MOMENTUM: AMOUNT OF MOTION :→

Linear momentum of a body is defined as product of its mass and velocity. It provides measure of amount of motion.

Linear momentum $\vec{p}$ of a body of mass m, moving with velocity by $\vec{v}$ is expressed by the following equation.

$$\boxed{\vec{p} = m\vec{v}}$$

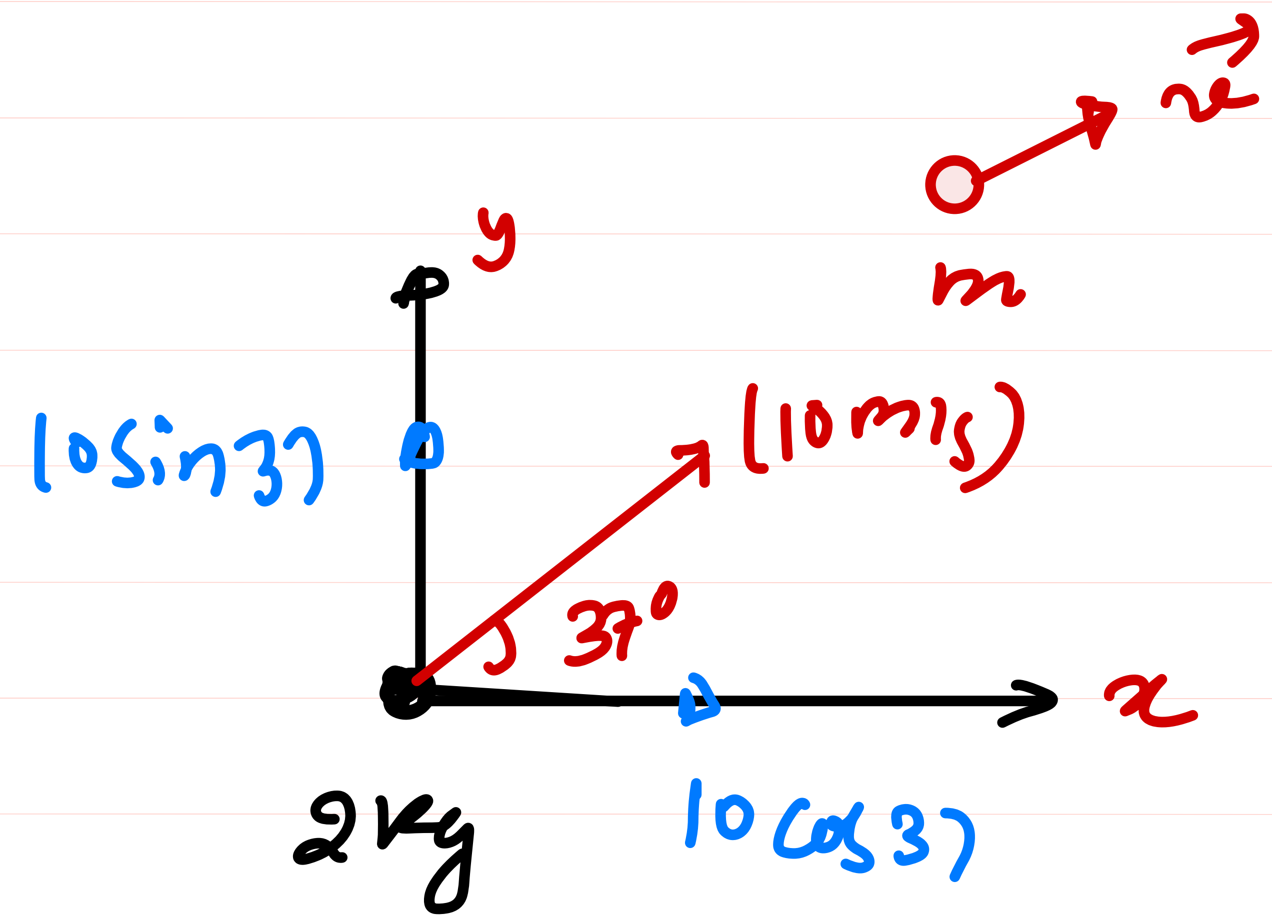
SI unit of momentum is kg-m/s.

Dimensions of momentum are MLT^{-1}

Linear momentum = $\vec{p}$ = mass × velocity

$$\boxed{\vec{p} = m\vec{v}}$$

Ex



Find linear momentum

$\vec{p} = m(10\cos 37\,\hat{i} + 10\sin 37\,\hat{j})$

$= 2(10 \times 4/5\,\hat{i} + 10 \times 3/5\,\hat{j})$

$\boxed{\vec{p} = 16\hat{i} + 12\hat{j}}$ Ans

$p = mv$

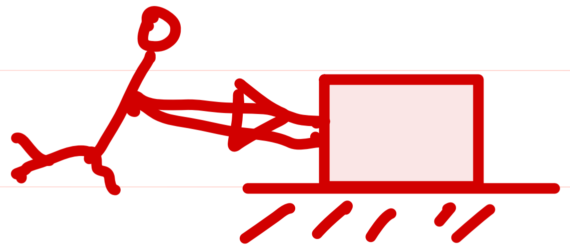
$= 2 \times 10$

$\boxed{p = 20 \text{ kg}\cdot\text{m/s}}$

Ans

FORCE :

The concept of force is used to explain mutual interaction between two material bodies as the action of one body on another in form of push or pull, which brings out or tries to bring out a change in the state of motion of the two bodies.



is vector Quantity
SI unit is Newton (N)

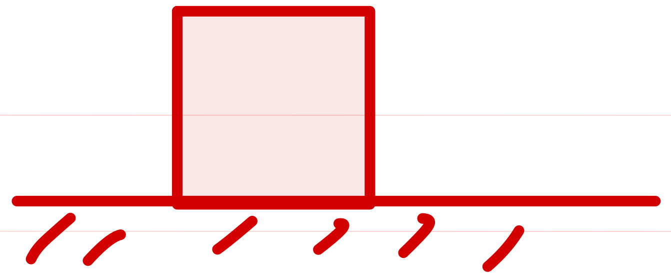
C.G.S Unit = dyne

$$\boxed{1N = 10^5 \text{ dyne}}$$

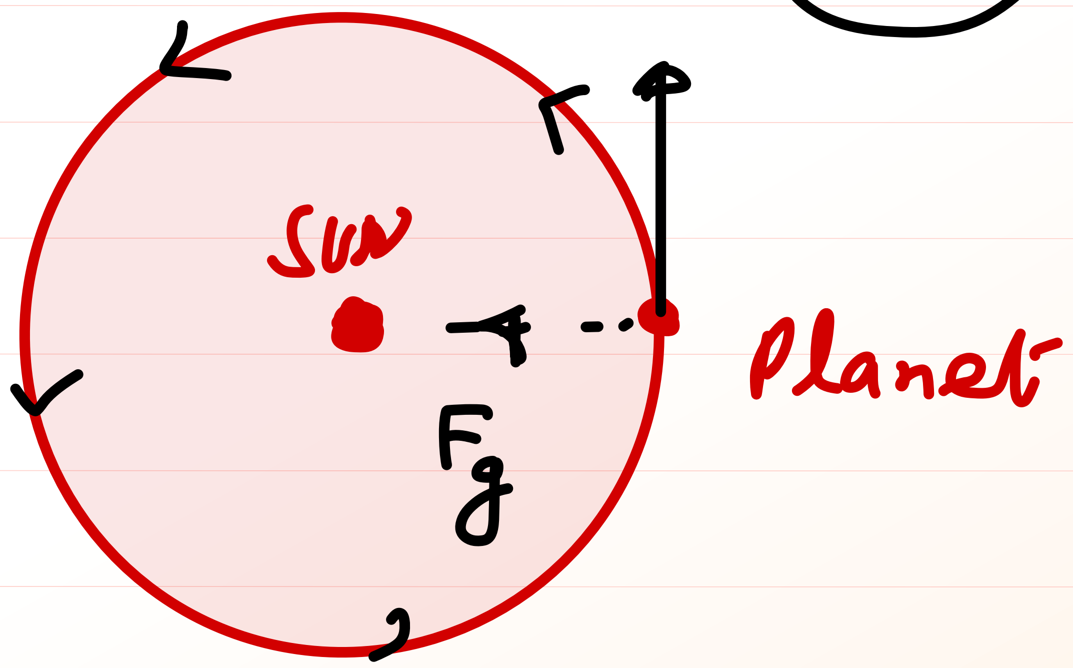
① Contact and Field Forces :

SL AL

When a body applies force on other by direct contact, the force is known as contact force. When two bodies apply force on each other without any contact between them, the force is known as field force.



force by box on surface
OR
Force by surface on box are
Contact force



① Gravitational force
② magnetic force
③ Nuclear force
④ Electrostatic force

NEWTON'S LAWS OF MOTION :→

The First Law : (law of INERTIA)

SL AL

"Every material body has tendency to preserve its state of rest, or of uniform motion in a straight line, unless it is compelled to change that state by external forces impressed on it."

Inertia :→

The tendency of a material body to preserve its present state of uniform motion or of rest is known as inertia of the body. It was first conceived by Galileo.

Inertia is a physical quantity and mass of a material body is measure of its inertia.

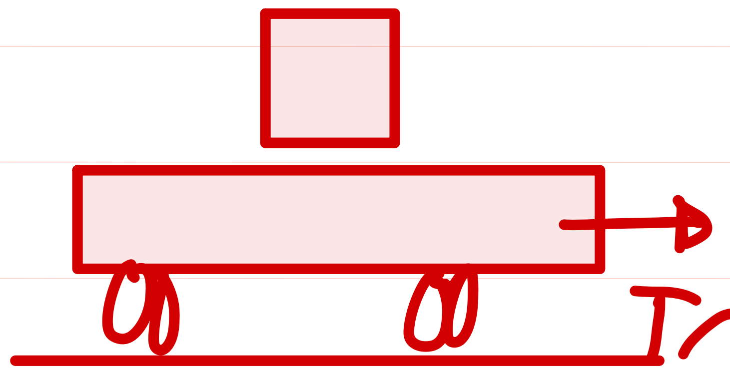
$\boxed{\text{Inertial} \propto \text{mass}}$

Inertial Frame of Reference :→

The first law requires a frame of reference in which only the forces acting on a body can be responsible for any acceleration produced in the body and not the acceleration of the frame of reference. These frames of reference are known as inertial frames.

Frame of Reference → ① uniform velocity or ($\vec{v}$ = constant)

② At Rest ($\vec{v} = 0$)



$v = 10\,m/s$

Trolley (Inertial frame)

The Second Law (law of force)

SL **AL**

The rate of change in momentum of a body is equal to, and occurs in the direction of the net applied force.

A body of mass m in translational motion with velocity $\vec{v}$, if acted upon with a net external force $\vec{F}$, the second law suggests:

$$\boxed{\vec{F} = \frac{d}{dt}(m\vec{v})}$$

If mass of the body is constant, the above equation relates the acceleration $\vec{a}$ of the body with the net force $\vec{F}$ acting on it.

$$\vec{F} = \frac{d}{dt}(m\vec{v}) = m\vec{a}$$

The first law provides concept of force and the second law provides the quantitative definition of force, therefore the second law is also valid only in inertial frames.

SI unit of force is newton. It is abbreviated as N. One newton equals to one kilogram-meter per second square.

$$\boxed{1\text{ N} = 1\text{ kg-m/s}^2}$$

Dimensions of force are MLT^{-2}

$$\boxed{\vec{F}_{net} \propto \frac{d\vec{p}}{dt}}$$

$$\vec{F}_{ext} = K\frac{d\vec{p}}{dt}$$

In S.I unit $K=1$

$$\boxed{\vec{F}_{ext} = \frac{d\vec{p}}{dt}}$$

$\because \vec{F}_{ext} = \frac{d\vec{p}}{dt}$

$\because \vec{p} = m\vec{v}$

$$\frac{d\vec{p}}{dt} = m\frac{d\vec{v}}{dt} + \vec{v}\frac{dm}{dt}$$

$$\boxed{\vec{F}_{ext} = m\vec{a} + \vec{v}\frac{dm}{dt}}$$

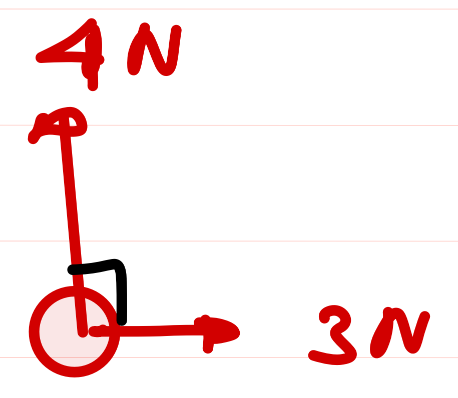
If mass of an object is equal to constant

$$\boxed{\frac{dm}{dt} = 0}$$

$$\boxed{\vec{F}_{ext} = m\vec{a}}$$

$$\boxed{\vec{a} = \frac{\vec{F}_{ext}}{m}}$$

Ex



find acc of object

$F_{ext} = \sqrt{3^2 + 4^2}$

$= 5N$

$\because \frac{F_{ext}}{m} = a$

$$\boxed{a = \frac{5}{2} = 2.5\ m/s^2}$$

Ex momentum of an object is changing as given Equation

$$P = t^2 - 2t \quad \text{along } x\text{-axis}$$

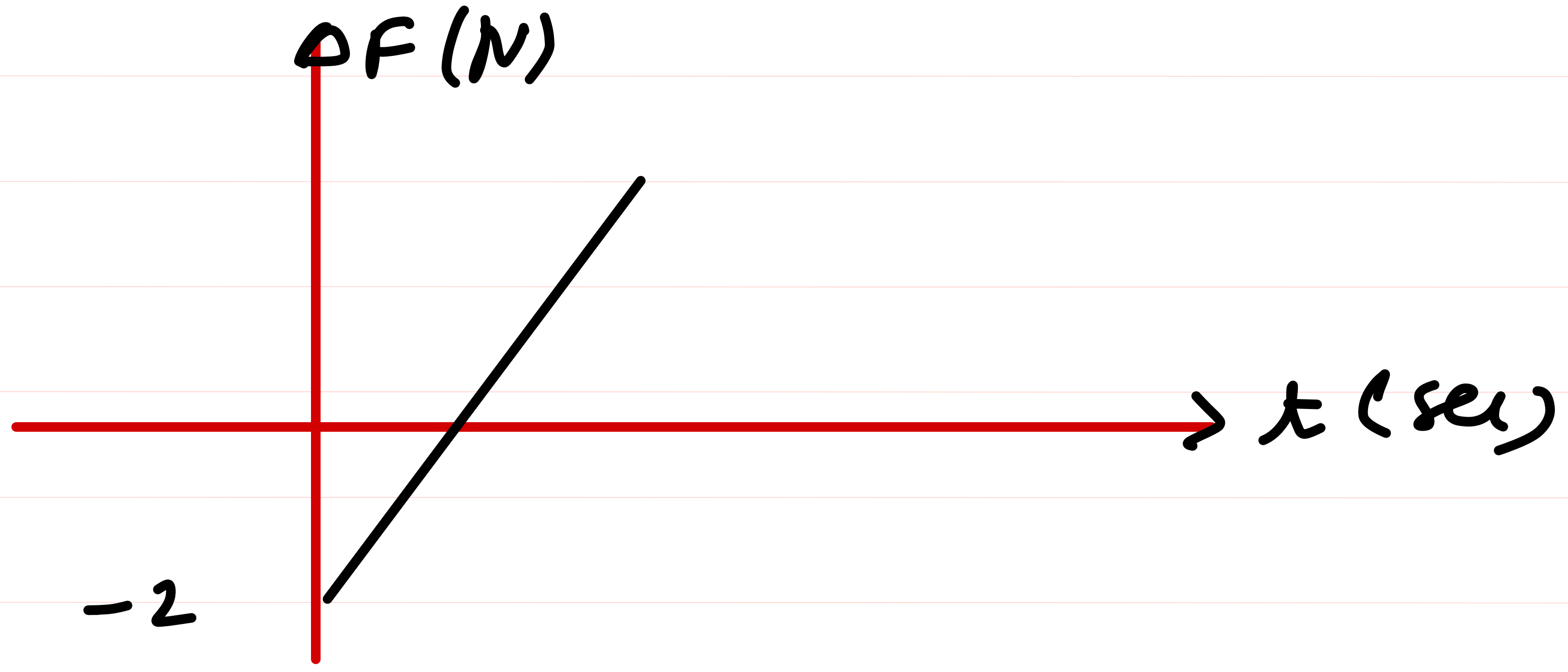
Find ① net force on object at any time t

② Avg force on object b/w $t=0$ to $t = 2$ sec

Sol ① $F = \dfrac{dP}{dt}$

$$\boxed{F = 2t - 2}$$

Draw F-t curve



② $\boxed{F_{avg} = \dfrac{\Delta P}{\Delta t}}$

$$F_{avg} = \frac{P(2) - P(0)}{2 - 0}$$

$$= \frac{(2^2 - 2\times 2) - (0^2 - 2\times 0)}{2 - 0}$$

$$= \frac{4 - 4}{2}$$

$$\boxed{F_{avg} = 0 \text{ N}}$$

Ex Force on object is given by an Equation

$$F = 3t^2$$

Find momentum of an object at any time t if initial is 2 kg.m/s

Sol

$$F = \frac{dp}{dt}$$

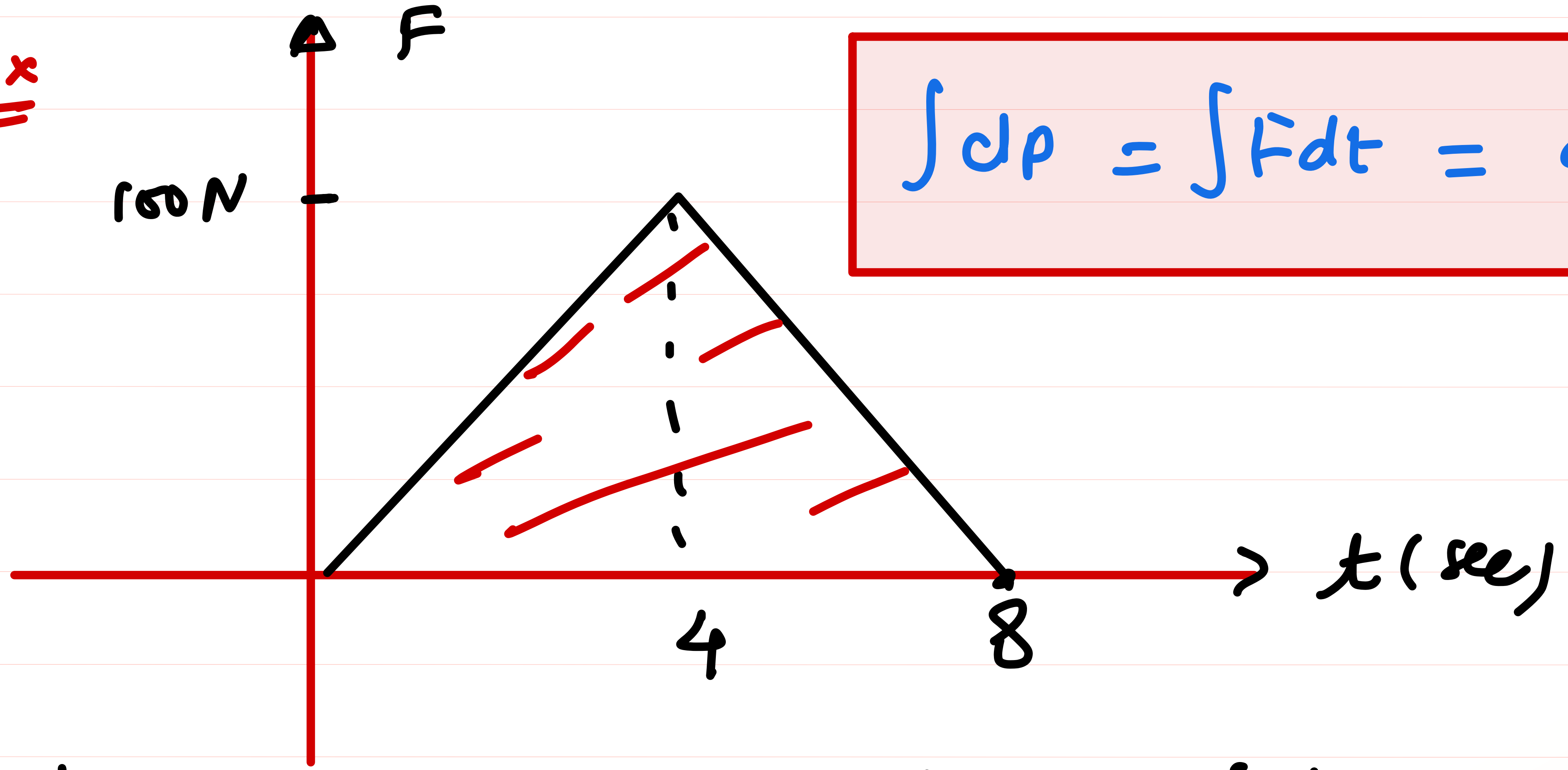
$$\int_2^P dp = \int_0^t F.dt$$

$$P - 2 = \int_0^t 3t^2 dt$$

$$P - 2 = 3\left[\frac{t^3}{3}\right]_0^t$$

$$\boxed{P = t^3 + 2}$$ Ans

Ex



$$\boxed{\int dp = \int F dt = \text{area of F-t curve}}$$

If initial momentum is 0 find momentum after 8 sec.

$$\Delta p = \text{Area}$$

$$= \frac{1}{2} \times 8 \times 100$$

$$p - 0 = 400$$

$$\boxed{P = 400 \text{ kg-m/s}}$$

Ans

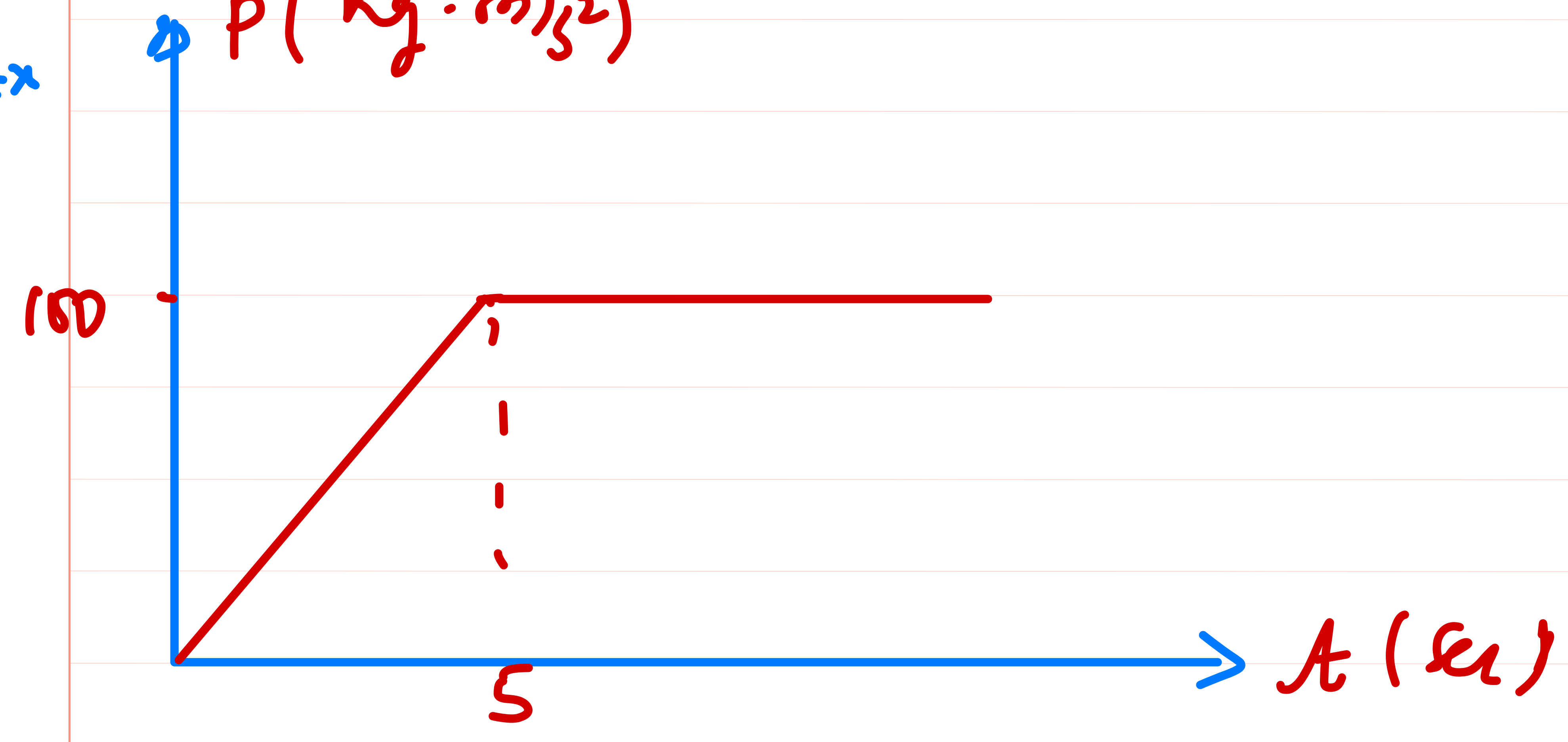
(ii) What is Avg force b/w $t=0$ to $t=8$

$F_{avg} = \frac{\Delta p}{\Delta t}$

$= \frac{400}{8}$

$\boxed{F_{avg} = 50\ N}$ Ans

Ex



Find acc of an object at $t = 3$ sec and at $t = 7$ sec

(mass = 2kg)

$\because$ $F = \frac{dp}{dt}$ = slope of p-t curve

b/w 0 to 5 sec :

$F = \frac{100}{5} = 20\ N$

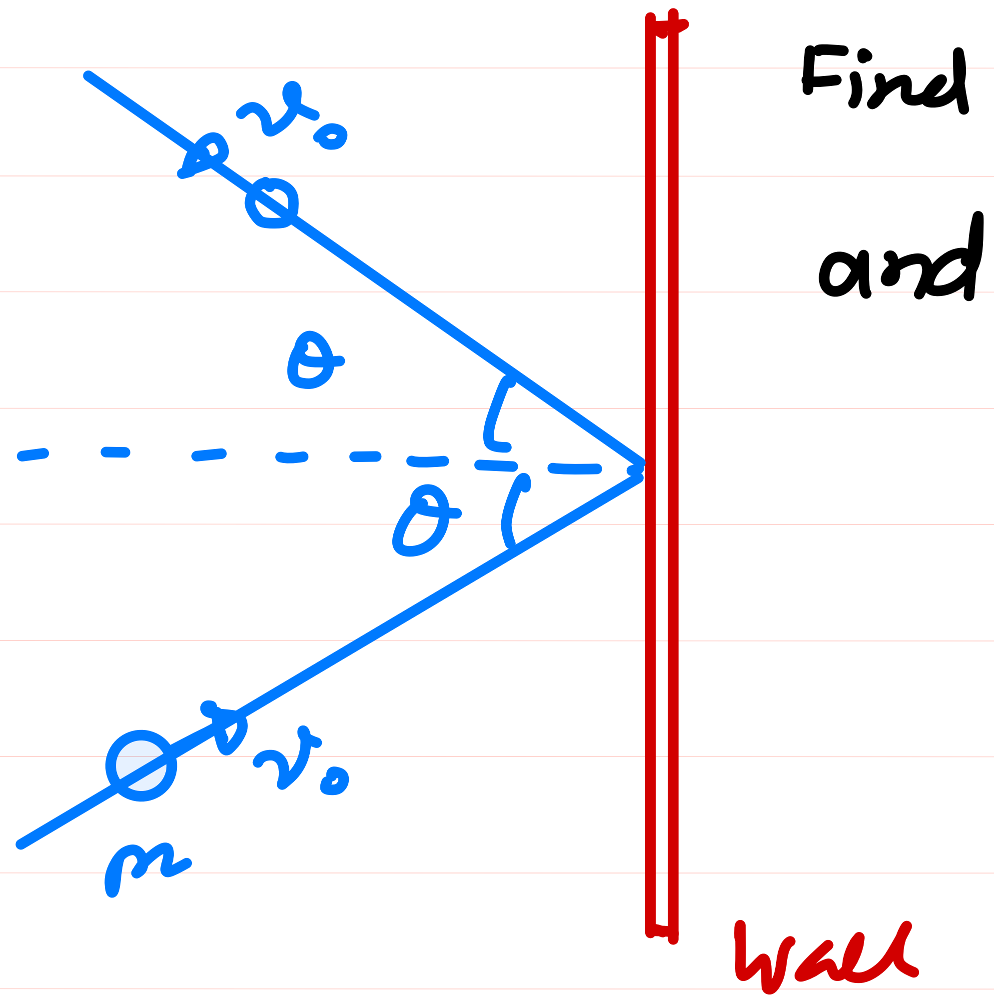
$\boxed{a = \frac{F}{m} = \frac{20}{2} = 10\ m/s^2}$ Ans

After 5 sec :

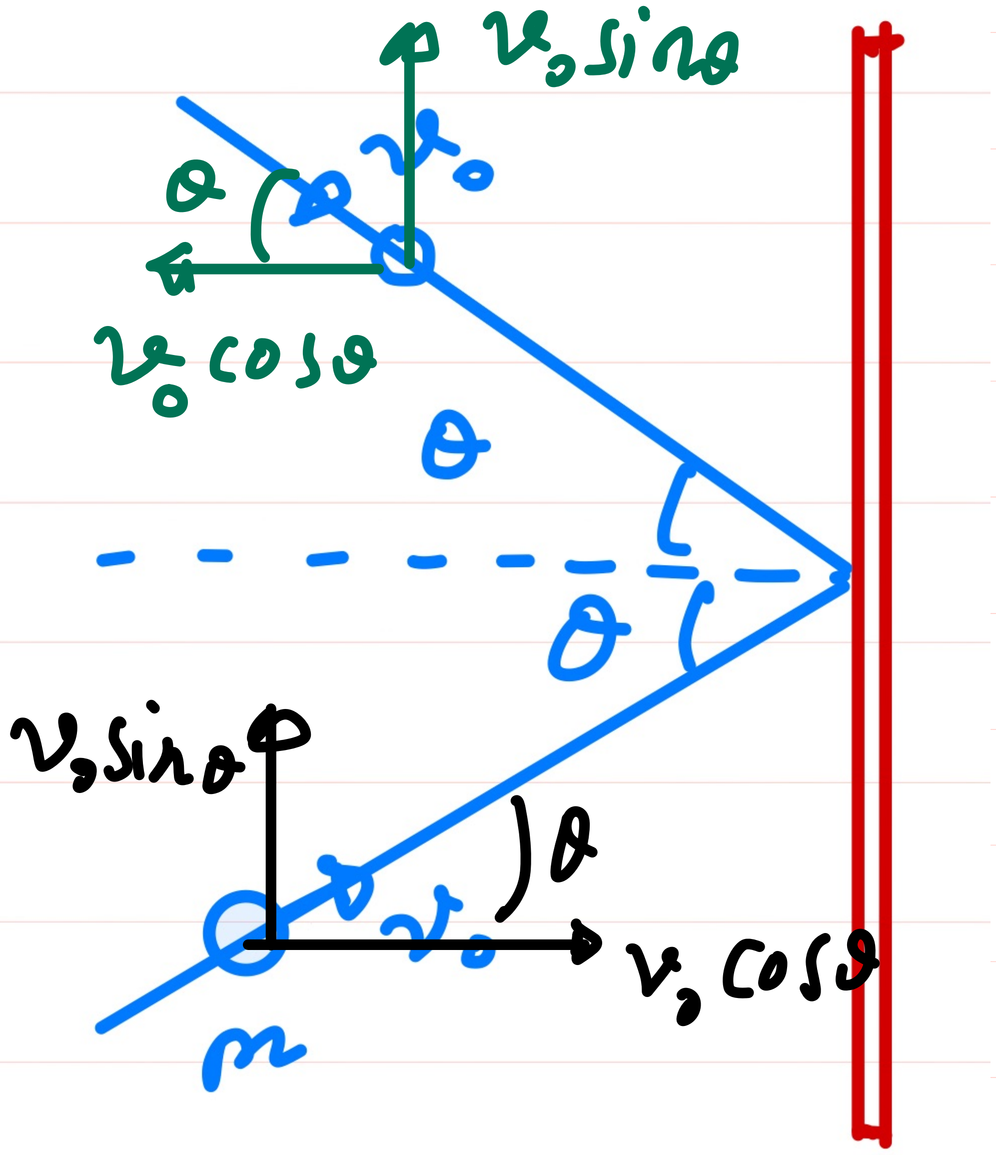
$F = 0\ N$

$a = \frac{F}{m} = \frac{0}{2} \Rightarrow \boxed{a = 0\ m/s^2}$ Ans

Ex



Find Avg force on ball of mass m due to wall if contact time b/w ball and wall is t_0



$$\because F_{avg} = \frac{\Delta P}{\Delta t}$$

$$= \frac{P_f - P_i}{t_0}$$

$$= \frac{m(\vec{v}_f - \vec{v}_i)}{t_0}$$

$$= \frac{m}{t_0}\left[(-v_0\cos\theta\,\hat{i} + v_0\sin\theta\,\hat{j}) - (v_0\cos\theta\,\hat{i} + v_0\sin\theta\,\hat{j})\right]$$

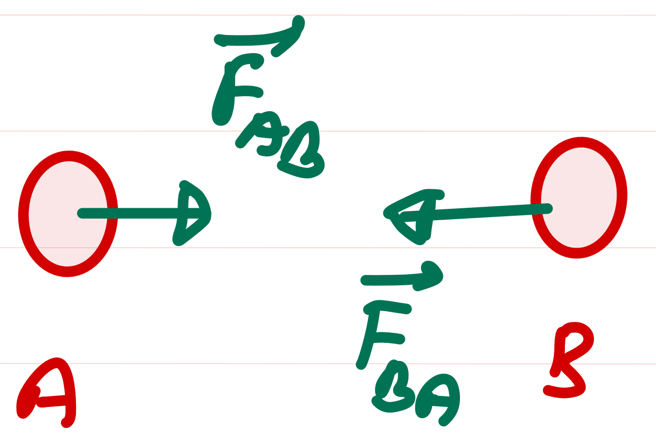
$$\boxed{\vec{F}_{avg} = -\frac{2mv_0\cos\theta}{t_0}\,\hat{i}}$$

The Third Law :→

SL AL

Force is always a two-body interaction. The first law describes qualitatively and the second law describes quantitatively what happens to a body if a force acts on it, but do not reveal anything about what happens to the other body participating in the interaction responsible for the force.

The third law accounts for this aspect of the force and states that every action on a body has equal and opposite reaction on the other body participating in the interaction.



$$\boxed{\vec{F}_{AB} = -\vec{F}_{BA}}$$

Net force on system of body-A and body-B

$$\boxed{\vec{F}_{net} = 0}$$

$$\boxed{\vec{F}_{AB} + \vec{F}_{BA} = 0}$$