例1 $\vdash (A \to (B \to C)) \to ((C \to D) \to (A \to (B \to D)))$

只须证　$A \to (B \to C) \vdash (C \to D) \to (A \to (B \to D))$

只须证　$A \to (B \to C), C \to D \vdash A \to (B \to D)$

只须证　$A \to (B \to C), C \to D, A \vdash B \to D$

只须证　$A \to (B \to C), C \to D, A, B \vdash D$

证明:

1	A	假设	5	$B \to C$	(1)与(4)用分离规则
2	B	假设	6	C	(2)与(5)用分离规则
3	$C \to D$	假设	7	D	(6)与(3)用分离规则
4	$A \to (B \to C)$	假设			

例2 $\vdash ((A \to B) \to (A \to C)) \to (A \to (B \to C))$

只须证　$(A \to B) \to (A \to C) \vdash A \to (B \to C)$

只须证　$(A \to B) \to (A \to C), A \vdash B \to C$

只须证　$(A \to B) \to (A \to C), A, B \vdash C$

证明:

1	A	假设	6	$A \to C$	(2)与(5)用分离规则
2	B	假设	7	C	(1)与(6)用分离规则
3	$(A \to B) \to (A \to C)$	假设			
4	$B \to (A \to B)$	公理1			
5	$B \to (A \to C)$	(4)与(3)用三段论定理8			