Modelling stock-recruitment relationships with the FLSR class

Iago Mosqueira, EC JRC G03 and Laurie Kell, ICCAT

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```
library(FLCore)
```

We have the result of an stock assesment (using VPA) in ple4

```
data(ple4)
```

and now want to fit an stock-recruitment relationship

```
rec(ple4)
```

```
## An object of class "FLQuant"
## , , unit = unique, season = all, area = unique
##
##    year
## age 1957    1958    1959    1960    1961    1962    1963    1964
##   1  457973  698110  863386  757299  860577  589154  688367 2231504
##    year
## age 1965    1966    1967    1968    1969    1970    1971    1972
##   1  694575  586779  401298  434281  648877  650584  410281  366633
##    year
## age 1973    1974    1975    1976    1977    1978    1979    1980
##   1 1312097 1132831  864875  692849  988889  913474  891160 1128822
##    year
## age 1981    1982    1983    1984    1985    1986    1987    1988
##   1  869640 2029493 1306601 1261067 1849179 4732214 1918256 1770637
##    year
## age 1989    1990    1991    1992    1993    1994    1995    1996
##   1 1184055 1033216  910370  773003  522410  434986 1153325 1283485
##    year
## age 1997    1998    1999    2000    2001    2002    2003    2004
##   1 2105676  765785  836929  927442  516739 1612473  505292 1159019
##    year
## age 2005    2006    2007    2008
##   1  714344  820006  949341  844041
##
## units:  10^3
```

```
ssb(ple4)
```

```
## An object of class "FLQuant"
## , , unit = unique, season = all, area = unique
##
##      year
```

```
## age   1957   1958   1959   1960   1961   1962   1963   1964   1965
##   all 274205 288540 296825 308164 321354 372863 370373 363077 344013
##      year
## age   1966   1967   1968   1969   1970   1971   1972   1973   1974
##   all 361549 416563 402521 377432 333933 316343 319062 268714 278648
##      year
## age   1975   1976   1977   1978   1979   1980   1981   1982   1983
##   all 293136 310954 316929 303433 297122 272416 262061 263998 314021
##      year
## age   1984   1985   1986   1987   1988   1989   1990   1991   1992
##   all 326341 348675 375392 445855 391254 408489 368969 335747 269528
##      year
## age   1993   1994   1995   1996   1997   1998   1999   2000   2001
##   all 228668 193093 174408 173903 185308 211327 184733 208393 234078
##      year
## age   2002   2003   2004   2005   2006   2007   2008
##   all 162725 179158 151508 167531 173783 166061 206480
##
## units:  t
```

We can convert and FLStock into an FLSR

```
p4sr <- as.FLSR(ple4)

summary(p4sr)
```

```
## An object of class "FLSR"
##
## Name: Plaice in IV
## Description: 'rec' and 'ssb' slots obtained from a 'FLStock' object
## Range:    min    minyear max maxyear
##  1   1958    1   2008
## Quant: age
##
## rec           : [ 1 51 1 1 1 1 ], units =  10^3
## ssb           : [ 1 51 1 1 1 1 ], units =  kg
## residuals     : [ 1 51 1 1 1 1 ], units =  NA
## fitted        : [ 1 51 1 1 1 1 ], units =  10^3
##
## Model:   list()
## <environment: 0x6693a80>
## Parameters:
##     params
## iter
##    1
##
## Log-likelihood:  NA(NA)
## Variance-covariance: <0 x 0 matrix>
```

As recruitts are of age=1, the lag between ssb and rec is also 1

```
rec(p4sr)
```

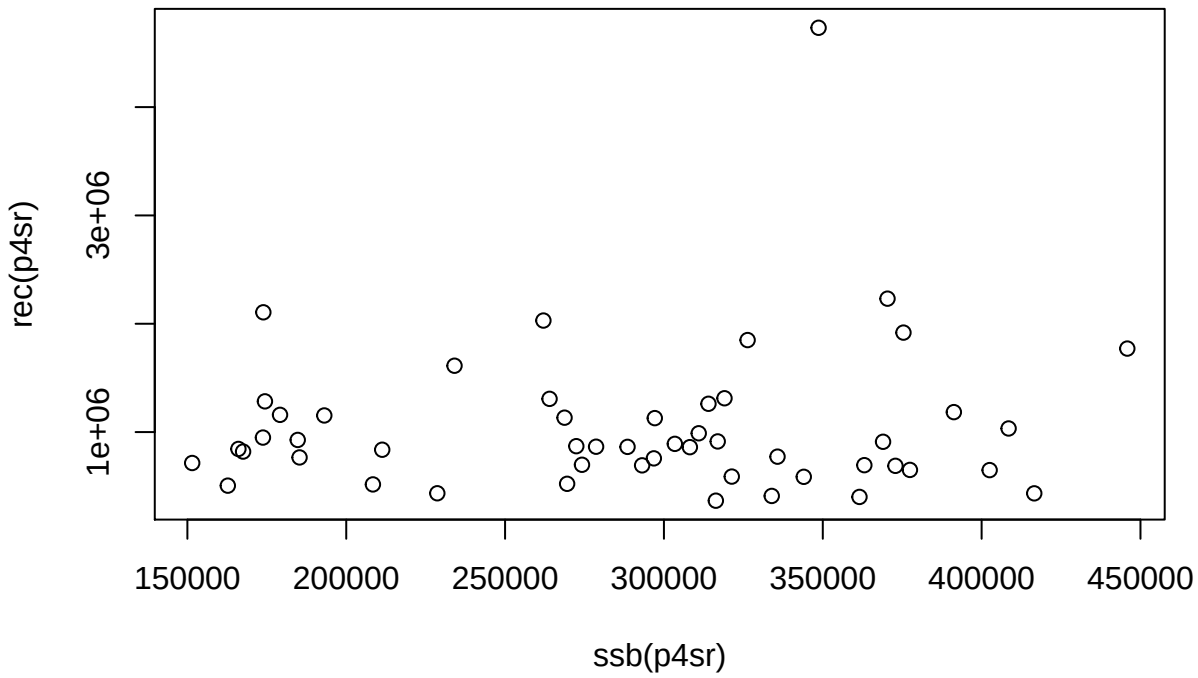
```
## An object of class "FLQuant"
## , , unit = unique, season = all, area = unique
##
##    year
## age 1958    1959    1960    1961    1962    1963    1964    1965
##   1  698110  863386  757299  860577  589154  688367 2231504  694575
##    year
## age 1966    1967    1968    1969    1970    1971    1972    1973
##   1  586779  401298  434281  648877  650584  410281  366633 1312097
##    year
## age 1974    1975    1976    1977    1978    1979    1980    1981
##   1 1132831  864875  692849  988889  913474  891160 1128822  869640
##    year
## age 1982    1983    1984    1985    1986    1987    1988    1989
##   1 2029493 1306601 1261067 1849179 4732214 1918256 1770637 1184055
##    year
## age 1990    1991    1992    1993    1994    1995    1996    1997
##   1 1033216  910370  773003  522410  434986 1153325 1283485 2105676
##    year
## age 1998    1999    2000    2001    2002    2003    2004    2005
##   1  765785  836929  927442  516739 1612473  505292 1159019  714344
##    year
## age 2006    2007    2008
##   1  820006  949341  844041
##
## units:  10^3
```

```
ssb(p4sr)
```

```
## An object of class "FLQuant"
## , , unit = unique, season = all, area = unique
##
##      year
## age   1957   1958   1959   1960   1961   1962   1963   1964   1965
##   all 274205 288540 296825 308164 321354 372863 370373 363077 344013
##      year
## age   1966   1967   1968   1969   1970   1971   1972   1973   1974
##   all 361549 416563 402521 377432 333933 316343 319062 268714 278648
##      year
## age   1975   1976   1977   1978   1979   1980   1981   1982   1983
##   all 293136 310954 316929 303433 297122 272416 262061 263998 314021
##      year
## age   1984   1985   1986   1987   1988   1989   1990   1991   1992
##   all 326341 348675 375392 445855 391254 408489 368969 335747 269528
##      year
## age   1993   1994   1995   1996   1997   1998   1999   2000   2001
##   all 228668 193093 174408 173903 185308 211327 184733 208393 234078
##      year
## age   2002   2003   2004   2005   2006   2007
##   all 162725 179158 151508 167531 173783 166061
```

```
##
## units:  kg
```

```
plot(ssb(p4sr), rec(p4sr))
```



To fit a model, we need to select a functional form

```
model(p4sr) <- ricker()
```

so p4sr is all set up

model formula

```
model(p4sr)
```

```
## rec ~ a * ssb * exp(-b * ssb)
## <environment: 0x4b59cc0>
```

log-likelihood function

```
logl(p4sr)
```

```
## function (a, b, rec, ssb)
## loglAR1(log(rec), log(a * ssb * exp(-b * ssb)))
## <environment: 0x4b59cc0>
```

initial values function

```
initial(p4sr)
```

```
## function (rec, ssb)
## {
##     res <- coefficients(lm(log(c(rec)/c(ssb)) ~ c(ssb)))
##     return(FLPar(a = max(exp(res[1])), b = -max(res[2])))
## }
## <environment: 0x4b59cc0>
## attr(,"lower")
## [1] -Inf -Inf
## attr(,"upper")
## [1] Inf Inf
```

and ouputs: params

```
params(p4sr)
```

```
## An object of class "FLPar"
## params
##  a  b
## NA NA
## units:  NA NA
```

Fitting thorugh MLE

```
p4sr <- fmle(p4sr)
```

```
##   Nelder-Mead direct search function minimizer
## function value for initial parameters = -21.363701
##   Scaled convergence tolerance is 3.18344e-07
## Stepsize computed as 0.916257
## BUILD              3 1000000000000000015902891109759918046836080856394528138978132755774783877217038
## SHRINK             7 1000000000000000015902891109759918046836080856394528138978132755774783877217038
## SHRINK            11 1000000000000000015902891109759918046836080856394528138978132755774783877217038
## SHRINK            15 1000000000000000015902891109759918046836080856394528138978132755774783877217038
## SHRINK            19 1000000000000000015902891109759918046836080856394528138978132755774783877217038
## SHRINK            23 1000000000000000015902891109759918046836080856394528138978132755774783877217038
## SHRINK            27 1000000000000000015902891109759918046836080856394528138978132755774783877217038
## SHRINK            31 1000000000000000015902891109759918046836080856394528138978132755774783877217038
## SHRINK            35 1000000000000000015902891109759918046836080856394528138978132755774783877217038
## SHRINK            39 1000000000000000015902891109759918046836080856394528138978132755774783877217038
## HI-REDUCTION      41 298.701908 -21.363701
## HI-REDUCTION      43 263.351655 -21.363701
## HI-REDUCTION      45 228.002195 -21.363701
## HI-REDUCTION      47 192.655949 -21.363701
## HI-REDUCTION      49 157.322617 -21.363701
## HI-REDUCTION      51 122.040982 -21.363701
## HI-REDUCTION      53 86.964680 -21.363701
## HI-REDUCTION      55 52.683513 -21.363701
## HI-REDUCTION      57 21.213301 -21.363701
## HI-REDUCTION      59 -2.719834 -21.363701
```

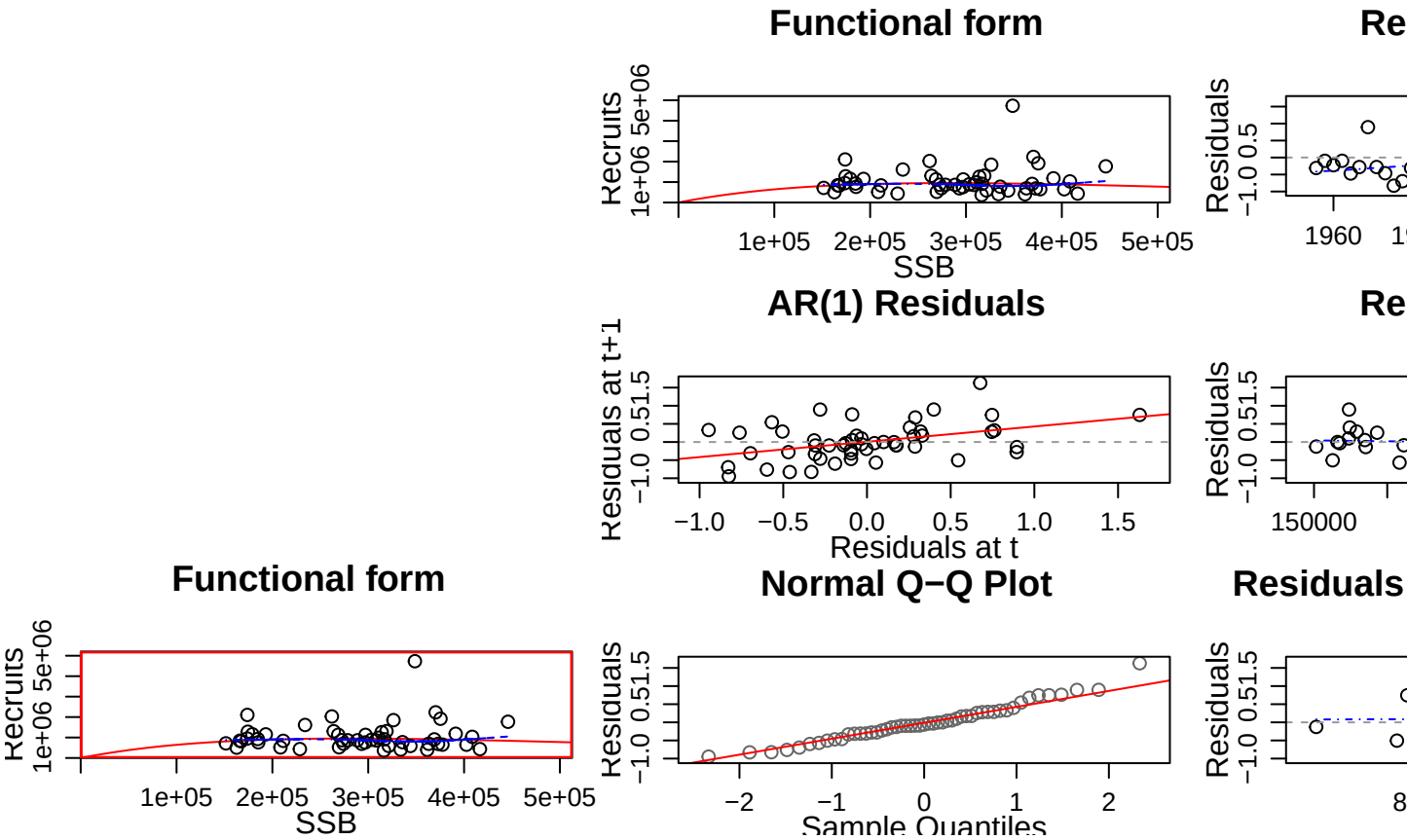
```
## HI-REDUCTION      61 -15.283622 -21.363701
## HI-REDUCTION      63 -19.703632 -21.363701
## HI-REDUCTION      65 -20.939051 -21.363701
## HI-REDUCTION      67 -21.257181 -21.363701
## HI-REDUCTION      69 -21.337183 -21.363701
## HI-REDUCTION      71 -21.357145 -21.363701
## HI-REDUCTION      73 -21.362100 -21.363701
## HI-REDUCTION      75 -21.363319 -21.363701
## HI-REDUCTION      77 -21.363614 -21.363701
## HI-REDUCTION      79 -21.363683 -21.363701
## HI-REDUCTION      81 -21.363697 -21.363701
## HI-REDUCTION      83 -21.363698 -21.363701
## HI-REDUCTION      85 -21.363700 -21.363701
## HI-REDUCTION      87 -21.363700 -21.363701
## Exiting from Nelder Mead minimizer
##     89 function evaluations used
```

gives us some results, not too good

```
summary(p4sr)
```

```
## An object of class "FLSR"
##
## Name: Plaice in IV
## Description: 'rec' and 'ssb' slots obtained from a 'FLStock' object
## Range:    min    minyear max maxyear
##  1   1958    1   2008
## Quant: age
##
## rec           : [ 1 51 1 1 1 1 ], units =  10^3
## ssb           : [ 1 51 1 1 1 1 ], units =  kg
## residuals     : [ 1 51 1 1 1 1 ], units =  NA
## fitted        : [ 1 51 1 1 1 1 ], units =  10^3
##
## Model:   rec ~ a * ssb * exp(-b * ssb)
## <environment: 0x4b59cc0>
## Parameters:
##     params
## iter     a         b
##    1 9.163 3.546e-06
##
## Log-likelihood:  21.364(0)
## Variance-covariance:
##      a  b
##   a NA NA
##   b NA NA
```

```
plot(p4sr)
```



default method is Nelder-Mead

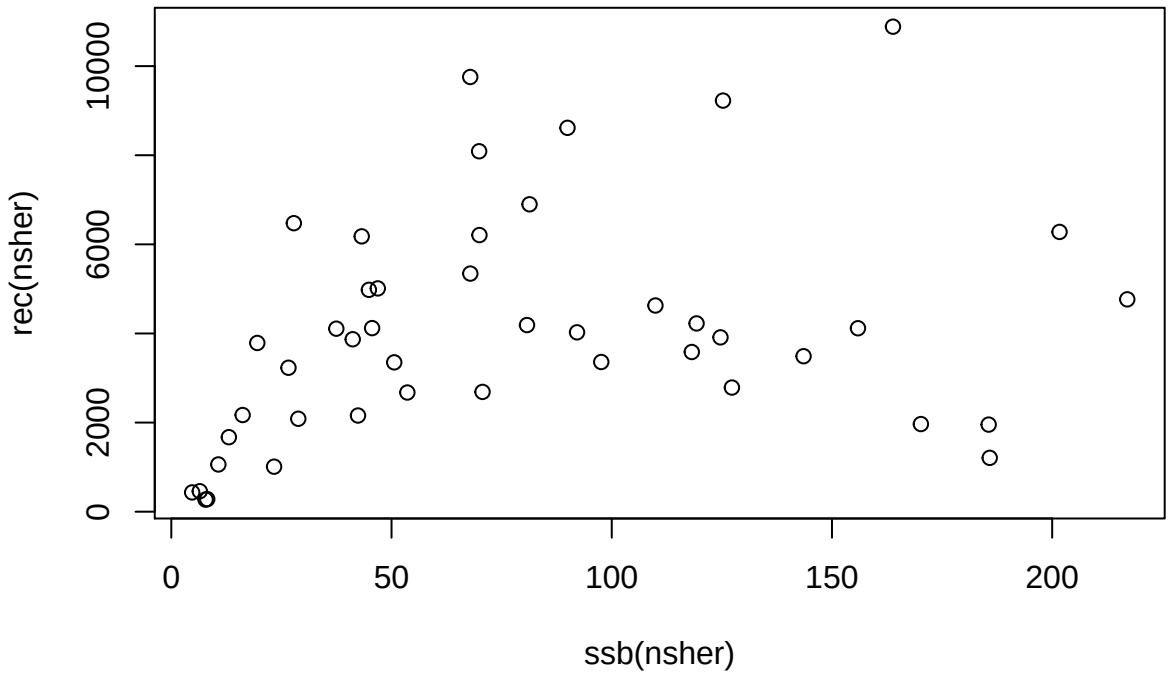
```
p4sr <- fmle(p4sr, method='L-BFGS-B')
```

```
## final  value -21.363701
## converged
```

The nhser dataset is perfect for FLSR

```
data(nsher)
```

```
plot(ssb(nsher), rec(nsher))
```



There are also other SR models to choose from

```
bevholt()
```

```
## $logl
## function (a, b, rec, ssb)
## loglAR1(log(rec), log(a * ssb/(b + ssb)))
## <environment: 0x6891420>
##
## $model
## rec ~ a * ssb/(b + ssb)
## <environment: 0x6891420>
##
## $initial
## function (rec, ssb)
## {
##     a <- max(quantile(c(rec), 0.75, na.rm = TRUE))
##     b <- max(quantile(c(rec)/c(ssb), 0.9, na.rm = TRUE))
##     return(FLPar(a = a, b = a/b))
## }
## <environment: 0x6891420>
## attr(,"lower")
## [1] -Inf -Inf
## attr(,"upper")
## [1] Inf Inf
```

```
shepherd()
```

```
## $logl
## function (a, b, c, rec, ssb)
## loglAR1(log(rec), log(a * ssb/(1 + (ssb/b)^c)))
```

```
## <environment: 0x6540c58>
##
## $model
## rec ~ a * ssb/(1 + (ssb/b)^c)
## <environment: 0x6540c58>
##
## $initial
## function (rec, ssb)
## {
##     c <- 1
##     x <- ssb^c
##     y <- ssb/rec
##     res <- coefficients(lm(c(y) ~ c(x)))
##     a <- max(1/res[1])
##     b <- max(b = 1/((res[2] * a)^(1/c)))
##     return(FLPar(a = a, b = b, c = c))
## }
## <environment: 0x6540c58>
## attr(,"lower")
## [1] 0 0 1
## attr(,"upper")
## [1] Inf Inf  10
```

```
cushing()
```

```
## $logl
## function (a, b, rec, ssb)
## loglAR1(log(rec), log(a * ssb^b))
## <environment: 0x60537e0>
##
## $model
## rec ~ a * ssb^b
## <environment: 0x60537e0>
##
## $initial
## function (rec, ssb)
## {
##     a <- mean(rec/ssb)
##     b <- 1
##     return(FLPar(a = a, b = b))
## }
## <environment: 0x60537e0>
## attr(,"lower")
## [1] -Inf -Inf
## attr(,"upper")
## [1] Inf Inf
```

```
geomean()
```

```
## $logl
## function (a, rec)
## loglAR1(log(rec), log(FLQuant(rep(a, length(rec)))))
## <environment: 0x4e9c6a8>
```

```
##
## $model
## rec ~ a + ssb/ssb - 1
## <environment: 0x4e9c6a8>
##
## $initial
## function (rec)
## {
##     return(FLPar(a = exp(mean(log(rec), na.rm = TRUE))))
## }
## <environment: 0x4e9c6a8>
## attr(,"lower")
## [1] 1e-08
## attr(,"upper")
## [1] Inf
```

```
segreg()
```

```
## $logl
## function (a, b, rec, ssb)
## {
##     loglAR1(log(rec), FLQuant(log(ifelse(c(ssb) <= b, a * c(ssb),
##         a * b)), dimnames = dimnames(ssb)))
## }
## <environment: 0x4cd5a10>
##
## $model
## rec ~ FLQuant(ifelse(c(ssb) <= b, a * c(ssb), a * b), dimnames = dimnames(ssb))
## <environment: 0x4cd5a10>
##
## $initial
## function (rec, ssb)
## {
##     return(FLPar(a = median(c(rec)/c(ssb), na.rm = TRUE), b = median(c(ssb),
##         na.rm = TRUE)))
## }
## <environment: 0x4cd5a10>
## attr(,"lower")
## numeric(0)
## attr(,"upper")
## [1] Inf Inf
```

```
rickerSV()
```

```
## $logl
## function (s, v, spr0, rec, ssb)
## {
##     pars <- abPars("ricker", s = s, v = v, spr0 = spr0)
##     loglAR1(log(rec), log(pars["a"] * ssb * exp(-pars["b"] *
##         ssb)))
## }
## <environment: 0x4c68cf0>
##
```

```
## $model
## rec ~ abPars("ricker", s = s, v = v, spr0 = spr0)["a"] * ssb *
##     exp(-abPars("ricker", s = s, v = v, spr0 = spr0)["b"] * ssb)
## <environment: 0x4c68cf0>
##
## $initial
## function (rec, ssb)
## {
##     s <- 0.75
##     spr0 <- quantile(c(ssb/rec), prob = 0.9, na.rm = TRUE, names = FALSE)
##     v <- mean(as.vector(ssb), na.rm = TRUE) * 2
##     return(FLPar(s = s, v = v, spr0 = spr0))
## }
## <environment: 0x4c68cf0>
## attr(,"lower")
## [1] 1e-08 1e-08 1e-08
## attr(,"upper")
## [1]  10 Inf Inf
```

```
bevholtSV()
```

```
## $logl
## function (s, v, spr0, rec, ssb)
## {
##     pars <- FLPar(abPars("bevholt", s = s, v = v, spr0 = spr0))
##     loglAR1(log(rec), log(pars["a"] %*% ssb/(pars["b"] %+% ssb)))
## }
## <environment: 0x4bf85d8>
##
## $model
## rec ~ FLPar(abPars("bevholt", s = s, v = v, spr0 = spr0))["a"] %*%
##     ssb%/%(FLPar(abPars("bevholt", s = s, v = v, spr0 = spr0))["b"] %+%
##     ssb)
## <environment: 0x4bf85d8>
##
## $initial
## function (rec, ssb)
## {
##     s <- 0.75
##     spr0 <- quantile(c(ssb/rec), prob = 0.9, na.rm = TRUE, names = FALSE)
##     v <- mean(as.vector(ssb), na.rm = TRUE) * 2
##     return(FLPar(s = s, v = v, spr0 = spr0))
## }
## <environment: 0x4bf85d8>
## attr(,"lower")
## [1] 2e-01 1e-07 1e-07
## attr(,"upper")
## [1] 0.999   Inf   Inf
```

```
shepherdSV()
```

```
## $logl
## function (s, v, spr0, c, rec, ssb)
```

```
## {
##     pars <- abPars("shepherd", s = s, v = v, spr0 = spr0, c = c)
##     loglAR1(log(rec), log(pars["a"] * ssb/(1 + (ssb/pars["b"])^c)))
## }
## <environment: 0x4b7ca88>
##
## $model
## rec ~ abPars("shepherd", s = s, v = v, spr0 = spr0, c = c)["a"] *
##     ssb/(1 + (ssb/abPars("shepherd", s = s, v = v, spr0 = spr0,
##     c = c)["b"])^c)
## <environment: 0x4b7ca88>
##
## $initial
## function (rec, ssb)
## {
##     s <- 0.75
##     spr0 <- quantile(c(ssb/rec), prob = 0.9, na.rm = TRUE, names = FALSE)
##     v <- mean(as.vector(ssb), na.rm = TRUE) * 2
##     return(FLPar(s = s, v = v, spr0 = spr0, c = 1))
## }
## <environment: 0x4b7ca88>
## attr(,"lower")
## [1] 2e-01 1e-07 1e-07 1e+00
## attr(,"upper")
## [1]  0.999    Inf    Inf 10.000
```

```
bevholtAR1()
```

```
## $logl
## function (a, b, rho, rec, ssb)
## loglAR1(log(rec), log(a * ssb/(b + ssb)), rho = rho)
## <environment: 0x4af9a00>
##
## $model
## rec ~ a * ssb/(b + ssb)
## <environment: 0x4af9a00>
##
## $initial
## function (rec, ssb)
## {
##     a <- max(quantile(c(rec), 0.75, na.rm = TRUE))
##     b <- max(quantile(c(rec)/c(ssb), 0.9, na.rm = TRUE))
##     return(FLPar(a = a, b = a/b, rho = 0))
## }
## <environment: 0x4af9a00>
## attr(,"lower")
## [1]  1e-07  1e-07 -1e+00
## attr(,"upper")
## [1] Inf Inf   1
```

Let's try ricker

```
model(nsher) <- 'ricker'

nhri <- fmle(nsher)
```

```
##   Nelder-Mead direct search function minimizer
## function value for initial parameters = -15.862252
##   Scaled convergence tolerance is 2.36366e-07
## Stepsize computed as 11.939303
## BUILD              3 1000000000000000015902891109759918046836080856394528138978132755774783877217038
## SHRINK             7 1000000000000000015902891109759918046836080856394528138978132755774783877217038
## HI-REDUCTION       9 267.632807 -15.862252
## HI-REDUCTION      11 236.438352 -15.862252
## HI-REDUCTION      13 205.241529 -15.862252
## HI-REDUCTION      15 174.041331 -15.862252
## HI-REDUCTION      17 142.839861 -15.862252
## HI-REDUCTION      19 111.657775 -15.862252
## HI-REDUCTION      21 80.601711 -15.862252
## HI-REDUCTION      23 50.133420 -15.862252
## HI-REDUCTION      25 21.954719 -15.862252
## HI-REDUCTION      27 0.340450 -15.862252
## HI-REDUCTION      29 -10.920787 -15.862252
## HI-REDUCTION      31 -14.696800 -15.862252
## HI-REDUCTION      33 -15.639120 -15.862252
## HI-REDUCTION      35 -15.676851 -15.862252
## HI-REDUCTION      37 -15.816477 -15.862252
## HI-REDUCTION      39 -15.825739 -15.862252
## LO-REDUCTION      41 -15.847989 -15.862252
## LO-REDUCTION      43 -15.852490 -15.862252
## LO-REDUCTION      45 -15.857566 -15.862252
## LO-REDUCTION      47 -15.860412 -15.862252
## LO-REDUCTION      49 -15.861062 -15.862252
## LO-REDUCTION      51 -15.861647 -15.862252
## LO-REDUCTION      53 -15.862025 -15.862252
## LO-REDUCTION      55 -15.862108 -15.862252
## LO-REDUCTION      57 -15.862175 -15.862252
## LO-REDUCTION      59 -15.862224 -15.862252
## LO-REDUCTION      61 -15.862235 -15.862252
## LO-REDUCTION      63 -15.862243 -15.862252
## LO-REDUCTION      65 -15.862249 -15.862252
## LO-REDUCTION      67 -15.862250 -15.862252
## LO-REDUCTION      69 -15.862251 -15.862252
## LO-REDUCTION      71 -15.862252 -15.862252
## LO-REDUCTION      73 -15.862252 -15.862252
## Exiting from Nelder Mead minimizer
##     75 function evaluations used
```

and then bevholt

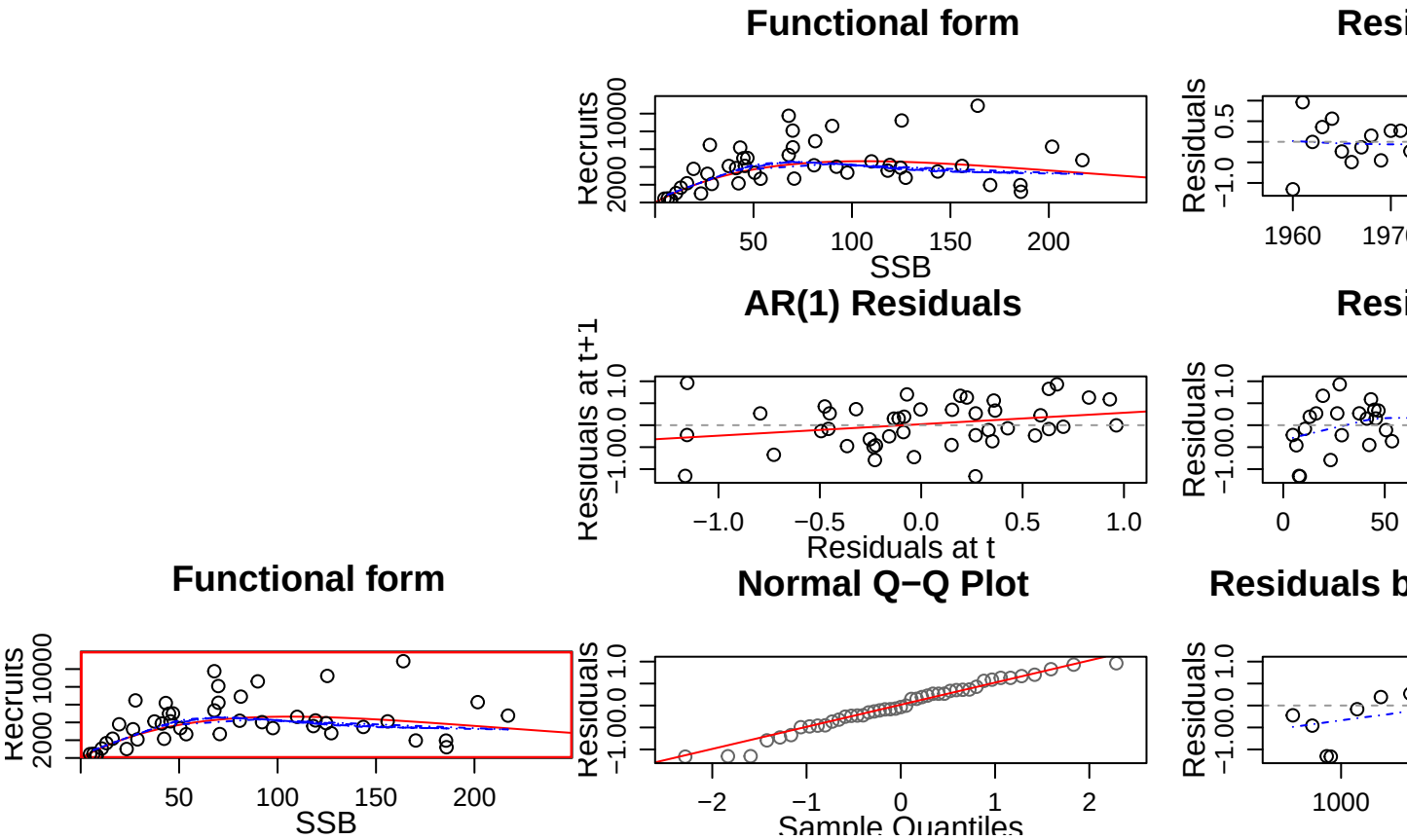
```
model(nsher) <- 'bevholt'

nhbh <- fmle(nsher)
```

```
##   Nelder-Mead direct search function minimizer
```

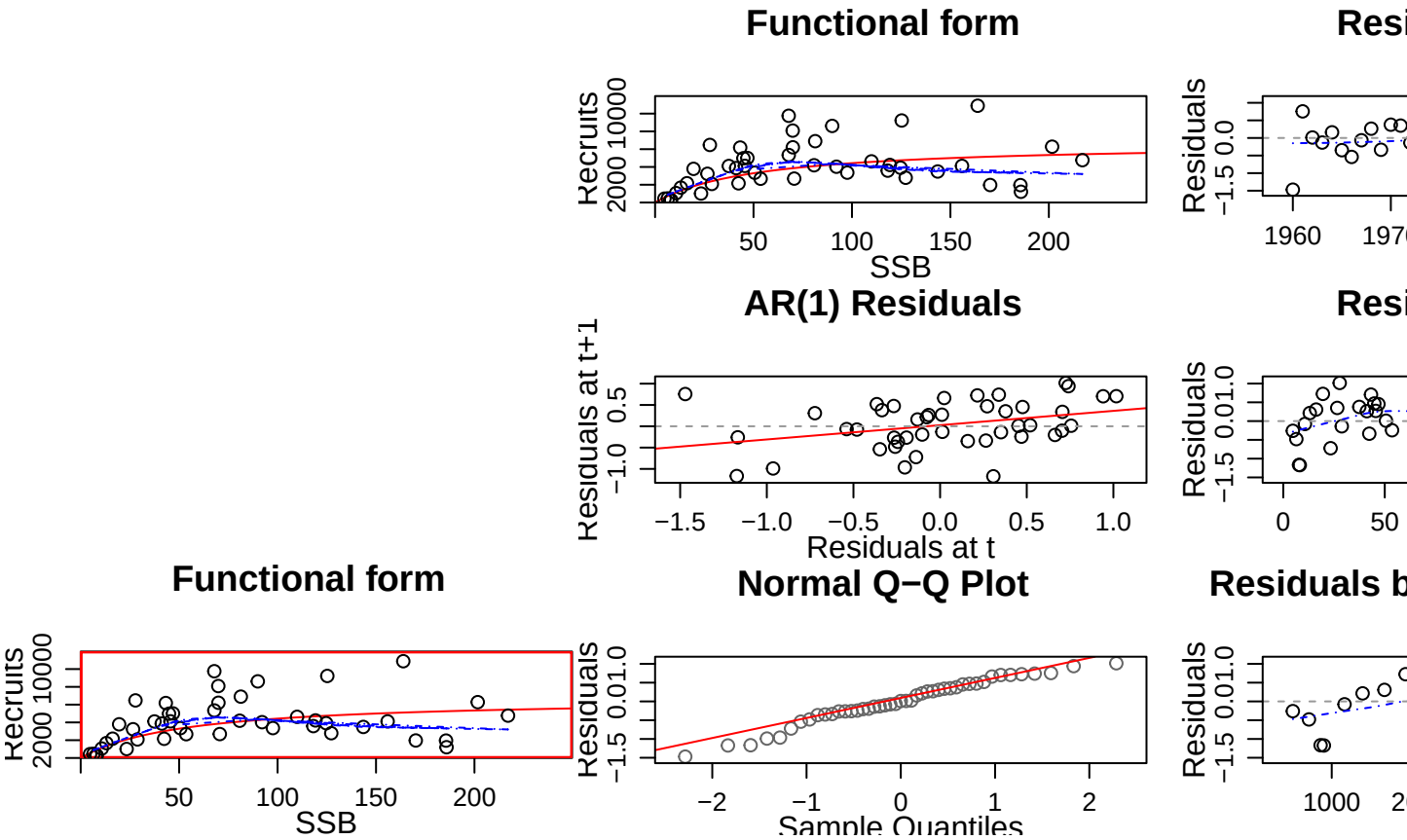
```
## function value for initial parameters = -10.336211
##   Scaled convergence tolerance is 1.54022e-07
## Stepsize computed as 501.110000
## BUILD              3 44.842344 -11.603908
## HI-REDUCTION       5 31.685209 -11.603908
## HI-REDUCTION       7 17.913114 -11.603908
## HI-REDUCTION       9 5.415279 -11.603908
## HI-REDUCTION      11 -3.412974 -11.603908
## HI-REDUCTION      13 -8.018030 -11.603908
## LO-REDUCTION      15 -10.336211 -11.603908
## LO-REDUCTION      17 -11.081040 -11.603908
## EXTENSION         19 -11.295930 -12.061705
## LO-REDUCTION      21 -11.603908 -12.061705
## REFLECTION        23 -11.813826 -12.087620
## REFLECTION        25 -12.061705 -12.199591
## LO-REDUCTION      27 -12.087620 -12.199591
## LO-REDUCTION      29 -12.158184 -12.199591
## LO-REDUCTION      31 -12.191726 -12.199591
## HI-REDUCTION      33 -12.192269 -12.199591
## HI-REDUCTION      35 -12.197784 -12.199591
## LO-REDUCTION      37 -12.198015 -12.199591
## HI-REDUCTION      39 -12.199555 -12.199776
## REFLECTION        41 -12.199591 -12.200058
## HI-REDUCTION      43 -12.199776 -12.200092
## HI-REDUCTION      45 -12.200058 -12.200142
## HI-REDUCTION      47 -12.200092 -12.200155
## HI-REDUCTION      49 -12.200142 -12.200160
## HI-REDUCTION      51 -12.200155 -12.200177
## HI-REDUCTION      53 -12.200160 -12.200177
## LO-REDUCTION      55 -12.200171 -12.200179
## HI-REDUCTION      57 -12.200177 -12.200179
## HI-REDUCTION      59 -12.200178 -12.200179
## HI-REDUCTION      61 -12.200179 -12.200179
## HI-REDUCTION      63 -12.200179 -12.200179
## HI-REDUCTION      65 -12.200179 -12.200179
## Exiting from Nelder Mead minimizer
##     67 function evaluations used
```

and compare the fits

```
plot(nhri)
```


Functional form
Recruits
2000 10000
50
100
150
200
SSB
Resi
Residuals
−1.0 0.5
1960
197
AR(1) Residuals
Residuals at t+1
−1.00.0 1.0
−1.0
−0.5
0.0
0.5
1.0
Residuals at t
Resi
Residuals
−1.00.0 1.0
0
50
Functional form
Recruits
2000 10000
50
100
150
200
SSB
Normal Q−Q Plot
Residuals
−1.00.0 1.0
−2
−1
0
1
2
Sample Quantiles
Residuals b
Residuals
−1.00.0 1.0
1000

```
plot(nhbh)
```



... using AIC

```
AIC(nhri)
```

```
## [1] -27.72
```

```
AIC(nhbh)
```

```
## [1] -20.4
```

or Schwarz's Bayesian Information Criterion

```
BIC(nhri)
```

```
## [1] -24.11
```

```
BIC(nhbh)
```

```
## [1] -16.79
```

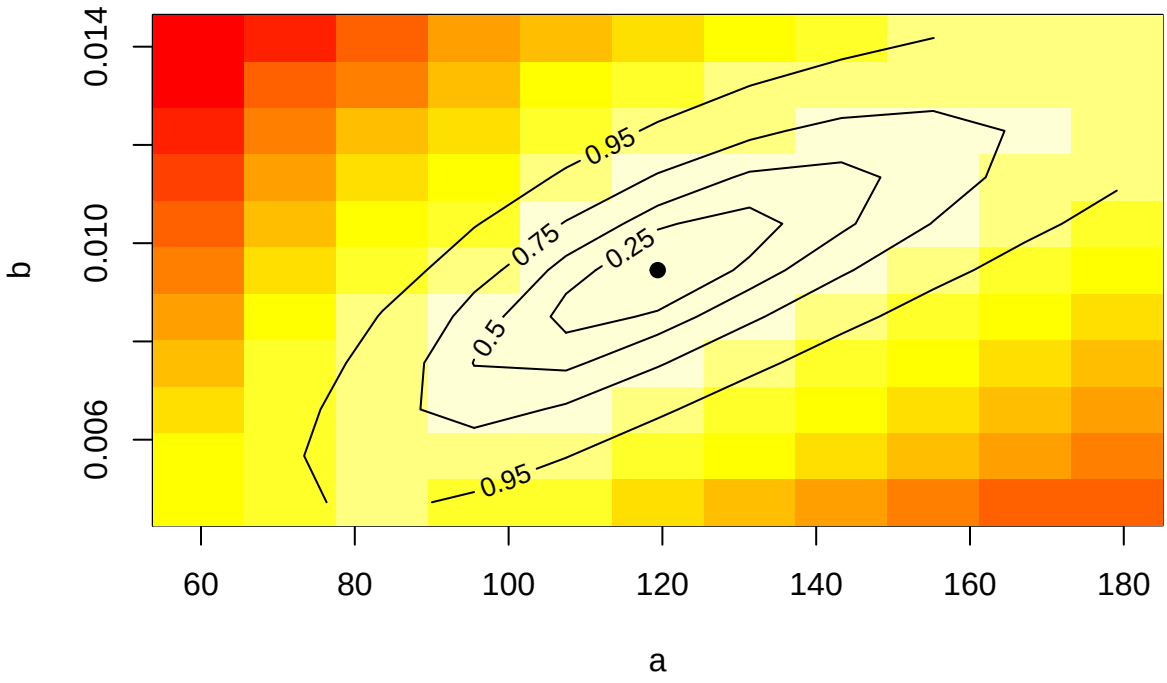
Predict recruitments using predict()

```
predict(nhri, ssb=FLQuant(seq(185, 500, length=10)))
```

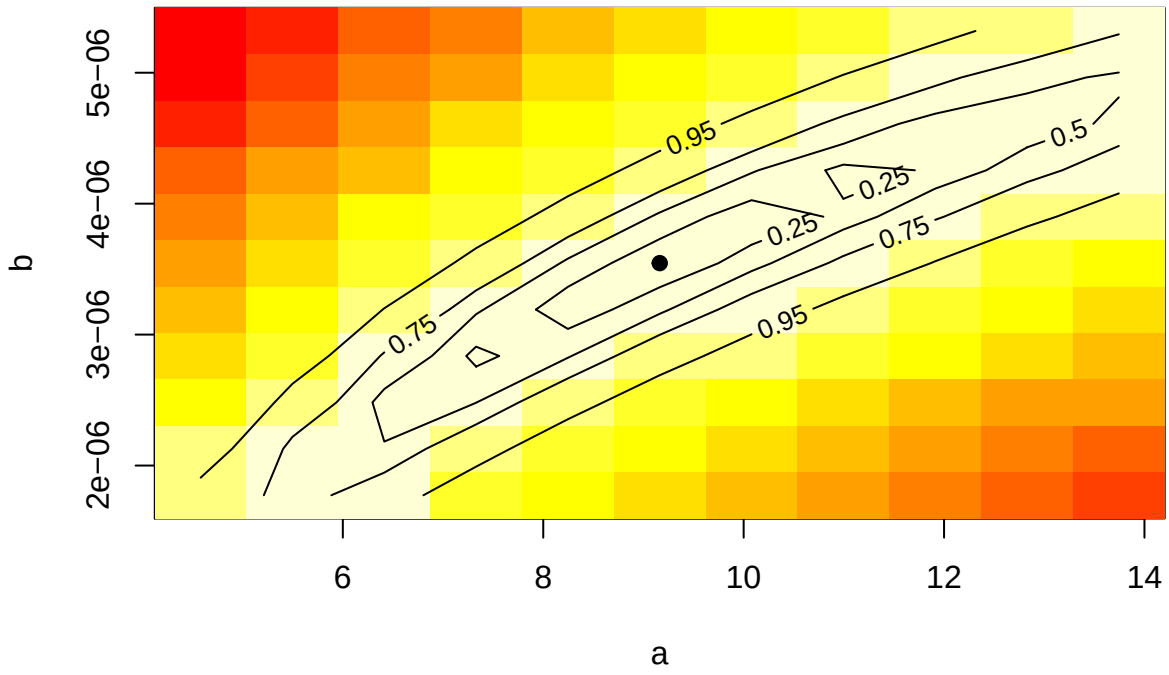
```
## An object of class "FLQuant"
## , , unit = unique, season = all, area = unique
##
##      year
## quant 1       2       3       4       5       6       7       8
##   all 3844.23 3283.99 2734.39 2233.88 1798.40 1431.02 1127.93  882.05
##      year
## quant 9       10
##   all  685.20  529.27
##
## units:  NA
```

Profile the likelihood to check the fit

```
profile(nhri)
```




```
profile(p4sr)
```



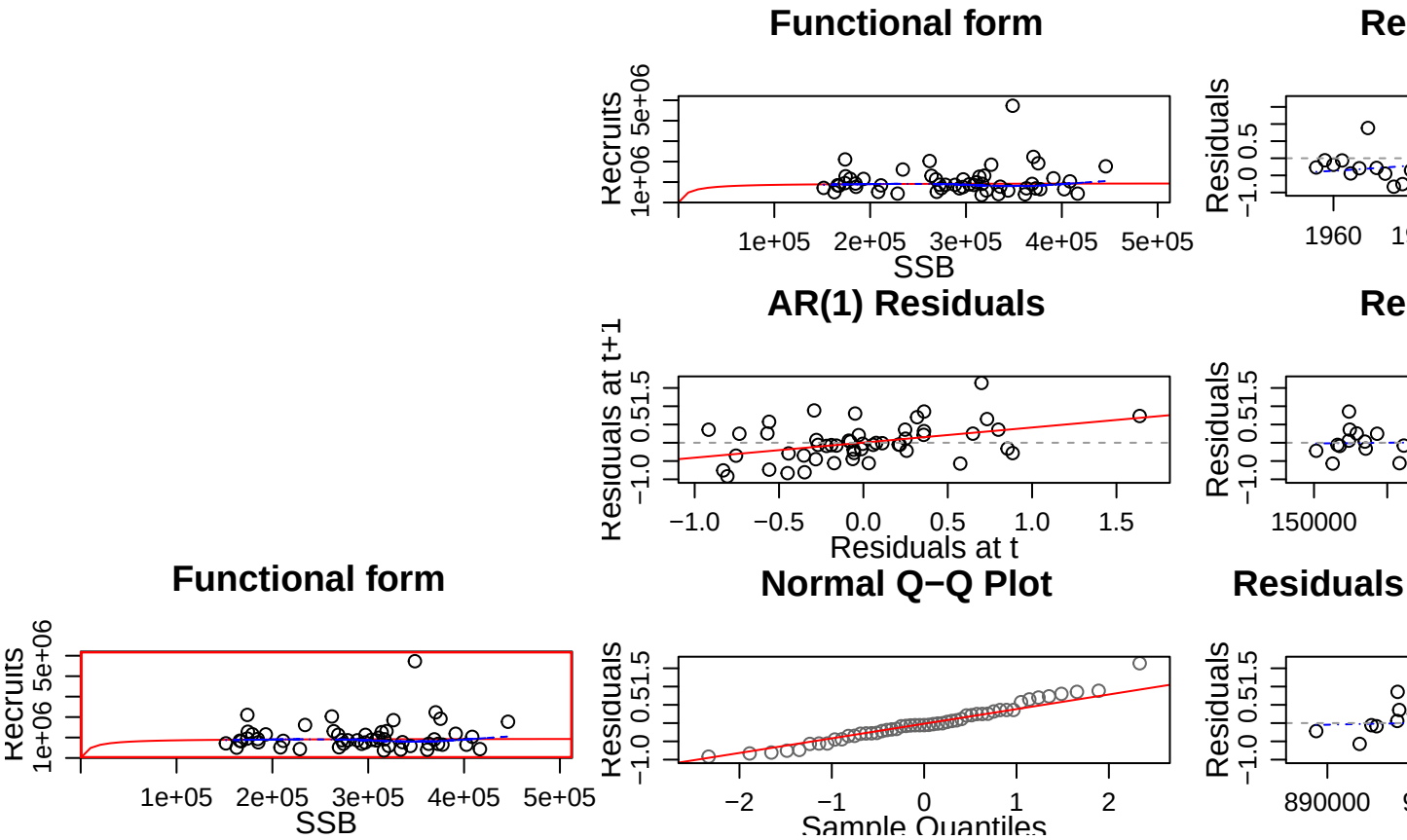
Fix some parameters, e.g. steepness

```
model(p4sr) <- bevholtSV

p4sr <- fmle(p4sr, fixed = list(s = 0.8))
```

```
##   Nelder-Mead direct search function minimizer
## function value for initial parameters = -21.555680
##   Scaled convergence tolerance is 3.21205e-07
## Stepsize computed as 57698.729975
## BUILD              3 138.239783 -21.555680
## HI-REDUCTION       5 134.971658 -21.555680
## HI-REDUCTION       7 131.535838 -21.555680
## HI-REDUCTION       9 127.881431 -21.555680
## HI-REDUCTION      11 123.961309 -21.555680
## HI-REDUCTION      13 119.724583 -21.555680
## HI-REDUCTION      15 115.111062 -21.555680
## HI-REDUCTION      17 110.045701 -21.555680
## HI-REDUCTION      19 104.431766 -21.555680
## HI-REDUCTION      21 98.141556 -21.555680
## HI-REDUCTION      23 91.003478 -21.555680
## HI-REDUCTION      25 82.784736 -21.555680
## HI-REDUCTION      27 73.171495 -21.555680
## HI-REDUCTION      29 61.758606 -21.555680
## HI-REDUCTION      31 48.093801 -21.555680
## HI-REDUCTION      33 31.909759 -21.555680
## HI-REDUCTION      35 13.835912 -21.555680
## HI-REDUCTION      37 -3.319845 -21.555680
## HI-REDUCTION      39 -15.017575 -21.555680
## HI-REDUCTION      41 -20.031655 -21.555680
## HI-REDUCTION      43 -20.760888 -21.555680
## HI-REDUCTION      45 -21.372072 -21.555680
```

```
## HI-REDUCTION      47 -21.422166 -21.555680
## HI-REDUCTION      49 -21.523998 -21.555680
## HI-REDUCTION      51 -21.537566 -21.555680
## HI-REDUCTION      53 -21.548170 -21.555680
## REFLECTION        55 -21.552329 -21.557356
## EXTENSION         57 -21.555680 -21.564655
## EXTENSION         59 -21.557356 -21.569964
## EXTENSION         61 -21.564655 -21.594889
## LO-REDUCTION      63 -21.569964 -21.594889
## EXTENSION         65 -21.590236 -21.626546
## LO-REDUCTION      67 -21.594889 -21.626546
## LO-REDUCTION      69 -21.605773 -21.633123
## HI-REDUCTION      71 -21.620052 -21.633123
## LO-REDUCTION      73 -21.626546 -21.633123
## LO-REDUCTION      75 -21.631055 -21.633123
## EXTENSION         77 -21.632799 -21.636830
## LO-REDUCTION      79 -21.633123 -21.636830
## REFLECTION        81 -21.635481 -21.637874
## HI-REDUCTION      83 -21.636830 -21.637874
## EXTENSION         85 -21.637041 -21.639180
## HI-REDUCTION      87 -21.637874 -21.639180
## LO-REDUCTION      89 -21.638130 -21.639180
## EXTENSION         91 -21.638504 -21.639587
## REFLECTION        93 -21.639180 -21.639887
## LO-REDUCTION      95 -21.639587 -21.640005
## HI-REDUCTION      97 -21.639887 -21.640005
## HI-REDUCTION      99 -21.639933 -21.640005
## HI-REDUCTION     101 -21.639993 -21.640005
## LO-REDUCTION     103 -21.640003 -21.640015
## HI-REDUCTION     105 -21.640005 -21.640017
## HI-REDUCTION     107 -21.640015 -21.640017
## LO-REDUCTION     109 -21.640017 -21.640018
## HI-REDUCTION     111 -21.640017 -21.640019
## Exiting from Nelder Mead minimizer
##     113 function evaluations used
```

```
plot(p4sr)
```




```
params(p4sr)
```

```
## An object of class "FLPar"
## params
##          s          v       spr0
## 8.0000e-01 1.4400e+05 1.6247e-01
## units:  NA
```

Other outputs include the covariance matrix

```
vcov(nhri)
```

```
## , , iter = 1
##
##
##              a         b
##   a 255.33882 1.809e-02
##   b   0.01809 1.993e-06
```

from which we can derive a correlation matrix

```
cov2cor(vcov(nhri)[,,1])
```

```
##
```

```
##          a      b
##   a 1.0000 0.8019
##   b 0.8019 1.0000
```

Bootstrapping

```
niter <- 10

model(p4sr) <- bevholt

p4sr <- fmle(p4sr)
```

```
##   Nelder-Mead direct search function minimizer
## function value for initial parameters = -14.431620
##   Scaled convergence tolerance is 2.15048e-07
## Stepsize computed as 117153.724404
## BUILD              3 -3.320691 -17.802770
## REFLECTION         5 -14.431620 -19.728903
## LO-REDUCTION       7 -17.802770 -20.347066
## HI-REDUCTION       9 -19.728903 -20.713125
## LO-REDUCTION      11 -20.347066 -20.761206
## HI-REDUCTION      13 -20.713125 -20.909622
## LO-REDUCTION      15 -20.761206 -20.909622
## HI-REDUCTION      17 -20.863918 -20.909622
## EXTENSION         19 -20.881239 -20.969985
## EXTENSION         21 -20.909622 -21.033815
## EXTENSION         23 -20.969985 -21.229026
## REFLECTION        25 -21.033815 -21.288651
## REFLECTION        27 -21.229026 -21.448177
## REFLECTION        29 -21.288651 -21.474796
## REFLECTION        31 -21.448177 -21.576403
## LO-REDUCTION      33 -21.474796 -21.597660
## HI-REDUCTION      35 -21.576403 -21.597660
## HI-REDUCTION      37 -21.586625 -21.626562
## REFLECTION        39 -21.597660 -21.639925
## HI-REDUCTION      41 -21.626562 -21.639925
## HI-REDUCTION      43 -21.627760 -21.639925
## LO-REDUCTION      45 -21.634753 -21.639925
## LO-REDUCTION      47 -21.638490 -21.639925
## HI-REDUCTION      49 -21.639074 -21.639925
## LO-REDUCTION      51 -21.639617 -21.639925
## HI-REDUCTION      53 -21.639825 -21.639946
## LO-REDUCTION      55 -21.639925 -21.640005
## HI-REDUCTION      57 -21.639946 -21.640008
## HI-REDUCTION      59 -21.640005 -21.640011
## HI-REDUCTION      61 -21.640008 -21.640016
## HI-REDUCTION      63 -21.640011 -21.640017
## HI-REDUCTION      65 -21.640016 -21.640018
## HI-REDUCTION      67 -21.640017 -21.640018
## HI-REDUCTION      69 -21.640018 -21.640019
## HI-REDUCTION      71 -21.640018 -21.640019
## LO-REDUCTION      73 -21.640019 -21.640019
## Exiting from Nelder Mead minimizer
##     75 function evaluations used
```

```
res.boot <- sample(c(residuals(p4sr)), niter * dims(p4sr)$year, replace=T)

p4sb <- propagate(p4sr, niter)

rec(p4sb) <- rec(p4sb) * exp(res.boot)

p4sb <- fmle(p4sb)
```

```
##   Nelder-Mead direct search function minimizer
## function value for initial parameters = -0.831891
##   Scaled convergence tolerance is 1.23961e-08
## Stepsize computed as 133099.592295
## BUILD              3 4.870909 -1.722579
## LO-REDUCTION       5 -0.831891 -1.722579
## HI-REDUCTION       7 -1.654253 -1.722579
## HI-REDUCTION       9 -1.677906 -1.853891
## HI-REDUCTION      11 -1.722579 -1.910093
## LO-REDUCTION      13 -1.853891 -1.919050
## HI-REDUCTION      15 -1.910093 -1.919050
## REFLECTION        17 -1.910187 -1.923050
## REFLECTION        19 -1.919050 -1.926593
## REFLECTION        21 -1.923050 -1.934913
## LO-REDUCTION      23 -1.926593 -1.937534
## LO-REDUCTION      25 -1.934913 -1.940025
## LO-REDUCTION      27 -1.937534 -1.941648
## LO-REDUCTION      29 -1.940025 -1.942336
## LO-REDUCTION      31 -1.941648 -1.943290
## LO-REDUCTION      33 -1.942336 -1.943408
## LO-REDUCTION      35 -1.943290 -1.943972
## HI-REDUCTION      37 -1.943408 -1.944062
## HI-REDUCTION      39 -1.943972 -1.944215
## HI-REDUCTION      41 -1.944062 -1.944215
## REFLECTION        43 -1.944168 -1.944285
## HI-REDUCTION      45 -1.944215 -1.944291
## EXTENSION         47 -1.944285 -1.944416
## HI-REDUCTION      49 -1.944291 -1.944416
## EXTENSION         51 -1.944350 -1.944600
## EXTENSION         53 -1.944416 -1.944713
## EXTENSION         55 -1.944600 -1.945228
## LO-REDUCTION      57 -1.944713 -1.945228
## EXTENSION         59 -1.945157 -1.946210
## EXTENSION         61 -1.945228 -1.946666
## EXTENSION         63 -1.946210 -1.949024
## EXTENSION         65 -1.946666 -1.950208
## EXTENSION         67 -1.949024 -1.955631
## LO-REDUCTION      69 -1.950208 -1.955631
## EXTENSION         71 -1.955103 -1.963638
## EXTENSION         73 -1.955631 -1.967953
## REFLECTION        75 -1.963638 -1.971487
## REFLECTION        77 -1.967953 -1.972537
## LO-REDUCTION      79 -1.971487 -1.972537
## HI-REDUCTION      81 -1.972406 -1.972664
## HI-REDUCTION      83 -1.972537 -1.972836
```

```
## HI-REDUCTION      85 -1.972664 -1.972881
## HI-REDUCTION      87 -1.972836 -1.972881
## HI-REDUCTION      89 -1.972853 -1.972891
## HI-REDUCTION      91 -1.972881 -1.972897
## HI-REDUCTION      93 -1.972891 -1.972906
## LO-REDUCTION      95 -1.972897 -1.972906
## HI-REDUCTION      97 -1.972902 -1.972906
## REFLECTION        99 -1.972904 -1.972906
## HI-REDUCTION     101 -1.972906 -1.972906
## HI-REDUCTION     103 -1.972906 -1.972906
## HI-REDUCTION     105 -1.972906 -1.972906
## HI-REDUCTION     107 -1.972906 -1.972906
## HI-REDUCTION     109 -1.972906 -1.972906
## HI-REDUCTION     111 -1.972906 -1.972906
## HI-REDUCTION     113 -1.972906 -1.972906
## Exiting from Nelder Mead minimizer
##     115 function evaluations used
##   Nelder-Mead direct search function minimizer
## function value for initial parameters = -13.547202
##   Scaled convergence tolerance is 2.01869e-07
## Stepsize computed as 133099.592295
## BUILD              3 -3.688776 -15.736196
## LO-REDUCTION       5 -13.547202 -16.478542
## LO-REDUCTION       7 -15.736196 -16.478542
## HI-REDUCTION       9 -16.436705 -16.478542
## HI-REDUCTION      11 -16.477038 -16.543592
## HI-REDUCTION      13 -16.478542 -16.572664
## LO-REDUCTION      15 -16.543592 -16.576352
## HI-REDUCTION      17 -16.571674 -16.576352
## HI-REDUCTION      19 -16.572664 -16.576352
## REFLECTION        21 -16.576297 -16.578494
## LO-REDUCTION      23 -16.576352 -16.578494
## LO-REDUCTION      25 -16.578448 -16.578929
## HI-REDUCTION      27 -16.578494 -16.578976
## REFLECTION        29 -16.578929 -16.579238
## HI-REDUCTION      31 -16.578976 -16.579238
## REFLECTION        33 -16.579127 -16.579313
## LO-REDUCTION      35 -16.579238 -16.579319
## LO-REDUCTION      37 -16.579313 -16.579332
## LO-REDUCTION      39 -16.579319 -16.579332
## LO-REDUCTION      41 -16.579331 -16.579334
## HI-REDUCTION      43 -16.579332 -16.579338
## LO-REDUCTION      45 -16.579334 -16.579338
## HI-REDUCTION      47 -16.579337 -16.579338
## HI-REDUCTION      49 -16.579337 -16.579338
## REFLECTION        51 -16.579338 -16.579338
## Exiting from Nelder Mead minimizer
##     53 function evaluations used
##   Nelder-Mead direct search function minimizer
## function value for initial parameters = -2.210507
##   Scaled convergence tolerance is 3.29391e-08
## Stepsize computed as 133099.592295
## BUILD              3 3.622481 -3.105648
## LO-REDUCTION       5 -2.210507 -3.105648
```

```
## HI-REDUCTION       7 -2.923991 -3.105648
## HI-REDUCTION       9 -3.026367 -3.245042
## LO-REDUCTION      11 -3.105648 -3.245042
## HI-REDUCTION      13 -3.212251 -3.245042
## HI-REDUCTION      15 -3.238277 -3.245042
## HI-REDUCTION      17 -3.240960 -3.245042
## REFLECTION        19 -3.243964 -3.248898
## HI-REDUCTION      21 -3.245042 -3.248898
## REFLECTION        23 -3.247286 -3.249763
## EXTENSION         25 -3.248898 -3.251535
## REFLECTION        27 -3.249763 -3.252009
## HI-REDUCTION      29 -3.251535 -3.252009
## HI-REDUCTION      31 -3.251719 -3.252301
## LO-REDUCTION      33 -3.252009 -3.252368
## HI-REDUCTION      35 -3.252301 -3.252368
## HI-REDUCTION      37 -3.252337 -3.252426
## LO-REDUCTION      39 -3.252368 -3.252430
## HI-REDUCTION      41 -3.252421 -3.252430
## HI-REDUCTION      43 -3.252426 -3.252433
## HI-REDUCTION      45 -3.252430 -3.252436
## HI-REDUCTION      47 -3.252433 -3.252436
## LO-REDUCTION      49 -3.252435 -3.252437
## HI-REDUCTION      51 -3.252436 -3.252437
## HI-REDUCTION      53 -3.252436 -3.252437
## HI-REDUCTION      55 -3.252437 -3.252437
## HI-REDUCTION      57 -3.252437 -3.252437
## Exiting from Nelder Mead minimizer
##     59 function evaluations used
##   Nelder-Mead direct search function minimizer
## function value for initial parameters = -0.037728
##   Scaled convergence tolerance is 5.62188e-10
## Stepsize computed as 133099.592295
## BUILD              3 5.782136 -1.033719
## LO-REDUCTION       5 -0.037728 -1.173865
## LO-REDUCTION       7 -1.033719 -1.173865
## HI-REDUCTION       9 -1.095001 -1.299348
## REFLECTION        11 -1.173865 -1.308018
## HI-REDUCTION      13 -1.299348 -1.308018
## REFLECTION        15 -1.302060 -1.340353
## HI-REDUCTION      17 -1.308018 -1.340353
## HI-REDUCTION      19 -1.327324 -1.340353
## EXTENSION         21 -1.331207 -1.346699
## REFLECTION        23 -1.340353 -1.358520
## LO-REDUCTION      25 -1.346699 -1.358520
## REFLECTION        27 -1.355308 -1.360269
## HI-REDUCTION      29 -1.358520 -1.360962
## REFLECTION        31 -1.360269 -1.361987
## HI-REDUCTION      33 -1.360962 -1.362153
## HI-REDUCTION      35 -1.361987 -1.362285
## HI-REDUCTION      37 -1.362153 -1.362438
## HI-REDUCTION      39 -1.362285 -1.362438
## HI-REDUCTION      41 -1.362413 -1.362454
## LO-REDUCTION      43 -1.362438 -1.362469
## HI-REDUCTION      45 -1.362454 -1.362479
```

```
## HI-REDUCTION      47 -1.362469 -1.362482
## LO-REDUCTION      49 -1.362479 -1.362485
## HI-REDUCTION      51 -1.362482 -1.362485
## HI-REDUCTION      53 -1.362485 -1.362485
## HI-REDUCTION      55 -1.362485 -1.362486
## HI-REDUCTION      57 -1.362485 -1.362486
## HI-REDUCTION      59 -1.362486 -1.362486
## HI-REDUCTION      61 -1.362486 -1.362486
## HI-REDUCTION      63 -1.362486 -1.362486
## HI-REDUCTION      65 -1.362486 -1.362486
## LO-REDUCTION      67 -1.362486 -1.362486
## HI-REDUCTION      69 -1.362486 -1.362486
## LO-REDUCTION      71 -1.362486 -1.362486
## HI-REDUCTION      73 -1.362486 -1.362486
## HI-REDUCTION      75 -1.362486 -1.362486
## Exiting from Nelder Mead minimizer
##     77 function evaluations used
##   Nelder-Mead direct search function minimizer
## function value for initial parameters = -1.947264
##   Scaled convergence tolerance is 2.90165e-08
## Stepsize computed as 133099.592295
## BUILD              3 4.812219 -3.289008
## LO-REDUCTION       5 -1.947264 -3.835560
## LO-REDUCTION       7 -3.289008 -3.835560
## HI-REDUCTION       9 -3.771681 -3.835560
## HI-REDUCTION      11 -3.775416 -3.840389
## HI-REDUCTION      13 -3.835560 -3.853578
## HI-REDUCTION      15 -3.840389 -3.853645
## REFLECTION        17 -3.853578 -3.865625
## REFLECTION        19 -3.853645 -3.865980
## REFLECTION        21 -3.865625 -3.876719
## REFLECTION        23 -3.865980 -3.877250
## REFLECTION        25 -3.876719 -3.886722
## REFLECTION        27 -3.877250 -3.887290
## REFLECTION        29 -3.886722 -3.895475
## REFLECTION        31 -3.887290 -3.895916
## REFLECTION        33 -3.895475 -3.902798
## REFLECTION        35 -3.895916 -3.902915
## REFLECTION        37 -3.902798 -3.908490
## LO-REDUCTION      39 -3.902915 -3.914513
## HI-REDUCTION      41 -3.908490 -3.914513
## HI-REDUCTION      43 -3.913867 -3.916414
## REFLECTION        45 -3.914513 -3.918543
## HI-REDUCTION      47 -3.916414 -3.918543
## LO-REDUCTION      49 -3.917314 -3.918543
## EXTENSION         51 -3.918439 -3.919743
## HI-REDUCTION      53 -3.918543 -3.919743
## EXTENSION         55 -3.919201 -3.921422
## LO-REDUCTION      57 -3.919743 -3.921422
## EXTENSION         59 -3.920714 -3.922722
## EXTENSION         61 -3.921422 -3.923850
## LO-REDUCTION      63 -3.922722 -3.923850
## LO-REDUCTION      65 -3.923557 -3.923850
## LO-REDUCTION      67 -3.923731 -3.923850
```

```
## HI-REDUCTION      69 -3.923843 -3.923850
## HI-REDUCTION      71 -3.923844 -3.923870
## HI-REDUCTION      73 -3.923850 -3.923871
## HI-REDUCTION      75 -3.923870 -3.923874
## HI-REDUCTION      77 -3.923871 -3.923875
## LO-REDUCTION      79 -3.923874 -3.923875
## HI-REDUCTION      81 -3.923875 -3.923876
## HI-REDUCTION      83 -3.923875 -3.923876
## LO-REDUCTION      85 -3.923876 -3.923876
## HI-REDUCTION      87 -3.923876 -3.923876
## LO-REDUCTION      89 -3.923876 -3.923876
## Exiting from Nelder Mead minimizer
##     91 function evaluations used
##   Nelder-Mead direct search function minimizer
## function value for initial parameters = 7.309599
##   Scaled convergence tolerance is 1.08922e-07
## Stepsize computed as 133099.592295
## BUILD              3 13.846930 5.711943
## REFLECTION         5 7.309599 4.005974
## LO-REDUCTION       7 5.711943 3.852642
## HI-REDUCTION       9 4.011695 3.852642
## HI-REDUCTION      11 4.005974 3.700266
## LO-REDUCTION      13 3.852642 3.700266
## HI-REDUCTION      15 3.792808 3.700266
## REFLECTION        17 3.760379 3.685979
## EXTENSION         19 3.700266 3.542770
## EXTENSION         21 3.685979 3.462920
## EXTENSION         23 3.542770 3.048340
## REFLECTION        25 3.462920 3.010221
## EXTENSION         27 3.048340 2.418697
## LO-REDUCTION      29 3.010221 2.418697
## HI-REDUCTION      31 2.577476 2.418697
## HI-REDUCTION      33 2.459831 2.418697
## HI-REDUCTION      35 2.425515 2.382023
## HI-REDUCTION      37 2.418697 2.374329
## HI-REDUCTION      39 2.382023 2.364068
## HI-REDUCTION      41 2.374329 2.364068
## LO-REDUCTION      43 2.367653 2.360575
## HI-REDUCTION      45 2.364068 2.360575
## LO-REDUCTION      47 2.360910 2.360575
## LO-REDUCTION      49 2.360594 2.360063
## HI-REDUCTION      51 2.360575 2.359934
## LO-REDUCTION      53 2.360063 2.359934
## HI-REDUCTION      55 2.359998 2.359934
## LO-REDUCTION      57 2.359936 2.359925
## HI-REDUCTION      59 2.359934 2.359919
## HI-REDUCTION      61 2.359925 2.359918
## HI-REDUCTION      63 2.359919 2.359918
## HI-REDUCTION      65 2.359918 2.359917
## HI-REDUCTION      67 2.359918 2.359917
## HI-REDUCTION      69 2.359917 2.359917
## HI-REDUCTION      71 2.359917 2.359917
## LO-REDUCTION      73 2.359917 2.359917
## Exiting from Nelder Mead minimizer
```

```
##     75 function evaluations used
##   Nelder-Mead direct search function minimizer
## function value for initial parameters = 1.512538
##   Scaled convergence tolerance is 2.25386e-08
## Stepsize computed as 133099.592295
## BUILD              3 6.140630 1.140290
## LO-REDUCTION       5 1.512538 1.140290
## HI-REDUCTION       7 1.492599 1.064441
## HI-REDUCTION       9 1.140290 1.064441
## EXTENSION         11 1.089455 0.903322
## REFLECTION        13 1.064441 0.816437
## REFLECTION        15 0.903322 0.739192
## REFLECTION        17 0.816437 0.546603
## LO-REDUCTION      19 0.739192 0.529950
## REFLECTION        21 0.546603 0.472926
## REFLECTION        23 0.529950 0.334884
## LO-REDUCTION      25 0.472926 0.334884
## REFLECTION        27 0.344848 0.309179
## HI-REDUCTION      29 0.334884 0.256801
## EXTENSION         31 0.309179 0.120245
## LO-REDUCTION      33 0.256801 0.120245
## EXTENSION         35 0.203472 0.028910
## REFLECTION        37 0.120245 -0.036684
## LO-REDUCTION      39 0.028910 -0.036684
## LO-REDUCTION      41 -0.013923 -0.036684
## HI-REDUCTION      43 -0.036111 -0.036684
## HI-REDUCTION      45 -0.036387 -0.041103
## HI-REDUCTION      47 -0.036684 -0.041801
## HI-REDUCTION      49 -0.041103 -0.041801
## HI-REDUCTION      51 -0.041374 -0.041963
## HI-REDUCTION      53 -0.041801 -0.042137
## HI-REDUCTION      55 -0.041963 -0.042137
## REFLECTION        57 -0.042125 -0.042189
## HI-REDUCTION      59 -0.042137 -0.042229
## HI-REDUCTION      61 -0.042189 -0.042229
## HI-REDUCTION      63 -0.042211 -0.042229
## LO-REDUCTION      65 -0.042226 -0.042233
## HI-REDUCTION      67 -0.042229 -0.042236
## HI-REDUCTION      69 -0.042233 -0.042236
## LO-REDUCTION      71 -0.042234 -0.042236
## HI-REDUCTION      73 -0.042236 -0.042236
## HI-REDUCTION      75 -0.042236 -0.042236
## LO-REDUCTION      77 -0.042236 -0.042236
## HI-REDUCTION      79 -0.042236 -0.042236
## HI-REDUCTION      81 -0.042236 -0.042236
## LO-REDUCTION      83 -0.042236 -0.042236
## Exiting from Nelder Mead minimizer
##     85 function evaluations used
##   Nelder-Mead direct search function minimizer
## function value for initial parameters = -1.882291
##   Scaled convergence tolerance is 2.80483e-08
## Stepsize computed as 133099.592295
## BUILD              3 3.255358 -2.270909
## LO-REDUCTION       5 -1.804931 -2.270909
```

```
## HI-REDUCTION       7 -1.882291 -2.375993
## HI-REDUCTION       9 -2.267491 -2.375993
## HI-REDUCTION      11 -2.270909 -2.375993
## EXTENSION         13 -2.342560 -2.553365
## HI-REDUCTION      15 -2.375993 -2.553365
## EXTENSION         17 -2.436915 -2.771703
## EXTENSION         19 -2.553365 -2.962433
## EXTENSION         21 -2.771703 -3.333399
## HI-REDUCTION      23 -2.962433 -3.333399
## REFLECTION        25 -3.110784 -3.480779
## HI-REDUCTION      27 -3.333399 -3.480779
## HI-REDUCTION      29 -3.385092 -3.480779
## HI-REDUCTION      31 -3.465729 -3.480779
## HI-REDUCTION      33 -3.470150 -3.489022
## HI-REDUCTION      35 -3.480779 -3.493491
## HI-REDUCTION      37 -3.489022 -3.496102
## LO-REDUCTION      39 -3.493491 -3.497350
## HI-REDUCTION      41 -3.496102 -3.497350
## HI-REDUCTION      43 -3.497139 -3.497427
## HI-REDUCTION      45 -3.497350 -3.497615
## HI-REDUCTION      47 -3.497427 -3.497615
## HI-REDUCTION      49 -3.497594 -3.497615
## HI-REDUCTION      51 -3.497609 -3.497643
## HI-REDUCTION      53 -3.497615 -3.497648
## HI-REDUCTION      55 -3.497643 -3.497650
## HI-REDUCTION      57 -3.497648 -3.497651
## HI-REDUCTION      59 -3.497650 -3.497653
## HI-REDUCTION      61 -3.497651 -3.497653
## LO-REDUCTION      63 -3.497653 -3.497653
## HI-REDUCTION      65 -3.497653 -3.497653
## HI-REDUCTION      67 -3.497653 -3.497653
## HI-REDUCTION      69 -3.497653 -3.497654
## HI-REDUCTION      71 -3.497653 -3.497654
## LO-REDUCTION      73 -3.497654 -3.497654
## Exiting from Nelder Mead minimizer
##     75 function evaluations used
##   Nelder-Mead direct search function minimizer
## function value for initial parameters = 2.162382
##   Scaled convergence tolerance is 3.2222e-08
## Stepsize computed as 133099.592295
## BUILD              3 11.204690 -0.638906
## REFLECTION         5 2.162382 -5.665541
## LO-REDUCTION       7 -0.638906 -5.665541
## HI-REDUCTION       9 -4.063061 -5.665541
## HI-REDUCTION      11 -5.185693 -5.665541
## LO-REDUCTION      13 -5.551541 -5.665541
## HI-REDUCTION      15 -5.558860 -5.665541
## LO-REDUCTION      17 -5.638888 -5.673454
## EXTENSION         19 -5.665541 -5.716925
## EXTENSION         21 -5.673454 -5.775224
## REFLECTION        23 -5.716925 -5.810233
## REFLECTION        25 -5.775224 -5.858372
## REFLECTION        27 -5.810233 -5.872387
## REFLECTION        29 -5.858372 -5.911380
```

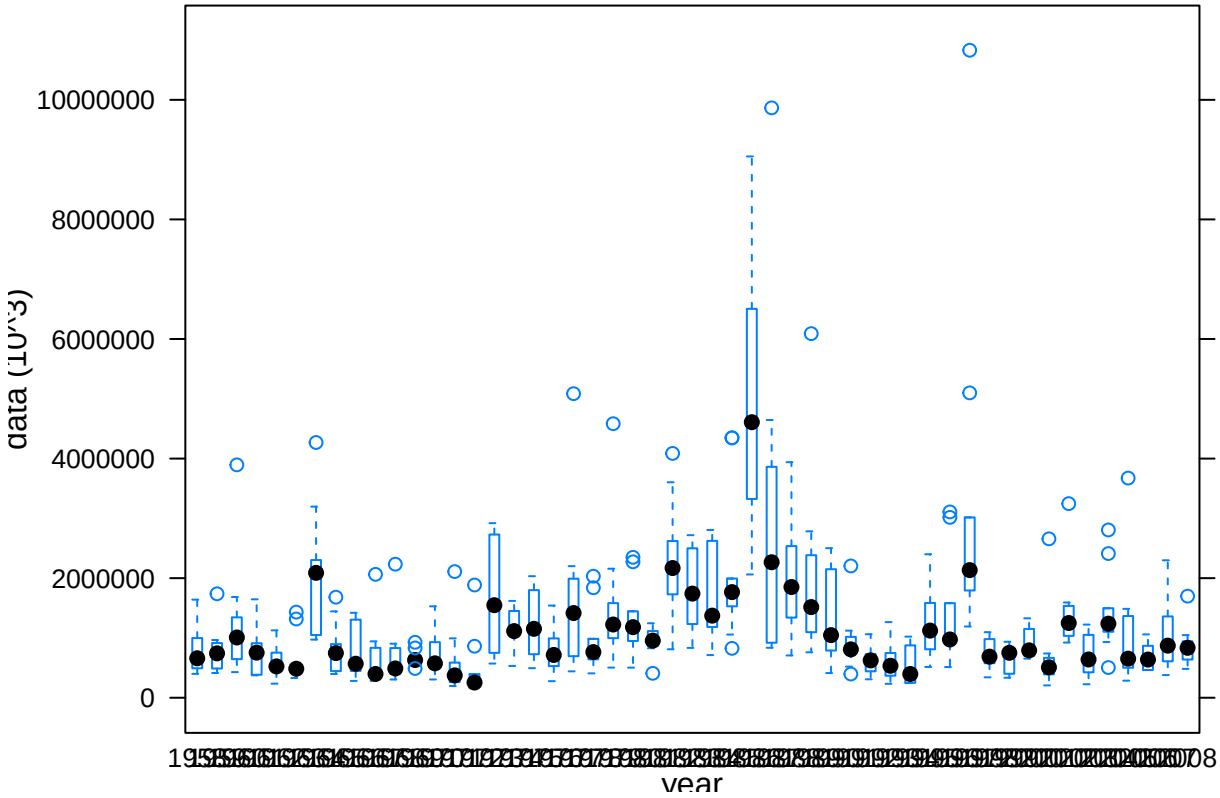
```
## LO-REDUCTION      31 -5.872387 -5.918749
## LO-REDUCTION      33 -5.911380 -5.925080
## LO-REDUCTION      35 -5.918749 -5.931686
## HI-REDUCTION      37 -5.925080 -5.933799
## HI-REDUCTION      39 -5.931686 -5.937241
## LO-REDUCTION      41 -5.933799 -5.937241
## HI-REDUCTION      43 -5.936322 -5.937241
## EXTENSION         45 -5.936736 -5.938016
## REFLECTION        47 -5.937241 -5.938734
## LO-REDUCTION      49 -5.938016 -5.938781
## LO-REDUCTION      51 -5.938734 -5.939067
## HI-REDUCTION      53 -5.938781 -5.939067
## HI-REDUCTION      55 -5.939048 -5.939109
## EXTENSION         57 -5.939067 -5.939228
## HI-REDUCTION      59 -5.939109 -5.939228
## LO-REDUCTION      61 -5.939184 -5.939238
## LO-REDUCTION      63 -5.939228 -5.939240
## LO-REDUCTION      65 -5.939238 -5.939245
## HI-REDUCTION      67 -5.939240 -5.939246
## HI-REDUCTION      69 -5.939245 -5.939247
## HI-REDUCTION      71 -5.939246 -5.939247
## LO-REDUCTION      73 -5.939247 -5.939247
## HI-REDUCTION      75 -5.939247 -5.939247
## LO-REDUCTION      77 -5.939247 -5.939247
## HI-REDUCTION      79 -5.939247 -5.939247
## LO-REDUCTION      81 -5.939247 -5.939247
## Exiting from Nelder Mead minimizer
##     83 function evaluations used
##   Nelder-Mead direct search function minimizer
## function value for initial parameters = -3.465494
##   Scaled convergence tolerance is 5.16399e-08
## Stepsize computed as 133099.592295
## BUILD              3 2.925128 -4.523474
## LO-REDUCTION       5 -3.465494 -4.575127
## LO-REDUCTION       7 -4.473094 -4.575127
## HI-REDUCTION       9 -4.523474 -4.707008
## HI-REDUCTION      11 -4.575127 -4.767325
## LO-REDUCTION      13 -4.707008 -4.767325
## HI-REDUCTION      15 -4.750826 -4.767325
## REFLECTION        17 -4.762170 -4.773276
## REFLECTION        19 -4.767325 -4.783045
## LO-REDUCTION      21 -4.773276 -4.786604
## LO-REDUCTION      23 -4.783045 -4.789605
## LO-REDUCTION      25 -4.786604 -4.791589
## LO-REDUCTION      27 -4.789605 -4.792447
## LO-REDUCTION      29 -4.791589 -4.793544
## LO-REDUCTION      31 -4.792447 -4.793714
## LO-REDUCTION      33 -4.793544 -4.794339
## HI-REDUCTION      35 -4.793714 -4.794371
## HI-REDUCTION      37 -4.794339 -4.794548
## HI-REDUCTION      39 -4.794371 -4.794548
## REFLECTION        41 -4.794515 -4.794655
## HI-REDUCTION      43 -4.794548 -4.794655
## EXTENSION         45 -4.794642 -4.794831
```

```
## EXTENSION         47 -4.794655 -4.794994
## EXTENSION         49 -4.794831 -4.795337
## EXTENSION         51 -4.794994 -4.796003
## EXTENSION         53 -4.795337 -4.796611
## EXTENSION         55 -4.796003 -4.798817
## EXTENSION         57 -4.796611 -4.799137
## EXTENSION         59 -4.798817 -4.805195
## LO-REDUCTION      61 -4.799137 -4.805195
## EXTENSION         63 -4.802578 -4.813563
## EXTENSION         65 -4.805195 -4.820095
## EXTENSION         67 -4.813563 -4.838123
## EXTENSION         69 -4.820095 -4.838680
## REFLECTION        71 -4.838123 -4.846981
## HI-REDUCTION      73 -4.838680 -4.846981
## EXTENSION         75 -4.845256 -4.859843
## LO-REDUCTION      77 -4.846981 -4.859843
## REFLECTION        79 -4.855857 -4.861252
## LO-REDUCTION      81 -4.859843 -4.861252
## HI-REDUCTION      83 -4.860610 -4.861335
## LO-REDUCTION      85 -4.861252 -4.861582
## HI-REDUCTION      87 -4.861335 -4.861610
## HI-REDUCTION      89 -4.861582 -4.861610
## HI-REDUCTION      91 -4.861602 -4.861638
## HI-REDUCTION      93 -4.861610 -4.861644
## HI-REDUCTION      95 -4.861638 -4.861644
## HI-REDUCTION      97 -4.861641 -4.861646
## HI-REDUCTION      99 -4.861644 -4.861646
## HI-REDUCTION     101 -4.861646 -4.861647
## LO-REDUCTION     103 -4.861646 -4.861647
## HI-REDUCTION     105 -4.861647 -4.861647
## LO-REDUCTION     107 -4.861647 -4.861647
## HI-REDUCTION     109 -4.861647 -4.861647
## Exiting from Nelder Mead minimizer
##     111 function evaluations used
```

```
params(p4sb)
```

```
## An object of class "FLPar"
## iters:  10
##
## params
##               a               b
## 1190338(163182)   81421(104892)
## units:  NA
```

```
plot(rec(p4sb))
```




```
plot(fitted(p4sb))
```