Trigonometric Ratios and Identities

Lecture - 10

Application of Trigonometry in maximising & minimizing :→

Type I :→

$$-1 \le \sin x \le 1$$

$$-1 \le \cos x \le 1$$

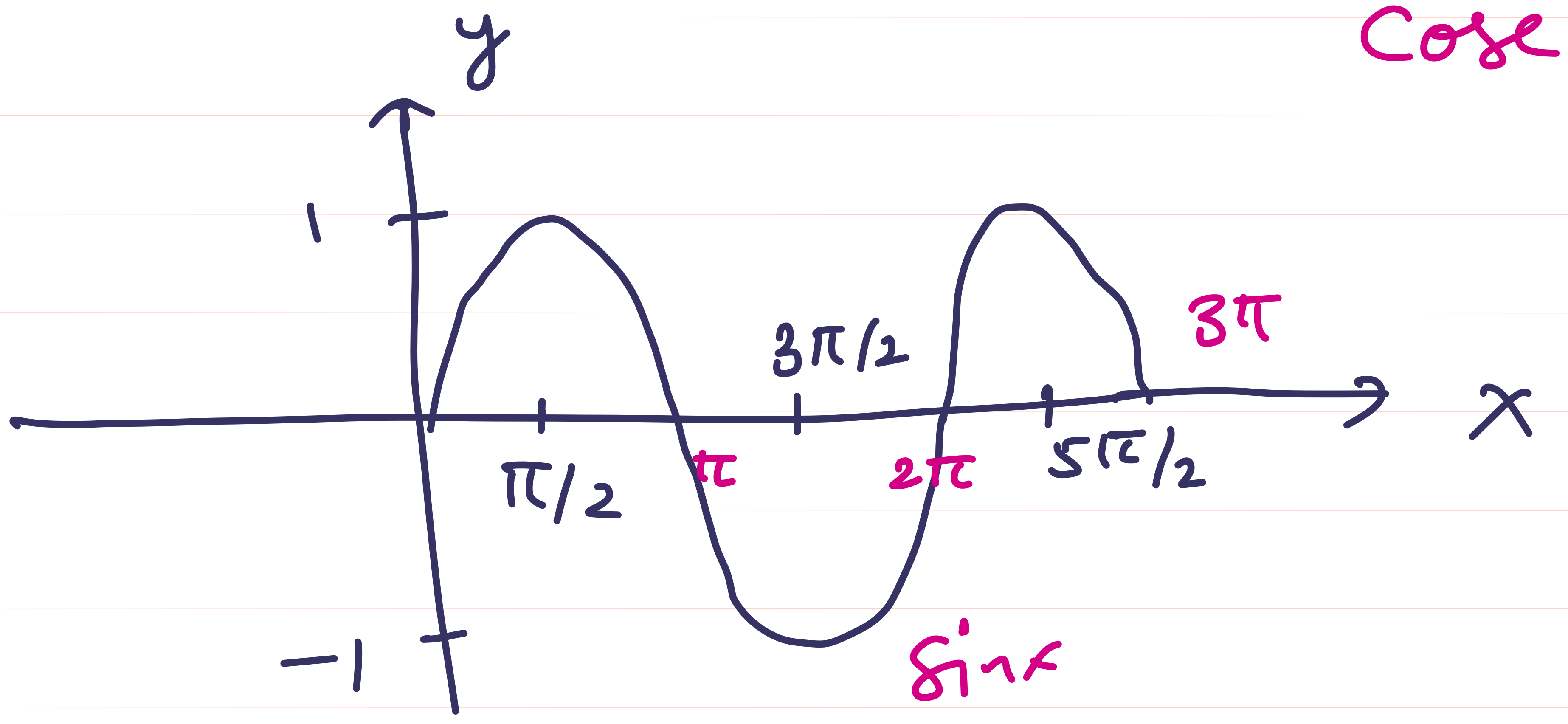
$$\tan x \in (-\infty, \infty)$$

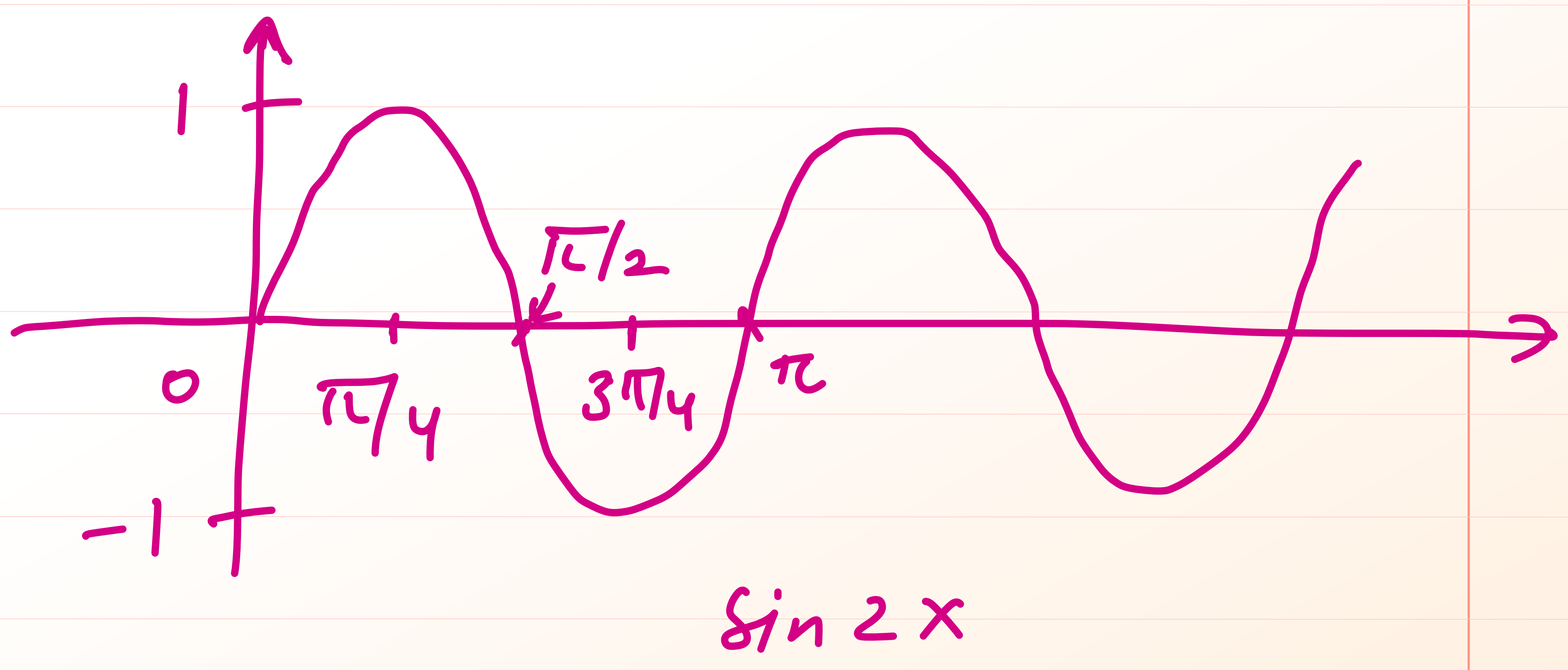
$$\cot x \in (-\infty, \infty)$$

$$\sec x \in (-\infty, -1] \cup [1, \infty)$$

$$\operatorname{cosec} x \in (-\infty, -1] \cup [1, \infty)$$

Range of $\sin x$ is $[-1, 1]$





① find range of $y = \cos^4 x - \sin^4 x$

$$= (\cos^2 x + \sin^2 x)(\cos^2 x - \sin^2 x)$$

$$= 1\,(\underbrace{\cos^2 x - \sin^2 x})$$

$$= \cos 2x$$

$$\boxed{y \in [-1, 1]}$$

(2) (a) $y = 4\tan x \cos x$

$y = 4\frac{\sin x}{\cos x}(\cos x)$

$y = 4\sin x$

~~$y \in [-4, 4]$~~

$y \in (-4, 4)$

$y = 4\sin x$

$y \in [-4, 4]$

3 (a) $y = \sin(\sqrt{x})$

$\boxed{y \in [-1, 1]}$

(b) $y = \sin^2 x$

$-1 \le \sin x \le 1$

$0 \le \sin^2 x \le 1$

$\boxed{y \in [0, 1]}$

(c) $y = \sin 3x$

$\boxed{y \in [-1, 1]}$

Q

$$y = \sin^2\left(\frac{15\pi}{8} - 4x\right) - \sin^2\left(\frac{17\pi}{8} - 4x\right)$$

Type II when argument of sine & cosine are same :→

$a\sin\underline{x} + b\cos\underline{x}$

$$y = \left(\frac{a\sin x}{\sqrt{a^2+b^2}} + \frac{b\cos x}{\sqrt{a^2+b^2}}\right)\sqrt{a^2+b^2}$$

$$y = \sqrt{a^2+b^2}\left[\frac{a}{\sqrt{a^2+b^2}}\sin x + \frac{b}{\sqrt{a^2+b^2}}\cos x\right]$$

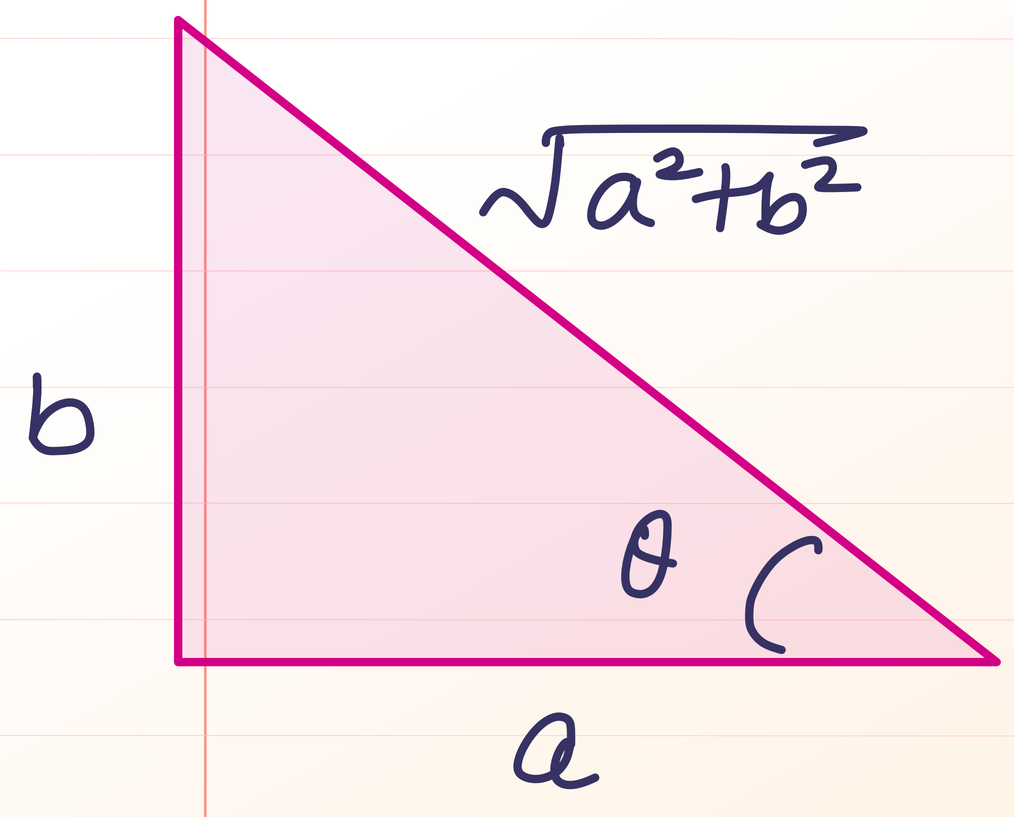
$$= \sqrt{a^2+b^2}\left[\cos\theta\cdot\sin x + \sin\theta\cos x\right]$$

$$= \sqrt{a^2+b^2}\left[\sin(x+\theta)\right]$$

$$-1 \le \sin(x+\theta) \le 1$$

$$-\sqrt{a^2+b^2} \le \underline{\sqrt{a^2+b^2}\,(\sin(x+\theta))} \le \sqrt{a^2+b^2}$$

$$\boxed{y \in \left[-\sqrt{a^2+b^2},\ +\sqrt{a^2+b^2}\right]}$$



$$\cos\theta = \frac{a}{\sqrt{a^2+b^2}}$$

$$\sin\theta = \frac{b}{\sqrt{a^2+b^2}}$$

find range

Q ① $y = \sin x + \cos x$

$-\sqrt{1^2+1^2} \le \underbrace{\sin x + \cos x} \le \sqrt{1^2+1^2}$

$-\sqrt{2} \le y \le \sqrt{2}$

$\boxed{y \in [-\sqrt{2}, \sqrt{2}]}$

Q2 $y = 3\sin x + 4\cos x + 5$

$-\sqrt{3^2+4^2} \le 3\sin x + 4\cos x \le +\sqrt{3^2+4^2}$

$-5 \le \underbrace{3\sin x + 4\cos x + 5} \le +5$

$+5 \qquad\qquad\qquad\qquad +5$

$0 \le y \le 10$

$y \in [0, 10]$

Q-3

$$y = \log_2\left(\frac{3\sin x - 4\cos x + 15}{10}\right)$$

$$-\sqrt{3^2+4^2} \le 3\sin x - 4\cos x \le \sqrt{3^2+4^2}$$

$$15 - 5 \le 3\sin x - 4\cos x + 15 \le 5 + 15$$

$$\frac{10}{10} \le \frac{3\sin x - 4\cos x + 15}{10} \le \frac{20}{10}$$

$$\underbrace{\log_2(1)} \le \log_2\left(\frac{3\sin x - 4\cos x + 15}{10}\right) \le \log_2 2$$

$$0 \le y \le 1$$

$$\boxed{y \in [0, 1]}$$

Q 4 If $b \le 3\sin^2 x + 6\cos^2 x \underbrace{- 4\sin x \cos x} + 5 \le a$ find a & b.

$$y = 3\sin^2 x + 6\cos^2 x \underbrace{- 4\sin x \cos x} + 5$$

$$= \underbrace{3\sin^2 x + 3\cos^2 x} + 3\cos^2 x - 2\sin 2x$$

$$= 3(\sin^2 x + \cos^2 x) + 3\cos^2 x - 2\sin 2x + 5$$

$$= 8 + 3\cos^2 x - 2\sin 2x$$

$$= 8 + 3\left(\frac{\cos 2x + 1}{2}\right) - 2\sin 2x$$

$$y = 8 + \frac{3}{2}\cos 2x + \frac{3}{2} - 2\sin 2x$$

$$\boxed{y = \frac{19}{2} + \frac{3}{2}\cos 2x - 2\sin 2x}$$

$\cos 2x$
$= 2\cos^2 x - 1$

$\cos^2 x = \frac{\cos 2x + 1}{2}$

$$\boxed{y = \frac{19}{2} + \frac{3}{2}\cos 2x - 2\sin 2x}$$

$$-\sqrt{\left(\frac{3}{2}\right)^2 + (2)^2} \le \frac{3}{2}\cos 2x - 2\sin 2x \le \sqrt{\left(\frac{3}{2}\right)^2 + (2)^2}$$

$$-\frac{5}{2} \le \frac{3}{2}\cos 2x - 2\sin 2x \le \frac{5}{2}$$

$$+\frac{19}{2} \qquad\qquad +\frac{19}{2} \qquad\qquad +\frac{19}{2}$$

$$7 \le y \le 12$$

$$y \in [7, 12]$$

Q 6 find range of $y = 5\sin\left(x + \frac{\pi}{6}\right) + 3\cos x$

Q-7 find range of $y = \sin\left(x + \frac{\pi}{6}\right) + 3\cos\left(x - \frac{\pi}{3}\right)$

Q-8 find maximum & minimum value of

$$y = \frac{17 + (5\sin x + 12\cos x)}{17 - (5\sin x + 12\cos x)}$$