Ex-1 The vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}+\vec{b}|=|\vec{a}-\vec{b}|$, then the angle between $\vec{a}$ and $\vec{b}$ will be :–

(1) $\frac{\pi}{3}$ (2) π (3) $\frac{\pi}{2}$ (4) zero

① $|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$

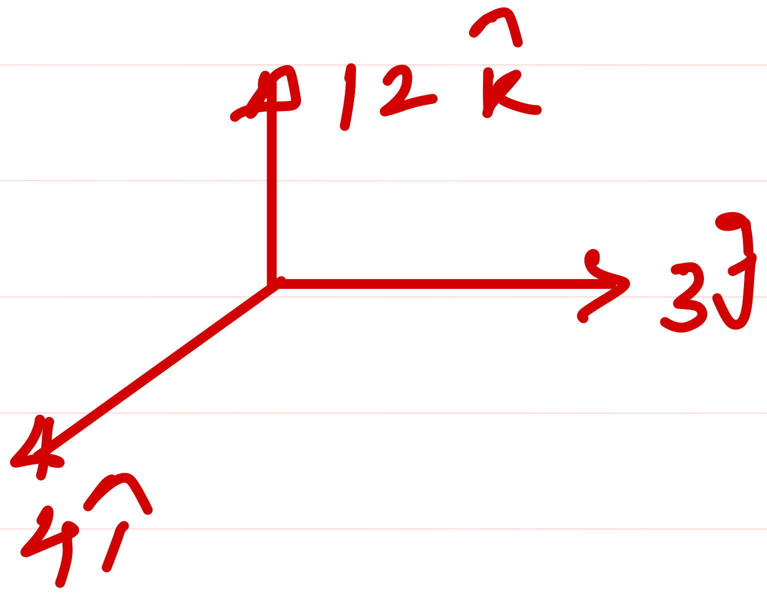
$\cancel{a^2+b^2} + 2ab\cos\theta = \cancel{a^2+b^2} - 2ab\cos\theta$

$4ab\cos\theta = 0 \Rightarrow \cos\theta = 0 \Rightarrow \theta = 90^\circ / \frac{\pi}{2}$

Ex-2 Force 3N, 4N and 12N act at a point in mutually perpendicular directions. The magnitude of the resultant force is :–

(1) 19 N (2) 13 N (3) 11 N (4) 5 N

②



$\vec{R} = 4\hat{i} + 3\hat{j} + 12\hat{k}$

$R = \sqrt{4^2 + 3^2 + 12^2} = 13\,N$

Ex-3 If $|\vec{A}+\vec{B}|=|\vec{A}|=|\vec{B}|$ then angle between A and B will be :–

(1) 90° (2) 120° (3) 0° (4) 60°

③ $R = \sqrt{A^2+B^2+2AB\cos\theta}$

$\cancel{A^2} = \cancel{A^2} + A^2 + 2A^2\cos\theta$

$0 = A^2 + 2A^2\cos\theta$

$\cos\theta = -\frac{A^2}{2A^2} = -\frac{1}{2}$

$\boxed{\theta = 120^\circ}$

Ex-4 The direction cosines of a vector $\hat{i}+\hat{j}+\sqrt{2}\hat{k}$ are :-

$\vec{A} =$

(1) $\frac{1}{2},\frac{1}{2},1$ (2) $\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}},\frac{1}{2}$

(3) $\frac{1}{2},\frac{1}{2},\frac{1}{\sqrt{2}}$ (4) $\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}$

$\cos\alpha = \frac{A_x}{A} = \frac{1}{2} \Rightarrow \alpha = 60^\circ$

$\cos\beta = \frac{A_y}{A} = \frac{1}{2} \Rightarrow \beta = 60^\circ$

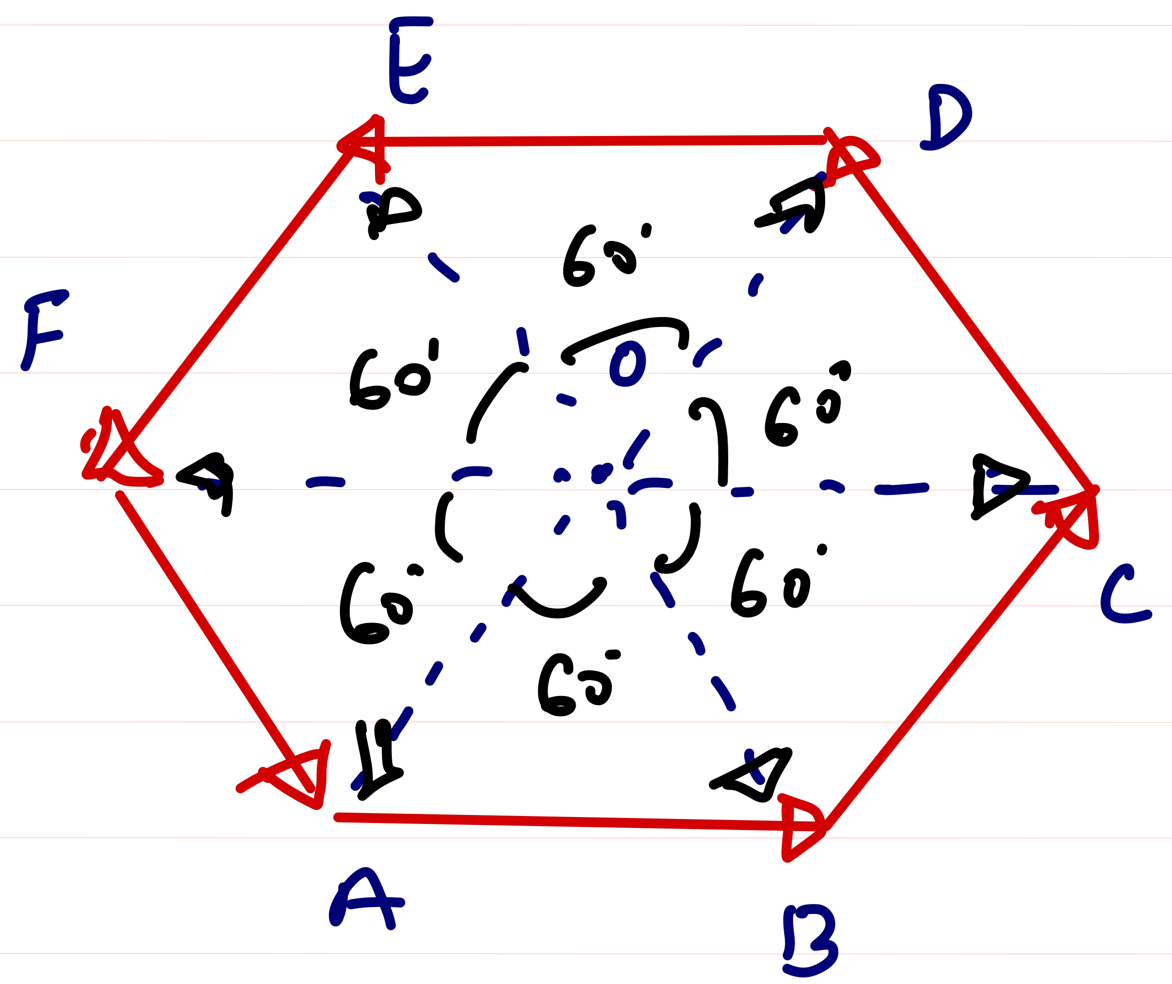
$\cos\gamma = \frac{A_z}{A} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}} \Rightarrow \gamma = 45^\circ$

Ex

ABCDEF is a regular hexagon. The centre of hexagon is a point O. Then the value of

$\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} + \overrightarrow{AE} + \overrightarrow{AF}$ is

(a) $2\overrightarrow{AO}$ (b) $4\overrightarrow{AO}$ (c) $6\overrightarrow{AO}$ (d) Zero



$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$, $\overrightarrow{AD} = \overrightarrow{OD} - \overrightarrow{OA}$, $\overrightarrow{AF} = \overrightarrow{OF} - \overrightarrow{OA}$

$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA}$, $\overrightarrow{AE} = \overrightarrow{OE} - \overrightarrow{OA}$

$\because \overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} + \overrightarrow{OD} + \overrightarrow{OE} + \overrightarrow{OF} = 0$

$(\overrightarrow{OB} - \overrightarrow{OA}) + (\overrightarrow{OC} - \overrightarrow{OA}) + (\overrightarrow{OD} - \overrightarrow{OA}) + (\overrightarrow{OE} - \overrightarrow{OA}) + (\overrightarrow{OF} - \overrightarrow{OA})$

$= \overrightarrow{OB} + \overrightarrow{OC} + \overrightarrow{OD} + \overrightarrow{OE} + \overrightarrow{OF} - 5\overrightarrow{OA}$

$= -6\overrightarrow{OA}$

$= +6\overrightarrow{AO}$

Ex

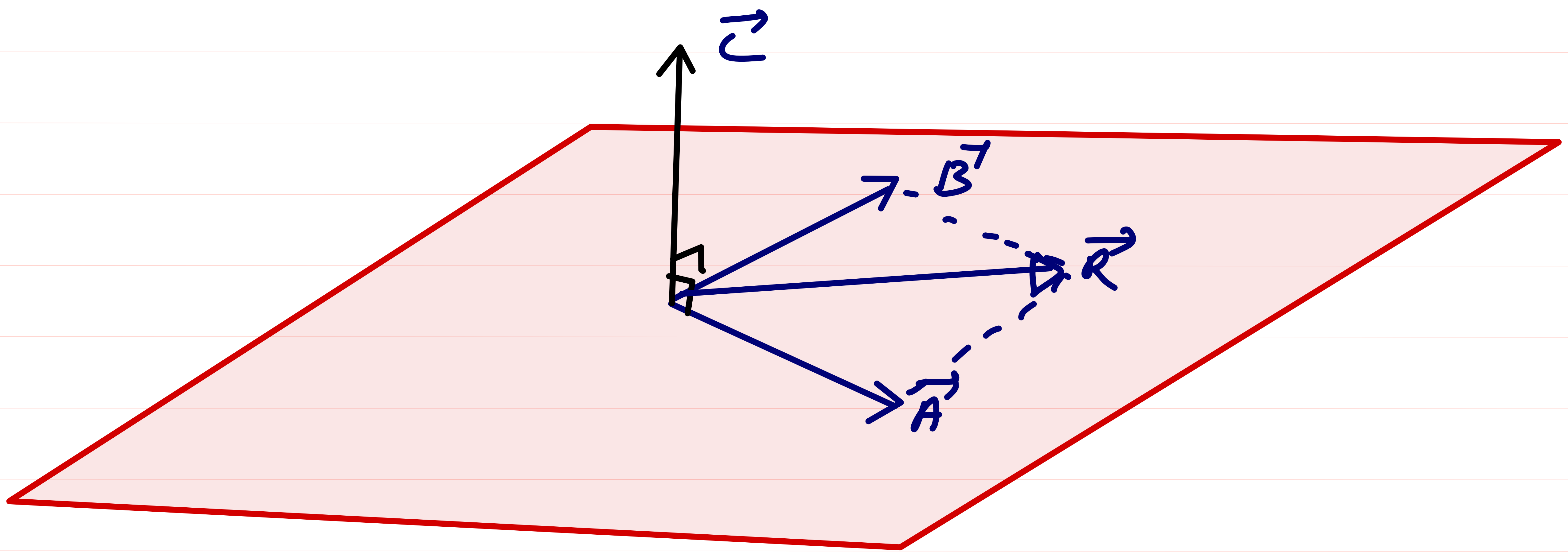
The vectors $\vec{A}$ and $\vec{B}$ lie in a plane. Another vector $\vec{C}$ lies outside this plane. The resultant $\vec{A}+\vec{B}+\vec{C}$ of these three vectors :

(A) Can be zero

(B) Cannot be zero

(C) Lies in the plane of $\vec{A}$ and $\vec{B}$

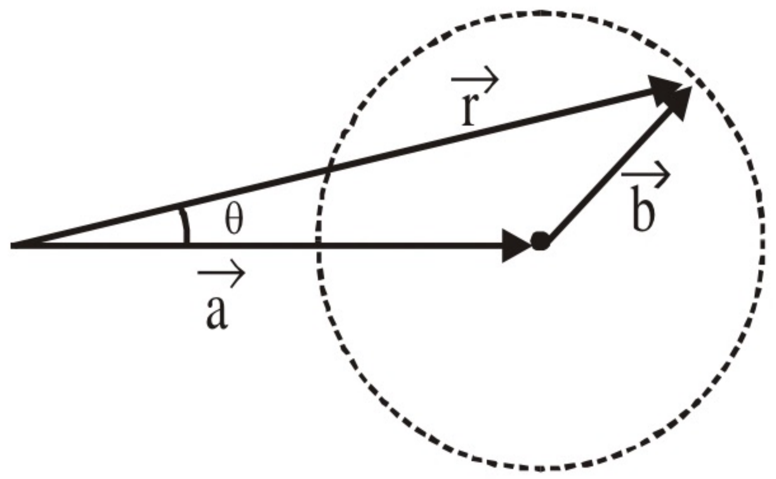
(D) Lies in the plane of $\vec{A}$ and $\vec{A}+\vec{B}$



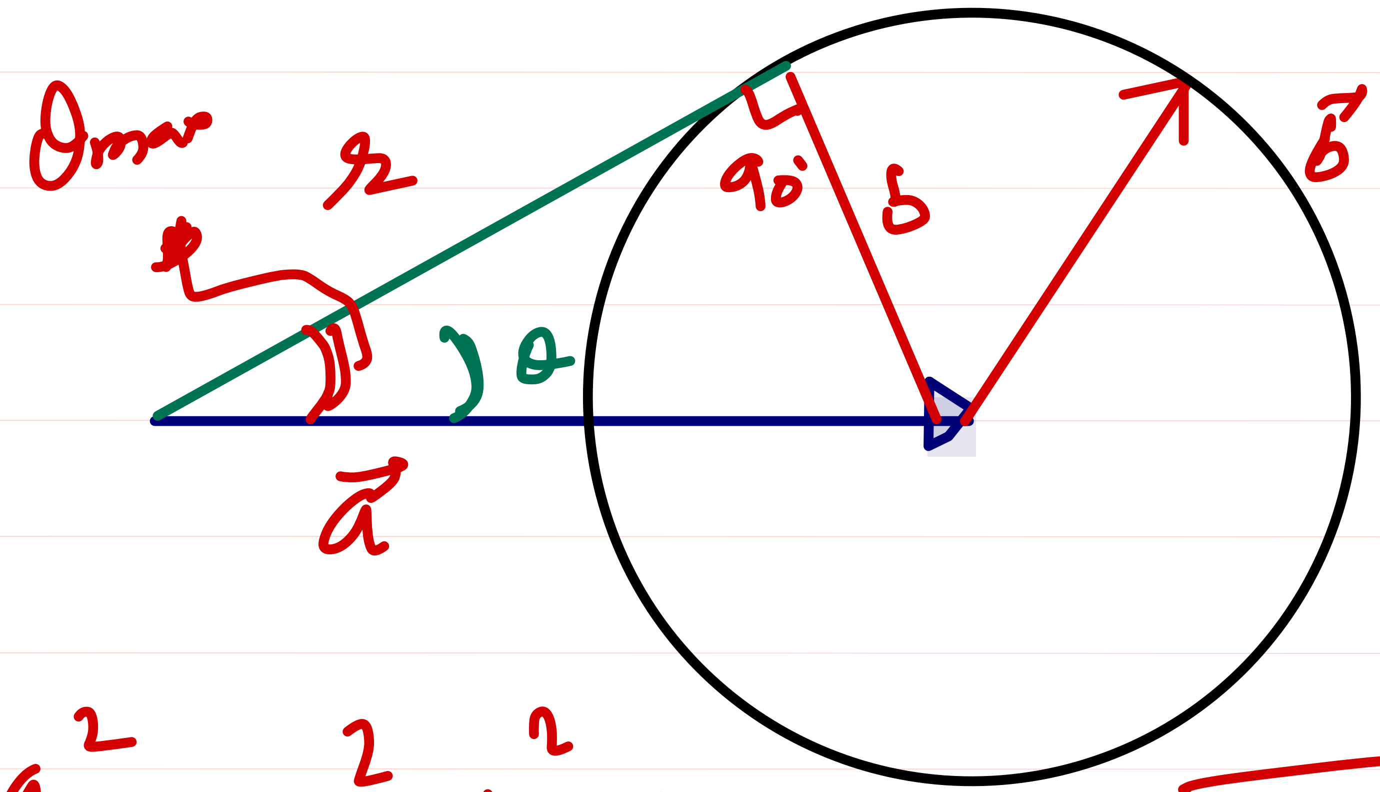
$\vec{R} = \vec{A}+\vec{B}+\vec{C}$

cannot be zero

11. Keeping one vector constant, if direction of other to be added in the first vector is changed continuously, tip of the resultant vector describes a circle. In the following figure vector $\vec{a}$ is kept constant. When vector $\vec{b}$ added to $\vec{a}$ changes its direction, the tip of the resultant vector $\vec{r} = \vec{a} + \vec{b}$ describes circle of radius b with its center at the tip of vector $\vec{a}$. Maximum angle between vector $\vec{a}$ and the resultant $\vec{r} = \vec{a} + \vec{b}$ is



(A) $\tan^{-1}\left(\frac{b}{r}\right)$ (B) $\tan^{-1}\left(\frac{b}{\sqrt{a^2 - b^2}}\right)$ (C) $\cos^{-1}(r/a)$ (D) $\cos^{-1}(a/r)$



$\cos\theta = \frac{r}{a} \Rightarrow \theta = \cos^{-1}(r/a)$

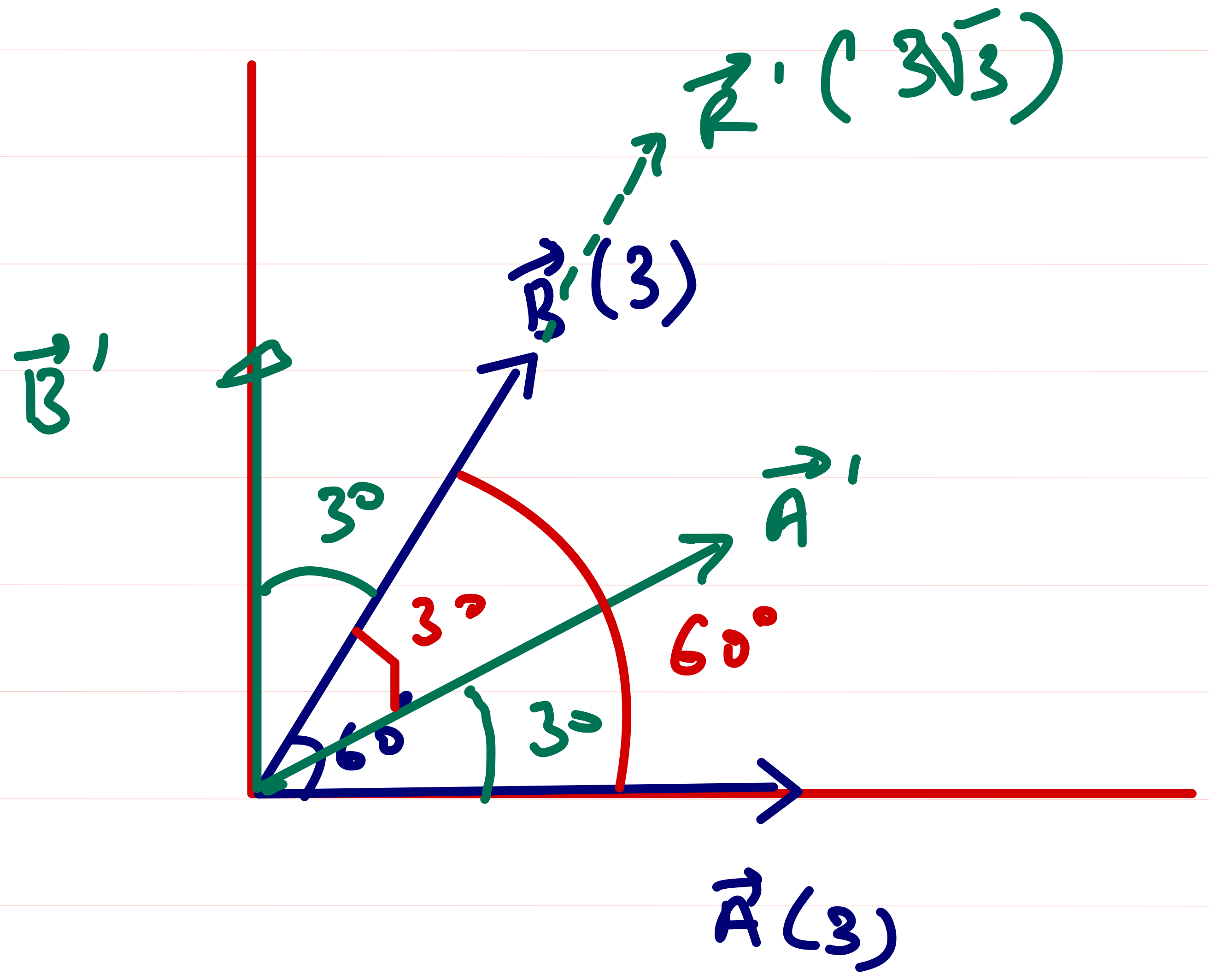
$\sin\theta = b/a \Rightarrow \theta = \sin^{-1}(b/a)$

$\tan\theta = b/r \Rightarrow \theta = \tan^{-1}(b/r)$

$a^2 = r^2 + b^2 \Rightarrow r = \sqrt{a^2 - b^2}$

9. Two vectors in the x–y plane of magnitude 3 units each make angle of 60° between them, where one is along x-axis. If the vectors are rotated by 30° each in same direction the x-component of their resultant will be

(A) $2\sqrt{3}$ units (B) $\frac{3\sqrt{3}}{2}$ units (C) $3\sqrt{3}$ units (D) 6 units



$R = 2A\cos(\theta/2)$

$= 2 \times 3 \cos(60/2)$

$= 2 \times 3 \frac{\sqrt{3}}{2}$

$R = 3\sqrt{3}$

$R'_x = R' \cos 60$

$= 3\sqrt{3} \times \frac{1}{2} = \frac{3\sqrt{3}}{2}$ units Ans

Paragraph for Question no. 16 to 19

If two vectors are represented by two adjacent sides of a parallelogram which are directed away from their common point then their sum (i.e. resultant vector) is given by the diagonal of the parallelogram passing away through that common point. On the basis of above theory, answer the following questions.

16. If two vectors of magnitude of 5 and 3 are added such that angle between resultant and vector of magnitude 5 is maximum and it will be

(A) 37° (B) 53° (C) 90° (D) 180°

17. If two vectors of magnitude of 5 and 3 are added such that angle between resultant and vector of magnitude 3 is maximum and it will be

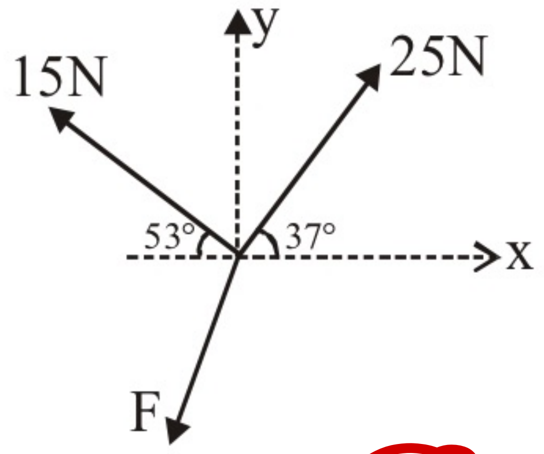
(A) 37° (B) 53° (C) 90° (D) 180°

18. A vector $\vec{A}$ of unknown magnitude makes 127° or 37° with another vector of magnitude 5. What is the minimum possible magnitude of resultant vector?

(A) 3 or 5 (B) 4 or 5 (C) 0 or 3 (D) Data insufficient

19. Three forces are acting on an object shown in diagram.

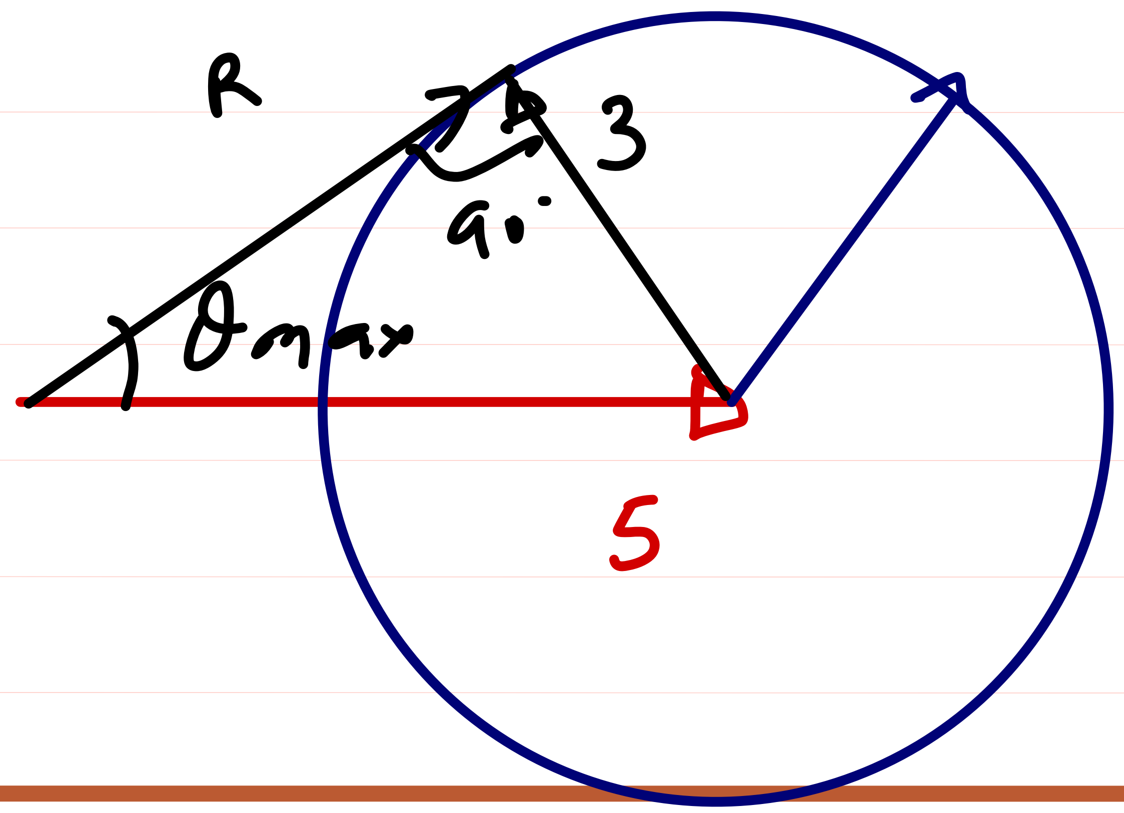
Their resultant is zero. The $\vec{F}$ is :-



(A) $-11\hat{i}-27\hat{j}$ (B) $-20\hat{i}-27\hat{j}$

(C) $11\hat{i}-3\hat{j}$ (D) $20\hat{i}-3\hat{j}$

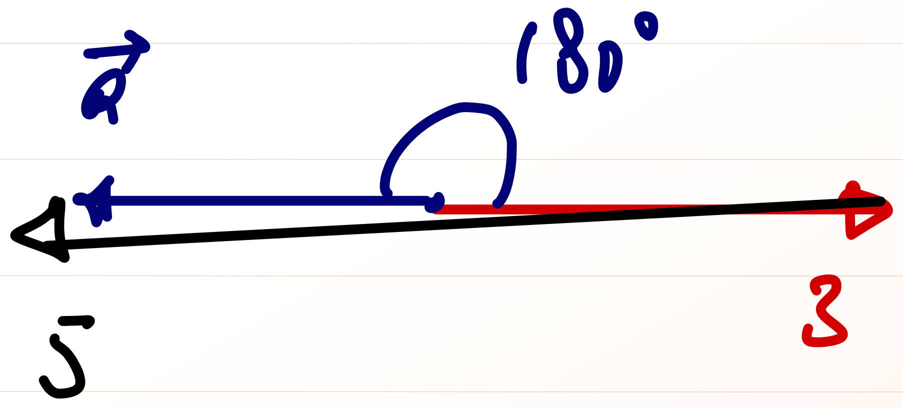
16



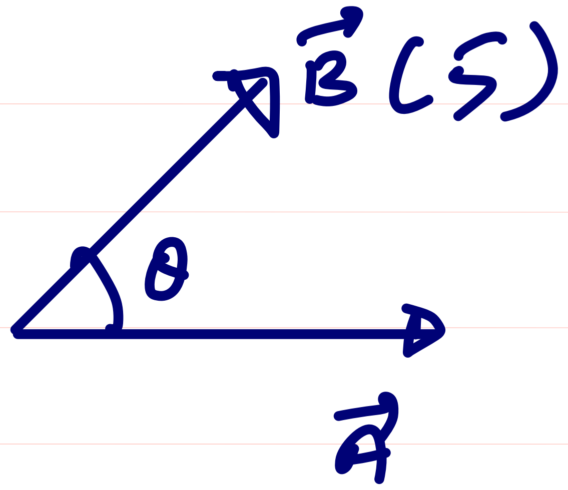
$\sin\theta_m = \frac{3}{5}$

$\theta_{max} = 37°$

17



18



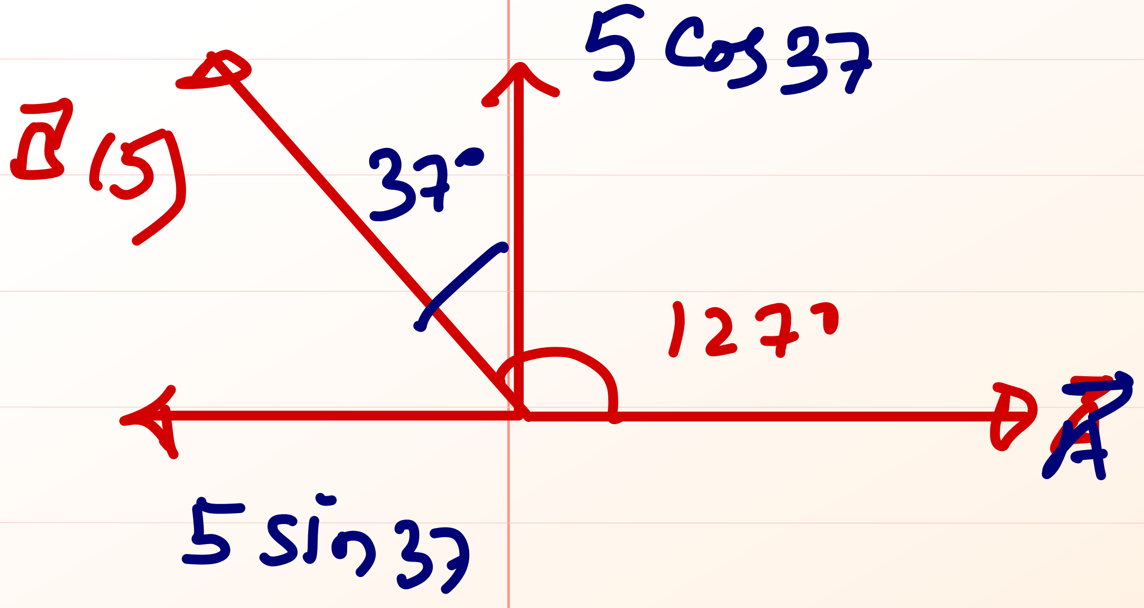
$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

If $\theta \le 90°$

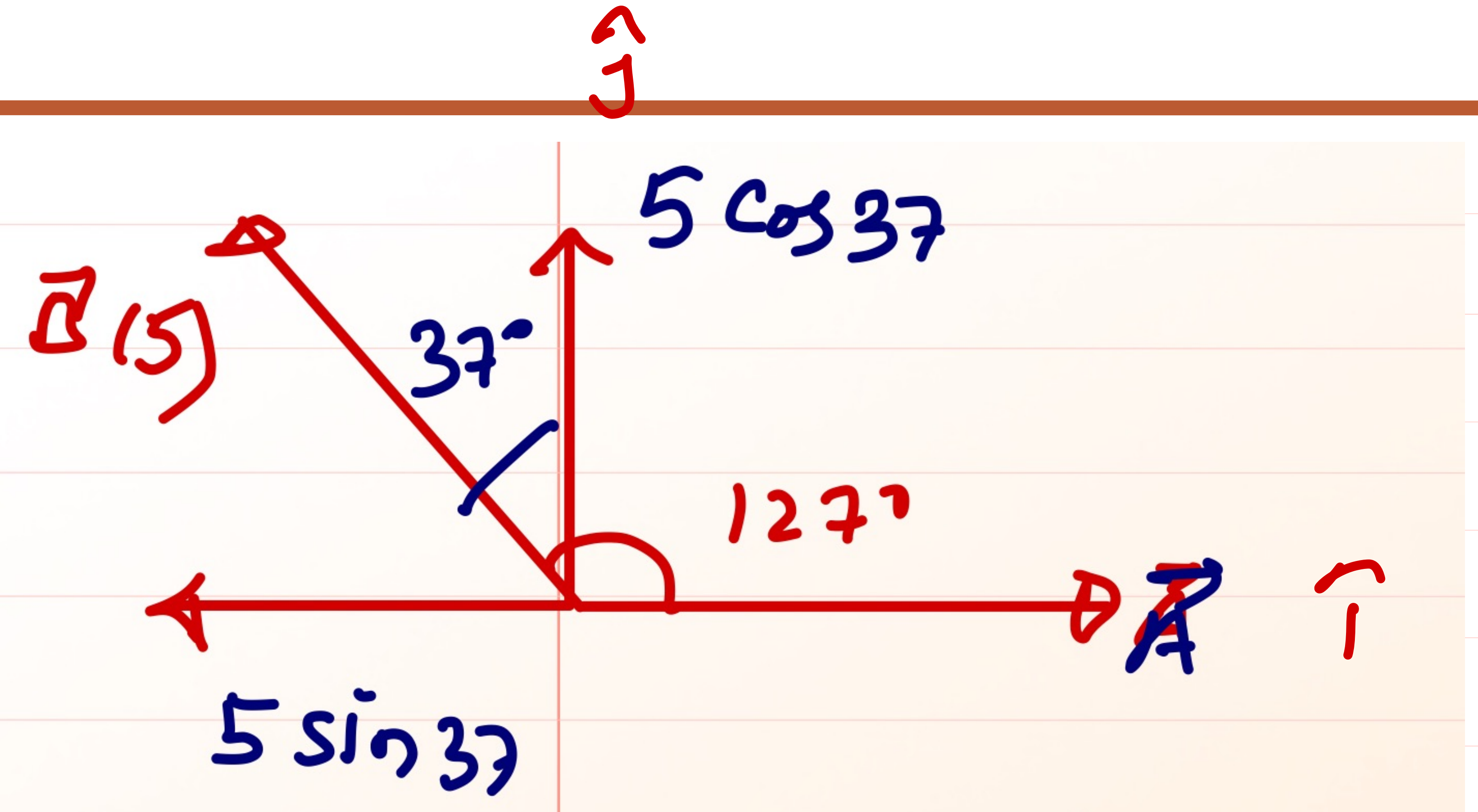
For R_{min} $A_{min} = 0$

$R_{min} = B = 5$

If $\theta > 90°$



$\vec{R} = (A - 5\sin 37)\hat{i} + 5\cos 37\hat{j}$



$\vec{R} = (A - 5\sin 37)\hat{i} + 5\cos 37\,\hat{j}$

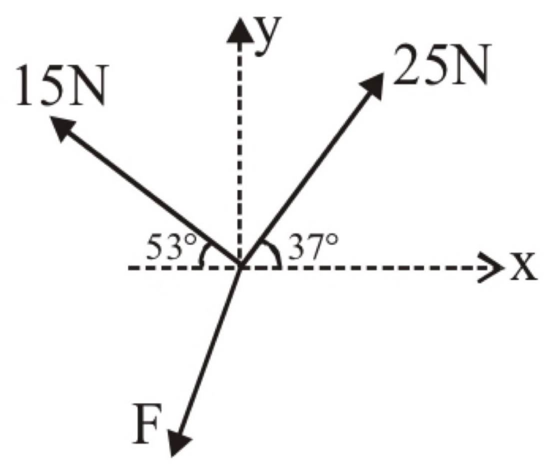
$R = \sqrt{(A - 5\sin 37)^2 + (5\cos 37)^2}$

For R_{min} $\quad A - 5\sin 37 = 0$

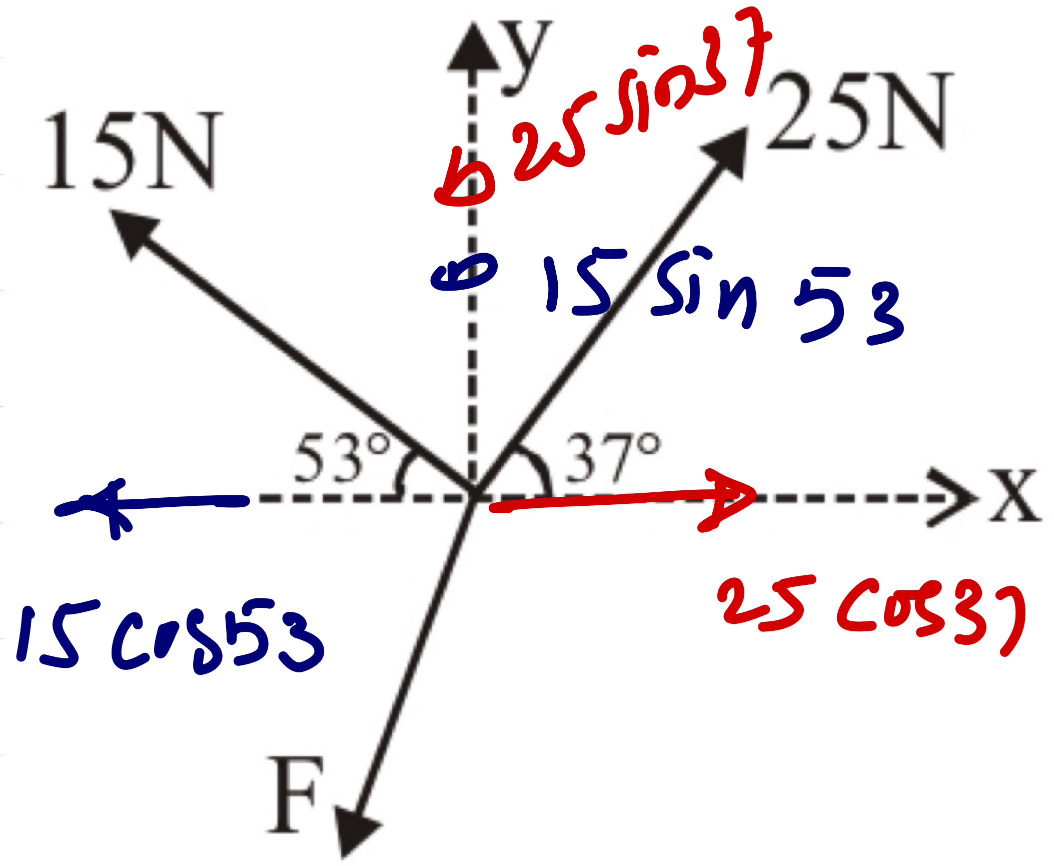
$R_{min} = 5\cos 37$

$= 5 \times \frac{4}{5} = 4$

19. Three forces are acting on an object shown in diagram. Their resultant is zero. The $\vec{F}$ is :-



(A) $-11\hat{i} - 27\hat{j}$ ✓
(B) $-20\hat{i} - 27\hat{j}$
(C) $11\hat{i} - 3\hat{j}$
(D) $20\hat{i} - 3\hat{j}$



$\vec{F} + (25\cos 37 - 15\cos 53)\hat{i} + (15\sin 53 + 25\sin 37)\hat{j} = 0$

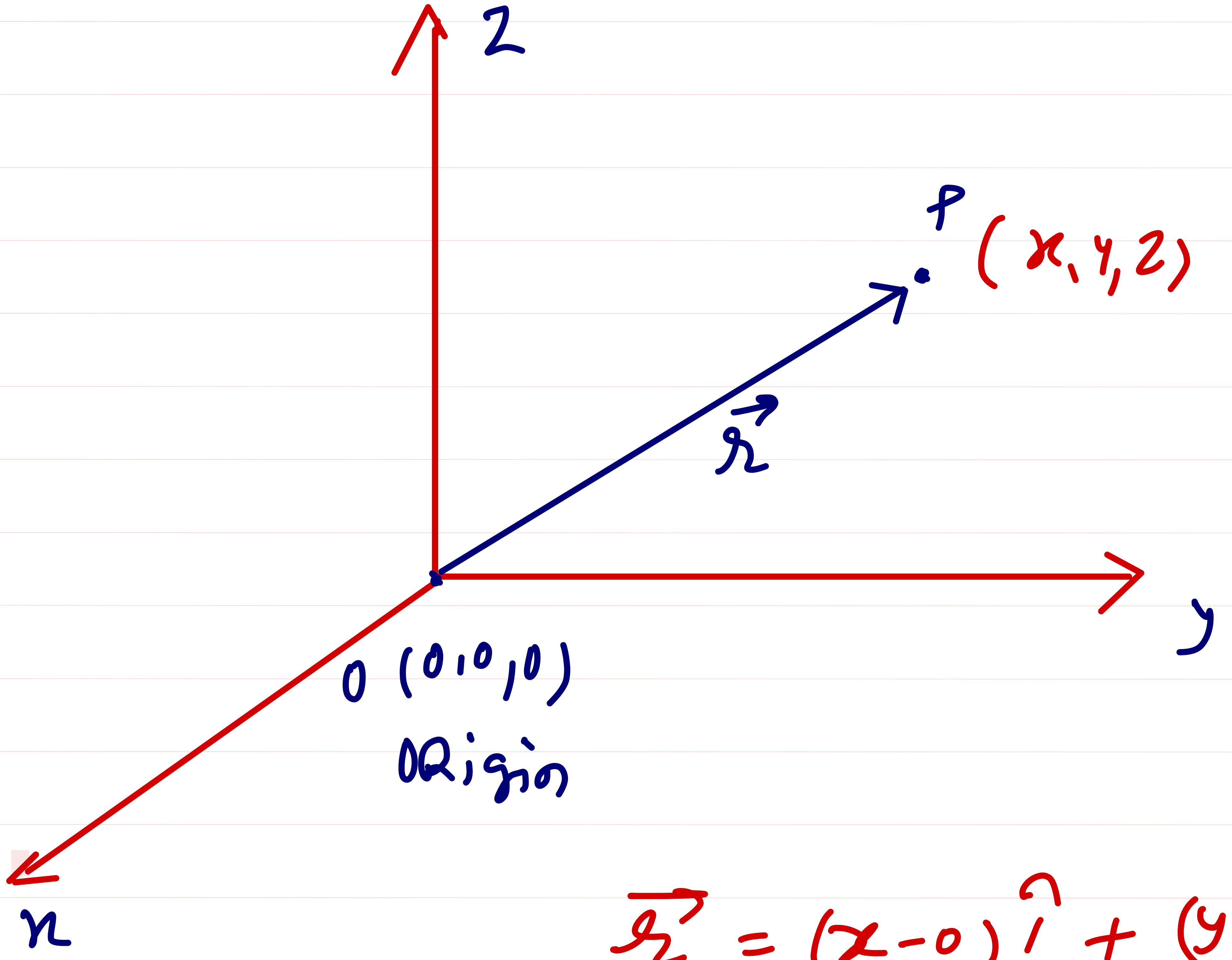
$\vec{F} + (25 \times 4/5 - 15 \times 3/5)\hat{i} + (15 \times 4/5 + 25 \times \frac{3}{5})\hat{j} = 0$

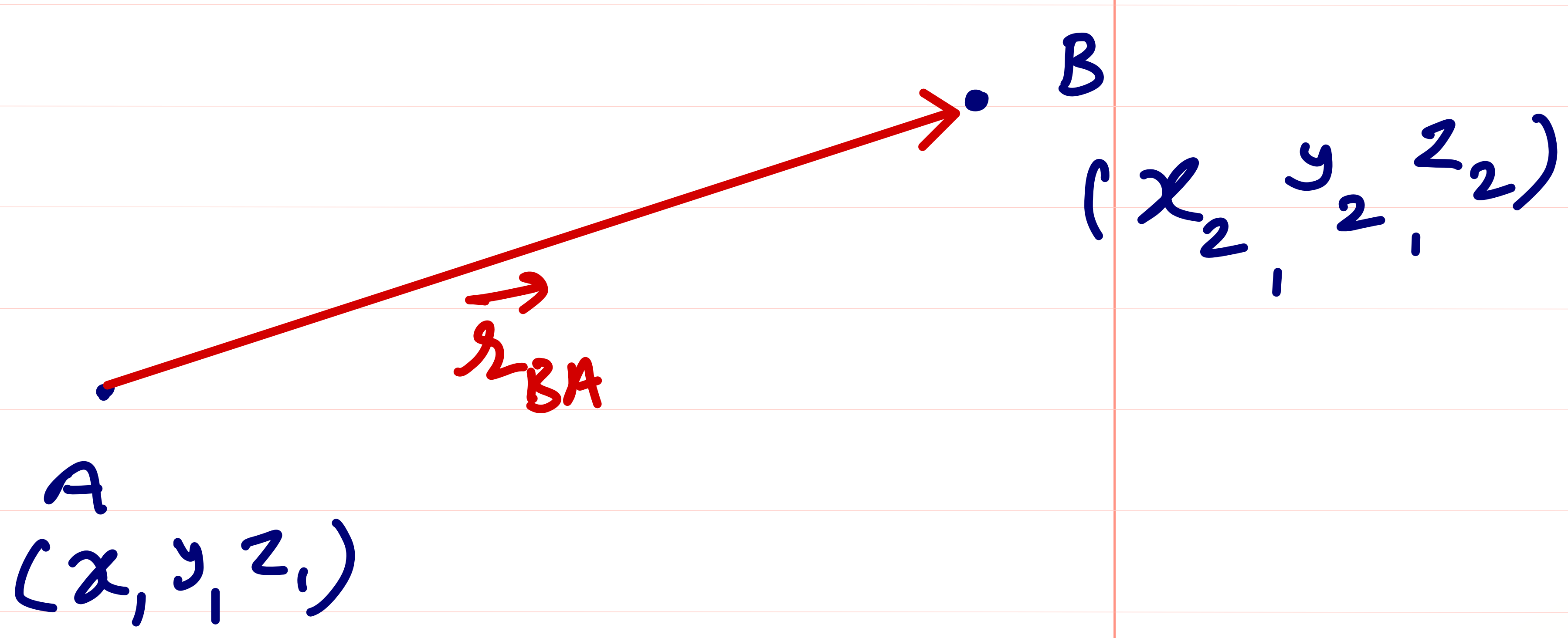
$\vec{F} + (20 - 9)\hat{i} + (12 + 15)\hat{j} = 0$

$\vec{F} = -11\hat{i} - 27\hat{j}$

Position Vector

The position vector of a particle describes its instantaneous position with respect to the origin of the chosen frame of reference. It is a vector joining the origin to the particle and is denoted by vector **r**.





$\vec{r}_{BA}$ = Position of B w.r.t A

$$\boxed{\vec{r}_{BA} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}}$$

$$\vec{r} = (x-0)\hat{i} + (y-0)\hat{j} + (z-0)\hat{k}$$

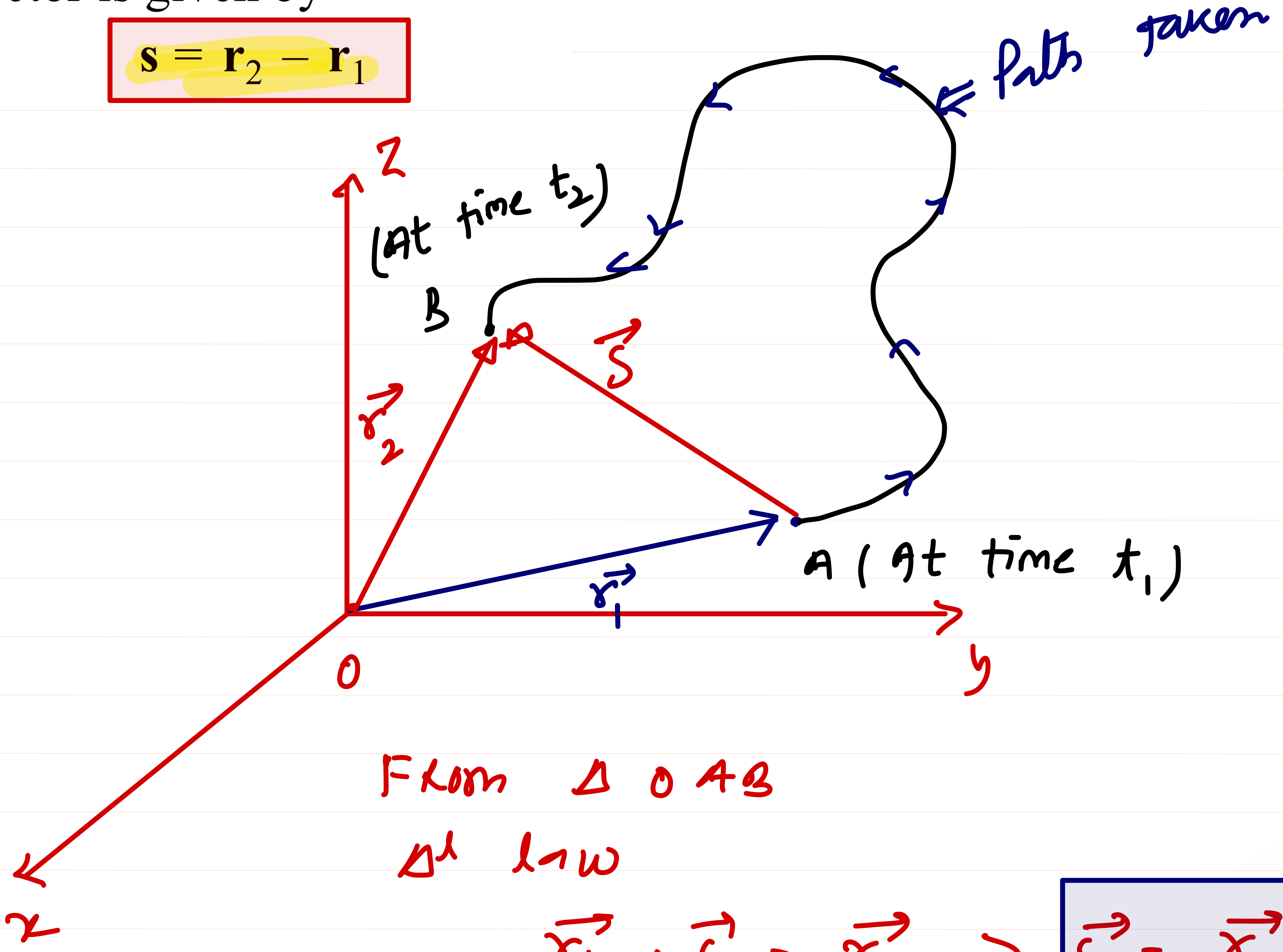
$$\boxed{\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}}$$

Displacement Vector : →

If $\mathbf{r}_1$ is the position vector of a particle at time t_1, and $\mathbf{r}_2$ at time t_2, then the displacement vector is given by

$$\boxed{\mathbf{s} = \mathbf{r}_2 - \mathbf{r}_1}$$

$\vec{S}$ = change in position vector



From Δ OAB

Δ

Δ law

$$\vec{r}_1 + \vec{S} = \vec{r}_2 \Rightarrow \boxed{\vec{S} = \vec{r}_2 - \vec{r}_1}$$

KINEMATICS

Distance & displacement :→

Distance (d)

⇒ Path length taken by particle

⇒ Scalar

⇒ Always +ve and zero

⇒ SI unit m

Displacement

⇒ Shortest path taken by particle

⇒ Vector

⇒ can be +ve or −ve or zero

⇒ SI unit m

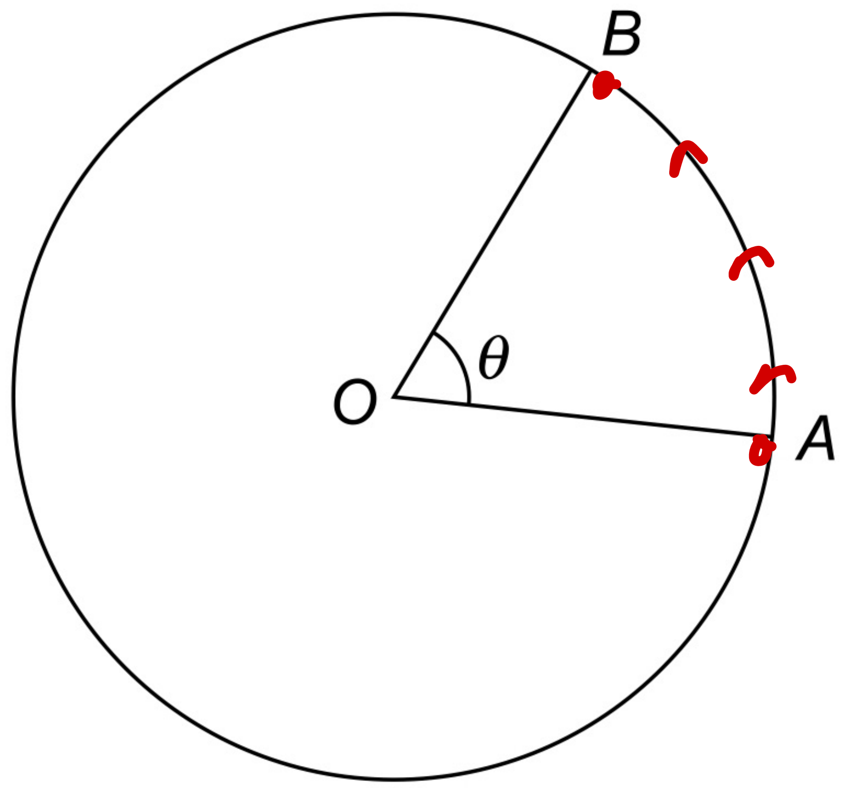
(i) $d \geq 0$

(ii) $\vec{s} \Rightarrow 0, +ve, -ve$

(iii) $\boxed{d \geq |\vec{s}|}$

Ex

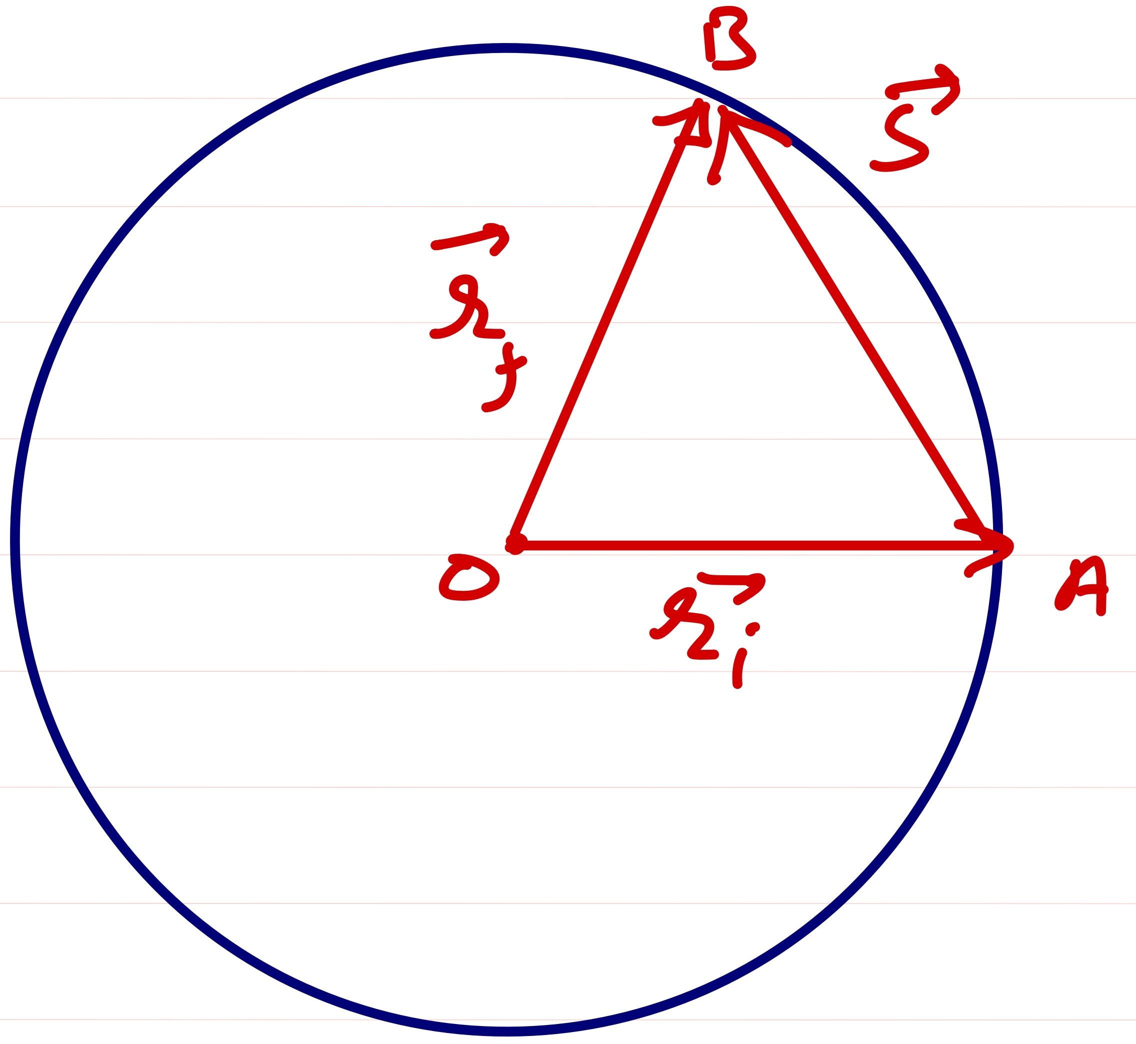
A particle moves from A to B (A.C.W) along a circle of radius R. Find the path length and the magnitude of the displacement in terms of R and θ. [see Fig. 2.2]



$$\theta = \frac{\text{Arc (AB)}}{R} = \frac{\text{Path length}}{R}$$

$$\boxed{\text{Path (length)} = \theta R}$$

$\theta \Rightarrow$ in Radian



$$\vec{S} = \vec{r}_f - \vec{r}_i$$

$$|\vec{r}_f| = |\vec{r}_i| = R$$

$$\boxed{S = 2R\sin(\theta/2)}$$

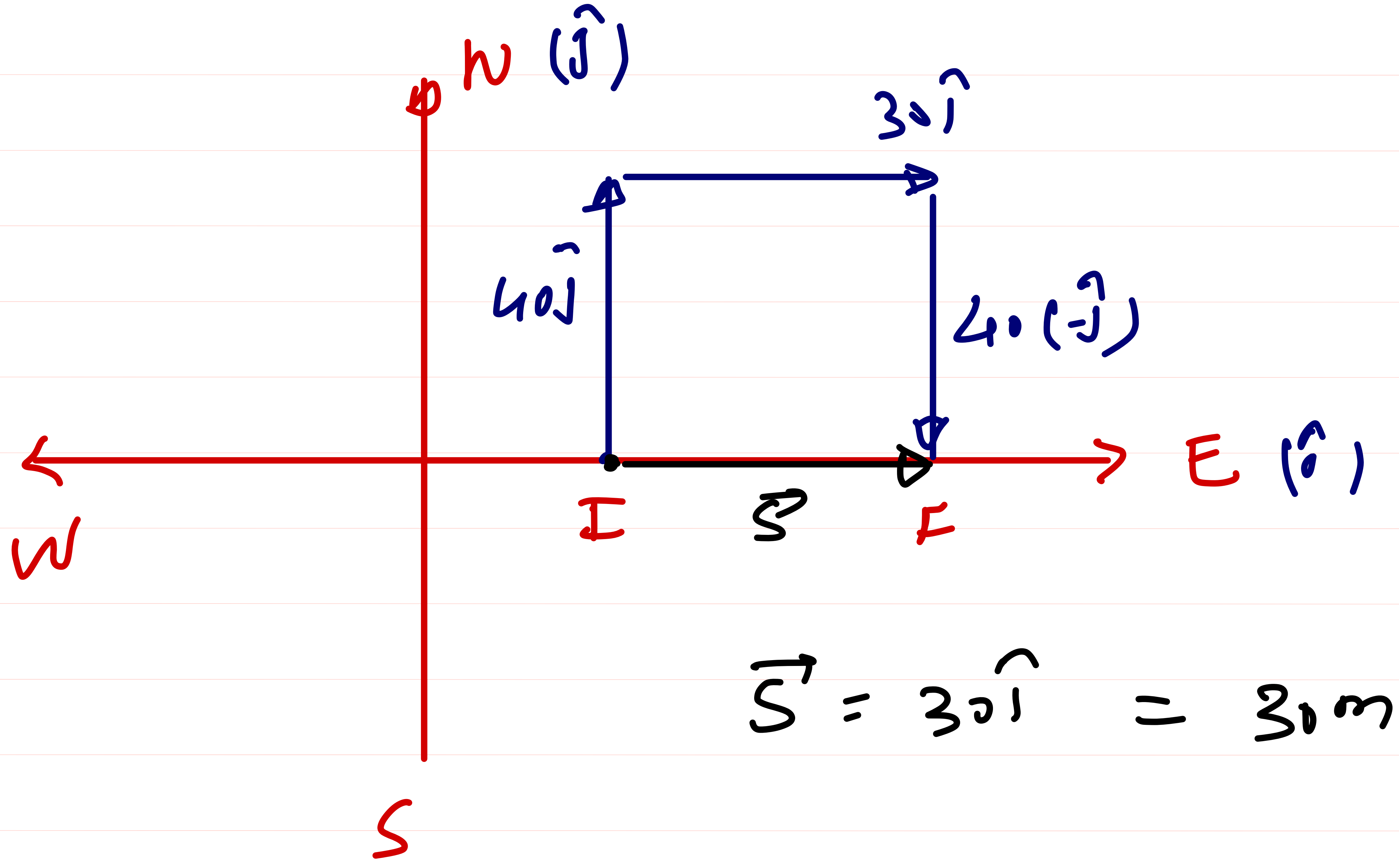
If $\theta = 120°$ then d and S

$$\boxed{d = 120 \times \frac{\pi}{180} \times R = \frac{2\pi R}{3}}$$ Ans

$$S = 2R\sin\left(\frac{120}{2}\right)$$

$$\boxed{S = \sqrt{3}R}$$ Ans

8. A man walks 40 m North, then 30 m East and then 40 m South. Find the displacement from the starting point?



$\vec{S} = 30\hat{i} = 30\text{ m due East}$

$d = 40 + 30 + 40$

$\boxed{d = 110\text{ m}}$