5 Solutions

1. You are given the function $y = f(x) = -2x + 10$.
   a. Draw the graph of the function (for x between 0 and 10). Hint: For the next exercises it might be helpful to consider a range from -10 to 30 for the y-axis!

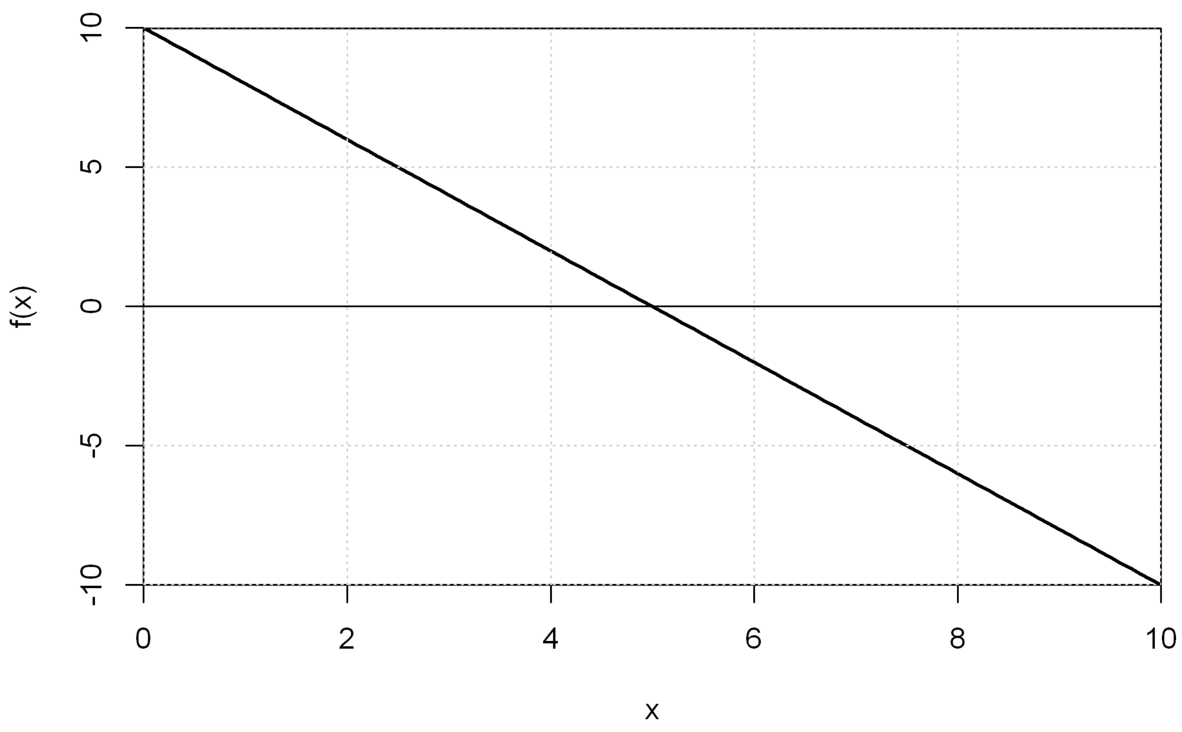

Figure 11: Graph of the function $f(x) = -2x + 10$

   b. What is the slope of the function?

   As $f(x)$ is a linear function of the form $f(x) = kx + d$ the slope can be direclty read from our function: $k = -2$.

   Otherwise we could calculate the slope with help of the derivative by

$$\frac{dy}{dx} = -2$$

   c. Is the relationship between the independent and the dependent variable positive or negative? What does this mean?

   As the slope of the function is negative also the relationship between the independent variable x and the dependent variable y is negative. I.e. an increase in x results in an decrease of y.

   d. Add the function

$$x(y) = \frac{y}{4} - 1$$

   to your graph. (Hint: The left hand side has to be identical to the first function!)

   To be able to add this function the left hand side has to be equal to the y-axis of our first chart. Therefore we have to find the inverse of $x(y)$ which is given by

$$y(x) = 4x + 4$$

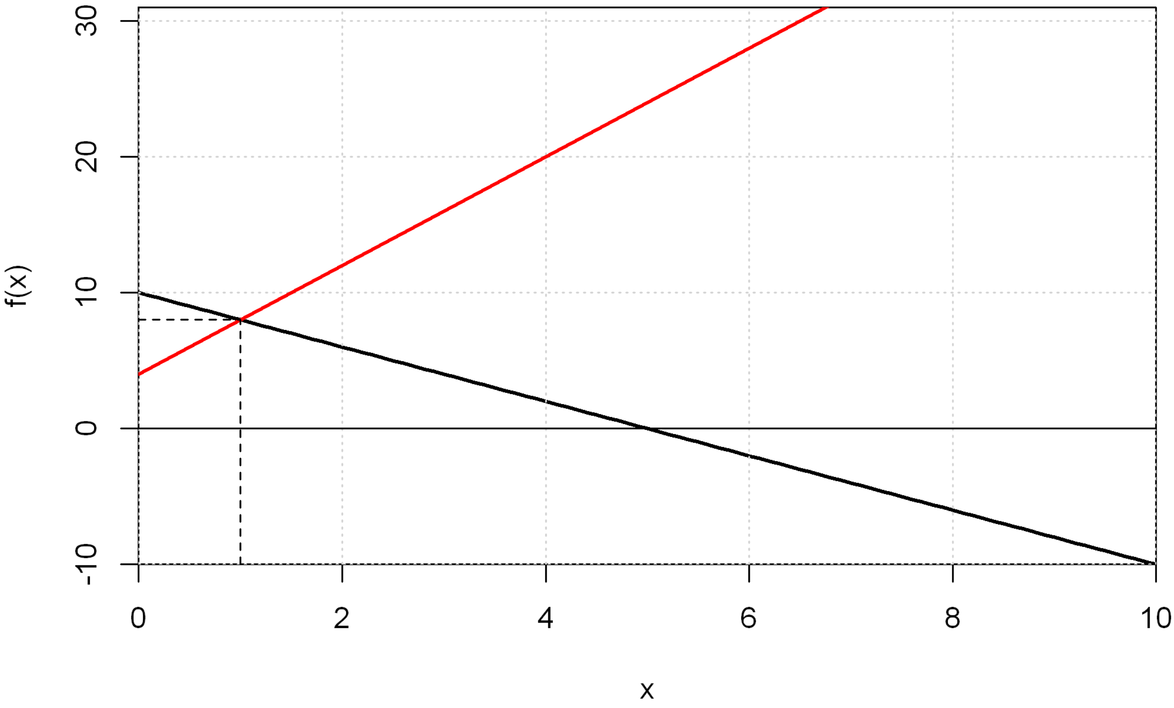

Figure 12: Graph of the function $f(x)=-2x+10$

e. Read from your graph where the two functions cross each other.

   From the chart we can see that the two lines cross at $(x,y)=(1,8)$.

f. Calculate the crossing point!

   At the crossing point the values of both functions must be identical, i.e.

$$-2x+10 = 4x + 4$$

   . By solving this equation for x we get $x=1$. Then we can insert $x=1$ in one of our functions and get e.g. $f(1) = -2 \cdot 1 + 10 = 8$. Hence the crossing point must be $(x, y) = (1,8)$.

2. You are given the function $f(x) = x^3 - 12x^2 + 60x + 100$.
   a. Draw the graph of the function. (Hint: Draw the graph for x between 0 and 10!)

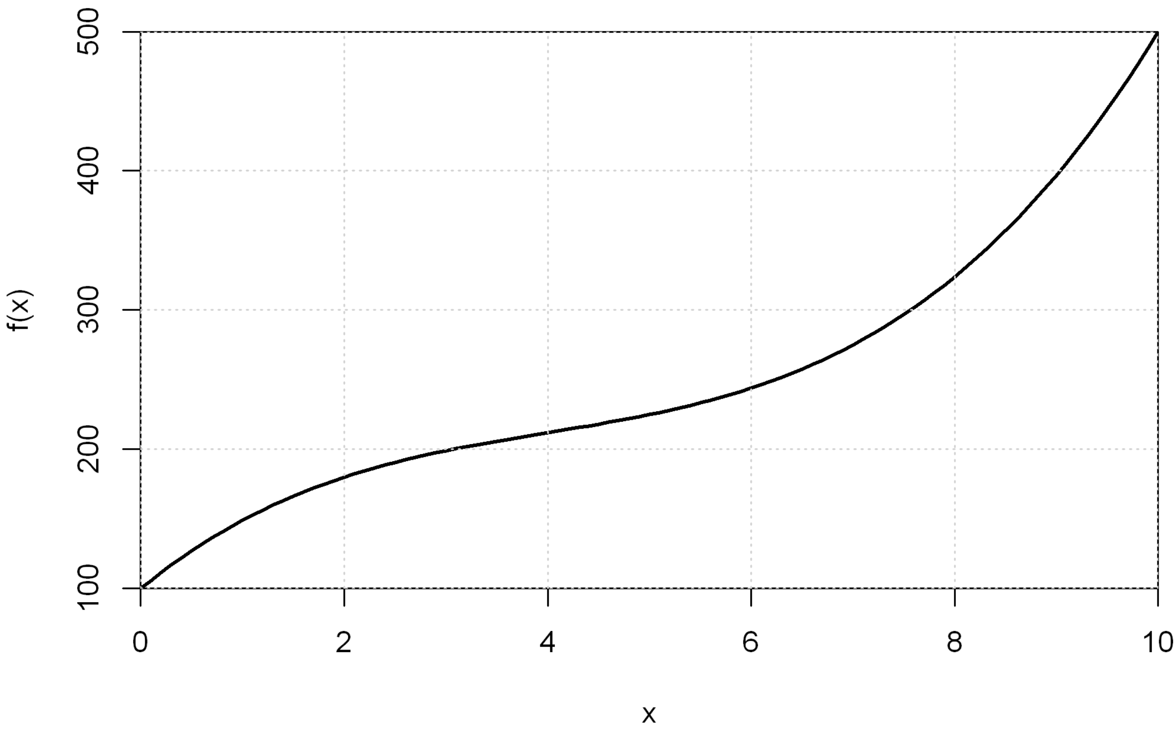

Figure 13: Graph of $f(x) = x^3 - 12x^2 + 60x + 100$

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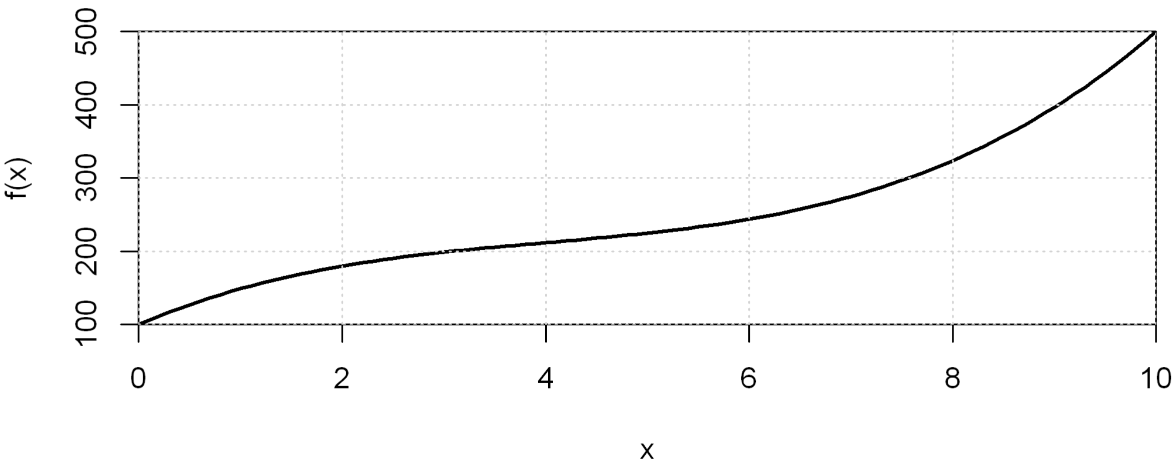
b. Starting from x=3:
   - Calculate the change in y if x increases by 5 units.
   $$\Delta y = f(x_1) - f(x_0) = f(8) - f(3) = 324 - 199 = 125$$
   - Calculate the difference quotient.
   $$\frac{\Delta y}{\Delta x} = \frac{f(x_1) - f(x_0)}{\Delta x} = \frac{125}{5} = 25$$
   - What is the interpretation of the difference quotient?

     Between x_0=3 and x_1=8 the average change in y per unit change of x is 25.

c. Calculate the derivative of the function.
$$\frac{dy}{dx} = 3x^2 - 24x + 60$$

d. Draw the graph of the derivative. (Hint: Use the same dimension for the x-axis as for the graph of the original function.)



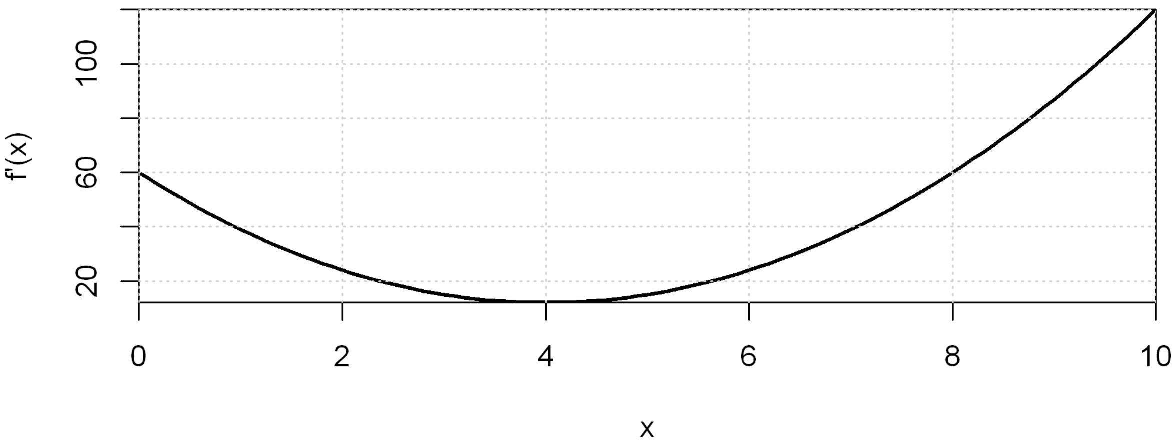


Figure 14: Graph of $f(x) = x^3 - 12x^2 + 60x + 100$

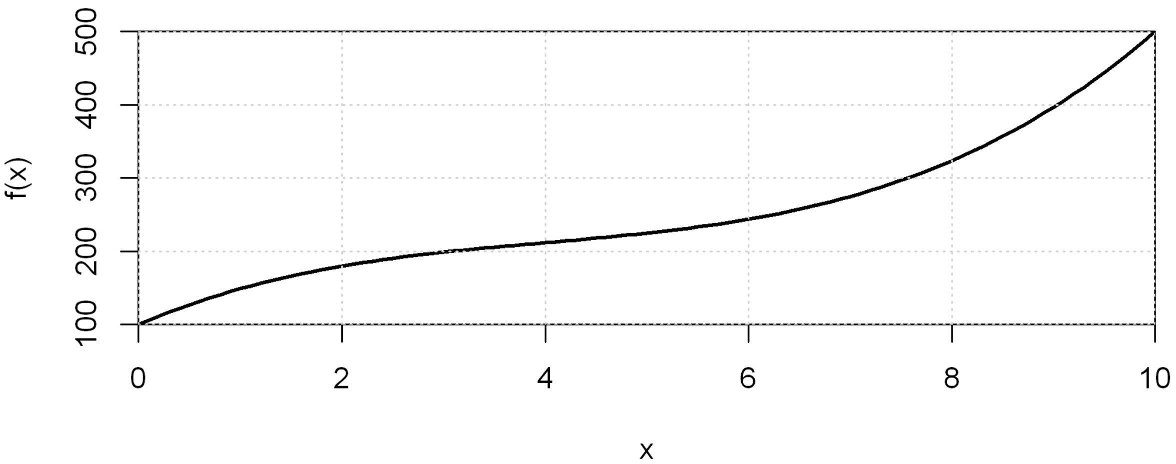
- Describe the relationship between the function and it's derivative. (Hint: Where is the derivative positive/negative, where is it increasing/decreasing, where is the slope of the function positive/negative, where is the slope increasing/decreasing, etc.)

  The derivative is always positive for x between 0 and 10. This implies that the slope of the function is also always positive in this range. Hence there is always a positive relationship between the dependent and the independent variable. In other words an increase in the independent variable results in an increase in the dependent variable.

  For x between 0 and 4 the derivative is decreasing, i.e. the slope of the function $f(x)$ is also decreasing. Graphically this means that the graph of the function $f(x)$ is becoming flatter. Or in other words, as the derivative is still positive but decreasing between 0 and 4, the **increase in *y*** is decreasing between 0 and 4.

  For x between 4 and 10 the derivative is increasing, i.e. the slope of the function $f(x)$ is increasing. Graphically the function $f(x)$ is becoming steeper. Or again in other words, by increasing x, the **increase in *y*** is increasing (in the range from 4 to 10).

e. Add the function $g(x) = x^2 - 12x + 60$ to the graph with the derivative.



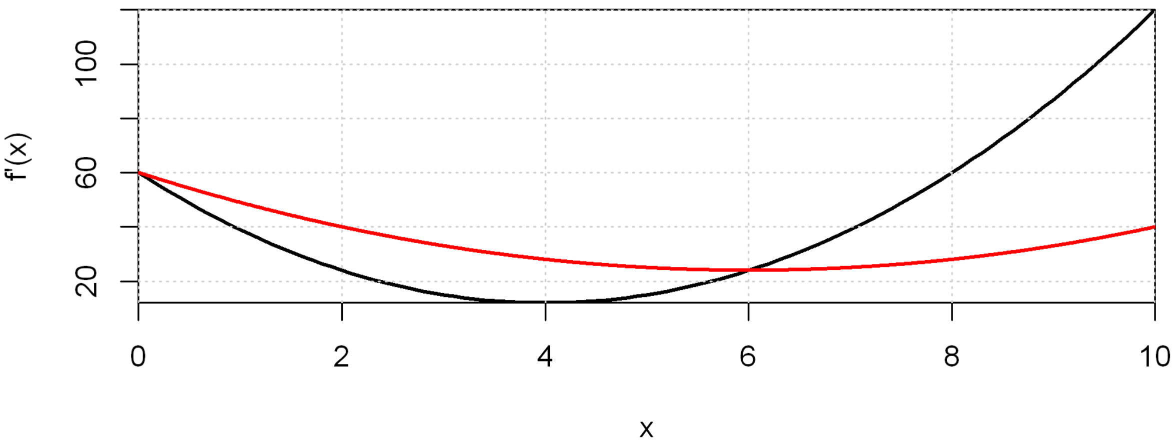


Figure 15: Graph of $f(x) = x^3 - 12x^2 + 60x + 100$