[SINGLE CORRECT CHOICE TYPE]

1. For all pairs of angles (A, B), measured in degrees such that $\sin A + \sin B = \sqrt{2}$ and $\cos A + \cos B = \sqrt{\sqrt{2}}$, both hold simultaneously. The smallest possible value of $|A - B|$ in degrees is

(A) 15 (B) 30 (C) 45 ✓ (D) 60

2. Which one of the following trigonometric statement does not hold good ?

(A) $\tan\left(\frac{\pi}{4}+x\right) = \cot\left(\frac{\pi}{4}-x\right)$ (B) $\tan\left(\frac{\pi}{4}+x\right) = \frac{1+\tan x}{1-\tan x}$

(C) $\tan\left(\frac{\pi}{4}+x\right) = \sec 2x + \tan 2x$ (D) ✓ $\tan\left(\frac{\pi}{4}+x\right) = \frac{\cos 2x}{1+\sin 2x}$

(1) $\sin A + \sin B = \sqrt{2} \Rightarrow \sin^2 A + \sin^2 B + 2\sin A \sin B = 2$

$\cos A + \cos B = \sqrt{\sqrt{2}} \Rightarrow \cos^2 A + \cos^2 B + 2\cos A \cos B = \sqrt{2}$

add

$1 + 1 + 2(\cos(A-B)) = 2+\sqrt{2}$

$\cos(A-B) = \frac{1}{\sqrt{2}}$

$|A-B| = 45^\circ$

(2) (A) $\tan\left(\frac{\pi}{4}+x\right) = \tan\left(\frac{\pi}{2} - \frac{\pi}{4} + x\right) = \tan\left(\frac{\pi}{2} - \left(\frac{\pi}{4}-x\right)\right)$

$= \cot\left(\frac{\pi}{4}-x\right)$

(B) $\tan\left(\frac{\pi}{4}+x\right) = \frac{1+\tan x}{1-\tan x}$ (from $\tan(A+B)$ Identy)

(C) $\tan\left(\frac{\pi}{4}+x\right) = \frac{1+\tan x}{1-\tan x} = \frac{\sin x + \cos x}{-\sin x + \cos x} \cdot \frac{\sin x + \cos x}{\cos x + \sin x}$

$= \frac{\sin^2 x + \cos^2 x + 2\sin x \cos x}{\cos^2 x - \sin^2 x}$

$= \frac{1+\sin 2x}{\cos 2x} = \sec 2x + \tan 2x$

3. $\tan(x+y) = a\ (a \neq 0)$ & $\tan(x-y) = b\ (b \neq 0)$ such that $\tan(x+y) + \tan(x-y) = \tan 2x$ then

(A) $a^2 + b = 0$ (B) $a + b = 0$ (C) $ab = 1$ (D) $a - b = 0$

4. Suppose $\sin\theta - \cos\theta = 1$ then the value of $\sin^3\theta - \cos^3\theta$ is $(\theta \in R)$

(A) 1 (B) -2 (C) -1 (D) 0

(3) $\tan(x+y) = a$

$\tan(x-y) = b$

$\tan(x+y) + \tan(x-y) = \tan 2x$

$\tan(x+y) + \tan(x-y) = \tan(x+y+x-y)$

$$\tan(x+y) + \tan(x-y) = \frac{\tan(x+y) + \tan(x-y)}{1 - \tan(x+y)\tan(x-y)}$$

$$(a+b) = \frac{a+b}{1-ab}$$

$$(a+b) - \frac{a+b}{1-ab} = 0$$

$$(a+b)\left[1 - \frac{1}{1-ab}\right] = 0$$

$\boxed{a+b=0}$

$$1 - \frac{1}{1-ab} = 0$$

$1 - ab - 1 = 0$

$\boxed{ab=0}$

(4) $\sin\theta - \cos\theta = 1 \Rightarrow \sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta = 1$

$\sin\theta\cos\theta = 0$

$\sin^3\theta - \cos^3\theta = (\sin\theta - \cos\theta)(\sin^2\theta + \cos^2\theta + \sin\theta\cos\theta)$

$= 1(1+0) = 1$

5. If $90^0 < \alpha < 180^0$ and $0 < \beta < 90^0$ such that $\sin\alpha = \frac{4}{5}$, $\cos\beta = \frac{5}{13}$ and $\tan\left(\frac{\alpha+\beta}{2}\right) = p$, $\tan\left(\frac{\alpha-\beta}{2}\right) = q$, then $\frac{p}{q}$ is equal to

(A) 14 (B) –14 (C) 16 (D) –16

6. If $\cos(\alpha - \beta) + \cos(\beta - \gamma) + \cos(\gamma - \alpha) = -\frac{3}{2}$, $\cos\alpha + \cos\beta + \cos\gamma = p$ & $\sin\alpha + \sin\beta + \sin\gamma = q$, then

(A) $p = q = 0$ (B) $p + q = 1$ (C) $p = q = 1$ (D) $p - q = 1$

(5) $$\frac{p}{q} = \frac{\tan\left(\frac{\alpha+\beta}{2}\right)}{\tan\left(\frac{\alpha-\beta}{2}\right)} = \frac{2\sin\left(\frac{\alpha+\beta}{2}\right)}{2\cos\left(\frac{\alpha+\beta}{2}\right)} \cdot \frac{\cos\left(\frac{\alpha-\beta}{2}\right)}{\sin\left(\frac{\alpha-\beta}{2}\right)}$$

$$= \frac{\sin\alpha + \sin\beta}{\sin\alpha - \sin\beta} = \frac{\frac{4}{5} + \frac{12}{13}}{\frac{4}{5} - \frac{12}{13}} = \frac{52+60}{52-60} = \frac{112}{-8}$$

$$= -14$$

(6) $$\cos(\alpha-\beta) + \cos(\beta-\gamma) + \cos(\gamma-\alpha) = -\frac{3}{2}$$

$$\cos\alpha + \cos\beta + \cos\gamma = p$$

$$\sin\alpha + \sin\beta + \sin\gamma = q$$

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma + 2(\cos\alpha\cos\beta + \cos\beta\cos\gamma + \cos\gamma\cos\alpha) = p^2$$

$$\sin^2\alpha + \sin^2\beta + \sin^2\gamma + 2(\sin\alpha\sin\beta + \sin\beta\sin\gamma + \sin\alpha\sin\gamma) = q^2$$

add

$$1 + 1 + 1 + 2[\cos(\alpha-\beta) + \cos(\beta-\gamma) + \cos(\gamma-\alpha)] = p^2 + q^2$$

$$3 + 2\left(-\frac{3}{2}\right) = p^2 + q^2$$

$$p^2 + q^2 = 0$$

$$\therefore \boxed{p = 0 = q}$$

7. If $\sin\theta + \sin\phi = a$ and $\cos\theta + \cos\phi = b$, then

(A) $\cos\left(\frac{\theta-\phi}{2}\right) = \pm\frac{1}{2}\sqrt{(a^2+b^2)}$ ✓

(B) $\cos\left(\frac{\theta-\phi}{2}\right) = \pm\frac{1}{2}\sqrt{(a^2-b^2)}$

(C) $\tan\left(\frac{\theta-\phi}{2}\right) = \pm\sqrt{\frac{4-a^2-b^2}{a^2+b^2}}$ ✓

(D) $\cos(\theta-\phi) = \frac{a^2+b^2-2}{2}$ ✓

$\sin\theta + \sin\phi = a \Rightarrow \sin^2\theta + \sin^2\phi + 2\sin\theta\sin\phi = a^2$

$\cos\theta + \cos\phi = b \Rightarrow \cos^2\theta + \cos^2\phi + 2\cos\theta\cos\phi = b^2$

$1 + 1 + 2(\cos(\theta-\phi)) = a^2+b^2$

$$\boxed{\cos(\theta-\phi) = \frac{a^2+b^2-2}{2}}$$

$$\cos\left(\frac{\theta-\phi}{2}\right) = \pm\sqrt{\frac{1+\cos(\theta-\phi)}{2}} = \pm\frac{\sqrt{a^2+b^2}}{2}$$

$$\tan\left(\frac{\theta-\phi}{2}\right) = \pm\sqrt{\frac{1-\cos(\theta-\phi)}{1+\cos(\theta-\phi)}} = \pm\sqrt{\frac{4-a^2-b^2}{a^2+b^2}}$$

8. If $\cos\alpha = \frac{3}{5}$ and $\cos\beta = \frac{5}{13}$, (where $\alpha, \beta, \in 1^{st}$ quadrant)

(A) $\cos(\alpha+\beta) = \frac{33}{65}$

✓(B) $\sin(\alpha+\beta) = \frac{56}{65}$

✓(C) $\sin^2\left(\frac{\alpha-\beta}{2}\right) = \frac{1}{65}$

✓(D) $\cos(\alpha-\beta) = \frac{63}{65}$

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$= \frac{3}{5}\cdot\frac{5}{13} - \frac{4}{5}\cdot\frac{12}{13}$$

$$= \frac{-33}{65}$$

$\cos\alpha = 3/5$

$\sin\alpha = 4/5$

$\cos\beta = 5/13$

$\sin\beta = 12/13$

$$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta = \frac{63}{65}$$

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$= \frac{4}{5}\cdot\frac{5}{13} + \frac{3}{5}\cdot\frac{12}{13} = \frac{56}{65}$$

$$\sin^2\left(\frac{\alpha-\beta}{2}\right) = \frac{1-\cos(\alpha-\beta)}{2} = \frac{1}{2}\left(1 - \frac{63}{65}\right) = \frac{1}{65}$$

9. **Column-I** **Column-II**

Column-I		Column-II
(A)	Number of real solutions of the equation $\lvert x-1 \rvert + \lvert x-3 \rvert = \frac{3}{2}$ is	(P) -1
(B)	If $\sin x + \sin^2 x = 1$ then the value of $\cos^2 x + \cos^4 x$ equals	(Q) 0
(C)	If $\log_{10}(x^2 + x) = \log_{10}(x^3 - x)$ then the product of all solutions of the equation is	(R) 1
(D)	If $1 + x + x^2 + x^3 = 0$ where $x \in R$ then the value of $1 + x + x^2 + x^3 + x^4 + ...+ x^{2018} + x^{2019}$ equals	(S) 2

(A) $|x-1| + |x-3| = \frac{3}{2}$

CI $\quad x < 1 \qquad -(x-1) - (x-3) = \frac{3}{2}$

$-2x + 4 = \frac{3}{2} \Rightarrow -2x = \frac{3}{2} - 4$

$\boxed{x = \frac{5}{4}}$ ✗

Rejected

CII $\quad 1 \leq x < 3$

$+(x-1) - (x-3) = \frac{3}{2}$ false

CIII $\quad x \geq 3 \qquad (x-1) + (x-3) = \frac{3}{2} \Rightarrow \boxed{x = \frac{11}{4}}$ ✗

Number of real solution = 0

(B) $\sin x = 1 - \sin^2 x \Rightarrow \sin x = \cos^2 x$

$\cos^2 x + \cos^4 x = \sin x + \sin^2 x = 1$

(C) $x^2 + x = x^3 - x \Rightarrow x^3 - x^2 - 2x = 0 \Rightarrow x(x+1)(x-2) = 0$

$x = 0, -1, 2$

$x = 0$ and -1 does not work for $\log_{10}(x^2+x)$

so $x = 2$ is the final answer.

(D) $1 + x + x^2 + x^3 = 0 \Rightarrow$

$1 + x + x^2 + x^3 + x^4 + x^5 + \cdots + x^{2016} + x^{2017} + x^{2018} + x^{2019}$

make 4-4 pairs to get answer 0.

10. **Column-I** **Column-II**

	Column-I	Column-II
(A)	If x, y, z be positive real numbers such that $\log_{2x}(z) = 3$, $\log_{5y}(z) = 6$ and $\log_{xy}(z) = 2/3$ then the value of z is in the form of m/n in lowest form then (n – m) is equal to	(P) 4
(B)	Let $0 \le a, b, c, d \le \pi$ where b and c are not complementary such that $2\cos a + 6\cos b + 7\cos c + 9\cos d = 0$ and $2\sin a - 6\sin b + 7\sin c - 9\sin d = 0$. If $\dfrac{\cos(a+d)}{\cos(b+c)} = \dfrac{m}{n}$ where m and n are relatively prime positive numbers, then the value of (m + n) is equal to	(Q) 8
(C)	Suppose A and B are two angles such that $A, B \in (0, \pi)$ and satisfy $\sin A + \sin B = 1$ and $\cos A + \cos B = 0$. The value of $12\cos 2A + 4\cos 2B$, is equal to	(R) 9
(D)	α and β are the positive acute angles and satisfying simultaneously the equation $5\sin 2\beta = 3\sin 2\alpha$ and $\tan\beta = 3\tan\alpha$. The value of $\tan\alpha + \tan\beta$ is	(S) 10

(A)

$$\log_{2x} z = 3 \Rightarrow \frac{\log(2x)}{\log z} = \frac{1}{3}$$ (Reciprocal)

$$\log_{5y} z = 6 \Rightarrow \frac{\log(5y)}{\log z} = \frac{1}{6}$$

add

$$\frac{\log 2x + \log 5y}{\log z} = \frac{1}{2}$$

$$\frac{\log(10xy)}{\log z} = \frac{1}{2}$$

$$\frac{\log 10 + \log xy}{\log z} = \frac{1}{2}$$

given as $\frac{3}{2}$

$$\frac{\log 10}{\log z} + \boxed{\log_z xy} = \frac{1}{2}$$

$$\frac{\log 10}{\log z} = \frac{1}{2} - \frac{3}{2} = -1$$

$\Leftarrow$ $\log z = -\log 10$

$\boxed{z = 1/10}$ $\boxed{n - m = 9}$

(B) $2\cos a + 6\cos b + 7\cos c + 9\cos d = 0$

and $2\sin a - 6\sin b + 7\sin c - 9\sin d = 0$.

$$2\cos a + 9\cos d = -(6\cos b + 7\cos c)$$
$$2\sin a - 9\sin d = 6\sin b - 7\sin c$$

$$4\cos^2 a + 81\cos^2 d + 36\cos a\cos d = 36\cos^2 b + 49\cos^2 c + 84\cos b\cos c$$

$$4\sin^2 a + 81\sin^2 d - 36\sin a\sin d = 36\sin^2 b + 49\sin^2 c - 84\sin b\sin c$$

$$4 + 81 + 36(\cos(a+d)) = 36 + 49 + 84\cos(b+c)$$

$$\frac{\cos(a+d)}{\cos(b+c)} = \frac{84}{36} = \frac{7}{3}$$

$m+n=10$

(C) $\cos A + \cos B = 0$

$\cos A = -\cos B$

$\cos A = \cos(\pi - B)$

$A + B = \pi$

$\sin A + \sin B = 1$

$\sin A + \sin(\pi - A) = 1$

$\sin A = \frac{1}{2}$ | $\sin B = \frac{1}{2}$

$\cos A = \frac{\sqrt{3}}{2}$

$12\cos 2A + 4\cos 2B$

$= 12(1 - 2\sin^2 A) + 4(1 - 2\sin^2 B)$

$= 12 - 24\left(\frac{1}{4}\right) + 4 - 8\left(\frac{1}{4}\right)$

$= 16 - 8 = 8$

ⓓ α and β are the positive acute angles and satisfying simultaneously the equation $5 \sin 2\beta = 3 \sin 2\alpha$ and $\tan \beta = 3 \tan \alpha$.

The value of $\tan \alpha + \tan \beta$ is

$$5 \sin 2\beta = 3 \sin 2\alpha$$

$$\frac{\sin 2\beta}{\sin 2\alpha} = \frac{3}{5}$$

$$\frac{\sin 2\alpha + \sin 2\beta}{\sin 2\alpha - \sin 2\beta} = \frac{3+5}{5-3}$$

$$\frac{2 \sin(\alpha+\beta) \cos(\alpha-\beta)}{2 \cos(\alpha+\beta) \sin(\alpha-\beta)} = 4$$

$$\tan(\alpha+\beta) = 4 \tan(\alpha-\beta)$$

$$\frac{\tan\alpha + \tan\beta}{1 - \tan\alpha \tan\beta} = 4\left[\frac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta}\right]$$

$$\frac{4\tan\alpha}{1-3\tan^2\alpha} = 4\left[\frac{-2\tan\alpha}{1+3\tan^2\alpha}\right]$$

$\tan\alpha = 0$

$$\frac{4}{1-3\tan^2\alpha} + \frac{8}{1+3\tan^2\alpha} = 0$$

$$\frac{4 + 12\tan^2\alpha + 8 - 24\tan^2\alpha}{(1-3\tan^2\alpha)(1+3\tan^2\alpha)} = 0$$

$$1 - \tan^2\alpha = 0$$

$$\tan\alpha = \pm 1$$

$\tan\alpha = 1$

$\tan\beta = 3$

$\tan\alpha + \tan\beta = 4$

11. If 5 sin x = sin(x + 2y), then prove that 2 tan(x + y) = 3 tan y

12. If sin(A + 2B) = cos(2A + B) & B – A = $\frac{\pi}{3}$ where A,B $\in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, then $\left|\frac{B}{A}\right|$ is

(11) $5\sin x = \sin(x+2y)$

$$\frac{5}{1} = \frac{\sin(x+2y)}{\sin x}$$

$$\frac{5+1}{5-1} = \frac{\sin(x+2y)+\sin x}{\sin(x+2y)-\sin x}$$

$$\frac{\overset{3}{\cancel{6}}}{\underset{2}{\cancel{4}}} = \frac{\cancel{2}\sin(x+y)\cos(y)}{\cancel{2}\cos(x+y)\sin(y)}$$

$$\frac{3}{2} = \tan(x+y)\cdot\cot y$$

$$2\tan(x+y) = 3\tan y$$

(12) $\sin\left(A + 2A + \frac{2\pi}{3}\right) = \cos\left(2A + A + \frac{\pi}{3}\right)$

$$\sin\left(3A + \frac{2\pi}{3}\right) = \cos\left(3A + \frac{\pi}{3}\right)$$

$$\sin 3A\cos\frac{2\pi}{3} + \cos 3A\sin\frac{2\pi}{3} = \cos 3A\cos\frac{\pi}{3} - \sin 3A\sin\frac{\pi}{3}$$

$$\sin 3A\left(-\frac{1}{2}\right) + \cos 3A\cdot\left(\frac{\sqrt{3}}{2}\right) = \cos 3A\left(\frac{1}{2}\right) - \sin 3A\cdot\frac{\sqrt{3}}{2}$$

$$\sin 3A\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}\right) = \cos 3A\left(\frac{1}{2}-\frac{\sqrt{3}}{2}\right)$$

$$\sin 3A = -\cos 3A \Rightarrow \tan 3A = -1 \Rightarrow 3A = -45$$

$$A = -15$$

$$B = 60 - 15 = 45$$

$$\left|\frac{B}{A}\right| = \left|\frac{45}{-15}\right| = 3$$

13. Prove that $\tan\left(\frac{\pi}{4}+\theta\right)\times\tan\left(\frac{3\pi}{4}+\theta\right)=-1$

14. If $\tan\theta=\frac{k}{k+1}$ and $\tan\phi=\frac{k}{k+1}$, find $\tan(\theta+\phi)$ and $\tan(\theta-\phi)$

13

$$\tan\left(\frac{\pi}{4}+\theta\right)\cdot\tan\left(\frac{3\pi}{4}+\theta\right)$$

$$=\tan\left(\frac{\pi}{4}+\theta\right)\cdot\tan\left(\frac{\pi}{2}+\left(\frac{\pi}{4}+\theta\right)\right)$$

$$=\tan\left(\frac{\pi}{4}+\theta\right)\cdot\left[-\cot\left(\frac{\pi}{4}+\theta\right)\right]=-1=\underline{RHS.}$$

14

$$\tan(\theta+\phi)=\frac{\tan\theta+\tan\phi}{1-\tan\theta\tan\phi}=\frac{\frac{k}{k+1}+\frac{k}{k+1}}{1-\frac{k}{k+1}\cdot\frac{k}{k+1}}$$

$$=\frac{2k}{k+1}\ \frac{(k+1)^2}{(k^2+1-k^2)}=2k(k+1)$$

$$\tan(\theta-\phi)=\frac{\tan\theta-\tan\phi}{1+\tan\theta\tan\phi}=\frac{\frac{k}{k+1}-\frac{k}{k+1}}{1+\frac{k}{k+1}\cdot\frac{k}{k+1}}$$

$$=0$$

15. If 2 $\sin\alpha \cos\beta \sin\gamma = \sin\beta \sin(\alpha + \gamma)$. Then show $\tan\alpha$, $\tan\beta$ and $\tan\gamma$ are in Harmonic Progression

16. If $3\tan\theta \tan\phi = 1$, then prove that $2\cos(\theta + \phi) = \cos(\theta - \phi)$

(15) $2 \sin\alpha \cos\beta \sin\gamma = \sin\beta (\sin\alpha \cos\gamma + \cos\alpha \sin\gamma)$

divide by $\sin\alpha \sin\beta \sin\gamma$

$2 \cot\beta = \cot\gamma + \cot\alpha$

$\therefore$ $\cot\alpha$, $\cot\beta$, $\cot\gamma$ are in AP.

$\Rightarrow$ $\tan\alpha$, $\tan\beta$, $\tan\gamma$ are in HP.

(16) $3 \tan\theta \tan\phi = 1$

$$\frac{\sin\theta \sin\phi}{\cos\theta \cos\phi} = \frac{1}{3}$$

apply componendo & dividendo

$$\frac{\sin\theta \sin\phi + \cos\theta \cos\phi}{\sin\theta \sin\phi - \cos\theta \cos\phi} = \frac{1+3}{1-3}$$

$$\frac{\cos(\theta - \phi)}{-\cos(\theta + \phi)} = \frac{4}{-2}$$

$$\cos(\theta - \phi) = 2\cos(\theta + \phi)$$

17. Show that $2\cos\theta = +\sqrt{2+\sqrt{2+2\cos 4\theta}}$ where $-\frac{\pi}{4} \le \theta \le \frac{\pi}{4}+$

18. Prove that $2\cos\left(\frac{A+3B}{2}\right)\cos\left(\frac{3A-B}{2}\right) = \cos(2A+B)+\cos(A-2B)$

(17)

$$\sqrt{2+\sqrt{2+2\cos 4\theta}}$$

$$= \sqrt{2+2\cos 2\theta}$$

$$= \sqrt{4\cos^2\theta} = 2\cos\theta$$

(18) $2\cos\left(\frac{A+3B}{2}\right)\cos\left(\frac{3A-B}{2}\right)$

$$= \cos\left(\frac{A+3B}{2}+\frac{3A-B}{2}\right) + \cos\left(\frac{A+3B}{2}-\frac{3A-B}{2}\right)$$

$$= \cos(2A+B) + \cos(-A+2B)$$

$$= \cos(2A+B) + \cos(A-2B) \quad \because \cos(-x) = \cos x$$

= RHS.

Hence Proved

19. Prove that $\sin(A+B).\sin(A-B)=\frac{1}{2}[2\sin(A+B).\sin(A-B)]=\frac{1}{2}(\cos 2B-\cos 2A)$

20. Prove the following:

(A) $\dfrac{\sin(x+y)-2\sin x+\sin(x-y)}{\cos(x+y)-2\cos x+\cos(x-y)}=\tan x$

(B) $\dfrac{\sin(a-c)+2\sin a+\sin(a+c)}{\sin(b-c)+2\sin b+\sin(b+c)}=\dfrac{\sin a}{\sin b}$

(C) $\dfrac{\sin A+\sin B}{\sin A-\sin B}=\tan\dfrac{A+B}{2}\cot\dfrac{A-B}{2}$

(D) $\dfrac{\cos A+\cos B}{\cos A-\cos B}=-\cot\dfrac{A+B}{2}\cot\dfrac{A-B}{2}$

(E) $\cos(-B+C+A)+\cos(-A+B+C)+\cos(A+B-C)+\cos(A+B+C = 4\cos A\cos B\cos C$

19 $\frac{1}{2}[2\sin(A+B)\cdot\sin(A-B)] = \frac{1}{2}[\cos(A+B-A+B) - \cos(A+B+A-B)]$

$= \frac{1}{2}(\cos 2B - \cos 2A)$

$=$ RHS.

20 a $\dfrac{\sin(x+y)-2\sin x+\sin(x-y)}{\cos(x+y)-2\cos x+\cos(x-y)}=\tan x$

LHS. $= \dfrac{\sin(x+y)-2\sin x+\sin(x-y)}{\cos(x+y)-2\cos x+\cos(x-y)}$

$= \dfrac{2\sin x\cos y - 2\sin x}{2\cos x\cos y - 2\cos x} = \dfrac{2\sin x}{2\cos x} = \tan x$

(b) $$\frac{\sin(a-c)+2\sin a+\sin(a+c)}{\sin(b-c)+2\sin b+\sin(b+c)}=\frac{\sin a}{\sin b}$$

$$LHS = \frac{\sin(a-c)+2\sin a+\sin(a+c)}{\sin(b-c)+2\sin b+\sin(b+c)}$$

$$= \frac{2\sin a\cos c + 2\sin a}{2\sin b\cos c + 2\sin b} = \frac{\sin a}{\sin b}$$

(c) $$\frac{\sin A+\sin B}{\sin A-\sin B}=\tan\frac{A+B}{2}\cot\frac{A-B}{2}$$

$$LHS = \frac{\sin A+\sin B}{\sin A-\sin B} = \frac{2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)}{2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)}$$

$$= \tan\left(\frac{A+B}{2}\right)\cot\left(\frac{A-B}{2}\right)$$

(d) $$\frac{\cos A+\cos B}{\cos A-\cos B}=-\cot\frac{A+B}{2}\cot\frac{A-B}{2}$$

$$LHS = \frac{\cos A+\cos B}{\cos A-\cos B} = \frac{2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)}{-2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)}$$

$$= -\cot\left(\frac{A+B}{2}\right)\cot\left(\frac{A-B}{2}\right)$$

(e) $\underbrace{\cos(-B + C + A) + \cos(-A + B + C)} + \cos(A + B - C) + \cos(A + B + C) = 4\cos A\cos B\cos C$

$$2\cos C\cos(A-B) + 2\cos(A+B)\cos C$$

$$= 2\cos C\left[\cos(A-B) + \cos(A+B)\right]$$

$$= 2\cos C\, 2\cos A\cos B$$

$$= 4\cos A\cos B\cos C$$