

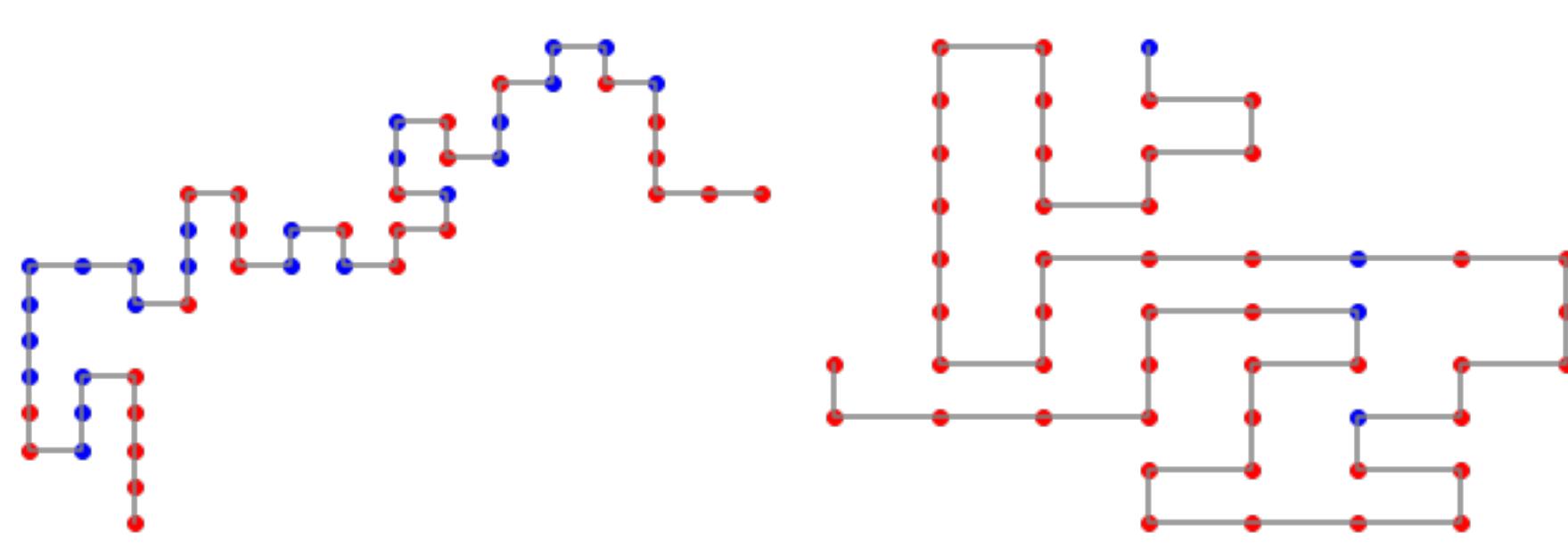
## Models

We consider several models: the first one is Ising model on interacting self-avoiding walk (further called Ising-ISAW). Considering the case of lack of outer magnetic field in this work, the Hamiltonian of the model of fixed conformation  $u$  with length  $N$  and strength of nearest-neighbors interaction  $J$  reads:

$$H_{u,N,\{\sigma\}} = - \sum_{\langle i,j \rangle} J \sigma_i \sigma_j, \quad i, j \in u, |u| = N$$

The summation runs through spins involved in conformation and only with the nearest neighbors. We also consider the parental model of interacting self-avoiding walk (further called ISAW), where all spins are equal 1.

Rules of being “the nearest neighbor” for monomers are defined by the used lattice. For example, a monomer on the square lattice has 4 potential nearest neighbors: on the left, right, front and back from the monomer, while on the triangle lattice two additional diagonal monomers are defined as nearest too.



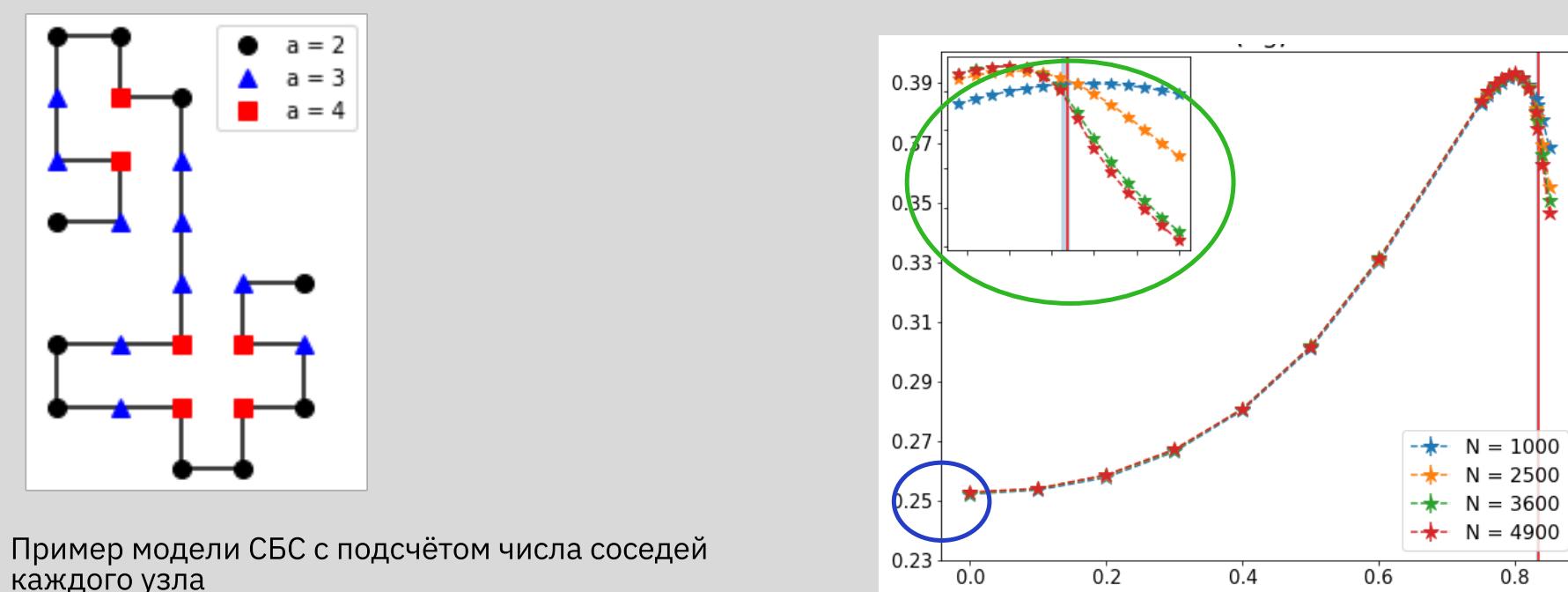
## Methods

lattice	$\sqrt{J_c}$
Square	0.6673(5) [5]
Cubic	0.2779 ± 0.0041[9]
Triangle	0.405 ± 0.07[10]

lattice	$J_c$
Square	0.8340(5)[3]
Cubic	0.5263 ± 0.055[7]

## Previous results - surface monomers



From work [1] it was established that in  $J=0$  mean fraction of “surface-like” monomers on Ising-ISAW conformation is **close to but numerically distinct from 0.25**. We performed same simulations of ISAW and Ising-ISAW models for triangular and cubic lattices with coordination number 6 to enumerate the distribution of monomers in a range of spin-spin coupling strength  $J$  with fixed number of neighbors 2-6. We also simulated the model on  $J=0$  for 2D-square, triangular, cubic and 4D-hypercubic lattices.

## Non-interacting limit of a simple SAW

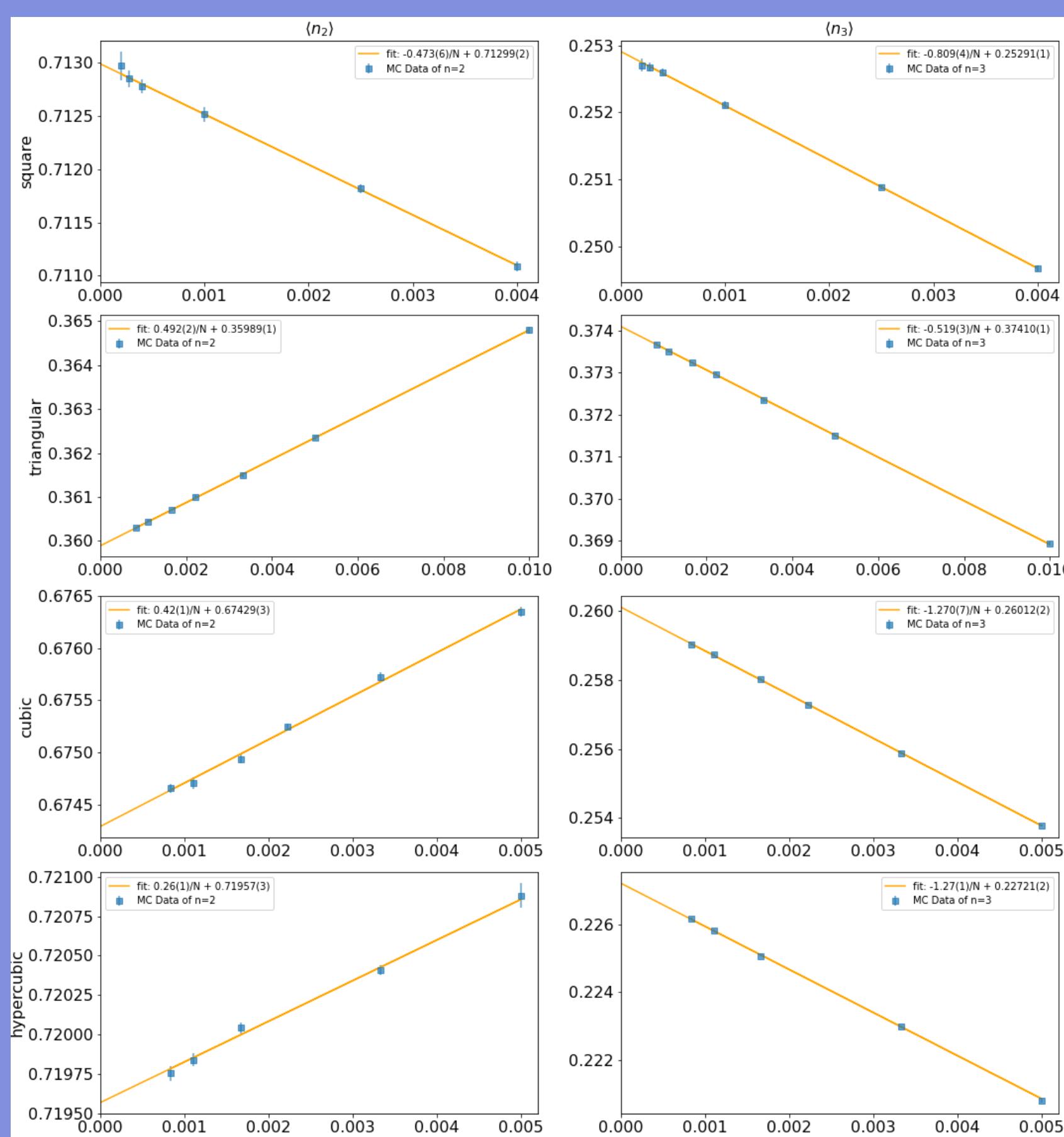
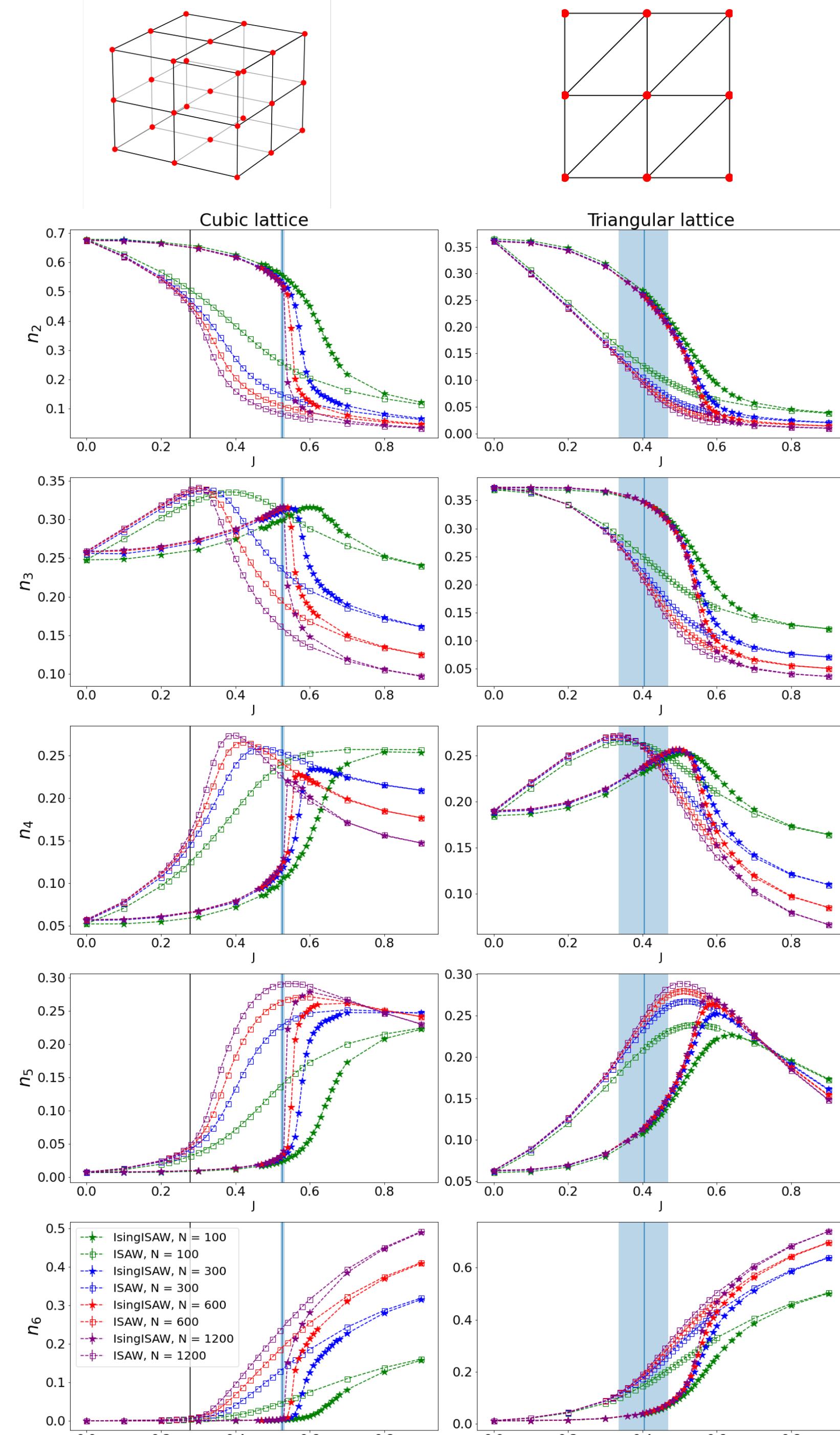


Figure shows results of Monte-Carlo simulations for mean fractions of monomers with 2 and 3 neighbors on a conformation of non-interacting SAW with length of  $N$  (monomers) as a function of  $1/N$ . As it seen, results, especially on triangular lattice, have strong linear approximation near 0, where length of the SAW-chain is close to infinite.

However, linear coefficients of approximation of the results for each lattice do not have any numerical similarities, except conformations of the triangular, cubic and hypercubic lattices have same signs of approximation coefficients. So, despite the unequal number of dimensions, the geometrical behaviour of conformations on a triangular lattice is more similar to ones on a lattice with same coordination number or higher.

## Cubic and triangular lattice comparison



As it is seen from left part of figure, increasing the strength of nearest-neighbors interaction  $J$  leads to conformation becoming more dense as proportions of monomers with higher numbers of close-range connections significantly increases after theta-transition located in a blue zone (for Ising-ISAW model on a cubic lattice  $T_c = 0.526(55)$ ).

We also repeated MC simulations for two-dimensional triangular lattice. Unlike the 3D-square lattice, the 5-th and 6-th possible neighbors located on the same plane as the 1st four, so conformations are expected to be more dense on this lattice.

Critical region of the model on a triangular lattice was not enumerated yet. But as it was suggested, density of conformations becomes higher as nearest-neighbors interaction  $J$  strengthen. Moreover, proportion of monomers with six neighbors on a triangular lattice is far higher than on a cubic one. It is also significant that “triangular” conformations with no inner interaction have almost twice shorter one-dimensional chains than conformations on a cubic lattice in the same conditions.

## Peaks convergence (Square vs Cubic)

