beta$

$$2\left(\frac{\sin\alpha\cancel{\cos\beta}}{\cos\alpha\cancel{\cos\beta}} - \frac{\cancel{\cos\alpha}\sin\beta}{\cancel{\cos\alpha}\cos\beta}\right) = \frac{\sin\alpha\cancel{\cos\beta}}{\cos\alpha\cancel{\cos\beta}} + \frac{\cancel{\cos\alpha}\sin\beta}{\cancel{\cos\alpha}\cos\beta}$$

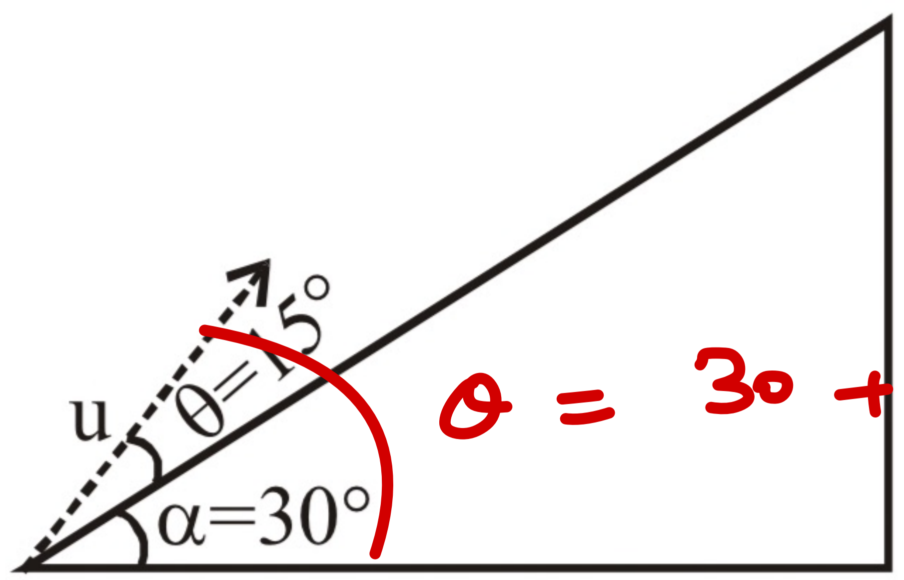
$$2(\tan\alpha - \tan\beta) = \tan\alpha + \tan\beta$$

$$2\tan\alpha - \tan\alpha = 2\tan\beta + \tan\beta$$

$$\boxed{\tan\alpha = 3\tan\beta}$$

11. A plane is inclined at an angle $\alpha = 30°$ with a respect to the horizontal. A particle is projected with a speed $u = 2\ ms^{-1}$ from the base of the plane, making an angle $\theta = 15°$ with respect to the plane as shown in the figure. The distance from the base, at which the particle hits the plane is close to : (Take $g = 10\ ms^{-2}$)

[JEE(Main) 2019]



$\theta = 30 + 15 = 45°$

(A) 14 cm (B) 20 cm (C) 18 cm (D) 26 cm

$$R = \frac{u^2}{g\cos^2\alpha}\left[\sin(2\theta - \alpha) - \sin\alpha\right]$$

$$= \frac{2^2}{10(\cos 30)^2}\left(\sin(90 - 30) - \sin 30\right)$$

$$= \frac{4}{10 \times 3} \times 4\left[\frac{\sqrt{3}}{2} - \frac{1}{2}\right]$$

$$R = \frac{16}{30} \times \frac{\sqrt{3}-1}{2}$$

$$= \frac{8}{30} \times (1.73 - 1)$$

$$= \frac{8}{30} \times 0.73\ m$$

$$R = \frac{8 \times 73}{30}$$

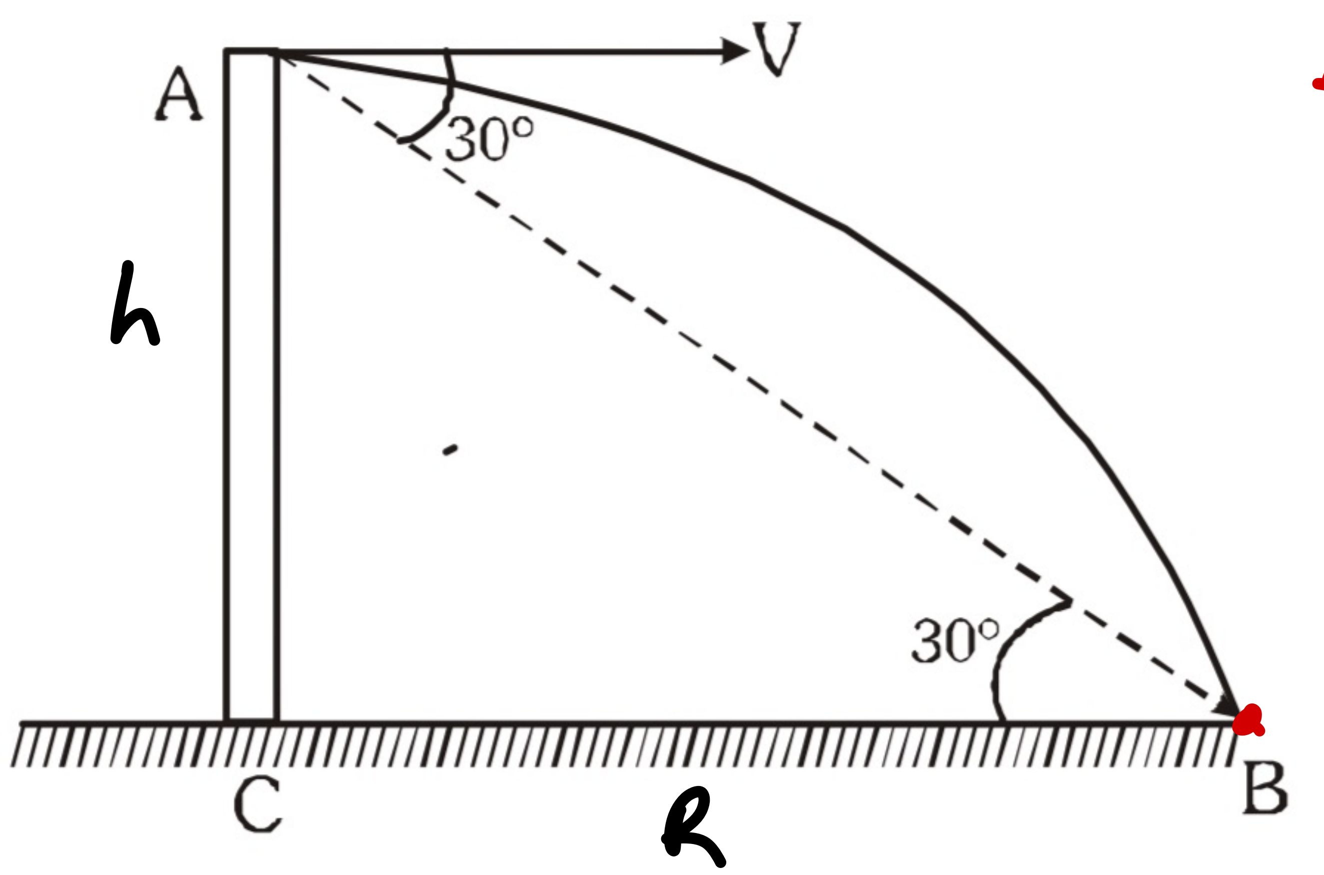
$$\boxed{R \simeq 20\,cm}$$

Ans

5. An object is thrown horizontally from a point 'A' from a tower and hits the ground 3s later at B. The line from 'A' to 'B' makes an angle of 30° with the horizontal. The initial velocity of the object is : (take $g = 10\ m/s^2$)

$t = 3 = \sqrt{\frac{2h}{g}}$

$h = \frac{9g}{2} = 45m$



ΔACB

$\tan 30 = \frac{h}{R}$

$\frac{1}{\sqrt{3}} = \frac{h}{R} \Rightarrow \boxed{R = 45\sqrt{3}}$

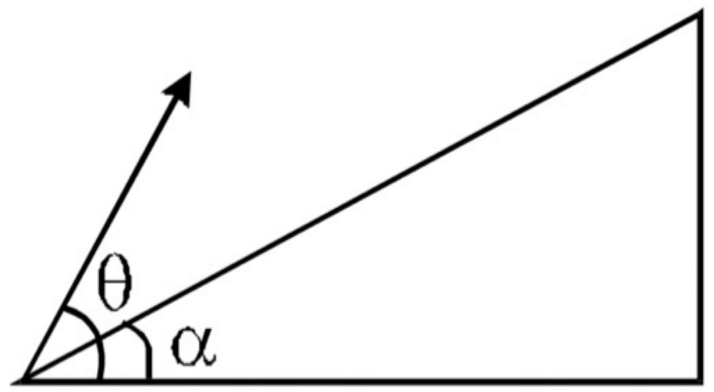
$V \times t = 45\sqrt{3}$

(A) $15\sqrt{3}$ m/s (B) 15 m/s (C) $10\sqrt{3}$ m/s (D) $25/\sqrt{3}$ m/s

$V \times 3 = 45\sqrt{3}$

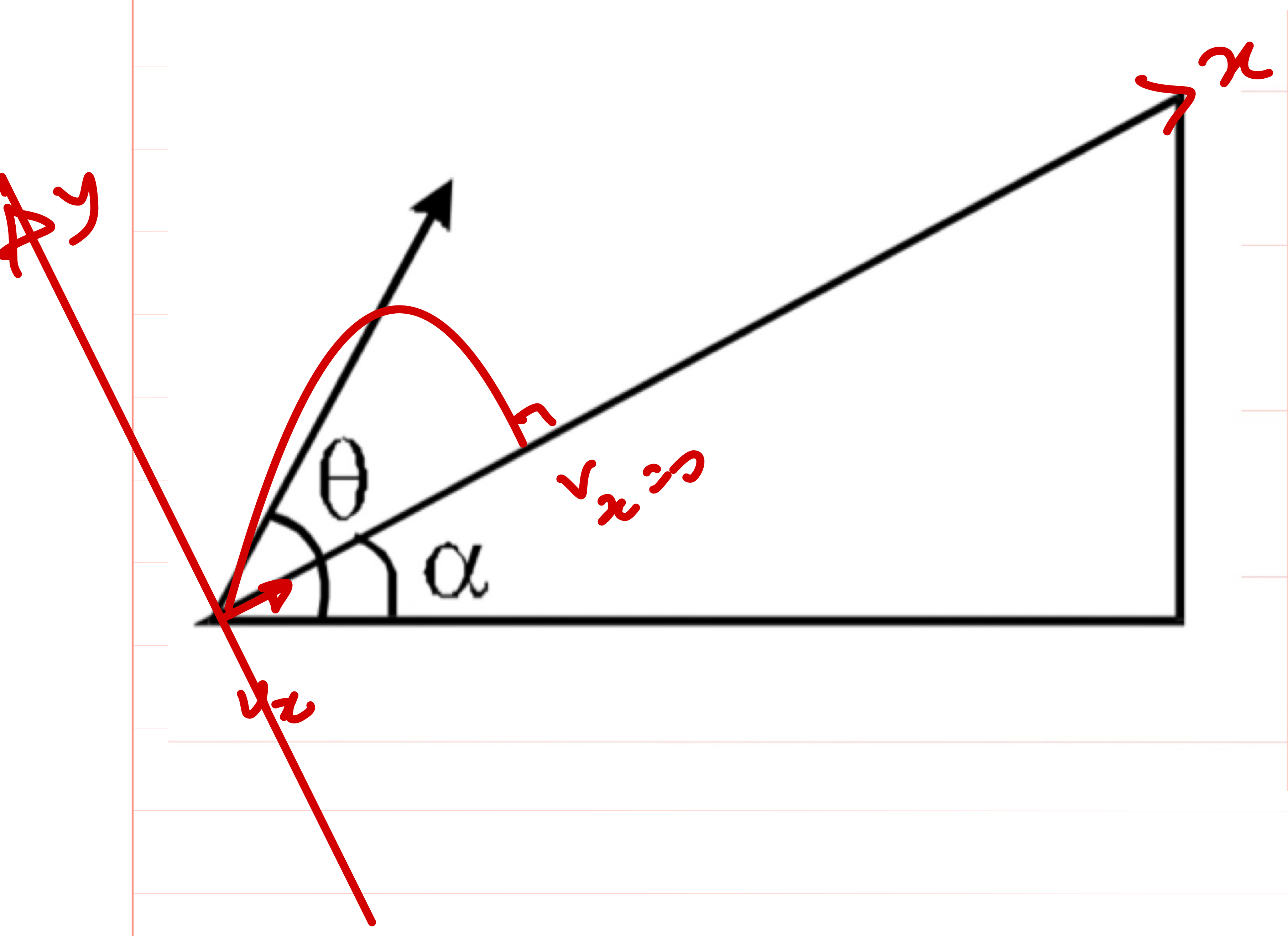
$\boxed{V = 15\sqrt{3}\ m/s}$

21. A projectile is fired at an angle θ with the horizontal. Find the condition under which it lands perpendicular on an inclined plane inclination α as shown in figure.



(A) $\sin\alpha = \cos(\theta-\alpha)$ (B) $\cos\alpha = \sin(\theta-\alpha)$

(C) $\tan\theta = \cot(\theta-\alpha)$ (D) $\cot(\theta-\alpha) = 2\tan\alpha$



$$T = \frac{2u\sin(\theta-\alpha)}{g\cos\alpha}$$

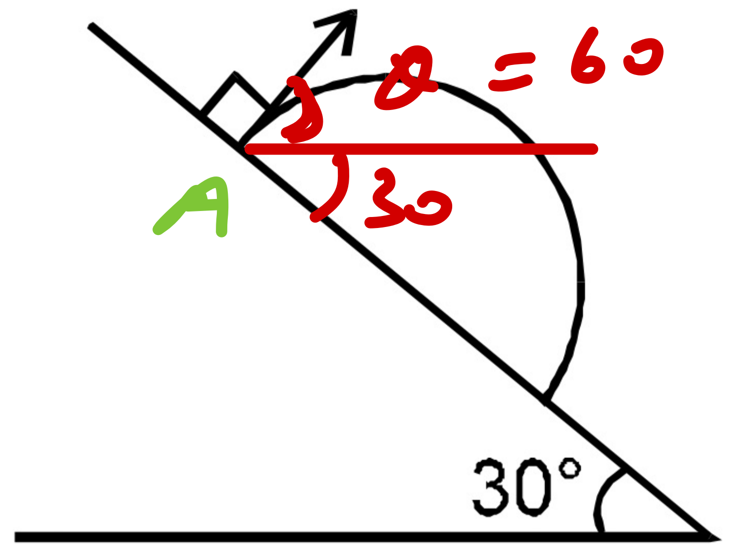
$$V_x = u_x + a_x t$$

$$0 = u\cos(\theta-\alpha) - g\sin\alpha\, T$$

$$\cancel{u}\cos(\theta-\alpha) = \cancel{g}\sin\alpha \times \frac{2\cancel{u}\sin(\theta-\alpha)}{\cancel{g}\cos\alpha}$$

$$\frac{\cos(\theta-\alpha)}{\sin(\theta-\alpha)} = 2\tan\alpha \Rightarrow \boxed{\cot(\theta-\alpha) = 2\tan\alpha}$$

3*. A ball is projected from point A with a velocity 10 m/s perpendicular to the inclined plane as shown in figure. Range of the ball on the inclined plane is :



(A) $\frac{40}{3}$ m ✓
(B) $\frac{20}{13}$ m
(C) $\frac{13}{20}$ m
(D) $\frac{13}{40}$ m

$$\boxed{\theta = 60^\circ \qquad \alpha = 30}$$

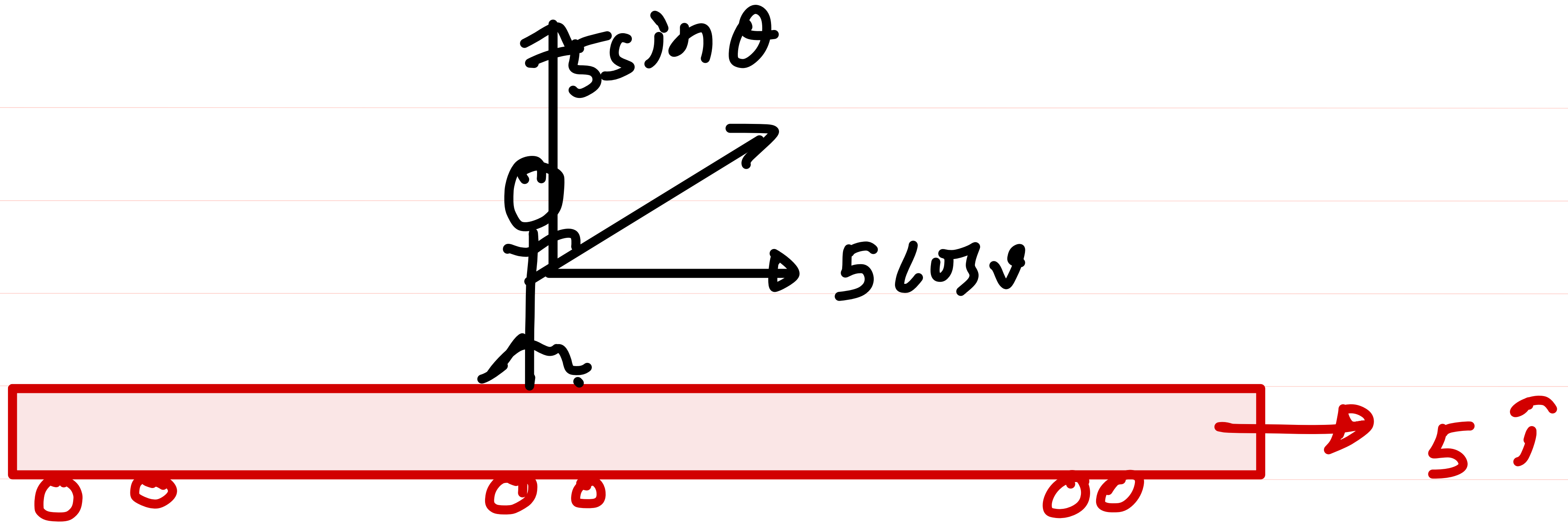
$$R_d = \frac{u^2}{g\cos^2\alpha}\left[\sin(2\theta+\alpha) + \sin\alpha\right]$$

$$= \frac{100 \times 4}{10 \times 3}\left[\sin(120+30) + \sin 30\right]$$

$$= \frac{40}{3}\left[\sin(90+60) + \frac{1}{2}\right] = \frac{40}{3}\,m$$

5. A person is standing on a trolley which moves horizontally with uniform velocity $5\hat{i}$ m/s. At t=0, person throws a ball with velocity $\left(5\cos\theta\hat{i} + 5\sin\theta\hat{j}\right)$ m/s wrt himself. He always sees ball overhead. Then

(A) $\theta = \frac{\pi}{2}$ (B) $\theta = \frac{\pi}{4}$ (C) $\theta = \pi$ (D) $\theta = \frac{\pi}{3}$



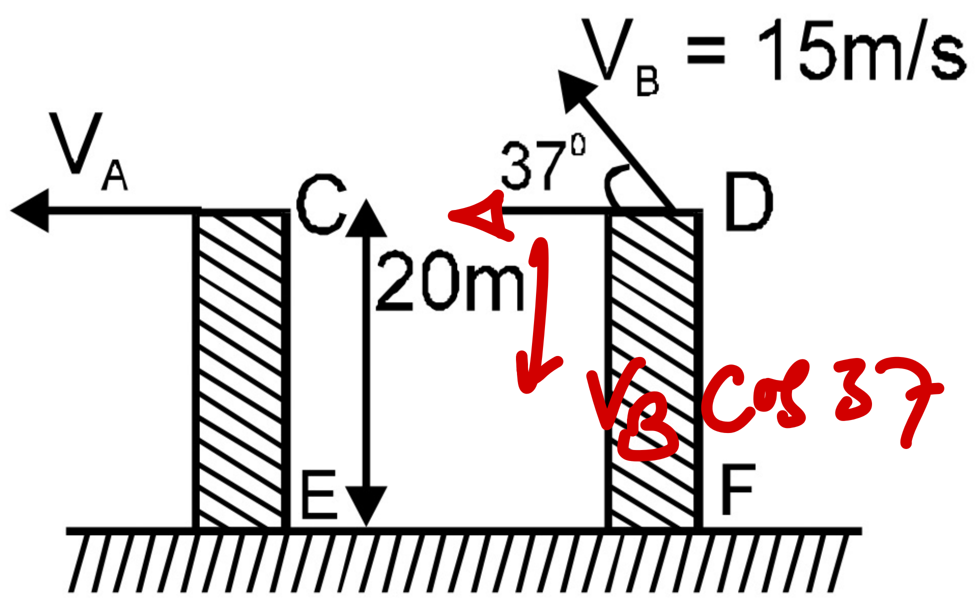
For see the ball overhead

$5\cos\theta = 0$

$\cos\theta = 0$

$\boxed{\theta = 90 \text{ or } \frac{\pi}{2}}$

13. CE and DF are two walls of equal height (20 meter) from which two particles A and B of same mass are projected as shown in the figure. A is projected horizontally towards left while B is projected at an angle 37^0 (with horizontal towards left) with velocity 15 m/sec. If A always sees B to be moving perpendicular to EF, then the range of A on ground is :



(A) 24 m (B) 30 m
(C) 26 m (D) 28 m

$V_A = V_B \cos 37 = 15 \times \frac{4}{5} = 12\ m/s$

$R = V_A \sqrt{\frac{2h}{g}}$

$= 12\sqrt{\frac{2\times 20}{10}} = 24m$

H.W

B.B # 3

Ex – 1
– 2
– 3