定理6: $\vdash A \wedge (B \vee C) \leftrightarrow (A \wedge B) \vee (A \wedge C)$

证明:

1 $A \wedge (B \vee C), B \vdash B$ $(\in)$

2 $A \wedge (B \vee C), B \vdash A \wedge (B \vee C)$ $(\in)$

3 $A \wedge (B \vee C), B \vdash A$ $(2)(\wedge -)$

4 $A \wedge (B \vee C), B \vdash A \wedge B$ $(3)(1)(\wedge +)$

5 $A \wedge (B \vee C), B \vdash (A \wedge B) \vee (A \wedge C)$ $(4)(\vee +)$

6 $A \wedge (B \vee C), C \vdash C$ $(\in)$

7 $A \wedge (B \vee C), C \vdash A \wedge (B \vee C)$ $(\in)$

8 $A \wedge (B \vee C), C \vdash A$ $(7)(\wedge -)$

9 $A \wedge (B \vee C), C \vdash A \wedge C$ $(6)(8)(\wedge +)$

10 $A \wedge (B \vee C), C \vdash (A \wedge B) \vee (A \wedge C)$ $(9)(\vee +)$

11 $A \wedge (B \vee C) \vdash A \wedge (B \vee C)$ $(\in)$

12 $A \wedge (B \vee C) \vdash B \vee C$ $(11)(\wedge -)$

13 $A \wedge (B \vee C) \vdash (A \wedge B) \vee (A \wedge C)$ $(5)(10)(11)(\vee -)$

14 $\vdash A \wedge (B \vee C) \rightarrow (A \wedge B) \vee (A \wedge C)$ $(13)(\rightarrow +)$

15 $(A \wedge B) \vee (A \wedge C), A \wedge B \vdash A$ $(\in)(\wedge -)$

16 $(A \wedge B) \vee (A \wedge C), A \wedge B \vdash B$ $(\in)(\wedge -)$

17 $(A \wedge B) \vee (A \wedge C), A \wedge B \vdash B \vee C$ $(16)(\vee +)$

18 $(A \wedge B) \vee (A \wedge C), A \wedge B \vdash A \wedge (B \vee C)$ $(15)(17)(\wedge +)$

19 $(A \wedge B) \vee (A \wedge C), A \wedge C \vdash A$ $(\in)(\wedge -)$

20 $(A \wedge B) \vee (A \wedge C), A \wedge C \vdash C$ $(\in)(\wedge -)$

21 $(A \wedge B) \vee (A \wedge C), A \wedge C \vdash B \vee C$ $(20)(\vee +)$

22 $(A \wedge B) \vee (A \wedge C), A \wedge C \vdash A \wedge (B \vee C)$ $(19)(21)(\wedge +)$

23 $(A \wedge B) \vee (A \wedge C) \vdash (A \wedge B) \vee (A \wedge C)$ $(\in)$

24 $(A \wedge B) \vee (A \wedge C) \vdash A \wedge (B \vee C)$ $(18)(22)(23)(\vee -)$

25 $\vdash (A \wedge B) \vee (A \wedge C) \rightarrow A \wedge (B \vee C)$ $(24)(\rightarrow +)$

26 $\vdash A \wedge (B \vee C) \leftrightarrow (A \wedge B) \vee (A \wedge C)$ $(14)(24)(\leftrightarrow +)$