

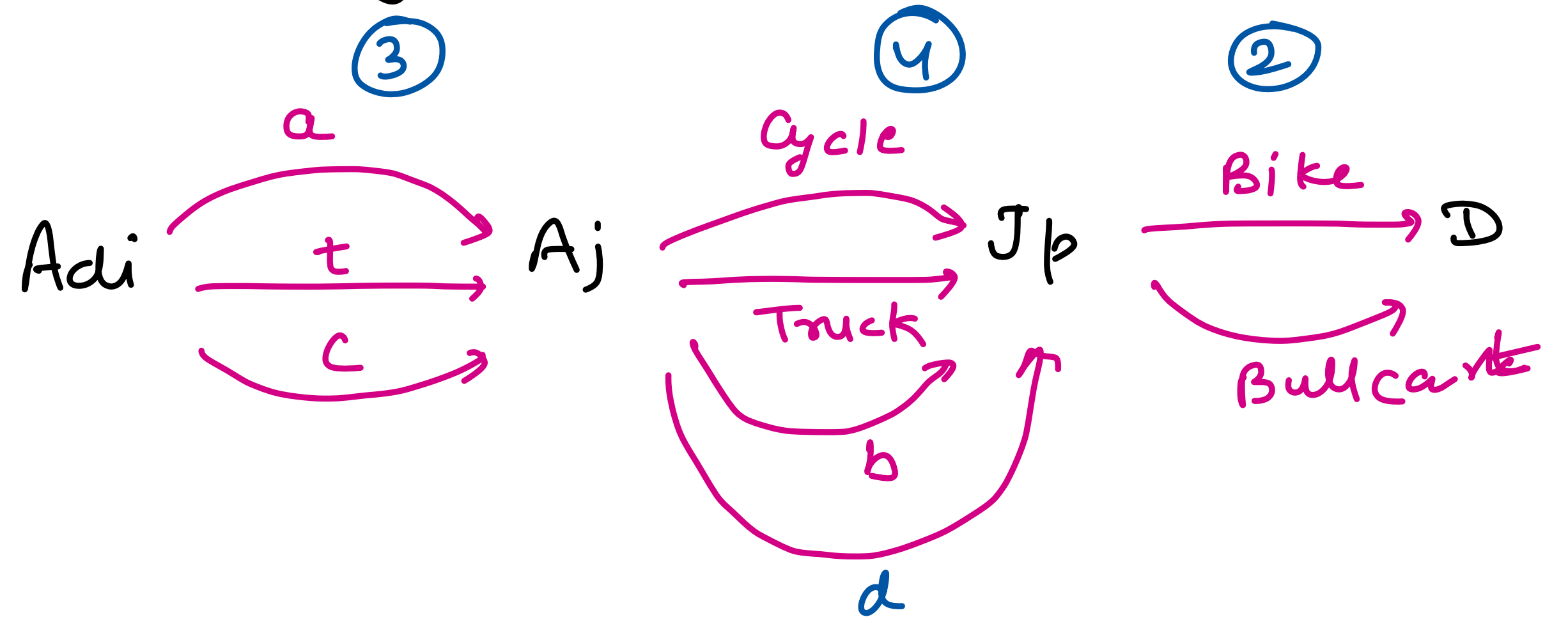
Permutation and combination

(1) fundamental principle of counting : →

Ex-1

(24)
 No. of ways.

←

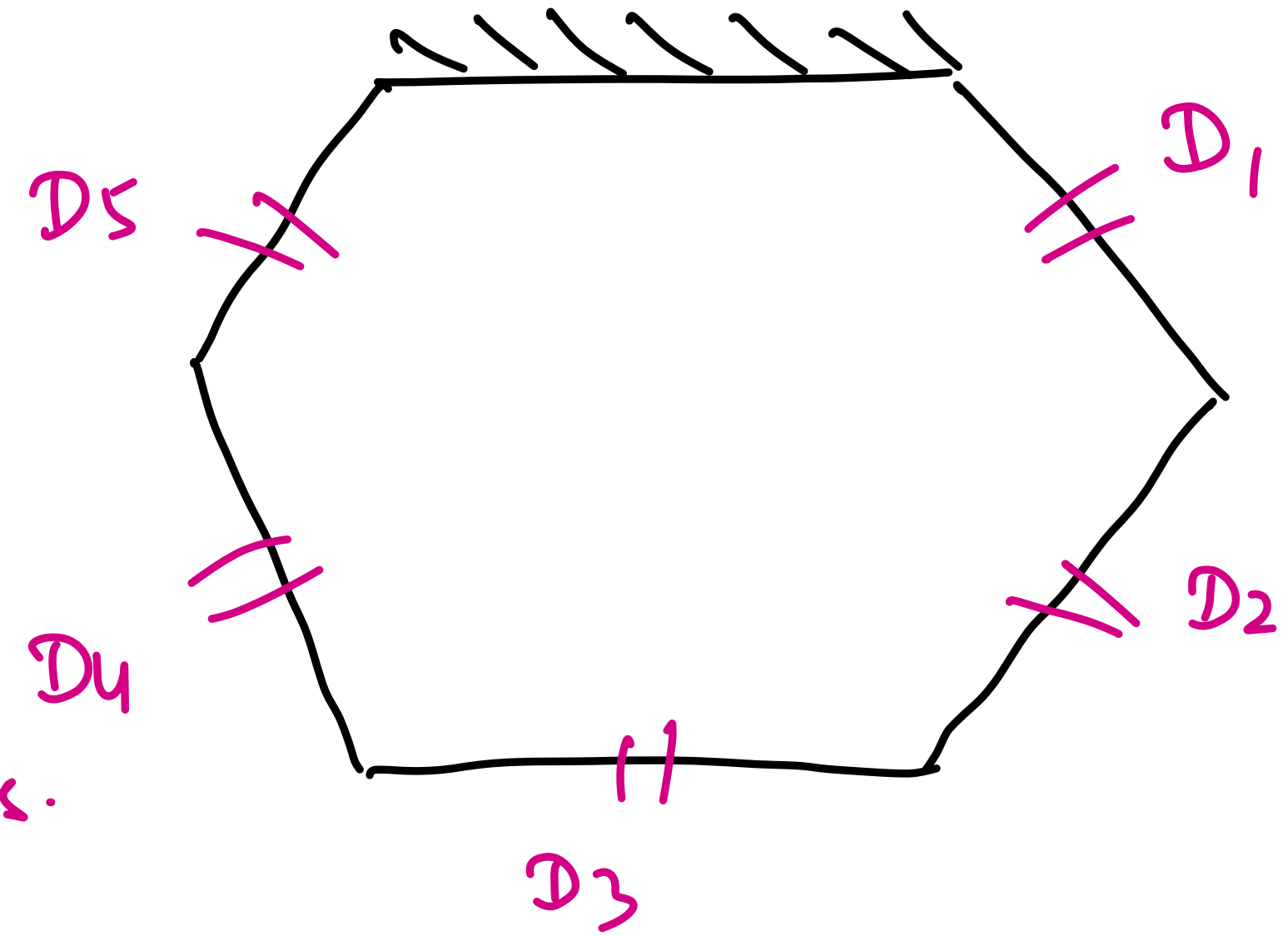


a cycle
 a cycle
 a truck
 " "
 a b
 a d

Bike
 Bullock
 Bike
 R
 ✓
 ✓
 ✓
 ✓

Ex-2 Cinema hall (5 doors)

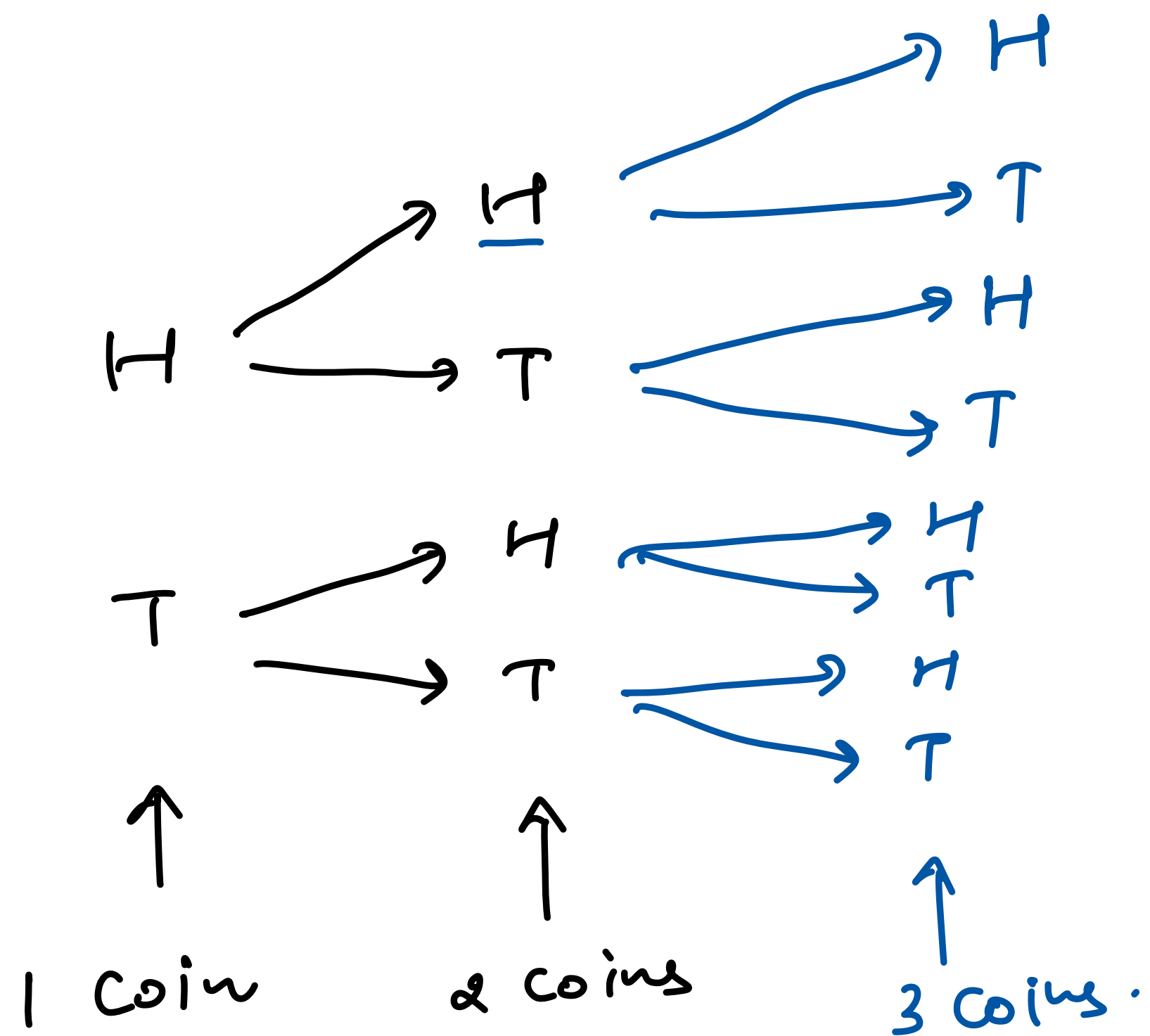
No. of ways in which
 he can enter and
 leave the cinema hall
 by a different door
 = $5 \times 4 = 20$ ways.



$$2^4 \rightarrow 16$$

E 3

Tossing a coin



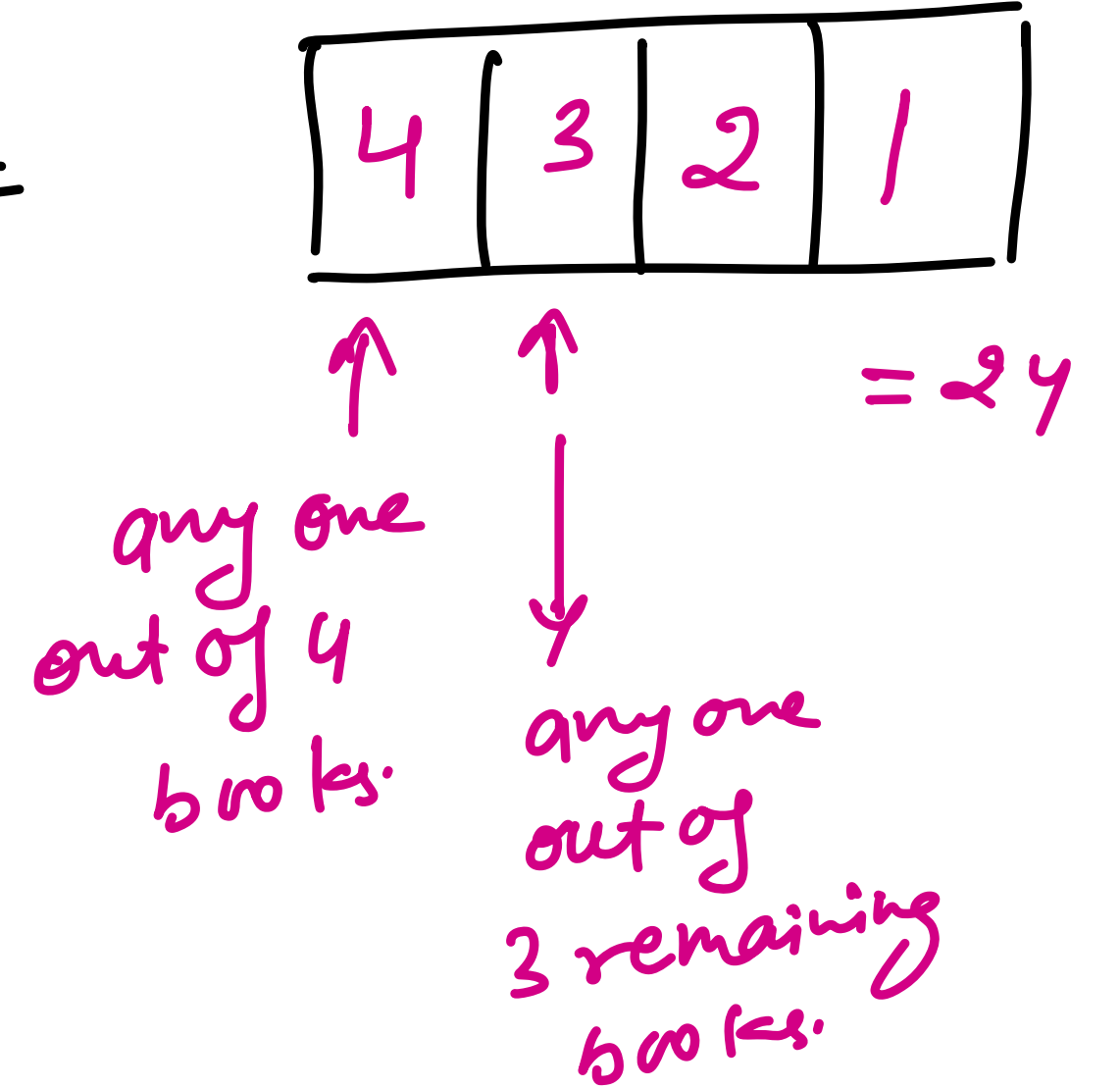
4 coins

H	H	H	H
H	H	H	T
H	H	T	H
H	H	T	T
H	T	H	H
H	T	H	T
H	T	T	H
H	T	T	T
T	H	H	H
T	H	H	T
T	H	T	H
T	H	T	T
T	T	H	H
T	T	H	T
T	T	T	H
T	T	T	T

E-4

P/C/M/B \rightarrow 4 books

No. of ways of arranging =



E(1) 3 digit numbers using the digits 1, 2, 3, 4, 5 without repetition.

$$\boxed{5} \boxed{4} \boxed{3} = 60 \text{ ways.}$$

E(2) 10 students compete in a swimming race. In how many ways can they occupy the first 3 positions.

$$\boxed{10} \boxed{9} \boxed{8} = 720$$

E(3) 7 flags of different colour. Number of different signals that can be transmitted by the use 2 flags one above the other.

$$\frac{7}{6} = 42 \text{ ways.}$$

E(4) Number of words which can be formed from the letters of the word **Machine/Miracle** if Vowels may occupy the

- (a) odd position
- (b) even position.

E(5) If the letters of the word "TOUGH" are written in all possible ways and then are arranged as in a dictionary, then find the rank of the word TOUGH.

E(6) 4 lettered word using only the letters from the word "DAUGHTER" if each word is to include "G".

D(7) 10 T/F questions. How many sequences of answers are possible.

$$2 \times 2 \times 2 \times 2 \times \dots \times 2 = 2^{10} = 1024$$

D(8) A letter lock consists of four rings marked 26 different english alphabet. The number of ways in which an unsuccessful attempt can be made.

$$(26 \times 26 \times 26 \times 26) - 1 = 26^4 - 1 = 456975$$

D(9) If there m monkeys & n available masters in how many ways monkeys can be given to the masters, if a master has any number of monkeys.

$m \rightarrow \text{monkey}$
 $n \rightarrow \text{masters.}$

$$\boxed{n}$$

$$\begin{matrix} 1 \rightarrow n \\ 2 \rightarrow n \end{matrix}$$

$$n^n$$

Ex 1

Repetition allowed

5	5	5
---	---	---

$$= 5^3 = 125$$

↑
1, 2, 3, 4, 5

Rep not allowed

5	4	3
---	---	---

$$= 60$$

↑

Q

TOUGH

Arrange alphabetic order

G H O T U

G _ _ _ _

$$= 4 \times 3 \times 2 \times 1 = 24$$

H _ _ _ _

$$= 4 \times 3 \times 2 \times 1 = 24$$

O _ _ _ _

$$= 4 \times 3 \times 2 \times 1 = 24$$

T G _ _ _

$$= 3 \times 2 \times 1 = 6$$

T H _ _ _

$$= 3 \times 2 \times 1 = 6$$

T O G _ _

$$= 2 \times 1 = 2$$

T O H _ _

$$= 2 \times 1 = 2$$

T O U G H

$$= 1$$

89th
Rank

G 4 3 2 1
 ↑ ↑
 H O T U

Q

PROBLEM

B E L M O P R

B	-	-	-	-	-	-	=	$6 \times 5 \times 4 \times 3 \times 2 \times 1$	=	720
E	-	-	-	-	-	-	=			720
L	-	-	-	-	-	-	=			720
M	-	-	-	-	-	-	=			720
O	-	-	-	-	-	-	=			720
P	B	-	-	-	-	-	=	$5 \times 4 \times 3 \times 2 \times 1$	=	120
P	E	-	-	-	-	-	=		=	120
P	L	-	-	-	-	-	=		=	120
P	M	-	-	-	-	-	=		=	120
P	O	-	-	-	-	-	=			24
P	R	B	-	-	-	-	=			

B E L M O P R

~~PROBLEM~~

P R E	_ _ _	= 24
P R L	_ _ _	= 24
P R M	_ _ _	= 24
P R O B E	_ _	= 2
P R O B L E M		= 1

4299th Rank

12

~~PERSON~~

E N O P R S

E _ _ _ _ _ = 120

N _ _ _ _ _ = 120

O _ _ _ _ _ = 120

P E N _ _ _ = 6

P E O _ _ _ = 6

P E R N _ _ = 2

P E R O _ _ = 2

P E R S N _ = 1

P E R S O N = 1

378th Rank

$$\underbrace{m}_{\text{input}} \times \underbrace{m}_{\text{input}} \times \underbrace{m}_{\text{input}} \dots \underbrace{m}_{\text{input}}$$

2

* Exponent of prime p in $n!$: \rightarrow

$$= \left[\frac{n}{p} \right] + \left[\frac{n}{p^2} \right] + \left[\frac{n}{p^3} \right] + \left[\frac{n}{p^4} \right] + \dots$$

where $[] \rightarrow$ G.I.F
 $p \rightarrow$ prime.

Q find exponent of 2 in 100!

$$\text{exp of 2} = \left[\frac{100}{2} \right] + \left[\frac{100}{2^2} \right] + \left[\frac{100}{2^3} \right] + \left[\frac{100}{2^4} \right] + \left[\frac{100}{2^5} \right] + \left[\frac{100}{2^6} \right] + \left[\frac{100}{2^7} \right] + \dots$$

$$= [50] + [25] + [12.5] + [6.25] + [3.125] + [1.5625] + [0.778125] + \dots$$

$$= \underline{50} + \underline{25} + \underline{12} + \underline{6} + \underline{3} + \underline{1} + 0 + 0 + \dots$$

$$= 97$$

Q find exponent of 3 in 100!

$$= \left[\frac{100}{3} \right] + \left[\frac{100}{3^2} \right] + \left[\frac{100}{3^3} \right] + \left[\frac{100}{3^4} \right] + \dots$$

$$= 33 + 11 + 3 + 1$$

$$= 48$$

Q find exponent of (i) 4 in 200!
 (ii) 8 in 200!

$$\begin{aligned} \text{exp of 2 in } 200! &= \left[\frac{200}{2} \right] + \left[\frac{200}{2^2} \right] + \left[\frac{200}{2^3} \right] + \dots \\ &= 100 + 50 + 25 + 12 + 6 + 3 + 1 \\ &= 197 \end{aligned}$$

$$\begin{aligned} \text{exponent of } 4 &= \left[\frac{197}{2} \right] = 98 \\ 8 &= \left[\frac{197}{3} \right] = 65 \end{aligned}$$

$$200! \rightarrow \frac{2^{197}}{3 \cdot 5 \cdot 7 \cdot \dots}$$

Q find exponent of 18 in 200!

$$2, 3^2$$

exp 2 in 200! \rightarrow 197 ✓

$$\text{exponent of 3 in } 200! = \left[\frac{200}{3} \right] + \left[\frac{200}{3^2} \right] + \left[\frac{200}{3^3} \right] + \left[\frac{200}{3^4} \right] + \dots$$

$$= 66 + 22 + 7 + 2$$

$$= 97$$

$$3^2 \text{ in } 200! = \left[\frac{97}{2} \right] = \underline{48} \checkmark$$

$$\text{exponent of } 18 \text{ in } 200! = \underline{48}$$

Q Exponent of 15 in 300!

$$\text{exp of 5 in } 300! = \left[\frac{300}{5} \right] + \left[\frac{300}{5^2} \right] + \left[\frac{300}{5^3} \right] + \dots$$

$$= 60 + 12 + 2 + \dots$$

$$= 74$$

Concept

$[x]$

$$[3.2] = 3$$

$$[5.5] = 5$$

$$[-5.5] = -6$$

$$[-100.53] = -101$$

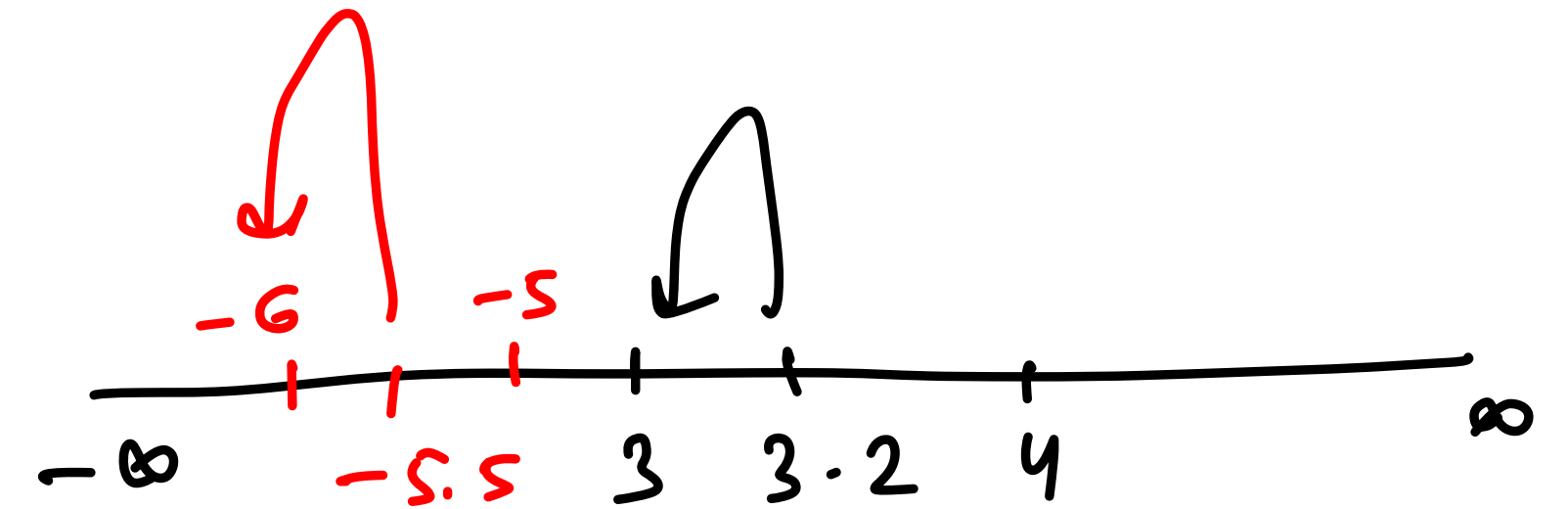
$$[-254.57] = -255$$

$$[255.36] = 255$$

$$[255] = 255$$

$$[-0.5] = -1$$

$$[0.5] = 0$$



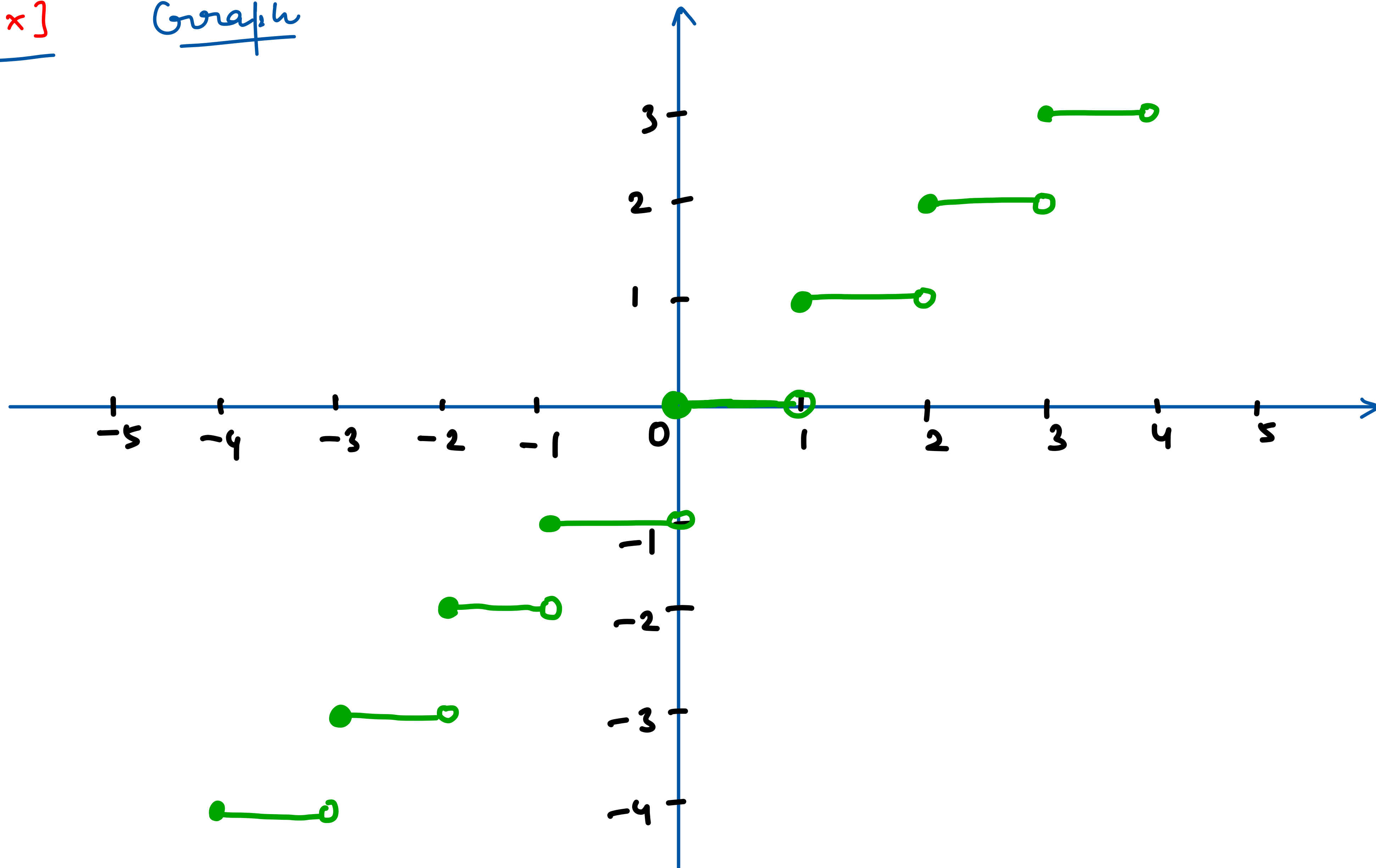
$$f(x) = [x]$$

Graph

$$[0] = 0$$

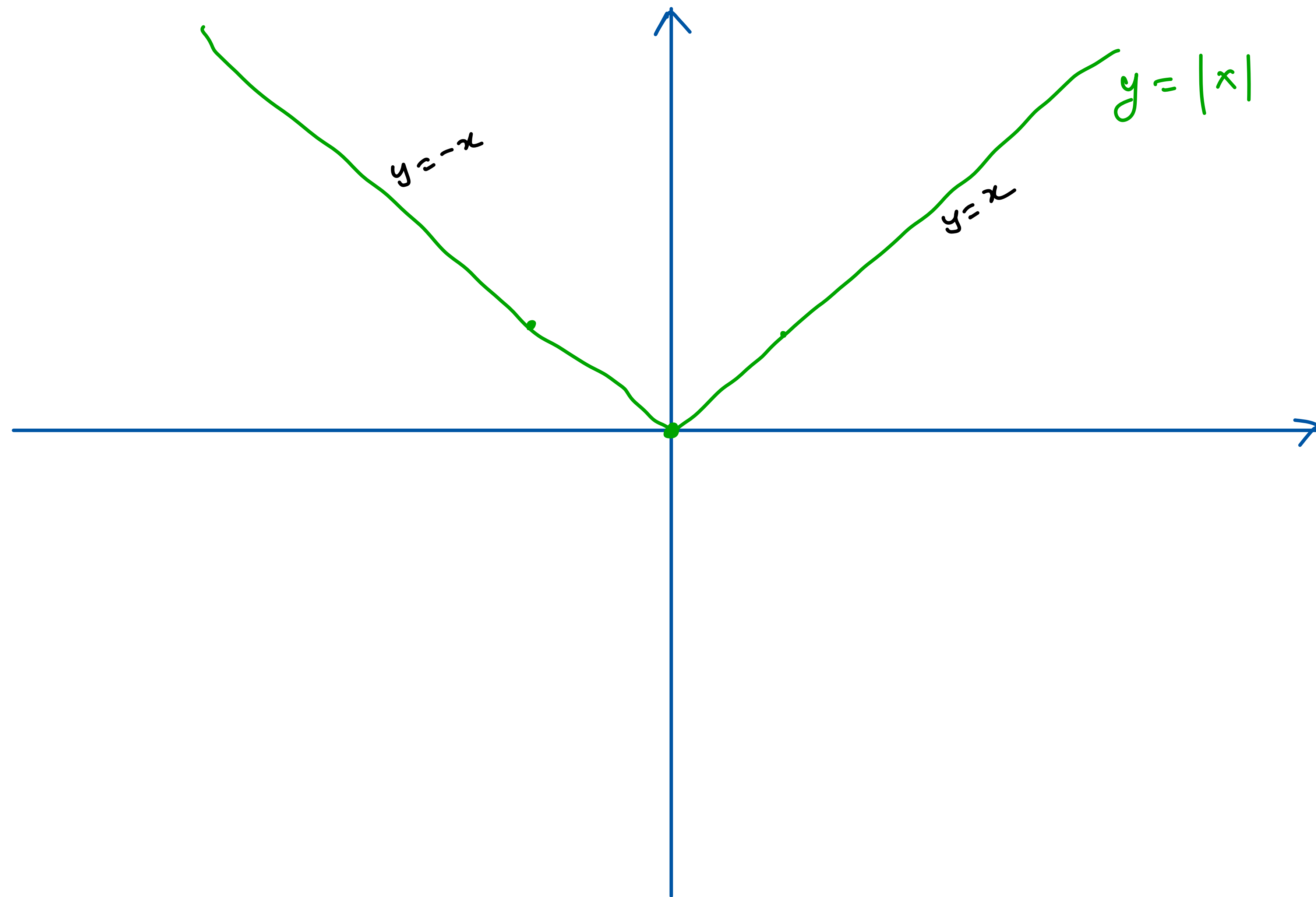
$$[1] = 1$$

$$[0.9999\ldots] = 0$$



x	$f(x)$
$[0, 1)$	0
$[1, 2)$	1
$[2, 3)$	2
$[3, 4)$	3
$[-1, 0)$	-1
$[-2, -1)$	-2

$$\underline{f(x) = |x|}$$



Q

$$|x|^2 - 3|x| + 2 = 0$$

$$(|x| - 1)(|x| - 2) = 0$$

$$|x| = 1$$

$$x = \pm 1$$

$$|x| = 2$$

$$x = \pm 2$$

$$x \in \{-2, -1, 1, 2\}$$