Homework 2

(1)**针对习题**2.3**中的**$(a),(c)$**和**(e):

- 简述各正规式所描述的语言

  $(a).\ \ 0(0|1)^*0$

  表示由$0,1$组成的以0开始并以0结尾的长度大于1的符号串全体

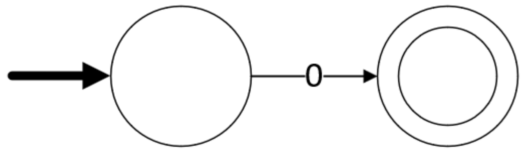
  $(c).\ \ (0|1)^*0(0|1)(0|1)$

  表示由$0,1$组成的倒数第3个字符为0的长度大于2的符号串全体

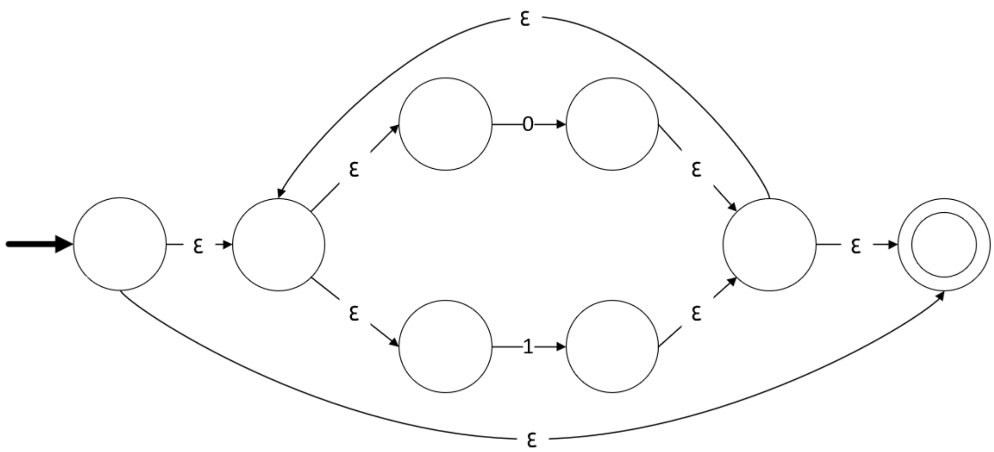
  $(e).\ \ (00|11)^*((01|10)(00|11)^*(01|10)(00|11)^*)^*$

  表示由$0,1$组成的$0,1$个数均为偶数的符号串全体

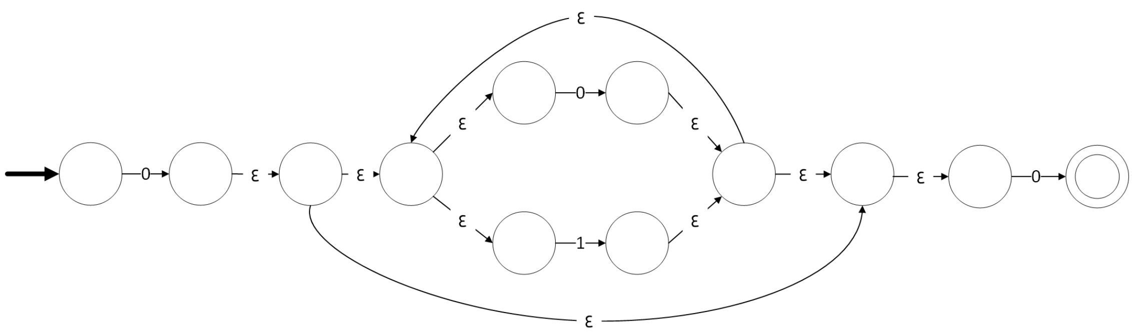
- 采用$Thopmson$方法，为正规式(a)构建非确定有限自动机
  - 识别0对应的FA



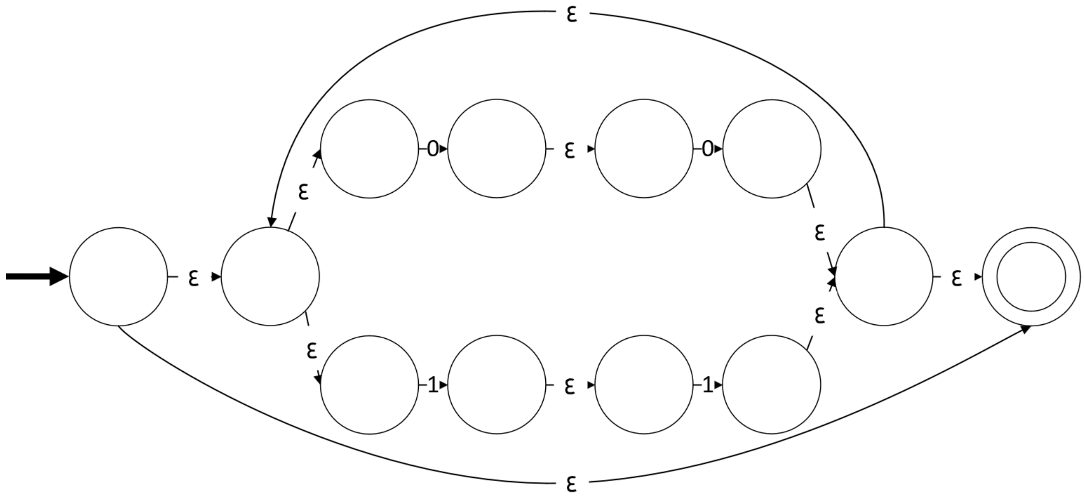
  - 识别$(0|1)^*$对应的FA



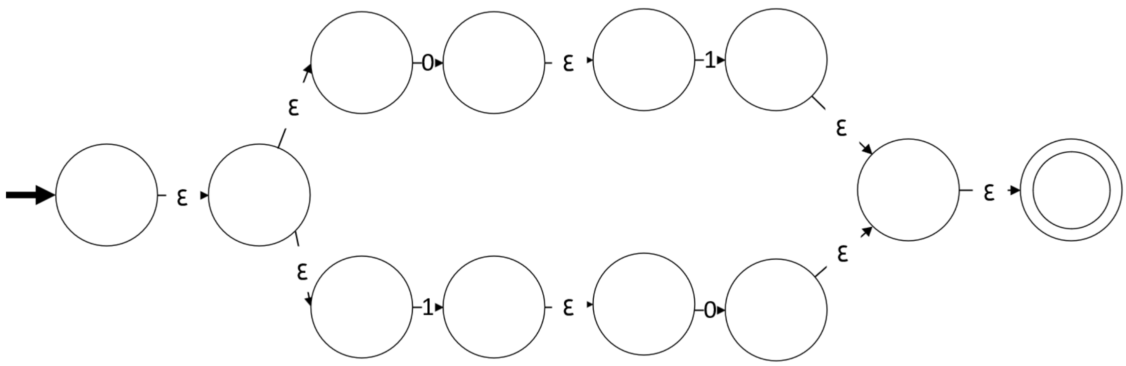
  - 最后得到识别正规式(a) $0(0|1)^*0$的NFA



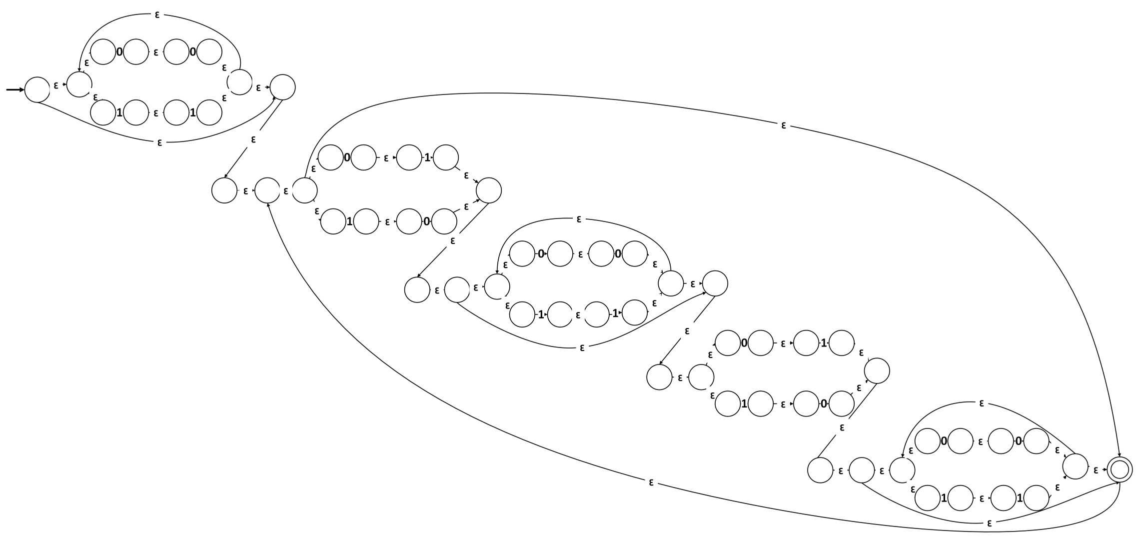
- 先为正规式(e)构建非确定有限自动机，再进行确定化和极小化。
    - 识别$(00|11)^*$的FA



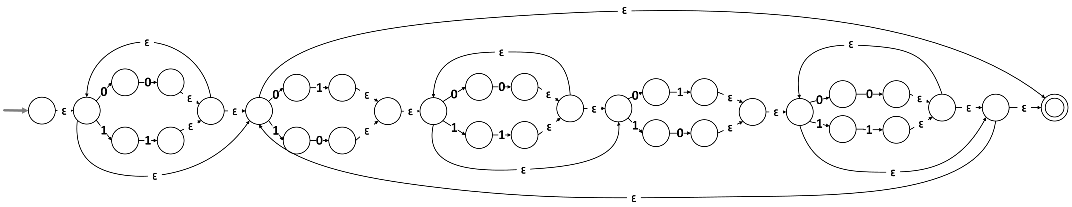
    - 识别$(01|10)$的FA



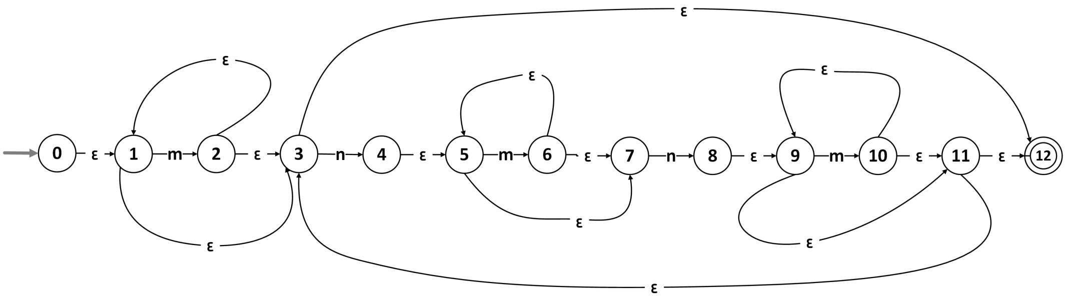
    - 最后得到识别正规式(e) $(00|11)^*((01|10)(00|11)^*(01|10)(00|11)^*)^*$的$NFA$



    - 简化后的NFA



- 进行确定化与极小化
- 进一步简化便于进行确定化，并标注状态$0,1,2,3,4,5,6$，假设$00|11,01|10$为m,n



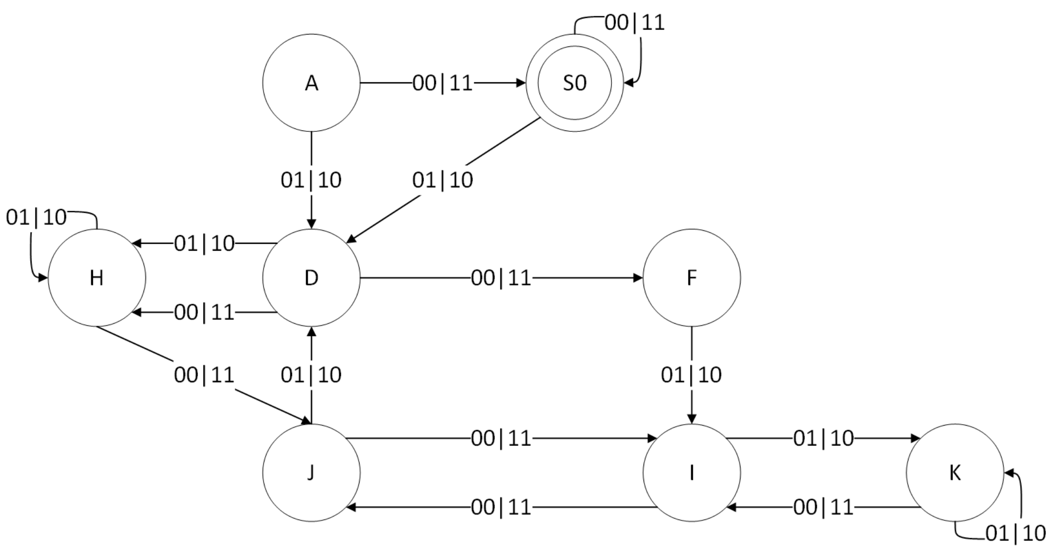
$$
\begin{aligned}
&\varepsilon\_closure(0) = \{0,1,3,12\}\\
&\varepsilon\_closure(1) = \{1,3,12\}\\
&\varepsilon\_closure(2) = \{1,2,3,12\}\\
&\varepsilon\_closure(3) = \{3,12\}\\
&\varepsilon\_closure(4) = \{4,5,7\}\\
&\varepsilon\_closure(5) = \{5,7\}\\
&\varepsilon\_closure(6) = \{6,7\}\\
&\varepsilon\_closure(7) = \{7\}\\
&\varepsilon\_closure(8) = \{3,8,9,11,12\}\\
&\varepsilon\_closure(9) = \{3,9,11,12\}\\
&\varepsilon\_closure(10) = \{3,9,10,11,12\}\\
&\varepsilon\_closure(11) = \{3,11\text{，}12\}\\
&\varepsilon\_closure(12) = \{12\}
\end{aligned}
$$

$$
\begin{aligned}
&\varepsilon\_closure(0) = \{0,1,3,12\} = A\\
&\varepsilon\_closure(move(A,m)) = \varepsilon\_closure(\{2\}) = \{1,3,12\} = B\\
&\varepsilon\_closure(move(A,n)) = \varepsilon\_closure(\{4\}) = \{4,5,7\} = D\\
&\varepsilon\_closure(move(B,m)) = \varepsilon\_closure(\{2\}) = \{1,3,12\} = B\\
&\varepsilon\_closure(move(B,n)) = \varepsilon\_closure(\{4\}) = \{4,5,7\} = D\\
&\varepsilon\_closure(move(D,m)) = \varepsilon\_closure(\{6\}) = \{6,7\} = F\\
&\varepsilon\_closure(move(D,n)) = \varepsilon\_closure(\{8\}) = \{3,8,9,11,12\} = H\\
&\varepsilon\_closure(move(F,m)) = \varepsilon\_closure(\{\varnothing\}) = \varnothing\\
&\varepsilon\_closure(move(F,n)) = \varepsilon\_closure(\{6,8\}) = \{3,6,7,8,9,11,12\} = I\\
&\varepsilon\_closure(move(H,m)) = \varepsilon\_closure(\{10\}) = \{3,9,10,11,12\} = J\\
&\varepsilon\_closure(move(H,n)) = \varepsilon\_closure(\{4\}) = \{3,8,9,11,12\} = H\\
&\varepsilon\_closure(move(I,m)) = \varepsilon\_closure(\{10\}) = \{3,9,10,11,12\} = J\\
&\varepsilon\_closure(move(I,n)) = \varepsilon\_closure(\{4,8\}) = \{3,4,5,7,8,9,11,12\} = K\\
&\varepsilon\_closure(move(J,m)) = \varepsilon\_closure(\{10\}) = \{3,6,7,8,9,11,12\} = I\\
&\varepsilon\_closure(move(J,n)) = \varepsilon\_closure(\{4\}) = \{4,5,7\} = D\\
&\varepsilon\_closure(move(K,m)) = \varepsilon\_closure(\{6,10\}) = \{3,6,7,8,9,11,12\} = I\\
&\varepsilon\_closure(move(K,n)) = \varepsilon\_closure(\{4,8\}) = \{3,4,5,7,8,9,11,12\} = K
\end{aligned}
$$

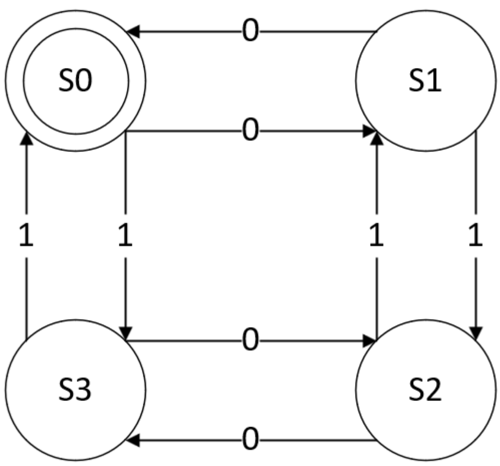
- 得到状态转换表如下

状态	m	n
A	B	D
B	B	D
D	F	H
F	$\varnothing$	I
H	J	H
I	J	K
J	I	D
K	I	K

- 得到DFA



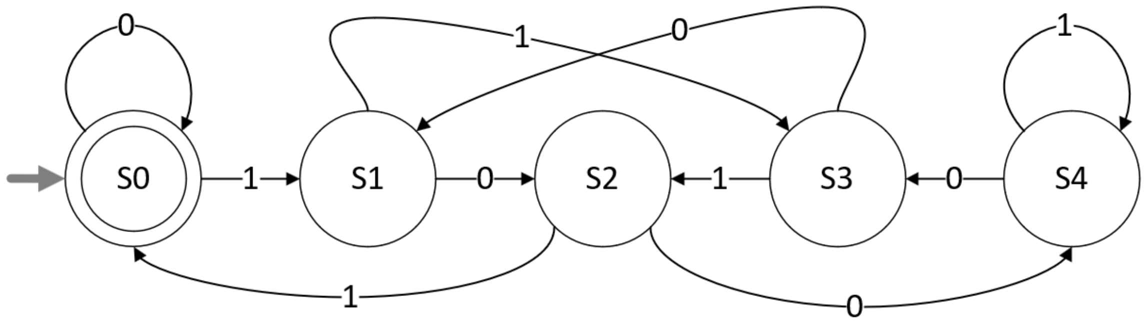
- 极小化后的DFA, $S0$表示偶数个0与偶数个1的状态，$S1, S2, S3$分别表示奇数个0偶数个1，奇数个0奇数个1，奇数个1偶数个0的状态。



(2)**针对习题**2.14：

构造一个DFA，它接受$\sum = \{0,1\}$上能被5整除的二进制数

- 构造相应$DFA\ \ M$
  - 构造DFA如下，$S0, S1, S2, S3, S4$分别代表能被5整除以及被5整除余$1, 2, 3, 4$。



- 给出正规式R，使得 $L(R) = L(M)$
  - 由于状态转换图中到达终态$S0$的状态转换为$S0 \to S0, S2 \to S0$

    因此首先消去$S1, S3, S4$得到$S2$的状态正规式：$1(10)^*(0|11)(01^*01|01^*00(10)^*(0|11))^*$

    再消去$S2$得到正规式R=$(0|1(10)^*(0|11)(01^*01|01^*00(10)^*(0|11))^*1)^*$。