WI.2.

$$I_n(\alpha) = \int_0^1 \frac{x^n}{x+\alpha}\, dx$$

1) $I_n(\alpha) = \int_0^1 \frac{x^n}{x+\alpha} dx = \int_0^1 x^{n-1}\left(\frac{x}{x+\alpha}\right) dx = \int_0^1 x^{n-1} dx - \int_0^1 \frac{x^{n-1}\alpha}{x+\alpha} dx = \frac{x^n}{n}\Big|_0^1 - \alpha I_{n-1}(\alpha) =$

$= \frac{1}{n} - \alpha I_{n-1}(\alpha)$

$\Downarrow$

$I_{n-1}(\alpha) = \frac{1 - n I_n(\alpha)}{\alpha n}$

2) $I_0(\alpha) = \int_0^1 \frac{dx}{x+\alpha} = \ln x+\alpha \Big|_0^1 = \ln\left(\frac{1+\alpha}{\alpha}\right) = \ln\left(1 + \frac{1}{\alpha}\right)$

3) $I_{25}(0.1)$, $n\uparrow$:

$I_0(0.1) = 2.3979$

$I_1(0.1) = 1 - 0.1\, I_0(0.1) = 0.76021$

$\vdots$

$I_{25}(0.1) = 0.036227$

4) $I_n \xrightarrow[n\to\infty]{} 0$

$\exists\, I_{50}(0.1) = 0$ $\quad n\downarrow$:

$I_{49}(0.1) = \frac{1}{49 \cdot 0.1} = 0.204$

$I_{48}(0.1) = \frac{1 - 48 \cdot 0.204}{48 \cdot 0.1} = -1.83 < 0$

5) $I_{25}(10)$:

$n\uparrow$ $I_0(10) = \ln(1.1) = 0.095$

$\vdots$

$I_{15}(10) = -0.069 < 0$

$n\downarrow$: $\exists\ I_{50}(10) = 0$

$I_{49} = 0.002$

$\vdots$

$I_{25} = 0.00364916$

Вывод:

В зависимости от того, больше или меньше α по модулю, чем 1, следует использовать разные методы:

I-ый для $|\alpha| < 1$ (умножение на α)

II-й для $|\alpha| > 1$ (деление на α)

№ I.3

$$a_n = -a_{n-1} + 6a_{n-2}$$

1. $a_0 = 1$

$a_1 = 2$

$$q^n = -q^{n-1} + 6q^{n-2}$$

$$q^2 + q - 6 = 0$$

$$q = 2; -3 \Rightarrow a_n = C_1 2^n + (-3)^n \cdot C_2$$

$$\left.\begin{aligned} a_0 &= C_1 + C_2 = 1 \\ a_1 &= 2C_1 - 3C_2 = 2 \end{aligned}\right\} \Rightarrow 5C_1 = 5;\ C_1 = 1,\ C_2 = 0$$

Тогда:

$a_n = 2^n$ и $\text{recur}(2021) = 2^{2021}$

2.

$a_0 = 1$

$a_1 = 2 + \varepsilon$

$$\Downarrow$$

$$\begin{cases} C_1 + C_2 = 1 \\ 2C_1 - 3C_2 = 2 + \varepsilon \end{cases} \Rightarrow 5C_1 = 5 + \varepsilon \Rightarrow C_1 = 1 + \frac{\varepsilon}{5} \Rightarrow C_2 = -\frac{\varepsilon}{5}$$

Тогда:

$a_n = 2^n - \frac{\varepsilon}{5}(-3)^n$ – при малых изменениях параметров решение меняется значительно.

I.4

$$A = \begin{pmatrix} 1 & 10 \\ \delta & 1 \end{pmatrix}$$

$$\varepsilon(\delta) = \max(\lambda_1, \lambda_2(A))$$

$$\det(A - \lambda E) = \begin{pmatrix} 1-\lambda & 10 \\ \delta & 1-\lambda \end{pmatrix} = (1-\lambda)^2 - 10\delta = 0$$

$$\lambda^2 - 2\lambda + 1 - 10\delta = 0$$

$$\mathcal{D} = 4 - 4 + 40\delta = 40\delta$$

$$\lambda_{1,2} = \frac{2 \pm 2\sqrt{10\delta}}{2} = 1 \pm \sqrt{10\delta} \Rightarrow \varepsilon(\delta) = 1 + \sqrt{10\delta}$$

$$k(10) = \frac{d\varepsilon}{d\delta}\Big/_{\delta=10} = \frac{1}{2\sqrt{10\delta}} \cdot 10\Big/_{\delta=10} = \frac{\sqrt{10}}{2\sqrt{\delta}}\Big/_{\delta=10} = \frac{1}{2}$$

$$k(0.1) = \frac{\sqrt{10}}{2\sqrt{\frac{1}{10}}} = 5$$

I.6

a) $M_1 = 40.00 \times 10^4 \pm 0.05 \times 10^4$

$M_2 = 30.0 \times 10^4 \pm 0.1 \times 10^4$

$r = 3.20 \pm 0.01\ m$

$$F = \frac{G M_1 M_2}{r^2} = \frac{6.67 \cdot 10^{-11} \cdot 40 \cdot 30 \cdot 10^8}{(3.2)^2} = 0.782$$

$$\Delta F = \left(\frac{\partial F}{\partial m_1} \cdot \Delta m_1 + \frac{\partial F}{\partial m_2} \cdot \Delta m_2 + \frac{\partial F}{\partial r} \cdot \Delta r \right) =$$

$$= \frac{G M_2 \Delta M_1}{r^2} + \frac{G M_1 \Delta M_2}{r^2} + 2 \frac{G M_1 M_2}{r^3} \Delta r \Big|_{M_1, M_2, r} = 0.001 + 0.003 + 0.005 = 0.009$$

б)