

Proj: report

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数学与应用数学 (强基计划)

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A.

$N = 6, 11, 21, 41, 81$, 对于每一个 N 对应的三阶 PP 曲线拟合, 区间中点误差最大值依次如图中所示:

0.0112402 0.00493075 0.00223354 0.00105424 0.00051124

图 1: the max-norm of the interpolation error at mid-points of subintervals for each N

对于每一个 N , 插值函数图像如下:

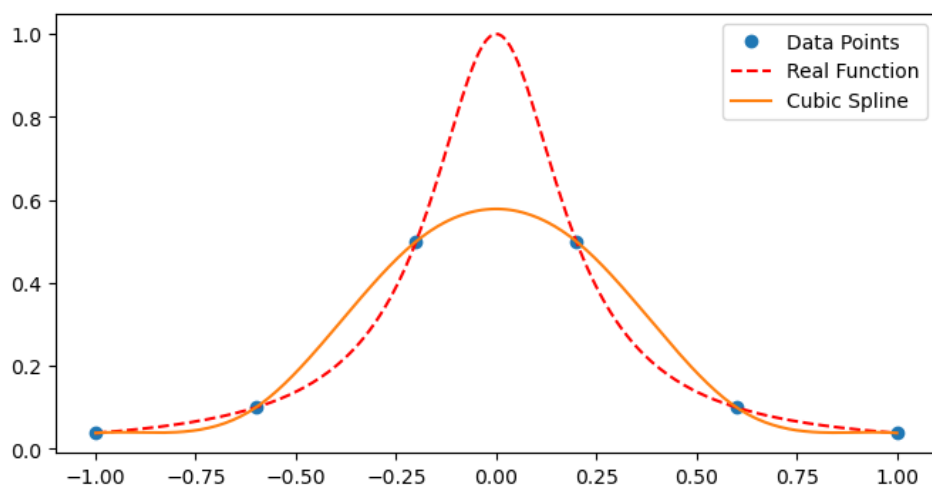


图 2: interpolation spline with $N=6$

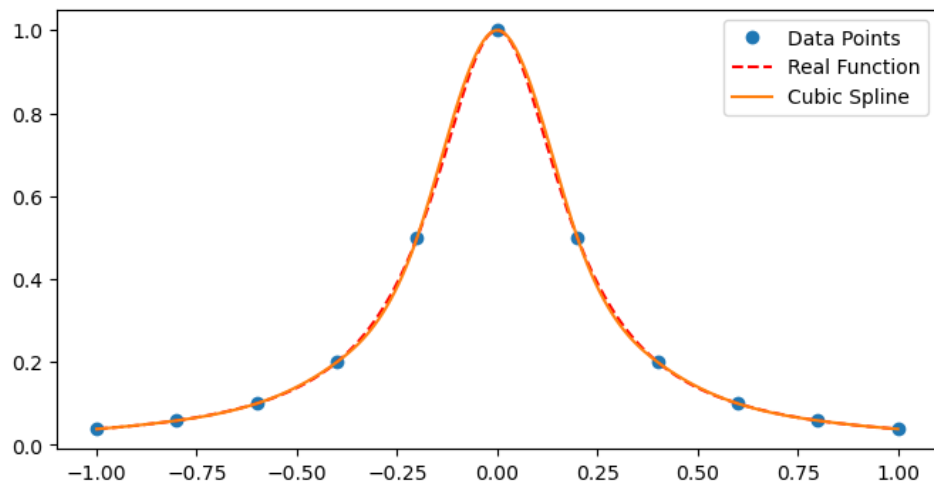


图 3: interpolation spline with $N=11$

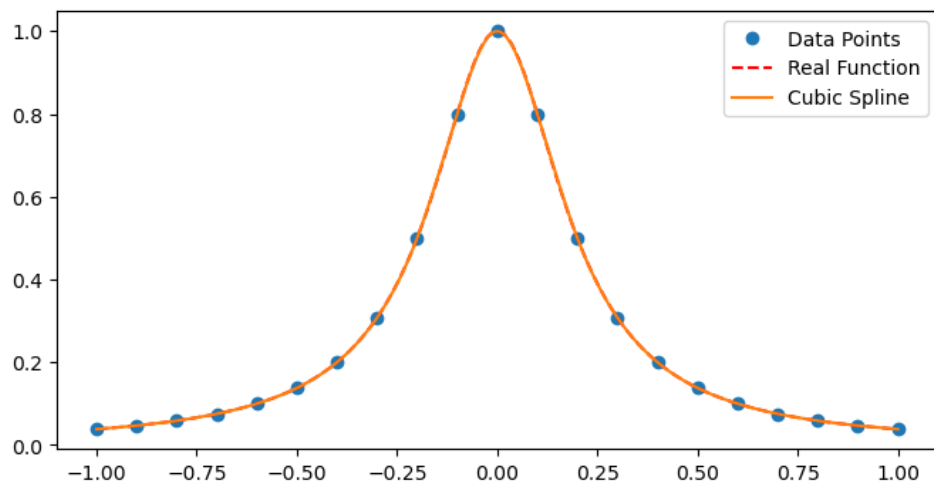


图 4: interpolation spline with $N=11$

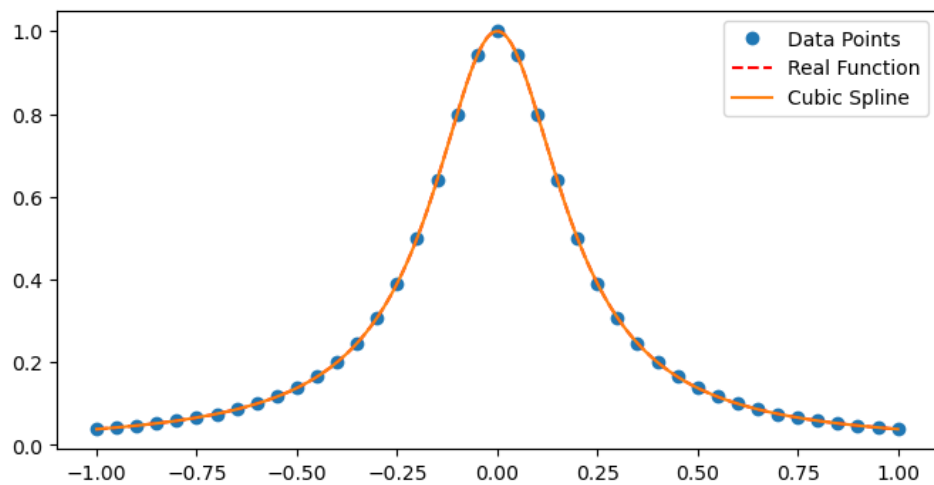


图 5: interpolation spline with $N=41$

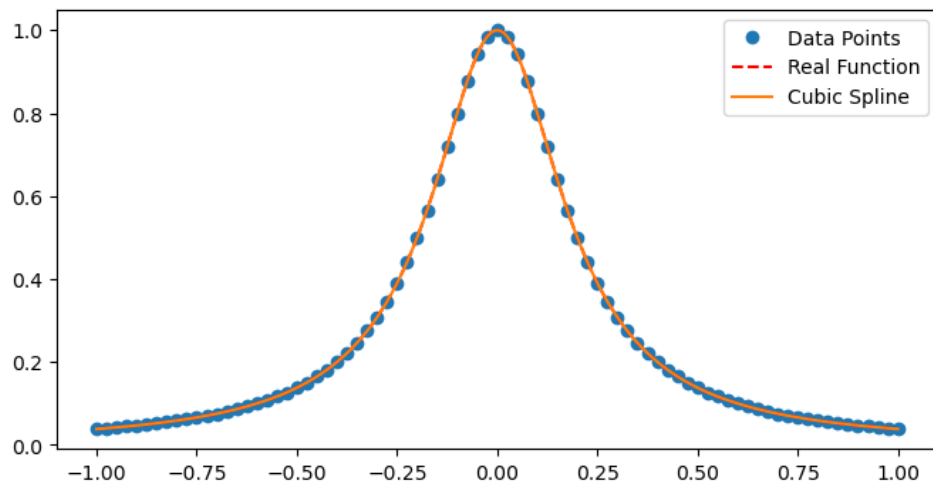


图 6: interpolation spline with $N=81$

综上可看出龙格现象被极大避免了.

C.

用二阶 B 样条在 $i - \frac{11}{2}, i = 1, \dots, 10$ 插值的函数图像为:

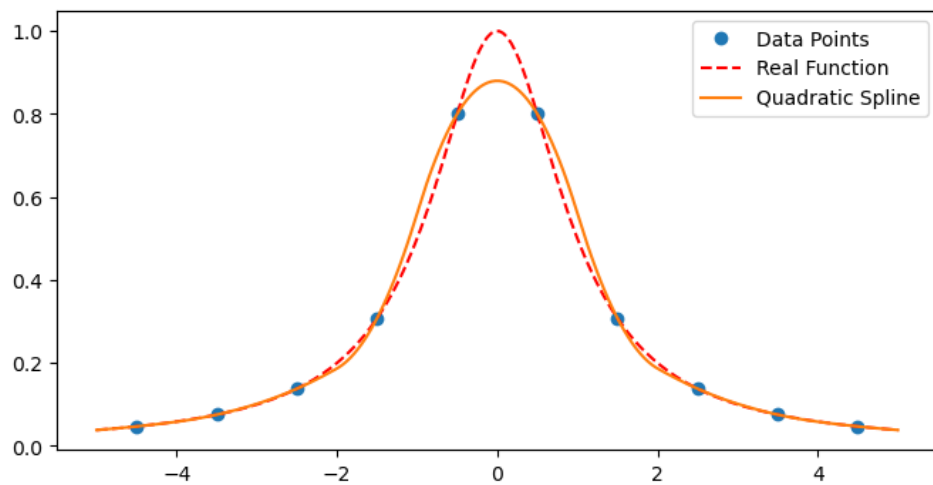


图 7: Quadratic Spline

用三阶 B 样条在 $i - 6, i = 1, \dots, 11$ 插值的函数图像为:

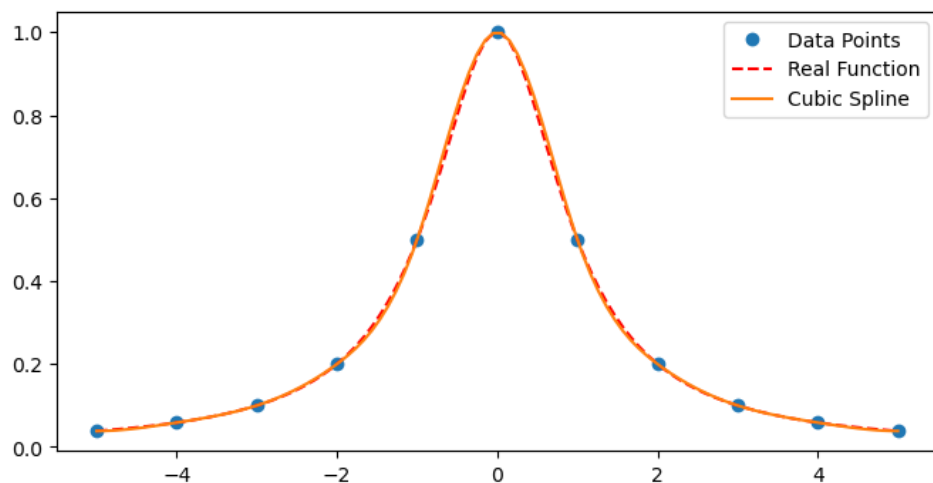


图 8: Cubic Spline

D.

用三阶 B 样条拟合后的曲线在给定点处的误差值:

```
0.000669605 5.74936e-08 0.0205289 4.47035e-08 0.0205289 2.01166e-08 0.000669561
```

图 9: the interpolation error of Cubic Spline at given sites

用二阶 B 样条拟合后的曲线在给定点处的误差值:

```
2.5339e-08 0.00141839 1.25542e-07 0.120238 9.23872e-08 0.00141838 6.24689e-09
```

图 10: the interpolation error of Quadratic Spline at given sites

可以看到, 用三阶 B 样条拟合后的曲线在 $-3, 0, 3$ 处的误差较小, 用二阶 B 样条拟合后的曲线在 $-3.5, -0.5, 0.5, 3.5$ 处的误差较小. 这是因为用三阶 B 样条曲线拟合时, 插值点选在 $-3, 0, 3$, 用二阶 B 样条曲线拟合时插值点选在 $-3.5, -0.5, 0.5, 3.5$. 总体来看用三阶 B 样条拟合后的曲线误差更小.

E.

对心形曲线, 等距节点拟合和使用累计曲线长度进行拟合的效果如下:

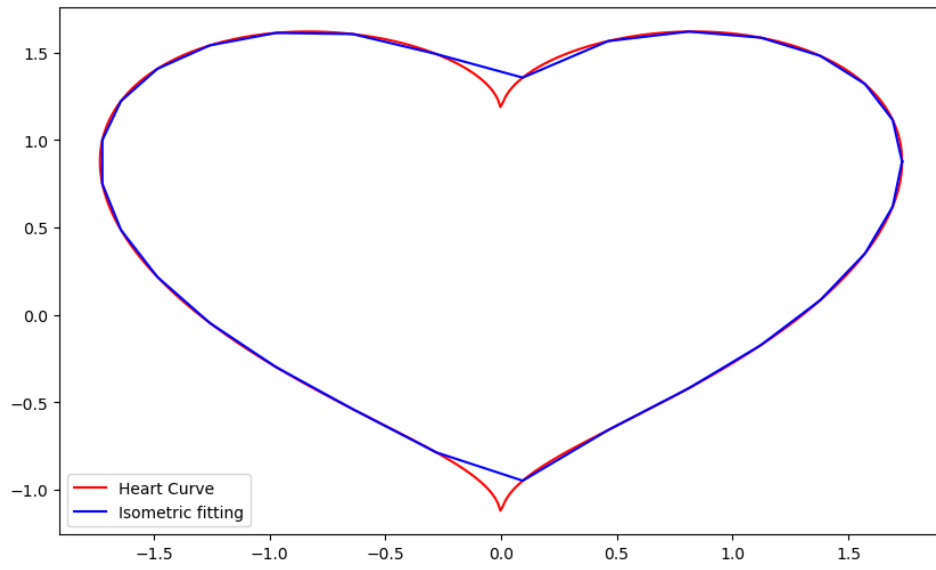


图 11: heart curve Isometric Fitting

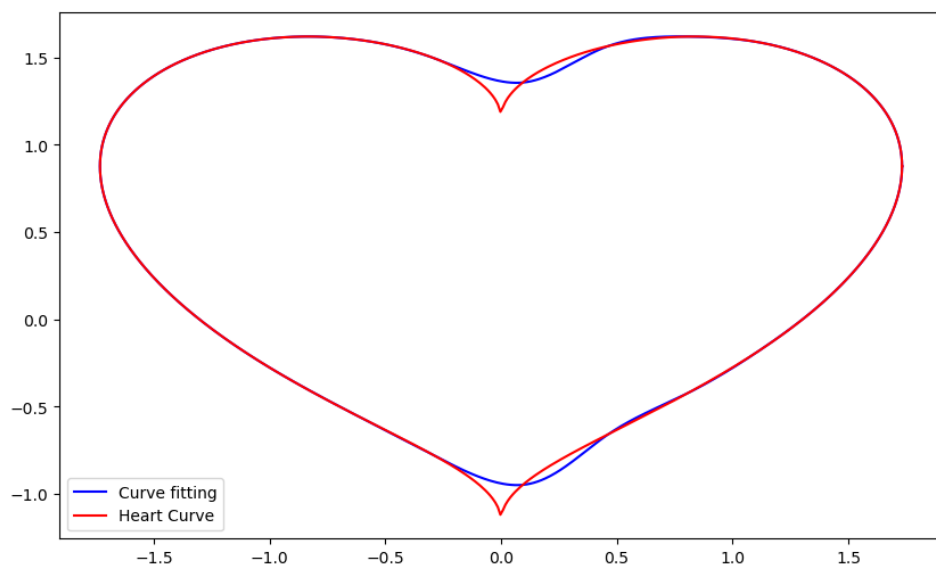


图 12: heart curve Curve Fitting

对第二种曲线, 等距节点拟合和使用累计曲线长度进行拟合的效果如下:

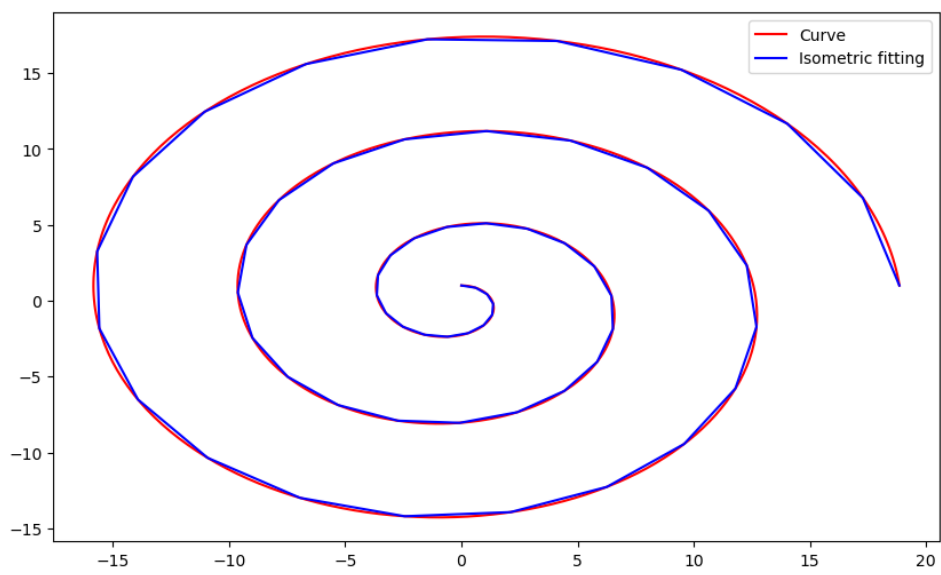


图 13: second curve Isometric Fitting

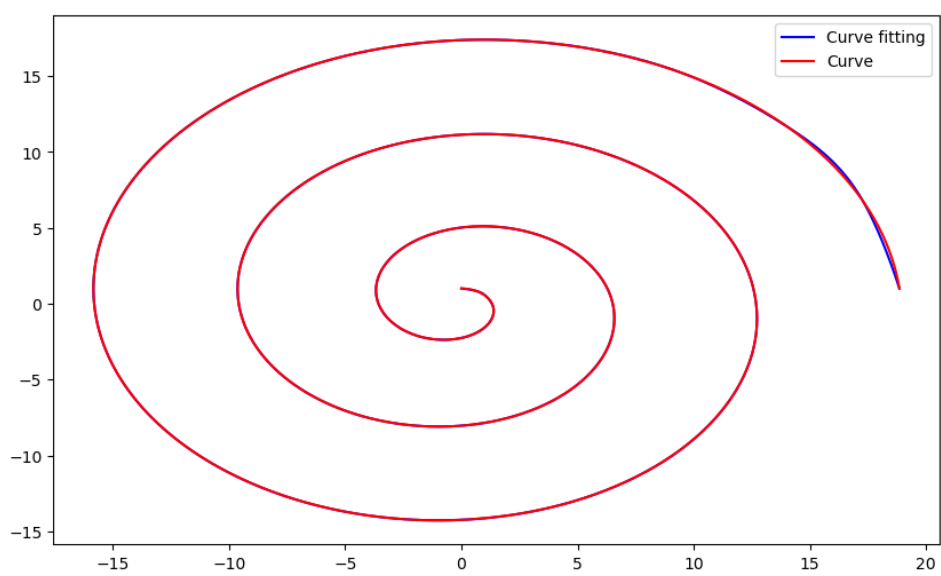


图 14: second curve Curve Fitting

对第三种曲线, 等距节点拟合和使用累计曲线长度进行拟合的效果如下:

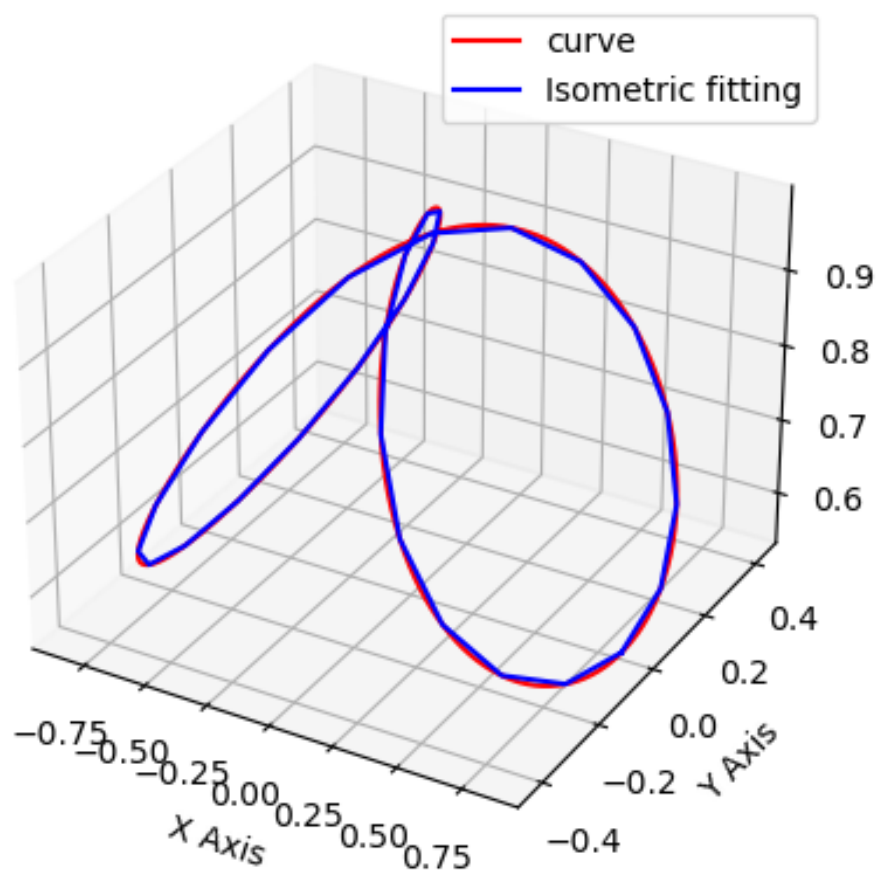


图 15: third curve Isometric Fitting

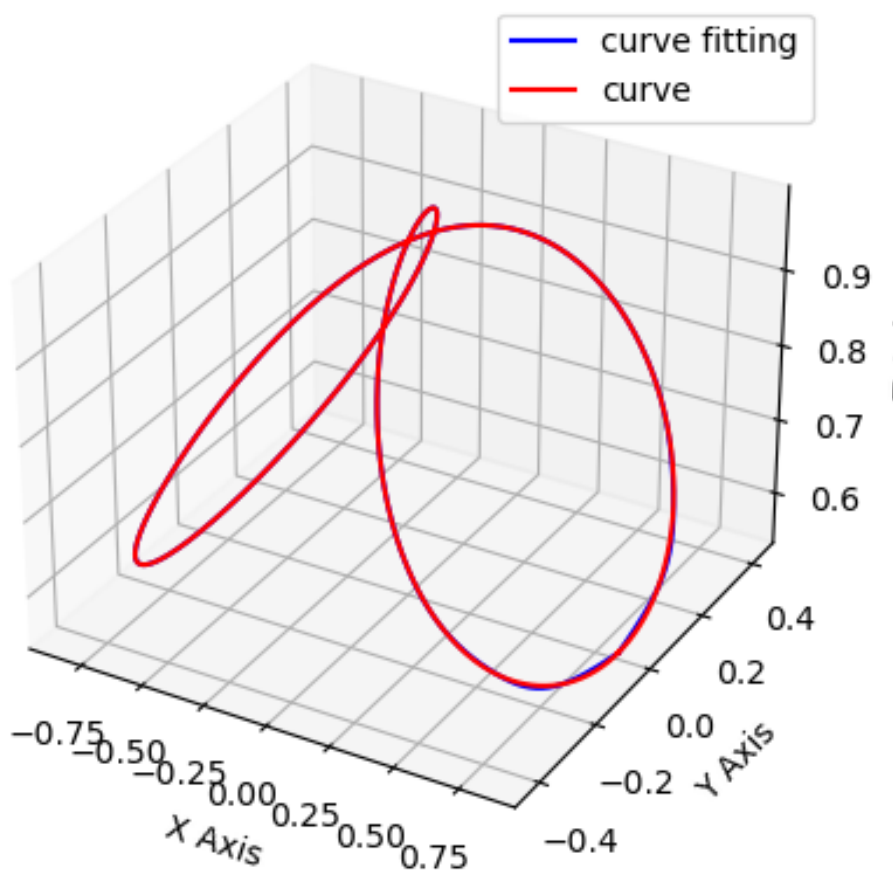


图 16: third curve Curve Fitting

F.

$n = 1$ 时, table of divided difference of truncated power functions 如下图所示:

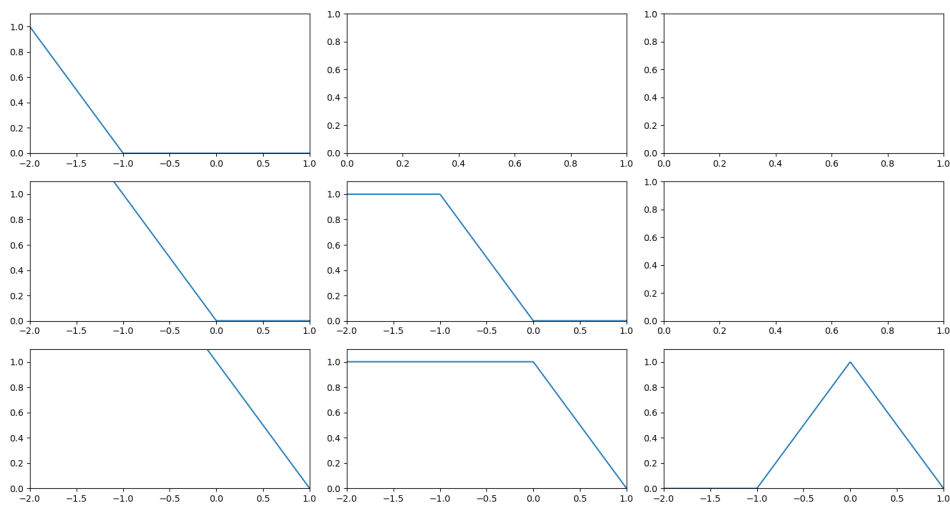


图 17: table of divided difference when $n = 1$

$n = 2$ 时, table of divided difference of truncated power functions 如下图所示:

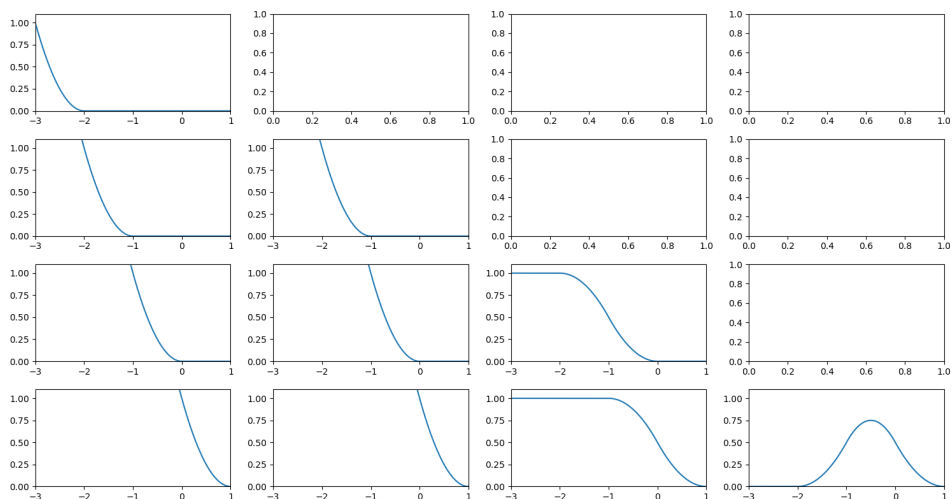


图 18: table of divided difference when $n = 2$