命题3 对公式 A，如果 $\Delta \vdash A$ 那么必有 n 使得 $\Delta_n \vdash A$

证明:

$A_1, A_2, \cdots, A_m (= A)$

$A_{i_1}, A_{i_2}, \cdots, A_{i_k} (\subseteq \Delta = \bigcup_{n=0}^{\infty} \Delta_n)$

$A_{i_1} \in \Delta_{n_{i_1}}, A_{i_2} \in \Delta_{n_{i_2}}, \cdots, A_{i_k} \in \Delta_{n_{i_k}}$

令 $M = max\{n_{i_1}, n_{i_2}, \cdots, n_{i_k}\}$

$n_{i_j} \leq M \quad j = 1, 2, \cdots, k$

$\Delta_{n_{i_j}} \subseteq \Delta_M$

$A_{i_1}, A_{i_2}, \cdots, A_{i_k} \in \Delta_M$

$\Delta_M \vdash A$