Some Imp Point's :→

For Two projectile's projected with same velocity at angle θ_1 and θ_2

If $\boxed{\theta_1 + \theta_2 = 90}$ then $\boxed{R_1 = R_2}$

let T_1 & T_2 are Time of flight and H_1, H_2 are maximum Heights

$$\boxed{\begin{aligned} T_1 T_2 &\propto R \\ H_1 H_2 &\propto R^2 \end{aligned}}$$

$$T_1 = \frac{2u\sin\theta_1}{g} \quad —(1)$$

$$T_2 = \frac{2u\sin\theta_2}{g}$$

$$\theta_2 = 90 - \theta_1$$

$$T_2 = \frac{2u\sin(90-\theta_1)}{g}$$

$$T_2 = \frac{2u\cos\theta_1}{g} \quad —(2)$$

multiply eq (1) & (2)

$$T_1 T_2 = \frac{4u^2 \sin\theta_1 \cos\theta_1}{g^2}$$

$$= 2\frac{u^2 \sin 2\theta_1}{g \cdot g}$$

$$\boxed{T_1 T_2 = \frac{2R}{g}}$$

$$H_1 = \frac{u^2 \sin^2\theta_1}{2g} \quad —(1)$$

$$H_2 = \frac{u^2 \sin^2\theta_2}{2g}$$

$$H_2 = \frac{u^2 \sin^2(90-\theta_1)}{2g}$$

$$H_2 = \frac{u^2 \cos^2\theta_1}{2g} \quad —(2)$$

multiply eq (1) & (2)

$$H_1 H_2 = \frac{u^4 \sin^2\theta_1 \cos^2\theta_1}{4g^2}$$

$$= \left(\frac{u^2\, 2\sin\theta_1 \cos\theta_1}{2 \times 2g}\right)^2$$

$$= \left(\frac{u^2 \sin 2\theta_1}{4g}\right)^2$$

$$\boxed{H_1 H_2 = \frac{R^2}{16}}$$

(2) For maximum Range

$$R = \frac{u^2 \sin 2\theta}{g}$$

$$\boxed{R_{max} = \frac{u^2}{g}}$$

$\sin 2\theta = 1$

$2\theta = 90^\circ$

$$\boxed{\theta = 45^\circ}$$

$$H = \frac{u^2 \sin^2 45}{2g} = \frac{u^2}{4g}$$

$$\boxed{H = \frac{R_{max}}{4}}$$

Ex A projectile is given an initial velocity $\mathbf{u} = (2\hat{\mathbf{i}} + \hat{\mathbf{j}})$ ms^{-1}. The cartesian equation of its trajectory is (take $g = 10\ ms^{-2}$)

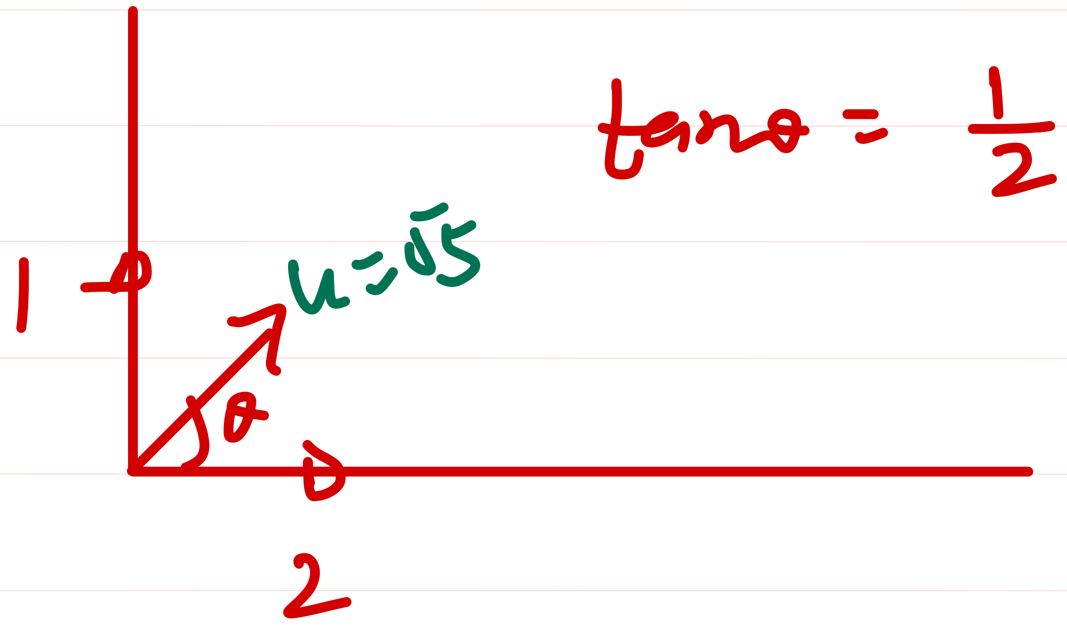
$|u| = \sqrt{5}$

(a) $y = 2x - 5x^2$ (b) $2y = 2x - 5x^2$

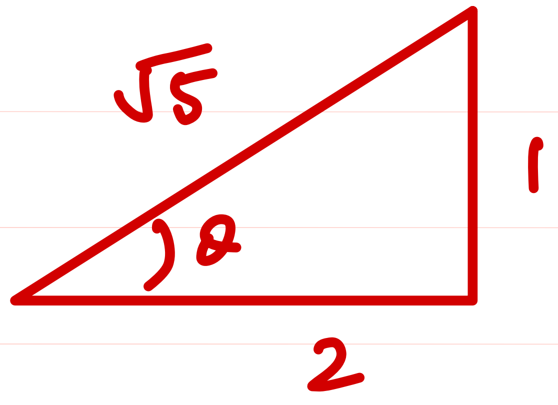
(c) $4y = 2x - 5x^2$ ✓ (d) $4y = x - 5x^2$

$$R = \frac{u^2 \sin 2\theta}{g} = \frac{2u^2 \sin\theta \cos\theta}{g}$$

$$R = \frac{2 \times (\sqrt{5})^2 \frac{1}{\sqrt{5}} \times \frac{2}{\sqrt{5}}}{10} = \frac{2}{5}$$



$\tan\theta = \frac{1}{2}$



$\cos\theta = \frac{2}{\sqrt{5}}$

$\sin\theta = \frac{1}{\sqrt{5}}$

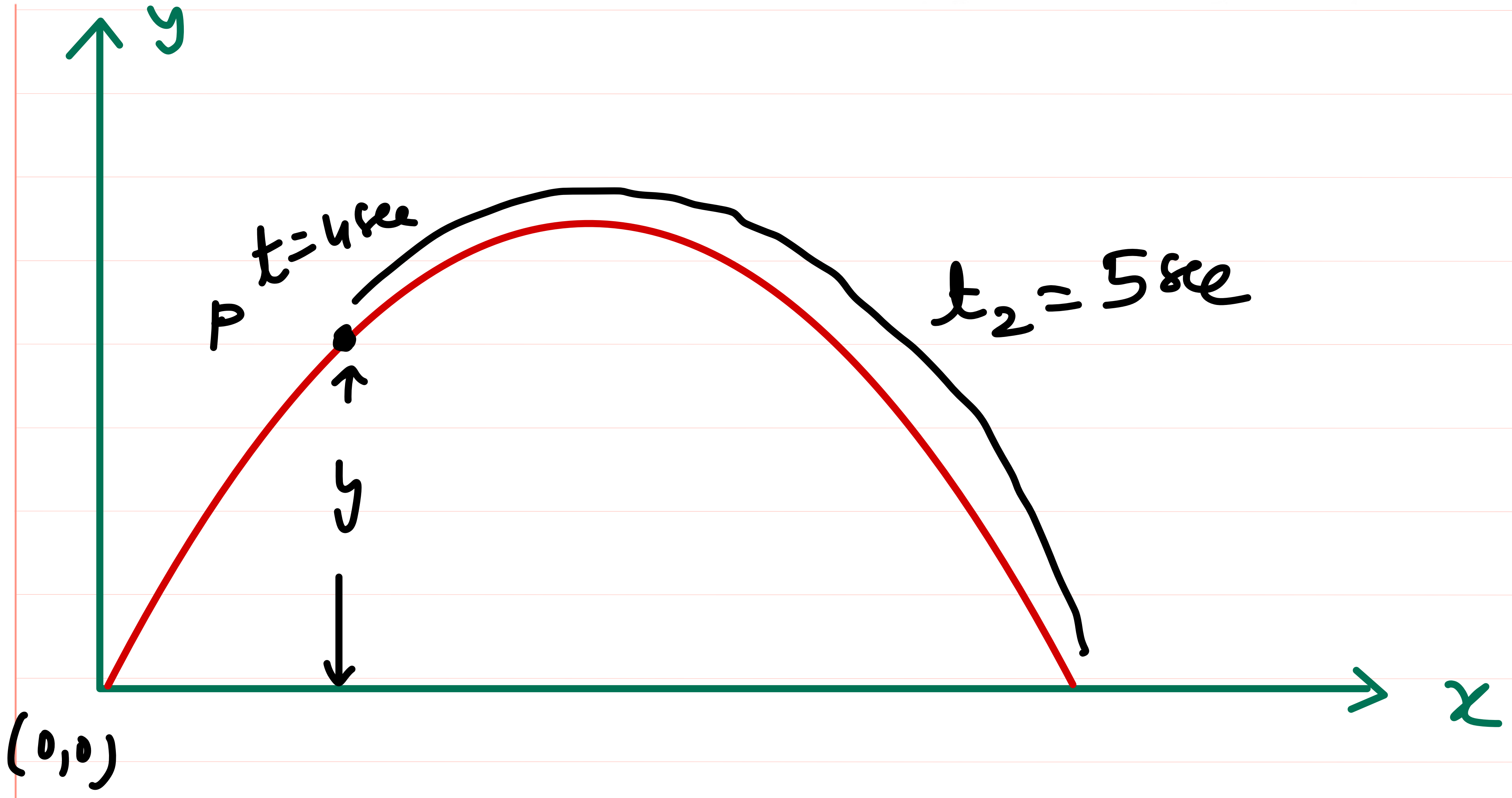
$$y = x \tan\theta \left[1 - \frac{x}{R}\right]$$

$$y = \frac{x}{2}\left[1 - \frac{5x}{2}\right]$$

$$2y = x - \frac{5x^2}{2}$$

$$\boxed{4y = 2x - 5x^2}$$

Illustration 2*. A ball 4 s after the instant it was thrown from the ground passes through a point P, and strikes the ground after 5 s from the instant it passes through the point P. Assuming acceleration due to gravity to be 9.8 m/s^2 find height of the point P above the ground.



Total time of flight = 4 + 5 = 9 sec

$$T = \frac{2u_y}{g}$$

$$\frac{9 \times 9.8}{2} = u_y$$

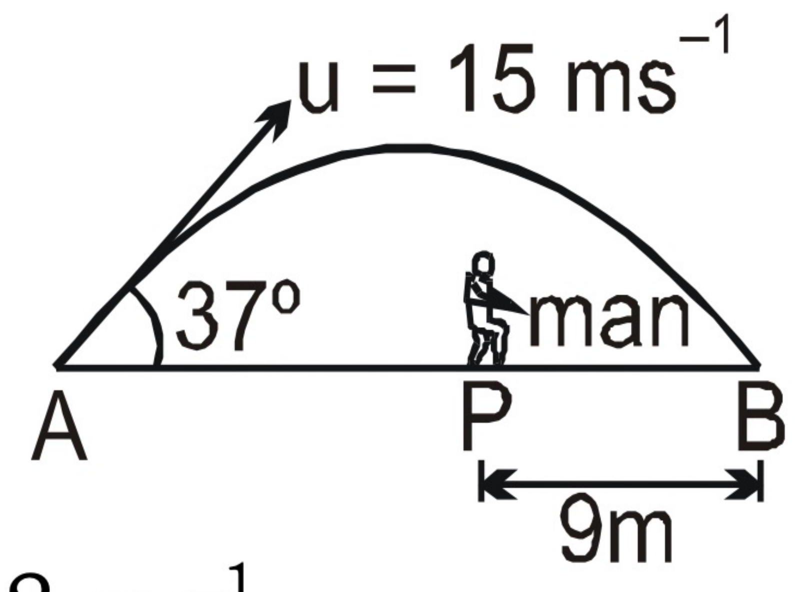
$$y = u_y t - \frac{1}{2} g t^2$$

$$= \frac{9}{2} \times 9.8 \times 4 - \frac{9.8}{2} 4^2$$

$$= 9.8 [18 - 8]$$

$$\boxed{y = 98 \text{ m}}$$ Ans

7*. A ball is hit by a batsman at an angle of 37° as shown in figure. The man standing at P should run at what minimum velocity so that he catches the ball before it strikes the ground. Assume that height of man is negligible in comparison to maximum height of projectile.



(A) 3 ms^{-1} ~~(B)~~ 5 ms^{-1} (C) 9 ms^{-1} (D) 12 ms^{-1}

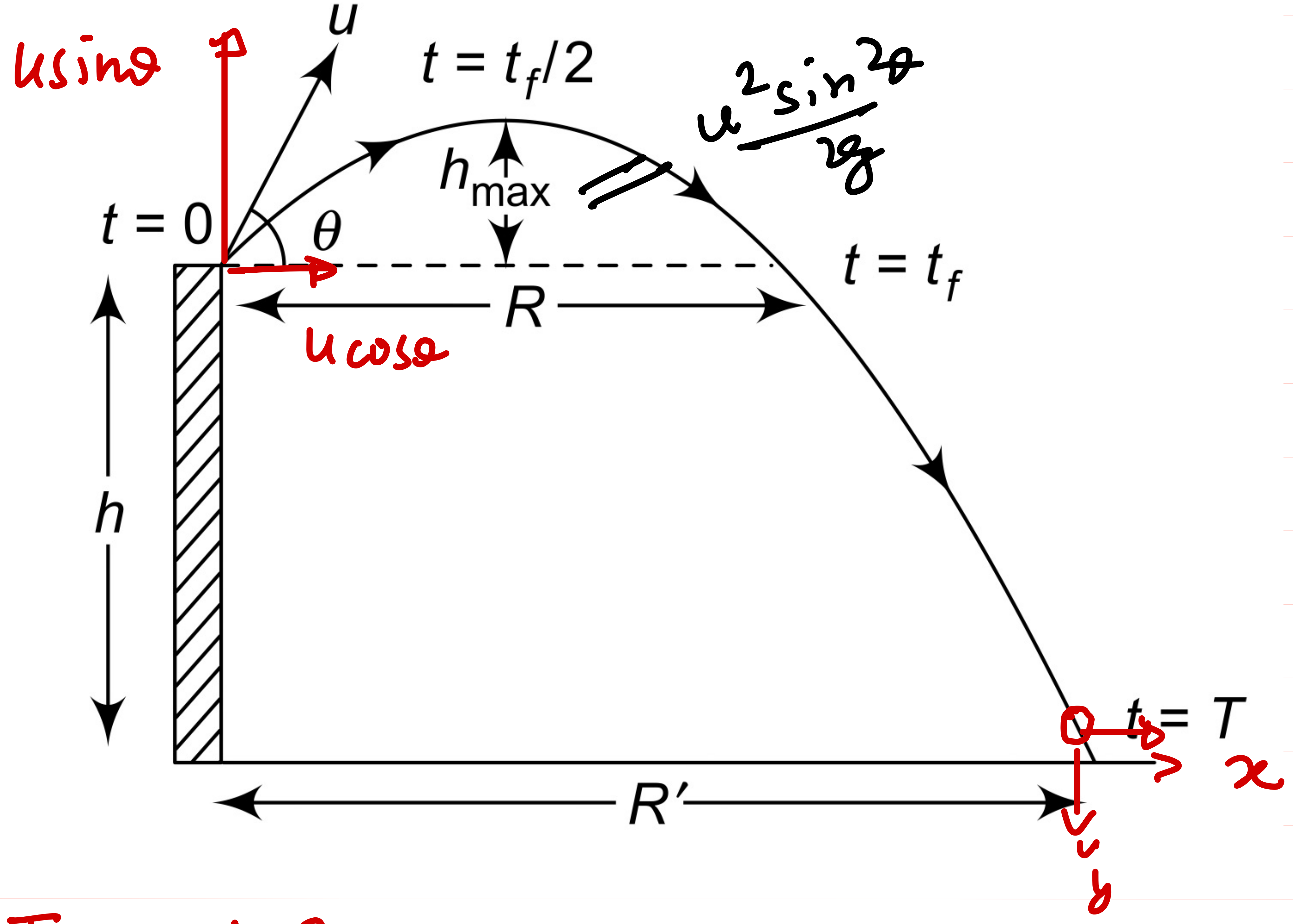
let minimum speed of man = v

$d = vT$ ← $T = \frac{2u\sin\theta}{g} = \frac{2\times 15\times \sin 37}{10} = \frac{30}{10}\times\frac{3}{5} = 1.8$ sec

$9 = v \times 1.8$

$$\boxed{v = \frac{9}{1.8} = \frac{90}{18} = 5\ m/s}$$

Case -2 *A body projected from a height h with a velocity u at an angle θ with the horizontal. (Fig. 3)*



Time of Flight :

$$S_y = u_y t + \frac{1}{2} a_y t^2$$

$$\boxed{-h = u\sin\theta\, T - \frac{g}{2} T^2}$$

$$\frac{g}{2} T^2 - u\sin\theta\, T - h = 0$$

$$T = \frac{u\sin\theta \pm \sqrt{(u\sin\theta)^2 - 4\frac{g}{2}(-h)}}{2 \times g/2}$$

$$\boxed{T = \frac{u\sin\theta \pm \sqrt{(u\sin\theta)^2 + 2gh}}{g}}$$

Range $(R') = u_x \cdot T$

$$\boxed{R' = u\cos\theta \times T}$$

Speed at Ground

$v_x = u\cos\theta$

$v_y^2 = u_y^2 + 2gh$

$$V = \sqrt{v_x^2 + v_y^2}$$

$$\boxed{V = \sqrt{u^2 + 2gh}}$$

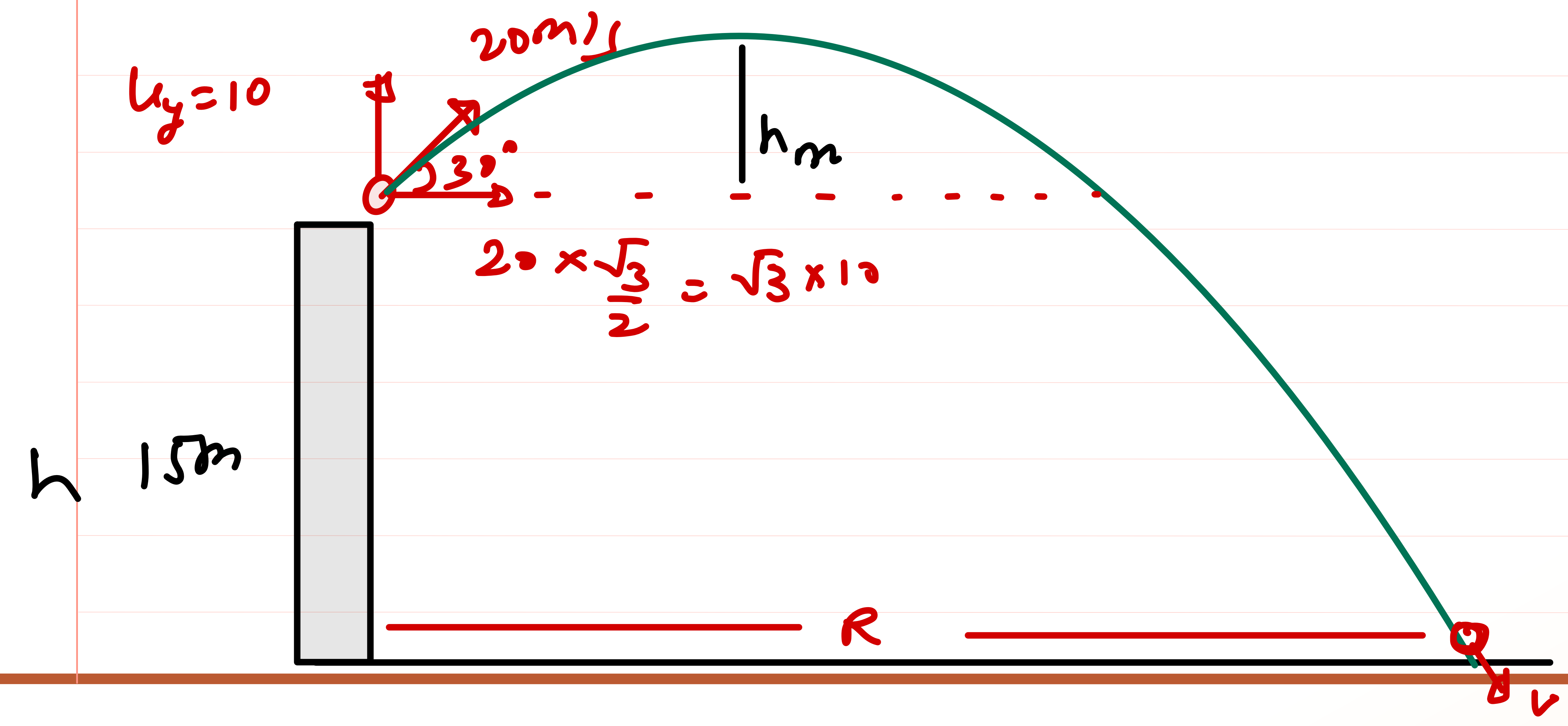
Ex A ball is thrown with a velocity of 20 ms^{-1} at an angle of 30° above the horizontal from the top of a building 15 m high. Find (take $g = 10$ ms^{-2})

(a) the time after which the ball hits the ground.

(b) the distance from the bottom of the building at which it hits the ground.

(c) the velocity with which the ball hits the ground.

(d) the maximum height attained by the ball above the ground.



(a) From y-dirn :

$$-15 = 10 \times T - \frac{10}{2} \times T^2$$

$$-15 = 10T - 5T^2$$

$$T^2 - 2T - 3 = 0$$

$$T^2 - 3T + T - 3 = 0$$

$$T(T-3) + 1(T-3) = 0$$

$$\boxed{T = 3\text{ sec}}$$

$$T = \frac{u\sin\theta \pm \sqrt{(u\sin\theta)^2 + 2gh}}{g}$$

$$= \frac{10 \pm \sqrt{10^2 + 2\times 10 \times 15}}{10}$$

$$= \frac{10 \pm 20}{10} = 3\text{ sec}$$

(b) $R = u_x T = 10\sqrt{3} \times 3$

$$\boxed{R = 30\sqrt{3}\text{ m}}$$

(c) $v = \sqrt{u^2 + 2gh}$

$$= \sqrt{(20)^2 + 2 \times 10 \times 15}$$

$$= \sqrt{700}$$

$$\boxed{v = 10\sqrt{7}\text{ m/s}}$$

(d) $H = h + h_m$

$$= 15 + \frac{u^2 \sin^2\theta}{2g}$$

$$= 15 + \frac{(20)^2}{20} \times \frac{1}{4}$$

$$\boxed{H = 20\text{ m}}$$

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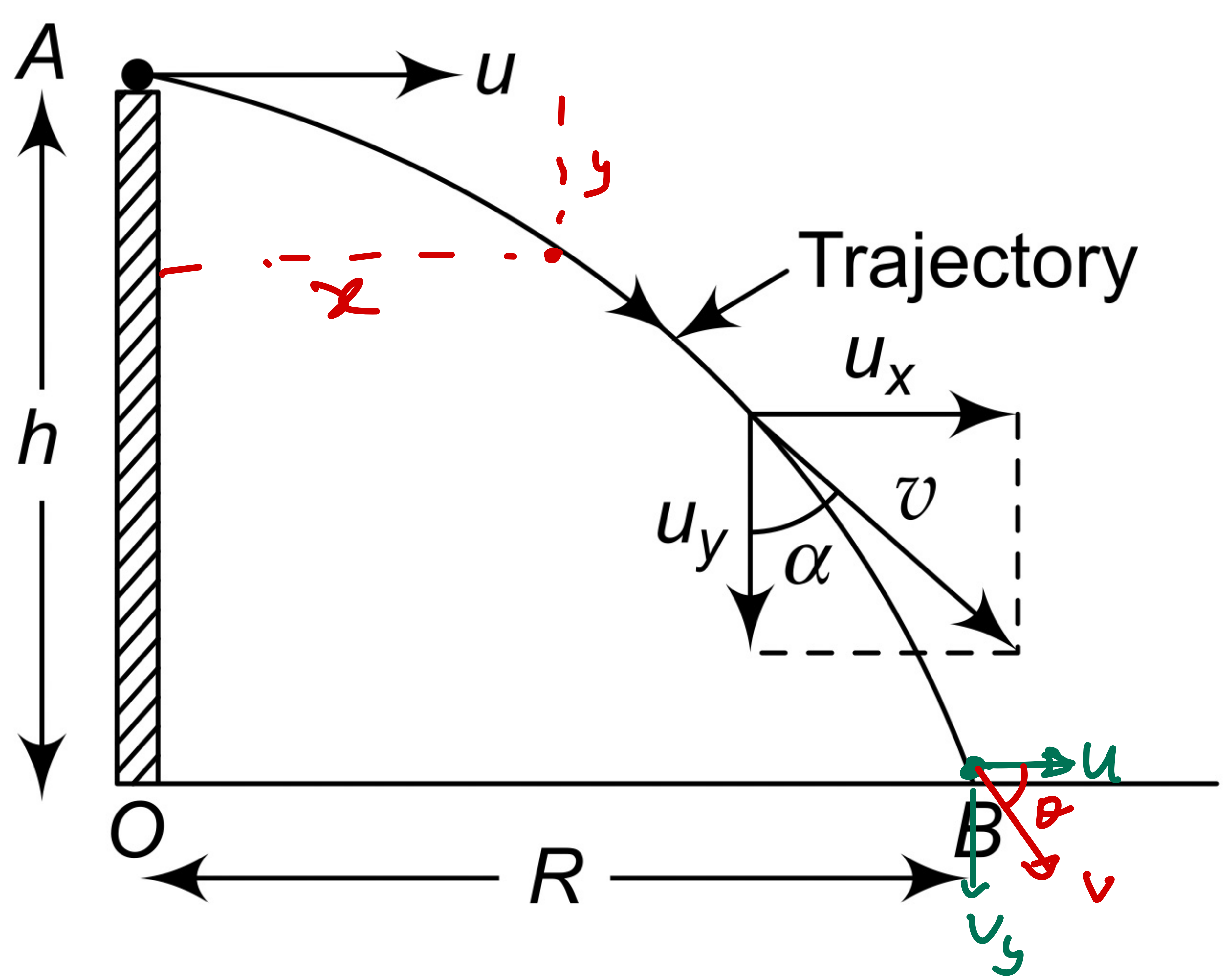
Case -3 A body projected horizontally with a velocity u from a height h.



$$\boxed{\tan\theta = \frac{v_y}{u} = \frac{\sqrt{2gh}}{u}}$$

Time of flight (from y-dirn)

$$-h = 0\cdot T - \frac{1}{2}gT^2$$

$$\boxed{T = \sqrt{\frac{2h}{g}}}$$

Range

$$R = u\cdot T$$

$$\boxed{R = u\sqrt{\frac{2h}{g}}}$$

Equation of path

$$y = -\frac{1}{2}gt^2$$

$$x = ut$$

$$\boxed{y = -\frac{1}{2}g\frac{x^2}{u^2}}$$

Speed at Ground

$$V = \sqrt{u^2 + v_y^2}$$

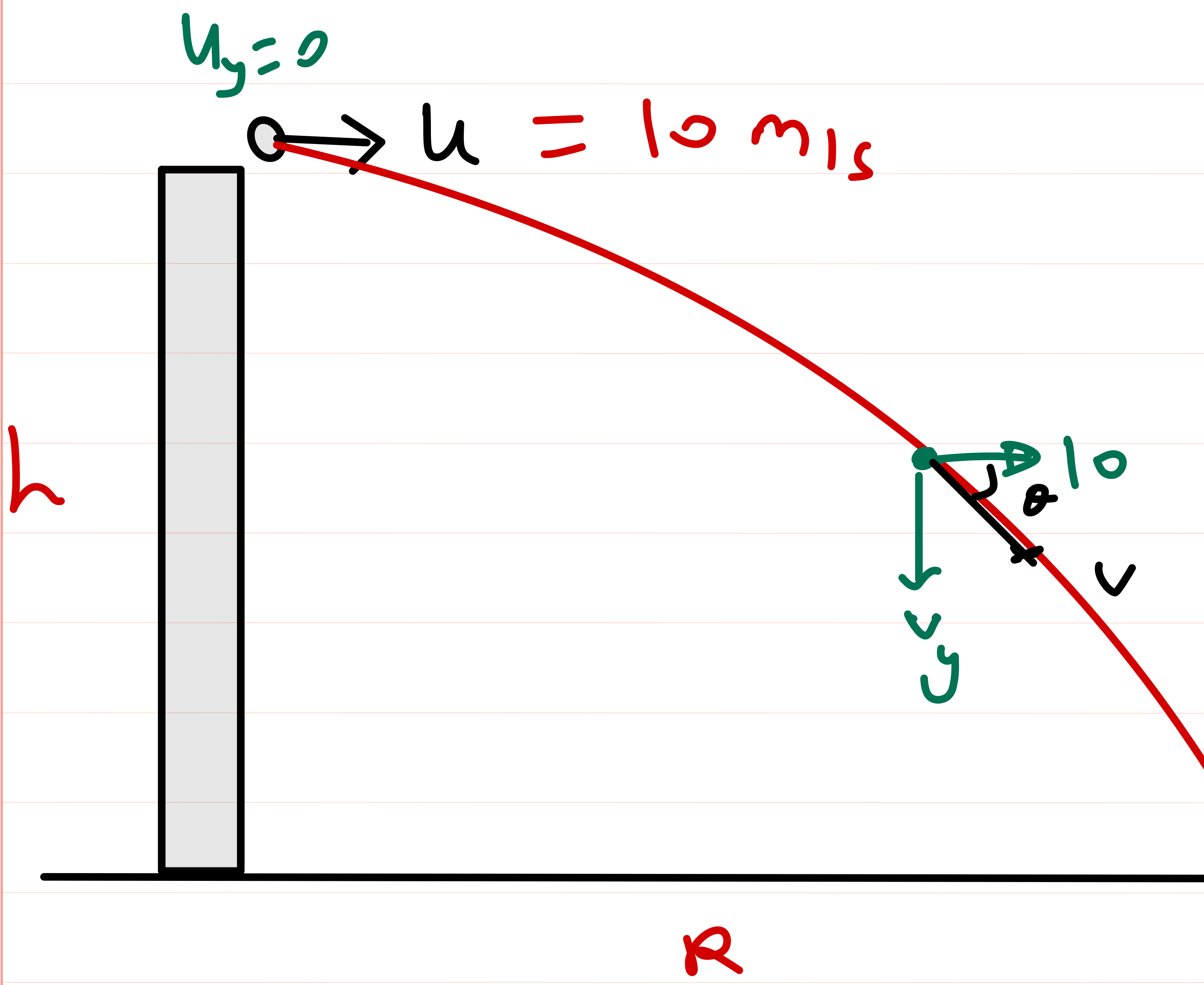
$$\because v_y^2 = 0^2 + 2(-g)(-h)$$

$$\boxed{V = \sqrt{u^2 + 2gh}}$$

Ex A body is projected horizontally with a velocity of 10 ms^{-1} from the top of building 20 m high. Find

(a) horizontal distance from the bottom of the building at which the body will strike the ground.

(b) the magnitude and direction of the velocity of the body 1 s after it is projected. Take $g = 10$ ms^{-2}.



$T = \sqrt{\frac{2h}{g}}$

$= \sqrt{\frac{2 \times 20}{10}}$

$T = 2$ sec

$R = uT$

$= 10 \times 2$

$\boxed{R = 20m}$

$v_y = u_y + a_y t$ (Along y-dir^n)

$v_y = 0 + (-10) \times 1$

$v_y = -10$ m/s

$\vec{V} = 10\hat{i} - 10\hat{j} \Rightarrow \boxed{V = 10\sqrt{2}\text{ m/s}}$

$\tan\theta = \frac{v_y}{v_x} = \frac{10}{10} = 1 \Rightarrow \boxed{\theta = 45^\circ}$

Ans

Speed at Ground

$V = \sqrt{u^2 + 2gh}$

$= \sqrt{10^2 + 2 \times 10 \times 20}$

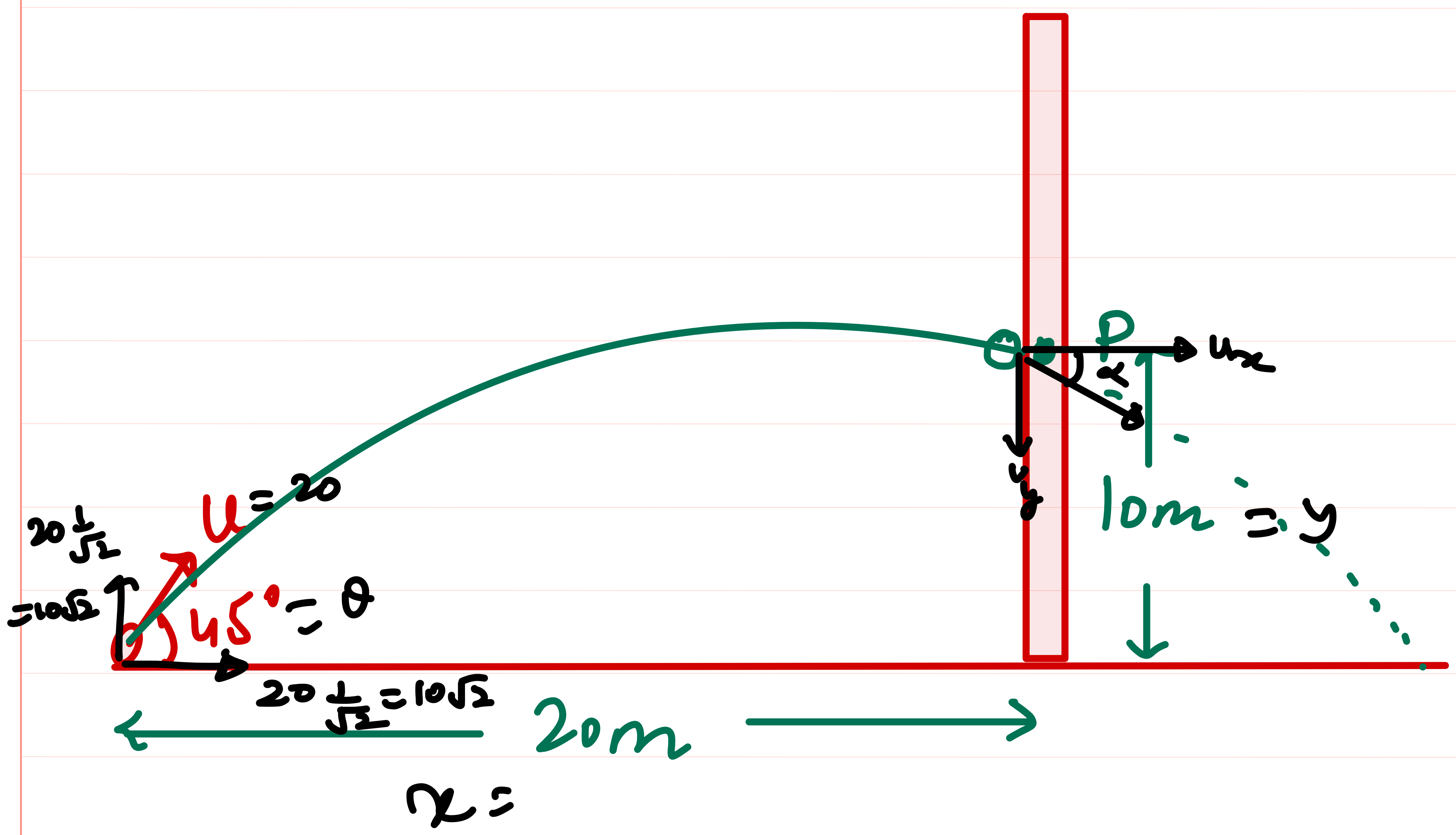
$\boxed{V = 10\sqrt{5}\text{ m/s}}$ Ans

Ex

A stone thrown from the ground at an angle of 45° above the horizontal strikes a vertical wall at a point 10 m above the ground. If the wall is at a distance of 20 m from the point of projection, find (take $g = 10\ \text{ms}^{-2}$)

(a) the speed with which the stone is projected,

(b) the magnitude and direction of the velocity of the stone when it strikes the wall.



(a) Eq. of Traj.

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

$$10 = 20\tan 45 - \frac{10\times(20)^2}{\cancel{2}u^2\times\frac{1}{\cancel{2}}}$$

$$10 = 20 - \frac{4000}{u^2} \Rightarrow 10 = \frac{4000}{u^2}$$

$$\boxed{u = 20\ \text{m/s}}$$

Ans

(b) $u_x = 10\sqrt{2}$

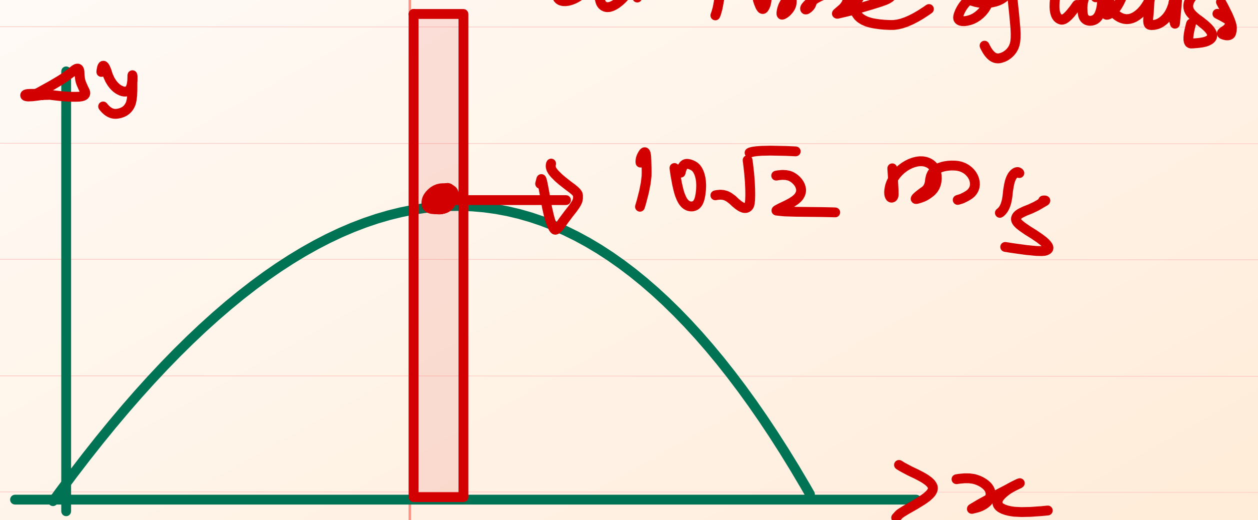
$$v_y^2 = u_y^2 + 2a_y s_y$$

$$= (10\sqrt{2})^2 - 2\times10\times10$$

$$v_y^2 = 200 - 200 = 0$$

$$\boxed{v_y = 0}$$

Projectile is at highest point of trajectory at time of collision



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