Magnetic and Geometric Properties of the Ising Model on Lattice Random Walks

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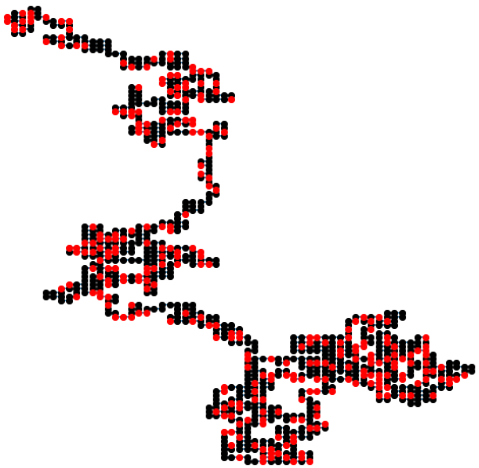
Ising-ISAW model

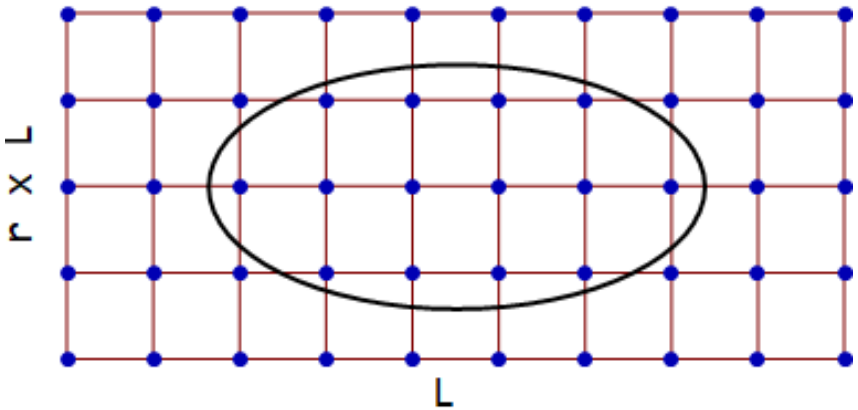
- Linear self-avoiding conformation (**SAW**-models)
- Spin subsistem inside of the monomers (regular **Ising model**)
- Close-range interaction (1)
- Tricriticality of the phase transition point [1]



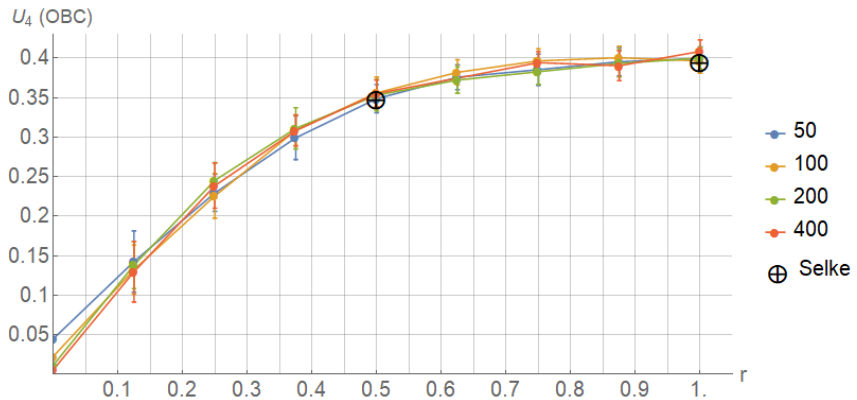
$$H_{u,N,\{\sigma\}} = -\sum_{\langle i,j \rangle} J\sigma_i\sigma_j, \quad i,j \in u, \ |u| = N \tag{1}$$

Universality of critical properties of the regular Ising model

$$H_{L,r,\{\sigma\}} = -\sum_{\langle i,j \rangle} J\sigma_i\sigma_j \qquad (2)$$



(a) Example of the regular Ising model and its gyration ellipse

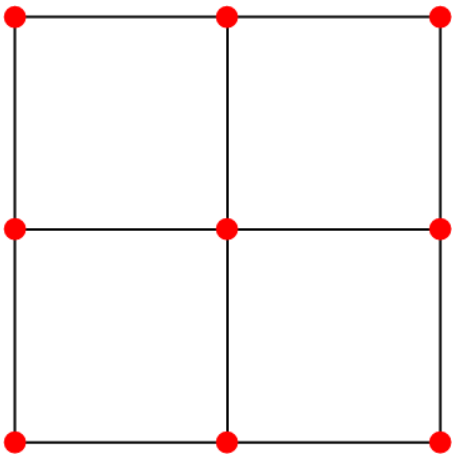


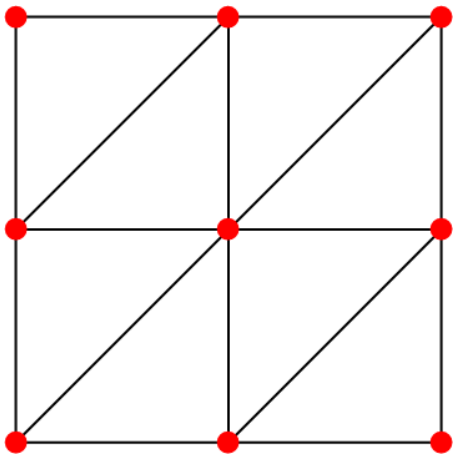
(b) Critical cumulant as a function of the lattice ratio [2]

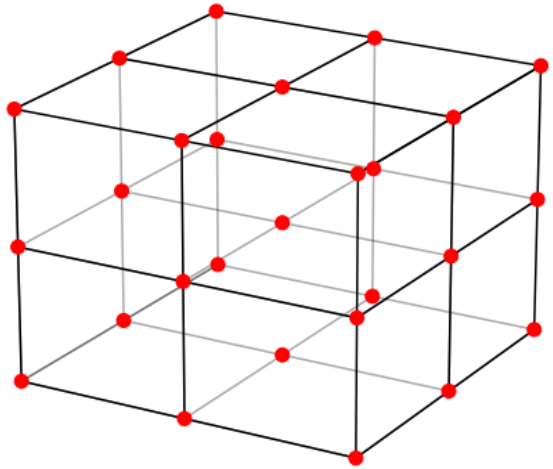
Question №1

Equal critical geometrical properties ⇒ equal critical magnetic ones?

Lattice nearest-neighbors modifications







Question №2

Universality of lattices with equal numbers of dimensions OR equal coordination numbers?

Observables

Critical cumulant:

$$U_4 = 1 - \frac{\langle m^4 \rangle}{3\langle m^2 \rangle^2} \tag{3}$$

Gyration tensor of SAW-system [3]:

$$Q_{N,\alpha\beta} = \frac{1}{N} \sum_{i=1}^{N} (w_{i,\alpha} - w_{c,\alpha})(w_{i,\beta} - w_{c,\beta}) \tag{4}$$

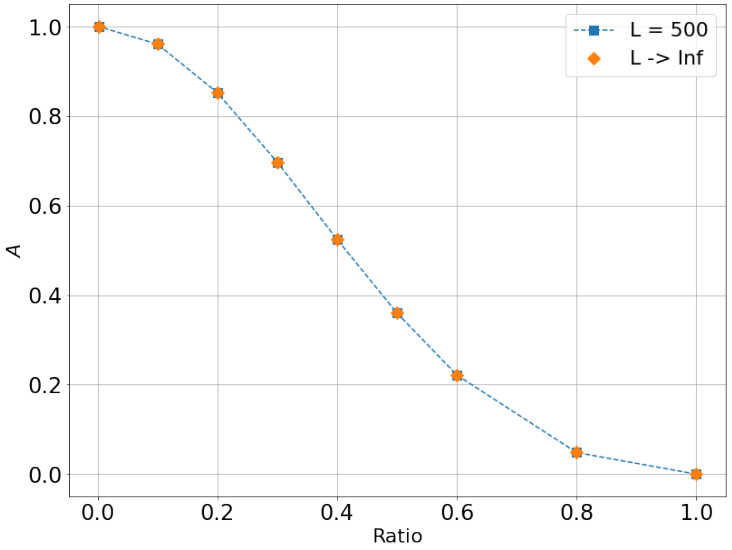
Aspect ratio of the SAW-system:

$$r = \sqrt{\frac{\langle q_1 \rangle_N}{\langle q_2 \rangle_N}} \tag{5}$$

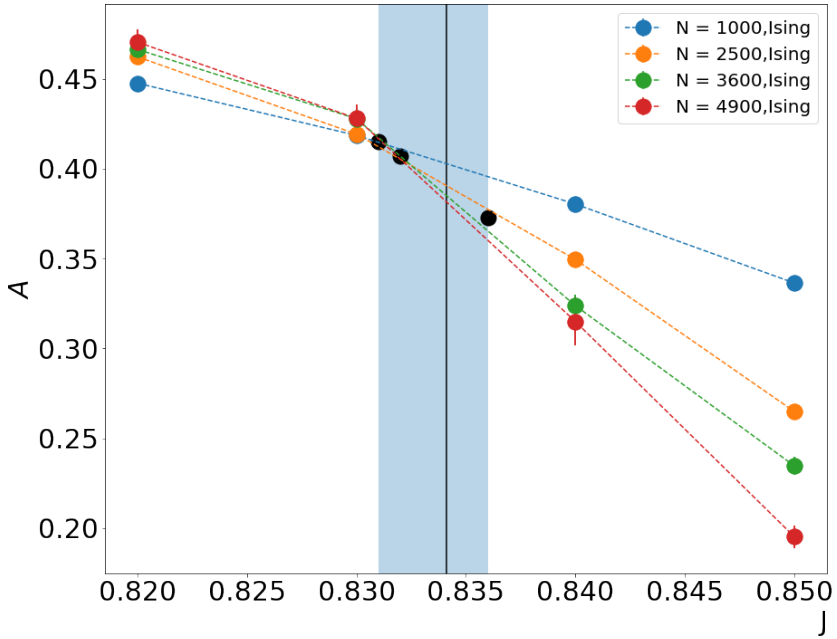
Asphericity:

$$\mathcal{A} = \left\langle \frac{(q_1 - q_2)^2}{(q_1 + q_2)^2} \right\rangle_N \tag{6}$$

Critical Asphericity



(a) Asphericity of the regular Ising as a function of the lattice ratio



(b) Asphericity of the Ising-ISAW as a function of J in the crit. region

Structure	lattice	J_c
SAW	Square	0.8340(5) [4]
lattice	Rectangular	$\ln(1+\sqrt{2})/2$ [5]

Results

Ising-ISAW			
J	$\mathcal{A}$	r	U_4 *Rectangular*
0.831	0.415	0.465	0.338 ± 0.006
0.832	0.4072	0.47	0.343 ± 0.006
0.836	0.373	0.492	0.349 ± 0.006

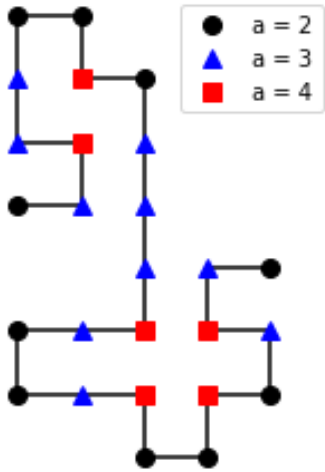
Result

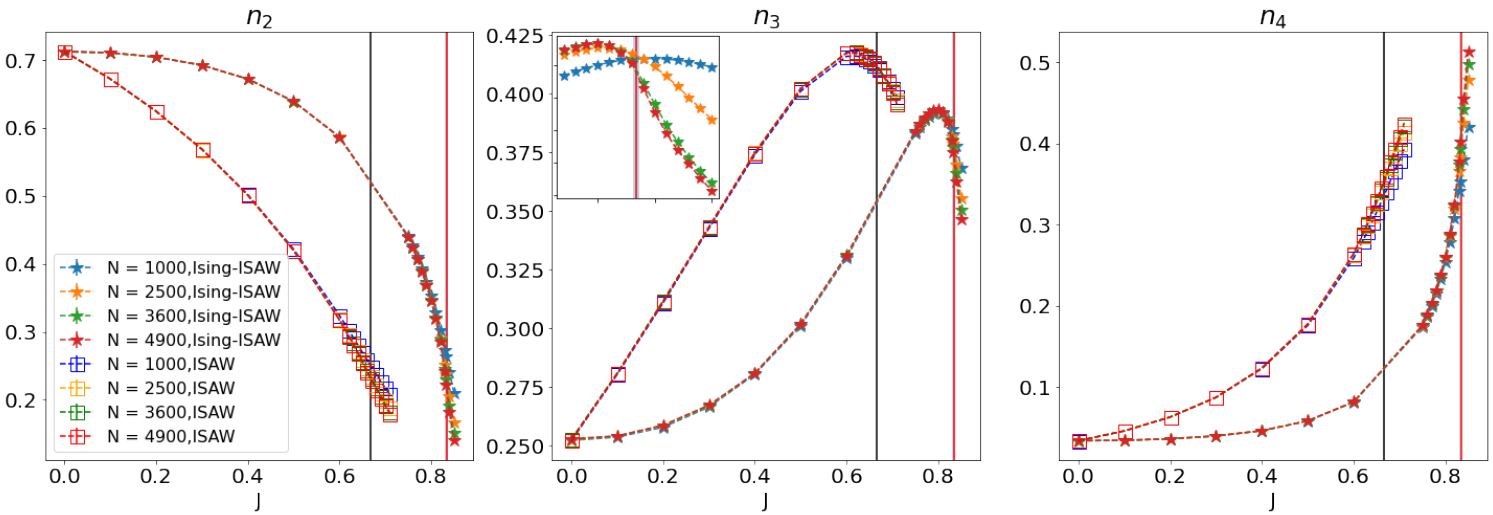
As for Ising-ISAW $U_4 = 0.308(8)$, critical cumulant showed complete mismatch.

Bulks of the Square lattice

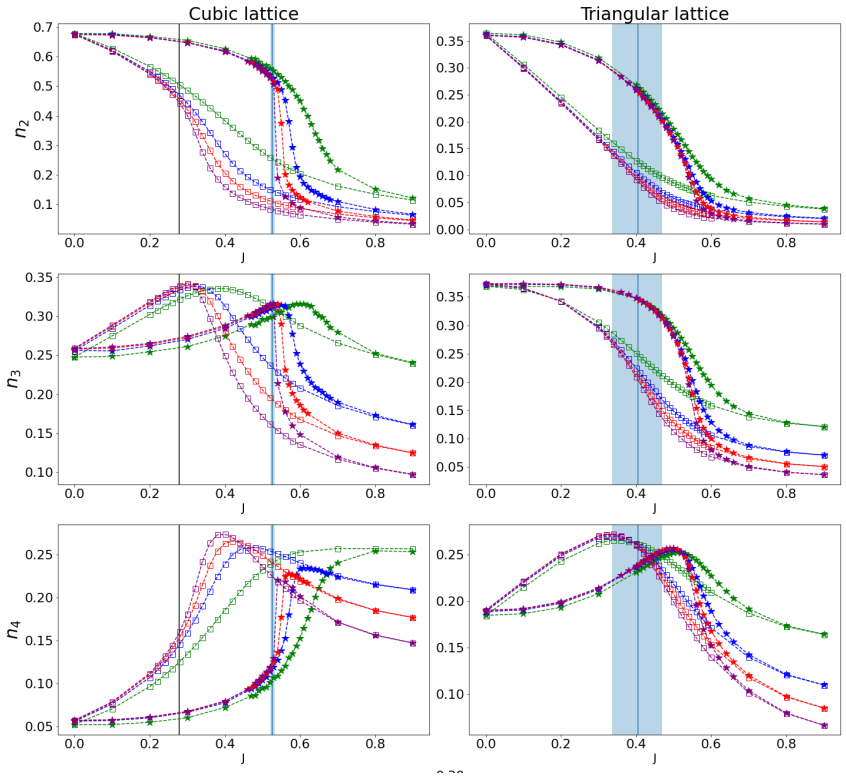
Monomers according to the number of interactions:

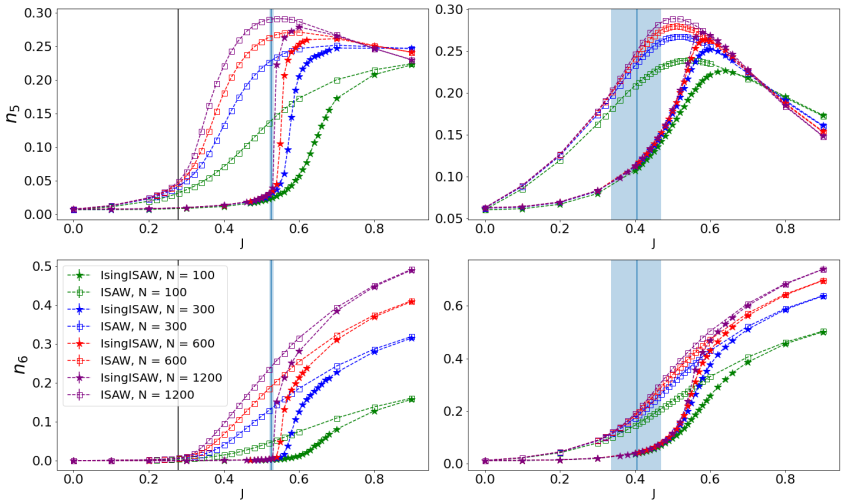
- 2 neighbors - 1D-chains
- 3 neighbors - boundaries of the cluster
- 4 neighbors - core of the cluster





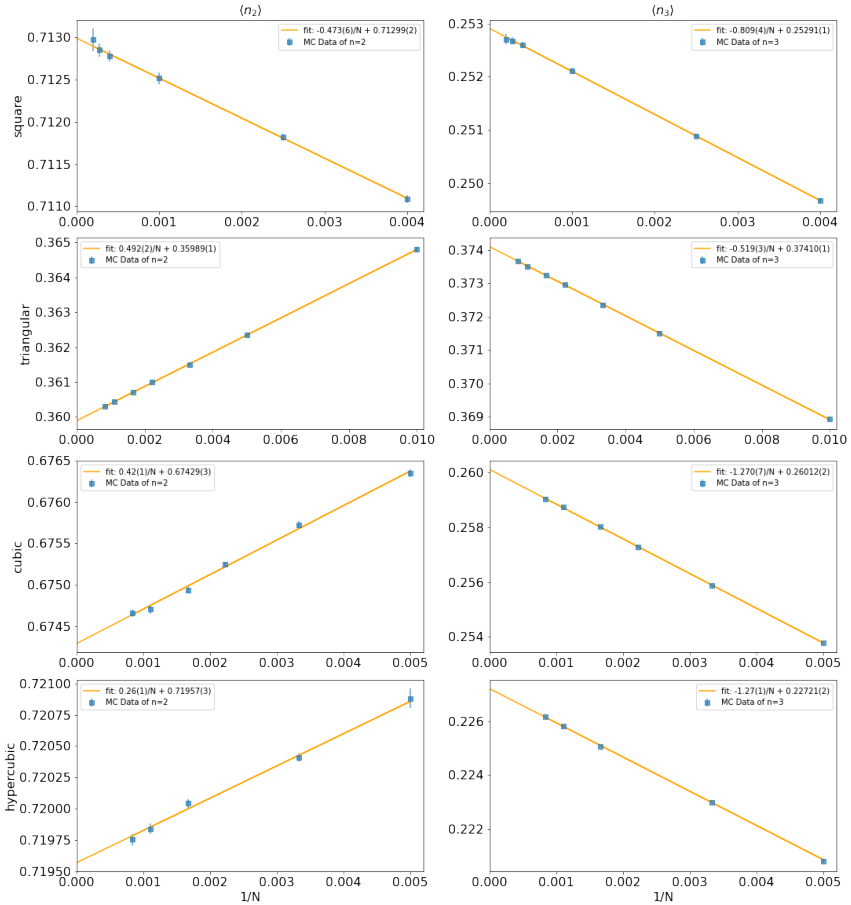
Bulk results





- First order transition was determined in cubic Ising-ISAW
- Continious transition in triangular Ising-ISAW
- Clear depiction of the chain consolidation

Anticipated results



J=0 case research:

- fractions scaling nature as functions of the chain length?
- comparability of the lattice modifications

References

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Thanks for watching!