Large-Scale Optimization for Games and Markets

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Game-theoretic and economic models provide powerful frameworks for reasoning about settings with multiple strategic agents whose interests do not always align. These models are utilized in the real world in high-stakes settings such as security (cybersecurity, airport security, etc.), markets (e.g. auctions), and recreational games (e.g. poker). My work facilitates the use of economic models in modern large-scale settings by developing optimization algorithms, as well as analytical results, both with practical applications and rigorous theoretical foundations. My work lies at the intersection of artificial intelligence, economics, and operations research. I am particularly focused on sequential games and decision making, practical algorithms for markets, and large-scale convex optimization. **My long-term research vision is the development of practical and principled optimization algorithms for adversarial or multiagent settings, and the development of general optimization methods inspired by progress in these areas.** In addition to this overarching goal, I am also interested in the application of my research to real-world problems. For large-scale game solving this includes recreational games such as poker, as well as physical and cybersecurity games. For market algorithms this includes advertisement markets such as those run by large Internet companies, and prediction markets for election or sport outcomes. Finally, I am interested in how data science and game-theoretic reasoning interact with each other. So far, this has manifested itself in ongoing research on predictive analytics for large-scale markets and robust optimization for machine-learning problems.

1 Thesis Research: Theoretical foundations and practical algorithms for solving large-scale sequential games

Game-theoretic equilibrium concepts provide a sound definition of how rational agents should act in multiagent settings. To operationalize them, they must be accompanied by techniques to compute equilibria. In my thesis, I study the computation of equilibria for extensive-form games, a broad game class that can model sequential interaction, imperfect information, and outcome uncertainty. Practical equilibrium computation in extensive-form games relies on two complementary methods: abstraction methods and sparse iterative equilibrium-finding algorithms. These methods are necessary in order to handle the size of many real-world games.

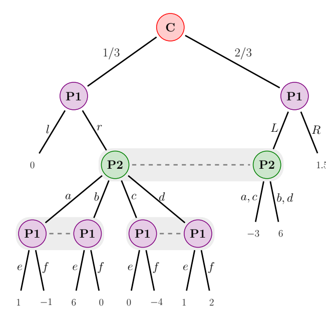

Figure 1: An example of an extensive-form game.

I developed new algorithmic and structural results for both parts of extensive-form game solving: I introduced state-of-the-art theoretical guarantees on the performance of my algorithms [1, 13, 14], and first-of-their-kind guarantees on the solution quality of abstractions for large games [7, 9, 11].

For abstraction, I developed new theoretical guarantees on the solution quality of equilibria computed in abstractions. I developed new results for several types of games and abstractions: discrete and continuous extensive-form games, and perfect and imperfect-recall abstractions. For all settings, my results are the first algorithm-agnostic solution-quality guarantees. Additionally, even compared to algorithm-specific results, my approach leads to exponentially stronger bounds than prior results, and extends to more general games and abstractions. My results are very broadly applicable: they can be used to reason about compact game representations even when numerically solving the extensive-form game representation is undesirable. For example, they have been used to reason about more compact security games. As a professor I plan to further develop the theory and practice of abstraction for practical settings including, but not limited to, security games.

For equilibrium computation, I have primarily focused on the formulation of an extensive-form two-player zero-sum Nash equilibrium as a bilinear saddle-point problem. This makes the problem amenable to methods from the convex optimization literature. I developed a smoothing method based on a dilated entropy function and proved bounds on the strong convexity and polytope diameter associated with this function that are significantly stronger than bounds for prior smoothing methods. This leads to the state-of-the-art in convergence rate for sparse iterative methods for computing a Nash equilibrium. My results can also be viewed more generally as strong convexity results for a class of convex polytopes called treeplexes. One of my papers in this line of work was a runner-up for the INFORMS Computing Society Student Paper Competition. As a professor, I will explore practical applications of zero-sum sequential games such as security games and poker. I will also explore applications of my convex-analytical results beyond games, e.g., in sequential decision making and machine learning.

In addition to my work on Nash equilibrium computation, I also developed the first scalable algorithms for computing Nash equilibrium refinements by adapting convex optimization [6] or reinforcement learning [4] algorithms.

Finally, I developed new solution concepts and associated algorithmic results for special game models with real-world application such as Stackelberg and limited-lookahead extensive-form games [10], jamming games [3], and sequential decision making for biological steering [12].

Convex optimization for solving zero-sum games

Sparse iterative methods, in particular, first-order methods, are known to be among the most effective in solving large-scale two-player zero-sum extensive-form games. The convergence rates of these methods depend heavily on the properties of the distance-generating function that they are based on.

In collaboration with Kevin Waugh (DeepMind), Fatma Kilinc-Karzan (CMU) and Tuomas Sandholm (CMU), I developed a new distance-generating function for extensive-form games and showed new tight state-of-the-art strong convexity bounds for this function [13, 14]. When coupled with an accelerated algorithm for bilinear saddle-point problems, a class of convex optimization problems that generalize zero-sum games, this leads to state-of-the-art convergence rate for sparse iterative methods on extensive-form games. Our strong convexity results are the first, and only, to have only a logarithmic dependence on the branching factor of a player's strategy space, and thus are the first to generalize the logarithmic dependence on dimension for the strong convexity and polytope diameter of the entropy function over the simplex (i.e. for normal-form games). We then showed that our distance-generating function, along with an appropriate first-order method, can also be used in practice to solve games at a significantly faster rate than the previous practical state-of-the-art (which has much worse theoretical convergence rate). In addition to these results, we also developed the first sampling-based approach for solving extensive-form games with first-order methods.

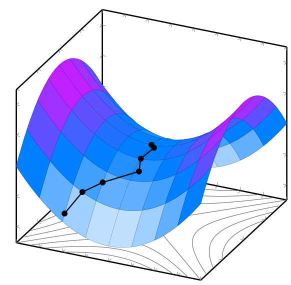

Figure 2: Convergence to a saddle point.

In collaboration with Noam Brown (CMU) and Tuomas Sandholm, I showed that first-order methods, as well as reinforcement learning-based algorithms, can prune low-probability sections of the game tree when computing updates, thereby reducing the per-iteration cost of these iterative methods [1].

It is well-known that Nash equilibria can lead to strategies that may seem irrational at information sets that are reached with probability zero in equilibrium. This can be important when applying equilibria to real-world scenarios, where other agents may not play according to an equilibrium. However, scalable algorithms for computing equilibrium refinements were not known. Together with Gabriele Farina (CMU) and Tuomas Sandholm, we developed the first variants of scalable iterative algorithms for computing equilibrium refinements, in particular extensive-form perfect equilibria. We showed both how to use reinforcement learning algorithms [4] and first-order methods [6] for computing refinements.

Appeared at EC-15, EC-17, and AAAI-17, IJCAI-17 and ICML-17. The EC-17 results are now under submission at Mathematical Programming.

Abstraction methods for dimensionality-reduction

Abstractions methods are used to complement iterative algorithms for computing equilibria. Abstractions are usually created algorithmically, by utilizing domain-dependent structure to set up a manageable optimization problem that produces a smaller game which retains as much of the original game structure as possible. No reasonable bounds on solution quality for solving an abstracted game rather than the full game existed before my work (strong previous results were known only for lossless abstraction). The only previous results of mildly comparable, though significantly lower, generality had a linear dependence on the number of information sets in the game, leading to very loose bounds. In contrast to this, I developed results with *no* dependence on the number of information sets, but rather a constant dependence on payoff error for perfect-recall abstraction [7] and linear dependence on game height (which is logarithmic in the number of information sets) for imperfect-recall abstraction [11]. In addition to this linear-to-constant or linear-to-logarithmic improvement, my results were also the first to allow any error in the modeling of stochastic outcomes. I verified experimentally that the theoretical bounds were within an order of magnitude of the actual error introduced by abstracting. I also introduced the first complexity-theoretic results for understanding the computationally hard problems embedded in constructing a good abstraction. I later extended these results to allow the first analysis of a fairly general class of discretizations of extensive-form games with continuous action spaces [9].

Appeared at EC-14, AAMAS-15, and EC-16. I am working on journal submissions for both EC papers.

Stackelberg games, limited lookahead, and other applications

I developed a model of limited-lookahead behavior in imperfect-information games, and showed several complexity results as well as gave algorithms for computing optimal strategies to commit to for a rational leader with a limited-lookahead follower. These results generalize Stackelberg extensive-form games and have application when modeling adversaries of limited rationality in security games or biological games [10]. In ongoing work with collaborators I am developing a new model of robust Stackelberg solutions in extensive-form games.

Together with Bruce DeBruhl, Patrick Tague, Anupam Datta, and Tuomas Sandholm, all from CMU, I developed a model of energy-constrained jamming of Wi-Fi signal as a two-player zero-sum game. We experimentally investigate the impact of jamming under game-theoretic and baseline behavior models in several models of software-defined radios [3].

Appeared at WiSec-14 and IJCAI-15.

2 Market algorithms

Concurrently with my thesis research, I have also investigated algorithmic and analytical questions pertaining to the facilitation of large-scale automated markets.

In a research collaboration with Facebook's Core Data Science group, I am investigating how Internet companies can run *auction markets* consisting of a large number of auctions and advertisers, with budget-sensitive constraints across auctions [2]. Understanding how to best allocate budgets in these markets is crucial; billions of dollars are spent in these markets every year, and a better understanding of how to allocate funds could lead to huge efficiency improvements. In a separate more preliminary investigation with that group, I am investigating how economic models can be combined with machine learning and predictive modeling for counterfactual outcome modeling in auction markets. Market shocks such as Brexit or currency crashes can have an enormous effect; better counterfactual forecasting allows companies to design better contingency plans for such events.

Previously, I have also worked on *prediction markets*, where we developed a convex-optimization approach that decomposes the computationally hard problem of pricing into a manageable series of mixed-integer programs. We showed on real-world betting data that this approach leads to better inference about market outcomes [5]. I have also worked on *bundling* in auctions, where I developed theoretical characterizations and an algorithmic approach for computing a revenue-maximizing bundling in a particular VCG setting [8].

Appeared at AAMAS-15 and EC-16. The work on pacing equilibrium is in submission to a conference, and will be submitted to a journal thereafter.

3 Ongoing & Future Research

In the long run, I intend to investigate problems that pertain to game-theoretic models, market design, data science, and optimization. I will be especially focused on large-scale problems that are amenable to optimization techniques such as first-order algorithms. Below I describe a few such directions that I am currently working on.

Better algorithms for game solving

While I have already developed a number of important results for large-scale game solving, there are still many interesting directions for future work. It is currently unknown how to utilize stochastic first-order methods well for extensive-form game solving, and this could be an area that could enable better scalability. On the theoretical side, I want to relate my strong-convexity results on the dilated entropy function to other distance functions, and develop a complete picture of the landscape of proximal algorithms for extensive-form game solving. For abstraction, there are many open questions around how to obtain algorithms that are feasible to run in practice, while still guaranteeing solution quality as in my theoretical work. There are also interesting problems around the relationship between the error introduced by abstracting and the first-order methods used to solve games.

General optimization and machine learning

During my work on game theory and markets I have come across several ideas for new convex optimization or machine learning problems. For convex optimization, in particular first-order methods, examples include novel optimization models such as ones with a changing domain and warm-starting questions. In the future I will investigate such general optimization algorithms, and apply them to domains beyond game theory, such as adversarial or robust machine learning. Along these lines, I am currently investigating how new methods from robust optimization can be used to solve robust machine learning problems. Secondly, my work on abstraction algorithms is tightly connected to that of clustering. In fact, independent authors already developed new clustering results inspired by a clustering problem I raised in [11]. In the future, I will investigate new general clustering and statistical problems inspired by and related to that of abstracting information in games.

Stackelberg games

Practical security concerns have lead to a great number of applications of game-theoretic models. I will explore the relationship between extensive-form games, first-order methods, and Stackelberg games, with a focus on extending techniques from the extensive-form game setting to security games. In current research I am introducing a robust notion of Stackelberg equilibrium n extensive-form games, and developing algorithms for computing optimal strategies to commit to in this setting.

Market design and large-scale algorithms

The continued evolution of online market places and their collision with real-world engineering constraints means that there is a rich space of research problems related to facilitating web-scale markets. I will continue to investigate both theoretical and practical aspects of such markets, with an eye towards practical models and algorithms that can be applied in real life. I also intend to maintain a relationship with industry leaders in this space, thus ensuring that I am working on problems with practical appeal.

4 Conclusion

My research has real-world relevance in both government and industry. Game-theoretic models are increasingly popular for modeling physical and cybersecurity settings, and sequential game models can compactly represent complicated security settings. My research on market algorithms is directly related to how large Internet companies generate revenue, and can help to improve the efficiency of such markets, improving outcomes for advertisers, users, and the seller. Finally, my work on large-scale game solving can lead to more general advances on large-scale convex optimization algorithms, and as such has numerous applications such as machine learning and robust optimization. As evidence of government relevance, over the course of my doctoral studies my research has been consistently funded with NSF sources, as well as attracting interest from the ARO. As evidence of industry relevance, I received a Facebook Fellowship, and I have an ongoing relationship with the Facebook Core Data Science group where we work on problems that are relevant to Facebook.

For more information and copies of published and working papers, please visit http://christiankroer.com.

Selected References

[1] Noam Brown, **Christian Kroer**, and Tuomas Sandholm. Dynamic thresholding and pruning for regret minimization. In *AAAI Conference on Artificial Intelligence (AAAI)*, 2017.

[2] Vincent Conitzer, **Christian Kroer**, Eric Sodomka, and Nicolas E Stier-Moses. Multiplicative pacing equilibria in auction markets. *arXiv, under submission*, 2017.

[3] Bruce DeBruhl, **Christian Kroer**, Anupam Datta, Tuomas Sandholm, and Patrick Tague. Power napping with loud neighbors: optimal energy-constrained jamming and anti-jamming. In *Proceedings of the ACM conference on Security and privacy in wireless & mobile networks (WiSec)*, 2014.

[4] Gabriele Farina, **Christian Kroer**, and Tuomas Sandholm. Regret minimization in behaviorally-constrained zero-sum games. In *International Conference on Machine Learning (ICML)*, 2017.

[5] **Christian Kroer**, Miroslav Dudík, Sébastien Lahaie, and Sivaraman Balakrishnan. Arbitrage-free combinatorial market making via integer programming. In *Proceedings of the 2016 ACM Conference on Economics and Computation*. ACM, 2016.

[6] **Christian Kroer**, Gabriele Farina, and Tuomas Sandholm. Smoothing method for approximate extensive-form perfect equilibrium. In *Proceedings of the International Joint Conference on Artificial Intelligence (IJCAI)*, 2017.

[7] **Christian Kroer** and Tuomas Sandholm. Extensive-form game abstraction with bounds. In *Proceedings of the ACM Conference on Economics and Computation (EC)*, 2014.

[8] **Christian Kroer** and Tuomas Sandholm. Computational bundling for auctions. In *International Conference on Autonomous Agents and Multi-Agent Systems (AAMAS)*, 2015.

[9] **Christian Kroer** and Tuomas Sandholm. Discretization of continuous action spaces in extensive-form games. In *International Conference on Autonomous Agents and Multi-Agent Systems (AAMAS)*, 2015.

[10] **Christian Kroer** and Tuomas Sandholm. Limited lookahead in imperfect-information games. In *Proceedings of the International Joint Conference on Artificial Intelligence (IJCAI)*, 2015.

[11] **Christian Kroer** and Tuomas Sandholm. Imperfect-recall abstractions with bounds in games. In *Proceedings of the ACM Conference on Economics and Computation (EC)*, 2016.

[12] **Christian Kroer** and Tuomas Sandholm. Sequential planning for steering immune system adaptation. In *Proceedings of the International Joint Conference on Artificial Intelligence (IJCAI)*, 2016.

[13] **Christian Kroer**, Kevin Waugh, Fatma Kilinc-Karzan, and Tuomas Sandholm. Theoretical and practical advances on smoothing for extensive-form games. In *Proceedings of the ACM Conference on Economics and Computation (EC)*, 2017.

[14] **Christian Kroer**, Kevin Waugh, Fatma Kılınç-Karzan, and Tuomas Sandholm. Faster first-order methods for extensive-form game solving. In *Proceedings of the ACM Conference on Economics and Computation (EC)*, 2015.