BB-1

7) $$a^2+b^2+c^2-ab-bc-ca = \frac{1}{2}\left[(a-b)^2+(b-c)^2+(c-a)^2\right]$$

$$a=b=c=k$$

$$\frac{a+b}{c} = \frac{k+k}{k} = 2$$

8) $$x = 4+2i$$

$$(x-4)^2 = (2i)^2$$

$$x^2-8x+16 = 4i^2$$

$$x^2-8x+16 = -4$$

$$x^2-8x+20=0$$

BB-2

(2) $3a^2 + 27b^2 + 5c^2 - 18ab - 30c + 237$

$= 3(a^2 + 9b^2 - 6ab) + 5(c^2 - 6c) + 237$

$+9-9$

$= 3(a-3b)^2 + 5(c^2 - 6c + 9 - 9) + 237$

$= 3(a-3b)^2 + 5(c-3)^2 - 45 + 237$

$= 3(a-3b)^2 + 5(c-3)^2 + 192$

BB-1	✗
BB-2	✗
BB-3	✗
BB-4	✗
BB-5	✗
BB-6	

(3) Dividend = divisor · quotient + Remainder

$P(x) = (x-2)(x-3)\,q(x) + (ax+b)$

$P(3) = 2 \qquad 2 = 3a + b$

$P(2) = 3 \qquad 3 = 2a + b$

$a = -1$

$b = 5$

Rem $= -1x + 5$

$= 5 - x$

(4) $P(-1) = 1$

$P(1) = 2$

⑥

$$P(x) = ax^7 + bx^5 + cx^3 + 3$$

$$P(7) = a(7^7) + b(7^5) + c(7^3) + 3$$

$$P(-7) = a(-7)^7 + b(-7)^5 + c(-7)^3 + 3$$

$$P(-7) = \cancel{-a(7)^7} \cancel{- b(7)^5} \cancel{- c(7)^3} + 3$$

$$P(7) = \cancel{a(7)^7} \cancel{+ b(7)^5} \cancel{+ c(7)^3} + 3$$

$$P(-7) + P(7) = 6$$

$$P(-7) + 2 = 6$$

$$\boxed{P(-7) = 4}$$

(5)

$$P(x) = \frac{1}{x+1} \quad \checkmark \qquad x = \{1, 2, 3, 4\}$$

$$\underbrace{P(x)(x+1) - 1}_{} = 0$$

$$\underbrace{P(x)\underbrace{(x+1)}_{}}_{\text{at } x=-1} - 1 = a(x-1)(x-2)(x-3)(x-4)$$

$$0 - 1 = a(-2)(-3)(-4)(-5)$$

$$\boxed{a = \frac{-1}{120}}$$

$$P(x)(x+1) - 1 = -\frac{1}{120}(x-1)(x-2)(x-3)(x-4)$$

put $x = 5$

$$P(5)(6) - 1 = -\frac{1}{120}(4)(3)(2)(1)$$

$$= -\frac{1}{120}(24)$$

$$6P(5) = -\frac{1}{5} + 1 \Rightarrow 6P(5) = \frac{4}{5} \Rightarrow \boxed{P(5) = \frac{2}{15}}$$

Rule

componendo & dividendo

if $\left[\frac{a}{b} = \frac{c}{d}\right.$

then $\frac{a+b}{a-b} = \frac{c+d}{c-d}$

$$\frac{a}{2} = \frac{y+1}{y-1}$$

$$\frac{a+2}{a-2} = \frac{y+1+y-1}{y+1-(y-1)}$$

$$\frac{a+2}{a-2} = \frac{\cancel{2}y}{\cancel{2}}$$

$$\boxed{y = \frac{a+2}{a-2}}$$

BB-3

(2) $\underbrace{x|x|} + 7x - 8 = 0$

$x < 0$

$-x^2 + 7x - 8 = 0$

$x^2 - 7x + 8 = 0$

$x \geq 0$

$x^2 + 7x - 8 = 0$

(9) $||x-1| - 2| = |x-3|$

$(|x-1| - 2)^2 = (x-3)^2$

$|x-1|^2 + 4 - 4|x-1| = x^2 - 6x + 9$

$(x-1)^2 + 4 - 4|x-1| = x^2 - 6x + 9$

$x < 1$

$x \geq 1$

Group

$$|x^2 - 3|x| + 2| = (x^2 - 2x)$$

$$x^2 - 3|x| + 2 = +(x^2 - 2x)$$

$$2x - 3|x| + 2 = 0$$

$x < 0$

$5x = -2$

$\boxed{x = -2/5}$ ✓

$x > 0$

$-x = -2$

$\boxed{x = 2}$ ✓

$$x^2 - 3|x| + 2 = -x^2 + 2x$$

$$2x^2 - 2x - 3|x| + 2 = 0$$

$x < 0$

$2x^2 - 2x + 3x + 2 = 0$

$2x^2 + x + 2 = 0$

$x \geq 0$

$2x^2 - 5x + 2 = 0$

$2x^2 - 4x - x + 2 = 0$

$\boxed{x = 2;\ 1/2}$

② B.B-4

$$\left|\frac{2x-1}{x-1}\right| > 2$$

$$\frac{2x-1}{x-1} < -2 \quad \text{OR} \quad \frac{2x-1}{x-1} > 2$$

$|x| > a$ Union

$x < -a$ OR $x > a$

$|x| < a$

$-a < x < a$

intersection

①

$$\big|\,|x-1| - 2\,\big| \le 4$$

$$\underbrace{-4 \le |x-1| - 2}_{} \le 4$$

$-4 \le |x-1| - 2$ $\quad \cap \quad$ $|x-1| - 2 \le 4$

$|x-1| - 2 \ge -4$ $\qquad$ $|x-1| \le 6$

$\boxed{|x-1|} \ge -2 \Rightarrow |x-1| + 2 \ge 0$ $\qquad$ $-6 \le x - 1 \le 6$

$x \in R$ $\qquad$ $+1 \quad +1 \quad +1$

$\boxed{-5 \le x \le 7}$

5

$$|x+1| + |x-1| = |2x|$$

$$|\underline{x+1}| + |\underline{x-1}| = |x+1+x-1|$$

$$|a| + |b| = |a+b|$$

$$a \cdot b \geq 0$$

$$(x+1)(x-1) \geq 0$$

$$\boxed{x \in (-\infty, -1] \cup [1, \infty)}$$

7

$$|x^2+x| < 5$$

$$\underbrace{-5 < \overbrace{x^2+x < 5}}$$

(10) $$\underline{|x-1|} + \underline{|y-2|} + \underline{(z-3)^2} \leq 0$$

$x = 1;\quad y = 2;\quad z = 3$

(9) $$\left| \frac{x^2-5x+4}{x^2-4} \right| \leq 1$$

$$-1 \leq \frac{x^2-5x+4}{x^2-4} \leq 1$$

5

b $\frac{43}{30} = 1.433333\ --$

c $\left(\frac{1}{2}\right)^{\log_2 5} = 2^{-1\log_2 5} = 2^{\log_2(5^{-1})} = 5^{-1} = \frac{1}{5}$

$$\boxed{\begin{aligned} \sin^2\theta + \cos^2\theta &= 1 \\ \sec^2\theta &= 1 + \tan^2\theta \\ \operatorname{cosec}^2\theta &= 1 + \cot^2\theta \end{aligned}}$$

I 36

$$\log_a x = p \Rightarrow \log_x a = \frac{1}{p}$$

$$2\log_b x = q \Rightarrow \log_x b = \frac{2}{q}$$

$$\frac{1}{2}\left[\log_x a + \log_x b\right] = \frac{1}{2}\left[\frac{1}{p} + \frac{2}{q}\right]$$

BB-6

Q-2

$$\log_e x - \log_e y = a \Rightarrow \log_e\left(\frac{x}{y}\right) = a \Rightarrow \frac{x}{y} = e^a$$

$$\frac{y}{z} = e^b$$

$$\frac{z}{x} = e^c$$

③

$$\frac{\log_3 135}{\log_{15} 3} - \frac{\log_3 5}{\log_{405}(3)} = \frac{\log(27(5))}{\log 3}\,\frac{\log(15)}{\log 3} - \frac{\log 5 \cdot \log 405}{\log 3 \cdot \log 3}$$

$$= \frac{(3\log 3 + \log 5)(\log 3 + \log 5) - (\log 5)(4\log 3 + \log 5)}{(\log 3)^2}$$

$$= \frac{3(\log 3)^2 + \cancel{\log 3 \cdot \log 5} + \cancel{3\log 3 \log 5} + \cancel{(\log 5)^2} - \cancel{4\log 3 \log 5} - \cancel{(\log 5)^2}}{(\log 3)^2}$$

$$= 3$$

7

$$[3.14] - [\underbrace{-2.71}]$$

$$= 3 - (-3)$$

$$= 3+3 = 6$$

2
10
15

9

15

$$2\,\frac{\log x/\log x}{(\log x - \log 2)/\log x} + \frac{4\log x}{\log 4 + \log x} = 9\,\frac{\log x}{(\log 2 + \log x)}$$

$$\frac{2}{1 - \log_x 2} + \frac{4}{\underbrace{\log_x 4} + 1} = \frac{9}{\log_x 2 + 1}$$

$$\log_x 2 = t$$

$$\frac{2}{1-t} + \frac{4}{2t+1} - \frac{9}{t+1} = 0$$

(10) $60^a = 3 \Rightarrow a = \log_{60} 3$

$60^b = 5 \Rightarrow b = \log_{60} 5$

$$\frac{1-a-b}{2(1-b)} = \frac{1-\log_{60}3-\log_{60}5}{2(1-\log_{60}5)} = \frac{1-\log_{60}15}{2(\log_{60}60-\log_{60}5)}$$

$$= \frac{\log_{60}60-\log_{60}15}{2(\log_{60}12)} = \frac{\log_{60}4}{2(\log_{60}12)}$$

$$= \frac{\log_{60}2}{\log_{60}12} = \log_{12}2$$

$$12^{\frac{1-a-b}{2(1-b)}} = 12^{\log_{12}2} = 2$$

14

$$B = \frac{12}{3+\sqrt{5}+\sqrt{8}} \cdot \frac{(3+\sqrt{5}) - (\sqrt{8})}{(3+\sqrt{5}) - (\sqrt{8})}$$

$$= \frac{12(3+\sqrt{5}-\sqrt{8})}{9+5+6\sqrt{5}-8} = \frac{\overset{2}{\cancel{12}}(3+\sqrt{5}-\sqrt{8})}{\cancel{6}(1+\sqrt{5})} \cdot \frac{(\sqrt{5}-1)}{(\sqrt{5}-1)}$$

$$= \frac{2}{4}\left(\qquad \right)$$