Chapter_4_FIF_Truncated

May 27, 2019

1 Non-Global self-similar approximation with moments

```
In [1]: import numpy as np
        import matplotlib.pyplot as plt
        from scipy import optimize
        from scipy.optimize import least_squares
        from scipy.spatial.distance import directed_hausdorff

        import Moment_Function as mf
        import FIF_Trun as FT
```

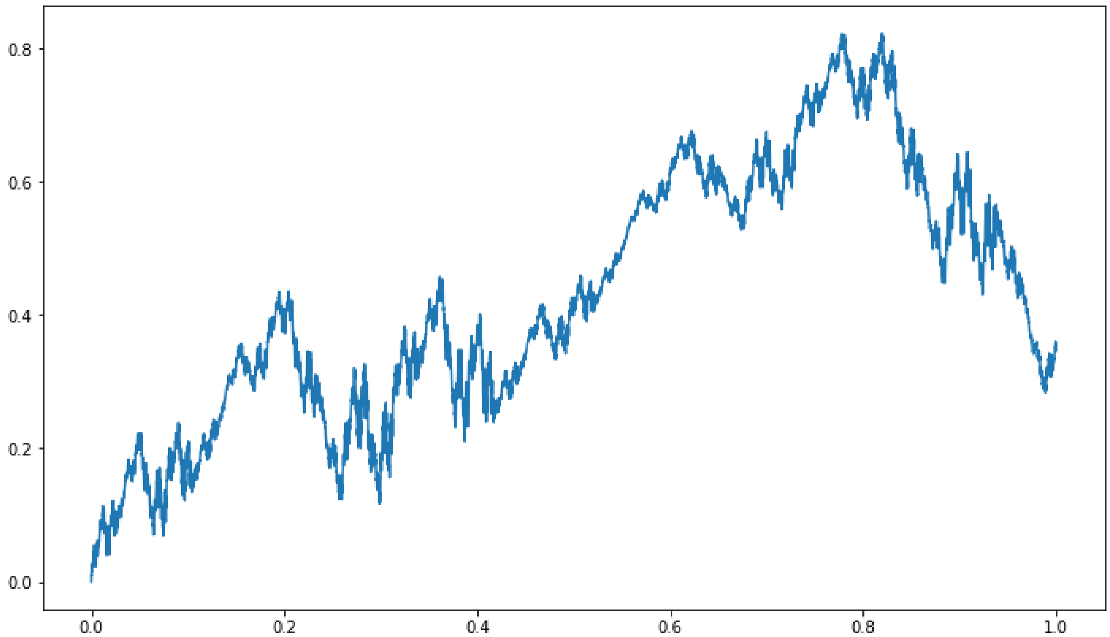
Load in data from Chapter 2

```
In [2]: A_data = np.load('A.npy')
        b_data = np.load('b.npy')
        d_data = np.load('d.npy')


        data_full = np.load('rough_data.npy')
        data = data_full[data_full[:,0]<=1,:]

        plt.figure(figsize=(12,7))
        plt.plot(data[:,0],data[:,1])

Out[2]: [<matplotlib.lines.Line2D at 0x135c42115c0>]
```



Calculate the first 36 moments with eltons theorem of the data above

```
In [3]: data_mom = mf.elt(data,size = 36)

        print(data_mom)

[1.         0.5        0.42553919 0.33335    0.25607629 0.217679
 0.250025   0.18113949 0.14462554 0.12451333 0.20003    0.13811088
 0.10577778 0.08746549 0.0765646  0.1667     0.11014834 0.08129486
 0.06522437 0.05556955 0.04945745 0.14289286 0.09063187 0.06454253
 0.05032914 0.04193625 0.03662755 0.03309413 0.1250375  0.07634642
 0.05250698 0.03979068 0.03243632 0.02786309 0.02483804 0.02273889]
```

Define a cost function to minimise with our moments and a regularisation term

```
In [4]: k1 = 0

        #D = 1.5

        def FIF_cost(x):

            global k1, D, N

            if k1%20 == 0 :
                print('Current parameters:')
                print('x points:')
```

```
            print(np.round(x[:N+1], 2))
            print('y points:')
            print(np.round(x[N+1:2*N+2], 2))
            print('d values:')
            print(np.round(x[2*N+2:], 2))

        k1+=1

        Phi_k, a_l = FT.FIF_wrap(x)

        d_l = x[2*N+2:]

        #dim_cost = np.dot( np.power(a_l, D - 1), np.abs(d_l) ) - 1

        mom_cost = Phi_k@data_mom - data_mom

        reg = 0.1*(x - x0)

        return  np.hstack((mom_cost,reg))
```

We approximate our data with a FIF made from 6 maps, this gives the same amount of free variables given in Chapter 2 with FIC from 16 maps

```
In [5]: #number of maps
        N = 6

        x0 = np.linspace(0,1,N+1)
        idx = FT.find_nearest_v(x0,data[:,0])

        x0_x = data[idx,0]
        x0_y = data[idx,1]

        #d values from chapter 2
        if N == 6:
            x0_d = np.array([0.34,-0.5,-0.42,0.5,0.46,-0.36])

        else:
            x0_d = np.zeros(N)

        x0 = np.hstack((x0_x,x0_y,x0_d))
```

We can restrict the interpolation points to lie in a ‘grid’, but instead we use regularisation to improve our previous model. This can be uncommented and experimented with should the reader desire. It is interesting to reduce N and let these points vary in an interval and discover what the moment values give.

```
In [6]: '''
        bnds_x_m = x0_x - (1/((2.01)*N))*np.ones(len(x0_x))
        bnds_x_p = x0_x + (1/(2*N))*np.ones(len(x0_x))
```

```
        bnds_y_m = -np.inf*np.ones(len(bnds_x_m))
        bnds_y_p =  np.inf*np.ones(len(bnds_x_p))

        bnds_d_m = -np.ones(len(bnds_x_m)-1)
        bnds_d_p =  np.ones(len(bnds_x_p)-1)

        bnds_m = np.hstack((bnds_x_m,bnds_y_m,bnds_d_m))
        bnds_p = np.hstack((bnds_x_p,bnds_y_p,bnds_d_p))

        bnds = (bnds_m,bnds_p)

        bnds
        '''
```

```
Out[6]: '\nbnds_x_m = x0_x - (1/((2.01)*N))*np.ones(len(x0_x))\nbnds_x_p = x0_x + (1/(2*N))*np
```

```
In [7]: k1 = 0

        sol_FIF = least_squares(FIF_cost,x0 ,verbose = 2)

        print(sol_FIF.x)
```

```
Current parameters:
x points:
[0.   0.17 0.33 0.5  0.67 0.83 1.  ]
y points:
[0.   0.31 0.34 0.42 0.57 0.73 0.35]
d values:
[ 0.34 -0.5  -0.42  0.5   0.46 -0.36]
Current parameters:
x points:
[0.   0.17 0.33 0.5  0.67 0.83 1.  ]
y points:
[0.   0.31 0.34 0.42 0.57 0.73 0.35]
d values:
[ 0.34 -0.5  -0.42  0.5   0.46 -0.36]
   Iteration     Total nfev        Cost      Cost reduction    Step norm     Optimality
       0              1         2.7362e-03                                    7.51e-02
Current parameters:
x points:
[0.   0.17 0.33 0.5  0.7  0.84 1.  ]
y points:
[0.01 0.32 0.36 0.42 0.53 0.71 0.41]
d values:
[ 0.34 -0.5  -0.42  0.49  0.44 -0.35]
       1              2         1.2275e-04      2.61e-03       9.29e-02       6.76e-03
Current parameters:
```

```
x points:
[0.   0.17 0.33 0.49 0.71 0.84 1.  ]
y points:
[0.01 0.32 0.36 0.42 0.53 0.7  0.43]
d values:
[ 0.34 -0.5  -0.41  0.49  0.44 -0.35]
       2              3         7.1684e-05      5.11e-05       2.23e-02       1.41e-04
Current parameters:
x points:
[0.   0.17 0.33 0.49 0.71 0.84 1.  ]
y points:
[0.01 0.32 0.36 0.42 0.53 0.7  0.43]
d values:
[ 0.34 -0.5  -0.42  0.49  0.44 -0.35]
       3              4         7.1623e-05      6.10e-08       3.16e-03       1.92e-05
Current parameters:
x points:
[0.   0.17 0.33 0.49 0.71 0.84 1.  ]
y points:
[0.01 0.32 0.36 0.42 0.53 0.7  0.43]
d values:
[ 0.34 -0.5  -0.42  0.49  0.44 -0.35]
       4              5         7.1620e-05      2.89e-09       7.08e-04       1.35e-06
Current parameters:
x points:
[0.   0.17 0.33 0.49 0.71 0.84 1.  ]
y points:
[0.01 0.32 0.36 0.42 0.53 0.7  0.43]
d values:
[ 0.34 -0.5  -0.42  0.49  0.44 -0.35]
       5              6         7.1620e-05      2.27e-10       2.09e-04       7.73e-07
Current parameters:
x points:
[0.   0.17 0.33 0.49 0.71 0.84 1.  ]
y points:
[0.01 0.32 0.36 0.42 0.53 0.7  0.43]
d values:
[ 0.34 -0.5  -0.42  0.49  0.44 -0.35]
       6              7         7.1620e-05      2.04e-11       5.99e-05       1.28e-07
Current parameters:
x points:
[0.   0.17 0.33 0.49 0.71 0.84 1.  ]
y points:
[0.01 0.32 0.36 0.42 0.53 0.7  0.43]
d values:
[ 0.34 -0.5  -0.42  0.49  0.44 -0.35]
       7              8         7.1620e-05      1.91e-12       1.84e-05       7.27e-08
Current parameters:
```

```
x points:
[0.   0.17 0.33 0.49 0.71 0.84 1.  ]
y points:
[0.01 0.32 0.36 0.42 0.53 0.7  0.43]
d values:
[ 0.34 -0.5  -0.42  0.49  0.44 -0.35]
       8              9         7.1620e-05      1.87e-13       5.52e-06       1.55e-08
`ftol` termination condition is satisfied.
Function evaluations 9, initial cost 2.7362e-03, final cost 7.1620e-05, first-order optimality
[ 2.10550494e-04  1.67726475e-01  3.25618284e-01  4.89619633e-01
  7.05609832e-01  8.41821780e-01  1.00036326e+00  5.21091044e-03
  3.20369889e-01  3.60311557e-01  4.18925223e-01  5.28548308e-01
  6.99462629e-01  4.30261426e-01  3.41646460e-01 -4.96453689e-01
 -4.15163331e-01  4.89471543e-01  4.39424699e-01 -3.47391504e-01]
```

Plot our initial guess from Chapter 2 and our moment refinement. Uncomment the `#plt.plot(initi[:,0],initi[:,1], label = '$L^2$')` to compare the approximant made in Chapter 2, otherwise the plot becomes crowded

```
In [10]: A_ans,b_ans,p_ans = FT.FIF_wrap(sol_FIF.x, maps = True)
         approx = FT.chaos(A_ans,b_ans,p = p_ans,it= 10000)

         A_in,b_in,p_in = FT.FIF_wrap(x0, maps = True)
         initi = FT.chaos(A_in,b_in,p = p_in,it= 10000)

         FIF_trun_apr_a = np.zeros(N)
         FIF_trun_apr_d = np.zeros(N)

         for i in range(N):

             FIF_trun_apr_a[i] = A_ans[i][0,0]
             FIF_trun_apr_d[i] = A_ans[i][1,1]

         def dim_FIF(a,d,D):

             d0 = d.flatten()
             aD = np.power(a,D-1)

             return np.dot(d0.flatten(),aD.flatten())-1

         dim_FIF_trun = np.round(optimize.minimize( lambda x : (dim_FIF(FIF_trun_apr_a,np.abs(F

In [11]: plt.figure(figsize=(16,9))
         plt.plot(data[:,0],data[:,1], label = 'Data')
         #plt.plot(initi[:,0],initi[:,1], label = '$L^2$')
         plt.scatter(sol_FIF.x[:N+1],sol_FIF.x[N+1:2*N+2],s = 80,color = 'red', label = 'Interp
         plt.plot(approx[:,0],approx[:,1], label = 'Approximant')
```

```
dH = np.round(max(directed_hausdorff(approx,data)[0],directed_hausdorff(data,approx)[(
plt.ylim(0,1)
plt.title("Approximant Dimension = " + str(dim_FIF_trun) +'     ' +'Hausdorff Distance
plt.legend(fontsize = 20)
```

Out[11]: <matplotlib.legend.Legend at 0x135c43611d0>