Homework 3 学号 姓名

1. Design a context-free grammar for $\{a^i b^j c^k \mid i \neq j \text{ or } j \neq k\}$, that is, the set of strings of a's followed by b's followed by c's, such that there are either a different number of a's and b's or a different number of b's and c's, or both.

   $S \to A_1C \mid A_2C \mid AB_1 \mid AB_2$
   $A_1 \to aA_1b \mid aA_1 \mid a$
   $A_2 \to aA_2b \mid A_2b \mid b$
   $C \to Cc \mid \varepsilon$
   $B_1 \to bB_1c \mid bB_1 \mid b$
   $B_2 \to bB_2c \mid B_2c \mid c$
   $A \to Aa \mid \varepsilon$

   (注意：$Cc \mid \varepsilon$ 若为 $Cc \mid c$ 则不能产生 a, c 同时为 0 个，或 b, c)

2. Design a context-free grammar for the set of all strings with twice as many 0's as 1's.

   $S \to 0S0S1S \mid 0S1S0S \mid 1S0S0S \mid \varepsilon$ 或
   $S \to 0S0S1S \mid 0S1S0S \mid 1S0S0S \mid 001 \mid 010 \mid 110$ (但此文法无法产生 ε) 或
   $S \to 0S0S1 \mid 0S1S0 \mid 1S0S0 \mid SS \mid \varepsilon$

   若仅为 $S \to 0S0S1 \mid 0S1S0 \mid 1S0S0 \mid \varepsilon$ (产生式中缺少 S 的)，则无法产生：001100 或开头结尾都是 1 的串.

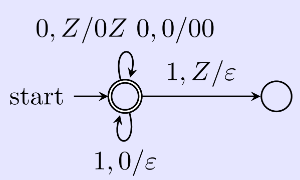
3. Design a context-free grammar for the language consisting of all strings over $\{a, b\}$ that are **not** of the form ww, for some string w. Explain how your grammar works. You needn't prove it's correctness formally.

   如果串长为奇数，显然不是 ww 形式 (对应下面文法中的 A 或 B)。而对于长度为偶数 ($2k$) 的串，一定有在 j ($1 \leq j \leq k$) 位置和 $(k + j)$ 位置不相同，且只要有一处即可。可以将偶数串拆分成两个奇数串，只要中间位置不同，分别通过 A 和 B 生成。

   $S \to A \mid B \mid AB \mid BA$
   $A \to XAX \mid a$
   $B \to XBX \mid b$
   $X \to a \mid b$

   任何偶数串，拆成两个奇数串，两个位于中间的字符不相同即可，如 $aaaabbbb = \underline{aa\check{a}ab}\ \underline{b\check{b}b}$ 或 $aabaaa = \underline{aa\check{b}aa}\ \underline{\check{a}}$.
   但 ww 形式无法生成，拆成任何两个奇数串，中间部分都相等，比如 $baabbaab = \underline{baa\hat{b}baa}\ \underline{\hat{b}} = \underline{b\hat{a}a}\ \underline{bb\hat{a}ab}$.

4. Design a PDA to accept the set of all strings of 0's and l's such that no prefix has more l's than 0's.



5. Design a PDA to accept: $\{0^n 1^m \mid n < m < 2n\}$. You may accept either by final state or by empty stack, whichever is more convenient.