Power, Fluence, and Gaussian Beams

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1 Gaussian Function

Gaussian function in terms of full width half maximum(FWHM) and in terms of $1/e^2$ waist w_0 radius

$$I(FWHM/2) = a/2 \quad I(w_0) = a/e^2 \tag{1}$$

$$I(x) = ae^{-4\ln 2(x-b)^2/FWHM^2} \quad I(x) = ae^{-2(x-b)^2/w_0^2} \tag{2}$$

The relationship between FWHM diameter and $1/e^2$ radius:

$$FWHM = w_0\sqrt{2\ln 2} \tag{3}$$

Rayleigh Range - Distance from focal spot to where waist is $\sqrt{2}$ times bigger, giving area 2x big or half the fluence. λ is wavelength and n is refractive index

$$z_r = \frac{n\pi w_0^2}{\lambda} \tag{4}$$

To measure the M^2 (M-squared parameter), fit data points to the following equation where W is half the $D4\sigma$ diameter. for gaussian beam, $D4\sigma\text{radius} = 1/e^2\text{radius}$

$$W^2(z) = W_0^2 + M^4(\frac{\lambda}{\pi W_0})^2(z - z_0)^2 \tag{5}$$

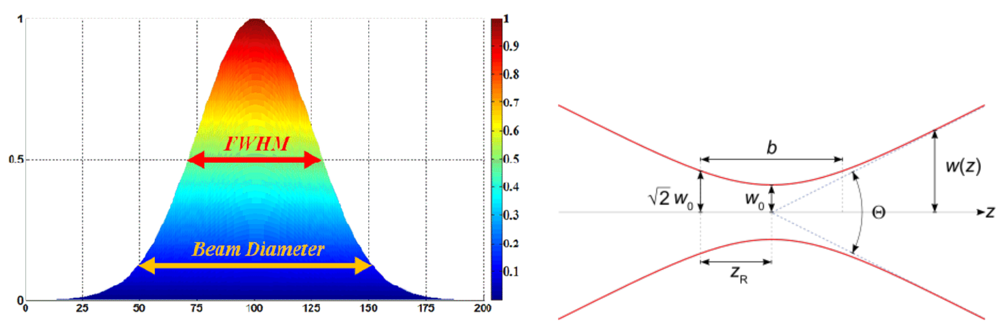

Figure 1: Left: Comparison of FWHM to $1/e^2$ beam diameter. Right: Rayleigh range and $1/e^2$ w_0 beam waist

2 Power, Rep Rate, Energy

Units: Pulse energy U (J), Rep Rate or frequency f (1/s) Power P $(J/s = W)$

Conversion between energy and power for pulsed laser. CW only has Power

$$P = u * f \qquad u = P/f \tag{6}$$

3 Fluence, Intensity

Units: Fluence F (J/cm^2), Intensity I $(J/cm^2/s = W/cm^2)$, Power P $(J/s$ or $W)$

Average fluence - Energy per beam area

$$F = \frac{u}{\pi w_{0x} w_{0y}} = \frac{2 \ln 2 u}{\pi FWHM_x FWHM_y} \tag{7}$$

peak fluence F_0 is twice that of $1/e^2$ fluence

$$F_0 = \frac{2u}{\pi w_{0x} w_{0y}} = \frac{4 \ln 2 u}{\pi FWHM_x FWHM_y} \tag{8}$$

for non-zero incidence angle ϕ

$$F_{angle} = F_{normal} \cos \phi \tag{9}$$

Peak Power, t is FWHM pulse duration

$$P_0 = \sqrt{\frac{4 \ln 2}{\pi}} \frac{u}{t} = 0.94 \frac{u}{t} \tag{10}$$

Peak intensity for temporally Gaussian pulse. Use 0.88 for $sech^2$ pulses

$$I_0 = \sqrt{\frac{4 \ln 2}{\pi}} \frac{F_0}{t} = 0.94 \frac{F_0}{t} \tag{11}$$

4 Good Sources

1. https://rtd.xfel.eu/docs/scs-documentation/en/latest/OL/GaussianBeam/gaussian_beam.html
2. https://lidaris.com/lidaris-online-calculator/
3. https://en.wikipedia.org/wiki/Gaussian_function
4. https://en.wikipedia.org/wiki/Gaussian_beam
5. https://www.gentec-eo.com/laser-calculators/peak-power-density