ALLEN CAREER INSTITUTE
KOTA (RAJASTHAN)

CONTENTS

S.NO.	TOPIC	PAGE NO.
01.	NEWTON'S LAWS OF MOTION	01 - 56
02.	FRICTION	57 - 87
03.	WORK, POWER & ENERGY	88 - 127
04.	CIRCULAR MOTION	128 - 166

NEWTON'S LAWS OF MOTION

Recap of Early Classes

To move a football at rest, someone must kick it. To throw a stone upwards, one has to give it an upward push. A breeze causes the branches of a tree to swing; a strong wind can even move heavy objects. A boat moves in a flowing river without anyone rowing it. Clearly, some external agency is needed to provide force to move a body from rest. Likewise, an external force is needed also to retard or stop motion. You can stop a ball rolling down an inclined plane by applying a force against the direction of its motion.

In these examples, the external agency of force (hands, wind, stream, etc) is in contact with the object. This is not always necessary. A stone released from the top of a building accelerates downward due to the gravitational pull of the earth. A bar magnet can attract an iron nail from a distance. This shows that external agencies (e.g. gravitational and magnetic forces) can exert force on a body even from a distance.

Index

1.0 MOTION

2.0 MOMENTUM: AMOUNT OF MOTION

3.0 FORCE

3.1 Contact and Field Forces

3.2 Basic Characteristics of a Force

4.0 NEWTON'S LAWS OF MOTION

4.1 The First Law

4.2 The Second Law

4.3 The Third Law

5.0 CONCEPT OF FREE BODY DIAGRAM (FBD)

5.1 Various Field Forces

5.2 Various Contact Forces

6.0 TRANSLATIONAL EQUILIBRIUM

7.0 DYNAMICS OF PARTICLES: TRANSLATION MOTION OF ACCELERATED BODIES

8.0 WEIGHING MACHINE

9.0 SPRING BALANCE

10.0 CONSTRAINED MOTION

10.1 String Constraint

10.2 Wedge Constraint

11.0 NEWTON'S LAW FOR A SYSTEM

12.0 INERTIAL AND NON-INERTIAL REFERENCE FRAMES

EXERCISE-1

EXERCISE-2

EXERCISE-3

EXERCISE-4

EXERCISE-5

CONTENTS

THEORY

1. Comprehensive theory covering all concepts & subtopics for ***excellence*** in both ***school level*** as well as ***competitive exams***.

Symbols used for categorization

SL $\Rightarrow$ Topics required for ***school level*** preparations.

AL $\Rightarrow$ Topics required for ***Advance level*** preparations useful for competitive exams.

2. ***Golden Key Points :*** Important points/formulaes or concepts summarized at the end to have a ***quick revision*** of the topic.

3. ***Illustrations : Subtopic based solved questions*** to get comfortable in problem solving. [Students should go through these after the topic is dealt]

4. ***Solved examples :*** A collection of ***miscellaneous solved question*** based on different concepts from the chapter at the end to be referred before exercise solving.

5. ***Beginner Boxes :*** Collection of ***elementary sub–topic*** based questions to be attempted on completion of each subtopic.

EXERCISE

6. ***EXERCISE-1***
EXERCISE-2
EXERCISE-3
EXERCISE-4
EXERCISE-5

NEWTON'S LAWS OF MOTION

1.0 MOTION

SL AL

Motion of a body is its movement and is identified by change in either its location or orientation or both, relative to other objects.

Location

Location of a rigid body tells us where it is placed and can be measured by position coordinates of any particle of the body or its mass center. It is also known as position.

Orientation

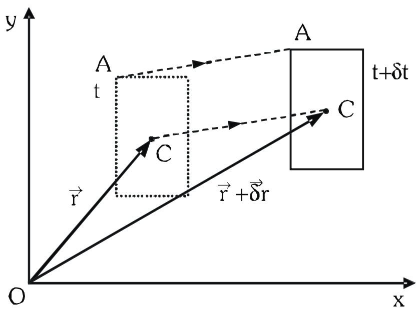
Orientation of a body tells us how it is placed with respect to the coordinate axes. Angles made with the coordinate axes by any linear dimension of the body or a straight line drawn on it, provide suitable measure of orientation.

Translation and Rotation Motion

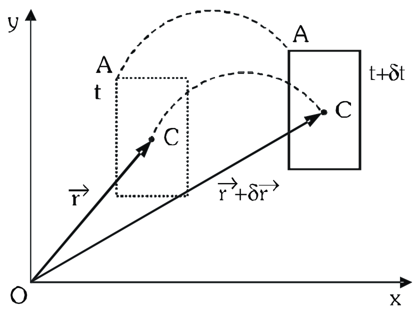
If a body changes its location without change in orientation, it is in pure translation motion and if it changes orientation without change in location, it is in pure rotation motion.

Translation Motion

Let us consider the motion of a plate, which involves only change in position without change in orientation. It is in pure translation motion. The plate is shown at two different instants t and t+δt. The coordinate axes shown are in the plane of the plate and represent the reference frame. A careful observation makes the following points obvious.



Rectilinear Translation



Curvilinear Translation

- None of the linear dimension or any line drawn on the body changes its angles with the coordinate. Therefore, there is no rotation motion.
- All the particles of the body including its mass center move on identical parallel trajectories. Here trajectories of corner A and center C are shown by dashed lines.
- All the particles and mass center of the body cover identical segments of their trajectories in a given time interval. Therefore, at any instant of time all of them have identical velocities and accelerations.

Pure translation motion of a body can be represented by motion of any of its particle. This is why, we usually consider a body in pure translation motion as a particle.

2.0 MOMENTUM: AMOUNT OF MOTION

SL AL

Amount of motion in a body depends on its velocity and mass.

Linear momentum of a body is defined as product of its mass and velocity. It provides measure of amount of motion.

Linear momentum $\vec{p}$ of a body of mass m, moving with velocity by $\vec{v}$ is expressed by the following equation.

$$\vec{p} = m\vec{v}$$

SI unit of momentum is kg-m/s.

Dimensions of momentum are MLT^{-1}

3.0 FORCE

SL AL

The concept of force is used to explain mutual interaction between two material bodies as the action of one body on another in form of push or pull, which brings out or tries to bring out a change in the state of motion of the two bodies. A mutual interaction between two bodies, which creates force on one body, also creates force on the other body. Force on body under study is known as *action* and the force applied by this body on the other is known as *reaction*.

3.1 Contact and Field Forces

SL AL

When a body applies force on other by direct contact, the force is known as contact force. When two bodies apply force on each other without any contact between them, the force is known as field force.

When you lift something, you first hold it to establish contact between your hand and that thing, and then you apply the necessary force to lift. When you pull bucket of water out of a well, the necessary force you apply on the rope by direct contact between your hand and the rope and the rope exerts the necessary force on the bucket through a direct contact. When you deform a spring, you have to hold the spring and establish contact between your hand and the spring and then you apply the necessary force. In this way, you can find countless examples of contact forces.

Things left free, fall on the ground, planets orbit around the sun, satellites orbit around a planet due to *gravitational force*, which can act without any contact between the concerned bodies. A plastic comb when rubbed with dry hair, becomes electrically charged. A charged plastic comb attracts small paper pieces without any physical contact due to *electrostatic force*. A bar magnet attracts iron nails without any physical contact between them. This force is known as *magnetic force*. The gravitation, electrostatic and magnetic forces are examples of field forces.

3.2 Basic Characteristics of a Force

SL AL

Force is a vector quantity therefore has *magnitude* as well as *direction*. To predict how a force affects motion of a body we must know its magnitude, direction and point on the body where the force is applied. This point is known a *point of application* of the force. The direction and the point of application of a force both decide *line of action* of the force. Magnitude and direction decide effect on translation motion and magnitude and line of action decides effects on rotation motion.

4.0 NEWTON'S LAWS OF MOTION

SL AL

Newton has published three laws, which describe how forces affect motion of a body on which they act. These laws are fundamental in nature in the sense that the first law gives concept of force, inertia and the inertial frames; the second law defines force and the third law action and reaction as two aspects of mutual interaction between two bodies.

4.1 The First Law

SL AL

Every material body has tendency to preserve its state of rest, or of uniform motion in a straight line, unless it is compelled to change that state by external forces impressed on it.

Inertia

The tendency of a material body to preserve its present state of uniform motion or of rest is known as inertia of the body. It was first conceived by Galileo.

Inertia is a physical quantity and mass of a material body is measure of its inertia.

Inertial Frame of Reference

The first law requires a frame of reference in which only the forces acting on a body can be responsible for any acceleration produced in the body and not the acceleration of the frame of reference. These frames of reference are known as inertial frames.

4.2 The Second Law

SL AL

The rate of change in momentum of a body is equal to, and occurs in the direction of the net applied force.

A body of mass m in translational motion with velocity $\vec{v}$, if acted upon with a net external force $\vec{F}$, the second law suggests:

$$\vec{F} = \frac{d}{dt}(m\vec{v})$$

If mass of the body is constant, the above equation relates the acceleration $\vec{a}$ of the body with the net force $\vec{F}$ acting on it.

$$\vec{F} = \frac{d}{dt}(m\vec{v}) = m\vec{a}$$

The first law provides concept of force and the second law provides the quantitative definition of force, therefore the second law is also valid only in inertial frames.

SI unit of force is newton. It is abbreviated as N. One newton equals to one kilogram-meter per second square.

$$1\text{ N} = 1\text{ kg-m/s}^2$$

Dimensions of force are MLT^{-2}

4.3 The Third Law

SL AL

Force is always a two-body interaction. The first law describes qualitatively and the second law describes quantitatively what happens to a body if a force acts on it, but do not reveal anything about what happens to the other body participating in the interaction responsible for the force.

The third law accounts for this aspect of the force and states that every action on a body has equal and opposite reaction on the other body participating in the interaction.

5.0 CONCEPT OF FREE BODY DIAGRAM (FBD)

SL AL

A force on a body can only exists when there is another body to create it, therefore in every physical situation of concern there must be two or more bodies applying forces on each other. On the other hand the three laws of Newton, describe motion of a single body under action of several forces, therefore, to analyze a given problem, we have to consider each of the bodies separately one by one. This idea provides us with the concept of free body diagram.

A free body diagram is a pictorial representation in which the body under study is assumed free from rest of the system i.e. assumed separated from rest of the interacting bodies and is drawn in its actual shape and orientation and all the forces acting on the body are shown.

How to draw a Free Body Diagram (FBD)

- Separate the body under consideration from the rest of the system and draw it separately in actual shape and orientation.
- Show all the forces whether known or unknown acting on the body at their respective points of application.

  For the purpose count every contact where we separate the body under study from other bodies. At every such point, there may be a contact force. After showing, all the contact forces show all the field forces.

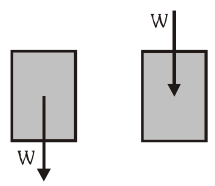
5.1 Various Field Forces

SL AL

Field forces include the gravitational force (weight) electrostatic forces and magnetic forces, which can easily be identified. At present, we consider only gravitational pull from the earth i.e. weight of the body.

Weight: The net gravitational pull of the Earth

The gravitational pull from the earth acts on every particle of the body hence it is a distributed force. The net gravitational pull of the Earth on a body may be considered as weight of the body. It is assumed to act on the center of gravity of the body. For terrestrial bodies or celestial bodies of small size, this force can be assumed uniform throughout its volume. Under such circumstances, center of gravity and center of mass coincide and the weight is assumed to act on them. Furthermore, center of mass of uniform bodies lies at their geometrical center. At present, we discuss only uniform bodies and assume their weight to act on their geometrical center. In the figure weight of a uniform block is shown acting on its geometrical centre that coincides with the center of mass and the centre of gravity of the body.



Weight can be shown in either of the above two ways

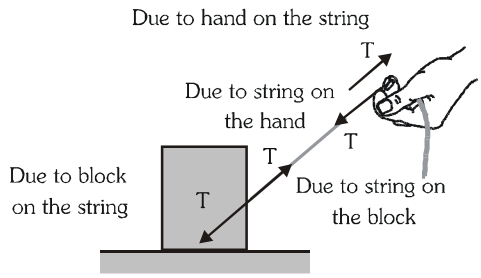
5.2 Various Contact Forces

SL AL

At every point where a body under consideration is supposed to be separated from other bodies to draw its free-body diagram, there may be a contact force. Most common contact forces, which we usually encounter, are tension force of a string, normal reaction on a surface in contact, friction, spring force etc.

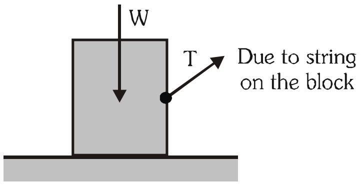
Tension Force of Strings

A string or similar flexible connecting links as a thread or a chain etc. we use to transmit a force. Due to flexibility, a string can be used only to pull a body connected to it by applying a force always along the string. According to the third law, the connected body must also apply an equal and opposite force on the string, which makes the string taut. Therefore, this force is known as tension force T of the string. In the given figure is shown a block pulled by a string, which is being pulled by a person.

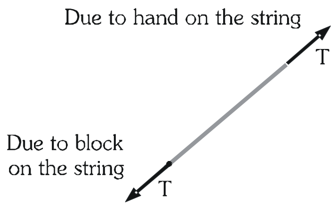


The tension force applied by string on the block and the force applied by the block on the string shown in the figure constitute a third law action-reaction pair. Similarly, tension force applied by the string on hand and force applied by the hand on string is another third law action-reaction pair.

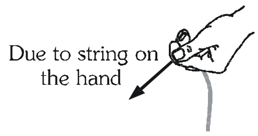
While studying motion of the block, the force applied by the string on it, weight of the block and a reaction from the floor has to be considered. In the figure only weight and tension of string are shown.



To study motion of the string, the force applied by the block on the string and the force applied by the hand on the string must be considered. These forces are shown in the FBD of string.

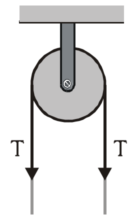


To study conditions of motion of the person, the force applied by the string on the hand has to be considered as shown in the figure.



String passing over a pulley

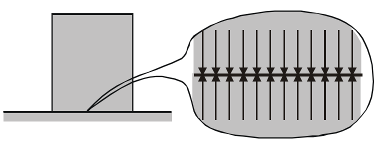
A pulley is a device consisting of a wheel, which can rotate freely on its axel. A single pulley changes direction of tension force. At present for simplicity, we discuss only ideal pulley, which is massless i.e. has negligible mass and rotates on its axel without any friction. An ideal pulley offers no resistance to its rotation, therefore tension force in the string on both sides of it are equal in magnitude. Such a pulley is known as ideal pulley.



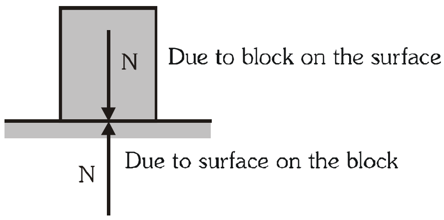
Normal Reaction

Two bodies in contact, when press each other, must apply equal and opposite forces on each other. These forces constitute a third law action-reaction pair. If surfaces of the bodies in contact are frictionless, this force acts along normal to the surface at the point of contact. Therefore, it is known as normal reaction.

Consider a block of weight W placed on a frictionless floor. Because of its weight it presses the floor at every point in contact and the floor also applies equal and opposite reaction forces on every point of contact. We show all of them by a single resultant N obtained by their vector addition.

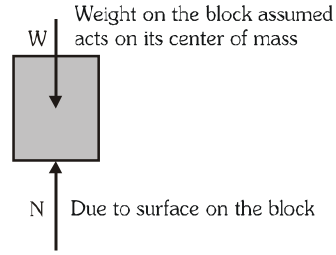


Normal contact forces on every contact point

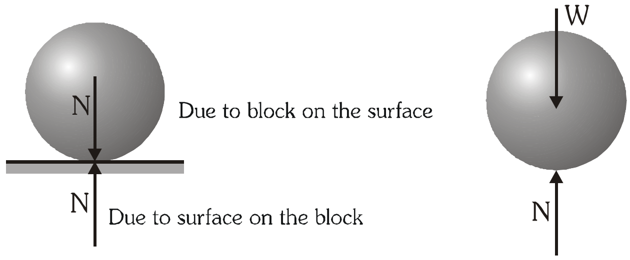


Normal contact forces on every contact point are represented by their resultant N

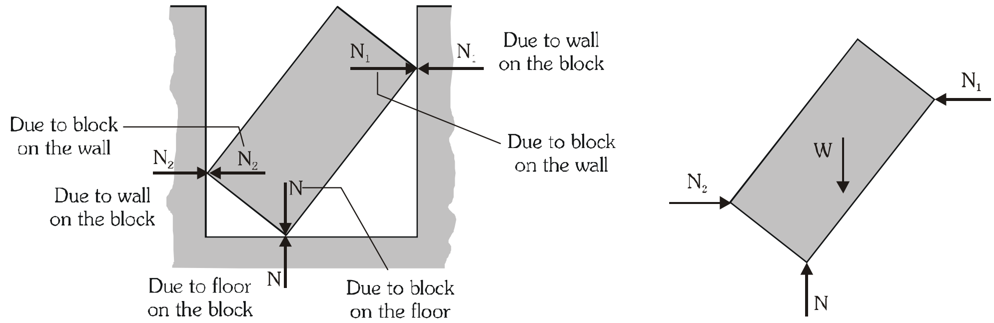
To apply Newton's laws of motion (NLM) on the block, its weight W and normal reaction N applied by the floor on the block must be considered as shown in the following figure. It is the FBD of the block.



Consider a spherical ball of weight W placed on a floor. The normal reaction from the floor on the ball and from the ball on the floor makes third law action-reaction pair. These forces are shown in the left figure.



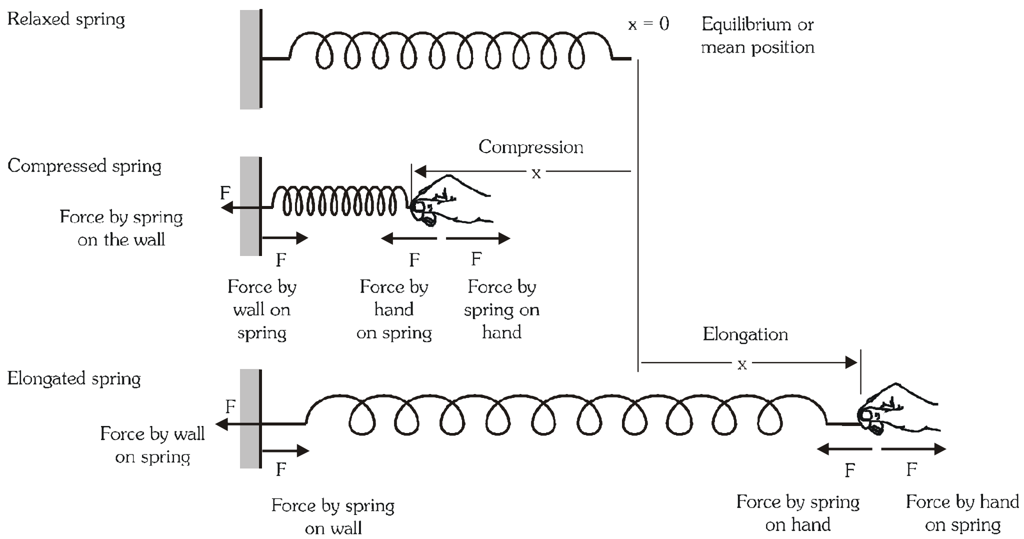
To apply Newton's laws of motion (NLM) on the ball, its weight W and normal reaction N applied by the floor on the ball must be considered as shown in the above right figure. It is the FBD of the ball.
When two surfaces make contact, the normal reaction acts along the common normal and when a surface and a sharp corner make a contact the normal reaction acts along the normal to the surface. Consider a block placed in a rectangular trough as shown in the figure.



To apply Newton's laws of motion (NLM) on the block, its free body diagram (FBD) is shown in the above right figure.

Spring Force

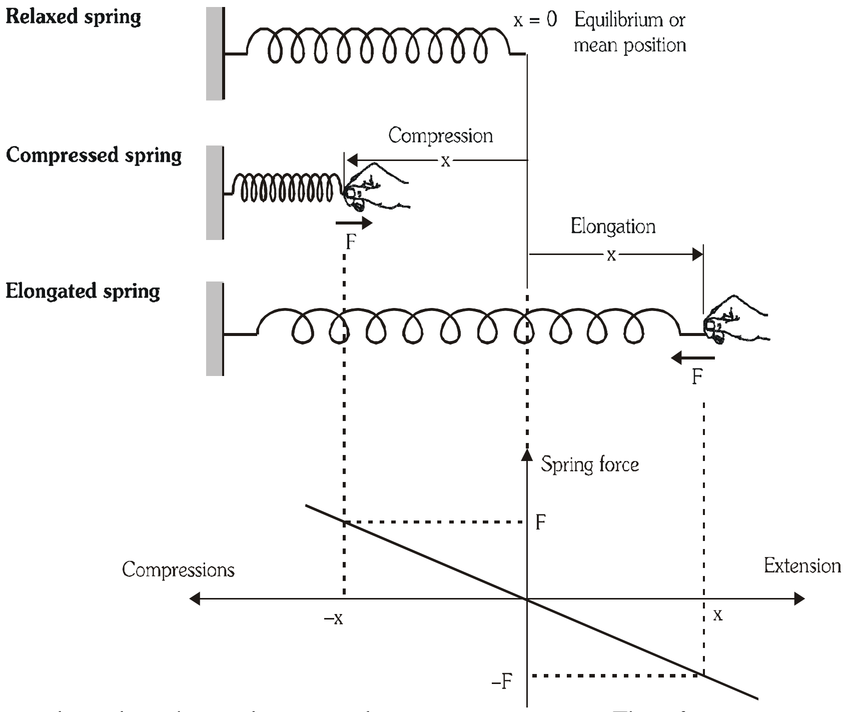
When no force acts on a spring, it is in relaxed condition i.e. neither compressed nor elongated. Consider a spring attached to a fixed support at one of its end and the other end is free. If we neglect gravity, it remains in relaxed state. When it is pushed by a force F, it is compressed and displacement x of its free end is called compression. When the spring is pulled by a force F, it is elongated and displacement x of its free end is called elongation. Various forces developed in these situations are shown in the following figure.



The force applied by the spring on the wall and the force applied by the wall on the spring make a third law action-reaction pair. Similarly, force by hand on the spring and the force by spring on the hand make another third law action-reaction pair.

Hooke's Law

How spring force varies with deformation in length x of the spring is also shown in the following figure.



The force F varies linearly with x and acts in a direction opposite to x. Therefore, it is expressed by the following equation

$$F = -kx$$

Here, the minus (–) sign represents the fact that force F is always opposite to x.

The constant of proportionality k is known as *force constant of the spring* or simply as *spring constant*. The slope modulus of the graph equals to the spring constant.

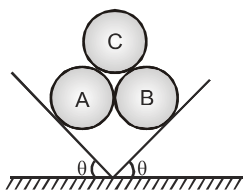
SI unit of spring constant is newton per meter or (N/m).

Dimensions of spring constant are MT^{-2}.

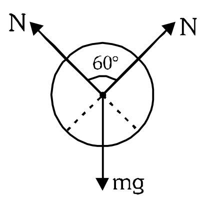
BEGINNER'S BOX-1

Free Body Diagram

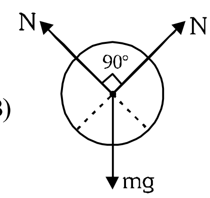
1. A, B & C are three identical smooth sphere placed on frictionless inclined plane as shown in figure then F.B.D. of sphere C is



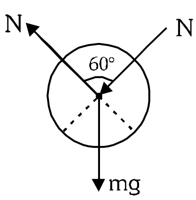
(A)



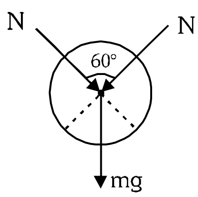
(B)



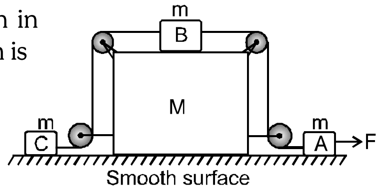
(C)

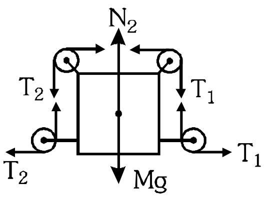


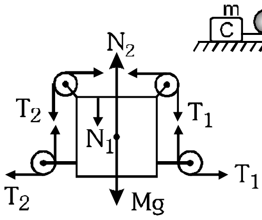
(D)

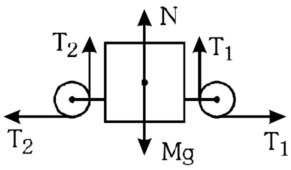


2. Block A, B & C connected with light string with larger block M as shown in figure. Pullies are light and frictionless. There is no friction anywhere. Which is the correct F.B.D. of larger block of mass M.



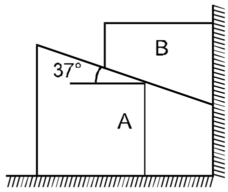
(A) 

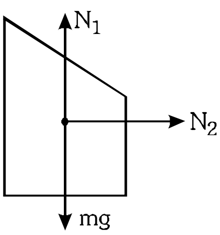
(B) 

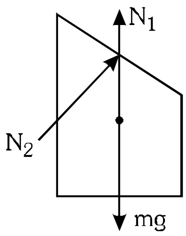
(C) 

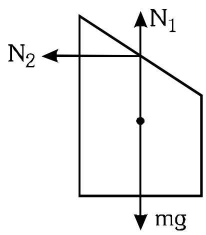
(D) None of these

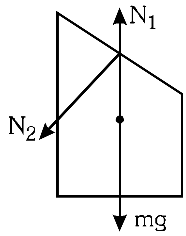
3. Two blocks A and B are placed as shown in figure. There is no friction anywhere then F.B.D. of block A



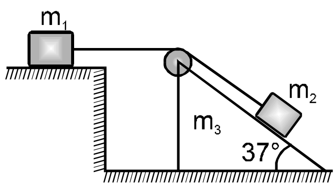
(A) 

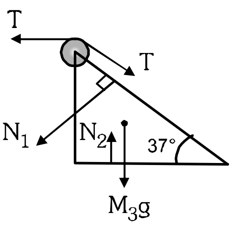
(B) 

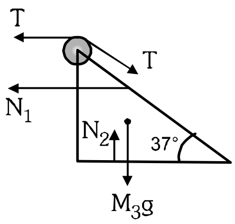
(C) 

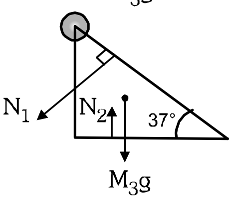
(D) 

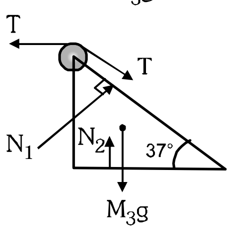
4. Masses m_1, m_2 and m_3 are connected light string as shown in figure. There is no friction at anywhere. Which is the F.B.D. of which of mass m_3



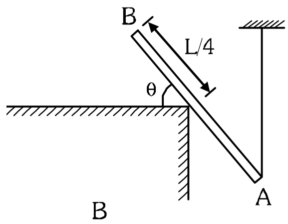
(A) 

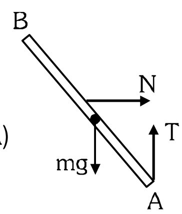
(B) 

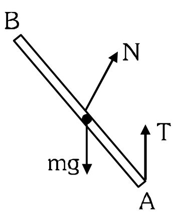
(C) 

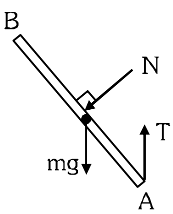
(D) 

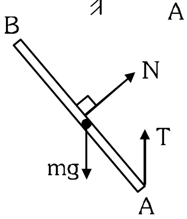
5. Rod B of mass m placed on smooth surface and connected with light string as shown in figure. Then which is the correct F.B.D. of rod



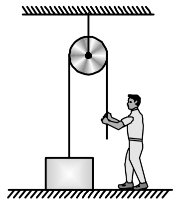
(A) 

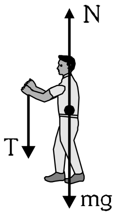
(B) 

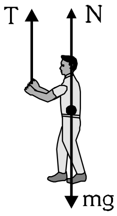
(C) 

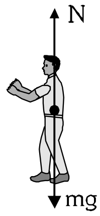
(D) 

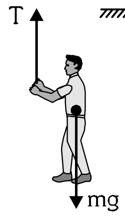
6. A man is pulling the string vertically downward. Then which is the correct F.B.D. of man assume there is no friction anywhere.



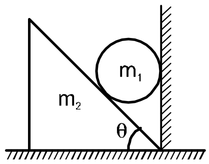
(A) 

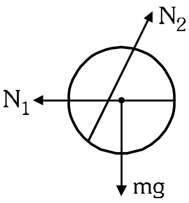
(B) 

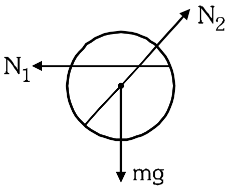
(C) 

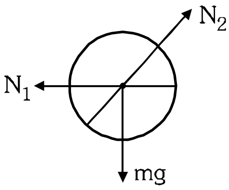
(D) 

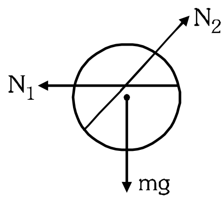
7. Two mass m_1 and m_2 are placed as shown in figure. Assume all surface are frictionless then the correct F.B.D. of m_1 is



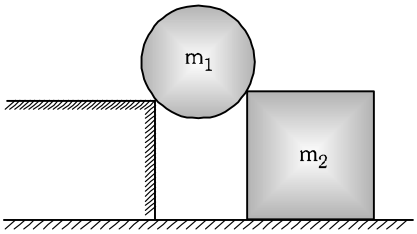
(A) 

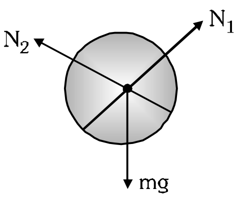
(B) 

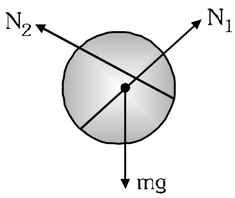
(C) 

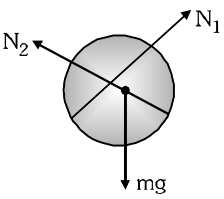
(D) 

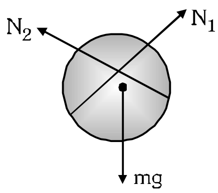
8. Masses m_1 and m_2 are placed as shown in figure. Assume there is no friction anywhere then correct F.B.D. of m_1 is



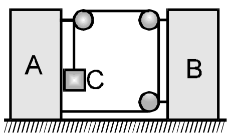
(A) 

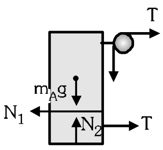
(B) 

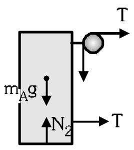
(C) 

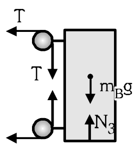
(D) 

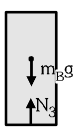
9. Block A and B are connected by light string as shown in figure. All surface are frictionless then which is the correct F.B.D. of block B and block A.



(A) 

(B) 

(C) 

(D) 

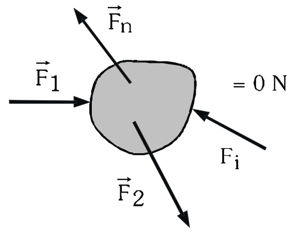
6.0 TRANSLATIONAL EQUILIBRIUM

A body in state of rest or moving with constant velocity is said to be in translational equilibrium. Thus if a body is in translational equilibrium in a particular inertial frame of reference, it must have no linear acceleration. When it is at rest, it is in *static equilibrium*, whereas if it is moving at constant velocity it is in *dynamic equilibrium*.

Conditions for translational equilibrium

For a body to be in translational equilibrium, no net force must act on it i.e. vector sum of all the forces acting on it must be zero.

If several external forces $\vec{F}_1$, $\vec{F}_2$ ..... $\vec{F}_i$ ..... and $\vec{F}_n$ act simultaneously on a body and the body is in translational equilibrium, the resultant of these forces must be zero.



$$\sum \vec{F}_i = \vec{0}$$

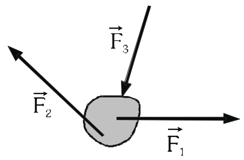
If the forces $\vec{F}_1$, $\vec{F}_2$ ..... $\vec{F}_i$ .....and $\vec{F}_n$ are expressed in Cartesian components, we have :

$$\sum F_{ix} = 0 \qquad \sum F_{iy} = 0 \qquad \sum F_{iz} = 0$$

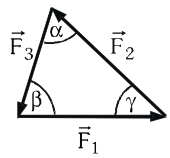
If a body is acted upon by a single external force, it cannot be in equilibrium.

If a body is in equilibrium under the action of only two external forces, the forces must be equal and opposite.

If a body is in equilibrium under action of three forces, their resultant must be zero; therefore, according to the triangle law of vector addition they must be coplanar and make a closed triangle.



$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = \vec{0} \quad \Rightarrow$$

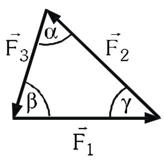


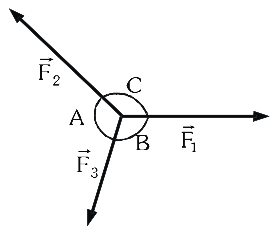
The situation can be analyzed by either graphical method or analytical method.

- Graphical method makes use of sine rule or Lami's theorem.

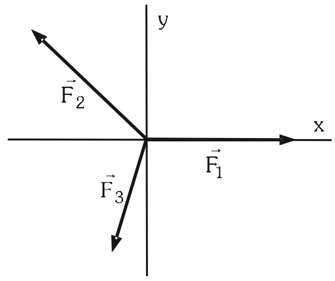
Sine rule : $\dfrac{F_1}{\sin\alpha} = \dfrac{F_2}{\sin\beta} = \dfrac{F_3}{\sin\gamma}$

Lami's theorem : $\dfrac{F_1}{\sin A} = \dfrac{F_2}{\sin B} = \dfrac{F_3}{\sin C}$





- Analytical method makes use of Cartesian components. Since the forces involved make a closed triangle, they lie in a plane and a two-dimensional Cartesian frame can be used to resolve the forces. As far as possible orientation of the x-y frame is selected in such a manner that angles made by forces with axes should have convenient values.



$$\sum F_x = 0 \Rightarrow F_{1x} + F_{2x} + F_{3x} = 0$$

$$\sum F_y = 0 \Rightarrow F_{1y} + F_{2y} + F_{3y} = 0$$

Problems involving more than three forces should be analyzed by analytical method. However, in some situations, there may be some parallel or anti-parallel forces and they should be combined first to minimize the number of forces. This may sometimes lead a problem involving more than three forces to a three-force system.

Illustrations

Illustration 1. Consider a box of mass 10 kg resting on a horizontal table and acceleration due to gravity to be 10 m/s^2.

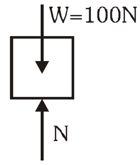
(a) Draw the free body diagram of the box.

(b) Find value of the force exerted by the table on the box.

(c) Find value of the force exerted by the box on the table.

(d) Are force exerted by table on the box and weight of the box third law action-reaction pair?

Solution (a) N : Force exerted by table on the box.



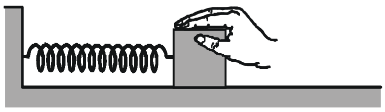
(b) The block is in equilibrium.

$$\sum \vec{F} = \vec{0} \Rightarrow W - N = 0 \Rightarrow N = 100\ N$$

(c) N = 100 N : Because force by table on the box and force by box on table make Newton's third law pair.

(d) No

Illustration 2. Consider a spring attached at one of its ends to a fixed support and at other end to a box, which rests on a smooth floor as shown in the figure. Denote mass of the box by m, force constant of the spring by k and acceleration due to gravity by g.

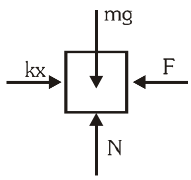


The box is pushed horizontally displacing it by distance x towards the fixed support and held at rest.

(a) Draw free body diagram of the box.

(b) Find force exerted by hand on the box.

(c) Write all the third law action-reaction pairs.



Solution (a) F is push by hand.

(b) Since the block is in equilibrium $\Sigma F_x = 0 \Rightarrow F = kx$

(c) (i) Force by hand on box and force by box on hand.

(ii) Force by spring on box and force by box on spring.

(iii) Normal reaction by box on floor and normal reaction by floor on box.

(iv) Weight of the box and the gravitational force by which box pulls the earth.

(v) Force by spring on support and force by support on spring.

Illustration 3. (a) A box of weight $10\sqrt{3}$ N is held in equilibrium with the help of two strings OA and OB as shown in figure-I. The string OA is horizontal. Find the tensions in both the strings.

Fig. I

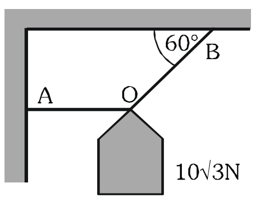
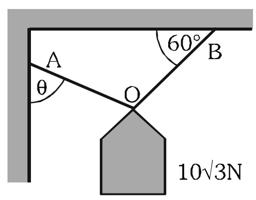
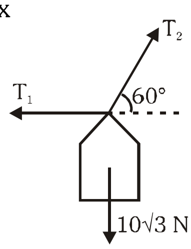


Fig. II



(b) If you can change location of the point A on the wall and hence the orientation of the string OA without altering the orientation of the string OB as shown in figure-II. What angle should the string OA make with the wall so that a minimum tension is developed in it?

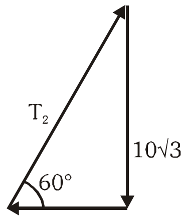
Solution (a) Free body diagram of the box



Graphical Method : Use triangle law

$T_2\sin60° = 10\sqrt{3} \Rightarrow T_2=20N$

$T_1\tan60° = 10\sqrt{3} \Rightarrow T_1=10N$



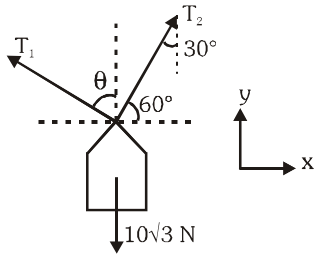
Analytical Method: Use Cartesian components

$\sum F_x = 0 \quad \Rightarrow \quad T_2 \cos 60° = T$ ...(i)

$\sum F_y = 0 \quad \Rightarrow \quad T_2 \sin 60° = 10\sqrt{3}$ ...(ii)

From equation (i) & (ii) we have $T_1=10N$ and $T_2=20N$

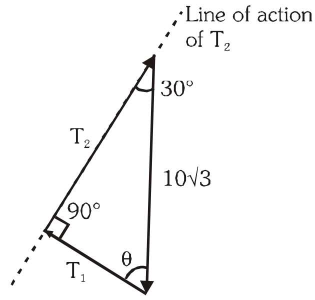
(b) Free body diagram of the box



Graphical Method : Use triangle law

For T_1 to be minimum, it must be perpendicular to T_2.

From figure $\theta = 60°$



Analytical Method : Use Cartesian components

$\sum F_x = 0 \quad \Rightarrow \quad T_2 \cos 60° = T_1 \sin\theta$ ...(i)

$\sum F_y = 0 \quad \Rightarrow \quad T_1 \cos\theta + T_2 \sin 60° = 10\sqrt{3}$ ...(ii)

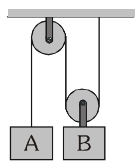
From equation (i) and (ii), we have $T_1 = \dfrac{10\sqrt{3}}{\sqrt{3}\sin\theta + \cos\theta}$

If T_1 is minimum, $\sqrt{3}\sin\theta + \cos\theta$ must be maximum. Maximum value of $\sqrt{3}\sin\theta + \cos\theta$ is 2.

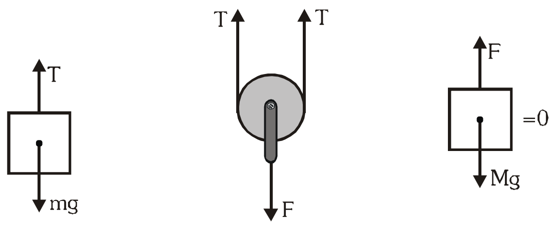
$$\sqrt{3}\sin\theta + \cos\theta = 2$$

Solving the above equation we get $\theta = 60°$

Illustration 4. Two boxes A and B of masses m and M are suspended by a system of pulleys are in equilibrium as shown. Express M in terms of m.



Solution Since tension on both sides of a pulley are equal and string is massless therefore tension everywhere on the string must have same magnitude.



FBD of block A FBD of pulley FBD of Block B

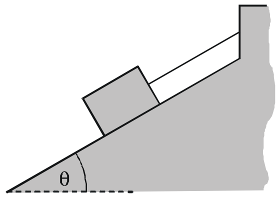
For equilibrium of block A $\sum \vec{F} = \vec{0} \Rightarrow \quad T = mg$ ...(i)

For equilibrium of pulley attached to block B $\sum \vec{F} = \vec{0} \Rightarrow \quad F = 2T$ ...(ii)

For equilibrium of block B $\sum \vec{F} = \vec{0} \Rightarrow \quad F = Mg$ ...(iii)

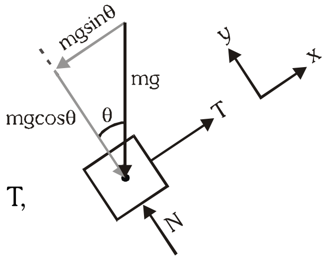
From equation (i), (ii) and (iii), we have M = 2m

Illustration 5. A box of mass m rests on a smooth slope with help of a thread as shown in the figure. The thread is parallel to the incline plane.



(a) Draw free body diagram of the box.
(b) Find tension in the thread.
(c) If the thread is replaced by a spring of force constant k, find extension in the spring.

Solution (a) Free body diagram of the block



(b) The block is in equilibrium, therefore

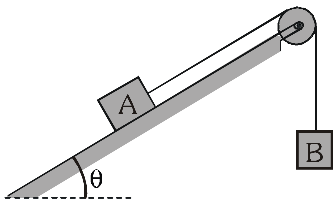
$$\sum F_x = 0 \quad \Rightarrow \quad T = mg\sin\theta \qquad \text{...(i)}$$

(c) If the thread is replaced by a spring, spring force must be equal to T,

therefore T = kx ...(ii)

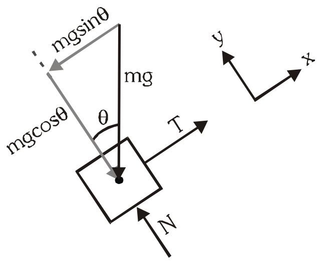
From equation (i) and (ii), we have $x = \dfrac{mg\sin\theta}{k}$

Illustration 6. Block A of mass m placed on a smooth slope is connected by a string with another block B of mass M as shown in the figure. If the system is in equilibrium, express M in terms of m.



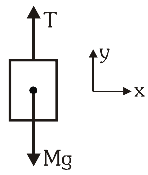
Solution For equilibrium of the block A, net force on it must be zero.

N: Normal reaction from slope



$$\sum F_x = 0 \quad \Rightarrow \quad T = mg\sin\theta \qquad \text{...(i)}$$

$$\sum F_y = 0 \quad \Rightarrow \quad N = mg\cos\theta \qquad \text{...(ii)}$$

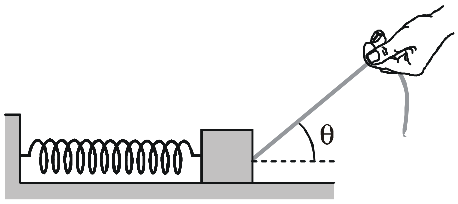


For equilibrium of block B, the net force on it must be zero.

$$\sum F_y = 0 \quad \Rightarrow \quad T = mg \qquad \text{... (ii)}$$

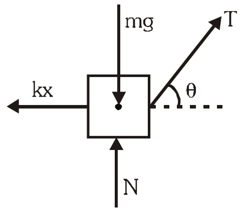
From equations (i) and (ii), we have M = m sinθ

Illustration 7. A block of mass m placed on a smooth floor is connected to a fixed support with the help of a spring of force constant k. It is pulled by a rope as shown in the figure. Tension force T of the rope is increased gradually without changing its direction, until the block losses contact from the floor. The increase in rope tension T is so gradual that acceleration in the block can be neglected.

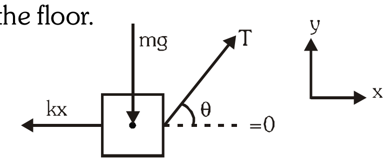


(a) Well before the block losses contact from the floor, draw its free body diagram.
(b) What is the necessary tension in the rope so that the block looses contact from the floor?
(c) What is the extension in the spring, when the block looses contact with the floor?

Solution (a) Free body diagram of the block, well before it looses contact with the floor.



(b) When the block is about to leave the floor, it is not pressing the floor. Therefore N = 0 and the block is in equilibrium.



$\sum F_x = 0 \quad \Rightarrow \quad T\cos\theta = kx \qquad ...(i)$

$\sum F_y = 0 \quad \Rightarrow \quad T\sin\theta = mg \qquad ...(ii)$

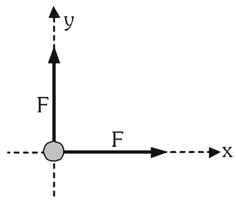
From equations (ii), we have $T = mg\operatorname{cosec}\theta$

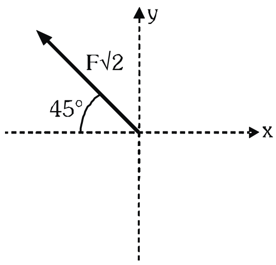
(c) From equation (i) and (ii), we have $x = \dfrac{mg\cot\theta}{k}$

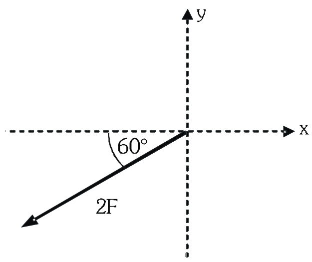
BEGINNER'S BOX-2

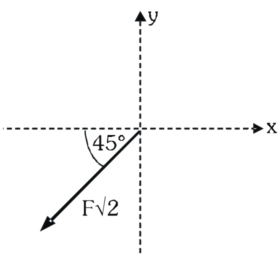
Equilibrium

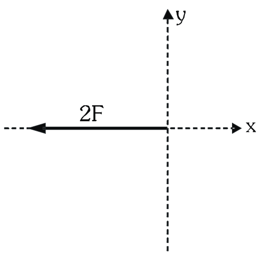
1. Two forces are simultaneously applied on an object. What third force would make the net force becomes zero ?



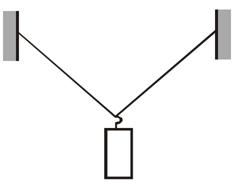
(A)


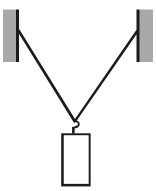
(B)


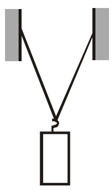
(C)


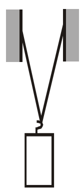
(D)


2. A block of weight W is suspended by a string of fixed length. The ends of the string are held at various positions as shown in the figures below. In which case, if any, is the magnitude of the tension along the string largest?

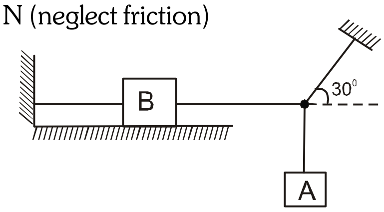
(A)


(B)


(C)


(D)


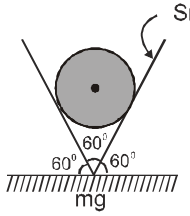
3. The breaking strength of the string connecting wall and block B is 175 N, find the maximum magnitude of weight of block A for which the system will be stationary. The block B weighs 700 N (neglect friction) ($g = 10\ m/s^2$)



(A) $\dfrac{175}{\sqrt{3}}$ N (B) $\dfrac{175\sqrt{3}}{2}$ N

(C) $175\sqrt{3}$ N (D) None of these

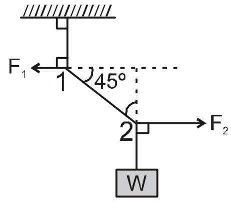
4. A cylinder of weight w is resting on a fixed V-groove as shown in figure. Calculate normal reactions between the cylinder and two inclined walls.



(A) w,w (B) w/2,w/2

(C) w,w/2 (D) w/2,w

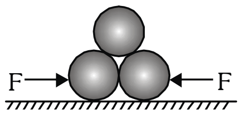
5. In the figure the tension in the string between 1 and 2 is 60 N. Find the magnitude of the horizontal force $\vec{F}_1$ and $\vec{F}_2$ that must be applied to hold the system in the position shown.



(A) $|\vec{F}_1| = |\vec{F}_2| = 40\sqrt{2}$ N (B) $|\vec{F}_1| = |\vec{F}_2| = 30\sqrt{2}$ N

(C) $|\vec{F}_1| = |\vec{F}_2| = 10\sqrt{2}$ N (D) $|\vec{F}_1| = |\vec{F}_2| = 20\sqrt{2}$ N

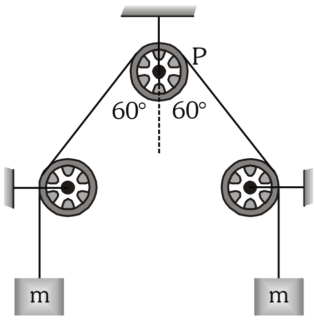
6. Two smooth cylindrical bars weighing W each lie next to each other in contact. A similar third bar is placed over the two bars as shown in figure. Neglecting friction, the minimum horizontal force on each lower bar necessary to keep them together is-



(A) $\dfrac{W}{2}$ (B) W

(C) $\dfrac{W}{\sqrt{3}}$ (D) $\dfrac{W}{2\sqrt{3}}$

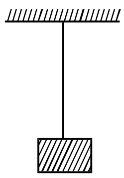
7. Two blocks of mass m each is connected with the string which passes over fixed pulleys, as shown in figure. The force exerted by the string on the pulley P is :



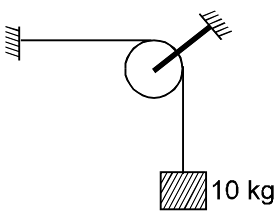
(A) mg (B) 2 mg

(C) $\sqrt{2}$ mg (D) 4 mg

8. A 'block' of mass 10 kg is suspended with massless string as shown in figure. Find tension in the string. ($g = 10\ m/s^2$)



9. Find magnitude of force exerted by string on pulley.

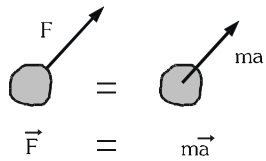


7.0 DYNAMICS OF PARTICLES: TRANSLATION MOTION OF ACCELERATED BODIES

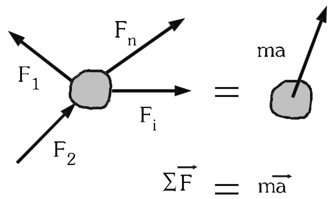
SL AL

Newton's laws are valid in inertial frames, which are un-accelerated frames. At present, we are interested in motion of terrestrial bodies and for this purpose; ground can be assumed a satisfactory inertial frame.

In particle dynamics, according to Newton's second law, forces acting on the body are considered as cause and rate of change in momentum as effect. For a rigid body of constant mass, the rate of change in momentum equals to product of mass and acceleration vector. Therefore, forces acting on it are the cause and product of mass and acceleration vector is the effect.



To write the equation of motion it is recommended to draw the free body diagram, put a sign of equality and in front of it draw the body attached with a vector equal to mass times acceleration produced. In the figure is shown a body of mass m on which a single force $\vec{F}$ acts and an observer in an inertial frame of reference observes the body moving with acceleration $\vec{a}$.



Acceleration imparted to a body by a force is independent of other forces, therefore when several forces $\vec{F}_1$, $\vec{F}_2$ and $\vec{F}_n$ act simultaneously on a body, the acceleration imparted to the body is the same as a single force equal to the vector sum of these forces could produce. The vector sum of these forces is known as the net resultant of these forces.

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \dots\dots\dots\dots + \vec{F}_n = m\vec{a}$$

$$\sum \vec{F} = m\vec{a}$$

In Cartesian coordinate system the vector quantities in the above equation is resolved into their components along x, y, and z axes as follows:

$$\sum F_x = ma_x \qquad \sum F_y = ma_y \qquad \sum F_z = ma_z$$

Illustrations

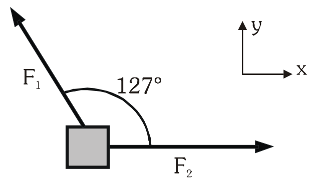
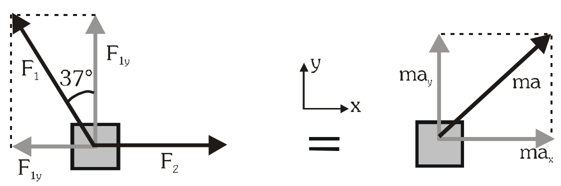


Illustration 8. Two forces F_1 and F_2 of magnitudes 50 N and 60 N act on a free body of mass m = 5 kg in directions shown in the figure. What is acceleration of object with respect to the free space?

Solution In an inertial frame of reference with its x-axis along the force F_2, the forces are expressed in Cartesian components.



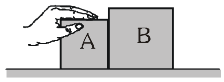
$\vec{F}_1 = (-30\hat{i} + 40\hat{j})$ N and $\vec{F}_2 = 60\hat{i}$ N

$$\sum F_x = ma_x \Rightarrow a_x = 6 \text{ m/s}^2$$

$$\sum F_y = ma_y \Rightarrow a_y = 8 \text{ m/s}^2$$

$$\vec{a} = (6\hat{i} + 8\hat{j}) \text{ m/s}^2$$

Illustration 9. Boxes A and B of mass $m_A = 1$ kg and $m_B = 2$ kg are placed on a smooth horizontal plane. A man pushes horizontally the 1 kg box with a force F = 6 N. Find the acceleration and the reaction force between the boxes.

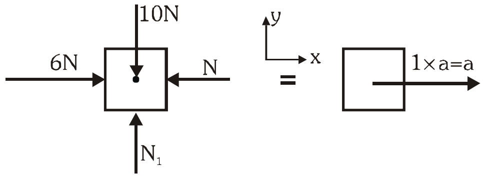


Solution Since both the blocks move in contact it is obvious that both of them have same acceleration. Say it is 'a'.

Applying NLM to block A

N: Normal reaction from B

N_1: Normal reaction from floor

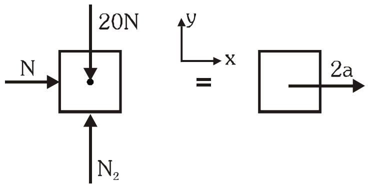


$\sum F_x = ma_x \quad \Rightarrow \quad 6 - N = a \quad$ ...(i)

$\sum F_y = 0 \quad \Rightarrow \quad N_1 = 10\text{ N} \quad$ ...(ii)

Applying NLM to block B

N_2: Normal reaction from ground



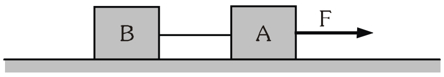
N : Normal reaction from A

$\sum F_x = ma_x \quad \Rightarrow \quad N = 2a \quad$ ...(iii)

$\sum F_y = 0 \quad \Rightarrow \quad N_2 = 20\text{ N} \quad$ ...(iv)

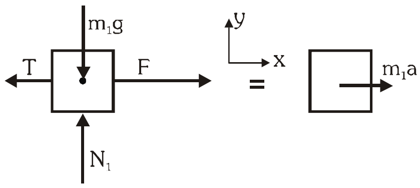
From equations (i) & (iii), we have $a = 2$ m/s^2 and N = 4 N

Illustration 10. Two blocks A and B of masses m_1 and m_2 connected by light strings are placed on a smooth floor as shown in the figure. If the block A is pulled by a constant force F, find accelerations of both the blocks and tension in the string connecting them.



Solution String connecting the blocks remain taut keeping separation between them constant. Therefore it is obvious that both of them move with the same acceleration. Say it is 'a'.

Applying NLM to block A



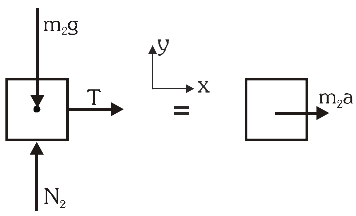
T: Tension of string

N_1 : Normal reaction from ground

$\sum F_x = ma \quad \Rightarrow \quad F - T = m_1 a \quad$ ...(i)

$\sum F_y = 0 \quad \Rightarrow \quad N_1 = m_1 g \quad$ ...(ii)

Applying NLM to block B.



T : Tension of string

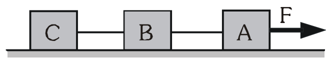
N_2 : Normal reaction from ground.

$\sum F_x = ma_x \quad \Rightarrow \quad T = m_2 a \quad$ . ..(iii)

$\sum F_y = 0 \quad \Rightarrow \quad N_2 = m_2 g \quad$ ...(iv)

From equation (i) and (iii), we have $a = \dfrac{F}{m_1 + m_2}$ and $T = \dfrac{m_2 F}{m_1 + m_2}$

Illustration 11. Three identical blocks A, B and C, each of mass 2.0 kg are connected by light strings as shown in the figure. If the block A is pulled by an unknown force F, the tension in the string connecting blocks A and B is measured to be 8.0 N. Calculate magnitude of the force F, tension in the string connecting blocks B and C, and accelerations of the blocks.

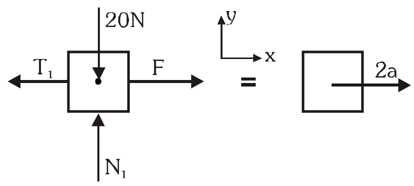


Solution It is obvious that all the three blocks move with the same acceleration. Say it is 'a'.

Applying NLM to the block A.

T_1: Tension of string connecting blocks A and B.

N_1 : Normal reaction from floor.



$\sum F_x = ma_x \quad \Rightarrow \quad F - T_1 = 2a$ ...(i)

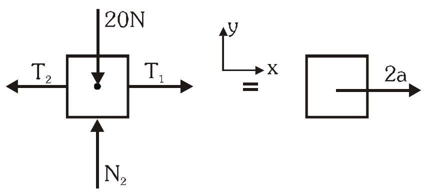
$\sum F_y = 0 \quad \Rightarrow \quad N_1 = 20N$ ...(ii)

Applying NLM to the block B.

T_1: Tension of string connecting blocks A and B.

T_2 : Tension of string connecting B & C.

N_1 : Normal reaction from floor.



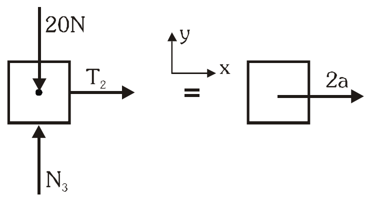
$\sum F_x = ma_x \quad \Rightarrow \quad T_1 - T_2 = 2a$ ...(iii)

$\sum F_y = 0 \quad \Rightarrow \quad N_2 = 20N$ ...(iv)

Applying NLM to the block C.

T_3 : Tension of string connecting B & C.

N_3 : Normal reaction from floor.



$\sum F_x = ma_x \quad \Rightarrow \quad T_2 = 2a$ ...(v)

$\sum F_y = 0 \quad \Rightarrow \quad N_3 = 20N$ ...(vi)

From equations (i), (iii) and (v), we have $F = 6a$ ...(vii)

Now using the fact that $T_1 = 8N$ with equation (i), We have $a = 2\ m/s^2$

Now from equation (i) we have $F = 12\ N$

From equation (iii), we have $T_2 = 4\ N$

Illustration 12. Two blocks A and B of masses m_1 and m_2 connected by uniform string of mass m and length ℓ are placed on smooth floor as shown in the figure. The string also lies on the floor. The block A is pulled by a constant force F.

(a) Find accelerations a of both the blocks and tensions T_A and T_B at the ends of the string.

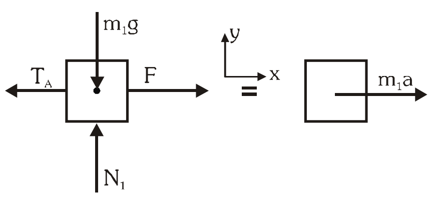
(b) Find an expression for tension T in the string at a distance x from the rear block in terms of T_A, T_B, m, ℓ and x.

Solution It is obvious that both the blocks and the whole string move with the same acceleration say it is 'a'. Since string has mass it may have different tensions at different points.

(a) Applying NLM to block A.

T_A : Tension of the string at end connected to block A.

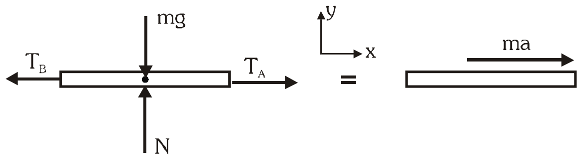
N_1 : Normal reaction of floor



$\sum F_x = ma_x \quad \Rightarrow \quad F - T_A = m_1a$ ...(i)

$\sum F_y = 0 \quad \Rightarrow \quad N_1 = m_1g$ ...(ii)

Applying NLM to the rope

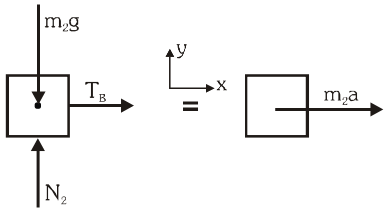


T_B : Tension of string at end connected to block B.
N : Normal reaction of floor

$\sum F_x = ma_x \quad \Rightarrow \quad T_A - T_B = ma \quad \ldots(iii)$

$\sum F_y = 0 \quad \Rightarrow \quad N = mg \quad \ldots(iv)$

Applying NLM to the block B
T_B : Tension of string
N_2 : Normal reaction from floor



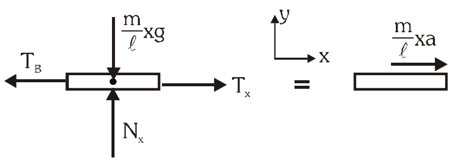
$\sum F_x = ma_x \quad \Rightarrow \quad T_B = m_2a \quad \ldots(v)$

$\sum F_y = 0 \quad \Rightarrow \quad N_2 = m_2g \quad \ldots(vi)$

From equations (i), (iii) and (v), we have

$a = \dfrac{F}{m+m_1+m_2} \ldots(vii) \qquad T_A = \dfrac{(m+m_2)F}{m+m_1+m_2} \ldots(viii) \qquad T_B = \dfrac{m_2F}{m+m_1+m_2} \ldots(ix)$

(b) To find tension at a point x distance away from block B, we can consider string of length x or ℓ–x.
Let as consider length of string x and apply NLM.



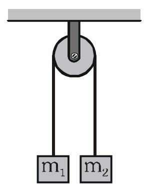
$\dfrac{mx}{\ell}$: mass of length x.

T_x = Tension at distance x
N_x = Normal reaction of floor

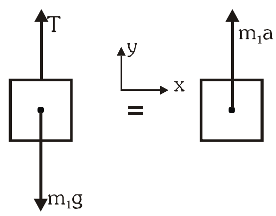
$\sum F_x = ma_x \quad \Rightarrow \quad T_x - T_B = \dfrac{mx}{\ell}a \ldots(x)$

From equation (vii), (viii), (ix) and (x), we have $T_x = \left(\dfrac{m}{\ell}x + m_2\right)\dfrac{F}{m+m_1+m_2}$

Illustration 13. The system shown in the figure is released from rest. Assuming mass m_2 more than the mass m_1, find the accelerations of the blocks and the tension in the string.



Solution It obvious that both blocks move with same acceleration magnitudes. Say it is 'a'.
Since m_2 is heavier, it moves downwards and m_1 moves upwards.
Tension at both the ends of the string has same magnitude. Say it is 'T'.
Apply NLM to block A of mass m_1



$\sum F_y = ma_y \quad \Rightarrow \quad T - m_1g = m_1a \quad \ldots(i)$

Apply NLM to block of mass m_2

$\sum F_y = ma_y \quad \Rightarrow \quad m_2g - T = m_2a \quad$ ...(ii)

From equations (i) & (ii), we have $a = \left(\dfrac{m_2 - m_1}{m_1 + m_2}\right)g$, $T = \left(\dfrac{2m_1m_2}{m_1 + m_2}\right)g$

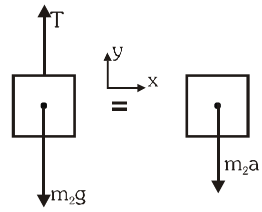
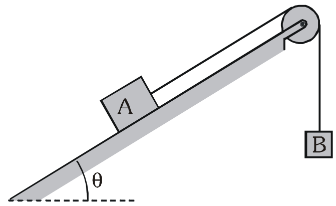


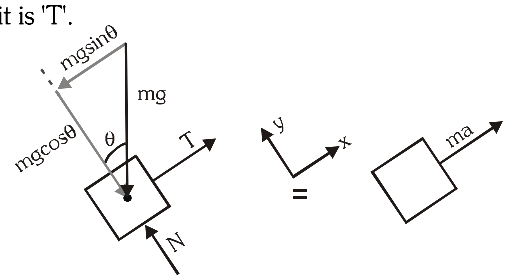
Illustration 14. Block A of mass m placed on a smooth slope is connected by a string with another block B of mass M (> msinθ) as shown in the figure. Initially the block A is held at rest and then let free. Find acceleration of the blocks and tension in the string.



Solution Both the blocks must move with the same magnitude of acceleration.
Since M > m sinθ, block B move downward pulling block A up the plane. Let acceleration magnitude is 'a'.
Tension at both the ends of the string is same. Say it is 'T'.
Apply NLM to block A
N: Normal reaction from slope
T : Tension of string

$\sum F_x = ma_x \quad \Rightarrow \quad T - mg\sin\theta = ma \quad$ ...(i)

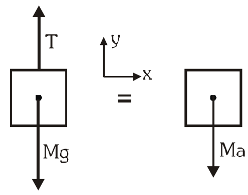
$\sum F_y = 0 \quad \Rightarrow \quad N = mg\cos\theta \quad$ ...(ii)



Apply NLM to block B
T: Tension of string

$\sum F_y = ma_y \quad \Rightarrow \quad Mg - T = Ma \quad$ ...(iii)

From equation (i) & (iii), we have $a = \dfrac{(M - m\sin\theta)g}{M + m}$, $T = \dfrac{(1 + \sin\theta)mMg}{m + M}$



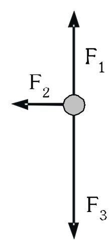
BEGINNER'S BOX-3

Basic Use of $\vec{F} = M\vec{a}$

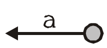
1. A monkey is descending from the branch of a tree with constant acceleration. If the breaking strength of branch is 75% of the weight of the monkey, the minimum acceleration with which the monkey can slide down without breaking the branch is

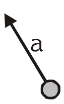
(A) g (B) $\dfrac{3g}{4}$ (C) $\dfrac{g}{4}$ (D) $\dfrac{g}{2}$

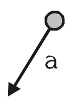
2. Three forces F_1, F_2 and F_3 act on an object simultaneously. These force vectors are shown in the following free-body diagram. In which direction does the object accelerate?



(A) 

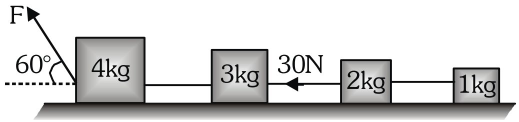
(B) 

(C) 

(D) 

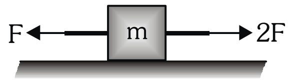
3. A monkey is sitting on the pan of a spring balance which is placed on an elevator. The maximum reading of the spring balance will be when :
(A) the elevator is stationary
(B) the string of the elevator breaks and it drops freely towards the earth
(C) the elevator is accelerated downwards
(D) the elevator is accelerated upwards.

4. Figure shows four blocks that are being pulled along a smooth horizontal surface. The masses of the blocks and tension in one cord are given. The pulling force F is :



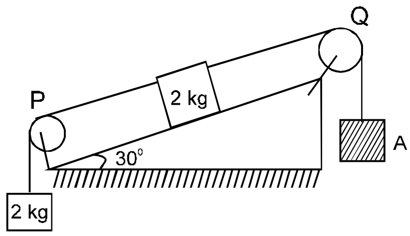
(A) 50 N (B) 100 N (C) 125 N (D) 200 N

5. A block of mass m is pulled on the smooth horizontal surface with the help of two ropes, each of mass m, connected to the opposite faces of the block. The forces on the ropes are F and 2F. The pulling force on the block is (Neglect gravity)

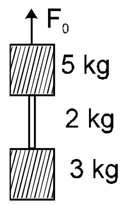


(A) F (B) 2F (C) F/3 (D) 3F/2

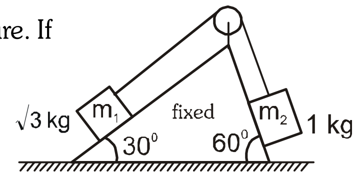
6. In the arrangement shown in figure, what should be the mass of block A so that the system remains at rest. Also find force exerted by string on the pulley Q. ($g = 10\ m/s^2$)



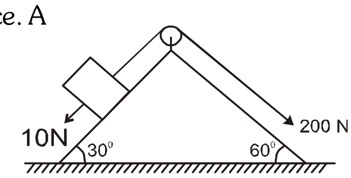
7. A 5 kg block has a uniform rope of mass 2 kg attached to its underside and a 3 kg block is suspended from the other end of the rope. The whole system is accelerated upward at $2\ m/s^2$ by an external force F_0.
(a) What is F_0 ?
(b) What is the net force on rope ?
(c) What is the tension at middle point of the rope ? ($g = 10\ m/s^2$)



8. Two blocks m_1 and m_2 are placed on a smooth inclined plane as shown in figure. If they are released from rest. Find :
(i) tension in the string
(ii) net force on pulley exerted by string



9. A 10 kg block kept on an inclined plane is pulled by a string applying 200 N force. A 10 N force is also applied on 10 kg block as shown in figure.
Find :
(a) tension in the string.
(b) acceleration of 10 kg block.
(c) net force on pulley exerted by string



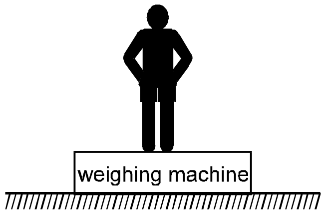
8.0 WEIGHING MACHINE

SL AL

A weighing machine does not measure the weight but measures the force exerted by object on its upper surface.

Illustrations

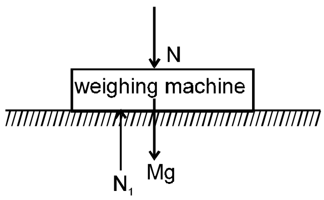
Illustration 15. A man of mass 60 Kg is standing on a weighing machine placed on ground. Calculate the reading of machine ($g = 10\ m/s^2$).



Solution For calculating the reading of weighing machine, we draw F.B.D. of man and machine separately.

F.B.D. of man F.B.D. of weighing machine

$N = Mg$



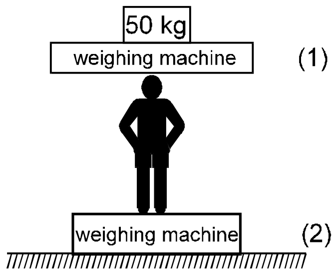
Here force exerted by object on upper surface is N

Reading of weighing machine

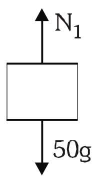
$N = Mg = 60 \times 10$

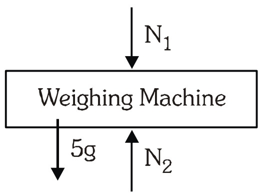
$N = 600\ N.$

Illustration 16. A man of mass 60 kg is standing on a weighing machine (2) of mass 5kg placed on ground. Another same weighing machine is placed over man's head. A block of mass 50kg is put on the weighing machine (1). Calculate the readings of weighing machines (1) and (2).

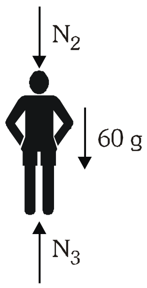


Solution





$N_1 = 50\ g;$ $N_2 = N_1 + 5g = 55\ g$



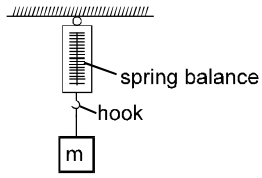
$N_3 = N_2 + 60\ g = 115\ g$

N_1 is the reading of weighing machine 1 and N_3 is the reading of weighing machine 2.

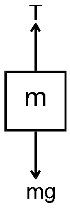
9.0 SPRING BALANCE

SL AL

It does not measure the weight. It measures the force exerted by the object at the hook.
Symbolically, it is represented as shown in figure.
A block of mass 'm' is suspended at hook.
When spring balance is in equilibrium, we draw the F.B.D. of mass m for calculating the reading of balance.



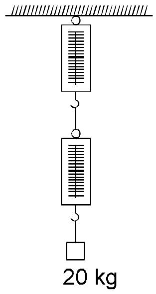
F.B.D. of 'm'.



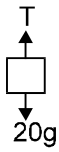
$mg - T = 0$
$T = mg$
Magnitude of T gives the reading of spring balance.

Illustrations

Illustration 17. A block of mass 20 kg is suspended through two light spring balances as shown in figure. Calculate the :
(1) reading of spring balance (1).
(2) reading of spring balance (2).



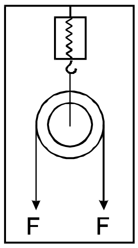
Solution: For calculating the reading, first we draw F.B.D. of 20 kg block.



F.B.D of 20 kg.
$mg - T = 0$
$T = 20\ g = 200\ N$

Since both balances are light so, both the scales will read 20 kg.

Illustration 18. Find the reading of spring balance in the adjoining figure, pulley and strings are ideal.

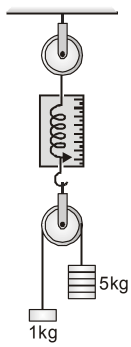


Ans. 2F

BEGINNER'S BOX-4

Weighing Machine and Spring Balance

1. A frictionless pulley of negligible weight is suspended from a spring balance. Masses of 1 kg and 5 kg are tied to the two ends of a string which passes over the pulley. The masses move due to gravity. During motion, the reading of the spring balance will be :

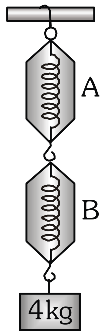


(A) $\frac{5}{3}$ kg wt (B) $\frac{10}{3}$ kg wt
(C) 6 kg wt (D) 3 kg wt

2. The tension in the spring is :

(A) zero (B) 2.5 N (C) 5 N (D) 10 N

3. A block of mass 4 kg is suspended through two light spring balances A and B. Then A and B will read respectively :



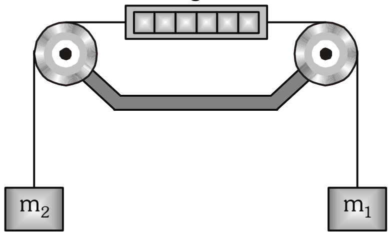
(A) 4 kg and zero kg

(B) zero kg and 4 kg

(C) 4 kg and 4 kg

(D) 2 kg and 2 kg

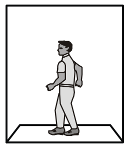
4. In the arrangement shown, the pulleys are fixed and ideal, the strings are light. $m_1 > m_2$ and S is a spring balance which is itself massless. The reading of S (in unit of mass) is :



(A) $(m_1 - m_2)$ (B) $\frac{1}{2}(m_1 - m_2)$

(C) $\frac{m_1 m_2}{m_1 + m_2}$ (D) $\frac{2m_1 m_2}{m_1 + m_2}$

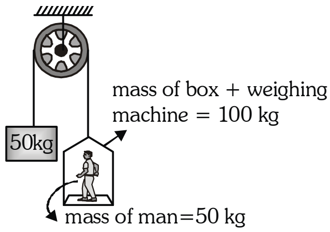
5. In the given figure, mass of man is 60 kg and reading of weighing machine is 90 kg the acceleration of lift is



(A) $\frac{g}{2}$ in upward direction (B) g in upward direction

(B) $\frac{g}{2}$ in downward direction (D) g in downward

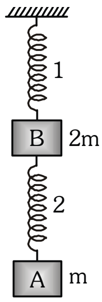
6. In the given figure pulley is smooth and string is light. Reading of weighing machine is



(A) 25 kg (B) 20 kg

(C) 250 newton (D) 200 newton

7. In the given figure, block are at rest and in equillibrium. Acceleration of block A is a and B is b just after spring (2) is cut, then.



(A) $a = g\downarrow, b = g\uparrow$ (B) $a = g\downarrow, b = \frac{g}{2}\uparrow$

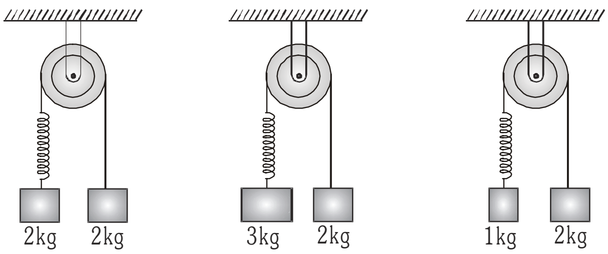
(C) $a = 0, b = g\uparrow$ (D) $a = 0, b = \frac{g}{2}\uparrow$

8. In the above question the value of a and b 'if spring (1) is cut instead of spring 2 is

(A) $a = 0, b = \frac{3g}{2}\downarrow$ (B) $a = 0, b = g\downarrow$

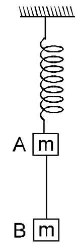
(C) $a = g\downarrow, b = g\downarrow$ (D) None of these

9. Same spring is attached with 2kg, 3kg and 1 kg blocks in three different cases as shown in figure. If x_1, x_2 and x_3 be the extensions in the spring in these three cases then



(A) $x_1 = 0, x_3 > x_2$ (B) $x_2 > x_1 > x_3$ (C) $x_3 > x_1 > x_2$ (D) $x_1 > x_2 > x_3$

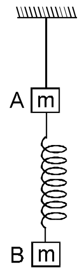
10. Two blocks 'A' and 'B' of same mass 'm' attached with a light string are suspended by a spring as shown in figure Find the acceleration of block 'A' just after string is cut :



(A) g/2 (upwards) (B) g(upwards)

(C) g/2 (downwards) (D) g (downwards)

11. Two blocks 'A' and 'B' of same mass 'm' attached with a light spring are suspended by a string as shown in figure. Find the acceleration of block 'A' and 'B' just after the string is cut.



10.0 CONSTRAINED MOTION

AL

10.1 String Constraint

When two objects are connected through a string and if the string have the following properties :

(a) The length of the string remains constant i.e. inextensible string.

(b) Always remains tight, does not slacks.

Then the parameters of the motion of the objects along the length of the string and in the direction of extension have a definite relation between them.

Steps for String Constraint

Step 1. Identify all the objects and number of strings in the problem.

Step 2. Assume variable to represent the parameters of motion such as displacement, velocity acceleration etc.

(i) Object which moves along a line can be specified by one variable.

(ii) Object moving in a plane are specified by two variables.

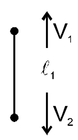
(iii) Objects moving in 3-D requires three variables to represent the motion.

Step 3. Identify a single string and divide it into different linear sections and write in the equation format.

$$\ell_1+\ell_2+\ell_3+\ell_4+\ell_5+\ell_6=\ell$$

Step 4. Differentiate with respect to time

$$\frac{d\ell_1}{dt}+\frac{d\ell_2}{dt}+\frac{d\ell_3}{dt}+....=0$$



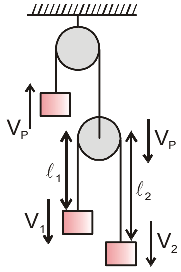
$\frac{d\ell_1}{dt}$ = represents the rate of increment of the portion 1, end points are always in contact with some object so take the velocity of the object along the length of the string

$$\frac{d\ell_1}{dt}=V_1+V_2$$

Take positive sign if it tends to increase the length and negative sign if it tends to decrease the length. Here $+V_1$ represents that upper end is tending to increase the length at rate V_1 and lower end is tending to increase the length at rate V_2.

Step 5. Repeat all above steps for different-different strings.

Let us consider a problem given below



Here $\ell_1 + \ell_2 =$ constant

$$\frac{d\ell_1}{dt} + \frac{d\ell_2}{dt} = 0$$

$$(V_1 - V_P) + (V_P - V_2) = 0$$

$$V_p = \frac{V_1 + V_2}{2}$$

Similarly,

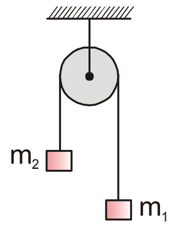
$$a_P = \frac{a_1 + a_2}{2}$$ Remember this result

Illustrations

Illustration 19. Two blocks of masses m_1 and m_2 are attached at the ends of an inextensible string which passes over a smooth massless pulley. If $m_1 > m_2$, find :

(i) the acceleration of each block

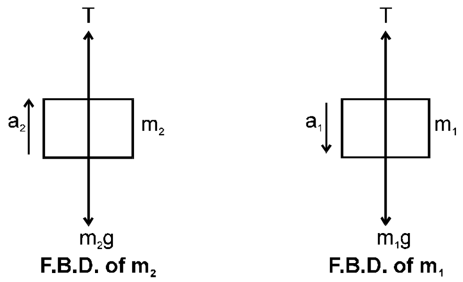
(ii) the tension in the string.



Solution The block m_1 is assumed to be moving downward and the block m_2 is assumed to be moving upward. It is merely an assumption and it does not imply the real direction. If the values of a_1 and a_2 come out to be positive then only the assumed directions are correct; otherwise the body moves in the opposite direction. Since the pulley is smooth and massless, therefore, the tension on each side of the pulley is same.

The free body diagram of each block is shown in the figure.

F.B.D. of m_2 ***F.B.D. of m_1***



Applying Newton's second Law on blocks m_1 and m_2

Block m_1 $\quad m_1g - T = m_1a_1 \quad$ ...(1)

Block m_2 $\quad -m_2g + T = m_2a_2 \quad$ ...(2)

Number of unknowns : T, a_1 and a_2 (three)

Number of equations: only two

Obviously, we require one more equation to solve the problem. Note that whenever one finds the number of equations less than the number of unknowns, one must think about the constraint relation. Now we are going to explain the mathematical procedure for this.

How to determine Constraint Relation ?

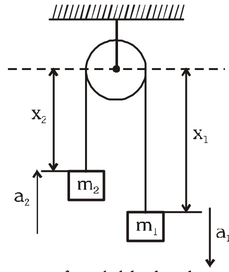
(1) Assume the direction of acceleration of each block, e.g. a_1 (downward) and a_2 (upward) in this case.

(2) Locate the position of each block from a fixed point (depending on convenience), e.g. centre of the pulley in this case.

(3) Identify the constraint and write down the equation of constraint in terms of the distance assumed. For example, in the chosen problem, the length of string remains constant is the constraint or restriction.

Thus, $x_1 + x_2 =$ constant

Differentiating both the sides w.r.t. time we get $\frac{dx_1}{dt} + \frac{dx_2}{dt} = 0$



Position of each block is located w.r.t. centre of the pulley

Each term on the left side represents the velocity of the blocks. Since we have to find a relation between accelerations, therefore we differentiate it once again w.r.t. time.

Thus $$\frac{d^2x_1}{dt^2} + \frac{d^2x_2}{dt^2} = 0$$

Since, the block m_1 is assumed to be moving downward (x_1 is increasing with time)

$$\therefore \quad \frac{d^2x_1}{dt^2} = +a_1$$

and block m_2 is assumed to be moving upward (x_2 is decreasing with time)

$$\therefore \quad \frac{d^2x_2}{dt^2} = -a_2$$

Thus $a_1 - a_2 = 0$

or $a_1 = a_2 = a$ (say) is the required constraint relation.

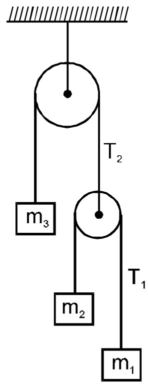
Substituting $a_1 = a_2 = a$ in equations (1) and (2) and solving them, we get

(i) $a = \left[\frac{m_1 - m_2}{m_1 + m_2}\right]g$ (ii) $T = \left[\frac{2m_1m_2}{m_1 + m_2}\right]g$

Illustration 20. A system of three masses m_1, m_2 and m_3 are shown in the figure. The pulleys are smooth and massless; the strings are massless and inextensible.

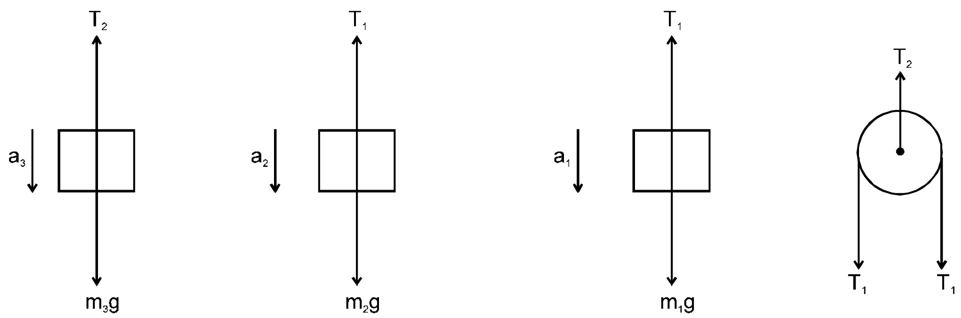
(i) Find the tensions in the strings.

(ii) Find the acceleration of each mass.



Solution All the blocks are assumed to be moving downward and the free body diagram of each block is shown in figure.

F.B.D. m_3 ***F.B.D. m_2*** ***F.B.D. m_1*** ***F.B.D. of pulley***



Applying Newton's Second Law to

Block m_1 : $m_1g - T_1 = m_1a_1$ ...(1)

Block m_2 : $m_2g - T_1 = m_2a_2$ ...(2)

Block m_3 : $m_3g - T_2 = m_3a_3$ ...(3)

Pulley : $T_2 = 2T_1$ ...(4)

Number of unknowns a_1, a_2, a_3, T_1 and T_2 (Five)

Number of equations: Four

The constraint relation among accelerations can be obtained as follows

For upper string $\quad x_3 + x_0 = c_1$

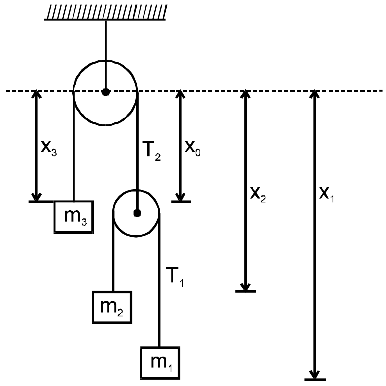
For lower string $\quad (x_2 - x_0) + (x_1 - x_0) = c_2$

$x_2 + x_1 - 2x_0 = c_2$

Eliminating x_0 from the above two relations,

we get $\quad x_1 + x_1 + 2x_3 = 2c_1 + c_2 = \text{constant.}$

Differentiating twice with respect to time,



we get $\quad \dfrac{d^2x_1}{dt^2} + \dfrac{d^2x_2}{dt^2} + 2\dfrac{d^2x_3}{dt^2} = 0$

or $\quad a_1 + a_2 + 2a_3 = 0 \qquad ...(5)$

Solving equations (1) to (5), we get

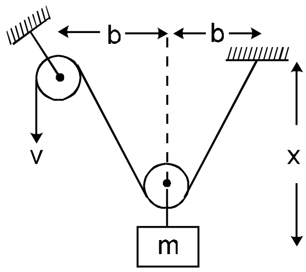
(i) $T_1 = \left[\dfrac{4m_1m_2m_3}{4m_1m_2 + m_3(m_1+m_2)}\right]g$; $\qquad T_2 = 2T_1$

(ii) $a_1 = \left[\dfrac{4m_1m_2 + m_1m_3 - 3m_2m_3}{4m_1m_2 + m_3(m_1+m_2)}\right]g$

$a_2 = \left[\dfrac{3m_1m_3 - m_2m_3 - 4m_1m_2}{4m_1m_2 + m_3(m_1+m_2)}\right]g$

$a_3 = \left[\dfrac{4m_1m_2 - m_3(m_1+m_2)}{4m_1m_2 + m_3(m_1+m_2)}\right]g$

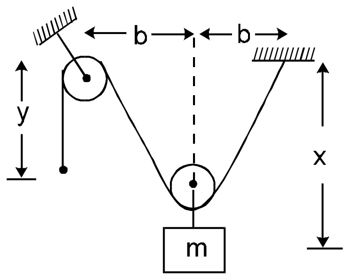
Illustration 21. The figure shows one end of a string being pulled down at constant velocity v. Find the velocity of mass 'm' as a function of 'x'.



Solution Using constraint equation

$2\sqrt{x^2+b^2} + y = \text{length of string} = \text{constant}$

Differentiating w.r.t. time :

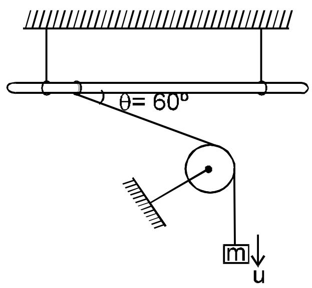


$$\frac{2}{2\sqrt{x^2+b^2}} \cdot 2x\left(\frac{dx}{dt}\right) + \left(\frac{dy}{dt}\right) = 0$$

$$\Rightarrow \left(\frac{dy}{dt}\right) = v$$

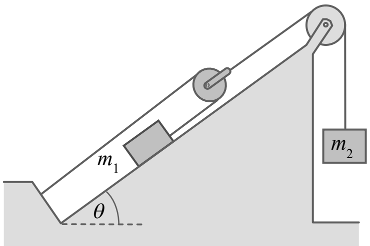
$$\therefore \left(\frac{dx}{dt}\right) = -\frac{v}{2x}\sqrt{x^2+b^2}$$

Illustration 22. The figure shows mass m moves with velocity u. Find the velocity of ring at that moment. Ring is restricted to move on smooth rod.

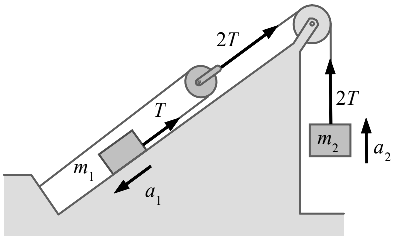


Solution $\quad V_R = \dfrac{u}{\cos\theta}, \quad V_R = 2u$

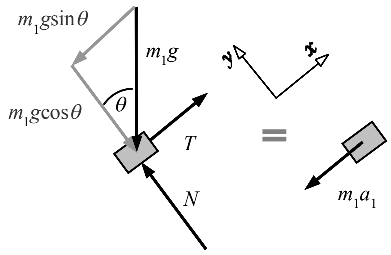
Illustration 23. In the system shown in figure, block m_1 slides down a friction less inclined plane. The pulleys and strings are ideal. Find the accelerations of the blocks.



Solution Tension forces applied by the strings are shown in the adjacent figure.



Let the block m_1 is moving down the plane with an acceleration a_1 and m_2 is moving upwards with accelerations a_2. Relation between accelerations a_1 and a_2 of the blocks can be obtained easily by method of virtual work.



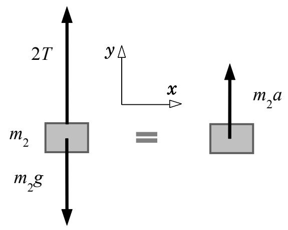
$a_1 = 2a_2$ ...(i)

Applying Newton's laws to analyze motion of block m_1

$\sum F_x = ma_x \rightarrow m_1 g\sin\theta - T = m_1 a_1$ ...(ii)

Applying Newton's laws to analyze motion of block m_2

$\sum F_y = ma_y \rightarrow 2T - m_2 g = m_2 a_2$ ...(iii)



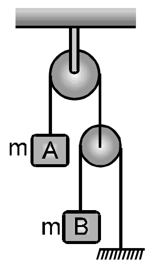
From equation (i), (ii) and (iii), we have

$$a_1 = \frac{2(2m_1\sin\theta - m_2)}{4m_1 + m_2}g \quad a_2 = \frac{2m_1 g\sin\theta - m_2 g}{4m_1 + m_2}$$

BEGINNER'S BOX-5

String Constraint

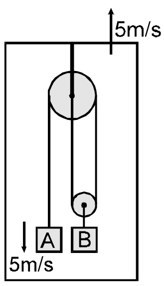
1. In the figure shown neglecting friction and mass of pulleys, what is the acceleration of mass B?



(A) $\frac{g}{3}$ (B) $\frac{5g}{2}$

(C) $\frac{2g}{2}$ (D) $\frac{2g}{5}$

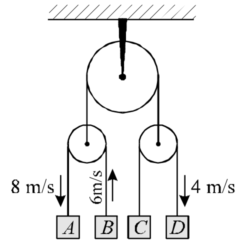
2. A system is as shown in the figure. All speeds shown are with respect to ground. Then the speed of Block B with respect to ground is :



(A) 5 m/s (B) 10 m/s

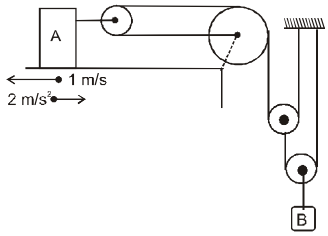
(C) 15 m/s (D) 7.5 m/s

3. In the figure shown the velocity of different blocks is shown. The speed of C is :



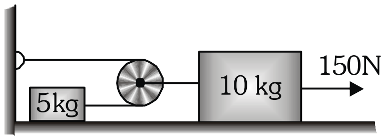
(A) 6 m/s
(B) 4 m/s
(C) 0 m/s
(D) none of these

4. In the given figure find the velocity and acceleration of B, if instantaneous velocity and acceleration of A are as shown in the figure.



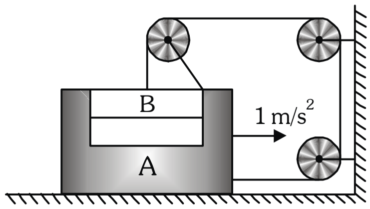
(A) v = 0.5 m/s, a = 1m/s²
(B) v = 1 m/s, a = 2m/s²
(C) v = 2 m/s, a = 4m/s²
(D) v = 1 m/s, a = 1m/s²

5. The acceleration of block of mass 5 kg is:



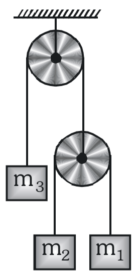
(A) 5 m/s² (B) 10 m/s² (C) 15 m/s² (D) 20 m/s²

6. If block A is moving with an acceleration of 1 m/s², the acceleration of B w.r.t. ground is:



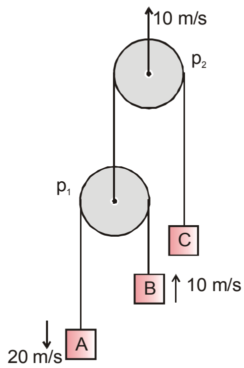
(A) 1 m/s² (B) $\sqrt{2}$ m/s²
(C) $\sqrt{5}$ m/s² (D) 2 m/s²

7. Three masses m_1, m_2 and m_3 are attached to a string as shown in the figure. All three masses are held at rest and then released. To keep m_3 at rest, the condition is:



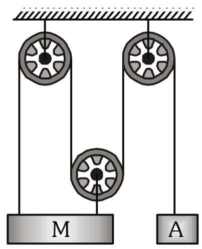
(A) $\frac{1}{m_3}=\frac{1}{m_1}+\frac{1}{m_2}$ (B) $m_1 + m_2 = m_3$
(C) $\frac{4}{m_3}=\frac{1}{m_1}+\frac{1}{m_2}$ (D) $\frac{1}{m_1}+\frac{2}{m_2}=\frac{3}{m_3}$

8. Velocities of blocks A, B and pulley p_2 are shown in figure. Find velocity of pulley p_1 and block C.



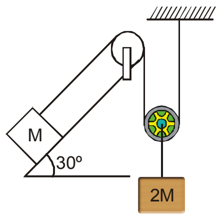
(A) $V_{P_1} = 10$ m/s $\downarrow$, $V_C = 25$ m/s $\uparrow$
(B) $V_{P_1} = 5$ m/s $\uparrow$, $V_C = 25$ m/s $\uparrow$
(C) $V_{P_1} = 5$ m/s $\downarrow$, $V_C = 25$ m/s $\downarrow$
(D) $V_{P_1} = 5$ m/s $\downarrow$, $V_C = 25$ m/s $\uparrow$

9. See the arrangement shown in figure. If the acceleration of block B is a, then the acceleration of block A in magnitude will be :



(A) a (B) 2a
(C) 3a (D) 4a

10. Find the acceleration of the block of mass M in the situation shown in figure. All the surfaces are frictionless and the pulleys and the string are light.



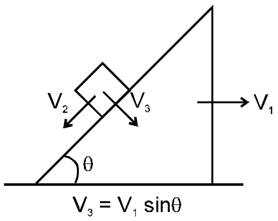
10.2 Wedge Constraint

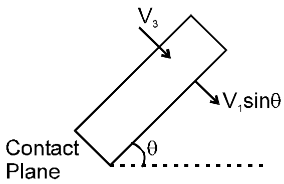
Conditions

(i) There is a regular contact between two objects.

(ii) Objects are rigid.

The relative velocity perpendicular to the contact plane of the two rigid objects is always zero if there is a regular contact between the objects. Wedge constraint is applied for each contact.



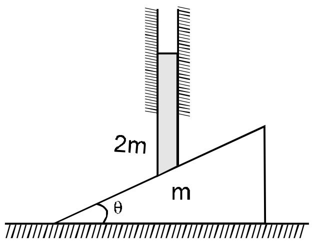


In other words,

Components of velocity along perpendicular direction to the contact plane of the two objects is always equal if there is no deformations and they remain in contact.

Illustrations

Illustration 24. A rod of mass 2m moves vertically downward on the surface of wedge of mass as shown in figure. Find the relation between velocity of rod and that of the wedge at any instant.



Solution Using wedge constraint.

Component of velocity of rod along perpendicular to inclined surface is equal to velocity of wedge along that direction.

$u \cos\theta = v \sin\theta$

$$\frac{u}{v} = \tan\theta$$

$u = v \tan\theta$

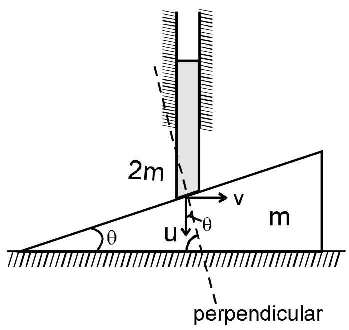


Illustration 25. In the above solved example, find a relation between acceleration of rod to that of the wedge.

Solution. $a_{rod} = a_{wedge} \tan\theta$.

11.0 NEWTON'S LAW FOR A SYSTEM

SL

$$\vec{F}_{ext} = m_1\vec{a}_1 + m_2\vec{a}_2 + m_3\vec{a}_3 + ...$$

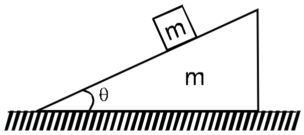
$\vec{F}_{ext}$ = Net external force on the system.

m_1, m_2, m_3 are the masses of the objects of the system and

$\vec{a}_1, \vec{a}_2, \vec{a}_3$ are the acceleration of the objects respectively.

Illustrations

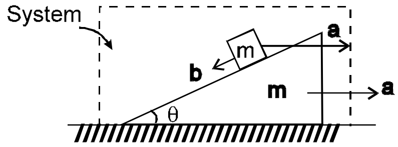
Illustration 26. The block of mass m slides on a wedge of mass 'm' which is free to move on the horizontal ground. Find the accelerations of wedge and block. (All surfaces are smooth).

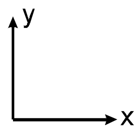


Solution Let .a $\Rightarrow$ acceleration of wedge

b $\Rightarrow$ acceleration of block with respect to wedge

Taking block and wedge as a system and applying Newton's law in the horizontal direction





$F_x = m_1\, \vec{a}_{1x} + m_2\, \vec{a}_{2x} = 0$

$0 = ma + m(a - b\cos\theta)$ ....(i)

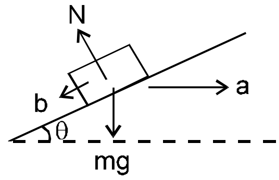
here 'a ' and 'b' are two unknowns , so for making second equation, we draw F.B.D. of block.

F.B.D of block.

using Newton's second law along inclined plane

$mg \sin\theta = m\,(b - a\cos\theta)$ ....(ii)

Now solving equations (1) and (2) we will get



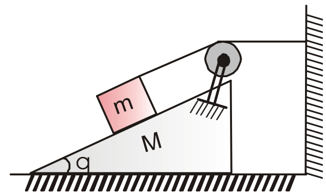
$$a = \frac{mg\sin\theta\cos\theta}{m(1+\sin^2\theta)} = \frac{g\sin\theta\cos\theta}{(1+\sin^2\theta)} \quad \text{and} \quad b = \frac{2g\sin\theta}{(1+\sin^2\theta)}$$

So in vector form :

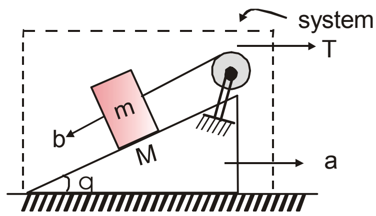
$$\vec{a}_{\text{wedge}} = a\hat{i} = \left(\frac{g\sin\theta\cos\theta}{1+\sin^2\theta}\right)\hat{i}\,; \qquad \vec{a}_{\text{block}} = (a - b\cos\theta)\,\hat{i} - b\sin\theta\,\hat{j}$$

$$\vec{a}_{\text{block}} = -\frac{g\sin\theta\cos\theta}{(1+\sin^2\theta)}\,\hat{i} - \frac{2g\sin^2\theta}{(1+\sin^2\theta)}\,\hat{j}\,.$$

Illustration 27. For the arrangement shown in figure when the system is released, find the acceleration of wedge. Pulley and string are ideal and friction is absent.



Solution Considering block and wedge as a system and using Newton' s law for the system along x– direction

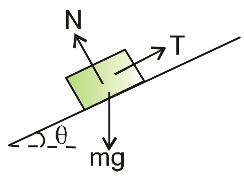


$T = Ma + m\,(a - b\cos\theta)$ ....(i)

F.B.D of m

along the inclined plane

$mg\sin\theta - T = m\,(b - a\cos\theta)$ ....(ii)



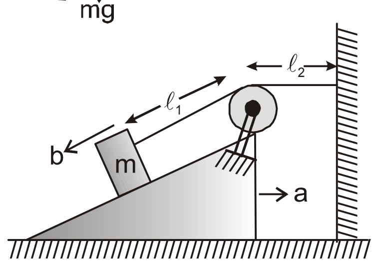
using string constraint equation.

$\ell_1 + \ell_2 = \text{constant}$

$$\frac{d^2\ell_1}{dt^2} + \frac{d^2\ell_2}{dt^2} = 0$$

$b - a = 0$ ....(iii)

Solving above equations (i),(ii) & (iii) , we get



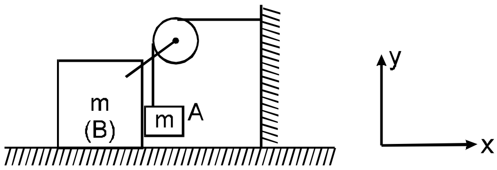
$$a = \frac{mg\sin\theta}{M + 2m(1-\cos\theta)}$$

Illustration 28. In the system shown in figure, the block A is released from rest. Find :

(i) the acceleration of both blocks 'A' and 'B'.

(ii) Tension in the string.

(iii) Contact force between 'A' and 'B'.

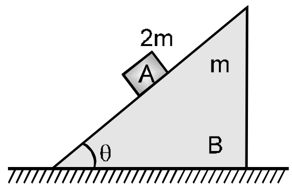


Solution (i) $\frac{g}{3}\hat{i}-\frac{g}{3}\hat{j},\ \frac{g}{3}\hat{i}$ (ii) $\frac{2mg}{3}$ (iii) $\frac{mg}{3}$.

BEGINNER'S BOX-6

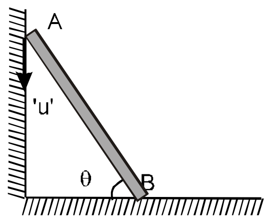
Wedge Constraint

1. In the given figure if blocks A and B will move with same acceleration due to external agent, there is no friction between A and B, then the magnitude of interaction force between the two blocks will be :



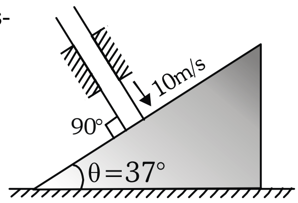
(A) 2 mg/cos θ
(B) 2 mg cos θ
(C) mg cos θ
(D) none of these

2. The velocity of end 'A' of rigid rod placed between two smooth vertical walls moves with velocity 'u' along vertical direction. Find out the velocity of end 'B' of that rod, rod always remains in contact with the vertical walls.



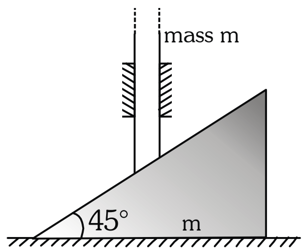
(A)u cot θ (B) u sec θ
(C) u cos θ (D) u tan θ

3. In the given situation when rod is moving with speed 10 m/s, speed of wedge is-



(A) $\frac{50}{3}$ m/s (B) $\frac{40}{3}$ m/s
(C) $\frac{20}{3}$ m/s (D) None of these

4. In the given figure all surface are smooth. Find acceleration of rod.

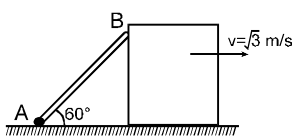


(A) 5 m/s^2 (B) 4 m/s^2
(C) 6 m/s^2 (D) None of these

5. In above question normal force between rod and wedge is.

(A) $\frac{mg}{\sqrt{2}}$ (B) mg (C) $\frac{mg}{2}$ (D) $\frac{mg}{2\sqrt{2}}$

6. A rod AB is shown in figure. End A of the rod is fixed on the ground. Block is moving with velocity $\sqrt{3}$ m/s towards right. Find the velocity of end B of rod when rod makes an angle of 60° with the ground.



12.0 INERTIAL AND NON-INERTIAL REFERENCE FRAMES

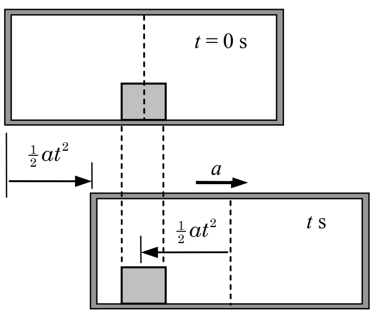
AL

A body is observed in motion, when it changes its position or orientation relative to another body or another set of bodies. A frame of reference consists of a set of coordinate axes fixed with the body relative to which the motion is observed and a clock. The coordinate axes are required to measure position of the moving body and the clock is required to measure time.

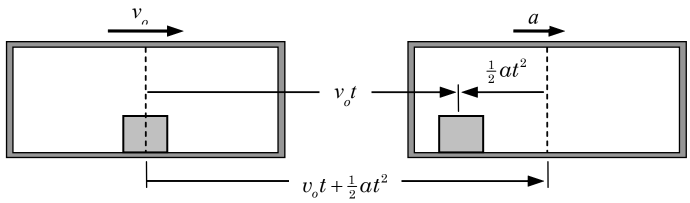
All the kinematical variables position, velocity, and acceleration are measured relative to a reference frame; therefore depend on the state of motion of the reference frame and we say that motion is essentially a relative concept. When the reference frame and a body both are stationary or move identically, the body is observed stationary relative to the reference frame. It is only when the reference frame and the body move in different manner, the body is observed moving relative the reference frame.

Now think about the whole universe where the planets, stars, galaxies and other celestial bodies all are in motion relative to each other. If any one of them can be assumed in state of rest, we can attach a reference frame to it and define motion of all other bodies relative to it. Such a body, which we assume in state of rest with respect to all other bodies in the universe, is known in absolute rest and the reference frame attached to it as most preferred reference frame. Unfortunately, the very notion of the reference frame and the idea motion as a relative concept, make it impossible to find a body anywhere in the universe at absolute rest. Therefore, the idea of absolute rest and a preferred reference frame become essentially meaningless. Now we can have only two categories of reference frames. In one category, we can have reference frames that move with uniform velocities and in the other category; we can have reference frames that are in accelerated motion.

To understand the above ideas let us think an experiment. Consider a closed container on a goods train either at rest or moving with constant velocity v_o on a level track. The floor of the container is smooth and a block is placed in the center of the container. We observe the situation relative to two reference frames, one fixed with the ground and other fixed with the container. Relative to the ground frame both the container and the block are at rest or move together with the same velocity and relative to the container frame the block is at rest as shown in the figure.



Container accelerates from rest



Uniformly moving container accelerates

Now let the driver of the train accelerates the container at uniform rate a. If the train were initially at rest, relative to the ground, the block remains at rest and the container moves forward. Relative to the container the block moves backwards with the same magnitude of acceleration as with the container moves forward. If the train were initially moving uniformly, relative to the ground the block continues to move with the same original velocity and train accelerates and becomes ahead in space. Relative to the container the block appears moving backward with acceleration that is equal in magnitude to the acceleration of the container.

Now consider a man sitting on a fixed chair in the container. He is always stationary relative to the container. If he does not look outs side, in no way he can know whether the container is at rest or moving uniformly. However, he can definitely say whether the container accelerates or not, by observing motion of the block relative to the container. Because net forces acting on the block are still zero, therefore observed acceleration of the block can only be due to acceleration of the container as per Newton's laws of motion.

Now we can conclude that there can be only two kinds of reference frames either non-accelerated or accelerated. The reference frames that are non-accelerated i.e. at rest or moving with uniform velocities are known as inertial reference frames and those in accelerated motion as non-inertial reference frames.

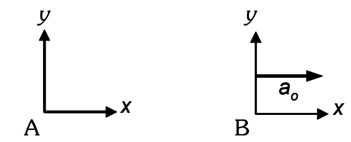
Inertial Reference Frames and Newton's laws of motion

In Newton's laws of motion, force is conceived as two-body interaction that can be the only agent producing acceleration in a body. As far as we observe motion of a body from an inertial frame, any acceleration observed in a body can only be due to some forces acting on the body. All the three laws are in perfect agreement with the observed facts and we say that all the laws holds true in inertial reference frames.

Non-Inertial Reference Frames and Newton's laws of motion

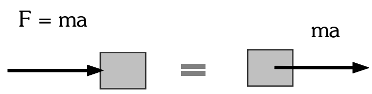
A body if at rest or in uniform velocity motion relative to some inertial frame net forces acting on it must be zero. Now if motion of the same body is observed relative to a non-inertial frame, it will be observed moving with acceleration that is equal in magnitude and opposite in direction to the acceleration of the non-inertial frame. This observed acceleration of the body is purely a kinematical effect. But to explain this observed acceleration relative to the non-inertial frame according to Newton's laws of motion, we have assume a force must be acting on the body. This force has to be taken equal to product of mass of the body and opposite of acceleration vector of the non-inertial frame. Since this force is purely an assumption and not a result of interaction of the body with any other body, it is a fictitious force. This fictitious force is known as pseudo force or inertial force.

Until now, we have learnt the idea that how we can apply Newton's laws of motion in non-inertial frame to a body in equilibrium in inertial frame. Now it is turn to discuss how we can apply Newton's laws of motion to analyze motion in non-inertial frame of a body, which is in accelerated motion relative to an inertial frame.



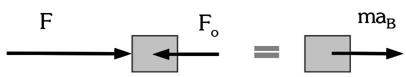
Reference frames *A* is an inertial frame and *B* is a non inertial frame.

Consider a net physical force $\vec{F}$ in positive x-direction applied on the box. Here by the term physical force, we refer forces produced by two body interactions. Relative to inertial frame A, the box is observed to have an acceleration $\vec{a} = \vec{F}/m$ defined by the second law of motion and a force equal in magnitude and opposite in direction to $\vec{F}$ can be assigned to the body exerting $\vec{F}$ on the box constituting Newton's third law pair. All the three laws of motion hold equally well in inertial frame A.



A net physical force imparts acceleration to the box in inertial frame *A*.

Relative to non-inertial frame B, the box is observed moving in x-direction with acceleration $\vec{a}_B = \vec{a} - \vec{a}_o$, which can satisfy Newton's second law, only if the fictitious force $\vec{F}_o = -m\vec{a}_o$ is assumed acting together with the net physical force $\vec{F}$ as shown in the figure. Now we can write modified equation of Newton's second law in non-inertial frame.



A net physical force and pseudo force imparts acceleration to the box in non-inertial frame B.

$$\vec{F} + \vec{F}_o = m\vec{a}_B \rightarrow F - F_o = ma_B$$

From the above discussion, we can conclude that in non-inertial frames Newton's second law is made applicable by introducing pseudo force in addition to physical forces. The pseudo force equals to the product of mass of the concerned body and the acceleration of the frame of reference in a direction opposite to the acceleration of the frame of reference.

Practical inertial frame of reference

The definition of an inertial frame of reference is based on the concept of a free body in uniform velocity motion or absolute rest. It is impossible to locate a body anywhere in the universe, where forces from all other bodies exactly balance themselves and lead to a situation of uniform velocity motion according to the first law. Thus, we cannot find anywhere in the universe a body, to which we attach a frame of reference and say that it is a perfect inertial frame of reference. It is the degree of accuracy, required in analyzing a particular physical situation that decides which body in the universe is to be selected a preferred inertial frame of reference.

The earth and other planets of the sun are rotating about their own axis and revolving around the sun and the sun is moving too. In fact, all celestial bodies in the universe the sun, its planets, other stars, our galaxy the

Milky Way, and other galaxies are in accelerated motion whose nature is not known exactly. Therefore, none of them can be used as a perfect inertial frame of reference. However, when the acceleration of any one of the above-mentioned celestial bodies becomes negligible as compared to the accelerations involved in a physical situation, a frame of reference attached with the corresponding celestial body may be approximated as an inertial frame of reference and the physical situation under consideration may be analyzed with acceptable degree of accuracy.

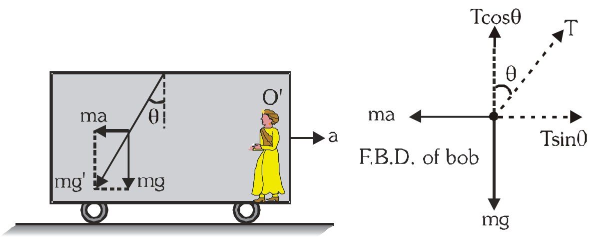
The centripetal acceleration due to rotation of the earth at any point on its surface varies from zero at the poles to a maximum value of approximately 0.034 m/s^2 at the equator. The physical phenomena, which we usually observe describe motion of a body on or near the earth surface such as motion of transport vehicles, short-range missiles, an oscillating pendulum etc. In these phenomena, the acceleration due to rotation of the earth may be neglected and a frame of reference attached at any point on the earth surface may be considered as a preferred inertial frame of reference.

If the moving body is at considerable distance from the earth as in the case of satellites, long-range missiles etc., the effect of rotation of the earth become significant. For these situations or like ones we can attach the frame of reference at the earth's center and consider it as an inertial frame of reference. In astronomical field and space exploration programs, we require very high accuracy. Therefore, a frame of reference attached to distant stars is used as an inertial frame of reference. These stars are situated at such a vast distance from the earth that they appear as a motionless point source of light thus closely approach to the condition of absolute rest.

Illustrations

Illustration 29. A pendulum of mass m is suspended from the ceiling of a train moving with an acceleration 'a' as shown in figure. Find the angle θ in equilibrium position.

Solution Non–inertial frame of reference (Train)

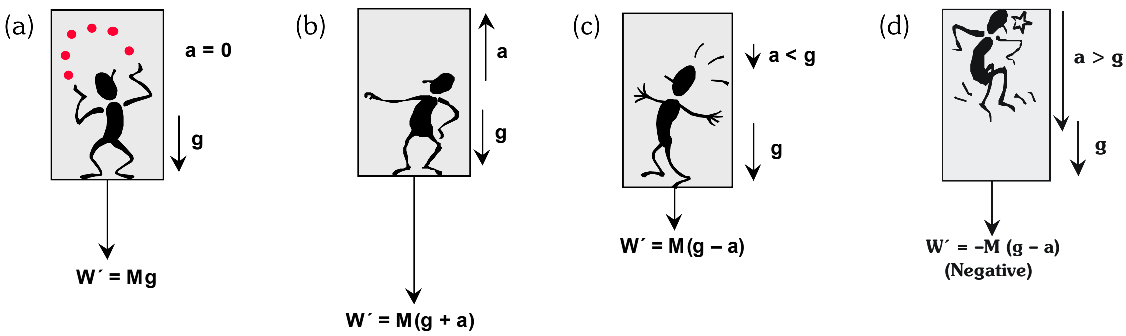


F.B.D. of bob w.r.t. train. (physical forces + pseudo force) : with respect to train, bob is in equilibrium

$\therefore\ \Sigma F_y = 0 \Rightarrow T\cos\theta = mg$ and $\Sigma F_x = 0$

$\Rightarrow T\sin\theta = ma \Rightarrow \tan\theta = \dfrac{a}{g} \Rightarrow \theta = \tan^{-1}\left(\dfrac{a}{g}\right)$

Illustration 30. The weight of a body is simply the force exerted by earth on the body. If body is on an accelerated platform, the body experiences fictitious force, so the weight of the body appears changed and this new weight is called apparent weight. Let a man of weight W = Mg be standing in a lift.
We consider the following cases :



Case :

(a) If the lift moving with constant velocity v upwards or downwards. In this case there is no accelerated motion hence no pseudo force experienced by observer inside the lift. So apparent weight $W' = Mg$ Actual weight.

(b) If the lift is accelerated upward with constant acceleration a. Then net forces acting on the man are (i) weight $W = Mg$ downward (ii) fictitious force $F_0=Ma$ downward. So apparent weight $W' = W + F_0 = Mg + Ma = M(g + a)$

(c) If the lift is accelerated downward with acceleration $a < g$. Then fictitious force $F_0 = Ma$ acts upward while weight of man $W = Mg$ always acts downward. So apparent weight $W'=W+F_0 = Mg - Ma = M(g-a)$

Special Case :

If $a = g$ then $W' = 0$ (condition of weightlessness).Thus, in a freely falling lift the man will experience weightlessness.

(d) If lift accelerates downward with acceleration $a > g$. Then as in Case c . Apparent weight $W' =M(g-a)$ is negative, i.e., the man will be accelerated upward and will stay at the ceiling of the lift.

Illustration 31. A spring weighing machine inside a stationary lift reads 50 kg when a man stands on it. What would happen to the scale reading if the lift is moving upward with (i) constant velocity, and (ii) constant acceleration?

Solution (i) In the case of constant velocity of lift, there is no fictitious force; therefore the apparent weight = actual weight. Hence the reading of machine is 50 kgwt.

(ii) In this case the acceleration is upward, the fictitious force ma acts downward, therefore apparent weight is more than actual weight i.e. $W' = m (g + a)$.

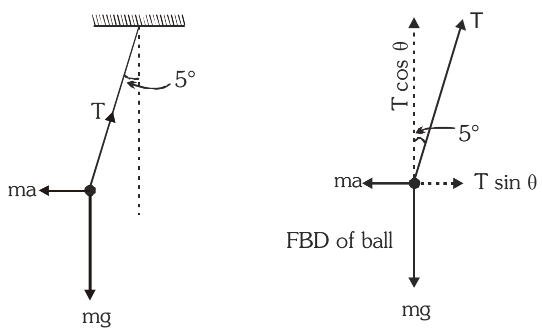
Hence scale shows a reading $= m (g + a) = \dfrac{mg\left(1+\frac{a}{g}\right)}{g} = \left(50+\dfrac{50a}{g}\right)$ kg wt.

Illustration 32. Two objects of equal mass rest on the opposite pans of an arm balance. Does the scale remain balanced when it is accelerated up or down in a lift?

Solution Yes, since both masses experience equal fictitious forces in magnitude as well as direction.

Illustration 33. A passenger on a large ship sailing in a quiet sea hangs a ball from the ceiling of her cabin by means of a long thread. Whenever the ship accelerates, she notes that the pendulum ball lags behind the point of suspension and so the pendulum no longer hangs vertically. How large is the ship's acceleration when the pendulum stands at an angle of 5° to the vertical?

Solution The ball is accelerated by the force T sin5°.



Therefore $T \sin 5° = ma$

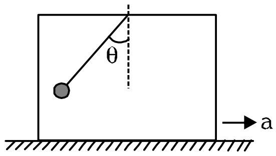
Vertical component $\Sigma F = 0$, so $T \cos 5° = mg$

By solving $a = g \tan 5° = 0.0875\ g = 0.86\ m/s^2$

BEGINNER'S BOX-7

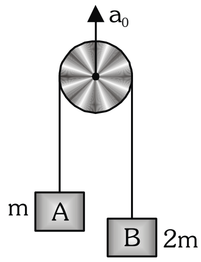
Newton's Laws from Non-Inertial Frame

1. In the given figure bob is at rest with respect to box, find acceleration of box a.



(A) g tanθ (B) g cotθ (C) g sinθ (D) g cosθ

2. In the given figure pulley is moving with an accelation a_0 in upward direction, acceleration of block A with respect to pulley is



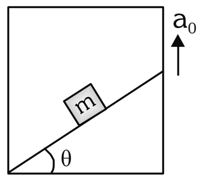
(A) $\frac{1}{3}(g+a_0)\uparrow$ (B) $\frac{1}{3}(g-a_0)\uparrow$

(C) $\frac{g+a_0}{2}\uparrow$ (D) $\left(\frac{g-a_0}{2}\right)\uparrow$

3. In the above question, thension in string is

(A) $\frac{4}{3}m(g+a_0)$ (B) $\frac{4}{3}m(g-a_0)$ (C) $\frac{4}{3}mg$ (D) $\frac{2mg}{3}$

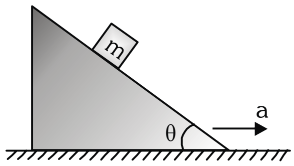
4. In the given figure box is moving vertically upward with acceleration a_0. (all surfaces are smooth), normal force between block and inclined is



(A) mg cos θ (B) $m(g + a_0)\cos\theta$

(C) $m(g - a_0)\cos\theta$ (D) $\frac{m}{2}(g+a_0)\sin\theta$

5. In the given figure if acceleration of block with respect to wedge is zero. Then find the value of 'a'

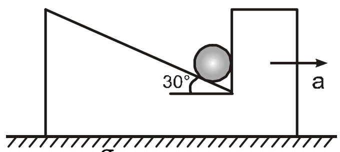


(A) g cotθ (B) g sinθ
(C) g tanθ (D) g cosθ

6. In above question find normal force between block and wedge.

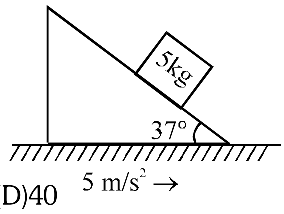
(A) mg cosθ (B) mg sec θ (C) mg tan θ (D) mg cosec θ

7. A heavy spherical ball is constrained in a frame as in figure. The inclined surface is smooth. The maximum acceleration with which the frame can move without causing the ball to leave the frame:



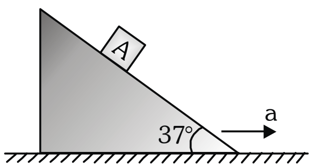
(A) $\frac{g}{2}$ (B) $g\sqrt{3}$ (C) $\frac{g}{\sqrt{3}}$ (D) $\frac{g}{\sqrt{2}}$

8. Inclined plane is moved towards right with an acceleration of 5 ms^{-2} as shown in figure. Find force in newton which block of mass 5 kg exerts on the incline plane. (All surfaces are smooth)



(A) 50 (B) 55 (C) 60 (D)40

9. In the given figure all surface are smooth. Find acceleration of block A with respect to wedge if acceleration of wedge w.r. to ground is



(i) a = 5 m/s^2

(ii) a = 10 m/s^2

(iii) a = 7.5 m/s^2

GOLDEN KEY POINTS

- The common problem which students often face is the resolution of forces. following two rules can be made in this regard.

  Rule 1 : If the body is in equilibrium, you can resolve the forces in any direction. Net force should be zero in all directions. A body moving with constant velocity is also in equilibrium.

  Rule 2 : If the body is accelerated, resolve the forces alongs acceleration and perpenndicular to it. Net force along acceleration = m $\vec{a}$ and net force perpendicular to acceleration is zero.

- Mathematically a body is said to be in equilibrium if

  (a) Net force acting on it is zero, i.e., $\vec{F}_{net} = 0$.

  (b) Net moments of all the forces acting on it about any axis is zero. Physically the body at rest is said to be in equilibrium, if it is permanently at rest (unless some other force is applied on it, which may disturb its equilibrium). If a body is at rest just for a moment, it does not mean it is in equilibrium.

  For example, when a ball is thrown upwards, at highest point of its journey it momentarily comes at rest, but there it is not in equilibrium. A net force (equal to its weight) is acting downward. Due to that force it moves downwards.

  If a problem is asked on equilibrium, check whether the body is in equilibrium (permanent rest) or it is at rest just for a moment.

  Now, if the body is in equilibrium, you may resolve the forces in any direction (x, y, z whatsoever). Net force on the body should be zero in all directions.

  But, if the body is ***momentarily at rest but not in equilibrium***; you will have to be very careful. First see in what direction will the body move just after few second. Obviously the net force on the body should point in that particular direction. Therefore components of all the forces in a direction perpendicular to the net force or perpendicular to the direction in which motion is likely to occur affter few second should be zero.

- The second law of motion given above is strictly applicable to a single point mass. This force ***F*** in the law stand for the net external force on the particle and ***a*** stands for the acceleration of the particle. Any internal forces in the system are not to be included in ***F***.

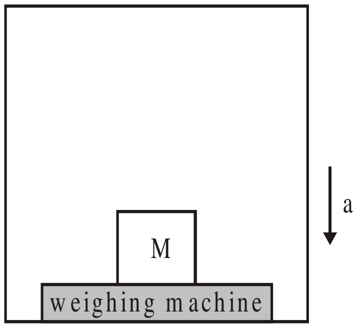
- The Second Law of motion is a local relation. What this means is that the force ***F*** at a point in space (location of the particle) at a certain instant of times is related to a at the same point at the same instant. That is acceleration here and now is determined by the force here and now not by any history of the motion of the particle.

SOME WORKED OUT ILLUSTRATIONS

Illustration 1.

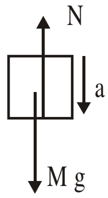
With what acceleration 'a' shown the elevator descends so that the block of mass M exerts a force of $\frac{Mg}{10}$ on the weighing machine? [g= acceleration due to gravity]

(A) 0.3 g
(B) 0.1 g
(C) 0.9 g
(D) 0.6 g



Ans. (C)
Solution

FBD of block :



$Mg - N = Ma$; Now according to question $N = \frac{Mg}{10}$ so $a = \frac{Mg - \frac{Mg}{10}}{M} = 0.9\,g$

Illustration 2.

An astronaut accidentally gets separated out of his small spaceship accelerating in inter-stellar space at a constant acceleration of 10 m/s^2. What is the acceleration of the astronaut at the instant he is outside the spaceship?

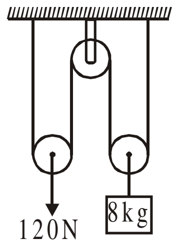
(A) 10 m/s^2 (B) 9.8 m/s^2 (C) $\approx$ 0 m/s^2 (D) could be anything

Ans. (C)
Solution

When the astronaut is outside the spaceship, the net external force (except negligible gravitational force due to spaceship) is zero as he is isolated from all interactions.

Illustration 3.

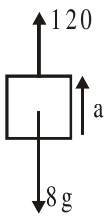
If the string is pulled down with a force of 120 N as shown in the figure, then the acceleration of 8 kg block would be



(A) 10 m/s^2 (B) 5 m/s^2
(C) 0 m/s^2 (D) 4 m/s^2

Ans. (B)
Solution

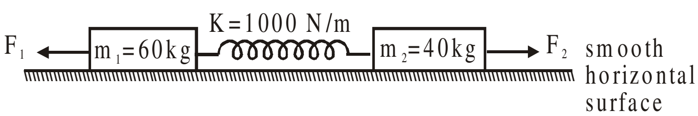
FBD of 8 kg block



$$a = \frac{120 - 80}{8} = 5\ \text{m/s}^2$$

Illustration 4.

In the shown situation, which of the following is/are possible? Assume length of spring remains constant with time.



(A) Spring force = 52 N, if F_1 = 40 N and F_2 = 60 N
(B) Spring force = 52 N, if F_1 = 60 N and F_2 = 40 N
(C) Spring force = 0, if $F_1 = F_2$ = 100 N
(D) Spring force $\neq$ 0, if F_1 = 0.2 N and F_2 = 0.3 N

Ans. (A,B,D)

Solution

If $F_1 \neq F_2$, then system will move with acceleration so spring force $\neq 0$

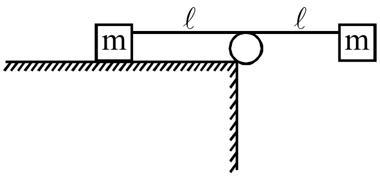
If $F_1 = 40$ N & $F_2 = 60$ N then $a = \dfrac{F_2 - F_1}{m_1 + m_2} = \dfrac{20}{100} = \dfrac{1}{5}$ m/s^2

and spring force $= F_1 + m_1 a = 40 + \dfrac{1}{5}(60) = 52$ N

If $F_1 = 60$ N & $F_2 = 40$ N then spring force $= 52$ N

Illustration 5.

As shown in figure, the left block rests on a table at distance ℓ from the edge while the right block is kept at the same level so that thread is unstretched and does not sag and then released. What will happen first ?



(A) Left block reach the edge of the table
(B) Right block hit the table
(C) Both (A) & (B) happens at the same time
(D) Can't say anything

Ans. (A)

Solution

Net force in horizontal direction is more for left block so it will reach the edge of the table first.

Illustration 6.

The force exerted by the floor of an elevator on the foot of a person standing there is less than the weight of the person if the elevator is

(A) going up and slowing down
(B) going up and speeding up
(C) going down and slowing down
(D) going down and speeding up

Ans. (A,D)

Solution

If $N < mg$ then $N = m(g-a) \Rightarrow$ elevator is going down with acceleration or elevator is going up with retardation.

Illustration 7.

If a body is placed on a rough inclined plane, the nature of forces acting on the body is(are)

(A) gravitational (B) electromagnetic (C) nuclear (D) weak

Ans. (A,B)

Solution

When a body is placed on a rough inclined plane, it is acted upon by a reactional force due to plane [electromagnetic in nature], a frictional force due to roughness of plane [electromagnetic in nature] and a gravitational force (due to its weight).

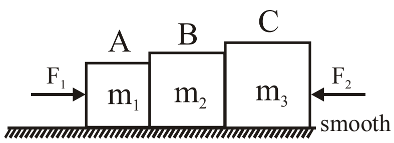
Illustration 8.

For shown situation let

N_1 = Normal reaction between A & B

N_2 = Normal reaction between B & C

Which of the following statement (s) is/are correct ?



(A) If $F_1 > F_2$ then $N_1 \neq N_2$ and $F_2 < \sqrt{N_1 N_2} < F_1$
(B) If $F_1 < F_2$ then $N_2 > N_1$
(C) If $F_1 = F_2$ then $N_1 = N_2$
(D) If $F_1 = F_2$ then $N_1 \neq N_2$

Ans. (A, B, C)

Solution

If $F_1 > F_2$, the system moves towards right so $N_1 < F_1$, $N_2 < N_1$ & $F_2 < N_2$

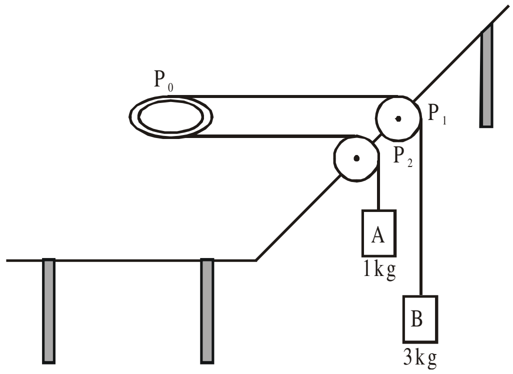
$$F_2 < N_1 \text{ or } N_2 < F_1$$

If $F_1 < F_2$, the system moves towards left so $N_1 < N_2$

If $F_1 = F_2$, the system does not move.

Illustration 9 to 11.

A smooth pulley P_0 of mass 2 kg is lying on a smooth table. A light string passes round the pulley and has masses 1 kg and 3kg attached to its ends. The two portions of the string being perpendicular to the edge of the table so that the masses hang vertically. Pulleys P_1 and P_2 are of negligible mass. [$g=10m/s^2$]



9. Tension in string is

(A) 12 N (B) 6 N (C) 24 N (D) 18 N

10. Acceleration of pulley P_0 is

(A) 2 m/s^2 (B) 4 m/s^2 (C) 3 m/s^2 (D) 6 m/s^2

11. Acceleration of block A is

(A) 6 m/s^2 (B) 4 m/s^2 (C) 3 m/s^2 (D) 8 m/s^2

Solution

9. ***Ans. (B)***

Let acceleration of P_0 = a_0; acceleration of A = a_1; acceleration of B = a_2

By constraint relations $a_0 = \dfrac{a_1 + a_2}{2}$ ...(i)

Now for pulley P_0 : $2T = 2a_0 \Rightarrow T = a_0$ ...(ii)

For block A : $1\,g - T = 1(a_1) \Rightarrow 10 - T = a_1$ ...(iii)

For block B : $3g - T = 3\,(a_2) \Rightarrow 30 - T = 3a_2$ ...(iv)

By putting the values of a_0, a_1 & a_2 in equation (i)

$$T = \frac{(10-T)+\left(10-\frac{T}{3}\right)}{2} \Rightarrow T = 6N$$

10. ***Ans. (D)***

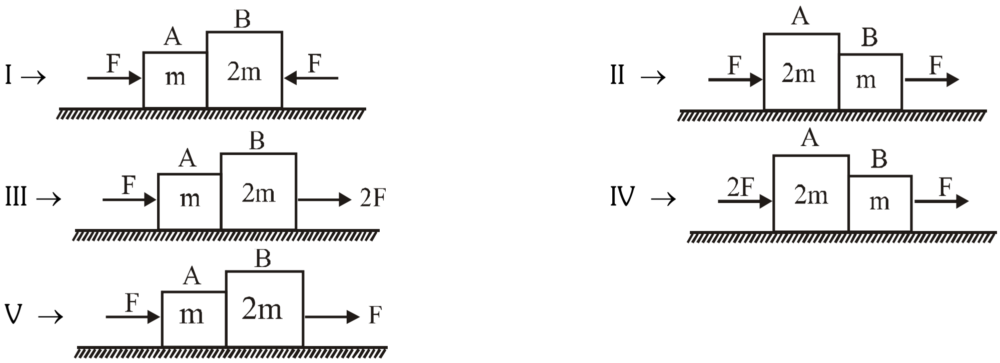
Acceleration of pulley; $a_0 = T = 6\ m/s^2$

11. ***Ans. (B)***

Acceleration of block A; $a_1 = 10 - T = 10 - 6 = 4\ m/s^2$

Illustration 12.

Five situations are given in the figure (All surfaces are smooth)



Column–I	**Column–II**
(A) Accelerations of A & B are same	(p) I
(B) Accelerations of A & B are different	(q) II
(C) Normal reaction between A & B is zero	(r) III
(D) Normal reaction between A & B is non zero	(s) IV
	(t) V

Ans. (A) p,r,s,t; (B) q; (C) q,r,s; (D) p,t

Solution

$\mathrm{I}: a_A = a_B = 0\ \&\ N \neq 0$

$\mathrm{II}: a_A = \dfrac{F}{2m},\ a_B = \dfrac{F}{m}\ \&\ N = 0$

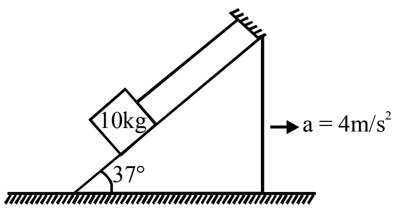
$\mathrm{III}: a_A = \dfrac{F}{m},\ a_B = \dfrac{2F}{2m} = \dfrac{F}{m}\ \&\ N = 0$

$\mathrm{IV}: a_A = \dfrac{2F}{2m} = \dfrac{F}{m},\ a_B = \dfrac{F}{m}\ \&\ N = 0$

$\mathrm{V}: a_A = a_B = \dfrac{2F}{3m}\ \&\ N \neq 0$

Illustration 13 to 15.

A body of mass 10 kg is placed on a smooth inclined plane as shown in figure. The inclined plane is moved with a horizontal acceleration a.



13. The normal reaction between block and inclined plane is :-

(A) 92 N (B) 44 N (C) 56 N (D) Can't be determined

14. The tension in thread is :-

(A) 92 N (B) 44 N (C) 56 N (D) Can't be determined

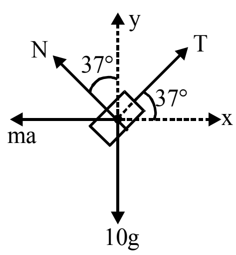
15. At what acceleration 'a' will the body lose contact with the inclined plane ?

(A) 10 m/s^2 (B) 13.33 m/s^2 (C) 3.33 m/s^2 (D) 6.66 m/s^2

Solution

13. ***Ans. (C)***

FBD of block w.r.t. inclined plane



$\sum F_x = 0 \qquad \Rightarrow T\cos 37° - N\sin 37° - ma = 0$

$\Rightarrow 4T - 3N = 200$ ....(i)

$\sum F_y = 0 \qquad \Rightarrow N\cos 37° + T\sin 37° - 10g = 0$

$\Rightarrow 4N + 3T = 500$ ....(ii)

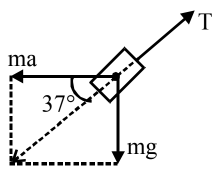
By solving equation (i) & (ii) , N = 56 newton

14. ***Ans. (A)***

From above equation, T = 92 newton

15. ***Ans. (B)***

To lose contact, N = 0



$\tan 37° = \dfrac{mg}{ma} \qquad \Rightarrow \dfrac{3}{4} = \dfrac{10}{9} \quad \Rightarrow a = \dfrac{40}{3} = 13.33 m/s^2$

ANSWERS

BEGINNER'S BOX-1

1. (A) **2.** (B) **3.** (D) **4.** (A) **5.** (D) **6.** (B) **7.** (C) **8.** (A)
9. (AC)

BEGINNER'S BOX-2

1. (C) **2.** (A) **3.** (A) **4.** (A) **5.** (B) **6.** (D) **7.** (A) **8.** T = 100 N
9. $100\sqrt{2}$ N

BEGINNER'S BOX-3

1. (C) **2.** (D) **3.** (D) **4.** (D) **5.** (C) **6.** m = 3 kg, $30\sqrt{3}$ N.

7. (a) 120 N. (b) 4 N (c) 48 N **8.** (i) $\frac{\sqrt{3}g}{2}$ (ii) $\sqrt{\frac{3}{2}}g$

9. (a) 200 N, (b) 14 m/s^2, (c) $200\sqrt{2}$ N

BEGINNER'S BOX-4

1. (B) **2.** (C) **3.** (C) **4.** (D) **5.** (A) **6.** (A) (C) **7.** (B) **8.** (A)
9. (B) **10.** (B) **11.** Acceleration of A = 2g, Acceleration of B = 0

BEGINNER'S BOX-5

1. (D) **2.** (B) **3.** (A) **4.** (A) **5.** (B) **6.** (C) **7.** (C) **8.** (D)
9. (C) **10.** g/3 up the plane

BEGINNER'S BOX-6

1. (A) **2.** (D) **3.** (A) **4.** (A) **5.** (A) **6.** 2 m/s

BEGINNER'S BOX-7

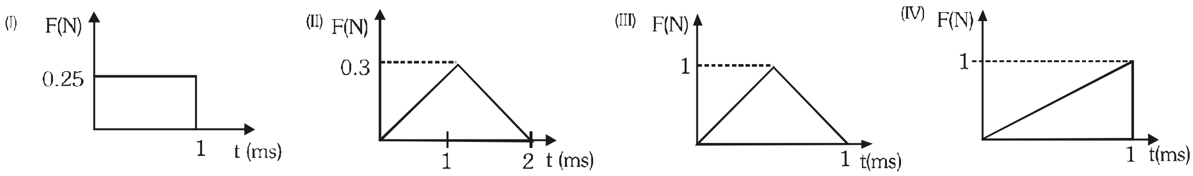
1. (A) **2.** (A) **3.** (A) **4.** (B) **5.** (C) **6.** (B) **7.** (C) **8.** (B)
9. (i) 2 m/s^2 down along the inclined plane; (ii) 2 m/s^2 up along the inclined palne; (iii) 0

EXERCISE – 1 MCQ (SINGLE CHOICE CORRECT)

1. A body of mass m_1 exerts a force on another body of mass m_2. If the magnitude of acceleration of m_2 is a_2, then the magnitude of the acceleration of m_1 is (considering only two bodies in space)

(A) Zero (B) $\frac{m_2a_2}{m_1}$ (C) $\frac{m_1a_2}{m_2}$ (D) a_2

2. Figures I, II, III and IV depicts variation of force with time



In which situation impulse will be maximum

(A) I & II (B) III & I (C) III & IV (D) Only IV

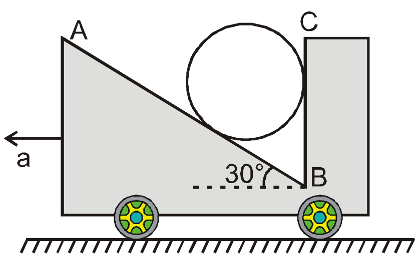
3. A body kept on a smooth inclined plane inclination 1 in x ($\sin\theta = 1/x$) will remain stationary relative to the inclined plane if the plane is given a horizontal acceleration equal to :–

(A) $\sqrt{x^2-1}g$ (B) $\frac{\sqrt{x^2-1}}{x}g$ (C) $\frac{gx}{\sqrt{x^2-1}}$ (D) $\frac{g}{\sqrt{x^2-1}}$

4. A pulley is attached to the ceiling of a lift moving upwards. Two particles are attached to the two ends of a massless string passing over the smooth pulley. The masses of the particles are in the ratio 2 : 1. If the acceleration of the particles is g/2 w.r.t. lift, then the acceleration of the lift will be

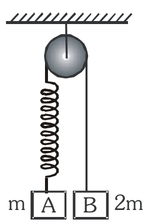
(A) g (B) $\frac{g}{2}$ (C) $\frac{g}{3}$ (D) $\frac{g}{4}$

5. A cylinder rests in a supporting carriage as shown. The side AB of carriage makes an angle 30° with the horizontal and side BC is vertical. The carriage lies on a fixed horizontal surface and is being pulled towards left with an horizontal acceleration 'a'. The magnitude of normal reactions exerted by sides AB and BC of carriage on the cylinder be N_{AB} and N_{BC} respectively. Neglect friction everywhere. Then as the magnitude of acceleration 'a' of the carriage is increased, pick up the correct statement:



(A) N_{AB} increases and N_{BC} decreases. (B) Both N_{AB} and N_{BC} increase.
(C) N_{AB} remains constant and N_{BC} increases. (D) N_{AB} increases and N_{BC} remains constant.

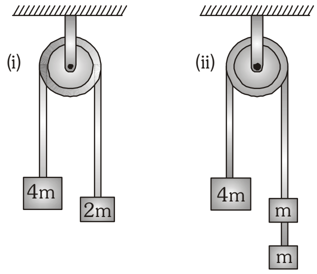
6. Two blocks A and B of masses m & 2m respectively are held at rest such that the spring is in natural length. What is the acceleration of both the blocks just after release?



(A) g ↓, g ↓ (B) $\frac{g}{3}\downarrow, \frac{g}{3}\uparrow$

(C) 0, 0 (D) g ↓ , 0

7. In the arrangement shown in figure, pulley is smooth and massless and all the strings are light. Let F_1 be the force exerted on the pulley by the string in case (i) and F_2 the force in case (ii). Then



(A) $F_1 > F_2$

(B) $F_1 < F_2$

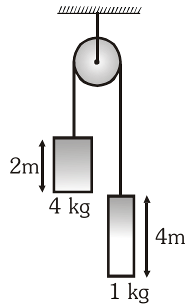
(C) $F_1 = F_2$

(D) $F_1 = 2F_2$

8. In figure shown, both blocks are released from rest. The time to cross each other is

(A) 2 second (B) 3 second

(C) 1 second (D) 4 second



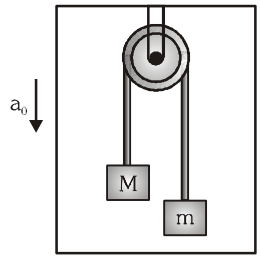
9. If the acceleration of the elevator $a_0 > g$, then

(A) the acceleration of the masses will be a_0

(B) the acceleration of the masses will be $(a_0 - g)$

(C) the tension in the string will be $\frac{mM}{M+m}(g - a_0)$

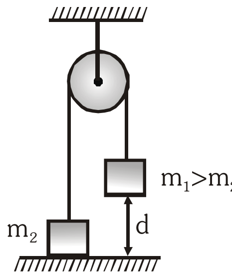
(D) tension in the string will be zero.



10. If masses are released from the position shown in figure then time elapsed before mass m_1 collides with the floor will be :

(A) $\sqrt{\frac{2m_1gd}{m_1+m_2}}$ (B) $\sqrt{\frac{2(m_1+m_2)d}{(m_1-m_2)g}}$

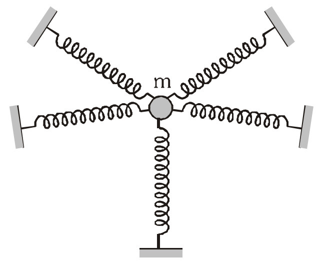
(C) $\sqrt{\frac{2(m_1-m_2)d}{(m_1+m_2)g}}$ (D) None of these



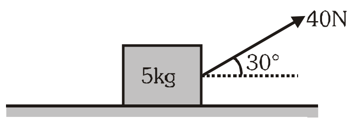
11. A sphere of mass m is kept in equilibrium with the help of several springs as shown in the figure. Measurement shows that one of the springs applies a force $\vec{F}$ on the sphere. With what acceleration the sphere will move immediately after this particular spring is cut?

(A) zero (B) $\vec{F}/m$

(C) $-\vec{F}/m$ (D) insufficient information



12. The adjoining figure shows a force of 40 N pulling a body of mass 5 kg in a direction 30° above the horizontal. The body is in rest on a smooth horizontal surface. Assuming acceleration of free-fall is 10 m/s^2. Which of the following statements I and II is/are correct?



I. The weight of the 5 kg mass acts vertically downwards

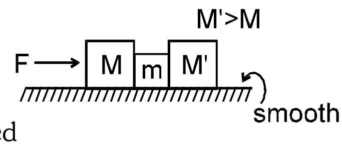
II. The net vertical force acting on the body is 30 N.

(A) Only I. (B) Only II. (C) Both I and II. (D) None of them

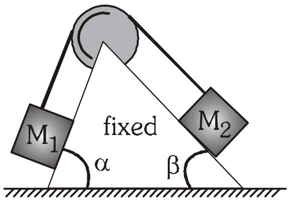
13. A constant force F is applied in horizontal direction as shown. Contact force between M and m is N and between m and M' is N' then

(A) N or N' equal (B) N > N'

(C) N' > N (D) cannot be determined



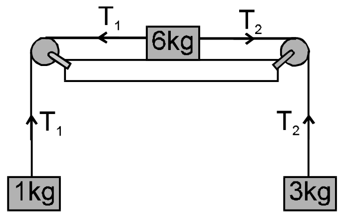
14. Two masses M_1 and M_2 are attached to the ends of a string which passes over a pulley attached to the top of a double inclined plane of angles of inclination α and β. If $M_2 > M_1$, the acceleration a of the system can be given by (neglect friction) :



(A) $\frac{M_2 g(\sin\beta)}{M_1+M_2}$ (B) $\frac{M_1 g(\sin\alpha)}{M_1+M_2}$

(C) $\left(\frac{M_2\sin\beta - M_1\sin\alpha}{M_1+M_2}\right)g$ (D) zero

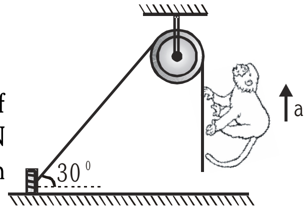
15. Three masses of 1 kg , 6 kg and 3 kg are connected to each other with threads and are placed on table as shown in figure, What is the acceleration with which the system is moving ? Take $g = 10 m\ s^{-2}$.



(A) Zero (B) $1\ m\ s^{-2}$

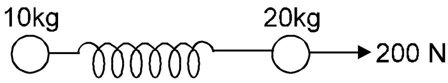
(C) $2\ m\ s^{-2}$ (D) $3\ m\ s^{-2}$

16. A light string fixed at one end to a clamp on ground passes over a fixed pulley and hangs at the other side. It makes an angle of 30° with the ground. A monkey of mass 5 kg climbs up the rope. The clamp can tolerate a vertical force of 40 N only. The maximum acceleration in upward direction with which the monkey can climb safely is (neglect friction and take $g = 10\ m/s^2$) :



(A) $2\ m/s^2$ (B) $4\ m/s^2$ (C) $6\ m/s^2$ (D) $8\ m/s^2$

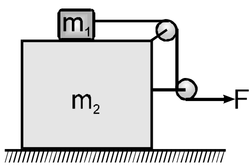
17. Two masses of 10 kg and 20 kg respectively are connected by a massless spring as shown in figure. A force of 200 N acts on the 20 kg mass at the instant when the 10 kg mass has an acceleration of $12\ ms^{-2}$ towards right, the acceleration of the 20 kg mass is :



(A) 2 ms–2 (B) $4ms^{-2}$

(C) $10ms^{-2}$ (D) $20ms^{-2}$

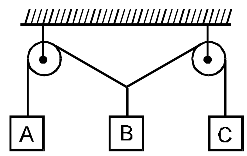
18. In the arrangement shown in the figure all surfaces are frictionless, the masses of the block are $m_1 = 20$ kg and $m_2 = 30$ kg. The accelerations of masses m_1 and m_2 will be if F = 180 N.



(A) $a_{m_1} = 9m/s^2, a_{m_2} = 0$ (B) $a_{m_1} = 9m/s^2, a_{m_2} = 9\ m/s^2$

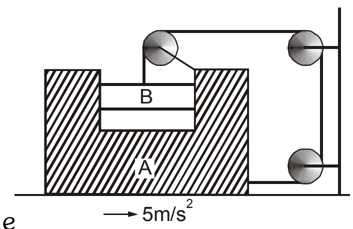
(C) $a_{m_1} = 0, a_{m_2} = 9m/s^2$ (D) None of these

19. Three blocks A , B and C are suspended as shown in the figure. Mass of each blocks A and C is m. If system is in equilibrium and mass of B is M, then :



(A) M = 2 m (B) M < 2 m
(C) M > 2 m (D) M = m

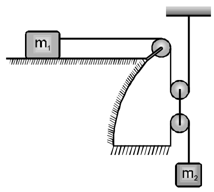
20. If block A is moving with an acceleration of $5\ m/s^2$, the acceleration of B w.r.t. ground is :



(A) $5\ m/s^2$ (B) $5\sqrt{2}\ m/s^2$

(C) $5\sqrt{5}\ m/s^2$ (D) $10\ m/s^2$

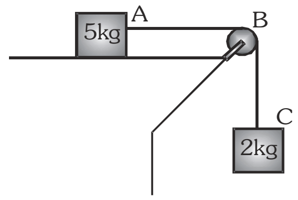
21. Two blocks of masses m_1 and m_2 are connected as shown in the figure. The acceleration of the block m_2 is:



(A) $\frac{m_2 g}{m_1+m_2}$ (B) $\frac{m_1 g}{m_1+m_2}$

(C) $\frac{4m_2 g - m_1 g}{m_1+m_2}$ (D) $\frac{m_2 g}{m_1+4m_2}$

22. A trolley of mass 5 kg on a horizontal smooth surface is pulled by a load of mass 2 kg by means of uniform rope ABC of length 2 m and mass 1 kg. As the load falls from BC=0 to BC = 2m. its acceleration in m/s^2 changes



(A) $\frac{20}{6}$ to $\frac{20}{5}$ (B) $\frac{20}{8}$ to $\frac{30}{8}$

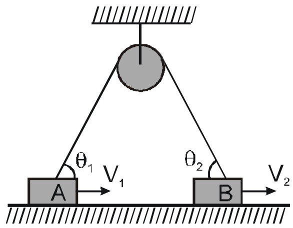
(C) $\frac{20}{5}$ to $\frac{30}{6}$ (D) None of the above

23. Figure shows a 5 kg ladder hanging from a string that is connected with a ceiling and is having a spring balance connected in between. A boy of mass 25 kg is climbing up the ladder at acceleration 1 m/s^2. Assuming the spring balance and the string to be massless and the spring to show a constant reading, the reading of the spring balance is : (Take g = 10 m/s^2)

(A) 30 kg (B) 32.5 kg

(C) 35 kg (D) 37.5 kg

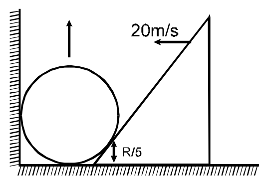
24. In the figure shown, blocks A and B move with velocities v_1 and v_2 along horizontal direction. Find the ratio of $\frac{v_1}{v_2}$.



(A) $\frac{\cos\theta_2}{\cos\theta_1}$ (B) $\frac{\cos\theta_1}{\cos\theta_2}$

(C) $\frac{\sin\theta_2}{\sin\theta_1}$ (D) $\frac{\tan\theta_2}{\tan\theta_1}$

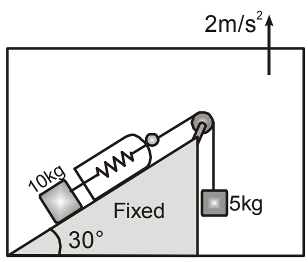
25. A sphere of radius R is in contact with a wedge. The point of contact is R/5 from the ground as shown in the figure. Wedge is moving with velocity 20 m/s, then the velocity of the sphere at this instant will be:



(A) 20 m/s (B) 15 m/s

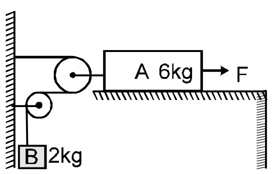
(C) 5 m/s (D) 10 m/s

26. In the given figure find the reading of the spring balance (Assuming that the extension of the spring is not changing) and the inclined plane is smooth. Also assume that the pulley, string, spring and the box of the spring are light. (g = $10m/s^2$)



(A) 2 kgf (B) 4 kgf (C) 6 kgf (D) 8 kgf

27. The system starts from rest and A attains a velocity of 5 m/s after it has moved 5 m towards right. Assuming the arrangement to be frictionless every where and pulley & strings to be light. Find the value of the constant force F applied on A.

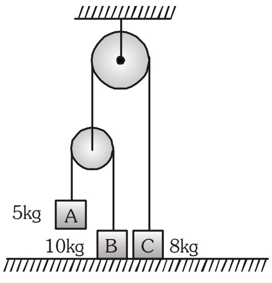


(A) 50 N (B) 75 N

(C) 100 N (D) 150 N

EXERCISE – 2 MCQ (ONE OR MORE CHOICE CORRECT)

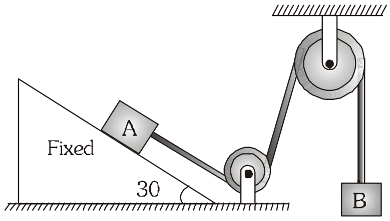
1*. In the following arrangement the system is initially at rest. The 5 kg block is now released. Assuming the pulleys and string to be massless and smooth, the acceleration of blocks is



(A) $a_A = \frac{g}{7}$ (B) $a_B = 0\ m/s^2$

(C) $a_c = \frac{g}{14}$ (D) $2a_C = a_A$

2. Two blocks A and B of equal mass m are connected through a massless string and arranged as shown in figure. Friction is absent everywhere. When the system is released from rest, then



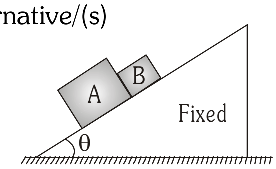
(A) tension in string is $\frac{mg}{2}$

(B) tension in string is $\frac{mg}{4}$

(C) acceleration of A is $\frac{g}{2}$

(D) acceleration of A is $\frac{3}{4}g$

3. In the arrangement shown in figure all surfaces are smooth. Select the correct alternative/(s)



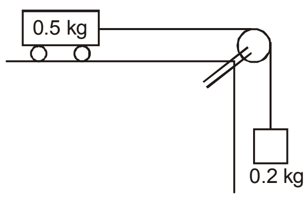
(A) for any value of θ acceleration of A and B are equal

(B) contact force between the two blocks is zero only if $\frac{m_A}{m_B} = \tan\theta$

(C) contact force between the two blocks is zero for any value of m_A or m_B

(D) normal reactions exerted by the wedge on the blocks are equal

4*. A cart of mass 0.5 kg is placed on a smooth surface and is connected by a string to a block of mass 0.2 kg. At the initial moment the cart moves to the left along a horizontal plane at a speed of 7 m/s. (Use $g = 9.8\ m/s^2$)



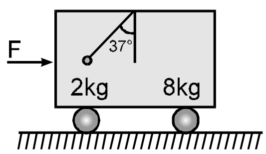
(A) The acceleration of the cart is $\frac{2g}{7}$ towards right.

(B) The cart comes to momentary rest after 2.5 s.

(C) The distance travelled by the cart in the first 5s is 17.5 m.

(D) The velocity of the cart after 5s will be same as initial velocity.

5. A trolley of mass 8 kg is standing on a frictionless surface inside which an object of mass 2 kg is suspended. A constant force F starts acting on the trolley as a result of which the string stood at an angle of 37^0 from the vertical. Then :



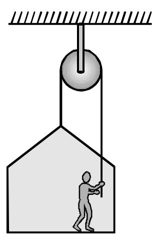
(A) acceleration of the trolley is $40/3\ m/sec^2$

(B) force applied in 60 N

(C) force applied is 75 N

(D) tension in the string is 25 N

6. A painter is applying force himself to raise him and the box with an acceleration of $5\ m/s^2$ by a massless rope and pulley arrangement as shown in figure. Mass of painter is 100 kg and that of box is 50 kg. If $g = 10\ m/s^2$, then:



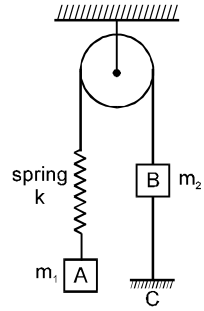
(A) tension in the rope is 1125 N

(B) tension in the rope is 2250 N

(C) force of contact between the painter and the floor is 375 N

(D) force of contact between the painter and the floor is 750 N

7. In the system shown in the figure $m_1 > m_2$. System is held at rest by thread BC. Just after the thread BC is burnt (neglect friction) :



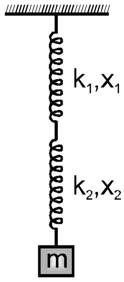
(A) acceleration of m_2 will be upwards

(B) magnitude of acceleration of both blocks will be equal to $\left(\frac{m_1 - m_2}{m_1 + m_2}\right)g$

(C) acceleration of m_1 will be equal to zero

(D) magnitude of acceleration of two blocks will be non–zero and unequal.

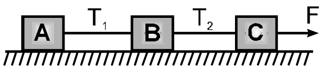
8. Two springs are in a series combination and are attached to a block of mass 'm' which is in equilibrium. The spring constants and the extensions in the springs are as shown in the figure. Then the value of force exerted by the spring on the block is :



(A) $\frac{k_1 k_2}{k_1 + k_2}(x_1 + x_2)$ (B) $k_1x_1 + k_2x_2$

(C) k_1x_1 (D) None of these

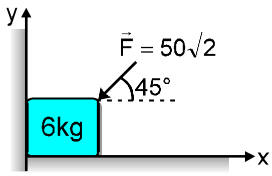
9. Three blocks are connected by strings as shown in figure and pulled by a force F = 60 N. If m_A = 10 kg, m_B = 20 kg and m_C = 30 kg, then :



(A) acceleration of the system is 2 m/s^2 (B) T_1 = 10 N

(C) T_2 = 30 N (D) T_1 = 20 N & T_2 = 40 N

10. A block of 6 kg is put between two smooth walls. If $\vec{F} = 50\sqrt{2}$ is also applied as shown in figure, then



(A) Interaction force on the block due to walls = $50\hat{i} + 110\hat{j}$

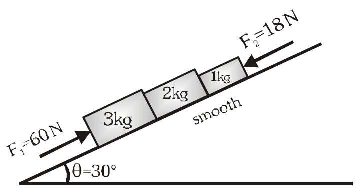
(B) Interaction force on the walls due to the block = $50\hat{i} + 50\hat{j}$

(C) If $\vec{F}$ were reversed, now interactron force on the block due to wall $= -50\hat{i} + 110\hat{j}$

(D) If $\vec{F}$ were reversed, now the acceleration of the block = $\frac{50}{6}\hat{i}$ m/sec^2.

Match the column

11. In the diagram shown in figure (g = 10 m/s^2)



Column I	*Column II*
(A) Acceleration of 2kg block	(p) 8 SI unit
(B) Net force on 3kg block	(q) 25 SI unit
(C) Normal reaction between 2kg and 1kg	(r) 2 SI unit
(D) Normal reaction between 3kg and 2kg	(s) 39 N
	(t) 6 N

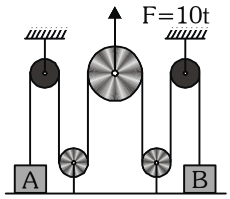
12. Velocity of three particles A, B and C varies with time t as, $\vec{v}_A = (2t\hat{i} + 6\hat{j})$ m/s $\vec{v}_B = (3\hat{i} + 4\hat{j})$ m/s and $\vec{v}_C = (6\hat{i} - 4t\hat{j})$ m/s. Regarding the pseudo force match the following table :

Column I	*Column II*
(A) On A as observed by B	(p) Along positive x–direction
(B) On B as observed by C	(q) Along negative x–direction
(C) On A as observed by C	(r) Along positive y–direction
(D) On C as observed by A	(s) Along negative y–direction
	(t) Zero

Comprehension Based Questions

Comprehension 1

In the arrangement shown in figure $m_A = m$ and $m_B = 2m$, while all the pulleys and string are massless and frictionless. At t = 0, a force F = 10t starts acting over central pulley in vertically upward direction. Find [Take all the units into S.I. system] [m = 1kg]

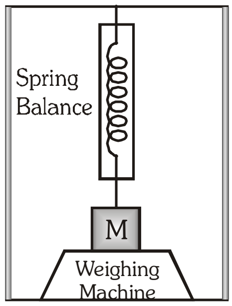


13. Time at which B loose contact with floor is
(A) 8 sec (B) 4 sec (C) 2 sec (D) 1 sec

14. Velocity of A when B loses contact with floor is
(A) 8 m/s (B) 6 m/s
(C) 10 m/s (D) 11 m/s

Comprehension 2

Figure shows a weighing machine kept in a lift. Lift is moving upwards with acceleration of 5 m/s^2. A block is kept on the weighing machine. Upper surface of block is attached with a spring balance. Reading shown by weighing machine and spring balance is 15 kg and 45 kg respectively.
Answer the following questions. Asume that the weighing machine can measure weight by having negligible deformation due to block, while the spring balance requires larger expansion :(take g = 10 m/s^2)



15. Mass of the object in kg is :
(A) 60 kg (B) 40 kg (C) 80 kg (D) 10 kg

16. In the above situation normal acting on the block as seen by an observer in the lift.
(A) 450 N (B) 150 N (C) 400 N (D) zero

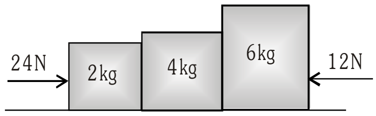
17. If lift is stopped and equilibrium is reached. Reading of weighing machine will be :
(A) 40 kg (B) 10 kg (C) 20 kg (D) zero

18. If lift is stopped and equilibrium is again reached. Reading of spring balance will be :
(A) zero (B) 20 kg (C) 10 kg (D) 40 kg

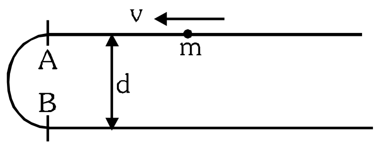
19. Find the acceleration of the lift such that the weighing machine shows its true weight.
(A) $\frac{45}{4}$ m/s^2 (B) $\frac{85}{4}$ m/s^2 (C) $\frac{11}{2}$ m/s^2 (D) 15 m/s^2

EXERCISE – 3 SUBJECTIVE

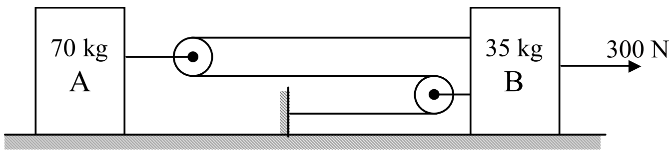
1. If contact force between 2kg and 4kg is f_1 and between 4kg and 6 kg is f_2. Find out f_1 and f_2.



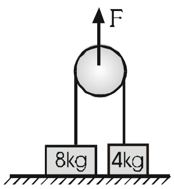
2. Fig. shows a bead of mass m moving with uniform speed v through a U–shaped smooth wire the wire has a semicircular bending between A and B. Calculate The average force exerted by the bead on the part AB of the wire.



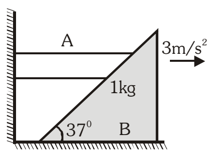
3. Find accelaration of block B. Assume the pulleys and string to be ideal and neglect any friction.



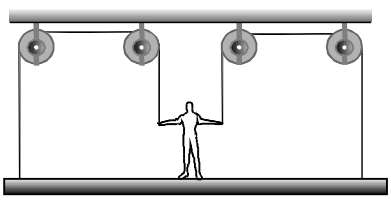
4. Two block of mass 8 kg and 4kg are connected by a string as shown. Calculate their acceleration if they are initially at rest on the floor, when a force of 100N is applied on the pulley in upward direction ($g = 10ms^{-2}$)



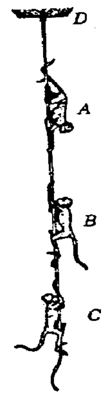
5. Find force in newton which mass A exerts on mass B if B is moving towards right with 3 ms^{-2}. All surfaces are smooth and $g=10m/s^2$.



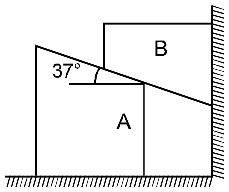
6. A painter of mass M stand on a platform of mass m and pulls himself up by two ropes which hang over pulley as shown. He pulls each rope with the force F and moves upward with uniform acceleration 'a'. Find 'a' (neglecting the fact that no one could do this for long time).



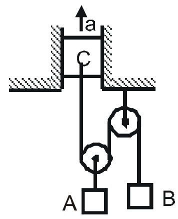
7. Three monkeys A, B and C with masses of 10 , 15 & 8 Kg respectively are climbing up & down the rope suspended from D . At the instant represented , A is descending the rope with an acceleration of 2 m/s^2 & C is pulling himself up with an acceleration of 1.5 m/s^2 . Monkeys B is climbing up with a constant speed of 0.8 m/s . Treat the rope and monkeys as a complete system & calculate the tension T in the rope at D. ($g = 10\ m/s^{-2}$)



8. The masses of blocks A and B are same and equal to m. Friction is absent everywhere. Find the magnitude of normal force with which block B presses on the wall and accelerations of the blocks A and B.



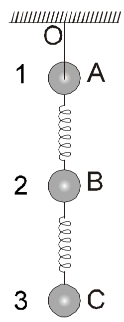
9*. The block C shown in the figure is ascending with an acceleration $a = 3\ m/s^2$ by means of some motor not shown here. Find the acceleration of the bodies A and B of masses 10 kg and 5 kg, respectively, assuming that pulleys are massless and friction is absent everywhere.



10. Three identical balls 1,2,3 are suspended on springs one below the other as shown in the figure. OA is a weightless thread.

(a) If the thread is cut, the system starts falling. Find the acceleration of all the balls at the initial instant

(b) Find the initial accelerations of all the balls if we cut the spring BC which is supporting ball 3 instead of cutting the thread.



EXERCISE – 4 RECAP OF AIEEE/JEE (MAIN)

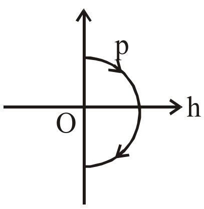
1. A mass of 10 kg is suspended vertically by a rope from the roof. When a horizontal force is applied on the rope at some point, the rope deviated at an angle of 45° at the roof point. If the suspended ass is at equilibrium, the magnitude of the force applied is ($g = 10\ ms^{-2}$) ***[JEE-MAIN-2019]***

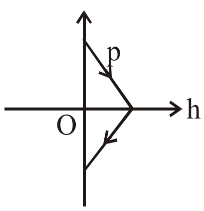
(A) 200 N (B) 100 N (C) 140 N (D) 70 N

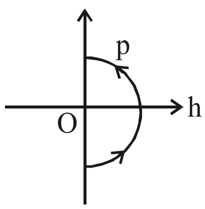
2. A particle of mass m is moving in a straight line with momentum p. Starting at time t = 0, a force F = kt acts in the same direction on the moving particle during time interval T so that its momentum changes from p to 3p. Here k is a constant. The value of T is : ***[JEE-MAIN-2019]***

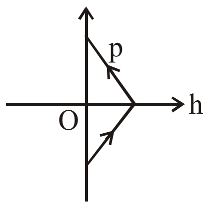
(A) $2\sqrt{\frac{p}{k}}$ (B) $\sqrt{\frac{2p}{k}}$ (C) $\sqrt{\frac{2k}{p}}$ (D) $2\sqrt{\frac{k}{p}}$

3. A ball is thrown vertically up (taken as +z-axis) from the ground. The correct momentum-height (p-h) diagram is : ***[JEE-MAIN-2019]***

(A) 

(B) 

(C) 

(D) 

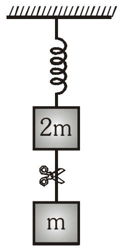
4. A spring whose unstretched length is l has a force constant k. The spring is cut into two pieces of unstretched lengths l_1 and l_2 where, $l_1 = nl_2$ and n is an integer. The ratio k_1/k_2 of the corresponding force constants, k_1 and k_2 will be : ***[JEE-MAIN-2019]***

(A) $\frac{1}{n^2}$ (B) n^2 (C) $\frac{1}{n}$ (D) n

EXERCISE – 5 RECAP OF IIT-JEE/JEE (ADVANCED)

Select the correct alternative (single correct answer)

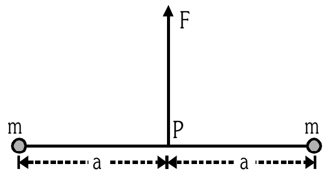
1. System shown in figure is in equilibrium and at rest. The spring and string are massless, now the string is cut. The acceleration of mass 2m and m just after the string is cut will be : ***[IIT-JEE 2006]***



(A) $\frac{g}{2}$ upwards, g downwards (B) g upwards, $\frac{g}{2}$ downwards

(C) g upwards, 2g downwards (D) 2g upwards, g downwards

2. Two particles of mass m each are tied at the ends of a light string of length 2a. The whole system is kept on a frictionless horizontal surface with the string held tight so that each mass is at the distance a from the centre P (as shown in the figure). Now, the mid–point of the string is pulled vertically upwards with a small but constant force F. As a result, the particles move towards each other on the surface. The magnitude of acceleration, when the separation between them become 2x, is : ***[IIT-JEE 2007]***



(A) $\frac{F}{2m}\frac{a}{\sqrt{a^2-x^2}}$ (B) $\frac{F}{2m}\frac{x}{\sqrt{a^2-x^2}}$ (C) $\frac{F}{2m}\frac{x}{a}$ (D) $\frac{F}{2m}\frac{\sqrt{a^2-x^2}}{x}$

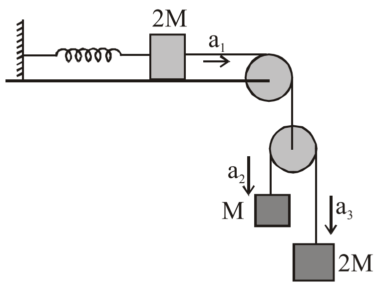
3. A particle moves in the X-Y plane under the influence of a force such that its linear momentum is $\vec{p}(t) = A[\hat{i}\cos(kt) - \hat{j}\sin(kt)]$, where A and k are constants. The angle between the force and the momentum is ***[IIT JEE 2007]***

(A) 0° (B) 30° (C) 45° (D) 90°

4. A piece of wire is bent in the shape of a parabola $y = kx^2$ (y-axis vertical) with a bead of mass m on it. The bead can slide on the wire without friction. It stays at the lowest point of the parabola when the wire is at rest. The wire is now accelerated parallel to the x-axis with a constant acceleration a. The distance of the new equilibrium position of the bead, where the bead can stay at rest with respect to the wire, from the y-axis is : ***[IIT-JEE 2009]***

(A) $\frac{a}{gk}$ (B) $\frac{a}{2gk}$ (C) $\frac{2a}{gk}$ (D) $\frac{a}{4gk}$

5. A block of mass 2M is attached to a massless spring with spring-constant k. This block is connected to two other blocks of masses M and 2M using two massless pulleys and strings. The accelerations of the blocks are a_1, a_2 and a_3 as shown in figure. The system is released from rest with the spring in its unstretched state. The maximum extension of the spring is x_0. Which of the following option(s) is/are correct ? [g is the acceleration due to gravity. Neglect friction]



(A) $x_0 = \frac{4Mg}{k}$

(B) When spring achieves an extension of $\frac{x_0}{2}$ for the first time, the speed of the block connected to the spring is $3g\sqrt{\frac{M}{5k}}$

(C) $a_2 - a_1 = a_1 - a_3$

(D) At an extension of $\frac{x_0}{4}$ of the spring, the magnitude of acceleration of the block connected to the spring is $\frac{3g}{10}$

Assertion-Reason

This question contains, statement I (assertion) and statement II (reason).

6. ***Statement–I :*** A cloth covers a table. Some dishes are kept on it. The cloth can be pulled out without dislodging the dishes from the table. [***IIT-JEE 2007***]

Because :

Statement–II : For every action there is an equal and opposite reaction.

(A) statement–I is true, statement–II is true; statement–II is a correct explanation for statement–I

(B) statement–I is true, statement–II is true, statement–II is NOT a correct explanation for statement–I

(C) statement–I is true, statement–II is false

(D) statement–I is false, statement–II is true

7. A particle of mass m is initially at rest at the origin. It is subjected to a force and starts moving along the x-axis. Its kinetic energy K changes with time as $dK/dt = \gamma t$, where γ is a positive constant of appropriate dimensions. Which of the following statements is (are) true ?

(A) The force applied on the particle is constant

(B) The speed of the particle is proportional to time

(C) The distance of the particle from the origin increses linerarly with time

(D) The force is conservative

* * * * *

ANSWERS

EXERCISE-1

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	B	C	D	B	C	A	C	C	D	B	C	A	B	C	C
Que.	16	17	18	19	20	21	22	23	24	25	26	27			
Ans.	C	B	A	B	C	A	B	B	A	B	C	B			

EXERCISE-2

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	A,B,C,D	B,D	A,C	A,B,C	C,D	A,C	A,C	A,C	B,C	A,D

- ***Match the column :*** **11.** (A) r, (B) t, (C) q, (D) s **12.** (A) t, (B) r, (C) r, (D) q
- ***Comprehension based Questions :***

Comprehension-1 **13.** (B) **14.** (C)

Comprehension-2 **15.** (B) **16.** (B) **17.** (D) **18.** (D) **19.** (A)

EXERCISE-3

1. 22N, 18N **2.** $\frac{4mv^2}{\pi d}$ **3.** $a_B = 1.56\ m/s^2$

4. For 4 kg $a = 2.5\ ms^{-2}$, For 8 kg $a=0$ **5.** 5N **6.** $a = \frac{4F}{M+m} - g.$

7. 322 N **8.** $a = \frac{12g}{25}$; $b = \frac{9g}{25}$; $N_{BW} = \frac{12mg}{25}$.

9. $a_A = a_B = 1 m/s^2$ upward **10.** (a) $3\,g\downarrow, 0, 0$, (b) $0, g\uparrow, g\downarrow$

EXERCISE-4

Que.	1	2	3	4
Ans.	B	A	A	C

EXERCISE-5

Que.	1	2	3	4	5	6	7
Ans.	A	B	D	B	C	B	ABD

* * * * *

FRICTION

Recap of Early Classes

In mechanics, we encounter several kinds of forces. the gravitational force is, of course, pervasive. energy object on the earth experiences the force of gravity due to the earth. Gravity also governs the motion of celestial bodies. The gravitational force can act at a distance without the need of any intervening medium. All the other forces common in mechanics are contact forces. As the name suggests, a contact force on an object arises due to contact with some other object: solid or fluid. When bodies are in contact (e.g. a book resting on a table, a system of rigid bodies connected by rods, hinges and other types of supports), there are mutual contact forces (for each pair of bodies) satisfying the third law. The component of contact force normal to the surfaces in contact is called normal reaction. The component parallel to the surfaces in contact is called friction.

Index

1.0 TYPES OF FRICTION

1.1 Dry Friction

1.2 Fluid Friction

1.3 Internal Friction

1.4 Types of Dry Friction

2.0 LAWS OF FRICTION

3.0 ANGLE OF FRICTION

4.0 ANGLE OF REPOSE (θ)

5.0 HOW TO SOLVE TWO BLOCK SYSTEM

EXERCISE-1

EXERCISE-2

EXERCISE-3

EXERCISE-4

EXERCISE-5

FRICTION

Whenever surfaces in contact are pressing each other slide or tend to slide over each other, opposing forces are generated tangentially to the surfaces in contact. These tangential forces, which oppose sliding or tendency of sliding between two surfaces are called frictional forces. Frictional forces on both bodies constitute third law action-reaction pair.

1.0 TYPES OF FRICTION

SL AL

Before we proceed further into detailed account of frictional phenomena, it is advisable to become familiar with different types of frictional forces. All types of frictional phenomenon can be categorized into dry friction, fluid friction, and internal friction.

1.1 Dry Friction

It exists when two solid un-lubricated surfaces are in contact under the condition of sliding or tendency of sliding. It is also known as Coulomb friction.

1.2 Fluid Friction

Fluid friction is developed when adjacent layers of a fluid move at different velocities and gives birth to phenomena, which we call viscosity of the fluid. Resistance offered to motion of a solid body in a fluid also comes in this category and commonly known as viscous drag. We will study this kind of friction in fluid mechanics.

1.3 Internal Friction

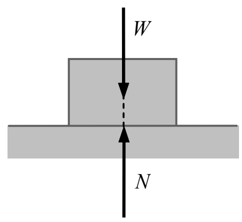
When solid materials are subjected to deformation, internal resistive forces developed because of relative movement of different parts of the solid. These internal resistive forces constitute a system of force, which is defined as internal friction. They always cause loss of energy.

Frictional forces exist everywhere in nature and result in loss of energy that is primarily dissipated in form of heat. Wear and tear of moving bodies is another unwanted result of friction. Therefore, sometimes, we try to reduce their effects – such as in bearings of all types, between piston and the inner walls of the cylinder of an IC engine, flow of fluid in pipes, and aircraft and missile propulsion through air. Though these Illustrations create a negative picture of frictional forces, yet there are other situations where frictional forces become essential and we try to maximize the effects. It is the friction between our feet and the earth surface, which enables us to walk and run. Both the traction and braking of wheeled vehicles depend on friction.

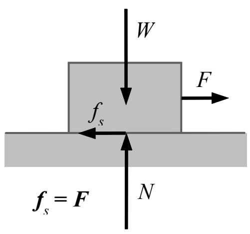
1.4 Types of Dry Friction

In mechanics of non-deformable bodies, we are always concerned with the dry friction. Therefore, we often drop the word "dry" and simply call it friction.

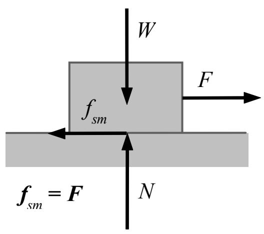
To understand nature of friction let us consider a box of weight W placed on a horizontal rough surface. The forces acting on the box are its weight and reaction from the horizontal surface. They are shown in the figure. The weight does not have any horizontal component, so the reaction of the horizontal surface on the box is normal to the surface. It is represented by N in the figure. The box is in equilibrium therefore both W and N are equal in magnitude, opposite in direction, and collinear.



Now suppose the box is being pulled by a gradually increasing horizontal force F to slide the box. Initially when the force F is small enough, the box does not slide. This can only be explained if we assume a frictional force, which is equal in magnitude and opposite in direction to the applied force F acts on the box. The force F produces in the box a tendency of sliding and the friction force is opposing this tendency of sliding. The frictional force developed before sliding initiates is defined as static friction. It opposes tendency of sliding.

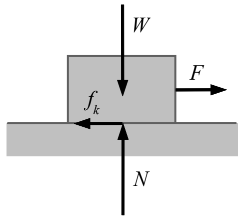


Static Friction



Limiting friction: The maximum Static Friction

As we increase F, the box remains stationary until a value of F is reached when the box starts sliding. Before the box starts sliding, the static friction increases with F and counterbalances F until the static friction reaches its maximum value known as limiting friction or maximum static friction f_{sm}.



Kinetic friction

When the box starts sliding, to maintain it sliding still a force F is needed to over come frictional force. This frictional force is known as kinetic friction (f_k). It always opposes sliding.

Dry Friction

Static Friction	**Kinetic Friction**
Opposes tendency of sliding	Opposes sliding and has a constant characteristic value

2.0 LAWS OF FRICTION

SL AL

When a normal force N exists between two surfaces, and we try to slide them over each other, the force of static friction (f_s) acts before sliding initiates. It can have a value maximum up to the limiting friction (f_{sm}).

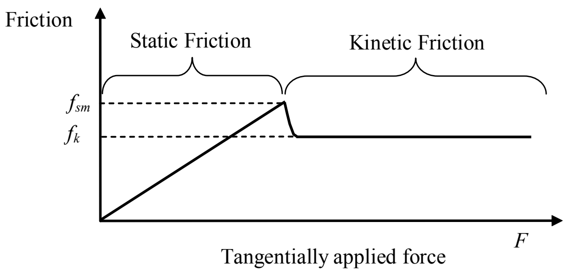
$$f_s \le f_{sm}$$

The limiting friction is experimentally observed proportional to the normal reaction between surfaces in contact.

$$f_{sm} = \mu_s N$$

Here μ_S is the constant of proportionality. It is known as the coefficient of static friction for the two surfaces involved.

When sliding starts between the surfaces, the frictionalforce rapidly drops to a characteristic value, which always opposes the sliding. This characteristic frictional force is known as kinetic friction (f_k). Kinetic friction is experimentally found proportional to the normal reaction between surfaces in contact.



$$f_k = \mu_k N$$

Here μ_k is the constant of proportionality. It is known as the coefficient of kinetic friction for the two surfaces involved.

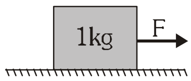
The frictional forces between any pair of surfaces are decided by the respective coefficients of friction. The coefficients of friction are dimensionless constants and have no units. The coefficient of static fiction (μ_s) is generally larger than the coefficient of kinetic friction (μ_k) but never become smaller; at the most both of them may be equal. Therefore, the magnitude of kinetic friction is usually smaller than the limiting static friction (f_{sm}) and sometimes kinetic friction becomes equal to the limiting static friction but it can never exceed the limiting friction.

The limiting static friction and the kinetic friction between any pair of solid surfaces follow these two empirical laws.

- Frictional forces are independent of measured area of contact.
- Both the limiting static friction and kinetic friction are proportional to the normal force pressing the surfaces in contact.

Illustrations

Illustration 1. A block of mass 1 kg is at rest on a rough horizontal surface, where coefficients of static and kinetic friction are 0.2 and 0.15. Find the frictional forces if a horizontal force



(a) F = 1N (b) F = 1.96 N (c) F = 2.5 N is applied on a block

Solution Maximum force of friction is the limiting friction $f_{sm} = 0.2 \times 1 \times 9.8\,N = 1.96\,N$

(a) For $F = 1\,N$, $F < f_{sm}$

So, body is in rest means static friction is present and hence $f_s = F = 1\,N$

(b) For $F = 1.96\,N$, $F = f_{sm} = 1.96\,N$. The block is about to slide, therefore $f = 1.96\,N$

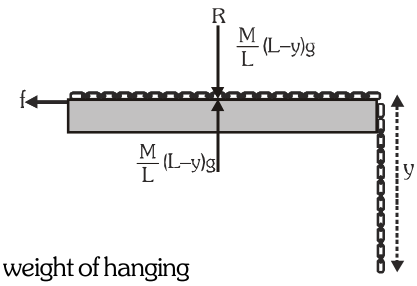
(c) For $F = 2.5\,N$, So $F > f_{sm}$

Now body is sliding and kinetic friction acts.

Therefore $f = f_k = \mu_k N = \mu_k mg = 0.15 \times 1 \times 9.8 = 1.47\,N$

Illustration 2. Length of a uniform chain is L and coefficient of static friction is μ between the chain and the table top. Calculate the maximum length of the chain which can hang from the table without sliding.

Solution Let y be the maximum length of the chain that can hang without causing the portion of chain on table to slide.



Length of chain on the table = (L – y)

Weight of part of the chain on table $= \frac{M}{L}(L-y)g$

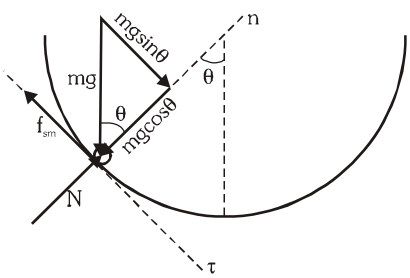
Weight of hanging part of the chain $= \frac{M}{L}yg$

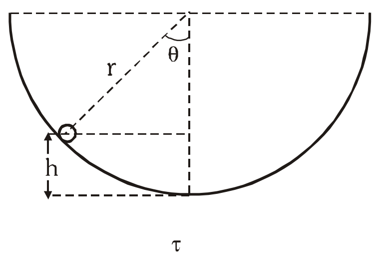
For equilibrium with maximum portion hanging, limiting friction = weight of hanging part of the chain

$$\mu\frac{M}{L}(L-y)g = \frac{M}{L}yg \Rightarrow y = \frac{\mu L}{1+\mu}$$

Illustration 3. An insect crawls on the inner surface of hemispherical bowl of radius r. If the coefficient of friction between an insect and bowl is μ and the radius of the bowl is r, find the maximum height to which the insect can crawl up.

Solution The insect can crawl up, the bowl till the component of its weight tangent to the bowl is balanced by limiting frictional force.





$\sum F_n = 0 \rightarrow$ $N = mg\cos\theta$ ...(i)

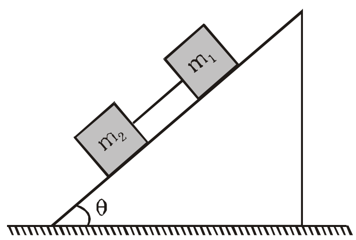
$\sum F_\tau = 0 \rightarrow$ $f_{sm} = mg\sin\theta$ ...(ii)

Force of limiting friction $f_{sm} = \mu N$ ...(iii)

From equation (i), (ii) and (iii), $\tan\theta = \mu$ ...(iv)

$$h = r - r\cos\theta = r\left[1 - \frac{1}{\sqrt{1+\mu^2}}\right]$$

Illustration 4. Two blocks with masses $m_1 = 1$ kg and $m_2 = 2$ kg are connected by a massless string and slide down a plane inclined at an angle $\theta = 45°$ with the horizontal. The coefficient of sliding friction between m_1 and plane is $\mu_1 = 0.4$ and that between m_2 and plane is $\mu_2 = 0.2$. Calculate the common acceleration of the two blocks and the tension in the string.



Solution As $\mu_2 < \mu_1$, block m_2 has greater acceleration than m_1 if we separately consider the motion of blocks. But they are connected so they move together as a system with common acceleration. So acceleration of the blocks :

$$a = \frac{(m_1+m_2)g\sin\theta - \mu_1 m_1 g\cos\theta - \mu_2 m_2 g\cos\theta}{m_1+m_2}$$

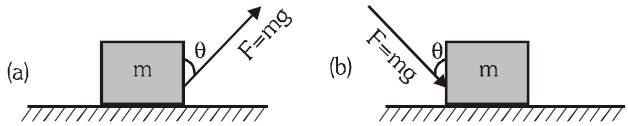
$$= \frac{(1+2)(10)\left(\frac{1}{\sqrt{2}}\right) - 0.4\times1\times10\times\frac{1}{\sqrt{2}} - 0.2\times2\times10\times\frac{1}{\sqrt{2}}}{1+2} = \frac{22}{3\sqrt{2}}\,ms^{-2}$$

For block m_2 : $m_2 g\sin\theta - \mu_2 m_2 g\cos\theta - T = m_2 a$

$\Rightarrow T = m_2 g\sin\theta - \mu_2 m_2 g\cos\theta - m_2 a$

$$= 2\times10\times\frac{1}{\sqrt{2}} - 0.2\times2\times10\times\frac{1}{\sqrt{2}} - 2\times\frac{22}{3\sqrt{2}} = \frac{4}{3\sqrt{2}}N$$

Illustration 5. A block of mass m rests on a rough horizontal surface as shown in figure (a) and (b). Coefficient of friction between block and surface is μ. A force F = mg acting at an angle θ with the vertical side of the block. Find the condition for which block will move along the surface.

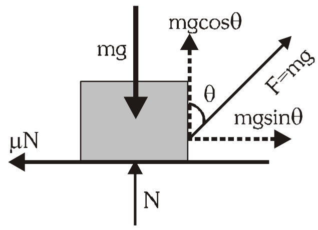


Solution ***For (a) :*** normal reaction $N = mg - mg\cos\theta$

frictional force $= \mu N = \mu(mg - mg\cos\theta)$

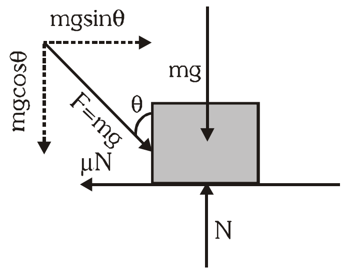
Now block can be pulled when : Horizontal component of force $\geq$ frictional force

i.e. $mg\sin\theta \geq \mu(mg - mg\cos\theta)$



or $2\sin\frac{\theta}{2}\cos\frac{\theta}{2} \geq \mu(1-\cos\theta)$

or $2\sin\frac{\theta}{2}\cos\frac{\theta}{2} \geq 2\mu\sin^2\frac{\theta}{2}$ or $\cot\frac{\theta}{2} \geq \mu$



For (b) : Normal reaction $N = mg + mg\cos\theta = mg(1+\cos\theta)$

Hence, block can be pushed along the horizontal surface when horizontal component of force $\geq$ frictional force

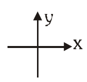
i.e. $mg\sin\theta \geq \mu\, mg(1+\cos\theta)$

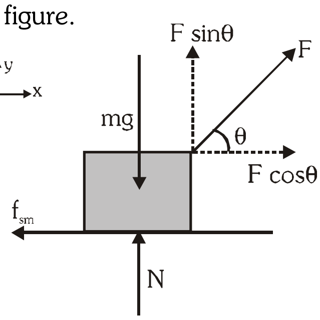
or $2\sin\frac{\theta}{2}\cos\frac{\theta}{2} \geq \mu\times2\cos^2\frac{\theta}{2} \Rightarrow \tan\frac{\theta}{2} \geq \mu$

Illustration 6. A body of mass m rests on a horizontal floor with which it has a coefficient of static friction μ. It is desired to make the body move by applying the minimum possible force F. Find the magnitude of F and the direction in which it has to be applied.

Solution Let the force F be applied at an angle θ with the horizontal as shown in figure.

$\sum F_y = 0 \rightarrow N = mg - F\sin\theta$ ...(i) $N = mg - F\sin\theta$ ...(i)





$\sum F_x = 0 \rightarrow F\cos\theta \geq f_{sm} \Rightarrow F\cos\theta \geq \mu N$ [as $f_{sm} = \mu N$] ...(ii)

Substituting value of N from equation (i) in (ii),

$$F \geq \frac{\mu mg}{(\cos\theta + \mu\sin\theta)} \qquad ...(iii)$$

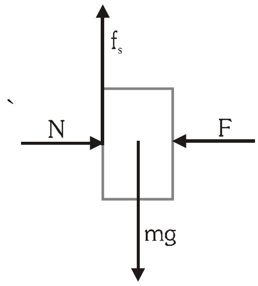
For the force F to be minimum $(\cos\theta + \mu\sin\theta)$ must be maximum,

maximum value of $\cos\theta + \mu\sin\theta$ is $\sqrt{1+\mu^2}$

so that $F_{min} = \dfrac{\mu mg}{\sqrt{1+\mu^2}}$ with $\theta = \tan^{-1}(\mu)$

Illustration 7. A book of 1 kg is held against a wall by applying a force F perpendicular to the wall. If $\mu_S = 0.2$, what is the minimum value of F ?

Solution The situation is shown in fig. The forces acting on the book are–



For book to be at rest it is essential that $Mg = f_s$

But $f_{s\,max} = \mu_S N$ and $N = F$

$\therefore\ Mg = \mu_S F \Rightarrow F = \dfrac{Mg}{\mu_s} = \dfrac{1\times 9.8}{0.2} = 49\text{ N}$

BEGINNER'S BOX–1

Static and Kinetic Friction

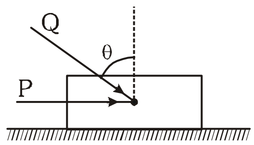
1. The maximum static frictional force is :

(A) Equal to twice the area of surface in contact
(B) Independent of the area of surface in contact
(C) Equal to the area of surface in contact
(D) None of the above

2. Starting from rest a body slides down a 45° inclined plane in twice the time it takes to slide down the same distance in the absence of friction. The co-efficient of friction between the body and the inclined plane is:

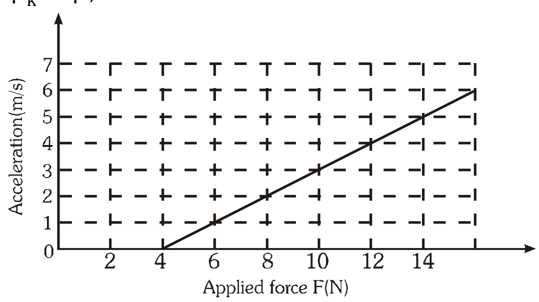
(A) 0.75 (B) 0.33 (C) 0.25 (D) 0.80

3. A block of mass m lying on a rough horizontal plane is acted upon by a horizontal force P and another force Q inclined an at an angle θ to the vertical. The minimum value of coefficient of friction between the block and the surface for which the block will remain in equilibrium is :



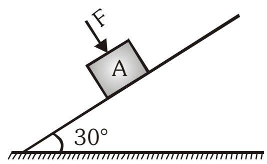
(A) $\dfrac{P+Q\sin\theta}{mg+Q\cos\theta}$ (B) $\dfrac{P\cos\theta+Q}{mg-Q\sin\theta}$ (C) $\dfrac{P+Q\cos\theta}{mg+Q\sin\theta}$ (D) $\dfrac{P\sin\theta-Q}{mg-Q\cos\theta}$

4. A block of unknown mass is at rest on a rough, horizontal surface. A horizontal force F is applied to the block. The graph in the figure shows the acceleration of the block with respect to the applied force. The mass of the block is $(\mu_s = \mu_k = \mu)$



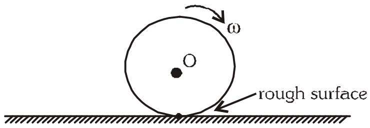
(A) 1.0 kg (B) 0.1 kg (C) 2.0 kg (D) 0.2 kg

5. A block of mass m = 2 kg is resting on a rough inclined plane of inclination 30° as shown in figure. The coefficient of friction between the block and the plane is $\mu = 0.5$. What minimum force F should be applied perpendicular to the plane on the block, so that block does not slip on the plane ? $(g=10\text{m/s}^2)$



(A) zero (B) 6.24 N (C) 2.68 N (D) 4.34 N

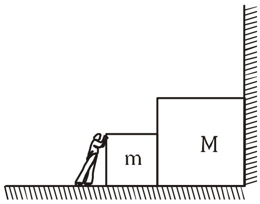
6. The wheel shown is fixed at 'O' and is in contact with a rough surface as shown. The wheel rotates with an angular velocity ω. What is the direction and nature of friction force on the wheel and on the ground.



7. An object is slowing down on a rough horizontal plane with a deceleration of $2 m/s^2$. What is the coefficient of kinetic friction ?

8. A block is shot with an initial velocity $5ms^{-1}$ on a rough horizontal plane. Find the distance covered by the block till it comes to rest. The coefficient of kinetic friction between the block and plane is 0.1.

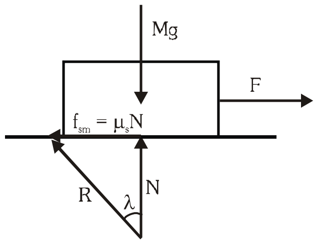
9. The person applies horizontal force F on the smaller block as shown in figure. The coefficient of static friction is μ between the blocks and the surface. Find the force exerted by the vertical wall on mass M . What is the value of action-reaction forces between m and M?



3.0 ANGLE OF FRICTION

SL

The angle of friction is the angle between resultant contact force of and normal reaction N, when sliding is initiating. It is denoted by λ



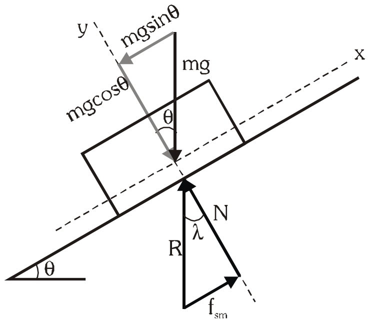
$$\tan\lambda = \frac{f_{sm}}{N} = \frac{\mu_s N}{N} = \mu_s$$

• For smooth surface $\lambda = 0$

4.0 ANGLE OF REPOSE (θ)

SL

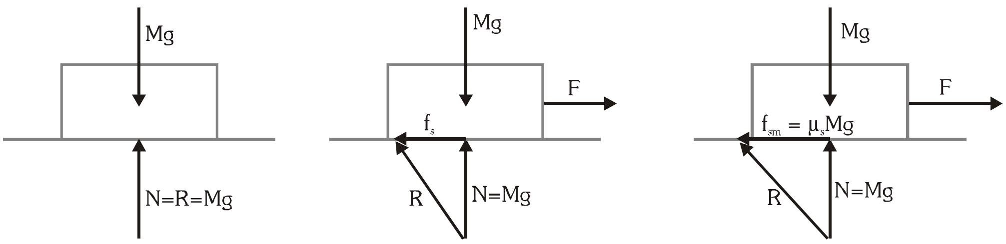
A body is placed on an inclined plane and the angle of inclination is gradually increased. At some angle of inclination θ the body starts sliding down the plane due to gravity. This angle of inclination is called angle of repose (θ).
Angle of repose is that minimum angle of inclination at which a body placed on the inclined starts sliding down due to its own weight.
Thus, angle of repose = angle of friction.



Illustrations

Illustration 8. A body of mass M is kept on a rough horizontal ground (static friction coefficient = μ_s). A person is trying to pull the body by applying a horizontal force F, but the body is not moving. What is the contact force between the ground and the block.

Solution

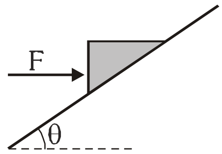


When F = 0   When $F < f_{sm}$ When $F = f_{sm}$

$R = Mg$ $R = \sqrt{(Mg)^2 + f_s^2}$ $R = Mg\sqrt{1+\mu_s^2}$

Therefore $\quad Mg \le R \le Mg\sqrt{1+\mu_s^2}$

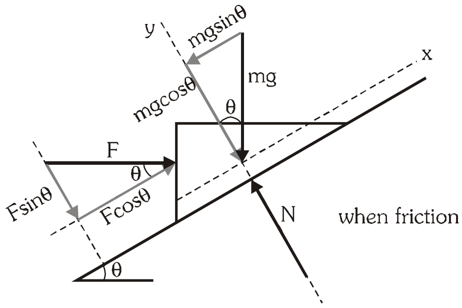
Illustration 9. A block rest on a rough inclined plane as shown in fig. A horizontal force F is applied to it (a) Find the force of normal reaction, (b) Can the force of friction be zero, if yes when? and (c) Assuming that friction is not zero find its magnitude and direction of its limiting value.



Solution (a) $\sum F_y = 0 \rightarrow \quad N = mg\cos\theta + F\sin\theta$

(b) $\sum F_x = 0 \rightarrow \quad F\cos\theta = mg\sin\theta \Rightarrow F = mg\tan\theta$

(c) Limiting friction $f_{sm} = \mu N = \mu\ (mg\cos\theta + F\sin\theta)$;



when friction is zero.

It acts down the plane if body has tendency to slide up and acts up the plane if body has tendency to slide down.

BEGINNER'S BOX–2

Angle of Friction and Angle of Repose

1. The coefficient of friction between a body and ground is $1/\sqrt{3}$ then (assuming only external horizontal force can act on the body) :

(A) The angle between contact force and normal force can vary from 60° to 90°

(B) The angle between contact force and normal force can vary from 0° to 30°

(C) The angle between contact force and normal force can vary from 0° to 60°

(D) The angle between contact force and normal force can be vary from 30° to 90°

2. A body of mass M is kept on a rough horizontal surface (friction coefficient = μ). A person is trying to pull the body by applying a horizontal force but the body is not moving. The force by the surface on A is F where

(A) $F = Mg$ (B) $F = \mu Mg$ (C) $Mg \le F \le Mg\sqrt{1+\mu^2}$ (D) $Mg \ge F \ge Mg\sqrt{1-\mu^2}$ 1

3. A block A kept on an inclined surface just begins to slide if the inclination is 30°. The block is replaced by another block B and it is found that it just begins to slide if the inclination is 40°.

(A) mass of A > mass of B (B) mass of A < mass of B

(C) mass of A = mass of B (D) all the three are possible.

4. A 500 kg horse pulls a cart of mass 1500 kg along a level horizontal road with an acceleration of 1 ms^{-2}. If the coefficient of sliding friction between the cart and ground is 0.2, then the force exerted by the horse on the cart in forward direction is : (Assume limiting friction is acting)

(A) 3000 N (B) 4500 N (C) 5000 N (D) 6000 N

5.* Let F, F_N and f denote the magnitudes of the contact force , normal force and the friction exerted by one surface on the other kept in contact. If none of these is zero,

(A) $F > F_N$ (B) $F > f$ (C) $F_N > f$ (D) $F_N - f < F < F_N + f$

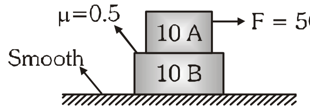
6.* The contact force exerted by one body on another body is equal to the normal force between the bodies. It can be said that :
(A) the surface must be frictionless
(B) the force of friction between the bodies is zero
(C) the magnitude of normal force equals that of friction
(D) it is possible that the bodies are rough and they do not slip on each other

7. A body of mass 400 g slides on a rough horizontal surface . If the frictional force is 3.0 N, find (a) the angle made by the contact force on the body with the vertical and (b) the magnitude of the force. Take g = 10 m/s^2.

5.0 HOW TO SOLVE TWO BLOCK SYSTEM

AL

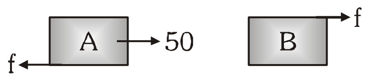
The given system initially at rest and the friction coefficient are as shown in the figure. For calculation of acceleration and friction



Step 1 : Make force diagram.
Step 2 : Show static friction force by f because value of friction is not known.
Step 3 : Calculate separately for two cases.

Case 1 : Move together
Step 4 : Calculate acceleration.
Step 5 : Check value of friction for above case.
Step 6 : If required friction is less than available it means they will move together else move separately.
Step 7 : (a) above acceleration will be common acceleration for both.

Case 2 : Move separately
Step 7(b) If they move separately then kinetic friction is involved, whose value is μN.
Step 8 : Calculate acceleration for above case.



$f_{max} = \mu N$

∴ $f \le 50$ N (available friction)

Move together

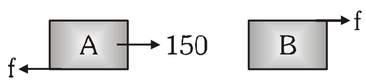
(i) $a = \dfrac{50}{10+10} = 2.5\,m/s^2$

(ii) check friction for B :
$f = 10 \times 2.5 = 25$
25 N is required which is less than available friction hence they will move together:
and $a_A = a_B = 2.5$ m/s^2

Move separately

No need to calculate

If the value of F is 150 N than



Move together

(i) $a = \dfrac{150}{20} = 7.5$

(ii) check friction for B :

$f = 10 \times 7.5 = 75$
75 N is required friction that is greater than available friction which is not possible so both the block move seperately

Move separately

$a_A = \dfrac{150-50}{10} = 10\,m/s^2$

$a_B = \dfrac{50}{10} = 5\,m/s^2$

Illustrations

Illustration 10. In the given figure block A is placed on block B and both are placed on a smooth horizontal plane. Assume lower block to be sufficiently long. The force F pulling the block B horizontally is increased according to law F = 10t N

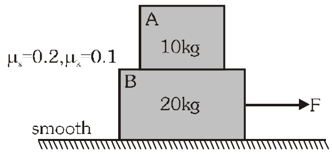
(a) When does block A start slipping on block B? What will be force F and acceleration just before slipping starts?

(b) When F is increased beyond the value obtained in part (a), what will be acceleration of A ?

(c) Draw acceleration-time graph.

Solution ***Direction of friction forces***

Block A moves forward always, due to friction, therefore friction on it must be in forward direction. Friction between two adjacent surfaces are equal and opposite because they make Newton's is third law action reaction pair.



Range of Value of friction

Before slipping starts, friction is static $f_s \leq 20$ N

After slipping starts, friction is kinetic $f_k = 10$ N

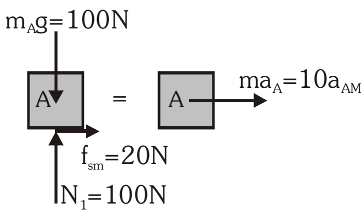
Maximum possible acceleration

A can accelerate only due to friction, its maximum possible acceleration is a_{AM}

(when $f_s = f_{sm} = 20$ N)

Block A

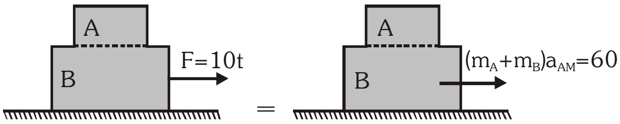
So $20 = 10a_{AM} \Rightarrow a_{AM} = 2$ m/s^2



Sequence of slipping :

Since ground is smooth, block B first starts slipping on the ground and carries A together with it. When acceleration of A & B becomes equal to a_{AM}, Block A starts slipping on B.

(a) Just before the moment A starts slipping, both were moving together with acceleration a_{AM}. Considering them as a one body.

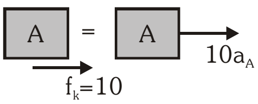


(On a smooth stationary surface we will not show the normal forces i.e. FBD of combined block showing horizontal forces only).

Value of F = 60 N

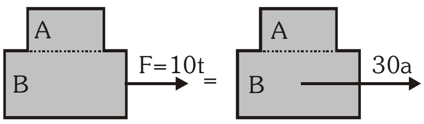
and Time $10t = 60 \Rightarrow t = 6$s

(b) If F is increased beyond 60 N, A slides and kinetic friction acts on it. Now acceleration of A



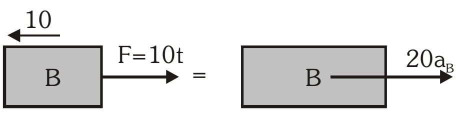
$\Rightarrow 10 = 10a_A \Rightarrow a_A = 1$ m/s^2

(c) When F $\leq$ 60 N, both are moving with same acceleration a. We treat them as one body.



So $10t = 30a \Rightarrow a = \frac{1}{3}t$

This acceleration increases to $a_{AM} = 2$ m/s^2, when F = 60 N at t = 6 s. Thereafter A starts slipping and its acceleration provided by kinetic friction, drops to a constant value $a_A = 1$ m/s^2. However acceleration of B keeps on increasing according to equation



$10t - 10 = 20a_B \Rightarrow a_B = \frac{1}{2}t - \frac{1}{2}$

Graph between acceleration and time

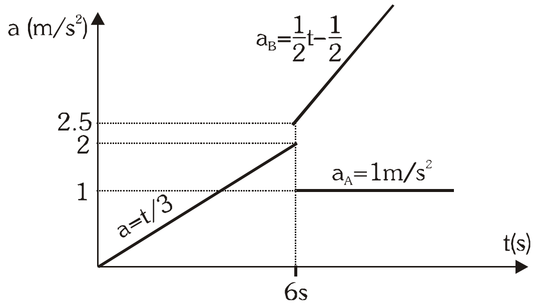
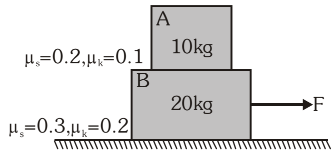
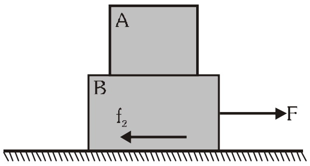


Illustration 11. Block A is placed on another block B, which rests on a rough horizontal ground. Horizontal force F pulling the block B is increased gradually.

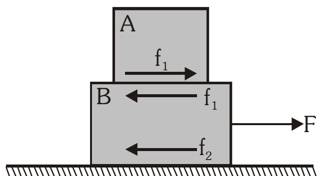


(a) Find the maximum value of F so that no motion occurs.
(b) Find maximum F so that A does not slide on B.
(c) If F is increased beyond the value obtained in part (b) what are acceleration of both the blocks? Explain your answer in terms of F.
(d) If F is increased according to law F = 10t N draw a–t graph

Solution ***Directions of friction forces***



Before B starts slipping



After B starts slipping

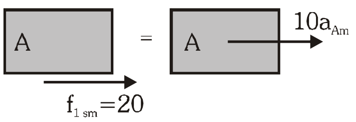
Range of values of frictional forces

$f_{1k} = 10$ N; $\quad f_{1s} \le 20$ N

$f_{2k} = 60$ N; $\quad f_{2s} \le 90$ N

Maximum possible acceleration : A can move only due to friction. Its maximum possible acceleration is

Block A



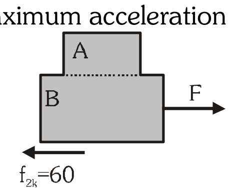
$\Rightarrow a_{Am}=2$ m/s^2

Sequence of slipping : When $F \geq f_{2s}$, block B starts slipping on ground and carries block A together with it till its acceleration reaches value a_{AM}. Thereafter A also starts slipping on B.

(a) F = 90 N

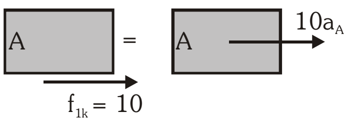
(b) When A does not slide on B, both move with the same acceleration (a_{Am}) and can be treated as one body, which can have maximum acceleration a_{AM} = 2 m/s^2.

$F - 60 = 60 \Rightarrow F = 120$ N



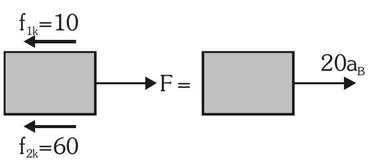
(c) When F is increased beyond F = 120 N, block A starts sliding and friction between A & B drops to f_{1K} = 10 N. Both the blocks now move with different acceleration so we treat them separate bodies. Now acceleration A also drops to a constant value a_A.

Acceleration of A :



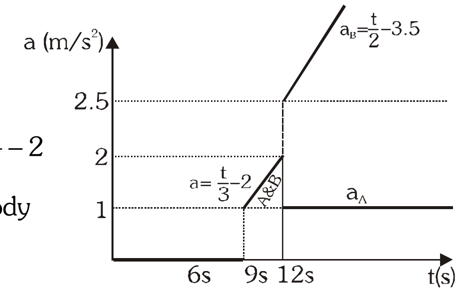
$\Rightarrow 10 = 10\, a_A \Rightarrow a_A = 1$ m/s^2

Acceleration of B :



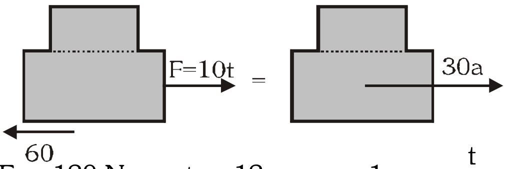
So $F-70 = 20a_B \Rightarrow a_B = \dfrac{F-70}{20}$

(d) If F = 10t, values of acceleration of both the blocks in different time intervals are as under:



- $F \le 90$ N $\Rightarrow t \le 9$ s $\quad a_A = a_B = 0$
- $90\text{ N} < F \le 120\text{ N} \Rightarrow 9\text{s} < t \le 12\text{ s} \quad a_A = a_B = \dfrac{t}{3} - 2$

  In the above interval both the blocks move as one body



$$a = \frac{10t - 60}{30} = \frac{t}{3} - 2$$

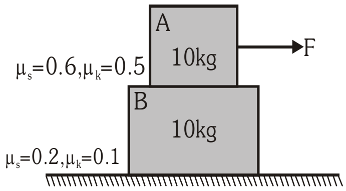
- $F > 120$ N $\Rightarrow t > 12$s $\quad a_A = 1;\ a_B = \dfrac{t}{2} - 3.5$

Illustration 12. Block A is placed on another block B, which rests on a rough horizontal ground. Horizontal force pulling A is increased gradually

(a) Find maximum F so that none of the blocks move. Which block starts sliding first ?

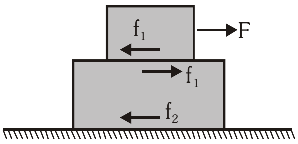
(b) Express acceleration of each block as function of F for all positive values of F.

(c) If F=10t draw a-t graph



Solution

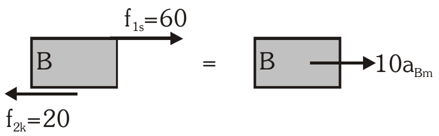
Directions of friction forces



Range of values of friction forces

$f_{1k} = 50$ N
$f_{1s} \le 60$ N
$f_{2k} = 20$ N
$f_{2s} \le 40$ N

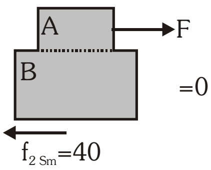
Maximum possible acceleration of B : Block B acceleration due to friction only. Its maximum acceleration is



$$60–20 = 10a_{Bm} \Rightarrow a_{Bm} = 4\text{ m/s}^2$$

Sequence of slipping : Smaller, limiting friction is between B and ground so it will start sliding first. Then both will move together till acceleration B reaches its maximum possible values 4 m/s^2. Thereafter A starts sliding on B

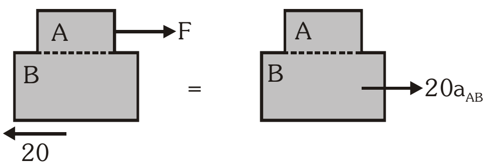
(a) Till the F reaches the limiting friction between block B and the ground none of the blocks move.



$\Rightarrow F = 40$ N

(b) If $F \le 40 \Rightarrow a_A = a_B = 0$ ...(i)

If F > 40 N, block B starts sliding and carries A together with it with the same acceleration till acceleration reach to 4 m/s^2. At this moment A starts slipping. Before this moment we may treat both of them as single body.

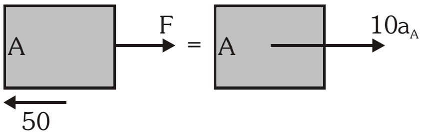


$$\Rightarrow F - 20 = 20a_{AB} \Rightarrow a_{AB} = a_A = a_B = \frac{F - 20}{20}$$

When A starts sliding on B, $a_A = a_B = 4$, from the above equation, we have F = 100 N.

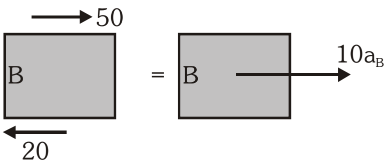
When $F \geq 100$ N block A also starts slipping on B and friction between A & B drops to value 50 N. Now since they move with different acceleration we treat them separately.

Block A



$$\Rightarrow a_A = \frac{F-50}{10}$$

Block B



$$\Rightarrow a_B = \frac{50-20}{10} = 3\ m/s^2$$

(c) $F \leq 40$ N $\quad t \leq 4$ s $\quad a_A = a_B = 0$

$40 < F \leq 100 \quad 4 < t \leq 10 \quad a_A = a_B = \frac{t}{2} - 1$

$100 < F \quad \Rightarrow t > 10 \quad a_A = t - 5,\ a_B = 3\ m/s^2$

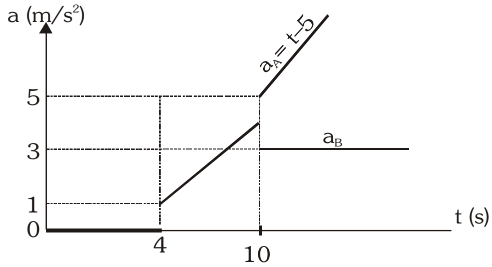
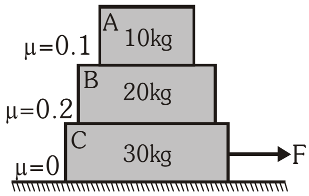


Illustration 13. Block A is placed on B and B is placed on block C, which rests on smooth horizontal ground as shown in the figure. Block A is pulled horizontally by a force F which increases gradually.

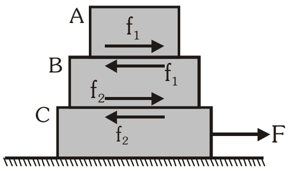
(a) Decide sequence of slipping.

(b) If F is increased gradually find acceleration of each block for all values of F.

(c) If F = 15t N, draw a–t graph.



Solution ***Direction of friction forces :***



Range of values of friction forces

$f_1 \leq 10$ N (A does not slides on B)

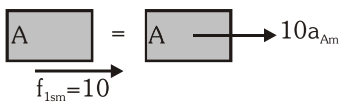
$f_1 = 10$ N (A slides on B)

$f_2 \leq 60$ N (B does not slides on C)

$f_2 = 60$ N (B slides on C)

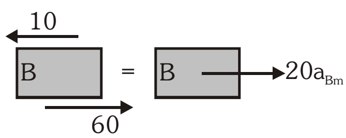
Maximum possible acceleration : Blocks A and B move due to friction forces only, we find their maximum possible acceleration.

Block A



$a_{Am} = 1\ m/s^2$

Block B



$$a_{Bm} = \frac{60-10}{20} = 2.5\ m/s^2$$

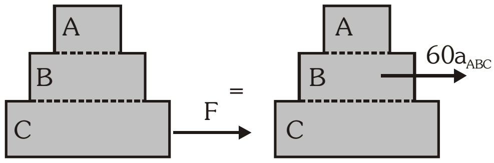
(a) Sequence of slipping

Since ground is smooth the block C starts sliding first

A starts slipping on B secondly till that moment all the three blocks move with same acceleration, which can achieve maximum value of $a_{Am} = 1\ m/s^2$.

Thirdly B starts sliding on C, till that moment B & C move with the same acceleration $a_{Bm} = 2.5\ m/s^2$

(b) Before A starts slipping, all the three were moving with the same acceleration $a_{Am} = 1\ m/s^2$. We therefore treat then as a single body.

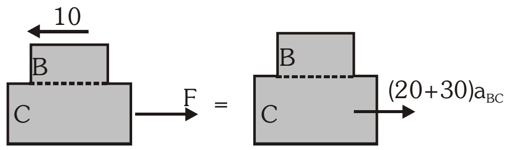


$$a_{ABC} = \frac{F}{60}$$

When A starts sliding $\quad a_{ABC} \le a_{Am} \Rightarrow \frac{F}{60} \le 1 \Rightarrow F \le 60\ N$

When $F \ge 60$ N, block A starts slipping on B and its acceleration decided by friction f_1, achieves a constant value $a_A = 1\ m/s^2$.

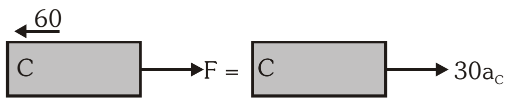
Now, F is increased beyond 60 N and B and C will continue to move together till their acceleration a_{BC} becomes $a_{Bm} = 2.5\ m/s^2$, when slipping between B and C starts. Till this moment, we treat B and C as one body.



$$\Rightarrow a_{BC} = \frac{F-10}{50}$$

When slipping between B & C starts : $a_{BC} = a_{Bm} \Rightarrow \frac{F-10}{50} = 2.5 \Rightarrow F \le 135\ N$

When $F > 135$ N, block B also starts slipping on C. Now acceleration of A & B achieves the maximum value $a_{Bm} = 2.5\ m/s^2$ and acceleration of block C is decided by F.

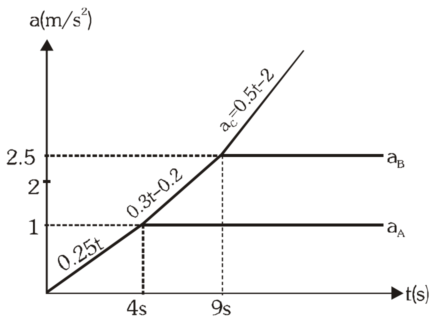


$$\Rightarrow F - 60 = 30\,a_C \Rightarrow a_C = \frac{F-60}{30}$$

Acceleration of blocks for different values of force.

- $F \le 60$ N $\qquad a_A = a_B = a_C = a_{ABC} = \frac{F}{60}$
- $60 < F \le 135$ N $\qquad a_A = a_{Am} = 1\ m/s^2,\ a_B = a_C = a_{BC} = \frac{F-10}{50}$
- $135 < F$ $\qquad a_A = a_{Am} = 1,\ a_B = a_{Bm} = 2.5$, and $a_C = \frac{F-60}{30}$

(c) If $F = 15t$



- $F \le 60 \qquad t \le 4$ s $\qquad a_A = a_B = a_C = \frac{F}{60} = \frac{t}{4} = 0.25t$
- $60 < F \le 135 \qquad 4 < t \le 9$

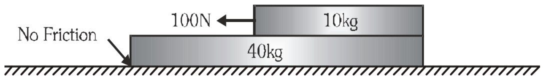
$$a_A = 1\ m/s^2\ ,\quad a_B = a_C = \frac{F-10}{50} = 0.3t - 0.2$$

- $135 < F \qquad 9 < t \qquad a_A = 1\ m/s^2,\ a_B = 2.5\ m/s^2,\ a_C = \frac{F-60}{30} = 0.5t - 2$

BEGINNER'S BOX-3

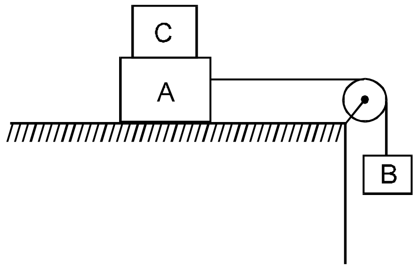
Two Block System

1. A 40 kg slab rests on a frictionless floor. A 10 kg block rests on top of the slab. The static coefficient of friction between the block and slab is 0.60 while the kinetic coefficient is 0.40. The 10 kg block is acted upon by a horizontal force of 100N. If g = 9.8 m/s^2, the resulting acceleration of the slab will be :



(A) 0.98 m/s^2 (B) 1.47 m/s^2 (C) 1.52 m/s^2 (D) 6.1 m/s^2

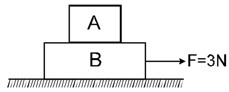
2. Two masses A and B of 10 kg and 5 kg respectively are connected with a string passing over a frictionless pulley fixed at the corner of a table as shown. The coefficient of static friction of A with table is 0.2. The minimum mass of C that may be placed on A to prevent it from moving is



(A) 15 kg (B) 10 kg

(C) 5 kg (D) 12 kg

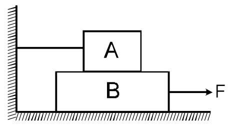
3. In the arrangement shown mass of A = 1 kg, mass of B = 2kg and coefficient of friction between A and B is 0.2. There is no friction between B and ground. The frictional force on A is (g = 10 m/s^2).



(A) 0 N (B) 2 N

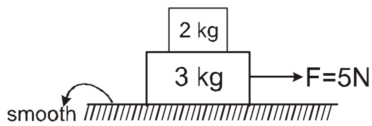
(C) 1.96 N (D) 1 N.

4. A is a 100 kg block and B is a 200 kg block. As shown in figure, the block A is attached to a string tied to a wall. The coefficient of friction between A and B is 0.2 and the coefficient of friction between B and floor is 0.3. Then the minimum force required to move the block B will be (g = 10 m/s^2)

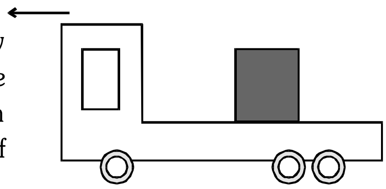


(A) 600 N (B) 800 N (C) 900 N (D) 1100 N

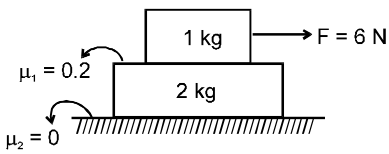
5. Determine the force and its direction on 2 kg block in the given situation. It is known that the two blocks move together. Can we determine the coefficient static friction between the two blocks. If yes then what is its value?



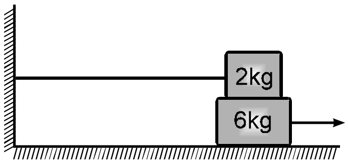
6. The rear side of a truck is open and a box of 40 kg mass is placed 5 m away from the open end as shown in figure. The coefficient of friction between the box and the surface below it is 0.15. On a straight road, the truck starts from rest and accelerates with 2 ms^{-2}. At what distance from the starting point of the truck does the box fall off the truck? (Ignore the size of the box).



7. In the situation shown find the accelerations of the blocks. Also find the accelerations if the force is shifted from the upper block to the lower block.



8. With reference to the figure shown, if the coefficient of friction at the surfaces is 0.42, then the force required to pull out the 6.0 kg block with an acceleration of 1.50 m/s^2 will be:



(A) 36 N (B) 24 N (C) 84 N (D) 51 N

GOLDEN KEY POINTS

- ***Reasons for Friction***
  (i) Inter-locking of extended parts of one object into the extended parts of the other object.
  (ii) Bonding between the molecules of the two surfaces or objects in contact.

- (i) Direction of friction is not opposite to the force applied it is opposite to the relative motion of the body considered which is in contact with the other surface.
  (ii) The static friction is involved when there is no relative motion between two surfaces.

- Minimum force required to start the motion of block kept on rough horizontal surface is

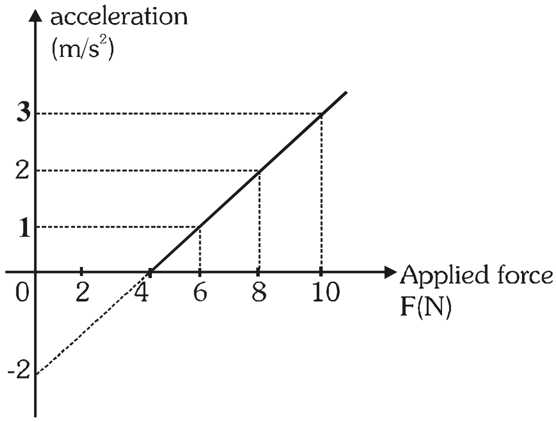
  $$F_{min.} = \frac{\mu_s mg}{\sqrt{1+\mu_s^2}}$$

  At an angle : $\theta = \tan^{-1} \mu_s$ with horizontal

SOME WORKED OUT ILLUSTRATIONS

Illustration 1.

A block of unknown mass is at rest on a rough horizontal surface. A force F is applied to the block. The graph in the figure shows the acceleration of the block w.r.t. the applied force.



The mass of the block and coefficient of friction are ($g = 10\ m/s^2$)

(A) 2 kg, 0.1 (B) 2 kg, 0.2 (C) 1 kg, 0.1 (D) can't be determined

Ans. (B)

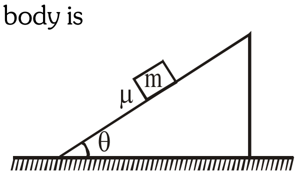
Solution

Acceleration of block, $a = \dfrac{F - \mu mg}{m} \Rightarrow a = \left(\dfrac{1}{m}\right)F - \mu g$

From graph ; slope $= \dfrac{1}{m} = \dfrac{1}{2} \Rightarrow m = 2kg$ and y-intercept; $-\mu g = -2 \Rightarrow \mu = 0.2$

Illustration 2.

A body is placed on an inclined plane. The coefficient of friction between the body and the plane is μ. The plane is gradually tilted up. If θ is the inclination of the plane, then frictional force on the body is



(A) constant upto $\theta = \tan^{1}(\mu)$ and decreases after that

(B) increases upto $\theta = \tan^{1}(\mu)$ and decreases after that

(C) decreases upto $\theta = \tan^{1}(\mu)$ and constant after that

(D) constant throughout

Ans. (B)

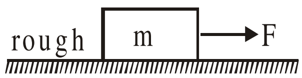
Solution

Friction force $F = mg\sin\theta$ if $\theta \le \tan^{-1}(\mu)$ and $F = \mu mg\cos\theta$ if $\theta \ge \tan^{-1}(\mu)$

which increases upto $\theta = \tan^{-1}(\mu)$ and then decreases.

Illustration 3.

A block is placed on a rough horizontal surface and a horizontal force F is applied to it as shown in figure. The force F is increased from zero in small steps. The graph between applied force and frictional force f is plotted by taking equal scales on axes. The graph is



(A) a straight line of slope 45°

(B) a straight line parallel to F-axis

(C) a straight line parallel to f-axis

(D) a straight line of slope 45° for small F and a straight line parallel to F-axis for large F.

Ans. (D)

Solution

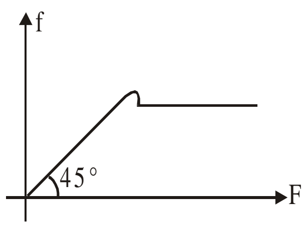
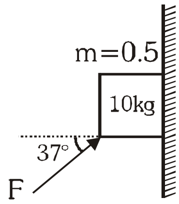


Illustration 4.

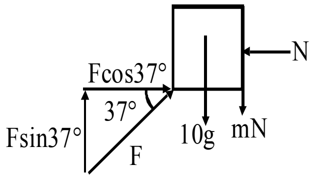
A force F pushes a block weighing 10 kg against a vertical wall as shown in the figure. The coefficient of friction between the block and wall is 0.5. The minimum value of F to start the upward motion of block is [g = 10 m/s^2]



(A) 100 N
(B) 500 N
(C) $\frac{500}{3}$ N
(D) can't be determined

Ans. ***(B)***

Solution

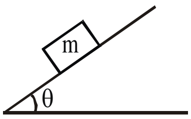


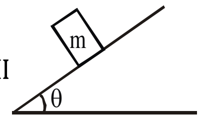
$N = F\cos 37° = \frac{4}{5}F$ and to start upward motion $F\sin 37° = 10\,g + \mu N$

$\frac{3}{5}F = 100 + (0.5)\left(\frac{4}{5}F\right) \Rightarrow F = 500\text{ N}$

Illustration 5.

A block is first placed on its long side and then on its short side on the same inclined plane (see figure). The block slides down in situation II but remains at rest in situation I. A possible explanation is

Situation–I 

Situation–II 

(A) The normal contact force is less in situation-II.
(B) The frictional force is less in situation-II because the contact area is less.
(C) The shorter side is smoother.
(D) In situation-I, frictional force is more.

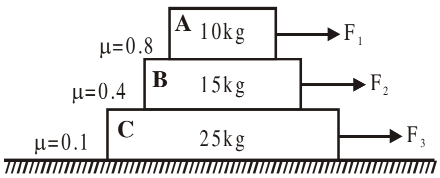
Ans. ***(C, D)***

Solution

This is due to less frictional force or low friction coefficient.

Illustration 6.

For the situation shown in the figure below, match the entries of column I with the entries of column II. [g = 10 m/s^2]



Column I

(A) If F_1 = 160 N, F_2 = 0 & F_3 = 0, then
(B) If F_1 = 140 N, F_2=0 & F_3 = 0 N, then
(C) If F_1 = F_2 = 0 & F_3 = 45 N, then
(D) If F_1 = 0, F_2 = 160 N and F_3 = 0, then

Column II

(p) There is no relative motion between A & B
(q) There is no relative motion between B & C
(r) There is a relative motion between A & B
(s) There is a relative motion between B & C
(t) There is a relative motion between C & ground

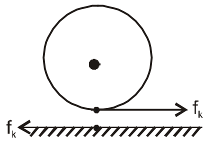
Ans. ***(A) q,r,t; (B) q,r,t; (C) p,q; (D) s,t***

Solution

Maximum frictional force between A and B = (0.8) (10) (10) = 80 N
Maximum frictional force between B and C = (0.4) (25) (10) = 100 N
Maximum frictional force between C and ground = (0.1) (50) (10) = 50 N
Therefore there is no relative motion between B and C if F_2 = 0 & F_3 =0
To start motion $(F_1 + F_2 + F_3)_{min}$ = 50 N
To start slipping between B and C (if F_3=0), $(F_1 + F_2)_{min}$ = 150 N
If F_2 = F_3 =0 then to start slipping beween A & B $F_{1\,min}$ = 87.5 N

ANSWERS

BEGINNER'S BOX-1

1. (B) **2.** (A) **3.** (A) **4.** (C) **5.** (C) **6.** 

7. 0.2

8. 12.5 m

9. N = 0; for $F \le \mu(M+m)g$; $N = F - \mu(M+m)g$ for $F > \mu(M+m)g$
action-reaction forces between m and M is $F - \mu mg$ for $F > \mu mg$ and 0 for $F \le \mu\, mg$

BEGINNER'S BOX-2

1.(B) **2.** (C) **3.** (D) **4.** (B) **5.** (ABD) **6.** (BD) **7.** (a) 37°, (b) 5.0 N

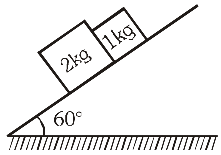
BEGINNER'S BOX-3

1. (A) **2.** (A) **3.** (D) **4.** (D) **5.** $\mu \ge 0.1$ **6.** 20 m

7. Upper block 4 m/s^2, lower block 1 m/s^2; Both blocks 2 m/s^2 **8.** (D)

EXERCISE – 1 MCQ (SINGLE CHOICE CORRECT)

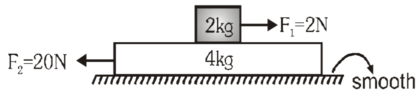
1. In the figure shown if friction coefficient of block 1kg and 2kg with inclined plane is μ_1=0.5 and μ_2 = 0.4 respectively, then



(A) both block will move together
(B) both block will move separately
(C) there is a non zero contact force between two blocks
(D) None of these

2. A block of mass 0.1 kg is held against a wall applying a horizontal force of 5N on the block. If the coefficient of friction between the block and the wall is 0.5, the magnitude of the frictional force acting on the block is :–
(A) 2.5 N (B) 0.98 N (C) 4.9 N (D) 0.49 N

3. In the arrangement shown in figure, coefficient of friction between the two blocks is $\mu = 1/2$. The force of friction acting between the two blocks is

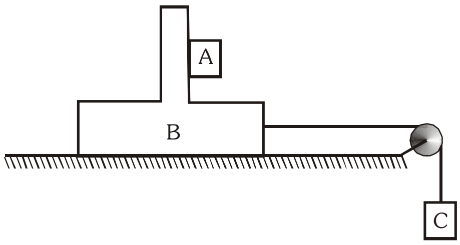


(A) 8 N (B) 10 N (C) 6 N (D) 4 N

4. ϕ is the angle of the incline when a block of mass m just starts slipping down. The distance covered by the block if thrown up the incline with an initial speed v_0 is :

(A) $\dfrac{v_0^2}{4g\sin\phi}$ (B) $\dfrac{4v_0^2}{g\sin\phi}$ (C) $\dfrac{v_0^2\sin\phi}{4g}$ (D) $\dfrac{4v_0^2\sin\phi}{g}$

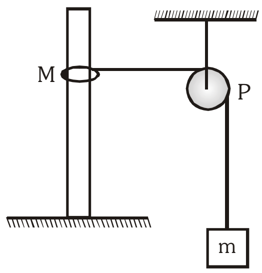
5*. In the arrangement shown in the figure, mass of the block B and A is 2m and m respectively. Surface between B and floor is smooth. The block B is connected to the block C by means of a string pulley system. If the whole system is released, then find the minimum value of mass of block C so that A remains stationary w.r.t. B. Coefficient of friction between A and B is μ.



(A) $\dfrac{m}{\mu}$ (B) $\dfrac{2m+1}{\mu+1}$

(C) $\dfrac{3m}{\mu-1}$ (D) $\dfrac{6m}{\mu+1}$

6. In the figure shown a ring of mass M and a block of mass m are in equilibrium. The string is light and pulley P does not offer any friction and coefficient of friction between pole and M is μ. The frictional force offered by the pole on M is

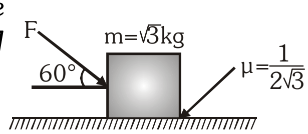


(A) Mg directed up (B) μ mg directed up
(C) (M – m) g directed down (D) μ mg direction down

7. If you want to pile up sand onto a circular area of radius R. The greatest height of the sand pile that can be erected without spilling the sand onto the surrounding area, if μ is the coefficient of friction between sand particle is :

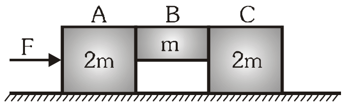
(A) R (B) μ^2R (C) μR (D) $\dfrac{R}{\mu}$

8. What is the maximum value of the force F such that the block shown in the arrangement, does not move: ***[IIT-JEE 2003]***



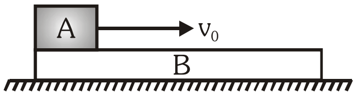
(A) 20 N (B) 10 N
(C) 12 N (D) 15 N

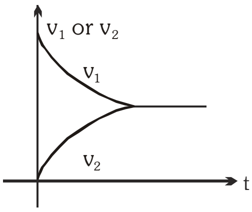
9*. The system is pushed by a force F as shown in figure. All surfaces are smooth except between B and C. Friction coefficient between B and C is μ. Minimum value of F to prevent block B from downward slipping is

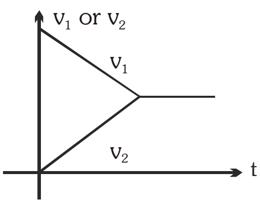


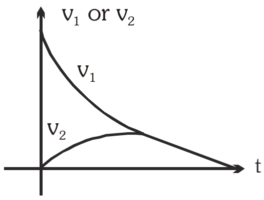
(A) $\left(\frac{3}{2\mu}\right)mg$ (B) $\left(\frac{5}{2\mu}\right)mg$ (C) $\left(\frac{5}{2}\right)\mu mg$ (D) $\left(\frac{3}{2}\right)\mu mg$

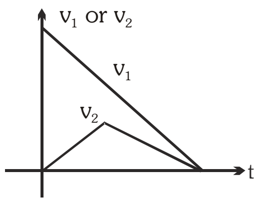
10. A block A is placed over a long rough plank B of same mass as shown in figure. The plank is placed over a smooth horizontal surface. At time t=0, block A is given a velocity v_0 in horizontal direction. Let v_1 and v_2 be the velocities of A and B at time t. Then choose the correct graph between v_1 or v_2 and t.



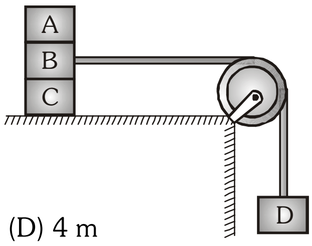
(A)


(B)


(C)


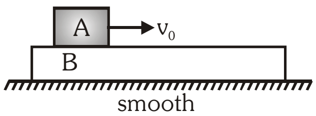
(D)


11*. Three blocks A , B and C of equal mass m are placed one over the other on a smooth horizontal ground as shown in figure. Coefficient of friction between any two blocks of A,B and C is $\frac{1}{2}$. The maximum value of mass of block D so that the blocks A, B and C move without slipping over each other is



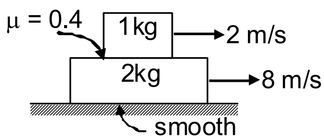
(A) 6 m (B) 5 m (C) 3 m (D) 4 m

12. A block A of mass m is placed over a plank B of mass 2m. Plank B is placed over a smooth horizontal surface. The coefficient of friction between A and B is 0.5. Block A is given a velocity v_0 towards right. Acceleration of B relative to A is



(A) $\frac{g}{2}$ (B) g (C) $\frac{3g}{4}$ (D) zero

13. The time when relative motion between the blocks will stop is



(A) 1 s (B) 2 s

(C) 3 s (D) 4s

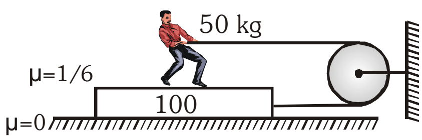
14*. A man of mass 50 kg is pulling on a plank of mass 100 kg kept on a smooth floor as shown with force of 100 N. If both man & plank move together, find force of friction acting on man.

(A) $\frac{100}{3}$ N towards left

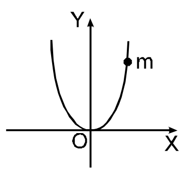
(B) $\frac{100}{3}$ N towards right

(C) $\frac{250}{3}$ N towards left

(D) $\frac{250}{3}$ N towards right

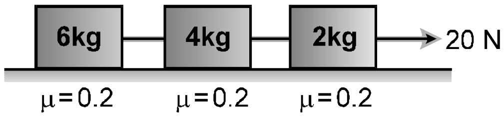


15*. A bead of mass m is located on a parabolic wire with its axis vertical and vertex directed towards downward as in figure and whose equation is $x^2 = ay$. If the coefficient of friction is μ, the highest distance above the x-axis at which the particle will be in equilibrium is



(A) $\frac{a\mu^2}{4}$ (B) $\frac{a\mu^2}{2}$ (C) $\frac{a}{2\mu^2}$ (D) $\frac{a}{4\mu^2}$

16. Three blocks of masses 6 kg, 4kg & 2 kg are pulled on a rough surface by applying a constant force 20N. The values of cofficient of friction between blocks & surface are shown in figrue.



(i) In the arrangement shown tension in the string connecting 4kg and 6kg masses is

(A) 8N (B) 12N (C) 6N (D) 4N

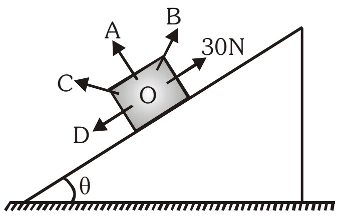
(ii) Friction force on 4 kg block is

(A) 4N (B) 6 N (C) 12 N (D) 8 N

(iii) Friction force on 6 kg block is

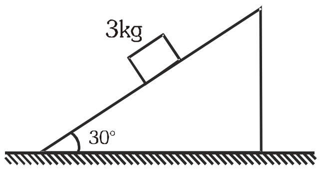
(A) 12 N (B) 8 N (C) 6 N (D) 4 N

17. A block of mass of 10 kg lies on a rough inclined plane of inclination $\theta = \sin^{-1}\left(\frac{3}{5}\right)$ with the horizontal when a force of 30N is applied on the block parallel to and upward the plane, the total force exerted by the plane on the block is nearly along (coefficient of friction is $\mu = \frac{3}{4}$) ($g = 10\ m/s^2$)



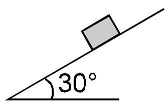
(A) OA (B) OB (C) OC (D) OD

18. A block of mass 3 kg is at rest on a rough inclined plane as shown in the figure. The magnitude of net force exerted by the surface on the block will be ($g=10\ m/s^2$)



(A) 26N (B) 19.5N
(C) 10N (D) 30 N

19. Figure shows a block kept on a rough inclined plane. The maximum external force down the incline for which the block remains at rest is 2N while the maximum external force up the incline for which the block is at rest is 10 N. The coefficient of static friction μ is :

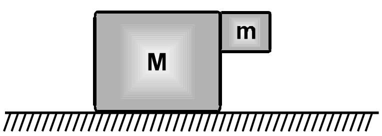


(A) $\frac{\sqrt{3}}{2}$ (B) $\frac{1}{\sqrt{6}}$ (C) $\sqrt{3}$ (D) $\frac{1}{\sqrt{3}}$

20. A block of mass 0.1 kg is held against a wall by applying a horizontal force of 5 N on the block. If the coefficient of friction between the block and the wall is 0.5, the magnitude of the frictional force acting on the block is:

(A) 2.5 N (B) 0.98 N (C) 4.9 N (D) 0.49 N

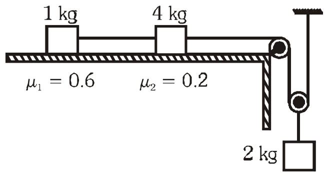
21. With what minimum acceleration mass M must be moved on frictionless surface so that m remains stick to it as shown. The co–efficient of friction between M & m is μ.



(A) μg (B) $\frac{g}{\mu}$ (C) $\frac{\mu mg}{M+m}$ (D) $\frac{\mu mg}{M}$

EXERCISE – 2 MCQ (ONE OR MORE CHOICE CORRECT)

1. 1 kg and 4 kg blocks lie on a rough horizontal surface. The coefficient of friction between 4 kg block and surface is 0.2 while the ocefficient of friction between 1 kg block and the surface is 0.6. All the pulleys shown in the figure are massless and frictionless and all strings are massless. [$g = 10\ m/s^2$]

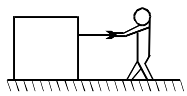


(A) The frictional force acting on 1 kg block is 2 N.
(B) The frictional force acting on 1 kg block is 6 N.
(C) The tension in the string connecting 4 kg block and 1 kg block is 2 N.
(D) The tension in the string connecting 1 kg block and 4 kg block is zero.

2. A block is placed over a large plank (plank is kept on horizontal surface). The coefficient of friction between the block and the plank is $\mu = 0.2$. Initially both are at rest, suddenly the plank starts moving horizontally with acceleration $a_0 = 4\ m/s^2$ with the help of some external force. The displacement of the block in 1s is ($g = 10\ m/s^2$)

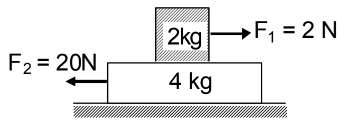
(A) 1 m relative to ground (B) 1 m relative to plank
(C) zero relative to plank (D) 2 m relative to ground

3. A man pulls a block heavier than himself with a light horizontal rope. The coefficient of friction is the same between the man and the ground, and between the block and the ground :



(A) The block will not move unless the man also moves
(B) The man can move even when the block is stationary
(C) If both move, the acceleration of the man is greater than the acceleration of the block
(D) None of the above assertions is correct

4. In the arrangement shown in figure, coefficient of friction between the two blocks is $\mu = 1/2$. The force of friction acting between the two blocks is

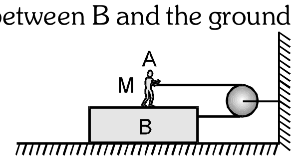


(A) 8 N
(B) 10 N
(C) 6 N
(D) 4 N

5. A variable force $F = 10\ t$ is applied to block B placed on a smooth surface. The coefficient of friction between A & B is 0.5. (t is time in seconds. Initial velocities are zero)

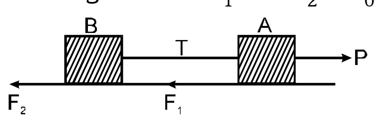
(A) block A starts sliding on B at t = 5 seconds (B) block A starts sliding on B at t = 10 seconds
(C) acceleration of A at 10 seconds is $15\ m/s^2$. (D) acceleration of A at 10 seconds is $5\ m/s^2$.

6. As shown in the figure, M is a man of mass 60 kg standing on a block of mass 40 kg kept on ground. The co-efficient of friction between the feet of the man and the block is 0.3 and that between B and the ground is 0.1. If the person pulls the string with 100 N force, then :



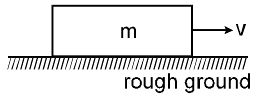
(A) B slides on ground.
(B) A and B both move together with acceleration $1\ m/s^2$.
(C) the friction force acting between A & B may be 40 N.
(D) the friction force acting between A & B will be 180 N.

7. Two blocks A and B of the same mass are joined by a light string and placed on a horizontal surface. An external horizontal force P acts on A. The tension in the string is T. The forces of friction acting on A and B are F_1 and F_2 respectively. The limiting value of F_1 and F_2 is F_0. As P is gradually increased,



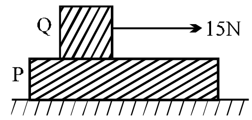
(A) for $P < F_0$, $T = 0$ (B) for $F_0 < P < 2F_0$, $T = P - F_0$
(C) for $P > 2F_0$, $T = P/2$ (D) none of the above

8. Find, which is the corret statement about the friction force :



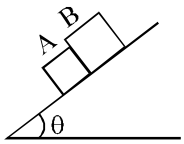
(A) friction will try to reduce relative motion
(B) friction will try to reduce velocity of mass m
(C) friction will try to increase velocity of mass m
(D) friction will try to increase velocity of ground

9. A long plank P of the mass 5 kg is placed on a smooth floor. On P is placed a block Q of mass 2 kg. The coefficient of friction between P and Q is 0.5. If a horizontal force 15N is applied to Q, as shown, and you may take g as 10N/kg.



(A) The reaction force on Q due to P is 10N
(B) The acceleration of Q relative to P is 2.5 m/s^2
(C) The acceleration of P relative to the Floor is 2.0 m/s^2
(D) The acceleration of centre of mass of P + Q system relative to the floor is $(15/7) m/s^2$

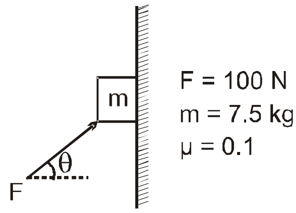
10. The two blocks A and B of equal mass are initially in contact when released from rest on the inclined plane. The coefficients of friction between the inclined plane A and B are μ_1 and μ_2 respectively.



(A) If $\mu_1 > \mu_2$, the blocks will always remain in contact.
(B) If $\mu_1 < \mu_2$, the blocks will slide down with different accelerations (if blocks slide)
(C) If $\mu_1 > \mu_2$, the blocks will have a common acceleration $\frac{1}{2}(\mu_1+\mu_2)\, g \sin\theta$.
(D) If $\mu_1 < \mu_2$, the blocks will have a common acceleration $\frac{\mu_1\mu_2 g}{\mu_1+\mu_2}\sin\theta$.

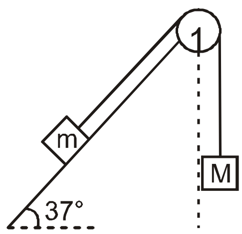
Match the column

11. Figure shows a block pressed against a rough vertical wall with a force F as shown in side view. Column-I shows angle at which force F and column-II gives information about corresponding friction force, match them.



Column I	*Column II*
(A) $\theta = 37°$	(p) friction by wall on block is upwards
(B) $\theta = 45°$	(q) friction by wall on block is downwards
(C) $\theta = 53°$	(r) friction by wall on block is static
(D) $\theta = 0°$	(s) friction by wall on block is kinetic

12 The inclined surface is rough with $\mu = \frac{1}{2}$. For different values of m and M, the system slides down or up the palne or remains stationary. Match the appropriate entries of column-I with those of column-II.



Column I	*Column II*
(A) Minimum value of $\frac{m}{M}$ so that m slides down	(p) $\frac{5}{3}$
(B) Minimum value of $\frac{M}{m}$ so that m slides up	(q) 1
(C) Value of $\frac{m}{M}$ so that friction force on m is zero	(r) $\frac{3}{5}$
(D) Ratio of vertical component of acceleration of m and acceleration of M (if both blocks move).	(s) 5

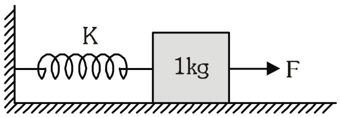
Comprehension Based Questions

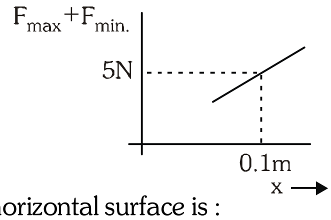
Comprehension 1.

A block of mass 1 kg is placed on a rough horizontal surface. A spring is attached to the block whose other end is joined to a rigid wall, as shown in the figure. A horizontal force is applied on the block so that it remains at rest while the spring is elongated by x, $x \ge \frac{\mu mg}{k}$. Let F_{max} and F_{min} be the maximum and minimum values of force F for which the block remains in equilibrium. For a particular x.

$$F_{max} - F_{min} = 2 \text{ N}$$

Also shown is the variation of $F_{max} + F_{min}$ versus x, the elongation of the spring.





13. The coefficient of friction between the block and the horizontal surface is :
(A) 0.1 (B) 0.2 (C) 0.3 (D) 0.4

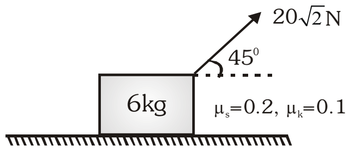
14. The spring constant of the spring is :
(A) 25 N/m (B) 20 N/m (C) 2.5 N/m (D) 50 N/m

15. The value of F_{min}, if x = 3 cm is :
(A) zero (B) 0.2 N (C) 5N (D) 1N

Comprehension 2.

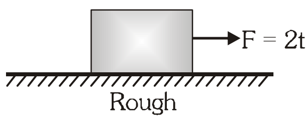
Contact force $\left(\vec{F}_c\right)$ between two bodies is the resultant of force of friction and normal reaction.

16. Contact force for shown position is ($g = 10 \text{ ms}^{-2}$)



(A) 40N (B) $\sqrt{1616}$ N
(C) 4N (D) None of these

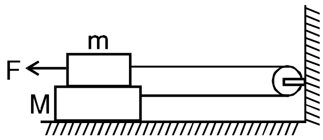
17. A time varying force is applied on a block placed over a rough surface as shown in figure. Let θ be the angle between contact force on the block and the normal reaction, then with time, θ will :



(A) Remain constant
(B) First increase to a maximum value (say θ_{max}) and then becomes constant in a value less then θ_{max}
(C) First decrease to a minimum value (say θ_{min}) and then becomes constant in a value more than θ_{min}
(D) None of the above

Comprehension 3.

The friction cofficient between the two blocks shown in figure is μ but floor is smooth.



18. What maximum horizontal force F can be applied without disturbing the equilibrium of the system?
(A) μ mg (B) 2μ mg (C) 3μ mg (D) 4μ mg

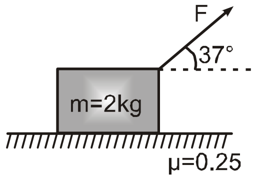
19. Suppose the horizontal force applied is double of that found in previous part Find the accelerations of the two masses.

(A) $\frac{2\mu Mg}{M+m}$ in same direction (B) $\frac{2\mu mg}{M+m}$ in same direction

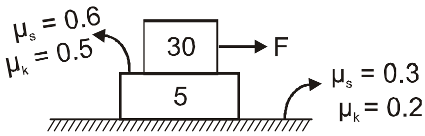
(C) $\frac{2\mu mg}{M+m}$ in opposite direction (D) $\frac{\mu Mg}{M+m}$ in opposite direction

EXERCISE – 3 SUBJECTIVE

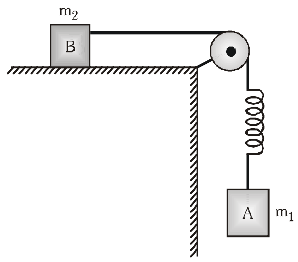
1. A force F = 20 N is applied to a block (at rest) as shown in figure. After the block has moved a distance of 8 m to the right, the direction of horizontal component of the force F is reversed. Find distance (in nearest meter) travelled before block stops after the F is reversed. ($g = 10\ m/s^2$, $\sin 37° = 3/5$).



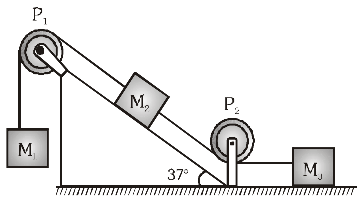
2. Find the maximum value of F for which both blocks will move together



3. If the two blocks moves with a constant uniform speed then find coefficient of friction between the surface of the block B and the table. The spring is massless and the pulley is frictionless.

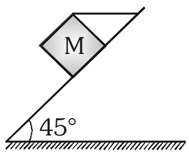


4. Masses M_1, M_2 and M_3 are connected by strings of negligible mass which pass over massless and frictionless pulleys P_1 and P_2 as shown in fig. The masses move such that the portion of the string between P_1 and P_2 is parallel to the inclined plane and the portion of the string between P_2 and M_3 is horizontal. The masses M_2 and M_3 are 4.0 kg each and the coefficient of kinetic friction between the masses and the surfaces is 0.25. The inclined plane makes an angle of 37° with the horizontal. If the mass M_1 moves downwards with a uniform velocity, find the mass of M_1.

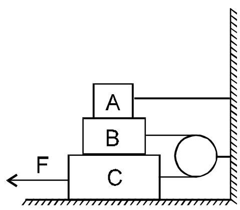


5. A block of mass 2 kg is pushed against a rough vertical wall with a force of 40 N, coefficient of static friction being 0.5. Another horizontal force of 15 N is applied on the block in a direction parallel to the wall. Will the block move ? If yes , in which direction ? If no, find the frictional force exerted by the wall on the block.

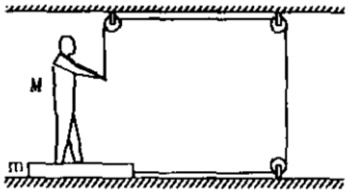
6. A block of mass 15kg is resting on a rough inclined plane as shown in figure. The block is tied up by a horizontal string which has a tension of 50N. Calculate the coefficient of friction between the block and inclined plane.



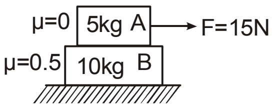
7. $M_A = 3$ kg, $M_B = 4$ kg and $M_C = 8$ kg. μ between any two surfaces is 0.25. Pulley is frictionless and string is massless Block. A is connected to the wall through a massless rigid rod as shown in figure.The value of F to keep C moving with constant speed is 10X N. Find value of X ($g = 10 m/s^2$).

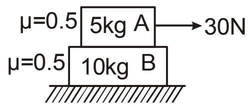


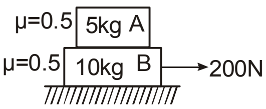
8. The friction coefficient between the board and the floor shown in figure is μ. Find the maximum force that the man can exert on the rope so that the board does not slip on the floor.

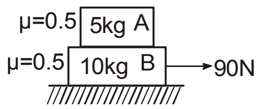


9. Find the accelerations and the friction forces involved :

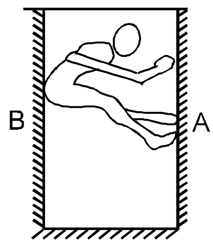
(a)


(b)


(c)


(d)


10. A person (40 kg) is managing to be at rest between two vertical walls by pressing one wall B by his back (figure). Assume that the friction cofficient between his body and the walls is 0.8 and that limiting friction acts at all the contacts. (a) Show that the person pushes the two walls with equal force. (b) Find the normal force exerted by either wall on the peson. Take g = 101 m/s^2.

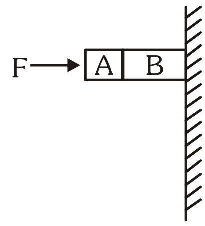


EXERCISE – 4 **RECAP OF AIEEE/JEE (MAIN)**

1. A block of mass m is placed on a surface with a vertical cross section given by $y = \frac{x^3}{6}$. If the coefficient of friction is 0.5, the maximum height above the ground at which the block can be placed without slipping is : ***[JEE(Main) - 2014]***

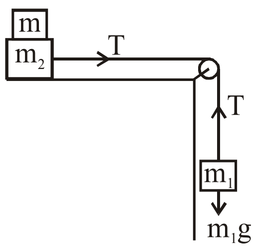
(A) $\frac{1}{6}$m (B) $\frac{2}{3}$m (C) $\frac{1}{3}$m (D) $\frac{1}{2}$m

2. Given in the figure are two blocks A and B of weight 20 N and 100 N, respectively. These are being pressed angainst a wall by a force F as shown. If the coefficient of friction between the blocks is 0.1 and between block B and the wall is 0.15, the frictional force applied by the wall on block B is ***[JEE(Main) - 2015]***



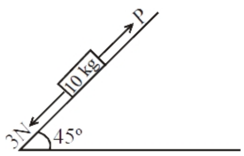
(A) 100 N (B) 80 N (C) 120 N (D) 150 N

3. Two masses m_1 = 5kg and m_2 = 10kg, connected by an inextensible string over a frictionless pulley, are moving as shown in the figure. The coefficient of friction of horizontal surface is 0.15. The minimum weight m that should be put on top of m_2 to stop the motion is :- ***[JEE(Main) - 2018]***



(A) 27.3 kg (B) 43.3 kg (C) 10.3 kg (D) 18.3 kg

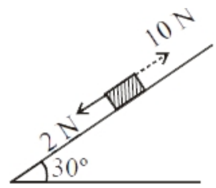
4. A block of mass 10 kg is kept on a rough inclined plane as shown in the figure. A force of 3 N is applied on the block. The coefficient of static friction between the plane and the block is 0.6. What should be the minimum value of force P, such that the block doesnot move downward ?



(take g = 10 ms^{-2}) ***[JEE(Main) - 2019]***

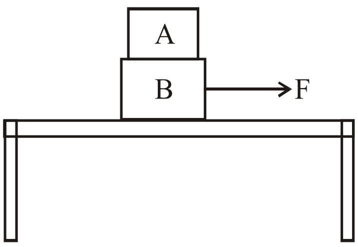
(A) 32 N (B) 25 N (C) 23 N (D) 18 N

5. A block kept on a rough inclined plane, as shown in the figure, remaias at rest upto a maximum force 2 N down the inclined plane. The maximum external force up the inclined plane that does not move the block is 10 N. The coefficient of static friction between the block and the plane is : [Take g = 10 m/s^2] ***[JEE(Main) - 2019]***



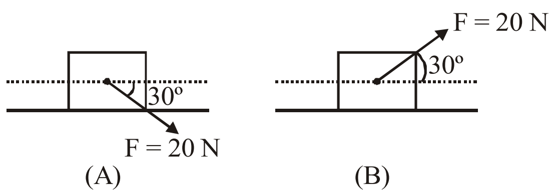
(A) $\frac{2}{3}$ (B) $\frac{\sqrt{3}}{2}$ (C) $\frac{\sqrt{3}}{4}$ (D) $\frac{1}{2}$

6. Two blocks A and B of masses m_A = 1 kg and m_B = 3 kg are kept on the table as shown in figure. The coefficient of friction between A and B is 0.2 and between B and the surface of the table is also 0.2. The maximum force F that can be applied on B horizontally, so that the block A does not slide over the block B is :
(Take g = 10 m/s^2) ***[JEE(Main) - 2019]***



(A) 16 N (B) 40 N (C) 12 N (D) 8 N

7. A block of mass 5 kg is (i) pushed in case (A) and (ii) pulled in case (B), by a force F = 20 N, making an angle of 30° with the horizontal, as shown in the figures. The coefficient of friction between the block and floor is μ = 0.2. The difference between the accelerations of the block, in case (B) and case (A) will be : (g = 10 ms^{-2}) ***[JEE(Main) - 2019]***

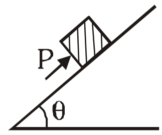


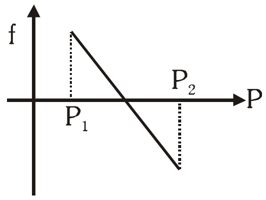
(A) 0 ms^{-2} (B) 0.8 ms^{-2} (C) 0.4 ms^{-2} (D) 3.2 ms^{-2}

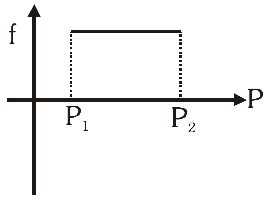
EXERCISE – 5 **RECAP OF IIT-JEE/JEE (ADVANCED)**

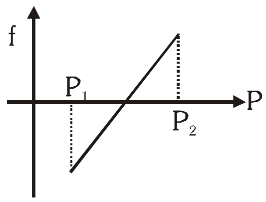
Single Choice Correct

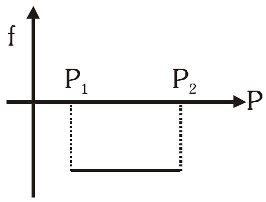
1. A block of mass m is on an inclined plane of angle θ. The coefficient of friction between the block and the plane is μ and $\tan\theta > \mu$. The block is held stationary by applying a force P parallel to the plane. The direction of force pointing up the plane is taken to be positive. As P is varied from $P_1 = mg(\sin\theta - \mu\cos\theta)$ to $P_2 = mg(\sin\theta + \mu\cos\theta)$, the frictional force f versus P graph will look like ***[IIT-JEE-2010]***



(A)


(B)


(C)


(D)


2. ***Statement -I :*** It is easier to pull a heavy object than to push it on a level ground. ***[IIT-JEE 2008]***

and

Statement-II: The magnitude of frictional force depends on the nature of the two surface in contact.

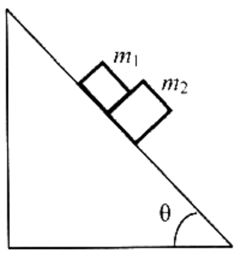
(A) statement–I is true, statement–II is true; statement–II is a correct explanation for statement–I

(B) statement–I is true, statement–II is true, statement–II is NOT a correct explanation for statement–I

(C) statement–I is true, statement–II is false

(D) statement–I is false, statement–II is true

3. A block of mass $m_1 = 1$ kg another mass $m_2 = 2$ kg, are placed together (see figure) on an inclined plane with angle of inclination θ. Various values of θ are given in List I. The coefficient of friction between the block m_1 and the plane is always zero. The coefficient of static and dynamic friction between the block m_2 and the plane are equal to $\mu = 0.3$. In List II expressions for the friction on block m_2 are given. Match the correct expression of the firction in List II with the angle given in List I, and choose the correct option. The acceleration due to gravity is denoted by g. [Useful information : $\tan(5.5°) \approx 0.1$; $\tan(11.5°) \approx 0.2$; $\tan(16.5°) \approx 0.3$] ***[JEE(Adv.) 2014]***



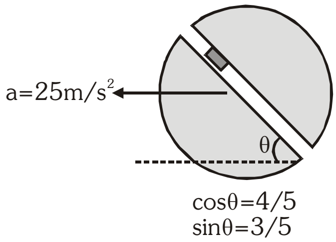
List I	***List II***
P. $\theta = 5°$	1. $m_2 g\sin\theta$
Q. $\theta = 10°$	2. $(m_1 + m_2)g\sin\theta$
R. $\theta = 15°$	3. $\mu m_2 g\cos\theta$
S. $\theta = 20°$	4. $\mu(m_1 + m_2)g\cos\theta$

Code :

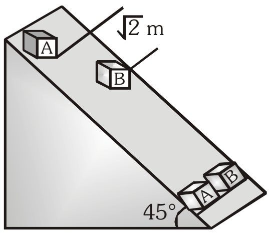
(A) P-1, Q-1, R-1, S-3 (B) P-2, Q-2, R-2, S-3 (C) P-2, Q-2, R-2, S-4 (D) P-2, Q-2, R-3, S-3

Subjective Questions

4. A circular disc with a groove along its diameter is placed horizontally. A block of mass 1 kg is placed as shown. The coefficient of friction between the block and all surface of groove in contact is $\mu = \frac{2}{5}$. The disc has an acceleration of 25 m/s^2. Find the acceleration of the block with respect to disc. ***[IIT-JEE 2006]***



5. Two blocks A and B of equal masses are release from an inclined plane of inclination 45° at t = 0. Both the blocks are initially at rest. The coefficient of kinetic friction between the block A and the inclined plane is 0.2 while it is 0.3 for block B. Initially the block A is $\sqrt{2}$ m behind the block B. When and where their front faces will come in a line. (Take g = 10 m/s^2) ***[IIT-JEE 2004]***



Integer type Questions

6. A block is moving on an inclined plane making an angle 45° with the horizontal and the coefficient of friction is μ. The force required to just push it up the inclined plane is 3 times the force required to just prevent it from sliding down. If we define N = 10μ, then N is ***[IIT-JEE-2011]***

* * * * *

ANSWERS

EXERCISE-1

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	B	B	A	A	C	A	C	A	B	B	C	C	A	A	A
Que.	16	17	18	19	20	21									
Ans.	(i) (A) (ii) (D) (iii) (B)	B	D	A	B	B									

EXERCISE-2

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	A,C	A,B	A,B,C	A	A,D	A,B,C	A,B,C	A,B,D	C,D	A,B

- ***Match the column :*** **11.** (A) p,s (B) p,r (C) q,r (D) p,s **12.** (A) s (B) q (C) p (D) r
- ***Comprehension based Questions :***
  Comprehension 1 : **13.** A **14.** A **15.** A
  Comprehension 2 : **16.** B **17.** B
  Comprehension 3 : **18.** B **19.** C

EXERCISE-3

1. 6 **2.** 840 **3.** $\frac{m_1}{m_2}$ **4.** 4.2 kg

5. It will move at an angle of 53° with the 15N force **6.** $\mu \geq 0.5$ **7.** 8

8. $\frac{\mu(M+m)g}{1+\mu}$

9. (a) $a_A = 3\ m/s^2$, $a_B = 0$, $f_A = 0$, $f_B = 0$ (b) $a_A = 1\ m/s^2$, $a_B = 0$, $f_A = 25N$, $f_B = 25N$
(c) $a_A = 5\ m/s^2$; $a_B = 10\ m/s^2$; $f_A = 25N$; $f_B = 75N$ (d) $a_A = 1 m/s^2$; $a_B = 1 m/s^2$; $f_A = 5N$; $f_B = 75N$

10. (b) 250N

EXERCISE-4

Que.	1	2	3	4	5	6	7
Ans.	A	C	A	A	B	A	B

EXERCISE-5

1. (A) **2.** (B) **3.** (D) **4.** 10 ms^{-2} **5.** (For A) $8\sqrt{2}$ m, 2s **6.** 5

* * * * *

WORK, POWER & ENERGY

Recap of Early Classes

In early classes we have studied all motions in terms of the forces that causes them. However, we will explain in this chapter, the conservation of energy greatly simplifies the description of motion in many instances. the principle of conservation of energy is a universal concept that is important not only in mechanics but also in other branches of physics. In order to understand the concept we first discuss the concept of work and the basic relation between force, work, and energy.

Index

1.0 WORK OF A FORCE

1.1 Work of a constant Force on a body in rectilinear motion

1.2 Work of a variable Force on a body in rectilinear motion

1.3 Work of a Variable Force on a Body in Curvilinear Translation Motion

1.4 Work of a force depends on frame of reference

2.0 WORK AND ENERGY

2.1 Potential, Kinetic and Mechanical Energy

2.2 Other forms of Energy

2.3 Work-Kinetic Energy Theorem

3.0 CONSERVATIVE AND NON-CONSERVATIVE FORCES

4.0 POTENTIAL ENERGY

4.1 Gravitational potential energy for uniform gravitational force

4.2 Gravitational potential energy for non-uniform gravitational force

4.3 Potential energy associated with spring force

5.0 CONSERVATION OF MECHANICAL ENERGY

6.0 POTENTIAL ENERGY AND THE ASSOCIATED CONSERVATIVE FORCE

7.0 POTENTIAL ENERGY AND NATURE OF EQUILIBRIUM

7.1 Stable Equilibrium

7.2 Unstable Equilibrium

7.3 Neutral Equilibrium

8.0 POWER

EXERCISE-1

EXERCISE-2

EXERCISE-3

EXERCISE-4

EXERCISE-5

WORK, POWER & ENERGY

1.0 WORK OF A FORCE

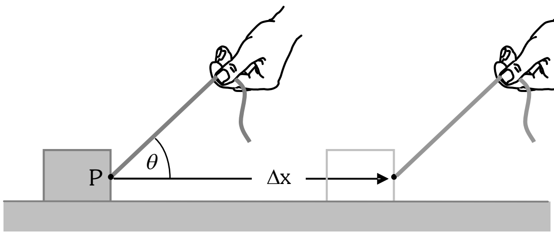
SL

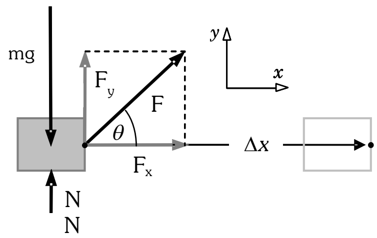
In everyday life by the word 'work', we refer to a vast category of jobs. This meaning is not precise enough to be used as a physical quantity. It was the practical need of scientists and engineers of the late 18^{th} century at the start of Industrial Revolution that made necessary to define work quantitatively as a physical quantity. Physical concept of work involves a force and displacement produced.

1.1 Work of a constant Force on a body in rectilinear motion

SL AL

To understand concept of work, consider a block being pulled with the help of a string on frictionless horizontal ground. Let pull $\vec{F}$ of the string on the box is constant in magnitude as well as direction. The vertical component F_y of $\vec{F}$, the weight (mg) and the normal reaction N all act on the box in vertical direction but none of them can move it unless F_y becomes greater than the weight (mg). Consider that F_y is smaller than the weight of the box. Under this condition, the box moves along the plane only due to the horizontal component F_x of the force $\vec{F}$. The weight mg, the normal reaction N from the ground and vertical component F_y all are perpendicular to the displacement therefore have no contribution in its displacement. Therefore, work is done on the box only by the horizontal component F_x of the force $\vec{F}$.





Here we must take care of one more point that is the box, which is a rigid body and undergoes translation motion therefore, displacement of every particle of the body including that on which the force is applied are equal. The particle of a body on which force acts is known as point of application of the force.

Now we observer that block is displaced & its speed is increased. And work W of the force $\vec{F}$ on the block is proportional to the product of its component in the direction of the displacement and the magnitude of the displacement Δx.

$$W \propto F_x \cdot \Delta x = F\cos\theta \cdot \Delta x$$

If we chose one unit of work as newton-meter, the constant of proportionality becomes unity and we have

$$W = F\cos\theta \cdot \Delta x = \vec{F} \cdot \Delta\vec{x}$$

The work W done by the force $\vec{F}$ is defined as scalar product of the force $\vec{F}$ and displacement $\Delta\vec{x}$ of point of application of the force.

For a generalization, work done by constant force $\vec{F}$ is

$$W_{i\to f} = \vec{F}\cdot(\vec{r}_f - \vec{r}_i) = \vec{F}\cdot\Delta\vec{r} \quad \text{where } \vec{r}_f = \text{final position vector, } \vec{r}_i = \text{initial positon vector}$$

Unit and Dimensions of Work of a Force

SI unit of work is "joule", named after famous scientist James Prescott Joule. It is abbreviated by letter J.

1 joule = 1 newton × 1 meter

CGS unit of work is "erg". Its name is derived from the Greek ergon, meaning work.

1 erg = 1 dyne × 1 centimeter, 1 erg = 10^{-7} Joule

Dimensions of work are ML^2T^{-2}

Illustrations

Illustration 1. Calculate work done by the force $\vec{F} = (3\hat{i} - 2\hat{j} + 4\hat{k})$ N in carrying a particle from point (–2 m, 1 m, 3 m) to (3 m, 6 m, –2 m).

Solution. The force $\vec{F}$ is a constant force, therefore we can use equation $W_{i\to f} = \vec{F}\cdot\Delta\vec{r}$.

$$W = \vec{F}.\Delta\vec{r} = (3\hat{i} - 2\hat{j} + 4\hat{k}).(5\hat{i} + 5\hat{j} - 5\hat{k}) = -15 \text{ J}$$

Illustration 2. A particle is shifted form point (0, 0, 1 m) to (1 m, 1 m, 2 m), under simultaneous action of several forces. Two of the forces are $\vec{F}_1 = (2i + 3\hat{j} - \hat{k})N$ and $\vec{F}_2 = (\hat{i} - 2\hat{j} + 2\hat{k})N$. Find work done by these two forces.

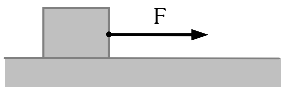
Solution. Work done by a constant force equals to dot product of the force and displacement vectors.

$$W = \vec{F} \cdot \Delta\vec{r} \rightarrow W = (\vec{F}_1 + \vec{F}_2) \cdot \Delta\vec{r}$$

Substituting given values, we have

$$W = (3\hat{i} + \hat{j} + \hat{k}) \cdot (\hat{i} + \hat{j} + \hat{k}) = 3 + 1 + 1 = 5\,J$$

Illustration 3. A 10 kg block placed on a rough horizontal floor is being pulled by a constant force 50 N. Coefficient of kinetic friction between the block and the floor is 0.4. Find work done by each individual force acting on the block over displacement of 5 m.



Solution. Forces acting on the block are its weight (mg = 100 N), normal reaction (N = 100 N) from the ground, force of kinetic friction (f = 40 N) and the applied force (F = 50 N) and displacement of the block are shown in the given figure.

All these force are constant force, therefore we use equation

$$W_{i\to f} = \vec{F} \cdot \Delta\vec{r}\,.$$

Work done W_g by the gravity i.e. weight of the block

$$W_g = 0\,J \qquad (\because m\vec{g} \perp \Delta\vec{x})$$

Work done W_N by the normal reaction

$$W_N = 0\,J \qquad (\because \vec{N} \perp \Delta\vec{x})$$

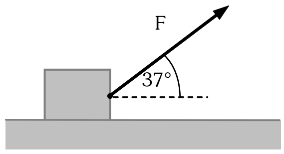
Work done W_F by the applied force

$$W_F = 250\,J \qquad (\because \vec{F} \parallel \Delta\vec{x})$$

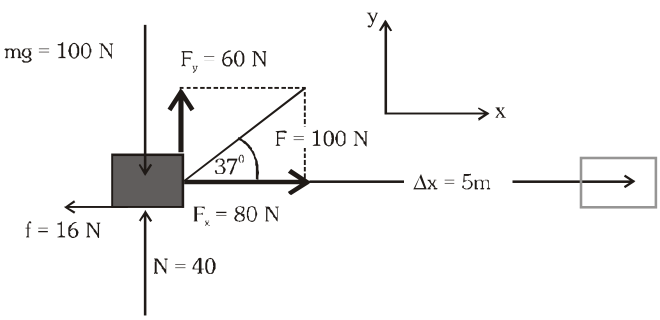
Work done W_f by the force of kinetic friction

$$W_f = -200\,J \qquad (\because \vec{f} \uparrow\downarrow \Delta\vec{x})$$

Illustration 4. A 10 kg block placed on a rough horizontal floor is being pulled by a constant force 100 N acting at angle 37°. Coefficient of kinetic friction between the block and the floor is 0.4. Find work done by each individual force acting on the block over displacement of 5 m.



Solution. Forces acting on the block are its weight (mg = 100 N), normal reaction (N = 40 N) from the ground, force of kinetic friction (f = 16 N) and the applied force (F = 100 N) and displacement of the block are shown in the given figure.



All these force are constant force, therefore we use equation

$$W_{i\to f} = \vec{F}\cdot\Delta\vec{r}\,.$$

Work done W_g by the gravity i.e. weight of the block

$$W_g = 0\text{ J} \qquad (\because m\vec{g} \perp \Delta\vec{x})$$

Work done W_N by the normal reaction

$$W_N = 0\text{ J} \qquad (\because \vec{N} \perp \Delta\vec{x})$$

Work done W_F by the applied force

$$W_F = \vec{F}\cdot\Delta\vec{x} = F_x\Delta x = 400\text{ J}$$

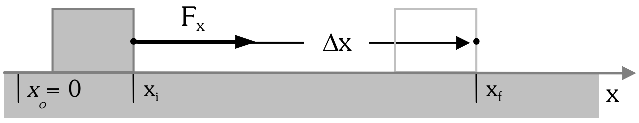
Work done W_f by the force of kinetic friction

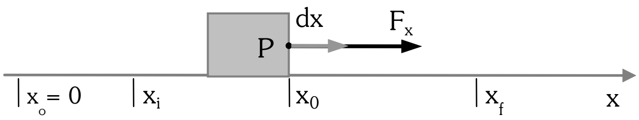
$$W_f = -80\text{ J} \qquad (\because \vec{f} \uparrow\downarrow \Delta\vec{x})$$

1.2 Work of a variable Force on a body in rectilinear motion

AL

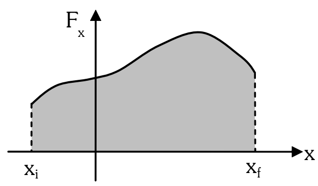
Usually a variable force does not vary appreciably during an infinitely small displacement of its point of application and therefore can be assumed constant in that infinitely small displacement.





Consider a box being pulled by a variable horizontal force F_x which is known as function of position x. We now calculate work done by this force in moving the box from position x_i to x_f. Over any infinitely small displacement dx the force does not vary appreciably and can be assumed constant. Therefore to calculate work done dW by the force F_x during infinitely small displacement dx is given by $dW = \vec{F}_x.d\vec{x} = F_x dx$. Integrating dW from x_i to x_f we obtain work done by the force in moving the box from position x_i to x_f.

$$W_{i\to f} = \int_{x_i}^{x_f} F_x.dx$$



The above equation also suggests that in rectilinear motion work done by a force equals to area under the force-position graph and the position axis.

In the given figure is shown how a force F_x varies with position coordinate x. Work done by this force in moving its point of application from position x_i to x_f equals to area of the shaded portion.

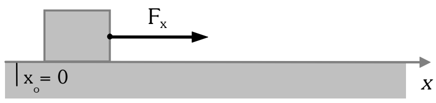
Illustrations

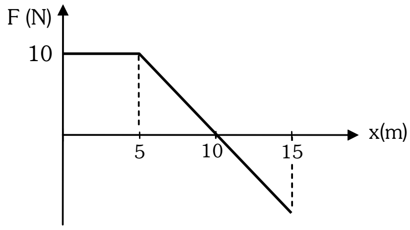
Illustration 5. A force which varies with position coordinate x according to equation F_x = (4x+2) N. Here x is in meters. Calculate work done by this force in carrying a particle from position x_i = 1 m to x_f = 2 m.

Solution. Using the equation $W_{i\to f} = \int_{x_i}^{x_f} F_x.dx$, we have $W_{i\to f} = \int_1^2 (4x+2).dx = 8\text{ J}$

The above problem can also be solved by using graph

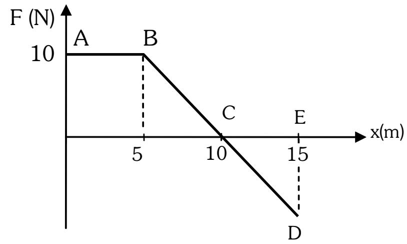
Illustration 6. A horizontal force F is used to pull a box placed on floor. Variation in the force with position coordinate x measured along the floor is shown in the graph.





(a) Calculate work done by the force in moving the box from x = 0 m to x = 10 m.
(b) Calculate work done by the force in moving the box from x = 10 m to x = 15 m.
(c) Calculate work done by the force in moving the box from x = 0 m to x = 15 m.

Solution. In rectilinear motion work done by a force equals to area under the force-position graph and the position axis



(a) $W_{0\to10}$ = Area of trapazium OABC = 75 J

(b) $W_{10\to15}$ = –Area of triangle CDE = –25 J

(c) $W_{0\to15}$ = Area of trapazium OABC – Area of triangle CDE = 50 J

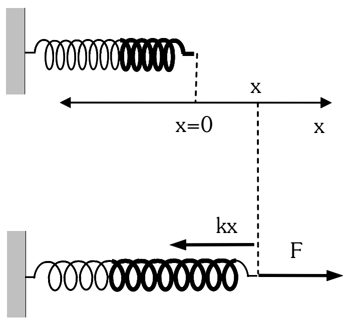
Illustration 7. A coiled spring with one end fixed has a realaxed length l_0 and a spring constant k. What amount of work must be done to stretch the spring by an amount s?

Solution. In order to stretch the free end of the spring to a point x, some agency must exert a force F, which must everywhere be equal to spring force.

$$F = kx$$

The applied force and the spring force are shown in the adjoining figure.

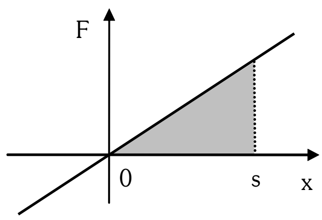
The work done W_F by the applied force in moving the free end of the spring from x = 0 to x = s be



$$W_F = \int_0^s \vec{F}\cdot d\vec{x} = \tfrac{1}{2}ks^2$$

Use of Graph

The variation in F with extension x in the spring is linear therefore area under the force extension graph can easily be calculated. This area equals to the work done by the applied force. The graph showing variation in F with x is shown in the adjoining figure.



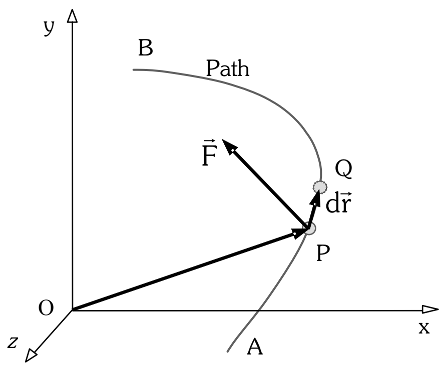
Variation in F with extension x.

$$W_F = \int_0^s \vec{F}\cdot d\vec{x} = \text{ Area of the shaded portion} = \tfrac{1}{2}ks^2$$

1.3 Work of a Variable Force on a Body in Curvilinear Translation Motion

SL AL

Till now we have learnt how to calculate work of a force in rectilinear motion. We can extend this idea to calculate work of a variable force on any curvilinear path. To understand this let us consider a particle moving from point A to B. There may be several forces acting on it but here we show only that force whose work we want to calculate. This force may be constant or variable. Let this force is denoted by $\vec{F}$. Consider an infinitely small path length PQ. Over this infinitely small path length, the force can be assumed constant. Work of this force $\vec{F}$ over this path length PQ is given by



$$dW = \vec{F}\cdot d\vec{r}$$

The whole path from A to B can be divided in several such infinitely small elements and work done by the force over the whole path from A to B is sum of work done over every such infinitely small element. This we can calculate by integration. Therefore, work done $W_{A\to B}$ by the force $\vec{F}$ is given by the following equation.

$$W_{A\to B} = \int_{\vec{r}_A}^{\vec{r}_B} \vec{F}\cdot d\vec{r}$$

Illustrations

Illustration 8. An object is displaced from point A(1, 2) to B(0, 1) by applying force $\vec{F} = x\hat{i} + 2y\hat{j}$. Find out work done by $\vec{F}$ to move the object from point A to B.

Solution

$$dW = \vec{F}\cdot\vec{dr}$$

$$W = \int_{x_i}^{x_f} F_x dx + \int_{y_i}^{y_f} F_y dy$$

$$W = \int_{1}^{0} x\,dx + \int_{2}^{1} 2y\,dy$$

$\therefore$ W = –3.5 J

1.4 Work of a force depends on frame of reference

AL

A force does not depend on frame of reference and assumes same value in all frame of references, but displacement depends on frame of reference and may assume different values relative to different reference frames. Therefore, work of a force depends on choice of reference frame. For Illustration , consider a man holding a suitcase stands in a lift that is moving up. In the reference frame attached with the lift, the man applies a force equal to weight of the bag but the displacement of the bag is zero, therefore work of this force on the bag is zero. However, in a reference frame attached with the ground the bag has displacement equal to that of the lift and the force applied by the man does a nonzero work.

BEGINNER'S BOX–1

Work done by Constant and Variable Forces

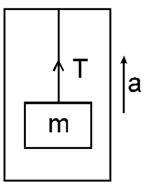
1. A rigid body of mass 6 kg is under a force which causes displacement in it given by $S = \frac{t^2}{4}$ metres where t is time. The work done by the force in first 2 seconds is

(A) 12 J (B) 9 J (C) 6 J (D) 3 J

2. A ball is released from the top of a tower. The ratio of work done by force of gravity in first, second and third second of the motion of the ball is

(A) 1 : 2 : 3 (B) 1 : 4 : 9 (C) 1 : 3 : 5 (D) 1 : 5 : 3

3. A block of mass m is suspended by a light thread from an elevator. The elevator start from rest and is accelerating upward with uniform acceleration a. The work done by tension on the block during first t seconds is :



(A) $\frac{m}{2}(g + a)\,at^2$ (B) $\frac{m}{2}(g - a)at^2$ (C) $\frac{m}{2}gat^2$ (D) 0

4. A force $\vec{F} = -K(y\hat{i} + x\hat{j})$ where K is a positive constant, acts on a particle moving in the x-y plane. Starting from the origin, the particle is taken along the positive x-axis to the point (a, 0) and then parallel to the y-axis to the point (a, a). The total work done by the force $\vec{F}$ on the particle is :
(A) $-2Ka^2$ (B) $2Ka^2$ (C) $-Ka^2$ (D) Ka^2

5. A chord is used to lower vertically a block of mass M a distance d at a constant downward acceleration of g/4. Then the work done by the cord on the block is :
(A) mgd/4 (B) 3 Mgd/4 (C) –3 Mgd/4 (D) Mgd

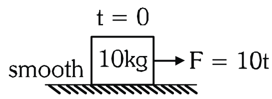
6. A body of mass m is moving in a circle of radius ' r ' with a constant speed ' v '. The force on the body is $\frac{mv^2}{r}$ and is directed towards the centre . What is the work done by the force in moving the body half the circumference of the circle .
(A) $\frac{mv^2}{r} \times \pi r$ (B) zero (C) $\frac{mv^2}{r^2}$ (D) $\frac{\pi r^2}{mv^2}$

7. Calculate the work done by a coolie in carrying a load of mass 10 kg on his head when he walks uniformly a distance of 5 m in the (i) horizontal direction (ii) vertical direction. (Take $g = 10\ m/s^2$)

8. A particle moves along the x-axis from x = 0 to x = 5 m under the influence of a force F(in N) given by $F = 3x^2 - 2x + 7$. Calculate the work done by this force.

9. A flexible chain of length ℓ and mass m is slowly pulled at constant speed up over the edge of a table by a force F parallel to the surface of the table. Assuming that there is no friction between the table and chain, calculate the work done by force F till the chain reaches to the horizontal surface of the table.

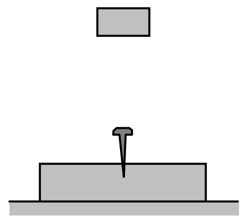
10. A time dependent force F = 10 t newton is applied on 10 kg block as shown in figure.
Find out the work done by F in first 2 seconds. (block starts from rests)



2.0 WORK AND ENERGY

SL

Suppose you have to push a heavy box on a rough horizontal floor. You apply a force on the box it moves and you do work. If you continue pushing, after some time you get tired and become unable to maintain your speed and eventually become unable to push the box further. You take rest and next day you can repeat the experiment and same thing happens. Why you get tired and eventually become unable to pull the box further? The explanation lies in fact that you have a capacity to do work, and when it is used up, you become unable to do work further. Next day you recollect this capacity and repeat the experiment. This capacity of doing work is known as energy. Here it comes from chemical reactions occurring with food in our body and is called chemical energy.



Consider another experiment in which we drop a block on a nail as shown in the figure. When set free, weight of the block accelerates it through the distance it falls and when it strikes the nail, its motion vanishes and what appear are the work that drives the nail, heat that increases temperature of the surrounding, and sound that causes air molecules to oscillate. If the block were placed on the nail and pressed hard, it would not have been so effective. Actually, the weight and the distance through which the hammer falls on the nail decide its effectiveness. We can explain these events by assuming that the block possesses energy due to its position at height against gravity.

This energy is known as gravitational potential energy. When the block falls, this potential energy is converted into another form that is energy due to motion. This energy is known as kinetic energy. Moreover, when the block strikes the nail this kinetic energy is converted into work driving the nail, increasing temperature and producing sound.

2.1 Potential, Kinetic and Mechanical Energy

If a material-body is moved against a force like gravitational, electrostatic, or spring, a work must be done. In addition, if the force continues to acts even after the displacement, the work done can be recovered in form of energy, if the body is set loose. This recoverable stored energy by virtue of position in a force field is defined as potential energy, a name given by William Rankine.

All material bodies have energy due to their motion. This energy is known as kinetic energy, a name given by Lord Kelvin.

These two forms of energies - the kinetic energy and the potential energy are directly connected with motion of the body and force acting on the body respectively. They are collectively known as mechanical energy.

2.2 Other forms of Energy

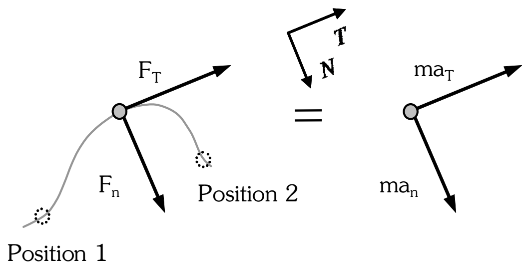
Thermal energy, sound energy, chemical energy, electrical energy and nuclear energy are Illustration s of some other forms of energy. Actually, in very fundamental way every form of energy is either kinetic or potential in nature. Thermal energy which is contribution of kinetic energy of chaotic motion of molecules in a body and potential energy due to intermolecular forces within the body. Sound energy is contribution kinetic energy of oscillating molecules and potential energy due intermolecular forces within the medium in which sound propagates. Chemical energy is contribution of potential energy due inter-atomic forces. Electric energy is kinetic energy of moving charge carries in conductors. In addition, nuclear energy is contribution of electrostatic potential energy of nucleons.

In fact, every physical phenomenon involves in some way conversion of one form of energy into other. Whenever mechanical energy is converted into other forms or vice versa it always occurs through forces and displacements of their point of applications i.e. work. Therefore, we can say that work is measure of transfer of mechanical energy from one body to other. That is why the unit of energy is usually chosen equal to the unit of work.

2.3 Work-Kinetic Energy Theorem

Consider the situation described in the figure. The body shown is in translation motion on a curvilinear path with increasing speed. The net force acting on the body must have two components – the tangential component necessary to increase the speed and the normal component necessary to change the direction of motion. Applying Newton's laws of motion in an inertial frame, we have

$$\sum F_T = ma_T \text{ and } \sum F_N = ma_N$$



Let the body starts at position 1 with speed v_1 and reaches position 2 with speed v_2. If an infinitely small path increment is represented by vector $d\vec{s}$, the work done by the net force during the process is

$$W_{1\to2} = \int_1^2 \vec{F}\cdot d\vec{s} = \int_1^2 \left(\vec{F}_T + \vec{F}_N\right)\cdot d\vec{s} = \int_1^2 F_T ds$$

$$W_{1\to2} = \int_1^2 ma_T ds = \int_{v_1}^{v_2} mvdv = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2$$

The terms $\frac{1}{2}mv_1^2$ and $\frac{1}{2}mv_2^2$ represent the kinetic energies K_1 and K_2 of the particle at position-1 and 2 respectively. With this information the above equation reduces to $W_{1\to2} = K_2 - K_1$

The above equation expresses that the work done by all external forces acting on a body in carrying it from one position to another equals to the change in the kinetic energy of the body between these positions. This statement is known as the work kinetic energy theorem.

How to apply work kinetic energy theorem

The work kinetic energy theorem is deduced here for a single body moving relative to an inertial frame, therefore it is recommended at present to use it for a single body in inertial frame. To use work kinetic energy theorem the following steps should be followed.

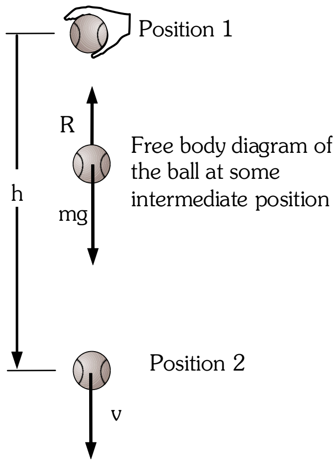
- Identify the initial and final positions as position 1 and 2 and write expressions for kinetic energies, whether known or unknown.

- Draw the free body diagram of the body at any intermediate stage between positions 1 and 2. The forces shown will help in deciding their work. Calculate work by each force and add them to obtain work done $W_{1\to2}$ by all the forces.
- $K_2 - K_1$ = Sum of work done by all force.

Illustrations

Illustration 9. A 5kg ball when falls through a height of 20 m acquires a speed of 10 m/s. Find the work done by air resistance.

Solution. The ball starts falling from position 1, where its speed is zero; hence, kinetic energy is also zero.



$$K_1 = 0\text{ J} \quad ...(i)$$

During downwards motion of the ball constant gravitational force mg acts downwards and air resistance R of unknown magnitude acts upwards as shown in the free body diagram. The ball reaches position 20 m below the position-1 with a speed v = 10 m/s, so the kinetic energy of the ball at position 2 is

$$K_2 = \tfrac{1}{2}mv^2 = 250\text{ J} \quad ...(ii)$$

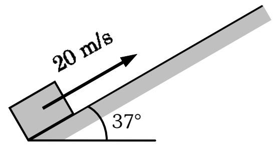
The work done by gravity

$$W_{g,1\to2} = mgh = 1000\text{ J} \quad ...(iii)$$

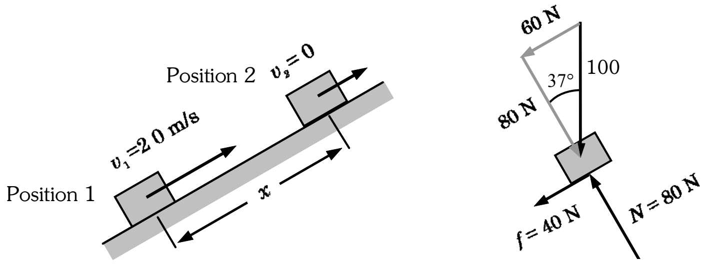
Denoting the work done by the air resistance $W_{R,1\to2}$ and making use of eq. 1, 2, and 3 in work kinetic energy theorem, we have

$$W_{1\to2} = K_2 - K_1 \to W_{g,1\to2} + W_{R,1\to2} = K_2 - K_1 \Rightarrow W_{R,1\to2} = -750\text{J}$$

Illustration 10. A box of mass m = 10 kg is projected up an inclined plane from its foot with a speed of 20 m/s as shown in the figure. The coefficient of friction μ between the box and the plane is 0.5. Find the distance traveled by the box on the plane before it stops first time.



Solution. The box starts from position 1 with speed v_1 = 20 m/s and stops at position 2.



Kinetic energy at position 1: $K_1 = \tfrac{1}{2}mv_1^2 = 2000\text{ J}$

Kinetic energy at position 2: $K_2 = 0$ J

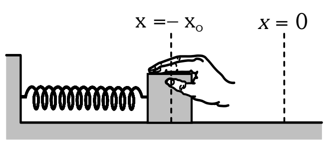
Work done by external forces as the box moves from position 1 to position 2:

$$W_{1\to2} = W_{g,1\to2} + W_{f,1\to2} = -60x - 40x = -100x\text{ J}$$

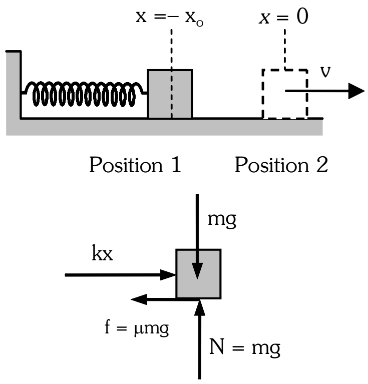
Applying work energy theorem for the motion of the box from position 1 to position 2, we have

$$W_{1\to2} = k_2 - k_1 \Rightarrow -100\,x = 0 - 2000 \Rightarrow x = 20\text{ m}$$

Illustration 11. A box of mass m is attached to one end of a coiled spring of force constant k. The other end of the spring is fixed and the box can slide on a rough horizontal surface, where the coefficient of friction is μ. The box is held against the spring force compressing the spring by a distance x_o. The spring force in this position is more than force of limiting friction. Find the speed of the box when it passes the equilibrium position, when released.



Solution. Before the equilibrium position, when the box passes the position coordinate –x, forces acting on it are its weight mg, normal reaction N from the horizontal surface, the force of kinetic friction f, and spring force F = kx as shown in the free body diagram. Let the box passes the equilibrium position with a speed v_o.

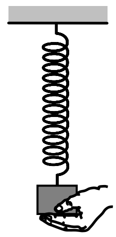


Applying work energy theorem on the box when it moves from position 1 ($-x_o$) to position 2 (x = 0), we have

$$W_{1\to2} = K_2 - K_1 \Rightarrow W_{F,1\to2} + W_{f,1\to2} = K_2 - K_1$$

$$\tfrac{1}{2}kx_o^2 - \mu mgx_o = \tfrac{1}{2}mv_o^2 - 0 \Rightarrow v_o = \sqrt{\left(kx_o^2 - 2\mu mgx_o\right)}$$

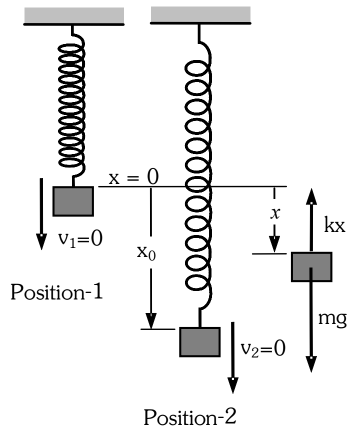
Illustration 12. A block of mass m is suspended from a spring of force constant k. it is held to keep the spring in its relaxed length as shown in the figure.



(a) The applied force is decreased gradually so that the block moves downward at negligible speed. How far below the initial position will the block stop?

(b) The applied force is removed suddenly. How far below the initial position, will the block come to an instantaneous rest?

Solution. (a) As the applied force (F) is decreased gradually, everywhere in its downward motion the block remains in the state of translational equilibrium and moves with negligible speed. Its weight (mg) is balanced by the upward spring force (kx) and the applied force. When the applied force becomes zero the spring force becomes equal to the weight and the block stops below a distance x_o from the initial position. The initial and final positions and free body diagram of the block at any intermediate position are shown in the adjoining figure. Applying the conditions of equilibrium, we have



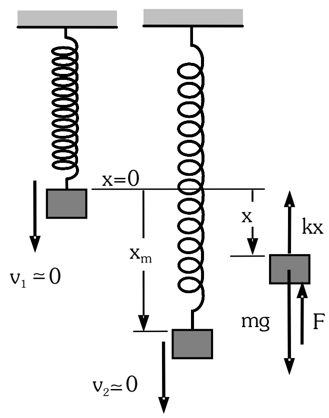
$$x_o = \frac{mg}{k}$$

(b) In the previous situation the applied force was decreased gradually keeping the block everywhere in equilibrium. If the applied force is removed suddenly, the block will accelerate downwards. As the block moves, the increase in spring extension increases the upward force, due to which acceleration decreases until extension becomes x_o. At this extension, the block will acquire its maximum speed and it will move further downward. When extension becomes more than x_o spring force becomes more than the weight (mg) and the block decelerates and ultimately stops at a distance x_m below the initial position. The initial position-1, the final position-2, and the free body diagram of the block at some intermediate position when spring extension is x are shown in the adjoining figure.

Kinetic energy in position-1 is $K_1 = 0$

Kinetic energy in position-2 is $K_2 = 0$

Work done $W_{1\to2}$ by gravity and the spring force is



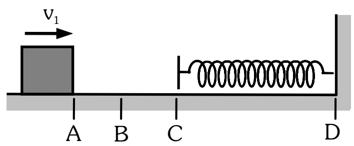
$$W_{1\to2} = W_{g,1\to2} + W_{spring,1\to2} = mgx_m - \tfrac{1}{2}kx_m^2$$

Using above values in the work energy theorem, we have

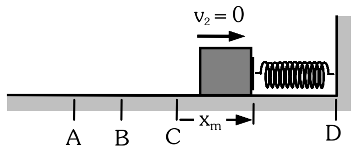
$$W_{1\to2} = K_2 - K_1 \to mgx_m - \tfrac{1}{2}kx_m^2 = 0$$

$$x_m = 2mg/k$$

Illustration 13. A block of mass m = 0.5 kg slides from the point A on a horizontal track with an initial sped of v_1 = 3 m/s towards a weightless horizontal spring of length 1 m and force constant k = 2 N/m. The part AB of the track is frictionless and the part BD has the coefficients of static and kinetic friction as 0.22 and 0.2 respectively. If the distance AB and BC are 2 m and 2.14 m respectively, find the total distance through which the block moves before it comes to rest completely. g = 10 m/s^2.



Solution Since portion AB of the track is smooth, the block reaches B with velocity v_1. Afterward force of kinetic friction starts opposing its motion. As the block passes the point C the spring force also starts opposing its motion in addition to the force of kinetic friction. The work done by these forces decrease the kinetic energy of the block and stop the block momentarily at a distance x_m after the point C.



Kinetic energy of the block at position-1 is $K_1 = \frac{1}{2}mv_1^2 = 2.25$ J.

Kinetic energy of the block at position-2 is $K_2 = \frac{1}{2}mv_2^2 = 0$ J.

Work $W_{f,1\to2}$ done by the frictional force before the block stops is

$$W_{f,1\to2} = \mu mg(BC + x_m) = 2.14 + x_m$$

Work $W_{s,1\to2}$ done by the spring force before the block stops is

$$W_{s,1\to2} = \int_{x=0}^{x_m} kx\,dx = \frac{1}{2}kx_m^2 = x_m^2$$

Using above information and the work energy principle, we have

$$W_{1\to2} = K_2 - K_1 \Rightarrow 2.14 + x_m + x_m^2 = 2.25 \Rightarrow x_m = 0.1 \text{ m.}$$

The motion of block after it stops momentarily at position-2 depends upon the condition whether the spring force is more than or less than the force of limiting friction. If the spring force in position-2 is more than the force of limiting friction the block will move back and if the spring force in position-2 is less than the force of static friction the block will not move back and stop permanently.

Spring force F_s at position-2 is $F_s = kx_m = 0.2$ N.

The force of limiting friction f_m is $f_m = \mu_s mg = 1.1$ N.

The force of limiting friction is more than the spring force therefore the block will stop at position-2 permanently.

The total distance traveled by the block = AB + BC + x_m =4.24 m.

BEGINNER'S BOX-2

Work Energy Theorem

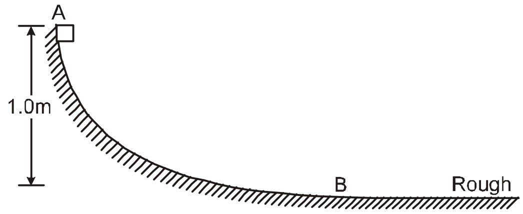
1. A body moving at 2 m/s can be stopped over a distance x. If its kinetic energy is doubled, how long will it go before coming to rest, if the retarding force remains unchanged ?

(A) x (B) 2x (C) 4x (D) 8x

2. A rod of length 1m and mass 0.5 kg hinged at one end, is initially hanging vertical. The other end is now raised slowly until it makes an angle 60° with the vertical. The required work is : (use g =9.8 m/s^2)

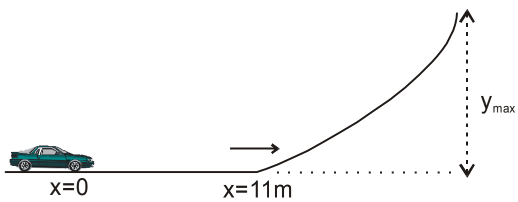
(A) 1.522 J (B) 1.225 J (C) 2.125 J (D) 3.125 K

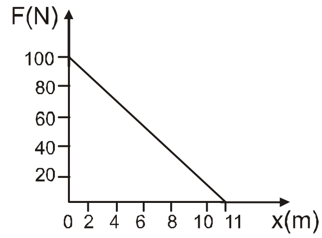
3. A block weighing 10 N travles down a smooth curved track AB joined to a rough horizontal surface (figure). The rough surface has a friction coefficient of 0.20 with the block. If the block starts slipping on the track from a point 1.0 m above the horizontal surface, the distance it will move on the rough surface is :



(A) 5.0 m (B) 10.0 m (C) 15.0 m (D) 20.0 m

4. A toy car of mass 5 kg moves up a ramp under the influence of force F plotted against displacement x. The maximum height attained is given by (neglect friction)





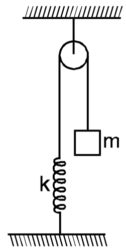
(A) $y_{max} = 20$ m (B) $y_{max} = 15$ m (C) $y_{max} = 11$ m (D) $y_{max} = 5$ m

5. A stone is projected with initial velocity u from a building of height h. After some time the stone falls on ground. Find out speed with it strikes the ground.

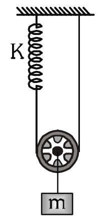
6. A force of 1000 N acts on a particle parallel to its direction of motion which is horizontal. Its velocity increases from 1 m s^{-1} to 10 m s^{-1}, when the force acts through a distance of 4 metre. Calculate the mass of the particle. Given : a force of 10 newton is necessary for overcoming friction.

7. A block of mass m moving at a speed v compresses a spring through a distance x before its speed is halved. Find the spring constant of the spring.

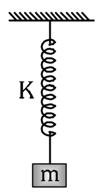
8. Consider the situation shown in figure. Initially the spring is unstretched when the system is released from rest. Assuming no friction in the pulley, find the maximum elongation of the spring.



9. In the given figure, spring, string and pulley are massless. System released from rest when spring in its natural length. Find maximum elongation in the spring.



10. In the given figure initially spring in its natural length and system released from rest. Finally block again comes to at rest at mean position due to air resistance. Find total work done by
(i) gravitational force
(ii) spring force
(iii) force due to air resistance.



3.0 CONSERVATIVE AND NON-CONSERVATIVE FORCES

SL AL

Gravitational, electrostatic, and restoring force of a spring are some of the natural forces with a property in common that work done by them in moving a particle from one point to another depends solely on the locations of the initial and final points and not on the path followed irrespective of pair of points selected. On the other hand, there are forces such as friction, whose work depends on path followed. Accordingly, forces are divided into two categories – one whose work is path independent and other whose work is path dependant. The forces of the former category are known as conservative forces and of the later one as non-conservative forces.

A force, whose finite non-zero work $W_{1\to2}$ expended in moving a particle from a position 1 to another position 2 is independent of the path followed, is defined as a conservative force.

Consider a particle moving from position 1 to position 2 along different paths A, B, and C under the action of a conservative force F as shown in figure. If work done by the force along path A, B, and C are $W_{1\to2,A}$, $W_{1\to2,B}$ and $W_{1\to2,C}$ respectively, we have

$$W_{1\to2,A} = W_{1\to2,B} = W_{1\to2,C}$$

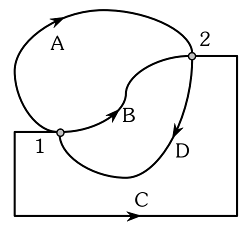
If these works are positive, the work done $W_{2\to1,D}$ by the same force in moving the particle from position 2 to 1 by any other path say D will have the same magnitude but negative sign. Hence, we have

$$W_{1\to2,A} = W_{1\to2,B} = W_{1\to2,C} = -W_{2\to1,D}$$

$$W_{1\to2,A} + W_{2\to1,D} = W_{1\to2,B} + W_{2\to1,D} = W_{1\to2,C} + W_{2\to1,D} = 0$$

The above equations are true for any path between any pair of points-1 and 2.

Representing the existing conservative force by $\vec{F}$, an infinitely path increment by $d\vec{r}$ and integration over a closed path by $\oint(\ )\cdot d\vec{r}$, the above equation can be represented in an alternative form as $\oint \vec{F}\cdot d\vec{r} = 0$



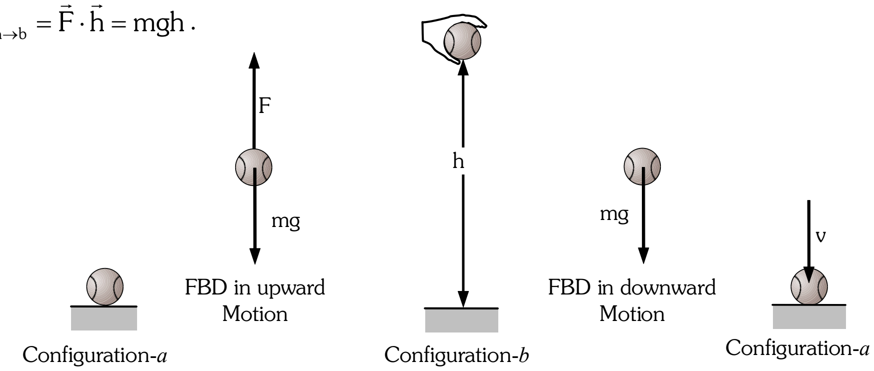
The equation () shows that the total work done by a conservative force in moving a particle from position 1 to another position 2 and moving it back to position 1 i.e. around a closed path is zero. It is used as a fundamental property of a conservative force.

- A conservative force must be function only of position not of velocity or time.
- All uniform and constant forces are conservative forces. Here the term uniform means same magnitude and direction everywhere in the space and the term constant means same magnitude and direction at all instants of time.
- All central forces are conservative forces. A central force at any point acts always towards or away from a fixed point and its magnitude depends on the distance from the fixed point.
- All forces, whose magnitude or direction depends on the velocity, are non-conservative. Sliding friction, which acts in opposite direction to that of velocity, and viscous drag of fluid depends in magnitude of velocity and acts in opposite direction to that of velocity are non-conservative.

4.0 POTENTIAL ENERGY

SL AL

Consider a ball of mass m placed on the ground and someone moves it at negligible speed through a height h above the ground as shown in figure. The ball remains in the state of equilibrium therefore the upward force F applied on it everywhere equals to the weight (mg) of the ball. The work $W_{a\to b}$ done by the applied force on the ball is $W_{a\to b} = \vec{F}\cdot\vec{h} = mgh$.

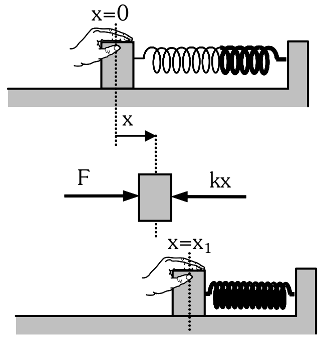


Now if the ball is dropped from the height h it starts moving downwards due to its weight and strikes the ground with speed v. The work $W_{b\to a}$ done by its weight during its downward motion imparts it a kinetic energy K_c, which is obtained by using work energy principle and the above equation as

$$W_{b\to a} = K_a - K_b \Rightarrow mgh = \tfrac{1}{2}mv^2 - 0 \Rightarrow K_a = \tfrac{1}{2}mv^2 = mgh$$

Instead of raising the ball to height h, if it were thrown upwards with a speed v it would have reached the height h and returned to the ground with the same speed. Now if we assume a new form of energy that depends on the separation between the ball and the ground, the above phenomena can be explained. This new form of energy is known as potential energy of the earth-ball system. When ball moves up, irrespective of the path or method how the ball has been moved, potential energy of the earth-ball system increases. This increase equals to work done by applied force F in moving the ball to height h or negative of work done by gravity. When the ball descends, potential energy of the earth ball system decreases; and is recovered as the kinetic energy of the ball when separation vanishes. During descend of the ball gravity does positive work, which equals to decrease in potential energy.

Potential energy of the earth ball system is due to gravitation force and therefore is call gravitation potential energy. Change in gravitational potential energy equals to negative of work done by gravitational force. It is denoted by ΔU.



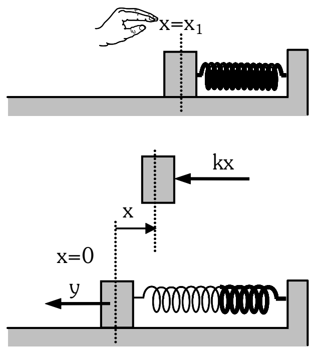
In fact, when the ball is released both the ball and the earth move towards each other and acquire momenta of equal magnitude but the mass of the earth is infinitely large as compared to that of the ball, the earth acquires negligible kinetic energy. It is the ball, that acquires almost all the kinetic energy and therefore sometimes the potential energy is erroneously assigned with the ball and called the potential energy of the ball. Nevertheless, it must be kept in mind that the potential energy belongs to the entire system.

As another instance, consider a block of mass m placed on a smooth horizontal plane and connected to one end of a spring of force constant k, whose other end is connected to a fixed support. Initially, when the spring is relaxed, no net force acts on the block and it is in equilibrium at position x = 0. If the block is pushed gradually against the spring force and moves at negligible speed without acceleration, at every position x, the applied force F balances the spring force kx. The work done $W_{0\to 1}$ by this force in moving the block from position x = 0 to x = x_1 is

$$W_{0\to 1} = \int_{x=0}^{x=x_1} \vec{F}\cdot d\vec{x} = \tfrac{1}{2}kx_1^2$$

If the applied force is removed, the block moves back and reaches its initial position with a kinetic energy K_0, which is obtained by applying work energy theorem together with the above equation.

$$W_{1\to 0} = K_0 - K_1 \Rightarrow K_0 = \tfrac{1}{2}mv^2 = \tfrac{1}{2}kx_1^2$$



The above equation shows that the work done on the block by the applied force in moving it from x = 0 to x = x_1 is stored in the spring block system as increase in potential energy and when the block returns to its initial position x = 0 this stored potential energy decreases and is recovered as the kinetic energy of the block. The same result would have been obtained if the block were pulled elongating the spring and then released. The change in potential energy of the spring block system when the spring length is increased or decreased by x equals to negative of work done by the spring force.

In both the above cases forces involved were conservative. In fact, work done against all conservative forces is recoverable. With every conservative force, we can associate a potential energy, whose change equals to negative of work done by the conservative force. For an infinitely small change in configuration, change in potential energy dU equals to the negative of work done dW_C by conservative forces.

$$dW = dU = -\,dW_C$$

Since a force is the interaction between two bodies, on very fundamental level potential energy is defined for every pair of bodies interacting with conservative forces. The potential energy of a system consisting of a large number of bodies thus will be sum of potential energies of all possible pairs of bodies constituting the system.

Because only change in potential energy has significance, we can chose potential energy of any configuration as reference value.

4.1 Gravitational potential energy for uniform gravitational force

Near the earth surface for heights small compared to the radius of the earth, the variation in the gravitational force between a body of mass m and the ground can be neglected. For such a system, change in gravitational potential energy in any vertically upward displacement h of mass m is given by ΔU = mgh and in vertical downward displacement h is given by $\Delta U = -$ mgh.

4.2 Gravitational potential energy for non-uniform gravitational force

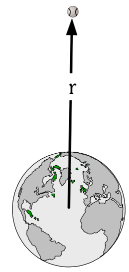
When motion of a body of mass m involves distances from the earth surface large enough, the variation in the gravitational force between the body and the earth cannot be neglected. For such physical situations the configuration, when the body is at infinitely large distance from the earth center is taken as the reference configuration and potential energy of this configuration is arbitrarily assumed zero ($U_\infty = 0$).

If the body is brought at negligible speed to a distance r from infinitely large distance from the earth center, the work done W_g by the gravitational force is given by the following equation.

$$W_g = \int_\infty^r \vec{F}_g \cdot d\vec{r} = \left[\frac{GMm}{r}\right]_\infty^r$$

Negative of this work done equals to change in potential energy of the system. Denoting potential energies in configuration of separation r and ∞ by U_r and U_∞, we have

$$U_r - U_\infty = -W_g \rightarrow U_r = -\frac{GMm}{r}$$



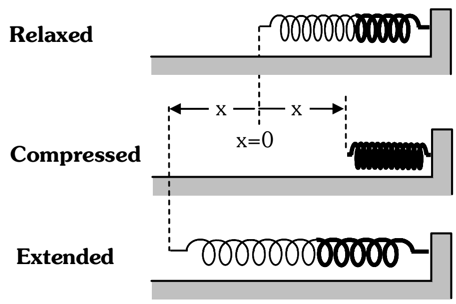
The ball at a distance *r* from the center of the earth.

4.3 Potential energy associated with spring force

The potential energy associated with a spring force of an ideal spring when compressed or elongated by a distance x from its natural length is defined by the following equation

$$U = \tfrac{1}{2}kx^2$$

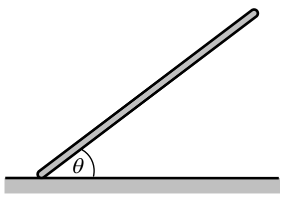
(assuming potential energy of spring is zero in natural length)

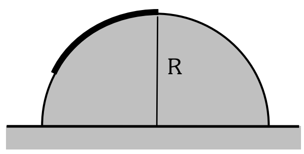


Illustrations

Illustration 14. Find the gravitational potential energies in the following physical situations. Assume the ground as the reference potential energy level.

(a) A thin rod of mass m and length L kept at angle θ with one of its end touching the ground.

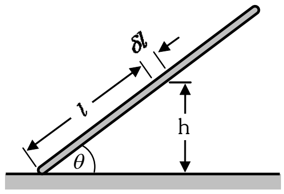




(b) A flexible rope of mass m and length L placed on a smooth hemisphere of radius R and one of the ends of the rope is fixed at the top of the hemisphere.

Solution In both the above situations, mass is distributed over a range of position coordinates. In such situations calculate potential energy of an infinitely small portion of the body and integrate the expression obtained over the entire range of position coordinates covered by the body.

(a) Assume a small portion of length $\delta\ell$ of the rod at distance ℓ from the bottom end and height of the midpoint of this portion from the ground is h. Mass of this portion is δm. When $\delta\ell$ approaches to zero, the gravitational potential energy dU of the assumed portion becomes

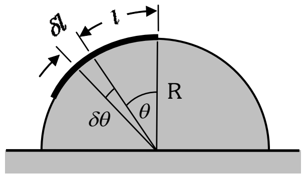


$$dU = \frac{m}{L} ghd\ell = \frac{m}{L} g\ell \sin\theta d\ell$$

The gravitational potential energy U of the rod is obtained by carrying integration of the above equation over the entire length of the rod.

$$U = \int_{\ell=0}^{L} \frac{m}{L} g\ell \sin\theta d\ell = \tfrac{1}{2} mgL \sin\theta$$

(b) The gravitational potential energy dU of a small portion of length $\delta\ell$ shown in the adjoining figure, when $\delta\ell$ approaches to zero is



$$dU = \frac{m}{L} gR^2 \cos\theta\, d\theta$$

The gravitational potential energy U of the rope is obtained by carrying integration of the above equation over the entire length of the rope.

$$U = \int_{\theta=0}^{\theta=\frac{L}{R}} \frac{m}{L} gR^2 \cos\theta d\theta = \frac{m}{L} gR^2 \left\{ \sin\left(\frac{L}{R}\right) \right\}$$

5.0 CONSERVATION OF MECHANICAL ENERGY

SL AL

The total potential energy of the system and the total kinetic energy of all the constituent bodies together are known as the mechanical energy of the system. If E, K, and U respectively denote the total mechanical energy, total kinetic energy, and the total potential energy of a system in any configuration, we have

$$E = K + U$$

Consider a system on which no external force acts and all the internal forces are conservative. If we apply work-kinetic energy $\left(W_{1\to2} = K_2 - K_1\right)$ theorem, the work $W_{1\to2}$ will be the work done by internal conservative forces, negative of which equals change in potential energy. Rearranging the kinetic energy and potential energy terms, we have

$$E = K_1 + U_1 = K_2 + U_2$$

The above equation takes the following forms

$$E = K + U = \text{constant}$$

$$\Delta E = 0 \Rightarrow \Delta K + \Delta U = 0$$

Above equations, express the principle of conservation of mechanical energy.

If there is no net work done by any external force or any internal non-conservative force, the total mechanical energy of a system is conserved.

The principle of conservation of mechanical energy is developed from the work energy principle for systems where change in configuration takes place under internal conservative forces only. Therefore, in physical situations, where external forces or non-conservative internal forces are involved, the use of work energy principle should be preferred.

In systems, where external forces or internal nonconservative forces do work, the net work done by these forces becomes equal to change in the mechanical energy of the system.

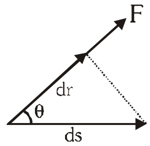
6.0 POTENTIAL ENERGY AND THE ASSOCIATED CONSERVATIVE FORCE

SL AL

We know how to find potential energy associated with a conservative force. Now we learn how to obtain the conservative force if potential energy function is known. Consider work done dW by a conservative force in moving a particle through an infinitely small path length $d\vec{s}$ as shown in the figure.

$$dU = -dW = -\vec{F} \cdot d\vec{s} = -Fds\cos\theta$$

From the above equation, the magnitude F of the conservative force can be expressed.



$$F = -\frac{dU}{ds\cos\theta} = -\frac{dU}{dr}$$

If we assume an infinitely small displacement $d\vec{r}$ in the direction of the force, magnitude of the force is given by the following equation.

$$F = -\frac{dU}{dr}$$

Here minus sign suggest that the force acts in the direction of decreasing potential energy. Therefore if we assume unit vector $\hat{e}_r$ in the direction of $d\vec{r}$, force vector $\vec{F}$ is given by the following equation.

$$\vec{F} = -\frac{dU}{dr}\hat{e}_r$$

Illustrations

Illustration 15. Force between the atoms of a diatomic molecule has its origin in the interactions between the electrons and the nuclei present in each atom. This force is conservative and associated potential energy U(r) is, to a good approximation, represented by the Lennard – Jones potential.

$$U(r) = U_o\left\{\left(\frac{a}{r}\right)^{12} - \left(\frac{a}{r}\right)^{6}\right\}$$

Here r is the distance between the two atoms and U_o and a are positive constants. Develop expression for the associated force and find the equilibrium separation between the atoms.

Solution. Using equation $F = -\frac{dU}{dr}$, we obtain the expression for the force

$$F = \frac{6U_0}{a}\left\{2\left(\frac{a}{r}\right)^{13} - \left(\frac{a}{r}\right)^{7}\right\}$$

At equilibrium the force must be zero. Therefore the equilibrium separation r_o is

$$r_o = 2^{\frac{1}{6}}a$$

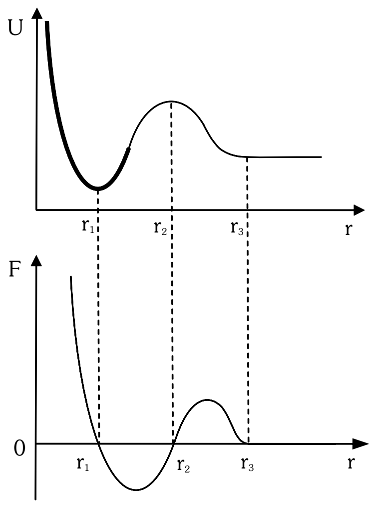
7.0 POTENTIAL ENERGY AND NATURE OF EQUILIBRIUM

SL AL

The above equation suggests that on every location where the potential energy function assumes a minimum or a maximum value or in every region where the potential energy function assumes a constant value, the associated conservative force becomes zero and a body under the action of only this conservative force must be in the state of equilibrium. Different status of potential energy function in the state of equilibrium suggests us to define three different types of equilibriums – the stable, unstable and neutral equilibrium.

The state of stable and unstable equilibrium is associated with a point location, where the potential energy function assumes a minimum and maximum value respectively, and the neutral equilibrium is associated with region of space, where the potential energy function assumes a constant value.

For the sake of simplicity, consider a one dimensional potential energy function U of a central force F. Here r is the radial coordinate of a particle. The central force F experienced by the particle equals to the negative of the slope of the potential energy function. Variation in the force with r is also shown in the figure.



Force is negative of the slope of the potential energy function.

At locations $r = r_1$, $r = r_2$, and in the region $r \geq r_3$, where potential energy function assumes a minimum, a maximum, and a constant value respectively, the force becomes zero and the particle is in the state of equilibrium.

7.1 Stable Equilibrium

At $r = r_1$ the potential energy function is a minima and the force on either side acts towards the point $r = r_1$. If the particle is displaced on either side and released, the force tries to restore it at $r = r_1$. At this location the particle is in the state of stable equilibrium. The dip in the potential energy curve at the location of stable equilibrium is known as potential well. A particle when disturbed from the state of stable within the potential well starts oscillations about the location of stable equilibrium. At the locations of stable equilibrium we have

$$F(r) = -\frac{\partial U}{\partial r} = 0\text{ ; and }\frac{\partial F}{\partial r} < 0\text{ ;}\quad\text{and }\frac{\partial^2 U}{\partial r^2} > 0$$

7.2 Unstable Equilibrium

At $r = r_2$ the potential energy function is a maxima, the force acts away from the point $r = r_2$. If the particle is displaces slightly on either side, it will not return to the location $r = r_2$. At this location, the particle is in the state of unstable equilibrium. At the locations of unstable equilibrium we have

$$F(r) = -\frac{\partial U}{\partial r} = 0\text{ therefore }\frac{\partial F}{\partial r} > 0\text{ ; and }\frac{\partial^2 U}{\partial r^2} < 0$$

7.3 Neutral Equilibrium

In the region $r \geq r_3$, the potential energy function is constant and the force is zero everywhere. In this region, the particle is in the state of neutral equilibrium. At the locations of neutral equilibrium we have

$$F(r) = -\frac{\partial U}{\partial r} = 0\text{ therefore }\frac{\partial F}{\partial r} = 0\text{ and }\frac{\partial^2 U}{\partial r^2} = 0$$

8.0 POWER

SL

When we purchase a car or jeep we are interested in the horsepower of its engine. We know that usually an engine with large horsepower is most effective in accelerating the automobile.

In many cases it is useful to know not just the total amount of work being done, but how fast the work is done. We define power as the rate at which work is being done.

$$\text{Average Power} = \frac{\text{Work done}}{\text{Time taken to do work}} = \frac{\text{Total change in kinetic energy}}{\text{Total change in time}}$$

If ΔW is the amount of work done in the time interval Δt. Then $P = \frac{\Delta W}{\Delta t} = \frac{W_2 - W_1}{t_2 - t_1}$

When work is measured in joules and t is in seconds, the unit for power is the joule per second, which is called watt. For motors and engines, power is usually measured in horsepower, where horsepower is 1 hp = 746 W. The definition of power is applicable to all types of work like mechanical, electrical, thermal.

$$\text{Instanteneous power } P = \frac{dW}{dt} = \frac{\vec{F}.d\vec{r}}{dt} = \vec{F}.\vec{v}$$

Where v is the instantaneous velocity of the particle and dot product is used as only that component of force will contribute to power which is acting in the direction of instantaneous velocity.

- Power is a scalar quantity with dimension $M^1L^2T^{-3}$
- SI unit of power is J/s or watt
- 1 horsepower = 746 watt

Illustrations

Illustration 16. What is the power of an engine which can lift 20 metric ton of coal per hour from a 20 metre deep mine?

Solution Mass, m = 20 metric ton = 20 × 1000 kg; Distance, S = 20 m; Time, t = 1 hour = 3600 s

$$\text{Power} = \frac{\text{Work}}{\text{Time}} = \frac{mg \times S}{t} = \frac{20 \times 1000 \times 9.8 \times 20}{3600} \text{ watt} = 1.09 \times 10^3 \text{ W}$$

Illustration 17. A large family uses 8 kW of power. Direct solar energy is incident on the horizontal surface at an average rate of 200 W per square metre. If 20% of this energy can be converted to useful electrical energy, how large an area is needed to supply 8kW?

Solution If A m^2 be the area, then power = 200 . A watts

Useful electrical energy produced/s = $\frac{20}{100}$ (200 A) = 40 . A watts

But 40 A = 8000 or A = 200 m^2

Illustration 18. An elevator of total mass (elevator + passenger) 1800 kg is moving up with a constant speed of 2 ms^{-1}. A frictional force of 4000 N oppose its motion. Determine the minimum power delivered by the motor to the elevator. Take g = 10 m s^{-2}.

Solution Weight of (elevator + passenger) = mg = 1800 × 10 N = 18000 N
Frictional force = 4000 N
Total downward force on the elevator = (18000 + 4000) N = 22000 N
Clearly, the motor must have enough power to balance this force.

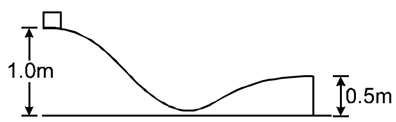
Now, power, P = Fv = 2200 N × 2 m s^{-1} = 4400 W = $\frac{44000}{746}$ hp = 58.98 hp

BEGINNER'S BOX–3

Potential Energy and Conservation of Energy

1. A stone projected up with a velocity u reaches a maximum height h. When it is at a height of 3h/4 from the ground, the ratio of KE and PE at that point is : (consider PE = 0 at the point of projectory)
(A) 1 : 1 (B) 1 : 2 (C) 1 : 3 (D) 3 : 1

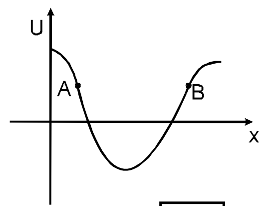
2. Figure shows a particle sliding on a frictionless track which terminates in a straight horizontal section. If the particle starts slipping from the point A, how far away from the track will the particle hit the ground?
(A) At a horizontal distance of 1 m from the end of the track.
(B) At a horizontal distance of 2 m from the end of the track.
(C) At a horizontal distance of 3 m from the end of the track.
(D) Insufficient information



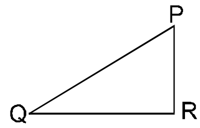
3. An electric motor creates a tension of 4500 N in hoisting cable and reels it at the rate of 2 m/s. What is the power of electric motor ?
(A) 9 W (B) 9 KW (C) 225 W (D) 9000 H.P.

4. A man M_1 of mass 80 kg runs up a staircase in 15 s. Another man M_2 also of mass 80 kg runs up the stair case in 20 s. The ratio of the power developed by them (P_1 / P_2) will be :
(A) 1 (B) 4/3 (C) 16/9 (D) None of the above

5. Potential energy v/s displacement curve for one dimensional conservative field is shown. Force at A and B is respectively.
(A) Positive, Positive (B) Positive, Negative
(C) Negative, Positive (D) Negative, Negative



6. For the path PQR in a conservative force field (fig.), the amounts work done in carrying a body from P to Q & from Q to R are 5 J & 2 J respectively . The work done in carrying the body from P to R will be

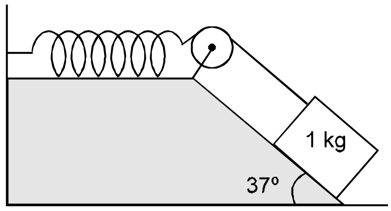


(A) 7 J
(B) 3 J
(C) $\sqrt{21}$ J
(D) zero

7. The negative of the work done by the conservative internal forces on a system equals the change in

(A) total energy
(B) kinetic energy
(C) potential energy
(D) none of these

8. A 1 kg block situated on a rough inclined plane is connected to a spring of spring constant 100 N m^{-1} as shown. The block is released from rest with the spring in the unstretched position. The block moves 10 cm down the incline before coming to rest. Find the coefficient of friction between the block and the incline assume that the spring has negligible mass and the pulley is frictionless. Take g = 10 ms^{-2}.

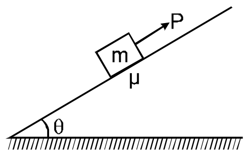


9. The potential energy function of a particle in a region of space is given as U = (2xy + yz)J
Here x, y and z are in metre. Find the force acting on the particle at a general point P(x, y, z).

10. A lift is designed to carry a load of 4000 kg in 10 seconds uniformly through 10 floors of a building averaging 6 metre per floor . Calculate the hourse power of the lift. (Take g = 10 m s^{-2} and 1 hp = 750 watts).

11. An engine lifts 90 metric ton of coal per hour from a mine whose depth is 200 metre. Calculate the power of the engine (use g = 9.8 m/s^2)

12. A block of mass m is being pulled up the rough incline by an agent delivering constant power P. The coefficient of friction between the block and the incline is μ. The
maximum speed of the block during the course of ascent is



(A) $v = \dfrac{P}{mg\sin\theta + \mu mg\cos\theta}$
(B) $v = \dfrac{P}{mg\sin\theta - \mu mg\cos\theta}$
(C) $v = \dfrac{2P}{mg\sin\theta - \mu mg\cos\theta}$
(D) $v = \dfrac{3P}{mg\sin\theta - \mu mg\cos\theta}$

GOLDEN KEY POINTS

- For a particular displacement work is independent of time. Work will be same for same displacement whether the time taken is small or large.
- For a particular displacement, work done by a force is independent of type of motion i.e. whether it moves with constant velocity, acceleration or retardation etc.
- A force is independent from reference frame. Its displacement depends on frame so work done by a force is frame dependent therefore work done by a force can be different in different reference frame.
- Total work done by internal forces on the rigid system is equal to zero.
- Sum of work done by all the forces is not always equal to work done by net force on the system.
- Work done by spring force is positive whenever block move towards natural length of spring.
- Work done by kinetic or static friction on the object may be positive, negative or zero, but total work by static friction on the system is zero and kinetic friction is negative.
- Change in kinetic energy of particle or of system is equal to sum of work done by all the forces on the particle or on the system.
- A force which is always perpendicular to velocity of particle can't change kinetic of particle but can change momentum of particle.
- K.E. $= \frac{P^2}{2m}$ for a particle not for system of particle.

  Where P = momentum of particle.
- It is possible momentum of system is zero but kinetic energy of that system is non-zero.
- Chang in potential energy defined only for conservative forces. It has no relevance for non-conservative forces.
- Change in potential energy = –Work done by conservative force.
- Change in kinetic energy + Change in potential energy = Work done by non-conservative force.
- Work done by or against a conservative force depends only initial and final position of the body not depend on the nature of path between initial and final position.

SOME WORKED OUT ILLUSTRATIONS

Illustration 1.

When a conservative force does positive work on a body, then

(A) its potential energy must increase. (B) its potential energy must decrease.

(C) its kinetic energy must increase. (D) its total energy must decrease.

Ans. (B)

Solution

Work done by conservative force = – ΔU = positive ⇒ ΔU↓

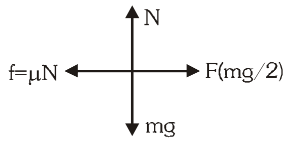
Illustration 2.

A box of mass m is initially at rest on a horizontal surface. A constant horizontal force of mg/2 is applied to the box directed to the right. The coefficient of friction of the surface changes with the distance pushed as $\mu = \mu_0 x$ where x is the distance from the initial location. For what distance is the box pushed until it comes to rest again?

(A) $\frac{2}{\mu_0}$ (B) $\frac{1}{\mu_0}$ (C) $\frac{1}{2\mu_0}$ (D) $\frac{1}{4\mu_0}$

Ans. (B)

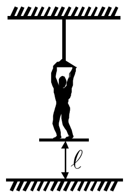
Solution



Net change in kinetic energy = 0 ⇒ net work W = 0

$$W = \int dW = \int Fdx - \int \mu N dx = \frac{mg}{2}x - mg\mu_0 \int_0^x x dx = 0 \Rightarrow x = \frac{1}{\mu_0}$$

Illustration 3.



One end of a light rope is tied directly to the ceiling. A man of mass M initially at rest on the ground starts climbing the rope hand over hand upto a height ℓ. From the time he starts at rest on the ground to the time he is hanging at rest at a height ℓ, how much work was done on the man by the rope?

(A) 0 (B) $Mg\ell$

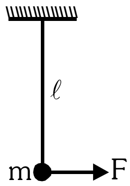
(C) $-Mg\ell$ (D) It depends on how fast the man goes up.

Ans. (B)

Solution

Total work done on man = 0 ⇒ Work done by string = – work done by gravity = $-(-Mg\ell) = Mg\ell$

Illustration 4.



A pendulum bob of mass m is suspended at rest. A cosntant horiozntal force F = mg/2 starts acting on it. The maximum angular deflection of the string is

(A) 90° (B) 53°

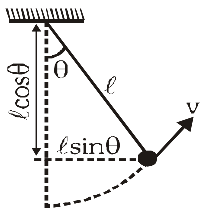
(C) 37° (D) None of these

Ans. (D)

Solution

Let at angular deflection θ its velocity be v then by work energy theorem W = ΔKE

$$\frac{1}{2}mv^2 = -mg(\ell - \ell\cos\theta) + F\ell\sin\theta$$



At maximum angular deflection, v = 0

$$0 = -mg\ell\,(1-\cos\theta) + \frac{mg\ell}{2}\sin\theta \Rightarrow 2-2\cos\theta = \sin\theta$$

$$\theta = 2\tan^{-1}\frac{1}{2}$$

Illustration 5.

The potential energy for the force $\vec{F} = yz\hat{i} + xz\hat{j} + xy\hat{k}$, if the zero of the potential energy is to be chosen at the point (2, 2, 2), is

(A) 8 + xyz (B) 8 – xyz (C) 4 – xyz (D) 4 + xyz

Ans. (B)

Solution

$\because \vec{F} = -\frac{\partial U}{\partial x}\hat{i} - \frac{\partial U}{\partial y}\hat{j} - \frac{\partial U}{\partial z}\hat{k} \quad \therefore \frac{\partial U}{\partial x} = -yz, \frac{\partial U}{\partial y} = -xz, \frac{\partial U}{\partial z} = -xy$

Therefore U = – xyz + C where C = constant As at (2, 2, 2), U =0 so C =8

OR

Objective question approach : Check that U =0 at (2, 2, 2)

Illustration 6.

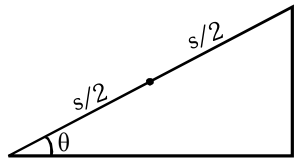
The upper half of an inclined plane with inclination θ is perfectly smooth while the lower half is rough. A body starting from rest at the top will again come to rest at the bottom if the coefficient of friction for the lower half is given by

(A) tanθ (B) 2tanθ (C) 2cosθ (D) 2sinθ

Ans. (B)

Solution

Refer to figure. In the journey over the upper half of incline, $v^2 - u^2 = 2as$



$v^2 - 0 = 2(g\sin\theta)\frac{s}{2} = g\sin\theta.s$

In the journey over the lower half of incline $v^2 - u^2 = 2as$

$0 - g\sin\theta.s = 2g(\sin\theta - \mu\cos\theta)\frac{s}{2} \Rightarrow -\sin\theta = \sin\theta - \mu\cos\theta \Rightarrow \mu = \frac{2\sin\theta}{\cos\theta} = 2\tan\theta$

Illustration 7.

Simple pendulums P_1 and P_2 have lengths ℓ_1 = 80 cm and ℓ_2 = 100 cm respectively. The bobs are of masses m_1 and m_2. Initially both are at rest in equilibrium position. If each of the bobs is given a displacement of 2 cm, the work done is W_1 and W_2 respectively. Then,

(A) $W_1 > W_2$ if $m_1 = m_2$ (B) $W_1 < W_2$ if $m_1 = m_2$ (C) $W_1 = W_2$ if $\frac{m_1}{m_2} = \frac{5}{4}$ (D) $W_1 = W_2$ if $\frac{m_1}{m_2} = \frac{4}{5}$

Ans. (A,D)

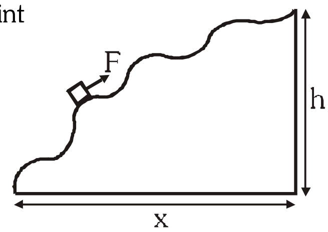
Solution

With usual notation, the height through which the bob falls is $h = \ell(1-\cos\theta) = \ell\left(2\sin^2\frac{\theta}{2}\right) = 2\ell\left(\frac{\theta^2}{4}\right)$ since θ is small. Therefore, we can write $h = \frac{\ell\theta^2}{2} = \frac{\ell}{2}\left(\frac{a}{\ell}\right)^2 = \frac{a^2}{2\ell}$. where a = amplitude

Thus, the work done W= P.E. = mgh $= \frac{mga^2}{2\ell} \Rightarrow W \propto \frac{1}{\ell}$

Illustration 8.

A body of mass m is slowly halved up the rough hill by a force F at which each point is directed along a tangent to the hill.



Work done by the force

(A) independent of shape of trajectory.

(B) depends upon x.

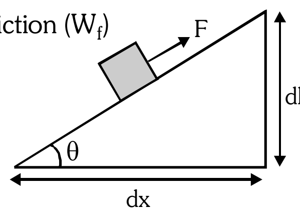
(C) depends upon h.

(D) depends upon coefficient of friction (μ)

Ans. (ABCD)

Solution

Work done by the force = Work done against gravity (W_g) + work done against friction (W_f)



$= W_g = \int (mg\sin\theta)ds = mg\int ds\sin\theta = mg\int dh = mgh$

and $W_f = \int (\mu mg\cos\theta)ds = \mu mg\int ds\cos\theta = \mu mg\int dx = \mu mgx$

Illustration 9.

The kinetic energy of a particle continuously increase with time. Then

(A) the magnitude of its linear momentum is increasing continuously.

(B) its height above the ground must continuously decrease.

(C) the work done by all forces acting on the particle must be positive.

(D) the resultant force on the particle must be parallel to the velocity at all times.

Ans. (A, C)

Solution

For (A) : $p = \sqrt{2mK}$ if K ↑ then p ↑

For (B) : Its height may ↑ or ↓

For (C) : W = ΔK if ΔK = positive then W = positive

For (D) : The resultant force on the particle must be at an angle less than 90° all times

Illustration 10.

A particle moves in one dimensional field with total mechanical energy E. If potential energy of particle is U(x), then

(A) Particle has zero speed where U(x) = E (B) Particle has zero acceleration where U(x) = E

(C) Particle has zero velocity where $\frac{dU(x)}{dx} = 0$ (D) Particle has zero acceleration where $\frac{dU(x)}{dx} = 0$

Ans. (A,D)

Solution

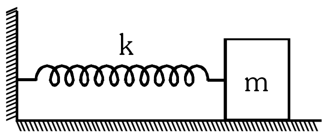
Mechanical energy = kinetic energy + potential energy $E = K + U(x)$ where $K = \frac{1}{2}mv^2$

If K = 0 then E = U(x)

If F = 0 then $F = -\frac{dU(x)}{dx} = 0 \Rightarrow \frac{dU(x)}{dx} = 0$

Illustration 11.

A spring block system is placed on a rough horizontal surface having coefficient of friction μ. The spring is given initial elongation $\frac{3\mu mg}{k}$ and the block is released from rest. For the subsequent motion



(A) Initial acceleration of block is $2\mu g$.

(B) Maximum compression in spring is $\frac{\mu mg}{k}$.

(C) Minimum compression in spring is zero.

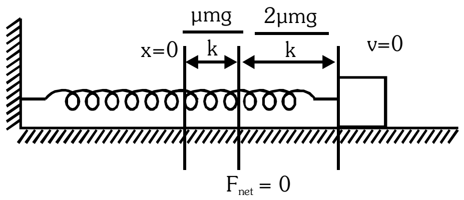
(D) Maximum speed of the block is $2\mu g\sqrt{\frac{m}{k}}$

Ans. (A,B,C,D)

Solution

For (A) : Initial acceleration $= \dfrac{k\left(\frac{3\mu mg}{k}\right) - \mu mg}{m} = 2\mu g$

For (B,C) :



Therefore maximum compression $= \frac{2\mu mg}{k} - \frac{\mu mg}{k} = \frac{\mu mg}{k}$ and minimum compression = 0

For (D) : At maximum speed $F_{net} = 0$ so by using work energy therorem

$$\frac{1}{2}mv^2 = \frac{1}{2}k\left(\frac{3\mu mg}{k}\right)^2 - \frac{1}{2}k\left(\frac{\mu mg}{k}\right)^2 - \mu mg\left(\frac{2\mu mg}{k}\right) \Rightarrow v = 2\mu g\sqrt{m/k}$$

Illustration 12 to 14.

A particle of mass m = 1 kg is moving along y-axis and a single conservative force F(y) acts on it. The potential energy of particle is given by $U(y) = (y^2 - 6y + 14)$ J where y is in meters. At y = 3 m the particle has kinetic energy of 15 J.

12. The total mechanical energy of the particle is

(A) 15 J (B) 5 J (C) 20 J (D) can't be determined

13. The maximum speed of the particle is

(A) 5 m/s (B) $\sqrt{30}$ m/s (C) $\sqrt{40}$ m/s (D) $\sqrt{10}$ m/s

14. The largest value of y (position of particle) is

(A) $3+\sqrt{5}$ (B) $3-\sqrt{5}$ (C) $3+\sqrt{15}$ (D) $6+\sqrt{15}$

Solution

12. *Ans. (C)*

Total mechanical energy = kinetic energy + potential energy = $15 + [3^2 - 6(3) + 14] = 15 + 5 = 20$ J

13. *Ans. (B)*

At maximum speed (i.e. maximum kinetic energy), potential energy is minimum

$U = y^2 - 6y + 14 = 5 + (y-3)^2$

which is minimum at y=3 m so $U_{min} = 5$J

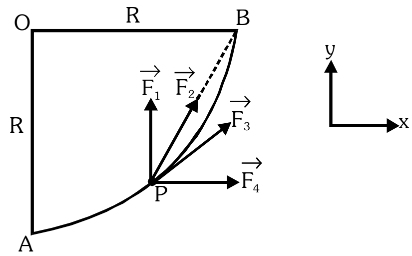
Therefore $K_{max} = 20 - 5 = 15 \text{ J} \Rightarrow \frac{1}{2}mv_{max}^2 = 15 \Rightarrow v_{max} = \sqrt{30}$ m/s

14. *Ans. (C)*

For particle $K \geq 0 \Rightarrow E - U \geq 0 \Rightarrow 20 - (5+(y-3)^2] \geq 0 \Rightarrow (y-3)^2 \leq 15 \Rightarrow y-3 \leq \sqrt{15} \Rightarrow y \leq 3 + \sqrt{15}$

Illustration 15

AB is a quarter of a smooth horizontal circular track of radius R. A particle P of mass m moves along the track from A to B under the action of following forces :



$\vec{F}_1$ = F (always towards y-axis)

$\vec{F}_2$ = F (always towards point B)

$\vec{F}_3$ = F (always along the tangent to path AB)

$\vec{F}_4$ = F (always towards x-axis)

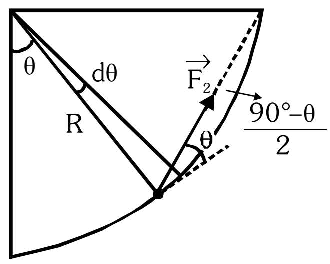
	Column I		Column II
(A)	Work done by $\vec{F}_1$	(P)	$\sqrt{2}$ FR
(B)	Work done by $\vec{F}_2$	(Q)	$\frac{1}{\sqrt{2}}$FR
(C)	Work done by $\vec{F}_3$	(R)	FR
(D)	Work done by $\vec{F}_4$	(S)	$\frac{\pi FR}{2}$
		(T)	$\frac{2FR}{\pi}$

Ans. (A) R; (B) P; (C) S; (D) R

Solution

For (A) : Work done by $\vec{F}_1 = FR$

For (B) : $dW = \vec{F}.d\vec{s} = (FRd\theta)\cos\left(45 - \frac{\theta}{2}\right) = FR\left(45 - \frac{\theta}{2}\right)d\theta$



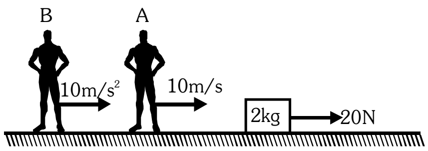
$$W = \int_0^{\pi/4} FR\cos\left(45 - \frac{\theta}{2}\right) = -2FR\left(\sin 45° - \frac{\theta}{2}\right)_0^{\pi/2} = \sqrt{2}FR$$

For (C) : $W = \int \vec{F}.d\vec{s} = F\left(\frac{\pi R}{2}\right) = \frac{\pi FR}{2}$

For (D) : $W = \int \vec{F}.d\vec{s} = (F)(R) = FR$

Illustration 16.

A block of mass 2 kg is dragged by a force of 20 N on a smooth horizontal surface. It is observed from three reference frames ground, observer A and observer B. Observer A is moving with constant velocity of 10 m/s and B is moving with constant acceleration of 10 m/s^2. The observer B and block starts from rest simultaneously at t =0.



Column I	***Column II***
(A) Work energy theorem is applicable in	(P) 100 J
(B) Work done on block in 1s as observed by ground is	(Q) – 100 J
(C) Work done on block is 1 s as observed by observer A is	(R) zero
(D) Work done on block in 1 s as observed by observer B is	(S) only ground & A
	(T) all frames ground, A & B

Ans. (A) T; (B) P; (C) Q; (D) R

Solution

For (A) : Work energy theorem is applicable in all reference frames.

For (B) : w.r.t. ground : At t =0, u =0 and t = 1 s, v = at = $\left(\frac{20}{2}\right)(1) = 10$ m/s

Work done = change in kinetic energy = $\frac{1}{2}(2)(10)^2 - \frac{1}{2}(2)(0)^2 = 100$ J

For (C) : w.r.t. observer A : Initial velocity = 0 – 10 = – 10 m/s, Final velocity = 10 – 10 = 0

Work done = $\frac{1}{2}(2)(0)^2 - \frac{1}{2}(2)(-10)^2 = -100$ J

For (D) : w.r.t. observer B : Initial velocity = 0 – 0 = 0

Final velocity = 10 – 10 = 0; Work done = 0

Illustration 17.

A particle of mass 0.5 kg travels in a straight line with velocity $v = ax^{3/2}$ where $a = 5\ m^{-1/2}\ s^{-1}$. What is the work done by the all force during its displacement from x = 0 to x = 2m?

Sol. $m = 0.5$ kg, $v = ax^{3/2}$, $a = 5\ m^{-1/2}\ s^{-1}$, W = ?

Initial velocity at x = 0, $v_0 = a \times 0 = 0$

Final velocity at x = 2, $v_2 = a \times 2^{3/2} = 5 \times 2^{3/2}$

Work done = Increase in kinetic energy = $\frac{1}{2}m(v_2^2 - v_0^2)$

$$= \frac{1}{2} \times 0.5\left[(5 \times 2^{3/2})^2 - 0\right] = 50\text{ J.}$$

ANSWERS

BEGINNER'S BOX-1

1. D **2.** C **3.** A **4.** C **5.** C **6.** B **7.** (i) Zero (ii) 500J

8. 135 J **9.** $\frac{mg\ell}{2}$ **10.** 20 J

BEGINNER'S BOX-2

1. B **2.** B **3.** A **4.** C **5.** $\sqrt{u^2+2gh}$ **6.** 80 kg **7.** $\frac{3mv^2}{4x^2}$

8. 2mg/k **9.** mg/K **10.** (i) $\frac{m^2g^2}{K}$ (ii) $-\frac{m^2g^2}{2K}$ (iii) $\frac{-m^2g^2}{2K}$

BEGINNER'S BOX-3

1. C **2.** A **3.** B **4.** B **5.** B **6.** A **7.** C **8.** 1/8

9. $\vec{F} = -[2y\hat{i} + (2x+z)\hat{j} + y\hat{k}]$ **10.** 320 hp **11.** 49 kW **12.** A

EXERCISE – 1 MCQ (SINGLE CHOICE CORRECT)

1. A machine gun fires 360 bullets per minute, each bullet moves with a velocity of 600 m/s. If the power of gun is 5.4 kW, mass of each bullet is

(A) 5 kg (B) 0.5 kg (C) 0.5×10^{-3} kg (D) 0.0025 kg

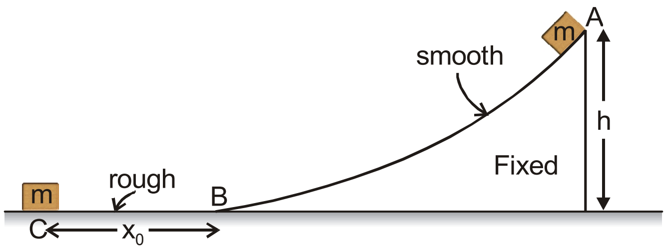
2. The work done by the frictional force on a pencil in drawing a complete circle of radius $r = 1/\pi$ metre on the surface by a pencil of negligible mass with a normal pressing force N = 5 N ($\mu = 0.5$) is :

(A) + 4J (B) –3 J (C) – 2 J (D) – 5J

3. A rope is used to lower vertically a block of mass M by a distance x with a constant downward acceleration g/2. The work done by the rope on the block is :

(A) Mgx (B) $\frac{1}{2}Mgx^2$ (C) $-\frac{1}{2}Mgx$ (D) Mgx^2

4. A small block of mass m is released from point A on a fixed smooth wedge as shown in figure. Bottom of wedge is marked as B and at a point C the block stops moving because the straight part of floor is rough.



(i) Work done by normal reaction is zero during the motion of the block

(A) from point A to B only (B) from point B to C only

(C) from A to C (D) None of these

(ii) The friction coefficient of the block with the floor is :

(A) $\frac{h}{x_0}$ (B) $\frac{x_0}{h}$ (C) zero (D) 1

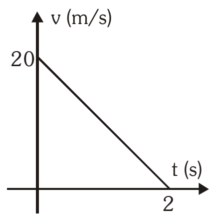
(iii) The velocity of the block at the midpoint between B to C will be :

(A) $\frac{\sqrt{2gh}}{2}$ (B) $\sqrt{2gh}$ (C) $\sqrt{gh}$ (D) $\frac{\sqrt{gh}}{2}$

5. A small mass 'm' slides down an inclined plane of angle θ with $\mu = \mu_0 x$ where 'x' is the distance through which the mass slides down and μ_0 is a constant. Then the distance covered by the mass before it stops is

(A) $\frac{2}{\mu_0}\tan\theta$ (B) $\frac{4}{\mu_0}\tan\theta$ (C) $\frac{1}{2\mu_0}\tan\theta$ (D) $\frac{1}{\mu_0}\tan\theta$

6. Velocity–time graph of a particle of mass 2 kg moving in a straight line is as shown in figure. Work done by all the forces in t = 0 to t = 2 on the particle is :



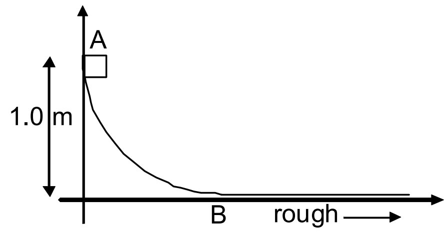
(A) 400 J (B) –400 J

(C) –200 J (D) 200 J

7. A particle moves on a rough horizontal ground with some initial velocity say v_0. If $\frac{3}{4}$ of its kinetic energy is lost due to friction in time t_0 then coefficient of friction between the particle and the ground is :

(A) $\frac{v_0}{2gt_0}$ (B) $\frac{v_0}{4gt_0}$ (C) $\frac{3v_0}{4gt_0}$ (D) $\frac{v_0}{gt_0}$

8. A block weighing 10 N travels down a smooth curved track AB joined to a rough horizontal surface. The rough surface has a friction coefficient 0.20 with the block. If the block starts slipping on the track from a point 1.0 m above the horizontal surface. The distance it will move on the rough surface is

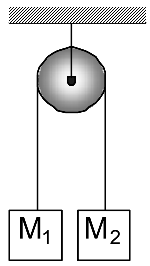


(A) 5.0 m (B) 10.0 m
(C) 15.0 m (D) 20.0 m

9 A force acts on a 3 gm particle in such a way that the position of the particle as a function of time is given by $x = 3t - 4t^2 + t^3$, where x is in meters and t is in seconds. The work done on the particle during the first 4 second is:
(A) 384 mJ (B) 168 mJ (C) 528 mJ (D) 541 mJ

10. A body is moved along a straight line by a machine delivering constant power. The distance moved by the body in time t is proportional to :
(A) $t^{1/2}$ (B) $t^{3/4}$ (C) $t^{3/2}$ (D) t^2

11. The masses M_1 and M_2 ($M_2 > M_1$) are released from rest, using work–energy theorem find out velocity of the blocks when they move a distance 'x'.

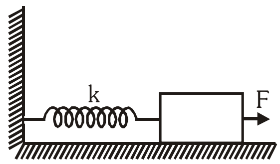


(A) $\sqrt{\dfrac{2(M_2 - M_1)gx}{M_1 + M_2}}$ (B) $\sqrt{\dfrac{2(M_2 + M_1)gx}{M_2 - M_1}}$

(C) $\sqrt{\dfrac{(M_2 + M_1)gx}{M_2 - M_1}}$ (D) $\sqrt{\dfrac{(M_2 - M_1)gx}{2(M_2 + M_1)}}$

12. A rope ladder with a length ℓ carrying a man of mass m at its end is attached to the basket of balloon with a mass M. The entire system is in equilibrium in the air. As the man climbs up the ladder into the balloon, the balloon descends by a height h. Then the potential energy of the man :
(A) Increases by mg (ℓ–h) (B) Increases by mgℓ
(C) Increases by mgh (D) Increases by mg (2ℓ–h)

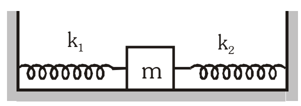
13. A block attached to a spring, pulled by a constant horizontal force, is kept on a smooth surface as shown in the figure. Initially, the spring is in the natural state. Then the maximum positive work that the applied force F can do is : [Given that spring does not break]



(A) $\dfrac{F^2}{k}$ (B) $\dfrac{2F^2}{k}$

(C) ∞ (D) $\dfrac{F^2}{2k}$

14. A block of mass m is attached to two spring of spring constant k_1 and k_2 as shown in figure. The block is displaced by x towards right and released. The velocity of the block when it is at x/2 will be :

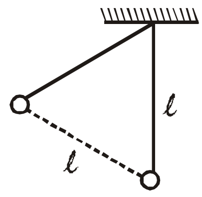


(A) $\sqrt{\dfrac{(k_1 + k_2)x^2}{2m}}$ (B) $\sqrt{\dfrac{3}{4}\dfrac{(k_1 + k_2)x^2}{m}}$

(C) $\sqrt{\dfrac{(k_1 + k_2)x^2}{m}}$ (D) $\sqrt{\dfrac{(k_1 + k_2)x^2}{4m}}$

15. An object of mass m slides down a hill of height h of arbitrary shape and after travelling a certain horizontal path stops because of friction. The friction coefficient is different for different segments for the entire path but is independent of the velocity and direction of motion. The work that a force must perform to return the object to its initial position along the same path is :
(A) mgh (B) 2mgh (C) 4mgh (D) –mgh

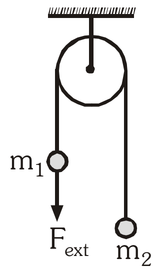
16. A bob hangs from a rigid support by an inextensible string of length ℓ. If it is displaced through a distance ℓ (from the lowest position) keeping the string straight & released, the speed of the bob at the lowest position is:



(A) $\sqrt{g\ell}$ (B) $\sqrt{3g\ell}$

(C) $\sqrt{2g\ell}$ (D) $\sqrt{5g\ell}$

17. Two bodies of mass m_1 and m_2 $(m_2 > m_1)$ are connected by a light inextensible string which passes through a smooth fixed pulley. The instantaneous power delivered by an external agent to pull m_1 with constant velocity v is :



(A) $(m_2 - m_1)$ g/v

(B) $(m_2 - m_1)$ v/g

(C) $(m_2 - m_1)$ gv

(D) $(m_1 - m_2)$ gv

18. A man places a chain of mass 'm' and length 'ℓ' on a table slowly. Initially the lower end of the chain just touches the table. The man drops the chain when half of the chain is in vertical position. Then work done by the man in this process is :

(A) $-mg\frac{\ell}{2}$ (B) $-\frac{mg\ell}{4}$ (C) $-\frac{3mg\ell}{8}$ (D) $-\frac{mg\ell}{8}$

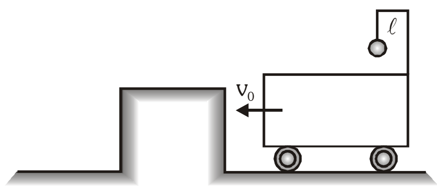
19. A force $\vec{F} = (3\hat{i} + 4\hat{j})$ N acts on a 2 kg movable object that moves from an initial position $\vec{d_i} = (-3\hat{i} - 2\hat{j})$ m to final position $\vec{d_f} = (5\hat{i} + 4\hat{j})$ in 6s. The average power delivered by the force during the interval is equal to :

(A) 8 watt (B) $\frac{25}{3}$ watt (C) 15 watt (D) $\frac{50}{3}$ watt

20. A particle is released from a height H. At certain height its kinetic energy is two times its potential energy. Height and speed of the particle at that instant are

(A) $\frac{H}{3}, \sqrt{\frac{2gH}{3}}$ (B) $\frac{H}{3}, 2\sqrt{\frac{gH}{3}}$ (C) $\frac{2H}{3}, \sqrt{\frac{2gH}{3}}$ (D) $\frac{H}{3}, \sqrt{2gH}$

21. A bob is suspended from a crane by a cable of length ℓ = 5 m. The crane and load are moving at a constant speed v_0. The crane is stopped by a bumper and the bob on the cable swings out an angle of 60°. The initial speed v_0 is– (g = 9.8 m/s^2)



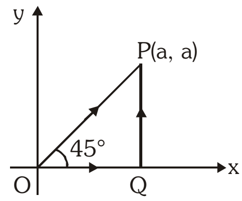
(A) 10 m/s (B) 7 m/s

(C) 4 m/s (D) 2 m/s

22. A 1.0 kg block collides with a horizontal weightless spring of force constant 2.75 Nm^{-1}. The block compresses the spring 4.0 m from the rest position. If the coefficient of kinetic friction between the block and horizontal surface is 0.25, the speed of the block at the instant of collision is :

(A) 0.4 ms^{-1} (B) 4 ms^{-1} (C) 0.8 ms^{-1} (D) 8 ms^{-1}

23. A particle is moved from (0, 0) to (a, a) under a force $\vec{F} = (3\hat{i} + 4\hat{j})$ from two paths. Path 1 is OP and path 2 is OQP. Let W_1 and W_2 be the work done by this force in these two paths. Then :



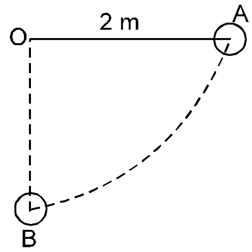
(A) $W_1 = W_2$ (B) $W_1 = 2W_2$

(C) $W_2 = 2W_1$ (D) $W_2 = 4W_1$

24. A spring of spring constant k placed horizontally on a rough horizontal surface is compressed against a block of mass m placed on the surface so as to store maximum energy in the spring. If the coefficient of friction between the block and the surface is μ, the potential energy stored the spring is :

(A) $\dfrac{\mu^2 m^2 g^2}{k}$ (B) $\dfrac{2\mu m^2 g^2}{k}$ (C) $\dfrac{\mu^2 m^2 g^2}{2k}$ (D) $\dfrac{3\mu^2 m g^2}{k}$

25. The bob of a pendulum is released from a horizontal position A as shown in figure. If the length of the pendulum is 2 m, what is the speed with which the bob arrives at the lowermost point B, given that it dissipated 10% of its initial energy against air resistance? (gravitational potential energy is zero at B).



(A) 3 m s^{-1} (B) 6 m s^{-1}

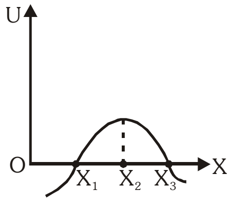
(C) 9 m s^{-1} (D) 12 m s^{-1}

EXERCISE – 2 MCQ (ONE OR MORE CHOICE CORRECT)

1. A particle of mass m = 1 kg lying on x–axis experiences a force given by law F=x(3x–2) Newton, where x is the x–coordinate of the particle in meters. The points on x–axis where the particle is in equilibrium are :
(A) x = 0 (B) x = 1/3 (C) x = 2/3 (D) x = 1

2. A heavy stone is thrown from a cliff of height h in a given direction. The speed with which it hits the ground
(A) must depend on the speed of projection (B) must be larger than the speed of projection
(C) must be independent of the speed of projection (D) may be smaller than the speed of projection

3. In the figure shown the potential energy (U) of a particle is plotted against its position 'x' from origin. Then which of the following statement is correct.
A particle at :



(A) x_1 is in stable equilibrium (B) x_2 is in stable equilibrium
(C) x_3 is in stable equilibrium (D) None of these

4. No work is done by a force on an object if
(A) the force is always perpendicular to its velocity
(B) the force is always perpendicular to its acceleration
(C) the object is stationary but the point of application of the force moves on the object
(D) the object moves in such a way that the point of application of the force remains fixed.

5. The kinetic energy of a particle continuously increases with time
(A) the resultant force on the particle must be parallel to the velocity at all instants.
(B) the resultant force on the particle must be at an angle less than 90° all the time
(C) its height above the ground level must continuously decrease
(D) the magnitude of its linear momentum is increasing continuously

6. A small sphere starts falling from a very large height and after falling a distance of 100 m it attains the terminal velocity and continues to fall with this velocity. The work done by the atmosphere during the first fall of 100m is :
(A) Greater than the work done for next fall of 100 m (B) Less than the work done for next fall of 100 m
(C) Equal to 100 mg (D) Greater than 100 mg

7. Work done by force of friction on the object :
(A) can be zero (B) can be positive (C) can be negative (D) information insufficient

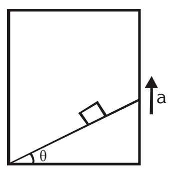
8. When work done by force of gravity is negative (Assume only gravitational force to be acting)
(A) KE increases (B) KE decreases (C) PE increases (D) PE stays constant

9. When total work done on a particle is positive
(A) KE remains constant (B) Momentum increases
(C) KE decreases (D) KE increases

10. When a man walks on a horizontal surface with constant velocity, work done by
(A) friction is zero (B) contact force is zero (C) gravity is zero (D) None of these

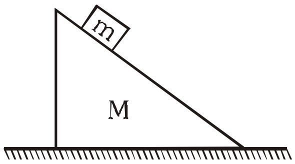
Match the column

11. A block of mass m is stationary with respect to a rough wedge as shown in figure. Starting from rest, in time t work done on the block : (m = 1kg, θ = 30°, a = 2m/s^2, t = 4s)



Column I	***Column II***
(A) By gravity	(p) 144 J
(B) By normal reaction	(q) 32 J
(C) By friction	(r) 56 J
(D) By all the forces	(s) 48 J
	(t) None

12. A block of mass m lies on wedge of mass M. The wedge in turn lies on smooth horizontal surface. Friction is absent everywhere. The wedge block system is released from rest. All situation given in column–I are to be estimated in duration the block undergoes a vertical displacement 'h' starting from rest (assume the block to be still on the wedge, g is acceleration due to gravity).

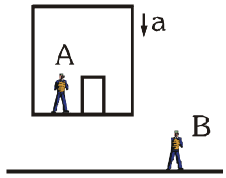


Column I	*Column II*
(A) Work done by normal reaction acting on the block is	(p) Positive
(B) Work done by normal reaction (exerted by block) acting on wedge is	(q) Negative
(C) The sum of work done by normal reaction on block and work done by normal reaction (exerted by block) on wedge is	(r) Zero
(D) Net work done by all forces on block is	(s) Less than mgh in magnitude

Comprehension Based Questions

Comprehension 1

A block of mass m is kept in an elevator which starts moving downward with an acceleration a as shown in figure. The block is observed by two observers A and B for a time interval t_0.



13. The observer B finds that the work done by gravity is

(A) $\frac{1}{2}mg^2t_0^2$ (B) $-\frac{1}{2}mg^2t_0^2$ (C) $\frac{1}{2}mgat_0^2$ (D) $-\frac{1}{2}mgat_0^2$

14. The observer B finds that work done by normal reaction N is

(A) zero (B) $-Nat_0^2$ (C) $+\frac{Nat_0^2}{2}$ (D) None of these

15. According to observer B, the net work done on the block is

(A) $-\frac{1}{2}ma^2t_0^2$ (B) $\frac{1}{2}ma^2t_0^2$ (C) $\frac{1}{2}mgat_0^2$ (D) $-\frac{1}{2}mgat_0^2$

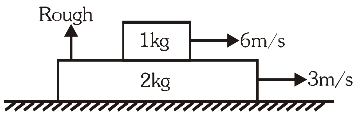
16. According to the observer A

(A) the work done by gravity is zero (B) the work done by normal reaction is zero

(C) the work done by pseudo force is zero (D) all the above

Comprehension 2.

In the figure shown upper block is given a velocity of 6 m/s and lower block 3 m/s. When relative motion between them is stopped.



17. Choose correct statement/(s) :

(A) Work done by friction on upper block is negative (B) Work done by friction on both the blocks is positive

(C) Work done by friction on lower block is negative (D) Work done by friction on both the blocks is negative

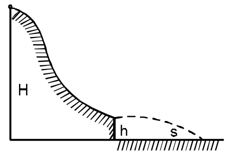
18*. Choose correct statement/(s) :

(A) Work done by friction on upper block is – 10 J (B) Work done by friction on lower block is + 10 J

(C) Net work done by friction is zero (D) All of the above

EXERCISE – 3 SUBJECTIVE

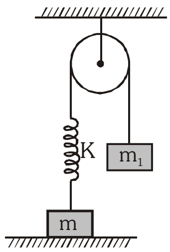
1. A small disc A slides down with initial velocity equal to zero from the top of a smooth hill of height H having a horizontal portion (as shown in fig.). What must be the height of the horizontal portion h to ensure the maximum distance s covered by the disc? What is it equal to?



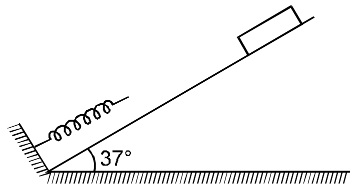
2. A particle is moving in x direction, under the influence of force F = $\pi \sin \pi x$. Find the work done by another external agent in slowly moving a particle from x=0 to x = 0.5m.

3. The potential energy of a particle of mass 1kg free to move along x-axis is given by $V(x) = \left(\frac{x^2}{2} - x\right)$ joule. If total mechanical energy of the particle is 2J, then find the maximum speed of the particle. (Assuming only conservative force acts on particle)

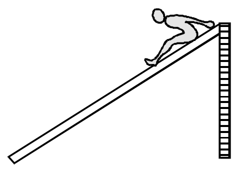
4. For what minimum value of m_1 the block of mass m will leave the contact with surface ?



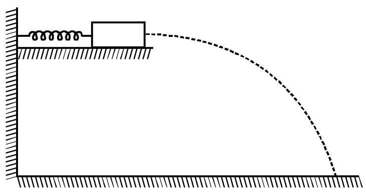
5. Figure shows a spring fixed at the bottom end of an incline of inclination 37°. A small block of mass 2 kg starts slipping down the incline from a point 4.8 m away from free end of the spring. The block compresses the spring by 20 cm, stops momentarily and then rebounds through a distance of 1 m up the incline. Find (a) The friction coefficient between the plane and the block and (b) the spring constant of the spring. Take g = 10 m/s^2.



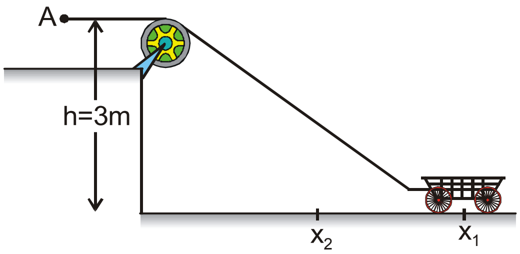
6. In a children's park, there is a slide which has a total length of 10 m and a height of 8.0 m (figure). A vertical ladder is provided to reach the top. a boy weighing 200 N climbs up the ladder to the top of the slide and slides down to the ground. The average friction offered by the slide is three tenth of his weight. Find (a) the work done by the ladder on the boy as he goes up, (b) the work done by the slide on the boy as he comes down. (c) Find the work done by forces inside the body of the boy.



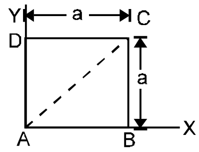
7. A small block of mass 100 g is pressed against a horizontal spring fixed at one end to compress the spring through 5.0 cm (figure). The spring constant is 100 N/m. When released, the block moves horizontally till it leaves the spring. Where will it hit the ground 2 m below the spring ?



8. Figure shows a light, inextensible string attached to a cart that can slide along a frictionless horizontal rail aligned along an x axis. The left and of the string is pulled over a pulley, of negligible mass and friction and fixed at height h = 3m from the ground level. The cart slides from $x_1 = 3\sqrt{3}$ m to $x_2 = 4$ m and during the move, tension in the string is kept constant 50 N. Find change in kinetic energy of the cart in joules. (Use $\sqrt{3} = 1.7$)



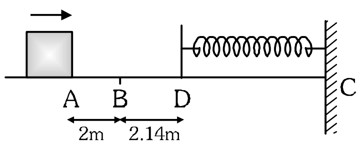
9. A force $\boldsymbol{F} = x^2y^2\boldsymbol{i} + x^2y^2\boldsymbol{j}$ (N) acts on a particle which moves in the XY plane.



(a) Determine if F is conservative and

(b) Ffind the work done by ***F*** as it moves the particle from A to C (fig.) along each of the paths ABC, ADC, and AC.

10. A 0.5 kg block slides from the points A on a horizontal frack with an initial speed of 3 m/s towards a weightless horizontal spring of length 1m and force constant 2 N/m. The part AB of the track is frictionless and the part BC has coefficient of static and kinetic friction as 0.22 and 0.2 respectively. Find the total distance through which the block moves before it comes to rest completely. ($g = 10\ m/s^2$)

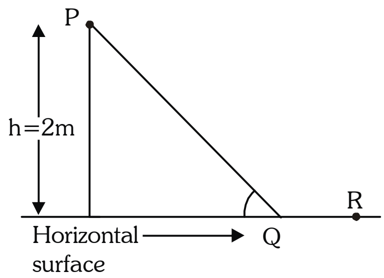


EXERCISE – 4 RECAP OF AIEEE/JEE (MAIN)

1. When a rubber-band is stretched by a distance x, it exerts a restoring force of magnitude $F = ax + bx^2$ where a and b are constants. the work done in stretching the unstretched rubber-band by L is : ***[JEE(Main)- 2014]***

(A) $aL^2 + bL^3$ (B) $\frac{1}{2}(aL^2 + bL^3)$ (C) $\frac{aL^2}{2} + \frac{bL^3}{3}$ (D) $\frac{1}{2}\left(\frac{aL^2}{2} + \frac{bL^3}{3}\right)$

2. A point particle of mass m, moves along the uniformly rough track PQR as shown in the figure. The coefficient of friction, between the particle and the rough track equals μ. The particle released, from rest, from the point P and it comes to rest at a point R. The energies, lost by the ball, over the parts, PQ and QR, of the track, are equal to each other, and no energy is lost when particle changes direction from PQ to QR The values of the coefficient of friction μ and the distance x(=QR), are, respectively close to: ***[JEE(Main)- 2016]***



(A) 0.2 and 6.5 m (B) 0.2 and 3.5 m (C) 0.29 and 3.5 m (D) 0.29 and 6.5 m

3. A person trying to lose weight by burning fat lifts a mass of 10 kg upto a height of 1 m 1000 times. Assume that the potential energy lost each time he lowers the mass is dissipated. How much fat will he use up considering the work done only when the weight is lifted up? Fat supplies 3.8×10^7 J of energy per kg which is converted to mechanical energy with a 20% efficiency rate. Take $g = 9.8\ ms^{-2}$: ***[JEE(Main)- 2016]***

(A) 2.45×10^{-3} kg (B) 6.45×10^{-3} kg (C) 9.89×10^{-3} kg (D) 12.89×10^{-3} kg

4. A time dependent force F = 6t acts on a particle of mass 1 kg. If the particle starts from rest, the work done by the force during the first 1 sec. will be : ***[JEE(Main)- 2017]***

(A) 9 J (B) 18 J (C) 4.5 J (D) 22 J

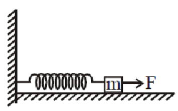
5. A particle is moving in a circular path of radius a under the action of an attractive potential $U = -\frac{k}{2r^2}$. Its total energy is :- ***[JEE(Main)- 2018]***

(A) $\frac{k}{2a^2}$ (B) Zero (C) $-\frac{3}{2}\frac{k}{a^2}$ (D) $-\frac{k}{4a^2}$

6. A force acts on a 2 kg object so that its position is given as a function of time as $x = 3t^2 + 5$. What is the work done by this force in first 5 seconds ? ***[JEE(Main)- 2019]***

(A) 850 J (B) 900 J (C) 950 J (D) 875 J

7. A block of mass m, lying on a smooth horizontal surface, is attached to a spring (of negligible mass) of spring constant k. The other end of the spring is fixed, as shown in the figure. The block is initally at rest in its equilibrium position. If now the block is pulled with a constant force F, the maximum speed of the block is : ***[JEE(Main)- 2019]***

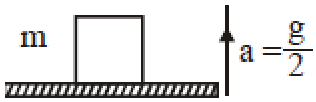


(A) $\frac{\pi F}{\sqrt{mk}}$ (B) $\frac{2F}{\sqrt{mk}}$ (C) $\frac{F}{\sqrt{mk}}$ (D) $\frac{F}{\pi\sqrt{mK}}$

8. A particle which is experiencing a force, given by $\vec{F} = 3\hat{i} - 12\hat{j}$, undergoes a displacement of $\vec{d} = 4\hat{i}$. If the particle had a kinetic energy of 3 J at the beginning of the displacement, what is its kinetic energy at the end of the displacement? ***[JEE(Main)- 2019]***

(A) 15 J (B) 10 J (C) 12 J (D) 9 J

9. A block of mass m is kept on a platform which starts from rest with constant acceleration g/2 upward, as shown in fig. Work done bv normal reaction on block in time t is : ***[JEE(Main)- 2019]***

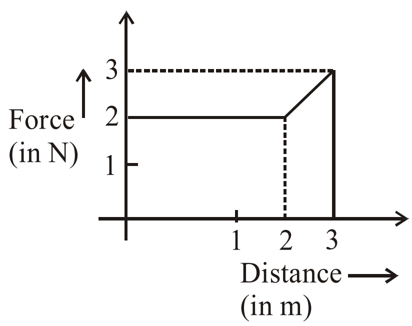


(A) 0 (B) $\frac{3mg^2t^2}{8}$ (C) $-\frac{mg^2t^2}{8}$ (D) $\frac{mg^2t^2}{8}$

10. A body of mass 1 kg falls freely from a height of 100 m on a platform of mass 3 kg which is mounted on a spring having spring constant k = 1.25 × 10^6 N/m. The body sticks to the platform and the spring's maximum compression is found to be x. Given that g = 10 ms^{-2}, the value of x will be close to : ***[JEE(Main)- 2019]***

(A) 4 cm (B) 8 cm (C) 80 cm (D) 40 cm

11. A particle moves in one dimension from rest under the influence of a force that varies with the distance travelled by the particle as shown in the figure. The kinetic energy of the particle after it has travelled 3m is : ***[JEE(Main)- 2019]***



(A) 6.5 J (B) 2.5 J (C) 4 J (D) 5 J

12. A uniform cable of mass 'M' and length 'L' is placed on a horizontal surface such that its $\left(\frac{1}{n}\right)^{th}$ part is hanging below the edge of the surface. To lift the hanging part of the cable upto the surface, the work done should be ***[JEE(Main)- 2019]***

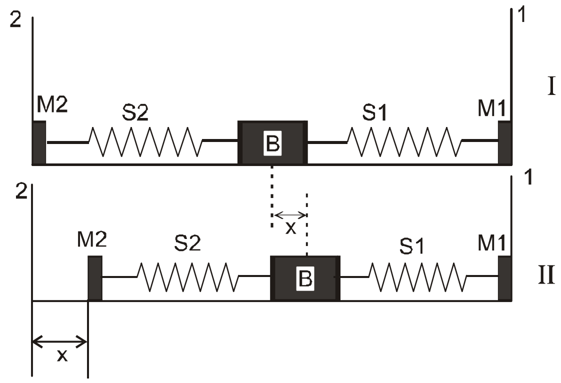
(A) $\frac{MgL}{n^2}$ (B) $\frac{MgL}{2n^2}$ (C) $\frac{2MgL}{n^2}$ (D) nMgL

EXERCISE – 5 RECAP OF IIT-JEE/JEE (ADVANCED)

MCQs with one correct answer

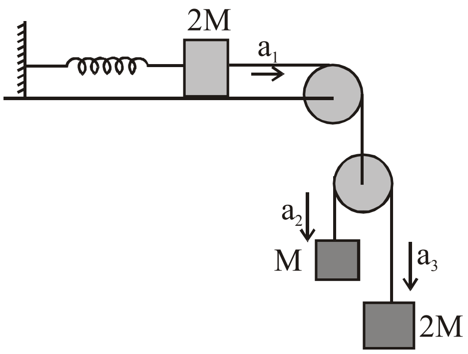
1. A block (B) is attached to two unstretched springs S_1 and S_2 with spring constants k and 4 k, respectively (see figure I). The other ends are attached to identical supports M_1 and M_2 not attached to the walls. The springs and supports have negligible mass. There is no friction anywhere. The block B is displaced towards wall 1 by a small distance x (figure II) and released. The block returns and moves a maximum distance y towards wall 2. Displacements x and y are measured with respect to the equilibrium position of the block B. The ratio y/x is

[IIT-JEE 2008]



(A) 4 (B) 2 (C) $\frac{1}{2}$ (D) $\frac{1}{4}$

2. A block of mass 2M is attached to a massless spring with spring-constant k. This block is connected to two other blocks of masses M and 2M using two massless pulleys and strings. The accelerations of the blocks are a_1, a_2 and a_3 as shown in figure. The system is released from rest with the spring in its unstretched state. The maximum extension of the spring is x_0. Which of the following option(s) is/are correct ? [g is the acceleration due to gravity. Neglect friction]



(A) $x_0 = \frac{4Mg}{k}$

(B) When spring achieves an extension of $\frac{x_0}{2}$ for the first time, the speed of the block connected to the spring is $3g\sqrt{\frac{M}{5k}}$

(C) $a_2 - a_1 = a_1 - a_3$

(D) At an extension of $\frac{x_0}{4}$ of the spring, the magnitude of acceleration of the block connected to the spring is $\frac{3g}{10}$

MCQS with one or more than one correct answer

3. A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of the particle. The motion of the particle takes place in a plane. It follows that :– ***[IIT-JEE 1987]***

(A) its velocity is constant

(B) its acceleration is constant

(C) its kinetic energy is constant

(D) it moves in a circular path

4. A particle of mass m is initially at rest at the origin. It is subjected to a force and starts moving along the x-axis. Its kinetic energy K changes with time as $dK/dt = \gamma t$, where γ is a positive constant of appropriate dimensions. Which of the following statements is (are) true ?

(A) The force applied on the particle is constant

(B) The speed of the particle is proportional to time

(C) The distance of the particle from the origin increses linerarly with time

(D) The force is conservative

Assertion & Reason

5. ***Statement–I :*** A block of mass m starts moving on a rough horizontal surface with a velocity v. It stops due to friction between the block and the surface after moving through a certain distance. The surface is now tilted to an angle of 30° with the horizontal and the same block is made to go up on the surface with the same initial velocity v. The decrease in the mechanical energy in the second situation is smaller than that in the first situation.

and ***[IIT-JEE 2007]***

Statement–II : The coefficient of friction between the block and the surface decreases with the increase in the angle of inclination.

(A) statement–I is true, statement–II is true; statement–II is a correct explanation for statement–I

(B) statement–I is true, statement–II is true, statement–II is NOT a correct explanation for statement–I

(C) statement–I is true, statement–II is false

(D) statement–I is false, statement–II is true

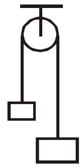
Matrix-Match Questions

6. A particle of unit mass is moving along the x-axis under the influence of a force and its total energy is conserved. Four possible forms of the potential energy of the particle are given in column I (a and U_0 are constants). Match the potential energies in column I to the corresponding statement(s) in column II. ***[IIT-JEE(Adv.) 2015]***

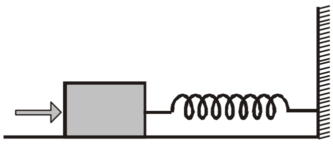
Column I	***Column II***
(A) $U_1(x) = \dfrac{U_0}{2}\left[1-\left(\dfrac{x}{a}\right)^2\right]^2$	(p) The force acting on the particle is zero at x = a
(B) $U_2(x) = \dfrac{U_0}{2}\left(\dfrac{x}{a}\right)^2$	(q) The force acting on the particle is zero at x = 0.
(C) $U_3(x) = \dfrac{U_0}{2}\left(\dfrac{x}{a}\right)^2 \exp\left[-\left(\dfrac{x}{a}\right)^2\right]$	(r) The force acting on the particle is zero at x = –a.
(D) $U_4(x) = \dfrac{U_0}{2}\left[\dfrac{x}{a}-\dfrac{1}{3}\left(\dfrac{x}{a}\right)^3\right]$	(s) The particle experiences an attractive force towards x = 0 in the region $\lvert x\rvert < a$.
	(t) The particle with total energy $\dfrac{U_0}{4}$ can oscillate about the point x = –a

Subjective Questions

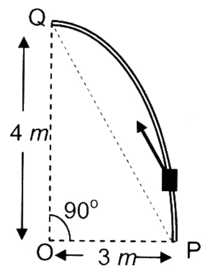
7. A light inextensible string that goes over a smooth fixed pulley as shown in the figure connects two blocks of masses 0.36 kg and 0.72 kg. Taking g = 10 m/s^2, find the work done (***in joules***) by the string on the block of mass 0.36 kg during the first second after the system is released from rest. ***[IIT-JEE 2009]***



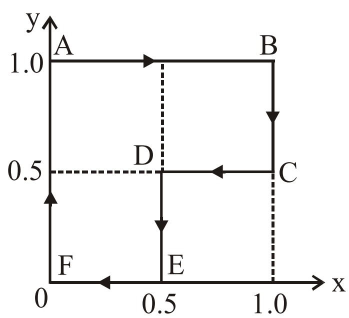
8. A block of mass 0.18 kg is attached to a spring of force-constant 2 N/m. The coefficient of friction between the block and the floor is 0.1. Initially the block is at rest and the spring is un-stretched. An impulse is given to the block as shown in the figure. The block slides a distance of 0.06 m and comes to rest for the first time. The initial velocity of the block in m/s is V= N/10. Then N is ***[IIT-JEE-2011]***



9. Condider an elliptically shaped rail PQ in the vertical plane with OP = 3 m and OQ = 4 m. A block of mass 1 kg is pulled along the rail from P to Q with a force of 18 N, which is always parallel to line PQ (see the figure given). Assuming no frictional losses, the kinetic energy of the block when it reaches Q is (n × 10) Joules. The value of n is (take acceleration due to gravity = 10 ms^{-2}) ***[JEE(Adv.)-2014]***



10. A particle is moved along a path AB-BC-CD-DE-EF-FA, as shown in figure, in presence of a force $\vec{F} = \left(\alpha y\hat{i} + 2\alpha x\hat{j}\right)N$, where x and y are in meter and $\alpha = -1$ N/m^{-1}. The work done on the particle by this force $\vec{F}$ will be ____ Joule. ***[JEE(Adv.)-2019]***



* * * * * *

ANSWERS

EXERCISE-1

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	C	D	C	(i) C (ii) A (iii) C	A	B	A	A	C	C	A	A	B	B	B
Que.	16	17	18	19	20	21	22	23	24	25					
Ans.	A	C	C	A	B	B	D	A	C	B					

EXERCISE-2

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	A,C	A,B	D	A,D	B,D	B	A,B,C	B,C	B,D	A,B,C

- ***Match the Column*** **11**. (A) t, (B) p, (C) s, (D) q
  12. (A) q,s; (B) p,s; (C) r,s; (D) p,s

- ***Comprehension Based Questions***
  Comprehension 1 : **13**. C **14**. D **15**.B **16**. D
  Comprehension 2 : **17**. A **18**. A

EXERCISE-3

1. $h = H/2$; $s_{max} = H$ **2**.–1J **3.** $\sqrt{5}$ ms^{-1} **4.** $m_1 = \frac{m}{2}$

5. (a) 0.5 (b) 1000 N/m **6.** (a) zero (b) – 600 J (c) 1600 J

7. At a horizontal distance of 1 m from the free end of the spring.

8. 50

9. (b) $W_{ABC} = W_{ADC} = \frac{a^5}{3}$ (J), $W_{AC} = \frac{2a^5}{5}$ (J)] **10.** 4.24 meters

EXERCISE-4

Que.	1	2	3	4	5	6	7	8	9	10	11	12
Ans.	C	C	D	C	B	B	C	A	B	A	A	B

EXERCISE-5

1. (C) **2.** (C) **3.** (C, D) **3.** (A B D)
5. (C) **6.** (A) pqrt, (B) qs, (C) pqrs, (D) prt **7.** 8J
8. 4 **9.** (5) **10.** (0.75)

* * * * *

CIRCULAR MOTION

Recap of Early Classes

In the previous chapter about motion we have studied one dimensional motion and projectile motion which is two dimensional. In this chapter we are going to discuss circular motion which is also a two dimensional motion. We will discuss some basic properties, concepts and different parameters of circular motion. We will also learn application of Newton's Laws in circular motion. Some part of the terminology learnt in this chapter will be useful in further chapters for describing other types of motion such as rotational motion and oscillations.

Index

1.0 DESCRIBING TRANSLATION MOTION BY ANGULAR VARIABLES

1.1 Angular Motion

1.2 Kinematics of Circular Motion

2.0 ANGULAR VELOCITY OF A PARTICLE RELATIVE TO ANOTHER PARTICLE OR POINT

3.0 RADIUS OF CURVATURE

4.0 DYNAMICS OF CIRCULAR MOTION

4.1 Conical Pendulum

5.0 CIRCULAR TURNING ON ROADS

6.0 CENTRIFUGAL FORCE

6.1 Effect of Earth's Rotation on Apparent weight

7.0 CIRCULAR MOTION IN VERTICAL PLANE

7.1 Condition of Looping the Loop

7.2 Condition of Leaving the Circle

7.3 Condition of Oscillation

EXERCISE-1

EXERCISE-2

EXERCISE-3

EXERCISE-4

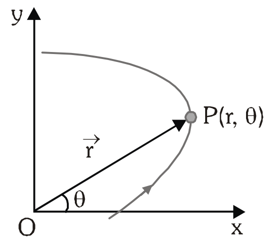
EXERCISE-5

CIRCULAR MOTION

1.0 DESCRIBING TRANSLATION MOTION BY ANGULAR VARIABLES

SL AL

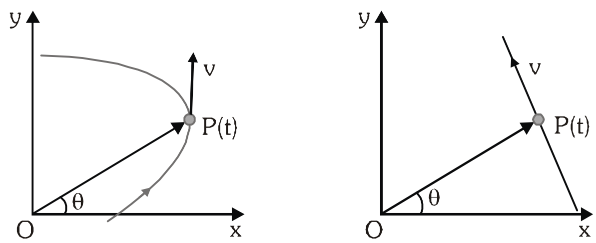
Position of particle can completely be specified by its position vector $\vec{r}$, if magnitude r of the position vector and its orientation relative to some fixed reference direction is known. In the given figure is shown a particle P at location shown by position vector $\vec{r} = \overrightarrow{OP}$. Magnitude of the position vector is distance r = OP of the particle from the origin O and orientation of the position vector is the angle θ made by line OP with the positive x-axis. We now specify position of a particle by these to variables r and θ, known as polar coordinates.



When the particle moves, either or both of these coordinates change with time. If a particle moves radially away from the origin, magnitude r of its position vector $\vec{r}$ increases without any change in angle θ. Similarly, if a particle moves radially towards the origin, r decreases without any change in angle θ. If a particle moves on a circular path with center at the origin, only the angle θ changes with time. If the particle moves on any path other than a radial straight line or circle centered at the origin, both of the coordinates r and θ change with time.

1.1 Angular Motion

Change in direction of position vector $\vec{r}$ is known as angular motion. It happens when a particle moves on a curvilinear path or straight-line path not containing the origin as shown in the following figures.



Angular Motion

Angular position : The coordinate angle θ at an instant is known as angular position of the particle.
Angular Displacement : A change in angular position θ in a time interval is known as angular displacement.
Angular Velocity : The instantaneous rate of change in angular position θ with respect to time is known as angular velocity.

We denote angular velocity by symbol ω. $\qquad \omega = \dfrac{d\theta}{dt}$

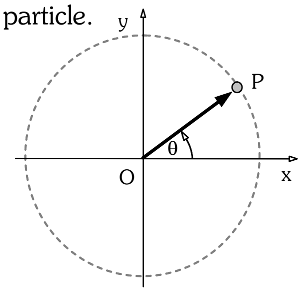
Angular Acceleration : The instantaneous rate of change in angular velocity ω with respect to time is known as angular acceleration.

We denote angular acceleration by symbol α. $\qquad \alpha = \dfrac{d\omega}{dt} = \dfrac{d^2\theta}{dt^2} = \omega\dfrac{d\omega}{d\theta}$

If a particle moves in a plane, the position vector turns either in clockwise or anticlockwise sense. Assuming one of these direction positive and other negative, problems of angular motion involving angular position θ, angular velocity ω and angular acceleration α can be solved in fashion similar to problems of rectilinear motion involving position x, velocity v, and acceleration a.

1.2 Kinematics of Circular Motion

A body in circular motion moves on a circular path. At present, we discuss only translation motion of a body on circular path and disregard any rotation; therefore, we represent the body as a particle. In the given figure, a particle P is shown moving on a circular path of radius r. Here only for simplicity center of the circular path is assumed at the origin of a coordinate system. In general, it is not necessary to assume center at the origin. Position vector of the particle is shown by a directed radius $\overline{OP} = \vec{r}$. Therefore, it is also known as radius vector. The radius vector is always normal to the path and has constant magnitude and as the particle moves, it is the angular position θ, which varies with time.



Angular Variables in Circular Motion

Angular position θ, angular velocity ω and angular acceleration α known as angular variables vary in different manner depending on how the particle moves.

Motion with uniform angular velocity

If a particle moves with constant angular velocity, its angular acceleration is zero and position vector turns at constant rate. It is analogous to uniform velocity motion on straight line. The angular position θ at any instant of time t is expressed by the following equation.

$$\theta = \theta_0 + \omega t$$

Motion with uniform angular acceleration

If a particle moves with constant angular acceleration, its angular velocity changes with time at a constant rate. The angular position θ, angular velocity ω and the angular acceleration α bear relations described by the following equations, which have forms similar to corresponding equations that describe uniform acceleration motion.

$$\omega = \omega_o + \alpha t$$

$$\theta = \theta_o + \omega_o t + \tfrac{1}{2}\alpha t^2$$

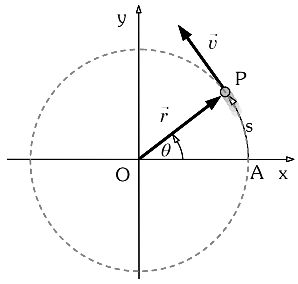
$$\theta = \theta_o + \tfrac{1}{2}\left(\omega_o + \omega\right)t$$

$$\omega^2 = \omega_o^2 + 2\alpha\left(\theta - \theta_o\right)$$

Motion with variable angular acceleration

Variable angular acceleration of a particle is generally specified as function of time, angular position or angular velocity. Problems involving variable angular acceleration can also be solved in a way analogous to corresponding rectilinear motion problems in which acceleration is specified as function of time, position or velocity.

Linear Velocity and Acceleration in Circular Motion



The instantaneous velocity $\vec{v}$ and the instantaneous acceleration $\vec{a}$ are also known as linear velocity and linear acceleration.

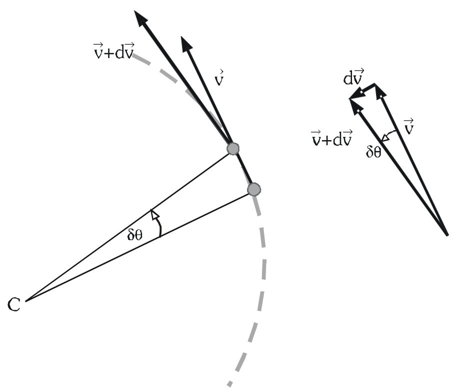
In the figure is shown a particle moving on a circular path. As it moves it covers a distance s (arc length).

$$s = \theta r$$

Linear velocity $\vec{v}$ is always along the path. Its magnitude known as linear speed is obtained by differentiating s with respect to time t.

$$v = \frac{d\theta}{dt}r = \omega r$$

If speed of the particle is uniform, the circular motion is known as uniform circular motion. In this kind of motion as the particle precedes further, only direction of velocity changes. Therefore, instantaneous acceleration or linear acceleration accounts for only change in direction of motion.

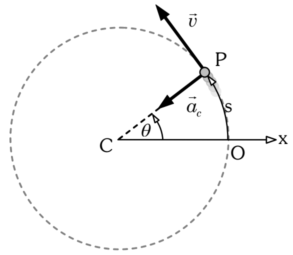


Consider a particle in uniform circular motion. It is shown at two infinitely close instants t and t + dt, where its velocity vectors are $\vec{v}$ and $\vec{v} + d\vec{v}$. These two velocity vectors are equal in magnitude and shown in adjacent figure. From this figure, it is obvious that the change $d\vec{v}$ in velocity vector is perpendicular to velocity vector $\vec{v}$ i.e towards the center. It can be approximated as arc of radius equal to magnitude of $\vec{v}$. Therefore we can write $|d\vec{v}| = d\theta \cdot v$. Hence acceleration of this particle is towards the center. It is known as normal component of acceleration or more commonly centripetal acceleration.

Dividing $|d\vec{v}|$ by time interval dt we get magnitude of centripetal acceleration a_c.

$$a_c = \frac{d\theta}{dt}v = \omega v = \omega^2 r = \frac{v^2}{r}$$

Acceleration and velocity of a particle in uniform circular motion are shown in the following figure.



To keep the particle in uniform circular motion net force acting on it must be towards the center, therefore it is known as centripetal force.

If particle moves with varying speed, the net force on it must have a component along the direction of velocity vector in addition to the centripetal force. This component force is along the tangent to the path and produces a component of acceleration in the tangential direction. This component known as tangential component of acceleration a_T, accounts for change in speed.

$$a_T = \frac{dv}{dt} = \frac{d\omega}{dt}r = \alpha r$$

Illustrations

Illustration 1. Angular position θ of a particle moving on a curvilinear path varies according to the equation $\theta = t^3 - 3t^2 + 4t - 2$, where θ is in radians and time t is in seconds. What is its average angular acceleration in the time interval t = 2s to t = 4s?

Solution Like average linear acceleration, the average angular acceleration α_{av} equals to ratio of change in angular velocity $\Delta\omega$ to the concerned time interval Δt.

$$\alpha_{av} = \frac{\Delta\omega}{\Delta t} = \frac{\omega_{final} - \omega_{initial}}{t_{final} - t_{initial}} \qquad \ldots(i)$$

The angular velocity ω being rate of change in angular position can be obtained by equation

$$\omega = \frac{d\theta}{dt}$$

Substituting the given expression of the angular position θ, we have

$$\omega = 3t^2 - 6t + 4 \ \ldots(ii)$$

From the above eq. (ii), angular velocities ω_2 and ω_4 at the given instants $t = 2$ s and 4s are

ω_4 = 4 rad/s and ω_4=28 rad/s.

Substituting the above values in eq. (1), we have α_{av}=12 rad/s^2

Illustration 2. A particle starts form rest and moves on a curve with constant angular acceleration of 3.0 rad/s^2. An observer starts his stopwatch at a certain instant and record that the particle covers an angular span of 120 rad at the end of 4^{th} second. How long the particle had moved when the observer started his stopwatch?

Solution Let the instants when the particle starts moving and the observer starts his stopwatch, are t_0=0 to t=t_1. Denoting angular positions and angular velocity at the instant $t = t_1$ by θ_1 and ω_1 and the angular position at the instant $t_2 = t_1 + 4$ s by θ_2, we can express the angular span covered during the interval from eq.

$$\theta - \theta_o = \omega_o t + \tfrac{1}{2}\alpha t^2 \rightarrow \theta_2 - \theta_1 = \omega_1\left(t_2 - t_1\right) + \tfrac{1}{2}\alpha\left(t_2 - t_1\right)^2$$

Substituting values θ_1, θ_2, $t_1$and t_2, we have $\omega_1 = 24$ rad/s

From eq. $\omega=\omega_0+at$, we have $\omega_1 = \omega_0+at_1$

Now substituting ω_0=0, ω_1= 24 and α = 3 rad/s^2, we have t_1 = 8.0 s

Illustration 3. A particle moves on a circular path of radius 8 m. Distance traveled by the particle in time t is given by the equation $s = \frac{2}{3}t^3$. Find its speed when tangential and normal accelerations have equal magnitude.

Solution The speed v, tangential acceleration a_τ and the normal acceleration a_n are expressed by the following equations.

$$v = \frac{ds}{dt}$$

Substituting the given expression for s, we have $v = 2t^2$

$$a_\tau = \frac{d^2s}{dt^2} \quad ...(i)$$

Substituting the given expression for s, we have $a_\tau = 4t$

$$a_n = \frac{v^2}{r} \quad ...(ii)$$

Substituting v from eq. (i) and r =8m, we have $a_n = \frac{1}{2}t^4$ ...(iii)

The instant when the tangential and the normal accelerations have equal magnitude, can be obtained by equating their expressions given in eq. (ii) and (iii).

$a_n = a_T \rightarrow \quad t = 2s$

Substituting the above value of t in eq. (i), we obtain v = 8 m/s

Illustration 4. A particle is moving on a circular path of radius 1.5 m at a constant angular acceleration of 2 rad/s^2. At the instant t = 0, angular speed is 60/π rpm. What are its angular speed, angular displacement, linear velocity, tangential acceleration and normal acceleration at the instant t = 2 s.

Solution Initial angular speed is given in rpm (revolution per minute). It is expressed in rad/s as

$$1 \text{ rpm} = \frac{2\pi \text{ rad}}{60 \text{ s}}$$

$$\omega_o = \left(\frac{60}{\pi}\right) \times \frac{2\pi \text{ rad}}{60 \text{ s}} = 2 \text{ rad/s}$$

At the instant t = 2 s, angular speed ω_2 and angular displacement θ_2 are calculated by using eq.

$$\omega_2 = \omega_o + \alpha t$$

Substituting values $\omega_o = 2$ rad/s, $\alpha = 2$ rad/s^2, $t = 2$ s, we have

$$\omega_2 = 6 \text{ rad/s}$$

$$\theta_2 = \theta_o + \tfrac{1}{2}(\omega_o + \omega_2)t$$

Substituting values $\theta_o = 0$ rad, $\omega_o = 2$ rad/s, $\omega_2 = 6$ rad/s and t =2 s, we have

$\theta_2 = 8$ rad

Linear velocity at t = 2 s, can be calculated by using eq.

$v_2 = r\omega_2$

Substituting $r = 1.5$ m and $\omega_2 = 6$ rad/s, we have

$v_2 = 9$ m/s

Tangential acceleration a_τ and normal acceleration a_n can be calculated by using

$a_\tau = r\alpha$

Substituting $r = 1.5$ m and $\alpha = 2$ rad/s^2, we have

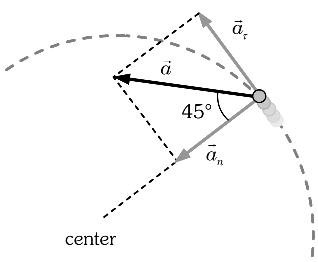
$a_\tau = 3$ m/s^2

$a_n = \omega^2 r$

Substituting $\omega_2 = 6$ rad/s and r = 1.5 m, we have

$a_n = 54$ m/s^2

Illustration 5. A particle is moving in a circular orbit with a constant tangential acceleration. After 2 s from the beginning of motion, angle between the total acceleration vector and the radius R becomes 45°. What is the angular acceleration of the particle?



Solution In the adjoining figure are shown the total acceleration vector $\vec{a}$ and its components the tangential accelerations $\vec{a}_\tau$ and normal accelerations $\vec{a}_n$ are shown. These two components are always mutually perpendicular to each other and act along the tangent to the circle and radius respectively. Therefore, if the total acceleration vector makes an angle of 45° with the radius, both the tangential and the normal components must be equal in magnitude. Now from eq. and , we have

$$a_\tau = a_n \;\rightarrow \alpha R = \omega^2 R \Rightarrow \alpha = \omega^2 \qquad ...(i)$$

Since angular acceleration is uniform, form eq., we have $\omega = \omega_o + \alpha t$

Substituting $\omega_0 = 0$ and t = 2 s, we have $\omega = 2\alpha$ ...(ii)

From eq. (i) and (ii), we have α = 0.25 rad/s^2

Illustration 6. A particle is moving in a circle of radius R in such a way that at any instant the normal and the tangential component of its acceleration are equal. If its speed at t=0 is v_0 then the time it takes to complete the first revolution is $\frac{R}{\alpha v_0}(1-e^{-\beta\pi})$. Find the value of $(\alpha+\beta)$.

Solution :

$$\frac{dv}{dt} = \frac{v^2}{R} \Rightarrow \int_{v_0}^{v}\frac{dv}{v^2} = \frac{1}{R}\int_0^t dt \Rightarrow \left(-\frac{1}{v}\right)_{v_0}^{v} = \frac{t}{R} \Rightarrow v = \frac{v_0}{1-\frac{v_0}{R}t} \Rightarrow \frac{ds}{dt} = \frac{v_0}{1-\frac{v_0}{R}t} \Rightarrow \int_0^{2\pi R} ds = \int_0^t \frac{v_0}{\left(1-\frac{v_0}{R}t\right)}dt$$

$$\Rightarrow 2\pi R = -R\left[\ell n\left(1-\frac{v_0}{R}t\right)\right]_0^t \Rightarrow 2\pi = -\ell n\left(1-\frac{v_0}{R}t\right) \Rightarrow 1-\frac{v_0}{R}t = e^{-2\pi} \Rightarrow t = \frac{R}{v_0}(1-e^{-2\pi})$$

$$\Rightarrow \alpha = 1, \beta = 2 \Rightarrow (\alpha+\beta) = (1+2) = 3$$

BEGINNER'S BOX–1

Kinematics of Circular Motion

1. If angular velocity of a disc depends an angle rotated θ as $\omega = \theta^2 + 2\theta$, then its angular acceleration α at $\theta = 1$ rad is :

(A) 8 rad/s^2 (B) 10rad/s^2 (C) 12 rad/s^2 (D) None of these

2. If the radii of circular path of two particles are in the ratio of 1 : 2, then in order to have same centripetal acceleration, their speeds should be in the ratio of :

(A) 1 : 4 (B) 4 : 1 (C) $1:\sqrt{2}$ (D) $\sqrt{2}:1$

3. A stone tied to the end of a string 80 cm long is whirled in a horizontal circle with a constant speed. If the stone makes 14 revolutions in 25 s, the magnitude of acceleration is :

(A) 20 ms^{-2} (B) 12 m/s^2 (C) 9.9 ms^{-2} (D) 8 ms^{-2}

4. A particle starts moving along a circle of radius $(20/\pi)$m with constant tangential acceleration. If speed of the particle is 50 m/s at the end of the second revolution after motion has began, the tangential acceleration in m/s^2 is :

(A) 1.6 (B) 4 (C) 15.6 (D) 13.2

5. For a body in circular motion with a constant angular velocity, the magnitude of the average acceleration over a period of half a revolution is.... times the magnitude of its instantaneous acceleration.

(A) $\frac{2}{\pi}$ (B) $\frac{\pi}{2}$ (C) π (D) 2

6. The second's hand of a watch has length 6 cm. Speed of end point and magnitude of difference of velocities at two perpendicular positions will be

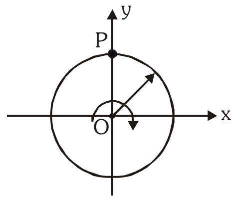
(A) 2π & 0 mm/s (B) $2\sqrt{2}\,\pi$ & 4.44 mm/s

(C) $2\sqrt{2}\,\pi$ & 2π mm/s (D) 2π & $2\sqrt{2}\pi$ mm/s

7. A particle is kept fixed on a turntable rotating uniformly. As seen from the ground, the particle goes in a circle, its speed is 20 cm/s and acceleration is 20 cm/s^2. The particle is now shifted to a new position to make the radius half of the original value. The new values of the speed and acceleration will be

(A) 10 cm/s, 10 cm/s^2 (B) 10 cm/s, 80 cm/s^2 (C) 40 cm/s, 10 cm/s^2 (D) 40 cm/s, 40 cm/s^2

8. A ring rotates about z axis as shown in figure. The plane of rotation is xy. At a certain instant the acceleration of a particle P (shown in figure) on the ring is $\left(6\hat{i}-8\hat{j}\right)m/s^2$. Find the angular acceleration of the ring & the angular velocity at that instant. Radius of the ring is 2m.



9. A particle is performing circular motion of radius 1 m. Its speed is v = $(2t^2)$ m/s. What will be magnitude of its acceleration at t = 1s.

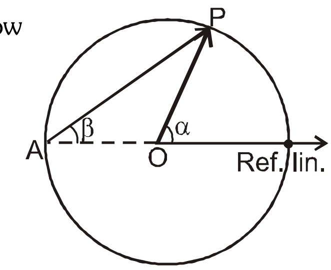
2.0 ANGULAR VELOCITY OF A PARTICLE RELATIVE TO ANOTHER PARTICLE OR POINT

AL

Just as velocities are always relative, similarly angular velocity is also always relative. There is no such thing as absolute angular velocity. Angular velocity is defined with respect to origin, the point from which the position vector of the moving particle is drawn.

Consider a particle P moving along a circular path shown in the figure given below

Here angular velocity of the particle P w.r.t. 'O' and 'A' will be different



Angular velocity of a particle P w.r.t. O, $\omega_{PO} = \dfrac{d\alpha}{dt}$

Angular velocity of a particle P w.r.t. A, $\omega_{PA} = \dfrac{d\beta}{dt}$

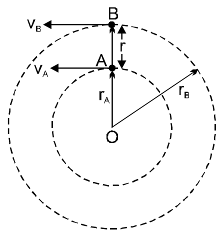
Definition :

Angular velocity of a particle 'A' with respect to the other moving particle 'B' is the rate at which position vector of 'A' with respect to 'B' rotates at that instant. (or it is simply, angular velocity of A with origin fixed at B). Angular velocity of A w.r.t. B, ω_{AB} is mathematically define as

$$\omega_{AB} = \frac{\text{Component of relative velocity of A w.r.t. B, perpendicular to line}}{\text{seperation between A and B}} = \frac{(V_{AB})_\perp}{r_{AB}}$$

Important points:

- If two particles are moving on two different concentric circles with different velocities then angular velocity of B as observed by A will depend on their positions and velocities. Consider the case when A and B are closest to each other moving in same direction as shown in figure. In this situation



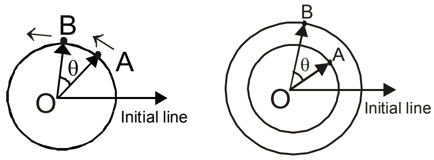
$(V_{AB})_\perp = v_B - v_A$

Separation between A and B is $r_{BA} = r_B - r_A$

so, $\omega_{AB} = \dfrac{(V_{AB})_\perp}{r_{AB}} = \dfrac{v_B - v_A}{r_B - r_A}$

- If two particles are moving on the same circle or different coplanar concentric circles in same direction with different uniform angular speed ω_A and ω_B respectively, the rate of change of angle between $\overrightarrow{OA}$ and $\overrightarrow{OB}$ is

$$\frac{d\theta}{dt} = \omega_B - \omega_A$$



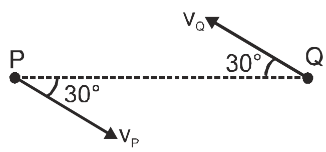
So the time taken by one to complete one revolution around O w.r.t. the other

$$T = \frac{2\pi}{\omega_{rel}} = \frac{2\pi}{\omega_2 - \omega_1} = \frac{T_1 T_2}{T_1 - T_2}$$

- $\omega_B - \omega_A$ is rate of change of angle between $\overline{OA}$ and $\overline{OB}$. This is not angular velocity of B w.r.t. A. (Which is rate at which line AB rotates)

Illustrations

Illustration 7. Two moving particles P and Q are 10 m apart at any instant. Velocity of P is 8 m/s at 30°, from line joining the P and Q and velocity of Q is 6m/s at 30°.Calculate the angular velocity of P w.r.t. Q



Solution $\omega_{PQ} = \frac{8\sin 30^\circ - (-6\sin 30^\circ)}{10} = 0.7$ rad/s.

3.0 RADIUS OF CURVATURE

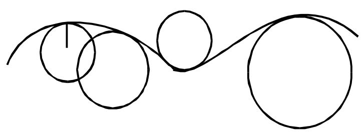
SL AL

Any curved path can be assume to be made of infinite circular arcs. Radius of curvature at a point is the radius of the circular arc at a particular point which fits the curve at that point.

$$F_c = \frac{mv^2}{R} \Rightarrow R = \frac{mv^2}{F_C} = \frac{mv^2}{F_\perp}$$

$F_\perp$ = Force perpendicular to velocity (centripetal force)
If the equation of trajectory of a particle is given we can
find the radius of curvature of the instantaneous circle by using the formula ,

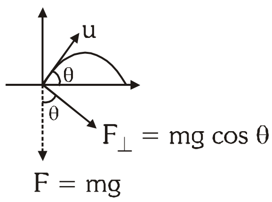
$$R = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}.$$



Illustrations

Illustration 8. A particle of mass m isprojected with speed u at an angle θ with the horizontal. Find the radius of curvature of the path traced out by the particle at the point of projection and also at the highest point of trajectory.

Solution At point of projection



$$R = \frac{mv^2}{F_\perp} = \frac{mu^2}{mg\cos\theta}$$

$$R = \frac{u^2}{g\cos\theta} \quad \text{Ans.}$$

at highest point

$$a_\perp = g,\ v = u\cos\theta : \ R = \frac{v^2}{a_\perp} = \frac{u^2\cos^2\theta}{g}$$

Illustration 9. A particle moves along the plane trajectory y(x) with constant speed v. Find the radius of curvature of the trajectory at the point x = 0 if the trajectory has the form of a parabola $y = ax^2$ where 'a' is a positive constant.

Solution If the equation of the trajectory of a particle is given we can find the radius of trajectory of the instantaneous circle by using the formula

$$R = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}$$

As; $y = ax^2 \Rightarrow \frac{dy}{dx} = 2ax = 0 \text{ (at } x = 0) \text{ and } \frac{d^2y}{dx^2} = 2a$

Now radius of trajectory is given by

$$R = \frac{[1+0]^{3/2}}{2a} = \frac{1}{2a}$$

Alter: This problem can also be solved by using the formula : $R = \frac{v^2}{a_\perp}$, $y = ax^2$, differentiate with respect to time $\frac{dy}{dt} = 2ax\frac{dx}{dt}$ .....(1)

at x = 0, $v_y = \frac{dy}{dt} = 0$ hence $v_x = v$

Now, differentiate (1) with respect to time $\frac{d^2y}{dt^2} = 2ax\frac{d^2x}{dt^2} + 2a\left(\frac{dx}{dt}\right)^2$

at x = 0, vx = v

$\therefore$ net acceleration, $a = a_y = 2av^2$ (since ax = 0) this acceleration is perpendicular to velocity (vx). Hence it is equal to centripetal acceleration

$$R = \frac{v^2}{a_\perp} = \frac{v^2}{2av^2} = \frac{1}{2a}$$ ***Ans.***

BEGINNER'S BOX-2

Relative Angular speed and Radius of Curvature

1. A particle is projected with a speed u at an angle θ with the horizontal. Consider a small part of its path near the highest position and take it approximately to be a circular arc. What is the radius of this circle?

(A) $\frac{u^2\sin^2\theta}{g}$ (B) $\frac{u^2\cos^2\theta}{g}$ (C) $\frac{u^2\tan^2\theta}{g}$ (D) $\frac{u^2}{g}$

2. A stone is thrown horizontally under gravity with a speed of 10m/sec. Find the radius of curvature of it's trajectory at the end of 3 sec after motion began.

(A) $10\sqrt{10}$ m (B) $100\sqrt{10}$ m (C) $\sqrt{10}$ m (D) 100 m

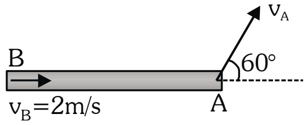
3. Particle is moving with speed 10 m/s parallel to x-axis start from (0, 2m). Find angular speed of particle about origin at t = 0.2 sec.

(A) 2.5 rad/s (B) 5 rad/s (C) 2 rad/s (D) None of these

4. In above question find angular speed of particle about origin initially.

(A) 2.5 rad/s (B) 5 rad/s (C) 2 rad/s (D) None of these

5. At any instant velocity of end A and B are given in figure with proper direction. Find angular speed of end A w.r. to B at given instant (length of rod = 1 m)



(A) 2 rad/s (B) $2\sqrt{3}$ rad/s

(C) $\sqrt{3}$ rad/s (D) None of these

6. A particle is projected horizontally with speed 10 m/s from tower, Radius of curvature of path of particle after one second is

(A) $20\sqrt{2}$ m (B) 100 m (C) $50\sqrt{2}$ m (D) None of these

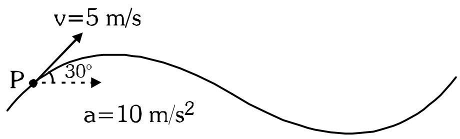
7. If equation of path of moving particle is given by $y = 4x^2$, radius of curvature of path of particle at origin is :

(A) $\frac{1}{4}$m (B) $\frac{1}{2}$m (C) $\frac{1}{8}$m (D) None of these

8. In above question radius of curvature when particle at (x, y) is

(A) $\frac{\left[1+(8x)^2\right]^{3/2}}{8}$ (B) $\frac{\left[1+(4x)^2\right]^{3/2}}{8}$ (C) $\frac{\left[1+(8x)^2\right]^{3/2}}{4}$ (D) $\frac{\left[1+(4x^2)^2\right]^{3/2}}{4}$

9. A particle is moving on given curve path. At certain instant its velocity and acceleration shown in figure



(i) Rate of change of speed of particle at P is

(A) $5\sqrt{3}\,m/s^2$ (B) 10 m/s² (C) 5 m/s² (D) None of these

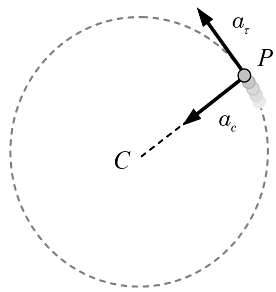
(ii) Radius of curvature at point P is :

(A) $5\sqrt{3}$ m (B) 10 m (C) 5 m (D) None of these

4.0 DYNAMICS OF CIRCULAR MOTION

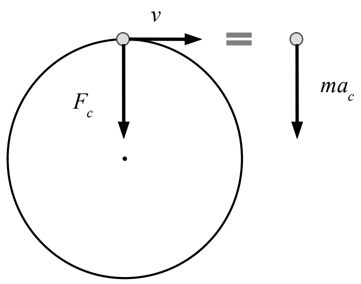
SL AL

Velocity vector points always tangent to the path and continuously change its direction, as a particle moves on a circular path even with constant speed and give rise to normal component of acceleration, which always points toward the center of the circular path. This component of acceleration is known as centripetal (center seeking) acceleration and denoted by a_c. Moreover, if speed also changes the particle will have an additional acceleration component along the tangent to the path. This component of acceleration is known as tangential acceleration and denoted by a_τ.



The centripetal and tangential accelerations

Application of Newton's Second law in Circular Motion

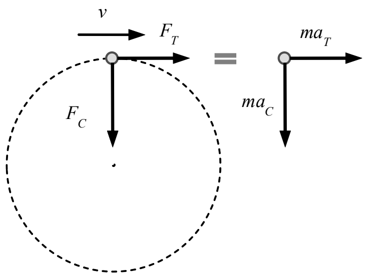


Consider a particle of mass m moving with uniform speed v in a circle of radius r as shown in figure. It necessarily posses a centripetal acceleration and hence there must be a net force $\left(\sum \vec{F} = \vec{F}_C\right)$ acting always towards the center according to the second law. This net force F_C acting towards the center is known as centripetal force.

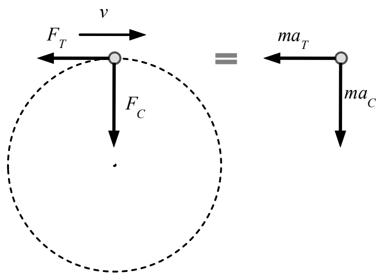
$$\sum \vec{F} = m\vec{a} \rightarrow F_C = ma_C$$

When a particle is whirled with the help of a string in a horizontal circle, the required centripetal force is the tension in the string. The gravitational attraction between a satellite and the earth, between moon and the earth, between the sun and its planets, and the electrostatic attraction between the nucleus and electrons are the centripetal forces and provide the necessary centripetal acceleration.

Now consider a particle moving on circular path with varying speed. The net acceleration has two components, the tangential acceleration and the centripetal acceleration. Therefore, the net force must also have two components, one component in tangential direction to provide the tangential acceleration and the other component towards the center to provide the centripetal acceleration. The former one is known as tangential force and the latter one as centripetal force.



Particle moving with increasing speed.



Particle moving with decreasing speed.

When the particle moves with increasing speed the tangential force acts in the direction of motion and when the particle moves with decreasing speed the tangential force acts in direction opposite to direction of motion.

To write equations according to the second law, we consider the tangential and the radial directions as two mutually perpendicular axes. The components along the tangential and the radial directions are designated by subscripts T and C.

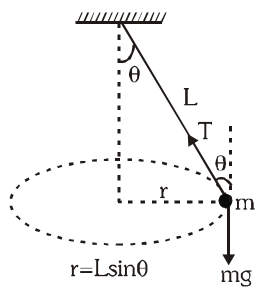
$$F_C = ma_C$$

$$F_T = ma_T$$

4.1 Conical Pendulum

SL AL

If a small particle of mass m tied to a string is whirled in a horizontal circle, as shown in figure. The arrangement is called the 'conical pendulum'. In case of conical pendulum the vertical component of tension balances the weight while its horizontal component provides the necessary centripetal force. Thus,,



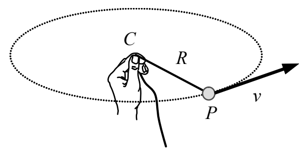
$$T\sin\theta = \frac{mv^2}{r} \quad \text{and} \quad T\cos\theta = mg \Rightarrow v = \sqrt{rg\tan\theta}$$

$\therefore$ Angular speed $\quad \omega = \dfrac{v}{r} = \sqrt{\dfrac{g\tan\theta}{r}}$

So, the time period of pendulum is $T = \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{r}{g\tan\theta}} = 2\pi\sqrt{\dfrac{L\cos\theta}{g}}$

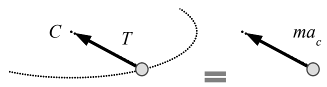
Illustrations

Illustration 10. In free space, a man whirls a small stone P of mass m with the help of a light string in a circle of radius R as shown in the figure. Establish the relation between the speed of the stone and the tension developed in the string. Also, find the force applied on the string by the man.



Solution The system is in free space therefore no force other than the tension acts on the stone to provide necessary centripetal force. The tension does not have any component in tangential direction therefore tangential component of acceleration is zero. In the adjoining figure it is shown that how tension (T) in the string produces necessary centripetal force.

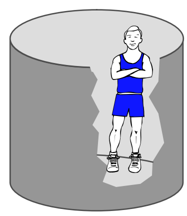
Applying Newton's second law of motion to the stone, we have $F_C = ma_C \rightarrow T = ma_C = \dfrac{mv^2}{R}$



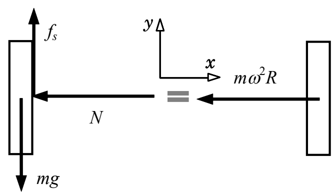
The end where man holds the string is stationary and tension applied by the string to this end is T towards the stone, therefore the man must apply a force equal to the tension in magnitude but in a direction away from the stone.

$$T = \frac{mv^2}{R}$$

Illustration 11. A boy stands on a horizontal platform inside a cylindrical container of radius R resting his back on the inner surface of the container. The container can be rotated about the vertical axis of symmetry. The coefficient of static friction between his back and the inner surface of the container is μ_s. The angular speed of the container is gradually increased. Find the minimum angular speed at which if the platform below his feet removed, the boy should not fall.



Solution As the container rotates at angular speed ω the boy moves in a circular path of radius R with a speed $v = \omega R$. Since the angular speed is increased gradually the angular acceleration can be ignored and hence the tangential acceleration of the boy too. Thus, the boy has a centripetal acceleration of $\omega^2 R$, provided by the normal reaction N applied by the wall of the container. The weight of the boy is balanced by the force of static friction. All these force are shown in the adjoining figure where the boy is shown schematically by a rectangular box of mass m.



$$\sum F_x = ma_x \rightarrow N = m\omega^2 R \quad \text{...(i)}$$

$$\sum F_y = ma_y \rightarrow f_s = mg \quad \text{...(ii)}$$

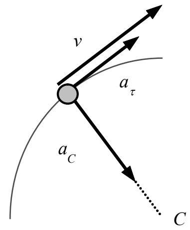
Since force of static friction cannot be greater than the limiting friction $\mu_S N$,

we have $f_s \le \mu_s N$ ...(iii)

From the above equations, the minimum angular speed is $\omega_{min} = \sqrt{\dfrac{g}{\mu_s R}}$

Illustration 12. A motorcyclist wishes to travel in circle of radius R on horizontal ground and increases speed at constant rate a. The coefficient of static and kinetic frictions between the wheels and the ground are μ_s and μ_k. What maximum speed can he achieve without slipping?

Solution The motorcyclist and the motorcycle always move together hence they can be assumed to behave as a single rigid body of mass equal to that of the motorcyclist and the motorcycle. Let the mass of this body is m. The external forces acting on it are its weight (mg), the normal reaction N on wheels from ground, and the force of static friction f_s. The body has no acceleration in vertical direction therefore; the normal reaction N balances the weight (mg).

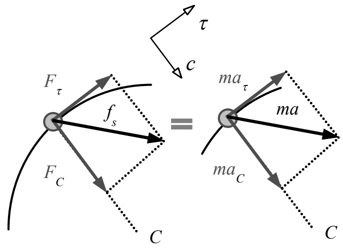


$$N = mg \qquad ...(i)$$

The frictional force cannot exceed the limiting friction.

$$f_{sm} \le \mu_s N \qquad ...(ii)$$

During its motion on circular path, the only external force in horizontal plane is the force of static friction, which is responsible to provide the body necessary centripetal and tangential acceleration. These conditions are shown in the adjoining figure where forces in vertical direction are not shown.



$$F_\tau = ma_\tau \qquad ...(iii)$$

$$F_C = ma_C = \frac{mv^2}{r} \qquad ...(iv)$$

The above two forces are components of the frictional force in tangential and normal directions. Therefore, we have

$$f_s = \sqrt{F_\tau^2 + F_C^2} = m\sqrt{a_\tau^2 + a_C^2} \qquad ...(v)$$

The centripetal acceleration increases with increase in speed and the tangential acceleration remains constant. Therefore, their resultant increases with speed. At maximum speed the frictional force achieves its maximum value (limiting friction f_{sm}), therefore from eq. (i), (ii), (iii), (iv), and (v), we have

$$v = \left[r^2\left\{(\mu_s g)^2 - a^2\right\}\right]^{\frac{1}{4}}$$

5.0 CIRCULAR TURNING ON ROADS

SL AL

When vehicles go through turnings, they travel along a nearly circular arc. There must be some force which will produce the required centripetal acceleration. If the vehicles travel in a horizontal circular path, this resultant force is also horizontal. The necessary centripetal force is being provided to the vehicles by following three ways.

1. By friction only.

2. By banking of roads only.

3. By friction and banking of roads both.

In real life the necessary centripetal force is provided by friction and banking of roads both. Now let us write equations of motion in each of the three cases separately and see what are the constant in each case.

By Friction Only

Suppose a car of mass m is moving at a speed v in a horizontal circular arc of radius r. In this case, the necessary centripetal force to the car will be provided by force of friction f acting towards center

Thus, $f = \frac{mv^2}{r}$

Further, limiting value of f is μN or $f_L = \mu N = \mu mg$ $(N = mg)$

Therefore, for a safe turn without sliding $\frac{mv^2}{r} \le f_L$ or $\frac{mv^2}{r} \le \mu mg$ or $\mu \ge \frac{v^2}{rg}$ or $v \le \sqrt{\mu rg}$

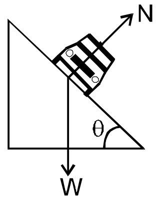
Here, two situations may arise. If μ and r are known to us, the speed of the vehicle should not exceed $\sqrt{\mu rg}$ and if v and r are known to us, the coefficient of friction should be greater than $\frac{v^2}{rg}$

By Banking of Roads Only

Friction is not always reliable at circular turns if high speeds and sharp turns are involved to avoid dependence on friction, the roads are banked at the turn so that the outer part of the road is some what lifted compared to the inner part.

Applying Newton's second law along the radial direction

$$N\sin\theta = \frac{mv^2}{r}$$



Net force along vertical direction should be zero

or $\quad N\cos\theta = mg$

from these two equations, we get

$$\tan\theta = \frac{v^2}{rg} \quad \text{or} \quad v = \sqrt{rg\tan\theta}$$

By Friction and Banking of Road Both

If a vehicle is moving on a circular road which is rough and banked also, then three forces may act on the vehicle, of these the first force, i.e., weight (mg) is fixed both in magnitude and direction.

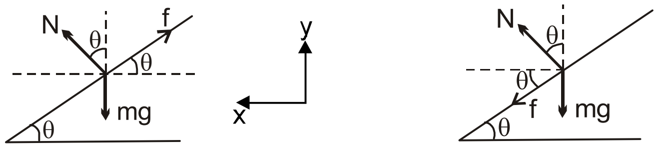


Figure (i) Figure (ii)

The direction of second force, i.e., normal reaction N is also fixed (perpendicular to road) while the direction of the third force i.e., friction f can be either inwards or outwards while its magnitude can be varied upto a maximum limit ($f_L = \mu N$). So the magnitude of normal reaction N and directions plus magnitude of friction f are so adjusted that the resultant of the three forces mentioned above is $\frac{mv^2}{r}$ towards the center. Of these m and r are also constant. Therefore, magnitude of N and directions plus magnitude of friction mainly depends on the speed of the vehicle v. Thus, situation varies from problem to problem. Even though we can see that :

(i) Friction f will be outwards if the vehicle is at rest v = 0. Because in that case the component of weight mg $\sin\theta$ is balanced by f.

(ii) Friction f will be inwards if $v > \sqrt{rg\tan\theta}$

(iii) Friction f will be outwards if $v < \sqrt{rg\tan\theta}$ and

(iv) Friction f will be zero if $v = \sqrt{rg\tan\theta}$

(v) For maximum safe speed (figure (ii)

$$N\sin\theta + f\cos\theta = \frac{mv^2}{r} \qquad \text{........(i)}$$

$$N\cos\theta - f\sin\theta = mg \qquad \text{.......(ii)}$$

As maximum value of friction $f = \mu N$

$$\therefore \quad \frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta} = \frac{v^2}{rg} \qquad \therefore \quad v_{max} = \sqrt{\frac{rg(\tan\theta + \mu)}{(1 - \mu\tan\theta)}}$$

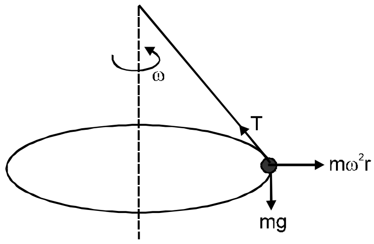
Similarly for minimum safe speed (see figure (i)); $v_{min} = \sqrt{\frac{rg(\tan\theta - \mu)}{(1 + \mu\tan\theta)}}$

6.0 CENTRIFUGAL FORCE

SL AL

When a body is rotating in a circular path and the centripetal force vanishes, the body would leave the circular path. To an observer A who is not sharing the motion along the circular path, the body appears to fly off tangentially at the point of release. To another observer B, who is sharing the motion along the circular path (i.e., the observer B is also rotating with the body which is released, it appears to B, as if it has been thrown off along the radius away from the centre by some force. This inertial force is called centrifugal force.)

Its magnitude is equal to that of the centripetal force. $= \frac{mv^2}{r} = m\omega^2 r$



Direction of cetrifugal force, it is always directed radially outward.

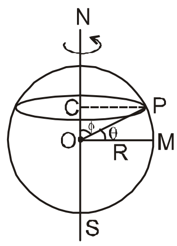
Centrifugal force is a fictitious force which has to be applied as a concept only in a rotating frame of reference to apply Newton's law of motion in that frame. FBD of ball w.r.t. non inertial frame rotating with the ball.

Suppose we are working from a frame of reference that is rotating at a constant, angular velocity ω with respect to an inertial frame. If we analyses the dynamics of a particle of mass m kept at a distance r from the axis of rotation, we have to assume that a force $mr\omega^2$ react radially outward on the particle. Only then we can apply Newton's laws of motion in the rotating frame. This radially outward pseudo force is called the centrifugal force.

6.1 Effect of Earth's Rotation on Apparent weight

SL AL

The earth rotates about its axis at an angular speed of one revolution per 24 hours. The line joining the north and the south poles is the axis of rotation.

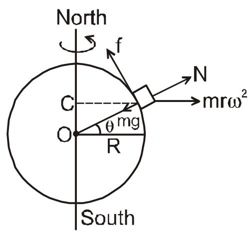


Every point on the earth moves in a circle. A point at equator moves in a circle of radius equal to the radius of the earth and the centre of the circle is same as the centre of the earth. For any other point on the earth, the circle of rotation is smaller than this. Consider a place P on the earth (figure).

Draw a perpendicular PC from P to the axis SN. The place P rotates in a circle with the centre at C. The radius of this circle is CP. The angle between the line OM and the radius OP through P is called the latitude of the place P. We have

$$CP = OP\cos\theta \qquad \text{or,} \qquad r = R\cos\theta$$

where R is the radius of the earth and ϕ is colatitude angle.



If we work from the frame of reference of the earth, we shall have to assume the existence of pseudo force. In particular, a centrifugal force mw^2r has to be assumed on any particle of mass m placed at P.

If we consider a block of mass m at point P then this block is at rest with respect to earth. If resolve the forces along and perpendicular the centre of earth then

$$N + mr\omega^2\cos\theta = mg$$

$$\Rightarrow \quad N = mg - mr\omega^2\cos\theta$$

$$\Rightarrow \quad N = mg - mR\omega^2\cos^2\theta$$

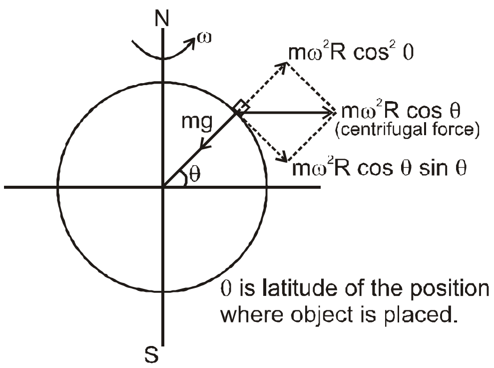


Figure (1) Earth's gravity & centrifugal force due rotation of Earth.

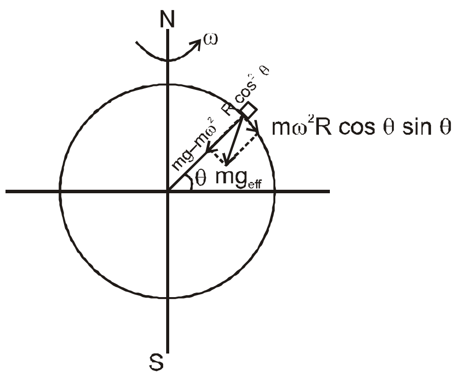


Figure (2) Resultant of Earth's gravity & centrifugal force is shown.

Illustrations

Illustration 13. Find the maximum speed at which a car can turn round a curve of 30 m radius on a level road if the coefficient of friction between the tyres and the road is 0.4 [acceleration due to gravity = 10 m/s^2]

Solution Here centripetal force is provided by friction so

$$\frac{mv^2}{r} \le \mu mg \Rightarrow v_{max} = \sqrt{\mu rg} = \sqrt{120} \approx 11\ ms^{-1}$$

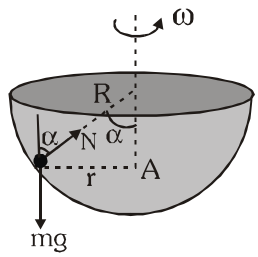
Illustration 14. For traffic moving at 60 km/hr, if the radius of the curve is 0.1 km, what is the correct angle of banking of the road ? (g = 10 m/s^2)

Solution In case of banking $\tan\theta = \frac{v^2}{rg}$

Here $v = 60$ km/hr $= 60 \times \frac{5}{18}\ ms^{-1} = \frac{50}{3}\ ms^{-1}$ $r = 0.1$ km $= 100$ m

So $\tan\theta = \frac{50/3 \times 50/3}{100 \times 10} = \frac{5}{18} \Rightarrow \theta = \tan^{-1}\left(\frac{5}{18}\right)$

Illustration 15. A hemispherical bowl of radius R is rotating about its axis of symmetry which is kept vertical. A small ball kept in the bowl rotates with the bowl without slipping on its surface. If the surface of the bowl is smooth and the angle made by the radius through the ball with the vertical is α. Find the angular speed at which the bowl is rotating.



Solution $N\cos\alpha = mg$ and $N\sin\alpha = mr\omega^2$ but $r = R\sin\alpha$

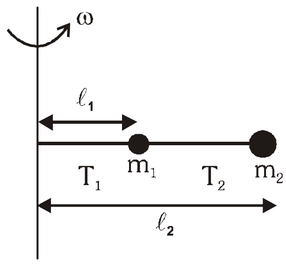
$\Rightarrow N\sin\alpha = mR\sin\alpha\omega^2 \Rightarrow N = mR\omega^2$

$\Rightarrow (mR\omega^2)\cos\alpha = mg \Rightarrow \omega = \sqrt{\frac{g}{R\cos\alpha}}$

BEGINNER'S BOX-3

Dynamics of Circular Motion and Banking of Roads

1. The whole set up shown in the figure is rotating with constant angular velocity ω on a horizontal frictionless table then the ratio of tensions $\frac{T_1}{T_2}$ is (Given $\frac{\ell_2}{\ell_1} = \frac{2}{1}$)



(A) $\frac{m_1}{m_2}$ (B) $\frac{(m_1 + 2m_2)}{2m_2}$

(C) $\frac{m_2}{m_1}$ (D) $\frac{(m_2 + m_1)}{m_2}$

2. A coin placed on a rotating turn table just slips if it is at a distance of 40 cm from the centre if the angular velocity of the turntable is doubled, it will just slip at a distance of

(A) 10 cm (B) 20 cm (C) 40 cm (D) 80 cm

3. Centrifugal force is considerd as pseudo force when

(A) An observer at the centre of circular motion

(B) An outside observer

(C) An observer who is moving with the particle which is experiencing the force

(D) None of the above

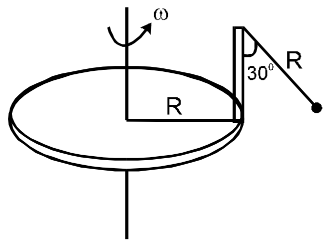
4. If the apparent weight of the bodies at the equator is to be zero, then the earth should rotate with angular velocity

(A) $\sqrt{\frac{g}{R}}$ rad/sec (B) $\sqrt{\frac{2g}{R}}$ rad/sec (C) $\sqrt{\frac{g}{2R}}$ rad/sec (D) $\sqrt{\frac{3g}{2R}}$ rad/sec

5. A vehicle can travel round a curve at a higher speed when the road is banked than when the road is level. This is because
(A) banking increases the coefficient of friction
(B) banking increases the radius,
(C) the normal reaction has a horizontal component,
(D) when the track is banked the weight of the car acts down the incline.

6. A stone of mass of 16 kg is attached to a string 144 m long and is whirled in a horizontal circle. The maximum tension the string can withstand is 16 newton. The maximum speed of revolution of the stone without breaking it, will be :
(A) 20 ms^{-1} (B) 16 ms^{-1} (C) 14 ms^{-1} (D) 12 ms^{-1}

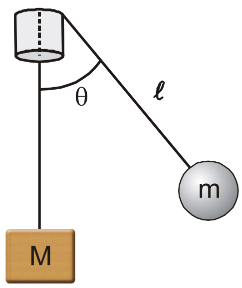
7. A disc of radius R has a light pole fixed perpendicular to the disc at the circumference which in turn has a pendulum of length R attached to its other end as shown in figure. The disc is rotated with a constant angular speed ω. The string is making an angle 30^0 with the rod. Then the angular speed ω of disc is:



(A) $\left(\frac{\sqrt{3}g}{R}\right)^{1/2}$ (B) $\left(\frac{\sqrt{3}g}{2R}\right)^{1/2}$

(C) $\left(\frac{g}{\sqrt{3}R}\right)^{1/2}$ (D) $\left(\frac{2g}{3\sqrt{3}R}\right)^{1/2}$

8. A mass M hangs stationary at the end of a light string that passes through a smooth fixed tube to a small mass m that moves around in a horizontal circular path. If ℓ is the length of the string from m to the top end of the tube and θ is angle between this part and vertical part of the string as shown in the figure, then time taken by m to complete one circle is equal to



(A) $2\pi\sqrt{\frac{\ell}{g\sin\theta}}$ (B) $2\pi\sqrt{\frac{\ell}{g\cos\theta}}$

(C) $2\pi\sqrt{\frac{m\ell}{gM\sin\theta}}$ (D) $2\pi\sqrt{\frac{\ell m}{gM}}$

9. A motorcyclist wants to drive on the vertical surface of wooden 'well' of radius 5 m, with a minimum speed of $5\sqrt{5}$ m/s. Find the minimum value of coefficient of friction between the tyres and the wall of the well . (Take g = 10 m/s^2)

10. A train has to negotiate a curve of radius 400 m. By how much height should the outer rail be raised with respect to inner rail for a speed of 48 km/hr ? The distance between the rails is 1 m.

7.0 CIRCULAR MOTION IN VERTICAL PLANE

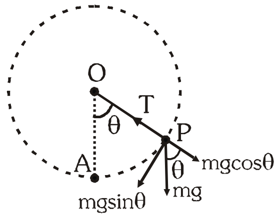
SL AL

Suppose a particle of mass m is attached to an inextensible light string of length R. The particle is moving in a vertical circle of radius R about a fixed point O. It is imparted a velocity u in horizontal direction at lowest point A. Let v be its velocity at point P of the circle as shown in figure.

Here, $h = R(1 - \cos\theta)$ ...(i)

From conservation of mechanical energy

$1/2m(u^2 - v^2) = mgh \Rightarrow v^2 = u^2 - 2gh$ ....(ii)



The necessary centripetal force is provided by the resultant of tension

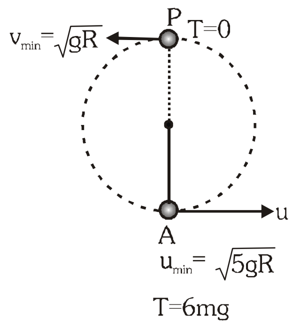
T and mg cosθ

$$T - mg\cos\theta = \frac{mv^2}{R} \quad ...(iii)$$

Since speed of the particle decreases with height, hence tension is maximum at the bottom, where $\cos\theta = 1$ (as $\theta = 0^0$)

$\Rightarrow T_{max} = \frac{mv^2}{R} + mg$; $T_{min} = \frac{mv'^2}{R} - mg$ at the top. Here, v' = speed of the particle at the top.

7.1 Condition of Looping the Loop $\left(u \geq \sqrt{5gR}\right)$



The particle will complete the circle if the string does not slack even at the highest point $(\theta = \pi)$. Thus, tension in the string should be greater than or equal to zero $(T \geq 0)$ at $\theta = \pi$. In critical case substituting T=0 and $\theta = \pi$ in Eq. (iii), we get $mg = \frac{mv_{min}^2}{R} \Rightarrow v_{min} = \sqrt{gR}$ (at highest point)

Substituting $\theta = \pi$ in Eq. (i), Therefore, from Eq. (ii)

$$u_{min}^2 = v_{min}^2 + 2gh = gR + 2g(2R) = 5gR \Rightarrow u_{min} = \sqrt{5gR}$$

Thus, if $u \geq \sqrt{5gR}$, the particle will complete the circle. At $u = \sqrt{5gR}$, velocity at highest point is $v = \sqrt{gR}$ and tension in the string is zero.

Substituting $\theta = 0^0$ and $v = \sqrt{5gR}$ in Eq. (iii), we get T =6 mg or in the critical condition tension in the string at lowest position is 6 mg. This is shown in figure. If $u < \sqrt{5gR}$, following two cases are possible.

7.2 Condition of Leaving the Circle $\left(\sqrt{2gR} < u < \sqrt{5gR}\right)$

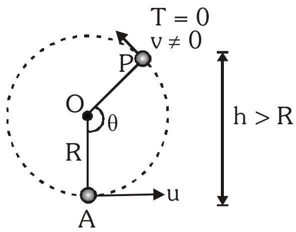
If $u < \sqrt{5gR}$, the tension in the string will become zero before reaching the highest point. From Eq. (iii), tension in the string becomes zero (T=0) where, $\cos\theta = \frac{-v^2}{Rg} \Rightarrow \cos\theta = \frac{2gh - u^2}{Rg}$

Substituting, this value of $\cos\theta$ in Eq. (i), we get $\frac{2gh - u^2}{Rg} = 1 - \frac{h}{R} \Rightarrow h = \frac{u^2 + Rg}{3g} = h_1$ (say) .....(iv)

or we can say that at height h_1 tension in the string becomes zero. Further, if $u < \sqrt{5gR}$, velocity of the particle becomes zero when $0 = u^2 - 2gh \Rightarrow h = \frac{u^2}{2g} = h_2$(say)...(v) i.e., at height h_2 velocity of particle becomes zero.

Now, the particle will leave the circle if tension in the string becomes zero but velocity is not zero. or T = 0 but $v \neq 0$. This is possible only when $h_1 < h_2$

$$\Rightarrow \frac{u^2 + Rg}{3g} < \frac{u^2}{2g} \Rightarrow 2u^2 + 2Rg < 3u^2 \Rightarrow u^2 > 2Rg \Rightarrow u > \sqrt{2Rg}$$



Therefore, if $\sqrt{2gR} < u < \sqrt{5gR}$, the particle leaves the circle.

From Eq. (iv), we can see that h > R if $u^2 > 2gR$. Thus, the particle, will leave the circle when h > R or $90^0 < \theta < 180^0$. This situation is shown in the figure

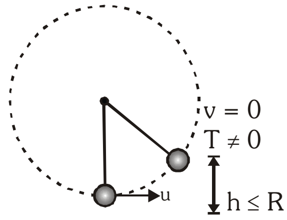
$$\sqrt{2gR} < u < \sqrt{5gR} \text{ or } 90^0 < \theta < 180^0$$

Note : After leaving the circle, the particle will follow a parabolic path.

7.3 Condition of Oscillation $\left(0 < u \le \sqrt{2gR}\right)$

The particle will oscillate if velocity of the particle becomes zero but tension in the string is not zero or v = 0, but $T \neq 0$. This is possible when $h_2 < h_1$

$\Rightarrow \frac{u^2}{2g} < \frac{u^2 + Rg}{3g} \Rightarrow 3u^2 < 2u^2 + 2Rg \Rightarrow u^2 < 2Rg \Rightarrow u < \sqrt{2Rg}$



Moreover, if $h_1 = h_2$, $u = \sqrt{2Rg}$ and tension and velocity both becomes zero simultaneously. Further, from Eq. (iv), we can see that $h \le R$ if $u \le \sqrt{2Rg}$.

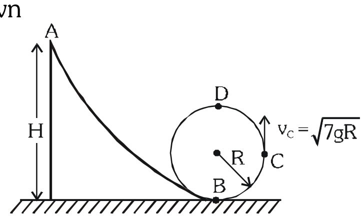
Thus, for $0 < u \le \sqrt{2gR}$, particle oscillates in lower half of the circle ($0^0 < \theta \le 90°$)

This situation is shown in the figure. $0 < u \le \sqrt{2gR}$ or $0° < \theta \le 90°$

Illustrations

Illustration 16. A particle is released from rest from A. Calculate following for shown situation (neglect friction) :

(a) Speed at D
(b) Normal reaction at D
(c) Height H



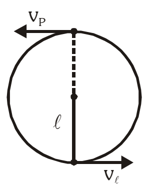
Solution (a) $v_D^2 = v_C^2 - 2gR = 5gR \Rightarrow v_D = \sqrt{5gR}$

(b) $mg + N_D = \frac{mv_D^2}{R} \Rightarrow N_D = \frac{m(5gR)}{R} - mg = 4mg$

(c) by energy conservation between point A & C

$$mgH = \frac{1}{2}mv_C^2 + mgR = \frac{1}{2}mv_D^2 + mg2R = \frac{1}{2}m(5gR) + mg2R = \frac{9}{2}mgR \Rightarrow H = \frac{9}{2}R$$

Illustration 17. A stone of mass 1 kg tied to a light string of length $\ell = \frac{10}{3}$ m is whirling in a circular path in vertical plane. If the ratio of the maximum to minimum tension in the string is 4, find the speed of the stone at the lowest and highest points



Solution $\because \frac{T_{max}}{T_{min}} = 4 \therefore \frac{\frac{mv_\ell^2}{\ell} + mg}{\frac{mv_p^2}{\ell} - mg} = 4 \Rightarrow \frac{v_\ell^2 + g\ell}{v_p^2 - g\ell} = 4$

We know $v_\ell^2 = v_p^2 + 4g\ell \Rightarrow \frac{v_P^2 + 5g\ell}{v_P^2 - g\ell} = 4 \Rightarrow 3v_P^2 = 9g\ell$

$\Rightarrow v_P = \sqrt{3g\ell} = \sqrt{3 \times 10 \times \frac{10}{3}} = 10\ ms^{-1} \Rightarrow v_\ell = \sqrt{7g\ell} = \sqrt{7 \times 10 \times \frac{10}{3}} = 15.2\ ms^{-1}$

$\Rightarrow T = (mg + ma) + 2m(g+a)(1-\cos\theta) = m(g+a)(3 - 2\cos\theta)$

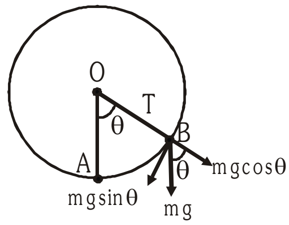
Illustration 18. A heavy particle hanging from a fixed point by a light inextensible string of length ℓ, is projected horizontally with speed $\sqrt{(g\ell)}$. Find the speed of the particle and the inclination of the string to the vertical at the instant of the motion when the tension in the string is equal to the weight of the particle.

Solution Let tension in the string becomes equal to the weight of the particle when particle reaches the point B and deflection of the string from vertical is θ. Resolving mg along the string and perpendicular to the string, we get net radial force on the particle at B i.e.

$F_R = T - mg\cos\theta$ ....(i)

If v_B be the speed of the particle at B, then

$$F_R = \frac{mv_B^2}{\ell} \quad ....(ii)$$



From (i) and (ii), we get, $T - mg\cos\theta = \frac{mv_B^2}{\ell} \quad ....(iii)$

Since at B, T = mg $\Rightarrow$ $mg(1-\cos\theta) = \frac{mv_B^2}{\ell}$

$\Rightarrow v_B^2 = g\ell(1-\cos\theta)$ ...(iv)

Applying conservation of mechanical energy of the particle at point A and B, we have

$$\frac{1}{2}mv_A^2 = mg\ell\ (1-\cos\theta) + \frac{1}{2}mv_B^2\text{; where } v_A = \sqrt{g\ell} \text{ and } v_B = \sqrt{g\ell(1-\cos\theta)}$$

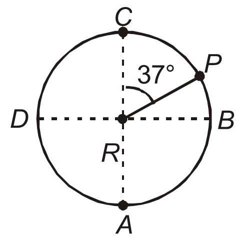
$$\Rightarrow g\ell = 2g\ell\ (1-\cos\theta) + g\ell\ (1-\cos\theta) \Rightarrow \cos\theta = \frac{2}{3} \Rightarrow \theta = \cos^{-1}\left(\frac{2}{3}\right)$$

Putting the value of $\cos\theta$ in equation (iv), we get : $v_B = \sqrt{\frac{g\ell}{3}}$

BEGINNER'S BOX-4

Vertical Circular Motion

1*. A particle is moving along a vertical circle of radius R. The velocity of particle at P will be (assume critical condition at C)



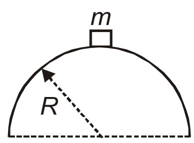
(A) $\sqrt{\frac{7}{5}gR}$ (B) $\sqrt{2gR}$

(C) $\sqrt{\frac{3gR}{5}}$ (D) $\sqrt{\frac{3}{2}gR}$

2. A particle is projected so as to just move along a vertical circle of radius r with the help of massless string. The ratio of the tension in the string when the particle is at the lowest and highest point on the circle is -

(A) 1 (B) finite but large (C) zero (D) Infinite

3. If a particle of mass m resting at the top of a fixed smooth hemisphere of radius R is displaced, then the angle with vertical at which it looses contact with the surface is

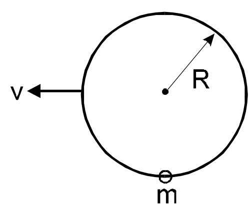


(A) $\tan^{-1}\left(\frac{1}{3}\right)$ (B) $\cos^{-1}\left(\frac{1}{3}\right)$ (C) $\tan^{-1}\left(\frac{2}{3}\right)$ (D) $\cos^{-1}\left(\frac{2}{3}\right)$

4. A bucket is whirled in a vertical circle with a string attached to it.The water in bucket does not fall down even when the bucket is inverted at the top of its path. We can say that in this position.

(A) $mg = \frac{mv^2}{r}$ (B) mg is greater than $\frac{mv^2}{r}$

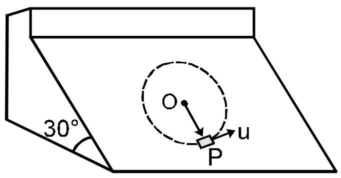
(C) mg is not greater than $\frac{mv^2}{r}$ (D) mg is not less than $\frac{mv^2}{r}$



5. A ring of radius R lies in vertical plane. A bead of mass 'm' can move along the ring without friction. Initially the bead is at rest at the bottom most point on ring. The minimum constant horizontal speed v with which the ring must be pulled such that the bead completes the vertical circle

(A) $\sqrt{3gR}$ (B) $\sqrt{4gR}$ (C) $\sqrt{5gR}$ (D) $\sqrt{5.5gR}$

6. A particle is attached with a string of length ℓ which is fixed at point O on an inclined plane what minimum velocity should be given to the particle along the incline so that it may complete a circle on inclined plane (plane is smooth and initially particle was resting on the inclined plane.)



(A) $\sqrt{5g\ell}$ (B) $\sqrt{\frac{5g\ell}{2}}$ (C) $\sqrt{\frac{5\sqrt{3}g\ell}{2}}$ (D) $\sqrt{4g\ell}$

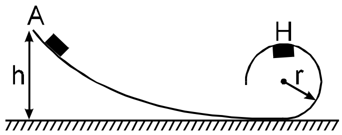
7. A ball suspended by a thread swings in a vertical plane so that its acceleration in the extreme position and lowest position are equal. The angle θ of thread deflection in the extreme position will be -

(A) $\tan^{-1}(2)$ (B) $\tan^{-1}(\sqrt{2})$ (C) $\tan^{-1}\left(\frac{1}{2}\right)$ (D) $2\tan^{-1}\left(\frac{1}{2}\right)$

8. A small body of mass m is allowed to slide on an inclined frictionless track from rest position 'A' as shown in the figure.

(i) Find the minimum height h, so that body may successfully complete the loop of radius 'r'.

(ii) If h is double of that minimum height, find the resultant force on the block at position H

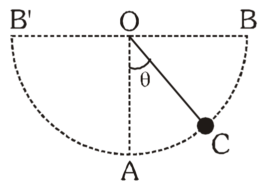


9*. A simple pendulum is vibrating with an angular amplitude of 90° as shown in the given figure. For what value of θ, is the acceleration directed

(i) vertically upwards

(ii) horizontally

(iii) vertically downwards



GOLDEN KEY POINTS

- For a rigid body, as all points will rotate through same angle in same time, angular velocity is a characteristic of the body as a whole, e.g., angular velocity of all points of earth about earth's axis is $(2\pi/24)$ rad/hr.
- Both average and instantaneous angular acceleration are axial vectors with dimension $[T^{-2}]$ and unit rad/s^2.

- Infinitesimally small angular displacement is a vector quantity, but finite angular displacement is a scalar, because while the addition of the infinitesimally small angular displacements obeys laws of vector addition, but addition of finite angular displacement do not obey laws of vector addition.

$$d\vec{\theta}_1 + d\vec{\theta}_2 = d\vec{\theta}_2 + d\vec{\theta}_1 \qquad \text{but} \qquad \theta_1 + \theta_2 \neq \theta_2 + \theta_1$$

- Direction of small angular displacement is decided by right hand thumb rule. When the fingers are directed along the motion of the point then thumb will represents the direction of angular displacement.

- In vector form

(i) $\vec{a}_t = \vec{\alpha} \times \vec{r}$

(ii) $\vec{a}_c = \vec{\omega} \times \vec{v}$

(iii) $\vec{v} = \vec{\omega} \times \vec{r}$

- (i) Differentiation of speed gives tangential acceleration.

(ii) Differentiation of velocity $(\vec{v})$ gives total acceleration.

(iii) $\left|\dfrac{d\vec{v}}{dt}\right|$ & $\dfrac{d|\vec{v}|}{dt}$ are not same physical quantity. $\left|\dfrac{d\vec{v}}{dt}\right|$ is the magnitude of rate of change of velocity, i.e. magnitude of total acceleration and $\dfrac{d|\vec{v}|}{dt}$ is a rate of change of speed, i.e. tangential acceleration.

- Remember $\dfrac{mv^2}{r}$ is not a force itself. It is just the value of the net force acting along the inside normal which is responsible for circular motion. This force may be friction, normal, tension, spring force, gravitational force or a combination of them.

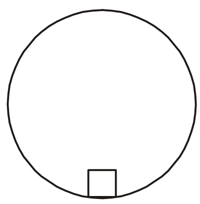
Condition for Looping the Loop in some other Cases

- ***Case 1 :*** A mass moving on a smooth vertical circular track.

Mass moving along a smooth vertical circular loop condition for just looping the loop, normal at highest point = 0

By calculation similar to article (motion in vertical circle)

Minimum horizontal velocity at lowest point = $\sqrt{5g\ell}$

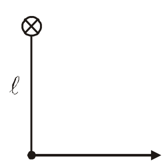


(B)

- ***Case 2 :*** A particle attached to a light rod rotated in vertical circle.

Condition for just looping the loop, velocity v = 0 at highest point (even if tension is zero, rod won't slack and a compressive force can appear in the rod).

By energy conservation, velocity at lowest point = $\sqrt{4g\ell}$



$V_{min} = \sqrt{4g\ell}$ (for completing the circle)

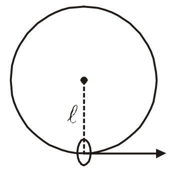
- ***Case 3 :*** A bead attached to a ring and rotated.

  Condition for just looping the loop, velocity v = 0 at highest point (even if normal is zero, the bead will not lose contact with the track, normal can act radially outward).

  By energy conservation,

  velocity at lowest point = $\sqrt{4g\ell}$

  $V_{min} = \sqrt{4g\ell}$ (for completing the circle)

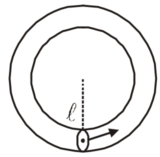


- ***Case-4 :*** A block rotated between smooth surfaces of a pipe.

  condition for just looping the loop, velocity v = 0 at highest point (even if normal is zero, the bead will not lose contact with the track, normal can act radially outward).

  By energy conservation,

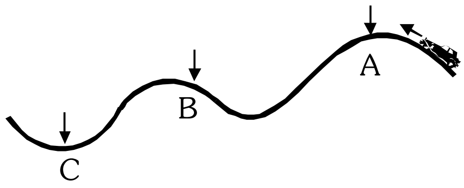
  $V_{min} = \sqrt{4g\ell}$ (for completing the circle)



SOME WORKED OUT ILLUSTRATIONS

Illustration 1.

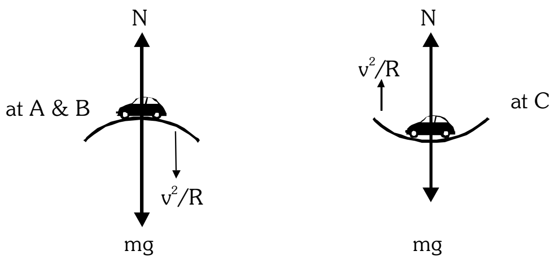
A car is moving along a hilly road as shown (side view). The coefficient of static friction between the tyres and pavement is constant and the car maintains a steady speed. If, at one of the points shown the driver applies the brakes as hard as possible without making the tyres slip, the magnitude of the frictional force immediately after the brakes are applied will be maximum if the car was at



(A) point A (B) point B
(C) point C (D) friction force same for positions A, B and C

Ans. (C)

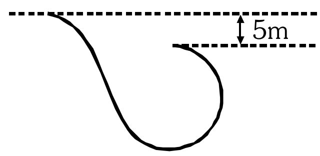
Solution



At A & B, $N = mg - mv^2/R$ & at C, $N = mg + mv^2/R$ $\therefore f_{max} = \mu_s N \rightarrow$ maximum for C

Illustration 2.

Consider a roller coaster with a circular loop. A roller coaster car starts from rest from the top of a hill which is 5 m higher than the top of the loop. It rolls down the hill and through the loop. What must the radius of the loop be so that the passengers of the car will feel at highest point, as if they have their normal weight?

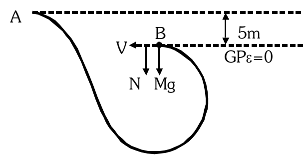


(A) 5 m (B) 10 m
(C) 15 m (D) 20 m

Ans. (A)

Solution

According to mechanical energy conservation between A and B



$$mg(5) = 0 + \frac{1}{2}mv^2 \Rightarrow v^2 = 10g \text{ ...(i)}$$

According to centripetal force equation

$$N + mg = \frac{mv^2}{r} \quad \text{for } N = mg;\ 2mg = \frac{mv^2}{r} \Rightarrow r = \frac{v^2}{2g} = \frac{10g}{2g} = 5m$$

Illustration 3.

A particle is projected along the inner surface of a smooth vertical circle of radius R, its velocity at the lowest point being $\frac{1}{5}\sqrt{95Rg}$. It will leave the circle at an angular distance.... from the highest point

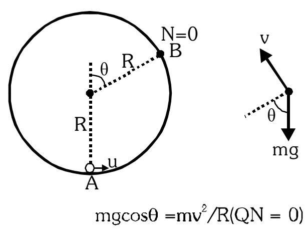
(A) 37° (B) 53° (C) 60° (D) 30°

Ans. (B)

Solution

By conservation of mechanical energy [between point A and B]

$$\frac{1}{2}mu^2 = mgR(1+\cos\theta) + \frac{1}{2}mv^2$$

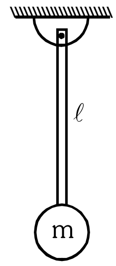


$$\frac{1}{2}m\left(\frac{1}{5}\sqrt{95Rg}\right)^2 = mgR(1+\cos\theta)+\frac{1}{2}mgR\cos\theta$$

$$\Rightarrow \frac{95}{25} = 2+2\cos\theta+\cos\theta \Rightarrow 3\cos\theta = \frac{45}{25} \Rightarrow \cos\theta = \frac{15}{25}=\frac{3}{5} \Rightarrow \theta = 53°$$

Illustration 4 to 6.

A rigid rod of length ℓ and negligible mass has a ball with mass m attached to one end and its other end fixed, to form a pendulum as shown in figure. The pendulum is inverted, with the rod straight up, and then released.



4. At the lowest point of trajectory, what is the ball's speed?

(A) $\sqrt{2g\ell}$ (B) $\sqrt{4g\ell}$ (C) $2\sqrt{2g\ell}$ (D) $\sqrt{8g\ell}$

5. What is the tension in the rod at the lowest point of trajectory of ball?

(A) 6 mg (B) 3 mg (C) 4 mg (D) 5 mg

6. Now, if the pendulum is released from rest from a horizontal position. At what angle from the vertical does the tension in the rod equal to the weight of the ball?

(A) $\cos^{-1}\left(\frac{2}{3}\right)$ (B) $\cos^{-1}\left(\frac{1}{3}\right)$ (C) $\cos^{-1}\left(\frac{1}{2}\right)$ (D) $\cos^{-1}\left(\frac{1}{4}\right)$

Solution

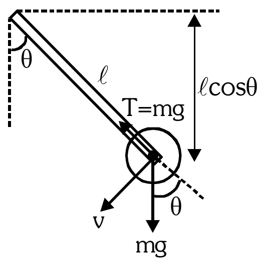
4. ***Ans. (B)***

From COME : $2mg\ell = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{4g\ell} = 2\sqrt{g\ell}$

5. ***Ans. (D)***

At the lowest point $T - mg = \frac{mv^2}{\ell} \Rightarrow T = mg + \frac{m}{\ell}(4g\ell) = 5mg$

6. ***Ans. (B)***



Force equation $T - mg\cos\theta = \frac{mv^2}{\ell}$

Energy equation $mg\ell\cos\theta = \frac{1}{2}mv^2$

Therefore $mg - mg\cos\theta = 2mg\cos\theta \Rightarrow 3\cos\theta = 1 \Rightarrow \cos\theta = \frac{1}{3}$

Illustration 7.

A point mass m connected to one end of inextensible string of length ℓ and other end of string is fixed at peg. String is free to rotate in vertical plane. Find the minimum velocity give to the mass in horizontal direction so that it hits the peg in its subsequent motion.

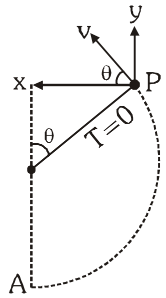
Solution

Tension in string is zero at point P in its subsequent motion, after this point its motion is projectile.

Velocity at point P, $T = 0 \Rightarrow mg\cos\theta = \frac{mv^2}{\ell} \Rightarrow v = \sqrt{g\ell\cos\theta}$

Assume its projectile motion start at point P and it passes through point C. So that equation of trajectory satisfy the co-ordinate of C (ℓ sin θ – ℓ cos θ)

Equation of trajectory



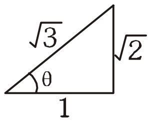
$$y = x\tan\theta = \frac{gx^2}{2v^2\cos^2\theta}$$

$$-\ell\cos\theta = \ell\sin\theta\tan\theta - \frac{g(\ell\sin\theta)^2}{2(g\ell\cos\theta)\cos^2\theta}$$

$$\Rightarrow \quad -\cos\theta = \frac{\sin^2\theta}{\cos\theta} - \frac{1}{2}\frac{\sin^2}{\cos^3\theta}$$

$\Rightarrow$ $-2\cos^4\theta = 2\sin^2\theta\cos^2\theta - \sin^2\theta \Rightarrow \sin^2\theta = 2\sin^2\theta\cos^2\theta + 2\cos4\theta$

$\Rightarrow$ $\sin^2\theta = 2\cos^2\theta(\sin^2\theta + \cos^2\theta) \Rightarrow \tan^2\theta = 2$

$\Rightarrow$ $\tan\theta = \sqrt{2} \Rightarrow$  $\Rightarrow \cos\theta = \frac{1}{\sqrt{3}}, \sin\theta = \sqrt{\frac{2}{3}}$

From energy conservation between point P and A.

$$\frac{1}{2}mu^2 - \frac{1}{2}mv^2 + mg\ell(1+\cos\theta)$$

$\Rightarrow$ $u^2 = v^2 + 2g\ell(1 + \cos\theta)$

$$\Rightarrow \quad u^2 = 2g\ell + 3g\ell + 3g\ell\frac{1}{\sqrt{3}} \Rightarrow u = \left[\left(2+\sqrt{3}\right)g\ell\right]^{1/2} \quad \textbf{Ans.}$$

Illustration 8.

A block of mass m is kept on rough horizontal turn table at a distance r from centre of table. Coefficient of friction between turn table and block is μ. Now turn table starts rotating with uniform angular acceleration α.

(i) Find the time after which slipping occurs between block and turn table.

(ii) Find angle made by friction force with velocity at the point of slipping

Solution

(i) $a_t = \alpha r$

speed after t time

$$\frac{dv}{dt} = \alpha r \Rightarrow v = 0 + \alpha rt$$

$$a_c = \frac{v^2}{r} = \alpha^2 rt^2$$

Net acceleration $a_{net} = \sqrt{a_t^2 + a_c^2} = \sqrt{\alpha^2 r^2 + \alpha^4 r^2 t^4}$

block just start slipping

$$\mu mg = ma_{net} = m\sqrt{\alpha^2 r^2 + \alpha^4 r^2 t^4}$$

$$t = \left(\frac{\mu^2 g^2 - \alpha^2 r^2}{\alpha^4 r^2}\right)^{1/4} \Rightarrow t = \left[\left(\frac{\mu g}{\alpha^2 r}\right)^2 - \left(\frac{1}{\alpha}\right)^2\right]^{1/4} \quad \text{Ans.}$$

(ii) $\tan\theta = \frac{a_c}{a_t} \Rightarrow \tan\theta = \frac{\alpha^2 rt^2}{\alpha r} \Rightarrow \theta = \tan^{-1}(\alpha t^2)$

ANSWERS

BEGINNER'S BOX-1

1. C　**2.** C　**3.** C　**4.** C　**5.** A　**6.** D

7. A　**8.** $-3\hat{k}\ \text{rad/s}^2$, $-2\hat{k}\ \text{rad/s}$　**9.** $4\sqrt{2}\ \text{ms}^{-2}$

BEGINNER'S BOX-2

1. (B)　**2.** (B)　**3.** (A)　**4.** (B)　**5.** (B)　**6.** (A)　**7.** (C)　**8.** (A)

9. (i) (A) (ii) (C)

BEGINNER'S BOX-3

1. B　**2.** A　**3.** C　**4.** A　**5.** C　**6.** D　**7.** D　**8.** D

9. 0.40　**10.** 4.5 cm

BEGINNER'S BOX-4

1. A　**2.** D　**3.** D　**4.** C　**5.** B　**6.** B　**7.** D

8. (i) $h_{min} = \frac{5}{2}r$ (ii) $F = 6\ mg$　**9.** (i) 0°, (ii) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$, (iii) 90°

EXERCISE – 1 MCQ (SINGLE CHOICE CORRECT)

1. When a particle moves in a circle with a uniform speed
(A) its velocity and acceleration are both constant
(B) its velocity is constant but the acceleration changes
(C) its acceleration is constant but the velocity changes
(D) its velocity and acceleration both change

2. A particle of mass m starts undergoing a circular motion of radius R. The speed of particle is increasing as V = Kt (K is a constant), (t is time in seconds). The magnitude of net acceleration after 1 second is

(A) K (B) $K\sqrt{1+\frac{K^2}{R^2}}$ (C) $\frac{K^2}{R}$ (D) $2K\sqrt{1+\frac{K^2}{R^2}}$

3. A particle of mass m is fixed to one end of a light rigid rod of length ℓ and rotated in a vertical circular path about its other end. The minimum speed of the particle at its highest point must be :

(A) zero (B) $\sqrt{g\ell}$ (C) $\sqrt{1.5g\ell}$ (D) $\sqrt{2g\ell}$

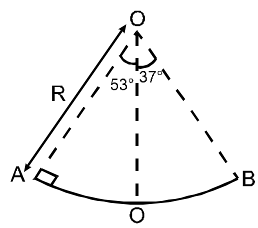
4. A marble of mass m and radius b is placed in a hemispherical bowl of radius r. The minimum velocity to be given to the marble so that it reaches the highest point is :

(A) $\sqrt{2g(r-b)}$ (B) $\sqrt{2gr}$ (C) $\sqrt{2g(r+b)}$ (D) $\sqrt{g(r-b)}$

5. A particle is moving in a circular path with a constant speed v. If θ is the angular displacement, then starting from $\theta = 0^0$, the maximum and minimum change in the linear momentum will occur when value of θ is respectively :
(A) 45° & 90° (B) 90° & 180° (C) 180° & 360° (D) 90° & 270°

6. A rod of length L is pivoted at one end and is rotated with a uniform angular velocity in a horizontal plane . Let T_1 and T_2 be the tensions at the points L/4 and 3L/4 away from the pivoted ends.
(A) $T_1 > T_2$ (B) $T_2 > T_1$
(C) $T_1 = T_2$
(D) The relation between T_1 and T_2 depends on whether the rod rotates clockwise or anticlockwise

7. A section of fixed smooth circular track of radius R in vertical plane is shown in the figure. A block is released from position A and leaves the track at B. The radius of curvature of its trajectory when it just leaves the track at B is -

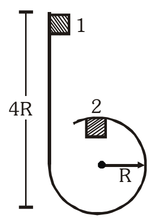


(A) R (B) $\frac{R}{4}$
(C) $\frac{R}{2}$ (D) none of these

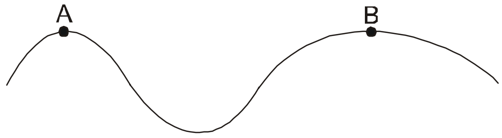
8. In a simple pendulum, the breaking strength of the string is double the weight of the bob. The bob is released from rest when the string is horizontal. The string breaks when it makes an angle θ with the vertical–

(A) $\theta = \cos^{-1}\left(\frac{1}{3}\right)$ (B) $\theta = 60°$ (C) $\theta = \cos^{-1}\left(\frac{2}{3}\right)$ (D) $\theta = 0$

9. A cube of mass M starts at rest from point 1 at a height 4R, where R is the radius of the circular track. The cube slides down the frictionless track and around the loop. The force which the track exerts on the cube at point 2 is:
(A) 3 mg (B) mg
(C) 2 mg (D) cube will not reach the point 2.

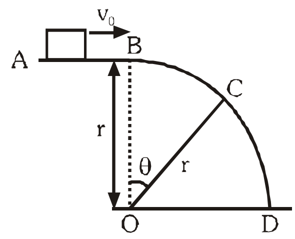


10. A car moves at a constant speed on a road as shown in figure. The normal force by the road on the car is N_A and N_B when when it is at the points A and B respectively.



(A) $N_A = N_B$ (B) $N_A > N_B$ (C) $N_A < N_B$ (D) insufficient information

11. A small block slides with velocity $v_0 = 0.5\sqrt{gr}$ on the horizontal frictionless surface as shown in the figure. The block leaves the surface at point C. The angle θ in the figure is :



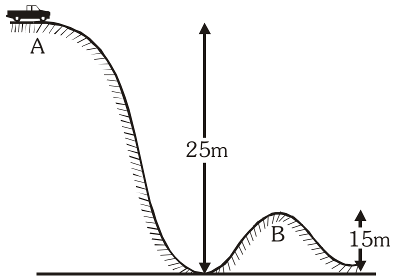
(A) $\cos^{-1}\frac{4}{9}$ (B) $\cos^{-1}\frac{3}{4}$

(C) $\cos^{-1}\frac{1}{2}$ (D) $\cos^{-1}\frac{4}{5}$

12. A particle suspended from a fixed point, by a light inextensible thread of length L is projected horizontally from its lowest position with velocity $\sqrt{\frac{7gL}{2}}$. The thread will slack after swinging through an angle θ, such that θ equal-

(A) 30° (B) 135° (C) 120° (D) 150°

13. Figure shows the roller coaster track. Each car will start from rest at point A and will roll with negligible friction. It is important that there should be at least some small positive normal force exerted by the track on the car at all points, otherwise the car would leave the track. With the above fact, the minimum safe value for the radius of curvature at point B is ($g = 10\ m/s^2$) :



(A) 20 m (B) 10 m

(C) 40 m (D) 25 m

14. A curved section of a road is banked for a speed v. If there is no friction between road and tyres of the car, then:

(A) car is more likely to slip at speeds higher than v than speeds lower than v

(B) car is in static equilibrium on the curved section

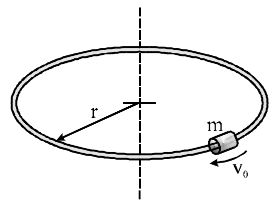
(C) car will not slip when moving with speed v

(D) none of the above

15. The kinetic energy K of a particle moving along a circle of radius R depends upon the distance s, as $K = as^2$. The force acting on the particle is–

(A) $2a\frac{s^2}{R}$ (B) $2as\left(1+\frac{s^2}{R^2}\right)^{\frac{1}{2}}$ (C) 2as (D) 2a

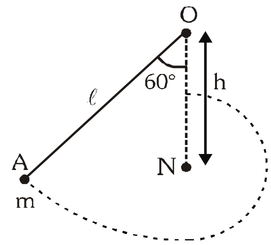
16. A small hoop of mass m is given an initial velocity of magnitude v_0 on the horizontal circular ring of radius 'r'. If the coefficient of kinetic friction is μ_k the tangential acceleration of the hoop immediately after its release is (assume that horizontal ring to be fixed and not in contact with any supporting surface)



(A) $\mu_k g$ (B) $\mu_k \frac{v_0^2}{r}$

(C) $\mu_k \sqrt{g^2 + \frac{v_0^2}{r}}$ (D) $\mu_k \sqrt{g^2 + \frac{v_0^4}{r^2}}$

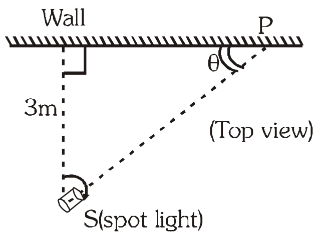
17. A particle of mass *m* attached to the end of string of length ℓ is released from the initial position *A* as shown in the figure. The particle moves in a vertical circular path about *O*. When it is vertically below *O*, the string makes contact with nail *N* placed directly below *O* at distance *h* and rotates around it. If the particle just complete the vertical circle about *N*, then



(A) $h = \frac{3l}{5}$ (B) $h = \frac{2l}{5}$

(C) $h = \frac{l}{5}$ (D) $h = \frac{4l}{5}$

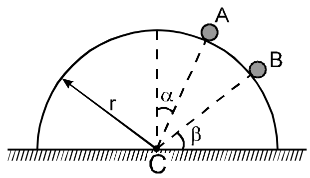
18*. A spot light S rotates in a horizontal plane with a constant angular velocity of 0.1 rad/s. The spot of light P moves along the wall at a distance 3m. What is the velocity of the spot P when $\theta = 45°$?



(A) 0.6 m/s   (B) 0.5 m/s

(C) 0.4 m/s   (D) 0.3 m/s

19. A particle initially at rest starts moving from point A on the surface of a fixed smooth hemisphere of radius r as shown. The particle looses its contact with hemisphere at point B. C is centre of the hemisphere. The equation relating α and β is



(A) $3 \sin \alpha = 2 \cos \beta$ (B) $2 \sin \alpha = 3 \cos \beta$

(C) $3 \sin \beta = 2 \cos \alpha$ (D) $2 \sin \beta = 3 \cos \alpha$

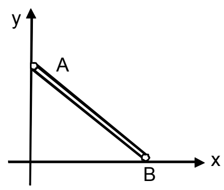
20. A rigid rod of length ℓ leans against a vertical wall (y-axis) as shown in figure. The other end of the rod is on the horizontal floor. Point A is pushed downwards with constant velocity. Path of the centre of the rod

(A) a straight line passing through origin

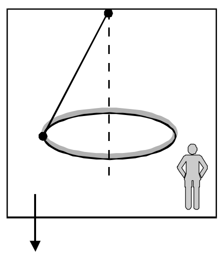
(B) a straight line not passing through origin

(C) a circle of radius $\left(\frac{\ell}{2}\right)$ and centre at origin

(D) a circle of radius $\left(\frac{\ell}{2}\right)$ but centre not at origin



21. In the figure shown a lift goes downwards with a constant retardation. An observer in the lift observers a conical pendulum in the lift, revolving in a horizontal circle with time period 2 seconds. The distance between the centre of the circle and the point of suspension is 2.0 m. Find the retardation of the lift in m/s^2.   Use $\pi^2 = 10$ and $g = 10\ m/s^2$



(A) 10   (B) 20   (C) 15   (D) 5

22. A particle moves with deceleration along the circle of radius R so that at any moment of time its tangential and normal accelerations are equal in moduli. At the initial moment t = 0 the speed of the particle equals v_0, then:

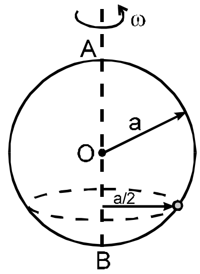
(i) the speed of the particle as a function of the distance covered s will be

(A) $v = v_0 e^{-s/R}$ (B) $v = v_0 e^{s/R}$ (C) $v = v_0 e^{-R/s}$ (D) $v = v_0 e^{R/s}$

(ii) the total acceleration of the particle as function of velocity and distance covered

(A) $a = \sqrt{2}\frac{v^2}{R}$ (B) $a = \sqrt{2}\frac{v}{R}$ (C) $a = \sqrt{2}\frac{R}{v}$ (D) $a = \frac{2R}{v}$

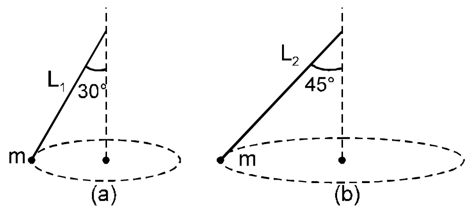
23. A smooth wire is bent into a vertical circle of radius ***a***. A bead P can slide smoothly on the wire. The circle is rotated about vertical diameter AB as axis with a constant speed ω as shown in figure. The bead P is at rest w.r.t. the wire in the position shown. Then ω^2 is equal to :



(A) $\frac{2g}{a}$ (B) $\frac{2g}{a\sqrt{3}}$

(C) $\frac{g\sqrt{3}}{a}$ (D) $\frac{2a}{g\sqrt{3}}$

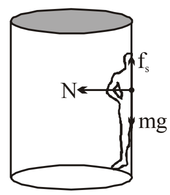
24. Two particles tied to different strings are whirled in a horizontal circle as shown in figure. The ratio of lengths of the strings so that they complete their circular path with equal time period is:



(A) $\sqrt{\frac{3}{2}}$ (B) $\sqrt{\frac{2}{3}}$

(C) 1 (D) None of these

25. In a rotor, a hollow vertical cyclindrical structure rotates about its axis and a person rests against the inner wall. At a particular speed of the rotor, the floor below the person is removed and the person hangs resting against the wall without any floor. If athe radius of the rotor is 2m and the coefficient of static friction between the wall and the person is 0.2, find the minimum speed at which the floor may be removed. Take g = 10 m/s^2.



(A) 10 m/s (B) 20 m/s (C) 50 m/s (D) 100 m/s

EXERCISE – 2 MCQ (ONE OR MORE CHOICE CORRECT)

1. One end of massless inextensible string of length ℓ is fixed and other end is tied to a small ball of mass m. The ball is performing a circular motion in vertical plane. At the lowest position, speed of ball is $\sqrt{20g\ell}$. Neglect any other forces on the ball except tension and gravitational force. Acceleration due to gravity is g.

(A) At the highest position of ball, tangential acceleration of ball is 0

(B) At the highest position of ball, tangential acceleration of ball is g

(C) During circular motion, minimum value of tension in the string is 15 mg

(D) During circular motion, maximum value of tension in the string is 21 mg

2. A simple pendulum of length L and mass (bob) M is oscillating in a plane about a vertical line between angular limits $-\phi$ and ϕ. For an angular displacement θ, $[|\theta|<\phi]$ the tension in the string and velocity of the bob are T and v respectively. The following relations hold good under the above conditions

(A) $T\cos\theta = Mg$

(B) $T - Mg\cos\theta = \frac{Mv^2}{L}$

(C) Tangential acceleration $= g\sin\theta$

(D) $T=Mg\cos\theta$

3. A car of mass moving horizontally on a circular path of radius r. At an instant its speed is v and is increasing at a rate a.

(A) The acceleration of the car is towards the centre of the path

(B) The magnitude of the frictional force on the car is greater than $\frac{mv^2}{r}$

(C) The friction coefficient between the ground and the car is greater than a/g.

(D) The friction coefficient between the ground and the car is $\mu = \tan^{-1}\frac{v^2}{rg}$

4. A circular road of radius r is banked for a speed v = 40 km/hr. A car of mass m attempts to go on the circular road. The friction coefficient between the tyre and the road is negligible.

(A) The car cannot make a turn without skidding.

(B) If the car turns at a speed less than 40 km/hr, it will slip down

(C) If the car turns at the current speed of 40 km/hr, the force by the road on the car is equal to $\frac{mv^2}{r}$

(D) If the car turns at the correct speed of 40 km/hr, the force by the road on the car is greater than mg as well as greater than $\frac{mv^2}{r}$

5. A person applies a constant force $\vec{F}$ on a particle of mass m and finds that the particle moves in a circle of radius r with a uniform speed v as seen from an inertial frame of reference.

(A) This is not possible.

(B) There are other forces on the particle

(C) The resultant of the other forces is $\frac{mv^2}{r}$ towards the centre.

(D) The resultant of the other forces varies in magnitude as well as in direction.

6. A stone is projected from level ground at t = 0 sec such that its horizontal and vertical components of initial velocity are 10 m/s and 20 m/s respectively. Then the instant of time at which magnitude of tangential and magnitude of normal components of acceleration of stone are same is : (neglect air resistance) $g = 10\ m/s^2$.

(A) 1/2sec (B) 1 sec (C) 3 sec (D) 4 sec.

7. A small object moves counter clockwise along the circular path whose centre is at origin as shown in figure. As it moves along the path, its acceleration vector continuously points towards point S. Then the object

(A) Speed up as it moves from A to C via B.
(B) Slows down as it moves from A to C via B.
(C) Slows down as it moves from C to A via D.
(D) Speed up as it moves from C to A via D.

8. A simple pendulum has a bob of mass m and swings with an angular amplitude ϕ. The tension in the thread is T. At a certain time, the string makes an angle θ with the vertical ($\theta \le \phi$).

(A) $T = mg\cos\theta$ for all values of θ
(B) $T = mg\cos\theta$ only for $\theta = \phi$
(C) $T = mg$ for $\theta = \cos^{-1}\left[\frac{1}{3}(2\cos\phi + 1)\right]$
(D) T will be larger for smaller values of θ.

9. A ball tied to the end of a string swings in a vertical circle under the influence of gravity

(A) when the string makes an angle 90° with the vertical the tangential acceleration is zero & radial acceleration is somewhere between maximum and minimum
(B) when the string makes an angle 90° with the vertical the tangential acceleration is maximum & radial acceleration is somewhere between maximum and minimum
(C) at no place in the circular motion, tangential acceleration is equal to radial acceleration
(D) throughout the path whenever radial acceleration has its extreme value, the tangential acceleration is zero.

10. A heavy particle is tied to the end A of a string of length 1.6 m. Its other end O is fixed. It revolves as a conical pendulum with the string making 60° with the vertical. Then : ($g = 9.8\ m/s^2$)

(A) its period of revolution is $\frac{4\pi}{7}$ sec.
(B) the tension in the string is doubled the weight of the particle
(C) the velocity of the particle $= 2.8\sqrt{3}$ m/s
(D) the centripetal acceleration of the particle is $9.8\sqrt{3}\ m/s^2$.

Match the column

11. In column-I condition on velocity, force and acceleration of a particle is given. Resultant motion is described in column-II. $\vec{u}$ = initial velocity, $\vec{F}$ = resultant force and $\vec{v}$ = instantaneous velocity.

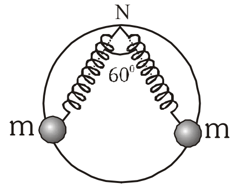
Column-I	***Column-II***
(A) $\vec{u}\times\vec{F} = 0$ and $\vec{F}$ = constant	(P) path will be circular path
(B) $\vec{u}\cdot\vec{F} = 0$ and $\vec{F}$ = constant	(Q) speed will increase
(C) $\vec{v}\cdot\vec{F} = 0$ and all the time and $\lvert\vec{F}\rvert$ = constant and the particle always remains in one plane.	(R) path will be straight line
(D) $\vec{u} = 2\hat{i} - 3\hat{j}$ and acceleration at all time $\vec{a} = 6\hat{i} - 9\hat{j}$	(S) path will be parabolic

12. A particle is moving with speed $v = 2t^2$ on the circumference of circle of radius R. Match the quantities given in column-I with corresponding results in column-II

Column-I	***Column-II***
(A) Magnitude of tangential acceleration of particle	(P) decreases with time.
(B) Magnitude of Centripetal acceleration of particle	(Q) increase with time
(C) Magnitude of angular speed of particle with respect to centre of circle	(R) remains constant
(D) Angle between the total acceleration vector and centripetal acceleration vector of particle	(S) proportional to R
	(T) inversely proportional to R

Comprehension Based Questions

Comprehension 1



Two identical beads are attached to free ends of two identical springs of spring constant $k = \frac{(2+\sqrt{3})mg}{\sqrt{3}\ \ R}$. Initially both springs make an angle of 60° at the fixed point N. Normal length of each spring is 2R. Where R is the radius of smooth ring over which bead is sliding. Ring is placed on vertical plane and beads are at symmetry with respect to vertical line as diameter.

13. Normal reaction on one of the bead at initial moment due to ring is

(A) mg/2 (B) $\sqrt{3}$ mg/2 (C) mg (D) Insufficient data

14. Relative acceleration between two beads at the initial moment

(A) g/2 vertically away from each other
(B) g/2 horizontally towards each other
(C) 2g/ $\sqrt{3}$ vertically away from each other
(D) 2g/ $\sqrt{3}$ horizontally towards each other

15. The speed of bead when spring is at normal length

(A) $\sqrt{\frac{\left(2+\sqrt{3}\right)gR}{\sqrt{3}}}$ (B) $\sqrt{\frac{\left(2-\sqrt{3}\right)gR}{\sqrt{3}}}$ (C) $\sqrt{\frac{2gR}{\sqrt{3}}}$ (D) $\sqrt{3gR}$

Comprehension 2

One end of massless inextensible string of length ℓ is fixed and other end is tied to a small ball of mass m. The ball is performing a circular motion in vertical plane. At the lowest position, speed of ball is $\sqrt{20g\ell}$. Neglect any other forces on the ball except tension and gravitational force. Acceleration due to gravity is g.

16. Motion of ball is in nature of

(A) circular motion with constant speed
(B) circular motion with variable speed
(C) circular motion with constant angular acceleration about centre of the circle.
(D) none of these

17. At the highest position of ball, tangential acceleration of ball is -

(A) 0 (B) g (C) 5 g (D) 16 g

Comprehension 3

A small sphere of mass m suspended by a thread is first taken aside so that the thread forms the right angle with the vertical and then released, then :

18. The total acceleration of the sphere and the thread tension as a function of θ, the angle of deflection of the thread from the vertical will be

(A) $g\sqrt{1+3\cos^2\theta}$, T = 3mg cos θ
(B) g cos θ, T = 3 mg cos θ.
(C) $g\sqrt{1+3\sin^2\theta}$, T = 5mg cos θ
(D) g sin θ , T = 5 mg cos θ.

19. The thread tension at the moment when the vertical component of the sphere's velocity is maximum will be

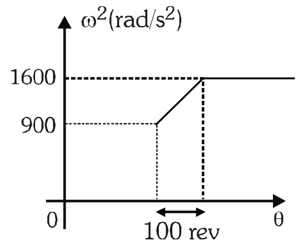
(A) mg (B) mg $\sqrt{2}$ (C) mg $\sqrt{3}$ (D) $\frac{mg}{\sqrt{3}}$

20. The angle θ between the thread and the vertical at the moment when the total acceleration vector of the sphere is directed horizontally will be

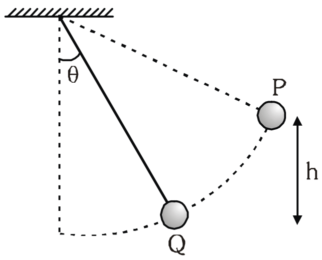
(A) $\cos\theta = \frac{1}{\sqrt{3}}$ (B) $\cos\theta = \frac{1}{3}$ (C) $\sin\theta = \frac{1}{\sqrt{3}}$ (D) $\sin\theta = \frac{1}{\sqrt{2}}$

EXERCISE – 3 SUBJECTIVE

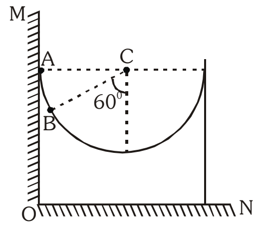
1. The square of the angular velocity ω of a certain wheel increases linearly with the angular displacement during 100 rev of the wheel's motion as shown. Compute the time t required for given 100 revolution.



2. The bob of a simple pendulum of length ℓ is released from point P. What is the angle made by the direction of net acceleration of the bob with the string at point Q.



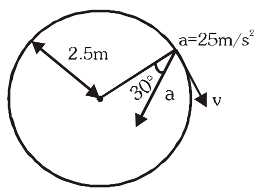
3. A ball of mass 1 kg is released from position, A inside a fixed wedge with a hemispherical cut of radius 0.5 m as shown in the figure. Find the force exerted by the vertical wall OM on wedge, when the ball is in position B. (neglect friction everywhere). (Take g = 10 m/s^2)



4. A particle is moving in a circular orbit with a constant tangential acceleration. After a certain time t has elapsed after the beginning of motion, the angle between the total acceleration a and the direction along the radius r becomes equal to 45°. What is the angular acceleration of the particle .

5. A particle moves clockwise in a circle of radius 1m with centre at (x,y) = (1m,0). It starts from rest at the origin at time t=0. Its speed increases at the constant rate of $\left(\frac{\pi}{2}\right)$ m/s^2. (i) How long does it take to travel halfway around the circle? (ii) What is the speed at that time?

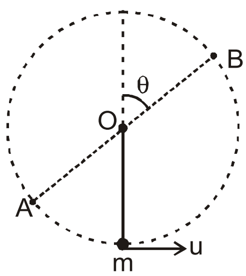
6. Figure shows the total acceleration and velocity of a particle moving clockwise in a circle of radius 2.5m at a given instant of time. At this instant, Find : (i) the radial acceleration, (ii) the speed of the particle and (iii) its tangential acceleration.



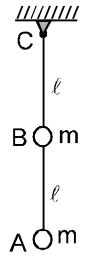
7. A ball is hanging vertically by a light inextensible string of length L from fixed point O. The ball of mass m is given a speed u at the lowest position such that it completes a vertical circle with centre at O as shown. Let AB be a diameter of circular path of ball making an angle θ with vertical as shown. (g is acceleration due to gravity)

(a) Let T_A and T_B be value of tension in string when ball is at A and B respectively, then find the value of $T_A - T_B$.

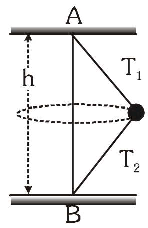
(b)* Let $\vec{a}_A$ and $\vec{a}_B$ be acceleration of ball when it is at A and B respectively, then find the value of $|\vec{a}_A + \vec{a}_B|$ is equal to



8. A weightless rod of length 2ℓ carries two equal masses 'm', one secured at lower end A and the other at the middle of the rod at B. The rod can rotate in vertical plane about a fixed horizontal axis passing through C. What horizontal velocity must be imparted to the mass at A so that it just completes the vertical circle.



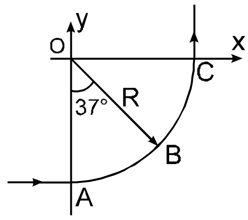
9. A particle is attached by means of two equal strings to points A and B in the same vertical line and decribe a horizontal circle with a uniform angular speed. If the angular speed of the particle is $2\sqrt{(2g/h)}$ with AB = h, show that the ratio of the tension of the string is 5 : 3.



10. A car initially traveling eastwards turns north by traveling in a quarter circular path of radius R metres at uniform speed as shown in figure. The car completes the turn in T second.

(a) What is the acceleration of the car when it is at B located at an angle of 37°. Express your answers in terms of unit vectors $\hat{i}$ and $\hat{j}$

(b) The magnitude of car's average acceleration during T second period.



EXERCISE – 4 RECAP OF AIEEE/JEE (MAIN)

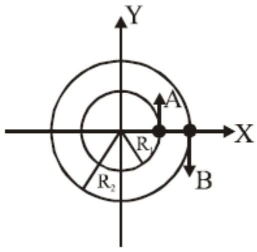
1. A body is projected at t = 0 with a velocity 10 ms^{-1} at an angle of 60° with the horizontal. The radius of curvature of its trajectory at t = 1s is R. Neglecting air resistance and taking acceleration due to gravity g = 10 ms^{-2}, the value of R is : ***[IIT-JEE 2019]***

(A) 2.5 m (B) 10.3 m (C) 2.8 m (D) 5.1 m

2. A particle is moving along a circular path with a constant speed of 10 ms^{-1}. What is the magnitude of the change is velocity of the particle, when it moves through an angle of 60° around the centre of the circle? ***[IIT-JEE 2019]***

(A) zero (B) 10 m/s (C) $10\sqrt{3}$ m/s (D) $10\sqrt{2}$ m/s

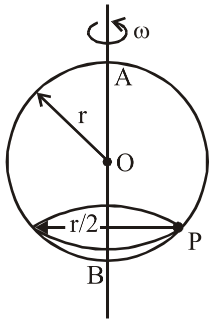
3. Two particles A, B are moving on two concentric circles of radii R_1 and R_2 with equal angular speed w. At t = 0, their positions and direction of motion are shown in the figure :



The relative velocity $\overrightarrow{v_a} - \overrightarrow{v_B}$ at $t = \frac{\pi}{2\omega}$ is given by : ***[IIT-JEE 2019]***

(A) $-\omega(R_1 + R_2)\hat{i}$ (B) $\omega(R_1 + R_2)\hat{i}$ (C) $\omega(R_1 - R_2)\hat{i}$ (D) $\omega(R_2 - R_1)\hat{i}$

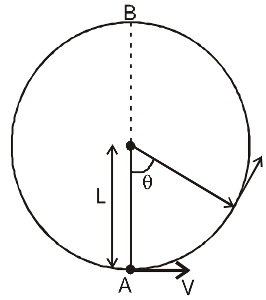
4. A smooth wire of length 2πr is bent into a circle and kept in a vertical plane. A bead can slide smoothly on the wire. When the circle is rotating with angular speed ω about the vertical diameter AB, as shown in figure, the bead is at rest with respect to the circular ring at position P as shown. Then the value of ω^2 is equal to : ***[IIT-JEE 2019]***



(A) $\left(g\sqrt{3}\right)/r$ (B) $\frac{\sqrt{3}g}{2r}$ (C) 2g/r (D) $2g/\left(r\sqrt{3}\right)$

EXERCISE – 5 RECAP OF IIT-JEE/JEE (ADVANCED)

1. A bob of mass M is suspended by a massless string of length L. The horizontal velocity V at position A is just sufficient to make it reach the point B. The angle θ at which the speed of the bob is half of that at A, satisfies Figure : ***[IIT-JEE 2008]***



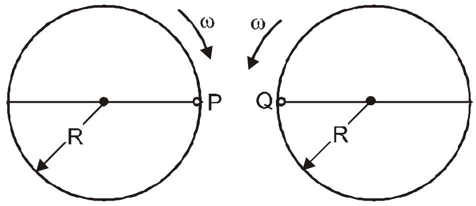
(A) $\theta = \frac{\pi}{4}$

(B) $\frac{\pi}{4} < \theta < \frac{\pi}{2}$

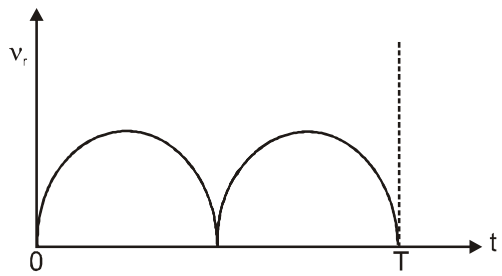
(C) $\frac{\pi}{2} < \theta < \frac{3\pi}{4}$

(D) $\frac{3\pi}{4} < \theta < \pi$

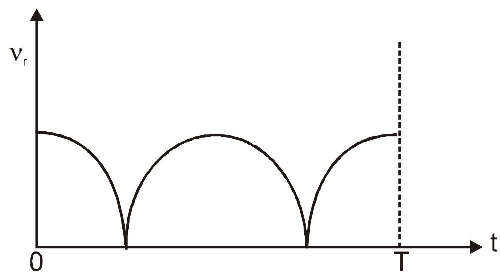
2. Two identical discs of same radius R are rotating about their axes in opposite directions with the same constant angular speed ω. The discs are in the same horizontal plane. At time t = 0, the points P and Q are facing each other as shown in the figure. The relative speed between the two points P and Q is v_r. In one time period (T) of rotaion of the discs v_r, as a function of time is best represented by ***[IIT-JEE 2012]***



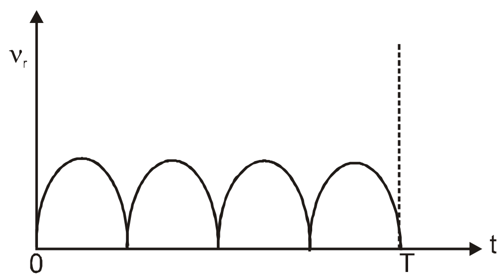
(A)



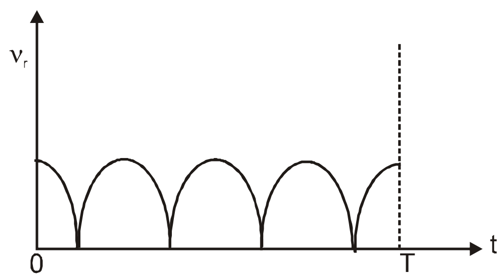
(B)



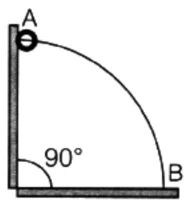
(C)



(D)



3. A wire, which passes through the hole in a small bead, is bent in the form of quarter of a circle. The wire is fixed vertically on ground as shown in the figure. The bead is released from near the top of the wire and it slides along the wire without friction. As the bead moves from A to B, the force it applies on the wire is– ***[JEE(Adv.) - 2014]***



(A) always radially outwards

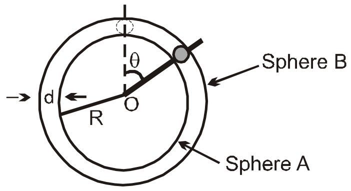
(B) always radially inwards

(C) radially outwards initially and radially inwards later

(D) radially inwards initially and radially outwards later

Subjective Type

4. A spherical ball of mass m is kept at the highest point in the space between two fixed, concentric spheres A and B (see figure). The smaller sphere A has a radius R and the space between the two spheres has a width d. The ball has a diameter very slightly less than d. All surfaces are frictionless. The ball given a gentle push (towards the right in the figure). The angle made by the radius vector of the ball with the upward vertical is represented by θ (shown in figure) ***[JEE 2002]***



(a) Express the total normal reaction force exerted by the spheres on the ball as a function of angle θ.

(b) Let N_A and N_B denote the magnitudes of the normal reaction force on the ball exerted by the spheres A and B, respectively. Sketch the variations of N_A and N_B as functions of $\cos\theta$ in the range $0 \le \theta \le \pi$ by drawing two separate graphs in your answer book, taking $\cos\theta$ on the horizontal axis

ANSWERS

EXERCISE-1

Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
Ans.	D	B	A	A	C	A	C	C	A	C	B	C	A	C	B
Que.	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**					
Ans.	D	D	A	C	C	A	(i) A (ii) A	B	B	A					

EXERCISE-2

Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Ans.	A,C,D	B,C	B,C	B,D	B,D	B,C	A,C	B,C,D	B,D	A,B,C,D

- ***Match the column :*** **11.** (A) R (B) Q, S (C) P (D) Q, R **12.** (A) Q, (B) Q, T (C) Q, T (D) P, S

- ***Comprehension based Questions :***

Comprehension 1 : **13.** C **14.** D **15.** C

Comprehension 5 : **16.** B **17.** A

Comprehension 6 : **18.** A **19.** C **20.** A

EXERCISE-3

1. $\frac{40\pi}{7}$ **2.** $\tan^{-1}\left(\frac{\ell\sin\theta}{2h}\right)$ **3.** $\frac{15\sqrt{3}}{2}$ **4.** $\frac{1}{t^2}$

5. (i) t= 2s, (ii) 3.14 m/s

6. (i) $25\frac{\sqrt{3}}{2}$ m/s² (ii) $\left(125\frac{\sqrt{3}}{4}\right)^{1/2}$ m/s (iii) $\frac{25}{2}$ m/s²

7. (a) 6 mg cosθ (b) $g\sqrt{12\cos^2\theta+4}$ **8.** $u = \sqrt{\frac{48}{5}g\ell}$

10. (a) $\frac{\pi^2 R}{20T^2}(-3\hat{i}+4\hat{j})$ m/s² (b) $\frac{\pi R}{\sqrt{2}T^2}$ m/s²

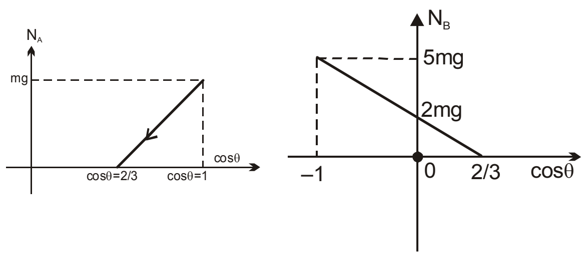
EXERCISE-4

Que.	**1**	**2**	**3**	**4**
Ans.	C	B	D	D

EXERCISE-5

1. D **2.** A **3.** D **4.** (a) $N_A = 3mg\cos\theta - 2mg$, $N_B = 2mg - 3mg\cos\theta$

(b)



* * * * *

Important Notes

Important Notes

Important Notes

Important Notes