Programming Homework of Stochastic Processes, Fall 2023

Due 11pm, Dec. 11, 2023.

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Problem 1.

Let $\mathcal{N} = \{1, 2, 3, ...\}$ be the set of natural numbers. Let $s[n]$ be a regular process. In particular,

$$s[n] + a_1 \cdot s[n-1] + a_2 \cdot s[n-2] + a_3 \cdot s[n-3] \;\; = \;\; b_0 \cdot i[n], \tag{1}$$

where $a_1 = -1.9$, $a_2 = 1.18$, $a_3 = -0.24$, $b_0 = 2$, and $\{i[n], -\infty < n < \infty\}$ is a white Gaussian noise process with $\mathbb{E}[(i[n])^2] = 1$, $\forall n \in \mathbb{Z}$.

Let M be a positive integer. In addition,

$$\hat{s}_M[n] \;\; = \;\; \hat{E}\{s[n] | s[n-k], 1 \le k \le M\}. \tag{2}$$

Define $\hat{P}[n]$ as follows.

$$\hat{P}_M[n] \;\; = \;\; \frac{1}{n} \sum_{k=1}^{n} (\hat{s}_M[k] - s[k])^2. \tag{3}$$

For each $\delta \in (0, 1)$, define $T(\delta)$ as the minimum positive integer n such that $|\hat{P}[n] - \hat{P}[n-1]| \le \delta \cdot \hat{P}[n-1]$ in your simulation.

Write a computer program (in Matlab, C/C++ or Python) to obtain the answers for the following tasks. Your figures should be in good quality. It is highly recommended that you use Matlab or the Matplotlib module in Python to plot the figures.

(1) Obtain and plot the values of $R_{ss}[m]$, $\forall 0 \le m \le 20$, in Figure 1.

(2) Plot $\{\hat{s}_{20}[n], 10000 \le n \le 10100\}$ and $\{s[n], 10000 \le n \le 10100\}$, in Figure 2.

(3) Plot $\hat{P}_{20}[n]$, $\forall n \in \{10, 10^2, 10^3, 10^4, 10^5, 10^6\}$, in Figure 3.

(4) Consider the case in which $\{i[n], -\infty < n < \infty\}$ is a white noise process but $i[n]$ has the following probability density function for each $n \in \mathbb{Z}$.

$$f_{i[n]}(t) = \begin{cases} 0.5e^{-t}, \forall t > 0 \\ 0.5e^{t}, \forall t < 0. \end{cases}$$

Plot $\hat{P}_{20}[n]$, $\forall n \in \{10, 10^2, 10^3, 10^4, 10^5, 10^6\}$, in Figure 4.

(5) Write a short report that includes your figures and codes. In addition, you should explain how you obtain the answers in English. Name your Matlab program RP-YourStudentId.m (or your Python program RP-YourStudentId.py). Compress your report and programs into a ZIP file named RP-YourStudentId.zip. Submit the ZIP file to the course webpage in NYCU E3 website.