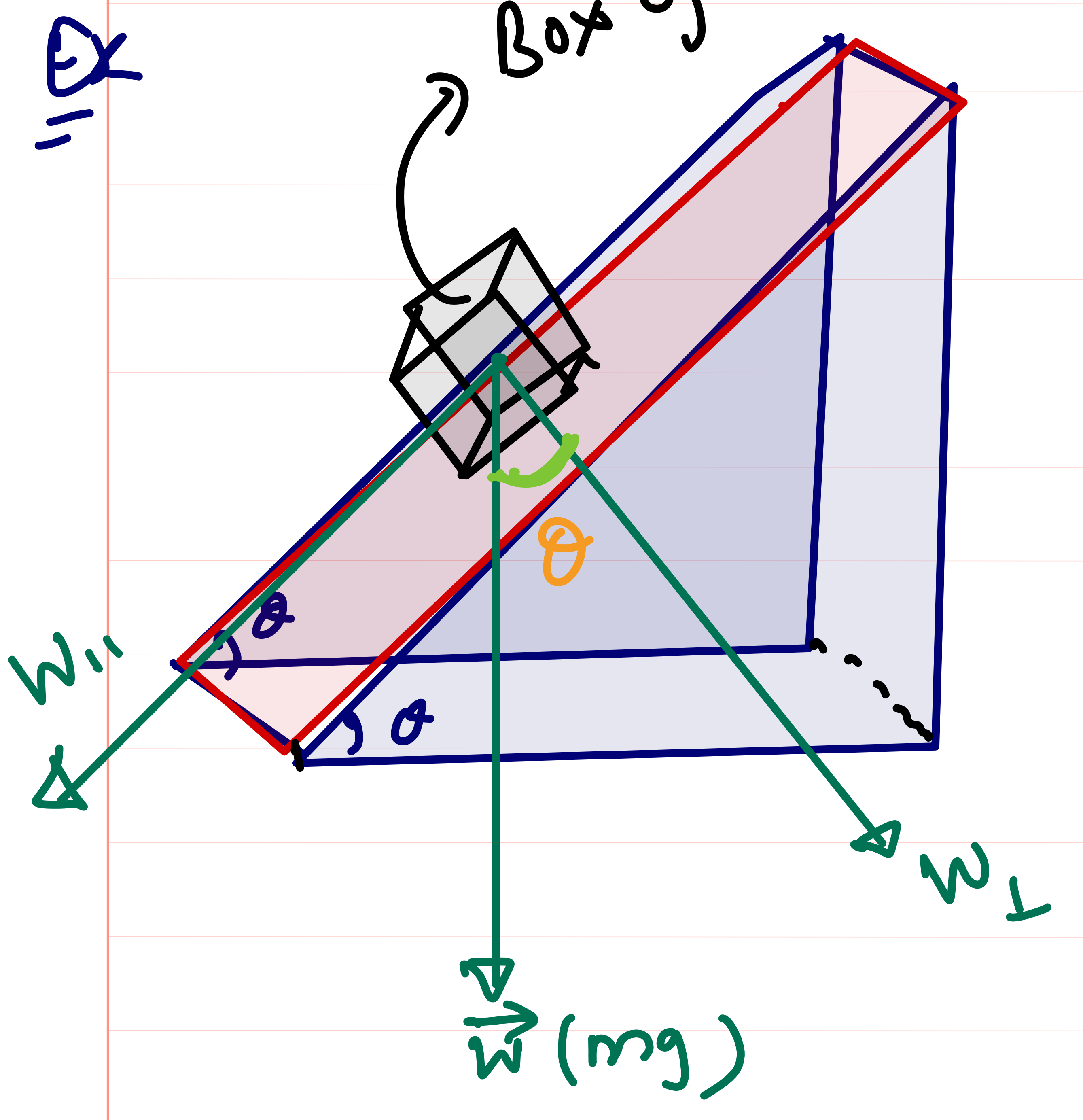
Ex



Find component of Weight along and ⊥r to Inclined Plane

$W_{\perp} = W\cos\theta = mg\cos\theta$ ⊥r

$W_{\parallel} = W\sin\theta = mg\sin\theta$ Along

let $m = 5g$ $\theta = 37^\circ$ $g = 10\ m/s$

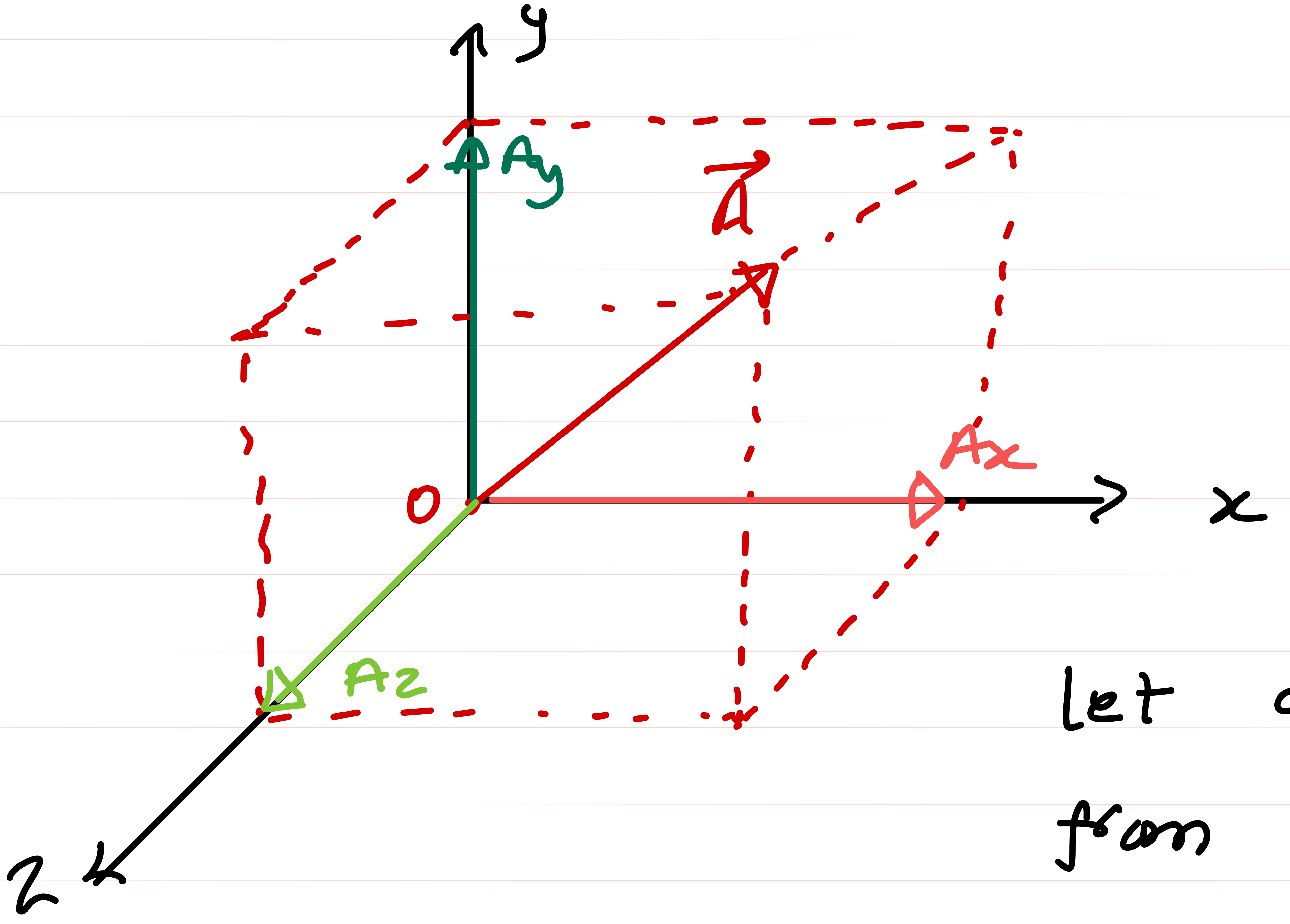
$W_{\perp} = 5\times10\cos37 = 40\ N$

$W_{\parallel} = 5\times10\sin37 = 30\ N$

Ans

3-D Component :→

"Dividing A vector along 3-mutually $\perp^r$ axes"



$$\vec{A} = \vec{A_x} + \vec{A_y} + \vec{A_z}$$

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

let α, β and γ are angle make by $\vec{A}$ from x-axis, y-axis and z-axis respectively

$\cos\alpha = \frac{A_x}{A} \Rightarrow A_x = A\cos\alpha$

$\cos\beta = \frac{A_y}{A} \Rightarrow A_y = A\cos\beta$

$\cos\gamma = \frac{A_z}{A} \Rightarrow A_z = A\cos\gamma$

$\cos\alpha$, $\cos\beta$ and $\cos\gamma$ are known as Direction cosines

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = \frac{A_x^2 + A_y^2 + A_z^2}{A^2}$$

$$\boxed{\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1}$$

Ex If $\vec{A} = 2\hat{i} + \hat{j} + 2\hat{k}$

$A = \sqrt{2^2 + 1^2 + 2^2}$

$A = 3$

① Find Direction cosines

$\cos\alpha = \frac{A_x}{A} = \frac{2}{3}$

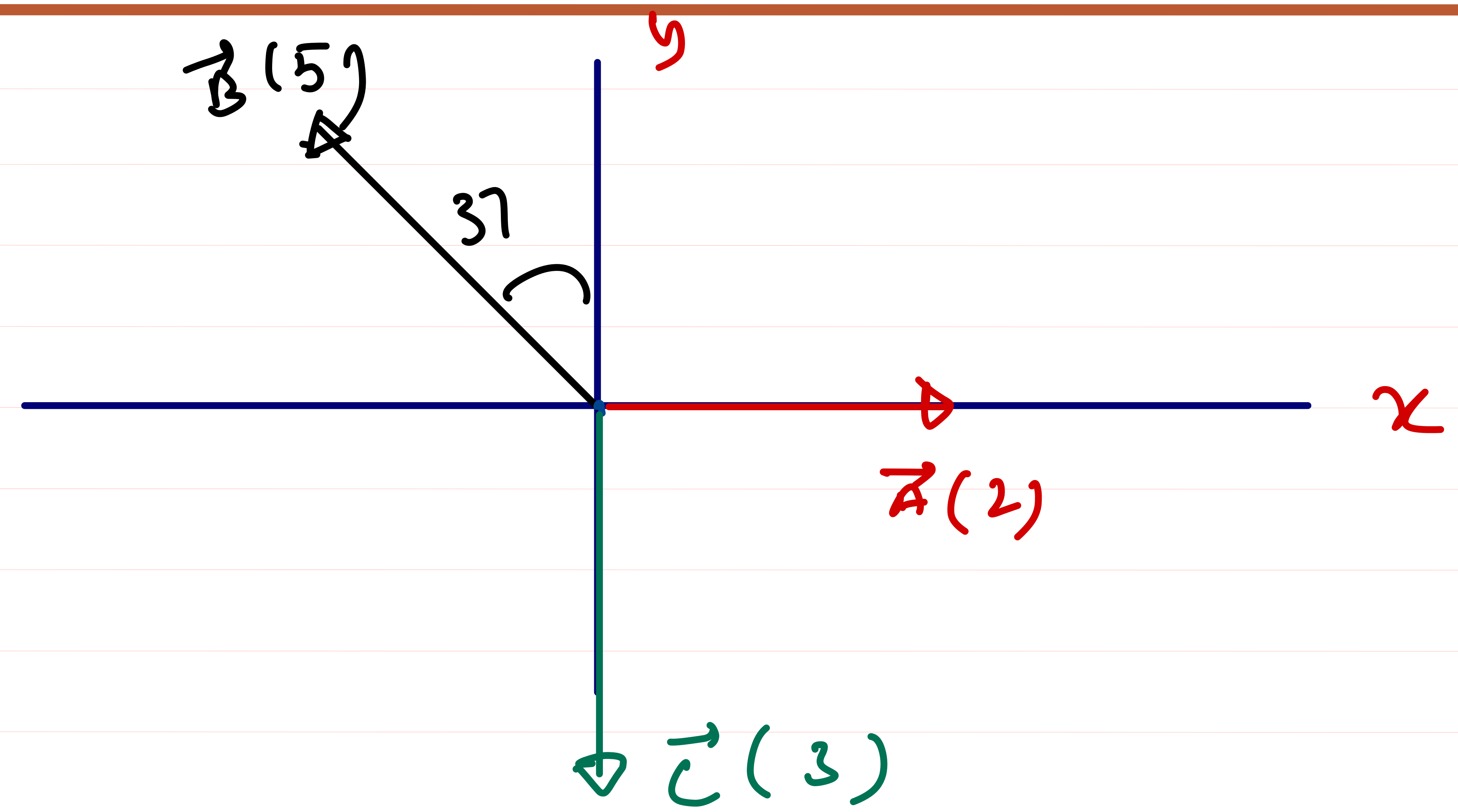
$\cos\beta = \frac{A_y}{A} = \frac{1}{3}$

$\cos\gamma = \frac{A_z}{A} = 2/3$

② Angles from axes

$\alpha = \cos^{-1}(2/3)$ $\beta = \cos^{-1}(1/3)$ $\gamma = \cos^{-1}(2/3)$

Ex



Find $\vec{A}+\vec{B}+\vec{C}$ and angle from +ve x-axis

let $\vec{R} = \vec{A}+\vec{B}+\vec{C}$

$$\vec{R} = (A_x + B_x + C_x)\hat{i} + (A_y + B_y + C_y)\hat{j}$$

$$\vec{R} = (2-3+0)\hat{i} + (0+4-3)\hat{j}$$

$$\boxed{\vec{R} = -\hat{i} + \hat{j}}$$ Ans

$A_x = 2$

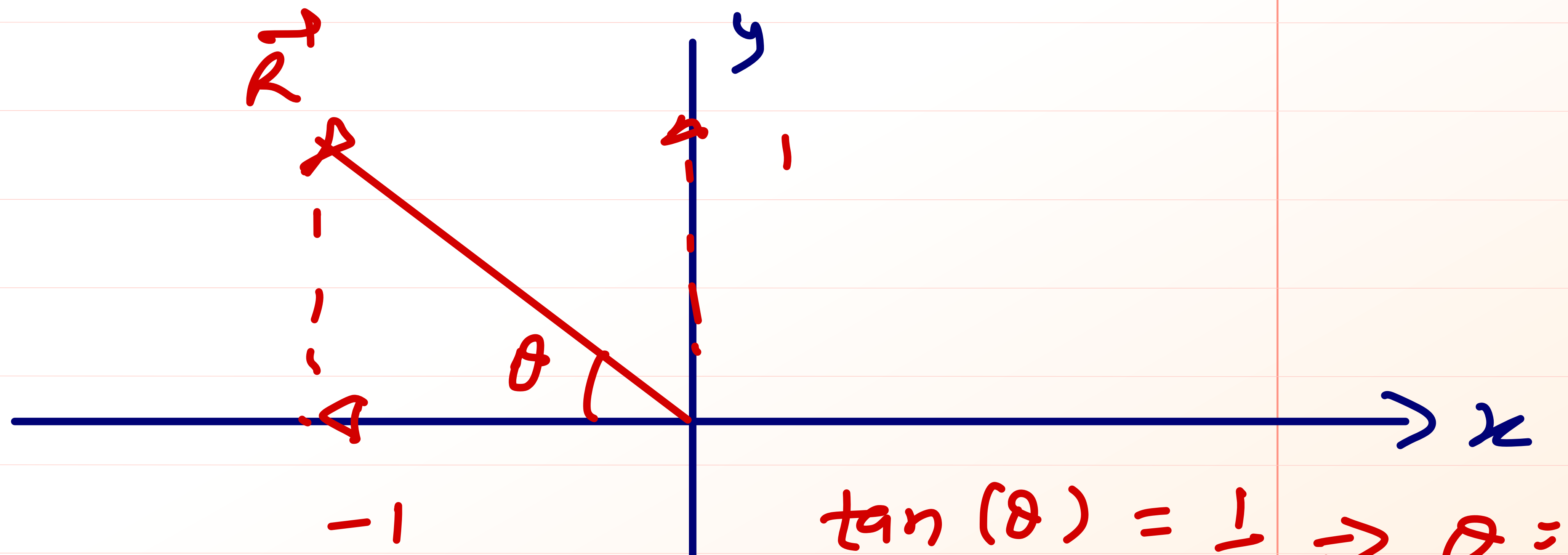
$A_y = 0$

$B_x = -5\sin 37 = -3$

$B_y = +5\cos 37 = 4$

$C_x = 0$

$C_y = -3$

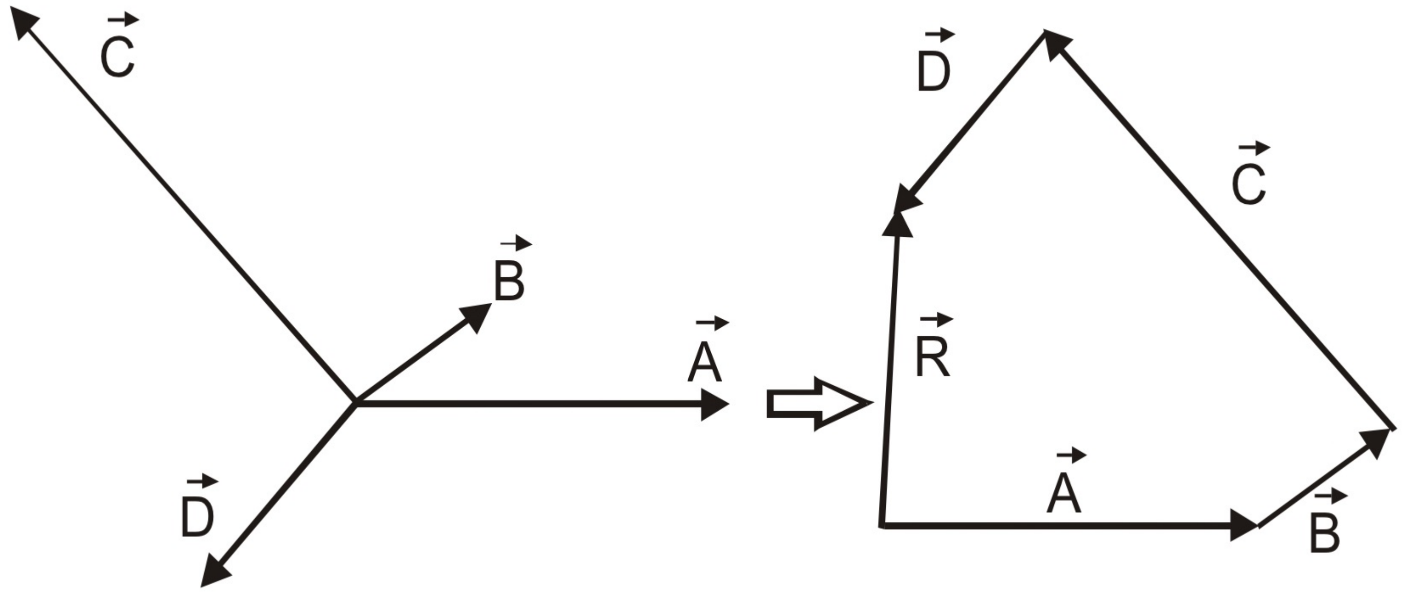


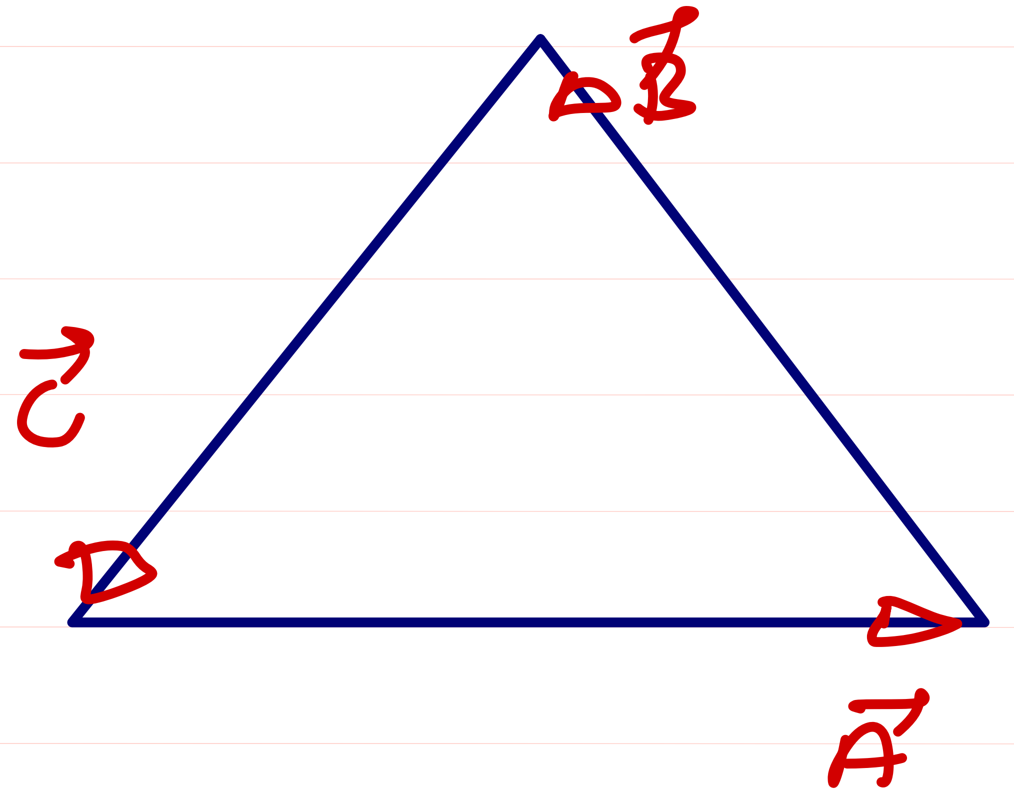
$\tan(\theta) = \frac{1}{1} \Rightarrow \theta = 45^\circ$

$\boxed{\text{from +ve x-axis} = 135^\circ}$ Ans

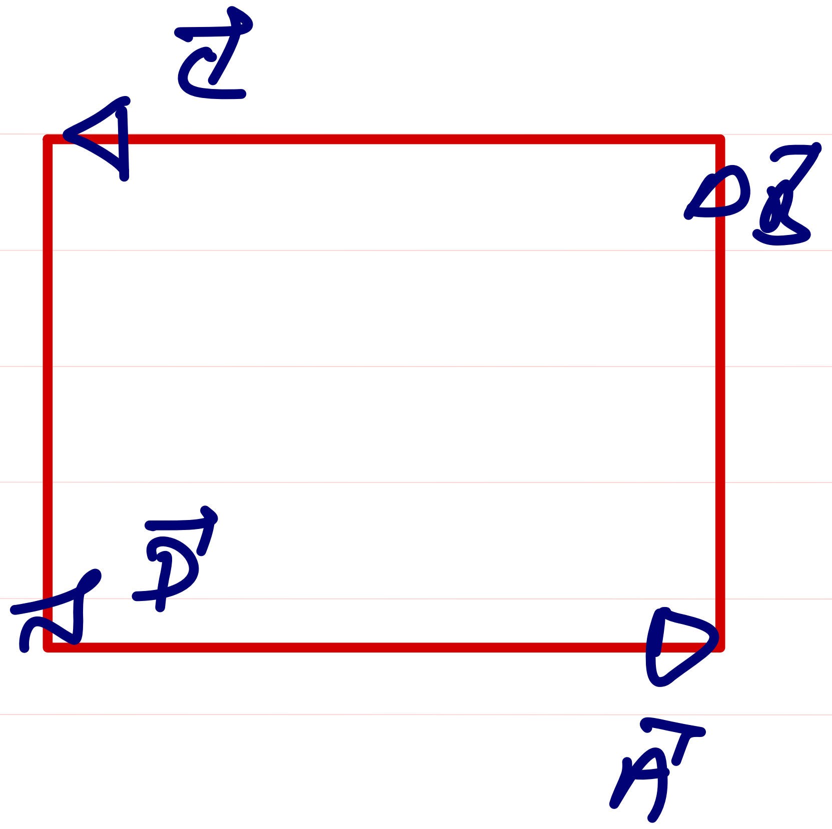
Addition of More Than Two Vectors (Law of Polygon)

If some vectors are represented by sides of a polygon in same order, then their resultant vector is represented by the closing side of polygon in the opposite order. $\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D}$

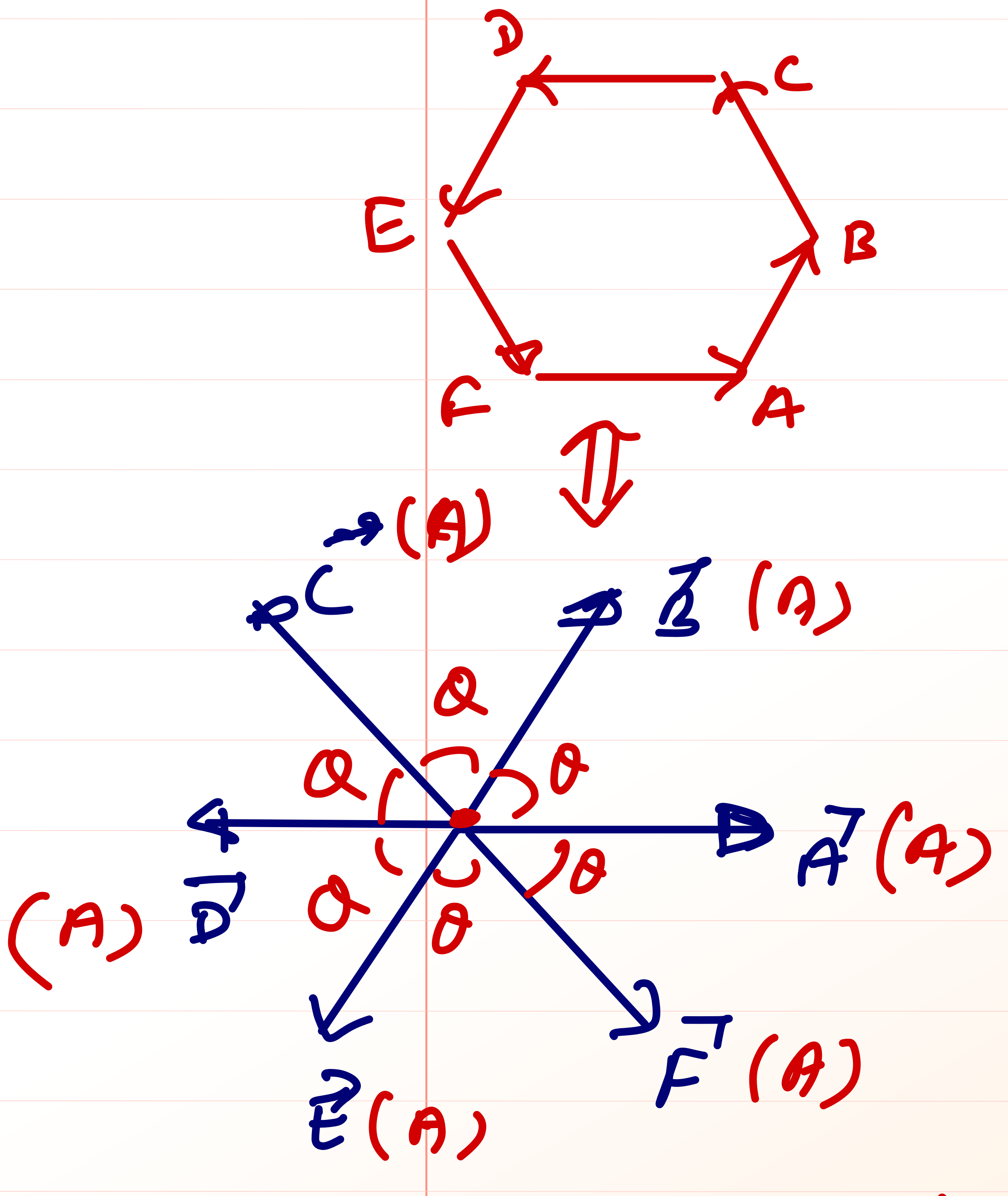




$\vec{A} + \vec{B} + \vec{C} = \vec{0}$



$\vec{A} + \vec{B} + \vec{C} + \vec{D} = \vec{0}$

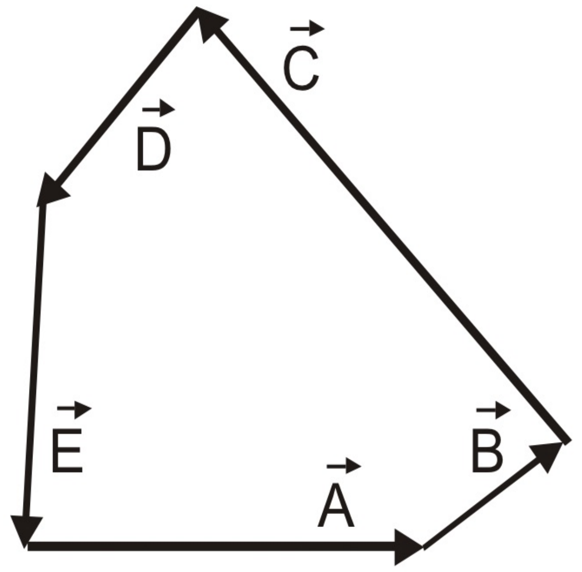


$\vec{A} + \vec{B} + \vec{C} + \vec{D} + \vec{E} + \vec{F} = 0$

NOTE

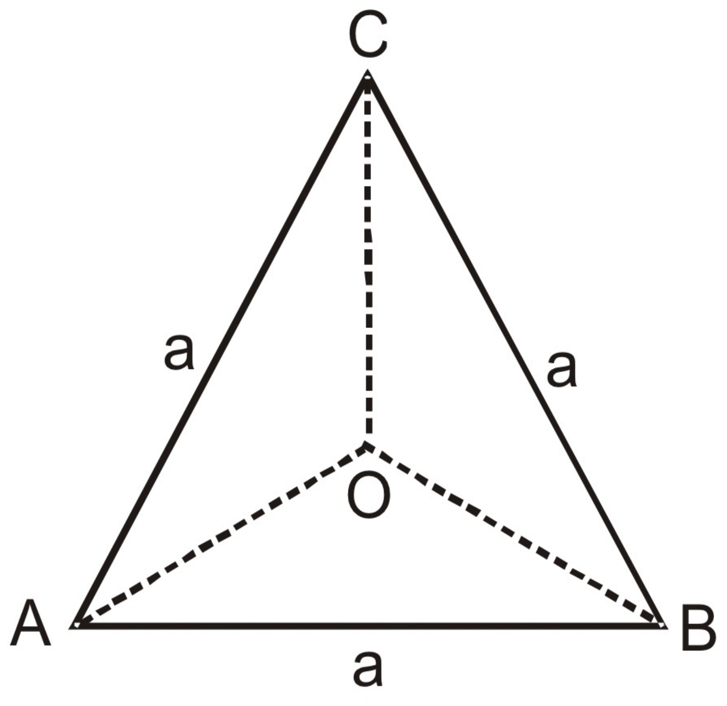
- In a polygon if all the vectors are in same order then their resultant is a null vector.

$$\boxed{\vec{A} + \vec{B} + \vec{C} + \vec{D} + \vec{E} = \vec{0}}$$



- If n vectors of equal magnitude are arranged at equal angles of separation then their resultant is always zero.

Ex. ABC is an equilateral triangle. Length of each side is 'a' and centroid is point



(i) $\vec{AB}+\vec{BC}+\vec{CA}=?$ (ii) $\vec{OA}+\vec{OB}+\vec{OC}=?$

(iii) If $|\vec{AB}+\vec{BC}+\vec{AC}| = n\,a$ then n = ?

(iv) If $\vec{AB}+\vec{AC} = n\,\vec{AO}$ then n = ?

(i) $\vec{AB}+\vec{BC}+\vec{CA}=\vec{0}$ Ans

(ii) $\vec{OA}+\vec{OB}+\vec{OC}=\vec{0}$ Ans

(iii) $|\vec{AB}+\vec{BC}+\vec{AC}|$

$\vec{AB}+\vec{BC}=\vec{AC}$ (Δ^l law)

$\therefore$ $|2\vec{AC}| = 2a$

$\boxed{n=2}$ Ans

Δ OAB

$\vec{OA}+\vec{AB}=\vec{OB} \Rightarrow \vec{AB}=\vec{OB}-\vec{OA}$

Δ OAC

$\vec{OA}+\vec{AC}=\vec{OC} \Rightarrow \vec{AC}=\vec{OC}-\vec{OA}$

(IV) $\vec{AB}+\vec{AC}=n\,\vec{AO}$

$=\vec{AB}+\vec{AC}$

$=\vec{OB}-\vec{OA}+\vec{OC}-\vec{OA}$

$=\vec{OB}+\vec{OC}-2\vec{OA}$

$\because \vec{OA}+\vec{OB}+\vec{OC}=\vec{0}$

$\vec{OB}+\vec{OC}=-\vec{OA}$

$=-3\vec{OA}$

$=+3\vec{AO}$

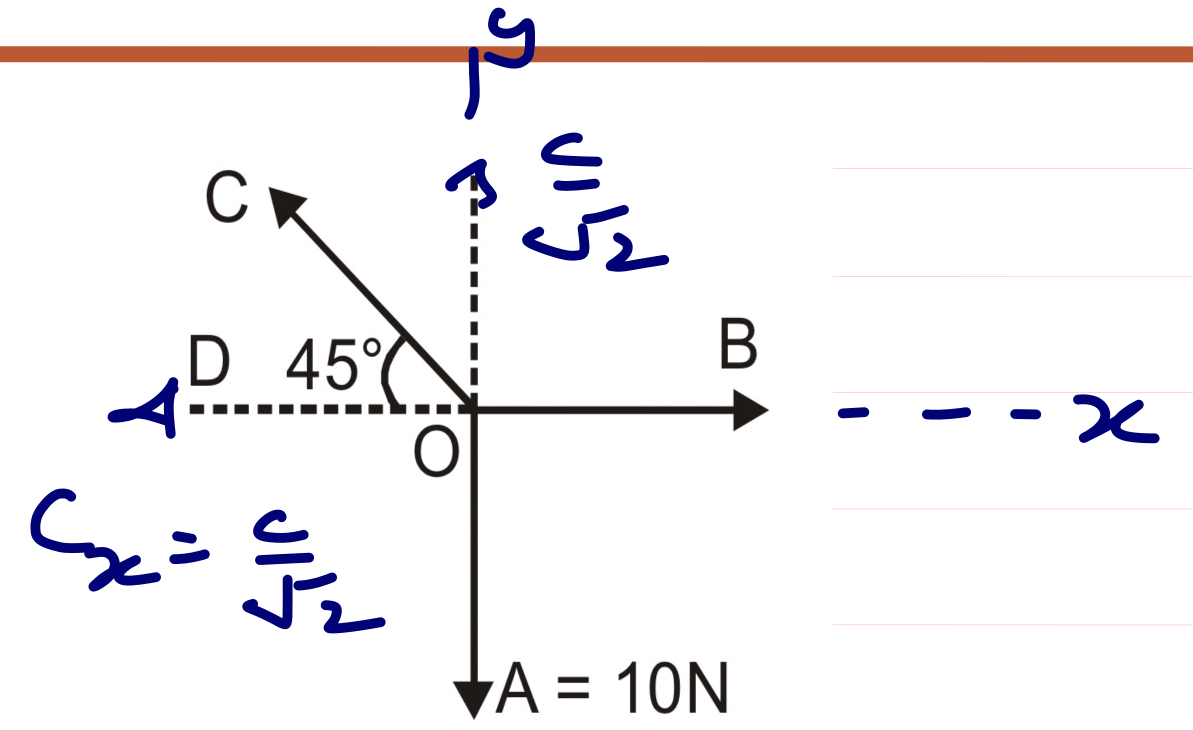
$\boxed{n=3}$

Ans

Ex. The sum of three vectors shown in figure, is zero.

(i) What is the magnitude of vector $\overrightarrow{OB}$?

(ii) What is the magnitude of vector $\overrightarrow{OC}$?



let $\vec{R} = \vec{A} + \vec{B} + \vec{C}$

$= (A_x + B_x + C_x)\hat{i} + (A_y + B_y + C_y)\hat{j}$

$\vec{R} = \left[0 + B + \left(-\frac{C}{\sqrt{2}}\right)\right]\hat{i} + \left(-10 + 0 + \frac{C}{\sqrt{2}}\right)\hat{j}$

Given $\vec{R} = \vec{0}$

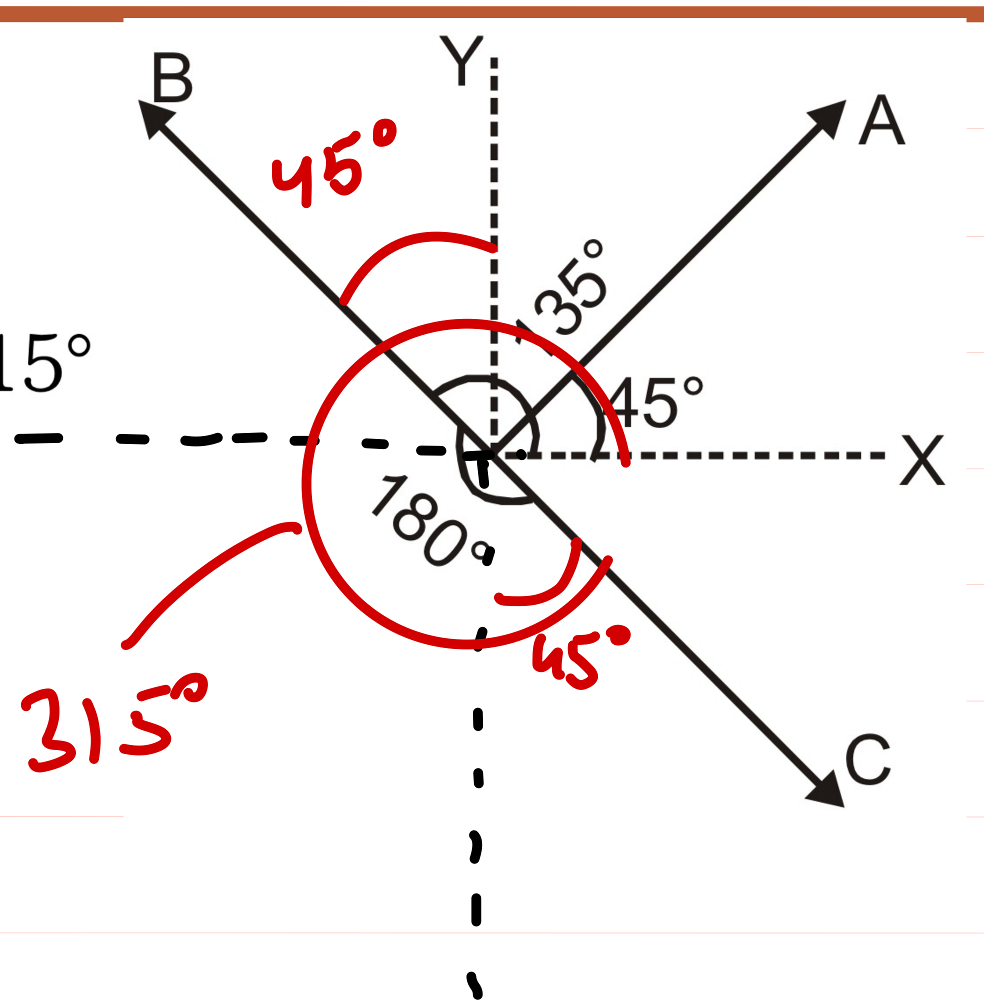
$0 = \left(B - \frac{C}{\sqrt{2}}\right)\hat{i} + \left(\frac{C}{\sqrt{2}} - 10\right)\hat{j}$

$B - \frac{C}{\sqrt{2}} = 0$ And $\frac{C}{\sqrt{2}} - 10 = 0 \Rightarrow \boxed{C = 10\sqrt{2}}$

$B = \frac{C}{\sqrt{2}} = \frac{10\sqrt{2}}{\sqrt{2}} = 10$

$\boxed{B = 10}$ Ans

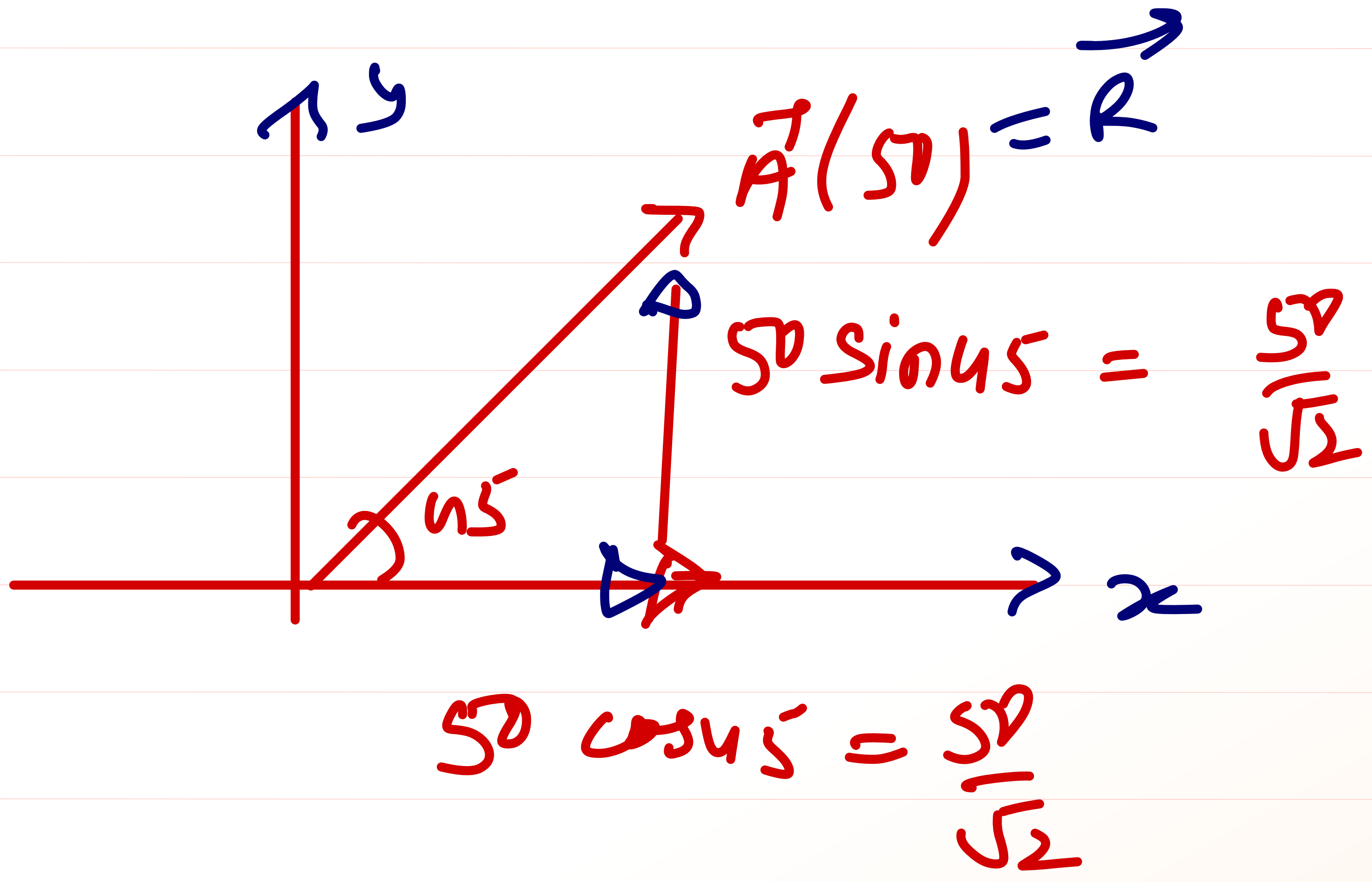
Ex. Add vectors $\vec{A}, \vec{B}$ and $\vec{C}$ which have equal magnitude of 50 unit and are inclined at angles of 45°, 135° and 315° respectively from x-axis.



Let $\vec{R} = \vec{A} + \vec{B} + \vec{C}$

$\because \vec{B} = -\vec{C}$

$\vec{R} = \vec{A}$ Ans



$$\boxed{\vec{R} = \frac{50}{\sqrt{2}}\hat{i} + \frac{50}{\sqrt{2}}\hat{j}}$$

Ans

H.W

Ex-1 The vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}+\vec{b}|=|\vec{a}-\vec{b}|$, then the angle between $\vec{a}$ and $\vec{b}$ will be :–

(1) $\frac{\pi}{3}$ (2) π (3) $\frac{\pi}{2}$ (4) zero

Ex-2 Force 3N, 4N and 12N act at a point in mutually perpendicular directions. The magnitude of the resultant force is :–

(1) 19 N (2) 13 N (3) 11 N (4) 5 N

Ex-3 If $|\vec{A}+\vec{B}|=|\vec{A}|=|\vec{B}|$ then angle between A and B will be :–

(1) 90° (2) 120° (3) 0° (4) 60°

Ex-4 The direction cosines of a vector $\hat{i}+\hat{j}+\sqrt{2}\hat{k}$ are :-

(1) $\frac{1}{2},\frac{1}{2},1$ (2) $\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}},\frac{1}{2}$

(3) $\frac{1}{2},\frac{1}{2},\frac{1}{\sqrt{2}}$ (4) $\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}$

B.B # 4 complete

Race #16 complete

Race #17 1, 2, 3, 4, 7, 8, 9