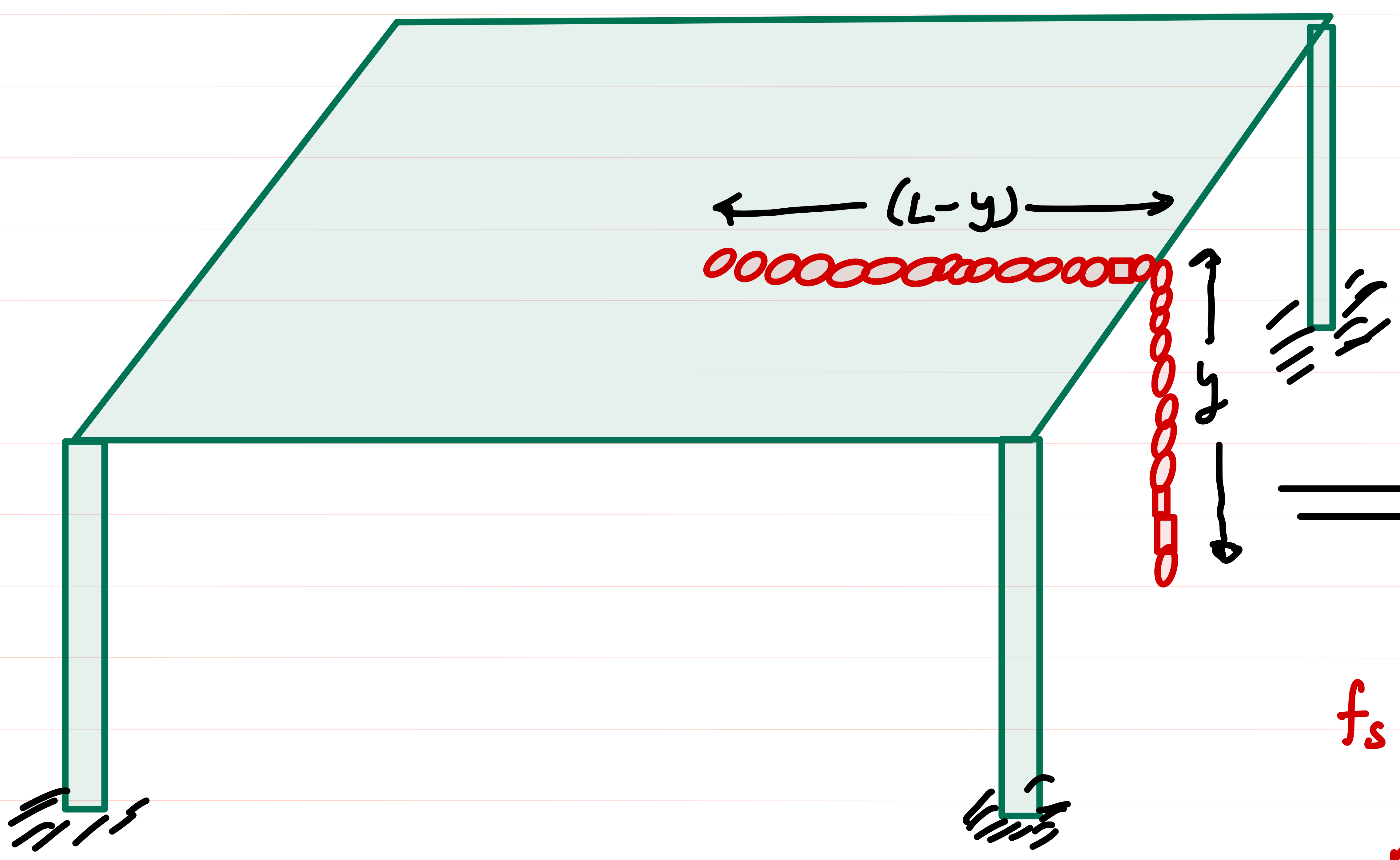
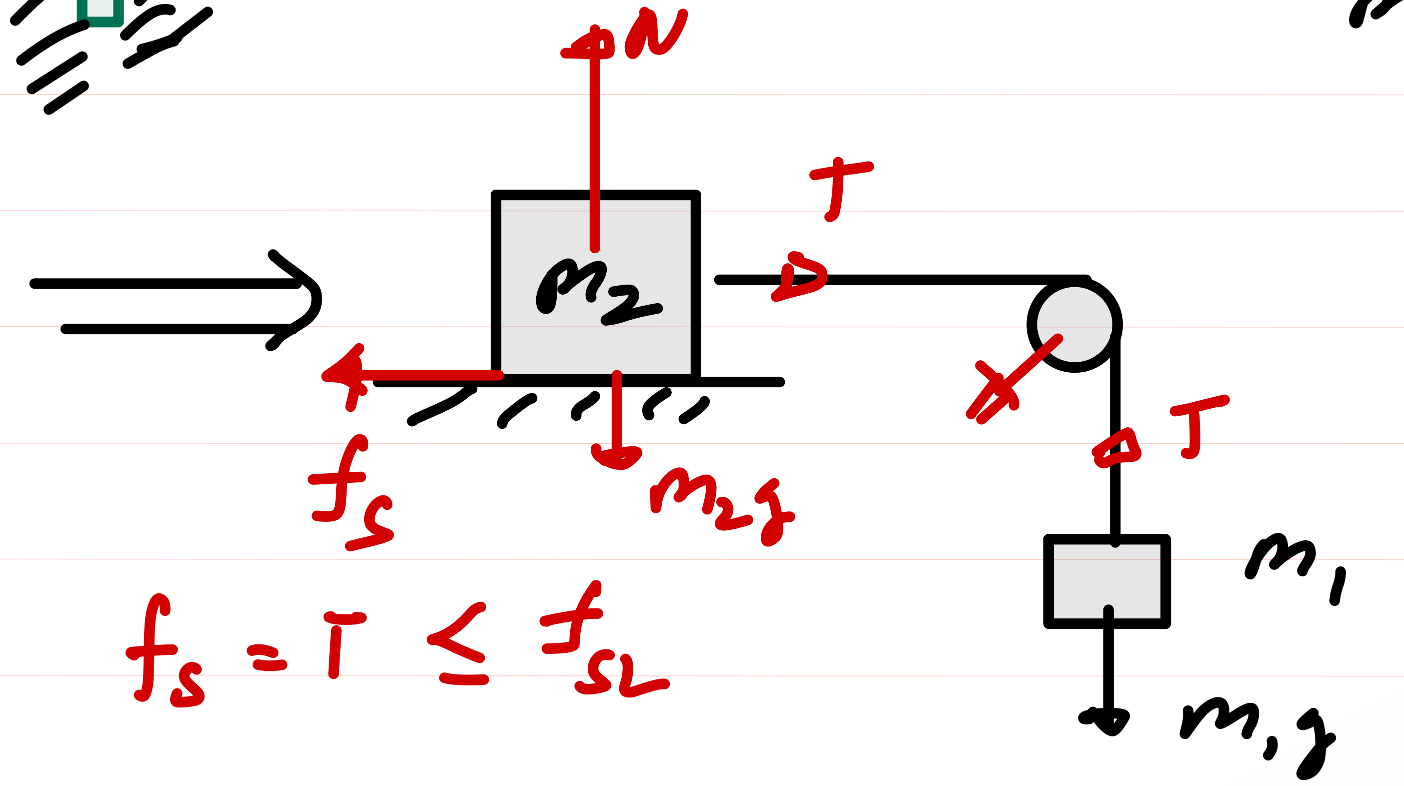
Illustration 2. Length of a uniform chain is L and coefficient of static friction is μ between the chain and the table top. Calculate the maximum length of the chain which can hang from the table without sliding.



Let mass of Hanging chain $= m_1 = \frac{M}{L} \cdot y$

mass of chain on Table $= m_2$

$$m_2 = \frac{M}{L}(L-y)$$



$$T = m_1 g$$

$$f_s = T \le f_{sL}$$

$$m_1 g \le \mu m_2 g$$

$$m_1 \le \mu m_2$$

$$\cancel{\frac{M}{L}} y \le \mu \cancel{\frac{M}{L}}(L-y)$$

$$y \le \mu(L-y)$$

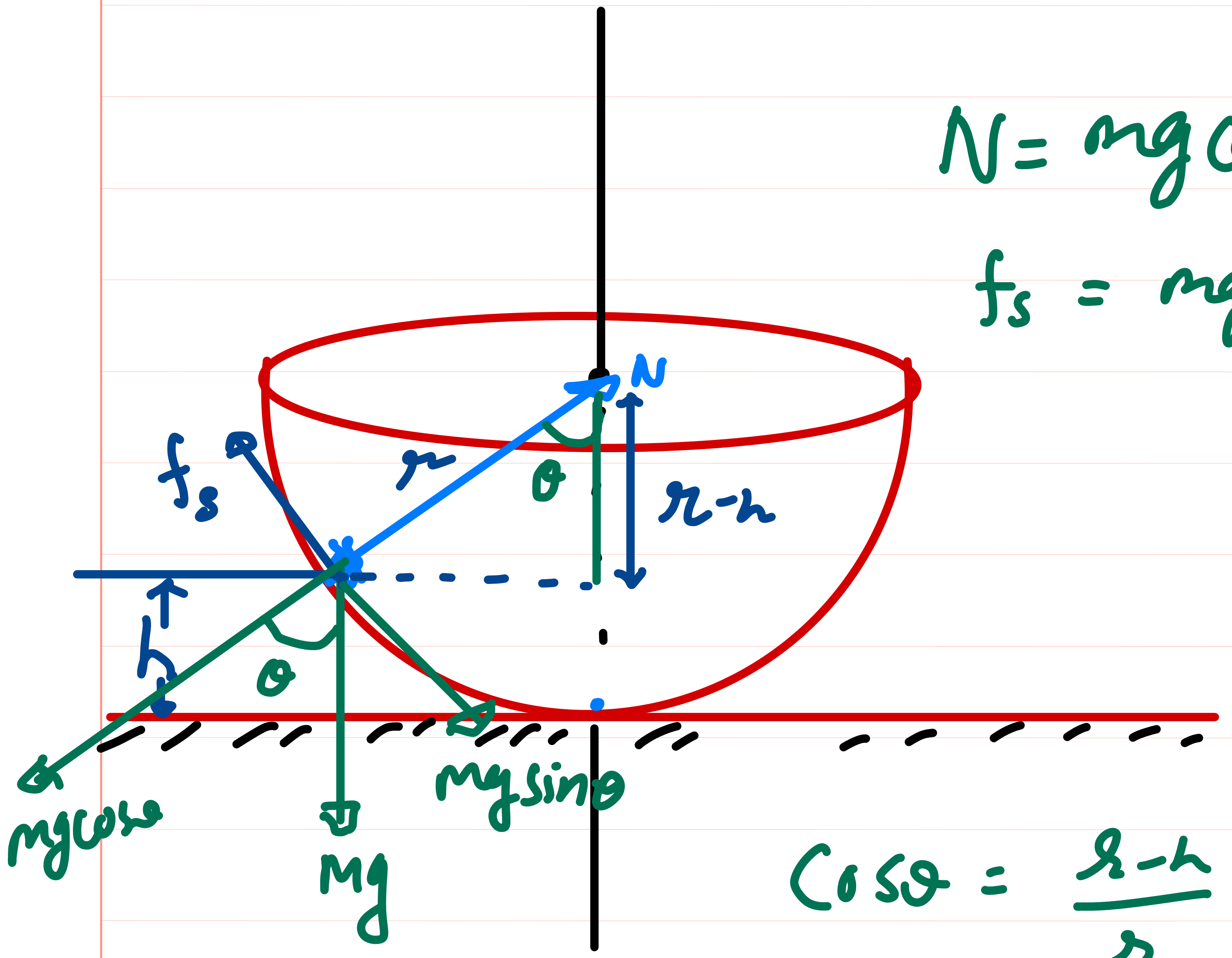
$$y + \mu y \le \mu L$$

$$y \le \frac{\mu}{1+\mu} \cdot L$$

$$\boxed{y_{max} = \frac{\mu}{1+\mu} L}$$

Ans

Illustration 3. An insect crawls on the inner surface of hemispherical bowl of radius r. If the coefficient of friction between an insect and bowl is μ and the radius of the bowl is r, find the maximum height to which the insect can crawl up.



$N = mg\cos\theta$

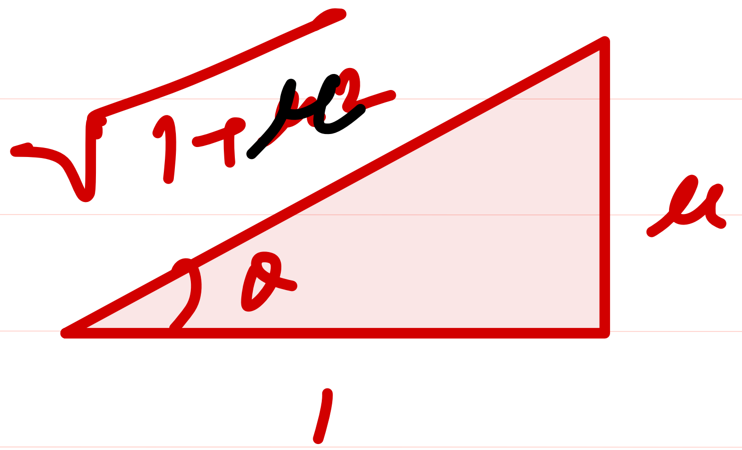
$f_s = mg\sin\theta$

$\cos\theta = \dfrac{r-h}{r}$

$f_s \le \mu N$ (condition for no slipping)

$\cancel{m}g\sin\theta \le \mu \cancel{m}g\cos\theta$

$\tan\theta \le \mu$



$\cos\theta = \dfrac{1}{\sqrt{1+\mu^2}} = \dfrac{r-h}{r}$

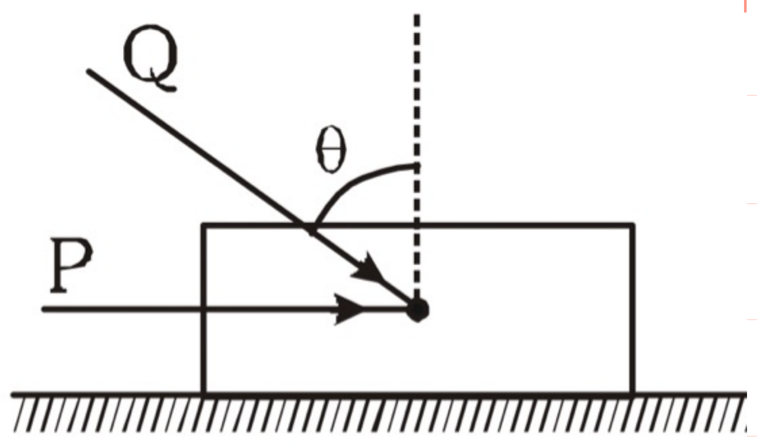
$\dfrac{r}{\sqrt{1+\mu^2}} = r - h$

$h = r - \dfrac{r}{\sqrt{1+\mu^2}}$

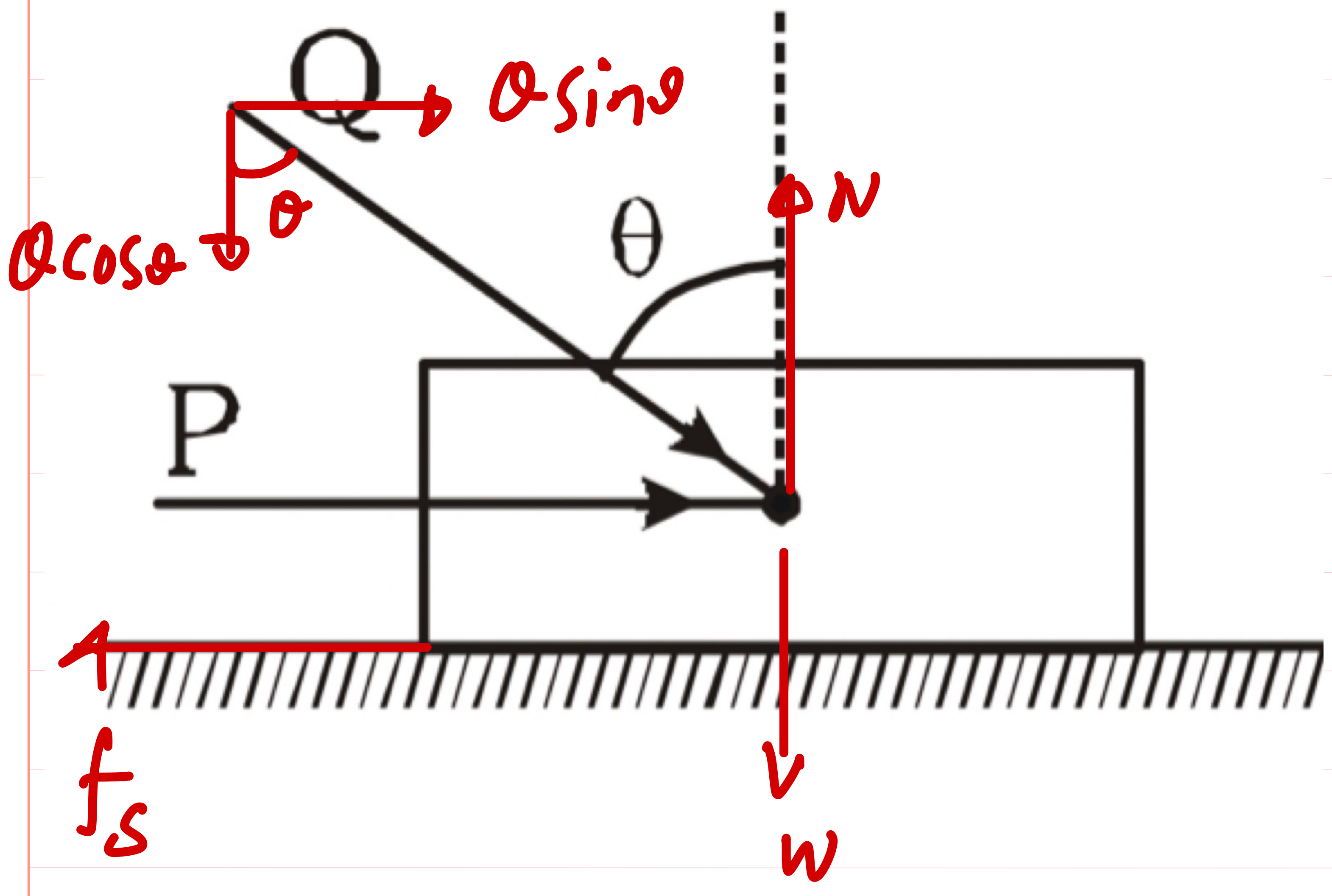
$\boxed{h = r\left(1 - \dfrac{1}{\sqrt{1+\mu^2}}\right)}$

Ans

3. A block of mass m lying on a rough horizontal plane is acted upon by a horizontal force P and another force Q inclined an at an angle θ to the vertical. The minimum value of coefficient of friction between the block and the surface for which the block will remain in equilibrium is :



(A) $\dfrac{P + Q\sin\theta}{mg + Q\cos\theta}$ (B) $\dfrac{P\cos\theta + Q}{mg - Q\sin\theta}$ (C) $\dfrac{P + Q\cos\theta}{mg + Q\sin\theta}$ (D) $\dfrac{P\sin\theta - Q}{mg - Q\cos\theta}$



$f_s = P + Q\sin\theta \leq \mu N$

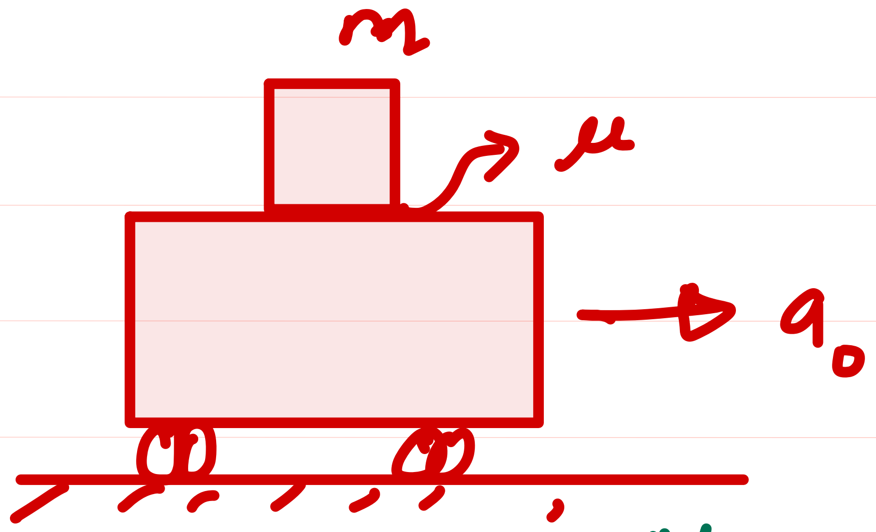
$N = W + Q\cos\theta$

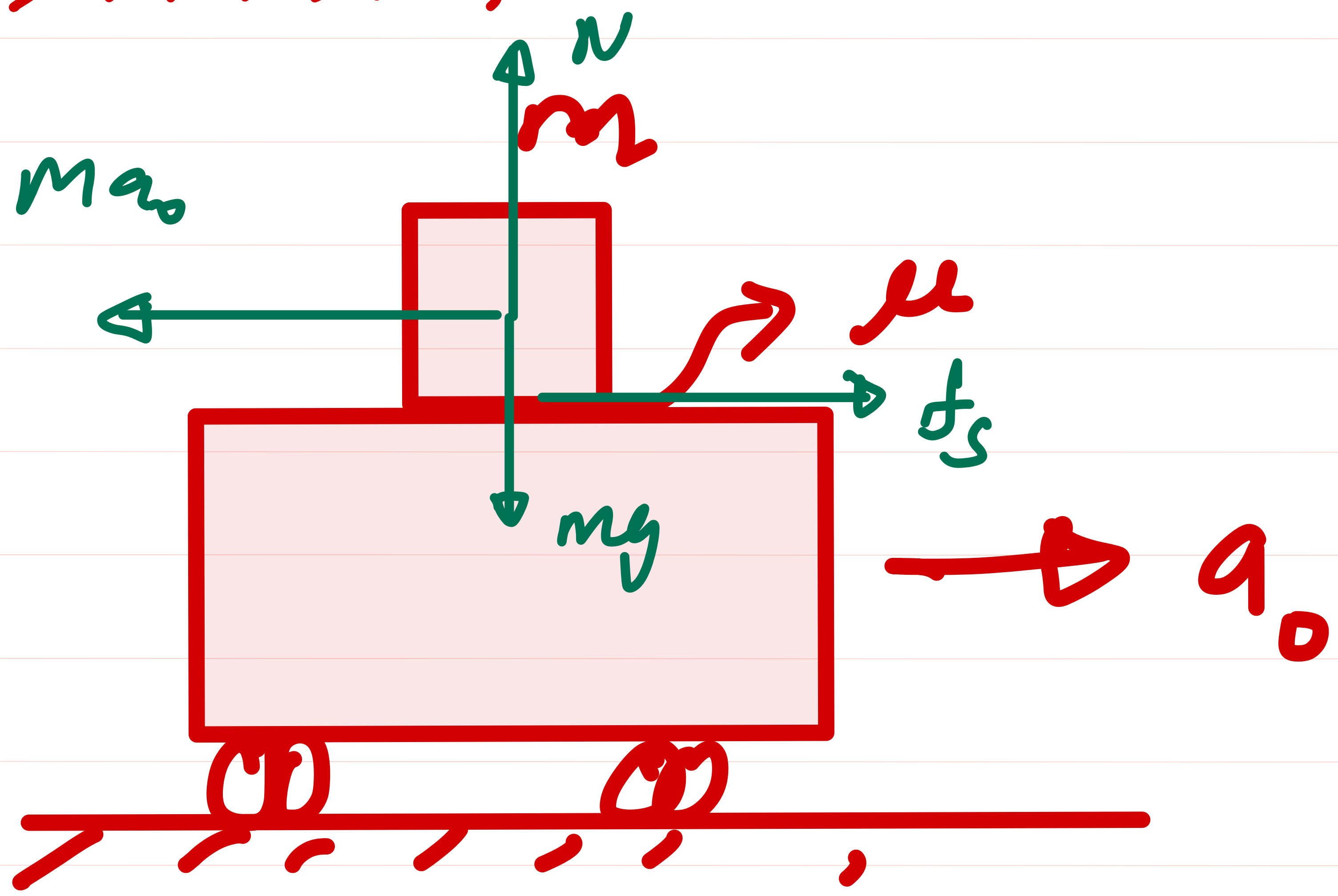
$\dfrac{P + Q\sin\theta}{W + Q\cos\theta} \leq \mu$

$\mu_{min} = \dfrac{P + Q\sin\theta}{mg + Q\cos\theta}$

Ex

Maximum acceleration of cart (a_0) for that Block does not move w.r.t cart (g = Acc. due to gravity)





$$f_s = ma_0 \le \mu N$$

$$\cancel{m}a_0 \le \mu \cancel{m}g$$

$$a_0 \le \mu g$$

$$\boxed{(a_0)_{max} = \mu g}$$ Ans

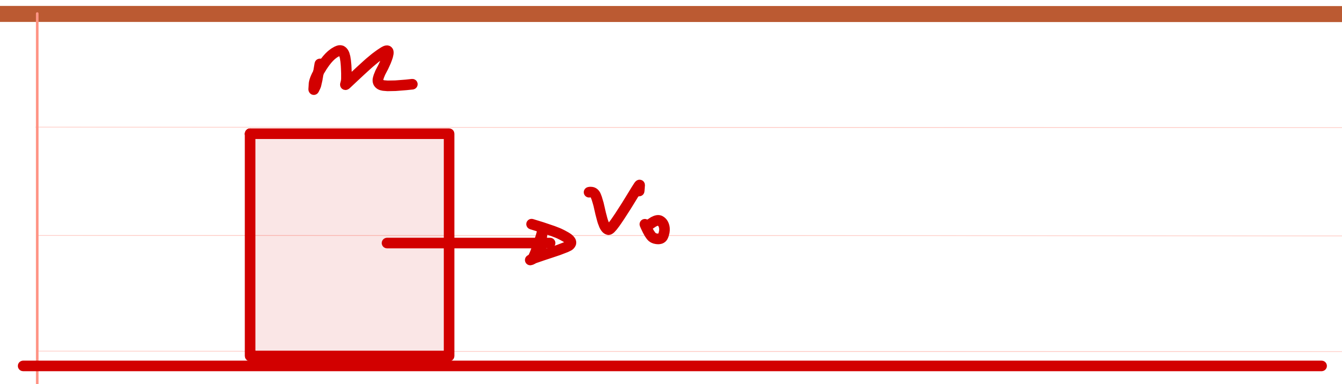
$a < \mu g$

Block will not slide

$a > \mu g$

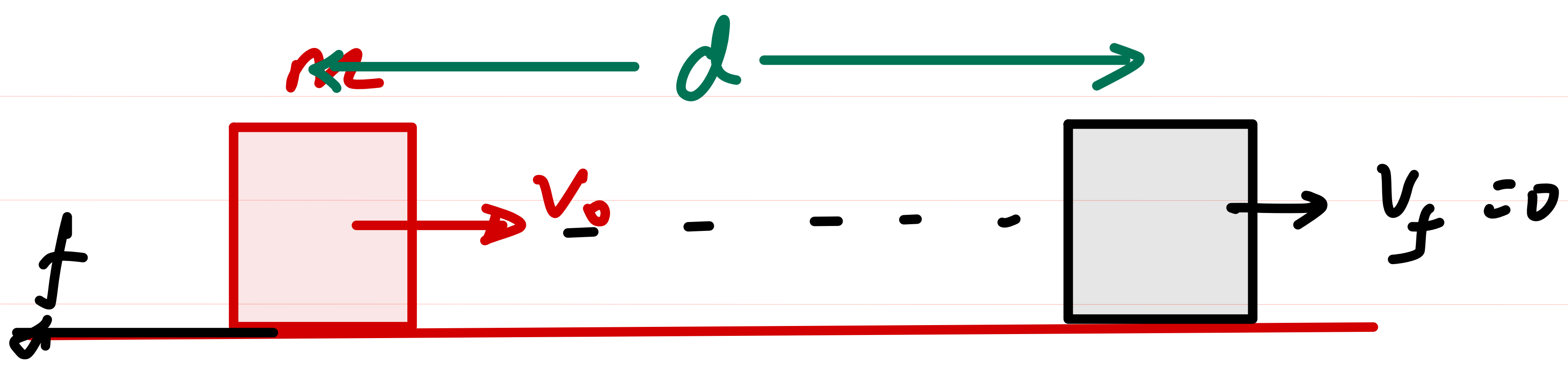
Block will slide

Ex



Rough (μ)

Find maximum distance covered by block



Rough (μ)

$$f = \mu N = ma$$

$$\mu \cancel{m} g = \cancel{m} a$$

$$\boxed{a = \mu g}$$

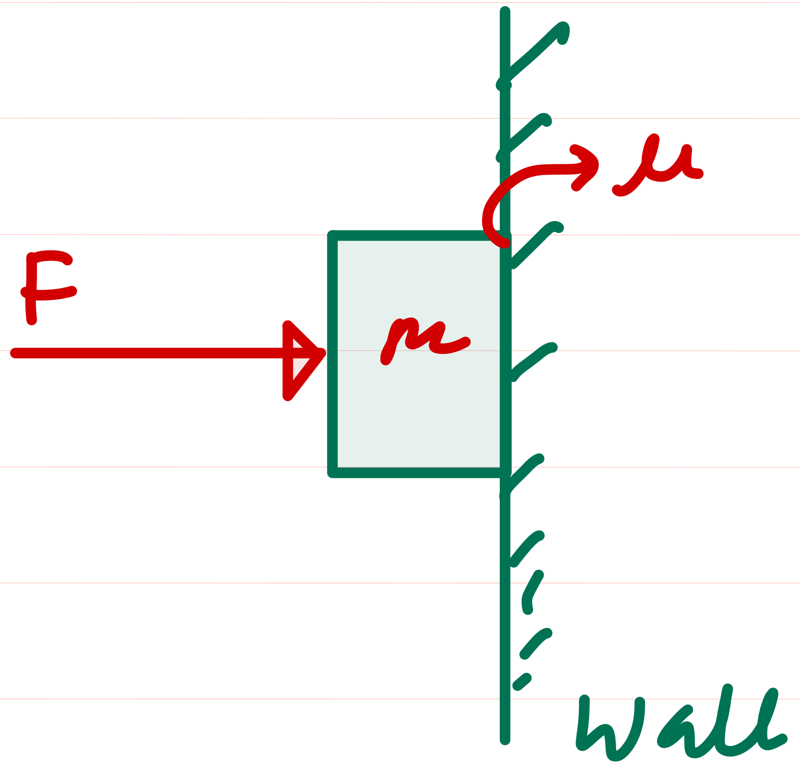
$$v^2 = u^2 + 2as$$

$$0^2 = V_0^2 + 2(-\mu g)\, d$$

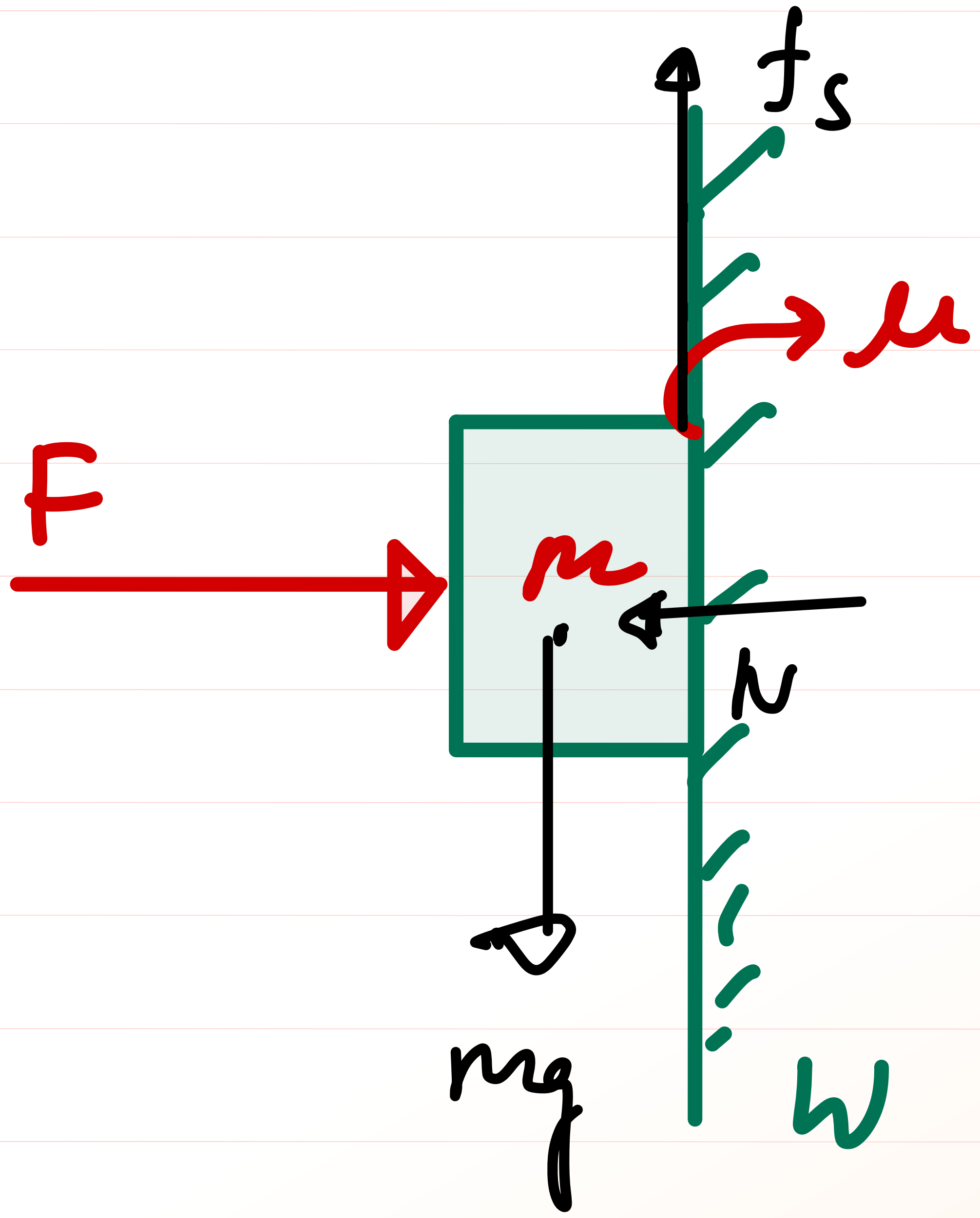
$$\boxed{d = \frac{V_0^2}{2\mu g}}$$

Ans

Ex



F_{min} for that block does not slide



$$\Sigma F_x = 0 \qquad \Sigma F_y = 0$$

$$N = F \qquad f_s = mg$$

No Sliding

$$f_s \le \mu N$$

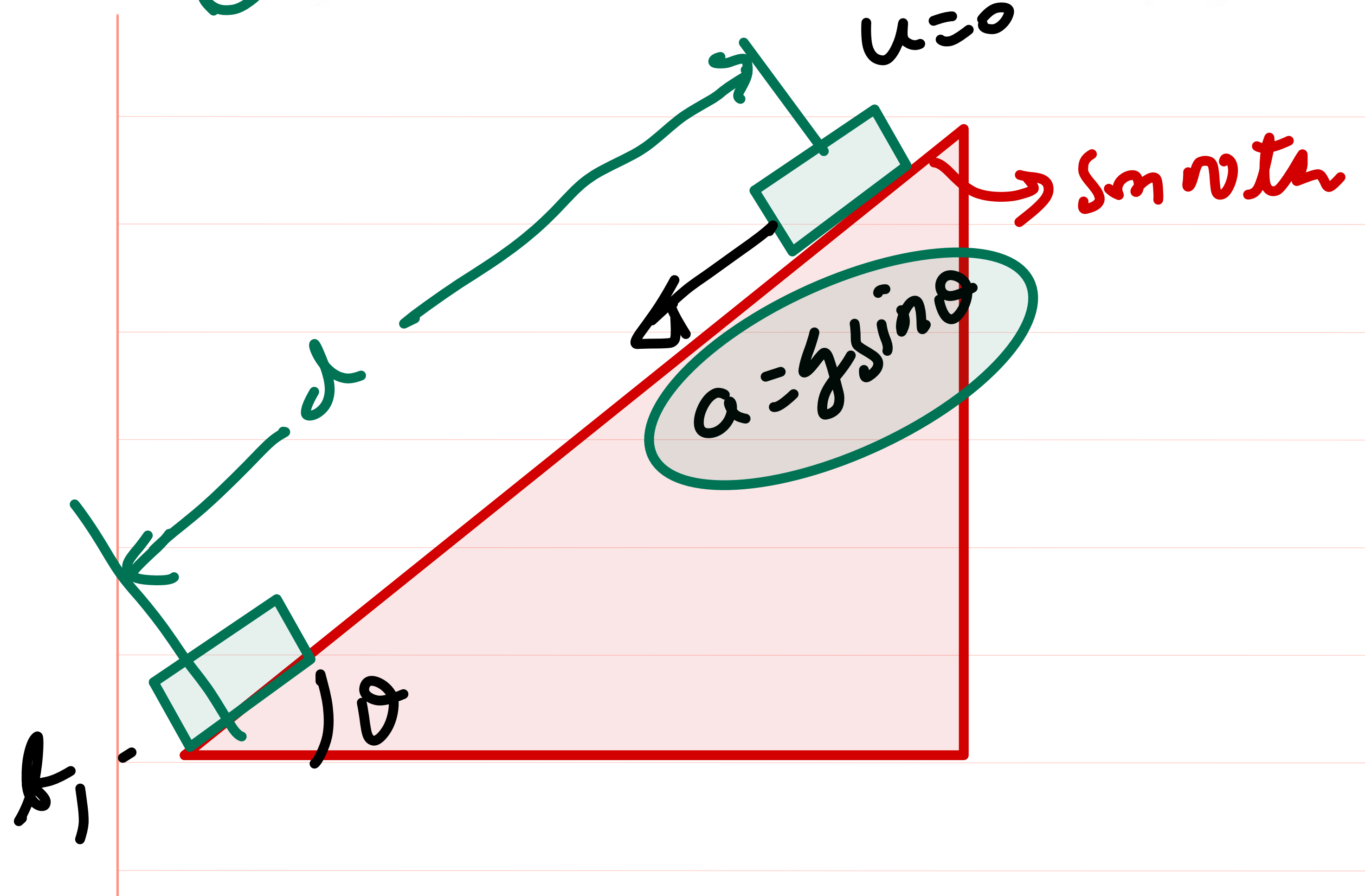
$$mg \le \mu F$$

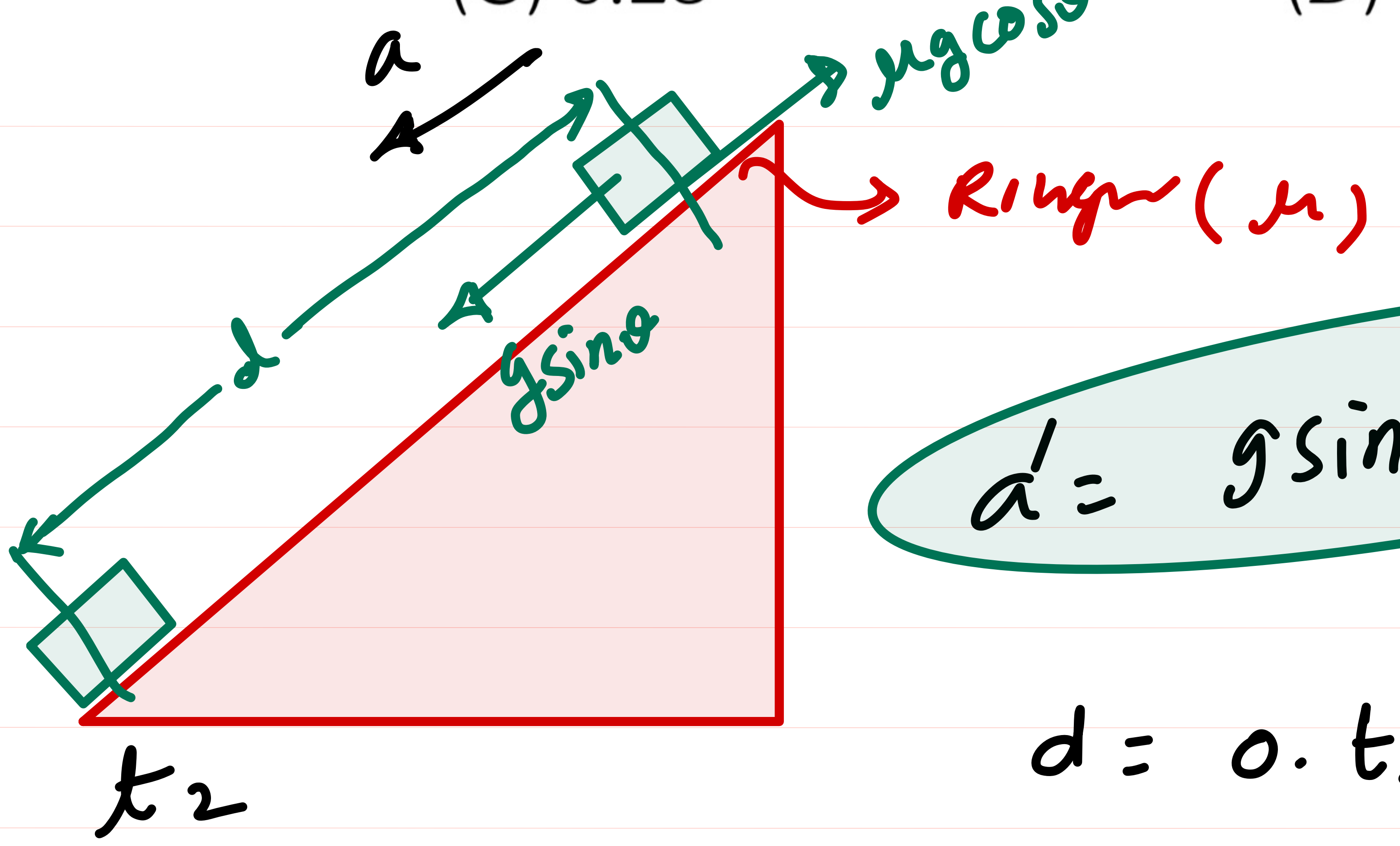
$$\boxed{F \geq \frac{mg}{\mu}}$$

Ans

2. Starting from rest a body slides down a 45° inclined plane in twice the time it takes to slide down the same distance in the absence of friction. The co-efficient of friction between the body and the inclined plane is:

(A) 0.75 ✓ (B) 0.33 (C) 0.25 (D) 0.80





$$\boxed{a' = g\sin\theta - \mu g\cos\theta}$$

$$d = 0\cdot t + \frac{1}{2}at^2$$

$$\sqrt{\frac{2d}{a}} = t_1$$

$$t_1^2 = \frac{2d}{a}$$

$$d = 0\cdot t_2 + \frac{1}{2}a't_2^2$$

$$t_2 = \sqrt{\frac{2d}{a'}} \Rightarrow t_2^2 = \frac{2d}{a'}$$

$$t_2 = 2t_1$$

$$t_2^2 = 4t_1^2$$

$$\frac{\cancel{2d}}{a'} = 4\frac{\cancel{2d}}{a}$$

$$a = 4a'$$

$$\cancel{g}\sin\theta = 4\cancel{g}(\sin\theta - \mu\cos\theta)$$

$$\frac{1}{\sqrt{2}} = 4\left(\frac{1}{\sqrt{2}} - \mu\frac{1}{\sqrt{2}}\right)$$

$$1 = 4(1-\mu)$$

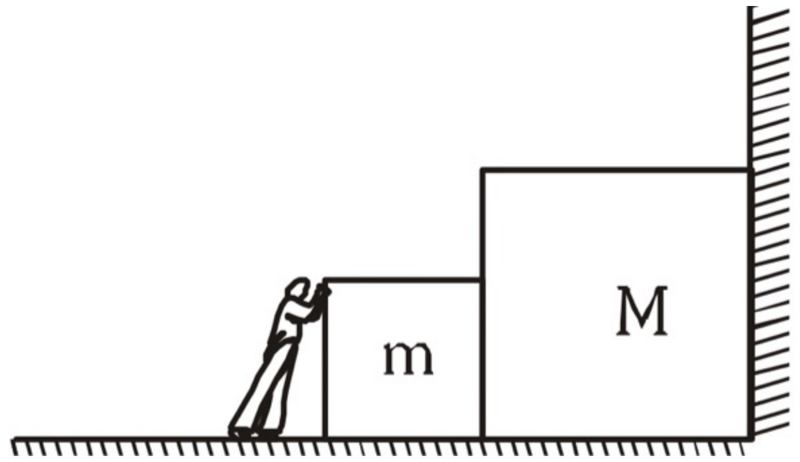
$$\frac{1}{4} = 1 - \mu$$

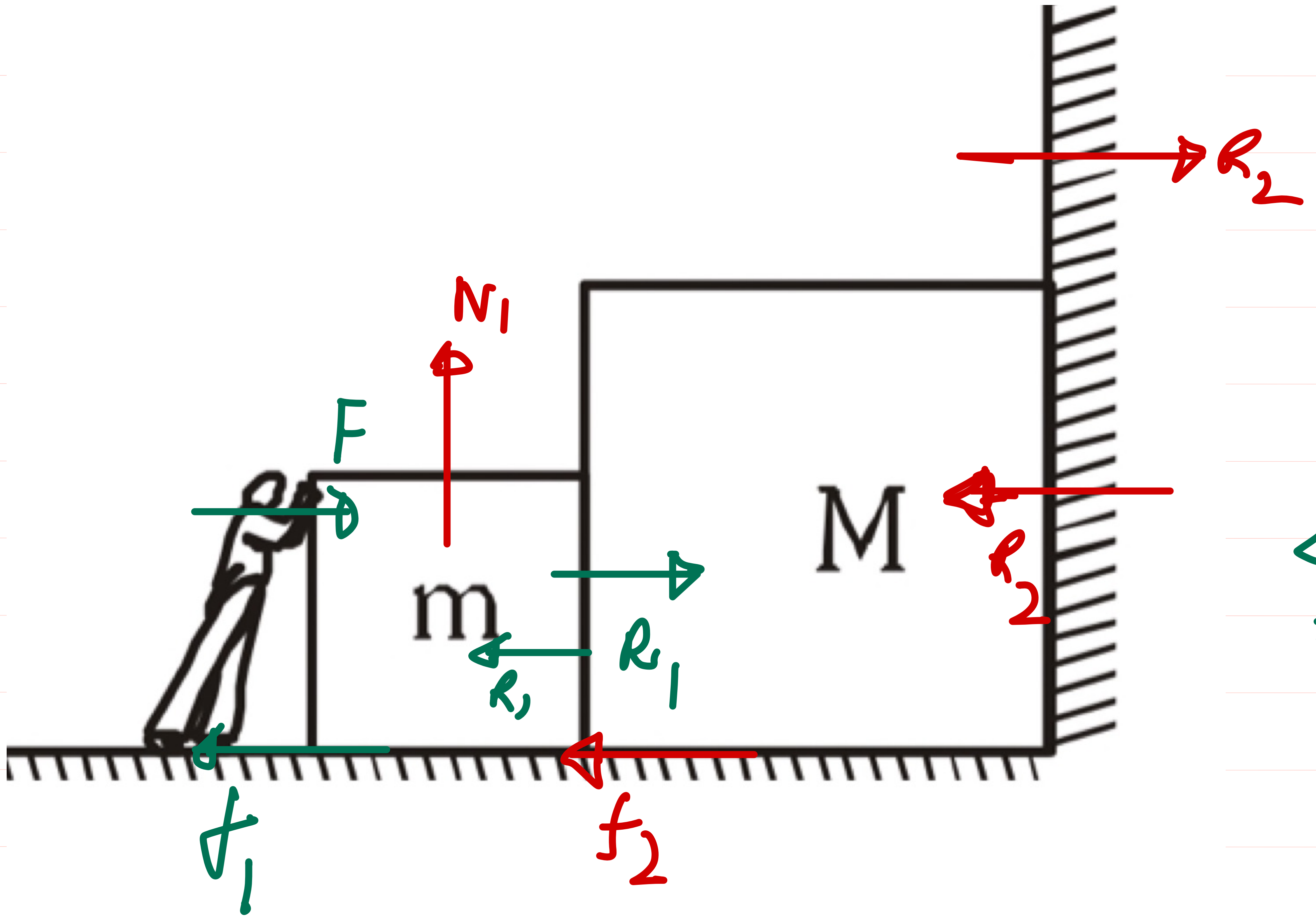
$$\mu = 1 - \frac{1}{4} = \frac{3}{4}$$

$$\boxed{\mu = 0.75}$$

Ans

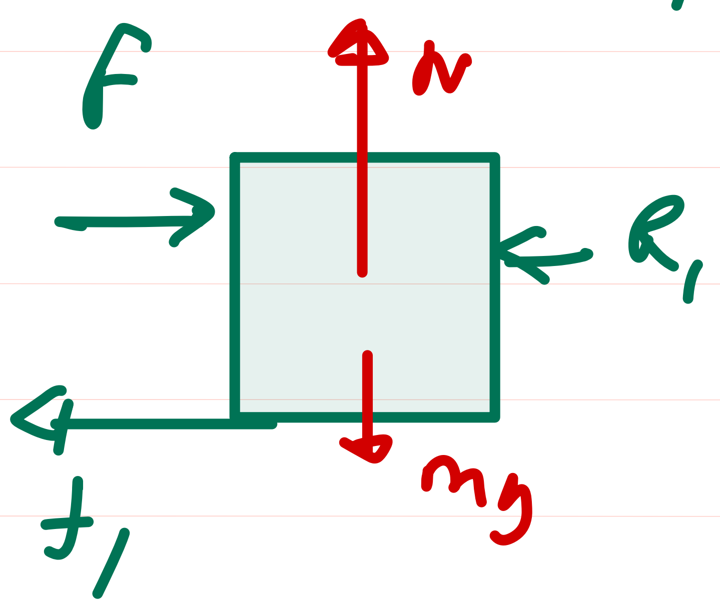
9. The person applies horizontal force F on the smaller block as shown in figure. The coefficient of static friction is μ between the blocks and the surface. Find the force exerted by the vertical wall on mass M. What is the value of action-reaction forces between m and M?





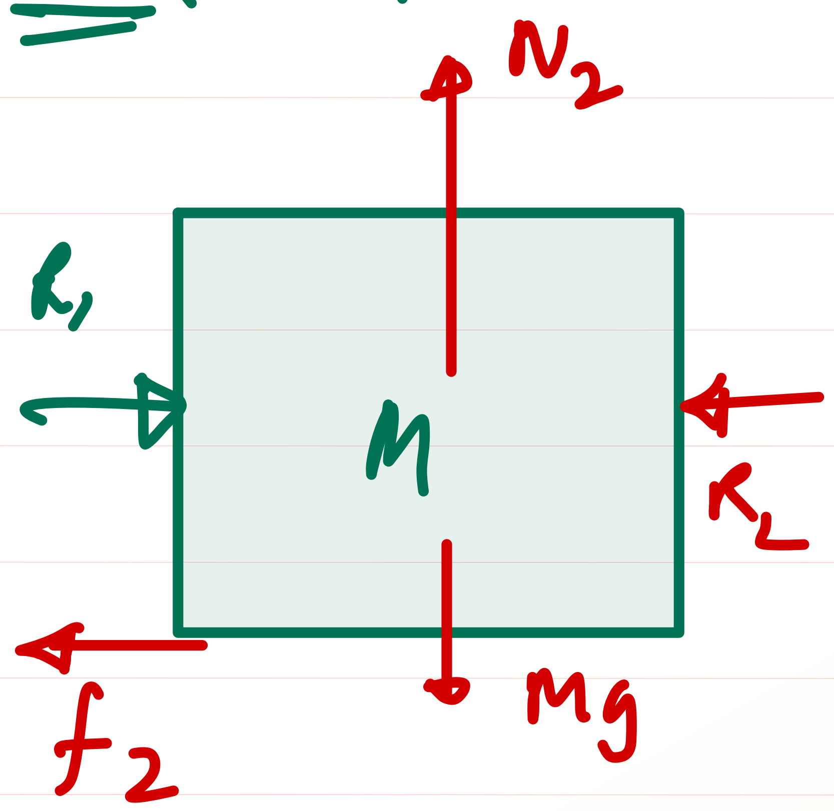
For m

$F = R_1 + f_1$

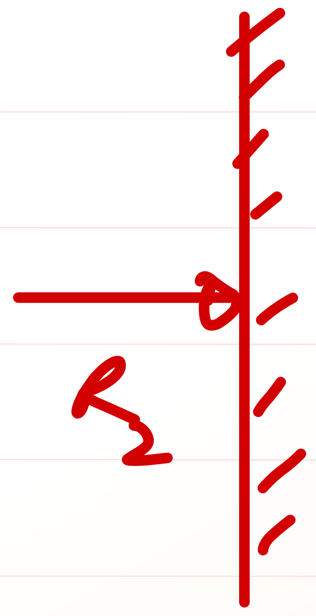


$f_1 = \mu N_1 = \mu mg$

For M



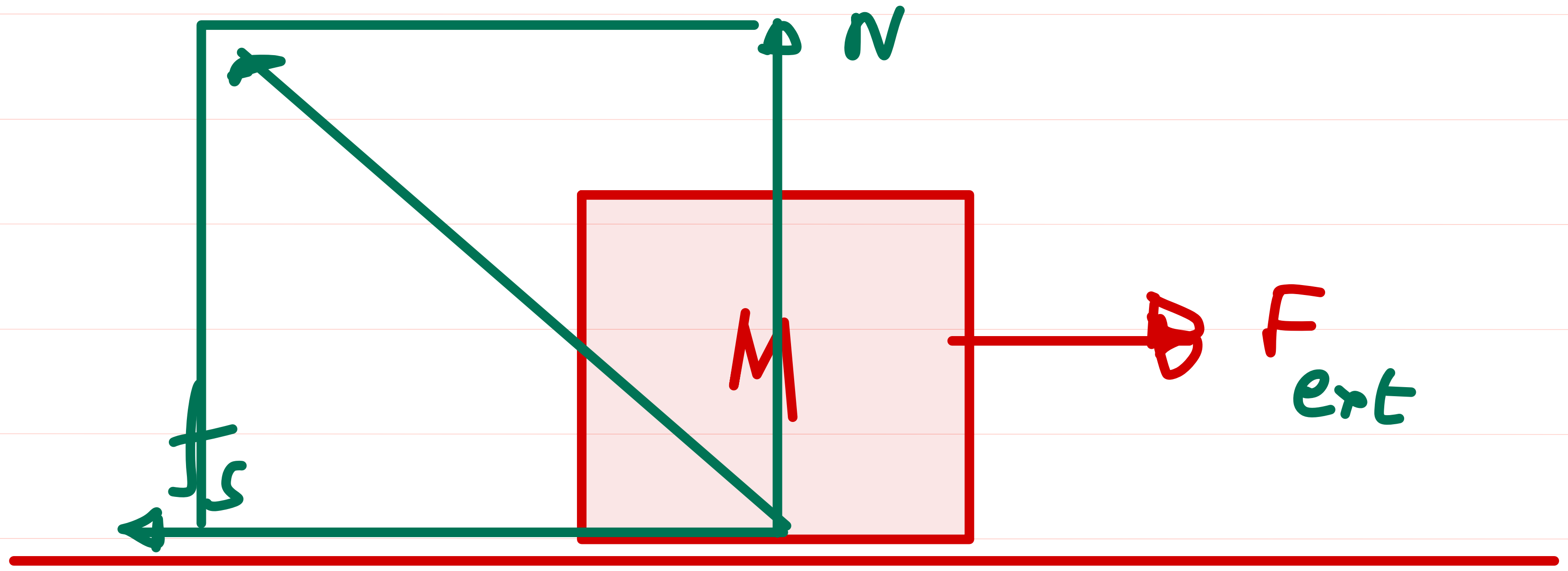
$R_1 = f_2 + R_2$



$f_2 = \mu Mg$

2. A body of mass M is kept on a rough horizontal surface (friction cofficient = μ). A person is trying to pull the body by applying a horizontal force but the body is not moving. The force by the surface on A is F where

(A) $F = Mg$ (B) $F = \mu Mg$ (C) $Mg \le F \le Mg\sqrt{1+\mu^2}$ (D) $Mg \ge F \ge Mg\sqrt{1-\mu^2}$ 1



Rough(μ)

$N = Mg$

$f_s \le \mu N$

$F_{net} = F = \sqrt{N^2 + f_s^2}$

$F_{min} = N = Mg$ when $F_{ext} = 0$

$f_s = 0$

$F_{max} = \sqrt{N^2 + (f_{sL})^2}$

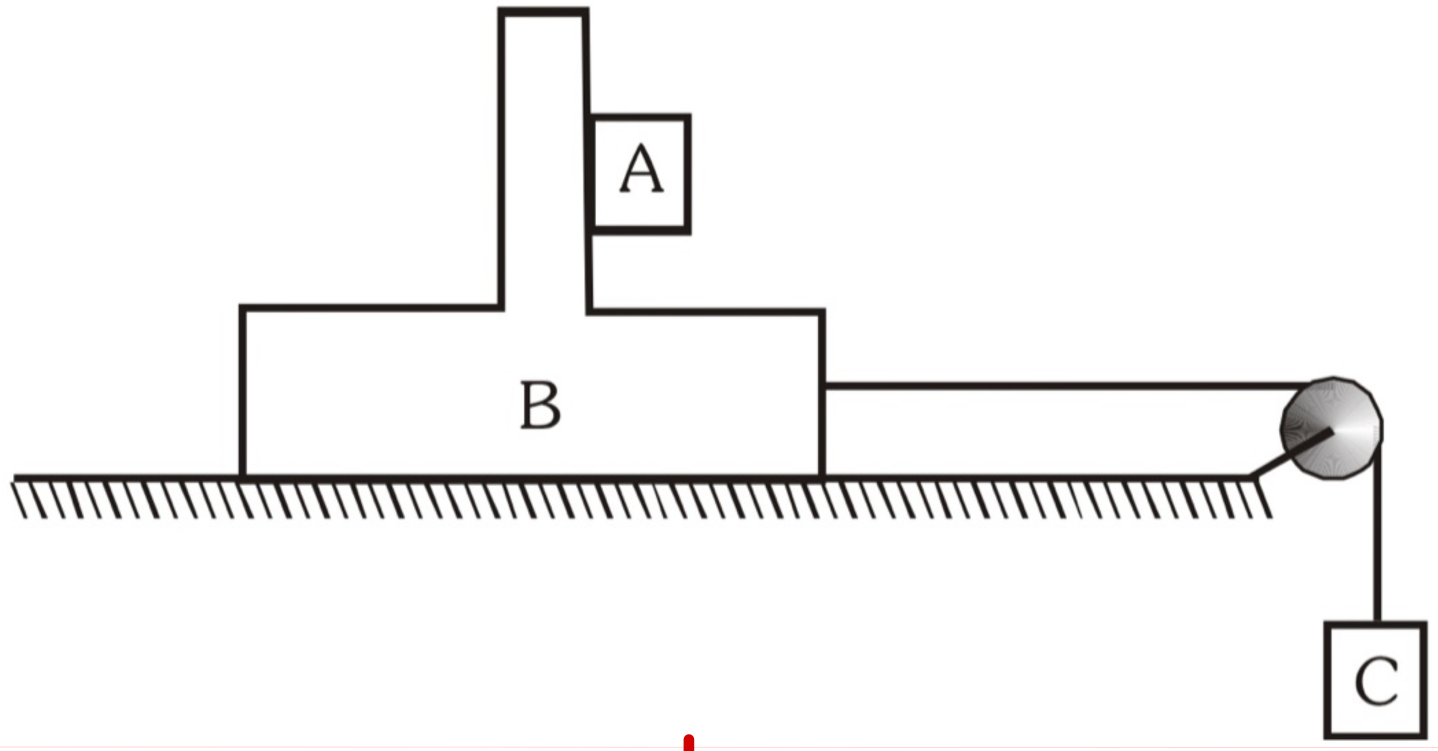
$= \sqrt{N^2 + (\mu N)^2}$

$= N\sqrt{1+\mu^2}$

$(F_{ext})_{max} = f_{sL}$

$$\boxed{Mg \le F \le Mg\sqrt{1+\mu^2}}$$

5*. In the arrangement shown in the figure, mass of the block B and A is 2m and m respectively. Surface between B and floor is smooth. The block B is connected to the block C by means of a string pulley system. If the whole system is released, then find the minimum value of mass of block C so that A remains stationary w.r.t. B. Coefficient of friction between A and B is μ.

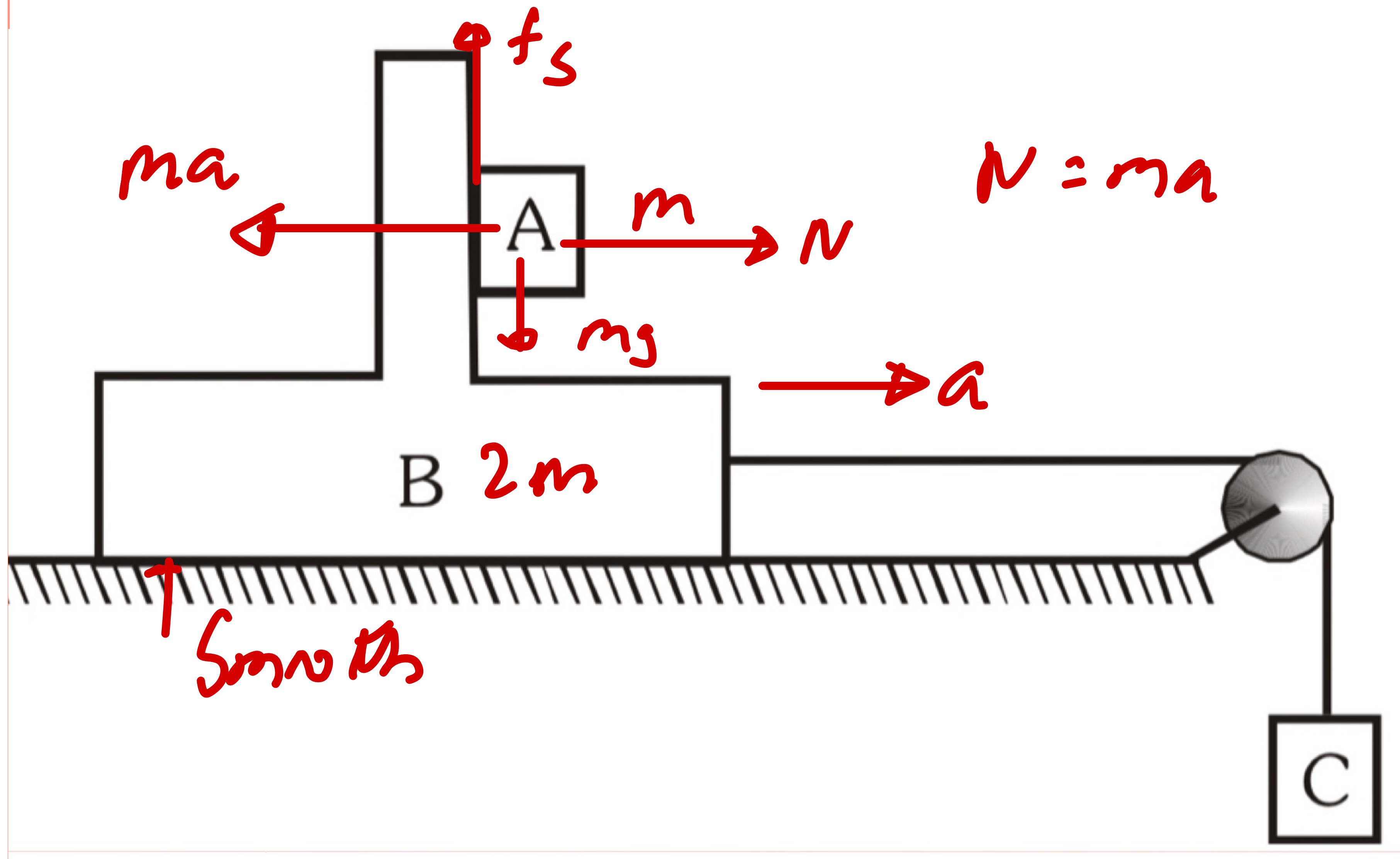


(A) $\frac{m}{\mu}$

(B) $\frac{2m+1}{\mu+1}$

(C) $\frac{3m}{\mu-1}$ ✓

(D) $\frac{6m}{\mu+1}$



m' ↓ a

$f_s = mg \le \mu N$

$\cancel{m}g \le \mu \cancel{m} a$

$\frac{g}{\mu} \le a$

$a = \frac{m'g}{2m+m+m'} = \frac{m'g}{3m+m'}$

$\frac{\cancel{g}}{\mu} \le \frac{m'\cancel{g}}{3m+m'}$

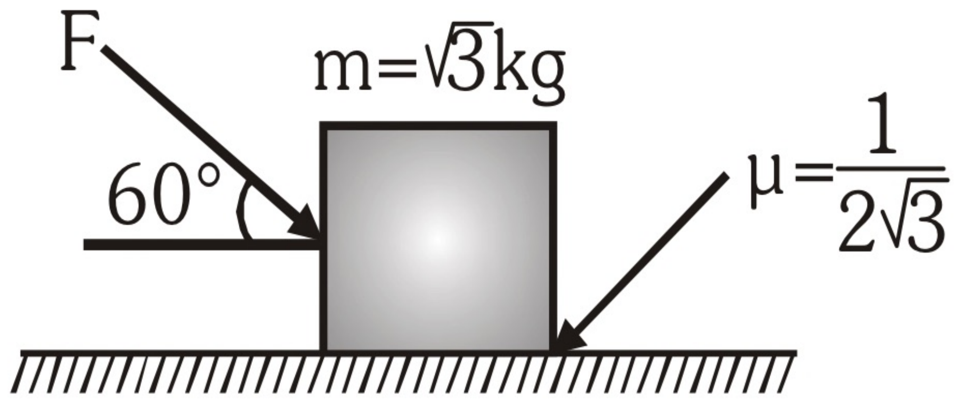
$3m+m' \le \mu m'$

$3m \le (\mu-1)m'$

$m' \ge \frac{3m}{\mu-1}$

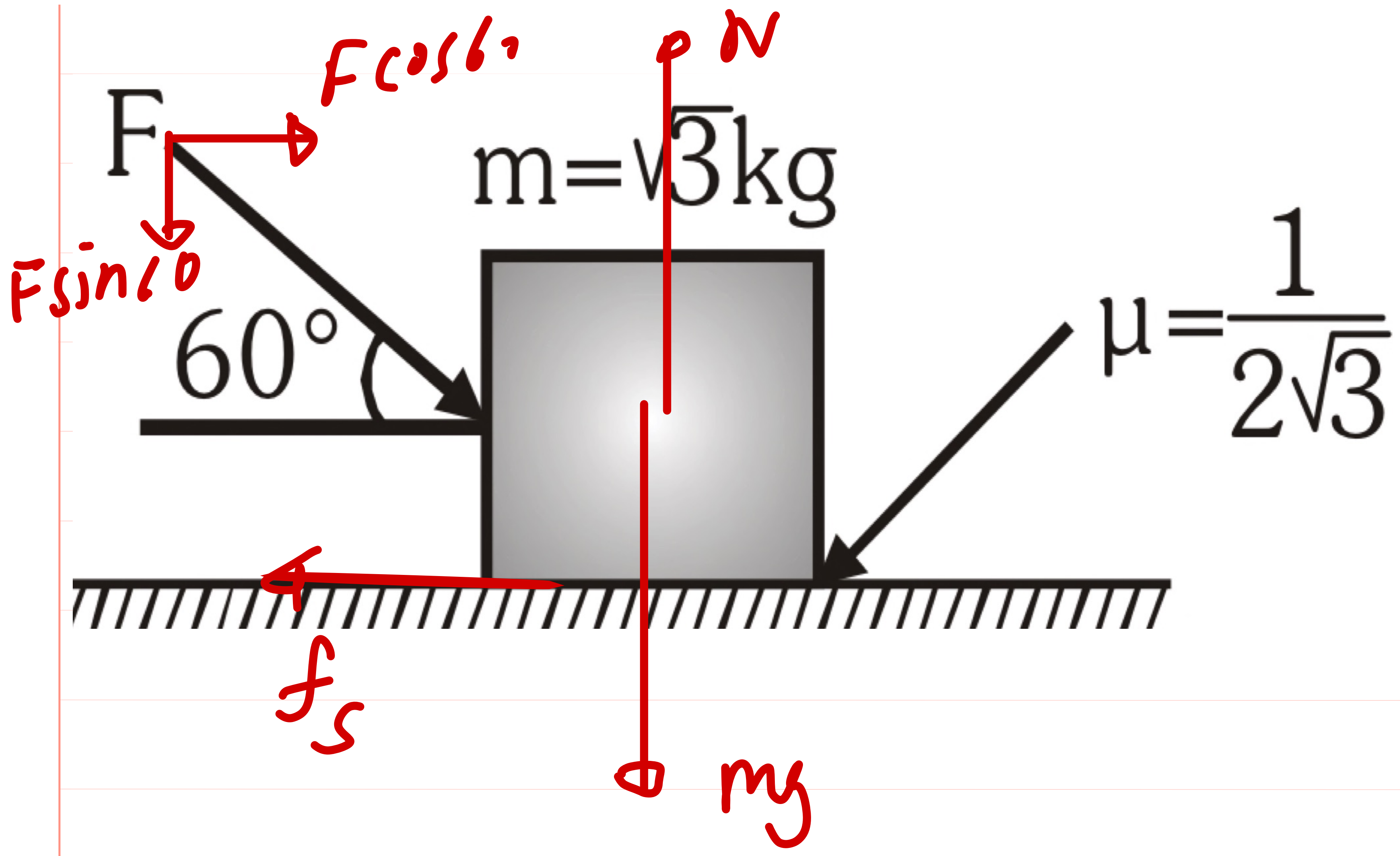
Ans

8. What is the maximum value of the force F such that the block shown in the arrangement, does not move: ***[IIT-JEE 2003]***



(A) 20 N ✓ (B) 10 N

(C) 12 N (D) 15 N



$$f_s = F\cos 60 \le \mu N$$

$$N = mg + F\sin 60$$

$$F\cos 60 \le \mu(mg + F\sin 60)$$

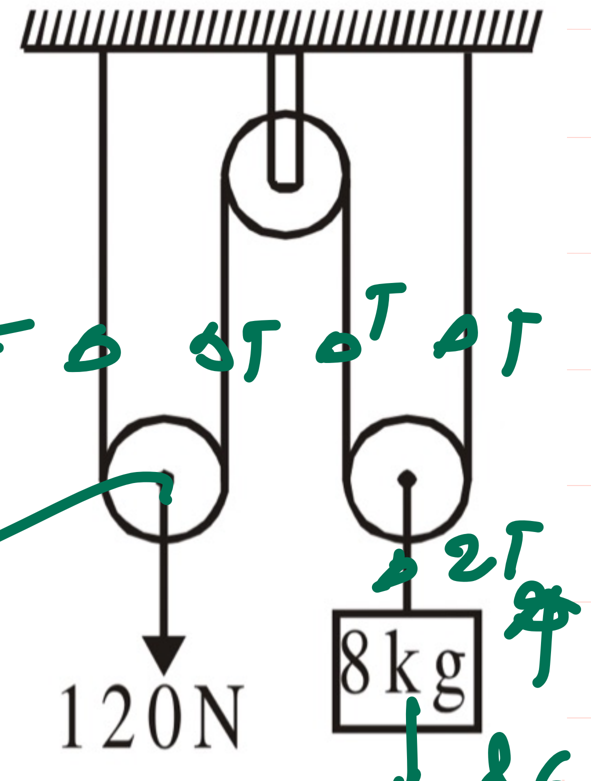
$$F\cos 60 - \mu F\sin 60 \le \mu mg$$

$$F \le \frac{\mu mg}{\cos 60 - \mu\sin 60}$$

$$F_{max} = \frac{\mu mg}{\cos 60 - \mu \sin 60} = \frac{\frac{1}{2\cancel{\sqrt3}} \times \cancel{\sqrt3} \times 10}{\frac{1}{2} - \frac{1}{2\cancel{\sqrt3}} \times \frac{\cancel{\sqrt3}}{2}} = \frac{5}{\frac{1}{4}} = 20\ N$$

Illustration 3.

If the string is pulled down with a force of 120 N as shown in the figure, then the acceleration of 8 kg block would be

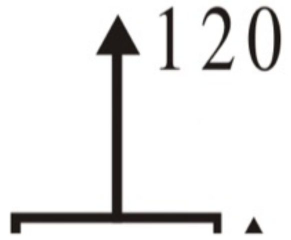


(A) 10 m/s^2 (B) 5 m/s^2

(C) 0 m/s^2 (D) 4 m/s^2

Ans. **(B)**

Solution



For 8 kg Block

$2T - 8g = 8a$

$120 - 80 = 8a$

$a = \frac{40}{8} = 5 \, m/s^2$

$120 - 2T = m_p a_p \quad \because m_p = 0$

$120 - 2T = 0$

$T = 60 \, N$