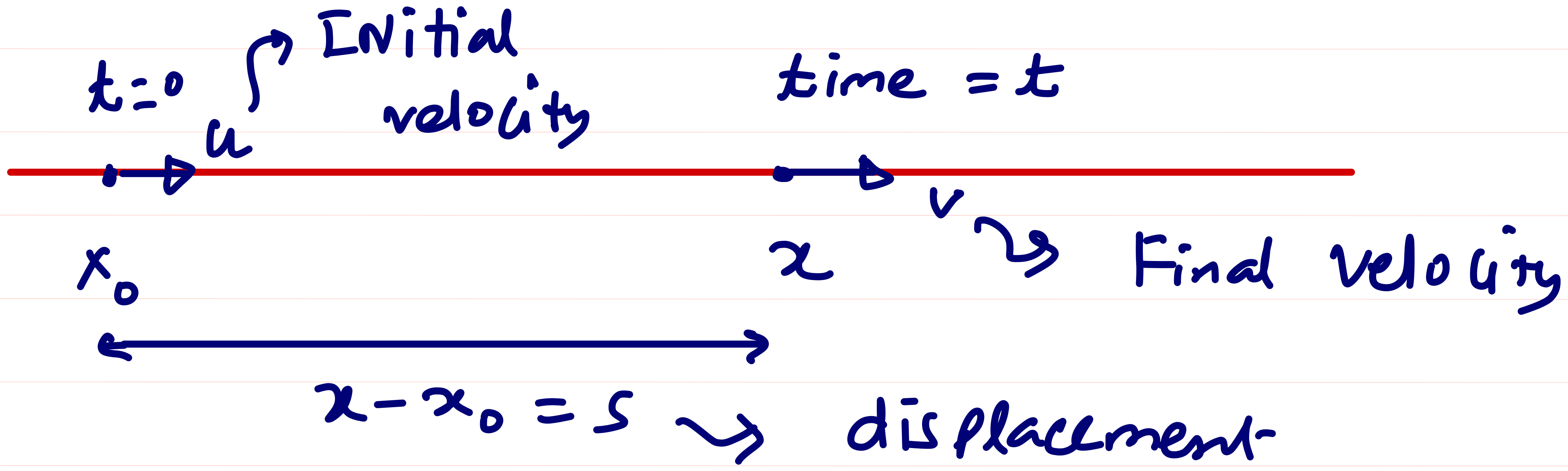
Uniform Acceleration Motion

{ Constant acceleration $\vec{a} = 2\hat{i}$ or $2\hat{i} - 2\hat{j}$ or $2\hat{i} + \hat{j} - 3\hat{k}$ }

Let x_0 be the position of a particle at time $t = 0$ and let u be its velocity at $t = 0$. It is given a constant acceleration a for time t. As a result it moves in a straight line to a position x and acquires a velocity v.

The particle suffers a displacement $s = x - x_0$ in time t. The equations of motion of the particle are



$V = u + at$ —— (i)

$S = x - x_0 = ut + \frac{1}{2}at^2$ —(ii)

$v^2 = u^2 + 2as$ —(iii)

Imp

$$\boxed{\vec{V}_{avg} = \frac{\vec{u} + \vec{v}}{2}}$$

If u is abs and v is known

$$\boxed{S = vt - \frac{1}{2}at^2}$$

Proof

$\because a = \frac{dv}{dt}$

$\int_u^v dv = \int_0^t a\,dt$

$v - u = at$

$\boxed{v = u + at}$

$\because v = \frac{dx}{dt}$

$dx = v\,dt$

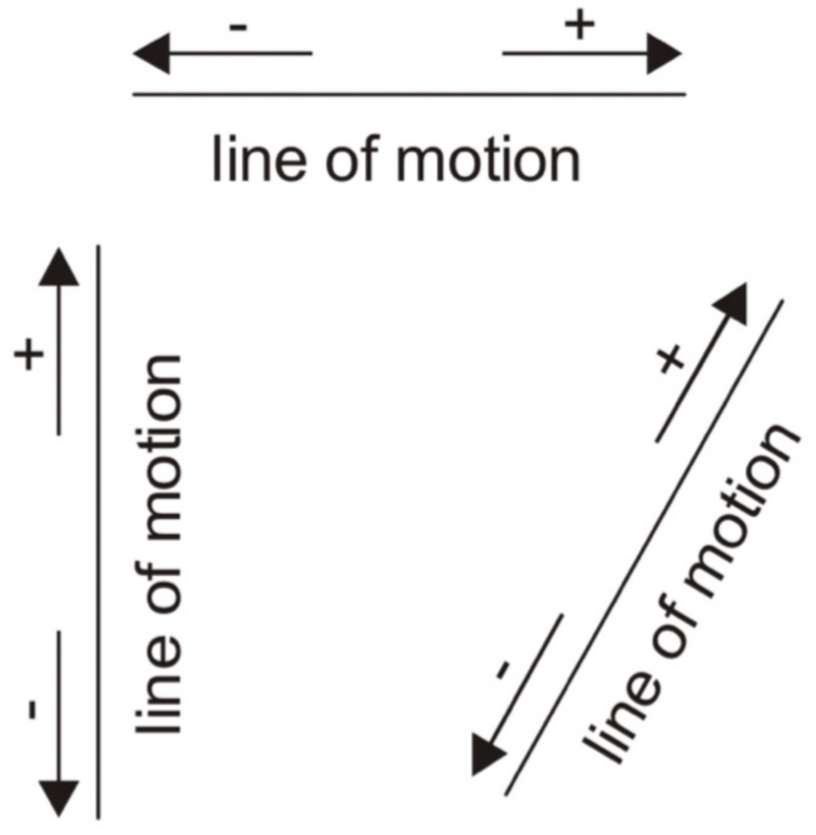
$\int_{x_0}^{x} dx = u\int_0^t dt + a\int_0^t t\,dt$

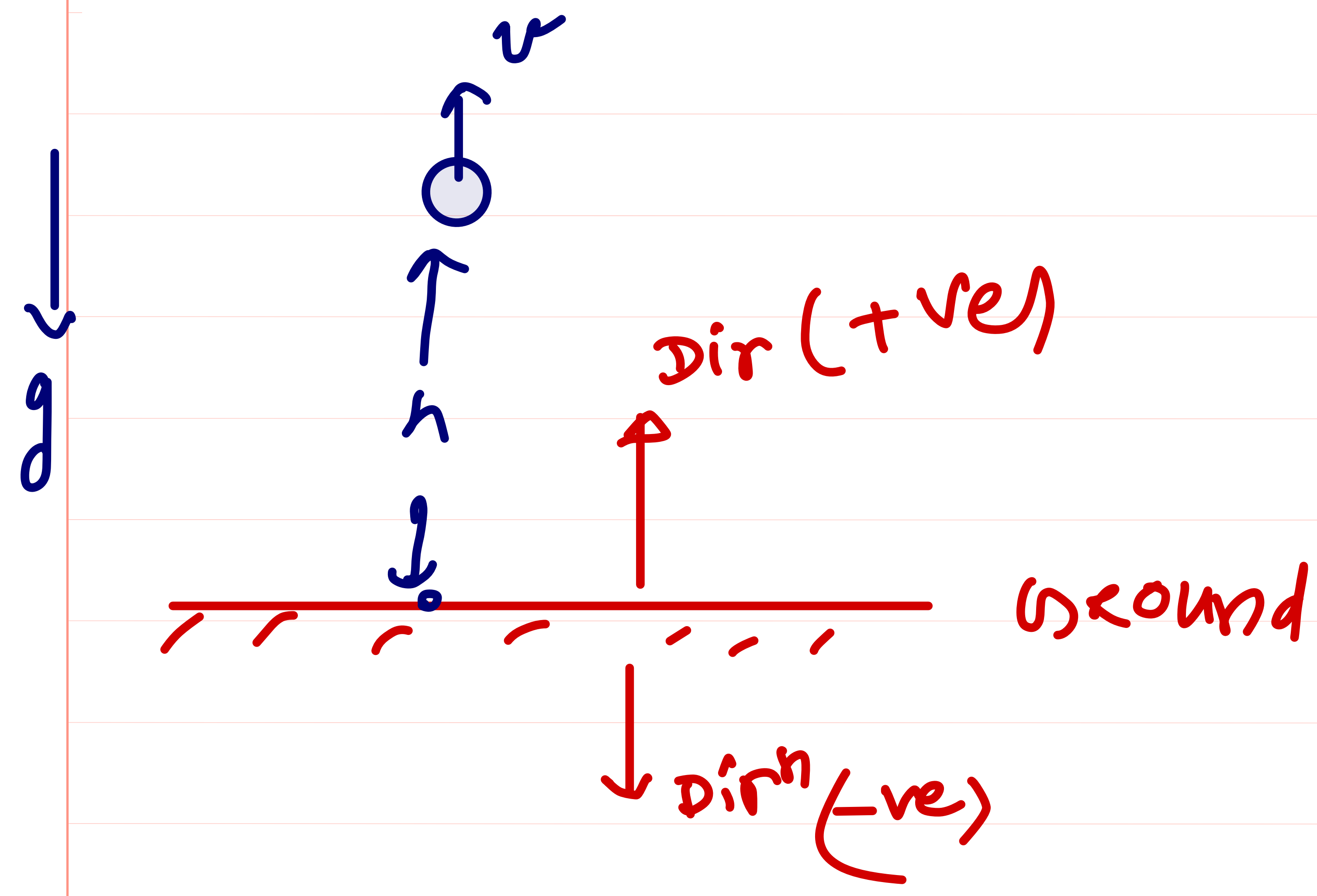
$\boxed{x - x_0 = ut + a\frac{t^2}{2}}$

Directions of Vectors in Straight Line Motion :->

For example, if a particle is moving in a horizontal line (x–axis), the two directions are right and left. Any vector directed towards right can be represented by a positive number and towards left can be represented by a negative number.

For vertical or inclined motion, upward direction can be taken +ve and downward as -ve

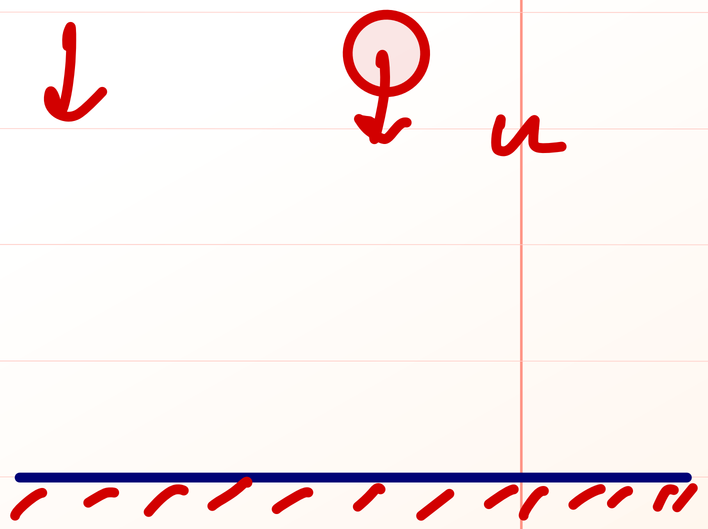




Initial position = +h

Initial velocity = +v

acceleration = a = -g



velocity = -u

acce = -g

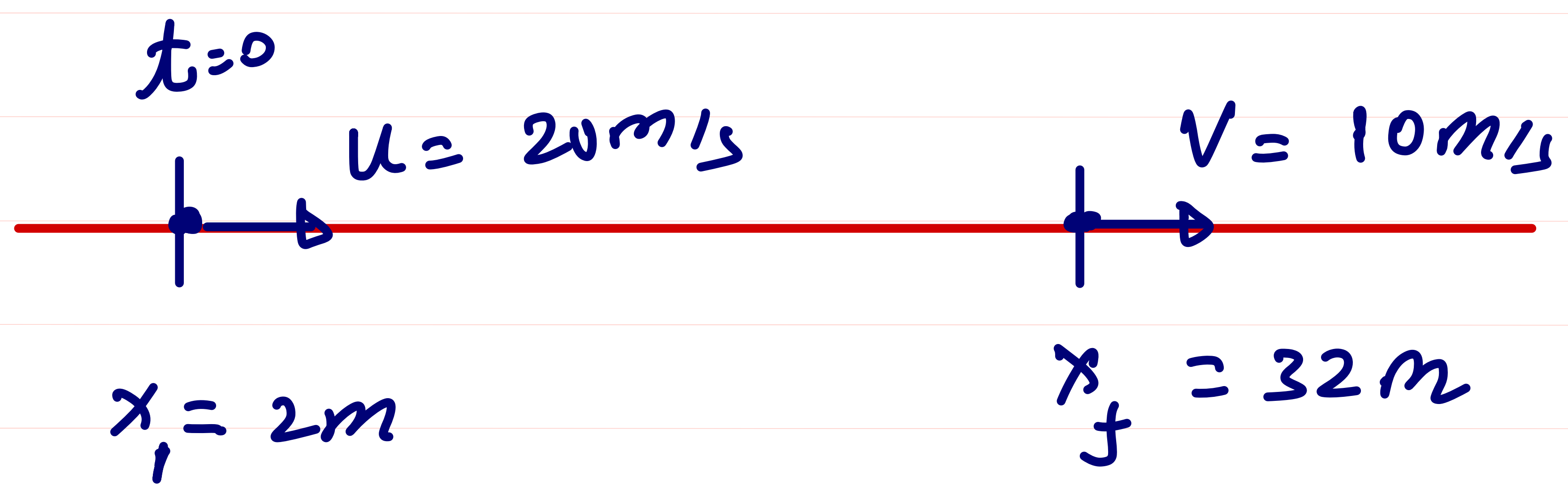
Acceleration due to gravity = g = 10 m/s² or 9.8 m/s²

Illustration 6. A particle moving with uniform acceleration passes the point $x = 2$ m with velocity 20 m/s at the instant $t = 0$. Some time latter it is observed at the point $x = 32$ m moving with velocity 10 m/s.

(a) What is its acceleration?

(b) Find its position and velocity at the instant $t = 8$ s.

(c) What is the distance travelled during the interval $t = 0$ to 8 s?



(a) $v^2 = u^2 + 2a(x_f - x_i)$

$(10)^2 = (20)^2 + 2 \times a(32-2)$

$10^2 - 20^2 = 2 \times 30\, a$

$100 - 400 = 2 \times 30 \times a$

$-300 = 2 \times 30 \times a$

$\boxed{a = -5\ m/s^2}$ Ans

(b) $x - x_0 = ut + \frac{1}{2}at^2$

$x - 2 = 20 \times 8 + \frac{1}{2}(-5) \times 8^2$

$= 160 - \frac{5}{2} \times 64$

$= 160 - 5 \times 32$

$x - 2 = 160 - 160$

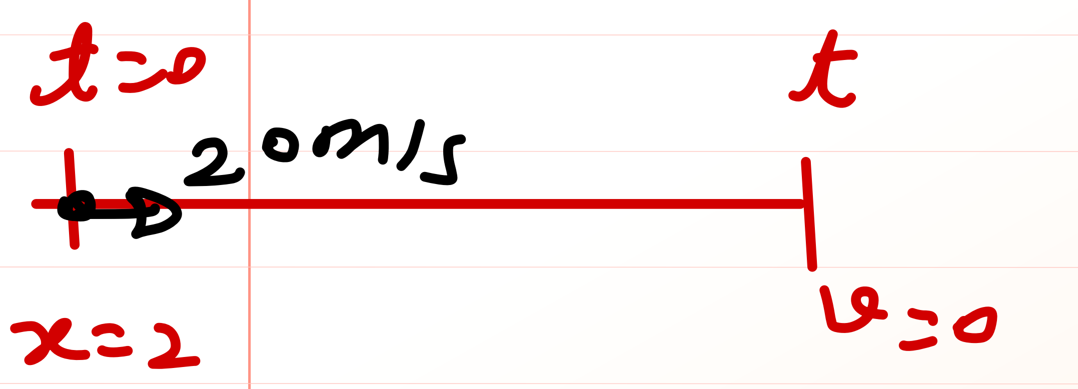
$\boxed{x = 2\ m}$

$v = u + at$

$v = 20 - 5 \times 8$

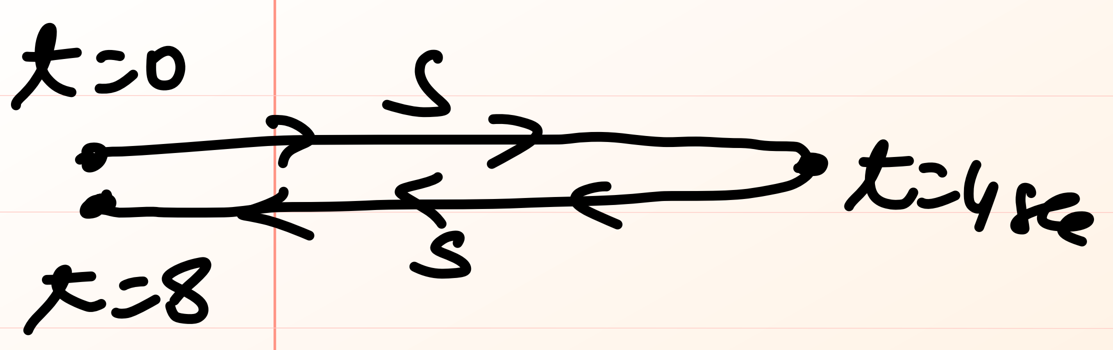
$\boxed{v = -20\ m/s}$

(c)



$0 = 20 - 5 \times t \Rightarrow$

$t = 4$ sec



$d = 2S$

$d = 2(20 \times 4 - \frac{1}{2} \times 5 \times 4^2)$

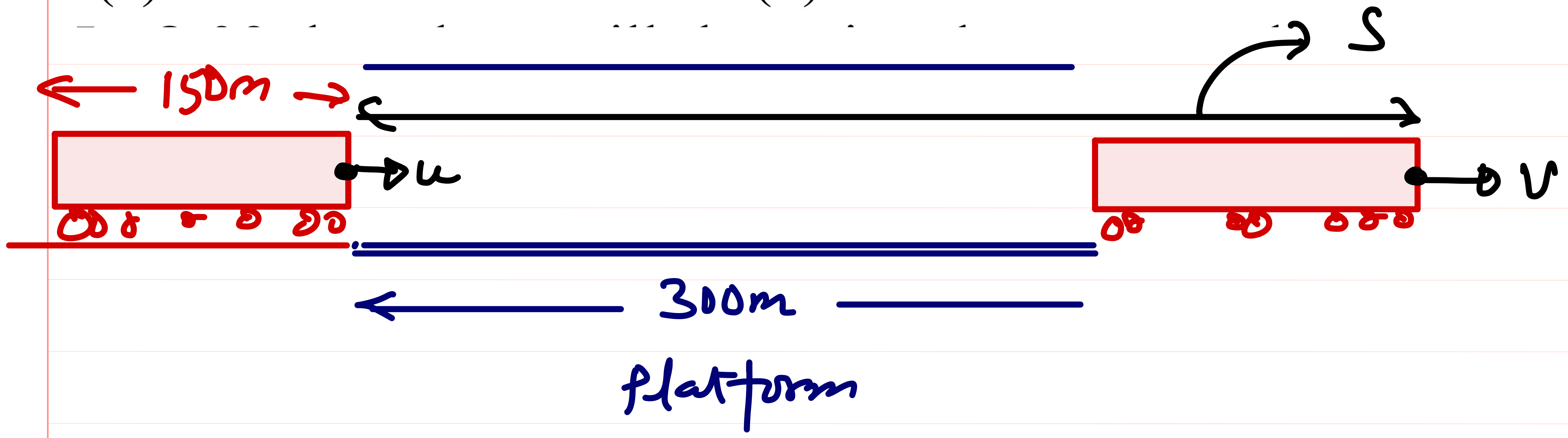
$\boxed{d = 80\ m}$ Ans

Ex

A 150 m long train having a constant acceleration crosses a 300 m long platform. It enters the platform at a speed of 40 ms^{-1} and leaves it at a speed of 50 ms^{-1}. What is the acceleration of the train?

(a) 0.6 ms^{-2} (b) 0.8 ms^{-2}

~~(c)~~ 1.0 ms^{-2} (d) 1.2 ms^{-2}



$$S = 300 + 150 = 450m$$

$$v^2 = u^2 + 2aS$$

$$(50)^2 = (40)^2 + 2a \times 450$$

$$(50)^2 - (40)^2 = 900a$$

$$30^2 = 900a$$

$$900 = 900 \times a$$

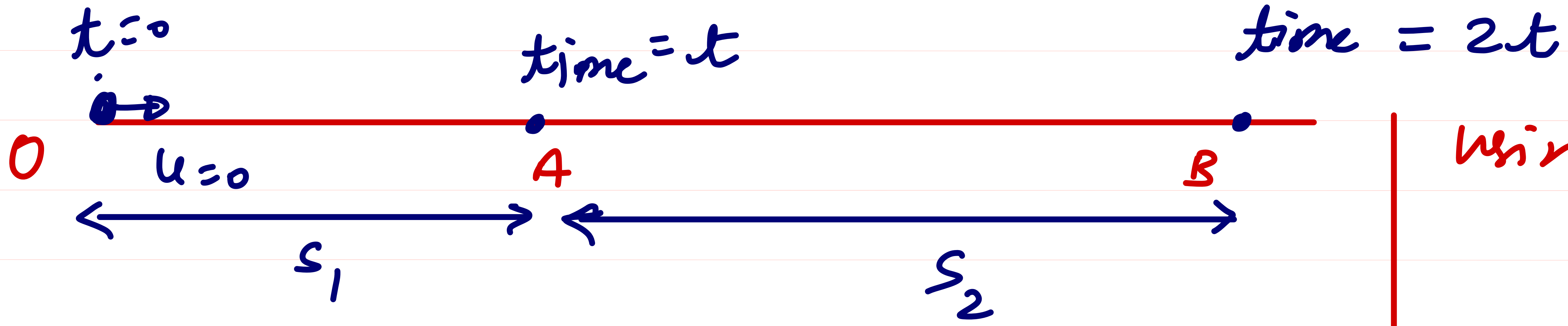
$$\boxed{a = 1 m/s^2}$$

Ex A car, starting from rest, at a constant acceleration covers a distance s_1 in a time interval t. It covers a distance of s_2 in the next time interval t at the same acceleration. Which of the following relations is true?

(a) $s_2 = s_1$ (b) $s_2 = 2s_1$

(c) $s_2 = 3s_1$ ✓ (d) $s_2 = 4s_1$

let acc = a



using $S = ut + \frac{1}{2}at^2$

$$s_1 = 0t + \frac{1}{2}at^2$$

$$\boxed{S_1 = \frac{1}{2}at^2}$$

using (B/w O to B)

$$S = ut + \frac{1}{2}at^2$$

$$s_1 + s_2 = 0 \times 2t + \frac{1}{2}a(2t)^2$$

$$s_1 + s_2 = \left(\frac{1}{2}at^2\right)4$$

$$s_1 + s_2 = 4s_1$$

$$\boxed{s_2 = 3s_1}$$

Illustration 7*. A ball is dropped from the top of a building. The ball takes 0.50 s to fall past the 3 m length of a window, which is some distance below the top of the building.

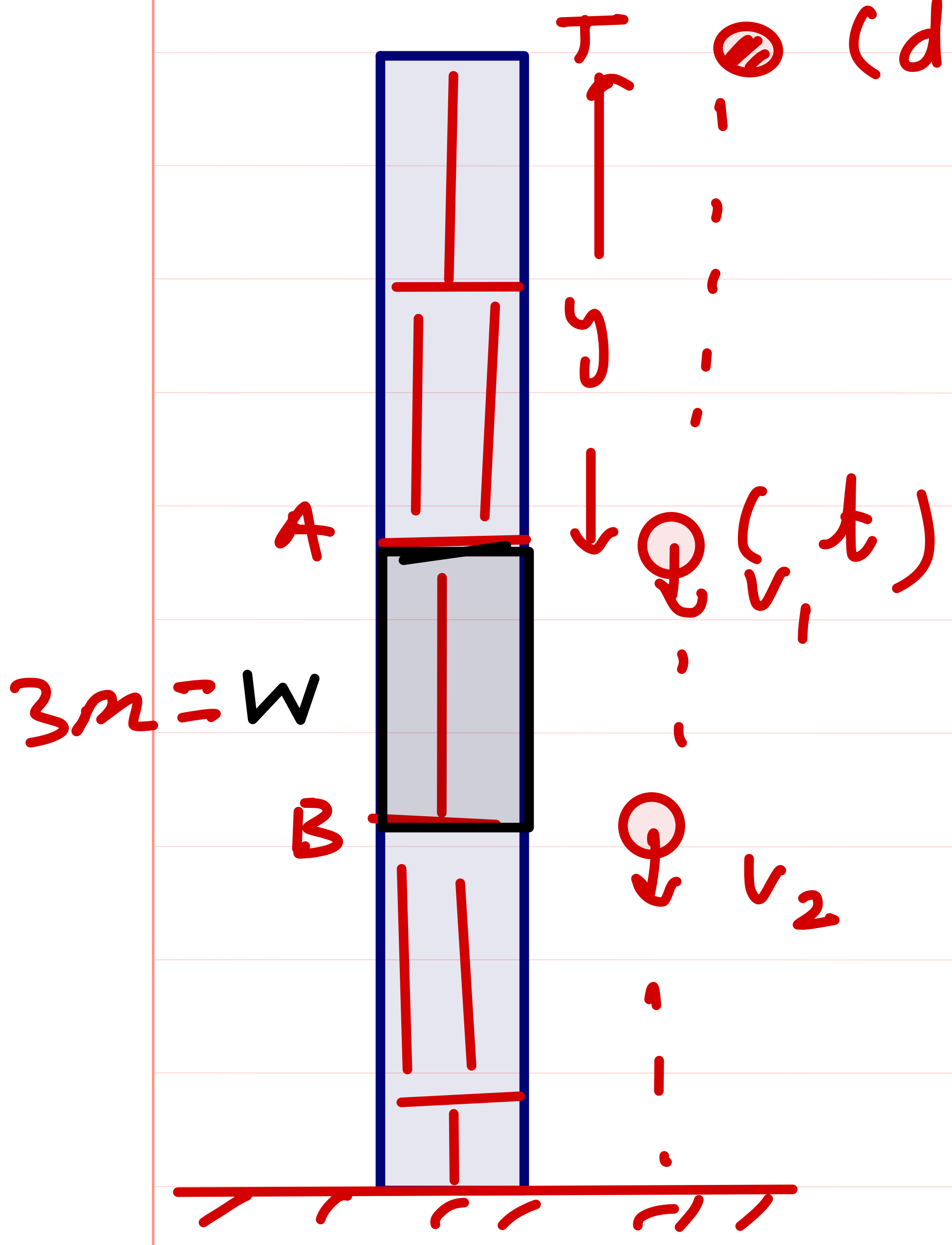
(a) How fast was the ball going as it passed the top of the window?

(b) How far is the top of the window from the point at which the ball was dropped?

Assume acceleration g in free fall due to gravity be 10 m/s^2 downwards.

+ve

-ve



Asked

$v_1 = ??$

$y = ??$

Using

$$s = ut + \frac{1}{2}at^2$$

B/w (T A)

$$-y = 0t + \frac{1}{2}(-g)t^2 \quad \text{— (1)}$$

B/w (T B)

$$-(y+3) = 0(t+0.5) + \frac{1}{2}(-g)(t+0.5)^2 \quad \text{— (2)}$$

(2) − (1)

$$-(y+3) + y = -\frac{g}{2}(t+0.5)^2 + \frac{g}{2}t^2$$

$$-3 = \cancel{-\frac{g}{2}t^2} - \frac{g}{2}(0.5)^2 - 0.5gt + \cancel{\frac{g}{2}t^2}$$

$$3 = \frac{g}{2}(0.5)^2 + 0.5gt$$

$$3 = 5 \times 0.25 + 5t$$

$$0.6 = 0.25 + t$$

$$\boxed{t = 0.35 \text{ sec}}$$

b/w T to A

$$v = u + at$$

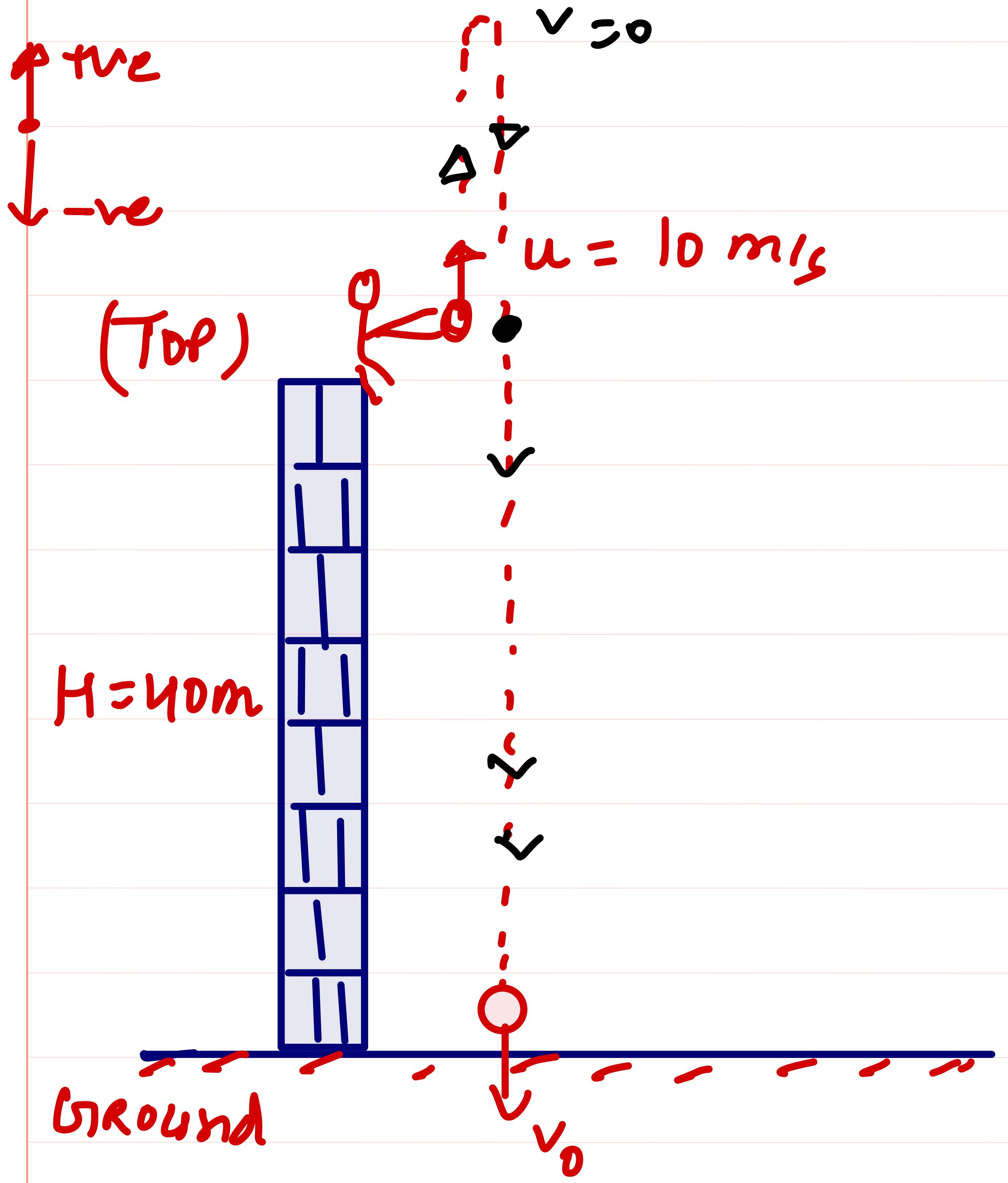
$$v_1 = 0 + (-10) \times .35$$

$$\boxed{v_1 = -3.5\ m/s}$$

$$-y = 0 \times 0.35 + \frac{1}{2}(-10)(0.35)^2$$

$$\boxed{y = 5(.35)^2\ m}$$

Ex From the top of a building 40 m tall, a ball a thrown vertically upwards with a velocity of 10 ms $^{-1}$. (a) After how long will the ball hit the ground? (b) After how long will the ball pass through the point from where it was projected? (c) With what velocity will it hit the ground? Take $g = 10$ ms^{-2}.



(a) using (B/w Top to Ground)

$S = ut + \frac{1}{2}at^2$

$-40 = +10t + \frac{1}{2}(-10)t^2$

$-40 = 10t - 5t^2$

$-8 = 2t - t^2$

$t^2 - 2t - 8 = 0$

$t^2 - 4t + 2t - 8 = 0$

$t(t-4) + 2(t-4) = 0$

$\boxed{t = 4 \text{ sec}}$ Ans $\quad t = \cancel{-2} \text{ sec}$

(b) using (Top to Top)

$S = ut + \frac{1}{2}at^2$

$0 = 10t - \frac{1}{2} \times 10 t^2$

$\boxed{t = 2 \text{ sec}}$ Ans

(c) Top to Ground

using $(v^2 = u^2 + 2aS)$

$v^2 = 10^2 + 2(-10)(-40)$

$v^2 = 100 + 800$

$\boxed{v = 30 \text{ m/s}}$ velocity = -30 m/s

Ex-1 A body is thrown vertically up with a velocity u. It passes a point at a height h above the ground at time t_1 while going up and at time t_2 while falling down. Then the relation between u, t_1 and t_2 is

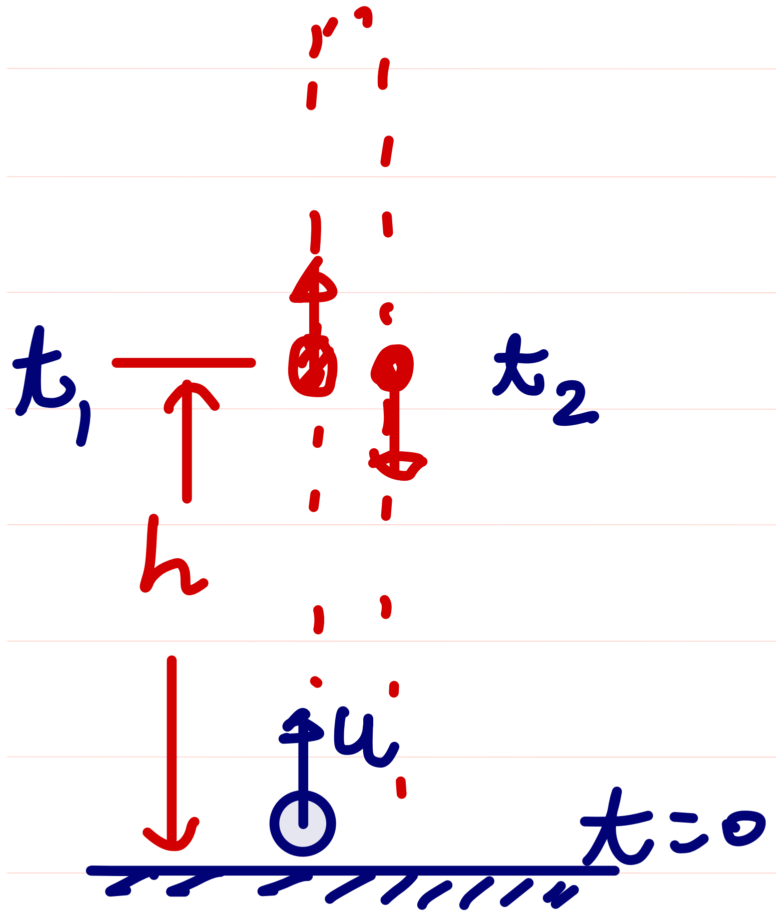
(a) $t_1 + t_2 = \dfrac{2u}{g}$ ✓ (b) $t_2 - t_1 = \dfrac{2u}{g}$

(c) $t_1 + t_2 = \dfrac{u}{g}$ (d) $t_2 - t_1 = \dfrac{u}{g}$

Ex-2 In Q. ~~59~~ 1 above, the relation between t_1, t_2 and h is

(a) $t_1 t_2 = \dfrac{2h}{g}$ ✓ (b) $t_1 t_2 = \dfrac{h}{g}$

(c) $(t_1 + t_2)^2 = \dfrac{2h}{g}$ (d) $(t_1 + t_2)^2 = \dfrac{h}{g}$



using $s = ut + \frac{1}{2}at^2$

$h = ut - \frac{1}{2}gt^2$

Roots of Quadratic Eq.

$\boxed{\alpha+\beta = -b/a \qquad \alpha\beta = \frac{c}{a}}$

$\left(\frac{g}{2}\right)t^2 - ut + h = 0$

$a = g/2 \qquad b = -u \qquad c = h$

$\boxed{t_1 + t_2 = -\frac{(-u)}{g/2} = \frac{2u}{g}}$

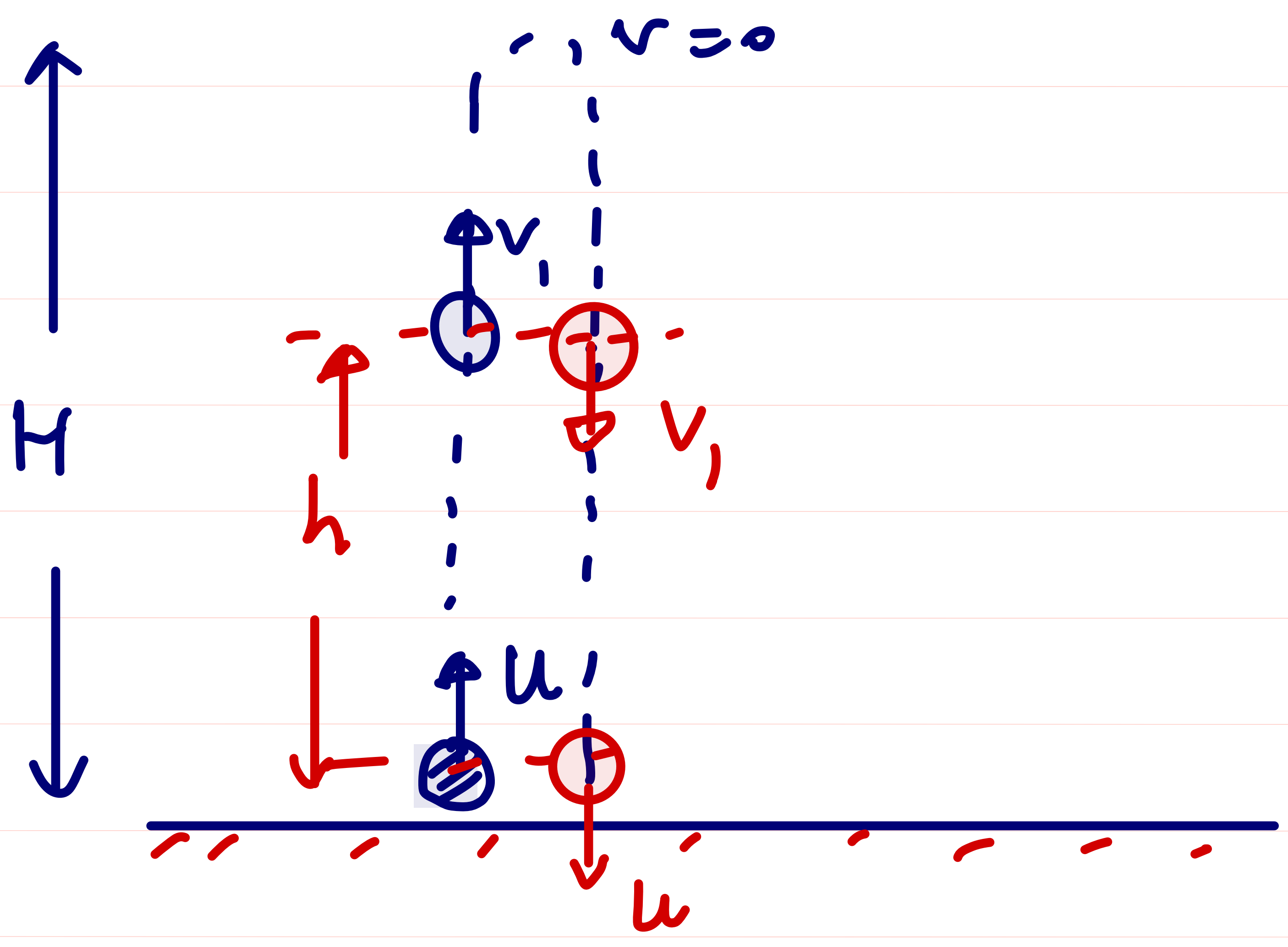
$t_1 t_2 = \dfrac{h}{g/2}$

$\boxed{t_1 t_2 = \frac{2h}{g}}$

Imp NOTE :

UNDER GRAVITY

① Speed at Same level will be Same



$$\boxed{v^2 = u^2 - 2gh}$$ always

$$\boxed{\text{Time of flight} = \frac{2u}{g}}$$

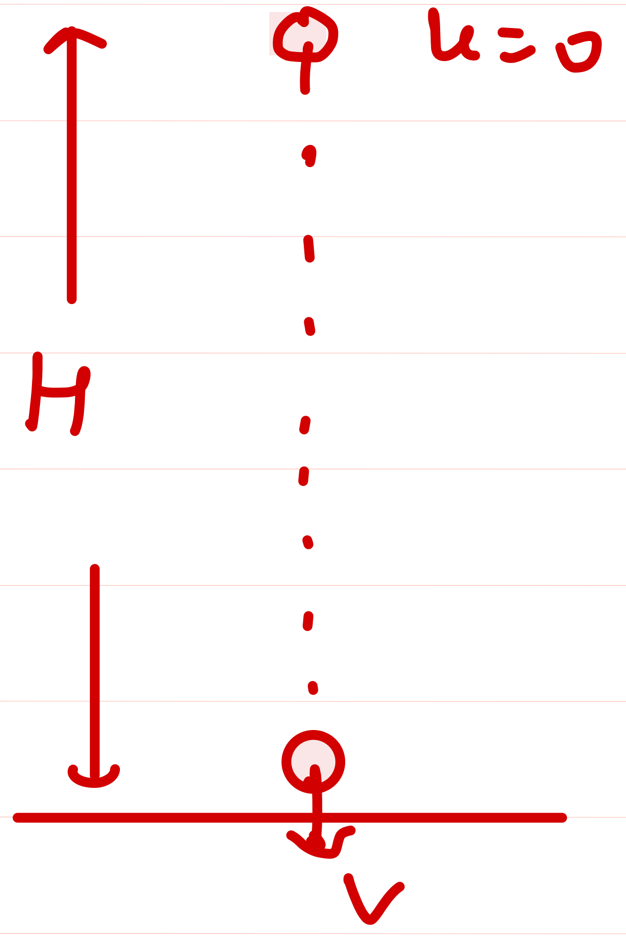
$t_{up} = \frac{u}{g}$ $t_{down} = \frac{u}{g}$

Time of flight $= t_{up} + t_{down}$

maximum Height

$$\boxed{H = \frac{u^2}{2g}}$$

Dropped



Speed at ground

$v^2 = 0^2 + 2(-g)(-H)$

$$\boxed{v = \sqrt{2gH}}$$

Time of flight

$S = ut + \frac{1}{2}at^2$

$-H = 0T + \frac{1}{2}(-g)T^2$

$$\boxed{T = \sqrt{\frac{2H}{g}}}$$