[NO NAME ARTICLE]

Ilya Pchelintsev, Kamilla Faizullina, and Evgeni Burovski
HSE University, 101000 Moscow, Russia

This is an abstract and that's really abstract.

I. INTRODUCTION

This is cite-checking.[1]

II. MODELS AND METHODS

In the paper we consider several models: the first one is Ising model on interacting self-avoiding walk from the [REF KAMILLA], on three different lattices: 2D-square lattice, 3D-square lattice and 2D-triangle lattice. The main difference between square and triangle lattice in defining two additional diagonal monomers on lattice as nearest too, see [FIGURE FOR 2D-SQUARE AND 2D-TRIANGLE LATTICE]. Considering the case of lack of outer magnetic field in this work, the Hamiltonian of the model of fixed conformation u with length N and strength of nearest-neighbors interaction J reads:

$$H_{u,N,\{\sigma\}} = -\sum_{\langle i,j\rangle} J\sigma_i\sigma_j, \quad i,j \in u, \ |u| = N \tag{1}$$

The summation runs through spins involved in conformation and only with the nearest neighbors.

The second model considered in this paper is the Ising model on the rectangular lattice from the [REF SELKE]. Simulated lattices has $L \times rL$ spins and the Hamiltonian is calculated through interaction between all spins and their nearest neighbors respectively:

$$H_{L,r,\{\sigma\}} = -\sum_{\langle i,j\rangle} J\sigma_i\sigma_j, \quad i,j \in [1..L] \times [1..rL] \tag{2}$$

For comparing magnetic properties of models with similar geometric ones we also define shape factors, such as gyration tensor [REF PELLISETTO]:

$$Q_{N,\alpha\beta} = \frac{1}{N+1}\sum_{i=0}^{N}(w_{i,\alpha} - w_{c,\alpha})(w_{i,\beta} - w_{c,\beta}) \tag{3}$$

where N is length of the system (number of monomers in conformations of Ising-ISAW models and number of spins in the lattice in rectangular Ising), and α, β are coordinates (so, $Q_{N,xx}$ and $Q_{N,yy}$ can be defined as mean squares of coordinates of the points of the model in the cartesian coordinate system with the center in the center of model). Eugen values $q1$, $q2$ of given tensor can be interpreted as $Q_{N,xx}$ and $Q_{N,yy}$ in the coordinate system of eugen vectors, or more important - as square of semi-axes of ellipse of inertia of given system. The proportion of them for systems with length N will be[REF PELLISSETTO]:

$$r = \sqrt{\frac{\langle q_1\rangle_N}{\langle q_2\rangle_N}} \tag{4}$$

Eugen values $q1$, $q2$ are also used in enumerating another important shape factor - mean asphericity[REF PELLISETTO]:

$$\mathcal{A} = \left\langle \frac{(q_1 - q_2)^2}{(q_1 + q_2)^2} \right\rangle_N \tag{5}$$

The compared magnetic property of our models is the fourth order cumulant of the magnetization of the Binder cumulant, defined as[REF SELKE]:

$$U_4 = 1 - \frac{\langle m^4\rangle}{3\langle m^2\rangle^2} \tag{6}$$

Where $\langle m^4\rangle$ and $\langle m^2\rangle$ are mean fourth and second order of mean magnetization per spin respectively.

We also need to define mean proportion of monomers with fixed number i of nearest neighbors $\langle n_i\rangle$, which is counted directly for every monomer in every simulated conformation of walk.

We are interested in comparing models in their respective critical regions. For each structure, critical temperatures of Ising models are known as[REF FOSTER][REF SELKE]:

Type of lattice	T_c
2D-square	1.199 ± 0.003
3D-square	1.90 ± 0.02
Rectangular	2.26918...

TABLE I. Known values of critical temperature of different modifications of Ising-ISAW model and normal Ising on the rectangular lattice

Method. All methods of simulating for Ising model on a SAW conformation are remained the same as in[REF KAMILLA]. For simulating Rectangular Ising, however, we used cluster update of Wolff Algorithm[REF NEWMAN BARKEMA][REF SCHRODINGER CAT].

III. RESULTS

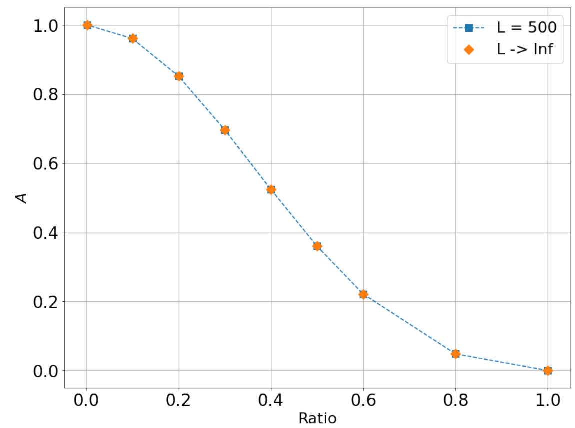

FIG. 1. Asphericity as function of aspect ratio r of the rectangular lattice

A. Mean Asphericity and Critical Cumulant

We attempted to learn how magnetic properties of Ising-like models depend on their geometrical ones and to define their comparability in critical region, where observable values of models don't depend on the length of conformation N. As we know, in the Ising model on rectangular lattice shape factors like aspect ratio r are the parameters, not observable values. Therefore, we can find value of asphericity $\mathcal{A}$ for any aspect ratio.

We found values of aspericity of Ising-ISAW and ISAW models in their respective critical regions. They are used to enumerate aspect ratio of the rectangular lattice with the same value of asphericity.

PolIsing			
J	$\mathcal{A}$	r	U_4 *Rectangular*
0.831	0.415	0.465	0.340 ± 0.006
0.832	0.4072	0.47	0.343 ± 0.006
0.836	0.373	0.490 ± 0.002	0.348 ± 0.006
ISAW			
0.667	0.375	0.49	0.349 ± 0.006

TABLE II. Values of critical cumulant for Ising model on rectangular lattice with mean asphericity related to models ISAW and Ising-ISAW in their critical regions

B. Bulk

IV. DISCUSSION

V. ACKNOWLEDGMENTS

[1] A. Papale and A. Rosa. The Ising model in swollen vs. compact polymers: Mean-field approach and computer simulations. *Eur. Phys. J. E*, 41, 12 2018.

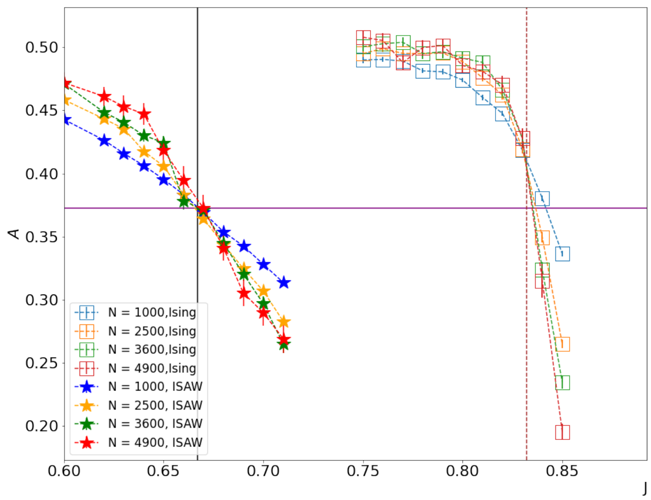

FIG. 2. Asphericity of Ising-ISAW (empty squares) and ISAW-only models (stars) as function of $J = 1/T$, varying lengths of conformations $N = 1000$ (blue), 2500 (yellow), 3600 (green) and 4900 (red)

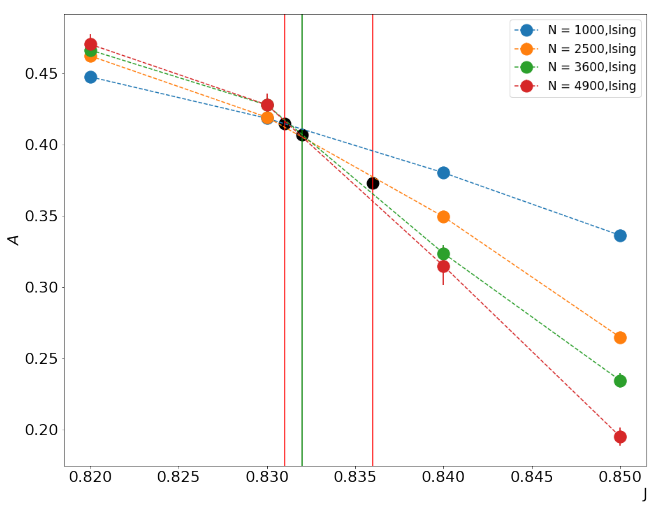

FIG. 3. Asphericity of Ising-ISAW model as function of zoomed in the critical region (red vertical lines), varying lengths of conformations N = 1000 (blue), 2500 (yellow), 3600 (green) and 4900 (red)

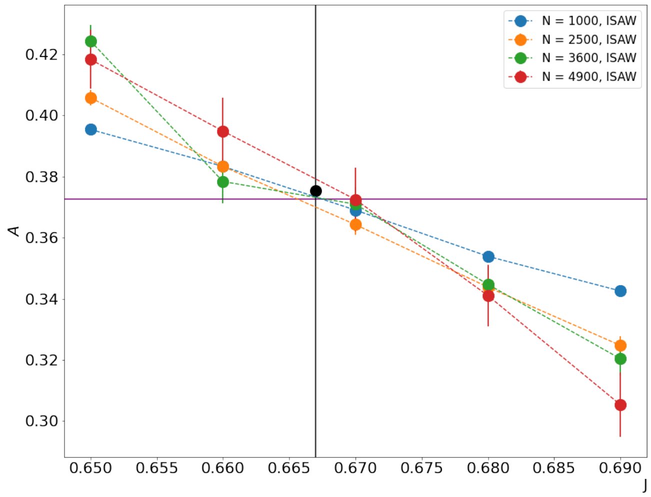

FIG. 4. Asphericity of ISAW model as function of zoomed in the critical region (red vertical lines), varying lengths of conformations N = 1000 (blue), 2500 (yellow), 3600 (green) and 4900 (red)