MODELS OF PEN-PL GAUSSIAN GAMES IN NON-COOPERATIVE COMMUNICATION

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Abstract

Consider non-cooperative pen games where both players act strategically and heavily influence each other. In spam and malware detection, players exploit randomization to obfuscate malicious data and increase their chances of evading detection at test time. The result shows Pen-PL Games have a probability distribution that approximates a Gaussian distribution according to some probability distribution defined over the respective strategy set. With quadratic cost functions and multivariate Gaussian processes, evolving according to first order autoregressive models, we show that Pen-PL "smooth" curve signaling rules are optimal. Finally, we show that computing a socially optimal Pen-PL network placement is NP-hard and that this result holds for all P-PL-G distributions.

1   Introduction

The Pen-PL game is an idealized model for competi- tive facility location, which was proposed by *cite*. The Pen-PL game is played on a bounded continuous arena by two players. Two players X (X_{kp}) and H (H_{ml}) put even number of pens (usually two) alternately, and the continuous field is subdivided according to the nearest neighbor rule. At the final step, the player whose pens hit all rival's pens out of the area wins.

The Pen-PL game is a natural game, but the general case seems to be very hard to analyze from the theoretical point of view. Hence, in *cite* investigated the case that the game field is a bounded 1-dimensional continuous domain. On the other hand, *cite*2, and *cite*3 deal with a 2- or 4- dimensional case, but they restrict themselves to the one-round game: first, X puts all pens, and next H puts all pens, but Ignoring the situations that pens flying out on own.

In this paper, we introduce the discrete Pen-PL game. Two players alternately occupy n pens on a graph, which is a bounded discrete arena. (Hence the graph contains at least 2n vertices.) This restriction seems to be appropriate since real estates are already bounded in general, and we have to build intercepting curve in the bounded area. More precisely, the discrete Pen-PL game is played on a given finite graph G, instead of a bounded continuous arena. Each vertex of G can be assigned to the nearest vertices occupied by X or H, according to the nearest neighbor rule. (Hence a vertex can be a "tie" when it has the same distance from a vertex occupied by X and another vertex occupied by H.) Finally, the player who whose pens hit all rival's pens out of the area (or a larger number of vertices) wins. We note that the two players can tie in some cases.

And then we set up a comparison Gaussian model of the front part of pens and make a prediction of the possibility trajectory of motion of the front part going out. At the same time, we experimented and organize the data to show the relationship between this non-cooperative communicating game and Gaussian distribution.

2   Gauss Models and System Analysis

In this section we first introduced the Gaussian model and use it for modeling. Then we analyze the Pen-PL game system. We start with a basic statement that enables the Pen-PL agents to anticipate the players' behavior.

2.1   Representation of Gaussian density

The pdf for a Pen-Pl normal in *D* dimensions is defined by the following:

$$\mathcal{N}(\vec{x}|\vec{\mu}, \Sigma) \triangleq \frac{1}{(2\pi)^{\frac{D}{2}} |\Sigma|^{\frac{1}{2}}} \exp\left[-\frac{1}{2}(\vec{x} - \vec{\mu})^T \Sigma^{-1}(\vec{x} - \vec{\mu})\right] \tag{1}$$

The expression inside the exponent is the Mahalanobis distance between a data vector $\vec{x}$ and the mean vector $\vec{\mu}$, We can gain a better understanding of this quantity by performing an eigendecomposition of $\vec{\Sigma}$. That is, we write $\vec{\Sigma} = \vec{U}\vec{\Lambda}\vec{U}^T$, where $\vec{U}$ is an orthonormal matrix of eigenvectors satsifying $\vec{U}^T\vec{U} = \vec{I}$, and $\vec{\Lambda}$ is a diagonal matrix of eigenvalues. Using the eigendecomposition, we have that

$$\vec{\Sigma}^{-1} = \vec{U}^{-T}\vec{\Lambda}^{-1}\vec{U}^{-1} = \vec{U}\vec{\Lambda}^{-1}\vec{U}^T = \sum_{i=1}^{D} \frac{1}{\lambda_i}\vec{u}_i\vec{u}_i^T \tag{2}$$

where $\vec{u}_i$ is the *i*'th column of $\vec{U}$, containing the *i*'th eigenvector. Hence we can rewrite the Mahalanobis distance as

follows:

$$(\vec{x} - \vec{\mu})^T \Sigma^{-1} (\vec{x} - \vec{\mu}) = (\vec{x} - \vec{\mu})^T \left(\sum_{i=1}^{D} \frac{1}{\lambda_i} \vec{u}_i \vec{u}_i^T \right) (\vec{x} - \vec{\mu}) \quad (3)$$

$$= \sum_{i=1}^{D} \frac{1}{\lambda_i} (\vec{x} - \vec{\mu})^T \vec{u}_i \vec{u}_i^T (\vec{x} - \vec{\mu}) \quad (4)$$

$$= \sum_{i=1}^{D} \frac{y_i^2}{\lambda_i} \quad (5)$$

where $y_i \triangleq \vec{u}_i^T (\vec{x} - \vec{\mu})$. Recall that the equation for an ellipse in 2d is

$$\frac{y_1^2}{\lambda_1} + \frac{y_2^2}{\lambda_2} = 1 \quad (6)$$

We see that both the marginal and conditional distributions are themselves Gaussian. For the marginals, we just extract the rows and columns corresponding to $\vec{x}_1$ or $\vec{x}_2$. For the conditional, we have to do a bit more work.

Hence we see that the contours of equal probability density of a Gaussian lie along ellipses. This is illustrated in Figure 1. The eigenvectors determine the orientation of the ellipse, and the eigenvalues determine how elogonated it is.
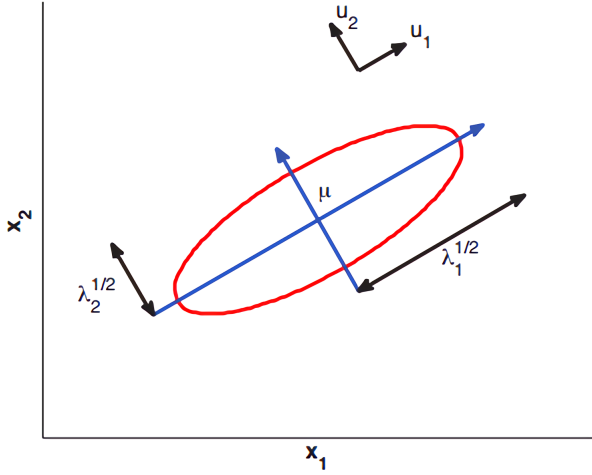

Figure 1: Visualization of a 2 dimensional Gaussian density.

In general, we see that the Mahalanobis distance corresponds to Euclidean distance in a transformed coordinate system, where we shift by $\vec{\mu}$ and rotate by $\vec{U}$.

If we have N iid samples $\vec{x}_i \sim \mathcal{N}(\vec{\mu}, \vec{\Sigma})$, then the MLE for the parameters is given by

$$\vec{\hat{\mu}} = \frac{1}{N} \sum_{i=1}^{N} \vec{x}_i \triangleq \vec{\bar{x}} \quad (7)$$

$$\vec{\hat{\Sigma}} = \frac{1}{N} \sum_{i=1}^{N} (\vec{x}_i - \vec{\bar{x}})(\vec{x}_i - \vec{\bar{x}})^T \quad (8)$$

$$= \frac{1}{N} \left(\sum_{i=1}^{N} \vec{x}_i \vec{x}_i^T \right) - \vec{\bar{x}} \vec{\bar{x}}^T \quad (9)$$

We then show that the multivariate Gaussian is the distribution with maximum entropy subject to having a specified mean and covariance. This is one reason the Gaussian is so widely used: the first two moments are usually all that we can reliably estimate from data, so we want a distribution that captures these properties, but otherwise makes as few addtional assumptions as possible.

To simplify notation, we will assume the mean is zero. The pdf has the form

$$f(\vec{x}) = \frac{1}{Z} \exp \left(-\frac{1}{2} \vec{x}^T \vec{\Sigma}^{-1} \vec{x} \right) \quad (10)$$

2.2 Gaussian discriminant analysis

One important application of MVNs is to define the the class conditional densities in a generative classifier, i.e.,

$$p(\vec{x}|y = c, \vec{\theta}) = \mathcal{N}(\vec{x}|\vec{\mu}_c, \vec{\Sigma}_c) \quad (11)$$

The resulting technique is called GDA (even though it is a generative, not discriminative, classifier). If $\vec{\Sigma}_c$ is diagonal, this is equivalent to naive Bayes.

We can classify a feature vector using the following decision rule, derived from:

$$y = \arg \max_c \left[\log p(y = c|\vec{\pi}) + \log p(\vec{x}|\vec{\theta}) \right] \quad (12)$$

When we compute the probability of $\vec{x}$ under each class conditional density, we are measuring the distance from $\vec{x}$ to the center of each class, $\vec{\mu}_c$, using Mahalanobis distance. This can be thought of as a nearest centroids classifier. In this way, We idealized part of the pen and treat it as a smooth curve or sphere. We now build two ideal models as examples:
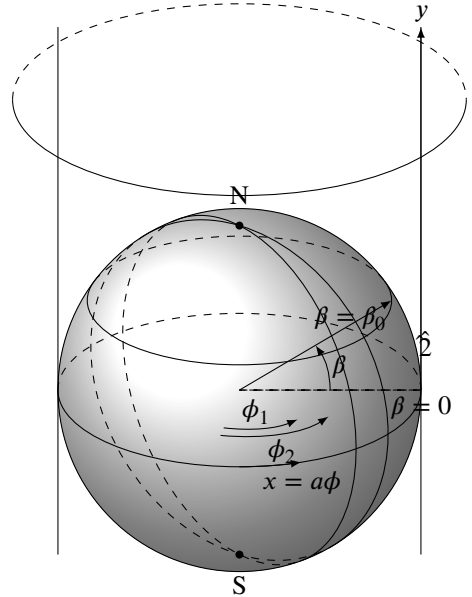

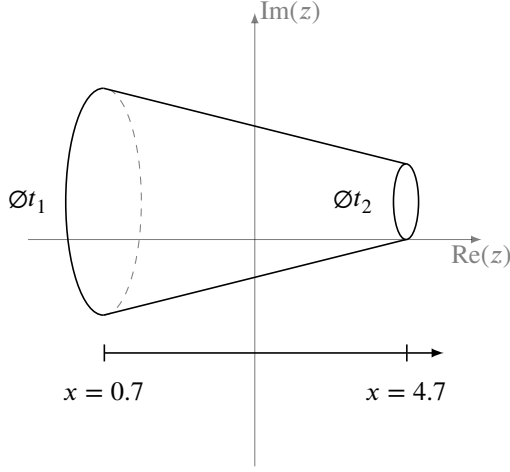Figure 2: The model of the pen point and its motion space.

Figure 3: An ideal model of a pen cap in C_{12}^8 space.

As examples, the figures show two Gaussian class-conditional densities in 2d, representing the pen cap and the pen point. We can see that the features are correlated, as is to be expected. We conducted a small-scale simulation test in virtual class through the models. The ellipses for each class contain 95% of the probability mass. If we have a uniform prior over classes, we can classify a new test vector as follows:

$$y = \arg\max_c (\vec{x} - \vec{\mu}_c)^T \vec{\Sigma}_c^{-1} (\vec{x} - \vec{\mu}_c)$$

3   Data Distribution and Probability

In this section, we conduct a series of experiments to evaluate the performance of our proposed method. Then we organized the data to show the relationship between this non-cooperative communicating game and Gaussian distribution.

3.1   Powerful tool for data network

Previous works on networks, i.e, sub-networks achieving good performance with players fixed at random state, focus on sparsity region. Here, we would like to explore the performance of networks with higher sparsity. We conduct experiments on modern architecture ReNet0 and dataset CzfXB-100, a harder task than CzfXB-10 where a large portion of previous experiments are conducted. In this experiment, weights are fixed at initialization state. Hyperparameters follow the same as previous CzfXB experiments. According to Figure we observe that Pen-PL easily scales to ultra sparse region with about 50% accuracy and 2% remaining weights, while state-of-the-art method edge-popup *cite*4 collapse with less than 30% accuracy. It is a surprising result that a subnet with 2% fixed random weights still succeeds in obtaining nearly 50% accuracy on a task with 100 categories. It shows that the structure in networks already provides valuable information for classification.

Dataset	Out %			
Ratio	90%	95%	98%	99%
ReNet0	77.01	-	-	-
PDD	69.44	56.84	22.46	5.98
MLPrem	60.98	30.89	3.16	0.77
APM	73.91	70.59	57.90	44.78
MPGG	74.00	68.30	58.20	-
MGT	74.31	70.40	61.46	50.35
Nash-P	72.00	67.50	-	-
Pen-PL	**74.68**	**71.50**	**66.83**	**61.07**

Table 1: Accuracy of Pen-PL at different pruning ratios.

There exist some divergences from This Gaussian position for our experimental CDF. Therefore, it constitutes possible that our residuals represent non-Gaussian,and therefore we shouldn't understand our information with a Gaussian error model. This led me to build out measures of uncertainty in our model using a bootstrap procedure with resampling of cases.

Pen-PL Games steadily beats previous state-of-the-art methods on Gaussian-based pruning, weight magnitude pruning, dense-to-sparse training and sparse-to-sparse training. MVN improves with the help of ERK (Erdós-Rényi-Kernel) but will result in doubling the PPLGs at inference time, so we put it in Figure 4:
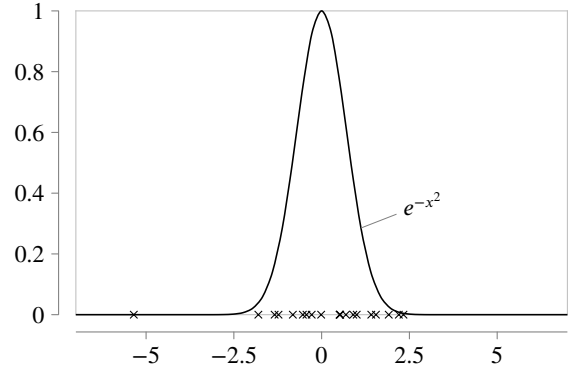

Figure 4: H-X distribution of $\sum_{i=1}^{10} x_i$, where $x_i \sim U(-1, 1)$

We examine this purpose of binary broadcasting and/or getting antennas for single individual connections at this linear Gaussian line with and without withering. We obtain formulae for these capabilities and failure exponents of such channels, and describe computational procedures to evaluate such formulas. We show that the potential gains of such multi-antenna systems over single-antenna systems is rather large under independence assumptions for the fades and noises at different receiving antennas.*cite*5

Through this usage of the molecular model method, these writers of the study depict that ways by which non-covalent methods perform focus transport in these ITZ. This general ballpoint pen constitutes the result of body output, with elements created individually on construction channels according to *cite*6. These simulations exist applied to determine adhesion energy from Van der Waal and electrostatic forces,calculate the increase in radial stress due to CTE mismatching, and establish the radial deformation of the CNT due

to atomic interactions. Though the report lacks any experimental data to validate the model,clear explanation is documented to connect theory to the data from the molecular simulations.

3.2 Avoidance and attack path

What we find at these 2 players are 3 other choices Pen-PL game concept acts at deep education.

(i) As The way of identifying and examining current DL-architectures.

(ii) As a way to construct a learning strategy.

(iii) a way to predict behavior of human participants.

This is how we can use machine learning to solve problems in real life.The problem with this approach is that it does not allow for any kind of data analysis, which would be very difficult to do. So, we need some sort of algorithm that can analyze the data. We have to find out what the data is about. This is where machine learning comes into play. It's a method of analyzing data from different sources such as text, images or videos. It allows us to understand the underlying. Pen-PL path patterns of behapredictions based on these patterns. It also helps us to predict future events by predicting how they will affectvior and then make path to attack or avoid. These stimulus-sampling beliefs translate well into numerical structure; they are an example of statistical education theory, a more common process in the quantitative treatment of education. The idea is that we can use mathematical models to model the path of two players' and their actions. This is called logistic regression, which has been used for many years to describe the way in which people learn. It is based on a simple equation:

$$P = PL(t)$$

where P is the number of steps taken by each player, t is the number of times played. The formula is as follows:

$$\frac{dPL}{dt} = \frac{\alpha^2 P_e' \cos^2 \frac{\theta_e}{2}}{4t_e^3 \sin^4 \frac{\theta_e}{2}} \left[G_P^2 + \frac{\tau}{\epsilon} G_{Mt}^2 \right] \left(\frac{1}{1+\tau} \right)$$

The problem with this approach is that it assumes that all players are equal.The solution is to assume that all players have equal numbers of steps taken by each other. Play concept constitutes this examination of science simulations of important action between logical decision-makers.It gets coatings at all areas of cultural field, also equally at philosophy and machine science. Originally, it addressed zero-sum games, in which one person's gains result in losses for the other participants.Today,game theory applies to a wide range of behavioral relations, and is now an umbrella term for the science of logical decision making by the two Pen-PL players.
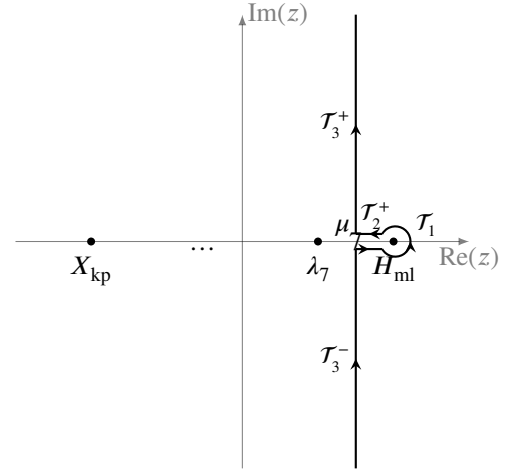

Figure 5: Contour of integration for avoidance path of H_{ml}

When Pen-PL goes cooperatively,neither participant can defect, because if he does, this different X_{kp} player can likewise defect, from Figure and they both would turn out worse away. Believing ahead, therefore, neither player will defect. Outcome:The players stay at the cooperative outcome. The result is that the player who has a better score than the one who did not defect wins. The player who has a better score than the one who did not defect. If the player who had a better score won, then he would be more likely to defect.