Informal Notes on Mathematics
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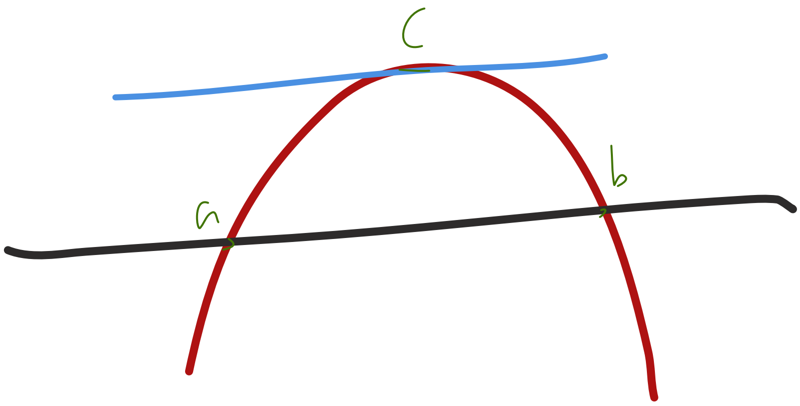
A Review of Calculus *(continue)*

First of all, let's review the end of the mean value theorem.

One corollary of the work in the previous section is Rolle's theorem.

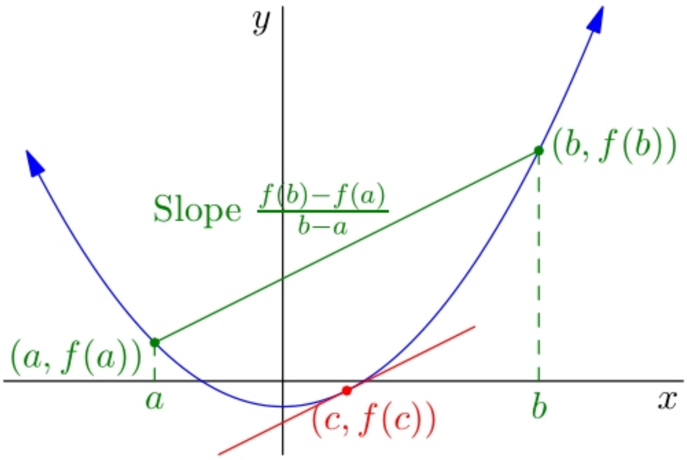
Theorem 28.4.1 (Rolle's theorem)
Suppose $f\colon [a,b] \to \mathbb{R}$ is a continuous function, which is differentiable on the open interval (a,b), such that $f(a) = f(b)$. Then there is a point $c \in (a,b)$ such that $f'(c) = 0$.



Obviously. Then one can adapt the theorem as follows.

Theorem 28.4.2 (Mean value theorem)
Suppose $f\colon [a,b] \to \mathbb{R}$ is a continuous function, which is differentiable on the open interval (a,b). Then there is a point $c \in (a,b)$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$



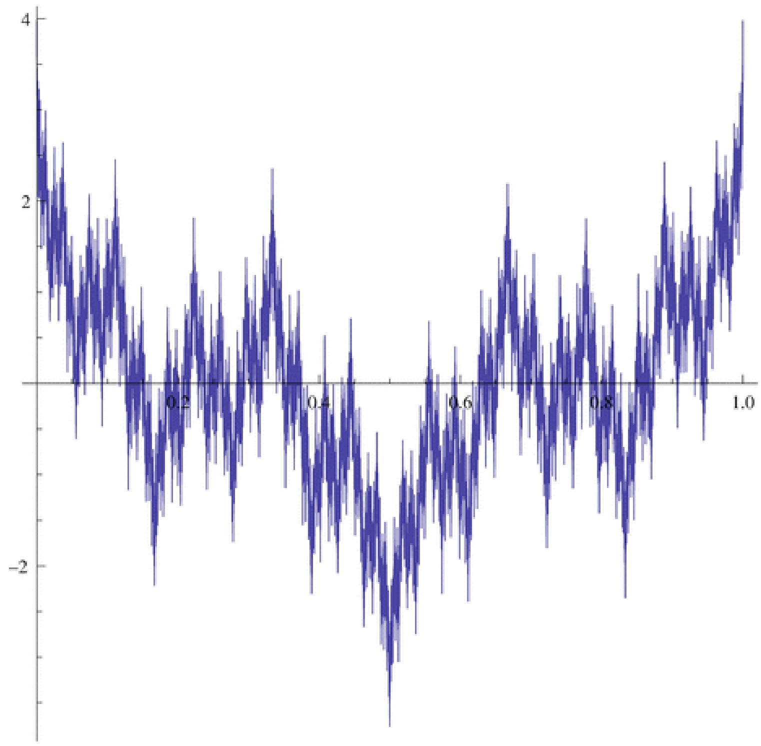
A triaval fact is that the distance from point $(c, f(c))$ to line AB is the longest of all the points under the straight line AB (in the case of the above figure)

Theorem 28.4.5 (Ratio mean value theorem)

Let $f, g\colon [a, b] \to \mathbb{R}$ be two continuous functions which are differentiable on (a, b), and such that $g(a) \neq g(b)$. Then there exists $c \in (a, b)$ such that

$$f'(c)(g(b) - g(a)) = g'(c)(f(b) - f(a))$$

Also called Cauchy's mean value theorem.
Here we are as an insignificant supplement. Suppose f is a real-valued function defined in a neighborhood of point $a \in \mathbb{R}$. Intuitively speaking, f is *continuous* at a means that $\lim_{x \to a} f(x)$ exists and equal to $f(a)$. It is obvious that a function is derivable at a certain point then it is continuous at this point, but the reverse is not always true. But something that might be counter-intuitive. Functions that are differentiable everywhere are continuous, but functions that are not differentiable anywhere can be continuous as well. (The word *differentiable* and *derivable* mean the same thing now, but not always.) Here it is Weierstrass function.



Weierstrass cosine function:

$$W_{\lambda,b}(x) = \sum_{n=0}^{\infty} \lambda^n \cos\left(2\pi b^n x\right)$$

The rough shape of the graph is determined by the $n = 0$ term in the series: $\cos(\pi x)$. The higher-order termscreate the smaller oscillations. With b carefully chosen as in the theorem, the graph becomes so jagged thatthere is no reasonable choice for a tangent line at any point; that is, the function is nowhere differentiable. So much for today's notes. Time for bed.