Trigonometric equations
CL04

Type-4 Solving equations with the use of boundness of the function sin x or cos x

① $\sin^4 x = 1 + \cos^6 y$

$-1 \le \sin x \le 1$

$0 \le \sin^4 x \le 1$

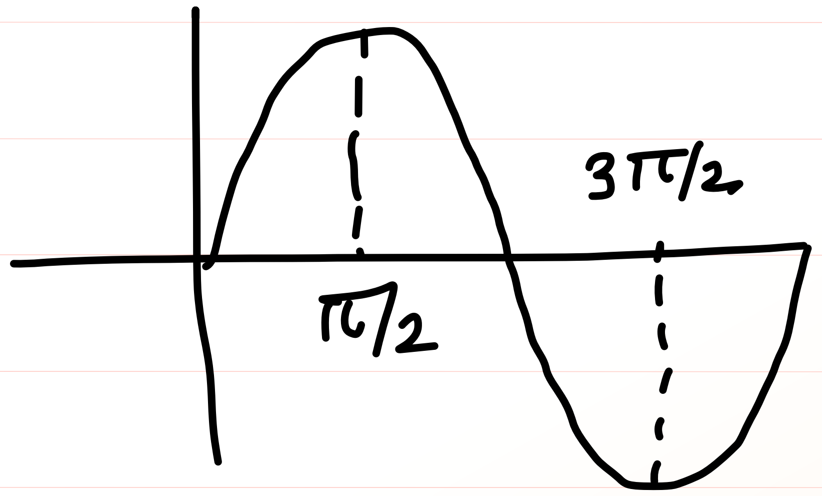
$-1 \le \cos y \le 1$

$0 \le \cos^6 y \le 1$

$1 \le 1 + \cos^6 y \le 2$

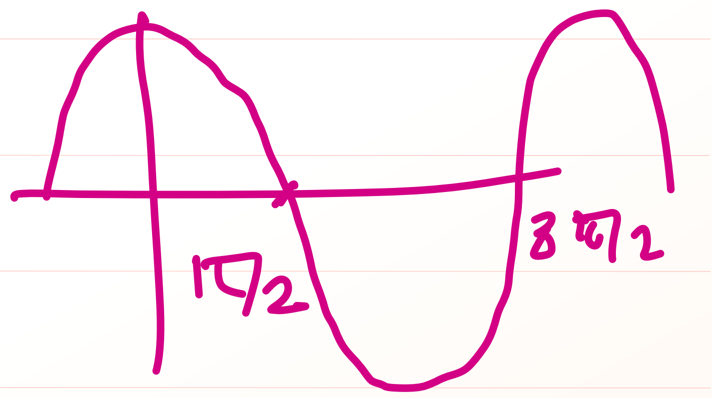
$\sin^4 x = 1$

$\sin x = \underbrace{\pm 1}$



$\boxed{x \in (2n+1)\frac{\pi}{2},\ n \in \mathbb{Z}}$

$\cos^6 y = 0$



$\boxed{y \in (2m+1)\frac{\pi}{2},\ m \in \mathbb{Z}}$

(2) $\cos x + \cos 2x + \cos 3x = 3$

$\cos x = 1$ $\qquad$ $\cos 2x = 1$ $\qquad$ $\cos 3x = 1$

$x = 2n\pi$ $\qquad$ $2x = 2n\pi$ $\qquad$ $3x = 2n\pi$ $\qquad$ $n \in Z.$

$x = \frac{2n\pi}{1}$ $\qquad$ $x = \frac{n\pi}{1}$ $\qquad$ $x = \frac{2n\pi}{3}$ $\qquad$ $n \in Z$

$$x = \frac{2n\pi}{1} \; ; \; n \in Z$$

(3) $\sin x\left(\cos\frac{x}{4} - 2\sin x\right) + \left(1 + \sin\frac{x}{4} - 2\cos x\right)\cos x = 0$

$\underbrace{\sin x\cdot\cos\frac{x}{4}} - 2\sin^2 x + \cos x + \underbrace{\sin\frac{x}{4}\cos x} - 2\cos^2 x = 0$

$\sin\left(x + \frac{x}{4}\right) - 2\left(\sin^2 x + \cos^2 x\right) + \cos x = 0$

$\sin\left(\frac{5x}{4}\right) + \cos x = 2$

$\sin\left(\frac{5x}{4}\right) = 1$ $\qquad$ $\cos x = 1$

$\frac{5x}{4} = (4n+1)\frac{\pi}{2}$ $\qquad$ $\boxed{x = 2m\pi \;;\; m\in Z}$ $\qquad$ $x \in \{0, 2\pi, 4\pi,$

$\boxed{x = (4n+1)\frac{2\pi}{5} \;;\; n\in Z}$ $\qquad$ common solutions $\{-\,-\,-6\pi, \underline{2\pi}, \underline{10\pi} \,-\,-\,-\}$

$x \in = \frac{2\pi}{5}, 2\pi, \frac{18\pi}{5}, \frac{26\pi}{5}, \frac{34\pi}{5}, \frac{42\pi}{5}, 10\pi$

$= (2 + (p-1)8)\pi \quad p\in z$

$= (8p-6)\pi \;;\; p\in Z$

④ Solve for x and y

$$1-2x-x^2 = \tan^2(x+y) + \cot^2(x+y)$$

$$-(x^2+2x)+1 = \tan^2(x+y) + \cot^2(x+y)$$

$$-(x^2+2x+1)+1+1 = \tan^2(x+y) + \cot^2(x+y)$$

$$\underbrace{-(x+1)^2}\ +2 = \tan^2(x+y) + \cot^2(x+y)$$

$$(x+1) = 0$$

$$\boxed{x=-1}$$

$$\tan^2(x+y) = 1$$

$$(x+y) = n\pi \pm \frac{\pi}{4}$$

$$y-1 = n\pi \pm \frac{\pi}{4}$$

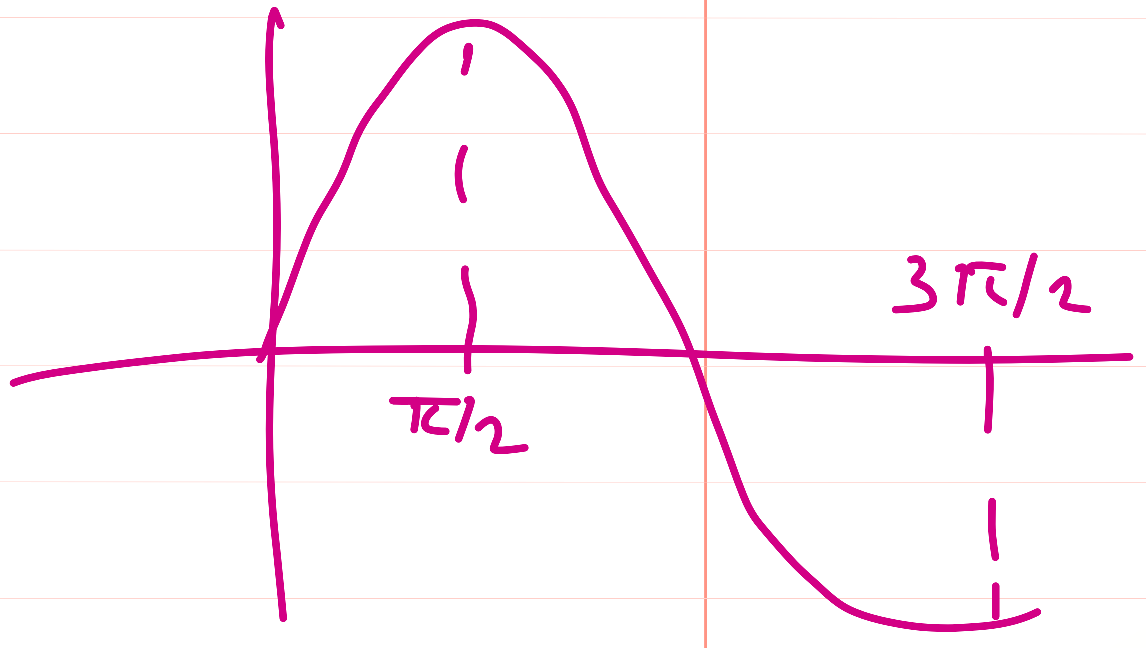
$$y = n\pi \pm \frac{\pi}{4} + 1$$

Type 5 Solution of trigo equations of the form $f(x) = \sqrt{\phi(x)}$

① $\sqrt{1-\cos x} = \sin x$ ✓

$1-\cos x = 1-\cos^2 x$

$\cos x = 0, 1$



$\cos x = 0$

$x = (2n+1)\frac{\pi}{2}$

$x = \{ \ldots, \cancel{-\frac{\pi}{2}}, \frac{\pi}{2}, \cancel{\frac{3\pi}{2}}, \frac{5\pi}{2}, \ldots \}$

$x = (4n+1)\frac{\pi}{2} \quad \forall\, n \in Z.$

$\cos x = 1$

$x = 2n\pi$

$x = \{ -2\pi, 0, 2\pi, \ldots \}$

$x = 2n\pi \quad \forall\, n \in Z$

(2)

$$2^{\frac{1}{\sin^2 x}} \cdot \sqrt{y^2-2y+2} \leq 2$$